Computer algebra independent integration tests

5-Inverse-trig-functions/5.3-Inverse-tangent/5.3.4-u-a+b-arctan-c-x-^p

Nasser M. Abbasi

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [1301]. This is test number [150].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (1301)	% 0. (0)
Mathematica	% 98.31 (1279)	% 1.69 (22)
Maple	% 92.24 (1200)	% 7.76 (101)
Maxima	% 29.9 (389)	% 70.1 (912)
Fricas	% 42.43 (552)	% 57.57 (749)
Sympy	% 30.36 (395)	% 69.64 (906)
Giac	% 55.8 (726)	% 44.2 (575)

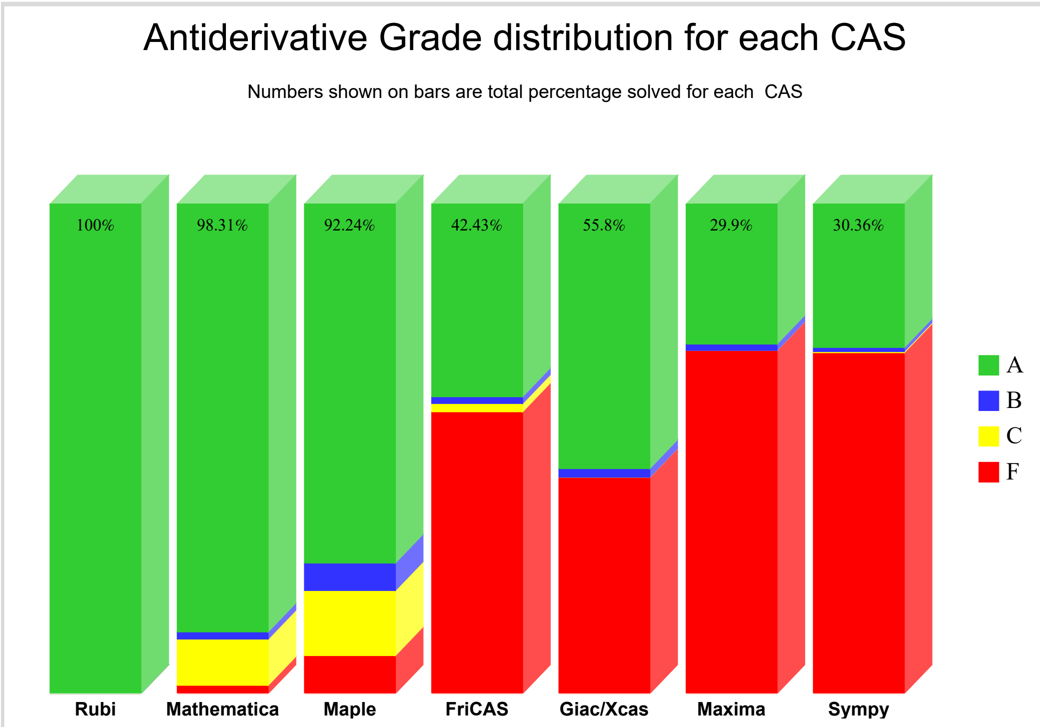
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

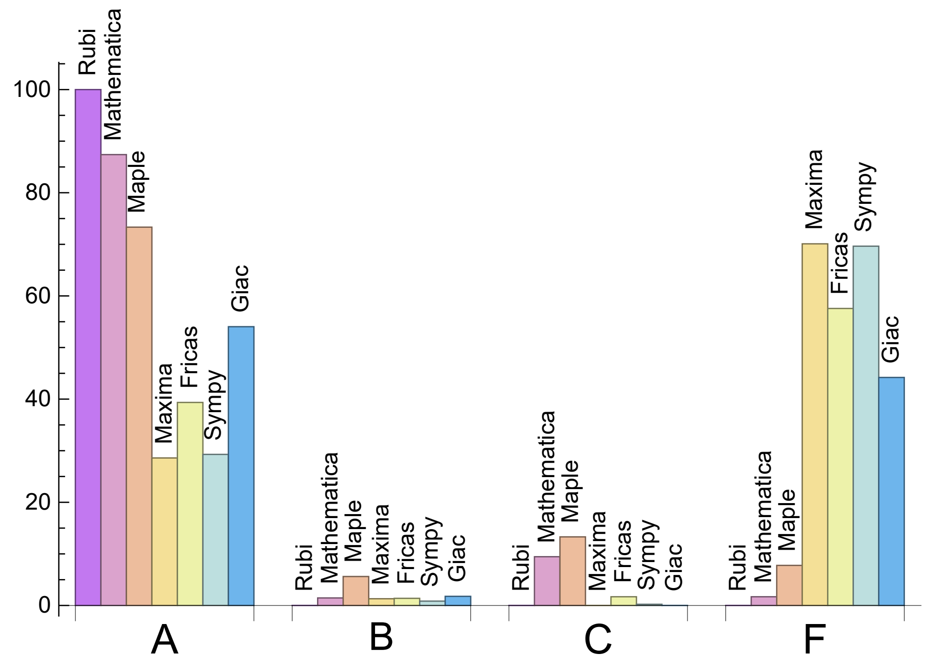
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	100.	0.	0.	0.
Mathematica	87.39	1.46	9.45	1.69
Maple	73.33	5.61	13.3	7.76
Maxima	28.59	1.31	0.	70.1
Fricas	39.35	1.38	1.69	57.57
Sympy	29.29	0.85	0.23	69.64
Giac	54.04	1.77	0.	44.2

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.37	136.61	0.55	69.	1.
Mathematica	2.58	148.47	0.58	55.	0.56
Maple	0.85	471.68	1.91	30.	0.59
Maxima	0.7	103.53	0.77	0.	0.
Fricas	1.47	232.15	1.72	0.	0.
Sympy	2.28	57.03	0.45	0.	0.
Giac	0.29	50.71	0.37	0.	0.

1.4 list of integrals that has no closed form antiderivative

{133, 148, 251, 252, 253, 254, 255, 256, 257, 355, 356, 357, 358, 359, 360, 361, 362, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 478, 479, 480, 481, 485, 486, 487, 488, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 512, 513, 514, 515, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 547, 548, 549, 550, 551, 555, 556, 557, 558, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 583, 584, 585, 586, 587, 588, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 623, 624, 625, 626, 627, 631, 632, 633, 634, 639, 640, 641, 642, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 662, 663, 664, 665, 666, 667, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 703, 704, 705, 706, 707, 708, 712, 713, 714, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 747, 748, 749, 750, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 779, 780, 781, 782, 783, 784, 788, 789, 790, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 826, 827, 828, 829, 830, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 860, 861, 862, 863, 864, 865, 869, 870, 871, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 906, 907, 908, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 929, 930, 931, 935, 936, 937, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 964, 965, 966, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 987, 988, 989, 990, 994, 995, 996, 997, 998, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1029, 1030, 1031, 1032, 1033, 1038, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1057, 1058, 1059, 1063, 1064, 1065, 1066, 1067, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1081, 1082, 1083, 1084, 1085, 1086, 1087, 1088, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1098, 1099, 1100, 1101, 1102, 1107, 1108, 1109, 1110, 1111, 1113, 1174, 1176, 1177, 1178, 1179, 1181, 1184, 1186, 1187, 1188, 1189, 1190, 1191, 1194, 1196, 1197, 1198, 1199, 1200, 1202, 1204, 1205, 1207, 1210, 1213, 1215, 1217, 1222, 1224, 1231, 1232, 1233, 1234, 1235, 1236, 1237, 1238, 1239, 1240, 1242, 1244, 1246, 1299}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {15, 16, 25, 26, 27, 36, 37, 38, 39, 43, 44, 45, 47, 48, 49, 50, 51, 52, 57, 58, 59, 65, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 99, 100, 101, 102, 103, 104, 105, 106, 108, 109, 110, 111, 112, 113, 116, 117, 119, 120, 121, 122, 127, 128, 130, 131, 132, 134, 135, 136, 137, 138, 139, 140, 155, 163, 171, 178, 180, 182, 184, 188, 190, 196, 198, 201, 203, 205, 206, 209, 211, 213, 214, 215, 217, 219, 221, 222, 223, 225, 227, 228, 230, 233, 236, 238, 241, 246, 259, 261, 263, 265, 267, 269, 270, 271, 272, 273, 275, 277, 278, 279, 280, 281, 282, 283, 284, 287, 288, 289, 290, 291, 295, 296, 297, 298, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 335, 336, 337, 338, 339, 340, 343, 344, 345, 346, 347, 348, 353, 354, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 392, 393, 394, 395, 396, 400, 401, 402, 403, 408, 409, 410, 411, 412, 413, 414, 415, 416, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 440, 441, 442, 443, 444, 445, 448, 449, 450, 451, 456, 457, 710, 711, 715, 716, 717, 718, 719, 745, 751, 753, 754, 785, 791, 792, 793, 794, 824, 825, 831, 832, 833, 867, 872, 873, 874, 875, 876, 904, 905, 909, 910, 911, 912, 932, 938, 939, 940, 941, 942, 962, 968, 969, 970, 999, 1000, 1001, 1002, 1028, 1034, 1035, 1036, 1037, 1060, 1062, 1068, 1069, 1070, 1071, 1096, 1097, 1103, 1104, 1105, 1106, 1120, 1131, 1133, 1143, 1145, 1147, 1151, 1152, 1153, 1154, 1155, 1157, 1162, 1163, 1164, 1170, 1171, 1172, 1241, 1248, 1250, 1251, 1252, 1253, 1255, 1257, 1258, 1259, 1260, 1261, 1263, 1265, 1267, 1270, 1297, 1300, 1301}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in-s

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

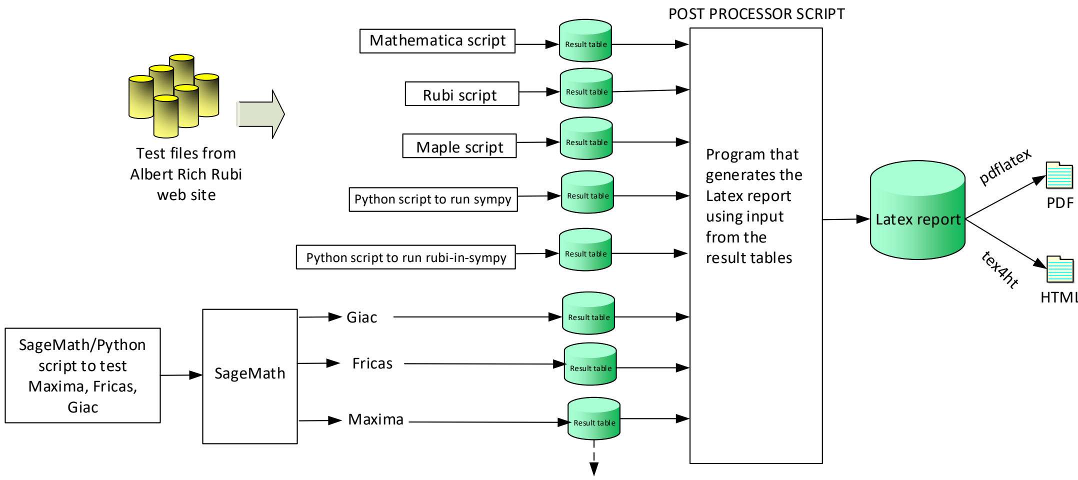
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntx.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. The optimal antiderivative in CAS own syntax.

High level overview of the CAS independent integration test build system

Nasser M. Abbasi
June 22, 2018

Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 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620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1081, 1082, 1083, 1084, 1085, 1086, 1087, 1088, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1104, 1105, 1106, 1107, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1126, 1127, 1128, 1129, 1130, 1131, 1132, 1133, 1134, 1135, 1136, 1137, 1138, 1139, 1140, 1141, 1142, 1143, 1144, 1145, 1146, 1147, 1148, 1149, 1150, 1151, 1152, 1153, 1154, 1155, 1156, 1157, 1158, 1159, 1160, 1161, 1162, 1163, 1164, 1165, 1166, 1167, 1168, 1169, 1170, 1171, 1172, 1173, 1174, 1175, 1176, 1177, 1178, 1179, 1180, 1181, 1182, 1183, 1184, 1185, 1186, 1187, 1188, 1189, 1190, 1191, 1192, 1193, 1194, 1195, 1196, 1197, 1198, 1199, 1200, 1201, 1202, 1203, 1204, 1205, 1206, 1207, 1208, 1209, 1210, 1211, 1212, 1213, 1214, 1215, 1216, 1217, 1218, 1219, 1220, 1221, 1222, 1223, 1224, 1225, 1226, 1227, 1228, 1229, 1230, 1231, 1232, 1233, 1234, 1235, 1236, 1237, 1238, 1239, 1240, 1241, 1242, 1243, 1244, 1245, 1246, 1247, 1248, 1249, 1250, 1251, 1252, 1253, 1254, 1255, 1256, 1257, 1258, 1259, 1260, 1261, 1262, 1263, 1264, 1265, 1266, 1267, 1268, 1269, 1270, 1271, 1272, 1273, 1274, 1275, 1276, 1277, 1278, 1279, 1280, 1281, 1282, 1283, 1284, 1285, 1286, 1287, 1288, 1289, 1290, 1291, 1292, 1293, 1294, 1295, 1296, 1297, 1298, 1299, 1300, 1301 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 10, 11, 12, 13, 14, 15, 20, 21, 22, 23, 24, 25, 26, 31, 32, 33, 34, 35, 36, 37, 38, 43, 44, 45, 46, 47, 48, 51, 52, 53, 54, 55, 56, 58, 59, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 131, 132, 133, 134, 135, 136, 137, 138, 139, 148, 149, 150, 151, 152, 153, 154, 156, 157, 158, 159, 160, 161, 162, 164, 165, 166, 167, 168, 169, 170, 172, 173, 174, 175, 176, 177, 178, 179, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 324, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 414, 415, 416, 417, 418, 419, 420, 422, 424, 425, 426, 427, 428, 430, 432, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 712, 713, 714, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 746, 747, 748, 749, 750, 752, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 786, 787, 788, 789, 790, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 826, 827, 828, 829, 830, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 868, 869, 870, 871, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 906, 907, 908, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 933, 934, 935, 936, 937, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 963, 964, 965, 966, 967, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1029, 1030, 1031, 1032, 1033, 1038, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1061, 1063, 1064, 1065, 1066, 1067, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1081, 1082, 1083, 1084, 1085, 1086, 1087, 1088, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1098, 1099, 1100, 1101, 1102, 1107, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1121, 1123, 1125, 1126, 1127, 1128, 1129, 1130, 1132, 1134, 1136, 1137, 1138, 1139, 1140, 1141, 1142, 1144, 1146, 1148, 1150, 1151, 1152, 1153, 1155, 1156, 1157, 1158, 1159, 1160, 1161, 1162, 1163, 1164, 1165, 1166, 1167, 1168, 1169, 1170, 1171, 1172, 1174, 1176, 1177, 1178, 1179, 1181, 1184, 1186, 1187, 1188, 1189, 1190, 1191, 1194, 1196, 1197, 1198, 1199, 1200, 1202, 1204, 1205, 1207, 1210, 1213, 1215, 1217, 1222, 1224, 1228, 1229, 1230, 1231, 1232, 1233, 1234, 1235, 1236, 1237, 1238, 1239, 1240, 1241, 1242, 1244, 1246, 1247, 1248, 1249, 1250, 1251, 1252, 1253, 1254, 1255, 1256, 1257, 1258, 1259, 1260, 1270, 1275, 1276, 1277, 1278, 1279, 1281, 1282, 1283, 1284, 1285, 1286, 1287, 1288, 1289, 1290, 1292, 1293, 1294, 1295, 1296, 1299, 1300 }

B grade: { 130, 217, 287, 323, 325, 392, 413, 421, 423, 429, 431, 433, 1261, 1263, 1265, 1267, 1297, 1298, 1301 }

C grade: { 7, 8, 9, 16, 17, 18, 19, 27, 28, 29, 30, 39, 40, 41, 42, 49, 50, 57, 65, 140, 155, 163, 171, 180, 710, 711, 715, 716, 717, 718, 719, 745, 751, 753, 785, 791, 792, 793, 794, 824, 825, 831, 832, 833, 867, 872, 873, 874, 875, 904, 905, 909, 910, 911, 912, 932, 938, 939, 940, 941, 942, 962, 968, 969, 970, 999, 1000, 1001, 1002, 1028, 1034, 1035, 1036, 1037, 1060, 1062, 1068, 1069, 1070, 1071, 1096, 1097, 1103, 1104, 1105, 1106, 1120, 1122, 1124, 1131, 1133, 1135, 1143, 1145, 1147, 1149, 1154, 1173, 1175, 1180, 1182, 1183, 1185, 1192, 1193, 1195, 1201, 1203, 1206, 1208, 1209, 1211, 1212, 1214, 1216, 1218, 1219, 1220, 1221, 1223, 1225, 1226, 1227 }

F grade: { 141, 142, 143, 144, 145, 146, 147, 509, 1027, 1243, 1245, 1262, 1264, 1266, 1268, 1269, 1271, 1272, 1273, 1274, 1280, 1291}

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 29, 30, 31, 32, 33, 35, 36, 37, 38, 39, 41, 42, 51, 54, 55, 56, 57, 58, 59, 61, 62, 63, 64, 65, 66, 76, 84, 94, 133, 134, 135, 136, 137, 138, 139, 140, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 175, 177, 179, 181, 182, 183, 185, 186, 187, 189, 191, 192, 193, 194, 195, 197, 199, 201, 203, 204, 205, 206, 209, 211, 212, 213, 214, 215, 217, 219, 220, 221, 222, 223, 225, 227, 228, 230, 233, 236, 238, 241, 246, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 266, 267, 268, 269, 271, 273, 274, 275, 276, 277, 279, 281, 282, 286, 292, 293, 294, 299, 300, 301, 302, 304, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 335, 336, 337, 338, 339, 343, 344, 345, 346, 347, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 365, 367, 371, 373, 375, 377, 379, 381, 383, 385, 388, 391, 397, 398, 399, 404, 405, 406, 407, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 440, 441, 442, 443, 448, 449, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 512, 513, 514, 515, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 583, 584, 585, 586, 587, 588, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 662, 663, 664, 665, 666, 667, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 747, 748, 749, 750, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 826, 827, 828, 829, 830, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 906, 907, 908, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 964, 965, 966, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1029, 1030, 1031, 1032, 1033, 1038, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1081, 1082, 1083, 1084, 1085, 1086, 1087, 1088, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1098, 1099, 1100, 1101, 1102, 1107, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1126, 1127, 1128, 1129, 1130, 1131, 1132, 1133, 1134, 1135, 1136, 1137, 1138, 1139, 1140, 1141, 1142, 1143, 1144, 1145, 1146, 1147, 1148, 1149, 1150, 1159, 1167, 1174, 1176, 1177, 1178, 1179, 1181, 1184, 1186, 1187, 1188, 1189, 1190, 1191, 1194, 1196, 1197, 1198, 1199, 1200, 1202, 1204, 1205, 1207, 1210, 1213, 1215, 1217, 1222, 1224, 1231, 1232, 1233, 1234, 1235, 1236, 1237, 1238, 1239, 1240, 1242, 1244, 1246, 1247, 1249, 1254, 1256, 1290, 1299 }

B grade: { 28, 34, 40, 43, 44, 45, 46, 47, 48, 49, 50, 52, 53, 60, 67, 68, 69, 70, 71, 74, 75, 77, 78, 79, 83, 85, 86, 87, 92, 93, 107, 114, 115, 118, 119, 124, 125, 126, 174, 176, 178, 180, 184, 188, 190, 196, 198, 261, 263, 265, 284, 288, 290, 296, 298, 369, 394, 402, 410, 1156, 1162, 1163, 1166, 1170, 1171, 1248, 1250, 1252, 1255, 1257, 1259, 1264, 1270 }

C grade: { 72, 73, 80, 81, 82, 88, 89, 90, 91, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 108, 109, 110, 111, 112, 113, 116, 117, 120, 121, 122, 123, 127, 128, 129, 130, 131, 132, 141, 142, 143, 144, 145, 146, 147, 200, 202, 207, 208, 210, 216, 218, 224, 226, 229, 231, 232, 234, 235, 237, 239, 240, 242, 243, 244, 245, 247, 248, 249, 250, 262, 264, 270, 272, 278, 280, 283, 285, 287, 289, 291, 295, 297, 303, 305, 341, 342, 349, 350, 351, 352, 364, 366, 368, 370, 372, 374, 376, 378, 380, 382, 384, 386, 387, 389, 390, 392, 393, 395, 396, 400, 401, 403, 408, 409, 411, 446, 447, 452, 453, 454, 455, 510, 511, 516, 517, 518, 519, 581, 582, 589, 590, 591, 592, 660, 661, 668, 669, 670, 671, 1151, 1152, 1153, 1154, 1155, 1157, 1158, 1160, 1161, 1164, 1165, 1168, 1169, 1172, 1251, 1253, 1258, 1260, 1269, 1271, 1275, 1276, 1277, 1278, 1279, 1280, 1286, 1287, 1288, 1289, 1291, 1297 }

F grade: { 334, 340, 348, 438, 439, 444, 445, 450, 451, 643, 745, 746, 751, 752, 753, 754, 824, 825, 831, 832, 833, 834, 904, 905, 909, 910, 911, 912, 962, 963, 967, 968, 969, 970, 1027, 1028, 1034, 1035, 1036, 1037, 1096, 1097, 1103, 1104, 1105, 1106, 1173, 1175, 1180, 1182, 1183, 1185, 1192, 1193, 1195, 1201, 1203, 1206, 1208, 1209, 1211, 1212, 1214, 1216, 1218, 1219, 1220, 1221, 1223, 1225, 1226, 1227, 1228, 1229, 1230, 1241, 1243, 1245, 1261, 1262, 1263, 1265, 1266, 1267, 1268, 1272, 1273, 1274, 1281, 1282, 1283, 1284, 1285, 1292, 1293, 1294, 1295, 1296, 1298, 1300, 1301 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 7, 8, 9, 10, 11, 12, 14, 17, 18, 19, 20, 21, 22, 24, 25, 29, 30, 31, 35, 36, 37, 42, 58, 59, 60, 61, 62, 66, 114, 115, 118, 125, 126, 133, 148, 149, 150, 151, 152, 153, 154, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 175, 177, 179, 181, 183, 185, 186, 187, 189, 191, 192, 193, 194, 195, 197, 199, 226, 234, 235, 242, 243, 244, 245, 251, 252, 253, 254, 255, 256, 257, 258, 260, 266, 268, 274, 276, 286, 292, 293, 294, 299, 300, 301, 302, 341, 342, 350, 352, 357, 358, 359, 360, 361, 362, 391, 397, 398, 399, 404, 405, 406, 407, 446, 447, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 485, 486, 487, 488, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 512, 513, 514, 515, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 555, 556, 557, 558, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 583, 584, 585, 586, 587, 588, 593, 594, 595, 596, 597, 601, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 631, 632, 633, 634, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 662, 663, 664, 665, 666, 667, 672, 673, 677, 678, 679, 680, 681, 682, 683, 684, 685, 1113, 1114, 1115, 1116, 1117, 1119, 1121, 1122, 1123, 1124, 1125, 1126, 1127, 1128, 1129, 1130, 1131, 1132, 1134, 1135, 1136, 1137, 1138, 1139, 1140, 1141, 1142, 1143, 1144, 1145, 1146, 1148, 1149, 1150, 1231, 1232, 1233, 1234, 1235, 1236, 1237, 1238, 1239, 1240, 1242, 1244, 1246, 1247, 1249, 1254, 1256, 1275, 1276, 1277, 1278, 1279, 1281, 1282, 1283, 1284, 1285, 1286, 1287, 1288, 1289, 1290, 1299

B grade: { 13, 23, 28, 32, 33, 34, 40, 41, 63, 64, 65, 67, 155, 202, 210, 218, 1118 }

C grade: { }

F grade: { 5, 6, 15, 16, 26, 27, 38, 39, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 116, 117, 119, 120, 121, 122, 123, 124, 127, 128, 129, 130, 131, 132, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 174, 176, 178, 180, 182, 184, 188, 190, 196, 198, 200, 201, 203, 204, 205, 206, 207, 208, 209, 211, 212, 213, 214, 215, 216, 217, 219, 220, 221, 222, 223, 224, 225, 227, 228, 229, 230, 231, 232, 233, 236, 237, 238, 239, 240, 241, 246, 247, 248, 249, 250, 259, 261, 262, 263, 264, 265, 267, 269, 270, 271, 272, 273, 275, 277, 278, 279, 280, 281, 282, 283, 284, 285, 287, 288, 289, 290, 291, 295, 296, 297, 298, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 343, 344, 345, 346, 347, 348, 349, 351, 353, 354, 355, 356, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 392, 393, 394, 395, 396, 400, 401, 402, 403, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 482, 483, 484, 489, 490, 491, 492, 493, 510, 511, 516, 517, 518, 519, 552, 553, 554, 559, 560, 561, 562, 581, 582, 589, 590, 591, 592, 598, 599, 600, 602, 603, 628, 629, 630, 635, 636, 637, 638, 660, 661, 668, 669, 670, 671, 674, 675, 676, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1081, 1082, 1083, 1084, 1085, 1086, 1087, 1088, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1104, 1105, 1106, 1107, 1108, 1109, 1110, 1111, 1112, 1120, 1133, 1147, 1151, 1152, 1153, 1154, 1155, 1156, 1157, 1158, 1159, 1160, 1161, 1162, 1163, 1164, 1165, 1166, 1167, 1168, 1169, 1170, 1171, 1172, 1173, 1174, 1175, 1176, 1177, 1178, 1179, 1180, 1181, 1182, 1183, 1184, 1185, 1186, 1187, 1188, 1189, 1190, 1191, 1192, 1193, 1194, 1195, 1196, 1197, 1198, 1199, 1200, 1201, 1202, 1203, 1204, 1205, 1206, 1207, 1208, 1209, 1210, 1211, 1212, 1213, 1214, 1215, 1216, 1217, 1218, 1219, 1220, 1221, 1222, 1223, 1224, 1225, 1226, 1227, 1228, 1229, 1230, 1241, 1243, 1245, 1248, 1250, 1251, 1252, 1253, 1255, 1257, 1258, 1259, 1260, 1261, 1262, 1263, 1264, 1265, 1266, 1267, 1268, 1269, 1270, 1271, 1272, 1273, 1274, 1280, 1291, 1292, 1293, 1294, 1295, 1296, 1297, 1298, 1300, 1301 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 7, 8, 9, 10, 11, 12, 13, 17, 18, 19, 20, 21, 22, 23, 29, 30, 31, 32, 33, 34, 41, 42, 47, 54, 61, 62, 66, 67, 107, 114, 115, 118, 124, 125, 126, 133, 148, 149, 150, 151, 152, 154, 156, 157, 158, 159, 160, 162, 164, 165, 166, 167, 168, 170, 172, 173, 175, 177, 179, 181, 183, 185, 186, 187, 189, 191, 192, 193, 194, 195, 197, 199, 200, 202, 207, 208, 210, 216, 218, 224, 226, 229, 231, 232, 234, 235, 237, 239, 240, 242, 243, 244, 245, 247, 251, 252, 253, 254, 255, 256, 257, 258, 260, 266, 268, 274, 276, 286, 292, 293, 294, 299, 300, 301, 302, 341, 342, 349, 350, 351, 352, 355, 356, 357, 358, 359, 360, 361, 362, 391, 397, 398, 399, 404, 405, 406, 407, 446, 447, 452, 453, 454, 455, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 485, 486, 487, 488, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 512, 513, 514, 515, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 555, 556, 557, 558, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 583, 584, 585, 586, 587, 588, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 631, 632, 633, 634, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 662, 663, 664, 665, 666, 667, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 690, 694, 698, 702, 707, 713, 721, 725, 729, 733, 742, 749, 756, 762, 768, 774, 778, 783, 789, 797, 802, 807, 812, 821, 828, 837, 843, 849, 855, 859, 864, 870, 878, 883, 888, 893, 902, 907, 914, 918, 922, 926, 928, 930, 936, 944, 948, 952, 956, 960, 965, 972, 976, 980, 984, 986, 988, 998, 1007, 1011, 1015, 1019, 1024, 1033, 1042, 1046, 1050, 1054, 1056, 1058, 1067, 1076, 1080, 1084, 1088, 1093, 1102, 1111, 1112, 1114, 1115, 1116, 1117, 1119, 1121, 1122, 1123, 1124, 1125, 1126, 1127, 1128, 1130, 1132, 1134, 1135, 1136, 1137, 1138, 1139, 1140, 1142, 1144, 1146, 1148, 1149, 1150, 1159, 1173, 1174, 1175, 1176, 1177, 1178, 1179, 1180, 1181, 1182, 1183, 1184, 1185, 1186, 1187, 1188, 1189, 1190, 1191, 1192, 1194, 1195, 1196, 1197, 1198, 1199, 1200, 1201, 1202, 1203, 1204, 1205, 1206, 1207, 1208, 1210, 1213, 1215, 1217, 1222, 1224, 1231, 1232, 1233, 1234, 1235, 1236, 1237, 1238, 1239, 1240, 1242, 1244, 1246, 1247, 1249, 1254, 1256, 1275, 1276, 1277, 1278, 1279, 1286, 1287, 1288, 1289, 1290, 1299 }

B grade: { 4, 28, 40, 1166, 1167, 1209, 1211, 1212, 1214, 1216, 1218, 1219, 1220, 1221, 1223, 1225, 1226, 1227 }

C grade: { 482, 483, 484, 489, 490, 491, 492, 493, 552, 553, 554, 559, 560, 561, 562, 628, 629, 630, 635, 636, 637, 638 }

F grade: { 5, 6, 14, 15, 16, 24, 25, 26, 27, 35, 36, 37, 38, 39, 43, 44, 45, 46, 48, 49, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 63, 64, 65, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 108, 109, 110, 111, 112, 113, 116, 117, 119, 120, 121, 122, 123, 127, 128, 129, 130, 131, 132, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 153, 155, 161, 163, 169, 171, 174, 176, 178, 180, 182, 184, 188, 190, 196, 198, 201, 203, 204, 205, 206, 209, 211, 212, 213, 214, 215, 217, 219, 220, 221, 222, 223, 225, 227, 228, 230, 233, 236, 238, 241, 246, 248, 249, 250, 259, 261, 262, 263, 264, 265, 267, 269, 270, 271, 272, 273, 275, 277, 278, 279, 280, 281, 282, 283, 284, 285, 287, 288, 289, 290, 291, 295, 296, 297, 298, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 343, 344, 345, 346, 347, 348, 353, 354, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 392, 393, 394, 395, 396, 400, 401, 402, 403, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 448, 449, 450, 451, 456, 457, 510, 511, 516, 517, 518, 519, 581, 582, 589, 590, 591, 592, 660, 661, 668, 669, 670, 671, 687, 688, 689, 691, 692, 693, 695, 696, 697, 699, 700, 701, 703, 704, 705, 706, 708, 709, 710, 711, 712, 714, 715, 716, 717, 718, 719, 720, 722, 723, 724, 726, 727, 728, 730, 731, 732, 734, 735, 736, 737, 738, 739, 740, 741, 743, 744, 745, 746, 747, 748, 750, 751, 752, 753, 754, 755, 757, 758, 759, 760, 761, 763, 764, 765, 766, 767, 769, 770, 771, 772, 773, 775, 776, 777, 779, 780, 781, 782, 784, 785, 786, 787, 788, 790, 791, 792, 793, 794, 795, 796, 798, 799, 800, 801, 803, 804, 805, 806, 808, 809, 810, 811, 813, 814, 815, 816, 817, 818, 819, 820, 822, 823, 824, 825, 826, 827, 829, 830, 831, 832, 833, 834, 835, 836, 838, 839, 840, 841, 842, 844, 845, 846, 847, 848, 850, 851, 852, 853, 854, 856, 857, 858, 860, 861, 862, 863, 865, 866, 867, 868, 869, 871, 872, 873, 874, 875, 876, 877, 879, 880, 881, 882, 884, 885, 886, 887, 889, 890, 891, 892, 894, 895, 896, 897, 898, 899, 900, 901, 903, 904, 905, 906, 908, 909, 910, 911, 912, 913, 915, 916, 917, 919, 920, 921, 923, 924, 925, 927, 929, 931, 932, 933, 934, 935, 937, 938, 939, 940, 941, 942, 943, 945, 946, 947, 949, 950, 951, 953, 954, 955, 957, 958, 959, 961, 962, 963, 964, 966, 967, 968, 969, 970, 971, 973, 974, 975, 977, 978, 979, 981, 982, 983, 985, 987, 989, 990, 991, 992, 993, 994, 995, 996, 997, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1008, 1009, 1010, 1012, 1013, 1014, 1016, 1017, 1018, 1020, 1021, 1022, 1023, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1034, 1035, 1036, 1037, 1038, 1039, 1040, 1041, 1043, 1044, 1045, 1047, 1048, 1049, 1051, 1052, 1053, 1055, 1057, 1059, 1060, 1061, 1062, 1063, 1064, 1065, 1066, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1077, 1078, 1079, 1081, 1082, 1083, 1085, 1086, 1087, 1089, 1090, 1091, 1092, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1101, 1103, 1104, 1105, 1106, 1107, 1108, 1109, 1110, 1113, 1118, 1120, 1129, 1131, 1133, 1141, 1143, 1145, 1147, 1151, 1152, 1153, 1154, 1155, 1156, 1157, 1158, 1160, 1161, 1162, 1163, 1164, 1165, 1168, 1169, 1170, 1171, 1172, 1193, 1228, 1229, 1230, 1241, 1243, 1245, 1248, 1250, 1251, 1252, 1253, 1255, 1257, 1258, 1259, 1260, 1261, 1262, 1263, 1264, 1265, 1266, 1267, 1268, 1269, 1270, 1271, 1272, 1273, 1274, 1280, 1281, 1282, 1283, 1284, 1285, 1291, 1292, 1293, 1294, 1295, 1296, 1297, 1298, 1300, 1301 }

2.1.6 Sympy

A grade: { 1, 2, 3, 10, 11, 12, 20, 21, 22, 31, 32, 148, 149, 150, 151, 152, 154, 156, 157, 158, 159, 160, 162, 164, 165, 166, 167, 168, 170, 172, 173, 175, 177, 179, 181, 183, 186, 192, 194, 251, 252, 255, 256, 258, 260, 266, 268, 274, 276, 355, 356, 357, 358, 361, 458, 459, 460, 461, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 478, 479, 480, 481, 485, 486, 487, 488, 494, 495, 496, 497, 498, 499, 500, 501, 503, 504, 505, 506, 507, 508, 509, 512, 513, 514, 515, 520, 521, 522, 523, 524, 525, 526, 527, 530, 531, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 547, 548, 549, 550, 551, 555, 556, 557, 558, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 574, 575, 576, 577, 578, 579, 580, 583, 584, 585, 587, 588, 593, 598, 599, 600, 601, 602, 603, 606, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 623, 624, 625, 626, 627, 631, 632, 633, 634, 639, 640, 641, 642, 644, 645, 646, 647, 648, 649, 651, 652, 653, 654, 655, 656, 657, 658, 659, 662, 663, 664, 666, 667, 674, 675, 676, 677, 678, 687, 688, 689, 691, 692, 693, 695, 696, 697, 698, 699, 700, 701, 703, 704, 705, 706, 708, 712, 714, 720, 722, 723, 724, 734, 735, 736, 737, 738, 739, 740, 743, 744, 747, 758, 759, 760, 761, 766, 767, 775, 776, 777, 779, 780, 781, 782, 784, 788, 790, 796, 858, 860, 861, 862, 863, 869, 915, 916, 917, 919, 920, 921, 923, 924, 925, 926, 927, 929, 931, 935, 937, 943, 945, 946, 947, 957, 958, 959, 961, 973, 974, 975, 977, 978, 979, 981, 982, 983, 985, 987, 989, 990, 994, 995, 996, 1003, 1008, 1009, 1010, 1020, 1021, 1043, 1044, 1045, 1047, 1048, 1049, 1052, 1053, 1055, 1057, 1111, 1114, 1115, 1116, 1117, 1119, 1121, 1122, 1123, 1124, 1125, 1126, 1127, 1128, 1130, 1132, 1134, 1135, 1136, 1137, 1138, 1139, 1140, 1142, 1144, 1146, 1148, 1150, 1174, 1176, 1177, 1178, 1179, 1181, 1184, 1186, 1187, 1188, 1189, 1190, 1191, 1202, 1204, 1205, 1207, 1247, 1249, 1254, 1256, 1275, 1276, 1277, 1278, 1279, 1286, 1287, 1288, 1289, 1290, 1292, 1294, 1296 }

B grade: { 4, 13, 23, 33, 34, 54, 189, 191, 197, 199, 1149 }

C grade: { 1281, 1283, 1285 }

F grade: { 5, 6, 7, 8, 9, 14, 15, 16, 17, 18, 19, 24, 25, 26, 27, 28, 29, 30, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 153, 155, 161, 163, 169, 171, 174, 176, 178, 180, 182, 184, 185, 187, 188, 190, 193, 195, 196, 198, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 253, 254, 257, 259, 261, 262, 263, 264, 265, 267, 269, 270, 271, 272, 273, 275, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 359, 360, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 462, 463, 464, 465, 477, 482, 483, 484, 489, 490, 491, 492, 493, 502, 510, 511, 516, 517, 518, 519, 528, 529, 532, 533, 546, 552, 553, 554, 559, 560, 561, 562, 573, 581, 582, 586, 589, 590, 591, 592, 594, 595, 596, 597, 604, 605, 607, 608, 609, 622, 628, 629, 630, 635, 636, 637, 638, 643, 650, 660, 661, 665, 668, 669, 670, 671, 672, 673, 679, 680, 681, 682, 683, 684, 685, 686, 690, 694, 702, 707, 709, 710, 711, 713, 715, 716, 717, 718, 719, 721, 725, 726, 727, 728, 729, 730, 731, 732, 733, 741, 742, 745, 746, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 762, 763, 764, 765, 768, 769, 770, 771, 772, 773, 774, 778, 783, 785, 786, 787, 789, 791, 792, 793, 794, 795, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 859, 864, 865, 866, 867, 868, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 918, 922, 928, 930, 932, 933, 934, 936, 938, 939, 940, 941, 942, 944, 948, 949, 950, 951, 952, 953, 954, 955, 956, 960, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 976, 980, 984, 986, 988, 991, 992, 993, 997, 998, 999, 1000, 1001, 1002, 1004, 1005, 1006, 1007, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1039, 1040, 1041, 1042, 1046, 1050, 1051, 1054, 1056, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1081, 1082, 1083, 1084, 1085, 1086, 1087, 1088, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1104, 1105, 1106, 1107, 1108, 1109, 1110, 1112, 1113, 1118, 1120, 1129, 1131, 1133, 1141, 1143, 1145, 1147, 1151, 1152, 1153, 1154, 1155, 1156, 1157, 1158, 1159, 1160, 1161, 1162, 1163, 1164, 1165, 1166, 1167, 1168, 1169, 1170, 1171, 1172, 1173, 1175, 1180, 1182, 1183, 1185, 1192, 1193, 1194, 1195, 1196, 1197, 1198, 1199, 1200, 1201, 1203, 1206, 1208, 1209, 1210, 1211, 1212, 1213, 1214, 1215, 1216, 1217, 1218, 1219, 1220, 1221, 1222, 1223, 1224, 1225, 1226, 1227, 1228, 1229, 1230, 1231, 1232, 1233, 1234, 1235, 1236, 1237, 1238, 1239, 1240, 1241, 1242, 1243, 1244, 1245, 1246, 1248, 1250, 1251, 1252, 1253, 1255, 1257, 1258, 1259, 1260, 1261, 1262, 1263, 1264, 1265, 1266, 1267, 1268, 1269, 1270, 1271, 1272, 1273, 1274, 1280, 1282, 1284, 1291, 1293, 1295, 1297, 1298, 1299, 1300, 1301 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 7, 8, 9, 10, 11, 12, 13, 17, 18, 19, 20, 21, 22, 29, 30, 31, 32, 33, 41, 42, 54, 62, 133, 148, 149, 150, 151, 152, 154, 156, 157, 158, 159, 160, 162, 164, 165, 166, 167, 168, 170, 172, 173, 175, 177, 186, 192, 194, 200, 202, 208, 210, 216, 218, 224, 226, 232, 234, 235, 240, 242, 243, 244, 245, 251, 252, 256, 257, 258, 260, 266, 268, 274, 276, 286, 341, 342, 349, 350, 351, 352, 355, 356, 357, 358, 361, 362, 391, 446, 447, 452, 453, 454, 455, 458, 459, 460, 461, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 485, 486, 487, 488, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 512, 513, 514, 515, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 555, 556, 557, 558, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 583, 584, 585, 586, 587, 588, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 631, 632, 633, 634, 639, 640, 641, 642, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 662, 663, 664, 665, 666, 667, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 712, 713, 714, 720, 722, 723, 724, 726, 727, 728, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 747, 748, 749, 750, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 788, 789, 790, 796, 798, 799, 800, 801, 803, 804, 805, 806, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 826, 827, 828, 829, 830, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 869, 870, 871, 877, 879, 880, 881, 882, 884, 885, 886, 887, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 906, 907, 908, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 935, 936, 937, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 964, 965, 966, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 994, 995, 996, 997, 998, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1029, 1030, 1031, 1032, 1033, 1038, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1063, 1064, 1065, 1066, 1067, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1081, 1082, 1083, 1084, 1085, 1086, 1087, 1088, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1098, 1099, 1100, 1101, 1102, 1107, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1119, 1121, 1122, 1123, 1124, 1125, 1126, 1127, 1128, 1130, 1132, 1134, 1135, 1136, 1137, 1138, 1140, 1142, 1144, 1146, 1148, 1150, 1159, 1173, 1174, 1175, 1176, 1177, 1178, 1179, 1181, 1184, 1186, 1187, 1188, 1189, 1190, 1191, 1194, 1196, 1197, 1198, 1199, 1200, 1202, 1204, 1205, 1207, 1210, 1213, 1215, 1217, 1222, 1224, 1226, 1227, 1231, 1232, 1233, 1234, 1235, 1236, 1237, 1238, 1239, 1240, 1242, 1244, 1246, 1247, 1249, 1254, 1256, 1275, 1276, 1286, 1287, 1288, 1299 }

B grade: { 23, 28, 34, 40, 61, 66, 107, 124, 229, 231, 1139, 1149, 1166, 1167, 1183, 1185, 1193, 1195, 1277, 1278, 1279, 1289, 1290 }

C grade: { }

F grade: { 5, 6, 14, 15, 16, 24, 25, 26, 27, 35, 36, 37, 38, 39, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 63, 64, 65, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 125, 126, 127, 128, 129, 130, 131, 132, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 153, 155, 161, 163, 169, 171, 174, 176, 178, 179, 180, 181, 182, 183, 184, 185, 187, 188, 189, 190, 191, 193, 195, 196, 197, 198, 199, 201, 203, 204, 205, 206, 207, 209, 211, 212, 213, 214, 215, 217, 219, 220, 221, 222, 223, 225, 227, 228, 230, 233, 236, 237, 238, 239, 241, 246, 247, 248, 249, 250, 253, 254, 255, 259, 261, 262, 263, 264, 265, 267, 269, 270, 271, 272, 273, 275, 277, 278, 279, 280, 281, 282, 283, 284, 285, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 343, 344, 345, 346, 347, 348, 353, 354, 359, 360, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 448, 449, 450, 451, 456, 457, 462, 463, 482, 483, 484, 489, 490, 491, 492, 493, 510, 511, 516, 517, 518, 519, 552, 553, 554, 559, 560, 561, 562, 581, 582, 589, 590, 591, 592, 628, 629, 630, 635, 636, 637, 638, 643, 660, 661, 668, 669, 670, 671, 709, 710, 711, 715, 716, 717, 718, 719, 721, 725, 729, 745, 746, 751, 752, 753, 754, 785, 786, 787, 791, 792, 793, 794, 795, 797, 802, 807, 824, 825, 831, 832, 833, 834, 866, 867, 868, 872, 873, 874, 875, 876, 878, 883, 888, 904, 905, 909, 910, 911, 912, 932, 933, 934, 938, 939, 940, 941, 942, 962, 963, 967, 968, 969, 970, 991, 992, 993, 999, 1000, 1001, 1002, 1027, 1028, 1034, 1035, 1036, 1037, 1060, 1061, 1062, 1068, 1069, 1070, 1071, 1096, 1097, 1103, 1104, 1105, 1106, 1118, 1120, 1129, 1131, 1133, 1141, 1143, 1145, 1147, 1151, 1152, 1153, 1154, 1155, 1156, 1157, 1158, 1160, 1161, 1162, 1163, 1164, 1165, 1168, 1169, 1170, 1171, 1172, 1180, 1182, 1192, 1201, 1203, 1206, 1208, 1209, 1211, 1212, 1214, 1216, 1218, 1219, 1220, 1221, 1223, 1225, 1228, 1229, 1230, 1241, 1243, 1245, 1248, 1250, 1251, 1252, 1253, 1255, 1257, 1258, 1259, 1260, 1261, 1262, 1263, 1264, 1265, 1266, 1267, 1268, 1269, 1270, 1271, 1272, 1273, 1274, 1280, 1281, 1282, 1283, 1284, 1285, 1291, 1292, 1293, 1294, 1295, 1296, 1297, 1298, 1300, 1301 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	98	108	147	302	153	162
normalized size	1	1.	0.84	0.92	1.26	2.58	1.31	1.38
time (sec)	N/A	0.103	0.074	0.027	1.483	2.865	1.848	1.31

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	88	98	134	262	131	142
normalized size	1	1.	0.84	0.93	1.28	2.5	1.25	1.35
time (sec)	N/A	0.094	0.06	0.026	1.503	2.785	1.786	1.165

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	76	87	119	243	128	134
normalized size	1	1.	0.84	0.96	1.31	2.67	1.41	1.47
time (sec)	N/A	0.077	0.047	0.028	1.469	2.806	1.78	1.16

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	84	71	99	204	102	104
normalized size	1	1.	1.58	1.34	1.87	3.85	1.92	1.96
time (sec)	N/A	0.031	0.005	0.027	1.478	2.812	1.68	1.148

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	76	113	0	0	0	0
normalized size	1	1.	1.	1.49	0.	0.	0.	0.
time (sec)	N/A	0.086	0.004	0.043	0.	0.	0.	0.

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	75	127	0	0	0	0
normalized size	1	1.	0.97	1.65	0.	0.	0.	0.
time (sec)	N/A	0.1	0.049	0.044	0.	0.	0.	0.

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	88	91	101	244	0	123
normalized size	1	1.	1.35	1.4	1.55	3.75	0.	1.89
time (sec)	N/A	0.055	0.055	0.034	1.493	3.019	0.	1.147

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	94	101	117	266	0	134
normalized size	1	1.	0.89	0.95	1.1	2.51	0.	1.26
time (sec)	N/A	0.091	0.052	0.035	1.488	2.958	0.	1.197

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	99	112	138	296	0	151
normalized size	1	1.	0.8	0.9	1.11	2.39	0.	1.22
time (sec)	N/A	0.096	0.051	0.036	1.493	2.86	0.	1.236

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	124	166	250	404	233	247
normalized size	1	1.	0.75	1.	1.51	2.43	1.4	1.49
time (sec)	N/A	0.163	0.129	0.027	1.51	2.834	2.776	1.207

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	116	154	235	359	206	221
normalized size	1	1.	0.76	1.01	1.55	2.36	1.36	1.45
time (sec)	N/A	0.15	0.109	0.027	1.511	2.824	2.632	1.217

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	101	141	209	333	202	207
normalized size	1	1.	0.74	1.04	1.54	2.45	1.49	1.52
time (sec)	N/A	0.127	0.091	0.027	1.472	2.798	2.719	1.152

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	57	133	186	279	165	176
normalized size	1	1.	0.69	1.6	2.24	3.36	1.99	2.12
time (sec)	N/A	0.046	0.039	0.028	1.492	2.688	2.607	1.158

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	103	177	204	0	0	0
normalized size	1	1.	0.8	1.37	1.58	0.	0.	0.
time (sec)	N/A	0.127	0.093	0.043	2.143	0.	0.	0.

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	89	89	79	152	0	0	0	0
normalized size	1	1.	0.89	1.71	0.	0.	0.	0.
time (sec)	N/A	0.138	0.092	0.042	0.	0.	0.	0.

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	152	152	139	217	0	0	0	0
normalized size	1	1.	0.91	1.43	0.	0.	0.	0.
time (sec)	N/A	0.154	0.071	0.05	0.	0.	0.	0.

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	114	145	194	316	0	194
normalized size	1	1.	1.31	1.67	2.23	3.63	0.	2.23
time (sec)	N/A	0.082	0.09	0.036	1.479	2.749	0.	1.222

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	152	160	205	366	0	217
normalized size	1	1.	0.94	0.99	1.27	2.27	0.	1.35
time (sec)	N/A	0.15	0.068	0.038	1.479	2.818	0.	1.202

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	124	172	247	387	0	228
normalized size	1	1.	0.73	1.01	1.44	2.26	0.	1.33
time (sec)	N/A	0.158	0.086	0.037	1.489	2.9	0.	1.385

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	248	209	352	481	292	301
normalized size	1	1.	1.21	1.02	1.72	2.35	1.42	1.47
time (sec)	N/A	0.184	0.106	0.03	1.481	2.762	3.456	1.196

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	191	234	197	327	447	275	279
normalized size	1	1.	1.23	1.03	1.71	2.34	1.44	1.46
time (sec)	N/A	0.171	0.086	0.027	1.477	2.793	3.319	1.199

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	132	184	300	404	260	266
normalized size	1	1.	0.84	1.17	1.91	2.57	1.66	1.69
time (sec)	N/A	0.097	0.101	0.027	1.485	2.762	3.362	1.178

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	77	176	266	367	228	235
normalized size	1	1.	0.77	1.76	2.66	3.67	2.28	2.35
time (sec)	N/A	0.054	0.039	0.026	1.476	2.943	3.247	1.156

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	139	220	259	0	0	0
normalized size	1	1.	0.82	1.29	1.52	0.	0.	0.
time (sec)	N/A	0.175	0.132	0.041	2.168	0.	0.	0.

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	162	162	150	223	281	0	0	0
normalized size	1	1.	0.93	1.38	1.73	0.	0.	0.
time (sec)	N/A	0.173	0.116	0.046	2.172	0.	0.	0.

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	180	180	164	243	0	0	0	0
normalized size	1	1.	0.91	1.35	0.	0.	0.	0.
time (sec)	N/A	0.178	0.121	0.047	0.	0.	0.	0.

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	189	189	170	255	0	0	0	0
normalized size	1	1.	0.9	1.35	0.	0.	0.	0.
time (sec)	N/A	0.203	0.094	0.051	0.	0.	0.	0.

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	165	190	273	410	0	258
normalized size	1	1.	1.6	1.84	2.65	3.98	0.	2.5
time (sec)	N/A	0.091	0.122	0.036	1.495	2.953	0.	1.258

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	185	200	302	427	0	270
normalized size	1	1.	1.23	1.33	2.01	2.85	0.	1.8
time (sec)	N/A	0.107	0.094	0.038	1.488	2.877	0.	1.568

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	188	215	335	481	0	292
normalized size	1	1.	0.88	1.	1.57	2.25	0.	1.36
time (sec)	N/A	0.178	0.112	0.039	1.492	3.167	0.	1.69

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	238	238	290	249	455	563	355	354
normalized size	1	1.	1.22	1.05	1.91	2.37	1.49	1.49
time (sec)	N/A	0.214	0.155	0.029	1.512	3.392	4.12	1.167

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	276	237	429	525	325	333
normalized size	1	1.	1.43	1.23	2.22	2.72	1.68	1.73
time (sec)	N/A	0.167	0.12	0.027	1.486	3.317	4.203	1.206

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	264	224	392	491	328	320
normalized size	1	1.	1.48	1.26	2.2	2.76	1.84	1.8
time (sec)	N/A	0.113	0.101	0.028	1.505	2.839	4.057	1.17

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	77	216	356	435	272	289
normalized size	1	1.	0.62	1.73	2.85	3.48	2.18	2.31
time (sec)	N/A	0.063	0.03	0.029	1.492	2.535	3.962	1.203

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	174	260	308	0	0	0
normalized size	1	1.	0.86	1.28	1.52	0.	0.	0.
time (sec)	N/A	0.211	0.141	0.041	2.201	0.	0.	0.

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	190	190	181	264	336	0	0	0
normalized size	1	1.	0.95	1.39	1.77	0.	0.	0.
time (sec)	N/A	0.207	0.147	0.046	2.159	0.	0.	0.

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	173	173	163	248	350	0	0	0
normalized size	1	1.	0.94	1.43	2.02	0.	0.	0.
time (sec)	N/A	0.199	0.14	0.047	2.161	0.	0.	0.

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	201	201	193	277	0	0	0	0
normalized size	1	1.	0.96	1.38	0.	0.	0.	0.
time (sec)	N/A	0.218	0.147	0.047	0.	0.	0.	0.

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	227	227	227	298	0	0	0	0
normalized size	1	1.	1.	1.31	0.	0.	0.	0.
time (sec)	N/A	0.23	0.113	0.052	0.	0.	0.	0.

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	191	230	371	475	0	308
normalized size	1	1.	1.63	1.97	3.17	4.06	0.	2.63
time (sec)	N/A	0.097	0.159	0.036	1.476	2.396	0.	1.445

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	235	243	392	527	0	329
normalized size	1	1.	1.4	1.45	2.33	3.14	0.	1.96
time (sec)	N/A	0.114	0.119	0.039	1.489	2.403	0.	1.57

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	243	243	293	255	444	560	0	342
normalized size	1	1.	1.21	1.05	1.83	2.3	0.	1.41
time (sec)	N/A	0.196	0.096	0.036	1.491	2.36	0.	2.575

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	196	196	166	353	0	0	0	0
normalized size	1	1.	0.85	1.8	0.	0.	0.	0.
time (sec)	N/A	0.286	0.442	0.053	0.	0.	0.	0.

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	156	156	132	308	0	0	0	0
normalized size	1	1.	0.85	1.97	0.	0.	0.	0.
time (sec)	N/A	0.181	0.187	0.05	0.	0.	0.	0.

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	110	110	108	244	0	0	0	0
normalized size	1	1.	0.98	2.22	0.	0.	0.	0.
time (sec)	N/A	0.104	0.168	0.049	0.	0.	0.	0.

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	60	142	0	0	0	0
normalized size	1	1.	1.02	2.41	0.	0.	0.	0.
time (sec)	N/A	0.047	0.014	0.036	0.	0.	0.	0.

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	54	54	102	193	0	109	0	0
normalized size	1	1.	1.89	3.57	0.	2.02	0.	0.
time (sec)	N/A	0.071	0.056	0.051	0.	2.598	0.	0.

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	100	100	149	252	0	0	0	0
normalized size	1	1.	1.49	2.52	0.	0.	0.	0.
time (sec)	N/A	0.155	0.091	0.056	0.	0.	0.	0.

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	161	161	178	335	0	0	0	0
normalized size	1	1.	1.11	2.08	0.	0.	0.	0.
time (sec)	N/A	0.236	0.171	0.064	0.	0.	0.	0.

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	197	197	254	369	0	0	0	0
normalized size	1	1.	1.29	1.87	0.	0.	0.	0.
time (sec)	N/A	0.339	0.137	0.055	0.	0.	0.	0.

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	203	203	186	367	0	0	0	0
normalized size	1	1.	0.92	1.81	0.	0.	0.	0.
time (sec)	N/A	0.222	0.97	0.06	0.	0.	0.	0.

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	167	167	153	316	0	0	0	0
normalized size	1	1.	0.92	1.89	0.	0.	0.	0.
time (sec)	N/A	0.19	0.753	0.056	0.	0.	0.	0.

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	128	293	0	0	0	0
normalized size	1	1.	1.05	2.4	0.	0.	0.	0.
time (sec)	N/A	0.147	0.09	0.055	0.	0.	0.	0.

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	42	76	0	112	1698	150
normalized size	1	1.	0.61	1.1	0.	1.62	24.61	2.17
time (sec)	N/A	0.047	0.033	0.039	0.	2.528	16.147	1.122

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	128	251	0	0	0	0
normalized size	1	1.	0.85	1.67	0.	0.	0.	0.
time (sec)	N/A	0.194	0.148	0.063	0.	0.	0.	0.

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	165	340	0	0	0	0
normalized size	1	1.	0.85	1.75	0.	0.	0.	0.
time (sec)	N/A	0.241	0.287	0.072	0.	0.	0.	0.

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	244	244	222	380	0	0	0	0
normalized size	1	1.	0.91	1.56	0.	0.	0.	0.
time (sec)	N/A	0.268	0.334	0.069	0.	0.	0.	0.

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	256	256	235	423	478	0	0	0
normalized size	1	1.	0.92	1.65	1.87	0.	0.	0.
time (sec)	N/A	0.285	1.018	0.062	1.351	0.	0.	0.

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	225	225	216	375	441	0	0	0
normalized size	1	1.	0.96	1.67	1.96	0.	0.	0.
time (sec)	N/A	0.243	0.789	0.056	1.28	0.	0.	0.

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	187	349	393	0	0	0
normalized size	1	1.	1.06	1.98	2.23	0.	0.	0.
time (sec)	N/A	0.217	0.155	0.059	1.236	0.	0.	0.

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	63	128	96	196	0	200
normalized size	1	1.	0.72	1.45	1.09	2.23	0.	2.27
time (sec)	N/A	0.077	0.082	0.041	1.045	2.263	0.	1.163

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	55	93	89	177	0	174
normalized size	1	1.	0.6	1.01	0.97	1.92	0.	1.89
time (sec)	N/A	0.055	0.04	0.036	1.033	2.247	0.	1.184

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	162	327	563	0	0	0
normalized size	1	1.	0.83	1.68	2.89	0.	0.	0.
time (sec)	N/A	0.242	0.209	0.066	1.338	0.	0.	0.

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	250	250	227	394	729	0	0	0
normalized size	1	1.	0.91	1.58	2.92	0.	0.	0.
time (sec)	N/A	0.288	0.302	0.071	1.422	0.	0.	0.

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	306	306	285	481	829	0	0	0
normalized size	1	1.	0.93	1.57	2.71	0.	0.	0.
time (sec)	N/A	0.32	0.542	0.075	1.464	0.	0.	0.

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	73	93	112	235	0	232
normalized size	1	1.	0.73	0.93	1.12	2.35	0.	2.32
time (sec)	N/A	0.051	0.044	0.04	1.066	2.207	0.	1.132

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	49	49	88	148	170	55	0	0
normalized size	1	1.	1.8	3.02	3.47	1.12	0.	0.
time (sec)	N/A	0.064	0.024	0.049	1.493	2.157	0.	0.

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	287	287	285	499	0	0	0	0
normalized size	1	1.	0.99	1.74	0.	0.	0.	0.
time (sec)	N/A	0.596	0.787	0.093	0.	0.	0.	0.

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	255	255	241	467	0	0	0	0
normalized size	1	1.	0.95	1.83	0.	0.	0.	0.
time (sec)	N/A	0.491	0.591	0.094	0.	0.	0.	0.

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	211	211	208	416	0	0	0	0
normalized size	1	1.	0.99	1.97	0.	0.	0.	0.
time (sec)	N/A	0.358	0.497	0.093	0.	0.	0.	0.

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	130	130	151	367	0	0	0	0
normalized size	1	1.	1.16	2.82	0.	0.	0.	0.
time (sec)	N/A	0.121	0.253	0.087	0.	0.	0.	0.

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	216	216	272	7034	0	0	0	0
normalized size	1	1.	1.26	32.56	0.	0.	0.	0.
time (sec)	N/A	0.421	0.453	0.599	0.	0.	0.	0.

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	228	228	289	5963	0	0	0	0
normalized size	1	1.	1.27	26.15	0.	0.	0.	0.
time (sec)	N/A	0.466	0.412	1.268	0.	0.	0.	0.

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	159	159	190	487	0	0	0	0
normalized size	1	1.	1.19	3.06	0.	0.	0.	0.
time (sec)	N/A	0.339	0.274	0.107	0.	0.	0.	0.

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	224	224	240	556	0	0	0	0
normalized size	1	1.	1.07	2.48	0.	0.	0.	0.
time (sec)	N/A	0.432	0.496	0.105	0.	0.	0.	0.

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	373	373	342	650	0	0	0	0
normalized size	1	1.	0.92	1.74	0.	0.	0.	0.
time (sec)	N/A	0.973	1.175	0.094	0.	0.	0.	0.

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	333	333	306	612	0	0	0	0
normalized size	1	1.	0.92	1.84	0.	0.	0.	0.
time (sec)	N/A	0.857	1.142	0.096	0.	0.	0.	0.

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	293	293	257	556	0	0	0	0
normalized size	1	1.	0.88	1.9	0.	0.	0.	0.
time (sec)	N/A	0.621	0.775	0.097	0.	0.	0.	0.

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	192	192	205	523	0	0	0	0
normalized size	1	1.	1.07	2.72	0.	0.	0.	0.
time (sec)	N/A	0.197	0.637	0.089	0.	0.	0.	0.

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	300	300	360	1542	0	0	0	0
normalized size	1	1.	1.2	5.14	0.	0.	0.	0.
time (sec)	N/A	0.578	0.651	1.217	0.	0.	0.	0.

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	317	317	378	11959	0	0	0	0
normalized size	1	1.	1.19	37.73	0.	0.	0.	0.
time (sec)	N/A	0.62	0.632	1.474	0.	0.	0.	0.

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	337	337	388	1647	0	0	0	0
normalized size	1	1.	1.15	4.89	0.	0.	0.	0.
time (sec)	N/A	0.645	0.84	3.639	0.	0.	0.	0.

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	267	267	253	669	0	0	0	0
normalized size	1	1.	0.95	2.51	0.	0.	0.	0.
time (sec)	N/A	0.268	0.658	0.116	0.	0.	0.	0.

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	438	438	408	750	0	0	0	0
normalized size	1	1.	0.93	1.71	0.	0.	0.	0.
time (sec)	N/A	1.366	1.739	0.096	0.	0.	0.	0.

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	402	402	369	712	0	0	0	0
normalized size	1	1.	0.92	1.77	0.	0.	0.	0.
time (sec)	N/A	1.196	1.373	0.1	0.	0.	0.	0.

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	307	307	325	656	0	0	0	0
normalized size	1	1.	1.06	2.14	0.	0.	0.	0.
time (sec)	N/A	0.614	1.287	0.1	0.	0.	0.	0.

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	226	226	267	620	0	0	0	0
normalized size	1	1.	1.18	2.74	0.	0.	0.	0.
time (sec)	N/A	0.206	0.895	0.094	0.	0.	0.	0.

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	385	385	465	1651	0	0	0	0
normalized size	1	1.	1.21	4.29	0.	0.	0.	0.
time (sec)	N/A	0.769	0.882	2.737	0.	0.	0.	0.

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	402	402	512	1739	0	0	0	0
normalized size	1	1.	1.27	4.33	0.	0.	0.	0.
time (sec)	N/A	0.736	0.593	4.881	0.	0.	0.	0.

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	416	416	500	1846	0	0	0	0
normalized size	1	1.	1.2	4.44	0.	0.	0.	0.
time (sec)	N/A	0.753	1.078	2.691	0.	0.	0.	0.

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	429	429	595	1814	0	0	0	0
normalized size	1	1.	1.39	4.23	0.	0.	0.	0.
time (sec)	N/A	0.89	0.746	2.547	0.	0.	0.	0.

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	293	293	322	757	0	0	0	0
normalized size	1	1.	1.1	2.58	0.	0.	0.	0.
time (sec)	N/A	0.322	0.835	0.111	0.	0.	0.	0.

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	384	384	363	816	0	0	0	0
normalized size	1	1.	0.95	2.12	0.	0.	0.	0.
time (sec)	N/A	0.367	1.251	0.115	0.	0.	0.	0.

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	513	513	401	853	0	0	0	0
normalized size	1	1.	0.78	1.66	0.	0.	0.	0.
time (sec)	N/A	0.517	1.461	0.115	0.	0.	0.	0.

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	356	356	421	1331	0	0	0	0
normalized size	1	1.	1.18	3.74	0.	0.	0.	0.
time (sec)	N/A	0.823	1.011	2.46	0.	0.	0.	0.

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	277	277	330	1212	0	0	0	0
normalized size	1	1.	1.19	4.38	0.	0.	0.	0.
time (sec)	N/A	0.513	0.554	1.643	0.	0.	0.	0.

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	192	192	239	4589	0	0	0	0
normalized size	1	1.	1.24	23.9	0.	0.	0.	0.
time (sec)	N/A	0.293	0.467	0.513	0.	0.	0.	0.

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	95	1062	0	0	0	0
normalized size	1	1.	0.97	10.84	0.	0.	0.	0.
time (sec)	N/A	0.133	0.034	0.305	0.	0.	0.	0.

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	88	88	113	1741	0	0	0	0
normalized size	1	1.	1.28	19.78	0.	0.	0.	0.
time (sec)	N/A	0.161	0.114	0.349	0.	0.	0.	0.

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	186	186	265	9235	0	0	0	0
normalized size	1	1.	1.42	49.65	0.	0.	0.	0.
time (sec)	N/A	0.398	0.86	0.783	0.	0.	0.	0.

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	273	273	372	2221	0	0	0	0
normalized size	1	1.	1.36	8.14	0.	0.	0.	0.
time (sec)	N/A	0.624	1.121	2.36	0.	0.	0.	0.

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	365	365	535	2380	0	0	0	0
normalized size	1	1.	1.47	6.52	0.	0.	0.	0.
time (sec)	N/A	0.975	1.147	4.619	0.	0.	0.	0.

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	433	433	502	1498	0	0	0	0
normalized size	1	1.	1.16	3.46	0.	0.	0.	0.
time (sec)	N/A	0.827	2.377	2.018	0.	0.	0.	0.

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	364	364	429	1395	0	0	0	0
normalized size	1	1.	1.18	3.83	0.	0.	0.	0.
time (sec)	N/A	0.614	1.801	1.612	0.	0.	0.	0.

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	292	292	362	4774	0	0	0	0
normalized size	1	1.	1.24	16.35	0.	0.	0.	0.
time (sec)	N/A	0.495	1.214	0.612	0.	0.	0.	0.

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	216	216	300	1059	0	0	0	0
normalized size	1	1.	1.39	4.9	0.	0.	0.	0.
time (sec)	N/A	0.342	0.76	0.316	0.	0.	0.	0.

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	72	344	0	231	0	405
normalized size	1	1.	0.59	2.82	0.	1.89	0.	3.32
time (sec)	N/A	0.122	0.175	0.073	0.	2.194	0.	1.165

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	221	221	299	1921	0	0	0	0
normalized size	1	1.	1.35	8.69	0.	0.	0.	0.
time (sec)	N/A	0.616	0.948	0.416	0.	0.	0.	0.

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	306	306	398	9420	0	0	0	0
normalized size	1	1.	1.3	30.78	0.	0.	0.	0.
time (sec)	N/A	0.767	2.571	0.996	0.	0.	0.	0.

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	403	403	491	2384	0	0	0	0
normalized size	1	1.	1.22	5.92	0.	0.	0.	0.
time (sec)	N/A	0.934	2.902	2.811	0.	0.	0.	0.

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	462	462	578	1618	0	0	0	0
normalized size	1	1.	1.25	3.5	0.	0.	0.	0.
time (sec)	N/A	0.833	2.502	1.728	0.	0.	0.	0.

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	383	383	507	5012	0	0	0	0
normalized size	1	1.	1.32	13.09	0.	0.	0.	0.
time (sec)	N/A	0.675	1.528	0.625	0.	0.	0.	0.

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	304	304	431	1276	0	0	0	0
normalized size	1	1.	1.42	4.2	0.	0.	0.	0.
time (sec)	N/A	0.56	1.226	0.347	0.	0.	0.	0.

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	117	464	192	378	0	0
normalized size	1	1.	0.66	2.61	1.08	2.12	0.	0.
time (sec)	N/A	0.217	0.367	0.084	1.206	2.311	0.	0.

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	110	405	182	363	0	0
normalized size	1	1.	0.61	2.25	1.01	2.02	0.	0.
time (sec)	N/A	0.18	0.178	0.072	1.175	2.357	0.	0.

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	299	299	435	2151	0	0	0	0
normalized size	1	1.	1.45	7.19	0.	0.	0.	0.
time (sec)	N/A	0.795	1.579	0.466	0.	0.	0.	0.

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	391	391	549	9659	0	0	0	0
normalized size	1	1.	1.4	24.7	0.	0.	0.	0.
time (sec)	N/A	0.976	3.509	1.066	0.	0.	0.	0.

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	207	155	404	248	502	0	0
normalized size	1	1.	0.75	1.95	1.2	2.43	0.	0.
time (sec)	N/A	0.222	0.209	0.075	1.321	2.294	0.	0.

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	76	76	82	183	0	0	0	0
normalized size	1	1.	1.08	2.41	0.	0.	0.	0.
time (sec)	N/A	0.138	0.253	0.207	0.	0.	0.	0.

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	382	382	693	2004	0	0	0	0
normalized size	1	1.	1.81	5.25	0.	0.	0.	0.
time (sec)	N/A	0.707	1.663	5.163	0.	0.	0.	0.

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	298	298	528	1815	0	0	0	0
normalized size	1	1.	1.77	6.09	0.	0.	0.	0.
time (sec)	N/A	0.478	0.98	1.902	0.	0.	0.	0.

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	220	220	367	7451	0	0	0	0
normalized size	1	1.	1.67	33.87	0.	0.	0.	0.
time (sec)	N/A	0.339	0.468	0.787	0.	0.	0.	0.

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	133	2044	0	0	0	0
normalized size	1	1.	0.96	14.71	0.	0.	0.	0.
time (sec)	N/A	0.23	0.079	0.345	0.	0.	0.	0.

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	182	121	551	0	401	0	782
normalized size	1	1.	0.66	3.03	0.	2.2	0.	4.3
time (sec)	N/A	0.22	0.222	0.319	0.	1.88	0.	1.16

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	271	271	183	711	313	618	0	0
normalized size	1	1.	0.68	2.62	1.15	2.28	0.	0.
time (sec)	N/A	0.403	0.273	0.395	1.48	1.911	0.	0.

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	360	360	269	881	439	900	0	0
normalized size	1	1.	0.75	2.45	1.22	2.5	0.	0.
time (sec)	N/A	0.674	0.297	0.467	1.864	2.369	0.	0.

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	410	410	541	1725	0	0	0	0
normalized size	1	1.	1.32	4.21	0.	0.	0.	0.
time (sec)	N/A	0.862	0.966	2.217	0.	0.	0.	0.

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	277	277	393	5478	0	0	0	0
normalized size	1	1.	1.42	19.78	0.	0.	0.	0.
time (sec)	N/A	0.503	0.737	0.645	0.	0.	0.	0.

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	133	2044	0	0	0	0
normalized size	1	1.	0.96	14.71	0.	0.	0.	0.
time (sec)	N/A	0.215	0.078	0.242	0.	0.	0.	0.

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	128	128	311	3393	0	0	0	0
normalized size	1	1.	2.43	26.51	0.	0.	0.	0.
time (sec)	N/A	0.232	0.172	0.442	0.	0.	0.	0.

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	263	263	436	11233	0	0	0	0
normalized size	1	1.	1.66	42.71	0.	0.	0.	0.
time (sec)	N/A	0.6	1.382	0.925	0.	0.	0.	0.

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	414	414	634	3058	0	0	0	0
normalized size	1	1.	1.53	7.39	0.	0.	0.	0.
time (sec)	N/A	1.02	2.524	2.979	0.	0.	0.	0.

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.037	3.122	0.671	0.	0.	0.	0.

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	297	297	484	394	0	0	0	0
normalized size	1	1.	1.63	1.33	0.	0.	0.	0.
time (sec)	N/A	0.27	3.287	0.053	0.	0.	0.	0.

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	237	237	404	305	0	0	0	0
normalized size	1	1.	1.7	1.29	0.	0.	0.	0.
time (sec)	N/A	0.208	1.53	0.05	0.	0.	0.	0.

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	179	179	329	235	0	0	0	0
normalized size	1	1.	1.84	1.31	0.	0.	0.	0.
time (sec)	N/A	0.159	1.458	0.045	0.	0.	0.	0.

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	138	138	138	168	0	0	0	0
normalized size	1	1.	1.	1.22	0.	0.	0.	0.
time (sec)	N/A	0.075	0.056	0.038	0.	0.	0.	0.

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	181	181	169	260	0	0	0	0
normalized size	1	1.	0.93	1.44	0.	0.	0.	0.
time (sec)	N/A	0.186	0.095	0.051	0.	0.	0.	0.

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	232	232	223	321	0	0	0	0
normalized size	1	1.	0.96	1.38	0.	0.	0.	0.
time (sec)	N/A	0.241	0.141	0.07	0.	0.	0.	0.

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	293	293	298	393	0	0	0	0
normalized size	1	1.	1.02	1.34	0.	0.	0.	0.
time (sec)	N/A	0.284	0.173	0.061	0.	0.	0.	0.

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	598	598	0	2136	0	0	0	0
normalized size	1	1.	0.	3.57	0.	0.	0.	0.
time (sec)	N/A	0.671	180.002	14.474	0.	0.	0.	0.

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	430	430	0	1784	0	0	0	0
normalized size	1	1.	0.	4.15	0.	0.	0.	0.
time (sec)	N/A	0.425	122.316	8.652	0.	0.	0.	0.

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	323	323	0	16024	0	0	0	0
normalized size	1	1.	0.	49.61	0.	0.	0.	0.
time (sec)	N/A	0.266	180.002	5.046	0.	0.	0.	0.

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	223	223	0	1297	0	0	0	0
normalized size	1	1.	0.	5.82	0.	0.	0.	0.
time (sec)	N/A	0.048	0.155	0.26	0.	0.	0.	0.

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	369	369	0	2363	0	0	0	0
normalized size	1	1.	0.	6.4	0.	0.	0.	0.
time (sec)	N/A	0.433	180.003	0.664	0.	0.	0.	0.

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	473	473	0	40579	0	0	0	0
normalized size	1	1.	0.	85.79	0.	0.	0.	0.
time (sec)	N/A	0.604	120.59	6.674	0.	0.	0.	0.

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	591	591	0	2861	0	0	0	0
normalized size	1	1.	0.	4.84	0.	0.	0.	0.
time (sec)	N/A	0.842	180.003	14.738	0.	0.	0.	0.

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.031	0.515	0.622	0.	0.	0.	0.

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	69	58	86	135	65	86
normalized size	1	1.	1.	0.84	1.25	1.96	0.94	1.25
time (sec)	N/A	0.086	0.005	0.023	1.483	1.679	1.86	1.127

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	66	57	85	143	61	85
normalized size	1	1.	1.	0.86	1.29	2.17	0.92	1.29
time (sec)	N/A	0.095	0.022	0.023	0.988	1.636	1.363	1.157

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	58	49	68	107	54	72
normalized size	1	1.	1.38	1.17	1.62	2.55	1.29	1.71
time (sec)	N/A	0.025	0.004	0.021	0.971	1.559	1.02	1.129

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	65	50	45	61	109	48	58
normalized size	1	1.3	1.	0.9	1.22	2.18	0.96	1.16
time (sec)	N/A	0.023	0.01	0.024	0.975	1.612	0.748	1.124

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	62	93	101	0	0	0
normalized size	1	1.	1.	1.5	1.63	0.	0.	0.
time (sec)	N/A	0.066	0.004	0.037	1.622	0.	0.	0.

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	40	43	54	100	41	55
normalized size	1	1.	1.	1.08	1.35	2.5	1.02	1.38
time (sec)	N/A	0.055	0.005	0.029	0.988	1.603	1.061	1.152

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	70	70	74	110	142	0	0	0
normalized size	1	1.	1.06	1.57	2.03	0.	0.	0.
time (sec)	N/A	0.071	0.005	0.041	1.638	0.	0.	0.

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	58	58	76	140	61	88
normalized size	1	1.	0.92	0.92	1.21	2.22	0.97	1.4
time (sec)	N/A	0.082	0.02	0.033	0.991	1.596	1.523	1.122

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	111	96	132	196	104	132
normalized size	1	1.	1.	0.86	1.19	1.77	0.94	1.19
time (sec)	N/A	0.156	0.095	0.025	1.441	1.562	3.458	1.147

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	106	93	128	211	105	128
normalized size	1	1.	1.	0.88	1.21	1.99	0.99	1.21
time (sec)	N/A	0.172	0.063	0.024	0.974	1.693	2.473	1.171

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	98	85	84	170	92	80
normalized size	1	1.	1.61	1.39	1.38	2.79	1.51	1.31
time (sec)	N/A	0.043	0.048	0.023	0.968	1.607	2.103	1.109

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	65	79	104	176	88	111
normalized size	1	1.	0.56	0.68	0.89	1.5	0.75	0.95
time (sec)	N/A	0.045	0.056	0.025	0.98	1.57	1.46	1.185

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	99	134	150	0	0	0
normalized size	1	1.	1.	1.35	1.52	0.	0.	0.
time (sec)	N/A	0.119	0.034	0.036	1.639	0.	0.	0.

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	62	78	96	167	82	97
normalized size	1	1.	0.77	0.96	1.19	2.06	1.01	1.2
time (sec)	N/A	0.117	0.051	0.03	0.968	1.706	1.851	1.109

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	90	90	103	139	182	0	0	0
normalized size	1	1.	1.14	1.54	2.02	0.	0.	0.
time (sec)	N/A	0.123	0.045	0.04	1.656	0.	0.	0.

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	68	80	103	177	87	120
normalized size	1	1.	0.8	0.94	1.21	2.08	1.02	1.41
time (sec)	N/A	0.124	0.055	0.032	1.007	1.729	1.928	1.126

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	141	122	162	261	138	162
normalized size	1	1.	1.	0.87	1.15	1.85	0.98	1.15
time (sec)	N/A	0.207	0.148	0.023	1.477	1.633	5.653	1.134

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	136	119	159	270	138	158
normalized size	1	1.	1.	0.88	1.17	1.99	1.01	1.16
time (sec)	N/A	0.233	0.084	0.026	0.979	1.723	4.325	1.178

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	128	111	99	216	124	95
normalized size	1	1.	1.73	1.5	1.34	2.92	1.68	1.28
time (sec)	N/A	0.05	0.088	0.025	0.972	1.708	3.503	1.121

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	83	104	134	223	117	140
normalized size	1	1.	0.52	0.65	0.83	1.39	0.73	0.87
time (sec)	N/A	0.076	0.077	0.026	0.967	1.658	2.761	1.136

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	132	161	181	0	0	0
normalized size	1	1.	1.	1.22	1.37	0.	0.	0.
time (sec)	N/A	0.154	0.053	0.037	1.636	0.	0.	0.

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	78	103	126	216	110	134
normalized size	1	1.	0.72	0.95	1.17	2.	1.02	1.24
time (sec)	N/A	0.156	0.07	0.031	0.972	1.609	3.106	1.163

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	138	138	154	177	220	0	0	0
normalized size	1	1.	1.12	1.28	1.59	0.	0.	0.
time (sec)	N/A	0.153	0.043	0.044	1.699	0.	0.	0.

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	83	107	130	216	117	149
normalized size	1	1.	0.72	0.92	1.12	1.86	1.01	1.28
time (sec)	N/A	0.158	0.071	0.033	1.129	1.747	2.941	1.188

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	56	73	100	131	110	77
normalized size	1	1.	0.7	0.91	1.25	1.64	1.38	0.96
time (sec)	N/A	0.155	0.056	0.033	1.634	1.775	2.597	1.286

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	120	238	0	0	0	0
normalized size	1	1.	1.06	2.11	0.	0.	0.	0.
time (sec)	N/A	0.142	0.034	0.091	0.	0.	0.	0.

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	49	46	73	92	75	50
normalized size	1	1.	1.	0.94	1.49	1.88	1.53	1.02
time (sec)	N/A	0.073	0.028	0.03	1.67	1.71	1.417	1.158

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	77	202	0	0	0	0
normalized size	1	1.	1.07	2.81	0.	0.	0.	0.
time (sec)	N/A	0.072	0.005	0.086	0.	0.	0.	0.

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	19	34	36	19
normalized size	1	1.	1.	0.94	1.19	2.12	2.25	1.19
time (sec)	N/A	0.017	0.003	0.028	1.548	1.62	2.426	1.113

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	64	64	103	251	0	0	0	0
normalized size	1	1.	1.61	3.92	0.	0.	0.	0.
time (sec)	N/A	0.1	0.024	0.092	0.	0.	0.	0.

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	52	51	72	116	68	0
normalized size	1	1.	1.	0.98	1.38	2.23	1.31	0.
time (sec)	N/A	0.087	0.008	0.036	1.602	1.699	2.077	0.

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	113	113	142	327	0	0	0	0
normalized size	1	1.	1.26	2.89	0.	0.	0.	0.
time (sec)	N/A	0.162	0.061	0.095	0.	0.	0.	0.

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	88	81	122	169	117	0
normalized size	1	1.	1.	0.92	1.39	1.92	1.33	0.
time (sec)	N/A	0.165	0.018	0.041	1.654	1.63	3.489	0.

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	157	157	90	281	0	0	0	0
normalized size	1	1.	0.57	1.79	0.	0.	0.	0.
time (sec)	N/A	0.363	0.232	0.095	0.	0.	0.	0.

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	79	89	154	185	291	0
normalized size	1	1.	0.82	0.93	1.6	1.93	3.03	0.
time (sec)	N/A	0.18	0.06	0.044	1.58	1.634	3.178	0.

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	133	133	77	257	0	0	0	0
normalized size	1	1.	0.58	1.93	0.	0.	0.	0.
time (sec)	N/A	0.163	0.121	0.095	0.	0.	0.	0.

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	47	59	112	113	0	0
normalized size	1	1.	0.73	0.92	1.75	1.77	0.	0.
time (sec)	N/A	0.064	0.043	0.036	1.613	1.613	0.	0.

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	39	57	80	85	107	77
normalized size	1	1.	0.63	0.92	1.29	1.37	1.73	1.24
time (sec)	N/A	0.041	0.028	0.026	1.595	1.652	2.045	1.166

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	44	56	105	109	0	0
normalized size	1	1.	0.72	0.92	1.72	1.79	0.	0.
time (sec)	N/A	0.026	0.022	0.034	1.666	1.654	0.	0.

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	117	117	72	298	0	0	0	0
normalized size	1	1.	0.62	2.55	0.	0.	0.	0.
time (sec)	N/A	0.184	0.157	0.049	0.	0.	0.	0.

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	94	92	161	221	299	0
normalized size	1	1.	0.97	0.95	1.66	2.28	3.08	0.
time (sec)	N/A	0.162	0.073	0.05	1.598	1.686	2.134	0.

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	156	156	93	369	0	0	0	0
normalized size	1	1.	0.6	2.37	0.	0.	0.	0.
time (sec)	N/A	0.406	0.392	0.108	0.	0.	0.	0.

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	124	125	216	279	360	0
normalized size	1	1.	0.91	0.92	1.59	2.05	2.65	0.
time (sec)	N/A	0.375	0.097	0.05	1.636	1.753	3.446	0.

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	58	102	146	146	243	104
normalized size	1	1.	0.67	1.19	1.7	1.7	2.83	1.21
time (sec)	N/A	0.066	0.142	0.036	1.542	1.649	3.959	1.155

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	64	101	174	176	0	0
normalized size	1	1.	0.58	0.91	1.57	1.59	0.	0.
time (sec)	N/A	0.075	0.053	0.042	1.706	1.596	0.	0.

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	55	77	116	146	235	92
normalized size	1	1.	0.65	0.92	1.38	1.74	2.8	1.1
time (sec)	N/A	0.05	0.045	0.029	1.482	1.542	3.816	1.188

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	68	96	174	189	0	0
normalized size	1	1.	0.65	0.91	1.66	1.8	0.	0.
time (sec)	N/A	0.046	0.026	0.038	1.634	1.637	0.	0.

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	159	159	90	340	0	0	0	0
normalized size	1	1.	0.57	2.14	0.	0.	0.	0.
time (sec)	N/A	0.284	0.222	0.102	0.	0.	0.	0.

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	118	135	244	333	602	0
normalized size	1	1.	0.83	0.95	1.72	2.35	4.24	0.
time (sec)	N/A	0.263	0.094	0.047	1.627	1.734	4.046	0.

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	205	205	111	415	0	0	0	0
normalized size	1	1.	0.54	2.02	0.	0.	0.	0.
time (sec)	N/A	0.76	0.593	0.103	0.	0.	0.	0.

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	142	170	301	394	763	0
normalized size	1	1.	0.78	0.93	1.64	2.15	4.17	0.
time (sec)	N/A	0.69	0.129	0.051	1.685	1.786	6.957	0.

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	105	176	0	228	0	144
normalized size	1	1.	0.66	1.1	0.	1.42	0.	0.9
time (sec)	N/A	0.271	0.124	0.776	0.	1.746	0.	1.18

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	298	298	278	199	0	0	0	0
normalized size	1	1.	0.93	0.67	0.	0.	0.	0.
time (sec)	N/A	0.27	2.831	0.559	0.	0.	0.	0.

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	86	156	351	188	0	107
normalized size	1	1.	1.	1.81	4.08	2.19	0.	1.24
time (sec)	N/A	0.061	0.111	0.408	1.86	1.739	0.	1.154

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	244	244	141	178	0	0	0	0
normalized size	1	1.	0.58	0.73	0.	0.	0.	0.
time (sec)	N/A	0.093	0.572	0.38	0.	0.	0.	0.

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	229	164	151	0	0	0	0
normalized size	1	1.	0.72	0.66	0.	0.	0.	0.
time (sec)	N/A	0.221	0.181	0.412	0.	0.	0.	0.

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	242	242	163	221	0	0	0	0
normalized size	1	1.	0.67	0.91	0.	0.	0.	0.
time (sec)	N/A	0.227	0.446	0.426	0.	0.	0.	0.

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	240	240	165	169	0	0	0	0
normalized size	1	1.	0.69	0.7	0.	0.	0.	0.
time (sec)	N/A	0.349	1.048	0.418	0.	0.	0.	0.

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	105	153	0	200	0	0
normalized size	1	1.	1.25	1.82	0.	2.38	0.	0.
time (sec)	N/A	0.102	0.116	0.552	0.	1.757	0.	0.

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	217	119	199	0	285	0	246
normalized size	1	1.	0.55	0.92	0.	1.31	0.	1.13
time (sec)	N/A	0.765	0.166	0.914	0.	1.87	0.	1.158

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	357	357	576	221	0	0	0	0
normalized size	1	1.	1.61	0.62	0.	0.	0.	0.
time (sec)	N/A	0.783	5.95	0.457	0.	0.	0.	0.

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	101	179	576	235	0	170
normalized size	1	1.	0.93	1.64	5.28	2.16	0.	1.56
time (sec)	N/A	0.075	0.162	0.3	2.052	1.765	0.	1.282

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	298	298	351	201	0	0	0	0
normalized size	1	1.	1.18	0.67	0.	0.	0.	0.
time (sec)	N/A	0.138	2.589	0.292	0.	0.	0.	0.

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	281	281	220	174	0	0	0	0
normalized size	1	1.	0.78	0.62	0.	0.	0.	0.
time (sec)	N/A	0.376	0.246	0.312	0.	0.	0.	0.

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	300	300	218	240	0	0	0	0
normalized size	1	1.	0.73	0.8	0.	0.	0.	0.
time (sec)	N/A	0.422	0.897	0.319	0.	0.	0.	0.

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	304	304	301	180	0	0	0	0
normalized size	1	1.	0.99	0.59	0.	0.	0.	0.
time (sec)	N/A	0.642	1.604	0.326	0.	0.	0.	0.

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	310	310	263	245	0	0	0	0
normalized size	1	1.	0.85	0.79	0.	0.	0.	0.
time (sec)	N/A	0.434	0.476	0.456	0.	0.	0.	0.

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	289	289	129	225	0	360	0	366
normalized size	1	1.	0.45	0.78	0.	1.25	0.	1.27
time (sec)	N/A	1.977	0.238	0.934	0.	2.298	0.	1.223

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F(-2)	F	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	418	418	1059	245	0	0	0	0
normalized size	1	1.	2.53	0.59	0.	0.	0.	0.
time (sec)	N/A	2.031	15.463	0.503	0.	0.	0.	0.

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	111	205	933	298	0	238
normalized size	1	1.	0.83	1.53	6.96	2.22	0.	1.78
time (sec)	N/A	0.085	0.211	0.329	2.356	2.487	0.	1.193

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	348	348	643	225	0	0	0	0
normalized size	1	1.	1.85	0.65	0.	0.	0.	0.
time (sec)	N/A	0.194	6.149	0.315	0.	0.	0.	0.

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	329	329	268	198	0	0	0	0
normalized size	1	1.	0.81	0.6	0.	0.	0.	0.
time (sec)	N/A	0.55	0.343	0.342	0.	0.	0.	0.

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	355	355	491	265	0	0	0	0
normalized size	1	1.	1.38	0.75	0.	0.	0.	0.
time (sec)	N/A	0.773	3.945	0.353	0.	0.	0.	0.

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	364	364	361	204	0	0	0	0
normalized size	1	1.	0.99	0.56	0.	0.	0.	0.
time (sec)	N/A	1.144	2.029	0.356	0.	0.	0.	0.

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	372	372	313	270	0	0	0	0
normalized size	1	1.	0.84	0.73	0.	0.	0.	0.
time (sec)	N/A	0.975	0.98	0.487	0.	0.	0.	0.

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	91	165	0	196	0	134
normalized size	1	1.	0.76	1.38	0.	1.63	0.	1.12
time (sec)	N/A	0.153	0.12	1.153	0.	2.471	0.	1.23

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	250	250	158	184	0	0	0	0
normalized size	1	1.	0.63	0.74	0.	0.	0.	0.
time (sec)	N/A	0.147	0.577	0.895	0.	0.	0.	0.

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	60	144	82	158	0	81
normalized size	1	1.	1.02	2.44	1.39	2.68	0.	1.37
time (sec)	N/A	0.058	0.068	0.431	1.775	2.426	0.	1.157

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	193	193	118	150	0	0	0	0
normalized size	1	1.	0.61	0.78	0.	0.	0.	0.
time (sec)	N/A	0.057	0.107	0.352	0.	0.	0.	0.

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	177	177	100	139	0	0	0	0
normalized size	1	1.	0.56	0.79	0.	0.	0.	0.
time (sec)	N/A	0.134	0.144	0.393	0.	0.	0.	0.

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	62	139	0	163	0	136
normalized size	1	1.	1.11	2.48	0.	2.91	0.	2.43
time (sec)	N/A	0.093	0.092	0.369	0.	2.4	0.	1.233

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	242	242	165	175	0	0	0	0
normalized size	1	1.	0.68	0.72	0.	0.	0.	0.
time (sec)	N/A	0.221	0.689	0.497	0.	0.	0.	0.

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	110	163	0	211	0	381
normalized size	1	1.	0.93	1.38	0.	1.79	0.	3.23
time (sec)	N/A	0.2	0.119	0.801	0.	2.45	0.	1.414

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	107	242	0	228	0	130
normalized size	1	1.	1.	2.26	0.	2.13	0.	1.21
time (sec)	N/A	0.202	0.126	1.029	0.	2.414	0.	1.356

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	251	251	155	247	0	0	0	0
normalized size	1	1.	0.62	0.98	0.	0.	0.	0.
time (sec)	N/A	0.159	0.24	0.766	0.	0.	0.	0.

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	42	100	38	88	0	61
normalized size	1	1.	0.86	2.04	0.78	1.8	0.	1.24
time (sec)	N/A	0.055	0.05	0.266	1.742	2.228	0.	1.369

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	38	98	55	88	0	55
normalized size	1	1.	0.84	2.18	1.22	1.96	0.	1.22
time (sec)	N/A	0.025	0.045	0.231	1.025	2.315	0.	1.38

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	229	229	141	232	0	0	0	0
normalized size	1	1.	0.62	1.01	0.	0.	0.	0.
time (sec)	N/A	0.28	0.218	0.281	0.	0.	0.	0.

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	122	231	0	234	0	0
normalized size	1	1.	1.18	2.24	0.	2.27	0.	0.
time (sec)	N/A	0.202	0.181	0.295	0.	2.692	0.	0.

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	300	300	258	273	0	0	0	0
normalized size	1	1.	0.86	0.91	0.	0.	0.	0.
time (sec)	N/A	0.614	1.238	0.367	0.	0.	0.	0.

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	143	259	0	282	0	0
normalized size	1	1.	0.87	1.57	0.	1.71	0.	0.
time (sec)	N/A	0.495	0.327	0.655	0.	2.559	0.	0.

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	131	386	0	316	0	177
normalized size	1	1.	0.77	2.27	0.	1.86	0.	1.04
time (sec)	N/A	0.433	0.202	1.507	0.	2.497	0.	1.247

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	308	308	177	389	0	0	0	0
normalized size	1	1.	0.57	1.26	0.	0.	0.	0.
time (sec)	N/A	0.37	0.386	0.756	0.	0.	0.	0.

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	65	244	88	158	0	99
normalized size	1	1.	0.58	2.18	0.79	1.41	0.	0.88
time (sec)	N/A	0.137	0.079	0.971	1.393	2.663	0.	1.217

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	57	240	126	142	0	78
normalized size	1	1.	0.74	3.12	1.64	1.84	0.	1.01
time (sec)	N/A	0.113	0.062	0.713	1.034	2.556	0.	1.241

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	51	244	89	136	0	78
normalized size	1	1.	0.65	3.09	1.13	1.72	0.	0.99
time (sec)	N/A	0.061	0.054	0.303	1.309	2.542	0.	1.192

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	63	240	116	155	0	95
normalized size	1	1.	0.62	2.38	1.15	1.53	0.	0.94
time (sec)	N/A	0.054	0.052	0.267	1.066	2.259	0.	1.215

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	279	279	168	370	0	0	0	0
normalized size	1	1.	0.6	1.33	0.	0.	0.	0.
time (sec)	N/A	0.43	0.366	0.323	0.	0.	0.	0.

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	151	369	0	317	0	0
normalized size	1	1.	0.96	2.34	0.	2.01	0.	0.
time (sec)	N/A	0.337	0.249	0.337	0.	2.781	0.	0.

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	270	234	600	0	0	0	0
normalized size	1	1.	0.87	2.22	0.	0.	0.	0.
time (sec)	N/A	0.225	0.338	0.747	0.	0.	0.	0.

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	175	376	0	0	0	0
normalized size	1	1.	0.87	1.87	0.	0.	0.	0.
time (sec)	N/A	0.157	0.138	0.577	0.	0.	0.	0.

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	111	222	0	0	0	0
normalized size	1	1.	0.9	1.79	0.	0.	0.	0.
time (sec)	N/A	0.076	0.111	0.454	0.	0.	0.	0.

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.048	0.852	0.499	0.	0.	0.	0.

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.05	0.605	1.122	0.	0.	0.	0.

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	F(-2)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.086	0.919	0.53	0.	0.	0.	0.

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	F(-2)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.081	0.495	0.482	0.	0.	0.	0.

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	F(-2)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	112	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.17	0.1	0.596	0.	0.	0.	0.

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.073	0.444	0.961	0.	0.	0.	0.

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.084	0.509	1.092	0.	0.	0.	0.

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	89	107	157	225	121	142
normalized size	1	1.	0.72	0.86	1.27	1.81	0.98	1.15
time (sec)	N/A	0.425	0.038	0.033	1.53	2.203	2.808	1.151

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	156	156	104	258	0	0	0	0
normalized size	1	1.	0.67	1.65	0.	0.	0.	0.
time (sec)	N/A	0.409	0.607	0.095	0.	0.	0.	0.

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	64	85	117	177	94	112
normalized size	1	1.	0.67	0.89	1.22	1.84	0.98	1.17
time (sec)	N/A	0.053	0.03	0.033	1.006	2.202	1.496	1.141

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	128	128	82	233	0	0	0	0
normalized size	1	1.	0.64	1.82	0.	0.	0.	0.
time (sec)	N/A	0.096	0.05	0.086	0.	0.	0.	0.

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	177	1078	0	0	0	0
normalized size	1	1.	1.05	6.38	0.	0.	0.	0.
time (sec)	N/A	0.311	0.044	1.577	0.	0.	0.	0.

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	113	113	123	262	0	0	0	0
normalized size	1	1.	1.09	2.32	0.	0.	0.	0.
time (sec)	N/A	0.223	0.148	0.092	0.	0.	0.	0.

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	196	208	1167	0	0	0	0
normalized size	1	1.	1.06	5.95	0.	0.	0.	0.
time (sec)	N/A	0.325	0.087	3.306	0.	0.	0.	0.

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	135	135	103	323	0	0	0	0
normalized size	1	1.	0.76	2.39	0.	0.	0.	0.
time (sec)	N/A	0.312	0.591	0.096	0.	0.	0.	0.

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	191	110	168	228	319	185	217
normalized size	1	1.	0.58	0.88	1.19	1.67	0.97	1.14
time (sec)	N/A	0.789	0.081	0.034	1.563	2.179	4.817	1.181

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	225	225	133	333	0	0	0	0
normalized size	1	1.	0.59	1.48	0.	0.	0.	0.
time (sec)	N/A	0.752	1.242	0.092	0.	0.	0.	0.

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	84	142	150	274	158	216
normalized size	1	1.	0.55	0.93	0.98	1.79	1.03	1.41
time (sec)	N/A	0.093	0.065	0.031	1.004	2.213	2.788	1.165

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	205	205	112	304	0	0	0	0
normalized size	1	1.	0.55	1.48	0.	0.	0.	0.
time (sec)	N/A	0.139	0.655	0.085	0.	0.	0.	0.

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	235	235	218	1173	0	0	0	0
normalized size	1	1.	0.93	4.99	0.	0.	0.	0.
time (sec)	N/A	0.515	0.316	3.003	0.	0.	0.	0.

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	205	205	167	346	0	0	0	0
normalized size	1	1.	0.81	1.69	0.	0.	0.	0.
time (sec)	N/A	0.423	0.353	0.096	0.	0.	0.	0.

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	207	207	226	1255	0	0	0	0
normalized size	1	1.	1.09	6.06	0.	0.	0.	0.
time (sec)	N/A	0.452	0.307	3.27	0.	0.	0.	0.

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	216	216	189	375	0	0	0	0
normalized size	1	1.	0.88	1.74	0.	0.	0.	0.
time (sec)	N/A	0.44	0.381	0.102	0.	0.	0.	0.

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	240	240	126	211	273	417	241	267
normalized size	1	1.	0.52	0.88	1.14	1.74	1.	1.11
time (sec)	N/A	1.227	0.094	0.033	1.525	2.228	7.62	1.178

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	274	274	157	376	0	0	0	0
normalized size	1	1.	0.57	1.37	0.	0.	0.	0.
time (sec)	N/A	1.154	2.221	0.089	0.	0.	0.	0.

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	100	185	180	343	207	301
normalized size	1	1.	0.5	0.92	0.9	1.72	1.03	1.5
time (sec)	N/A	0.121	0.08	0.034	0.995	2.232	4.817	1.163

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	268	268	137	346	0	0	0	0
normalized size	1	1.	0.51	1.29	0.	0.	0.	0.
time (sec)	N/A	0.184	1.165	0.069	0.	0.	0.	0.

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	287	287	252	1217	0	0	0	0
normalized size	1	1.	0.88	4.24	0.	0.	0.	0.
time (sec)	N/A	0.743	0.529	3.927	0.	0.	0.	0.

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	251	251	202	388	0	0	0	0
normalized size	1	1.	0.8	1.55	0.	0.	0.	0.
time (sec)	N/A	0.645	0.705	0.097	0.	0.	0.	0.

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	299	299	333	1333	0	0	0	0
normalized size	1	1.	1.11	4.46	0.	0.	0.	0.
time (sec)	N/A	0.603	0.386	4.543	0.	0.	0.	0.

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	250	250	221	417	0	0	0	0
normalized size	1	1.	0.88	1.67	0.	0.	0.	0.
time (sec)	N/A	0.608	0.574	0.106	0.	0.	0.	0.

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	166	166	90	284	0	0	0	0
normalized size	1	1.	0.54	1.71	0.	0.	0.	0.
time (sec)	N/A	0.387	0.281	0.088	0.	0.	0.	0.

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	169	169	123	1695	0	0	0	0
normalized size	1	1.	0.73	10.03	0.	0.	0.	0.
time (sec)	N/A	0.294	0.112	0.727	0.	0.	0.	0.

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	98	98	69	230	0	0	0	0
normalized size	1	1.	0.7	2.35	0.	0.	0.	0.
time (sec)	N/A	0.167	0.173	0.093	0.	0.	0.	0.

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	110	897	0	0	0	0
normalized size	1	1.	1.08	8.79	0.	0.	0.	0.
time (sec)	N/A	0.146	0.01	0.346	0.	0.	0.	0.

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	19	34	0	19
normalized size	1	1.	1.	0.94	1.19	2.12	0.	1.19
time (sec)	N/A	0.024	0.003	0.024	1.529	2.127	0.	1.14

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	91	91	243	1767	0	0	0	0
normalized size	1	1.	2.67	19.42	0.	0.	0.	0.
time (sec)	N/A	0.179	0.05	0.283	0.	0.	0.	0.

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	92	92	73	292	0	0	0	0
normalized size	1	1.	0.79	3.17	0.	0.	0.	0.
time (sec)	N/A	0.195	0.179	0.099	0.	0.	0.	0.

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	178	178	142	5491	0	0	0	0
normalized size	1	1.	0.8	30.85	0.	0.	0.	0.
time (sec)	N/A	0.336	0.305	0.715	0.	0.	0.	0.

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	166	166	120	374	0	0	0	0
normalized size	1	1.	0.72	2.25	0.	0.	0.	0.
time (sec)	N/A	0.436	0.346	0.108	0.	0.	0.	0.

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	192	192	117	1092	0	0	0	0
normalized size	1	1.	0.61	5.69	0.	0.	0.	0.
time (sec)	N/A	0.29	0.179	0.423	0.	0.	0.	0.

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	68	97	204	166	0	0
normalized size	1	1.	0.64	0.92	1.92	1.57	0.	0.
time (sec)	N/A	0.11	0.09	0.035	1.636	2.123	0.	0.

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	47	84	140	112	0	0
normalized size	1	1.	0.52	0.92	1.54	1.23	0.	0.
time (sec)	N/A	0.07	0.031	0.032	1.527	2.172	0.	0.

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	65	91	197	162	0	0
normalized size	1	1.	0.65	0.91	1.97	1.62	0.	0.
time (sec)	N/A	0.069	0.04	0.029	1.605	2.125	0.	0.

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	170	170	119	1936	0	0	0	0
normalized size	1	1.	0.7	11.39	0.	0.	0.	0.
time (sec)	N/A	0.311	0.199	0.53	0.	0.	0.	0.

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	177	177	109	369	0	0	0	0
normalized size	1	1.	0.62	2.08	0.	0.	0.	0.
time (sec)	N/A	0.34	0.325	0.11	0.	0.	0.	0.

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	250	250	183	5115	0	0	0	0
normalized size	1	1.	0.73	20.46	0.	0.	0.	0.
time (sec)	N/A	0.74	0.618	3.573	0.	0.	0.	0.

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	242	242	166	444	0	0	0	0
normalized size	1	1.	0.69	1.83	0.	0.	0.	0.
time (sec)	N/A	0.873	0.419	0.12	0.	0.	0.	0.

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	74	154	250	192	0	0
normalized size	1	1.	0.53	1.1	1.79	1.37	0.	0.
time (sec)	N/A	0.185	0.086	0.047	1.588	2.185	0.	0.

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	95	164	313	255	0	0
normalized size	1	1.	0.52	0.91	1.73	1.41	0.	0.
time (sec)	N/A	0.266	0.108	0.041	1.664	2.147	0.	0.

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	71	127	220	192	0	0
normalized size	1	1.	0.51	0.92	1.59	1.39	0.	0.
time (sec)	N/A	0.096	0.039	0.041	1.59	2.127	0.	0.

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	98	159	313	261	0	0
normalized size	1	1.	0.58	0.94	1.85	1.54	0.	0.
time (sec)	N/A	0.119	0.045	0.041	1.671	2.156	0.	0.

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	236	236	156	2176	0	0	0	0
normalized size	1	1.	0.66	9.22	0.	0.	0.	0.
time (sec)	N/A	0.485	0.255	0.559	0.	0.	0.	0.

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	250	250	139	440	0	0	0	0
normalized size	1	1.	0.56	1.76	0.	0.	0.	0.
time (sec)	N/A	0.551	0.4	0.115	0.	0.	0.	0.

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	322	322	226	2421	0	0	0	0
normalized size	1	1.	0.7	7.52	0.	0.	0.	0.
time (sec)	N/A	1.333	0.771	2.632	0.	0.	0.	0.

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	317	317	189	517	0	0	0	0
normalized size	1	1.	0.6	1.63	0.	0.	0.	0.
time (sec)	N/A	1.526	0.764	0.116	0.	0.	0.	0.

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	385	385	360	235	0	0	0	0
normalized size	1	1.	0.94	0.61	0.	0.	0.	0.
time (sec)	N/A	1.425	1.142	0.999	0.	0.	0.	0.

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	436	436	267	302	0	0	0	0
normalized size	1	1.	0.61	0.69	0.	0.	0.	0.
time (sec)	N/A	1.135	1.216	0.59	0.	0.	0.	0.

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	279	279	260	198	0	0	0	0
normalized size	1	1.	0.93	0.71	0.	0.	0.	0.
time (sec)	N/A	0.176	0.581	0.404	0.	0.	0.	0.

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	340	340	201	268	0	0	0	0
normalized size	1	1.	0.59	0.79	0.	0.	0.	0.
time (sec)	N/A	0.215	0.381	0.405	0.	0.	0.	0.

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	439	439	250	337	0	0	0	0
normalized size	1	1.	0.57	0.77	0.	0.	0.	0.
time (sec)	N/A	0.513	0.255	0.454	0.	0.	0.	0.

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	458	458	265	309	0	0	0	0
normalized size	1	1.	0.58	0.67	0.	0.	0.	0.
time (sec)	N/A	0.529	0.749	0.453	0.	0.	0.	0.

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	328	328	222	255	0	0	0	0
normalized size	1	1.	0.68	0.78	0.	0.	0.	0.
time (sec)	N/A	0.856	1.792	0.414	0.	0.	0.	0.

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	275	275	239	195	0	0	0	0
normalized size	1	1.	0.87	0.71	0.	0.	0.	0.
time (sec)	N/A	0.427	1.663	0.549	0.	0.	0.	0.

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	476	476	797	271	0	0	0	0
normalized size	1	1.	1.67	0.57	0.	0.	0.	0.
time (sec)	N/A	4.073	4.567	0.909	0.	0.	0.	0.

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	531	531	527	338	0	0	0	0
normalized size	1	1.	0.99	0.64	0.	0.	0.	0.
time (sec)	N/A	3.186	3.463	0.487	0.	0.	0.	0.

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	334	334	601	237	0	0	0	0
normalized size	1	1.	1.8	0.71	0.	0.	0.	0.
time (sec)	N/A	0.232	4.045	0.309	0.	0.	0.	0.

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	438	438	439	304	0	0	0	0
normalized size	1	1.	1.	0.69	0.	0.	0.	0.
time (sec)	N/A	0.311	0.935	0.317	0.	0.	0.	0.

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	530	530	496	365	0	0	0	0
normalized size	1	1.	0.94	0.69	0.	0.	0.	0.
time (sec)	N/A	0.882	3.178	0.371	0.	0.	0.	0.

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	556	556	376	356	0	0	0	0
normalized size	1	1.	0.68	0.64	0.	0.	0.	0.
time (sec)	N/A	0.968	1.029	0.383	0.	0.	0.	0.

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	567	567	455	412	0	0	0	0
normalized size	1	1.	0.8	0.73	0.	0.	0.	0.
time (sec)	N/A	1.655	2.95	0.385	0.	0.	0.	0.

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	579	579	537	343	0	0	0	0
normalized size	1	1.	0.93	0.59	0.	0.	0.	0.
time (sec)	N/A	1.146	7.294	0.5	0.	0.	0.	0.

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F(-2)	F	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	578	578	1320	309	0	0	0	0
normalized size	1	1.	2.28	0.53	0.	0.	0.	0.
time (sec)	N/A	10.701	8.257	0.957	0.	0.	0.	0.

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	638	638	759	376	0	0	0	0
normalized size	1	1.	1.19	0.59	0.	0.	0.	0.
time (sec)	N/A	8.348	4.831	0.569	0.	0.	0.	0.

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	F	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	387	387	1087	275	0	0	0	0
normalized size	1	1.	2.81	0.71	0.	0.	0.	0.
time (sec)	N/A	0.281	7.786	0.351	0.	0.	0.	0.

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	516	516	771	342	0	0	0	0
normalized size	1	1.	1.49	0.66	0.	0.	0.	0.
time (sec)	N/A	0.39	1.577	0.364	0.	0.	0.	0.

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	605	605	889	404	0	0	0	0
normalized size	1	1.	1.47	0.67	0.	0.	0.	0.
time (sec)	N/A	1.263	7.093	0.422	0.	0.	0.	0.

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	655	655	626	399	0	0	0	0
normalized size	1	1.	0.96	0.61	0.	0.	0.	0.
time (sec)	N/A	1.408	1.723	0.426	0.	0.	0.	0.

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	661	661	761	454	0	0	0	0
normalized size	1	1.	1.15	0.69	0.	0.	0.	0.
time (sec)	N/A	2.621	7.723	0.425	0.	0.	0.	0.

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	675	675	644	401	0	0	0	0
normalized size	1	1.	0.95	0.59	0.	0.	0.	0.
time (sec)	N/A	2.309	4.565	0.556	0.	0.	0.	0.

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	315	315	279	206	0	0	0	0
normalized size	1	1.	0.89	0.65	0.	0.	0.	0.
time (sec)	N/A	0.426	0.676	1.143	0.	0.	0.	0.

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	344	344	175	271	0	0	0	0
normalized size	1	1.	0.51	0.79	0.	0.	0.	0.
time (sec)	N/A	0.334	0.344	0.898	0.	0.	0.	0.

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	220	220	126	180	0	0	0	0
normalized size	1	1.	0.57	0.82	0.	0.	0.	0.
time (sec)	N/A	0.143	0.239	0.43	0.	0.	0.	0.

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	256	256	140	0	0	0	0	0
normalized size	1	1.	0.55	0.	0.	0.	0.	0.
time (sec)	N/A	0.154	0.115	0.45	0.	0.	0.	0.

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	227	227	145	198	0	0	0	0
normalized size	1	1.	0.64	0.87	0.	0.	0.	0.
time (sec)	N/A	0.253	0.165	0.393	0.	0.	0.	0.

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	208	208	128	171	0	0	0	0
normalized size	1	1.	0.62	0.82	0.	0.	0.	0.
time (sec)	N/A	0.252	0.419	0.369	0.	0.	0.	0.

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	328	328	231	261	0	0	0	0
normalized size	1	1.	0.7	0.8	0.	0.	0.	0.
time (sec)	N/A	0.475	1.168	0.493	0.	0.	0.	0.

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	311	311	228	206	0	0	0	0
normalized size	1	1.	0.73	0.66	0.	0.	0.	0.
time (sec)	N/A	0.628	2.61	0.798	0.	0.	0.	0.

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	305	305	209	294	0	0	0	0
normalized size	1	1.	0.69	0.96	0.	0.	0.	0.
time (sec)	N/A	0.396	0.842	1.	0.	0.	0.	0.

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	349	349	228	0	0	0	0	0
normalized size	1	1.	0.65	0.	0.	0.	0.	0.
time (sec)	N/A	0.341	0.348	0.873	0.	0.	0.	0.

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	50	116	99	115	0	97
normalized size	1	1.	0.64	1.49	1.27	1.47	0.	1.24
time (sec)	N/A	0.111	0.075	0.265	2.537	2.245	0.	1.226

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	49	114	72	117	0	97
normalized size	1	1.	0.68	1.58	1.	1.62	0.	1.35
time (sec)	N/A	0.046	0.062	0.223	1.765	2.299	0.	1.232

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	310	310	204	307	0	0	0	0
normalized size	1	1.	0.66	0.99	0.	0.	0.	0.
time (sec)	N/A	0.506	0.308	0.297	0.	0.	0.	0.

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	293	293	226	279	0	0	0	0
normalized size	1	1.	0.77	0.95	0.	0.	0.	0.
time (sec)	N/A	0.431	1.048	0.306	0.	0.	0.	0.

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	422	422	371	376	0	0	0	0
normalized size	1	1.	0.88	0.89	0.	0.	0.	0.
time (sec)	N/A	1.128	2.239	0.393	0.	0.	0.	0.

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	397	397	270	318	0	0	0	0
normalized size	1	1.	0.68	0.8	0.	0.	0.	0.
time (sec)	N/A	1.202	3.415	0.683	0.	0.	0.	0.

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	400	400	229	454	0	0	0	0
normalized size	1	1.	0.57	1.14	0.	0.	0.	0.
time (sec)	N/A	0.818	1.315	1.515	0.	0.	0.	0.

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	444	444	239	0	0	0	0	0
normalized size	1	1.	0.54	0.	0.	0.	0.	0.
time (sec)	N/A	0.769	0.574	0.904	0.	0.	0.	0.

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	81	276	0	208	0	151
normalized size	1	1.	0.47	1.6	0.	1.21	0.	0.88
time (sec)	N/A	0.283	0.113	0.982	0.	2.253	0.	1.263

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	80	272	158	197	0	138
normalized size	1	1.	0.58	1.96	1.14	1.42	0.	0.99
time (sec)	N/A	0.267	0.087	0.733	1.393	2.329	0.	1.246

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	71	276	0	186	0	131
normalized size	1	1.	0.52	2.01	0.	1.36	0.	0.96
time (sec)	N/A	0.142	0.082	0.294	0.	2.296	0.	1.256

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	86	272	150	212	0	146
normalized size	1	1.	0.55	1.73	0.96	1.35	0.	0.93
time (sec)	N/A	0.102	0.07	0.271	1.464	2.244	0.	1.255

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	389	389	246	461	0	0	0	0
normalized size	1	1.	0.63	1.19	0.	0.	0.	0.
time (sec)	N/A	0.783	0.501	0.345	0.	0.	0.	0.

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	381	381	296	433	0	0	0	0
normalized size	1	1.	0.78	1.14	0.	0.	0.	0.
time (sec)	N/A	0.686	1.608	0.345	0.	0.	0.	0.

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.054	1.769	0.588	0.	0.	0.	0.

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.035	0.877	0.462	0.	0.	0.	0.

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.062	0.855	0.464	0.	0.	0.	0.

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.063	0.676	1.135	0.	0.	0.	0.

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	F(-2)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.109	0.951	0.49	0.	0.	0.	0.

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	F(-2)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.098	0.155	0.616	0.	0.	0.	0.

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.102	0.502	0.925	0.	0.	0.	0.

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.115	0.63	1.092	0.	0.	0.	0.

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	219	219	135	313	0	0	0	0
normalized size	1	1.	0.62	1.43	0.	0.	0.	0.
time (sec)	N/A	1.114	0.612	0.097	0.	0.	0.	0.

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	211	211	171	2555	0	0	0	0
normalized size	1	1.	0.81	12.11	0.	0.	0.	0.
time (sec)	N/A	0.882	0.532	2.629	0.	0.	0.	0.

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	160	160	101	276	0	0	0	0
normalized size	1	1.	0.63	1.72	0.	0.	0.	0.
time (sec)	N/A	0.129	0.067	0.097	0.	0.	0.	0.

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	172	172	144	1635	0	0	0	0
normalized size	1	1.	0.84	9.51	0.	0.	0.	0.
time (sec)	N/A	0.182	0.049	1.237	0.	0.	0.	0.

Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	276	276	284	460	0	0	0	0
normalized size	1	1.	1.03	1.67	0.	0.	0.	0.
time (sec)	N/A	0.523	0.071	1.325	0.	0.	0.	0.

Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	169	169	181	1826	0	0	0	0
normalized size	1	1.	1.07	10.8	0.	0.	0.	0.
time (sec)	N/A	0.395	0.175	0.431	0.	0.	0.	0.

Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	310	310	337	568	0	0	0	0
normalized size	1	1.	1.09	1.83	0.	0.	0.	0.
time (sec)	N/A	0.557	0.241	2.037	0.	0.	0.	0.

Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	189	189	177	5426	0	0	0	0
normalized size	1	1.	0.94	28.71	0.	0.	0.	0.
time (sec)	N/A	0.585	0.384	1.928	0.	0.	0.	0.

Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	313	313	165	411	0	0	0	0
normalized size	1	1.	0.53	1.31	0.	0.	0.	0.
time (sec)	N/A	2.283	1.224	0.099	0.	0.	0.	0.

Problem 372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	321	321	233	1121	0	0	0	0
normalized size	1	1.	0.73	3.49	0.	0.	0.	0.
time (sec)	N/A	1.795	1.089	1.699	0.	0.	0.	0.

Problem 373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	242	242	131	368	0	0	0	0
normalized size	1	1.	0.54	1.52	0.	0.	0.	0.
time (sec)	N/A	0.188	0.743	0.11	0.	0.	0.	0.

Problem 374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	289	289	195	2691	0	0	0	0
normalized size	1	1.	0.67	9.31	0.	0.	0.	0.
time (sec)	N/A	0.245	0.573	2.737	0.	0.	0.	0.

Problem 375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	370	370	302	566	0	0	0	0
normalized size	1	1.	0.82	1.53	0.	0.	0.	0.
time (sec)	N/A	0.97	0.558	1.497	0.	0.	0.	0.

Problem 376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	284	284	246	5486	0	0	0	0
normalized size	1	1.	0.87	19.32	0.	0.	0.	0.
time (sec)	N/A	0.751	0.388	2.39	0.	0.	0.	0.

Problem 377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	399	399	302	682	0	0	0	0
normalized size	1	1.	0.76	1.71	0.	0.	0.	0.
time (sec)	N/A	0.796	0.601	2.365	0.	0.	0.	0.

Problem 378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	311	311	289	5651	0	0	0	0
normalized size	1	1.	0.93	18.17	0.	0.	0.	0.
time (sec)	N/A	0.789	0.502	2.881	0.	0.	0.	0.

Problem 379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	381	381	191	471	0	0	0	0
normalized size	1	1.	0.5	1.24	0.	0.	0.	0.
time (sec)	N/A	3.725	2.	0.099	0.	0.	0.	0.

Problem 380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	389	389	281	1181	0	0	0	0
normalized size	1	1.	0.72	3.04	0.	0.	0.	0.
time (sec)	N/A	3.039	1.866	8.589	0.	0.	0.	0.

Problem 381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	308	308	157	428	0	0	0	0
normalized size	1	1.	0.51	1.39	0.	0.	0.	0.
time (sec)	N/A	0.254	1.317	0.096	0.	0.	0.	0.

Problem 382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	388	388	243	1134	0	0	0	0
normalized size	1	1.	0.63	2.92	0.	0.	0.	0.
time (sec)	N/A	0.34	1.127	2.551	0.	0.	0.	0.

Problem 383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	447	447	350	664	0	0	0	0
normalized size	1	1.	0.78	1.49	0.	0.	0.	0.
time (sec)	N/A	1.655	1.01	1.26	0.	0.	0.	0.

Problem 384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	354	354	298	10139	0	0	0	0
normalized size	1	1.	0.84	28.64	0.	0.	0.	0.
time (sec)	N/A	1.28	0.722	10.748	0.	0.	0.	0.

Problem 385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	503	503	464	790	0	0	0	0
normalized size	1	1.	0.92	1.57	0.	0.	0.	0.
time (sec)	N/A	1.192	0.75	6.651	0.	0.	0.	0.

Problem 386	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	336	336	331	7948	0	0	0	0
normalized size	1	1.	0.99	23.65	0.	0.	0.	0.
time (sec)	N/A	1.109	0.706	18.202	0.	0.	0.	0.

Problem 387	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	217	217	154	1740	0	0	0	0
normalized size	1	1.	0.71	8.02	0.	0.	0.	0.
time (sec)	N/A	0.626	0.247	3.701	0.	0.	0.	0.

Problem 388	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	260	260	162	292	0	0	0	0
normalized size	1	1.	0.62	1.12	0.	0.	0.	0.
time (sec)	N/A	0.447	0.29	5.585	0.	0.	0.	0.

Problem 389	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	130	130	93	925	0	0	0	0
normalized size	1	1.	0.72	7.12	0.	0.	0.	0.
time (sec)	N/A	0.245	0.209	1.033	0.	0.	0.	0.

Problem 390	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	149	936	0	0	0	0
normalized size	1	1.	1.08	6.78	0.	0.	0.	0.
time (sec)	N/A	0.218	0.012	1.132	0.	0.	0.	0.

Problem 391	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	19	34	0	19
normalized size	1	1.	1.	0.94	1.19	2.12	0.	1.19
time (sec)	N/A	0.024	0.004	0.078	1.491	1.788	0.	1.174

Problem 392	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	124	124	354	1834	0	0	0	0
normalized size	1	1.	2.85	14.79	0.	0.	0.	0.
time (sec)	N/A	0.231	0.057	1.253	0.	0.	0.	0.

Problem 393	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	122	122	108	1829	0	0	0	0
normalized size	1	1.	0.89	14.99	0.	0.	0.	0.
time (sec)	N/A	0.286	0.164	0.959	0.	0.	0.	0.

Problem 394	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	262	262	189	479	0	0	0	0
normalized size	1	1.	0.72	1.83	0.	0.	0.	0.
time (sec)	N/A	0.511	0.397	7.375	0.	0.	0.	0.

Problem 395	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	227	227	180	5574	0	0	0	0
normalized size	1	1.	0.79	24.56	0.	0.	0.	0.
time (sec)	N/A	0.722	0.483	7.244	0.	0.	0.	0.

Problem 396	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	270	270	156	1227	0	0	0	0
normalized size	1	1.	0.58	4.54	0.	0.	0.	0.
time (sec)	N/A	0.413	0.188	1.636	0.	0.	0.	0.

Problem 397	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	74	124	294	186	0	0
normalized size	1	1.	0.55	0.92	2.18	1.38	0.	0.
time (sec)	N/A	0.144	0.06	0.158	1.735	1.928	0.	0.

Problem 398	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	68	122	235	163	0	0
normalized size	1	1.	0.51	0.92	1.77	1.23	0.	0.
time (sec)	N/A	0.122	0.042	0.126	1.632	1.999	0.	0.

Problem 399	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	71	118	288	182	0	0
normalized size	1	1.	0.55	0.91	2.23	1.41	0.	0.
time (sec)	N/A	0.104	0.031	0.144	1.768	1.856	0.	0.

Problem 400	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	240	240	156	2089	0	0	0	0
normalized size	1	1.	0.65	8.7	0.	0.	0.	0.
time (sec)	N/A	0.428	0.222	1.752	0.	0.	0.	0.

Problem 401	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	234	234	157	2038	0	0	0	0
normalized size	1	1.	0.67	8.71	0.	0.	0.	0.
time (sec)	N/A	0.468	0.34	1.958	0.	0.	0.	0.

Problem 402	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	374	374	243	815	0	0	0	0
normalized size	1	1.	0.65	2.18	0.	0.	0.	0.
time (sec)	N/A	1.026	0.737	7.276	0.	0.	0.	0.

Problem 403	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	332	332	243	5190	0	0	0	0
normalized size	1	1.	0.73	15.63	0.	0.	0.	0.
time (sec)	N/A	1.29	0.942	8.694	0.	0.	0.	0.

Problem 404	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	212	105	220	390	271	0	0
normalized size	1	1.	0.5	1.04	1.84	1.28	0.	0.
time (sec)	N/A	0.294	0.242	0.261	1.68	1.808	0.	0.

Problem 405	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	237	111	216	451	289	0	0
normalized size	1	1.	0.47	0.91	1.9	1.22	0.	0.
time (sec)	N/A	0.389	0.073	0.144	1.866	1.968	0.	0.

Problem 406	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	208	103	191	367	271	0	0
normalized size	1	1.	0.5	0.92	1.76	1.3	0.	0.
time (sec)	N/A	0.177	0.078	0.133	1.692	1.882	0.	0.

Problem 407	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	225	114	211	452	315	0	0
normalized size	1	1.	0.51	0.94	2.01	1.4	0.	0.
time (sec)	N/A	0.196	0.056	0.302	1.885	2.001	0.	0.

Problem 408	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	332	332	208	2463	0	0	0	0
normalized size	1	1.	0.63	7.42	0.	0.	0.	0.
time (sec)	N/A	0.709	0.313	3.697	0.	0.	0.	0.

Problem 409	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	332	332	232	2315	0	0	0	0
normalized size	1	1.	0.7	6.97	0.	0.	0.	0.
time (sec)	N/A	0.754	0.547	3.306	0.	0.	0.	0.

Problem 410	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	478	478	295	1339	0	0	0	0
normalized size	1	1.	0.62	2.8	0.	0.	0.	0.
time (sec)	N/A	1.838	1.047	13.75	0.	0.	0.	0.

Problem 411	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	432	432	301	2523	0	0	0	0
normalized size	1	1.	0.7	5.84	0.	0.	0.	0.
time (sec)	N/A	2.16	1.163	11.602	0.	0.	0.	0.

Problem 412	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	523	523	262	417	0	0	0	0
normalized size	1	1.	0.5	0.8	0.	0.	0.	0.
time (sec)	N/A	2.437	1.164	4.634	0.	0.	0.	0.

Problem 413	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F(-2)	F	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	747	747	1844	460	0	0	0	0
normalized size	1	1.	2.47	0.62	0.	0.	0.	0.
time (sec)	N/A	1.846	12.133	2.878	0.	0.	0.	0.

Problem 414	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	373	373	206	370	0	0	0	0
normalized size	1	1.	0.55	0.99	0.	0.	0.	0.
time (sec)	N/A	0.355	0.581	1.884	0.	0.	0.	0.

Problem 415	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	626	626	258	422	0	0	0	0
normalized size	1	1.	0.41	0.67	0.	0.	0.	0.
time (sec)	N/A	0.361	0.798	2.117	0.	0.	0.	0.

Problem 416	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	600	600	366	453	0	0	0	0
normalized size	1	1.	0.61	0.76	0.	0.	0.	0.
time (sec)	N/A	0.727	0.586	2.316	0.	0.	0.	0.

Problem 417	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	622	622	768	466	0	0	0	0
normalized size	1	1.	1.23	0.75	0.	0.	0.	0.
time (sec)	N/A	0.774	3.175	2.294	0.	0.	0.	0.

Problem 418	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	602	602	345	404	0	0	0	0
normalized size	1	1.	0.57	0.67	0.	0.	0.	0.
time (sec)	N/A	1.238	5.517	1.722	0.	0.	0.	0.

Problem 419	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	361	361	341	462	0	0	0	0
normalized size	1	1.	0.94	1.28	0.	0.	0.	0.
time (sec)	N/A	1.016	3.491	2.786	0.	0.	0.	0.

Problem 420	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	652	652	538	469	0	0	0	0
normalized size	1	1.	0.83	0.72	0.	0.	0.	0.
time (sec)	N/A	7.371	3.265	4.163	0.	0.	0.	0.

Problem 421	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F(-2)	F	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	882	882	4015	514	0	0	0	0
normalized size	1	1.	4.55	0.58	0.	0.	0.	0.
time (sec)	N/A	5.47	18.256	2.363	0.	0.	0.	0.

Problem 422	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	477	477	441	421	0	0	0	0
normalized size	1	1.	0.92	0.88	0.	0.	0.	0.
time (sec)	N/A	0.416	3.85	1.543	0.	0.	0.	0.

Problem 423	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F(-2)	F	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	760	760	2105	466	0	0	0	0
normalized size	1	1.	2.77	0.61	0.	0.	0.	0.
time (sec)	N/A	0.524	12.947	1.434	0.	0.	0.	0.

Problem 424	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	726	726	555	511	0	0	0	0
normalized size	1	1.	0.76	0.7	0.	0.	0.	0.
time (sec)	N/A	1.144	1.369	1.52	0.	0.	0.	0.

Problem 425	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	901	901	1387	602	0	0	0	0
normalized size	1	1.	1.54	0.67	0.	0.	0.	0.
time (sec)	N/A	1.237	6.502	2.009	0.	0.	0.	0.

Problem 426	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	919	919	691	592	0	0	0	0
normalized size	1	1.	0.75	0.64	0.	0.	0.	0.
time (sec)	N/A	2.016	9.573	1.964	0.	0.	0.	0.

Problem 427	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	788	788	1508	557	0	0	0	0
normalized size	1	1.	1.91	0.71	0.	0.	0.	0.
time (sec)	N/A	1.879	10.086	2.326	0.	0.	0.	0.

Problem 428	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	798	798	850	525	0	0	0	0
normalized size	1	1.	1.07	0.66	0.	0.	0.	0.
time (sec)	N/A	19.664	6.475	4.576	0.	0.	0.	0.

Problem 429	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F(-2)	F	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1019	1019	6517	566	0	0	0	0
normalized size	1	1.	6.4	0.56	0.	0.	0.	0.
time (sec)	N/A	15.417	24.402	2.773	0.	0.	0.	0.

Problem 430	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	561	561	718	477	0	0	0	0
normalized size	1	1.	1.28	0.85	0.	0.	0.	0.
time (sec)	N/A	0.531	5.545	1.486	0.	0.	0.	0.

Problem 431	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F(-2)	F	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	870	870	4281	518	0	0	0	0
normalized size	1	1.	4.92	0.6	0.	0.	0.	0.
time (sec)	N/A	0.792	18.904	2.278	0.	0.	0.	0.

Problem 432	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	845	845	723	562	0	0	0	0
normalized size	1	1.	0.86	0.67	0.	0.	0.	0.
time (sec)	N/A	1.784	6.599	1.787	0.	0.	0.	0.

Problem 433	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1027	1027	3267	655	0	0	0	0
normalized size	1	1.	3.18	0.64	0.	0.	0.	0.
time (sec)	N/A	2.111	15.936	1.996	0.	0.	0.	0.

Problem 434	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1043	1043	1128	660	0	0	0	0
normalized size	1	1.	1.08	0.63	0.	0.	0.	0.
time (sec)	N/A	3.537	10.206	1.978	0.	0.	0.	0.

Problem 435	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1061	1061	1771	694	0	0	0	0
normalized size	1	1.	1.67	0.65	0.	0.	0.	0.
time (sec)	N/A	3.375	11.421	2.281	0.	0.	0.	0.

Problem 436	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	408	408	220	380	0	0	0	0
normalized size	1	1.	0.54	0.93	0.	0.	0.	0.
time (sec)	N/A	0.728	0.79	4.271	0.	0.	0.	0.

Problem 437	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	625	625	812	430	0	0	0	0
normalized size	1	1.	1.3	0.69	0.	0.	0.	0.
time (sec)	N/A	0.489	6.007	3.78	0.	0.	0.	0.

Problem 438	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	283	283	168	0	0	0	0	0
normalized size	1	1.	0.59	0.	0.	0.	0.	0.
time (sec)	N/A	0.234	0.275	1.898	0.	0.	0.	0.

Problem 439	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	368	368	190	0	0	0	0	0
normalized size	1	1.	0.52	0.	0.	0.	0.	0.
time (sec)	N/A	0.186	0.139	1.368	0.	0.	0.	0.

Problem 440	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	327	327	208	261	0	0	0	0
normalized size	1	1.	0.64	0.8	0.	0.	0.	0.
time (sec)	N/A	0.281	0.217	0.404	0.	0.	0.	0.

Problem 441	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	260	260	174	230	0	0	0	0
normalized size	1	1.	0.67	0.88	0.	0.	0.	0.
time (sec)	N/A	0.372	0.326	0.38	0.	0.	0.	0.

Problem 442	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	597	597	345	410	0	0	0	0
normalized size	1	1.	0.58	0.69	0.	0.	0.	0.
time (sec)	N/A	0.685	3.788	0.508	0.	0.	0.	0.

Problem 443	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	396	396	343	487	0	0	0	0
normalized size	1	1.	0.87	1.23	0.	0.	0.	0.
time (sec)	N/A	0.986	5.656	0.842	0.	0.	0.	0.

Problem 444	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	403	403	308	0	0	0	0	0
normalized size	1	1.	0.76	0.	0.	0.	0.	0.
time (sec)	N/A	0.516	0.795	1.137	0.	0.	0.	0.

Problem 445	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	495	495	639	0	0	0	0	0
normalized size	1	1.	1.29	0.	0.	0.	0.	0.
time (sec)	N/A	0.412	1.827	0.885	0.	0.	0.	0.

Problem 446	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	61	134	132	144	0	134
normalized size	1	1.	0.57	1.25	1.23	1.35	0.	1.25
time (sec)	N/A	0.131	0.093	0.275	3.285	1.751	0.	1.298

Problem 447	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	56	132	134	142	0	134
normalized size	1	1.	0.56	1.32	1.34	1.42	0.	1.34
time (sec)	N/A	0.069	0.067	0.236	2.551	1.788	0.	1.274

Problem 448	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	443	443	295	388	0	0	0	0
normalized size	1	1.	0.67	0.88	0.	0.	0.	0.
time (sec)	N/A	0.558	0.424	0.309	0.	0.	0.	0.

Problem 449	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	377	377	301	356	0	0	0	0
normalized size	1	1.	0.8	0.94	0.	0.	0.	0.
time (sec)	N/A	0.584	1.473	0.319	0.	0.	0.	0.

Problem 450	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	534	534	367	0	0	0	0	0
normalized size	1	1.	0.69	0.	0.	0.	0.	0.
time (sec)	N/A	1.111	2.564	1.638	0.	0.	0.	0.

Problem 451	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	622	622	691	0	0	0	0	0
normalized size	1	1.	1.11	0.	0.	0.	0.	0.
time (sec)	N/A	0.998	2.629	0.884	0.	0.	0.	0.

Problem 452	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	237	104	312	0	266	0	207
normalized size	1	1.	0.44	1.32	0.	1.12	0.	0.87
time (sec)	N/A	0.412	0.133	0.969	0.	2.006	0.	1.358

Problem 453	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	199	95	308	0	243	0	192
normalized size	1	1.	0.48	1.55	0.	1.22	0.	0.96
time (sec)	N/A	0.407	0.102	0.733	0.	2.127	0.	1.308

Problem 454	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	199	91	312	0	239	0	184
normalized size	1	1.	0.46	1.57	0.	1.2	0.	0.92
time (sec)	N/A	0.194	0.093	0.311	0.	1.959	0.	1.307

Problem 455	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	215	104	308	0	263	0	198
normalized size	1	1.	0.48	1.43	0.	1.22	0.	0.92
time (sec)	N/A	0.179	0.085	0.27	0.	1.723	0.	1.44

Problem 456	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	553	553	347	560	0	0	0	0
normalized size	1	1.	0.63	1.01	0.	0.	0.	0.
time (sec)	N/A	0.922	0.777	0.353	0.	0.	0.	0.

Problem 457	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	493	493	399	528	0	0	0	0
normalized size	1	1.	0.81	1.07	0.	0.	0.	0.
time (sec)	N/A	0.952	2.328	0.362	0.	0.	0.	0.

Problem 458	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.054	1.85	0.588	0.	0.	0.	0.

Problem 459	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.034	0.924	0.445	0.	0.	0.	0.

Problem 460	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.065	0.865	0.443	0.	0.	0.	0.

Problem 461	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.062	0.681	1.095	0.	0.	0.	0.

Problem 462	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	F(-2)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.109	0.974	0.478	0.	0.	0.	0.

Problem 463	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	F(-2)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.095	0.152	0.602	0.	0.	0.	0.

Problem 464	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.103	0.504	0.925	0.	0.	0.	0.

Problem 465	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.117	0.644	1.05	0.	0.	0.	0.

Problem 466	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.024	0.415	0.745	0.	0.	0.	0.

Problem 467	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.013	0.371	0.698	0.	0.	0.	0.

Problem 468	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.034	0.619	0.816	0.	0.	0.	0.

Problem 469	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.037	0.558	0.905	0.	0.	0.	0.

Problem 470	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.024	0.433	0.861	0.	0.	0.	0.

Problem 471	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.052	0.766	0.921	0.	0.	0.	0.

Problem 472	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.038	0.532	1.16	0.	0.	0.	0.

Problem 473	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.023	0.454	1.009	0.	0.	0.	0.

Problem 474	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.05	0.738	0.994	0.	0.	0.	0.

Problem 475	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.066	1.452	0.386	0.	0.	0.	0.

Problem 476	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.045	0.52	0.123	0.	0.	0.	0.

Problem 477	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	20	32	0	18
normalized size	1	1.	1.	1.08	1.67	2.67	0.	1.5
time (sec)	N/A	0.026	0.014	0.05	1.055	1.858	0.	1.158

Problem 478	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.066	0.183	0.119	0.	0.	0.	0.

Problem 479	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.068	0.202	0.187	0.	0.	0.	0.

Problem 480	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.069	3.654	0.46	0.	0.	0.	0.

Problem 481	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.069	2.064	0.372	0.	0.	0.	0.

Problem 482	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	25	30	0	197	0	0
normalized size	1	1.	0.76	0.91	0.	5.97	0.	0.
time (sec)	N/A	0.108	0.109	0.058	0.	1.733	0.	0.

Problem 483	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	0	174	0	0
normalized size	1	1.	1.	0.94	0.	10.24	0.	0.
time (sec)	N/A	0.071	0.038	0.055	0.	1.756	0.	0.

Problem 484	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	23	30	0	194	0	0
normalized size	1	1.	0.7	0.91	0.	5.88	0.	0.
time (sec)	N/A	0.066	0.027	0.058	0.	1.647	0.	0.

Problem 485	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.061	0.715	0.214	0.	0.	0.	0.

Problem 486	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.068	1.087	0.25	0.	0.	0.	0.

Problem 487	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.067	6.677	0.514	0.	0.	0.	0.

Problem 488	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.067	7.697	0.403	0.	0.	0.	0.

Problem 489	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	34	45	0	452	0	0
normalized size	1	1.	0.68	0.9	0.	9.04	0.	0.
time (sec)	N/A	0.127	0.145	0.059	0.	1.672	0.	0.

Problem 490	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	27	32	0	436	0	0
normalized size	1	1.	0.77	0.91	0.	12.46	0.	0.
time (sec)	N/A	0.113	0.115	0.06	0.	1.691	0.	0.

Problem 491	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	25	30	0	298	0	0
normalized size	1	1.	0.76	0.91	0.	9.03	0.	0.
time (sec)	N/A	0.11	0.065	0.065	0.	1.645	0.	0.

Problem 492	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	27	32	0	435	0	0
normalized size	1	1.	0.77	0.91	0.	12.43	0.	0.
time (sec)	N/A	0.09	0.081	0.058	0.	1.612	0.	0.

Problem 493	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	34	45	0	450	0	0
normalized size	1	1.	0.68	0.9	0.	9.	0.	0.
time (sec)	N/A	0.087	0.03	0.056	0.	1.732	0.	0.

Problem 494	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.065	0.878	0.323	0.	0.	0.	0.

Problem 495	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.078	1.272	0.282	0.	0.	0.	0.

Problem 496	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.074	2.126	0.635	0.	0.	0.	0.

Problem 497	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.039	0.261	0.638	0.	0.	0.	0.

Problem 498	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.105	1.19	0.695	0.	0.	0.	0.

Problem 499	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.085	2.211	0.599	0.	0.	0.	0.

Problem 500	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.044	0.345	0.575	0.	0.	0.	0.

Problem 501	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.129	1.374	0.671	0.	0.	0.	0.

Problem 502	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.095	2.272	0.72	0.	0.	0.	0.

Problem 503	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.068	0.426	0.766	0.	0.	0.	0.

Problem 504	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.135	1.434	0.729	0.	0.	0.	0.

Problem 505	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.082	0.795	0.596	0.	0.	0.	0.

Problem 506	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.043	0.201	0.483	0.	0.	0.	0.

Problem 507	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.114	0.581	0.538	0.	0.	0.	0.

Problem 508	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.133	4.755	1.117	0.	0.	0.	0.

Problem 509	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	F	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.131	180.002	0.847	0.	0.	0.	0.

Problem 510	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	37	82	0	0	0	0
normalized size	1	1.	0.95	2.1	0.	0.	0.	0.
time (sec)	N/A	0.169	0.116	0.296	0.	0.	0.	0.

Problem 511	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	39	136	0	0	0	0
normalized size	1	1.	1.	3.49	0.	0.	0.	0.
time (sec)	N/A	0.09	0.167	0.287	0.	0.	0.	0.

Problem 512	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.125	1.09	0.456	0.	0.	0.	0.

Problem 513	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.125	1.013	0.493	0.	0.	0.	0.

Problem 514	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.138	5.775	1.595	0.	0.	0.	0.

Problem 515	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.138	20.074	0.915	0.	0.	0.	0.

Problem 516	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	52	125	0	0	0	0
normalized size	1	1.	0.6	1.44	0.	0.	0.	0.
time (sec)	N/A	0.276	0.142	0.928	0.	0.	0.	0.

Problem 517	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	53	84	0	0	0	0
normalized size	1	1.	0.61	0.97	0.	0.	0.	0.
time (sec)	N/A	0.273	0.109	0.655	0.	0.	0.	0.

Problem 518	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	51	125	0	0	0	0
normalized size	1	1.	0.59	1.44	0.	0.	0.	0.
time (sec)	N/A	0.199	0.125	0.283	0.	0.	0.	0.

Problem 519	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	50	179	0	0	0	0
normalized size	1	1.	0.57	2.06	0.	0.	0.	0.
time (sec)	N/A	0.133	0.043	0.283	0.	0.	0.	0.

Problem 520	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.125	1.296	0.47	0.	0.	0.	0.

Problem 521	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.128	1.345	0.535	0.	0.	0.	0.

Problem 522	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.057	0.672	0.572	0.	0.	0.	0.

Problem 523	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.054	0.766	0.507	0.	0.	0.	0.

Problem 524	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.035	0.498	0.385	0.	0.	0.	0.

Problem 525	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.064	0.295	0.4	0.	0.	0.	0.

Problem 526	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.063	0.394	1.061	0.	0.	0.	0.

Problem 527	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.068	0.415	1.09	0.	0.	0.	0.

Problem 528	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.115	1.073	0.531	0.	0.	0.	0.

Problem 529	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.114	0.578	0.503	0.	0.	0.	0.

Problem 530	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.099	0.152	0.634	0.	0.	0.	0.

Problem 531	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.107	0.431	0.983	0.	0.	0.	0.

Problem 532	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.121	0.487	1.065	0.	0.	0.	0.

Problem 533	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.124	0.527	1.069	0.	0.	0.	0.

Problem 534	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.023	0.722	0.787	0.	0.	0.	0.

Problem 535	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.012	0.543	0.757	0.	0.	0.	0.

Problem 536	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.032	0.865	0.842	0.	0.	0.	0.

Problem 537	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.036	0.893	0.934	0.	0.	0.	0.

Problem 538	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.023	1.107	0.871	0.	0.	0.	0.

Problem 539	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.049	1.101	0.873	0.	0.	0.	0.

Problem 540	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.036	0.921	1.217	0.	0.	0.	0.

Problem 541	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.021	0.72	1.079	0.	0.	0.	0.

Problem 542	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.051	1.124	1.024	0.	0.	0.	0.

Problem 543	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	38	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.082	0.973	1.025	0.	0.	0.	0.

Problem 544	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	36	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.073	0.55	0.394	0.	0.	0.	0.

Problem 545	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.047	0.281	0.134	0.	0.	0.	0.

Problem 546	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	19	30	0	19
normalized size	1	1.	1.	1.07	1.36	2.14	0.	1.36
time (sec)	N/A	0.025	0.003	0.055	1.009	1.603	0.	1.16

Problem 547	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	38	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.078	0.479	0.122	0.	0.	0.	0.

Problem 548	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	38	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.082	0.627	0.391	0.	0.	0.	0.

Problem 549	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	38	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.082	0.953	0.842	0.	0.	0.	0.

Problem 550	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	38	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.082	1.141	0.892	0.	0.	0.	0.

Problem 551	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	71	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.332	4.063	0.482	0.	0.	0.	0.

Problem 552	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	40	37	0	301	0	0
normalized size	1	1.	0.93	0.86	0.	7.	0.	0.
time (sec)	N/A	0.139	0.109	0.065	0.	1.971	0.	0.

Problem 553	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	36	38	0	288	0	0
normalized size	1	1.	0.88	0.93	0.	7.02	0.	0.
time (sec)	N/A	0.21	0.07	0.058	0.	1.989	0.	0.

Problem 554	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	34	37	0	286	0	0
normalized size	1	1.	0.83	0.9	0.	6.98	0.	0.
time (sec)	N/A	0.093	0.064	0.061	0.	2.071	0.	0.

Problem 555	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	73	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.357	1.488	0.246	0.	0.	0.	0.

Problem 556	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	72	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.244	2.543	0.29	0.	0.	0.	0.

Problem 557	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	109	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.499	3.093	0.986	0.	0.	0.	0.

Problem 558	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	110	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.396	2.946	0.945	0.	0.	0.	0.

Problem 559	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	83	58	0	699	0	0
normalized size	1	1.	0.97	0.67	0.	8.13	0.	0.
time (sec)	N/A	0.517	0.133	0.061	0.	2.013	0.	0.

Problem 560	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	59	37	0	460	0	0
normalized size	1	1.	0.88	0.55	0.	6.87	0.	0.
time (sec)	N/A	0.28	0.171	0.071	0.	2.085	0.	0.

Problem 561	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	75	60	0	693	0	0
normalized size	1	1.	1.23	0.98	0.	11.36	0.	0.
time (sec)	N/A	0.246	0.067	0.061	0.	2.078	0.	0.

Problem 562	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	45	59	0	720	0	0
normalized size	1	1.	0.78	1.02	0.	12.41	0.	0.
time (sec)	N/A	0.114	0.094	0.061	0.	2.011	0.	0.

Problem 563	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	112	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.662	1.601	0.396	0.	0.	0.	0.

Problem 564	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	110	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.426	2.123	0.343	0.	0.	0.	0.

Problem 565	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	157	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	1.22	2.913	1.204	0.	0.	0.	0.

Problem 566	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	153	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.897	3.517	1.036	0.	0.	0.	0.

Problem 567	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.068	1.555	0.686	0.	0.	0.	0.

Problem 568	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.035	0.423	0.645	0.	0.	0.	0.

Problem 569	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.096	3.241	0.759	0.	0.	0.	0.

Problem 570	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.077	3.829	0.653	0.	0.	0.	0.

Problem 571	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.037	0.93	0.63	0.	0.	0.	0.

Problem 572	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.109	4.077	0.751	0.	0.	0.	0.

Problem 573	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.078	1.946	0.776	0.	0.	0.	0.

Problem 574	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.036	0.823	0.728	0.	0.	0.	0.

Problem 575	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.111	2.188	0.756	0.	0.	0.	0.

Problem 576	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.076	1.145	0.628	0.	0.	0.	0.

Problem 577	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.035	0.596	0.714	0.	0.	0.	0.

Problem 578	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	63	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.217	1.177	0.638	0.	0.	0.	0.

Problem 579	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	100	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.372	9.776	1.271	0.	0.	0.	0.

Problem 580	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	97	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.366	8.762	1.124	0.	0.	0.	0.

Problem 581	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	55	210	0	0	0	0
normalized size	1	1.	0.8	3.04	0.	0.	0.	0.
time (sec)	N/A	0.174	0.101	0.309	0.	0.	0.	0.

Problem 582	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	53	212	0	0	0	0
normalized size	1	1.	0.77	3.07	0.	0.	0.	0.
time (sec)	N/A	0.205	0.1	0.279	0.	0.	0.	0.

Problem 583	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	129	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.506	1.853	0.509	0.	0.	0.	0.

Problem 584	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	93	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.425	3.458	0.534	0.	0.	0.	0.

Problem 585	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	158	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.74	5.809	0.698	0.	0.	0.	0.

Problem 586	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	133	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.665	6.58	1.17	0.	0.	0.	0.

Problem 587	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	176	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.912	12.313	1.791	0.	0.	0.	0.

Problem 588	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	173	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	1.081	11.226	1.131	0.	0.	0.	0.

Problem 589	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	82	582	0	0	0	0
normalized size	1	1.	0.69	4.93	0.	0.	0.	0.
time (sec)	N/A	0.403	0.205	1.072	0.	0.	0.	0.

Problem 590	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	99	586	0	0	0	0
normalized size	1	1.	0.7	4.13	0.	0.	0.	0.
time (sec)	N/A	0.581	0.267	0.977	0.	0.	0.	0.

Problem 591	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	95	582	0	0	0	0
normalized size	1	1.	0.82	5.02	0.	0.	0.	0.
time (sec)	N/A	0.501	0.18	0.379	0.	0.	0.	0.

Problem 592	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	61	586	0	0	0	0
normalized size	1	1.	0.53	5.1	0.	0.	0.	0.
time (sec)	N/A	0.245	0.186	0.341	0.	0.	0.	0.

Problem 593	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	198	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	1.137	2.207	0.577	0.	0.	0.	0.

Problem 594	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	163	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.804	3.959	0.596	0.	0.	0.	0.

Problem 595	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	236	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	2.048	6.799	0.762	0.	0.	0.	0.

Problem 596	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	207	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	1.607	6.344	1.217	0.	0.	0.	0.

Problem 597	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	32	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.1	28.961	1.223	0.	0.	0.	0.

Problem 598	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.055	0.739	0.565	0.	0.	0.	0.

Problem 599	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.055	0.806	0.524	0.	0.	0.	0.

Problem 600	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.034	0.53	0.392	0.	0.	0.	0.

Problem 601	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	40	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.085	0.332	0.437	0.	0.	0.	0.

Problem 602	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.063	0.45	1.14	0.	0.	0.	0.

Problem 603	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.063	0.466	1.261	0.	0.	0.	0.

Problem 604	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.112	1.221	0.615	0.	0.	0.	0.

Problem 605	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.114	0.657	0.586	0.	0.	0.	0.

Problem 606	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.096	0.177	0.703	0.	0.	0.	0.

Problem 607	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.105	0.469	1.109	0.	0.	0.	0.

Problem 608	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.119	0.534	1.208	0.	0.	0.	0.

Problem 609	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.118	0.581	1.236	0.	0.	0.	0.

Problem 610	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.024	1.096	0.871	0.	0.	0.	0.

Problem 611	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.013	1.151	0.793	0.	0.	0.	0.

Problem 612	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.032	1.506	0.96	0.	0.	0.	0.

Problem 613	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.035	0.98	1.066	0.	0.	0.	0.

Problem 614	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.022	0.698	0.929	0.	0.	0.	0.

Problem 615	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.048	1.233	0.949	0.	0.	0.	0.

Problem 616	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.036	1.018	1.348	0.	0.	0.	0.

Problem 617	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.022	1.15	1.2	0.	0.	0.	0.

Problem 618	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.049	1.212	1.163	0.	0.	0.	0.

Problem 619	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	42	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.096	0.861	0.963	0.	0.	0.	0.

Problem 620	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	37	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.075	0.574	0.463	0.	0.	0.	0.

Problem 621	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	36	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.049	0.45	0.158	0.	0.	0.	0.

Problem 622	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	19	35	0	19
normalized size	1	1.	1.	0.94	1.19	2.19	0.	1.19
time (sec)	N/A	0.025	0.004	0.059	0.992	1.566	0.	1.123

Problem 623	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	42	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.077	0.44	0.132	0.	0.	0.	0.

Problem 624	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	40	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.082	1.044	0.441	0.	0.	0.	0.

Problem 625	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	42	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.081	1.238	0.947	0.	0.	0.	0.

Problem 626	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	40	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.082	2.322	0.974	0.	0.	0.	0.

Problem 627	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	115	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.244	9.382	0.525	0.	0.	0.	0.

Problem 628	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	51	52	0	327	0	0
normalized size	1	1.	0.72	0.73	0.	4.61	0.	0.
time (sec)	N/A	0.288	0.106	0.075	0.	1.741	0.	0.

Problem 629	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	70	51	0	338	0	0
normalized size	1	1.	0.86	0.63	0.	4.17	0.	0.
time (sec)	N/A	0.119	0.052	0.065	0.	1.77	0.	0.

Problem 630	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	58	52	0	316	0	0
normalized size	1	1.	0.89	0.8	0.	4.86	0.	0.
time (sec)	N/A	0.246	0.065	0.071	0.	1.742	0.	0.

Problem 631	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	113	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.26	1.773	0.275	0.	0.	0.	0.

Problem 632	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	103	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.39	2.527	0.319	0.	0.	0.	0.

Problem 633	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	159	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.408	2.067	1.105	0.	0.	0.	0.

Problem 634	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	141	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.547	6.365	1.066	0.	0.	0.	0.

Problem 635	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	72	90	0	799	0	0
normalized size	1	1.	0.41	0.51	0.	4.51	0.	0.
time (sec)	N/A	0.642	0.233	0.069	0.	1.747	0.	0.

Problem 636	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	60	52	0	497	0	0
normalized size	1	1.	0.5	0.43	0.	4.14	0.	0.
time (sec)	N/A	0.596	0.133	0.075	0.	1.732	0.	0.

Problem 637	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	98	88	0	784	0	0
normalized size	1	1.	0.87	0.78	0.	6.94	0.	0.
time (sec)	N/A	0.449	0.163	0.066	0.	1.874	0.	0.

Problem 638	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	89	89	0	726	0	0
normalized size	1	1.	1.1	1.1	0.	8.96	0.	0.
time (sec)	N/A	0.266	0.093	0.067	0.	1.759	0.	0.

Problem 639	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	199	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.765	2.574	0.436	0.	0.	0.	0.

Problem 640	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	167	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.733	3.649	0.391	0.	0.	0.	0.

Problem 641	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	246	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	1.281	4.82	1.508	0.	0.	0.	0.

Problem 642	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	206	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	1.403	9.029	1.156	0.	0.	0.	0.

Problem 643	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	0	19	38	0	0
normalized size	1	1.	1.	0.	1.19	2.38	0.	0.
time (sec)	N/A	0.091	0.152	1.108	1.364	1.585	0.	0.

Problem 644	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.067	1.794	0.77	0.	0.	0.	0.

Problem 645	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.035	1.025	0.762	0.	0.	0.	0.

Problem 646	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.115	3.377	0.877	0.	0.	0.	0.

Problem 647	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.08	6.042	0.696	0.	0.	0.	0.

Problem 648	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.037	1.072	0.68	0.	0.	0.	0.

Problem 649	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.113	4.077	0.696	0.	0.	0.	0.

Problem 650	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.099	2.191	0.825	0.	0.	0.	0.

Problem 651	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.047	1.176	0.77	0.	0.	0.	0.

Problem 652	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.126	2.535	0.77	0.	0.	0.	0.

Problem 653	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.082	1.266	0.693	0.	0.	0.	0.

Problem 654	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.037	0.678	0.803	0.	0.	0.	0.

Problem 655	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	67	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.216	3.575	0.838	0.	0.	0.	0.

Problem 656	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.108	5.111	0.629	0.	0.	0.	0.

Problem 657	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.111	6.436	0.859	0.	0.	0.	0.

Problem 658	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	135	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.507	3.376	1.299	0.	0.	0.	0.

Problem 659	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	134	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.412	3.166	1.103	0.	0.	0.	0.

Problem 660	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	63	294	0	0	0	0
normalized size	1	1.	0.61	2.83	0.	0.	0.	0.
time (sec)	N/A	0.322	0.13	0.331	0.	0.	0.	0.

Problem 661	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	65	292	0	0	0	0
normalized size	1	1.	0.64	2.89	0.	0.	0.	0.
time (sec)	N/A	0.224	0.075	0.303	0.	0.	0.	0.

Problem 662	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	165	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.63	2.405	0.559	0.	0.	0.	0.

Problem 663	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	130	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.462	2.48	0.595	0.	0.	0.	0.

Problem 664	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	199	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.878	3.135	0.764	0.	0.	0.	0.

Problem 665	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	168	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.684	6.98	1.14	0.	0.	0.	0.

Problem 666	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	240	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	1.358	7.71	1.859	0.	0.	0.	0.

Problem 667	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	234	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	1.443	7.416	1.25	0.	0.	0.	0.

Problem 668	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	114	848	0	0	0	0
normalized size	1	1.	0.63	4.71	0.	0.	0.	0.
time (sec)	N/A	0.715	0.312	1.096	0.	0.	0.	0.

Problem 669	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	119	844	0	0	0	0
normalized size	1	1.	0.57	4.04	0.	0.	0.	0.
time (sec)	N/A	0.907	0.227	0.961	0.	0.	0.	0.

Problem 670	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	118	848	0	0	0	0
normalized size	1	1.	0.67	4.85	0.	0.	0.	0.
time (sec)	N/A	0.931	0.27	0.401	0.	0.	0.	0.

Problem 671	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	102	844	0	0	0	0
normalized size	1	1.	0.7	5.82	0.	0.	0.	0.
time (sec)	N/A	0.551	0.195	0.389	0.	0.	0.	0.

Problem 672	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	262	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	1.72	3.466	0.653	0.	0.	0.	0.

Problem 673	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	231	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	1.152	5.093	0.689	0.	0.	0.	0.

Problem 674	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.056	0.755	0.573	0.	0.	0.	0.

Problem 675	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.054	0.825	0.535	0.	0.	0.	0.

Problem 676	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.034	0.554	0.372	0.	0.	0.	0.

Problem 677	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	45	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.082	0.375	0.422	0.	0.	0.	0.

Problem 678	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.067	0.492	1.17	0.	0.	0.	0.

Problem 679	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.063	0.511	1.22	0.	0.	0.	0.

Problem 680	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.114	1.471	0.618	0.	0.	0.	0.

Problem 681	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.113	0.771	0.585	0.	0.	0.	0.

Problem 682	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.117	0.213	0.753	0.	0.	0.	0.

Problem 683	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.104	0.515	1.19	0.	0.	0.	0.

Problem 684	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.116	0.571	1.271	0.	0.	0.	0.

Problem 685	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.117	0.617	1.25	0.	0.	0.	0.

Problem 686	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.034	1.958	0.765	0.	0.	0.	0.

Problem 687	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	56	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.046	2.281	0.377	0.	0.	0.	0.

Problem 688	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.013	3.107	0.256	0.	0.	0.	0.

Problem 689	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.04	1.612	0.401	0.	0.	0.	0.

Problem 690	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.085	1.26	0.904	0.	0.	0.	0.

Problem 691	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	60	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.072	1.773	0.527	0.	0.	0.	0.

Problem 692	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.023	1.644	0.413	0.	0.	0.	0.

Problem 693	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.061	1.261	0.493	0.	0.	0.	0.

Problem 694	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.058	0.841	0.951	0.	0.	0.	0.

Problem 695	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	60	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.064	1.816	0.734	0.	0.	0.	0.

Problem 696	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.024	1.712	0.599	0.	0.	0.	0.

Problem 697	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.052	1.318	0.672	0.	0.	0.	0.

Problem 698	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.064	0.775	0.572	0.	0.	0.	0.

Problem 699	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	59	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.122	2.644	0.555	0.	0.	0.	0.

Problem 700	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	36	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.098	1.065	0.257	0.	0.	0.	0.

Problem 701	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	40	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.049	0.974	0.142	0.	0.	0.	0.

Problem 702	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	0	39	0	19
normalized size	1	1.	1.	0.83	0.	2.17	0.	1.06
time (sec)	N/A	0.024	0.003	0.092	0.	1.717	0.	1.089

Problem 703	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	48	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.107	0.584	0.139	0.	0.	0.	0.

Problem 704	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	35	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.104	1.452	0.248	0.	0.	0.	0.

Problem 705	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	72	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.194	2.151	0.605	0.	0.	0.	0.

Problem 706	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	59	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.187	6.028	0.5	0.	0.	0.	0.

Problem 707	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.064	1.637	0.786	0.	0.	0.	0.

Problem 708	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.064	3.681	0.562	0.	0.	0.	0.

Problem 709	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	66	60	0	0	0	0
normalized size	1	1.	0.82	0.75	0.	0.	0.	0.
time (sec)	N/A	0.147	0.204	0.105	0.	0.	0.	0.

Problem 710	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	79	79	136	46	0	0	0	0
normalized size	1	1.	1.72	0.58	0.	0.	0.	0.
time (sec)	N/A	0.121	0.28	0.112	0.	0.	0.	0.

Problem 711	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	77	77	89	60	0	0	0	0
normalized size	1	1.	1.16	0.78	0.	0.	0.	0.
time (sec)	N/A	0.104	0.159	0.108	0.	0.	0.	0.

Problem 712	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.06	1.686	0.54	0.	0.	0.	0.

Problem 713	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.065	2.047	0.849	0.	0.	0.	0.

Problem 714	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.066	5.138	0.584	0.	0.	0.	0.

Problem 715	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	139	139	181	102	0	0	0	0
normalized size	1	1.	1.3	0.73	0.	0.	0.	0.
time (sec)	N/A	0.179	0.483	0.125	0.	0.	0.	0.

Problem 716	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	118	118	230	93	0	0	0	0
normalized size	1	1.	1.95	0.79	0.	0.	0.	0.
time (sec)	N/A	0.215	0.65	0.114	0.	0.	0.	0.

Problem 717	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	83	83	141	66	0	0	0	0
normalized size	1	1.	1.7	0.8	0.	0.	0.	0.
time (sec)	N/A	0.136	0.405	0.115	0.	0.	0.	0.

Problem 718	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	118	118	230	94	0	0	0	0
normalized size	1	1.	1.95	0.8	0.	0.	0.	0.
time (sec)	N/A	0.15	0.642	0.115	0.	0.	0.	0.

Problem 719	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	139	139	192	102	0	0	0	0
normalized size	1	1.	1.38	0.73	0.	0.	0.	0.
time (sec)	N/A	0.145	0.42	0.118	0.	0.	0.	0.

Problem 720	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.062	2.204	0.813	0.	0.	0.	0.

Problem 721	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	F(-2)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.101	0.922	1.26	0.	0.	0.	0.

Problem 722	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.103	3.609	1.5	0.	0.	0.	0.

Problem 723	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	65	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.104	6.864	0.974	0.	0.	0.	0.

Problem 724	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.032	0.417	0.862	0.	0.	0.	0.

Problem 725	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	F(-2)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.109	1.001	0.987	0.	0.	0.	0.

Problem 726	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.124	3.964	1.306	0.	0.	0.	0.

Problem 727	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	65	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.122	7.829	0.783	0.	0.	0.	0.

Problem 728	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.036	1.689	0.691	0.	0.	0.	0.

Problem 729	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	F(-2)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.109	1.345	1.094	0.	0.	0.	0.

Problem 730	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.121	3.422	1.474	0.	0.	0.	0.

Problem 731	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	65	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.117	7.406	0.915	0.	0.	0.	0.

Problem 732	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.037	0.486	0.791	0.	0.	0.	0.

Problem 733	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.105	0.877	1.339	0.	0.	0.	0.

Problem 734	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	135	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.332	4.247	4.891	0.	0.	0.	0.

Problem 735	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	99	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.216	2.514	3.52	0.	0.	0.	0.

Problem 736	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	62	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.109	0.797	1.201	0.	0.	0.	0.

Problem 737	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.034	0.204	0.94	0.	0.	0.	0.

Problem 738	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.107	1.048	1.052	0.	0.	0.	0.

Problem 739	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	64	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.214	1.731	0.801	0.	0.	0.	0.

Problem 740	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	101	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.329	3.517	1.257	0.	0.	0.	0.

Problem 741	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	136	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.424	19.234	3.262	0.	0.	0.	0.

Problem 742	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.115	1.015	0.981	0.	0.	0.	0.

Problem 743	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.124	24.94	3.121	0.	0.	0.	0.

Problem 744	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.117	3.622	2.974	0.	0.	0.	0.

Problem 745	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	93	93	121	0	0	0	0	0
normalized size	1	1.	1.3	0.	0.	0.	0.	0.
time (sec)	N/A	0.182	0.151	0.882	0.	0.	0.	0.

Problem 746	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	78	0	0	0	0	0
normalized size	1	1.	0.86	0.	0.	0.	0.	0.
time (sec)	N/A	0.111	0.072	0.705	0.	0.	0.	0.

Problem 747	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.116	1.943	0.733	0.	0.	0.	0.

Problem 748	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.118	5.757	0.776	0.	0.	0.	0.

Problem 749	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.114	1.681	0.947	0.	0.	0.	0.

Problem 750	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.122	3.854	2.079	0.	0.	0.	0.

Problem 751	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	215	215	324	0	0	0	0	0
normalized size	1	1.	1.51	0.	0.	0.	0.	0.
time (sec)	N/A	0.347	0.504	3.116	0.	0.	0.	0.

Problem 752	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	133	0	0	0	0	0
normalized size	1	1.	0.82	0.	0.	0.	0.	0.
time (sec)	N/A	0.425	0.263	2.87	0.	0.	0.	0.

Problem 753	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	163	163	167	0	0	0	0	0
normalized size	1	1.	1.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.25	0.417	0.823	0.	0.	0.	0.

Problem 754	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	213	213	137	0	0	0	0	0
normalized size	1	1.	0.64	0.	0.	0.	0.	0.
time (sec)	N/A	0.188	0.166	0.709	0.	0.	0.	0.

Problem 755	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.121	2.233	0.721	0.	0.	0.	0.

Problem 756	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.035	1.823	0.819	0.	0.	0.	0.

Problem 757	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.036	4.042	0.64	0.	0.	0.	0.

Problem 758	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	56	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.038	1.26	0.39	0.	0.	0.	0.

Problem 759	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	79	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.024	3.922	0.317	0.	0.	0.	0.

Problem 760	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.032	1.717	0.413	0.	0.	0.	0.

Problem 761	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.035	1.58	0.234	0.	0.	0.	0.

Problem 762	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.054	1.254	0.944	0.	0.	0.	0.

Problem 763	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.056	3.053	0.934	0.	0.	0.	0.

Problem 764	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	60	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.062	1.276	0.543	0.	0.	0.	0.

Problem 765	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	169	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.066	2.125	0.431	0.	0.	0.	0.

Problem 766	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.05	1.698	0.517	0.	0.	0.	0.

Problem 767	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.056	2.078	0.444	0.	0.	0.	0.

Problem 768	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.057	0.846	1.001	0.	0.	0.	0.

Problem 769	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.057	2.91	1.284	0.	0.	0.	0.

Problem 770	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	60	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.062	1.366	0.778	0.	0.	0.	0.

Problem 771	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	255	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.124	2.196	0.671	0.	0.	0.	0.

Problem 772	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.052	1.639	0.822	0.	0.	0.	0.

Problem 773	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.057	2.477	0.557	0.	0.	0.	0.

Problem 774	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.063	0.671	0.566	0.	0.	0.	0.

Problem 775	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	59	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.123	3.953	0.562	0.	0.	0.	0.

Problem 776	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	36	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.096	1.167	0.255	0.	0.	0.	0.

Problem 777	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	40	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.048	0.972	0.141	0.	0.	0.	0.

Problem 778	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	0	39	0	19
normalized size	1	1.	1.	0.83	0.	2.17	0.	1.06
time (sec)	N/A	0.025	0.003	0.083	0.	1.699	0.	1.099

Problem 779	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	48	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.108	0.575	0.14	0.	0.	0.	0.

Problem 780	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	35	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.102	1.236	0.244	0.	0.	0.	0.

Problem 781	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	72	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.192	1.719	0.622	0.	0.	0.	0.

Problem 782	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	59	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.186	3.739	0.498	0.	0.	0.	0.

Problem 783	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.064	1.357	0.766	0.	0.	0.	0.

Problem 784	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.064	3.877	0.523	0.	0.	0.	0.

Problem 785	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	127	127	187	81	0	0	0	0
normalized size	1	1.	1.47	0.64	0.	0.	0.	0.
time (sec)	N/A	0.188	0.354	0.102	0.	0.	0.	0.

Problem 786	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	75	67	0	0	0	0
normalized size	1	1.	0.69	0.61	0.	0.	0.	0.
time (sec)	N/A	0.152	0.092	0.099	0.	0.	0.	0.

Problem 787	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	90	75	0	0	0	0
normalized size	1	1.	0.73	0.6	0.	0.	0.	0.
time (sec)	N/A	0.148	0.167	0.109	0.	0.	0.	0.

Problem 788	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.059	1.735	0.575	0.	0.	0.	0.

Problem 789	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.063	1.76	0.835	0.	0.	0.	0.

Problem 790	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.064	6.337	0.762	0.	0.	0.	0.

Problem 791	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	230	230	355	132	0	0	0	0
normalized size	1	1.	1.54	0.57	0.	0.	0.	0.
time (sec)	N/A	0.411	0.776	0.168	0.	0.	0.	0.

Problem 792	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	168	168	350	124	0	0	0	0
normalized size	1	1.	2.08	0.74	0.	0.	0.	0.
time (sec)	N/A	0.247	0.281	0.12	0.	0.	0.	0.

Problem 793	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	108	108	353	81	0	0	0	0
normalized size	1	1.	3.27	0.75	0.	0.	0.	0.
time (sec)	N/A	0.163	0.745	0.111	0.	0.	0.	0.

Problem 794	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	168	168	347	124	0	0	0	0
normalized size	1	1.	2.07	0.74	0.	0.	0.	0.
time (sec)	N/A	0.186	0.234	0.14	0.	0.	0.	0.

Problem 795	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	219	123	132	0	0	0	0
normalized size	1	1.	0.56	0.6	0.	0.	0.	0.
time (sec)	N/A	0.291	0.571	0.118	0.	0.	0.	0.

Problem 796	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.061	2.309	0.801	0.	0.	0.	0.

Problem 797	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	F(-2)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.098	0.65	1.234	0.	0.	0.	0.

Problem 798	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.101	2.97	1.598	0.	0.	0.	0.

Problem 799	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	65	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.106	6.064	1.02	0.	0.	0.	0.

Problem 800	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	117	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.105	0.278	0.773	0.	0.	0.	0.

Problem 801	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.097	2.885	0.886	0.	0.	0.	0.

Problem 802	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	F(-2)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.11	0.985	0.963	0.	0.	0.	0.

Problem 803	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.115	3.769	1.235	0.	0.	0.	0.

Problem 804	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	65	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.118	2.388	0.757	0.	0.	0.	0.

Problem 805	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	209	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.178	1.424	0.642	0.	0.	0.	0.

Problem 806	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.113	1.997	0.671	0.	0.	0.	0.

Problem 807	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	F(-2)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.113	1.286	0.938	0.	0.	0.	0.

Problem 808	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.114	3.105	1.398	0.	0.	0.	0.

Problem 809	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	65	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.116	6.805	0.884	0.	0.	0.	0.

Problem 810	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	303	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.274	0.455	0.737	0.	0.	0.	0.

Problem 811	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.112	2.159	0.842	0.	0.	0.	0.

Problem 812	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.102	0.806	1.44	0.	0.	0.	0.

Problem 813	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	166	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.45	3.489	4.641	0.	0.	0.	0.

Problem 814	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	130	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.259	2.334	3.53	0.	0.	0.	0.

Problem 815	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	62	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.108	0.654	1.283	0.	0.	0.	0.

Problem 816	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.035	0.207	0.915	0.	0.	0.	0.

Problem 817	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.105	1.219	0.986	0.	0.	0.	0.

Problem 818	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	64	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.213	1.245	0.8	0.	0.	0.	0.

Problem 819	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	136	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.423	3.955	1.401	0.	0.	0.	0.

Problem 820	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	171	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.656	18.204	3.905	0.	0.	0.	0.

Problem 821	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.115	0.931	1.016	0.	0.	0.	0.

Problem 822	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.135	6.279	3.514	0.	0.	0.	0.

Problem 823	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.152	3.677	3.1	0.	0.	0.	0.

Problem 824	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	129	129	128	0	0	0	0	0
normalized size	1	1.	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.211	0.172	0.906	0.	0.	0.	0.

Problem 825	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	125	125	104	0	0	0	0	0
normalized size	1	1.	0.83	0.	0.	0.	0.	0.
time (sec)	N/A	0.142	0.167	0.885	0.	0.	0.	0.

Problem 826	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.117	2.208	0.743	0.	0.	0.	0.

Problem 827	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.118	7.111	0.762	0.	0.	0.	0.

Problem 828	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.118	1.419	0.97	0.	0.	0.	0.

Problem 829	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.124	8.91	4.625	0.	0.	0.	0.

Problem 830	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.123	3.829	2.106	0.	0.	0.	0.

Problem 831	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	263	263	272	0	0	0	0	0
normalized size	1	1.	1.03	0.	0.	0.	0.	0.
time (sec)	N/A	0.641	1.055	3.464	0.	0.	0.	0.

Problem 832	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	247	247	338	0	0	0	0	0
normalized size	1	1.	1.37	0.	0.	0.	0.	0.
time (sec)	N/A	0.476	0.546	2.95	0.	0.	0.	0.

Problem 833	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	248	248	261	0	0	0	0	0
normalized size	1	1.	1.05	0.	0.	0.	0.	0.
time (sec)	N/A	0.281	1.01	0.772	0.	0.	0.	0.

Problem 834	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	252	252	153	0	0	0	0	0
normalized size	1	1.	0.61	0.	0.	0.	0.	0.
time (sec)	N/A	0.349	0.203	0.67	0.	0.	0.	0.

Problem 835	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.116	2.397	0.743	0.	0.	0.	0.

Problem 836	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.117	8.213	0.721	0.	0.	0.	0.

Problem 837	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.034	1.818	0.743	0.	0.	0.	0.

Problem 838	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.035	3.423	0.657	0.	0.	0.	0.

Problem 839	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	115	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.055	1.925	0.377	0.	0.	0.	0.

Problem 840	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	79	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.025	3.567	0.323	0.	0.	0.	0.

Problem 841	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.032	2.849	0.386	0.	0.	0.	0.

Problem 842	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.033	1.617	0.23	0.	0.	0.	0.

Problem 843	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.054	1.313	0.914	0.	0.	0.	0.

Problem 844	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.056	2.525	0.905	0.	0.	0.	0.

Problem 845	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	213	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.114	1.428	0.532	0.	0.	0.	0.

Problem 846	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	169	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.07	2.19	0.419	0.	0.	0.	0.

Problem 847	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.048	1.819	0.504	0.	0.	0.	0.

Problem 848	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.054	2.52	0.422	0.	0.	0.	0.

Problem 849	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.055	0.884	0.954	0.	0.	0.	0.

Problem 850	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.054	2.415	1.273	0.	0.	0.	0.

Problem 851	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	305	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.187	1.507	0.76	0.	0.	0.	0.

Problem 852	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	255	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.126	2.173	0.592	0.	0.	0.	0.

Problem 853	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.05	1.515	0.669	0.	0.	0.	0.

Problem 854	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.054	2.544	0.56	0.	0.	0.	0.

Problem 855	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.063	0.677	0.556	0.	0.	0.	0.

Problem 856	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	59	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.12	4.256	0.609	0.	0.	0.	0.

Problem 857	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	36	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.091	1.198	0.244	0.	0.	0.	0.

Problem 858	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	40	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.054	0.992	0.136	0.	0.	0.	0.

Problem 859	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	0	39	0	19
normalized size	1	1.	1.	0.83	0.	2.17	0.	1.06
time (sec)	N/A	0.025	0.003	0.079	0.	1.619	0.	1.084

Problem 860	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	48	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.115	0.594	0.14	0.	0.	0.	0.

Problem 861	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	35	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.104	1.267	0.242	0.	0.	0.	0.

Problem 862	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	72	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.187	1.984	0.604	0.	0.	0.	0.

Problem 863	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	59	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.179	3.456	0.487	0.	0.	0.	0.

Problem 864	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.064	1.364	0.74	0.	0.	0.	0.

Problem 865	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.066	4.306	0.507	0.	0.	0.	0.

Problem 866	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	111	102	0	0	0	0
normalized size	1	1.	0.71	0.65	0.	0.	0.	0.
time (sec)	N/A	0.224	0.2	0.106	0.	0.	0.	0.

Problem 867	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	156	156	234	88	0	0	0	0
normalized size	1	1.	1.5	0.56	0.	0.	0.	0.
time (sec)	N/A	0.199	0.169	0.103	0.	0.	0.	0.

Problem 868	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	85	102	0	0	0	0
normalized size	1	1.	0.56	0.68	0.	0.	0.	0.
time (sec)	N/A	0.184	0.267	0.109	0.	0.	0.	0.

Problem 869	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.059	1.92	0.526	0.	0.	0.	0.

Problem 870	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.063	1.749	0.805	0.	0.	0.	0.

Problem 871	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.062	8.172	0.566	0.	0.	0.	0.

Problem 872	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	310	310	287	194	0	0	0	0
normalized size	1	1.	0.93	0.63	0.	0.	0.	0.
time (sec)	N/A	0.486	0.599	0.122	0.	0.	0.	0.

Problem 873	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	256	256	359	180	0	0	0	0
normalized size	1	1.	1.4	0.7	0.	0.	0.	0.
time (sec)	N/A	0.492	0.702	0.12	0.	0.	0.	0.

Problem 874	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	133	133	185	96	0	0	0	0
normalized size	1	1.	1.39	0.72	0.	0.	0.	0.
time (sec)	N/A	0.175	0.44	0.114	0.	0.	0.	0.

Problem 875	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	254	254	359	180	0	0	0	0
normalized size	1	1.	1.41	0.71	0.	0.	0.	0.
time (sec)	N/A	0.338	0.651	0.133	0.	0.	0.	0.

Problem 876	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	296	296	162	194	0	0	0	0
normalized size	1	1.	0.55	0.66	0.	0.	0.	0.
time (sec)	N/A	0.369	0.361	0.117	0.	0.	0.	0.

Problem 877	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.057	3.199	0.78	0.	0.	0.	0.

Problem 878	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	F(-2)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.095	0.662	1.262	0.	0.	0.	0.

Problem 879	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.109	3.276	1.536	0.	0.	0.	0.

Problem 880	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	158	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.179	4.821	0.995	0.	0.	0.	0.

Problem 881	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	117	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.103	0.294	0.869	0.	0.	0.	0.

Problem 882	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.094	2.284	0.938	0.	0.	0.	0.

Problem 883	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	F(-2)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.107	1.037	1.021	0.	0.	0.	0.

Problem 884	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.116	3.96	1.33	0.	0.	0.	0.

Problem 885	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	256	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.272	2.559	0.796	0.	0.	0.	0.

Problem 886	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	209	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.179	1.51	0.741	0.	0.	0.	0.

Problem 887	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.108	2.069	0.74	0.	0.	0.	0.

Problem 888	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	F(-2)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.108	1.395	1.03	0.	0.	0.	0.

Problem 889	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.114	3.379	1.628	0.	0.	0.	0.

Problem 890	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	356	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.364	5.802	0.964	0.	0.	0.	0.

Problem 891	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	303	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.267	0.485	0.844	0.	0.	0.	0.

Problem 892	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.109	2.36	0.842	0.	0.	0.	0.

Problem 893	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.102	0.806	1.338	0.	0.	0.	0.

Problem 894	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	197	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.472	3.855	4.998	0.	0.	0.	0.

Problem 895	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	130	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.254	2.466	3.563	0.	0.	0.	0.

Problem 896	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	62	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.109	0.662	1.2	0.	0.	0.	0.

Problem 897	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.035	0.209	0.927	0.	0.	0.	0.

Problem 898	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.103	1.223	1.101	0.	0.	0.	0.

Problem 899	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	64	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.21	1.37	0.918	0.	0.	0.	0.

Problem 900	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	136	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.423	4.517	1.404	0.	0.	0.	0.

Problem 901	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	206	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.744	16.054	3.984	0.	0.	0.	0.

Problem 902	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.114	0.936	1.059	0.	0.	0.	0.

Problem 903	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.122	3.654	2.978	0.	0.	0.	0.

Problem 904	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	161	161	139	0	0	0	0	0
normalized size	1	1.	0.86	0.	0.	0.	0.	0.
time (sec)	N/A	0.227	0.191	0.946	0.	0.	0.	0.

Problem 905	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	155	155	94	0	0	0	0	0
normalized size	1	1.	0.61	0.	0.	0.	0.	0.
time (sec)	N/A	0.158	0.15	0.693	0.	0.	0.	0.

Problem 906	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.119	2.227	0.757	0.	0.	0.	0.

Problem 907	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.117	1.406	0.953	0.	0.	0.	0.

Problem 908	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.119	4.122	2.063	0.	0.	0.	0.

Problem 909	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	350	350	370	0	0	0	0	0
normalized size	1	1.	1.06	0.	0.	0.	0.	0.
time (sec)	N/A	0.708	0.553	3.497	0.	0.	0.	0.

Problem 910	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	295	295	287	0	0	0	0	0
normalized size	1	1.	0.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.775	1.082	2.843	0.	0.	0.	0.

Problem 911	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	293	293	356	0	0	0	0	0
normalized size	1	1.	1.22	0.	0.	0.	0.	0.
time (sec)	N/A	0.451	0.533	0.799	0.	0.	0.	0.

Problem 912	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	337	337	293	0	0	0	0	0
normalized size	1	1.	0.87	0.	0.	0.	0.	0.
time (sec)	N/A	0.414	1.091	0.741	0.	0.	0.	0.

Problem 913	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.119	2.537	0.807	0.	0.	0.	0.

Problem 914	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.035	1.948	0.745	0.	0.	0.	0.

Problem 915	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.023	0.99	0.382	0.	0.	0.	0.

Problem 916	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.012	0.108	0.274	0.	0.	0.	0.

Problem 917	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.032	1.381	0.427	0.	0.	0.	0.

Problem 918	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.055	1.315	0.883	0.	0.	0.	0.

Problem 919	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.039	1.173	0.514	0.	0.	0.	0.

Problem 920	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.022	0.723	0.406	0.	0.	0.	0.

Problem 921	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.05	1.301	0.461	0.	0.	0.	0.

Problem 922	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.056	0.793	0.959	0.	0.	0.	0.

Problem 923	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.037	1.201	0.696	0.	0.	0.	0.

Problem 924	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.022	0.742	0.531	0.	0.	0.	0.

Problem 925	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.052	1.338	0.631	0.	0.	0.	0.

Problem 926	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.068	0.603	0.566	0.	0.	0.	0.

Problem 927	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	36	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.049	0.814	0.148	0.	0.	0.	0.

Problem 928	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	0	36	0	19
normalized size	1	1.	1.	0.94	0.	2.25	0.	1.19
time (sec)	N/A	0.024	0.003	0.087	0.	1.882	0.	1.107

Problem 929	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.065	0.252	0.137	0.	0.	0.	0.

Problem 930	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.067	1.457	0.779	0.	0.	0.	0.

Problem 931	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.066	3.555	0.645	0.	0.	0.	0.

Problem 932	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	47	47	122	38	0	0	0	0
normalized size	1	1.	2.6	0.81	0.	0.	0.	0.
time (sec)	N/A	0.107	0.204	0.099	0.	0.	0.	0.

Problem 933	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	24	0	0	0	0
normalized size	1	1.	1.	0.77	0.	0.	0.	0.
time (sec)	N/A	0.074	0.053	0.079	0.	0.	0.	0.

Problem 934	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	43	38	0	0	0	0
normalized size	1	1.	0.91	0.81	0.	0.	0.	0.
time (sec)	N/A	0.07	0.233	0.138	0.	0.	0.	0.

Problem 935	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.06	1.353	0.688	0.	0.	0.	0.

Problem 936	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.064	1.877	0.941	0.	0.	0.	0.

Problem 937	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.063	4.368	0.641	0.	0.	0.	0.

Problem 938	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	89	89	230	68	0	0	0	0
normalized size	1	1.	2.58	0.76	0.	0.	0.	0.
time (sec)	N/A	0.137	0.495	0.117	0.	0.	0.	0.

Problem 939	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	71	71	131	54	0	0	0	0
normalized size	1	1.	1.85	0.76	0.	0.	0.	0.
time (sec)	N/A	0.128	0.143	0.11	0.	0.	0.	0.

Problem 940	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	58	58	229	45	0	0	0	0
normalized size	1	1.	3.95	0.78	0.	0.	0.	0.
time (sec)	N/A	0.119	0.453	0.117	0.	0.	0.	0.

Problem 941	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	71	71	133	54	0	0	0	0
normalized size	1	1.	1.87	0.76	0.	0.	0.	0.
time (sec)	N/A	0.106	0.106	0.09	0.	0.	0.	0.

Problem 942	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	89	89	147	68	0	0	0	0
normalized size	1	1.	1.65	0.76	0.	0.	0.	0.
time (sec)	N/A	0.096	0.266	0.115	0.	0.	0.	0.

Problem 943	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.059	1.634	0.808	0.	0.	0.	0.

Problem 944	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.095	0.798	1.248	0.	0.	0.	0.

Problem 945	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.067	1.857	1.023	0.	0.	0.	0.

Problem 946	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.034	0.265	0.912	0.	0.	0.	0.

Problem 947	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.097	2.782	1.029	0.	0.	0.	0.

Problem 948	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.109	0.98	1.023	0.	0.	0.	0.

Problem 949	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.08	2.906	0.858	0.	0.	0.	0.

Problem 950	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.036	0.364	0.668	0.	0.	0.	0.

Problem 951	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.11	2.406	0.724	0.	0.	0.	0.

Problem 952	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.111	1.319	1.014	0.	0.	0.	0.

Problem 953	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.078	2.493	0.899	0.	0.	0.	0.

Problem 954	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.037	0.44	0.739	0.	0.	0.	0.

Problem 955	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.12	2.429	0.753	0.	0.	0.	0.

Problem 956	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.106	0.84	1.307	0.	0.	0.	0.

Problem 957	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.071	1.08	1.086	0.	0.	0.	0.

Problem 958	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.035	0.223	0.868	0.	0.	0.	0.

Problem 959	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.107	0.947	1.012	0.	0.	0.	0.

Problem 960	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.116	0.97	0.923	0.	0.	0.	0.

Problem 961	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.118	3.479	3.299	0.	0.	0.	0.

Problem 962	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	60	60	97	0	0	0	0	0
normalized size	1	1.	1.62	0.	0.	0.	0.	0.
time (sec)	N/A	0.165	0.134	0.859	0.	0.	0.	0.

Problem 963	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	60	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.093	0.254	0.719	0.	0.	0.	0.

Problem 964	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.117	1.8	0.704	0.	0.	0.	0.

Problem 965	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.118	1.557	0.961	0.	0.	0.	0.

Problem 966	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.12	3.644	2.194	0.	0.	0.	0.

Problem 967	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	95	0	0	0	0	0
normalized size	1	1.	0.73	0.	0.	0.	0.	0.
time (sec)	N/A	0.291	0.208	3.53	0.	0.	0.	0.

Problem 968	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	131	131	159	0	0	0	0	0
normalized size	1	1.	1.21	0.	0.	0.	0.	0.
time (sec)	N/A	0.296	0.192	2.98	0.	0.	0.	0.

Problem 969	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	131	131	156	0	0	0	0	0
normalized size	1	1.	1.19	0.	0.	0.	0.	0.
time (sec)	N/A	0.214	0.208	0.815	0.	0.	0.	0.

Problem 970	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	131	131	159	0	0	0	0	0
normalized size	1	1.	1.21	0.	0.	0.	0.	0.
time (sec)	N/A	0.146	0.215	0.7	0.	0.	0.	0.

Problem 971	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.118	2.078	0.727	0.	0.	0.	0.

Problem 972	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.035	1.964	0.722	0.	0.	0.	0.

Problem 973	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.023	1.515	0.386	0.	0.	0.	0.

Problem 974	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.012	1.412	0.268	0.	0.	0.	0.

Problem 975	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.032	1.92	0.399	0.	0.	0.	0.

Problem 976	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.054	1.346	0.932	0.	0.	0.	0.

Problem 977	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.036	1.632	0.558	0.	0.	0.	0.

Problem 978	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.022	1.494	0.44	0.	0.	0.	0.

Problem 979	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.049	2.032	0.49	0.	0.	0.	0.

Problem 980	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.056	0.856	0.989	0.	0.	0.	0.

Problem 981	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.037	1.671	0.837	0.	0.	0.	0.

Problem 982	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.022	1.53	0.657	0.	0.	0.	0.

Problem 983	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.05	2.088	0.697	0.	0.	0.	0.

Problem 984	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	45	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.081	0.701	0.572	0.	0.	0.	0.

Problem 985	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	36	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.048	1.019	0.141	0.	0.	0.	0.

Problem 986	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	0	38	0	19
normalized size	1	1.	1.	0.94	0.	2.38	0.	1.19
time (sec)	N/A	0.025	0.005	0.087	0.	1.615	0.	1.112

Problem 987	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	42	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.076	0.201	0.145	0.	0.	0.	0.

Problem 988	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.062	1.116	0.765	0.	0.	0.	0.

Problem 989	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	91	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.199	4.164	0.501	0.	0.	0.	0.

Problem 990	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	105	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.246	4.959	0.505	0.	0.	0.	0.

Problem 991	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	60	46	0	0	0	0
normalized size	1	1.	1.	0.77	0.	0.	0.	0.
time (sec)	N/A	0.145	0.182	0.107	0.	0.	0.	0.

Problem 992	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	48	47	0	0	0	0
normalized size	1	1.	0.35	0.34	0.	0.	0.	0.
time (sec)	N/A	0.17	0.092	0.098	0.	0.	0.	0.

Problem 993	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	52	47	0	0	0	0
normalized size	1	1.	0.91	0.82	0.	0.	0.	0.
time (sec)	N/A	0.104	0.15	0.102	0.	0.	0.	0.

Problem 994	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	101	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.199	4.551	0.525	0.	0.	0.	0.

Problem 995	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	91	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.193	4.483	0.51	0.	0.	0.	0.

Problem 996	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	91	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.199	6.727	0.869	0.	0.	0.	0.

Problem 997	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	91	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.201	6.773	0.66	0.	0.	0.	0.

Problem 998	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.063	1.355	0.839	0.	0.	0.	0.

Problem 999	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	96	96	148	86	0	0	0	0
normalized size	1	1.	1.54	0.9	0.	0.	0.	0.
time (sec)	N/A	0.334	0.347	0.112	0.	0.	0.	0.

Problem 1000	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	67	67	112	53	0	0	0	0
normalized size	1	1.	1.67	0.79	0.	0.	0.	0.
time (sec)	N/A	0.313	0.391	0.115	0.	0.	0.	0.

Problem 1001	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	93	93	156	84	0	0	0	0
normalized size	1	1.	1.68	0.9	0.	0.	0.	0.
time (sec)	N/A	0.271	0.236	0.113	0.	0.	0.	0.

Problem 1002	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	94	94	144	85	0	0	0	0
normalized size	1	1.	1.53	0.9	0.	0.	0.	0.
time (sec)	N/A	0.138	0.299	0.112	0.	0.	0.	0.

Problem 1003	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	139	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.233	5.082	0.777	0.	0.	0.	0.

Problem 1004	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	91	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.195	6.083	0.562	0.	0.	0.	0.

Problem 1005	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	91	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.217	7.285	1.398	0.	0.	0.	0.

Problem 1006	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	91	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.204	7.032	0.754	0.	0.	0.	0.

Problem 1007	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.098	0.689	1.216	0.	0.	0.	0.

Problem 1008	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.07	1.815	1.009	0.	0.	0.	0.

Problem 1009	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.036	0.425	0.902	0.	0.	0.	0.

Problem 1010	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.1	4.702	0.931	0.	0.	0.	0.

Problem 1011	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.112	1.038	1.01	0.	0.	0.	0.

Problem 1012	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.077	6.781	0.806	0.	0.	0.	0.

Problem 1013	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.036	1.348	0.679	0.	0.	0.	0.

Problem 1014	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.112	9.478	0.714	0.	0.	0.	0.

Problem 1015	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.112	1.4	1.028	0.	0.	0.	0.

Problem 1016	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.079	2.653	1.01	0.	0.	0.	0.

Problem 1017	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.038	1.122	0.837	0.	0.	0.	0.

Problem 1018	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.113	4.709	0.924	0.	0.	0.	0.

Problem 1019	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.108	0.84	1.389	0.	0.	0.	0.

Problem 1020	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.072	1.03	1.2	0.	0.	0.	0.

Problem 1021	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.035	0.632	0.914	0.	0.	0.	0.

Problem 1022	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	67	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.211	5.142	1.042	0.	0.	0.	0.

Problem 1023	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.105	10.359	0.832	0.	0.	0.	0.

Problem 1024	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.117	0.988	0.945	0.	0.	0.	0.

Problem 1025	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	98	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.373	5.979	2.855	0.	0.	0.	0.

Problem 1026	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	126	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.431	2.217	2.893	0.	0.	0.	0.

Problem 1027	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F(-2)	F(-2)	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	93	93	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.178	0.342	0.83	0.	0.	0.	0.

Problem 1028	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	92	92	107	0	0	0	0	0
normalized size	1	1.	1.16	0.	0.	0.	0.	0.
time (sec)	N/A	0.213	0.127	0.723	0.	0.	0.	0.

Problem 1029	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	125	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.332	5.294	0.809	0.	0.	0.	0.

Problem 1030	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	98	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.36	19.438	0.81	0.	0.	0.	0.

Problem 1031	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	98	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.365	16.243	0.94	0.	0.	0.	0.

Problem 1032	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	98	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.363	18.258	1.993	0.	0.	0.	0.

Problem 1033	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.115	1.076	1.112	0.	0.	0.	0.

Problem 1034	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	160	160	182	0	0	0	0	0
normalized size	1	1.	1.14	0.	0.	0.	0.	0.
time (sec)	N/A	0.434	0.517	4.371	0.	0.	0.	0.

Problem 1035	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	281	281	241	0	0	0	0	0
normalized size	1	1.	0.86	0.	0.	0.	0.	0.
time (sec)	N/A	0.666	0.504	3.001	0.	0.	0.	0.

Problem 1036	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	280	280	299	0	0	0	0	0
normalized size	1	1.	1.07	0.	0.	0.	0.	0.
time (sec)	N/A	0.546	0.483	0.855	0.	0.	0.	0.

Problem 1037	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	157	157	158	0	0	0	0	0
normalized size	1	1.	1.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.265	0.383	0.763	0.	0.	0.	0.

Problem 1038	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	185	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.397	5.545	0.746	0.	0.	0.	0.

Problem 1039	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	98	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.369	14.574	0.797	0.	0.	0.	0.

Problem 1040	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	98	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.365	24.679	0.909	0.	0.	0.	0.

Problem 1041	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	98	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.369	20.791	2.022	0.	0.	0.	0.

Problem 1042	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.036	1.955	0.764	0.	0.	0.	0.

Problem 1043	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.023	2.541	0.389	0.	0.	0.	0.

Problem 1044	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.013	1.955	0.266	0.	0.	0.	0.

Problem 1045	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.032	5.295	0.391	0.	0.	0.	0.

Problem 1046	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.056	1.36	0.922	0.	0.	0.	0.

Problem 1047	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.037	2.029	0.559	0.	0.	0.	0.

Problem 1048	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.022	1.493	0.437	0.	0.	0.	0.

Problem 1049	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.05	2.992	0.506	0.	0.	0.	0.

Problem 1050	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.056	0.867	0.964	0.	0.	0.	0.

Problem 1051	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.037	2.056	0.817	0.	0.	0.	0.

Problem 1052	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.022	1.536	0.638	0.	0.	0.	0.

Problem 1053	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.051	3.783	0.705	0.	0.	0.	0.

Problem 1054	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	49	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.082	0.726	0.575	0.	0.	0.	0.

Problem 1055	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	40	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.049	1.377	0.142	0.	0.	0.	0.

Problem 1056	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	0	41	0	19
normalized size	1	1.	1.	0.83	0.	2.28	0.	1.06
time (sec)	N/A	0.024	0.006	0.08	0.	1.605	0.	1.107

Problem 1057	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	46	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.077	1.298	0.142	0.	0.	0.	0.

Problem 1058	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.063	1.156	0.77	0.	0.	0.	0.

Problem 1059	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	184	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.415	4.558	0.496	0.	0.	0.	0.

Problem 1060	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	180	180	162	62	0	0	0	0
normalized size	1	1.	0.9	0.34	0.	0.	0.	0.
time (sec)	N/A	0.263	0.342	0.11	0.	0.	0.	0.

Problem 1061	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	88	59	0	0	0	0
normalized size	1	1.	0.87	0.58	0.	0.	0.	0.
time (sec)	N/A	0.122	0.078	0.102	0.	0.	0.	0.

Problem 1062	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	174	174	170	62	0	0	0	0
normalized size	1	1.	0.98	0.36	0.	0.	0.	0.
time (sec)	N/A	0.206	0.417	0.106	0.	0.	0.	0.

Problem 1063	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	178	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.36	4.25	0.535	0.	0.	0.	0.

Problem 1064	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	196	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.483	6.392	0.51	0.	0.	0.	0.

Problem 1065	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	186	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.464	5.284	0.867	0.	0.	0.	0.

Problem 1066	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	186	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.471	11.	0.662	0.	0.	0.	0.

Problem 1067	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.065	1.365	0.826	0.	0.	0.	0.

Problem 1068	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	160	160	227	112	0	0	0	0
normalized size	1	1.	1.42	0.7	0.	0.	0.	0.
time (sec)	N/A	0.59	0.442	0.119	0.	0.	0.	0.

Problem 1069	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	129	129	259	68	0	0	0	0
normalized size	1	1.	2.01	0.53	0.	0.	0.	0.
time (sec)	N/A	0.688	0.9	0.115	0.	0.	0.	0.

Problem 1070	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	155	155	220	110	0	0	0	0
normalized size	1	1.	1.42	0.71	0.	0.	0.	0.
time (sec)	N/A	0.5	0.38	0.116	0.	0.	0.	0.

Problem 1071	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	125	125	186	113	0	0	0	0
normalized size	1	1.	1.49	0.9	0.	0.	0.	0.
time (sec)	N/A	0.297	0.697	0.121	0.	0.	0.	0.

Problem 1072	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	214	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.412	5.769	0.816	0.	0.	0.	0.

Problem 1073	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	229	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.508	7.283	0.584	0.	0.	0.	0.

Problem 1074	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	184	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.461	5.785	1.507	0.	0.	0.	0.

Problem 1075	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	188	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.469	12.762	0.755	0.	0.	0.	0.

Problem 1076	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.098	0.687	1.264	0.	0.	0.	0.

Problem 1077	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.067	2.568	1.01	0.	0.	0.	0.

Problem 1078	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.034	1.507	0.878	0.	0.	0.	0.

Problem 1079	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.099	17.757	0.918	0.	0.	0.	0.

Problem 1080	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.109	1.056	0.98	0.	0.	0.	0.

Problem 1081	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.076	2.595	0.799	0.	0.	0.	0.

Problem 1082	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.036	1.63	0.678	0.	0.	0.	0.

Problem 1083	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.111	10.906	0.741	0.	0.	0.	0.

Problem 1084	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.11	1.394	1.077	0.	0.	0.	0.

Problem 1085	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.079	3.024	0.974	0.	0.	0.	0.

Problem 1086	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.043	1.735	0.803	0.	0.	0.	0.

Problem 1087	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.114	11.05	0.921	0.	0.	0.	0.

Problem 1088	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.114	0.849	1.395	0.	0.	0.	0.

Problem 1089	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.075	1.738	1.203	0.	0.	0.	0.

Problem 1090	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.036	0.798	0.962	0.	0.	0.	0.

Problem 1091	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	71	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.216	3.351	1.277	0.	0.	0.	0.

Problem 1092	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.109	13.098	0.898	0.	0.	0.	0.

Problem 1093	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.118	0.989	1.246	0.	0.	0.	0.

Problem 1094	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	227	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.928	2.574	3.161	0.	0.	0.	0.

Problem 1095	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	229	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.68	2.106	3.109	0.	0.	0.	0.

Problem 1096	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	129	129	124	0	0	0	0	0
normalized size	1	1.	0.96	0.	0.	0.	0.	0.
time (sec)	N/A	0.303	0.186	0.904	0.	0.	0.	0.

Problem 1097	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	126	126	120	0	0	0	0	0
normalized size	1	1.	0.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.228	0.144	0.82	0.	0.	0.	0.

Problem 1098	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	225	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.697	11.486	0.805	0.	0.	0.	0.

Problem 1099	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	225	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.833	13.035	0.803	0.	0.	0.	0.

Problem 1100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	199	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.849	15.086	0.983	0.	0.	0.	0.

Problem 1101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	203	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.852	26.87	2.049	0.	0.	0.	0.

Problem 1102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.119	1.085	1.023	0.	0.	0.	0.

Problem 1103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	190	190	255	0	0	0	0	0
normalized size	1	1.	1.34	0.	0.	0.	0.	0.
time (sec)	N/A	0.808	0.852	3.796	0.	0.	0.	0.

Problem 1104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	224	224	311	0	0	0	0	0
normalized size	1	1.	1.39	0.	0.	0.	0.	0.
time (sec)	N/A	1.14	0.815	3.125	0.	0.	0.	0.

Problem 1105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	222	222	261	0	0	0	0	0
normalized size	1	1.	1.18	0.	0.	0.	0.	0.
time (sec)	N/A	1.057	0.83	0.898	0.	0.	0.	0.

Problem 1106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	183	183	300	0	0	0	0	0
normalized size	1	1.	1.64	0.	0.	0.	0.	0.
time (sec)	N/A	0.617	0.513	0.761	0.	0.	0.	0.

Problem 1107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	285	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.768	12.181	0.783	0.	0.	0.	0.

Problem 1108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	290	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.932	13.988	0.834	0.	0.	0.	0.

Problem 1109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	195	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.899	16.518	0.951	0.	0.	0.	0.

Problem 1110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	203	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.918	31.13	2.015	0.	0.	0.	0.

Problem 1111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	45	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.085	0.892	0.602	0.	0.	0.	0.

Problem 1112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	21	0	55	0	27
normalized size	1	1.	1.	1.05	0.	2.75	0.	1.35
time (sec)	N/A	0.04	0.006	0.066	0.	1.738	0.	1.077

Problem 1113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.099	0.63	5.934	0.	0.	0.	0.

Problem 1114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	127	106	146	258	138	180
normalized size	1	1.	1.19	0.99	1.36	2.41	1.29	1.68
time (sec)	N/A	0.11	0.005	0.038	1.478	1.643	2.868	1.182

Problem 1115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	119	102	142	244	128	162
normalized size	1	1.	1.27	1.09	1.51	2.6	1.36	1.72
time (sec)	N/A	0.139	0.021	0.037	1.083	1.75	2.056	1.09

Problem 1116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	103	86	119	197	114	154
normalized size	1	1.	1.26	1.05	1.45	2.4	1.39	1.88
time (sec)	N/A	0.07	0.004	0.037	1.418	1.751	1.563	1.188

Problem 1117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	85	76	108	181	94	127
normalized size	1	1.	1.25	1.12	1.59	2.66	1.38	1.87
time (sec)	N/A	0.072	0.01	0.035	1.058	1.699	1.032	1.106

Problem 1118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	83	117	158	0	0	0
normalized size	1	1.	1.08	1.52	2.05	0.	0.	0.
time (sec)	N/A	0.093	0.004	0.049	2.152	0.	0.	0.

Problem 1119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	73	72	99	174	80	120
normalized size	1	1.	1.28	1.26	1.74	3.05	1.4	2.11
time (sec)	N/A	0.077	0.005	0.04	0.945	1.667	1.435	1.1

Problem 1120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	77	77	86	117	0	0	0	0
normalized size	1	1.	1.12	1.52	0.	0.	0.	0.
time (sec)	N/A	0.099	0.005	0.053	0.	0.	0.	0.

Problem 1121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	98	97	126	204	116	142
normalized size	1	1.	1.18	1.17	1.52	2.46	1.4	1.71
time (sec)	N/A	0.119	0.034	0.046	0.942	1.729	1.85	1.097

Problem 1122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	97	86	108	177	99	143
normalized size	1	1.	1.18	1.05	1.32	2.16	1.21	1.74
time (sec)	N/A	0.09	0.005	0.044	1.437	1.681	1.49	1.161

Problem 1123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	123	120	157	269	153	174
normalized size	1	1.	1.12	1.09	1.43	2.45	1.39	1.58
time (sec)	N/A	0.129	0.042	0.044	0.956	1.854	3.179	1.11

Problem 1124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	97	106	139	238	122	174
normalized size	1	1.	0.92	1.01	1.32	2.27	1.16	1.66
time (sec)	N/A	0.109	0.005	0.044	1.458	1.77	2.253	1.196

Problem 1125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	174	203	248	455	260	315
normalized size	1	1.	0.94	1.1	1.34	2.46	1.41	1.7
time (sec)	N/A	0.193	0.14	0.038	1.473	1.772	5.688	1.364

Problem 1126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	162	192	244	424	245	284
normalized size	1	1.	1.01	1.19	1.52	2.63	1.52	1.76
time (sec)	N/A	0.246	0.117	0.036	0.973	1.687	4.097	1.1

Problem 1127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	140	168	211	363	219	281
normalized size	1	1.	1.22	1.46	1.83	3.16	1.9	2.44
time (sec)	N/A	0.114	0.089	0.038	1.447	1.632	3.535	1.269

Problem 1128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	130	151	198	339	194	231
normalized size	1	1.	1.05	1.22	1.6	2.73	1.56	1.86
time (sec)	N/A	0.16	0.092	0.038	0.972	1.533	2.322	1.13

Problem 1129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	123	187	251	0	0	0
normalized size	1	1.	0.9	1.36	1.83	0.	0.	0.
time (sec)	N/A	0.18	0.101	0.051	2.148	0.	0.	0.

Problem 1130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	114	138	176	302	165	220
normalized size	1	1.	1.05	1.27	1.61	2.77	1.51	2.02
time (sec)	N/A	0.156	0.106	0.046	0.971	1.593	2.509	1.082

Problem 1131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	128	128	118	178	223	0	0	0
normalized size	1	1.	0.92	1.39	1.74	0.	0.	0.
time (sec)	N/A	0.163	0.102	0.058	2.146	0.	0.	0.

Problem 1132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	119	147	182	311	180	232
normalized size	1	1.	1.03	1.28	1.58	2.7	1.57	2.02
time (sec)	N/A	0.168	0.116	0.046	0.964	1.382	2.526	1.097

Problem 1133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	139	139	130	190	0	0	0	0
normalized size	1	1.	0.94	1.37	0.	0.	0.	0.
time (sec)	N/A	0.168	0.095	0.059	0.	0.	0.	0.

Problem 1134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	149	186	224	379	235	265
normalized size	1	1.	0.99	1.24	1.49	2.53	1.57	1.77
time (sec)	N/A	0.185	0.128	0.049	0.982	1.494	3.436	1.098

Problem 1135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	112	168	196	329	192	258
normalized size	1	1.	1.01	1.51	1.77	2.96	1.73	2.32
time (sec)	N/A	0.148	0.096	0.048	1.535	1.459	2.516	1.34

Problem 1136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	186	177	224	266	462	289	315
normalized size	1	1.	0.95	1.2	1.43	2.48	1.55	1.69
time (sec)	N/A	0.232	0.173	0.047	0.975	1.568	5.408	1.096

Problem 1137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	240	285	262	315	362	711	411	479
normalized size	1	1.19	1.09	1.31	1.51	2.96	1.71	2.
time (sec)	N/A	0.459	0.227	0.038	1.484	1.424	9.673	1.65

Problem 1138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	239	239	252	297	358	647	389	424
normalized size	1	1.	1.05	1.24	1.5	2.71	1.63	1.77
time (sec)	N/A	0.384	0.209	0.039	1.041	1.353	6.921	1.1

Problem 1139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	217	265	313	559	350	416
normalized size	1	1.	1.37	1.68	1.98	3.54	2.22	2.63
time (sec)	N/A	0.147	0.16	0.037	1.464	1.542	6.384	1.468

Problem 1140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	192	239	300	513	306	352
normalized size	1	1.	1.02	1.27	1.6	2.73	1.63	1.87
time (sec)	N/A	0.151	0.141	0.038	0.962	1.786	4.239	1.112

Problem 1141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	228	228	190	272	356	0	0	0
normalized size	1	1.	0.83	1.19	1.56	0.	0.	0.
time (sec)	N/A	0.221	0.17	0.052	2.226	0.	0.	0.

Problem 1142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	169	211	266	446	258	325
normalized size	1	1.	1.06	1.32	1.66	2.79	1.61	2.03
time (sec)	N/A	0.258	0.149	0.044	0.966	1.951	4.543	1.115

Problem 1143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	200	200	170	251	321	0	0	0
normalized size	1	1.	0.85	1.25	1.6	0.	0.	0.
time (sec)	N/A	0.211	0.16	0.056	2.209	0.	0.	0.

Problem 1144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	166	213	261	433	272	340
normalized size	1	1.	1.05	1.35	1.65	2.74	1.72	2.15
time (sec)	N/A	0.265	0.165	0.048	0.981	1.73	4.606	1.101

Problem 1145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	200	200	169	251	316	0	0	0
normalized size	1	1.	0.84	1.25	1.58	0.	0.	0.
time (sec)	N/A	0.207	0.206	0.056	2.164	0.	0.	0.

Problem 1146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	184	236	281	473	289	359
normalized size	1	1.	1.04	1.33	1.59	2.67	1.63	2.03
time (sec)	N/A	0.285	0.172	0.047	0.998	1.783	4.915	1.104

Problem 1147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	228	228	175	272	0	0	0	0
normalized size	1	1.	0.77	1.19	0.	0.	0.	0.
time (sec)	N/A	0.231	0.138	0.06	0.	0.	0.	0.

Problem 1148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	224	230	290	333	567	362	405
normalized size	1	1.	1.03	1.29	1.49	2.53	1.62	1.81
time (sec)	N/A	0.327	0.172	0.047	1.034	2.003	6.391	1.104

Problem 1149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	154	265	294	512	309	410
normalized size	1	1.	1.01	1.74	1.93	3.37	2.03	2.7
time (sec)	N/A	0.195	0.177	0.049	1.471	1.822	4.462	2.115

Problem 1150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	244	244	212	279	305	541	314	315
normalized size	1	1.	0.87	1.14	1.25	2.22	1.29	1.29
time (sec)	N/A	0.176	0.174	0.039	0.979	1.771	6.627	1.098

Problem 1151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	361	361	503	703	0	0	0	0
normalized size	1	1.	1.39	1.95	0.	0.	0.	0.
time (sec)	N/A	0.37	0.26	0.211	0.	0.	0.	0.

Problem 1152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	311	311	441	646	0	0	0	0
normalized size	1	1.	1.42	2.08	0.	0.	0.	0.
time (sec)	N/A	0.243	0.113	0.193	0.	0.	0.	0.

Problem 1153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	353	353	429	736	0	0	0	0
normalized size	1	1.	1.22	2.08	0.	0.	0.	0.
time (sec)	N/A	0.386	0.242	0.208	0.	0.	0.	0.

Problem 1154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	409	409	504	801	0	0	0	0
normalized size	1	1.	1.23	1.96	0.	0.	0.	0.
time (sec)	N/A	0.482	0.273	0.2	0.	0.	0.	0.

Problem 1155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	555	555	776	2409	0	0	0	0
normalized size	1	1.	1.4	4.34	0.	0.	0.	0.
time (sec)	N/A	0.631	3.387	0.501	0.	0.	0.	0.

Problem 1156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	517	517	461	886	0	0	0	0
normalized size	1	1.	0.89	1.71	0.	0.	0.	0.
time (sec)	N/A	0.406	0.233	0.219	0.	0.	0.	0.

Problem 1157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	561	561	468	2439	0	0	0	0
normalized size	1	1.	0.83	4.35	0.	0.	0.	0.
time (sec)	N/A	0.528	0.777	0.453	0.	0.	0.	0.

Problem 1158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	403	403	522	760	0	0	0	0
normalized size	1	1.	1.3	1.89	0.	0.	0.	0.
time (sec)	N/A	0.449	7.91	0.203	0.	0.	0.	0.

Problem 1159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	98	109	0	501	0	157
normalized size	1	1.	1.08	1.2	0.	5.51	0.	1.73
time (sec)	N/A	0.066	0.148	0.043	0.	1.888	0.	1.283

Problem 1160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	443	443	590	847	0	0	0	0
normalized size	1	1.	1.33	1.91	0.	0.	0.	0.
time (sec)	N/A	0.489	5.876	0.212	0.	0.	0.	0.

Problem 1161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	489	489	643	925	0	0	0	0
normalized size	1	1.	1.31	1.89	0.	0.	0.	0.
time (sec)	N/A	0.513	12.401	0.217	0.	0.	0.	0.

Problem 1162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1335	1335	881	2315	0	0	0	0
normalized size	1	1.	0.66	1.73	0.	0.	0.	0.
time (sec)	N/A	1.964	10.29	0.815	0.	0.	0.	0.

Problem 1163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	819	819	861	2315	0	0	0	0
normalized size	1	1.	1.05	2.83	0.	0.	0.	0.
time (sec)	N/A	0.893	10.05	0.767	0.	0.	0.	0.

Problem 1164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1382	1382	982	3851	0	0	0	0
normalized size	1	1.	0.71	2.79	0.	0.	0.	0.
time (sec)	N/A	1.58	12.95	0.638	0.	0.	0.	0.

Problem 1165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	532	532	589	959	0	0	0	0
normalized size	1	1.	1.11	1.8	0.	0.	0.	0.
time (sec)	N/A	0.65	12.464	0.229	0.	0.	0.	0.

Problem 1166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	158	297	0	1418	0	506
normalized size	1	1.	1.22	2.28	0.	10.91	0.	3.89
time (sec)	N/A	0.186	3.184	0.058	0.	2.749	0.	9.283

Problem 1167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	131	216	0	1288	0	518
normalized size	1	1.	1.	1.65	0.	9.83	0.	3.95
time (sec)	N/A	0.113	1.026	0.046	0.	2.522	0.	2.045

Problem 1168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	574	574	645	1041	0	0	0	0
normalized size	1	1.	1.12	1.81	0.	0.	0.	0.
time (sec)	N/A	0.631	12.844	0.219	0.	0.	0.	0.

Problem 1169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	629	629	723	1128	0	0	0	0
normalized size	1	1.	1.15	1.79	0.	0.	0.	0.
time (sec)	N/A	0.68	15.912	0.232	0.	0.	0.	0.

Problem 1170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	966	966	1894	3801	0	0	0	0
normalized size	1	1.	1.96	3.93	0.	0.	0.	0.
time (sec)	N/A	2.26	12.969	0.75	0.	0.	0.	0.

Problem 1171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	893	893	1745	4027	0	0	0	0
normalized size	1	1.	1.95	4.51	0.	0.	0.	0.
time (sec)	N/A	0.949	12.194	0.971	0.	0.	0.	0.

Problem 1172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1518	1518	1985	6655	0	0	0	0
normalized size	1	1.	1.31	4.38	0.	0.	0.	0.
time (sec)	N/A	2.636	13.326	0.939	0.	0.	0.	0.

Problem 1173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	391	0	0	2657	0	362
normalized size	1	1.	1.75	0.	0.	11.91	0.	1.62
time (sec)	N/A	0.368	0.491	0.965	0.	26.306	0.	1.323

Problem 1174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	96	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.158	10.949	0.787	0.	0.	0.	0.

Problem 1175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	279	0	0	1947	0	251
normalized size	1	1.	1.99	0.	0.	13.91	0.	1.79
time (sec)	N/A	0.142	0.517	0.806	0.	6.768	0.	1.293

Problem 1176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.024	4.702	1.524	0.	0.	0.	0.

Problem 1177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	63	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.163	71.161	0.771	0.	0.	0.	0.

Problem 1178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	67	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.147	8.546	0.758	0.	0.	0.	0.

Problem 1179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	72	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.165	47.667	0.77	0.	0.	0.	0.

Problem 1180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	288	0	0	1940	0	0
normalized size	1	1.	2.1	0.	0.	14.16	0.	0.
time (sec)	N/A	0.279	0.637	0.829	0.	3.74	0.	0.

Problem 1181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	97	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.176	50.348	0.785	0.	0.	0.	0.

Problem 1182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	224	413	0	0	2593	0	0
normalized size	1	1.	1.84	0.	0.	11.58	0.	0.
time (sec)	N/A	0.352	0.509	0.881	0.	6.679	0.	0.

Problem 1183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	279	279	418	0	0	3487	0	852
normalized size	1	1.	1.5	0.	0.	12.5	0.	3.05
time (sec)	N/A	0.461	0.653	0.654	0.	117.58	0.	1.58

Problem 1184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	118	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.195	10.74	0.593	0.	0.	0.	0.

Problem 1185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	313	0	0	2603	0	620
normalized size	1	1.	1.73	0.	0.	14.38	0.	3.43
time (sec)	N/A	0.233	0.425	0.619	0.	29.711	0.	1.505

Problem 1186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.028	4.92	1.197	0.	0.	0.	0.

Problem 1187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	80	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.193	71.74	0.568	0.	0.	0.	0.

Problem 1188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	89	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.174	8.687	0.574	0.	0.	0.	0.

Problem 1189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	89	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.204	47.919	0.579	0.	0.	0.	0.

Problem 1190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	87	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.175	31.165	0.586	0.	0.	0.	0.

Problem 1191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	94	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.196	49.644	0.594	0.	0.	0.	0.

Problem 1192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	334	0	0	2538	0	0
normalized size	1	1.	1.88	0.	0.	14.26	0.	0.
time (sec)	N/A	0.325	0.464	0.682	0.	6.771	0.	0.

Problem 1193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	345	345	470	0	0	0	0	1481
normalized size	1	1.	1.36	0.	0.	0.	0.	4.29
time (sec)	N/A	0.583	0.87	0.652	0.	0.	0.	1.936

Problem 1194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	140	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.227	11.443	0.586	0.	0.	0.	0.

Problem 1195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	233	233	353	0	0	3430	0	1115
normalized size	1	1.	1.52	0.	0.	14.72	0.	4.79
time (sec)	N/A	0.331	0.521	0.606	0.	95.808	0.	1.821

Problem 1196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.028	5.003	1.19	0.	0.	0.	0.

Problem 1197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	99	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.203	71.182	0.566	0.	0.	0.	0.

Problem 1198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	110	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.187	9.015	0.585	0.	0.	0.	0.

Problem 1199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	107	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.208	48.548	0.587	0.	0.	0.	0.

Problem 1200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	113	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.187	8.942	0.595	0.	0.	0.	0.

Problem 1201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	377	0	0	1976	0	0
normalized size	1	1.	2.14	0.	0.	11.23	0.	0.
time (sec)	N/A	0.248	0.485	0.831	0.	7.632	0.	0.

Problem 1202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	74	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.156	9.116	0.785	0.	0.	0.	0.

Problem 1203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	251	0	0	1461	0	0
normalized size	1	1.	2.44	0.	0.	14.18	0.	0.
time (sec)	N/A	0.099	0.372	0.806	0.	2.575	0.	0.

Problem 1204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.025	3.432	1.367	0.	0.	0.	0.

Problem 1205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	50	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.163	5.319	0.783	0.	0.	0.	0.

Problem 1206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	247	0	0	1504	0	0
normalized size	1	1.	2.47	0.	0.	15.04	0.	0.
time (sec)	N/A	0.177	0.418	0.81	0.	2.444	0.	0.

Problem 1207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	75	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.172	50.955	0.776	0.	0.	0.	0.

Problem 1208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	372	0	0	1971	0	0
normalized size	1	1.	2.08	0.	0.	11.01	0.	0.
time (sec)	N/A	0.266	0.486	0.837	0.	2.886	0.	0.

Problem 1209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	321	0	0	2724	0	0
normalized size	1	1.	2.34	0.	0.	19.88	0.	0.
time (sec)	N/A	0.184	0.616	0.605	0.	5.297	0.	0.

Problem 1210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	68	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.168	18.218	0.598	0.	0.	0.	0.

Problem 1211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	210	0	0	791	0	0
normalized size	1	1.	2.96	0.	0.	11.14	0.	0.
time (sec)	N/A	0.074	0.367	0.611	0.	2.349	0.	0.

Problem 1212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	202	0	0	810	0	0
normalized size	1	1.	2.89	0.	0.	11.57	0.	0.
time (sec)	N/A	0.077	0.265	1.188	0.	2.356	0.	0.

Problem 1213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	66	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.176	49.962	0.558	0.	0.	0.	0.

Problem 1214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	306	0	0	2758	0	0
normalized size	1	1.	2.27	0.	0.	20.43	0.	0.
time (sec)	N/A	0.23	0.641	0.599	0.	3.609	0.	0.

Problem 1215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	95	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.196	53.028	0.599	0.	0.	0.	0.

Problem 1216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	249	249	405	0	0	3976	0	0
normalized size	1	1.	1.63	0.	0.	15.97	0.	0.
time (sec)	N/A	0.874	0.711	0.598	0.	5.394	0.	0.

Problem 1217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	90	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.18	11.438	0.608	0.	0.	0.	0.

Problem 1218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	326	0	0	1781	0	0
normalized size	1	1.	2.28	0.	0.	12.45	0.	0.
time (sec)	N/A	0.209	0.538	0.601	0.	5.673	0.	0.

Problem 1219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	252	0	0	1386	0	0
normalized size	1	1.	2.31	0.	0.	12.72	0.	0.
time (sec)	N/A	0.199	1.063	0.604	0.	3.369	0.	0.

Problem 1220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	259	0	0	1385	0	0
normalized size	1	1.	2.35	0.	0.	12.59	0.	0.
time (sec)	N/A	0.094	0.725	0.607	0.	5.248	0.	0.

Problem 1221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	317	0	0	1778	0	0
normalized size	1	1.	2.2	0.	0.	12.35	0.	0.
time (sec)	N/A	0.314	0.553	1.197	0.	5.921	0.	0.

Problem 1222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	85	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.193	50.611	0.599	0.	0.	0.	0.

Problem 1223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	274	274	418	0	0	5621	0	0
normalized size	1	1.	1.53	0.	0.	20.51	0.	0.
time (sec)	N/A	0.932	1.218	0.584	0.	12.238	0.	0.

Problem 1224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	115	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.209	53.748	0.595	0.	0.	0.	0.

Problem 1225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	423	425	510	0	0	7200	0	0
normalized size	1	1.	1.21	0.	0.	17.02	0.	0.
time (sec)	N/A	1.095	2.139	0.601	0.	20.367	0.	0.

Problem 1226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	208	345	0	0	2568	0	275
normalized size	1	1.	1.66	0.	0.	12.35	0.	1.32
time (sec)	N/A	0.996	0.814	0.763	0.	2.713	0.	1.184

Problem 1227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	293	293	450	0	0	4096	0	454
normalized size	1	1.	1.54	0.	0.	13.98	0.	1.55
time (sec)	N/A	1.231	1.35	0.664	0.	6.073	0.	1.218

Problem 1228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	378	374	264	0	0	0	0	0
normalized size	1	0.99	0.7	0.	0.	0.	0.	0.
time (sec)	N/A	1.983	0.582	1.137	0.	0.	0.	0.

Problem 1229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	230	226	193	0	0	0	0	0
normalized size	1	0.98	0.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.294	0.205	0.935	0.	0.	0.	0.

Problem 1230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	119	0	0	0	0	0
normalized size	1	1.	0.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.127	0.173	0.678	0.	0.	0.	0.

Problem 1231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	62	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.121	2.225	1.5	0.	0.	0.	0.

Problem 1232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	62	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.117	5.574	0.701	0.	0.	0.	0.

Problem 1233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	75	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.176	3.944	0.615	0.	0.	0.	0.

Problem 1234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	75	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.172	0.124	0.607	0.	0.	0.	0.

Problem 1235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	75	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.154	0.09	0.816	0.	0.	0.	0.

Problem 1236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	75	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.156	3.408	0.797	0.	0.	0.	0.

Problem 1237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	73	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.174	4.371	0.619	0.	0.	0.	0.

Problem 1238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	75	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.176	5.77	0.606	0.	0.	0.	0.

Problem 1239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	76	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.118	3.057	0.886	0.	0.	0.	0.

Problem 1240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	80	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.142	3.01	0.871	0.	0.	0.	0.

Problem 1241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	129	129	166	0	0	0	0	0
normalized size	1	1.	1.29	0.	0.	0.	0.	0.
time (sec)	N/A	0.166	0.434	0.886	0.	0.	0.	0.

Problem 1242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	80	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.14	3.301	0.874	0.	0.	0.	0.

Problem 1243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	285	285	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.375	4.084	0.898	0.	0.	0.	0.

Problem 1244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	80	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.144	3.427	0.881	0.	0.	0.	0.

Problem 1245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	466	466	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	1.426	4.457	0.874	0.	0.	0.	0.

Problem 1246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	80	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.141	2.752	0.904	0.	0.	0.	0.

Problem 1247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	271	271	240	329	413	652	398	490
normalized size	1	1.	0.89	1.21	1.52	2.41	1.47	1.81
time (sec)	N/A	0.651	0.235	0.053	1.599	1.816	6.143	1.424

Problem 1248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	323	323	287	667	0	0	0	0
normalized size	1	1.	0.89	2.07	0.	0.	0.	0.
time (sec)	N/A	0.59	0.809	0.132	0.	0.	0.	0.

Problem 1249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	199	179	249	333	471	296	382
normalized size	1	1.	0.9	1.25	1.67	2.37	1.49	1.92
time (sec)	N/A	0.398	0.174	0.049	1.614	2.042	3.615	1.286

Problem 1250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	231	231	208	570	0	0	0	0
normalized size	1	1.	0.9	2.47	0.	0.	0.	0.
time (sec)	N/A	0.358	0.434	0.128	0.	0.	0.	0.

Problem 1251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	217	217	263	1284	0	0	0	0
normalized size	1	1.	1.21	5.92	0.	0.	0.	0.
time (sec)	N/A	0.442	0.364	1.72	0.	0.	0.	0.

Problem 1252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	172	172	204	597	0	0	0	0
normalized size	1	1.	1.19	3.47	0.	0.	0.	0.
time (sec)	N/A	0.327	0.273	0.132	0.	0.	0.	0.

Problem 1253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	220	220	273	1313	0	0	0	0
normalized size	1	1.	1.24	5.97	0.	0.	0.	0.
time (sec)	N/A	0.461	0.346	4.318	0.	0.	0.	0.

Problem 1254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	502	502	414	621	697	1168	758	879
normalized size	1	1.	0.82	1.24	1.39	2.33	1.51	1.75
time (sec)	N/A	1.14	0.456	0.053	1.71	2.131	14.175	2.532

Problem 1255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	580	580	513	1158	0	0	0	0
normalized size	1	1.	0.88	2.	0.	0.	0.	0.
time (sec)	N/A	1.068	1.57	0.139	0.	0.	0.	0.

Problem 1256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	380	380	317	484	585	894	575	716
normalized size	1	1.	0.83	1.27	1.54	2.35	1.51	1.88
time (sec)	N/A	0.754	0.316	0.053	1.621	2.308	7.655	1.625

Problem 1257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	442	442	391	1005	0	0	0	0
normalized size	1	1.	0.88	2.27	0.	0.	0.	0.
time (sec)	N/A	0.689	1.086	0.129	0.	0.	0.	0.

Problem 1258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	355	355	389	1549	0	0	0	0
normalized size	1	1.	1.1	4.36	0.	0.	0.	0.
time (sec)	N/A	0.69	0.668	4.632	0.	0.	0.	0.

Problem 1259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	343	343	349	997	0	0	0	0
normalized size	1	1.	1.02	2.91	0.	0.	0.	0.
time (sec)	N/A	0.576	0.767	0.145	0.	0.	0.	0.

Problem 1260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	320	320	367	1511	0	0	0	0
normalized size	1	1.	1.15	4.72	0.	0.	0.	0.
time (sec)	N/A	0.609	0.585	3.498	0.	0.	0.	0.

Problem 1261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	590	590	1567	0	0	0	0	0
normalized size	1	1.	2.66	0.	0.	0.	0.	0.
time (sec)	N/A	0.499	9.599	18.961	0.	0.	0.	0.

Problem 1262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	554	554	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.48	180.001	2.909	0.	0.	0.	0.

Problem 1263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	492	492	1527	0	0	0	0	0
normalized size	1	1.	3.1	0.	0.	0.	0.	0.
time (sec)	N/A	0.25	8.218	6.373	0.	0.	0.	0.

Problem 1264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	B	F(-2)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	460	460	0	2600	0	0	0	0
normalized size	1	1.	0.	5.65	0.	0.	0.	0.
time (sec)	N/A	0.247	180.002	0.359	0.	0.	0.	0.

Problem 1265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	637	637	1410	0	0	0	0	0
normalized size	1	1.	2.21	0.	0.	0.	0.	0.
time (sec)	N/A	0.667	6.852	9.102	0.	0.	0.	0.

Problem 1266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	553	553	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.516	180.002	3.365	0.	0.	0.	0.

Problem 1267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	745	745	1555	0	0	0	0	0
normalized size	1	1.	2.09	0.	0.	0.	0.	0.
time (sec)	N/A	0.923	10.915	26.595	0.	0.	0.	0.

Problem 1268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	943	943	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	1.759	18.153	9.468	0.	0.	0.	0.

Problem 1269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F(-2)	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1033	1033	0	6575	0	0	0	0
normalized size	1	1.	0.	6.36	0.	0.	0.	0.
time (sec)	N/A	1.948	42.98	1.473	0.	0.	0.	0.

Problem 1270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	457	457	885	1185	0	0	0	0
normalized size	1	1.	1.94	2.59	0.	0.	0.	0.
time (sec)	N/A	1.088	8.778	0.363	0.	0.	0.	0.

Problem 1271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F(-2)	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1039	1039	0	6575	0	0	0	0
normalized size	1	1.	0.	6.33	0.	0.	0.	0.
time (sec)	N/A	1.329	23.03	1.447	0.	0.	0.	0.

Problem 1272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1087	1087	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	1.861	15.904	10.853	0.	0.	0.	0.

Problem 1273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1141	1141	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	2.047	180.004	3.653	0.	0.	0.	0.

Problem 1274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1181	1181	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	2.017	28.907	16.436	0.	0.	0.	0.

Problem 1275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	79	3626	108	227	107	227
normalized size	1	1.	0.71	32.67	0.97	2.05	0.96	2.05
time (sec)	N/A	0.444	0.025	1.822	1.647	1.188	8.405	1.104

Problem 1276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	56	2849	84	154	83	167
normalized size	1	1.	0.64	32.38	0.95	1.75	0.94	1.9
time (sec)	N/A	0.118	0.023	1.192	1.522	1.375	4.916	1.112

Problem 1277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	64	3041	88	174	78	182
normalized size	1	1.	0.78	37.09	1.07	2.12	0.95	2.22
time (sec)	N/A	0.325	0.02	1.144	1.488	1.32	2.891	1.117

Problem 1278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	38	2240	53	107	56	116
normalized size	1	1.	0.78	45.71	1.08	2.18	1.14	2.37
time (sec)	N/A	0.05	0.017	0.581	1.892	1.403	1.589	1.108

Problem 1279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	1913	57	113	39	124
normalized size	1	1.	1.	50.34	1.5	2.97	1.03	3.26
time (sec)	N/A	0.106	0.008	0.582	1.445	1.433	0.942	1.091

Problem 1280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	189	189	0	5237	0	0	0	0
normalized size	1	1.	0.	27.71	0.	0.	0.	0.
time (sec)	N/A	0.181	2.642	0.974	0.	0.	0.	0.

Problem 1281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	0	78	0	37	0
normalized size	1	1.	1.	0.	1.9	0.	0.9	0.
time (sec)	N/A	0.125	0.01	1.354	1.48	0.	87.837	0.

Problem 1282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	49	0	95	0	0	0
normalized size	1	1.	0.71	0.	1.38	0.	0.	0.
time (sec)	N/A	0.075	0.029	2.244	1.639	0.	0.	0.

Problem 1283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	81	0	128	0	97	0
normalized size	1	1.	1.	0.	1.58	0.	1.2	0.
time (sec)	N/A	0.21	0.013	1.478	1.65	0.	35.9	0.

Problem 1284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	98	0	120	0	0	0
normalized size	1	1.	0.96	0.	1.18	0.	0.	0.
time (sec)	N/A	0.129	0.035	3.61	1.688	0.	0.	0.

Problem 1285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	114	0	155	0	134	0
normalized size	1	1.	1.	0.	1.36	0.	1.18	0.
time (sec)	N/A	0.279	0.024	1.354	1.5	0.	85.702	0.

Problem 1286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	278	278	214	4941	346	541	338	618
normalized size	1	1.	0.77	17.77	1.24	1.95	1.22	2.22
time (sec)	N/A	0.693	0.173	2.08	1.506	1.377	35.491	1.655

Problem 1287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	221	164	3897	302	405	279	479
normalized size	1	1.	0.74	17.63	1.37	1.83	1.26	2.17
time (sec)	N/A	0.244	0.152	1.831	1.492	1.386	22.374	1.549

Problem 1288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	213	171	4146	286	406	258	486
normalized size	1	1.	0.8	19.46	1.34	1.91	1.21	2.28
time (sec)	N/A	0.569	0.131	1.43	1.502	1.295	12.912	1.338

Problem 1289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	105	3074	201	262	202	398
normalized size	1	1.	0.77	22.44	1.47	1.91	1.47	2.91
time (sec)	N/A	0.111	0.089	0.785	1.45	1.449	6.082	1.315

Problem 1290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	138	192	207	263	148	354
normalized size	1	1.	1.38	1.92	2.07	2.63	1.48	3.54
time (sec)	N/A	0.189	0.016	0.156	1.493	1.44	2.977	1.197

Problem 1291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	282	282	0	6931	0	0	0	0
normalized size	1	1.	0.	24.58	0.	0.	0.	0.
time (sec)	N/A	0.342	0.204	2.036	0.	0.	0.	0.

Problem 1292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	92	113	0	0	0	160	0
normalized size	1	0.92	1.13	0.	0.	0.	1.6	0.
time (sec)	N/A	0.25	0.09	3.487	0.	0.	140.782	0.

Problem 1293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	189	0	0	0	0	0
normalized size	1	1.	1.23	0.	0.	0.	0.	0.
time (sec)	N/A	0.14	0.118	11.976	0.	0.	0.	0.

Problem 1294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	186	181	0	0	0	428	0
normalized size	1	0.98	0.96	0.	0.	0.	2.26	0.
time (sec)	N/A	0.428	0.14	6.51	0.	0.	53.457	0.

Problem 1295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	225	260	0	0	0	0	0
normalized size	1	1.	1.16	0.	0.	0.	0.	0.
time (sec)	N/A	0.256	0.175	9.67	0.	0.	0.	0.

Problem 1296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	248	245	259	0	0	0	474	0
normalized size	1	0.99	1.04	0.	0.	0.	1.91	0.
time (sec)	N/A	0.625	0.247	14.653	0.	0.	87.364	0.

Problem 1297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	562	562	1138	21442	0	0	0	0
normalized size	1	1.	2.02	38.15	0.	0.	0.	0.
time (sec)	N/A	0.712	6.824	3.667	0.	0.	0.	0.

Problem 1298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	656	656	1362	0	0	0	0	0
normalized size	1	1.	2.08	0.	0.	0.	0.	0.
time (sec)	N/A	0.831	3.485	3.373	0.	0.	0.	0.

Problem 1299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	101	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.281	0.193	1.146	0.	0.	0.	0.

Problem 1300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	672	672	552	0	0	0	0	0
normalized size	1	1.	0.82	0.	0.	0.	0.	0.
time (sec)	N/A	0.757	0.903	1.717	0.	0.	0.	0.

Problem 1301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	528	528	1213	0	0	0	0	0
normalized size	1	1.	2.3	0.	0.	0.	0.	0.
time (sec)	N/A	0.769	5.674	4.83	0.	0.	0.	0.

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [1283] had the largest ratio of [1.25]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	7	7	1.	21	0.333
2	A	7	7	1.	21	0.333
3	A	7	7	1.	19	0.368
4	A	4	3	1.	18	0.167
5	A	8	5	1.	21	0.238
6	A	10	8	1.	21	0.381
7	A	4	4	1.	21	0.19
8	A	4	4	1.	21	0.19
9	A	4	4	1.	21	0.19
10	A	7	7	1.	23	0.304
11	A	7	7	1.	23	0.304
12	A	7	7	1.	21	0.333
13	A	4	3	1.	20	0.15
14	A	11	8	1.	23	0.348
15	A	13	10	1.	23	0.435

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
16	A	13	10	1.	23	0.435
17	A	4	4	1.	23	0.174
18	A	4	4	1.	23	0.174
19	A	4	4	1.	23	0.174
20	A	7	7	1.	23	0.304
21	A	7	7	1.	23	0.304
22	A	4	4	1.	21	0.19
23	A	4	3	1.	20	0.15
24	A	15	10	1.	23	0.435
25	A	16	12	1.	23	0.522
26	A	16	12	1.	23	0.522
27	A	17	11	1.	23	0.478
28	A	4	4	1.	23	0.174
29	A	4	5	1.	23	0.217
30	A	4	4	1.	23	0.174
31	A	7	7	1.	23	0.304
32	A	4	4	1.	23	0.174
33	A	4	4	1.	21	0.19
34	A	4	3	1.	20	0.15
35	A	19	11	1.	23	0.478
36	A	20	13	1.	23	0.565
37	A	19	13	1.	23	0.565
38	A	20	13	1.	23	0.565
39	A	21	11	1.	23	0.478
40	A	4	4	1.	23	0.174
41	A	4	5	1.	23	0.217
42	A	4	4	1.	23	0.174
43	A	16	11	1.	23	0.478
44	A	11	9	1.	23	0.391
45	A	7	6	1.	21	0.286
46	A	3	3	1.	20	0.15
47	A	2	2	1.	23	0.087

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
48	A	8	8	1.	23	0.348
49	A	12	10	1.	23	0.435
50	A	17	11	1.	23	0.478
51	A	16	12	1.	23	0.522
52	A	13	10	1.	23	0.435
53	A	10	8	1.	21	0.381
54	A	5	4	1.	20	0.2
55	A	13	10	1.	23	0.435
56	A	18	15	1.	23	0.652
57	A	21	16	1.	23	0.696
58	A	21	12	1.	23	0.522
59	A	18	10	1.	23	0.435
60	A	15	8	1.	23	0.348
61	A	5	5	1.	21	0.238
62	A	5	4	1.	20	0.2
63	A	18	10	1.	23	0.435
64	A	23	15	1.	23	0.652
65	A	26	16	1.	23	0.696
66	A	5	4	1.	19	0.21
67	A	3	3	1.	20	0.15
68	A	27	15	1.	23	0.652
69	A	22	14	1.	23	0.609
70	A	17	12	1.	21	0.571
71	A	9	7	1.	20	0.35
72	A	13	11	1.	23	0.478
73	A	12	10	1.	23	0.435
74	A	14	11	1.	23	0.478
75	A	18	13	1.	23	0.565
76	A	43	15	1.	25	0.6
77	A	36	15	1.	25	0.6
78	A	28	14	1.	23	0.609
79	A	12	10	1.	22	0.454

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
80	A	19	14	1.	25	0.56
81	A	17	15	1.	25	0.6
82	A	20	15	1.	25	0.6
83	A	16	14	1.	25	0.56
84	A	62	15	1.	25	0.6
85	A	52	15	1.	25	0.6
86	A	38	14	1.	23	0.609
87	A	16	12	1.	22	0.546
88	A	28	16	1.	25	0.64
89	A	23	17	1.	25	0.68
90	A	25	20	1.	25	0.8
91	A	28	17	1.	25	0.68
92	A	20	15	1.	25	0.6
93	A	24	16	1.	25	0.64
94	A	31	15	1.	25	0.6
95	A	26	14	1.	25	0.56
96	A	16	12	1.	25	0.48
97	A	9	9	1.	23	0.391
98	A	3	4	1.	22	0.182
99	A	3	4	1.	25	0.16
100	A	8	8	1.	25	0.32
101	A	17	13	1.	25	0.52
102	A	26	15	1.	25	0.6
103	A	33	18	1.	25	0.72
104	A	24	17	1.	25	0.68
105	A	18	14	1.	25	0.56
106	A	13	10	1.	23	0.435
107	A	8	6	1.	22	0.273
108	A	19	12	1.	25	0.48
109	A	23	16	1.	25	0.64
110	A	31	21	1.	25	0.84
111	A	37	17	1.	25	0.68

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
112	A	31	14	1.	25	0.56
113	A	26	10	1.	25	0.4
114	A	13	7	1.	23	0.304
115	A	13	6	1.	22	0.273
116	A	32	12	1.	25	0.48
117	A	36	16	1.	25	0.64
118	A	18	6	1.	21	0.286
119	A	4	5	1.	22	0.227
120	A	26	15	1.	22	0.682
121	A	17	13	1.	22	0.591
122	A	11	10	1.	20	0.5
123	A	4	5	1.	22	0.227
124	A	11	6	1.	22	0.273
125	A	24	6	1.	22	0.273
126	A	42	6	1.	22	0.273
127	A	19	12	1.	25	0.48
128	A	10	8	1.	23	0.348
129	A	4	5	1.	22	0.227
130	A	4	5	1.	25	0.2
131	A	10	9	1.	25	0.36
132	A	18	11	1.	25	0.44
133	A	0	0	0.	0	0.
134	A	16	12	1.	19	0.632
135	A	12	10	1.	19	0.526
136	A	9	7	1.	17	0.412
137	A	4	4	1.	16	0.25
138	A	9	7	1.	19	0.368
139	A	14	12	1.	19	0.632
140	A	17	14	1.	19	0.737
141	A	23	13	1.	21	0.619
142	A	14	11	1.	21	0.524
143	A	8	7	1.	19	0.368

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
144	A	1	1	1.	18	0.056
145	A	9	7	1.	21	0.333
146	A	13	11	1.	21	0.524
147	A	21	16	1.	21	0.762
148	A	0	0	0.	0	0.
149	A	9	4	1.	18	0.222
150	A	9	4	1.	18	0.222
151	A	2	1	1.	16	0.062
152	A	3	3	1.3	15	0.2
153	A	7	6	1.	18	0.333
154	A	8	8	1.	18	0.444
155	A	7	6	1.	18	0.333
156	A	10	7	1.	18	0.389
157	A	14	4	1.	20	0.2
158	A	14	4	1.	20	0.2
159	A	3	2	1.	18	0.111
160	A	4	3	1.	17	0.176
161	A	12	7	1.	20	0.35
162	A	13	9	1.	20	0.45
163	A	11	7	1.	20	0.35
164	A	13	9	1.	20	0.45
165	A	18	4	1.	20	0.2
166	A	18	4	1.	20	0.2
167	A	3	2	1.	18	0.111
168	A	5	3	1.	17	0.176
169	A	16	7	1.	20	0.35
170	A	17	9	1.	20	0.45
171	A	15	8	1.	20	0.4
172	A	17	10	1.	20	0.5
173	A	9	7	1.	20	0.35
174	A	8	8	1.	20	0.4
175	A	4	4	1.	20	0.2

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
176	A	4	4	1.	18	0.222
177	A	1	1	1.	17	0.059
178	A	3	3	1.	20	0.15
179	A	7	7	1.	20	0.35
180	A	7	7	1.	20	0.35
181	A	12	8	1.	20	0.4
182	A	17	12	1.	20	0.6
183	A	7	6	1.	20	0.3
184	A	8	8	1.	20	0.4
185	A	2	2	1.	20	0.1
186	A	3	3	1.	18	0.167
187	A	2	2	1.	17	0.118
188	A	7	7	1.	20	0.35
189	A	10	10	1.	20	0.5
190	A	15	11	1.	20	0.55
191	A	23	11	1.	20	0.55
192	A	4	3	1.	20	0.15
193	A	3	3	1.	20	0.15
194	A	4	3	1.	18	0.167
195	A	3	3	1.	17	0.176
196	A	12	7	1.	20	0.35
197	A	14	11	1.	20	0.55
198	A	28	11	1.	20	0.55
199	A	38	12	1.	20	0.6
200	A	12	6	1.	22	0.273
201	A	8	7	1.	22	0.318
202	A	4	4	1.	20	0.2
203	A	3	3	1.	19	0.158
204	A	5	5	1.	22	0.227
205	A	7	7	1.	22	0.318
206	A	6	5	1.	22	0.227
207	A	5	5	1.	22	0.227

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
208	A	31	7	1.	22	0.318
209	A	21	8	1.	22	0.364
210	A	5	4	1.	20	0.2
211	A	4	3	1.	19	0.158
212	A	10	8	1.	22	0.364
213	A	11	8	1.	22	0.364
214	A	12	8	1.	22	0.364
215	A	13	8	1.	22	0.364
216	A	76	7	1.	22	0.318
217	A	51	8	1.	22	0.364
218	A	6	4	1.	20	0.2
219	A	5	3	1.	19	0.158
220	A	16	8	1.	22	0.364
221	A	16	8	1.	22	0.364
222	A	23	10	1.	22	0.454
223	A	25	9	1.	22	0.409
224	A	7	5	1.	22	0.227
225	A	4	4	1.	22	0.182
226	A	3	3	1.	20	0.15
227	A	2	2	1.	19	0.105
228	A	2	2	1.	22	0.091
229	A	4	4	1.	22	0.182
230	A	4	4	1.	22	0.182
231	A	9	6	1.	22	0.273
232	A	6	5	1.	22	0.227
233	A	3	3	1.	22	0.136
234	A	2	2	1.	20	0.1
235	A	1	1	1.	19	0.053
236	A	5	5	1.	22	0.227
237	A	6	6	1.	22	0.273
238	A	10	7	1.	22	0.318
239	A	16	8	1.	22	0.364

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
240	A	10	6	1.	22	0.273
241	A	8	7	1.	22	0.318
242	A	3	3	1.	22	0.136
243	A	4	3	1.	22	0.136
244	A	3	3	1.	20	0.15
245	A	2	2	1.	19	0.105
246	A	9	6	1.	22	0.273
247	A	9	7	1.	22	0.318
248	A	10	3	1.	20	0.15
249	A	8	3	1.	20	0.15
250	A	5	3	1.	18	0.167
251	A	0	0	0.	0	0.
252	A	0	0	0.	0	0.
253	A	0	0	0.	0	0.
254	A	0	0	0.	0	0.
255	A	0	0	0.	0	0.
256	A	0	0	0.	0	0.
257	A	0	0	0.	0	0.
258	A	26	8	1.	20	0.4
259	A	24	10	1.	20	0.5
260	A	4	4	1.	18	0.222
261	A	7	7	1.	17	0.412
262	A	12	10	1.	20	0.5
263	A	10	10	1.	20	0.5
264	A	15	12	1.	20	0.6
265	A	13	8	1.	20	0.4
266	A	47	8	1.	22	0.364
267	A	44	10	1.	22	0.454
268	A	5	4	1.	20	0.2
269	A	9	7	1.	19	0.368
270	A	23	12	1.	22	0.546
271	A	20	13	1.	22	0.591

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
272	A	21	15	1.	22	0.682
273	A	19	13	1.	22	0.591
274	A	72	8	1.	22	0.364
275	A	68	10	1.	22	0.454
276	A	6	4	1.	20	0.2
277	A	12	8	1.	19	0.421
278	A	38	12	1.	22	0.546
279	A	34	14	1.	22	0.636
280	A	31	16	1.	22	0.727
281	A	28	15	1.	22	0.682
282	A	17	10	1.	22	0.454
283	A	10	9	1.	22	0.409
284	A	7	7	1.	22	0.318
285	A	4	5	1.	20	0.25
286	A	1	1	1.	19	0.053
287	A	4	5	1.	22	0.227
288	A	6	6	1.	22	0.273
289	A	13	11	1.	22	0.5
290	A	15	8	1.	22	0.364
291	A	8	9	1.	22	0.409
292	A	4	4	1.	22	0.182
293	A	3	3	1.	20	0.15
294	A	4	4	1.	19	0.21
295	A	8	9	1.	22	0.409
296	A	11	11	1.	22	0.5
297	A	22	15	1.	22	0.682
298	A	27	13	1.	22	0.591
299	A	4	4	1.	22	0.182
300	A	13	6	1.	22	0.273
301	A	4	4	1.	20	0.2
302	A	8	5	1.	19	0.263
303	A	13	10	1.	22	0.454

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
304	A	20	12	1.	22	0.546
305	A	36	16	1.	22	0.727
306	A	48	14	1.	22	0.636
307	A	26	8	1.	24	0.333
308	A	35	12	1.	24	0.5
309	A	4	4	1.	22	0.182
310	A	12	9	1.	21	0.429
311	A	13	10	1.	24	0.417
312	A	13	10	1.	24	0.417
313	A	24	12	1.	24	0.5
314	A	7	6	1.	24	0.25
315	A	75	8	1.	24	0.333
316	A	92	12	1.	24	0.5
317	A	5	4	1.	22	0.182
318	A	16	10	1.	21	0.476
319	A	18	11	1.	24	0.458
320	A	26	13	1.	24	0.542
321	A	38	15	1.	24	0.625
322	A	21	13	1.	24	0.542
323	A	203	8	1.	24	0.333
324	A	238	12	1.	24	0.5
325	A	6	4	1.	22	0.182
326	A	21	10	1.	21	0.476
327	A	24	11	1.	24	0.458
328	A	43	14	1.	24	0.583
329	A	57	16	1.	24	0.667
330	A	48	16	1.	24	0.667
331	A	8	5	1.	24	0.208
332	A	13	10	1.	24	0.417
333	A	3	3	1.	22	0.136
334	A	9	6	1.	21	0.286
335	A	9	6	1.	24	0.25

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
336	A	3	3	1.	24	0.125
337	A	14	11	1.	24	0.458
338	A	8	5	1.	24	0.208
339	A	6	5	1.	24	0.208
340	A	12	9	1.	24	0.375
341	A	2	2	1.	22	0.091
342	A	2	2	1.	21	0.095
343	A	12	9	1.	24	0.375
344	A	6	6	1.	24	0.25
345	A	27	14	1.	24	0.583
346	A	15	8	1.	24	0.333
347	A	13	8	1.	24	0.333
348	A	17	12	1.	24	0.5
349	A	6	5	1.	24	0.208
350	A	4	4	1.	24	0.167
351	A	3	3	1.	22	0.136
352	A	5	4	1.	21	0.19
353	A	16	10	1.	24	0.417
354	A	12	8	1.	24	0.333
355	A	0	0	0.	0	0.
356	A	0	0	0.	0	0.
357	A	0	0	0.	0	0.
358	A	0	0	0.	0	0.
359	A	0	0	0.	0	0.
360	A	0	0	0.	0	0.
361	A	0	0	0.	0	0.
362	A	0	0	0.	0	0.
363	A	52	12	1.	20	0.6
364	A	34	12	1.	20	0.6
365	A	8	8	1.	18	0.444
366	A	8	8	1.	17	0.471
367	A	17	14	1.	20	0.7

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
368	A	11	11	1.	20	0.55
369	A	16	12	1.	20	0.6
370	A	20	12	1.	20	0.6
371	A	106	12	1.	22	0.546
372	A	73	12	1.	22	0.546
373	A	10	8	1.	20	0.4
374	A	12	9	1.	19	0.474
375	A	36	16	1.	22	0.727
376	A	23	13	1.	22	0.591
377	A	25	18	1.	22	0.818
378	A	26	16	1.	22	0.727
379	A	184	12	1.	22	0.546
380	A	132	12	1.	22	0.546
381	A	13	9	1.	20	0.45
382	A	17	9	1.	19	0.474
383	A	69	17	1.	22	0.773
384	A	45	15	1.	22	0.682
385	A	43	20	1.	22	0.909
386	A	37	18	1.	22	0.818
387	A	19	9	1.	22	0.409
388	A	14	11	1.	22	0.5
389	A	7	7	1.	22	0.318
390	A	5	6	1.	20	0.3
391	A	1	1	1.	19	0.053
392	A	5	6	1.	22	0.273
393	A	7	7	1.	22	0.318
394	A	13	9	1.	22	0.409
395	A	22	11	1.	22	0.5
396	A	11	11	1.	22	0.5
397	A	4	4	1.	22	0.182
398	A	5	4	1.	20	0.2
399	A	4	3	1.	19	0.158

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
400	A	11	11	1.	22	0.5
401	A	12	11	1.	22	0.5
402	A	25	14	1.	22	0.636
403	A	35	15	1.	22	0.682
404	A	9	7	1.	22	0.318
405	A	13	6	1.	22	0.273
406	A	9	5	1.	20	0.25
407	A	8	5	1.	19	0.263
408	A	21	12	1.	22	0.546
409	A	21	13	1.	22	0.591
410	A	47	15	1.	22	0.682
411	A	57	17	1.	22	0.773
412	A	71	12	1.	24	0.5
413	A	40	12	1.	24	0.5
414	A	13	10	1.	22	0.454
415	A	14	9	1.	21	0.429
416	A	22	12	1.	24	0.5
417	A	22	12	1.	24	0.5
418	A	27	11	1.	24	0.458
419	A	25	12	1.	24	0.5
420	A	200	12	1.	24	0.5
421	A	108	14	1.	24	0.583
422	A	17	11	1.	22	0.5
423	A	18	10	1.	21	0.476
424	A	36	15	1.	24	0.625
425	A	37	14	1.	24	0.583
426	A	50	15	1.	24	0.625
427	A	48	16	1.	24	0.667
428	A	547	12	1.	24	0.5
429	A	293	14	1.	24	0.583
430	A	22	11	1.	22	0.5
431	A	23	10	1.	21	0.476

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
432	A	54	16	1.	24	0.667
433	A	56	15	1.	24	0.625
434	A	87	18	1.	24	0.75
435	A	86	18	1.	24	0.75
436	A	24	10	1.	24	0.417
437	A	15	10	1.	24	0.417
438	A	10	7	1.	22	0.318
439	A	11	7	1.	21	0.333
440	A	11	7	1.	24	0.292
441	A	10	7	1.	24	0.292
442	A	15	10	1.	24	0.417
443	A	25	11	1.	24	0.458
444	A	14	10	1.	24	0.417
445	A	14	10	1.	24	0.417
446	A	3	3	1.	22	0.136
447	A	2	2	1.	21	0.095
448	A	15	11	1.	24	0.458
449	A	13	10	1.	24	0.417
450	A	22	12	1.	24	0.5
451	A	22	15	1.	24	0.625
452	A	7	5	1.	24	0.208
453	A	7	6	1.	24	0.25
454	A	6	5	1.	22	0.227
455	A	5	4	1.	21	0.19
456	A	22	13	1.	24	0.542
457	A	19	12	1.	24	0.5
458	A	0	0	0.	0	0.
459	A	0	0	0.	0	0.
460	A	0	0	0.	0	0.
461	A	0	0	0.	0	0.
462	A	0	0	0.	0	0.
463	A	0	0	0.	0	0.

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
464	A	0	0	0.	0	0.
465	A	0	0	0.	0	0.
466	A	0	0	0.	0	0.
467	A	0	0	0.	0	0.
468	A	0	0	0.	0	0.
469	A	0	0	0.	0	0.
470	A	0	0	0.	0	0.
471	A	0	0	0.	0	0.
472	A	0	0	0.	0	0.
473	A	0	0	0.	0	0.
474	A	0	0	0.	0	0.
475	A	0	0	0.	0	0.
476	A	0	0	0.	0	0.
477	A	1	1	1.	19	0.053
478	A	0	0	0.	0	0.
479	A	0	0	0.	0	0.
480	A	0	0	0.	0	0.
481	A	0	0	0.	0	0.
482	A	4	3	1.	22	0.136
483	A	4	4	1.	20	0.2
484	A	4	3	1.	19	0.158
485	A	0	0	0.	0	0.
486	A	0	0	0.	0	0.
487	A	0	0	0.	0	0.
488	A	0	0	0.	0	0.
489	A	5	3	1.	22	0.136
490	A	5	3	1.	22	0.136
491	A	4	3	1.	22	0.136
492	A	5	3	1.	20	0.15
493	A	5	3	1.	19	0.158
494	A	0	0	0.	0	0.
495	A	0	0	0.	0	0.

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
496	A	0	0	0.	0	0.
497	A	0	0	0.	0	0.
498	A	0	0	0.	0	0.
499	A	0	0	0.	0	0.
500	A	0	0	0.	0	0.
501	A	0	0	0.	0	0.
502	A	0	0	0.	0	0.
503	A	0	0	0.	0	0.
504	A	0	0	0.	0	0.
505	A	0	0	0.	0	0.
506	A	0	0	0.	0	0.
507	A	0	0	0.	0	0.
508	A	0	0	0.	0	0.
509	A	0	0	0.	0	0.
510	A	3	3	1.	22	0.136
511	A	3	3	1.	21	0.143
512	A	0	0	0.	0	0.
513	A	0	0	0.	0	0.
514	A	0	0	0.	0	0.
515	A	0	0	0.	0	0.
516	A	6	4	1.	24	0.167
517	A	6	4	1.	24	0.167
518	A	6	4	1.	22	0.182
519	A	6	4	1.	21	0.19
520	A	0	0	0.	0	0.
521	A	0	0	0.	0	0.
522	A	0	0	0.	0	0.
523	A	0	0	0.	0	0.
524	A	0	0	0.	0	0.
525	A	0	0	0.	0	0.
526	A	0	0	0.	0	0.
527	A	0	0	0.	0	0.

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
528	A	0	0	0.	0	0.
529	A	0	0	0.	0	0.
530	A	0	0	0.	0	0.
531	A	0	0	0.	0	0.
532	A	0	0	0.	0	0.
533	A	0	0	0.	0	0.
534	A	0	0	0.	0	0.
535	A	0	0	0.	0	0.
536	A	0	0	0.	0	0.
537	A	0	0	0.	0	0.
538	A	0	0	0.	0	0.
539	A	0	0	0.	0	0.
540	A	0	0	0.	0	0.
541	A	0	0	0.	0	0.
542	A	0	0	0.	0	0.
543	A	0	0	0.	0	0.
544	A	0	0	0.	0	0.
545	A	0	0	0.	0	0.
546	A	1	1	1.	19	0.053
547	A	0	0	0.	0	0.
548	A	0	0	0.	0	0.
549	A	0	0	0.	0	0.
550	A	0	0	0.	0	0.
551	A	0	0	0.	0	0.
552	A	5	5	1.	22	0.227
553	A	9	5	1.	20	0.25
554	A	5	5	1.	19	0.263
555	A	0	0	0.	0	0.
556	A	0	0	0.	0	0.
557	A	0	0	0.	0	0.
558	A	0	0	0.	0	0.
559	A	20	7	1.	22	0.318

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
560	A	12	6	1.	22	0.273
561	A	10	6	1.	20	0.3
562	A	6	4	1.	19	0.21
563	A	0	0	0.	0	0.
564	A	0	0	0.	0	0.
565	A	0	0	0.	0	0.
566	A	0	0	0.	0	0.
567	A	0	0	0.	0	0.
568	A	0	0	0.	0	0.
569	A	0	0	0.	0	0.
570	A	0	0	0.	0	0.
571	A	0	0	0.	0	0.
572	A	0	0	0.	0	0.
573	A	0	0	0.	0	0.
574	A	0	0	0.	0	0.
575	A	0	0	0.	0	0.
576	A	0	0	0.	0	0.
577	A	0	0	0.	0	0.
578	A	0	0	0.	0	0.
579	A	0	0	0.	0	0.
580	A	0	0	0.	0	0.
581	A	4	4	1.	22	0.182
582	A	4	4	1.	21	0.19
583	A	0	0	0.	0	0.
584	A	0	0	0.	0	0.
585	A	0	0	0.	0	0.
586	A	0	0	0.	0	0.
587	A	0	0	0.	0	0.
588	A	0	0	0.	0	0.
589	A	7	5	1.	24	0.208
590	A	12	6	1.	24	0.25
591	A	13	8	1.	22	0.364

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
592	A	7	5	1.	21	0.238
593	A	0	0	0.	0	0.
594	A	0	0	0.	0	0.
595	A	0	0	0.	0	0.
596	A	0	0	0.	0	0.
597	A	0	0	0.	0	0.
598	A	0	0	0.	0	0.
599	A	0	0	0.	0	0.
600	A	0	0	0.	0	0.
601	A	0	0	0.	0	0.
602	A	0	0	0.	0	0.
603	A	0	0	0.	0	0.
604	A	0	0	0.	0	0.
605	A	0	0	0.	0	0.
606	A	0	0	0.	0	0.
607	A	0	0	0.	0	0.
608	A	0	0	0.	0	0.
609	A	0	0	0.	0	0.
610	A	0	0	0.	0	0.
611	A	0	0	0.	0	0.
612	A	0	0	0.	0	0.
613	A	0	0	0.	0	0.
614	A	0	0	0.	0	0.
615	A	0	0	0.	0	0.
616	A	0	0	0.	0	0.
617	A	0	0	0.	0	0.
618	A	0	0	0.	0	0.
619	A	0	0	0.	0	0.
620	A	0	0	0.	0	0.
621	A	0	0	0.	0	0.
622	A	1	1	1.	19	0.053
623	A	0	0	0.	0	0.

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
624	A	0	0	0.	0	0.
625	A	0	0	0.	0	0.
626	A	0	0	0.	0	0.
627	A	0	0	0.	0	0.
628	A	10	6	1.	22	0.273
629	A	5	5	1.	20	0.25
630	A	10	6	1.	19	0.316
631	A	0	0	0.	0	0.
632	A	0	0	0.	0	0.
633	A	0	0	0.	0	0.
634	A	0	0	0.	0	0.
635	A	25	8	1.	22	0.364
636	A	22	8	1.	22	0.364
637	A	19	7	1.	20	0.35
638	A	11	7	1.	19	0.368
639	A	0	0	0.	0	0.
640	A	0	0	0.	0	0.
641	A	0	0	0.	0	0.
642	A	0	0	0.	0	0.
643	A	2	1	1.	38	0.026
644	A	0	0	0.	0	0.
645	A	0	0	0.	0	0.
646	A	0	0	0.	0	0.
647	A	0	0	0.	0	0.
648	A	0	0	0.	0	0.
649	A	0	0	0.	0	0.
650	A	0	0	0.	0	0.
651	A	0	0	0.	0	0.
652	A	0	0	0.	0	0.
653	A	0	0	0.	0	0.
654	A	0	0	0.	0	0.
655	A	0	0	0.	0	0.

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
656	A	0	0	0.	0	0.
657	A	0	0	0.	0	0.
658	A	0	0	0.	0	0.
659	A	0	0	0.	0	0.
660	A	5	5	1.	22	0.227
661	A	5	5	1.	21	0.238
662	A	0	0	0.	0	0.
663	A	0	0	0.	0	0.
664	A	0	0	0.	0	0.
665	A	0	0	0.	0	0.
666	A	0	0	0.	0	0.
667	A	0	0	0.	0	0.
668	A	13	7	1.	24	0.292
669	A	20	11	1.	24	0.458
670	A	20	7	1.	22	0.318
671	A	14	9	1.	21	0.429
672	A	0	0	0.	0	0.
673	A	0	0	0.	0	0.
674	A	0	0	0.	0	0.
675	A	0	0	0.	0	0.
676	A	0	0	0.	0	0.
677	A	0	0	0.	0	0.
678	A	0	0	0.	0	0.
679	A	0	0	0.	0	0.
680	A	0	0	0.	0	0.
681	A	0	0	0.	0	0.
682	A	0	0	0.	0	0.
683	A	0	0	0.	0	0.
684	A	0	0	0.	0	0.
685	A	0	0	0.	0	0.
686	A	0	0	0.	0	0.
687	A	0	0	0.	0	0.

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
688	A	0	0	0.	0	0.
689	A	0	0	0.	0	0.
690	A	0	0	0.	0	0.
691	A	0	0	0.	0	0.
692	A	0	0	0.	0	0.
693	A	0	0	0.	0	0.
694	A	0	0	0.	0	0.
695	A	0	0	0.	0	0.
696	A	0	0	0.	0	0.
697	A	0	0	0.	0	0.
698	A	0	0	0.	0	0.
699	A	0	0	0.	0	0.
700	A	0	0	0.	0	0.
701	A	0	0	0.	0	0.
702	A	1	1	1.	21	0.048
703	A	0	0	0.	0	0.
704	A	0	0	0.	0	0.
705	A	0	0	0.	0	0.
706	A	0	0	0.	0	0.
707	A	0	0	0.	0	0.
708	A	0	0	0.	0	0.
709	A	6	6	1.	24	0.25
710	A	6	5	1.	22	0.227
711	A	6	6	1.	21	0.286
712	A	0	0	0.	0	0.
713	A	0	0	0.	0	0.
714	A	0	0	0.	0	0.
715	A	9	5	1.	24	0.208
716	A	8	5	1.	24	0.208
717	A	6	5	1.	24	0.208
718	A	8	5	1.	22	0.227
719	A	9	5	1.	21	0.238

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
720	A	0	0	0.	0	0.
721	A	0	0	0.	0	0.
722	A	0	0	0.	0	0.
723	A	0	0	0.	0	0.
724	A	0	0	0.	0	0.
725	A	0	0	0.	0	0.
726	A	0	0	0.	0	0.
727	A	0	0	0.	0	0.
728	A	0	0	0.	0	0.
729	A	0	0	0.	0	0.
730	A	0	0	0.	0	0.
731	A	0	0	0.	0	0.
732	A	0	0	0.	0	0.
733	A	0	0	0.	0	0.
734	A	0	0	0.	0	0.
735	A	0	0	0.	0	0.
736	A	0	0	0.	0	0.
737	A	0	0	0.	0	0.
738	A	0	0	0.	0	0.
739	A	0	0	0.	0	0.
740	A	0	0	0.	0	0.
741	A	0	0	0.	0	0.
742	A	0	0	0.	0	0.
743	A	0	0	0.	0	0.
744	A	0	0	0.	0	0.
745	A	5	5	1.	24	0.208
746	A	5	5	1.	23	0.217
747	A	0	0	0.	0	0.
748	A	0	0	0.	0	0.
749	A	0	0	0.	0	0.
750	A	0	0	0.	0	0.
751	A	10	6	1.	26	0.231

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
752	A	9	6	1.	26	0.231
753	A	9	6	1.	24	0.25
754	A	10	6	1.	23	0.261
755	A	0	0	0.	0	0.
756	A	0	0	0.	0	0.
757	A	0	0	0.	0	0.
758	A	0	0	0.	0	0.
759	A	0	0	0.	0	0.
760	A	0	0	0.	0	0.
761	A	0	0	0.	0	0.
762	A	0	0	0.	0	0.
763	A	0	0	0.	0	0.
764	A	0	0	0.	0	0.
765	A	0	0	0.	0	0.
766	A	0	0	0.	0	0.
767	A	0	0	0.	0	0.
768	A	0	0	0.	0	0.
769	A	0	0	0.	0	0.
770	A	0	0	0.	0	0.
771	A	0	0	0.	0	0.
772	A	0	0	0.	0	0.
773	A	0	0	0.	0	0.
774	A	0	0	0.	0	0.
775	A	0	0	0.	0	0.
776	A	0	0	0.	0	0.
777	A	0	0	0.	0	0.
778	A	1	1	1.	21	0.048
779	A	0	0	0.	0	0.
780	A	0	0	0.	0	0.
781	A	0	0	0.	0	0.
782	A	0	0	0.	0	0.
783	A	0	0	0.	0	0.

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
784	A	0	0	0.	0	0.
785	A	7	6	1.	24	0.25
786	A	7	7	1.	22	0.318
787	A	7	6	1.	21	0.286
788	A	0	0	0.	0	0.
789	A	0	0	0.	0	0.
790	A	0	0	0.	0	0.
791	A	15	8	1.	24	0.333
792	A	10	6	1.	24	0.25
793	A	7	5	1.	24	0.208
794	A	10	6	1.	22	0.273
795	A	15	7	1.	21	0.333
796	A	0	0	0.	0	0.
797	A	0	0	0.	0	0.
798	A	0	0	0.	0	0.
799	A	0	0	0.	0	0.
800	A	0	0	0.	0	0.
801	A	0	0	0.	0	0.
802	A	0	0	0.	0	0.
803	A	0	0	0.	0	0.
804	A	0	0	0.	0	0.
805	A	0	0	0.	0	0.
806	A	0	0	0.	0	0.
807	A	0	0	0.	0	0.
808	A	0	0	0.	0	0.
809	A	0	0	0.	0	0.
810	A	0	0	0.	0	0.
811	A	0	0	0.	0	0.
812	A	0	0	0.	0	0.
813	A	0	0	0.	0	0.
814	A	0	0	0.	0	0.
815	A	0	0	0.	0	0.

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
816	A	0	0	0.	0	0.
817	A	0	0	0.	0	0.
818	A	0	0	0.	0	0.
819	A	0	0	0.	0	0.
820	A	0	0	0.	0	0.
821	A	0	0	0.	0	0.
822	A	0	0	0.	0	0.
823	A	0	0	0.	0	0.
824	A	6	6	1.	24	0.25
825	A	5	5	1.	23	0.217
826	A	0	0	0.	0	0.
827	A	0	0	0.	0	0.
828	A	0	0	0.	0	0.
829	A	0	0	0.	0	0.
830	A	0	0	0.	0	0.
831	A	15	10	1.	26	0.385
832	A	11	7	1.	26	0.269
833	A	11	7	1.	24	0.292
834	A	14	7	1.	23	0.304
835	A	0	0	0.	0	0.
836	A	0	0	0.	0	0.
837	A	0	0	0.	0	0.
838	A	0	0	0.	0	0.
839	A	0	0	0.	0	0.
840	A	0	0	0.	0	0.
841	A	0	0	0.	0	0.
842	A	0	0	0.	0	0.
843	A	0	0	0.	0	0.
844	A	0	0	0.	0	0.
845	A	0	0	0.	0	0.
846	A	0	0	0.	0	0.
847	A	0	0	0.	0	0.

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
848	A	0	0	0.	0	0.
849	A	0	0	0.	0	0.
850	A	0	0	0.	0	0.
851	A	0	0	0.	0	0.
852	A	0	0	0.	0	0.
853	A	0	0	0.	0	0.
854	A	0	0	0.	0	0.
855	A	0	0	0.	0	0.
856	A	0	0	0.	0	0.
857	A	0	0	0.	0	0.
858	A	0	0	0.	0	0.
859	A	1	1	1.	21	0.048
860	A	0	0	0.	0	0.
861	A	0	0	0.	0	0.
862	A	0	0	0.	0	0.
863	A	0	0	0.	0	0.
864	A	0	0	0.	0	0.
865	A	0	0	0.	0	0.
866	A	8	8	1.	24	0.333
867	A	8	6	1.	22	0.273
868	A	8	7	1.	21	0.333
869	A	0	0	0.	0	0.
870	A	0	0	0.	0	0.
871	A	0	0	0.	0	0.
872	A	18	11	1.	24	0.458
873	A	16	9	1.	24	0.375
874	A	8	5	1.	24	0.208
875	A	16	7	1.	22	0.318
876	A	18	11	1.	21	0.524
877	A	0	0	0.	0	0.
878	A	0	0	0.	0	0.
879	A	0	0	0.	0	0.

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
880	A	0	0	0.	0	0.
881	A	0	0	0.	0	0.
882	A	0	0	0.	0	0.
883	A	0	0	0.	0	0.
884	A	0	0	0.	0	0.
885	A	0	0	0.	0	0.
886	A	0	0	0.	0	0.
887	A	0	0	0.	0	0.
888	A	0	0	0.	0	0.
889	A	0	0	0.	0	0.
890	A	0	0	0.	0	0.
891	A	0	0	0.	0	0.
892	A	0	0	0.	0	0.
893	A	0	0	0.	0	0.
894	A	0	0	0.	0	0.
895	A	0	0	0.	0	0.
896	A	0	0	0.	0	0.
897	A	0	0	0.	0	0.
898	A	0	0	0.	0	0.
899	A	0	0	0.	0	0.
900	A	0	0	0.	0	0.
901	A	0	0	0.	0	0.
902	A	0	0	0.	0	0.
903	A	0	0	0.	0	0.
904	A	6	6	1.	24	0.25
905	A	6	6	1.	23	0.261
906	A	0	0	0.	0	0.
907	A	0	0	0.	0	0.
908	A	0	0	0.	0	0.
909	A	17	11	1.	26	0.423
910	A	16	11	1.	26	0.423
911	A	15	8	1.	24	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
912	A	17	8	1.	23	0.348
913	A	0	0	0.	0	0.
914	A	0	0	0.	0	0.
915	A	0	0	0.	0	0.
916	A	0	0	0.	0	0.
917	A	0	0	0.	0	0.
918	A	0	0	0.	0	0.
919	A	0	0	0.	0	0.
920	A	0	0	0.	0	0.
921	A	0	0	0.	0	0.
922	A	0	0	0.	0	0.
923	A	0	0	0.	0	0.
924	A	0	0	0.	0	0.
925	A	0	0	0.	0	0.
926	A	0	0	0.	0	0.
927	A	0	0	0.	0	0.
928	A	1	1	1.	21	0.048
929	A	0	0	0.	0	0.
930	A	0	0	0.	0	0.
931	A	0	0	0.	0	0.
932	A	5	4	1.	24	0.167
933	A	5	5	1.	22	0.227
934	A	5	4	1.	21	0.19
935	A	0	0	0.	0	0.
936	A	0	0	0.	0	0.
937	A	0	0	0.	0	0.
938	A	7	4	1.	24	0.167
939	A	7	4	1.	24	0.167
940	A	5	4	1.	24	0.167
941	A	7	4	1.	22	0.182
942	A	7	4	1.	21	0.19
943	A	0	0	0.	0	0.

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
944	A	0	0	0.	0	0.
945	A	0	0	0.	0	0.
946	A	0	0	0.	0	0.
947	A	0	0	0.	0	0.
948	A	0	0	0.	0	0.
949	A	0	0	0.	0	0.
950	A	0	0	0.	0	0.
951	A	0	0	0.	0	0.
952	A	0	0	0.	0	0.
953	A	0	0	0.	0	0.
954	A	0	0	0.	0	0.
955	A	0	0	0.	0	0.
956	A	0	0	0.	0	0.
957	A	0	0	0.	0	0.
958	A	0	0	0.	0	0.
959	A	0	0	0.	0	0.
960	A	0	0	0.	0	0.
961	A	0	0	0.	0	0.
962	A	4	4	1.	24	0.167
963	A	4	4	1.	23	0.174
964	A	0	0	0.	0	0.
965	A	0	0	0.	0	0.
966	A	0	0	0.	0	0.
967	A	8	5	1.	26	0.192
968	A	8	5	1.	26	0.192
969	A	8	5	1.	24	0.208
970	A	8	5	1.	23	0.217
971	A	0	0	0.	0	0.
972	A	0	0	0.	0	0.
973	A	0	0	0.	0	0.
974	A	0	0	0.	0	0.
975	A	0	0	0.	0	0.

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
976	A	0	0	0.	0	0.
977	A	0	0	0.	0	0.
978	A	0	0	0.	0	0.
979	A	0	0	0.	0	0.
980	A	0	0	0.	0	0.
981	A	0	0	0.	0	0.
982	A	0	0	0.	0	0.
983	A	0	0	0.	0	0.
984	A	0	0	0.	0	0.
985	A	0	0	0.	0	0.
986	A	1	1	1.	21	0.048
987	A	0	0	0.	0	0.
988	A	0	0	0.	0	0.
989	A	0	0	0.	0	0.
990	A	0	0	0.	0	0.
991	A	6	6	1.	24	0.25
992	A	7	6	1.	22	0.273
993	A	6	6	1.	21	0.286
994	A	0	0	0.	0	0.
995	A	0	0	0.	0	0.
996	A	0	0	0.	0	0.
997	A	0	0	0.	0	0.
998	A	0	0	0.	0	0.
999	A	13	6	1.	24	0.25
1000	A	15	5	1.	24	0.208
1001	A	13	7	1.	22	0.318
1002	A	8	5	1.	21	0.238
1003	A	0	0	0.	0	0.
1004	A	0	0	0.	0	0.
1005	A	0	0	0.	0	0.
1006	A	0	0	0.	0	0.
1007	A	0	0	0.	0	0.

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1008	A	0	0	0.	0	0.
1009	A	0	0	0.	0	0.
1010	A	0	0	0.	0	0.
1011	A	0	0	0.	0	0.
1012	A	0	0	0.	0	0.
1013	A	0	0	0.	0	0.
1014	A	0	0	0.	0	0.
1015	A	0	0	0.	0	0.
1016	A	0	0	0.	0	0.
1017	A	0	0	0.	0	0.
1018	A	0	0	0.	0	0.
1019	A	0	0	0.	0	0.
1020	A	0	0	0.	0	0.
1021	A	0	0	0.	0	0.
1022	A	0	0	0.	0	0.
1023	A	0	0	0.	0	0.
1024	A	0	0	0.	0	0.
1025	A	0	0	0.	0	0.
1026	A	0	0	0.	0	0.
1027	A	5	5	1.	24	0.208
1028	A	5	5	1.	23	0.217
1029	A	0	0	0.	0	0.
1030	A	0	0	0.	0	0.
1031	A	0	0	0.	0	0.
1032	A	0	0	0.	0	0.
1033	A	0	0	0.	0	0.
1034	A	9	6	1.	26	0.231
1035	A	17	7	1.	26	0.269
1036	A	17	9	1.	24	0.375
1037	A	9	6	1.	23	0.261
1038	A	0	0	0.	0	0.
1039	A	0	0	0.	0	0.

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1040	A	0	0	0.	0	0.
1041	A	0	0	0.	0	0.
1042	A	0	0	0.	0	0.
1043	A	0	0	0.	0	0.
1044	A	0	0	0.	0	0.
1045	A	0	0	0.	0	0.
1046	A	0	0	0.	0	0.
1047	A	0	0	0.	0	0.
1048	A	0	0	0.	0	0.
1049	A	0	0	0.	0	0.
1050	A	0	0	0.	0	0.
1051	A	0	0	0.	0	0.
1052	A	0	0	0.	0	0.
1053	A	0	0	0.	0	0.
1054	A	0	0	0.	0	0.
1055	A	0	0	0.	0	0.
1056	A	1	1	1.	21	0.048
1057	A	0	0	0.	0	0.
1058	A	0	0	0.	0	0.
1059	A	0	0	0.	0	0.
1060	A	8	7	1.	24	0.292
1061	A	6	6	1.	22	0.273
1062	A	8	7	1.	21	0.333
1063	A	0	0	0.	0	0.
1064	A	0	0	0.	0	0.
1065	A	0	0	0.	0	0.
1066	A	0	0	0.	0	0.
1067	A	0	0	0.	0	0.
1068	A	24	6	1.	24	0.25
1069	A	27	7	1.	24	0.292
1070	A	24	6	1.	22	0.273
1071	A	14	8	1.	21	0.381

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1072	A	0	0	0.	0	0.
1073	A	0	0	0.	0	0.
1074	A	0	0	0.	0	0.
1075	A	0	0	0.	0	0.
1076	A	0	0	0.	0	0.
1077	A	0	0	0.	0	0.
1078	A	0	0	0.	0	0.
1079	A	0	0	0.	0	0.
1080	A	0	0	0.	0	0.
1081	A	0	0	0.	0	0.
1082	A	0	0	0.	0	0.
1083	A	0	0	0.	0	0.
1084	A	0	0	0.	0	0.
1085	A	0	0	0.	0	0.
1086	A	0	0	0.	0	0.
1087	A	0	0	0.	0	0.
1088	A	0	0	0.	0	0.
1089	A	0	0	0.	0	0.
1090	A	0	0	0.	0	0.
1091	A	0	0	0.	0	0.
1092	A	0	0	0.	0	0.
1093	A	0	0	0.	0	0.
1094	A	0	0	0.	0	0.
1095	A	0	0	0.	0	0.
1096	A	6	6	1.	24	0.25
1097	A	6	6	1.	23	0.261
1098	A	0	0	0.	0	0.
1099	A	0	0	0.	0	0.
1100	A	0	0	0.	0	0.
1101	A	0	0	0.	0	0.
1102	A	0	0	0.	0	0.
1103	A	18	8	1.	26	0.308

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1104	A	27	10	1.	26	0.385
1105	A	27	8	1.	24	0.333
1106	A	18	10	1.	23	0.435
1107	A	0	0	0.	0	0.
1108	A	0	0	0.	0	0.
1109	A	0	0	0.	0	0.
1110	A	0	0	0.	0	0.
1111	A	0	0	0.	0	0.
1112	A	1	1	1.	19	0.053
1113	A	0	0	0.	0	0.
1114	A	5	5	1.	19	0.263
1115	A	4	4	1.	19	0.21
1116	A	4	3	1.	17	0.176
1117	A	5	4	1.	16	0.25
1118	A	8	6	1.	19	0.316
1119	A	4	4	1.	19	0.21
1120	A	8	6	1.	19	0.316
1121	A	5	5	1.	19	0.263
1122	A	5	6	1.	19	0.316
1123	A	5	5	1.	19	0.263
1124	A	6	6	1.	19	0.316
1125	A	4	5	1.	21	0.238
1126	A	5	5	1.	21	0.238
1127	A	4	3	1.	19	0.158
1128	A	5	5	1.	18	0.278
1129	A	12	7	1.	21	0.333
1130	A	4	4	1.	21	0.19
1131	A	11	7	1.	21	0.333
1132	A	5	5	1.	21	0.238
1133	A	12	6	1.	21	0.286
1134	A	5	5	1.	21	0.238
1135	A	5	5	1.	21	0.238

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1136	A	5	5	1.	21	0.238
1137	A	8	7	1.19	21	0.333
1138	A	5	5	1.	21	0.238
1139	A	4	3	1.	19	0.158
1140	A	4	4	1.	18	0.222
1141	A	16	7	1.	21	0.333
1142	A	4	4	1.	21	0.19
1143	A	15	8	1.	21	0.381
1144	A	5	5	1.	21	0.238
1145	A	15	7	1.	21	0.333
1146	A	5	5	1.	21	0.238
1147	A	17	6	1.	21	0.286
1148	A	5	5	1.	21	0.238
1149	A	5	5	1.	21	0.238
1150	A	4	4	1.	14	0.286
1151	A	14	9	1.	21	0.429
1152	A	10	5	1.	19	0.263
1153	A	15	8	1.	21	0.381
1154	A	19	12	1.	21	0.571
1155	A	23	10	1.	21	0.476
1156	A	19	7	1.	18	0.389
1157	A	25	13	1.	21	0.619
1158	A	16	9	1.	21	0.429
1159	A	4	4	1.	19	0.21
1160	A	19	11	1.	21	0.524
1161	A	22	13	1.	21	0.619
1162	A	45	14	1.	21	0.667
1163	A	24	12	1.	18	0.667
1164	A	50	17	1.	21	0.81
1165	A	21	11	1.	21	0.524
1166	A	6	6	1.	21	0.286
1167	A	5	5	1.	19	0.263

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1168	A	24	13	1.	21	0.619
1169	A	27	15	1.	21	0.714
1170	A	49	15	1.	21	0.714
1171	A	23	11	1.	18	0.611
1172	A	73	19	1.	21	0.905
1173	A	9	10	1.	23	0.435
1174	A	0	0	0.	0	0.
1175	A	7	7	1.	21	0.333
1176	A	0	0	0.	0	0.
1177	A	0	0	0.	0	0.
1178	A	0	0	0.	0	0.
1179	A	0	0	0.	0	0.
1180	A	9	8	1.	23	0.348
1181	A	0	0	0.	0	0.
1182	A	10	9	1.	23	0.391
1183	A	10	10	1.	23	0.435
1184	A	0	0	0.	0	0.
1185	A	8	8	1.	21	0.381
1186	A	0	0	0.	0	0.
1187	A	0	0	0.	0	0.
1188	A	0	0	0.	0	0.
1189	A	0	0	0.	0	0.
1190	A	0	0	0.	0	0.
1191	A	0	0	0.	0	0.
1192	A	10	9	1.	23	0.391
1193	A	11	10	1.	23	0.435
1194	A	0	0	0.	0	0.
1195	A	9	8	1.	21	0.381
1196	A	0	0	0.	0	0.
1197	A	0	0	0.	0	0.
1198	A	0	0	0.	0	0.
1199	A	0	0	0.	0	0.

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1200	A	0	0	0.	0	0.
1201	A	8	10	1.	23	0.435
1202	A	0	0	0.	0	0.
1203	A	6	6	1.	21	0.286
1204	A	0	0	0.	0	0.
1205	A	0	0	0.	0	0.
1206	A	7	6	1.	23	0.261
1207	A	0	0	0.	0	0.
1208	A	9	9	1.	23	0.391
1209	A	7	9	1.	23	0.391
1210	A	0	0	0.	0	0.
1211	A	3	3	1.	21	0.143
1212	A	5	6	1.	20	0.3
1213	A	0	0	0.	0	0.
1214	A	8	8	1.	23	0.348
1215	A	0	0	0.	0	0.
1216	A	14	10	1.	23	0.435
1217	A	0	0	0.	0	0.
1218	A	6	7	1.	23	0.304
1219	A	5	6	1.	23	0.261
1220	A	4	4	1.	21	0.19
1221	A	7	9	1.	20	0.45
1222	A	0	0	0.	0	0.
1223	A	13	12	1.	23	0.522
1224	A	0	0	0.	0	0.
1225	A	18	12	1.	23	0.522
1226	A	8	9	1.	16	0.562
1227	A	8	9	1.	16	0.562
1228	A	4	4	0.99	21	0.19
1229	A	4	4	0.98	21	0.19
1230	A	3	4	1.	19	0.21
1231	A	0	0	0.	0	0.

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1232	A	0	0	0.	0	0.
1233	A	0	0	0.	0	0.
1234	A	0	0	0.	0	0.
1235	A	0	0	0.	0	0.
1236	A	0	0	0.	0	0.
1237	A	0	0	0.	0	0.
1238	A	0	0	0.	0	0.
1239	A	0	0	0.	0	0.
1240	A	0	0	0.	0	0.
1241	A	4	5	1.	25	0.2
1242	A	0	0	0.	0	0.
1243	A	8	9	1.	25	0.36
1244	A	0	0	0.	0	0.
1245	A	10	9	1.	25	0.36
1246	A	0	0	0.	0	0.
1247	A	29	8	1.	21	0.381
1248	A	25	10	1.	21	0.476
1249	A	19	8	1.	19	0.421
1250	A	16	10	1.	18	0.556
1251	A	14	10	1.	21	0.476
1252	A	11	10	1.	21	0.476
1253	A	16	12	1.	21	0.571
1254	A	50	8	1.	23	0.348
1255	A	44	10	1.	23	0.435
1256	A	35	8	1.	21	0.381
1257	A	30	11	1.	20	0.55
1258	A	25	12	1.	23	0.522
1259	A	20	13	1.	23	0.565
1260	A	22	15	1.	23	0.652
1261	A	11	7	1.	23	0.304
1262	A	10	8	1.	23	0.348
1263	A	4	2	1.	21	0.095

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1264	A	4	2	1.	20	0.1
1265	A	12	7	1.	23	0.304
1266	A	9	7	1.	23	0.304
1267	A	21	13	1.	23	0.565
1268	A	33	12	1.	23	0.522
1269	A	38	12	1.	23	0.522
1270	A	27	10	1.	21	0.476
1271	A	32	11	1.	20	0.55
1272	A	39	16	1.	23	0.696
1273	A	42	15	1.	23	0.652
1274	A	47	22	1.	23	0.956
1275	A	24	14	1.	12	1.167
1276	A	14	12	1.	12	1.
1277	A	19	12	1.	12	1.
1278	A	7	8	1.	10	0.8
1279	A	8	8	1.	9	0.889
1280	A	12	7	1.	12	0.583
1281	A	8	12	1.	12	1.
1282	A	6	6	1.	12	0.5
1283	A	18	15	1.	12	1.25
1284	A	12	8	1.	12	0.667
1285	A	26	15	1.	12	1.25
1286	A	26	15	1.	26	0.577
1287	A	14	11	1.	26	0.423
1288	A	21	15	1.	26	0.577
1289	A	7	7	1.	24	0.292
1290	A	9	8	1.	23	0.348
1291	A	18	9	1.	26	0.346
1292	A	8	8	0.92	26	0.308
1293	A	10	9	1.	26	0.346
1294	A	17	16	0.98	26	0.615
1295	A	15	10	1.	26	0.385

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1296	A	26	18	0.99	26	0.692
1297	A	21	16	1.	22	0.727
1298	A	28	12	1.	21	0.571
1299	A	0	0	0.	0	0.
1300	A	28	14	1.	24	0.583
1301	A	22	18	1.	24	0.75

Chapter 3

Listing of integrals

3.1 $\int x^3(d+icdx)\left(a+b\tan^{-1}(cx)\right)\,dx$

Optimal. Leaf size=117

$$\frac{1}{5}icdx^5\left(a+b\tan^{-1}(cx)\right)+\frac{1}{4}dx^4\left(a+b\tan^{-1}(cx)\right)+\frac{ibdx^2}{10c^2}-\frac{ibd\log\left(c^2x^2+1\right)}{10c^4}+\frac{bdx}{4c^3}-\frac{bd\tan^{-1}(cx)}{4c^4}-\frac{bdx^3}{12c}-\frac{1}{20}ibd$$

```
[Out] (b*d*x)/(4*c^3) + ((I/10)*b*d*x^2)/c^2 - (b*d*x^3)/(12*c) - (I/20)*b*d*x^4
- (b*d*ArcTan[c*x])/(4*c^4) + (d*x^4*(a + b*ArcTan[c*x]))/4 + (I/5)*c*d*x^5
*(a + b*ArcTan[c*x]) - ((I/10)*b*d*Log[1 + c^2*x^2])/c^4
```

Rubi [A] time = 0.103317, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {43, 4872, 12, 801, 635, 203, 260}

$$\frac{1}{5}icdx^5\left(a+b\tan^{-1}(cx)\right)+\frac{1}{4}dx^4\left(a+b\tan^{-1}(cx)\right)+\frac{ibdx^2}{10c^2}-\frac{ibd\log\left(c^2x^2+1\right)}{10c^4}+\frac{bdx}{4c^3}-\frac{bd\tan^{-1}(cx)}{4c^4}-\frac{bdx^3}{12c}-\frac{1}{20}ibd$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + I*c*d*x)*(a + b*ArcTan[c*x]),x]
```


```
[Out] (b*d*x)/(4*c^3) + ((I/10)*b*d*x^2)/c^2 - (b*d*x^3)/(12*c) - (I/20)*b*d*x^4
- (b*d*ArcTan[c*x])/(4*c^4) + (d*x^4*(a + b*ArcTan[c*x]))/4 + (I/5)*c*d*x^5
*(a + b*ArcTan[c*x]) - ((I/10)*b*d*Log[1 + c^2*x^2])/c^4
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 4872

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
+ b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2), x
], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2*m
] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int x^3(d+icdx)\left(a+b\tan^{-1}(cx)\right)dx = \frac{1}{4}dx^4\left(a+b\tan^{-1}(cx)\right)+\frac{1}{5}icdx^5\left(a+b\tan^{-1}(cx)\right)-(bc)\int\frac{dx^4(5+4icx)}{20\left(1+c^2x^2\right)}dx$$
$$=\frac{1}{4}dx^4\left(a+b\tan^{-1}(cx)\right)+\frac{1}{5}icdx^5\left(a+b\tan^{-1}(cx)\right)-\frac{1}{20}(bcd)\int\frac{x^4(5+4icx)}{1+c^2x^2}dx$$
$$=\frac{1}{4}dx^4\left(a+b\tan^{-1}(cx)\right)+\frac{1}{5}icdx^5\left(a+b\tan^{-1}(cx)\right)-\frac{1}{20}(bcd)\int\left(-\frac{5}{c^4}-\frac{4ix}{c^3}+\right.$$
$$=\frac{bdx}{4c^3}+\frac{ibdx^2}{10c^2}-\frac{bdx^3}{12c}-\frac{1}{20}ibdx^4+\frac{1}{4}dx^4\left(a+b\tan^{-1}(cx)\right)+\frac{1}{5}icdx^5\left(a+b\tan^{-1}\right.$$
$$=\frac{bdx}{4c^3}+\frac{ibdx^2}{10c^2}-\frac{bdx^3}{12c}-\frac{1}{20}ibdx^4+\frac{1}{4}dx^4\left(a+b\tan^{-1}(cx)\right)+\frac{1}{5}icdx^5\left(a+b\tan^{-1}\right.$$
$$=\frac{bdx}{4c^3}+\frac{ibdx^2}{10c^2}-\frac{bdx^3}{12c}-\frac{1}{20}ibdx^4-\frac{bd\tan^{-1}(cx)}{4c^4}+\frac{1}{4}dx^4\left(a+b\tan^{-1}(cx)\right)+\frac{1}{5}icd$$

Mathematica [A] time = 0.074168, size = 98, normalized size = 0.84

$$\frac{d\left(3ac^4x^4(5+4icx)+bcx\left(-3ic^3x^3-5c^2x^2+6icx+15\right)-6ib\log\left(c^2x^2+1\right)+3b\left(4ic^5x^5+5c^4x^4-5\right)\tan^{-1}(cx)\right)}{60c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + I*c*d*x)*(a + b*ArcTan[c*x]),x]
```

```
[Out] (d*(3*a*c^4*x^4*(5 + (4*I)*c*x) + b*c*x*(15 + (6*I)*c*x - 5*c^2*x^2 - (3*I)
*c^3*x^3) + 3*b*(-5 + 5*c^4*x^4 + (4*I)*c^5*x^5)*ArcTan[c*x] - (6*I)*b*Log[
1 + c^2*x^2]))/(60*c^4)
```

Maple [A] time = 0.027, size = 108, normalized size = 0.9

$$\frac{i}{5}cdax^5+\frac{dax^4}{4}+\frac{i}{5}cdb\arctan(cx)x^5+\frac{db\arctan(cx)x^4}{4}+\frac{dbx}{4c^3}-\frac{i}{20}bdx^4-\frac{dbx^3}{12c}+\frac{\frac{i}{10}bdx^2}{c^2}-\frac{\frac{i}{10}bd\ln\left(c^2x^2+1\right)}{c^4}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d+I*c*d*x)*(a+b*arctan(c*x)),x)
```

```
[Out] 1/5*I*c*d*a*x^5+1/4*d*a*x^4+1/5*I*c*d*b*arctan(c*x)*x^5+1/4*d*b*arctan(c*x)
*x^4+1/4*b*d*x/c^3-1/20*I*b*d*x^4-1/12*b*d*x^3/c+1/10*I*b*d*x^2/c^2-1/10*I*
```

```
b*d*ln(c^2*x^2+1)/c^4-1/4*b*d*arctan(c*x)/c^4
```

Maxima [A] time = 1.48333, size = 147, normalized size = 1.26

$$\frac{1}{5}i\,acdx^5+\frac{1}{4}\,adx^4+\frac{1}{20}i\left(4\,x^5\arctan\left(cx\right)-c\left(\frac{c^2x^4-2\,x^2}{c^4}+\frac{2\,\log\left(c^2x^2+1\right)}{c^6}\right)\right)bcd+\frac{1}{12}\left(3\,x^4\arctan\left(cx\right)-c\left(\frac{c^2x^3-3\,x}{c^4}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d+I*c*d*x)*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] 1/5*I*a*c*d*x^5 + 1/4*a*d*x^4 + 1/20*I*(4*x^5*arctan(c*x) - c*((c^2*x^4 - 2
*x^2)/c^4 + 2*log(c^2*x^2 + 1)/c^6))*b*c*d + 1/12*(3*x^4*arctan(c*x) - c*((
c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5))*b*d
```

Fricas [A] time = 2.86457, size = 302, normalized size = 2.58

$$\frac{24i\,ac^5dx^5+6\,(5\,a-i\,b)c^4dx^4-10\,bc^3dx^3+12i\,bc^2dx^2+30\,bcdx-27i\,bd\log\left(\frac{cx+i}{c}\right)+3i\,bd\log\left(\frac{cx-i}{c}\right)-\left(12\,bc^5dx^5-15i\right.}{120\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d+I*c*d*x)*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] 1/120*(24*I*a*c^5*d*x^5 + 6*(5*a - I*b)*c^4*d*x^4 - 10*b*c^3*d*x^3 + 12*I*b
*c^2*d*x^2 + 30*b*c*d*x - 27*I*b*d*log((c*x + I)/c) + 3*I*b*d*log((c*x - I)
/c) - (12*b*c^5*d*x^5 - 15*I*b*c^4*d*x^4)*log(-(c*x + I)/(c*x - I)))/c^4
```

Sympy [A] time = 1.84808, size = 153, normalized size = 1.31

$$\frac{iacdx^5}{5}-\frac{bdx^3}{12c}+\frac{ibdx^2}{10c^2}+\frac{bdx}{4c^3}+\frac{ibd\log\left(x-\frac{i}{c}\right)}{40c^4}-\frac{9ibd\log\left(x+\frac{i}{c}\right)}{40c^4}+x^4\left(\frac{ad}{4}-\frac{ibd}{20}\right)+\left(-\frac{bcdx^5}{10}+\frac{ibdx^4}{8}\right)\log\left(-icx+1\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(d+I*c*d*x)*(a+b*atan(c*x)),x)
```

```
[Out] I*a*c*d*x**5/5 - b*d*x**3/(12*c) + I*b*d*x**2/(10*c**2) + b*d*x/(4*c**3) +
I*b*d*log(x - I/c)/(40*c**4) - 9*I*b*d*log(x + I/c)/(40*c**4) + x**4*(a*d/4
 - I*b*d/20) + (-b*c*d*x**5/10 + I*b*d*x**4/8)*log(-I*c*x + 1) + (b*c*d*x**
5/10 - I*b*d*x**4/8)*log(I*c*x + 1)
```

Giac [A] time = 1.30961, size = 162, normalized size = 1.38

$$-\frac{24\,bc^5dx^5\arctan\left(cx\right)+24\,ac^5dx^5-30\,bc^4dix^4\arctan\left(cx\right)-30\,ac^4dix^4-6\,bc^4dx^4+10\,bc^3dix^3+12\,bc^2dx^2-30\,bcdi}{120\,c^4i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d+I*c*d*x)*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] -1/120*(24*b*c^5*d*x^5*arctan(c*x) + 24*a*c^5*d*x^5 - 30*b*c^4*d*i*x^4*arct
an(c*x) - 30*a*c^4*d*i*x^4 - 6*b*c^4*d*x^4 + 10*b*c^3*d*i*x^3 + 12*b*c^2*d*
x^2 - 30*b*c*d*i*x + 3*b*d*log(c*i*x + 1) - 27*b*d*log(-c*i*x + 1))/(c^4*i)
```

3.2 $\int x^2(d+icdx)\left(a+b\tan^{-1}(cx)\right)\,dx$

Optimal. Leaf size=105

$$\frac{1}{4}icdx^4\left(a+b\tan^{-1}(cx)\right)+\frac{1}{3}dx^3\left(a+b\tan^{-1}(cx)\right)+\frac{bd\log\left(c^2x^2+1\right)}{6c^3}+\frac{ibdx}{4c^2}-\frac{ibd\tan^{-1}(cx)}{4c^3}-\frac{bdx^2}{6c}-\frac{1}{12}ibdx^3$$

```
[Out] ((I/4)*b*d*x)/c^2 - (b*d*x^2)/(6*c) - (I/12)*b*d*x^3 - ((I/4)*b*d*ArcTan[c*
x])/c^3 + (d*x^3*(a + b*ArcTan[c*x]))/3 + (I/4)*c*d*x^4*(a + b*ArcTan[c*x])
 + (b*d*Log[1 + c^2*x^2])/(6*c^3)
```

Rubi [A] time = 0.0942691, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {43, 4872, 12, 801, 635, 203, 260}

$$\frac{1}{4}icdx^4\left(a+b\tan^{-1}(cx)\right)+\frac{1}{3}dx^3\left(a+b\tan^{-1}(cx)\right)+\frac{bd\log\left(c^2x^2+1\right)}{6c^3}+\frac{ibdx}{4c^2}-\frac{ibd\tan^{-1}(cx)}{4c^3}-\frac{bdx^2}{6c}-\frac{1}{12}ibdx^3$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + I*c*d*x)*(a + b*ArcTan[c*x]),x]
```

```
[Out] ((I/4)*b*d*x)/c^2 - (b*d*x^2)/(6*c) - (I/12)*b*d*x^3 - ((I/4)*b*d*ArcTan[c*
x])/c^3 + (d*x^3*(a + b*ArcTan[c*x]))/3 + (I/4)*c*d*x^4*(a + b*ArcTan[c*x])
 + (b*d*Log[1 + c^2*x^2])/(6*c^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 4872

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
+ b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2), x
], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2*m
] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int x^2(d+icdx)\left(a+b\tan^{-1}(cx)\right)dx=\frac{1}{3}dx^3\left(a+b\tan^{-1}(cx)\right)+\frac{1}{4}icdx^4\left(a+b\tan^{-1}(cx)\right)-(bc)\int\frac{dx^3(4+3icx)}{12\left(1+c^2x^2\right)}dx$$
$$=\frac{1}{3}dx^3\left(a+b\tan^{-1}(cx)\right)+\frac{1}{4}icdx^4\left(a+b\tan^{-1}(cx)\right)-\frac{1}{12}(bcd)\int\frac{x^3(4+3icx)}{1+c^2x^2}dx$$
$$=\frac{1}{3}dx^3\left(a+b\tan^{-1}(cx)\right)+\frac{1}{4}icdx^4\left(a+b\tan^{-1}(cx)\right)-\frac{1}{12}(bcd)\int\left(-\frac{3i}{c^3}+\frac{4x}{c^2}+\frac{3ix}{c}\right.$$
$$=\frac{ibdx}{4c^2}-\frac{bdx^2}{6c}-\frac{1}{12}ibdx^3+\frac{1}{3}dx^3\left(a+b\tan^{-1}(cx)\right)+\frac{1}{4}icdx^4\left(a+b\tan^{-1}(cx)\right)-\frac{(b}{}$$
$$=\frac{ibdx}{4c^2}-\frac{bdx^2}{6c}-\frac{1}{12}ibdx^3+\frac{1}{3}dx^3\left(a+b\tan^{-1}(cx)\right)+\frac{1}{4}icdx^4\left(a+b\tan^{-1}(cx)\right)-\frac{(ib}{}$$
$$=\frac{ibdx}{4c^2}-\frac{bdx^2}{6c}-\frac{1}{12}ibdx^3-\frac{ibd\tan^{-1}(cx)}{4c^3}+\frac{1}{3}dx^3\left(a+b\tan^{-1}(cx)\right)+\frac{1}{4}icdx^4\left(a+b\right.$$

Mathematica [A] time = 0.0602777, size = 88, normalized size = 0.84

$$\frac{d\left(ac^3x^3(4+3icx)+bcx\left(-ic^2x^2-2cx+3i\right)+2b\log\left(c^2x^2+1\right)+b\left(3ic^4x^4+4c^3x^3-3i\right)\tan^{-1}(cx)\right)}{12c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + I*c*d*x)*(a + b*ArcTan[c*x]),x]
```

```
[Out] (d*(a*c^3*x^3*(4 + (3*I)*c*x) + b*c*x*(3*I - 2*c*x - I*c^2*x^2) + b*(-3*I +
 4*c^3*x^3 + (3*I)*c^4*x^4)*ArcTan[c*x] + 2*b*Log[1 + c^2*x^2]))/(12*c^3)
```

Maple [A] time = 0.026, size = 98, normalized size = 0.9

$$\frac{i}{4}cdax^4+\frac{dax^3}{3}+\frac{i}{4}cdb\arctan(cx)x^4+\frac{db\arctan(cx)x^3}{3}+\frac{\frac{i}{4}bdx}{c^2}-\frac{i}{12}bdx^3-\frac{dbx^2}{6c}+\frac{db\ln\left(c^2x^2+1\right)}{6c^3}-\frac{\frac{i}{4}bd\arctan(c}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d+I*c*d*x)*(a+b*arctan(c*x)),x)
```

```
[Out] 1/4*I*c*d*a*x^4+1/3*d*a*x^3+1/4*I*c*d*b*arctan(c*x)*x^4+1/3*d*b*arctan(c*x)
*x^3+1/4*I*b*d*x/c^2-1/12*I*b*d*x^3-1/6*b*d*x^2/c+1/6*b*d*ln(c^2*x^2+1)/c^3
```

```
-1/4*I*b*d*arctan(c*x)/c^3
```

Maxima [A] time = 1.50341, size = 134, normalized size = 1.28

$$\frac{1}{4}i\,acdx^4 + \frac{1}{3}\,adx^3 + \frac{1}{12}i\left(3\,x^4\arctan\left(cx\right) - c\left(\frac{c^2x^3 - 3\,x}{c^4} + \frac{3\,\arctan\left(cx\right)}{c^5}\right)\right)bcd + \frac{1}{6}\left(2\,x^3\arctan\left(cx\right) - c\left(\frac{x^2}{c^2} - \frac{\log\left(c^2x^2+1\right)}{c^4}\right)\right)bd$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d+I*c*d*x)*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] 1/4*I*a*c*d*x^4 + 1/3*a*d*x^3 + 1/12*I*(3*x^4*arctan(c*x) - c*((c^2*x^3 - 3
*x)/c^4 + 3*arctan(c*x)/c^5))*b*c*d + 1/6*(2*x^3*arctan(c*x) - c*(x^2/c^2 -
 log(c^2*x^2 + 1)/c^4))*b*d
```

Fricas [A] time = 2.78507, size = 262, normalized size = 2.5

$$\frac{6i\,ac^4dx^4 + 2\,(4\,a - i\,b)c^3dx^3 - 4\,bc^2dx^2 + 6i\,bcdx + 7\,bd\log\left(\frac{cx+i}{c}\right) + bd\log\left(\frac{cx-i}{c}\right) - \left(3\,bc^4dx^4 - 4i\,bc^3dx^3\right)\log\left(-\frac{cx+i}{cx-i}\right)}{24\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d+I*c*d*x)*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] 1/24*(6*I*a*c^4*d*x^4 + 2*(4*a - I*b)*c^3*d*x^3 - 4*b*c^2*d*x^2 + 6*I*b*c*d
*x + 7*b*d*log((c*x + I)/c) + b*d*log((c*x - I)/c) - (3*b*c^4*d*x^4 - 4*I*b
*c^3*d*x^3)*log(-(c*x + I)/(c*x - I)))/c^3
```

Sympy [A] time = 1.78637, size = 131, normalized size = 1.25

$$\frac{iacdx^4}{4} - \frac{bdx^2}{6c} + \frac{ibdx}{4c^2} + \frac{bd\left(\frac{\log\left(x-\frac{i}{c}\right)}{24} + \frac{7\log\left(x+\frac{i}{c}\right)}{24}\right)}{c^3} + x^3\left(\frac{ad}{3} - \frac{ibd}{12}\right) + \left(-\frac{bcdx^4}{8} + \frac{ibdx^3}{6}\right)\log\left(-icx+1\right) + \left(\frac{bcdx^4}{8} - \frac{ibd}{6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d+I*c*d*x)*(a+b*atan(c*x)),x)
```

```
[Out] I*a*c*d*x**4/4 - b*d*x**2/(6*c) + I*b*d*x/(4*c**2) + b*d*(log(x - I/c)/24 +
 7*log(x + I/c)/24)/c**3 + x**3*(a*d/3 - I*b*d/12) + (-b*c*d*x**4/8 + I*b*d
*x**3/6)*log(-I*c*x + 1) + (b*c*d*x**4/8 - I*b*d*x**3/6)*log(I*c*x + 1)
```

Giac [A] time = 1.16483, size = 142, normalized size = 1.35

$$\frac{6\,bc^4dix^4\arctan(cx) + 6\,ac^4dix^4 - 2\,bc^3dix^3 + 8\,bc^3dx^3\arctan(cx) + 8\,ac^3dx^3 - 4\,bc^2dx^2 + 6\,bcdix + 7\,bd\log(cx+i) +}{24\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d+I*c*d*x)*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] 1/24*(6*b*c^4*d*i*x^4*arctan(c*x) + 6*a*c^4*d*i*x^4 - 2*b*c^3*d*i*x^3 + 8*b
*c^3*d*x^3*arctan(c*x) + 8*a*c^3*d*x^3 - 4*b*c^2*d*x^2 + 6*b*c*d*i*x + 7*b*
d*log(c*x + i) + b*d*log(c*x - i))/c^3
```

3.3 $\int x(d+icdx)\left(a+b\tan^{-1}(cx)\right)\,dx$

Optimal. Leaf size=91

$$\frac{1}{3}icdx^3\left(a+b\tan^{-1}(cx)\right)+\frac{1}{2}dx^2\left(a+b\tan^{-1}(cx)\right)+\frac{ibd\log\left(c^2x^2+1\right)}{6c^2}+\frac{bd\tan^{-1}(cx)}{2c^2}-\frac{bdx}{2c}-\frac{1}{6}ibdx^2$$

```
[Out] -(b*d*x)/(2*c) - (I/6)*b*d*x^2 + (b*d*ArcTan[c*x])/(2*c^2) + (d*x^2*(a + b*
ArcTan[c*x]))/2 + (I/3)*c*d*x^3*(a + b*ArcTan[c*x]) + ((I/6)*b*d*Log[1 + c^
2*x^2])/c^2
```

Rubi [A] time = 0.0773123, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {43, 4872, 12, 801, 635, 203, 260}

$$\frac{1}{3}icdx^3\left(a+b\tan^{-1}(cx)\right)+\frac{1}{2}dx^2\left(a+b\tan^{-1}(cx)\right)+\frac{ibd\log\left(c^2x^2+1\right)}{6c^2}+\frac{bd\tan^{-1}(cx)}{2c^2}-\frac{bdx}{2c}-\frac{1}{6}ibdx^2$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + I*c*d*x)*(a + b*ArcTan[c*x]),x]
```

```
[Out] -(b*d*x)/(2*c) - (I/6)*b*d*x^2 + (b*d*ArcTan[c*x])/(2*c^2) + (d*x^2*(a + b*
ArcTan[c*x]))/2 + (I/3)*c*d*x^3*(a + b*ArcTan[c*x]) + ((I/6)*b*d*Log[1 + c^
2*x^2])/c^2
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 4872

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
+ b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2), x
], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2*m
] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int x(d+icdx)\left(a+b\tan^{-1}(cx)\right)dx = \frac{1}{2}dx^2\left(a+b\tan^{-1}(cx)\right)+\frac{1}{3}icdx^3\left(a+b\tan^{-1}(cx)\right)-(bc)\int\frac{dx^2(3+2icx)}{6+6c^2x^2}dx$$
$$= \frac{1}{2}dx^2\left(a+b\tan^{-1}(cx)\right)+\frac{1}{3}icdx^3\left(a+b\tan^{-1}(cx)\right)-(bcd)\int\frac{x^2(3+2icx)}{6+6c^2x^2}dx$$
$$= \frac{1}{2}dx^2\left(a+b\tan^{-1}(cx)\right)+\frac{1}{3}icdx^3\left(a+b\tan^{-1}(cx)\right)-(bcd)\int\left(\frac{1}{2c^2}+\frac{ix}{3c}+\frac{i(3i}{c^2(6}\right.$$
$$= -\frac{bdx}{2c}-\frac{1}{6}ibdx^2+\frac{1}{2}dx^2\left(a+b\tan^{-1}(cx)\right)+\frac{1}{3}icdx^3\left(a+b\tan^{-1}(cx)\right)-\frac{(ibd)\int\frac{3}{6+}}{c}$$
$$= -\frac{bdx}{2c}-\frac{1}{6}ibdx^2+\frac{1}{2}dx^2\left(a+b\tan^{-1}(cx)\right)+\frac{1}{3}icdx^3\left(a+b\tan^{-1}(cx)\right)+(2ibd)\int$$
$$= -\frac{bdx}{2c}-\frac{1}{6}ibdx^2+\frac{bd\tan^{-1}(cx)}{2c^2}+\frac{1}{2}dx^2\left(a+b\tan^{-1}(cx)\right)+\frac{1}{3}icdx^3\left(a+b\tan^{-1}(\right.$$

Mathematica [A] time = 0.0468314, size = 76, normalized size = 0.84

$$\frac{d\left(cx(acx(3+2icx)+b(-3-icx))+ib\log\left(c^2x^2+1\right)+b\left(2ic^3x^3+3c^2x^2+3\right)\tan^{-1}(cx)\right)}{6c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + I*c*d*x)*(a + b*ArcTan[c*x]),x]
```

```
[Out] (d*(c*x*(b*(-3 - I*c*x) + a*c*x*(3 + (2*I)*c*x)) + b*(3 + 3*c^2*x^2 + (2*I)
*c^3*x^3)*ArcTan[c*x] + I*b*Log[1 + c^2*x^2]))/(6*c^2)
```

Maple [A] time = 0.028, size = 87, normalized size = 1.

$$\frac{i}{3}cdax^3+\frac{dax^2}{2}+\frac{i}{3}cdb\arctan(cx)x^3+\frac{db\arctan(cx)x^2}{2}-\frac{i}{6}bdx^2-\frac{dbx}{2c}+\frac{\frac{i}{6}db\ln\left(c^2x^2+1\right)}{c^2}+\frac{db\arctan(cx)}{2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d+I*c*d*x)*(a+b*arctan(c*x)),x)
```

```
[Out] 1/3*I*c*d*a*x^3+1/2*d*a*x^2+1/3*I*c*d*b*arctan(c*x)*x^3+1/2*d*b*arctan(c*x)
*x^2-1/6*I*b*d*x^2-1/2*b*d*x/c+1/6*I*b*d*ln(c^2*x^2+1)/c^2+1/2*b*d*arctan(c
```

```
*x)/c^2
```

Maxima [A] time = 1.46907, size = 119, normalized size = 1.31

$$\frac{1}{3}i\,acdx^3+\frac{1}{6}i\left(2\,x^3\arctan\left(cx\right)-c\left(\frac{x^2}{c^2}-\frac{\log\left(c^2x^2+1\right)}{c^4}\right)\right)bcd+\frac{1}{2}\,adx^2+\frac{1}{2}\left(x^2\arctan\left(cx\right)-c\left(\frac{x}{c^2}-\frac{\arctan\left(cx\right)}{c^3}\right)\right)bd$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d+I*c*d*x)*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] 1/3*I*a*c*d*x^3 + 1/6*I*(2*x^3*arctan(c*x) - c*(x^2/c^2 - log(c^2*x^2 + 1)/
c^4))*b*c*d + 1/2*a*d*x^2 + 1/2*(x^2*arctan(c*x) - c*(x/c^2 - arctan(c*x)/c
^3))*b*d
```

Fricas [A] time = 2.80572, size = 243, normalized size = 2.67

$$\frac{4i\,ac^3dx^3+2\,(3\,a-i\,b)c^2dx^2-6\,bcdx+5i\,bd\log\left(\frac{cx+i}{c}\right)-i\,bd\log\left(\frac{cx-i}{c}\right)-\left(2\,bc^3dx^3-3i\,bc^2dx^2\right)\log\left(-\frac{cx+i}{cx-i}\right)}{12\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d+I*c*d*x)*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] 1/12*(4*I*a*c^3*d*x^3 + 2*(3*a - I*b)*c^2*d*x^2 - 6*b*c*d*x + 5*I*b*d*log((
c*x + I)/c) - I*b*d*log((c*x - I)/c) - (2*b*c^3*d*x^3 - 3*I*b*c^2*d*x^2)*lo
g(-(c*x + I)/(c*x - I)))/c^2
```

Sympy [A] time = 1.7801, size = 128, normalized size = 1.41

$$\frac{iacdx^3}{3}-\frac{bdx}{2c}-\frac{ibd\log\left(x-\frac{i}{c}\right)}{12c^2}+\frac{5ibd\log\left(x+\frac{i}{c}\right)}{12c^2}+x^2\left(\frac{ad}{2}-\frac{ibd}{6}\right)+\left(-\frac{bcdx^3}{6}+\frac{ibdx^2}{4}\right)\log\left(-icx+1\right)+\left(\frac{bcdx^3}{6}-\frac{ibdx^2}{4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d+I*c*d*x)*(a+b*atan(c*x)),x)
```

```
[Out] I*a*c*d*x**3/3 - b*d*x/(2*c) - I*b*d*log(x - I/c)/(12*c**2) + 5*I*b*d*log(x
 + I/c)/(12*c**2) + x**2*(a*d/2 - I*b*d/6) + (-b*c*d*x**3/6 + I*b*d*x**2/4)
*log(-I*c*x + 1) + (b*c*d*x**3/6 - I*b*d*x**2/4)*log(I*c*x + 1)
```

Giac [A] time = 1.1604, size = 134, normalized size = 1.47

$$-\frac{4\,bc^3dx^3\arctan\left(cx\right)+4\,ac^3dx^3-6\,bc^2dix^2\arctan\left(cx\right)-6\,ac^2dix^2-2\,bc^2dx^2+6\,bcdix+5\,bd\log\left(cix-1\right)-bd\log\left(-\right.}{12\,c^2i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d+I*c*d*x)*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] -1/12*(4*b*c^3*d*x^3*arctan(c*x) + 4*a*c^3*d*x^3 - 6*b*c^2*d*i*x^2*arctan(c
*x) - 6*a*c^2*d*i*x^2 - 2*b*c^2*d*x^2 + 6*b*c*d*i*x + 5*b*d*log(c*i*x - 1)
- b*d*log(-c*i*x - 1))/(c^2*i)
```

3.4 $\int (d+icdx)\left(a+b\tan^{-1}(cx)\right)\,dx$

Optimal. Leaf size=53

$$-\frac{id(1+icx)^2\left(a+b\tan^{-1}(cx)\right)}{2c}-\frac{bd\log(cx+i)}{c}-\frac{1}{2}ibdx$$

```
[Out] (-I/2)*b*d*x - ((I/2)*d*(1 + I*c*x)^2*(a + b*ArcTan[c*x]))/c - (b*d*Log[I +
 c*x])/c
```

Rubi [A] time = 0.0312274, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4862, 627, 43}

$$-\frac{id(1+icx)^2\left(a+b\tan^{-1}(cx)\right)}{2c}-\frac{bd\log(cx+i)}{c}-\frac{1}{2}ibdx$$

Antiderivative was successfully verified.

```
[In] Int[(d + I*c*d*x)*(a + b*ArcTan[c*x]),x]
```

```
[Out] (-I/2)*b*d*x - ((I/2)*d*(1 + I*c*x)^2*(a + b*ArcTan[c*x]))/c - (b*d*Log[I +
 c*x])/c
```

Rule 4862

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x]))/(e*(q + 1)), x] - Dist[(b*
c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (d+icdx)\left(a+b\tan^{-1}(cx)\right)dx &= -\frac{id(1+icx)^2\left(a+b\tan^{-1}(cx)\right)}{2c}+\frac{(ib)\int\frac{(d+icdx)^2}{1+c^2x^2}\,dx}{2d}\\ &= -\frac{id(1+icx)^2\left(a+b\tan^{-1}(cx)\right)}{2c}+\frac{(ib)\int\frac{d+icdx}{\frac{1}{d}-\frac{icx}{d}}\,dx}{2d}\\ &= -\frac{id(1+icx)^2\left(a+b\tan^{-1}(cx)\right)}{2c}+\frac{(ib)\int\left(-d^2+\frac{2id^2}{i+cx}\right)dx}{2d}\\ &= -\frac{1}{2}ibdx-\frac{id(1+icx)^2\left(a+b\tan^{-1}(cx)\right)}{2c}-\frac{bd\log(i+cx)}{c}\end{aligned}$$

Mathematica [A] time = 0.0053182, size = 84, normalized size = 1.58

$$\frac{1}{2}iacdx^2+adx-\frac{bd\log\left(c^2x^2+1\right)}{2c}+\frac{1}{2}ibcdx^2\tan^{-1}(cx)+bdx\tan^{-1}(cx)+\frac{ibd\tan^{-1}(cx)}{2c}-\frac{1}{2}ibdx$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + I*c*d*x)*(a + b*ArcTan[c*x]),x]
```

```
[Out] a*d*x - (I/2)*b*d*x + (I/2)*a*c*d*x^2 + ((I/2)*b*d*ArcTan[c*x])/c + b*d*x*A
rcTan[c*x] + (I/2)*b*c*d*x^2*ArcTan[c*x] - (b*d*Log[1 + c^2*x^2])/(2*c)
```

Maple [A] time = 0.027, size = 71, normalized size = 1.3

$$adx+\frac{i}{2}cdax^2+b\arctan(cx)\,xd+\frac{i}{2}cdb\arctan(cx)\,x^2-\frac{i}{2}bdx-\frac{db\ln\left(c^2x^2+1\right)}{2\,c}+\frac{\frac{i}{2}db\arctan(cx)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)*(a+b*arctan(c*x)),x)
```

```
[Out] a*d*x+1/2*I*c*d*a*x^2+b*arctan(c*x)*x*d+1/2*I*c*d*b*arctan(c*x)*x^2-1/2*I*b
*d*x-1/2*b*d*ln(c^2*x^2+1)/c+1/2*I/c*d*b*arctan(c*x)
```

Maxima [A] time = 1.47815, size = 99, normalized size = 1.87

$$\frac{1}{2}i\,acdx^2+\frac{1}{2}i\left(x^2\arctan\left(cx\right)-c\left(\frac{x}{c^2}-\frac{\arctan\left(cx\right)}{c^3}\right)\right)bcd+adx+\frac{\left(2\,cx\arctan\left(cx\right)-\log\left(c^2x^2+1\right)\right)bd}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] 1/2*I*a*c*d*x^2 + 1/2*I*(x^2*arctan(c*x) - c*(x/c^2 - arctan(c*x)/c^3))*b*c
*d + a*d*x + 1/2*(2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*b*d/c
```

Fricas [B] time = 2.8117, size = 204, normalized size = 3.85

$$\frac{2i\,ac^2dx^2+2\left(2\,a-i\,b\right)cdx-3\,bd\log\left(\frac{cx+i}{c}\right)-bd\log\left(\frac{cx-i}{c}\right)-\left(bc^2dx^2-2i\,bcdx\right)\log\left(-\frac{cx+i}{cx-i}\right)}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*I*a*c^2*d*x^2 + 2*(2*a - I*b)*c*d*x - 3*b*d*log((c*x + I)/c) - b*d*l
og((c*x - I)/c) - (b*c^2*d*x^2 - 2*I*b*c*d*x)*log(-(c*x + I)/(c*x - I)))/c
```

Sympy [B] time = 1.68031, size = 102, normalized size = 1.92

$$\frac{iacdx^2}{2}+\frac{bd\left(-\frac{\log\left(x-\frac{i}{c}\right)}{4}-\frac{3\log\left(x+\frac{i}{c}\right)}{4}\right)}{c}+x\left(ad-\frac{ibd}{2}\right)+\left(-\frac{bcdx^2}{4}+\frac{ibdx}{2}\right)\log\left(-icx+1\right)+\left(\frac{bcdx^2}{4}-\frac{ibdx}{2}\right)\log\left(icx+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)*(a+b*atan(c*x)),x)
```

```
[Out] I*a*c*d*x**2/2 + b*d*(-log(x - I/c)/4 - 3*log(x + I/c)/4)/c + x*(a*d - I*b*
d/2) + (-b*c*d*x**2/4 + I*b*d*x/2)*log(-I*c*x + 1) + (b*c*d*x**2/4 - I*b*d*
x/2)*log(I*c*x + 1)
```

Giac [A] time = 1.14796, size = 104, normalized size = 1.96

$$\frac{2\,bc^2dix^2\arctan\left(cx\right)+2\,ac^2dix^2-2\,bcdix+4\,bcdx\arctan\left(cx\right)+4\,acdx-3\,bd\log\left(cx+i\right)-bd\log\left(cx-i\right)}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] 1/4*(2*b*c^2*d*i*x^2*arctan(c*x) + 2*a*c^2*d*i*x^2 - 2*b*c*d*i*x + 4*b*c*d*
x*arctan(c*x) + 4*a*c*d*x - 3*b*d*log(c*x + i) - b*d*log(c*x - i))/c
```

3.5 $\int \frac{(d+icdx)\left(a+b\tan^{-1}(cx)\right)}{x} dx$

Optimal. Leaf size=76

$$\frac{1}{2}ibd\text{PolyLog}(2,-icx)-\frac{1}{2}ibd\text{PolyLog}(2,icx)+iacdx+ad\log(x)-\frac{1}{2}ibd\log\left(c^2x^2+1\right)+ibcdx\tan^{-1}(cx)$$

```
[Out] I*a*c*d*x + I*b*c*d*x*ArcTan[c*x] + a*d*Log[x] - (I/2)*b*d*Log[1 + c^2*x^2]
 + (I/2)*b*d*PolyLog[2, (-I)*c*x] - (I/2)*b*d*PolyLog[2, I*c*x]
```

Rubi [A] time = 0.0858881, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {4876, 4846, 260, 4848, 2391}

$$\frac{1}{2}ibd\text{PolyLog}(2,-icx)-\frac{1}{2}ibd\text{PolyLog}(2,icx)+iacdx+ad\log(x)-\frac{1}{2}ibd\log\left(c^2x^2+1\right)+ibcdx\tan^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)*(a + b*ArcTan[c*x]))/x,x]
```

```
[Out] I*a*c*d*x + I*b*c*d*x*ArcTan[c*x] + a*d*Log[x] - (I/2)*b*d*Log[1 + c^2*x^2]
 + (I/2)*b*d*PolyLog[2, (-I)*c*x] - (I/2)*b*d*PolyLog[2, I*c*x]
```

Rule 4876

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{(d+icdx)\left(a+b\tan^{-1}(cx)\right)}{x}\,dx = \int \left(icd\left(a+b\tan^{-1}(cx)\right)+\frac{d\left(a+b\tan^{-1}(cx)\right)}{x}\right)dx$$
$$= d\int \frac{a+b\tan^{-1}(cx)}{x}\,dx+(icd)\int \left(a+b\tan^{-1}(cx)\right)dx$$
$$= iacdx+ad\log(x)+\frac{1}{2}(ibd)\int \frac{\log(1-icx)}{x}\,dx-\frac{1}{2}(ibd)\int \frac{\log(1+icx)}{x}\,dx+(ibcd)$$
$$= iacdx+ibcdx\tan^{-1}(cx)+ad\log(x)+\frac{1}{2}ibd\mathrm{Li}_2(-icx)-\frac{1}{2}ibd\mathrm{Li}_2(icx)-\left(ibc^2d\right)\int$$
$$= iacdx+ibcdx\tan^{-1}(cx)+ad\log(x)-\frac{1}{2}ibd\log\left(1+c^2x^2\right)+\frac{1}{2}ibd\mathrm{Li}_2(-icx)-\frac{1}{2}ibd$$

Mathematica [A] time = 0.0044113, size = 76, normalized size = 1.

$$\frac{1}{2}ibd\mathrm{PolyLog}(2,-icx)-\frac{1}{2}ibd\mathrm{PolyLog}(2,icx)+iacdx+ad\log(x)-\frac{1}{2}ibd\log\left(c^2x^2+1\right)+ibcdx\tan^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + I*c*d*x)*(a + b*ArcTan[c*x]))/x,x]
```

```
[Out] I*a*c*d*x + I*b*c*d*x*ArcTan[c*x] + a*d*Log[x] - (I/2)*b*d*Log[1 + c^2*x^2]
 + (I/2)*b*d*PolyLog[2, (-I)*c*x] - (I/2)*b*d*PolyLog[2, I*c*x]
```

Maple [A] time = 0.043, size = 113, normalized size = 1.5

$$iacdx+ad\ln(cx)+ibcdx\arctan(cx)+db\arctan(cx)\ln(cx)+\frac{i}{2}db\ln(cx)\ln(1+icx)-\frac{i}{2}db\ln(cx)\ln(1-icx)+\frac{i}{2}db$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)*(a+b*arctan(c*x))/x,x)
```

```
[Out] I*a*c*d*x+a*d*ln(c*x)+I*b*c*d*x*arctan(c*x)+d*b*arctan(c*x)*ln(c*x)+1/2*I*d
*b*ln(c*x)*ln(1+I*c*x)-1/2*I*d*b*ln(c*x)*ln(1-I*c*x)+1/2*I*d*b*dilog(1+I*c*
x)-1/2*I*d*b*dilog(1-I*c*x)-1/2*I*b*d*ln(c^2*x^2+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$i\,acdx + \frac{1}{2}i\left(2\,cx\arctan\left(cx\right) - \log\left(c^{2}x^{2} + 1\right)\right)bd + bd\int\frac{\arctan\left(cx\right)}{x}\,dx + ad\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)*(a+b*arctan(c*x))/x,x, algorithm="maxima")
```

```
[Out] I*a*c*d*x + 1/2*I*(2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*b*d + b*d*integrat
e(arctan(c*x)/x, x) + a*d*log(x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{2i\,acdx + 2\,ad - (bcdx - i\,bd)\log\left(-\frac{cx+i}{cx-i}\right)}{2\,x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)*(a+b*arctan(c*x))/x,x, algorithm="fricas")
```

```
[Out] integral(1/2*(2*I*a*c*d*x + 2*a*d - (b*c*d*x - I*b*d)*log(-(c*x + I)/(c*x -
 I)))/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d\left(\int\frac{a}{x}\,dx + \int iac\,dx + \int\frac{b\operatorname{atan}\left(cx\right)}{x}\,dx + \int ibc\operatorname{atan}\left(cx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)*(a+b*atan(c*x))/x,x)
```

```
[Out] d*(Integral(a/x, x) + Integral(I*a*c, x) + Integral(b*atan(c*x)/x, x) + Int
egral(I*b*c*atan(c*x), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(icdx + d)(b\arctan(cx) + a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)*(a+b*arctan(c*x))/x,x, algorithm="giac")
```

```
[Out] integrate((I*c*d*x + d)*(b*arctan(c*x) + a)/x, x)
```

3.6 $\int \frac{(d+icdx)\left(a+b\tan^{-1}(cx)\right)}{x^2}\,dx$

Optimal. Leaf size=77

$$-\frac{1}{2}bcd\text{PolyLog}(2,-icx)+\frac{1}{2}bcd\text{PolyLog}(2,icx)-\frac{d\left(a+b\tan^{-1}(cx)\right)}{x}+iacd\log(x)-\frac{1}{2}bcd\log\left(c^2x^2+1\right)+bcd\log(x)$$

```
[Out] -((d*(a + b*ArcTan[c*x]))/x) + I*a*c*d*Log[x] + b*c*d*Log[x] - (b*c*d*Log[1
 + c^2*x^2])/2 - (b*c*d*PolyLog[2, (-I)*c*x])/2 + (b*c*d*PolyLog[2, I*c*x])
/2
```

Rubi [A] time = 0.100228, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {4876, 4852, 266, 36, 29, 31, 4848, 2391}

$$-\frac{1}{2}bcd\text{PolyLog}(2,-icx)+\frac{1}{2}bcd\text{PolyLog}(2,icx)-\frac{d\left(a+b\tan^{-1}(cx)\right)}{x}+iacd\log(x)-\frac{1}{2}bcd\log\left(c^2x^2+1\right)+bcd\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)*(a + b*ArcTan[c*x]))/x^2,x]
```

```
[Out] -((d*(a + b*ArcTan[c*x]))/x) + I*a*c*d*Log[x] + b*c*d*Log[x] - (b*c*d*Log[1
 + c^2*x^2])/2 - (b*c*d*PolyLog[2, (-I)*c*x])/2 + (b*c*d*PolyLog[2, I*c*x])
/2
```

Rule 4876

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{(d+icdx)\left(a+b\tan^{-1}(cx)\right)}{x^2}\,dx = \int \left(\frac{d\left(a+b\tan^{-1}(cx)\right)}{x^2}+\frac{icd\left(a+b\tan^{-1}(cx)\right)}{x}\right)dx$$
$$= d\int \frac{a+b\tan^{-1}(cx)}{x^2}\,dx+(icd)\int \frac{a+b\tan^{-1}(cx)}{x}\,dx$$
$$= -\frac{d\left(a+b\tan^{-1}(cx)\right)}{x}+iacd\log(x)-\frac{1}{2}(bcd)\int \frac{\log(1-icx)}{x}\,dx+\frac{1}{2}(bcd)\int \frac{\log(1+$$
$$= -\frac{d\left(a+b\tan^{-1}(cx)\right)}{x}+iacd\log(x)-\frac{1}{2}bcd\mathrm{Li}_2(-icx)+\frac{1}{2}bcd\mathrm{Li}_2(icx)+\frac{1}{2}(bcd)\,\mathrm{Subst}$$
$$= -\frac{d\left(a+b\tan^{-1}(cx)\right)}{x}+iacd\log(x)-\frac{1}{2}bcd\mathrm{Li}_2(-icx)+\frac{1}{2}bcd\mathrm{Li}_2(icx)+\frac{1}{2}(bcd)\,\mathrm{Subst}$$
$$= -\frac{d\left(a+b\tan^{-1}(cx)\right)}{x}+iacd\log(x)+bcd\log(x)-\frac{1}{2}bcd\log\left(1+c^2x^2\right)-\frac{1}{2}bcd\mathrm{Li}_2(-i$$

Mathematica [A] time = 0.048796, size = 75, normalized size = 0.97

$$\frac{d\left(-bcx\mathrm{PolyLog}(2,-icx)+bcx\mathrm{PolyLog}(2,icx)+2iacx\log(x)-2a-bcx\log\left(c^2x^2+1\right)+2bcx\log(x)-2b\tan^{-1}(cx)\right)}{2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + I*c*d*x)*(a + b*ArcTan[c*x]))/x^2,x]
```

```
[Out] (d*(-2*a - 2*b*ArcTan[c*x] + (2*I)*a*c*x*Log[x] + 2*b*c*x*Log[x] - b*c*x*Lo
g[1 + c^2*x^2] - b*c*x*PolyLog[2, (-I)*c*x] + b*c*x*PolyLog[2, I*c*x]))/(2*
x)
```

Maple [A] time = 0.044, size = 127, normalized size = 1.7

$$-\frac{da}{x}+icda\ln(cx)-\frac{db\arctan(cx)}{x}+icdb\arctan(cx)\ln(cx)-\frac{cdb\ln(cx)\ln(1+icx)}{2}+\frac{cdb\ln(cx)\ln(1-icx)}{2}-\frac{cdb\mathrm{dil}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)*(a+b*arctan(c*x))/x^2,x)
```

```
[Out] -d*a/x+I*c*d*a*ln(c*x)-d*b*arctan(c*x)/x+I*c*d*b*arctan(c*x)*ln(c*x)-1/2*c*
d*b*ln(c*x)*ln(1+I*c*x)+1/2*c*d*b*ln(c*x)*ln(1-I*c*x)-1/2*c*d*b*dilog(1+I*c
*x)+1/2*c*d*b*dilog(1-I*c*x)-1/2*b*c*d*ln(c^2*x^2+1)+c*d*b*ln(c*x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$i\,bcd\int \frac{\arctan(cx)}{x}\,dx + i\,acd\log(x) - \frac{1}{2}\left(c\left(\log\left(c^2x^2+1\right) - \log\left(x^2\right)\right) + \frac{2\,\arctan(cx)}{x}\right)bd - \frac{ad}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)*(a+b*arctan(c*x))/x^2,x, algorithm="maxima")
```

```
[Out] I*b*c*d*integrate(arctan(c*x)/x, x) + I*a*c*d*log(x) - 1/2*(c*(log(c^2*x^2
+ 1) - log(x^2)) + 2*arctan(c*x)/x)*b*d - a*d/x
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{2i\,acdx + 2\,ad - (bcdx - i\,bd)\log\left(-\frac{cx+i}{cx-i}\right)}{2\,x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)*(a+b*arctan(c*x))/x^2,x, algorithm="fricas")
```

```
[Out] integral(1/2*(2*I*a*c*d*x + 2*a*d - (b*c*d*x - I*b*d)*log(-(c*x + I)/(c*x -
 I)))/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d\left(\int \frac{a}{x^2}\,dx + \int \frac{b\,\text{atan}(cx)}{x^2}\,dx + \int \frac{iac}{x}\,dx + \int \frac{ibc\,\text{atan}(cx)}{x}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)*(a+b*atan(c*x))/x**2,x)
```

```
[Out] d*(Integral(a/x**2, x) + Integral(b*atan(c*x)/x**2, x) + Integral(I*a*c/x,
x) + Integral(I*b*c*atan(c*x)/x, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(icdx + d)(b \arctan(cx) + a)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)*(a+b*arctan(c*x))/x^2,x, algorithm="giac")
```

```
[Out] integrate((I*c*d*x + d)*(b*arctan(c*x) + a)/x^2, x)
```

3.7 $\int \frac{(d+icdx)\left(a+b\tan^{-1}(cx)\right)}{x^3}\,dx$

Optimal. Leaf size=65

$$-\frac{d(1+icx)^2\left(a+b\tan^{-1}(cx)\right)}{2x^2}+ibc^2d\log(x)-ibc^2d\log(cx+i)-\frac{bcd}{2x}$$

```
[Out] -(b*c*d)/(2*x) - (d*(1 + I*c*x)^2*(a + b*ArcTan[c*x]))/(2*x^2) + I*b*c^2*d*
Log[x] - I*b*c^2*d*Log[I + c*x]
```

Rubi [A] time = 0.0552368, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {37, 4872, 12, 77}

$$-\frac{d(1+icx)^2\left(a+b\tan^{-1}(cx)\right)}{2x^2}+ibc^2d\log(x)-ibc^2d\log(cx+i)-\frac{bcd}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)*(a + b*ArcTan[c*x]))/x^3,x]
```

```
[Out] -(b*c*d)/(2*x) - (d*(1 + I*c*x)^2*(a + b*ArcTan[c*x]))/(2*x^2) + I*b*c^2*d*
Log[x] - I*b*c^2*d*Log[I + c*x]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 4872

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
+ b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2), x
], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2*m
] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int \frac{(d+icdx)\left(a+b\tan^{-1}(cx)\right)}{x^3}\,dx &= -\frac{d(1+icx)^2\left(a+b\tan^{-1}(cx)\right)}{2x^2}-(bc)\int \frac{d(-i+cx)}{2x^2(i+cx)}\,dx\\ &= -\frac{d(1+icx)^2\left(a+b\tan^{-1}(cx)\right)}{2x^2}-\frac{1}{2}(bcd)\int \frac{-i+cx}{x^2(i+cx)}\,dx\\ &= -\frac{d(1+icx)^2\left(a+b\tan^{-1}(cx)\right)}{2x^2}-\frac{1}{2}(bcd)\int \left(-\frac{1}{x^2}-\frac{2ic}{x}+\frac{2ic^2}{i+cx}\right)dx\\ &= -\frac{bcd}{2x}-\frac{d(1+icx)^2\left(a+b\tan^{-1}(cx)\right)}{2x^2}+ibc^2d\log(x)-ibc^2d\log(i+cx)\end{aligned}$$

Mathematica [C] time = 0.0547187, size = 88, normalized size = 1.35

$$-\frac{bcd\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-c^2x^2\right)}{2x}-\frac{d\left(a+b\tan^{-1}(cx)\right)}{2x^2}-\frac{icd\left(a+b\tan^{-1}(cx)\right)}{x}+\frac{1}{2}ibc^2d\left(2\log(x)-\log\left(c^2x^2\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + I*c*d*x)*(a + b*ArcTan[c*x]))/x^3,x]
```

```
[Out] -(d*(a + b*ArcTan[c*x]))/(2*x^2) - (I*c*d*(a + b*ArcTan[c*x]))/x - (b*c*d*H
ypergeometric2F1[-1/2, 1, 1/2, -(c^2*x^2)])/(2*x) + (I/2)*b*c^2*d*(2*Log[x]
 - Log[1 + c^2*x^2])
```

Maple [A] time = 0.034, size = 91, normalized size = 1.4

$$-\frac{da}{2x^2}-\frac{idca}{x}-\frac{db\arctan(cx)}{2x^2}-\frac{idcb\arctan(cx)}{x}-\frac{i}{2}c^2db\ln\left(c^2x^2+1\right)-\frac{bc^2d\arctan(cx)}{2}-\frac{bcd}{2x}+ic^2db\ln(cx)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)*(a+b*arctan(c*x))/x^3,x)
```

```
[Out] -1/2*a*d/x^2-I*c*d*a/x-1/2*d*b*arctan(c*x)/x^2-I*c*d*b*arctan(c*x)/x-1/2*I*
c^2*d*b*ln(c^2*x^2+1)-1/2*b*c^2*d*arctan(c*x)-1/2*b*c*d/x+I*c^2*d*b*ln(c*x)
```

Maxima [A] time = 1.49313, size = 101, normalized size = 1.55

$$-\frac{1}{2}i\left(c\left(\log\left(c^2x^2+1\right)-\log\left(x^2\right)\right)+\frac{2\arctan(cx)}{x}\right)bcd-\frac{1}{2}\left(\left(c\arctan(cx)+\frac{1}{x}\right)c+\frac{\arctan(cx)}{x^2}\right)bd-\frac{iacd}{x}-\frac{ad}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)*(a+b*arctan(c*x))/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*I*(c*(log(c^2*x^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*b*c*d - 1/2*((c*
arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*b*d - I*a*c*d/x - 1/2*a*d/x^2
```

Fricas [A] time = 3.01925, size = 244, normalized size = 3.75

$$\frac{4i\,bc^2dx^2\log(x)-3i\,bc^2dx^2\log\left(\frac{cx+i}{c}\right)-i\,bc^2dx^2\log\left(\frac{cx-i}{c}\right)+(-4i\,a-2\,b)cdx-2\,ad+(2\,bcdx-i\,bd)\log\left(-\frac{cx+i}{cx-i}\right)}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)*(a+b*arctan(c*x))/x^3,x, algorithm="fricas")
```

```
[Out] 1/4*(4*I*b*c^2*d*x^2*log(x) - 3*I*b*c^2*d*x^2*log((c*x + I)/c) - I*b*c^2*d*
x^2*log((c*x - I)/c) + (-4*I*a - 2*b)*c*d*x - 2*a*d + (2*b*c*d*x - I*b*d)*l
og(-(c*x + I)/(c*x - I)))/x^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)*(a+b*atan(c*x))/x**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.14698, size = 123, normalized size = 1.89

$$-\frac{bc^2dix^2\log(cix+1) + 3\,bc^2dix^2\log(-cix+1) - 4\,bc^2dix^2\log(x) + 4\,bcdix\arctan(cx) + 4\,acdix + 2\,bcdx + 2\,bd\,\mathrm{arcta}}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)*(a+b*arctan(c*x))/x^3,x, algorithm="giac")
```

```
[Out] -1/4*(b*c^2*d*i*x^2*log(c*i*x + 1) + 3*b*c^2*d*i*x^2*log(-c*i*x + 1) - 4*b*
c^2*d*i*x^2*log(x) + 4*b*c*d*i*x*arctan(c*x) + 4*a*c*d*i*x + 2*b*c*d*x + 2*
b*d*arctan(c*x) + 2*a*d)/x^2
```

3.8 $\int \frac{(d+icdx)\left(a+b\tan^{-1}(cx)\right)}{x^4}\,dx$

Optimal. Leaf size=106

$$-\frac{icd\left(a+b\tan^{-1}(cx)\right)}{2x^2}-\frac{d\left(a+b\tan^{-1}(cx)\right)}{3x^3}-\frac{ibc^2d}{2x}-\frac{1}{3}bc^3d\log(x)-\frac{1}{12}bc^3d\log(-cx+i)+\frac{5}{12}bc^3d\log(cx+i)-\frac{bcd}{6x^2}$$

```
[Out] -(b*c*d)/(6*x^2) - ((I/2)*b*c^2*d)/x - (d*(a + b*ArcTan[c*x]))/(3*x^3) - ((
I/2)*c*d*(a + b*ArcTan[c*x]))/x^2 - (b*c^3*d*Log[x])/3 - (b*c^3*d*Log[I - c
*x])/12 + (5*b*c^3*d*Log[I + c*x])/12
```

Rubi [A] time = 0.0908906, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {43, 4872, 12, 801}

$$-\frac{icd\left(a+b\tan^{-1}(cx)\right)}{2x^2}-\frac{d\left(a+b\tan^{-1}(cx)\right)}{3x^3}-\frac{ibc^2d}{2x}-\frac{1}{3}bc^3d\log(x)-\frac{1}{12}bc^3d\log(-cx+i)+\frac{5}{12}bc^3d\log(cx+i)-\frac{bcd}{6x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)*(a + b*ArcTan[c*x]))/x^4,x]
```

```
[Out] -(b*c*d)/(6*x^2) - ((I/2)*b*c^2*d)/x - (d*(a + b*ArcTan[c*x]))/(3*x^3) - ((
I/2)*c*d*(a + b*ArcTan[c*x]))/x^2 - (b*c^3*d*Log[x])/3 - (b*c^3*d*Log[I - c
*x])/12 + (5*b*c^3*d*Log[I + c*x])/12
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 4872

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
+ b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2), x
], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2*m
] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{(d+icdx)\left(a+b\tan^{-1}(cx)\right)}{x^4}\,dx = -\frac{d\left(a+b\tan^{-1}(cx)\right)}{3x^3}-\frac{icd\left(a+b\tan^{-1}(cx)\right)}{2x^2}-(bc)\int \frac{d(-2-3icx)}{6x^3\left(1+c^2x^2\right)}\,dx$$
$$= -\frac{d\left(a+b\tan^{-1}(cx)\right)}{3x^3}-\frac{icd\left(a+b\tan^{-1}(cx)\right)}{2x^2}-\frac{1}{6}(bcd)\int \frac{-2-3icx}{x^3\left(1+c^2x^2\right)}\,dx$$
$$= -\frac{d\left(a+b\tan^{-1}(cx)\right)}{3x^3}-\frac{icd\left(a+b\tan^{-1}(cx)\right)}{2x^2}-\frac{1}{6}(bcd)\int \left(-\frac{2}{x^3}-\frac{3ic}{x^2}+\frac{2c^2}{x}+\frac{c}{2(-i}\right.$$
$$= -\frac{bcd}{6x^2}-\frac{ibc^2d}{2x}-\frac{d\left(a+b\tan^{-1}(cx)\right)}{3x^3}-\frac{icd\left(a+b\tan^{-1}(cx)\right)}{2x^2}-\frac{1}{3}bc^3d\log(x)-\frac{1}{12}bc^3$$

Mathematica [C] time = 0.0520788, size = 94, normalized size = 0.89

$$-\frac{d\left(3ibc^2x^2\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-c^2x^2\right)+3iacx+2a+2bc^3x^3\log(x)-bc^3x^3\log\left(c^2x^2+1\right)+bcx+b(2+3ic\right.}{6x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + I*c*d*x)*(a + b*ArcTan[c*x]))/x^4,x]
```

```
[Out] -(d*(2*a + (3*I)*a*c*x + b*c*x + b*(2 + (3*I)*c*x)*ArcTan[c*x] + (3*I)*b*c^
2*x^2*Hypergeometric2F1[-1/2, 1, 1/2, -(c^2*x^2)] + 2*b*c^3*x^3*Log[x] - b*
c^3*x^3*Log[1 + c^2*x^2]))/(6*x^3)
```

Maple [A] time = 0.035, size = 101, normalized size = 1.

$$\frac{-\frac{i}{2}cda}{x^2}-\frac{da}{3\,x^3}-\frac{\frac{i}{2}cdb\arctan(cx)}{x^2}-\frac{db\arctan(cx)}{3\,x^3}+\frac{c^3db\ln\left(c^2x^2+1\right)}{6}-\frac{i}{2}c^3db\arctan(cx)-\frac{\frac{i}{2}c^2bd}{x}-\frac{bcd}{6\,x^2}-\frac{c^3db\ln(}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)*(a+b*arctan(c*x))/x^4,x)
```

```
[Out] -1/2*I*c*d*a/x^2-1/3*d*a/x^3-1/2*I*c*d*b*arctan(c*x)/x^2-1/3*d*b*arctan(c*x
)/x^3+1/6*c^3*d*b*ln(c^2*x^2+1)-1/2*I*c^3*d*b*arctan(c*x)-1/2*I*b*c^2*d/x-1
/6*b*c*d/x^2-1/3*c^3*d*b*ln(c*x)
```

Maxima [A] time = 1.48848, size = 117, normalized size = 1.1

$$-\frac{1}{2}i\left(\left(c\arctan(cx)+\frac{1}{x}\right)c+\frac{\arctan(cx)}{x^2}\right)bcd+\frac{1}{6}\left(\left(c^2\log\left(c^2x^2+1\right)-c^2\log\left(x^2\right)-\frac{1}{x^2}\right)c-\frac{2\arctan(cx)}{x^3}\right)bd-\frac{iacd}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)*(a+b*arctan(c*x))/x^4,x, algorithm="maxima")
```

```
[Out] -1/2*I*((c*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*b*c*d + 1/6*((c^2*log(c^
2*x^2 + 1) - c^2*log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^3)*b*d - 1/2*I*a*c*d
/x^2 - 1/3*a*d/x^3
```

Fricas [A] time = 2.95781, size = 266, normalized size = 2.51

$$-\frac{4\,bc^3dx^3\log(x)-5\,bc^3dx^3\log\left(\frac{cx+i}{c}\right)+bc^3dx^3\log\left(\frac{cx-i}{c}\right)+6i\,bc^2dx^2-(-6i\,a-2\,b)cdx+4\,ad-(3\,bcdx-2i\,bd)\log\left(-\right.}{12\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)*(a+b*arctan(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] -1/12*(4*b*c^3*d*x^3*log(x) - 5*b*c^3*d*x^3*log((c*x + I)/c) + b*c^3*d*x^3*
log((c*x - I)/c) + 6*I*b*c^2*d*x^2 - (-6*I*a - 2*b)*c*d*x + 4*a*d - (3*b*c*
d*x - 2*I*b*d)*log(-(c*x + I)/(c*x - I)))/x^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)*(a+b*atan(c*x))/x**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19741, size = 134, normalized size = 1.26

$$\frac{5\,bc^3dx^3\log\left(cx+i\right)-bc^3dx^3\log\left(cx-i\right)-4\,bc^3dx^3\log\left(x\right)-6\,bc^2dix^2-6\,bcdix\arctan\left(cx\right)-6\,acdix-2\,bcdx-4\,bd\,\mathrm{arc}}{12\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)*(a+b*arctan(c*x))/x^4,x, algorithm="giac")
```

```
[Out] 1/12*(5*b*c^3*d*x^3*log(c*x + i) - b*c^3*d*x^3*log(c*x - i) - 4*b*c^3*d*x^3
*log(x) - 6*b*c^2*d*i*x^2 - 6*b*c*d*i*x*arctan(c*x) - 6*a*c*d*i*x - 2*b*c*d
*x - 4*b*d*arctan(c*x) - 4*a*d)/x^3
```

3.9 $\int \frac{(d+icdx)\left(a+b\tan^{-1}(cx)\right)}{x^5}\,dx$

Optimal. Leaf size=124

$$-\frac{icd\left(a+b\tan^{-1}(cx)\right)}{3x^3}-\frac{d\left(a+b\tan^{-1}(cx)\right)}{4x^4}-\frac{ibc^2d}{6x^2}+\frac{bc^3d}{4x}-\frac{1}{3}ibc^4d\log(x)+\frac{1}{24}ibc^4d\log(-cx+i)+\frac{7}{24}ibc^4d\log(cx$$

```
[Out] -(b*c*d)/(12*x^3) - ((I/6)*b*c^2*d)/x^2 + (b*c^3*d)/(4*x) - (d*(a + b*ArcTa
n[c*x]))/(4*x^4) - ((I/3)*c*d*(a + b*ArcTan[c*x]))/x^3 - (I/3)*b*c^4*d*Log[
x] + (I/24)*b*c^4*d*Log[I - c*x] + ((7*I)/24)*b*c^4*d*Log[I + c*x]
```

Rubi [A] time = 0.0964044, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {43, 4872, 12, 801}

$$-\frac{icd\left(a+b\tan^{-1}(cx)\right)}{3x^3}-\frac{d\left(a+b\tan^{-1}(cx)\right)}{4x^4}-\frac{ibc^2d}{6x^2}+\frac{bc^3d}{4x}-\frac{1}{3}ibc^4d\log(x)+\frac{1}{24}ibc^4d\log(-cx+i)+\frac{7}{24}ibc^4d\log(cx$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)*(a + b*ArcTan[c*x]))/x^5,x]
```


```
[Out] -(b*c*d)/(12*x^3) - ((I/6)*b*c^2*d)/x^2 + (b*c^3*d)/(4*x) - (d*(a + b*ArcTa
n[c*x]))/(4*x^4) - ((I/3)*c*d*(a + b*ArcTan[c*x]))/x^3 - (I/3)*b*c^4*d*Log[
x] + (I/24)*b*c^4*d*Log[I - c*x] + ((7*I)/24)*b*c^4*d*Log[I + c*x]
```


Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 4872

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
+ b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2), x
], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2*m
] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+icdx)\left(a+b\tan^{-1}(cx)\right)}{x^5}\,dx &= -\frac{d\left(a+b\tan^{-1}(cx)\right)}{4x^4}-\frac{icd\left(a+b\tan^{-1}(cx)\right)}{3x^3}-(bc)\int \frac{d(-3-4icx)}{12x^4\left(1+c^2x^2\right)}\,dx\\ &= -\frac{d\left(a+b\tan^{-1}(cx)\right)}{4x^4}-\frac{icd\left(a+b\tan^{-1}(cx)\right)}{3x^3}-\frac{1}{12}(bcd)\int \frac{-3-4icx}{x^4\left(1+c^2x^2\right)}\,dx\\ &= -\frac{d\left(a+b\tan^{-1}(cx)\right)}{4x^4}-\frac{icd\left(a+b\tan^{-1}(cx)\right)}{3x^3}-\frac{1}{12}(bcd)\int \left(-\frac{3}{x^4}-\frac{4ic}{x^3}+\frac{3c^2}{x^2}+\frac{4ic^3}{x}\right.\\ &= -\frac{bcd}{12x^3}-\frac{ibc^2d}{6x^2}+\frac{bc^3d}{4x}-\frac{d\left(a+b\tan^{-1}(cx)\right)}{4x^4}-\frac{icd\left(a+b\tan^{-1}(cx)\right)}{3x^3}-\frac{1}{3}ibc^4d\log(x\end{aligned}$$

Mathematica [C] time = 0.0513447, size = 99, normalized size = 0.8

$$-\frac{bcd\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-c^2x^2\right)}{12x^3}-\frac{icd\left(a+b\tan^{-1}(cx)\right)}{3x^3}-\frac{d\left(a+b\tan^{-1}(cx)\right)}{4x^4}-\frac{1}{6}ibc^2d\left(-c^2\log\left(c^2x^2+1\right)\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + I*c*d*x)*(a + b*ArcTan[c*x]))/x^5,x]
```

```
[Out] -(d*(a + b*ArcTan[c*x]))/(4*x^4) - ((I/3)*c*d*(a + b*ArcTan[c*x]))/x^3 - (b
*c*d*Hypergeometric2F1[-3/2, 1, -1/2, -(c^2*x^2)]])/(12*x^3) - (I/6)*b*c^2*d
*(x^(-2) + 2*c^2*Log[x] - c^2*Log[1 + c^2*x^2])
```

Maple [A] time = 0.036, size = 112, normalized size = 0.9

$$-\frac{da}{4x^4}-\frac{\frac{i}{3}cda}{x^3}-\frac{db\arctan(cx)}{4x^4}-\frac{\frac{i}{3}cdb\arctan(cx)}{x^3}+\frac{i}{6}c^4db\ln\left(c^2x^2+1\right)+\frac{c^4db\arctan(cx)}{4}-\frac{\frac{i}{6}bc^2d}{x^2}-\frac{i}{3}c^4db\ln(cx)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)*(a+b*arctan(c*x))/x^5,x)
```

```
[Out] -1/4*d*a/x^4-1/3*I*c*d*a/x^3-1/4*d*b*arctan(c*x)/x^4-1/3*I*c*d*b*arctan(c*x
)/x^3+1/6*I*c^4*d*b*ln(c^2*x^2+1)+1/4*c^4*d*b*arctan(c*x)-1/6*I*b*c^2*d/x^2
-1/3*I*c^4*d*b*ln(c*x)-1/12*b*c*d/x^3+1/4*b*c^3*d/x
```

Maxima [A] time = 1.49316, size = 138, normalized size = 1.11

$$\frac{1}{6}i\left(\left(c^2\log\left(c^2x^2+1\right)-c^2\log\left(x^2\right)-\frac{1}{x^2}\right)c-\frac{2\arctan\left(cx\right)}{x^3}\right)bcd+\frac{1}{12}\left(\left(3\,c^3\arctan\left(cx\right)+\frac{3\,c^2x^2-1}{x^3}\right)c-\frac{3\arctan\left(cx\right)}{x^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)*(a+b*arctan(c*x))/x^5,x, algorithm="maxima")
```

```
[Out] 1/6*I*((c^2*log(c^2*x^2 + 1) - c^2*log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^3)
*b*c*d + 1/12*((3*c^3*arctan(c*x) + (3*c^2*x^2 - 1)/x^3)*c - 3*arctan(c*x)/
x^4)*b*d - 1/3*I*a*c*d/x^3 - 1/4*a*d/x^4
```

Fricas [A] time = 2.85988, size = 296, normalized size = 2.39

$$\frac{-8i\,bc^4dx^4\log\left(x\right)+7i\,bc^4dx^4\log\left(\frac{cx+i}{c}\right)+i\,bc^4dx^4\log\left(\frac{cx-i}{c}\right)+6\,bc^3dx^3-4i\,bc^2dx^2+\left(-8i\,a-2\,b\right)cdx-6\,ad+\left(4\,bcdx\right.}{24\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)*(a+b*arctan(c*x))/x^5,x, algorithm="fricas")
```

```
[Out] 1/24*(-8*I*b*c^4*d*x^4*log(x) + 7*I*b*c^4*d*x^4*log((c*x + I)/c) + I*b*c^4*
d*x^4*log((c*x - I)/c) + 6*b*c^3*d*x^3 - 4*I*b*c^2*d*x^2 + (-8*I*a - 2*b)*c
*d*x - 6*a*d + (4*b*c*d*x - 3*I*b*d)*log(-(c*x + I)/(c*x - I)))/x^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)*(a+b*atan(c*x))/x**5,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.23585, size = 151, normalized size = 1.22

$$\frac{7\,bc^4dix^4\log\left(cix-1\right)+bc^4dix^4\log\left(-cix-1\right)-8\,bc^4dix^4\log\left(x\right)+6\,bc^3dx^3-4\,bc^2dix^2-8\,bcdix\arctan\left(cx\right)-8\,acdix-}{24\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)*(a+b*arctan(c*x))/x^5,x, algorithm="giac")
```

```
[Out] 1/24*(7*b*c^4*d*i*x^4*log(c*i*x - 1) + b*c^4*d*i*x^4*log(-c*i*x - 1) - 8*b*
c^4*d*i*x^4*log(x) + 6*b*c^3*d*x^3 - 4*b*c^2*d*i*x^2 - 8*b*c*d*i*x*arctan(c
*x) - 8*a*c*d*i*x - 2*b*c*d*x - 6*b*d*arctan(c*x) - 6*a*d)/x^4
```

3.10 $\int x^3(d+icdx)^2\left(a+b\tan^{-1}(cx)\right)\,dx$

Optimal. Leaf size=166

$$-\frac{1}{6}c^2d^2x^6\left(a+b\tan^{-1}(cx)\right)+\frac{2}{5}icd^2x^5\left(a+b\tan^{-1}(cx)\right)+\frac{1}{4}d^2x^4\left(a+b\tan^{-1}(cx)\right)+\frac{ibd^2x^2}{5c^2}-\frac{ibd^2\log\left(c^2x^2+1\right)}{5c^4}+\frac{5b}{1}$$

```
[Out] (5*b*d^2*x)/(12*c^3) + ((I/5)*b*d^2*x^2)/c^2 - (5*b*d^2*x^3)/(36*c) - (I/10
)*b*d^2*x^4 + (b*c*d^2*x^5)/30 - (5*b*d^2*ArcTan[c*x])/(12*c^4) + (d^2*x^4*
(a + b*ArcTan[c*x]))/4 + ((2*I)/5)*c*d^2*x^5*(a + b*ArcTan[c*x]) - (c^2*d^2
*x^6*(a + b*ArcTan[c*x]))/6 - ((I/5)*b*d^2*Log[1 + c^2*x^2])/c^4
```

Rubi [A] time = 0.16337, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {43, 4872, 12, 1802, 635, 203, 260}

$$-\frac{1}{6}c^2d^2x^6\left(a+b\tan^{-1}(cx)\right)+\frac{2}{5}icd^2x^5\left(a+b\tan^{-1}(cx)\right)+\frac{1}{4}d^2x^4\left(a+b\tan^{-1}(cx)\right)+\frac{ibd^2x^2}{5c^2}-\frac{ibd^2\log\left(c^2x^2+1\right)}{5c^4}+\frac{5b}{1}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + I*c*d*x)^2*(a + b*ArcTan[c*x]),x]
```

```
[Out] (5*b*d^2*x)/(12*c^3) + ((I/5)*b*d^2*x^2)/c^2 - (5*b*d^2*x^3)/(36*c) - (I/10
)*b*d^2*x^4 + (b*c*d^2*x^5)/30 - (5*b*d^2*ArcTan[c*x])/(12*c^4) + (d^2*x^4*
(a + b*ArcTan[c*x]))/4 + ((2*I)/5)*c*d^2*x^5*(a + b*ArcTan[c*x]) - (c^2*d^2
*x^6*(a + b*ArcTan[c*x]))/6 - ((I/5)*b*d^2*Log[1 + c^2*x^2])/c^4
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 4872

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
+ b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2), x
], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2*m
```

```
] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int x^3(d+icdx)^2\left(a+b\tan^{-1}(cx)\right)dx = \frac{1}{4}d^2x^4\left(a+b\tan^{-1}(cx)\right)+\frac{2}{5}icd^2x^5\left(a+b\tan^{-1}(cx)\right)-\frac{1}{6}c^2d^2x^6\left(a+b\tan^{-1}(cx\right.$$

$$=\frac{1}{4}d^2x^4\left(a+b\tan^{-1}(cx)\right)+\frac{2}{5}icd^2x^5\left(a+b\tan^{-1}(cx)\right)-\frac{1}{6}c^2d^2x^6\left(a+b\tan^{-1}(cx\right.$$

$$=\frac{1}{4}d^2x^4\left(a+b\tan^{-1}(cx)\right)+\frac{2}{5}icd^2x^5\left(a+b\tan^{-1}(cx)\right)-\frac{1}{6}c^2d^2x^6\left(a+b\tan^{-1}(cx\right.$$

$$=\frac{5bd^2x}{12c^3}+\frac{ibd^2x^2}{5c^2}-\frac{5bd^2x^3}{36c}-\frac{1}{10}ibd^2x^4+\frac{1}{30}bcd^2x^5+\frac{1}{4}d^2x^4\left(a+b\tan^{-1}(cx)\right)+$$

$$=\frac{5bd^2x}{12c^3}+\frac{ibd^2x^2}{5c^2}-\frac{5bd^2x^3}{36c}-\frac{1}{10}ibd^2x^4+\frac{1}{30}bcd^2x^5+\frac{1}{4}d^2x^4\left(a+b\tan^{-1}(cx)\right)+$$

$$=\frac{5bd^2x}{12c^3}+\frac{ibd^2x^2}{5c^2}-\frac{5bd^2x^3}{36c}-\frac{1}{10}ibd^2x^4+\frac{1}{30}bcd^2x^5-\frac{5bd^2\tan^{-1}(cx)}{12c^4}+\frac{1}{4}d^2x^4\Big($$

Mathematica [A] time = 0.128977, size = 124, normalized size = 0.75

$$\frac{d^2\left(3ac^4x^4\left(-10c^2x^2+24icx+15\right)+bcx\left(6c^4x^4-18ic^3x^3-25c^2x^2+36icx+75\right)-36ib\log\left(c^2x^2+1\right)+3b\left(-10c^6x^6+\right.\right.}{180c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + I*c*d*x)^2*(a + b*ArcTan[c*x]),x]
```

```
[Out] (d^2*(3*a*c^4*x^4*(15 + (24*I)*c*x - 10*c^2*x^2) + b*c*x*(75 + (36*I)*c*x -
 25*c^2*x^2 - (18*I)*c^3*x^3 + 6*c^4*x^4) + 3*b*(-25 + 15*c^4*x^4 + (24*I)*
c^5*x^5 - 10*c^6*x^6)*ArcTan[c*x] - (36*I)*b*Log[1 + c^2*x^2]))/(180*c^4)
```

Maple [A] time = 0.027, size = 166, normalized size = 1.

$$-\frac{c^2d^2ax^6}{6}+\frac{2\,i}{5}cd^2ax^5+\frac{d^2ax^4}{4}-\frac{c^2d^2b\arctan(cx)\,x^6}{6}+\frac{2\,i}{5}cd^2b\arctan(cx)\,x^5+\frac{d^2b\arctan(cx)\,x^4}{4}+\frac{5\,d^2bx}{12\,c^3}+\frac{bcd^2x^5}{30}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d+I*c*d*x)^2*(a+b*arctan(c*x)),x)
```

```
[Out] -1/6*c^2*d^2*a*x^6+2/5*I*c*d^2*a*x^5+1/4*d^2*a*x^4-1/6*c^2*d^2*b*arctan(c*x
)*x^6+2/5*I*c*d^2*b*arctan(c*x)*x^5+1/4*d^2*b*arctan(c*x)*x^4+5/12*b*d^2*x/
c^3+1/30*b*c*d^2*x^5-1/10*I*b*d^2*x^4-5/36*b*d^2*x^3/c+1/5*I*b*d^2*x^2/c^2-
1/5*I*b*d^2*ln(c^2*x^2+1)/c^4-5/12*b*d^2*arctan(c*x)/c^4
```

Maxima [A] time = 1.50962, size = 250, normalized size = 1.51

$$-\frac{1}{6}ac^2d^2x^6+\frac{2}{5}iacd^2x^5+\frac{1}{4}ad^2x^4-\frac{1}{90}\left(15x^6\arctan(cx)-c\left(\frac{3c^4x^5-5c^2x^3+15x}{c^6}-\frac{15\arctan(cx)}{c^7}\right)\right)bc^2d^2+\frac{1}{10}i\Big(4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d+I*c*d*x)^2*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] -1/6*a*c^2*d^2*x^6 + 2/5*I*a*c*d^2*x^5 + 1/4*a*d^2*x^4 - 1/90*(15*x^6*arcta
n(c*x) - c*((3*c^4*x^5 - 5*c^2*x^3 + 15*x)/c^6 - 15*arctan(c*x)/c^7))*b*c^2
*d^2 + 1/10*I*(4*x^5*arctan(c*x) - c*((c^2*x^4 - 2*x^2)/c^4 + 2*log(c^2*x^2
 + 1)/c^6))*b*c*d^2 + 1/12*(3*x^4*arctan(c*x) - c*((c^2*x^3 - 3*x)/c^4 + 3*
arctan(c*x)/c^5))*b*d^2
```

Fricas [A] time = 2.83353, size = 404, normalized size = 2.43

$$-\frac{60ac^6d^2x^6-(144ia+12b)c^5d^2x^5-18(5a-2ib)c^4d^2x^4+50bc^3d^2x^3-72ibc^2d^2x^2-150bcd^2x+147ibd^2\log\left(\frac{cx+i}{c}\right)-}{360c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d+I*c*d*x)^2*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] -1/360*(60*a*c^6*d^2*x^6 - (144*I*a + 12*b)*c^5*d^2*x^5 - 18*(5*a - 2*I*b)*
c^4*d^2*x^4 + 50*b*c^3*d^2*x^3 - 72*I*b*c^2*d^2*x^2 - 150*b*c*d^2*x + 147*I
*b*d^2*log((c*x + I)/c) - 3*I*b*d^2*log((c*x - I)/c) - (-30*I*b*c^6*d^2*x^6
 - 72*b*c^5*d^2*x^5 + 45*I*b*c^4*d^2*x^4)*log(-(c*x + I)/(c*x - I)))/c^4
```

Sympy [A] time = 2.7756, size = 233, normalized size = 1.4

$$-\frac{ac^2d^2x^6}{6}-\frac{5bd^2x^3}{36c}+\frac{ibd^2x^2}{5c^2}+\frac{5bd^2x}{12c^3}+\frac{ibd^2\log\left(x-\frac{i}{c}\right)}{120c^4}-\frac{49ibd^2\log\left(x+\frac{i}{c}\right)}{120c^4}-x^5\left(-\frac{2iacd^2}{5}-\frac{bcd^2}{30}\right)-x^4\left(-\frac{ad^2}{4}+\frac{ibd^2}{10}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(d+I*c*d*x)**2*(a+b*atan(c*x)),x)
```

```
[Out] -a*c**2*d**2*x**6/6 - 5*b*d**2*x**3/(36*c) + I*b*d**2*x**2/(5*c**2) + 5*b*d
**2*x/(12*c**3) + I*b*d**2*log(x - I/c)/(120*c**4) - 49*I*b*d**2*log(x + I/
c)/(120*c**4) - x**5*(-2*I*a*c*d**2/5 - b*c*d**2/30) - x**4*(-a*d**2/4 + I*
b*d**2/10) + (-I*b*c**2*d**2*x**6/12 - b*c*d**2*x**5/5 + I*b*d**2*x**4/8)*l
og(-I*c*x + 1) + (I*b*c**2*d**2*x**6/12 + b*c*d**2*x**5/5 - I*b*d**2*x**4/8
)*log(I*c*x + 1)
```

Giac [A] time = 1.20699, size = 247, normalized size = 1.49

$$-\frac{60\,bc^6d^2ix^6\arctan(cx)+60\,ac^6d^2ix^6-12\,bc^5d^2ix^5+144\,bc^5d^2x^5\arctan(cx)+144\,ac^5d^2x^5-90\,bc^4d^2ix^4\arctan(cx}{360\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d+I*c*d*x)^2*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] -1/360*(60*b*c^6*d^2*i*x^6*arctan(c*x) + 60*a*c^6*d^2*i*x^6 - 12*b*c^5*d^2*
i*x^5 + 144*b*c^5*d^2*x^5*arctan(c*x) + 144*a*c^5*d^2*x^5 - 90*b*c^4*d^2*i*
x^4*arctan(c*x) - 90*a*c^4*d^2*i*x^4 - 36*b*c^4*d^2*x^4 + 50*b*c^3*d^2*i*x^
3 + 72*b*c^2*d^2*x^2 - 150*b*c*d^2*i*x + 3*b*d^2*log(c*i*x + 1) - 147*b*d^2
*log(-c*i*x + 1))/(c^4*i)
```

3.11 $\int x^2(d+icdx)^2\left(a+b\tan^{-1}(cx)\right)\,dx$

Optimal. Leaf size=152

$$-\frac{1}{5}c^2d^2x^5\left(a+b\tan^{-1}(cx)\right)+\frac{1}{2}icd^2x^4\left(a+b\tan^{-1}(cx)\right)+\frac{1}{3}d^2x^3\left(a+b\tan^{-1}(cx)\right)+\frac{4bd^2\log\left(c^2x^2+1\right)}{15c^3}+\frac{ibd^2x}{2c^2}-\frac{ibd^2}{}$$

```
[Out] ((I/2)*b*d^2*x)/c^2 - (4*b*d^2*x^2)/(15*c) - (I/6)*b*d^2*x^3 + (b*c*d^2*x^4
)/20 - ((I/2)*b*d^2*ArcTan[c*x])/c^3 + (d^2*x^3*(a + b*ArcTan[c*x]))/3 + (I
/2)*c*d^2*x^4*(a + b*ArcTan[c*x]) - (c^2*d^2*x^5*(a + b*ArcTan[c*x]))/5 + (
4*b*d^2*Log[1 + c^2*x^2])/(15*c^3)
```

Rubi [A] time = 0.150438, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {43, 4872, 12, 1802, 635, 203, 260}

$$-\frac{1}{5}c^2d^2x^5\left(a+b\tan^{-1}(cx)\right)+\frac{1}{2}icd^2x^4\left(a+b\tan^{-1}(cx)\right)+\frac{1}{3}d^2x^3\left(a+b\tan^{-1}(cx)\right)+\frac{4bd^2\log\left(c^2x^2+1\right)}{15c^3}+\frac{ibd^2x}{2c^2}-\frac{ibd^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + I*c*d*x)^2*(a + b*ArcTan[c*x]),x]
```

```
[Out] ((I/2)*b*d^2*x)/c^2 - (4*b*d^2*x^2)/(15*c) - (I/6)*b*d^2*x^3 + (b*c*d^2*x^4
)/20 - ((I/2)*b*d^2*ArcTan[c*x])/c^3 + (d^2*x^3*(a + b*ArcTan[c*x]))/3 + (I
/2)*c*d^2*x^4*(a + b*ArcTan[c*x]) - (c^2*d^2*x^5*(a + b*ArcTan[c*x]))/5 + (
4*b*d^2*Log[1 + c^2*x^2])/(15*c^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 4872

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
+ b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2), x
], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2*m
```

```
] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int x^2(d+icdx)^2\left(a+b\tan^{-1}(cx)\right)dx=\frac{1}{3}d^2x^3\left(a+b\tan^{-1}(cx)\right)+\frac{1}{2}icd^2x^4\left(a+b\tan^{-1}(cx)\right)-\frac{1}{5}c^2d^2x^5\left(a+b\tan^{-1}(cx)\right.$$

$$=\frac{1}{3}d^2x^3\left(a+b\tan^{-1}(cx)\right)+\frac{1}{2}icd^2x^4\left(a+b\tan^{-1}(cx)\right)-\frac{1}{5}c^2d^2x^5\left(a+b\tan^{-1}(cx)\right.$$

$$=\frac{1}{3}d^2x^3\left(a+b\tan^{-1}(cx)\right)+\frac{1}{2}icd^2x^4\left(a+b\tan^{-1}(cx)\right)-\frac{1}{5}c^2d^2x^5\left(a+b\tan^{-1}(cx)\right.$$

$$=\frac{ibd^2x}{2c^2}-\frac{4bd^2x^2}{15c}-\frac{1}{6}ibd^2x^3+\frac{1}{20}bcd^2x^4+\frac{1}{3}d^2x^3\left(a+b\tan^{-1}(cx)\right)+\frac{1}{2}icd^2x^4\left(a\right.$$

$$=\frac{ibd^2x}{2c^2}-\frac{4bd^2x^2}{15c}-\frac{1}{6}ibd^2x^3+\frac{1}{20}bcd^2x^4+\frac{1}{3}d^2x^3\left(a+b\tan^{-1}(cx)\right)+\frac{1}{2}icd^2x^4\left(a\right.$$

$$=\frac{ibd^2x}{2c^2}-\frac{4bd^2x^2}{15c}-\frac{1}{6}ibd^2x^3+\frac{1}{20}bcd^2x^4-\frac{ibd^2\tan^{-1}(cx)}{2c^3}+\frac{1}{3}d^2x^3\left(a+b\tan^{-1}(c\right.$$

Mathematica [A] time = 0.108917, size = 116, normalized size = 0.76

$$\frac{d^2\left(2ac^3x^3\left(-6c^2x^2+15icx+10\right)+bcx\left(3c^3x^3-10ic^2x^2-16cx+30i\right)+16b\log\left(c^2x^2+1\right)+2b\left(-6c^5x^5+15ic^4x^4+10c\right.\right.}{60c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + I*c*d*x)^2*(a + b*ArcTan[c*x]),x]
```

```
[Out] (d^2*(2*a*c^3*x^3*(10 + (15*I)*c*x - 6*c^2*x^2) + b*c*x*(30*I - 16*c*x - (1
0*I)*c^2*x^2 + 3*c^3*x^3) + 2*b*(-15*I + 10*c^3*x^3 + (15*I)*c^4*x^4 - 6*c^
5*x^5)*ArcTan[c*x] + 16*b*Log[1 + c^2*x^2]))/(60*c^3)
```

Maple [A] time = 0.027, size = 154, normalized size = 1.

$$-\frac{c^2d^2ax^5}{5}+\frac{i}{2}cd^2ax^4+\frac{d^2ax^3}{3}-\frac{c^2d^2b\arctan(cx)x^5}{5}+\frac{i}{2}cd^2b\arctan(cx)x^4+\frac{d^2b\arctan(cx)x^3}{3}+\frac{\frac{i}{2}bd^2x}{c^2}+\frac{bcd^2x^4}{20}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d+I*c*d*x)^2*(a+b*arctan(c*x)),x)
```

```
[Out] -1/5*c^2*d^2*a*x^5+1/2*I*c*d^2*a*x^4+1/3*d^2*a*x^3-1/5*c^2*d^2*b*arctan(c*x
)*x^5+1/2*I*c*d^2*b*arctan(c*x)*x^4+1/3*d^2*b*arctan(c*x)*x^3+1/2*I*b*d^2*x
/c^2+1/20*b*c*d^2*x^4-1/6*I*b*d^2*x^3-4/15*b*d^2*x^2/c+4/15*b*d^2*ln(c^2*x^
2+1)/c^3-1/2*I*b*d^2*arctan(c*x)/c^3
```

Maxima [A] time = 1.51143, size = 235, normalized size = 1.55

$$-\frac{1}{5}ac^2d^2x^5+\frac{1}{2}i\,acd^2x^4-\frac{1}{20}\left(4x^5\arctan(cx)-c\left(\frac{c^2x^4-2x^2}{c^4}+\frac{2\log\left(c^2x^2+1\right)}{c^6}\right)\right)bc^2d^2+\frac{1}{3}ad^2x^3+\frac{1}{6}i\left(3x^4\arctan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d+I*c*d*x)^2*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] -1/5*a*c^2*d^2*x^5 + 1/2*I*a*c*d^2*x^4 - 1/20*(4*x^5*arctan(c*x) - c*((c^2*
x^4 - 2*x^2)/c^4 + 2*log(c^2*x^2 + 1)/c^6))*b*c^2*d^2 + 1/3*a*d^2*x^3 + 1/6
*I*(3*x^4*arctan(c*x) - c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5))*b*c*d^
2 + 1/6*(2*x^3*arctan(c*x) - c*(x^2/c^2 - log(c^2*x^2 + 1)/c^4))*b*d^2
```

Fricas [A] time = 2.82369, size = 359, normalized size = 2.36

$$-\frac{12\,ac^5d^2x^5-(30i\,a+3\,b)c^4d^2x^4-10\,(2\,a-i\,b)c^3d^2x^3+16\,bc^2d^2x^2-30i\,bcd^2x-31\,bd^2\log\left(\frac{cx+i}{c}\right)-bd^2\log\left(\frac{cx-i}{c}\right)-(\cdots}{60\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d+I*c*d*x)^2*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] -1/60*(12*a*c^5*d^2*x^5 - (30*I*a + 3*b)*c^4*d^2*x^4 - 10*(2*a - I*b)*c^3*d
^2*x^3 + 16*b*c^2*d^2*x^2 - 30*I*b*c*d^2*x - 31*b*d^2*log((c*x + I)/c) - b*
d^2*log((c*x - I)/c) - (-6*I*b*c^5*d^2*x^5 - 15*b*c^4*d^2*x^4 + 10*I*b*c^3*
d^2*x^3)*log(-(c*x + I)/(c*x - I)))/c^3
```

Sympy [A] time = 2.63168, size = 206, normalized size = 1.36

$$-\frac{ac^2d^2x^5}{5}-\frac{4bd^2x^2}{15c}+\frac{ibd^2x}{2c^2}-\frac{bd^2\left(-\frac{\log\left(x-\frac{i}{c}\right)}{60}-\frac{31\log\left(x+\frac{i}{c}\right)}{60}\right)}{c^3}-x^4\left(-\frac{iacd^2}{2}-\frac{bcd^2}{20}\right)-x^3\left(-\frac{ad^2}{3}+\frac{ibd^2}{6}\right)+\left(-\frac{ibc^2d^2x^5}{10}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d+I*c*d*x)**2*(a+b*atan(c*x)),x)
```

```
[Out] -a*c**2*d**2*x**5/5 - 4*b*d**2*x**2/(15*c) + I*b*d**2*x/(2*c**2) - b*d**2*(
-log(x - I/c)/60 - 31*log(x + I/c)/60)/c**3 - x**4*(-I*a*c*d**2/2 - b*c*d**
2/20) - x**3*(-a*d**2/3 + I*b*d**2/6) + (-I*b*c**2*d**2*x**5/10 - b*c*d**2*
x**4/4 + I*b*d**2*x**3/6)*log(-I*c*x + 1) + (I*b*c**2*d**2*x**5/10 + b*c*d*
*2*x**4/4 - I*b*d**2*x**3/6)*log(I*c*x + 1)
```

Giac [A] time = 1.21708, size = 221, normalized size = 1.45

$$-\frac{12\,bc^5d^2x^5\arctan(cx)+12\,ac^5d^2x^5-30\,bc^4d^2ix^4\arctan(cx)-30\,ac^4d^2ix^4-3\,bc^4d^2x^4+10\,bc^3d^2ix^3-20\,bc^3d^2x^3\,\mathrm{arct}}{60\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d+I*c*d*x)^2*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] -1/60*(12*b*c^5*d^2*x^5*arctan(c*x) + 12*a*c^5*d^2*x^5 - 30*b*c^4*d^2*i*x^4
*arctan(c*x) - 30*a*c^4*d^2*i*x^4 - 3*b*c^4*d^2*x^4 + 10*b*c^3*d^2*i*x^3 -
20*b*c^3*d^2*x^3*arctan(c*x) - 20*a*c^3*d^2*x^3 + 16*b*c^2*d^2*x^2 - 30*b*c
*d^2*i*x - 31*b*d^2*log(c*x + i) - b*d^2*log(c*x - i))/c^3
```

3.12 $\int x(d+icdx)^2\left(a+b\tan^{-1}(cx)\right)\,dx$

Optimal. Leaf size=136

$$-\frac{1}{4}c^2d^2x^4\left(a+b\tan^{-1}(cx)\right)+\frac{2}{3}icd^2x^3\left(a+b\tan^{-1}(cx)\right)+\frac{1}{2}d^2x^2\left(a+b\tan^{-1}(cx)\right)+\frac{ibd^2\log\left(c^2x^2+1\right)}{3c^2}+\frac{3bd^2\tan^{-1}(}{4c^2}$$

```
[Out] (-3*b*d^2*x)/(4*c) - (I/3)*b*d^2*x^2 + (b*c*d^2*x^3)/12 + (3*b*d^2*ArcTan[c
*x])/(4*c^2) + (d^2*x^2*(a + b*ArcTan[c*x]))/2 + ((2*I)/3)*c*d^2*x^3*(a + b
*ArcTan[c*x]) - (c^2*d^2*x^4*(a + b*ArcTan[c*x]))/4 + ((I/3)*b*d^2*Log[1 +
c^2*x^2])/c^2
```

Rubi [A] time = 0.127064, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {43, 4872, 12, 1802, 635, 203, 260}

$$-\frac{1}{4}c^2d^2x^4\left(a+b\tan^{-1}(cx)\right)+\frac{2}{3}icd^2x^3\left(a+b\tan^{-1}(cx)\right)+\frac{1}{2}d^2x^2\left(a+b\tan^{-1}(cx)\right)+\frac{ibd^2\log\left(c^2x^2+1\right)}{3c^2}+\frac{3bd^2\tan^{-1}(}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + I*c*d*x)^2*(a + b*ArcTan[c*x]),x]
```

```
[Out] (-3*b*d^2*x)/(4*c) - (I/3)*b*d^2*x^2 + (b*c*d^2*x^3)/12 + (3*b*d^2*ArcTan[c
*x])/(4*c^2) + (d^2*x^2*(a + b*ArcTan[c*x]))/2 + ((2*I)/3)*c*d^2*x^3*(a + b
*ArcTan[c*x]) - (c^2*d^2*x^4*(a + b*ArcTan[c*x]))/4 + ((I/3)*b*d^2*Log[1 +
c^2*x^2])/c^2
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 4872

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
+ b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2), x
], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2*m
```

```
] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int x(d+icdx)^2\left(a+b\tan^{-1}(cx)\right)dx=\frac{1}{2}d^2x^2\left(a+b\tan^{-1}(cx)\right)+\frac{2}{3}icd^2x^3\left(a+b\tan^{-1}(cx)\right)-\frac{1}{4}c^2d^2x^4\left(a+b\tan^{-1}(cx\right.$$

$$=\frac{1}{2}d^2x^2\left(a+b\tan^{-1}(cx)\right)+\frac{2}{3}icd^2x^3\left(a+b\tan^{-1}(cx)\right)-\frac{1}{4}c^2d^2x^4\left(a+b\tan^{-1}(cx\right.$$

$$=\frac{1}{2}d^2x^2\left(a+b\tan^{-1}(cx)\right)+\frac{2}{3}icd^2x^3\left(a+b\tan^{-1}(cx)\right)-\frac{1}{4}c^2d^2x^4\left(a+b\tan^{-1}(cx\right.$$

$$=-\frac{3bd^2x}{4c}-\frac{1}{3}ibd^2x^2+\frac{1}{12}bcd^2x^3+\frac{1}{2}d^2x^2\left(a+b\tan^{-1}(cx)\right)+\frac{2}{3}icd^2x^3\left(a+b\tan^{-\right.$$

$$=-\frac{3bd^2x}{4c}-\frac{1}{3}ibd^2x^2+\frac{1}{12}bcd^2x^3+\frac{1}{2}d^2x^2\left(a+b\tan^{-1}(cx)\right)+\frac{2}{3}icd^2x^3\left(a+b\tan^{-\right.$$

$$=-\frac{3bd^2x}{4c}-\frac{1}{3}ibd^2x^2+\frac{1}{12}bcd^2x^3+\frac{3bd^2\tan^{-1}(cx)}{4c^2}+\frac{1}{2}d^2x^2\left(a+b\tan^{-1}(cx)\right)+\frac{2}{3}$$

Mathematica [A] time = 0.0910559, size = 101, normalized size = 0.74

$$\frac{d^2\left(cx\left(acx\left(-3c^2x^2+8icx+6\right)+b\left(c^2x^2-4icx-9\right)\right)+4ib\log\left(c^2x^2+1\right)+b\left(-3c^4x^4+8ic^3x^3+6c^2x^2+9\right)\tan^{-1}(cx)\right)}{12c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + I*c*d*x)^2*(a + b*ArcTan[c*x]),x]
```

```
[Out] (d^2*(c*x*(a*c*x*(6 + (8*I)*c*x - 3*c^2*x^2) + b*(-9 - (4*I)*c*x + c^2*x^2)
) + b*(9 + 6*c^2*x^2 + (8*I)*c^3*x^3 - 3*c^4*x^4)*ArcTan[c*x] + (4*I)*b*Log
[1 + c^2*x^2]))/(12*c^2)
```

Maple [A] time = 0.027, size = 141, normalized size = 1.

$$-\frac{c^2d^2ax^4}{4}+\frac{2i}{3}cd^2ax^3+\frac{d^2ax^2}{2}-\frac{c^2d^2b\arctan(cx)x^4}{4}+\frac{2i}{3}cd^2b\arctan(cx)x^3+\frac{d^2b\arctan(cx)x^2}{2}-\frac{3d^2bx}{4c}+\frac{bcd^2x^3}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d+I*c*d*x)^2*(a+b*arctan(c*x)),x)
```

```
[Out] -1/4*c^2*d^2*a*x^4+2/3*I*c*d^2*a*x^3+1/2*d^2*a*x^2-1/4*c^2*d^2*b*arctan(c*x
)*x^4+2/3*I*c*d^2*b*arctan(c*x)*x^3+1/2*d^2*b*arctan(c*x)*x^2-3/4*b*d^2*x/c
+1/12*b*c*d^2*x^3-1/3*I*b*d^2*x^2+1/3*I*b*d^2*ln(c^2*x^2+1)/c^2+3/4*b*d^2*a
rctan(c*x)/c^2
```

Maxima [A] time = 1.47229, size = 209, normalized size = 1.54

$$-\frac{1}{4}\,ac^2d^2x^4+\frac{2}{3}i\,acd^2x^3-\frac{1}{12}\left(3\,x^4\arctan\left(cx\right)-c\left(\frac{c^2x^3-3\,x}{c^4}+\frac{3\,\arctan\left(cx\right)}{c^5}\right)\right)bc^2d^2+\frac{1}{3}i\left(2\,x^3\arctan\left(cx\right)-c\left(\frac{x^2}{c^2}-\frac{\mathrm{lo}}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d+I*c*d*x)^2*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] -1/4*a*c^2*d^2*x^4 + 2/3*I*a*c*d^2*x^3 - 1/12*(3*x^4*arctan(c*x) - c*((c^2*
x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5))*b*c^2*d^2 + 1/3*I*(2*x^3*arctan(c*x) -
 c*(x^2/c^2 - log(c^2*x^2 + 1)/c^4))*b*c*d^2 + 1/2*a*d^2*x^2 + 1/2*(x^2*arc
tan(c*x) - c*(x/c^2 - arctan(c*x)/c^3))*b*d^2
```

Fricas [A] time = 2.79827, size = 333, normalized size = 2.45

$$-\frac{6\,ac^4d^2x^4-(16i\,a+2\,b)c^3d^2x^3-4\,(3\,a-2i\,b)c^2d^2x^2+18\,bcd^2x-17i\,bd^2\log\left(\frac{cx+i}{c}\right)+i\,bd^2\log\left(\frac{cx-i}{c}\right)-\left(-3i\,bc^4d^2x^4-8\right.}{24\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d+I*c*d*x)^2*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] -1/24*(6*a*c^4*d^2*x^4 - (16*I*a + 2*b)*c^3*d^2*x^3 - 4*(3*a - 2*I*b)*c^2*d
^2*x^2 + 18*b*c*d^2*x - 17*I*b*d^2*log((c*x + I)/c) + I*b*d^2*log((c*x - I)
/c) - (-3*I*b*c^4*d^2*x^4 - 8*b*c^3*d^2*x^3 + 6*I*b*c^2*d^2*x^2)*log(-(c*x
+ I)/(c*x - I)))/c^2
```

Sympy [A] time = 2.71854, size = 202, normalized size = 1.49

$$-\frac{ac^2d^2x^4}{4}-\frac{3bd^2x}{4c}-\frac{ibd^2\log\left(x-\frac{i}{c}\right)}{24c^2}+\frac{17ibd^2\log\left(x+\frac{i}{c}\right)}{24c^2}-x^3\left(-\frac{2iacd^2}{3}-\frac{bcd^2}{12}\right)-x^2\left(-\frac{ad^2}{2}+\frac{ibd^2}{3}\right)+\left(-\frac{ibc^2d^2x^4}{8}-\frac{b}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d+I*c*d*x)**2*(a+b*atan(c*x)),x)
```

```
[Out] -a*c**2*d**2*x**4/4 - 3*b*d**2*x/(4*c) - I*b*d**2*log(x - I/c)/(24*c**2) +
17*I*b*d**2*log(x + I/c)/(24*c**2) - x**3*(-2*I*a*c*d**2/3 - b*c*d**2/12) -
 x**2*(-a*d**2/2 + I*b*d**2/3) + (-I*b*c**2*d**2*x**4/8 - b*c*d**2*x**3/3 +
 I*b*d**2*x**2/4)*log(-I*c*x + 1) + (I*b*c**2*d**2*x**4/8 + b*c*d**2*x**3/3
 - I*b*d**2*x**2/4)*log(I*c*x + 1)
```

Giac [A] time = 1.15196, size = 207, normalized size = 1.52

$$-\frac{6\,bc^4d^2x^4\arctan(cx) + 6\,ac^4d^2x^4 - 16\,bc^3d^2ix^3\arctan(cx) - 16\,ac^3d^2ix^3 - 2\,bc^3d^2x^3 + 8\,bc^2d^2ix^2 - 12\,bc^2d^2x^2\,\mathrm{arcta}}{24\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d+I*c*d*x)^2*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] -1/24*(6*b*c^4*d^2*x^4*arctan(c*x) + 6*a*c^4*d^2*x^4 - 16*b*c^3*d^2*i*x^3*a
rctan(c*x) - 16*a*c^3*d^2*i*x^3 - 2*b*c^3*d^2*x^3 + 8*b*c^2*d^2*i*x^2 - 12*
b*c^2*d^2*x^2*arctan(c*x) - 12*a*c^2*d^2*x^2 + 18*b*c*d^2*x - 17*b*d^2*i*lo
g(c*i*x - 1) + b*d^2*i*log(-c*i*x - 1))/c^2
```

3.13 $\int (d+icdx)^2 \left(a+b\tan^{-1}(cx)\right) \, dx$

Optimal. Leaf size=83

$$-\frac{id^2(1+icx)^3\left(a+b\tan^{-1}(cx)\right)}{3c}-\frac{bd^2(1+icx)^2}{6c}-\frac{4bd^2\log(1-icx)}{3c}-\frac{2}{3}ibd^2x$$

```
[Out] ((-2*I)/3)*b*d^2*x - (b*d^2*(1 + I*c*x)^2)/(6*c) - ((I/3)*d^2*(1 + I*c*x)^3
*(a + b*ArcTan[c*x]))/c - (4*b*d^2*Log[1 - I*c*x])/(3*c)
```

Rubi [A] time = 0.0456393, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {4862, 627, 43}

$$-\frac{id^2(1+icx)^3\left(a+b\tan^{-1}(cx)\right)}{3c}-\frac{bd^2(1+icx)^2}{6c}-\frac{4bd^2\log(1-icx)}{3c}-\frac{2}{3}ibd^2x$$

Antiderivative was successfully verified.

```
[In] Int[(d + I*c*d*x)^2*(a + b*ArcTan[c*x]),x]
```

```
[Out] ((-2*I)/3)*b*d^2*x - (b*d^2*(1 + I*c*x)^2)/(6*c) - ((I/3)*d^2*(1 + I*c*x)^3
*(a + b*ArcTan[c*x]))/c - (4*b*d^2*Log[1 - I*c*x])/(3*c)
```

Rule 4862

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x]))/(e*(q + 1)), x] - Dist[(b*
c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (d+icdx)^2\left(a+b\tan^{-1}(cx)\right)dx &= -\frac{id^2(1+icx)^3\left(a+b\tan^{-1}(cx)\right)}{3c}+\frac{(ib)\int\frac{(d+icdx)^3}{1+c^2x^2}\,dx}{3d}\\ &= -\frac{id^2(1+icx)^3\left(a+b\tan^{-1}(cx)\right)}{3c}+\frac{(ib)\int\frac{(d+icdx)^2}{\frac{1}{d}-\frac{icx}{d}}\,dx}{3d}\\ &= -\frac{id^2(1+icx)^3\left(a+b\tan^{-1}(cx)\right)}{3c}+\frac{(ib)\int\left(-2d^3+\frac{4d^2}{\frac{1}{d}-\frac{icx}{d}}-d^2(d+icdx)\right)dx}{3d}\\ &= -\frac{2}{3}ibd^2x-\frac{bd^2(1+icx)^2}{6c}-\frac{id^2(1+icx)^3\left(a+b\tan^{-1}(cx)\right)}{3c}-\frac{4bd^2\log(1-icx)}{3c}\end{aligned}$$

Mathematica [A] time = 0.0385873, size = 57, normalized size = 0.69

$$\frac{1}{3}d^2\left(-\frac{(cx-i)^3\left(a+b\tan^{-1}(cx)\right)}{c}+\frac{1}{2}bx(cx-6i)-\frac{4b\log(cx+i)}{c}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + I*c*d*x)^2*(a + b*ArcTan[c*x]),x]
```

```
[Out] (d^2*((b*x*(-6*I + c*x))/2 - ((-I + c*x)^3*(a + b*ArcTan[c*x]))/c - (4*b*Lo
g[I + c*x])/c))/3
```

Maple [A] time = 0.028, size = 133, normalized size = 1.6

$$-\frac{c^2x^3ad^2}{3}+icx^2ad^2+axd^2-\frac{\frac{i}{3}d^2a}{c}-\frac{c^2d^2b\arctan(cx)\,x^3}{3}+icd^2b\arctan(cx)\,x^2+d^2bx\arctan(cx)+\frac{id^2b\arctan(cx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^2*(a+b*arctan(c*x)),x)
```

```
[Out] -1/3*c^2*x^3*a*d^2+I*c*x^2*a*d^2+a*x*d^2-1/3*I/c*d^2*a-1/3*c^2*d^2*b*arctan
(c*x)*x^3+I*c*d^2*b*arctan(c*x)*x^2+d^2*b*x*arctan(c*x)+I/c*d^2*b*arctan(c*
x)-I*d^2*b*x+1/6*c*d^2*b*x^2-2/3/c*d^2*b*ln(c^2*x^2+1)
```

Maxima [B] time = 1.49232, size = 186, normalized size = 2.24

$$-\frac{1}{3}\,ac^2d^2x^3-\frac{1}{6}\left(2\,x^3\arctan\left(cx\right)-c\left(\frac{x^2}{c^2}-\frac{\log\left(c^2x^2+1\right)}{c^4}\right)\right)bc^2d^2+i\,acd^2x^2+i\left(x^2\arctan\left(cx\right)-c\left(\frac{x}{c^2}-\frac{\arctan\left(cx\right)}{c^3}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^2*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] -1/3*a*c^2*d^2*x^3 - 1/6*(2*x^3*arctan(c*x) - c*(x^2/c^2 - log(c^2*x^2 + 1)
/c^4))*b*c^2*d^2 + I*a*c*d^2*x^2 + I*(x^2*arctan(c*x) - c*(x/c^2 - arctan(c
*x)/c^3))*b*c*d^2 + a*d^2*x + 1/2*(2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*b*
d^2/c
```

Fricas [A] time = 2.68781, size = 279, normalized size = 3.36

$$-\frac{2\,ac^3d^2x^3-(6i\,a+b)c^2d^2x^2-6\,(a-i\,b)cd^2x+7\,bd^2\log\left(\frac{cx+i}{c}\right)+bd^2\log\left(\frac{cx-i}{c}\right)-\left(-i\,bc^3d^2x^3-3\,bc^2d^2x^2+3i\,bcd^2x\right)\log}{6\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^2*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] -1/6*(2*a*c^3*d^2*x^3 - (6*I*a + b)*c^2*d^2*x^2 - 6*(a - I*b)*c*d^2*x + 7*b
*d^2*log((c*x + I)/c) + b*d^2*log((c*x - I)/c) - (-I*b*c^3*d^2*x^3 - 3*b*c^
2*d^2*x^2 + 3*I*b*c*d^2*x)*log(-(c*x + I)/(c*x - I)))/c
```

Sympy [B] time = 2.60729, size = 165, normalized size = 1.99

$$-\frac{ac^2d^2x^3}{3}-\frac{bd^2\left(\frac{\log\left(x-\frac{i}{c}\right)}{6}+\frac{7\log\left(x+\frac{i}{c}\right)}{6}\right)}{c}-x^2\left(-iacd^2-\frac{bcd^2}{6}\right)-x\left(-ad^2+ibd^2\right)+\left(-\frac{ibc^2d^2x^3}{6}-\frac{bcd^2x^2}{2}+\frac{ibd^2x}{2}\right)\log\left(-ic\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**2*(a+b*atan(c*x)),x)
```

```
[Out] -a*c**2*d**2*x**3/3 - b*d**2*(log(x - I/c)/6 + 7*log(x + I/c)/6)/c - x**2*(
-I*a*c*d**2 - b*c*d**2/6) - x*(-a*d**2 + I*b*d**2) + (-I*b*c**2*d**2*x**3/6
 - b*c*d**2*x**2/2 + I*b*d**2*x/2)*log(-I*c*x + 1) + (I*b*c**2*d**2*x**3/6
+ b*c*d**2*x**2/2 - I*b*d**2*x/2)*log(I*c*x + 1)
```

Giac [A] time = 1.15826, size = 176, normalized size = 2.12

$$-\frac{2\,bc^3d^2x^3\arctan\left(cx\right)+2\,ac^3d^2x^3-6\,bc^2d^2ix^2\arctan\left(cx\right)-6\,ac^2d^2ix^2-bc^2d^2x^2+6\,bcd^2ix-6\,bcd^2x\arctan\left(cx\right)-6}{6\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^2*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] -1/6*(2*b*c^3*d^2*x^3*arctan(c*x) + 2*a*c^3*d^2*x^3 - 6*b*c^2*d^2*i*x^2*arc
tan(c*x) - 6*a*c^2*d^2*i*x^2 - b*c^2*d^2*x^2 + 6*b*c*d^2*i*x - 6*b*c*d^2*x*
arctan(c*x) - 6*a*c*d^2*x + 7*b*d^2*log(c*x + i) + b*d^2*log(c*x - i))/c
```

3.14 $\int \frac{(d+icdx)^2\left(a+b\tan^{-1}(cx)\right)}{x}\,dx$

Optimal. Leaf size=129

$$\frac{1}{2}ibd^2\text{PolyLog}(2,-icx)-\frac{1}{2}ibd^2\text{PolyLog}(2,icx)-\frac{1}{2}c^2d^2x^2\left(a+b\tan^{-1}(cx)\right)+2iacd^2x+ad^2\log(x)-ibd^2\log\left(c^2x^2+1\right)$$

```
[Out] (2*I)*a*c*d^2*x + (b*c*d^2*x)/2 - (b*d^2*ArcTan[c*x])/2 + (2*I)*b*c*d^2*x*A
rcTan[c*x] - (c^2*d^2*x^2*(a + b*ArcTan[c*x]))/2 + a*d^2*Log[x] - I*b*d^2*L
og[1 + c^2*x^2] + (I/2)*b*d^2*PolyLog[2, (-I)*c*x] - (I/2)*b*d^2*PolyLog[2,
 I*c*x]
```

Rubi [A] time = 0.12686, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {4876, 4846, 260, 4848, 2391, 4852, 321, 203}

$$\frac{1}{2}ibd^2\text{PolyLog}(2,-icx)-\frac{1}{2}ibd^2\text{PolyLog}(2,icx)-\frac{1}{2}c^2d^2x^2\left(a+b\tan^{-1}(cx)\right)+2iacd^2x+ad^2\log(x)-ibd^2\log\left(c^2x^2+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)^2*(a + b*ArcTan[c*x]))/x,x]
```

```
[Out] (2*I)*a*c*d^2*x + (b*c*d^2*x)/2 - (b*d^2*ArcTan[c*x])/2 + (2*I)*b*c*d^2*x*A
rcTan[c*x] - (c^2*d^2*x^2*(a + b*ArcTan[c*x]))/2 + a*d^2*Log[x] - I*b*d^2*L
og[1 + c^2*x^2] + (I/2)*b*d^2*PolyLog[2, (-I)*c*x] - (I/2)*b*d^2*PolyLog[2,
 I*c*x]
```

Rule 4876

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{(d+icdx)^2\left(a+b\tan^{-1}(cx)\right)}{x}\,dx = \int\left(2icd^2\left(a+b\tan^{-1}(cx)\right)+\frac{d^2\left(a+b\tan^{-1}(cx)\right)}{x}-c^2d^2x\left(a+b\tan^{-1}(cx)\right)\right)dx$$

$$= d^2\int\frac{a+b\tan^{-1}(cx)}{x}\,dx+\left(2icd^2\right)\int\left(a+b\tan^{-1}(cx)\right)dx-\left(c^2d^2\right)\int x\left(a+b\tan\right.$$

$$= 2iacd^2x-\frac{1}{2}c^2d^2x^2\left(a+b\tan^{-1}(cx)\right)+ad^2\log(x)+\frac{1}{2}\left(ibd^2\right)\int\frac{\log(1-icx)}{x}\,dx-\frac{1}{2}$$

$$= 2iacd^2x+\frac{1}{2}bcd^2x+2ibcd^2x\tan^{-1}(cx)-\frac{1}{2}c^2d^2x^2\left(a+b\tan^{-1}(cx)\right)+ad^2\log(x)+$$

$$= 2iacd^2x+\frac{1}{2}bcd^2x-\frac{1}{2}bd^2\tan^{-1}(cx)+2ibcd^2x\tan^{-1}(cx)-\frac{1}{2}c^2d^2x^2\left(a+b\tan^{-1}(c\right.$$

Mathematica [A] time = 0.0925627, size = 103, normalized size = 0.8

$$-\frac{1}{2}d^2\left(-ib\text{PolyLog}(2,-icx)+ib\text{PolyLog}(2,icx)+ac^2x^2-4iacx-2a\log(x)+2ib\log\left(c^2x^2+1\right)+bc^2x^2\tan^{-1}(cx)-bc\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + I*c*d*x)^2*(a + b*ArcTan[c*x]))/x,x]
```

```
[Out] -(d^2*((-4*I)*a*c*x - b*c*x + a*c^2*x^2 + b*ArcTan[c*x] - (4*I)*b*c*x*ArcTa
n[c*x] + b*c^2*x^2*ArcTan[c*x] - 2*a*Log[x] + (2*I)*b*Log[1 + c^2*x^2] - I*
b*PolyLog[2, (-I)*c*x] + I*b*PolyLog[2, I*c*x]))/2
```

Maple [A] time = 0.043, size = 177, normalized size = 1.4

$$2\,iacd^2x-\frac{d^2ac^2x^2}{2}+d^2a\ln(cx)+2\,ibcd^2x\arctan(cx)-\frac{d^2b\arctan(cx)\,c^2x^2}{2}+d^2b\arctan(cx)\ln(cx)+\frac{bcd^2x}{2}-ibd^2\ln$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^2*(a+b*arctan(c*x))/x,x)
```

```
[Out] 2*I*a*c*d^2*x-1/2*d^2*a*c^2*x^2+d^2*a*ln(c*x)+2*I*b*c*d^2*x*arctan(c*x)-1/2
*d^2*b*arctan(c*x)*c^2*x^2+d^2*b*arctan(c*x)*ln(c*x)+1/2*b*c*d^2*x-I*b*d^2*
ln(c^2*x^2+1)-1/2*b*d^2*arctan(c*x)+1/2*I*d^2*b*ln(c*x)*ln(1+I*c*x)-1/2*I*d
^2*b*ln(c*x)*ln(1-I*c*x)+1/2*I*d^2*b*dilog(1+I*c*x)-1/2*I*d^2*b*dilog(1-I*c
*x)
```

Maxima [A] time = 2.14319, size = 204, normalized size = 1.58

$$-\frac{1}{2}\,ac^2d^2x^2+2i\,acd^2x+\frac{1}{2}\,bcd^2x-\frac{1}{4}\,\pi bd^2\log\left(c^2x^2+1\right)+bd^2\arctan\left(cx\right)\log\left(x|c|\right)+i\left(2\,cx\arctan\left(cx\right)-\log\left(c^2x^2+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))/x,x, algorithm="maxima")
```

```
[Out] -1/2*a*c^2*d^2*x^2 + 2*I*a*c*d^2*x + 1/2*b*c*d^2*x - 1/4*pi*b*d^2*log(c^2*x
^2 + 1) + b*d^2*arctan(c*x)*log(x*abs(c)) + I*(2*c*x*arctan(c*x) - log(c^2*
x^2 + 1))*b*d^2 - 1/2*I*b*d^2*dilog(I*c*x + 1) + 1/2*I*b*d^2*dilog(-I*c*x +
 1) + a*d^2*log(x) - 1/2*(b*c^2*d^2*x^2 - b*d^2*(2*I*arctan2(0, c) - 1))*ar
ctan(c*x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{2\,ac^2d^2x^2-4i\,acd^2x-2\,ad^2-\left(-i\,bc^2d^2x^2-2\,bcd^2x+i\,bd^2\right)\log\left(-\frac{cx+i}{cx-i}\right)}{2\,x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))/x,x, algorithm="fricas")
```

```
[Out] integral(-1/2*(2*a*c^2*d^2*x^2 - 4*I*a*c*d^2*x - 2*a*d^2 - (-I*b*c^2*d^2*x^
2 - 2*b*c*d^2*x + I*b*d^2)*log(-(c*x + I)/(c*x - I)))/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d^2\left(\int\frac{a}{x}\,dx+\int 2iac\,dx+\int -ac^2x\,dx+\int\frac{b\,\mathrm{atan}\,(cx)}{x}\,dx+\int 2ibc\,\mathrm{atan}\,(cx)\,dx+\int -bc^2x\,\mathrm{atan}\,(cx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**2*(a+b*atan(c*x))/x,x)
```

```
[Out] d**2*(Integral(a/x, x) + Integral(2*I*a*c, x) + Integral(-a*c**2*x, x) + In
tegral(b*atan(c*x)/x, x) + Integral(2*I*b*c*atan(c*x), x) + Integral(-b*c**
2*x*atan(c*x), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(icdx+d)^2(b\arctan(cx)+a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))/x,x, algorithm="giac")
```

```
[Out] integrate((I*c*d*x + d)^2*(b*arctan(c*x) + a)/x, x)
```

3.15 $\int \frac{(d+icdx)^2\left(a+b\tan^{-1}(cx)\right)}{x^2}\,dx$

Optimal. Leaf size=89

$$-bcd^2\text{PolyLog}(2,-icx)+bcd^2\text{PolyLog}(2,icx)-\frac{d^2\left(a+b\tan^{-1}(cx)\right)}{x}-ac^2d^2x+2iacd^2\log(x)-bc^2d^2x\tan^{-1}(cx)+bc$$

```
[Out] -(a*c^2*d^2*x) - b*c^2*d^2*x*ArcTan[c*x] - (d^2*(a + b*ArcTan[c*x]))/x + (2
*I)*a*c*d^2*Log[x] + b*c*d^2*Log[x] - b*c*d^2*PolyLog[2, (-I)*c*x] + b*c*d^
2*PolyLog[2, I*c*x]
```

Rubi [A] time = 0.137576, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {4876, 4846, 260, 4852, 266, 36, 29, 31, 4848, 2391}

$$-bcd^2\text{PolyLog}(2,-icx)+bcd^2\text{PolyLog}(2,icx)-\frac{d^2\left(a+b\tan^{-1}(cx)\right)}{x}-ac^2d^2x+2iacd^2\log(x)-bc^2d^2x\tan^{-1}(cx)+bc$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)^2*(a + b*ArcTan[c*x]))/x^2,x]
```


```
[Out] -(a*c^2*d^2*x) - b*c^2*d^2*x*ArcTan[c*x] - (d^2*(a + b*ArcTan[c*x]))/x + (2
*I)*a*c*d^2*Log[x] + b*c*d^2*Log[x] - b*c*d^2*PolyLog[2, (-I)*c*x] + b*c*d^
2*PolyLog[2, I*c*x]
```


Rule 4876

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{(d+icdx)^2\left(a+b\tan^{-1}(cx)\right)}{x^2}\,dx = \int \left(-c^2d^2\left(a+b\tan^{-1}(cx)\right) + \frac{d^2\left(a+b\tan^{-1}(cx)\right)}{x^2} + \frac{2icd^2\left(a+b\tan^{-1}(cx)\right)}{x}\right) dx$$

$$= d^2\int \frac{a+b\tan^{-1}(cx)}{x^2}\,dx + \left(2icd^2\right)\int \frac{a+b\tan^{-1}(cx)}{x}\,dx - \left(c^2d^2\right)\int \left(a+b\tan^{-1}\right.$$

$$= -ac^2d^2x - \frac{d^2\left(a+b\tan^{-1}(cx)\right)}{x} + 2iacd^2\log(x) + \left(bcd^2\right)\int \frac{1}{x\left(1+c^2x^2\right)}\,dx - \left(bc\right.$$

$$= -ac^2d^2x - bc^2d^2x\tan^{-1}(cx) - \frac{d^2\left(a+b\tan^{-1}(cx)\right)}{x} + 2iacd^2\log(x) - bcd^2\text{Li}_2(-ic$$

$$= -ac^2d^2x - bc^2d^2x\tan^{-1}(cx) - \frac{d^2\left(a+b\tan^{-1}(cx)\right)}{x} + 2iacd^2\log(x) + \frac{1}{2}bcd^2\log\left(\right.$$

$$= -ac^2d^2x - bc^2d^2x\tan^{-1}(cx) - \frac{d^2\left(a+b\tan^{-1}(cx)\right)}{x} + 2iacd^2\log(x) + bcd^2\log(x)$$

Mathematica [A] time = 0.0918685, size = 79, normalized size = 0.89

$$-\frac{d^2\left(bcx\text{PolyLog}(2,-icx) - bcx\text{PolyLog}(2,icx) + ac^2x^2 - 2iacx\log(x) + a + bc^2x^2\tan^{-1}(cx) - bcx\log(cx) + b\tan^{-1}(\right.}{x}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + I*c*d*x)^2*(a + b*ArcTan[c*x]))/x^2,x]
```

```
[Out] -((d^2*(a + a*c^2*x^2 + b*ArcTan[c*x] + b*c^2*x^2*ArcTan[c*x] - (2*I)*a*c*x
*Log[x] - b*c*x*Log[c*x] + b*c*x*PolyLog[2, (-I)*c*x] - b*c*x*PolyLog[2, I*
c*x]))/x)
```

Maple [A] time = 0.042, size = 152, normalized size = 1.7

$$-ac^2d^2x - \frac{d^2a}{x} + 2\,icd^2a\ln(cx) - bc^2d^2x\arctan(cx) - \frac{bd^2\arctan(cx)}{x} + 2\,icd^2b\arctan(cx)\ln(cx) - cd^2b\ln(cx)\ln(1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^2*(a+b*arctan(c*x))/x^2,x)
```

```
[Out] -a*c^2*d^2*x-d^2*a/x+2*I*c*d^2*a*ln(c*x)-b*c^2*d^2*x*arctan(c*x)-d^2*b*arct
an(c*x)/x+2*I*c*d^2*b*arctan(c*x)*ln(c*x)-c*d^2*b*ln(c*x)*ln(1+I*c*x)+c*d^2
```

```
*b*ln(c*x)*ln(1-I*c*x)-c*d^2*b*dilog(1+I*c*x)+c*d^2*b*dilog(1-I*c*x)+c*d^2*
b*ln(c*x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-ac^2d^2x - \frac{1}{2}\left(2\,cx\arctan\left(cx\right) - \log\left(c^2x^2+1\right)\right)bcd^2 + 2i\,bcd^2\int\frac{\arctan\left(cx\right)}{x}\,dx + 2i\,acd^2\log\left(x\right) - \frac{1}{2}\left(c\left(\log\left(c^2x^2+1\right) - \right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))/x^2,x, algorithm="maxima")
```

```
[Out] -a*c^2*d^2*x - 1/2*(2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*b*c*d^2 + 2*I*b*c
*d^2*integrate(arctan(c*x)/x, x) + 2*I*a*c*d^2*log(x) - 1/2*(c*(log(c^2*x^2
 + 1) - log(x^2)) + 2*arctan(c*x)/x)*b*d^2 - a*d^2/x
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{2\,ac^2d^2x^2 - 4i\,acd^2x - 2\,ad^2 - \left(-i\,bc^2d^2x^2 - 2\,bcd^2x + i\,bd^2\right)\log\left(-\frac{cx+i}{cx-i}\right)}{2\,x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))/x^2,x, algorithm="fricas")
```

```
[Out] integral(-1/2*(2*a*c^2*d^2*x^2 - 4*I*a*c*d^2*x - 2*a*d^2 - (-I*b*c^2*d^2*x^
2 - 2*b*c*d^2*x + I*b*d^2)*log(-(c*x + I)/(c*x - I)))/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d^2\left(\int -ac^2\,dx + \int\frac{a}{x^2}\,dx + \int -bc^2\operatorname{atan}\left(cx\right)dx + \int\frac{b\operatorname{atan}\left(cx\right)}{x^2}\,dx + \int\frac{2iac}{x}\,dx + \int\frac{2ibc\operatorname{atan}\left(cx\right)}{x}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**2*(a+b*atan(c*x))/x**2,x)
```

```
[Out] d**2*(Integral(-a*c**2, x) + Integral(a/x**2, x) + Integral(-b*c**2*atan(c*
x), x) + Integral(b*atan(c*x)/x**2, x) + Integral(2*I*a*c/x, x) + Integral(
2*I*b*c*atan(c*x)/x, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(icdx + d)^2 (b \arctan(cx) + a)}{x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))/x^2,x, algorithm="giac")
```

```
[Out] integrate((I*c*d*x + d)^2*(b*arctan(c*x) + a)/x^2, x)
```

3.16 $\int \frac{(d+icdx)^2\left(a+b\tan^{-1}(cx)\right)}{x^3}\,dx$

Optimal. Leaf size=152

$$-\frac{1}{2}ibc^2d^2\text{PolyLog}(2,-icx)+\frac{1}{2}ibc^2d^2\text{PolyLog}(2,icx)-\frac{d^2\left(a+b\tan^{-1}(cx)\right)}{2x^2}-\frac{2icd^2\left(a+b\tan^{-1}(cx)\right)}{x}-ac^2d^2\log(x)-$$

```
[Out] -(b*c*d^2)/(2*x) - (b*c^2*d^2*ArcTan[c*x])/2 - (d^2*(a + b*ArcTan[c*x]))/(2
*x^2) - ((2*I)*c*d^2*(a + b*ArcTan[c*x]))/x - a*c^2*d^2*Log[x] + (2*I)*b*c^
2*d^2*Log[x] - I*b*c^2*d^2*Log[1 + c^2*x^2] - (I/2)*b*c^2*d^2*PolyLog[2, (-
I)*c*x] + (I/2)*b*c^2*d^2*PolyLog[2, I*c*x]
```

Rubi [A] time = 0.153578, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {4876, 4852, 325, 203, 266, 36, 29, 31, 4848, 2391}

$$-\frac{1}{2}ibc^2d^2\text{PolyLog}(2,-icx)+\frac{1}{2}ibc^2d^2\text{PolyLog}(2,icx)-\frac{d^2\left(a+b\tan^{-1}(cx)\right)}{2x^2}-\frac{2icd^2\left(a+b\tan^{-1}(cx)\right)}{x}-ac^2d^2\log(x)-$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)^2*(a + b*ArcTan[c*x]))/x^3,x]
```

```
[Out] -(b*c*d^2)/(2*x) - (b*c^2*d^2*ArcTan[c*x])/2 - (d^2*(a + b*ArcTan[c*x]))/(2
*x^2) - ((2*I)*c*d^2*(a + b*ArcTan[c*x]))/x - a*c^2*d^2*Log[x] + (2*I)*b*c^
2*d^2*Log[x] - I*b*c^2*d^2*Log[1 + c^2*x^2] - (I/2)*b*c^2*d^2*PolyLog[2, (-
I)*c*x] + (I/2)*b*c^2*d^2*PolyLog[2, I*c*x]
```

Rule 4876

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
```

```
erQ[m]) && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{(d+icdx)^2\left(a+b\tan^{-1}(cx)\right)}{x^3}\,dx=\int\left(\frac{d^2\left(a+b\tan^{-1}(cx)\right)}{x^3}+\frac{2icd^2\left(a+b\tan^{-1}(cx)\right)}{x^2}-\frac{c^2d^2\left(a+b\tan^{-1}(cx)\right)}{x}\right)dx$$

$$=d^2\int\frac{a+b\tan^{-1}(cx)}{x^3}\,dx+\left(2icd^2\right)\int\frac{a+b\tan^{-1}(cx)}{x^2}\,dx-\left(c^2d^2\right)\int\frac{a+b\tan^{-1}(c$$

$$=-\frac{d^2\left(a+b\tan^{-1}(cx)\right)}{2x^2}-\frac{2icd^2\left(a+b\tan^{-1}(cx)\right)}{x}-ac^2d^2\log(x)+\frac{1}{2}\left(bcd^2\right)\int\frac{}{x^2\left(1\right.}$$

$$=-\frac{bcd^2}{2x}-\frac{d^2\left(a+b\tan^{-1}(cx)\right)}{2x^2}-\frac{2icd^2\left(a+b\tan^{-1}(cx)\right)}{x}-ac^2d^2\log(x)-\frac{1}{2}ibc^2d^2\mathrm{L}$$

$$=-\frac{bcd^2}{2x}-\frac{1}{2}bc^2d^2\tan^{-1}(cx)-\frac{d^2\left(a+b\tan^{-1}(cx)\right)}{2x^2}-\frac{2icd^2\left(a+b\tan^{-1}(cx)\right)}{x}-ac^2d$$

$$=-\frac{bcd^2}{2x}-\frac{1}{2}bc^2d^2\tan^{-1}(cx)-\frac{d^2\left(a+b\tan^{-1}(cx)\right)}{2x^2}-\frac{2icd^2\left(a+b\tan^{-1}(cx)\right)}{x}-ac^2d$$

Mathematica [C] time = 0.0710135, size = 139, normalized size = 0.91

$$-\frac{d^2\left(bcx\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-c^2x^2\right)+ibc^2x^2\text{PolyLog}(2,-icx)-ibc^2x^2\text{PolyLog}(2,icx)+2ac^2x^2\log(x)+4i\right.}{2x^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + I*c*d*x)^2*(a + b*ArcTan[c*x]))/x^3,x]
```

```
[Out] -(d^2*(a + (4*I)*a*c*x + b*ArcTan[c*x] + (4*I)*b*c*x*ArcTan[c*x] + b*c*x*Hy
pergeometric2F1[-1/2, 1, 1/2, -(c^2*x^2)] + 2*a*c^2*x^2*Log[x] - (4*I)*b*c^
2*x^2*Log[x] + (2*I)*b*c^2*x^2*Log[1 + c^2*x^2] + I*b*c^2*x^2*PolyLog[2, (-
I)*c*x] - I*b*c^2*x^2*PolyLog[2, I*c*x]))/(2*x^2)
```

Maple [A] time = 0.05, size = 217, normalized size = 1.4

$$-\frac{d^2a}{2x^2}-\frac{2icd^2a}{x}-c^2d^2a\ln(cx)-\frac{bd^2\arctan(cx)}{2x^2}-\frac{2icd^2b\arctan(cx)}{x}-c^2d^2b\arctan(cx)\ln(cx)-\frac{i}{2}c^2d^2b\ln(cx)\ln(1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^2*(a+b*arctan(c*x))/x^3,x)
```

```
[Out] -1/2*a*d^2/x^2-2*I*c*d^2*a/x-c^2*d^2*a*ln(c*x)-1/2*d^2*b*arctan(c*x)/x^2-2*
I*c*d^2*b*arctan(c*x)/x-c^2*d^2*b*arctan(c*x)*ln(c*x)-1/2*I*c^2*d^2*b*ln(c*
x)*ln(1+I*c*x)+1/2*I*c^2*d^2*b*ln(c*x)*ln(1-I*c*x)-1/2*I*c^2*d^2*b*dilog(1+
I*c*x)+1/2*I*c^2*d^2*b*dilog(1-I*c*x)-I*b*c^2*d^2*ln(c^2*x^2+1)-1/2*b*c^2*d
^2*arctan(c*x)-1/2*b*c*d^2/x+2*I*c^2*d^2*b*ln(c*x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-bc^2d^2\int \frac{\arctan(cx)}{x}\,dx - ac^2d^2\log(x) - i\left(c\left(\log\left(c^2x^2+1\right)-\log\left(x^2\right)\right)+\frac{2\arctan(cx)}{x}\right)bcd^2 - \frac{1}{2}\left(\left(c\arctan(cx)+\frac{1}{x}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))/x^3,x, algorithm="maxima")
```

```
[Out] -b*c^2*d^2*integrate(arctan(c*x)/x, x) - a*c^2*d^2*log(x) - I*(c*(log(c^2*x
^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*b*c*d^2 - 1/2*((c*arctan(c*x) + 1/x)
*c + arctan(c*x)/x^2)*b*d^2 - 2*I*a*c*d^2/x - 1/2*a*d^2/x^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{2\,ac^2d^2x^2 - 4i\,acd^2x - 2\,ad^2 - \left(-i\,bc^2d^2x^2 - 2\,bcd^2x + i\,bd^2\right)\log\left(-\frac{cx+i}{cx-i}\right)}{2\,x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))/x^3,x, algorithm="fricas")
```

```
[Out] integral(-1/2*(2*a*c^2*d^2*x^2 - 4*I*a*c*d^2*x - 2*a*d^2 - (-I*b*c^2*d^2*x^
2 - 2*b*c*d^2*x + I*b*d^2)*log(-(c*x + I)/(c*x - I)))/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d^2\left(\int \frac{a}{x^3}\,dx + \int -\frac{ac^2}{x}\,dx + \int \frac{b\,\mathrm{atan}\,(cx)}{x^3}\,dx + \int \frac{2iac}{x^2}\,dx + \int -\frac{bc^2\,\mathrm{atan}\,(cx)}{x}\,dx + \int \frac{2ibc\,\mathrm{atan}\,(cx)}{x^2}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**2*(a+b*atan(c*x))/x**3,x)
```

```
[Out] d**2*(Integral(a/x**3, x) + Integral(-a*c**2/x, x) + Integral(b*atan(c*x)/x
**3, x) + Integral(2*I*a*c/x**2, x) + Integral(-b*c**2*atan(c*x)/x, x) + In
tegral(2*I*b*c*atan(c*x)/x**2, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(i\,cdx + d)^2(b\arctan\,(cx) + a)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))/x^3,x, algorithm="giac")
```

```
[Out] integrate((I*c*d*x + d)^2*(b*arctan(c*x) + a)/x^3, x)
```

3.17 $\int \frac{(d+icdx)^2\left(a+b\tan^{-1}(cx)\right)}{x^4}\,dx$

Optimal. Leaf size=87

$$-\frac{d^2(1+icx)^3\left(a+b\tan^{-1}(cx)\right)}{3x^3}-\frac{ibc^2d^2}{x}-\frac{4}{3}bc^3d^2\log(x)+\frac{4}{3}bc^3d^2\log(cx+i)-\frac{bcd^2}{6x^2}$$

```
[Out] -(b*c*d^2)/(6*x^2) - (I*b*c^2*d^2)/x - (d^2*(1 + I*c*x)^3*(a + b*ArcTan[c*x
]))/(3*x^3) - (4*b*c^3*d^2*Log[x])/3 + (4*b*c^3*d^2*Log[I + c*x])/3
```

Rubi [A] time = 0.0818295, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {37, 4872, 12, 88}

$$-\frac{d^2(1+icx)^3\left(a+b\tan^{-1}(cx)\right)}{3x^3}-\frac{ibc^2d^2}{x}-\frac{4}{3}bc^3d^2\log(x)+\frac{4}{3}bc^3d^2\log(cx+i)-\frac{bcd^2}{6x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)^2*(a + b*ArcTan[c*x]))/x^4,x]
```

```
[Out] -(b*c*d^2)/(6*x^2) - (I*b*c^2*d^2)/x - (d^2*(1 + I*c*x)^3*(a + b*ArcTan[c*x
]))/(3*x^3) - (4*b*c^3*d^2*Log[x])/3 + (4*b*c^3*d^2*Log[I + c*x])/3
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 4872

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
+ b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2), x
], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2*m
] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int \frac{(d+icdx)^2\left(a+b\tan^{-1}(cx)\right)}{x^4}\,dx &= -\frac{d^2(1+icx)^3\left(a+b\tan^{-1}(cx)\right)}{3x^3}-(bc)\int \frac{id^2(i-cx)^2}{3x^3(i+cx)}\,dx\\ &= -\frac{d^2(1+icx)^3\left(a+b\tan^{-1}(cx)\right)}{3x^3}-\frac{1}{3}\left(ibcd^2\right)\int \frac{(i-cx)^2}{x^3(i+cx)}\,dx\\ &= -\frac{d^2(1+icx)^3\left(a+b\tan^{-1}(cx)\right)}{3x^3}-\frac{1}{3}\left(ibcd^2\right)\int \left(\frac{i}{x^3}-\frac{3c}{x^2}-\frac{4ic^2}{x}+\frac{4ic^3}{i+cx}\right)dx\\ &= -\frac{bcd^2}{6x^2}-\frac{ibc^2d^2}{x}-\frac{d^2(1+icx)^3\left(a+b\tan^{-1}(cx)\right)}{3x^3}-\frac{4}{3}bc^3d^2\log(x)+\frac{4}{3}bc^3d^2\log(i+\end{aligned}$$

Mathematica [C] time = 0.0903624, size = 114, normalized size = 1.31

$$-\frac{d^2\left(6ibc^2x^2\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-c^2x^2\right)-6ac^2x^2+6iacx+2a+8bc^3x^3\log(x)-4bc^3x^3\log\left(c^2x^2+1\right)+2b\right.}{6x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + I*c*d*x)^2*(a + b*ArcTan[c*x]))/x^4,x]
```

```
[Out] -(d^2*(2*a + (6*I)*a*c*x + b*c*x - 6*a*c^2*x^2 + 2*b*(1 + (3*I)*c*x - 3*c^2
*x^2)*ArcTan[c*x] + (6*I)*b*c^2*x^2*Hypergeometric2F1[-1/2, 1, 1/2, -(c^2*x
^2)] + 8*b*c^3*x^3*Log[x] - 4*b*c^3*x^3*Log[1 + c^2*x^2]))/(6*x^3)
```

Maple [A] time = 0.036, size = 145, normalized size = 1.7

$$\frac{-icd^2a}{x^2}+\frac{c^2d^2a}{x}-\frac{d^2a}{3\,x^3}-\frac{icd^2b\arctan(cx)}{x^2}+\frac{bc^2d^2\arctan(cx)}{x}-\frac{bd^2\arctan(cx)}{3\,x^3}+\frac{2\,c^3d^2b\ln\left(c^2x^2+1\right)}{3}-ic^3d^2b\,\text{arcta}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^2*(a+b*arctan(c*x))/x^4,x)
```

```
[Out] -I*c*d^2*a/x^2+c^2*d^2*a/x-1/3*d^2*a/x^3-I*c*d^2*b*arctan(c*x)/x^2+c^2*d^2*
b*arctan(c*x)/x-1/3*d^2*b*arctan(c*x)/x^3+2/3*c^3*d^2*b*ln(c^2*x^2+1)-I*c^3
*d^2*b*arctan(c*x)-I*b*c^2*d^2/x-1/6*b*c*d^2/x^2-4/3*c^3*d^2*b*ln(c*x)
```

Maxima [A] time = 1.47882, size = 194, normalized size = 2.23

$$\frac{1}{2}\left(c\left(\log\left(c^2x^2+1\right)-\log\left(x^2\right)\right)+\frac{2\arctan\left(cx\right)}{x}\right)bc^2d^2-i\left(\left(c\arctan\left(cx\right)+\frac{1}{x}\right)c+\frac{\arctan\left(cx\right)}{x^2}\right)bcd^2+\frac{1}{6}\left(\left(c^2\log\left(c^2x^2\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))/x^4,x, algorithm="maxima")
```

```
[Out] 1/2*(c*(log(c^2*x^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*b*c^2*d^2 - I*((c*a
rctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*b*c*d^2 + 1/6*((c^2*log(c^2*x^2 + 1)
 - c^2*log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^3)*b*d^2 + a*c^2*d^2/x - I*a*c
*d^2/x^2 - 1/3*a*d^2/x^3
```

Fricas [A] time = 2.74852, size = 316, normalized size = 3.63

$$-\frac{8\,bc^3d^2x^3\log\left(x\right)-7\,bc^3d^2x^3\log\left(\frac{cx+i}{c}\right)-bc^3d^2x^3\log\left(\frac{cx-i}{c}\right)-6\left(a-i\,b\right)c^2d^2x^2-\left(-6i\,a-b\right)cd^2x+2\,ad^2-\left(3i\,bc^2d^2x^2\right.}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] -1/6*(8*b*c^3*d^2*x^3*log(x) - 7*b*c^3*d^2*x^3*log((c*x + I)/c) - b*c^3*d^2
*x^3*log((c*x - I)/c) - 6*(a - I*b)*c^2*d^2*x^2 - (-6*I*a - b)*c*d^2*x + 2*
a*d^2 - (3*I*b*c^2*d^2*x^2 + 3*b*c*d^2*x - I*b*d^2)*log(-(c*x + I)/(c*x - I
)))/x^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**2*(a+b*atan(c*x))/x**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.22176, size = 194, normalized size = 2.23

$$\frac{7\,bc^3d^2x^3\log\left(cx+i\right)+bc^3d^2x^3\log\left(cx-i\right)-8\,bc^3d^2x^3\log\left(x\right)-6\,bc^2d^2ix^2+6\,bc^2d^2x^2\arctan\left(cx\right)+6\,ac^2d^2x^2-6\,bcd^2i}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))/x^4,x, algorithm="giac")
```

```
[Out] 1/6*(7*b*c^3*d^2*x^3*log(c*x + i) + b*c^3*d^2*x^3*log(c*x - i) - 8*b*c^3*d^
2*x^3*log(x) - 6*b*c^2*d^2*i*x^2 + 6*b*c^2*d^2*x^2*arctan(c*x) + 6*a*c^2*d^
2*x^2 - 6*b*c*d^2*i*x*arctan(c*x) - 6*a*c*d^2*i*x - b*c*d^2*x - 2*b*d^2*arc
tan(c*x) - 2*a*d^2)/x^3
```

3.18 $\int \frac{(d+icdx)^2\left(a+b\tan^{-1}(cx)\right)}{x^5}\,dx$

Optimal. Leaf size=161

$$\frac{c^2d^2\left(a+b\tan^{-1}(cx)\right)}{2x^2}-\frac{2icd^2\left(a+b\tan^{-1}(cx)\right)}{3x^3}-\frac{d^2\left(a+b\tan^{-1}(cx)\right)}{4x^4}-\frac{ibc^2d^2}{3x^2}+\frac{3bc^3d^2}{4x}-\frac{2}{3}ibc^4d^2\log(x)-\frac{1}{24}ibc^4d$$

```
[Out] -(b*c*d^2)/(12*x^3) - ((I/3)*b*c^2*d^2)/x^2 + (3*b*c^3*d^2)/(4*x) - (d^2*(a
 + b*ArcTan[c*x]))/(4*x^4) - (((2*I)/3)*c*d^2*(a + b*ArcTan[c*x]))/x^3 + (c
^2*d^2*(a + b*ArcTan[c*x]))/(2*x^2) - ((2*I)/3)*b*c^4*d^2*Log[x] - (I/24)*b
*c^4*d^2*Log[I - c*x] + ((17*I)/24)*b*c^4*d^2*Log[I + c*x]
```

Rubi [A] time = 0.149881, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {43, 4872, 12, 1802}

$$\frac{c^2d^2\left(a+b\tan^{-1}(cx)\right)}{2x^2}-\frac{2icd^2\left(a+b\tan^{-1}(cx)\right)}{3x^3}-\frac{d^2\left(a+b\tan^{-1}(cx)\right)}{4x^4}-\frac{ibc^2d^2}{3x^2}+\frac{3bc^3d^2}{4x}-\frac{2}{3}ibc^4d^2\log(x)-\frac{1}{24}ibc^4d$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)^2*(a + b*ArcTan[c*x]))/x^5,x]
```

```
[Out] -(b*c*d^2)/(12*x^3) - ((I/3)*b*c^2*d^2)/x^2 + (3*b*c^3*d^2)/(4*x) - (d^2*(a
 + b*ArcTan[c*x]))/(4*x^4) - (((2*I)/3)*c*d^2*(a + b*ArcTan[c*x]))/x^3 + (c
^2*d^2*(a + b*ArcTan[c*x]))/(2*x^2) - ((2*I)/3)*b*c^4*d^2*Log[x] - (I/24)*b
*c^4*d^2*Log[I - c*x] + ((17*I)/24)*b*c^4*d^2*Log[I + c*x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 4872

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
+ b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2), x
], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2*m
```

```
] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(d+icdx)^2\left(a+b\tan^{-1}(cx)\right)}{x^5}\,dx = -\frac{d^2\left(a+b\tan^{-1}(cx)\right)}{4x^4}-\frac{2icd^2\left(a+b\tan^{-1}(cx)\right)}{3x^3}+\frac{c^2d^2\left(a+b\tan^{-1}(cx)\right)}{2x^2}-(bc)\int$$

$$= -\frac{d^2\left(a+b\tan^{-1}(cx)\right)}{4x^4}-\frac{2icd^2\left(a+b\tan^{-1}(cx)\right)}{3x^3}+\frac{c^2d^2\left(a+b\tan^{-1}(cx)\right)}{2x^2}-\frac{1}{12}\left(bc\right.$$

$$= -\frac{d^2\left(a+b\tan^{-1}(cx)\right)}{4x^4}-\frac{2icd^2\left(a+b\tan^{-1}(cx)\right)}{3x^3}+\frac{c^2d^2\left(a+b\tan^{-1}(cx)\right)}{2x^2}-\frac{1}{12}\left(bc\right.$$

$$= -\frac{bcd^2}{12x^3}-\frac{ibc^2d^2}{3x^2}+\frac{3bc^3d^2}{4x}-\frac{d^2\left(a+b\tan^{-1}(cx)\right)}{4x^4}-\frac{2icd^2\left(a+b\tan^{-1}(cx)\right)}{3x^3}+\frac{c^2d^2}{}$$

Mathematica [C] time = 0.0679123, size = 152, normalized size = 0.94

$$\frac{d^2\left(6bc^3x^3\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-c^2x^2\right)-bcx\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-c^2x^2\right)+6ac^2x^2-8iacx-3a\right.}{12x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + I*c*d*x)^2*(a + b*ArcTan[c*x]))/x^5,x]
```

```
[Out] (d^2*(-3*a - (8*I)*a*c*x + 6*a*c^2*x^2 - (4*I)*b*c^2*x^2 - 3*b*ArcTan[c*x]
- (8*I)*b*c*x*ArcTan[c*x] + 6*b*c^2*x^2*ArcTan[c*x] - b*c*x*Hypergeometric2
F1[-3/2, 1, -1/2, -(c^2*x^2)] + 6*b*c^3*x^3*Hypergeometric2F1[-1/2, 1, 1/2,
 -(c^2*x^2)] - (8*I)*b*c^4*x^4*Log[x] + (4*I)*b*c^4*x^4*Log[1 + c^2*x^2]))/
(12*x^4)
```

Maple [A] time = 0.038, size = 160, normalized size = 1.

$$\frac{c^2d^2a}{2\,x^2}-\frac{d^2a}{4\,x^4}-\frac{\frac{2\,i}{3}cd^2a}{x^3}+\frac{bc^2d^2\arctan\left(cx\right)}{2\,x^2}-\frac{bd^2\arctan\left(cx\right)}{4\,x^4}-\frac{\frac{2\,i}{3}cd^2b\arctan\left(cx\right)}{x^3}+\frac{i}{3}c^4d^2b\ln\left(c^2x^2+1\right)+\frac{3\,bc^4d^2\,\mathrm{a}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^2*(a+b*arctan(c*x))/x^5,x)
```

```
[Out] 1/2*c^2*d^2*a/x^2-1/4*d^2*a/x^4-2/3*I*c*d^2*a/x^3+1/2*c^2*d^2*b*arctan(c*x)
/x^2-1/4*d^2*b*arctan(c*x)/x^4-2/3*I*c*d^2*b*arctan(c*x)/x^3+1/3*I*c^4*d^2*
b*ln(c^2*x^2+1)+3/4*b*c^4*d^2*arctan(c*x)-1/3*I*b*c^2*d^2/x^2-2/3*I*c^4*d^2
*b*ln(c*x)-1/12*b*c*d^2/x^3+3/4*b*c^3*d^2/x
```

Maxima [A] time = 1.47861, size = 205, normalized size = 1.27

$$\frac{1}{2}\left(\left(c\arctan\left(cx\right)+\frac{1}{x}\right)c+\frac{\arctan\left(cx\right)}{x^2}\right)bc^2d^2+\frac{1}{3}i\left(\left(c^2\log\left(c^2x^2+1\right)-c^2\log\left(x^2\right)-\frac{1}{x^2}\right)c-\frac{2\arctan\left(cx\right)}{x^3}\right)bcd^2+\frac{1}{12}\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))/x^5,x, algorithm="maxima")
```

```
[Out] 1/2*((c*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*b*c^2*d^2 + 1/3*I*((c^2*log
(c^2*x^2 + 1) - c^2*log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^3)*b*c*d^2 + 1/12
*((3*c^3*arctan(c*x) + (3*c^2*x^2 - 1)/x^3)*c - 3*arctan(c*x)/x^4)*b*d^2 +
1/2*a*c^2*d^2/x^2 - 2/3*I*a*c*d^2/x^3 - 1/4*a*d^2/x^4
```

Fricas [A] time = 2.81822, size = 366, normalized size = 2.27

$$\frac{-16i\,bc^4d^2x^4\log\left(x\right)+17i\,bc^4d^2x^4\log\left(\frac{cx+i}{c}\right)-i\,bc^4d^2x^4\log\left(\frac{cx-i}{c}\right)+18\,bc^3d^2x^3+4\left(3\,a-2i\,b\right)c^2d^2x^2+\left(-16i\,a-2\,b\right)cd^2}{24\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))/x^5,x, algorithm="fricas")
```

```
[Out] 1/24*(-16*I*b*c^4*d^2*x^4*log(x) + 17*I*b*c^4*d^2*x^4*log((c*x + I)/c) - I*
b*c^4*d^2*x^4*log((c*x - I)/c) + 18*b*c^3*d^2*x^3 + 4*(3*a - 2*I*b)*c^2*d^2
*x^2 + (-16*I*a - 2*b)*c*d^2*x - 6*a*d^2 + (6*I*b*c^2*d^2*x^2 + 8*b*c*d^2*x
 - 3*I*b*d^2)*log(-(c*x + I)/(c*x - I)))/x^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**2*(a+b*atan(c*x))/x**5,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20183, size = 217, normalized size = 1.35

$$\frac{17\,bc^4d^2ix^4\log\left(cix-1\right)-bc^4d^2ix^4\log\left(-cix-1\right)-16\,bc^4d^2ix^4\log\left(x\right)+18\,bc^3d^2x^3-8\,bc^2d^2ix^2+12\,bc^2d^2x^2\arctan\left(cx\right)}{24\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))/x^5,x, algorithm="giac")
```

```
[Out] 1/24*(17*b*c^4*d^2*i*x^4*log(c*i*x - 1) - b*c^4*d^2*i*x^4*log(-c*i*x - 1) -
 16*b*c^4*d^2*i*x^4*log(x) + 18*b*c^3*d^2*x^3 - 8*b*c^2*d^2*i*x^2 + 12*b*c^
2*d^2*x^2*arctan(c*x) + 12*a*c^2*d^2*x^2 - 16*b*c*d^2*i*x*arctan(c*x) - 16*
a*c*d^2*i*x - 2*b*c*d^2*x - 6*b*d^2*arctan(c*x) - 6*a*d^2)/x^4
```

3.19 $\int \frac{(d+icdx)^2\left(a+b\tan^{-1}(cx)\right)}{x^6}\,dx$

Optimal. Leaf size=171

$$\frac{c^2d^2\left(a+b\tan^{-1}(cx)\right)}{3x^3}-\frac{icd^2\left(a+b\tan^{-1}(cx)\right)}{2x^4}-\frac{d^2\left(a+b\tan^{-1}(cx)\right)}{5x^5}+\frac{4bc^3d^2}{15x^2}-\frac{ibc^2d^2}{6x^3}+\frac{ibc^4d^2}{2x}+\frac{8}{15}bc^5d^2\log(x)-$$

```
[Out] -(b*c*d^2)/(20*x^4) - ((I/6)*b*c^2*d^2)/x^3 + (4*b*c^3*d^2)/(15*x^2) + ((I/
2)*b*c^4*d^2)/x - (d^2*(a + b*ArcTan[c*x]))/(5*x^5) - ((I/2)*c*d^2*(a + b*A
rcTan[c*x]))/x^4 + (c^2*d^2*(a + b*ArcTan[c*x]))/(3*x^3) + (8*b*c^5*d^2*Log
[x])/15 - (b*c^5*d^2*Log[I - c*x])/60 - (31*b*c^5*d^2*Log[I + c*x])/60
```

Rubi [A] time = 0.158001, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {43, 4872, 12, 1802}

$$\frac{c^2d^2\left(a+b\tan^{-1}(cx)\right)}{3x^3}-\frac{icd^2\left(a+b\tan^{-1}(cx)\right)}{2x^4}-\frac{d^2\left(a+b\tan^{-1}(cx)\right)}{5x^5}+\frac{4bc^3d^2}{15x^2}-\frac{ibc^2d^2}{6x^3}+\frac{ibc^4d^2}{2x}+\frac{8}{15}bc^5d^2\log(x)-$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)^2*(a + b*ArcTan[c*x]))/x^6,x]
```

```
[Out] -(b*c*d^2)/(20*x^4) - ((I/6)*b*c^2*d^2)/x^3 + (4*b*c^3*d^2)/(15*x^2) + ((I/
2)*b*c^4*d^2)/x - (d^2*(a + b*ArcTan[c*x]))/(5*x^5) - ((I/2)*c*d^2*(a + b*A
rcTan[c*x]))/x^4 + (c^2*d^2*(a + b*ArcTan[c*x]))/(3*x^3) + (8*b*c^5*d^2*Log
[x])/15 - (b*c^5*d^2*Log[I - c*x])/60 - (31*b*c^5*d^2*Log[I + c*x])/60
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 4872

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
+ b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2), x
], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2*m
```

```
] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(d+icdx)^2\left(a+b\tan^{-1}(cx)\right)}{x^6}\,dx = -\frac{d^2\left(a+b\tan^{-1}(cx)\right)}{5x^5}-\frac{icd^2\left(a+b\tan^{-1}(cx)\right)}{2x^4}+\frac{c^2d^2\left(a+b\tan^{-1}(cx)\right)}{3x^3}-(bc)\int$$

$$= -\frac{d^2\left(a+b\tan^{-1}(cx)\right)}{5x^5}-\frac{icd^2\left(a+b\tan^{-1}(cx)\right)}{2x^4}+\frac{c^2d^2\left(a+b\tan^{-1}(cx)\right)}{3x^3}-\frac{1}{30}\left(bcd\right.$$

$$= -\frac{d^2\left(a+b\tan^{-1}(cx)\right)}{5x^5}-\frac{icd^2\left(a+b\tan^{-1}(cx)\right)}{2x^4}+\frac{c^2d^2\left(a+b\tan^{-1}(cx)\right)}{3x^3}-\frac{1}{30}\left(bcd\right.$$

$$= -\frac{bcd^2}{20x^4}-\frac{ibc^2d^2}{6x^3}+\frac{4bc^3d^2}{15x^2}+\frac{ibc^4d^2}{2x}-\frac{d^2\left(a+b\tan^{-1}(cx)\right)}{5x^5}-\frac{icd^2\left(a+b\tan^{-1}(cx)\right)}{2x^4}$$

Mathematica [C] time = 0.0862631, size = 124, normalized size = 0.73

$$\frac{d^2\left(-10ibc^2x^2\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-c^2x^2\right)+20ac^2x^2-30iacx-12a+16bc^3x^3+32bc^5x^5\log(x)-16bc^5x^5\,\mathrm{l}\right.}{60x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + I*c*d*x)^2*(a + b*ArcTan[c*x]))/x^6,x]
```

```
[Out] (d^2*(-12*a - (30*I)*a*c*x - 3*b*c*x + 20*a*c^2*x^2 + 16*b*c^3*x^3 + 2*b*(-
6 - (15*I)*c*x + 10*c^2*x^2)*ArcTan[c*x] - (10*I)*b*c^2*x^2*Hypergeometric2
F1[-3/2, 1, -1/2, -(c^2*x^2)] + 32*b*c^5*x^5*Log[x] - 16*b*c^5*x^5*Log[1 +
c^2*x^2]))/(60*x^5)
```

Maple [A] time = 0.037, size = 172, normalized size = 1.

$$-\frac{\frac{i}{2}cd^2a}{x^4}-\frac{d^2a}{5x^5}+\frac{c^2d^2a}{3x^3}-\frac{\frac{i}{2}cd^2b\arctan(cx)}{x^4}-\frac{bd^2\arctan(cx)}{5x^5}+\frac{bc^2d^2\arctan(cx)}{3x^3}-\frac{4c^5d^2b\ln(c^2x^2+1)}{15}+\frac{i}{2}c^5d^2b\,\mathrm{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^2*(a+b*arctan(c*x))/x^6,x)
```

```
[Out] -1/2*I*c*d^2*a/x^4-1/5*d^2*a/x^5+1/3*c^2*d^2*a/x^3-1/2*I*c*d^2*b*arctan(c*x
)/x^4-1/5*d^2*b*arctan(c*x)/x^5+1/3*c^2*d^2*b*arctan(c*x)/x^3-4/15*c^5*d^2*
b*ln(c^2*x^2+1)+1/2*I*c^5*d^2*b*arctan(c*x)-1/6*I*b*c^2*d^2/x^3+1/2*I*b*c^4
*d^2/x-1/20*b*c*d^2/x^4+4/15*b*c^3*d^2/x^2+8/15*c^5*d^2*b*ln(c*x)
```

Maxima [A] time = 1.48874, size = 247, normalized size = 1.44

$$-\frac{1}{6}\left(\left(c^2\log\left(c^2x^2+1\right)-c^2\log\left(x^2\right)-\frac{1}{x^2}\right)c-\frac{2\arctan(cx)}{x^3}\right)bc^2d^2+\frac{1}{6}i\left(\left(3c^3\arctan(cx)+\frac{3c^2x^2-1}{x^3}\right)c-\frac{3\arctan(c}{x^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))/x^6,x, algorithm="maxima")
```

```
[Out] -1/6*((c^2*log(c^2*x^2 + 1) - c^2*log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^3)*
b*c^2*d^2 + 1/6*I*((3*c^3*arctan(c*x) + (3*c^2*x^2 - 1)/x^3)*c - 3*arctan(c
*x)/x^4)*b*c*d^2 - 1/20*((2*c^4*log(c^2*x^2 + 1) - 2*c^4*log(x^2) - (2*c^2*
x^2 - 1)/x^4)*c + 4*arctan(c*x)/x^5)*b*d^2 + 1/3*a*c^2*d^2/x^3 - 1/2*I*a*c*
d^2/x^4 - 1/5*a*d^2/x^5
```

Fricas [A] time = 2.89968, size = 387, normalized size = 2.26

$$\frac{32bc^5d^2x^5\log(x)-31bc^5d^2x^5\log\left(\frac{cx+i}{c}\right)-bc^5d^2x^5\log\left(\frac{cx-i}{c}\right)+30i\,bc^4d^2x^4+16bc^3d^2x^3+10(2a-ib)c^2d^2x^2+(-30i}{60x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))/x^6,x, algorithm="fricas")
```

```
[Out] 1/60*(32*b*c^5*d^2*x^5*log(x) - 31*b*c^5*d^2*x^5*log((c*x + I)/c) - b*c^5*d
^2*x^5*log((c*x - I)/c) + 30*I*b*c^4*d^2*x^4 + 16*b*c^3*d^2*x^3 + 10*(2*a -
 I*b)*c^2*d^2*x^2 + (-30*I*a - 3*b)*c*d^2*x - 12*a*d^2 + (10*I*b*c^2*d^2*x^
2 + 15*b*c*d^2*x - 6*I*b*d^2)*log(-(c*x + I)/(c*x - I)))/x^5
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**2*(a+b*atan(c*x))/x**6,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.38457, size = 228, normalized size = 1.33

$$-\frac{31\,bc^5d^2x^5\log\left(cx+i\right)+bc^5d^2x^5\log\left(cx-i\right)-32\,bc^5d^2x^5\log\left(x\right)-30\,bc^4d^2ix^4-16\,bc^3d^2x^3+10\,bc^2d^2ix^2-20\,bc^2d^2x^2}{60\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))/x^6,x, algorithm="giac")
```

```
[Out] -1/60*(31*b*c^5*d^2*x^5*log(c*x + i) + b*c^5*d^2*x^5*log(c*x - i) - 32*b*c^
5*d^2*x^5*log(x) - 30*b*c^4*d^2*i*x^4 - 16*b*c^3*d^2*x^3 + 10*b*c^2*d^2*i*x
^2 - 20*b*c^2*d^2*x^2*arctan(c*x) - 20*a*c^2*d^2*x^2 + 30*b*c*d^2*i*x*arcta
n(c*x) + 30*a*c*d^2*i*x + 3*b*c*d^2*x + 12*b*d^2*arctan(c*x) + 12*a*d^2)/x^
5
```

3.20 $\int x^3(d+icdx)^3 \left(a+b\tan^{-1}(cx)\right) dx$

Optimal. Leaf size=205

$$-\frac{1}{7}ic^3d^3x^7\left(a+b\tan^{-1}(cx)\right)-\frac{1}{2}c^2d^3x^6\left(a+b\tan^{-1}(cx)\right)+\frac{3}{5}icd^3x^5\left(a+b\tan^{-1}(cx)\right)+\frac{1}{4}d^3x^4\left(a+b\tan^{-1}(cx)\right)+\frac{1}{42}il$$

```
[Out] (3*b*d^3*x)/(4*c^3) + (((13*I)/35)*b*d^3*x^2)/c^2 - (b*d^3*x^3)/(4*c) - ((1
3*I)/70)*b*d^3*x^4 + (b*c*d^3*x^5)/10 + (I/42)*b*c^2*d^3*x^6 - (3*b*d^3*Arc
Tan[c*x])/(4*c^4) + (d^3*x^4*(a + b*ArcTan[c*x]))/4 + ((3*I)/5)*c*d^3*x^5*(
a + b*ArcTan[c*x]) - (c^2*d^3*x^6*(a + b*ArcTan[c*x]))/2 - (I/7)*c^3*d^3*x^
7*(a + b*ArcTan[c*x]) - (((13*I)/35)*b*d^3*Log[1 + c^2*x^2])/c^4
```

Rubi [A] time = 0.183864, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {43, 4872, 12, 1802, 635, 203, 260}

$$-\frac{1}{7}ic^3d^3x^7\left(a+b\tan^{-1}(cx)\right)-\frac{1}{2}c^2d^3x^6\left(a+b\tan^{-1}(cx)\right)+\frac{3}{5}icd^3x^5\left(a+b\tan^{-1}(cx)\right)+\frac{1}{4}d^3x^4\left(a+b\tan^{-1}(cx)\right)+\frac{1}{42}il$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + I*c*d*x)^3*(a + b*ArcTan[c*x]),x]
```

```
[Out] (3*b*d^3*x)/(4*c^3) + (((13*I)/35)*b*d^3*x^2)/c^2 - (b*d^3*x^3)/(4*c) - ((1
3*I)/70)*b*d^3*x^4 + (b*c*d^3*x^5)/10 + (I/42)*b*c^2*d^3*x^6 - (3*b*d^3*Arc
Tan[c*x])/(4*c^4) + (d^3*x^4*(a + b*ArcTan[c*x]))/4 + ((3*I)/5)*c*d^3*x^5*(
a + b*ArcTan[c*x]) - (c^2*d^3*x^6*(a + b*ArcTan[c*x]))/2 - (I/7)*c^3*d^3*x^
7*(a + b*ArcTan[c*x]) - (((13*I)/35)*b*d^3*Log[1 + c^2*x^2])/c^4
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 4872

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
+ b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2), x
```

```
], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2*m
] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int x^3(d+icdx)^3\left(a+b\tan^{-1}(cx)\right)\,dx=\frac{1}{4}d^3x^4\left(a+b\tan^{-1}(cx)\right)+\frac{3}{5}icd^3x^5\left(a+b\tan^{-1}(cx)\right)-\frac{1}{2}c^2d^3x^6\left(a+b\tan^{-1}(cx\right.$$

$$=\frac{1}{4}d^3x^4\left(a+b\tan^{-1}(cx)\right)+\frac{3}{5}icd^3x^5\left(a+b\tan^{-1}(cx)\right)-\frac{1}{2}c^2d^3x^6\left(a+b\tan^{-1}(cx\right.$$

$$=\frac{1}{4}d^3x^4\left(a+b\tan^{-1}(cx)\right)+\frac{3}{5}icd^3x^5\left(a+b\tan^{-1}(cx)\right)-\frac{1}{2}c^2d^3x^6\left(a+b\tan^{-1}(cx\right.$$

$$=\frac{3bd^3x}{4c^3}+\frac{13ibd^3x^2}{35c^2}-\frac{bd^3x^3}{4c}-\frac{13}{70}ibd^3x^4+\frac{1}{10}bcd^3x^5+\frac{1}{42}ibc^2d^3x^6+\frac{1}{4}d^3x^4\left(a+\right.$$

$$=\frac{3bd^3x}{4c^3}+\frac{13ibd^3x^2}{35c^2}-\frac{bd^3x^3}{4c}-\frac{13}{70}ibd^3x^4+\frac{1}{10}bcd^3x^5+\frac{1}{42}ibc^2d^3x^6+\frac{1}{4}d^3x^4\left(a+\right.$$

$$=\frac{3bd^3x}{4c^3}+\frac{13ibd^3x^2}{35c^2}-\frac{bd^3x^3}{4c}-\frac{13}{70}ibd^3x^4+\frac{1}{10}bcd^3x^5+\frac{1}{42}ibc^2d^3x^6-\frac{3bd^3\tan^{-}}{4c^4}$$

Mathematica [A] time = 0.105529, size = 248, normalized size = 1.21

$$-\frac{1}{7}iac^3d^3x^7-\frac{1}{2}ac^2d^3x^6+\frac{3}{5}iacd^3x^5+\frac{1}{4}ad^3x^4+\frac{1}{42}ibc^2d^3x^6+\frac{13ibd^3x^2}{35c^2}-\frac{13ibd^3\log\left(c^2x^2+1\right)}{35c^4}-\frac{1}{7}ibc^3d^3x^7\tan^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + I*c*d*x)^3*(a + b*ArcTan[c*x]),x]
```

```
[Out] (3*b*d^3*x)/(4*c^3) + (((13*I)/35)*b*d^3*x^2)/c^2 - (b*d^3*x^3)/(4*c) + (a*
d^3*x^4)/4 - ((13*I)/70)*b*d^3*x^4 + ((3*I)/5)*a*c*d^3*x^5 + (b*c*d^3*x^5)/
10 - (a*c^2*d^3*x^6)/2 + (I/42)*b*c^2*d^3*x^6 - (I/7)*a*c^3*d^3*x^7 - (3*b*
d^3*ArcTan[c*x])/(4*c^4) + (b*d^3*x^4*ArcTan[c*x])/4 + ((3*I)/5)*b*c*d^3*x^
5*ArcTan[c*x] - (b*c^2*d^3*x^6*ArcTan[c*x])/2 - (I/7)*b*c^3*d^3*x^7*ArcTan[
c*x] - (((13*I)/35)*b*d^3*Log[1 + c^2*x^2])/c^4
```

Maple [A] time = 0.03, size = 209, normalized size = 1.

$$-\frac{i}{7}c^3d^3b\arctan(cx)\,x^7-\frac{c^2d^3ax^6}{2}-\frac{\frac{13\,i}{35}bd^3\ln\left(c^2x^2+1\right)}{c^4}+\frac{d^3ax^4}{4}-\frac{13\,i}{70}bd^3x^4-\frac{c^2d^3b\arctan(cx)\,x^6}{2}+\frac{3\,i}{5}cd^3b\arctan$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d+I*c*d*x)^3*(a+b*arctan(c*x)),x)
```

```
[Out] -1/7*I*c^3*d^3*b*arctan(c*x)*x^7-1/2*c^2*d^3*a*x^6-13/35*I*b*d^3*ln(c^2*x^2
+1)/c^4+1/4*d^3*a*x^4-13/70*I*b*d^3*x^4-1/2*c^2*d^3*b*arctan(c*x)*x^6+3/5*I
*c*d^3*b*arctan(c*x)*x^5+1/4*d^3*b*arctan(c*x)*x^4+3/4*b*d^3*x/c^3+13/35*I*
b*d^3*x^2/c^2+1/10*b*c*d^3*x^5+1/42*I*b*c^2*d^3*x^6-1/4*b*d^3*x^3/c-1/7*I*c
^3*d^3*a*x^7+3/5*I*c*d^3*a*x^5-3/4*b*d^3*arctan(c*x)/c^4
```

Maxima [A] time = 1.48081, size = 352, normalized size = 1.72

$$-\frac{1}{7}i\,ac^3d^3x^7-\frac{1}{2}\,ac^2d^3x^6+\frac{3}{5}i\,acd^3x^5-\frac{1}{84}i\left(12\,x^7\arctan\left(cx\right)-c\left(\frac{2\,c^4x^6-3\,c^2x^4+6\,x^2}{c^6}-\frac{6\,\log\left(c^2x^2+1\right)}{c^8}\right)\right)bc^3d^3+\frac{1}{4}\,a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d+I*c*d*x)^3*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] -1/7*I*a*c^3*d^3*x^7 - 1/2*a*c^2*d^3*x^6 + 3/5*I*a*c*d^3*x^5 - 1/84*I*(12*x
^7*arctan(c*x) - c*((2*c^4*x^6 - 3*c^2*x^4 + 6*x^2)/c^6 - 6*log(c^2*x^2 + 1
)/c^8))*b*c^3*d^3 + 1/4*a*d^3*x^4 - 1/30*(15*x^6*arctan(c*x) - c*((3*c^4*x^
5 - 5*c^2*x^3 + 15*x)/c^6 - 15*arctan(c*x)/c^7))*b*c^2*d^3 + 3/20*I*(4*x^5*
arctan(c*x) - c*((c^2*x^4 - 2*x^2)/c^4 + 2*log(c^2*x^2 + 1)/c^6))*b*c*d^3 +
 1/12*(3*x^4*arctan(c*x) - c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5))*b*d
^3
```

Fricas [A] time = 2.76209, size = 481, normalized size = 2.35

$$\frac{-120i\,ac^7d^3x^7-20\left(21\,a-i\,b\right)c^6d^3x^6+\left(504i\,a+84\,b\right)c^5d^3x^5+6\left(35\,a-26i\,b\right)c^4d^3x^4-210\,bc^3d^3x^3+312i\,bc^2d^3x^2+630}{840\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d+I*c*d*x)^3*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] 1/840*(-120*I*a*c^7*d^3*x^7 - 20*(21*a - I*b)*c^6*d^3*x^6 + (504*I*a + 84*b
)*c^5*d^3*x^5 + 6*(35*a - 26*I*b)*c^4*d^3*x^4 - 210*b*c^3*d^3*x^3 + 312*I*b
*c^2*d^3*x^2 + 630*b*c*d^3*x - 627*I*b*d^3*log((c*x + I)/c) + 3*I*b*d^3*log
((c*x - I)/c) + (60*b*c^7*d^3*x^7 - 210*I*b*c^6*d^3*x^6 - 252*b*c^5*d^3*x^5
```

```
+ 105*I*b*c^4*d^3*x^4)*log(-(c*x + I)/(c*x - I)))/c^4
```

Sympy [A] time = 3.45616, size = 292, normalized size = 1.42

$$-\frac{iac^3d^3x^7}{7}-\frac{bd^3x^3}{4c}+\frac{13ibd^3x^2}{35c^2}+\frac{3bd^3x}{4c^3}+\frac{ibd^3\log\left(x-\frac{i}{c}\right)}{280c^4}-\frac{209ibd^3\log\left(x+\frac{i}{c}\right)}{280c^4}-x^6\left(\frac{ac^2d^3}{2}-\frac{ibc^2d^3}{42}\right)-x^5\left(-\frac{3iacd^3}{5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(d+I*c*d*x)**3*(a+b*atan(c*x)),x)
```

```
[Out] -I*a*c**3*d**3*x**7/7 - b*d**3*x**3/(4*c) + 13*I*b*d**3*x**2/(35*c**2) + 3*
b*d**3*x/(4*c**3) + I*b*d**3*log(x - I/c)/(280*c**4) - 209*I*b*d**3*log(x +
 I/c)/(280*c**4) - x**6*(a*c**2*d**3/2 - I*b*c**2*d**3/42) - x**5*(-3*I*a*c
*d**3/5 - b*c*d**3/10) - x**4*(-a*d**3/4 + 13*I*b*d**3/70) + (-b*c**3*d**3*
x**7/14 + I*b*c**2*d**3*x**6/4 + 3*b*c*d**3*x**5/10 - I*b*d**3*x**4/8)*log(
I*c*x + 1) + (b*c**3*d**3*x**7/14 - I*b*c**2*d**3*x**6/4 - 3*b*c*d**3*x**5/
10 + I*b*d**3*x**4/8)*log(-I*c*x + 1)
```

Giac [A] time = 1.196, size = 301, normalized size = 1.47

$$\frac{120\,bc^7d^3x^7\arctan(cx)+120\,ac^7d^3x^7-420\,bc^6d^3ix^6\arctan(cx)-420\,ac^6d^3ix^6-20\,bc^6d^3x^6+84\,bc^5d^3ix^5-504\,bc^5d}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d+I*c*d*x)^3*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] 1/840*(120*b*c^7*d^3*x^7*arctan(c*x) + 120*a*c^7*d^3*x^7 - 420*b*c^6*d^3*i*
x^6*arctan(c*x) - 420*a*c^6*d^3*i*x^6 - 20*b*c^6*d^3*x^6 + 84*b*c^5*d^3*i*x
^5 - 504*b*c^5*d^3*x^5*arctan(c*x) - 504*a*c^5*d^3*x^5 + 210*b*c^4*d^3*i*x^
4*arctan(c*x) + 210*a*c^4*d^3*i*x^4 + 156*b*c^4*d^3*x^4 - 210*b*c^3*d^3*i*x
^3 - 312*b*c^2*d^3*x^2 + 630*b*c*d^3*i*x - 3*b*d^3*log(c*i*x + 1) + 627*b*d
^3*log(-c*i*x + 1))/(c^4*i)
```

3.21 $\int x^2(d+icdx)^3\left(a+b\tan^{-1}(cx)\right)dx$

Optimal. Leaf size=191

$$-\frac{1}{6}ic^3d^3x^6\left(a+b\tan^{-1}(cx)\right)-\frac{3}{5}c^2d^3x^5\left(a+b\tan^{-1}(cx)\right)+\frac{3}{4}icd^3x^4\left(a+b\tan^{-1}(cx)\right)+\frac{1}{3}d^3x^3\left(a+b\tan^{-1}(cx)\right)+\frac{1}{30}ibc$$

```
[Out] ((((11*I)/12)*b*d^3*x)/c^2 - (7*b*d^3*x^2)/(15*c) - ((11*I)/36)*b*d^3*x^3 +
(3*b*c*d^3*x^4)/20 + (I/30)*b*c^2*d^3*x^5 - ((((11*I)/12)*b*d^3*ArcTan[c*x])
/c^3 + (d^3*x^3*(a + b*ArcTan[c*x]))/3 + ((3*I)/4)*c*d^3*x^4*(a + b*ArcTan[
c*x]) - (3*c^2*d^3*x^5*(a + b*ArcTan[c*x]))/5 - (I/6)*c^3*d^3*x^6*(a + b*Ar
cTan[c*x]) + (7*b*d^3*Log[1 + c^2*x^2])/(15*c^3)
```

Rubi [A] time = 0.171228, antiderivative size = 191, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {43, 4872, 12, 1802, 635, 203, 260}

$$-\frac{1}{6}ic^3d^3x^6\left(a+b\tan^{-1}(cx)\right)-\frac{3}{5}c^2d^3x^5\left(a+b\tan^{-1}(cx)\right)+\frac{3}{4}icd^3x^4\left(a+b\tan^{-1}(cx)\right)+\frac{1}{3}d^3x^3\left(a+b\tan^{-1}(cx)\right)+\frac{1}{30}ibc$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + I*c*d*x)^3*(a + b*ArcTan[c*x]),x]
```


```
[Out] ((((11*I)/12)*b*d^3*x)/c^2 - (7*b*d^3*x^2)/(15*c) - ((11*I)/36)*b*d^3*x^3 +
(3*b*c*d^3*x^4)/20 + (I/30)*b*c^2*d^3*x^5 - ((((11*I)/12)*b*d^3*ArcTan[c*x])
/c^3 + (d^3*x^3*(a + b*ArcTan[c*x]))/3 + ((3*I)/4)*c*d^3*x^4*(a + b*ArcTan[
c*x]) - (3*c^2*d^3*x^5*(a + b*ArcTan[c*x]))/5 - (I/6)*c^3*d^3*x^6*(a + b*Ar
cTan[c*x]) + (7*b*d^3*Log[1 + c^2*x^2])/(15*c^3)
```


Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 4872

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
+ b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2), x
```

```
], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2*m
] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int x^2(d+icdx)^3\left(a+b\tan^{-1}(cx)\right)dx = \frac{1}{3}d^3x^3\left(a+b\tan^{-1}(cx)\right)+\frac{3}{4}icd^3x^4\left(a+b\tan^{-1}(cx)\right)-\frac{3}{5}c^2d^3x^5\left(a+b\tan^{-1}(cx)\right.$$

$$=\frac{1}{3}d^3x^3\left(a+b\tan^{-1}(cx)\right)+\frac{3}{4}icd^3x^4\left(a+b\tan^{-1}(cx)\right)-\frac{3}{5}c^2d^3x^5\left(a+b\tan^{-1}(cx)\right.$$

$$=\frac{1}{3}d^3x^3\left(a+b\tan^{-1}(cx)\right)+\frac{3}{4}icd^3x^4\left(a+b\tan^{-1}(cx)\right)-\frac{3}{5}c^2d^3x^5\left(a+b\tan^{-1}(cx)\right.$$

$$=\frac{11ibd^3x}{12c^2}-\frac{7bd^3x^2}{15c}-\frac{11}{36}ibd^3x^3+\frac{3}{20}bcd^3x^4+\frac{1}{30}ibc^2d^3x^5+\frac{1}{3}d^3x^3\left(a+b\tan^{-1}(c\right.$$

$$=\frac{11ibd^3x}{12c^2}-\frac{7bd^3x^2}{15c}-\frac{11}{36}ibd^3x^3+\frac{3}{20}bcd^3x^4+\frac{1}{30}ibc^2d^3x^5+\frac{1}{3}d^3x^3\left(a+b\tan^{-1}(c\right.$$

$$=\frac{11ibd^3x}{12c^2}-\frac{7bd^3x^2}{15c}-\frac{11}{36}ibd^3x^3+\frac{3}{20}bcd^3x^4+\frac{1}{30}ibc^2d^3x^5-\frac{11ibd^3\tan^{-1}(cx)}{12c^3}+\frac{1}{3}$$

Mathematica [A] time = 0.0856371, size = 234, normalized size = 1.23

$$-\frac{1}{6}iac^3d^3x^6-\frac{3}{5}ac^2d^3x^5+\frac{3}{4}iacd^3x^4+\frac{1}{3}ad^3x^3+\frac{1}{30}ibc^2d^3x^5+\frac{7bd^3\log\left(c^2x^2+1\right)}{15c^3}-\frac{1}{6}ibc^3d^3x^6\tan^{-1}(cx)-\frac{3}{5}bc^2d^3x^5\tan$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + I*c*d*x)^3*(a + b*ArcTan[c*x]),x]
```

```
[Out] (((11*I)/12)*b*d^3*x)/c^2 - (7*b*d^3*x^2)/(15*c) + (a*d^3*x^3)/3 - ((11*I)/
36)*b*d^3*x^3 + ((3*I)/4)*a*c*d^3*x^4 + (3*b*c*d^3*x^4)/20 - (3*a*c^2*d^3*x
^5)/5 + (I/30)*b*c^2*d^3*x^5 - (I/6)*a*c^3*d^3*x^6 - (((11*I)/12)*b*d^3*Arc
Tan[c*x])/c^3 + (b*d^3*x^3*ArcTan[c*x])/3 + ((3*I)/4)*b*c*d^3*x^4*ArcTan[c*
x] - (3*b*c^2*d^3*x^5*ArcTan[c*x])/5 - (I/6)*b*c^3*d^3*x^6*ArcTan[c*x] + (7
*b*d^3*Log[1 + c^2*x^2])/(15*c^3)
```

Maple [A] time = 0.027, size = 197, normalized size = 1.

$$-\frac{i}{6}c^3d^3ax^6-\frac{3c^2d^3ax^5}{5}+\frac{3i}{4}cd^3ax^4+\frac{d^3ax^3}{3}-\frac{i}{6}c^3d^3b\arctan(cx)x^6-\frac{3c^2d^3b\arctan(cx)x^5}{5}+\frac{3i}{4}cd^3b\arctan(cx)x^4+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d+I*c*d*x)^3*(a+b*arctan(c*x)),x)
```

```
[Out] -1/6*I*c^3*d^3*a*x^6-3/5*c^2*d^3*a*x^5+3/4*I*c*d^3*a*x^4+1/3*d^3*a*x^3-1/6*
I*c^3*d^3*b*arctan(c*x)*x^6-3/5*c^2*d^3*b*arctan(c*x)*x^5+3/4*I*c*d^3*b*arc
tan(c*x)*x^4+1/3*d^3*b*arctan(c*x)*x^3+11/12*I*b*d^3*x/c^2+1/30*I*b*c^2*d^3
*x^5+3/20*b*c*d^3*x^4-11/36*I*b*d^3*x^3-7/15*b*d^3*x^2/c+7/15*b*d^3*ln(c^2*
x^2+1)/c^3-11/12*I*b*d^3*arctan(c*x)/c^3
```

Maxima [A] time = 1.4773, size = 327, normalized size = 1.71

$$-\frac{1}{6} i\, ac^3d^3x^6 - \frac{3}{5}\, ac^2d^3x^5 + \frac{3}{4} i\, acd^3x^4 - \frac{1}{90} i \left(15\, x^6 \arctan\left(cx\right) - c\left(\frac{3\, c^4x^5 - 5\, c^2x^3 + 15\, x}{c^6} - \frac{15\, \arctan\left(cx\right)}{c^7}\right)\right)bc^3d^3 - \frac{3}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d+I*c*d*x)^3*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] -1/6*I*a*c^3*d^3*x^6 - 3/5*a*c^2*d^3*x^5 + 3/4*I*a*c*d^3*x^4 - 1/90*I*(15*x
^6*arctan(c*x) - c*((3*c^4*x^5 - 5*c^2*x^3 + 15*x)/c^6 - 15*arctan(c*x)/c^7
))*b*c^3*d^3 - 3/20*(4*x^5*arctan(c*x) - c*((c^2*x^4 - 2*x^2)/c^4 + 2*log(c
^2*x^2 + 1)/c^6))*b*c^2*d^3 + 1/3*a*d^3*x^3 + 1/4*I*(3*x^4*arctan(c*x) - c*
((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5))*b*c*d^3 + 1/6*(2*x^3*arctan(c*x)
 - c*(x^2/c^2 - log(c^2*x^2 + 1)/c^4))*b*d^3
```

Fricas [A] time = 2.79283, size = 447, normalized size = 2.34

$$\frac{-60 i\, ac^6d^3x^6 - 12\, (18\, a - i\, b)c^5d^3x^5 + (270 i\, a + 54\, b)c^4d^3x^4 + 10\, (12\, a - 11 i\, b)c^3d^3x^3 - 168\, bc^2d^3x^2 + 330 i\, bcd^3x + 333\, b}{360\, c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d+I*c*d*x)^3*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] 1/360*(-60*I*a*c^6*d^3*x^6 - 12*(18*a - I*b)*c^5*d^3*x^5 + (270*I*a + 54*b)
*c^4*d^3*x^4 + 10*(12*a - 11*I*b)*c^3*d^3*x^3 - 168*b*c^2*d^3*x^2 + 330*I*b
*c*d^3*x + 333*b*d^3*log((c*x + I)/c) + 3*b*d^3*log((c*x - I)/c) + (30*b*c^
6*d^3*x^6 - 108*I*b*c^5*d^3*x^5 - 135*b*c^4*d^3*x^4 + 60*I*b*c^3*d^3*x^3)*l
og(-(c*x + I)/(c*x - I)))/c^3
```

Sympy [A] time = 3.31909, size = 275, normalized size = 1.44

$$-\frac{iac^3d^3x^6}{6} - \frac{7bd^3x^2}{15c} + \frac{11ibd^3x}{12c^2} - \frac{bd^3\left(-\frac{\log\left(x-\frac{i}{c}\right)}{120} - \frac{37\log\left(x+\frac{i}{c}\right)}{40}\right)}{c^3} - x^5\left(\frac{3ac^2d^3}{5} - \frac{ibc^2d^3}{30}\right) - x^4\left(-\frac{3iacd^3}{4} - \frac{3bcd^3}{20}\right) - x^3\left(-\frac{a}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d+I*c*d*x)**3*(a+b*atan(c*x)),x)
```

```
[Out] -I*a*c**3*d**3*x**6/6 - 7*b*d**3*x**2/(15*c) + 11*I*b*d**3*x/(12*c**2) - b*
d**3*(-log(x - I/c)/120 - 37*log(x + I/c)/40)/c**3 - x**5*(3*a*c**2*d**3/5
- I*b*c**2*d**3/30) - x**4*(-3*I*a*c*d**3/4 - 3*b*c*d**3/20) - x**3*(-a*d**
3/3 + 11*I*b*d**3/36) + (-b*c**3*d**3*x**6/12 + 3*I*b*c**2*d**3*x**5/10 + 3
*b*c*d**3*x**4/8 - I*b*d**3*x**3/6)*log(I*c*x + 1) + (b*c**3*d**3*x**6/12 -
 3*I*b*c**2*d**3*x**5/10 - 3*b*c*d**3*x**4/8 + I*b*d**3*x**3/6)*log(-I*c*x
+ 1)
```

Giac [A] time = 1.19851, size = 279, normalized size = 1.46

$$-\frac{60\,bc^6d^3ix^6\arctan(cx) + 60\,ac^6d^3ix^6 - 12\,bc^5d^3ix^5 + 216\,bc^5d^3x^5\arctan(cx) + 216\,ac^5d^3x^5 - 270\,bc^4d^3ix^4\arctan(cx)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d+I*c*d*x)^3*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] -1/360*(60*b*c^6*d^3*i*x^6*arctan(c*x) + 60*a*c^6*d^3*i*x^6 - 12*b*c^5*d^3*
i*x^5 + 216*b*c^5*d^3*x^5*arctan(c*x) + 216*a*c^5*d^3*x^5 - 270*b*c^4*d^3*i
*x^4*arctan(c*x) - 270*a*c^4*d^3*i*x^4 - 54*b*c^4*d^3*x^4 + 110*b*c^3*d^3*i
*x^3 - 120*b*c^3*d^3*x^3*arctan(c*x) - 120*a*c^3*d^3*x^3 + 168*b*c^2*d^3*x^
2 - 330*b*c*d^3*i*x - 333*b*d^3*log(c*x + i) - 3*b*d^3*log(c*x - i))/c^3
```

3.22 $\int x(d+icdx)^3 \left(a+b\tan^{-1}(cx)\right) dx$

Optimal. Leaf size=157

$$-\frac{d^3(1+icx)^5\left(a+b\tan^{-1}(cx)\right)}{5c^2}+\frac{d^3(1+icx)^4\left(a+b\tan^{-1}(cx)\right)}{4c^2}+\frac{ibd^3(-cx+i)^4}{20c^2}-\frac{bd^3(-cx+i)^3}{20c^2}-\frac{3ibd^3(-cx+i)^2}{20c^2}+$$

```
[Out] (-3*b*d^3*x)/(5*c) - (((3*I)/20)*b*d^3*(I - c*x)^2)/c^2 - (b*d^3*(I - c*x)^
3)/(20*c^2) + ((I/20)*b*d^3*(I - c*x)^4)/c^2 + (d^3*(1 + I*c*x)^4*(a + b*Ar
cTan[c*x]))/(4*c^2) - (d^3*(1 + I*c*x)^5*(a + b*ArcTan[c*x]))/(5*c^2) + (((
6*I)/5)*b*d^3*Log[I + c*x])/c^2
```

Rubi [A] time = 0.0968673, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {43, 4872, 12, 77}

$$-\frac{d^3(1+icx)^5\left(a+b\tan^{-1}(cx)\right)}{5c^2}+\frac{d^3(1+icx)^4\left(a+b\tan^{-1}(cx)\right)}{4c^2}+\frac{ibd^3(-cx+i)^4}{20c^2}-\frac{bd^3(-cx+i)^3}{20c^2}-\frac{3ibd^3(-cx+i)^2}{20c^2}+$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + I*c*d*x)^3*(a + b*ArcTan[c*x]),x]
```


```
[Out] (-3*b*d^3*x)/(5*c) - (((3*I)/20)*b*d^3*(I - c*x)^2)/c^2 - (b*d^3*(I - c*x)^
3)/(20*c^2) + ((I/20)*b*d^3*(I - c*x)^4)/c^2 + (d^3*(1 + I*c*x)^4*(a + b*Ar
cTan[c*x]))/(4*c^2) - (d^3*(1 + I*c*x)^5*(a + b*ArcTan[c*x]))/(5*c^2) + (((
6*I)/5)*b*d^3*Log[I + c*x])/c^2
```


Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 4872

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
+ b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2), x
], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2*m
```

```
] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int x(d+icdx)^3\left(a+b\tan^{-1}(cx)\right)dx=\frac{d^3(1+icx)^4\left(a+b\tan^{-1}(cx)\right)}{4c^2}-\frac{d^3(1+icx)^5\left(a+b\tan^{-1}(cx)\right)}{5c^2}-(bc)\int\frac{d^3(i-cx)}{20c^2}$$

$$=\frac{d^3(1+icx)^4\left(a+b\tan^{-1}(cx)\right)}{4c^2}-\frac{d^3(1+icx)^5\left(a+b\tan^{-1}(cx)\right)}{5c^2}-\frac{\left(bd^3\right)\int\frac{(i-cx)^3(-1}{i+cx}}{20c}$$

$$=\frac{d^3(1+icx)^4\left(a+b\tan^{-1}(cx)\right)}{4c^2}-\frac{d^3(1+icx)^5\left(a+b\tan^{-1}(cx)\right)}{5c^2}-\frac{\left(bd^3\right)\int\left(12+4i\right.}{}$$

$$=-\frac{3bd^3x}{5c}-\frac{3ibd^3(i-cx)^2}{20c^2}-\frac{bd^3(i-cx)^3}{20c^2}+\frac{ibd^3(i-cx)^4}{20c^2}+\frac{d^3(1+icx)^4\left(a+b\tan^{-1}\right.}{4c^2}$$

Mathematica [A] time = 0.100708, size = 132, normalized size = 0.84

$$\frac{d^3\left(cx\left(acx\left(-4ic^3x^3-15c^2x^2+20icx+10\right)+b\left(ic^3x^3+5c^2x^2-12icx-25\right)\right)+12ib\log\left(c^2x^2+1\right)+b\left(-4ic^5x^5-15c^4x^4\right.\right.}{20c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + I*c*d*x)^3*(a + b*ArcTan[c*x]),x]
```

```
[Out] (d^3*(c*x*(b*(-25 - (12*I)*c*x + 5*c^2*x^2 + I*c^3*x^3) + a*c*x*(10 + (20*I
)*c*x - 15*c^2*x^2 - (4*I)*c^3*x^3)) + b*(25 + 10*c^2*x^2 + (20*I)*c^3*x^3
- 15*c^4*x^4 - (4*I)*c^5*x^5)*ArcTan[c*x] + (12*I)*b*Log[1 + c^2*x^2]))/(20
```

```
*c^2)
```

Maple [A] time = 0.027, size = 184, normalized size = 1.2

$$-\frac{i}{5}c^3d^3ax^5-\frac{3\,c^2d^3ax^4}{4}+icd^3ax^3+\frac{d^3ax^2}{2}-\frac{i}{5}c^3d^3b\arctan\left(cx\right)x^5-\frac{3\,c^2d^3b\arctan\left(cx\right)x^4}{4}+icd^3b\arctan\left(cx\right)x^3+\frac{d}{\quad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d+I*c*d*x)^3*(a+b*arctan(c*x)),x)
```

```
[Out] -1/5*I*c^3*d^3*a*x^5-3/4*c^2*d^3*a*x^4+I*c*d^3*a*x^3+1/2*d^3*a*x^2-1/5*I*c^
3*d^3*b*arctan(c*x)*x^5-3/4*c^2*d^3*b*arctan(c*x)*x^4+I*c*d^3*b*arctan(c*x)
*x^3+1/2*d^3*b*arctan(c*x)*x^2-5/4*b*d^3*x/c+1/20*I*c^2*d^3*b*x^4+1/4*c*d^3
*b*x^3-3/5*I*d^3*b*x^2+3/5*I/c^2*d^3*b*ln(c^2*x^2+1)+5/4/c^2*d^3*b*arctan(c
*x)
```

Maxima [A] time = 1.48549, size = 300, normalized size = 1.91

$$-\frac{1}{5}i\,ac^3d^3x^5-\frac{3}{4}\,ac^2d^3x^4-\frac{1}{20}i\left(4\,x^5\arctan\left(cx\right)-c\left(\frac{c^2x^4-2\,x^2}{c^4}+\frac{2\,\log\left(c^2x^2+1\right)}{c^6}\right)\right)bc^3d^3+i\,acd^3x^3-\frac{1}{4}\left(3\,x^4\arctan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d+I*c*d*x)^3*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] -1/5*I*a*c^3*d^3*x^5 - 3/4*a*c^2*d^3*x^4 - 1/20*I*(4*x^5*arctan(c*x) - c*((
c^2*x^4 - 2*x^2)/c^4 + 2*log(c^2*x^2 + 1)/c^6))*b*c^3*d^3 + I*a*c*d^3*x^3 -
 1/4*(3*x^4*arctan(c*x) - c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5))*b*c^
2*d^3 + 1/2*I*(2*x^3*arctan(c*x) - c*(x^2/c^2 - log(c^2*x^2 + 1)/c^4))*b*c*
d^3 + 1/2*a*d^3*x^2 + 1/2*(x^2*arctan(c*x) - c*(x/c^2 - arctan(c*x)/c^3))*b
*d^3
```

Fricas [A] time = 2.76153, size = 404, normalized size = 2.57

$$\frac{-8i\,ac^5d^3x^5-2\left(15\,a-i\,b\right)c^4d^3x^4+\left(40i\,a+10\,b\right)c^3d^3x^3+4\left(5\,a-6i\,b\right)c^2d^3x^2-50\,bcd^3x+49i\,bd^3\log\left(\frac{cx+i}{c}\right)-i\,bd^3\log}{40\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d+I*c*d*x)^3*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] 1/40*(-8*I*a*c^5*d^3*x^5 - 2*(15*a - I*b)*c^4*d^3*x^4 + (40*I*a + 10*b)*c^3
*d^3*x^3 + 4*(5*a - 6*I*b)*c^2*d^3*x^2 - 50*b*c*d^3*x + 49*I*b*d^3*log((c*x
 + I)/c) - I*b*d^3*log((c*x - I)/c) + (4*b*c^5*d^3*x^5 - 15*I*b*c^4*d^3*x^4
 - 20*b*c^3*d^3*x^3 + 10*I*b*c^2*d^3*x^2)*log(-(c*x + I)/(c*x - I)))/c^2
```

Sympy [A] time = 3.36243, size = 260, normalized size = 1.66

$$-\frac{iac^3d^3x^5}{5} - \frac{5bd^3x}{4c} - \frac{ibd^3\log\left(x - \frac{i}{c}\right)}{40c^2} + \frac{49ibd^3\log\left(x + \frac{i}{c}\right)}{40c^2} - x^4\left(\frac{3ac^2d^3}{4} - \frac{ibc^2d^3}{20}\right) - x^3\left(-iacd^3 - \frac{bcd^3}{4}\right) - x^2\left(-\frac{ad^3}{2} + \cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d+I*c*d*x)**3*(a+b*atan(c*x)),x)
```

```
[Out] -I*a*c**3*d**3*x**5/5 - 5*b*d**3*x/(4*c) - I*b*d**3*log(x - I/c)/(40*c**2)
+ 49*I*b*d**3*log(x + I/c)/(40*c**2) - x**4*(3*a*c**2*d**3/4 - I*b*c**2*d**
3/20) - x**3*(-I*a*c*d**3 - b*c*d**3/4) - x**2*(-a*d**3/2 + 3*I*b*d**3/5) +
 (-b*c**3*d**3*x**5/10 + 3*I*b*c**2*d**3*x**4/8 + b*c*d**3*x**3/2 - I*b*d**
3*x**2/4)*log(I*c*x + 1) + (b*c**3*d**3*x**5/10 - 3*I*b*c**2*d**3*x**4/8 -
b*c*d**3*x**3/2 + I*b*d**3*x**2/4)*log(-I*c*x + 1)
```

Giac [A] time = 1.17778, size = 266, normalized size = 1.69

$$\frac{8\,bc^5d^3x^5\arctan(cx) + 8\,ac^5d^3x^5 - 30\,bc^4d^3ix^4\arctan(cx) - 30\,ac^4d^3ix^4 - 2\,bc^4d^3x^4 + 10\,bc^3d^3ix^3 - 40\,bc^3d^3x^3\arctan\cdots}{4\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d+I*c*d*x)^3*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] 1/40*(8*b*c^5*d^3*x^5*arctan(c*x) + 8*a*c^5*d^3*x^5 - 30*b*c^4*d^3*i*x^4*ar
ctan(c*x) - 30*a*c^4*d^3*i*x^4 - 2*b*c^4*d^3*x^4 + 10*b*c^3*d^3*i*x^3 - 40*
b*c^3*d^3*x^3*arctan(c*x) - 40*a*c^3*d^3*x^3 + 20*b*c^2*d^3*i*x^2*arctan(c*
x) + 20*a*c^2*d^3*i*x^2 + 24*b*c^2*d^3*x^2 - 50*b*c*d^3*i*x - 49*b*d^3*log(
c*i*x - 1) + b*d^3*log(-c*i*x - 1))/(c^2*i)
```

3.23 $\int (d+icdx)^3 \left(a+b\tan^{-1}(cx)\right) dx$

Optimal. Leaf size=100

$$-\frac{id^3(1+icx)^4\left(a+b\tan^{-1}(cx)\right)}{4c}-\frac{bd^3(1+icx)^3}{12c}-\frac{bd^3(1+icx)^2}{4c}-\frac{2bd^3\log(1-icx)}{c}-ibd^3x$$

```
[Out] (-I)*b*d^3*x - (b*d^3*(1 + I*c*x)^2)/(4*c) - (b*d^3*(1 + I*c*x)^3)/(12*c) -
 ((I/4)*d^3*(1 + I*c*x)^4*(a + b*ArcTan[c*x]))/c - (2*b*d^3*Log[1 - I*c*x])
/c
```

Rubi [A] time = 0.0537698, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {4862, 627, 43}

$$-\frac{id^3(1+icx)^4\left(a+b\tan^{-1}(cx)\right)}{4c}-\frac{bd^3(1+icx)^3}{12c}-\frac{bd^3(1+icx)^2}{4c}-\frac{2bd^3\log(1-icx)}{c}-ibd^3x$$

Antiderivative was successfully verified.

```
[In] Int[(d + I*c*d*x)^3*(a + b*ArcTan[c*x]),x]
```

```
[Out] (-I)*b*d^3*x - (b*d^3*(1 + I*c*x)^2)/(4*c) - (b*d^3*(1 + I*c*x)^3)/(12*c) -
 ((I/4)*d^3*(1 + I*c*x)^4*(a + b*ArcTan[c*x]))/c - (2*b*d^3*Log[1 - I*c*x])
/c
```

Rule 4862

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x]))/(e*(q + 1)), x] - Dist[(b*
c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (d+icdx)^3\left(a+b\tan^{-1}(cx)\right)dx &= -\frac{id^3(1+icx)^4\left(a+b\tan^{-1}(cx)\right)}{4c}+\frac{(ib)\int\frac{(d+icdx)^4}{1+c^2x^2}\,dx}{4d}\\ &= -\frac{id^3(1+icx)^4\left(a+b\tan^{-1}(cx)\right)}{4c}+\frac{(ib)\int\frac{(d+icdx)^3}{\frac{1}{d}-\frac{icx}{d}}\,dx}{4d}\\ &= -\frac{id^3(1+icx)^4\left(a+b\tan^{-1}(cx)\right)}{4c}+\frac{(ib)\int\left(-4d^4+\frac{8d^3}{\frac{1}{d}-\frac{icx}{d}}-2d^3(d+icdx)-d^2(d+ic\right.}{4d}\\ &= -ibd^3x-\frac{bd^3(1+icx)^2}{4c}-\frac{bd^3(1+icx)^3}{12c}-\frac{id^3(1+icx)^4\left(a+b\tan^{-1}(cx)\right)}{4c}-\frac{2bd^3\log}{c}\end{aligned}$$

Mathematica [A] time = 0.0387604, size = 77, normalized size = 0.77

$$-\frac{i\left(3(d+icdx)^4\left(a+b\tan^{-1}(cx)\right)-bd^4\left(c^3x^3-6ic^2x^2-21cx+24i\log(cx+i)+4i\right)\right)}{12cd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + I*c*d*x)^3*(a + b*ArcTan[c*x]),x]
```

```
[Out] ((-I/12)*(3*(d + I*c*d*x)^4*(a + b*ArcTan[c*x]) - b*d^4*(4*I - 21*c*x - (6*
I)*c^2*x^2 + c^3*x^3 + (24*I)*Log[I + c*x])))/(c*d)
```

Maple [A] time = 0.026, size = 176, normalized size = 1.8

$$-\frac{i}{4}c^3x^4ad^3-c^2x^3ad^3+\frac{3\,i}{2}cx^2ad^3+xad^3-\frac{\frac{i}{4}d^3a}{c}-\frac{i}{4}c^3d^3b\arctan(cx)\,x^4-c^2d^3b\arctan(cx)\,x^3+\frac{3\,i}{2}cd^3b\arctan(cx)\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^3*(a+b*arctan(c*x)),x)
```

```
[Out] -1/4*I*c^3*x^4*a*d^3-c^2*x^3*a*d^3+3/2*I*c*x^2*a*d^3+x*a*d^3-1/4*I/c*d^3*a-
1/4*I*c^3*d^3*b*arctan(c*x)*x^4-c^2*d^3*b*arctan(c*x)*x^3+3/2*I*c*d^3*b*arc
tan(c*x)*x^2+d^3*b*x*arctan(c*x)+7/4*I/c*d^3*b*arctan(c*x)-7/4*I*d^3*b*x+1/
12*I*c^2*d^3*b*x^3+1/2*c*d^3*b*x^2-1/c*d^3*b*ln(c^2*x^2+1)
```

Maxima [B] time = 1.47576, size = 266, normalized size = 2.66

$$-\frac{1}{4}i\,ac^3d^3x^4-ac^2d^3x^3-\frac{1}{12}i\left(3\,x^4\arctan\left(cx\right)-c\left(\frac{c^2x^3-3\,x}{c^4}+\frac{3\,\arctan\left(cx\right)}{c^5}\right)\right)bc^3d^3-\frac{1}{2}\left(2\,x^3\arctan\left(cx\right)-c\left(\frac{x^2}{c^2}-\frac{\mathrm{lo}}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^3*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] -1/4*I*a*c^3*d^3*x^4 - a*c^2*d^3*x^3 - 1/12*I*(3*x^4*arctan(c*x) - c*((c^2*
x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5))*b*c^3*d^3 - 1/2*(2*x^3*arctan(c*x) - c
*(x^2/c^2 - log(c^2*x^2 + 1)/c^4))*b*c^2*d^3 + 3/2*I*a*c*d^3*x^2 + 3/2*I*(x
^2*arctan(c*x) - c*(x/c^2 - arctan(c*x)/c^3))*b*c*d^3 + a*d^3*x + 1/2*(2*c*
x*arctan(c*x) - log(c^2*x^2 + 1))*b*d^3/c
```

Fricas [A] time = 2.94267, size = 367, normalized size = 3.67

$$\frac{-6i\,ac^4d^3x^4-2\left(12\,a-i\,b\right)c^3d^3x^3+\left(36i\,a+12\,b\right)c^2d^3x^2+6\left(4\,a-7i\,b\right)cd^3x-45\,bd^3\log\left(\frac{cx+i}{c}\right)-3\,bd^3\log\left(\frac{cx-i}{c}\right)+\left(3\,bc\right.}{24\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^3*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] 1/24*(-6*I*a*c^4*d^3*x^4 - 2*(12*a - I*b)*c^3*d^3*x^3 + (36*I*a + 12*b)*c^2
*d^3*x^2 + 6*(4*a - 7*I*b)*c*d^3*x - 45*b*d^3*log((c*x + I)/c) - 3*b*d^3*lo
g((c*x - I)/c) + (3*b*c^4*d^3*x^4 - 12*I*b*c^3*d^3*x^3 - 18*b*c^2*d^3*x^2 +
 12*I*b*c*d^3*x)*log(-(c*x + I)/(c*x - I)))/c
```

Sympy [B] time = 3.24745, size = 228, normalized size = 2.28

$$-\frac{iac^3d^3x^4}{4} - \frac{bd^3\left(\frac{\log\left(x-\frac{i}{c}\right)}{8} + \frac{15\log\left(x+\frac{i}{c}\right)}{8}\right)}{c} - x^3\left(ac^2d^3 - \frac{ibc^2d^3}{12}\right) - x^2\left(-\frac{3iacd^3}{2} - \frac{bcd^3}{2}\right) - x\left(-ad^3 + \frac{7ibd^3}{4}\right) + \left(-\frac{bc^3d^3x^4}{8}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**3*(a+b*atan(c*x)),x)
```

```
[Out] -I*a*c**3*d**3*x**4/4 - b*d**3*(log(x - I/c)/8 + 15*log(x + I/c)/8)/c - x**
3*(a*c**2*d**3 - I*b*c**2*d**3/12) - x**2*(-3*I*a*c*d**3/2 - b*c*d**3/2) -
x*(-a*d**3 + 7*I*b*d**3/4) + (-b*c**3*d**3*x**4/8 + I*b*c**2*d**3*x**3/2 +
3*b*c*d**3*x**2/4 - I*b*d**3*x/2)*log(I*c*x + 1) + (b*c**3*d**3*x**4/8 - I*
b*c**2*d**3*x**3/2 - 3*b*c*d**3*x**2/4 + I*b*d**3*x/2)*log(-I*c*x + 1)
```

Giac [B] time = 1.15569, size = 235, normalized size = 2.35

$$-\frac{6\,bc^4d^3ix^4\arctan(cx) + 6\,ac^4d^3ix^4 - 2\,bc^3d^3ix^3 + 24\,bc^3d^3x^3\arctan(cx) + 24\,ac^3d^3x^3 - 36\,bc^2d^3ix^2\arctan(cx) - 36\,a}{24\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^3*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] -1/24*(6*b*c^4*d^3*i*x^4*arctan(c*x) + 6*a*c^4*d^3*i*x^4 - 2*b*c^3*d^3*i*x^
3 + 24*b*c^3*d^3*x^3*arctan(c*x) + 24*a*c^3*d^3*x^3 - 36*b*c^2*d^3*i*x^2*ar
ctan(c*x) - 36*a*c^2*d^3*i*x^2 - 12*b*c^2*d^3*x^2 + 42*b*c*d^3*i*x - 24*b*c
*d^3*x*arctan(c*x) - 24*a*c*d^3*x + 45*b*d^3*log(c*x + i) + 3*b*d^3*log(c*x
 - i))/c
```

3.24 $\int \frac{(d+icdx)^3\left(a+b\tan^{-1}(cx)\right)}{x}\,dx$

Optimal. Leaf size=170

$$\frac{1}{2}ibd^3\text{PolyLog}(2,-icx)-\frac{1}{2}ibd^3\text{PolyLog}(2,icx)-\frac{1}{3}ic^3d^3x^3\left(a+b\tan^{-1}(cx)\right)-\frac{3}{2}c^2d^3x^2\left(a+b\tan^{-1}(cx)\right)+3iacd^3x+$$

```
[Out] (3*I)*a*c*d^3*x + (3*b*c*d^3*x)/2 + (I/6)*b*c^2*d^3*x^2 - (3*b*d^3*ArcTan[c
*x])/2 + (3*I)*b*c*d^3*x*ArcTan[c*x] - (3*c^2*d^3*x^2*(a + b*ArcTan[c*x]))/
2 - (I/3)*c^3*d^3*x^3*(a + b*ArcTan[c*x]) + a*d^3*Log[x] - ((5*I)/3)*b*d^3*
Log[1 + c^2*x^2] + (I/2)*b*d^3*PolyLog[2, (-I)*c*x] - (I/2)*b*d^3*PolyLog[2
, I*c*x]
```

Rubi [A] time = 0.17451, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {4876, 4846, 260, 4848, 2391, 4852, 321, 203, 266, 43}

$$\frac{1}{2}ibd^3\text{PolyLog}(2,-icx)-\frac{1}{2}ibd^3\text{PolyLog}(2,icx)-\frac{1}{3}ic^3d^3x^3\left(a+b\tan^{-1}(cx)\right)-\frac{3}{2}c^2d^3x^2\left(a+b\tan^{-1}(cx)\right)+3iacd^3x+$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)^3*(a + b*ArcTan[c*x]))/x,x]
```



```
[Out] (3*I)*a*c*d^3*x + (3*b*c*d^3*x)/2 + (I/6)*b*c^2*d^3*x^2 - (3*b*d^3*ArcTan[c
*x])/2 + (3*I)*b*c*d^3*x*ArcTan[c*x] - (3*c^2*d^3*x^2*(a + b*ArcTan[c*x]))/
2 - (I/3)*c^3*d^3*x^3*(a + b*ArcTan[c*x]) + a*d^3*Log[x] - ((5*I)/3)*b*d^3*
Log[1 + c^2*x^2] + (I/2)*b*d^3*PolyLog[2, (-I)*c*x] - (I/2)*b*d^3*PolyLog[2
, I*c*x]
```


Rule 4876

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
```

```
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(d+icdx)^3\left(a+b\tan^{-1}(cx)\right)}{x}\,dx = \int \left(3icd^3\left(a+b\tan^{-1}(cx)\right)+\frac{d^3\left(a+b\tan^{-1}(cx)\right)}{x}-3c^2d^3x\left(a+b\tan^{-1}(cx)\right)-ic\right.$$

$$= d^3\int \frac{a+b\tan^{-1}(cx)}{x}\,dx+\left(3icd^3\right)\int \left(a+b\tan^{-1}(cx)\right)dx-\left(3c^2d^3\right)\int x\left(a+b\,\mathrm{t}\right.$$

$$= 3iacd^3x-\frac{3}{2}c^2d^3x^2\left(a+b\tan^{-1}(cx)\right)-\frac{1}{3}ic^3d^3x^3\left(a+b\tan^{-1}(cx)\right)+ad^3\log(x)+$$

$$= 3iacd^3x+\frac{3}{2}bcd^3x+3ibcd^3x\tan^{-1}(cx)-\frac{3}{2}c^2d^3x^2\left(a+b\tan^{-1}(cx)\right)-\frac{1}{3}ic^3d^3x^3\left(a\right.$$

$$= 3iacd^3x+\frac{3}{2}bcd^3x-\frac{3}{2}bd^3\tan^{-1}(cx)+3ibcd^3x\tan^{-1}(cx)-\frac{3}{2}c^2d^3x^2\left(a+b\tan^{-1}(\right.$$

$$= 3iacd^3x+\frac{3}{2}bcd^3x+\frac{1}{6}ibc^2d^3x^2-\frac{3}{2}bd^3\tan^{-1}(cx)+3ibcd^3x\tan^{-1}(cx)-\frac{3}{2}c^2d^3x^2\left(\right.$$

Mathematica [A] time = 0.13178, size = 139, normalized size = 0.82

$$-\frac{1}{6}id^3\left(-3b\mathrm{PolyLog}(2,-icx)+3b\mathrm{PolyLog}(2,icx)+2ac^3x^3-9iac^2x^2-18acx+6ia\log(x)-bc^2x^2+10b\log\left(c^2x^2+1\right)\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + I*c*d*x)^3*(a + b*ArcTan[c*x]))/x,x]
```

```
[Out] (-I/6)*d^3*(-18*a*c*x + (9*I)*b*c*x - (9*I)*a*c^2*x^2 - b*c^2*x^2 + 2*a*c^3
*x^3 - (9*I)*b*ArcTan[c*x] - 18*b*c*x*ArcTan[c*x] - (9*I)*b*c^2*x^2*ArcTan[
c*x] + 2*b*c^3*x^3*ArcTan[c*x] + (6*I)*a*Log[x] + 10*b*Log[1 + c^2*x^2] - 3
*b*PolyLog[2, (-I)*c*x] + 3*b*PolyLog[2, I*c*x])
```

Maple [A] time = 0.041, size = 220, normalized size = 1.3

$$3\,iacd^3x-\frac{i}{2}d^3b\ln\left(cx\right)\ln\left(1-icx\right)-\frac{3\,d^3ac^2x^2}{2}+d^3a\ln\left(cx\right)+\frac{i}{2}d^3b\mathrm{dilog}\left(1+icx\right)+\frac{i}{2}d^3b\ln\left(cx\right)\ln\left(1+icx\right)-\frac{3\,d^3b\,\mathrm{arct}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^3*(a+b*arctan(c*x))/x,x)
```

```
[Out] 3*I*a*c*d^3*x-1/2*I*d^3*b*ln(c*x)*ln(1-I*c*x)-3/2*d^3*a*c^2*x^2+d^3*a*ln(c*
x)+1/2*I*d^3*b*dilog(1+I*c*x)+1/2*I*d^3*b*ln(c*x)*ln(1+I*c*x)-3/2*d^3*b*arc
tan(c*x)*c^2*x^2+d^3*b*arctan(c*x)*ln(c*x)+3*I*b*c*d^3*x*arctan(c*x)-1/3*I*
d^3*a*c^3*x^3+1/6*I*b*c^2*d^3*x^2-1/3*I*d^3*b*arctan(c*x)*c^3*x^3+3/2*b*c*d
^3*x-1/2*I*d^3*b*dilog(1-I*c*x)-5/3*I*b*d^3*ln(c^2*x^2+1)-3/2*b*d^3*arctan(
c*x)
```

Maxima [A] time = 2.16782, size = 259, normalized size = 1.52

$$-\frac{1}{3}i\,ac^3d^3x^3-\frac{3}{2}\,ac^2d^3x^2+\frac{1}{6}i\,bc^2d^3x^2+3i\,acd^3x+\frac{3}{2}\,bcd^3x-\frac{1}{12}\,(3\,\pi+2i)bd^3\log\left(c^2x^2+1\right)+bd^3\arctan\left(cx\right)\log\left(x|c|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))/x,x, algorithm="maxima")
```

```
[Out] -1/3*I*a*c^3*d^3*x^3 - 3/2*a*c^2*d^3*x^2 + 1/6*I*b*c^2*d^3*x^2 + 3*I*a*c*d^
3*x + 3/2*b*c*d^3*x - 1/12*(3*pi + 2*I)*b*d^3*log(c^2*x^2 + 1) + b*d^3*arct
an(c*x)*log(x*abs(c)) + 3/2*I*(2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*b*d^3
- 1/2*I*b*d^3*dilog(I*c*x + 1) + 1/2*I*b*d^3*dilog(-I*c*x + 1) + a*d^3*log(
x) - 1/12*(4*I*b*c^3*d^3*x^3 + 18*b*c^2*d^3*x^2 - 6*b*d^3*(2*I*arctan2(0, c
) - 3))*arctan(c*x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{-2i\,ac^3d^3x^3-6\,ac^2d^3x^2+6i\,acd^3x+2\,ad^3+\left(bc^3d^3x^3-3i\,bc^2d^3x^2-3\,bcd^3x+i\,bd^3\right)\log\left(-\frac{cx+i}{cx-i}\right)}{2\,x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))/x,x, algorithm="fricas")
```

```
[Out] integral(1/2*(-2*I*a*c^3*d^3*x^3 - 6*a*c^2*d^3*x^2 + 6*I*a*c*d^3*x + 2*a*d^
3 + (b*c^3*d^3*x^3 - 3*I*b*c^2*d^3*x^2 - 3*b*c*d^3*x + I*b*d^3)*log(-(c*x +
 I)/(c*x - I)))/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d^3\left(\int \frac{a}{x}\,dx + \int 3iac\,dx + \int -3ac^2x\,dx + \int \frac{b\operatorname{atan}(cx)}{x}\,dx + \int -iac^3x^2\,dx + \int 3ibc\operatorname{atan}(cx)\,dx + \int -3bc^2x\operatorname{atan}(c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**3*(a+b*atan(c*x))/x,x)
```

```
[Out] d**3*(Integral(a/x, x) + Integral(3*I*a*c, x) + Integral(-3*a*c**2*x, x) +
Integral(b*atan(c*x)/x, x) + Integral(-I*a*c**3*x**2, x) + Integral(3*I*b*c
*atan(c*x), x) + Integral(-3*b*c**2*x*atan(c*x), x) + Integral(-I*b*c**3*x*
*2*atan(c*x), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(icdx+d)^3(b\arctan(cx)+a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))/x,x, algorithm="giac")
```

```
[Out] integrate((I*c*d*x + d)^3*(b*arctan(c*x) + a)/x, x)
```

3.25 $\int \frac{(d+icdx)^3\left(a+b\tan^{-1}(cx)\right)}{x^2}\,dx$

Optimal. Leaf size=162

$$-\frac{3}{2}bcd^3\text{PolyLog}(2,-icx)+\frac{3}{2}bcd^3\text{PolyLog}(2,icx)-\frac{1}{2}ic^3d^3x^2\left(a+b\tan^{-1}(cx)\right)-\frac{d^3\left(a+b\tan^{-1}(cx)\right)}{x}-3ac^2d^3x+3iac$$

```
[Out] -3*a*c^2*d^3*x + (I/2)*b*c^2*d^3*x - (I/2)*b*c*d^3*ArcTan[c*x] - 3*b*c^2*d^
3*x*ArcTan[c*x] - (d^3*(a + b*ArcTan[c*x]))/x - (I/2)*c^3*d^3*x^2*(a + b*Ar
cTan[c*x]) + (3*I)*a*c*d^3*Log[x] + b*c*d^3*Log[x] + b*c*d^3*Log[1 + c^2*x^
2] - (3*b*c*d^3*PolyLog[2, (-I)*c*x])/2 + (3*b*c*d^3*PolyLog[2, I*c*x])/2
```

Rubi [A] time = 0.173482, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 12, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.522, Rules used = {4876, 4846, 260, 4852, 266, 36, 29, 31, 4848, 2391, 321, 203}

$$-\frac{3}{2}bcd^3\text{PolyLog}(2,-icx)+\frac{3}{2}bcd^3\text{PolyLog}(2,icx)-\frac{1}{2}ic^3d^3x^2\left(a+b\tan^{-1}(cx)\right)-\frac{d^3\left(a+b\tan^{-1}(cx)\right)}{x}-3ac^2d^3x+3iac$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)^3*(a + b*ArcTan[c*x]))/x^2,x]
```

```
[Out] -3*a*c^2*d^3*x + (I/2)*b*c^2*d^3*x - (I/2)*b*c*d^3*ArcTan[c*x] - 3*b*c^2*d^
3*x*ArcTan[c*x] - (d^3*(a + b*ArcTan[c*x]))/x - (I/2)*c^3*d^3*x^2*(a + b*Ar
cTan[c*x]) + (3*I)*a*c*d^3*Log[x] + b*c*d^3*Log[x] + b*c*d^3*Log[1 + c^2*x^
2] - (3*b*c*d^3*PolyLog[2, (-I)*c*x])/2 + (3*b*c*d^3*PolyLog[2, I*c*x])/2
```

Rule 4876

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+icdx)^3\left(a+b\tan^{-1}(cx)\right)}{x^2}\,dx &= \int \left(-3c^2d^3\left(a+b\tan^{-1}(cx)\right)+\frac{d^3\left(a+b\tan^{-1}(cx)\right)}{x^2}+\frac{3icd^3\left(a+b\tan^{-1}(cx)\right)}{x}-ic\right.\\
&= d^3\int\frac{a+b\tan^{-1}(cx)}{x^2}\,dx+\left(3icd^3\right)\int\frac{a+b\tan^{-1}(cx)}{x}\,dx-\left(3c^2d^3\right)\int\left(a+b\tan^{-1}(\right.\\
&= -3ac^2d^3x-\frac{d^3\left(a+b\tan^{-1}(cx)\right)}{x}-\frac{1}{2}ic^3d^3x^2\left(a+b\tan^{-1}(cx)\right)+3iacd^3\log(x)+\left(b\right.\\
&= -3ac^2d^3x+\frac{1}{2}ibc^2d^3x-3bc^2d^3x\tan^{-1}(cx)-\frac{d^3\left(a+b\tan^{-1}(cx)\right)}{x}-\frac{1}{2}ic^3d^3x^2\left(a+b\right.\\
&= -3ac^2d^3x+\frac{1}{2}ibc^2d^3x-\frac{1}{2}ibcd^3\tan^{-1}(cx)-3bc^2d^3x\tan^{-1}(cx)-\frac{d^3\left(a+b\tan^{-1}(cx\right.}{x}\\
&= -3ac^2d^3x+\frac{1}{2}ibc^2d^3x-\frac{1}{2}ibcd^3\tan^{-1}(cx)-3bc^2d^3x\tan^{-1}(cx)-\frac{d^3\left(a+b\tan^{-1}(cx\right.}{x}
\end{aligned}$$

Mathematica [A] time = 0.115984, size = 150, normalized size = 0.93

$$\frac{d^3\left(-3bcx\text{PolyLog}(2,-icx)+3bcx\text{PolyLog}(2,icx)-iac^3x^3-6ac^2x^2+6iacx\log(x)-2a+ibc^2x^2+2bcx\log\left(c^2x^2+1\right)\right.}{2x}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + I*c*d*x)^3*(a + b*ArcTan[c*x]))/x^2,x]
```

```
[Out] (d^3*(-2*a - 6*a*c^2*x^2 + I*b*c^2*x^2 - I*a*c^3*x^3 - 2*b*ArcTan[c*x] - I*
b*c*x*ArcTan[c*x] - 6*b*c^2*x^2*ArcTan[c*x] - I*b*c^3*x^3*ArcTan[c*x] + (6*
```

```
I)*a*c*x*Log[x] + 2*b*c*x*Log[c*x] + 2*b*c*x*Log[1 + c^2*x^2] - 3*b*c*x*Pol
yLog[2, (-I)*c*x] + 3*b*c*x*PolyLog[2, I*c*x]))/(2*x)
```

Maple [A] time = 0.046, size = 223, normalized size = 1.4

$$-3\,ac^2d^3x+\frac{i}{2}bc^2d^3x-\frac{d^3a}{x}+3\,icd^3a\ln(cx)-3\,bc^2d^3x\arctan(cx)-\frac{i}{2}bcd^3\arctan(cx)-\frac{bd^3\arctan(cx)}{x}+3\,icd^3b\,\mathrm{arc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^3*(a+b*arctan(c*x))/x^2,x)
```

```
[Out] -3*a*c^2*d^3*x+1/2*I*b*c^2*d^3*x-d^3*a/x+3*I*c*d^3*a*ln(c*x)-3*b*c^2*d^3*x*
arctan(c*x)-1/2*I*b*c*d^3*arctan(c*x)-d^3*b*arctan(c*x)/x+3*I*c*d^3*b*arcta
n(c*x)*ln(c*x)-3/2*c*d^3*b*ln(c*x)*ln(1+I*c*x)+3/2*c*d^3*b*ln(c*x)*ln(1-I*c
*x)-3/2*c*d^3*b*dilog(1+I*c*x)+3/2*c*d^3*b*dilog(1-I*c*x)-1/2*I*d^3*b*arcta
n(c*x)*c^3*x^2+b*c*d^3*ln(c^2*x^2+1)-1/2*I*d^3*a*c^3*x^2+c*d^3*b*ln(c*x)
```

Maxima [A] time = 2.17171, size = 281, normalized size = 1.73

$$-\frac{1}{2}i\,ac^3d^3x^2-3\,ac^2d^3x+\frac{1}{2}i\,bc^2d^3x-\frac{3}{4}i\,\pi bcd^3\log\left(c^2x^2+1\right)+3i\,bcd^3\arctan(cx)\log(x|c|)-\frac{3}{2}\left(2\,cx\arctan(cx)-\log\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))/x^2,x, algorithm="maxima")
```

```
[Out] -1/2*I*a*c^3*d^3*x^2 - 3*a*c^2*d^3*x + 1/2*I*b*c^2*d^3*x - 3/4*I*pi*b*c*d^3
*log(c^2*x^2 + 1) + 3*I*b*c*d^3*arctan(c*x)*log(x*abs(c)) - 3/2*(2*c*x*arct
an(c*x) - log(c^2*x^2 + 1))*b*c*d^3 + 3/2*b*c*d^3*dilog(I*c*x + 1) - 3/2*b*
c*d^3*dilog(-I*c*x + 1) + 3*I*a*c*d^3*log(x) - 1/2*(c*(log(c^2*x^2 + 1) - l
og(x^2)) + 2*arctan(c*x)/x)*b*d^3 - a*d^3/x - 1/4*(2*I*b*c^3*d^3*x^2 + b*c*
d^3*(12*arctan2(0, c) + 2*I))*arctan(c*x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{-2i\,ac^3d^3x^3-6\,ac^2d^3x^2+6i\,acd^3x+2\,ad^3+\left(bc^3d^3x^3-3i\,bc^2d^3x^2-3\,bcd^3x+i\,bd^3\right)\log\left(-\frac{cx+i}{cx-i}\right)}{2\,x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))/x^2,x, algorithm="fricas")
```

```
[Out] integral(1/2*(-2*I*a*c^3*d^3*x^3 - 6*a*c^2*d^3*x^2 + 6*I*a*c*d^3*x + 2*a*d^
3 + (b*c^3*d^3*x^3 - 3*I*b*c^2*d^3*x^2 - 3*b*c*d^3*x + I*b*d^3)*log(-(c*x +
 I)/(c*x - I)))/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d^3\left(\int -3ac^2\,dx + \int \frac{a}{x^2}\,dx + \int -3bc^2\operatorname{atan}(cx)\,dx + \int \frac{b\operatorname{atan}(cx)}{x^2}\,dx + \int \frac{3iac}{x}\,dx + \int -iac^3x\,dx + \int \frac{3ibc\operatorname{atan}(cx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**3*(a+b*atan(c*x))/x**2,x)
```

```
[Out] d**3*(Integral(-3*a*c**2, x) + Integral(a/x**2, x) + Integral(-3*b*c**2*ata
n(c*x), x) + Integral(b*atan(c*x)/x**2, x) + Integral(3*I*a*c/x, x) + Integ
ral(-I*a*c**3*x, x) + Integral(3*I*b*c*atan(c*x)/x, x) + Integral(-I*b*c**3
*x*atan(c*x), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(icdx + d)^3 (b\arctan(cx) + a)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))/x^2,x, algorithm="giac")
```

```
[Out] integrate((I*c*d*x + d)^3*(b*arctan(c*x) + a)/x^2, x)
```

3.26 $\int \frac{(d+icdx)^3\left(a+b\tan^{-1}(cx)\right)}{x^3}\,dx$

Optimal. Leaf size=180

$$-\frac{3}{2}ibc^2d^3\text{PolyLog}(2,-icx)+\frac{3}{2}ibc^2d^3\text{PolyLog}(2,icx)-\frac{d^3\left(a+b\tan^{-1}(cx)\right)}{2x^2}-\frac{3icd^3\left(a+b\tan^{-1}(cx)\right)}{x}-iac^3d^3x-3ac$$

```
[Out] -(b*c*d^3)/(2*x) - I*a*c^3*d^3*x - (b*c^2*d^3*ArcTan[c*x])/2 - I*b*c^3*d^3*
x*ArcTan[c*x] - (d^3*(a + b*ArcTan[c*x]))/(2*x^2) - ((3*I)*c*d^3*(a + b*Arc
Tan[c*x]))/x - 3*a*c^2*d^3*Log[x] + (3*I)*b*c^2*d^3*Log[x] - I*b*c^2*d^3*Lo
g[1 + c^2*x^2] - ((3*I)/2)*b*c^2*d^3*PolyLog[2, (-I)*c*x] + ((3*I)/2)*b*c^2
*d^3*PolyLog[2, I*c*x]
```

Rubi [A] time = 0.178091, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 12, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.522, Rules used = {4876, 4846, 260, 4852, 325, 203, 266, 36, 29, 31, 4848, 2391}

$$-\frac{3}{2}ibc^2d^3\text{PolyLog}(2,-icx)+\frac{3}{2}ibc^2d^3\text{PolyLog}(2,icx)-\frac{d^3\left(a+b\tan^{-1}(cx)\right)}{2x^2}-\frac{3icd^3\left(a+b\tan^{-1}(cx)\right)}{x}-iac^3d^3x-3ac$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)^3*(a + b*ArcTan[c*x]))/x^3,x]
```


```
[Out] -(b*c*d^3)/(2*x) - I*a*c^3*d^3*x - (b*c^2*d^3*ArcTan[c*x])/2 - I*b*c^3*d^3*
x*ArcTan[c*x] - (d^3*(a + b*ArcTan[c*x]))/(2*x^2) - ((3*I)*c*d^3*(a + b*Arc
Tan[c*x]))/x - 3*a*c^2*d^3*Log[x] + (3*I)*b*c^2*d^3*Log[x] - I*b*c^2*d^3*Lo
g[1 + c^2*x^2] - ((3*I)/2)*b*c^2*d^3*PolyLog[2, (-I)*c*x] + ((3*I)/2)*b*c^2
*d^3*PolyLog[2, I*c*x]
```


Rule 4876

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
```

```
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{(d+icdx)^3\left(a+b\tan^{-1}(cx)\right)}{x^3}\,dx = \int\left(-ic^3d^3\left(a+b\tan^{-1}(cx)\right)+\frac{d^3\left(a+b\tan^{-1}(cx)\right)}{x^3}+\frac{3icd^3\left(a+b\tan^{-1}(cx)\right)}{x^2}-\right.$$

$$= d^3\int\frac{a+b\tan^{-1}(cx)}{x^3}\,dx+\left(3icd^3\right)\int\frac{a+b\tan^{-1}(cx)}{x^2}\,dx-\left(3c^2d^3\right)\int\frac{a+b\tan^{-1}}{x}$$

$$= -iac^3d^3x-\frac{d^3\left(a+b\tan^{-1}(cx)\right)}{2x^2}-\frac{3icd^3\left(a+b\tan^{-1}(cx)\right)}{x}-3ac^2d^3\log(x)+\frac{1}{2}\left(b\right.$$

$$= -\frac{bcd^3}{2x}-iac^3d^3x-ibc^3d^3x\tan^{-1}(cx)-\frac{d^3\left(a+b\tan^{-1}(cx)\right)}{2x^2}-\frac{3icd^3\left(a+b\tan^{-1}\right.}{x}$$

$$= -\frac{bcd^3}{2x}-iac^3d^3x-\frac{1}{2}bc^2d^3\tan^{-1}(cx)-ibc^3d^3x\tan^{-1}(cx)-\frac{d^3\left(a+b\tan^{-1}(cx)\right)}{2x^2}-$$

$$= -\frac{bcd^3}{2x}-iac^3d^3x-\frac{1}{2}bc^2d^3\tan^{-1}(cx)-ibc^3d^3x\tan^{-1}(cx)-\frac{d^3\left(a+b\tan^{-1}(cx)\right)}{2x^2}-$$

Mathematica [A] time = 0.121194, size = 164, normalized size = 0.91

$$-\frac{id^3\left(3bc^2x^2\text{PolyLog}(2,-icx)-3bc^2x^2\text{PolyLog}(2,icx)+2ac^3x^3-6iac^2x^2\log(x)+6acx-ia-6bc^2x^2\log(cx)+2bc^2x\right.}{2x^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + I*c*d*x)^3*(a + b*ArcTan[c*x]))/x^3,x]
```

```
[Out] ((-I/2)*d^3*((-I)*a + 6*a*c*x - I*b*c*x + 2*a*c^3*x^3 - I*b*ArcTan[c*x] + 6
*b*c*x*ArcTan[c*x] - I*b*c^2*x^2*ArcTan[c*x] + 2*b*c^3*x^3*ArcTan[c*x] - (6
*I)*a*c^2*x^2*Log[x] - 6*b*c^2*x^2*Log[c*x] + 2*b*c^2*x^2*Log[1 + c^2*x^2]
+ 3*b*c^2*x^2*PolyLog[2, (-I)*c*x] - 3*b*c^2*x^2*PolyLog[2, I*c*x]))/x^2
```

Maple [A] time = 0.047, size = 243, normalized size = 1.4

$$3\,ic^2d^3b\ln(cx)-\frac{d^3a}{2\,x^2}+\frac{3\,i}{2}c^2d^3b\,\mathrm{dilog}(1-icx)-3\,c^2d^3a\ln(cx)-\frac{3\,icd^3b\arctan(cx)}{x}-\frac{bd^3\arctan(cx)}{2\,x^2}-\frac{3\,i}{2}c^2d^3b\ln(cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^3*(a+b*arctan(c*x))/x^3,x)
```

```
[Out] 3*I*c^2*d^3*b*ln(c*x)-1/2*d^3*a/x^2+3/2*I*c^2*d^3*b*dilog(1-I*c*x)-3*c^2*d^
3*a*ln(c*x)-3*I*c*d^3*b*arctan(c*x)/x-1/2*d^3*b*arctan(c*x)/x^2-3/2*I*c^2*d
^3*b*ln(c*x)*ln(1+I*c*x)-3*c^2*d^3*b*arctan(c*x)*ln(c*x)-I*b*c^2*d^3*ln(c^2
*x^2+1)-I*b*c^3*d^3*x*arctan(c*x)-3/2*I*c^2*d^3*b*dilog(1+I*c*x)-3*I*c*d^3*
a/x+3/2*I*c^2*d^3*b*ln(c*x)*ln(1-I*c*x)-1/2*b*c^2*d^3*arctan(c*x)-1/2*b*c*d
^3/x-I*a*c^3*d^3*x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-i\,ac^3d^3x-\frac{1}{2}i\left(2\,cx\arctan(cx)-\log\left(c^2x^2+1\right)\right)bc^2d^3-3\,bc^2d^3\int\frac{\arctan(cx)}{x}\,dx-3\,ac^2d^3\log(x)-\frac{3}{2}i\left(c\left(\log\left(c^2x^2+1\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))/x^3,x, algorithm="maxima")
```

```
[Out] -I*a*c^3*d^3*x - 1/2*I*(2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*b*c^2*d^3 - 3
*b*c^2*d^3*integrate(arctan(c*x)/x, x) - 3*a*c^2*d^3*log(x) - 3/2*I*(c*(log
(c^2*x^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*b*c*d^3 - 1/2*((c*arctan(c*x)
+ 1/x)*c + arctan(c*x)/x^2)*b*d^3 - 3*I*a*c*d^3/x - 1/2*a*d^3/x^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{-2i\,ac^3d^3x^3-6\,ac^2d^3x^2+6i\,acd^3x+2\,ad^3+\left(bc^3d^3x^3-3i\,bc^2d^3x^2-3\,bcd^3x+i\,bd^3\right)\log\left(-\frac{cx+i}{cx-i}\right)}{2\,x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))/x^3,x, algorithm="fricas")
```

```
[Out] integral(1/2*(-2*I*a*c^3*d^3*x^3 - 6*a*c^2*d^3*x^2 + 6*I*a*c*d^3*x + 2*a*d^
3 + (b*c^3*d^3*x^3 - 3*I*b*c^2*d^3*x^2 - 3*b*c*d^3*x + I*b*d^3)*log(-(c*x +
 I)/(c*x - I)))/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d^3\left(\int \frac{a}{x^3}\,dx + \int -iac^3\,dx + \int -\frac{3ac^2}{x}\,dx + \int \frac{b\,\mathrm{atan}\,(cx)}{x^3}\,dx + \int \frac{3iac}{x^2}\,dx + \int -ibc^3\,\mathrm{atan}\,(cx)\,dx + \int -\frac{3bc^2\,\mathrm{atan}\,(cx}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**3*(a+b*atan(c*x))/x**3,x)
```

```
[Out] d**3*(Integral(a/x**3, x) + Integral(-I*a*c**3, x) + Integral(-3*a*c**2/x,
x) + Integral(b*atan(c*x)/x**3, x) + Integral(3*I*a*c/x**2, x) + Integral(-
I*b*c**3*atan(c*x), x) + Integral(-3*b*c**2*atan(c*x)/x, x) + Integral(3*I*
b*c*atan(c*x)/x**2, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(i\,cdx + d)^3(b\arctan(cx) + a)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))/x^3,x, algorithm="giac")
```

```
[Out] integrate((I*c*d*x + d)^3*(b*arctan(c*x) + a)/x^3, x)
```

3.27 $\int \frac{(d+icdx)^3\left(a+b\tan^{-1}(cx)\right)}{x^4}\,dx$

Optimal. Leaf size=189

$$\frac{1}{2}bc^3d^3\text{PolyLog}(2,-icx)-\frac{1}{2}bc^3d^3\text{PolyLog}(2,icx)+\frac{3c^2d^3\left(a+b\tan^{-1}(cx)\right)}{x}-\frac{3icd^3\left(a+b\tan^{-1}(cx)\right)}{2x^2}-\frac{d^3\left(a+b\tan^{-\cdots}\right.}{3x^3}$$

```
[Out] -(b*c*d^3)/(6*x^2) - (((3*I)/2)*b*c^2*d^3)/x - ((3*I)/2)*b*c^3*d^3*ArcTan[c
*x] - (d^3*(a + b*ArcTan[c*x]))/(3*x^3) - (((3*I)/2)*c*d^3*(a + b*ArcTan[c*
x]))/x^2 + (3*c^2*d^3*(a + b*ArcTan[c*x]))/x - I*a*c^3*d^3*Log[x] - (10*b*c
^3*d^3*Log[x])/3 + (5*b*c^3*d^3*Log[1 + c^2*x^2])/3 + (b*c^3*d^3*PolyLog[2,
 (-I)*c*x])/2 - (b*c^3*d^3*PolyLog[2, I*c*x])/2
```

Rubi [A] time = 0.203295, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {4876, 4852, 266, 44, 325, 203, 36, 29, 31, 4848, 2391}

$$\frac{1}{2}bc^3d^3\text{PolyLog}(2,-icx)-\frac{1}{2}bc^3d^3\text{PolyLog}(2,icx)+\frac{3c^2d^3\left(a+b\tan^{-1}(cx)\right)}{x}-\frac{3icd^3\left(a+b\tan^{-1}(cx)\right)}{2x^2}-\frac{d^3\left(a+b\tan^{-\cdots}\right.}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)^3*(a + b*ArcTan[c*x]))/x^4,x]
```

```
[Out] -(b*c*d^3)/(6*x^2) - (((3*I)/2)*b*c^2*d^3)/x - ((3*I)/2)*b*c^3*d^3*ArcTan[c
*x] - (d^3*(a + b*ArcTan[c*x]))/(3*x^3) - (((3*I)/2)*c*d^3*(a + b*ArcTan[c*
x]))/x^2 + (3*c^2*d^3*(a + b*ArcTan[c*x]))/x - I*a*c^3*d^3*Log[x] - (10*b*c
^3*d^3*Log[x])/3 + (5*b*c^3*d^3*Log[1 + c^2*x^2])/3 + (b*c^3*d^3*PolyLog[2,
 (-I)*c*x])/2 - (b*c^3*d^3*PolyLog[2, I*c*x])/2
```

Rule 4876

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
```

```
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{(d+icdx)^3\left(a+b\tan^{-1}(cx)\right)}{x^4}\,dx = \int \left(\frac{d^3\left(a+b\tan^{-1}(cx)\right)}{x^4}+\frac{3icd^3\left(a+b\tan^{-1}(cx)\right)}{x^3}-\frac{3c^2d^3\left(a+b\tan^{-1}(cx)\right)}{x^2}-\frac{ic^3\right.$$

$$= d^3\int \frac{a+b\tan^{-1}(cx)}{x^4}\,dx+\left(3icd^3\right)\int \frac{a+b\tan^{-1}(cx)}{x^3}\,dx-\left(3c^2d^3\right)\int \frac{a+b\tan^{-1}(c}{x^2}$$

$$= -\frac{d^3\left(a+b\tan^{-1}(cx)\right)}{3x^3}-\frac{3icd^3\left(a+b\tan^{-1}(cx)\right)}{2x^2}+\frac{3c^2d^3\left(a+b\tan^{-1}(cx)\right)}{x}-iac^3d^3$$

$$= -\frac{3ibc^2d^3}{2x}-\frac{d^3\left(a+b\tan^{-1}(cx)\right)}{3x^3}-\frac{3icd^3\left(a+b\tan^{-1}(cx)\right)}{2x^2}+\frac{3c^2d^3\left(a+b\tan^{-1}(cx\right.}{x}$$

$$= -\frac{3ibc^2d^3}{2x}-\frac{3}{2}ibc^3d^3\tan^{-1}(cx)-\frac{d^3\left(a+b\tan^{-1}(cx)\right)}{3x^3}-\frac{3icd^3\left(a+b\tan^{-1}(cx)\right)}{2x^2}+$$

$$= -\frac{bcd^3}{6x^2}-\frac{3ibc^2d^3}{2x}-\frac{3}{2}ibc^3d^3\tan^{-1}(cx)-\frac{d^3\left(a+b\tan^{-1}(cx)\right)}{3x^3}-\frac{3icd^3\left(a+b\tan^{-1}(\right.}{2x^2}$$

Mathematica [C] time = 0.094224, size = 170, normalized size = 0.9

$$\frac{d^3\left(-9ibc^2x^2\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-c^2x^2\right)+3bc^3x^3\text{PolyLog}(2,-icx)-3bc^3x^3\text{PolyLog}(2,icx)+18ac^2x^2-6i\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + I*c*d*x)^3*(a + b*ArcTan[c*x]))/x^4,x]
```

```
[Out] (d^3*(-2*a - (9*I)*a*c*x - b*c*x + 18*a*c^2*x^2 - 2*b*ArcTan[c*x] - (9*I)*b
*c*x*ArcTan[c*x] + 18*b*c^2*x^2*ArcTan[c*x] - (9*I)*b*c^2*x^2*Hypergeometri
```

```
c2F1[-1/2, 1, 1/2, -(c^2*x^2)] - (6*I)*a*c^3*x^3*Log[x] - 20*b*c^3*x^3*Log[
x] + 10*b*c^3*x^3*Log[1 + c^2*x^2] + 3*b*c^3*x^3*PolyLog[2, (-I)*c*x] - 3*b
*c^3*x^3*PolyLog[2, I*c*x]))/(6*x^3)
```

Maple [A] time = 0.051, size = 255, normalized size = 1.4

$$\frac{-\frac{3i}{2}cd^3b\arctan(cx)}{x^2}+3\frac{c^2d^3a}{x}-\frac{d^3a}{3x^3}-ic^3d^3b\arctan(cx)\ln(cx)-\frac{\frac{3i}{2}bc^2d^3}{x}+3\frac{bc^2d^3\arctan(cx)}{x}-\frac{bd^3\arctan(cx)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^3*(a+b*arctan(c*x))/x^4,x)
```

```
[Out] -3/2*I*c*d^3*b*arctan(c*x)/x^2+3*c^2*d^3*a/x-1/3*d^3*a/x^3-I*c^3*d^3*b*arct
an(c*x)*ln(c*x)-3/2*I*b*c^2*d^3/x+3*c^2*d^3*b*arctan(c*x)/x-1/3*d^3*b*arcta
n(c*x)/x^3-3/2*I*c*d^3*a/x^2+1/2*c^3*d^3*b*ln(c*x)*ln(1+I*c*x)-1/2*c^3*d^3*
b*ln(c*x)*ln(1-I*c*x)+1/2*c^3*d^3*b*dilog(1+I*c*x)-1/2*c^3*d^3*b*dilog(1-I*
c*x)+5/3*b*c^3*d^3*ln(c^2*x^2+1)-I*c^3*d^3*a*ln(c*x)-3/2*I*b*c^3*d^3*arctan
(c*x)-1/6*b*c*d^3/x^2-10/3*c^3*d^3*b*ln(c*x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-i\,bc^3d^3\int\frac{\arctan(cx)}{x}\,dx-i\,ac^3d^3\log(x)+\frac{3}{2}\left(c\left(\log\left(c^2x^2+1\right)-\log\left(x^2\right)\right)+\frac{2\arctan(cx)}{x}\right)bc^2d^3-\frac{3}{2}i\left(\left(c\arctan(c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))/x^4,x, algorithm="maxima")
```

```
[Out] -I*b*c^3*d^3*integrate(arctan(c*x)/x, x) - I*a*c^3*d^3*log(x) + 3/2*(c*(log
(c^2*x^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*b*c^2*d^3 - 3/2*I*((c*arctan(c
*x) + 1/x)*c + arctan(c*x)/x^2)*b*c*d^3 + 1/6*((c^2*log(c^2*x^2 + 1) - c^2*
log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^3)*b*d^3 + 3*a*c^2*d^3/x - 3/2*I*a*c*
d^3/x^2 - 1/3*a*d^3/x^3
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{-2i\,ac^3d^3x^3 - 6\,ac^2d^3x^2 + 6i\,acd^3x + 2\,ad^3 + \left(bc^3d^3x^3 - 3i\,bc^2d^3x^2 - 3\,bcd^3x + i\,bd^3\right)\log\left(-\frac{cx+i}{cx-i}\right)}{2\,x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] integral(1/2*(-2*I*a*c^3*d^3*x^3 - 6*a*c^2*d^3*x^2 + 6*I*a*c*d^3*x + 2*a*d^
3 + (b*c^3*d^3*x^3 - 3*I*b*c^2*d^3*x^2 - 3*b*c*d^3*x + I*b*d^3)*log(-(c*x +
 I)/(c*x - I)))/x^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d^3\left(\int \frac{a}{x^4}\,dx + \int -\frac{3ac^2}{x^2}\,dx + \int \frac{b\,\text{atan}\,(cx)}{x^4}\,dx + \int \frac{3iac}{x^3}\,dx + \int -\frac{iac^3}{x}\,dx + \int -\frac{3bc^2\,\text{atan}\,(cx)}{x^2}\,dx + \int \frac{3ibc\,\text{atan}\,(cx)}{x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**3*(a+b*atan(c*x))/x**4,x)
```

```
[Out] d**3*(Integral(a/x**4, x) + Integral(-3*a*c**2/x**2, x) + Integral(b*atan(c
*x)/x**4, x) + Integral(3*I*a*c/x**3, x) + Integral(-I*a*c**3/x, x) + Integ
ral(-3*b*c**2*atan(c*x)/x**2, x) + Integral(3*I*b*c*atan(c*x)/x**3, x) + In
tegral(-I*b*c**3*atan(c*x)/x, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(i\,cdx + d)^3(b\,\text{arctan}\,(cx) + a)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))/x^4,x, algorithm="giac")
```

```
[Out] integrate((I*c*d*x + d)^3*(b*arctan(c*x) + a)/x^4, x)
```

3.28 $\int \frac{(d+icdx)^3\left(a+b\tan^{-1}(cx)\right)}{x^5}\,dx$

Optimal. Leaf size=103

$$-\frac{d^3(1+icx)^4\left(a+b\tan^{-1}(cx)\right)}{4x^4}-\frac{ibc^2d^3}{2x^2}+\frac{7bc^3d^3}{4x}-2ibc^4d^3\log(x)+2ibc^4d^3\log(cx+i)-\frac{bcd^3}{12x^3}$$

```
[Out] -(b*c*d^3)/(12*x^3) - ((I/2)*b*c^2*d^3)/x^2 + (7*b*c^3*d^3)/(4*x) - (d^3*(1
 + I*c*x)^4*(a + b*ArcTan[c*x]))/(4*x^4) - (2*I)*b*c^4*d^3*Log[x] + (2*I)*b
*c^4*d^3*Log[I + c*x]
```

Rubi [A] time = 0.0906254, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {37, 4872, 12, 88}

$$-\frac{d^3(1+icx)^4\left(a+b\tan^{-1}(cx)\right)}{4x^4}-\frac{ibc^2d^3}{2x^2}+\frac{7bc^3d^3}{4x}-2ibc^4d^3\log(x)+2ibc^4d^3\log(cx+i)-\frac{bcd^3}{12x^3}$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)^3*(a + b*ArcTan[c*x]))/x^5,x]
```

```
[Out] -(b*c*d^3)/(12*x^3) - ((I/2)*b*c^2*d^3)/x^2 + (7*b*c^3*d^3)/(4*x) - (d^3*(1
 + I*c*x)^4*(a + b*ArcTan[c*x]))/(4*x^4) - (2*I)*b*c^4*d^3*Log[x] + (2*I)*b
*c^4*d^3*Log[I + c*x]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 4872

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
+ b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2), x
], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2*m
] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int \frac{(d+icdx)^3\left(a+b\tan^{-1}(cx)\right)}{x^5}\,dx &= -\frac{d^3(1+icx)^4\left(a+b\tan^{-1}(cx)\right)}{4x^4}-(bc)\int \frac{d^3(i-cx)^3}{4x^4(i+cx)}\,dx\\ &= -\frac{d^3(1+icx)^4\left(a+b\tan^{-1}(cx)\right)}{4x^4}-\frac{1}{4}\left(bcd^3\right)\int \frac{(i-cx)^3}{x^4(i+cx)}\,dx\\ &= -\frac{d^3(1+icx)^4\left(a+b\tan^{-1}(cx)\right)}{4x^4}-\frac{1}{4}\left(bcd^3\right)\int \left(-\frac{1}{x^4}-\frac{4ic}{x^3}+\frac{7c^2}{x^2}+\frac{8ic^3}{x}-\frac{8ic^4}{i+cx}\right) d\\ &= -\frac{bcd^3}{12x^3}-\frac{ibc^2d^3}{2x^2}+\frac{7bc^3d^3}{4x}-\frac{d^3(1+icx)^4\left(a+b\tan^{-1}(cx)\right)}{4x^4}-2ibc^4d^3\log(x)+2ibc^4\end{aligned}$$

Mathematica [C] time = 0.122469, size = 165, normalized size = 1.6

$$\frac{d^3\left(-bcx\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-c^2x^2\right)-3i\left(6ibc^3x^3\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-c^2x^2\right)-4ac^3x^3+6iac^2x\right.\right.}{12x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + I*c*d*x)^3*(a + b*ArcTan[c*x]))/x^5,x]
```

```
[Out] (d^3*(-(b*c*x*Hypergeometric2F1[-3/2, 1, -1/2, -(c^2*x^2)]) - (3*I)*((-I)*a
 + 4*a*c*x + (6*I)*a*c^2*x^2 + 2*b*c^2*x^2 - 4*a*c^3*x^3 + b*(-I + 4*c*x +
(6*I)*c^2*x^2 - 4*c^3*x^3)*ArcTan[c*x] + (6*I)*b*c^3*x^3*Hypergeometric2F1[
-1/2, 1, 1/2, -(c^2*x^2)] + 8*b*c^4*x^4*Log[x] - 4*b*c^4*x^4*Log[1 + c^2*x^
2])))/(12*x^4)
```

Maple [B] time = 0.036, size = 190, normalized size = 1.8

$$\frac{3\,c^2d^3a}{2\,x^2}-\frac{d^3a}{4\,x^4}+\frac{ic^3d^3a}{x}-\frac{icd^3a}{x^3}+\frac{3\,bc^2d^3\arctan\left(cx\right)}{2\,x^2}-\frac{bd^3\arctan\left(cx\right)}{4\,x^4}+\frac{ic^3d^3b\arctan\left(cx\right)}{x}-\frac{icd^3b\arctan\left(cx\right)}{x^3}+ic$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^3*(a+b*arctan(c*x))/x^5,x)
```

```
[Out] 3/2*c^2*d^3*a/x^2-1/4*d^3*a/x^4+I*c^3*d^3*a/x-I*c*d^3*a/x^3+3/2*c^2*d^3*b*a
rctan(c*x)/x^2-1/4*d^3*b*arctan(c*x)/x^4+I*c^3*d^3*b*arctan(c*x)/x-I*c*d^3*
b*arctan(c*x)/x^3+I*c^4*d^3*b*ln(c^2*x^2+1)+7/4*b*c^4*d^3*arctan(c*x)-1/2*I
*b*c^2*d^3/x^2-2*I*c^4*d^3*b*ln(c*x)-1/12*b*c*d^3/x^3+7/4*b*c^3*d^3/x
```

Maxima [B] time = 1.49499, size = 273, normalized size = 2.65

$$\frac{1}{2}i\left(c\left(\log\left(c^2x^2+1\right)-\log\left(x^2\right)\right)+\frac{2\arctan\left(cx\right)}{x}\right)bc^3d^3+\frac{3}{2}\left(\left(c\arctan\left(cx\right)+\frac{1}{x}\right)c+\frac{\arctan\left(cx\right)}{x^2}\right)bc^2d^3+\frac{1}{2}i\left(\left(c^2\log\left(c\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))/x^5,x, algorithm="maxima")
```

```
[Out] 1/2*I*(c*(log(c^2*x^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*b*c^3*d^3 + 3/2*(
(c*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*b*c^2*d^3 + 1/2*I*((c^2*log(c^2*
x^2 + 1) - c^2*log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^3)*b*c*d^3 + I*a*c^3*d
^3/x + 1/12*((3*c^3*arctan(c*x) + (3*c^2*x^2 - 1)/x^3)*c - 3*arctan(c*x)/x^
4)*b*d^3 + 3/2*a*c^2*d^3/x^2 - I*a*c*d^3/x^3 - 1/4*a*d^3/x^4
```

Fricas [B] time = 2.9531, size = 410, normalized size = 3.98

$$\frac{-48i\,bc^4d^3x^4\log\left(x\right)+45i\,bc^4d^3x^4\log\left(\frac{cx+i}{c}\right)+3i\,bc^4d^3x^4\log\left(\frac{cx-i}{c}\right)+\left(24i\,a+42\,b\right)c^3d^3x^3+12\left(3\,a-i\,b\right)c^2d^3x^2+\left(-24\right.}{24\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))/x^5,x, algorithm="fricas")
```

```
[Out] 1/24*(-48*I*b*c^4*d^3*x^4*log(x) + 45*I*b*c^4*d^3*x^4*log((c*x + I)/c) + 3*
I*b*c^4*d^3*x^4*log((c*x - I)/c) + (24*I*a + 42*b)*c^3*d^3*x^3 + 12*(3*a -
I*b)*c^2*d^3*x^2 + (-24*I*a - 2*b)*c*d^3*x - 6*a*d^3 - (12*b*c^3*d^3*x^3 -
18*I*b*c^2*d^3*x^2 - 12*b*c*d^3*x + 3*I*b*d^3)*log(-(c*x + I)/(c*x - I)))/x
^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**3*(a+b*atan(c*x))/x**5,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.2576, size = 258, normalized size = 2.5

$$\frac{45\,bc^4d^3ix^4\log\left(cix-1\right)+3\,bc^4d^3ix^4\log\left(-cix-1\right)-48\,bc^4d^3ix^4\log\left(x\right)+24\,bc^3d^3ix^3\arctan\left(cx\right)+24\,ac^3d^3ix^3+42\,bc^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))/x^5,x, algorithm="giac")
```

```
[Out] 1/24*(45*b*c^4*d^3*i*x^4*log(c*i*x - 1) + 3*b*c^4*d^3*i*x^4*log(-c*i*x - 1)
 - 48*b*c^4*d^3*i*x^4*log(x) + 24*b*c^3*d^3*i*x^3*arctan(c*x) + 24*a*c^3*d^
3*i*x^3 + 42*b*c^3*d^3*x^3 - 12*b*c^2*d^3*i*x^2 + 36*b*c^2*d^3*x^2*arctan(c
*x) + 36*a*c^2*d^3*x^2 - 24*b*c*d^3*i*x*arctan(c*x) - 24*a*c*d^3*i*x - 2*b*
c*d^3*x - 6*b*d^3*arctan(c*x) - 6*a*d^3)/x^4
```

3.29 $\int \frac{(d+icdx)^3\left(a+b\tan^{-1}(cx)\right)}{x^6}\,dx$

Optimal. Leaf size=150

$$\frac{icd^3(1+icx)^4\left(a+b\tan^{-1}(cx)\right)}{20x^4}-\frac{d^3(1+icx)^4\left(a+b\tan^{-1}(cx)\right)}{5x^5}+\frac{3bc^3d^3}{5x^2}-\frac{ibc^2d^3}{4x^3}+\frac{5ibc^4d^3}{4x}+\frac{6}{5}bc^5d^3\log(x)-\frac{6}{5}bc^5$$

```
[Out] -(b*c*d^3)/(20*x^4) - ((I/4)*b*c^2*d^3)/x^3 + (3*b*c^3*d^3)/(5*x^2) + (((5*
I)/4)*b*c^4*d^3)/x - (d^3*(1 + I*c*x)^4*(a + b*ArcTan[c*x]))/(5*x^5) + ((I/
20)*c*d^3*(1 + I*c*x)^4*(a + b*ArcTan[c*x]))/x^4 + (6*b*c^5*d^3*Log[x])/5 -
 (6*b*c^5*d^3*Log[I + c*x])/5
```

Rubi [A] time = 0.106787, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {45, 37, 4872, 12, 148}

$$\frac{icd^3(1+icx)^4\left(a+b\tan^{-1}(cx)\right)}{20x^4}-\frac{d^3(1+icx)^4\left(a+b\tan^{-1}(cx)\right)}{5x^5}+\frac{3bc^3d^3}{5x^2}-\frac{ibc^2d^3}{4x^3}+\frac{5ibc^4d^3}{4x}+\frac{6}{5}bc^5d^3\log(x)-\frac{6}{5}bc^5$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)^3*(a + b*ArcTan[c*x]))/x^6,x]
```


```
[Out] -(b*c*d^3)/(20*x^4) - ((I/4)*b*c^2*d^3)/x^3 + (3*b*c^3*d^3)/(5*x^2) + (((5*
I)/4)*b*c^4*d^3)/x - (d^3*(1 + I*c*x)^4*(a + b*ArcTan[c*x]))/(5*x^5) + ((I/
20)*c*d^3*(1 + I*c*x)^4*(a + b*ArcTan[c*x]))/x^4 + (6*b*c^5*d^3*Log[x])/5 -
 (6*b*c^5*d^3*Log[I + c*x])/5
```


Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
```

```
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 4872

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
+ b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2), x
], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2*m
] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 148

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(
c + d*x)^n*(e + f*x)^p*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
m}, x] && (IntegersQ[m, n, p] || (IGtQ[n, 0] && IGtQ[p, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{(d+icdx)^3\left(a+b\tan^{-1}(cx)\right)}{x^6}\,dx &= -\frac{d^3(1+icx)^4\left(a+b\tan^{-1}(cx)\right)}{5x^5}+\frac{icd^3(1+icx)^4\left(a+b\tan^{-1}(cx)\right)}{20x^4}-(bc)\int\frac{d^3(-4i\\ &= -\frac{d^3(1+icx)^4\left(a+b\tan^{-1}(cx)\right)}{5x^5}+\frac{icd^3(1+icx)^4\left(a+b\tan^{-1}(cx)\right)}{20x^4}-\frac{1}{20}\left(bcd^3\right)\int\\ &= -\frac{d^3(1+icx)^4\left(a+b\tan^{-1}(cx)\right)}{5x^5}+\frac{icd^3(1+icx)^4\left(a+b\tan^{-1}(cx)\right)}{20x^4}-\frac{1}{20}\left(bcd^3\right)\int\Big(\\ &= -\frac{bcd^3}{20x^4}-\frac{ibc^2d^3}{4x^3}+\frac{3bc^3d^3}{5x^2}+\frac{5ibc^4d^3}{4x}-\frac{d^3(1+icx)^4\left(a+b\tan^{-1}(cx)\right)}{5x^5}+\frac{icd^3(1+ic\end{aligned}$$

Mathematica [C] time = 0.094492, size = 185, normalized size = 1.23

$$\frac{d^3\left(10ibc^4x^4\text{Hypergeometric2F1}\left(-\tfrac{1}{2},1,\tfrac{1}{2},-c^2x^2\right)-5ibc^2x^2\text{Hypergeometric2F1}\left(-\tfrac{3}{2},1,-\tfrac{1}{2},-c^2x^2\right)+10iac^3x^3+20a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + I*c*d*x)^3*(a + b*ArcTan[c*x]))/x^6,x]
```

```
[Out] (d^3*(-4*a - (15*I)*a*c*x - b*c*x + 20*a*c^2*x^2 + (10*I)*a*c^3*x^3 + 12*b*
c^3*x^3 - 4*b*ArcTan[c*x] - (15*I)*b*c*x*ArcTan[c*x] + 20*b*c^2*x^2*ArcTan[
c*x] + (10*I)*b*c^3*x^3*ArcTan[c*x] - (5*I)*b*c^2*x^2*Hypergeometric2F1[-3/
2, 1, -1/2, -(c^2*x^2)] + (10*I)*b*c^4*x^4*Hypergeometric2F1[-1/2, 1, 1/2,
-(c^2*x^2)] + 24*b*c^5*x^5*Log[x] - 12*b*c^5*x^5*Log[1 + c^2*x^2]))/(20*x^5
)
```

Maple [A] time = 0.038, size = 200, normalized size = 1.3

$$\frac{\frac{i}{2}c^3d^3a}{x^2} - \frac{\frac{3i}{4}cd^3a}{x^4} - \frac{d^3a}{5x^5} + \frac{c^2d^3a}{x^3} + \frac{\frac{i}{2}c^3d^3b\arctan(cx)}{x^2} - \frac{\frac{3i}{4}cd^3b\arctan(cx)}{x^4} - \frac{bd^3\arctan(cx)}{5x^5} + \frac{bc^2d^3\arctan(cx)}{x^3} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^3*(a+b*arctan(c*x))/x^6,x)
```

```
[Out] 1/2*I*c^3*d^3*a/x^2-3/4*I*c*d^3*a/x^4-1/5*d^3*a/x^5+c^2*d^3*a/x^3+1/2*I*c^3
*d^3*b*arctan(c*x)/x^2-3/4*I*c*d^3*b*arctan(c*x)/x^4-1/5*d^3*b*arctan(c*x)/
x^5+c^2*d^3*b*arctan(c*x)/x^3-3/5*c^5*d^3*b*ln(c^2*x^2+1)+5/4*I*c^5*d^3*b*a
rctan(c*x)-1/4*I*b*c^2*d^3/x^3+5/4*I*b*c^4*d^3/x-1/20*b*c*d^3/x^4+3/5*b*c^3
*d^3/x^2+6/5*c^5*d^3*b*ln(c*x)
```

Maxima [A] time = 1.48809, size = 302, normalized size = 2.01

$$\frac{1}{2}i\left(\left(c\arctan(cx) + \frac{1}{x}\right)c + \frac{\arctan(cx)}{x^2}\right)bc^3d^3 - \frac{1}{2}\left(\left(c^2\log\left(c^2x^2+1\right) - c^2\log\left(x^2\right) - \frac{1}{x^2}\right)c - \frac{2\arctan(cx)}{x^3}\right)bc^2d^3 + \frac{1}{4}i$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))/x^6,x, algorithm="maxima")
```

```
[Out] 1/2*I*((c*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*b*c^3*d^3 - 1/2*((c^2*log
(c^2*x^2 + 1) - c^2*log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^3)*b*c^2*d^3 + 1/
4*I*((3*c^3*arctan(c*x) + (3*c^2*x^2 - 1)/x^3)*c - 3*arctan(c*x)/x^4)*b*c*d
^3 - 1/20*((2*c^4*log(c^2*x^2 + 1) - 2*c^4*log(x^2) - (2*c^2*x^2 - 1)/x^4)*
c + 4*arctan(c*x)/x^5)*b*d^3 + 1/2*I*a*c^3*d^3/x^2 + a*c^2*d^3/x^3 - 3/4*I*
```

```
a*c*d^3/x^4 - 1/5*a*d^3/x^5
```

Fricas [A] time = 2.87688, size = 427, normalized size = 2.85

$$\frac{48\,bc^5d^3x^5\log\left(x\right)-49\,bc^5d^3x^5\log\left(\frac{cx+i}{c}\right)+bc^5d^3x^5\log\left(\frac{cx-i}{c}\right)+50i\,bc^4d^3x^4+(20i\,a+24\,b)c^3d^3x^3+10\,(4\,a-i\,b)c^2d^3x^2}{40\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))/x^6,x, algorithm="fricas")
```

```
[Out] 1/40*(48*b*c^5*d^3*x^5*log(x) - 49*b*c^5*d^3*x^5*log((c*x + I)/c) + b*c^5*d
^3*x^5*log((c*x - I)/c) + 50*I*b*c^4*d^3*x^4 + (20*I*a + 24*b)*c^3*d^3*x^3
+ 10*(4*a - I*b)*c^2*d^3*x^2 + (-30*I*a - 2*b)*c*d^3*x - 8*a*d^3 - (10*b*c^
3*d^3*x^3 - 20*I*b*c^2*d^3*x^2 - 15*b*c*d^3*x + 4*I*b*d^3)*log(-(c*x + I)/(
c*x - I)))/x^5
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**3*(a+b*atan(c*x))/x**6,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.56752, size = 270, normalized size = 1.8

$$-\frac{49\,bc^5d^3x^5\log\left(cx+i\right)-bc^5d^3x^5\log\left(cx-i\right)-48\,bc^5d^3x^5\log\left(x\right)-50\,bc^4d^3ix^4-20\,bc^3d^3ix^3\arctan\left(cx\right)-20\,ac^3d^3ix^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))/x^6,x, algorithm="giac")
```

```
[Out] -1/40*(49*b*c^5*d^3*x^5*log(c*x + i) - b*c^5*d^3*x^5*log(c*x - i) - 48*b*c^
5*d^3*x^5*log(x) - 50*b*c^4*d^3*i*x^4 - 20*b*c^3*d^3*i*x^3*arctan(c*x) - 20
*a*c^3*d^3*i*x^3 - 24*b*c^3*d^3*x^3 + 10*b*c^2*d^3*i*x^2 - 40*b*c^2*d^3*x^2
*arctan(c*x) - 40*a*c^2*d^3*x^2 + 30*b*c*d^3*i*x*arctan(c*x) + 30*a*c*d^3*i
*x + 2*b*c*d^3*x + 8*b*d^3*arctan(c*x) + 8*a*d^3)/x^5
```

3.30 $\int \frac{(d+icdx)^3\left(a+b\tan^{-1}(cx)\right)}{x^7}\,dx$

Optimal. Leaf size=214

$$\frac{ic^3d^3\left(a+b\tan^{-1}(cx)\right)}{3x^3}+\frac{3c^2d^3\left(a+b\tan^{-1}(cx)\right)}{4x^4}-\frac{3icd^3\left(a+b\tan^{-1}(cx)\right)}{5x^5}-\frac{d^3\left(a+b\tan^{-1}(cx)\right)}{6x^6}+\frac{7ibc^4d^3}{15x^2}+\frac{11bc^3d^3}{36x^3}$$

```
[Out] -(b*c*d^3)/(30*x^5) - (((3*I)/20)*b*c^2*d^3)/x^4 + (11*b*c^3*d^3)/(36*x^3)
+ (((7*I)/15)*b*c^4*d^3)/x^2 - (11*b*c^5*d^3)/(12*x) - (d^3*(a + b*ArcTan[c
*x]))/(6*x^6) - (((3*I)/5)*c*d^3*(a + b*ArcTan[c*x]))/x^5 + (3*c^2*d^3*(a +
 b*ArcTan[c*x]))/(4*x^4) + ((I/3)*c^3*d^3*(a + b*ArcTan[c*x]))/x^3 + ((14*I
)/15)*b*c^6*d^3*Log[x] - (I/120)*b*c^6*d^3*Log[I - c*x] - ((37*I)/40)*b*c^6
*d^3*Log[I + c*x]
```

Rubi [A] time = 0.178493, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {43, 4872, 12, 1802}

$$\frac{ic^3d^3\left(a+b\tan^{-1}(cx)\right)}{3x^3}+\frac{3c^2d^3\left(a+b\tan^{-1}(cx)\right)}{4x^4}-\frac{3icd^3\left(a+b\tan^{-1}(cx)\right)}{5x^5}-\frac{d^3\left(a+b\tan^{-1}(cx)\right)}{6x^6}+\frac{7ibc^4d^3}{15x^2}+\frac{11bc^3d^3}{36x^3}$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)^3*(a + b*ArcTan[c*x]))/x^7,x]
```



```
[Out] -(b*c*d^3)/(30*x^5) - (((3*I)/20)*b*c^2*d^3)/x^4 + (11*b*c^3*d^3)/(36*x^3)
+ (((7*I)/15)*b*c^4*d^3)/x^2 - (11*b*c^5*d^3)/(12*x) - (d^3*(a + b*ArcTan[c
*x]))/(6*x^6) - (((3*I)/5)*c*d^3*(a + b*ArcTan[c*x]))/x^5 + (3*c^2*d^3*(a +
 b*ArcTan[c*x]))/(4*x^4) + ((I/3)*c^3*d^3*(a + b*ArcTan[c*x]))/x^3 + ((14*I
)/15)*b*c^6*d^3*Log[x] - (I/120)*b*c^6*d^3*Log[I - c*x] - ((37*I)/40)*b*c^6
*d^3*Log[I + c*x]
```


Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 4872

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
+ b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2), x
], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2*m
] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(d+icdx)^3\left(a+b\tan^{-1}(cx)\right)}{x^7}\,dx = -\frac{d^3\left(a+b\tan^{-1}(cx)\right)}{6x^6}-\frac{3icd^3\left(a+b\tan^{-1}(cx)\right)}{5x^5}+\frac{3c^2d^3\left(a+b\tan^{-1}(cx)\right)}{4x^4}+\frac{ic^3d}{}$$

$$= -\frac{d^3\left(a+b\tan^{-1}(cx)\right)}{6x^6}-\frac{3icd^3\left(a+b\tan^{-1}(cx)\right)}{5x^5}+\frac{3c^2d^3\left(a+b\tan^{-1}(cx)\right)}{4x^4}+\frac{ic^3d}{}$$

$$= -\frac{d^3\left(a+b\tan^{-1}(cx)\right)}{6x^6}-\frac{3icd^3\left(a+b\tan^{-1}(cx)\right)}{5x^5}+\frac{3c^2d^3\left(a+b\tan^{-1}(cx)\right)}{4x^4}+\frac{ic^3d}{}$$

$$= -\frac{bcd^3}{30x^5}-\frac{3ibc^2d^3}{20x^4}+\frac{11bc^3d^3}{36x^3}+\frac{7ibc^4d^3}{15x^2}-\frac{11bc^5d^3}{12x}-\frac{d^3\left(a+b\tan^{-1}(cx)\right)}{6x^6}-\frac{3icd^3}{}$$

Mathematica [C] time = 0.111934, size = 188, normalized size = 0.88

$$\frac{d^3\left(-2bcx\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},-c^2x^2\right)+i\left(-15ibc^3x^3\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-c^2x^2\right)+20ac^3x^3-\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + I*c*d*x)^3*(a + b*ArcTan[c*x]))/x^7,x]
```

```
[Out] (d^3*(-2*b*c*x*Hypergeometric2F1[-5/2, 1, -3/2, -(c^2*x^2)] + I*((10*I)*a -
 36*a*c*x - (45*I)*a*c^2*x^2 - 9*b*c^2*x^2 + 20*a*c^3*x^3 + 28*b*c^4*x^4 +
```

(10*I)*b*ArcTan[c*x] - 36*b*c*x*ArcTan[c*x] - (45*I)*b*c^2*x^2*ArcTan[c*x] + 20*b*c^3*x^3*ArcTan[c*x] - (15*I)*b*c^3*x^3*Hypergeometric2F1[-3/2, 1, -1/2, -(c^2*x^2)] + 56*b*c^6*x^6*Log[x] - 28*b*c^6*x^6*Log[1 + c^2*x^2])))/(60*x^6)

Maple [A] time = 0.039, size = 215, normalized size = 1.

$$\frac{3\,c^2d^3a}{4\,x^4}-\frac{d^3a}{6\,x^6}+\frac{14\,i}{15}c^6d^3b\ln(cx)+\frac{\frac{i}{3}c^3d^3a}{x^3}+\frac{3\,bc^2d^3\arctan(cx)}{4\,x^4}-\frac{bd^3\arctan(cx)}{6\,x^6}+\frac{\frac{7\,i}{15}bc^4d^3}{x^2}-\frac{7\,i}{15}c^6d^3b\ln\left(c^2x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d+I*c*d*x)^3*(a+b*arctan(c*x))/x^7,x)

[Out] 3/4*c^2*d^3*a/x^4-1/6*d^3*a/x^6+14/15*I*c^6*d^3*b*ln(c*x)+1/3*I*c^3*d^3*a/x^3+3/4*c^2*d^3*b*arctan(c*x)/x^4-1/6*d^3*b*arctan(c*x)/x^6+7/15*I*b*c^4*d^3/x^2-7/15*I*c^6*d^3*b*ln(c^2*x^2+1)-3/5*I*c*d^3*b*arctan(c*x)/x^5-11/12*b*c^6*d^3*arctan(c*x)-3/5*I*c*d^3*a/x^5+1/3*I*c^3*d^3*b*arctan(c*x)/x^3-3/20*I*b*c^2*d^3/x^4-1/30*b*c*d^3/x^5+11/36*b*c^3*d^3/x^3-11/12*b*c^5*d^3/x

---

Maxima [A] time = 1.49242, size = 335, normalized size = 1.57

$$-\frac{1}{6}i\left(\left(c^2\log\left(c^2x^2+1\right)-c^2\log\left(x^2\right)-\frac{1}{x^2}\right)c-\frac{2\arctan(cx)}{x^3}\right)bc^3d^3-\frac{1}{4}\left(\left(3\,c^3\arctan(cx)+\frac{3\,c^2x^2-1}{x^3}\right)c-\frac{3\arctan(cx)}{x^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))/x^7,x, algorithm="maxima")

[Out] -1/6*I*((c^2*log(c^2*x^2 + 1) - c^2*log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^3)*b*c^3*d^3 - 1/4*((3*c^3*arctan(c*x) + (3*c^2*x^2 - 1)/x^3)*c - 3*arctan(c*x)/x^4)*b*c^2*d^3 - 3/20*I*((2*c^4*log(c^2*x^2 + 1) - 2*c^4*log(x^2) - (2*c^2*x^2 - 1)/x^4)*c + 4*arctan(c*x)/x^5)*b*c*d^3 - 1/90*((15*c^5*arctan(c*x) + (15*c^4*x^4 - 5*c^2*x^2 + 3)/x^5)*c + 15*arctan(c*x)/x^6)*b*d^3 + 1/3*I*a*c^3*d^3/x^3 + 3/4*a*c^2*d^3/x^4 - 3/5*I*a*c*d^3/x^5 - 1/6*a*d^3/x^6

Fricas [A] time = 3.16693, size = 481, normalized size = 2.25

$$\frac{336i\,bc^6d^3x^6\log(x) - 333i\,bc^6d^3x^6\log\left(\frac{cx+i}{c}\right) - 3i\,bc^6d^3x^6\log\left(\frac{cx-i}{c}\right) - 330\,bc^5d^3x^5 + 168i\,bc^4d^3x^4 + (120i\,a + 110\,b)c^3d^3}{36}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))/x^7,x, algorithm="fricas")
```

```
[Out] 1/360*(336*I*b*c^6*d^3*x^6*log(x) - 333*I*b*c^6*d^3*x^6*log((c*x + I)/c) -
3*I*b*c^6*d^3*x^6*log((c*x - I)/c) - 330*b*c^5*d^3*x^5 + 168*I*b*c^4*d^3*x^
4 + (120*I*a + 110*b)*c^3*d^3*x^3 + 54*(5*a - I*b)*c^2*d^3*x^2 + (-216*I*a
- 12*b)*c*d^3*x - 60*a*d^3 - (60*b*c^3*d^3*x^3 - 135*I*b*c^2*d^3*x^2 - 108*
b*c*d^3*x + 30*I*b*d^3)*log(-(c*x + I)/(c*x - I)))/x^6
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**3*(a+b*atan(c*x))/x**7,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.69019, size = 292, normalized size = 1.36

$$-\frac{3\,bc^6d^3ix^6\log(cix+1) + 333\,bc^6d^3ix^6\log(-cix+1) - 336\,bc^6d^3ix^6\log(x) + 330\,bc^5d^3x^5 - 168\,bc^4d^3ix^4 - 120\,bc^3d^3i}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))/x^7,x, algorithm="giac")
```

```
[Out] -1/360*(3*b*c^6*d^3*i*x^6*log(c*i*x + 1) + 333*b*c^6*d^3*i*x^6*log(-c*i*x +
 1) - 336*b*c^6*d^3*i*x^6*log(x) + 330*b*c^5*d^3*x^5 - 168*b*c^4*d^3*i*x^4
- 120*b*c^3*d^3*i*x^3*arctan(c*x) - 120*a*c^3*d^3*i*x^3 - 110*b*c^3*d^3*x^3
 + 54*b*c^2*d^3*i*x^2 - 270*b*c^2*d^3*x^2*arctan(c*x) - 270*a*c^2*d^3*x^2 +
 216*b*c*d^3*i*x*arctan(c*x) + 216*a*c*d^3*i*x + 12*b*c*d^3*x + 60*b*d^3*ar
ctan(c*x) + 60*a*d^3)/x^6
```

3.31 $\int x^3(d+icdx)^4\left(a+b\tan^{-1}(cx)\right)dx$

Optimal. Leaf size=238

$$\frac{1}{8}c^4d^4x^8\left(a+b\tan^{-1}(cx)\right)-\frac{4}{7}ic^3d^4x^7\left(a+b\tan^{-1}(cx)\right)-c^2d^4x^6\left(a+b\tan^{-1}(cx)\right)+\frac{4}{5}icd^4x^5\left(a+b\tan^{-1}(cx)\right)+\frac{1}{4}d^4x^4($$

```
[Out] (11*b*d^4*x)/(8*c^3) + (((24*I)/35)*b*d^4*x^2)/c^2 - (11*b*d^4*x^3)/(24*c)
- ((12*I)/35)*b*d^4*x^4 + (9*b*c*d^4*x^5)/40 + ((2*I)/21)*b*c^2*d^4*x^6 - (
b*c^3*d^4*x^7)/56 - (11*b*d^4*ArcTan[c*x])/(8*c^4) + (d^4*x^4*(a + b*ArcTan
[c*x]))/4 + ((4*I)/5)*c*d^4*x^5*(a + b*ArcTan[c*x]) - c^2*d^4*x^6*(a + b*Ar
cTan[c*x]) - ((4*I)/7)*c^3*d^4*x^7*(a + b*ArcTan[c*x]) + (c^4*d^4*x^8*(a +
b*ArcTan[c*x]))/8 - (((24*I)/35)*b*d^4*Log[1 + c^2*x^2])/c^4
```

Rubi [A] time = 0.214227, antiderivative size = 238, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {43, 4872, 12, 1802, 635, 203, 260}

$$\frac{1}{8}c^4d^4x^8\left(a+b\tan^{-1}(cx)\right)-\frac{4}{7}ic^3d^4x^7\left(a+b\tan^{-1}(cx)\right)-c^2d^4x^6\left(a+b\tan^{-1}(cx)\right)+\frac{4}{5}icd^4x^5\left(a+b\tan^{-1}(cx)\right)+\frac{1}{4}d^4x^4($$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + I*c*d*x)^4*(a + b*ArcTan[c*x]),x]
```


```
[Out] (11*b*d^4*x)/(8*c^3) + (((24*I)/35)*b*d^4*x^2)/c^2 - (11*b*d^4*x^3)/(24*c)
- ((12*I)/35)*b*d^4*x^4 + (9*b*c*d^4*x^5)/40 + ((2*I)/21)*b*c^2*d^4*x^6 - (
b*c^3*d^4*x^7)/56 - (11*b*d^4*ArcTan[c*x])/(8*c^4) + (d^4*x^4*(a + b*ArcTan
[c*x]))/4 + ((4*I)/5)*c*d^4*x^5*(a + b*ArcTan[c*x]) - c^2*d^4*x^6*(a + b*Ar
cTan[c*x]) - ((4*I)/7)*c^3*d^4*x^7*(a + b*ArcTan[c*x]) + (c^4*d^4*x^8*(a +
b*ArcTan[c*x]))/8 - (((24*I)/35)*b*d^4*Log[1 + c^2*x^2])/c^4
```


Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 4872

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
+ b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2), x
], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2*m
] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int x^3(d+icdx)^4\left(a+b\tan^{-1}(cx)\right)dx = \frac{1}{4}d^4x^4\left(a+b\tan^{-1}(cx)\right)+\frac{4}{5}icd^4x^5\left(a+b\tan^{-1}(cx)\right)-c^2d^4x^6\left(a+b\tan^{-1}(cx)\right)$$

$$= \frac{1}{4}d^4x^4\left(a+b\tan^{-1}(cx)\right)+\frac{4}{5}icd^4x^5\left(a+b\tan^{-1}(cx)\right)-c^2d^4x^6\left(a+b\tan^{-1}(cx)\right)$$

$$= \frac{1}{4}d^4x^4\left(a+b\tan^{-1}(cx)\right)+\frac{4}{5}icd^4x^5\left(a+b\tan^{-1}(cx)\right)-c^2d^4x^6\left(a+b\tan^{-1}(cx)\right)$$

$$= \frac{11bd^4x}{8c^3}+\frac{24ibd^4x^2}{35c^2}-\frac{11bd^4x^3}{24c}-\frac{12}{35}ibd^4x^4+\frac{9}{40}bcd^4x^5+\frac{2}{21}ibc^2d^4x^6-\frac{1}{56}bc^3d^4$$

$$= \frac{11bd^4x}{8c^3}+\frac{24ibd^4x^2}{35c^2}-\frac{11bd^4x^3}{24c}-\frac{12}{35}ibd^4x^4+\frac{9}{40}bcd^4x^5+\frac{2}{21}ibc^2d^4x^6-\frac{1}{56}bc^3d^4$$

$$= \frac{11bd^4x}{8c^3}+\frac{24ibd^4x^2}{35c^2}-\frac{11bd^4x^3}{24c}-\frac{12}{35}ibd^4x^4+\frac{9}{40}bcd^4x^5+\frac{2}{21}ibc^2d^4x^6-\frac{1}{56}bc^3d^4$$

Mathematica [A] time = 0.155058, size = 290, normalized size = 1.22

$$\frac{1}{8}ac^4d^4x^8-\frac{4}{7}iac^3d^4x^7-ac^2d^4x^6+\frac{4}{5}iacd^4x^5+\frac{1}{4}ad^4x^4-\frac{1}{56}bc^3d^4x^7+\frac{2}{21}ibc^2d^4x^6+\frac{24ibd^4x^2}{35c^2}-\frac{24ibd^4\log\left(c^2x^2+1\right)}{35c^4}+$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + I*c*d*x)^4*(a + b*ArcTan[c*x]),x]
```

```
[Out] (11*b*d^4*x)/(8*c^3) + (((24*I)/35)*b*d^4*x^2)/c^2 - (11*b*d^4*x^3)/(24*c)
+ (a*d^4*x^4)/4 - ((12*I)/35)*b*d^4*x^4 + ((4*I)/5)*a*c*d^4*x^5 + (9*b*c*d^
4*x^5)/40 - a*c^2*d^4*x^6 + ((2*I)/21)*b*c^2*d^4*x^6 - ((4*I)/7)*a*c^3*d^4*
x^7 - (b*c^3*d^4*x^7)/56 + (a*c^4*d^4*x^8)/8 - (11*b*d^4*ArcTan[c*x])/(8*c^
4) + (b*d^4*x^4*ArcTan[c*x])/4 + ((4*I)/5)*b*c*d^4*x^5*ArcTan[c*x] - b*c^2*
d^4*x^6*ArcTan[c*x] - ((4*I)/7)*b*c^3*d^4*x^7*ArcTan[c*x] + (b*c^4*d^4*x^8*
ArcTan[c*x])/8 - (((24*I)/35)*b*d^4*Log[1 + c^2*x^2])/c^4
```

Maple [A] time = 0.029, size = 249, normalized size = 1.1

$$\frac{c^4d^4ax^8}{8}-\frac{\frac{24i}{35}bd^4\ln\left(c^2x^2+1\right)}{c^4}-c^2d^4ax^6-\frac{12i}{35}bd^4x^4+\frac{d^4ax^4}{4}+\frac{c^4d^4b\arctan(cx)x^8}{8}+\frac{2i}{21}bc^2d^4x^6-c^2d^4b\arctan(cx)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d+I*c*d*x)^4*(a+b*arctan(c*x)),x)
```

```
[Out] 1/8*c^4*d^4*a*x^8-24/35*I*b*d^4*ln(c^2*x^2+1)/c^4-c^2*d^4*a*x^6-12/35*I*b*d
^4*x^4+1/4*d^4*a*x^4+1/8*c^4*d^4*b*arctan(c*x)*x^8+2/21*I*b*c^2*d^4*x^6-c^2
*d^4*b*arctan(c*x)*x^6-4/7*I*c^3*d^4*b*arctan(c*x)*x^7+1/4*d^4*b*arctan(c*x
)*x^4+11/8*b*d^4*x/c^3-1/56*b*c^3*d^4*x^7+4/5*I*c*d^4*a*x^5+9/40*b*c*d^4*x^
5+4/5*I*c*d^4*b*arctan(c*x)*x^5-11/24*b*d^4*x^3/c+24/35*I*b*d^4*x^2/c^2-4/7
*I*c^3*d^4*a*x^7-11/8*b*d^4*arctan(c*x)/c^4
```

Maxima [A] time = 1.51219, size = 455, normalized size = 1.91

$$\frac{1}{8}ac^4d^4x^8 - \frac{4}{7}i\,ac^3d^4x^7 - ac^2d^4x^6 + \frac{4}{5}i\,acd^4x^5 + \frac{1}{840}\left(105\,x^8\arctan\left(cx\right) - c\left(\frac{15\,c^6x^7 - 21\,c^4x^5 + 35\,c^2x^3 - 105\,x}{c^8} + \frac{105}{\ldots}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d+I*c*d*x)^4*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] 1/8*a*c^4*d^4*x^8 - 4/7*I*a*c^3*d^4*x^7 - a*c^2*d^4*x^6 + 4/5*I*a*c*d^4*x^5
 + 1/840*(105*x^8*arctan(c*x) - c*((15*c^6*x^7 - 21*c^4*x^5 + 35*c^2*x^3 -
105*x)/c^8 + 105*arctan(c*x)/c^9))*b*c^4*d^4 - 1/21*I*(12*x^7*arctan(c*x) -
 c*((2*c^4*x^6 - 3*c^2*x^4 + 6*x^2)/c^6 - 6*log(c^2*x^2 + 1)/c^8))*b*c^3*d^
4 + 1/4*a*d^4*x^4 - 1/15*(15*x^6*arctan(c*x) - c*((3*c^4*x^5 - 5*c^2*x^3 +
15*x)/c^6 - 15*arctan(c*x)/c^7))*b*c^2*d^4 + 1/5*I*(4*x^5*arctan(c*x) - c*(
(c^2*x^4 - 2*x^2)/c^4 + 2*log(c^2*x^2 + 1)/c^6))*b*c*d^4 + 1/12*(3*x^4*arct
an(c*x) - c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5))*b*d^4
```

Fricas [A] time = 3.39207, size = 563, normalized size = 2.37

$$\frac{210\,ac^8d^4x^8 + (-960i\,a - 30\,b)c^7d^4x^7 - 80\,(21\,a - 2i\,b)c^6d^4x^6 + (1344i\,a + 378\,b)c^5d^4x^5 + 12\,(35\,a - 48i\,b)c^4d^4x^4 - 770\ldots}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d+I*c*d*x)^4*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] 1/1680*(210*a*c^8*d^4*x^8 + (-960*I*a - 30*b)*c^7*d^4*x^7 - 80*(21*a - 2*I*
b)*c^6*d^4*x^6 + (1344*I*a + 378*b)*c^5*d^4*x^5 + 12*(35*a - 48*I*b)*c^4*d^
```

```
4*x^4 - 770*b*c^3*d^4*x^3 + 1152*I*b*c^2*d^4*x^2 + 2310*b*c*d^4*x - 2307*I*
b*d^4*log((c*x + I)/c) + 3*I*b*d^4*log((c*x - I)/c) + (105*I*b*c^8*d^4*x^8
+ 480*b*c^7*d^4*x^7 - 840*I*b*c^6*d^4*x^6 - 672*b*c^5*d^4*x^5 + 210*I*b*c^4
*d^4*x^4)*log(-(c*x + I)/(c*x - I)))/c^4
```

Sympy [A] time = 4.11981, size = 355, normalized size = 1.49

$$\frac{ac^4d^4x^8}{8} - \frac{11bd^4x^3}{24c} + \frac{24ibd^4x^2}{35c^2} + \frac{11bd^4x}{8c^3} + \frac{ibd^4\log\left(x-\frac{i}{c}\right)}{560c^4} - \frac{769ibd^4\log\left(x+\frac{i}{c}\right)}{560c^4} + x^7\left(-\frac{4iac^3d^4}{7} - \frac{bc^3d^4}{56}\right) + x^6\left(-ac^2d\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(d+I*c*d*x)**4*(a+b*atan(c*x)),x)
```

```
[Out] a*c**4*d**4*x**8/8 - 11*b*d**4*x**3/(24*c) + 24*I*b*d**4*x**2/(35*c**2) + 1
1*b*d**4*x/(8*c**3) + I*b*d**4*log(x - I/c)/(560*c**4) - 769*I*b*d**4*log(x
 + I/c)/(560*c**4) + x**7*(-4*I*a*c**3*d**4/7 - b*c**3*d**4/56) + x**6*(-a*
c**2*d**4 + 2*I*b*c**2*d**4/21) + x**5*(4*I*a*c*d**4/5 + 9*b*c*d**4/40) + x
**4*(a*d**4/4 - 12*I*b*d**4/35) + (-I*b*c**4*d**4*x**8/16 - 2*b*c**3*d**4*x
**7/7 + I*b*c**2*d**4*x**6/2 + 2*b*c*d**4*x**5/5 - I*b*d**4*x**4/8)*log(I*c
*x + 1) + (I*b*c**4*d**4*x**8/16 + 2*b*c**3*d**4*x**7/7 - I*b*c**2*d**4*x**
6/2 - 2*b*c*d**4*x**5/5 + I*b*d**4*x**4/8)*log(-I*c*x + 1)
```

Giac [A] time = 1.16692, size = 354, normalized size = 1.49

$$\frac{210\,bc^8d^4x^8\arctan\left(cx\right) + 210\,ac^8d^4x^8 - 960\,bc^7d^4ix^7\arctan\left(cx\right) - 960\,ac^7d^4ix^7 - 30\,bc^7d^4x^7 + 160\,bc^6d^4ix^6 - 1680\,bc^6d}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d+I*c*d*x)^4*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] 1/1680*(210*b*c^8*d^4*x^8*arctan(c*x) + 210*a*c^8*d^4*x^8 - 960*b*c^7*d^4*i
*x^7*arctan(c*x) - 960*a*c^7*d^4*i*x^7 - 30*b*c^7*d^4*x^7 + 160*b*c^6*d^4*i
*x^6 - 1680*b*c^6*d^4*x^6*arctan(c*x) - 1680*a*c^6*d^4*x^6 + 1344*b*c^5*d^4
*i*x^5*arctan(c*x) + 1344*a*c^5*d^4*i*x^5 + 378*b*c^5*d^4*x^5 - 576*b*c^4*d
^4*i*x^4 + 420*b*c^4*d^4*x^4*arctan(c*x) + 420*a*c^4*d^4*x^4 - 770*b*c^3*d^
4*x^3 + 1152*b*c^2*d^4*i*x^2 + 2310*b*c*d^4*x + 3*b*d^4*i*log(c*i*x + 1) -
2307*b*d^4*i*log(-c*i*x + 1))/c^4
```

3.32 $\int x^2(d+icdx)^4\left(a+b\tan^{-1}(cx)\right)dx$

Optimal. Leaf size=193

$$\frac{id^4(1+icx)^7\left(a+b\tan^{-1}(cx)\right)}{7c^3}-\frac{id^4(1+icx)^6\left(a+b\tan^{-1}(cx)\right)}{3c^3}+\frac{id^4(1+icx)^5\left(a+b\tan^{-1}(cx)\right)}{5c^3}-\frac{1}{42}bc^3d^4x^6+\frac{2}{15}ibc$$

```
[Out] (((5*I)/3)*b*d^4*x)/c^2 - (88*b*d^4*x^2)/(105*c) - ((5*I)/9)*b*d^4*x^3 + (4
7*b*c*d^4*x^4)/140 + ((2*I)/15)*b*c^2*d^4*x^5 - (b*c^3*d^4*x^6)/42 + ((I/5)
*d^4*(1 + I*c*x)^5*(a + b*ArcTan[c*x]))/c^3 - ((I/3)*d^4*(1 + I*c*x)^6*(a +
 b*ArcTan[c*x]))/c^3 + ((I/7)*d^4*(1 + I*c*x)^7*(a + b*ArcTan[c*x]))/c^3 +
(176*b*d^4*Log[I + c*x])/(105*c^3)
```

Rubi [A] time = 0.166718, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {43, 4872, 12, 893}

$$\frac{id^4(1+icx)^7\left(a+b\tan^{-1}(cx)\right)}{7c^3}-\frac{id^4(1+icx)^6\left(a+b\tan^{-1}(cx)\right)}{3c^3}+\frac{id^4(1+icx)^5\left(a+b\tan^{-1}(cx)\right)}{5c^3}-\frac{1}{42}bc^3d^4x^6+\frac{2}{15}ibc$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + I*c*d*x)^4*(a + b*ArcTan[c*x]),x]
```


```
[Out] (((5*I)/3)*b*d^4*x)/c^2 - (88*b*d^4*x^2)/(105*c) - ((5*I)/9)*b*d^4*x^3 + (4
7*b*c*d^4*x^4)/140 + ((2*I)/15)*b*c^2*d^4*x^5 - (b*c^3*d^4*x^6)/42 + ((I/5)
*d^4*(1 + I*c*x)^5*(a + b*ArcTan[c*x]))/c^3 - ((I/3)*d^4*(1 + I*c*x)^6*(a +
 b*ArcTan[c*x]))/c^3 + ((I/7)*d^4*(1 + I*c*x)^7*(a + b*ArcTan[c*x]))/c^3 +
(176*b*d^4*Log[I + c*x])/(105*c^3)
```


Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 4872

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
+ b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2), x
```

```
], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2*m
] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rubi steps

$$\int x^2(d+icdx)^4\left(a+b\tan^{-1}(cx)\right)dx = \frac{id^4(1+icx)^5\left(a+b\tan^{-1}(cx)\right)}{5c^3} - \frac{id^4(1+icx)^6\left(a+b\tan^{-1}(cx)\right)}{3c^3} + \frac{id^4(1+icx)^7\left(\right.}{}$$

$$= \frac{id^4(1+icx)^5\left(a+b\tan^{-1}(cx)\right)}{5c^3} - \frac{id^4(1+icx)^6\left(a+b\tan^{-1}(cx)\right)}{3c^3} + \frac{id^4(1+icx)^7\left(\right.}{}$$

$$= \frac{id^4(1+icx)^5\left(a+b\tan^{-1}(cx)\right)}{5c^3} - \frac{id^4(1+icx)^6\left(a+b\tan^{-1}(cx)\right)}{3c^3} + \frac{id^4(1+icx)^7\left(\right.}{}$$

$$= \frac{5ibd^4x}{3c^2} - \frac{88bd^4x^2}{105c} - \frac{5}{9}ibd^4x^3 + \frac{47}{140}bcd^4x^4 + \frac{2}{15}ibc^2d^4x^5 - \frac{1}{42}bc^3d^4x^6 + \frac{id^4(1+}{}$$

Mathematica [A] time = 0.120197, size = 276, normalized size = 1.43

$$\frac{1}{7}ac^4d^4x^7 - \frac{2}{3}iac^3d^4x^6 - \frac{6}{5}ac^2d^4x^5 + iacd^4x^4 + \frac{1}{3}ad^4x^3 - \frac{1}{42}bc^3d^4x^6 + \frac{2}{15}ibc^2d^4x^5 + \frac{88bd^4\log\left(c^2x^2+1\right)}{105c^3} + \frac{1}{7}bc^4d^4x^7\,\mathrm{ta}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + I*c*d*x)^4*(a + b*ArcTan[c*x]),x]
```

```
[Out] (((5*I)/3)*b*d^4*x)/c^2 - (88*b*d^4*x^2)/(105*c) + (a*d^4*x^3)/3 - ((5*I)/9
)*b*d^4*x^3 + I*a*c*d^4*x^4 + (47*b*c*d^4*x^4)/140 - (6*a*c^2*d^4*x^5)/5 +
```

```
((2*I)/15)*b*c^2*d^4*x^5 - ((2*I)/3)*a*c^3*d^4*x^6 - (b*c^3*d^4*x^6)/42 + (
a*c^4*d^4*x^7)/7 - (((5*I)/3)*b*d^4*ArcTan[c*x])/c^3 + (b*d^4*x^3*ArcTan[c*
x])/3 + I*b*c*d^4*x^4*ArcTan[c*x] - (6*b*c^2*d^4*x^5*ArcTan[c*x])/5 - ((2*I
)/3)*b*c^3*d^4*x^6*ArcTan[c*x] + (b*c^4*d^4*x^7*ArcTan[c*x])/7 + (88*b*d^4*
Log[1 + c^2*x^2])/(105*c^3)
```

Maple [A] time = 0.027, size = 237, normalized size = 1.2

$$\frac{c^4d^4ax^7}{7}+\frac{2\,i}{15}bc^2d^4x^5-\frac{6\,c^2d^4ax^5}{5}+icd^4b\arctan\left(cx\right)x^4+\frac{d^4ax^3}{3}+\frac{c^4d^4b\arctan\left(cx\right)x^7}{7}-\frac{2\,i}{3}c^3d^4b\arctan\left(cx\right)x^6-\frac{6}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d+I*c*d*x)^4*(a+b*arctan(c*x)),x)
```

```
[Out] 1/7*c^4*d^4*a*x^7+2/15*I*b*c^2*d^4*x^5-6/5*c^2*d^4*a*x^5+I*c*d^4*b*arctan(c
*x)*x^4+1/3*d^4*a*x^3+1/7*c^4*d^4*b*arctan(c*x)*x^7-2/3*I*c^3*d^4*b*arctan(
c*x)*x^6-6/5*c^2*d^4*b*arctan(c*x)*x^5-5/3*I/c^3*d^4*b*arctan(c*x)+1/3*d^4*
b*arctan(c*x)*x^3+5/3*I*b*d^4*x/c^2-1/42*b*c^3*d^4*x^6-2/3*I*c^3*d^4*a*x^6+
47/140*b*c*d^4*x^4+I*c*d^4*a*x^4-88/105*b*d^4*x^2/c+88/105/c^3*d^4*b*ln(c^2
*x^2+1)-5/9*I*b*d^4*x^3
```

Maxima [B] time = 1.48581, size = 429, normalized size = 2.22

$$\frac{1}{7}\,ac^4d^4x^7-\frac{2}{3}i\,ac^3d^4x^6-\frac{6}{5}\,ac^2d^4x^5+\frac{1}{84}\left(12\,x^7\arctan\left(cx\right)-c\left(\frac{2\,c^4x^6-3\,c^2x^4+6\,x^2}{c^6}-\frac{6\,\log\left(c^2x^2+1\right)}{c^8}\right)\right)bc^4d^4+i\,ac$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d+I*c*d*x)^4*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] 1/7*a*c^4*d^4*x^7 - 2/3*I*a*c^3*d^4*x^6 - 6/5*a*c^2*d^4*x^5 + 1/84*(12*x^7*
arctan(c*x) - c*((2*c^4*x^6 - 3*c^2*x^4 + 6*x^2)/c^6 - 6*log(c^2*x^2 + 1)/c
^8))*b*c^4*d^4 + I*a*c*d^4*x^4 - 2/45*I*(15*x^6*arctan(c*x) - c*((3*c^4*x^5
 - 5*c^2*x^3 + 15*x)/c^6 - 15*arctan(c*x)/c^7))*b*c^3*d^4 - 3/10*(4*x^5*arc
tan(c*x) - c*((c^2*x^4 - 2*x^2)/c^4 + 2*log(c^2*x^2 + 1)/c^6))*b*c^2*d^4 +
1/3*a*d^4*x^3 + 1/3*I*(3*x^4*arctan(c*x) - c*((c^2*x^3 - 3*x)/c^4 + 3*arcta
n(c*x)/c^5))*b*c*d^4 + 1/6*(2*x^3*arctan(c*x) - c*(x^2/c^2 - log(c^2*x^2 +
```

```
1)/c^4))*b*d^4
```

Fricas [A] time = 3.31694, size = 525, normalized size = 2.72

$$\frac{180\,ac^7d^4x^7 + (-840i\,a - 30\,b)c^6d^4x^6 - 168\,(9\,a - i\,b)c^5d^4x^5 + (1260i\,a + 423\,b)c^4d^4x^4 + 140\,(3\,a - 5i\,b)c^3d^4x^3 - 1056\,bc^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d+I*c*d*x)^4*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] 1/1260*(180*a*c^7*d^4*x^7 + (-840*I*a - 30*b)*c^6*d^4*x^6 - 168*(9*a - I*b)
*c^5*d^4*x^5 + (1260*I*a + 423*b)*c^4*d^4*x^4 + 140*(3*a - 5*I*b)*c^3*d^4*x
^3 - 1056*b*c^2*d^4*x^2 + 2100*I*b*c*d^4*x + 2106*b*d^4*log((c*x + I)/c) +
6*b*d^4*log((c*x - I)/c) + (90*I*b*c^7*d^4*x^7 + 420*b*c^6*d^4*x^6 - 756*I*
b*c^5*d^4*x^5 - 630*b*c^4*d^4*x^4 + 210*I*b*c^3*d^4*x^3)*log(-(c*x + I)/(c*
x - I)))/c^3
```

Sympy [A] time = 4.20341, size = 325, normalized size = 1.68

$$\frac{ac^4d^4x^7}{7} - \frac{88bd^4x^2}{105c} + \frac{5ibd^4x}{3c^2} + \frac{bd^4\left(\frac{\log\left(x-\frac{i}{c}\right)}{210} + \frac{117\log\left(x+\frac{i}{c}\right)}{70}\right)}{c^3} + x^6\left(-\frac{2iac^3d^4}{3} - \frac{bc^3d^4}{42}\right) + x^5\left(-\frac{6ac^2d^4}{5} + \frac{2ibc^2d^4}{15}\right) + x^4\left(i\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d+I*c*d*x)**4*(a+b*atan(c*x)),x)
```

```
[Out] a*c**4*d**4*x**7/7 - 88*b*d**4*x**2/(105*c) + 5*I*b*d**4*x/(3*c**2) + b*d**
4*(log(x - I/c)/210 + 117*log(x + I/c)/70)/c**3 + x**6*(-2*I*a*c**3*d**4/3
- b*c**3*d**4/42) + x**5*(-6*a*c**2*d**4/5 + 2*I*b*c**2*d**4/15) + x**4*(I*
a*c*d**4 + 47*b*c*d**4/140) + x**3*(a*d**4/3 - 5*I*b*d**4/9) + (-I*b*c**4*d
**4*x**7/14 - b*c**3*d**4*x**6/3 + 3*I*b*c**2*d**4*x**5/5 + b*c*d**4*x**4/2
 - I*b*d**4*x**3/6)*log(I*c*x + 1) + (I*b*c**4*d**4*x**7/14 + b*c**3*d**4*x
**6/3 - 3*I*b*c**2*d**4*x**5/5 - b*c*d**4*x**4/2 + I*b*d**4*x**3/6)*log(-I*
c*x + 1)
```

Giac [A] time = 1.20647, size = 333, normalized size = 1.73

$$\frac{180\,bc^7d^4x^7\arctan(cx) + 180\,ac^7d^4x^7 - 840\,bc^6d^4ix^6\arctan(cx) - 840\,ac^6d^4ix^6 - 30\,bc^6d^4x^6 + 168\,bc^5d^4ix^5 - 1512\,bc}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d+I*c*d*x)^4*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] 1/1260*(180*b*c^7*d^4*x^7*arctan(c*x) + 180*a*c^7*d^4*x^7 - 840*b*c^6*d^4*i
*x^6*arctan(c*x) - 840*a*c^6*d^4*i*x^6 - 30*b*c^6*d^4*x^6 + 168*b*c^5*d^4*i
*x^5 - 1512*b*c^5*d^4*x^5*arctan(c*x) - 1512*a*c^5*d^4*x^5 + 1260*b*c^4*d^4
*i*x^4*arctan(c*x) + 1260*a*c^4*d^4*i*x^4 + 423*b*c^4*d^4*x^4 - 700*b*c^3*d
^4*i*x^3 + 420*b*c^3*d^4*x^3*arctan(c*x) + 420*a*c^3*d^4*x^3 - 1056*b*c^2*d
^4*x^2 + 2100*b*c*d^4*i*x + 2106*b*d^4*log(c*x + i) + 6*b*d^4*log(c*x - i))
/c^3
```

3.33 $\int x(d+icdx)^4\left(a+b\tan^{-1}(cx)\right)dx$

Optimal. Leaf size=178

$$-\frac{d^4(1+icx)^6\left(a+b\tan^{-1}(cx)\right)}{6c^2}+\frac{d^4(1+icx)^5\left(a+b\tan^{-1}(cx)\right)}{5c^2}+\frac{bd^4(-cx+i)^5}{30c^2}+\frac{ibd^4(-cx+i)^4}{30c^2}-\frac{4bd^4(-cx+i)^3}{45c^2}-\frac{4i}{}$$

```
[Out] (-16*b*d^4*x)/(15*c) - (((4*I)/15)*b*d^4*(I - c*x)^2)/c^2 - (4*b*d^4*(I - c
*x)^3)/(45*c^2) + ((I/30)*b*d^4*(I - c*x)^4)/c^2 + (b*d^4*(I - c*x)^5)/(30*
c^2) + (d^4*(1 + I*c*x)^5*(a + b*ArcTan[c*x]))/(5*c^2) - (d^4*(1 + I*c*x)^6
*(a + b*ArcTan[c*x]))/(6*c^2) + (((32*I)/15)*b*d^4*Log[I + c*x])/c^2
```

Rubi [A] time = 0.11255, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {43, 4872, 12, 77}

$$-\frac{d^4(1+icx)^6\left(a+b\tan^{-1}(cx)\right)}{6c^2}+\frac{d^4(1+icx)^5\left(a+b\tan^{-1}(cx)\right)}{5c^2}+\frac{bd^4(-cx+i)^5}{30c^2}+\frac{ibd^4(-cx+i)^4}{30c^2}-\frac{4bd^4(-cx+i)^3}{45c^2}-\frac{4i}{}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + I*c*d*x)^4*(a + b*ArcTan[c*x]),x]
```


```
[Out] (-16*b*d^4*x)/(15*c) - (((4*I)/15)*b*d^4*(I - c*x)^2)/c^2 - (4*b*d^4*(I - c
*x)^3)/(45*c^2) + ((I/30)*b*d^4*(I - c*x)^4)/c^2 + (b*d^4*(I - c*x)^5)/(30*
c^2) + (d^4*(1 + I*c*x)^5*(a + b*ArcTan[c*x]))/(5*c^2) - (d^4*(1 + I*c*x)^6
*(a + b*ArcTan[c*x]))/(6*c^2) + (((32*I)/15)*b*d^4*Log[I + c*x])/c^2
```


Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 4872

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
+ b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2), x
], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2*m
```

```
] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int x(d+icdx)^4\left(a+b\tan^{-1}(cx)\right)dx=\frac{d^4(1+icx)^5\left(a+b\tan^{-1}(cx)\right)}{5c^2}-\frac{d^4(1+icx)^6\left(a+b\tan^{-1}(cx)\right)}{6c^2}-(bc)\int\frac{d^4(i-c}{30c}$$

$$=\frac{d^4(1+icx)^5\left(a+b\tan^{-1}(cx)\right)}{5c^2}-\frac{d^4(1+icx)^6\left(a+b\tan^{-1}(cx)\right)}{6c^2}-\frac{\left(bd^4\right)\int\frac{(i-cx)^4(}{i+c}}{30c}$$

$$=\frac{d^4(1+icx)^5\left(a+b\tan^{-1}(cx)\right)}{5c^2}-\frac{d^4(1+icx)^6\left(a+b\tan^{-1}(cx)\right)}{6c^2}-\frac{\left(bd^4\right)\int\left(32+\right.}{}$$

$$=-\frac{16bd^4x}{15c}-\frac{4ibd^4(i-cx)^2}{15c^2}-\frac{4bd^4(i-cx)^3}{45c^2}+\frac{ibd^4(i-cx)^4}{30c^2}+\frac{bd^4(i-cx)^5}{30c^2}+\frac{d^4(1}{}$$

Mathematica [A] time = 0.101014, size = 264, normalized size = 1.48

$$\frac{1}{6}ac^4d^4x^6-\frac{4}{5}iac^3d^4x^5-\frac{3}{2}ac^2d^4x^4+\frac{4}{3}iacd^4x^3+\frac{1}{2}ad^4x^2-\frac{1}{30}bc^3d^4x^5+\frac{1}{5}ibc^2d^4x^4+\frac{16ibd^4\log\left(c^2x^2+1\right)}{15c^2}+\frac{1}{6}bc^4d^4x^6$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + I*c*d*x)^4*(a + b*ArcTan[c*x]),x]
```

```
[Out] (-13*b*d^4*x)/(6*c) + (a*d^4*x^2)/2 - ((16*I)/15)*b*d^4*x^2 + ((4*I)/3)*a*c
*d^4*x^3 + (5*b*c*d^4*x^3)/9 - (3*a*c^2*d^4*x^4)/2 + (I/5)*b*c^2*d^4*x^4 -
((4*I)/5)*a*c^3*d^4*x^5 - (b*c^3*d^4*x^5)/30 + (a*c^4*d^4*x^6)/6 + (13*b*d^
```

```
4*ArcTan[c*x])/(6*c^2) + (b*d^4*x^2*ArcTan[c*x])/2 + ((4*I)/3)*b*c*d^4*x^3*
ArcTan[c*x] - (3*b*c^2*d^4*x^4*ArcTan[c*x])/2 - ((4*I)/5)*b*c^3*d^4*x^5*Arc
Tan[c*x] + (b*c^4*d^4*x^6*ArcTan[c*x])/6 + (((16*I)/15)*b*d^4*Log[1 + c^2*x
^2])/c^2
```

Maple [A] time = 0.028, size = 224, normalized size = 1.3

$$\frac{c^4d^4ax^6}{6}+\frac{\frac{16i}{15}d^4b\ln\left(c^2x^2+1\right)}{c^2}-\frac{3c^2d^4ax^4}{2}+\frac{i}{5}c^2d^4bx^4+\frac{d^4ax^2}{2}+\frac{c^4d^4b\arctan(cx)x^6}{6}-\frac{4i}{5}c^3d^4ax^5-\frac{3c^2d^4b\arctan(}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d+I*c*d*x)^4*(a+b*arctan(c*x)),x)
```

```
[Out] 1/6*c^4*d^4*a*x^6+16/15*I/c^2*d^4*b*ln(c^2*x^2+1)-3/2*c^2*d^4*a*x^4+1/5*I*c
^2*d^4*b*x^4+1/2*d^4*a*x^2+1/6*c^4*d^4*b*arctan(c*x)*x^6-4/5*I*c^3*d^4*a*x^
5-3/2*c^2*d^4*b*arctan(c*x)*x^4-16/15*I*d^4*b*x^2+1/2*d^4*b*arctan(c*x)*x^2
-13/6*b*d^4*x/c-1/30*c^3*d^4*b*x^5-4/5*I*c^3*d^4*b*arctan(c*x)*x^5+5/9*c*d^
4*b*x^3+4/3*I*c*d^4*b*arctan(c*x)*x^3+4/3*I*c*d^4*a*x^3+13/6/c^2*d^4*b*arct
an(c*x)
```

Maxima [B] time = 1.50462, size = 392, normalized size = 2.2

$$\frac{1}{6}ac^4d^4x^6-\frac{4}{5}i\,ac^3d^4x^5-\frac{3}{2}ac^2d^4x^4+\frac{1}{90}\left(15\,x^6\arctan(cx)-c\left(\frac{3\,c^4x^5-5\,c^2x^3+15\,x}{c^6}-\frac{15\arctan(cx)}{c^7}\right)\right)bc^4d^4-\frac{1}{5}i\left(4\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d+I*c*d*x)^4*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] 1/6*a*c^4*d^4*x^6 - 4/5*I*a*c^3*d^4*x^5 - 3/2*a*c^2*d^4*x^4 + 1/90*(15*x^6*
arctan(c*x) - c*((3*c^4*x^5 - 5*c^2*x^3 + 15*x)/c^6 - 15*arctan(c*x)/c^7))*
b*c^4*d^4 - 1/5*I*(4*x^5*arctan(c*x) - c*((c^2*x^4 - 2*x^2)/c^4 + 2*log(c^2
*x^2 + 1)/c^6))*b*c^3*d^4 + 4/3*I*a*c*d^4*x^3 - 1/2*(3*x^4*arctan(c*x) - c*
((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5))*b*c^2*d^4 + 2/3*I*(2*x^3*arctan(
c*x) - c*(x^2/c^2 - log(c^2*x^2 + 1)/c^4))*b*c*d^4 + 1/2*a*d^4*x^2 + 1/2*(x
^2*arctan(c*x) - c*(x/c^2 - arctan(c*x)/c^3))*b*d^4
```

Fricas [A] time = 2.83865, size = 491, normalized size = 2.76

$$\frac{30\,ac^6d^4x^6 + (-144i\,a - 6\,b)c^5d^4x^5 - 18\,(15\,a - 2i\,b)c^4d^4x^4 + (240i\,a + 100\,b)c^3d^4x^3 + 6\,(15\,a - 32i\,b)c^2d^4x^2 - 390\,bcd^4x}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d+I*c*d*x)^4*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] 1/180*(30*a*c^6*d^4*x^6 + (-144*I*a - 6*b)*c^5*d^4*x^5 - 18*(15*a - 2*I*b)*
c^4*d^4*x^4 + (240*I*a + 100*b)*c^3*d^4*x^3 + 6*(15*a - 32*I*b)*c^2*d^4*x^2
 - 390*b*c*d^4*x + 387*I*b*d^4*log((c*x + I)/c) - 3*I*b*d^4*log((c*x - I)/c
) + (15*I*b*c^6*d^4*x^6 + 72*b*c^5*d^4*x^5 - 135*I*b*c^4*d^4*x^4 - 120*b*c^
3*d^4*x^3 + 45*I*b*c^2*d^4*x^2)*log(-(c*x + I)/(c*x - I)))/c^2
```

Sympy [B] time = 4.05707, size = 328, normalized size = 1.84

$$\frac{ac^4d^4x^6}{6} - \frac{13bd^4x}{6c} - \frac{ibd^4\log\left(x - \frac{i}{c}\right)}{60c^2} + \frac{43ibd^4\log\left(x + \frac{i}{c}\right)}{20c^2} + x^5\left(-\frac{4iac^3d^4}{5} - \frac{bc^3d^4}{30}\right) + x^4\left(-\frac{3ac^2d^4}{2} + \frac{ibc^2d^4}{5}\right) + x^3\left(\frac{4i}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d+I*c*d*x)**4*(a+b*atan(c*x)),x)
```

```
[Out] a*c**4*d**4*x**6/6 - 13*b*d**4*x/(6*c) - I*b*d**4*log(x - I/c)/(60*c**2) +
43*I*b*d**4*log(x + I/c)/(20*c**2) + x**5*(-4*I*a*c**3*d**4/5 - b*c**3*d**4
/30) + x**4*(-3*a*c**2*d**4/2 + I*b*c**2*d**4/5) + x**3*(4*I*a*c*d**4/3 + 5
*b*c*d**4/9) + x**2*(a*d**4/2 - 16*I*b*d**4/15) + (-I*b*c**4*d**4*x**6/12 -
 2*b*c**3*d**4*x**5/5 + 3*I*b*c**2*d**4*x**4/4 + 2*b*c*d**4*x**3/3 - I*b*d*
*4*x**2/4)*log(I*c*x + 1) + (I*b*c**4*d**4*x**6/12 + 2*b*c**3*d**4*x**5/5 -
 3*I*b*c**2*d**4*x**4/4 - 2*b*c*d**4*x**3/3 + I*b*d**4*x**2/4)*log(-I*c*x +
 1)
```

Giac [A] time = 1.1696, size = 320, normalized size = 1.8

$$\frac{30\,bc^6d^4x^6\arctan(cx) + 30\,ac^6d^4x^6 - 144\,bc^5d^4ix^5\arctan(cx) - 144\,ac^5d^4ix^5 - 6\,bc^5d^4x^5 + 36\,bc^4d^4ix^4 - 270\,bc^4d^4x^4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d+I*c*d*x)^4*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] 1/180*(30*b*c^6*d^4*x^6*arctan(c*x) + 30*a*c^6*d^4*x^6 - 144*b*c^5*d^4*i*x^
5*arctan(c*x) - 144*a*c^5*d^4*i*x^5 - 6*b*c^5*d^4*x^5 + 36*b*c^4*d^4*i*x^4
- 270*b*c^4*d^4*x^4*arctan(c*x) - 270*a*c^4*d^4*x^4 + 240*b*c^3*d^4*i*x^3*a
rctan(c*x) + 240*a*c^3*d^4*i*x^3 + 100*b*c^3*d^4*x^3 - 192*b*c^2*d^4*i*x^2
+ 90*b*c^2*d^4*x^2*arctan(c*x) + 90*a*c^2*d^4*x^2 - 390*b*c*d^4*x + 387*b*d
^4*i*log(c*i*x - 1) - 3*b*d^4*i*log(-c*i*x - 1))/c^2
```

3.34 $\int (d+icdx)^4 \left(a+b\tan^{-1}(cx)\right) dx$

Optimal. Leaf size=125

$$-\frac{id^4(1+icx)^5\left(a+b\tan^{-1}(cx)\right)}{5c}-\frac{bd^4(1+icx)^4}{20c}-\frac{2bd^4(1+icx)^3}{15c}-\frac{2bd^4(1+icx)^2}{5c}-\frac{16bd^4\log(1-icx)}{5c}-\frac{8}{5}ibd^4x$$

```
[Out] ((-8*I)/5)*b*d^4*x - (2*b*d^4*(1 + I*c*x)^2)/(5*c) - (2*b*d^4*(1 + I*c*x)^3
)/(15*c) - (b*d^4*(1 + I*c*x)^4)/(20*c) - ((I/5)*d^4*(1 + I*c*x)^5*(a + b*A
rcTan[c*x]))/c - (16*b*d^4*Log[1 - I*c*x])/(5*c)
```

Rubi [A] time = 0.063127, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {4862, 627, 43}

$$-\frac{id^4(1+icx)^5\left(a+b\tan^{-1}(cx)\right)}{5c}-\frac{bd^4(1+icx)^4}{20c}-\frac{2bd^4(1+icx)^3}{15c}-\frac{2bd^4(1+icx)^2}{5c}-\frac{16bd^4\log(1-icx)}{5c}-\frac{8}{5}ibd^4x$$

Antiderivative was successfully verified.

```
[In] Int[(d + I*c*d*x)^4*(a + b*ArcTan[c*x]),x]
```

```
[Out] ((-8*I)/5)*b*d^4*x - (2*b*d^4*(1 + I*c*x)^2)/(5*c) - (2*b*d^4*(1 + I*c*x)^3
)/(15*c) - (b*d^4*(1 + I*c*x)^4)/(20*c) - ((I/5)*d^4*(1 + I*c*x)^5*(a + b*A
rcTan[c*x]))/c - (16*b*d^4*Log[1 - I*c*x])/(5*c)
```

Rule 4862

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x]))/(e*(q + 1)), x] - Dist[(b*
c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (d+icdx)^4\left(a+b\tan^{-1}(cx)\right)dx = -\frac{id^4(1+icx)^5\left(a+b\tan^{-1}(cx)\right)}{5c}+\frac{(ib)\int\frac{(d+icdx)^5}{1+c^2x^2}dx}{5d}$$

$$= -\frac{id^4(1+icx)^5\left(a+b\tan^{-1}(cx)\right)}{5c}+\frac{(ib)\int\frac{(d+icdx)^4}{\frac{1}{d}-\frac{icx}{d}}dx}{5d}$$

$$= -\frac{id^4(1+icx)^5\left(a+b\tan^{-1}(cx)\right)}{5c}+\frac{(ib)\int\left(-8d^5+\frac{16d^4}{\frac{1}{d}-\frac{icx}{d}}-4d^4(d+icdx)-2d^3(d+i\right.}{5d}$$

$$= -\frac{8}{5}ibd^4x-\frac{2bd^4(1+icx)^2}{5c}-\frac{2bd^4(1+icx)^3}{15c}-\frac{bd^4(1+icx)^4}{20c}-\frac{id^4(1+icx)^5\left(a+b\,\mathrm{ta}\right.}{5c}$$

Mathematica [A] time = 0.0298645, size = 77, normalized size = 0.62

$$\frac{d^4\left(12(cx-i)^5\left(a+b\tan^{-1}(cx)\right)-b\left(3c^4x^4-20ic^3x^3-66c^2x^2+180icx+192\log(cx+i)+35\right)\right)}{60c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + I*c*d*x)^4*(a + b*ArcTan[c*x]),x]
```

```
[Out] (d^4*(12*(-I + c*x)^5*(a + b*ArcTan[c*x]) - b*(35 + (180*I)*c*x - 66*c^2*x^
2 - (20*I)*c^3*x^3 + 3*c^4*x^4 + 192*Log[I + c*x])))/(60*c)
```

Maple [B] time = 0.029, size = 216, normalized size = 1.7

$$\frac{c^4x^5ad^4}{5}+2\,icd^4b\arctan(cx)\,x^2-2\,c^2x^3ad^4-\frac{\frac{i}{5}d^4a}{c}+xad^4-ic^3d^4b\arctan(cx)\,x^4+\frac{c^4d^4b\arctan(cx)\,x^5}{5}+2\,icx^2ad^4-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^4*(a+b*arctan(c*x)),x)
```

```
[Out] 1/5*c^4*x^5*a*d^4+2*I*c*d^4*b*arctan(c*x)*x^2-2*c^2*x^3*a*d^4-1/5*I/c*d^4*a
+x*a*d^4-I*c^3*d^4*b*arctan(c*x)*x^4+1/5*c^4*d^4*b*arctan(c*x)*x^5+2*I*c*x^
2*a*d^4-2*c^2*d^4*b*arctan(c*x)*x^3-3*I*d^4*b*x+d^4*b*x*arctan(c*x)-I*c^3*x
^4*a*d^4+1/3*I*c^2*d^4*b*x^3-1/20*c^3*d^4*b*x^4+3*I/c*d^4*b*arctan(c*x)+11/
10*c*d^4*b*x^2-8/5/c*d^4*b*ln(c^2*x^2+1)
```

Maxima [B] time = 1.49166, size = 356, normalized size = 2.85

$$\frac{1}{5}ac^4d^4x^5 - i\,ac^3d^4x^4 + \frac{1}{20}\left(4\,x^5\arctan(cx) - c\left(\frac{c^2x^4 - 2\,x^2}{c^4} + \frac{2\,\log\left(c^2x^2+1\right)}{c^6}\right)\right)bc^4d^4 - 2\,ac^2d^4x^3 - \frac{1}{3}i\left(3\,x^4\arctan(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^4*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] 1/5*a*c^4*d^4*x^5 - I*a*c^3*d^4*x^4 + 1/20*(4*x^5*arctan(c*x) - c*((c^2*x^4
 - 2*x^2)/c^4 + 2*log(c^2*x^2 + 1)/c^6))*b*c^4*d^4 - 2*a*c^2*d^4*x^3 - 1/3*
I*(3*x^4*arctan(c*x) - c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5))*b*c^3*d
^4 - (2*x^3*arctan(c*x) - c*(x^2/c^2 - log(c^2*x^2 + 1)/c^4))*b*c^2*d^4 + 2
*I*a*c*d^4*x^2 + 2*I*(x^2*arctan(c*x) - c*(x/c^2 - arctan(c*x)/c^3))*b*c*d^
4 + a*d^4*x + 1/2*(2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*b*d^4/c
```

Fricas [A] time = 2.53474, size = 435, normalized size = 3.48

$$\frac{12\,ac^5d^4x^5 + (-60i\,a - 3\,b)c^4d^4x^4 - 20\,(6\,a - i\,b)c^3d^4x^3 + (120i\,a + 66\,b)c^2d^4x^2 + 60\,(a - 3i\,b)cd^4x - 186\,bd^4\log\left(\frac{cx+i}{c}\right) -}{60\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^4*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] 1/60*(12*a*c^5*d^4*x^5 + (-60*I*a - 3*b)*c^4*d^4*x^4 - 20*(6*a - I*b)*c^3*d
^4*x^3 + (120*I*a + 66*b)*c^2*d^4*x^2 + 60*(a - 3*I*b)*c*d^4*x - 186*b*d^4*
log((c*x + I)/c) - 6*b*d^4*log((c*x - I)/c) + (6*I*b*c^5*d^4*x^5 + 30*b*c^4
*d^4*x^4 - 60*I*b*c^3*d^4*x^3 - 60*b*c^2*d^4*x^2 + 30*I*b*c*d^4*x)*log(-(c*
x + I)/(c*x - I)))/c
```

Sympy [B] time = 3.96163, size = 272, normalized size = 2.18

$$\frac{ac^4d^4x^5}{5} + \frac{bd^4\left(-\frac{\log\left(x-\frac{i}{c}\right)}{10} - \frac{31\log\left(x+\frac{i}{c}\right)}{10}\right)}{c} + x^4\left(-iac^3d^4 - \frac{bc^3d^4}{20}\right) + x^3\left(-2ac^2d^4 + \frac{ibc^2d^4}{3}\right) + x^2\left(2iacd^4 + \frac{11bcd^4}{10}\right) + x\left(ad\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**4*(a+b*atan(c*x)),x)
```

```
[Out] a*c**4*d**4*x**5/5 + b*d**4*(-log(x - I/c)/10 - 31*log(x + I/c)/10)/c + x**
4*(-I*a*c**3*d**4 - b*c**3*d**4/20) + x**3*(-2*a*c**2*d**4 + I*b*c**2*d**4/
3) + x**2*(2*I*a*c*d**4 + 11*b*c*d**4/10) + x*(a*d**4 - 3*I*b*d**4) + (-I*b
*c**4*d**4*x**5/10 - b*c**3*d**4*x**4/2 + I*b*c**2*d**4*x**3 + b*c*d**4*x**
2 - I*b*d**4*x/2)*log(I*c*x + 1) + (I*b*c**4*d**4*x**5/10 + b*c**3*d**4*x**
4/2 - I*b*c**2*d**4*x**3 - b*c*d**4*x**2 + I*b*d**4*x/2)*log(-I*c*x + 1)
```

Giac [B] time = 1.20288, size = 289, normalized size = 2.31

$$\frac{12\,bc^5d^4x^5\arctan(cx) + 12\,ac^5d^4x^5 - 60\,bc^4d^4ix^4\arctan(cx) - 60\,ac^4d^4ix^4 - 3\,bc^4d^4x^4 + 20\,bc^3d^4ix^3 - 120\,bc^3d^4x^3\,\mathrm{arct}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^4*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] 1/60*(12*b*c^5*d^4*x^5*arctan(c*x) + 12*a*c^5*d^4*x^5 - 60*b*c^4*d^4*i*x^4*
arctan(c*x) - 60*a*c^4*d^4*i*x^4 - 3*b*c^4*d^4*x^4 + 20*b*c^3*d^4*i*x^3 - 1
20*b*c^3*d^4*x^3*arctan(c*x) - 120*a*c^3*d^4*x^3 + 120*b*c^2*d^4*i*x^2*arct
an(c*x) + 120*a*c^2*d^4*i*x^2 + 66*b*c^2*d^4*x^2 - 180*b*c*d^4*i*x + 60*b*c
*d^4*x*arctan(c*x) + 60*a*c*d^4*x - 186*b*d^4*log(c*x + i) - 6*b*d^4*log(c*
x - i))/c
```

3.35 $\int \frac{(d+icdx)^4\left(a+b\tan^{-1}(cx)\right)}{x} dx$

Optimal. Leaf size=203

$$\frac{1}{2}ibd^4\text{PolyLog}(2,-icx)-\frac{1}{2}ibd^4\text{PolyLog}(2,icx)+\frac{1}{4}c^4d^4x^4\left(a+b\tan^{-1}(cx)\right)-\frac{4}{3}ic^3d^4x^3\left(a+b\tan^{-1}(cx)\right)-3c^2d^4x^2\left(a\right.$$

```
[Out] (4*I)*a*c*d^4*x + (13*b*c*d^4*x)/4 + ((2*I)/3)*b*c^2*d^4*x^2 - (b*c^3*d^4*x
^3)/12 - (13*b*d^4*ArcTan[c*x])/4 + (4*I)*b*c*d^4*x*ArcTan[c*x] - 3*c^2*d^4
*x^2*(a + b*ArcTan[c*x]) - ((4*I)/3)*c^3*d^4*x^3*(a + b*ArcTan[c*x]) + (c^4
*d^4*x^4*(a + b*ArcTan[c*x]))/4 + a*d^4*Log[x] - ((8*I)/3)*b*d^4*Log[1 + c^
2*x^2] + (I/2)*b*d^4*PolyLog[2, (-I)*c*x] - (I/2)*b*d^4*PolyLog[2, I*c*x]
```

Rubi [A] time = 0.210705, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {4876, 4846, 260, 4848, 2391, 4852, 321, 203, 266, 43, 302}

$$\frac{1}{2}ibd^4\text{PolyLog}(2,-icx)-\frac{1}{2}ibd^4\text{PolyLog}(2,icx)+\frac{1}{4}c^4d^4x^4\left(a+b\tan^{-1}(cx)\right)-\frac{4}{3}ic^3d^4x^3\left(a+b\tan^{-1}(cx)\right)-3c^2d^4x^2\left(a\right.$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)^4*(a + b*ArcTan[c*x]))/x,x]
```


```
[Out] (4*I)*a*c*d^4*x + (13*b*c*d^4*x)/4 + ((2*I)/3)*b*c^2*d^4*x^2 - (b*c^3*d^4*x
^3)/12 - (13*b*d^4*ArcTan[c*x])/4 + (4*I)*b*c*d^4*x*ArcTan[c*x] - 3*c^2*d^4
*x^2*(a + b*ArcTan[c*x]) - ((4*I)/3)*c^3*d^4*x^3*(a + b*ArcTan[c*x]) + (c^4
*d^4*x^4*(a + b*ArcTan[c*x]))/4 + a*d^4*Log[x] - ((8*I)/3)*b*d^4*Log[1 + c^
2*x^2] + (I/2)*b*d^4*PolyLog[2, (-I)*c*x] - (I/2)*b*d^4*PolyLog[2, I*c*x]
```


Rule 4876

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
```

```
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\int \frac{(d+icdx)^4\left(a+b\tan^{-1}(cx)\right)}{x}\,dx = \int \left(4icd^4\left(a+b\tan^{-1}(cx)\right)+\frac{d^4\left(a+b\tan^{-1}(cx)\right)}{x}-6c^2d^4x\left(a+b\tan^{-1}(cx)\right)-4\right.$$

$$= d^4\int \frac{a+b\tan^{-1}(cx)}{x}\,dx+\left(4icd^4\right)\int \left(a+b\tan^{-1}(cx)\right)\,dx-\left(6c^2d^4\right)\int x\left(a+b\,\mathrm{t}\right.$$

$$= 4iacd^4x-3c^2d^4x^2\left(a+b\tan^{-1}(cx)\right)-\frac{4}{3}ic^3d^4x^3\left(a+b\tan^{-1}(cx)\right)+\frac{1}{4}c^4d^4x^4\left(a+\right.$$

$$= 4iacd^4x+3bcd^4x+4ibcd^4x\tan^{-1}(cx)-3c^2d^4x^2\left(a+b\tan^{-1}(cx)\right)-\frac{4}{3}ic^3d^4x^3\left(a\right.$$

$$= 4iacd^4x+\frac{13}{4}bcd^4x-\frac{1}{12}bc^3d^4x^3-3bd^4\tan^{-1}(cx)+4ibcd^4x\tan^{-1}(cx)-3c^2d^4x^2$$

$$= 4iacd^4x+\frac{13}{4}bcd^4x+\frac{2}{3}ibc^2d^4x^2-\frac{1}{12}bc^3d^4x^3-\frac{13}{4}bd^4\tan^{-1}(cx)+4ibcd^4x\tan^{-1}$$

Mathematica [A] time = 0.141025, size = 174, normalized size = 0.86

$$\frac{1}{12}d^4\left(6ib\text{PolyLog}(2,-icx)-6ib\text{PolyLog}(2,icx)+3ac^4x^4-16iac^3x^3-36ac^2x^2+48iacx+12a\log(x)-bc^3x^3+8ibc^2\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + I*c*d*x)^4*(a + b*ArcTan[c*x]))/x,x]
```

```
[Out] (d^4*((48*I)*a*c*x + 39*b*c*x - 36*a*c^2*x^2 + (8*I)*b*c^2*x^2 - (16*I)*a*c
^3*x^3 - b*c^3*x^3 + 3*a*c^4*x^4 - 39*b*ArcTan[c*x] + (48*I)*b*c*x*ArcTan[c
*x] - 36*b*c^2*x^2*ArcTan[c*x] - (16*I)*b*c^3*x^3*ArcTan[c*x] + 3*b*c^4*x^4
*ArcTan[c*x] + 12*a*Log[x] - (32*I)*b*Log[1 + c^2*x^2] + (6*I)*b*PolyLog[2,
 (-I)*c*x] - (6*I)*b*PolyLog[2, I*c*x]))/12
```

Maple [A] time = 0.041, size = 260, normalized size = 1.3

$$\frac{i}{2}d^4b\mathrm{dilog}\,(1+icx)+\frac{d^4ac^4x^4}{4}+\frac{i}{2}d^4b\ln(cx)\ln(1+icx)-3\,d^4ac^2x^2+d^4a\ln(cx)+\frac{2\,i}{3}bc^2d^4x^2+\frac{d^4b\arctan(cx)\,c^4x^4}{4}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^4*(a+b*arctan(c*x))/x,x)
```

```
[Out] 1/2*I*d^4*b*dilog(1+I*c*x)+1/4*d^4*a*c^4*x^4+1/2*I*d^4*b*ln(c*x)*ln(1+I*c*x
)-3*d^4*a*c^2*x^2+d^4*a*ln(c*x)+2/3*I*b*c^2*d^4*x^2+1/4*d^4*b*arctan(c*x)*c
^4*x^4-4/3*I*d^4*a*c^3*x^3-3*d^4*b*arctan(c*x)*c^2*x^2+d^4*b*arctan(c*x)*ln
(c*x)-8/3*I*b*d^4*ln(c^2*x^2+1)+4*I*a*c*d^4*x-1/2*I*d^4*b*ln(c*x)*ln(1-I*c*
x)+4*I*b*c*d^4*x*arctan(c*x)+13/4*b*c*d^4*x-1/12*b*c^3*d^4*x^3-4/3*I*d^4*b*
arctan(c*x)*c^3*x^3-1/2*I*d^4*b*dilog(1-I*c*x)-13/4*b*d^4*arctan(c*x)
```

Maxima [A] time = 2.20149, size = 308, normalized size = 1.52

$$\frac{1}{4}\,ac^4d^4x^4-\frac{4}{3}i\,ac^3d^4x^3-\frac{1}{12}\,bc^3d^4x^3-3\,ac^2d^4x^2+\frac{2}{3}i\,bc^2d^4x^2+4i\,acd^4x+\frac{13}{4}\,bcd^4x-\frac{1}{12}\,(3\,\pi+8i)bd^4\log\left(c^2x^2+1\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^4*(a+b*arctan(c*x))/x,x, algorithm="maxima")
```

```
[Out] 1/4*a*c^4*d^4*x^4 - 4/3*I*a*c^3*d^4*x^3 - 1/12*b*c^3*d^4*x^3 - 3*a*c^2*d^4*
x^2 + 2/3*I*b*c^2*d^4*x^2 + 4*I*a*c*d^4*x + 13/4*b*c*d^4*x - 1/12*(3*pi + 8
*I)*b*d^4*log(c^2*x^2 + 1) + b*d^4*arctan(c*x)*log(x*abs(c)) + 2*I*(2*c*x*a
rctan(c*x) - log(c^2*x^2 + 1))*b*d^4 - 1/2*I*b*d^4*dilog(I*c*x + 1) + 1/2*I
*b*d^4*dilog(-I*c*x + 1) + a*d^4*log(x) + 1/12*(3*b*c^4*d^4*x^4 - 16*I*b*c^
3*d^4*x^3 - 36*b*c^2*d^4*x^2 + 3*b*d^4*(4*I*arctan2(0, c) - 13))*arctan(c*x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{2\,ac^4d^4x^4 - 8i\,ac^3d^4x^3 - 12\,ac^2d^4x^2 + 8i\,acd^4x + 2\,ad^4 + \left(i\,bc^4d^4x^4 + 4\,bc^3d^4x^3 - 6i\,bc^2d^4x^2 - 4\,bcd^4x + i\,bd^4\right.}{2\,x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^4*(a+b*arctan(c*x))/x,x, algorithm="fricas")
```

```
[Out] integral(1/2*(2*a*c^4*d^4*x^4 - 8*I*a*c^3*d^4*x^3 - 12*a*c^2*d^4*x^2 + 8*I*
a*c*d^4*x + 2*a*d^4 + (I*b*c^4*d^4*x^4 + 4*b*c^3*d^4*x^3 - 6*I*b*c^2*d^4*x^
2 - 4*b*c*d^4*x + I*b*d^4)*log(-(c*x + I)/(c*x - I)))/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d^4\left(\int \frac{a}{x}\,dx + \int 4iac\,dx + \int -6ac^2x\,dx + \int ac^4x^3\,dx + \int \frac{b\,\text{atan}\,(cx)}{x}\,dx + \int -4iac^3x^2\,dx + \int 4ibc\,\text{atan}\,(cx)\,dx + \int\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**4*(a+b*atan(c*x))/x,x)
```

```
[Out] d**4*(Integral(a/x, x) + Integral(4*I*a*c, x) + Integral(-6*a*c**2*x, x) +
Integral(a*c**4*x**3, x) + Integral(b*atan(c*x)/x, x) + Integral(-4*I*a*c**
3*x**2, x) + Integral(4*I*b*c*atan(c*x), x) + Integral(-6*b*c**2*x*atan(c*x
), x) + Integral(b*c**4*x**3*atan(c*x), x) + Integral(-4*I*b*c**3*x**2*atan
(c*x), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(i\,cdx + d)^4(b\arctan\,(cx) + a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^4*(a+b*arctan(c*x))/x,x, algorithm="giac")
```

```
[Out] integrate((I*c*d*x + d)^4*(b*arctan(c*x) + a)/x, x)
```

3.36 $\int \frac{(d+icdx)^4\left(a+b\tan^{-1}(cx)\right)}{x^2}\,dx$

Optimal. Leaf size=190

$$-2bcd^4\text{PolyLog}(2,-icx)+2bcd^4\text{PolyLog}(2,icx)+\frac{1}{3}c^4d^4x^3\left(a+b\tan^{-1}(cx)\right)-2ic^3d^4x^2\left(a+b\tan^{-1}(cx)\right)-\frac{d^4\left(a+b\tan^{-1}(cx)\right)}{x}$$

```
[Out] -6*a*c^2*d^4*x + (2*I)*b*c^2*d^4*x - (b*c^3*d^4*x^2)/6 - (2*I)*b*c*d^4*ArcT
an[c*x] - 6*b*c^2*d^4*x*ArcTan[c*x] - (d^4*(a + b*ArcTan[c*x]))/x - (2*I)*c
^3*d^4*x^2*(a + b*ArcTan[c*x]) + (c^4*d^4*x^3*(a + b*ArcTan[c*x]))/3 + (4*I
)*a*c*d^4*Log[x] + b*c*d^4*Log[x] + (8*b*c*d^4*Log[1 + c^2*x^2])/3 - 2*b*c*
d^4*PolyLog[2, (-I)*c*x] + 2*b*c*d^4*PolyLog[2, I*c*x]
```

Rubi [A] time = 0.206559, antiderivative size = 190, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 13, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.565, Rules used = {4876, 4846, 260, 4852, 266, 36, 29, 31, 4848, 2391, 321, 203, 43}

$$-2bcd^4\text{PolyLog}(2,-icx)+2bcd^4\text{PolyLog}(2,icx)+\frac{1}{3}c^4d^4x^3\left(a+b\tan^{-1}(cx)\right)-2ic^3d^4x^2\left(a+b\tan^{-1}(cx)\right)-\frac{d^4\left(a+b\tan^{-1}(cx)\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)^4*(a + b*ArcTan[c*x]))/x^2,x]
```



```
[Out] -6*a*c^2*d^4*x + (2*I)*b*c^2*d^4*x - (b*c^3*d^4*x^2)/6 - (2*I)*b*c*d^4*ArcT
an[c*x] - 6*b*c^2*d^4*x*ArcTan[c*x] - (d^4*(a + b*ArcTan[c*x]))/x - (2*I)*c
^3*d^4*x^2*(a + b*ArcTan[c*x]) + (c^4*d^4*x^3*(a + b*ArcTan[c*x]))/3 + (4*I
)*a*c*d^4*Log[x] + b*c*d^4*Log[x] + (8*b*c*d^4*Log[1 + c^2*x^2])/3 - 2*b*c*
d^4*PolyLog[2, (-I)*c*x] + 2*b*c*d^4*PolyLog[2, I*c*x]
```


Rule 4876

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
```

```
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(d+icdx)^4\left(a+b\tan^{-1}(cx)\right)}{x^2}\,dx = \int\left(-6c^2d^4\left(a+b\tan^{-1}(cx)\right)+\frac{d^4\left(a+b\tan^{-1}(cx)\right)}{x^2}+\frac{4icd^4\left(a+b\tan^{-1}(cx)\right)}{x}-4i\right.$$

$$= d^4\int\frac{a+b\tan^{-1}(cx)}{x^2}\,dx+\left(4icd^4\right)\int\frac{a+b\tan^{-1}(cx)}{x}\,dx-\left(6c^2d^4\right)\int\left(a+b\tan^{-1}(\right.$$

$$= -6ac^2d^4x-\frac{d^4\left(a+b\tan^{-1}(cx)\right)}{x}-2ic^3d^4x^2\left(a+b\tan^{-1}(cx)\right)+\frac{1}{3}c^4d^4x^3\left(a+b\tan\right.$$

$$= -6ac^2d^4x+2ibc^2d^4x-6bc^2d^4x\tan^{-1}(cx)-\frac{d^4\left(a+b\tan^{-1}(cx)\right)}{x}-2ic^3d^4x^2\left(a+b\right.$$

$$= -6ac^2d^4x+2ibc^2d^4x-2ibcd^4\tan^{-1}(cx)-6bc^2d^4x\tan^{-1}(cx)-\frac{d^4\left(a+b\tan^{-1}(cx)\right)}{x}$$

$$= -6ac^2d^4x+2ibc^2d^4x-\frac{1}{6}bc^3d^4x^2-2ibcd^4\tan^{-1}(cx)-6bc^2d^4x\tan^{-1}(cx)-\frac{d^4\left(a+\right.}{}$$

Mathematica [A] time = 0.146726, size = 181, normalized size = 0.95

$$d^4\left(-12bcx\text{PolyLog}(2,-icx)+12bcx\text{PolyLog}(2,icx)+2ac^4x^4-12iac^3x^3-36ac^2x^2+24iacx\log(x)-6a-bc^3x^3+12$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + I*c*d*x)^4*(a + b*ArcTan[c*x]))/x^2,x]
```

```
[Out] (d^4*(-6*a - 36*a*c^2*x^2 + (12*I)*b*c^2*x^2 - (12*I)*a*c^3*x^3 - b*c^3*x^3
 + 2*a*c^4*x^4 - 6*b*ArcTan[c*x] - (12*I)*b*c*x*ArcTan[c*x] - 36*b*c^2*x^2*
ArcTan[c*x] - (12*I)*b*c^3*x^3*ArcTan[c*x] + 2*b*c^4*x^4*ArcTan[c*x] + (24*
I)*a*c*x*Log[x] + 6*b*c*x*Log[c*x] + 16*b*c*x*Log[1 + c^2*x^2] - 12*b*c*x*P
olyLog[2, (-I)*c*x] + 12*b*c*x*PolyLog[2, I*c*x]))/(6*x)
```

Maple [A] time = 0.046, size = 264, normalized size = 1.4

$$-6\,ac^2d^4x+\frac{d^4ac^4x^3}{3}-2\,id^4ac^3x^2-\frac{d^4a}{x}+4\,icd^4a\ln(cx)-6\,bc^2d^4x\arctan(cx)+\frac{d^4b\arctan(cx)\,c^4x^3}{3}+4\,icd^4b\,\text{arcta}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^2,x)
```

```
[Out] -6*a*c^2*d^4*x+1/3*d^4*a*c^4*x^3-2*I*d^4*a*c^3*x^2-d^4*a/x+4*I*c*d^4*a*ln(c
*x)-6*b*c^2*d^4*x*arctan(c*x)+1/3*d^4*b*arctan(c*x)*c^4*x^3+4*I*c*d^4*b*arc
tan(c*x)*ln(c*x)-d^4*b*arctan(c*x)/x-2*I*d^4*b*arctan(c*x)*c^3*x^2-2*c*d^4*
b*ln(c*x)*ln(1+I*c*x)+2*c*d^4*b*ln(c*x)*ln(1-I*c*x)-2*c*d^4*b*dilog(1+I*c*x
)+2*c*d^4*b*dilog(1-I*c*x)-2*I*b*c*d^4*arctan(c*x)-1/6*b*c^3*d^4*x^2+8/3*b*
c*d^4*ln(c^2*x^2+1)+2*I*b*c^2*d^4*x+c*d^4*b*ln(c*x)
```

Maxima [A] time = 2.15905, size = 336, normalized size = 1.77

$$\frac{1}{3}\,ac^4d^4x^3-2i\,ac^3d^4x^2-\frac{1}{6}\,bc^3d^4x^2-6\,ac^2d^4x+2i\,bc^2d^4x-\frac{1}{6}\,(6i\,\pi-1)bcd^4\log\left(c^2x^2+1\right)+4i\,bcd^4\arctan(cx)\log(x|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^2,x, algorithm="maxima")
```

```
[Out] 1/3*a*c^4*d^4*x^3 - 2*I*a*c^3*d^4*x^2 - 1/6*b*c^3*d^4*x^2 - 6*a*c^2*d^4*x +
 2*I*b*c^2*d^4*x - 1/6*(6*I*pi - 1)*b*c*d^4*log(c^2*x^2 + 1) + 4*I*b*c*d^4*
arctan(c*x)*log(x*abs(c)) - 3*(2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*b*c*d^
4 + 2*b*c*d^4*dilog(I*c*x + 1) - 2*b*c*d^4*dilog(-I*c*x + 1) + 4*I*a*c*d^4*
log(x) - 1/2*(c*(log(c^2*x^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*b*d^4 - a*
d^4/x + 1/6*(2*b*c^4*d^4*x^3 - 12*I*b*c^3*d^4*x^2 - b*c*d^4*(24*arctan2(0,
c) + 12*I))*arctan(c*x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{2\,ac^4d^4x^4 - 8i\,ac^3d^4x^3 - 12\,ac^2d^4x^2 + 8i\,acd^4x + 2\,ad^4 + \left(i\,bc^4d^4x^4 + 4\,bc^3d^4x^3 - 6i\,bc^2d^4x^2 - 4\,bcd^4x + i\,bd^4\right)\mathrm{l}}{2\,x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^2,x, algorithm="fricas")
```

```
[Out] integral(1/2*(2*a*c^4*d^4*x^4 - 8*I*a*c^3*d^4*x^3 - 12*a*c^2*d^4*x^2 + 8*I*
a*c*d^4*x + 2*a*d^4 + (I*b*c^4*d^4*x^4 + 4*b*c^3*d^4*x^3 - 6*I*b*c^2*d^4*x^
2 - 4*b*c*d^4*x + I*b*d^4)*log(-(c*x + I)/(c*x - I)))/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d^4\left(\int -6ac^2\,dx + \int \frac{a}{x^2}\,dx + \int ac^4x^2\,dx + \int -6bc^2\operatorname{atan}(cx)\,dx + \int \frac{b\operatorname{atan}(cx)}{x^2}\,dx + \int \frac{4iac}{x}\,dx + \int -4iac^3x\,dx + \int\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**4*(a+b*atan(c*x))/x**2,x)
```

```
[Out] d**4*(Integral(-6*a*c**2, x) + Integral(a/x**2, x) + Integral(a*c**4*x**2,
x) + Integral(-6*b*c**2*atan(c*x), x) + Integral(b*atan(c*x)/x**2, x) + Int
egral(4*I*a*c/x, x) + Integral(-4*I*a*c**3*x, x) + Integral(b*c**4*x**2*ata
n(c*x), x) + Integral(4*I*b*c*atan(c*x)/x, x) + Integral(-4*I*b*c**3*x*atan
(c*x), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(icdx + d)^4 (b \arctan (cx) + a)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^2,x, algorithm="giac")
```

```
[Out] integrate((I*c*d*x + d)^4*(b*arctan(c*x) + a)/x^2, x)
```

3.37 $\int \frac{(d+icdx)^4\left(a+b\tan^{-1}(cx)\right)}{x^3}\,dx$

Optimal. Leaf size=173

$$-3ibc^2d^4\text{PolyLog}(2,-icx)+3ibc^2d^4\text{PolyLog}(2,icx)+\frac{1}{2}c^4d^4x^2\left(a+b\tan^{-1}(cx)\right)-\frac{d^4\left(a+b\tan^{-1}(cx)\right)}{2x^2}-\frac{4icd^4\left(a+b\,\mathrm{t}\right.}{x}$$

```
[Out] -(b*c*d^4)/(2*x) - (4*I)*a*c^3*d^4*x - (b*c^3*d^4*x)/2 - (4*I)*b*c^3*d^4*x*
ArcTan[c*x] - (d^4*(a + b*ArcTan[c*x]))/(2*x^2) - ((4*I)*c*d^4*(a + b*ArcTa
n[c*x]))/x + (c^4*d^4*x^2*(a + b*ArcTan[c*x]))/2 - 6*a*c^2*d^4*Log[x] + (4*
I)*b*c^2*d^4*Log[x] - (3*I)*b*c^2*d^4*PolyLog[2, (-I)*c*x] + (3*I)*b*c^2*d^
4*PolyLog[2, I*c*x]
```

Rubi [A] time = 0.199452, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 13, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.565, Rules used = {4876, 4846, 260, 4852, 325, 203, 266, 36, 29, 31, 4848, 2391, 321}

$$-3ibc^2d^4\text{PolyLog}(2,-icx)+3ibc^2d^4\text{PolyLog}(2,icx)+\frac{1}{2}c^4d^4x^2\left(a+b\tan^{-1}(cx)\right)-\frac{d^4\left(a+b\tan^{-1}(cx)\right)}{2x^2}-\frac{4icd^4\left(a+b\,\mathrm{t}\right.}{x}$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)^4*(a + b*ArcTan[c*x]))/x^3,x]
```


```
[Out] -(b*c*d^4)/(2*x) - (4*I)*a*c^3*d^4*x - (b*c^3*d^4*x)/2 - (4*I)*b*c^3*d^4*x*
ArcTan[c*x] - (d^4*(a + b*ArcTan[c*x]))/(2*x^2) - ((4*I)*c*d^4*(a + b*ArcTa
n[c*x]))/x + (c^4*d^4*x^2*(a + b*ArcTan[c*x]))/2 - 6*a*c^2*d^4*Log[x] + (4*
I)*b*c^2*d^4*Log[x] - (3*I)*b*c^2*d^4*PolyLog[2, (-I)*c*x] + (3*I)*b*c^2*d^
4*PolyLog[2, I*c*x]
```


Rule 4876

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
```

```
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{(d+icdx)^4\left(a+b\tan^{-1}(cx)\right)}{x^3}\,dx = \int\left(-4ic^3d^4\left(a+b\tan^{-1}(cx)\right)+\frac{d^4\left(a+b\tan^{-1}(cx)\right)}{x^3}+\frac{4icd^4\left(a+b\tan^{-1}(cx)\right)}{x^2}-\frac{6\right.$$

$$= d^4\int\frac{a+b\tan^{-1}(cx)}{x^3}\,dx+\left(4icd^4\right)\int\frac{a+b\tan^{-1}(cx)}{x^2}\,dx-\left(6c^2d^4\right)\int\frac{a+b\tan^{-1}(c}{x}$$

$$= -4iac^3d^4x-\frac{d^4\left(a+b\tan^{-1}(cx)\right)}{2x^2}-\frac{4icd^4\left(a+b\tan^{-1}(cx)\right)}{x}+\frac{1}{2}c^4d^4x^2\left(a+b\tan^{-1}\right.$$

$$= -\frac{bcd^4}{2x}-4iac^3d^4x-\frac{1}{2}bc^3d^4x-4ibc^3d^4x\tan^{-1}(cx)-\frac{d^4\left(a+b\tan^{-1}(cx)\right)}{2x^2}-\frac{4icd^4\left(\right.}{}$$

$$= -\frac{bcd^4}{2x}-4iac^3d^4x-\frac{1}{2}bc^3d^4x-4ibc^3d^4x\tan^{-1}(cx)-\frac{d^4\left(a+b\tan^{-1}(cx)\right)}{2x^2}-\frac{4icd^4\left(\right.}{}$$

$$= -\frac{bcd^4}{2x}-4iac^3d^4x-\frac{1}{2}bc^3d^4x-4ibc^3d^4x\tan^{-1}(cx)-\frac{d^4\left(a+b\tan^{-1}(cx)\right)}{2x^2}-\frac{4icd^4\left(\right.}{}$$

Mathematica [A] time = 0.139878, size = 163, normalized size = 0.94

$$\frac{d^4\left(-6ibc^2x^2\text{PolyLog}(2,-icx)+6ibc^2x^2\text{PolyLog}(2,icx)+ac^4x^4-8iac^3x^3-12ac^2x^2\log(x)-8iacx-a-bc^3x^3+8ibc^2\right.}{2x^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + I*c*d*x)^4*(a + b*ArcTan[c*x]))/x^3,x]
```

```
[Out] (d^4*(-a - (8*I)*a*c*x - b*c*x - (8*I)*a*c^3*x^3 - b*c^3*x^3 + a*c^4*x^4 -
b*ArcTan[c*x] - (8*I)*b*c*x*ArcTan[c*x] - (8*I)*b*c^3*x^3*ArcTan[c*x] + b*c
^4*x^4*ArcTan[c*x] - 12*a*c^2*x^2*Log[x] + (8*I)*b*c^2*x^2*Log[c*x] - (6*I)
*b*c^2*x^2*PolyLog[2, (-I)*c*x] + (6*I)*b*c^2*x^2*PolyLog[2, I*c*x]))/(2*x^
2)
```

Maple [A] time = 0.047, size = 248, normalized size = 1.4

$$3\,ic^2d^4b\ln(cx)\ln(1-icx)+\frac{c^4d^4ax^2}{2}-\frac{d^4a}{2\,x^2}-\frac{4\,icd^4a}{x}-6\,c^2d^4a\ln(cx)+3\,ic^2d^4b\text{dilog}(1-icx)+\frac{c^4d^4b\arctan(cx)\,x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^3,x)
```

```
[Out] 3*I*c^2*d^4*b*ln(c*x)*ln(1-I*c*x)+1/2*c^4*d^4*a*x^2-1/2*d^4*a/x^2-4*I*c*d^4
*a/x-6*c^2*d^4*a*ln(c*x)+3*I*c^2*d^4*b*dilog(1-I*c*x)+1/2*c^4*d^4*b*arctan(
c*x)*x^2-1/2*d^4*b*arctan(c*x)/x^2-3*I*c^2*d^4*b*ln(c*x)*ln(1+I*c*x)-6*c^2*
d^4*b*arctan(c*x)*ln(c*x)-1/2*b*c^3*d^4*x-1/2*b*c*d^4/x-4*I*b*c^3*d^4*x*arc
tan(c*x)-4*I*c*d^4*b*arctan(c*x)/x-3*I*c^2*d^4*b*dilog(1+I*c*x)+4*I*c^2*d^4
*b*ln(c*x)-4*I*a*c^3*d^4*x
```

Maxima [A] time = 2.16075, size = 350, normalized size = 2.02

$$\frac{1}{2}\,ac^4d^4x^2-4i\,ac^3d^4x-\frac{1}{2}\,bc^3d^4x+\frac{3}{2}\,\pi bc^2d^4\log\left(c^2x^2+1\right)-6\,bc^2d^4\arctan(cx)\log(x|c|)-2i\left(2\,cx\arctan(cx)-\log\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*a*c^4*d^4*x^2 - 4*I*a*c^3*d^4*x - 1/2*b*c^3*d^4*x + 3/2*pi*b*c^2*d^4*lo
g(c^2*x^2 + 1) - 6*b*c^2*d^4*arctan(c*x)*log(x*abs(c)) - 2*I*(2*c*x*arctan(
c*x) - log(c^2*x^2 + 1))*b*c^2*d^4 + 3*I*b*c^2*d^4*dilog(I*c*x + 1) - 3*I*b
*c^2*d^4*dilog(-I*c*x + 1) - 6*a*c^2*d^4*log(x) - 2*I*(c*(log(c^2*x^2 + 1)
- log(x^2)) + 2*arctan(c*x)/x)*b*c*d^4 - 1/2*((c*arctan(c*x) + 1/x)*c + arc
tan(c*x)/x^2)*b*d^4 - 4*I*a*c*d^4/x - 1/2*a*d^4/x^2 + 1/2*(b*c^4*d^4*x^2 +
b*c^2*d^4*(-12*I*arctan2(0, c) + 1))*arctan(c*x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{2\,ac^4d^4x^4 - 8i\,ac^3d^4x^3 - 12\,ac^2d^4x^2 + 8i\,acd^4x + 2\,ad^4 + \left(i\,bc^4d^4x^4 + 4\,bc^3d^4x^3 - 6i\,bc^2d^4x^2 - 4\,bcd^4x + i\,bd^4\right)\,\mathrm{l}}{2\,x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^3,x, algorithm="fricas")
```

```
[Out] integral(1/2*(2*a*c^4*d^4*x^4 - 8*I*a*c^3*d^4*x^3 - 12*a*c^2*d^4*x^2 + 8*I*
a*c*d^4*x + 2*a*d^4 + (I*b*c^4*d^4*x^4 + 4*b*c^3*d^4*x^3 - 6*I*b*c^2*d^4*x^
2 - 4*b*c*d^4*x + I*b*d^4)*log(-(c*x + I)/(c*x - I)))/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d^4\left(\int \frac{a}{x^3}\,dx + \int -4iac^3\,dx + \int -\frac{6ac^2}{x}\,dx + \int ac^4x\,dx + \int \frac{b\,\mathrm{atan}\,(cx)}{x^3}\,dx + \int \frac{4iac}{x^2}\,dx + \int -4ibc^3\,\mathrm{atan}\,(cx)\,dx + \int -\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**4*(a+b*atan(c*x))/x**3,x)
```

```
[Out] d**4*(Integral(a/x**3, x) + Integral(-4*I*a*c**3, x) + Integral(-6*a*c**2/x
, x) + Integral(a*c**4*x, x) + Integral(b*atan(c*x)/x**3, x) + Integral(4*I
*a*c/x**2, x) + Integral(-4*I*b*c**3*atan(c*x), x) + Integral(-6*b*c**2*ata
n(c*x)/x, x) + Integral(b*c**4*x*atan(c*x), x) + Integral(4*I*b*c*atan(c*x)
/x**2, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(icdx+d)^4(b\arctan(cx)+a)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^3,x, algorithm="giac")
```

```
[Out] integrate((I*c*d*x + d)^4*(b*arctan(c*x) + a)/x^3, x)
```

3.38 $\int \frac{(d+icdx)^4\left(a+b\tan^{-1}(cx)\right)}{x^4}\,dx$

Optimal. Leaf size=201

$$2bc^3d^4\text{PolyLog}(2,-icx)-2bc^3d^4\text{PolyLog}(2,icx)+\frac{6c^2d^4\left(a+b\tan^{-1}(cx)\right)}{x}-\frac{2icd^4\left(a+b\tan^{-1}(cx)\right)}{x^2}-\frac{d^4\left(a+b\tan^{-1}(cx)\right)}{3x^3}$$

```
[Out] -(b*c*d^4)/(6*x^2) - ((2*I)*b*c^2*d^4)/x + a*c^4*d^4*x - (2*I)*b*c^3*d^4*Ar
cTan[c*x] + b*c^4*d^4*x*ArcTan[c*x] - (d^4*(a + b*ArcTan[c*x]))/(3*x^3) - (
(2*I)*c*d^4*(a + b*ArcTan[c*x]))/x^2 + (6*c^2*d^4*(a + b*ArcTan[c*x]))/x -
(4*I)*a*c^3*d^4*Log[x] - (19*b*c^3*d^4*Log[x])/3 + (8*b*c^3*d^4*Log[1 + c^2
*x^2])/3 + 2*b*c^3*d^4*PolyLog[2, (-I)*c*x] - 2*b*c^3*d^4*PolyLog[2, I*c*x]
```

Rubi [A] time = 0.217667, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 13, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.565, Rules used = {4876, 4846, 260, 4852, 266, 44, 325, 203, 36, 29, 31, 4848, 2391}

$$2bc^3d^4\text{PolyLog}(2,-icx)-2bc^3d^4\text{PolyLog}(2,icx)+\frac{6c^2d^4\left(a+b\tan^{-1}(cx)\right)}{x}-\frac{2icd^4\left(a+b\tan^{-1}(cx)\right)}{x^2}-\frac{d^4\left(a+b\tan^{-1}(cx)\right)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)^4*(a + b*ArcTan[c*x]))/x^4,x]
```

```
[Out] -(b*c*d^4)/(6*x^2) - ((2*I)*b*c^2*d^4)/x + a*c^4*d^4*x - (2*I)*b*c^3*d^4*Ar
cTan[c*x] + b*c^4*d^4*x*ArcTan[c*x] - (d^4*(a + b*ArcTan[c*x]))/(3*x^3) - (
(2*I)*c*d^4*(a + b*ArcTan[c*x]))/x^2 + (6*c^2*d^4*(a + b*ArcTan[c*x]))/x -
(4*I)*a*c^3*d^4*Log[x] - (19*b*c^3*d^4*Log[x])/3 + (8*b*c^3*d^4*Log[1 + c^2
*x^2])/3 + 2*b*c^3*d^4*PolyLog[2, (-I)*c*x] - 2*b*c^3*d^4*PolyLog[2, I*c*x]
```

Rule 4876

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
```

```
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{(d+icdx)^4\left(a+b\tan^{-1}(cx)\right)}{x^4}\,dx = \int\left(c^4d^4\left(a+b\tan^{-1}(cx)\right)+\frac{d^4\left(a+b\tan^{-1}(cx)\right)}{x^4}+\frac{4icd^4\left(a+b\tan^{-1}(cx)\right)}{x^3}-\frac{6c^2d\right.$$

$$= d^4\int\frac{a+b\tan^{-1}(cx)}{x^4}\,dx+\left(4icd^4\right)\int\frac{a+b\tan^{-1}(cx)}{x^3}\,dx-\left(6c^2d^4\right)\int\frac{a+b\tan^{-1}(c}{x^2}$$

$$= ac^4d^4x-\frac{d^4\left(a+b\tan^{-1}(cx)\right)}{3x^3}-\frac{2icd^4\left(a+b\tan^{-1}(cx)\right)}{x^2}+\frac{6c^2d^4\left(a+b\tan^{-1}(cx)\right)}{x}$$

$$= -\frac{2ibc^2d^4}{x}+ac^4d^4x+bc^4d^4x\tan^{-1}(cx)-\frac{d^4\left(a+b\tan^{-1}(cx)\right)}{3x^3}-\frac{2icd^4\left(a+b\tan^{-1}\right.}{x^2}$$

$$= -\frac{2ibc^2d^4}{x}+ac^4d^4x-2ibc^3d^4\tan^{-1}(cx)+bc^4d^4x\tan^{-1}(cx)-\frac{d^4\left(a+b\tan^{-1}(cx)\right)}{3x^3}$$

$$= -\frac{bcd^4}{6x^2}-\frac{2ibc^2d^4}{x}+ac^4d^4x-2ibc^3d^4\tan^{-1}(cx)+bc^4d^4x\tan^{-1}(cx)-\frac{d^4\left(a+b\tan^{-1}\right.}{3x^3}$$

Mathematica [A] time = 0.147416, size = 193, normalized size = 0.96

$$\frac{d^4\left(12bc^3x^3\text{PolyLog}(2,-icx)-12bc^3x^3\text{PolyLog}(2,icx)+6ac^4x^4+36ac^2x^2-24iac^3x^3\log(x)-12iacx-2a-12ibc^2x^2\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + I*c*d*x)^4*(a + b*ArcTan[c*x]))/x^4,x]
```

```
[Out] (d^4*(-2*a - (12*I)*a*c*x - b*c*x + 36*a*c^2*x^2 - (12*I)*b*c^2*x^2 + 6*a*c
^4*x^4 - 2*b*ArcTan[c*x] - (12*I)*b*c*x*ArcTan[c*x] + 36*b*c^2*x^2*ArcTan[c
*x] - (12*I)*b*c^3*x^3*ArcTan[c*x] + 6*b*c^4*x^4*ArcTan[c*x] - (24*I)*a*c^3
*x^3*Log[x] - 38*b*c^3*x^3*Log[c*x] + 16*b*c^3*x^3*Log[1 + c^2*x^2] + 12*b*
c^3*x^3*PolyLog[2, (-I)*c*x] - 12*b*c^3*x^3*PolyLog[2, I*c*x]))/(6*x^3)
```

Maple [A] time = 0.047, size = 277, normalized size = 1.4

$$ac^4d^4x-4\,ic^3d^4a\ln(cx)+6\,\frac{c^2d^4a}{x}-\frac{d^4a}{3\,x^3}-\frac{2\,icd^4b\arctan(cx)}{x^2}+bc^4d^4x\arctan(cx)-\frac{2\,ibc^2d^4}{x}+6\,\frac{c^2d^4b\arctan(cx)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^4,x)
```

```
[Out] a*c^4*d^4*x-4*I*c^3*d^4*a*ln(c*x)+6*c^2*d^4*a/x-1/3*d^4*a/x^3-2*I*c*d^4*b*a
rctan(c*x)/x^2+b*c^4*d^4*x*arctan(c*x)-2*I*b*c^2*d^4/x+6*c^2*d^4*b*arctan(c
*x)/x-1/3*d^4*b*arctan(c*x)/x^3-4*I*c^3*d^4*b*arctan(c*x)*ln(c*x)+2*c^3*d^4
*b*ln(c*x)*ln(1+I*c*x)-2*c^3*d^4*b*ln(c*x)*ln(1-I*c*x)+2*c^3*d^4*b*dilog(1+
I*c*x)-2*c^3*d^4*b*dilog(1-I*c*x)+8/3*b*c^3*d^4*ln(c^2*x^2+1)-2*I*c*d^4*a/x
^2-1/6*b*c*d^4/x^2-2*I*b*c^3*d^4*arctan(c*x)-19/3*c^3*d^4*b*ln(c*x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$ac^4d^4x+\frac{1}{2}\left(2\,cx\arctan(cx)-\log\left(c^2x^2+1\right)\right)bc^3d^4-4i\,bc^3d^4\int\frac{\arctan(cx)}{x}\,dx-4i\,ac^3d^4\log(x)+3\left(c\left(\log\left(c^2x^2+1\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^4,x, algorithm="maxima")
```

```
[Out] a*c^4*d^4*x + 1/2*(2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*b*c^3*d^4 - 4*I*b*
c^3*d^4*integrate(arctan(c*x)/x, x) - 4*I*a*c^3*d^4*log(x) + 3*(c*(log(c^2*
x^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*b*c^2*d^4 - 2*I*((c*arctan(c*x) + 1
/x)*c + arctan(c*x)/x^2)*b*c*d^4 + 1/6*((c^2*log(c^2*x^2 + 1) - c^2*log(x^2
) - 1/x^2)*c - 2*arctan(c*x)/x^3)*b*d^4 + 6*a*c^2*d^4/x - 2*I*a*c*d^4/x^2 -
 1/3*a*d^4/x^3
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{2\,ac^4d^4x^4 - 8i\,ac^3d^4x^3 - 12\,ac^2d^4x^2 + 8i\,acd^4x + 2\,ad^4 + \left(i\,bc^4d^4x^4 + 4\,bc^3d^4x^3 - 6i\,bc^2d^4x^2 - 4\,bcd^4x + i\,bd^4\right)\mathrm{l}}{2\,x^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] integral(1/2*(2*a*c^4*d^4*x^4 - 8*I*a*c^3*d^4*x^3 - 12*a*c^2*d^4*x^2 + 8*I*
a*c*d^4*x + 2*a*d^4 + (I*b*c^4*d^4*x^4 + 4*b*c^3*d^4*x^3 - 6*I*b*c^2*d^4*x^
2 - 4*b*c*d^4*x + I*b*d^4)*log(-(c*x + I)/(c*x - I)))/x^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d^4\left(\int ac^4\,dx + \int \frac{a}{x^4}\,dx + \int -\frac{6ac^2}{x^2}\,dx + \int bc^4\operatorname{atan}(cx)\,dx + \int \frac{b\operatorname{atan}(cx)}{x^4}\,dx + \int \frac{4iac}{x^3}\,dx + \int -\frac{4iac^3}{x}\,dx + \int -\frac{6bc}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**4*(a+b*atan(c*x))/x**4,x)
```

```
[Out] d**4*(Integral(a*c**4, x) + Integral(a/x**4, x) + Integral(-6*a*c**2/x**2,
x) + Integral(b*c**4*atan(c*x), x) + Integral(b*atan(c*x)/x**4, x) + Integr
al(4*I*a*c/x**3, x) + Integral(-4*I*a*c**3/x, x) + Integral(-6*b*c**2*atan(
c*x)/x**2, x) + Integral(4*I*b*c*atan(c*x)/x**3, x) + Integral(-4*I*b*c**3*
atan(c*x)/x, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(icdx + d)^4 (b \arctan (cx) + a)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^4,x, algorithm="giac")
```

```
[Out] integrate((I*c*d*x + d)^4*(b*arctan(c*x) + a)/x^4, x)
```

3.39 $\int \frac{(d+icdx)^4\left(a+b\tan^{-1}(cx)\right)}{x^5}\,dx$

Optimal. Leaf size=227

$$\frac{1}{2}ibc^4d^4\text{PolyLog}(2,-icx)-\frac{1}{2}ibc^4d^4\text{PolyLog}(2,icx)+\frac{3c^2d^4\left(a+b\tan^{-1}(cx)\right)}{x^2}+\frac{4ic^3d^4\left(a+b\tan^{-1}(cx)\right)}{x}-\frac{4icd^4\left(a+b\right.}{3x}$$

```
[Out] -(b*c*d^4)/(12*x^3) - (((2*I)/3)*b*c^2*d^4)/x^2 + (13*b*c^3*d^4)/(4*x) + (1
3*b*c^4*d^4*ArcTan[c*x])/4 - (d^4*(a + b*ArcTan[c*x]))/(4*x^4) - (((4*I)/3)
*c*d^4*(a + b*ArcTan[c*x]))/x^3 + (3*c^2*d^4*(a + b*ArcTan[c*x]))/x^2 + ((4
*I)*c^3*d^4*(a + b*ArcTan[c*x]))/x + a*c^4*d^4*Log[x] - ((16*I)/3)*b*c^4*d^
4*Log[x] + ((8*I)/3)*b*c^4*d^4*Log[1 + c^2*x^2] + (I/2)*b*c^4*d^4*PolyLog[2
, (-I)*c*x] - (I/2)*b*c^4*d^4*PolyLog[2, I*c*x]
```

Rubi [A] time = 0.229686, antiderivative size = 227, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {4876, 4852, 325, 203, 266, 44, 36, 29, 31, 4848, 2391}

$$\frac{1}{2}ibc^4d^4\text{PolyLog}(2,-icx)-\frac{1}{2}ibc^4d^4\text{PolyLog}(2,icx)+\frac{3c^2d^4\left(a+b\tan^{-1}(cx)\right)}{x^2}+\frac{4ic^3d^4\left(a+b\tan^{-1}(cx)\right)}{x}-\frac{4icd^4\left(a+b\right.}{3x}$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)^4*(a + b*ArcTan[c*x]))/x^5,x]
```


```
[Out] -(b*c*d^4)/(12*x^3) - (((2*I)/3)*b*c^2*d^4)/x^2 + (13*b*c^3*d^4)/(4*x) + (1
3*b*c^4*d^4*ArcTan[c*x])/4 - (d^4*(a + b*ArcTan[c*x]))/(4*x^4) - (((4*I)/3)
*c*d^4*(a + b*ArcTan[c*x]))/x^3 + (3*c^2*d^4*(a + b*ArcTan[c*x]))/x^2 + ((4
*I)*c^3*d^4*(a + b*ArcTan[c*x]))/x + a*c^4*d^4*Log[x] - ((16*I)/3)*b*c^4*d^
4*Log[x] + ((8*I)/3)*b*c^4*d^4*Log[1 + c^2*x^2] + (I/2)*b*c^4*d^4*PolyLog[2
, (-I)*c*x] - (I/2)*b*c^4*d^4*PolyLog[2, I*c*x]
```


Rule 4876

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{(d+icdx)^4\left(a+b\tan^{-1}(cx)\right)}{x^5}\,dx=\int\left(\frac{d^4\left(a+b\tan^{-1}(cx)\right)}{x^5}+\frac{4icd^4\left(a+b\tan^{-1}(cx)\right)}{x^4}-\frac{6c^2d^4\left(a+b\tan^{-1}(cx)\right)}{x^3}-\frac{4ic\right.$$

$$=d^4\int\frac{a+b\tan^{-1}(cx)}{x^5}\,dx+\left(4icd^4\right)\int\frac{a+b\tan^{-1}(cx)}{x^4}\,dx-\left(6c^2d^4\right)\int\frac{a+b\tan^{-1}(c}{x^3}$$

$$=-\frac{d^4\left(a+b\tan^{-1}(cx)\right)}{4x^4}-\frac{4icd^4\left(a+b\tan^{-1}(cx)\right)}{3x^3}+\frac{3c^2d^4\left(a+b\tan^{-1}(cx)\right)}{x^2}+\frac{4ic^3d}{}$$

$$=-\frac{bcd^4}{12x^3}+\frac{3bc^3d^4}{x}-\frac{d^4\left(a+b\tan^{-1}(cx)\right)}{4x^4}-\frac{4icd^4\left(a+b\tan^{-1}(cx)\right)}{3x^3}+\frac{3c^2d^4\left(a+b\,\mathrm{ta}\right.}{x^2}$$

$$=-\frac{bcd^4}{12x^3}+\frac{13bc^3d^4}{4x}+3bc^4d^4\tan^{-1}(cx)-\frac{d^4\left(a+b\tan^{-1}(cx)\right)}{4x^4}-\frac{4icd^4\left(a+b\tan^{-1}(c\right.}{3x^3}$$

$$=-\frac{bcd^4}{12x^3}-\frac{2ibc^2d^4}{3x^2}+\frac{13bc^3d^4}{4x}+\frac{13}{4}bc^4d^4\tan^{-1}(cx)-\frac{d^4\left(a+b\tan^{-1}(cx)\right)}{4x^4}-\frac{4icd^4\left(a\right.}{}$$

Mathematica [C] time = 0.113119, size = 227, normalized size = 1.

$$\frac{d^4\left(36bc^3x^3\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-c^2x^2\right)-bcx\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-c^2x^2\right)+6ibc^4x^4\text{PolyLog}(2,\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + I*c*d*x)^4*(a + b*ArcTan[c*x]))/x^5,x]
```

```
[Out] (d^4*(-3*a - (16*I)*a*c*x + 36*a*c^2*x^2 - (8*I)*b*c^2*x^2 + (48*I)*a*c^3*x
^3 - 3*b*ArcTan[c*x] - (16*I)*b*c*x*ArcTan[c*x] + 36*b*c^2*x^2*ArcTan[c*x]
+ (48*I)*b*c^3*x^3*ArcTan[c*x] - b*c*x*Hypergeometric2F1[-3/2, 1, -1/2, -(c
^2*x^2)] + 36*b*c^3*x^3*Hypergeometric2F1[-1/2, 1, 1/2, -(c^2*x^2)] + 12*a*
c^4*x^4*Log[x] - (64*I)*b*c^4*x^4*Log[x] + (32*I)*b*c^4*x^4*Log[1 + c^2*x^2
] + (6*I)*b*c^4*x^4*PolyLog[2, (-I)*c*x] - (6*I)*b*c^4*x^4*PolyLog[2, I*c*x
]))/(12*x^4)
```

Maple [A] time = 0.052, size = 298, normalized size = 1.3

$$3\,\frac{c^2d^4a}{x^2}-\frac{d^4a}{4\,x^4}+\frac{4\,ic^3d^4a}{x}-\frac{\frac{2\,i}{3}bc^2d^4}{x^2}+c^4d^4a\ln(cx)+3\,\frac{c^2d^4b\arctan(cx)}{x^2}-\frac{bd^4\arctan(cx)}{4\,x^4}-\frac{\frac{4\,i}{3}cd^4a}{x^3}-\frac{i}{2}c^4d^4b\mathrm{dilog}($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^5,x)
```

```
[Out] 3*c^2*d^4*a/x^2-1/4*d^4*a/x^4+4*I*c^3*d^4*a/x-2/3*I*b*c^2*d^4/x^2+c^4*d^4*a
*ln(c*x)+3*c^2*d^4*b*arctan(c*x)/x^2-1/4*d^4*b*arctan(c*x)/x^4-4/3*I*c*d^4*
a/x^3-1/2*I*c^4*d^4*b*dilog(1-I*c*x)+c^4*d^4*b*arctan(c*x)*ln(c*x)+8/3*I*b*
c^4*d^4*ln(c^2*x^2+1)+13/4*b*c^4*d^4*arctan(c*x)-4/3*I*c*d^4*b*arctan(c*x)/
x^3-1/2*I*c^4*d^4*b*ln(c*x)*ln(1-I*c*x)-1/12*b*c*d^4/x^3+13/4*b*c^3*d^4/x+1
/2*I*c^4*d^4*b*ln(c*x)*ln(1+I*c*x)+4*I*c^3*d^4*b*arctan(c*x)/x-16/3*I*c^4*d
^4*b*ln(c*x)+1/2*I*c^4*d^4*b*dilog(1+I*c*x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$bc^4d^4\int\frac{\arctan(cx)}{x}\,dx+ac^4d^4\log(x)+2i\left(c\left(\log\left(c^2x^2+1\right)-\log\left(x^2\right)\right)+\frac{2\arctan(cx)}{x}\right)bc^3d^4+3\left(\left(c\arctan(cx)+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^5,x, algorithm="maxima")
```

```
[Out] b*c^4*d^4*integrate(arctan(c*x)/x, x) + a*c^4*d^4*log(x) + 2*I*(c*(log(c^2*
x^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*b*c^3*d^4 + 3*((c*arctan(c*x) + 1/x
)*c + arctan(c*x)/x^2)*b*c^2*d^4 + 2/3*I*((c^2*log(c^2*x^2 + 1) - c^2*log(x
^2) - 1/x^2)*c - 2*arctan(c*x)/x^3)*b*c*d^4 + 4*I*a*c^3*d^4/x + 1/12*((3*c^
3*arctan(c*x) + (3*c^2*x^2 - 1)/x^3)*c - 3*arctan(c*x)/x^4)*b*d^4 + 3*a*c^2
```

```
*d^4/x^2 - 4/3*I*a*c*d^4/x^3 - 1/4*a*d^4/x^4
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{2\,ac^4d^4x^4 - 8i\,ac^3d^4x^3 - 12\,ac^2d^4x^2 + 8i\,acd^4x + 2\,ad^4 + \left(i\,bc^4d^4x^4 + 4\,bc^3d^4x^3 - 6i\,bc^2d^4x^2 - 4\,bcd^4x + i\,bd^4\right)\right.}{2\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^5,x, algorithm="fricas")
```

```
[Out] integral(1/2*(2*a*c^4*d^4*x^4 - 8*I*a*c^3*d^4*x^3 - 12*a*c^2*d^4*x^2 + 8*I*
a*c*d^4*x + 2*a*d^4 + (I*b*c^4*d^4*x^4 + 4*b*c^3*d^4*x^3 - 6*I*b*c^2*d^4*x^
2 - 4*b*c*d^4*x + I*b*d^4)*log(-(c*x + I)/(c*x - I)))/x^5, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d^4\left(\int \frac{a}{x^5}\,dx + \int -\frac{6ac^2}{x^3}\,dx + \int \frac{ac^4}{x}\,dx + \int \frac{b\,\mathrm{atan}\,(cx)}{x^5}\,dx + \int \frac{4iac}{x^4}\,dx + \int -\frac{4iac^3}{x^2}\,dx + \int -\frac{6bc^2\,\mathrm{atan}\,(cx)}{x^3}\,dx + \int \frac{b}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**4*(a+b*atan(c*x))/x**5,x)
```

```
[Out] d**4*(Integral(a/x**5, x) + Integral(-6*a*c**2/x**3, x) + Integral(a*c**4/x
, x) + Integral(b*atan(c*x)/x**5, x) + Integral(4*I*a*c/x**4, x) + Integral
(-4*I*a*c**3/x**2, x) + Integral(-6*b*c**2*atan(c*x)/x**3, x) + Integral(b*
c**4*atan(c*x)/x, x) + Integral(4*I*b*c*atan(c*x)/x**4, x) + Integral(-4*I*
b*c**3*atan(c*x)/x**2, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(i\,cdx + d)^4(b\arctan(cx) + a)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^5,x, algorithm="giac")
```

```
[Out] integrate((I*c*d*x + d)^4*(b*arctan(c*x) + a)/x^5, x)
```

3.40 $\int \frac{(d+icdx)^4\left(a+b\tan^{-1}(cx)\right)}{x^6}\,dx$

Optimal. Leaf size=117

$$-\frac{d^4(1+icx)^5\left(a+b\tan^{-1}(cx)\right)}{5x^5}+\frac{11bc^3d^4}{10x^2}-\frac{ibc^2d^4}{3x^3}+\frac{3ibc^4d^4}{x}+\frac{16}{5}bc^5d^4\log(x)-\frac{16}{5}bc^5d^4\log(cx+i)-\frac{bcd^4}{20x^4}$$

```
[Out] -(b*c*d^4)/(20*x^4) - ((I/3)*b*c^2*d^4)/x^3 + (11*b*c^3*d^4)/(10*x^2) + ((3
*I)*b*c^4*d^4)/x - (d^4*(1 + I*c*x)^5*(a + b*ArcTan[c*x]))/(5*x^5) + (16*b*
c^5*d^4*Log[x])/5 - (16*b*c^5*d^4*Log[I + c*x])/5
```

Rubi [A] time = 0.0972312, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {37, 4872, 12, 88}

$$-\frac{d^4(1+icx)^5\left(a+b\tan^{-1}(cx)\right)}{5x^5}+\frac{11bc^3d^4}{10x^2}-\frac{ibc^2d^4}{3x^3}+\frac{3ibc^4d^4}{x}+\frac{16}{5}bc^5d^4\log(x)-\frac{16}{5}bc^5d^4\log(cx+i)-\frac{bcd^4}{20x^4}$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)^4*(a + b*ArcTan[c*x]))/x^6,x]
```

```
[Out] -(b*c*d^4)/(20*x^4) - ((I/3)*b*c^2*d^4)/x^3 + (11*b*c^3*d^4)/(10*x^2) + ((3
*I)*b*c^4*d^4)/x - (d^4*(1 + I*c*x)^5*(a + b*ArcTan[c*x]))/(5*x^5) + (16*b*
c^5*d^4*Log[x])/5 - (16*b*c^5*d^4*Log[I + c*x])/5
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 4872

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
+ b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2), x
], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2*m
] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{(d+icdx)^4\left(a+b\tan^{-1}(cx)\right)}{x^6}\,dx = -\frac{d^4(1+icx)^5\left(a+b\tan^{-1}(cx)\right)}{5x^5}-(bc)\int -\frac{id^4(i-cx)^4}{5x^5(i+cx)}\,dx$$
$$= -\frac{d^4(1+icx)^5\left(a+b\tan^{-1}(cx)\right)}{5x^5}+\frac{1}{5}\left(ibcd^4\right)\int \frac{(i-cx)^4}{x^5(i+cx)}\,dx$$
$$= -\frac{d^4(1+icx)^5\left(a+b\tan^{-1}(cx)\right)}{5x^5}+\frac{1}{5}\left(ibcd^4\right)\int \left(-\frac{i}{x^5}+\frac{5c}{x^4}+\frac{11ic^2}{x^3}-\frac{15c^3}{x^2}-\frac{16ic^4}{x}\right.$$
$$= -\frac{bcd^4}{20x^4}-\frac{ibc^2d^4}{3x^3}+\frac{11bc^3d^4}{10x^2}+\frac{3ibc^4d^4}{x}-\frac{d^4(1+icx)^5\left(a+b\tan^{-1}(cx)\right)}{5x^5}+\frac{16}{5}bc^5d^4$$

Mathematica [C] time = 0.159117, size = 191, normalized size = 1.63

$$-\frac{d^4\left(3\left(-40ibc^4x^4\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-c^2x^2\right)+20ac^4x^4-40iac^3x^3-40ac^2x^2+20iacx+4a-22bc^3x^3-6\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + I*c*d*x)^4*(a + b*ArcTan[c*x]))/x^6,x]
```

```
[Out] -(d^4*((20*I)*b*c^2*x^2*Hypergeometric2F1[-3/2, 1, -1/2, -(c^2*x^2)] + 3*(4
*a + (20*I)*a*c*x + b*c*x - 40*a*c^2*x^2 - (40*I)*a*c^3*x^3 - 22*b*c^3*x^3
+ 20*a*c^4*x^4 + 4*b*(1 + (5*I)*c*x - 10*c^2*x^2 - (10*I)*c^3*x^3 + 5*c^4*x
^4)*ArcTan[c*x] - (40*I)*b*c^4*x^4*Hypergeometric2F1[-1/2, 1, 1/2, -(c^2*x^
2)] - 64*b*c^5*x^5*Log[x] + 32*b*c^5*x^5*Log[1 + c^2*x^2])))/(60*x^5)
```

Maple [B] time = 0.036, size = 230, normalized size = 2.

$$\frac{-icd^4a}{x^4}+\frac{2\,ic^3d^4b\arctan\left(cx\right)}{x^2}-\frac{d^4c^4a}{x}-\frac{d^4a}{5\,x^5}+2\,\frac{c^2d^4a}{x^3}+3\,ic^5d^4b\arctan\left(cx\right)-\frac{icd^4b\arctan\left(cx\right)}{x^4}-\frac{bc^4d^4\arctan\left(cx\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^6,x)
```

```
[Out] -I*c*d^4*a/x^4+2*I*c^3*d^4*b*arctan(c*x)/x^2-c^4*d^4*a/x-1/5*d^4*a/x^5+2*c^
2*d^4*a/x^3+3*I*c^5*d^4*b*arctan(c*x)-I*c*d^4*b*arctan(c*x)/x^4-c^4*d^4*b*a
rctan(c*x)/x-1/5*d^4*b*arctan(c*x)/x^5+2*c^2*d^4*b*arctan(c*x)/x^3-8/5*c^5*
d^4*b*ln(c^2*x^2+1)+2*I*c^3*d^4*a/x^2-1/3*I*b*c^2*d^4/x^3+3*I*b*c^4*d^4/x-1
/20*b*c*d^4/x^4+11/10*b*c^3*d^4/x^2+16/5*c^5*d^4*b*ln(c*x)
```

Maxima [B] time = 1.4758, size = 371, normalized size = 3.17

$$-\frac{1}{2}\left(c\left(\log\left(c^2x^2+1\right)-\log\left(x^2\right)\right)+\frac{2\arctan\left(cx\right)}{x}\right)bc^4d^4+2i\left(\left(c\arctan\left(cx\right)+\frac{1}{x}\right)c+\frac{\arctan\left(cx\right)}{x^2}\right)bc^3d^4-\left(\left(c^2\log\left(c^2x^2\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^6,x, algorithm="maxima")
```

```
[Out] -1/2*(c*(log(c^2*x^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*b*c^4*d^4 + 2*I*((
c*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*b*c^3*d^4 - ((c^2*log(c^2*x^2 + 1
) - c^2*log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^3)*b*c^2*d^4 - a*c^4*d^4/x +
1/3*I*((3*c^3*arctan(c*x) + (3*c^2*x^2 - 1)/x^3)*c - 3*arctan(c*x)/x^4)*b*c
*d^4 - 1/20*((2*c^4*log(c^2*x^2 + 1) - 2*c^4*log(x^2) - (2*c^2*x^2 - 1)/x^4
)*c + 4*arctan(c*x)/x^5)*b*d^4 + 2*I*a*c^3*d^4/x^2 + 2*a*c^2*d^4/x^3 - I*a*
c*d^4/x^4 - 1/5*a*d^4/x^5
```

Fricas [B] time = 2.39585, size = 475, normalized size = 4.06

$$\frac{192\,bc^5d^4x^5\log\left(x\right)-186\,bc^5d^4x^5\log\left(\frac{cx+i}{c}\right)-6\,bc^5d^4x^5\log\left(\frac{cx-i}{c}\right)-60\left(a-3i\,b\right)c^4d^4x^4+\left(120i\,a+66\,b\right)c^3d^4x^3+20\left(6\,a\right.}{60\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^6,x, algorithm="fricas")
```

```
[Out] 1/60*(192*b*c^5*d^4*x^5*log(x) - 186*b*c^5*d^4*x^5*log((c*x + I)/c) - 6*b*c
^5*d^4*x^5*log((c*x - I)/c) - 60*(a - 3*I*b)*c^4*d^4*x^4 + (120*I*a + 66*b)
*c^3*d^4*x^3 + 20*(6*a - I*b)*c^2*d^4*x^2 + (-60*I*a - 3*b)*c*d^4*x - 12*a*
d^4 + (-30*I*b*c^4*d^4*x^4 - 60*b*c^3*d^4*x^3 + 60*I*b*c^2*d^4*x^2 + 30*b*c
*d^4*x - 6*I*b*d^4)*log(-(c*x + I)/(c*x - I)))/x^5
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**4*(a+b*atan(c*x))/x**6,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.4451, size = 308, normalized size = 2.63

$$-\frac{186\,bc^5d^4x^5\log\left(cx+i\right)+6\,bc^5d^4x^5\log\left(cx-i\right)-192\,bc^5d^4x^5\log\left(x\right)-180\,bc^4d^4ix^4+60\,bc^4d^4x^4\arctan\left(cx\right)+60\,ac^4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^6,x, algorithm="giac")
```

```
[Out] -1/60*(186*b*c^5*d^4*x^5*log(c*x + i) + 6*b*c^5*d^4*x^5*log(c*x - i) - 192*
b*c^5*d^4*x^5*log(x) - 180*b*c^4*d^4*i*x^4 + 60*b*c^4*d^4*x^4*arctan(c*x) +
 60*a*c^4*d^4*x^4 - 120*b*c^3*d^4*i*x^3*arctan(c*x) - 120*a*c^3*d^4*i*x^3 -
 66*b*c^3*d^4*x^3 + 20*b*c^2*d^4*i*x^2 - 120*b*c^2*d^4*x^2*arctan(c*x) - 12
0*a*c^2*d^4*x^2 + 60*b*c*d^4*i*x*arctan(c*x) + 60*a*c*d^4*i*x + 3*b*c*d^4*x
 + 12*b*d^4*arctan(c*x) + 12*a*d^4)/x^5
```

3.41 $\int \frac{(d+icdx)^4\left(a+b\tan^{-1}(cx)\right)}{x^7}\,dx$

Optimal. Leaf size=168

$$\frac{icd^4(1+icx)^5\left(a+b\tan^{-1}(cx)\right)}{30x^5}-\frac{d^4(1+icx)^5\left(a+b\tan^{-1}(cx)\right)}{6x^6}+\frac{16ibc^4d^4}{15x^2}+\frac{5bc^3d^4}{9x^3}-\frac{ibc^2d^4}{5x^4}-\frac{13bc^5d^4}{6x}+\frac{32}{15}ibc^6d^4\text{lo}$$

```
[Out] -(b*c*d^4)/(30*x^5) - ((I/5)*b*c^2*d^4)/x^4 + (5*b*c^3*d^4)/(9*x^3) + (((16
*I)/15)*b*c^4*d^4)/x^2 - (13*b*c^5*d^4)/(6*x) - (d^4*(1 + I*c*x)^5*(a + b*A
rcTan[c*x]))/(6*x^6) + ((I/30)*c*d^4*(1 + I*c*x)^5*(a + b*ArcTan[c*x]))/x^5
 + ((32*I)/15)*b*c^6*d^4*Log[x] - ((32*I)/15)*b*c^6*d^4*Log[I + c*x]
```

Rubi [A] time = 0.113769, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {45, 37, 4872, 12, 148}

$$\frac{icd^4(1+icx)^5\left(a+b\tan^{-1}(cx)\right)}{30x^5}-\frac{d^4(1+icx)^5\left(a+b\tan^{-1}(cx)\right)}{6x^6}+\frac{16ibc^4d^4}{15x^2}+\frac{5bc^3d^4}{9x^3}-\frac{ibc^2d^4}{5x^4}-\frac{13bc^5d^4}{6x}+\frac{32}{15}ibc^6d^4\text{lo}$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)^4*(a + b*ArcTan[c*x]))/x^7,x]
```

```
[Out] -(b*c*d^4)/(30*x^5) - ((I/5)*b*c^2*d^4)/x^4 + (5*b*c^3*d^4)/(9*x^3) + (((16
*I)/15)*b*c^4*d^4)/x^2 - (13*b*c^5*d^4)/(6*x) - (d^4*(1 + I*c*x)^5*(a + b*A
rcTan[c*x]))/(6*x^6) + ((I/30)*c*d^4*(1 + I*c*x)^5*(a + b*ArcTan[c*x]))/x^5
 + ((32*I)/15)*b*c^6*d^4*Log[x] - ((32*I)/15)*b*c^6*d^4*Log[I + c*x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
```

```
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 4872

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
+ b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2), x
], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2*m
] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 148

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(
c + d*x)^n*(e + f*x)^p*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
m}, x] && (IntegersQ[m, n, p] || (IGtQ[n, 0] && IGtQ[p, 0]))
```

Rubi steps

$$\int \frac{(d+icdx)^4\left(a+b\tan^{-1}(cx)\right)}{x^7}\,dx = -\frac{d^4(1+icx)^5\left(a+b\tan^{-1}(cx)\right)}{6x^6}+\frac{icd^4(1+icx)^5\left(a+b\tan^{-1}(cx)\right)}{30x^5}-(bc)\int \frac{d^4(i-$$

$$= -\frac{d^4(1+icx)^5\left(a+b\tan^{-1}(cx)\right)}{6x^6}+\frac{icd^4(1+icx)^5\left(a+b\tan^{-1}(cx)\right)}{30x^5}-\frac{1}{30}\left(bcd^4\right)\int$$

$$= -\frac{d^4(1+icx)^5\left(a+b\tan^{-1}(cx)\right)}{6x^6}+\frac{icd^4(1+icx)^5\left(a+b\tan^{-1}(cx)\right)}{30x^5}-\frac{1}{30}\left(bcd^4\right)\int$$

$$= -\frac{bcd^4}{30x^5}-\frac{ibc^2d^4}{5x^4}+\frac{5bc^3d^4}{9x^3}+\frac{16ibc^4d^4}{15x^2}-\frac{13bc^5d^4}{6x}-\frac{d^4(1+icx)^5\left(a+b\tan^{-1}(cx)\right)}{6x^6}$$

Mathematica [C] time = 0.119118, size = 235, normalized size = 1.4

$$-\frac{d^4\left(15bc^5x^5\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-c^2x^2\right)-15bc^3x^3\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-c^2x^2\right)+bcx\text{Hyperg}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + I*c*d*x)^4*(a + b*ArcTan[c*x]))/x^7,x]
```

```
[Out] -(d^4*(5*a + (24*I)*a*c*x - 45*a*c^2*x^2 + (6*I)*b*c^2*x^2 - (40*I)*a*c^3*x
^3 + 15*a*c^4*x^4 - (32*I)*b*c^4*x^4 + 5*b*ArcTan[c*x] + (24*I)*b*c*x*ArcTa
n[c*x] - 45*b*c^2*x^2*ArcTan[c*x] - (40*I)*b*c^3*x^3*ArcTan[c*x] + 15*b*c^4
*x^4*ArcTan[c*x] + b*c*x*Hypergeometric2F1[-5/2, 1, -3/2, -(c^2*x^2)] - 15*
b*c^3*x^3*Hypergeometric2F1[-3/2, 1, -1/2, -(c^2*x^2)] + 15*b*c^5*x^5*Hyper
geometric2F1[-1/2, 1, 1/2, -(c^2*x^2)] - (64*I)*b*c^6*x^6*Log[x] + (32*I)*b
*c^6*x^6*Log[1 + c^2*x^2]))/(30*x^6)
```

Maple [A] time = 0.039, size = 243, normalized size = 1.5

$$-\frac{d^4c^4a}{2\,x^2}+\frac{3\,c^2d^4a}{2\,x^4}-\frac{d^4a}{6\,x^6}+\frac{\frac{4\,i}{3}c^3d^4a}{x^3}-\frac{\frac{4\,i}{5}cd^4b\arctan\left(cx\right)}{x^5}-\frac{bc^4d^4\arctan\left(cx\right)}{2\,x^2}+\frac{3\,c^2d^4b\arctan\left(cx\right)}{2\,x^4}-\frac{bd^4\arctan\left(cx\right)}{6\,x^6}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^7,x)
```

```
[Out] -1/2*c^4*d^4*a/x^2+3/2*c^2*d^4*a/x^4-1/6*d^4*a/x^6+4/3*I*c^3*d^4*a/x^3-4/5*
I*c*d^4*b*arctan(c*x)/x^5-1/2*c^4*d^4*b*arctan(c*x)/x^2+3/2*c^2*d^4*b*arcta
n(c*x)/x^4-1/6*d^4*b*arctan(c*x)/x^6-4/5*I*c*d^4*a/x^5-1/5*I*b*c^2*d^4/x^4-
16/15*I*c^6*d^4*b*ln(c^2*x^2+1)-13/6*c^6*d^4*b*arctan(c*x)+32/15*I*c^6*d^4*
b*ln(c*x)+16/15*I*b*c^4*d^4/x^2+4/3*I*c^3*d^4*b*arctan(c*x)/x^3-1/30*b*c*d^
4/x^5+5/9*b*c^3*d^4/x^3-13/6*b*c^5*d^4/x
```

Maxima [B] time = 1.48864, size = 392, normalized size = 2.33

$$-\frac{1}{2}\left(\left(c\arctan\left(cx\right)+\frac{1}{x}\right)c+\frac{\arctan\left(cx\right)}{x^2}\right)bc^4d^4-\frac{2}{3}i\left(\left(c^2\log\left(c^2x^2+1\right)-c^2\log\left(x^2\right)-\frac{1}{x^2}\right)c-\frac{2\arctan\left(cx\right)}{x^3}\right)bc^3d^4-\frac{1}{2}\Bigg($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^7,x, algorithm="maxima")
```

```
[Out] -1/2*((c*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*b*c^4*d^4 - 2/3*I*((c^2*lo
g(c^2*x^2 + 1) - c^2*log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^3)*b*c^3*d^4 - 1
/2*((3*c^3*arctan(c*x) + (3*c^2*x^2 - 1)/x^3)*c - 3*arctan(c*x)/x^4)*b*c^2*
d^4 - 1/5*I*((2*c^4*log(c^2*x^2 + 1) - 2*c^4*log(x^2) - (2*c^2*x^2 - 1)/x^4
```

```
)*c + 4*arctan(c*x)/x^5)*b*c*d^4 - 1/2*a*c^4*d^4/x^2 - 1/90*((15*c^5*arctan
(c*x) + (15*c^4*x^4 - 5*c^2*x^2 + 3)/x^5)*c + 15*arctan(c*x)/x^6)*b*d^4 + 4
/3*I*a*c^3*d^4/x^3 + 3/2*a*c^2*d^4/x^4 - 4/5*I*a*c*d^4/x^5 - 1/6*a*d^4/x^6
```

Fricas [A] time = 2.40283, size = 527, normalized size = 3.14

$$\frac{384i\,bc^6d^4x^6\log(x) - 387i\,bc^6d^4x^6\log\left(\frac{cx+i}{c}\right) + 3i\,bc^6d^4x^6\log\left(\frac{cx-i}{c}\right) - 390\,bc^5d^4x^5 - 6\,(15\,a - 32i\,b)c^4d^4x^4 + (240i\,a + \cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^7,x, algorithm="fricas")
```

```
[Out] 1/180*(384*I*b*c^6*d^4*x^6*log(x) - 387*I*b*c^6*d^4*x^6*log((c*x + I)/c) +
3*I*b*c^6*d^4*x^6*log((c*x - I)/c) - 390*b*c^5*d^4*x^5 - 6*(15*a - 32*I*b)*
c^4*d^4*x^4 + (240*I*a + 100*b)*c^3*d^4*x^3 + 18*(15*a - 2*I*b)*c^2*d^4*x^2
 + (-144*I*a - 6*b)*c*d^4*x - 30*a*d^4 + (-45*I*b*c^4*d^4*x^4 - 120*b*c^3*d
^4*x^3 + 135*I*b*c^2*d^4*x^2 + 72*b*c*d^4*x - 15*I*b*d^4)*log(-(c*x + I)/(c
*x - I)))/x^6
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**4*(a+b*atan(c*x))/x**7,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.56953, size = 329, normalized size = 1.96

$$\frac{3\,bc^6d^4ix^6\log(cix+1) - 387\,bc^6d^4ix^6\log(-cix+1) + 384\,bc^6d^4ix^6\log(x) - 390\,bc^5d^4x^5 + 192\,bc^4d^4ix^4 - 90\,bc^4d^4x^4\,a\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^7,x, algorithm="giac")
```

```
[Out] 1/180*(3*b*c^6*d^4*i*x^6*log(c*i*x + 1) - 387*b*c^6*d^4*i*x^6*log(-c*i*x +
1) + 384*b*c^6*d^4*i*x^6*log(x) - 390*b*c^5*d^4*x^5 + 192*b*c^4*d^4*i*x^4 -
 90*b*c^4*d^4*x^4*arctan(c*x) - 90*a*c^4*d^4*x^4 + 240*b*c^3*d^4*i*x^3*arct
an(c*x) + 240*a*c^3*d^4*i*x^3 + 100*b*c^3*d^4*x^3 - 36*b*c^2*d^4*i*x^2 + 27
0*b*c^2*d^4*x^2*arctan(c*x) + 270*a*c^2*d^4*x^2 - 144*b*c*d^4*i*x*arctan(c*
x) - 144*a*c*d^4*i*x - 6*b*c*d^4*x - 30*b*d^4*arctan(c*x) - 30*a*d^4)/x^6
```

3.42 $\int \frac{(d+icdx)^4\left(a+b\tan^{-1}(cx)\right)}{x^8}\,dx$

Optimal. Leaf size=243

$$-\frac{c^4d^4\left(a+b\tan^{-1}(cx)\right)}{3x^3}+\frac{ic^3d^4\left(a+b\tan^{-1}(cx)\right)}{x^4}+\frac{6c^2d^4\left(a+b\tan^{-1}(cx)\right)}{5x^5}-\frac{2icd^4\left(a+b\tan^{-1}(cx)\right)}{3x^6}-\frac{d^4\left(a+b\tan\right.}{7x^7}$$

```
[Out] -(b*c*d^4)/(42*x^6) - (((2*I)/15)*b*c^2*d^4)/x^5 + (47*b*c^3*d^4)/(140*x^4)
 + (((5*I)/9)*b*c^4*d^4)/x^3 - (88*b*c^5*d^4)/(105*x^2) - (((5*I)/3)*b*c^6*
d^4)/x - (d^4*(a + b*ArcTan[c*x]))/(7*x^7) - (((2*I)/3)*c*d^4*(a + b*ArcTan
[c*x]))/x^6 + (6*c^2*d^4*(a + b*ArcTan[c*x]))/(5*x^5) + (I*c^3*d^4*(a + b*A
rcTan[c*x]))/x^4 - (c^4*d^4*(a + b*ArcTan[c*x]))/(3*x^3) - (176*b*c^7*d^4*L
og[x])/105 + (b*c^7*d^4*Log[I - c*x])/210 + (117*b*c^7*d^4*Log[I + c*x])/70
```

Rubi [A] time = 0.195779, antiderivative size = 243, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {43, 4872, 12, 1802}

$$-\frac{c^4d^4\left(a+b\tan^{-1}(cx)\right)}{3x^3}+\frac{ic^3d^4\left(a+b\tan^{-1}(cx)\right)}{x^4}+\frac{6c^2d^4\left(a+b\tan^{-1}(cx)\right)}{5x^5}-\frac{2icd^4\left(a+b\tan^{-1}(cx)\right)}{3x^6}-\frac{d^4\left(a+b\tan\right.}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)^4*(a + b*ArcTan[c*x]))/x^8,x]
```

```
[Out] -(b*c*d^4)/(42*x^6) - (((2*I)/15)*b*c^2*d^4)/x^5 + (47*b*c^3*d^4)/(140*x^4)
 + (((5*I)/9)*b*c^4*d^4)/x^3 - (88*b*c^5*d^4)/(105*x^2) - (((5*I)/3)*b*c^6*
d^4)/x - (d^4*(a + b*ArcTan[c*x]))/(7*x^7) - (((2*I)/3)*c*d^4*(a + b*ArcTan
[c*x]))/x^6 + (6*c^2*d^4*(a + b*ArcTan[c*x]))/(5*x^5) + (I*c^3*d^4*(a + b*A
rcTan[c*x]))/x^4 - (c^4*d^4*(a + b*ArcTan[c*x]))/(3*x^3) - (176*b*c^7*d^4*L
og[x])/105 + (b*c^7*d^4*Log[I - c*x])/210 + (117*b*c^7*d^4*Log[I + c*x])/70
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 4872

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
+ b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2), x
], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2*m
] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(d+icdx)^4 \left(a+b\tan^{-1}(cx)\right)}{x^8}\,dx = -\frac{d^4\left(a+b\tan^{-1}(cx)\right)}{7x^7} - \frac{2icd^4\left(a+b\tan^{-1}(cx)\right)}{3x^6} + \frac{6c^2d^4\left(a+b\tan^{-1}(cx)\right)}{5x^5} + \frac{ic^3d^4}{}$$

$$= -\frac{d^4\left(a+b\tan^{-1}(cx)\right)}{7x^7} - \frac{2icd^4\left(a+b\tan^{-1}(cx)\right)}{3x^6} + \frac{6c^2d^4\left(a+b\tan^{-1}(cx)\right)}{5x^5} + \frac{ic^3d^4}{}$$

$$= -\frac{d^4\left(a+b\tan^{-1}(cx)\right)}{7x^7} - \frac{2icd^4\left(a+b\tan^{-1}(cx)\right)}{3x^6} + \frac{6c^2d^4\left(a+b\tan^{-1}(cx)\right)}{5x^5} + \frac{ic^3d^4}{}$$

$$= -\frac{bcd^4}{42x^6} - \frac{2ibc^2d^4}{15x^5} + \frac{47bc^3d^4}{140x^4} + \frac{5ibc^4d^4}{9x^3} - \frac{88bc^5d^4}{105x^2} - \frac{5ibc^6d^4}{3x} - \frac{d^4\left(a+b\tan^{-1}(cx)\right)}{7x^7}$$

Mathematica [C] time = 0.0962663, size = 293, normalized size = 1.21

$$\frac{ibc^4d^4\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-c^2x^2\right)}{3x^3} - \frac{2ibc^2d^4\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},-c^2x^2\right)}{15x^5} - \frac{ac^4d^4}{3x^3} + \frac{iac^3d^4}{x^4} +$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + I*c*d*x)^4*(a + b*ArcTan[c*x]))/x^8,x]
```

```
[Out] -(a*d^4)/(7*x^7) - (((2*I)/3)*a*c*d^4)/x^6 - (b*c*d^4)/(42*x^6) + (6*a*c^2*
d^4)/(5*x^5) + (I*a*c^3*d^4)/x^4 + (47*b*c^3*d^4)/(140*x^4) - (a*c^4*d^4)/(
```

```
3*x^3) - (88*b*c^5*d^4)/(105*x^2) - (b*d^4*ArcTan[c*x])/(7*x^7) - (((2*I)/3
)*b*c*d^4*ArcTan[c*x])/x^6 + (6*b*c^2*d^4*ArcTan[c*x])/(5*x^5) + (I*b*c^3*d
^4*ArcTan[c*x])/x^4 - (b*c^4*d^4*ArcTan[c*x])/(3*x^3) - (((2*I)/15)*b*c^2*d
^4*Hypergeometric2F1[-5/2, 1, -3/2, -(c^2*x^2)])/x^5 + ((I/3)*b*c^4*d^4*Hyp
ergeometric2F1[-3/2, 1, -1/2, -(c^2*x^2)])/x^3 - (176*b*c^7*d^4*Log[x])/105
 + (88*b*c^7*d^4*Log[1 + c^2*x^2])/105
```

Maple [A] time = 0.036, size = 255, normalized size = 1.1

$$-\frac{d^4a}{7\,x^7}-\frac{\frac{2\,i}{3}cd^4a}{x^6}-\frac{\frac{5\,i}{3}bc^6d^4}{x}+\frac{6\,c^2d^4a}{5\,x^5}-\frac{d^4c^4a}{3\,x^3}-\frac{bd^4\arctan(cx)}{7\,x^7}-\frac{\frac{2\,i}{15}bc^2d^4}{x^5}+\frac{ic^3d^4b\arctan(cx)}{x^4}+\frac{6\,c^2d^4b\arctan(cx)}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^8,x)
```

```
[Out] -1/7*d^4*a/x^7-2/3*I*c*d^4*a/x^6-5/3*I*b*c^6*d^4/x+6/5*c^2*d^4*a/x^5-1/3*c^
4*d^4*a/x^3-1/7*d^4*b*arctan(c*x)/x^7-2/15*I*b*c^2*d^4/x^5+I*c^3*d^4*b*arct
an(c*x)/x^4+6/5*c^2*d^4*b*arctan(c*x)/x^5-1/3*c^4*d^4*b*arctan(c*x)/x^3+88/
105*c^7*d^4*b*ln(c^2*x^2+1)+5/9*I*b*c^4*d^4/x^3-5/3*I*c^7*d^4*b*arctan(c*x)
-2/3*I*c*d^4*b*arctan(c*x)/x^6+I*c^3*d^4*a/x^4-1/42*b*c*d^4/x^6+47/140*b*c^
3*d^4/x^4-88/105*b*c^5*d^4/x^2-176/105*c^7*d^4*b*ln(c*x)
```

Maxima [A] time = 1.4912, size = 444, normalized size = 1.83

$$\frac{1}{6}\left(\left(\left(c^2\log\left(c^2x^2+1\right)-c^2\log\left(x^2\right)-\frac{1}{x^2}\right)c-\frac{2\arctan(cx)}{x^3}\right)bc^4d^4-\frac{1}{3}i\left(\left(3\,c^3\arctan(cx)+\frac{3\,c^2x^2-1}{x^3}\right)c-\frac{3\arctan(cx)}{x^4}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^8,x, algorithm="maxima")
```

```
[Out] 1/6*((c^2*log(c^2*x^2 + 1) - c^2*log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^3)*b
*c^4*d^4 - 1/3*I*((3*c^3*arctan(c*x) + (3*c^2*x^2 - 1)/x^3)*c - 3*arctan(c*
x)/x^4)*b*c^3*d^4 + 3/10*((2*c^4*log(c^2*x^2 + 1) - 2*c^4*log(x^2) - (2*c^2
*x^2 - 1)/x^4)*c + 4*arctan(c*x)/x^5)*b*c^2*d^4 - 2/45*I*((15*c^5*arctan(c*
x) + (15*c^4*x^4 - 5*c^2*x^2 + 3)/x^5)*c + 15*arctan(c*x)/x^6)*b*c*d^4 + 1/
84*((6*c^6*log(c^2*x^2 + 1) - 6*c^6*log(x^2) - (6*c^4*x^4 - 3*c^2*x^2 + 2)/
x^6)*c - 12*arctan(c*x)/x^7)*b*d^4 - 1/3*a*c^4*d^4/x^3 + I*a*c^3*d^4/x^4 +
```

```
6/5*a*c^2*d^4/x^5 - 2/3*I*a*c*d^4/x^6 - 1/7*a*d^4/x^7
```

Fricas [A] time = 2.35951, size = 560, normalized size = 2.3

$$-\frac{2112\,bc^7d^4x^7\log(x) - 2106\,bc^7d^4x^7\log\left(\frac{cx+i}{c}\right) - 6\,bc^7d^4x^7\log\left(\frac{cx-i}{c}\right) + 2100i\,bc^6d^4x^6 + 1056\,bc^5d^4x^5 + 140\,(3\,a - 5i\,b)c^4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^8,x, algorithm="fricas")
```

```
[Out] -1/1260*(2112*b*c^7*d^4*x^7*log(x) - 2106*b*c^7*d^4*x^7*log((c*x + I)/c) -
6*b*c^7*d^4*x^7*log((c*x - I)/c) + 2100*I*b*c^6*d^4*x^6 + 1056*b*c^5*d^4*x^
5 + 140*(3*a - 5*I*b)*c^4*d^4*x^4 - (1260*I*a + 423*b)*c^3*d^4*x^3 - 168*(9
*a - I*b)*c^2*d^4*x^2 - (-840*I*a - 30*b)*c*d^4*x + 180*a*d^4 - (-210*I*b*c
^4*d^4*x^4 - 630*b*c^3*d^4*x^3 + 756*I*b*c^2*d^4*x^2 + 420*b*c*d^4*x - 90*I
*b*d^4)*log(-(c*x + I)/(c*x - I)))/x^7
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**4*(a+b*atan(c*x))/x**8,x)
```

```
[Out] Timed out
```

Giac [A] time = 2.57516, size = 342, normalized size = 1.41

$$\frac{2106\,bc^7d^4x^7\log(cx+i) + 6\,bc^7d^4x^7\log(cx-i) - 2112\,bc^7d^4x^7\log(x) - 2100\,bc^6d^4ix^6 - 1056\,bc^5d^4x^5 + 700\,bc^4d^4ix^4 -}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^8,x, algorithm="giac")
```

```
[Out] 1/1260*(2106*b*c^7*d^4*x^7*log(c*x + i) + 6*b*c^7*d^4*x^7*log(c*x - i) - 21
12*b*c^7*d^4*x^7*log(x) - 2100*b*c^6*d^4*i*x^6 - 1056*b*c^5*d^4*x^5 + 700*b
*c^4*d^4*i*x^4 - 420*b*c^4*d^4*x^4*arctan(c*x) - 420*a*c^4*d^4*x^4 + 1260*b
*c^3*d^4*i*x^3*arctan(c*x) + 1260*a*c^3*d^4*i*x^3 + 423*b*c^3*d^4*x^3 - 168
*b*c^2*d^4*i*x^2 + 1512*b*c^2*d^4*x^2*arctan(c*x) + 1512*a*c^2*d^4*x^2 - 84
0*b*c*d^4*i*x*arctan(c*x) - 840*a*c*d^4*i*x - 30*b*c*d^4*x - 180*b*d^4*arct
an(c*x) - 180*a*d^4)/x^7
```

3.43 $\int \frac{x^3(a+b\tan^{-1}(cx))}{d+icdx}\,dx$

Optimal. Leaf size=196

$$\frac{ib\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{2c^4d}+\frac{x^2\left(a+b\tan^{-1}(cx)\right)}{2c^2d}+\frac{\log\left(\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{c^4d}-\frac{ix^3\left(a+b\tan^{-1}(cx)\right)}{3cd}+\frac{iax}{c^3d}+\frac{ibx^2}{6c^2d}-$$

```
[Out] (I*a*x)/(c^3*d) - (b*x)/(2*c^3*d) + ((I/6)*b*x^2)/(c^2*d) + (b*ArcTan[c*x])
/(2*c^4*d) + (I*b*x*ArcTan[c*x])/(c^3*d) + (x^2*(a + b*ArcTan[c*x]))/(2*c^2
*d) - ((I/3)*x^3*(a + b*ArcTan[c*x]))/(c*d) + ((a + b*ArcTan[c*x])*Log[2/(1
 + I*c*x)])/(c^4*d) - (((2*I)/3)*b*Log[1 + c^2*x^2])/(c^4*d) + ((I/2)*b*Pol
yLog[2, 1 - 2/(1 + I*c*x)])/(c^4*d)
```

Rubi [A] time = 0.286152, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {4866, 4852, 266, 43, 321, 203, 4846, 260, 4854, 2402, 2315}

$$\frac{ib\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{2c^4d}+\frac{x^2\left(a+b\tan^{-1}(cx)\right)}{2c^2d}+\frac{\log\left(\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{c^4d}-\frac{ix^3\left(a+b\tan^{-1}(cx)\right)}{3cd}+\frac{iax}{c^3d}+\frac{ibx^2}{6c^2d}-$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcTan[c*x]))/(d + I*c*d*x),x]
```


```
[Out] (I*a*x)/(c^3*d) - (b*x)/(2*c^3*d) + ((I/6)*b*x^2)/(c^2*d) + (b*ArcTan[c*x])
/(2*c^4*d) + (I*b*x*ArcTan[c*x])/(c^3*d) + (x^2*(a + b*ArcTan[c*x]))/(2*c^2
*d) - ((I/3)*x^3*(a + b*ArcTan[c*x]))/(c*d) + ((a + b*ArcTan[c*x])*Log[2/(1
 + I*c*x)])/(c^4*d) - (((2*I)/3)*b*Log[1 + c^2*x^2])/(c^4*d) + ((I/2)*b*Pol
yLog[2, 1 - 2/(1 + I*c*x)])/(c^4*d)
```


Rule 4866

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_) + (
e_.)*(x_)), x_Symbol] :> Dist[f/e, Int[(f*x)^(m - 1)*(a + b*ArcTan[c*x])^p,
 x], x] - Dist[(d*f)/e, Int[((f*x)^(m - 1)*(a + b*ArcTan[c*x])^p)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2
, 0] && GtQ[m, 0]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3\left(a+b\tan^{-1}(cx)\right)}{d+icdx}\,dx &= \frac{i\int \frac{x^2\left(a+b\tan^{-1}(cx)\right)}{d+icdx}\,dx}{c}-\frac{i\int x^2\left(a+b\tan^{-1}(cx)\right)dx}{cd}\\
&= -\frac{ix^3\left(a+b\tan^{-1}(cx)\right)}{3cd}-\frac{\int \frac{x\left(a+b\tan^{-1}(cx)\right)}{d+icdx}\,dx}{c^2}+\frac{(ib)\int \frac{x^3}{1+c^2x^2}\,dx}{3d}+\frac{\int x\left(a+b\tan^{-1}(cx)\right)dx}{c^2d}\\
&= \frac{x^2\left(a+b\tan^{-1}(cx)\right)}{2c^2d}-\frac{ix^3\left(a+b\tan^{-1}(cx)\right)}{3cd}-\frac{i\int \frac{a+b\tan^{-1}(cx)}{d+icdx}\,dx}{c^3}+\frac{(ib)\,\mathrm{Subst}\left(\int \frac{x}{1+c^2x}\,dx,\right.}{6d}\\
&= \frac{iax}{c^3d}-\frac{bx}{2c^3d}+\frac{x^2\left(a+b\tan^{-1}(cx)\right)}{2c^2d}-\frac{ix^3\left(a+b\tan^{-1}(cx)\right)}{3cd}+\frac{\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2}{1+icx}\right)}{c^4d}\\
&= \frac{iax}{c^3d}-\frac{bx}{2c^3d}+\frac{ibx^2}{6c^2d}+\frac{b\tan^{-1}(cx)}{2c^4d}+\frac{ibx\tan^{-1}(cx)}{c^3d}+\frac{x^2\left(a+b\tan^{-1}(cx)\right)}{2c^2d}-\frac{ix^3\left(a+b\tan^{-1}\right.}{3cd}\\
&= \frac{iax}{c^3d}-\frac{bx}{2c^3d}+\frac{ibx^2}{6c^2d}+\frac{b\tan^{-1}(cx)}{2c^4d}+\frac{ibx\tan^{-1}(cx)}{c^3d}+\frac{x^2\left(a+b\tan^{-1}(cx)\right)}{2c^2d}-\frac{ix^3\left(a+b\tan^{-1}\right.}{3cd}
\end{aligned}$$

Mathematica [A] time = 0.442145, size = 166, normalized size = 0.85

$$-\frac{i\left(3b\mathrm{PolyLog}\left(2,-e^{2i\tan^{-1}(cx)}\right)+\tan^{-1}(cx)\left(6a+b\left(2c^3x^3+3ic^2x^2-6cx+3i\right)+6ib\log\left(1+e^{2i\tan^{-1}(cx)}\right)\right)+2ac^3x^3+3\right.}{6c^4d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(a + b*ArcTan[c*x]))/(d + I*c*d*x),x]
```

```
[Out] ((-I/6)*(-b - 6*a*c*x - (3*I)*b*c*x + (3*I)*a*c^2*x^2 - b*c^2*x^2 + 2*a*c^3
*x^3 + 6*b*ArcTan[c*x]^2 + ArcTan[c*x]*(6*a + b*(3*I - 6*c*x + (3*I)*c^2*x^
2 + 2*c^3*x^3) + (6*I)*b*Log[1 + E^((2*I)*ArcTan[c*x])]) - (3*I)*a*Log[1 +
c^2*x^2] + 4*b*Log[1 + c^2*x^2] + 3*b*PolyLog[2, -E^((2*I)*ArcTan[c*x])]))/
(c^4*d)
```

Maple [B] time = 0.053, size = 353, normalized size = 1.8

$$\frac{-\frac{i}{3}ax^3}{dc}+\frac{\frac{i}{6}bx^2}{c^2d}+\frac{ax^2}{2\,c^2d}-\frac{a\ln\left(c^2x^2+1\right)}{2\,dc^4}-\frac{ia\arctan\left(cx\right)}{dc^4}+\frac{iax}{dc^3}+\frac{\frac{i}{2}b\mathrm{dilog}\left(-\frac{i}{2}\left(cx+i\right)\right)}{dc^4}+\frac{bx^2\arctan\left(cx\right)}{2\,c^2d}-\frac{b\arctan$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctan(c*x))/(d+I*c*d*x),x)
```

```
[Out] -1/3*I/c*a/d*x^3+1/6*I*b*x^2/c^2/d+1/2/c^2*a/d*x^2-1/2/c^4*a/d*ln(c^2*x^2+1
)-I/c^4*a/d*arctan(c*x)+I*a*x/d/c^3+1/2*I/c^4*b/d*dilog(-1/2*I*(c*x+I))+1/2
/c^2*b/d*arctan(c*x)*x^2-1/c^4*b/d*arctan(c*x)*ln(c*x-I)-1/3*I/c*b/d*arctan
(c*x)*x^3+I*b*x*arctan(c*x)/d/c^3+1/2*I/c^4*b/d*ln(-1/2*I*(c*x+I))*ln(c*x-I
)-1/2*b*x/d/c^3-1/4*I/c^4*b/d*ln(c*x-I)^2-11/24*I/c^4*b/d*ln(c^2*x^2+1)+2/3
*I/c^4*b/d+5/24/c^4*b/d*arctan(1/2*c*x)-5/24/c^4*b/d*arctan(1/6*c^3*x^3+7/6
*c*x)-5/12/c^4*b/d*arctan(1/2*c*x-1/2*I)-5/48*I/c^4*b/d*ln(c^4*x^4+10*c^2*x
^2+9)+11/12*b*arctan(c*x)/d/c^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{6}a\left(\frac{i\left(2\,c^2x^3+3i\,cx^2-6\,x\right)}{c^3d}+\frac{6\,\log\left(i\,cx+1\right)}{c^4d}\right)-\frac{-\frac{1}{2}\left(-12i\left(2\left(\frac{c^2x^3-3\,x}{c^7d}+\frac{3\,\arctan(cx)}{c^8d}\right)\arctan\left(cx\right)-\frac{c^2x^2+3\,\arctan(cx)^2-4\,\mathrm{l}}{c^8d}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))/(d+I*c*d*x),x, algorithm="maxima")
```

```
[Out] -1/6*a*(I*(2*c^2*x^3 + 3*I*c*x^2 - 6*x)/(c^3*d) + 6*log(I*c*x + 1)/(c^4*d))
 - 1/72*(432*I*c^8*d*integrate(1/12*x^4*arctan(c*x)/(c^5*d*x^2 + c^3*d), x)
 + 216*c^8*d*integrate(1/12*x^4*log(c^2*x^2 + 1)/(c^5*d*x^2 + c^3*d), x) -
432*c^7*d*integrate(1/12*x^3*arctan(c*x)/(c^5*d*x^2 + c^3*d), x) + 216*I*c^
7*d*integrate(1/12*x^3*log(c^2*x^2 + 1)/(c^5*d*x^2 + c^3*d), x) + 432*c^5*d
*integrate(1/12*x*arctan(c*x)/(c^5*d*x^2 + c^3*d), x) - 216*I*c^5*d*integra
te(1/12*x*log(c^2*x^2 + 1)/(c^5*d*x^2 + c^3*d), x) + 4*c^3*x^3 - 216*c^4*d*
integrate(1/12*log(c^2*x^2 + 1)/(c^5*d*x^2 + c^3*d), x) + 3*I*c^2*x^2 - 30*
c*x + (12*I*c^3*x^3 - 18*c^2*x^2 - 36*I*c*x + 30)*arctan(c*x) + 18*I*arctan
(c*x)^2 - (6*c^3*x^3 + 9*I*c^2*x^2 - 18*c*x + 3*I)*log(c^2*x^2 + 1) + 9*I*l
og(c^2*x^2 + 1)^2 + 18*I*log(12*c^5*d*x^2 + 12*c^3*d))*b/(c^4*d)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{bx^3\log\left(-\frac{cx+i}{cx-i}\right)-2i\,ax^3}{2\,cdx-2i\,d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))/(d+I*c*d*x),x, algorithm="fricas")
```

```
[Out] integral((b*x^3*log(-(c*x + I)/(c*x - I)) - 2*I*a*x^3)/(2*c*d*x - 2*I*d), x
)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atan(c*x))/(d+I*c*d*x),x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(b\arctan(cx)+a)x^3}{i\,cdx+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))/(d+I*c*d*x),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)*x^3/(I*c*d*x + d), x)
```

3.44 $\int \frac{x^2(a+b\tan^{-1}(cx))}{d+icdx} dx$

Optimal. Leaf size=156

$$\frac{b\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{2c^3d}-\frac{i\log\left(\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{c^3d}-\frac{ix^2\left(a+b\tan^{-1}(cx)\right)}{2cd}+\frac{ax}{c^2d}-\frac{b\log\left(c^2x^2+1\right)}{2c^3d}+\frac{ibx}{2c^2d}+\frac{bx\,\mathrm{t}}{}$$

```
[Out] (a*x)/(c^2*d) + ((I/2)*b*x)/(c^2*d) - ((I/2)*b*ArcTan[c*x])/(c^3*d) + (b*x*
ArcTan[c*x])/(c^2*d) - ((I/2)*x^2*(a + b*ArcTan[c*x]))/(c*d) - (I*(a + b*Ar
cTan[c*x])*Log[2/(1 + I*c*x)])/(c^3*d) - (b*Log[1 + c^2*x^2])/(2*c^3*d) + (
b*PolyLog[2, 1 - 2/(1 + I*c*x)])/(2*c^3*d)
```

Rubi [A] time = 0.181002, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {4866, 4852, 321, 203, 4846, 260, 4854, 2402, 2315}

$$\frac{b\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{2c^3d}-\frac{i\log\left(\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{c^3d}-\frac{ix^2\left(a+b\tan^{-1}(cx)\right)}{2cd}+\frac{ax}{c^2d}-\frac{b\log\left(c^2x^2+1\right)}{2c^3d}+\frac{ibx}{2c^2d}+\frac{bx\,\mathrm{t}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcTan[c*x]))/(d + I*c*d*x),x]
```


```
[Out] (a*x)/(c^2*d) + ((I/2)*b*x)/(c^2*d) - ((I/2)*b*ArcTan[c*x])/(c^3*d) + (b*x*
ArcTan[c*x])/(c^2*d) - ((I/2)*x^2*(a + b*ArcTan[c*x]))/(c*d) - (I*(a + b*Ar
cTan[c*x])*Log[2/(1 + I*c*x)])/(c^3*d) - (b*Log[1 + c^2*x^2])/(2*c^3*d) + (
b*PolyLog[2, 1 - 2/(1 + I*c*x)])/(2*c^3*d)
```


Rule 4866

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_) + (
e_.)*(x_)), x_Symbol] :> Dist[f/e, Int[(f*x)^(m - 1)*(a + b*ArcTan[c*x])^p,
 x], x] - Dist[(d*f)/e, Int[((f*x)^(m - 1)*(a + b*ArcTan[c*x])^p)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2
, 0] && GtQ[m, 0]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
```

```
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int \frac{x^2\left(a+b\tan^{-1}(cx)\right)}{d+icdx}\,dx = \frac{i\int \frac{x\left(a+b\tan^{-1}(cx)\right)}{d+icdx}\,dx}{c} - \frac{i\int x\left(a+b\tan^{-1}(cx)\right)\,dx}{cd}$$

$$= -\frac{ix^2\left(a+b\tan^{-1}(cx)\right)}{2cd} - \frac{\int \frac{a+b\tan^{-1}(cx)}{d+icdx}\,dx}{c^2} + \frac{(ib)\int \frac{x^2}{1+c^2x^2}\,dx}{2d} + \frac{\int\left(a+b\tan^{-1}(cx)\right)\,dx}{c^2d}$$

$$= \frac{ax}{c^2d} + \frac{ibx}{2c^2d} - \frac{ix^2\left(a+b\tan^{-1}(cx)\right)}{2cd} - \frac{i\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2}{1+icx}\right)}{c^3d} - \frac{(ib)\int \frac{1}{1+c^2x^2}\,dx}{2c^2d} + \frac{(ib}{}$$

$$= \frac{ax}{c^2d} + \frac{ibx}{2c^2d} - \frac{ib\tan^{-1}(cx)}{2c^3d} + \frac{bx\tan^{-1}(cx)}{c^2d} - \frac{ix^2\left(a+b\tan^{-1}(cx)\right)}{2cd} - \frac{i\left(a+b\tan^{-1}(cx)\right)\log}{c^3d}$$

$$= \frac{ax}{c^2d} + \frac{ibx}{2c^2d} - \frac{ib\tan^{-1}(cx)}{2c^3d} + \frac{bx\tan^{-1}(cx)}{c^2d} - \frac{ix^2\left(a+b\tan^{-1}(cx)\right)}{2cd} - \frac{i\left(a+b\tan^{-1}(cx)\right)\log}{c^3d}$$

Mathematica [A] time = 0.186967, size = 132, normalized size = 0.85

$$-\frac{b\text{PolyLog}\left(2,-e^{2i\tan^{-1}(cx)}\right) + i\tan^{-1}(cx)\left(-2ia + bc^2x^2 + 2ibcx + 2b\log\left(1+e^{2i\tan^{-1}(cx)}\right) + b\right) + iac^2x^2 - ia\log\left(c^2x^2 +\right.}{2c^3d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(a + b*ArcTan[c*x]))/(d + I*c*d*x),x]
```

```
[Out] -(-2*a*c*x - I*b*c*x + I*a*c^2*x^2 + 2*b*ArcTan[c*x]^2 + I*ArcTan[c*x]*((-2
*I)*a + b + (2*I)*b*c*x + b*c^2*x^2 + 2*b*Log[1 + E^((2*I)*ArcTan[c*x])]) -
 I*a*Log[1 + c^2*x^2] + b*Log[1 + c^2*x^2] + b*PolyLog[2, -E^((2*I)*ArcTan[
c*x])])/(2*c^3*d)
```

Maple [B] time = 0.05, size = 308, normalized size = 2.

$$\frac{ax}{c^2d} + \frac{\frac{i}{2}a\ln\left(c^2x^2+1\right)}{dc^3} + \frac{\frac{i}{4}b}{dc^3}\arctan\left(\frac{cx}{2}-\frac{i}{2}\right) - \frac{a\arctan(cx)}{dc^3} + \frac{bx\arctan(cx)}{c^2d} + \frac{\frac{i}{2}bx}{c^2d} + \frac{\frac{i}{8}b}{dc^3}\arctan\left(\frac{c^3x^3}{6}+\frac{7cx}{6}\right) + \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctan(c*x))/(d+I*c*d*x),x)
```

```
[Out] 1/c^2*a/d*x+1/2*I/c^3*a/d*ln(c^2*x^2+1)+1/4*I/c^3*b/d*arctan(1/2*c*x-1/2*I)
-1/c^3*a/d*arctan(c*x)+b*x*arctan(c*x)/c^2/d+1/2*I*b*x/c^2/d+1/8*I/c^3*b/d*
arctan(1/6*c^3*x^3+7/6*c*x)+1/2/c^3*b/d*ln(c*x-I)*ln(-1/2*I*(c*x+I))+1/2/c^
3*b/d*dilog(-1/2*I*(c*x+I))-1/4/c^3*b/d*ln(c*x-I)^2-3/4*I/c^3*b/d*arctan(c*
x)+1/2/c^3*b/d-1/16/c^3*b/d*ln(c^4*x^4+10*c^2*x^2+9)-1/2*I/c*b/d*arctan(c*x
)*x^2+I/c^3*b/d*arctan(c*x)*ln(c*x-I)-1/8*I/c^3*b/d*arctan(1/2*c*x)-3/8*b*l
n(c^2*x^2+1)/d/c^3-1/2*I/c*a/d*x^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2}\,a\left(\frac{i\,cx^2-2\,x}{c^2d}-\frac{2i\,\log\left(i\,cx+1\right)}{c^3d}\right)-\frac{\frac{1}{2}\left(\left(\left(2\left(\frac{x^2}{c^4d}-\frac{\log\left(c^2x^2+1\right)}{c^6d}\right)\log\left(c^2x^2+1\right)-\frac{2\,c^2x^2-\log\left(c^2x^2+1\right)^2-2\,\log\left(c^2x^2+1\right)}{c^6d}\right)c^6d+8i\,c^6\cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))/(d+I*c*d*x),x, algorithm="maxima")
```

```
[Out] -1/2*a*((I*c*x^2 - 2*x)/(c^2*d) - 2*I*log(I*c*x + 1)/(c^3*d)) - 1/8*(32*I*c
^6*d*integrate(1/8*x^3*arctan(c*x)/(c^4*d*x^2 + c^2*d), x) + 16*c^6*d*integ
rate(1/8*x^3*log(c^2*x^2 + 1)/(c^4*d*x^2 + c^2*d), x) - 32*c^5*d*integrate(
1/8*x^2*arctan(c*x)/(c^4*d*x^2 + c^2*d), x) + 16*I*c^5*d*integrate(1/8*x^2*
log(c^2*x^2 + 1)/(c^4*d*x^2 + c^2*d), x) - 32*I*c^4*d*integrate(1/8*x*arcta
n(c*x)/(c^4*d*x^2 + c^2*d), x) - 16*c^4*d*integrate(1/8*x*log(c^2*x^2 + 1)/
(c^4*d*x^2 + c^2*d), x) + 16*I*c^3*d*integrate(1/8*log(c^2*x^2 + 1)/(c^4*d*
x^2 + c^2*d), x) + c^2*x^2 + 2*I*c*x + (2*I*c^2*x^2 - 4*c*x - 2*I)*arctan(c
*x) + 2*arctan(c*x)^2 - (c^2*x^2 + 2*I*c*x + 1)*log(c^2*x^2 + 1) + log(c^2*
x^2 + 1)^2 + 2*log(8*c^4*d*x^2 + 8*c^2*d))*b/(c^3*d)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{bx^2\log\left(-\frac{cx+i}{cx-i}\right)-2i\,ax^2}{2\,cdx-2i\,d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))/(d+I*c*d*x),x, algorithm="fricas")
```

```
[Out] integral((b*x^2*log(-(c*x + I)/(c*x - I)) - 2*I*a*x^2)/(2*c*d*x - 2*I*d), x
)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atan(c*x))/(d+I*c*d*x),x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \arctan(cx) + a)x^2}{i\,cdx + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))/(d+I*c*d*x),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)*x^2/(I*c*d*x + d), x)
```

3.45 $\int \frac{x\left(a+b\tan^{-1}(cx)\right)}{d+icdx}\,dx$

Optimal. Leaf size=110

$$-\frac{ib\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{2c^2d}-\frac{\log\left(\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{c^2d}-\frac{iax}{cd}+\frac{ib\log\left(c^2x^2+1\right)}{2c^2d}-\frac{ibx\tan^{-1}(cx)}{cd}$$

```
[Out] ((-I)*a*x)/(c*d) - (I*b*x*ArcTan[c*x])/(c*d) - ((a + b*ArcTan[c*x])*Log[2/(
1 + I*c*x)])/(c^2*d) + ((I/2)*b*Log[1 + c^2*x^2])/(c^2*d) - ((I/2)*b*PolyLo
g[2, 1 - 2/(1 + I*c*x)])/(c^2*d)
```

Rubi [A] time = 0.103856, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4866, 4846, 260, 4854, 2402, 2315}

$$-\frac{ib\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{2c^2d}-\frac{\log\left(\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{c^2d}-\frac{iax}{cd}+\frac{ib\log\left(c^2x^2+1\right)}{2c^2d}-\frac{ibx\tan^{-1}(cx)}{cd}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcTan[c*x]))/(d + I*c*d*x),x]
```

```
[Out] ((-I)*a*x)/(c*d) - (I*b*x*ArcTan[c*x])/(c*d) - ((a + b*ArcTan[c*x])*Log[2/(
1 + I*c*x)])/(c^2*d) + ((I/2)*b*Log[1 + c^2*x^2])/(c^2*d) - ((I/2)*b*PolyLo
g[2, 1 - 2/(1 + I*c*x)])/(c^2*d)
```

Rule 4866

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_) + (
e_.)*(x_)), x_Symbol] :> Dist[f/e, Int[(f*x)^(m - 1)*(a + b*ArcTan[c*x])^p,
 x], x] - Dist[(d*f)/e, Int[((f*x)^(m - 1)*(a + b*ArcTan[c*x])^p)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2
, 0] && GtQ[m, 0]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x\left(a+b\tan^{-1}(cx)\right)}{d+icdx}\,dx &= \frac{i\int \frac{a+b\tan^{-1}(cx)}{d+icdx}\,dx}{c}-\frac{i\int \left(a+b\tan^{-1}(cx)\right)dx}{cd}\\
&= -\frac{iax}{cd}-\frac{\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2}{1+icx}\right)}{c^2d}-\frac{(ib)\int \tan^{-1}(cx)\,dx}{cd}+\frac{b\int \frac{\log\left(\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx}{cd}\\
&= -\frac{iax}{cd}-\frac{ibx\tan^{-1}(cx)}{cd}-\frac{\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2}{1+icx}\right)}{c^2d}+\frac{(ib)\int \frac{x}{1+c^2x^2}\,dx}{d}-\frac{(ib)\operatorname{Subst}\left(\int \frac{\log(2}{1-2}\right.}{c^2d}\\
&= -\frac{iax}{cd}-\frac{ibx\tan^{-1}(cx)}{cd}-\frac{\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2}{1+icx}\right)}{c^2d}+\frac{ib\log\left(1+c^2x^2\right)}{2c^2d}-\frac{ib\text{Li}_2\left(1-\frac{2}{1+icx}\right)}{2c^2d}\end{aligned}$$

Mathematica [A] time = 0.167911, size = 108, normalized size = 0.98

$$\frac{ib\text{PolyLog}\left(2,-e^{2i\tan^{-1}(cx)}\right)+2i\tan^{-1}(cx)\left(a-bcx+ib\log\left(1+e^{2i\tan^{-1}(cx)}\right)\right)+a\log\left(c^2x^2+1\right)-2iacx+ib\log\left(c^2x^2+\right.}{2c^2d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(a + b*ArcTan[c*x]))/(d + I*c*d*x),x]
```

```
[Out] ((-2*I)*a*c*x + (2*I)*b*ArcTan[c*x]^2 + (2*I)*ArcTan[c*x]*(a - b*c*x + I*b*
Log[1 + E^((2*I)*ArcTan[c*x])]) + a*Log[1 + c^2*x^2] + I*b*Log[1 + c^2*x^2]
 + I*b*PolyLog[2, -E^((2*I)*ArcTan[c*x])])/(2*c^2*d)
```

Maple [B] time = 0.049, size = 244, normalized size = 2.2

$$\frac{-iax}{dc}+\frac{a\ln\left(c^2x^2+1\right)}{2\,c^2d}+\frac{ia\arctan(cx)}{c^2d}-\frac{ibx\arctan(cx)}{dc}+\frac{b\arctan(cx)\ln(cx-i)}{c^2d}-\frac{\frac{i}{2}b\ln\left(-\frac{i}{2}(cx+i)\right)\ln(cx-i)}{c^2d}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctan(c*x))/(d+I*c*d*x),x)
```

```
[Out] -I*a*x/d/c+1/2/c^2*a/d*ln(c^2*x^2+1)+I/c^2*a/d*arctan(c*x)-I*b*x*arctan(c*x
)/d/c+1/c^2*b/d*arctan(c*x)*ln(c*x-I)-1/2*I/c^2*b/d*ln(-1/2*I*(c*x+I))*ln(c
*x-I)-1/2*I/c^2*b/d*dilog(-1/2*I*(c*x+I))+1/4*I/c^2*b/d*ln(c*x-I)^2+1/8*I/c
^2*b/d*ln(c^8*x^8+12*c^6*x^6+30*c^4*x^4+28*c^2*x^2+9)-1/4/c^2*b/d*arctan(1/
12*c^3*x^3+13/12*c*x)-1/4/c^2*b/d*arctan(1/4*c*x)+1/2/c^2*b/d*arctan(1/2*c*
x-1/2*I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$a\left(-\frac{ix}{cd}+\frac{\log(icx+1)}{c^2d}\right)-\frac{\left(2i\left(2\left(\frac{x}{c^3d}-\frac{\arctan(cx)}{c^4d}\right)\arctan(cx)+\frac{\arctan(cx)^2-\log\left(c^2x^2+1\right)}{c^4d}\right)c^4d+2\,c^4d\int\frac{x^2\log\left(c^2x^2+1\right)}{c^3dx^2+cd}\,dx-8\,c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))/(d+I*c*d*x),x, algorithm="maxima")
```

```
[Out] a*(-I*x/(c*d) + log(I*c*x + 1)/(c^2*d)) - 1/8*(8*I*c^4*d*integrate(1/2*x^2*
arctan(c*x)/(c^3*d*x^2 + c*d), x) + 4*c^4*d*integrate(1/2*x^2*log(c^2*x^2 +
 1)/(c^3*d*x^2 + c*d), x) - 16*c^3*d*integrate(1/2*x*arctan(c*x)/(c^3*d*x^2
 + c*d), x) + 8*I*c^3*d*integrate(1/2*x*log(c^2*x^2 + 1)/(c^3*d*x^2 + c*d),
 x) + 4*c^2*d*integrate(1/2*log(c^2*x^2 + 1)/(c^3*d*x^2 + c*d), x) - 2*c*x*
log(c^2*x^2 + 1) + 4*c*x - 4*(-I*c*x + 1)*arctan(c*x) - 2*I*arctan(c*x)^2 -
```

```
I*log(c^2*x^2 + 1)^2 - 2*I*log(2*c^3*d*x^2 + 2*c*d))*b/(c^2*d)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{bx\log\left(-\frac{cx+i}{cx-i}\right)-2i\,ax}{2\,cdx-2i\,d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))/(d+I*c*d*x),x, algorithm="fricas")

[Out] integral((b*x*log(-(c*x + I)/(c*x - I)) - 2*I*a*x)/(2*c*d*x - 2*I*d), x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atan(c*x))/(d+I*c*d*x),x)

[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(b\arctan(cx)+a)x}{i\,cdx+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))/(d+I*c*d*x),x, algorithm="giac")

[Out] integrate((b*arctan(c*x) + a)*x/(I*c*d*x + d), x)
```

3.46 $\int \frac{a+b\tan^{-1}(cx)}{d+icdx}\,dx$

Optimal. Leaf size=59

$$\frac{i\log\left(\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{cd}-\frac{b\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{2cd}$$

```
[Out] (I*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(c*d) - (b*PolyLog[2, 1 - 2/(1 +
 I*c*x)])/(2*c*d)
```

Rubi [A] time = 0.047172, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {4854, 2402, 2315}

$$\frac{i\log\left(\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{cd}-\frac{b\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{2cd}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])/(d + I*c*d*x),x]
```

```
[Out] (I*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(c*d) - (b*PolyLog[2, 1 - 2/(1 +
 I*c*x)])/(2*c*d)
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int \frac{a+b\tan^{-1}(cx)}{d+icdx}\,dx = \frac{i\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2}{1+icx}\right)}{cd} - \frac{(ib)\int \frac{\log\left(\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx}{d}$$
$$= \frac{i\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2}{1+icx}\right)}{cd} - \frac{b\,\mathrm{Subst}\left(\int \frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1+icx}\right)}{cd}$$
$$= \frac{i\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2}{1+icx}\right)}{cd} - \frac{b\mathrm{Li}_2\left(1-\frac{2}{1+icx}\right)}{2cd}$$

Mathematica [A] time = 0.0140577, size = 60, normalized size = 1.02

$$\frac{2i\log\left(\frac{2d}{d+icdx}\right)\left(a+b\tan^{-1}(cx)\right) - b\mathrm{PolyLog}\left(2,\frac{cx+i}{cx-i}\right)}{2cd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])/(d + I*c*d*x),x]
```

```
[Out] ((2*I)*(a + b*ArcTan[c*x])*Log[(2*d)/(d + I*c*d*x)] - b*PolyLog[2, (I + c*x
)/(-I + c*x)])/(2*c*d)
```

Maple [B] time = 0.036, size = 142, normalized size = 2.4

$$\frac{-\frac{i}{2}a\ln\left(c^2x^2+1\right)}{dc} + \frac{a\arctan(cx)}{dc} - \frac{ib\ln(1+icx)\arctan(cx)}{dc} - \frac{b\ln(1+icx)}{2\,dc}\ln\left(\frac{1}{2}-\frac{i}{2}cx\right) + \frac{b}{2\,dc}\ln\left(\frac{1}{2}-\frac{i}{2}cx\right)\ln\left(\frac{i}{2}cx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/(d+I*c*d*x),x)
```

```
[Out] -1/2*I/c*a/d*ln(c^2*x^2+1)+1/c*a/d*arctan(c*x)-I/c*b/d*ln(1+I*c*x)*arctan(c
*x)-1/2/c*b/d*ln(1/2-1/2*I*c*x)*ln(1+I*c*x)+1/2/c*b/d*ln(1/2-1/2*I*c*x)*ln(
```

```
1/2*I*c*x+1/2)+1/2/c*b/d*dilog(1/2*I*c*x+1/2)+1/4/c*b/d*ln(1+I*c*x)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{-4\left(-2i\,c^2\int\frac{x\arctan(cx)}{c^2x^2+1}\,dx+\arctan(cx)^2\right)b}{8\,cd}-\frac{i\,a\log(i\,cdx+d)}{cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/(d+I*c*d*x),x, algorithm="maxima")
```

```
[Out] -1/8*(8*I*c^2*d*integrate(x*arctan(c*x)/(c^2*d*x^2 + d), x) + 4*c^2*d*integ
rate(x*log(c^2*x^2 + 1)/(c^2*d*x^2 + d), x) - 4*arctan(c*x)^2 - log(c^2*x^2
 + 1)^2)*b/(c*d) - I*a*log(I*c*d*x + d)/(c*d)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b\log\left(-\frac{cx+i}{cx-i}\right)-2i\,a}{2\,cdx-2i\,d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/(d+I*c*d*x),x, algorithm="fricas")
```

```
[Out] integral((b*log(-(c*x + I)/(c*x - I)) - 2*I*a)/(2*c*d*x - 2*I*d), x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/(d+I*c*d*x),x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\arctan(cx) + a}{icdx + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/(d+I*c*d*x),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)/(I*c*d*x + d), x)
```

3.47 $\int \frac{a+b\tan^{-1}(cx)}{x(d+icdx)}\,dx$

Optimal. Leaf size=54

$$\frac{ib\text{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)}{2d}+\frac{\log\left(2-\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{d}$$

```
[Out] ((a + b*ArcTan[c*x])*Log[2 - 2/(1 + I*c*x)])/d + ((I/2)*b*PolyLog[2, -1 + 2
/(1 + I*c*x)])/d
```

Rubi [A] time = 0.0711642, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {4868, 2447}

$$\frac{ib\text{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)}{2d}+\frac{\log\left(2-\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])/(x*(d + I*c*d*x)),x]
```

```
[Out] ((a + b*ArcTan[c*x])*Log[2 - 2/(1 + I*c*x)])/d + ((I/2)*b*PolyLog[2, -1 + 2
/(1 + I*c*x)])/d
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{a+b\tan^{-1}(cx)}{x(d+icdx)}\,dx = \frac{\left(a+b\tan^{-1}(cx)\right)\log\left(2-\frac{2}{1+icx}\right)}{d} - \frac{(bc)\int \frac{\log\left(2-\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx}{d}$$
$$= \frac{\left(a+b\tan^{-1}(cx)\right)\log\left(2-\frac{2}{1+icx}\right)}{d} + \frac{ib\text{Li}_2\left(-1+\frac{2}{1+icx}\right)}{2d}$$

Mathematica [A] time = 0.0556677, size = 102, normalized size = 1.89

$$\frac{ib\text{PolyLog}(2,-icx)}{2d} - \frac{ib\text{PolyLog}(2,icx)}{2d} + \frac{ib\text{PolyLog}\left(2,-\frac{cx+i}{-cx+i}\right)}{2d} + \frac{\log\left(\frac{2i}{-cx+i}\right)\left(a+b\tan^{-1}(cx)\right)}{d} + \frac{a\log(x)}{d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c*x])/(x*(d + I*c*d*x)),x]
```

```
[Out] (a*Log[x])/d + ((a + b*ArcTan[c*x])*Log[(2*I)/(I - c*x)])/d + ((I/2)*b*Poly
Log[2, (-I)*c*x])/d - ((I/2)*b*PolyLog[2, I*c*x])/d + ((I/2)*b*PolyLog[2, -
((I + c*x)/(I - c*x))])/d
```

Maple [B] time = 0.051, size = 193, normalized size = 3.6

$$-\frac{a\ln\left(c^2x^2+1\right)}{2\,d} - \frac{ia\arctan(cx)}{d} + \frac{a\ln(cx)}{d} - \frac{b\arctan(cx)\ln(cx-i)}{d} + \frac{b\arctan(cx)\ln(cx)}{d} + \frac{\frac{i}{2}b\ln(cx)\ln(1+icx)}{d} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/x/(d+I*c*d*x),x)
```

```
[Out] -1/2*a/d*ln(c^2*x^2+1)-I*a/d*arctan(c*x)+a/d*ln(c*x)-b/d*arctan(c*x)*ln(c*x
-I)+b/d*arctan(c*x)*ln(c*x)+1/2*I*b/d*ln(c*x)*ln(1+I*c*x)-1/2*I*b/d*ln(c*x)
*ln(1-I*c*x)+1/2*I*b/d*dilog(1+I*c*x)-1/2*I*b/d*dilog(1-I*c*x)+1/2*I*b/d*ln
(-1/2*I*(c*x+I))*ln(c*x-I)+1/2*I*b/d*dilog(-1/2*I*(c*x+I))-1/4*I*b/d*ln(c*x
-I)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2}\,b\left(\frac{i\arctan\left(cx\right)^{2}}{d}-2\int\frac{\arctan\left(cx\right)}{c^{2}dx^{3}+dx}\,dx\right)-a\left(\frac{\log\left(icx+1\right)}{d}-\frac{\log\left(x\right)}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x/(d+I*c*d*x),x, algorithm="maxima")
```

```
[Out] -1/2*b*(I*arctan(c*x)^2/d - 2*integrate(arctan(c*x)/(c^2*d*x^3 + d*x), x))
 - a*(log(I*c*x + 1)/d - log(x)/d)
```

Fricas [A] time = 2.59776, size = 109, normalized size = 2.02

$$\frac{-i\,b\mathrm{Li}_2\left(\frac{cx+i}{cx-i}+1\right)+2\,a\log\left(x\right)-2\,a\log\left(\frac{cx-i}{c}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x/(d+I*c*d*x),x, algorithm="fricas")
```

```
[Out] 1/2*(-I*b*dilog((c*x + I)/(c*x - I) + 1) + 2*a*log(x) - 2*a*log((c*x - I)/c
))/d
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/x/(d+I*c*d*x),x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{b\arctan\left(cx\right)+a}{\left(icdx+d\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x/(d+I*c*d*x),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)/((I*c*d*x + d)*x), x)
```

3.48 $\int \frac{a+b\tan^{-1}(cx)}{x^2(d+icdx)}\,dx$

Optimal. Leaf size=100

$$\frac{bc\text{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)}{2d}-\frac{a+b\tan^{-1}(cx)}{dx}-\frac{ic\log\left(2-\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{d}-\frac{bc\log\left(c^2x^2+1\right)}{2d}+\frac{bc\log(x)}{d}$$

```
[Out] -((a + b*ArcTan[c*x])/(d*x)) + (b*c*Log[x])/d - (b*c*Log[1 + c^2*x^2])/(2*d
) - (I*c*(a + b*ArcTan[c*x])*Log[2 - 2/(1 + I*c*x)])/d + (b*c*PolyLog[2, -1
 + 2/(1 + I*c*x)])/(2*d)
```

Rubi [A] time = 0.15467, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {4870, 4852, 266, 36, 29, 31, 4868, 2447}

$$\frac{bc\text{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)}{2d}-\frac{a+b\tan^{-1}(cx)}{dx}-\frac{ic\log\left(2-\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{d}-\frac{bc\log\left(c^2x^2+1\right)}{2d}+\frac{bc\log(x)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])/(x^2*(d + I*c*d*x)),x]
```

```
[Out] -((a + b*ArcTan[c*x])/(d*x)) + (b*c*Log[x])/d - (b*c*Log[1 + c^2*x^2])/(2*d
) - (I*c*(a + b*ArcTan[c*x])*Log[2 - 2/(1 + I*c*x)])/d + (b*c*PolyLog[2, -1
 + 2/(1 + I*c*x)])/(2*d)
```

Rule 4870

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x], x]
 - Dist[e/(d*f), Int[((f*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d + e*x), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0] &&
 LtQ[m, -1]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
```

```
erQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{a+b\tan^{-1}(cx)}{x^2(d+icdx)}\,dx = -\left((ic)\int \frac{a+b\tan^{-1}(cx)}{x(d+icdx)}\,dx\right) + \frac{\int \frac{a+b\tan^{-1}(cx)}{x^2}\,dx}{d}$$

$$= -\frac{a+b\tan^{-1}(cx)}{dx} - \frac{ic\left(a+b\tan^{-1}(cx)\right)\log\left(2-\frac{2}{1+icx}\right)}{d} + \frac{(bc)\int \frac{1}{x(1+c^2x^2)}\,dx}{d} + \frac{\left(ibc^2\right)\int \frac{\log\left(2-\frac{2}{1-}\right.}{1+c^2x}}{d}$$

$$= -\frac{a+b\tan^{-1}(cx)}{dx} - \frac{ic\left(a+b\tan^{-1}(cx)\right)\log\left(2-\frac{2}{1+icx}\right)}{d} + \frac{bc\text{Li}_2\left(-1+\frac{2}{1+icx}\right)}{2d} + \frac{(bc)\,\text{Subst}\left(\int\right.}{2}$$

$$= -\frac{a+b\tan^{-1}(cx)}{dx} - \frac{ic\left(a+b\tan^{-1}(cx)\right)\log\left(2-\frac{2}{1+icx}\right)}{d} + \frac{bc\text{Li}_2\left(-1+\frac{2}{1+icx}\right)}{2d} + \frac{(bc)\,\text{Subst}\left(\int\right.}{2d}$$

$$= -\frac{a+b\tan^{-1}(cx)}{dx} + \frac{bc\log(x)}{d} - \frac{bc\log\left(1+c^2x^2\right)}{2d} - \frac{ic\left(a+b\tan^{-1}(cx)\right)\log\left(2-\frac{2}{1+icx}\right)}{d} + \frac{bc\text{Li}}{}$$

Mathematica [A] time = 0.0906333, size = 149, normalized size = 1.49

$$\frac{bc\text{PolyLog}(2,-icx)}{2d} - \frac{bc\text{PolyLog}(2,icx)}{2d} + \frac{bc\text{PolyLog}\left(2,-\frac{cx+i}{-cx+i}\right)}{2d} - \frac{a+b\tan^{-1}(cx)}{dx} - \frac{ic\log\left(\frac{2i}{-cx+i}\right)\left(a+b\tan^{-1}(cx)\right)}{d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c*x])/(x^2*(d + I*c*d*x)),x]
```

```
[Out] -((a + b*ArcTan[c*x])/(d*x)) - (I*a*c*Log[x])/d - (I*c*(a + b*ArcTan[c*x])*
Log[(2*I)/(I - c*x)])/d + (b*c*(2*Log[x] - Log[1 + c^2*x^2]))/(2*d) + (b*c*
PolyLog[2, (-I)*c*x])/(2*d) - (b*c*PolyLog[2, I*c*x])/(2*d) + (b*c*PolyLog[
2, -((I + c*x)/(I - c*x))])/(2*d)
```

Maple [B] time = 0.056, size = 252, normalized size = 2.5

$$\frac{-ica\ln(cx)}{d} - \frac{ca\arctan(cx)}{d} - \frac{a}{dx} + \frac{icb\arctan(cx)\ln(cx-i)}{d} - \frac{icb\arctan(cx)\ln(cx)}{d} - \frac{b\arctan(cx)}{dx} + \frac{\frac{i}{2}ca\ln\left(c^2x^2\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/x^2/(d+I*c*d*x),x)
```

```
[Out] -I*c*a/d*ln(c*x)-c*a/d*arctan(c*x)-a/d/x+I*c*b/d*arctan(c*x)*ln(c*x-I)-I*c*
b/d*arctan(c*x)*ln(c*x)-b/d*arctan(c*x)/x+1/2*I*c*a/d*ln(c^2*x^2+1)+1/2*c*b
/d*ln(c*x)*ln(1+I*c*x)-1/2*c*b/d*ln(c*x)*ln(1-I*c*x)+1/2*c*b/d*dilog(1+I*c*
x)-1/2*c*b/d*dilog(1-I*c*x)+1/2*c*b/d*ln(c*x-I)*ln(-1/2*I*(c*x+I))+1/2*c*b/
d*dilog(-1/2*I*(c*x+I))-1/4*c*b/d*ln(c*x-I)^2-1/2*b*c*ln(c^2*x^2+1)/d+c*b/d
*ln(c*x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\left(-i\,c\int\frac{\arctan\left(cx\right)}{c^2dx^3+dx}\,dx+\int\frac{\arctan\left(cx\right)}{c^2dx^4+dx^2}\,dx\right)b+a\left(\frac{i\,c\log\left(i\,cx+1\right)}{d}-\frac{i\,c\log\left(x\right)}{d}-\frac{1}{dx}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^2/(d+I*c*d*x),x, algorithm="maxima")
```

```
[Out] (-I*c*integrate(arctan(c*x)/(c^2*d*x^3 + d*x), x) + integrate(arctan(c*x)/(
c^2*d*x^4 + d*x^2), x))*b + a*(I*c*log(I*c*x + 1)/d - I*c*log(x)/d - 1/(d*x
))
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b\log\left(-\frac{cx+i}{cx-i}\right)-2i\,a}{2\left(cdx^3-i\,dx^2\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^2/(d+I*c*d*x),x, algorithm="fricas")
```

```
[Out] integral(1/2*(b*log(-(c*x + I)/(c*x - I)) - 2*I*a)/(c*d*x^3 - I*d*x^2), x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/x**2/(d+I*c*d*x),x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \arctan(cx) + a}{(icdx + d)x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^2/(d+I*c*d*x),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)/((I*c*d*x + d)*x^2), x)
```

3.49 $\int \frac{a+b\tan^{-1}(cx)}{x^3(d+icdx)}\,dx$

Optimal. Leaf size=161

$$-\frac{ibc^2\text{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)}{2d}-\frac{c^2\log\left(2-\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{d}-\frac{a+b\tan^{-1}(cx)}{2dx^2}+\frac{ic\left(a+b\tan^{-1}(cx)\right)}{dx}+\frac{ibc^2\log}{2}$$

```
[Out] -(b*c)/(2*d*x) - (b*c^2*ArcTan[c*x])/(2*d) - (a + b*ArcTan[c*x])/(2*d*x^2)
+ (I*c*(a + b*ArcTan[c*x]))/(d*x) - (I*b*c^2*Log[x])/d + ((I/2)*b*c^2*Log[1
 + c^2*x^2])/d - (c^2*(a + b*ArcTan[c*x])*Log[2 - 2/(1 + I*c*x)])/d - ((I/2
)*b*c^2*PolyLog[2, -1 + 2/(1 + I*c*x)])/d
```

Rubi [A] time = 0.235719, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {4870, 4852, 325, 203, 266, 36, 29, 31, 4868, 2447}

$$-\frac{ibc^2\text{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)}{2d}-\frac{c^2\log\left(2-\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{d}-\frac{a+b\tan^{-1}(cx)}{2dx^2}+\frac{ic\left(a+b\tan^{-1}(cx)\right)}{dx}+\frac{ibc^2\log}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])/(x^3*(d + I*c*d*x)),x]
```

```
[Out] -(b*c)/(2*d*x) - (b*c^2*ArcTan[c*x])/(2*d) - (a + b*ArcTan[c*x])/(2*d*x^2)
+ (I*c*(a + b*ArcTan[c*x]))/(d*x) - (I*b*c^2*Log[x])/d + ((I/2)*b*c^2*Log[1
 + c^2*x^2])/d - (c^2*(a + b*ArcTan[c*x])*Log[2 - 2/(1 + I*c*x)])/d - ((I/2
)*b*c^2*PolyLog[2, -1 + 2/(1 + I*c*x)])/d
```

Rule 4870

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x], x]
 - Dist[e/(d*f), Int[((f*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d + e*x), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0] &&
 LtQ[m, -1]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
```

```
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
```

```
^2 + e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{a+b\tan^{-1}(cx)}{x^3(d+icdx)}\,dx = -\left((ic)\int \frac{a+b\tan^{-1}(cx)}{x^2(d+icdx)}\,dx\right)+\frac{\int \frac{a+b\tan^{-1}(cx)}{x^3}\,dx}{d}$$

$$= -\frac{a+b\tan^{-1}(cx)}{2dx^2}-c^2\int \frac{a+b\tan^{-1}(cx)}{x(d+icdx)}\,dx-\frac{(ic)\int \frac{a+b\tan^{-1}(cx)}{x^2}\,dx}{d}+\frac{(bc)\int \frac{1}{x^2(1+c^2x^2)}\,dx}{2d}$$

$$= -\frac{bc}{2dx}-\frac{a+b\tan^{-1}(cx)}{2dx^2}+\frac{ic\left(a+b\tan^{-1}(cx)\right)}{dx}-\frac{c^2\left(a+b\tan^{-1}(cx)\right)\log\left(2-\frac{2}{1+icx}\right)}{d}-\frac{\left(ibc^2\right)\int}{}$$

$$= -\frac{bc}{2dx}-\frac{bc^2\tan^{-1}(cx)}{2d}-\frac{a+b\tan^{-1}(cx)}{2dx^2}+\frac{ic\left(a+b\tan^{-1}(cx)\right)}{dx}-\frac{c^2\left(a+b\tan^{-1}(cx)\right)\log\left(2-\right.}{d}$$

$$= -\frac{bc}{2dx}-\frac{bc^2\tan^{-1}(cx)}{2d}-\frac{a+b\tan^{-1}(cx)}{2dx^2}+\frac{ic\left(a+b\tan^{-1}(cx)\right)}{dx}-\frac{c^2\left(a+b\tan^{-1}(cx)\right)\log\left(2-\right.}{d}$$

$$= -\frac{bc}{2dx}-\frac{bc^2\tan^{-1}(cx)}{2d}-\frac{a+b\tan^{-1}(cx)}{2dx^2}+\frac{ic\left(a+b\tan^{-1}(cx)\right)}{dx}-\frac{ibc^2\log(x)}{d}+\frac{ibc^2\log\left(1+c^2x\right.}{2d}$$

Mathematica [C] time = 0.170923, size = 178, normalized size = 1.11

$$-\frac{\frac{bc\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-c^2x^2\right)}{x}+ibc^2\text{PolyLog}(2,-icx)-ibc^2\text{PolyLog}(2,icx)+ibc^2\text{PolyLog}\left(2,\frac{cx+i}{cx-i}\right)+2c^2\log\left(\frac{2i}{-cx+i}\right.}{2d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c*x])/(x^3*(d + I*c*d*x)),x]
```

```
[Out] -((a + b*ArcTan[c*x])/x^2 - ((2*I)*c*(a + b*ArcTan[c*x]))/x + (b*c*Hypergeo
metric2F1[-1/2, 1, 1/2, -(c^2*x^2)])/x + 2*a*c^2*Log[x] + 2*c^2*(a + b*ArcT
```

```
an[c*x])*Log[(2*I)/(I - c*x)] + I*b*c^2*(2*Log[x] - Log[1 + c^2*x^2]) + I*b
*c^2*PolyLog[2, (-I)*c*x] - I*b*c^2*PolyLog[2, I*c*x] + I*b*c^2*PolyLog[2,
(I + c*x)/(-I + c*x)])/(2*d)
```

Maple [B] time = 0.064, size = 335, normalized size = 2.1

$$\frac{c^2 a \ln\left(c^2x^2+1\right)}{2d}+\frac{\frac{i}{2}c^2 b \ln(cx)\ln(1-icx)}{d}-\frac{a}{2dx^2}+\frac{\frac{i}{2}c^2 b\ln\left(c^2x^2+1\right)}{d}-\frac{c^2 a\ln(cx)}{d}+\frac{c^2 b\arctan(cx)\ln(cx-i)}{d}-\frac{b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/x^3/(d+I*c*d*x),x)
```

```
[Out] 1/2*c^2*a/d*ln(c^2*x^2+1)+1/2*I*c^2*b/d*ln(c*x)*ln(1-I*c*x)-1/2*a/d/x^2+1/2
*I*b*c^2*ln(c^2*x^2+1)/d-c^2*a/d*ln(c*x)+c^2*b/d*arctan(c*x)*ln(c*x-I)-1/2*
b/d*arctan(c*x)/x^2-1/2*I*c^2*b/d*ln(-1/2*I*(c*x+I))*ln(c*x-I)-c^2*b/d*arct
an(c*x)*ln(c*x)+I*c^2*a/d*arctan(c*x)-1/2*b*c^2*arctan(c*x)/d+I*c*a/d/x-1/2
*b*c/d/x+1/4*I*c^2*b/d*ln(c*x-I)^2-1/2*I*c^2*b/d*ln(c*x)*ln(1+I*c*x)+1/2*I*
c^2*b/d*dilog(1-I*c*x)-1/2*I*c^2*b/d*dilog(-1/2*I*(c*x+I))+I*c*b/d*arctan(c
*x)/x-I*c^2*b/d*ln(c*x)-1/2*I*c^2*b/d*dilog(1+I*c*x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}\left(\frac{2\,c^2\log(i\,cx+1)}{d}-\frac{2\,c^2\log(x)}{d}+\frac{2i\,cx-1}{dx^2}\right)a+\left(-i\,c\int\frac{\arctan(cx)}{c^2dx^4+dx^2}\,dx+\int\frac{\arctan(cx)}{c^2dx^5+dx^3}\,dx\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^3/(d+I*c*d*x),x, algorithm="maxima")
```

```
[Out] 1/2*(2*c^2*log(I*c*x + 1)/d - 2*c^2*log(x)/d + (2*I*c*x - 1)/(d*x^2))*a + (
-I*c*integrate(arctan(c*x)/(c^2*d*x^4 + d*x^2), x) + integrate(arctan(c*x)/
(c^2*d*x^5 + d*x^3), x))*b
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b\log\left(-\frac{cx+i}{cx-i}\right)-2i\,a}{2\left(cdx^4-i\,dx^3\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^3/(d+I*c*d*x),x, algorithm="fricas")
```

```
[Out] integral(1/2*(b*log(-(c*x + I)/(c*x - I)) - 2*I*a)/(c*d*x^4 - I*d*x^3), x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/x**3/(d+I*c*d*x),x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \arctan(cx) + a}{(i\,cdx + d)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^3/(d+I*c*d*x),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)/((I*c*d*x + d)*x^3), x)
```

3.50 $\int \frac{a+b\tan^{-1}(cx)}{x^4(d+icdx)}\,dx$

Optimal. Leaf size=197

$$-\frac{bc^3\text{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)}{2d}+\frac{c^2\left(a+b\tan^{-1}(cx)\right)}{dx}+\frac{ic^3\log\left(2-\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{d}+\frac{ic\left(a+b\tan^{-1}(cx)\right)}{2dx^2}-\frac{a}{}$$

```
[Out] -(b*c)/(6*d*x^2) + ((I/2)*b*c^2)/(d*x) + ((I/2)*b*c^3*ArcTan[c*x])/d - (a +
 b*ArcTan[c*x])/(3*d*x^3) + ((I/2)*c*(a + b*ArcTan[c*x]))/(d*x^2) + (c^2*(a
 + b*ArcTan[c*x]))/(d*x) - (4*b*c^3*Log[x])/(3*d) + (2*b*c^3*Log[1 + c^2*x^
2])/(3*d) + (I*c^3*(a + b*ArcTan[c*x])*Log[2 - 2/(1 + I*c*x)])/d - (b*c^3*P
olyLog[2, -1 + 2/(1 + I*c*x)])/(2*d)
```

Rubi [A] time = 0.339208, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {4870, 4852, 266, 44, 325, 203, 36, 29, 31, 4868, 2447}

$$-\frac{bc^3\text{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)}{2d}+\frac{c^2\left(a+b\tan^{-1}(cx)\right)}{dx}+\frac{ic^3\log\left(2-\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{d}+\frac{ic\left(a+b\tan^{-1}(cx)\right)}{2dx^2}-\frac{a}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])/(x^4*(d + I*c*d*x)),x]
```

```
[Out] -(b*c)/(6*d*x^2) + ((I/2)*b*c^2)/(d*x) + ((I/2)*b*c^3*ArcTan[c*x])/d - (a +
 b*ArcTan[c*x])/(3*d*x^3) + ((I/2)*c*(a + b*ArcTan[c*x]))/(d*x^2) + (c^2*(a
 + b*ArcTan[c*x]))/(d*x) - (4*b*c^3*Log[x])/(3*d) + (2*b*c^3*Log[1 + c^2*x^
2])/(3*d) + (I*c^3*(a + b*ArcTan[c*x])*Log[2 - 2/(1 + I*c*x)])/d - (b*c^3*P
olyLog[2, -1 + 2/(1 + I*c*x)])/(2*d)
```

Rule 4870

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x], x]
 - Dist[e/(d*f), Int[((f*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d + e*x), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0] &&
 LtQ[m, -1]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{a+b\tan^{-1}(cx)}{x^4(d+icdx)}\,dx = -\left((ic)\int \frac{a+b\tan^{-1}(cx)}{x^3(d+icdx)}\,dx\right)+\frac{\int \frac{a+b\tan^{-1}(cx)}{x^4}\,dx}{d}$$

$$= -\frac{a+b\tan^{-1}(cx)}{3dx^3} - c^2\int \frac{a+b\tan^{-1}(cx)}{x^2(d+icdx)}\,dx - \frac{(ic)\int \frac{a+b\tan^{-1}(cx)}{x^3}\,dx}{d} + \frac{(bc)\int \frac{1}{x^3(1+c^2x^2)}\,dx}{3d}$$

$$= -\frac{a+b\tan^{-1}(cx)}{3dx^3} + \frac{ic\left(a+b\tan^{-1}(cx)\right)}{2dx^2} + \left(ic^3\right)\int \frac{a+b\tan^{-1}(cx)}{x(d+icdx)}\,dx + \frac{(bc)\,\text{Subst}\left(\int \frac{1}{x^2(1+c^2x}\right.}{6d}$$

$$= \frac{ibc^2}{2dx} - \frac{a+b\tan^{-1}(cx)}{3dx^3} + \frac{ic\left(a+b\tan^{-1}(cx)\right)}{2dx^2} + \frac{c^2\left(a+b\tan^{-1}(cx)\right)}{dx} + \frac{ic^3\left(a+b\tan^{-1}(cx)\right)\text{lo}}{d}$$

$$= -\frac{bc}{6dx^2} + \frac{ibc^2}{2dx} + \frac{ibc^3\tan^{-1}(cx)}{2d} - \frac{a+b\tan^{-1}(cx)}{3dx^3} + \frac{ic\left(a+b\tan^{-1}(cx)\right)}{2dx^2} + \frac{c^2\left(a+b\tan^{-1}(cx)\right.}{dx}$$

$$= -\frac{bc}{6dx^2} + \frac{ibc^2}{2dx} + \frac{ibc^3\tan^{-1}(cx)}{2d} - \frac{a+b\tan^{-1}(cx)}{3dx^3} + \frac{ic\left(a+b\tan^{-1}(cx)\right)}{2dx^2} + \frac{c^2\left(a+b\tan^{-1}(cx)\right.}{dx}$$

$$= -\frac{bc}{6dx^2} + \frac{ibc^2}{2dx} + \frac{ibc^3\tan^{-1}(cx)}{2d} - \frac{a+b\tan^{-1}(cx)}{3dx^3} + \frac{ic\left(a+b\tan^{-1}(cx)\right)}{2dx^2} + \frac{c^2\left(a+b\tan^{-1}(cx)\right.}{dx}$$

Mathematica [C] time = 0.136916, size = 254, normalized size = 1.29

$$\frac{3ibc^2x^2\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-c^2x^2\right)-3bc^3x^3\text{PolyLog}(2,-icx)+3bc^3x^3\text{PolyLog}(2,icx)-3bc^3x^3\text{PolyLog}\left(2\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c*x])/(x^4*(d + I*c*d*x)),x]
```

```
[Out] (-2*a + (3*I)*a*c*x - b*c*x + 6*a*c^2*x^2 - 2*b*ArcTan[c*x] + (3*I)*b*c*x*A
rcTan[c*x] + 6*b*c^2*x^2*ArcTan[c*x] + (3*I)*b*c^2*x^2*Hypergeometric2F1[-1
/2, 1, 1/2, -(c^2*x^2)] + (6*I)*a*c^3*x^3*Log[x] - 8*b*c^3*x^3*Log[x] + (6*
I)*a*c^3*x^3*Log[(2*I)/(I - c*x)] + (6*I)*b*c^3*x^3*ArcTan[c*x]*Log[(2*I)/(
I - c*x)] + 4*b*c^3*x^3*Log[1 + c^2*x^2] - 3*b*c^3*x^3*PolyLog[2, (-I)*c*x]
 + 3*b*c^3*x^3*PolyLog[2, I*c*x] - 3*b*c^3*x^3*PolyLog[2, (I + c*x)/(-I + c
*x)])/(6*d*x^3)
```

Maple [B] time = 0.055, size = 369, normalized size = 1.9

$$\frac{ic^3b\arctan(cx)\ln(cx)}{d}+\frac{ac^3\arctan(cx)}{d}-\frac{a}{3\,dx^3}+\frac{\frac{i}{2}cb\arctan(cx)}{dx^2}+\frac{\frac{i}{2}bc^3\arctan(cx)}{d}+\frac{c^2a}{dx}-\frac{ic^3b\arctan(cx)\ln(cx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/x^4/(d+I*c*d*x),x)
```

```
[Out] I*c^3*b/d*arctan(c*x)*ln(c*x)+c^3*a/d*arctan(c*x)-1/3*a/d/x^3+1/2*I*c*b/d*a
rctan(c*x)/x^2+1/2*I*b*c^3*arctan(c*x)/d+c^2*a/d/x-I*c^3*b/d*arctan(c*x)*ln
(c*x-I)-1/3*b/d*arctan(c*x)/x^3+I*c^3*a/d*ln(c*x)+1/2*I*c*a/d/x^2+c^2*b/d*a
rctan(c*x)/x-1/2*c^3*b/d*ln(c*x-I)*ln(-1/2*I*(c*x+I))-1/2*c^3*b/d*dilog(-1/
2*I*(c*x+I))+1/4*c^3*b/d*ln(c*x-I)^2-1/2*c^3*b/d*ln(c*x)*ln(1+I*c*x)+1/2*c^
3*b/d*ln(c*x)*ln(1-I*c*x)+1/2*c^3*b/d*dilog(1-I*c*x)-1/2*c^3*b/d*dilog(1+I*
c*x)+2/3*b*c^3*ln(c^2*x^2+1)/d+1/2*I*b*c^2/d/x-1/2*I*c^3*a/d*ln(c^2*x^2+1)-
1/6*b*c/d/x^2-4/3*c^3*b/d*ln(c*x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{6}\left(\frac{6i\,c^3\log(i\,cx+1)}{d}-\frac{6i\,c^3\log(x)}{d}-\frac{6\,c^2x^2+3i\,cx-2}{dx^3}\right)a+\left(-i\,c\int\frac{\arctan(cx)}{c^2dx^5+dx^3}\,dx+\int\frac{\arctan(cx)}{c^2dx^6+dx^4}\,dx\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^4/(d+I*c*d*x),x, algorithm="maxima")
```

```
[Out] -1/6*(6*I*c^3*log(I*c*x + 1)/d - 6*I*c^3*log(x)/d - (6*c^2*x^2 + 3*I*c*x -
2)/(d*x^3))*a + (-I*c*integrate(arctan(c*x)/(c^2*d*x^5 + d*x^3), x) + integ
rate(arctan(c*x)/(c^2*d*x^6 + d*x^4), x))*b
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b\log\left(-\frac{cx+i}{cx-i}\right)-2i\,a}{2\left(cdx^5-i\,dx^4\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^4/(d+I*c*d*x),x, algorithm="fricas")
```

```
[Out] integral(1/2*(b*log(-(c*x + I)/(c*x - I)) - 2*I*a)/(c*d*x^5 - I*d*x^4), x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/x**4/(d+I*c*d*x),x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{b\arctan\left(cx\right)+a}{\left(i\,cdx+d\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^4/(d+I*c*d*x),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)/((I*c*d*x + d)*x^4), x)
```

3.51 $\int \frac{x^3(a+b\tan^{-1}(cx))}{(d+icdx)^2}\,dx$

Optimal. Leaf size=203

$$-\frac{3ib\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{2c^4d^2}-\frac{x^2\left(a+b\tan^{-1}(cx)\right)}{2c^2d^2}+\frac{i\left(a+b\tan^{-1}(cx)\right)}{c^4d^2(-cx+i)}-\frac{3\log\left(\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{c^4d^2}-\frac{2iax}{c^3d^2}+\frac{ib}{}$$

```
[Out] ((-2*I)*a*x)/(c^3*d^2) + (b*x)/(2*c^3*d^2) + b/(2*c^4*d^2*(I - c*x)) - (b*A
rcTan[c*x])/(c^4*d^2) - ((2*I)*b*x*ArcTan[c*x])/(c^3*d^2) - (x^2*(a + b*Arc
Tan[c*x]))/(2*c^2*d^2) + (I*(a + b*ArcTan[c*x]))/(c^4*d^2*(I - c*x)) - (3*(
a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(c^4*d^2) + (I*b*Log[1 + c^2*x^2])/(
c^4*d^2) - (((3*I)/2)*b*PolyLog[2, 1 - 2/(1 + I*c*x)])/(c^4*d^2)
```

Rubi [A] time = 0.221916, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 12, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.522, Rules used = {4876, 4846, 260, 4852, 321, 203, 4862, 627, 44, 4854, 2402, 2315}

$$-\frac{3ib\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{2c^4d^2}-\frac{x^2\left(a+b\tan^{-1}(cx)\right)}{2c^2d^2}+\frac{i\left(a+b\tan^{-1}(cx)\right)}{c^4d^2(-cx+i)}-\frac{3\log\left(\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{c^4d^2}-\frac{2iax}{c^3d^2}+\frac{ib}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcTan[c*x]))/(d + I*c*d*x)^2,x]
```


```
[Out] ((-2*I)*a*x)/(c^3*d^2) + (b*x)/(2*c^3*d^2) + b/(2*c^4*d^2*(I - c*x)) - (b*A
rcTan[c*x])/(c^4*d^2) - ((2*I)*b*x*ArcTan[c*x])/(c^3*d^2) - (x^2*(a + b*Arc
Tan[c*x]))/(2*c^2*d^2) + (I*(a + b*ArcTan[c*x]))/(c^4*d^2*(I - c*x)) - (3*(
a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(c^4*d^2) + (I*b*Log[1 + c^2*x^2])/(
c^4*d^2) - (((3*I)/2)*b*PolyLog[2, 1 - 2/(1 + I*c*x)])/(c^4*d^2)
```


Rule 4876

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4862

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x]))/(e*(q + 1)), x] - Dist[(b*
c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int \frac{x^3 \left(a+b\tan^{-1}(cx)\right)}{(d+icdx)^2}\,dx = \int \left(-\frac{2i\left(a+b\tan^{-1}(cx)\right)}{c^3d^2} - \frac{x\left(a+b\tan^{-1}(cx)\right)}{c^2d^2} + \frac{i\left(a+b\tan^{-1}(cx)\right)}{c^3d^2(-i+cx)^2} + \frac{3\left(a+b\tan^{-1}(cx)\right)}{c^3d^2(-i+cx)}\right.$$

$$= \frac{i\int \frac{a+b\tan^{-1}(cx)}{(-i+cx)^2}\,dx}{c^3d^2} - \frac{(2i)\int \left(a+b\tan^{-1}(cx)\right)dx}{c^3d^2} + \frac{3\int \frac{a+b\tan^{-1}(cx)}{-i+cx}\,dx}{c^3d^2} - \frac{\int x\left(a+b\tan^{-1}(cx\right.}{c^2d^2}$$

$$= -\frac{2iax}{c^3d^2} - \frac{x^2\left(a+b\tan^{-1}(cx)\right)}{2c^2d^2} + \frac{i\left(a+b\tan^{-1}(cx)\right)}{c^4d^2(i-cx)} - \frac{3\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2}{1+icx}\right)}{c^4d^2} + \frac{(ib)}{}$$

$$= -\frac{2iax}{c^3d^2} + \frac{bx}{2c^3d^2} - \frac{2ibx\tan^{-1}(cx)}{c^3d^2} - \frac{x^2\left(a+b\tan^{-1}(cx)\right)}{2c^2d^2} + \frac{i\left(a+b\tan^{-1}(cx)\right)}{c^4d^2(i-cx)} - \frac{3\left(a+b\,\mathrm{ta}\right.}{}$$

$$= -\frac{2iax}{c^3d^2} + \frac{bx}{2c^3d^2} - \frac{b\tan^{-1}(cx)}{2c^4d^2} - \frac{2ibx\tan^{-1}(cx)}{c^3d^2} - \frac{x^2\left(a+b\tan^{-1}(cx)\right)}{2c^2d^2} + \frac{i\left(a+b\tan^{-1}(cx)\right)}{c^4d^2(i-cx)}$$

$$= -\frac{2iax}{c^3d^2} + \frac{bx}{2c^3d^2} + \frac{b}{2c^4d^2(i-cx)} - \frac{b\tan^{-1}(cx)}{2c^4d^2} - \frac{2ibx\tan^{-1}(cx)}{c^3d^2} - \frac{x^2\left(a+b\tan^{-1}(cx)\right)}{2c^2d^2} + \frac{i\left(\right.}{}$$

$$= -\frac{2iax}{c^3d^2} + \frac{bx}{2c^3d^2} + \frac{b}{2c^4d^2(i-cx)} - \frac{b\tan^{-1}(cx)}{c^4d^2} - \frac{2ibx\tan^{-1}(cx)}{c^3d^2} - \frac{x^2\left(a+b\tan^{-1}(cx)\right)}{2c^2d^2} + \frac{i\left(\right.}{}$$

Mathematica [A] time = 0.96972, size = 186, normalized size = 0.92

$$-\frac{b\left(-6i\mathrm{PolyLog}\left(2,-e^{2i\tan^{-1}(cx)}\right) - 4i\log\left(c^2x^2+1\right) + 2\tan^{-1}(cx)\left(c^2x^2+4icx+6\log\left(1+e^{2i\tan^{-1}(cx)}\right) + i\sin\left(2\tan^{-1}(\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(a + b*ArcTan[c*x]))/(d + I*c*d*x)^2,x]
```

```
[Out] -((8*I)*a*c*x + 2*a*c^2*x^2 + ((4*I)*a)/(-I + c*x) - (12*I)*a*ArcTan[c*x] -
 6*a*Log[1 + c^2*x^2] + b*(-2*c*x - (12*I)*ArcTan[c*x]^2 + I*Cos[2*ArcTan[c
*x]] - (4*I)*Log[1 + c^2*x^2] - (6*I)*PolyLog[2, -E^((2*I)*ArcTan[c*x])] +
2*ArcTan[c*x]*(1 + (4*I)*c*x + c^2*x^2 - Cos[2*ArcTan[c*x]] + 6*Log[1 + E^(
(2*I)*ArcTan[c*x])] + I*Sin[2*ArcTan[c*x]]) + Sin[2*ArcTan[c*x]]))/(4*c^4*d
^2)
```

Maple [A] time = 0.06, size = 367, normalized size = 1.8

$$-\frac{2\,iax}{c^3d^2}-\frac{ax^2}{2\,c^2d^2}+\frac{3\,a\ln\left(c^2x^2+1\right)}{2\,d^2c^4}-\frac{2\,ibx\arctan\left(cx\right)}{c^3d^2}-\frac{ia}{d^2c^4\left(cx-i\right)}-\frac{ib\arctan\left(cx\right)}{d^2c^4\left(cx-i\right)}-\frac{bx^2\arctan\left(cx\right)}{2\,c^2d^2}+3\,\frac{b\arctan}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctan(c*x))/(d+I*c*d*x)^2,x)
```

```
[Out] -2*I*a*x/c^3/d^2-1/2/c^2*a/d^2*x^2+3/2/c^4*a/d^2*ln(c^2*x^2+1)-2*I*b*x*arct
an(c*x)/c^3/d^2-I/c^4*a/d^2/(c*x-I)-I/c^4*b/d^2*arctan(c*x)/(c*x-I)-1/2/c^2
*b/d^2*arctan(c*x)*x^2+3/c^4*b/d^2*arctan(c*x)*ln(c*x-I)+1/8*I/c^4*b/d^2*ln
(c^4*x^4+10*c^2*x^2+9)-1/2*I/c^4*b/d^2-3/2*I/c^4*b/d^2*ln(-1/2*I*(c*x+I))*l
n(c*x-I)-3/2*I/c^4*b/d^2*dilog(-1/2*I*(c*x+I))+1/2*b*x/c^3/d^2+3*I/c^4*a/d^
2*arctan(c*x)+3/4*I/c^4*b/d^2*ln(c*x-I)^2-1/4/c^4*b/d^2*arctan(1/2*c*x)+1/4
/c^4*b/d^2*arctan(1/6*c^3*x^3+7/6*c*x)+1/2/c^4*b/d^2*arctan(1/2*c*x-1/2*I)-
1/2/c^4*b/d^2/(c*x-I)+3/4*I/c^4*b/d^2*ln(c^2*x^2+1)-3/2*b*arctan(c*x)/d^2/c
^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))/(d+I*c*d*x)^2,x, algorithm="maxima")
```

```
[Out] -1/2*a*(2*I/(c^5*d^2*x - I*c^4*d^2) + (c*x^2 + 4*I*x)/(c^3*d^2) - 6*log(c*x
 - I)/(c^4*d^2)) - 1/16*(-16*I*c^3*x^3 + 80*c^2*x^2 + 16*c*x*(2*arctan2(1,
c*x) - 6*I) + 192*(-I*c*x - 1)*arctan(c*x)^2 + 48*(-I*c*x - 1)*log(c^2*x^2
+ 1)^2 + (48*c^5*d^2*x - 48*I*c^4*d^2)*((c*(x/(c^7*d^2*x^2 + c^5*d^2) + arc
tan(c*x)/(c^6*d^2)) - 2*arctan(c*x)/(c^7*d^2*x^2 + c^5*d^2))*c + 16*integra
te(1/8*log(c^2*x^2 + 1)/(c^7*d^2*x^4 + 2*c^5*d^2*x^2 + c^3*d^2), x)) - 48*(
-I*c^5*d^2*x - c^4*d^2)*(c*(c^2/(c^9*d^2*x^2 + c^7*d^2) + log(c^2*x^2 + 1)/
(c^7*d^2*x^2 + c^5*d^2)) + 32*integrate(1/8*arctan(c*x)/(c^7*d^2*x^4 + 2*c^
5*d^2*x^2 + c^3*d^2), x)) - (96*c^6*d^2*x - 96*I*c^5*d^2)*(c*(x/(c^7*d^2*x^
2 + c^5*d^2) + arctan(c*x)/(c^6*d^2)) - 16*c*integrate(1/8*x^2*log(c^2*x^2
+ 1)/(c^7*d^2*x^4 + 2*c^5*d^2*x^2 + c^3*d^2), x) - 2*arctan(c*x)/(c^7*d^2*x
^2 + c^5*d^2)) + 96*(I*c^6*d^2*x + c^5*d^2)*(32*c*integrate(1/8*x^2*arctan(
c*x)/(c^7*d^2*x^4 + 2*c^5*d^2*x^2 + c^3*d^2), x) - c^2/(c^9*d^2*x^2 + c^7*d
^2) - log(c^2*x^2 + 1)/(c^7*d^2*x^2 + c^5*d^2)) + 16*(2*c^3*x^3 + 6*I*c^2*x
```

```
^2 + 8*c*x + 4*I)*arctan(c*x) + 16*(16*c^9*d^2*x - 16*I*c^8*d^2)*integrate(
1/8*(2*c*x^5*arctan(c*x) + x^4*log(c^2*x^2 + 1))/(c^7*d^2*x^4 + 2*c^5*d^2*x
^2 + c^3*d^2), x) + 256*(-I*c^9*d^2*x - c^8*d^2)*integrate(1/8*(c*x^5*log(c
^2*x^2 + 1) - 2*x^4*arctan(c*x))/(c^7*d^2*x^4 + 2*c^5*d^2*x^2 + c^3*d^2), x
) + 256*(I*c^8*d^2*x + c^7*d^2)*integrate(1/8*(2*c*x^4*arctan(c*x) + x^3*lo
g(c^2*x^2 + 1))/(c^7*d^2*x^4 + 2*c^5*d^2*x^2 + c^3*d^2), x) + 16*(16*c^8*d^
2*x - 16*I*c^7*d^2)*integrate(1/8*(c*x^4*log(c^2*x^2 + 1) - 2*x^3*arctan(c*
x))/(c^7*d^2*x^4 + 2*c^5*d^2*x^2 + c^3*d^2), x) - 16*(48*c^7*d^2*x - 48*I*c
^6*d^2)*integrate(1/8*(2*c*x^3*arctan(c*x) + x^2*log(c^2*x^2 + 1))/(c^7*d^2
*x^4 + 2*c^5*d^2*x^2 + c^3*d^2), x) + 768*(I*c^7*d^2*x + c^6*d^2)*integrate
(1/8*(c*x^3*log(c^2*x^2 + 1) - 2*x^2*arctan(c*x))/(c^7*d^2*x^4 + 2*c^5*d^2*
x^2 + c^3*d^2), x) - 16*(-I*c^3*x^3 + 3*c^2*x^2 - I*c*x + 5)*log(c^2*x^2 +
1) - 32*I*arctan2(1, c*x))*b/(8*c^5*d^2*x - 8*I*c^4*d^2)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{-i\,bx^3\log\left(-\frac{cx+i}{cx-i}\right)-2\,ax^3}{2\,c^2d^2x^2-4i\,cd^2x-2\,d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))/(d+I*c*d*x)^2,x, algorithm="fricas")
```

```
[Out] integral((-I*b*x^3*log(-(c*x + I)/(c*x - I)) - 2*a*x^3)/(2*c^2*d^2*x^2 - 4*
I*c*d^2*x - 2*d^2), x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atan(c*x))/(d+I*c*d*x)**2,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \arctan(cx) + a)x^3}{(icdx + d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))/(d+I*c*d*x)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)*x^3/(I*c*d*x + d)^2, x)
```

3.52 $\int \frac{x^2\left(a+b\tan^{-1}(cx)\right)}{(d+icdx)^2}\,dx$

Optimal. Leaf size=167

$$-\frac{b\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c^3d^2}+\frac{a+b\tan^{-1}(cx)}{c^3d^2(-cx+i)}+\frac{2i\log\left(\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{c^3d^2}-\frac{ax}{c^2d^2}+\frac{b\log\left(c^2x^2+1\right)}{2c^3d^2}-\frac{ib}{2c^3d^2(-cx+$$

```
[Out] -((a*x)/(c^2*d^2)) - ((I/2)*b)/(c^3*d^2*(I - c*x)) + ((I/2)*b*ArcTan[c*x])/
(c^3*d^2) - (b*x*ArcTan[c*x])/(c^2*d^2) + (a + b*ArcTan[c*x])/(c^3*d^2*(I -
 c*x)) + ((2*I)*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(c^3*d^2) + (b*Log[
1 + c^2*x^2])/(2*c^3*d^2) - (b*PolyLog[2, 1 - 2/(1 + I*c*x)])/(c^3*d^2)
```

Rubi [A] time = 0.190293, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {4876, 4846, 260, 4862, 627, 44, 203, 4854, 2402, 2315}

$$-\frac{b\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c^3d^2}+\frac{a+b\tan^{-1}(cx)}{c^3d^2(-cx+i)}+\frac{2i\log\left(\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{c^3d^2}-\frac{ax}{c^2d^2}+\frac{b\log\left(c^2x^2+1\right)}{2c^3d^2}-\frac{ib}{2c^3d^2(-cx+$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcTan[c*x]))/(d + I*c*d*x)^2,x]
```

```
[Out] -((a*x)/(c^2*d^2)) - ((I/2)*b)/(c^3*d^2*(I - c*x)) + ((I/2)*b*ArcTan[c*x])/
(c^3*d^2) - (b*x*ArcTan[c*x])/(c^2*d^2) + (a + b*ArcTan[c*x])/(c^3*d^2*(I -
 c*x)) + ((2*I)*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(c^3*d^2) + (b*Log[
1 + c^2*x^2])/(2*c^3*d^2) - (b*PolyLog[2, 1 - 2/(1 + I*c*x)])/(c^3*d^2)
```

Rule 4876

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
```

```
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4862

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x]))/(e*(q + 1)), x] - Dist[(b*
c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
```

```
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int \frac{x^2 \left(a+b\tan^{-1}(cx)\right)}{(d+icdx)^2}\,dx = \int \left(-\frac{a+b\tan^{-1}(cx)}{c^2d^2} + \frac{a+b\tan^{-1}(cx)}{c^2d^2(-i+cx)^2} - \frac{2i\left(a+b\tan^{-1}(cx)\right)}{c^2d^2(-i+cx)}\right) dx$$

$$= -\frac{(2i)\int \frac{a+b\tan^{-1}(cx)}{-i+cx}\,dx}{c^2d^2} - \frac{\int \left(a+b\tan^{-1}(cx)\right) dx}{c^2d^2} + \frac{\int \frac{a+b\tan^{-1}(cx)}{(-i+cx)^2}\,dx}{c^2d^2}$$

$$= -\frac{ax}{c^2d^2} + \frac{a+b\tan^{-1}(cx)}{c^3d^2(i-cx)} + \frac{2i\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2}{1+icx}\right)}{c^3d^2} - \frac{(2ib)\int \frac{\log\left(\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx}{c^2d^2} + \frac{b\int \frac{}{(-i+}}{}$$

$$= -\frac{ax}{c^2d^2} - \frac{bx\tan^{-1}(cx)}{c^2d^2} + \frac{a+b\tan^{-1}(cx)}{c^3d^2(i-cx)} + \frac{2i\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2}{1+icx}\right)}{c^3d^2} - \frac{(2b)\,\mathrm{Subst}\left(\int\right.}{}$$

$$= -\frac{ax}{c^2d^2} - \frac{bx\tan^{-1}(cx)}{c^2d^2} + \frac{a+b\tan^{-1}(cx)}{c^3d^2(i-cx)} + \frac{2i\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2}{1+icx}\right)}{c^3d^2} + \frac{b\log\left(1+c^2x\right.}{2c^3d^2}$$

$$= -\frac{ax}{c^2d^2} - \frac{ib}{2c^3d^2(i-cx)} - \frac{bx\tan^{-1}(cx)}{c^2d^2} + \frac{a+b\tan^{-1}(cx)}{c^3d^2(i-cx)} + \frac{2i\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2}{1+icx}\right)}{c^3d^2}$$

$$= -\frac{ax}{c^2d^2} - \frac{ib}{2c^3d^2(i-cx)} + \frac{ib\tan^{-1}(cx)}{2c^3d^2} - \frac{bx\tan^{-1}(cx)}{c^2d^2} + \frac{a+b\tan^{-1}(cx)}{c^3d^2(i-cx)} + \frac{2i\left(a+b\tan^{-1}(c\right.}{c^3d}$$

Mathematica [A] time = 0.75328, size = 153, normalized size = 0.92

$$-\frac{b\left(-4\mathrm{PolyLog}\left(2,-e^{2i\tan^{-1}(cx)}\right)-2\log\left(c^2x^2+1\right)-8\tan^{-1}(cx)^2-i\sin\left(2\tan^{-1}(cx)\right)+\cos\left(2\tan^{-1}(cx)\right)+2\tan^{-1}(cx\right.}{4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(a + b*ArcTan[c*x]))/(d + I*c*d*x)^2,x]
```

```
[Out] -(4*a*c*x + (4*a)/(-I + c*x) - 8*a*ArcTan[c*x] + (4*I)*a*Log[1 + c^2*x^2] +
 b*(-8*ArcTan[c*x]^2 + Cos[2*ArcTan[c*x]] - 2*Log[1 + c^2*x^2] - 4*PolyLog[
```

```
2, -E^((2*I)*ArcTan[c*x])] - I*Sin[2*ArcTan[c*x]] + 2*ArcTan[c*x]*(2*c*x +
I*Cos[2*ArcTan[c*x]] - (4*I)*Log[1 + E^((2*I)*ArcTan[c*x])] + Sin[2*ArcTan[
c*x]])))/(4*c^3*d^2)
```

Maple [B] time = 0.056, size = 316, normalized size = 1.9

$$-\frac{ax}{c^2d^2}+2\frac{a\arctan(cx)}{c^3d^2}-\frac{ia\ln\left(c^2x^2+1\right)}{c^3d^2}-\frac{a}{c^3d^2(cx-i)}-\frac{bx\arctan(cx)}{c^2d^2}-\frac{\frac{i}{4}b}{c^3d^2}\arctan\left(\frac{cx}{2}-\frac{i}{2}\right)-\frac{b\arctan(cx)}{c^3d^2(cx-i)}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctan(c*x))/(d+I*c*d*x)^2,x)
```

```
[Out] -a*x/c^2/d^2+2/c^3*a/d^2*arctan(c*x)-I/c^3*a/d^2*ln(c^2*x^2+1)-1/c^3*a/d^2/
(c*x-I)-b*x*arctan(c*x)/c^2/d^2-1/4*I/c^3*b/d^2*arctan(1/2*c*x-1/2*I)-1/c^3
*b/d^2*arctan(c*x)/(c*x-I)-1/c^3*b/d^2*ln(c*x-I)*ln(-1/2*I*(c*x+I))-1/c^3*b
/d^2*dilog(-1/2*I*(c*x+I))+1/2/c^3*b/d^2*ln(c*x-I)^2+1/16/c^3*b/d^2*ln(c^4*
x^4+10*c^2*x^2+9)+1/2*I/c^3*b/d^2/(c*x-I)+1/8*I/c^3*b/d^2*arctan(1/2*c*x)-1
/8*I/c^3*b/d^2*arctan(1/6*c^3*x^3+7/6*c*x)-2*I/c^3*b/d^2*arctan(c*x)*ln(c*x
-I)+3/8*b*ln(c^2*x^2+1)/c^3/d^2+3/4*I/c^3*b/d^2*arctan(c*x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))/(d+I*c*d*x)^2,x, algorithm="maxima")
```

```
[Out] -a*(1/(c^4*d^2*x - I*c^3*d^2) + x/(c^2*d^2) + 2*I*log(c*x - I)/(c^3*d^2)) -
 1/8*(-16*I*c^2*x^2 - 8*(4*c*x - 4*I)*arctan(c*x)^2 - 8*(c*x - I)*log(c^2*x
^2 + 1)^2 + 8*(-I*c^4*d^2*x - c^3*d^2)*((c*(x/(c^6*d^2*x^2 + c^4*d^2) + arc
tan(c*x)/(c^5*d^2)) - 2*arctan(c*x)/(c^6*d^2*x^2 + c^4*d^2))*c + 8*integrat
e(1/4*log(c^2*x^2 + 1)/(c^6*d^2*x^4 + 2*c^4*d^2*x^2 + c^2*d^2), x)) + (8*c^
4*d^2*x - 8*I*c^3*d^2)*(c*(c^2/(c^8*d^2*x^2 + c^6*d^2) + log(c^2*x^2 + 1)/(
c^6*d^2*x^2 + c^4*d^2)) + 16*integrate(1/4*arctan(c*x)/(c^6*d^2*x^4 + 2*c^4
*d^2*x^2 + c^2*d^2), x)) - 16*(-I*c^5*d^2*x - c^4*d^2)*(c*(x/(c^6*d^2*x^2 +
 c^4*d^2) + arctan(c*x)/(c^5*d^2)) - 8*c*integrate(1/4*x^2*log(c^2*x^2 + 1)
/(c^6*d^2*x^4 + 2*c^4*d^2*x^2 + c^2*d^2), x) - 2*arctan(c*x)/(c^6*d^2*x^2 +
```

```
c^4*d^2)) + (16*c^5*d^2*x - 16*I*c^4*d^2)*(16*c*integrate(1/4*x^2*arctan(c
*x)/(c^6*d^2*x^4 + 2*c^4*d^2*x^2 + c^2*d^2), x) - c^2/(c^8*d^2*x^2 + c^6*d^
2) - log(c^2*x^2 + 1)/(c^6*d^2*x^2 + c^4*d^2)) - 16*c*x + 16*(c^2*x^2 - I*c
*x + 1)*arctan(c*x) + 8*(4*c^7*d^2*x - 4*I*c^6*d^2)*integrate(1/4*(2*c*x^4*
arctan(c*x) + x^3*log(c^2*x^2 + 1))/(c^6*d^2*x^4 + 2*c^4*d^2*x^2 + c^2*d^2)
, x) + 32*(-I*c^7*d^2*x - c^6*d^2)*integrate(1/4*(c*x^4*log(c^2*x^2 + 1) -
2*x^3*arctan(c*x))/(c^6*d^2*x^4 + 2*c^4*d^2*x^2 + c^2*d^2), x) + 96*(I*c^6*
d^2*x + c^5*d^2)*integrate(1/4*(2*c*x^3*arctan(c*x) + x^2*log(c^2*x^2 + 1))
/(c^6*d^2*x^4 + 2*c^4*d^2*x^2 + c^2*d^2), x) + 8*(12*c^6*d^2*x - 12*I*c^5*d
^2)*integrate(1/4*(c*x^3*log(c^2*x^2 + 1) - 2*x^2*arctan(c*x))/(c^6*d^2*x^4
 + 2*c^4*d^2*x^2 + c^2*d^2), x) - 8*(-I*c^2*x^2 - 2*I)*log(c^2*x^2 + 1))*b/
(4*c^4*d^2*x - 4*I*c^3*d^2)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{-i\,bx^2\log\left(-\frac{cx+i}{cx-i}\right)-2\,ax^2}{2\,c^2d^2x^2-4i\,cd^2x-2\,d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))/(d+I*c*d*x)^2,x, algorithm="fricas")
```

```
[Out] integral((-I*b*x^2*log(-(c*x + I)/(c*x - I)) - 2*a*x^2)/(2*c^2*d^2*x^2 - 4*
I*c*d^2*x - 2*d^2), x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atan(c*x))/(d+I*c*d*x)**2,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\arctan(cx)+a)x^2}{(icdx+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))/(d+I*c*d*x)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)*x^2/(I*c*d*x + d)^2, x)
```

3.53 $\int \frac{x\left(a+b\tan^{-1}(cx)\right)}{(d+icdx)^2}\,dx$

Optimal. Leaf size=122

$$\frac{ib\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{2c^2d^2}-\frac{i\left(a+b\tan^{-1}(cx)\right)}{c^2d^2(-cx+i)}+\frac{\log\left(\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{c^2d^2}-\frac{b}{2c^2d^2(-cx+i)}+\frac{b\tan^{-1}(cx)}{2c^2d^2}$$

```
[Out] -b/(2*c^2*d^2*(I - c*x)) + (b*ArcTan[c*x])/(2*c^2*d^2) - (I*(a + b*ArcTan[c
*x]))/(c^2*d^2*(I - c*x)) + ((a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(c^2*d
^2) + ((I/2)*b*PolyLog[2, 1 - 2/(1 + I*c*x)])/(c^2*d^2)
```

Rubi [A] time = 0.146809, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {4876, 4862, 627, 44, 203, 4854, 2402, 2315}

$$\frac{ib\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{2c^2d^2}-\frac{i\left(a+b\tan^{-1}(cx)\right)}{c^2d^2(-cx+i)}+\frac{\log\left(\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{c^2d^2}-\frac{b}{2c^2d^2(-cx+i)}+\frac{b\tan^{-1}(cx)}{2c^2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcTan[c*x]))/(d + I*c*d*x)^2,x]
```

```
[Out] -b/(2*c^2*d^2*(I - c*x)) + (b*ArcTan[c*x])/(2*c^2*d^2) - (I*(a + b*ArcTan[c
*x]))/(c^2*d^2*(I - c*x)) + ((a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(c^2*d
^2) + ((I/2)*b*PolyLog[2, 1 - 2/(1 + I*c*x)])/(c^2*d^2)
```

Rule 4876

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 4862

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x]))/(e*(q + 1)), x] - Dist[(b*
c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int \frac{x\left(a+b\tan^{-1}(cx)\right)}{(d+icdx)^2}\,dx = \int\left(-\frac{i\left(a+b\tan^{-1}(cx)\right)}{cd^2(-i+cx)^2}-\frac{a+b\tan^{-1}(cx)}{cd^2(-i+cx)}\right)dx$$

$$= -\frac{i\int\frac{a+b\tan^{-1}(cx)}{(-i+cx)^2}\,dx}{cd^2}-\frac{\int\frac{a+b\tan^{-1}(cx)}{-i+cx}\,dx}{cd^2}$$

$$= -\frac{i\left(a+b\tan^{-1}(cx)\right)}{c^2d^2(i-cx)}+\frac{\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2}{1+icx}\right)}{c^2d^2}-\frac{(ib)\int\frac{1}{(-i+cx)\left(1+c^2x^2\right)}\,dx}{cd^2}-\frac{b\int\frac{\log\left(\frac{2}{1+icx}\right)}{1+c^2x^2}}{cd^2}$$

$$= -\frac{i\left(a+b\tan^{-1}(cx)\right)}{c^2d^2(i-cx)}+\frac{\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2}{1+icx}\right)}{c^2d^2}+\frac{(ib)\,\text{Subst}\left(\int\frac{\log(2x)}{1-2x}\,dx,x,\frac{1}{1+icx}\right)}{c^2d^2}-\frac{(i}{}$$

$$= -\frac{i\left(a+b\tan^{-1}(cx)\right)}{c^2d^2(i-cx)}+\frac{\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2}{1+icx}\right)}{c^2d^2}+\frac{ib\text{Li}_2\left(1-\frac{2}{1+icx}\right)}{2c^2d^2}-\frac{(ib)\int\left(-\frac{i}{2(-i+cx)^2}\right.}{cd^2}$$

$$= -\frac{b}{2c^2d^2(i-cx)}-\frac{i\left(a+b\tan^{-1}(cx)\right)}{c^2d^2(i-cx)}+\frac{\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2}{1+icx}\right)}{c^2d^2}+\frac{ib\text{Li}_2\left(1-\frac{2}{1+icx}\right)}{2c^2d^2}+\frac{b}{}$$

$$= -\frac{b}{2c^2d^2(i-cx)}+\frac{b\tan^{-1}(cx)}{2c^2d^2}-\frac{i\left(a+b\tan^{-1}(cx)\right)}{c^2d^2(i-cx)}+\frac{\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2}{1+icx}\right)}{c^2d^2}+\frac{ib\text{Li}_2\left(\right.}{2}$$

Mathematica [A] time = 0.0897185, size = 128, normalized size = 1.05

$$\frac{ib\text{PolyLog}\left(2,-\frac{cx+i}{-cx+i}\right)}{2c^2d^2}-\frac{i\left(a+b\tan^{-1}(cx)\right)}{c^2d^2(-cx+i)}+\frac{\log\left(\frac{2i}{-cx+i}\right)\left(a+b\tan^{-1}(cx)\right)}{c^2d^2}-\frac{b\left(-\frac{\tan^{-1}(cx)}{c}+\frac{1}{c(-cx+i)}\right)}{2cd^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcTan[c*x]))/(d + I*c*d*x)^2,x]
```

```
[Out] ((-I)*(a + b*ArcTan[c*x]))/(c^2*d^2*(I - c*x)) - (b*(1/(c*(I - c*x)) - ArcT
an[c*x]/c))/(2*c*d^2) + ((a + b*ArcTan[c*x])*Log[(2*I)/(I - c*x)])/(c^2*d^2
) + ((I/2)*b*PolyLog[2, -((I + c*x)/(I - c*x))])/(c^2*d^2)
```

Maple [B] time = 0.055, size = 293, normalized size = 2.4

$$-\frac{a\ln\left(c^2x^2+1\right)}{2\,c^2d^2}-\frac{ia\arctan(cx)}{c^2d^2}+\frac{ia}{c^2d^2\,(cx-i)}-\frac{b\arctan(cx)\ln(cx-i)}{c^2d^2}+\frac{ib\arctan(cx)}{c^2d^2\,(cx-i)}+\frac{\frac{i}{16}b\ln\left(c^4x^4+10\,c^2x^2+9\right.}{c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctan(c*x))/(d+I*c*d*x)^2,x)
```

```
[Out] -1/2/c^2*a/d^2*ln(c^2*x^2+1)-I/c^2*a/d^2*arctan(c*x)+I/c^2*a/d^2/(c*x-I)-1/
c^2*b/d^2*arctan(c*x)*ln(c*x-I)+I/c^2*b/d^2*arctan(c*x)/(c*x-I)+1/16*I/c^2*
b/d^2*ln(c^4*x^4+10*c^2*x^2+9)+1/8/c^2*b/d^2*arctan(1/6*c^3*x^3+7/6*c*x)-1/
8/c^2*b/d^2*arctan(1/2*c*x)+1/4/c^2*b/d^2*arctan(1/2*c*x-1/2*I)+1/2/c^2*b/d
^2/(c*x-I)-1/8*I/c^2*b/d^2*ln(c^2*x^2+1)+1/4*b*arctan(c*x)/c^2/d^2+1/2*I/c^
2*b/d^2*ln(-1/2*I*(c*x+I))*ln(c*x-I)+1/2*I/c^2*b/d^2*dilog(-1/2*I*(c*x+I))-
1/4*I/c^2*b/d^2*ln(c*x-I)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))/(d+I*c*d*x)^2,x, algorithm="maxima")
```

```
[Out] a*(I/(c^3*d^2*x - I*c^2*d^2) - log(c*x - I)/(c^2*d^2)) - 1/8*(32*(I*c*x + 1
)*arctan(c*x)^2 + 32*c*x*arctan2(1, c*x) - 8*(-I*c*x - 1)*log(c^2*x^2 + 1)^
2 - (8*c^3*d^2*x - 8*I*c^2*d^2)*((c*(x/(c^5*d^2*x^2 + c^3*d^2) + arctan(c*x
)/(c^4*d^2)) - 2*arctan(c*x)/(c^5*d^2*x^2 + c^3*d^2))*c + 8*integrate(1/4*l
og(c^2*x^2 + 1)/(c^5*d^2*x^4 + 2*c^3*d^2*x^2 + c*d^2), x)) - 8*(I*c^3*d^2*x
 + c^2*d^2)*(c*(c^2/(c^7*d^2*x^2 + c^5*d^2) + log(c^2*x^2 + 1)/(c^5*d^2*x^2
 + c^3*d^2)) + 16*integrate(1/4*arctan(c*x)/(c^5*d^2*x^4 + 2*c^3*d^2*x^2 +
c*d^2), x)) + (8*c^4*d^2*x - 8*I*c^3*d^2)*(c*(x/(c^5*d^2*x^2 + c^3*d^2) + a
rctan(c*x)/(c^4*d^2)) - 8*c*integrate(1/4*x^2*log(c^2*x^2 + 1)/(c^5*d^2*x^4
 + 2*c^3*d^2*x^2 + c*d^2), x) - 2*arctan(c*x)/(c^5*d^2*x^2 + c^3*d^2)) + 8*
(-I*c^4*d^2*x - c^3*d^2)*(16*c*integrate(1/4*x^2*arctan(c*x)/(c^5*d^2*x^4 +
 2*c^3*d^2*x^2 + c*d^2), x) - c^2/(c^7*d^2*x^2 + c^5*d^2) - log(c^2*x^2 + 1
)/(c^5*d^2*x^2 + c^3*d^2)) + 8*(16*c^5*d^2*x - 16*I*c^4*d^2)*integrate(1/4*
(2*c*x^3*arctan(c*x) + x^2*log(c^2*x^2 + 1))/(c^5*d^2*x^4 + 2*c^3*d^2*x^2 +
 c*d^2), x) + 128*(-I*c^5*d^2*x - c^4*d^2)*integrate(1/4*(c*x^3*log(c^2*x^2
 + 1) - 2*x^2*arctan(c*x))/(c^5*d^2*x^4 + 2*c^3*d^2*x^2 + c*d^2), x) - 32*I
*arctan(c*x) - 32*I*arctan2(1, c*x) + 16*log(c^2*x^2 + 1))*b/(8*c^3*d^2*x -
 8*I*c^2*d^2)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{-i\,bx\log\left(-\frac{cx+i}{cx-i}\right)-2\,ax}{2\,c^2d^2x^2-4i\,cd^2x-2\,d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))/(d+I*c*d*x)^2,x, algorithm="fricas")
```

```
[Out] integral((-I*b*x*log(-(c*x + I)/(c*x - I)) - 2*a*x)/(2*c^2*d^2*x^2 - 4*I*c*
d^2*x - 2*d^2), x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atan(c*x))/(d+I*c*d*x)**2,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\arctan(cx)+a)x}{(i\,cdx+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))/(d+I*c*d*x)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)*x/(I*c*d*x + d)^2, x)
```

3.54 $\int \frac{a+b\tan^{-1}(cx)}{(d+icdx)^2} \, dx$

Optimal. Leaf size=69

$$\frac{i\left(a+b\tan^{-1}(cx)\right)}{cd^2(1+icx)}+\frac{ib}{2cd^2(-cx+i)}-\frac{ib\tan^{-1}(cx)}{2cd^2}$$

```
[Out] ((I/2)*b)/(c*d^2*(I - c*x)) - ((I/2)*b*ArcTan[c*x])/(c*d^2) + (I*(a + b*Arc
Tan[c*x]))/(c*d^2*(1 + I*c*x))
```

Rubi [A] time = 0.0472076, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4862, 627, 44, 203}

$$\frac{i\left(a+b\tan^{-1}(cx)\right)}{cd^2(1+icx)}+\frac{ib}{2cd^2(-cx+i)}-\frac{ib\tan^{-1}(cx)}{2cd^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])/(d + I*c*d*x)^2,x]
```


```
[Out] ((I/2)*b)/(c*d^2*(I - c*x)) - ((I/2)*b*ArcTan[c*x])/(c*d^2) + (I*(a + b*Arc
Tan[c*x]))/(c*d^2*(1 + I*c*x))
```


Rule 4862

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x]))/(e*(q + 1)), x] - Dist[(b*
c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\tan^{-1}(cx)}{(d+icdx)^2}\,dx &= \frac{i\left(a+b\tan^{-1}(cx)\right)}{cd^2(1+icx)}-\frac{(ib)\int \frac{1}{(d+icdx)\left(1+c^2x^2\right)}\,dx}{d}\\
&= \frac{i\left(a+b\tan^{-1}(cx)\right)}{cd^2(1+icx)}-\frac{(ib)\int \frac{1}{\left(\frac{1}{d}-\frac{icx}{d}\right)(d+icdx)^2}\,dx}{d}\\
&= \frac{i\left(a+b\tan^{-1}(cx)\right)}{cd^2(1+icx)}-\frac{(ib)\int \left(-\frac{1}{2d(-i+cx)^2}+\frac{1}{2d\left(1+c^2x^2\right)}\right)dx}{d}\\
&= \frac{ib}{2cd^2(i-cx)}+\frac{i\left(a+b\tan^{-1}(cx)\right)}{cd^2(1+icx)}-\frac{(ib)\int \frac{1}{1+c^2x^2}\,dx}{2d^2}\\
&= \frac{ib}{2cd^2(i-cx)}-\frac{ib\tan^{-1}(cx)}{2cd^2}+\frac{i\left(a+b\tan^{-1}(cx)\right)}{cd^2(1+icx)}
\end{aligned}$$

Mathematica [A] time = 0.032655, size = 42, normalized size = 0.61

$$\frac{2a+(b-ibcx)\tan^{-1}(cx)-ib}{2cd^2(cx-i)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])/(d + I*c*d*x)^2,x]
```

```
[Out] (2*a - I*b + (b - I*b*c*x)*ArcTan[c*x])/(2*c*d^2*(-I + c*x))
```

Maple [A] time = 0.039, size = 76, normalized size = 1.1

$$\frac{ia}{cd^2(1+icx)}+\frac{i\arctan(cx)\,b}{cd^2(1+icx)}-\frac{\frac{i}{2}b\arctan(cx)}{cd^2}-\frac{\frac{i}{2}b}{cd^2(cx-i)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/(d+I*c*d*x)^2,x)
```

```
[Out] I/c*a/d^2/(1+I*c*x)+I/c*b/d^2/(1+I*c*x)*arctan(c*x)-1/2*I*b*arctan(c*x)/c/d
^2-1/2*I/c*b/d^2/(c*x-I)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/(d+I*c*d*x)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 2.52786, size = 112, normalized size = 1.62

$$\frac{(bcx+ib)\log\left(-\frac{cx+i}{cx-i}\right)+4a-2ib}{4\left(c^2d^2x-icd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/(d+I*c*d*x)^2,x, algorithm="fricas")
```

```
[Out] 1/4*((b*c*x + I*b)*log(-(c*x + I)/(c*x - I)) + 4*a - 2*I*b)/(c^2*d^2*x - I*
c*d^2)
```

Sympy [B] time = 16.1471, size = 1698, normalized size = 24.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/(d+I*c*d*x)**2,x)
```

```
[Out] I*b*log(-I*c*x + 1)/(2*c**2*d**2*x - 2*I*c*d**2) - I*b*log(I*c*x + 1)/(2*c*
*2*d**2*x - 2*I*c*d**2) + b*(log(-b*(262144*a**16*d**2 - 2097152*I*a**15*b*
d**2 - 7864320*a**14*b**2*d**2 + 18350080*I*a**13*b**3*d**2 + 29818880*a**1
2*b**4*d**2 - 35782656*I*a**11*b**5*d**2 - 32800768*a**10*b**6*d**2 + 23429
120*I*a**9*b**7*d**2 + 13178880*a**8*b**8*d**2 - 5857280*I*a**7*b**9*d**2 -
 2050048*a**6*b**10*d**2 + 559104*I*a**5*b**11*d**2 + 116480*a**4*b**12*d**
2 - 17920*I*a**3*b**13*d**2 - 1920*a**2*b**14*d**2 + 128*I*a*b**15*d**2 + 4
*b**16*d**2)/(4*c*d**2*(65536*I*a**16*b + 524288*a**15*b**2 - 1966080*I*a**
14*b**3 - 4587520*a**13*b**4 + 7454720*I*a**12*b**5 + 8945664*a**11*b**6 -
8200192*I*a**10*b**7 - 5857280*a**9*b**8 + 3294720*I*a**8*b**9 + 1464320*a*
*7*b**10 - 512512*I*a**6*b**11 - 139776*a**5*b**12 + 29120*I*a**4*b**13 + 4
480*a**3*b**14 - 480*I*a**2*b**15 - 32*a*b**16 + I*b**17)) + x)/4 - log(b*(
262144*a**16*d**2 - 2097152*I*a**15*b*d**2 - 7864320*a**14*b**2*d**2 + 1835
0080*I*a**13*b**3*d**2 + 29818880*a**12*b**4*d**2 - 35782656*I*a**11*b**5*d
**2 - 32800768*a**10*b**6*d**2 + 23429120*I*a**9*b**7*d**2 + 13178880*a**8*
b**8*d**2 - 5857280*I*a**7*b**9*d**2 - 2050048*a**6*b**10*d**2 + 559104*I*a
**5*b**11*d**2 + 116480*a**4*b**12*d**2 - 17920*I*a**3*b**13*d**2 - 1920*a*
*2*b**14*d**2 + 128*I*a*b**15*d**2 + 4*b**16*d**2)/(4*c*d**2*(65536*I*a**16
*b + 524288*a**15*b**2 - 1966080*I*a**14*b**3 - 4587520*a**13*b**4 + 745472
0*I*a**12*b**5 + 8945664*a**11*b**6 - 8200192*I*a**10*b**7 - 5857280*a**9*b
**8 + 3294720*I*a**8*b**9 + 1464320*a**7*b**10 - 512512*I*a**6*b**11 - 1397
76*a**5*b**12 + 29120*I*a**4*b**13 + 4480*a**3*b**14 - 480*I*a**2*b**15 - 3
2*a*b**16 + I*b**17)) + x)/4)/(c*d**2) + (2097152*I*a**21*c + 22020096*a**2
0*b*c - 110100480*I*a**19*b**2*c - 348651520*a**18*b**3*c + 784465920*I*a**
17*b**4*c + 1333592064*a**16*b**5*c - 1778122752*I*a**15*b**6*c - 190513152
0*a**14*b**7*c + 1666990080*I*a**13*b**8*c + 1203937280*a**12*b**9*c - 7223
62368*I*a**11*b**10*c - 361181184*a**10*b**11*c + 150492160*I*a**9*b**12*c
+ 52093440*a**8*b**13*c - 14883840*I*a**7*b**14*c - 3472896*a**6*b**15*c +
651168*I*a**5*b**16*c + 95760*a**4*b**17*c - 10640*I*a**3*b**18*c - 840*a**
2*b**19*c + 42*I*a*b**20*c + b**21*c)/(2097152*a**20*c**2*d**2 - 20971520*I
*a**19*b*c**2*d**2 - 99614720*a**18*b**2*c**2*d**2 + 298844160*I*a**17*b**3
*c**2*d**2 + 635043840*a**16*b**4*c**2*d**2 - 1016070144*I*a**15*b**5*c**2*
d**2 - 1270087680*a**14*b**6*c**2*d**2 + 1270087680*I*a**13*b**7*c**2*d**2
+ 1031946240*a**12*b**8*c**2*d**2 - 687964160*I*a**11*b**9*c**2*d**2 - 3783
80288*a**10*b**10*c**2*d**2 + 171991040*I*a**9*b**11*c**2*d**2 + 64496640*a
**8*b**12*c**2*d**2 - 19845120*I*a**7*b**13*c**2*d**2 - 4961280*a**6*b**14*
c**2*d**2 + 992256*I*a**5*b**15*c**2*d**2 + 155040*a**4*b**16*c**2*d**2 - 1
8240*I*a**3*b**17*c**2*d**2 - 1520*a**2*b**18*c**2*d**2 + 80*I*a*b**19*c**2
*d**2 + 2*b**20*c**2*d**2 + x*(2097152*I*a**20*c**3*d**2 + 20971520*a**19*b
*c**3*d**2 - 99614720*I*a**18*b**2*c**3*d**2 - 298844160*a**17*b**3*c**3*d*
```

```
*2 + 635043840*I*a**16*b**4*c**3*d**2 + 1016070144*a**15*b**5*c**3*d**2 - 1
270087680*I*a**14*b**6*c**3*d**2 - 1270087680*a**13*b**7*c**3*d**2 + 103194
6240*I*a**12*b**8*c**3*d**2 + 687964160*a**11*b**9*c**3*d**2 - 378380288*I*
a**10*b**10*c**3*d**2 - 171991040*a**9*b**11*c**3*d**2 + 64496640*I*a**8*b*
*12*c**3*d**2 + 19845120*a**7*b**13*c**3*d**2 - 4961280*I*a**6*b**14*c**3*d
**2 - 992256*a**5*b**15*c**3*d**2 + 155040*I*a**4*b**16*c**3*d**2 + 18240*a
**3*b**17*c**3*d**2 - 1520*I*a**2*b**18*c**3*d**2 - 80*a*b**19*c**3*d**2 +
2*I*b**20*c**3*d**2))
```

Giac [A] time = 1.12151, size = 150, normalized size = 2.17

$$-\frac{1}{4}\left(cd^2 i\left(\frac{i\log\left(\frac{2\,di}{cdix+d}-i\right)}{c^2d^4}+\frac{2\,i}{(cdix+d)c^2d^3}\right)-\frac{4\,i\arctan\left(\frac{\left((cdix+d)i^2+d\right)i}{d}\right)}{(cdix+d)cd}\right)b+\frac{ai}{(cdix+d)cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/(d+I*c*d*x)^2,x, algorithm="giac")
```

```
[Out] -1/4*(c*d^2*i*(i*log(2*d*i/(c*d*i*x + d) - i)/(c^2*d^4) + 2*i/((c*d*i*x + d
)*c^2*d^3)) - 4*i*arctan(((c*d*i*x + d)*i^2 + d)*i/d)/((c*d*i*x + d)*c*d))*
b + a*i/((c*d*i*x + d)*c*d)
```

3.55 $\int \frac{a+b\tan^{-1}(cx)}{x(d+icdx)^2}\,dx$

Optimal. Leaf size=150

$$\frac{ib\text{PolyLog}(2,-icx)}{2d^2}-\frac{ib\text{PolyLog}(2,icx)}{2d^2}+\frac{ib\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{2d^2}+\frac{i\left(a+b\tan^{-1}(cx)\right)}{d^2(-cx+i)}+\frac{\log\left(\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{d^2}$$

```
[Out] b/(2*d^2*(I - c*x)) - (b*ArcTan[c*x])/(2*d^2) + (I*(a + b*ArcTan[c*x]))/(d^
2*(I - c*x)) + (a*Log[x])/d^2 + ((a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/d^
2 + ((I/2)*b*PolyLog[2, (-I)*c*x])/d^2 - ((I/2)*b*PolyLog[2, I*c*x])/d^2 +
((I/2)*b*PolyLog[2, 1 - 2/(1 + I*c*x)])/d^2
```

Rubi [A] time = 0.193568, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {4876, 4848, 2391, 4862, 627, 44, 203, 4854, 2402, 2315}

$$\frac{ib\text{PolyLog}(2,-icx)}{2d^2}-\frac{ib\text{PolyLog}(2,icx)}{2d^2}+\frac{ib\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{2d^2}+\frac{i\left(a+b\tan^{-1}(cx)\right)}{d^2(-cx+i)}+\frac{\log\left(\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])/(x*(d + I*c*d*x)^2),x]
```

```
[Out] b/(2*d^2*(I - c*x)) - (b*ArcTan[c*x])/(2*d^2) + (I*(a + b*ArcTan[c*x]))/(d^
2*(I - c*x)) + (a*Log[x])/d^2 + ((a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/d^
2 + ((I/2)*b*PolyLog[2, (-I)*c*x])/d^2 - ((I/2)*b*PolyLog[2, I*c*x])/d^2 +
((I/2)*b*PolyLog[2, 1 - 2/(1 + I*c*x)])/d^2
```

Rule 4876

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
```

```
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4862

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x]))/(e*(q + 1)), x] - Dist[(b*
c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
```

```
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int \frac{a+b\tan^{-1}(cx)}{x(d+icdx)^2}\,dx = \int \left(\frac{a+b\tan^{-1}(cx)}{d^2x} + \frac{ic\left(a+b\tan^{-1}(cx)\right)}{d^2(-i+cx)^2} - \frac{c\left(a+b\tan^{-1}(cx)\right)}{d^2(-i+cx)}\right) dx$$

$$= \frac{\int \frac{a+b\tan^{-1}(cx)}{x}\,dx}{d^2} + \frac{(ic)\int \frac{a+b\tan^{-1}(cx)}{(-i+cx)^2}\,dx}{d^2} - \frac{c\int \frac{a+b\tan^{-1}(cx)}{-i+cx}\,dx}{d^2}$$

$$= \frac{i\left(a+b\tan^{-1}(cx)\right)}{d^2(i-cx)} + \frac{a\log(x)}{d^2} + \frac{\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2}{1+icx}\right)}{d^2} + \frac{(ib)\int \frac{\log(1-icx)}{x}\,dx}{2d^2} - \frac{(ib)\int \frac{\log(1}{x}}{2d^2}$$

$$= \frac{i\left(a+b\tan^{-1}(cx)\right)}{d^2(i-cx)} + \frac{a\log(x)}{d^2} + \frac{\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2}{1+icx}\right)}{d^2} + \frac{ib\text{Li}_2(-icx)}{2d^2} - \frac{ib\text{Li}_2(icx)}{2d^2} + \frac{(ib)}{}$$

$$= \frac{i\left(a+b\tan^{-1}(cx)\right)}{d^2(i-cx)} + \frac{a\log(x)}{d^2} + \frac{\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2}{1+icx}\right)}{d^2} + \frac{ib\text{Li}_2(-icx)}{2d^2} - \frac{ib\text{Li}_2(icx)}{2d^2} + \frac{ib\text{L}}{}$$

$$= \frac{b}{2d^2(i-cx)} + \frac{i\left(a+b\tan^{-1}(cx)\right)}{d^2(i-cx)} + \frac{a\log(x)}{d^2} + \frac{\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2}{1+icx}\right)}{d^2} + \frac{ib\text{Li}_2(-icx)}{2d^2} - \frac{ib\text{l}}{}$$

$$= \frac{b}{2d^2(i-cx)} - \frac{b\tan^{-1}(cx)}{2d^2} + \frac{i\left(a+b\tan^{-1}(cx)\right)}{d^2(i-cx)} + \frac{a\log(x)}{d^2} + \frac{\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2}{1+icx}\right)}{d^2} + \frac{ib\text{l}}{}$$

Mathematica [A] time = 0.148046, size = 128, normalized size = 0.85

$$\frac{ib\text{PolyLog}(2,-icx) - ib\text{PolyLog}(2,icx) + ib\text{PolyLog}\left(2,\frac{cx+i}{cx-i}\right) - \frac{2i\left(a+b\tan^{-1}(cx)\right)}{cx-i} + 2\log\left(\frac{2i}{-cx+i}\right)\left(a+b\tan^{-1}(cx)\right) + 2a\log}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])/(x*(d + I*c*d*x)^2),x]
```

```
[Out] (b*((I - c*x)^(-1) - ArcTan[c*x]) - ((2*I)*(a + b*ArcTan[c*x]))/(-I + c*x)
+ 2*a*Log[x] + 2*(a + b*ArcTan[c*x])*Log[(2*I)/(I - c*x)] + I*b*PolyLog[2,
```

```
(-I)*c*x] - I*b*PolyLog[2, I*c*x] + I*b*PolyLog[2, (I + c*x)/(-I + c*x)])/(
2*d^2)
```

Maple [A] time = 0.063, size = 251, normalized size = 1.7

$$\frac{-ia}{d^2(cx-i)}-\frac{a\ln\left(c^2x^2+1\right)}{2d^2}+\frac{\frac{i}{2}b\operatorname{dilog}(1+icx)}{d^2}+\frac{a\ln(cx)}{d^2}-\frac{ib\arctan(cx)}{d^2(cx-i)}-\frac{b\arctan(cx)\ln(cx-i)}{d^2}+\frac{b\arctan(cx)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/x/(d+I*c*d*x)^2,x)
```

```
[Out] -I*a/d^2/(c*x-I)-1/2*a/d^2*ln(c^2*x^2+1)+1/2*I*b/d^2*dilog(1+I*c*x)+a/d^2*l
n(c*x)-I*b/d^2*arctan(c*x)/(c*x-I)-b/d^2*arctan(c*x)*ln(c*x-I)+b/d^2*arctan
(c*x)*ln(c*x)-I*a/d^2*arctan(c*x)-1/2*I*b/d^2*dilog(1-I*c*x)+1/2*I*b/d^2*ln
(-1/2*I*(c*x+I))*ln(c*x-I)+1/2*I*b/d^2*ln(c*x)*ln(1+I*c*x)-1/2*b/d^2/(c*x-I
)-1/2*b*arctan(c*x)/d^2-1/4*I*b/d^2*ln(c*x-I)^2+1/2*I*b/d^2*dilog(-1/2*I*(c
*x+I))-1/2*I*b/d^2*ln(c*x)*ln(1-I*c*x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\left(-2ic\int\frac{\arctan(cx)}{c^4d^2x^4+2c^2d^2x^2+d^2}\,dx-\int\frac{\left(c^2x^2-1\right)\arctan(cx)}{c^4d^2x^5+2c^2d^2x^3+d^2x}\,dx\right)b+a\left(-\frac{i}{cd^2x-id^2}-\frac{\log(cx-i)}{d^2}+\frac{\log(x)}{d^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x/(d+I*c*d*x)^2,x, algorithm="maxima")
```

```
[Out] (-2*I*c*integrate(arctan(c*x)/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2), x) - int
egrate((c^2*x^2 - 1)*arctan(c*x)/(c^4*d^2*x^5 + 2*c^2*d^2*x^3 + d^2*x), x))
*b + a*(-I/(c*d^2*x - I*d^2) - log(c*x - I)/d^2 + log(x)/d^2)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{-ib\log\left(-\frac{cx+i}{cx-i}\right)-2a}{2c^2d^2x^3-4icd^2x^2-2d^2x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x/(d+I*c*d*x)^2,x, algorithm="fricas")
```

```
[Out] integral((-I*b*log(-(c*x + I)/(c*x - I)) - 2*a)/(2*c^2*d^2*x^3 - 4*I*c*d^2*
x^2 - 2*d^2*x), x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/x/(d+I*c*d*x)**2,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \arctan(cx) + a}{(i\,cdx + d)^2 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x/(d+I*c*d*x)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)/((I*c*d*x + d)^2*x), x)
```

3.56 $\int \frac{a+b\tan^{-1}(cx)}{x^2(d+icdx)^2}\,dx$

Optimal. Leaf size=194

$$\frac{bc\text{PolyLog}(2,-icx)}{d^2}-\frac{bc\text{PolyLog}(2,icx)}{d^2}+\frac{bc\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{d^2}+\frac{c\left(a+b\tan^{-1}(cx)\right)}{d^2(-cx+i)}-\frac{a+b\tan^{-1}(cx)}{d^2x}-\frac{2ic\log}{}$$

```
[Out] ((-I/2)*b*c)/(d^2*(I - c*x)) + ((I/2)*b*c*ArcTan[c*x])/d^2 - (a + b*ArcTan[
c*x])/(d^2*x) + (c*(a + b*ArcTan[c*x]))/(d^2*(I - c*x)) - ((2*I)*a*c*Log[x]
)/d^2 + (b*c*Log[x])/d^2 - ((2*I)*c*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])
/d^2 - (b*c*Log[1 + c^2*x^2])/(2*d^2) + (b*c*PolyLog[2, (-I)*c*x])/d^2 - (b
*c*PolyLog[2, I*c*x])/d^2 + (b*c*PolyLog[2, 1 - 2/(1 + I*c*x)])/d^2
```

Rubi [A] time = 0.241161, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 15, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.652, Rules used = {4876, 4852, 266, 36, 29, 31, 4848, 2391, 4862, 627, 44, 203, 4854, 2402, 2315}

$$\frac{bc\text{PolyLog}(2,-icx)}{d^2}-\frac{bc\text{PolyLog}(2,icx)}{d^2}+\frac{bc\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{d^2}+\frac{c\left(a+b\tan^{-1}(cx)\right)}{d^2(-cx+i)}-\frac{a+b\tan^{-1}(cx)}{d^2x}-\frac{2ic\log}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])/(x^2*(d + I*c*d*x)^2),x]
```


```
[Out] ((-I/2)*b*c)/(d^2*(I - c*x)) + ((I/2)*b*c*ArcTan[c*x])/d^2 - (a + b*ArcTan[
c*x])/(d^2*x) + (c*(a + b*ArcTan[c*x]))/(d^2*(I - c*x)) - ((2*I)*a*c*Log[x]
)/d^2 + (b*c*Log[x])/d^2 - ((2*I)*c*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])
/d^2 - (b*c*Log[1 + c^2*x^2])/(2*d^2) + (b*c*PolyLog[2, (-I)*c*x])/d^2 - (b
*c*PolyLog[2, I*c*x])/d^2 + (b*c*PolyLog[2, 1 - 2/(1 + I*c*x)])/d^2
```


Rule 4876

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4862

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x]))/(e*(q + 1)), x] - Dist[(b*
c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int \frac{a+b\tan^{-1}(cx)}{x^2(d+icdx)^2}\,dx = \int \left(\frac{a+b\tan^{-1}(cx)}{d^2x^2} - \frac{2ic\left(a+b\tan^{-1}(cx)\right)}{d^2x} + \frac{c^2\left(a+b\tan^{-1}(cx)\right)}{d^2(-i+cx)^2} + \frac{2ic^2\left(a+b\tan^{-1}(cx)\right)}{d^2(-i+cx)}\right) dx$$

$$= \frac{\int \frac{a+b\tan^{-1}(cx)}{x^2}\,dx}{d^2} - \frac{(2ic)\int \frac{a+b\tan^{-1}(cx)}{x}\,dx}{d^2} + \frac{\left(2ic^2\right)\int \frac{a+b\tan^{-1}(cx)}{-i+cx}\,dx}{d^2} + \frac{c^2\int \frac{a+b\tan^{-1}(cx)}{(-i+cx)^2}\,dx}{d^2}$$

$$= -\frac{a+b\tan^{-1}(cx)}{d^2x} + \frac{c\left(a+b\tan^{-1}(cx)\right)}{d^2(i-cx)} - \frac{2iac\log(x)}{d^2} - \frac{2ic\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2}{1+icx}\right)}{d^2} + \frac{(bc)\ldots}{}$$

$$= -\frac{a+b\tan^{-1}(cx)}{d^2x} + \frac{c\left(a+b\tan^{-1}(cx)\right)}{d^2(i-cx)} - \frac{2iac\log(x)}{d^2} - \frac{2ic\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2}{1+icx}\right)}{d^2} + \frac{bc\text{Li}\ldots}{}$$

$$= -\frac{a+b\tan^{-1}(cx)}{d^2x} + \frac{c\left(a+b\tan^{-1}(cx)\right)}{d^2(i-cx)} - \frac{2iac\log(x)}{d^2} - \frac{2ic\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2}{1+icx}\right)}{d^2} + \frac{bc\text{Li}\ldots}{}$$

$$= -\frac{ibc}{2d^2(i-cx)} - \frac{a+b\tan^{-1}(cx)}{d^2x} + \frac{c\left(a+b\tan^{-1}(cx)\right)}{d^2(i-cx)} - \frac{2iac\log(x)}{d^2} + \frac{bc\log(x)}{d^2} - \frac{2ic\left(a+b\tan\ldots\right.}{}$$

$$= -\frac{ibc}{2d^2(i-cx)} + \frac{ibc\tan^{-1}(cx)}{2d^2} - \frac{a+b\tan^{-1}(cx)}{d^2x} + \frac{c\left(a+b\tan^{-1}(cx)\right)}{d^2(i-cx)} - \frac{2iac\log(x)}{d^2} + \frac{bc\log(x)}{d^2}\ldots$$

Mathematica [A] time = 0.287197, size = 165, normalized size = 0.85

$$-\frac{-2bc\text{PolyLog}(2,-icx) + 2bc\text{PolyLog}(2,icx) - 2bc\text{PolyLog}\left(2,\frac{cx+i}{cx-i}\right) + \frac{2\left(a+b\tan^{-1}(cx)\right)}{x} + \frac{2c\left(a+b\tan^{-1}(cx)\right)}{cx-i} + 4ic\log\left(\frac{2i}{-cx+i}\right)\ldots}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])/(x^2*(d + I*c*d*x)^2),x]
```

```
[Out] -(I*b*c*((I - c*x)^(-1) - ArcTan[c*x]) + (2*(a + b*ArcTan[c*x]))/x + (2*c*(
a + b*ArcTan[c*x]))/(-I + c*x) + (4*I)*a*c*Log[x] + (4*I)*c*(a + b*ArcTan[c
*x])*Log[(2*I)/(I - c*x)] + b*c*(-2*Log[x] + Log[1 + c^2*x^2]) - 2*b*c*Poly
Log[2, (-I)*c*x] + 2*b*c*PolyLog[2, I*c*x] - 2*b*c*PolyLog[2, (I + c*x)/(-I
 + c*x)])/(2*d^2)
```

Maple [A] time = 0.072, size = 340, normalized size = 1.8

$$-\frac{ca}{d^2(cx-i)}-2\frac{ca\arctan(cx)}{d^2}+\frac{ica\ln(c^2x^2+1)}{d^2}-\frac{a}{d^2x}-\frac{2icb\arctan(cx)\ln(cx)}{d^2}-\frac{bc\arctan(cx)}{d^2(cx-i)}+\frac{\frac{i}{2}bc\arctan(cx)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/x^2/(d+I*c*d*x)^2,x)
```

```
[Out] -c*a/d^2/(c*x-I)-2*c*a/d^2*arctan(c*x)+I*c*a/d^2*ln(c^2*x^2+1)-a/d^2/x-2*I*
c*b/d^2*arctan(c*x)*ln(c*x)-c*b/d^2*arctan(c*x)/(c*x-I)+1/2*I*b*c*arctan(c*
x)/d^2-b/d^2*arctan(c*x)/x+2*I*c*b/d^2*arctan(c*x)*ln(c*x-I)-c*b/d^2*ln(-I*
(-c*x+I))*ln(-I*c*x)+c*b/d^2*ln(-I*(-c*x+I))*ln(c*x)-c*b/d^2*dilog(-I*c*x)-
c*b/d^2*dilog(-I*(c*x+I))-c*b/d^2*ln(c*x)*ln(-I*(c*x+I))+c*b/d^2*dilog(-1/2
*I*(c*x+I))+c*b/d^2*ln(c*x-I)*ln(-1/2*I*(c*x+I))-1/2*c*b/d^2*ln(c*x-I)^2-2*
I*c*a/d^2*ln(c*x)-1/2*b*c*ln(c^2*x^2+1)/d^2+1/2*I*c*b/d^2/(c*x-I)+c*b/d^2*l
n(c*x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\left(-2ic\int\frac{\arctan(cx)}{c^4d^2x^5+2c^2d^2x^3+d^2x}\,dx-\int\frac{(c^2x^2-1)\arctan(cx)}{c^4d^2x^6+2c^2d^2x^4+d^2x^2}\,dx\right)b-a\left(\frac{c}{cd^2x-id^2}-\frac{2ic\log(cx-i)}{d^2}+\frac{2ic\log(x)}{d^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^2/(d+I*c*d*x)^2,x, algorithm="maxima")
```

```
[Out] (-2*I*c*integrate(arctan(c*x)/(c^4*d^2*x^5 + 2*c^2*d^2*x^3 + d^2*x), x) - i
ntegrate((c^2*x^2 - 1)*arctan(c*x)/(c^4*d^2*x^6 + 2*c^2*d^2*x^4 + d^2*x^2),
 x))*b - a*(c/(c*d^2*x - I*d^2) - 2*I*c*log(c*x - I)/d^2 + 2*I*c*log(x)/d^2
 + 1/(d^2*x))
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{-ib\log\left(-\frac{cx+i}{cx-i}\right)-2a}{2c^2d^2x^4-4icd^2x^3-2d^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^2/(d+I*c*d*x)^2,x, algorithm="fricas")
```

```
[Out] integral((-I*b*log(-(c*x + I)/(c*x - I)) - 2*a)/(2*c^2*d^2*x^4 - 4*I*c*d^2*
x^3 - 2*d^2*x^2), x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/x**2/(d+I*c*d*x)**2,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \arctan(cx) + a}{(icdx + d)^2 x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^2/(d+I*c*d*x)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)/((I*c*d*x + d)^2*x^2), x)
```

3.57 $\int \frac{a+b\tan^{-1}(cx)}{x^3(d+icdx)^2}\,dx$

Optimal. Leaf size=244

$$-\frac{3ibc^2\text{PolyLog}(2,-icx)}{2d^2}+\frac{3ibc^2\text{PolyLog}(2,icx)}{2d^2}-\frac{3ibc^2\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{2d^2}-\frac{ic^2\left(a+b\tan^{-1}(cx)\right)}{d^2(-cx+i)}-\frac{3c^2\log\left(\frac{2}{1+ic}\right.}{}$$

```
[Out] -(b*c)/(2*d^2*x) - (b*c^2)/(2*d^2*(I - c*x)) - (a + b*ArcTan[c*x])/(2*d^2*x
^2) + ((2*I)*c*(a + b*ArcTan[c*x]))/(d^2*x) - (I*c^2*(a + b*ArcTan[c*x]))/(
d^2*(I - c*x)) - (3*a*c^2*Log[x])/d^2 - ((2*I)*b*c^2*Log[x])/d^2 - (3*c^2*(
a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/d^2 + (I*b*c^2*Log[1 + c^2*x^2])/d^2
 - (((3*I)/2)*b*c^2*PolyLog[2, (-I)*c*x])/d^2 + (((3*I)/2)*b*c^2*PolyLog[2,
 I*c*x])/d^2 - (((3*I)/2)*b*c^2*PolyLog[2, 1 - 2/(1 + I*c*x)])/d^2
```

Rubi [A] time = 0.267544, antiderivative size = 244, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 16, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.696, Rules used = {4876, 4852, 325, 203, 266, 36, 29, 31, 4848, 2391, 4862, 627, 44, 4854, 2402, 2315}

$$-\frac{3ibc^2\text{PolyLog}(2,-icx)}{2d^2}+\frac{3ibc^2\text{PolyLog}(2,icx)}{2d^2}-\frac{3ibc^2\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{2d^2}-\frac{ic^2\left(a+b\tan^{-1}(cx)\right)}{d^2(-cx+i)}-\frac{3c^2\log\left(\frac{2}{1+ic}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])/(x^3*(d + I*c*d*x)^2),x]
```


```
[Out] -(b*c)/(2*d^2*x) - (b*c^2)/(2*d^2*(I - c*x)) - (a + b*ArcTan[c*x])/(2*d^2*x
^2) + ((2*I)*c*(a + b*ArcTan[c*x]))/(d^2*x) - (I*c^2*(a + b*ArcTan[c*x]))/(
d^2*(I - c*x)) - (3*a*c^2*Log[x])/d^2 - ((2*I)*b*c^2*Log[x])/d^2 - (3*c^2*(
a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/d^2 + (I*b*c^2*Log[1 + c^2*x^2])/d^2
 - (((3*I)/2)*b*c^2*PolyLog[2, (-I)*c*x])/d^2 + (((3*I)/2)*b*c^2*PolyLog[2,
 I*c*x])/d^2 - (((3*I)/2)*b*c^2*PolyLog[2, 1 - 2/(1 + I*c*x)])/d^2
```


Rule 4876

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4862

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x]))/(e*(q + 1)), x] - Dist[(b*
c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int \frac{a+b\tan^{-1}(cx)}{x^3(d+icdx)^2}\,dx = \int \left(\frac{a+b\tan^{-1}(cx)}{d^2x^3} - \frac{2ic\left(a+b\tan^{-1}(cx)\right)}{d^2x^2} - \frac{3c^2\left(a+b\tan^{-1}(cx)\right)}{d^2x} - \frac{ic^3\left(a+b\tan^{-1}(cx)\right)}{d^2(-i+cx)^2} + \frac{3c\ldots}{\ldots}\right.$$

$$= \frac{\int \frac{a+b\tan^{-1}(cx)}{x^3}\,dx}{d^2} - \frac{(2ic)\int \frac{a+b\tan^{-1}(cx)}{x^2}\,dx}{d^2} - \frac{\left(3c^2\right)\int \frac{a+b\tan^{-1}(cx)}{x}\,dx}{d^2} - \frac{\left(ic^3\right)\int \frac{a+b\tan^{-1}(cx)}{(-i+cx)^2}\,dx}{d^2} + \frac{(3\ldots}{\ldots}$$

$$= -\frac{a+b\tan^{-1}(cx)}{2d^2x^2} + \frac{2ic\left(a+b\tan^{-1}(cx)\right)}{d^2x} - \frac{ic^2\left(a+b\tan^{-1}(cx)\right)}{d^2(i-cx)} - \frac{3ac^2\log(x)}{d^2} - \frac{3c^2\left(a+b\tan\ldots\right.}{\ldots}$$

$$= -\frac{bc}{2d^2x} - \frac{a+b\tan^{-1}(cx)}{2d^2x^2} + \frac{2ic\left(a+b\tan^{-1}(cx)\right)}{d^2x} - \frac{ic^2\left(a+b\tan^{-1}(cx)\right)}{d^2(i-cx)} - \frac{3ac^2\log(x)}{d^2} - \frac{3c^2\left(a\ldots\right.}{\ldots}$$

$$= -\frac{bc}{2d^2x} - \frac{bc^2\tan^{-1}(cx)}{2d^2} - \frac{a+b\tan^{-1}(cx)}{2d^2x^2} + \frac{2ic\left(a+b\tan^{-1}(cx)\right)}{d^2x} - \frac{ic^2\left(a+b\tan^{-1}(cx)\right)}{d^2(i-cx)} - \frac{3ac\ldots}{\ldots}$$

$$= -\frac{bc}{2d^2x} - \frac{bc^2}{2d^2(i-cx)} - \frac{bc^2\tan^{-1}(cx)}{2d^2} - \frac{a+b\tan^{-1}(cx)}{2d^2x^2} + \frac{2ic\left(a+b\tan^{-1}(cx)\right)}{d^2x} - \frac{ic^2\left(a+b\tan\ldots\right.}{d^2(i-c\ldots}$$

$$= -\frac{bc}{2d^2x} - \frac{bc^2}{2d^2(i-cx)} - \frac{a+b\tan^{-1}(cx)}{2d^2x^2} + \frac{2ic\left(a+b\tan^{-1}(cx)\right)}{d^2x} - \frac{ic^2\left(a+b\tan^{-1}(cx)\right)}{d^2(i-cx)} - \frac{3ac^2\,\mathrm{lo}\ldots}{d^2\ldots}$$

Mathematica [C] time = 0.334188, size = 222, normalized size = 0.91

$$-\frac{\frac{bc\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-c^2x^2\right)}{x} + 3ibc^2\text{PolyLog}(2,-icx) - 3ibc^2\text{PolyLog}(2,icx) + 3ibc^2\text{PolyLog}\left(2,\frac{cx+i}{cx-i}\right) - \frac{2ic^2\left(a+b\tan\ldots\right.}{cx-i}}{\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c*x])/(x^3*(d + I*c*d*x)^2),x]
```

```
[Out] -(-(b*c^2*((-I + c*x)^(-1) + ArcTan[c*x])) + (a + b*ArcTan[c*x])/x^2 - ((4*
I)*c*(a + b*ArcTan[c*x]))/x - ((2*I)*c^2*(a + b*ArcTan[c*x]))/(-I + c*x) +
(b*c*Hypergeometric2F1[-1/2, 1, 1/2, -(c^2*x^2)])/x + 6*a*c^2*Log[x] + 6*c^
2*(a + b*ArcTan[c*x])*Log[(2*I)/(I - c*x)] + (2*I)*b*c^2*(2*Log[x] - Log[1
+ c^2*x^2]) + (3*I)*b*c^2*PolyLog[2, (-I)*c*x] - (3*I)*b*c^2*PolyLog[2, I*c
```

```
*x] + (3*I)*b*c^2*PolyLog[2, (I + c*x)/(-I + c*x)])/(2*d^2)
```

Maple [A] time = 0.069, size = 380, normalized size = 1.6

$$\frac{\frac{3i}{2}c^2b\ln(cx)\ln(1-icx)}{d^2}+\frac{3c^2a\ln\left(c^2x^2+1\right)}{2d^2}+\frac{\frac{3i}{4}c^2b\left(\ln(cx-i)\right)^2}{d^2}-\frac{a}{2d^2x^2}-\frac{\frac{3i}{2}c^2b\,\mathrm{dilog}\left(-\frac{i}{2}(cx+i)\right)}{d^2}-3\frac{c^2a\ln(cx)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/x^3/(d+I*c*d*x)^2,x)
```

```
[Out] 3/2*I*c^2*b/d^2*ln(c*x)*ln(1-I*c*x)+3/2*c^2*a/d^2*ln(c^2*x^2+1)+3/4*I*c^2*b
/d^2*ln(c*x-I)^2-1/2*a/d^2/x^2-3/2*I*c^2*b/d^2*dilog(-1/2*I*(c*x+I))-3*c^2*
a/d^2*ln(c*x)-3/2*I*c^2*b/d^2*ln(-1/2*I*(c*x+I))*ln(c*x-I)+3*c^2*b/d^2*arct
an(c*x)*ln(c*x-I)-1/2*b/d^2*arctan(c*x)/x^2-3/2*I*c^2*b/d^2*dilog(1+I*c*x)-
3*c^2*b/d^2*arctan(c*x)*ln(c*x)-2*I*c^2*b/d^2*ln(c*x)+2*I*c*b/d^2*arctan(c*
x)/x+I*c^2*a/d^2/(c*x-I)+2*I*c*a/d^2/x+3*I*c^2*a/d^2*arctan(c*x)+I*c^2*b/d^
2*arctan(c*x)/(c*x-I)+I*b*c^2*ln(c^2*x^2+1)/d^2-3/2*I*c^2*b/d^2*ln(c*x)*ln(
1+I*c*x)+1/2*c^2*b/d^2/(c*x-I)-1/2*b*c/d^2/x+3/2*I*c^2*b/d^2*dilog(1-I*c*x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\left(-2ic\int\frac{\arctan(cx)}{c^4d^2x^6+2c^2d^2x^4+d^2x^2}\,dx-\int\frac{\left(c^2x^2-1\right)\arctan(cx)}{c^4d^2x^7+2c^2d^2x^5+d^2x^3}\,dx\right)b-\frac{1}{2}a\left(-\frac{2ic^2}{cd^2x-id^2}-\frac{6c^2\log(cx-i)}{d^2}+\frac{6c^2\log\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^3/(d+I*c*d*x)^2,x, algorithm="maxima")
```

```
[Out] (-2*I*c*integrate(arctan(c*x)/(c^4*d^2*x^6 + 2*c^2*d^2*x^4 + d^2*x^2), x) -
 integrate((c^2*x^2 - 1)*arctan(c*x)/(c^4*d^2*x^7 + 2*c^2*d^2*x^5 + d^2*x^3
), x))*b - 1/2*a*(-2*I*c^2/(c*d^2*x - I*d^2) - 6*c^2*log(c*x - I)/d^2 + 6*c
^2*log(x)/d^2 - (4*I*c*x - 1)/(d^2*x^2))
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{-i\,b\log\left(-\frac{cx+i}{cx-i}\right)-2\,a}{2\,c^2d^2x^5-4i\,cd^2x^4-2\,d^2x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^3/(d+I*c*d*x)^2,x, algorithm="fricas")
```

```
[Out] integral((-I*b*log(-(c*x + I)/(c*x - I)) - 2*a)/(2*c^2*d^2*x^5 - 4*I*c*d^2*
x^4 - 2*d^2*x^3), x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/x**3/(d+I*c*d*x)**2,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\arctan(cx)+a}{(i\,cdx+d)^2x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^3/(d+I*c*d*x)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)/((I*c*d*x + d)^2*x^3), x)
```

3.58 $\int \frac{x^4(a+b\tan^{-1}(cx))}{(d+icdx)^3} dx$

Optimal. Leaf size=256

$$-\frac{3b\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c^5d^3}+\frac{ix^2\left(a+b\tan^{-1}(cx)\right)}{2c^3d^3}+\frac{4\left(a+b\tan^{-1}(cx)\right)}{c^5d^3(-cx+i)}-\frac{i\left(a+b\tan^{-1}(cx)\right)}{2c^5d^3(-cx+i)^2}+\frac{6i\log\left(\frac{2}{1+icx}\right)\left(a+b\text{t}\right.}{c^5d^3}$$

```
[Out] (-3*a*x)/(c^4*d^3) - ((I/2)*b*x)/(c^4*d^3) - b/(8*c^5*d^3*(I - c*x)^2) - ((
(15*I)/8)*b)/(c^5*d^3*(I - c*x)) + (((19*I)/8)*b*ArcTan[c*x])/(c^5*d^3) - (
3*b*x*ArcTan[c*x])/(c^4*d^3) + ((I/2)*x^2*(a + b*ArcTan[c*x]))/(c^3*d^3) -
((I/2)*(a + b*ArcTan[c*x]))/(c^5*d^3*(I - c*x)^2) + (4*(a + b*ArcTan[c*x]))
/(c^5*d^3*(I - c*x)) + ((6*I)*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(c^5*
d^3) + (3*b*Log[1 + c^2*x^2])/(2*c^5*d^3) - (3*b*PolyLog[2, 1 - 2/(1 + I*c*
x)])/(c^5*d^3)
```

Rubi [A] time = 0.284669, antiderivative size = 256, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 12, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.522, Rules used = {4876, 4846, 260, 4852, 321, 203, 4862, 627, 44, 4854, 2402, 2315}

$$-\frac{3b\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c^5d^3}+\frac{ix^2\left(a+b\tan^{-1}(cx)\right)}{2c^3d^3}+\frac{4\left(a+b\tan^{-1}(cx)\right)}{c^5d^3(-cx+i)}-\frac{i\left(a+b\tan^{-1}(cx)\right)}{2c^5d^3(-cx+i)^2}+\frac{6i\log\left(\frac{2}{1+icx}\right)\left(a+b\text{t}\right.}{c^5d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcTan[c*x]))/(d + I*c*d*x)^3,x]
```


```
[Out] (-3*a*x)/(c^4*d^3) - ((I/2)*b*x)/(c^4*d^3) - b/(8*c^5*d^3*(I - c*x)^2) - ((
(15*I)/8)*b)/(c^5*d^3*(I - c*x)) + (((19*I)/8)*b*ArcTan[c*x])/(c^5*d^3) - (
3*b*x*ArcTan[c*x])/(c^4*d^3) + ((I/2)*x^2*(a + b*ArcTan[c*x]))/(c^3*d^3) -
((I/2)*(a + b*ArcTan[c*x]))/(c^5*d^3*(I - c*x)^2) + (4*(a + b*ArcTan[c*x]))
/(c^5*d^3*(I - c*x)) + ((6*I)*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(c^5*
d^3) + (3*b*Log[1 + c^2*x^2])/(2*c^5*d^3) - (3*b*PolyLog[2, 1 - 2/(1 + I*c*
x)])/(c^5*d^3)
```


Rule 4876

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
```

```
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4862

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x]))/(e*(q + 1)), x] - Dist[(b*
c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int \frac{x^4\left(a+b\tan^{-1}(cx)\right)}{(d+icdx)^3}\,dx = \int\left(-\frac{3\left(a+b\tan^{-1}(cx)\right)}{c^4d^3}+\frac{ix\left(a+b\tan^{-1}(cx)\right)}{c^3d^3}+\frac{i\left(a+b\tan^{-1}(cx)\right)}{c^4d^3(-i+cx)^3}+\frac{4\left(a+b\tan^{-1}(cx)\right)}{c^4d^3(-i+cx)^2}\right.$$

$$=\frac{i\int\frac{a+b\tan^{-1}(cx)}{(-i+cx)^3}\,dx}{c^4d^3}-\frac{(6i)\int\frac{a+b\tan^{-1}(cx)}{-i+cx}\,dx}{c^4d^3}-\frac{3\int\left(a+b\tan^{-1}(cx)\right)dx}{c^4d^3}+\frac{4\int\frac{a+b\tan^{-1}(cx)}{(-i+cx)^2}\,dx}{c^4d^3}$$

$$=-\frac{3ax}{c^4d^3}+\frac{ix^2\left(a+b\tan^{-1}(cx)\right)}{2c^3d^3}-\frac{i\left(a+b\tan^{-1}(cx)\right)}{2c^5d^3(i-cx)^2}+\frac{4\left(a+b\tan^{-1}(cx)\right)}{c^5d^3(i-cx)}+\frac{6i\left(a+b\tan^{-1}\right.}{c}$$

$$=-\frac{3ax}{c^4d^3}-\frac{ibx}{2c^4d^3}-\frac{3bx\tan^{-1}(cx)}{c^4d^3}+\frac{ix^2\left(a+b\tan^{-1}(cx)\right)}{2c^3d^3}-\frac{i\left(a+b\tan^{-1}(cx)\right)}{2c^5d^3(i-cx)^2}+\frac{4\left(a+b\tan\right.}{c^5d^3(i-}$$

$$=-\frac{3ax}{c^4d^3}-\frac{ibx}{2c^4d^3}+\frac{ib\tan^{-1}(cx)}{2c^5d^3}-\frac{3bx\tan^{-1}(cx)}{c^4d^3}+\frac{ix^2\left(a+b\tan^{-1}(cx)\right)}{2c^3d^3}-\frac{i\left(a+b\tan^{-1}(cx)\right.}{2c^5d^3(i-cx)^2}$$

$$=-\frac{3ax}{c^4d^3}-\frac{ibx}{2c^4d^3}-\frac{b}{8c^5d^3(i-cx)^2}-\frac{15ib}{8c^5d^3(i-cx)}+\frac{ib\tan^{-1}(cx)}{2c^5d^3}-\frac{3bx\tan^{-1}(cx)}{c^4d^3}+\frac{ix^2\left(a+\right.}{2}$$

$$=-\frac{3ax}{c^4d^3}-\frac{ibx}{2c^4d^3}-\frac{b}{8c^5d^3(i-cx)^2}-\frac{15ib}{8c^5d^3(i-cx)}+\frac{19ib\tan^{-1}(cx)}{8c^5d^3}-\frac{3bx\tan^{-1}(cx)}{c^4d^3}+\frac{ix^2\left(a\right.}{}$$

Mathematica [A] time = 1.01811, size = 235, normalized size = 0.92

$$\frac{b\left(96\text{PolyLog}\left(2,-e^{2i\tan^{-1}(cx)}\right)+48\log\left(c^2x^2+1\right)+4i\tan^{-1}(cx)\left(4c^2x^2+24icx+48\log\left(1+e^{2i\tan^{-1}(cx)}\right)+14i\sin\left(2\tan\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(a + b*ArcTan[c*x]))/(d + I*c*d*x)^3,x]
```

```
[Out] (-96*a*c*x + (16*I)*a*c^2*x^2 - ((16*I)*a)/(-I + c*x)^2 - (128*a)/(-I + c*x
) + 192*a*ArcTan[c*x] - (96*I)*a*Log[1 + c^2*x^2] + b*((-16*I)*c*x + 192*Ar
cTan[c*x]^2 - 28*Cos[2*ArcTan[c*x]] + Cos[4*ArcTan[c*x]] + 48*Log[1 + c^2*x
^2] + 96*PolyLog[2, -E^((2*I)*ArcTan[c*x])] + (28*I)*Sin[2*ArcTan[c*x]] + (
4*I)*ArcTan[c*x]*(4 + (24*I)*c*x + 4*c^2*x^2 - 14*Cos[2*ArcTan[c*x]] + Cos[
4*ArcTan[c*x]] + 48*Log[1 + E^((2*I)*ArcTan[c*x])] + (14*I)*Sin[2*ArcTan[c*
x]] - I*Sin[4*ArcTan[c*x]]) - I*Sin[4*ArcTan[c*x]]))/(32*c^5*d^3)
```

Maple [A] time = 0.062, size = 423, normalized size = 1.7

$$-3\frac{ax}{c^4d^3}+\frac{\frac{i}{2}b\arctan(cx)x^2}{c^3d^3}+6\frac{a\arctan(cx)}{c^5d^3}+\frac{\frac{15i}{8}b}{c^5d^3(cx-i)}+\frac{\frac{43i}{16}b\arctan(cx)}{c^5d^3}-4\frac{a}{c^5d^3(cx-i)}-3\frac{bx\arctan(cx)}{c^4d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arctan(c*x))/(d+I*c*d*x)^3,x)
```

```
[Out] -3*a*x/c^4/d^3+1/2*I/c^3*b/d^3*arctan(c*x)*x^2+6/c^5*a/d^3*arctan(c*x)+15/8
*I/c^5*b/d^3/(c*x-I)+43/16*I/c^5*b/d^3*arctan(c*x)-4/c^5*a/d^3/(c*x-I)-3*b*
x*arctan(c*x)/c^4/d^3-5/32*I/c^5*b/d^3*arctan(1/6*c^3*x^3+7/6*c*x)-3*I/c^5*
a/d^3*ln(c^2*x^2+1)-1/2*I*b*x/c^4/d^3-4/c^5*b/d^3*arctan(c*x)/(c*x-I)-1/2/c
^5*b/d^3+1/2*I/c^3*a/d^3*x^2+5/64/c^5*b/d^3*ln(c^4*x^4+10*c^2*x^2+9)-1/2*I/
c^5*b/d^3*arctan(c*x)/(c*x-I)^2+5/32*I/c^5*b/d^3*arctan(1/2*c*x)-6*I/c^5*b/
d^3*arctan(c*x)*ln(c*x-I)-1/2*I/c^5*a/d^3/(c*x-I)^2-1/8/c^5*b/d^3/(c*x-I)^2
+43/32*b*ln(c^2*x^2+1)/c^5/d^3-5/16*I/c^5*b/d^3*arctan(1/2*c*x-1/2*I)-3/c^5
*b/d^3*ln(c*x-I)*ln(-1/2*I*(c*x+I))+3/2/c^5*b/d^3*ln(c*x-I)^2-3/c^5*b/d^3*d
ilog(-1/2*I*(c*x+I))
```

Maxima [A] time = 1.35084, size = 478, normalized size = 1.87

$$\frac{8i\,ac^4x^4-8(4a+ib)c^3x^3+(b(5i\arctan(1,cx)-16)+88i\,a)c^2x^2+(b(10\arctan(1,cx)+38i)-16a)cx+24\left(bc^2x^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctan(c*x))/(d+I*c*d*x)^3,x, algorithm="maxima")
```

```
[Out] (8*I*a*c^4*x^4 - 8*(4*a + I*b)*c^3*x^3 + (b*(5*I*arctan2(1, c*x) - 16) + 88
*I*a)*c^2*x^2 + (b*(10*arctan2(1, c*x) + 38*I) - 16*a)*c*x + 24*(b*c^2*x^2
- 2*I*b*c*x - b)*arctan(c*x)^2 + 6*(b*c^2*x^2 - 2*I*b*c*x - b)*log(c^2*x^2
+ 1)^2 + (-24*I*b*c^2*x^2 - 48*b*c*x + 24*I*b)*arctan(c*x)*log(1/4*c^2*x^2
+ 1/4) + b*(-5*I*arctan2(1, c*x) + 28) + (8*I*b*c^4*x^4 - 32*b*c^3*x^3 + (9
6*a + 131*I*b)*c^2*x^2 + (-192*I*a + 70*b)*c*x - 96*a + 13*I*b)*arctan(c*x)
 - 48*(b*c^2*x^2 - 2*I*b*c*x - b)*dilog(1/2*I*c*x + 1/2) + ((-48*I*a + 24*b
)*c^2*x^2 - 48*(2*a + I*b)*c*x - 12*(b*c^2*x^2 - 2*I*b*c*x - b)*log(1/4*c^2
*x^2 + 1/4) + 48*I*a - 24*b)*log(c^2*x^2 + 1) + 56*I*a)/(16*c^7*d^3*x^2 - 3
2*I*c^6*d^3*x - 16*c^5*d^3)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{bx^4\log\left(-\frac{cx+i}{cx-i}\right)-2i\,ax^4}{2\,c^3d^3x^3-6i\,c^2d^3x^2-6\,cd^3x+2i\,d^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctan(c*x))/(d+I*c*d*x)^3,x, algorithm="fricas")
```

```
[Out] integral(-(b*x^4*log(-(c*x + I)/(c*x - I)) - 2*I*a*x^4)/(2*c^3*d^3*x^3 - 6*
I*c^2*d^3*x^2 - 6*c*d^3*x + 2*I*d^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*atan(c*x))/(d+I*c*d*x)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(b\arctan(cx)+a)x^4}{(i\,cdx+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctan(c*x))/(d+I*c*d*x)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)*x^4/(I*c*d*x + d)^3, x)
```

3.59 $\int \frac{x^3\left(a+b\tan^{-1}(cx)\right)}{(d+icdx)^3}\,dx$

Optimal. Leaf size=225

$$\frac{3ib\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{2c^4d^3}-\frac{3i\left(a+b\tan^{-1}(cx)\right)}{c^4d^3(-cx+i)}-\frac{a+b\tan^{-1}(cx)}{2c^4d^3(-cx+i)^2}+\frac{3\log\left(\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{c^4d^3}+\frac{iax}{c^3d^3}-\frac{ib\log\left(c\right.}{2c}$$

```
[Out] (I*a*x)/(c^3*d^3) + ((I/8)*b)/(c^4*d^3*(I - c*x)^2) - (11*b)/(8*c^4*d^3*(I
- c*x)) + (11*b*ArcTan[c*x])/(8*c^4*d^3) + (I*b*x*ArcTan[c*x])/(c^3*d^3) -
(a + b*ArcTan[c*x])/(2*c^4*d^3*(I - c*x)^2) - ((3*I)*(a + b*ArcTan[c*x]))/(
c^4*d^3*(I - c*x)) + (3*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(c^4*d^3) -
 ((I/2)*b*Log[1 + c^2*x^2])/(c^4*d^3) + (((3*I)/2)*b*PolyLog[2, 1 - 2/(1 +
I*c*x)])/(c^4*d^3)
```

Rubi [A] time = 0.24278, antiderivative size = 225, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {4876, 4846, 260, 4862, 627, 44, 203, 4854, 2402, 2315}

$$\frac{3ib\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{2c^4d^3}-\frac{3i\left(a+b\tan^{-1}(cx)\right)}{c^4d^3(-cx+i)}-\frac{a+b\tan^{-1}(cx)}{2c^4d^3(-cx+i)^2}+\frac{3\log\left(\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{c^4d^3}+\frac{iax}{c^3d^3}-\frac{ib\log\left(c\right.}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcTan[c*x]))/(d + I*c*d*x)^3,x]
```


```
[Out] (I*a*x)/(c^3*d^3) + ((I/8)*b)/(c^4*d^3*(I - c*x)^2) - (11*b)/(8*c^4*d^3*(I
- c*x)) + (11*b*ArcTan[c*x])/(8*c^4*d^3) + (I*b*x*ArcTan[c*x])/(c^3*d^3) -
(a + b*ArcTan[c*x])/(2*c^4*d^3*(I - c*x)^2) - ((3*I)*(a + b*ArcTan[c*x]))/(
c^4*d^3*(I - c*x)) + (3*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(c^4*d^3) -
 ((I/2)*b*Log[1 + c^2*x^2])/(c^4*d^3) + (((3*I)/2)*b*PolyLog[2, 1 - 2/(1 +
I*c*x)])/(c^4*d^3)
```


Rule 4876

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4862

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x]))/(e*(q + 1)), x] - Dist[(b*
c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int \frac{x^3\left(a+b\tan^{-1}(cx)\right)}{(d+icdx)^3}\,dx=\int\left(\frac{i\left(a+b\tan^{-1}(cx)\right)}{c^3d^3}+\frac{a+b\tan^{-1}(cx)}{c^3d^3(-i+cx)^3}-\frac{3i\left(a+b\tan^{-1}(cx)\right)}{c^3d^3(-i+cx)^2}-\frac{3\left(a+b\tan^{-1}(cx)\right)}{c^3d^3(-i+cx)}\right)d$$

$$=\frac{i\int\left(a+b\tan^{-1}(cx)\right)dx}{c^3d^3}-\frac{(3i)\int\frac{a+b\tan^{-1}(cx)}{(-i+cx)^2}dx}{c^3d^3}+\frac{\int\frac{a+b\tan^{-1}(cx)}{(-i+cx)^3}dx}{c^3d^3}-\frac{3\int\frac{a+b\tan^{-1}(cx)}{-i+cx}dx}{c^3d^3}$$

$$=\frac{iax}{c^3d^3}-\frac{a+b\tan^{-1}(cx)}{2c^4d^3(i-cx)^2}-\frac{3i\left(a+b\tan^{-1}(cx)\right)}{c^4d^3(i-cx)}+\frac{3\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2}{1+icx}\right)}{c^4d^3}+\frac{(ib)\int t}{}$$

$$=\frac{iax}{c^3d^3}+\frac{ibx\tan^{-1}(cx)}{c^3d^3}-\frac{a+b\tan^{-1}(cx)}{2c^4d^3(i-cx)^2}-\frac{3i\left(a+b\tan^{-1}(cx)\right)}{c^4d^3(i-cx)}+\frac{3\left(a+b\tan^{-1}(cx)\right)\log\left(\right.}{c^4d^3}$$

$$=\frac{iax}{c^3d^3}+\frac{ibx\tan^{-1}(cx)}{c^3d^3}-\frac{a+b\tan^{-1}(cx)}{2c^4d^3(i-cx)^2}-\frac{3i\left(a+b\tan^{-1}(cx)\right)}{c^4d^3(i-cx)}+\frac{3\left(a+b\tan^{-1}(cx)\right)\log\left(\right.}{c^4d^3}$$

$$=\frac{iax}{c^3d^3}+\frac{ib}{8c^4d^3(i-cx)^2}-\frac{11b}{8c^4d^3(i-cx)}+\frac{ibx\tan^{-1}(cx)}{c^3d^3}-\frac{a+b\tan^{-1}(cx)}{2c^4d^3(i-cx)^2}-\frac{3i\left(a+b\tan^{-1}\right.}{c^4d^3(i-c}$$

$$=\frac{iax}{c^3d^3}+\frac{ib}{8c^4d^3(i-cx)^2}-\frac{11b}{8c^4d^3(i-cx)}+\frac{11b\tan^{-1}(cx)}{8c^4d^3}+\frac{ibx\tan^{-1}(cx)}{c^3d^3}-\frac{a+b\tan^{-1}(cx)}{2c^4d^3(i-cx)^2}$$

Mathematica [A] time = 0.788684, size = 216, normalized size = 0.96

$$\frac{ib\left(-48\text{PolyLog}\left(2,-e^{2i\tan^{-1}(cx)}\right)-16\log\left(c^2x^2+1\right)-96\tan^{-1}(cx)^2-20i\sin\left(2\tan^{-1}(cx)\right)+i\sin\left(4\tan^{-1}(cx)\right)+20\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(a + b*ArcTan[c*x]))/(d + I*c*d*x)^3,x]
```

```
[Out] ((32*I)*a*c*x - (16*a)/(-I + c*x)^2 + ((96*I)*a)/(-I + c*x) - (96*I)*a*ArcT
an[c*x] - 48*a*Log[1 + c^2*x^2] + I*b*(-96*ArcTan[c*x]^2 + 20*Cos[2*ArcTan[
c*x]] - Cos[4*ArcTan[c*x]] - 16*Log[1 + c^2*x^2] - 48*PolyLog[2, -E^((2*I)*
ArcTan[c*x])] - (20*I)*Sin[2*ArcTan[c*x]] + 4*ArcTan[c*x]*(8*c*x + (10*I)*C
os[2*ArcTan[c*x]] - I*Cos[4*ArcTan[c*x]] - (24*I)*Log[1 + E^((2*I)*ArcTan[c
*x])] + 10*Sin[2*ArcTan[c*x]] - Sin[4*ArcTan[c*x]]) + I*Sin[4*ArcTan[c*x]])
)/(32*c^4*d^3)
```

Maple [A] time = 0.056, size = 375, normalized size = 1.7

$$\frac{iax}{c^3d^3} - \frac{3a\ln\left(c^2x^2+1\right)}{2c^4d^3} + \frac{3ib\arctan(cx)}{c^4d^3(cx-i)} - \frac{a}{2c^4d^3(cx-i)^2} + \frac{\frac{3i}{2}b\ln\left(-\frac{i}{2}(cx+i)\right)\ln(cx-i)}{c^4d^3} - \frac{\frac{19i}{32}b\ln\left(c^2x^2+1\right)}{c^4d^3} - 3\frac{b\,a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctan(c*x))/(d+I*c*d*x)^3,x)
```

```
[Out] I*a*x/c^3/d^3-3/2/c^4*a/d^3*ln(c^2*x^2+1)+3*I/c^4*b/d^3*arctan(c*x)/(c*x-I)
-1/2/c^4*a/d^3/(c*x-I)^2+3/2*I/c^4*b/d^3*ln(-1/2*I*(c*x+I))*ln(c*x-I)-19/32
*I/c^4*b/d^3*ln(c^2*x^2+1)-3/c^4*b/d^3*arctan(c*x)*ln(c*x-I)-1/2/c^4*b/d^3*
arctan(c*x)/(c*x-I)^2+1/8*I/c^4*b/d^3/(c*x-I)^2+3/64*I/c^4*b/d^3*ln(c^4*x^4
+10*c^2*x^2+9)+3/32/c^4*b/d^3*arctan(1/6*c^3*x^3+7/6*c*x)-3/32/c^4*b/d^3*ar
ctan(1/2*c*x)+3/16/c^4*b/d^3*arctan(1/2*c*x-1/2*I)+3*I/c^4*a/d^3/(c*x-I)+I*
b*x*arctan(c*x)/c^3/d^3+19/16*b*arctan(c*x)/c^4/d^3+11/8/c^4*b/d^3/(c*x-I)+
3/2*I/c^4*b/d^3*dilog(-1/2*I*(c*x+I))-3/4*I/c^4*b/d^3*ln(c*x-I)^2-3*I/c^4*a
/d^3*arctan(c*x)
```

Maxima [A] time = 1.27976, size = 441, normalized size = 1.96

$$-\frac{-16iac^3x^3 - 32ac^2x^2 + (-32ia - 22b)cx + \left(12ibc^2x^2 + 24bcx - 12ib\right)\arctan(cx)^2 + \left(3ibc^2x^2 + 6bcx - 3ib\right)\log\left(c^2x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))/(d+I*c*d*x)^3,x, algorithm="maxima")
```

```
[Out] -(-16*I*a*c^3*x^3 - 32*a*c^2*x^2 + (-32*I*a - 22*b)*c*x + (12*I*b*c^2*x^2 +
 24*b*c*x - 12*I*b)*arctan(c*x)^2 + (3*I*b*c^2*x^2 + 6*b*c*x - 3*I*b)*log(c
^2*x^2 + 1)^2 + 12*(b*c^2*x^2 - 2*I*b*c*x - b)*arctan(c*x)*log(1/4*c^2*x^2
+ 1/4) + (-16*I*b*c^3*x^3 + (48*I*a - 51*b)*c^2*x^2 + 6*(16*a + I*b)*c*x -
48*I*a - 21*b)*arctan(c*x) + 3*(b*c^2*x^2 - 2*I*b*c*x - b)*arctan2(c*x, -1)
 + (-24*I*b*c^2*x^2 - 48*b*c*x + 24*I*b)*dilog(1/2*I*c*x + 1/2) + (8*(3*a +
 I*b)*c^2*x^2 + (-48*I*a + 16*b)*c*x + (-6*I*b*c^2*x^2 - 12*b*c*x + 6*I*b)*
log(1/4*c^2*x^2 + 1/4) - 24*a - 8*I*b)*log(c^2*x^2 + 1) - 40*a + 20*I*b)/(1
6*c^6*d^3*x^2 - 32*I*c^5*d^3*x - 16*c^4*d^3)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{bx^3\log\left(-\frac{cx+i}{cx-i}\right)-2i\,ax^3}{2\,c^3d^3x^3-6i\,c^2d^3x^2-6\,cd^3x+2i\,d^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))/(d+I*c*d*x)^3,x, algorithm="fricas")
```

```
[Out] integral(-(b*x^3*log(-(c*x + I)/(c*x - I)) - 2*I*a*x^3)/(2*c^3*d^3*x^3 - 6*
I*c^2*d^3*x^2 - 6*c*d^3*x + 2*I*d^3), x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atan(c*x))/(d+I*c*d*x)**3,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(b\arctan(cx)+a)x^3}{(i\,cdx+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))/(d+I*c*d*x)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)*x^3/(I*c*d*x + d)^3, x)
```

3.60 $\int \frac{x^2\left(a+b\tan^{-1}(cx)\right)}{(d+icdx)^3}\,dx$

Optimal. Leaf size=176

$$\frac{b\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{2c^3d^3}-\frac{2\left(a+b\tan^{-1}(cx)\right)}{c^3d^3(-cx+i)}+\frac{i\left(a+b\tan^{-1}(cx)\right)}{2c^3d^3(-cx+i)^2}-\frac{i\log\left(\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{c^3d^3}+\frac{7ib}{8c^3d^3(-cx+i)}$$

```
[Out] b/(8*c^3*d^3*(I - c*x)^2) + (((7*I)/8)*b)/(c^3*d^3*(I - c*x)) - (((7*I)/8)*
b*ArcTan[c*x])/(c^3*d^3) + ((I/2)*(a + b*ArcTan[c*x]))/(c^3*d^3*(I - c*x)^2
) - (2*(a + b*ArcTan[c*x]))/(c^3*d^3*(I - c*x)) - (I*(a + b*ArcTan[c*x])*Lo
g[2/(1 + I*c*x)])/(c^3*d^3) + (b*PolyLog[2, 1 - 2/(1 + I*c*x)])/(2*c^3*d^3)
```

Rubi [A] time = 0.21721, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {4876, 4862, 627, 44, 203, 4854, 2402, 2315}

$$\frac{b\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{2c^3d^3}-\frac{2\left(a+b\tan^{-1}(cx)\right)}{c^3d^3(-cx+i)}+\frac{i\left(a+b\tan^{-1}(cx)\right)}{2c^3d^3(-cx+i)^2}-\frac{i\log\left(\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{c^3d^3}+\frac{7ib}{8c^3d^3(-cx+i)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcTan[c*x]))/(d + I*c*d*x)^3,x]
```


```
[Out] b/(8*c^3*d^3*(I - c*x)^2) + (((7*I)/8)*b)/(c^3*d^3*(I - c*x)) - (((7*I)/8)*
b*ArcTan[c*x])/(c^3*d^3) + ((I/2)*(a + b*ArcTan[c*x]))/(c^3*d^3*(I - c*x)^2
) - (2*(a + b*ArcTan[c*x]))/(c^3*d^3*(I - c*x)) - (I*(a + b*ArcTan[c*x])*Lo
g[2/(1 + I*c*x)])/(c^3*d^3) + (b*PolyLog[2, 1 - 2/(1 + I*c*x)])/(2*c^3*d^3)
```


Rule 4876

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 4862

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x]))/(e*(q + 1)), x] - Dist[(b*
c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
```

```
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int \frac{x^2\left(a+b\tan^{-1}(cx)\right)}{(d+icdx)^3}\,dx = \int\left(-\frac{i\left(a+b\tan^{-1}(cx)\right)}{c^2d^3(-i+cx)^3} - \frac{2\left(a+b\tan^{-1}(cx)\right)}{c^2d^3(-i+cx)^2} + \frac{i\left(a+b\tan^{-1}(cx)\right)}{c^2d^3(-i+cx)}\right)dx$$

$$= -\frac{i\int\frac{a+b\tan^{-1}(cx)}{(-i+cx)^3}\,dx}{c^2d^3} + \frac{i\int\frac{a+b\tan^{-1}(cx)}{-i+cx}\,dx}{c^2d^3} - \frac{2\int\frac{a+b\tan^{-1}(cx)}{(-i+cx)^2}\,dx}{c^2d^3}$$

$$= \frac{i\left(a+b\tan^{-1}(cx)\right)}{2c^3d^3(i-cx)^2} - \frac{2\left(a+b\tan^{-1}(cx)\right)}{c^3d^3(i-cx)} - \frac{i\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2}{1+icx}\right)}{c^3d^3} - \frac{(ib)\int\frac{1}{(-i+cx)^2(}}{2c^2d}$$

$$= \frac{i\left(a+b\tan^{-1}(cx)\right)}{2c^3d^3(i-cx)^2} - \frac{2\left(a+b\tan^{-1}(cx)\right)}{c^3d^3(i-cx)} - \frac{i\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2}{1+icx}\right)}{c^3d^3} + \frac{b\,\mathrm{Subst}\left(\int\frac{\mathrm{lo}}{}\right.}{}$$

$$= \frac{i\left(a+b\tan^{-1}(cx)\right)}{2c^3d^3(i-cx)^2} - \frac{2\left(a+b\tan^{-1}(cx)\right)}{c^3d^3(i-cx)} - \frac{i\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2}{1+icx}\right)}{c^3d^3} + \frac{b\mathrm{Li}_2\left(1-\frac{2}{1+i}\right.}{2c^3d^3}$$

$$= \frac{b}{8c^3d^3(i-cx)^2} + \frac{7ib}{8c^3d^3(i-cx)} + \frac{i\left(a+b\tan^{-1}(cx)\right)}{2c^3d^3(i-cx)^2} - \frac{2\left(a+b\tan^{-1}(cx)\right)}{c^3d^3(i-cx)} - \frac{i\left(a+b\tan^{-}\right.}{}$$

$$= \frac{b}{8c^3d^3(i-cx)^2} + \frac{7ib}{8c^3d^3(i-cx)} - \frac{7ib\tan^{-1}(cx)}{8c^3d^3} + \frac{i\left(a+b\tan^{-1}(cx)\right)}{2c^3d^3(i-cx)^2} - \frac{2\left(a+b\tan^{-1}(cx)\right)}{c^3d^3(i-cx)}$$

Mathematica [A] time = 0.155007, size = 187, normalized size = 1.06

$$-\frac{i\left(4ib(cx-i)^2\mathrm{PolyLog}\left(2,\frac{cx+i}{cx-i}\right) + 8ac^2x^2\log\left(\frac{2i}{-cx+i}\right) + 16iacx - 16iacx\log\left(\frac{2i}{-cx+i}\right) - 8a\log\left(\frac{2i}{-cx+i}\right) + 12a + b\left(7c^2x^2 +\right.\right.}{8c^3d^3(cx-i)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*ArcTan[c*x]))/(d + I*c*d*x)^3,x]
```

```
[Out] ((-I/8)*(12*a - (6*I)*b + (16*I)*a*c*x + 7*b*c*x - 8*a*Log[(2*I)/(I - c*x)]
 - (16*I)*a*c*x*Log[(2*I)/(I - c*x)] + 8*a*c^2*x^2*Log[(2*I)/(I - c*x)] + b
*ArcTan[c*x]*(5 + (2*I)*c*x + 7*c^2*x^2 + 8*(-I + c*x)^2*Log[(2*I)/(I - c*x
)]) + (4*I)*b*(-I + c*x)^2*PolyLog[2, (I + c*x)/(-I + c*x)]))/(c^3*d^3*(-I
+ c*x)^2)
```

Maple [B] time = 0.059, size = 349, normalized size = 2.

$$\frac{\frac{7i}{32}b}{c^3d^3}\arctan\left(\frac{cx}{2}\right)-\frac{\frac{7i}{32}b}{c^3d^3}\arctan\left(\frac{c^3x^3}{6}+\frac{7cx}{6}\right)-\frac{a\arctan(cx)}{c^3d^3}+2\frac{a}{c^3d^3(cx-i)}-\frac{\frac{7i}{16}b\arctan(cx)}{c^3d^3}+\frac{\frac{i}{2}a\ln\left(c^2x^2+1\right)}{c^3d^3}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctan(c*x))/(d+I*c*d*x)^3,x)
```

```
[Out] 7/32*I/c^3*b/d^3*arctan(1/2*c*x)-7/32*I/c^3*b/d^3*arctan(1/6*c^3*x^3+7/6*c*
x)-1/c^3*a/d^3*arctan(c*x)+2/c^3*a/d^3/(c*x-I)-7/16*I/c^3*b/d^3*arctan(c*x)
+1/2*I/c^3*a/d^3*ln(c^2*x^2+1)+2/c^3*b/d^3*arctan(c*x)/(c*x-I)+7/64/c^3*b/d
^3*ln(c^4*x^4+10*c^2*x^2+9)+1/2*I/c^3*a/d^3/(c*x-I)^2+I/c^3*b/d^3*arctan(c*
x)*ln(c*x-I)+1/2*I/c^3*b/d^3*arctan(c*x)/(c*x-I)^2-7/8*I/c^3*b/d^3/(c*x-I)+
1/8/c^3*b/d^3/(c*x-I)^2-7/32/c^3*b/d^3*ln(c^2*x^2+1)-7/16*I/c^3*b/d^3*arcta
n(1/2*c*x-1/2*I)+1/2/c^3*b/d^3*ln(c*x-I)*ln(-1/2*I*(c*x+I))+1/2/c^3*b/d^3*d
ilog(-1/2*I*(c*x+I))-1/4/c^3*b/d^3*ln(c*x-I)^2
```

Maxima [A] time = 1.23644, size = 393, normalized size = 2.23

$$-\frac{-7i\,bc^2x^2\arctan(1,cx)-(b(14\arctan(1,cx)-14i)+32\,a)cx+4\left(bc^2x^2-2i\,bcx-b\right)\arctan(cx)^2+\left(bc^2x^2-2i\,bcx-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))/(d+I*c*d*x)^3,x, algorithm="maxima")
```

```
[Out] -(-7*I*b*c^2*x^2*arctan2(1, c*x) - (b*(14*arctan2(1, c*x) - 14*I) + 32*a)*c
*x + 4*(b*c^2*x^2 - 2*I*b*c*x - b)*arctan(c*x)^2 + (b*c^2*x^2 - 2*I*b*c*x -
 b)*log(c^2*x^2 + 1)^2 + (-4*I*b*c^2*x^2 - 8*b*c*x + 4*I*b)*arctan(c*x)*log
(1/4*c^2*x^2 + 1/4) + b*(7*I*arctan2(1, c*x) + 12) + ((16*a + 7*I*b)*c^2*x^
2 + (-32*I*a - 18*b)*c*x - 16*a + 17*I*b)*arctan(c*x) - 8*(b*c^2*x^2 - 2*I*
b*c*x - b)*dilog(1/2*I*c*x + 1/2) + (-8*I*a*c^2*x^2 - 16*a*c*x - 2*(b*c^2*x
^2 - 2*I*b*c*x - b)*log(1/4*c^2*x^2 + 1/4) + 8*I*a)*log(c^2*x^2 + 1) + 24*I
*a)/(16*c^5*d^3*x^2 - 32*I*c^4*d^3*x - 16*c^3*d^3)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{bx^2\log\left(-\frac{cx+i}{cx-i}\right)-2i\,ax^2}{2\,c^3d^3x^3-6i\,c^2d^3x^2-6\,cd^3x+2i\,d^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))/(d+I*c*d*x)^3,x, algorithm="fricas")
```

```
[Out] integral(-(b*x^2*log(-(c*x + I)/(c*x - I)) - 2*I*a*x^2)/(2*c^3*d^3*x^3 - 6*
I*c^2*d^3*x^2 - 6*c*d^3*x + 2*I*d^3), x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atan(c*x))/(d+I*c*d*x)**3,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\arctan(cx)+a)x^2}{(i\,cdx+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))/(d+I*c*d*x)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)*x^2/(I*c*d*x + d)^3, x)
```

3.61 $\int \frac{x\left(a+b\tan^{-1}(cx)\right)}{(d+icdx)^3}\,dx$

Optimal. Leaf size=88

$$\frac{x^2\left(a+b\tan^{-1}(cx)\right)}{2d^3(1+icx)^2}+\frac{3b}{8c^2d^3(-cx+i)}-\frac{ib}{8c^2d^3(-cx+i)^2}+\frac{b\tan^{-1}(cx)}{8c^2d^3}$$

```
[Out] ((-I/8)*b)/(c^2*d^3*(I - c*x)^2) + (3*b)/(8*c^2*d^3*(I - c*x)) + (b*ArcTan[
c*x])/(8*c^2*d^3) + (x^2*(a + b*ArcTan[c*x]))/(2*d^3*(1 + I*c*x)^2)
```

Rubi [A] time = 0.0767148, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {37, 4872, 12, 88, 203}

$$\frac{x^2\left(a+b\tan^{-1}(cx)\right)}{2d^3(1+icx)^2}+\frac{3b}{8c^2d^3(-cx+i)}-\frac{ib}{8c^2d^3(-cx+i)^2}+\frac{b\tan^{-1}(cx)}{8c^2d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcTan[c*x]))/(d + I*c*d*x)^3,x]
```

```
[Out] ((-I/8)*b)/(c^2*d^3*(I - c*x)^2) + (3*b)/(8*c^2*d^3*(I - c*x)) + (b*ArcTan[
c*x])/(8*c^2*d^3) + (x^2*(a + b*ArcTan[c*x]))/(2*d^3*(1 + I*c*x)^2)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 4872

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
+ b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2), x
], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2*m
] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x\left(a+b\tan^{-1}(cx)\right)}{(d+icdx)^3}\,dx &= \frac{x^2\left(a+b\tan^{-1}(cx)\right)}{2d^3(1+icx)^2}-(bc)\int \frac{x^2}{2d^3(i-cx)^3(i+cx)}\,dx\\
&= \frac{x^2\left(a+b\tan^{-1}(cx)\right)}{2d^3(1+icx)^2}-\frac{(bc)\int \frac{x^2}{(i-cx)^3(i+cx)}\,dx}{2d^3}\\
&= \frac{x^2\left(a+b\tan^{-1}(cx)\right)}{2d^3(1+icx)^2}-\frac{(bc)\int \left(-\frac{i}{2c^2(-i+cx)^3}-\frac{3}{4c^2(-i+cx)^2}-\frac{1}{4c^2\left(1+c^2x^2\right)}\right)dx}{2d^3}\\
&= -\frac{ib}{8c^2d^3(i-cx)^2}+\frac{3b}{8c^2d^3(i-cx)}+\frac{x^2\left(a+b\tan^{-1}(cx)\right)}{2d^3(1+icx)^2}+\frac{b\int \frac{1}{1+c^2x^2}\,dx}{8cd^3}\\
&= -\frac{ib}{8c^2d^3(i-cx)^2}+\frac{3b}{8c^2d^3(i-cx)}+\frac{b\tan^{-1}(cx)}{8c^2d^3}+\frac{x^2\left(a+b\tan^{-1}(cx)\right)}{2d^3(1+icx)^2}
\end{aligned}
$$

Mathematica [A] time = 0.0818942, size = 63, normalized size = 0.72

$$
\frac{a(-4-8icx)-b\left(3c^2x^2+2icx+1\right)\tan^{-1}(cx)+b(-3cx+2i)}{8c^2d^3(cx-i)^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcTan[c*x]))/(d + I*c*d*x)^3,x]
```

```
[Out] (b*(2*I - 3*c*x) + a*(-4 - (8*I)*c*x) - b*(1 + (2*I)*c*x + 3*c^2*x^2)*ArcTa
n[c*x])/(8*c^2*d^3*(-I + c*x)^2)
```

Maple [A] time = 0.041, size = 128, normalized size = 1.5

$$\frac{a}{2\,c^2d^3\,(cx-i)^2}-\frac{ia}{c^2d^3\,(cx-i)}+\frac{b\arctan{(cx)}}{2\,c^2d^3\,(cx-i)^2}-\frac{ib\arctan{(cx)}}{c^2d^3\,(cx-i)}-\frac{3\,b\arctan{(cx)}}{8\,c^2d^3}-\frac{\frac{i}{8}b}{c^2d^3\,(cx-i)^2}-\frac{3\,b}{8\,c^2d^3\,(cx-i)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctan(c*x))/(d+I*c*d*x)^3,x)
```

```
[Out] 1/2/c^2*a/d^3/(c*x-I)^2-I/c^2*a/d^3/(c*x-I)+1/2/c^2*b/d^3*arctan(c*x)/(c*x-
I)^2-I/c^2*b/d^3*arctan(c*x)/(c*x-I)-3/8*b*arctan(c*x)/c^2/d^3-1/8*I/c^2*b/
d^3/(c*x-I)^2-3/8/c^2*b/d^3/(c*x-I)
```

Maxima [A] time = 1.04489, size = 96, normalized size = 1.09

$$-\frac{(8i\,a+3\,b)cx+\left(3\,bc^2x^2+2i\,bcx+b\right)\arctan{(cx)}+4\,a-2i\,b}{8\,c^4d^3x^2-16i\,c^3d^3x-8\,c^2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))/(d+I*c*d*x)^3,x, algorithm="maxima")
```

```
[Out] -((8*I*a + 3*b)*c*x + (3*b*c^2*x^2 + 2*I*b*c*x + b)*arctan(c*x) + 4*a - 2*I
*b)/(8*c^4*d^3*x^2 - 16*I*c^3*d^3*x - 8*c^2*d^3)
```

Fricas [A] time = 2.26326, size = 196, normalized size = 2.23

$$\frac{(-16i\,a-6\,b)cx+\left(-3i\,bc^2x^2+2\,bcx-i\,b\right)\log\left(-\frac{cx+i}{cx-i}\right)-8\,a+4i\,b}{16\,c^4d^3x^2-32i\,c^3d^3x-16\,c^2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))/(d+I*c*d*x)^3,x, algorithm="fricas")
```

```
[Out] ((-16*I*a - 6*b)*c*x + (-3*I*b*c^2*x^2 + 2*b*c*x - I*b)*log(-(c*x + I)/(c*x
 - I)) - 8*a + 4*I*b)/(16*c^4*d^3*x^2 - 32*I*c^3*d^3*x - 16*c^2*d^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atan(c*x))/(d+I*c*d*x)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.16286, size = 200, normalized size = 2.27

$$\frac{3\,bc^2x^2\log\left(cx+i\right)-3\,bc^2x^2\log\left(cx-i\right)-6\,bcix\log\left(cx+i\right)+6\,bcix\log\left(cx-i\right)-6\,bcix+16\,bcx\arctan\left(cx\right)+16\,acx}{16\left(c^4d^3ix^2+2\,c^3d^3x-c^2d^3i\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))/(d+I*c*d*x)^3,x, algorithm="giac")
```

```
[Out] 1/16*(3*b*c^2*x^2*log(c*x + i) - 3*b*c^2*x^2*log(c*x - i) - 6*b*c*i*x*log(c
*x + i) + 6*b*c*i*x*log(c*x - i) - 6*b*c*i*x + 16*b*c*x*arctan(c*x) + 16*a*
c*x - 8*b*i*arctan(c*x) - 8*a*i - 3*b*log(c*x + i) + 3*b*log(c*x - i) - 4*b
)/(c^4*d^3*i*x^2 + 2*c^3*d^3*x - c^2*d^3*i)
```

3.62 $\int \frac{a+b\tan^{-1}(cx)}{(d+icdx)^3}\,dx$

Optimal. Leaf size=92

$$\frac{i\left(a+b\tan^{-1}(cx)\right)}{2cd^3(1+icx)^2}+\frac{ib}{8cd^3(-cx+i)}-\frac{b}{8cd^3(-cx+i)^2}-\frac{ib\tan^{-1}(cx)}{8cd^3}$$

```
[Out] -b/(8*c*d^3*(I - c*x)^2) + ((I/8)*b)/(c*d^3*(I - c*x)) - ((I/8)*b*ArcTan[c*
x])/(c*d^3) + ((I/2)*(a + b*ArcTan[c*x]))/(c*d^3*(1 + I*c*x)^2)
```

Rubi [A] time = 0.0549023, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4862, 627, 44, 203}

$$\frac{i\left(a+b\tan^{-1}(cx)\right)}{2cd^3(1+icx)^2}+\frac{ib}{8cd^3(-cx+i)}-\frac{b}{8cd^3(-cx+i)^2}-\frac{ib\tan^{-1}(cx)}{8cd^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])/(d + I*c*d*x)^3,x]
```

```
[Out] -b/(8*c*d^3*(I - c*x)^2) + ((I/8)*b)/(c*d^3*(I - c*x)) - ((I/8)*b*ArcTan[c*
x])/(c*d^3) + ((I/2)*(a + b*ArcTan[c*x]))/(c*d^3*(1 + I*c*x)^2)
```

Rule 4862

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x]))/(e*(q + 1)), x] - Dist[(b*
c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\tan^{-1}(cx)}{(d+icdx)^3}\,dx &= \frac{i\left(a+b\tan^{-1}(cx)\right)}{2cd^3(1+icx)^2}-\frac{(ib)\int \frac{1}{(d+icdx)^2\left(1+c^2x^2\right)}\,dx}{2d}\\
&= \frac{i\left(a+b\tan^{-1}(cx)\right)}{2cd^3(1+icx)^2}-\frac{(ib)\int \frac{1}{\left(\frac{1}{d}-\frac{icx}{d}\right)(d+icdx)^3}\,dx}{2d}\\
&= \frac{i\left(a+b\tan^{-1}(cx)\right)}{2cd^3(1+icx)^2}-\frac{(ib)\int \left(\frac{i}{2d^2(-i+cx)^3}-\frac{1}{4d^2(-i+cx)^2}+\frac{1}{4d^2\left(1+c^2x^2\right)}\right)dx}{2d}\\
&= -\frac{b}{8cd^3(i-cx)^2}+\frac{ib}{8cd^3(i-cx)}+\frac{i\left(a+b\tan^{-1}(cx)\right)}{2cd^3(1+icx)^2}-\frac{(ib)\int \frac{1}{1+c^2x^2}\,dx}{8d^3}\\
&= -\frac{b}{8cd^3(i-cx)^2}+\frac{ib}{8cd^3(i-cx)}-\frac{ib\tan^{-1}(cx)}{8cd^3}+\frac{i\left(a+b\tan^{-1}(cx)\right)}{2cd^3(1+icx)^2}
\end{aligned}$$

Mathematica [A] time = 0.0397565, size = 55, normalized size = 0.6

$$-\frac{i\left(4a+b\left(c^2x^2-2icx+3\right)\tan^{-1}(cx)+b(cx-2i)\right)}{8cd^3(cx-i)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])/(d + I*c*d*x)^3,x]
```

```
[Out] ((-I/8)*(4*a + b*(-2*I + c*x) + b*(3 - (2*I)*c*x + c^2*x^2)*ArcTan[c*x]))/(
c*d^3*(-I + c*x)^2)
```

Maple [A] time = 0.036, size = 93, normalized size = 1.

$$\frac{\frac{i}{2}a}{cd^3(1+icx)^2}+\frac{\frac{i}{2}b\arctan(cx)}{cd^3(1+icx)^2}-\frac{\frac{i}{8}b\arctan(cx)}{cd^3}-\frac{b}{8\,cd^3(cx-i)^2}-\frac{\frac{i}{8}b}{cd^3(cx-i)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/(d+I*c*d*x)^3,x)
```

```
[Out] 1/2*I/c*a/d^3/(1+I*c*x)^2+1/2*I/c*b/d^3/(1+I*c*x)^2*arctan(c*x)-1/8*I*b*arc
tan(c*x)/c/d^3-1/8/c*b/d^3/(c*x-I)^2-1/8*I/c*b/d^3/(c*x-I)
```

Maxima [A] time = 1.03283, size = 89, normalized size = 0.97

$$-\frac{i\,bcx+\left(i\,bc^2x^2+2\,bcx+3i\,b\right)\arctan(cx)+4i\,a+2\,b}{8\,c^3d^3x^2-16i\,c^2d^3x-8\,cd^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/(d+I*c*d*x)^3,x, algorithm="maxima")
```

```
[Out] -(I*b*c*x + (I*b*c^2*x^2 + 2*b*c*x + 3*I*b)*arctan(c*x) + 4*I*a + 2*b)/(8*c
^3*d^3*x^2 - 16*I*c^2*d^3*x - 8*c*d^3)
```

Fricas [A] time = 2.24684, size = 177, normalized size = 1.92

$$\frac{-2i\,bcx+\left(bc^2x^2-2i\,bcx+3\,b\right)\log\left(-\frac{cx+i}{cx-i}\right)-8i\,a-4\,b}{16\,c^3d^3x^2-32i\,c^2d^3x-16\,cd^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/(d+I*c*d*x)^3,x, algorithm="fricas")
```

```
[Out] (-2*I*b*c*x + (b*c^2*x^2 - 2*I*b*c*x + 3*b)*log(-(c*x + I)/(c*x - I)) - 8*I
*a - 4*b)/(16*c^3*d^3*x^2 - 32*I*c^2*d^3*x - 16*c*d^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/(d+I*c*d*x)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.18408, size = 174, normalized size = 1.89

$$\frac{bc^2x^2\log(cx+i) - bc^2x^2\log(cx-i) - 2\,bcix\log(cx+i) + 2\,bcix\log(cx-i) - 2\,bcix - 8\,bi\arctan(cx) - 8\,ai - b\log(c}{16\left(c^3d^3x^2 - 2\,c^2d^3ix - cd^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/(d+I*c*d*x)^3,x, algorithm="giac")
```

```
[Out] 1/16*(b*c^2*x^2*log(c*x + i) - b*c^2*x^2*log(c*x - i) - 2*b*c*i*x*log(c*x +
 i) + 2*b*c*i*x*log(c*x - i) - 2*b*c*i*x - 8*b*i*arctan(c*x) - 8*a*i - b*lo
g(c*x + i) + b*log(c*x - i) - 4*b)/(c^3*d^3*x^2 - 2*c^2*d^3*i*x - c*d^3)
```

3.63 $\int \frac{a+b\tan^{-1}(cx)}{x(d+icdx)^3}\,dx$

Optimal. Leaf size=195

$$\frac{ib\text{PolyLog}(2,-icx)}{2d^3}-\frac{ib\text{PolyLog}(2,icx)}{2d^3}+\frac{ib\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{2d^3}+\frac{i\left(a+b\tan^{-1}(cx)\right)}{d^3(-cx+i)}-\frac{a+b\tan^{-1}(cx)}{2d^3(-cx+i)^2}+\frac{\log\left(\frac{2}{1+ic}\right.}{}$$

```
[Out] ((I/8)*b)/(d^3*(I - c*x)^2) + (5*b)/(8*d^3*(I - c*x)) - (5*b*ArcTan[c*x])/(
8*d^3) - (a + b*ArcTan[c*x])/(2*d^3*(I - c*x)^2) + (I*(a + b*ArcTan[c*x]))/
(d^3*(I - c*x)) + (a*Log[x])/d^3 + ((a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])
/d^3 + ((I/2)*b*PolyLog[2, (-I)*c*x])/d^3 - ((I/2)*b*PolyLog[2, I*c*x])/d^3
 + ((I/2)*b*PolyLog[2, 1 - 2/(1 + I*c*x)])/d^3
```

Rubi [A] time = 0.241664, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {4876, 4848, 2391, 4862, 627, 44, 203, 4854, 2402, 2315}

$$\frac{ib\text{PolyLog}(2,-icx)}{2d^3}-\frac{ib\text{PolyLog}(2,icx)}{2d^3}+\frac{ib\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{2d^3}+\frac{i\left(a+b\tan^{-1}(cx)\right)}{d^3(-cx+i)}-\frac{a+b\tan^{-1}(cx)}{2d^3(-cx+i)^2}+\frac{\log\left(\frac{2}{1+ic}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])/(x*(d + I*c*d*x)^3),x]
```



```
[Out] ((I/8)*b)/(d^3*(I - c*x)^2) + (5*b)/(8*d^3*(I - c*x)) - (5*b*ArcTan[c*x])/(
8*d^3) - (a + b*ArcTan[c*x])/(2*d^3*(I - c*x)^2) + (I*(a + b*ArcTan[c*x]))/
(d^3*(I - c*x)) + (a*Log[x])/d^3 + ((a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])
/d^3 + ((I/2)*b*PolyLog[2, (-I)*c*x])/d^3 - ((I/2)*b*PolyLog[2, I*c*x])/d^3
 + ((I/2)*b*PolyLog[2, 1 - 2/(1 + I*c*x)])/d^3
```


Rule 4876

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4862

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x]))/(e*(q + 1)), x] - Dist[(b*
c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int \frac{a+b\tan^{-1}(cx)}{x(d+icdx)^3}\,dx = \int \left(\frac{a+b\tan^{-1}(cx)}{d^3x}+\frac{c\left(a+b\tan^{-1}(cx)\right)}{d^3(-i+cx)^3}+\frac{ic\left(a+b\tan^{-1}(cx)\right)}{d^3(-i+cx)^2}-\frac{c\left(a+b\tan^{-1}(cx)\right)}{d^3(-i+cx)}\right)dx$$

$$=\frac{\int \frac{a+b\tan^{-1}(cx)}{x}\,dx}{d^3}+\frac{(ic)\int \frac{a+b\tan^{-1}(cx)}{(-i+cx)^2}\,dx}{d^3}+\frac{c\int \frac{a+b\tan^{-1}(cx)}{(-i+cx)^3}\,dx}{d^3}-\frac{c\int \frac{a+b\tan^{-1}(cx)}{-i+cx}\,dx}{d^3}$$

$$=-\frac{a+b\tan^{-1}(cx)}{2d^3(i-cx)^2}+\frac{i\left(a+b\tan^{-1}(cx)\right)}{d^3(i-cx)}+\frac{a\log(x)}{d^3}+\frac{\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2}{1+icx}\right)}{d^3}+\frac{(ib)\int \frac{\log(1-}{x}}{2d^3}$$

$$=-\frac{a+b\tan^{-1}(cx)}{2d^3(i-cx)^2}+\frac{i\left(a+b\tan^{-1}(cx)\right)}{d^3(i-cx)}+\frac{a\log(x)}{d^3}+\frac{\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2}{1+icx}\right)}{d^3}+\frac{ib\mathrm{Li}_2(-icx}{2d^3}$$

$$=-\frac{a+b\tan^{-1}(cx)}{2d^3(i-cx)^2}+\frac{i\left(a+b\tan^{-1}(cx)\right)}{d^3(i-cx)}+\frac{a\log(x)}{d^3}+\frac{\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2}{1+icx}\right)}{d^3}+\frac{ib\mathrm{Li}_2(-icx}{2d^3}$$

$$=\frac{ib}{8d^3(i-cx)^2}+\frac{5b}{8d^3(i-cx)}-\frac{a+b\tan^{-1}(cx)}{2d^3(i-cx)^2}+\frac{i\left(a+b\tan^{-1}(cx)\right)}{d^3(i-cx)}+\frac{a\log(x)}{d^3}+\frac{\left(a+b\tan^{-1}(cx\right.}{d^3}$$

$$=\frac{ib}{8d^3(i-cx)^2}+\frac{5b}{8d^3(i-cx)}-\frac{5b\tan^{-1}(cx)}{8d^3}-\frac{a+b\tan^{-1}(cx)}{2d^3(i-cx)^2}+\frac{i\left(a+b\tan^{-1}(cx)\right)}{d^3(i-cx)}+\frac{a\log(x)}{d^3}+$$

Mathematica [A] time = 0.209265, size = 162, normalized size = 0.83

$$\frac{4ib\mathrm{PolyLog}(2,-icx)-4ib\mathrm{PolyLog}(2,icx)+4ib\mathrm{PolyLog}\left(2,\frac{cx+i}{cx-i}\right)-\frac{8i\left(a+b\tan^{-1}(cx)\right)}{cx-i}-\frac{4\left(a+b\tan^{-1}(cx)\right)}{(cx-i)^2}+8\log\left(\frac{2i}{-cx+i}\right)\left(a+\right.}{8d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])/(x*(d + I*c*d*x)^3),x]
```

```
[Out] ((5*b)/(I - c*x) + (I*b)/(-I + c*x)^2 - 5*b*ArcTan[c*x] - (4*(a + b*ArcTan[
c*x]))/(-I + c*x)^2 - ((8*I)*(a + b*ArcTan[c*x]))/(-I + c*x) + 8*a*Log[x] +
 8*(a + b*ArcTan[c*x])*Log[(2*I)/(I - c*x)] + (4*I)*b*PolyLog[2, (-I)*c*x]
- (4*I)*b*PolyLog[2, I*c*x] + (4*I)*b*PolyLog[2, (I + c*x)/(-I + c*x)])/(8*
d^3)
```

Maple [A] time = 0.066, size = 327, normalized size = 1.7

$$-\frac{a}{2\,d^3\,(cx-i)^2}-\frac{\frac{i}{2}b\mathrm{dilog}\,(-i\,(cx+i))}{d^3}-\frac{a\ln\left(c^2x^2+1\right)}{2\,d^3}-\frac{ia\arctan\,(cx)}{d^3}+\frac{a\ln\,(cx)}{d^3}-\frac{b\arctan\,(cx)}{2\,d^3\,(cx-i)^2}+\frac{\frac{i}{8}b}{d^3\,(cx-i)^2}-\frac{b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/x/(d+I*c*d*x)^3,x)
```

```
[Out] -1/2*a/d^3/(c*x-I)^2-1/2*I*b/d^3*dilog(-I*(c*x+I))-1/2*a/d^3*ln(c^2*x^2+1)-
I*a/d^3*arctan(c*x)+a/d^3*ln(c*x)-1/2*b/d^3*arctan(c*x)/(c*x-I)^2+1/8*I*b/d
^3/(c*x-I)^2-b/d^3*arctan(c*x)*ln(c*x-I)+b/d^3*arctan(c*x)*ln(c*x)-1/4*I*b/
d^3*ln(c*x-I)^2-5/8*b*arctan(c*x)/d^3-5/8*b/d^3/(c*x-I)-I*b/d^3*arctan(c*x)
/(c*x-I)+1/2*I*b/d^3*ln(-I*(-c*x+I))*ln(c*x)-1/2*I*b/d^3*ln(-I*(c*x+I))*ln(
c*x)-1/2*I*b/d^3*ln(-I*c*x)*ln(-I*(-c*x+I))+1/2*I*b/d^3*ln(-1/2*I*(c*x+I))*
ln(c*x-I)-I*a/d^3/(c*x-I)-1/2*I*b/d^3*dilog(-I*c*x)+1/2*I*b/d^3*dilog(-1/2*
I*(c*x+I))
```

Maxima [B] time = 1.33817, size = 563, normalized size = 2.89

$$-\frac{(16i\,a+10\,b)cx+\left(4i\,bc^2x^2+8\,bcx-4i\,b\right)\arctan\,(cx)^2+\left(i\,bc^2x^2+2\,bcx-i\,b\right)\log\left(c^2x^2+1\right)^2+4\left(bc^2x^2-2i\,bcx-b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x/(d+I*c*d*x)^3,x, algorithm="maxima")
```

```
[Out] -((16*I*a + 10*b)*c*x + (4*I*b*c^2*x^2 + 8*b*c*x - 4*I*b)*arctan(c*x)^2 + (
I*b*c^2*x^2 + 2*b*c*x - I*b)*log(c^2*x^2 + 1)^2 + 4*(b*c^2*x^2 - 2*I*b*c*x
- b)*arctan(c*x)*log(1/4*c^2*x^2 + 1/4) - 16*(b*c^2*x^2 - 2*I*b*c*x - b)*ar
ctan(c*x)*log(x*abs(c)) + ((b*(-16*I*arctan2(0, c) + 5) + 16*I*a)*c^2*x^2 -
 (b*(32*arctan2(0, c) - 6*I) - 32*a)*c*x + b*(16*I*arctan2(0, c) + 19) - 16
```

```
*I*a)*arctan(c*x) - 5*(b*c^2*x^2 - 2*I*b*c*x - b)*arctan2(c*x, -1) + (8*I*b
*c^2*x^2 + 16*b*c*x - 8*I*b)*dilog(I*c*x + 1) + (-8*I*b*c^2*x^2 - 16*b*c*x
+ 8*I*b)*dilog(1/2*I*c*x + 1/2) + (-8*I*b*c^2*x^2 - 16*b*c*x + 8*I*b)*dilog
(-I*c*x + 1) + (4*(pi*b + 2*a)*c^2*x^2 + (-8*I*pi*b - 16*I*a)*c*x - 4*pi*b
+ (-2*I*b*c^2*x^2 - 4*b*c*x + 2*I*b)*log(1/4*c^2*x^2 + 1/4) - 8*a)*log(c^2*
x^2 + 1) - (16*a*c^2*x^2 - 32*I*a*c*x - 16*a)*log(x) + 24*a - 12*I*b)/(16*c
^2*d^3*x^2 - 32*I*c*d^3*x - 16*d^3)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{b\log\left(-\frac{cx+i}{cx-i}\right)-2i\,a}{2\,c^3d^3x^4-6i\,c^2d^3x^3-6\,cd^3x^2+2i\,d^3x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x/(d+I*c*d*x)^3,x, algorithm="fricas")
```

```
[Out] integral(-(b*log(-(c*x + I)/(c*x - I)) - 2*I*a)/(2*c^3*d^3*x^4 - 6*I*c^2*d^
3*x^3 - 6*c*d^3*x^2 + 2*I*d^3*x), x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/x/(d+I*c*d*x)**3,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\arctan(cx)+a}{(i\,cdx+d)^3x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x/(d+I*c*d*x)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)/((I*c*d*x + d)^3*x), x)
```

3.64 $\int \frac{a+b\tan^{-1}(cx)}{x^2(d+icdx)^3}\,dx$

Optimal. Leaf size=250

$$\frac{3bc\text{PolyLog}(2,-icx)}{2d^3}-\frac{3bc\text{PolyLog}(2,icx)}{2d^3}+\frac{3bc\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{2d^3}+\frac{2c\left(a+b\tan^{-1}(cx)\right)}{d^3(-cx+i)}+\frac{ic\left(a+b\tan^{-1}(cx)\right)}{2d^3(-cx+i)^2}$$

```
[Out] (b*c)/(8*d^3*(I - c*x)^2) - (((9*I)/8)*b*c)/(d^3*(I - c*x)) + (((9*I)/8)*b*
c*ArcTan[c*x])/d^3 - (a + b*ArcTan[c*x])/(d^3*x) + ((I/2)*c*(a + b*ArcTan[c
*x]))/(d^3*(I - c*x)^2) + (2*c*(a + b*ArcTan[c*x]))/(d^3*(I - c*x)) - ((3*I
)*a*c*Log[x])/d^3 + (b*c*Log[x])/d^3 - ((3*I)*c*(a + b*ArcTan[c*x])*Log[2/(
1 + I*c*x)])/d^3 - (b*c*Log[1 + c^2*x^2])/(2*d^3) + (3*b*c*PolyLog[2, (-I)*
c*x])/(2*d^3) - (3*b*c*PolyLog[2, I*c*x])/(2*d^3) + (3*b*c*PolyLog[2, 1 - 2
/(1 + I*c*x)])/(2*d^3)
```

Rubi [A] time = 0.287827, antiderivative size = 250, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 15, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.652, Rules used = {4876, 4852, 266, 36, 29, 31, 4848, 2391, 4862, 627, 44, 203, 4854, 2402, 2315}

$$\frac{3bc\text{PolyLog}(2,-icx)}{2d^3}-\frac{3bc\text{PolyLog}(2,icx)}{2d^3}+\frac{3bc\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{2d^3}+\frac{2c\left(a+b\tan^{-1}(cx)\right)}{d^3(-cx+i)}+\frac{ic\left(a+b\tan^{-1}(cx)\right)}{2d^3(-cx+i)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])/(x^2*(d + I*c*d*x)^3),x]
```


```
[Out] (b*c)/(8*d^3*(I - c*x)^2) - (((9*I)/8)*b*c)/(d^3*(I - c*x)) + (((9*I)/8)*b*
c*ArcTan[c*x])/d^3 - (a + b*ArcTan[c*x])/(d^3*x) + ((I/2)*c*(a + b*ArcTan[c
*x]))/(d^3*(I - c*x)^2) + (2*c*(a + b*ArcTan[c*x]))/(d^3*(I - c*x)) - ((3*I
)*a*c*Log[x])/d^3 + (b*c*Log[x])/d^3 - ((3*I)*c*(a + b*ArcTan[c*x])*Log[2/(
1 + I*c*x)])/d^3 - (b*c*Log[1 + c^2*x^2])/(2*d^3) + (3*b*c*PolyLog[2, (-I)*
c*x])/(2*d^3) - (3*b*c*PolyLog[2, I*c*x])/(2*d^3) + (3*b*c*PolyLog[2, 1 - 2
/(1 + I*c*x)])/(2*d^3)
```


Rule 4876

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
```

```
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4862

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x]))/(e*(q + 1)), x] - Dist[(b*
c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int \frac{a+b\tan^{-1}(cx)}{x^2(d+icdx)^3}\,dx = \int \left(\frac{a+b\tan^{-1}(cx)}{d^3x^2} - \frac{3ic\left(a+b\tan^{-1}(cx)\right)}{d^3x} - \frac{ic^2\left(a+b\tan^{-1}(cx)\right)}{d^3(-i+cx)^3} + \frac{2c^2\left(a+b\tan^{-1}(cx)\right)}{d^3(-i+cx)^2} +\right.$$

$$= \frac{\int \frac{a+b\tan^{-1}(cx)}{x^2}\,dx}{d^3} - \frac{(3ic)\int \frac{a+b\tan^{-1}(cx)}{x}\,dx}{d^3} - \frac{\left(ic^2\right)\int \frac{a+b\tan^{-1}(cx)}{(-i+cx)^3}\,dx}{d^3} + \frac{\left(3ic^2\right)\int \frac{a+b\tan^{-1}(cx)}{-i+cx}\,dx}{d^3} +$$

$$= -\frac{a+b\tan^{-1}(cx)}{d^3x} + \frac{ic\left(a+b\tan^{-1}(cx)\right)}{2d^3(i-cx)^2} + \frac{2c\left(a+b\tan^{-1}(cx)\right)}{d^3(i-cx)} - \frac{3iac\log(x)}{d^3} - \frac{3ic\left(a+b\tan^{\text{-}}\right.}{}$$

$$= -\frac{a+b\tan^{-1}(cx)}{d^3x} + \frac{ic\left(a+b\tan^{-1}(cx)\right)}{2d^3(i-cx)^2} + \frac{2c\left(a+b\tan^{-1}(cx)\right)}{d^3(i-cx)} - \frac{3iac\log(x)}{d^3} - \frac{3ic\left(a+b\tan^{\text{-}}\right.}{}$$

$$= -\frac{a+b\tan^{-1}(cx)}{d^3x} + \frac{ic\left(a+b\tan^{-1}(cx)\right)}{2d^3(i-cx)^2} + \frac{2c\left(a+b\tan^{-1}(cx)\right)}{d^3(i-cx)} - \frac{3iac\log(x)}{d^3} - \frac{3ic\left(a+b\tan^{\text{-}}\right.}{}$$

$$= \frac{bc}{8d^3(i-cx)^2} - \frac{9ibc}{8d^3(i-cx)} - \frac{a+b\tan^{-1}(cx)}{d^3x} + \frac{ic\left(a+b\tan^{-1}(cx)\right)}{2d^3(i-cx)^2} + \frac{2c\left(a+b\tan^{-1}(cx)\right)}{d^3(i-cx)} - \frac{3}{}$$

$$= \frac{bc}{8d^3(i-cx)^2} - \frac{9ibc}{8d^3(i-cx)} + \frac{9ibc\tan^{-1}(cx)}{8d^3} - \frac{a+b\tan^{-1}(cx)}{d^3x} + \frac{ic\left(a+b\tan^{-1}(cx)\right)}{2d^3(i-cx)^2} + \frac{2c\left(a+\right.}{d}$$

Mathematica [A] time = 0.301583, size = 227, normalized size = 0.91

$$\frac{12bc\text{PolyLog}(2,-icx) - 12bc\text{PolyLog}(2,icx) + 12bc\text{PolyLog}\left(2,\frac{cx+i}{cx-i}\right) - \frac{8\left(a+b\tan^{-1}(cx)\right)}{x} - \frac{16c\left(a+b\tan^{-1}(cx)\right)}{cx-i} + \frac{4ic\left(a+b\tan^{-1}\right.}{(cx-i)^2}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])/(x^2*(d + I*c*d*x)^3),x]
```

```
[Out] ((-8*I)*b*c*((I - c*x)^(-1) - ArcTan[c*x]) - (8*(a + b*ArcTan[c*x]))/x + ((
4*I)*c*(a + b*ArcTan[c*x]))/(-I + c*x)^2 - (16*c*(a + b*ArcTan[c*x]))/(-I +
 c*x) + (b*c*(2 + I*c*x + I*(-I + c*x)^2*ArcTan[c*x]))/(-I + c*x)^2 - (24*I
)*a*c*Log[x] - (24*I)*c*(a + b*ArcTan[c*x])*Log[(2*I)/(I - c*x)] + 4*b*c*(2
*Log[x] - Log[1 + c^2*x^2]) + 12*b*c*PolyLog[2, (-I)*c*x] - 12*b*c*PolyLog[
2, I*c*x] + 12*b*c*PolyLog[2, (I + c*x)/(-I + c*x)])/(8*d^3)
```

Maple [A] time = 0.071, size = 394, normalized size = 1.6

$$\frac{\frac{9i}{8}cb}{d^3(cx-i)}+\frac{\frac{9i}{8}cb\arctan(cx)}{d^3}-3\frac{ca\arctan(cx)}{d^3}-2\frac{ca}{d^3(cx-i)}-\frac{a}{d^3x}+\frac{\frac{i}{2}ca}{d^3(cx-i)^2}-\frac{3icb\arctan(cx)\ln(cx)}{d^3}-\frac{3ica\ln}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/x^2/(d+I*c*d*x)^3,x)
```

```
[Out] 9/8*I*c*b/d^3/(c*x-I)+9/8*I*b*c*arctan(c*x)/d^3-3*c*a/d^3*arctan(c*x)-2*c*a
/d^3/(c*x-I)-a/d^3/x+1/2*I*c*a/d^3/(c*x-I)^2-3*I*c*b/d^3*arctan(c*x)*ln(c*x
)-3*I*c*a/d^3*ln(c*x)-2*c*b/d^3*arctan(c*x)/(c*x-I)-b/d^3*arctan(c*x)/x+3/2
*I*c*a/d^3*ln(c^2*x^2+1)+3*I*c*b/d^3*arctan(c*x)*ln(c*x-I)+1/8*c*b/d^3/(c*x
-I)^2-1/2*b*c*ln(c^2*x^2+1)/d^3+1/2*I*c*b/d^3*arctan(c*x)/(c*x-I)^2+c*b/d^3
*ln(c*x)-3/2*c*b/d^3*ln(-I*(-c*x+I))*ln(-I*c*x)+3/2*c*b/d^3*ln(-I*(-c*x+I))
*ln(c*x)-3/2*c*b/d^3*dilog(-I*c*x)-3/2*c*b/d^3*dilog(-I*(c*x+I))-3/2*c*b/d^
3*ln(c*x)*ln(-I*(c*x+I))+3/2*c*b/d^3*dilog(-1/2*I*(c*x+I))+3/2*c*b/d^3*ln(c
*x-I)*ln(-1/2*I*(c*x+I))-3/4*c*b/d^3*ln(c*x-I)^2
```

Maxima [B] time = 1.42188, size = 729, normalized size = 2.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^2/(d+I*c*d*x)^3,x, algorithm="maxima")
```

```
[Out] -1/16*(17*I*b*c^3*x^3*arctan2(1, c*x) + (b*(34*arctan2(1, c*x) - 18*I) + 48
*a)*c^2*x^2 + (b*(-17*I*arctan2(1, c*x) - 20) - 72*I*a)*c*x + (12*b*c^3*x^3
 - 24*I*b*c^2*x^2 - 12*b*c*x)*arctan(c*x)^2 + (3*b*c^3*x^3 - 6*I*b*c^2*x^2
- 3*b*c*x)*log(c^2*x^2 + 1)^2 + (-12*I*b*c^3*x^3 - 24*b*c^2*x^2 + 12*I*b*c*
x)*arctan(c*x)*log(1/4*c^2*x^2 + 1/4) + (48*I*b*c^3*x^3 + 96*b*c^2*x^2 - 48
*I*b*c*x)*arctan(c*x)*log(x*abs(c)) - ((b*(48*arctan2(0, c) + I) - 48*a)*c^
3*x^3 + (2*b*(-48*I*arctan2(0, c) - 23) + 96*I*a)*c^2*x^2 - (b*(48*arctan2(
0, c) - 71*I) - 48*a)*c*x + 16*b)*arctan(c*x) + (24*b*c^3*x^3 - 48*I*b*c^2*
x^2 - 24*b*c*x)*dilog(I*c*x + 1) - (24*b*c^3*x^3 - 48*I*b*c^2*x^2 - 24*b*c*
x)*dilog(1/2*I*c*x + 1/2) - (24*b*c^3*x^3 - 48*I*b*c^2*x^2 - 24*b*c*x)*dilo
g(-I*c*x + 1) - ((4*(3*I*pi - 2)*b + 24*I*a)*c^3*x^3 + ((24*pi + 16*I)*b +
48*a)*c^2*x^2 + (4*(-3*I*pi + 2)*b - 24*I*a)*c*x + (6*b*c^3*x^3 - 12*I*b*c^
2*x^2 - 6*b*c*x)*log(1/4*c^2*x^2 + 1/4))*log(c^2*x^2 + 1) + ((48*I*a - 16*b
)*c^3*x^3 + 32*(3*a + I*b)*c^2*x^2 + (-48*I*a + 16*b)*c*x)*log(x) - 16*a)/(
```

```
c^2*d^3*x^3 - 2*I*c*d^3*x^2 - d^3*x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{b\log\left(-\frac{cx+i}{cx-i}\right)-2i\,a}{2\,c^3d^3x^5-6i\,c^2d^3x^4-6\,cd^3x^3+2i\,d^3x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^2/(d+I*c*d*x)^3,x, algorithm="fricas")
```

```
[Out] integral(-(b*log(-(c*x + I)/(c*x - I)) - 2*I*a)/(2*c^3*d^3*x^5 - 6*I*c^2*d^
3*x^4 - 6*c*d^3*x^3 + 2*I*d^3*x^2), x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/x**2/(d+I*c*d*x)**3,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\arctan(cx)+a}{(i\,cdx+d)^3x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^2/(d+I*c*d*x)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)/((I*c*d*x + d)^3*x^2), x)
```

3.65 $\int \frac{a+b\tan^{-1}(cx)}{x^3(d+icdx)^3}\,dx$

Optimal. Leaf size=306

$$-\frac{3ibc^2\text{PolyLog}(2,-icx)}{d^3}+\frac{3ibc^2\text{PolyLog}(2,icx)}{d^3}-\frac{3ibc^2\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{d^3}-\frac{3ic^2\left(a+b\tan^{-1}(cx)\right)}{d^3(-cx+i)}+\frac{c^2\left(a+b\tan\right.}{2d^3(-cx\ }$$

```
[Out] -(b*c)/(2*d^3*x) - ((I/8)*b*c^2)/(d^3*(I - c*x)^2) - (13*b*c^2)/(8*d^3*(I -
 c*x)) + (9*b*c^2*ArcTan[c*x])/(8*d^3) - (a + b*ArcTan[c*x])/(2*d^3*x^2) +
((3*I)*c*(a + b*ArcTan[c*x]))/(d^3*x) + (c^2*(a + b*ArcTan[c*x]))/(2*d^3*(I
 - c*x)^2) - ((3*I)*c^2*(a + b*ArcTan[c*x]))/(d^3*(I - c*x)) - (6*a*c^2*Log
[x])/d^3 - ((3*I)*b*c^2*Log[x])/d^3 - (6*c^2*(a + b*ArcTan[c*x])*Log[2/(1 +
 I*c*x)])/d^3 + (((3*I)/2)*b*c^2*Log[1 + c^2*x^2])/d^3 - ((3*I)*b*c^2*PolyL
og[2, (-I)*c*x])/d^3 + ((3*I)*b*c^2*PolyLog[2, I*c*x])/d^3 - ((3*I)*b*c^2*P
olyLog[2, 1 - 2/(1 + I*c*x)])/d^3
```

Rubi [A] time = 0.320179, antiderivative size = 306, normalized size of antiderivative = 1., number of steps used = 26, number of rules used = 16, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.696, Rules used = {4876, 4852, 325, 203, 266, 36, 29, 31, 4848, 2391, 4862, 627, 44, 4854, 2402, 2315}

$$-\frac{3ibc^2\text{PolyLog}(2,-icx)}{d^3}+\frac{3ibc^2\text{PolyLog}(2,icx)}{d^3}-\frac{3ibc^2\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{d^3}-\frac{3ic^2\left(a+b\tan^{-1}(cx)\right)}{d^3(-cx+i)}+\frac{c^2\left(a+b\tan\right.}{2d^3(-cx\ }$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])/(x^3*(d + I*c*d*x)^3),x]
```

```
[Out] -(b*c)/(2*d^3*x) - ((I/8)*b*c^2)/(d^3*(I - c*x)^2) - (13*b*c^2)/(8*d^3*(I -
 c*x)) + (9*b*c^2*ArcTan[c*x])/(8*d^3) - (a + b*ArcTan[c*x])/(2*d^3*x^2) +
((3*I)*c*(a + b*ArcTan[c*x]))/(d^3*x) + (c^2*(a + b*ArcTan[c*x]))/(2*d^3*(I
 - c*x)^2) - ((3*I)*c^2*(a + b*ArcTan[c*x]))/(d^3*(I - c*x)) - (6*a*c^2*Log
[x])/d^3 - ((3*I)*b*c^2*Log[x])/d^3 - (6*c^2*(a + b*ArcTan[c*x])*Log[2/(1 +
 I*c*x)])/d^3 + (((3*I)/2)*b*c^2*Log[1 + c^2*x^2])/d^3 - ((3*I)*b*c^2*PolyL
og[2, (-I)*c*x])/d^3 + ((3*I)*b*c^2*PolyLog[2, I*c*x])/d^3 - ((3*I)*b*c^2*P
olyLog[2, 1 - 2/(1 + I*c*x)])/d^3
```

Rule 4876

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4862

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x]))/(e*(q + 1)), x] - Dist[(b*
c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int \frac{a+b\tan^{-1}(cx)}{x^3(d+icdx)^3}\,dx = \int \left(\frac{a+b\tan^{-1}(cx)}{d^3x^3} - \frac{3ic\left(a+b\tan^{-1}(cx)\right)}{d^3x^2} - \frac{6c^2\left(a+b\tan^{-1}(cx)\right)}{d^3x} - \frac{c^3\left(a+b\tan^{-1}(cx)\right)}{d^3(-i+cx)^3} - \right.$$

$$= \frac{\int \frac{a+b\tan^{-1}(cx)}{x^3}\,dx}{d^3} - \frac{(3ic)\int \frac{a+b\tan^{-1}(cx)}{x^2}\,dx}{d^3} - \frac{\left(6c^2\right)\int \frac{a+b\tan^{-1}(cx)}{x}\,dx}{d^3} - \frac{\left(3ic^3\right)\int \frac{a+b\tan^{-1}(cx)}{(-i+cx)^2}\,dx}{d^3} -$$

$$= -\frac{a+b\tan^{-1}(cx)}{2d^3x^2} + \frac{3ic\left(a+b\tan^{-1}(cx)\right)}{d^3x} + \frac{c^2\left(a+b\tan^{-1}(cx)\right)}{2d^3(i-cx)^2} - \frac{3ic^2\left(a+b\tan^{-1}(cx)\right)}{d^3(i-cx)} - 6a$$

$$= -\frac{bc}{2d^3x} - \frac{a+b\tan^{-1}(cx)}{2d^3x^2} + \frac{3ic\left(a+b\tan^{-1}(cx)\right)}{d^3x} + \frac{c^2\left(a+b\tan^{-1}(cx)\right)}{2d^3(i-cx)^2} - \frac{3ic^2\left(a+b\tan^{-1}(c\right.}{d^3(i-cx)}$$

$$= -\frac{bc}{2d^3x} - \frac{bc^2\tan^{-1}(cx)}{2d^3} - \frac{a+b\tan^{-1}(cx)}{2d^3x^2} + \frac{3ic\left(a+b\tan^{-1}(cx)\right)}{d^3x} + \frac{c^2\left(a+b\tan^{-1}(cx)\right)}{2d^3(i-cx)^2} - 3ic$$

$$= -\frac{bc}{2d^3x} - \frac{ibc^2}{8d^3(i-cx)^2} - \frac{13bc^2}{8d^3(i-cx)} - \frac{bc^2\tan^{-1}(cx)}{2d^3} - \frac{a+b\tan^{-1}(cx)}{2d^3x^2} + \frac{3ic\left(a+b\tan^{-1}(cx)\right)}{d^3x}$$

$$= -\frac{bc}{2d^3x} - \frac{ibc^2}{8d^3(i-cx)^2} - \frac{13bc^2}{8d^3(i-cx)} + \frac{9bc^2\tan^{-1}(cx)}{8d^3} - \frac{a+b\tan^{-1}(cx)}{2d^3x^2} + \frac{3ic\left(a+b\tan^{-1}(cx\right.}{d^3x}$$

Mathematica [C] time = 0.54173, size = 285, normalized size = 0.93

$$-\frac{\frac{4bc\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-c^2x^2\right)}{x} + 24ibc^2\text{PolyLog}(2,-icx) - 24ibc^2\text{PolyLog}(2,icx) + 24ibc^2\text{PolyLog}\left(2,\frac{cx+i}{cx-i}\right) - \frac{24ic^2}{}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c*x])/(x^3*(d + I*c*d*x)^3),x]
```

```
[Out] -(12*b*c^2*((I - c*x)^(-1) - ArcTan[c*x]) + (4*(a + b*ArcTan[c*x]))/x^2 - (
(24*I)*c*(a + b*ArcTan[c*x]))/x - (4*c^2*(a + b*ArcTan[c*x]))/(-I + c*x)^2
- ((24*I)*c^2*(a + b*ArcTan[c*x]))/(-I + c*x) - (b*c^2*(-2*I + c*x + (-I +
c*x)^2*ArcTan[c*x]))/(-I + c*x)^2 + (4*b*c*Hypergeometric2F1[-1/2, 1, 1/2,
-(c^2*x^2)])/x + 48*a*c^2*Log[x] + 48*c^2*(a + b*ArcTan[c*x])*Log[(2*I)/(I
- c*x)] + (12*I)*b*c^2*(2*Log[x] - Log[1 + c^2*x^2]) + (24*I)*b*c^2*PolyLog
[2, (-I)*c*x] - (24*I)*b*c^2*PolyLog[2, I*c*x] + (24*I)*b*c^2*PolyLog[2, (I
 + c*x)/(-I + c*x)])/(8*d^3)
```

Maple [A] time = 0.075, size = 481, normalized size = 1.6

$$\frac{-3\,ic^2b\ln\left(-\frac{i}{2}\,(cx+i)\right)\ln(cx-i)}{d^3}+\frac{3\,icb\arctan(cx)}{d^3x}+\frac{3\,ic^2b\ln(-i\,(cx+i))\ln(cx)}{d^3}+6\,\frac{c^2b\arctan(cx)\ln(cx-i)}{d^3}-\frac{bc}{2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/x^3/(d+I*c*d*x)^3,x)
```

```
[Out] -3*I*c^2*b/d^3*ln(-1/2*I*(c*x+I))*ln(c*x-I)+3*I*c*b/d^3*arctan(c*x)/x+3*I*c
^2*b/d^3*ln(-I*(c*x+I))*ln(c*x)+6*c^2*b/d^3*arctan(c*x)*ln(c*x-I)-1/2*b*c/d
^3/x+9/8*b*c^2*arctan(c*x)/d^3-1/2*b/d^3*arctan(c*x)/x^2+3*c^2*a/d^3*ln(c^2
*x^2+1)+13/8*c^2*b/d^3/(c*x-I)-6*c^2*a/d^3*ln(c*x)-1/2*a/d^3/x^2+3/2*I*c^2*
b/d^3*ln(c*x-I)^2-3*I*c^2*b/d^3*dilog(-1/2*I*(c*x+I))+3*I*c^2*b/d^3*dilog(-
I*c*x)+6*I*c^2*a/d^3*arctan(c*x)+3*I*c^2*b/d^3*dilog(-I*(c*x+I))+1/2*c^2*a/
d^3/(c*x-I)^2+1/2*c^2*b/d^3*arctan(c*x)/(c*x-I)^2-6*c^2*b/d^3*arctan(c*x)*l
n(c*x)+3*I*c*a/d^3/x+3*I*c^2*a/d^3/(c*x-I)-3*I*c^2*b/d^3*ln(c*x)+3*I*c^2*b/
d^3*ln(-I*c*x)*ln(-I*(-c*x+I))+3/2*I*b*c^2*ln(c^2*x^2+1)/d^3-3*I*c^2*b/d^3*
ln(-I*(-c*x+I))*ln(c*x)-1/8*I*c^2*b/d^3/(c*x-I)^2+3*I*c^2*b/d^3*arctan(c*x)
/(c*x-I)
```

Maxima [B] time = 1.46372, size = 829, normalized size = 2.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^3/(d+I*c*d*x)^3,x, algorithm="maxima")
```

```
[Out] -(33*b*c^4*x^4*arctan2(1, c*x) + (6*b*(-11*I*arctan2(1, c*x) - 3) - 96*I*a)
*c^3*x^3 - (b*(33*arctan2(1, c*x) - 12*I) + 144*a)*c^2*x^2 - (-32*I*a + 8*b
```

```
)*c*x - (24*I*b*c^4*x^4 + 48*b*c^3*x^3 - 24*I*b*c^2*x^2)*arctan(c*x)^2 - (6
*I*b*c^4*x^4 + 12*b*c^3*x^3 - 6*I*b*c^2*x^2)*log(c^2*x^2 + 1)^2 - 24*(b*c^4
*x^4 - 2*I*b*c^3*x^3 - b*c^2*x^2)*arctan(c*x)*log(1/4*c^2*x^2 + 1/4) + 96*(
b*c^4*x^4 - 2*I*b*c^3*x^3 - b*c^2*x^2)*arctan(c*x)*log(x*abs(c)) + ((3*b*(3
2*I*arctan2(0, c) + 5) - 96*I*a)*c^4*x^4 + (b*(192*arctan2(0, c) - 126*I) -
 192*a)*c^3*x^3 + (3*b*(-32*I*arctan2(0, c) - 53) + 96*I*a)*c^2*x^2 + 32*I*
b*c*x - 8*b)*arctan(c*x) - (48*I*b*c^4*x^4 + 96*b*c^3*x^3 - 48*I*b*c^2*x^2)
*dilog(I*c*x + 1) - (-48*I*b*c^4*x^4 - 96*b*c^3*x^3 + 48*I*b*c^2*x^2)*dilog
(1/2*I*c*x + 1/2) - (-48*I*b*c^4*x^4 - 96*b*c^3*x^3 + 48*I*b*c^2*x^2)*dilog
(-I*c*x + 1) - (((24*pi + 24*I)*b + 48*a)*c^4*x^4 - (48*(I*pi - 1)*b + 96*I
*a)*c^3*x^3 - ((24*pi + 24*I)*b + 48*a)*c^2*x^2 + (-12*I*b*c^4*x^4 - 24*b*c
^3*x^3 + 12*I*b*c^2*x^2)*log(1/4*c^2*x^2 + 1/4))*log(c^2*x^2 + 1) + (48*(2*
a + I*b)*c^4*x^4 - (192*I*a - 96*b)*c^3*x^3 - 48*(2*a + I*b)*c^2*x^2)*log(x
) - 8*a)/(16*c^2*d^3*x^4 - 32*I*c*d^3*x^3 - 16*d^3*x^2)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{b\log\left(-\frac{cx+i}{cx-i}\right)-2i\,a}{2\,c^3d^3x^6-6i\,c^2d^3x^5-6\,cd^3x^4+2i\,d^3x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^3/(d+I*c*d*x)^3,x, algorithm="fricas")
```

```
[Out] integral(-(b*log(-(c*x + I)/(c*x - I)) - 2*I*a)/(2*c^3*d^3*x^6 - 6*I*c^2*d^
3*x^5 - 6*c*d^3*x^4 + 2*I*d^3*x^3), x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/x**3/(d+I*c*d*x)**3,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\arctan(cx) + a}{(icdx + d)^3 x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^3/(d+I*c*d*x)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)/((I*c*d*x + d)^3*x^3), x)
```

3.66 $\int \frac{a+b\tan^{-1}(cx)}{(1+icx)^4}\,dx$

Optimal. Leaf size=100

$$\frac{i\left(a+b\tan^{-1}(cx)\right)}{3c(1+icx)^3}+\frac{ib}{24c(-cx+i)}-\frac{b}{24c(-cx+i)^2}-\frac{ib}{18c(-cx+i)^3}-\frac{ib\tan^{-1}(cx)}{24c}$$

```
[Out] ((-I/18)*b)/(c*(I - c*x)^3) - b/(24*c*(I - c*x)^2) + ((I/24)*b)/(c*(I - c*x
)) - ((I/24)*b*ArcTan[c*x])/c + ((I/3)*(a + b*ArcTan[c*x]))/(c*(1 + I*c*x)^
3)
```

Rubi [A] time = 0.0512029, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {4862, 627, 44, 203}

$$\frac{i\left(a+b\tan^{-1}(cx)\right)}{3c(1+icx)^3}+\frac{ib}{24c(-cx+i)}-\frac{b}{24c(-cx+i)^2}-\frac{ib}{18c(-cx+i)^3}-\frac{ib\tan^{-1}(cx)}{24c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])/(1 + I*c*x)^4,x]
```



```
[Out] ((-I/18)*b)/(c*(I - c*x)^3) - b/(24*c*(I - c*x)^2) + ((I/24)*b)/(c*(I - c*x
)) - ((I/24)*b*ArcTan[c*x])/c + ((I/3)*(a + b*ArcTan[c*x]))/(c*(1 + I*c*x)^
3)
```


Rule 4862

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x]))/(e*(q + 1)), x] - Dist[(b*
c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+b\tan^{-1}(cx)}{(1+icx)^4}\,dx &= \frac{i\left(a+b\tan^{-1}(cx)\right)}{3c(1+icx)^3}-\frac{1}{3}(ib)\int \frac{1}{(1+icx)^3\left(1+c^2x^2\right)}\,dx\\
&= \frac{i\left(a+b\tan^{-1}(cx)\right)}{3c(1+icx)^3}-\frac{1}{3}(ib)\int \frac{1}{(1-icx)(1+icx)^4}\,dx\\
&= \frac{i\left(a+b\tan^{-1}(cx)\right)}{3c(1+icx)^3}-\frac{1}{3}(ib)\int \left(\frac{1}{2(-i+cx)^4}+\frac{i}{4(-i+cx)^3}-\frac{1}{8(-i+cx)^2}+\frac{1}{8\left(1+c^2x^2\right)}\right)dx\\
&= -\frac{ib}{18c(i-cx)^3}-\frac{b}{24c(i-cx)^2}+\frac{ib}{24c(i-cx)}+\frac{i\left(a+b\tan^{-1}(cx)\right)}{3c(1+icx)^3}-\frac{1}{24}(ib)\int \frac{1}{1+c^2x^2}\,dx\\
&= -\frac{ib}{18c(i-cx)^3}-\frac{b}{24c(i-cx)^2}+\frac{ib}{24c(i-cx)}-\frac{ib\tan^{-1}(cx)}{24c}+\frac{i\left(a+b\tan^{-1}(cx)\right)}{3c(1+icx)^3}
\end{aligned}
$$

Mathematica [A] time = 0.0438442, size = 73, normalized size = 0.73

$$
\frac{-24a+b\left(-3ic^2x^2-9cx+10i\right)+3b\left(-ic^3x^3-3c^2x^2+3icx-7\right)\tan^{-1}(cx)}{72c(cx-i)^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])/(1 + I*c*x)^4,x]
```

```
[Out] (-24*a + b*(10*I - 9*c*x - (3*I)*c^2*x^2) + 3*b*(-7 + (3*I)*c*x - 3*c^2*x^2
 - I*c^3*x^3)*ArcTan[c*x])/(72*c*(-I + c*x)^3)
```

Maple [A] time = 0.04, size = 93, normalized size = 0.9

$$\frac{\frac{i}{3}a}{c\,(1+icx)^3}+\frac{\frac{i}{3}b\arctan(cx)}{c\,(1+icx)^3}-\frac{\frac{i}{24}b\arctan(cx)}{c}-\frac{b}{24\,c\,(cx-i)^2}+\frac{\frac{i}{18}b}{c\,(cx-i)^3}-\frac{\frac{i}{24}b}{c\,(cx-i)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/(1+I*c*x)^4,x)
```

```
[Out] 1/3*I/c*a/(1+I*c*x)^3+1/3*I/c*b/(1+I*c*x)^3*arctan(c*x)-1/24*I*b*arctan(c*x
)/c-1/24/c*b/(c*x-I)^2+1/18*I/c*b/(c*x-I)^3-1/24*I/c*b/(c*x-I)
```

Maxima [A] time = 1.06591, size = 112, normalized size = 1.12

$$-\frac{3i\,bc^2x^2+9\,bcx+\left(3i\,bc^3x^3+9\,bc^2x^2-9i\,bcx+21\,b\right)\arctan(cx)+24\,a-10i\,b}{72\,c^4x^3-216i\,c^3x^2-216\,c^2x+72i\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/(1+I*c*x)^4,x, algorithm="maxima")
```

```
[Out] -(3*I*b*c^2*x^2 + 9*b*c*x + (3*I*b*c^3*x^3 + 9*b*c^2*x^2 - 9*I*b*c*x + 21*b
)*arctan(c*x) + 24*a - 10*I*b)/(72*c^4*x^3 - 216*I*c^3*x^2 - 216*c^2*x + 72
*I*c)
```

Fricas [A] time = 2.20688, size = 235, normalized size = 2.35

$$\frac{-6i\,bc^2x^2-18\,bcx+\left(3\,bc^3x^3-9i\,bc^2x^2-9\,bcx-21i\,b\right)\log\left(-\frac{cx+i}{cx-i}\right)-48\,a+20i\,b}{144\,c^4x^3-432i\,c^3x^2-432\,c^2x+144i\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/(1+I*c*x)^4,x, algorithm="fricas")
```

```
[Out] (-6*I*b*c^2*x^2 - 18*b*c*x + (3*b*c^3*x^3 - 9*I*b*c^2*x^2 - 9*b*c*x - 21*I*
b)*log(-(c*x + I)/(c*x - I)) - 48*a + 20*I*b)/(144*c^4*x^3 - 432*I*c^3*x^2
- 432*c^2*x + 144*I*c)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/(1+I*c*x)**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.13206, size = 232, normalized size = 2.32

$$\frac{3\,bc^3x^3\log\left(cx+i\right)-3\,bc^3x^3\log\left(cx-i\right)-9\,bc^2ix^2\log\left(cx+i\right)+9\,bc^2ix^2\log\left(cx-i\right)-6\,bc^2ix^2-9\,bcx\log\left(cx+i\right)+9\,bcx}{144\left(c^4x^3-3\,c^3ix^2-3\,c^2x+ci\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/(1+I*c*x)^4,x, algorithm="giac")
```

```
[Out] 1/144*(3*b*c^3*x^3*log(c*x + i) - 3*b*c^3*x^3*log(c*x - i) - 9*b*c^2*i*x^2*
log(c*x + i) + 9*b*c^2*i*x^2*log(c*x - i) - 6*b*c^2*i*x^2 - 9*b*c*x*log(c*x
 + i) + 9*b*c*x*log(c*x - i) - 18*b*c*x + 3*b*i*log(c*x + i) - 3*b*i*log(c*
x - i) + 20*b*i - 48*b*arctan(c*x) - 48*a)/(c^4*x^3 - 3*c^3*i*x^2 - 3*c^2*x
 + c*i)
```

3.67 $\int \frac{\tan^{-1}(ax)}{cx+iacx^2}\,dx$

Optimal. Leaf size=49

$$\frac{i\text{PolyLog}\left(2,-1+\frac{2}{1+iax}\right)}{2c}+\frac{\log\left(2-\frac{2}{1+iax}\right)\tan^{-1}(ax)}{c}$$

```
[Out] (ArcTan[a*x]*Log[2 - 2/(1 + I*a*x)])/c + ((I/2)*PolyLog[2, -1 + 2/(1 + I*a*
x)])/c
```

Rubi [A] time = 0.0636605, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {1593, 4868, 2447}

$$\frac{i\text{PolyLog}\left(2,-1+\frac{2}{1+iax}\right)}{2c}+\frac{\log\left(2-\frac{2}{1+iax}\right)\tan^{-1}(ax)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]/(c*x + I*a*c*x^2),x]
```

```
[Out] (ArcTan[a*x]*Log[2 - 2/(1 + I*a*x)])/c + ((I/2)*PolyLog[2, -1 + 2/(1 + I*a*
x)])/c
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^{-1}(ax)}{cx+iacx^2}\,dx &= \int \frac{\tan^{-1}(ax)}{x(c+iacx)}\,dx\\ &= \frac{\tan^{-1}(ax)\log\left(2-\frac{2}{1+iax}\right)}{c} - \frac{a\int \frac{\log\left(2-\frac{2}{1+iax}\right)}{1+a^2x^2}\,dx}{c}\\ &= \frac{\tan^{-1}(ax)\log\left(2-\frac{2}{1+iax}\right)}{c} + \frac{i\text{Li}_2\left(-1+\frac{2}{1+iax}\right)}{2c}\end{aligned}$$

Mathematica [A] time = 0.0244393, size = 88, normalized size = 1.8

$$\frac{i\text{PolyLog}(2,-iax)}{2c} - \frac{i\text{PolyLog}(2,iax)}{2c} + \frac{i\text{PolyLog}\left(2,-\frac{ax+i}{-ax+i}\right)}{2c} + \frac{\log\left(\frac{2i}{-ax+i}\right)\tan^{-1}(ax)}{c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTan[a*x]/(c*x + I*a*c*x^2),x]
```

```
[Out] (ArcTan[a*x]*Log[(2*I)/(I - a*x)])/c + ((I/2)*PolyLog[2, (-I)*a*x])/c - ((I
/2)*PolyLog[2, I*a*x])/c + ((I/2)*PolyLog[2, -((I + a*x)/(I - a*x))])/c
```

Maple [B] time = 0.049, size = 148, normalized size = 3.

$$-\frac{\arctan(ax)\ln(ax-i)}{c} + \frac{\arctan(ax)\ln(ax)}{c} + \frac{\frac{i}{2}\ln(ax)\ln(1+iax)}{c} - \frac{\frac{i}{2}\ln(ax)\ln(1-iax)}{c} + \frac{\frac{i}{2}\text{dilog}(1+iax)}{c} - \frac{\frac{i}{2}\text{dilc}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)/(c*x+I*a*c*x^2),x)
```

```
[Out] -1/c*arctan(a*x)*ln(a*x-I)+1/c*arctan(a*x)*ln(a*x)+1/2*I/c*ln(a*x)*ln(1+I*a
*x)-1/2*I/c*ln(a*x)*ln(1-I*a*x)+1/2*I/c*dilog(1+I*a*x)-1/2*I/c*dilog(1-I*a*
```

```
x)+1/2*I/c*ln(-1/2*I*(a*x+I))*ln(a*x-I)+1/2*I/c*dilog(-1/2*I*(a*x+I))-1/4*I
/c*ln(a*x-I)^2
```

Maxima [B] time = 1.49263, size = 170, normalized size = 3.47

$$\frac{1}{4}a\left(-\frac{i\log\left(iax+1\right)^2}{ac}+\frac{2i\left(\log\left(iax+1\right)\log\left(-\frac{1}{2}iax+\frac{1}{2}\right)+\operatorname{Li}_2\left(\frac{1}{2}iax+\frac{1}{2}\right)\right)}{ac}+\frac{2i\left(\log\left(iax+1\right)\log\left(x\right)+\operatorname{Li}_2\left(-iax\right)\right)}{ac}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/(c*x+I*a*c*x^2),x, algorithm="maxima")
```

```
[Out] 1/4*a*(-I*log(I*a*x + 1)^2/(a*c) + 2*I*(log(I*a*x + 1)*log(-1/2*I*a*x + 1/2
) + dilog(1/2*I*a*x + 1/2))/(a*c) + 2*I*(log(I*a*x + 1)*log(x) + dilog(-I*a
*x))/(a*c) - 2*I*(log(-I*a*x + 1)*log(x) + dilog(I*a*x))/(a*c)) - (log(I*a*
x + 1)/c - log(x)/c)*arctan(a*x)
```

Fricas [A] time = 2.1571, size = 55, normalized size = 1.12

$$-\frac{i\operatorname{Li}_2\left(\frac{ax+i}{ax-i}+1\right)}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/(c*x+I*a*c*x^2),x, algorithm="fricas")
```

```
[Out] -1/2*I*dilog((a*x + I)/(a*x - I) + 1)/c
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)/(c*x+I*a*c*x**2),x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(ax)}{i\,acx^2+cx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/(c*x+I*a*c*x^2),x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)/(I*a*c*x^2 + c*x), x)
```

3.68 $\int x^3(d+icdx)\left(a+b\tan^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=287

$$-\frac{b^2d\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{5c^4}+\frac{ibdx^2\left(a+b\tan^{-1}(cx)\right)}{5c^2}+\frac{abdx}{2c^3}-\frac{9d\left(a+b\tan^{-1}(cx)\right)^2}{20c^4}+\frac{2ibd\log\left(\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(\right.}{5c^4}$$

```
[Out] (a*b*d*x)/(2*c^3) - (((3*I)/10)*b^2*d*x)/c^3 + (b^2*d*x^2)/(12*c^2) + ((I/3
0)*b^2*d*x^3)/c + (((3*I)/10)*b^2*d*ArcTan[c*x])/c^4 + (b^2*d*x*ArcTan[c*x]
)/(2*c^3) + ((I/5)*b*d*x^2*(a + b*ArcTan[c*x]))/c^2 - (b*d*x^3*(a + b*ArcTa
n[c*x]))/(6*c) - (I/10)*b*d*x^4*(a + b*ArcTan[c*x]) - (9*d*(a + b*ArcTan[c*
x])^2)/(20*c^4) + (d*x^4*(a + b*ArcTan[c*x])^2)/4 + (I/5)*c*d*x^5*(a + b*Ar
cTan[c*x])^2 + (((2*I)/5)*b*d*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/c^4 -
 (b^2*d*Log[1 + c^2*x^2])/(3*c^4) - (b^2*d*PolyLog[2, 1 - 2/(1 + I*c*x)])/(
5*c^4)
```

Rubi [A] time = 0.596383, antiderivative size = 287, normalized size of antiderivative = 1., number of steps used = 27, number of rules used = 15, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.652, Rules used = {4876, 4852, 4916, 266, 43, 4846, 260, 4884, 302, 203, 321, 4920, 4854, 2402, 2315}

$$-\frac{b^2d\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{5c^4}+\frac{ibdx^2\left(a+b\tan^{-1}(cx)\right)}{5c^2}+\frac{abdx}{2c^3}-\frac{9d\left(a+b\tan^{-1}(cx)\right)^2}{20c^4}+\frac{2ibd\log\left(\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(\right.}{5c^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + I*c*d*x)*(a + b*ArcTan[c*x])^2,x]
```


```
[Out] (a*b*d*x)/(2*c^3) - (((3*I)/10)*b^2*d*x)/c^3 + (b^2*d*x^2)/(12*c^2) + ((I/3
0)*b^2*d*x^3)/c + (((3*I)/10)*b^2*d*ArcTan[c*x])/c^4 + (b^2*d*x*ArcTan[c*x]
)/(2*c^3) + ((I/5)*b*d*x^2*(a + b*ArcTan[c*x]))/c^2 - (b*d*x^3*(a + b*ArcTa
n[c*x]))/(6*c) - (I/10)*b*d*x^4*(a + b*ArcTan[c*x]) - (9*d*(a + b*ArcTan[c*
x])^2)/(20*c^4) + (d*x^4*(a + b*ArcTan[c*x])^2)/4 + (I/5)*c*d*x^5*(a + b*Ar
cTan[c*x])^2 + (((2*I)/5)*b*d*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/c^4 -
 (b^2*d*Log[1 + c^2*x^2])/(3*c^4) - (b^2*d*PolyLog[2, 1 - 2/(1 + I*c*x)])/(
5*c^4)
```


Rule 4876

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
```

```
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int x^3(d+icdx)\left(a+b\tan^{-1}(cx)\right)^2\,dx=\int\left(dx^3\left(a+b\tan^{-1}(cx)\right)^2+icdx^4\left(a+b\tan^{-1}(cx)\right)^2\right)dx$$

$$=d\int x^3\left(a+b\tan^{-1}(cx)\right)^2\,dx+(icd)\int x^4\left(a+b\tan^{-1}(cx)\right)^2\,dx$$

$$=\frac{1}{4}dx^4\left(a+b\tan^{-1}(cx)\right)^2+\frac{1}{5}icdx^5\left(a+b\tan^{-1}(cx)\right)^2-\frac{1}{2}(bcd)\int\frac{x^4\left(a+b\tan^{-1}\right.}{1+c^2x^2}$$

$$=\frac{1}{4}dx^4\left(a+b\tan^{-1}(cx)\right)^2+\frac{1}{5}icdx^5\left(a+b\tan^{-1}(cx)\right)^2-\frac{1}{5}(2ibd)\int x^3\left(a+b\tan^{-1}\right.$$

$$=-\frac{bdx^3\left(a+b\tan^{-1}(cx)\right)}{6c}-\frac{1}{10}ibdx^4\left(a+b\tan^{-1}(cx)\right)+\frac{1}{4}dx^4\left(a+b\tan^{-1}(cx)\right)^2$$

$$=\frac{abdx}{2c^3}+\frac{ibdx^2\left(a+b\tan^{-1}(cx)\right)}{5c^2}-\frac{bdx^3\left(a+b\tan^{-1}(cx)\right)}{6c}-\frac{1}{10}ibdx^4\left(a+b\tan^{-1}\right.$$

$$=\frac{abdx}{2c^3}-\frac{3ib^2dx}{10c^3}+\frac{ib^2dx^3}{30c}+\frac{b^2dx\tan^{-1}(cx)}{2c^3}+\frac{ibdx^2\left(a+b\tan^{-1}(cx)\right)}{5c^2}-\frac{bdx^3\left(a\right.}{}$$

$$=\frac{abdx}{2c^3}-\frac{3ib^2dx}{10c^3}+\frac{b^2dx^2}{12c^2}+\frac{ib^2dx^3}{30c}+\frac{3ib^2d\tan^{-1}(cx)}{10c^4}+\frac{b^2dx\tan^{-1}(cx)}{2c^3}+\frac{ibdx^2\left(\right.}{}$$

$$=\frac{abdx}{2c^3}-\frac{3ib^2dx}{10c^3}+\frac{b^2dx^2}{12c^2}+\frac{ib^2dx^3}{30c}+\frac{3ib^2d\tan^{-1}(cx)}{10c^4}+\frac{b^2dx\tan^{-1}(cx)}{2c^3}+\frac{ibdx^2\left(\right.}{}$$

Mathematica [A] time = 0.786761, size = 285, normalized size = 0.99

$$\frac{d\left(12b^2\text{PolyLog}\left(2,-e^{2i\tan^{-1}(cx)}\right)+12ia^2c^5x^5+15a^2c^4x^4-6iabc^4x^4-10abc^3x^3+12iabc^2x^2-12iab\log\left(c^2x^2+1\right)+2b\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*(d + I*c*d*x)*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] (d*((18*I)*a*b + 5*b^2 + 30*a*b*c*x - (18*I)*b^2*c*x + (12*I)*a*b*c^2*x^2 +
 5*b^2*c^2*x^2 - 10*a*b*c^3*x^3 + (2*I)*b^2*c^3*x^3 + 15*a^2*c^4*x^4 - (6*I
```

```
)*a*b*c^4*x^4 + (12*I)*a^2*c^5*x^5 + 3*b^2*(-1 + 5*c^4*x^4 + (4*I)*c^5*x^5)
*ArcTan[c*x]^2 + 2*b*ArcTan[c*x]*(b*(9*I + 15*c*x + (6*I)*c^2*x^2 - 5*c^3*x
^3 - (3*I)*c^4*x^4) + 3*a*(-5 + 5*c^4*x^4 + (4*I)*c^5*x^5) + (12*I)*b*Log[1
 + E^((2*I)*ArcTan[c*x])]) - (12*I)*a*b*Log[1 + c^2*x^2] - 20*b^2*Log[1 + c
^2*x^2] + 12*b^2*PolyLog[2, -E^((2*I)*ArcTan[c*x])]))/(60*c^4)
```

Maple [B] time = 0.093, size = 499, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d+I*c*d*x)*(a+b*arctan(c*x))^2,x)
```

```
[Out] -1/20/c^4*d*b^2*ln(c*x-I)^2-1/10/c^4*d*b^2*dilog(-1/2*I*(c*x+I))+1/20/c^4*d
*b^2*ln(c*x+I)^2+1/10/c^4*d*b^2*dilog(1/2*I*(c*x-I))+1/4*d*b^2*arctan(c*x)^
2*x^4-1/4/c^4*d*b^2*arctan(c*x)^2-1/3*b^2*d*ln(c^2*x^2+1)/c^4-1/10*I*d*a*b*
x^4+1/5*I/c^2*d*a*b*x^2+1/2*d*a*b*arctan(c*x)*x^4+1/5*I*c*d*a^2*x^5-1/10*I*
d*b^2*arctan(c*x)*x^4-1/6/c*d*a*b*x^3-1/10/c^4*d*b^2*ln(c*x+I)*ln(c^2*x^2+1
)+1/10/c^4*d*b^2*ln(c*x+I)*ln(1/2*I*(c*x-I))+1/10/c^4*d*b^2*ln(c*x-I)*ln(c^
2*x^2+1)-1/10/c^4*d*b^2*ln(c*x-I)*ln(-1/2*I*(c*x+I))-1/2/c^4*d*a*b*arctan(c
*x)-1/6/c*d*b^2*arctan(c*x)*x^3+1/4*d*a^2*x^4+1/5*I*c*d*b^2*arctan(c*x)^2*x
^5-1/5*I/c^4*d*b^2*arctan(c*x)*ln(c^2*x^2+1)+1/2*a*b*d*x/c^3+1/2*b^2*d*x*ar
ctan(c*x)/c^3-3/10*I*b^2*d*x/c^3+1/30*I*b^2*d*x^3/c+3/10*I*b^2*d*arctan(c*x
)/c^4+1/5*I/c^2*d*b^2*arctan(c*x)*x^2+2/5*I*c*d*a*b*arctan(c*x)*x^5-1/5*I/c
^4*d*a*b*ln(c^2*x^2+1)+1/12*b^2*d*x^2/c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{5} i a^2 c d x^5 + \frac{1}{4} b^2 d x^4 \arctan(cx)^2 + \frac{1}{4} a^2 d x^4 + \frac{1}{10} i \left(4 x^5 \arctan(cx) - c\left(\frac{c^2 x^4 - 2 x^2}{c^4} + \frac{2 \log\left(c^2 x^2 + 1\right)}{c^6}\right)\right) abcd + \frac{1}{80} i \left(4\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d+I*c*d*x)*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/5*I*a^2*c*d*x^5 + 1/4*b^2*d*x^4*arctan(c*x)^2 + 1/4*a^2*d*x^4 + 1/10*I*(4
*x^5*arctan(c*x) - c*((c^2*x^4 - 2*x^2)/c^4 + 2*log(c^2*x^2 + 1)/c^6))*a*b*
c*d + 1/80*I*(4*x^5*arctan(c*x)^2 - x^5*log(c^2*x^2 + 1)^2 + 80*integrate(1
```

```
/80*(4*c^2*x^6*log(c^2*x^2 + 1) - 8*c*x^5*arctan(c*x) + 60*(c^2*x^6 + x^4)*
arctan(c*x)^2 + 5*(c^2*x^6 + x^4)*log(c^2*x^2 + 1)^2)/(c^2*x^2 + 1), x))*b^
2*c*d + 1/6*(3*x^4*arctan(c*x) - c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5
))*a*b*d - 1/12*(2*c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5)*arctan(c*x)
- (c^2*x^2 + 3*arctan(c*x)^2 - 4*log(c^2*x^2 + 1))/c^4)*b^2*d
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{80}\left(-4i\,b^2cdx^5 - 5\,b^2dx^4\right)\log\left(-\frac{cx+i}{cx-i}\right)^2 + \text{integral}\left(\frac{20i\,a^2c^3dx^6 + 20\,a^2c^2dx^5 + 20i\,a^2cdx^4 + 20\,a^2dx^3 - \left(20\,abc^3dx^6 - \right.}{20\left(c^2x\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d+I*c*d*x)*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/80*(-4*I*b^2*c*d*x^5 - 5*b^2*d*x^4)*log(-(c*x + I)/(c*x - I))^2 + integra
l(1/20*(20*I*a^2*c^3*d*x^6 + 20*a^2*c^2*d*x^5 + 20*I*a^2*c*d*x^4 + 20*a^2*d
*x^3 - (20*a*b*c^3*d*x^6 - (20*I*a*b + 4*b^2)*c^2*d*x^5 + 5*(4*a*b + I*b^2)
*c*d*x^4 - 20*I*a*b*d*x^3)*log(-(c*x + I)/(c*x - I)))/(c^2*x^2 + 1), x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(d+I*c*d*x)*(a+b*atan(c*x))**2,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (i\,cdx + d)(b\arctan(cx) + a)^2x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d+I*c*d*x)*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((I*c*d*x + d)*(b*arctan(c*x) + a)^2*x^3, x)
```

3.69 $\int x^2(d+icdx)\left(a+b\tan^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=255

$$-\frac{ib^2d\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{3c^3}+\frac{iabdx}{2c^2}-\frac{7id\left(a+b\tan^{-1}(cx)\right)^2}{12c^3}-\frac{2bd\log\left(\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{3c^3}+\frac{1}{4}icdx^4\left(a+b\tan^{-1}\right.$$

```
[Out] ((I/2)*a*b*d*x)/c^2 + (b^2*d*x)/(3*c^2) + ((I/12)*b^2*d*x^2)/c - (b^2*d*Arc
Tan[c*x])/(3*c^3) + ((I/2)*b^2*d*x*ArcTan[c*x])/c^2 - (b*d*x^2*(a + b*ArcTa
n[c*x]))/(3*c) - (I/6)*b*d*x^3*(a + b*ArcTan[c*x]) - (((7*I)/12)*d*(a + b*A
rcTan[c*x])^2)/c^3 + (d*x^3*(a + b*ArcTan[c*x])^2)/3 + (I/4)*c*d*x^4*(a + b
*ArcTan[c*x])^2 - (2*b*d*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(3*c^3) -
((I/3)*b^2*d*Log[1 + c^2*x^2])/c^3 - ((I/3)*b^2*d*PolyLog[2, 1 - 2/(1 + I*c
*x)])/c^3
```

Rubi [A] time = 0.491, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 14, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.609, Rules used = {4876, 4852, 4916, 321, 203, 4920, 4854, 2402, 2315, 266, 43, 4846, 260, 4884}

$$-\frac{ib^2d\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{3c^3}+\frac{iabdx}{2c^2}-\frac{7id\left(a+b\tan^{-1}(cx)\right)^2}{12c^3}-\frac{2bd\log\left(\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{3c^3}+\frac{1}{4}icdx^4\left(a+b\tan^{-1}\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + I*c*d*x)*(a + b*ArcTan[c*x])^2,x]
```


```
[Out] ((I/2)*a*b*d*x)/c^2 + (b^2*d*x)/(3*c^2) + ((I/12)*b^2*d*x^2)/c - (b^2*d*Arc
Tan[c*x])/(3*c^3) + ((I/2)*b^2*d*x*ArcTan[c*x])/c^2 - (b*d*x^2*(a + b*ArcTa
n[c*x]))/(3*c) - (I/6)*b*d*x^3*(a + b*ArcTan[c*x]) - (((7*I)/12)*d*(a + b*A
rcTan[c*x])^2)/c^3 + (d*x^3*(a + b*ArcTan[c*x])^2)/3 + (I/4)*c*d*x^4*(a + b
*ArcTan[c*x])^2 - (2*b*d*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(3*c^3) -
((I/3)*b^2*d*Log[1 + c^2*x^2])/c^3 - ((I/3)*b^2*d*PolyLog[2, 1 - 2/(1 + I*c
*x)])/c^3
```


Rule 4876

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
```

```
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rubi steps

$$\int x^2(d+icdx)\left(a+b\tan^{-1}(cx)\right)^2\,dx = \int \left(dx^2\left(a+b\tan^{-1}(cx)\right)^2+icdx^3\left(a+b\tan^{-1}(cx)\right)^2\right)dx$$
$$= d\int x^2\left(a+b\tan^{-1}(cx)\right)^2\,dx+(icd)\int x^3\left(a+b\tan^{-1}(cx)\right)^2\,dx$$
$$= \frac{1}{3}dx^3\left(a+b\tan^{-1}(cx)\right)^2+\frac{1}{4}icdx^4\left(a+b\tan^{-1}(cx)\right)^2-\frac{1}{3}(2bcd)\int\frac{x^3\left(a+b\tan\right.}{1+c^2}$$
$$= \frac{1}{3}dx^3\left(a+b\tan^{-1}(cx)\right)^2+\frac{1}{4}icdx^4\left(a+b\tan^{-1}(cx)\right)^2-\frac{1}{2}(ibd)\int x^2\left(a+b\tan^{-}\right.$$
$$= -\frac{bdx^2\left(a+b\tan^{-1}(cx)\right)}{3c}-\frac{1}{6}ibdx^3\left(a+b\tan^{-1}(cx)\right)-\frac{id\left(a+b\tan^{-1}(cx)\right)^2}{3c^3}+$$
$$= \frac{iabdx}{2c^2}+\frac{b^2dx}{3c^2}-\frac{bdx^2\left(a+b\tan^{-1}(cx)\right)}{3c}-\frac{1}{6}ibdx^3\left(a+b\tan^{-1}(cx)\right)-\frac{7id\left(a+b\right.}{}$$
$$= \frac{iabdx}{2c^2}+\frac{b^2dx}{3c^2}-\frac{b^2d\tan^{-1}(cx)}{3c^3}+\frac{ib^2dx\tan^{-1}(cx)}{2c^2}-\frac{bdx^2\left(a+b\tan^{-1}(cx)\right)}{3c}-\frac{1}{6}$$
$$= \frac{iabdx}{2c^2}+\frac{b^2dx}{3c^2}+\frac{ib^2dx^2}{12c}-\frac{b^2d\tan^{-1}(cx)}{3c^3}+\frac{ib^2dx\tan^{-1}(cx)}{2c^2}-\frac{bdx^2\left(a+b\tan^{-}\right.}{3c}$$

Mathematica [A] time = 0.591374, size = 241, normalized size = 0.95

$$\frac{id\left(4b^2\text{PolyLog}\left(2,-e^{2i\tan^{-1}(cx)}\right)+3a^2c^4x^4-4ia^2c^3x^3-2abc^3x^3+4iabc^2x^2-4iab\log\left(c^2x^2+1\right)+2b\tan^{-1}(cx)\left(a\left(3c\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(d + I*c*d*x)*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] ((I/12)*d*(b^2 + 6*a*b*c*x - (4*I)*b^2*c*x + (4*I)*a*b*c^2*x^2 + b^2*c^2*x^
2 - (4*I)*a^2*c^3*x^3 - 2*a*b*c^3*x^3 + 3*a^2*c^4*x^4 + b^2*(1 - (4*I)*c^3*
x^3 + 3*c^4*x^4)*ArcTan[c*x]^2 + 2*b*ArcTan[c*x]*(b*(2*I + 3*c*x + (2*I)*c^
2*x^2 - c^3*x^3) + a*(-3 - (4*I)*c^3*x^3 + 3*c^4*x^4) + (4*I)*b*Log[1 + E^(
(2*I)*ArcTan[c*x])]) - (4*I)*a*b*Log[1 + c^2*x^2] - 4*b^2*Log[1 + c^2*x^2]
+ 4*b^2*PolyLog[2, -E^((2*I)*ArcTan[c*x])]))/c^3
```

Maple [B] time = 0.094, size = 467, normalized size = 1.8

$$\frac{\frac{i}{12}db^2x^2}{c}+\frac{\frac{i}{2}b^2dx\arctan(cx)}{c^2}+\frac{dab\ln\left(c^2x^2+1\right)}{3c^3}+\frac{i}{2}cdab\arctan(cx)x^4+\frac{db^2\arctan(cx)\ln\left(c^2x^2+1\right)}{3c^3}-\frac{db^2\arctan(}{3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d+I*c*d*x)*(a+b*arctan(c*x))^2,x)
```

```
[Out] 1/12*I*b^2*d*x^2/c+1/2*I*b^2*d*x*arctan(c*x)/c^2+1/3/c^3*d*a*b*ln(c^2*x^2+1
)+1/2*I*c*d*a*b*arctan(c*x)*x^4+1/3/c^3*d*b^2*arctan(c*x)*ln(c^2*x^2+1)-1/3
/c*d*b^2*arctan(c*x)*x^2-1/6*I*d*a*b*x^3-1/6*I*d*b^2*arctan(c*x)*x^3+1/4*I*
c*d*a^2*x^4-1/6*I/c^3*d*b^2*dilog(-1/2*I*(c*x+I))-1/4*I/c^3*d*b^2*arctan(c*
x)^2-1/12*I/c^3*d*b^2*ln(c*x-I)^2+1/12*I/c^3*d*b^2*ln(c*x+I)^2+1/6*I/c^3*d*
b^2*dilog(1/2*I*(c*x-I))-1/3/c*d*a*b*x^2+2/3*d*a*b*arctan(c*x)*x^3+1/3*d*a^
2*x^3-1/3*b^2*d*arctan(c*x)/c^3+1/3*d*b^2*arctan(c*x)^2*x^3+1/2*I*a*b*d*x/c
^2-1/3*I*b^2*d*ln(c^2*x^2+1)/c^3+1/6*I/c^3*d*b^2*ln(c^2*x^2+1)*ln(c*x-I)+1/
3*b^2*d*x/c^2-1/6*I/c^3*d*b^2*ln(c^2*x^2+1)*ln(c*x+I)+1/4*I*c*d*b^2*arctan(
c*x)^2*x^4-1/2*I/c^3*d*a*b*arctan(c*x)+1/6*I/c^3*d*b^2*ln(c*x+I)*ln(1/2*I*(
c*x-I))-1/6*I/c^3*d*b^2*ln(c*x-I)*ln(-1/2*I*(c*x+I))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{4}i\,a^2cdx^4+\frac{1}{3}\,a^2dx^3+\frac{1}{6}i\left(3\,x^4\arctan(cx)-c\left(\frac{c^2x^3-3\,x}{c^4}+\frac{3\,\arctan(cx)}{c^5}\right)\right)abcd+\frac{1}{3}\left(2\,x^3\arctan(cx)-c\left(\frac{x^2}{c^2}-\frac{\log\left(c^2\right.}{c}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d+I*c*d*x)*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/4*I*a^2*c*d*x^4 + 1/3*a^2*d*x^3 + 1/6*I*(3*x^4*arctan(c*x) - c*((c^2*x^3
- 3*x)/c^4 + 3*arctan(c*x)/c^5))*a*b*c*d + 1/3*(2*x^3*arctan(c*x) - c*(x^2/
c^2 - log(c^2*x^2 + 1)/c^4))*a*b*d + 1/192*(12*I*b^2*c*d*x^4 + 16*b^2*d*x^3
)*arctan(c*x)^2 - 1/48*(3*b^2*c*d*x^4 - 4*I*b^2*d*x^3)*arctan(c*x)*log(c^2*
x^2 + 1) + 1/192*(-3*I*b^2*c*d*x^4 - 4*b^2*d*x^3)*log(c^2*x^2 + 1)^2 + I*in
tegrate(-1/48*(14*b^2*c^2*d*x^4*arctan(c*x) - 36*(b^2*c^3*d*x^5 + b^2*c*d*x
^3)*arctan(c*x)^2 - 3*(b^2*c^3*d*x^5 + b^2*c*d*x^3)*log(c^2*x^2 + 1)^2 - (3
*b^2*c^3*d*x^5 - 4*b^2*c*d*x^3 - 12*(b^2*c^2*d*x^4 + b^2*d*x^2)*arctan(c*x)
)*log(c^2*x^2 + 1))/(c^2*x^2 + 1), x) + integrate(1/48*(36*(b^2*c^2*d*x^4 +
 b^2*d*x^2)*arctan(c*x)^2 + 3*(b^2*c^2*d*x^4 + b^2*d*x^2)*log(c^2*x^2 + 1)^
```

```
2 + 2*(3*b^2*c^3*d*x^5 - 4*b^2*c*d*x^3)*arctan(c*x) + (7*b^2*c^2*d*x^4 + 12
*(b^2*c^3*d*x^5 + b^2*c*d*x^3)*arctan(c*x))*log(c^2*x^2 + 1))/(c^2*x^2 + 1)
, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{48}\left(-3i\,b^2cdx^4-4\,b^2dx^3\right)\log\left(-\frac{cx+i}{cx-i}\right)^2+\mathrm{integral}\left(\frac{12i\,a^2c^3dx^5+12\,a^2c^2dx^4+12i\,a^2cdx^3+12\,a^2dx^2-\left(12\,abc^3dx^5-\right.}{12\left(c^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d+I*c*d*x)*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/48*(-3*I*b^2*c*d*x^4 - 4*b^2*d*x^3)*log(-(c*x + I)/(c*x - I))^2 + integra
l(1/12*(12*I*a^2*c^3*d*x^5 + 12*a^2*c^2*d*x^4 + 12*I*a^2*c*d*x^3 + 12*a^2*d
*x^2 - (12*a*b*c^3*d*x^5 - (12*I*a*b + 3*b^2)*c^2*d*x^4 + 4*(3*a*b + I*b^2)
*c*d*x^3 - 12*I*a*b*d*x^2)*log(-(c*x + I)/(c*x - I)))/(c^2*x^2 + 1), x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d+I*c*d*x)*(a+b*atan(c*x))**2,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int(i\,cdx+d)(b\arctan(cx)+a)^2x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d+I*c*d*x)*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((I*c*d*x + d)*(b*arctan(c*x) + a)^2*x^2, x)
```

3.70 $\int x(d+icdx)\left(a+b\tan^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=211

$$\frac{b^2 d\,\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{3c^2}+\frac{5d\left(a+b\tan^{-1}(cx)\right)^2}{6c^2}-\frac{2ibd\log\left(\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{3c^2}+\frac{1}{3}icdx^3\left(a+b\tan^{-1}(cx)\right)^2+$$

```
[Out] -((a*b*d*x)/c) + ((I/3)*b^2*d*x)/c - ((I/3)*b^2*d*ArcTan[c*x])/c^2 - (b^2*d
*x*ArcTan[c*x])/c - (I/3)*b*d*x^2*(a + b*ArcTan[c*x]) + (5*d*(a + b*ArcTan[
c*x])^2)/(6*c^2) + (d*x^2*(a + b*ArcTan[c*x])^2)/2 + (I/3)*c*d*x^3*(a + b*A
rcTan[c*x])^2 - (((2*I)/3)*b*d*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/c^2
+ (b^2*d*Log[1 + c^2*x^2])/(2*c^2) + (b^2*d*PolyLog[2, 1 - 2/(1 + I*c*x)])/
(3*c^2)
```

Rubi [A] time = 0.358016, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 12, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {4876, 4852, 4916, 4846, 260, 4884, 321, 203, 4920, 4854, 2402, 2315}

$$\frac{b^2 d\,\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{3c^2}+\frac{5d\left(a+b\tan^{-1}(cx)\right)^2}{6c^2}-\frac{2ibd\log\left(\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{3c^2}+\frac{1}{3}icdx^3\left(a+b\tan^{-1}(cx)\right)^2+$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + I*c*d*x)*(a + b*ArcTan[c*x])^2,x]
```


```
[Out] -((a*b*d*x)/c) + ((I/3)*b^2*d*x)/c - ((I/3)*b^2*d*ArcTan[c*x])/c^2 - (b^2*d
*x*ArcTan[c*x])/c - (I/3)*b*d*x^2*(a + b*ArcTan[c*x]) + (5*d*(a + b*ArcTan[
c*x])^2)/(6*c^2) + (d*x^2*(a + b*ArcTan[c*x])^2)/2 + (I/3)*c*d*x^3*(a + b*A
rcTan[c*x])^2 - (((2*I)/3)*b*d*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/c^2
+ (b^2*d*Log[1 + c^2*x^2])/(2*c^2) + (b^2*d*PolyLog[2, 1 - 2/(1 + I*c*x)])/
(3*c^2)
```


Rule 4876

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int x(d+icdx)\left(a+b\tan^{-1}(cx)\right)^2\,dx = \int \left(dx\left(a+b\tan^{-1}(cx)\right)^2+icdx^2\left(a+b\tan^{-1}(cx)\right)^2\right)dx$$

$$= d\int x\left(a+b\tan^{-1}(cx)\right)^2\,dx+(icd)\int x^2\left(a+b\tan^{-1}(cx)\right)^2\,dx$$

$$= \frac{1}{2}dx^2\left(a+b\tan^{-1}(cx)\right)^2+\frac{1}{3}icdx^3\left(a+b\tan^{-1}(cx)\right)^2-(bcd)\int \frac{x^2\left(a+b\tan^{-1}(cx\right.}{1+c^2x^2}$$

$$= \frac{1}{2}dx^2\left(a+b\tan^{-1}(cx)\right)^2+\frac{1}{3}icdx^3\left(a+b\tan^{-1}(cx)\right)^2-\frac{1}{3}(2ibd)\int x\left(a+b\tan^{-1}(\right.$$

$$= -\frac{abdx}{c}-\frac{1}{3}ibdx^2\left(a+b\tan^{-1}(cx)\right)+\frac{5d\left(a+b\tan^{-1}(cx)\right)^2}{6c^2}+\frac{1}{2}dx^2\left(a+b\tan^{-1}(c\right.$$

$$= -\frac{abdx}{c}+\frac{ib^2dx}{3c}-\frac{b^2dx\tan^{-1}(cx)}{c}-\frac{1}{3}ibdx^2\left(a+b\tan^{-1}(cx)\right)+\frac{5d\left(a+b\tan^{-1}(c\right.}{6c^2}$$

$$= -\frac{abdx}{c}+\frac{ib^2dx}{3c}-\frac{ib^2d\tan^{-1}(cx)}{3c^2}-\frac{b^2dx\tan^{-1}(cx)}{c}-\frac{1}{3}ibdx^2\left(a+b\tan^{-1}(cx)\right)+$$

$$= -\frac{abdx}{c}+\frac{ib^2dx}{3c}-\frac{ib^2d\tan^{-1}(cx)}{3c^2}-\frac{b^2dx\tan^{-1}(cx)}{c}-\frac{1}{3}ibdx^2\left(a+b\tan^{-1}(cx)\right)+$$

Mathematica [A] time = 0.496672, size = 208, normalized size = 0.99

$$\frac{d\left(-2b^2\text{PolyLog}\left(2,-e^{2i\tan^{-1}(cx)}\right)+2ia^2c^3x^3+3a^2c^2x^2-2iabc^2x^2+2iab\log\left(c^2x^2+1\right)+2b\tan^{-1}(cx)\left(a\left(2ic^3x^3+3c^2x\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*(d + I*c*d*x)*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] (d*(-6*a*b*c*x + (2*I)*b^2*c*x + 3*a^2*c^2*x^2 - (2*I)*a*b*c^2*x^2 + (2*I)*
a^2*c^3*x^3 + b^2*(1 + 3*c^2*x^2 + (2*I)*c^3*x^3)*ArcTan[c*x]^2 + 2*b*ArcTa
n[c*x]*((-I)*b*(1 - (3*I)*c*x + c^2*x^2) + a*(3 + 3*c^2*x^2 + (2*I)*c^3*x^3
) - (2*I)*b*Log[1 + E^((2*I)*ArcTan[c*x])]) + (2*I)*a*b*Log[1 + c^2*x^2] +
3*b^2*Log[1 + c^2*x^2] - 2*b^2*PolyLog[2, -E^((2*I)*ArcTan[c*x])]))/(6*c^2)
```

Maple [B] time = 0.093, size = 416, normalized size = 2.

$$\frac{\frac{i}{3}dab\ln\left(c^2x^2+1\right)}{c^2}+\frac{db^2\ln\left(c^2x^2+1\right)}{2c^2}+\frac{2i}{3}cdab\arctan(cx)x^3-\frac{i}{3}dabx^2+\frac{i}{3}cda^2x^3-\frac{i}{3}db^2\arctan(cx)x^2+\frac{dab\arctan}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d+I*c*d*x)*(a+b*arctan(c*x))^2,x)
```

```
[Out] 1/3*I/c^2*d*a*b*ln(c^2*x^2+1)+1/2*b^2*d*ln(c^2*x^2+1)/c^2+2/3*I*c*d*a*b*arc
tan(c*x)*x^3-1/3*I*d*a*b*x^2+1/3*I*c*d*a^2*x^3-1/3*I*d*b^2*arctan(c*x)*x^2+
1/c^2*d*a*b*arctan(c*x)+1/6/c^2*d*b^2*ln(c*x+I)*ln(c^2*x^2+1)-1/6/c^2*d*b^2
*ln(c*x+I)*ln(1/2*I*(c*x-I))-1/6/c^2*d*b^2*ln(c*x-I)*ln(c^2*x^2+1)+1/6/c^2*
d*b^2*ln(c*x-I)*ln(-1/2*I*(c*x+I))+d*a*b*arctan(c*x)*x^2+1/3*I/c^2*d*b^2*ar
ctan(c*x)*ln(c^2*x^2+1)+1/2*d*a^2*x^2+1/2/c^2*d*b^2*arctan(c*x)^2+1/12/c^2*
d*b^2*ln(c*x-I)^2+1/2*d*b^2*arctan(c*x)^2*x^2+1/6/c^2*d*b^2*dilog(-1/2*I*(c
*x+I))-1/12/c^2*d*b^2*ln(c*x+I)^2-1/6/c^2*d*b^2*dilog(1/2*I*(c*x-I))+1/3*I*
b^2*d*x/c-1/3*I*b^2*d*arctan(c*x)/c^2-a*b*d*x/c-b^2*d*x*arctan(c*x)/c+1/3*I
*c*d*b^2*arctan(c*x)^2*x^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{3} i a^2 c d x^3 + \frac{1}{2} b^2 d x^2 \arctan(cx)^2 + \frac{1}{3} i \left(2 x^3 \arctan(cx) - c\left(\frac{x^2}{c^2} - \frac{\log(c^2x^2+1)}{c^4}\right)\right) abcd + \frac{1}{48} i \left(4 x^3 \arctan(cx)^2 - x^3 \text{lo}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d+I*c*d*x)*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/3*I*a^2*c*d*x^3 + 1/2*b^2*d*x^2*arctan(c*x)^2 + 1/3*I*(2*x^3*arctan(c*x)
- c*(x^2/c^2 - log(c^2*x^2 + 1)/c^4))*a*b*c*d + 1/48*I*(4*x^3*arctan(c*x)^2
 - x^3*log(c^2*x^2 + 1)^2 + 48*integrate(1/48*(4*c^2*x^4*log(c^2*x^2 + 1) -
 8*c*x^3*arctan(c*x) + 36*(c^2*x^4 + x^2)*arctan(c*x)^2 + 3*(c^2*x^4 + x^2)
*log(c^2*x^2 + 1)^2)/(c^2*x^2 + 1), x))*b^2*c*d + 1/2*a^2*d*x^2 + (x^2*arct
an(c*x) - c*(x/c^2 - arctan(c*x)/c^3))*a*b*d - 1/2*(2*c*(x/c^2 - arctan(c*x
)/c^3)*arctan(c*x) + (arctan(c*x)^2 - log(c^2*x^2 + 1))/c^2)*b^2*d
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{24}\left(-2i b^2 c d x^3 - 3 b^2 d x^2\right)\log\left(-\frac{cx+i}{cx-i}\right)^2 + \text{integral}\left(\frac{6i a^2c^3dx^4 + 6 a^2c^2dx^3 + 6i a^2cdx^2 + 6 a^2dx - \left(6 abc^3dx^4 - \left(6i ab\right.\right.}{6\left(c^2x^2+1\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d+I*c*d*x)*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/24*(-2*I*b^2*c*d*x^3 - 3*b^2*d*x^2)*log(-(c*x + I)/(c*x - I))^2 + integra
l(1/6*(6*I*a^2*c^3*d*x^4 + 6*a^2*c^2*d*x^3 + 6*I*a^2*c*d*x^2 + 6*a^2*d*x -
(6*a*b*c^3*d*x^4 - (6*I*a*b + 2*b^2)*c^2*d*x^3 + 3*(2*a*b + I*b^2)*c*d*x^2
- 6*I*a*b*d*x)*log(-(c*x + I)/(c*x - I)))/(c^2*x^2 + 1), x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d+I*c*d*x)*(a+b*atan(c*x))**2,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (icdx + d)(b \arctan (cx) + a)^2 x \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d+I*c*d*x)*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((I*c*d*x + d)*(b*arctan(c*x) + a)^2*x, x)
```

3.71 $\int (d+icdx)\left(a+b\tan^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=130

$$-\frac{ib^2d\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{c}-\frac{id(1+icx)^2\left(a+b\tan^{-1}(cx)\right)^2}{2c}+\frac{2bd\log\left(\frac{2}{1-icx}\right)\left(a+b\tan^{-1}(cx)\right)}{c}-iabdx+\frac{ib^2d\log\left(c\right.}{2c}$$

```
[Out] (-I)*a*b*d*x - I*b^2*d*x*ArcTan[c*x] - ((I/2)*d*(1 + I*c*x)^2*(a + b*ArcTan
[c*x])^2)/c + (2*b*d*(a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/c + ((I/2)*b^2
*d*Log[1 + c^2*x^2])/c - (I*b^2*d*PolyLog[2, 1 - 2/(1 - I*c*x)])/c
```

Rubi [A] time = 0.121163, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {4864, 4846, 260, 1586, 4854, 2402, 2315}

$$-\frac{ib^2d\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{c}-\frac{id(1+icx)^2\left(a+b\tan^{-1}(cx)\right)^2}{2c}+\frac{2bd\log\left(\frac{2}{1-icx}\right)\left(a+b\tan^{-1}(cx)\right)}{c}-iabdx+\frac{ib^2d\log\left(c\right.}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + I*c*d*x)*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] (-I)*a*b*d*x - I*b^2*d*x*ArcTan[c*x] - ((I/2)*d*(1 + I*c*x)^2*(a + b*ArcTan
[c*x])^2)/c + (2*b*d*(a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/c + ((I/2)*b^2
*d*Log[1 + c^2*x^2])/c - (I*b^2*d*PolyLog[2, 1 - 2/(1 - I*c*x)])/c
```

Rule 4864

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x])^p)/(e*(q + 1)), x] - D
ist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int (d+icdx)\left(a+b\tan^{-1}(cx)\right)^2\,dx = -\frac{id(1+icx)^2\left(a+b\tan^{-1}(cx)\right)^2}{2c} + \frac{(ib)\int\left(-d^2\left(a+b\tan^{-1}(cx)\right)-\frac{2i\left(id^2-cd^2x\right)\left(a+b\,\mathrm{ta}\right.}{1+c^2x^2}\right.}{d}$$

$$= -\frac{id(1+icx)^2\left(a+b\tan^{-1}(cx)\right)^2}{2c} + \frac{(2b)\int\frac{\left(id^2-cd^2x\right)\left(a+b\tan^{-1}(cx)\right)}{1+c^2x^2}\,dx}{d} - (ibd)\int\left(a+\right.$$

$$= -iabdx - \frac{id(1+icx)^2\left(a+b\tan^{-1}(cx)\right)^2}{2c} + \frac{(2b)\int\frac{a+b\tan^{-1}(cx)}{-\frac{i}{d^2}-\frac{cx}{d^2}}\,dx}{d} - \left(ib^2d\right)\int\tan^{-}$$

$$= -iabdx - ib^2dx\tan^{-1}(cx) - \frac{id(1+icx)^2\left(a+b\tan^{-1}(cx)\right)^2}{2c} + \frac{2bd\left(a+b\tan^{-1}(cx)\right.}{c}$$

$$= -iabdx - ib^2dx\tan^{-1}(cx) - \frac{id(1+icx)^2\left(a+b\tan^{-1}(cx)\right)^2}{2c} + \frac{2bd\left(a+b\tan^{-1}(cx)\right.}{c}$$

$$= -iabdx - ib^2dx\tan^{-1}(cx) - \frac{id(1+icx)^2\left(a+b\tan^{-1}(cx)\right)^2}{2c} + \frac{2bd\left(a+b\tan^{-1}(cx)\right.}{c}$$

Mathematica [A] time = 0.252557, size = 151, normalized size = 1.16

$$\frac{id\left(-2b^2\mathrm{PolyLog}\left(2,-e^{2i\tan^{-1}(cx)}\right)+a^2c^2x^2-2ia^2cx+2iab\log\left(c^2x^2+1\right)+2b\tan^{-1}(cx)\left(ac^2x^2-2iacx+a-bcx-2i\right.\right.}{2c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + I*c*d*x)*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] ((I/2)*d*((-2*I)*a^2*c*x - 2*a*b*c*x + a^2*c^2*x^2 + b^2*(-I + c*x)^2*ArcTa
n[c*x]^2 + 2*b*ArcTan[c*x]*(a - (2*I)*a*c*x - b*c*x + a*c^2*x^2 - (2*I)*b*L
og[1 + E^((2*I)*ArcTan[c*x])]) + (2*I)*a*b*Log[1 + c^2*x^2] + b^2*Log[1 + c
^2*x^2] - 2*b^2*PolyLog[2, -E^((2*I)*ArcTan[c*x])]))/c
```

Maple [B] time = 0.087, size = 367, normalized size = 2.8

$$da^2x + \frac{\frac{i}{2}db^2\left(\arctan(cx)\right)^2}{c} + db^2\left(\arctan(cx)\right)^2x + \frac{\frac{i}{2}db^2\ln\left(c^2x^2+1\right)}{c} - iabdx + \frac{\frac{i}{4}db^2\left(\ln(cx-i)\right)^2}{c} - \frac{db^2\arctan(cx)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)*(a+b*arctan(c*x))^2,x)
```

```
[Out] d*a^2*x+1/2*I/c*d*b^2*arctan(c*x)^2+d*b^2*arctan(c*x)^2*x+1/2*I*b^2*d*ln(c^
2*x^2+1)/c-I*a*b*d*x+1/4*I/c*d*b^2*ln(c*x-I)^2-1/c*d*b^2*arctan(c*x)*ln(c^2
*x^2+1)-1/4*I/c*d*b^2*ln(c*x+I)^2-1/2*I/c*d*b^2*ln(c*x+I)*ln(1/2*I*(c*x-I))
-1/2*I/c*d*b^2*dilog(1/2*I*(c*x-I))+1/2*I*c*d*b^2*arctan(c*x)^2*x^2+1/2*I*c
*d*a^2*x^2+I*c*d*a*b*arctan(c*x)*x^2-I*b^2*d*x*arctan(c*x)+1/2*I/c*d*b^2*di
log(-1/2*I*(c*x+I))+1/2*I/c*d*b^2*ln(c*x-I)*ln(-1/2*I*(c*x+I))+I/c*d*a*b*ar
ctan(c*x)+1/2*I/c*d*b^2*ln(c^2*x^2+1)*ln(c*x+I)+2*d*a*b*arctan(c*x)*x-1/2*I
/c*d*b^2*ln(c^2*x^2+1)*ln(c*x-I)-1/c*d*a*b*ln(c^2*x^2+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

```
[Out] 4*b^2*c^3*d*integrate(1/16*x^3*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^2 + 1),
x) + 4*b^2*c^3*d*integrate(1/16*x^3*arctan(c*x)/(c^2*x^2 + 1), x) + 1/2*I*a
^2*c*d*x^2 + 12*b^2*c^2*d*integrate(1/16*x^2*arctan(c*x)^2/(c^2*x^2 + 1), x
) + b^2*c^2*d*integrate(1/16*x^2*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) + 6*b
^2*c^2*d*integrate(1/16*x^2*log(c^2*x^2 + 1)/(c^2*x^2 + 1), x) + I*(x^2*arc
tan(c*x) - c*(x/c^2 - arctan(c*x)/c^3))*a*b*c*d + 1/4*b^2*d*arctan(c*x)^3/c
 + 4*b^2*c*d*integrate(1/16*x*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^2 + 1), x
) - 8*b^2*c*d*integrate(1/16*x*arctan(c*x)/(c^2*x^2 + 1), x) + a^2*d*x + b^
2*d*integrate(1/16*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) + (2*c*x*arctan(c*x
) - log(c^2*x^2 + 1))*a*b*d/c + 1/32*(4*I*b^2*c*d*x^2 + 8*b^2*d*x)*arctan(c
*x)^2 - 1/8*(b^2*c*d*x^2 - 2*I*b^2*d*x)*arctan(c*x)*log(c^2*x^2 + 1) + 1/32
*(-I*b^2*c*d*x^2 - 2*b^2*d*x)*log(c^2*x^2 + 1)^2 + I*integrate(-1/16*(12*b^
2*c^2*d*x^2*arctan(c*x) - 12*(b^2*c^3*d*x^3 + b^2*c*d*x)*arctan(c*x)^2 - (b
^2*c^3*d*x^3 + b^2*c*d*x)*log(c^2*x^2 + 1)^2 - 2*(b^2*c^3*d*x^3 - 2*b^2*c*d
*x - 2*(b^2*c^2*d*x^2 + b^2*d)*arctan(c*x))*log(c^2*x^2 + 1))/(c^2*x^2 + 1)
, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{8}\left(-i\,b^2cdx^2 - 2\,b^2dx\right)\log\left(-\frac{cx+i}{cx-i}\right)^2 + \text{integral}\left(\frac{2i\,a^2c^3dx^3 + 2\,a^2c^2dx^2 + 2i\,a^2cdx + 2\,a^2d - \left(2\,abc^3dx^3 - \left(2i\,ab + b^2\right)c^2\right.}{2\left(c^2x^2+1\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/8*(-I*b^2*c*d*x^2 - 2*b^2*d*x)*log(-(c*x + I)/(c*x - I))^2 + integral(1/2
*(2*I*a^2*c^3*d*x^3 + 2*a^2*c^2*d*x^2 + 2*I*a^2*c*d*x + 2*a^2*d - (2*a*b*c^
3*d*x^3 - (2*I*a*b + b^2)*c^2*d*x^2 + 2*(a*b + I*b^2)*c*d*x - 2*I*a*b*d)*lo
g(-(c*x + I)/(c*x - I)))/(c^2*x^2 + 1), x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)*(a+b*atan(c*x))**2,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (icdx + d)(b \arctan(cx) + a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((I*c*d*x + d)*(b*arctan(c*x) + a)^2, x)
```

3.72 $\int \frac{(d+icdx)\left(a+b\tan^{-1}(cx)\right)^2}{x}\,dx$

Optimal. Leaf size=216

$$-ibd\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)+ibd\text{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)+b^2(-d)\text{PolyLog}\left(2,\right.$$

```
[Out] -(d*(a + b*ArcTan[c*x])^2) + I*c*d*x*(a + b*ArcTan[c*x])^2 + 2*d*(a + b*Arc
Tan[c*x])^2*ArcTanh[1 - 2/(1 + I*c*x)] + (2*I)*b*d*(a + b*ArcTan[c*x])*Log[
2/(1 + I*c*x)] - b^2*d*PolyLog[2, 1 - 2/(1 + I*c*x)] - I*b*d*(a + b*ArcTan[
c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)] + I*b*d*(a + b*ArcTan[c*x])*PolyLog[2,
-1 + 2/(1 + I*c*x)] - (b^2*d*PolyLog[3, 1 - 2/(1 + I*c*x)])/2 + (b^2*d*Poly
Log[3, -1 + 2/(1 + I*c*x)])/2
```

Rubi [A] time = 0.420807, antiderivative size = 216, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {4876, 4846, 4920, 4854, 2402, 2315, 4850, 4988, 4884, 4994, 6610}

$$-ibd\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)+ibd\text{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)+b^2(-d)\text{PolyLog}\left(2,\right.$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)*(a + b*ArcTan[c*x])^2)/x,x]
```


```
[Out] -(d*(a + b*ArcTan[c*x])^2) + I*c*d*x*(a + b*ArcTan[c*x])^2 + 2*d*(a + b*Arc
Tan[c*x])^2*ArcTanh[1 - 2/(1 + I*c*x)] + (2*I)*b*d*(a + b*ArcTan[c*x])*Log[
2/(1 + I*c*x)] - b^2*d*PolyLog[2, 1 - 2/(1 + I*c*x)] - I*b*d*(a + b*ArcTan[
c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)] + I*b*d*(a + b*ArcTan[c*x])*PolyLog[2,
-1 + 2/(1 + I*c*x)] - (b^2*d*PolyLog[3, 1 - 2/(1 + I*c*x)])/2 + (b^2*d*Poly
Log[3, -1 + 2/(1 + I*c*x)])/2
```


Rule 4876

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 4850

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTan[c*x])^(p - 1)*ArcTanh[1 - 2/(1 + I*c*x)])/(1 + c^2*x^2), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4988

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTan[c*x])^p)/(d + e
*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTan[c*x])^p)/(d + e*x^2
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{(d+icdx)\left(a+b\tan^{-1}(cx)\right)^2}{x}\,dx = \int \left(icd\left(a+b\tan^{-1}(cx)\right)^2+\frac{d\left(a+b\tan^{-1}(cx)\right)^2}{x}\right)dx$$

$$= d\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x}\,dx+(icd)\int \left(a+b\tan^{-1}(cx)\right)^2\,dx$$

$$= icdx\left(a+b\tan^{-1}(cx)\right)^2+2d\left(a+b\tan^{-1}(cx)\right)^2\tanh^{-1}\left(1-\frac{2}{1+icx}\right)-(4bcd)\int \frac{(a}{}$$

$$= -d\left(a+b\tan^{-1}(cx)\right)^2+icdx\left(a+b\tan^{-1}(cx)\right)^2+2d\left(a+b\tan^{-1}(cx)\right)^2\tanh^{-1}\left(1-\right.$$

$$= -d\left(a+b\tan^{-1}(cx)\right)^2+icdx\left(a+b\tan^{-1}(cx)\right)^2+2d\left(a+b\tan^{-1}(cx)\right)^2\tanh^{-1}\left(1-\right.$$

$$= -d\left(a+b\tan^{-1}(cx)\right)^2+icdx\left(a+b\tan^{-1}(cx)\right)^2+2d\left(a+b\tan^{-1}(cx)\right)^2\tanh^{-1}\left(1-\right.$$

$$= -d\left(a+b\tan^{-1}(cx)\right)^2+icdx\left(a+b\tan^{-1}(cx)\right)^2+2d\left(a+b\tan^{-1}(cx)\right)^2\tanh^{-1}\left(1-\right.$$

Mathematica [A] time = 0.453498, size = 272, normalized size = 1.26

$$d\left(iab(\text{PolyLog}(2,-icx)-\text{PolyLog}(2,icx))+b^2\left(\text{PolyLog}\left(2,-e^{2i\tan^{-1}(cx)}\right)+\tan^{-1}(cx)\left((1+icx)\tan^{-1}(cx)+2i\log\left(\right.\right.\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + I*c*d*x)*(a + b*ArcTan[c*x])^2)/x,x]
```

```
[Out] d*(I*a^2*c*x + a^2*Log[c*x] + I*a*b*(2*c*x*ArcTan[c*x] - Log[1 + c^2*x^2])
+ b^2*(ArcTan[c*x]*((1 + I*c*x)*ArcTan[c*x] + (2*I)*Log[1 + E^((2*I)*ArcTan
[c*x])]) + PolyLog[2, -E^((2*I)*ArcTan[c*x])]) + I*a*b*(PolyLog[2, (-I)*c*x
] - PolyLog[2, I*c*x]) + b^2*((-I/24)*Pi^3 + ((2*I)/3)*ArcTan[c*x]^3 + ArcT
an[c*x]^2*Log[1 - E^((-2*I)*ArcTan[c*x])] - ArcTan[c*x]^2*Log[1 + E^((2*I)*
ArcTan[c*x])] + I*ArcTan[c*x]*PolyLog[2, E^((-2*I)*ArcTan[c*x])] + I*ArcTan
[c*x]*PolyLog[2, -E^((2*I)*ArcTan[c*x])] + PolyLog[3, E^((-2*I)*ArcTan[c*x]
)]/2 - PolyLog[3, -E^((2*I)*ArcTan[c*x])]/2))
```

Maple [C] time = 0.599, size = 7034, normalized size = 32.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)*(a+b*arctan(c*x))^2/x,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)*(a+b*arctan(c*x))^2/x,x, algorithm="maxima")
```

```
[Out] 1/4*I*b^2*c*d*x*arctan(c*x)^2 + 12*I*b^2*c^3*d*integrate(1/16*x^3*arctan(c*
x)^2/(c^2*x^3 + x), x) + 4*b^2*c^3*d*integrate(1/16*x^3*arctan(c*x)*log(c^2
*x^2 + 1)/(c^2*x^3 + x), x) + I*b^2*c^3*d*integrate(1/16*x^3*log(c^2*x^2 +
```

```
1)^2/(c^2*x^3 + x), x) + 8*b^2*c^3*d*integrate(1/16*x^3*arctan(c*x)/(c^2*x^
3 + x), x) + 4*I*b^2*c^3*d*integrate(1/16*x^3*log(c^2*x^2 + 1)/(c^2*x^3 + x
), x) - 1/4*b^2*c*d*x*arctan(c*x)*log(c^2*x^2 + 1) - 1/16*I*b^2*c*d*x*log(c
^2*x^2 + 1)^2 + 1/4*I*b^2*d*arctan(c*x)^3 + 12*b^2*c^2*d*integrate(1/16*x^2
*arctan(c*x)^2/(c^2*x^3 + x), x) - 4*I*b^2*c^2*d*integrate(1/16*x^2*arctan(
c*x)*log(c^2*x^2 + 1)/(c^2*x^3 + x), x) + 32*a*b*c^2*d*integrate(1/16*x^2*a
rctan(c*x)/(c^2*x^3 + x), x) - 8*I*b^2*c^2*d*integrate(1/16*x^2*arctan(c*x)
/(c^2*x^3 + x), x) + 1/96*b^2*d*log(c^2*x^2 + 1)^3 + I*a^2*c*d*x + 4*b^2*c*
d*integrate(1/16*x*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^3 + x), x) + I*b^2*c
*d*integrate(1/16*x*log(c^2*x^2 + 1)^2/(c^2*x^3 + x), x) + 1/16*b^2*d*log(c
^2*x^2 + 1)^2 + I*(2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*a*b*d + 12*b^2*d*i
ntegrate(1/16*arctan(c*x)^2/(c^2*x^3 + x), x) - 4*I*b^2*d*integrate(1/16*ar
ctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^3 + x), x) + b^2*d*integrate(1/16*log(c^2
*x^2 + 1)^2/(c^2*x^3 + x), x) + 32*a*b*d*integrate(1/16*arctan(c*x)/(c^2*x^
3 + x), x) + a^2*d*log(x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{4i\,a^2cdx + 4\,a^2d + \left(-i\,b^2cdx - b^2d\right)\log\left(-\frac{cx+i}{cx-i}\right)^2 - 4\,(abcdx - i\,abd)\log\left(-\frac{cx+i}{cx-i}\right)}{4\,x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)*(a+b*arctan(c*x))^2/x,x, algorithm="fricas")
```

```
[Out] integral(1/4*(4*I*a^2*c*d*x + 4*a^2*d + (-I*b^2*c*d*x - b^2*d)*log(-(c*x +
I)/(c*x - I))^2 - 4*(a*b*c*d*x - I*a*b*d)*log(-(c*x + I)/(c*x - I)))/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d\left(\int\frac{a^2}{x}\,dx + \int ia^2c\,dx + \int\frac{b^2\,\mathrm{atan}^2\,(cx)}{x}\,dx + \int ib^2c\,\mathrm{atan}^2\,(cx)\,dx + \int\frac{2ab\,\mathrm{atan}\,(cx)}{x}\,dx + \int 2iabc\,\mathrm{atan}\,(cx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)*(a+b*atan(c*x))**2/x,x)
```

```
[Out] d*(Integral(a**2/x, x) + Integral(I*a**2*c, x) + Integral(b**2*atan(c*x)**2
/x, x) + Integral(I*b**2*c*atan(c*x)**2, x) + Integral(2*a*b*atan(c*x)/x, x
```

```
) + Integral(2*I*a*b*c*atan(c*x), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(icdx + d)(b\arctan(cx) + a)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)*(a+b*arctan(c*x))^2/x,x, algorithm="giac")
```

```
[Out] integrate((I*c*d*x + d)*(b*arctan(c*x) + a)^2/x, x)
```

3.73 $\int \frac{(d+icdx)\left(a+b\tan^{-1}(cx)\right)^2}{x^2} dx$

Optimal. Leaf size=228

$$bcd\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)-bcd\text{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)-ib^2cd\text{PolyLog}\left(2,-1\right.$$

```
[Out] (-I)*c*d*(a + b*ArcTan[c*x])^2 - (d*(a + b*ArcTan[c*x])^2)/x + (2*I)*c*d*(a
 + b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1 + I*c*x)] + 2*b*c*d*(a + b*ArcTan[c*x]
)*Log[2 - 2/(1 - I*c*x)] - I*b^2*c*d*PolyLog[2, -1 + 2/(1 - I*c*x)] + b*c*d
*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)] - b*c*d*(a + b*ArcTan[c*
x])*PolyLog[2, -1 + 2/(1 + I*c*x)] - (I/2)*b^2*c*d*PolyLog[3, 1 - 2/(1 + I*
c*x)] + (I/2)*b^2*c*d*PolyLog[3, -1 + 2/(1 + I*c*x)]
```

Rubi [A] time = 0.466255, antiderivative size = 228, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {4876, 4852, 4924, 4868, 2447, 4850, 4988, 4884, 4994, 6610}

$$bcd\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)-bcd\text{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)-ib^2cd\text{PolyLog}\left(2,-1\right.$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)*(a + b*ArcTan[c*x])^2)/x^2,x]
```


```
[Out] (-I)*c*d*(a + b*ArcTan[c*x])^2 - (d*(a + b*ArcTan[c*x])^2)/x + (2*I)*c*d*(a
 + b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1 + I*c*x)] + 2*b*c*d*(a + b*ArcTan[c*x]
)*Log[2 - 2/(1 - I*c*x)] - I*b^2*c*d*PolyLog[2, -1 + 2/(1 - I*c*x)] + b*c*d
*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)] - b*c*d*(a + b*ArcTan[c*
x])*PolyLog[2, -1 + 2/(1 + I*c*x)] - (I/2)*b^2*c*d*PolyLog[3, 1 - 2/(1 + I*
c*x)] + (I/2)*b^2*c*d*PolyLog[3, -1 + 2/(1 + I*c*x)]
```


Rule 4876

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4850

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTan[c*x])^(p - 1)*ArcTanh[1 - 2/(1 + I*c*x)])/(1 + c^2*x^2), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4988

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTan[c*x])^p)/(d + e
*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTan[c*x])^p)/(d + e*x^2
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{(d+icdx)\left(a+b\tan^{-1}(cx)\right)^2}{x^2}\,dx = \int\left(\frac{d\left(a+b\tan^{-1}(cx)\right)^2}{x^2}+\frac{icd\left(a+b\tan^{-1}(cx)\right)^2}{x}\right)dx$$

$$= d\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^2}\,dx + (icd)\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x}\,dx$$

$$= -\frac{d\left(a+b\tan^{-1}(cx)\right)^2}{x} + 2icd\left(a+b\tan^{-1}(cx)\right)^2\tanh^{-1}\left(1-\frac{2}{1+icx}\right) + (2bcd)\int$$

$$= -icd\left(a+b\tan^{-1}(cx)\right)^2 - \frac{d\left(a+b\tan^{-1}(cx)\right)^2}{x} + 2icd\left(a+b\tan^{-1}(cx)\right)^2\tanh^{-1}\left(1\right.$$

$$= -icd\left(a+b\tan^{-1}(cx)\right)^2 - \frac{d\left(a+b\tan^{-1}(cx)\right)^2}{x} + 2icd\left(a+b\tan^{-1}(cx)\right)^2\tanh^{-1}\left(1\right.$$

$$= -icd\left(a+b\tan^{-1}(cx)\right)^2 - \frac{d\left(a+b\tan^{-1}(cx)\right)^2}{x} + 2icd\left(a+b\tan^{-1}(cx)\right)^2\tanh^{-1}\left(1\right.$$

Mathematica [A] time = 0.411697, size = 289, normalized size = 1.27

$$id\left(iabcx(\text{PolyLog}(2,-icx)-\text{PolyLog}(2,icx))+ib^2\left(icx\left(\tan^{-1}(cx)^2+\text{PolyLog}\left(2,e^{2i\tan^{-1}(cx)}\right)\right)+\tan^{-1}(cx)^2-2cx\tan\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + I*c*d*x)*(a + b*ArcTan[c*x])^2)/x^2,x]
```

```
[Out] (I*d*(I*a^2 + a^2*c*x*Log[x] + I*a*b*(2*ArcTan[c*x] + c*x*(-2*Log[c*x] + Lo
g[1 + c^2*x^2])) + I*b^2*(ArcTan[c*x]^2 - 2*c*x*ArcTan[c*x]*Log[1 - E^((2*I
)*ArcTan[c*x])] + I*c*x*(ArcTan[c*x]^2 + PolyLog[2, E^((2*I)*ArcTan[c*x])])
) + I*a*b*c*x*(PolyLog[2, (-I)*c*x] - PolyLog[2, I*c*x]) + (b^2*c*x*((-I)*P
i^3 + (16*I)*ArcTan[c*x]^3 + 24*ArcTan[c*x]^2*Log[1 - E^((-2*I)*ArcTan[c*x]
)] - 24*ArcTan[c*x]^2*Log[1 + E^((2*I)*ArcTan[c*x])] + (24*I)*ArcTan[c*x]*P
olyLog[2, E^((-2*I)*ArcTan[c*x])] + (24*I)*ArcTan[c*x]*PolyLog[2, -E^((2*I)
*ArcTan[c*x])] + 12*PolyLog[3, E^((-2*I)*ArcTan[c*x])] - 12*PolyLog[3, -E^(
(2*I)*ArcTan[c*x])]))/24))/x
```

Maple [C] time = 1.268, size = 5963, normalized size = 26.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)*(a+b*arctan(c*x))^2/x^2,x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)*(a+b*arctan(c*x))^2/x^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{4i\,a^2cdx+4\,a^2d+\left(-i\,b^2cdx-b^2d\right)\log\left(-\frac{cx+i}{cx-i}\right)^2-4\,(abcdx-i\,abd)\log\left(-\frac{cx+i}{cx-i}\right)}{4\,x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)*(a+b*arctan(c*x))^2/x^2,x, algorithm="fricas")
```

```
[Out] integral(1/4*(4*I*a^2*c*d*x + 4*a^2*d + (-I*b^2*c*d*x - b^2*d)*log(-(c*x +
I)/(c*x - I))^2 - 4*(a*b*c*d*x - I*a*b*d)*log(-(c*x + I)/(c*x - I)))/x^2, x
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d\left(\int\frac{a^2}{x^2}\,dx+\int\frac{b^2\,\mathrm{atan}^2\,(cx)}{x^2}\,dx+\int\frac{ia^2c}{x}\,dx+\int\frac{2ab\,\mathrm{atan}\,(cx)}{x^2}\,dx+\int\frac{ib^2c\,\mathrm{atan}^2\,(cx)}{x}\,dx+\int\frac{2iabc\,\mathrm{atan}\,(cx)}{x}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)*(a+b*atan(c*x))**2/x**2,x)
```

```
[Out] d*(Integral(a**2/x**2, x) + Integral(b**2*atan(c*x)**2/x**2, x) + Integral(
I*a**2*c/x, x) + Integral(2*a*b*atan(c*x)/x**2, x) + Integral(I*b**2*c*atan
(c*x)**2/x, x) + Integral(2*I*a*b*c*atan(c*x)/x, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(i\,cdx+d)(b\arctan\,(cx)+a)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)*(a+b*arctan(c*x))^2/x^2,x, algorithm="giac")
```

```
[Out] integrate((I*c*d*x + d)*(b*arctan(c*x) + a)^2/x^2, x)
```

3.74 $\int \frac{(d+icdx)\left(a+b\tan^{-1}(cx)\right)^2}{x^3}\,dx$

Optimal. Leaf size=159

$$b^2c^2d\text{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)+\frac{1}{2}c^2d\left(a+b\tan^{-1}(cx)\right)^2+2ibc^2d\log\left(2-\frac{2}{1-icx}\right)\left(a+b\tan^{-1}(cx)\right)-\frac{d\left(a+b\tan^{-1}\right.}{2x^2}$$

```
[Out] -((b*c*d*(a + b*ArcTan[c*x]))/x) + (c^2*d*(a + b*ArcTan[c*x])^2)/2 - (d*(a
+ b*ArcTan[c*x])^2)/(2*x^2) - (I*c*d*(a + b*ArcTan[c*x])^2)/x + b^2*c^2*d*L
og[x] - (b^2*c^2*d*Log[1 + c^2*x^2])/2 + (2*I)*b*c^2*d*(a + b*ArcTan[c*x])*
Log[2 - 2/(1 - I*c*x)] + b^2*c^2*d*PolyLog[2, -1 + 2/(1 - I*c*x)]
```

Rubi [A] time = 0.339214, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {4876, 4852, 4918, 266, 36, 29, 31, 4884, 4924, 4868, 2447}

$$b^2c^2d\text{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)+\frac{1}{2}c^2d\left(a+b\tan^{-1}(cx)\right)^2+2ibc^2d\log\left(2-\frac{2}{1-icx}\right)\left(a+b\tan^{-1}(cx)\right)-\frac{d\left(a+b\tan^{-1}\right.}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)*(a + b*ArcTan[c*x])^2)/x^3,x]
```


```
[Out] -((b*c*d*(a + b*ArcTan[c*x]))/x) + (c^2*d*(a + b*ArcTan[c*x])^2)/2 - (d*(a
+ b*ArcTan[c*x])^2)/(2*x^2) - (I*c*d*(a + b*ArcTan[c*x])^2)/x + b^2*c^2*d*L
og[x] - (b^2*c^2*d*Log[1 + c^2*x^2])/2 + (2*I)*b*c^2*d*(a + b*ArcTan[c*x])*
Log[2 - 2/(1 - I*c*x)] + b^2*c^2*d*PolyLog[2, -1 + 2/(1 - I*c*x)]
```


Rule 4876

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
```

```
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4918

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/(d + e*x^2),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{(d+icdx)\left(a+b\tan^{-1}(cx)\right)^2}{x^3}\,dx = \int \left(\frac{d\left(a+b\tan^{-1}(cx)\right)^2}{x^3}+\frac{icd\left(a+b\tan^{-1}(cx)\right)^2}{x^2}\right)dx$$

$$= d\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^3}\,dx+(icd)\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^2}\,dx$$

$$= -\frac{d\left(a+b\tan^{-1}(cx)\right)^2}{2x^2}-\frac{icd\left(a+b\tan^{-1}(cx)\right)^2}{x}+(bcd)\int \frac{a+b\tan^{-1}(cx)}{x^2\left(1+c^2x^2\right)}\,dx+\Big(2$$

$$= c^2d\left(a+b\tan^{-1}(cx)\right)^2-\frac{d\left(a+b\tan^{-1}(cx)\right)^2}{2x^2}-\frac{icd\left(a+b\tan^{-1}(cx)\right)^2}{x}+(bcd)\int$$

$$= -\frac{bcd\left(a+b\tan^{-1}(cx)\right)}{x}+\frac{1}{2}c^2d\left(a+b\tan^{-1}(cx)\right)^2-\frac{d\left(a+b\tan^{-1}(cx)\right)^2}{2x^2}-\frac{icd\left(a\right.}{}$$

$$= -\frac{bcd\left(a+b\tan^{-1}(cx)\right)}{x}+\frac{1}{2}c^2d\left(a+b\tan^{-1}(cx)\right)^2-\frac{d\left(a+b\tan^{-1}(cx)\right)^2}{2x^2}-\frac{icd\left(a\right.}{}$$

$$= -\frac{bcd\left(a+b\tan^{-1}(cx)\right)}{x}+\frac{1}{2}c^2d\left(a+b\tan^{-1}(cx)\right)^2-\frac{d\left(a+b\tan^{-1}(cx)\right)^2}{2x^2}-\frac{icd\left(a\right.}{}$$

$$= -\frac{bcd\left(a+b\tan^{-1}(cx)\right)}{x}+\frac{1}{2}c^2d\left(a+b\tan^{-1}(cx)\right)^2-\frac{d\left(a+b\tan^{-1}(cx)\right)^2}{2x^2}-\frac{icd\left(a\right.}{}$$

Mathematica [A] time = 0.274071, size = 190, normalized size = 1.19

$$-\frac{d\left(-2b^2c^2x^2\text{PolyLog}\left(2,e^{2i\tan^{-1}(cx)}\right)+2ia^2cx+a^2-4iabc^2x^2\log(cx)+2iabc^2x^2\log\left(c^2x^2+1\right)+2b\tan^{-1}(cx)\left(ac^2x^2+\right.\right.}{2x^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + I*c*d*x)*(a + b*ArcTan[c*x])^2)/x^3,x]
```

```
[Out] -(d*(a^2 + (2*I)*a^2*c*x + 2*a*b*c*x - b^2*(-I + c*x)^2*ArcTan[c*x]^2 + 2*b
*ArcTan[c*x]*(a + (2*I)*a*c*x + b*c*x + a*c^2*x^2 - (2*I)*b*c^2*x^2*Log[1 -
 E^((2*I)*ArcTan[c*x])]) - (4*I)*a*b*c^2*x^2*Log[c*x] - 2*b^2*c^2*x^2*Log[(
c*x)/Sqrt[1 + c^2*x^2]] + (2*I)*a*b*c^2*x^2*Log[1 + c^2*x^2] - 2*b^2*c^2*x^
2*PolyLog[2, E^((2*I)*ArcTan[c*x])]))/(2*x^2)
```

Maple [B] time = 0.107, size = 487, normalized size = 3.1

$$-\frac{b^2c^2d\ln\left(c^2x^2+1\right)}{2}-c^2db^2\text{dilog}\left(1+icx\right)+c^2db^2\text{dilog}\left(1-icx\right)-\frac{c^2db^2\left(\arctan\left(cx\right)\right)^2}{2}-\frac{c^2db^2\left(\ln\left(cx-i\right)\right)^2}{4}-\frac{c^2db^2\text{di}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)*(a+b*arctan(c*x))^2/x^3,x)
```

```
[Out] -1/2*b^2*c^2*d*ln(c^2*x^2+1)-c^2*d*b^2*dilog(1+I*c*x)+c^2*d*b^2*dilog(1-I*c
*x)-1/2*c^2*d*b^2*arctan(c*x)^2-1/4*c^2*d*b^2*ln(c*x-I)^2-1/2*c^2*d*b^2*dil
og(-1/2*I*(c*x+I))+1/4*c^2*d*b^2*ln(c*x+I)^2+1/2*c^2*d*b^2*dilog(1/2*I*(c*x
-I))+c^2*d*b^2*ln(c*x)-1/2*d*b^2*arctan(c*x)^2/x^2-2*I*c*d*a*b*arctan(c*x)/
x-I*c*d*b^2*arctan(c*x)^2/x+2*I*c^2*d*b^2*arctan(c*x)*ln(c*x)+2*I*c^2*d*a*b
*ln(c*x)-I*c^2*d*a*b*ln(c^2*x^2+1)-I*c^2*d*b^2*arctan(c*x)*ln(c^2*x^2+1)-d*
a*b*arctan(c*x)/x^2-I*c*d*a^2/x-c*d*a*b/x-c^2*d*b^2*ln(c*x)*ln(1+I*c*x)+c^2
*d*b^2*ln(c*x)*ln(1-I*c*x)+1/2*c^2*d*b^2*ln(c*x-I)*ln(c^2*x^2+1)+1/2*c^2*d*
b^2*ln(c*x+I)*ln(1/2*I*(c*x-I))-1/2*c^2*d*b^2*ln(c*x-I)*ln(-1/2*I*(c*x+I))-
c^2*d*a*b*arctan(c*x)-c*d*b^2*arctan(c*x)/x-1/2*c^2*d*b^2*ln(c*x+I)*ln(c^2*
x^2+1)-1/2*d*a^2/x^2
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)*(a+b*arctan(c*x))^2/x^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{8\,x^2\text{integral}\left(\frac{2i\,a^2c^3dx^3+2\,a^2c^2dx^2+2i\,a^2cdx+2\,a^2d-\left(2\,abc^3dx^3-\left(2i\,ab-2\,b^2\right)c^2dx^2+\left(2\,ab-i\,b^2\right)cdx-2i\,abd\right)\log\left(-\frac{cx+i}{cx-i}\right)}{2\left(c^2x^5+x^3\right)},x\right)+\left(2i\,b^2cdx+b^2d\right)\log}{8\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)*(a+b*arctan(c*x))^2/x^3,x, algorithm="fricas")
```

```
[Out] 1/8*(8*x^2*integral(1/2*(2*I*a^2*c^3*d*x^3 + 2*a^2*c^2*d*x^2 + 2*I*a^2*c*d*
x + 2*a^2*d - (2*a*b*c^3*d*x^3 - (2*I*a*b - 2*b^2)*c^2*d*x^2 + (2*a*b - I*b
^2)*c*d*x - 2*I*a*b*d)*log(-(c*x + I)/(c*x - I)))/(c^2*x^5 + x^3), x) + (2*
I*b^2*c*d*x + b^2*d)*log(-(c*x + I)/(c*x - I))^2)/x^2
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)*(a+b*atan(c*x))**2/x**3,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(i\,cdx+d)(b\arctan(cx)+a)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)*(a+b*arctan(c*x))^2/x^3,x, algorithm="giac")
```

```
[Out] integrate((I*c*d*x + d)*(b*arctan(c*x) + a)^2/x^3, x)
```

3.75 $\int \frac{(d+icdx)\left(a+b\tan^{-1}(cx)\right)^2}{x^4}\,dx$

Optimal. Leaf size=224

$$\frac{1}{3}ib^2c^3d\text{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)-\frac{1}{6}ic^3d\left(a+b\tan^{-1}(cx)\right)^2-\frac{ibc^2d\left(a+b\tan^{-1}(cx)\right)}{x}-\frac{2}{3}bc^3d\log\left(2-\frac{2}{1-icx}\right)(a+$$

```
[Out] -(b^2*c^2*d)/(3*x) - (b^2*c^3*d*ArcTan[c*x])/3 - (b*c*d*(a + b*ArcTan[c*x])
)/(3*x^2) - (I*b*c^2*d*(a + b*ArcTan[c*x]))/x - (I/6)*c^3*d*(a + b*ArcTan[c
*x])^2 - (d*(a + b*ArcTan[c*x])^2)/(3*x^3) - ((I/2)*c*d*(a + b*ArcTan[c*x])
^2)/x^2 + I*b^2*c^3*d*Log[x] - (I/2)*b^2*c^3*d*Log[1 + c^2*x^2] - (2*b*c^3*
d*(a + b*ArcTan[c*x])*Log[2 - 2/(1 - I*c*x)])/3 + (I/3)*b^2*c^3*d*PolyLog[2
, -1 + 2/(1 - I*c*x)]
```

Rubi [A] time = 0.432427, antiderivative size = 224, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 13, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.565, Rules used = {4876, 4852, 4918, 325, 203, 4924, 4868, 2447, 266, 36, 29, 31, 4884}

$$\frac{1}{3}ib^2c^3d\text{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)-\frac{1}{6}ic^3d\left(a+b\tan^{-1}(cx)\right)^2-\frac{ibc^2d\left(a+b\tan^{-1}(cx)\right)}{x}-\frac{2}{3}bc^3d\log\left(2-\frac{2}{1-icx}\right)(a+$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)*(a + b*ArcTan[c*x])^2)/x^4,x]
```


```
[Out] -(b^2*c^2*d)/(3*x) - (b^2*c^3*d*ArcTan[c*x])/3 - (b*c*d*(a + b*ArcTan[c*x])
)/(3*x^2) - (I*b*c^2*d*(a + b*ArcTan[c*x]))/x - (I/6)*c^3*d*(a + b*ArcTan[c
*x])^2 - (d*(a + b*ArcTan[c*x])^2)/(3*x^3) - ((I/2)*c*d*(a + b*ArcTan[c*x])
^2)/x^2 + I*b^2*c^3*d*Log[x] - (I/2)*b^2*c^3*d*Log[1 + c^2*x^2] - (2*b*c^3*
d*(a + b*ArcTan[c*x])*Log[2 - 2/(1 - I*c*x)])/3 + (I/3)*b^2*c^3*d*PolyLog[2
, -1 + 2/(1 - I*c*x)]
```


Rule 4876

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4918

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/(d + e*x^2),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{(d+icdx)\left(a+b\tan^{-1}(cx)\right)^2}{x^4}\,dx = \int \left(\frac{d\left(a+b\tan^{-1}(cx)\right)^2}{x^4} + \frac{icd\left(a+b\tan^{-1}(cx)\right)^2}{x^3}\right)dx$$

$$= d\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^4}\,dx + (icd)\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^3}\,dx$$

$$= -\frac{d\left(a+b\tan^{-1}(cx)\right)^2}{3x^3} - \frac{icd\left(a+b\tan^{-1}(cx)\right)^2}{2x^2} + \frac{1}{3}(2bcd)\int \frac{a+b\tan^{-1}(cx)}{x^3\left(1+c^2x^2\right)}\,dx +$$

$$= -\frac{d\left(a+b\tan^{-1}(cx)\right)^2}{3x^3} - \frac{icd\left(a+b\tan^{-1}(cx)\right)^2}{2x^2} + \frac{1}{3}(2bcd)\int \frac{a+b\tan^{-1}(cx)}{x^3}\,dx +$$

$$= -\frac{bcd\left(a+b\tan^{-1}(cx)\right)}{3x^2} - \frac{ibc^2d\left(a+b\tan^{-1}(cx)\right)}{x} - \frac{1}{6}ic^3d\left(a+b\tan^{-1}(cx)\right)^2 - \frac{d(a}{}$$

$$= -\frac{b^2c^2d}{3x} - \frac{bcd\left(a+b\tan^{-1}(cx)\right)}{3x^2} - \frac{ibc^2d\left(a+b\tan^{-1}(cx)\right)}{x} - \frac{1}{6}ic^3d\left(a+b\tan^{-1}(cx)\right.$$

$$= -\frac{b^2c^2d}{3x} - \frac{1}{3}b^2c^3d\tan^{-1}(cx) - \frac{bcd\left(a+b\tan^{-1}(cx)\right)}{3x^2} - \frac{ibc^2d\left(a+b\tan^{-1}(cx)\right)}{x} - \frac{1}{6}i$$

$$= -\frac{b^2c^2d}{3x} - \frac{1}{3}b^2c^3d\tan^{-1}(cx) - \frac{bcd\left(a+b\tan^{-1}(cx)\right)}{3x^2} - \frac{ibc^2d\left(a+b\tan^{-1}(cx)\right)}{x} - \frac{1}{6}i$$

Mathematica [A] time = 0.495674, size = 240, normalized size = 1.07

$$\frac{d\left(2ib^2c^3x^3\text{PolyLog}\left(2,e^{2i\tan^{-1}(cx)}\right) - 3ia^2cx - 2a^2 - 6iabc^2x^2 - 4abc^3x^3\log(cx) + 2abc^3x^3\log\left(c^2x^2+1\right) - 2b\tan^{-1}(cx)\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + I*c*d*x)*(a + b*ArcTan[c*x])^2)/x^4,x]
```

```
[Out] (d*(-2*a^2 - (3*I)*a^2*c*x - 2*a*b*c*x - (6*I)*a*b*c^2*x^2 - 2*b^2*c^2*x^2
- I*b^2*(-2*I + 3*c*x + c^3*x^3)*ArcTan[c*x]^2 - 2*b*ArcTan[c*x]*(b*c*x*(1
+ (3*I)*c*x + c^2*x^2) + a*(2 + (3*I)*c*x + (3*I)*c^3*x^3) + 2*b*c^3*x^3*Lo
g[1 - E^((2*I)*ArcTan[c*x])]) - 4*a*b*c^3*x^3*Log[c*x] + (6*I)*b^2*c^3*x^3*
Log[(c*x)/Sqrt[1 + c^2*x^2]] + 2*a*b*c^3*x^3*Log[1 + c^2*x^2] + (2*I)*b^2*c
^3*x^3*PolyLog[2, E^((2*I)*ArcTan[c*x])]))/(6*x^3)
```

Maple [B] time = 0.105, size = 556, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)*(a+b*arctan(c*x))^2/x^4,x)
```

```
[Out] -1/3*c*d*a*b/x^2+1/12*I*c^3*d*b^2*ln(c*x+I)^2+1/3*c^3*d*a*b*ln(c^2*x^2+1)-1
/3*c*d*b^2*arctan(c*x)/x^2-1/3*I*c^3*d*b^2*ln(c*x)*ln(1+I*c*x)+1/3*I*c^3*d*
b^2*ln(c*x)*ln(1-I*c*x)+1/6*I*c^3*d*b^2*ln(c^2*x^2+1)*ln(c*x-I)-1/6*I*c^3*d
*b^2*ln(c*x-I)*ln(-1/2*I*(c*x+I))-1/6*I*c^3*d*b^2*ln(c^2*x^2+1)*ln(c*x+I)+1
/6*I*c^3*d*b^2*ln(c*x+I)*ln(1/2*I*(c*x-I))-I*c^3*d*a*b*arctan(c*x)-I*c^2*d*
b^2*arctan(c*x)/x-1/3*d*a^2/x^3+1/3*c^3*d*b^2*arctan(c*x)*ln(c^2*x^2+1)+I*c
^3*d*b^2*ln(c*x)-2/3*d*a*b*arctan(c*x)/x^3-2/3*c^3*d*b^2*arctan(c*x)*ln(c*x
)-2/3*c^3*d*a*b*ln(c*x)-1/2*I*c*d*a^2/x^2-1/3*I*c^3*d*b^2*dilog(1+I*c*x)-1/
12*I*c^3*d*b^2*ln(c*x-I)^2-1/2*I*c^3*d*b^2*arctan(c*x)^2-1/6*I*c^3*d*b^2*di
log(-1/2*I*(c*x+I))+1/6*I*c^3*d*b^2*dilog(1/2*I*(c*x-I))+1/3*I*c^3*d*b^2*di
log(1-I*c*x)-1/2*I*c*d*b^2*arctan(c*x)^2/x^2-I*c^2*d*a*b/x-1/3*d*b^2*arctan
(c*x)^2/x^3-I*c*d*a*b*arctan(c*x)/x^2-1/3*b^2*c^2*d/x-1/3*b^2*c^3*d*arctan(
c*x)-1/2*I*b^2*c^3*d*ln(c^2*x^2+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-i\left(\left(c\arctan\left(cx\right)+\frac{1}{x}\right)c+\frac{\arctan\left(cx\right)}{x^2}\right)abcd+\frac{1}{3}\left(\left(c^2\log\left(c^2x^2+1\right)-c^2\log\left(x^2\right)-\frac{1}{x^2}\right)c-\frac{2\arctan\left(cx\right)}{x^3}\right)abd-\frac{i\,a^2cd}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)*(a+b*arctan(c*x))^2/x^4,x, algorithm="maxima")
```

```
[Out] -I*((c*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*a*b*c*d + 1/3*((c^2*log(c^2*
x^2 + 1) - c^2*log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^3)*a*b*d - 1/2*I*a^2*c
*d/x^2 - 1/3*a^2*d/x^3 + 1/96*(96*I*x^3*integrate(1/48*(20*b^2*c^2*d*x^2*ar
ctan(c*x) + 36*(b^2*c^3*d*x^3 + b^2*c*d*x)*arctan(c*x)^2 + 3*(b^2*c^3*d*x^3
 + b^2*c*d*x)*log(c^2*x^2 + 1)^2 - 2*(3*b^2*c^3*d*x^3 - 2*b^2*c*d*x + 6*(b^
2*c^2*d*x^2 + b^2*d)*arctan(c*x))*log(c^2*x^2 + 1))/(c^2*x^6 + x^4), x) + 9
6*x^3*integrate(1/48*(36*(b^2*c^2*d*x^2 + b^2*d)*arctan(c*x)^2 + 3*(b^2*c^2
*d*x^2 + b^2*d)*log(c^2*x^2 + 1)^2 - 4*(3*b^2*c^3*d*x^3 - 2*b^2*c*d*x)*arct
an(c*x) - 2*(5*b^2*c^2*d*x^2 - 6*(b^2*c^3*d*x^3 + b^2*c*d*x)*arctan(c*x))*l
```

```
og(c^2*x^2 + 1))/(c^2*x^6 + x^4), x) + (-12*I*b^2*c*d*x - 8*b^2*d)*arctan(c
*x)^2 + 4*(3*b^2*c*d*x - 2*I*b^2*d)*arctan(c*x)*log(c^2*x^2 + 1) + (3*I*b^2
*c*d*x + 2*b^2*d)*log(c^2*x^2 + 1)^2)/x^3
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{24\,x^3\text{integral}\left(\frac{6i\,a^2c^3dx^3+6\,a^2c^2dx^2+6i\,a^2cdx+6\,a^2d-\left(6\,abc^3dx^3-\left(6i\,ab-3\,b^2\right)c^2dx^2+2\,\left(3\,ab-i\,b^2\right)cdx-6i\,abd\right)\log\left(-\frac{cx+i}{cx-i}\right)}{6\,\left(c^2x^6+x^4\right)},x\right)+\left(3i\,b^2cdx+2\,b^2d\right)\text{lo}}{24\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)*(a+b*arctan(c*x))^2/x^4,x, algorithm="fricas")
```

```
[Out] 1/24*(24*x^3*integral(1/6*(6*I*a^2*c^3*d*x^3 + 6*a^2*c^2*d*x^2 + 6*I*a^2*c*
d*x + 6*a^2*d - (6*a*b*c^3*d*x^3 - (6*I*a*b - 3*b^2)*c^2*d*x^2 + 2*(3*a*b -
 I*b^2)*c*d*x - 6*I*a*b*d)*log(-(c*x + I)/(c*x - I)))/(c^2*x^6 + x^4), x) +
 (3*I*b^2*c*d*x + 2*b^2*d)*log(-(c*x + I)/(c*x - I))^2)/x^3
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)*(a+b*atan(c*x))**2/x**4,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(i\,cdx+d)(b\arctan(cx)+a)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)*(a+b*arctan(c*x))^2/x^4,x, algorithm="giac")
```

```
[Out] integrate((I*c*d*x + d)*(b*arctan(c*x) + a)^2/x^4, x)
```

3.76 $\int x^3(d+icdx)^2\left(a+b\tan^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=373

$$-\frac{2b^2d^2\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{5c^4}-\frac{1}{6}c^2d^2x^6\left(a+b\tan^{-1}(cx)\right)^2+\frac{2ibd^2x^2\left(a+b\tan^{-1}(cx)\right)}{5c^2}+\frac{5abd^2x}{6c^3}-\frac{49d^2\left(a+b\tan^{-1}(\right.}{60c^4}$$

```
[Out] (5*a*b*d^2*x)/(6*c^3) - (((3*I)/5)*b^2*d^2*x)/c^3 + (31*b^2*d^2*x^2)/(180*c
^2) + ((I/15)*b^2*d^2*x^3)/c - (b^2*d^2*x^4)/60 + (((3*I)/5)*b^2*d^2*ArcTan
[c*x])/c^4 + (5*b^2*d^2*x*ArcTan[c*x])/(6*c^3) + (((2*I)/5)*b*d^2*x^2*(a +
b*ArcTan[c*x]))/c^2 - (5*b*d^2*x^3*(a + b*ArcTan[c*x]))/(18*c) - (I/5)*b*d^
2*x^4*(a + b*ArcTan[c*x]) + (b*c*d^2*x^5*(a + b*ArcTan[c*x]))/15 - (49*d^2*
(a + b*ArcTan[c*x])^2)/(60*c^4) + (d^2*x^4*(a + b*ArcTan[c*x])^2)/4 + ((2*I
)/5)*c*d^2*x^5*(a + b*ArcTan[c*x])^2 - (c^2*d^2*x^6*(a + b*ArcTan[c*x])^2)/
6 + (((4*I)/5)*b*d^2*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/c^4 - (53*b^2*
d^2*Log[1 + c^2*x^2])/(90*c^4) - (2*b^2*d^2*PolyLog[2, 1 - 2/(1 + I*c*x)])/
(5*c^4)
```

Rubi [A] time = 0.973352, antiderivative size = 373, normalized size of antiderivative = 1., number of steps used = 43, number of rules used = 15, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {4876, 4852, 4916, 266, 43, 4846, 260, 4884, 302, 203, 321, 4920, 4854, 2402, 2315}

$$-\frac{2b^2d^2\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{5c^4}-\frac{1}{6}c^2d^2x^6\left(a+b\tan^{-1}(cx)\right)^2+\frac{2ibd^2x^2\left(a+b\tan^{-1}(cx)\right)}{5c^2}+\frac{5abd^2x}{6c^3}-\frac{49d^2\left(a+b\tan^{-1}(\right.}{60c^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + I*c*d*x)^2*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] (5*a*b*d^2*x)/(6*c^3) - (((3*I)/5)*b^2*d^2*x)/c^3 + (31*b^2*d^2*x^2)/(180*c
^2) + ((I/15)*b^2*d^2*x^3)/c - (b^2*d^2*x^4)/60 + (((3*I)/5)*b^2*d^2*ArcTan
[c*x])/c^4 + (5*b^2*d^2*x*ArcTan[c*x])/(6*c^3) + (((2*I)/5)*b*d^2*x^2*(a +
b*ArcTan[c*x]))/c^2 - (5*b*d^2*x^3*(a + b*ArcTan[c*x]))/(18*c) - (I/5)*b*d^
2*x^4*(a + b*ArcTan[c*x]) + (b*c*d^2*x^5*(a + b*ArcTan[c*x]))/15 - (49*d^2*
(a + b*ArcTan[c*x])^2)/(60*c^4) + (d^2*x^4*(a + b*ArcTan[c*x])^2)/4 + ((2*I
)/5)*c*d^2*x^5*(a + b*ArcTan[c*x])^2 - (c^2*d^2*x^6*(a + b*ArcTan[c*x])^2)/
6 + (((4*I)/5)*b*d^2*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/c^4 - (53*b^2*
d^2*Log[1 + c^2*x^2])/(90*c^4) - (2*b^2*d^2*PolyLog[2, 1 - 2/(1 + I*c*x)])/
(5*c^4)
```

Rule 4876

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
```

```
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int x^3(d+icdx)^2\left(a+b\tan^{-1}(cx)\right)^2\,dx = \int \left(d^2x^3\left(a+b\tan^{-1}(cx)\right)^2+2icd^2x^4\left(a+b\tan^{-1}(cx)\right)^2-c^2d^2x^5\left(a+b\tan^{-1}\right.\right.$$

$$= d^2\int x^3\left(a+b\tan^{-1}(cx)\right)^2\,dx+\left(2icd^2\right)\int x^4\left(a+b\tan^{-1}(cx)\right)^2\,dx-\left(c^2d^2\right)\int$$

$$= \frac{1}{4}d^2x^4\left(a+b\tan^{-1}(cx)\right)^2+\frac{2}{5}icd^2x^5\left(a+b\tan^{-1}(cx)\right)^2-\frac{1}{6}c^2d^2x^6\left(a+b\tan^{-1}\right.$$

$$= \frac{1}{4}d^2x^4\left(a+b\tan^{-1}(cx)\right)^2+\frac{2}{5}icd^2x^5\left(a+b\tan^{-1}(cx)\right)^2-\frac{1}{6}c^2d^2x^6\left(a+b\tan^{-1}\right.$$

$$= -\frac{bd^2x^3\left(a+b\tan^{-1}(cx)\right)}{6c}-\frac{1}{5}ibd^2x^4\left(a+b\tan^{-1}(cx)\right)+\frac{1}{15}bcd^2x^5\left(a+b\tan\right.$$

$$= \frac{abd^2x}{2c^3}+\frac{2ibd^2x^2\left(a+b\tan^{-1}(cx)\right)}{5c^2}-\frac{5bd^2x^3\left(a+b\tan^{-1}(cx)\right)}{18c}-\frac{1}{5}ibd^2x^4\left(a+\right.$$

$$= \frac{5abd^2x}{6c^3}-\frac{3ib^2d^2x}{5c^3}+\frac{ib^2d^2x^3}{15c}+\frac{b^2d^2x\tan^{-1}(cx)}{2c^3}+\frac{2ibd^2x^2\left(a+b\tan^{-1}(cx)\right)}{5c^2}$$

$$= \frac{5abd^2x}{6c^3}-\frac{3ib^2d^2x}{5c^3}+\frac{7b^2d^2x^2}{60c^2}+\frac{ib^2d^2x^3}{15c}-\frac{1}{60}b^2d^2x^4+\frac{3ib^2d^2\tan^{-1}(cx)}{5c^4}+\frac{5b}{}$$

$$= \frac{5abd^2x}{6c^3}-\frac{3ib^2d^2x}{5c^3}+\frac{31b^2d^2x^2}{180c^2}+\frac{ib^2d^2x^3}{15c}-\frac{1}{60}b^2d^2x^4+\frac{3ib^2d^2\tan^{-1}(cx)}{5c^4}+\frac{5}{}$$

Mathematica [A] time = 1.17504, size = 342, normalized size = 0.92

$$\frac{d^2\left(72b^2\text{PolyLog}\left(2,-e^{2i\tan^{-1}(cx)}\right)-30a^2c^6x^6+72ia^2c^5x^5+45a^2c^4x^4+12abc^5x^5-36iabc^4x^4-50abc^3x^3+72iabc^2x^2\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*(d + I*c*d*x)^2*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] (d^2*((108*I)*a*b + 34*b^2 + 150*a*b*c*x - (108*I)*b^2*c*x + (72*I)*a*b*c^2
*x^2 + 31*b^2*c^2*x^2 - 50*a*b*c^3*x^3 + (12*I)*b^2*c^3*x^3 + 45*a^2*c^4*x^
4 - (36*I)*a*b*c^4*x^4 - 3*b^2*c^4*x^4 + (72*I)*a^2*c^5*x^5 + 12*a*b*c^5*x^
5 - 30*a^2*c^6*x^6 - 3*b^2*(1 - 15*c^4*x^4 - (24*I)*c^5*x^5 + 10*c^6*x^6)*A
rcTan[c*x]^2 + 2*b*ArcTan[c*x]*(b*(54*I + 75*c*x + (36*I)*c^2*x^2 - 25*c^3*
x^3 - (18*I)*c^4*x^4 + 6*c^5*x^5) + a*(-75 + 45*c^4*x^4 + (72*I)*c^5*x^5 -
30*c^6*x^6) + (72*I)*b*Log[1 + E^((2*I)*ArcTan[c*x])]) - (72*I)*a*b*Log[1 +
 c^2*x^2] - 106*b^2*Log[1 + c^2*x^2] + 72*b^2*PolyLog[2, -E^((2*I)*ArcTan[c
*x])]))/(180*c^4)
```

Maple [A] time = 0.094, size = 650, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d+I*c*d*x)^2*(a+b*arctan(c*x))^2,x)
```

```
[Out] -2/5*I/c^4*d^2*a*b*ln(c^2*x^2+1)-1/60*b^2*d^2*x^4+1/5/c^4*d^2*b^2*dilog(1/2
*I*(c*x-I))-1/5/c^4*d^2*b^2*dilog(-1/2*I*(c*x+I))+1/4*d^2*b^2*arctan(c*x)^2
*x^4-1/6*c^2*d^2*a^2*x^6-5/12/c^4*d^2*b^2*arctan(c*x)^2+1/10/c^4*d^2*b^2*ln
(c*x+I)^2-1/10/c^4*d^2*b^2*ln(c*x-I)^2+2/5*I/c^2*d^2*b^2*arctan(c*x)*x^2+2/
5*I*c*d^2*b^2*arctan(c*x)^2*x^5+2/5*I/c^2*d^2*a*b*x^2-5/18/c*d^2*a*b*x^3-1/
6*c^2*d^2*b^2*arctan(c*x)^2*x^6-5/6/c^4*d^2*a*b*arctan(c*x)+1/15*c*d^2*b^2*
arctan(c*x)*x^5-1/5/c^4*d^2*b^2*ln(c*x+I)*ln(c^2*x^2+1)-1/5/c^4*d^2*b^2*ln(
c*x-I)*ln(-1/2*I*(c*x+I))+1/5/c^4*d^2*b^2*ln(c*x-I)*ln(c^2*x^2+1)+1/5/c^4*d
^2*b^2*ln(c*x+I)*ln(1/2*I*(c*x-I))+1/2*d^2*a*b*arctan(c*x)*x^4+1/15*c*d^2*a
*b*x^5+2/5*I*c*d^2*a^2*x^5-1/5*I*d^2*b^2*arctan(c*x)*x^4-5/18/c*d^2*b^2*arc
tan(c*x)*x^3-1/5*I*d^2*a*b*x^4-1/3*c^2*d^2*a*b*arctan(c*x)*x^6+1/4*d^2*a^2*
x^4+5/6*a*b*d^2*x/c^3+5/6*b^2*d^2*x*arctan(c*x)/c^3-3/5*I*b^2*d^2*x/c^3-2/5
*I/c^4*d^2*b^2*arctan(c*x)*ln(c^2*x^2+1)+1/15*I*b^2*d^2*x^3/c+3/5*I*b^2*d^2
*arctan(c*x)/c^4+4/5*I*c*d^2*a*b*arctan(c*x)*x^5+31/180*b^2*d^2*x^2/c^2-53/
90*b^2*d^2*ln(c^2*x^2+1)/c^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d+I*c*d*x)^2*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

```
[Out] -1/6*a^2*c^2*d^2*x^6 + 2/5*I*a^2*c*d^2*x^5 + 1/4*b^2*d^2*x^4*arctan(c*x)^2
+ 1/4*a^2*d^2*x^4 - 1/45*(15*x^6*arctan(c*x) - c*((3*c^4*x^5 - 5*c^2*x^3 +
15*x)/c^6 - 15*arctan(c*x)/c^7))*a*b*c^2*d^2 + 1/5*I*(4*x^5*arctan(c*x) - c
*((c^2*x^4 - 2*x^2)/c^4 + 2*log(c^2*x^2 + 1)/c^6))*a*b*c*d^2 + 1/6*(3*x^4*a
rctan(c*x) - c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5))*a*b*d^2 - 1/12*(2
*c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5)*arctan(c*x) - (c^2*x^2 + 3*arc
tan(c*x)^2 - 4*log(c^2*x^2 + 1))/c^4)*b^2*d^2 - 1/120*(5*b^2*c^2*d^2*x^6 -
12*I*b^2*c*d^2*x^5)*arctan(c*x)^2 - 1/480*(20*I*b^2*c^2*d^2*x^6 + 48*b^2*c*
d^2*x^5)*arctan(c*x)*log(c^2*x^2 + 1) + 1/480*(5*b^2*c^2*d^2*x^6 - 12*I*b^2
*c*d^2*x^5)*log(c^2*x^2 + 1)^2 - integrate(-1/240*(68*b^2*c^3*d^2*x^6*arcta
n(c*x) - 180*(b^2*c^4*d^2*x^7 + b^2*c^2*d^2*x^5)*arctan(c*x)^2 - 15*(b^2*c^
4*d^2*x^7 + b^2*c^2*d^2*x^5)*log(c^2*x^2 + 1)^2 - 2*(5*b^2*c^4*d^2*x^7 - 12
*b^2*c^2*d^2*x^5 - 60*(b^2*c^3*d^2*x^6 + b^2*c*d^2*x^4)*arctan(c*x))*log(c^
2*x^2 + 1))/(c^2*x^2 + 1), x) + I*integrate(1/120*(180*(b^2*c^3*d^2*x^6 + b
^2*c*d^2*x^4)*arctan(c*x)^2 + 15*(b^2*c^3*d^2*x^6 + b^2*c*d^2*x^4)*log(c^2*
x^2 + 1)^2 + 2*(5*b^2*c^4*d^2*x^7 - 12*b^2*c^2*d^2*x^5)*arctan(c*x) + (17*b
^2*c^3*d^2*x^6 + 30*(b^2*c^4*d^2*x^7 + b^2*c^2*d^2*x^5)*arctan(c*x))*log(c^
2*x^2 + 1))/(c^2*x^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{240}\left(10\,b^2c^2d^2x^6-24i\,b^2cd^2x^5-15\,b^2d^2x^4\right)\log\left(-\frac{cx+i}{cx-i}\right)^2+\mathrm{integral}\left(-\frac{60\,a^2c^4d^2x^7-120i\,a^2c^3d^2x^6-120i\,a^2cd^2x^4-\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d+I*c*d*x)^2*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/240*(10*b^2*c^2*d^2*x^6 - 24*I*b^2*c*d^2*x^5 - 15*b^2*d^2*x^4)*log(-(c*x
+ I)/(c*x - I))^2 + integral(-1/60*(60*a^2*c^4*d^2*x^7 - 120*I*a^2*c^3*d^2*
x^6 - 120*I*a^2*c*d^2*x^4 - 60*a^2*d^2*x^3 - (-60*I*a*b*c^4*d^2*x^7 - 10*(1
2*a*b - I*b^2)*c^3*d^2*x^6 + 24*b^2*c^2*d^2*x^5 - 15*(8*a*b + I*b^2)*c*d^2*
x^4 + 60*I*a*b*d^2*x^3)*log(-(c*x + I)/(c*x - I)))/(c^2*x^2 + 1), x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(d+I*c*d*x)**2*(a+b*atan(c*x))**2,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (icdx+d)^2(b\arctan(cx)+a)^2x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d+I*c*d*x)^2*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((I*c*d*x + d)^2*(b*arctan(c*x) + a)^2*x^3, x)
```

3.77 $\int x^2(d + icdx)^2 \left(a + b\tan^{-1}(cx)\right)^2 dx$

Optimal. Leaf size=333

$$-\frac{8ib^2d^2\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{15c^3}-\frac{1}{5}c^2d^2x^5\left(a+b\tan^{-1}(cx)\right)^2+\frac{iabd^2x}{c^2}-\frac{31id^2\left(a+b\tan^{-1}(cx)\right)^2}{30c^3}-\frac{16bd^2\log\left(\frac{2}{1+icx}\right)(}{15c}$$

```
[Out] (I*a*b*d^2*x)/c^2 + (19*b^2*d^2*x)/(30*c^2) + ((I/6)*b^2*d^2*x^2)/c - (b^2*
d^2*x^3)/30 - (19*b^2*d^2*ArcTan[c*x])/(30*c^3) + (I*b^2*d^2*x*ArcTan[c*x])
/c^2 - (8*b*d^2*x^2*(a + b*ArcTan[c*x]))/(15*c) - (I/3)*b*d^2*x^3*(a + b*Ar
cTan[c*x]) + (b*c*d^2*x^4*(a + b*ArcTan[c*x]))/10 - (((31*I)/30)*d^2*(a + b
*ArcTan[c*x])^2)/c^3 + (d^2*x^3*(a + b*ArcTan[c*x])^2)/3 + (I/2)*c*d^2*x^4*
(a + b*ArcTan[c*x])^2 - (c^2*d^2*x^5*(a + b*ArcTan[c*x])^2)/5 - (16*b*d^2*(
a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(15*c^3) - (((2*I)/3)*b^2*d^2*Log[1
+ c^2*x^2])/c^3 - (((8*I)/15)*b^2*d^2*PolyLog[2, 1 - 2/(1 + I*c*x)])/c^3
```

Rubi [A] time = 0.856662, antiderivative size = 333, normalized size of antiderivative = 1., number of steps used = 36, number of rules used = 15, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {4876, 4852, 4916, 321, 203, 4920, 4854, 2402, 2315, 266, 43, 4846, 260, 4884, 302}

$$-\frac{8ib^2d^2\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{15c^3}-\frac{1}{5}c^2d^2x^5\left(a+b\tan^{-1}(cx)\right)^2+\frac{iabd^2x}{c^2}-\frac{31id^2\left(a+b\tan^{-1}(cx)\right)^2}{30c^3}-\frac{16bd^2\log\left(\frac{2}{1+icx}\right)(}{15c}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + I*c*d*x)^2*(a + b*ArcTan[c*x])^2,x]
```


```
[Out] (I*a*b*d^2*x)/c^2 + (19*b^2*d^2*x)/(30*c^2) + ((I/6)*b^2*d^2*x^2)/c - (b^2*
d^2*x^3)/30 - (19*b^2*d^2*ArcTan[c*x])/(30*c^3) + (I*b^2*d^2*x*ArcTan[c*x])
/c^2 - (8*b*d^2*x^2*(a + b*ArcTan[c*x]))/(15*c) - (I/3)*b*d^2*x^3*(a + b*Ar
cTan[c*x]) + (b*c*d^2*x^4*(a + b*ArcTan[c*x]))/10 - (((31*I)/30)*d^2*(a + b
*ArcTan[c*x])^2)/c^3 + (d^2*x^3*(a + b*ArcTan[c*x])^2)/3 + (I/2)*c*d^2*x^4*
(a + b*ArcTan[c*x])^2 - (c^2*d^2*x^5*(a + b*ArcTan[c*x])^2)/5 - (16*b*d^2*(
a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(15*c^3) - (((2*I)/3)*b^2*d^2*Log[1
+ c^2*x^2])/c^3 - (((8*I)/15)*b^2*d^2*PolyLog[2, 1 - 2/(1 + I*c*x)])/c^3
```


Rule 4876

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
```

```
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
```

```
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\int x^2(d+icdx)^2\left(a+b\tan^{-1}(cx)\right)^2\,dx=\int\left(d^2x^2\left(a+b\tan^{-1}(cx)\right)^2+2icd^2x^3\left(a+b\tan^{-1}(cx)\right)^2-c^2d^2x^4\left(a+b\tan^{-1}(c\right.\right.$$

$$=d^2\int x^2\left(a+b\tan^{-1}(cx)\right)^2dx+\left(2icd^2\right)\int x^3\left(a+b\tan^{-1}(cx)\right)^2dx-\left(c^2d^2\right)\int$$

$$=\frac{1}{3}d^2x^3\left(a+b\tan^{-1}(cx)\right)^2+\frac{1}{2}icd^2x^4\left(a+b\tan^{-1}(cx)\right)^2-\frac{1}{5}c^2d^2x^5\left(a+b\tan^{-1}(c\right.$$

$$=\frac{1}{3}d^2x^3\left(a+b\tan^{-1}(cx)\right)^2+\frac{1}{2}icd^2x^4\left(a+b\tan^{-1}(cx)\right)^2-\frac{1}{5}c^2d^2x^5\left(a+b\tan^{-1}(c\right.$$

$$=-\frac{bd^2x^2\left(a+b\tan^{-1}(cx)\right)}{3c}-\frac{1}{3}ibd^2x^3\left(a+b\tan^{-1}(cx)\right)+\frac{1}{10}bcd^2x^4\left(a+b\tan^{-1}(\right.$$

$$=\frac{iabd^2x}{c^2}+\frac{b^2d^2x}{3c^2}-\frac{8bd^2x^2\left(a+b\tan^{-1}(cx)\right)}{15c}-\frac{1}{3}ibd^2x^3\left(a+b\tan^{-1}(cx)\right)+\frac{1}{10}bc$$

$$=\frac{iabd^2x}{c^2}+\frac{19b^2d^2x}{30c^2}-\frac{1}{30}b^2d^2x^3-\frac{b^2d^2\tan^{-1}(cx)}{3c^3}+\frac{ib^2d^2x\tan^{-1}(cx)}{c^2}-\frac{8bd^2x^2(}{}$$

$$=\frac{iabd^2x}{c^2}+\frac{19b^2d^2x}{30c^2}+\frac{ib^2d^2x^2}{6c}-\frac{1}{30}b^2d^2x^3-\frac{19b^2d^2\tan^{-1}(cx)}{30c^3}+\frac{ib^2d^2x\tan^{-1}(cx)}{c^2}$$

$$=\frac{iabd^2x}{c^2}+\frac{19b^2d^2x}{30c^2}+\frac{ib^2d^2x^2}{6c}-\frac{1}{30}b^2d^2x^3-\frac{19b^2d^2\tan^{-1}(cx)}{30c^3}+\frac{ib^2d^2x\tan^{-1}(cx)}{c^2}$$

Mathematica [A] time = 1.14217, size = 306, normalized size = 0.92

$$-\frac{d^2\left(-16ib^2\text{PolyLog}\left(2,-e^{2i\tan^{-1}(cx)}\right)+6a^2c^5x^5-15ia^2c^4x^4-10a^2c^3x^3-3abc^4x^4+10iabc^3x^3+16abc^2x^2-16ab\log\left(c^2\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(d + I*c*d*x)^2*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] -(d^2*(9*a*b - (5*I)*b^2 - (30*I)*a*b*c*x - 19*b^2*c*x + 16*a*b*c^2*x^2 - (
5*I)*b^2*c^2*x^2 - 10*a^2*c^3*x^3 + (10*I)*a*b*c^3*x^3 + b^2*c^3*x^3 - (15*
```

```
I)*a^2*c^4*x^4 - 3*a*b*c^4*x^4 + 6*a^2*c^5*x^5 + b^2*(-I + c*x)^3*(-1 + (3*
I)*c*x + 6*c^2*x^2)*ArcTan[c*x]^2 + b*ArcTan[c*x]*(b*(19 - (30*I)*c*x + 16*
c^2*x^2 + (10*I)*c^3*x^3 - 3*c^4*x^4) + 2*a*(15*I - 10*c^3*x^3 - (15*I)*c^4
*x^4 + 6*c^5*x^5) + 32*b*Log[1 + E^((2*I)*ArcTan[c*x])]) - 16*a*b*Log[1 + c
^2*x^2] + (20*I)*b^2*Log[1 + c^2*x^2] - (16*I)*b^2*PolyLog[2, -E^((2*I)*Arc
Tan[c*x])]))/(30*c^3)
```

Maple [B] time = 0.096, size = 612, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d+I*c*d*x)^2*(a+b*arctan(c*x))^2,x)
```

```
[Out] -1/5*c^2*d^2*a^2*x^5+1/3*d^2*b^2*arctan(c*x)^2*x^3-1/30*b^2*d^2*x^3+I*c*d^2
*a*b*arctan(c*x)*x^4+4/15*I/c^3*d^2*b^2*dilog(1/2*I*(c*x-I))-4/15*I/c^3*d^2
*b^2*dilog(-1/2*I*(c*x+I))+8/15/c^3*d^2*b^2*arctan(c*x)*ln(c^2*x^2+1)+2/3*d
^2*a*b*arctan(c*x)*x^3+2/15*I/c^3*d^2*b^2*ln(c*x+I)^2+8/15/c^3*d^2*a*b*ln(c
^2*x^2+1)+1/10*c*d^2*b^2*arctan(c*x)*x^4-8/15/c*d^2*b^2*arctan(c*x)*x^2-1/5
*c^2*d^2*b^2*arctan(c*x)^2*x^5-1/3*I*d^2*a*b*x^3+1/10*c*d^2*a*b*x^4+1/2*I*c
*d^2*a^2*x^4-1/3*I*d^2*b^2*arctan(c*x)*x^3-2/15*I/c^3*d^2*b^2*ln(c*x-I)^2-1
/2*I/c^3*d^2*b^2*arctan(c*x)^2-8/15/c*d^2*a*b*x^2+1/3*d^2*a^2*x^3+I*a*b*d^2
*x/c^2+I*b^2*d^2*x*arctan(c*x)/c^2+4/15*I/c^3*d^2*b^2*ln(c*x+I)*ln(1/2*I*(c
*x-I))+1/6*I*b^2*d^2*x^2/c-2/3*I*b^2*d^2*ln(c^2*x^2+1)/c^3-2/5*c^2*d^2*a*b*
arctan(c*x)*x^5-4/15*I/c^3*d^2*b^2*ln(c*x+I)*ln(c^2*x^2+1)+4/15*I/c^3*d^2*b
^2*ln(c*x-I)*ln(c^2*x^2+1)-I/c^3*d^2*a*b*arctan(c*x)+19/30*b^2*d^2*x/c^2-19
/30*b^2*d^2*arctan(c*x)/c^3+1/2*I*c*d^2*b^2*arctan(c*x)^2*x^4-4/15*I/c^3*d^
2*b^2*ln(c*x-I)*ln(-1/2*I*(c*x+I))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d+I*c*d*x)^2*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

```
[Out] -1/5*a^2*c^2*d^2*x^5 + 1/2*I*a^2*c*d^2*x^4 - 1/10*(4*x^5*arctan(c*x) - c*((
c^2*x^4 - 2*x^2)/c^4 + 2*log(c^2*x^2 + 1)/c^6))*a*b*c^2*d^2 + 1/3*a^2*d^2*x
```

```
^3 + 1/3*I*(3*x^4*arctan(c*x) - c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5)
)*a*b*c*d^2 + 1/3*(2*x^3*arctan(c*x) - c*(x^2/c^2 - log(c^2*x^2 + 1)/c^4))*
a*b*d^2 - 1/480*(24*b^2*c^2*d^2*x^5 - 60*I*b^2*c*d^2*x^4 - 40*b^2*d^2*x^3)*
arctan(c*x)^2 - 1/480*(24*I*b^2*c^2*d^2*x^5 + 60*b^2*c*d^2*x^4 - 40*I*b^2*d
^2*x^3)*arctan(c*x)*log(c^2*x^2 + 1) + 1/480*(6*b^2*c^2*d^2*x^5 - 15*I*b^2*
c*d^2*x^4 - 10*b^2*d^2*x^3)*log(c^2*x^2 + 1)^2 - integrate(1/240*(180*(b^2*
c^4*d^2*x^6 - b^2*d^2*x^2)*arctan(c*x)^2 + 15*(b^2*c^4*d^2*x^6 - b^2*d^2*x^
2)*log(c^2*x^2 + 1)^2 - 4*(21*b^2*c^3*d^2*x^5 - 10*b^2*c*d^2*x^3)*arctan(c*
x) + 2*(6*b^2*c^4*d^2*x^6 - 25*b^2*c^2*d^2*x^4 - 60*(b^2*c^3*d^2*x^5 + b^2*
c*d^2*x^3)*arctan(c*x))*log(c^2*x^2 + 1))/(c^2*x^2 + 1), x) + I*integrate(1
/120*(180*(b^2*c^3*d^2*x^5 + b^2*c*d^2*x^3)*arctan(c*x)^2 + 15*(b^2*c^3*d^2
*x^5 + b^2*c*d^2*x^3)*log(c^2*x^2 + 1)^2 + 2*(6*b^2*c^4*d^2*x^6 - 25*b^2*c^
2*d^2*x^4)*arctan(c*x) + (21*b^2*c^3*d^2*x^5 - 10*b^2*c*d^2*x^3 + 30*(b^2*c
^4*d^2*x^6 - b^2*d^2*x^2)*arctan(c*x))*log(c^2*x^2 + 1))/(c^2*x^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{120}\left(6\,b^2c^2d^2x^5-15i\,b^2cd^2x^4-10\,b^2d^2x^3\right)\log\left(-\frac{cx+i}{cx-i}\right)^2+\text{integral}\left(-\frac{30\,a^2c^4d^2x^6-60i\,a^2c^3d^2x^5-60i\,a^2cd^2x^3-30\,a^2}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d+I*c*d*x)^2*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/120*(6*b^2*c^2*d^2*x^5 - 15*I*b^2*c*d^2*x^4 - 10*b^2*d^2*x^3)*log(-(c*x +
 I)/(c*x - I))^2 + integral(-1/30*(30*a^2*c^4*d^2*x^6 - 60*I*a^2*c^3*d^2*x^
5 - 60*I*a^2*c*d^2*x^3 - 30*a^2*d^2*x^2 - (-30*I*a*b*c^4*d^2*x^6 - 6*(10*a*
b - I*b^2)*c^3*d^2*x^5 + 15*b^2*c^2*d^2*x^4 - 10*(6*a*b + I*b^2)*c*d^2*x^3
+ 30*I*a*b*d^2*x^2)*log(-(c*x + I)/(c*x - I)))/(c^2*x^2 + 1), x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d+I*c*d*x)**2*(a+b*atan(c*x))**2,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (icdx + d)^2 (b \arctan (cx) + a)^2 x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d+I*c*d*x)^2*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((I*c*d*x + d)^2*(b*arctan(c*x) + a)^2*x^2, x)
```

3.78 $\int x(d+icdx)^2\left(a+b\tan^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=293

$$\frac{2b^2d^2\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{3c^2}-\frac{1}{4}c^2d^2x^4\left(a+b\tan^{-1}(cx)\right)^2+\frac{17d^2\left(a+b\tan^{-1}(cx)\right)^2}{12c^2}-\frac{4ibd^2\log\left(\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx\right.}{3c^2}$$

```
[Out] (-3*a*b*d^2*x)/(2*c) + (((2*I)/3)*b^2*d^2*x)/c - (b^2*d^2*x^2)/12 - (((2*I)
/3)*b^2*d^2*ArcTan[c*x])/c^2 - (3*b^2*d^2*x*ArcTan[c*x])/(2*c) - ((2*I)/3)*
b*d^2*x^2*(a + b*ArcTan[c*x]) + (b*c*d^2*x^3*(a + b*ArcTan[c*x]))/6 + (17*d
^2*(a + b*ArcTan[c*x])^2)/(12*c^2) + (d^2*x^2*(a + b*ArcTan[c*x])^2)/2 + ((
2*I)/3)*c*d^2*x^3*(a + b*ArcTan[c*x])^2 - (c^2*d^2*x^4*(a + b*ArcTan[c*x])^
2)/4 - (((4*I)/3)*b*d^2*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/c^2 + (5*b^
2*d^2*Log[1 + c^2*x^2])/(6*c^2) + (2*b^2*d^2*PolyLog[2, 1 - 2/(1 + I*c*x)])
/(3*c^2)
```

Rubi [A] time = 0.621398, antiderivative size = 293, normalized size of antiderivative = 1., number of steps used = 28, number of rules used = 14, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.609, Rules used = {4876, 4852, 4916, 4846, 260, 4884, 321, 203, 4920, 4854, 2402, 2315, 266, 43}

$$\frac{2b^2d^2\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{3c^2}-\frac{1}{4}c^2d^2x^4\left(a+b\tan^{-1}(cx)\right)^2+\frac{17d^2\left(a+b\tan^{-1}(cx)\right)^2}{12c^2}-\frac{4ibd^2\log\left(\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx\right.}{3c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + I*c*d*x)^2*(a + b*ArcTan[c*x])^2,x]
```


```
[Out] (-3*a*b*d^2*x)/(2*c) + (((2*I)/3)*b^2*d^2*x)/c - (b^2*d^2*x^2)/12 - (((2*I)
/3)*b^2*d^2*ArcTan[c*x])/c^2 - (3*b^2*d^2*x*ArcTan[c*x])/(2*c) - ((2*I)/3)*
b*d^2*x^2*(a + b*ArcTan[c*x]) + (b*c*d^2*x^3*(a + b*ArcTan[c*x]))/6 + (17*d
^2*(a + b*ArcTan[c*x])^2)/(12*c^2) + (d^2*x^2*(a + b*ArcTan[c*x])^2)/2 + ((
2*I)/3)*c*d^2*x^3*(a + b*ArcTan[c*x])^2 - (c^2*d^2*x^4*(a + b*ArcTan[c*x])^
2)/4 - (((4*I)/3)*b*d^2*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/c^2 + (5*b^
2*d^2*Log[1 + c^2*x^2])/(6*c^2) + (2*b^2*d^2*PolyLog[2, 1 - 2/(1 + I*c*x)])
/(3*c^2)
```


Rule 4876

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
```

```
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x(d+icdx)^2\left(a+b\tan^{-1}(cx)\right)^2\,dx=\int\left(d^2x\left(a+b\tan^{-1}(cx)\right)^2+2icd^2x^2\left(a+b\tan^{-1}(cx)\right)^2-c^2d^2x^3\left(a+b\tan^{-1}(c\right.\right.$$
$$=d^2\int x\left(a+b\tan^{-1}(cx)\right)^2\,dx+\left(2icd^2\right)\int x^2\left(a+b\tan^{-1}(cx)\right)^2\,dx-\left(c^2d^2\right)\int$$
$$=\frac{1}{2}d^2x^2\left(a+b\tan^{-1}(cx)\right)^2+\frac{2}{3}icd^2x^3\left(a+b\tan^{-1}(cx)\right)^2-\frac{1}{4}c^2d^2x^4\left(a+b\tan^{-1}(\right.$$
$$=\frac{1}{2}d^2x^2\left(a+b\tan^{-1}(cx)\right)^2+\frac{2}{3}icd^2x^3\left(a+b\tan^{-1}(cx)\right)^2-\frac{1}{4}c^2d^2x^4\left(a+b\tan^{-1}(\right.$$
$$=-\frac{abd^2x}{c}-\frac{2}{3}ibd^2x^2\left(a+b\tan^{-1}(cx)\right)+\frac{1}{6}bcd^2x^3\left(a+b\tan^{-1}(cx)\right)+\frac{7d^2\left(a+b\right.}{6}$$
$$=-\frac{3abd^2x}{2c}+\frac{2ib^2d^2x}{3c}-\frac{b^2d^2x\tan^{-1}(cx)}{c}-\frac{2}{3}ibd^2x^2\left(a+b\tan^{-1}(cx)\right)+\frac{1}{6}bcd^2x^3$$
$$=-\frac{3abd^2x}{2c}+\frac{2ib^2d^2x}{3c}-\frac{2ib^2d^2\tan^{-1}(cx)}{3c^2}-\frac{3b^2d^2x\tan^{-1}(cx)}{2c}-\frac{2}{3}ibd^2x^2\left(a+b\,t\right.$$
$$=-\frac{3abd^2x}{2c}+\frac{2ib^2d^2x}{3c}-\frac{1}{12}b^2d^2x^2-\frac{2ib^2d^2\tan^{-1}(cx)}{3c^2}-\frac{3b^2d^2x\tan^{-1}(cx)}{2c}-\frac{2}{3}ib$$

Mathematica [A] time = 0.775093, size = 257, normalized size = 0.88

$$-\frac{d^2\left(8b^2\text{PolyLog}\left(2,-e^{2i\tan^{-1}(cx)}\right)+3a^2c^4x^4-8ia^2c^3x^3-6a^2c^2x^2-2abc^3x^3+8iabc^2x^2-8iab\log\left(c^2x^2+1\right)+2b\tan\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*(d + I*c*d*x)^2*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] -(d^2*(b^2 + 18*a*b*c*x - (8*I)*b^2*c*x - 6*a^2*c^2*x^2 + (8*I)*a*b*c^2*x^2
 + b^2*c^2*x^2 - (8*I)*a^2*c^3*x^3 - 2*a*b*c^3*x^3 + 3*a^2*c^4*x^4 + b^2*(-
I + c*x)^3*(I + 3*c*x)*ArcTan[c*x]^2 + 2*b*ArcTan[c*x]*(b*(4*I + 9*c*x + (4
*I)*c^2*x^2 - c^3*x^3) + a*(-9 - 6*c^2*x^2 - (8*I)*c^3*x^3 + 3*c^4*x^4) + (
8*I)*b*Log[1 + E^((2*I)*ArcTan[c*x])]) - (8*I)*a*b*Log[1 + c^2*x^2] - 10*b^
2*Log[1 + c^2*x^2] + 8*b^2*PolyLog[2, -E^((2*I)*ArcTan[c*x])]))/(12*c^2)
```

Maple [B] time = 0.097, size = 556, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d+I*c*d*x)^2*(a+b*arctan(c*x))^2,x)
```

```
[Out] -1/4*c^2*d^2*a^2*x^4+1/3/c^2*d^2*b^2*dilog(-1/2*I*(c*x+I))+3/4/c^2*d^2*b^2*
arctan(c*x)^2-1/6/c^2*d^2*b^2*ln(c*x+I)^2+1/6/c^2*d^2*b^2*ln(c*x-I)^2-1/3/c
^2*d^2*b^2*dilog(1/2*I*(c*x-I))+1/2*d^2*b^2*arctan(c*x)^2*x^2+4/3*I*c*d^2*a
*b*arctan(c*x)*x^3+2/3*I*c*d^2*a^2*x^3+d^2*a*b*arctan(c*x)*x^2-2/3*I*d^2*a*
b*x^2+1/3/c^2*d^2*b^2*ln(c*x+I)*ln(c^2*x^2+1)+1/3/c^2*d^2*b^2*ln(c*x-I)*ln(
-1/2*I*(c*x+I))-2/3*I*d^2*b^2*arctan(c*x)*x^2-1/4*c^2*d^2*b^2*arctan(c*x)^2
*x^4+3/2/c^2*d^2*a*b*arctan(c*x)-1/3/c^2*d^2*b^2*ln(c*x-I)*ln(c^2*x^2+1)+1/
6*c*d^2*b^2*arctan(c*x)*x^3-1/3/c^2*d^2*b^2*ln(c*x+I)*ln(1/2*I*(c*x-I))+1/6
*c*d^2*a*b*x^3-1/12*b^2*d^2*x^2+2/3*I/c^2*d^2*b^2*arctan(c*x)*ln(c^2*x^2+1)
+2/3*I/c^2*d^2*a*b*ln(c^2*x^2+1)+2/3*I*c*d^2*b^2*arctan(c*x)^2*x^3-1/2*c^2*
d^2*a*b*arctan(c*x)*x^4+1/2*d^2*a^2*x^2+2/3*I*b^2*d^2*x/c-2/3*I*b^2*d^2*arc
tan(c*x)/c^2-3/2*a*b*d^2*x/c-3/2*b^2*d^2*x*arctan(c*x)/c+5/6*b^2*d^2*ln(c^2
*x^2+1)/c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d+I*c*d*x)^2*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

```
[Out] -1/4*a^2*c^2*d^2*x^4 + 2/3*I*a^2*c*d^2*x^3 + 1/2*b^2*d^2*x^2*arctan(c*x)^2
- 1/6*(3*x^4*arctan(c*x) - c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5))*a*b
*c^2*d^2 + 2/3*I*(2*x^3*arctan(c*x) - c*(x^2/c^2 - log(c^2*x^2 + 1)/c^4))*a
*b*c*d^2 + 1/2*a^2*d^2*x^2 + (x^2*arctan(c*x) - c*(x/c^2 - arctan(c*x)/c^3)
)*a*b*d^2 - 1/2*(2*c*(x/c^2 - arctan(c*x)/c^3)*arctan(c*x) + (arctan(c*x)^2
 - log(c^2*x^2 + 1))/c^2)*b^2*d^2 - 1/48*(3*b^2*c^2*d^2*x^4 - 8*I*b^2*c*d^2
*x^3)*arctan(c*x)^2 - 1/192*(12*I*b^2*c^2*d^2*x^4 + 32*b^2*c*d^2*x^3)*arcta
n(c*x)*log(c^2*x^2 + 1) + 1/192*(3*b^2*c^2*d^2*x^4 - 8*I*b^2*c*d^2*x^3)*log
(c^2*x^2 + 1)^2 - integrate(-1/48*(22*b^2*c^3*d^2*x^4*arctan(c*x) - 36*(b^2
*c^4*d^2*x^5 + b^2*c^2*d^2*x^3)*arctan(c*x)^2 - 3*(b^2*c^4*d^2*x^5 + b^2*c^
2*d^2*x^3)*log(c^2*x^2 + 1)^2 - (3*b^2*c^4*d^2*x^5 - 8*b^2*c^2*d^2*x^3 - 24
*(b^2*c^3*d^2*x^4 + b^2*c*d^2*x^2)*arctan(c*x))*log(c^2*x^2 + 1))/(c^2*x^2
+ 1), x) + I*integrate(1/48*(72*(b^2*c^3*d^2*x^4 + b^2*c*d^2*x^2)*arctan(c*
x)^2 + 6*(b^2*c^3*d^2*x^4 + b^2*c*d^2*x^2)*log(c^2*x^2 + 1)^2 + 2*(3*b^2*c^
4*d^2*x^5 - 8*b^2*c^2*d^2*x^3)*arctan(c*x) + (11*b^2*c^3*d^2*x^4 + 12*(b^2*
c^4*d^2*x^5 + b^2*c^2*d^2*x^3)*arctan(c*x))*log(c^2*x^2 + 1))/(c^2*x^2 + 1)
```

```
, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{48}\left(3\,b^2c^2d^2x^4 - 8i\,b^2cd^2x^3 - 6\,b^2d^2x^2\right)\log\left(-\frac{cx+i}{cx-i}\right)^2 + \mathrm{integral}\left(-\frac{12\,a^2c^4d^2x^5 - 24i\,a^2c^3d^2x^4 - 24i\,a^2cd^2x^2 - 12\,a^2d^2}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d+I*c*d*x)^2*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/48*(3*b^2*c^2*d^2*x^4 - 8*I*b^2*c*d^2*x^3 - 6*b^2*d^2*x^2)*log(-(c*x + I)
/(c*x - I))^2 + integral(-1/12*(12*a^2*c^4*d^2*x^5 - 24*I*a^2*c^3*d^2*x^4 -
 24*I*a^2*c*d^2*x^2 - 12*a^2*d^2*x - (-12*I*a*b*c^4*d^2*x^5 - 3*(8*a*b - I*
b^2)*c^3*d^2*x^4 + 8*b^2*c^2*d^2*x^3 - 6*(4*a*b + I*b^2)*c*d^2*x^2 + 12*I*a
*b*d^2*x)*log(-(c*x + I)/(c*x - I)))/(c^2*x^2 + 1), x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d+I*c*d*x)**2*(a+b*atan(c*x))**2,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (i\,cdx + d)^2(b\arctan(cx) + a)^2x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d+I*c*d*x)^2*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((I*c*d*x + d)^2*(b*arctan(c*x) + a)^2*x, x)
```

3.79 $\int (d+icdx)^2 \left(a+b\tan^{-1}(cx)\right)^2 dx$

Optimal. Leaf size=192

$$-\frac{4ib^2d^2\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{3c}+\frac{1}{3}bcd^2x^2\left(a+b\tan^{-1}(cx)\right)-\frac{id^2(1+icx)^3\left(a+b\tan^{-1}(cx)\right)^2}{3c}+\frac{8bd^2\log\left(\frac{2}{1-icx}\right)\left(a+b\text{t}\right.}{3c}$$

```
[Out] (-2*I)*a*b*d^2*x - (b^2*d^2*x)/3 + (b^2*d^2*ArcTan[c*x])/(3*c) - (2*I)*b^2*
d^2*x*ArcTan[c*x] + (b*c*d^2*x^2*(a + b*ArcTan[c*x]))/3 - ((I/3)*d^2*(1 + I
*c*x)^3*(a + b*ArcTan[c*x])^2)/c + (8*b*d^2*(a + b*ArcTan[c*x])*Log[2/(1 -
I*c*x)])/(3*c) + (I*b^2*d^2*Log[1 + c^2*x^2])/c - (((4*I)/3)*b^2*d^2*PolyLo
g[2, 1 - 2/(1 - I*c*x)])/c
```

Rubi [A] time = 0.196765, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {4864, 4846, 260, 4852, 321, 203, 1586, 4854, 2402, 2315}

$$-\frac{4ib^2d^2\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{3c}+\frac{1}{3}bcd^2x^2\left(a+b\tan^{-1}(cx)\right)-\frac{id^2(1+icx)^3\left(a+b\tan^{-1}(cx)\right)^2}{3c}+\frac{8bd^2\log\left(\frac{2}{1-icx}\right)\left(a+b\text{t}\right.}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + I*c*d*x)^2*(a + b*ArcTan[c*x])^2,x]
```


```
[Out] (-2*I)*a*b*d^2*x - (b^2*d^2*x)/3 + (b^2*d^2*ArcTan[c*x])/(3*c) - (2*I)*b^2*
d^2*x*ArcTan[c*x] + (b*c*d^2*x^2*(a + b*ArcTan[c*x]))/3 - ((I/3)*d^2*(1 + I
*c*x)^3*(a + b*ArcTan[c*x])^2)/c + (8*b*d^2*(a + b*ArcTan[c*x])*Log[2/(1 -
I*c*x)])/(3*c) + (I*b^2*d^2*Log[1 + c^2*x^2])/c - (((4*I)/3)*b^2*d^2*PolyLo
g[2, 1 - 2/(1 - I*c*x)])/c
```


Rule 4864

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x])^p)/(e*(q + 1)), x] - D
ist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int (d+icdx)^2\left(a+b\tan^{-1}(cx)\right)^2\,dx=-\frac{id^2(1+icx)^3\left(a+b\tan^{-1}(cx)\right)^2}{3c}+\frac{(2ib)\int\left(-3d^3\left(a+b\tan^{-1}(cx)\right)-icd^3x\left(a+b\right.\right.}{3d}$$

$$=-\frac{id^2(1+icx)^3\left(a+b\tan^{-1}(cx)\right)^2}{3c}+\frac{(8b)\int\frac{\left(id^3-cd^3x\right)\left(a+b\tan^{-1}(cx)\right)}{1+c^2x^2}\,dx}{3d}-\left(2ibd^2\right)\int\left(a\right.$$

$$=-2iabd^2x+\frac{1}{3}bcd^2x^2\left(a+b\tan^{-1}(cx)\right)-\frac{id^2(1+icx)^3\left(a+b\tan^{-1}(cx)\right)^2}{3c}+\frac{(8b)\int}{}$$

$$=-2iabd^2x-\frac{1}{3}b^2d^2x-2ib^2d^2x\tan^{-1}(cx)+\frac{1}{3}bcd^2x^2\left(a+b\tan^{-1}(cx)\right)-\frac{id^2(1+icx}{}$$

$$=-2iabd^2x-\frac{1}{3}b^2d^2x+\frac{b^2d^2\tan^{-1}(cx)}{3c}-2ib^2d^2x\tan^{-1}(cx)+\frac{1}{3}bcd^2x^2\left(a+b\tan^{-1}(\right.$$

$$=-2iabd^2x-\frac{1}{3}b^2d^2x+\frac{b^2d^2\tan^{-1}(cx)}{3c}-2ib^2d^2x\tan^{-1}(cx)+\frac{1}{3}bcd^2x^2\left(a+b\tan^{-1}(\right.$$

Mathematica [A] time = 0.636959, size = 205, normalized size = 1.07

$$-\frac{d^2\left(4ib^2\text{PolyLog}\left(2,-e^{2i\tan^{-1}(cx)}\right)+a^2c^3x^3-3ia^2c^2x^2-3a^2cx-abc^2x^2+4ab\log\left(c^2x^2+1\right)-b\tan^{-1}(cx)\left(a\left(-2c^3x^3+\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + I*c*d*x)^2*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] -(d^2*(-3*a^2*c*x + (6*I)*a*b*c*x + b^2*c*x - (3*I)*a^2*c^2*x^2 - a*b*c^2*x
^2 + a^2*c^3*x^3 + b^2*(-I + c*x)^3*ArcTan[c*x]^2 - b*ArcTan[c*x]*(b*(1 - (
```

```
6*I)*c*x + c^2*x^2) + a*(6*I + 6*c*x + (6*I)*c^2*x^2 - 2*c^3*x^3) + 8*b*Log
[1 + E^((2*I)*ArcTan[c*x])]) + 4*a*b*Log[1 + c^2*x^2] - (3*I)*b^2*Log[1 + c
^2*x^2] + (4*I)*b^2*PolyLog[2, -E^((2*I)*ArcTan[c*x])]))/(3*c)
```

Maple [B] time = 0.089, size = 523, normalized size = 2.7

$$\frac{d^2b^2\arctan(cx)}{3c} - \frac{\frac{i}{3}d^2b^2(\ln(cx+i))^2}{c} + \frac{cd^2abx^2}{3} - \frac{4d^2ab\ln(c^2x^2+1)}{3c} - \frac{4d^2b^2\arctan(cx)\ln(c^2x^2+1)}{3c} + \frac{id^2b^2(\mathrm{ar}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^2*(a+b*arctan(c*x))^2,x)
```

```
[Out] 1/3*b^2*d^2*arctan(c*x)/c-1/3*I/c*d^2*b^2*ln(c*x+I)^2+1/3*c*d^2*a*b*x^2-4/3
/c*d^2*a*b*ln(c^2*x^2+1)-4/3/c*d^2*b^2*arctan(c*x)*ln(c^2*x^2+1)+I/c*d^2*b^
2*arctan(c*x)^2+I*c*x^2*a^2*d^2-1/3*c^2*d^2*b^2*arctan(c*x)^2*x^3+2*d^2*a*b
*arctan(c*x)*x-2/3*I/c*d^2*b^2*dilog(1/2*I*(c*x-I))+1/3*I/c*d^2*b^2*ln(c*x-
I)^2+2/3*I/c*d^2*b^2*dilog(-1/2*I*(c*x+I))+2*I*c*d^2*a*b*arctan(c*x)*x^2-1/
3*c^2*x^3*a^2*d^2+d^2*b^2*arctan(c*x)^2*x-1/3*I/c*d^2*a^2+1/3*c*d^2*b^2*arc
tan(c*x)*x^2-1/3*b^2*d^2*x+I*c*d^2*b^2*arctan(c*x)^2*x^2-2/3*c^2*d^2*a*b*ar
ctan(c*x)*x^3+2/3*I/c*d^2*b^2*ln(c*x-I)*ln(-1/2*I*(c*x+I))-2/3*I/c*d^2*b^2*
ln(c*x+I)*ln(1/2*I*(c*x-I))-2/3*I/c*d^2*b^2*ln(c^2*x^2+1)*ln(c*x-I)+2/3*I/c
*d^2*b^2*ln(c^2*x^2+1)*ln(c*x+I)+2*I/c*d^2*a*b*arctan(c*x)+x*a^2*d^2-2*I*a*
b*d^2*x-2*I*b^2*d^2*x*arctan(c*x)+I*b^2*d^2*ln(c^2*x^2+1)/c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

```
[Out] -1/3*a^2*c^2*d^2*x^3 - 36*b^2*c^4*d^2*integrate(1/48*x^4*arctan(c*x)^2/(c^2
*x^2 + 1), x) - 3*b^2*c^4*d^2*integrate(1/48*x^4*log(c^2*x^2 + 1)^2/(c^2*x^
2 + 1), x) - 4*b^2*c^4*d^2*integrate(1/48*x^4*log(c^2*x^2 + 1)/(c^2*x^2 + 1
), x) + 24*b^2*c^3*d^2*integrate(1/48*x^3*arctan(c*x)*log(c^2*x^2 + 1)/(c^2
*x^2 + 1), x) + 32*b^2*c^3*d^2*integrate(1/48*x^3*arctan(c*x)/(c^2*x^2 + 1)
, x) - 1/3*(2*x^3*arctan(c*x) - c*(x^2/c^2 - log(c^2*x^2 + 1)/c^4))*a*b*c^2
```

```
*d^2 + I*a^2*c*d^2*x^2 + 24*b^2*c^2*d^2*integrate(1/48*x^2*log(c^2*x^2 + 1)
/(c^2*x^2 + 1), x) + 2*I*(x^2*arctan(c*x) - c*(x/c^2 - arctan(c*x)/c^3))*a*
b*c*d^2 + 1/4*b^2*d^2*arctan(c*x)^3/c + 24*b^2*c*d^2*integrate(1/48*x*arcta
n(c*x)*log(c^2*x^2 + 1)/(c^2*x^2 + 1), x) - 24*b^2*c*d^2*integrate(1/48*x*a
rctan(c*x)/(c^2*x^2 + 1), x) + a^2*d^2*x + 3*b^2*d^2*integrate(1/48*log(c^2
*x^2 + 1)^2/(c^2*x^2 + 1), x) + (2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*a*b*
d^2/c - 1/48*(4*b^2*c^2*d^2*x^3 - 12*I*b^2*c*d^2*x^2 - 12*b^2*d^2*x)*arctan
(c*x)^2 - 1/48*(4*I*b^2*c^2*d^2*x^3 + 12*b^2*c*d^2*x^2 - 12*I*b^2*d^2*x)*ar
ctan(c*x)*log(c^2*x^2 + 1) + 1/48*(b^2*c^2*d^2*x^3 - 3*I*b^2*c*d^2*x^2 - 3*
b^2*d^2*x)*log(c^2*x^2 + 1)^2 + I*integrate(1/24*(36*(b^2*c^3*d^2*x^3 + b^2
*c*d^2*x)*arctan(c*x)^2 + 3*(b^2*c^3*d^2*x^3 + b^2*c*d^2*x)*log(c^2*x^2 + 1
)^2 + 4*(b^2*c^4*d^2*x^4 - 6*b^2*c^2*d^2*x^2)*arctan(c*x) + 2*(4*b^2*c^3*d^
2*x^3 - 3*b^2*c*d^2*x + 3*(b^2*c^4*d^2*x^4 - b^2*d^2)*arctan(c*x))*log(c^2*
x^2 + 1))/(c^2*x^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{12}\left(b^2c^2d^2x^3 - 3i\,b^2cd^2x^2 - 3\,b^2d^2x\right)\log\left(-\frac{cx+i}{cx-i}\right)^2 + \text{integral}\left(-\frac{3\,a^2c^4d^2x^4 - 6i\,a^2c^3d^2x^3 - 6i\,a^2cd^2x - 3\,a^2d^2 - \left(-3i\,ab\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/12*(b^2*c^2*d^2*x^3 - 3*I*b^2*c*d^2*x^2 - 3*b^2*d^2*x)*log(-(c*x + I)/(c*
x - I))^2 + integral(-1/3*(3*a^2*c^4*d^2*x^4 - 6*I*a^2*c^3*d^2*x^3 - 6*I*a^
2*c*d^2*x - 3*a^2*d^2 - (-3*I*a*b*c^4*d^2*x^4 - (6*a*b - I*b^2)*c^3*d^2*x^3
 + 3*b^2*c^2*d^2*x^2 - 3*(2*a*b + I*b^2)*c*d^2*x + 3*I*a*b*d^2)*log(-(c*x +
 I)/(c*x - I)))/(c^2*x^2 + 1), x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**2*(a+b*atan(c*x))**2,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (icdx+d)^2(b\arctan(cx)+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((I*c*d*x + d)^2*(b*arctan(c*x) + a)^2, x)
```

3.80 $\int \frac{(d+icdx)^2\left(a+b\tan^{-1}(cx)\right)^2}{x}\,dx$

Optimal. Leaf size=300

$$-ibd^2\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)+ibd^2\text{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)-2b^2d^2\text{PolyLog}\left(2\right.$$

```
[Out] a*b*c*d^2*x + b^2*c*d^2*x*ArcTan[c*x] - (5*d^2*(a + b*ArcTan[c*x])^2)/2 + (
2*I)*c*d^2*x*(a + b*ArcTan[c*x])^2 - (c^2*d^2*x^2*(a + b*ArcTan[c*x])^2)/2
+ 2*d^2*(a + b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1 + I*c*x)] + (4*I)*b*d^2*(a +
 b*ArcTan[c*x])*Log[2/(1 + I*c*x)] - (b^2*d^2*Log[1 + c^2*x^2])/2 - 2*b^2*d
^2*PolyLog[2, 1 - 2/(1 + I*c*x)] - I*b*d^2*(a + b*ArcTan[c*x])*PolyLog[2, 1
 - 2/(1 + I*c*x)] + I*b*d^2*(a + b*ArcTan[c*x])*PolyLog[2, -1 + 2/(1 + I*c*
x)] - (b^2*d^2*PolyLog[3, 1 - 2/(1 + I*c*x)])/2 + (b^2*d^2*PolyLog[3, -1 +
2/(1 + I*c*x)])/2
```

Rubi [A] time = 0.57812, antiderivative size = 300, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 14, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.56, Rules used = {4876, 4846, 4920, 4854, 2402, 2315, 4850, 4988, 4884, 4994, 6610, 4852, 4916, 260}

$$-ibd^2\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)+ibd^2\text{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)-2b^2d^2\text{PolyLog}\left(2\right.$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)^2*(a + b*ArcTan[c*x])^2)/x,x]
```


```
[Out] a*b*c*d^2*x + b^2*c*d^2*x*ArcTan[c*x] - (5*d^2*(a + b*ArcTan[c*x])^2)/2 + (
2*I)*c*d^2*x*(a + b*ArcTan[c*x])^2 - (c^2*d^2*x^2*(a + b*ArcTan[c*x])^2)/2
+ 2*d^2*(a + b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1 + I*c*x)] + (4*I)*b*d^2*(a +
 b*ArcTan[c*x])*Log[2/(1 + I*c*x)] - (b^2*d^2*Log[1 + c^2*x^2])/2 - 2*b^2*d
^2*PolyLog[2, 1 - 2/(1 + I*c*x)] - I*b*d^2*(a + b*ArcTan[c*x])*PolyLog[2, 1
 - 2/(1 + I*c*x)] + I*b*d^2*(a + b*ArcTan[c*x])*PolyLog[2, -1 + 2/(1 + I*c*
x)] - (b^2*d^2*PolyLog[3, 1 - 2/(1 + I*c*x)])/2 + (b^2*d^2*PolyLog[3, -1 +
2/(1 + I*c*x)])/2
```


Rule 4876

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
```

```
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 4850

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTan[c*x])^(p - 1)*ArcTanh[1 - 2/(1 + I*c*x)])/(1 + c^2*x^2), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4988

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTan[c*x])^p)/(d + e
```

```
*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTan[c*x])^p)/(d + e*x^2
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{(d+icdx)^2\left(a+b\tan^{-1}(cx)\right)^2}{x}\,dx = \int\left(2icd^2\left(a+b\tan^{-1}(cx)\right)^2+\frac{d^2\left(a+b\tan^{-1}(cx)\right)^2}{x}-c^2d^2x\left(a+b\tan^{-1}(cx)\right)^2\right)$$

$$= d^2\int\frac{\left(a+b\tan^{-1}(cx)\right)^2}{x}\,dx+\left(2icd^2\right)\int\left(a+b\tan^{-1}(cx)\right)^2\,dx-\left(c^2d^2\right)\int x\left(a+\right.$$

$$= 2icd^2x\left(a+b\tan^{-1}(cx)\right)^2-\frac{1}{2}c^2d^2x^2\left(a+b\tan^{-1}(cx)\right)^2+2d^2\left(a+b\tan^{-1}(cx)\right)^2$$

$$= -2d^2\left(a+b\tan^{-1}(cx)\right)^2+2icd^2x\left(a+b\tan^{-1}(cx)\right)^2-\frac{1}{2}c^2d^2x^2\left(a+b\tan^{-1}(cx)\right)$$

$$= abcd^2x-\frac{5}{2}d^2\left(a+b\tan^{-1}(cx)\right)^2+2icd^2x\left(a+b\tan^{-1}(cx)\right)^2-\frac{1}{2}c^2d^2x^2\left(a+b\,\mathrm{ta}\right.$$

$$= abcd^2x+b^2cd^2x\tan^{-1}(cx)-\frac{5}{2}d^2\left(a+b\tan^{-1}(cx)\right)^2+2icd^2x\left(a+b\tan^{-1}(cx)\right)^2$$

$$= abcd^2x+b^2cd^2x\tan^{-1}(cx)-\frac{5}{2}d^2\left(a+b\tan^{-1}(cx)\right)^2+2icd^2x\left(a+b\tan^{-1}(cx)\right)^2$$

Mathematica [A] time = 0.651142, size = 360, normalized size = 1.2

$$\frac{1}{2}d^2\left(2iab(\text{PolyLog}(2,-icx)-\text{PolyLog}(2,icx))+4b^2\left(\text{PolyLog}\left(2,-e^{2i\tan^{-1}(cx)}\right)+\tan^{-1}(cx)\left((1+icx)\tan^{-1}(cx)+2i\right.\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + I*c*d*x)^2*(a + b*ArcTan[c*x])^2)/x,x]
```

```
[Out] (d^2*((4*I)*a^2*c*x - a^2*c^2*x^2 + 2*b^2*c*x*ArcTan[c*x] - b^2*(1 + c^2*x^
2)*ArcTan[c*x]^2 - 2*a*b*(-(c*x) + (1 + c^2*x^2)*ArcTan[c*x]) + 2*a^2*Log[c
*x] + (4*I)*a*b*(2*c*x*ArcTan[c*x] - Log[1 + c^2*x^2]) - b^2*Log[1 + c^2*x^
2] + 4*b^2*(ArcTan[c*x]*((1 + I*c*x)*ArcTan[c*x] + (2*I)*Log[1 + E^((2*I)*A
rcTan[c*x])]) + PolyLog[2, -E^((2*I)*ArcTan[c*x])]) + (2*I)*a*b*(PolyLog[2,
 (-I)*c*x] - PolyLog[2, I*c*x]) + 2*b^2*((-I/24)*Pi^3 + ((2*I)/3)*ArcTan[c*
x]^3 + ArcTan[c*x]^2*Log[1 - E^((-2*I)*ArcTan[c*x])] - ArcTan[c*x]^2*Log[1
+ E^((2*I)*ArcTan[c*x])] + I*ArcTan[c*x]*PolyLog[2, E^((-2*I)*ArcTan[c*x])]
 + I*ArcTan[c*x]*PolyLog[2, -E^((2*I)*ArcTan[c*x])] + PolyLog[3, E^((-2*I)*
ArcTan[c*x])]/2 - PolyLog[3, -E^((2*I)*ArcTan[c*x])]/2))/2
```

Maple [C] time = 1.217, size = 1542, normalized size = 5.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^2*(a+b*arctan(c*x))^2/x,x)
```

```
[Out] d^2*a^2*ln(c*x)+2*d^2*b^2*polylog(3,(1+I*c*x)/(c^2*x^2+1)^(1/2))+4*d^2*b^2*
dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+4*d^2*b^2*dilog(1-I*(1+I*c*x)/(c^2*x
^2+1)^(1/2))+d^2*b^2*ln((1+I*c*x)^2/(c^2*x^2+1)+1)+2*d^2*b^2*polylog(3,-(1+
I*c*x)/(c^2*x^2+1)^(1/2))-1/2*d^2*b^2*polylog(3,-(1+I*c*x)^2/(c^2*x^2+1))+a
*b*c*d^2*x+b^2*c*d^2*x*arctan(c*x)-1/2*I*d^2*b^2*Pi*csgn(I/((1+I*c*x)^2/(c^
2*x^2+1)+1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1)
)^2*arctan(c*x)^2-1/2*I*d^2*b^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I
*c*x)^2/(c^2*x^2+1)+1))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*
x^2+1)+1))^2*arctan(c*x)^2+4*I*d^2*a*b*arctan(c*x)*c*x+1/2*I*d^2*b^2*Pi*csg
n(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(((1+I*c*x
)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)^2-1/2*I*d^2*b^2
*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/
((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2+1/2*I*d^2*b^2*Pi*csgn(((1+I*c*
x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*arctan(c*x)^2+1/2*I*d^2*
b^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+
1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan(
c*x)^2+3/2*d^2*b^2*arctan(c*x)^2+I*d^2*a*b*ln(c*x)*ln(1+I*c*x)-I*d^2*a*b*ln
(c*x)*ln(1-I*c*x)-d^2*a*b*arctan(c*x)*c^2*x^2+2*I*d^2*b^2*arctan(c*x)^2*c*x
-1/2*I*d^2*b^2*Pi*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)
+1))^2*arctan(c*x)^2+1/2*I*d^2*b^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((
1+I*c*x)^2/(c^2*x^2+1)+1))^3*arctan(c*x)^2-1/2*d^2*a^2*c^2*x^2-d^2*b^2*arct
an(c*x)^2*ln((1+I*c*x)^2/(c^2*x^2+1)-1)+d^2*b^2*arctan(c*x)^2*ln(1+(1+I*c*x
)/(c^2*x^2+1)^(1/2))+d^2*b^2*arctan(c*x)^2*ln(c*x)+d^2*b^2*arctan(c*x)^2*ln
(1-(1+I*c*x)/(c^2*x^2+1)^(1/2))-I*d^2*b^2*arctan(c*x)-d^2*a*b*arctan(c*x)-1
/2*d^2*b^2*arctan(c*x)^2*c^2*x^2+2*I*d^2*a^2*c*x+I*d^2*b^2*arctan(c*x)*poly
log(2,-(1+I*c*x)^2/(c^2*x^2+1))+1/2*I*d^2*b^2*Pi*arctan(c*x)^2-2*I*d^2*b^2*
arctan(c*x)*polylog(2,-(1+I*c*x)/(c^2*x^2+1)^(1/2))-2*I*d^2*b^2*arctan(c*x)
*polylog(2,(1+I*c*x)/(c^2*x^2+1)^(1/2))+4*I*d^2*b^2*arctan(c*x)*ln(1+I*(1+I
*c*x)/(c^2*x^2+1)^(1/2))+4*I*d^2*b^2*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+
1)^(1/2))+I*d^2*a*b*dilog(1+I*c*x)+2*d^2*a*b*arctan(c*x)*ln(c*x)-I*d^2*a*b*
dilog(1-I*c*x)-2*I*d^2*a*b*ln(c^2*x^2+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))^2/x,x, algorithm="maxima")
```

```
[Out] -12*b^2*c^4*d^2*integrate(1/16*x^4*arctan(c*x)^2/(c^2*x^3 + x), x) + 2*I*b^
2*c^4*d^2*integrate(1/8*x^4*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^3 + x), x)
- b^2*c^4*d^2*integrate(1/16*x^4*log(c^2*x^2 + 1)^2/(c^2*x^3 + x), x) + 2*I
*b^2*c^4*d^2*integrate(1/8*x^4*arctan(c*x)/(c^2*x^3 + x), x) - 32*a*b*c^4*d
^2*integrate(1/16*x^4*arctan(c*x)/(c^2*x^3 + x), x) - 2*b^2*c^4*d^2*integra
te(1/16*x^4*log(c^2*x^2 + 1)/(c^2*x^3 + x), x) - 1/2*a^2*c^2*d^2*x^2 + 12*I
*b^2*c^3*d^2*integrate(1/8*x^3*arctan(c*x)^2/(c^2*x^3 + x), x) + 8*b^2*c^3*
d^2*integrate(1/16*x^3*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^3 + x), x) + I*b
^2*c^3*d^2*integrate(1/8*x^3*log(c^2*x^2 + 1)^2/(c^2*x^3 + x), x) + 20*b^2*
c^3*d^2*integrate(1/16*x^3*arctan(c*x)/(c^2*x^3 + x), x) + 5*I*b^2*c^3*d^2*
integrate(1/8*x^3*log(c^2*x^2 + 1)/(c^2*x^3 + x), x) + 1/2*I*b^2*d^2*arctan
(c*x)^3 - 8*I*b^2*c^2*d^2*integrate(1/8*x^2*arctan(c*x)/(c^2*x^3 + x), x) +
 2*I*a^2*c*d^2*x + 8*b^2*c*d^2*integrate(1/16*x*arctan(c*x)*log(c^2*x^2 + 1
)/(c^2*x^3 + x), x) + I*b^2*c*d^2*integrate(1/8*x*log(c^2*x^2 + 1)^2/(c^2*x
^3 + x), x) + 1/8*b^2*d^2*log(c^2*x^2 + 1)^2 + 2*I*(2*c*x*arctan(c*x) - log
(c^2*x^2 + 1))*a*b*d^2 + 12*b^2*d^2*integrate(1/16*arctan(c*x)^2/(c^2*x^3 +
 x), x) - 2*I*b^2*d^2*integrate(1/8*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^3 +
 x), x) + b^2*d^2*integrate(1/16*log(c^2*x^2 + 1)^2/(c^2*x^3 + x), x) + 32*
a*b*d^2*integrate(1/16*arctan(c*x)/(c^2*x^3 + x), x) + a^2*d^2*log(x) - 1/8
*(b^2*c^2*d^2*x^2 - 4*I*b^2*c*d^2*x)*arctan(c*x)^2 - 1/32*(4*I*b^2*c^2*d^2*
x^2 + 16*b^2*c*d^2*x)*arctan(c*x)*log(c^2*x^2 + 1) + 1/32*(b^2*c^2*d^2*x^2
- 4*I*b^2*c*d^2*x)*log(c^2*x^2 + 1)^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{4\,a^2c^2d^2x^2 - 8i\,a^2cd^2x - 4\,a^2d^2 - \left(b^2c^2d^2x^2 - 2i\,b^2cd^2x - b^2d^2\right)\log\left(-\frac{cx+i}{cx-i}\right)^2 - \left(-4i\,abc^2d^2x^2 - 8\,abcd^2x + 4\right.}{4\,x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))^2/x,x, algorithm="fricas")
```

```
[Out] integral(-1/4*(4*a^2*c^2*d^2*x^2 - 8*I*a^2*c*d^2*x - 4*a^2*d^2 - (b^2*c^2*d
^2*x^2 - 2*I*b^2*c*d^2*x - b^2*d^2)*log(-(c*x + I)/(c*x - I))^2 - (-4*I*a*b
*c^2*d^2*x^2 - 8*a*b*c*d^2*x + 4*I*a*b*d^2)*log(-(c*x + I)/(c*x - I)))/x, x
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d^2\left(\int\frac{a^2}{x}\,dx+\int 2ia^2c\,dx+\int -a^2c^2x\,dx+\int\frac{b^2\operatorname{atan}^2(cx)}{x}\,dx+\int 2ib^2c\operatorname{atan}^2(cx)\,dx+\int\frac{2ab\operatorname{atan}(cx)}{x}\,dx+\int -b^2c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**2*(a+b*atan(c*x))**2/x,x)
```

```
[Out] d**2*(Integral(a**2/x, x) + Integral(2*I*a**2*c, x) + Integral(-a**2*c**2*x
, x) + Integral(b**2*atan(c*x)**2/x, x) + Integral(2*I*b**2*c*atan(c*x)**2,
 x) + Integral(2*a*b*atan(c*x)/x, x) + Integral(-b**2*c**2*x*atan(c*x)**2,
x) + Integral(4*I*a*b*c*atan(c*x), x) + Integral(-2*a*b*c**2*x*atan(c*x), x
))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(icdx+d)^2(b\arctan(cx)+a)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))^2/x,x, algorithm="giac")
```

```
[Out] integrate((I*c*d*x + d)^2*(b*arctan(c*x) + a)^2/x, x)
```

3.81 $\int \frac{(d+icdx)^2 \left(a+b\tan^{-1}(cx)\right)^2}{x^2} \, dx$

Optimal. Leaf size=317

$$2bcd^2\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)-2bcd^2\text{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)-ib^2cd^2\text{PolyLog}$$

```
[Out] (-2*I)*c*d^2*(a + b*ArcTan[c*x])^2 - (d^2*(a + b*ArcTan[c*x])^2)/x - c^2*d^
2*x*(a + b*ArcTan[c*x])^2 + (4*I)*c*d^2*(a + b*ArcTan[c*x])^2*ArcTanh[1 - 2
/(1 + I*c*x)] - 2*b*c*d^2*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)] + 2*b*c*d^
2*(a + b*ArcTan[c*x])*Log[2 - 2/(1 - I*c*x)] - I*b^2*c*d^2*PolyLog[2, -1 +
2/(1 - I*c*x)] - I*b^2*c*d^2*PolyLog[2, 1 - 2/(1 + I*c*x)] + 2*b*c*d^2*(a +
 b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)] - 2*b*c*d^2*(a + b*ArcTan[c*x
])*PolyLog[2, -1 + 2/(1 + I*c*x)] - I*b^2*c*d^2*PolyLog[3, 1 - 2/(1 + I*c*x
)] + I*b^2*c*d^2*PolyLog[3, -1 + 2/(1 + I*c*x)]
```

Rubi [A] time = 0.619669, antiderivative size = 317, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 15, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {4876, 4846, 4920, 4854, 2402, 2315, 4852, 4924, 4868, 2447, 4850, 4988, 4884, 4994, 6610}

$$2bcd^2\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)-2bcd^2\text{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)-ib^2cd^2\text{PolyLog}$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)^2*(a + b*ArcTan[c*x])^2)/x^2,x]
```

```
[Out] (-2*I)*c*d^2*(a + b*ArcTan[c*x])^2 - (d^2*(a + b*ArcTan[c*x])^2)/x - c^2*d^
2*x*(a + b*ArcTan[c*x])^2 + (4*I)*c*d^2*(a + b*ArcTan[c*x])^2*ArcTanh[1 - 2
/(1 + I*c*x)] - 2*b*c*d^2*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)] + 2*b*c*d^
2*(a + b*ArcTan[c*x])*Log[2 - 2/(1 - I*c*x)] - I*b^2*c*d^2*PolyLog[2, -1 +
2/(1 - I*c*x)] - I*b^2*c*d^2*PolyLog[2, 1 - 2/(1 + I*c*x)] + 2*b*c*d^2*(a +
 b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)] - 2*b*c*d^2*(a + b*ArcTan[c*x
])*PolyLog[2, -1 + 2/(1 + I*c*x)] - I*b^2*c*d^2*PolyLog[3, 1 - 2/(1 + I*c*x
)] + I*b^2*c*d^2*PolyLog[3, -1 + 2/(1 + I*c*x)]
```

Rule 4876

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
```

```
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4850

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTan[c*x])^(p - 1)*ArcTanh[1 - 2/(1 + I*c*x)])/(1 + c^2*x^2), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4988

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTan[c*x])^p)/(d + e
*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTan[c*x])^p)/(d + e*x^2
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
```

```
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{(d+icdx)^2\left(a+b\tan^{-1}(cx)\right)^2}{x^2}\,dx = \int\left(-c^2d^2\left(a+b\tan^{-1}(cx)\right)^2+\frac{d^2\left(a+b\tan^{-1}(cx)\right)^2}{x^2}+\frac{2icd^2\left(a+b\tan^{-1}(cx)\right)^2}{x}\right)$$

$$= d^2\int\frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^2}\,dx+\left(2icd^2\right)\int\frac{\left(a+b\tan^{-1}(cx)\right)^2}{x}\,dx-\left(c^2d^2\right)\int\left(a+b\t\right.$$

$$= -\frac{d^2\left(a+b\tan^{-1}(cx)\right)^2}{x}-c^2d^2x\left(a+b\tan^{-1}(cx)\right)^2+4icd^2\left(a+b\tan^{-1}(cx)\right)^2\tanh$$

$$= -2icd^2\left(a+b\tan^{-1}(cx)\right)^2-\frac{d^2\left(a+b\tan^{-1}(cx)\right)^2}{x}-c^2d^2x\left(a+b\tan^{-1}(cx)\right)^2+4ic$$

$$= -2icd^2\left(a+b\tan^{-1}(cx)\right)^2-\frac{d^2\left(a+b\tan^{-1}(cx)\right)^2}{x}-c^2d^2x\left(a+b\tan^{-1}(cx)\right)^2+4ic$$

$$= -2icd^2\left(a+b\tan^{-1}(cx)\right)^2-\frac{d^2\left(a+b\tan^{-1}(cx)\right)^2}{x}-c^2d^2x\left(a+b\tan^{-1}(cx)\right)^2+4ic$$

$$= -2icd^2\left(a+b\tan^{-1}(cx)\right)^2-\frac{d^2\left(a+b\tan^{-1}(cx)\right)^2}{x}-c^2d^2x\left(a+b\tan^{-1}(cx)\right)^2+4ic$$

Mathematica [A] time = 0.632202, size = 378, normalized size = 1.19

$$-\frac{d^2\left(24abcx\text{PolyLog}(2,-icx)-24abcx\text{PolyLog}(2,icx)+24b^2cx\tan^{-1}(cx)\text{PolyLog}\left(2,e^{-2i\tan^{-1}(cx)}\right)+12b^2cx\left(2\tan^{-1}(\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + I*c*d*x)^2*(a + b*ArcTan[c*x])^2)/x^2,x]
```

```
[Out] -(d^2*(12*a^2 - b^2*c*Pi^3*x + 12*a^2*c^2*x^2 + 24*a*b*ArcTan[c*x] + 24*a*b
*c^2*x^2*ArcTan[c*x] + 12*b^2*ArcTan[c*x]^2 + 12*b^2*c^2*x^2*ArcTan[c*x]^2
+ 16*b^2*c*x*ArcTan[c*x]^3 - (24*I)*b^2*c*x*ArcTan[c*x]^2*Log[1 - E^((-2*I)
*ArcTan[c*x])] - 24*b^2*c*x*ArcTan[c*x]*Log[1 - E^((2*I)*ArcTan[c*x])] + 24
*b^2*c*x*ArcTan[c*x]*Log[1 + E^((2*I)*ArcTan[c*x])] + (24*I)*b^2*c*x*ArcTan
[c*x]^2*Log[1 + E^((2*I)*ArcTan[c*x])] - (24*I)*a^2*c*x*Log[c*x] - 24*a*b*c
*x*Log[c*x] + 24*b^2*c*x*ArcTan[c*x]*PolyLog[2, E^((-2*I)*ArcTan[c*x])] + 1
2*b^2*c*x*(-I + 2*ArcTan[c*x])*PolyLog[2, -E^((2*I)*ArcTan[c*x])] + (12*I)*
b^2*c*x*PolyLog[2, E^((2*I)*ArcTan[c*x])] + 24*a*b*c*x*PolyLog[2, (-I)*c*x]
 - 24*a*b*c*x*PolyLog[2, I*c*x] - (12*I)*b^2*c*x*PolyLog[3, E^((-2*I)*ArcTa
n[c*x])] + (12*I)*b^2*c*x*PolyLog[3, -E^((2*I)*ArcTan[c*x])]))/(12*x)
```

Maple [C] time = 1.474, size = 11959, normalized size = 37.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^2*(a+b*arctan(c*x))^2/x^2,x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))^2/x^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{4\,a^2c^2d^2x^2 - 8i\,a^2cd^2x - 4\,a^2d^2 - \left(b^2c^2d^2x^2 - 2i\,b^2cd^2x - b^2d^2\right)\log\left(-\frac{cx+i}{cx-i}\right)^2 - \left(-4i\,abc^2d^2x^2 - 8\,abcd^2x + 4\right.}{4\,x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))^2/x^2,x, algorithm="fricas")
```

```
[Out] integral(-1/4*(4*a^2*c^2*d^2*x^2 - 8*I*a^2*c*d^2*x - 4*a^2*d^2 - (b^2*c^2*d
^2*x^2 - 2*I*b^2*c*d^2*x - b^2*d^2)*log(-(c*x + I)/(c*x - I))^2 - (-4*I*a*b
*c^2*d^2*x^2 - 8*a*b*c*d^2*x + 4*I*a*b*d^2)*log(-(c*x + I)/(c*x - I)))/x^2,
 x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d^2\left(\int -a^2c^2\,dx + \int \frac{a^2}{x^2}\,dx + \int -b^2c^2\,\mathrm{atan}^2\,(cx)\,dx + \int \frac{b^2\,\mathrm{atan}^2\,(cx)}{x^2}\,dx + \int \frac{2ia^2c}{x}\,dx + \int -2abc^2\,\mathrm{atan}\,(cx)\,dx + \int \frac{2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**2*(a+b*atan(c*x))**2/x**2,x)
```

```
[Out] d**2*(Integral(-a**2*c**2, x) + Integral(a**2/x**2, x) + Integral(-b**2*c**
2*atan(c*x)**2, x) + Integral(b**2*atan(c*x)**2/x**2, x) + Integral(2*I*a**
2*c/x, x) + Integral(-2*a*b*c**2*atan(c*x), x) + Integral(2*a*b*atan(c*x)/x
**2, x) + Integral(2*I*b**2*c*atan(c*x)**2/x, x) + Integral(4*I*a*b*c*atan(
c*x)/x, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(i\,cdx + d)^2(b\arctan\,(cx) + a)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))^2/x^2,x, algorithm="giac")
```

```
[Out] integrate((I*c*d*x + d)^2*(b*arctan(c*x) + a)^2/x^2, x)
```

3.82 $\int \frac{(d+icdx)^2\left(a+b\tan^{-1}(cx)\right)^2}{x^3}\,dx$

Optimal. Leaf size=337

$$ibc^2d^2\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)-ibc^2d^2\text{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)+2b^2c^2d^2\text{PolyL}$$

```
[Out] -((b*c*d^2*(a + b*ArcTan[c*x]))/x) + (3*c^2*d^2*(a + b*ArcTan[c*x])^2)/2 -
(d^2*(a + b*ArcTan[c*x])^2)/(2*x^2) - ((2*I)*c*d^2*(a + b*ArcTan[c*x])^2)/x
 - 2*c^2*d^2*(a + b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1 + I*c*x)] + b^2*c^2*d^2
*Log[x] - (b^2*c^2*d^2*Log[1 + c^2*x^2])/2 + (4*I)*b*c^2*d^2*(a + b*ArcTan[
c*x])*Log[2 - 2/(1 - I*c*x)] + 2*b^2*c^2*d^2*PolyLog[2, -1 + 2/(1 - I*c*x)]
 + I*b*c^2*d^2*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)] - I*b*c^2*
d^2*(a + b*ArcTan[c*x])*PolyLog[2, -1 + 2/(1 + I*c*x)] + (b^2*c^2*d^2*PolyL
og[3, 1 - 2/(1 + I*c*x)])/2 - (b^2*c^2*d^2*PolyLog[3, -1 + 2/(1 + I*c*x)])/
2
```

Rubi [A] time = 0.645457, antiderivative size = 337, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 15, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {4876, 4852, 4918, 266, 36, 29, 31, 4884, 4924, 4868, 2447, 4850, 4988, 4994, 6610}

$$ibc^2d^2\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)-ibc^2d^2\text{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)+2b^2c^2d^2\text{PolyL}$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)^2*(a + b*ArcTan[c*x])^2)/x^3,x]
```


```
[Out] -((b*c*d^2*(a + b*ArcTan[c*x]))/x) + (3*c^2*d^2*(a + b*ArcTan[c*x])^2)/2 -
(d^2*(a + b*ArcTan[c*x])^2)/(2*x^2) - ((2*I)*c*d^2*(a + b*ArcTan[c*x])^2)/x
 - 2*c^2*d^2*(a + b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1 + I*c*x)] + b^2*c^2*d^2
*Log[x] - (b^2*c^2*d^2*Log[1 + c^2*x^2])/2 + (4*I)*b*c^2*d^2*(a + b*ArcTan[
c*x])*Log[2 - 2/(1 - I*c*x)] + 2*b^2*c^2*d^2*PolyLog[2, -1 + 2/(1 - I*c*x)]
 + I*b*c^2*d^2*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)] - I*b*c^2*
d^2*(a + b*ArcTan[c*x])*PolyLog[2, -1 + 2/(1 + I*c*x)] + (b^2*c^2*d^2*PolyL
og[3, 1 - 2/(1 + I*c*x)])/2 - (b^2*c^2*d^2*PolyLog[3, -1 + 2/(1 + I*c*x)])/
2
```


Rule 4876

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4918

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/(d + e*x^2),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
```

```
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4850

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTan[c*x])^(p - 1)*ArcTanh[1 - 2/(1 + I*c*x)])/(1 + c^2*x^2), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4988

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTan[c*x])^p)/(d + e
*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTan[c*x])^p)/(d + e*x^2
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
```

```
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{(d+icdx)^2\left(a+b\tan^{-1}(cx)\right)^2}{x^3}\,dx = \int \left(\frac{d^2\left(a+b\tan^{-1}(cx)\right)^2}{x^3}+\frac{2icd^2\left(a+b\tan^{-1}(cx)\right)^2}{x^2}-\frac{c^2d^2\left(a+b\tan^{-1}(cx)\right)^2}{x}\right) d$$

$$= d^2\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^3}\,dx+\left(2icd^2\right)\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^2}\,dx-\left(c^2d^2\right)\int \frac{(a+b}{}$$

$$= -\frac{d^2\left(a+b\tan^{-1}(cx)\right)^2}{2x^2}-\frac{2icd^2\left(a+b\tan^{-1}(cx)\right)^2}{x}-2c^2d^2\left(a+b\tan^{-1}(cx)\right)^2\tanh$$

$$= 2c^2d^2\left(a+b\tan^{-1}(cx)\right)^2-\frac{d^2\left(a+b\tan^{-1}(cx)\right)^2}{2x^2}-\frac{2icd^2\left(a+b\tan^{-1}(cx)\right)^2}{x}-2c^2$$

$$= -\frac{bcd^2\left(a+b\tan^{-1}(cx)\right)}{x}+\frac{3}{2}c^2d^2\left(a+b\tan^{-1}(cx)\right)^2-\frac{d^2\left(a+b\tan^{-1}(cx)\right)^2}{2x^2}-\frac{2ic}{}$$

$$= -\frac{bcd^2\left(a+b\tan^{-1}(cx)\right)}{x}+\frac{3}{2}c^2d^2\left(a+b\tan^{-1}(cx)\right)^2-\frac{d^2\left(a+b\tan^{-1}(cx)\right)^2}{2x^2}-\frac{2ic}{}$$

$$= -\frac{bcd^2\left(a+b\tan^{-1}(cx)\right)}{x}+\frac{3}{2}c^2d^2\left(a+b\tan^{-1}(cx)\right)^2-\frac{d^2\left(a+b\tan^{-1}(cx)\right)^2}{2x^2}-\frac{2ic}{}$$

$$= -\frac{bcd^2\left(a+b\tan^{-1}(cx)\right)}{x}+\frac{3}{2}c^2d^2\left(a+b\tan^{-1}(cx)\right)^2-\frac{d^2\left(a+b\tan^{-1}(cx)\right)^2}{2x^2}-\frac{2ic}{}$$

Mathematica [A] time = 0.83999, size = 388, normalized size = 1.15

$$-\frac{d^2\left(2iabc^2x^2(\text{PolyLog}(2,-icx)-\text{PolyLog}(2,icx))+\frac{1}{12}b^2c^2x^2\left(24i\tan^{-1}(cx)\text{PolyLog}\left(2,e^{-2i\tan^{-1}(cx)}\right)+24i\tan^{-1}(cx)\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + I*c*d*x)^2*(a + b*ArcTan[c*x])^2)/x^3,x]
```

```
[Out] -(d^2*(a^2 + (4*I)*a^2*c*x + 2*a*b*(ArcTan[c*x] + c*x*(1 + c*x*ArcTan[c*x])
) + 2*a^2*c^2*x^2*Log[x] + b^2*(2*c*x*ArcTan[c*x] + (1 + c^2*x^2)*ArcTan[c*
x]^2 - 2*c^2*x^2*Log[(c*x)/Sqrt[1 + c^2*x^2]]) + (4*I)*a*b*c*x*(2*ArcTan[c*
x] + c*x*(-2*Log[c*x] + Log[1 + c^2*x^2])) + (4*I)*b^2*c*x*(ArcTan[c*x]^2 -
 2*c*x*ArcTan[c*x]*Log[1 - E^((2*I)*ArcTan[c*x])] + I*c*x*(ArcTan[c*x]^2 +
PolyLog[2, E^((2*I)*ArcTan[c*x])])) + (2*I)*a*b*c^2*x^2*(PolyLog[2, (-I)*c*
x] - PolyLog[2, I*c*x]) + (b^2*c^2*x^2*((-I)*Pi^3 + (16*I)*ArcTan[c*x]^3 +
24*ArcTan[c*x]^2*Log[1 - E^((-2*I)*ArcTan[c*x])] - 24*ArcTan[c*x]^2*Log[1 +
 E^((2*I)*ArcTan[c*x])] + (24*I)*ArcTan[c*x]*PolyLog[2, E^((-2*I)*ArcTan[c*
x])] + (24*I)*ArcTan[c*x]*PolyLog[2, -E^((2*I)*ArcTan[c*x])] + 12*PolyLog[3
, E^((-2*I)*ArcTan[c*x])] - 12*PolyLog[3, -E^((2*I)*ArcTan[c*x])]))/12))/(2
*x^2)
```

Maple [C] time = 3.639, size = 1647, normalized size = 4.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^2*(a+b*arctan(c*x))^2/x^3,x)
```

```
[Out] -1/2*I*c^2*d^2*b^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I/((1+I*c*x)
^2/(c^2*x^2+1)+1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2
+1)+1))*arctan(c*x)^2-c*d^2*b^2*arctan(c*x)/x-c^2*d^2*a*b*arctan(c*x)+c^2*d
^2*b^2*arctan(c*x)^2*ln((1+I*c*x)^2/(c^2*x^2+1)-1)-c^2*d^2*b^2*arctan(c*x)^
2*ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))-c^2*d^2*b^2*arctan(c*x)^2*ln(c*x)-c^2*d
^2*b^2*arctan(c*x)^2*ln(1-(1+I*c*x)/(c^2*x^2+1)^(1/2))-d^2*a*b*arctan(c*x)/
x^2-2*I*c*d^2*a^2/x-I*c^2*d^2*b^2*arctan(c*x)-1/2*d^2*b^2*arctan(c*x)^2/x^2
+4*c^2*d^2*b^2*dilog(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))+c^2*d^2*b^2*ln(1+(1+I*c
*x)/(c^2*x^2+1)^(1/2))-4*c^2*d^2*b^2*dilog((1+I*c*x)/(c^2*x^2+1)^(1/2))+c^2
*d^2*b^2*ln((1+I*c*x)/(c^2*x^2+1)^(1/2)-1)+3/2*c^2*d^2*b^2*arctan(c*x)^2-c^
2*d^2*a^2*ln(c*x)-2*c^2*d^2*b^2*polylog(3,(1+I*c*x)/(c^2*x^2+1)^(1/2))-2*c^
2*d^2*b^2*polylog(3,-(1+I*c*x)/(c^2*x^2+1)^(1/2))+1/2*c^2*d^2*b^2*polylog(3
,-(1+I*c*x)^2/(c^2*x^2+1))-c*d^2*a*b/x-1/2*I*c^2*d^2*b^2*Pi*csgn(I*((1+I*c*
x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*arctan(c*x)^2-1/2*I*c^2*
d^2*b^2*Pi*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*
arctan(c*x)^2+1/2*I*c^2*d^2*b^2*Pi*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c
*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2-I*c^2*d^2*a*b*ln(c*x)*ln(1+I*c*x)+I*c
^2*d^2*a*b*ln(c*x)*ln(1-I*c*x)-4*I*c*d^2*a*b*arctan(c*x)/x+1/2*I*c^2*d^2*b^
2*Pi*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)
/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2+1/2*I*c^2*d^2*b^2*Pi*csgn(I*(
(1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(((1+I*c*x)^2/(
```

```
c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2-1/2*I*c^2*d^2*b^
2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn((
(1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)^2+1/2*I
*c^2*d^2*b^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I*((1+I*c*x)^2/(c^
2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2-1/2*d^2*a^2/x^2-I*
c^2*d^2*a*b*dilog(1+I*c*x)+4*I*c^2*d^2*a*b*ln(c*x)-2*I*c*d^2*b^2*arctan(c*x
)^2/x-2*c^2*d^2*a*b*arctan(c*x)*ln(c*x)+I*c^2*d^2*a*b*dilog(1-I*c*x)-2*I*c^
2*d^2*a*b*ln(c^2*x^2+1)-1/2*I*c^2*d^2*b^2*Pi*arctan(c*x)^2+4*I*c^2*d^2*b^2*
arctan(c*x)*ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))+2*I*c^2*d^2*b^2*arctan(c*x)*p
olylog(2,(1+I*c*x)/(c^2*x^2+1)^(1/2))-I*c^2*d^2*b^2*arctan(c*x)*polylog(2,-
(1+I*c*x)^2/(c^2*x^2+1))+2*I*c^2*d^2*b^2*arctan(c*x)*polylog(2,-(1+I*c*x)/(
c^2*x^2+1)^(1/2))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))^2/x^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{4\,a^2c^2d^2x^2-8i\,a^2cd^2x-4\,a^2d^2-\left(b^2c^2d^2x^2-2i\,b^2cd^2x-b^2d^2\right)\log\left(-\frac{cx+i}{cx-i}\right)^2-\left(-4i\,abc^2d^2x^2-8\,abcd^2x+4i\,a\right.}{4\,x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))^2/x^3,x, algorithm="fricas")
```

```
[Out] integral(-1/4*(4*a^2*c^2*d^2*x^2 - 8*I*a^2*c*d^2*x - 4*a^2*d^2 - (b^2*c^2*d
^2*x^2 - 2*I*b^2*c*d^2*x - b^2*d^2)*log(-(c*x + I)/(c*x - I))^2 - (-4*I*a*b
*c^2*d^2*x^2 - 8*a*b*c*d^2*x + 4*I*a*b*d^2)*log(-(c*x + I)/(c*x - I)))/x^3,
 x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d^2\left(\int \frac{a^2}{x^3}\,dx + \int -\frac{a^2c^2}{x}\,dx + \int \frac{b^2\operatorname{atan}^2(cx)}{x^3}\,dx + \int \frac{2ia^2c}{x^2}\,dx + \int \frac{2ab\operatorname{atan}(cx)}{x^3}\,dx + \int -\frac{b^2c^2\operatorname{atan}^2(cx)}{x}\,dx + \int \frac{2}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**2*(a+b*atan(c*x))**2/x**3,x)
```

```
[Out] d**2*(Integral(a**2/x**3, x) + Integral(-a**2*c**2/x, x) + Integral(b**2*at
an(c*x)**2/x**3, x) + Integral(2*I*a**2*c/x**2, x) + Integral(2*a*b*atan(c*
x)/x**3, x) + Integral(-b**2*c**2*atan(c*x)**2/x, x) + Integral(2*I*b**2*c*
atan(c*x)**2/x**2, x) + Integral(-2*a*b*c**2*atan(c*x)/x, x) + Integral(4*I
*a*b*c*atan(c*x)/x**2, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(icdx + d)^2(b\arctan(cx) + a)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))^2/x^3,x, algorithm="giac")
```

```
[Out] integrate((I*c*d*x + d)^2*(b*arctan(c*x) + a)^2/x^3, x)
```

3.83 $\int \frac{(d+icdx)^2\left(a+b\tan^{-1}(cx)\right)^2}{x^4}\,dx$

Optimal. Leaf size=267

$$-\frac{4}{3}ib^2c^3d^2\text{PolyLog}(2,-icx)+\frac{4}{3}ib^2c^3d^2\text{PolyLog}(2,icx)+\frac{4}{3}ib^2c^3d^2\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)-\frac{8}{3}abc^3d^2\log(x)-\frac{2ibc^2d^2}{}$$

```
[Out] -(b^2*c^2*d^2)/(3*x) - (b^2*c^3*d^2*ArcTan[c*x])/3 - (b*c*d^2*(a + b*ArcTan
[c*x]))/(3*x^2) - ((2*I)*b*c^2*d^2*(a + b*ArcTan[c*x]))/x - (d^2*(1 + I*c*x
)^3*(a + b*ArcTan[c*x])^2)/(3*x^3) - (8*a*b*c^3*d^2*Log[x])/3 + (2*I)*b^2*c
^3*d^2*Log[x] - (8*b*c^3*d^2*(a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/3 - I*
b^2*c^3*d^2*Log[1 + c^2*x^2] - ((4*I)/3)*b^2*c^3*d^2*PolyLog[2, (-I)*c*x] +
 ((4*I)/3)*b^2*c^3*d^2*PolyLog[2, I*c*x] + ((4*I)/3)*b^2*c^3*d^2*PolyLog[2,
 1 - 2/(1 - I*c*x)]
```

Rubi [A] time = 0.268309, antiderivative size = 267, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 14, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.56, Rules used = {37, 4874, 4852, 325, 203, 266, 36, 29, 31, 4848, 2391, 4854, 2402, 2315}

$$-\frac{4}{3}ib^2c^3d^2\text{PolyLog}(2,-icx)+\frac{4}{3}ib^2c^3d^2\text{PolyLog}(2,icx)+\frac{4}{3}ib^2c^3d^2\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)-\frac{8}{3}abc^3d^2\log(x)-\frac{2ibc^2d^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)^2*(a + b*ArcTan[c*x])^2)/x^4,x]
```


```
[Out] -(b^2*c^2*d^2)/(3*x) - (b^2*c^3*d^2*ArcTan[c*x])/3 - (b*c*d^2*(a + b*ArcTan
[c*x]))/(3*x^2) - ((2*I)*b*c^2*d^2*(a + b*ArcTan[c*x]))/x - (d^2*(1 + I*c*x
)^3*(a + b*ArcTan[c*x])^2)/(3*x^3) - (8*a*b*c^3*d^2*Log[x])/3 + (2*I)*b^2*c
^3*d^2*Log[x] - (8*b*c^3*d^2*(a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/3 - I*
b^2*c^3*d^2*Log[1 + c^2*x^2] - ((4*I)/3)*b^2*c^3*d^2*PolyLog[2, (-I)*c*x] +
 ((4*I)/3)*b^2*c^3*d^2*PolyLog[2, I*c*x] + ((4*I)/3)*b^2*c^3*d^2*PolyLog[2,
 1 - 2/(1 - I*c*x)]
```


Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
```

```
1]
```

Rule 4874

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_.) + (e_
.)*(x_))^(q_), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dis
t[(a + b*ArcTan[c*x])^p, u, x] - Dist[b*c*p, Int[ExpandIntegrand[(a + b*Arc
Tan[c*x])^(p - 1), u/(1 + c^2*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
 q}, x] && IGtQ[p, 1] && EqQ[c^2*d^2 + e^2, 0] && IntegersQ[m, q] && NeQ[m,
 -1] && NeQ[q, -1] && ILtQ[m + q + 1, 0] && LtQ[m*q, 0]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int \frac{(d+icdx)^2\left(a+b\tan^{-1}(cx)\right)^2}{x^4}\,dx = -\frac{d^2(1+icx)^3\left(a+b\tan^{-1}(cx)\right)^2}{3x^3} - (2bc)\int\left(-\frac{d^2\left(a+b\tan^{-1}(cx)\right)}{3x^3} - \frac{icd^2\left(a+b\right.}{x}\right.$$

$$= -\frac{d^2(1+icx)^3\left(a+b\tan^{-1}(cx)\right)^2}{3x^3} + \frac{1}{3}\left(2bcd^2\right)\int\frac{a+b\tan^{-1}(cx)}{x^3}\,dx + \left(2ibc^2d^2\right)$$

$$= -\frac{bcd^2\left(a+b\tan^{-1}(cx)\right)}{3x^2} - \frac{2ibc^2d^2\left(a+b\tan^{-1}(cx)\right)}{x} - \frac{d^2(1+icx)^3\left(a+b\tan^{-1}\right.}{3x^3}$$

$$= -\frac{b^2c^2d^2}{3x} - \frac{bcd^2\left(a+b\tan^{-1}(cx)\right)}{3x^2} - \frac{2ibc^2d^2\left(a+b\tan^{-1}(cx)\right)}{x} - \frac{d^2(1+icx)^3\left(a\right.}{3x}$$

$$= -\frac{b^2c^2d^2}{3x} - \frac{1}{3}b^2c^3d^2\tan^{-1}(cx) - \frac{bcd^2\left(a+b\tan^{-1}(cx)\right)}{3x^2} - \frac{2ibc^2d^2\left(a+b\tan^{-1}(c\right.}{x}$$

$$= -\frac{b^2c^2d^2}{3x} - \frac{1}{3}b^2c^3d^2\tan^{-1}(cx) - \frac{bcd^2\left(a+b\tan^{-1}(cx)\right)}{3x^2} - \frac{2ibc^2d^2\left(a+b\tan^{-1}(c\right.}{x}$$

Mathematica [A] time = 0.65824, size = 253, normalized size = 0.95

$$\frac{d^2\left(4ib^2c^3x^3\text{PolyLog}\left(2,e^{2i\tan^{-1}(cx)}\right)+3a^2c^2x^2-3ia^2cx-a^2-6iabc^2x^2-8abc^3x^3\log(cx)+4abc^3x^3\log\left(c^2x^2+1\right)-b\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + I*c*d*x)^2*(a + b*ArcTan[c*x])^2)/x^4,x]
```

```
[Out] (d^2*(-a^2 - (3*I)*a^2*c*x - a*b*c*x + 3*a^2*c^2*x^2 - (6*I)*a*b*c^2*x^2 -
b^2*c^2*x^2 + b^2*(-1 - I*c*x)^3*ArcTan[c*x]^2 - b*ArcTan[c*x]*(b*c*x*(1 +
(6*I)*c*x + c^2*x^2) + a*(2 + (6*I)*c*x - 6*c^2*x^2 + (6*I)*c^3*x^3) + 8*b*
c^3*x^3*Log[1 - E^((2*I)*ArcTan[c*x])]) - 8*a*b*c^3*x^3*Log[c*x] + (6*I)*b^
2*c^3*x^3*Log[(c*x)/Sqrt[1 + c^2*x^2]] + 4*a*b*c^3*x^3*Log[1 + c^2*x^2] + (
4*I)*b^2*c^3*x^3*PolyLog[2, E^((2*I)*ArcTan[c*x])]))/(3*x^3)
```

Maple [B] time = 0.116, size = 669, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^2*(a+b*arctan(c*x))^2/x^4,x)
```

```
[Out] c^2*d^2*a^2/x-1/3*d^2*b^2*arctan(c*x)^2/x^3-I*c*d^2*a^2/x^2-2/3*d^2*a*b*arc
tan(c*x)/x^3+4/3*c^3*d^2*b^2*arctan(c*x)*ln(c^2*x^2+1)-8/3*c^3*d^2*a*b*ln(c
*x)-8/3*c^3*d^2*b^2*arctan(c*x)*ln(c*x)+4/3*c^3*d^2*a*b*ln(c^2*x^2+1)-1/3*c
*d^2*b^2*arctan(c*x)/x^2+c^2*d^2*b^2*arctan(c*x)^2/x+2*I*c^3*d^2*b^2*ln(c*x
)-I*c^3*d^2*b^2*arctan(c*x)^2-4/3*I*c^3*d^2*b^2*dilog(1+I*c*x)+2/3*I*c^3*d^
2*b^2*dilog(1/2*I*(c*x-I))+4/3*I*c^3*d^2*b^2*dilog(1-I*c*x)-1/3*I*c^3*d^2*b
^2*ln(c*x-I)^2+1/3*I*c^3*d^2*b^2*ln(c*x+I)^2-2/3*I*c^3*d^2*b^2*dilog(-1/2*I
*(c*x+I))-I*c*d^2*b^2*arctan(c*x)^2/x^2-2*I*c^2*d^2*a*b/x-2*I*c^3*d^2*a*b*a
rctan(c*x)-2/3*I*c^3*d^2*b^2*ln(c*x-I)*ln(-1/2*I*(c*x+I))-2/3*I*c^3*d^2*b^2
*ln(c*x+I)*ln(c^2*x^2+1)+2/3*I*c^3*d^2*b^2*ln(c*x+I)*ln(1/2*I*(c*x-I))-4/3*
I*c^3*d^2*b^2*ln(c*x)*ln(1+I*c*x)+2*c^2*d^2*a*b*arctan(c*x)/x+2/3*I*c^3*d^2
*b^2*ln(c*x-I)*ln(c^2*x^2+1)+4/3*I*c^3*d^2*b^2*ln(c*x)*ln(1-I*c*x)-2*I*c^2*
d^2*b^2*arctan(c*x)/x-1/3*c*d^2*a*b/x^2-1/3*d^2*a^2/x^3-I*b^2*c^3*d^2*ln(c^
2*x^2+1)-2*I*c*d^2*a*b*arctan(c*x)/x^2-1/3*b^2*c^2*d^2/x-1/3*b^2*c^3*d^2*ar
ctan(c*x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))^2/x^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{12\,x^{3}\mathrm{integral}\left(-\frac{3\,a^{2}c^{4}d^{2}x^{4}-6i\,a^{2}c^{3}d^{2}x^{3}-6i\,a^{2}cd^{2}x-3\,a^{2}d^{2}-\left(-3i\,abc^{4}d^{2}x^{4}-3\left(2\,ab+i\,b^{2}\right)c^{3}d^{2}x^{3}-3\,b^{2}c^{2}d^{2}x^{2}-\left(6\,ab-i\,b^{2}\right)cd^{2}x+3i\,abd^{2}\right)\log\left(-\frac{cx+i}{cx-i}\right)}{3\left(c^{2}x^{6}+x^{4}\right)},x\right)-(3}{12\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))^2/x^4,x, algorithm="fricas")
```

```
[Out] 1/12*(12*x^3*integral(-1/3*(3*a^2*c^4*d^2*x^4 - 6*I*a^2*c^3*d^2*x^3 - 6*I*a
^2*c*d^2*x - 3*a^2*d^2 - (-3*I*a*b*c^4*d^2*x^4 - 3*(2*a*b + I*b^2)*c^3*d^2*
```

```
x^3 - 3*b^2*c^2*d^2*x^2 - (6*a*b - I*b^2)*c*d^2*x + 3*I*a*b*d^2)*log(-(c*x
+ I)/(c*x - I)))/(c^2*x^6 + x^4), x) - (3*b^2*c^2*d^2*x^2 - 3*I*b^2*c*d^2*x
 - b^2*d^2)*log(-(c*x + I)/(c*x - I))^2)/x^3
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**2*(a+b*atan(c*x))**2/x**4,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(icdx + d)^2 (b \arctan(cx) + a)^2}{x^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))^2/x^4,x, algorithm="giac")
```

```
[Out] integrate((I*c*d*x + d)^2*(b*arctan(c*x) + a)^2/x^4, x)
```

3.84 $\int x^3(d+icdx)^3\left(a+b\tan^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=438

$$-\frac{26b^2d^3\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{35c^4}-\frac{1}{7}ic^3d^3x^7\left(a+b\tan^{-1}(cx)\right)^2-\frac{1}{2}c^2d^3x^6\left(a+b\tan^{-1}(cx)\right)^2+\frac{1}{21}ibc^2d^3x^6\left(a+b\tan^{-1}(c\right.$$

```
[Out] (3*a*b*d^3*x)/(2*c^3) - (((122*I)/105)*b^2*d^3*x)/c^3 + (7*b^2*d^3*x^2)/(20
*c^2) + (((44*I)/315)*b^2*d^3*x^3)/c - (b^2*d^3*x^4)/20 - (I/105)*b^2*c*d^3
*x^5 + (((122*I)/105)*b^2*d^3*ArcTan[c*x])/c^4 + (3*b^2*d^3*x*ArcTan[c*x])/
(2*c^3) + (((26*I)/35)*b*d^3*x^2*(a + b*ArcTan[c*x]))/c^2 - (b*d^3*x^3*(a +
 b*ArcTan[c*x]))/(2*c) - ((13*I)/35)*b*d^3*x^4*(a + b*ArcTan[c*x]) + (b*c*d
^3*x^5*(a + b*ArcTan[c*x]))/5 + (I/21)*b*c^2*d^3*x^6*(a + b*ArcTan[c*x]) -
(209*d^3*(a + b*ArcTan[c*x])^2)/(140*c^4) + (d^3*x^4*(a + b*ArcTan[c*x])^2)
/4 + ((3*I)/5)*c*d^3*x^5*(a + b*ArcTan[c*x])^2 - (c^2*d^3*x^6*(a + b*ArcTan
[c*x])^2)/2 - (I/7)*c^3*d^3*x^7*(a + b*ArcTan[c*x])^2 + (((52*I)/35)*b*d^3*
(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/c^4 - (11*b^2*d^3*Log[1 + c^2*x^2])
/(10*c^4) - (26*b^2*d^3*PolyLog[2, 1 - 2/(1 + I*c*x)])/(35*c^4)
```

Rubi [A] time = 1.36562, antiderivative size = 438, normalized size of antiderivative = 1., number of steps used = 62, number of rules used = 15, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {4876, 4852, 4916, 266, 43, 4846, 260, 4884, 302, 203, 321, 4920, 4854, 2402, 2315}

$$-\frac{26b^2d^3\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{35c^4}-\frac{1}{7}ic^3d^3x^7\left(a+b\tan^{-1}(cx)\right)^2-\frac{1}{2}c^2d^3x^6\left(a+b\tan^{-1}(cx)\right)^2+\frac{1}{21}ibc^2d^3x^6\left(a+b\tan^{-1}(c\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + I*c*d*x)^3*(a + b*ArcTan[c*x])^2,x]
```


```
[Out] (3*a*b*d^3*x)/(2*c^3) - (((122*I)/105)*b^2*d^3*x)/c^3 + (7*b^2*d^3*x^2)/(20
*c^2) + (((44*I)/315)*b^2*d^3*x^3)/c - (b^2*d^3*x^4)/20 - (I/105)*b^2*c*d^3
*x^5 + (((122*I)/105)*b^2*d^3*ArcTan[c*x])/c^4 + (3*b^2*d^3*x*ArcTan[c*x])/
(2*c^3) + (((26*I)/35)*b*d^3*x^2*(a + b*ArcTan[c*x]))/c^2 - (b*d^3*x^3*(a +
 b*ArcTan[c*x]))/(2*c) - ((13*I)/35)*b*d^3*x^4*(a + b*ArcTan[c*x]) + (b*c*d
^3*x^5*(a + b*ArcTan[c*x]))/5 + (I/21)*b*c^2*d^3*x^6*(a + b*ArcTan[c*x]) -
(209*d^3*(a + b*ArcTan[c*x])^2)/(140*c^4) + (d^3*x^4*(a + b*ArcTan[c*x])^2)
/4 + ((3*I)/5)*c*d^3*x^5*(a + b*ArcTan[c*x])^2 - (c^2*d^3*x^6*(a + b*ArcTan
[c*x])^2)/2 - (I/7)*c^3*d^3*x^7*(a + b*ArcTan[c*x])^2 + (((52*I)/35)*b*d^3*
```

```
(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/c^4 - (11*b^2*d^3*Log[1 + c^2*x^2])
/(10*c^4) - (26*b^2*d^3*PolyLog[2, 1 - 2/(1 + I*c*x)])/(35*c^4)
```

Rule 4876

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int x^3(d+icdx)^3\left(a+b\tan^{-1}(cx)\right)^2\,dx=\int\left(d^3x^3\left(a+b\tan^{-1}(cx)\right)^2+3icd^3x^4\left(a+b\tan^{-1}(cx)\right)^2-3c^2d^3x^5\left(a+b\tan\right.\right.$$

$$=d^3\int x^3\left(a+b\tan^{-1}(cx)\right)^2\,dx+\left(3icd^3\right)\int x^4\left(a+b\tan^{-1}(cx)\right)^2\,dx-\left(3c^2d^3\right)$$

$$=\frac{1}{4}d^3x^4\left(a+b\tan^{-1}(cx)\right)^2+\frac{3}{5}icd^3x^5\left(a+b\tan^{-1}(cx)\right)^2-\frac{1}{2}c^2d^3x^6\left(a+b\tan^{-1}\right.$$

$$=\frac{1}{4}d^3x^4\left(a+b\tan^{-1}(cx)\right)^2+\frac{3}{5}icd^3x^5\left(a+b\tan^{-1}(cx)\right)^2-\frac{1}{2}c^2d^3x^6\left(a+b\tan^{-1}\right.$$

$$=-\frac{bd^3x^3\left(a+b\tan^{-1}(cx)\right)}{6c}-\frac{3}{10}ibd^3x^4\left(a+b\tan^{-1}(cx)\right)+\frac{1}{5}bcd^3x^5\left(a+b\tan\right.$$

$$=\frac{abd^3x}{2c^3}+\frac{3ibd^3x^2\left(a+b\tan^{-1}(cx)\right)}{5c^2}-\frac{bd^3x^3\left(a+b\tan^{-1}(cx)\right)}{2c}-\frac{13}{35}ibd^3x^4\left(a+\right.$$

$$=\frac{3abd^3x}{2c^3}-\frac{199ib^2d^3x}{210c^3}+\frac{73ib^2d^3x^3}{630c}-\frac{1}{105}ib^2cd^3x^5+\frac{b^2d^3x\tan^{-1}(cx)}{2c^3}+\frac{26ibd^3}{}$$

$$=\frac{3abd^3x}{2c^3}-\frac{122ib^2d^3x}{105c^3}+\frac{11b^2d^3x^2}{60c^2}+\frac{44ib^2d^3x^3}{315c}-\frac{1}{20}b^2d^3x^4-\frac{1}{105}ib^2cd^3x^5+\frac{19}{}$$

$$=\frac{3abd^3x}{2c^3}-\frac{122ib^2d^3x}{105c^3}+\frac{7b^2d^3x^2}{20c^2}+\frac{44ib^2d^3x^3}{315c}-\frac{1}{20}b^2d^3x^4-\frac{1}{105}ib^2cd^3x^5+\frac{12}{}$$

$$=\frac{3abd^3x}{2c^3}-\frac{122ib^2d^3x}{105c^3}+\frac{7b^2d^3x^2}{20c^2}+\frac{44ib^2d^3x^3}{315c}-\frac{1}{20}b^2d^3x^4-\frac{1}{105}ib^2cd^3x^5+\frac{12}{}$$

Mathematica [A] time = 1.73905, size = 408, normalized size = 0.93

$$\frac{d^3\left(936b^2\text{PolyLog}\left(2,-e^{2i\tan^{-1}(cx)}\right)-180ia^2c^7x^7-630a^2c^6x^6+756ia^2c^5x^5+315a^2c^4x^4+60iabc^6x^6+252abc^5x^5-468\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*(d + I*c*d*x)^3*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] (d^3*((1464*I)*a*b + 504*b^2 + 1890*a*b*c*x - (1464*I)*b^2*c*x + (936*I)*a*
b*c^2*x^2 + 441*b^2*c^2*x^2 - 630*a*b*c^3*x^3 + (176*I)*b^2*c^3*x^3 + 315*a
^2*c^4*x^4 - (468*I)*a*b*c^4*x^4 - 63*b^2*c^4*x^4 + (756*I)*a^2*c^5*x^5 + 2
52*a*b*c^5*x^5 - (12*I)*b^2*c^5*x^5 - 630*a^2*c^6*x^6 + (60*I)*a*b*c^6*x^6
- (180*I)*a^2*c^7*x^7 + 9*b^2*(-I + c*x)^4*(-1 + (4*I)*c*x + 10*c^2*x^2 - (
20*I)*c^3*x^3)*ArcTan[c*x]^2 + 6*b*ArcTan[c*x]*(b*(244*I + 315*c*x + (156*I
)*c^2*x^2 - 105*c^3*x^3 - (78*I)*c^4*x^4 + 42*c^5*x^5 + (10*I)*c^6*x^6) + 3
*a*(-105 + 35*c^4*x^4 + (84*I)*c^5*x^5 - 70*c^6*x^6 - (20*I)*c^7*x^7) + (31
2*I)*b*Log[1 + E^((2*I)*ArcTan[c*x])]) - (936*I)*a*b*Log[1 + c^2*x^2] - 138
6*b^2*Log[1 + c^2*x^2] + 936*b^2*PolyLog[2, -E^((2*I)*ArcTan[c*x])]))/(1260
*c^4)
```

Maple [A] time = 0.096, size = 750, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d+I*c*d*x)^3*(a+b*arctan(c*x))^2,x)
```

```
[Out] -13/35*I*d^3*a*b*x^4-13/35/c^4*d^3*b^2*dilog(-1/2*I*(c*x+I))+13/70/c^4*d^3*
b^2*ln(c*x+I)^2+1/4*d^3*b^2*arctan(c*x)^2*x^4-1/2*c^2*d^3*a^2*x^6-13/70/c^4
*d^3*b^2*ln(c*x-I)^2-3/4/c^4*d^3*b^2*arctan(c*x)^2+13/35/c^4*d^3*b^2*dilog(
1/2*I*(c*x-I))+26/35*I/c^2*d^3*b^2*arctan(c*x)*x^2-26/35*I/c^4*d^3*b^2*arct
an(c*x)*ln(c^2*x^2+1)-26/35*I/c^4*d^3*a*b*ln(c^2*x^2+1)+26/35*I/c^2*d^3*a*b
*x^2-c^2*d^3*a*b*arctan(c*x)*x^6+1/21*I*c^2*d^3*a*b*x^6+1/21*I*c^2*d^3*b^2*
arctan(c*x)*x^6+3/5*I*c*d^3*b^2*arctan(c*x)^2*x^5-1/7*I*c^3*d^3*b^2*arctan(
c*x)^2*x^7-1/2*c^2*d^3*b^2*arctan(c*x)^2*x^6+13/35/c^4*d^3*b^2*ln(c*x-I)*ln
(c^2*x^2+1)-13/35/c^4*d^3*b^2*ln(c*x-I)*ln(-1/2*I*(c*x+I))-3/2/c^4*d^3*a*b*
arctan(c*x)+1/5*c*d^3*b^2*arctan(c*x)*x^5-1/2/c*d^3*b^2*arctan(c*x)*x^3-13/
35*I*d^3*b^2*arctan(c*x)*x^4-13/35/c^4*d^3*b^2*ln(c*x+I)*ln(c^2*x^2+1)+13/3
5/c^4*d^3*b^2*ln(c*x+I)*ln(1/2*I*(c*x-I))+1/2*d^3*a*b*arctan(c*x)*x^4-1/7*I
```

```
*c^3*d^3*a^2*x^7+3/5*I*c*d^3*a^2*x^5+1/5*c*d^3*a*b*x^5-1/2/c*d^3*a*b*x^3-1/
20*b^2*d^3*x^4+1/4*d^3*a^2*x^4-122/105*I*b^2*d^3*x/c^3-1/105*I*b^2*c*d^3*x^
5+3/2*a*b*d^3*x/c^3+3/2*b^2*d^3*x*arctan(c*x)/c^3+44/315*I*b^2*d^3*x^3/c+12
2/105*I*b^2*d^3*arctan(c*x)/c^4-2/7*I*c^3*d^3*a*b*arctan(c*x)*x^7+6/5*I*c*d
^3*a*b*arctan(c*x)*x^5-11/10*b^2*d^3*ln(c^2*x^2+1)/c^4+7/20*b^2*d^3*x^2/c^2
```

<u>Maxima</u> [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d+I*c*d*x)^3*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

```
[Out] -1/7*I*a^2*c^3*d^3*x^7 - 1/2*a^2*c^2*d^3*x^6 + 3/5*I*a^2*c*d^3*x^5 + 1/4*b^
2*d^3*x^4*arctan(c*x)^2 - 1/42*I*(12*x^7*arctan(c*x) - c*((2*c^4*x^6 - 3*c^
2*x^4 + 6*x^2)/c^6 - 6*log(c^2*x^2 + 1)/c^8))*a*b*c^3*d^3 + 1/4*a^2*d^3*x^4
 - 1/15*(15*x^6*arctan(c*x) - c*((3*c^4*x^5 - 5*c^2*x^3 + 15*x)/c^6 - 15*ar
ctan(c*x)/c^7))*a*b*c^2*d^3 + 3/10*I*(4*x^5*arctan(c*x) - c*((c^2*x^4 - 2*x
^2)/c^4 + 2*log(c^2*x^2 + 1)/c^6))*a*b*c*d^3 + 1/6*(3*x^4*arctan(c*x) - c*(
(c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5))*a*b*d^3 - 1/12*(2*c*((c^2*x^3 - 3
*x)/c^4 + 3*arctan(c*x)/c^5)*arctan(c*x) - (c^2*x^2 + 3*arctan(c*x)^2 - 4*l
og(c^2*x^2 + 1))/c^4)*b^2*d^3 - 1/1120*(40*I*b^2*c^3*d^3*x^7 + 140*b^2*c^2*
d^3*x^6 - 168*I*b^2*c*d^3*x^5)*arctan(c*x)^2 + 1/1120*(40*b^2*c^3*d^3*x^7 -
 140*I*b^2*c^2*d^3*x^6 - 168*b^2*c*d^3*x^5)*arctan(c*x)*log(c^2*x^2 + 1) -
1/1120*(-10*I*b^2*c^3*d^3*x^7 - 35*b^2*c^2*d^3*x^6 + 42*I*b^2*c*d^3*x^5)*lo
g(c^2*x^2 + 1)^2 - I*integrate(1/560*(420*(b^2*c^5*d^3*x^8 - 2*b^2*c^3*d^3*
x^6 - 3*b^2*c*d^3*x^4)*arctan(c*x)^2 + 35*(b^2*c^5*d^3*x^8 - 2*b^2*c^3*d^3*
x^6 - 3*b^2*c*d^3*x^4)*log(c^2*x^2 + 1)^2 - 12*(15*b^2*c^4*d^3*x^7 - 14*b^2
*c^2*d^3*x^5)*arctan(c*x) + 2*(10*b^2*c^5*d^3*x^8 - 77*b^2*c^3*d^3*x^6 - 21
0*(b^2*c^4*d^3*x^7 + b^2*c^2*d^3*x^5)*arctan(c*x))*log(c^2*x^2 + 1))/(c^2*x
^2 + 1), x) - integrate(1/560*(1260*(b^2*c^4*d^3*x^7 + b^2*c^2*d^3*x^5)*arc
tan(c*x)^2 + 105*(b^2*c^4*d^3*x^7 + b^2*c^2*d^3*x^5)*log(c^2*x^2 + 1)^2 + 4
*(10*b^2*c^5*d^3*x^8 - 77*b^2*c^3*d^3*x^6)*arctan(c*x) + 2*(45*b^2*c^4*d^3*
x^7 - 42*b^2*c^2*d^3*x^5 + 70*(b^2*c^5*d^3*x^8 - 2*b^2*c^3*d^3*x^6 - 3*b^2*
c*d^3*x^4)*arctan(c*x))*log(c^2*x^2 + 1))/(c^2*x^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{560}\left(20i\,b^2c^3d^3x^7+70\,b^2c^2d^3x^6-84i\,b^2cd^3x^5-35\,b^2d^3x^4\right)\log\left(-\frac{cx+i}{cx-i}\right)^2+\text{integral}\left(\frac{-140i\,a^2c^5d^3x^8-420\,a^2c^4d^3x^7+}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d+I*c*d*x)^3*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/560*(20*I*b^2*c^3*d^3*x^7 + 70*b^2*c^2*d^3*x^6 - 84*I*b^2*c*d^3*x^5 - 35*
b^2*d^3*x^4)*log(-(c*x + I)/(c*x - I))^2 + integral(1/140*(-140*I*a^2*c^5*d
^3*x^8 - 420*a^2*c^4*d^3*x^7 + 280*I*a^2*c^3*d^3*x^6 - 280*a^2*c^2*d^3*x^5
+ 420*I*a^2*c*d^3*x^4 + 140*a^2*d^3*x^3 + (140*a*b*c^5*d^3*x^8 + (-420*I*a*
b - 20*b^2)*c^4*d^3*x^7 - 70*(4*a*b - I*b^2)*c^3*d^3*x^6 + (-280*I*a*b + 84
*b^2)*c^2*d^3*x^5 - 35*(12*a*b + I*b^2)*c*d^3*x^4 + 140*I*a*b*d^3*x^3)*log(
-(c*x + I)/(c*x - I)))/(c^2*x^2 + 1), x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(d+I*c*d*x)**3*(a+b*atan(c*x))**2,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (i\,cdx+d)^3(b\arctan(cx)+a)^2x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d+I*c*d*x)^3*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((I*c*d*x + d)^3*(b*arctan(c*x) + a)^2*x^3, x)
```

3.85 $\int x^2(d+icdx)^3\left(a+b\tan^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=402

$$-\frac{14ib^2d^3\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{15c^3}-\frac{1}{6}ic^3d^3x^6\left(a+b\tan^{-1}(cx)\right)^2-\frac{3}{5}c^2d^3x^5\left(a+b\tan^{-1}(cx)\right)^2+\frac{1}{15}ibc^2d^3x^5\left(a+b\tan^{-1}\right.$$

```
[Out] (((11*I)/6)*a*b*d^3*x)/c^2 + (37*b^2*d^3*x)/(30*c^2) + (((61*I)/180)*b^2*d^
3*x^2)/c - (b^2*d^3*x^3)/10 - (I/60)*b^2*c*d^3*x^4 - (37*b^2*d^3*ArcTan[c*x
])/(30*c^3) + (((11*I)/6)*b^2*d^3*x*ArcTan[c*x])/c^2 - (14*b*d^3*x^2*(a + b
*ArcTan[c*x]))/(15*c) - ((11*I)/18)*b*d^3*x^3*(a + b*ArcTan[c*x]) + (3*b*c*
d^3*x^4*(a + b*ArcTan[c*x]))/10 + (I/15)*b*c^2*d^3*x^5*(a + b*ArcTan[c*x])
- (((37*I)/20)*d^3*(a + b*ArcTan[c*x])^2)/c^3 + (d^3*x^3*(a + b*ArcTan[c*x]
)^2)/3 + ((3*I)/4)*c*d^3*x^4*(a + b*ArcTan[c*x])^2 - (3*c^2*d^3*x^5*(a + b*
ArcTan[c*x])^2)/5 - (I/6)*c^3*d^3*x^6*(a + b*ArcTan[c*x])^2 - (28*b*d^3*(a
+ b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(15*c^3) - (((113*I)/90)*b^2*d^3*Log[1
 + c^2*x^2])/c^3 - (((14*I)/15)*b^2*d^3*PolyLog[2, 1 - 2/(1 + I*c*x)])/c^3
```

Rubi [A] time = 1.19617, antiderivative size = 402, normalized size of antiderivative = 1., number of steps used = 52, number of rules used = 15, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {4876, 4852, 4916, 321, 203, 4920, 4854, 2402, 2315, 266, 43, 4846, 260, 4884, 302}

$$-\frac{14ib^2d^3\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{15c^3}-\frac{1}{6}ic^3d^3x^6\left(a+b\tan^{-1}(cx)\right)^2-\frac{3}{5}c^2d^3x^5\left(a+b\tan^{-1}(cx)\right)^2+\frac{1}{15}ibc^2d^3x^5\left(a+b\tan^{-1}\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + I*c*d*x)^3*(a + b*ArcTan[c*x])^2,x]
```


```
[Out] (((11*I)/6)*a*b*d^3*x)/c^2 + (37*b^2*d^3*x)/(30*c^2) + (((61*I)/180)*b^2*d^
3*x^2)/c - (b^2*d^3*x^3)/10 - (I/60)*b^2*c*d^3*x^4 - (37*b^2*d^3*ArcTan[c*x
])/(30*c^3) + (((11*I)/6)*b^2*d^3*x*ArcTan[c*x])/c^2 - (14*b*d^3*x^2*(a + b
*ArcTan[c*x]))/(15*c) - ((11*I)/18)*b*d^3*x^3*(a + b*ArcTan[c*x]) + (3*b*c*
d^3*x^4*(a + b*ArcTan[c*x]))/10 + (I/15)*b*c^2*d^3*x^5*(a + b*ArcTan[c*x])
- (((37*I)/20)*d^3*(a + b*ArcTan[c*x])^2)/c^3 + (d^3*x^3*(a + b*ArcTan[c*x]
)^2)/3 + ((3*I)/4)*c*d^3*x^4*(a + b*ArcTan[c*x])^2 - (3*c^2*d^3*x^5*(a + b*
ArcTan[c*x])^2)/5 - (I/6)*c^3*d^3*x^6*(a + b*ArcTan[c*x])^2 - (28*b*d^3*(a
+ b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(15*c^3) - (((113*I)/90)*b^2*d^3*Log[1
 + c^2*x^2])/c^3 - (((14*I)/15)*b^2*d^3*PolyLog[2, 1 - 2/(1 + I*c*x)])/c^3
```

Rule 4876

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
```

```
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\int x^2(d+icdx)^3\left(a+b\tan^{-1}(cx)\right)^2\,dx=\int\left(d^3x^2\left(a+b\tan^{-1}(cx)\right)^2+3icd^3x^3\left(a+b\tan^{-1}(cx)\right)^2-3c^2d^3x^4\left(a+b\tan^{-1}\right.\right.$$

$$=d^3\int x^2\left(a+b\tan^{-1}(cx)\right)^2\,dx+\left(3icd^3\right)\int x^3\left(a+b\tan^{-1}(cx)\right)^2\,dx-\left(3c^2d^3\right)\int$$

$$=\frac{1}{3}d^3x^3\left(a+b\tan^{-1}(cx)\right)^2+\frac{3}{4}icd^3x^4\left(a+b\tan^{-1}(cx)\right)^2-\frac{3}{5}c^2d^3x^5\left(a+b\tan^{-1}(c\right.$$

$$=\frac{1}{3}d^3x^3\left(a+b\tan^{-1}(cx)\right)^2+\frac{3}{4}icd^3x^4\left(a+b\tan^{-1}(cx)\right)^2-\frac{3}{5}c^2d^3x^5\left(a+b\tan^{-1}(c\right.$$

$$=-\frac{bd^3x^2\left(a+b\tan^{-1}(cx)\right)}{3c}-\frac{1}{2}ibd^3x^3\left(a+b\tan^{-1}(cx)\right)+\frac{3}{10}bcd^3x^4\left(a+b\tan^{-1}(\right.$$

$$=\frac{3iabd^3x}{2c^2}+\frac{b^2d^3x}{3c^2}-\frac{14bd^3x^2\left(a+b\tan^{-1}(cx)\right)}{15c}-\frac{11}{18}ibd^3x^3\left(a+b\tan^{-1}(cx)\right)+\frac{3}{1}$$

$$=\frac{11iabd^3x}{6c^2}+\frac{37b^2d^3x}{30c^2}-\frac{1}{10}b^2d^3x^3-\frac{b^2d^3\tan^{-1}(cx)}{3c^3}+\frac{3ib^2d^3x\tan^{-1}(cx)}{2c^2}-\frac{14bd}{}$$

$$=\frac{11iabd^3x}{6c^2}+\frac{37b^2d^3x}{30c^2}+\frac{17ib^2d^3x^2}{60c}-\frac{1}{10}b^2d^3x^3-\frac{1}{60}ib^2cd^3x^4-\frac{37b^2d^3\tan^{-1}(cx)}{30c^3}$$

$$=\frac{11iabd^3x}{6c^2}+\frac{37b^2d^3x}{30c^2}+\frac{61ib^2d^3x^2}{180c}-\frac{1}{10}b^2d^3x^3-\frac{1}{60}ib^2cd^3x^4-\frac{37b^2d^3\tan^{-1}(cx)}{30c^3}$$

Mathematica [A] time = 1.3733, size = 369, normalized size = 0.92

$$d^3\left(168ib^2\text{PolyLog}\left(2,-e^{2i\tan^{-1}(cx)}\right)-30ia^2c^6x^6-108a^2c^5x^5+135ia^2c^4x^4+60a^2c^3x^3+12iabc^5x^5+54abc^4x^4-110iabc\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(d + I*c*d*x)^3*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] (d^3*(-162*a*b + (64*I)*b^2 + (330*I)*a*b*c*x + 222*b^2*c*x - 168*a*b*c^2*x
^2 + (61*I)*b^2*c^2*x^2 + 60*a^2*c^3*x^3 - (110*I)*a*b*c^3*x^3 - 18*b^2*c^3
*x^3 + (135*I)*a^2*c^4*x^4 + 54*a*b*c^4*x^4 - (3*I)*b^2*c^4*x^4 - 108*a^2*c
^5*x^5 + (12*I)*a*b*c^5*x^5 - (30*I)*a^2*c^6*x^6 + 3*b^2*(-I + c*x)^4*(I +
4*c*x - (10*I)*c^2*x^2)*ArcTan[c*x]^2 + 2*b*ArcTan[c*x]*(b*(-111 + (165*I)*
c*x - 84*c^2*x^2 - (55*I)*c^3*x^3 + 27*c^4*x^4 + (6*I)*c^5*x^5) + 3*a*(-55*
I + 20*c^3*x^3 + (45*I)*c^4*x^4 - 36*c^5*x^5 - (10*I)*c^6*x^6) - 168*b*Log[
1 + E^((2*I)*ArcTan[c*x])]) + 168*a*b*Log[1 + c^2*x^2] - (226*I)*b^2*Log[1
+ c^2*x^2] + (168*I)*b^2*PolyLog[2, -E^((2*I)*ArcTan[c*x])]))/(180*c^3)
```

Maple [B] time = 0.1, size = 712, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d+I*c*d*x)^3*(a+b*arctan(c*x))^2,x)
```

```
[Out] -7/15*I/c^3*d^3*b^2*ln(c*x-I)*ln(-1/2*I*(c*x+I))+1/15*I*c^2*d^3*a*b*x^5+7/1
5*I/c^3*d^3*b^2*ln(c*x-I)*ln(c^2*x^2+1)-6/5*c^2*d^3*a*b*arctan(c*x)*x^5+1/1
5*I*c^2*d^3*b^2*arctan(c*x)*x^5-1/6*I*c^3*d^3*b^2*arctan(c*x)^2*x^6+3/4*I*c
*d^3*b^2*arctan(c*x)^2*x^4+7/15*I/c^3*d^3*b^2*ln(c*x+I)*ln(1/2*I*(c*x-I))-1
1/6*I/c^3*d^3*a*b*arctan(c*x)-7/15*I/c^3*d^3*b^2*ln(c*x+I)*ln(c^2*x^2+1)+1/
3*d^3*b^2*arctan(c*x)^2*x^3-3/5*c^2*d^3*a^2*x^5+3/2*I*c*d^3*a*b*arctan(c*x)
*x^4-1/3*I*c^3*d^3*a*b*arctan(c*x)*x^6+2/3*d^3*a*b*arctan(c*x)*x^3-14/15/c*
d^3*b^2*arctan(c*x)*x^2-3/5*c^2*d^3*b^2*arctan(c*x)^2*x^5+14/15/c^3*d^3*b^2
*arctan(c*x)*ln(c^2*x^2+1)-11/18*I*d^3*a*b*x^3-1/6*I*c^3*d^3*a^2*x^6+3/4*I*
c*d^3*a^2*x^4+7/30*I/c^3*d^3*b^2*ln(c*x+I)^2+7/15*I/c^3*d^3*b^2*dilog(1/2*I
*(c*x-I))-7/15*I/c^3*d^3*b^2*dilog(-1/2*I*(c*x+I))+3/10*c*d^3*a*b*x^4-14/15
/c*d^3*a*b*x^2+3/10*c*d^3*b^2*arctan(c*x)*x^4-11/18*I*d^3*b^2*arctan(c*x)*x
^3-11/12*I/c^3*d^3*b^2*arctan(c*x)^2-7/30*I/c^3*d^3*b^2*ln(c*x-I)^2+14/15/c
^3*d^3*a*b*ln(c^2*x^2+1)+1/3*d^3*a^2*x^3-1/10*b^2*d^3*x^3+37/30*b^2*d^3*x/c
^2-37/30*b^2*d^3*arctan(c*x)/c^3+11/6*I*a*b*d^3*x/c^2+61/180*I*b^2*d^3*x^2/
c+11/6*I*b^2*d^3*x*arctan(c*x)/c^2-1/60*I*b^2*c*d^3*x^4-113/90*I*b^2*d^3*ln
(c^2*x^2+1)/c^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d+I*c*d*x)^3*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

```
[Out] -1/6*I*a^2*c^3*d^3*x^6 - 3/5*a^2*c^2*d^3*x^5 + 3/4*I*a^2*c*d^3*x^4 - 1/45*I
*(15*x^6*arctan(c*x) - c*((3*c^4*x^5 - 5*c^2*x^3 + 15*x)/c^6 - 15*arctan(c*
x)/c^7))*a*b*c^3*d^3 - 3/10*(4*x^5*arctan(c*x) - c*((c^2*x^4 - 2*x^2)/c^4 +
 2*log(c^2*x^2 + 1)/c^6))*a*b*c^2*d^3 + 1/3*a^2*d^3*x^3 + 1/2*I*(3*x^4*arct
an(c*x) - c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5))*a*b*c*d^3 + 1/3*(2*x
^3*arctan(c*x) - c*(x^2/c^2 - log(c^2*x^2 + 1)/c^4))*a*b*d^3 - 1/960*(40*I*
b^2*c^3*d^3*x^6 + 144*b^2*c^2*d^3*x^5 - 180*I*b^2*c*d^3*x^4 - 80*b^2*d^3*x^
3)*arctan(c*x)^2 + 1/960*(40*b^2*c^3*d^3*x^6 - 144*I*b^2*c^2*d^3*x^5 - 180*
b^2*c*d^3*x^4 + 80*I*b^2*d^3*x^3)*arctan(c*x)*log(c^2*x^2 + 1) - 1/960*(-10
*I*b^2*c^3*d^3*x^6 - 36*b^2*c^2*d^3*x^5 + 45*I*b^2*c*d^3*x^4 + 20*b^2*d^3*x
^3)*log(c^2*x^2 + 1)^2 - I*integrate(1/240*(180*(b^2*c^5*d^3*x^7 - 2*b^2*c^
3*d^3*x^5 - 3*b^2*c*d^3*x^3)*arctan(c*x)^2 + 15*(b^2*c^5*d^3*x^7 - 2*b^2*c^
3*d^3*x^5 - 3*b^2*c*d^3*x^3)*log(c^2*x^2 + 1)^2 - 2*(46*b^2*c^4*d^3*x^6 - 6
5*b^2*c^2*d^3*x^4)*arctan(c*x) + (10*b^2*c^5*d^3*x^7 - 81*b^2*c^3*d^3*x^5 +
 20*b^2*c*d^3*x^3 - 60*(3*b^2*c^4*d^3*x^6 + 2*b^2*c^2*d^3*x^4 - b^2*d^3*x^2
)*arctan(c*x))*log(c^2*x^2 + 1))/(c^2*x^2 + 1), x) - integrate(1/240*(180*(
3*b^2*c^4*d^3*x^6 + 2*b^2*c^2*d^3*x^4 - b^2*d^3*x^2)*arctan(c*x)^2 + 15*(3*
b^2*c^4*d^3*x^6 + 2*b^2*c^2*d^3*x^4 - b^2*d^3*x^2)*log(c^2*x^2 + 1)^2 + 2*(
10*b^2*c^5*d^3*x^7 - 81*b^2*c^3*d^3*x^5 + 20*b^2*c*d^3*x^3)*arctan(c*x) + (
46*b^2*c^4*d^3*x^6 - 65*b^2*c^2*d^3*x^4 + 60*(b^2*c^5*d^3*x^7 - 2*b^2*c^3*d
^3*x^5 - 3*b^2*c*d^3*x^3)*arctan(c*x))*log(c^2*x^2 + 1))/(c^2*x^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{240}\left(10i\,b^2c^3d^3x^6+36\,b^2c^2d^3x^5-45i\,b^2cd^3x^4-20\,b^2d^3x^3\right)\log\left(-\frac{cx+i}{cx-i}\right)^2+\text{integral}\left(\frac{-60i\,a^2c^5d^3x^7-180\,a^2c^4d^3x^6+1}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d+I*c*d*x)^3*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/240*(10*I*b^2*c^3*d^3*x^6 + 36*b^2*c^2*d^3*x^5 - 45*I*b^2*c*d^3*x^4 - 20*
b^2*d^3*x^3)*log(-(c*x + I)/(c*x - I))^2 + integral(1/60*(-60*I*a^2*c^5*d^3
```

```
*x^7 - 180*a^2*c^4*d^3*x^6 + 120*I*a^2*c^3*d^3*x^5 - 120*a^2*c^2*d^3*x^4 +
180*I*a^2*c*d^3*x^3 + 60*a^2*d^3*x^2 + (60*a*b*c^5*d^3*x^7 + (-180*I*a*b -
10*b^2)*c^4*d^3*x^6 - 12*(10*a*b - 3*I*b^2)*c^3*d^3*x^5 + (-120*I*a*b + 45*
b^2)*c^2*d^3*x^4 - 20*(9*a*b + I*b^2)*c*d^3*x^3 + 60*I*a*b*d^3*x^2)*log(-(c
*x + I)/(c*x - I)))/(c^2*x^2 + 1), x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d+I*c*d*x)**3*(a+b*atan(c*x))**2,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (icdx + d)^3 (b \arctan(cx) + a)^2 x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d+I*c*d*x)^3*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((I*c*d*x + d)^3*(b*arctan(c*x) + a)^2*x^2, x)
```

3.86 $\int x(d+icdx)^3\left(a+b\tan^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=307

$$-\frac{6b^2d^3\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{5c^2}+\frac{1}{10}ibc^2d^3x^4\left(a+b\tan^{-1}(cx)\right)-\frac{d^3(1+icx)^5\left(a+b\tan^{-1}(cx)\right)^2}{5c^2}+\frac{d^3(1+icx)^4\left(a+b\tan\right.}{4c^2}$$

```
[Out] (-5*a*b*d^3*x)/(2*c) + (((13*I)/10)*b^2*d^3*x)/c - (b^2*d^3*x^2)/4 - (I/30)
*b^2*c*d^3*x^3 - (((13*I)/10)*b^2*d^3*ArcTan[c*x])/c^2 - (5*b^2*d^3*x*ArcTa
n[c*x])/(2*c) - ((6*I)/5)*b*d^3*x^2*(a + b*ArcTan[c*x]) + (b*c*d^3*x^3*(a +
 b*ArcTan[c*x]))/2 + (I/10)*b*c^2*d^3*x^4*(a + b*ArcTan[c*x]) + (d^3*(1 + I
*c*x)^4*(a + b*ArcTan[c*x])^2)/(4*c^2) - (d^3*(1 + I*c*x)^5*(a + b*ArcTan[c
*x])^2)/(5*c^2) - (((12*I)/5)*b*d^3*(a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])
/c^2 + (3*b^2*d^3*Log[1 + c^2*x^2])/(2*c^2) - (6*b^2*d^3*PolyLog[2, 1 - 2/(
1 - I*c*x)])/(5*c^2)
```

Rubi [A] time = 0.613652, antiderivative size = 307, normalized size of antiderivative = 1., number of steps used = 38, number of rules used = 14, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.609, Rules used = {4876, 4864, 4846, 260, 4852, 321, 203, 266, 43, 1586, 4854, 2402, 2315, 302}

$$-\frac{6b^2d^3\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{5c^2}+\frac{1}{10}ibc^2d^3x^4\left(a+b\tan^{-1}(cx)\right)-\frac{d^3(1+icx)^5\left(a+b\tan^{-1}(cx)\right)^2}{5c^2}+\frac{d^3(1+icx)^4\left(a+b\tan\right.}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + I*c*d*x)^3*(a + b*ArcTan[c*x])^2,x]
```


```
[Out] (-5*a*b*d^3*x)/(2*c) + (((13*I)/10)*b^2*d^3*x)/c - (b^2*d^3*x^2)/4 - (I/30)
*b^2*c*d^3*x^3 - (((13*I)/10)*b^2*d^3*ArcTan[c*x])/c^2 - (5*b^2*d^3*x*ArcTa
n[c*x])/(2*c) - ((6*I)/5)*b*d^3*x^2*(a + b*ArcTan[c*x]) + (b*c*d^3*x^3*(a +
 b*ArcTan[c*x]))/2 + (I/10)*b*c^2*d^3*x^4*(a + b*ArcTan[c*x]) + (d^3*(1 + I
*c*x)^4*(a + b*ArcTan[c*x])^2)/(4*c^2) - (d^3*(1 + I*c*x)^5*(a + b*ArcTan[c
*x])^2)/(5*c^2) - (((12*I)/5)*b*d^3*(a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])
/c^2 + (3*b^2*d^3*Log[1 + c^2*x^2])/(2*c^2) - (6*b^2*d^3*PolyLog[2, 1 - 2/(
1 - I*c*x)])/(5*c^2)
```


Rule 4876

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
```

```
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 4864

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x])^p)/(e*(q + 1)), x] - D
ist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\int x(d+icdx)^3\left(a+b\tan^{-1}(cx)\right)^2\,dx=\int\left(\frac{i(d+icdx)^3\left(a+b\tan^{-1}(cx)\right)^2}{c}-\frac{i(d+icdx)^4\left(a+b\tan^{-1}(cx)\right)^2}{cd}\right)dx$$

$$=\frac{i\int(d+icdx)^3\left(a+b\tan^{-1}(cx)\right)^2\,dx}{c}-\frac{i\int(d+icdx)^4\left(a+b\tan^{-1}(cx)\right)^2\,dx}{cd}$$

$$=\frac{d^3(1+icx)^4\left(a+b\tan^{-1}(cx)\right)^2}{4c^2}-\frac{d^3(1+icx)^5\left(a+b\tan^{-1}(cx)\right)^2}{5c^2}+\frac{(2b)\int\left(-15\right.}{}$$

$$=\frac{d^3(1+icx)^4\left(a+b\tan^{-1}(cx)\right)^2}{4c^2}-\frac{d^3(1+icx)^5\left(a+b\tan^{-1}(cx)\right)^2}{5c^2}-(32ib)\int\frac{(id^5}{}$$

$$=-\frac{5abd^3x}{2c}-\frac{6}{5}ibd^3x^2\left(a+b\tan^{-1}(cx)\right)+\frac{1}{2}bcd^3x^3\left(a+b\tan^{-1}(cx)\right)+\frac{1}{10}ibc^2d^3x$$

$$=-\frac{5abd^3x}{2c}+\frac{6ib^2d^3x}{5c}-\frac{5b^2d^3x\tan^{-1}(cx)}{2c}-\frac{6}{5}ibd^3x^2\left(a+b\tan^{-1}(cx)\right)+\frac{1}{2}bcd^3x$$

$$=-\frac{5abd^3x}{2c}+\frac{13ib^2d^3x}{10c}-\frac{1}{30}ib^2cd^3x^3-\frac{6ib^2d^3\tan^{-1}(cx)}{5c^2}-\frac{5b^2d^3x\tan^{-1}(cx)}{2c}-\frac{6}{5}$$

$$=-\frac{5abd^3x}{2c}+\frac{13ib^2d^3x}{10c}-\frac{1}{4}b^2d^3x^2-\frac{1}{30}ib^2cd^3x^3-\frac{13ib^2d^3\tan^{-1}(cx)}{10c^2}-\frac{5b^2d^3x\,\mathrm{ta}}{2c}$$

Mathematica [A] time = 1.28737, size = 325, normalized size = 1.06

$$\frac{d^3\left(-72b^2\mathrm{PolyLog}\left(2,-e^{2i\tan^{-1}(cx)}\right)-12ia^2c^5x^5-45a^2c^4x^4+60ia^2c^3x^3+30a^2c^2x^2+6iabc^4x^4+30abc^3x^3-72iabc^2x^2\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*(d + I*c*d*x)^3*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] (d^3*((-18*I)*a*b - 15*b^2 - 150*a*b*c*x + (78*I)*b^2*c*x + 30*a^2*c^2*x^2
- (72*I)*a*b*c^2*x^2 - 15*b^2*c^2*x^2 + (60*I)*a^2*c^3*x^3 + 30*a*b*c^3*x^3
 - (2*I)*b^2*c^3*x^3 - 45*a^2*c^4*x^4 + (6*I)*a*b*c^4*x^4 - (12*I)*a^2*c^5*
x^5 + 3*b^2*(1 - (4*I)*c*x)*(-I + c*x)^4*ArcTan[c*x]^2 + 6*b*ArcTan[c*x]*(b
*(-13*I - 25*c*x - (12*I)*c^2*x^2 + 5*c^3*x^3 + I*c^4*x^4) + a*(25 + 10*c^2
*x^2 + (20*I)*c^3*x^3 - 15*c^4*x^4 - (4*I)*c^5*x^5) - (24*I)*b*Log[1 + E^((
2*I)*ArcTan[c*x])]) + (72*I)*a*b*Log[1 + c^2*x^2] + 90*b^2*Log[1 + c^2*x^2]
```

```
- 72*b^2*PolyLog[2, -E^((2*I)*ArcTan[c*x])]))/(60*c^2)
```

Maple [B] time = 0.1, size = 656, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d+I*c*d*x)^3*(a+b*arctan(c*x))^2,x)
```

```
[Out] I*c*d^3*b^2*arctan(c*x)^2*x^3-1/5*I*c^3*d^3*b^2*arctan(c*x)^2*x^5+1/10*I*c^
2*d^3*b^2*arctan(c*x)*x^4+6/5*I/c^2*d^3*b^2*ln(c^2*x^2+1)*arctan(c*x)+1/10*
I*c^2*d^3*a*b*x^4+6/5*I/c^2*d^3*a*b*ln(c^2*x^2+1)-3/2*c^2*d^3*a*b*arctan(c*
x)*x^4-3/4*c^2*d^3*a^2*x^4+1/2*d^3*b^2*arctan(c*x)^2*x^2-3/10/c^2*d^3*b^2*l
n(c*x+I)^2+3/10/c^2*d^3*b^2*ln(c*x-I)^2+5/4/c^2*d^3*b^2*arctan(c*x)^2-3/5/c
^2*d^3*b^2*dilog(1/2*I*(c*x-I))+3/5/c^2*d^3*b^2*dilog(-1/2*I*(c*x+I))-3/5/c
^2*d^3*b^2*ln(c*x-I)*ln(c^2*x^2+1)+3/5/c^2*d^3*b^2*ln(c*x-I)*ln(-1/2*I*(c*x
+I))+3/5/c^2*d^3*b^2*ln(c*x+I)*ln(c^2*x^2+1)-3/5/c^2*d^3*b^2*ln(c*x+I)*ln(1
/2*I*(c*x-I))-1/5*I*c^3*d^3*a^2*x^5-6/5*I*d^3*b^2*arctan(c*x)*x^2-6/5*I*d^3
*a*b*x^2+1/2*c*d^3*b^2*arctan(c*x)*x^3-3/4*c^2*d^3*b^2*arctan(c*x)^2*x^4+d^
3*a*b*arctan(c*x)*x^2+5/2/c^2*d^3*a*b*arctan(c*x)+I*c*d^3*a^2*x^3+1/2*c*d^3
*a*b*x^3-2/5*I*c^3*d^3*a*b*arctan(c*x)*x^5+2*I*c*d^3*a*b*arctan(c*x)*x^3+1/
2*d^3*a^2*x^2-1/4*b^2*d^3*x^2+13/10*I*b^2*d^3*x/c-5/2*a*b*d^3*x/c-5/2*b^2*d
^3*x*arctan(c*x)/c-1/30*I*b^2*c*d^3*x^3-13/10*I*b^2*d^3*arctan(c*x)/c^2+3/2
*b^2*d^3*ln(c^2*x^2+1)/c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d+I*c*d*x)^3*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

```
[Out] -1/5*I*a^2*c^3*d^3*x^5 - 3/4*a^2*c^2*d^3*x^4 - 1/10*I*(4*x^5*arctan(c*x) -
c*((c^2*x^4 - 2*x^2)/c^4 + 2*log(c^2*x^2 + 1)/c^6))*a*b*c^3*d^3 + I*a^2*c*d
^3*x^3 + 1/2*b^2*d^3*x^2*arctan(c*x)^2 - 1/2*(3*x^4*arctan(c*x) - c*((c^2*x
^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5))*a*b*c^2*d^3 + I*(2*x^3*arctan(c*x) - c*
(x^2/c^2 - log(c^2*x^2 + 1)/c^4))*a*b*c*d^3 + 1/2*a^2*d^3*x^2 + (x^2*arctan
```

```
(c*x) - c*(x/c^2 - arctan(c*x)/c^3))*a*b*d^3 - 1/2*(2*c*(x/c^2 - arctan(c*x
)/c^3)*arctan(c*x) + (arctan(c*x)^2 - log(c^2*x^2 + 1))/c^2)*b^2*d^3 - 1/32
0*(16*I*b^2*c^3*d^3*x^5 + 60*b^2*c^2*d^3*x^4 - 80*I*b^2*c*d^3*x^3)*arctan(c
*x)^2 + 1/320*(16*b^2*c^3*d^3*x^5 - 60*I*b^2*c^2*d^3*x^4 - 80*b^2*c*d^3*x^3
)*arctan(c*x)*log(c^2*x^2 + 1) - 1/320*(-4*I*b^2*c^3*d^3*x^5 - 15*b^2*c^2*d
^3*x^4 + 20*I*b^2*c*d^3*x^3)*log(c^2*x^2 + 1)^2 - I*integrate(1/80*(60*(b^2
*c^5*d^3*x^6 - 2*b^2*c^3*d^3*x^4 - 3*b^2*c*d^3*x^2)*arctan(c*x)^2 + 5*(b^2*
c^5*d^3*x^6 - 2*b^2*c^3*d^3*x^4 - 3*b^2*c*d^3*x^2)*log(c^2*x^2 + 1)^2 - 2*(
19*b^2*c^4*d^3*x^5 - 20*b^2*c^2*d^3*x^3)*arctan(c*x) + (4*b^2*c^5*d^3*x^6 -
 35*b^2*c^3*d^3*x^4 - 60*(b^2*c^4*d^3*x^5 + b^2*c^2*d^3*x^3)*arctan(c*x))*l
og(c^2*x^2 + 1))/(c^2*x^2 + 1), x) - integrate(1/80*(180*(b^2*c^4*d^3*x^5 +
 b^2*c^2*d^3*x^3)*arctan(c*x)^2 + 15*(b^2*c^4*d^3*x^5 + b^2*c^2*d^3*x^3)*lo
g(c^2*x^2 + 1)^2 + 2*(4*b^2*c^5*d^3*x^6 - 35*b^2*c^3*d^3*x^4)*arctan(c*x) +
 (19*b^2*c^4*d^3*x^5 - 20*b^2*c^2*d^3*x^3 + 20*(b^2*c^5*d^3*x^6 - 2*b^2*c^3
*d^3*x^4 - 3*b^2*c*d^3*x^2)*arctan(c*x))*log(c^2*x^2 + 1))/(c^2*x^2 + 1), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{80}\left(4i\,b^2c^3d^3x^5+15\,b^2c^2d^3x^4-20i\,b^2cd^3x^3-10\,b^2d^3x^2\right)\log\left(-\frac{cx+i}{cx-i}\right)^2+\text{integral}\left(\frac{-20i\,a^2c^5d^3x^6-60\,a^2c^4d^3x^5+40i}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d+I*c*d*x)^3*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/80*(4*I*b^2*c^3*d^3*x^5 + 15*b^2*c^2*d^3*x^4 - 20*I*b^2*c*d^3*x^3 - 10*b^
2*d^3*x^2)*log(-(c*x + I)/(c*x - I))^2 + integral(1/20*(-20*I*a^2*c^5*d^3*x
^6 - 60*a^2*c^4*d^3*x^5 + 40*I*a^2*c^3*d^3*x^4 - 40*a^2*c^2*d^3*x^3 + 60*I*
a^2*c*d^3*x^2 + 20*a^2*d^3*x + (20*a*b*c^5*d^3*x^6 + (-60*I*a*b - 4*b^2)*c^
4*d^3*x^5 - 5*(8*a*b - 3*I*b^2)*c^3*d^3*x^4 + (-40*I*a*b + 20*b^2)*c^2*d^3*
x^3 - 10*(6*a*b + I*b^2)*c*d^3*x^2 + 20*I*a*b*d^3*x)*log(-(c*x + I)/(c*x -
I)))/(c^2*x^2 + 1), x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d+I*c*d*x)**3*(a+b*atan(c*x))**2,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (icdx + d)^3 (b \arctan (cx) + a)^2 x \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d+I*c*d*x)^3*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((I*c*d*x + d)^3*(b*arctan(c*x) + a)^2*x, x)
```

3.87 $\int (d+icdx)^3 \left(a+b\tan^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=226

$$-\frac{2ib^2d^3\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{c}+\frac{1}{6}ibc^2d^3x^3\left(a+b\tan^{-1}(cx)\right)+bcd^3x^2\left(a+b\tan^{-1}(cx)\right)-\frac{id^3(1+icx)^4\left(a+b\tan^{-1}(c\right.}{4c}$$

```
[Out] ((-7*I)/2)*a*b*d^3*x - b^2*d^3*x - (I/12)*b^2*c*d^3*x^2 + (b^2*d^3*ArcTan[c
*x])/c - ((7*I)/2)*b^2*d^3*x*ArcTan[c*x] + b*c*d^3*x^2*(a + b*ArcTan[c*x])
+ (I/6)*b*c^2*d^3*x^3*(a + b*ArcTan[c*x]) - ((I/4)*d^3*(1 + I*c*x)^4*(a + b
*ArcTan[c*x])^2)/c + (4*b*d^3*(a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/c + (
((11*I)/6)*b^2*d^3*Log[1 + c^2*x^2])/c - ((2*I)*b^2*d^3*PolyLog[2, 1 - 2/(1
 - I*c*x)])/c
```

Rubi [A] time = 0.20649, antiderivative size = 226, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 12, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {4864, 4846, 260, 4852, 321, 203, 266, 43, 1586, 4854, 2402, 2315}

$$-\frac{2ib^2d^3\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{c}+\frac{1}{6}ibc^2d^3x^3\left(a+b\tan^{-1}(cx)\right)+bcd^3x^2\left(a+b\tan^{-1}(cx)\right)-\frac{id^3(1+icx)^4\left(a+b\tan^{-1}(c\right.}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + I*c*d*x)^3*(a + b*ArcTan[c*x])^2,x]
```


```
[Out] ((-7*I)/2)*a*b*d^3*x - b^2*d^3*x - (I/12)*b^2*c*d^3*x^2 + (b^2*d^3*ArcTan[c
*x])/c - ((7*I)/2)*b^2*d^3*x*ArcTan[c*x] + b*c*d^3*x^2*(a + b*ArcTan[c*x])
+ (I/6)*b*c^2*d^3*x^3*(a + b*ArcTan[c*x]) - ((I/4)*d^3*(1 + I*c*x)^4*(a + b
*ArcTan[c*x])^2)/c + (4*b*d^3*(a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/c + (
((11*I)/6)*b^2*d^3*Log[1 + c^2*x^2])/c - ((2*I)*b^2*d^3*PolyLog[2, 1 - 2/(1
 - I*c*x)])/c
```


Rule 4864

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x])^p)/(e*(q + 1)), x] - D
ist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int (d+icdx)^3 \left(a+b\tan^{-1}(cx)\right)^2\,dx = -\frac{id^3(1+icx)^4\left(a+b\tan^{-1}(cx)\right)^2}{4c} + \frac{(ib)\int \left(-7d^4\left(a+b\tan^{-1}(cx)\right) - 4icd^4x\left(a+b\right.\right.}{}$$

$$= -\frac{id^3(1+icx)^4\left(a+b\tan^{-1}(cx)\right)^2}{4c} + \frac{(4b)\int \frac{\left(id^4-cd^4x\right)\left(a+b\tan^{-1}(cx)\right)}{1+c^2x^2}\,dx}{d} - \frac{1}{2}\left(7ibd^3\right)\int$$

$$= -\frac{7}{2}iabd^3x + bcd^3x^2\left(a+b\tan^{-1}(cx)\right) + \frac{1}{6}ibc^2d^3x^3\left(a+b\tan^{-1}(cx)\right) - \frac{id^3(1+icx)}{}$$

$$= -\frac{7}{2}iabd^3x - b^2d^3x - \frac{7}{2}ib^2d^3x\tan^{-1}(cx) + bcd^3x^2\left(a+b\tan^{-1}(cx)\right) + \frac{1}{6}ibc^2d^3x^3\left(a\right.$$

$$= -\frac{7}{2}iabd^3x - b^2d^3x + \frac{b^2d^3\tan^{-1}(cx)}{c} - \frac{7}{2}ib^2d^3x\tan^{-1}(cx) + bcd^3x^2\left(a+b\tan^{-1}(cx\right.$$

$$= -\frac{7}{2}iabd^3x - b^2d^3x - \frac{1}{12}ib^2cd^3x^2 + \frac{b^2d^3\tan^{-1}(cx)}{c} - \frac{7}{2}ib^2d^3x\tan^{-1}(cx) + bcd^3x^2\left(a\right.$$

Mathematica [A] time = 0.895227, size = 267, normalized size = 1.18

$$-\frac{id^3\left(24b^2\text{PolyLog}\left(2,-e^{2i\tan^{-1}(cx)}\right) + 3a^2c^4x^4 - 12ia^2c^3x^3 - 18a^2c^2x^2 + 12ia^2cx - 2abc^3x^3 + 12iabc^2x^2 - 24iab\log\left(c^2x\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + I*c*d*x)^3*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] ((-I/12)*d^3*(b^2 + (12*I)*a^2*c*x + 42*a*b*c*x - (12*I)*b^2*c*x - 18*a^2*c
^2*x^2 + (12*I)*a*b*c^2*x^2 + b^2*c^2*x^2 - (12*I)*a^2*c^3*x^3 - 2*a*b*c^3*
x^3 + 3*a^2*c^4*x^4 + 3*b^2*(-I + c*x)^4*ArcTan[c*x]^2 + 2*b*ArcTan[c*x]*(b
*(6*I + 21*c*x + (6*I)*c^2*x^2 - c^3*x^3) + 3*a*(-7 + (4*I)*c*x - 6*c^2*x^2
 - (4*I)*c^3*x^3 + c^4*x^4) + (24*I)*b*Log[1 + E^((2*I)*ArcTan[c*x])]) - (2
4*I)*a*b*Log[1 + c^2*x^2] - 22*b^2*Log[1 + c^2*x^2] + 24*b^2*PolyLog[2, -E^
((2*I)*ArcTan[c*x])]))/c
```

Maple [B] time = 0.094, size = 620, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^3*(a+b*arctan(c*x))^2,x)
```

```
[Out] -1/4*I/c*d^3*a^2+d^3*b^2*arctan(c*x)^2*x-c^2*x^3*a^2*d^3-1/2*I*c^3*d^3*a*b*
arctan(c*x)*x^4+3*I*c*d^3*a*b*arctan(c*x)*x^2-c^2*d^3*b^2*arctan(c*x)^2*x^3
+c*d^3*b^2*arctan(c*x)*x^2-2/c*d^3*b^2*arctan(c*x)*ln(c^2*x^2+1)+I/c*d^3*b^
2*dilog(-1/2*I*(c*x+I))-2/c*d^3*a*b*ln(c^2*x^2+1)+1/2*I/c*d^3*b^2*ln(c*x-I)
^2+7/4*I/c*d^3*b^2*arctan(c*x)^2-1/4*I*c^3*x^4*a^2*d^3+3/2*I*c*x^2*a^2*d^3-
I/c*d^3*b^2*dilog(1/2*I*(c*x-I))-1/2*I/c*d^3*b^2*ln(c*x+I)^2+2*d^3*a*b*arct
an(c*x)*x-1/4*I*c^3*d^3*b^2*arctan(c*x)^2*x^4+c*d^3*a*b*x^2-I/c*d^3*b^2*ln(
c*x+I)*ln(1/2*I*(c*x-I))-I/c*d^3*b^2*ln(c^2*x^2+1)*ln(c*x-I)+7/2*I/c*d^3*a*
b*arctan(c*x)+I/c*d^3*b^2*ln(c*x-I)*ln(-1/2*I*(c*x+I))+I/c*d^3*b^2*ln(c^2*x
^2+1)*ln(c*x+I)-2*c^2*d^3*a*b*arctan(c*x)*x^3+1/6*I*c^2*d^3*b^2*arctan(c*x)
*x^3+3/2*I*c*d^3*b^2*arctan(c*x)^2*x^2+1/6*I*c^2*d^3*a*b*x^3+x*a^2*d^3-7/2*
I*a*b*d^3*x-1/12*I*b^2*c*d^3*x^2-7/2*I*b^2*d^3*x*arctan(c*x)+11/6*I*b^2*d^3
*ln(c^2*x^2+1)/c-b^2*d^3*x+b^2*d^3*arctan(c*x)/c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

```
[Out] -1/4*I*a^2*c^3*d^3*x^4 - 4*b^2*c^5*d^3*integrate(1/16*x^5*arctan(c*x)*log(c
^2*x^2 + 1)/(c^2*x^2 + 1), x) - 2*b^2*c^5*d^3*integrate(1/16*x^5*arctan(c*x
)/(c^2*x^2 + 1), x) - a^2*c^2*d^3*x^3 - 36*b^2*c^4*d^3*integrate(1/16*x^4*a
rctan(c*x)^2/(c^2*x^2 + 1), x) - 3*b^2*c^4*d^3*integrate(1/16*x^4*log(c^2*x
^2 + 1)^2/(c^2*x^2 + 1), x) - 5*b^2*c^4*d^3*integrate(1/16*x^4*log(c^2*x^2
+ 1)/(c^2*x^2 + 1), x) - 1/6*I*(3*x^4*arctan(c*x) - c*((c^2*x^3 - 3*x)/c^4
+ 3*arctan(c*x)/c^5))*a*b*c^3*d^3 + 8*b^2*c^3*d^3*integrate(1/16*x^3*arctan
(c*x)*log(c^2*x^2 + 1)/(c^2*x^2 + 1), x) + 20*b^2*c^3*d^3*integrate(1/16*x^
3*arctan(c*x)/(c^2*x^2 + 1), x) - (2*x^3*arctan(c*x) - c*(x^2/c^2 - log(c^2
*x^2 + 1)/c^4))*a*b*c^2*d^3 + 3/2*I*a^2*c*d^3*x^2 - 24*b^2*c^2*d^3*integrat
e(1/16*x^2*arctan(c*x)^2/(c^2*x^2 + 1), x) - 2*b^2*c^2*d^3*integrate(1/16*x
^2*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) + 10*b^2*c^2*d^3*integrate(1/16*x^2
*log(c^2*x^2 + 1)/(c^2*x^2 + 1), x) + 3*I*(x^2*arctan(c*x) - c*(x/c^2 - arc
tan(c*x)/c^3))*a*b*c*d^3 + 1/4*b^2*d^3*arctan(c*x)^3/c + 12*b^2*c*d^3*integ
rate(1/16*x*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^2 + 1), x) - 8*b^2*c*d^3*in
tegrate(1/16*x*arctan(c*x)/(c^2*x^2 + 1), x) + a^2*d^3*x + b^2*d^3*integrat
```

```
e(1/16*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) + (2*c*x*arctan(c*x) - log(c^2*
x^2 + 1))*a*b*d^3/c - 1/64*(4*I*b^2*c^3*d^3*x^4 + 16*b^2*c^2*d^3*x^3 - 24*I
*b^2*c*d^3*x^2 - 16*b^2*d^3*x)*arctan(c*x)^2 + 1/64*(4*b^2*c^3*d^3*x^4 - 16
*I*b^2*c^2*d^3*x^3 - 24*b^2*c*d^3*x^2 + 16*I*b^2*d^3*x)*arctan(c*x)*log(c^2
*x^2 + 1) - 1/64*(-I*b^2*c^3*d^3*x^4 - 4*b^2*c^2*d^3*x^3 + 6*I*b^2*c*d^3*x^
2 + 4*b^2*d^3*x)*log(c^2*x^2 + 1)^2 - I*integrate(1/16*(12*(b^2*c^5*d^3*x^5
 - 2*b^2*c^3*d^3*x^3 - 3*b^2*c*d^3*x)*arctan(c*x)^2 + (b^2*c^5*d^3*x^5 - 2*
b^2*c^3*d^3*x^3 - 3*b^2*c*d^3*x)*log(c^2*x^2 + 1)^2 - 10*(b^2*c^4*d^3*x^4 -
 2*b^2*c^2*d^3*x^2)*arctan(c*x) + (b^2*c^5*d^3*x^5 - 10*b^2*c^3*d^3*x^3 + 4
*b^2*c*d^3*x - 4*(3*b^2*c^4*d^3*x^4 + 2*b^2*c^2*d^3*x^2 - b^2*d^3)*arctan(c
*x))*log(c^2*x^2 + 1))/(c^2*x^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{16}\left(i\,b^2c^3d^3x^4+4\,b^2c^2d^3x^3-6i\,b^2cd^3x^2-4\,b^2d^3x\right)\log\left(-\frac{cx+i}{cx-i}\right)^2+\text{integral}\left(\frac{-4i\,a^2c^5d^3x^5-12\,a^2c^4d^3x^4+8i\,a^2c^3d^3x^3 -}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/16*(I*b^2*c^3*d^3*x^4 + 4*b^2*c^2*d^3*x^3 - 6*I*b^2*c*d^3*x^2 - 4*b^2*d^3
*x)*log(-(c*x + I)/(c*x - I))^2 + integral(1/4*(-4*I*a^2*c^5*d^3*x^5 - 12*a
^2*c^4*d^3*x^4 + 8*I*a^2*c^3*d^3*x^3 - 8*a^2*c^2*d^3*x^2 + 12*I*a^2*c*d^3*x
 + 4*a^2*d^3 + (4*a*b*c^5*d^3*x^5 + (-12*I*a*b - b^2)*c^4*d^3*x^4 - 4*(2*a*
b - I*b^2)*c^3*d^3*x^3 + (-8*I*a*b + 6*b^2)*c^2*d^3*x^2 - 4*(3*a*b + I*b^2)
*c*d^3*x + 4*I*a*b*d^3)*log(-(c*x + I)/(c*x - I)))/(c^2*x^2 + 1), x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**3*(a+b*atan(c*x))**2,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (icdx+d)^3(b\arctan(cx)+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((I*c*d*x + d)^3*(b*arctan(c*x) + a)^2, x)
```

3.88 $\int \frac{(d+icdx)^3\left(a+b\tan^{-1}(cx)\right)^2}{x}\,dx$

Optimal. Leaf size=385

$$-ibd^3\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)+ibd^3\text{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)-\frac{10}{3}b^2d^3\text{PolyLog}\left(\right.$$

```
[Out] 3*a*b*c*d^3*x - (I/3)*b^2*c*d^3*x + (I/3)*b^2*d^3*ArcTan[c*x] + 3*b^2*c*d^3
*x*ArcTan[c*x] + (I/3)*b*c^2*d^3*x^2*(a + b*ArcTan[c*x]) - (29*d^3*(a + b*A
rcTan[c*x])^2)/6 + (3*I)*c*d^3*x*(a + b*ArcTan[c*x])^2 - (3*c^2*d^3*x^2*(a
+ b*ArcTan[c*x])^2)/2 - (I/3)*c^3*d^3*x^3*(a + b*ArcTan[c*x])^2 + 2*d^3*(a
+ b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1 + I*c*x)] + ((20*I)/3)*b*d^3*(a + b*Arc
Tan[c*x])*Log[2/(1 + I*c*x)] - (3*b^2*d^3*Log[1 + c^2*x^2])/2 - (10*b^2*d^3
*PolyLog[2, 1 - 2/(1 + I*c*x)])/3 - I*b*d^3*(a + b*ArcTan[c*x])*PolyLog[2,
1 - 2/(1 + I*c*x)] + I*b*d^3*(a + b*ArcTan[c*x])*PolyLog[2, -1 + 2/(1 + I*c
*x)] - (b^2*d^3*PolyLog[3, 1 - 2/(1 + I*c*x)])/2 + (b^2*d^3*PolyLog[3, -1 +
 2/(1 + I*c*x)])/2
```

Rubi [A] time = 0.769411, antiderivative size = 385, normalized size of antiderivative = 1., number of steps used = 28, number of rules used = 16, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.64, Rules used = {4876, 4846, 4920, 4854, 2402, 2315, 4850, 4988, 4884, 4994, 6610, 4852, 4916, 260, 321, 203}

$$-ibd^3\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)+ibd^3\text{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)-\frac{10}{3}b^2d^3\text{PolyLog}\left(\right.$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)^3*(a + b*ArcTan[c*x])^2)/x,x]
```

```
[Out] 3*a*b*c*d^3*x - (I/3)*b^2*c*d^3*x + (I/3)*b^2*d^3*ArcTan[c*x] + 3*b^2*c*d^3
*x*ArcTan[c*x] + (I/3)*b*c^2*d^3*x^2*(a + b*ArcTan[c*x]) - (29*d^3*(a + b*A
rcTan[c*x])^2)/6 + (3*I)*c*d^3*x*(a + b*ArcTan[c*x])^2 - (3*c^2*d^3*x^2*(a
+ b*ArcTan[c*x])^2)/2 - (I/3)*c^3*d^3*x^3*(a + b*ArcTan[c*x])^2 + 2*d^3*(a
+ b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1 + I*c*x)] + ((20*I)/3)*b*d^3*(a + b*Arc
Tan[c*x])*Log[2/(1 + I*c*x)] - (3*b^2*d^3*Log[1 + c^2*x^2])/2 - (10*b^2*d^3
*PolyLog[2, 1 - 2/(1 + I*c*x)])/3 - I*b*d^3*(a + b*ArcTan[c*x])*PolyLog[2,
1 - 2/(1 + I*c*x)] + I*b*d^3*(a + b*ArcTan[c*x])*PolyLog[2, -1 + 2/(1 + I*c
*x)] - (b^2*d^3*PolyLog[3, 1 - 2/(1 + I*c*x)])/2 + (b^2*d^3*PolyLog[3, -1 +
 2/(1 + I*c*x)])/2
```

Rule 4876

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 4850

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTan[c*x])^(p - 1)*ArcTanh[1 - 2/(1 + I*c*x)])/(1 + c^2*x^2), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4988

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTan[c*x])^p)/(d + e
*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTan[c*x])^p)/(d + e*x^2
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
```

```
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{(d+icdx)^3\left(a+b\tan^{-1}(cx)\right)^2}{x}\,dx = \int\left(3icd^3\left(a+b\tan^{-1}(cx)\right)^2+\frac{d^3\left(a+b\tan^{-1}(cx)\right)^2}{x}-3c^2d^3x\left(a+b\tan^{-1}(cx)\right)^2\right.$$

$$= d^3\int\frac{\left(a+b\tan^{-1}(cx)\right)^2}{x}\,dx+\left(3icd^3\right)\int\left(a+b\tan^{-1}(cx)\right)^2\,dx-\left(3c^2d^3\right)\int x\left(a\right.$$

$$= 3icd^3x\left(a+b\tan^{-1}(cx)\right)^2-\frac{3}{2}c^2d^3x^2\left(a+b\tan^{-1}(cx)\right)^2-\frac{1}{3}ic^3d^3x^3\left(a+b\tan^{-1}(\right.$$

$$= -3d^3\left(a+b\tan^{-1}(cx)\right)^2+3icd^3x\left(a+b\tan^{-1}(cx)\right)^2-\frac{3}{2}c^2d^3x^2\left(a+b\tan^{-1}(cx)\right)$$

$$= 3abcd^3x+\frac{1}{3}ibc^2d^3x^2\left(a+b\tan^{-1}(cx)\right)-\frac{29}{6}d^3\left(a+b\tan^{-1}(cx)\right)^2+3icd^3x\left(a+\right.$$

$$= 3abcd^3x-\frac{1}{3}ib^2cd^3x+3b^2cd^3x\tan^{-1}(cx)+\frac{1}{3}ibc^2d^3x^2\left(a+b\tan^{-1}(cx)\right)-\frac{29}{6}d^3\left(\right.$$

$$= 3abcd^3x-\frac{1}{3}ib^2cd^3x+\frac{1}{3}ib^2d^3\tan^{-1}(cx)+3b^2cd^3x\tan^{-1}(cx)+\frac{1}{3}ibc^2d^3x^2\left(a+b\right.$$

$$= 3abcd^3x-\frac{1}{3}ib^2cd^3x+\frac{1}{3}ib^2d^3\tan^{-1}(cx)+3b^2cd^3x\tan^{-1}(cx)+\frac{1}{3}ibc^2d^3x^2\left(a+b\right.$$

Mathematica [A] time = 0.88216, size = 465, normalized size = 1.21

$$-\frac{1}{24}id^3\left(-24ab\text{PolyLog}(2,-icx)+24ab\text{PolyLog}(2,icx)-24b^2\tan^{-1}(cx)\text{PolyLog}\left(2,e^{-2i\tan^{-1}(cx)}\right)-8b^2\left(3\tan^{-1}(cx)-\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + I*c*d*x)^3*(a + b*ArcTan[c*x])^2)/x,x]
```

```
[Out] (-I/24)*d^3*(b^2*Pi^3 - 72*a^2*c*x + (72*I)*a*b*c*x + 8*b^2*c*x - (36*I)*a^
2*c^2*x^2 - 8*a*b*c^2*x^2 + 8*a^2*c^3*x^3 - (72*I)*a*b*ArcTan[c*x] - 8*b^2*
ArcTan[c*x] - 144*a*b*c*x*ArcTan[c*x] + (72*I)*b^2*c*x*ArcTan[c*x] - (72*I)
*a*b*c^2*x^2*ArcTan[c*x] - 8*b^2*c^2*x^2*ArcTan[c*x] + 16*a*b*c^3*x^3*ArcTa
n[c*x] + (44*I)*b^2*ArcTan[c*x]^2 - 72*b^2*c*x*ArcTan[c*x]^2 - (36*I)*b^2*c
^2*x^2*ArcTan[c*x]^2 + 8*b^2*c^3*x^3*ArcTan[c*x]^2 - 16*b^2*ArcTan[c*x]^3 +
 (24*I)*b^2*ArcTan[c*x]^2*Log[1 - E^((-2*I)*ArcTan[c*x])] - 160*b^2*ArcTan[
c*x]*Log[1 + E^((2*I)*ArcTan[c*x])] - (24*I)*b^2*ArcTan[c*x]^2*Log[1 + E^((
2*I)*ArcTan[c*x])] + (24*I)*a^2*Log[c*x] + 80*a*b*Log[1 + c^2*x^2] - (36*I)
*b^2*Log[1 + c^2*x^2] - 24*b^2*ArcTan[c*x]*PolyLog[2, E^((-2*I)*ArcTan[c*x]
)] - 8*b^2*(-10*I + 3*ArcTan[c*x])*PolyLog[2, -E^((2*I)*ArcTan[c*x])] - 24*
a*b*PolyLog[2, (-I)*c*x] + 24*a*b*PolyLog[2, I*c*x] + (12*I)*b^2*PolyLog[3,
 E^((-2*I)*ArcTan[c*x])] - (12*I)*b^2*PolyLog[3, -E^((2*I)*ArcTan[c*x])])
```

Maple [C] time = 2.737, size = 1651, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x,x)
```

```
[Out] 1/3*d^3*b^2+11/6*d^3*b^2*arctan(c*x)^2-10/3*I*d^3*a*b*ln(c^2*x^2+1)+2*d^3*a
*b*arctan(c*x)*ln(c*x)+3*I*d^3*a^2*c*x-1/3*I*d^3*a^2*c^3*x^3+3*a*b*c*d^3*x+
3*b^2*c*d^3*x*arctan(c*x)-1/3*I*b^2*c*d^3*x-3/2*d^3*b^2*arctan(c*x)^2*c^2*x
^2+I*d^3*b^2*arctan(c*x)*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))-2*I*d^3*b^2*ar
ctan(c*x)*polylog(2,-(1+I*c*x)/(c^2*x^2+1)^(1/2))+20/3*I*d^3*b^2*arctan(c*x
)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-2*I*d^3*b^2*arctan(c*x)*polylog(2,(1+
I*c*x)/(c^2*x^2+1)^(1/2))+1/2*I*d^3*b^2*Pi*arctan(c*x)^2+20/3*I*d^3*b^2*arc
tan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+I*d^3*a*b*dilog(1+I*c*x)-I*d^3
*a*b*dilog(1-I*c*x)+1/2*I*d^3*b^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*cs
gn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*
c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)^2+2*d^3*b^2*polylog(3,(1+I*c*x)/(c^2*x^2
```

```
+1)^(1/2))+20/3*d^3*b^2*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-1/2*d^3*b^2*
polylog(3,-(1+I*c*x)^2/(c^2*x^2+1))+2*d^3*b^2*polylog(3,-(1+I*c*x)/(c^2*x^2
+1)^(1/2))+3*d^3*b^2*ln((1+I*c*x)^2/(c^2*x^2+1)+1)+20/3*d^3*b^2*dilog(1-I*(
1+I*c*x)/(c^2*x^2+1)^(1/2))+d^3*a^2*ln(c*x)+1/2*I*d^3*b^2*Pi*csgn(I*((1+I*c
*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(((1+I*c*x)^2/(c^2*x^
2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)^2-1/2*I*d^3*b^2*Pi*csgn(I*
((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^
2/(c^2*x^2+1)+1))^2*arctan(c*x)^2-1/2*I*d^3*b^2*Pi*csgn(I/((1+I*c*x)^2/(c^2
*x^2+1)+1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))
^2*arctan(c*x)^2-1/2*I*d^3*b^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*
c*x)^2/(c^2*x^2+1)+1))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x
^2+1)+1))^2*arctan(c*x)^2+6*I*d^3*a*b*arctan(c*x)*c*x-2/3*I*d^3*a*b*arctan(
c*x)*c^3*x^3-3/2*d^3*a^2*c^2*x^2+d^3*b^2*arctan(c*x)^2*ln(c*x)+d^3*b^2*arct
an(c*x)^2*ln(1-(1+I*c*x)/(c^2*x^2+1)^(1/2))-d^3*b^2*arctan(c*x)^2*ln((1+I*c
*x)^2/(c^2*x^2+1)-1)+d^3*b^2*arctan(c*x)^2*ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/2)
)-8/3*I*d^3*b^2*arctan(c*x)-3*d^3*a*b*arctan(c*x)-3*d^3*a*b*arctan(c*x)*c^2
*x^2+1/3*I*d^3*b^2*arctan(c*x)*c^2*x^2+3*I*d^3*b^2*arctan(c*x)^2*c*x-1/3*I*
d^3*b^2*arctan(c*x)^2*c^3*x^3+1/3*I*d^3*a*b*c^2*x^2+I*d^3*a*b*ln(c*x)*ln(1+
I*c*x)-I*d^3*a*b*ln(c*x)*ln(1-I*c*x)-1/2*I*d^3*b^2*Pi*csgn(((1+I*c*x)^2/(c^
2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2+1/2*I*d^3*b^2*Pi*c
sgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*arctan(c*x)^
2+1/2*I*d^3*b^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2
+1)+1))^3*arctan(c*x)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x,x, algorithm="maxima")
```

```
[Out] -1/3*I*a^2*c^3*d^3*x^3 - 36*I*b^2*c^5*d^3*integrate(1/48*x^5*arctan(c*x)^2/
(c^2*x^3 + x), x) - 12*b^2*c^5*d^3*integrate(1/48*x^5*arctan(c*x)*log(c^2*x
^2 + 1)/(c^2*x^3 + x), x) - 3*I*b^2*c^5*d^3*integrate(1/48*x^5*log(c^2*x^2
+ 1)^2/(c^2*x^3 + x), x) - 96*I*a*b*c^5*d^3*integrate(1/48*x^5*arctan(c*x)/
(c^2*x^3 + x), x) - 8*b^2*c^5*d^3*integrate(1/48*x^5*arctan(c*x)/(c^2*x^3 +
 x), x) - 4*I*b^2*c^5*d^3*integrate(1/48*x^5*log(c^2*x^2 + 1)/(c^2*x^3 + x)
, x) - 108*b^2*c^4*d^3*integrate(1/48*x^4*arctan(c*x)^2/(c^2*x^3 + x), x) +
 36*I*b^2*c^4*d^3*integrate(1/48*x^4*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^3
+ x), x) - 9*b^2*c^4*d^3*integrate(1/48*x^4*log(c^2*x^2 + 1)^2/(c^2*x^3 + x
), x) - 288*a*b*c^4*d^3*integrate(1/48*x^4*arctan(c*x)/(c^2*x^3 + x), x) +
```

44*I*b^2*c^4*d^3*integrate(1/48*x^4*arctan(c*x)/(c^2*x^3 + x), x) - 22*b^2*
c^4*d^3*integrate(1/48*x^4*log(c^2*x^2 + 1)/(c^2*x^3 + x), x) - 3/2*a^2*c^2
*d^3*x^2 + 72*I*b^2*c^3*d^3*integrate(1/48*x^3*arctan(c*x)^2/(c^2*x^3 + x),
 x) + 24*b^2*c^3*d^3*integrate(1/48*x^3*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x
^3 + x), x) + 6*I*b^2*c^3*d^3*integrate(1/48*x^3*log(c^2*x^2 + 1)^2/(c^2*x^
3 + x), x) - 96*I*a*b*c^3*d^3*integrate(1/48*x^3*arctan(c*x)/(c^2*x^3 + x),
 x) + 108*b^2*c^3*d^3*integrate(1/48*x^3*arctan(c*x)/(c^2*x^3 + x), x) + 54
*I*b^2*c^3*d^3*integrate(1/48*x^3*log(c^2*x^2 + 1)/(c^2*x^3 + x), x) + 3/4*
I*b^2*d^3*arctan(c*x)^3 - 72*b^2*c^2*d^3*integrate(1/48*x^2*arctan(c*x)^2/(
c^2*x^3 + x), x) + 24*I*b^2*c^2*d^3*integrate(1/48*x^2*arctan(c*x)*log(c^2*
x^2 + 1)/(c^2*x^3 + x), x) - 192*a*b*c^2*d^3*integrate(1/48*x^2*arctan(c*x)
/(c^2*x^3 + x), x) - 72*I*b^2*c^2*d^3*integrate(1/48*x^2*arctan(c*x)/(c^2*x
^3 + x), x) - 1/48*b^2*d^3*log(c^2*x^2 + 1)^3 + 3*I*a^2*c*d^3*x + 36*b^2*c*
d^3*integrate(1/48*x*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^3 + x), x) + 9*I*b
^2*c*d^3*integrate(1/48*x*log(c^2*x^2 + 1)^2/(c^2*x^3 + x), x) + 3/16*b^2*d
^3*log(c^2*x^2 + 1)^2 + 3*I*(2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*a*b*d^3
 + 36*b^2*d^3*integrate(1/48*arctan(c*x)^2/(c^2*x^3 + x), x) - 12*I*b^2*d^3*
integrate(1/48*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^3 + x), x) + 3*b^2*d^3*i
ntegrate(1/48*log(c^2*x^2 + 1)^2/(c^2*x^3 + x), x) + 96*a*b*d^3*integrate(1
/48*arctan(c*x)/(c^2*x^3 + x), x) + a^2*d^3*log(x) - 1/96*(8*I*b^2*c^3*d^3*
x^3 + 36*b^2*c^2*d^3*x^2 - 72*I*b^2*c*d^3*x)*arctan(c*x)^2 + 1/96*(8*b^2*c^
3*d^3*x^3 - 36*I*b^2*c^2*d^3*x^2 - 72*b^2*c*d^3*x)*arctan(c*x)*log(c^2*x^2
 + 1) - 1/96*(-2*I*b^2*c^3*d^3*x^3 - 9*b^2*c^2*d^3*x^2 + 18*I*b^2*c*d^3*x)*l
og(c^2*x^2 + 1)^2

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{-4i\,a^2c^3d^3x^3-12\,a^2c^2d^3x^2+12i\,a^2cd^3x+4\,a^2d^3+\left(i\,b^2c^3d^3x^3+3\,b^2c^2d^3x^2-3i\,b^2cd^3x-b^2d^3\right)\log\left(-\frac{cx+i}{cx-i}\right)^2+}{4\,x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x,x, algorithm="fricas")
```

```
[Out] integral(1/4*(-4*I*a^2*c^3*d^3*x^3 - 12*a^2*c^2*d^3*x^2 + 12*I*a^2*c*d^3*x
+ 4*a^2*d^3 + (I*b^2*c^3*d^3*x^3 + 3*b^2*c^2*d^3*x^2 - 3*I*b^2*c*d^3*x - b^
2*d^3)*log(-(c*x + I)/(c*x - I))^2 + (4*a*b*c^3*d^3*x^3 - 12*I*a*b*c^2*d^3*
x^2 - 12*a*b*c*d^3*x + 4*I*a*b*d^3)*log(-(c*x + I)/(c*x - I)))/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d^3\left(\int \frac{a^2}{x}\,dx + \int 3ia^2c\,dx + \int -3a^2c^2x\,dx + \int \frac{b^2\,\mathrm{atan}^2\,(cx)}{x}\,dx + \int -ia^2c^3x^2\,dx + \int 3ib^2c\,\mathrm{atan}^2\,(cx)\,dx + \int \frac{2ab\,\mathrm{at}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**3*(a+b*atan(c*x))**2/x,x)
```

```
[Out] d**3*(Integral(a**2/x, x) + Integral(3*I*a**2*c, x) + Integral(-3*a**2*c**2
*x, x) + Integral(b**2*atan(c*x)**2/x, x) + Integral(-I*a**2*c**3*x**2, x)
+ Integral(3*I*b**2*c*atan(c*x)**2, x) + Integral(2*a*b*atan(c*x)/x, x) + I
ntegral(-3*b**2*c**2*x*atan(c*x)**2, x) + Integral(6*I*a*b*c*atan(c*x), x)
+ Integral(-I*b**2*c**3*x**2*atan(c*x)**2, x) + Integral(-6*a*b*c**2*x*atan
(c*x), x) + Integral(-2*I*a*b*c**3*x**2*atan(c*x), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(i\,cdx + d)^3(b\arctan\,(cx) + a)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x,x, algorithm="giac")
```

```
[Out] integrate((I*c*d*x + d)^3*(b*arctan(c*x) + a)^2/x, x)
```

3.89 $\int \frac{(d+icdx)^3\left(a+b\tan^{-1}(cx)\right)^2}{x^2}\,dx$

Optimal. Leaf size=402

$$3bcd^3\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)-3bcd^3\text{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)-ib^2cd^3\text{PolyLog}\left(\right.$$

```
[Out] I*a*b*c^2*d^3*x + I*b^2*c^2*d^3*x*ArcTan[c*x] - ((9*I)/2)*c*d^3*(a + b*ArcT
an[c*x])^2 - (d^3*(a + b*ArcTan[c*x])^2)/x - 3*c^2*d^3*x*(a + b*ArcTan[c*x]
)^2 - (I/2)*c^3*d^3*x^2*(a + b*ArcTan[c*x])^2 + (6*I)*c*d^3*(a + b*ArcTan[c
*x])^2*ArcTanh[1 - 2/(1 + I*c*x)] - 6*b*c*d^3*(a + b*ArcTan[c*x])*Log[2/(1
+ I*c*x)] - (I/2)*b^2*c*d^3*Log[1 + c^2*x^2] + 2*b*c*d^3*(a + b*ArcTan[c*x]
)*Log[2 - 2/(1 - I*c*x)] - I*b^2*c*d^3*PolyLog[2, -1 + 2/(1 - I*c*x)] - (3*
I)*b^2*c*d^3*PolyLog[2, 1 - 2/(1 + I*c*x)] + 3*b*c*d^3*(a + b*ArcTan[c*x])*
PolyLog[2, 1 - 2/(1 + I*c*x)] - 3*b*c*d^3*(a + b*ArcTan[c*x])*PolyLog[2, -1
 + 2/(1 + I*c*x)] - ((3*I)/2)*b^2*c*d^3*PolyLog[3, 1 - 2/(1 + I*c*x)] + ((3
*I)/2)*b^2*c*d^3*PolyLog[3, -1 + 2/(1 + I*c*x)]
```

Rubi [A] time = 0.736219, antiderivative size = 402, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 17, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.68, Rules used = {4876, 4846, 4920, 4854, 2402, 2315, 4852, 4924, 4868, 2447, 4850, 4988, 4884, 4994, 6610, 4916, 260}

$$3bcd^3\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)-3bcd^3\text{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)-ib^2cd^3\text{PolyLog}\left(\right.$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)^3*(a + b*ArcTan[c*x])^2)/x^2,x]
```


```
[Out] I*a*b*c^2*d^3*x + I*b^2*c^2*d^3*x*ArcTan[c*x] - ((9*I)/2)*c*d^3*(a + b*ArcT
an[c*x])^2 - (d^3*(a + b*ArcTan[c*x])^2)/x - 3*c^2*d^3*x*(a + b*ArcTan[c*x]
)^2 - (I/2)*c^3*d^3*x^2*(a + b*ArcTan[c*x])^2 + (6*I)*c*d^3*(a + b*ArcTan[c
*x])^2*ArcTanh[1 - 2/(1 + I*c*x)] - 6*b*c*d^3*(a + b*ArcTan[c*x])*Log[2/(1
+ I*c*x)] - (I/2)*b^2*c*d^3*Log[1 + c^2*x^2] + 2*b*c*d^3*(a + b*ArcTan[c*x]
)*Log[2 - 2/(1 - I*c*x)] - I*b^2*c*d^3*PolyLog[2, -1 + 2/(1 - I*c*x)] - (3*
I)*b^2*c*d^3*PolyLog[2, 1 - 2/(1 + I*c*x)] + 3*b*c*d^3*(a + b*ArcTan[c*x])*
PolyLog[2, 1 - 2/(1 + I*c*x)] - 3*b*c*d^3*(a + b*ArcTan[c*x])*PolyLog[2, -1
 + 2/(1 + I*c*x)] - ((3*I)/2)*b^2*c*d^3*PolyLog[3, 1 - 2/(1 + I*c*x)] + ((3
```

```
*I)/2)*b^2*c*d^3*PolyLog[3, -1 + 2/(1 + I*c*x)]
```

Rule 4876

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
```

```
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4850

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTan[c*x])^(p - 1)*ArcTanh[1 - 2/(1 + I*c*x)])/(1 + c^2*x^2), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4988

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTan[c*x])^p)/(d + e
*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTan[c*x])^p)/(d + e*x^2
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
```

```
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{(d+icdx)^3 \left(a+b\tan^{-1}(cx)\right)^2}{x^2}\,dx = \int \left(-3c^2d^3\left(a+b\tan^{-1}(cx)\right)^2 + \frac{d^3\left(a+b\tan^{-1}(cx)\right)^2}{x^2} + \frac{3icd^3\left(a+b\tan^{-1}(cx)\right)^2}{x}\right.$$

$$= d^3\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^2}\,dx + \left(3icd^3\right)\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x}\,dx - \left(3c^2d^3\right)\int \left(a+b\right.$$

$$= -\frac{d^3\left(a+b\tan^{-1}(cx)\right)^2}{x} - 3c^2d^3x\left(a+b\tan^{-1}(cx)\right)^2 - \frac{1}{2}ic^3d^3x^2\left(a+b\tan^{-1}(cx)\right)^2$$

$$= -4icd^3\left(a+b\tan^{-1}(cx)\right)^2 - \frac{d^3\left(a+b\tan^{-1}(cx)\right)^2}{x} - 3c^2d^3x\left(a+b\tan^{-1}(cx)\right)^2 - \frac{1}{2}$$

$$= iabc^2d^3x - \frac{9}{2}icd^3\left(a+b\tan^{-1}(cx)\right)^2 - \frac{d^3\left(a+b\tan^{-1}(cx)\right)^2}{x} - 3c^2d^3x\left(a+b\tan^{-1}\right.$$

$$= iabc^2d^3x + ib^2c^2d^3x\tan^{-1}(cx) - \frac{9}{2}icd^3\left(a+b\tan^{-1}(cx)\right)^2 - \frac{d^3\left(a+b\tan^{-1}(cx)\right)^2}{x}$$

$$= iabc^2d^3x + ib^2c^2d^3x\tan^{-1}(cx) - \frac{9}{2}icd^3\left(a+b\tan^{-1}(cx)\right)^2 - \frac{d^3\left(a+b\tan^{-1}(cx)\right)^2}{x}$$

Mathematica [A] time = 0.592865, size = 512, normalized size = 1.27

$$\frac{d^3\left(-24abcx\text{PolyLog}(2,-icx) + 24abcx\text{PolyLog}(2,icx) - 24b^2cx\tan^{-1}(cx)\text{PolyLog}\left(2,e^{-2i\tan^{-1}(cx)}\right) - 24b^2cx\left(\tan^{-1}(c\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + I*c*d*x)^3*(a + b*ArcTan[c*x])^2)/x^2,x]
```

```
[Out] (d^3*(-8*a^2 + b^2*c*Pi^3*x - 24*a^2*c^2*x^2 + (8*I)*a*b*c^2*x^2 - (4*I)*a^
2*c^3*x^3 - 16*a*b*ArcTan[c*x] - (8*I)*a*b*c*x*ArcTan[c*x] - 48*a*b*c^2*x^2
*ArcTan[c*x] + (8*I)*b^2*c^2*x^2*ArcTan[c*x] - (8*I)*a*b*c^3*x^3*ArcTan[c*x
] - 8*b^2*ArcTan[c*x]^2 + (12*I)*b^2*c*x*ArcTan[c*x]^2 - 24*b^2*c^2*x^2*Arc
Tan[c*x]^2 - (4*I)*b^2*c^3*x^3*ArcTan[c*x]^2 - 16*b^2*c*x*ArcTan[c*x]^3 + (
24*I)*b^2*c*x*ArcTan[c*x]^2*Log[1 - E^((-2*I)*ArcTan[c*x])] + 16*b^2*c*x*Ar
cTan[c*x]*Log[1 - E^((2*I)*ArcTan[c*x])] - 48*b^2*c*x*ArcTan[c*x]*Log[1 + E
^((2*I)*ArcTan[c*x])] - (24*I)*b^2*c*x*ArcTan[c*x]^2*Log[1 + E^((2*I)*ArcTa
n[c*x])] + (24*I)*a^2*c*x*Log[x] + 16*a*b*c*x*Log[c*x] + 16*a*b*c*x*Log[1 +
 c^2*x^2] - (4*I)*b^2*c*x*Log[1 + c^2*x^2] - 24*b^2*c*x*ArcTan[c*x]*PolyLog
[2, E^((-2*I)*ArcTan[c*x])] - 24*b^2*c*x*(-I + ArcTan[c*x])*PolyLog[2, -E^(
```

```
(2*I)*ArcTan[c*x])] - (8*I)*b^2*c*x*PolyLog[2, E^((2*I)*ArcTan[c*x])] - 24*
a*b*c*x*PolyLog[2, (-I)*c*x] + 24*a*b*c*x*PolyLog[2, I*c*x] + (12*I)*b^2*c*
x*PolyLog[3, E^((-2*I)*ArcTan[c*x])] - (12*I)*b^2*c*x*PolyLog[3, -E^((2*I)*
ArcTan[c*x])]))/(8*x)
```

Maple [C] time = 4.881, size = 1739, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x^2,x)
```

```
[Out] -3/2*c*d^3*b^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I/((1+I*c*x)^2/(
c^2*x^2+1)+1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+
1))*arctan(c*x)^2-3*d^3*b^2*arctan(c*x)^2*c^2*x-3/2*c*d^3*b^2*Pi*arctan(c*x
)^2+I*c*d^3*b^2*ln((1+I*c*x)^2/(c^2*x^2+1)+1)-6*c*d^3*b^2*arctan(c*x)*ln(1+
I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-6*c*d^3*b^2*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^
2*x^2+1)^(1/2))+6*c*d^3*b^2*arctan(c*x)*polylog(2,(1+I*c*x)/(c^2*x^2+1)^(1/
2))-3*c*d^3*b^2*arctan(c*x)*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))+6*c*d^3*b^2
*arctan(c*x)*polylog(2,-(1+I*c*x)/(c^2*x^2+1)^(1/2))+2*c*d^3*b^2*arctan(c*x
)*ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))+2*c*d^3*a*b*ln(c^2*x^2+1)+2*c*d^3*a*b*l
n(c*x)-3*c*d^3*a*b*dilog(1+I*c*x)+3*c*d^3*a*b*dilog(1-I*c*x)-2*d^3*a*b*arct
an(c*x)/x-1/2*I*d^3*a^2*c^3*x^2-3/2*I*c*d^3*b^2*polylog(3,-(1+I*c*x)^2/(c^2
*x^2+1))+6*I*c*d^3*b^2*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+3*I*c*d^3*a^2
*ln(c*x)+6*I*c*d^3*b^2*polylog(3,-(1+I*c*x)/(c^2*x^2+1)^(1/2))-2*I*c*d^3*b^
2*dilog(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))+2*I*c*d^3*b^2*dilog((1+I*c*x)/(c^2*x
^2+1)^(1/2))+3/2*I*c*d^3*b^2*arctan(c*x)^2+I*a*b*c^2*d^3*x+I*b^2*c^2*d^3*x*
arctan(c*x)-d^3*a^2/x+6*I*c*d^3*b^2*polylog(3,(1+I*c*x)/(c^2*x^2+1)^(1/2))+
6*I*c*d^3*b^2*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+3/2*c*d^3*b^2*Pi*csgn(
((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2-3/
2*c*d^3*b^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+
1))^3*arctan(c*x)^2-3/2*c*d^3*b^2*Pi*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I
*c*x)^2/(c^2*x^2+1)+1))^3*arctan(c*x)^2-3*c*d^3*a*b*ln(c*x)*ln(1+I*c*x)+3*c
*d^3*a*b*ln(c*x)*ln(1-I*c*x)-6*d^3*a*b*arctan(c*x)*c^2*x-I*c*d^3*a*b*arctan
(c*x)+3*I*c*d^3*b^2*arctan(c*x)^2*ln(c*x)-3*I*c*d^3*b^2*arctan(c*x)^2*ln((1
+I*c*x)^2/(c^2*x^2+1)-1)+3*I*c*d^3*b^2*arctan(c*x)^2*ln(1-(1+I*c*x)/(c^2*x^
2+1)^(1/2))+3*I*c*d^3*b^2*arctan(c*x)^2*ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))-1
/2*I*d^3*b^2*arctan(c*x)^2*c^3*x^2-3*c^2*x*a^2*d^3-d^3*b^2*arctan(c*x)^2/x+
c*d^3*b^2*arctan(c*x)+3/2*c*d^3*b^2*Pi*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*
csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*
x)^2+3/2*c*d^3*b^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*
```

```
x^2+1)+1))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*
arctan(c*x)^2-3/2*c*d^3*b^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x
)^2/(c^2*x^2+1)+1))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+
1)+1))*arctan(c*x)^2+3/2*c*d^3*b^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*c
sgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x
)^2+6*I*c*d^3*a*b*arctan(c*x)*ln(c*x)-I*d^3*a*b*arctan(c*x)*c^3*x^2
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{-4i\,a^2c^3d^3x^3-12\,a^2c^2d^3x^2+12i\,a^2cd^3x+4\,a^2d^3+\left(i\,b^2c^3d^3x^3+3\,b^2c^2d^3x^2-3i\,b^2cd^3x-b^2d^3\right)\log\left(-\frac{cx+i}{cx-i}\right)^2+}{4\,x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x^2,x, algorithm="fricas")
```

```
[Out] integral(1/4*(-4*I*a^2*c^3*d^3*x^3 - 12*a^2*c^2*d^3*x^2 + 12*I*a^2*c*d^3*x
+ 4*a^2*d^3 + (I*b^2*c^3*d^3*x^3 + 3*b^2*c^2*d^3*x^2 - 3*I*b^2*c*d^3*x - b^
2*d^3)*log(-(c*x + I)/(c*x - I))^2 + (4*a*b*c^3*d^3*x^3 - 12*I*a*b*c^2*d^3*
x^2 - 12*a*b*c*d^3*x + 4*I*a*b*d^3)*log(-(c*x + I)/(c*x - I)))/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d^3\left(\int -3a^2c^2\,dx+\int \frac{a^2}{x^2}\,dx+\int -3b^2c^2\,\text{atan}^2\,(cx)\,dx+\int \frac{b^2\,\text{atan}^2\,(cx)}{x^2}\,dx+\int \frac{3ia^2c}{x}\,dx+\int -ia^2c^3x\,dx+\int -6abc^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**3*(a+b*atan(c*x))**2/x**2,x)
```

```
[Out] d**3*(Integral(-3*a**2*c**2, x) + Integral(a**2/x**2, x) + Integral(-3*b**2
*c**2*atan(c*x)**2, x) + Integral(b**2*atan(c*x)**2/x**2, x) + Integral(3*I
*a**2*c/x, x) + Integral(-I*a**2*c**3*x, x) + Integral(-6*a*b*c**2*atan(c*x
), x) + Integral(2*a*b*atan(c*x)/x**2, x) + Integral(3*I*b**2*c*atan(c*x)**
2/x, x) + Integral(-I*b**2*c**3*x*atan(c*x)**2, x) + Integral(6*I*a*b*c*ata
n(c*x)/x, x) + Integral(-2*I*a*b*c**3*x*atan(c*x), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(icdx+d)^3(b\arctan(cx)+a)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x^2,x, algorithm="giac")
```

```
[Out] integrate((I*c*d*x + d)^3*(b*arctan(c*x) + a)^2/x^2, x)
```

3.90 $\int \frac{(d+icdx)^3\left(a+b\tan^{-1}(cx)\right)^2}{x^3}\,dx$

Optimal. Leaf size=416

$$3ibc^2d^3\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)-3ibc^2d^3\text{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)+3b^2c^2d^3\text{PolyL}$$

```
[Out] -((b*c*d^3*(a + b*ArcTan[c*x]))/x) + (7*c^2*d^3*(a + b*ArcTan[c*x])^2)/2 -
(d^3*(a + b*ArcTan[c*x])^2)/(2*x^2) - ((3*I)*c*d^3*(a + b*ArcTan[c*x])^2)/x
 - I*c^3*d^3*x*(a + b*ArcTan[c*x])^2 - 6*c^2*d^3*(a + b*ArcTan[c*x])^2*ArcT
anh[1 - 2/(1 + I*c*x)] + b^2*c^2*d^3*Log[x] - (2*I)*b*c^2*d^3*(a + b*ArcTan
[c*x])*Log[2/(1 + I*c*x)] - (b^2*c^2*d^3*Log[1 + c^2*x^2])/2 + (6*I)*b*c^2*
d^3*(a + b*ArcTan[c*x])*Log[2 - 2/(1 - I*c*x)] + 3*b^2*c^2*d^3*PolyLog[2, -
1 + 2/(1 - I*c*x)] + b^2*c^2*d^3*PolyLog[2, 1 - 2/(1 + I*c*x)] + (3*I)*b*c^
2*d^3*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)] - (3*I)*b*c^2*d^3*(
a + b*ArcTan[c*x])*PolyLog[2, -1 + 2/(1 + I*c*x)] + (3*b^2*c^2*d^3*PolyLog[
3, 1 - 2/(1 + I*c*x)])/2 - (3*b^2*c^2*d^3*PolyLog[3, -1 + 2/(1 + I*c*x)])/2
```

Rubi [A] time = 0.752747, antiderivative size = 416, normalized size of antiderivative = 1., number of steps used = 25, number of rules used = 20, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.8, Rules used = {4876, 4846, 4920, 4854, 2402, 2315, 4852, 4918, 266, 36, 29, 31, 4884, 4924, 4868, 2447, 4850, 4988, 4994, 6610}

$$3ibc^2d^3\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)-3ibc^2d^3\text{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)+3b^2c^2d^3\text{PolyL}$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)^3*(a + b*ArcTan[c*x])^2)/x^3,x]
```


```
[Out] -((b*c*d^3*(a + b*ArcTan[c*x]))/x) + (7*c^2*d^3*(a + b*ArcTan[c*x])^2)/2 -
(d^3*(a + b*ArcTan[c*x])^2)/(2*x^2) - ((3*I)*c*d^3*(a + b*ArcTan[c*x])^2)/x
 - I*c^3*d^3*x*(a + b*ArcTan[c*x])^2 - 6*c^2*d^3*(a + b*ArcTan[c*x])^2*ArcT
anh[1 - 2/(1 + I*c*x)] + b^2*c^2*d^3*Log[x] - (2*I)*b*c^2*d^3*(a + b*ArcTan
[c*x])*Log[2/(1 + I*c*x)] - (b^2*c^2*d^3*Log[1 + c^2*x^2])/2 + (6*I)*b*c^2*
d^3*(a + b*ArcTan[c*x])*Log[2 - 2/(1 - I*c*x)] + 3*b^2*c^2*d^3*PolyLog[2, -
1 + 2/(1 - I*c*x)] + b^2*c^2*d^3*PolyLog[2, 1 - 2/(1 + I*c*x)] + (3*I)*b*c^
2*d^3*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)] - (3*I)*b*c^2*d^3*(
a + b*ArcTan[c*x])*PolyLog[2, -1 + 2/(1 + I*c*x)] + (3*b^2*c^2*d^3*PolyLog[
```

```
3, 1 - 2/(1 + I*c*x)])/2 - (3*b^2*c^2*d^3*PolyLog[3, -1 + 2/(1 + I*c*x)])/2
```

Rule 4876

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
```

```
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4918

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/(d + e*x^2),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4850

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTan[c*x])^(p - 1)*ArcTanh[1 - 2/(1 + I*c*x)])/(1 + c^2*x^2), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4988

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTan[c*x])^p)/(d + e
*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTan[c*x])^p)/(d + e*x^2
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{(d+icdx)^3 \left(a+b\tan^{-1}(cx)\right)^2}{x^3}\,dx = \int \left(-ic^3d^3\left(a+b\tan^{-1}(cx)\right)^2 + \frac{d^3\left(a+b\tan^{-1}(cx)\right)^2}{x^3} + \frac{3icd^3\left(a+b\tan^{-1}(cx)\right)^2}{x^2}\right.$$

$$= d^3\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^3}\,dx + \left(3icd^3\right)\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^2}\,dx - \left(3c^2d^3\right)\int \frac{\left(a+b\right.}{}$$

$$= -\frac{d^3\left(a+b\tan^{-1}(cx)\right)^2}{2x^2} - \frac{3icd^3\left(a+b\tan^{-1}(cx)\right)^2}{x} - ic^3d^3x\left(a+b\tan^{-1}(cx)\right)^2 - 6$$

$$= 4c^2d^3\left(a+b\tan^{-1}(cx)\right)^2 - \frac{d^3\left(a+b\tan^{-1}(cx)\right)^2}{2x^2} - \frac{3icd^3\left(a+b\tan^{-1}(cx)\right)^2}{x} - ic^3d$$

$$= -\frac{bcd^3\left(a+b\tan^{-1}(cx)\right)}{x} + \frac{7}{2}c^2d^3\left(a+b\tan^{-1}(cx)\right)^2 - \frac{d^3\left(a+b\tan^{-1}(cx)\right)^2}{2x^2} - \frac{3ic}{}$$

$$= -\frac{bcd^3\left(a+b\tan^{-1}(cx)\right)}{x} + \frac{7}{2}c^2d^3\left(a+b\tan^{-1}(cx)\right)^2 - \frac{d^3\left(a+b\tan^{-1}(cx)\right)^2}{2x^2} - \frac{3ic}{}$$

$$= -\frac{bcd^3\left(a+b\tan^{-1}(cx)\right)}{x} + \frac{7}{2}c^2d^3\left(a+b\tan^{-1}(cx)\right)^2 - \frac{d^3\left(a+b\tan^{-1}(cx)\right)^2}{2x^2} - \frac{3ic}{}$$

$$= -\frac{bcd^3\left(a+b\tan^{-1}(cx)\right)}{x} + \frac{7}{2}c^2d^3\left(a+b\tan^{-1}(cx)\right)^2 - \frac{d^3\left(a+b\tan^{-1}(cx)\right)^2}{2x^2} - \frac{3ic}{}$$

Mathematica [A] time = 1.07768, size = 500, normalized size = 1.2

$$\frac{1}{2}d^3\left(-6iabc^2(\text{PolyLog}(2,-icx)-\text{PolyLog}(2,icx)) - 2ib^2c^2\left(\tan^{-1}(cx)\left((cx-i)\tan^{-1}(cx)+2\log\left(1+e^{2i\tan^{-1}(cx)}\right)\right) - i\text{P}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + I*c*d*x)^3*(a + b*ArcTan[c*x])^2)/x^3,x]
```

```
[Out] (d^3*(-(a^2/x^2) - ((6*I)*a^2*c)/x - (2*I)*a^2*c^3*x - (2*a*b*(ArcTan[c*x]
+ c*x*(1 + c*x*ArcTan[c*x])))/x^2 - 6*a^2*c^2*Log[x] - (b^2*(2*c*x*ArcTan[c
*x] + (1 + c^2*x^2)*ArcTan[c*x]^2 - 2*c^2*x^2*Log[(c*x)/Sqrt[1 + c^2*x^2]])
)/x^2 - (2*I)*a*b*c^2*(2*c*x*ArcTan[c*x] - Log[1 + c^2*x^2]) - ((6*I)*a*b*c
*(2*ArcTan[c*x] + c*x*(-2*Log[c*x] + Log[1 + c^2*x^2])))/x - (2*I)*b^2*c^2*
(ArcTan[c*x]*((-I + c*x)*ArcTan[c*x] + 2*Log[1 + E^((2*I)*ArcTan[c*x])]) -
I*PolyLog[2, -E^((2*I)*ArcTan[c*x])]) + (6*b^2*c*(ArcTan[c*x]*((-I + c*x)*A
```

```
rcTan[c*x] + (2*I)*c*x*Log[1 - E^((2*I)*ArcTan[c*x])]) + c*x*PolyLog[2, E^(
(2*I)*ArcTan[c*x])]))/x - (6*I)*a*b*c^2*(PolyLog[2, (-I)*c*x] - PolyLog[2,
I*c*x]) + 6*b^2*c^2*((I/24)*Pi^3 - ((2*I)/3)*ArcTan[c*x]^3 - ArcTan[c*x]^2*
Log[1 - E^((-2*I)*ArcTan[c*x])] + ArcTan[c*x]^2*Log[1 + E^((2*I)*ArcTan[c*x
])] - I*ArcTan[c*x]*PolyLog[2, E^((-2*I)*ArcTan[c*x])] - I*ArcTan[c*x]*Poly
Log[2, -E^((2*I)*ArcTan[c*x])] - PolyLog[3, E^((-2*I)*ArcTan[c*x])]/2 + Pol
yLog[3, -E^((2*I)*ArcTan[c*x])]/2)))/2
```

Maple [C] time = 2.691, size = 1846, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x^3,x)
```

```
[Out] 3/2*I*c^2*d^3*b^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I*((1+I*c*x)^
2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2-3/2*I*c^2*d^3
*b^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csg
n(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)^2+3/
2*I*c^2*d^3*b^2*Pi*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*((1+I*c*x)^2/
(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2+3/2*I*c^2*d^3*b
^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(
((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2-2*
I*c^3*d^3*a*b*arctan(c*x)*x-6*I*c*d^3*a*b*arctan(c*x)/x-3*I*c^2*d^3*a*b*ln(
c*x)*ln(1+I*c*x)+3*I*c^2*d^3*a*b*ln(c*x)*ln(1-I*c*x)-3/2*I*c^2*d^3*b^2*Pi*c
sgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*arctan(c*x
)^2-1/2*d^3*a^2/x^2-3/2*I*c^2*d^3*b^2*Pi*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/(
(1+I*c*x)^2/(c^2*x^2+1)+1))^3*arctan(c*x)^2+3/2*I*c^2*d^3*b^2*Pi*csgn(((1+I
*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2-3/2*I*c
^2*d^3*b^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I/((1+I*c*x)^2/(c^2*
x^2+1)+1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*
arctan(c*x)^2-3*I*c*d^3*a^2/x-I*c^2*d^3*b^2*arctan(c*x)-c*d^3*a*b/x-c*d^3*b
^2*arctan(c*x)/x-c^2*d^3*a*b*arctan(c*x)-3*c^2*d^3*b^2*arctan(c*x)^2*ln(1-(
1+I*c*x)/(c^2*x^2+1)^(1/2))-3*c^2*d^3*b^2*arctan(c*x)^2*ln(c*x)-3*c^2*d^3*b
^2*arctan(c*x)^2*ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))+3*c^2*d^3*b^2*arctan(c*x
)^2*ln((1+I*c*x)^2/(c^2*x^2+1)-1)-d^3*a*b*arctan(c*x)/x^2-I*c^3*d^3*a^2*x-1
/2*d^3*b^2*arctan(c*x)^2/x^2+3/2*c^2*d^3*b^2*arctan(c*x)^2+6*c^2*d^3*b^2*di
log(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))-6*c^2*d^3*b^2*polylog(3,-(1+I*c*x)/(c^2*
x^2+1)^(1/2))+3/2*c^2*d^3*b^2*polylog(3,-(1+I*c*x)^2/(c^2*x^2+1))-6*c^2*d^3
*b^2*polylog(3,(1+I*c*x)/(c^2*x^2+1)^(1/2))+c^2*d^3*b^2*ln(1+(1+I*c*x)/(c^2
*x^2+1)^(1/2))-6*c^2*d^3*b^2*dilog((1+I*c*x)/(c^2*x^2+1)^(1/2))+c^2*d^3*b^2
```

```
*ln((1+I*c*x)/(c^2*x^2+1)^(1/2)-1)-2*c^2*d^3*b^2*dilog(1-I*(1+I*c*x)/(c^2*x
^2+1)^(1/2))-2*c^2*d^3*b^2*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-3*c^2*d^3
*a^2*ln(c*x)-6*c^2*d^3*a*b*arctan(c*x)*ln(c*x)+6*I*c^2*d^3*a*b*ln(c*x)-3*I*
c*d^3*b^2*arctan(c*x)^2/x-I*c^3*d^3*b^2*arctan(c*x)^2*x-3*I*c^2*d^3*a*b*dil
og(1+I*c*x)+3*I*c^2*d^3*a*b*dilog(1-I*c*x)-3/2*I*c^2*d^3*b^2*Pi*arctan(c*x)
^2+6*I*c^2*d^3*b^2*arctan(c*x)*ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))+6*I*c^2*d^
3*b^2*arctan(c*x)*polylog(2,-(1+I*c*x)/(c^2*x^2+1)^(1/2))-3*I*c^2*d^3*b^2*a
rctan(c*x)*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))-2*I*c^2*d^3*b^2*arctan(c*x)*
ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+6*I*c^2*d^3*b^2*arctan(c*x)*polylog(2,(
1+I*c*x)/(c^2*x^2+1)^(1/2))-2*I*c^2*d^3*b^2*arctan(c*x)*ln(1-I*(1+I*c*x)/(c
^2*x^2+1)^(1/2))-2*I*c^2*d^3*a*b*ln(c^2*x^2+1)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{-4i\,a^2c^3d^3x^3 - 12\,a^2c^2d^3x^2 + 12i\,a^2cd^3x + 4\,a^2d^3 + \left(i\,b^2c^3d^3x^3 + 3\,b^2c^2d^3x^2 - 3i\,b^2cd^3x - b^2d^3\right)\log\left(-\frac{cx+i}{cx-i}\right)^2 + }{4\,x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x^3,x, algorithm="fricas")
```

```
[Out] integral(1/4*(-4*I*a^2*c^3*d^3*x^3 - 12*a^2*c^2*d^3*x^2 + 12*I*a^2*c*d^3*x
+ 4*a^2*d^3 + (I*b^2*c^3*d^3*x^3 + 3*b^2*c^2*d^3*x^2 - 3*I*b^2*c*d^3*x - b^
2*d^3)*log(-(c*x + I)/(c*x - I))^2 + (4*a*b*c^3*d^3*x^3 - 12*I*a*b*c^2*d^3*
x^2 - 12*a*b*c*d^3*x + 4*I*a*b*d^3)*log(-(c*x + I)/(c*x - I)))/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d^3\left(\int \frac{a^2}{x^3}\,dx + \int -ia^2c^3\,dx + \int -\frac{3a^2c^2}{x}\,dx + \int \frac{b^2\operatorname{atan}^2(cx)}{x^3}\,dx + \int \frac{3ia^2c}{x^2}\,dx + \int -ib^2c^3\operatorname{atan}^2(cx)\,dx + \int \frac{2ab\operatorname{ata}}{x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**3*(a+b*atan(c*x))**2/x**3,x)
```

```
[Out] d**3*(Integral(a**2/x**3, x) + Integral(-I*a**2*c**3, x) + Integral(-3*a**2
*c**2/x, x) + Integral(b**2*atan(c*x)**2/x**3, x) + Integral(3*I*a**2*c/x**
2, x) + Integral(-I*b**2*c**3*atan(c*x)**2, x) + Integral(2*a*b*atan(c*x)/x
**3, x) + Integral(-3*b**2*c**2*atan(c*x)**2/x, x) + Integral(-2*I*a*b*c**3
*atan(c*x), x) + Integral(3*I*b**2*c*atan(c*x)**2/x**2, x) + Integral(-6*a*
b*c**2*atan(c*x)/x, x) + Integral(6*I*a*b*c*atan(c*x)/x**2, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(icdx+d)^3(b\arctan(cx)+a)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x^3,x, algorithm="giac")
```

```
[Out] integrate((I*c*d*x + d)^3*(b*arctan(c*x) + a)^2/x^3, x)
```

3.91 $\int \frac{(d+icdx)^3\left(a+b\tan^{-1}(cx)\right)^2}{x^4}\,dx$

Optimal. Leaf size=429

$$-bc^3d^3\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)+bc^3d^3\text{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)+\frac{10}{3}ib^2c^3d^3\text{Poly}$$

```
[Out] -(b^2*c^2*d^3)/(3*x) - (b^2*c^3*d^3*ArcTan[c*x])/3 - (b*c*d^3*(a + b*ArcTan
[c*x]))/(3*x^2) - ((3*I)*b*c^2*d^3*(a + b*ArcTan[c*x]))/x + ((11*I)/6)*c^3*
d^3*(a + b*ArcTan[c*x])^2 - (d^3*(a + b*ArcTan[c*x])^2)/(3*x^3) - (((3*I)/2
)*c*d^3*(a + b*ArcTan[c*x])^2)/x^2 + (3*c^2*d^3*(a + b*ArcTan[c*x])^2)/x -
(2*I)*c^3*d^3*(a + b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1 + I*c*x)] + (3*I)*b^2*
c^3*d^3*Log[x] - ((3*I)/2)*b^2*c^3*d^3*Log[1 + c^2*x^2] - (20*b*c^3*d^3*(a
+ b*ArcTan[c*x])*Log[2 - 2/(1 - I*c*x)])/3 + ((10*I)/3)*b^2*c^3*d^3*PolyLog
[2, -1 + 2/(1 - I*c*x)] - b*c^3*d^3*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1
 + I*c*x)] + b*c^3*d^3*(a + b*ArcTan[c*x])*PolyLog[2, -1 + 2/(1 + I*c*x)] +
 (I/2)*b^2*c^3*d^3*PolyLog[3, 1 - 2/(1 + I*c*x)] - (I/2)*b^2*c^3*d^3*PolyLo
g[3, -1 + 2/(1 + I*c*x)]
```

Rubi [A] time = 0.890253, antiderivative size = 429, normalized size of antiderivative = 1., number of steps used = 28, number of rules used = 17, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.68, Rules used = {4876, 4852, 4918, 325, 203, 4924, 4868, 2447, 266, 36, 29, 31, 4884, 4850, 4988, 4994, 6610}

$$-bc^3d^3\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)+bc^3d^3\text{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)+\frac{10}{3}ib^2c^3d^3\text{Poly}$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)^3*(a + b*ArcTan[c*x])^2)/x^4,x]
```


```
[Out] -(b^2*c^2*d^3)/(3*x) - (b^2*c^3*d^3*ArcTan[c*x])/3 - (b*c*d^3*(a + b*ArcTan
[c*x]))/(3*x^2) - ((3*I)*b*c^2*d^3*(a + b*ArcTan[c*x]))/x + ((11*I)/6)*c^3*
d^3*(a + b*ArcTan[c*x])^2 - (d^3*(a + b*ArcTan[c*x])^2)/(3*x^3) - (((3*I)/2
)*c*d^3*(a + b*ArcTan[c*x])^2)/x^2 + (3*c^2*d^3*(a + b*ArcTan[c*x])^2)/x -
(2*I)*c^3*d^3*(a + b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1 + I*c*x)] + (3*I)*b^2*
c^3*d^3*Log[x] - ((3*I)/2)*b^2*c^3*d^3*Log[1 + c^2*x^2] - (20*b*c^3*d^3*(a
+ b*ArcTan[c*x])*Log[2 - 2/(1 - I*c*x)])/3 + ((10*I)/3)*b^2*c^3*d^3*PolyLog
[2, -1 + 2/(1 - I*c*x)] - b*c^3*d^3*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1
 + I*c*x)] + b*c^3*d^3*(a + b*ArcTan[c*x])*PolyLog[2, -1 + 2/(1 + I*c*x)] +
```

```
(I/2)*b^2*c^3*d^3*PolyLog[3, 1 - 2/(1 + I*c*x)] - (I/2)*b^2*c^3*d^3*PolyLo
g[3, -1 + 2/(1 + I*c*x)]
```

Rule 4876

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4918

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/(d + e*x^2),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4850

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTan[c*x])^(p - 1)*ArcTanh[1 - 2/(1 + I*c*x)])/(1 + c^2*x^2), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4988

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTan[c*x])^p)/(d + e
*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTan[c*x])^p)/(d + e*x^2
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{(d+icdx)^3 \left(a+b\tan^{-1}(cx)\right)^2}{x^4}\,dx = \int \left(\frac{d^3 \left(a+b\tan^{-1}(cx)\right)^2}{x^4} + \frac{3icd^3 \left(a+b\tan^{-1}(cx)\right)^2}{x^3} - \frac{3c^2d^3 \left(a+b\tan^{-1}(cx)\right)^2}{x^2} -$$

$$= d^3 \int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^4}\,dx + \left(3icd^3\right) \int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^3}\,dx - \left(3c^2d^3\right) \int \frac{(a+b}{}$$

$$= -\frac{d^3 \left(a+b\tan^{-1}(cx)\right)^2}{3x^3} - \frac{3icd^3 \left(a+b\tan^{-1}(cx)\right)^2}{2x^2} + \frac{3c^2d^3 \left(a+b\tan^{-1}(cx)\right)^2}{x} - 2$$

$$= 3ic^3d^3 \left(a+b\tan^{-1}(cx)\right)^2 - \frac{d^3 \left(a+b\tan^{-1}(cx)\right)^2}{3x^3} - \frac{3icd^3 \left(a+b\tan^{-1}(cx)\right)^2}{2x^2} + \frac{3c}{}$$

$$= -\frac{bcd^3 \left(a+b\tan^{-1}(cx)\right)}{3x^2} - \frac{3ibc^2d^3 \left(a+b\tan^{-1}(cx)\right)}{x} + \frac{11}{6}ic^3d^3 \left(a+b\tan^{-1}(cx)\right)^2$$

$$= -\frac{b^2c^2d^3}{3x} - \frac{bcd^3 \left(a+b\tan^{-1}(cx)\right)}{3x^2} - \frac{3ibc^2d^3 \left(a+b\tan^{-1}(cx)\right)}{x} + \frac{11}{6}ic^3d^3 \left(a+b\,\mathrm{ta}\right.$$

$$= -\frac{b^2c^2d^3}{3x} - \frac{1}{3}b^2c^3d^3\tan^{-1}(cx) - \frac{bcd^3 \left(a+b\tan^{-1}(cx)\right)}{3x^2} - \frac{3ibc^2d^3 \left(a+b\tan^{-1}(cx)\right.}{x}$$

$$= -\frac{b^2c^2d^3}{3x} - \frac{1}{3}b^2c^3d^3\tan^{-1}(cx) - \frac{bcd^3 \left(a+b\tan^{-1}(cx)\right)}{3x^2} - \frac{3ibc^2d^3 \left(a+b\tan^{-1}(cx)\right.}{x}$$

Mathematica [A] time = 0.746361, size = 595, normalized size = 1.39

$$\frac{d^3 \left(24abc^3x^3\mathrm{PolyLog}(2,-icx) - 24abc^3x^3\mathrm{PolyLog}(2,icx) + 24b^2c^3x^3\tan^{-1}(cx)\mathrm{PolyLog}\left(2,e^{-2i\tan^{-1}(cx)}\right) + 24b^2c^3x^3\,\mathrm{ta}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + I*c*d*x)^3*(a + b*ArcTan[c*x])^2)/x^4,x]
```

```
[Out] (d^3*(-8*a^2 - (36*I)*a^2*c*x - 8*a*b*c*x + 72*a^2*c^2*x^2 - (72*I)*a*b*c^2
*x^2 - 8*b^2*c^2*x^2 - b^2*c^3*Pi^3*x^3 - 16*a*b*ArcTan[c*x] - (72*I)*a*b*c
*x*ArcTan[c*x] - 8*b^2*c*x*ArcTan[c*x] + 144*a*b*c^2*x^2*ArcTan[c*x] - (72*
I)*b^2*c^2*x^2*ArcTan[c*x] - (72*I)*a*b*c^3*x^3*ArcTan[c*x] - 8*b^2*c^3*x^3
*ArcTan[c*x] - 8*b^2*ArcTan[c*x]^2 - (36*I)*b^2*c*x*ArcTan[c*x]^2 + 72*b^2*
c^2*x^2*ArcTan[c*x]^2 + (44*I)*b^2*c^3*x^3*ArcTan[c*x]^2 + 16*b^2*c^3*x^3*A
rcTan[c*x]^3 - (24*I)*b^2*c^3*x^3*ArcTan[c*x]^2*Log[1 - E^((-2*I)*ArcTan[c*
```

```
x])] - 160*b^2*c^3*x^3*ArcTan[c*x]*Log[1 - E^((2*I)*ArcTan[c*x])] + (24*I)*
b^2*c^3*x^3*ArcTan[c*x]^2*Log[1 + E^((2*I)*ArcTan[c*x])] - (24*I)*a^2*c^3*x
^3*Log[x] - 160*a*b*c^3*x^3*Log[c*x] + (72*I)*b^2*c^3*x^3*Log[(c*x)/Sqrt[1
+ c^2*x^2]] + 80*a*b*c^3*x^3*Log[1 + c^2*x^2] + 24*b^2*c^3*x^3*ArcTan[c*x]*
PolyLog[2, E^((-2*I)*ArcTan[c*x])] + 24*b^2*c^3*x^3*ArcTan[c*x]*PolyLog[2,
-E^((2*I)*ArcTan[c*x])] + (80*I)*b^2*c^3*x^3*PolyLog[2, E^((2*I)*ArcTan[c*x
])] + 24*a*b*c^3*x^3*PolyLog[2, (-I)*c*x] - 24*a*b*c^3*x^3*PolyLog[2, I*c*x
] - (12*I)*b^2*c^3*x^3*PolyLog[3, E^((-2*I)*ArcTan[c*x])] + (12*I)*b^2*c^3*
x^3*PolyLog[3, -E^((2*I)*ArcTan[c*x])]))/(24*x^3)
```

Maple [C] time = 2.547, size = 1814, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x^4,x)
```

```
[Out] 8/3*b^2*c^3*d^3*arctan(c*x)+1/2*c^3*d^3*b^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2
+1)-1))*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)
-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)^2-3/2*I*c*d^3*a^2/x^2-I*c^3*d^
3*a^2*ln(c*x)+3*I*c^3*d^3*b^2*ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))+11/6*I*c^3*
d^3*b^2*arctan(c*x)^2+3*I*c^3*d^3*b^2*ln((1+I*c*x)/(c^2*x^2+1)^(1/2)-1)-2*I
*c^3*d^3*b^2*polylog(3,(1+I*c*x)/(c^2*x^2+1)^(1/2))-20/3*I*c^3*d^3*b^2*dilo
g((1+I*c*x)/(c^2*x^2+1)^(1/2))+20/3*I*c^3*d^3*b^2*dilog(1+(1+I*c*x)/(c^2*x^
2+1)^(1/2))+1/2*I*c^3*d^3*b^2*polylog(3,-(1+I*c*x)^2/(c^2*x^2+1))-2*I*c^3*d
^3*b^2*polylog(3,-(1+I*c*x)/(c^2*x^2+1)^(1/2))-2/3*d^3*a*b*arctan(c*x)/x^3+
c^3*d^3*a*b*dilog(1+I*c*x)-c^3*d^3*a*b*dilog(1-I*c*x)+10/3*c^3*d^3*a*b*ln(c
^2*x^2+1)-20/3*c^3*d^3*a*b*ln(c*x)+6*c^2*d^3*a*b*arctan(c*x)/x+I*c^3*d^3*b^
2*arctan(c*x)^2*ln((1+I*c*x)^2/(c^2*x^2+1)-1)-1/2*c^3*d^3*b^2*Pi*csgn(((1+I
*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2+1/2*c^3
*d^3*b^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))
^3*arctan(c*x)^2+1/2*c^3*d^3*b^2*Pi*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*
c*x)^2/(c^2*x^2+1)+1))^3*arctan(c*x)^2+c^3*d^3*a*b*ln(c*x)*ln(1+I*c*x)-c^3*
d^3*a*b*ln(c*x)*ln(1-I*c*x)-3/2*I*c*d^3*b^2*arctan(c*x)^2/x^2-3*I*c^2*d^3*b
^2*arctan(c*x)/x-3*I*c^2*d^3*a*b/x-3*I*c^3*d^3*a*b*arctan(c*x)-I*c^3*d^3*b^
2*arctan(c*x)^2*ln(c*x)-I*c^3*d^3*b^2*arctan(c*x)^2*ln(1-(1+I*c*x)/(c^2*x^2
+1)^(1/2))-I*c^3*d^3*b^2*arctan(c*x)^2*ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))+1/
3*I*c^3*d^3*b^2/(I*c*x+(c^2*x^2+1)^(1/2)+1)*(c^2*x^2+1)^(1/2)-1/3*I*c^3*d^3
*b^2/(I*c*x-(c^2*x^2+1)^(1/2)+1)*(c^2*x^2+1)^(1/2)-1/3*d^3*a^2/x^3+3*c^2*d^
3*a^2/x-1/3*c*d^3*a*b/x^2+1/2*c^3*d^3*b^2*Pi*arctan(c*x)^2-2*c^3*d^3*b^2*ar
ctan(c*x)*polylog(2,(1+I*c*x)/(c^2*x^2+1)^(1/2))+c^3*d^3*b^2*arctan(c*x)*po
```

```
lylog(2,-(1+I*c*x)^2/(c^2*x^2+1))-2*c^3*d^3*b^2*arctan(c*x)*polylog(2,-(1+I
*c*x)/(c^2*x^2+1)^(1/2))-20/3*c^3*d^3*b^2*arctan(c*x)*ln(1+(1+I*c*x)/(c^2*x
^2+1)^(1/2))-1/3*c*d^3*b^2*arctan(c*x)/x^2+3*c^2*d^3*b^2*arctan(c*x)^2/x-1/
3*d^3*b^2*arctan(c*x)^2/x^3-1/2*c^3*d^3*b^2*Pi*csgn(I/((1+I*c*x)^2/(c^2*x^2
+1)+1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*a
rctan(c*x)^2-1/2*c^3*d^3*b^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*
x)^2/(c^2*x^2+1)+1))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2
+1)+1))^2*arctan(c*x)^2+1/2*c^3*d^3*b^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-
1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)
^2/(c^2*x^2+1)+1))*arctan(c*x)^2-1/2*c^3*d^3*b^2*Pi*csgn(I*((1+I*c*x)^2/(c^
2*x^2+1)-1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1)
)^2*arctan(c*x)^2-2*I*c^3*d^3*a*b*arctan(c*x)*ln(c*x)-3*I*c*d^3*a*b*arctan(
c*x)/x^2
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{-4i\,a^2c^3d^3x^3-12\,a^2c^2d^3x^2+12i\,a^2cd^3x+4\,a^2d^3+\left(i\,b^2c^3d^3x^3+3\,b^2c^2d^3x^2-3i\,b^2cd^3x-b^2d^3\right)\log\left(-\frac{cx+i}{cx-i}\right)^2+}{4\,x^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x^4,x, algorithm="fricas")
```

```
[Out] integral(1/4*(-4*I*a^2*c^3*d^3*x^3 - 12*a^2*c^2*d^3*x^2 + 12*I*a^2*c*d^3*x
+ 4*a^2*d^3 + (I*b^2*c^3*d^3*x^3 + 3*b^2*c^2*d^3*x^2 - 3*I*b^2*c*d^3*x - b^
2*d^3)*log(-(c*x + I)/(c*x - I))^2 + (4*a*b*c^3*d^3*x^3 - 12*I*a*b*c^2*d^3*
x^2 - 12*a*b*c*d^3*x + 4*I*a*b*d^3)*log(-(c*x + I)/(c*x - I)))/x^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d^3\left(\int \frac{a^2}{x^4}\,dx + \int -\frac{3a^2c^2}{x^2}\,dx + \int \frac{b^2\,\mathrm{atan}^2\,(cx)}{x^4}\,dx + \int \frac{3ia^2c}{x^3}\,dx + \int -\frac{ia^2c^3}{x}\,dx + \int \frac{2ab\,\mathrm{atan}\,(cx)}{x^4}\,dx + \int -\frac{3b^2c^2\,\mathrm{ata}}{x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**3*(a+b*atan(c*x))**2/x**4,x)
```

```
[Out] d**3*(Integral(a**2/x**4, x) + Integral(-3*a**2*c**2/x**2, x) + Integral(b*
*2*atan(c*x)**2/x**4, x) + Integral(3*I*a**2*c/x**3, x) + Integral(-I*a**2*
c**3/x, x) + Integral(2*a*b*atan(c*x)/x**4, x) + Integral(-3*b**2*c**2*atan
(c*x)**2/x**2, x) + Integral(3*I*b**2*c*atan(c*x)**2/x**3, x) + Integral(-I
*b**2*c**3*atan(c*x)**2/x, x) + Integral(-6*a*b*c**2*atan(c*x)/x**2, x) + I
ntegral(6*I*a*b*c*atan(c*x)/x**3, x) + Integral(-2*I*a*b*c**3*atan(c*x)/x,
x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(icdx + d)^3 (b \arctan(cx) + a)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x^4,x, algorithm="giac")
```

```
[Out] integrate((I*c*d*x + d)^3*(b*arctan(c*x) + a)^2/x^4, x)
```

3.92 $\int \frac{(d+icdx)^3\left(a+b\tan^{-1}(cx)\right)^2}{x^5}\,dx$

Optimal. Leaf size=293

$$2b^2c^4d^3\text{PolyLog}(2,-icx)-2b^2c^4d^3\text{PolyLog}(2,icx)-2b^2c^4d^3\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)-\frac{ibc^2d^3\left(a+b\tan^{-1}(cx)\right)}{x^2}-4iab$$

```
[Out] -(b^2*c^2*d^3)/(12*x^2) - (I*b^2*c^3*d^3)/x - I*b^2*c^4*d^3*ArcTan[c*x] - (
b*c*d^3*(a + b*ArcTan[c*x]))/(6*x^3) - (I*b*c^2*d^3*(a + b*ArcTan[c*x]))/x^
2 + (7*b*c^3*d^3*(a + b*ArcTan[c*x]))/(2*x) - (d^3*(1 + I*c*x)^4*(a + b*Arc
Tan[c*x])^2)/(4*x^4) - (4*I)*a*b*c^4*d^3*Log[x] - (11*b^2*c^4*d^3*Log[x])/3
 - (4*I)*b*c^4*d^3*(a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)] + (11*b^2*c^4*d^3
*Log[1 + c^2*x^2])/6 + 2*b^2*c^4*d^3*PolyLog[2, (-I)*c*x] - 2*b^2*c^4*d^3*P
olyLog[2, I*c*x] - 2*b^2*c^4*d^3*PolyLog[2, 1 - 2/(1 - I*c*x)]
```

Rubi [A] time = 0.321967, antiderivative size = 293, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 15, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {37, 4874, 4852, 266, 44, 325, 203, 36, 29, 31, 4848, 2391, 4854, 2402, 2315}

$$2b^2c^4d^3\text{PolyLog}(2,-icx)-2b^2c^4d^3\text{PolyLog}(2,icx)-2b^2c^4d^3\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)-\frac{ibc^2d^3\left(a+b\tan^{-1}(cx)\right)}{x^2}-4iab$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)^3*(a + b*ArcTan[c*x])^2)/x^5,x]
```


```
[Out] -(b^2*c^2*d^3)/(12*x^2) - (I*b^2*c^3*d^3)/x - I*b^2*c^4*d^3*ArcTan[c*x] - (
b*c*d^3*(a + b*ArcTan[c*x]))/(6*x^3) - (I*b*c^2*d^3*(a + b*ArcTan[c*x]))/x^
2 + (7*b*c^3*d^3*(a + b*ArcTan[c*x]))/(2*x) - (d^3*(1 + I*c*x)^4*(a + b*Arc
Tan[c*x])^2)/(4*x^4) - (4*I)*a*b*c^4*d^3*Log[x] - (11*b^2*c^4*d^3*Log[x])/3
 - (4*I)*b*c^4*d^3*(a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)] + (11*b^2*c^4*d^3
*Log[1 + c^2*x^2])/6 + 2*b^2*c^4*d^3*PolyLog[2, (-I)*c*x] - 2*b^2*c^4*d^3*P
olyLog[2, I*c*x] - 2*b^2*c^4*d^3*PolyLog[2, 1 - 2/(1 - I*c*x)]
```


Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
```

```
1]
```

Rule 4874

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_.) + (e_
.)*(x_))^(q_), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dis
t[(a + b*ArcTan[c*x])^p, u, x] - Dist[b*c*p, Int[ExpandIntegrand[(a + b*Arc
Tan[c*x])^(p - 1), u/(1 + c^2*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
 q}, x] && IGtQ[p, 1] && EqQ[c^2*d^2 + e^2, 0] && IntegersQ[m, q] && NeQ[m,
 -1] && NeQ[q, -1] && ILtQ[m + q + 1, 0] && LtQ[m*q, 0]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int \frac{(d+icdx)^3\left(a+b\tan^{-1}(cx)\right)^2}{x^5}\,dx = -\frac{d^3(1+icx)^4\left(a+b\tan^{-1}(cx)\right)^2}{4x^4} - (2bc)\int\left(-\frac{d^3\left(a+b\tan^{-1}(cx)\right)}{4x^4} - \frac{icd^3\left(a+b\right.}{x}\right.$$

$$= -\frac{d^3(1+icx)^4\left(a+b\tan^{-1}(cx)\right)^2}{4x^4} + \frac{1}{2}\left(bcd^3\right)\int\frac{a+b\tan^{-1}(cx)}{x^4}\,dx + \left(2ibc^2d^3\right)\int$$

$$= -\frac{bcd^3\left(a+b\tan^{-1}(cx)\right)}{6x^3} - \frac{ibc^2d^3\left(a+b\tan^{-1}(cx)\right)}{x^2} + \frac{7bc^3d^3\left(a+b\tan^{-1}(cx)\right)}{2x}$$

$$= -\frac{ib^2c^3d^3}{x} - \frac{bcd^3\left(a+b\tan^{-1}(cx)\right)}{6x^3} - \frac{ibc^2d^3\left(a+b\tan^{-1}(cx)\right)}{x^2} + \frac{7bc^3d^3\left(a+b\,\mathrm{ta}\right.}{2x}$$

$$= -\frac{ib^2c^3d^3}{x} - ib^2c^4d^3\tan^{-1}(cx) - \frac{bcd^3\left(a+b\tan^{-1}(cx)\right)}{6x^3} - \frac{ibc^2d^3\left(a+b\tan^{-1}(cx)\right.}{x^2}$$

$$= -\frac{b^2c^2d^3}{12x^2} - \frac{ib^2c^3d^3}{x} - ib^2c^4d^3\tan^{-1}(cx) - \frac{bcd^3\left(a+b\tan^{-1}(cx)\right)}{6x^3} - \frac{ibc^2d^3\left(a+b\right.}{x^2}$$

Mathematica [A] time = 0.834639, size = 322, normalized size = 1.1

$$d^3\left(-24b^2c^4x^4\mathrm{PolyLog}\left(2,e^{2i\tan^{-1}(cx)}\right)+12ia^2c^3x^3+18a^2c^2x^2-12ia^2cx-3a^2+42abc^3x^3-12iabc^2x^2-48iabc^4x^4\log\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + I*c*d*x)^3*(a + b*ArcTan[c*x])^2)/x^5,x]
```

```
[Out] (d^3*(-3*a^2 - (12*I)*a^2*c*x - 2*a*b*c*x + 18*a^2*c^2*x^2 - (12*I)*a*b*c^2
*x^2 - b^2*c^2*x^2 + (12*I)*a^2*c^3*x^3 + 42*a*b*c^3*x^3 - (12*I)*b^2*c^3*x
^3 - b^2*c^4*x^4 - 3*b^2*(-I + c*x)^4*ArcTan[c*x]^2 + 2*b*ArcTan[c*x]*(b*c*
x*(-1 - (6*I)*c*x + 21*c^2*x^2 - (6*I)*c^3*x^3) + 3*a*(-1 - (4*I)*c*x + 6*c
^2*x^2 + (4*I)*c^3*x^3 + 7*c^4*x^4) - (24*I)*b*c^4*x^4*Log[1 - E^((2*I)*Arc
Tan[c*x])]) - (48*I)*a*b*c^4*x^4*Log[c*x] - 44*b^2*c^4*x^4*Log[(c*x)/Sqrt[1
 + c^2*x^2]] + (24*I)*a*b*c^4*x^4*Log[1 + c^2*x^2] - 24*b^2*c^4*x^4*PolyLog
[2, E^((2*I)*ArcTan[c*x])]))/(12*x^4)
```

Maple [B] time = 0.111, size = 757, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x^5,x)
```

```
[Out] c^4*d^3*b^2*dilog(-1/2*I*(c*x+I))-1/2*c^4*d^3*b^2*ln(c*x+I)^2+1/2*c^4*d^3*b
^2*ln(c*x-I)^2+7/4*c^4*d^3*b^2*arctan(c*x)^2-11/3*c^4*d^3*b^2*ln(c*x)+2*c^4
*d^3*b^2*dilog(1+I*c*x)-2*c^4*d^3*b^2*dilog(1-I*c*x)-c^4*d^3*b^2*dilog(1/2*
I*(c*x-I))-1/4*d^3*b^2*arctan(c*x)^2/x^4+3/2*c^2*d^3*a^2/x^2-1/12*b^2*c^2*d
^3/x^2+2*c^4*d^3*b^2*ln(c*x)*ln(1+I*c*x)+7/2*c^3*d^3*b^2*arctan(c*x)/x-1/2*
d^3*a*b*arctan(c*x)/x^4-I*c*d^3*a^2/x^3-1/6*c*d^3*a*b/x^3+7/2*c^3*d^3*a*b/x
-2*c^4*d^3*b^2*ln(c*x)*ln(1-I*c*x)+c^4*d^3*b^2*ln(c*x+I)*ln(c^2*x^2+1)-c^4*
d^3*b^2*ln(c*x+I)*ln(1/2*I*(c*x-I))-c^4*d^3*b^2*ln(c*x-I)*ln(c^2*x^2+1)+c^4
*d^3*b^2*ln(c*x-I)*ln(-1/2*I*(c*x+I))+7/2*c^4*d^3*a*b*arctan(c*x)+I*c^3*d^3
*a^2/x+3/2*c^2*d^3*b^2*arctan(c*x)^2/x^2-1/6*c*d^3*b^2*arctan(c*x)/x^3+2*I*
c^4*d^3*a*b*ln(c^2*x^2+1)-I*c^2*d^3*b^2*arctan(c*x)/x^2-I*c*d^3*b^2*arctan(
c*x)^2/x^3-I*c^2*d^3*a*b/x^2+2*I*c^4*d^3*b^2*arctan(c*x)*ln(c^2*x^2+1)-4*I*
c^4*d^3*a*b*ln(c*x)+I*c^3*d^3*b^2*arctan(c*x)^2/x+3*c^2*d^3*a*b*arctan(c*x)
/x^2-4*I*c^4*d^3*b^2*arctan(c*x)*ln(c*x)-1/4*d^3*a^2/x^4-I*b^2*c^3*d^3/x-I*
b^2*c^4*d^3*arctan(c*x)-2*I*c*d^3*a*b*arctan(c*x)/x^3+2*I*c^3*d^3*a*b*arcta
n(c*x)/x+11/6*b^2*c^4*d^3*ln(c^2*x^2+1)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x^5,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{16\,x^4 \text{integral}\left(\frac{-4i\,a^2c^5d^3x^5-12\,a^2c^4d^3x^4+8i\,a^2c^3d^3x^3-8\,a^2c^2d^3x^2+12i\,a^2cd^3x+4\,a^2d^3+\left(4\,abc^5d^3x^5+\left(-12i\,ab+4\,b^2\right)c^4d^3x^4-2\left(4\,ab+3i\,b^2\right)c^3d^3x^3+\left(-8i\,ab-4\right.\right.}{4\left(c^2x^7+x^5\right)}\right.}{16\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x^5,x, algorithm="fricas")
```

```
[Out] 1/16*(16*x^4*integral(1/4*(-4*I*a^2*c^5*d^3*x^5 - 12*a^2*c^4*d^3*x^4 + 8*I*
a^2*c^3*d^3*x^3 - 8*a^2*c^2*d^3*x^2 + 12*I*a^2*c*d^3*x + 4*a^2*d^3 + (4*a*b
*c^5*d^3*x^5 + (-12*I*a*b + 4*b^2)*c^4*d^3*x^4 - 2*(4*a*b + 3*I*b^2)*c^3*d^
3*x^3 + (-8*I*a*b - 4*b^2)*c^2*d^3*x^2 - (12*a*b - I*b^2)*c*d^3*x + 4*I*a*b
*d^3)*log(-(c*x + I)/(c*x - I)))/(c^2*x^7 + x^5), x) + (-4*I*b^2*c^3*d^3*x^
3 - 6*b^2*c^2*d^3*x^2 + 4*I*b^2*c*d^3*x + b^2*d^3)*log(-(c*x + I)/(c*x - I)
)^2)/x^4
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**3*(a+b*atan(c*x))**2/x**5,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(icdx + d)^3 (b \arctan(cx) + a)^2}{x^5} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x^5,x, algorithm="giac")
```

```
[Out] integrate((I*c*d*x + d)^3*(b*arctan(c*x) + a)^2/x^5, x)
```

3.93 $\int \frac{(d+icdx)^3\left(a+b\tan^{-1}(cx)\right)^2}{x^6}\,dx$

Optimal. Leaf size=384

$$\frac{6}{5}ib^2c^5d^3\text{PolyLog}(2,-icx)-\frac{6}{5}ib^2c^5d^3\text{PolyLog}(2,icx)-\frac{6}{5}ib^2c^5d^3\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)+\frac{6bc^3d^3\left(a+b\tan^{-1}(cx)\right)}{5x^2}-$$

```
[Out] -(b^2*c^2*d^3)/(30*x^3) - ((I/4)*b^2*c^3*d^3)/x^2 + (13*b^2*c^4*d^3)/(10*x)
 + (13*b^2*c^5*d^3*ArcTan[c*x])/10 - (b*c*d^3*(a + b*ArcTan[c*x]))/(10*x^4)
 - ((I/2)*b*c^2*d^3*(a + b*ArcTan[c*x]))/x^3 + (6*b*c^3*d^3*(a + b*ArcTan[c
*x]))/(5*x^2) + (((5*I)/2)*b*c^4*d^3*(a + b*ArcTan[c*x]))/x - (d^3*(1 + I*c
*x)^4*(a + b*ArcTan[c*x])^2)/(5*x^5) + ((I/20)*c*d^3*(1 + I*c*x)^4*(a + b*A
rcTan[c*x])^2)/x^4 + (12*a*b*c^5*d^3*Log[x])/5 - (3*I)*b^2*c^5*d^3*Log[x] +
 (12*b*c^5*d^3*(a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/5 + ((3*I)/2)*b^2*c^
5*d^3*Log[1 + c^2*x^2] + ((6*I)/5)*b^2*c^5*d^3*PolyLog[2, (-I)*c*x] - ((6*I
)/5)*b^2*c^5*d^3*PolyLog[2, I*c*x] - ((6*I)/5)*b^2*c^5*d^3*PolyLog[2, 1 - 2
/(1 - I*c*x)]
```

Rubi [A] time = 0.366611, antiderivative size = 384, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 16, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.64, Rules used = {45, 37, 4874, 4852, 325, 203, 266, 44, 36, 29, 31, 4848, 2391, 4854, 2402, 2315}

$$\frac{6}{5}ib^2c^5d^3\text{PolyLog}(2,-icx)-\frac{6}{5}ib^2c^5d^3\text{PolyLog}(2,icx)-\frac{6}{5}ib^2c^5d^3\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)+\frac{6bc^3d^3\left(a+b\tan^{-1}(cx)\right)}{5x^2}-$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)^3*(a + b*ArcTan[c*x])^2)/x^6,x]
```


```
[Out] -(b^2*c^2*d^3)/(30*x^3) - ((I/4)*b^2*c^3*d^3)/x^2 + (13*b^2*c^4*d^3)/(10*x)
 + (13*b^2*c^5*d^3*ArcTan[c*x])/10 - (b*c*d^3*(a + b*ArcTan[c*x]))/(10*x^4)
 - ((I/2)*b*c^2*d^3*(a + b*ArcTan[c*x]))/x^3 + (6*b*c^3*d^3*(a + b*ArcTan[c
*x]))/(5*x^2) + (((5*I)/2)*b*c^4*d^3*(a + b*ArcTan[c*x]))/x - (d^3*(1 + I*c
*x)^4*(a + b*ArcTan[c*x])^2)/(5*x^5) + ((I/20)*c*d^3*(1 + I*c*x)^4*(a + b*A
rcTan[c*x])^2)/x^4 + (12*a*b*c^5*d^3*Log[x])/5 - (3*I)*b^2*c^5*d^3*Log[x] +
 (12*b*c^5*d^3*(a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/5 + ((3*I)/2)*b^2*c^
5*d^3*Log[1 + c^2*x^2] + ((6*I)/5)*b^2*c^5*d^3*PolyLog[2, (-I)*c*x] - ((6*I
)/5)*b^2*c^5*d^3*PolyLog[2, I*c*x] - ((6*I)/5)*b^2*c^5*d^3*PolyLog[2, 1 - 2
```

```
/(1 - I*c*x)]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 4874

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_.) + (e_
.)*(x_))^(q_), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dis
t[(a + b*ArcTan[c*x])^p, u, x] - Dist[b*c*p, Int[ExpandIntegrand[(a + b*Arc
Tan[c*x])^(p - 1), u/(1 + c^2*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
 q}, x] && IGtQ[p, 1] && EqQ[c^2*d^2 + e^2, 0] && IntegersQ[m, q] && NeQ[m,
 -1] && NeQ[q, -1] && ILtQ[m + q + 1, 0] && LtQ[m*q, 0]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int \frac{(d+icdx)^3\left(a+b\tan^{-1}(cx)\right)^2}{x^6}\,dx = -\frac{d^3(1+icx)^4\left(a+b\tan^{-1}(cx)\right)^2}{5x^5}+\frac{icd^3(1+icx)^4\left(a+b\tan^{-1}(cx)\right)^2}{20x^4}-(2bc)\int\Big($$

$$= -\frac{d^3(1+icx)^4\left(a+b\tan^{-1}(cx)\right)^2}{5x^5}+\frac{icd^3(1+icx)^4\left(a+b\tan^{-1}(cx)\right)^2}{20x^4}+\frac{1}{5}\Big(2bcd^3$$

$$= -\frac{bcd^3\left(a+b\tan^{-1}(cx)\right)}{10x^4}-\frac{ibc^2d^3\left(a+b\tan^{-1}(cx)\right)}{2x^3}+\frac{6bc^3d^3\left(a+b\tan^{-1}(cx)\right)}{5x^2}$$

$$= -\frac{b^2c^2d^3}{30x^3}+\frac{6b^2c^4d^3}{5x}-\frac{bcd^3\left(a+b\tan^{-1}(cx)\right)}{10x^4}-\frac{ibc^2d^3\left(a+b\tan^{-1}(cx)\right)}{2x^3}+\frac{6bc^3d}{}$$

$$= -\frac{b^2c^2d^3}{30x^3}+\frac{13b^2c^4d^3}{10x}+\frac{6}{5}b^2c^5d^3\tan^{-1}(cx)-\frac{bcd^3\left(a+b\tan^{-1}(cx)\right)}{10x^4}-\frac{ibc^2d^3\left(a+\right.}{}$$

$$= -\frac{b^2c^2d^3}{30x^3}-\frac{ib^2c^3d^3}{4x^2}+\frac{13b^2c^4d^3}{10x}+\frac{13}{10}b^2c^5d^3\tan^{-1}(cx)-\frac{bcd^3\left(a+b\tan^{-1}(cx)\right)}{10x^4}$$

Mathematica [A] time = 1.25055, size = 363, normalized size = 0.95

$$\frac{d^3\left(-72ib^2c^5x^5\text{PolyLog}\left(2,e^{2i\tan^{-1}(cx)}\right)+30ia^2c^3x^3+60a^2c^2x^2-45ia^2cx-12a^2+150iabc^4x^4+72abc^3x^3-30iabc^2x^2\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + I*c*d*x)^3*(a + b*ArcTan[c*x])^2)/x^6,x]
```

```
[Out] (d^3*(-12*a^2 - (45*I)*a^2*c*x - 6*a*b*c*x + 60*a^2*c^2*x^2 - (30*I)*a*b*c^
2*x^2 - 2*b^2*c^2*x^2 + (30*I)*a^2*c^3*x^3 + 72*a*b*c^3*x^3 - (15*I)*b^2*c^
3*x^3 + (150*I)*a*b*c^4*x^4 + 78*b^2*c^4*x^4 - (15*I)*b^2*c^5*x^5 + (3*I)*b
^2*(-I + c*x)^4*(4*I + c*x)*ArcTan[c*x]^2 + 6*b*ArcTan[c*x]*(b*c*x*(-1 - (5
*I)*c*x + 12*c^2*x^2 + (25*I)*c^3*x^3 + 13*c^4*x^4) + a*(-4 - (15*I)*c*x +
20*c^2*x^2 + (10*I)*c^3*x^3 + (25*I)*c^5*x^5) + 24*b*c^5*x^5*Log[1 - E^((2*
I)*ArcTan[c*x])]) + 144*a*b*c^5*x^5*Log[c*x] - (180*I)*b^2*c^5*x^5*Log[(c*x
)/Sqrt[1 + c^2*x^2]] - 72*a*b*c^5*x^5*Log[1 + c^2*x^2] - (72*I)*b^2*c^5*x^5
*PolyLog[2, E^((2*I)*ArcTan[c*x])]))/(60*x^5)
```

Maple [B] time = 0.115, size = 816, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x^6,x)
```

```
[Out] 5/4*I*c^5*d^3*b^2*arctan(c*x)^2+6/5*c^3*d^3*a*b/x^2-1/10*c*d^3*a*b/x^4-6/5*
c^5*d^3*b^2*arctan(c*x)*ln(c^2*x^2+1)+12/5*c^5*d^3*b^2*arctan(c*x)*ln(c*x)+
6/5*I*c^5*d^3*b^2*dilog(1+I*c*x)-6/5*I*c^5*d^3*b^2*dilog(1-I*c*x)-3*I*c^5*d
^3*b^2*ln(c*x)-3/5*I*c^5*d^3*b^2*dilog(1/2*I*(c*x-I))+3/10*I*c^5*d^3*b^2*ln
(c*x-I)^2+3/5*I*c^5*d^3*b^2*dilog(-1/2*I*(c*x+I))+1/2*I*c^3*d^3*a^2/x^2-3/4
*I*c*d^3*a^2/x^4-6/5*c^5*d^3*a*b*ln(c^2*x^2+1)+12/5*c^5*d^3*a*b*ln(c*x)+6/5
*c^3*d^3*b^2*arctan(c*x)/x^2+c^2*d^3*b^2*arctan(c*x)^2/x^3-1/10*c*d^3*b^2*a
rctan(c*x)/x^4-2/5*d^3*a*b*arctan(c*x)/x^5+3/5*I*c^5*d^3*b^2*ln(c*x-I)*ln(-
1/2*I*(c*x+I))-6/5*I*c^5*d^3*b^2*ln(c*x)*ln(1-I*c*x)+2*c^2*d^3*a*b*arctan(c
*x)/x^3+1/2*I*c^3*d^3*b^2*arctan(c*x)^2/x^2+6/5*I*c^5*d^3*b^2*ln(c*x)*ln(1+
I*c*x)+5/2*I*c^5*d^3*a*b*arctan(c*x)+5/2*I*c^4*d^3*a*b/x-1/2*I*c^2*d^3*a*b/
x^3-1/2*I*c^2*d^3*b^2*arctan(c*x)/x^3+5/2*I*c^4*d^3*b^2*arctan(c*x)/x-3/4*I
*c*d^3*b^2*arctan(c*x)^2/x^4-3/5*I*c^5*d^3*b^2*ln(c*x-I)*ln(c^2*x^2+1)-3/5*
I*c^5*d^3*b^2*ln(c*x+I)*ln(1/2*I*(c*x-I))+3/5*I*c^5*d^3*b^2*ln(c*x+I)*ln(c^
2*x^2+1)-3/10*I*c^5*d^3*b^2*ln(c*x+I)^2-1/5*d^3*a^2/x^5-1/4*I*b^2*c^3*d^3/x
^2+c^2*d^3*a^2/x^3-1/5*d^3*b^2*arctan(c*x)^2/x^5+I*c^3*d^3*a*b*arctan(c*x)/
x^2-3/2*I*c*d^3*a*b*arctan(c*x)/x^4+3/2*I*b^2*c^5*d^3*ln(c^2*x^2+1)-1/30*b^
2*c^2*d^3/x^3+13/10*b^2*c^4*d^3/x+13/10*b^2*c^5*d^3*arctan(c*x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x^6,x, algorithm="maxima")
```

```
[Out] I*((c*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*a*b*c^3*d^3 - ((c^2*log(c^2*x
^2 + 1) - c^2*log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^3)*a*b*c^2*d^3 + 1/2*I*
((3*c^3*arctan(c*x) + (3*c^2*x^2 - 1)/x^3)*c - 3*arctan(c*x)/x^4)*a*b*c*d^3
 - 1/10*((2*c^4*log(c^2*x^2 + 1) - 2*c^4*log(x^2) - (2*c^2*x^2 - 1)/x^4)*c
+ 4*arctan(c*x)/x^5)*a*b*d^3 + 1/2*I*a^2*c^3*d^3/x^2 + a^2*c^2*d^3/x^3 - 3/
4*I*a^2*c*d^3/x^4 - 1/5*a^2*d^3/x^5 - 1/320*(320*I*x^5*integrate(1/80*(60*(
b^2*c^5*d^3*x^5 - 2*b^2*c^3*d^3*x^3 - 3*b^2*c*d^3*x)*arctan(c*x)^2 + 5*(b^2
*c^5*d^3*x^5 - 2*b^2*c^3*d^3*x^3 - 3*b^2*c*d^3*x)*log(c^2*x^2 + 1)^2 + 2*(3
0*b^2*c^4*d^3*x^4 - 19*b^2*c^2*d^3*x^2)*arctan(c*x) - (10*b^2*c^5*d^3*x^5 -
 35*b^2*c^3*d^3*x^3 + 4*b^2*c*d^3*x + 20*(3*b^2*c^4*d^3*x^4 + 2*b^2*c^2*d^3
*x^2 - b^2*d^3)*arctan(c*x))*log(c^2*x^2 + 1))/(c^2*x^8 + x^6), x) + 320*x^
5*integrate(1/80*(60*(3*b^2*c^4*d^3*x^4 + 2*b^2*c^2*d^3*x^2 - b^2*d^3)*arct
an(c*x)^2 + 5*(3*b^2*c^4*d^3*x^4 + 2*b^2*c^2*d^3*x^2 - b^2*d^3)*log(c^2*x^2
 + 1)^2 - 2*(10*b^2*c^5*d^3*x^5 - 35*b^2*c^3*d^3*x^3 + 4*b^2*c*d^3*x)*arcta
n(c*x) - (30*b^2*c^4*d^3*x^4 - 19*b^2*c^2*d^3*x^2 - 20*(b^2*c^5*d^3*x^5 - 2
*b^2*c^3*d^3*x^3 - 3*b^2*c*d^3*x)*arctan(c*x))*log(c^2*x^2 + 1))/(c^2*x^8 +
 x^6), x) + (-40*I*b^2*c^3*d^3*x^3 - 80*b^2*c^2*d^3*x^2 + 60*I*b^2*c*d^3*x
+ 16*b^2*d^3)*arctan(c*x)^2 + (40*b^2*c^3*d^3*x^3 - 80*I*b^2*c^2*d^3*x^2 -
60*b^2*c*d^3*x + 16*I*b^2*d^3)*arctan(c*x)*log(c^2*x^2 + 1) + (10*I*b^2*c^3
*d^3*x^3 + 20*b^2*c^2*d^3*x^2 - 15*I*b^2*c*d^3*x - 4*b^2*d^3)*log(c^2*x^2 +
 1)^2)/x^5
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{80\,x^5\text{integral}\left(\frac{-20i\,a^2c^5d^3x^5-60\,a^2c^4d^3x^4+40i\,a^2c^3d^3x^3-40\,a^2c^2d^3x^2+60i\,a^2cd^3x+20\,a^2d^3+\left(20\,abc^5d^3x^5+\left(-60i\,ab+10\,b^2\right)c^4d^3x^4-20\left(2\,ab+i\,b^2\right)c^3d^3x^3+\left(\right.}{20\left(c^2x^8+x^6\right)}\right.}{80}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x^6,x, algorithm="fricas")
```

```
[Out] 1/80*(80*x^5*integral(1/20*(-20*I*a^2*c^5*d^3*x^5 - 60*a^2*c^4*d^3*x^4 + 40
*I*a^2*c^3*d^3*x^3 - 40*a^2*c^2*d^3*x^2 + 60*I*a^2*c*d^3*x + 20*a^2*d^3 + (
20*a*b*c^5*d^3*x^5 + (-60*I*a*b + 10*b^2)*c^4*d^3*x^4 - 20*(2*a*b + I*b^2)*
c^3*d^3*x^3 + (-40*I*a*b - 15*b^2)*c^2*d^3*x^2 - 4*(15*a*b - I*b^2)*c*d^3*x
 + 20*I*a*b*d^3)*log(-(c*x + I)/(c*x - I)))/(c^2*x^8 + x^6), x) + (-10*I*b^
2*c^3*d^3*x^3 - 20*b^2*c^2*d^3*x^2 + 15*I*b^2*c*d^3*x + 4*b^2*d^3)*log(-(c*
x + I)/(c*x - I))^2)/x^5
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**3*(a+b*atan(c*x))**2/x**6,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(icdx+d)^3(b\arctan(cx)+a)^2}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x^6,x, algorithm="giac")
```

```
[Out] integrate((I*c*d*x + d)^3*(b*arctan(c*x) + a)^2/x^6, x)
```

3.94 $\int \frac{(d+icdx)^3\left(a+b\tan^{-1}(cx)\right)^2}{x^7}\,dx$

Optimal. Leaf size=513

$$-\frac{14}{15}b^2c^6d^3\text{PolyLog}(2,-icx)+\frac{14}{15}b^2c^6d^3\text{PolyLog}(2,icx)+\frac{37}{40}b^2c^6d^3\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)-\frac{1}{120}b^2c^6d^3\text{PolyLog}\Big(2$$

```
[Out] -(b^2*c^2*d^3)/(60*x^4) - ((I/10)*b^2*c^3*d^3)/x^3 + (61*b^2*c^4*d^3)/(180*
x^2) + (((37*I)/30)*b^2*c^5*d^3)/x + ((37*I)/30)*b^2*c^6*d^3*ArcTan[c*x] -
(b*c*d^3*(a + b*ArcTan[c*x]))/(15*x^5) - (((3*I)/10)*b*c^2*d^3*(a + b*ArcTa
n[c*x]))/x^4 + (11*b*c^3*d^3*(a + b*ArcTan[c*x]))/(18*x^3) + (((14*I)/15)*b
*c^4*d^3*(a + b*ArcTan[c*x]))/x^2 - (11*b*c^5*d^3*(a + b*ArcTan[c*x]))/(6*x
) - (d^3*(a + b*ArcTan[c*x])^2)/(6*x^6) - (((3*I)/5)*c*d^3*(a + b*ArcTan[c*
x])^2)/x^5 + (3*c^2*d^3*(a + b*ArcTan[c*x])^2)/(4*x^4) + ((I/3)*c^3*d^3*(a
+ b*ArcTan[c*x])^2)/x^3 + ((28*I)/15)*a*b*c^6*d^3*Log[x] + (113*b^2*c^6*d^3
*Log[x])/45 + ((37*I)/20)*b*c^6*d^3*(a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)]
+ (I/60)*b*c^6*d^3*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)] - (113*b^2*c^6*d^
3*Log[1 + c^2*x^2])/90 - (14*b^2*c^6*d^3*PolyLog[2, (-I)*c*x])/15 + (14*b^2
*c^6*d^3*PolyLog[2, I*c*x])/15 + (37*b^2*c^6*d^3*PolyLog[2, 1 - 2/(1 - I*c*
x)])/40 - (b^2*c^6*d^3*PolyLog[2, 1 - 2/(1 + I*c*x)])/120
```

Rubi [A] time = 0.516808, antiderivative size = 513, normalized size of antiderivative = 1., number of steps used = 31, number of rules used = 15, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {43, 4874, 4852, 266, 44, 325, 203, 36, 29, 31, 4848, 2391, 4854, 2402, 2315}

$$-\frac{14}{15}b^2c^6d^3\text{PolyLog}(2,-icx)+\frac{14}{15}b^2c^6d^3\text{PolyLog}(2,icx)+\frac{37}{40}b^2c^6d^3\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)-\frac{1}{120}b^2c^6d^3\text{PolyLog}\Big(2$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)^3*(a + b*ArcTan[c*x])^2)/x^7,x]
```

```
[Out] -(b^2*c^2*d^3)/(60*x^4) - ((I/10)*b^2*c^3*d^3)/x^3 + (61*b^2*c^4*d^3)/(180*
x^2) + (((37*I)/30)*b^2*c^5*d^3)/x + ((37*I)/30)*b^2*c^6*d^3*ArcTan[c*x] -
(b*c*d^3*(a + b*ArcTan[c*x]))/(15*x^5) - (((3*I)/10)*b*c^2*d^3*(a + b*ArcTa
n[c*x]))/x^4 + (11*b*c^3*d^3*(a + b*ArcTan[c*x]))/(18*x^3) + (((14*I)/15)*b
*c^4*d^3*(a + b*ArcTan[c*x]))/x^2 - (11*b*c^5*d^3*(a + b*ArcTan[c*x]))/(6*x
) - (d^3*(a + b*ArcTan[c*x])^2)/(6*x^6) - (((3*I)/5)*c*d^3*(a + b*ArcTan[c*
x])^2)/x^5 + (3*c^2*d^3*(a + b*ArcTan[c*x])^2)/(4*x^4) + ((I/3)*c^3*d^3*(a
```

```
+ b*ArcTan[c*x])^2)/x^3 + ((28*I)/15)*a*b*c^6*d^3*Log[x] + (113*b^2*c^6*d^3
*Log[x])/45 + ((37*I)/20)*b*c^6*d^3*(a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)]
+ (I/60)*b*c^6*d^3*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)] - (113*b^2*c^6*d^
3*Log[1 + c^2*x^2])/90 - (14*b^2*c^6*d^3*PolyLog[2, (-I)*c*x])/15 + (14*b^2
*c^6*d^3*PolyLog[2, I*c*x])/15 + (37*b^2*c^6*d^3*PolyLog[2, 1 - 2/(1 - I*c*
x)])/40 - (b^2*c^6*d^3*PolyLog[2, 1 - 2/(1 + I*c*x)])/120
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 4874

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_.) + (e_
.)*(x_))^(q_), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dis
t[(a + b*ArcTan[c*x])^p, u, x] - Dist[b*c*p, Int[ExpandIntegrand[(a + b*Arc
Tan[c*x])^(p - 1), u/(1 + c^2*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
 q}, x] && IGtQ[p, 1] && EqQ[c^2*d^2 + e^2, 0] && IntegersQ[m, q] && NeQ[m,
 -1] && NeQ[q, -1] && ILtQ[m + q + 1, 0] && LtQ[m*q, 0]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
```

```
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int \frac{(d+icdx)^3\left(a+b\tan^{-1}(cx)\right)^2}{x^7}\,dx = -\frac{d^3\left(a+b\tan^{-1}(cx)\right)^2}{6x^6} - \frac{3icd^3\left(a+b\tan^{-1}(cx)\right)^2}{5x^5} + \frac{3c^2d^3\left(a+b\tan^{-1}(cx)\right)^2}{4x^4} + \cdots$$

$$= -\frac{d^3\left(a+b\tan^{-1}(cx)\right)^2}{6x^6} - \frac{3icd^3\left(a+b\tan^{-1}(cx)\right)^2}{5x^5} + \frac{3c^2d^3\left(a+b\tan^{-1}(cx)\right)^2}{4x^4} + \cdots$$

$$= -\frac{bcd^3\left(a+b\tan^{-1}(cx)\right)}{15x^5} - \frac{3ibc^2d^3\left(a+b\tan^{-1}(cx)\right)}{10x^4} + \frac{11bc^3d^3\left(a+b\tan^{-1}(cx)\right)}{18x^3} \cdots$$

$$= -\frac{ib^2c^3d^3}{10x^3} + \frac{14ib^2c^5d^3}{15x} - \frac{bcd^3\left(a+b\tan^{-1}(cx)\right)}{15x^5} - \frac{3ibc^2d^3\left(a+b\tan^{-1}(cx)\right)}{10x^4} + \cdots$$

$$= -\frac{ib^2c^3d^3}{10x^3} + \frac{37ib^2c^5d^3}{30x} + \frac{14}{15}ib^2c^6d^3\tan^{-1}(cx) - \frac{bcd^3\left(a+b\tan^{-1}(cx)\right)}{15x^5} - \frac{3ibc^2d^3\left(\cdots\right.}{\cdots}$$

$$= -\frac{b^2c^2d^3}{60x^4} - \frac{ib^2c^3d^3}{10x^3} + \frac{61b^2c^4d^3}{180x^2} + \frac{37ib^2c^5d^3}{30x} + \frac{37}{30}ib^2c^6d^3\tan^{-1}(cx) - \frac{bcd^3\left(a+b\cdots\right.}{15x\cdots}$$

Mathematica [A] time = 1.46127, size = 401, normalized size = 0.78

$$\frac{d^3\left(168b^2c^6x^6\text{PolyLog}\left(2,e^{2i\tan^{-1}(cx)}\right)+60ia^2c^3x^3+135a^2c^2x^2-108ia^2cx-30a^2-330abc^5x^5+168iabc^4x^4+110abc^3x\cdots\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + I*c*d*x)^3*(a + b*ArcTan[c*x])^2)/x^7,x]
```

```
[Out] (d^3*(-30*a^2 - (108*I)*a^2*c*x - 12*a*b*c*x + 135*a^2*c^2*x^2 - (54*I)*a*b
*c^2*x^2 - 3*b^2*c^2*x^2 + (60*I)*a^2*c^3*x^3 + 110*a*b*c^3*x^3 - (18*I)*b^
2*c^3*x^3 + (168*I)*a*b*c^4*x^4 + 61*b^2*c^4*x^4 - 330*a*b*c^5*x^5 + (222*I
)*b^2*c^5*x^5 + 64*b^2*c^6*x^6 + 3*b^2*(-I + c*x)^4*(-10 + (4*I)*c*x + c^2*
x^2)*ArcTan[c*x]^2 + 2*b*ArcTan[c*x]*(b*c*x*(-6 - (27*I)*c*x + 55*c^2*x^2 +
 (84*I)*c^3*x^3 - 165*c^4*x^4 + (111*I)*c^5*x^5) - 3*a*(10 + (36*I)*c*x - 4
5*c^2*x^2 - (20*I)*c^3*x^3 + 55*c^6*x^6) + (168*I)*b*c^6*x^6*Log[1 - E^((2*
I)*ArcTan[c*x])]) + (336*I)*a*b*c^6*x^6*Log[c*x] + 452*b^2*c^6*x^6*Log[(c*x
)/Sqrt[1 + c^2*x^2]] - (168*I)*a*b*c^6*x^6*Log[1 + c^2*x^2] + 168*b^2*c^6*x
^6*PolyLog[2, E^((2*I)*ArcTan[c*x])]))/(180*x^6)
```

Maple [A] time = 0.115, size = 853, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x^7,x)
```

```
[Out] 37/30*I*b^2*c^6*d^3*arctan(c*x)-3/5*I*c*d^3*a^2/x^5+1/3*I*c^3*d^3*a^2/x^3-1
/3*d^3*a*b*arctan(c*x)/x^6+11/18*c^3*d^3*b^2*arctan(c*x)/x^3-11/6*c^5*d^3*b
^2*arctan(c*x)/x-14/15*c^6*d^3*b^2*ln(c*x)*ln(1+I*c*x)+14/15*c^6*d^3*b^2*ln
(c*x)*ln(1-I*c*x)+7/15*c^6*d^3*b^2*ln(c*x-I)*ln(c^2*x^2+1)-7/15*c^6*d^3*b^2
*ln(c*x+I)*ln(c^2*x^2+1)-1/15*c*d^3*a*b/x^5+11/18*c^3*d^3*a*b/x^3-11/6*c^5*
d^3*a*b/x-7/15*c^6*d^3*b^2*ln(c*x-I)*ln(-1/2*I*(c*x+I))-11/6*c^6*d^3*a*b*ar
ctan(c*x)-1/15*c*d^3*b^2*arctan(c*x)/x^5+3/4*c^2*d^3*b^2*arctan(c*x)^2/x^4+
7/15*c^6*d^3*b^2*ln(c*x+I)*ln(1/2*I*(c*x-I))-1/6*d^3*a^2/x^6-1/10*I*b^2*c^3
*d^3/x^3-3/5*I*c*d^3*b^2*arctan(c*x)^2/x^5+1/3*I*c^3*d^3*b^2*arctan(c*x)^2/
x^3-3/10*I*c^2*d^3*b^2*arctan(c*x)/x^4+14/15*I*c^4*d^3*a*b/x^2+28/15*I*c^6*
d^3*b^2*arctan(c*x)*ln(c*x)-14/15*I*c^6*d^3*b^2*arctan(c*x)*ln(c^2*x^2+1)+3
/2*c^2*d^3*a*b*arctan(c*x)/x^4-14/15*I*c^6*d^3*a*b*ln(c^2*x^2+1)+28/15*I*c^
6*d^3*a*b*ln(c*x)-3/10*I*c^2*d^3*a*b/x^4+14/15*I*c^4*d^3*b^2*arctan(c*x)/x^
2-1/6*d^3*b^2*arctan(c*x)^2/x^6+14/15*c^6*d^3*b^2*dilog(1-I*c*x)+7/15*c^6*d
^3*b^2*dilog(1/2*I*(c*x-I))-7/15*c^6*d^3*b^2*dilog(-1/2*I*(c*x+I))+113/45*c
^6*d^3*b^2*ln(c*x)+7/30*c^6*d^3*b^2*ln(c*x+I)^2-7/30*c^6*d^3*b^2*ln(c*x-I)^
2-11/12*c^6*d^3*b^2*arctan(c*x)^2-14/15*c^6*d^3*b^2*dilog(1+I*c*x)+3/4*c^2*
d^3*a^2/x^4-6/5*I*c*d^3*a*b*arctan(c*x)/x^5+2/3*I*c^3*d^3*a*b*arctan(c*x)/x
^3+37/30*I*b^2*c^5*d^3/x-1/60*b^2*c^2*d^3/x^4+61/180*b^2*c^4*d^3/x^2-113/90
*b^2*c^6*d^3*ln(c^2*x^2+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x^7,x, algorithm="maxima")
```

```
[Out] -1/3*I*((c^2*log(c^2*x^2 + 1) - c^2*log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^3
)*a*b*c^3*d^3 - 1/2*((3*c^3*arctan(c*x) + (3*c^2*x^2 - 1)/x^3)*c - 3*arctan
(c*x)/x^4)*a*b*c^2*d^3 - 3/10*I*((2*c^4*log(c^2*x^2 + 1) - 2*c^4*log(x^2) -
 (2*c^2*x^2 - 1)/x^4)*c + 4*arctan(c*x)/x^5)*a*b*c*d^3 - 1/45*((15*c^5*arct
an(c*x) + (15*c^4*x^4 - 5*c^2*x^2 + 3)/x^5)*c + 15*arctan(c*x)/x^6)*a*b*d^3
 - 1/180*(4*(15*c^5*arctan(c*x) + (15*c^4*x^4 - 5*c^2*x^2 + 3)/x^5)*c*arcta
n(c*x) - (30*c^4*x^4*arctan(c*x)^2 - 46*c^4*x^4*log(c^2*x^2 + 1) + 92*c^4*x
^4*log(x) + 16*c^2*x^2 - 3)*c^2/x^4)*b^2*d^3 + 1/3*I*a^2*c^3*d^3/x^3 + 3/4*
a^2*c^2*d^3/x^4 - 3/5*I*a^2*c*d^3/x^5 - 1/6*b^2*d^3*arctan(c*x)^2/x^6 - 1/6
*a^2*d^3/x^6 - 1/960*(960*I*x^5*integrate(1/240*(180*(b^2*c^5*d^3*x^4 - 2*b
^2*c^3*d^3*x^2 - 3*b^2*c*d^3)*arctan(c*x)^2 + 15*(b^2*c^5*d^3*x^4 - 2*b^2*c
^3*d^3*x^2 - 3*b^2*c*d^3)*log(c^2*x^2 + 1)^2 + 2*(65*b^2*c^4*d^3*x^3 - 36*b
^2*c^2*d^3*x)*arctan(c*x) - (20*b^2*c^5*d^3*x^4 - 81*b^2*c^3*d^3*x^2 + 180*
(b^2*c^4*d^3*x^3 + b^2*c^2*d^3*x)*arctan(c*x))*log(c^2*x^2 + 1))/(c^2*x^8 +
 x^6), x) + 960*x^5*integrate(1/240*(540*(b^2*c^4*d^3*x^3 + b^2*c^2*d^3*x)*
arctan(c*x)^2 + 45*(b^2*c^4*d^3*x^3 + b^2*c^2*d^3*x)*log(c^2*x^2 + 1)^2 - 2
*(20*b^2*c^5*d^3*x^4 - 81*b^2*c^3*d^3*x^2)*arctan(c*x) - (65*b^2*c^4*d^3*x^
3 - 36*b^2*c^2*d^3*x - 60*(b^2*c^5*d^3*x^4 - 2*b^2*c^3*d^3*x^2 - 3*b^2*c*d^
3)*arctan(c*x))*log(c^2*x^2 + 1))/(c^2*x^8 + x^6), x) + (-80*I*b^2*c^3*d^3*
x^2 - 180*b^2*c^2*d^3*x + 144*I*b^2*c*d^3)*arctan(c*x)^2 + (80*b^2*c^3*d^3*
x^2 - 180*I*b^2*c^2*d^3*x - 144*b^2*c*d^3)*arctan(c*x)*log(c^2*x^2 + 1) + (
20*I*b^2*c^3*d^3*x^2 + 45*b^2*c^2*d^3*x - 36*I*b^2*c*d^3)*log(c^2*x^2 + 1)^
2)/x^5
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$240\,x^6\mathrm{integral}\left(\frac{-60i\,a^2c^5d^3x^5-180\,a^2c^4d^3x^4+120i\,a^2c^3d^3x^3-120\,a^2c^2d^3x^2+180i\,a^2cd^3x+60\,a^2d^3+\left(60\,abc^5d^3x^5+\left(-180i\,ab+20\,b^2\right)c^4d^3x^4-15\left(8\,ab+3i\,b^2\right)c^3}{60\left(c^2x^9+x^7\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x^7,x, algorithm="fricas")
```

```
[Out] 1/240*(240*x^6*integral(1/60*(-60*I*a^2*c^5*d^3*x^5 - 180*a^2*c^4*d^3*x^4 +
 120*I*a^2*c^3*d^3*x^3 - 120*a^2*c^2*d^3*x^2 + 180*I*a^2*c*d^3*x + 60*a^2*d
^3 + (60*a*b*c^5*d^3*x^5 + (-180*I*a*b + 20*b^2)*c^4*d^3*x^4 - 15*(8*a*b +
3*I*b^2)*c^3*d^3*x^3 + (-120*I*a*b - 36*b^2)*c^2*d^3*x^2 - 10*(18*a*b - I*b
^2)*c*d^3*x + 60*I*a*b*d^3)*log(-(c*x + I)/(c*x - I)))/(c^2*x^9 + x^7), x)
+ (-20*I*b^2*c^3*d^3*x^3 - 45*b^2*c^2*d^3*x^2 + 36*I*b^2*c*d^3*x + 10*b^2*d
^3)*log(-(c*x + I)/(c*x - I))^2)/x^6
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**3*(a+b*atan(c*x))**2/x**7,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(icdx + d)^3(b\arctan(cx) + a)^2}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x^7,x, algorithm="giac")
```

```
[Out] integrate((I*c*d*x + d)^3*(b*arctan(c*x) + a)^2/x^7, x)
```

3.95 $\int \frac{x^3(a+b\tan^{-1}(cx))^2}{d+icdx}\,dx$

Optimal. Leaf size=356

$$\frac{ib\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{c^4d}-\frac{4b^2\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{3c^4d}+\frac{b^2\text{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{2c^4d}+\frac{x^2\left(a+b\tan^{-1}(cx\right.}{2c^2d}$$

```
[Out] -((a*b*x)/(c^3*d)) - ((I/3)*b^2*x)/(c^3*d) + ((I/3)*b^2*ArcTan[c*x])/(c^4*d
) - (b^2*x*ArcTan[c*x])/(c^3*d) + ((I/3)*b*x^2*(a + b*ArcTan[c*x]))/(c^2*d)
 - (5*(a + b*ArcTan[c*x])^2)/(6*c^4*d) + (I*x*(a + b*ArcTan[c*x])^2)/(c^3*d
) + (x^2*(a + b*ArcTan[c*x])^2)/(2*c^2*d) - ((I/3)*x^3*(a + b*ArcTan[c*x])^
2)/(c*d) + (((8*I)/3)*b*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(c^4*d) + (
(a + b*ArcTan[c*x])^2*Log[2/(1 + I*c*x)])/(c^4*d) + (b^2*Log[1 + c^2*x^2])/
(2*c^4*d) - (4*b^2*PolyLog[2, 1 - 2/(1 + I*c*x)])/(3*c^4*d) + (I*b*(a + b*A
rcTan[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)])/(c^4*d) + (b^2*PolyLog[3, 1 - 2/
(1 + I*c*x)])/(2*c^4*d)
```

Rubi [A] time = 0.823159, antiderivative size = 356, normalized size of antiderivative = 1., number of steps used = 26, number of rules used = 14, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.56, Rules used = {4866, 4852, 4916, 321, 203, 4920, 4854, 2402, 2315, 4846, 260, 4884, 4994, 6610}

$$\frac{ib\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{c^4d}-\frac{4b^2\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{3c^4d}+\frac{b^2\text{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{2c^4d}+\frac{x^2\left(a+b\tan^{-1}(cx\right.}{2c^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcTan[c*x])^2)/(d + I*c*d*x),x]
```


```
[Out] -((a*b*x)/(c^3*d)) - ((I/3)*b^2*x)/(c^3*d) + ((I/3)*b^2*ArcTan[c*x])/(c^4*d
) - (b^2*x*ArcTan[c*x])/(c^3*d) + ((I/3)*b*x^2*(a + b*ArcTan[c*x]))/(c^2*d)
 - (5*(a + b*ArcTan[c*x])^2)/(6*c^4*d) + (I*x*(a + b*ArcTan[c*x])^2)/(c^3*d
) + (x^2*(a + b*ArcTan[c*x])^2)/(2*c^2*d) - ((I/3)*x^3*(a + b*ArcTan[c*x])^
2)/(c*d) + (((8*I)/3)*b*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(c^4*d) + (
(a + b*ArcTan[c*x])^2*Log[2/(1 + I*c*x)])/(c^4*d) + (b^2*Log[1 + c^2*x^2])/
(2*c^4*d) - (4*b^2*PolyLog[2, 1 - 2/(1 + I*c*x)])/(3*c^4*d) + (I*b*(a + b*A
rcTan[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)])/(c^4*d) + (b^2*PolyLog[3, 1 - 2/
(1 + I*c*x)])/(2*c^4*d)
```


Rule 4866

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_) + (
e_.)*(x_)), x_Symbol] :> Dist[f/e, Int[(f*x)^(m - 1)*(a + b*ArcTan[c*x])^p,
 x], x] - Dist[(d*f)/e, Int[((f*x)^(m - 1)*(a + b*ArcTan[c*x])^p)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2
, 0] && GtQ[m, 0]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{x^3\left(a+b\tan^{-1}(cx)\right)^2}{d+icdx}\,dx = \frac{i\int \frac{x^2\left(a+b\tan^{-1}(cx)\right)^2}{d+icdx}\,dx}{c} - \frac{i\int x^2\left(a+b\tan^{-1}(cx)\right)^2\,dx}{cd}$$

$$= -\frac{ix^3\left(a+b\tan^{-1}(cx)\right)^2}{3cd} - \frac{\int \frac{x\left(a+b\tan^{-1}(cx)\right)^2}{d+icdx}\,dx}{c^2} + \frac{(2ib)\int \frac{x^3\left(a+b\tan^{-1}(cx)\right)}{1+c^2x^2}\,dx}{3d} + \frac{\int x\left(a+b\,\mathrm{t}\right.}{}$$

$$= \frac{x^2\left(a+b\tan^{-1}(cx)\right)^2}{2c^2d} - \frac{ix^3\left(a+b\tan^{-1}(cx)\right)^2}{3cd} - \frac{i\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{d+icdx}\,dx}{c^3} + \frac{i\int\left(a+b\tan^{-1}(\right.}{c^3d}$$

$$= \frac{ibx^2\left(a+b\tan^{-1}(cx)\right)}{3c^2d} - \frac{\left(a+b\tan^{-1}(cx)\right)^2}{3c^4d} + \frac{ix\left(a+b\tan^{-1}(cx)\right)^2}{c^3d} + \frac{x^2\left(a+b\tan^{-1}(cx\right.}{2c^2d}$$

$$= -\frac{abx}{c^3d} - \frac{ib^2x}{3c^3d} + \frac{ibx^2\left(a+b\tan^{-1}(cx)\right)}{3c^2d} - \frac{5\left(a+b\tan^{-1}(cx)\right)^2}{6c^4d} + \frac{ix\left(a+b\tan^{-1}(cx)\right)^2}{c^3d} +$$

$$= -\frac{abx}{c^3d} - \frac{ib^2x}{3c^3d} + \frac{ib^2\tan^{-1}(cx)}{3c^4d} - \frac{b^2x\tan^{-1}(cx)}{c^3d} + \frac{ibx^2\left(a+b\tan^{-1}(cx)\right)}{3c^2d} - \frac{5\left(a+b\tan^{-1}\right.}{6c^4d}$$

$$= -\frac{abx}{c^3d} - \frac{ib^2x}{3c^3d} + \frac{ib^2\tan^{-1}(cx)}{3c^4d} - \frac{b^2x\tan^{-1}(cx)}{c^3d} + \frac{ibx^2\left(a+b\tan^{-1}(cx)\right)}{3c^2d} - \frac{5\left(a+b\tan^{-1}\right.}{6c^4d}$$

$$= -\frac{abx}{c^3d} - \frac{ib^2x}{3c^3d} + \frac{ib^2\tan^{-1}(cx)}{3c^4d} - \frac{b^2x\tan^{-1}(cx)}{c^3d} + \frac{ibx^2\left(a+b\tan^{-1}(cx)\right)}{3c^2d} - \frac{5\left(a+b\tan^{-1}\right.}{6c^4d}$$

Mathematica [A] time = 1.01068, size = 421, normalized size = 1.18

$$-\frac{iab\left(3\mathrm{PolyLog}\left(2,-e^{2i\tan^{-1}(cx)}\right)-8\log\left(\frac{1}{\sqrt{c^2x^2+1}}\right)+\left(c^2x^2+1\right)\left(2cx\tan^{-1}(cx)+3i\tan^{-1}(cx)-1\right)-3icx+6\tan^{-1}(cx\right.}{3c^4d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(a + b*ArcTan[c*x])^2)/(d + I*c*d*x),x]
```

```
[Out] (I*a^2*x)/(c^3*d) + (a^2*x^2)/(2*c^2*d) - ((I/3)*a^2*x^3)/(c*d) - (I*a^2*Ar
cTan[c*x])/(c^4*d) - (a^2*Log[1 + c^2*x^2])/(2*c^4*d) - ((I/3)*a*b*((-3*I)*
c*x - 8*c*x*ArcTan[c*x] + 6*ArcTan[c*x]^2 + (1 + c^2*x^2)*(-1 + (3*I)*ArcTa
n[c*x] + 2*c*x*ArcTan[c*x]) + (6*I)*ArcTan[c*x]*Log[1 + E^((2*I)*ArcTan[c*x
])] - 8*Log[1/Sqrt[1 + c^2*x^2]] + 3*PolyLog[2, -E^((2*I)*ArcTan[c*x])]))/(
```

```
c^4*d) - ((I/6)*b^2*(2*c*x - (6*I)*c*x*ArcTan[c*x] - 2*(1 + c^2*x^2)*ArcTan
[c*x] + (8*I)*ArcTan[c*x]^2 - 8*c*x*ArcTan[c*x]^2 + (3*I)*(1 + c^2*x^2)*Arc
Tan[c*x]^2 + 2*c*x*(1 + c^2*x^2)*ArcTan[c*x]^2 + 4*ArcTan[c*x]^3 - 16*ArcTa
n[c*x]*Log[1 + E^((2*I)*ArcTan[c*x])] + (6*I)*ArcTan[c*x]^2*Log[1 + E^((2*I
)*ArcTan[c*x])] - (6*I)*Log[1/Sqrt[1 + c^2*x^2]] + (8*I + 6*ArcTan[c*x])*Po
lyLog[2, -E^((2*I)*ArcTan[c*x])] + (3*I)*PolyLog[3, -E^((2*I)*ArcTan[c*x])]
))/(c^4*d)
```

Maple [C] time = 2.46, size = 1331, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctan(c*x))^2/(d+I*c*d*x),x)
```

```
[Out] -1/2*I/c^4*b^2/d*Pi*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn((1+I*c*x)^2/(c
^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan
(c*x)^2+1/2*I/c^4*b^2/d*Pi*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn((1+I*c*
x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2-1/2*I/c^4*b^2
/d*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)
^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2+1/2/c^2*b^2/d*arctan(c*x)^2*x^2+I/c^3*a^
2/d*x+5/12/c^4*a*b/d*arctan(1/2*c*x)-5/12/c^4*a*b/d*arctan(1/6*c^3*x^3+7/6*
c*x)-5/6/c^4*a*b/d*arctan(1/2*c*x-1/2*I)+11/6/c^4*a*b/d*arctan(c*x)+1/c^4*b
^2/d*arctan(c*x)^2*ln(2*I*(1+I*c*x)^2/(c^2*x^2+1))-1/c^4*b^2/d*arctan(c*x)^
2*ln(c*x-I)-1/3*I/c*a^2/d*x^3-I/c^4*a^2/d*arctan(c*x)-2/3*I/c^4*b^2/d*arcta
n(c*x)^3+4/3*I/c^4*b^2/d*arctan(c*x)+4/3*I/c^4*a*b/d-a*b*x/d/c^3-b^2*x*arct
an(c*x)/d/c^3-1/3*I*b^2*x/d/c^3-I/c^4*b^2/d*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)
/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2+I/c^4*a*b/d*ln(c*x-I)*ln(-1/2
*I*(c*x+I))+2*I/c^3*a*b/d*arctan(c*x)*x+1/3/c^4*b^2/d-2/3*I/c*a*b/d*arctan(
c*x)*x^3-1/2*I/c^4*b^2/d*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*
x^2+1)+1))^3*arctan(c*x)^2+1/2/c^2*a^2/d*x^2-1/2/c^4*a^2/d*ln(c^2*x^2+1)+11
/6/c^4*b^2/d*arctan(c*x)^2-1/c^4*b^2/d*ln((1+I*c*x)^2/(c^2*x^2+1)+1)+1/2/c^
4*b^2/d*polylog(3,-(1+I*c*x)^2/(c^2*x^2+1))+8/3/c^4*b^2/d*dilog(1+I*(1+I*c*
x)/(c^2*x^2+1)^(1/2))+8/3/c^4*b^2/d*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+
I/c^4*b^2/d*Pi*arctan(c*x)^2+I/c^4*a*b/d*dilog(-1/2*I*(c*x+I))-2/c^4*a*b/d*
arctan(c*x)*ln(c*x-I)+I/c^3*b^2/d*arctan(c*x)^2*x+1/c^2*a*b/d*arctan(c*x)*x
^2-11/12*I/c^4*a*b/d*ln(c^2*x^2+1)-1/2*I/c^4*a*b/d*ln(c*x-I)^2-5/24*I/c^4*a
*b/d*ln(c^4*x^4+10*c^2*x^2+9)+8/3*I/c^4*b^2/d*arctan(c*x)*ln(1-I*(1+I*c*x)/
(c^2*x^2+1)^(1/2))-I/c^4*b^2/d*arctan(c*x)*polylog(2,-(1+I*c*x)^2/(c^2*x^2+
1))+8/3*I/c^4*b^2/d*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+1/3*I/c
^2*a*b/d*x^2-1/3*I/c*b^2/d*arctan(c*x)^2*x^3+1/3*I/c^2*b^2/d*arctan(c*x)*x^
```

2

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))^2/(d+I*c*d*x),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{i\,b^2x^3\log\left(-\frac{cx+i}{cx-i}\right)^2+4\,abx^3\log\left(-\frac{cx+i}{cx-i}\right)-4i\,a^2x^3}{4\,cdx-4i\,d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))^2/(d+I*c*d*x),x, algorithm="fricas")
```

```
[Out] integral((I*b^2*x^3*log(-(c*x + I)/(c*x - I))^2 + 4*a*b*x^3*log(-(c*x + I)/
(c*x - I)) - 4*I*a^2*x^3)/(4*c*d*x - 4*I*d), x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atan(c*x))**2/(d+I*c*d*x),x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\arctan(cx) + a)^2 x^3}{icdx + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))^2/(d+I*c*d*x),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)^2*x^3/(I*c*d*x + d), x)
```

3.96 $\int \frac{x^2\left(a+b\tan^{-1}(cx)\right)^2}{d+icdx}\,dx$

Optimal. Leaf size=277

$$\frac{b\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{c^3d}+\frac{ib^2\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c^3d}-\frac{ib^2\,\mathrm{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{2c^3d}+\frac{iabx}{c^2d}+\frac{x\left(a+b\,\mathrm{t}\right.}{c}$$

```
[Out] (I*a*b*x)/(c^2*d) + (I*b^2*x*ArcTan[c*x])/(c^2*d) + ((I/2)*(a + b*ArcTan[c*
x])^2)/(c^3*d) + (x*(a + b*ArcTan[c*x])^2)/(c^2*d) - ((I/2)*x^2*(a + b*ArcT
an[c*x])^2)/(c*d) + (2*b*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(c^3*d) -
(I*(a + b*ArcTan[c*x])^2*Log[2/(1 + I*c*x)])/(c^3*d) - ((I/2)*b^2*Log[1 + c
^2*x^2])/(c^3*d) + (I*b^2*PolyLog[2, 1 - 2/(1 + I*c*x)])/(c^3*d) + (b*(a +
b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)])/(c^3*d) - ((I/2)*b^2*PolyLog[
3, 1 - 2/(1 + I*c*x)])/(c^3*d)
```

Rubi [A] time = 0.513029, antiderivative size = 277, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.48, Rules used = {4866, 4852, 4916, 4846, 260, 4884, 4920, 4854, 2402, 2315, 4994, 6610}

$$\frac{b\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{c^3d}+\frac{ib^2\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c^3d}-\frac{ib^2\,\mathrm{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{2c^3d}+\frac{iabx}{c^2d}+\frac{x\left(a+b\,\mathrm{t}\right.}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcTan[c*x])^2)/(d + I*c*d*x),x]
```


```
[Out] (I*a*b*x)/(c^2*d) + (I*b^2*x*ArcTan[c*x])/(c^2*d) + ((I/2)*(a + b*ArcTan[c*
x])^2)/(c^3*d) + (x*(a + b*ArcTan[c*x])^2)/(c^2*d) - ((I/2)*x^2*(a + b*ArcT
an[c*x])^2)/(c*d) + (2*b*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(c^3*d) -
(I*(a + b*ArcTan[c*x])^2*Log[2/(1 + I*c*x)])/(c^3*d) - ((I/2)*b^2*Log[1 + c
^2*x^2])/(c^3*d) + (I*b^2*PolyLog[2, 1 - 2/(1 + I*c*x)])/(c^3*d) + (b*(a +
b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)])/(c^3*d) - ((I/2)*b^2*PolyLog[
3, 1 - 2/(1 + I*c*x)])/(c^3*d)
```


Rule 4866

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_) + (
e_.)*(x_)), x_Symbol] :> Dist[f/e, Int[(f*x)^(m - 1)*(a + b*ArcTan[c*x])^p,
 x], x] - Dist[(d*f)/e, Int[((f*x)^(m - 1)*(a + b*ArcTan[c*x])^p)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2
```

```
, 0] && GtQ[m, 0]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
```

```
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{x^2\left(a+b\tan^{-1}(cx)\right)^2}{d+icdx}\,dx = \frac{i\int \frac{x\left(a+b\tan^{-1}(cx)\right)^2}{d+icdx}\,dx}{c} - \frac{i\int x\left(a+b\tan^{-1}(cx)\right)^2\,dx}{cd}$$

$$= -\frac{ix^2\left(a+b\tan^{-1}(cx)\right)^2}{2cd} - \frac{\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{d+icdx}\,dx}{c^2} + \frac{(ib)\int \frac{x^2\left(a+b\tan^{-1}(cx)\right)}{1+c^2x^2}\,dx}{d} + \frac{\int\left(a+b\tan^{-1}(\right.}{c^2d}$$

$$= \frac{x\left(a+b\tan^{-1}(cx)\right)^2}{c^2d} - \frac{ix^2\left(a+b\tan^{-1}(cx)\right)^2}{2cd} - \frac{i\left(a+b\tan^{-1}(cx)\right)^2\log\left(\frac{2}{1+icx}\right)}{c^3d} + \frac{(ib)\int\left(a\right.}{}$$

$$= \frac{iabx}{c^2d} + \frac{i\left(a+b\tan^{-1}(cx)\right)^2}{2c^3d} + \frac{x\left(a+b\tan^{-1}(cx)\right)^2}{c^2d} - \frac{ix^2\left(a+b\tan^{-1}(cx)\right)^2}{2cd} - \frac{i\left(a+b\tan\right.}{}$$

$$= \frac{iabx}{c^2d} + \frac{ib^2x\tan^{-1}(cx)}{c^2d} + \frac{i\left(a+b\tan^{-1}(cx)\right)^2}{2c^3d} + \frac{x\left(a+b\tan^{-1}(cx)\right)^2}{c^2d} - \frac{ix^2\left(a+b\tan^{-1}(cx\right.}{2cd}$$

$$= \frac{iabx}{c^2d} + \frac{ib^2x\tan^{-1}(cx)}{c^2d} + \frac{i\left(a+b\tan^{-1}(cx)\right)^2}{2c^3d} + \frac{x\left(a+b\tan^{-1}(cx)\right)^2}{c^2d} - \frac{ix^2\left(a+b\tan^{-1}(cx\right.}{2cd}$$

$$= \frac{iabx}{c^2d} + \frac{ib^2x\tan^{-1}(cx)}{c^2d} + \frac{i\left(a+b\tan^{-1}(cx)\right)^2}{2c^3d} + \frac{x\left(a+b\tan^{-1}(cx)\right)^2}{c^2d} - \frac{ix^2\left(a+b\tan^{-1}(cx\right.}{2cd}$$

Mathematica [A] time = 0.554076, size = 330, normalized size = 1.19

$$-\frac{i\left(6b\text{PolyLog}\left(2,-e^{2i\tan^{-1}(cx)}\right)\left(-ia-ib\tan^{-1}(cx)+b\right)+3b^2\text{PolyLog}\left(3,-e^{2i\tan^{-1}(cx)}\right)+3a^2c^2x^2-3a^2\log\left(c^2x^2+1\right)\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(a + b*ArcTan[c*x])^2)/(d + I*c*d*x),x]
```

```
[Out] ((-I/6)*((6*I)*a^2*c*x - 6*a*b*c*x + 3*a^2*c^2*x^2 - (6*I)*a^2*ArcTan[c*x]
+ 6*a*b*ArcTan[c*x] + (12*I)*a*b*c*x*ArcTan[c*x] - 6*b^2*c*x*ArcTan[c*x] +
6*a*b*c^2*x^2*ArcTan[c*x] - (12*I)*a*b*ArcTan[c*x]^2 + 9*b^2*ArcTan[c*x]^2
+ (6*I)*b^2*c*x*ArcTan[c*x]^2 + 3*b^2*c^2*x^2*ArcTan[c*x]^2 - (4*I)*b^2*Arc
Tan[c*x]^3 + 12*a*b*ArcTan[c*x]*Log[1 + E^((2*I)*ArcTan[c*x])] + (12*I)*b^2
*ArcTan[c*x]*Log[1 + E^((2*I)*ArcTan[c*x])] + 6*b^2*ArcTan[c*x]^2*Log[1 + E
^((2*I)*ArcTan[c*x])] - 3*a^2*Log[1 + c^2*x^2] - (6*I)*a*b*Log[1 + c^2*x^2]
 + 3*b^2*Log[1 + c^2*x^2] + 6*b*((-I)*a + b - I*b*ArcTan[c*x])*PolyLog[2, -
E^((2*I)*ArcTan[c*x])] + 3*b^2*PolyLog[3, -E^((2*I)*ArcTan[c*x])]))/(c^3*d)
```

Maple [C] time = 1.643, size = 1212, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctan(c*x))^2/(d+I*c*d*x),x)
```

```
[Out] 1/c^3*a*b/d-1/2/c^3*a*b/d*ln(c*x-I)^2-1/8/c^3*a*b/d*ln(c^4*x^4+10*c^2*x^2+9
)-3/4/c^3*a*b/d*ln(c^2*x^2+1)+1/c^2*b^2/d*arctan(c*x)^2*x+I/c^3*b^2/d*ln((1
+I*c*x)^2/(c^2*x^2+1)+1)+1/c^3*b^2/d*Pi*arctan(c*x)^2-1/c^3*b^2/d*arctan(c*
x)*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))+2/c^3*b^2/d*arctan(c*x)*ln(1+I*(1+I*
c*x)/(c^2*x^2+1)^(1/2))+2/c^3*b^2/d*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1
)^(1/2))+1/c^3*a*b/d*dilog(-1/2*I*(c*x+I))+1/2*I/c^3*a^2/d*ln(c^2*x^2+1)-1/
2*I/c*a^2/d*x^2-2*I/c^3*b^2/d*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-2*I/c^
3*b^2/d*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-1/2*I/c^3*b^2/d*polylog(3,-(
1+I*c*x)^2/(c^2*x^2+1))-3/2*I/c^3*b^2/d*arctan(c*x)^2-1/2/c^3*b^2/d*Pi*csgn
(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x
)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)^2+1/c^2*a^2/d*x-1/
c^3*a^2/d*arctan(c*x)+1/c^3*b^2/d*arctan(c*x)-2/3/c^3*b^2/d*arctan(c*x)^3+1
/2/c^3*b^2/d*Pi*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn((1+I*c*x)^2/(c^2*x
^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2-1/2/c^3*b^2/d*Pi*csgn((1
+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1
)+1))^2*arctan(c*x)^2+2*I/c^3*a*b/d*arctan(c*x)*ln(c*x-I)-I/c*a*b/d*arctan(
c*x)*x^2+I*a*b*x/c^2/d+I*b^2*x*arctan(c*x)/c^2/d+2/c^2*a*b/d*arctan(c*x)*x-
1/2/c^3*b^2/d*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^
3*arctan(c*x)^2-1/c^3*b^2/d*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c
^2*x^2+1)+1))^2*arctan(c*x)^2+I/c^3*b^2/d*arctan(c*x)^2*ln(c*x-I)+1/c^3*a*b
/d*ln(c*x-I)*ln(-1/2*I*(c*x+I))-1/2*I/c*b^2/d*arctan(c*x)^2*x^2+1/2*I/c^3*a
*b/d*arctan(1/2*c*x-1/2*I)-3/2*I/c^3*a*b/d*arctan(c*x)-I/c^3*b^2/d*arctan(c
*x)^2*ln(2*I*(1+I*c*x)^2/(c^2*x^2+1))-1/4*I/c^3*a*b/d*arctan(1/2*c*x)+1/4*I
/c^3*a*b/d*arctan(1/6*c^3*x^3+7/6*c*x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))^2/(d+I*c*d*x),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{i\,b^2x^2\log\left(-\frac{cx+i}{cx-i}\right)^2+4\,abx^2\log\left(-\frac{cx+i}{cx-i}\right)-4i\,a^2x^2}{4\,cdx-4i\,d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))^2/(d+I*c*d*x),x, algorithm="fricas")
```

```
[Out] integral((I*b^2*x^2*log(-(c*x + I)/(c*x - I))^2 + 4*a*b*x^2*log(-(c*x + I)/
(c*x - I)) - 4*I*a^2*x^2)/(4*c*d*x - 4*I*d), x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atan(c*x))**2/(d+I*c*d*x),x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\arctan(cx)+a)^2x^2}{i\,cdx+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))^2/(d+I*c*d*x),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)^2*x^2/(I*c*d*x + d), x)
```

3.97 $\int \frac{x\left(a+b\tan^{-1}(cx)\right)^2}{d+icdx}\,dx$

Optimal. Leaf size=192

$$-\frac{ib\mathrm{PolyLog}\left(2,1-\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{c^2d}+\frac{b^2\mathrm{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c^2d}-\frac{b^2\mathrm{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{2c^2d}+\frac{\left(a+b\tan^{-1}(cx\right.}{c^2d}$$

```
[Out] (a + b*ArcTan[c*x])^2/(c^2*d) - (I*x*(a + b*ArcTan[c*x])^2)/(c*d) - ((2*I)*
b*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(c^2*d) - ((a + b*ArcTan[c*x])^2*
Log[2/(1 + I*c*x)])/(c^2*d) + (b^2*PolyLog[2, 1 - 2/(1 + I*c*x)])/(c^2*d) -
 (I*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)])/(c^2*d) - (b^2*Pol
yLog[3, 1 - 2/(1 + I*c*x)])/(2*c^2*d)
```

Rubi [A] time = 0.292561, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {4866, 4846, 4920, 4854, 2402, 2315, 4884, 4994, 6610}

$$-\frac{ib\mathrm{PolyLog}\left(2,1-\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{c^2d}+\frac{b^2\mathrm{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c^2d}-\frac{b^2\mathrm{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{2c^2d}+\frac{\left(a+b\tan^{-1}(cx\right.}{c^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcTan[c*x])^2)/(d + I*c*d*x),x]
```

```
[Out] (a + b*ArcTan[c*x])^2/(c^2*d) - (I*x*(a + b*ArcTan[c*x])^2)/(c*d) - ((2*I)*
b*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(c^2*d) - ((a + b*ArcTan[c*x])^2*
Log[2/(1 + I*c*x)])/(c^2*d) + (b^2*PolyLog[2, 1 - 2/(1 + I*c*x)])/(c^2*d) -
 (I*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)])/(c^2*d) - (b^2*Pol
yLog[3, 1 - 2/(1 + I*c*x)])/(2*c^2*d)
```

Rule 4866

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_) + (
e_.)*(x_)), x_Symbol] :> Dist[f/e, Int[(f*x)^(m - 1)*(a + b*ArcTan[c*x])^p,
 x], x] - Dist[(d*f)/e, Int[((f*x)^(m - 1)*(a + b*ArcTan[c*x])^p)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2
, 0] && GtQ[m, 0]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{x\left(a+b\tan^{-1}(cx)\right)^2}{d+icdx}\,dx = \frac{i\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{d+icdx}\,dx}{c} - \frac{i\int \left(a+b\tan^{-1}(cx)\right)^2\,dx}{cd}$$

$$= -\frac{ix\left(a+b\tan^{-1}(cx)\right)^2}{cd} - \frac{\left(a+b\tan^{-1}(cx)\right)^2\log\left(\frac{2}{1+icx}\right)}{c^2d} + \frac{(2ib)\int \frac{x\left(a+b\tan^{-1}(cx)\right)}{1+c^2x^2}\,dx}{d} + \frac{(2b)}{}$$

$$= \frac{\left(a+b\tan^{-1}(cx)\right)^2}{c^2d} - \frac{ix\left(a+b\tan^{-1}(cx)\right)^2}{cd} - \frac{\left(a+b\tan^{-1}(cx)\right)^2\log\left(\frac{2}{1+icx}\right)}{c^2d} - \frac{ib\left(a+b\tan\right.}{}$$

$$= \frac{\left(a+b\tan^{-1}(cx)\right)^2}{c^2d} - \frac{ix\left(a+b\tan^{-1}(cx)\right)^2}{cd} - \frac{2ib\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2}{1+icx}\right)}{c^2d} - \frac{\left(a+b\tan\right.}{}$$

$$= \frac{\left(a+b\tan^{-1}(cx)\right)^2}{c^2d} - \frac{ix\left(a+b\tan^{-1}(cx)\right)^2}{cd} - \frac{2ib\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2}{1+icx}\right)}{c^2d} - \frac{\left(a+b\tan\right.}{}$$

$$= \frac{\left(a+b\tan^{-1}(cx)\right)^2}{c^2d} - \frac{ix\left(a+b\tan^{-1}(cx)\right)^2}{cd} - \frac{2ib\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2}{1+icx}\right)}{c^2d} - \frac{\left(a+b\tan\right.}{}$$

Mathematica [A] time = 0.466956, size = 239, normalized size = 1.24

$$-\frac{i\left(-6b\text{PolyLog}\left(2,-e^{2i\tan^{-1}(cx)}\right)\left(a+b\tan^{-1}(cx)+ib\right)-3ib^2\text{PolyLog}\left(3,-e^{2i\tan^{-1}(cx)}\right)+3ia^2\log\left(c^2x^2+1\right)+6a^2cx\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(a + b*ArcTan[c*x])^2)/(d + I*c*d*x),x]
```

```
[Out] ((-I/6)*(6*a^2*c*x - 6*a^2*ArcTan[c*x] + 12*a*b*c*x*ArcTan[c*x] - 12*a*b*Ar
cTan[c*x]^2 - (6*I)*b^2*ArcTan[c*x]^2 + 6*b^2*c*x*ArcTan[c*x]^2 - 4*b^2*Arc
Tan[c*x]^3 - (12*I)*a*b*ArcTan[c*x]*Log[1 + E^((2*I)*ArcTan[c*x])] + 12*b^2
*ArcTan[c*x]*Log[1 + E^((2*I)*ArcTan[c*x])] - (6*I)*b^2*ArcTan[c*x]^2*Log[1
 + E^((2*I)*ArcTan[c*x])] + (3*I)*a^2*Log[1 + c^2*x^2] - 6*a*b*Log[1 + c^2*
x^2] - 6*b*(a + I*b + b*ArcTan[c*x])*PolyLog[2, -E^((2*I)*ArcTan[c*x])] - (
3*I)*b^2*PolyLog[3, -E^((2*I)*ArcTan[c*x])]))/(c^2*d)
```

Maple [C] time = 0.513, size = 4589, normalized size = 23.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctan(c*x))^2/(d+I*c*d*x),x)
```

```
[Out] -1/2/c^2*a*b/d*arctan(1/12*c^3*x^3+13/12*c*x)-1/2/c^2*a*b/d*arctan(1/4*c*x)
+1/c^2*a*b/d*arctan(1/2*c*x-1/2*I)-1/c^2*b^2/d*arctan(c*x)^2*ln(2*I*(1+I*c*
x)^2/(c^2*x^2+1))+1/c^2*b^2/d*arctan(c*x)^2*ln(c*x-I)-I/c*a^2/d*x+2/3*I/c^2
*b^2/d*arctan(c*x)^3+I/c^2*a^2/d*arctan(c*x)-1/2*I/c^2*b^2/d*Pi*csgn((1+I*c
*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*dilog(1+I*(1+I*c*x)/(c^2*x
^2+1)^(1/2))+1/4*I/c^2*b^2/d*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(
c^2*x^2+1)+1))^3*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))+1/2*I/c^2*b^2/d*Pi*csg
n((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*polylog(2,-(1+I*c*
x)^2/(c^2*x^2+1))-1/2*I/c^2*b^2/d*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x
)^2/(c^2*x^2+1)+1))^3*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+1/2*I/c^2*b^2/
d*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*arctan(c*x
)^2+2/c^2*a*b/d*arctan(c*x)*ln(c*x-I)+1/c^2*b^2/d*Pi*arctan(c*x)*ln((1+I*c*
x)^2/(c^2*x^2+1)+1)-1/c^2*b^2/d*Pi*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)
^(1/2))+I/c^2*b^2/d*Pi*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+I/c^2*b^2/d*P
i*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-1/c^2*b^2/d*Pi*arctan(c*x)*ln(1+I*
(1+I*c*x)/(c^2*x^2+1)^(1/2))+I/c^2*b^2/d*arctan(c*x)*polylog(2,-(1+I*c*x)^2
/(c^2*x^2+1))-I/c^2*b^2/d*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+1
/4*I/c^2*a*b/d*ln(c^8*x^8+12*c^6*x^6+30*c^4*x^4+28*c^2*x^2+9)-I/c^2*b^2/d*P
i*arctan(c*x)^2-1/2*I/c^2*b^2/d*Pi*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))-I/c^
2*b^2/d*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-I/c^2*b^2/d*arctan(
c*x)*ln((1+I*c*x)^2/(c^2*x^2+1)+1)+1/2/c^2*b^2/d*Pi*csgn(I/((1+I*c*x)^2/(c^
2*x^2+1)+1))*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1
+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-1
/2*I/c^2*b^2/d*Pi*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn((1+I*c*x)^2/(c^2
*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*dilog(1+
I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+1/2*I/c^2*b^2/d*Pi*csgn(I/((1+I*c*x)^2/(c^2*
x^2+1)+1))*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I
*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)^2-1/2*I/c^2*b^2/d*Pi*csgn(I/((1+I*c*x)^
2/(c^2*x^2+1)+1))*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1
)/((1+I*c*x)^2/(c^2*x^2+1)+1))*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+1/4*I
/c^2*b^2/d*Pi*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn((1+I*c*x)^2/(c^2*x^2
+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*polylog(2,-(
1+I*c*x)^2/(c^2*x^2+1))+1/2/c^2*b^2/d*Pi*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1)
)*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(
c^2*x^2+1)+1))*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-1/2/c^2*b^2/
```

```
d*Pi*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn
((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)*ln((1+I*c
*x)^2/(c^2*x^2+1)+1)-I/c*b^2/d*arctan(c*x)^2*x-I/c^2*a*b/d*dilog(-1/2*I*(c*
x+I))+1/2*I/c^2*a*b/d*ln(c*x-I)^2-1/2/c^2*b^2/d*polylog(3,-(1+I*c*x)^2/(c^2
*x^2+1))-1/c^2*b^2/d*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-1/c^2*b^2/d*dil
og(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+1/2/c^2*a^2/d*ln(c^2*x^2+1)-1/2/c^2*b^2
/d*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))-1/c^2*b^2/d*arctan(c*x)^2+1/2/c^2*b^
2/d*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*arctan(c
*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+1/c^2*b^2/d*Pi*csgn((1+I*c*x)^2/(c^
2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x
^2+1)^(1/2))-1/c^2*b^2/d*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*
x^2+1)+1))^2*arctan(c*x)*ln((1+I*c*x)^2/(c^2*x^2+1)+1)+1/c^2*b^2/d*Pi*csgn(
(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)*ln(1+I*(
1+I*c*x)/(c^2*x^2+1)^(1/2))-I/c^2*b^2/d*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1
+I*c*x)^2/(c^2*x^2+1)+1))^2*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-I/c^2*b^
2/d*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*dilog(1+
I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-2*I/c*a*b/d*arctan(c*x)*x-I/c^2*a*b/d*ln(c*x
-I)*ln(-1/2*I*(c*x+I))-1/2*I/c^2*b^2/d*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1))*csg
n((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*dilog(1+I*(1+I*c*x
)/(c^2*x^2+1)^(1/2))+1/2*I/c^2*b^2/d*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn(
(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2+1/2*I/
c^2*b^2/d*Pi*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn((1+I*c*x)^2/(c^2*x^2+
1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-1/
2*I/c^2*b^2/d*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)
/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-1/4*
I/c^2*b^2/d*Pi*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn((1+I*c*x)^2/(c^2*x^
2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))+1/2
*I/c^2*b^2/d*Pi*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn((1+I*c*x)^2/(c^2*x
^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))
+1/2/c^2*b^2/d*Pi*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn((1+I*c*x)^2/(c^2
*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)*ln((1+I*c*x)^2/(c^2*x^2+
1)+1)-1/2/c^2*b^2/d*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*
x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)*ln((1+I*c*x)^2/(c^2*x^2+1
)+1)-1/2/c^2*b^2/d*Pi*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn((1+I*c*x)^2/
(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^
2*x^2+1)^(1/2))-1/2/c^2*b^2/d*Pi*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn((
1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)*ln(1+I*(1
+I*c*x)/(c^2*x^2+1)^(1/2))+1/2/c^2*b^2/d*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1))*c
sgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)*ln(1
-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+1/2/c^2*b^2/d*Pi*csgn((1+I*c*x)^2/(c^2*x^2+
1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)
*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-1/2*I/c^2*b^2/d*Pi*csgn(I/((1+I*c*x)^2
/(c^2*x^2+1)+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^
2*arctan(c*x)^2+1/4*I/c^2*b^2/d*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*
c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*polylog(2,-(1+I*c*x)^2/(c
```

```
^2*x^2+1))+I/c^2*b^2/d*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^
2+1)+1))^2*arctan(c*x)^2+1/2/c^2*b^2/d*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+
I*c*x)^2/(c^2*x^2+1)+1))^3*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-
1/2/c^2*b^2/d*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^
3*arctan(c*x)*ln((1+I*c*x)^2/(c^2*x^2+1)+1)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))^2/(d+I*c*d*x),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{i\,b^2x\log\left(-\frac{cx+i}{cx-i}\right)^2 + 4\,abx\log\left(-\frac{cx+i}{cx-i}\right) - 4i\,a^2x}{4\,cdx - 4i\,d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))^2/(d+I*c*d*x),x, algorithm="fricas")
```

```
[Out] integral((I*b^2*x*log(-(c*x + I)/(c*x - I))^2 + 4*a*b*x*log(-(c*x + I)/(c*x
 - I)) - 4*I*a^2*x)/(4*c*d*x - 4*I*d), x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atan(c*x))**2/(d+I*c*d*x),x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \arctan(cx) + a)^2 x}{i\, cdx + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))^2/(d+I*c*d*x),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)^2*x/(I*c*d*x + d), x)
```

3.98 $\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{d+icdx}\,dx$

Optimal. Leaf size=98

$$-\frac{b\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{cd}+\frac{ib^2\text{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{2cd}+\frac{i\log\left(\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)^2}{cd}$$

```
[Out] (I*(a + b*ArcTan[c*x])^2*Log[2/(1 + I*c*x)])/(c*d) - (b*(a + b*ArcTan[c*x])
*PolyLog[2, 1 - 2/(1 + I*c*x)])/(c*d) + ((I/2)*b^2*PolyLog[3, 1 - 2/(1 + I*
c*x)])/(c*d)
```

Rubi [A] time = 0.133452, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4854, 4884, 4994, 6610}

$$-\frac{b\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{cd}+\frac{ib^2\text{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{2cd}+\frac{i\log\left(\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)^2}{cd}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^2/(d + I*c*d*x),x]
```

```
[Out] (I*(a + b*ArcTan[c*x])^2*Log[2/(1 + I*c*x)])/(c*d) - (b*(a + b*ArcTan[c*x])
*PolyLog[2, 1 - 2/(1 + I*c*x)])/(c*d) + ((I/2)*b^2*PolyLog[3, 1 - 2/(1 + I*
c*x)])/(c*d)
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{d+icdx}\,dx &= \frac{i\left(a+b\tan^{-1}(cx)\right)^2\log\left(\frac{2}{1+icx}\right)}{cd}-\frac{(2ib)\int \frac{\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx}{d}\\ &= \frac{i\left(a+b\tan^{-1}(cx)\right)^2\log\left(\frac{2}{1+icx}\right)}{cd}-\frac{b\left(a+b\tan^{-1}(cx)\right)\mathrm{Li}_2\left(1-\frac{2}{1+icx}\right)}{cd}+\frac{b^2\int \frac{\mathrm{Li}_2\left(1-\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx}{d}\\ &= \frac{i\left(a+b\tan^{-1}(cx)\right)^2\log\left(\frac{2}{1+icx}\right)}{cd}-\frac{b\left(a+b\tan^{-1}(cx)\right)\mathrm{Li}_2\left(1-\frac{2}{1+icx}\right)}{cd}+\frac{ib^2\mathrm{Li}_3\left(1-\frac{2}{1+icx}\right)}{2cd}\end{aligned}$$

Mathematica [A] time = 0.0337886, size = 95, normalized size = 0.97

$$\frac{i\left(2ib\text{PolyLog}\left(2,\frac{cx+i}{cx-i}\right)\left(a+b\tan^{-1}(cx)\right)+b^2\text{PolyLog}\left(3,\frac{cx+i}{cx-i}\right)+2\log\left(\frac{2d}{d+icdx}\right)\left(a+b\tan^{-1}(cx)\right)^2\right)}{2cd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])^2/(d + I*c*d*x),x]
```

```
[Out] ((I/2)*(2*(a + b*ArcTan[c*x])^2*Log[(2*d)/(d + I*c*d*x)] + (2*I)*b*(a + b*A
rcTan[c*x])*PolyLog[2, (I + c*x)/(-I + c*x)] + b^2*PolyLog[3, (I + c*x)/(-I
 + c*x)]))/(c*d)
```

Maple [C] time = 0.305, size = 1062, normalized size = 10.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^2/(d+I*c*d*x),x)
```

```
[Out] -I/c*b^2/d*ln(1+I*c*x)*arctan(c*x)^2+1/c*a^2/d*arctan(c*x)-1/2*I/c*a^2/d*ln
(c^2*x^2+1)+I/c*b^2/d*arctan(c*x)^2*ln(2*I*(1+I*c*x)^2/(c^2*x^2+1))-1/2/c*b
^2/d*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*arctan(
c*x)^2-1/2/c*b^2/d*Pi*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn((1+I*c*x)^2/
(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2+1/2/c*b^2/d*Pi*csg
n((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x
^2+1)+1))^2*arctan(c*x)^2+1/2/c*b^2/d*Pi*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1)
)*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(
c^2*x^2+1)+1))*arctan(c*x)^2+1/2/c*b^2/d*arctan(c*x)^2*csgn((1+I*c*x)^2/(c^
2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*
c*x)^2/(c^2*x^2+1)+1))^2*Pi+1/2/c*b^2/d*arctan(c*x)^2*csgn(I*(1+I*c*x)^2/(c
^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*Pi-1/2/c*b^2/d*arctan(c*x)^2*csgn(
(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*(1+I*c*x)^2/(c^
2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*Pi-1/2/c*b^2/d*arctan(c*x)^2*csgn(I*(
1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*Pi+1/2/c*b^2/d*Pi*arc
tan(c*x)^2+1/c*b^2/d*arctan(c*x)*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))+1/2*I/
c*b^2/d*polylog(3,-(1+I*c*x)^2/(c^2*x^2+1))+2/3/c*b^2/d*arctan(c*x)^3-2*I/c
*a*b/d*ln(1+I*c*x)*arctan(c*x)-1/c*a*b/d*ln(1/2-1/2*I*c*x)*ln(1+I*c*x)+1/c*
a*b/d*ln(1/2-1/2*I*c*x)*ln(1/2*I*c*x+1/2)+1/c*a*b/d*dilog(1/2*I*c*x+1/2)+1/
2/c*a*b/d*ln(1+I*c*x)^2
```

<u>Maxima</u> [F] time = 0., size = 0, normalized size = 0.

$$-\frac{i a^2 \log (i cdx + d)}{cd} + \frac{24 b^2 \arctan (cx)^3 + 6 b^2 \arctan (cx) \log \left(c^2x^2 + 1\right)^2 - 2\left(12 b^2 c \int \frac{x \arctan(cx) \log\left(c^2x^2+1\right)}{c^2dx^2+d}\, dx - \frac{4 b^2 \operatorname{arct}}{c}\right.}{96\, cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/(d+I*c*d*x),x, algorithm="maxima")
```

```
[Out] -I*a^2*log(I*c*d*x + d)/(c*d) + 1/96*(24*b^2*arctan(c*x)^3 + 12*I*b^2*arcta
n(c*x)^2*log(c^2*x^2 + 1) + 6*b^2*arctan(c*x)*log(c^2*x^2 + 1)^2 + 3*I*b^2*
log(c^2*x^2 + 1)^3 - 8*(48*b^2*c*integrate(1/16*x*arctan(c*x)*log(c^2*x^2 +
 1)/(c^2*d*x^2 + d), x) - b^2*arctan(c*x)^3/(c*d) + 12*b^2*integrate(1/16*l
og(c^2*x^2 + 1)^2/(c^2*d*x^2 + d), x) - 12*a*b*arctan(c*x)^2/(c*d))*c*d - 9
6*I*c*d*integrate(1/16*(20*b^2*c*x*arctan(c*x)^2 + 3*b^2*c*x*log(c^2*x^2 +
1)^2 + 32*a*b*c*x*arctan(c*x) + 4*b^2*arctan(c*x)*log(c^2*x^2 + 1))/(c^2*d*
```

```
x^2 + d), x))/(c*d)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{i\,b^2\log\left(-\frac{cx+i}{cx-i}\right)^2+4\,ab\log\left(-\frac{cx+i}{cx-i}\right)-4i\,a^2}{4\,cdx-4i\,d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/(d+I*c*d*x),x, algorithm="fricas")
```

```
[Out] integral((I*b^2*log(-(c*x + I)/(c*x - I))^2 + 4*a*b*log(-(c*x + I)/(c*x - I
)) - 4*I*a^2)/(4*c*d*x - 4*I*d), x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**2/(d+I*c*d*x),x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\arctan(cx)+a)^2}{i\,cdx+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/(d+I*c*d*x),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)^2/(I*c*d*x + d), x)
```

3.99 $\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x(d+icdx)}\,dx$

Optimal. Leaf size=88

$$\frac{ib\text{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{d}+\frac{b^2\text{PolyLog}\left(3,-1+\frac{2}{1+icx}\right)}{2d}+\frac{\log\left(2-\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)^2}{d}$$

```
[Out] ((a + b*ArcTan[c*x])^2*Log[2 - 2/(1 + I*c*x)])/d + (I*b*(a + b*ArcTan[c*x])
*PolyLog[2, -1 + 2/(1 + I*c*x)])/d + (b^2*PolyLog[3, -1 + 2/(1 + I*c*x)])/(
2*d)
```

Rubi [A] time = 0.160682, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {4868, 4884, 4994, 6610}

$$\frac{ib\text{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{d}+\frac{b^2\text{PolyLog}\left(3,-1+\frac{2}{1+icx}\right)}{2d}+\frac{\log\left(2-\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)^2}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^2/(x*(d + I*c*d*x)),x]
```

```
[Out] ((a + b*ArcTan[c*x])^2*Log[2 - 2/(1 + I*c*x)])/d + (I*b*(a + b*ArcTan[c*x])
*PolyLog[2, -1 + 2/(1 + I*c*x)])/d + (b^2*PolyLog[3, -1 + 2/(1 + I*c*x)])/(
2*d)
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x(d+icdx)}\,dx=\frac{\left(a+b\tan^{-1}(cx)\right)^2\log\left(2-\frac{2}{1+icx}\right)}{d}-\frac{(2bc)\int \frac{\left(a+b\tan^{-1}(cx)\right)\log\left(2-\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx}{d}$$
$$=\frac{\left(a+b\tan^{-1}(cx)\right)^2\log\left(2-\frac{2}{1+icx}\right)}{d}+\frac{ib\left(a+b\tan^{-1}(cx)\right)\mathrm{Li}_2\left(-1+\frac{2}{1+icx}\right)}{d}-\frac{\left(ib^2c\right)\int \frac{\mathrm{Li}_2\left(-1+\right.}{1+c^2}}{d}$$
$$=\frac{\left(a+b\tan^{-1}(cx)\right)^2\log\left(2-\frac{2}{1+icx}\right)}{d}+\frac{ib\left(a+b\tan^{-1}(cx)\right)\mathrm{Li}_2\left(-1+\frac{2}{1+icx}\right)}{d}+\frac{b^2\mathrm{Li}_3\left(-1+\frac{2}{1+}\right.}{2d}$$

Mathematica [A] time = 0.113741, size = 113, normalized size = 1.28

$$\frac{2ib\text{PolyLog}\left(2,\frac{cx+i}{-cx+i}\right)\left(a+b\tan^{-1}(cx)\right)+b^2\text{PolyLog}\left(3,\frac{cx+i}{-cx+i}\right)+2\left(\log\left(\frac{2i}{-cx+i}\right)+2\tanh^{-1}\left(\frac{cx+i}{cx-i}\right)\right)\left(a+b\tan^{-1}(cx)\right)^2}{2d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c*x])^2/(x*(d + I*c*d*x)),x]
```

```
[Out] (2*(a + b*ArcTan[c*x])^2*(2*ArcTanh[(I + c*x)/(-I + c*x)] + Log[(2*I)/(I -
c*x)]) + (2*I)*b*(a + b*ArcTan[c*x])*PolyLog[2, (I + c*x)/(I - c*x)] + b^2*
PolyLog[3, (I + c*x)/(I - c*x)])/(2*d)
```

Maple [C] time = 0.349, size = 1741, normalized size = 19.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^2/x/(d+I*c*d*x),x)
```

```
[Out] -I*a^2/d*arctan(c*x)-b^2/d*arctan(c*x)^2*ln((1+I*c*x)^2/(c^2*x^2+1)-1)+b^2/
d*arctan(c*x)^2*ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))+b^2/d*arctan(c*x)^2*ln(c*
x)+b^2/d*arctan(c*x)^2*ln(1-(1+I*c*x)/(c^2*x^2+1)^(1/2))-2/3*I*b^2/d*arctan
(c*x)^3+b^2/d*arctan(c*x)^2*ln(2*I*(1+I*c*x)^2/(c^2*x^2+1))-b^2/d*arctan(c*
x)^2*ln(c*x-I)-1/2*I*b^2/d*Pi*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn((1+I
*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+
1))*arctan(c*x)^2+1/2*I*b^2/d*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I
/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)
^2/(c^2*x^2+1)+1))*arctan(c*x)^2+2*b^2/d*polylog(3,(1+I*c*x)/(c^2*x^2+1)^(1
/2))+2*b^2/d*polylog(3,-(1+I*c*x)/(c^2*x^2+1)^(1/2))+a^2/d*ln(c*x)+1/2*I*b^
2/d*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*ar
ctan(c*x)^2+1/2*I*b^2/d*Pi*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c
^2*x^2+1)+1))^3*arctan(c*x)^2-1/2*I*b^2/d*Pi*csgn(((1+I*c*x)^2/(c^2*x^2+1)-
1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2+I*a*b/d*ln(c*x)*ln(1+I*c*x)
-I*a*b/d*ln(c*x)*ln(1-I*c*x)+I*a*b/d*ln(c*x-I)*ln(-1/2*I*(c*x+I))-1/2*I*b^2
/d*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*arctan(c*
x)^2-I*b^2/d*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2
*arctan(c*x)^2-2*I*b^2/d*arctan(c*x)*polylog(2,-(1+I*c*x)/(c^2*x^2+1)^(1/2)
)+3/2*I*b^2/d*Pi*arctan(c*x)^2-2*I*b^2/d*arctan(c*x)*polylog(2,(1+I*c*x)/(c
^2*x^2+1)^(1/2))+2*a*b/d*arctan(c*x)*ln(c*x)+I*a*b/d*dilog(1+I*c*x)-I*a*b/d
*dilog(1-I*c*x)+I*a*b/d*dilog(-1/2*I*(c*x+I))-1/2*I*a*b/d*ln(c*x-I)^2-1/2*I
*b^2/d*Pi*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+
1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2+1/2*I*b^2/d*Pi*csgn(I*((
1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(((1+I*c*x)^2/(c
^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)^2+1/2*I*b^2/d*Pi*csgn
(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(
c^2*x^2+1)+1))^2*arctan(c*x)^2-1/2*I*b^2/d*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+
1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c
*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2-1/2*I*b^2/d*Pi*csgn(I*((1+I*c*x)^2/(c
^2*x^2+1)-1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1
))^2*arctan(c*x)^2-1/2*I*b^2/d*Pi*arctan(c*x)^2*csgn((1+I*c*x)^2/(c^2*x^2+1
))*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2-1/2*a^2/d*ln
(c^2*x^2+1)-2*a*b/d*arctan(c*x)*ln(c*x-I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-a^2\left(\frac{\log(icx+1)}{d}-\frac{\log(x)}{d}\right)+\frac{-24i\,b^2\arctan(cx)^3+12\,b^2\arctan(cx)^2\log\left(c^2x^2+1\right)-6i\,b^2\arctan(cx)\log\left(c^2x^2+1\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x/(d+I*c*d*x),x, algorithm="maxima")
```

```
[Out] -a^2*(log(I*c*x + 1)/d - log(x)/d) + 1/96*(-24*I*b^2*arctan(c*x)^3 + 12*b^2
*arctan(c*x)^2*log(c^2*x^2 + 1) - 6*I*b^2*arctan(c*x)*log(c^2*x^2 + 1)^2 +
3*b^2*log(c^2*x^2 + 1)^3 - 2*(384*b^2*c^2*integrate(1/16*x^2*arctan(c*x)^2/
(c^2*d*x^3 + d*x), x) + 192*b^2*c*integrate(1/16*x*arctan(c*x)*log(c^2*x^2
+ 1)/(c^2*d*x^3 + d*x), x) + b^2*log(c^2*x^2 + 1)^3/d - 576*b^2*integrate(1
/16*arctan(c*x)^2/(c^2*d*x^3 + d*x), x) - 48*b^2*integrate(1/16*log(c^2*x^2
 + 1)^2/(c^2*d*x^3 + d*x), x) - 1536*a*b*integrate(1/16*arctan(c*x)/(c^2*d*
x^3 + d*x), x))*d - 8*I*(b^2*arctan(c*x)^3/d - 12*b^2*c*integrate(1/16*x*lo
g(c^2*x^2 + 1)^2/(c^2*d*x^3 + d*x), x) + 12*a*b*arctan(c*x)^2/d + 48*b^2*in
tegrate(1/16*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*d*x^3 + d*x), x))*d)/d
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{i\,b^2\log\left(-\frac{cx+i}{cx-i}\right)^2+4\,ab\log\left(-\frac{cx+i}{cx-i}\right)-4i\,a^2}{4\left(cdx^2-i\,dx\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x/(d+I*c*d*x),x, algorithm="fricas")
```

```
[Out] integral(1/4*(I*b^2*log(-(c*x + I)/(c*x - I))^2 + 4*a*b*log(-(c*x + I)/(c*x
 - I)) - 4*I*a^2)/(c*d*x^2 - I*d*x), x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**2/x/(d+I*c*d*x),x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\arctan(cx)+a)^2}{(icdx+d)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x/(d+I*c*d*x),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)^2/((I*c*d*x + d)*x), x)
```

3.100 $\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^2(d+icdx)}\,dx$

Optimal. Leaf size=186

$$\frac{bc\text{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{d}-\frac{ib^2c\text{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)}{d}-\frac{ib^2c\text{PolyLog}\left(3,-1+\frac{2}{1+icx}\right)}{2d}-\frac{ic\left(a+b\right.}{}$$

```
[Out] ((-I)*c*(a + b*ArcTan[c*x])^2)/d - (a + b*ArcTan[c*x])^2/(d*x) + (2*b*c*(a
+ b*ArcTan[c*x])*Log[2 - 2/(1 - I*c*x)])/d - (I*c*(a + b*ArcTan[c*x])^2*Log
[2 - 2/(1 + I*c*x)])/d - (I*b^2*c*PolyLog[2, -1 + 2/(1 - I*c*x)])/d + (b*c*
(a + b*ArcTan[c*x])*PolyLog[2, -1 + 2/(1 + I*c*x)])/d - ((I/2)*b^2*c*PolyLo
g[3, -1 + 2/(1 + I*c*x)])/d
```

Rubi [A] time = 0.398114, antiderivative size = 186, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {4870, 4852, 4924, 4868, 2447, 4884, 4994, 6610}

$$\frac{bc\text{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{d}-\frac{ib^2c\text{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)}{d}-\frac{ib^2c\text{PolyLog}\left(3,-1+\frac{2}{1+icx}\right)}{2d}-\frac{ic\left(a+b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^2/(x^2*(d + I*c*d*x)),x]
```

```
[Out] ((-I)*c*(a + b*ArcTan[c*x])^2)/d - (a + b*ArcTan[c*x])^2/(d*x) + (2*b*c*(a
+ b*ArcTan[c*x])*Log[2 - 2/(1 - I*c*x)])/d - (I*c*(a + b*ArcTan[c*x])^2*Log
[2 - 2/(1 + I*c*x)])/d - (I*b^2*c*PolyLog[2, -1 + 2/(1 - I*c*x)])/d + (b*c*
(a + b*ArcTan[c*x])*PolyLog[2, -1 + 2/(1 + I*c*x)])/d - ((I/2)*b^2*c*PolyLo
g[3, -1 + 2/(1 + I*c*x)])/d
```

Rule 4870

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x], x]
 - Dist[e/(d*f), Int[((f*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d + e*x), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0] &&
 LtQ[m, -1]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^2(d+icdx)}\,dx = -\left((ic)\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x(d+icdx)}\,dx\right)+\frac{\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^2}\,dx}{d}$$

$$= -\frac{\left(a+b\tan^{-1}(cx)\right)^2}{dx}-\frac{ic\left(a+b\tan^{-1}(cx)\right)^2\log\left(2-\frac{2}{1+icx}\right)}{d}+\frac{(2bc)\int \frac{a+b\tan^{-1}(cx)}{x\left(1+c^2x^2\right)}\,dx}{d}+\frac{(2ib\cdots}{}$$

$$= -\frac{ic\left(a+b\tan^{-1}(cx)\right)^2}{d}-\frac{\left(a+b\tan^{-1}(cx)\right)^2}{dx}-\frac{ic\left(a+b\tan^{-1}(cx)\right)^2\log\left(2-\frac{2}{1+icx}\right)}{d}+\frac{bc\left(a\cdots\right.}{}$$

$$= -\frac{ic\left(a+b\tan^{-1}(cx)\right)^2}{d}-\frac{\left(a+b\tan^{-1}(cx)\right)^2}{dx}+\frac{2bc\left(a+b\tan^{-1}(cx)\right)\log\left(2-\frac{2}{1-icx}\right)}{d}-\frac{ic\left(a\cdots\right.}{}$$

$$= -\frac{ic\left(a+b\tan^{-1}(cx)\right)^2}{d}-\frac{\left(a+b\tan^{-1}(cx)\right)^2}{dx}+\frac{2bc\left(a+b\tan^{-1}(cx)\right)\log\left(2-\frac{2}{1-icx}\right)}{d}-\frac{ic\left(a\cdots\right.}{}$$

Mathematica [A] time = 0.859642, size = 265, normalized size = 1.42

$$-\frac{2abc\left(\text{PolyLog}\left(2,e^{2i\tan^{-1}(cx)}\right)+2\left(-\log\left(\frac{cx}{\sqrt{c^2x^2+1}}\right)+\tan^{-1}(cx)^2+\tan^{-1}(cx)\left(\frac{1}{cx}+i\log\left(1-e^{2i\tan^{-1}(cx)}\right)\right)\right)\right)+2ib^2c\left(\cdots\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c*x])^2/(x^2*(d + I*c*d*x)),x]
```

```
[Out] -((2*a^2)/x + 2*a^2*c*ArcTan[c*x] + (2*I)*a^2*c*Log[x] - I*a^2*c*Log[1 + c^
2*x^2] + 2*a*b*c*(2*(ArcTan[c*x]^2 + ArcTan[c*x]*(1/(c*x) + I*Log[1 - E^((2
*I)*ArcTan[c*x])])) - Log[(c*x)/Sqrt[1 + c^2*x^2]]) + PolyLog[2, E^((2*I)*Ar
cTan[c*x])]) + (2*I)*b^2*c*((-I/24)*Pi^3 + ArcTan[c*x]^2 - (I*ArcTan[c*x]^2
)/(c*x) + ArcTan[c*x]^2*Log[1 - E^((-2*I)*ArcTan[c*x])] + (2*I)*ArcTan[c*x]
*Log[1 - E^((2*I)*ArcTan[c*x])] + I*ArcTan[c*x]*PolyLog[2, E^((-2*I)*ArcTan
[c*x])] + PolyLog[2, E^((2*I)*ArcTan[c*x])] + PolyLog[3, E^((-2*I)*ArcTan[c
*x])]/2))/(2*d)
```

Maple [C] time = 0.783, size = 9235, normalized size = 49.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^2/x^2/(d+I*c*d*x),x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^2/(d+I*c*d*x),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{i\,b^2\log\left(-\frac{cx+i}{cx-i}\right)^2 + 4\,ab\log\left(-\frac{cx+i}{cx-i}\right) - 4i\,a^2}{4\left(cdx^3 - i\,dx^2\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^2/(d+I*c*d*x),x, algorithm="fricas")
```

```
[Out] integral(1/4*(I*b^2*log(-(c*x + I)/(c*x - I))^2 + 4*a*b*log(-(c*x + I)/(c*x
 - I)) - 4*I*a^2)/(c*d*x^3 - I*d*x^2), x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**2/x**2/(d+I*c*d*x),x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \arctan(cx) + a)^2}{(icdx + d)x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^2/(d+I*c*d*x),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)^2/((I*c*d*x + d)*x^2), x)
```

3.101 $\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^3(d+icdx)}\,dx$

Optimal. Leaf size=273

$$-\frac{ibc^2\text{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{d}-\frac{b^2c^2\text{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)}{d}-\frac{b^2c^2\text{PolyLog}\left(3,-1+\frac{2}{1+icx}\right)}{2d}-\frac{3c^2\left(a\right.}{}$$

```
[Out] -((b*c*(a + b*ArcTan[c*x]))/(d*x)) - (3*c^2*(a + b*ArcTan[c*x])^2)/(2*d) -
(a + b*ArcTan[c*x])^2/(2*d*x^2) + (I*c*(a + b*ArcTan[c*x])^2)/(d*x) + (b^2*
c^2*Log[x])/d - (b^2*c^2*Log[1 + c^2*x^2])/(2*d) - ((2*I)*b*c^2*(a + b*ArcT
an[c*x])*Log[2 - 2/(1 - I*c*x)])/d - (c^2*(a + b*ArcTan[c*x])^2*Log[2 - 2/(
1 + I*c*x)])/d - (b^2*c^2*PolyLog[2, -1 + 2/(1 - I*c*x)])/d - (I*b*c^2*(a +
 b*ArcTan[c*x])*PolyLog[2, -1 + 2/(1 + I*c*x)])/d - (b^2*c^2*PolyLog[3, -1
+ 2/(1 + I*c*x)])/(2*d)
```

Rubi [A] time = 0.624345, antiderivative size = 273, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.52, Rules used = {4870, 4852, 4918, 266, 36, 29, 31, 4884, 4924, 4868, 2447, 4994, 6610}

$$-\frac{ibc^2\text{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{d}-\frac{b^2c^2\text{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)}{d}-\frac{b^2c^2\text{PolyLog}\left(3,-1+\frac{2}{1+icx}\right)}{2d}-\frac{3c^2\left(a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^2/(x^3*(d + I*c*d*x)),x]
```


```
[Out] -((b*c*(a + b*ArcTan[c*x]))/(d*x)) - (3*c^2*(a + b*ArcTan[c*x])^2)/(2*d) -
(a + b*ArcTan[c*x])^2/(2*d*x^2) + (I*c*(a + b*ArcTan[c*x])^2)/(d*x) + (b^2*
c^2*Log[x])/d - (b^2*c^2*Log[1 + c^2*x^2])/(2*d) - ((2*I)*b*c^2*(a + b*ArcT
an[c*x])*Log[2 - 2/(1 - I*c*x)])/d - (c^2*(a + b*ArcTan[c*x])^2*Log[2 - 2/(
1 + I*c*x)])/d - (b^2*c^2*PolyLog[2, -1 + 2/(1 - I*c*x)])/d - (I*b*c^2*(a +
 b*ArcTan[c*x])*PolyLog[2, -1 + 2/(1 + I*c*x)])/d - (b^2*c^2*PolyLog[3, -1
+ 2/(1 + I*c*x)])/(2*d)
```


Rule 4870

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x], x]
 - Dist[e/(d*f), Int[((f*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d + e*x), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0] &&
```

```
LtQ[m, -1]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4918

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/(d + e*x^2),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^3(d+icdx)}\,dx = -\left((ic)\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^2(d+icdx)}\,dx\right) + \frac{\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^3}\,dx}{d}$$

$$= -\frac{\left(a+b\tan^{-1}(cx)\right)^2}{2dx^2} - c^2\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x(d+icdx)}\,dx - \frac{(ic)\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^2}\,dx}{d} + \frac{(bc)\int \frac{a+b\tan}{x^2(1+c}}{d}$$

$$= -\frac{\left(a+b\tan^{-1}(cx)\right)^2}{2dx^2} + \frac{ic\left(a+b\tan^{-1}(cx)\right)^2}{dx} - \frac{c^2\left(a+b\tan^{-1}(cx)\right)^2\log\left(2-\frac{2}{1+icx}\right)}{d} + \frac{(bc)\int}{}$$

$$= -\frac{bc\left(a+b\tan^{-1}(cx)\right)}{dx} - \frac{3c^2\left(a+b\tan^{-1}(cx)\right)^2}{2d} - \frac{\left(a+b\tan^{-1}(cx)\right)^2}{2dx^2} + \frac{ic\left(a+b\tan^{-1}(cx)\right)^2}{dx}$$

$$= -\frac{bc\left(a+b\tan^{-1}(cx)\right)}{dx} - \frac{3c^2\left(a+b\tan^{-1}(cx)\right)^2}{2d} - \frac{\left(a+b\tan^{-1}(cx)\right)^2}{2dx^2} + \frac{ic\left(a+b\tan^{-1}(cx)\right)^2}{dx}$$

$$= -\frac{bc\left(a+b\tan^{-1}(cx)\right)}{dx} - \frac{3c^2\left(a+b\tan^{-1}(cx)\right)^2}{2d} - \frac{\left(a+b\tan^{-1}(cx)\right)^2}{2dx^2} + \frac{ic\left(a+b\tan^{-1}(cx)\right)^2}{dx}$$

$$= -\frac{bc\left(a+b\tan^{-1}(cx)\right)}{dx} - \frac{3c^2\left(a+b\tan^{-1}(cx)\right)^2}{2d} - \frac{\left(a+b\tan^{-1}(cx)\right)^2}{2dx^2} + \frac{ic\left(a+b\tan^{-1}(cx)\right)^2}{dx}$$

Mathematica [A] time = 1.12109, size = 372, normalized size = 1.36

$$\frac{\frac{2iab\left(c^2x^2\operatorname{PolyLog}\left(2,e^{2i\tan^{-1}(cx)}\right)+cx\left(-2cx\log\left(\frac{cx}{\sqrt{c^2x^2+1}}\right)+i\right)+2c^2x^2\tan^{-1}(cx)^2+\tan^{-1}(cx)\left(ic^2x^2+2ic^2x^2\log\left(1-e^{2i\tan^{-1}(cx)}\right)+2cx+i\right)\right)}{x^2} + 2b^2c^2\left(-i\tan\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c*x])^2/(x^3*(d + I*c*d*x)),x]
```

```
[Out] (-(a^2/x^2) + ((2*I)*a^2*c)/x + (2*I)*a^2*c^2*ArcTan[c*x] - 2*a^2*c^2*Log[x
] + a^2*c^2*Log[1 + c^2*x^2] + ((2*I)*a*b*(2*c^2*x^2*ArcTan[c*x]^2 + ArcTan
[c*x]*(I + 2*c*x + I*c^2*x^2 + (2*I)*c^2*x^2*Log[1 - E^((2*I)*ArcTan[c*x])]
) + c*x*(I - 2*c*x*Log[(c*x)/Sqrt[1 + c^2*x^2]]) + c^2*x^2*PolyLog[2, E^((2
*I)*ArcTan[c*x])]))/x^2 + 2*b^2*c^2*((I/24)*Pi^3 - ArcTan[c*x]/(c*x) - (3*A
rcTan[c*x]^2)/2 - ArcTan[c*x]^2/(2*c^2*x^2) + (I*ArcTan[c*x]^2)/(c*x) - Arc
Tan[c*x]^2*Log[1 - E^((-2*I)*ArcTan[c*x])] - (2*I)*ArcTan[c*x]*Log[1 - E^((
2*I)*ArcTan[c*x])] + Log[(c*x)/Sqrt[1 + c^2*x^2]] - I*ArcTan[c*x]*PolyLog[2
, E^((-2*I)*ArcTan[c*x])] - PolyLog[2, E^((2*I)*ArcTan[c*x])] - PolyLog[3,
```

```
E^((-2*I)*ArcTan[c*x])]/2))/(2*d)
```

Maple [C] time = 2.36, size = 2221, normalized size = 8.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^2/x^3/(d+I*c*d*x),x)
```

```
[Out] 2*c^2*b^2/d*dilog((1+I*c*x)/(c^2*x^2+1)^(1/2))+c^2*b^2/d*ln((1+I*c*x)/(c^2*
x^2+1)^(1/2)-1)-1/2*b^2/d*arctan(c*x)^2/x^2-c^2*a^2/d*ln(c*x)+1/2*c^2*a^2/d
*ln(c^2*x^2+1)-2*c^2*b^2/d*polylog(3,(1+I*c*x)/(c^2*x^2+1)^(1/2))-2*c^2*b^2
/d*polylog(3,-(1+I*c*x)/(c^2*x^2+1)^(1/2))-3/2*c^2*b^2/d*arctan(c*x)^2-2*c^
2*b^2/d*dilog(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))+c^2*b^2/d*ln(1+(1+I*c*x)/(c^2*
x^2+1)^(1/2))-1/2*I*c^2*b^2/d*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I
/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)
^2/(c^2*x^2+1)+1))*arctan(c*x)^2+1/2*I*c^2*b^2/d*Pi*csgn(I/((1+I*c*x)^2/(c^
2*x^2+1)+1))*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1
+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)^2-1/2*a^2/d/x^2-1/2*I*c^2*b^2/d*Pi*cs
gn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2
/(c^2*x^2+1)+1))^2*arctan(c*x)^2+1/2*I*c^2*b^2/d*Pi*csgn((1+I*c*x)^2/(c^2*x
^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c
*x)^2+1/2*I*c^2*b^2/d*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I*((1+I*c
*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2-1/2*I*c^2
*b^2/d*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*c
sgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)^2+
1/2*I*c^2*b^2/d*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2
+1)+1))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arc
tan(c*x)^2+1/2*I*c^2*b^2/d*Pi*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*((
1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2+I*c^
2*b^2/d*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arct
an(c*x)^2+I*c^2*a*b/d*ln(c*x)*ln(1-I*c*x)+2*I*c*a*b/d*arctan(c*x)/x-I*c^2*a
*b/d*ln(c*x-I)*ln(-1/2*I*(c*x+I))+1/2*I*c^2*b^2/d*Pi*csgn((1+I*c*x)^2/(c^2*
x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*arctan(c*x)^2-I*c^2*a*b/d*ln(c*x)*ln(
1+I*c*x)-1/2*I*c^2*b^2/d*Pi*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(
c^2*x^2+1)+1))^3*arctan(c*x)^2+1/2*I*c^2*b^2/d*Pi*csgn(((1+I*c*x)^2/(c^2*x^
2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2-1/2*I*c^2*b^2/d*Pi*csg
n(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*arctan(c*x)^
2-c*a*b/d/x+c^2*b^2/d*arctan(c*x)^2*ln((1+I*c*x)^2/(c^2*x^2+1)-1)-c^2*b^2/d
*arctan(c*x)^2*ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))-c^2*b^2/d*arctan(c*x)^2*ln
(c*x)-c^2*b^2/d*arctan(c*x)^2*ln(1-(1+I*c*x)/(c^2*x^2+1)^(1/2))-c^2*b^2/d*a
```

```
rctan(c*x)^2*ln(2*I*(1+I*c*x)^2/(c^2*x^2+1))+c^2*b^2/d*arctan(c*x)^2*ln(c*x
-I)+I*c^2*a^2/d*arctan(c*x)-c^2*a*b/d*arctan(c*x)-c*b^2/d*arctan(c*x)/x+I*c
*a^2/d/x-a*b/d*arctan(c*x)/x^2+2/3*I*c^2*b^2/d*arctan(c*x)^3-I*c^2*b^2/d*ar
ctan(c*x)+I*c^2*a*b/d*dilog(1-I*c*x)+2*c^2*a*b/d*arctan(c*x)*ln(c*x-I)-2*c^
2*a*b/d*arctan(c*x)*ln(c*x)+I*c*b^2/d*arctan(c*x)^2/x+I*c^2*a*b/d*ln(c^2*x^
2+1)+2*I*c^2*b^2/d*arctan(c*x)*polylog(2,(1+I*c*x)/(c^2*x^2+1)^(1/2))+2*I*c
^2*b^2/d*arctan(c*x)*polylog(2,-(1+I*c*x)/(c^2*x^2+1)^(1/2))-2*I*c^2*b^2/d*
arctan(c*x)*ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))-2*I*c^2*a*b/d*ln(c*x)-I*c^2*a
*b/d*dilog(1+I*c*x)-I*c^2*a*b/d*dilog(-1/2*I*(c*x+I))+1/2*I*c^2*a*b/d*ln(c*
x-I)^2-3/2*I*c^2*b^2/d*Pi*arctan(c*x)^2
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^3/(d+I*c*d*x),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{i\,b^2\log\left(-\frac{cx+i}{cx-i}\right)^2 + 4\,ab\log\left(-\frac{cx+i}{cx-i}\right) - 4i\,a^2}{4\left(cdx^4 - i\,dx^3\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^3/(d+I*c*d*x),x, algorithm="fricas")
```

```
[Out] integral(1/4*(I*b^2*log(-(c*x + I)/(c*x - I))^2 + 4*a*b*log(-(c*x + I)/(c*x
 - I)) - 4*I*a^2)/(c*d*x^4 - I*d*x^3), x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**2/x**3/(d+I*c*d*x),x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\arctan{(cx)} + a)^2}{(i\,cdx + d)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^3/(d+I*c*d*x),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)^2/((I*c*d*x + d)*x^3), x)
```

3.102 $\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^4(d+icdx)}\,dx$

Optimal. Leaf size=365

$$-\frac{bc^3\text{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{d}+\frac{4ib^2c^3\text{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)}{3d}+\frac{ib^2c^3\text{PolyLog}\left(3,-1+\frac{2}{1+icx}\right)}{2d}+11i$$

```
[Out] -(b^2*c^2)/(3*d*x) - (b^2*c^3*ArcTan[c*x])/(3*d) - (b*c*(a + b*ArcTan[c*x])
)/(3*d*x^2) + (I*b*c^2*(a + b*ArcTan[c*x]))/(d*x) + (((11*I)/6)*c^3*(a + b*
ArcTan[c*x])^2)/d - (a + b*ArcTan[c*x])^2/(3*d*x^3) + ((I/2)*c*(a + b*ArcTa
n[c*x])^2)/(d*x^2) + (c^2*(a + b*ArcTan[c*x])^2)/(d*x) - (I*b^2*c^3*Log[x])
/d + ((I/2)*b^2*c^3*Log[1 + c^2*x^2])/d - (8*b*c^3*(a + b*ArcTan[c*x])*Log[
2 - 2/(1 - I*c*x)])/(3*d) + (I*c^3*(a + b*ArcTan[c*x])^2*Log[2 - 2/(1 + I*c
*x)])/d + (((4*I)/3)*b^2*c^3*PolyLog[2, -1 + 2/(1 - I*c*x)])/d - (b*c^3*(a
+ b*ArcTan[c*x])*PolyLog[2, -1 + 2/(1 + I*c*x)])/d + ((I/2)*b^2*c^3*PolyLog
[3, -1 + 2/(1 + I*c*x)])/d
```

Rubi [A] time = 0.974899, antiderivative size = 365, normalized size of antiderivative = 1., number of steps used = 26, number of rules used = 15, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {4870, 4852, 4918, 325, 203, 4924, 4868, 2447, 266, 36, 29, 31, 4884, 4994, 6610}

$$-\frac{bc^3\text{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{d}+\frac{4ib^2c^3\text{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)}{3d}+\frac{ib^2c^3\text{PolyLog}\left(3,-1+\frac{2}{1+icx}\right)}{2d}+11i$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^2/(x^4*(d + I*c*d*x)),x]
```

```
[Out] -(b^2*c^2)/(3*d*x) - (b^2*c^3*ArcTan[c*x])/(3*d) - (b*c*(a + b*ArcTan[c*x])
)/(3*d*x^2) + (I*b*c^2*(a + b*ArcTan[c*x]))/(d*x) + (((11*I)/6)*c^3*(a + b*
ArcTan[c*x])^2)/d - (a + b*ArcTan[c*x])^2/(3*d*x^3) + ((I/2)*c*(a + b*ArcTa
n[c*x])^2)/(d*x^2) + (c^2*(a + b*ArcTan[c*x])^2)/(d*x) - (I*b^2*c^3*Log[x])
/d + ((I/2)*b^2*c^3*Log[1 + c^2*x^2])/d - (8*b*c^3*(a + b*ArcTan[c*x])*Log[
2 - 2/(1 - I*c*x)])/(3*d) + (I*c^3*(a + b*ArcTan[c*x])^2*Log[2 - 2/(1 + I*c
*x)])/d + (((4*I)/3)*b^2*c^3*PolyLog[2, -1 + 2/(1 - I*c*x)])/d - (b*c^3*(a
+ b*ArcTan[c*x])*PolyLog[2, -1 + 2/(1 + I*c*x)])/d + ((I/2)*b^2*c^3*PolyLog
[3, -1 + 2/(1 + I*c*x)])/d
```

Rule 4870

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x], x]
 - Dist[e/(d*f), Int[((f*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d + e*x), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0] &&
 LtQ[m, -1]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4918

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/(d + e*x^2),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
```

```
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^4(d+icdx)}\,dx = -\left((ic)\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^3(d+icdx)}\,dx\right)+\frac{\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^4}\,dx}{d}$$

$$= -\frac{\left(a+b\tan^{-1}(cx)\right)^2}{3dx^3}-c^2\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^2(d+icdx)}\,dx-\frac{(ic)\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^3}\,dx}{d}+\frac{(2bc)\int \frac{a+b\tan^{-1}}{x^3(1+c}}{3d}$$

$$= -\frac{\left(a+b\tan^{-1}(cx)\right)^2}{3dx^3}+\frac{ic\left(a+b\tan^{-1}(cx)\right)^2}{2dx^2}+\left(ic^3\right)\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x(d+icdx)}\,dx+\frac{(2bc)\int \frac{a+b\tan}{x}}{3d}$$

$$= -\frac{bc\left(a+b\tan^{-1}(cx)\right)}{3dx^2}+\frac{ic^3\left(a+b\tan^{-1}(cx)\right)^2}{3d}-\frac{\left(a+b\tan^{-1}(cx)\right)^2}{3dx^3}+\frac{ic\left(a+b\tan^{-1}(cx)\right)^2}{2dx^2}$$

$$= -\frac{b^2c^2}{3dx}-\frac{bc\left(a+b\tan^{-1}(cx)\right)}{3dx^2}+\frac{ibc^2\left(a+b\tan^{-1}(cx)\right)}{dx}+\frac{11ic^3\left(a+b\tan^{-1}(cx)\right)^2}{6d}-\frac{\left(a+b\tan\right.}{3d}$$

$$= -\frac{b^2c^2}{3dx}-\frac{b^2c^3\tan^{-1}(cx)}{3d}-\frac{bc\left(a+b\tan^{-1}(cx)\right)}{3dx^2}+\frac{ibc^2\left(a+b\tan^{-1}(cx)\right)}{dx}+\frac{11ic^3\left(a+b\tan^{-1}(\right.}{6d}$$

$$= -\frac{b^2c^2}{3dx}-\frac{b^2c^3\tan^{-1}(cx)}{3d}-\frac{bc\left(a+b\tan^{-1}(cx)\right)}{3dx^2}+\frac{ibc^2\left(a+b\tan^{-1}(cx)\right)}{dx}+\frac{11ic^3\left(a+b\tan^{-1}(\right.}{6d}$$

$$= -\frac{b^2c^2}{3dx}-\frac{b^2c^3\tan^{-1}(cx)}{3d}-\frac{bc\left(a+b\tan^{-1}(cx)\right)}{3dx^2}+\frac{ibc^2\left(a+b\tan^{-1}(cx)\right)}{dx}+\frac{11ic^3\left(a+b\tan^{-1}(\right.}{6d}$$

Mathematica [A] time = 1.1467, size = 535, normalized size = 1.47

$$-\frac{2iabc^3\left(\frac{1}{2}i\left(\tan^{-1}(cx)^2+\text{PolyLog}\left(2,e^{2i\tan^{-1}(cx)}\right)\right)-\frac{i\left(c^2x^2+1\right)}{6c^2x^2}-\frac{4}{3}i\log\left(\frac{cx}{\sqrt{c^2x^2+1}}\right)-\frac{\left(c^2x^2+1\right)\tan^{-1}(cx)}{2c^2x^2}-\frac{i\left(c^2x^2+1\right)\tan^{-1}(cx)}{3c^3x^3}-\right.}{d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c*x])^2/(x^4*(d + I*c*d*x)),x]
```

```
[Out] -a^2/(3*d*x^3) + ((I/2)*a^2*c)/(d*x^2) + (a^2*c^2)/(d*x) + (a^2*c^3*ArcTan[
c*x])/d + (I*a^2*c^3*Log[x])/d - ((I/2)*a^2*c^3*Log[1 + c^2*x^2])/d - ((2*I
)*a*b*c^3*(-1/(2*c*x) - ((I/6)*(1 + c^2*x^2))/(c^2*x^2) + (((4*I)/3)*ArcTan
[c*x])/(c*x) - ((I/3)*(1 + c^2*x^2)*ArcTan[c*x])/(c^3*x^3) - ((1 + c^2*x^2)
*ArcTan[c*x])/(2*c^2*x^2) + (I/2)*ArcTan[c*x]^2 - ArcTan[c*x]*Log[1 - E^((2
*I)*ArcTan[c*x])] - ((4*I)/3)*Log[(c*x)/Sqrt[1 + c^2*x^2]] + (I/2)*(ArcTan[
c*x]^2 + PolyLog[2, E^((2*I)*ArcTan[c*x])])))/d + (b^2*c^3*(Pi^3 - 8/(c*x)
+ ((24*I)*ArcTan[c*x])/(c*x) - (8*(1 + c^2*x^2)*ArcTan[c*x])/(c^2*x^2) + (3
2*I)*ArcTan[c*x]^2 + (32*ArcTan[c*x]^2)/(c*x) - (8*(1 + c^2*x^2)*ArcTan[c*x
]^2)/(c^3*x^3) + ((12*I)*(1 + c^2*x^2)*ArcTan[c*x]^2)/(c^2*x^2) + (24*I)*Ar
cTan[c*x]^2*Log[1 - E^((-2*I)*ArcTan[c*x])] - 64*ArcTan[c*x]*Log[1 - E^((2*
I)*ArcTan[c*x])] - (24*I)*Log[(c*x)/Sqrt[1 + c^2*x^2]] - 24*ArcTan[c*x]*Pol
yLog[2, E^((-2*I)*ArcTan[c*x])] + (32*I)*PolyLog[2, E^((2*I)*ArcTan[c*x])]
+ (12*I)*PolyLog[3, E^((-2*I)*ArcTan[c*x])]))/(24*d)
```

Maple [C] time = 4.619, size = 2380, normalized size = 6.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^2/x^4/(d+I*c*d*x),x)
```

```
[Out] 2*c^2*a*b/d*arctan(c*x)/x+I*c^2*b^2/d*arctan(c*x)/x+I*c^3*a*b/d*arctan(c*x)
+1/2*c^3*b^2/d*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))
^3*arctan(c*x)^2-c^3*a*b/d*ln(c*x)*ln(1+I*c*x)+c^3*a*b/d*ln(c*x)*ln(1-I*c*x
)-1/2*c^3*b^2/d*Pi*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1
)+1))^3*arctan(c*x)^2+c^3*b^2/d*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^
2/(c^2*x^2+1)+1))^2*arctan(c*x)^2-1/2*c^3*b^2/d*Pi*csgn(I*((1+I*c*x)^2/(c^2
*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*arctan(c*x)^2+1/2*c^3*b^2/d*Pi*cs
gn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2
-c^3*a*b/d*ln(c*x-I)*ln(-1/2*I*(c*x+I))+I*c^3*b^2/d*arctan(c*x)^2*ln(1+(1+I
*c*x)/(c^2*x^2+1)^(1/2))+I*c^3*b^2/d*arctan(c*x)^2*ln(2*I*(1+I*c*x)^2/(c^2*
x^2+1))+I*c^3*b^2/d*arctan(c*x)^2*ln(1-(1+I*c*x)/(c^2*x^2+1)^(1/2))+I*c^3*b
^2/d*arctan(c*x)^2*ln(c*x)+I*c^2*a*b/d/x+1/2*I*c*b^2/d*arctan(c*x)^2/x^2-1/
3*I*c^3*b^2/d/(I*c*x-(c^2*x^2+1)^(1/2)+1)*(c^2*x^2+1)^(1/2)+1/3*I*c^3*b^2/d
/(I*c*x+(c^2*x^2+1)^(1/2)+1)*(c^2*x^2+1)^(1/2)-I*c^3*b^2/d*arctan(c*x)^2*ln
((1+I*c*x)^2/(c^2*x^2+1)-1)-I*c^3*b^2/d*arctan(c*x)^2*ln(c*x-I)+11/6*I*c^3*
b^2/d*arctan(c*x)^2+2*I*c^3*b^2/d*polylog(3,(1+I*c*x)/(c^2*x^2+1)^(1/2))+8/
3*I*c^3*b^2/d*dilog(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))-I*c^3*b^2/d*ln((1+I*c*x)
```

```
/(c^2*x^2+1)^(1/2)-1)-1/3*c*a*b/d/x^2-2/3*a*b/d*arctan(c*x)/x^3-3/2*c^3*b^2
/d*Pi*arctan(c*x)^2-c^3*a*b/d*dilog(-1/2*I*(c*x+I))+1/2*c^3*a*b/d*ln(c*x-I)
^2+4/3*c^3*a*b/d*ln(c^2*x^2+1)-c^3*a*b/d*dilog(1+I*c*x)+c^3*a*b/d*dilog(1-I
*c*x)-8/3*c^3*a*b/d*ln(c*x)+I*c^3*a^2/d*ln(c*x)+2*c^3*b^2/d*arctan(c*x)*pol
ylog(2,(1+I*c*x)/(c^2*x^2+1)^(1/2))-8/3*c^3*b^2/d*arctan(c*x)*ln(1+(1+I*c*x
)/(c^2*x^2+1)^(1/2))+2*c^3*b^2/d*arctan(c*x)*polylog(2,-(1+I*c*x)/(c^2*x^2+
1)^(1/2))+c^2*b^2/d*arctan(c*x)^2/x-1/3*c*b^2/d*arctan(c*x)/x^2-I*c^3*b^2/d
*ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))-8/3*I*c^3*b^2/d*dilog((1+I*c*x)/(c^2*x^2
+1)^(1/2))+2*I*c^3*b^2/d*polylog(3,-(1+I*c*x)/(c^2*x^2+1)^(1/2))-1/2*I*c^3*
a^2/d*ln(c^2*x^2+1)+1/2*I*c*a^2/d/x^2-4/3*b^2*c^3*arctan(c*x)/d-1/2*c^3*b^2
/d*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(
((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)^2+1/2*
c^3*b^2/d*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I*((1+I*c*x)^2/(c^2*x
^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2-1/2*c^3*b^2/d*Pi*csgn
(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(
c^2*x^2+1)+1))^2*arctan(c*x)^2+1/2*c^3*b^2/d*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1
))*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^
2+1/2*c^3*b^2/d*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2
+1)+1))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arc
tan(c*x)^2+1/2*c^3*b^2/d*Pi*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*((1+
I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2+I*c*a*
b/d*arctan(c*x)/x^2+2*I*c^3*a*b/d*arctan(c*x)*ln(c*x)-2*I*c^3*a*b/d*arctan(
c*x)*ln(c*x-I)+1/2*c^3*b^2/d*Pi*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn((1
+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1
)+1))*arctan(c*x)^2-1/2*c^3*b^2/d*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*cs
gn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*
c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)^2+c^2*a^2/d/x+2/3*c^3*b^2/d*arctan(c*x)^
3+c^3*a^2/d*arctan(c*x)-1/3*b^2/d*arctan(c*x)^2/x^3-1/3*a^2/d/x^3
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^4/(d+I*c*d*x),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{i\,b^2\log\left(-\frac{cx+i}{cx-i}\right)^2+4\,ab\log\left(-\frac{cx+i}{cx-i}\right)-4i\,a^2}{4\left(cdx^5-i\,dx^4\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^4/(d+I*c*d*x),x, algorithm="fricas")
```

```
[Out] integral(1/4*(I*b^2*log(-(c*x + I)/(c*x - I))^2 + 4*a*b*log(-(c*x + I)/(c*x
 - I)) - 4*I*a^2)/(c*d*x^5 - I*d*x^4), x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**2/x**4/(d+I*c*d*x),x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\arctan(cx)+a)^2}{(i\,cdx+d)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^4/(d+I*c*d*x),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)^2/((I*c*d*x + d)*x^4), x)
```

3.103 $\int \frac{x^4\left(a+b\tan^{-1}(cx)\right)^2}{(d+icdx)^2}\,dx$

Optimal. Leaf size=433

$$\frac{4b\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{c^5d^2}+\frac{10ib^2\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{3c^5d^2}-\frac{2ib^2\text{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{c^5d^2}-\frac{x^3\left(a+b\tan\right.}{3c^2d^2}$$

```
[Out] ((2*I)*a*b*x)/(c^4*d^2) - (b^2*x)/(3*c^4*d^2) + b^2/(2*c^5*d^2*(I - c*x)) -
 (b^2*ArcTan[c*x])/(6*c^5*d^2) + ((2*I)*b^2*x*ArcTan[c*x])/(c^4*d^2) + (b*x
^2*(a + b*ArcTan[c*x]))/(3*c^3*d^2) + (I*b*(a + b*ArcTan[c*x]))/(c^5*d^2*(I
 - c*x)) + (((11*I)/6)*(a + b*ArcTan[c*x])^2)/(c^5*d^2) + (3*x*(a + b*ArcTa
n[c*x])^2)/(c^4*d^2) - (I*x^2*(a + b*ArcTan[c*x])^2)/(c^3*d^2) - (x^3*(a +
b*ArcTan[c*x])^2)/(3*c^2*d^2) - (a + b*ArcTan[c*x])^2/(c^5*d^2*(I - c*x)) +
 (20*b*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(3*c^5*d^2) - ((4*I)*(a + b*
ArcTan[c*x])^2*Log[2/(1 + I*c*x)])/(c^5*d^2) - (I*b^2*Log[1 + c^2*x^2])/(c^
5*d^2) + (((10*I)/3)*b^2*PolyLog[2, 1 - 2/(1 + I*c*x)])/(c^5*d^2) + (4*b*(a
 + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)])/(c^5*d^2) - ((2*I)*b^2*Pol
yLog[3, 1 - 2/(1 + I*c*x)])/(c^5*d^2)
```

Rubi [A] time = 0.82717, antiderivative size = 433, normalized size of antiderivative = 1., number of steps used = 33, number of rules used = 18, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.72, Rules used = {4876, 4846, 4920, 4854, 2402, 2315, 4852, 4916, 260, 4884, 321, 203, 4864, 4862, 627, 44, 4994, 6610}

$$\frac{4b\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{c^5d^2}+\frac{10ib^2\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{3c^5d^2}-\frac{2ib^2\text{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{c^5d^2}-\frac{x^3\left(a+b\tan\right.}{3c^2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcTan[c*x])^2)/(d + I*c*d*x)^2,x]
```

```
[Out] ((2*I)*a*b*x)/(c^4*d^2) - (b^2*x)/(3*c^4*d^2) + b^2/(2*c^5*d^2*(I - c*x)) -
 (b^2*ArcTan[c*x])/(6*c^5*d^2) + ((2*I)*b^2*x*ArcTan[c*x])/(c^4*d^2) + (b*x
^2*(a + b*ArcTan[c*x]))/(3*c^3*d^2) + (I*b*(a + b*ArcTan[c*x]))/(c^5*d^2*(I
 - c*x)) + (((11*I)/6)*(a + b*ArcTan[c*x])^2)/(c^5*d^2) + (3*x*(a + b*ArcTa
n[c*x])^2)/(c^4*d^2) - (I*x^2*(a + b*ArcTan[c*x])^2)/(c^3*d^2) - (x^3*(a +
b*ArcTan[c*x])^2)/(3*c^2*d^2) - (a + b*ArcTan[c*x])^2/(c^5*d^2*(I - c*x)) +
 (20*b*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(3*c^5*d^2) - ((4*I)*(a + b*
ArcTan[c*x])^2*Log[2/(1 + I*c*x)])/(c^5*d^2) - (I*b^2*Log[1 + c^2*x^2])/(c^
```

```
5*d^2) + (((10*I)/3)*b^2*PolyLog[2, 1 - 2/(1 + I*c*x)])/(c^5*d^2) + (4*b*(a
 + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)])/(c^5*d^2) - ((2*I)*b^2*Pol
yLog[3, 1 - 2/(1 + I*c*x)])/(c^5*d^2)
```

Rule 4876

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4864

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x])^p)/(e*(q + 1)), x] - D
ist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
```

```
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 4862

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x]))/(e*(q + 1)), x] - Dist[(b*
c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{x^4 \left(a+b\tan^{-1}(cx)\right)^2}{(d+icdx)^2}\,dx = \int \left(\frac{3\left(a+b\tan^{-1}(cx)\right)^2}{c^4d^2} - \frac{2ix\left(a+b\tan^{-1}(cx)\right)^2}{c^3d^2} - \frac{x^2\left(a+b\tan^{-1}(cx)\right)^2}{c^2d^2} - \frac{\left(a+b\tan^{-1}(c\right.}{c^4d^2(-i+cx}\right.$$

$$= \frac{(4i)\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{-i+cx}\,dx}{c^4d^2} - \frac{\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{(-i+cx)^2}\,dx}{c^4d^2} + \frac{3\int \left(a+b\tan^{-1}(cx)\right)^2\,dx}{c^4d^2} - \frac{(2i)\int x\left(a+b\right.}{c}$$

$$= \frac{3x\left(a+b\tan^{-1}(cx)\right)^2}{c^4d^2} - \frac{ix^2\left(a+b\tan^{-1}(cx)\right)^2}{c^3d^2} - \frac{x^3\left(a+b\tan^{-1}(cx)\right)^2}{3c^2d^2} - \frac{\left(a+b\tan^{-1}(cx)\right.}{c^5d^2(i-cx)}$$

$$= \frac{3i\left(a+b\tan^{-1}(cx)\right)^2}{c^5d^2} + \frac{3x\left(a+b\tan^{-1}(cx)\right)^2}{c^4d^2} - \frac{ix^2\left(a+b\tan^{-1}(cx)\right)^2}{c^3d^2} - \frac{x^3\left(a+b\tan^{-1}(c\right.}{3c^2d^2}$$

$$= \frac{2iabx}{c^4d^2} + \frac{bx^2\left(a+b\tan^{-1}(cx)\right)}{3c^3d^2} + \frac{ib\left(a+b\tan^{-1}(cx)\right)}{c^5d^2(i-cx)} + \frac{11i\left(a+b\tan^{-1}(cx)\right)^2}{6c^5d^2} + \frac{3x\left(a+b\right.}{c}$$

$$= \frac{2iabx}{c^4d^2} - \frac{b^2x}{3c^4d^2} + \frac{2ib^2x\tan^{-1}(cx)}{c^4d^2} + \frac{bx^2\left(a+b\tan^{-1}(cx)\right)}{3c^3d^2} + \frac{ib\left(a+b\tan^{-1}(cx)\right)}{c^5d^2(i-cx)} + \frac{11i\left(a\right.}{}$$

$$= \frac{2iabx}{c^4d^2} - \frac{b^2x}{3c^4d^2} + \frac{b^2\tan^{-1}(cx)}{3c^5d^2} + \frac{2ib^2x\tan^{-1}(cx)}{c^4d^2} + \frac{bx^2\left(a+b\tan^{-1}(cx)\right)}{3c^3d^2} + \frac{ib\left(a+b\tan^{-1}\right.}{c^5d^2(i-c}$$

$$= \frac{2iabx}{c^4d^2} - \frac{b^2x}{3c^4d^2} + \frac{b^2}{2c^5d^2(i-cx)} + \frac{b^2\tan^{-1}(cx)}{3c^5d^2} + \frac{2ib^2x\tan^{-1}(cx)}{c^4d^2} + \frac{bx^2\left(a+b\tan^{-1}(cx)\right)}{3c^3d^2}$$

$$= \frac{2iabx}{c^4d^2} - \frac{b^2x}{3c^4d^2} + \frac{b^2}{2c^5d^2(i-cx)} - \frac{b^2\tan^{-1}(cx)}{6c^5d^2} + \frac{2ib^2x\tan^{-1}(cx)}{c^4d^2} + \frac{bx^2\left(a+b\tan^{-1}(cx)\right)}{3c^3d^2}$$

Mathematica [A] time = 2.37743, size = 502, normalized size = 1.16

$$-\frac{2ab\left(24\mathrm{PolyLog}\left(2,-e^{2i\tan^{-1}(cx)}\right) - 2c^2x^2 + 20\log\left(c^2x^2+1\right) + 2\tan^{-1}(cx)\left(2c^3x^3 + 6ic^2x^2 - 18cx + 24i\log\left(1+e^{2i\tan}\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(a + b*ArcTan[c*x])^2)/(d + I*c*d*x)^2,x]
```

```
[Out] -(-36*a^2*c*x + (12*I)*a^2*c^2*x^2 + 4*a^2*c^3*x^3 - (12*a^2)/(-I + c*x) +
48*a^2*ArcTan[c*x] - (24*I)*a^2*Log[1 + c^2*x^2] + 2*a*b*(-2 - (12*I)*c*x -
 2*c^2*x^2 + 48*ArcTan[c*x]^2 - 3*Cos[2*ArcTan[c*x]] + 20*Log[1 + c^2*x^2]
```

```
+ 24*PolyLog[2, -E^((2*I)*ArcTan[c*x])] + 2*ArcTan[c*x]*(6*I - 18*c*x + (6*
I)*c^2*x^2 + 2*c^3*x^3 - (3*I)*Cos[2*ArcTan[c*x]] + (24*I)*Log[1 + E^((2*I)
*ArcTan[c*x])] - 3*Sin[2*ArcTan[c*x]]) + (3*I)*Sin[2*ArcTan[c*x]]) + b^2*(4
*c*x - 4*ArcTan[c*x] - (24*I)*c*x*ArcTan[c*x] - 4*c^2*x^2*ArcTan[c*x] + (52
*I)*ArcTan[c*x]^2 - 36*c*x*ArcTan[c*x]^2 + (12*I)*c^2*x^2*ArcTan[c*x]^2 + 4
*c^3*x^3*ArcTan[c*x]^2 + 32*ArcTan[c*x]^3 + (3*I)*Cos[2*ArcTan[c*x]] - 6*Ar
cTan[c*x]*Cos[2*ArcTan[c*x]] - (6*I)*ArcTan[c*x]^2*Cos[2*ArcTan[c*x]] - 80*
ArcTan[c*x]*Log[1 + E^((2*I)*ArcTan[c*x])] + (48*I)*ArcTan[c*x]^2*Log[1 + E
^((2*I)*ArcTan[c*x])] + (12*I)*Log[1 + c^2*x^2] + 8*(5*I + 6*ArcTan[c*x])*P
olyLog[2, -E^((2*I)*ArcTan[c*x])] + (24*I)*PolyLog[3, -E^((2*I)*ArcTan[c*x]
)] + 3*Sin[2*ArcTan[c*x]] + (6*I)*ArcTan[c*x]*Sin[2*ArcTan[c*x]] - 6*ArcTan
[c*x]^2*Sin[2*ArcTan[c*x]]))/(12*c^5*d^2)
```

Maple [C] time = 2.018, size = 1498, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arctan(c*x))^2/(d+I*c*d*x)^2,x)
```

```
[Out] -2/c^5*b^2/d^2*Pi*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn((1+I*c*x)^2/(c^2
*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c
*x)^2+2*I*a*b*x/d^2/c^4+2*I*b^2*x*arctan(c*x)/d^2/c^4-1/3*b^2*x/d^2/c^4+7/3
*b^2*arctan(c*x)/d^2/c^5-20/3*I/c^5*b^2/d^2*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)
^(1/2))-29/6*I/c^5*b^2/d^2*arctan(c*x)^2+2/c^5*b^2/d^2*Pi*csgn(I/((1+I*c*x)
^2/(c^2*x^2+1)+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)
)^2*arctan(c*x)^2-2/c^5*b^2/d^2*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*
c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2+3/c^4*a^2/d
^2*x-1/3/c^2*a^2/d^2*x^3-4/c^5*a^2/d^2*arctan(c*x)+1/c^5*a^2/d^2/(c*x-I)-2/
c^5*b^2/d^2/(8*c*x-8*I)-8/3/c^5*b^2/d^2*arctan(c*x)^3+8*I/c^5*a*b/d^2*arcta
n(c*x)*ln(c*x-I)-2*I/c^3*a*b/d^2*arctan(c*x)*x^2+7/3/c^5*a*b/d^2-1/3*I/c^5*
b^2/d^2-2/3/c^2*a*b/d^2*arctan(c*x)*x^3-1/2/c^4*b^2/d^2*arctan(c*x)/(c*x-I)
*x+6/c^4*a*b/d^2*arctan(c*x)*x+2/c^5*a*b/d^2*arctan(c*x)/(c*x-I)+4/c^5*a*b/
d^2*ln(c*x-I)*ln(-1/2*I*(c*x+I))-2/c^5*b^2/d^2*Pi*csgn((1+I*c*x)^2/(c^2*x^2
+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*arctan(c*x)^2-4/c^5*b^2/d^2*Pi*csgn((1+I
*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2-I/c^3*b^2/
d^2*arctan(c*x)^2*x^2+2*I/c^4*b^2/d^2/(8*c*x-8*I)*x-11/12*I/c^5*a*b/d^2*arc
tan(1/2*c*x)+11/6*I/c^5*a*b/d^2*arctan(1/2*c*x-1/2*I)+11/12*I/c^5*a*b/d^2*a
rctan(1/6*c^3*x^3+7/6*c*x)-29/6*I/c^5*a*b/d^2*arctan(c*x)-I/c^5*a*b/d^2/(c*
x-I)+4*I/c^5*b^2/d^2*arctan(c*x)^2*ln(c*x-I)-1/2*I/c^5*b^2/d^2*arctan(c*x)/
(c*x-I)-4*I/c^5*b^2/d^2*arctan(c*x)^2*ln(2*I*(1+I*c*x)^2/(c^2*x^2+1))-20/3*
```

```
I/c^5*b^2/d^2*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-2*I/c^5*b^2/d^2*polylo
g(3,-(1+I*c*x)^2/(c^2*x^2+1))+2*I/c^5*b^2/d^2*ln((1+I*c*x)^2/(c^2*x^2+1)+1)
+2*I/c^5*a^2/d^2*ln(c^2*x^2+1)-I/c^3*a^2/d^2*x^2+3/c^4*b^2/d^2*arctan(c*x)^
2*x-1/3/c^2*b^2/d^2*arctan(c*x)^2*x^3+1/3/c^3*b^2/d^2*arctan(c*x)*x^2-2/c^5
*a*b/d^2*ln(c*x-I)^2+4/c^5*a*b/d^2*dilog(-1/2*I*(c*x+I))-11/24/c^5*a*b/d^2*
ln(c^4*x^4+10*c^2*x^2+9)-29/12/c^5*a*b/d^2*ln(c^2*x^2+1)+4/c^5*b^2/d^2*Pi*a
rctan(c*x)^2-4/c^5*b^2/d^2*arctan(c*x)*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))+
1/c^5*b^2/d^2*arctan(c*x)^2/(c*x-I)+20/3/c^5*b^2/d^2*arctan(c*x)*ln(1+I*(1+
I*c*x)/(c^2*x^2+1)^(1/2))+20/3/c^5*b^2/d^2*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^
2*x^2+1)^(1/2))+1/3/c^3*a*b/d^2*x^2
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctan(c*x))^2/(d+I*c*d*x)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2x^4\log\left(-\frac{cx+i}{cx-i}\right)^2-4i\,abx^4\log\left(-\frac{cx+i}{cx-i}\right)-4\,a^2x^4}{4\,c^2d^2x^2-8i\,cd^2x-4\,d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctan(c*x))^2/(d+I*c*d*x)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4*log(-(c*x + I)/(c*x - I))^2 - 4*I*a*b*x^4*log(-(c*x + I)/
(c*x - I)) - 4*a^2*x^4)/(4*c^2*d^2*x^2 - 8*I*c*d^2*x - 4*d^2), x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*atan(c*x))**2/(d+I*c*d*x)**2,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\arctan(cx)+a)^2x^4}{(icdx+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctan(c*x))^2/(d+I*c*d*x)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)^2*x^4/(I*c*d*x + d)^2, x)
```

3.104 $\int \frac{x^3\left(a+b\tan^{-1}(cx)\right)^2}{(d+icdx)^2}\,dx$

Optimal. Leaf size=364

$$-\frac{3ib\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{c^4d^2}+\frac{2b^2\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c^4d^2}-\frac{3b^2\text{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{2c^4d^2}-\frac{x^2\left(a+b\tan^{-1}\right.}{2c^2d^2}$$

```
[Out] (a*b*x)/(c^3*d^2) - ((I/2)*b^2)/(c^4*d^2*(I - c*x)) + ((I/2)*b^2*ArcTan[c*x
])/(c^4*d^2) + (b^2*x*ArcTan[c*x])/(c^3*d^2) + (b*(a + b*ArcTan[c*x]))/(c^4
*d^2*(I - c*x)) + (a + b*ArcTan[c*x])^2/(c^4*d^2) - ((2*I)*x*(a + b*ArcTan[
c*x])^2)/(c^3*d^2) - (x^2*(a + b*ArcTan[c*x])^2)/(2*c^2*d^2) + (I*(a + b*Ar
cTan[c*x])^2)/(c^4*d^2*(I - c*x)) - ((4*I)*b*(a + b*ArcTan[c*x])*Log[2/(1 +
 I*c*x)])/(c^4*d^2) - (3*(a + b*ArcTan[c*x])^2*Log[2/(1 + I*c*x)])/(c^4*d^2
) - (b^2*Log[1 + c^2*x^2])/(2*c^4*d^2) + (2*b^2*PolyLog[2, 1 - 2/(1 + I*c*x
)])/(c^4*d^2) - ((3*I)*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)])
/(c^4*d^2) - (3*b^2*PolyLog[3, 1 - 2/(1 + I*c*x)])/(2*c^4*d^2)
```

Rubi [A] time = 0.614199, antiderivative size = 364, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 17, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.68, Rules used = {4876, 4846, 4920, 4854, 2402, 2315, 4852, 4916, 260, 4884, 4864, 4862, 627, 44, 203, 4994, 6610}

$$-\frac{3ib\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{c^4d^2}+\frac{2b^2\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c^4d^2}-\frac{3b^2\text{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{2c^4d^2}-\frac{x^2\left(a+b\tan^{-1}\right.}{2c^2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcTan[c*x])^2)/(d + I*c*d*x)^2,x]
```


```
[Out] (a*b*x)/(c^3*d^2) - ((I/2)*b^2)/(c^4*d^2*(I - c*x)) + ((I/2)*b^2*ArcTan[c*x
])/(c^4*d^2) + (b^2*x*ArcTan[c*x])/(c^3*d^2) + (b*(a + b*ArcTan[c*x]))/(c^4
*d^2*(I - c*x)) + (a + b*ArcTan[c*x])^2/(c^4*d^2) - ((2*I)*x*(a + b*ArcTan[
c*x])^2)/(c^3*d^2) - (x^2*(a + b*ArcTan[c*x])^2)/(2*c^2*d^2) + (I*(a + b*Ar
cTan[c*x])^2)/(c^4*d^2*(I - c*x)) - ((4*I)*b*(a + b*ArcTan[c*x])*Log[2/(1 +
 I*c*x)])/(c^4*d^2) - (3*(a + b*ArcTan[c*x])^2*Log[2/(1 + I*c*x)])/(c^4*d^2
) - (b^2*Log[1 + c^2*x^2])/(2*c^4*d^2) + (2*b^2*PolyLog[2, 1 - 2/(1 + I*c*x
)])/(c^4*d^2) - ((3*I)*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)])
/(c^4*d^2) - (3*b^2*PolyLog[3, 1 - 2/(1 + I*c*x)])/(2*c^4*d^2)
```

Rule 4876

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
```

```
erQ[m]) && NeQ[m, -1]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4864

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x])^p)/(e*(q + 1)), x] - D
ist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 4862

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x]))/(e*(q + 1)), x] - Dist[(b*
c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{x^3 \left(a+b\tan^{-1}(cx)\right)^2}{(d+icdx)^2}\,dx = \int \left(-\frac{2i\left(a+b\tan^{-1}(cx)\right)^2}{c^3d^2} - \frac{x\left(a+b\tan^{-1}(cx)\right)^2}{c^2d^2} + \frac{i\left(a+b\tan^{-1}(cx)\right)^2}{c^3d^2(-i+cx)^2} + \frac{3\left(a+b\tan^{-1}(\right.}{c^3d^2(-i+c}\right.$$

$$= \frac{i\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{(-i+cx)^2}dx}{c^3d^2} - \frac{(2i)\int\left(a+b\tan^{-1}(cx)\right)^2 dx}{c^3d^2} + \frac{3\int\frac{\left(a+b\tan^{-1}(cx)\right)^2}{-i+cx}dx}{c^3d^2} - \frac{\int x\left(a+b\,\mathrm{ta}\right.}{c^2}$$

$$= -\frac{2ix\left(a+b\tan^{-1}(cx)\right)^2}{c^3d^2} - \frac{x^2\left(a+b\tan^{-1}(cx)\right)^2}{2c^2d^2} + \frac{i\left(a+b\tan^{-1}(cx)\right)^2}{c^4d^2(i-cx)} - \frac{3\left(a+b\tan^{-1}(c\right.}{c^4d}$$

$$= \frac{2\left(a+b\tan^{-1}(cx)\right)^2}{c^4d^2} - \frac{2ix\left(a+b\tan^{-1}(cx)\right)^2}{c^3d^2} - \frac{x^2\left(a+b\tan^{-1}(cx)\right)^2}{2c^2d^2} + \frac{i\left(a+b\tan^{-1}(cx)\right.}{c^4d^2(i-cx)}$$

$$= \frac{abx}{c^3d^2} + \frac{b\left(a+b\tan^{-1}(cx)\right)}{c^4d^2(i-cx)} + \frac{\left(a+b\tan^{-1}(cx)\right)^2}{c^4d^2} - \frac{2ix\left(a+b\tan^{-1}(cx)\right)^2}{c^3d^2} - \frac{x^2\left(a+b\tan^{-}\right.}{2c^2d^2}$$

$$= \frac{abx}{c^3d^2} + \frac{b^2x\tan^{-1}(cx)}{c^3d^2} + \frac{b\left(a+b\tan^{-1}(cx)\right)}{c^4d^2(i-cx)} + \frac{\left(a+b\tan^{-1}(cx)\right)^2}{c^4d^2} - \frac{2ix\left(a+b\tan^{-1}(cx)\right)^2}{c^3d^2}$$

$$= \frac{abx}{c^3d^2} + \frac{b^2x\tan^{-1}(cx)}{c^3d^2} + \frac{b\left(a+b\tan^{-1}(cx)\right)}{c^4d^2(i-cx)} + \frac{\left(a+b\tan^{-1}(cx)\right)^2}{c^4d^2} - \frac{2ix\left(a+b\tan^{-1}(cx)\right)^2}{c^3d^2}$$

$$= \frac{abx}{c^3d^2} - \frac{ib^2}{2c^4d^2(i-cx)} + \frac{b^2x\tan^{-1}(cx)}{c^3d^2} + \frac{b\left(a+b\tan^{-1}(cx)\right)}{c^4d^2(i-cx)} + \frac{\left(a+b\tan^{-1}(cx)\right)^2}{c^4d^2} - \frac{2ix\left(a\right.}{}$$

$$= \frac{abx}{c^3d^2} - \frac{ib^2}{2c^4d^2(i-cx)} + \frac{ib^2\tan^{-1}(cx)}{2c^4d^2} + \frac{b^2x\tan^{-1}(cx)}{c^3d^2} + \frac{b\left(a+b\tan^{-1}(cx)\right)}{c^4d^2(i-cx)} + \frac{\left(a+b\tan^{-}\right.}{c^4d^2}$$

Mathematica [A] time = 1.80082, size = 429, normalized size = 1.18

$$-\frac{2ab\left(-6i\mathrm{PolyLog}\left(2,-e^{2i\tan^{-1}(cx)}\right) - 4i\log\left(c^2x^2+1\right) + 2\tan^{-1}(cx)\left(c^2x^2+4icx+6\log\left(1+e^{2i\tan^{-1}(cx)}\right) + i\sin\left(2\tan\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(a + b*ArcTan[c*x])^2)/(d + I*c*d*x)^2,x]
```

```
[Out] -((8*I)*a^2*c*x + 2*a^2*c^2*x^2 + ((4*I)*a^2)/(-I + c*x) - (12*I)*a^2*ArcTa
n[c*x] - 6*a^2*Log[1 + c^2*x^2] + 2*a*b*(-2*c*x - (12*I)*ArcTan[c*x]^2 + I*
Cos[2*ArcTan[c*x]] - (4*I)*Log[1 + c^2*x^2] - (6*I)*PolyLog[2, -E^((2*I)*Ar
```

```
cTan[c*x])] + 2*ArcTan[c*x]*(1 + (4*I)*c*x + c^2*x^2 - Cos[2*ArcTan[c*x]] +
 6*Log[1 + E^((2*I)*ArcTan[c*x])] + I*Sin[2*ArcTan[c*x]]) + Sin[2*ArcTan[c*
x]]) + b^2*(-4*c*x*ArcTan[c*x] + 10*ArcTan[c*x]^2 + (8*I)*c*x*ArcTan[c*x]^2
 + 2*c^2*x^2*ArcTan[c*x]^2 - (8*I)*ArcTan[c*x]^3 + Cos[2*ArcTan[c*x]] + (2*
I)*ArcTan[c*x]*Cos[2*ArcTan[c*x]] - 2*ArcTan[c*x]^2*Cos[2*ArcTan[c*x]] + (1
6*I)*ArcTan[c*x]*Log[1 + E^((2*I)*ArcTan[c*x])] + 12*ArcTan[c*x]^2*Log[1 +
E^((2*I)*ArcTan[c*x])] + 2*Log[1 + c^2*x^2] + 4*(2 - (3*I)*ArcTan[c*x])*Pol
yLog[2, -E^((2*I)*ArcTan[c*x])] + 6*PolyLog[3, -E^((2*I)*ArcTan[c*x])] - I*
Sin[2*ArcTan[c*x]] + 2*ArcTan[c*x]*Sin[2*ArcTan[c*x]] + (2*I)*ArcTan[c*x]^2
*Sin[2*ArcTan[c*x]]))/(4*c^4*d^2)
```

Maple [C] time = 1.612, size = 1395, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctan(c*x))^2/(d+I*c*d*x)^2,x)
```

```
[Out] -3/2*I/c^4*b^2/d^2*Pi*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn((1+I*c*x)^2/
(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2+3/2*I/c^4*b^2/d^2*
Pi*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/
(c^2*x^2+1)+1))^2*arctan(c*x)^2+a*b*x/c^3/d^2+b^2*x*arctan(c*x)/c^3/d^2-4*I
/c^3*a*b/d^2*arctan(c*x)*x+3*I/c^4*b^2/d^2*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/
((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2+3/2*I/c^4*b^2/d^2*Pi*csgn((1+I
*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*arctan(c*x)^2-3*I/c^4*a*
b/d^2*ln(c*x-I)*ln(-1/2*I*(c*x+I))-2*I/c^4*a*b/d^2*arctan(c*x)/(c*x-I)+3/2*
I/c^4*b^2/d^2*Pi*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn((1+I*c*x)^2/(c^2*
x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*
x)^2-I/c^4*a*b/d^2-1/c^2*a*b/d^2*arctan(c*x)*x^2-4*I/c^4*b^2/d^2*arctan(c*x
)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-4*I/c^4*b^2/d^2*arctan(c*x)*ln(1-I*(1
+I*c*x)/(c^2*x^2+1)^(1/2))-I/c^4*b^2/d^2*arctan(c*x)^2/(c*x-I)+3*I/c^4*b^2/
d^2*arctan(c*x)*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))-3*I/c^4*b^2/d^2*Pi*arct
an(c*x)^2+3/2*I/c^4*a*b/d^2*ln(c^2*x^2+1)-2*I/c^3*b^2/d^2*arctan(c*x)^2*x+3
/2*I/c^4*a*b/d^2*ln(c*x-I)^2+1/4*I/c^4*a*b/d^2*ln(c^4*x^4+10*c^2*x^2+9)-3*I
/c^4*a*b/d^2*dilog(-1/2*I*(c*x+I))+6/c^4*a*b/d^2*arctan(c*x)*ln(c*x-I)+2*I/
c^4*b^2/d^2*arctan(c*x)^3-I/c^4*b^2/d^2*arctan(c*x)+1/4*I/c^4*b^2/d^2/(c*x-
I)-1/2/c^2*b^2/d^2*arctan(c*x)^2*x^2+1/4/c^3*b^2/d^2/(c*x-I)*x+3/c^4*b^2/d^
2*arctan(c*x)^2*ln(c*x-I)-3/c^4*b^2/d^2*arctan(c*x)^2*ln(2*I*(1+I*c*x)^2/(c
^2*x^2+1))-1/c^4*b^2/d^2*arctan(c*x)/(2*c*x-2*I)-1/2/c^4*a*b/d^2*arctan(1/2
*c*x)+1/2/c^4*a*b/d^2*arctan(1/6*c^3*x^3+7/6*c*x)+1/c^4*a*b/d^2*arctan(1/2*
c*x-1/2*I)-1/c^4*a*b/d^2/(c*x-I)-3/c^4*a*b/d^2*arctan(c*x)-2*I/c^3*a^2/d^2*
```

```
x-I/c^4*a^2/d^2/(c*x-I)+3*I/c^4*a^2/d^2*arctan(c*x)-1/2/c^2*a^2/d^2*x^2-3/c
^4*b^2/d^2*arctan(c*x)^2+1/c^4*b^2/d^2*ln((1+I*c*x)^2/(c^2*x^2+1)+1)-3/2/c^
4*b^2/d^2*polylog(3,-(1+I*c*x)^2/(c^2*x^2+1))-4/c^4*b^2/d^2*dilog(1-I*(1+I*
c*x)/(c^2*x^2+1)^(1/2))-4/c^4*b^2/d^2*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2)
)+3/2/c^4*a^2/d^2*ln(c^2*x^2+1)+I/c^3*b^2/d^2*arctan(c*x)/(2*c*x-2*I)*x
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))^2/(d+I*c*d*x)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b^2x^3\log\left(-\frac{cx+i}{cx-i}\right)^2-4i\,abx^3\log\left(-\frac{cx+i}{cx-i}\right)-4\,a^2x^3}{4\,c^2d^2x^2-8i\,cd^2x-4\,d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))^2/(d+I*c*d*x)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^3*log(-(c*x + I)/(c*x - I))^2 - 4*I*a*b*x^3*log(-(c*x + I)/
(c*x - I)) - 4*a^2*x^3)/(4*c^2*d^2*x^2 - 8*I*c*d^2*x - 4*d^2), x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atan(c*x))**2/(d+I*c*d*x)**2,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\arctan(cx)+a)^2x^3}{(icdx+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))^2/(d+I*c*d*x)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)^2*x^3/(I*c*d*x + d)^2, x)
```

3.105 $\int \frac{x^2\left(a+b\tan^{-1}(cx)\right)^2}{(d+icdx)^2}\,dx$

Optimal. Leaf size=292

$$-\frac{2b\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{c^3d^2}-\frac{ib^2\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c^3d^2}+\frac{ib^2\text{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{c^3d^2}-\frac{ib\left(a+b\tan^{-1}(\right.}{c^3d^2(-cx+i}$$

```
[Out] -b^2/(2*c^3*d^2*(I - c*x)) + (b^2*ArcTan[c*x])/(2*c^3*d^2) - (I*b*(a + b*Ar
cTan[c*x]))/(c^3*d^2*(I - c*x)) - ((I/2)*(a + b*ArcTan[c*x])^2)/(c^3*d^2) -
 (x*(a + b*ArcTan[c*x])^2)/(c^2*d^2) + (a + b*ArcTan[c*x])^2/(c^3*d^2*(I -
c*x)) - (2*b*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(c^3*d^2) + ((2*I)*(a
+ b*ArcTan[c*x])^2*Log[2/(1 + I*c*x)])/(c^3*d^2) - (I*b^2*PolyLog[2, 1 - 2/
(1 + I*c*x)])/(c^3*d^2) - (2*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 + I*
c*x)])/(c^3*d^2) + (I*b^2*PolyLog[3, 1 - 2/(1 + I*c*x)])/(c^3*d^2)
```

Rubi [A] time = 0.494662, antiderivative size = 292, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 14, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.56, Rules used = {4876, 4846, 4920, 4854, 2402, 2315, 4864, 4862, 627, 44, 203, 4884, 4994, 6610}

$$-\frac{2b\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{c^3d^2}-\frac{ib^2\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c^3d^2}+\frac{ib^2\text{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{c^3d^2}-\frac{ib\left(a+b\tan^{-1}(\right.}{c^3d^2(-cx+i}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcTan[c*x])^2)/(d + I*c*d*x)^2,x]
```

```
[Out] -b^2/(2*c^3*d^2*(I - c*x)) + (b^2*ArcTan[c*x])/(2*c^3*d^2) - (I*b*(a + b*Ar
cTan[c*x]))/(c^3*d^2*(I - c*x)) - ((I/2)*(a + b*ArcTan[c*x])^2)/(c^3*d^2) -
 (x*(a + b*ArcTan[c*x])^2)/(c^2*d^2) + (a + b*ArcTan[c*x])^2/(c^3*d^2*(I -
c*x)) - (2*b*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(c^3*d^2) + ((2*I)*(a
+ b*ArcTan[c*x])^2*Log[2/(1 + I*c*x)])/(c^3*d^2) - (I*b^2*PolyLog[2, 1 - 2/
(1 + I*c*x)])/(c^3*d^2) - (2*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 + I*
c*x)])/(c^3*d^2) + (I*b^2*PolyLog[3, 1 - 2/(1 + I*c*x)])/(c^3*d^2)
```

Rule 4876

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
```

```
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 4864

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x])^p)/(e*(q + 1)), x] - D
ist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 4862

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x]))/(e*(q + 1)), x] - Dist[(b*
c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{x^2 \left(a+b\tan^{-1}(cx)\right)^2}{(d+icdx)^2}\,dx = \int \left(-\frac{\left(a+b\tan^{-1}(cx)\right)^2}{c^2d^2}+\frac{\left(a+b\tan^{-1}(cx)\right)^2}{c^2d^2(-i+cx)^2}-\frac{2i\left(a+b\tan^{-1}(cx)\right)^2}{c^2d^2(-i+cx)}\right)dx$$

$$= -\frac{(2i)\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{-i+cx}\,dx}{c^2d^2}-\frac{\int \left(a+b\tan^{-1}(cx)\right)^2\,dx}{c^2d^2}+\frac{\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{(-i+cx)^2}\,dx}{c^2d^2}$$

$$= -\frac{x\left(a+b\tan^{-1}(cx)\right)^2}{c^2d^2}+\frac{\left(a+b\tan^{-1}(cx)\right)^2}{c^3d^2(i-cx)}+\frac{2i\left(a+b\tan^{-1}(cx)\right)^2\log\left(\frac{2}{1+icx}\right)}{c^3d^2}-\frac{(4ib)\int$$

$$= -\frac{i\left(a+b\tan^{-1}(cx)\right)^2}{c^3d^2}-\frac{x\left(a+b\tan^{-1}(cx)\right)^2}{c^2d^2}+\frac{\left(a+b\tan^{-1}(cx)\right)^2}{c^3d^2(i-cx)}+\frac{2i\left(a+b\tan^{-1}(cx)\right)}{c^3d^2}$$

$$= -\frac{ib\left(a+b\tan^{-1}(cx)\right)}{c^3d^2(i-cx)}-\frac{i\left(a+b\tan^{-1}(cx)\right)^2}{2c^3d^2}-\frac{x\left(a+b\tan^{-1}(cx)\right)^2}{c^2d^2}+\frac{\left(a+b\tan^{-1}(cx)\right)^2}{c^3d^2(i-cx)}$$

$$= -\frac{ib\left(a+b\tan^{-1}(cx)\right)}{c^3d^2(i-cx)}-\frac{i\left(a+b\tan^{-1}(cx)\right)^2}{2c^3d^2}-\frac{x\left(a+b\tan^{-1}(cx)\right)^2}{c^2d^2}+\frac{\left(a+b\tan^{-1}(cx)\right)^2}{c^3d^2(i-cx)}$$

$$= -\frac{ib\left(a+b\tan^{-1}(cx)\right)}{c^3d^2(i-cx)}-\frac{i\left(a+b\tan^{-1}(cx)\right)^2}{2c^3d^2}-\frac{x\left(a+b\tan^{-1}(cx)\right)^2}{c^2d^2}+\frac{\left(a+b\tan^{-1}(cx)\right)^2}{c^3d^2(i-cx)}$$

$$= -\frac{b^2}{2c^3d^2(i-cx)}-\frac{ib\left(a+b\tan^{-1}(cx)\right)}{c^3d^2(i-cx)}-\frac{i\left(a+b\tan^{-1}(cx)\right)^2}{2c^3d^2}-\frac{x\left(a+b\tan^{-1}(cx)\right)^2}{c^2d^2}+\frac{(a$$

$$= -\frac{b^2}{2c^3d^2(i-cx)}+\frac{b^2\tan^{-1}(cx)}{2c^3d^2}-\frac{ib\left(a+b\tan^{-1}(cx)\right)}{c^3d^2(i-cx)}-\frac{i\left(a+b\tan^{-1}(cx)\right)^2}{2c^3d^2}-\frac{x\left(a+b\,\mathrm{ta}\right.}{c^2}$$

Mathematica [A] time = 1.21438, size = 362, normalized size = 1.24

$$-\frac{6ab\left(-4\text{PolyLog}\left(2,-e^{2i\tan^{-1}(cx)}\right)-2\log\left(c^2x^2+1\right)-8\tan^{-1}(cx)^2-i\sin\left(2\tan^{-1}(cx)\right)+\cos\left(2\tan^{-1}(cx)\right)+2\tan^{-}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(a + b*ArcTan[c*x])^2)/(d + I*c*d*x)^2,x]
```

```
[Out] -(12*a^2*c*x + (12*a^2)/(-I + c*x) - 24*a^2*ArcTan[c*x] + (12*I)*a^2*Log[1
+ c^2*x^2] + b^2*((-12*I)*ArcTan[c*x]^2 + 12*c*x*ArcTan[c*x]^2 - 16*ArcTan[
```

```
c*x]^3 - (3*I)*Cos[2*ArcTan[c*x]] + 6*ArcTan[c*x]*Cos[2*ArcTan[c*x]] + (6*I
)*ArcTan[c*x]^2*Cos[2*ArcTan[c*x]] + 24*ArcTan[c*x]*Log[1 + E^((2*I)*ArcTan
[c*x])] - (24*I)*ArcTan[c*x]^2*Log[1 + E^((2*I)*ArcTan[c*x])] - 12*(I + 2*A
rcTan[c*x])*PolyLog[2, -E^((2*I)*ArcTan[c*x])] - (12*I)*PolyLog[3, -E^((2*I
)*ArcTan[c*x])] - 3*Sin[2*ArcTan[c*x]] - (6*I)*ArcTan[c*x]*Sin[2*ArcTan[c*x
]] + 6*ArcTan[c*x]^2*Sin[2*ArcTan[c*x]]) + 6*a*b*(-8*ArcTan[c*x]^2 + Cos[2*
ArcTan[c*x]] - 2*Log[1 + c^2*x^2] - 4*PolyLog[2, -E^((2*I)*ArcTan[c*x])] -
I*Sin[2*ArcTan[c*x]] + 2*ArcTan[c*x]*(2*c*x + I*Cos[2*ArcTan[c*x]] - (4*I)*
Log[1 + E^((2*I)*ArcTan[c*x])] + Sin[2*ArcTan[c*x]])))/(12*c^3*d^2)
```

Maple [C] time = 0.612, size = 4774, normalized size = 16.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctan(c*x))^2/(d+I*c*d*x)^2,x)
```

```
[Out] I/c^3*b^2/d^2*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)
/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)*ln((1+I*c*x)^2/(c^2*x^2+1)+1)+I
/c^3*b^2/d^2*Pi*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn((1+I*c*x)^2/(c^2*x
^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+
1)^(1/2))+I/c^3*b^2/d^2*Pi*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn((1+I*c*
x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)*ln(1-I*(1+I*c*x
)/(c^2*x^2+1)^(1/2))-I/c^3*b^2/d^2*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1
+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)*ln(1-I*(1+
I*c*x)/(c^2*x^2+1)^(1/2))-I/c^3*b^2/d^2*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1))*cs
gn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)*ln(1+
I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+2/c^3*b^2/d^2/(8*c*x-8*I)+2/c^3*a^2/d^2*arct
an(c*x)-1/c^3*a^2/d^2/(c*x-I)-1/c^2*b^2/d^2*arctan(c*x)^2*x-3/2/c^3*b^2/d^2
*arctan(c*x)*ln((1+I*c*x)^2/(c^2*x^2+1)+1)-2/c^3*a*b/d^2*dilog(-1/2*I*(c*x+
I))+1/c^3*a*b/d^2*ln(c*x-I)^2+1/8/c^3*a*b/d^2*ln(c^4*x^4+10*c^2*x^2+9)+3/4/
c^3*a*b/d^2*ln(c^2*x^2+1)+2/c^3*b^2/d^2*arctan(c*x)*polylog(2,-(1+I*c*x)^2/
(c^2*x^2+1))-1/2/c^3*b^2/d^2*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2)
)-1/2/c^3*b^2/d^2*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-1/c^3*b^2
/d^2*arctan(c*x)^2/(c*x-I)-1/c^3*b^2/d^2*Pi*polylog(2,-(1+I*c*x)^2/(c^2*x^2
+1))+2/c^3*b^2/d^2*Pi*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+2/c^3*b^2/d^2*
Pi*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+I/c^3*b^2/d^2*polylog(3,-(1+I*c*x
)^2/(c^2*x^2+1))-2/c^3*b^2/d^2*Pi*arctan(c*x)^2-1/c^2*a^2/d^2*x+4/3/c^3*b^2
/d^2*arctan(c*x)^3+3/2*I/c^3*a*b/d^2*arctan(c*x)-2/c^3*a*b/d^2*arctan(c*x)/
(c*x-I)-2/c^3*a*b/d^2*ln(c*x-I)*ln(-1/2*I*(c*x+I))-2/c^3*b^2/d^2*Pi*csgn((1
+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*dilog(1-I*(1+I*c*x)/(c
```

^2*x^2+1)^(1/2))+1/2/c^3*b^2/d^2*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)
^2/(c^2*x^2+1)+1))^3*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))+2/c^3*b^2/d^2*Pi*c
sgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2+1/
c^3*b^2/d^2*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*
polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))-2/c^3*b^2/d^2*Pi*csgn((1+I*c*x)^2/(c^2*
x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2)
)-1/c^3*b^2/d^2*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)
)^3*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+1/c^3*b^2/d^2*Pi*csgn((1+I*c*x)^
2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*arctan(c*x)^2-1/c^3*b^2/d^2*Pi
*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x
)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2+1/c^3*b^2/d^2*Pi*csgn((1+I*c*x)^2/(c^2*
x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(
c*x)^2-1/c^3*b^2/d^2*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2
*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2
))-1/c^3*b^2/d^2*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2
+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-1
/2/c^3*b^2/d^2*Pi*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn((1+I*c*x)^2/(c^2
*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))+
1/2/c^3*b^2/d^2*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+
1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))+3/2*I
/c^3*b^2/d^2*arctan(c*x)^2-I/c^3*a^2/d^2*ln(c^2*x^2+1)+1/2*I/c^3*b^2/d^2*di
log(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+1/2*I/c^3*b^2/d^2*dilog(1-I*(1+I*c*x)/
(c^2*x^2+1)^(1/2))+3/4*I/c^3*b^2/d^2*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))+1/
2/c^2*b^2/d^2*arctan(c*x)/(c*x-I)*x-2/c^2*a*b/d^2*arctan(c*x)*x-1/c^3*b^2/d
^2*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*dilog(1+I
*(1+I*c*x)/(c^2*x^2+1)^(1/2))+I/c^3*a*b/d^2/(c*x-I)+2*I/c^3*b^2/d^2*arctan(
c*x)^2*ln(2*I*(1+I*c*x)^2/(c^2*x^2+1))-2*I/c^3*b^2/d^2*arctan(c*x)^2*ln(c*x
-I)+1/2*I/c^3*b^2/d^2*arctan(c*x)/(c*x-I)-2*I/c^2*b^2/d^2/(8*c*x-8*I)*x-1/4
*I/c^3*a*b/d^2*arctan(1/6*c^3*x^3+7/6*c*x)+1/4*I/c^3*a*b/d^2*arctan(1/2*c*x
)-1/2*I/c^3*a*b/d^2*arctan(1/2*c*x-1/2*I)+1/c^3*b^2/d^2*Pi*csgn(I/((1+I*c*x
)^2/(c^2*x^2+1)+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1
))^2*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+1/c^3*b^2/d^2*Pi*csgn(I/((1+I*c
*x)^2/(c^2*x^2+1)+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)
+1))^2*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-4*I/c^3*a*b/d^2*arctan(c*x)*l
n(c*x-I)+2*I/c^3*b^2/d^2*Pi*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))
-2*I/c^3*b^2/d^2*Pi*arctan(c*x)*ln((1+I*c*x)^2/(c^2*x^2+1)+1)+2*I/c^3*b^2/d
^2*Pi*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+1/2/c^3*b^2/d^2*Pi*cs
gn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c
*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*polylog(2,-(1+I*c*x)^2/(c^2*
x^2+1))+I/c^3*b^2/d^2*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2
+1)+1))^3*arctan(c*x)*ln((1+I*c*x)^2/(c^2*x^2+1)+1)+1/c^3*b^2/d^2*Pi*csgn(I
/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^
2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)^2-2*I/c^3*b^2/d^2*Pi
*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)*ln
(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+2*I/c^3*b^2/d^2*Pi*csgn((1+I*c*x)^2/(c^2*

```
x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)*ln((1+I*c*x)^2/(c^2*x^2+1
)+1)-I/c^3*b^2/d^2*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)
+1))^3*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-I/c^3*b^2/d^2*Pi*csg
n((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*arctan(c*x)*ln(1-I
*(1+I*c*x)/(c^2*x^2+1)^(1/2))-2*I/c^3*b^2/d^2*Pi*csgn((1+I*c*x)^2/(c^2*x^2+
1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^
(1/2))-1/c^3*b^2/d^2*Pi*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn((1+I*c*x)^
2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*di
log(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-1/c^3*b^2/d^2*Pi*csgn(I/((1+I*c*x)^2/(
c^2*x^2+1)+1))*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/(
(1+I*c*x)^2/(c^2*x^2+1)+1))*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+I/c^3*b^
2/d^2*Pi*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn((1+I*c*x)^2/(c^2*x^2+1))*
csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)*ln((1
+I*c*x)^2/(c^2*x^2+1)+1)-I/c^3*b^2/d^2*Pi*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1
))*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/
(c^2*x^2+1)+1))*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-I/c^3*b^2/d
^2*Pi*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn((1+I*c*x)^2/(c^2*x^2+1))*csg
n((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)*ln(1-I*(
1+I*c*x)/(c^2*x^2+1)^(1/2))-I/c^3*b^2/d^2*Pi*csgn(I/((1+I*c*x)^2/(c^2*x^2+1
)+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*
x)*ln((1+I*c*x)^2/(c^2*x^2+1)+1)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))^2/(d+I*c*d*x)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{b^2x^2\log\left(-\frac{cx+i}{cx-i}\right)^2 - 4i\,abx^2\log\left(-\frac{cx+i}{cx-i}\right) - 4\,a^2x^2}{4\,c^2d^2x^2 - 8i\,cd^2x - 4\,d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))^2/(d+I*c*d*x)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^2*log(-(c*x + I)/(c*x - I))^2 - 4*I*a*b*x^2*log(-(c*x + I)/
(c*x - I)) - 4*a^2*x^2)/(4*c^2*d^2*x^2 - 8*I*c*d^2*x - 4*d^2), x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atan(c*x))**2/(d+I*c*d*x)**2,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\arctan(cx) + a)^2 x^2}{(i\,cdx + d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))^2/(d+I*c*d*x)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)^2*x^2/(I*c*d*x + d)^2, x)
```

3.106 $\int \frac{x\left(a+b\tan^{-1}(cx)\right)^2}{(d+icdx)^2}\,dx$

Optimal. Leaf size=216

$$\frac{ib\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{c^2d^2}+\frac{b^2\text{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{2c^2d^2}-\frac{b\left(a+b\tan^{-1}(cx)\right)}{c^2d^2(-cx+i)}-\frac{i\left(a+b\tan^{-1}(cx)\right)^2}{c^2d^2(-cx+i)}+\frac{(a}{}$$

```
[Out] ((I/2)*b^2)/(c^2*d^2*(I - c*x)) - ((I/2)*b^2*ArcTan[c*x])/(c^2*d^2) - (b*(a
 + b*ArcTan[c*x]))/(c^2*d^2*(I - c*x)) + (a + b*ArcTan[c*x])^2/(2*c^2*d^2)
- (I*(a + b*ArcTan[c*x])^2)/(c^2*d^2*(I - c*x)) + ((a + b*ArcTan[c*x])^2*Lo
g[2/(1 + I*c*x)])/(c^2*d^2) + (I*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1
+ I*c*x)])/(c^2*d^2) + (b^2*PolyLog[3, 1 - 2/(1 + I*c*x)])/(2*c^2*d^2)
```

Rubi [A] time = 0.342402, antiderivative size = 216, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {4876, 4864, 4862, 627, 44, 203, 4884, 4854, 4994, 6610}

$$\frac{ib\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{c^2d^2}+\frac{b^2\text{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{2c^2d^2}-\frac{b\left(a+b\tan^{-1}(cx)\right)}{c^2d^2(-cx+i)}-\frac{i\left(a+b\tan^{-1}(cx)\right)^2}{c^2d^2(-cx+i)}+\frac{(a}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcTan[c*x])^2)/(d + I*c*d*x)^2,x]
```


```
[Out] ((I/2)*b^2)/(c^2*d^2*(I - c*x)) - ((I/2)*b^2*ArcTan[c*x])/(c^2*d^2) - (b*(a
 + b*ArcTan[c*x]))/(c^2*d^2*(I - c*x)) + (a + b*ArcTan[c*x])^2/(2*c^2*d^2)
- (I*(a + b*ArcTan[c*x])^2)/(c^2*d^2*(I - c*x)) + ((a + b*ArcTan[c*x])^2*Lo
g[2/(1 + I*c*x)])/(c^2*d^2) + (I*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1
+ I*c*x)])/(c^2*d^2) + (b^2*PolyLog[3, 1 - 2/(1 + I*c*x)])/(2*c^2*d^2)
```


Rule 4876

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 4864

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x])^p)/(e*(q + 1)), x] - D
ist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 4862

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x]))/(e*(q + 1)), x] - Dist[(b*
c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
```

```
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{x\left(a+b\tan^{-1}(cx)\right)^2}{(d+icdx)^2}\,dx = \int \left(-\frac{i\left(a+b\tan^{-1}(cx)\right)^2}{cd^2(-i+cx)^2} - \frac{\left(a+b\tan^{-1}(cx)\right)^2}{cd^2(-i+cx)}\right)dx$$

$$= -\frac{i\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{(-i+cx)^2}\,dx}{cd^2} - \frac{\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{-i+cx}\,dx}{cd^2}$$

$$= -\frac{i\left(a+b\tan^{-1}(cx)\right)^2}{c^2d^2(i-cx)} + \frac{\left(a+b\tan^{-1}(cx)\right)^2\log\left(\frac{2}{1+icx}\right)}{c^2d^2} - \frac{(2ib)\int \left(-\frac{i\left(a+b\tan^{-1}(cx)\right)}{2(-i+cx)^2} + \frac{i\left(a+b\tan\right.}{2\left(1+\right.}\right.}{cd^2}$$

$$= -\frac{i\left(a+b\tan^{-1}(cx)\right)^2}{c^2d^2(i-cx)} + \frac{\left(a+b\tan^{-1}(cx)\right)^2\log\left(\frac{2}{1+icx}\right)}{c^2d^2} + \frac{ib\left(a+b\tan^{-1}(cx)\right)\mathrm{Li}_2\left(1-\frac{2}{1+ic}\right.}{c^2d^2}$$

$$= -\frac{b\left(a+b\tan^{-1}(cx)\right)}{c^2d^2(i-cx)} + \frac{\left(a+b\tan^{-1}(cx)\right)^2}{2c^2d^2} - \frac{i\left(a+b\tan^{-1}(cx)\right)^2}{c^2d^2(i-cx)} + \frac{\left(a+b\tan^{-1}(cx)\right)^2\log}{c^2d^2}$$

$$= -\frac{b\left(a+b\tan^{-1}(cx)\right)}{c^2d^2(i-cx)} + \frac{\left(a+b\tan^{-1}(cx)\right)^2}{2c^2d^2} - \frac{i\left(a+b\tan^{-1}(cx)\right)^2}{c^2d^2(i-cx)} + \frac{\left(a+b\tan^{-1}(cx)\right)^2\log}{c^2d^2}$$

$$= -\frac{b\left(a+b\tan^{-1}(cx)\right)}{c^2d^2(i-cx)} + \frac{\left(a+b\tan^{-1}(cx)\right)^2}{2c^2d^2} - \frac{i\left(a+b\tan^{-1}(cx)\right)^2}{c^2d^2(i-cx)} + \frac{\left(a+b\tan^{-1}(cx)\right)^2\log}{c^2d^2}$$

$$= \frac{ib^2}{2c^2d^2(i-cx)} - \frac{b\left(a+b\tan^{-1}(cx)\right)}{c^2d^2(i-cx)} + \frac{\left(a+b\tan^{-1}(cx)\right)^2}{2c^2d^2} - \frac{i\left(a+b\tan^{-1}(cx)\right)^2}{c^2d^2(i-cx)} + \frac{(a+b\,\mathrm{t}}{}$$

$$= \frac{ib^2}{2c^2d^2(i-cx)} - \frac{ib^2\tan^{-1}(cx)}{2c^2d^2} - \frac{b\left(a+b\tan^{-1}(cx)\right)}{c^2d^2(i-cx)} + \frac{\left(a+b\tan^{-1}(cx)\right)^2}{2c^2d^2} - \frac{i\left(a+b\tan^{-1}\right.}{c^2d^2(i-c}$$

Mathematica [A] time = 0.760157, size = 300, normalized size = 1.39

$$\frac{-6iab\left(2\mathrm{PolyLog}\left(2,-e^{2i\tan^{-1}(cx)}\right)+4\tan^{-1}(cx)^2+i\sin\left(2\tan^{-1}(cx)\right)-\cos\left(2\tan^{-1}(cx)\right)-2i\tan^{-1}(cx)\left(-2\log\left(1+\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(a + b*ArcTan[c*x])^2)/(d + I*c*d*x)^2,x]
```

```
[Out] (((12*I)*a^2)/(-I + c*x) - (12*I)*a^2*ArcTan[c*x] - 6*a^2*Log[1 + c^2*x^2]
- (6*I)*a*b*(4*ArcTan[c*x]^2 - Cos[2*ArcTan[c*x]] + 2*PolyLog[2, -E^((2*I)*
ArcTan[c*x])] - (2*I)*ArcTan[c*x]*(Cos[2*ArcTan[c*x]] - 2*Log[1 + E^((2*I)*
ArcTan[c*x])] - I*Sin[2*ArcTan[c*x]]) + I*Sin[2*ArcTan[c*x]]) + b^2*((-8*I)
```

```
*ArcTan[c*x]^3 + 3*Cos[2*ArcTan[c*x]] + (6*I)*ArcTan[c*x]*Cos[2*ArcTan[c*x]
] - 6*ArcTan[c*x]^2*Cos[2*ArcTan[c*x]] + 12*ArcTan[c*x]^2*Log[1 + E^((2*I)*
ArcTan[c*x])] - (12*I)*ArcTan[c*x]*PolyLog[2, -E^((2*I)*ArcTan[c*x])] + 6*P
olyLog[3, -E^((2*I)*ArcTan[c*x])] - (3*I)*Sin[2*ArcTan[c*x]] + 6*ArcTan[c*x
]*Sin[2*ArcTan[c*x]] + (6*I)*ArcTan[c*x]^2*Sin[2*ArcTan[c*x]]))/(12*c^2*d^2
)
```

Maple [C] time = 0.316, size = 1059, normalized size = 4.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctan(c*x))^2/(d+I*c*d*x)^2,x)
```

```
[Out] -1/2*I/c^2*b^2/d^2*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)
+1))^2*csgn((1+I*c*x)^2/(c^2*x^2+1))*arctan(c*x)^2+1/2*I/c^2*b^2/d^2*Pi*csg
n((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*csgn(I/((1+I*c*x)^
2/(c^2*x^2+1)+1))*arctan(c*x)^2-1/2*I/c^2*a*b/d^2*ln(c*x-I)^2+I/c^2*b^2/d^2
*arctan(c*x)^2/(c*x-I)+I/c^2*b^2/d^2*Pi*arctan(c*x)^2+I/c^2*a*b/d^2*dilog(-
1/2*I*(c*x+I))-2/c^2*a*b/d^2*arctan(c*x)*ln(c*x-I)-2*I/c*b^2/d^2*arctan(c*x
)/(4*c*x-4*I)*x+2*I/c^2*a*b/d^2*arctan(c*x)/(c*x-I)-1/2*I/c^2*b^2/d^2*Pi*cs
gn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*arctan(c*x)^2-I/c
^2*b^2/d^2*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*a
rctan(c*x)^2+I/c^2*a*b/d^2*ln(-1/2*I*(c*x+I))*ln(c*x-I)-I/c^2*b^2/d^2*arcta
n(c*x)*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))+1/8*I/c^2*a*b/d^2*ln(c^4*x^4+10*
c^2*x^2+9)-1/4*I/c^2*a*b/d^2*ln(c^2*x^2+1)-1/c^2*b^2/d^2*arctan(c*x)^2*ln(c
*x-I)+1/c^2*b^2/d^2*arctan(c*x)^2*ln(2*I*(1+I*c*x)^2/(c^2*x^2+1))-I/c^2*a^2
/d^2*arctan(c*x)-2/3*I/c^2*b^2/d^2*arctan(c*x)^3-1/4*I/c^2*b^2/d^2/(c*x-I)-
1/4/c*b^2/d^2/(c*x-I)*x+I/c^2*a^2/d^2/(c*x-I)-1/4/c^2*a*b/d^2*arctan(1/2*c*
x)+1/4/c^2*a*b/d^2*arctan(1/6*c^3*x^3+7/6*c*x)+1/2/c^2*a*b/d^2*arctan(1/2*c
*x-1/2*I)+1/c^2*a*b/d^2/(c*x-I)+1/2/c^2*a*b/d^2*arctan(c*x)+2/c^2*b^2/d^2*a
rctan(c*x)/(4*c*x-4*I)+1/2/c^2*b^2/d^2*polylog(3,-(1+I*c*x)^2/(c^2*x^2+1))-
1/2/c^2*a^2/d^2*ln(c^2*x^2+1)+1/2/c^2*b^2/d^2*arctan(c*x)^2-1/2*I/c^2*b^2/d
^2*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I/((1+
I*c*x)^2/(c^2*x^2+1)+1))*csgn((1+I*c*x)^2/(c^2*x^2+1))*arctan(c*x)^2
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))^2/(d+I*c*d*x)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2x\log\left(-\frac{cx+i}{cx-i}\right)^2-4i\,abx\log\left(-\frac{cx+i}{cx-i}\right)-4\,a^2x}{4\,c^2d^2x^2-8i\,cd^2x-4\,d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))^2/(d+I*c*d*x)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*x*log(-(c*x + I)/(c*x - I))^2 - 4*I*a*b*x*log(-(c*x + I)/(c*x
 - I)) - 4*a^2*x)/(4*c^2*d^2*x^2 - 8*I*c*d^2*x - 4*d^2), x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atan(c*x))**2/(d+I*c*d*x)**2,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\arctan(cx)+a)^2x}{(i\,cdx+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))^2/(d+I*c*d*x)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)^2*x/(I*c*d*x + d)^2, x)
```

3.107 $\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{(d+icdx)^2}\,dx$

Optimal. Leaf size=122

$$\frac{ib\left(a+b\tan^{-1}(cx)\right)}{cd^2(-cx+i)}+\frac{i\left(a+b\tan^{-1}(cx)\right)^2}{cd^2(1+icx)}-\frac{i\left(a+b\tan^{-1}(cx)\right)^2}{2cd^2}+\frac{b^2}{2cd^2(-cx+i)}-\frac{b^2\tan^{-1}(cx)}{2cd^2}$$

```
[Out] b^2/(2*c*d^2*(I - c*x)) - (b^2*ArcTan[c*x])/(2*c*d^2) + (I*b*(a + b*ArcTan[
c*x]))/(c*d^2*(I - c*x)) - ((I/2)*(a + b*ArcTan[c*x])^2)/(c*d^2) + (I*(a +
b*ArcTan[c*x])^2)/(c*d^2*(1 + I*c*x))
```

Rubi [A] time = 0.121582, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4864, 4862, 627, 44, 203, 4884}

$$\frac{ib\left(a+b\tan^{-1}(cx)\right)}{cd^2(-cx+i)}+\frac{i\left(a+b\tan^{-1}(cx)\right)^2}{cd^2(1+icx)}-\frac{i\left(a+b\tan^{-1}(cx)\right)^2}{2cd^2}+\frac{b^2}{2cd^2(-cx+i)}-\frac{b^2\tan^{-1}(cx)}{2cd^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^2/(d + I*c*d*x)^2,x]
```

```
[Out] b^2/(2*c*d^2*(I - c*x)) - (b^2*ArcTan[c*x])/(2*c*d^2) + (I*b*(a + b*ArcTan[
c*x]))/(c*d^2*(I - c*x)) - ((I/2)*(a + b*ArcTan[c*x])^2)/(c*d^2) + (I*(a +
b*ArcTan[c*x])^2)/(c*d^2*(1 + I*c*x))
```

Rule 4864

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x])^p)/(e*(q + 1)), x] - D
ist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 4862

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x]))/(e*(q + 1)), x] - Dist[(b*
c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
```

```
c, d, e, q}, x] && NeQ[q, -1]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{(d+icdx)^2}\,dx = \frac{i\left(a+b\tan^{-1}(cx)\right)^2}{cd^2(1+icx)} - \frac{(2ib)\int\left(-\frac{a+b\tan^{-1}(cx)}{2d(-i+cx)^2}+\frac{a+b\tan^{-1}(cx)}{2d\left(1+c^2x^2\right)}\right)dx}{d}$$

$$= \frac{i\left(a+b\tan^{-1}(cx)\right)^2}{cd^2(1+icx)} + \frac{(ib)\int\frac{a+b\tan^{-1}(cx)}{(-i+cx)^2}\,dx}{d^2} - \frac{(ib)\int\frac{a+b\tan^{-1}(cx)}{1+c^2x^2}\,dx}{d^2}$$

$$= \frac{ib\left(a+b\tan^{-1}(cx)\right)}{cd^2(i-cx)} - \frac{i\left(a+b\tan^{-1}(cx)\right)^2}{2cd^2} + \frac{i\left(a+b\tan^{-1}(cx)\right)^2}{cd^2(1+icx)} + \frac{\left(ib^2\right)\int\frac{1}{(-i+cx)\left(1+c^2x^2\right)}\,dx}{d^2}$$

$$= \frac{ib\left(a+b\tan^{-1}(cx)\right)}{cd^2(i-cx)} - \frac{i\left(a+b\tan^{-1}(cx)\right)^2}{2cd^2} + \frac{i\left(a+b\tan^{-1}(cx)\right)^2}{cd^2(1+icx)} + \frac{\left(ib^2\right)\int\frac{1}{(-i+cx)^2(i+cx)}\,dx}{d^2}$$

$$= \frac{ib\left(a+b\tan^{-1}(cx)\right)}{cd^2(i-cx)} - \frac{i\left(a+b\tan^{-1}(cx)\right)^2}{2cd^2} + \frac{i\left(a+b\tan^{-1}(cx)\right)^2}{cd^2(1+icx)} + \frac{\left(ib^2\right)\int\left(-\frac{i}{2(-i+cx)^2}+\frac{}{2(}\right.}{d^2}$$

$$= \frac{b^2}{2cd^2(i-cx)} + \frac{ib\left(a+b\tan^{-1}(cx)\right)}{cd^2(i-cx)} - \frac{i\left(a+b\tan^{-1}(cx)\right)^2}{2cd^2} + \frac{i\left(a+b\tan^{-1}(cx)\right)^2}{cd^2(1+icx)} - \frac{b^2\int\frac{1}{1+c}}{2d^2}$$

$$= \frac{b^2}{2cd^2(i-cx)} - \frac{b^2\tan^{-1}(cx)}{2cd^2} + \frac{ib\left(a+b\tan^{-1}(cx)\right)}{cd^2(i-cx)} - \frac{i\left(a+b\tan^{-1}(cx)\right)^2}{2cd^2} + \frac{i\left(a+b\tan^{-1}(c\right.}{cd^2(1+icx)}$$

Mathematica [A] time = 0.174562, size = 72, normalized size = 0.59

$$-\frac{-2a^2+b(b+2ia)(cx+i)\tan^{-1}(cx)+2iab+b^2(-1+icx)\tan^{-1}(cx)^2+b^2}{2cd^2(cx-i)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])^2/(d + I*c*d*x)^2,x]
```

```
[Out] -(-2*a^2 + (2*I)*a*b + b^2 + b*((2*I)*a + b)*(I + c*x)*ArcTan[c*x] + b^2*(-
1 + I*c*x)*ArcTan[c*x]^2)/(2*c*d^2*(-I + c*x))
```

Maple [B] time = 0.073, size = 344, normalized size = 2.8

$$\frac{ia^2}{cd^2(1+icx)} + \frac{ib^2(\arctan(cx))^2}{cd^2(1+icx)} - \frac{b^2\arctan(cx)\ln(cx-i)}{2cd^2} - \frac{ib^2\arctan(cx)}{cd^2(cx-i)} + \frac{b^2\arctan(cx)\ln(cx+i)}{2cd^2} + \frac{\frac{i}{4}b^2\ln(c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^2/(d+I*c*d*x)^2,x)
```

```
[Out] I/c*a^2/d^2/(1+I*c*x)+I/c*b^2/d^2/(1+I*c*x)*arctan(c*x)^2-1/2/c*b^2/d^2*arc
tan(c*x)*ln(c*x-I)-I/c*b^2/d^2*arctan(c*x)/(c*x-I)+1/2/c*b^2/d^2*arctan(c*x
)*ln(c*x+I)+1/4*I/c*b^2/d^2*ln(c*x-I)*ln(-1/2*I*(c*x+I))-1/8*I/c*b^2/d^2*ln
(c*x-I)^2-1/2/c*b^2/d^2/(c*x-I)-1/2*b^2*arctan(c*x)/c/d^2-1/4*I/c*b^2/d^2*l
n(-1/2*I*(-c*x+I))*ln(-1/2*I*(c*x+I))+1/4*I/c*b^2/d^2*ln(-1/2*I*(-c*x+I))*l
n(c*x+I)-1/8*I/c*b^2/d^2*ln(c*x+I)^2+2*I/c*a*b/d^2/(1+I*c*x)*arctan(c*x)-I/
c*a*b/d^2*arctan(c*x)-I/c*a*b/d^2/(c*x-I)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/(d+I*c*d*x)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 2.194, size = 231, normalized size = 1.89

$$\frac{\left(i\,b^2cx-b^2\right)\log\left(-\frac{cx+i}{cx-i}\right)^2+8\,a^2-8i\,ab-4\,b^2+\left(2\left(2\,ab-i\,b^2\right)cx+4i\,ab+2\,b^2\right)\log\left(-\frac{cx+i}{cx-i}\right)}{8\left(c^2d^2x-i\,cd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/(d+I*c*d*x)^2,x, algorithm="fricas")
```

```
[Out] 1/8*((I*b^2*c*x - b^2)*log(-(c*x + I)/(c*x - I))^2 + 8*a^2 - 8*I*a*b - 4*b^
2 + (2*(2*a*b - I*b^2)*c*x + 4*I*a*b + 2*b^2)*log(-(c*x + I)/(c*x - I)))/(c
^2*d^2*x - I*c*d^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**2/(d+I*c*d*x)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.16505, size = 405, normalized size = 3.32

$$\frac{\frac{2\,b^2di^2\arctan\left(\frac{(cdix+d)\left(\frac{di^2}{cdix+d}+1\right)i}{d}\right)}{cdix+d}-b^2i\arctan\left(\frac{(cdix+d)\left(\frac{di^2}{cdix+d}+1\right)i}{d}\right)^2+\frac{2\,b^2di\arctan\left(\frac{(cdix+d)\left(\frac{di^2}{cdix+d}+1\right)i}{d}\right)^2}{cdix+d}-\frac{2\,abdi^2}{cdix+d}+2\,abi\arctan\left(\frac{(cdix\cdots}{\cdots}\right.}{2\,cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/(d+I*c*d*x)^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*b^2*d*i^2*arctan((c*d*i*x + d)*(d*i^2/(c*d*i*x + d) + 1)*i/d)/(c*d*i
*x + d) - b^2*i*arctan((c*d*i*x + d)*(d*i^2/(c*d*i*x + d) + 1)*i/d)^2 + 2*b
^2*d*i*arctan((c*d*i*x + d)*(d*i^2/(c*d*i*x + d) + 1)*i/d)^2/(c*d*i*x + d)
- 2*a*b*d*i^2/(c*d*i*x + d) + 2*a*b*i*arctan((c*d*i*x + d)*(d*i^2/(c*d*i*x
+ d) + 1)*i/d) - 4*a*b*d*i*arctan((c*d*i*x + d)*(d*i^2/(c*d*i*x + d) + 1)*i
/d)/(c*d*i*x + d) + 2*a^2*d*i/(c*d*i*x + d) - b^2*d*i/(c*d*i*x + d) + b^2*a
rctan((c*d*i*x + d)*(d*i^2/(c*d*i*x + d) + 1)*i/d))/(c*d^2)
```

3.108 $\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x(d+icdx)^2}\,dx$

Optimal. Leaf size=221

$$\frac{ib\text{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{d^2}+\frac{b^2\text{PolyLog}\left(3,-1+\frac{2}{1+icx}\right)}{2d^2}+\frac{b\left(a+b\tan^{-1}(cx)\right)}{d^2(-cx+i)}+\frac{i\left(a+b\tan^{-1}(cx)\right)^2}{d^2(-cx+i)}-$$

```
[Out] ((-I/2)*b^2)/(d^2*(I - c*x)) + ((I/2)*b^2*ArcTan[c*x])/d^2 + (b*(a + b*ArcT
an[c*x]))/(d^2*(I - c*x)) - (a + b*ArcTan[c*x])^2/(2*d^2) + (I*(a + b*ArcTa
n[c*x])^2)/(d^2*(I - c*x)) + (2*(a + b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1 + I*
c*x)])/d^2 + ((a + b*ArcTan[c*x])^2*Log[2/(1 + I*c*x)])/d^2 + (I*b*(a + b*A
rcTan[c*x])*PolyLog[2, -1 + 2/(1 + I*c*x)])/d^2 + (b^2*PolyLog[3, -1 + 2/(1
 + I*c*x)])/(2*d^2)
```

Rubi [A] time = 0.616117, antiderivative size = 221, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.48, Rules used = {4876, 4850, 4988, 4884, 4994, 6610, 4864, 4862, 627, 44, 203, 4854}

$$\frac{ib\text{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{d^2}+\frac{b^2\text{PolyLog}\left(3,-1+\frac{2}{1+icx}\right)}{2d^2}+\frac{b\left(a+b\tan^{-1}(cx)\right)}{d^2(-cx+i)}+\frac{i\left(a+b\tan^{-1}(cx)\right)^2}{d^2(-cx+i)}-$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^2/(x*(d + I*c*d*x)^2),x]
```

```
[Out] ((-I/2)*b^2)/(d^2*(I - c*x)) + ((I/2)*b^2*ArcTan[c*x])/d^2 + (b*(a + b*ArcT
an[c*x]))/(d^2*(I - c*x)) - (a + b*ArcTan[c*x])^2/(2*d^2) + (I*(a + b*ArcTa
n[c*x])^2)/(d^2*(I - c*x)) + (2*(a + b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1 + I*
c*x)])/d^2 + ((a + b*ArcTan[c*x])^2*Log[2/(1 + I*c*x)])/d^2 + (I*b*(a + b*A
rcTan[c*x])*PolyLog[2, -1 + 2/(1 + I*c*x)])/d^2 + (b^2*PolyLog[3, -1 + 2/(1
 + I*c*x)])/(2*d^2)
```

Rule 4876

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 4850

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTan[c*x])^(p - 1)*ArcTanh[1 - 2/(1 + I*c*x)])/(1 + c^2*x^2), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4988

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTan[c*x])^p)/(d + e
*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTan[c*x])^p)/(d + e*x^2
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 4864

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x])^p)/(e*(q + 1)), x] - D
ist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 4862

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
```

```
:> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x]))/(e*(q + 1)), x] - Dist[(b*
c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x(d+icdx)^2}\,dx = \int \left(\frac{\left(a+b\tan^{-1}(cx)\right)^2}{d^2x} + \frac{ic\left(a+b\tan^{-1}(cx)\right)^2}{d^2(-i+cx)^2} - \frac{c\left(a+b\tan^{-1}(cx)\right)^2}{d^2(-i+cx)}\right)dx$$

$$= \frac{\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x}\,dx}{d^2} + \frac{(ic)\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{(-i+cx)^2}\,dx}{d^2} - \frac{c\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{-i+cx}\,dx}{d^2}$$

$$= \frac{i\left(a+b\tan^{-1}(cx)\right)^2}{d^2(i-cx)} + \frac{2\left(a+b\tan^{-1}(cx)\right)^2\tanh^{-1}\left(1-\frac{2}{1+icx}\right)}{d^2} + \frac{\left(a+b\tan^{-1}(cx)\right)^2\log\left(\frac{2}{1+i\ldots}\right.}{d^2}$$

$$= \frac{i\left(a+b\tan^{-1}(cx)\right)^2}{d^2(i-cx)} + \frac{2\left(a+b\tan^{-1}(cx)\right)^2\tanh^{-1}\left(1-\frac{2}{1+icx}\right)}{d^2} + \frac{\left(a+b\tan^{-1}(cx)\right)^2\log\left(\frac{2}{1+i\ldots}\right.}{d^2}$$

$$= \frac{b\left(a+b\tan^{-1}(cx)\right)}{d^2(i-cx)} - \frac{\left(a+b\tan^{-1}(cx)\right)^2}{2d^2} + \frac{i\left(a+b\tan^{-1}(cx)\right)^2}{d^2(i-cx)} + \frac{2\left(a+b\tan^{-1}(cx)\right)^2\tanh\ldots}{d^2}$$

$$= \frac{b\left(a+b\tan^{-1}(cx)\right)}{d^2(i-cx)} - \frac{\left(a+b\tan^{-1}(cx)\right)^2}{2d^2} + \frac{i\left(a+b\tan^{-1}(cx)\right)^2}{d^2(i-cx)} + \frac{2\left(a+b\tan^{-1}(cx)\right)^2\tanh\ldots}{d^2}$$

$$= \frac{b\left(a+b\tan^{-1}(cx)\right)}{d^2(i-cx)} - \frac{\left(a+b\tan^{-1}(cx)\right)^2}{2d^2} + \frac{i\left(a+b\tan^{-1}(cx)\right)^2}{d^2(i-cx)} + \frac{2\left(a+b\tan^{-1}(cx)\right)^2\tanh\ldots}{d^2}$$

$$= -\frac{ib^2}{2d^2(i-cx)} + \frac{b\left(a+b\tan^{-1}(cx)\right)}{d^2(i-cx)} - \frac{\left(a+b\tan^{-1}(cx)\right)^2}{2d^2} + \frac{i\left(a+b\tan^{-1}(cx)\right)^2}{d^2(i-cx)} + \frac{2\left(a+b\,t\ldots\right.}{}$$

$$= -\frac{ib^2}{2d^2(i-cx)} + \frac{ib^2\tan^{-1}(cx)}{2d^2} + \frac{b\left(a+b\tan^{-1}(cx)\right)}{d^2(i-cx)} - \frac{\left(a+b\tan^{-1}(cx)\right)^2}{2d^2} + \frac{i\left(a+b\tan^{-1}(c\ldots\right.}{d^2(i-cx)}$$

Mathematica [A] time = 0.948125, size = 299, normalized size = 1.35

$$\frac{-12ab\left(2i\text{PolyLog}\left(2,e^{2i\tan^{-1}(cx)}\right)+4i\tan^{-1}(cx)^2+\sin\left(2\tan^{-1}(cx)\right)+i\cos\left(2\tan^{-1}(cx)\right)-2\tan^{-1}(cx)\left(2\log\left(1-e^{2\ldots}\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c*x])^2/(x*(d + I*c*d*x)^2),x]
```

```
[Out] (((-24*I)*a^2)/(-I + c*x) - (24*I)*a^2*ArcTan[c*x] + 24*a^2*Log[c*x] - 12*a
^2*Log[1 + c^2*x^2] - 12*a*b*((4*I)*ArcTan[c*x]^2 + I*Cos[2*ArcTan[c*x]] +
(2*I)*PolyLog[2, E^((2*I)*ArcTan[c*x])] - 2*ArcTan[c*x]*(Cos[2*ArcTan[c*x]]
```

```
 + 2*Log[1 - E^((2*I)*ArcTan[c*x])] - I*Sin[2*ArcTan[c*x]]) + Sin[2*ArcTan[
c*x]]) + b^2*((-I)*Pi^3 - 6*Cos[2*ArcTan[c*x]] - (12*I)*ArcTan[c*x]*Cos[2*A
rcTan[c*x]] + 12*ArcTan[c*x]^2*Cos[2*ArcTan[c*x]] + 24*ArcTan[c*x]^2*Log[1
- E^((-2*I)*ArcTan[c*x])] + (24*I)*ArcTan[c*x]*PolyLog[2, E^((-2*I)*ArcTan[
c*x])] + 12*PolyLog[3, E^((-2*I)*ArcTan[c*x])] + (6*I)*Sin[2*ArcTan[c*x]] -
 12*ArcTan[c*x]*Sin[2*ArcTan[c*x]] - (12*I)*ArcTan[c*x]^2*Sin[2*ArcTan[c*x]
]))/(24*d^2)
```

Maple [C] time = 0.416, size = 1921, normalized size = 8.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^2/x/(d+I*c*d*x)^2,x)
```

```
[Out] 1/2*I*b^2/d^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I/((1+I*c*x)^2/(c
^2*x^2+1)+1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1
))*arctan(c*x)^2-1/2*I*b^2/d^2*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2
/(c^2*x^2+1)+1))*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn((1+I*c*x)^2/(c^2*
x^2+1))*arctan(c*x)^2+a^2/d^2*ln(c*x)+2*b^2/d^2*polylog(3,(1+I*c*x)/(c^2*x^
2+1)^(1/2))+2*b^2/d^2*polylog(3,-(1+I*c*x)/(c^2*x^2+1)^(1/2))-1/2*a^2/d^2*l
n(c^2*x^2+1)-1/2*I*b^2/d^2*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^
2*x^2+1)+1))^3*arctan(c*x)^2-I*b^2/d^2*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+
I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2+I*a*b/d^2*ln(-1/2*I*(c*x+I))*ln(c*
x-I)+1/2*I*b^2/d^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*
x^2+1)+1))^3*arctan(c*x)^2+1/2*I*b^2/d^2*Pi*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1
)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*arctan(c*x)^2-I*a^2/d^2/(c*x-I)+b^2/d^2*ar
ctan(c*x)^2*ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))+b^2/d^2*arctan(c*x)^2*ln(1-(1
+I*c*x)/(c^2*x^2+1)^(1/2))-b^2/d^2*arctan(c*x)^2*ln((1+I*c*x)^2/(c^2*x^2+1)
-1)+b^2/d^2*arctan(c*x)^2*ln(c*x)+1/4*I*b^2/d^2/(c*x-I)+1/2*I*b^2/d^2*Pi*cs
gn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*csgn(I/((1+I*c*x)
^2/(c^2*x^2+1)+1))*arctan(c*x)^2-1/2*I*b^2/d^2*Pi*csgn((1+I*c*x)^2/(c^2*x^2
+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*csgn((1+I*c*x)^2/(c^2*x^2+1))*arctan(c*x
)^2+1/2*I*b^2/d^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x
^2+1)+1))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*arc
tan(c*x)^2-1/2*I*b^2/d^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I*((1+
I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2-1/2*I*
b^2/d^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*
csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)
^2-1/2*I*b^2/d^2*Pi*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*((1+I*c*x)^2
/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2-1/2*I*b^2/d^2*
```

```
Pi*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c
*x)^2-2*I*a*b/d^2*arctan(c*x)/(c*x-I)-I*a*b/d^2*ln(c*x)*ln(1-I*c*x)+I*a*b/d
^2*ln(c*x)*ln(1+I*c*x)+2*I*b^2/d^2*arctan(c*x)/(4*c*x-4*I)*c*x-1/2*b^2/d^2*
arctan(c*x)^2-I*a^2/d^2*arctan(c*x)-2*b^2/d^2*arctan(c*x)/(4*c*x-4*I)-2/3*I
*b^2/d^2*arctan(c*x)^3-a*b/d^2/(c*x-I)-a*b/d^2*arctan(c*x)-b^2/d^2*arctan(c
*x)^2*ln(c*x-I)+b^2/d^2*arctan(c*x)^2*ln(2*I*(1+I*c*x)^2/(c^2*x^2+1))-2*I*b
^2/d^2*arctan(c*x)*polylog(2,-(1+I*c*x)/(c^2*x^2+1)^(1/2))-2*I*b^2/d^2*arct
an(c*x)*polylog(2,(1+I*c*x)/(c^2*x^2+1)^(1/2))+3/2*I*b^2/d^2*Pi*arctan(c*x)
^2-I*a*b/d^2*dilog(1-I*c*x)-2*a*b/d^2*arctan(c*x)*ln(c*x-I)+1/4*b^2/d^2/(c*
x-I)*c*x+2*a*b/d^2*arctan(c*x)*ln(c*x)+I*a*b/d^2*dilog(1+I*c*x)-I*b^2/d^2*a
rctan(c*x)^2/(c*x-I)+I*a*b/d^2*dilog(-1/2*I*(c*x+I))-1/2*I*a*b/d^2*ln(c*x-I
)^2
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x/(d+I*c*d*x)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2\log\left(-\frac{cx+i}{cx-i}\right)^2 - 4i\,ab\log\left(-\frac{cx+i}{cx-i}\right) - 4\,a^2}{4\,c^2d^2x^3 - 8i\,cd^2x^2 - 4\,d^2x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x/(d+I*c*d*x)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*log(-(c*x + I)/(c*x - I))^2 - 4*I*a*b*log(-(c*x + I)/(c*x - I
)) - 4*a^2)/(4*c^2*d^2*x^3 - 8*I*c*d^2*x^2 - 4*d^2*x), x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**2/x/(d+I*c*d*x)**2,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\arctan(cx) + a)^2}{(icdx + d)^2 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x/(d+I*c*d*x)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)^2/((I*c*d*x + d)^2*x), x)
```

3.109 $\int \frac{(a+b\tan^{-1}(cx))^2}{x^2(d+icdx)^2}\,dx$

Optimal. Leaf size=306

$$\frac{2bc\text{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{d^2}-\frac{ib^2c\text{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)}{d^2}-\frac{ib^2c\text{PolyLog}\left(3,-1+\frac{2}{1+icx}\right)}{d^2}-\frac{ibc\left(a+\right.}{d^2(}$$

```
[Out] -(b^2*c)/(2*d^2*(I - c*x)) + (b^2*c*ArcTan[c*x])/(2*d^2) - (I*b*c*(a + b*Ar
cTan[c*x]))/(d^2*(I - c*x)) - ((I/2)*c*(a + b*ArcTan[c*x])^2)/d^2 - (a + b*
ArcTan[c*x])^2/(d^2*x) + (c*(a + b*ArcTan[c*x])^2)/(d^2*(I - c*x)) - ((4*I)
*c*(a + b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1 + I*c*x)])/d^2 - ((2*I)*c*(a + b*
ArcTan[c*x])^2*Log[2/(1 + I*c*x)])/d^2 + (2*b*c*(a + b*ArcTan[c*x])*Log[2 -
 2/(1 - I*c*x)])/d^2 - (I*b^2*c*PolyLog[2, -1 + 2/(1 - I*c*x)])/d^2 + (2*b*
c*(a + b*ArcTan[c*x])*PolyLog[2, -1 + 2/(1 + I*c*x)])/d^2 - (I*b^2*c*PolyLo
g[3, -1 + 2/(1 + I*c*x)])/d^2
```

Rubi [A] time = 0.767048, antiderivative size = 306, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 16, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.64, Rules used = {4876, 4852, 4924, 4868, 2447, 4850, 4988, 4884, 4994, 6610, 4864, 4862, 627, 44, 203, 4854}

$$\frac{2bc\text{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{d^2}-\frac{ib^2c\text{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)}{d^2}-\frac{ib^2c\text{PolyLog}\left(3,-1+\frac{2}{1+icx}\right)}{d^2}-\frac{ibc\left(a+\right.}{d^2(}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^2/(x^2*(d + I*c*d*x)^2),x]
```


```
[Out] -(b^2*c)/(2*d^2*(I - c*x)) + (b^2*c*ArcTan[c*x])/(2*d^2) - (I*b*c*(a + b*Ar
cTan[c*x]))/(d^2*(I - c*x)) - ((I/2)*c*(a + b*ArcTan[c*x])^2)/d^2 - (a + b*
ArcTan[c*x])^2/(d^2*x) + (c*(a + b*ArcTan[c*x])^2)/(d^2*(I - c*x)) - ((4*I)
*c*(a + b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1 + I*c*x)])/d^2 - ((2*I)*c*(a + b*
ArcTan[c*x])^2*Log[2/(1 + I*c*x)])/d^2 + (2*b*c*(a + b*ArcTan[c*x])*Log[2 -
 2/(1 - I*c*x)])/d^2 - (I*b^2*c*PolyLog[2, -1 + 2/(1 - I*c*x)])/d^2 + (2*b*
c*(a + b*ArcTan[c*x])*PolyLog[2, -1 + 2/(1 + I*c*x)])/d^2 - (I*b^2*c*PolyLo
g[3, -1 + 2/(1 + I*c*x)])/d^2
```


Rule 4876

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4850

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTan[c*x])^(p - 1)*ArcTanh[1 - 2/(1 + I*c*x)])/(1 + c^2*x^2), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4988

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTan[c*x])^p)/(d + e
*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTan[c*x])^p)/(d + e*x^2
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 4864

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x])^p)/(e*(q + 1)), x] - D
ist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 4862

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x]))/(e*(q + 1)), x] - Dist[(b*
c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
```

```
rQ[m + p]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^2(d+icdx)^2}\,dx = \int\left(\frac{\left(a+b\tan^{-1}(cx)\right)^2}{d^2x^2} - \frac{2ic\left(a+b\tan^{-1}(cx)\right)^2}{d^2x} + \frac{c^2\left(a+b\tan^{-1}(cx)\right)^2}{d^2(-i+cx)^2} + \frac{2ic^2\left(a+b\tan^{-1}\right.}{d^2(-i+cx)}\right.$$

$$= \frac{\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^2}\,dx}{d^2} - \frac{(2ic)\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x}\,dx}{d^2} + \frac{\left(2ic^2\right)\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{-i+cx}\,dx}{d^2} + \frac{c^2\int \frac{\left(a+b\tan^{-1}(\right.}{(-i+cx)^2}}{d^2}$$

$$= -\frac{\left(a+b\tan^{-1}(cx)\right)^2}{d^2x} + \frac{c\left(a+b\tan^{-1}(cx)\right)^2}{d^2(i-cx)} - \frac{4ic\left(a+b\tan^{-1}(cx)\right)^2\tanh^{-1}\left(1-\frac{2}{1+icx}\right)}{d^2} - \frac{2}{}$$

$$= -\frac{ic\left(a+b\tan^{-1}(cx)\right)^2}{d^2} - \frac{\left(a+b\tan^{-1}(cx)\right)^2}{d^2x} + \frac{c\left(a+b\tan^{-1}(cx)\right)^2}{d^2(i-cx)} - \frac{4ic\left(a+b\tan^{-1}(cx)\right)^2}{d^2}$$

$$= -\frac{ibc\left(a+b\tan^{-1}(cx)\right)}{d^2(i-cx)} - \frac{ic\left(a+b\tan^{-1}(cx)\right)^2}{2d^2} - \frac{\left(a+b\tan^{-1}(cx)\right)^2}{d^2x} + \frac{c\left(a+b\tan^{-1}(cx)\right)^2}{d^2(i-cx)}$$

$$= -\frac{ibc\left(a+b\tan^{-1}(cx)\right)}{d^2(i-cx)} - \frac{ic\left(a+b\tan^{-1}(cx)\right)^2}{2d^2} - \frac{\left(a+b\tan^{-1}(cx)\right)^2}{d^2x} + \frac{c\left(a+b\tan^{-1}(cx)\right)^2}{d^2(i-cx)}$$

$$= -\frac{ibc\left(a+b\tan^{-1}(cx)\right)}{d^2(i-cx)} - \frac{ic\left(a+b\tan^{-1}(cx)\right)^2}{2d^2} - \frac{\left(a+b\tan^{-1}(cx)\right)^2}{d^2x} + \frac{c\left(a+b\tan^{-1}(cx)\right)^2}{d^2(i-cx)}$$

$$= -\frac{b^2c}{2d^2(i-cx)} - \frac{ibc\left(a+b\tan^{-1}(cx)\right)}{d^2(i-cx)} - \frac{ic\left(a+b\tan^{-1}(cx)\right)^2}{2d^2} - \frac{\left(a+b\tan^{-1}(cx)\right)^2}{d^2x} + \frac{c\left(a+\right.}{d^2}$$

$$= -\frac{b^2c}{2d^2(i-cx)} + \frac{b^2c\tan^{-1}(cx)}{2d^2} - \frac{ibc\left(a+b\tan^{-1}(cx)\right)}{d^2(i-cx)} - \frac{ic\left(a+b\tan^{-1}(cx)\right)^2}{2d^2} - \frac{\left(a+b\tan^{-1}\right.}{d^2x}$$

Mathematica [A] time = 2.57128, size = 398, normalized size = 1.3

$$-\frac{6abc\left(4\text{PolyLog}\left(2,e^{2i\tan^{-1}(cx)}\right)-4\log\left(\frac{cx}{\sqrt{c^2x^2+1}}\right)+8\tan^{-1}(cx)^2-i\sin\left(2\tan^{-1}(cx)\right)+\cos\left(2\tan^{-1}(cx)\right)+\tan^{-1}(cx\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c*x])^2/(x^2*(d + I*c*d*x)^2),x]
```

```
[Out] -((12*a^2)/x + (12*a^2*c)/(-I + c*x) + 24*a^2*c*ArcTan[c*x] + (24*I)*a^2*c*
Log[c*x] - (12*I)*a^2*c*Log[1 + c^2*x^2] + b^2*c*(Pi^3 + (12*I)*ArcTan[c*x]
^2 + (12*ArcTan[c*x]^2)/(c*x) - (3*I)*Cos[2*ArcTan[c*x]] + 6*ArcTan[c*x]*Co
```

```
s[2*ArcTan[c*x]] + (6*I)*ArcTan[c*x]^2*Cos[2*ArcTan[c*x]] + (24*I)*ArcTan[c
*x]^2*Log[1 - E^((-2*I)*ArcTan[c*x])] - 24*ArcTan[c*x]*Log[1 - E^((2*I)*Arc
Tan[c*x])] - 24*ArcTan[c*x]*PolyLog[2, E^((-2*I)*ArcTan[c*x])] + (12*I)*Pol
yLog[2, E^((2*I)*ArcTan[c*x])] + (12*I)*PolyLog[3, E^((-2*I)*ArcTan[c*x])]
- 3*Sin[2*ArcTan[c*x]] - (6*I)*ArcTan[c*x]*Sin[2*ArcTan[c*x]] + 6*ArcTan[c*
x]^2*Sin[2*ArcTan[c*x]]) + 6*a*b*c*(8*ArcTan[c*x]^2 + Cos[2*ArcTan[c*x]] -
4*Log[(c*x)/Sqrt[1 + c^2*x^2]] + 4*PolyLog[2, E^((2*I)*ArcTan[c*x])] - I*Si
n[2*ArcTan[c*x]] + ArcTan[c*x]*(4/(c*x) + (2*I)*Cos[2*ArcTan[c*x]] + (8*I)*
Log[1 - E^((2*I)*ArcTan[c*x])] + 2*Sin[2*ArcTan[c*x]])))/(12*d^2)
```

Maple [C] time = 0.996, size = 9420, normalized size = 30.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^2/x^2/(d+I*c*d*x)^2,x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^2/(d+I*c*d*x)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2\log\left(-\frac{cx+i}{cx-i}\right)^2 - 4i\,ab\log\left(-\frac{cx+i}{cx-i}\right) - 4\,a^2}{4\,c^2d^2x^4 - 8i\,cd^2x^3 - 4\,d^2x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^2/(d+I*c*d*x)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*log(-(c*x + I)/(c*x - I))^2 - 4*I*a*b*log(-(c*x + I)/(c*x - I
)) - 4*a^2)/(4*c^2*d^2*x^4 - 8*I*c*d^2*x^3 - 4*d^2*x^2), x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**2/x**2/(d+I*c*d*x)**2,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \arctan(cx) + a)^2}{(icdx + d)^2 x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^2/(d+I*c*d*x)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)^2/((I*c*d*x + d)^2*x^2), x)
```

3.110 $\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^3(d+icdx)^2}\,dx$

Optimal. Leaf size=403

$$-\frac{3ibc^2\text{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{d^2}-\frac{2b^2c^2\text{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)}{d^2}-\frac{3b^2c^2\text{PolyLog}\left(3,-1+\frac{2}{1+icx}\right)}{2d^2}-\frac{ic^2}{}$$

```
[Out] ((I/2)*b^2*c^2)/(d^2*(I - c*x)) - ((I/2)*b^2*c^2*ArcTan[c*x])/d^2 - (b*c*(a
 + b*ArcTan[c*x]))/(d^2*x) - (b*c^2*(a + b*ArcTan[c*x]))/(d^2*(I - c*x)) -
(2*c^2*(a + b*ArcTan[c*x])^2)/d^2 - (a + b*ArcTan[c*x])^2/(2*d^2*x^2) + ((2
*I)*c*(a + b*ArcTan[c*x])^2)/(d^2*x) - (I*c^2*(a + b*ArcTan[c*x])^2)/(d^2*(
I - c*x)) - (6*c^2*(a + b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1 + I*c*x)])/d^2 +
(b^2*c^2*Log[x])/d^2 - (3*c^2*(a + b*ArcTan[c*x])^2*Log[2/(1 + I*c*x)])/d^2
 - (b^2*c^2*Log[1 + c^2*x^2])/(2*d^2) - ((4*I)*b*c^2*(a + b*ArcTan[c*x])*Lo
g[2 - 2/(1 - I*c*x)])/d^2 - (2*b^2*c^2*PolyLog[2, -1 + 2/(1 - I*c*x)])/d^2
- ((3*I)*b*c^2*(a + b*ArcTan[c*x])*PolyLog[2, -1 + 2/(1 + I*c*x)])/d^2 - (3
*b^2*c^2*PolyLog[3, -1 + 2/(1 + I*c*x)])/(2*d^2)
```

Rubi [A] time = 0.933897, antiderivative size = 403, normalized size of antiderivative = 1., number of steps used = 31, number of rules used = 21, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.84, Rules used = {4876, 4852, 4918, 266, 36, 29, 31, 4884, 4924, 4868, 2447, 4850, 4988, 4994, 6610, 4864, 4862, 627, 44, 203, 4854}

$$-\frac{3ibc^2\text{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{d^2}-\frac{2b^2c^2\text{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)}{d^2}-\frac{3b^2c^2\text{PolyLog}\left(3,-1+\frac{2}{1+icx}\right)}{2d^2}-\frac{ic^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^2/(x^3*(d + I*c*d*x)^2),x]
```


```
[Out] ((I/2)*b^2*c^2)/(d^2*(I - c*x)) - ((I/2)*b^2*c^2*ArcTan[c*x])/d^2 - (b*c*(a
 + b*ArcTan[c*x]))/(d^2*x) - (b*c^2*(a + b*ArcTan[c*x]))/(d^2*(I - c*x)) -
(2*c^2*(a + b*ArcTan[c*x])^2)/d^2 - (a + b*ArcTan[c*x])^2/(2*d^2*x^2) + ((2
*I)*c*(a + b*ArcTan[c*x])^2)/(d^2*x) - (I*c^2*(a + b*ArcTan[c*x])^2)/(d^2*(
I - c*x)) - (6*c^2*(a + b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1 + I*c*x)])/d^2 +
(b^2*c^2*Log[x])/d^2 - (3*c^2*(a + b*ArcTan[c*x])^2*Log[2/(1 + I*c*x)])/d^2
 - (b^2*c^2*Log[1 + c^2*x^2])/(2*d^2) - ((4*I)*b*c^2*(a + b*ArcTan[c*x])*Lo
g[2 - 2/(1 - I*c*x)])/d^2 - (2*b^2*c^2*PolyLog[2, -1 + 2/(1 - I*c*x)])/d^2
- ((3*I)*b*c^2*(a + b*ArcTan[c*x])*PolyLog[2, -1 + 2/(1 + I*c*x)])/d^2 - (3
```

```
*b^2*c^2*PolyLog[3, -1 + 2/(1 + I*c*x)])/(2*d^2)
```

Rule 4876

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4918

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/(d + e*x^2),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4850

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTan[c*x])^(p - 1)*ArcTanh[1 - 2/(1 + I*c*x)])/(1 + c^2*x^2), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4988

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTan[c*x])^p)/(d + e
*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTan[c*x])^p)/(d + e*x^2
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 4864

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x])^p)/(e*(q + 1)), x] - D
ist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 4862

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x]))/(e*(q + 1)), x] - Dist[(b*
c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^3(d+icdx)^2}\,dx = \int\left(\frac{\left(a+b\tan^{-1}(cx)\right)^2}{d^2x^3} - \frac{2ic\left(a+b\tan^{-1}(cx)\right)^2}{d^2x^2} - \frac{3c^2\left(a+b\tan^{-1}(cx)\right)^2}{d^2x} - \frac{ic^3\left(a+b\tan^{-1}(c\right.}{d^2(-i+cx)^2}\right.$$

$$= \frac{\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^3}\,dx}{d^2} - \frac{(2ic)\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^2}\,dx}{d^2} - \frac{\left(3c^2\right)\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x}\,dx}{d^2} - \frac{\left(ic^3\right)\int \frac{\left(a+b\tan^{-1}(\right.}{(-i+cx)^2}}{d^2}$$

$$= -\frac{\left(a+b\tan^{-1}(cx)\right)^2}{2d^2x^2} + \frac{2ic\left(a+b\tan^{-1}(cx)\right)^2}{d^2x} - \frac{ic^2\left(a+b\tan^{-1}(cx)\right)^2}{d^2(i-cx)} - \frac{6c^2\left(a+b\tan^{-1}(cx)\right.}{d}$$

$$= -\frac{2c^2\left(a+b\tan^{-1}(cx)\right)^2}{d^2} - \frac{\left(a+b\tan^{-1}(cx)\right)^2}{2d^2x^2} + \frac{2ic\left(a+b\tan^{-1}(cx)\right)^2}{d^2x} - \frac{ic^2\left(a+b\tan^{-1}(cx)\right.}{d^2(i-cx)}$$

$$= -\frac{bc\left(a+b\tan^{-1}(cx)\right)}{d^2x} - \frac{bc^2\left(a+b\tan^{-1}(cx)\right)}{d^2(i-cx)} - \frac{2c^2\left(a+b\tan^{-1}(cx)\right)^2}{d^2} - \frac{\left(a+b\tan^{-1}(cx)\right)^2}{2d^2x^2}$$

$$= -\frac{bc\left(a+b\tan^{-1}(cx)\right)}{d^2x} - \frac{bc^2\left(a+b\tan^{-1}(cx)\right)}{d^2(i-cx)} - \frac{2c^2\left(a+b\tan^{-1}(cx)\right)^2}{d^2} - \frac{\left(a+b\tan^{-1}(cx)\right)^2}{2d^2x^2}$$

$$= -\frac{bc\left(a+b\tan^{-1}(cx)\right)}{d^2x} - \frac{bc^2\left(a+b\tan^{-1}(cx)\right)}{d^2(i-cx)} - \frac{2c^2\left(a+b\tan^{-1}(cx)\right)^2}{d^2} - \frac{\left(a+b\tan^{-1}(cx)\right)^2}{2d^2x^2}$$

$$= \frac{ib^2c^2}{2d^2(i-cx)} - \frac{bc\left(a+b\tan^{-1}(cx)\right)}{d^2x} - \frac{bc^2\left(a+b\tan^{-1}(cx)\right)}{d^2(i-cx)} - \frac{2c^2\left(a+b\tan^{-1}(cx)\right)^2}{d^2} - \frac{\left(a+b\right.}{2}$$

$$= \frac{ib^2c^2}{2d^2(i-cx)} - \frac{ib^2c^2\tan^{-1}(cx)}{2d^2} - \frac{bc\left(a+b\tan^{-1}(cx)\right)}{d^2x} - \frac{bc^2\left(a+b\tan^{-1}(cx)\right)}{d^2(i-cx)} - \frac{2c^2\left(a+b\tan\right.}{d^2}$$

Mathematica [A] time = 2.90155, size = 491, normalized size = 1.22

$$\frac{4iabc^2\left(6\text{PolyLog}\left(2,e^{2i\tan^{-1}(cx)}\right)-8\log\left(\frac{cx}{\sqrt{c^2x^2+1}}\right)+2\tan^{-1}(cx)\left(\frac{i}{c^2x^2}+\frac{4}{cx}+6i\log\left(1-e^{2i\tan^{-1}(cx)}\right)+\sin\left(2\tan^{-1}(cx\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c*x])^2/(x^3*(d + I*c*d*x)^2),x]
```

```
[Out] ((-4*a^2)/x^2 + ((16*I)*a^2*c)/x + ((8*I)*a^2*c^2)/(-I + c*x) + (24*I)*a^2*
c^2*ArcTan[c*x] - 24*a^2*c^2*Log[x] + 12*a^2*c^2*Log[1 + c^2*x^2] - b^2*c^2
*((-I)*Pi^3 + (8*ArcTan[c*x])/(c*x) + 20*ArcTan[c*x]^2 + (4*ArcTan[c*x]^2)/
(c^2*x^2) - ((16*I)*ArcTan[c*x]^2)/(c*x) - 2*Cos[2*ArcTan[c*x]] - (4*I)*Arc
Tan[c*x]*Cos[2*ArcTan[c*x]] + 4*ArcTan[c*x]^2*Cos[2*ArcTan[c*x]] + 24*ArcTa
n[c*x]^2*Log[1 - E^((-2*I)*ArcTan[c*x])] + (32*I)*ArcTan[c*x]*Log[1 - E^((2
*I)*ArcTan[c*x])] - 8*Log[(c*x)/Sqrt[1 + c^2*x^2]] + (24*I)*ArcTan[c*x]*Pol
yLog[2, E^((-2*I)*ArcTan[c*x])] + 16*PolyLog[2, E^((2*I)*ArcTan[c*x])] + 12
*PolyLog[3, E^((-2*I)*ArcTan[c*x])] + (2*I)*Sin[2*ArcTan[c*x]] - 4*ArcTan[c
*x]*Sin[2*ArcTan[c*x]] - (4*I)*ArcTan[c*x]^2*Sin[2*ArcTan[c*x]]) + (4*I)*a*
b*c^2*((2*I)/(c*x) + 12*ArcTan[c*x]^2 + Cos[2*ArcTan[c*x]] - 8*Log[(c*x)/Sq
rt[1 + c^2*x^2]] + 6*PolyLog[2, E^((2*I)*ArcTan[c*x])] - I*Sin[2*ArcTan[c*x
]] + 2*ArcTan[c*x]*(I + I/(c^2*x^2) + 4/(c*x) + I*Cos[2*ArcTan[c*x]] + (6*I
)*Log[1 - E^((2*I)*ArcTan[c*x])] + Sin[2*ArcTan[c*x]])))/(8*d^2)
```

Maple [C] time = 2.811, size = 2384, normalized size = 5.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^2/x^3/(d+I*c*d*x)^2,x)
```

```
[Out] -I*c^2*b^2/d^2*arctan(c*x)+2*I*c^2*b^2/d^2*arctan(c*x)^3+2*I*c*a^2/d^2/x-1/
4*I*c^2*b^2/d^2/(c*x-I)-3/2*I*c^2*b^2/d^2*Pi*csgn(I/((1+I*c*x)^2/(c^2*x^2+1
)+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*
x)^2+3/2*I*c^2*b^2/d^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(
c^2*x^2+1)+1))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1)
)^2*arctan(c*x)^2+3/2*I*c^2*b^2/d^2*Pi*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*
csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*
x)^2+3/2*I*c^2*b^2/d^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I*((1+I*
c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2-3/2*I*c^
```

```
2*b^2/d^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1)
)*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)
^2+3/2*I*c^2*b^2/d^2*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2
*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2+3/2*I*c^2*b^2/d^2*Pi*c
sgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*
c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)^2-3/2*I*c^2*b^2
/d^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)
+1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan
(c*x)^2-3/2*I*c^2*b^2/d^2*Pi*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/
(c^2*x^2+1)+1))^3*arctan(c*x)^2+3*I*c^2*b^2/d^2*Pi*csgn((1+I*c*x)^2/(c^2*x^
2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2+3/2*I*c^2*b^2/d^2*Pi*csgn
((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*arctan(c*x)^2-3*I*c
^2*a*b/d^2*ln(c*x)*ln(1+I*c*x)-3*I*c^2*a*b/d^2*ln(-1/2*I*(c*x+I))*ln(c*x-I)
+3*I*c^2*a*b/d^2*ln(c*x)*ln(1-I*c*x)-I*c^3*b^2/d^2*arctan(c*x)/(2*c*x-2*I)*
x+4*I*c*a*b/d^2*arctan(c*x)/x+2*I*c^2*a*b/d^2*arctan(c*x)/(c*x-I)+3/2*I*c^2
*b^2/d^2*Pi*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2
*arctan(c*x)^2-1/2*b^2/d^2*arctan(c*x)^2/x^2-2*c^2*b^2/d^2*arctan(c*x)^2-3*
c^2*a^2/d^2*ln(c*x)-6*c^2*b^2/d^2*polylog(3,(1+I*c*x)/(c^2*x^2+1)^(1/2))-6*
c^2*b^2/d^2*polylog(3,-(1+I*c*x)/(c^2*x^2+1)^(1/2))+3/2*c^2*a^2/d^2*ln(c^2*
x^2+1)+c^2*b^2/d^2*ln((1+I*c*x)/(c^2*x^2+1)^(1/2)-1)+c^2*b^2/d^2*ln(1+(1+I*
c*x)/(c^2*x^2+1)^(1/2))-4*c^2*b^2/d^2*dilog(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))+
4*c^2*b^2/d^2*dilog((1+I*c*x)/(c^2*x^2+1)^(1/2))-3/2*I*c^2*b^2/d^2*Pi*csgn(
I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*arctan(c*x)^2-
c*a*b/d^2/x-3*c^2*b^2/d^2*arctan(c*x)^2*ln(1-(1+I*c*x)/(c^2*x^2+1)^(1/2))+3
*c^2*b^2/d^2*arctan(c*x)^2*ln((1+I*c*x)^2/(c^2*x^2+1)-1)-3*c^2*b^2/d^2*arct
an(c*x)^2*ln(c*x)+3*c^2*b^2/d^2*arctan(c*x)^2*ln(c*x-I)-3*c^2*b^2/d^2*arcta
n(c*x)^2*ln(2*I*(1+I*c*x)^2/(c^2*x^2+1))+c^2*b^2/d^2*arctan(c*x)/(2*c*x-2*I
)-3*c^2*b^2/d^2*arctan(c*x)^2*ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))-c*b^2/d^2*a
rctan(c*x)/x-1/4*c^3*b^2/d^2/(c*x-I)*x+I*c^2*a^2/d^2/(c*x-I)+c^2*a*b/d^2/(c
*x-I)-a*b/d^2*arctan(c*x)/x^2+3*I*c^2*a^2/d^2*arctan(c*x)-1/2*a^2/d^2/x^2-6
*c^2*a*b/d^2*arctan(c*x)*ln(c*x)+6*c^2*a*b/d^2*arctan(c*x)*ln(c*x-I)+I*c^2*
b^2/d^2*arctan(c*x)^2/(c*x-I)+3/2*I*c^2*a*b/d^2*ln(c*x-I)^2-4*I*c^2*a*b/d^2
*ln(c*x)-3*I*c^2*a*b/d^2*dilog(-1/2*I*(c*x+I))-3*I*c^2*a*b/d^2*dilog(1+I*c*
x)+3*I*c^2*a*b/d^2*dilog(1-I*c*x)-9/2*I*c^2*b^2/d^2*Pi*arctan(c*x)^2+6*I*c^
2*b^2/d^2*arctan(c*x)*polylog(2,-(1+I*c*x)/(c^2*x^2+1)^(1/2))+6*I*c^2*b^2/d
^2*arctan(c*x)*polylog(2,(1+I*c*x)/(c^2*x^2+1)^(1/2))-4*I*c^2*b^2/d^2*arcta
n(c*x)*ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))+2*I*c^2*a*b/d^2*ln(c^2*x^2+1)+2*I*
c*b^2/d^2*arctan(c*x)^2/x
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^3/(d+I*c*d*x)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2\log\left(-\frac{cx+i}{cx-i}\right)^2 - 4i\,ab\log\left(-\frac{cx+i}{cx-i}\right) - 4\,a^2}{4\,c^2d^2x^5 - 8i\,cd^2x^4 - 4\,d^2x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^3/(d+I*c*d*x)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*log(-(c*x + I)/(c*x - I))^2 - 4*I*a*b*log(-(c*x + I)/(c*x - I
)) - 4*a^2)/(4*c^2*d^2*x^5 - 8*I*c*d^2*x^4 - 4*d^2*x^3), x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**2/x**3/(d+I*c*d*x)**2,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\arctan(cx) + a)^2}{(i\,cdx + d)^2x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^3/(d+I*c*d*x)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)^2/((I*c*d*x + d)^2*x^3), x)
```

3.111 $\int \frac{x^4\left(a+b\tan^{-1}(cx)\right)^2}{(d+icdx)^3}\,dx$

Optimal. Leaf size=462

$$-\frac{6b\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{c^5d^3}-\frac{3ib^2\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c^5d^3}+\frac{3ib^2\text{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{c^5d^3}+\frac{ix^2\left(a+b\tan\right.}{2c^3}$$

```
[Out] ((-I)*a*b*x)/(c^4*d^3) + ((I/16)*b^2)/(c^5*d^3*(I - c*x)^2) - (29*b^2)/(16*
c^5*d^3*(I - c*x)) + (29*b^2*ArcTan[c*x])/(16*c^5*d^3) - (I*b^2*x*ArcTan[c*
x])/(c^4*d^3) - (b*(a + b*ArcTan[c*x]))/(4*c^5*d^3*(I - c*x)^2) - (((15*I)/
4)*b*(a + b*ArcTan[c*x]))/(c^5*d^3*(I - c*x)) - (((5*I)/8)*(a + b*ArcTan[c*
x])^2)/(c^5*d^3) - (3*x*(a + b*ArcTan[c*x])^2)/(c^4*d^3) + ((I/2)*x^2*(a +
b*ArcTan[c*x])^2)/(c^3*d^3) - ((I/2)*(a + b*ArcTan[c*x])^2)/(c^5*d^3*(I - c
*x)^2) + (4*(a + b*ArcTan[c*x])^2)/(c^5*d^3*(I - c*x)) - (6*b*(a + b*ArcTan
[c*x])*Log[2/(1 + I*c*x)])/(c^5*d^3) + ((6*I)*(a + b*ArcTan[c*x])^2*Log[2/(
1 + I*c*x)])/(c^5*d^3) + ((I/2)*b^2*Log[1 + c^2*x^2])/(c^5*d^3) - ((3*I)*b^
2*PolyLog[2, 1 - 2/(1 + I*c*x)])/(c^5*d^3) - (6*b*(a + b*ArcTan[c*x])*PolyL
og[2, 1 - 2/(1 + I*c*x)])/(c^5*d^3) + ((3*I)*b^2*PolyLog[3, 1 - 2/(1 + I*c*
x)])/(c^5*d^3)
```

Rubi [A] time = 0.832922, antiderivative size = 462, normalized size of antiderivative = 1., number of steps used = 37, number of rules used = 17, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.68, Rules used = {4876, 4846, 4920, 4854, 2402, 2315, 4852, 4916, 260, 4884, 4864, 4862, 627, 44, 203, 4994, 6610}

$$-\frac{6b\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{c^5d^3}-\frac{3ib^2\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c^5d^3}+\frac{3ib^2\text{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{c^5d^3}+\frac{ix^2\left(a+b\tan\right.}{2c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcTan[c*x])^2)/(d + I*c*d*x)^3,x]
```


```
[Out] ((-I)*a*b*x)/(c^4*d^3) + ((I/16)*b^2)/(c^5*d^3*(I - c*x)^2) - (29*b^2)/(16*
c^5*d^3*(I - c*x)) + (29*b^2*ArcTan[c*x])/(16*c^5*d^3) - (I*b^2*x*ArcTan[c*
x])/(c^4*d^3) - (b*(a + b*ArcTan[c*x]))/(4*c^5*d^3*(I - c*x)^2) - (((15*I)/
4)*b*(a + b*ArcTan[c*x]))/(c^5*d^3*(I - c*x)) - (((5*I)/8)*(a + b*ArcTan[c*
x])^2)/(c^5*d^3) - (3*x*(a + b*ArcTan[c*x])^2)/(c^4*d^3) + ((I/2)*x^2*(a +
b*ArcTan[c*x])^2)/(c^3*d^3) - ((I/2)*(a + b*ArcTan[c*x])^2)/(c^5*d^3*(I - c
*x)^2) + (4*(a + b*ArcTan[c*x])^2)/(c^5*d^3*(I - c*x)) - (6*b*(a + b*ArcTan
```

```
[c*x])*Log[2/(1 + I*c*x)])/(c^5*d^3) + ((6*I)*(a + b*ArcTan[c*x])^2*Log[2/(
1 + I*c*x)])/(c^5*d^3) + ((I/2)*b^2*Log[1 + c^2*x^2])/(c^5*d^3) - ((3*I)*b^
2*PolyLog[2, 1 - 2/(1 + I*c*x)])/(c^5*d^3) - (6*b*(a + b*ArcTan[c*x])*PolyL
og[2, 1 - 2/(1 + I*c*x)])/(c^5*d^3) + ((3*I)*b^2*PolyLog[3, 1 - 2/(1 + I*c*
x)])/(c^5*d^3)
```

Rule 4876

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4864

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x])^p)/(e*(q + 1)), x] - D
ist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 4862

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x]))/(e*(q + 1)), x] - Dist[(b*
c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
```

```
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{x^4 \left(a+b\tan^{-1}(cx)\right)^2}{(d+icdx)^3}\,dx = \int \left(-\frac{3\left(a+b\tan^{-1}(cx)\right)^2}{c^4d^3} + \frac{ix\left(a+b\tan^{-1}(cx)\right)^2}{c^3d^3} + \frac{i\left(a+b\tan^{-1}(cx)\right)^2}{c^4d^3(-i+cx)^3} + \frac{4\left(a+b\tan^{-1}\right.}{c^4d^3(-i+\right.}$$

$$= \frac{i\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{(-i+cx)^3}\,dx}{c^4d^3} - \frac{(6i)\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{-i+cx}\,dx}{c^4d^3} - \frac{3\int \left(a+b\tan^{-1}(cx)\right)^2\,dx}{c^4d^3} + \frac{4\int \frac{\left(a+b\tan^{-1}\right.}{(-i+\right.}}{c^4d}$$

$$= -\frac{3x\left(a+b\tan^{-1}(cx)\right)^2}{c^4d^3} + \frac{ix^2\left(a+b\tan^{-1}(cx)\right)^2}{2c^3d^3} - \frac{i\left(a+b\tan^{-1}(cx)\right)^2}{2c^5d^3(i-cx)^2} + \frac{4\left(a+b\tan^{-1}(\right.}{c^5d^3(i-cx}$$

$$= -\frac{3i\left(a+b\tan^{-1}(cx)\right)^2}{c^5d^3} - \frac{3x\left(a+b\tan^{-1}(cx)\right)^2}{c^4d^3} + \frac{ix^2\left(a+b\tan^{-1}(cx)\right)^2}{2c^3d^3} - \frac{i\left(a+b\tan^{-1}\right.}{2c^5d^3(i-c}$$

$$= -\frac{iabx}{c^4d^3} - \frac{b\left(a+b\tan^{-1}(cx)\right)}{4c^5d^3(i-cx)^2} - \frac{15ib\left(a+b\tan^{-1}(cx)\right)}{4c^5d^3(i-cx)} - \frac{5i\left(a+b\tan^{-1}(cx)\right)^2}{8c^5d^3} - \frac{3x\left(a+b\right.}{}$$

$$= -\frac{iabx}{c^4d^3} - \frac{ib^2x\tan^{-1}(cx)}{c^4d^3} - \frac{b\left(a+b\tan^{-1}(cx)\right)}{4c^5d^3(i-cx)^2} - \frac{15ib\left(a+b\tan^{-1}(cx)\right)}{4c^5d^3(i-cx)} - \frac{5i\left(a+b\tan^{-1}\right.}{8c^5d^3}$$

$$= -\frac{iabx}{c^4d^3} - \frac{ib^2x\tan^{-1}(cx)}{c^4d^3} - \frac{b\left(a+b\tan^{-1}(cx)\right)}{4c^5d^3(i-cx)^2} - \frac{15ib\left(a+b\tan^{-1}(cx)\right)}{4c^5d^3(i-cx)} - \frac{5i\left(a+b\tan^{-1}\right.}{8c^5d^3}$$

$$= -\frac{iabx}{c^4d^3} + \frac{ib^2}{16c^5d^3(i-cx)^2} - \frac{29b^2}{16c^5d^3(i-cx)} - \frac{ib^2x\tan^{-1}(cx)}{c^4d^3} - \frac{b\left(a+b\tan^{-1}(cx)\right)}{4c^5d^3(i-cx)^2} - \frac{15ib\,(}{4}$$

$$= -\frac{iabx}{c^4d^3} + \frac{ib^2}{16c^5d^3(i-cx)^2} - \frac{29b^2}{16c^5d^3(i-cx)} + \frac{29b^2\tan^{-1}(cx)}{16c^5d^3} - \frac{ib^2x\tan^{-1}(cx)}{c^4d^3} - \frac{b\left(a+b\,\mathrm{t}\right.}{4c^5d^3(}$$

Mathematica [A] time = 2.50212, size = 578, normalized size = 1.25

$$\frac{ab\left(96\mathrm{PolyLog}\left(2,-e^{2i\tan^{-1}(cx)}\right) + 48\log\left(c^2x^2+1\right) + 4i\tan^{-1}(cx)\left(4c^2x^2+24icx+48\log\left(1+e^{2i\tan^{-1}(cx)}\right)+14i\sin\left(\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(a + b*ArcTan[c*x])^2)/(d + I*c*d*x)^3,x]
```

```
[Out] (-48*a^2*c*x + (8*I)*a^2*c^2*x^2 - ((8*I)*a^2)/(-I + c*x)^2 - (64*a^2)/(-I
+ c*x) + 96*a^2*ArcTan[c*x] - (48*I)*a^2*Log[1 + c^2*x^2] + a*b*((-16*I)*c*
x + 192*ArcTan[c*x]^2 - 28*Cos[2*ArcTan[c*x]] + Cos[4*ArcTan[c*x]] + 48*Log
```

```
[1 + c^2*x^2] + 96*PolyLog[2, -E^((2*I)*ArcTan[c*x])] + (28*I)*Sin[2*ArcTan
[c*x]] + (4*I)*ArcTan[c*x]*(4 + (24*I)*c*x + 4*c^2*x^2 - 14*Cos[2*ArcTan[c*
x]] + Cos[4*ArcTan[c*x]] + 48*Log[1 + E^((2*I)*ArcTan[c*x])] + (14*I)*Sin[2
*ArcTan[c*x]] - I*Sin[4*ArcTan[c*x]]) - I*Sin[4*ArcTan[c*x]]) + (16*I)*b^2*
(-(c*x*ArcTan[c*x]) + 3*ArcTan[c*x]^2 + (3*I)*c*x*ArcTan[c*x]^2 + ((1 + c^2
*x^2)*ArcTan[c*x]^2)/2 - (4*I)*ArcTan[c*x]^3 - (7*(-1 - (2*I)*ArcTan[c*x] +
 2*ArcTan[c*x]^2)*Cos[2*ArcTan[c*x]])/8 - Cos[4*ArcTan[c*x]]/64 - (I/16)*Ar
cTan[c*x]*Cos[4*ArcTan[c*x]] + (ArcTan[c*x]^2*Cos[4*ArcTan[c*x]])/8 + (6*I)
*ArcTan[c*x]*Log[1 + E^((2*I)*ArcTan[c*x])] + 6*ArcTan[c*x]^2*Log[1 + E^((2
*I)*ArcTan[c*x])] + Log[1 + c^2*x^2]/2 + (3 - (6*I)*ArcTan[c*x])*PolyLog[2,
 -E^((2*I)*ArcTan[c*x])] + 3*PolyLog[3, -E^((2*I)*ArcTan[c*x])] - ((7*I)/8)
*Sin[2*ArcTan[c*x]] + (7*ArcTan[c*x]*Sin[2*ArcTan[c*x]])/4 + ((7*I)/4)*ArcT
an[c*x]^2*Sin[2*ArcTan[c*x]] + (I/64)*Sin[4*ArcTan[c*x]] - (ArcTan[c*x]*Sin
[4*ArcTan[c*x]])/16 - (I/8)*ArcTan[c*x]^2*Sin[4*ArcTan[c*x]]))/(16*c^5*d^3)
```

Maple [C] time = 1.728, size = 1618, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arctan(c*x))^2/(d+I*c*d*x)^3,x)
```

```
[Out] 6/c^5*b^2/d^3*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^
2*arctan(c*x)^2+3/c^5*b^2/d^3*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/
(c^2*x^2+1)+1))^3*arctan(c*x)^2-8/c^5*a*b/d^3*arctan(c*x)/(c*x-I)-6/c^5*a*b
/d^3*ln(c*x-I)*ln(-1/2*I*(c*x+I))+7/4/c^4*b^2/d^3*arctan(c*x)/(c*x-I)*x+1/1
6/c^3*b^2/d^3*arctan(c*x)/(c*x-I)^2*x^2-6/c^4*a*b/d^3*arctan(c*x)*x+43/8*I/
c^5*a*b/d^3*arctan(c*x)+15/4*I/c^5*a*b/d^3/(c*x-I)-1/2*I/c^5*b^2/d^3*arctan
(c*x)^2/(c*x-I)^2+7/4*I/c^5*b^2/d^3*arctan(c*x)/(c*x-I)+6*I/c^5*b^2/d^3*arc
tan(c*x)^2*ln(2*I*(1+I*c*x)^2/(c^2*x^2+1))-6*I/c^5*b^2/d^3*arctan(c*x)^2*ln
(c*x-I)+5/16*I/c^5*a*b/d^3*arctan(1/2*c*x)-5/8*I/c^5*a*b/d^3*arctan(1/2*c*x
-1/2*I)-5/16*I/c^5*a*b/d^3*arctan(1/6*c^3*x^3+7/6*c*x)-7*I/c^4*b^2/d^3/(8*c
*x-8*I)*x-1/64*I/c^3*b^2/d^3/(c*x-I)^2*x^2+1/2*I/c^3*b^2/d^3*arctan(c*x)^2*
x^2-1/4/c^5*a*b/d^3/(c*x-I)^2+43/16/c^5*a*b/d^3*ln(c^2*x^2+1)+3/c^5*a*b/d^3
*ln(c*x-I)^2-6/c^5*a*b/d^3*dilog(-1/2*I*(c*x+I))-6/c^5*b^2/d^3*Pi*arctan(c*
x)^2+6/c^5*b^2/d^3*arctan(c*x)*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))-6/c^5*b^
2/d^3*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-6/c^5*b^2/d^3*arctan(
c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-1/16/c^5*b^2/d^3*arctan(c*x)/(c*x-
I)^2-1/2*I/c^5*a^2/d^3/(c*x-I)^2-3*I/c^5*a^2/d^3*ln(c^2*x^2+1)+1/2*I/c^3*a^
2/d^3*x^2-I/c^5*b^2/d^3*ln((1+I*c*x)^2/(c^2*x^2+1)+1)+43/8*I/c^5*b^2/d^3*ar
ctan(c*x)^2+1/64*I/c^5*b^2/d^3/(c*x-I)^2+3*I/c^5*b^2/d^3*polylog(3,-(1+I*c*
```

```
x)^2/(c^2*x^2+1))+6*I/c^5*b^2/d^3*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+6*
I/c^5*b^2/d^3*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-1/c^5*a*b/d^3-3/c^4*a^
2/d^3*x+4/c^5*b^2/d^3*arctan(c*x)^3-b^2*arctan(c*x)/c^5/d^3+3/c^5*b^2/d^3*P
i*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1
+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)^2-I/c^5*a*b/
d^3*arctan(c*x)/(c*x-I)^2+1/8*I/c^4*b^2/d^3*arctan(c*x)/(c*x-I)^2*x-3/c^5*b
^2/d^3*Pi*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/
((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2+3/c^5*b^2/d^3*Pi*csgn((1+I*c*x
)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^
2*arctan(c*x)^2+I/c^3*a*b/d^3*arctan(c*x)*x^2-12*I/c^5*a*b/d^3*arctan(c*x)*
ln(c*x-I)-I*a*b*x/c^4/d^3-I*b^2*x*arctan(c*x)/c^4/d^3-4/c^5*b^2/d^3*arctan(
c*x)^2/(c*x-I)+1/32/c^4*b^2/d^3/(c*x-I)^2*x-3/c^4*b^2/d^3*arctan(c*x)^2*x+5
/32/c^5*a*b/d^3*ln(c^4*x^4+10*c^2*x^2+9)+7/c^5*b^2/d^3/(8*c*x-8*I)-4/c^5*a^
2/d^3/(c*x-I)+6/c^5*a^2/d^3*arctan(c*x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctan(c*x))^2/(d+I*c*d*x)^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{-i\,b^2x^4\log\left(-\frac{cx+i}{cx-i}\right)^2-4\,abx^4\log\left(-\frac{cx+i}{cx-i}\right)+4i\,a^2x^4}{4\,c^3d^3x^3-12i\,c^2d^3x^2-12\,cd^3x+4i\,d^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctan(c*x))^2/(d+I*c*d*x)^3,x, algorithm="fricas")
```

```
[Out] integral((-I*b^2*x^4*log(-(c*x + I)/(c*x - I))^2 - 4*a*b*x^4*log(-(c*x + I)
/(c*x - I)) + 4*I*a^2*x^4)/(4*c^3*d^3*x^3 - 12*I*c^2*d^3*x^2 - 12*c*d^3*x +
 4*I*d^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*atan(c*x))**2/(d+I*c*d*x)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\arctan(cx) + a)^2 x^4}{(icdx + d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctan(c*x))^2/(d+I*c*d*x)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)^2*x^4/(I*c*d*x + d)^3, x)
```

3.112 $\int \frac{x^3(a+b\tan^{-1}(cx))^2}{(d+icdx)^3}\,dx$

Optimal. Leaf size=383

$$\frac{3ib\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{c^4d^3}-\frac{b^2\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c^4d^3}+\frac{3b^2\text{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{2c^4d^3}-\frac{11b\left(a+b\tan^{-1}\right.}{4c^4d^3(-cx+}$$

```
[Out] b^2/(16*c^4*d^3*(I - c*x)^2) + (((21*I)/16)*b^2)/(c^4*d^3*(I - c*x)) - (((2
1*I)/16)*b^2*ArcTan[c*x])/(c^4*d^3) + ((I/4)*b*(a + b*ArcTan[c*x]))/(c^4*d^
3*(I - c*x)^2) - (11*b*(a + b*ArcTan[c*x]))/(4*c^4*d^3*(I - c*x)) + (3*(a +
 b*ArcTan[c*x])^2)/(8*c^4*d^3) + (I*x*(a + b*ArcTan[c*x])^2)/(c^3*d^3) - (a
 + b*ArcTan[c*x])^2/(2*c^4*d^3*(I - c*x)^2) - ((3*I)*(a + b*ArcTan[c*x])^2)
/(c^4*d^3*(I - c*x)) + ((2*I)*b*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(c^
4*d^3) + (3*(a + b*ArcTan[c*x])^2*Log[2/(1 + I*c*x)])/(c^4*d^3) - (b^2*Poly
Log[2, 1 - 2/(1 + I*c*x)])/(c^4*d^3) + ((3*I)*b*(a + b*ArcTan[c*x])*PolyLog
[2, 1 - 2/(1 + I*c*x)])/(c^4*d^3) + (3*b^2*PolyLog[3, 1 - 2/(1 + I*c*x)])/(
2*c^4*d^3)
```

Rubi [A] time = 0.67463, antiderivative size = 383, normalized size of antiderivative = 1., number of steps used = 31, number of rules used = 14, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.56, Rules used = {4876, 4846, 4920, 4854, 2402, 2315, 4864, 4862, 627, 44, 203, 4884, 4994, 6610}

$$\frac{3ib\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{c^4d^3}-\frac{b^2\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c^4d^3}+\frac{3b^2\text{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{2c^4d^3}-\frac{11b\left(a+b\tan^{-1}\right.}{4c^4d^3(-cx+}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcTan[c*x])^2)/(d + I*c*d*x)^3,x]
```


```
[Out] b^2/(16*c^4*d^3*(I - c*x)^2) + (((21*I)/16)*b^2)/(c^4*d^3*(I - c*x)) - (((2
1*I)/16)*b^2*ArcTan[c*x])/(c^4*d^3) + ((I/4)*b*(a + b*ArcTan[c*x]))/(c^4*d^
3*(I - c*x)^2) - (11*b*(a + b*ArcTan[c*x]))/(4*c^4*d^3*(I - c*x)) + (3*(a +
 b*ArcTan[c*x])^2)/(8*c^4*d^3) + (I*x*(a + b*ArcTan[c*x])^2)/(c^3*d^3) - (a
 + b*ArcTan[c*x])^2/(2*c^4*d^3*(I - c*x)^2) - ((3*I)*(a + b*ArcTan[c*x])^2)
/(c^4*d^3*(I - c*x)) + ((2*I)*b*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(c^
4*d^3) + (3*(a + b*ArcTan[c*x])^2*Log[2/(1 + I*c*x)])/(c^4*d^3) - (b^2*Poly
Log[2, 1 - 2/(1 + I*c*x)])/(c^4*d^3) + ((3*I)*b*(a + b*ArcTan[c*x])*PolyLog
[2, 1 - 2/(1 + I*c*x)])/(c^4*d^3) + (3*b^2*PolyLog[3, 1 - 2/(1 + I*c*x)])/(
```

```
2*c^4*d^3)
```

Rule 4876

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 4864

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x])^p)/(e*(q + 1)), x] - D
ist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1), (
```

```
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 4862

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x]))/(e*(q + 1)), x] - Dist[(b*
c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{x^3\left(a+b\tan^{-1}(cx)\right)^2}{(d+icdx)^3}\,dx = \int\left(\frac{i\left(a+b\tan^{-1}(cx)\right)^2}{c^3d^3} + \frac{\left(a+b\tan^{-1}(cx)\right)^2}{c^3d^3(-i+cx)^3} - \frac{3i\left(a+b\tan^{-1}(cx)\right)^2}{c^3d^3(-i+cx)^2} - \frac{3\left(a+b\tan^{-1}(cx)\right.}{c^3d^3(-i+cx)}\right.$$

$$= \frac{i\int\left(a+b\tan^{-1}(cx)\right)^2\,dx}{c^3d^3} - \frac{(3i)\int\frac{\left(a+b\tan^{-1}(cx)\right)^2}{(-i+cx)^2}\,dx}{c^3d^3} + \frac{\int\frac{\left(a+b\tan^{-1}(cx)\right)^2}{(-i+cx)^3}\,dx}{c^3d^3} - \frac{3\int\frac{\left(a+b\tan^{-1}(c\right.}{-i+cx}}{c^3d^3}$$

$$= \frac{ix\left(a+b\tan^{-1}(cx)\right)^2}{c^3d^3} - \frac{\left(a+b\tan^{-1}(cx)\right)^2}{2c^4d^3(i-cx)^2} - \frac{3i\left(a+b\tan^{-1}(cx)\right)^2}{c^4d^3(i-cx)} + \frac{3\left(a+b\tan^{-1}(cx)\right)^2}{c^4d^3}$$

$$= -\frac{\left(a+b\tan^{-1}(cx)\right)^2}{c^4d^3} + \frac{ix\left(a+b\tan^{-1}(cx)\right)^2}{c^3d^3} - \frac{\left(a+b\tan^{-1}(cx)\right)^2}{2c^4d^3(i-cx)^2} - \frac{3i\left(a+b\tan^{-1}(cx)\right)^2}{c^4d^3(i-cx)}$$

$$= \frac{ib\left(a+b\tan^{-1}(cx)\right)}{4c^4d^3(i-cx)^2} - \frac{11b\left(a+b\tan^{-1}(cx)\right)}{4c^4d^3(i-cx)} + \frac{3\left(a+b\tan^{-1}(cx)\right)^2}{8c^4d^3} + \frac{ix\left(a+b\tan^{-1}(cx)\right)}{c^3d^3}$$

$$= \frac{ib\left(a+b\tan^{-1}(cx)\right)}{4c^4d^3(i-cx)^2} - \frac{11b\left(a+b\tan^{-1}(cx)\right)}{4c^4d^3(i-cx)} + \frac{3\left(a+b\tan^{-1}(cx)\right)^2}{8c^4d^3} + \frac{ix\left(a+b\tan^{-1}(cx)\right)}{c^3d^3}$$

$$= \frac{ib\left(a+b\tan^{-1}(cx)\right)}{4c^4d^3(i-cx)^2} - \frac{11b\left(a+b\tan^{-1}(cx)\right)}{4c^4d^3(i-cx)} + \frac{3\left(a+b\tan^{-1}(cx)\right)^2}{8c^4d^3} + \frac{ix\left(a+b\tan^{-1}(cx)\right)}{c^3d^3}$$

$$= \frac{b^2}{16c^4d^3(i-cx)^2} + \frac{21ib^2}{16c^4d^3(i-cx)} + \frac{ib\left(a+b\tan^{-1}(cx)\right)}{4c^4d^3(i-cx)^2} - \frac{11b\left(a+b\tan^{-1}(cx)\right)}{4c^4d^3(i-cx)} + \frac{3\left(a+b\right.}{8}$$

$$= \frac{b^2}{16c^4d^3(i-cx)^2} + \frac{21ib^2}{16c^4d^3(i-cx)} - \frac{21ib^2\tan^{-1}(cx)}{16c^4d^3} + \frac{ib\left(a+b\tan^{-1}(cx)\right)}{4c^4d^3(i-cx)^2} - \frac{11b\left(a+b\tan\right.}{4c^4d^3(i-}$$

Mathematica [A] time = 1.52805, size = 507, normalized size = 1.32

$$\frac{4iab\left(-48\text{PolyLog}\left(2,-e^{2i\tan^{-1}(cx)}\right)-16\log\left(c^2x^2+1\right)-96\tan^{-1}(cx)^2-20i\sin\left(2\tan^{-1}(cx)\right)+i\sin\left(4\tan^{-1}(cx)\right)+20\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(a + b*ArcTan[c*x])^2)/(d + I*c*d*x)^3,x]
```

```
[Out] ((64*I)*a^2*c*x - (32*a^2)/(-I + c*x)^2 + ((192*I)*a^2)/(-I + c*x) - (192*I
)*a^2*ArcTan[c*x] - 96*a^2*Log[1 + c^2*x^2] + (4*I)*a*b*(-96*ArcTan[c*x]^2
+ 20*Cos[2*ArcTan[c*x]] - Cos[4*ArcTan[c*x]] - 16*Log[1 + c^2*x^2] - 48*Pol
yLog[2, -E^((2*I)*ArcTan[c*x])] - (20*I)*Sin[2*ArcTan[c*x]] + 4*ArcTan[c*x]
*(8*c*x + (10*I)*Cos[2*ArcTan[c*x]] - I*Cos[4*ArcTan[c*x]] - (24*I)*Log[1 +
 E^((2*I)*ArcTan[c*x])] + 10*Sin[2*ArcTan[c*x]] - Sin[4*ArcTan[c*x]]) + I*S
in[4*ArcTan[c*x]]) + I*b^2*((-64*I)*ArcTan[c*x]^2 + 64*c*x*ArcTan[c*x]^2 -
128*ArcTan[c*x]^3 - (40*I)*Cos[2*ArcTan[c*x]] + 80*ArcTan[c*x]*Cos[2*ArcTan
[c*x]] + (80*I)*ArcTan[c*x]^2*Cos[2*ArcTan[c*x]] + I*Cos[4*ArcTan[c*x]] - 4
*ArcTan[c*x]*Cos[4*ArcTan[c*x]] - (8*I)*ArcTan[c*x]^2*Cos[4*ArcTan[c*x]] +
128*ArcTan[c*x]*Log[1 + E^((2*I)*ArcTan[c*x])] - (192*I)*ArcTan[c*x]^2*Log[
1 + E^((2*I)*ArcTan[c*x])] - 64*(I + 3*ArcTan[c*x])*PolyLog[2, -E^((2*I)*Ar
cTan[c*x])] - (96*I)*PolyLog[3, -E^((2*I)*ArcTan[c*x])] - 40*Sin[2*ArcTan[c
*x]] - (80*I)*ArcTan[c*x]*Sin[2*ArcTan[c*x]] + 80*ArcTan[c*x]^2*Sin[2*ArcTa
n[c*x]] + Sin[4*ArcTan[c*x]] + (4*I)*ArcTan[c*x]*Sin[4*ArcTan[c*x]] - 8*Arc
Tan[c*x]^2*Sin[4*ArcTan[c*x]]))/(64*c^4*d^3)
```

Maple [C] time = 0.625, size = 5012, normalized size = 13.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctan(c*x))^2/(d+I*c*d*x)^3,x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))^2/(d+I*c*d*x)^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{-i\,b^2x^3\log\left(-\frac{cx+i}{cx-i}\right)^2-4\,abx^3\log\left(-\frac{cx+i}{cx-i}\right)+4i\,a^2x^3}{4\,c^3d^3x^3-12i\,c^2d^3x^2-12\,cd^3x+4i\,d^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))^2/(d+I*c*d*x)^3,x, algorithm="fricas")
```

```
[Out] integral((-I*b^2*x^3*log(-(c*x + I)/(c*x - I))^2 - 4*a*b*x^3*log(-(c*x + I)
/(c*x - I)) + 4*I*a^2*x^3)/(4*c^3*d^3*x^3 - 12*I*c^2*d^3*x^2 - 12*c*d^3*x +
 4*I*d^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atan(c*x))**2/(d+I*c*d*x)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\arctan(cx)+a)^2x^3}{(i\,cdx+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))^2/(d+I*c*d*x)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)^2*x^3/(I*c*d*x + d)^3, x)
```

3.113 $\int \frac{x^2(a+b\tan^{-1}(cx))^2}{(d+icdx)^3}\,dx$

Optimal. Leaf size=304

$$\frac{b\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{c^3d^3}-\frac{ib^2\text{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{2c^3d^3}+\frac{7ib\left(a+b\tan^{-1}(cx)\right)}{4c^3d^3(-cx+i)}+\frac{b\left(a+b\tan^{-1}(cx)\right)}{4c^3d^3(-cx+i)^2}-$$

```
[Out] ((-I/16)*b^2)/(c^3*d^3*(I - c*x)^2) + (13*b^2)/(16*c^3*d^3*(I - c*x)) - (13
*b^2*ArcTan[c*x])/(16*c^3*d^3) + (b*(a + b*ArcTan[c*x]))/(4*c^3*d^3*(I - c*
x)^2) + (((7*I)/4)*b*(a + b*ArcTan[c*x]))/(c^3*d^3*(I - c*x)) - (((7*I)/8)*
(a + b*ArcTan[c*x])^2)/(c^3*d^3) + ((I/2)*(a + b*ArcTan[c*x])^2)/(c^3*d^3*(
I - c*x)^2) - (2*(a + b*ArcTan[c*x])^2)/(c^3*d^3*(I - c*x)) - (I*(a + b*Arc
Tan[c*x])^2*Log[2/(1 + I*c*x)])/(c^3*d^3) + (b*(a + b*ArcTan[c*x])*PolyLog[
2, 1 - 2/(1 + I*c*x)])/(c^3*d^3) - ((I/2)*b^2*PolyLog[3, 1 - 2/(1 + I*c*x)]
)/(c^3*d^3)
```

Rubi [A] time = 0.560428, antiderivative size = 304, normalized size of antiderivative = 1., number of steps used = 26, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {4876, 4864, 4862, 627, 44, 203, 4884, 4854, 4994, 6610}

$$\frac{b\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{c^3d^3}-\frac{ib^2\text{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{2c^3d^3}+\frac{7ib\left(a+b\tan^{-1}(cx)\right)}{4c^3d^3(-cx+i)}+\frac{b\left(a+b\tan^{-1}(cx)\right)}{4c^3d^3(-cx+i)^2}-$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcTan[c*x])^2)/(d + I*c*d*x)^3,x]
```


```
[Out] ((-I/16)*b^2)/(c^3*d^3*(I - c*x)^2) + (13*b^2)/(16*c^3*d^3*(I - c*x)) - (13
*b^2*ArcTan[c*x])/(16*c^3*d^3) + (b*(a + b*ArcTan[c*x]))/(4*c^3*d^3*(I - c*
x)^2) + (((7*I)/4)*b*(a + b*ArcTan[c*x]))/(c^3*d^3*(I - c*x)) - (((7*I)/8)*
(a + b*ArcTan[c*x])^2)/(c^3*d^3) + ((I/2)*(a + b*ArcTan[c*x])^2)/(c^3*d^3*(
I - c*x)^2) - (2*(a + b*ArcTan[c*x])^2)/(c^3*d^3*(I - c*x)) - (I*(a + b*Arc
Tan[c*x])^2*Log[2/(1 + I*c*x)])/(c^3*d^3) + (b*(a + b*ArcTan[c*x])*PolyLog[
2, 1 - 2/(1 + I*c*x)])/(c^3*d^3) - ((I/2)*b^2*PolyLog[3, 1 - 2/(1 + I*c*x)]
)/(c^3*d^3)
```


Rule 4876

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
```

```
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 4864

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x])^p)/(e*(q + 1)), x] - D
ist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 4862

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x]))/(e*(q + 1)), x] - Dist[(b*
c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{x^2\left(a+b\tan^{-1}(cx)\right)^2}{(d+icdx)^3}\,dx = \int\left(-\frac{i\left(a+b\tan^{-1}(cx)\right)^2}{c^2d^3(-i+cx)^3} - \frac{2\left(a+b\tan^{-1}(cx)\right)^2}{c^2d^3(-i+cx)^2} + \frac{i\left(a+b\tan^{-1}(cx)\right)^2}{c^2d^3(-i+cx)}\right)dx$$

$$= -\frac{i\int\frac{\left(a+b\tan^{-1}(cx)\right)^2}{(-i+cx)^3}dx}{c^2d^3} + \frac{i\int\frac{\left(a+b\tan^{-1}(cx)\right)^2}{-i+cx}dx}{c^2d^3} - \frac{2\int\frac{\left(a+b\tan^{-1}(cx)\right)^2}{(-i+cx)^2}dx}{c^2d^3}$$

$$= \frac{i\left(a+b\tan^{-1}(cx)\right)^2}{2c^3d^3(i-cx)^2} - \frac{2\left(a+b\tan^{-1}(cx)\right)^2}{c^3d^3(i-cx)} - \frac{i\left(a+b\tan^{-1}(cx)\right)^2\log\left(\frac{2}{1+icx}\right)}{c^3d^3} - \frac{(ib)\int\left(-\frac{i(a}{}\right.}{}$$

$$= \frac{i\left(a+b\tan^{-1}(cx)\right)^2}{2c^3d^3(i-cx)^2} - \frac{2\left(a+b\tan^{-1}(cx)\right)^2}{c^3d^3(i-cx)} - \frac{i\left(a+b\tan^{-1}(cx)\right)^2\log\left(\frac{2}{1+icx}\right)}{c^3d^3} + \frac{b\left(a+b\,\mathrm{ta}\right.}{}$$

$$= \frac{b\left(a+b\tan^{-1}(cx)\right)}{4c^3d^3(i-cx)^2} + \frac{7ib\left(a+b\tan^{-1}(cx)\right)}{4c^3d^3(i-cx)} - \frac{7i\left(a+b\tan^{-1}(cx)\right)^2}{8c^3d^3} + \frac{i\left(a+b\tan^{-1}(cx)\right)^2}{2c^3d^3(i-cx)^2}$$

$$= \frac{b\left(a+b\tan^{-1}(cx)\right)}{4c^3d^3(i-cx)^2} + \frac{7ib\left(a+b\tan^{-1}(cx)\right)}{4c^3d^3(i-cx)} - \frac{7i\left(a+b\tan^{-1}(cx)\right)^2}{8c^3d^3} + \frac{i\left(a+b\tan^{-1}(cx)\right)^2}{2c^3d^3(i-cx)^2}$$

$$= \frac{b\left(a+b\tan^{-1}(cx)\right)}{4c^3d^3(i-cx)^2} + \frac{7ib\left(a+b\tan^{-1}(cx)\right)}{4c^3d^3(i-cx)} - \frac{7i\left(a+b\tan^{-1}(cx)\right)^2}{8c^3d^3} + \frac{i\left(a+b\tan^{-1}(cx)\right)^2}{2c^3d^3(i-cx)^2}$$

$$= -\frac{ib^2}{16c^3d^3(i-cx)^2} + \frac{13b^2}{16c^3d^3(i-cx)} + \frac{b\left(a+b\tan^{-1}(cx)\right)}{4c^3d^3(i-cx)^2} + \frac{7ib\left(a+b\tan^{-1}(cx)\right)}{4c^3d^3(i-cx)} - \frac{7i\left(a+\right.}{}$$

$$= -\frac{ib^2}{16c^3d^3(i-cx)^2} + \frac{13b^2}{16c^3d^3(i-cx)} - \frac{13b^2\tan^{-1}(cx)}{16c^3d^3} + \frac{b\left(a+b\tan^{-1}(cx)\right)}{4c^3d^3(i-cx)^2} + \frac{7ib\left(a+b\,\mathrm{tan}\right.}{4c^3d^3(i-}$$

Mathematica [A] time = 1.22569, size = 431, normalized size = 1.42

$$\frac{-12ab\left(16\text{PolyLog}\left(2,-e^{2i\tan^{-1}(cx)}\right)+32\tan^{-1}(cx)^2+12i\sin\left(2\tan^{-1}(cx)\right)-i\sin\left(4\tan^{-1}(cx)\right)-12\cos\left(2\tan^{-1}(cx)\right)\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(a + b*ArcTan[c*x])^2)/(d + I*c*d*x)^3,x]
```

```
[Out] (((96*I)*a^2)/(-I + c*x)^2 + (384*a^2)/(-I + c*x) - 192*a^2*ArcTan[c*x] + (
96*I)*a^2*Log[1 + c^2*x^2] - b^2*(128*ArcTan[c*x]^3 + (72*I)*Cos[2*ArcTan[c
*x]] - 144*ArcTan[c*x]*Cos[2*ArcTan[c*x]] - (144*I)*ArcTan[c*x]^2*Cos[2*Arc
Tan[c*x]] - (3*I)*Cos[4*ArcTan[c*x]] + 12*ArcTan[c*x]*Cos[4*ArcTan[c*x]] +
```

```
(24*I)*ArcTan[c*x]^2*Cos[4*ArcTan[c*x]] + (192*I)*ArcTan[c*x]^2*Log[1 + E^(
(2*I)*ArcTan[c*x])] + 192*ArcTan[c*x]*PolyLog[2, -E^((2*I)*ArcTan[c*x])] +
(96*I)*PolyLog[3, -E^((2*I)*ArcTan[c*x])] + 72*Sin[2*ArcTan[c*x]] + (144*I)
*ArcTan[c*x]*Sin[2*ArcTan[c*x]] - 144*ArcTan[c*x]^2*Sin[2*ArcTan[c*x]] - 3*
Sin[4*ArcTan[c*x]] - (12*I)*ArcTan[c*x]*Sin[4*ArcTan[c*x]] + 24*ArcTan[c*x]
^2*Sin[4*ArcTan[c*x]]) - 12*a*b*(32*ArcTan[c*x]^2 - 12*Cos[2*ArcTan[c*x]] +
 Cos[4*ArcTan[c*x]] + 16*PolyLog[2, -E^((2*I)*ArcTan[c*x])] + (12*I)*Sin[2*
ArcTan[c*x]] - I*Sin[4*ArcTan[c*x]] + 4*ArcTan[c*x]*((-6*I)*Cos[2*ArcTan[c*
x]] + I*Cos[4*ArcTan[c*x]] + (8*I)*Log[1 + E^((2*I)*ArcTan[c*x])] - 6*Sin[2
*ArcTan[c*x]] + Sin[4*ArcTan[c*x]])))/(192*c^3*d^3)
```

Maple [C] time = 0.347, size = 1276, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctan(c*x))^2/(d+I*c*d*x)^3,x)
```

```
[Out] -1/2/c^3*b^2/d^3*Pi*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn((1+I*c*x)^2/(c
^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan
(c*x)^2-2/3/c^3*b^2/d^3*arctan(c*x)^3-3/c^3*b^2/d^3/(8*c*x-8*I)+2/c^3*a^2/d
^3/(c*x-I)-1/c^3*a^2/d^3*arctan(c*x)-1/8*I/c^2*b^2/d^3*arctan(c*x)/(c*x-I)^
2*x+2*I/c^3*a*b/d^3*arctan(c*x)*ln(c*x-I)+7/32/c^3*a*b/d^3*ln(c^4*x^4+10*c^
2*x^2+9)+1/4/c^3*a*b/d^3/(c*x-I)^2-7/16/c^3*a*b/d^3*ln(c^2*x^2+1)-1/2/c^3*a
*b/d^3*ln(c*x-I)^2+1/c^3*a*b/d^3*dilog(-1/2*I*(c*x+I))+1/c^3*b^2/d^3*Pi*arc
tan(c*x)^2-1/c^3*b^2/d^3*arctan(c*x)*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))+1/
16/c^3*b^2/d^3*arctan(c*x)/(c*x-I)^2+2/c^3*b^2/d^3*arctan(c*x)^2/(c*x-I)-1/
32/c^2*b^2/d^3/(c*x-I)^2*x-1/2*I/c^3*b^2/d^3*polylog(3,-(1+I*c*x)^2/(c^2*x^
2+1))+1/2*I/c^3*a^2/d^3/(c*x-I)^2+1/2*I/c^3*a^2/d^3*ln(c^2*x^2+1)-7/8*I/c^3
*b^2/d^3*arctan(c*x)^2-1/64*I/c^3*b^2/d^3/(c*x-I)^2+1/2/c^3*b^2/d^3*Pi*csgn
(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(
c^2*x^2+1)+1))^2*arctan(c*x)^2+I/c^3*a*b/d^3*arctan(c*x)/(c*x-I)^2-1/2/c^3*
b^2/d^3*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I
*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2+1/64*I/c*b^2/d^3/(c*x-I)^2*x^2-7/4*
I/c^3*a*b/d^3/(c*x-I)-7/8*I/c^3*a*b/d^3*arctan(c*x)-3/4*I/c^3*b^2/d^3*arcta
n(c*x)/(c*x-I)-I/c^3*b^2/d^3*arctan(c*x)^2*ln(2*I*(1+I*c*x)^2/(c^2*x^2+1))+
1/2*I/c^3*b^2/d^3*arctan(c*x)^2/(c*x-I)^2+7/16*I/c^3*a*b/d^3*arctan(1/2*c*x
)-7/16*I/c^3*a*b/d^3*arctan(1/6*c^3*x^3+7/6*c*x)-7/8*I/c^3*a*b/d^3*arctan(1
/2*c*x-1/2*I)+3*I/c^2*b^2/d^3/(8*c*x-8*I)*x-1/16/c*b^2/d^3*arctan(c*x)/(c*x
-I)^2*x^2+I/c^3*b^2/d^3*arctan(c*x)^2*ln(c*x-I)-1/c^3*b^2/d^3*Pi*csgn((1+I*
c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2-1/2/c^3*b^2
```

```
/d^3*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*arctan(
c*x)^2+4/c^3*a*b/d^3*arctan(c*x)/(c*x-I)+1/c^3*a*b/d^3*ln(c*x-I)*ln(-1/2*I*
(c*x+I))-3/4/c^2*b^2/d^3*arctan(c*x)/(c*x-I)*x
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))^2/(d+I*c*d*x)^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{-i\,b^2x^2\log\left(-\frac{cx+i}{cx-i}\right)^2-4\,abx^2\log\left(-\frac{cx+i}{cx-i}\right)+4i\,a^2x^2}{4\,c^3d^3x^3-12i\,c^2d^3x^2-12\,cd^3x+4i\,d^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))^2/(d+I*c*d*x)^3,x, algorithm="fricas")
```

```
[Out] integral((-I*b^2*x^2*log(-(c*x + I)/(c*x - I))^2 - 4*a*b*x^2*log(-(c*x + I)
/(c*x - I)) + 4*I*a^2*x^2)/(4*c^3*d^3*x^3 - 12*I*c^2*d^3*x^2 - 12*c*d^3*x +
 4*I*d^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atan(c*x))**2/(d+I*c*d*x)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\arctan(cx)+a)^2x^2}{(icdx+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))^2/(d+I*c*d*x)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)^2*x^2/(I*c*d*x + d)^3, x)
```

3.114 $\int \frac{x\left(a+b\tan^{-1}(cx)\right)^2}{(d+icdx)^3}\,dx$

Optimal. Leaf size=178

$$\frac{3b\left(a+b\tan^{-1}(cx)\right)}{4c^2d^3(-cx+i)}-\frac{ib\left(a+b\tan^{-1}(cx)\right)}{4c^2d^3(-cx+i)^2}+\frac{\left(a+b\tan^{-1}(cx)\right)^2}{8c^2d^3}+\frac{x^2\left(a+b\tan^{-1}(cx)\right)^2}{2d^3(1+icx)^2}-\frac{5ib^2}{16c^2d^3(-cx+i)}-\frac{b^2}{16c^2d^3(-c$$

```
[Out] -b^2/(16*c^2*d^3*(I - c*x)^2) - (((5*I)/16)*b^2)/(c^2*d^3*(I - c*x)) + (((5
*I)/16)*b^2*ArcTan[c*x])/(c^2*d^3) - ((I/4)*b*(a + b*ArcTan[c*x]))/(c^2*d^3
*(I - c*x)^2) + (3*b*(a + b*ArcTan[c*x]))/(4*c^2*d^3*(I - c*x)) + (a + b*Ar
cTan[c*x])^2/(8*c^2*d^3) + (x^2*(a + b*ArcTan[c*x])^2)/(2*d^3*(1 + I*c*x)^2
)
```

Rubi [A] time = 0.216778, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {37, 4874, 4862, 627, 44, 203, 4884}

$$\frac{3b\left(a+b\tan^{-1}(cx)\right)}{4c^2d^3(-cx+i)}-\frac{ib\left(a+b\tan^{-1}(cx)\right)}{4c^2d^3(-cx+i)^2}+\frac{\left(a+b\tan^{-1}(cx)\right)^2}{8c^2d^3}+\frac{x^2\left(a+b\tan^{-1}(cx)\right)^2}{2d^3(1+icx)^2}-\frac{5ib^2}{16c^2d^3(-cx+i)}-\frac{b^2}{16c^2d^3(-c$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcTan[c*x])^2)/(d + I*c*d*x)^3,x]
```

```
[Out] -b^2/(16*c^2*d^3*(I - c*x)^2) - (((5*I)/16)*b^2)/(c^2*d^3*(I - c*x)) + (((5
*I)/16)*b^2*ArcTan[c*x])/(c^2*d^3) - ((I/4)*b*(a + b*ArcTan[c*x]))/(c^2*d^3
*(I - c*x)^2) + (3*b*(a + b*ArcTan[c*x]))/(4*c^2*d^3*(I - c*x)) + (a + b*Ar
cTan[c*x])^2/(8*c^2*d^3) + (x^2*(a + b*ArcTan[c*x])^2)/(2*d^3*(1 + I*c*x)^2
)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 4874

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_.) + (e_
.)*(x_))^(q_), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dis
t[(a + b*ArcTan[c*x])^p, u, x] - Dist[b*c*p, Int[ExpandIntegrand[(a + b*Arc
Tan[c*x])^(p - 1), u/(1 + c^2*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
 q}, x] && IGtQ[p, 1] && EqQ[c^2*d^2 + e^2, 0] && IntegersQ[m, q] && NeQ[m,
 -1] && NeQ[q, -1] && ILtQ[m + q + 1, 0] && LtQ[m*q, 0]
```

Rule 4862

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x]))/(e*(q + 1)), x] - Dist[(b*
c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{x\left(a+b\tan^{-1}(cx)\right)^2}{(d+icdx)^3}\,dx = \frac{x^2\left(a+b\tan^{-1}(cx)\right)^2}{2d^3(1+icx)^2} - (2bc)\int\left(-\frac{i\left(a+b\tan^{-1}(cx)\right)}{4c^2d^3(-i+cx)^3} - \frac{3\left(a+b\tan^{-1}(cx)\right)}{8c^2d^3(-i+cx)^2} - \frac{a+b\tan^{-1}(}{8c^2d^3\left(1+c^2\right.}$$

$$= \frac{x^2\left(a+b\tan^{-1}(cx)\right)^2}{2d^3(1+icx)^2} + \frac{(ib)\int\frac{a+b\tan^{-1}(cx)}{(-i+cx)^3}\,dx}{2cd^3} + \frac{b\int\frac{a+b\tan^{-1}(cx)}{1+c^2x^2}\,dx}{4cd^3} + \frac{(3b)\int\frac{a+b\tan^{-1}(cx)}{(-i+cx)^2}\,dx}{4cd^3}$$

$$= -\frac{ib\left(a+b\tan^{-1}(cx)\right)}{4c^2d^3(i-cx)^2} + \frac{3b\left(a+b\tan^{-1}(cx)\right)}{4c^2d^3(i-cx)} + \frac{\left(a+b\tan^{-1}(cx)\right)^2}{8c^2d^3} + \frac{x^2\left(a+b\tan^{-1}(cx)\right)^2}{2d^3(1+icx)^2}$$

$$= -\frac{ib\left(a+b\tan^{-1}(cx)\right)}{4c^2d^3(i-cx)^2} + \frac{3b\left(a+b\tan^{-1}(cx)\right)}{4c^2d^3(i-cx)} + \frac{\left(a+b\tan^{-1}(cx)\right)^2}{8c^2d^3} + \frac{x^2\left(a+b\tan^{-1}(cx)\right)^2}{2d^3(1+icx)^2}$$

$$= -\frac{ib\left(a+b\tan^{-1}(cx)\right)}{4c^2d^3(i-cx)^2} + \frac{3b\left(a+b\tan^{-1}(cx)\right)}{4c^2d^3(i-cx)} + \frac{\left(a+b\tan^{-1}(cx)\right)^2}{8c^2d^3} + \frac{x^2\left(a+b\tan^{-1}(cx)\right)^2}{2d^3(1+icx)^2}$$

$$= -\frac{b^2}{16c^2d^3(i-cx)^2} - \frac{5ib^2}{16c^2d^3(i-cx)} - \frac{ib\left(a+b\tan^{-1}(cx)\right)}{4c^2d^3(i-cx)^2} + \frac{3b\left(a+b\tan^{-1}(cx)\right)}{4c^2d^3(i-cx)} + \frac{\left(a+b\,\mathrm{ta}\right.}{8c}$$

$$= -\frac{b^2}{16c^2d^3(i-cx)^2} - \frac{5ib^2}{16c^2d^3(i-cx)} + \frac{5ib^2\tan^{-1}(cx)}{16c^2d^3} - \frac{ib\left(a+b\tan^{-1}(cx)\right)}{4c^2d^3(i-cx)^2} + \frac{3b\left(a+b\tan^{-\right.}}{4c^2d^3(i-c}$$

Mathematica [A] time = 0.367463, size = 117, normalized size = 0.66

$$\frac{a^2(-8-16icx)+4ab(-3cx+2i)+b(cx+i)\tan^{-1}(cx)(a(-12cx+4i)+b(3+5icx))-2b^2\left(3c^2x^2+2icx+1\right)\tan^{-1}(cx)^2+}{16c^2d^3(cx-i)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcTan[c*x])^2)/(d + I*c*d*x)^3,x]
```

```
[Out] (4*a*b*(2*I - 3*c*x) + b^2*(4 + (5*I)*c*x) + a^2*(-8 - (16*I)*c*x) + b*(I +
 c*x)*(a*(4*I - 12*c*x) + b*(3 + (5*I)*c*x))*ArcTan[c*x] - 2*b^2*(1 + (2*I)
*c*x + 3*c^2*x^2)*ArcTan[c*x]^2)/(16*c^2*d^3*(-I + c*x)^2)
```

Maple [B] time = 0.084, size = 464, normalized size = 2.6

$$\frac{-ib^2(\arctan(cx))^2}{c^2d^3(cx-i)} + \frac{a^2}{2c^2d^3(cx-i)^2} - \frac{ia^2}{c^2d^3(cx-i)} + \frac{b^2(\arctan(cx))^2}{2c^2d^3(cx-i)^2} - \frac{\frac{i}{4}ab}{c^2d^3(cx-i)^2} + \frac{\frac{5i}{16}b^2\arctan(cx)}{c^2d^3} - \frac{3b^2\,\mathrm{arcta}}{4c^2d^3(c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctan(c*x))^2/(d+I*c*d*x)^3,x)
```

```
[Out] -I/c^2*b^2/d^3*arctan(c*x)^2/(c*x-I)+1/2/c^2*a^2/d^3/(c*x-I)^2-I/c^2*a^2/d^
3/(c*x-I)+1/2/c^2*b^2/d^3*arctan(c*x)^2/(c*x-I)^2-1/4*I/c^2*a*b/d^3/(c*x-I)
^2+5/16*I*b^2*arctan(c*x)/c^2/d^3-3/4/c^2*b^2/d^3*arctan(c*x)/(c*x-I)+3/8*I
/c^2*b^2/d^3*arctan(c*x)*ln(c*x-I)-3/8*I/c^2*b^2/d^3*arctan(c*x)*ln(c*x+I)-
1/16/c^2*b^2/d^3/(c*x-I)^2-2*I/c^2*a*b/d^3*arctan(c*x)/(c*x-I)+3/16/c^2*b^2
/d^3*ln(c*x-I)*ln(-1/2*I*(c*x+I))-3/32/c^2*b^2/d^3*ln(c*x-I)^2-3/16/c^2*b^2
/d^3*ln(-1/2*I*(-c*x+I))*ln(-1/2*I*(c*x+I))+3/16/c^2*b^2/d^3*ln(-1/2*I*(-c*
x+I))*ln(c*x+I)-3/32/c^2*b^2/d^3*ln(c*x+I)^2+5/16*I/c^2*b^2/d^3/(c*x-I)+1/c
^2*a*b/d^3*arctan(c*x)/(c*x-I)^2-1/4*I/c^2*b^2/d^3*arctan(c*x)/(c*x-I)^2-3/
4/c^2*a*b/d^3*arctan(c*x)-3/4/c^2*a*b/d^3/(c*x-I)
```

<u>Maxima</u> [A] time = 1.20614, size = 192, normalized size = 1.08

$$-\frac{\left(16i\,a^2+12\,ab-5i\,b^2\right)cx+2\left(3\,b^2c^2x^2+2i\,b^2cx+b^2\right)\arctan\left(cx\right)^2+8\,a^2-8i\,ab-4\,b^2+\left(\left(12\,ab-5i\,b^2\right)c^2x^2-2\left(-4\right.\right.}{16\,c^4d^3x^2-32i\,c^3d^3x-16\,c^2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))^2/(d+I*c*d*x)^3,x, algorithm="maxima")
```

```
[Out] -((16*I*a^2 + 12*a*b - 5*I*b^2)*c*x + 2*(3*b^2*c^2*x^2 + 2*I*b^2*c*x + b^2)
*arctan(c*x)^2 + 8*a^2 - 8*I*a*b - 4*b^2 + ((12*a*b - 5*I*b^2)*c^2*x^2 - 2*
(-4*I*a*b - b^2)*c*x + 4*a*b - 3*I*b^2)*arctan(c*x))/(16*c^4*d^3*x^2 - 32*I
*c^3*d^3*x - 16*c^2*d^3)
```

<u>Fricas</u> [A] time = 2.31147, size = 378, normalized size = 2.12

$$\frac{\left(-32i\,a^2-24\,ab+10i\,b^2\right)cx+\left(3\,b^2c^2x^2+2i\,b^2cx+b^2\right)\log\left(-\frac{cx+i}{cx-i}\right)^2-16\,a^2+16i\,ab+8\,b^2+\left(\left(-12i\,ab-5\,b^2\right)c^2x^2+2\right.}{32\,c^4d^3x^2-64i\,c^3d^3x-32\,c^2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))^2/(d+I*c*d*x)^3,x, algorithm="fricas")
```

```
[Out] ((-32*I*a^2 - 24*a*b + 10*I*b^2)*c*x + (3*b^2*c^2*x^2 + 2*I*b^2*c*x + b^2)*
log(-(c*x + I)/(c*x - I))^2 - 16*a^2 + 16*I*a*b + 8*b^2 + ((-12*I*a*b - 5*b
^2)*c^2*x^2 + 2*(4*a*b - I*b^2)*c*x - 4*I*a*b - 3*b^2)*log(-(c*x + I)/(c*x
- I)))/(32*c^4*d^3*x^2 - 64*I*c^3*d^3*x - 32*c^2*d^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atan(c*x))**2/(d+I*c*d*x)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\arctan(cx) + a)^2 x}{(icdx + d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))^2/(d+I*c*d*x)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)^2*x/(I*c*d*x + d)^3, x)
```

3.115 $\int \frac{(a+b\tan^{-1}(cx))^2}{(d+icdx)^3}\,dx$

Optimal. Leaf size=180

$$\frac{ib\left(a+b\tan^{-1}(cx)\right)}{4cd^3(-cx+i)}-\frac{b\left(a+b\tan^{-1}(cx)\right)}{4cd^3(-cx+i)^2}+\frac{i\left(a+b\tan^{-1}(cx)\right)^2}{2cd^3(1+icx)^2}-\frac{i\left(a+b\tan^{-1}(cx)\right)^2}{8cd^3}+\frac{3b^2}{16cd^3(-cx+i)}+\frac{ib^2}{16cd^3(-cx$$

```
[Out] ((I/16)*b^2)/(c*d^3*(I - c*x)^2) + (3*b^2)/(16*c*d^3*(I - c*x)) - (3*b^2*Ar
cTan[c*x])/(16*c*d^3) - (b*(a + b*ArcTan[c*x]))/(4*c*d^3*(I - c*x)^2) + ((I
/4)*b*(a + b*ArcTan[c*x]))/(c*d^3*(I - c*x)) - ((I/8)*(a + b*ArcTan[c*x])^2
)/(c*d^3) + ((I/2)*(a + b*ArcTan[c*x])^2)/(c*d^3*(1 + I*c*x)^2)
```

Rubi [A] time = 0.17955, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4864, 4862, 627, 44, 203, 4884}

$$\frac{ib\left(a+b\tan^{-1}(cx)\right)}{4cd^3(-cx+i)}-\frac{b\left(a+b\tan^{-1}(cx)\right)}{4cd^3(-cx+i)^2}+\frac{i\left(a+b\tan^{-1}(cx)\right)^2}{2cd^3(1+icx)^2}-\frac{i\left(a+b\tan^{-1}(cx)\right)^2}{8cd^3}+\frac{3b^2}{16cd^3(-cx+i)}+\frac{ib^2}{16cd^3(-cx$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^2/(d + I*c*d*x)^3,x]
```


```
[Out] ((I/16)*b^2)/(c*d^3*(I - c*x)^2) + (3*b^2)/(16*c*d^3*(I - c*x)) - (3*b^2*Ar
cTan[c*x])/(16*c*d^3) - (b*(a + b*ArcTan[c*x]))/(4*c*d^3*(I - c*x)^2) + ((I
/4)*b*(a + b*ArcTan[c*x]))/(c*d^3*(I - c*x)) - ((I/8)*(a + b*ArcTan[c*x])^2
)/(c*d^3) + ((I/2)*(a + b*ArcTan[c*x])^2)/(c*d^3*(1 + I*c*x)^2)
```


Rule 4864

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x])^p)/(e*(q + 1)), x] - D
ist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 4862

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x]))/(e*(q + 1)), x] - Dist[(b*
```

```
c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{(d+icdx)^3}\,dx = \frac{i\left(a+b\tan^{-1}(cx)\right)^2}{2cd^3(1+icx)^2} - \frac{(ib)\int\left(\frac{i\left(a+b\tan^{-1}(cx)\right)}{2d^2(-i+cx)^3} - \frac{a+b\tan^{-1}(cx)}{4d^2(-i+cx)^2} + \frac{a+b\tan^{-1}(cx)}{4d^2\left(1+c^2x^2\right)}\right)dx}{d}$$

$$= \frac{i\left(a+b\tan^{-1}(cx)\right)^2}{2cd^3(1+icx)^2} + \frac{(ib)\int\frac{a+b\tan^{-1}(cx)}{(-i+cx)^2}dx}{4d^3} - \frac{(ib)\int\frac{a+b\tan^{-1}(cx)}{1+c^2x^2}dx}{4d^3} + \frac{b\int\frac{a+b\tan^{-1}(cx)}{(-i+cx)^3}dx}{2d^3}$$

$$= -\frac{b\left(a+b\tan^{-1}(cx)\right)}{4cd^3(i-cx)^2} + \frac{ib\left(a+b\tan^{-1}(cx)\right)}{4cd^3(i-cx)} - \frac{i\left(a+b\tan^{-1}(cx)\right)^2}{8cd^3} + \frac{i\left(a+b\tan^{-1}(cx)\right)^2}{2cd^3(1+icx)^2} +$$

$$= -\frac{b\left(a+b\tan^{-1}(cx)\right)}{4cd^3(i-cx)^2} + \frac{ib\left(a+b\tan^{-1}(cx)\right)}{4cd^3(i-cx)} - \frac{i\left(a+b\tan^{-1}(cx)\right)^2}{8cd^3} + \frac{i\left(a+b\tan^{-1}(cx)\right)^2}{2cd^3(1+icx)^2} +$$

$$= -\frac{b\left(a+b\tan^{-1}(cx)\right)}{4cd^3(i-cx)^2} + \frac{ib\left(a+b\tan^{-1}(cx)\right)}{4cd^3(i-cx)} - \frac{i\left(a+b\tan^{-1}(cx)\right)^2}{8cd^3} + \frac{i\left(a+b\tan^{-1}(cx)\right)^2}{2cd^3(1+icx)^2} +$$

$$= \frac{ib^2}{16cd^3(i-cx)^2} + \frac{3b^2}{16cd^3(i-cx)} - \frac{b\left(a+b\tan^{-1}(cx)\right)}{4cd^3(i-cx)^2} + \frac{ib\left(a+b\tan^{-1}(cx)\right)}{4cd^3(i-cx)} - \frac{i\left(a+b\tan^{-1}\right.}{8cd^3}$$

$$= \frac{ib^2}{16cd^3(i-cx)^2} + \frac{3b^2}{16cd^3(i-cx)} - \frac{3b^2\tan^{-1}(cx)}{16cd^3} - \frac{b\left(a+b\tan^{-1}(cx)\right)}{4cd^3(i-cx)^2} + \frac{ib\left(a+b\tan^{-1}(cx)\right)}{4cd^3(i-cx)}$$

Mathematica [A] time = 0.178431, size = 110, normalized size = 0.61

$$-\frac{i\left(8a^2+4ab(cx-2i)+b(cx+i)\tan^{-1}(cx)(4a(cx-3i)+b(-5-3icx))+2b^2\left(c^2x^2-2icx+3\right)\tan^{-1}(cx)^2+b^2(-4-3i\right.}{16cd^3(cx-i)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])^2/(d + I*c*d*x)^3,x]
```

```
[Out] ((-I/16)*(8*a^2 + b^2*(-4 - (3*I)*c*x) + 4*a*b*(-2*I + c*x) + b*(I + c*x)*(
b*(-5 - (3*I)*c*x) + 4*a*(-3*I + c*x))*ArcTan[c*x] + 2*b^2*(3 - (2*I)*c*x +
 c^2*x^2)*ArcTan[c*x]^2))/(c*d^3*(-I + c*x)^2)
```

Maple [B] time = 0.072, size = 405, normalized size = 2.3

$$\frac{\frac{i}{16}b^2\ln\left(-\frac{i}{2}(-cx+i)\right)\ln(cx+i)}{cd^3} - \frac{\frac{i}{32}b^2(\ln(cx-i))^2}{cd^3} - \frac{b^2\arctan(cx)\ln(cx-i)}{8\,cd^3} - \frac{b^2\arctan(cx)}{4\,cd^3(cx-i)^2} + \frac{\frac{i}{2}a^2}{cd^3(1+icx)^2} + \frac{b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^2/(d+I*c*d*x)^3,x)
```

```
[Out] 1/16*I/c*b^2/d^3*ln(-1/2*I*(-c*x+I))*ln(c*x+I)-1/32*I/c*b^2/d^3*ln(c*x-I)^2
-1/8/c*b^2/d^3*arctan(c*x)*ln(c*x-I)-1/4/c*b^2/d^3*arctan(c*x)/(c*x-I)^2+1/
2*I/c*a^2/d^3/(1+I*c*x)^2+1/8/c*b^2/d^3*arctan(c*x)*ln(c*x+I)+I/c*a*b/d^3/(
1+I*c*x)^2*arctan(c*x)-1/16*I/c*b^2/d^3*ln(-1/2*I*(-c*x+I))*ln(-1/2*I*(c*x+
I))-1/4*I/c*a*b/d^3/(c*x-I)-3/16/c*b^2/d^3/(c*x-I)-1/4*I/c*a*b/d^3*arctan(c
*x)-3/16*b^2*arctan(c*x)/c/d^3+1/16*I/c*b^2/d^3/(c*x-I)^2-1/32*I/c*b^2/d^3*
ln(c*x+I)^2+1/16*I/c*b^2/d^3*ln(c*x-I)*ln(-1/2*I*(c*x+I))-1/4*I/c*b^2/d^3*a
rctan(c*x)/(c*x-I)-1/4/c*a*b/d^3/(c*x-I)^2+1/2*I/c*b^2/d^3/(1+I*c*x)^2*arct
an(c*x)^2
```

Maxima [A] time = 1.17536, size = 182, normalized size = 1.01

$$-\frac{\left(4i\,ab+3\,b^2\right)cx+\left(2i\,b^2c^2x^2+4\,b^2cx+6i\,b^2\right)\arctan\left(cx\right)^2+8i\,a^2+8\,ab-4i\,b^2+\left(\left(4i\,ab+3\,b^2\right)c^2x^2+\left(8\,ab-2i\,b^2\right)cx\cdots}{16\,c^3d^3x^2-32i\,c^2d^3x-16\,cd^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/(d+I*c*d*x)^3,x, algorithm="maxima")
```

```
[Out] -((4*I*a*b + 3*b^2)*c*x + (2*I*b^2*c^2*x^2 + 4*b^2*c*x + 6*I*b^2)*arctan(c*
x)^2 + 8*I*a^2 + 8*a*b - 4*I*b^2 + ((4*I*a*b + 3*b^2)*c^2*x^2 + (8*a*b - 2*
I*b^2)*c*x + 12*I*a*b + 5*b^2)*arctan(c*x))/(16*c^3*d^3*x^2 - 32*I*c^2*d^3*
x - 16*c*d^3)
```

Fricas [A] time = 2.35746, size = 363, normalized size = 2.02

$$\frac{\left(-8i\,ab-6\,b^2\right)cx+\left(i\,b^2c^2x^2+2\,b^2cx+3i\,b^2\right)\log\left(-\frac{cx+i}{cx-i}\right)^2-16i\,a^2-16\,ab+8i\,b^2+\left(\left(4\,ab-3i\,b^2\right)c^2x^2+\left(-8i\,ab-2\,b^2\right)c\cdots}{32\,c^3d^3x^2-64i\,c^2d^3x-32\,cd^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/(d+I*c*d*x)^3,x, algorithm="fricas")
```

```
[Out] ((-8*I*a*b - 6*b^2)*c*x + (I*b^2*c^2*x^2 + 2*b^2*c*x + 3*I*b^2)*log(-(c*x +
 I)/(c*x - I))^2 - 16*I*a^2 - 16*a*b + 8*I*b^2 + ((4*a*b - 3*I*b^2)*c^2*x^2
```

```
 + (-8*I*a*b - 2*b^2)*c*x + 12*a*b - 5*I*b^2)*log(-(c*x + I)/(c*x - I)))/(3
2*c^3*d^3*x^2 - 64*I*c^2*d^3*x - 32*c*d^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**2/(d+I*c*d*x)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\arctan(cx) + a)^2}{(icdx + d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/(d+I*c*d*x)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)^2/(I*c*d*x + d)^3, x)
```

3.116 $\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x(d+icdx)^3}\,dx$

Optimal. Leaf size=299

$$\frac{ib\text{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{d^3}+\frac{b^2\text{PolyLog}\left(3,-1+\frac{2}{1+icx}\right)}{2d^3}+\frac{5b\left(a+b\tan^{-1}(cx)\right)}{4d^3(-cx+i)}+\frac{ib\left(a+b\tan^{-1}(cx)\right)}{4d^3(-cx+i)^2}$$

```
[Out] b^2/(16*d^3*(I - c*x)^2) - (((11*I)/16)*b^2)/(d^3*(I - c*x)) + (((11*I)/16)
*b^2*ArcTan[c*x])/d^3 + ((I/4)*b*(a + b*ArcTan[c*x]))/(d^3*(I - c*x)^2) + (
5*b*(a + b*ArcTan[c*x]))/(4*d^3*(I - c*x)) - (5*(a + b*ArcTan[c*x])^2)/(8*d
^3) - (a + b*ArcTan[c*x])^2/(2*d^3*(I - c*x)^2) + (I*(a + b*ArcTan[c*x])^2)
/(d^3*(I - c*x)) + (2*(a + b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1 + I*c*x)])/d^3
 + ((a + b*ArcTan[c*x])^2*Log[2/(1 + I*c*x)])/d^3 + (I*b*(a + b*ArcTan[c*x]
)*PolyLog[2, -1 + 2/(1 + I*c*x)])/d^3 + (b^2*PolyLog[3, -1 + 2/(1 + I*c*x)]
)/(2*d^3)
```

Rubi [A] time = 0.794715, antiderivative size = 299, normalized size of antiderivative = 1., number of steps used = 32, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.48, Rules used = {4876, 4850, 4988, 4884, 4994, 6610, 4864, 4862, 627, 44, 203, 4854}

$$\frac{ib\text{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{d^3}+\frac{b^2\text{PolyLog}\left(3,-1+\frac{2}{1+icx}\right)}{2d^3}+\frac{5b\left(a+b\tan^{-1}(cx)\right)}{4d^3(-cx+i)}+\frac{ib\left(a+b\tan^{-1}(cx)\right)}{4d^3(-cx+i)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^2/(x*(d + I*c*d*x)^3),x]
```

```
[Out] b^2/(16*d^3*(I - c*x)^2) - (((11*I)/16)*b^2)/(d^3*(I - c*x)) + (((11*I)/16)
*b^2*ArcTan[c*x])/d^3 + ((I/4)*b*(a + b*ArcTan[c*x]))/(d^3*(I - c*x)^2) + (
5*b*(a + b*ArcTan[c*x]))/(4*d^3*(I - c*x)) - (5*(a + b*ArcTan[c*x])^2)/(8*d
^3) - (a + b*ArcTan[c*x])^2/(2*d^3*(I - c*x)^2) + (I*(a + b*ArcTan[c*x])^2)
/(d^3*(I - c*x)) + (2*(a + b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1 + I*c*x)])/d^3
 + ((a + b*ArcTan[c*x])^2*Log[2/(1 + I*c*x)])/d^3 + (I*b*(a + b*ArcTan[c*x]
)*PolyLog[2, -1 + 2/(1 + I*c*x)])/d^3 + (b^2*PolyLog[3, -1 + 2/(1 + I*c*x)]
)/(2*d^3)
```

Rule 4876

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
```

```
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 4850

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTan[c*x])^(p - 1)*ArcTanh[1 - 2/(1 + I*c*x)])/(1 + c^2*x^2), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4988

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTan[c*x])^p)/(d + e
*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTan[c*x])^p)/(d + e*x^2
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 4864

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x])^p)/(e*(q + 1)), x] - D
ist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 4862

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x]))/(e*(q + 1)), x] - Dist[(b*
c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x(d+icdx)^3}\,dx = \int \left(\frac{\left(a+b\tan^{-1}(cx)\right)^2}{d^3x} + \frac{c\left(a+b\tan^{-1}(cx)\right)^2}{d^3(-i+cx)^3} + \frac{ic\left(a+b\tan^{-1}(cx)\right)^2}{d^3(-i+cx)^2} - \frac{c\left(a+b\tan^{-1}(cx)\right)}{d^3(-i+cx)}\right.$$

$$= \frac{\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x}\,dx}{d^3} + \frac{(ic)\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{(-i+cx)^2}\,dx}{d^3} + \frac{c\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{(-i+cx)^3}\,dx}{d^3} - \frac{c\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{-i+cx}\,dx}{d^3}$$

$$= -\frac{\left(a+b\tan^{-1}(cx)\right)^2}{2d^3(i-cx)^2} + \frac{i\left(a+b\tan^{-1}(cx)\right)^2}{d^3(i-cx)} + \frac{2\left(a+b\tan^{-1}(cx)\right)^2\tanh^{-1}\left(1-\frac{2}{1+icx}\right)}{d^3} + \frac{(a}{}$$

$$= -\frac{\left(a+b\tan^{-1}(cx)\right)^2}{2d^3(i-cx)^2} + \frac{i\left(a+b\tan^{-1}(cx)\right)^2}{d^3(i-cx)} + \frac{2\left(a+b\tan^{-1}(cx)\right)^2\tanh^{-1}\left(1-\frac{2}{1+icx}\right)}{d^3} + \frac{(a}{}$$

$$= \frac{ib\left(a+b\tan^{-1}(cx)\right)}{4d^3(i-cx)^2} + \frac{5b\left(a+b\tan^{-1}(cx)\right)}{4d^3(i-cx)} - \frac{5\left(a+b\tan^{-1}(cx)\right)^2}{8d^3} - \frac{\left(a+b\tan^{-1}(cx)\right)^2}{2d^3(i-cx)^2} +$$

$$= \frac{ib\left(a+b\tan^{-1}(cx)\right)}{4d^3(i-cx)^2} + \frac{5b\left(a+b\tan^{-1}(cx)\right)}{4d^3(i-cx)} - \frac{5\left(a+b\tan^{-1}(cx)\right)^2}{8d^3} - \frac{\left(a+b\tan^{-1}(cx)\right)^2}{2d^3(i-cx)^2} +$$

$$= \frac{ib\left(a+b\tan^{-1}(cx)\right)}{4d^3(i-cx)^2} + \frac{5b\left(a+b\tan^{-1}(cx)\right)}{4d^3(i-cx)} - \frac{5\left(a+b\tan^{-1}(cx)\right)^2}{8d^3} - \frac{\left(a+b\tan^{-1}(cx)\right)^2}{2d^3(i-cx)^2} +$$

$$= \frac{b^2}{16d^3(i-cx)^2} - \frac{11ib^2}{16d^3(i-cx)} + \frac{ib\left(a+b\tan^{-1}(cx)\right)}{4d^3(i-cx)^2} + \frac{5b\left(a+b\tan^{-1}(cx)\right)}{4d^3(i-cx)} - \frac{5\left(a+b\tan^{-1}\right.}{8d^3}$$

$$= \frac{b^2}{16d^3(i-cx)^2} - \frac{11ib^2}{16d^3(i-cx)} + \frac{11ib^2\tan^{-1}(cx)}{16d^3} + \frac{ib\left(a+b\tan^{-1}(cx)\right)}{4d^3(i-cx)^2} + \frac{5b\left(a+b\tan^{-1}(cx\right.}{4d^3(i-cx)}$$

Mathematica [A] time = 1.57874, size = 435, normalized size = 1.45

$$\frac{12iab\left(-16\text{PolyLog}\left(2,e^{2i\tan^{-1}(cx)}\right)-32\tan^{-1}(cx)^2+12i\sin\left(2\tan^{-1}(cx)\right)+i\sin\left(4\tan^{-1}(cx)\right)-12\cos\left(2\tan^{-1}(cx)\right)\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c*x])^2/(x*(d + I*c*d*x)^3),x]
```

```
[Out] ((-96*a^2)/(-I + c*x)^2 - ((192*I)*a^2)/(-I + c*x) - (192*I)*a^2*ArcTan[c*x
] + 192*a^2*Log[c*x] - 96*a^2*Log[1 + c^2*x^2] + (12*I)*a*b*(-32*ArcTan[c*x
]^2 - 12*Cos[2*ArcTan[c*x]] - Cos[4*ArcTan[c*x]] - 16*PolyLog[2, E^((2*I)*A
```

```
rcTan[c*x])] + (12*I)*Sin[2*ArcTan[c*x]] - (4*I)*ArcTan[c*x]*(6*Cos[2*ArcTa
n[c*x]] + Cos[4*ArcTan[c*x]] + 8*Log[1 - E^((2*I)*ArcTan[c*x])] - (6*I)*Sin
[2*ArcTan[c*x]] - I*Sin[4*ArcTan[c*x]]) + I*Sin[4*ArcTan[c*x]]) + b^2*((-8*
I)*Pi^3 - 72*Cos[2*ArcTan[c*x]] - (144*I)*ArcTan[c*x]*Cos[2*ArcTan[c*x]] +
144*ArcTan[c*x]^2*Cos[2*ArcTan[c*x]] - 3*Cos[4*ArcTan[c*x]] - (12*I)*ArcTan
[c*x]*Cos[4*ArcTan[c*x]] + 24*ArcTan[c*x]^2*Cos[4*ArcTan[c*x]] + 192*ArcTan
[c*x]^2*Log[1 - E^((-2*I)*ArcTan[c*x])] + (192*I)*ArcTan[c*x]*PolyLog[2, E^
((-2*I)*ArcTan[c*x])] + 96*PolyLog[3, E^((-2*I)*ArcTan[c*x])] + (72*I)*Sin[
2*ArcTan[c*x]] - 144*ArcTan[c*x]*Sin[2*ArcTan[c*x]] - (144*I)*ArcTan[c*x]^2
*Sin[2*ArcTan[c*x]] + (3*I)*Sin[4*ArcTan[c*x]] - 12*ArcTan[c*x]*Sin[4*ArcTa
n[c*x]] - (24*I)*ArcTan[c*x]^2*Sin[4*ArcTan[c*x]]))/(192*d^3)
```

Maple [C] time = 0.466, size = 2151, normalized size = 7.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^2/x/(d+I*c*d*x)^3,x)
```

```
[Out] -2*I*b^2/d^3*arctan(c*x)*polylog(2,(1+I*c*x)/(c^2*x^2+1)^(1/2))-3*b^2/d^3*a
rctan(c*x)/(4*c*x-4*I)-1/2*b^2/d^3*arctan(c*x)^2/(c*x-I)^2+b^2/d^3*arctan(c
*x)^2*ln(2*I*(1+I*c*x)^2/(c^2*x^2+1))-b^2/d^3*arctan(c*x)^2*ln(c*x-I)-5/4*a
*b/d^3*arctan(c*x)-5/8*b^2/d^3*arctan(c*x)^2+1/64*b^2/d^3/(c*x-I)^2-1/2*a^2
/d^3*ln(c^2*x^2+1)-1/2*a^2/d^3/(c*x-I)^2-2*I*a*b/d^3*arctan(c*x)/(c*x-I)+1/
8*b^2/d^3*arctan(c*x)/(c*x-I)^2*c*x-1/32*I*b^2/d^3/(c*x-I)^2*c*x+3/2*I*b^2/
d^3*Pi*arctan(c*x)^2-2*I*b^2/d^3*arctan(c*x)*polylog(2,-(1+I*c*x)/(c^2*x^2+
1)^(1/2))+2*a*b/d^3*arctan(c*x)*ln(c*x)+I*a*b/d^3*dilog(-1/2*I*(c*x+I))-I*a
*b/d^3*dilog(-I*c*x)-I*a*b/d^3*dilog(-I*(c*x+I))-1/2*I*a*b/d^3*ln(c*x-I)^2-
I*b^2/d^3*arctan(c*x)^2/(c*x-I)-2*a*b/d^3*arctan(c*x)*ln(c*x-I)-a*b/d^3*arc
tan(c*x)/(c*x-I)^2+1/4*I*a*b/d^3/(c*x-I)^2-1/2*I*b^2/d^3*arctan(c*x)^2*Pi*c
sgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*
c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))+1/2*I*b^2/d^3*Pi*csgn(I*((1
+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*((1+I*
c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)^2+1/16*I*b^2
/d^3*arctan(c*x)/(c*x-I)^2-1/2*I*b^2/d^3*Pi*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)
+1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arct
an(c*x)^2-1/2*I*b^2/d^3*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I*((1+I
*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2+1/2*I*b
^2/d^3*Pi*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/
((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2+3*I*b^2/d^3*arctan(c*x)/(4*c*x
-4*I)*c*x+1/2*I*b^2/d^3*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/
```

```
(c^2*x^2+1)+1))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1
))*arctan(c*x)^2-1/2*I*b^2/d^3*arctan(c*x)^2*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1
))*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2-1/2*I*b^2/d^
3*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn((
(1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2-1/1
6*I*b^2/d^3*arctan(c*x)/(c*x-I)^2*c^2*x^2-1/2*I*b^2/d^3*Pi*csgn(((1+I*c*x)^
2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2-1/2*I*b^2/d^3
*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*arctan(c*x)
^2+1/2*I*b^2/d^3*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^
2+1)+1))^3*arctan(c*x)^2-I*b^2/d^3*Pi*arctan(c*x)^2*csgn((1+I*c*x)^2/(c^2*x
^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2+1/2*I*b^2/d^3*Pi*csgn(((1+I*c*x)^2/(c^
2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*arctan(c*x)^2+I*a*b/d^3*ln(c*x)*
ln(-I*(-c*x+I))+I*a*b/d^3*ln(-1/2*I*(c*x+I))*ln(c*x-I)-I*a*b/d^3*ln(-I*c*x)
*ln(-I*(-c*x+I))-I*a*b/d^3*ln(-I*(c*x+I))*ln(c*x)+2*b^2/d^3*polylog(3,(1+I*
c*x)/(c^2*x^2+1)^(1/2))+2*b^2/d^3*polylog(3,-(1+I*c*x)/(c^2*x^2+1)^(1/2))+a
^2/d^3*ln(c*x)-5/4*a*b/d^3/(c*x-I)-I*a^2/d^3/(c*x-I)-I*a^2/d^3*arctan(c*x)-
b^2/d^3*arctan(c*x)^2*ln((1+I*c*x)^2/(c^2*x^2+1)-1)+b^2/d^3*arctan(c*x)^2*l
n(1-(1+I*c*x)/(c^2*x^2+1)^(1/2))+b^2/d^3*arctan(c*x)^2*ln(1+(1+I*c*x)/(c^2*
x^2+1)^(1/2))+b^2/d^3*arctan(c*x)^2*ln(c*x)+3/8*I*b^2/d^3/(c*x-I)-2/3*I*b^2
/d^3*arctan(c*x)^3+3/8*b^2/d^3/(c*x-I)*c*x-1/64*b^2/d^3/(c*x-I)^2*c^2*x^2
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x/(d+I*c*d*x)^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{-i\,b^2\log\left(-\frac{cx+i}{cx-i}\right)^2 - 4\,ab\log\left(-\frac{cx+i}{cx-i}\right) + 4i\,a^2}{4\,c^3d^3x^4 - 12i\,c^2d^3x^3 - 12\,cd^3x^2 + 4i\,d^3x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x/(d+I*c*d*x)^3,x, algorithm="fricas")
```

```
[Out] integral((-I*b^2*log(-(c*x + I)/(c*x - I))^2 - 4*a*b*log(-(c*x + I)/(c*x -
I)) + 4*I*a^2)/(4*c^3*d^3*x^4 - 12*I*c^2*d^3*x^3 - 12*c*d^3*x^2 + 4*I*d^3*x
), x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**2/x/(d+I*c*d*x)**3,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\arctan(cx) + a)^2}{(i\,cdx + d)^3 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x/(d+I*c*d*x)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)^2/((I*c*d*x + d)^3*x), x)
```

3.117 $\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^2(d+icdx)^3}\,dx$

Optimal. Leaf size=391

$$\frac{3bc\text{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{d^3}-\frac{ib^2c\text{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)}{d^3}-\frac{3ib^2c\text{PolyLog}\left(3,-1+\frac{2}{1+icx}\right)}{2d^3}-\frac{9ibc\left(a\cdots\right.}{4\cdots}$$

```
[Out] ((-I/16)*b^2*c)/(d^3*(I - c*x)^2) - (19*b^2*c)/(16*d^3*(I - c*x)) + (19*b^2
*c*ArcTan[c*x])/(16*d^3) + (b*c*(a + b*ArcTan[c*x]))/(4*d^3*(I - c*x)^2) -
(((9*I)/4)*b*c*(a + b*ArcTan[c*x]))/(d^3*(I - c*x)) + ((I/8)*c*(a + b*ArcTa
n[c*x])^2)/d^3 - (a + b*ArcTan[c*x])^2/(d^3*x) + ((I/2)*c*(a + b*ArcTan[c*x
])^2)/(d^3*(I - c*x)^2) + (2*c*(a + b*ArcTan[c*x])^2)/(d^3*(I - c*x)) - ((6
*I)*c*(a + b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1 + I*c*x)])/d^3 - ((3*I)*c*(a +
 b*ArcTan[c*x])^2*Log[2/(1 + I*c*x)])/d^3 + (2*b*c*(a + b*ArcTan[c*x])*Log[
2 - 2/(1 - I*c*x)])/d^3 - (I*b^2*c*PolyLog[2, -1 + 2/(1 - I*c*x)])/d^3 + (3
*b*c*(a + b*ArcTan[c*x])*PolyLog[2, -1 + 2/(1 + I*c*x)])/d^3 - (((3*I)/2)*b
^2*c*PolyLog[3, -1 + 2/(1 + I*c*x)])/d^3
```

Rubi [A] time = 0.976482, antiderivative size = 391, normalized size of antiderivative = 1., number of steps used = 36, number of rules used = 16, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.64, Rules used = {4876, 4852, 4924, 4868, 2447, 4850, 4988, 4884, 4994, 6610, 4864, 4862, 627, 44, 203, 4854}

$$\frac{3bc\text{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{d^3}-\frac{ib^2c\text{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)}{d^3}-\frac{3ib^2c\text{PolyLog}\left(3,-1+\frac{2}{1+icx}\right)}{2d^3}-\frac{9ibc\left(a\cdots\right.}{4\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^2/(x^2*(d + I*c*d*x)^3),x]
```

```
[Out] ((-I/16)*b^2*c)/(d^3*(I - c*x)^2) - (19*b^2*c)/(16*d^3*(I - c*x)) + (19*b^2
*c*ArcTan[c*x])/(16*d^3) + (b*c*(a + b*ArcTan[c*x]))/(4*d^3*(I - c*x)^2) -
(((9*I)/4)*b*c*(a + b*ArcTan[c*x]))/(d^3*(I - c*x)) + ((I/8)*c*(a + b*ArcTa
n[c*x])^2)/d^3 - (a + b*ArcTan[c*x])^2/(d^3*x) + ((I/2)*c*(a + b*ArcTan[c*x
])^2)/(d^3*(I - c*x)^2) + (2*c*(a + b*ArcTan[c*x])^2)/(d^3*(I - c*x)) - ((6
*I)*c*(a + b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1 + I*c*x)])/d^3 - ((3*I)*c*(a +
 b*ArcTan[c*x])^2*Log[2/(1 + I*c*x)])/d^3 + (2*b*c*(a + b*ArcTan[c*x])*Log[
2 - 2/(1 - I*c*x)])/d^3 - (I*b^2*c*PolyLog[2, -1 + 2/(1 - I*c*x)])/d^3 + (3
*b*c*(a + b*ArcTan[c*x])*PolyLog[2, -1 + 2/(1 + I*c*x)])/d^3 - (((3*I)/2)*b
```

```
^2*c*PolyLog[3, -1 + 2/(1 + I*c*x)])/d^3
```

Rule 4876

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4850

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTan[c*x])^(p - 1)*ArcTanh[1 - 2/(1 + I*c*x)])/(1 + c^2*x^2), x], x] /;
```

```
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4988

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTan[c*x])^p)/(d + e
*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTan[c*x])^p)/(d + e*x^2
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 4864

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x])^p)/(e*(q + 1)), x] - D
ist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 4862

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x]))/(e*(q + 1)), x] - Dist[(b*
c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^2(d+icdx)^3}\,dx = \int \left(\frac{\left(a+b\tan^{-1}(cx)\right)^2}{d^3x^2} - \frac{3ic\left(a+b\tan^{-1}(cx)\right)^2}{d^3x} - \frac{ic^2\left(a+b\tan^{-1}(cx)\right)^2}{d^3(-i+cx)^3} + \frac{2c^2\left(a+b\tan^{-1}\right.}{d^3(-i+cx)}\right.$$

$$= \frac{\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^2}\,dx}{d^3} - \frac{(3ic)\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x}\,dx}{d^3} - \frac{\left(ic^2\right)\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{(-i+cx)^3}\,dx}{d^3} + \frac{\left(3ic^2\right)\int \frac{\left(a+b\tan\right.}{-i+}}{d^3}$$

$$= -\frac{\left(a+b\tan^{-1}(cx)\right)^2}{d^3x} + \frac{ic\left(a+b\tan^{-1}(cx)\right)^2}{2d^3(i-cx)^2} + \frac{2c\left(a+b\tan^{-1}(cx)\right)^2}{d^3(i-cx)} - \frac{6ic\left(a+b\tan^{-1}(cx)\right.}{d}$$

$$= -\frac{ic\left(a+b\tan^{-1}(cx)\right)^2}{d^3} - \frac{\left(a+b\tan^{-1}(cx)\right)^2}{d^3x} + \frac{ic\left(a+b\tan^{-1}(cx)\right)^2}{2d^3(i-cx)^2} + \frac{2c\left(a+b\tan^{-1}(cx)\right)^2}{d^3(i-cx)}$$

$$= \frac{bc\left(a+b\tan^{-1}(cx)\right)}{4d^3(i-cx)^2} - \frac{9ibc\left(a+b\tan^{-1}(cx)\right)}{4d^3(i-cx)} + \frac{ic\left(a+b\tan^{-1}(cx)\right)^2}{8d^3} - \frac{\left(a+b\tan^{-1}(cx)\right)^2}{d^3x}$$

$$= \frac{bc\left(a+b\tan^{-1}(cx)\right)}{4d^3(i-cx)^2} - \frac{9ibc\left(a+b\tan^{-1}(cx)\right)}{4d^3(i-cx)} + \frac{ic\left(a+b\tan^{-1}(cx)\right)^2}{8d^3} - \frac{\left(a+b\tan^{-1}(cx)\right)^2}{d^3x}$$

$$= \frac{bc\left(a+b\tan^{-1}(cx)\right)}{4d^3(i-cx)^2} - \frac{9ibc\left(a+b\tan^{-1}(cx)\right)}{4d^3(i-cx)} + \frac{ic\left(a+b\tan^{-1}(cx)\right)^2}{8d^3} - \frac{\left(a+b\tan^{-1}(cx)\right)^2}{d^3x}$$

$$= -\frac{ib^2c}{16d^3(i-cx)^2} - \frac{19b^2c}{16d^3(i-cx)} + \frac{bc\left(a+b\tan^{-1}(cx)\right)}{4d^3(i-cx)^2} - \frac{9ibc\left(a+b\tan^{-1}(cx)\right)}{4d^3(i-cx)} + \frac{ic\left(a+b\,\mathrm{ta}\right.}{8d}$$

$$= -\frac{ib^2c}{16d^3(i-cx)^2} - \frac{19b^2c}{16d^3(i-cx)} + \frac{19b^2c\tan^{-1}(cx)}{16d^3} + \frac{bc\left(a+b\tan^{-1}(cx)\right)}{4d^3(i-cx)^2} - \frac{9ibc\left(a+b\tan^{-1}\right.}{4d^3(i-cx)}$$

Mathematica [A] time = 3.50866, size = 549, normalized size = 1.4

$$-\frac{\frac{4ab\left(48cx\,\mathrm{PolyLog}\left(2,e^{2i\tan^{-1}(cx)}\right)+cx\left(-32\log\left(\frac{cx}{\sqrt{c^2x^2+1}}\right)-20i\sin\left(2\tan^{-1}(cx)\right)-i\sin\left(4\tan^{-1}(cx)\right)+20\cos\left(2\tan^{-1}(cx)\right)+\cos\left(4\tan^{-1}(cx)\right)\right)+96cx\tan^{-1}(cx)\right.}{x}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c*x])^2/(x^2*(d + I*c*d*x)^3),x]
```

```
[Out] -((64*a^2)/x - ((32*I)*a^2*c)/(-I + c*x)^2 + (128*a^2*c)/(-I + c*x) + 192*a
^2*c*ArcTan[c*x] + (192*I)*a^2*c*Log[x] - (96*I)*a^2*c*Log[1 + c^2*x^2] - I
*b^2*c*((8*I)*Pi^3 - 64*ArcTan[c*x]^2 + ((64*I)*ArcTan[c*x]^2)/(c*x) + 40*C
```

```
os[2*ArcTan[c*x]] + (80*I)*ArcTan[c*x]*Cos[2*ArcTan[c*x]] - 80*ArcTan[c*x]^
2*Cos[2*ArcTan[c*x]] + Cos[4*ArcTan[c*x]] + (4*I)*ArcTan[c*x]*Cos[4*ArcTan[
c*x]] - 8*ArcTan[c*x]^2*Cos[4*ArcTan[c*x]] - 192*ArcTan[c*x]^2*Log[1 - E^((
-2*I)*ArcTan[c*x])] - (128*I)*ArcTan[c*x]*Log[1 - E^((2*I)*ArcTan[c*x])] -
(192*I)*ArcTan[c*x]*PolyLog[2, E^((-2*I)*ArcTan[c*x])] - 64*PolyLog[2, E^((
2*I)*ArcTan[c*x])] - 96*PolyLog[3, E^((-2*I)*ArcTan[c*x])] - (40*I)*Sin[2*A
rcTan[c*x]] + 80*ArcTan[c*x]*Sin[2*ArcTan[c*x]] + (80*I)*ArcTan[c*x]^2*Sin[
2*ArcTan[c*x]] - I*Sin[4*ArcTan[c*x]] + 4*ArcTan[c*x]*Sin[4*ArcTan[c*x]] +
(8*I)*ArcTan[c*x]^2*Sin[4*ArcTan[c*x]]) + (4*a*b*(96*c*x*ArcTan[c*x]^2 + 48
*c*x*PolyLog[2, E^((2*I)*ArcTan[c*x])] + c*x*(20*Cos[2*ArcTan[c*x]] + Cos[4
*ArcTan[c*x]] - 32*Log[(c*x)/Sqrt[1 + c^2*x^2]] - (20*I)*Sin[2*ArcTan[c*x]]
 - I*Sin[4*ArcTan[c*x]]) + 4*ArcTan[c*x]*(8 + (10*I)*c*x*Cos[2*ArcTan[c*x]]
 + I*c*x*Cos[4*ArcTan[c*x]] + (24*I)*c*x*Log[1 - E^((2*I)*ArcTan[c*x])] + 1
0*c*x*Sin[2*ArcTan[c*x]] + c*x*Sin[4*ArcTan[c*x]])))/x)/(64*d^3)
```

Maple [C] time = 1.066, size = 9659, normalized size = 24.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^2/x^2/(d+I*c*d*x)^3,x)

[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^2/(d+I*c*d*x)^3,x, algorithm="maxima")

[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{-i\,b^2\log\left(-\frac{cx+i}{cx-i}\right)^2-4\,ab\log\left(-\frac{cx+i}{cx-i}\right)+4i\,a^2}{4\,c^3d^3x^5-12i\,c^2d^3x^4-12\,cd^3x^3+4i\,d^3x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^2/(d+I*c*d*x)^3,x, algorithm="fricas")
```

```
[Out] integral((-I*b^2*log(-(c*x + I)/(c*x - I))^2 - 4*a*b*log(-(c*x + I)/(c*x -
I)) + 4*I*a^2)/(4*c^3*d^3*x^5 - 12*I*c^2*d^3*x^4 - 12*c*d^3*x^3 + 4*I*d^3*x
^2), x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**2/x**2/(d+I*c*d*x)**3,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\arctan(cx)+a)^2}{(i\,cdx+d)^3x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^2/(d+I*c*d*x)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)^2/((I*c*d*x + d)^3*x^2), x)
```

3.118 $\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{(1+icx)^4}\,dx$

Optimal. Leaf size=207

$$\frac{ib\left(a+b\tan^{-1}(cx)\right)}{12c(-cx+i)}-\frac{b\left(a+b\tan^{-1}(cx)\right)}{12c(-cx+i)^2}-\frac{ib\left(a+b\tan^{-1}(cx)\right)}{9c(-cx+i)^3}-\frac{i\left(a+b\tan^{-1}(cx)\right)^2}{24c}+\frac{i\left(a+b\tan^{-1}(cx)\right)^2}{3c(1+icx)^3}+\frac{11b}{144c(-c}$$

```
[Out] -b^2/(54*c*(I - c*x)^3) + (((5*I)/144)*b^2)/(c*(I - c*x)^2) + (11*b^2)/(144
*c*(I - c*x)) - (11*b^2*ArcTan[c*x])/(144*c) - ((I/9)*b*(a + b*ArcTan[c*x])
)/(c*(I - c*x)^3) - (b*(a + b*ArcTan[c*x]))/(12*c*(I - c*x)^2) + ((I/12)*b*
(a + b*ArcTan[c*x]))/(c*(I - c*x)) - ((I/24)*(a + b*ArcTan[c*x])^2)/c + ((I
/3)*(a + b*ArcTan[c*x])^2)/(c*(1 + I*c*x)^3)
```

Rubi [A] time = 0.222115, antiderivative size = 207, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4864, 4862, 627, 44, 203, 4884}

$$\frac{ib\left(a+b\tan^{-1}(cx)\right)}{12c(-cx+i)}-\frac{b\left(a+b\tan^{-1}(cx)\right)}{12c(-cx+i)^2}-\frac{ib\left(a+b\tan^{-1}(cx)\right)}{9c(-cx+i)^3}-\frac{i\left(a+b\tan^{-1}(cx)\right)^2}{24c}+\frac{i\left(a+b\tan^{-1}(cx)\right)^2}{3c(1+icx)^3}+\frac{11b}{144c(-c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^2/(1 + I*c*x)^4,x]
```


```
[Out] -b^2/(54*c*(I - c*x)^3) + (((5*I)/144)*b^2)/(c*(I - c*x)^2) + (11*b^2)/(144
*c*(I - c*x)) - (11*b^2*ArcTan[c*x])/(144*c) - ((I/9)*b*(a + b*ArcTan[c*x])
)/(c*(I - c*x)^3) - (b*(a + b*ArcTan[c*x]))/(12*c*(I - c*x)^2) + ((I/12)*b*
(a + b*ArcTan[c*x]))/(c*(I - c*x)) - ((I/24)*(a + b*ArcTan[c*x])^2)/c + ((I
/3)*(a + b*ArcTan[c*x])^2)/(c*(1 + I*c*x)^3)
```


Rule 4864

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x])^p)/(e*(q + 1)), x] - D
ist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 4862

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x]))/(e*(q + 1)), x] - Dist[(b*
c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{(1+icx)^4}\,dx = \frac{i\left(a+b\tan^{-1}(cx)\right)^2}{3c(1+icx)^3} - \frac{1}{3}(2ib)\int\left(\frac{a+b\tan^{-1}(cx)}{2(-i+cx)^4} + \frac{i\left(a+b\tan^{-1}(cx)\right)}{4(-i+cx)^3} - \frac{a+b\tan^{-1}(cx)}{8(-i+cx)^2} + \cdots\right.$$

$$= \frac{i\left(a+b\tan^{-1}(cx)\right)^2}{3c(1+icx)^3} + \frac{1}{12}(ib)\int\frac{a+b\tan^{-1}(cx)}{(-i+cx)^2}\,dx - \frac{1}{12}(ib)\int\frac{a+b\tan^{-1}(cx)}{1+c^2x^2}\,dx - \frac{1}{3}(ib)\int \cdots$$

$$= -\frac{ib\left(a+b\tan^{-1}(cx)\right)}{9c(i-cx)^3} - \frac{b\left(a+b\tan^{-1}(cx)\right)}{12c(i-cx)^2} + \frac{ib\left(a+b\tan^{-1}(cx)\right)}{12c(i-cx)} - \frac{i\left(a+b\tan^{-1}(cx)\right)^2}{24c} + \cdots$$

$$= -\frac{ib\left(a+b\tan^{-1}(cx)\right)}{9c(i-cx)^3} - \frac{b\left(a+b\tan^{-1}(cx)\right)}{12c(i-cx)^2} + \frac{ib\left(a+b\tan^{-1}(cx)\right)}{12c(i-cx)} - \frac{i\left(a+b\tan^{-1}(cx)\right)^2}{24c} + \cdots$$

$$= -\frac{ib\left(a+b\tan^{-1}(cx)\right)}{9c(i-cx)^3} - \frac{b\left(a+b\tan^{-1}(cx)\right)}{12c(i-cx)^2} + \frac{ib\left(a+b\tan^{-1}(cx)\right)}{12c(i-cx)} - \frac{i\left(a+b\tan^{-1}(cx)\right)^2}{24c} + \cdots$$

$$= -\frac{b^2}{54c(i-cx)^3} + \frac{5ib^2}{144c(i-cx)^2} + \frac{11b^2}{144c(i-cx)} - \frac{ib\left(a+b\tan^{-1}(cx)\right)}{9c(i-cx)^3} - \frac{b\left(a+b\tan^{-1}(cx)\right)}{12c(i-cx)^2} + \cdots$$

$$= -\frac{b^2}{54c(i-cx)^3} + \frac{5ib^2}{144c(i-cx)^2} + \frac{11b^2}{144c(i-cx)} - \frac{11b^2\tan^{-1}(cx)}{144c} - \frac{ib\left(a+b\tan^{-1}(cx)\right)}{9c(i-cx)^3} - \frac{b\left(a+\cdots\right.}{1\cdots}$$

Mathematica [A] time = 0.2092, size = 155, normalized size = 0.75

$$-\frac{144a^2+12ab\left(3ic^2x^2+9cx-10i\right)+3b(cx+i)\tan^{-1}(cx)\left(12a\left(ic^2x^2+4cx-7i\right)+b\left(11c^2x^2-32icx-29\right)\right)+b^2\left(33c^2x^2\cdots\right.}{432c(cx-i)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])^2/(1 + I*c*x)^4,x]
```

```
[Out] -(144*a^2 + 12*a*b*(-10*I + 9*c*x + (3*I)*c^2*x^2) + b^2*(-56 - (81*I)*c*x
+ 33*c^2*x^2) + 3*b*(I + c*x)*(12*a*(-7*I + 4*c*x + I*c^2*x^2) + b*(-29 - (
32*I)*c*x + 11*c^2*x^2))*ArcTan[c*x] + 18*b^2*(7 - (3*I)*c*x + 3*c^2*x^2 +
I*c^3*x^3)*ArcTan[c*x]^2)/(432*c*(-I + c*x)^3)
```

Maple [B] time = 0.075, size = 404, normalized size = 2.

$$\frac{\frac{i}{9}ab}{c\,(cx-i)^3} + \frac{\frac{2i}{3}ab\arctan(cx)}{c\,(1+icx)^3} - \frac{b^2\arctan(cx)\ln(cx-i)}{24\,c} - \frac{b^2\arctan(cx)}{12\,c\,(cx-i)^2} + \frac{\frac{i}{9}b^2\arctan(cx)}{c\,(cx-i)^3} - \frac{\frac{i}{12}ab}{c\,(cx-i)} + \frac{b^2\arctan(c\cdots}{24\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^2/(1+I*c*x)^4,x)
```

```
[Out] 1/9*I/c*a*b/(c*x-I)^3+2/3*I/c*a*b/(1+I*c*x)^3*arctan(c*x)-1/24/c*b^2*arctan
(c*x)*ln(c*x-I)-1/12/c*b^2*arctan(c*x)/(c*x-I)^2+1/9*I/c*b^2*arctan(c*x)/(c
*x-I)^3-1/12*I/c*a*b/(c*x-I)+1/24/c*b^2*arctan(c*x)*ln(c*x+I)+1/54/c*b^2/(c
*x-I)^3-11/144/c*b^2/(c*x-I)-1/12*I/c*b^2*arctan(c*x)/(c*x-I)-11/144*b^2*ar
ctan(c*x)/c-1/48*I/c*b^2*ln(-1/2*I*(-c*x+I))*ln(-1/2*I*(c*x+I))+1/3*I/c*b^2
/(1+I*c*x)^3*arctan(c*x)^2-1/96*I/c*b^2*ln(c*x+I)^2-1/12*I/c*a*b*arctan(c*x
)+1/48*I/c*b^2*ln(c*x-I)*ln(-1/2*I*(c*x+I))+1/48*I/c*b^2*ln(-1/2*I*(-c*x+I)
)*ln(c*x+I)-1/96*I/c*b^2*ln(c*x-I)^2-1/12/c*a*b/(c*x-I)^2+1/3*I/c*a^2/(1+I*
c*x)^3+5/144*I/c*b^2/(c*x-I)^2
```

Maxima [A] time = 1.32133, size = 248, normalized size = 1.2

$$\frac{3\left(-12i\,ab-11\,b^2\right)c^2x^2-\left(108\,ab-81i\,b^2\right)cx-\left(18i\,b^2c^3x^3+54\,b^2c^2x^2-54i\,b^2cx+126\,b^2\right)\arctan\left(cx\right)^2-144\,a^2+120i}{432\,c^4x^3-1296i\,c^3x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/(1+I*c*x)^4,x, algorithm="maxima")
```

```
[Out] (3*(-12*I*a*b - 11*b^2)*c^2*x^2 - (108*a*b - 81*I*b^2)*c*x - (18*I*b^2*c^3*
x^3 + 54*b^2*c^2*x^2 - 54*I*b^2*c*x + 126*b^2)*arctan(c*x)^2 - 144*a^2 + 12
0*I*a*b + 56*b^2 + (3*(-12*I*a*b - 11*b^2)*c^3*x^3 - (108*a*b - 63*I*b^2)*c
^2*x^2 + 9*(12*I*a*b - b^2)*c*x - 252*a*b + 87*I*b^2)*arctan(c*x))/(432*c^4
*x^3 - 1296*I*c^3*x^2 - 1296*c^2*x + 432*I*c)
```

Fricas [A] time = 2.29416, size = 502, normalized size = 2.43

$$\frac{\left(-72i\,ab-66\,b^2\right)c^2x^2-54\left(4\,ab-3i\,b^2\right)cx+\left(9i\,b^2c^3x^3+27\,b^2c^2x^2-27i\,b^2cx+63\,b^2\right)\log\left(-\frac{cx+i}{cx-i}\right)^2-288\,a^2+240i\,ab}{864\,c^4x^3-2592i\,c^3x^2-25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/(1+I*c*x)^4,x, algorithm="fricas")
```

```
[Out] ((-72*I*a*b - 66*b^2)*c^2*x^2 - 54*(4*a*b - 3*I*b^2)*c*x + (9*I*b^2*c^3*x^3
 + 27*b^2*c^2*x^2 - 27*I*b^2*c*x + 63*b^2)*log(-(c*x + I)/(c*x - I))^2 - 28
8*a^2 + 240*I*a*b + 112*b^2 + (3*(12*a*b - 11*I*b^2)*c^3*x^3 + (-108*I*a*b
- 63*b^2)*c^2*x^2 - 9*(12*a*b + I*b^2)*c*x - 252*I*a*b - 87*b^2)*log(-(c*x
+ I)/(c*x - I)))/(864*c^4*x^3 - 2592*I*c^3*x^2 - 2592*c^2*x + 864*I*c)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**2/(1+I*c*x)**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \arctan(cx) + a)^2}{(i\,cx + 1)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/(1+I*c*x)^4,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)^2/(I*c*x + 1)^4, x)
```

3.119 $\int \frac{\tan^{-1}(ax)^2}{cx - iacx^2} \, dx$

Optimal. Leaf size=76

$$\frac{\text{PolyLog}\left(3, -1 + \frac{2}{1-iax}\right)}{2c} - \frac{i \tan^{-1}(ax) \text{PolyLog}\left(2, -1 + \frac{2}{1-iax}\right)}{c} + \frac{\log\left(2 - \frac{2}{1-iax}\right) \tan^{-1}(ax)^2}{c}$$

```
[Out] (ArcTan[a*x]^2*Log[2 - 2/(1 - I*a*x)])/c - (I*ArcTan[a*x]*PolyLog[2, -1 + 2
/(1 - I*a*x)])/c + PolyLog[3, -1 + 2/(1 - I*a*x)]/(2*c)
```

Rubi [A] time = 0.137644, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1593, 4868, 4884, 4992, 6610}

$$\frac{\text{PolyLog}\left(3, -1 + \frac{2}{1-iax}\right)}{2c} - \frac{i \tan^{-1}(ax) \text{PolyLog}\left(2, -1 + \frac{2}{1-iax}\right)}{c} + \frac{\log\left(2 - \frac{2}{1-iax}\right) \tan^{-1}(ax)^2}{c}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^2/(c*x - I*a*c*x^2),x]
```

```
[Out] (ArcTan[a*x]^2*Log[2 - 2/(1 - I*a*x)])/c - (I*ArcTan[a*x]*PolyLog[2, -1 + 2
/(1 - I*a*x)])/c + PolyLog[3, -1 + 2/(1 - I*a*x)]/(2*c)
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4992

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d
] && EqQ[(1 - u)^2 - (1 - (2*I)/(I + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^{-1}(ax)^2}{cx-iacx^2}\,dx &= \int \frac{\tan^{-1}(ax)^2}{x(c-iacx)}\,dx\\
&= \frac{\tan^{-1}(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{c}-\frac{(2a)\int \frac{\tan^{-1}(ax)\log\left(2-\frac{2}{1-iax}\right)}{1+a^2x^2}\,dx}{c}\\
&= \frac{\tan^{-1}(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{c}-\frac{i\tan^{-1}(ax)\mathrm{Li}_2\left(-1+\frac{2}{1-iax}\right)}{c}+\frac{(ia)\int \frac{\mathrm{Li}_2\left(-1+\frac{2}{1-iax}\right)}{1+a^2x^2}\,dx}{c}\\
&= \frac{\tan^{-1}(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{c}-\frac{i\tan^{-1}(ax)\mathrm{Li}_2\left(-1+\frac{2}{1-iax}\right)}{c}+\frac{\mathrm{Li}_3\left(-1+\frac{2}{1-iax}\right)}{2c}
\end{aligned}$$

Mathematica [A] time = 0.253494, size = 82, normalized size = 1.08

$$\frac{24i\tan^{-1}(ax)\text{PolyLog}\left(2,e^{-2i\tan^{-1}(ax)}\right)+12\text{PolyLog}\left(3,e^{-2i\tan^{-1}(ax)}\right)+16i\tan^{-1}(ax)^3+24\tan^{-1}(ax)^2\log\left(1-e^{-2i\tan^{-}}\right.}{24c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTan[a*x]^2/(c*x - I*a*c*x^2),x]
```

```
[Out] ((-I)*Pi^3 + (16*I)*ArcTan[a*x]^3 + 24*ArcTan[a*x]^2*Log[1 - E^((-2*I)*ArcT
an[a*x])] + (24*I)*ArcTan[a*x]*PolyLog[2, E^((-2*I)*ArcTan[a*x])] + 12*Poly
```

```
Log[3, E^((-2*I)*ArcTan[a*x])])/(24*c)
```

Maple [B] time = 0.207, size = 183, normalized size = 2.4

$$\frac{(\arctan(ax))^2}{c}\ln\left(1-(1+iax)\frac{1}{\sqrt{a^2x^2+1}}\right)-\frac{2\,i\arctan(ax)}{c}\mathrm{polylog}\left(2,(1+iax)\frac{1}{\sqrt{a^2x^2+1}}\right)+2\,\frac{1}{c}\mathrm{polylog}\left(3,\frac{1+ia}{\sqrt{a^2x^2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^2/(c*x-I*a*c*x^2),x)
```

```
[Out] 1/c*arctan(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*I/c*arctan(a*x)*polyl
og(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+2/c*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2)
)+1/c*arctan(a*x)^2*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*I/c*arctan(a*x)*pol
ylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+2/c*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(
1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{8i\arctan(ax)^3-12\arctan(ax)^2\log\left(a^2x^2+1\right)-6i\arctan(ax)\log\left(a^2x^2+1\right)^2+3\log\left(a^2x^2+1\right)^2\log\left(-a^2x^2\right)+6i}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/(c*x-I*a*c*x^2),x, algorithm="maxima")
```

```
[Out] 1/96*(8*I*arctan(a*x)^3 - 12*arctan(a*x)^2*log(a^2*x^2 + 1) - 6*I*arctan(a*
x)*log(a^2*x^2 + 1)^2 + log(a^2*x^2 + 1)^3 + 24*I*(arctan(a*x)^3/c + 4*a*in
tegrate(1/16*x*log(a^2*x^2 + 1)^2/(a^2*c*x^3 + c*x), x) - 16*integrate(1/16
*arctan(a*x)*log(a^2*x^2 + 1)/(a^2*c*x^3 + c*x), x))*c + 96*c*integrate(1/1
6*(4*a*x*arctan(a*x)*log(a^2*x^2 + 1) + 12*arctan(a*x)^2 + log(a^2*x^2 + 1)
^2)/(a^2*c*x^3 + c*x), x))/c
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{i\,\log\left(-\frac{ax+i}{ax-i}\right)^2}{4\,acx^2+4i\,cx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/(c*x-I*a*c*x^2),x, algorithm="fricas")
```

```
[Out] integral(-I*log(-(a*x + I)/(a*x - I))^2/(4*a*c*x^2 + 4*I*c*x), x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**2/(c*x-I*a*c*x**2),x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(ax)^2}{-i\,acx^2+cx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/(c*x-I*a*c*x^2),x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)^2/(-I*a*c*x^2 + c*x), x)
```

3.120 $\int (d+icdx)^3 \left(a+b\tan^{-1}(cx)\right)^3 \, dx$

Optimal. Leaf size=382

$$-\frac{6ib^2d^3\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)\left(a+b\tan^{-1}(cx)\right)}{c}+\frac{11b^3d^3\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{2c}+\frac{3b^3d^3\text{PolyLog}\left(3,1-\frac{2}{1-icx}\right)}{c}-\frac{1}{4}ib$$

```
[Out] -3*a*b^2*d^3*x + (I/4)*b^3*d^3*x - ((I/4)*b^3*d^3*ArcTan[c*x])/c - 3*b^3*d^
3*x*ArcTan[c*x] - (I/4)*b^2*c*d^3*x^2*(a + b*ArcTan[c*x]) + (7*b*d^3*(a + b
*ArcTan[c*x])^2)/c - ((21*I)/4)*b*d^3*x*(a + b*ArcTan[c*x])^2 + (3*b*c*d^3*
x^2*(a + b*ArcTan[c*x])^2)/2 + (I/4)*b*c^2*d^3*x^3*(a + b*ArcTan[c*x])^2 -
((I/4)*d^3*(1 + I*c*x)^4*(a + b*ArcTan[c*x])^3)/c + (6*b*d^3*(a + b*ArcTan[
c*x])^2*Log[2/(1 - I*c*x)])/c - ((11*I)*b^2*d^3*(a + b*ArcTan[c*x])*Log[2/(
1 + I*c*x)])/c + (3*b^3*d^3*Log[1 + c^2*x^2])/(2*c) - ((6*I)*b^2*d^3*(a + b
*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 - I*c*x)])/c + (11*b^3*d^3*PolyLog[2, 1 -
 2/(1 + I*c*x)])/(2*c) + (3*b^3*d^3*PolyLog[3, 1 - 2/(1 - I*c*x)])/c
```

Rubi [A] time = 0.706987, antiderivative size = 382, normalized size of antiderivative = 1., number of steps used = 26, number of rules used = 15, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.682, Rules used = {4864, 4846, 4920, 4854, 2402, 2315, 4852, 4916, 260, 4884, 321, 203, 1586, 4992, 6610}

$$-\frac{6ib^2d^3\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)\left(a+b\tan^{-1}(cx)\right)}{c}+\frac{11b^3d^3\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{2c}+\frac{3b^3d^3\text{PolyLog}\left(3,1-\frac{2}{1-icx}\right)}{c}-\frac{1}{4}ib$$

Antiderivative was successfully verified.

```
[In] Int[(d + I*c*d*x)^3*(a + b*ArcTan[c*x])^3,x]
```


```
[Out] -3*a*b^2*d^3*x + (I/4)*b^3*d^3*x - ((I/4)*b^3*d^3*ArcTan[c*x])/c - 3*b^3*d^
3*x*ArcTan[c*x] - (I/4)*b^2*c*d^3*x^2*(a + b*ArcTan[c*x]) + (7*b*d^3*(a + b
*ArcTan[c*x])^2)/c - ((21*I)/4)*b*d^3*x*(a + b*ArcTan[c*x])^2 + (3*b*c*d^3*
x^2*(a + b*ArcTan[c*x])^2)/2 + (I/4)*b*c^2*d^3*x^3*(a + b*ArcTan[c*x])^2 -
((I/4)*d^3*(1 + I*c*x)^4*(a + b*ArcTan[c*x])^3)/c + (6*b*d^3*(a + b*ArcTan[
c*x])^2*Log[2/(1 - I*c*x)])/c - ((11*I)*b^2*d^3*(a + b*ArcTan[c*x])*Log[2/(
1 + I*c*x)])/c + (3*b^3*d^3*Log[1 + c^2*x^2])/(2*c) - ((6*I)*b^2*d^3*(a + b
*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 - I*c*x)])/c + (11*b^3*d^3*PolyLog[2, 1 -
 2/(1 + I*c*x)])/(2*c) + (3*b^3*d^3*PolyLog[3, 1 - 2/(1 - I*c*x)])/c
```


Rule 4864

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x])^p)/(e*(q + 1)), x] - D
ist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 4992

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d
```

```
] && EqQ[(1 - u)^2 - (1 - (2*I)/(I + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int (d+icdx)^3 \left(a+b\tan^{-1}(cx)\right)^3 dx = -\frac{id^3(1+icx)^4\left(a+b\tan^{-1}(cx)\right)^3}{4c} + \frac{(3ib)\int \left(-7d^4\left(a+b\tan^{-1}(cx)\right)^2 - 4icd^4x\left(a+\right.\right.}{}$$

$$= -\frac{id^3(1+icx)^4\left(a+b\tan^{-1}(cx)\right)^3}{4c} + \frac{(6b)\int \frac{\left(id^4-cd^4x\right)\left(a+b\tan^{-1}(cx)\right)^2}{1+c^2x^2}dx}{d} - \frac{1}{4}\left(21ibd^3\right)$$

$$= -\frac{21}{4}ibd^3x\left(a+b\tan^{-1}(cx)\right)^2 + \frac{3}{2}bcd^3x^2\left(a+b\tan^{-1}(cx)\right)^2 + \frac{1}{4}ibc^2d^3x^3\left(a+b\tan\right.$$

$$= \frac{21bd^3\left(a+b\tan^{-1}(cx)\right)^2}{4c} - \frac{21}{4}ibd^3x\left(a+b\tan^{-1}(cx)\right)^2 + \frac{3}{2}bcd^3x^2\left(a+b\tan^{-1}(cx\right.$$

$$= -3ab^2d^3x - \frac{1}{4}ib^2cd^3x^2\left(a+b\tan^{-1}(cx)\right) + \frac{7bd^3\left(a+b\tan^{-1}(cx)\right)^2}{c} - \frac{21}{4}ibd^3x\left(a+\right.$$

$$= -3ab^2d^3x + \frac{1}{4}ib^3d^3x - 3b^3d^3x\tan^{-1}(cx) - \frac{1}{4}ib^2cd^3x^2\left(a+b\tan^{-1}(cx)\right) + \frac{7bd^3\left(a\right.}{}$$

$$= -3ab^2d^3x + \frac{1}{4}ib^3d^3x - \frac{ib^3d^3\tan^{-1}(cx)}{4c} - 3b^3d^3x\tan^{-1}(cx) - \frac{1}{4}ib^2cd^3x^2\left(a+b\tan\right.$$

$$= -3ab^2d^3x + \frac{1}{4}ib^3d^3x - \frac{ib^3d^3\tan^{-1}(cx)}{4c} - 3b^3d^3x\tan^{-1}(cx) - \frac{1}{4}ib^2cd^3x^2\left(a+b\tan\right.$$

Mathematica [A] time = 1.66321, size = 693, normalized size = 1.81

$$-\frac{id^3\left(2b^2\text{PolyLog}\left(2,-e^{2i\tan^{-1}(cx)}\right)\left(12a+12b\tan^{-1}(cx)-11ib\right)+12ib^3\text{PolyLog}\left(3,-e^{2i\tan^{-1}(cx)}\right)-a^2bc^3x^3+6ia^2bc^2x^2\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + I*c*d*x)^3*(a + b*ArcTan[c*x])^3,x]
```

```
[Out] ((-I/4)*d^3*(a*b^2 + (4*I)*a^3*c*x + 21*a^2*b*c*x - (12*I)*a*b^2*c*x - b^3*
c*x - 6*a^3*c^2*x^2 + (6*I)*a^2*b*c^2*x^2 + a*b^2*c^2*x^2 - (4*I)*a^3*c^3*x
^3 - a^2*b*c^3*x^3 + a^3*c^4*x^4 - 21*a^2*b*ArcTan[c*x] + (12*I)*a*b^2*ArcT
an[c*x] + b^3*ArcTan[c*x] + (12*I)*a^2*b*c*x*ArcTan[c*x] + 42*a*b^2*c*x*Arc
Tan[c*x] - (12*I)*b^3*c*x*ArcTan[c*x] - 18*a^2*b*c^2*x^2*ArcTan[c*x] + (12*
I)*a*b^2*c^2*x^2*ArcTan[c*x] + b^3*c^2*x^2*ArcTan[c*x] - (12*I)*a^2*b*c^3*x
^3*ArcTan[c*x] - 2*a*b^2*c^3*x^3*ArcTan[c*x] + 3*a^2*b*c^4*x^4*ArcTan[c*x]
+ 3*a*b^2*ArcTan[c*x]^2 - (16*I)*b^3*ArcTan[c*x]^2 + (12*I)*a*b^2*c*x*ArcTa
n[c*x]^2 + 21*b^3*c*x*ArcTan[c*x]^2 - 18*a*b^2*c^2*x^2*ArcTan[c*x]^2 + (6*I
)*b^3*c^2*x^2*ArcTan[c*x]^2 - (12*I)*a*b^2*c^3*x^3*ArcTan[c*x]^2 - b^3*c^3*
x^3*ArcTan[c*x]^2 + 3*a*b^2*c^4*x^4*ArcTan[c*x]^2 + b^3*ArcTan[c*x]^3 + (4*
I)*b^3*c*x*ArcTan[c*x]^3 - 6*b^3*c^2*x^2*ArcTan[c*x]^3 - (4*I)*b^3*c^3*x^3*
ArcTan[c*x]^3 + b^3*c^4*x^4*ArcTan[c*x]^3 + (48*I)*a*b^2*ArcTan[c*x]*Log[1
+ E^((2*I)*ArcTan[c*x])] + 44*b^3*ArcTan[c*x]*Log[1 + E^((2*I)*ArcTan[c*x])
] + (24*I)*b^3*ArcTan[c*x]^2*Log[1 + E^((2*I)*ArcTan[c*x])] - (12*I)*a^2*b*
Log[1 + c^2*x^2] - 22*a*b^2*Log[1 + c^2*x^2] + (6*I)*b^3*Log[1 + c^2*x^2] +
 2*b^2*(12*a - (11*I)*b + 12*b*ArcTan[c*x])*PolyLog[2, -E^((2*I)*ArcTan[c*x
])] + (12*I)*b^3*PolyLog[3, -E^((2*I)*ArcTan[c*x])]))/c
```

Maple [C] time = 5.163, size = 2004, normalized size = 5.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^3*(a+b*arctan(c*x))^3,x)
```

```
[Out] -3/2*I/c*d^3*b^3*Pi*csgn(I*(1+I*c*x)/(c^2*x^2+1)^(1/2))^2*csgn(I*(1+I*c*x)^
2/(c^2*x^2+1))*arctan(c*x)^2+3*I/c*d^3*b^3*Pi*csgn(I*(1+I*c*x)/(c^2*x^2+1)^
(1/2))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))^2*arctan(c*x)^2+3/2*I/c*d^3*b^3*Pi*c
sgn(I*(1+I*c*x)^2/(c^2*x^2+1))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/
(c^2*x^2+1)+1)^2)^2*arctan(c*x)^2+3/2*I/c*d^3*b^3*Pi*csgn(I/((1+I*c*x)^2/(c
^2*x^2+1)+1)^2)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^
2)^2*arctan(c*x)^2+3/2*I/c*d^3*b^3*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1))^2
*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*arctan(c*x)^2-3*I/c*d^3*b^3*Pi*csgn(
I*((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^2*arct
an(c*x)^2+1/4*I*b^3*d^3*x-3/4*I*c^3*d^3*a*b^2*arctan(c*x)^2*x^4+1/2*I*c^2*d
^3*a*b^2*arctan(c*x)*x^3+9/2*I*c*d^3*a*b^2*arctan(c*x)^2*x^2-3/4*I*c^3*d^3*
a^2*b*arctan(c*x)*x^4+9/2*I*c*d^3*a^2*b*arctan(c*x)*x^2-3/2*I/c*d^3*b^3*Pi*
csgn(I*(1+I*c*x)^2/(c^2*x^2+1))^3*arctan(c*x)^2-3/2*I/c*d^3*b^3*Pi*csgn(I*(
1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^3*arctan(c*x)^2+3/2*I
/c*d^3*b^3*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^3*arctan(c*x)^2-3*I/c*d
```

```
^3*a*b^2*ln(c^2*x^2+1)*ln(c*x-I)-3*I/c*d^3*a*b^2*ln(c*x+I)*ln(1/2*I*(c*x-I)
)+3*I/c*d^3*a*b^2*ln(c*x-I)*ln(-1/2*I*(c*x+I))+3*I/c*d^3*a*b^2*ln(c*x+I)*ln
(c^2*x^2+1)+3/2*c*d^3*a^2*b*x^2+3/2*c*d^3*b^3*arctan(c*x)^2*x^2-c^2*d^3*b^3
*arctan(c*x)^3*x^3+6/c*d^3*b^3*ln(2)*arctan(c*x)^2-3/c*d^3*a^2*b*ln(c^2*x^2
+1)-3/c*d^3*b^3*arctan(c*x)^2*ln(c^2*x^2+1)+6/c*d^3*b^3*arctan(c*x)^2*ln((1
+I*c*x)/(c^2*x^2+1)^(1/2))+3/c*d^3*a*b^2*arctan(c*x)+3*d^3*a*b^2*arctan(c*x
)^2*x+3*d^3*a^2*b*arctan(c*x)*x-1/4*I*c^3*x^4*a^3*d^3+3/2*I*c*x^2*a^3*d^3-1
/4*I/c*d^3*b^3*arctan(c*x)^3+11/4*I/c*d^3*b^3*arctan(c*x)-21/4*I*d^3*b^3*ar
ctan(c*x)^2*x-21/4*I*d^3*a^2*b*x-3/2*I/c*d^3*b^3*Pi*csgn(I/((1+I*c*x)^2/(c^
2*x^2+1)+1)^2)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*csgn(I*(1+I*c*x)^2/(c^2*x^2+
1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*arctan(c*x)^2-1/4*I/c*d^3*a^3-1/4/c*d^3*b
^3-3*a*b^2*d^3*x-3*b^3*d^3*x*arctan(c*x)-c^2*x^3*a^3*d^3-4/c*d^3*b^3*arctan
(c*x)^2-3/c*d^3*b^3*ln((1+I*c*x)^2/(c^2*x^2+1)+1)-11/c*d^3*b^3*dilog(1-I*(1
+I*c*x)/(c^2*x^2+1)^(1/2))-11/c*d^3*b^3*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/
2))+3/c*d^3*b^3*polylog(3,-(1+I*c*x)^2/(c^2*x^2+1))+d^3*b^3*arctan(c*x)^3*x
+x*a^3*d^3-6/c*d^3*a*b^2*arctan(c*x)*ln(c^2*x^2+1)-3*c^2*d^3*a*b^2*arctan(c
*x)^2*x^3+3*c*d^3*a*b^2*arctan(c*x)*x^2-3*c^2*d^3*a^2*b*arctan(c*x)*x^3-21/
2*I*d^3*a*b^2*arctan(c*x)*x+21/4*I/c*d^3*a^2*b*arctan(c*x)-6*I/c*d^3*b^3*ar
ctan(c*x)*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))-11*I/c*d^3*b^3*arctan(c*x)*ln
(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-11*I/c*d^3*b^3*arctan(c*x)*ln(1+I*(1+I*c*
x)/(c^2*x^2+1)^(1/2))+11/2*I/c*d^3*a*b^2*ln(c^2*x^2+1)-3/2*I/c*d^3*a*b^2*ln
(c*x+I)^2+3/2*I/c*d^3*a*b^2*ln(c*x-I)^2+21/4*I/c*d^3*a*b^2*arctan(c*x)^2+3*
I/c*d^3*a*b^2*dilog(-1/2*I*(c*x+I))-3*I/c*d^3*a*b^2*dilog(1/2*I*(c*x-I))-1/
4*I*c*d^3*a*b^2*x^2+1/4*I*c^2*d^3*a^2*b*x^3+1/4*I*c^2*d^3*b^3*arctan(c*x)^2
*x^3-1/4*I*c*d^3*b^3*arctan(c*x)*x^2-1/4*I*c^3*d^3*b^3*arctan(c*x)^3*x^4+3/
2*I*c*d^3*b^3*arctan(c*x)^3*x^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^3,x, algorithm="maxima")
```

```
[Out] -1/4*I*a^3*c^3*d^3*x^4 - 24*b^3*c^5*d^3*integrate(1/128*x^5*arctan(c*x)^2*l
og(c^2*x^2 + 1)/(c^2*x^2 + 1), x) + 2*b^3*c^5*d^3*integrate(1/128*x^5*log(c
^2*x^2 + 1)^3/(c^2*x^2 + 1), x) - 12*b^3*c^5*d^3*integrate(1/128*x^5*arctan
(c*x)^2/(c^2*x^2 + 1), x) + 3*b^3*c^5*d^3*integrate(1/128*x^5*log(c^2*x^2 +
 1)^2/(c^2*x^2 + 1), x) - a^3*c^2*d^3*x^3 - 336*b^3*c^4*d^3*integrate(1/128
*x^4*arctan(c*x)^3/(c^2*x^2 + 1), x) - 36*b^3*c^4*d^3*integrate(1/128*x^4*a
rctan(c*x)*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) - 1152*a*b^2*c^4*d^3*integr
```

```
ate(1/128*x^4*arctan(c*x)^2/(c^2*x^2 + 1), x) - 60*b^3*c^4*d^3*integrate(1/
128*x^4*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^2 + 1), x) - 1/4*I*(3*x^4*arcta
n(c*x) - c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5))*a^2*b*c^3*d^3 + 48*b^
3*c^3*d^3*integrate(1/128*x^3*arctan(c*x)^2*log(c^2*x^2 + 1)/(c^2*x^2 + 1),
 x) - 4*b^3*c^3*d^3*integrate(1/128*x^3*log(c^2*x^2 + 1)^3/(c^2*x^2 + 1), x
) + 120*b^3*c^3*d^3*integrate(1/128*x^3*arctan(c*x)^2/(c^2*x^2 + 1), x) - 3
0*b^3*c^3*d^3*integrate(1/128*x^3*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) - 3/
2*(2*x^3*arctan(c*x) - c*(x^2/c^2 - log(c^2*x^2 + 1)/c^4))*a^2*b*c^2*d^3 +
3/2*I*a^3*c*d^3*x^2 + 7/32*b^3*d^3*arctan(c*x)^4/c - 224*b^3*c^2*d^3*integr
ate(1/128*x^2*arctan(c*x)^3/(c^2*x^2 + 1), x) - 24*b^3*c^2*d^3*integrate(1/
128*x^2*arctan(c*x)*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) - 768*a*b^2*c^2*d^
3*integrate(1/128*x^2*arctan(c*x)^2/(c^2*x^2 + 1), x) + 120*b^3*c^2*d^3*int
egrate(1/128*x^2*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^2 + 1), x) + 9/2*I*(x^
2*arctan(c*x) - c*(x/c^2 - arctan(c*x)/c^3))*a^2*b*c*d^3 + a*b^2*d^3*arctan
(c*x)^3/c + 72*b^3*c*d^3*integrate(1/128*x*arctan(c*x)^2*log(c^2*x^2 + 1)/(
c^2*x^2 + 1), x) - 6*b^3*c*d^3*integrate(1/128*x*log(c^2*x^2 + 1)^3/(c^2*x^
2 + 1), x) - 48*b^3*c*d^3*integrate(1/128*x*arctan(c*x)^2/(c^2*x^2 + 1), x)
 + 12*b^3*c*d^3*integrate(1/128*x*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) + a^
3*d^3*x + 12*b^3*d^3*integrate(1/128*arctan(c*x)*log(c^2*x^2 + 1)^2/(c^2*x^
2 + 1), x) + 3/2*(2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*a^2*b*d^3/c + 1/256
*(-8*I*b^3*c^3*d^3*x^4 - 32*b^3*c^2*d^3*x^3 + 48*I*b^3*c*d^3*x^2 + 32*b^3*d
^3*x)*arctan(c*x)^3 + 1/256*(12*b^3*c^3*d^3*x^4 - 48*I*b^3*c^2*d^3*x^3 - 72
*b^3*c*d^3*x^2 + 48*I*b^3*d^3*x)*arctan(c*x)^2*log(c^2*x^2 + 1) + 1/256*(6*
I*b^3*c^3*d^3*x^4 + 24*b^3*c^2*d^3*x^3 - 36*I*b^3*c*d^3*x^2 - 24*b^3*d^3*x)
*arctan(c*x)*log(c^2*x^2 + 1)^2 - 1/256*(b^3*c^3*d^3*x^4 - 4*I*b^3*c^2*d^3*
x^3 - 6*b^3*c*d^3*x^2 + 4*I*b^3*d^3*x)*log(c^2*x^2 + 1)^3 - I*integrate(1/1
28*(112*(b^3*c^5*d^3*x^5 - 2*b^3*c^3*d^3*x^3 - 3*b^3*c*d^3*x)*arctan(c*x)^3
 + 2*(3*b^3*c^4*d^3*x^4 + 2*b^3*c^2*d^3*x^2 - b^3*d^3)*log(c^2*x^2 + 1)^3 +
 12*(32*a*b^2*c^5*d^3*x^5 - 5*b^3*c^4*d^3*x^4 - 64*a*b^2*c^3*d^3*x^3 + 10*b
^3*c^2*d^3*x^2 - 96*a*b^2*c*d^3*x)*arctan(c*x)^2 + 3*(5*b^3*c^4*d^3*x^4 - 1
0*b^3*c^2*d^3*x^2 + 4*(b^3*c^5*d^3*x^5 - 2*b^3*c^3*d^3*x^3 - 3*b^3*c*d^3*x)
*arctan(c*x))*log(c^2*x^2 + 1)^2 - 12*(2*(3*b^3*c^4*d^3*x^4 + 2*b^3*c^2*d^3
*x^2 - b^3*d^3)*arctan(c*x)^2 - (b^3*c^5*d^3*x^5 - 10*b^3*c^3*d^3*x^3 + 4*b
^3*c*d^3*x)*arctan(c*x))*log(c^2*x^2 + 1))/(c^2*x^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{32}\left(b^{3}c^{3}d^{3}x^{4}-4i\,b^{3}c^{2}d^{3}x^{3}-6\,b^{3}cd^{3}x^{2}+4i\,b^{3}d^{3}x\right)\log\left(-\frac{cx+i}{cx-i}\right)^{3}+\text{integral}\left(\frac{-16i\,a^{3}c^{5}d^{3}x^{5}-48\,a^{3}c^{4}d^{3}x^{4}+32i\,a^{3}c^{3}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^3,x, algorithm="fricas")
```

```
[Out] -1/32*(b^3*c^3*d^3*x^4 - 4*I*b^3*c^2*d^3*x^3 - 6*b^3*c*d^3*x^2 + 4*I*b^3*d^
3*x)*log(-(c*x + I)/(c*x - I))^3 + integral(1/16*(-16*I*a^3*c^5*d^3*x^5 - 4
8*a^3*c^4*d^3*x^4 + 32*I*a^3*c^3*d^3*x^3 - 32*a^3*c^2*d^3*x^2 + 48*I*a^3*c*
d^3*x + 16*a^3*d^3 + (12*I*a*b^2*c^5*d^3*x^5 + 3*(12*a*b^2 - I*b^3)*c^4*d^3
*x^4 + (-24*I*a*b^2 - 12*b^3)*c^3*d^3*x^3 + 6*(4*a*b^2 + 3*I*b^3)*c^2*d^3*x
^2 - 12*a*b^2*d^3 + (-36*I*a*b^2 + 12*b^3)*c*d^3*x)*log(-(c*x + I)/(c*x - I
))^2 + (24*a^2*b*c^5*d^3*x^5 - 72*I*a^2*b*c^4*d^3*x^4 - 48*a^2*b*c^3*d^3*x^
3 - 48*I*a^2*b*c^2*d^3*x^2 - 72*a^2*b*c*d^3*x + 24*I*a^2*b*d^3)*log(-(c*x +
 I)/(c*x - I)))/(c^2*x^2 + 1), x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**3*(a+b*atan(c*x))**3,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (i\,cdx + d)^3 (b\arctan(cx) + a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^3,x, algorithm="giac")
```

```
[Out] integrate((I*c*d*x + d)^3*(b*arctan(c*x) + a)^3, x)
```

3.121 $\int (d+icdx)^2 \left(a+b\tan^{-1}(cx)\right)^3 dx$

Optimal. Leaf size=298

$$-\frac{4ib^2d^2\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)\left(a+b\tan^{-1}(cx)\right)}{c}+\frac{3b^3d^2\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c}+\frac{2b^3d^2\text{PolyLog}\left(3,1-\frac{2}{1-icx}\right)}{c}-\frac{6ib^2\ldots}{}$$

```
[Out] -(a*b^2*d^2*x) - b^3*d^2*x*ArcTan[c*x] + (7*b*d^2*(a + b*ArcTan[c*x])^2)/(2
*c) - (3*I)*b*d^2*x*(a + b*ArcTan[c*x])^2 + (b*c*d^2*x^2*(a + b*ArcTan[c*x]
)^2)/2 - ((I/3)*d^2*(1 + I*c*x)^3*(a + b*ArcTan[c*x])^3)/c + (4*b*d^2*(a +
b*ArcTan[c*x])^2*Log[2/(1 - I*c*x)])/c - ((6*I)*b^2*d^2*(a + b*ArcTan[c*x])
*Log[2/(1 + I*c*x)])/c + (b^3*d^2*Log[1 + c^2*x^2])/(2*c) - ((4*I)*b^2*d^2*
(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 - I*c*x)])/c + (3*b^3*d^2*PolyLog[2
, 1 - 2/(1 + I*c*x)])/c + (2*b^3*d^2*PolyLog[3, 1 - 2/(1 - I*c*x)])/c
```

Rubi [A] time = 0.478184, antiderivative size = 298, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 13, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.591, Rules used = {4864, 4846, 4920, 4854, 2402, 2315, 4852, 4916, 260, 4884, 1586, 4992, 6610}

$$-\frac{4ib^2d^2\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)\left(a+b\tan^{-1}(cx)\right)}{c}+\frac{3b^3d^2\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c}+\frac{2b^3d^2\text{PolyLog}\left(3,1-\frac{2}{1-icx}\right)}{c}-\frac{6ib^2\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + I*c*d*x)^2*(a + b*ArcTan[c*x])^3,x]
```


```
[Out] -(a*b^2*d^2*x) - b^3*d^2*x*ArcTan[c*x] + (7*b*d^2*(a + b*ArcTan[c*x])^2)/(2
*c) - (3*I)*b*d^2*x*(a + b*ArcTan[c*x])^2 + (b*c*d^2*x^2*(a + b*ArcTan[c*x]
)^2)/2 - ((I/3)*d^2*(1 + I*c*x)^3*(a + b*ArcTan[c*x])^3)/c + (4*b*d^2*(a +
b*ArcTan[c*x])^2*Log[2/(1 - I*c*x)])/c - ((6*I)*b^2*d^2*(a + b*ArcTan[c*x])
*Log[2/(1 + I*c*x)])/c + (b^3*d^2*Log[1 + c^2*x^2])/(2*c) - ((4*I)*b^2*d^2*
(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 - I*c*x)])/c + (3*b^3*d^2*PolyLog[2
, 1 - 2/(1 + I*c*x)])/c + (2*b^3*d^2*PolyLog[3, 1 - 2/(1 - I*c*x)])/c
```


Rule 4864

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x])^p)/(e*(q + 1)), x] - D
ist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1), (
```

```
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 4992

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d
] && EqQ[(1 - u)^2 - (1 - (2*I)/(I + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int (d+icdx)^2 \left(a+b\tan^{-1}(cx)\right)^3 dx = -\frac{id^2(1+icx)^3 \left(a+b\tan^{-1}(cx)\right)^3}{3c} + \frac{(ib)\int \left(-3d^3\left(a+b\tan^{-1}(cx)\right)^2 - icd^3x\left(a+b\right.\right.}{d}$$

$$= -\frac{id^2(1+icx)^3 \left(a+b\tan^{-1}(cx)\right)^3}{3c} + \frac{(4b)\int \frac{\left(id^3-cd^3x\right)\left(a+b\tan^{-1}(cx)\right)^2}{1+c^2x^2}dx}{d} - \left(3ibd^2\right)\int \left(\right.$$

$$= -3ibd^2x\left(a+b\tan^{-1}(cx)\right)^2 + \frac{1}{2}bcd^2x^2\left(a+b\tan^{-1}(cx)\right)^2 - \frac{id^2(1+icx)^3\left(a+b\tan\right.}{3c}$$

$$= \frac{3bd^2\left(a+b\tan^{-1}(cx)\right)^2}{c} - 3ibd^2x\left(a+b\tan^{-1}(cx)\right)^2 + \frac{1}{2}bcd^2x^2\left(a+b\tan^{-1}(cx)\right)^2$$

$$= -ab^2d^2x + \frac{7bd^2\left(a+b\tan^{-1}(cx)\right)^2}{2c} - 3ibd^2x\left(a+b\tan^{-1}(cx)\right)^2 + \frac{1}{2}bcd^2x^2\left(a+b\,\mathrm{t}\right.$$

$$= -ab^2d^2x - b^3d^2x\tan^{-1}(cx) + \frac{7bd^2\left(a+b\tan^{-1}(cx)\right)^2}{2c} - 3ibd^2x\left(a+b\tan^{-1}(cx)\right)^2$$

$$= -ab^2d^2x - b^3d^2x\tan^{-1}(cx) + \frac{7bd^2\left(a+b\tan^{-1}(cx)\right)^2}{2c} - 3ibd^2x\left(a+b\tan^{-1}(cx)\right)^2$$

Mathematica [A] time = 0.979934, size = 528, normalized size = 1.77

$$-\frac{d^2\left(6b^2\mathrm{PolyLog}\left(2,-e^{2i\tan^{-1}(cx)}\right)\left(4ia+4ib\tan^{-1}(cx)+3b\right) - 12b^3\mathrm{PolyLog}\left(3,-e^{2i\tan^{-1}(cx)}\right) - 3a^2bc^2x^2 + 12a^2b\log\left(c\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + I*c*d*x)^2*(a + b*ArcTan[c*x])^3,x]
```

```
[Out] -(d^2*(-6*a^3*c*x + (18*I)*a^2*b*c*x + 6*a*b^2*c*x - (6*I)*a^3*c^2*x^2 - 3*
a^2*b*c^2*x^2 + 2*a^3*c^3*x^3 - (18*I)*a^2*b*ArcTan[c*x] - 6*a*b^2*ArcTan[c
*x] - 18*a^2*b*c*x*ArcTan[c*x] + (36*I)*a*b^2*c*x*ArcTan[c*x] + 6*b^3*c*x*A
rcTan[c*x] - (18*I)*a^2*b*c^2*x^2*ArcTan[c*x] - 6*a*b^2*c^2*x^2*ArcTan[c*x]
 + 6*a^2*b*c^3*x^3*ArcTan[c*x] + (6*I)*a*b^2*ArcTan[c*x]^2 + 15*b^3*ArcTan[
c*x]^2 - 18*a*b^2*c*x*ArcTan[c*x]^2 + (18*I)*b^3*c*x*ArcTan[c*x]^2 - (18*I)
*a*b^2*c^2*x^2*ArcTan[c*x]^2 - 3*b^3*c^2*x^2*ArcTan[c*x]^2 + 6*a*b^2*c^3*x^
3*ArcTan[c*x]^2 + (2*I)*b^3*ArcTan[c*x]^3 - 6*b^3*c*x*ArcTan[c*x]^3 - (6*I)
*b^3*c^2*x^2*ArcTan[c*x]^3 + 2*b^3*c^3*x^3*ArcTan[c*x]^3 - 48*a*b^2*ArcTan[
c*x]*Log[1 + E^((2*I)*ArcTan[c*x])] + (36*I)*b^3*ArcTan[c*x]*Log[1 + E^((2*
```

```
I)*ArcTan[c*x])] - 24*b^3*ArcTan[c*x]^2*Log[1 + E^((2*I)*ArcTan[c*x])] + 12
*a^2*b*Log[1 + c^2*x^2] - (18*I)*a*b^2*Log[1 + c^2*x^2] - 3*b^3*Log[1 + c^2
*x^2] + 6*b^2*((4*I)*a + 3*b + (4*I)*b*ArcTan[c*x])*PolyLog[2, -E^((2*I)*Ar
cTan[c*x])] - 12*b^3*PolyLog[3, -E^((2*I)*ArcTan[c*x])]))/(6*c)
```

Maple [C] time = 1.902, size = 1815, normalized size = 6.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^2*(a+b*arctan(c*x))^3,x)
```

```
[Out] I/c*d^2*b^3*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1))^2*csgn(I*((1+I*c*x)^2/(c
^2*x^2+1)+1)^2)*arctan(c*x)^2+I/c*d^2*b^3*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)
)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^2*arctan(c*
x)^2+I/c*d^2*b^3*Pi*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*csgn(I*(1+I*c*x)^
2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^2*arctan(c*x)^2+2*I/c*d^2*b^3*
Pi*csgn(I*(1+I*c*x)/(c^2*x^2+1)^(1/2))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))^2*ar
ctan(c*x)^2-2*I/c*d^2*b^3*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*((1
+I*c*x)^2/(c^2*x^2+1)+1)^2)^2*arctan(c*x)^2-I/c*d^2*b^3*Pi*csgn(I*(1+I*c*x)
/(c^2*x^2+1)^(1/2))^2*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*arctan(c*x)^2+x*a^3*d
^2-a*b^2*d^2*x-b^3*d^2*x*arctan(c*x)-2*I/c*d^2*a*b^2*ln(c*x+I)*ln(1/2*I*(c*
x-I))-I/c*d^2*b^3*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))^3*arctan(c*x)^2-I/c*d^
2*b^3*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^3*ar
ctan(c*x)^2+I/c*d^2*b^3*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^3*arctan(c
*x)^2+3*I*c*d^2*a*b^2*arctan(c*x)^2*x^2+3*I*c*d^2*a^2*b*arctan(c*x)*x^2-2*I
/c*d^2*a*b^2*ln(c^2*x^2+1)*ln(c*x-I)+2*I/c*d^2*a*b^2*ln(c^2*x^2+1)*ln(c*x+I
)+2*I/c*d^2*a*b^2*ln(c*x-I)*ln(-1/2*I*(c*x+I))+I*c*d^2*b^3*arctan(c*x)^3*x^
2-c^2*d^2*a*b^2*arctan(c*x)^2*x^3+c*d^2*a*b^2*arctan(c*x)*x^2-c^2*d^2*a^2*b
*arctan(c*x)*x^3+3*I/c*d^2*a^2*b*arctan(c*x)-I/c*d^2*a*b^2*ln(c*x+I)^2+3*I/
c*d^2*a*b^2*ln(c^2*x^2+1)+2*I/c*d^2*a*b^2*dilog(-1/2*I*(c*x+I))-I/c*d^2*b^3
*Pi*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*c
sgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*arctan(c*x)^2+
3*I/c*d^2*a*b^2*arctan(c*x)^2-6*I*d^2*a*b^2*arctan(c*x)*x-4/c*d^2*a*b^2*arc
tan(c*x)*ln(c^2*x^2+1)+I/c*d^2*a*b^2*ln(c*x-I)^2-6*I/c*d^2*b^3*arctan(c*x)*
ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-6*I/c*d^2*b^3*arctan(c*x)*ln(1-I*(1+I*c
*x)/(c^2*x^2+1)^(1/2))-4*I/c*d^2*b^3*arctan(c*x)*polylog(2,-(1+I*c*x)^2/(c^
2*x^2+1))-2*I/c*d^2*a*b^2*dilog(1/2*I*(c*x-I))-1/3*I/c*d^2*a^3-1/3*c^2*x^3*
a^3*d^2-5/2/c*d^2*b^3*arctan(c*x)^2+2/c*d^2*b^3*polylog(3,-(1+I*c*x)^2/(c^2
*x^2+1))-6/c*d^2*b^3*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-1/c*d^2*b^3*ln(
(1+I*c*x)^2/(c^2*x^2+1)+1)-6/c*d^2*b^3*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2
```

```
))+d^2*b^3*arctan(c*x)^3*x+1/2*c*d^2*a^2*b*x^2+3*d^2*a*b^2*arctan(c*x)^2*x+
3*d^2*a^2*b*arctan(c*x)*x+1/2*c*d^2*b^3*arctan(c*x)^2*x^2-1/3*c^2*d^2*b^3*a
rctan(c*x)^3*x^3+I*c*x^2*a^3*d^2+4/c*d^2*b^3*arctan(c*x)^2*ln((1+I*c*x)/(c^
2*x^2+1)^(1/2))-2/c*d^2*b^3*arctan(c*x)^2*ln(c^2*x^2+1)-2/c*d^2*a^2*b*ln(c^
2*x^2+1)+1/c*d^2*a*b^2*arctan(c*x)+I/c*d^2*b^3*arctan(c*x)+4/c*d^2*b^3*ln(2
)*arctan(c*x)^2-3*I*d^2*b^3*arctan(c*x)^2*x-1/3*I/c*d^2*b^3*arctan(c*x)^3-3
*I*d^2*a^2*b*x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))^3,x, algorithm="maxima")
```

```
[Out] -1/3*a^3*c^2*d^2*x^3 - 28*b^3*c^4*d^2*integrate(1/32*x^4*arctan(c*x)^3/(c^2
*x^2 + 1), x) - 3*b^3*c^4*d^2*integrate(1/32*x^4*arctan(c*x)*log(c^2*x^2 +
1)^2/(c^2*x^2 + 1), x) - 96*a*b^2*c^4*d^2*integrate(1/32*x^4*arctan(c*x)^2/
(c^2*x^2 + 1), x) - 4*b^3*c^4*d^2*integrate(1/32*x^4*arctan(c*x)*log(c^2*x^
2 + 1)/(c^2*x^2 + 1), x) + 12*b^3*c^3*d^2*integrate(1/32*x^3*arctan(c*x)^2*
log(c^2*x^2 + 1)/(c^2*x^2 + 1), x) - b^3*c^3*d^2*integrate(1/32*x^3*log(c^2
*x^2 + 1)^3/(c^2*x^2 + 1), x) + 16*b^3*c^3*d^2*integrate(1/32*x^3*arctan(c*
x)^2/(c^2*x^2 + 1), x) - 4*b^3*c^3*d^2*integrate(1/32*x^3*log(c^2*x^2 + 1)^
2/(c^2*x^2 + 1), x) - 1/2*(2*x^3*arctan(c*x) - c*(x^2/c^2 - log(c^2*x^2 + 1
)/c^4))*a^2*b*c^2*d^2 + I*a^3*c*d^2*x^2 + 7/32*b^3*d^2*arctan(c*x)^4/c + 24
*b^3*c^2*d^2*integrate(1/32*x^2*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^2 + 1),
 x) + 3*I*(x^2*arctan(c*x) - c*(x/c^2 - arctan(c*x)/c^3))*a^2*b*c*d^2 + a*b
^2*d^2*arctan(c*x)^3/c + 12*b^3*c*d^2*integrate(1/32*x*arctan(c*x)^2*log(c^
2*x^2 + 1)/(c^2*x^2 + 1), x) - b^3*c*d^2*integrate(1/32*x*log(c^2*x^2 + 1)^
3/(c^2*x^2 + 1), x) - 12*b^3*c*d^2*integrate(1/32*x*arctan(c*x)^2/(c^2*x^2
+ 1), x) + 3*b^3*c*d^2*integrate(1/32*x*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x
) + a^3*d^2*x + 3*b^3*d^2*integrate(1/32*arctan(c*x)*log(c^2*x^2 + 1)^2/(c^
2*x^2 + 1), x) + 3/2*(2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*a^2*b*d^2/c - 1
/192*(8*b^3*c^2*d^2*x^3 - 24*I*b^3*c*d^2*x^2 - 24*b^3*d^2*x)*arctan(c*x)^3
- 1/192*(12*I*b^3*c^2*d^2*x^3 + 36*b^3*c*d^2*x^2 - 36*I*b^3*d^2*x)*arctan(c
*x)^2*log(c^2*x^2 + 1) + 1/192*(6*b^3*c^2*d^2*x^3 - 18*I*b^3*c*d^2*x^2 - 18
*b^3*d^2*x)*arctan(c*x)*log(c^2*x^2 + 1)^2 - 1/192*(-I*b^3*c^2*d^2*x^3 - 3*
b^3*c*d^2*x^2 + 3*I*b^3*d^2*x)*log(c^2*x^2 + 1)^3 - I*integrate(-1/64*(112*
(b^3*c^3*d^2*x^3 + b^3*c*d^2*x)*arctan(c*x)^3 - (b^3*c^4*d^2*x^4 - b^3*d^2)
*log(c^2*x^2 + 1)^3 + 8*(b^3*c^4*d^2*x^4 + 48*a*b^2*c^3*d^2*x^3 - 6*b^3*c^2
*d^2*x^2 + 48*a*b^2*c*d^2*x)*arctan(c*x)^2 - 2*(b^3*c^4*d^2*x^4 - 6*b^3*c^2
```

```
*d^2*x^2 - 6*(b^3*c^3*d^2*x^3 + b^3*c*d^2*x)*arctan(c*x))*log(c^2*x^2 + 1)^
2 + 4*(3*(b^3*c^4*d^2*x^4 - b^3*d^2)*arctan(c*x)^2 + 2*(4*b^3*c^3*d^2*x^3 -
 3*b^3*c*d^2*x)*arctan(c*x))*log(c^2*x^2 + 1))/(c^2*x^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{24}\left(i\,b^3c^2d^2x^3+3\,b^3cd^2x^2-3i\,b^3d^2x\right)\log\left(-\frac{cx+i}{cx-i}\right)^3+\text{integral}\left(-\frac{4\,a^3c^4d^2x^4-8i\,a^3c^3d^2x^3-8i\,a^3cd^2x-4\,a^3d^2-\left(3\,a\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))^3,x, algorithm="fricas")
```

```
[Out] 1/24*(I*b^3*c^2*d^2*x^3 + 3*b^3*c*d^2*x^2 - 3*I*b^3*d^2*x)*log(-(c*x + I)/(
c*x - I))^3 + integral(-1/4*(4*a^3*c^4*d^2*x^4 - 8*I*a^3*c^3*d^2*x^3 - 8*I*
a^3*c*d^2*x - 4*a^3*d^2 - (3*a*b^2*c^4*d^2*x^4 + 3*I*b^3*c^2*d^2*x^2 + (-6*
I*a*b^2 - b^3)*c^3*d^2*x^3 - 3*a*b^2*d^2 + (-6*I*a*b^2 + 3*b^3)*c*d^2*x)*lo
g(-(c*x + I)/(c*x - I))^2 - (-6*I*a^2*b*c^4*d^2*x^4 - 12*a^2*b*c^3*d^2*x^3
- 12*a^2*b*c*d^2*x + 6*I*a^2*b*d^2)*log(-(c*x + I)/(c*x - I)))/(c^2*x^2 + 1
), x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**2*(a+b*atan(c*x))**3,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int(i\,cdx+d)^2(b\arctan(cx)+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))^3,x, algorithm="giac")
```

```
[Out] integrate((I*c*d*x + d)^2*(b*arctan(c*x) + a)^3, x)
```

3.122 $\int (d+icdx)\left(a+b\tan^{-1}(cx)\right)^3\,dx$

Optimal. Leaf size=220

$$-\frac{3ib^2d\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)\left(a+b\tan^{-1}(cx)\right)}{c}+\frac{3b^3d\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{2c}+\frac{3b^3d\text{PolyLog}\left(3,1-\frac{2}{1-icx}\right)}{2c}-\frac{3ib^2d\,\mathrm{lo}}{}$$

```
[Out] (3*b*d*(a + b*ArcTan[c*x])^2)/(2*c) - ((3*I)/2)*b*d*x*(a + b*ArcTan[c*x])^2
 - ((I/2)*d*(1 + I*c*x)^2*(a + b*ArcTan[c*x])^3)/c + (3*b*d*(a + b*ArcTan[c
*x])^2*Log[2/(1 - I*c*x)])/c - ((3*I)*b^2*d*(a + b*ArcTan[c*x])*Log[2/(1 +
I*c*x)])/c - ((3*I)*b^2*d*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 - I*c*x)]
)/c + (3*b^3*d*PolyLog[2, 1 - 2/(1 + I*c*x)])/(2*c) + (3*b^3*d*PolyLog[3, 1
 - 2/(1 - I*c*x)])/(2*c)
```

Rubi [A] time = 0.339436, antiderivative size = 220, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4864, 4846, 4920, 4854, 2402, 2315, 1586, 4884, 4992, 6610}

$$-\frac{3ib^2d\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)\left(a+b\tan^{-1}(cx)\right)}{c}+\frac{3b^3d\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{2c}+\frac{3b^3d\text{PolyLog}\left(3,1-\frac{2}{1-icx}\right)}{2c}-\frac{3ib^2d\,\mathrm{lo}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + I*c*d*x)*(a + b*ArcTan[c*x])^3,x]
```

```
[Out] (3*b*d*(a + b*ArcTan[c*x])^2)/(2*c) - ((3*I)/2)*b*d*x*(a + b*ArcTan[c*x])^2
 - ((I/2)*d*(1 + I*c*x)^2*(a + b*ArcTan[c*x])^3)/c + (3*b*d*(a + b*ArcTan[c
*x])^2*Log[2/(1 - I*c*x)])/c - ((3*I)*b^2*d*(a + b*ArcTan[c*x])*Log[2/(1 +
I*c*x)])/c - ((3*I)*b^2*d*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 - I*c*x)]
)/c + (3*b^3*d*PolyLog[2, 1 - 2/(1 + I*c*x)])/(2*c) + (3*b^3*d*PolyLog[3, 1
 - 2/(1 - I*c*x)])/(2*c)
```

Rule 4864

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x])^p)/(e*(q + 1)), x] - D
ist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4992

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d
] && EqQ[(1 - u)^2 - (1 - (2*I)/(I + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int (d+icdx)\left(a+b\tan^{-1}(cx)\right)^3\,dx = -\frac{id(1+icx)^2\left(a+b\tan^{-1}(cx)\right)^3}{2c}+\frac{(3ib)\int\left(-d^2\left(a+b\tan^{-1}(cx)\right)^2-\frac{2i\left(id^2-cd^2x\right)\left(a+\right.}{1+c^2}\right.}{2d}$$

$$= -\frac{id(1+icx)^2\left(a+b\tan^{-1}(cx)\right)^3}{2c}+\frac{(3b)\int\frac{\left(id^2-cd^2x\right)\left(a+b\tan^{-1}(cx)\right)^2}{1+c^2x^2}\,dx}{d}-\frac{1}{2}(3ibd)\int\left(\right.$$

$$= -\frac{3}{2}ibdx\left(a+b\tan^{-1}(cx)\right)^2-\frac{id(1+icx)^2\left(a+b\tan^{-1}(cx)\right)^3}{2c}+\frac{(3b)\int\frac{\left(a+b\tan^{-1}(cx)\right)}{-\frac{i}{d^2}-\frac{cx}{d^2}}}{d}$$

$$= \frac{3bd\left(a+b\tan^{-1}(cx)\right)^2}{2c}-\frac{3}{2}ibdx\left(a+b\tan^{-1}(cx)\right)^2-\frac{id(1+icx)^2\left(a+b\tan^{-1}(cx)\right.}{2c}$$

$$= \frac{3bd\left(a+b\tan^{-1}(cx)\right)^2}{2c}-\frac{3}{2}ibdx\left(a+b\tan^{-1}(cx)\right)^2-\frac{id(1+icx)^2\left(a+b\tan^{-1}(cx)\right.}{2c}$$

$$= \frac{3bd\left(a+b\tan^{-1}(cx)\right)^2}{2c}-\frac{3}{2}ibdx\left(a+b\tan^{-1}(cx)\right)^2-\frac{id(1+icx)^2\left(a+b\tan^{-1}(cx)\right.}{2c}$$

$$= \frac{3bd\left(a+b\tan^{-1}(cx)\right)^2}{2c}-\frac{3}{2}ibdx\left(a+b\tan^{-1}(cx)\right)^2-\frac{id(1+icx)^2\left(a+b\tan^{-1}(cx)\right.}{2c}$$

Mathematica [A] time = 0.467632, size = 367, normalized size = 1.67

$$\frac{id\left(-3b^2\text{PolyLog}\left(2,-e^{2i\tan^{-1}(cx)}\right)\left(2a+2b\tan^{-1}(cx)-ib\right)-3ib^3\text{PolyLog}\left(3,-e^{2i\tan^{-1}(cx)}\right)+3ia^2b\log\left(c^2x^2+1\right)+3a\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + I*c*d*x)*(a + b*ArcTan[c*x])^3,x]
```

```
[Out] ((I/2)*d*((-2*I)*a^3*c*x - 3*a^2*b*c*x + a^3*c^2*x^2 + 3*a^2*b*ArcTan[c*x]
- (6*I)*a^2*b*c*x*ArcTan[c*x] - 6*a*b^2*c*x*ArcTan[c*x] + 3*a^2*b*c^2*x^2*A
rcTan[c*x] - 3*a*b^2*ArcTan[c*x]^2 + (3*I)*b^3*ArcTan[c*x]^2 - (6*I)*a*b^2*
c*x*ArcTan[c*x]^2 - 3*b^3*c*x*ArcTan[c*x]^2 + 3*a*b^2*c^2*x^2*ArcTan[c*x]^2
 - b^3*ArcTan[c*x]^3 - (2*I)*b^3*c*x*ArcTan[c*x]^3 + b^3*c^2*x^2*ArcTan[c*x
]^3 - (12*I)*a*b^2*ArcTan[c*x]*Log[1 + E^((2*I)*ArcTan[c*x])] - 6*b^3*ArcTa
n[c*x]*Log[1 + E^((2*I)*ArcTan[c*x])] - (6*I)*b^3*ArcTan[c*x]^2*Log[1 + E^(
(2*I)*ArcTan[c*x])] + (3*I)*a^2*b*Log[1 + c^2*x^2] + 3*a*b^2*Log[1 + c^2*x^
2] - 3*b^2*(2*a - I*b + 2*b*ArcTan[c*x])*PolyLog[2, -E^((2*I)*ArcTan[c*x])]
 - (3*I)*b^3*PolyLog[3, -E^((2*I)*ArcTan[c*x])]))/c
```

Maple [C] time = 0.787, size = 7451, normalized size = 33.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)*(a+b*arctan(c*x))^3,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)*(a+b*arctan(c*x))^3,x, algorithm="maxima")
```

```
[Out] 12*b^3*c^3*d*integrate(1/64*x^3*arctan(c*x)^2*log(c^2*x^2 + 1)/(c^2*x^2 + 1
), x) - b^3*c^3*d*integrate(1/64*x^3*log(c^2*x^2 + 1)^3/(c^2*x^2 + 1), x) +
 12*b^3*c^3*d*integrate(1/64*x^3*arctan(c*x)^2/(c^2*x^2 + 1), x) - 3*b^3*c^
3*d*integrate(1/64*x^3*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) + 1/2*I*a^3*c*d
*x^2 + 7/32*b^3*d*arctan(c*x)^4/c + 56*b^3*c^2*d*integrate(1/64*x^2*arctan(
c*x)^3/(c^2*x^2 + 1), x) + 6*b^3*c^2*d*integrate(1/64*x^2*arctan(c*x)*log(c
^2*x^2 + 1)^2/(c^2*x^2 + 1), x) + 192*a*b^2*c^2*d*integrate(1/64*x^2*arctan
(c*x)^2/(c^2*x^2 + 1), x) + 36*b^3*c^2*d*integrate(1/64*x^2*arctan(c*x)*log
```

```
(c^2*x^2 + 1)/(c^2*x^2 + 1), x) + 3/2*I*(x^2*arctan(c*x) - c*(x/c^2 - arcta
n(c*x)/c^3))*a^2*b*c*d + a*b^2*d*arctan(c*x)^3/c + 12*b^3*c*d*integrate(1/6
4*x*arctan(c*x)^2*log(c^2*x^2 + 1)/(c^2*x^2 + 1), x) - b^3*c*d*integrate(1/
64*x*log(c^2*x^2 + 1)^3/(c^2*x^2 + 1), x) - 24*b^3*c*d*integrate(1/64*x*arc
tan(c*x)^2/(c^2*x^2 + 1), x) + 6*b^3*c*d*integrate(1/64*x*log(c^2*x^2 + 1)^
2/(c^2*x^2 + 1), x) + a^3*d*x + 6*b^3*d*integrate(1/64*arctan(c*x)*log(c^2*
x^2 + 1)^2/(c^2*x^2 + 1), x) + 3/2*(2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*a
^2*b*d/c - 1/128*(-8*I*b^3*c*d*x^2 - 16*b^3*d*x)*arctan(c*x)^3 - 3/32*(b^3*
c*d*x^2 - 2*I*b^3*d*x)*arctan(c*x)^2*log(c^2*x^2 + 1) - 1/128*(6*I*b^3*c*d*
x^2 + 12*b^3*d*x)*arctan(c*x)*log(c^2*x^2 + 1)^2 + 1/128*(b^3*c*d*x^2 - 2*I
*b^3*d*x)*log(c^2*x^2 + 1)^3 + I*integrate(1/64*(56*(b^3*c^3*d*x^3 + b^3*c*
d*x)*arctan(c*x)^3 + (b^3*c^2*d*x^2 + b^3*d)*log(c^2*x^2 + 1)^3 + 12*(16*a*
b^2*c^3*d*x^3 - 3*b^3*c^2*d*x^2 + 16*a*b^2*c*d*x)*arctan(c*x)^2 + 3*(3*b^3*
c^2*d*x^2 + 2*(b^3*c^3*d*x^3 + b^3*c*d*x)*arctan(c*x))*log(c^2*x^2 + 1)^2 -
 12*((b^3*c^2*d*x^2 + b^3*d)*arctan(c*x)^2 - (b^3*c^3*d*x^3 - 2*b^3*c*d*x)*
arctan(c*x))*log(c^2*x^2 + 1))/(c^2*x^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{16}\left(b^3cdx^2 - 2i\,b^3dx\right)\log\left(-\frac{cx+i}{cx-i}\right)^3 + \text{integral}\left(\frac{8i\,a^3c^3dx^3 + 8\,a^3c^2dx^2 + 8i\,a^3cdx + 8\,a^3d + \left(-6i\,ab^2c^3dx^3 - 3\left(2\,ab^2 -\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)*(a+b*arctan(c*x))^3,x, algorithm="fricas")
```

```
[Out] 1/16*(b^3*c*d*x^2 - 2*I*b^3*d*x)*log(-(c*x + I)/(c*x - I))^3 + integral(1/8
*(8*I*a^3*c^3*d*x^3 + 8*a^3*c^2*d*x^2 + 8*I*a^3*c*d*x + 8*a^3*d + (-6*I*a*b
^2*c^3*d*x^3 - 3*(2*a*b^2 - I*b^3)*c^2*d*x^2 - 6*a*b^2*d + (-6*I*a*b^2 + 6*
b^3)*c*d*x)*log(-(c*x + I)/(c*x - I))^2 - (12*a^2*b*c^3*d*x^3 - 12*I*a^2*b*
c^2*d*x^2 + 12*a^2*b*c*d*x - 12*I*a^2*b*d)*log(-(c*x + I)/(c*x - I)))/(c^2*
x^2 + 1), x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)*(a+b*atan(c*x))**3,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (icdx + d)(b\arctan(cx) + a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)*(a+b*arctan(c*x))^3,x, algorithm="giac")
```

```
[Out] integrate((I*c*d*x + d)*(b*arctan(c*x) + a)^3, x)
```

3.123 $\int \frac{(a+b\tan^{-1}(cx))^3}{d+icdx}\,dx$

Optimal. Leaf size=139

$$\frac{3ib^2\text{PolyLog}\left(3,1-\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{2cd}-\frac{3b\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)^2}{2cd}+\frac{3b^3\text{PolyLog}\left(4,1-\frac{2}{1+ic}\right.}{4cd}$$

```
[Out] (I*(a + b*ArcTan[c*x])^3*Log[2/(1 + I*c*x)])/(c*d) - (3*b*(a + b*ArcTan[c*x
])^2*PolyLog[2, 1 - 2/(1 + I*c*x)])/(2*c*d) + (((3*I)/2)*b^2*(a + b*ArcTan[
c*x])*PolyLog[3, 1 - 2/(1 + I*c*x)])/(c*d) + (3*b^3*PolyLog[4, 1 - 2/(1 + I
*c*x)])/(4*c*d)
```

Rubi [A] time = 0.229956, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {4854, 4884, 4994, 4998, 6610}

$$\frac{3ib^2\text{PolyLog}\left(3,1-\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{2cd}-\frac{3b\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)^2}{2cd}+\frac{3b^3\text{PolyLog}\left(4,1-\frac{2}{1+ic}\right.}{4cd}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^3/(d + I*c*d*x),x]
```

```
[Out] (I*(a + b*ArcTan[c*x])^3*Log[2/(1 + I*c*x)])/(c*d) - (3*b*(a + b*ArcTan[c*x
])^2*PolyLog[2, 1 - 2/(1 + I*c*x)])/(2*c*d) + (((3*I)/2)*b^2*(a + b*ArcTan[
c*x])*PolyLog[3, 1 - 2/(1 + I*c*x)])/(c*d) + (3*b^3*PolyLog[4, 1 - 2/(1 + I
*c*x)])/(4*c*d)
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
```

```
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 4998

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[k + 1, u])/(2
*c*d), x] - Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[k + 1,
 u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
qQ[e, c^2*d] && EqQ[u^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}(cx)\right)^3}{d+icdx}\,dx = \frac{i\left(a+b\tan^{-1}(cx)\right)^3\log\left(\frac{2}{1+icx}\right)}{cd} - \frac{(3ib)\int \frac{\left(a+b\tan^{-1}(cx)\right)^2\log\left(\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx}{d}$$

$$= \frac{i\left(a+b\tan^{-1}(cx)\right)^3\log\left(\frac{2}{1+icx}\right)}{cd} - \frac{3b\left(a+b\tan^{-1}(cx)\right)^2\text{Li}_2\left(1-\frac{2}{1+icx}\right)}{2cd} + \frac{\left(3b^2\right)\int \frac{\left(a+b\tan^{-1}(cx)\right.}{1+c}}{d}$$

$$= \frac{i\left(a+b\tan^{-1}(cx)\right)^3\log\left(\frac{2}{1+icx}\right)}{cd} - \frac{3b\left(a+b\tan^{-1}(cx)\right)^2\text{Li}_2\left(1-\frac{2}{1+icx}\right)}{2cd} + \frac{3ib^2\left(a+b\tan^{-1}(cx\right.}{2ca}$$

$$= \frac{i\left(a+b\tan^{-1}(cx)\right)^3\log\left(\frac{2}{1+icx}\right)}{cd} - \frac{3b\left(a+b\tan^{-1}(cx)\right)^2\text{Li}_2\left(1-\frac{2}{1+icx}\right)}{2cd} + \frac{3ib^2\left(a+b\tan^{-1}(cx\right.}{2ca}$$

Mathematica [A] time = 0.0786268, size = 133, normalized size = 0.96

$$\frac{i\left(3ib\left(2\text{PolyLog}\left(2,\frac{cx+i}{cx-i}\right)\left(a+b\tan^{-1}(cx)\right)^2-b\left(2i\text{PolyLog}\left(3,\frac{cx+i}{cx-i}\right)\left(a+b\tan^{-1}(cx)\right)+b\text{PolyLog}\left(4,\frac{cx+i}{cx-i}\right)\right)\right)+4\log\left(\right.}{4cd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])^3/(d + I*c*d*x),x]
```

```
[Out] ((I/4)*(4*(a + b*ArcTan[c*x])^3*Log[(2*d)/(d + I*c*d*x)] + (3*I)*b*(2*(a +
b*ArcTan[c*x])^2*PolyLog[2, (I + c*x)/(-I + c*x)] - b*((2*I)*(a + b*ArcTan[
c*x])*PolyLog[3, (I + c*x)/(-I + c*x)] + b*PolyLog[4, (I + c*x)/(-I + c*x)]
))))/(c*d)
```

Maple [C] time = 0.345, size = 2044, normalized size = 14.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^3/(d+I*c*d*x),x)
```

```
[Out] 3/2/c*a*b^2/d*arctan(c*x)^2*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*
x^2+1)+1))^2*csgn((1+I*c*x)^2/(c^2*x^2+1))*Pi+3/2/c*a*b^2/d*arctan(c*x)^2*c
sgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*(1+I*c*x)^2
/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*Pi-3/2/c*a*b^2/d*arctan(c*x)^2*
csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*(1+I*c*x)^
2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*Pi+1/2/c*b^3/d*arctan(c*x)^3*csg
n((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I/((1+I*c*x)^2/
(c^2*x^2+1)+1))*csgn((1+I*c*x)^2/(c^2*x^2+1))*Pi-3/2/c*a*b^2/d*arctan(c*x)^
2*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*csgn(I/((1+I*
c*x)^2/(c^2*x^2+1)+1))*Pi-1/2/c*b^3/d*arctan(c*x)^3*csgn((1+I*c*x)^2/(c^2*x
^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*Pi
+1/2/c*b^3/d*arctan(c*x)^3*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x
^2+1)+1))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*Pi+
3/2/c*a*b^2/d*arctan(c*x)^2*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*
x^2+1)+1))*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)
)*Pi+3/4/c*a^2*b/d*ln(1+I*c*x)^2+3/2/c*b^3/d*arctan(c*x)^2*polylog(2,-(1+I*
c*x)^2/(c^2*x^2+1))+1/2/c*b^3/d*Pi*arctan(c*x)^3-1/2*I/c*a^3/d*ln(c^2*x^2+1
)+2/c*a*b^2/d*arctan(c*x)^3+3/2/c*a^2*b/d*dilog(1/2*I*c*x+1/2)-1/2/c*b^3/d*
arctan(c*x)^3*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csg
n(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*Pi-3/2/c*a*b^2/d*a
rctan(c*x)^2*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*Pi
+3/2/c*a*b^2/d*arctan(c*x)^2*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c
^2*x^2+1)+1))^3*Pi-3/2/c*a*b^2/d*arctan(c*x)^2*csgn(I*(1+I*c*x)^2/(c^2*x^2+
1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*Pi-3*I/c*a^2*b/d*ln(1+I*c*x)*arctan(c*x)-
3*I/c*a*b^2/d*ln(1+I*c*x)*arctan(c*x)^2+3*I/c*a*b^2/d*arctan(c*x)^2*ln(2*I*
```

```
(1+I*c*x)^2/(c^2*x^2+1))+1/2/c*b^3/d*arctan(c*x)^3*csgn((1+I*c*x)^2/(c^2*x^
2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*csgn((1+I*c*x)^2/(c^2*x^2+1))*Pi+1/c*a^
3/d*arctan(c*x)-3/4/c*b^3/d*polylog(4,-(1+I*c*x)^2/(c^2*x^2+1))+1/2/c*b^3/d
*arctan(c*x)^4+3/c*a*b^2/d*arctan(c*x)*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))+
3/2/c*a*b^2/d*Pi*arctan(c*x)^2-I/c*b^3/d*ln(1+I*c*x)*arctan(c*x)^3+3/2*I/c*
b^3/d*arctan(c*x)*polylog(3,-(1+I*c*x)^2/(c^2*x^2+1))+3/2*I/c*a*b^2/d*polyl
og(3,-(1+I*c*x)^2/(c^2*x^2+1))+I/c*b^3/d*arctan(c*x)^3*ln(2*I*(1+I*c*x)^2/(
c^2*x^2+1))-1/2/c*b^3/d*arctan(c*x)^3*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*
x)^2/(c^2*x^2+1)+1))^3*Pi+1/2/c*b^3/d*arctan(c*x)^3*csgn(I*(1+I*c*x)^2/(c^2
*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*Pi-1/2/c*b^3/d*arctan(c*x)^3*csgn(I*
(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*Pi-3/2/c*a^2*b/d*ln(
1/2-1/2*I*c*x)*ln(1+I*c*x)+3/2/c*a^2*b/d*ln(1/2-1/2*I*c*x)*ln(1/2*I*c*x+1/2
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{i\,a^3\log\left(i\,cdx+d\right)}{cd}+\frac{16\,b^3\arctan\left(cx\right)^4-b^3\log\left(c^2x^2+1\right)^4+\left(b^3c\left(\frac{4\,\log\left(c^2dx^2+d\right)\log\left(c^2x^2+1\right)^3}{c^2d}+\frac{4\left(\log\left(c^2x^2+1\right)^3+3\,\log\left(c^2x^2+1\right)^2\log(\cdots}{c^2}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3/(d+I*c*d*x),x, algorithm="maxima")
```

```
[Out] -I*a^3*log(I*c*d*x + d)/(c*d) + 1/128*(16*b^3*arctan(c*x)^4 + 16*I*b^3*arct
an(c*x)^3*log(c^2*x^2 + 1) + 4*I*b^3*arctan(c*x)*log(c^2*x^2 + 1)^3 - b^3*l
og(c^2*x^2 + 1)^4 + 16*(b^3*arctan(c*x)^4/(c*d) + 8*b^3*c*integrate(1/16*x*
log(c^2*x^2 + 1)^3/(c^2*d*x^2 + d), x) + 8*a*b^2*arctan(c*x)^3/(c*d) + 12*a
^2*b*arctan(c*x)^2/(c*d))*c*d - 128*I*c*d*integrate(1/32*(40*b^3*c*x*arctan
(c*x)^3 + 6*b^3*c*x*arctan(c*x)*log(c^2*x^2 + 1)^2 + 96*a*b^2*c*x*arctan(c*
x)^2 + 96*a^2*b*c*x*arctan(c*x) + 12*b^3*arctan(c*x)^2*log(c^2*x^2 + 1) + b
^3*log(c^2*x^2 + 1)^3)/(c^2*d*x^2 + d), x))/(c*d)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{b^3\log\left(-\frac{cx+i}{cx-i}\right)^3-6i\,ab^2\log\left(-\frac{cx+i}{cx-i}\right)^2-12\,a^2b\log\left(-\frac{cx+i}{cx-i}\right)+8i\,a^3}{8\,cdx-8i\,d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3/(d+I*c*d*x),x, algorithm="fricas")
```

```
[Out] integral(-(b^3*log(-(c*x + I)/(c*x - I))^3 - 6*I*a*b^2*log(-(c*x + I)/(c*x
- I))^2 - 12*a^2*b*log(-(c*x + I)/(c*x - I)) + 8*I*a^3)/(8*c*d*x - 8*I*d),
x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**3/(d+I*c*d*x),x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\arctan(cx) + a)^3}{icdx + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3/(d+I*c*d*x),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)^3/(I*c*d*x + d), x)
```

3.124 $\int \frac{\left(a+b\tan^{-1}(cx)\right)^3}{(d+icdx)^2}\,dx$

Optimal. Leaf size=182

$$\frac{3b^2\left(a+b\tan^{-1}(cx)\right)}{2cd^2(-cx+i)}+\frac{3ib\left(a+b\tan^{-1}(cx)\right)^2}{2cd^2(-cx+i)}-\frac{3b\left(a+b\tan^{-1}(cx)\right)^2}{4cd^2}+\frac{i\left(a+b\tan^{-1}(cx)\right)^3}{cd^2(1+icx)}-\frac{i\left(a+b\tan^{-1}(cx)\right)^3}{2cd^2}-$$

```
[Out] (((-3*I)/4)*b^3)/(c*d^2*(I - c*x)) + (((3*I)/4)*b^3*ArcTan[c*x])/(c*d^2) +
(3*b^2*(a + b*ArcTan[c*x]))/(2*c*d^2*(I - c*x)) - (3*b*(a + b*ArcTan[c*x])^
2)/(4*c*d^2) + (((3*I)/2)*b*(a + b*ArcTan[c*x])^2)/(c*d^2*(I - c*x)) - ((I/
2)*(a + b*ArcTan[c*x])^3)/(c*d^2) + (I*(a + b*ArcTan[c*x])^3)/(c*d^2*(1 + I
*c*x))
```

Rubi [A] time = 0.219803, antiderivative size = 182, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4864, 4862, 627, 44, 203, 4884}

$$\frac{3b^2\left(a+b\tan^{-1}(cx)\right)}{2cd^2(-cx+i)}+\frac{3ib\left(a+b\tan^{-1}(cx)\right)^2}{2cd^2(-cx+i)}-\frac{3b\left(a+b\tan^{-1}(cx)\right)^2}{4cd^2}+\frac{i\left(a+b\tan^{-1}(cx)\right)^3}{cd^2(1+icx)}-\frac{i\left(a+b\tan^{-1}(cx)\right)^3}{2cd^2}-$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^3/(d + I*c*d*x)^2,x]
```

```
[Out] (((-3*I)/4)*b^3)/(c*d^2*(I - c*x)) + (((3*I)/4)*b^3*ArcTan[c*x])/(c*d^2) +
(3*b^2*(a + b*ArcTan[c*x]))/(2*c*d^2*(I - c*x)) - (3*b*(a + b*ArcTan[c*x])^
2)/(4*c*d^2) + (((3*I)/2)*b*(a + b*ArcTan[c*x])^2)/(c*d^2*(I - c*x)) - ((I/
2)*(a + b*ArcTan[c*x])^3)/(c*d^2) + (I*(a + b*ArcTan[c*x])^3)/(c*d^2*(1 + I
*c*x))
```

Rule 4864

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x])^p)/(e*(q + 1)), x] - D
ist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 4862

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x]))/(e*(q + 1)), x] - Dist[(b*
c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}(cx)\right)^3}{(d+icdx)^2}\,dx = \frac{i\left(a+b\tan^{-1}(cx)\right)^3}{cd^2(1+icx)} - \frac{(3ib)\int\left(-\frac{\left(a+b\tan^{-1}(cx)\right)^2}{2d(-i+cx)^2}+\frac{\left(a+b\tan^{-1}(cx)\right)^2}{2d\left(1+c^2x^2\right)}\right)dx}{d}$$

$$= \frac{i\left(a+b\tan^{-1}(cx)\right)^3}{cd^2(1+icx)} + \frac{(3ib)\int\frac{\left(a+b\tan^{-1}(cx)\right)^2}{(-i+cx)^2}\,dx}{2d^2} - \frac{(3ib)\int\frac{\left(a+b\tan^{-1}(cx)\right)^2}{1+c^2x^2}\,dx}{2d^2}$$

$$= \frac{3ib\left(a+b\tan^{-1}(cx)\right)^2}{2cd^2(i-cx)} - \frac{i\left(a+b\tan^{-1}(cx)\right)^3}{2cd^2} + \frac{i\left(a+b\tan^{-1}(cx)\right)^3}{cd^2(1+icx)} + \frac{\left(3ib^2\right)\int\left(-\frac{i\left(a+b\tan^{-1}(c\right.}{2(-i+cx)^2}\right.}{d}$$

$$= \frac{3ib\left(a+b\tan^{-1}(cx)\right)^2}{2cd^2(i-cx)} - \frac{i\left(a+b\tan^{-1}(cx)\right)^3}{2cd^2} + \frac{i\left(a+b\tan^{-1}(cx)\right)^3}{cd^2(1+icx)} + \frac{\left(3b^2\right)\int\frac{a+b\tan^{-1}(cx)}{(-i+cx)^2}\,dx}{2d^2}$$

$$= \frac{3b^2\left(a+b\tan^{-1}(cx)\right)}{2cd^2(i-cx)} - \frac{3b\left(a+b\tan^{-1}(cx)\right)^2}{4cd^2} + \frac{3ib\left(a+b\tan^{-1}(cx)\right)^2}{2cd^2(i-cx)} - \frac{i\left(a+b\tan^{-1}(cx)\right)^3}{2cd^2}$$

$$= \frac{3b^2\left(a+b\tan^{-1}(cx)\right)}{2cd^2(i-cx)} - \frac{3b\left(a+b\tan^{-1}(cx)\right)^2}{4cd^2} + \frac{3ib\left(a+b\tan^{-1}(cx)\right)^2}{2cd^2(i-cx)} - \frac{i\left(a+b\tan^{-1}(cx)\right)^3}{2cd^2}$$

$$= \frac{3b^2\left(a+b\tan^{-1}(cx)\right)}{2cd^2(i-cx)} - \frac{3b\left(a+b\tan^{-1}(cx)\right)^2}{4cd^2} + \frac{3ib\left(a+b\tan^{-1}(cx)\right)^2}{2cd^2(i-cx)} - \frac{i\left(a+b\tan^{-1}(cx)\right)^3}{2cd^2}$$

$$= -\frac{3ib^3}{4cd^2(i-cx)} + \frac{3b^2\left(a+b\tan^{-1}(cx)\right)}{2cd^2(i-cx)} - \frac{3b\left(a+b\tan^{-1}(cx)\right)^2}{4cd^2} + \frac{3ib\left(a+b\tan^{-1}(cx)\right)^2}{2cd^2(i-cx)} - \frac{i(}{}$$

$$= -\frac{3ib^3}{4cd^2(i-cx)} + \frac{3ib^3\tan^{-1}(cx)}{4cd^2} + \frac{3b^2\left(a+b\tan^{-1}(cx)\right)}{2cd^2(i-cx)} - \frac{3b\left(a+b\tan^{-1}(cx)\right)^2}{4cd^2} + \frac{3ib\left(a+b\right.}{2cd^2(}$$

Mathematica [A] time = 0.222033, size = 121, normalized size = 0.66

$$\frac{3ib\left(-2a^2+2iab+b^2\right)(cx+i)\tan^{-1}(cx)-6ia^2b+4a^3-3b^2(b+2ia)(cx+i)\tan^{-1}(cx)^2-6ab^2+2b^3(1-icx)\tan^{-1}(cx)^3}{4cd^2(cx-i)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])^3/(d + I*c*d*x)^2,x]
```

```
[Out] (4*a^3 - (6*I)*a^2*b - 6*a*b^2 + (3*I)*b^3 + (3*I)*b*(-2*a^2 + (2*I)*a*b +
b^2)*(I + c*x)*ArcTan[c*x] - 3*b^2*((2*I)*a + b)*(I + c*x)*ArcTan[c*x]^2 +
2*b^3*(1 - I*c*x)*ArcTan[c*x]^3)/(4*c*d^2*(-I + c*x))
```

Maple [B] time = 0.319, size = 551, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^3/(d+I*c*d*x)^2,x)
```

```
[Out] 3/4*I/c*a*b^2/d^2*ln(-1/2*I*(-c*x+I))*ln(c*x+I)+3*I/c*a^2*b/d^2/(1+I*c*x)*a
rctan(c*x)-1/2/c*b^3/d^2/(c*x-I)*arctan(c*x)^3-3/8*I/c*a*b^2/d^2*ln(c*x-I)^
2-3/4*b^3/d^2/(c*x-I)*arctan(c*x)^2*x-3/4*I/c*b^3/d^2/(c*x-I)*arctan(c*x)^2
-3/4/c*b^3/d^2/(c*x-I)*arctan(c*x)+I/c*b^3/d^2/(1+I*c*x)*arctan(c*x)^3-3*I/
c*a*b^2/d^2*arctan(c*x)/(c*x-I)+3/4*I/c*a*b^2/d^2*ln(c*x-I)*ln(-1/2*I*(c*x+
I))-3/2/c*a*b^2/d^2*arctan(c*x)*ln(c*x-I)-1/2*I*b^3/d^2/(c*x-I)*arctan(c*x)
^3*x+3/2/c*a*b^2/d^2*arctan(c*x)*ln(c*x+I)-3/4*I/c*a*b^2/d^2*ln(-1/2*I*(-c*
x+I))*ln(-1/2*I*(c*x+I))-3/2*I/c*a^2*b/d^2*arctan(c*x)-3/2/c*a*b^2/d^2/(c*x
-I)-3/2/c*a*b^2/d^2*arctan(c*x)+3/4*I/c*b^3/d^2/(c*x-I)-3/2*I/c*a^2*b/d^2/(
c*x-I)-3/8*I/c*a*b^2/d^2*ln(c*x+I)^2+3*I/c*a*b^2/d^2/(1+I*c*x)*arctan(c*x)^
2+3/4*I*b^3/d^2/(c*x-I)*arctan(c*x)*x+I/c*a^3/d^2/(1+I*c*x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3/(d+I*c*d*x)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 1.88012, size = 401, normalized size = 2.2

$$-\frac{\left(b^3cx + i\,b^3\right)\log\left(-\frac{cx+i}{cx-i}\right)^3 - 16\,a^3 + 24i\,a^2b + 24\,ab^2 - 12i\,b^3 + \left(6\,ab^2 - 3i\,b^3 - \left(6i\,ab^2 + 3\,b^3\right)cx\right)\log\left(-\frac{cx+i}{cx-i}\right)^2 - \left(12i\,a^2 \cdots\right.}{16\left(c^2d^2x - i\,cd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3/(d+I*c*d*x)^2,x, algorithm="fricas")
```

```
[Out] -1/16*((b^3*c*x + I*b^3)*log(-(c*x + I)/(c*x - I))^3 - 16*a^3 + 24*I*a^2*b
+ 24*a*b^2 - 12*I*b^3 + (6*a*b^2 - 3*I*b^3 - (6*I*a*b^2 + 3*b^3)*c*x)*log(-
(c*x + I)/(c*x - I))^2 - (12*I*a^2*b + 12*a*b^2 - 6*I*b^3 + (12*a^2*b - 12*
I*a*b^2 - 6*b^3)*c*x)*log(-(c*x + I)/(c*x - I)))/(c^2*d^2*x - I*c*d^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**3/(d+I*c*d*x)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.16034, size = 782, normalized size = 4.3

$$-\frac{\frac{6\,b^3 d i^2 \arctan\left(\frac{(cdix+d)\left(\frac{di^2}{cdix+d}+1\right)i}{d}\right)^2}{cdix+d} - 2\,b^3 i \arctan\left(\frac{(cdix+d)\left(\frac{di^2}{cdix+d}+1\right)i}{d}\right)^3 + \frac{4\,b^3 d i \arctan\left(\frac{(cdix+d)\left(\frac{di^2}{cdix+d}+1\right)i}{d}\right)^3}{cdix+d} - \frac{12\,ab^2 d i^2 \arctan\left(\frac{(cdix+d)\left(\frac{di^2}{cdix+d}+1\right)i}{d}\right)}{cdix+d}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3/(d+I*c*d*x)^2,x, algorithm="giac")
```

```
[Out] -1/4*(6*b^3*d*i^2*arctan((c*d*i*x + d)*(d*i^2/(c*d*i*x + d) + 1)*i/d)^2/(c*
d*i*x + d) - 2*b^3*i*arctan((c*d*i*x + d)*(d*i^2/(c*d*i*x + d) + 1)*i/d)^3
+ 4*b^3*d*i*arctan((c*d*i*x + d)*(d*i^2/(c*d*i*x + d) + 1)*i/d)^3/(c*d*i*x
+ d) - 12*a*b^2*d*i^2*arctan((c*d*i*x + d)*(d*i^2/(c*d*i*x + d) + 1)*i/d)/(
c*d*i*x + d) + 6*a*b^2*i*arctan((c*d*i*x + d)*(d*i^2/(c*d*i*x + d) + 1)*i/d
)^2 - 12*a*b^2*d*i*arctan((c*d*i*x + d)*(d*i^2/(c*d*i*x + d) + 1)*i/d)^2/(c
*d*i*x + d) + 6*a^2*b*d*i^2/(c*d*i*x + d) - 3*b^3*d*i^2/(c*d*i*x + d) - 6*a
^2*b*i*arctan((c*d*i*x + d)*(d*i^2/(c*d*i*x + d) + 1)*i/d) + 3*b^3*i*arctan
((c*d*i*x + d)*(d*i^2/(c*d*i*x + d) + 1)*i/d) + 12*a^2*b*d*i*arctan((c*d*i*
x + d)*(d*i^2/(c*d*i*x + d) + 1)*i/d)/(c*d*i*x + d) - 6*b^3*d*i*arctan((c*d
```

```
*i*x + d)*(d*i^2/(c*d*i*x + d) + 1)*i/d)/(c*d*i*x + d) + 3*b^3*arctan((c*d*
i*x + d)*(d*i^2/(c*d*i*x + d) + 1)*i/d)^2 - 4*a^3*d*i/(c*d*i*x + d) + 6*a*b
^2*d*i/(c*d*i*x + d) - 6*a*b^2*arctan((c*d*i*x + d)*(d*i^2/(c*d*i*x + d) +
1)*i/d))/(c*d^2)
```

3.125 $\int \frac{\left(a+b\tan^{-1}(cx)\right)^3}{(d+icdx)^3}\,dx$

Optimal. Leaf size=271

$$\frac{9b^2\left(a+b\tan^{-1}(cx)\right)}{16cd^3(-cx+i)}+\frac{3ib^2\left(a+b\tan^{-1}(cx)\right)}{16cd^3(-cx+i)^2}+\frac{3ib\left(a+b\tan^{-1}(cx)\right)^2}{8cd^3(-cx+i)}-\frac{3b\left(a+b\tan^{-1}(cx)\right)^2}{8cd^3(-cx+i)^2}-\frac{9b\left(a+b\tan^{-1}(cx)\right)^2}{32cd^3}$$

```
[Out] (3*b^3)/(64*c*d^3*(I - c*x)^2) - (((21*I)/64)*b^3)/(c*d^3*(I - c*x)) + (((2
1*I)/64)*b^3*ArcTan[c*x])/(c*d^3) + (((3*I)/16)*b^2*(a + b*ArcTan[c*x]))/(c
*d^3*(I - c*x)^2) + (9*b^2*(a + b*ArcTan[c*x]))/(16*c*d^3*(I - c*x)) - (9*b
*(a + b*ArcTan[c*x])^2)/(32*c*d^3) - (3*b*(a + b*ArcTan[c*x])^2)/(8*c*d^3*(
I - c*x)^2) + (((3*I)/8)*b*(a + b*ArcTan[c*x])^2)/(c*d^3*(I - c*x)) - ((I/8
)*(a + b*ArcTan[c*x])^3)/(c*d^3) + ((I/2)*(a + b*ArcTan[c*x])^3)/(c*d^3*(1
+ I*c*x)^2)
```

Rubi [A] time = 0.403001, antiderivative size = 271, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4864, 4862, 627, 44, 203, 4884}

$$\frac{9b^2\left(a+b\tan^{-1}(cx)\right)}{16cd^3(-cx+i)}+\frac{3ib^2\left(a+b\tan^{-1}(cx)\right)}{16cd^3(-cx+i)^2}+\frac{3ib\left(a+b\tan^{-1}(cx)\right)^2}{8cd^3(-cx+i)}-\frac{3b\left(a+b\tan^{-1}(cx)\right)^2}{8cd^3(-cx+i)^2}-\frac{9b\left(a+b\tan^{-1}(cx)\right)^2}{32cd^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^3/(d + I*c*d*x)^3,x]
```



```
[Out] (3*b^3)/(64*c*d^3*(I - c*x)^2) - (((21*I)/64)*b^3)/(c*d^3*(I - c*x)) + (((2
1*I)/64)*b^3*ArcTan[c*x])/(c*d^3) + (((3*I)/16)*b^2*(a + b*ArcTan[c*x]))/(c
*d^3*(I - c*x)^2) + (9*b^2*(a + b*ArcTan[c*x]))/(16*c*d^3*(I - c*x)) - (9*b
*(a + b*ArcTan[c*x])^2)/(32*c*d^3) - (3*b*(a + b*ArcTan[c*x])^2)/(8*c*d^3*(
I - c*x)^2) + (((3*I)/8)*b*(a + b*ArcTan[c*x])^2)/(c*d^3*(I - c*x)) - ((I/8
)*(a + b*ArcTan[c*x])^3)/(c*d^3) + ((I/2)*(a + b*ArcTan[c*x])^3)/(c*d^3*(1
+ I*c*x)^2)
```


Rule 4864

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x])^p)/(e*(q + 1)), x] - D
ist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
```

```
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 4862

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x]))/(e*(q + 1)), x] - Dist[(b*
c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}(cx)\right)^3}{(d+icdx)^3}\,dx = \frac{i\left(a+b\tan^{-1}(cx)\right)^3}{2cd^3(1+icx)^2} - \frac{(3ib)\int\left(\frac{i\left(a+b\tan^{-1}(cx)\right)^2}{2d^2(-i+cx)^3} - \frac{\left(a+b\tan^{-1}(cx)\right)^2}{4d^2(-i+cx)^2} + \frac{\left(a+b\tan^{-1}(cx)\right)^2}{4d^2(1+c^2x^2)}\right)dx}{2d}$$

$$= \frac{i\left(a+b\tan^{-1}(cx)\right)^3}{2cd^3(1+icx)^2} + \frac{(3ib)\int\frac{\left(a+b\tan^{-1}(cx)\right)^2}{(-i+cx)^2}dx}{8d^3} - \frac{(3ib)\int\frac{\left(a+b\tan^{-1}(cx)\right)^2}{1+c^2x^2}dx}{8d^3} + \frac{(3b)\int\frac{\left(a+b\tan^{-1}(\right.}{(-i+cx)^3}}{4d^3}$$

$$= -\frac{3b\left(a+b\tan^{-1}(cx)\right)^2}{8cd^3(i-cx)^2} + \frac{3ib\left(a+b\tan^{-1}(cx)\right)^2}{8cd^3(i-cx)} - \frac{i\left(a+b\tan^{-1}(cx)\right)^3}{8cd^3} + \frac{i\left(a+b\tan^{-1}(cx)\right)^3}{2cd^3(1+icx)^2}$$

$$= -\frac{3b\left(a+b\tan^{-1}(cx)\right)^2}{8cd^3(i-cx)^2} + \frac{3ib\left(a+b\tan^{-1}(cx)\right)^2}{8cd^3(i-cx)} - \frac{i\left(a+b\tan^{-1}(cx)\right)^3}{8cd^3} + \frac{i\left(a+b\tan^{-1}(cx)\right)^3}{2cd^3(1+icx)^2}$$

$$= \frac{3ib^2\left(a+b\tan^{-1}(cx)\right)}{16cd^3(i-cx)^2} + \frac{9b^2\left(a+b\tan^{-1}(cx)\right)}{16cd^3(i-cx)} - \frac{9b\left(a+b\tan^{-1}(cx)\right)^2}{32cd^3} - \frac{3b\left(a+b\tan^{-1}(cx\right.}{8cd^3(i-cx)^2}$$

$$= \frac{3ib^2\left(a+b\tan^{-1}(cx)\right)}{16cd^3(i-cx)^2} + \frac{9b^2\left(a+b\tan^{-1}(cx)\right)}{16cd^3(i-cx)} - \frac{9b\left(a+b\tan^{-1}(cx)\right)^2}{32cd^3} - \frac{3b\left(a+b\tan^{-1}(cx\right.}{8cd^3(i-cx)^2}$$

$$= \frac{3ib^2\left(a+b\tan^{-1}(cx)\right)}{16cd^3(i-cx)^2} + \frac{9b^2\left(a+b\tan^{-1}(cx)\right)}{16cd^3(i-cx)} - \frac{9b\left(a+b\tan^{-1}(cx)\right)^2}{32cd^3} - \frac{3b\left(a+b\tan^{-1}(cx\right.}{8cd^3(i-cx)^2}$$

$$= \frac{3b^3}{64cd^3(i-cx)^2} - \frac{21ib^3}{64cd^3(i-cx)} + \frac{3ib^2\left(a+b\tan^{-1}(cx)\right)}{16cd^3(i-cx)^2} + \frac{9b^2\left(a+b\tan^{-1}(cx)\right)}{16cd^3(i-cx)} - \frac{9b\left(a+b\,\mathrm{t}\right.}{32}$$

$$= \frac{3b^3}{64cd^3(i-cx)^2} - \frac{21ib^3}{64cd^3(i-cx)} + \frac{21ib^3\tan^{-1}(cx)}{64cd^3} + \frac{3ib^2\left(a+b\tan^{-1}(cx)\right)}{16cd^3(i-cx)^2} + \frac{9b^2\left(a+b\tan^{-}\right.}{16cd^3(i-c}$$

Mathematica [A] time = 0.273388, size = 183, normalized size = 0.68

$$-\frac{i\left(3b(cx+i)\tan^{-1}(cx)\left(8a^2(cx-3i)+4ab(-5-3icx)+b^2(-7cx+9i)\right)+24a^2b(cx-2i)+32a^3+12ab^2(-4-3icx)+6b\right.}{64cd^3(cx-i)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])^3/(d + I*c*d*x)^3,x]
```

```
[Out] ((-I/64)*(32*a^3 + 3*b^3*(8*I - 7*c*x) + 12*a*b^2*(-4 - (3*I)*c*x) + 24*a^2
*b*(-2*I + c*x) + 3*b*(I + c*x)*(b^2*(9*I - 7*c*x) + 4*a*b*(-5 - (3*I)*c*x)
 + 8*a^2*(-3*I + c*x))*ArcTan[c*x] + 6*b^2*(I + c*x)*(b*(-5 - (3*I)*c*x) +
4*a*(-3*I + c*x))*ArcTan[c*x]^2 + 8*b^3*(3 - (2*I)*c*x + c^2*x^2)*ArcTan[c*
```

```
x]^3))/(c*d^3*(-I + c*x)^2)
```

Maple [B] time = 0.395, size = 711, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^3/(d+I*c*d*x)^3,x)
```

```
[Out] 3/8/c*b^3/d^3/(c*x-I)^2+1/2*I/c*a^3/d^3/(1+I*c*x)^2+21/64*I*b^3/d^3/(c*x-I)
^2*x-15/32/c*b^3/d^3/(c*x-I)^2*arctan(c*x)^2+3/32*b^3/d^3/(c*x-I)^2*arctan(
c*x)*x+3/2*I/c*a*b^2/d^3/(1+I*c*x)^2*arctan(c*x)^2-3/4*I/c*a*b^2/d^3*arctan
(c*x)/(c*x-I)+3/16*I/c*a*b^2/d^3*ln(-1/2*I*(-c*x+I))*ln(c*x+I)-3/16*I/c*a*b
^2/d^3*ln(-1/2*I*(-c*x+I))*ln(-1/2*I*(c*x+I))+3/16*I/c*a*b^2/d^3*ln(c*x-I)*
ln(-1/2*I*(c*x+I))+3/2*I/c*a^2*b/d^3/(1+I*c*x)^2*arctan(c*x)-1/8*I*c*b^3/d^
3/(c*x-I)^2*arctan(c*x)^3*x^2+21/64*I*c*b^3/d^3/(c*x-I)^2*x^2*arctan(c*x)-3
/8/c*a^2*b/d^3/(c*x-I)^2-9/16/c*a*b^2/d^3/(c*x-I)-9/16/c*a*b^2/d^3*arctan(c
*x)-1/4*b^3/d^3/(c*x-I)^2*arctan(c*x)^3*x-3/32*I/c*a*b^2/d^3*ln(c*x-I)^2-3/
8*I/c*a^2*b/d^3*arctan(c*x)-3/8*I/c*a^2*b/d^3/(c*x-I)+1/2*I/c*b^3/d^3/(1+I*
c*x)^2*arctan(c*x)^3-3/8/c*a*b^2/d^3*arctan(c*x)*ln(c*x-I)-3/4/c*a*b^2/d^3/
(c*x-I)^2*arctan(c*x)+3/8/c*a*b^2/d^3*arctan(c*x)*ln(c*x+I)-9/32*c*b^3/d^3/
(c*x-I)^2*arctan(c*x)^2*x^2+3/16*I*b^3/d^3/(c*x-I)^2*arctan(c*x)^2*x+1/8*I/
c*b^3/d^3/(c*x-I)^2*arctan(c*x)^3+27/64*I/c*b^3/d^3/(c*x-I)^2*arctan(c*x)-3
/32*I/c*a*b^2/d^3*ln(c*x+I)^2+3/16*I/c*a*b^2/d^3/(c*x-I)^2
```

Maxima [A] time = 1.48003, size = 313, normalized size = 1.15

$$-\frac{\left(8i\,b^3c^2x^2+16\,b^3cx+24i\,b^3\right)\arctan\left(cx\right)^3+32i\,a^3+48\,a^2b-48i\,ab^2-24\,b^3+\left(24i\,a^2b+36\,ab^2-21i\,b^3\right)cx-\left(6\left(-4\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3/(d+I*c*d*x)^3,x, algorithm="maxima")
```

```
[Out] -((8*I*b^3*c^2*x^2 + 16*b^3*c*x + 24*I*b^3)*arctan(c*x)^3 + 32*I*a^3 + 48*a
^2*b - 48*I*a*b^2 - 24*b^3 + (24*I*a^2*b + 36*a*b^2 - 21*I*b^3)*c*x - (6*(-
4*I*a*b^2 - 3*b^3)*c^2*x^2 - 72*I*a*b^2 - 30*b^3 - (48*a*b^2 - 12*I*b^3)*c*
x)*arctan(c*x)^2 + ((24*I*a^2*b + 36*a*b^2 - 21*I*b^3)*c^2*x^2 + 72*I*a^2*b
```

```
 + 60*a*b^2 - 27*I*b^3 + 6*(8*a^2*b - 4*I*a*b^2 - b^3)*c*x)*arctan(c*x))/(6
4*c^3*d^3*x^2 - 128*I*c^2*d^3*x - 64*c*d^3)
```

Fricas [A] time = 1.9108, size = 618, normalized size = 2.28

$$-\frac{\left(2\,b^3c^2x^2-4i\,b^3cx+6\,b^3\right)\log\left(-\frac{cx+i}{cx-i}\right)^3+64i\,a^3+96\,a^2b-96i\,ab^2-48\,b^3-\left(-48i\,a^2b-72\,ab^2+42i\,b^3\right)cx-\left(\left(12i\,ab^2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3/(d+I*c*d*x)^3,x, algorithm="fricas")
```

```
[Out] -((2*b^3*c^2*x^2 - 4*I*b^3*c*x + 6*b^3)*log(-(c*x + I)/(c*x - I))^3 + 64*I*
a^3 + 96*a^2*b - 96*I*a*b^2 - 48*b^3 - (-48*I*a^2*b - 72*a*b^2 + 42*I*b^3)*
c*x - ((12*I*a*b^2 + 9*b^3)*c^2*x^2 + 36*I*a*b^2 + 15*b^3 + 6*(4*a*b^2 - I*
b^3)*c*x)*log(-(c*x + I)/(c*x - I))^2 - ((24*a^2*b - 36*I*a*b^2 - 21*b^3)*c
^2*x^2 + 72*a^2*b - 60*I*a*b^2 - 27*b^3 + (-48*I*a^2*b - 24*a*b^2 + 6*I*b^3
)*c*x)*log(-(c*x + I)/(c*x - I)))/(128*c^3*d^3*x^2 - 256*I*c^2*d^3*x - 128*
c*d^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**3/(d+I*c*d*x)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\arctan(cx)+a)^3}{(i\,cdx+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3/(d+I*c*d*x)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)^3/(I*c*d*x + d)^3, x)
```

3.126 $\int \frac{\left(a+b\tan^{-1}(cx)\right)^3}{(d+icdx)^4}\,dx$

Optimal. Leaf size=360

$$\frac{11b^2\left(a+b\tan^{-1}(cx)\right)}{48cd^4(-cx+i)}+\frac{5ib^2\left(a+b\tan^{-1}(cx)\right)}{48cd^4(-cx+i)^2}-\frac{b^2\left(a+b\tan^{-1}(cx)\right)}{18cd^4(-cx+i)^3}+\frac{ib\left(a+b\tan^{-1}(cx)\right)^2}{8cd^4(-cx+i)}-\frac{b\left(a+b\tan^{-1}(cx)\right)^2}{8cd^4(-cx+i)^2}-$$

```
[Out] ((I/108)*b^3)/(c*d^4*(I - c*x)^3) + (19*b^3)/(576*c*d^4*(I - c*x)^2) - (((8
5*I)/576)*b^3)/(c*d^4*(I - c*x)) + (((85*I)/576)*b^3*ArcTan[c*x])/(c*d^4) -
 (b^2*(a + b*ArcTan[c*x]))/(18*c*d^4*(I - c*x)^3) + (((5*I)/48)*b^2*(a + b*
ArcTan[c*x]))/(c*d^4*(I - c*x)^2) + (11*b^2*(a + b*ArcTan[c*x]))/(48*c*d^4*
(I - c*x)) - (11*b*(a + b*ArcTan[c*x])^2)/(96*c*d^4) - ((I/6)*b*(a + b*ArcT
an[c*x])^2)/(c*d^4*(I - c*x)^3) - (b*(a + b*ArcTan[c*x])^2)/(8*c*d^4*(I - c
*x)^2) + ((I/8)*b*(a + b*ArcTan[c*x])^2)/(c*d^4*(I - c*x)) - ((I/24)*(a + b
*ArcTan[c*x])^3)/(c*d^4) + ((I/3)*(a + b*ArcTan[c*x])^3)/(c*d^4*(1 + I*c*x)
^3)
```

Rubi [A] time = 0.673546, antiderivative size = 360, normalized size of antiderivative = 1., number of steps used = 42, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4864, 4862, 627, 44, 203, 4884}

$$\frac{11b^2\left(a+b\tan^{-1}(cx)\right)}{48cd^4(-cx+i)}+\frac{5ib^2\left(a+b\tan^{-1}(cx)\right)}{48cd^4(-cx+i)^2}-\frac{b^2\left(a+b\tan^{-1}(cx)\right)}{18cd^4(-cx+i)^3}+\frac{ib\left(a+b\tan^{-1}(cx)\right)^2}{8cd^4(-cx+i)}-\frac{b\left(a+b\tan^{-1}(cx)\right)^2}{8cd^4(-cx+i)^2}-$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^3/(d + I*c*d*x)^4,x]
```

```
[Out] ((I/108)*b^3)/(c*d^4*(I - c*x)^3) + (19*b^3)/(576*c*d^4*(I - c*x)^2) - (((8
5*I)/576)*b^3)/(c*d^4*(I - c*x)) + (((85*I)/576)*b^3*ArcTan[c*x])/(c*d^4) -
 (b^2*(a + b*ArcTan[c*x]))/(18*c*d^4*(I - c*x)^3) + (((5*I)/48)*b^2*(a + b*
ArcTan[c*x]))/(c*d^4*(I - c*x)^2) + (11*b^2*(a + b*ArcTan[c*x]))/(48*c*d^4*
(I - c*x)) - (11*b*(a + b*ArcTan[c*x])^2)/(96*c*d^4) - ((I/6)*b*(a + b*ArcT
an[c*x])^2)/(c*d^4*(I - c*x)^3) - (b*(a + b*ArcTan[c*x])^2)/(8*c*d^4*(I - c
*x)^2) + ((I/8)*b*(a + b*ArcTan[c*x])^2)/(c*d^4*(I - c*x)) - ((I/24)*(a + b
*ArcTan[c*x])^3)/(c*d^4) + ((I/3)*(a + b*ArcTan[c*x])^3)/(c*d^4*(1 + I*c*x)
^3)
```

Rule 4864

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x])^p)/(e*(q + 1)), x] - D
ist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 4862

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x]))/(e*(q + 1)), x] - Dist[(b*
c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}(cx)\right)^3}{(d+icdx)^4}\,dx = \frac{i\left(a+b\tan^{-1}(cx)\right)^3}{3cd^4(1+icx)^3} - \frac{(ib)\int\left(\frac{\left(a+b\tan^{-1}(cx)\right)^2}{2d^3(-i+cx)^4} + \frac{i\left(a+b\tan^{-1}(cx)\right)^2}{4d^3(-i+cx)^3} - \frac{\left(a+b\tan^{-1}(cx)\right)^2}{8d^3(-i+cx)^2} + \frac{\left(a+b\tan^{-1}(cx)\right)^2}{8d^3\left(1+c^2x^2\right)}\right.}{d}$$

$$= \frac{i\left(a+b\tan^{-1}(cx)\right)^3}{3cd^4(1+icx)^3} + \frac{(ib)\int\frac{\left(a+b\tan^{-1}(cx)\right)^2}{(-i+cx)^2}dx}{8d^4} - \frac{(ib)\int\frac{\left(a+b\tan^{-1}(cx)\right)^2}{1+c^2x^2}dx}{8d^4} - \frac{(ib)\int\frac{\left(a+b\tan^{-1}(cx)\right)^2}{(-i+cx)^4}}{2d^4}$$

$$= -\frac{ib\left(a+b\tan^{-1}(cx)\right)^2}{6cd^4(i-cx)^3} - \frac{b\left(a+b\tan^{-1}(cx)\right)^2}{8cd^4(i-cx)^2} + \frac{ib\left(a+b\tan^{-1}(cx)\right)^2}{8cd^4(i-cx)} - \frac{i\left(a+b\tan^{-1}(cx)\right)^3}{24cd^4}$$

$$= -\frac{ib\left(a+b\tan^{-1}(cx)\right)^2}{6cd^4(i-cx)^3} - \frac{b\left(a+b\tan^{-1}(cx)\right)^2}{8cd^4(i-cx)^2} + \frac{ib\left(a+b\tan^{-1}(cx)\right)^2}{8cd^4(i-cx)} - \frac{i\left(a+b\tan^{-1}(cx)\right)^3}{24cd^4}$$

$$= -\frac{b^2\left(a+b\tan^{-1}(cx)\right)}{18cd^4(i-cx)^3} + \frac{5ib^2\left(a+b\tan^{-1}(cx)\right)}{48cd^4(i-cx)^2} + \frac{11b^2\left(a+b\tan^{-1}(cx)\right)}{48cd^4(i-cx)} - \frac{11b\left(a+b\tan^{-1}(c\right.}{96cd^4}$$

$$= -\frac{b^2\left(a+b\tan^{-1}(cx)\right)}{18cd^4(i-cx)^3} + \frac{5ib^2\left(a+b\tan^{-1}(cx)\right)}{48cd^4(i-cx)^2} + \frac{11b^2\left(a+b\tan^{-1}(cx)\right)}{48cd^4(i-cx)} - \frac{11b\left(a+b\tan^{-1}(c\right.}{96cd^4}$$

$$= -\frac{b^2\left(a+b\tan^{-1}(cx)\right)}{18cd^4(i-cx)^3} + \frac{5ib^2\left(a+b\tan^{-1}(cx)\right)}{48cd^4(i-cx)^2} + \frac{11b^2\left(a+b\tan^{-1}(cx)\right)}{48cd^4(i-cx)} - \frac{11b\left(a+b\tan^{-1}(c\right.}{96cd^4}$$

$$= \frac{ib^3}{108cd^4(i-cx)^3} + \frac{19b^3}{576cd^4(i-cx)^2} - \frac{85ib^3}{576cd^4(i-cx)} - \frac{b^2\left(a+b\tan^{-1}(cx)\right)}{18cd^4(i-cx)^3} + \frac{5ib^2\left(a+b\tan^{-1}\right.}{48cd^4(i-cx}$$

$$= \frac{ib^3}{108cd^4(i-cx)^3} + \frac{19b^3}{576cd^4(i-cx)^2} - \frac{85ib^3}{576cd^4(i-cx)} + \frac{85ib^3\tan^{-1}(cx)}{576cd^4} - \frac{b^2\left(a+b\tan^{-1}(cx)\right)}{18cd^4(i-cx)^3}$$

Mathematica [A] time = 0.296543, size = 269, normalized size = 0.75

$$\frac{3b(cx+i)\tan^{-1}(cx)\left(-72ia^2\left(c^2x^2-4icx-7\right)+12ab\left(-11c^2x^2+32icx+29\right)+b^2\left(85ic^2x^2+208cx-139i\right)\right)-72ia^2b\left(3c\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])^3/(d + I*c*d*x)^4,x]
```

```
[Out] (-576*a^3 + 12*a*b^2*(56 + (81*I)*c*x - 33*c^2*x^2) + b^3*(-328*I + 567*c*x
 + (255*I)*c^2*x^2) - (72*I)*a^2*b*(-10 - (9*I)*c*x + 3*c^2*x^2) + 3*b*(I +
 c*x)*(12*a*b*(29 + (32*I)*c*x - 11*c^2*x^2) + b^2*(-139*I + 208*c*x + (85*
I)*c^2*x^2) - (72*I)*a^2*(-7 - (4*I)*c*x + c^2*x^2))*ArcTan[c*x] - (18*I)*b
```

```
^2*(I + c*x)*(b*(29*I - 32*c*x - (11*I)*c^2*x^2) + 12*a*(-7 - (4*I)*c*x + c
^2*x^2))*ArcTan[c*x]^2 - (72*I)*b^3*(-7*I - 3*c*x - (3*I)*c^2*x^2 + c^3*x^3
)*ArcTan[c*x]^3)/(1728*c*d^4*(-I + c*x)^3)
```

Maple [B] time = 0.467, size = 881, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^3/(d+I*c*d*x)^4,x)
```

```
[Out] 139/576/c*b^3/d^4/(c*x-I)^3*arctan(c*x)-11/48/c*a*b^2/d^4*arctan(c*x)-1/8/c
*a^2*b/d^4/(c*x-I)^2-41/216*I/c*b^3/d^4/(c*x-I)^3+1/3*I/c*a^3/d^4/(1+I*c*x)
^3-1/32*b^3/d^4/(c*x-I)^3*arctan(c*x)^2*x+1/18/c*a*b^2/d^4/(c*x-I)^3-11/48/
c*a*b^2/d^4/(c*x-I)-1/8/c*a*b^2/d^4*arctan(c*x)*ln(c*x-I)-1/4/c*a*b^2/d^4*a
rctan(c*x)/(c*x-I)^2-1/8*I/c*a^2*b/d^4*arctan(c*x)-1/32*I/c*a*b^2/d^4*ln(c*
x+I)^2+1/6*I/c*a^2*b/d^4/(c*x-I)^3+1/8*I*b^3/d^4/(c*x-I)^3*arctan(c*x)^3*x+
23/192*I*b^3/d^4/(c*x-I)^3*arctan(c*x)*x+41/192*c*b^3/d^4/(c*x-I)^3*x^2*arc
tan(c*x)-1/8*c*b^3/d^4/(c*x-I)^3*arctan(c*x)^3*x^2-11/96*c^2*b^3/d^4/(c*x-I
)^3*arctan(c*x)^2*x^3+1/8/c*a*b^2/d^4*arctan(c*x)*ln(c*x+I)-1/8*I/c*a^2*b/d
^4/(c*x-I)+1/3*I/c*b^3/d^4/(1+I*c*x)^3*arctan(c*x)^3+29/96*I/c*b^3/d^4/(c*x
-I)^3*arctan(c*x)^2+5/48*I/c*a*b^2/d^4/(c*x-I)^2-1/32*I/c*a*b^2/d^4*ln(c*x-
I)^2+85/576*I*c*b^3/d^4/(c*x-I)^3*x^2+1/24/c*b^3/d^4/(c*x-I)^3*arctan(c*x)^
3+7/32*I*c*b^3/d^4/(c*x-I)^3*arctan(c*x)^2*x^2+85/576*I*c^2*b^3/d^4/(c*x-I)
^3*arctan(c*x)*x^3+I/c*a^2*b/d^4/(1+I*c*x)^3*arctan(c*x)+I/c*a*b^2/d^4/(1+I
*c*x)^3*arctan(c*x)^2-1/16*I/c*a*b^2/d^4*ln(-1/2*I*(-c*x+I))*ln(-1/2*I*(c*x
+I))+1/3*I/c*a*b^2/d^4*arctan(c*x)/(c*x-I)^3-1/4*I/c*a*b^2/d^4*arctan(c*x)/
(c*x-I)+1/16*I/c*a*b^2/d^4*ln(c*x-I)*ln(-1/2*I*(c*x+I))+1/16*I/c*a*b^2/d^4*
ln(-1/2*I*(-c*x+I))*ln(c*x+I)-1/24*I*c^2*b^3/d^4/(c*x-I)^3*arctan(c*x)^3*x^
3+21/64*b^3/d^4/(c*x-I)^3*x
```

Maxima [A] time = 1.86376, size = 439, normalized size = 1.22

$$-\frac{\left(216i\,a^2b + 396\,ab^2 - 255i\,b^3\right)c^2x^2 + \left(72i\,b^3c^3x^3 + 216\,b^3c^2x^2 - 216i\,b^3cx + 504\,b^3\right)\arctan\left(cx\right)^3 + 576\,a^3 - 720i\,a^2b - \cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3/(d+I*c*d*x)^4,x, algorithm="maxima")
```

```
[Out] -((216*I*a^2*b + 396*a*b^2 - 255*I*b^3)*c^2*x^2 + (72*I*b^3*c^3*x^3 + 216*b
^3*c^2*x^2 - 216*I*b^3*c*x + 504*b^3)*arctan(c*x)^3 + 576*a^3 - 720*I*a^2*b
 - 672*a*b^2 + 328*I*b^3 + 81*(8*a^2*b - 12*I*a*b^2 - 7*b^3)*c*x - (18*(-12
*I*a*b^2 - 11*b^3)*c^3*x^3 - (648*a*b^2 - 378*I*b^3)*c^2*x^2 - 1512*a*b^2 +
 522*I*b^3 + 54*(12*I*a*b^2 - b^3)*c*x)*arctan(c*x)^2 + ((216*I*a^2*b + 396
*a*b^2 - 255*I*b^3)*c^3*x^3 + 9*(72*a^2*b - 84*I*a*b^2 - 41*b^3)*c^2*x^2 +
1512*a^2*b - 1044*I*a*b^2 - 417*b^3 + (-648*I*a^2*b + 108*a*b^2 - 207*I*b^3
)*c*x)*arctan(c*x))/(1728*c^4*d^4*x^3 - 5184*I*c^3*d^4*x^2 - 5184*c^2*d^4*x
 + 1728*I*c*d^4)
```

Fricas [A] time = 2.36914, size = 900, normalized size = 2.5

$$\left(-432i\,a^2b - 792\,ab^2 + 510i\,b^3\right)c^2x^2 - \left(18\,b^3c^3x^3 - 54i\,b^3c^2x^2 - 54\,b^3cx - 126i\,b^3\right)\log\left(-\frac{cx+i}{cx-i}\right)^3 - 1152\,a^3 + 1440i\,a^2b + 13$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3/(d+I*c*d*x)^4,x, algorithm="fricas")
```

```
[Out] ((-432*I*a^2*b - 792*a*b^2 + 510*I*b^3)*c^2*x^2 - (18*b^3*c^3*x^3 - 54*I*b^
3*c^2*x^2 - 54*b^3*c*x - 126*I*b^3)*log(-(c*x + I)/(c*x - I))^3 - 1152*a^3
+ 1440*I*a^2*b + 1344*a*b^2 - 656*I*b^3 - (1296*a^2*b - 1944*I*a*b^2 - 1134
*b^3)*c*x + ((108*I*a*b^2 + 99*b^3)*c^3*x^3 + 27*(12*a*b^2 - 7*I*b^3)*c^2*x
^2 + 756*a*b^2 - 261*I*b^3 + (-324*I*a*b^2 + 27*b^3)*c*x)*log(-(c*x + I)/(c
*x - I))^2 + ((216*a^2*b - 396*I*a*b^2 - 255*b^3)*c^3*x^3 + (-648*I*a^2*b -
 756*a*b^2 + 369*I*b^3)*c^2*x^2 - 1512*I*a^2*b - 1044*a*b^2 + 417*I*b^3 - (
648*a^2*b + 108*I*a*b^2 + 207*b^3)*c*x)*log(-(c*x + I)/(c*x - I)))/(3456*c^
4*d^4*x^3 - 10368*I*c^3*d^4*x^2 - 10368*c^2*d^4*x + 3456*I*c*d^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**3/(d+I*c*d*x)**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\arctan(cx) + a)^3}{(icdx + d)^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3/(d+I*c*d*x)^4,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)^3/(I*c*d*x + d)^4, x)
```

3.127 $\int \frac{x^2(a+b\tan^{-1}(cx))^3}{d+icdx}\,dx$

Optimal. Leaf size=410

$$\frac{3ib^2\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{c^3d}-\frac{3ib^2\text{PolyLog}\left(3,1-\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{2c^3d}+\frac{3b\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{2c^3d}$$

```
[Out] (-3*b*(a + b*ArcTan[c*x])^2)/(2*c^3*d) + (((3*I)/2)*b*x*(a + b*ArcTan[c*x])
^2)/(c^2*d) + ((I/2)*(a + b*ArcTan[c*x])^3)/(c^3*d) + (x*(a + b*ArcTan[c*x]
)^3)/(c^2*d) - ((I/2)*x^2*(a + b*ArcTan[c*x])^3)/(c*d) + ((3*I)*b^2*(a + b*
ArcTan[c*x])*Log[2/(1 + I*c*x)])/(c^3*d) + (3*b*(a + b*ArcTan[c*x])^2*Log[2
/(1 + I*c*x)])/(c^3*d) - (I*(a + b*ArcTan[c*x])^3*Log[2/(1 + I*c*x)])/(c^3*
d) - (3*b^3*PolyLog[2, 1 - 2/(1 + I*c*x)])/(2*c^3*d) + ((3*I)*b^2*(a + b*Ar
cTan[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)])/(c^3*d) + (3*b*(a + b*ArcTan[c*x]
)^2*PolyLog[2, 1 - 2/(1 + I*c*x)])/(2*c^3*d) + (3*b^3*PolyLog[3, 1 - 2/(1 +
 I*c*x)])/(2*c^3*d) - (((3*I)/2)*b^2*(a + b*ArcTan[c*x])*PolyLog[3, 1 - 2/(
1 + I*c*x)])/(c^3*d) - (3*b^3*PolyLog[4, 1 - 2/(1 + I*c*x)])/(4*c^3*d)
```

Rubi [A] time = 0.861898, antiderivative size = 410, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.48, Rules used = {4866, 4852, 4916, 4846, 4920, 4854, 2402, 2315, 4884, 4994, 6610, 4998}

$$\frac{3ib^2\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{c^3d}-\frac{3ib^2\text{PolyLog}\left(3,1-\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{2c^3d}+\frac{3b\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{2c^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcTan[c*x])^3)/(d + I*c*d*x),x]
```


```
[Out] (-3*b*(a + b*ArcTan[c*x])^2)/(2*c^3*d) + (((3*I)/2)*b*x*(a + b*ArcTan[c*x])
^2)/(c^2*d) + ((I/2)*(a + b*ArcTan[c*x])^3)/(c^3*d) + (x*(a + b*ArcTan[c*x]
)^3)/(c^2*d) - ((I/2)*x^2*(a + b*ArcTan[c*x])^3)/(c*d) + ((3*I)*b^2*(a + b*
ArcTan[c*x])*Log[2/(1 + I*c*x)])/(c^3*d) + (3*b*(a + b*ArcTan[c*x])^2*Log[2
/(1 + I*c*x)])/(c^3*d) - (I*(a + b*ArcTan[c*x])^3*Log[2/(1 + I*c*x)])/(c^3*
d) - (3*b^3*PolyLog[2, 1 - 2/(1 + I*c*x)])/(2*c^3*d) + ((3*I)*b^2*(a + b*Ar
cTan[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)])/(c^3*d) + (3*b*(a + b*ArcTan[c*x]
)^2*PolyLog[2, 1 - 2/(1 + I*c*x)])/(2*c^3*d) + (3*b^3*PolyLog[3, 1 - 2/(1 +
 I*c*x)])/(2*c^3*d) - (((3*I)/2)*b^2*(a + b*ArcTan[c*x])*PolyLog[3, 1 - 2/(
1 + I*c*x)])/(c^3*d) - (3*b^3*PolyLog[4, 1 - 2/(1 + I*c*x)])/(4*c^3*d)
```

Rule 4866

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_) + (
e_.)*(x_)), x_Symbol] :> Dist[f/e, Int[(f*x)^(m - 1)*(a + b*ArcTan[c*x])^p,
 x], x] - Dist[(d*f)/e, Int[((f*x)^(m - 1)*(a + b*ArcTan[c*x])^p)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2
, 0] && GtQ[m, 0]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 4998

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[k + 1, u])/(2
*c*d), x] - Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[k + 1,
 u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
qQ[e, c^2*d] && EqQ[u^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rubi steps

$$\int \frac{x^2\left(a+b\tan^{-1}(cx)\right)^3}{d+icdx}\,dx = \frac{i\int \frac{x\left(a+b\tan^{-1}(cx)\right)^3}{d+icdx}dx}{c} - \frac{i\int x\left(a+b\tan^{-1}(cx)\right)^3\,dx}{cd}$$

$$= -\frac{ix^2\left(a+b\tan^{-1}(cx)\right)^3}{2cd} - \frac{\int \frac{\left(a+b\tan^{-1}(cx)\right)^3}{d+icdx}dx}{c^2} + \frac{(3ib)\int \frac{x^2\left(a+b\tan^{-1}(cx)\right)^2}{1+c^2x^2}dx}{2d} + \frac{\int\left(a+b\tan\right.}{c}$$

$$= \frac{x\left(a+b\tan^{-1}(cx)\right)^3}{c^2d} - \frac{ix^2\left(a+b\tan^{-1}(cx)\right)^3}{2cd} - \frac{i\left(a+b\tan^{-1}(cx)\right)^3\log\left(\frac{2}{1+icx}\right)}{c^3d} + \frac{(3ib)\int}{}$$

$$= \frac{3ibx\left(a+b\tan^{-1}(cx)\right)^2}{2c^2d} + \frac{i\left(a+b\tan^{-1}(cx)\right)^3}{2c^3d} + \frac{x\left(a+b\tan^{-1}(cx)\right)^3}{c^2d} - \frac{ix^2\left(a+b\tan^{-1}\right.}{2cd}$$

$$= -\frac{3b\left(a+b\tan^{-1}(cx)\right)^2}{2c^3d} + \frac{3ibx\left(a+b\tan^{-1}(cx)\right)^2}{2c^2d} + \frac{i\left(a+b\tan^{-1}(cx)\right)^3}{2c^3d} + \frac{x\left(a+b\tan^{-1}\right.}{c^2d}$$

$$= -\frac{3b\left(a+b\tan^{-1}(cx)\right)^2}{2c^3d} + \frac{3ibx\left(a+b\tan^{-1}(cx)\right)^2}{2c^2d} + \frac{i\left(a+b\tan^{-1}(cx)\right)^3}{2c^3d} + \frac{x\left(a+b\tan^{-1}\right.}{c^2d}$$

$$= -\frac{3b\left(a+b\tan^{-1}(cx)\right)^2}{2c^3d} + \frac{3ibx\left(a+b\tan^{-1}(cx)\right)^2}{2c^2d} + \frac{i\left(a+b\tan^{-1}(cx)\right)^3}{2c^3d} + \frac{x\left(a+b\tan^{-1}\right.}{c^2d}$$

$$= -\frac{3b\left(a+b\tan^{-1}(cx)\right)^2}{2c^3d} + \frac{3ibx\left(a+b\tan^{-1}(cx)\right)^2}{2c^2d} + \frac{i\left(a+b\tan^{-1}(cx)\right)^3}{2c^3d} + \frac{x\left(a+b\tan^{-1}\right.}{c^2d}$$

Mathematica [A] time = 0.965524, size = 541, normalized size = 1.32

$$-\frac{i\left(6b^2\text{PolyLog}\left(3,-e^{2i\tan^{-1}(cx)}\right)\left(a+b\tan^{-1}(cx)+ib\right)-6ib\text{PolyLog}\left(2,-e^{2i\tan^{-1}(cx)}\right)\left(a+b\tan^{-1}(cx)+ib\right)^2+3ib^3\text{P}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(a + b*ArcTan[c*x])^3)/(d + I*c*d*x),x]
```

```
[Out] ((-I/4)*((4*I)*a^3*c*x - 6*a^2*b*c*x + 2*a^3*c^2*x^2 - (4*I)*a^3*ArcTan[c*x
] + 6*a^2*b*ArcTan[c*x] + (12*I)*a^2*b*c*x*ArcTan[c*x] - 12*a*b^2*c*x*ArcTa
n[c*x] + 6*a^2*b*c^2*x^2*ArcTan[c*x] - (12*I)*a^2*b*ArcTan[c*x]^2 + 18*a*b^
2*ArcTan[c*x]^2 + (6*I)*b^3*ArcTan[c*x]^2 + (12*I)*a*b^2*c*x*ArcTan[c*x]^2
- 6*b^3*c*x*ArcTan[c*x]^2 + 6*a*b^2*c^2*x^2*ArcTan[c*x]^2 - (8*I)*a*b^2*Arc
Tan[c*x]^3 + 6*b^3*ArcTan[c*x]^3 + (4*I)*b^3*c*x*ArcTan[c*x]^3 + 2*b^3*c^2*
x^2*ArcTan[c*x]^3 - (2*I)*b^3*ArcTan[c*x]^4 + 12*a^2*b*ArcTan[c*x]*Log[1 +
E^((2*I)*ArcTan[c*x])] + (24*I)*a*b^2*ArcTan[c*x]*Log[1 + E^((2*I)*ArcTan[c
```

```
*x])] - 12*b^3*ArcTan[c*x]*Log[1 + E^((2*I)*ArcTan[c*x])] + 12*a*b^2*ArcTan
[c*x]^2*Log[1 + E^((2*I)*ArcTan[c*x])] + (12*I)*b^3*ArcTan[c*x]^2*Log[1 + E
^((2*I)*ArcTan[c*x])] + 4*b^3*ArcTan[c*x]^3*Log[1 + E^((2*I)*ArcTan[c*x])]
- 2*a^3*Log[1 + c^2*x^2] - (6*I)*a^2*b*Log[1 + c^2*x^2] + 6*a*b^2*Log[1 + c
^2*x^2] - (6*I)*b*(a + I*b + b*ArcTan[c*x])^2*PolyLog[2, -E^((2*I)*ArcTan[c
*x])] + 6*b^2*(a + I*b + b*ArcTan[c*x])*PolyLog[3, -E^((2*I)*ArcTan[c*x])]
+ (3*I)*b^3*PolyLog[4, -E^((2*I)*ArcTan[c*x])]))/(c^3*d)
```

Maple [C] time = 2.217, size = 1725, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctan(c*x))^3/(d+I*c*d*x),x)
```

```
[Out] -3/2/c^3*a*b^2/d*arctan(c*x)^2*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c
^2*x^2+1)+1))*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn((1+I*c*x)^2/(c^2*x^2
+1))*Pi+3/2/c^3*a*b^2/d*arctan(c*x)^2*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*
x)^2/(c^2*x^2+1)+1))^2*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*Pi-3/2/c^3*a*b^2
/d*arctan(c*x)^2*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^
2*csgn((1+I*c*x)^2/(c^2*x^2+1))*Pi-3/c^3*a*b^2/d*Pi*csgn((1+I*c*x)^2/(c^2*x
^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2+1/c^2*b^3/d*arctan(c*x)^
3*x+1/2*I/c^3*a^3/d*ln(c^2*x^2+1)-3/2*I/c^3*b^3/d*arctan(c*x)^3-1/2*I/c*a^3
/d*x^2-3/c^3*a*b^2/d*arctan(c*x)*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))+3/2/c^
3*b*a^2/d*ln(c*x-I)*ln(-1/2*I*(c*x+I))+6/c^3*a*b^2/d*arctan(c*x)*ln(1+I*(1+
I*c*x)/(c^2*x^2+1)^(1/2))+6/c^3*a*b^2/d*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x
^2+1)^(1/2))+3/c^2*b*a^2/d*arctan(c*x)*x+3/c^2*a*b^2/d*arctan(c*x)^2*x+3/c^
3*a*b^2/d*Pi*arctan(c*x)^2-3/2*I/c^3*a*b^2/d*polylog(3,-(1+I*c*x)^2/(c^2*x^
2+1))+3*I/c^3*a*b^2/d*ln((1+I*c*x)^2/(c^2*x^2+1)+1)-I/c^3*b^3/d*arctan(c*x)
^3*ln((1+I*c*x)^2/(c^2*x^2+1)+1)-3/2*I/c^3*b^3/d*arctan(c*x)*polylog(3,-(1+
I*c*x)^2/(c^2*x^2+1))+3*I/c^3*b^3/d*arctan(c*x)*ln((1+I*c*x)^2/(c^2*x^2+1)+
1)-3*I/c^3*b^3/d*arctan(c*x)*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))+3/2*I/c^2*
b*a^2/d*x-1/2*I/c*b^3/d*arctan(c*x)^3*x^2+3/2*I/c^2*b^3/d*arctan(c*x)^2*x-9
/4*I/c^3*b*a^2/d*arctan(c*x)+3/8*I/c^3*b*a^2/d*arctan(1/6*c^3*x^3+7/6*c*x)-
3/8*I/c^3*b*a^2/d*arctan(1/2*c*x)+3/4*I/c^3*b*a^2/d*arctan(1/2*c*x-1/2*I)-6
*I/c^3*a*b^2/d*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-9/2*I/c^3*a*b^2/d*arc
tan(c*x)^2-6*I/c^3*a*b^2/d*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-3/2/c^3*a
*b^2/d*arctan(c*x)^2*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+
1))^3*Pi-3*I/c^3*a*b^2/d*arctan(c*x)^2*ln(2*I*(1+I*c*x)^2/(c^2*x^2+1))+3*I/
c^3*a*b^2/d*arctan(c*x)^2*ln(c*x-I)+3*I/c^3*b*a^2/d*arctan(c*x)*ln(c*x-I)+3
*I/c^2*a*b^2/d*arctan(c*x)*x-3/2*I/c*a*b^2/d*arctan(c*x)^2*x^2-3/2*I/c*b*a^
```

```
2/d*arctan(c*x)*x^2+3/2/c^3*b*a^2/d*dilog(-1/2*I*(c*x+I))-3/4/c^3*b*a^2/d*l
n(c*x-I)^2-2/c^3*a*b^2/d*arctan(c*x)^3-3/16/c^3*b*a^2/d*ln(c^4*x^4+10*c^2*x
^2+9)+3/c^3*a*b^2/d*arctan(c*x)-9/8/c^3*b*a^2/d*ln(c^2*x^2+1)-3/2/c^3*b^3/d
*arctan(c*x)^2*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))+3/c^3*b^3/d*arctan(c*x)^
2*ln((1+I*c*x)^2/(c^2*x^2+1)+1)+3/2/c^3*b^3/d*polylog(3,-(1+I*c*x)^2/(c^2*x
^2+1))+3/4/c^3*b^3/d*polylog(4,-(1+I*c*x)^2/(c^2*x^2+1))-1/2/c^3*b^3/d*arct
an(c*x)^4-1/c^3*a^3/d*arctan(c*x)+3/2/c^3*b^3/d*arctan(c*x)^2+3/2/c^3*b^3/d
*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))+3/2/c^3*b*a^2/d+1/c^2*a^3/d*x
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))^3/(d+I*c*d*x),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{b^3x^2\log\left(-\frac{cx+i}{cx-i}\right)^3-6i\,ab^2x^2\log\left(-\frac{cx+i}{cx-i}\right)^2-12\,a^2bx^2\log\left(-\frac{cx+i}{cx-i}\right)+8i\,a^3x^2}{8\,cdx-8i\,d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))^3/(d+I*c*d*x),x, algorithm="fricas")
```

```
[Out] integral(-(b^3*x^2*log(-(c*x + I)/(c*x - I))^3 - 6*I*a*b^2*x^2*log(-(c*x +
I)/(c*x - I))^2 - 12*a^2*b*x^2*log(-(c*x + I)/(c*x - I)) + 8*I*a^3*x^2)/(8*
c*d*x - 8*I*d), x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atan(c*x))**3/(d+I*c*d*x),x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\arctan(cx) + a)^3 x^2}{icdx + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))^3/(d+I*c*d*x),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)^3*x^2/(I*c*d*x + d), x)
```

3.128 $\int \frac{x\left(a+b\tan^{-1}(cx)\right)^3}{d+icdx}\,dx$

Optimal. Leaf size=277

$$\frac{3b^2\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{c^2d}-\frac{3b^2\text{PolyLog}\left(3,1-\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{2c^2d}-\frac{3ib\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right.}{2c^2d}$$

```
[Out] (a + b*ArcTan[c*x])^3/(c^2*d) - (I*x*(a + b*ArcTan[c*x])^3)/(c*d) - ((3*I)*
b*(a + b*ArcTan[c*x])^2*Log[2/(1 + I*c*x)])/(c^2*d) - ((a + b*ArcTan[c*x])^
3*Log[2/(1 + I*c*x)])/(c^2*d) + (3*b^2*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2
/(1 + I*c*x)])/(c^2*d) - (((3*I)/2)*b*(a + b*ArcTan[c*x])^2*PolyLog[2, 1 -
2/(1 + I*c*x)])/(c^2*d) - (((3*I)/2)*b^3*PolyLog[3, 1 - 2/(1 + I*c*x)])/(c^
2*d) - (3*b^2*(a + b*ArcTan[c*x])*PolyLog[3, 1 - 2/(1 + I*c*x)])/(2*c^2*d)
+ (((3*I)/4)*b^3*PolyLog[4, 1 - 2/(1 + I*c*x)])/(c^2*d)
```

Rubi [A] time = 0.503495, antiderivative size = 277, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {4866, 4846, 4920, 4854, 4884, 4994, 6610, 4998}

$$\frac{3b^2\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{c^2d}-\frac{3b^2\text{PolyLog}\left(3,1-\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{2c^2d}-\frac{3ib\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right.}{2c^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcTan[c*x])^3)/(d + I*c*d*x),x]
```


```
[Out] (a + b*ArcTan[c*x])^3/(c^2*d) - (I*x*(a + b*ArcTan[c*x])^3)/(c*d) - ((3*I)*
b*(a + b*ArcTan[c*x])^2*Log[2/(1 + I*c*x)])/(c^2*d) - ((a + b*ArcTan[c*x])^
3*Log[2/(1 + I*c*x)])/(c^2*d) + (3*b^2*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2
/(1 + I*c*x)])/(c^2*d) - (((3*I)/2)*b*(a + b*ArcTan[c*x])^2*PolyLog[2, 1 -
2/(1 + I*c*x)])/(c^2*d) - (((3*I)/2)*b^3*PolyLog[3, 1 - 2/(1 + I*c*x)])/(c^
2*d) - (3*b^2*(a + b*ArcTan[c*x])*PolyLog[3, 1 - 2/(1 + I*c*x)])/(2*c^2*d)
+ (((3*I)/4)*b^3*PolyLog[4, 1 - 2/(1 + I*c*x)])/(c^2*d)
```


Rule 4866

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_) + (
e_.)*(x_)), x_Symbol] :> Dist[f/e, Int[(f*x)^(m - 1)*(a + b*ArcTan[c*x])^p,
 x], x] - Dist[(d*f)/e, Int[((f*x)^(m - 1)*(a + b*ArcTan[c*x])^p)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2
```

```
, 0] && GtQ[m, 0]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 4998

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[k + 1, u])/(2
```

```
*c*d), x] - Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[k + 1,
 u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
qQ[e, c^2*d] && EqQ[u^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rubi steps

$$\int \frac{x\left(a+b\tan^{-1}(cx)\right)^3}{d+icdx}\,dx = \frac{i\int \frac{\left(a+b\tan^{-1}(cx)\right)^3}{d+icdx}\,dx}{c} - \frac{i\int \left(a+b\tan^{-1}(cx)\right)^3\,dx}{cd}$$

$$= -\frac{ix\left(a+b\tan^{-1}(cx)\right)^3}{cd} - \frac{\left(a+b\tan^{-1}(cx)\right)^3\log\left(\frac{2}{1+icx}\right)}{c^2d} + \frac{(3ib)\int \frac{x\left(a+b\tan^{-1}(cx)\right)^2}{1+c^2x^2}\,dx}{d} + \frac{(3b$$

$$= \frac{\left(a+b\tan^{-1}(cx)\right)^3}{c^2d} - \frac{ix\left(a+b\tan^{-1}(cx)\right)^3}{cd} - \frac{\left(a+b\tan^{-1}(cx)\right)^3\log\left(\frac{2}{1+icx}\right)}{c^2d} - \frac{3ib\left(a+b\,\mathrm{ta}\right.}{}$$

$$= \frac{\left(a+b\tan^{-1}(cx)\right)^3}{c^2d} - \frac{ix\left(a+b\tan^{-1}(cx)\right)^3}{cd} - \frac{3ib\left(a+b\tan^{-1}(cx)\right)^2\log\left(\frac{2}{1+icx}\right)}{c^2d} - \frac{\left(a+b\,\mathrm{ta}\right.}{}$$

$$= \frac{\left(a+b\tan^{-1}(cx)\right)^3}{c^2d} - \frac{ix\left(a+b\tan^{-1}(cx)\right)^3}{cd} - \frac{3ib\left(a+b\tan^{-1}(cx)\right)^2\log\left(\frac{2}{1+icx}\right)}{c^2d} - \frac{\left(a+b\,\mathrm{ta}\right.}{}$$

$$= \frac{\left(a+b\tan^{-1}(cx)\right)^3}{c^2d} - \frac{ix\left(a+b\tan^{-1}(cx)\right)^3}{cd} - \frac{3ib\left(a+b\tan^{-1}(cx)\right)^2\log\left(\frac{2}{1+icx}\right)}{c^2d} - \frac{\left(a+b\,\mathrm{ta}\right.}{}$$

Mathematica [A] time = 0.736524, size = 393, normalized size = 1.42

$$-\frac{i\left(-6b\,\mathrm{PolyLog}\left(2,-e^{2i\tan^{-1}(cx)}\right)\left(2b(a+ib)\tan^{-1}(cx)+a(a+2ib)+b^2\tan^{-1}(cx)^2\right)+6b^2\,\mathrm{PolyLog}\left(3,-e^{2i\tan^{-1}(cx)}\right)\left(\cdots\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(a + b*ArcTan[c*x])^3)/(d + I*c*d*x),x]
```

```
[Out] ((-I/4)*(4*a^3*c*x - 4*a^3*ArcTan[c*x] + 12*a^2*b*c*x*ArcTan[c*x] - 12*a^2*
b*ArcTan[c*x]^2 - (12*I)*a*b^2*ArcTan[c*x]^2 + 12*a*b^2*c*x*ArcTan[c*x]^2 -
 8*a*b^2*ArcTan[c*x]^3 - (4*I)*b^3*ArcTan[c*x]^3 + 4*b^3*c*x*ArcTan[c*x]^3
- 2*b^3*ArcTan[c*x]^4 - (12*I)*a^2*b*ArcTan[c*x]*Log[1 + E^((2*I)*ArcTan[c*
x])] + 24*a*b^2*ArcTan[c*x]*Log[1 + E^((2*I)*ArcTan[c*x])] - (12*I)*a*b^2*A
rcTan[c*x]^2*Log[1 + E^((2*I)*ArcTan[c*x])] + 12*b^3*ArcTan[c*x]^2*Log[1 +
E^((2*I)*ArcTan[c*x])] - (4*I)*b^3*ArcTan[c*x]^3*Log[1 + E^((2*I)*ArcTan[c*
```

```
x])] + (2*I)*a^3*Log[1 + c^2*x^2] - 6*a^2*b*Log[1 + c^2*x^2] - 6*b*(a*(a +
(2*I)*b) + 2*(a + I*b)*b*ArcTan[c*x] + b^2*ArcTan[c*x]^2)*PolyLog[2, -E^((2
*I)*ArcTan[c*x])] + 6*b^2*((-I)*a + b - I*b*ArcTan[c*x])*PolyLog[3, -E^((2*
I)*ArcTan[c*x])] + 3*b^3*PolyLog[4, -E^((2*I)*ArcTan[c*x])]))/(c^2*d)
```

Maple [C] time = 0.645, size = 5478, normalized size = 19.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctan(c*x))^3/(d+I*c*d*x),x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))^3/(d+I*c*d*x),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{b^3 x \log\left(-\frac{cx+i}{cx-i}\right)^3 - 6i\,ab^2 x \log\left(-\frac{cx+i}{cx-i}\right)^2 - 12\,a^2 bx \log\left(-\frac{cx+i}{cx-i}\right) + 8i\,a^3 x}{8\,cdx - 8i\,d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))^3/(d+I*c*d*x),x, algorithm="fricas")
```

```
[Out] integral(-(b^3*x*log(-(c*x + I)/(c*x - I))^3 - 6*I*a*b^2*x*log(-(c*x + I)/(
c*x - I))^2 - 12*a^2*b*x*log(-(c*x + I)/(c*x - I)) + 8*I*a^3*x)/(8*c*d*x -
```

```
8*I*d), x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atan(c*x))**3/(d+I*c*d*x),x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \arctan (cx) + a)^3 x}{i\, cdx + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))^3/(d+I*c*d*x),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)^3*x/(I*c*d*x + d), x)
```

3.129 $\int \frac{(a+b\tan^{-1}(cx))^3}{d+icdx}\,dx$

Optimal. Leaf size=139

$$\frac{3ib^2\text{PolyLog}\left(3,1-\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{2cd}-\frac{3b\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)^2}{2cd}+\frac{3b^3\text{PolyLog}\left(4,1-\frac{2}{1+icx}\right)}{4cd}$$

```
[Out] (I*(a + b*ArcTan[c*x])^3*Log[2/(1 + I*c*x)])/(c*d) - (3*b*(a + b*ArcTan[c*x
])^2*PolyLog[2, 1 - 2/(1 + I*c*x)])/(2*c*d) + (((3*I)/2)*b^2*(a + b*ArcTan[
c*x])*PolyLog[3, 1 - 2/(1 + I*c*x)])/(c*d) + (3*b^3*PolyLog[4, 1 - 2/(1 + I
*c*x)])/(4*c*d)
```

Rubi [A] time = 0.215199, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {4854, 4884, 4994, 4998, 6610}

$$\frac{3ib^2\text{PolyLog}\left(3,1-\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{2cd}-\frac{3b\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)^2}{2cd}+\frac{3b^3\text{PolyLog}\left(4,1-\frac{2}{1+icx}\right)}{4cd}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^3/(d + I*c*d*x),x]
```


```
[Out] (I*(a + b*ArcTan[c*x])^3*Log[2/(1 + I*c*x)])/(c*d) - (3*b*(a + b*ArcTan[c*x
])^2*PolyLog[2, 1 - 2/(1 + I*c*x)])/(2*c*d) + (((3*I)/2)*b^2*(a + b*ArcTan[
c*x])*PolyLog[3, 1 - 2/(1 + I*c*x)])/(c*d) + (3*b^3*PolyLog[4, 1 - 2/(1 + I
*c*x)])/(4*c*d)
```


Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
```

```
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 4998

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[k + 1, u])/(2
*c*d), x] - Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[k + 1,
 u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
qQ[e, c^2*d] && EqQ[u^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}(cx)\right)^3}{d+icdx}\,dx = \frac{i\left(a+b\tan^{-1}(cx)\right)^3\log\left(\frac{2}{1+icx}\right)}{cd} - \frac{(3ib)\int\frac{\left(a+b\tan^{-1}(cx)\right)^2\log\left(\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx}{d}$$

$$= \frac{i\left(a+b\tan^{-1}(cx)\right)^3\log\left(\frac{2}{1+icx}\right)}{cd} - \frac{3b\left(a+b\tan^{-1}(cx)\right)^2\text{Li}_2\left(1-\frac{2}{1+icx}\right)}{2cd} + \frac{\left(3b^2\right)\int\frac{\left(a+b\tan^{-1}(c\right.}{1\cdots}}{d}$$

$$= \frac{i\left(a+b\tan^{-1}(cx)\right)^3\log\left(\frac{2}{1+icx}\right)}{cd} - \frac{3b\left(a+b\tan^{-1}(cx)\right)^2\text{Li}_2\left(1-\frac{2}{1+icx}\right)}{2cd} + \frac{3ib^2\left(a+b\tan^{-1}(\cdots\right.}{2\cdots}$$

$$= \frac{i\left(a+b\tan^{-1}(cx)\right)^3\log\left(\frac{2}{1+icx}\right)}{cd} - \frac{3b\left(a+b\tan^{-1}(cx)\right)^2\text{Li}_2\left(1-\frac{2}{1+icx}\right)}{2cd} + \frac{3ib^2\left(a+b\tan^{-1}(\cdots\right.}{2\cdots}$$

Mathematica [A] time = 0.0779158, size = 133, normalized size = 0.96

$$\frac{i\left(3ib\left(2\text{PolyLog}\left(2,\frac{cx+i}{cx-i}\right)\left(a+b\tan^{-1}(cx)\right)^2-b\left(2i\text{PolyLog}\left(3,\frac{cx+i}{cx-i}\right)\left(a+b\tan^{-1}(cx)\right)+b\text{PolyLog}\left(4,\frac{cx+i}{cx-i}\right)\right)\right)+4\log\cdots}{4cd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])^3/(d + I*c*d*x),x]
```

```
[Out] ((I/4)*(4*(a + b*ArcTan[c*x])^3*Log[(2*d)/(d + I*c*d*x)] + (3*I)*b*(2*(a +
b*ArcTan[c*x])^2*PolyLog[2, (I + c*x)/(-I + c*x)] - b*((2*I)*(a + b*ArcTan[
c*x])*PolyLog[3, (I + c*x)/(-I + c*x)] + b*PolyLog[4, (I + c*x)/(-I + c*x)]
))))/(c*d)
```

Maple [C] time = 0.242, size = 2044, normalized size = 14.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^3/(d+I*c*d*x),x)
```

```
[Out] 3/2/c*a*b^2/d*arctan(c*x)^2*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*
x^2+1)+1))^2*csgn((1+I*c*x)^2/(c^2*x^2+1))*Pi+3/2/c*a*b^2/d*arctan(c*x)^2*c
sgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*(1+I*c*x)^2
/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*Pi-3/2/c*a*b^2/d*arctan(c*x)^2*
csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*(1+I*c*x)^
2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*Pi+1/2/c*b^3/d*arctan(c*x)^3*csg
n((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I/((1+I*c*x)^2/
(c^2*x^2+1)+1))*csgn((1+I*c*x)^2/(c^2*x^2+1))*Pi-3/2/c*a*b^2/d*arctan(c*x)^
2*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*csgn(I/((1+I*
c*x)^2/(c^2*x^2+1)+1))*Pi-1/2/c*b^3/d*arctan(c*x)^3*csgn((1+I*c*x)^2/(c^2*x
^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*Pi
+1/2/c*b^3/d*arctan(c*x)^3*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x
^2+1)+1))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*Pi+
3/2/c*a*b^2/d*arctan(c*x)^2*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*
x^2+1)+1))*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)
)*Pi+3/4/c*a^2*b/d*ln(1+I*c*x)^2+3/2/c*b^3/d*arctan(c*x)^2*polylog(2,-(1+I*
c*x)^2/(c^2*x^2+1))+1/2/c*b^3/d*Pi*arctan(c*x)^3-1/2*I/c*a^3/d*ln(c^2*x^2+1
)+2/c*a*b^2/d*arctan(c*x)^3+3/2/c*a^2*b/d*dilog(1/2*I*c*x+1/2)-1/2/c*b^3/d*
arctan(c*x)^3*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csg
n(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*Pi-3/2/c*a*b^2/d*a
rctan(c*x)^2*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*Pi
+3/2/c*a*b^2/d*arctan(c*x)^2*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c
^2*x^2+1)+1))^3*Pi-3/2/c*a*b^2/d*arctan(c*x)^2*csgn(I*(1+I*c*x)^2/(c^2*x^2+
1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*Pi-3*I/c*a^2*b/d*ln(1+I*c*x)*arctan(c*x)-
3*I/c*a*b^2/d*ln(1+I*c*x)*arctan(c*x)^2+3*I/c*a*b^2/d*arctan(c*x)^2*ln(2*I*
```

```
(1+I*c*x)^2/(c^2*x^2+1))+1/2/c*b^3/d*arctan(c*x)^3*csgn((1+I*c*x)^2/(c^2*x^
2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*csgn((1+I*c*x)^2/(c^2*x^2+1))*Pi+1/c*a^
3/d*arctan(c*x)-3/4/c*b^3/d*polylog(4,-(1+I*c*x)^2/(c^2*x^2+1))+1/2/c*b^3/d
*arctan(c*x)^4+3/c*a*b^2/d*arctan(c*x)*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))+
3/2/c*a*b^2/d*Pi*arctan(c*x)^2-I/c*b^3/d*ln(1+I*c*x)*arctan(c*x)^3+3/2*I/c*
b^3/d*arctan(c*x)*polylog(3,-(1+I*c*x)^2/(c^2*x^2+1))+3/2*I/c*a*b^2/d*polyl
og(3,-(1+I*c*x)^2/(c^2*x^2+1))+I/c*b^3/d*arctan(c*x)^3*ln(2*I*(1+I*c*x)^2/(
c^2*x^2+1))-1/2/c*b^3/d*arctan(c*x)^3*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*
x)^2/(c^2*x^2+1)+1))^3*Pi+1/2/c*b^3/d*arctan(c*x)^3*csgn(I*(1+I*c*x)^2/(c^2
*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*Pi-1/2/c*b^3/d*arctan(c*x)^3*csgn(I*
(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*Pi-3/2/c*a^2*b/d*ln(
1/2-1/2*I*c*x)*ln(1+I*c*x)+3/2/c*a^2*b/d*ln(1/2-1/2*I*c*x)*ln(1/2*I*c*x+1/2
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{i a^3 \log(icdx+d)}{cd}+\frac{16 b^3 \arctan(cx)^4 - b^3 \log\left(c^2x^2+1\right)^4 + \left(b^3 c\left(\frac{4 \log\left(c^2dx^2+d\right)\log\left(c^2x^2+1\right)^3}{c^2 d} + \frac{4\left(\log\left(c^2x^2+1\right)^3 + 3\log\left(c^2x^2+1\right)^2 \log \cdots\right.}{c^2}\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3/(d+I*c*d*x),x, algorithm="maxima")
```

```
[Out] -I*a^3*log(I*c*d*x + d)/(c*d) + 1/128*(16*b^3*arctan(c*x)^4 + 16*I*b^3*arct
an(c*x)^3*log(c^2*x^2 + 1) + 4*I*b^3*arctan(c*x)*log(c^2*x^2 + 1)^3 - b^3*l
og(c^2*x^2 + 1)^4 + 16*(b^3*arctan(c*x)^4/(c*d) + 8*b^3*c*integrate(1/16*x*
log(c^2*x^2 + 1)^3/(c^2*d*x^2 + d), x) + 8*a*b^2*arctan(c*x)^3/(c*d) + 12*a
^2*b*arctan(c*x)^2/(c*d))*c*d - 128*I*c*d*integrate(1/32*(40*b^3*c*x*arctan
(c*x)^3 + 6*b^3*c*x*arctan(c*x)*log(c^2*x^2 + 1)^2 + 96*a*b^2*c*x*arctan(c*
x)^2 + 96*a^2*b*c*x*arctan(c*x) + 12*b^3*arctan(c*x)^2*log(c^2*x^2 + 1) + b
^3*log(c^2*x^2 + 1)^3)/(c^2*d*x^2 + d), x))/(c*d)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{b^3 \log\left(-\frac{cx+i}{cx-i}\right)^3 - 6i\,ab^2 \log\left(-\frac{cx+i}{cx-i}\right)^2 - 12\,a^2 b \log\left(-\frac{cx+i}{cx-i}\right) + 8i\,a^3}{8\,cdx - 8i\,d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3/(d+I*c*d*x),x, algorithm="fricas")
```

```
[Out] integral(-(b^3*log(-(c*x + I)/(c*x - I))^3 - 6*I*a*b^2*log(-(c*x + I)/(c*x
- I))^2 - 12*a^2*b*log(-(c*x + I)/(c*x - I)) + 8*I*a^3)/(8*c*d*x - 8*I*d),
x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**3/(d+I*c*d*x),x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\arctan(cx) + a)^3}{i\,cdx + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3/(d+I*c*d*x),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)^3/(I*c*d*x + d), x)
```

3.130 $\int \frac{(a+b\tan^{-1}(cx))^3}{x(d+icdx)}\,dx$

Optimal. Leaf size=128

$$\frac{3b^2\text{PolyLog}\left(3,-1+\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{2d}+\frac{3ib\text{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)^2}{2d}-\frac{3ib^3\text{PolyLog}\left(4,-1+\dots\right.}{4d}$$

```
[Out] ((a + b*ArcTan[c*x])^3*Log[2 - 2/(1 + I*c*x)])/d + (((3*I)/2)*b*(a + b*ArcT
an[c*x])^2*PolyLog[2, -1 + 2/(1 + I*c*x)])/d + (3*b^2*(a + b*ArcTan[c*x])*P
olyLog[3, -1 + 2/(1 + I*c*x)])/(2*d) - (((3*I)/4)*b^3*PolyLog[4, -1 + 2/(1
+ I*c*x)])/d
```

Rubi [A] time = 0.232061, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4868, 4884, 4994, 4998, 6610}

$$\frac{3b^2\text{PolyLog}\left(3,-1+\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{2d}+\frac{3ib\text{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)^2}{2d}-\frac{3ib^3\text{PolyLog}\left(4,-1+\dots\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^3/(x*(d + I*c*d*x)),x]
```

```
[Out] ((a + b*ArcTan[c*x])^3*Log[2 - 2/(1 + I*c*x)])/d + (((3*I)/2)*b*(a + b*ArcT
an[c*x])^2*PolyLog[2, -1 + 2/(1 + I*c*x)])/d + (3*b^2*(a + b*ArcTan[c*x])*P
olyLog[3, -1 + 2/(1 + I*c*x)])/(2*d) - (((3*I)/4)*b^3*PolyLog[4, -1 + 2/(1
+ I*c*x)])/d
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
```

```
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 4998

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[k + 1, u])/(2
*c*d), x] - Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[k + 1,
 u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
qQ[e, c^2*d] && EqQ[u^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}(cx)\right)^3}{x(d+icdx)}\,dx = \frac{\left(a+b\tan^{-1}(cx)\right)^3\log\left(2-\frac{2}{1+icx}\right)}{d} - \frac{(3bc)\int\frac{\left(a+b\tan^{-1}(cx)\right)^2\log\left(2-\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx}{d}$$

$$= \frac{\left(a+b\tan^{-1}(cx)\right)^3\log\left(2-\frac{2}{1+icx}\right)}{d} + \frac{3ib\left(a+b\tan^{-1}(cx)\right)^2\mathrm{Li}_2\left(-1+\frac{2}{1+icx}\right)}{2d} - \frac{\left(3ib^2c\right)\int\frac{(a+b\,t}{}}{}$$

$$= \frac{\left(a+b\tan^{-1}(cx)\right)^3\log\left(2-\frac{2}{1+icx}\right)}{d} + \frac{3ib\left(a+b\tan^{-1}(cx)\right)^2\mathrm{Li}_2\left(-1+\frac{2}{1+icx}\right)}{2d} + \frac{3b^2\left(a+b\tan\right.}{}$$

$$= \frac{\left(a+b\tan^{-1}(cx)\right)^3\log\left(2-\frac{2}{1+icx}\right)}{d} + \frac{3ib\left(a+b\tan^{-1}(cx)\right)^2\mathrm{Li}_2\left(-1+\frac{2}{1+icx}\right)}{2d} + \frac{3b^2\left(a+b\tan\right.}{}$$

Mathematica [B] time = 0.172057, size = 311, normalized size = 2.43

$$\frac{6b^2\mathrm{PolyLog}\left(3,\frac{cx+i}{-cx+i}\right)\left(a+b\tan^{-1}(cx)\right)+6ib\mathrm{PolyLog}\left(2,\frac{cx+i}{-cx+i}\right)\left(a+b\tan^{-1}(cx)\right)^2-3ib^3\mathrm{PolyLog}\left(4,\frac{cx+i}{-cx+i}\right)+12a^2b\,\mathrm{lo}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c*x])^3/(x*(d + I*c*d*x)),x]
```

```
[Out] (8*a^3*ArcTanh[(I + c*x)/(-I + c*x)] + 24*a^2*b*ArcTan[c*x]*ArcTanh[(I + c*
x)/(-I + c*x)] + 24*a*b^2*ArcTan[c*x]^2*ArcTanh[(I + c*x)/(-I + c*x)] + 8*b
^3*ArcTan[c*x]^3*ArcTanh[(I + c*x)/(-I + c*x)] + 4*a^3*Log[(2*I)/(I - c*x)]
 + 12*a^2*b*ArcTan[c*x]*Log[(2*I)/(I - c*x)] + 12*a*b^2*ArcTan[c*x]^2*Log[(
2*I)/(I - c*x)] + 4*b^3*ArcTan[c*x]^3*Log[(2*I)/(I - c*x)] + (6*I)*b*(a + b
*ArcTan[c*x])^2*PolyLog[2, (I + c*x)/(I - c*x)] + 6*b^2*(a + b*ArcTan[c*x])
*PolyLog[3, (I + c*x)/(I - c*x)] - (3*I)*b^3*PolyLog[4, (I + c*x)/(I - c*x)
])/(4*d)
```

Maple [C] time = 0.442, size = 3393, normalized size = 26.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^3/x/(d+I*c*d*x),x)
```

```
[Out] 3/2*I*a*b^2/d*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1
)+1))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan(
c*x)^2-3/2*I*a*b^2/d*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^
2*x^2+1)+1))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^
2*arctan(c*x)^2-3/2*I*a*b^2/d*Pi*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I
*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2+3
/2*I*a*b^2/d*Pi*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn((1+I*c*x)^2/(c^2*x
^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2-3/2*I*a*b^2/d*Pi*arctan(
c*x)^2*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x
)^2/(c^2*x^2+1)+1))^2-3/2*I*a*b^2/d*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*
csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*
x)^2+1/2*I*b^3/d*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I/((1+I*c*x)^2
/(c^2*x^2+1)+1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1
)+1))*arctan(c*x)^3-1/2*I*b^3/d*Pi*arctan(c*x)^3*csgn(I/((1+I*c*x)^2/(c^2*x
^2+1)+1))*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*
c*x)^2/(c^2*x^2+1)+1))-3*a*b^2/d*arctan(c*x)^2*ln(c*x-I)+3*a*b^2/d*arctan(c
*x)^2*ln(2*I*(1+I*c*x)^2/(c^2*x^2+1))+3*a*b^2/d*arctan(c*x)^2*ln(c*x)+3*a*b
^2/d*arctan(c*x)^2*ln(1-(1+I*c*x)/(c^2*x^2+1)^(1/2))-3*a*b^2/d*arctan(c*x)^
2*ln((1+I*c*x)^2/(c^2*x^2+1)-1)+3*a*b^2/d*arctan(c*x)^2*ln(1+(1+I*c*x)/(c^2
*x^2+1)^(1/2))-2*I*a*b^2/d*arctan(c*x)^3-3*I*b^3/d*arctan(c*x)^2*polylog(2,
-(1+I*c*x)/(c^2*x^2+1)^(1/2))-3*I*b^3/d*arctan(c*x)^2*polylog(2,(1+I*c*x)/(
```

c^2*x^2+1)^(1/2))+3/2*I*b^3/d*Pi*arctan(c*x)^3+3*b*a^2/d*arctan(c*x)*ln(c*x
)-1/2*I*b^3/d*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^
3*arctan(c*x)^3+1/2*I*b^3/d*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x
)^2/(c^2*x^2+1)+1))^3*arctan(c*x)^3+1/2*I*b^3/d*Pi*csgn(((1+I*c*x)^2/(c^2*x
^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*arctan(c*x)^3-I*b^3/d*Pi*csgn((1+I*
c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^3-3*I*a*b^2/d
*Pi*arctan(c*x)^2*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))
^2+3/2*I*a*b^2/d*Pi*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+
1)+1))^3*arctan(c*x)^2+3/2*I*a*b^2/d*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/
((1+I*c*x)^2/(c^2*x^2+1)+1))^3*arctan(c*x)^2-1/2*I*b^3/d*Pi*csgn(I/((1+I*c*
x)^2/(c^2*x^2+1)+1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x
^2+1)+1))^2*arctan(c*x)^3-1/2*I*b^3/d*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)
)*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(
c*x)^3-1/2*I*b^3/d*Pi*arctan(c*x)^3*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I
*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2+1/2*I*b^3/d*Pi*csgn(I*((
1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(((1+I*c*x)^2/(c
^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)^3-1/2*I*b^3/d*Pi*csgn
(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(((1+I*c*x)
^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^3+1/2*I*b^3/d*
Pi*arctan(c*x)^3*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn((1+I*c*x)^2/(c^2*
x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2-3/2*I*a*b^2/d*Pi*csgn((1+I*c*x)^2/(c^
2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*arctan(c*x)^2-3/2*I*a*b^2/d*Pi*csgn
(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2-1
/2*a^3/d*ln(c^2*x^2+1)+a^3/d*ln(c*x)-3/2*I*b*a^2/d*ln(c*x)*ln(1-I*c*x)+3/2*
I*b*a^2/d*ln(c*x-I)*ln(-1/2*I*(c*x+I))+9/2*I*a*b^2/d*Pi*arctan(c*x)^2-6*I*a
*b^2/d*arctan(c*x)*polylog(2,-(1+I*c*x)/(c^2*x^2+1)^(1/2))-6*I*a*b^2/d*arct
an(c*x)*polylog(2,(1+I*c*x)/(c^2*x^2+1)^(1/2))-1/2*I*b^3/d*Pi*csgn(((1+I*c*
x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^3+3/2*I*b*a^
2/d*ln(c*x)*ln(1+I*c*x)+3/2*I*a*b^2/d*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)
)*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((
1+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)^2-b^3/d*arctan(c*x)^3*ln((1+I*c*x)^2
/(c^2*x^2+1)-1)+b^3/d*arctan(c*x)^3*ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))+b^3/d
*arctan(c*x)^3*ln(1-(1+I*c*x)/(c^2*x^2+1)^(1/2))+6*b^3/d*arctan(c*x)*polylo
g(3,(1+I*c*x)/(c^2*x^2+1)^(1/2))+6*b^3/d*arctan(c*x)*polylog(3,-(1+I*c*x)/(
c^2*x^2+1)^(1/2))-I*a^3/d*arctan(c*x)+6*a*b^2/d*polylog(3,(1+I*c*x)/(c^2*x^
2+1)^(1/2))+6*a*b^2/d*polylog(3,-(1+I*c*x)/(c^2*x^2+1)^(1/2))+b^3/d*arctan(
c*x)^3*ln(c*x)+6*I*b^3/d*polylog(4,-(1+I*c*x)/(c^2*x^2+1)^(1/2))-1/2*I*b^3/
d*arctan(c*x)^4+6*I*b^3/d*polylog(4,(1+I*c*x)/(c^2*x^2+1)^(1/2))-3/2*I*a*b^
2/d*Pi*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn((1+I*c*x)^2/(c^2*x^2+1))*cs
gn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)^2-b^3/d
*arctan(c*x)^3*ln(c*x-I)+b^3/d*arctan(c*x)^3*ln(2*I*(1+I*c*x)^2/(c^2*x^2+1)
)-3*b*a^2/d*arctan(c*x)*ln(c*x-I)+3/2*I*b*a^2/d*dilog(1+I*c*x)-3/2*I*b*a^2/
d*dilog(1-I*c*x)+3/2*I*b*a^2/d*dilog(-1/2*I*(c*x+I))-3/4*I*b*a^2/d*ln(c*x-I
)^2

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-a^3\left(\frac{\log(icx+1)}{d}-\frac{\log(x)}{d}\right)+\frac{-64i\,b^3\arctan(cx)^4+64\,b^3\arctan(cx)^3\log\left(c^2x^2+1\right)+4i\,b^3\log\left(c^2x^2+1\right)^4-i\left(\frac{64\,b^3}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3/x/(d+I*c*d*x),x, algorithm="maxima")
```

```
[Out] -a^3*(log(I*c*x + 1)/d - log(x)/d) + 1/512*(-64*I*b^3*arctan(c*x)^4 + 64*b^
3*arctan(c*x)^3*log(c^2*x^2 + 1) + 16*b^3*arctan(c*x)*log(c^2*x^2 + 1)^3 +
4*I*b^3*log(c^2*x^2 + 1)^4 - I*(64*b^3*arctan(c*x)^4/d + 6144*b^3*c^2*integ
rate(1/64*x^2*arctan(c*x)^2*log(c^2*x^2 + 1)/(c^2*d*x^3 + d*x), x) + 3*b^3*
log(c^2*x^2 + 1)^4/d + 512*a*b^2*arctan(c*x)^3/d + 768*a^2*b*arctan(c*x)^2/
d + 6144*b^3*integrate(1/64*arctan(c*x)^2*log(c^2*x^2 + 1)/(c^2*d*x^3 + d*x
), x) - 512*b^3*integrate(1/64*log(c^2*x^2 + 1)^3/(c^2*d*x^3 + d*x), x))*d
- 512*d*integrate(1/32*(12*b^3*c*x*arctan(c*x)^2*log(c^2*x^2 + 1) + b^3*c*x
*log(c^2*x^2 + 1)^3 - 96*a*b^2*arctan(c*x)^2 - 96*a^2*b*arctan(c*x) + 4*(3*
b^3*c^2*x^2 - 7*b^3)*arctan(c*x)^3 + 3*(b^3*c^2*x^2 - b^3)*arctan(c*x)*log(
c^2*x^2 + 1)^2)/(c^2*d*x^3 + d*x), x))/d
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{b^3\log\left(-\frac{cx+i}{cx-i}\right)^3-6i\,ab^2\log\left(-\frac{cx+i}{cx-i}\right)^2-12\,a^2b\log\left(-\frac{cx+i}{cx-i}\right)+8i\,a^3}{8\left(cdx^2-i\,dx\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3/x/(d+I*c*d*x),x, algorithm="fricas")
```

```
[Out] integral(-1/8*(b^3*log(-(c*x + I)/(c*x - I))^3 - 6*I*a*b^2*log(-(c*x + I)/(
c*x - I))^2 - 12*a^2*b*log(-(c*x + I)/(c*x - I)) + 8*I*a^3)/(c*d*x^2 - I*d*
x), x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**3/x/(d+I*c*d*x),x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \arctan(cx) + a)^3}{(icdx + d)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3/x/(d+I*c*d*x),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)^3/((I*c*d*x + d)*x), x)
```

3.131 $\int \frac{\left(a+b\tan^{-1}(cx)\right)^3}{x^2(d+icdx)}\,dx$

Optimal. Leaf size=263

$$-\frac{3ib^2c\text{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)\left(a+b\tan^{-1}(cx)\right)}{d}-\frac{3ib^2c\text{PolyLog}\left(3,-1+\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{2d}+\frac{3bc\text{PolyLog}\left(2,\right.}{}$$

```
[Out] ((-I)*c*(a + b*ArcTan[c*x])^3)/d - (a + b*ArcTan[c*x])^3/(d*x) + (3*b*c*(a
+ b*ArcTan[c*x])^2*Log[2 - 2/(1 - I*c*x)])/d - (I*c*(a + b*ArcTan[c*x])^3*L
og[2 - 2/(1 + I*c*x)])/d - ((3*I)*b^2*c*(a + b*ArcTan[c*x])*PolyLog[2, -1 +
 2/(1 - I*c*x)])/d + (3*b*c*(a + b*ArcTan[c*x])^2*PolyLog[2, -1 + 2/(1 + I*
c*x)])/(2*d) + (3*b^3*c*PolyLog[3, -1 + 2/(1 - I*c*x)])/(2*d) - (((3*I)/2)*
b^2*c*(a + b*ArcTan[c*x])*PolyLog[3, -1 + 2/(1 + I*c*x)])/d - (3*b^3*c*Poly
Log[4, -1 + 2/(1 + I*c*x)])/(4*d)
```

Rubi [A] time = 0.60013, antiderivative size = 263, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {4870, 4852, 4924, 4868, 4884, 4992, 6610, 4994, 4998}

$$-\frac{3ib^2c\text{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)\left(a+b\tan^{-1}(cx)\right)}{d}-\frac{3ib^2c\text{PolyLog}\left(3,-1+\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{2d}+\frac{3bc\text{PolyLog}\left(2,\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^3/(x^2*(d + I*c*d*x)),x]
```


```
[Out] ((-I)*c*(a + b*ArcTan[c*x])^3)/d - (a + b*ArcTan[c*x])^3/(d*x) + (3*b*c*(a
+ b*ArcTan[c*x])^2*Log[2 - 2/(1 - I*c*x)])/d - (I*c*(a + b*ArcTan[c*x])^3*L
og[2 - 2/(1 + I*c*x)])/d - ((3*I)*b^2*c*(a + b*ArcTan[c*x])*PolyLog[2, -1 +
 2/(1 - I*c*x)])/d + (3*b*c*(a + b*ArcTan[c*x])^2*PolyLog[2, -1 + 2/(1 + I*
c*x)])/(2*d) + (3*b^3*c*PolyLog[3, -1 + 2/(1 - I*c*x)])/(2*d) - (((3*I)/2)*
b^2*c*(a + b*ArcTan[c*x])*PolyLog[3, -1 + 2/(1 + I*c*x)])/d - (3*b^3*c*Poly
Log[4, -1 + 2/(1 + I*c*x)])/(4*d)
```


Rule 4870

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x], x]
 - Dist[e/(d*f), Int[((f*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d + e*x), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0] &&
```

```
LtQ[m, -1]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4992

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d
] && EqQ[(1 - u)^2 - (1 - (2*I)/(I + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 4998

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[k + 1, u])/(2
*c*d), x] - Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[k + 1,
 u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
qQ[e, c^2*d] && EqQ[u^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}(cx)\right)^3}{x^2(d+icdx)}\,dx = -\left((ic)\int \frac{\left(a+b\tan^{-1}(cx)\right)^3}{x(d+icdx)}\,dx\right)+\frac{\int \frac{\left(a+b\tan^{-1}(cx)\right)^3}{x^2}\,dx}{d}$$

$$= -\frac{\left(a+b\tan^{-1}(cx)\right)^3}{dx}-\frac{ic\left(a+b\tan^{-1}(cx)\right)^3\log\left(2-\frac{2}{1+icx}\right)}{d}+\frac{(3bc)\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x\left(1+c^2x^2\right)}\,dx}{d}+\frac{(3$$

$$= -\frac{ic\left(a+b\tan^{-1}(cx)\right)^3}{d}-\frac{\left(a+b\tan^{-1}(cx)\right)^3}{dx}-\frac{ic\left(a+b\tan^{-1}(cx)\right)^3\log\left(2-\frac{2}{1+icx}\right)}{d}+\frac{3bc\,(a$$

$$= -\frac{ic\left(a+b\tan^{-1}(cx)\right)^3}{d}-\frac{\left(a+b\tan^{-1}(cx)\right)^3}{dx}+\frac{3bc\left(a+b\tan^{-1}(cx)\right)^2\log\left(2-\frac{2}{1-icx}\right)}{d}-\frac{ic\,(a$$

$$= -\frac{ic\left(a+b\tan^{-1}(cx)\right)^3}{d}-\frac{\left(a+b\tan^{-1}(cx)\right)^3}{dx}+\frac{3bc\left(a+b\tan^{-1}(cx)\right)^2\log\left(2-\frac{2}{1-icx}\right)}{d}-\frac{ic\,(a$$

$$= -\frac{ic\left(a+b\tan^{-1}(cx)\right)^3}{d}-\frac{\left(a+b\tan^{-1}(cx)\right)^3}{dx}+\frac{3bc\left(a+b\tan^{-1}(cx)\right)^2\log\left(2-\frac{2}{1-icx}\right)}{d}-\frac{ic\,(a$$

Mathematica [A] time = 1.38152, size = 436, normalized size = 1.66

$$-\frac{3a^2bc\left(\text{PolyLog}\left(2,e^{2i\tan^{-1}(cx)}\right)+2\left(-\log\left(\frac{cx}{\sqrt{c^2x^2+1}}\right)+\tan^{-1}(cx)^2+\tan^{-1}(cx)\left(\frac{1}{cx}+i\log\left(1-e^{2i\tan^{-1}(cx)}\right)\right)\right)\right)+6iab^2c$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c*x])^3/(x^2*(d + I*c*d*x)),x]
```

```
[Out] -((2*a^3)/x + 2*a^3*c*ArcTan[c*x] + (2*I)*a^3*c*Log[x] - I*a^3*c*Log[1 + c^
2*x^2] + 3*a^2*b*c*(2*(ArcTan[c*x]^2 + ArcTan[c*x]*(1/(c*x) + I*Log[1 - E^(
(2*I)*ArcTan[c*x])]) - Log[(c*x)/Sqrt[1 + c^2*x^2]]) + PolyLog[2, E^((2*I)*
ArcTan[c*x])]) + (6*I)*a*b^2*c*((-I/24)*Pi^3 + ArcTan[c*x]^2 - (I*ArcTan[c*
x]^2)/(c*x) + ArcTan[c*x]^2*Log[1 - E^((-2*I)*ArcTan[c*x])] + (2*I)*ArcTan[
c*x]*Log[1 - E^((2*I)*ArcTan[c*x])] + I*ArcTan[c*x]*PolyLog[2, E^((-2*I)*Ar
cTan[c*x])] + PolyLog[2, E^((2*I)*ArcTan[c*x])] + PolyLog[3, E^((-2*I)*ArcT
an[c*x])]/2) + (2*I)*b^3*c*(Pi^3/8 - (I/64)*Pi^4 - ArcTan[c*x]^3 - (I*ArcTa
n[c*x]^3)/(c*x) + (3*I)*ArcTan[c*x]^2*Log[1 - E^((-2*I)*ArcTan[c*x])] + Arc
Tan[c*x]^3*Log[1 - E^((-2*I)*ArcTan[c*x])] + ((3*I)/2)*ArcTan[c*x]*(2*I + A
rcTan[c*x])*PolyLog[2, E^((-2*I)*ArcTan[c*x])] + (3*(I + ArcTan[c*x])*PolyL
og[3, E^((-2*I)*ArcTan[c*x])])/2 - ((3*I)/4)*PolyLog[4, E^((-2*I)*ArcTan[c*
x])]))/(2*d)
```

Maple [C] time = 0.925, size = 11233, normalized size = 42.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^3/x^2/(d+I*c*d*x),x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3/x^2/(d+I*c*d*x),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{b^3\log\left(-\frac{cx+i}{cx-i}\right)^3-6i\,ab^2\log\left(-\frac{cx+i}{cx-i}\right)^2-12\,a^2b\log\left(-\frac{cx+i}{cx-i}\right)+8i\,a^3}{8\left(cdx^3-i\,dx^2\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3/x^2/(d+I*c*d*x),x, algorithm="fricas")
```

```
[Out] integral(-1/8*(b^3*log(-(c*x + I)/(c*x - I))^3 - 6*I*a*b^2*log(-(c*x + I)/(
c*x - I))^2 - 12*a^2*b*log(-(c*x + I)/(c*x - I)) + 8*I*a^3)/(c*d*x^3 - I*d*
x^2), x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**3/x**2/(d+I*c*d*x),x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(b\arctan(cx)+a)^3}{(i\,cdx+d)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3/x^2/(d+I*c*d*x),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)^3/((I*c*d*x + d)*x^2), x)
```

3.132 $\int \frac{\left(a+b\tan^{-1}(cx)\right)^3}{x^3(d+icdx)}\,dx$

Optimal. Leaf size=414

$$-\frac{3b^2c^2\text{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)\left(a+b\tan^{-1}(cx)\right)}{d}-\frac{3b^2c^2\text{PolyLog}\left(3,-1+\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{2d}-\frac{3ibc^2\text{PolyLog}\left(2,\right.}{}$$

```
[Out] (((-3*I)/2)*b*c^2*(a + b*ArcTan[c*x])^2)/d - (3*b*c*(a + b*ArcTan[c*x])^2)/
(2*d*x) - (3*c^2*(a + b*ArcTan[c*x])^3)/(2*d) - (a + b*ArcTan[c*x])^3/(2*d*
x^2) + (I*c*(a + b*ArcTan[c*x])^3)/(d*x) + (3*b^2*c^2*(a + b*ArcTan[c*x])*L
og[2 - 2/(1 - I*c*x)])/d - ((3*I)*b*c^2*(a + b*ArcTan[c*x])^2*Log[2 - 2/(1
- I*c*x)])/d - (c^2*(a + b*ArcTan[c*x])^3*Log[2 - 2/(1 + I*c*x)])/d - (((3*
I)/2)*b^3*c^2*PolyLog[2, -1 + 2/(1 - I*c*x)])/d - (3*b^2*c^2*(a + b*ArcTan[
c*x])*PolyLog[2, -1 + 2/(1 - I*c*x)])/d - (((3*I)/2)*b*c^2*(a + b*ArcTan[c*
x])^2*PolyLog[2, -1 + 2/(1 + I*c*x)])/d - (((3*I)/2)*b^3*c^2*PolyLog[3, -1
+ 2/(1 - I*c*x)])/d - (3*b^2*c^2*(a + b*ArcTan[c*x])*PolyLog[3, -1 + 2/(1 +
 I*c*x)])/(2*d) + (((3*I)/4)*b^3*c^2*PolyLog[4, -1 + 2/(1 + I*c*x)])/d
```

Rubi [A] time = 1.01954, antiderivative size = 414, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.44, Rules used = {4870, 4852, 4918, 4924, 4868, 2447, 4884, 4992, 6610, 4994, 4998}

$$-\frac{3b^2c^2\text{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)\left(a+b\tan^{-1}(cx)\right)}{d}-\frac{3b^2c^2\text{PolyLog}\left(3,-1+\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{2d}-\frac{3ibc^2\text{PolyLog}\left(2,\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^3/(x^3*(d + I*c*d*x)),x]
```


```
[Out] (((-3*I)/2)*b*c^2*(a + b*ArcTan[c*x])^2)/d - (3*b*c*(a + b*ArcTan[c*x])^2)/
(2*d*x) - (3*c^2*(a + b*ArcTan[c*x])^3)/(2*d) - (a + b*ArcTan[c*x])^3/(2*d*
x^2) + (I*c*(a + b*ArcTan[c*x])^3)/(d*x) + (3*b^2*c^2*(a + b*ArcTan[c*x])*L
og[2 - 2/(1 - I*c*x)])/d - ((3*I)*b*c^2*(a + b*ArcTan[c*x])^2*Log[2 - 2/(1
- I*c*x)])/d - (c^2*(a + b*ArcTan[c*x])^3*Log[2 - 2/(1 + I*c*x)])/d - (((3*
I)/2)*b^3*c^2*PolyLog[2, -1 + 2/(1 - I*c*x)])/d - (3*b^2*c^2*(a + b*ArcTan[
c*x])*PolyLog[2, -1 + 2/(1 - I*c*x)])/d - (((3*I)/2)*b*c^2*(a + b*ArcTan[c*
x])^2*PolyLog[2, -1 + 2/(1 + I*c*x)])/d - (((3*I)/2)*b^3*c^2*PolyLog[3, -1
+ 2/(1 - I*c*x)])/d - (3*b^2*c^2*(a + b*ArcTan[c*x])*PolyLog[3, -1 + 2/(1 +
 I*c*x)])/(2*d) + (((3*I)/4)*b^3*c^2*PolyLog[4, -1 + 2/(1 + I*c*x)])/d
```

Rule 4870

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x], x]
 - Dist[e/(d*f), Int[((f*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d + e*x), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0] &&
 LtQ[m, -1]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4918

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/(d + e*x^2),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4992

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d
] && EqQ[(1 - u)^2 - (1 - (2*I)/(I + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 4998

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[k + 1, u])/(2
*c*d), x] - Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[k + 1,
 u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
qQ[e, c^2*d] && EqQ[u^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}(cx)\right)^3}{x^3(d+icdx)}\,dx = -\left((ic)\int \frac{\left(a+b\tan^{-1}(cx)\right)^3}{x^2(d+icdx)}\,dx\right) + \frac{\int \frac{\left(a+b\tan^{-1}(cx)\right)^3}{x^3}\,dx}{d}$$

$$= -\frac{\left(a+b\tan^{-1}(cx)\right)^3}{2dx^2} - c^2\int \frac{\left(a+b\tan^{-1}(cx)\right)^3}{x(d+icdx)}\,dx - \frac{(ic)\int \frac{\left(a+b\tan^{-1}(cx)\right)^3}{x^2}\,dx}{d} + \frac{(3bc)\int \frac{(a+b\mathrm{t}}{x^2(}}{2c}$$

$$= -\frac{\left(a+b\tan^{-1}(cx)\right)^3}{2dx^2} + \frac{ic\left(a+b\tan^{-1}(cx)\right)^3}{dx} - \frac{c^2\left(a+b\tan^{-1}(cx)\right)^3\log\left(2-\frac{2}{1+icx}\right)}{d} + (3bc)$$

$$= -\frac{3bc\left(a+b\tan^{-1}(cx)\right)^2}{2dx} - \frac{3c^2\left(a+b\tan^{-1}(cx)\right)^3}{2d} - \frac{\left(a+b\tan^{-1}(cx)\right)^3}{2dx^2} + \frac{ic\left(a+b\tan^{-1}(cx\right.}{dx}$$

$$= -\frac{3ibc^2\left(a+b\tan^{-1}(cx)\right)^2}{2d} - \frac{3bc\left(a+b\tan^{-1}(cx)\right)^2}{2dx} - \frac{3c^2\left(a+b\tan^{-1}(cx)\right)^3}{2d} - \frac{\left(a+b\tan^{-1}\right.}{2dx^2}$$

$$= -\frac{3ibc^2\left(a+b\tan^{-1}(cx)\right)^2}{2d} - \frac{3bc\left(a+b\tan^{-1}(cx)\right)^2}{2dx} - \frac{3c^2\left(a+b\tan^{-1}(cx)\right)^3}{2d} - \frac{\left(a+b\tan^{-1}\right.}{2dx^2}$$

$$= -\frac{3ibc^2\left(a+b\tan^{-1}(cx)\right)^2}{2d} - \frac{3bc\left(a+b\tan^{-1}(cx)\right)^2}{2dx} - \frac{3c^2\left(a+b\tan^{-1}(cx)\right)^3}{2d} - \frac{\left(a+b\tan^{-1}\right.}{2dx^2}$$

Mathematica [A] time = 2.52415, size = 634, normalized size = 1.53

$$\frac{\frac{3ia^2b\left(c^2x^2\text{PolyLog}\left(2,e^{2i\tan^{-1}(cx)}\right)+cx\left(-2cx\log\left(\frac{cx}{\sqrt{c^2x^2+1}}\right)+i\right)+2c^2x^2\tan^{-1}(cx)^2+\tan^{-1}(cx)\left(ic^2x^2+2ic^2x^2\log\left(1-e^{2i\tan^{-1}(cx)}\right)+2cx+i\right)\right)}{x^2} + 6ab^2c^2\left(-i\,\mathrm{ta}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c*x])^3/(x^3*(d + I*c*d*x)),x]
```

```
[Out] (-(a^3/x^2) + ((2*I)*a^3*c)/x + (2*I)*a^3*c^2*ArcTan[c*x] - 2*a^3*c^2*Log[x
] + a^3*c^2*Log[1 + c^2*x^2] + ((3*I)*a^2*b*(2*c^2*x^2*ArcTan[c*x]^2 + ArcT
an[c*x]*(I + 2*c*x + I*c^2*x^2 + (2*I)*c^2*x^2*Log[1 - E^((2*I)*ArcTan[c*x]
)]) + c*x*(I - 2*c*x*Log[(c*x)/Sqrt[1 + c^2*x^2]]) + c^2*x^2*PolyLog[2, E^(
(2*I)*ArcTan[c*x])]))/x^2 + 6*a*b^2*c^2*((I/24)*Pi^3 - ArcTan[c*x]/(c*x) -
(3*ArcTan[c*x]^2)/2 - ArcTan[c*x]^2/(2*c^2*x^2) + (I*ArcTan[c*x]^2)/(c*x) -
 ArcTan[c*x]^2*Log[1 - E^((-2*I)*ArcTan[c*x])] - (2*I)*ArcTan[c*x]*Log[1 -
E^((2*I)*ArcTan[c*x])] + Log[(c*x)/Sqrt[1 + c^2*x^2]] - I*ArcTan[c*x]*PolyL
```

```
og[2, E^((-2*I)*ArcTan[c*x])] - PolyLog[2, E^((2*I)*ArcTan[c*x])] - PolyLog
[3, E^((-2*I)*ArcTan[c*x])]/2) + 2*b^3*c^2*(-Pi^3/8 + (I/64)*Pi^4 - ((3*I)/
2)*ArcTan[c*x]^2 - (3*ArcTan[c*x]^2)/(2*c*x) + ArcTan[c*x]^3 + (I*ArcTan[c*
x]^3)/(c*x) - ((1 + c^2*x^2)*ArcTan[c*x]^3)/(2*c^2*x^2) - (3*I)*ArcTan[c*x]
^2*Log[1 - E^((-2*I)*ArcTan[c*x])] - ArcTan[c*x]^3*Log[1 - E^((-2*I)*ArcTan
[c*x])] + 3*ArcTan[c*x]*Log[1 - E^((2*I)*ArcTan[c*x])] + (3*(2 - I*ArcTan[c
*x])*ArcTan[c*x]*PolyLog[2, E^((-2*I)*ArcTan[c*x])])/2 - ((3*I)/2)*PolyLog[
2, E^((2*I)*ArcTan[c*x])] - (3*(I + ArcTan[c*x])*PolyLog[3, E^((-2*I)*ArcTa
n[c*x])])/2 + ((3*I)/4)*PolyLog[4, E^((-2*I)*ArcTan[c*x])]))/(2*d)
```

Maple [C] time = 2.979, size = 3058, normalized size = 7.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^3/x^3/(d+I*c*d*x),x)
```

```
[Out] 3/2*I*c^2*a*b^2/d*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I*((1+I*c*x)^
2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2+3/2*I*c^2*a*b
^2/d*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csg
n(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2+
3/2*I*c^2*a*b^2/d*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^
2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2-3/2*I*c^2*a*b^2/d*Pi*csgn
(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(
c^2*x^2+1)+1))^2*arctan(c*x)^2-3/2*I*c^2*a*b^2/d*Pi*csgn(I*((1+I*c*x)^2/(c^
2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/(
(1+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)^2+3/2*I*c^2*a*b^2/d*Pi*csgn(I/((1+I
*c*x)^2/(c^2*x^2+1)+1))*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2
*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)^2-3/2*I*c^2*a*b^2/d*Pi*csg
n(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I
*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)^2+1/2
*c^2*a^3/d*ln(c^2*x^2+1)-3/2*c^2*b^3/d*arctan(c*x)^3-c^2*a^3/d*ln(c*x)-1/2*
b^3/d*arctan(c*x)^3/x^2+3/2*I*c^2*a*b^2/d*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/(
(1+I*c*x)^2/(c^2*x^2+1)+1))^3*arctan(c*x)^2+3*I*c^2*a*b^2/d*Pi*csgn((1+I*c*
x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2+3/2*I*c^2*a*b
^2/d*Pi*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arc
tan(c*x)^2-3/2*I*c^2*a*b^2/d*Pi*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)
^2/(c^2*x^2+1)+1))^3*arctan(c*x)^2-3/2*I*c^2*a*b^2/d*Pi*csgn(I*((1+I*c*x)^2
/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*arctan(c*x)^2+3/2*I*c^2*a*b^
2/d*Pi*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-
1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2+I*c*b^3/d*arctan(c*x)^3/x+3
```

```
*c^2*b*a^2/d*arctan(c*x)*ln(c*x-I)-3*I*c^2*a*b^2/d*arctan(c*x)+3/2*I*c^2*a^
2*b/d*ln(c^2*x^2+1)-3/2*I*c^2*b*a^2/d*dilog(-1/2*I*(c*x+I))+3/4*I*c^2*b*a^2
/d*ln(c*x-I)^2+2*I*c^2*a*b^2/d*arctan(c*x)^3-3*I*c^2*b^3/d*arctan(c*x)^2*ln
(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))+3*I*c^2*b^3/d*arctan(c*x)^2*polylog(2,-(1+I
*c*x)/(c^2*x^2+1)^(1/2))-3*I*c^2*b^3/d*arctan(c*x)^2*ln(1-(1+I*c*x)/(c^2*x^
2+1)^(1/2))+3*I*c^2*b^3/d*arctan(c*x)^2*polylog(2,(1+I*c*x)/(c^2*x^2+1)^(1/
2))-3*I*c^2*a^2*b/d*ln(c*x)-3/2*I*c^2*a^2*b/d*dilog(1+I*c*x)+3/2*I*c^2*a^2*
b/d*dilog(1-I*c*x)+3*c^2*a*b^2/d*arctan(c*x)^2*ln(c*x-I)-3*c^2*a*b^2/d*arct
an(c*x)^2*ln(2*I*(1+I*c*x)^2/(c^2*x^2+1))-3*c^2*a*b^2/d*arctan(c*x)^2*ln(c*
x)-3*c^2*a*b^2/d*arctan(c*x)^2*ln(1-(1+I*c*x)/(c^2*x^2+1)^(1/2))+3*c^2*a*b^
2/d*arctan(c*x)^2*ln((1+I*c*x)^2/(c^2*x^2+1)-1)-3*c^2*a*b^2/d*arctan(c*x)^2
*ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))-3*c^2*b*a^2/d*arctan(c*x)*ln(c*x)-3*c*a*
b^2/d*arctan(c*x)/x-3/2*I*c^2*b*a^2/d*ln(-1/2*I*(c*x+I))*ln(c*x-I)-6*I*c^2*
a*b^2/d*arctan(c*x)*ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))+6*I*c^2*a*b^2/d*arcta
n(c*x)*polylog(2,-(1+I*c*x)/(c^2*x^2+1)^(1/2))+6*I*c^2*a*b^2/d*arctan(c*x)*
polylog(2,(1+I*c*x)/(c^2*x^2+1)^(1/2))-3/2*I*c^2*a^2*b/d*ln(c*x)*ln(1+I*c*x
)+3/2*I*c^2*a^2*b/d*ln(c*x)*ln(1-I*c*x)+3*I*c*a*b^2/d*arctan(c*x)^2/x+3*I*c
*a^2*b/d*arctan(c*x)/x-9/2*I*c^2*a*b^2/d*Pi*arctan(c*x)^2-1/2*a^3/d/x^2-3/2
*c*a^2*b/d/x-3/2*a*b^2/d*arctan(c*x)^2/x^2-3/2*a^2*b/d*arctan(c*x)/x^2-3/2*
c*b^3/d*arctan(c*x)^2/x+3*c^2*a*b^2/d*ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))+6*c
^2*a*b^2/d*dilog((1+I*c*x)/(c^2*x^2+1)^(1/2))-6*c^2*a*b^2/d*dilog(1+(1+I*c*
x)/(c^2*x^2+1)^(1/2))+3*c^2*a*b^2/d*ln((1+I*c*x)/(c^2*x^2+1)^(1/2)-1)-c^2*b
^3/d*arctan(c*x)^3*ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))-c^2*b^3/d*arctan(c*x)^
3*ln(1-(1+I*c*x)/(c^2*x^2+1)^(1/2))-6*c^2*b^3/d*arctan(c*x)*polylog(3,(1+I*
c*x)/(c^2*x^2+1)^(1/2))-6*c^2*b^3/d*arctan(c*x)*polylog(3,-(1+I*c*x)/(c^2*x
^2+1)^(1/2))-6*c^2*b^3/d*arctan(c*x)*polylog(2,(1+I*c*x)/(c^2*x^2+1)^(1/2))
-6*c^2*b^3/d*arctan(c*x)*polylog(2,-(1+I*c*x)/(c^2*x^2+1)^(1/2))+3*c^2*b^3/
d*arctan(c*x)*ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))+3*c^2*b^3/d*arctan(c*x)*ln(
1-(1+I*c*x)/(c^2*x^2+1)^(1/2))-6*c^2*a*b^2/d*polylog(3,(1+I*c*x)/(c^2*x^2+1
)^(1/2))-6*c^2*a*b^2/d*polylog(3,-(1+I*c*x)/(c^2*x^2+1)^(1/2))-3/2*c^2*a^2*
b/d*arctan(c*x)-9/2*c^2*a*b^2/d*arctan(c*x)^2+I*c^2*a^3/d*arctan(c*x)+I*c*a
^3/d/x-6*I*c^2*b^3/d*polylog(4,-(1+I*c*x)/(c^2*x^2+1)^(1/2))-3*I*c^2*b^3/d*
polylog(2,-(1+I*c*x)/(c^2*x^2+1)^(1/2))-6*I*c^2*b^3/d*polylog(3,-(1+I*c*x)/
(c^2*x^2+1)^(1/2))+1/2*I*c^2*b^3/d*arctan(c*x)^4-6*I*c^2*b^3/d*polylog(4,(1
+I*c*x)/(c^2*x^2+1)^(1/2))-3/2*I*c^2*b^3/d*arctan(c*x)^2-6*I*c^2*b^3/d*poly
log(3,(1+I*c*x)/(c^2*x^2+1)^(1/2))-3*I*c^2*b^3/d*polylog(2,(1+I*c*x)/(c^2*x
^2+1)^(1/2))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3/x^3/(d+I*c*d*x),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{b^3 \log\left(-\frac{cx+i}{cx-i}\right)^3 - 6i\,ab^2 \log\left(-\frac{cx+i}{cx-i}\right)^2 - 12\,a^2 b \log\left(-\frac{cx+i}{cx-i}\right) + 8i\,a^3}{8\left(cdx^4 - i\,dx^3\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3/x^3/(d+I*c*d*x),x, algorithm="fricas")
```

```
[Out] integral(-1/8*(b^3*log(-(c*x + I)/(c*x - I))^3 - 6*I*a*b^2*log(-(c*x + I)/(
c*x - I))^2 - 12*a^2*b*log(-(c*x + I)/(c*x - I)) + 8*I*a^3)/(c*d*x^4 - I*d*
x^3), x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**3/x**3/(d+I*c*d*x),x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \arctan(cx) + a)^3}{(i\,cdx + d)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3/x^3/(d+I*c*d*x),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)^3/((I*c*d*x + d)*x^3), x)
```

3.133 $\int \frac{1}{(d+icdx)\left(a+b\tan^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{1}{(d+icdx)\left(a+b\tan^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable[1/((d + I*c*d*x)*(a + b*ArcTan[c*x])), x]
```

Rubi [A] time = 0.0373987, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{(d+icdx)\left(a+b\tan^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((d + I*c*d*x)*(a + b*ArcTan[c*x])),x]
```

```
[Out] Defer[Int][1/((d + I*c*d*x)*(a + b*ArcTan[c*x])), x]
```

Rubi steps

$$\int \frac{1}{(d+icdx)\left(a+b\tan^{-1}(cx)\right)}\,dx=\int \frac{1}{(d+icdx)\left(a+b\tan^{-1}(cx)\right)}\,dx$$

Mathematica [A] time = 3.12181, size = 0, normalized size = 0.

$$\int \frac{1}{(d+icdx)\left(a+b\tan^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((d + I*c*d*x)*(a + b*ArcTan[c*x])),x]
```

```
[Out] Integrate[1/((d + I*c*d*x)*(a + b*ArcTan[c*x])), x]
```

Maple [A] time = 0.671, size = 0, normalized size = 0.

$$\int \frac{1}{(d+icdx)(a+b\arctan(cx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d+I*c*d*x)/(a+b*arctan(c*x)),x)
```

```
[Out] int(1/(d+I*c*d*x)/(a+b*arctan(c*x)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(i\,cdx+d)(b\arctan(cx)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+I*c*d*x)/(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(1/((I*c*d*x + d)*(b*arctan(c*x) + a)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{2}{-2i\,acdx-2\,ad+(bcdx-i\,bd)\log\left(-\frac{cx+i}{cx-i}\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+I*c*d*x)/(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] integral(-2/(-2*I*a*c*d*x - 2*a*d + (b*c*d*x - I*b*d)*log(-(c*x + I)/(c*x -
 I))), x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+I*c*d*x)/(a+b*atan(c*x)),x)
```

```
[Out] Exception raised: AttributeError
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(icdx + d)(b\arctan(cx) + a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+I*c*d*x)/(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] integrate(1/((I*c*d*x + d)*(b*arctan(c*x) + a)), x)
```

3.134 $\int \frac{x^3(a+b\tan^{-1}(cx))}{d+ex}\,dx$

Optimal. Leaf size=297

$$-\frac{ibd^3\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2e^4}+\frac{ibd^3\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(1-icx)(cd+ie)}\right)}{2e^4}+\frac{d^3\log\left(\frac{2}{1-icx}\right)\left(a+b\tan^{-1}(cx)\right)}{e^4}-\frac{d^3\left(a+b\tan^{-1}(c\right.}{}$$

```
[Out] (a*d^2*x)/e^3 + (b*d*x)/(2*c*e^2) - (b*x^2)/(6*c*e) - (b*d*ArcTan[c*x])/(2*
c^2*e^2) + (b*d^2*x*ArcTan[c*x])/e^3 - (d*x^2*(a + b*ArcTan[c*x]))/(2*e^2)
+ (x^3*(a + b*ArcTan[c*x]))/(3*e) + (d^3*(a + b*ArcTan[c*x])*Log[2/(1 - I*c
*x)])/e^4 - (d^3*(a + b*ArcTan[c*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 -
I*c*x))])/e^4 - (b*d^2*Log[1 + c^2*x^2])/(2*c*e^3) + (b*Log[1 + c^2*x^2])/(
6*c^3*e) - ((I/2)*b*d^3*PolyLog[2, 1 - 2/(1 - I*c*x)])/e^4 + ((I/2)*b*d^3*P
olyLog[2, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e^4
```

Rubi [A] time = 0.270136, antiderivative size = 297, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 12, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.632, Rules used = {4876, 4846, 260, 4852, 321, 203, 266, 43, 4856, 2402, 2315, 2447}

$$-\frac{ibd^3\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2e^4}+\frac{ibd^3\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(1-icx)(cd+ie)}\right)}{2e^4}+\frac{d^3\log\left(\frac{2}{1-icx}\right)\left(a+b\tan^{-1}(cx)\right)}{e^4}-\frac{d^3\left(a+b\tan^{-1}(c\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcTan[c*x]))/(d + e*x),x]
```


```
[Out] (a*d^2*x)/e^3 + (b*d*x)/(2*c*e^2) - (b*x^2)/(6*c*e) - (b*d*ArcTan[c*x])/(2*
c^2*e^2) + (b*d^2*x*ArcTan[c*x])/e^3 - (d*x^2*(a + b*ArcTan[c*x]))/(2*e^2)
+ (x^3*(a + b*ArcTan[c*x]))/(3*e) + (d^3*(a + b*ArcTan[c*x])*Log[2/(1 - I*c
*x)])/e^4 - (d^3*(a + b*ArcTan[c*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 -
I*c*x))])/e^4 - (b*d^2*Log[1 + c^2*x^2])/(2*c*e^3) + (b*Log[1 + c^2*x^2])/(
6*c^3*e) - ((I/2)*b*d^3*PolyLog[2, 1 - 2/(1 - I*c*x)])/e^4 + ((I/2)*b*d^3*P
olyLog[2, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e^4
```


Rule 4876

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{x^3 \left(a+b\tan^{-1}(cx)\right)}{d+ex}\,dx = \int \left(\frac{d^2\left(a+b\tan^{-1}(cx)\right)}{e^3} - \frac{dx\left(a+b\tan^{-1}(cx)\right)}{e^2} + \frac{x^2\left(a+b\tan^{-1}(cx)\right)}{e} - \frac{d^3\left(a+b\tan^{-1}(c\right.}{e^3(d+ex)}\right.$$

$$= \frac{d^2\int\left(a+b\tan^{-1}(cx)\right)dx}{e^3} - \frac{d^3\int\frac{a+b\tan^{-1}(cx)}{d+ex}dx}{e^3} - \frac{d\int x\left(a+b\tan^{-1}(cx)\right)dx}{e^2} + \frac{\int x^2\left(a+b\right.}{}$$

$$= \frac{ad^2x}{e^3} - \frac{dx^2\left(a+b\tan^{-1}(cx)\right)}{2e^2} + \frac{x^3\left(a+b\tan^{-1}(cx)\right)}{3e} + \frac{d^3\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2}{1-icx}\right)}{e^4} - \frac{d}{}$$

$$= \frac{ad^2x}{e^3} + \frac{bdx}{2ce^2} + \frac{bd^2x\tan^{-1}(cx)}{e^3} - \frac{dx^2\left(a+b\tan^{-1}(cx)\right)}{2e^2} + \frac{x^3\left(a+b\tan^{-1}(cx)\right)}{3e} + \frac{d^3\left(a+b\right.}{}$$

$$= \frac{ad^2x}{e^3} + \frac{bdx}{2ce^2} - \frac{bd\tan^{-1}(cx)}{2c^2e^2} + \frac{bd^2x\tan^{-1}(cx)}{e^3} - \frac{dx^2\left(a+b\tan^{-1}(cx)\right)}{2e^2} + \frac{x^3\left(a+b\tan^{-1}(c\right.}{3e}$$

$$= \frac{ad^2x}{e^3} + \frac{bdx}{2ce^2} - \frac{bx^2}{6ce} - \frac{bd\tan^{-1}(cx)}{2c^2e^2} + \frac{bd^2x\tan^{-1}(cx)}{e^3} - \frac{dx^2\left(a+b\tan^{-1}(cx)\right)}{2e^2} + \frac{x^3\left(a+b\,\mathrm{ta}\right.}{3e}$$

Mathematica [A] time = 3.28679, size = 484, normalized size = 1.63

$$-\frac{-3ibd^3\mathrm{PolyLog}\left(2,e^{2i\left(\tan^{-1}\left(\frac{cd}{e}\right)+\tan^{-1}(cx)\right)}\right)+3ibd^3\mathrm{PolyLog}\left(2,-e^{2i\tan^{-1}(cx)}\right)-6ad^2ex+6ad^3\log(d+ex)+3ade^2x^2-2}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(a + b*ArcTan[c*x]))/(d + e*x),x]
```

```
[Out] -((b*e^3)/c^3 - 6*a*d^2*e*x - (3*b*d*e^2*x)/c + 3*a*d*e^2*x^2 + (b*e^3*x^2)
/c - 2*a*e^3*x^3 + (3*b*d*e^2*ArcTan[c*x])/c^2 + (3*I)*b*d^3*Pi*ArcTan[c*x]
 - 6*b*d^2*e*x*ArcTan[c*x] + 3*b*d*e^2*x^2*ArcTan[c*x] - 2*b*e^3*x^3*ArcTan
[c*x] - (6*I)*b*d^3*ArcTan[(c*d)/e]*ArcTan[c*x] + (3*I)*b*d^3*ArcTan[c*x]^2
 + (3*b*d^2*e*ArcTan[c*x]^2)/c - (3*b*d^2*Sqrt[1 + (c^2*d^2)/e^2]*e*E^(I*Ar
cTan[(c*d)/e])*ArcTan[c*x]^2)/c + 3*b*d^3*Pi*Log[1 + E^((-2*I)*ArcTan[c*x])
] - 6*b*d^3*ArcTan[c*x]*Log[1 + E^((2*I)*ArcTan[c*x])] + 6*b*d^3*ArcTan[(c*
d)/e]*Log[1 - E^((2*I)*(ArcTan[(c*d)/e] + ArcTan[c*x]))] + 6*b*d^3*ArcTan[c
*x]*Log[1 - E^((2*I)*(ArcTan[(c*d)/e] + ArcTan[c*x]))] + 6*a*d^3*Log[d + e*
x] + (3*b*d^2*e*Log[1 + c^2*x^2])/c - (b*e^3*Log[1 + c^2*x^2])/c^3 + (3*b*d
^3*Pi*Log[1 + c^2*x^2])/2 - 6*b*d^3*ArcTan[(c*d)/e]*Log[Sin[ArcTan[(c*d)/e]
 + ArcTan[c*x]]] + (3*I)*b*d^3*PolyLog[2, -E^((2*I)*ArcTan[c*x])] - (3*I)*b
*d^3*PolyLog[2, E^((2*I)*(ArcTan[(c*d)/e] + ArcTan[c*x]))])/(6*e^4)
```

Maple [A] time = 0.053, size = 394, normalized size = 1.3

$$\frac{x^3a}{3e}-\frac{adx^2}{2e^2}+\frac{ad^2x}{e^3}-\frac{ad^3\ln(ecx+dc)}{e^4}+\frac{bx^3\arctan(cx)}{3e}-\frac{\arctan(cx)\,bdx^2}{2e^2}+\frac{bd^2x\arctan(cx)}{e^3}-\frac{bd^3\arctan(cx)\ln}{e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctan(c*x))/(e*x+d),x)
```

```
[Out] 1/3*a/e*x^3-1/2*a/e^2*d*x^2+a*d^2*x/e^3-a*d^3/e^4*ln(c*e*x+c*d)+1/3*b*arcta
n(c*x)/e*x^3-1/2*b*arctan(c*x)/e^2*d*x^2+b*d^2*x*arctan(c*x)/e^3-b*arctan(c
*x)*d^3/e^4*ln(c*e*x+c*d)-1/2/c*b/e^3*ln(c^2*d^2-2*(c*e*x+c*d)*c*d+(c*e*x+c
*d)^2+e^2)*d^2-1/2*b*d*arctan(c*x)/c^2/e^2+1/6/c^3*b/e*ln(c^2*d^2-2*(c*e*x+
c*d)*c*d+(c*e*x+c*d)^2+e^2)+1/2*b*d*x/c/e^2+2/3/c*b*d^2/e^3-1/6*b*x^2/c/e-1
/2*I*b/e^4*d^3*dilog((I*e-e*c*x)/(d*c+I*e))+1/2*I*b/e^4*d^3*ln(c*e*x+c*d)*l
n((I*e+e*c*x)/(I*e-d*c))+1/2*I*b/e^4*d^3*dilog((I*e+e*c*x)/(I*e-d*c))-1/2*I
*b/e^4*d^3*ln(c*e*x+c*d)*ln((I*e-e*c*x)/(d*c+I*e))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{6}\,a\left(\frac{6\,d^3\log(ex+d)}{e^4}-\frac{2\,e^2x^3-3\,dex^2+6\,d^2x}{e^3}\right)+2\,b\int\frac{x^3\arctan(cx)}{2\,(ex+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))/(e*x+d),x, algorithm="maxima")
```

```
[Out] -1/6*a*(6*d^3*log(e*x + d)/e^4 - (2*e^2*x^3 - 3*d*e*x^2 + 6*d^2*x)/e^3) + 2
*b*integrate(1/2*x^3*arctan(c*x)/(e*x + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{bx^3\arctan(cx)+ax^3}{ex+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b*x^3*arctan(c*x) + a*x^3)/(e*x + d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atan(c*x))/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \arctan(cx) + a)x^3}{ex + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)*x^3/(e*x + d), x)
```

3.135 $\int \frac{x^2(a+b\tan^{-1}(cx))}{d+ex}\,dx$

Optimal. Leaf size=237

$$\frac{ibd^2\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2e^3}-\frac{ibd^2\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(1-icx)(cd+ie)}\right)}{2e^3}-\frac{d^2\log\left(\frac{2}{1-icx}\right)\left(a+b\tan^{-1}(cx)\right)}{e^3}+\frac{d^2\left(a+b\tan^{-1}(cx\right.}{e}$$

```
[Out] -((a*d*x)/e^2) - (b*x)/(2*c*e) + (b*ArcTan[c*x])/(2*c^2*e) - (b*d*x*ArcTan[
c*x])/e^2 + (x^2*(a + b*ArcTan[c*x]))/(2*e) - (d^2*(a + b*ArcTan[c*x])*Log[
2/(1 - I*c*x)])/e^3 + (d^2*(a + b*ArcTan[c*x])*Log[(2*c*(d + e*x))/((c*d +
I*e)*(1 - I*c*x))])/e^3 + (b*d*Log[1 + c^2*x^2])/(2*c*e^2) + ((I/2)*b*d^2*P
olyLog[2, 1 - 2/(1 - I*c*x)])/e^3 - ((I/2)*b*d^2*PolyLog[2, 1 - (2*c*(d + e
*x))/((c*d + I*e)*(1 - I*c*x))])/e^3
```

Rubi [A] time = 0.208499, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {4876, 4846, 260, 4852, 321, 203, 4856, 2402, 2315, 2447}

$$\frac{ibd^2\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2e^3}-\frac{ibd^2\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(1-icx)(cd+ie)}\right)}{2e^3}-\frac{d^2\log\left(\frac{2}{1-icx}\right)\left(a+b\tan^{-1}(cx)\right)}{e^3}+\frac{d^2\left(a+b\tan^{-1}(cx\right.}{e}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcTan[c*x]))/(d + e*x),x]
```

```
[Out] -((a*d*x)/e^2) - (b*x)/(2*c*e) + (b*ArcTan[c*x])/(2*c^2*e) - (b*d*x*ArcTan[
c*x])/e^2 + (x^2*(a + b*ArcTan[c*x]))/(2*e) - (d^2*(a + b*ArcTan[c*x])*Log[
2/(1 - I*c*x)])/e^3 + (d^2*(a + b*ArcTan[c*x])*Log[(2*c*(d + e*x))/((c*d +
I*e)*(1 - I*c*x))])/e^3 + (b*d*Log[1 + c^2*x^2])/(2*c*e^2) + ((I/2)*b*d^2*P
olyLog[2, 1 - 2/(1 - I*c*x)])/e^3 - ((I/2)*b*d^2*PolyLog[2, 1 - (2*c*(d + e
*x))/((c*d + I*e)*(1 - I*c*x))])/e^3
```

Rule 4876

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
```

```
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{x^2\left(a+b\tan^{-1}(cx)\right)}{d+ex}\,dx=\int\left(-\frac{d\left(a+b\tan^{-1}(cx)\right)}{e^2}+\frac{x\left(a+b\tan^{-1}(cx)\right)}{e}+\frac{d^2\left(a+b\tan^{-1}(cx)\right)}{e^2(d+ex)}\right)dx$$

$$=-\frac{d\int\left(a+b\tan^{-1}(cx)\right)dx}{e^2}+\frac{d^2\int\frac{a+b\tan^{-1}(cx)}{d+ex}\,dx}{e^2}+\frac{\int x\left(a+b\tan^{-1}(cx)\right)dx}{e}$$

$$=-\frac{adx}{e^2}+\frac{x^2\left(a+b\tan^{-1}(cx)\right)}{2e}-\frac{d^2\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2}{1-icx}\right)}{e^3}+\frac{d^2\left(a+b\tan^{-1}(cx)\right)\log}{e^3}$$

$$=-\frac{adx}{e^2}-\frac{bx}{2ce}-\frac{bdx\tan^{-1}(cx)}{e^2}+\frac{x^2\left(a+b\tan^{-1}(cx)\right)}{2e}-\frac{d^2\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2}{1-icx}\right)}{e^3}+$$

$$=-\frac{adx}{e^2}-\frac{bx}{2ce}+\frac{b\tan^{-1}(cx)}{2c^2e}-\frac{bdx\tan^{-1}(cx)}{e^2}+\frac{x^2\left(a+b\tan^{-1}(cx)\right)}{2e}-\frac{d^2\left(a+b\tan^{-1}(cx)\right)}{e^3}$$

Mathematica [A] time = 1.52991, size = 404, normalized size = 1.7

$$\frac{-ibd^2\text{PolyLog}\left(2,e^{2i\left(\tan^{-1}\left(\frac{cd}{e}\right)+\tan^{-1}(cx)\right)}\right)+ibd^2\text{PolyLog}\left(2,-e^{2i\tan^{-1}(cx)}\right)+2ad^2\log(d+ex)-2adex+ae^2x^2-\frac{bde\sqrt{\frac{c^2d}{e^2}}}{}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(a + b*ArcTan[c*x]))/(d + e*x),x]
```

```
[Out] (-2*a*d*e*x - (b*e^2*x)/c + a*e^2*x^2 + (b*e^2*ArcTan[c*x])/c^2 + I*b*d^2*P
i*ArcTan[c*x] - 2*b*d*e*x*ArcTan[c*x] + b*e^2*x^2*ArcTan[c*x] - (2*I)*b*d^2
*ArcTan[(c*d)/e]*ArcTan[c*x] + I*b*d^2*ArcTan[c*x]^2 + (b*d*e*ArcTan[c*x]^2
)/c - (b*d*Sqrt[1 + (c^2*d^2)/e^2]*e*E^(I*ArcTan[(c*d)/e])*ArcTan[c*x]^2)/c
 + b*d^2*Pi*Log[1 + E^((-2*I)*ArcTan[c*x])] - 2*b*d^2*ArcTan[c*x]*Log[1 + E
^((2*I)*ArcTan[c*x])] + 2*b*d^2*ArcTan[(c*d)/e]*Log[1 - E^((2*I)*(ArcTan[(c
*d)/e] + ArcTan[c*x]))] + 2*b*d^2*ArcTan[c*x]*Log[1 - E^((2*I)*(ArcTan[(c*d
)/e] + ArcTan[c*x]))] + 2*a*d^2*Log[d + e*x] + (b*d*e*Log[1 + c^2*x^2])/c +
 (b*d^2*Pi*Log[1 + c^2*x^2])/2 - 2*b*d^2*ArcTan[(c*d)/e]*Log[Sin[ArcTan[(c*
d)/e] + ArcTan[c*x]]] + I*b*d^2*PolyLog[2, -E^((2*I)*ArcTan[c*x])] - I*b*d^
2*PolyLog[2, E^((2*I)*(ArcTan[(c*d)/e] + ArcTan[c*x]))])/(2*e^3)
```

Maple [A] time = 0.05, size = 305, normalized size = 1.3

$$\frac{ax^2}{2\,e}-\frac{adx}{e^2}+\frac{ad^2\ln\left(ecx+dc\right)}{e^3}+\frac{bx^2\arctan\left(cx\right)}{2\,e}-\frac{bdx\arctan\left(cx\right)}{e^2}+\frac{bd^2\arctan\left(cx\right)\ln\left(ecx+dc\right)}{e^3}+\frac{\frac{i}{2}bd^2\ln\left(ecx+dc\right)}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctan(c*x))/(e*x+d),x)
```

```
[Out] 1/2*a*x^2/e-a*d*x/e^2+a*d^2/e^3*ln(c*e*x+c*d)+1/2*b*arctan(c*x)*x^2/e-b*d*x
*arctan(c*x)/e^2+b*arctan(c*x)*d^2/e^3*ln(c*e*x+c*d)+1/2*I*b/e^3*d^2*ln(c*e
*x+c*d)*ln((I*e-e*c*x)/(d*c+I*e))-1/2*I*b/e^3*d^2*ln(c*e*x+c*d)*ln((I*e+e*c
*x)/(I*e-d*c))+1/2*I*b/e^3*d^2*dilog((I*e-e*c*x)/(d*c+I*e))-1/2*I*b/e^3*d^2
*dilog((I*e+e*c*x)/(I*e-d*c))+1/2/c*b/e^2*d*ln(c^2*d^2-2*(c*e*x+c*d)*c*d+(c
*e*x+c*d)^2+e^2)+1/2*b*arctan(c*x)/c^2/e-1/2*b*x/c/e-1/2/c*b*d/e^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}\,a\left(\frac{2\,d^2\log\left(ex+d\right)}{e^3}+\frac{ex^2-2\,dx}{e^2}\right)+2\,b\int\frac{x^2\arctan\left(cx\right)}{2\left(ex+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))/(e*x+d),x, algorithm="maxima")
```

```
[Out] 1/2*a*(2*d^2*log(e*x + d)/e^3 + (e*x^2 - 2*d*x)/e^2) + 2*b*integrate(1/2*x^
2*arctan(c*x)/(e*x + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{bx^2 \arctan(cx) + ax^2}{ex + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b*x^2*arctan(c*x) + a*x^2)/(e*x + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 (a + b \operatorname{atan}(cx))}{d + ex}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atan(c*x))/(e*x+d),x)
```

```
[Out] Integral(x**2*(a + b*atan(c*x))/(d + e*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \arctan(cx) + a)x^2}{ex + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)*x^2/(e*x + d), x)
```

3.136 $\int \frac{x(a+b\tan^{-1}(cx))}{d+ex}\,dx$

Optimal. Leaf size=179

$$-\frac{ibd\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2e^2}+\frac{ibd\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(1-icx)(cd+ie)}\right)}{2e^2}+\frac{d\log\left(\frac{2}{1-icx}\right)\left(a+b\tan^{-1}(cx)\right)}{e^2}-\frac{d\left(a+b\tan^{-1}(cx)\right)\text{lo}}{e^2}$$

```
[Out] (a*x)/e + (b*x*ArcTan[c*x])/e + (d*(a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/
e^2 - (d*(a + b*ArcTan[c*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))]
)/e^2 - (b*Log[1 + c^2*x^2])/(2*c*e) - ((I/2)*b*d*PolyLog[2, 1 - 2/(1 - I*c
*x)])/e^2 + ((I/2)*b*d*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c
*x))])/e^2
```

Rubi [A] time = 0.158595, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {4876, 4846, 260, 4856, 2402, 2315, 2447}

$$-\frac{ibd\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2e^2}+\frac{ibd\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(1-icx)(cd+ie)}\right)}{2e^2}+\frac{d\log\left(\frac{2}{1-icx}\right)\left(a+b\tan^{-1}(cx)\right)}{e^2}-\frac{d\left(a+b\tan^{-1}(cx)\right)\text{lo}}{e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcTan[c*x]))/(d + e*x),x]
```


```
[Out] (a*x)/e + (b*x*ArcTan[c*x])/e + (d*(a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/
e^2 - (d*(a + b*ArcTan[c*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))]
)/e^2 - (b*Log[1 + c^2*x^2])/(2*c*e) - ((I/2)*b*d*PolyLog[2, 1 - 2/(1 - I*c
*x)])/e^2 + ((I/2)*b*d*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c
*x))])/e^2
```


Rule 4876

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
```

```
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{x\left(a+b\tan^{-1}(cx)\right)}{d+ex}\,dx = \int \left(\frac{a+b\tan^{-1}(cx)}{e} - \frac{d\left(a+b\tan^{-1}(cx)\right)}{e(d+ex)}\right) dx$$

$$= \frac{\int \left(a+b\tan^{-1}(cx)\right)dx}{e} - \frac{d\int \frac{a+b\tan^{-1}(cx)}{d+ex}\,dx}{e}$$

$$= \frac{ax}{e} + \frac{d\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2}{1-icx}\right)}{e^2} - \frac{d\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{e^2} - \frac{(bcd)\int \frac{\log\left(\frac{2}{1-i\cdots}\right)}{1+c^2x\cdots}}{e^2}$$

$$= \frac{ax}{e} + \frac{bx\tan^{-1}(cx)}{e} + \frac{d\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2}{1-icx}\right)}{e^2} - \frac{d\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{e^2}$$

$$= \frac{ax}{e} + \frac{bx\tan^{-1}(cx)}{e} + \frac{d\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2}{1-icx}\right)}{e^2} - \frac{d\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{e^2}$$

Mathematica [A] time = 1.45844, size = 329, normalized size = 1.84

$$\frac{b\left(icd\,\text{PolyLog}\left(2,e^{2i\left(\tan^{-1}\left(\frac{cd}{e}\right)+\tan^{-1}(cx)\right)}\right)-icd\,\text{PolyLog}\left(2,-e^{2i\tan^{-1}(cx)}\right)+e\sqrt{\frac{c^2d^2}{e^2}+1}\tan^{-1}(cx)^2e^{i\tan^{-1}\left(\frac{cd}{e}\right)}-\frac{1}{2}\pi cd\log\left(c^2x^2+1\right)-e\log\left(c^2x^2+1\right)+2icd\tan^{-1}(cx)\cdots\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(a + b*ArcTan[c*x]))/(d + e*x),x]
```

```
[Out] (2*a*e*x - 2*a*d*Log[d + e*x] + (b*((-I)*c*d*Pi*ArcTan[c*x] + 2*c*e*x*ArcTa
n[c*x] + (2*I)*c*d*ArcTan[(c*d)/e]*ArcTan[c*x] - I*c*d*ArcTan[c*x]^2 - e*Ar
cTan[c*x]^2 + Sqrt[1 + (c^2*d^2)/e^2]*e*E^(I*ArcTan[(c*d)/e])*ArcTan[c*x]^2
 - c*d*Pi*Log[1 + E^((-2*I)*ArcTan[c*x])] + 2*c*d*ArcTan[c*x]*Log[1 + E^((2
*I)*ArcTan[c*x])] - 2*c*d*ArcTan[(c*d)/e]*Log[1 - E^((2*I)*(ArcTan[(c*d)/e]
 + ArcTan[c*x]))] - 2*c*d*ArcTan[c*x]*Log[1 - E^((2*I)*(ArcTan[(c*d)/e] + A
rcTan[c*x]))] - e*Log[1 + c^2*x^2] - (c*d*Pi*Log[1 + c^2*x^2])/2 + 2*c*d*Ar
cTan[(c*d)/e]*Log[Sin[ArcTan[(c*d)/e] + ArcTan[c*x]]] - I*c*d*PolyLog[2, -E
^((2*I)*ArcTan[c*x])] + I*c*d*PolyLog[2, E^((2*I)*(ArcTan[(c*d)/e] + ArcTan
[c*x]))]))/c)/(2*e^2)
```

Maple [A] time = 0.045, size = 235, normalized size = 1.3

$$\frac{ax}{e} - \frac{ad\ln(ecx+dc)}{e^2} + \frac{bx\arctan(cx)}{e} - \frac{\arctan(cx)\,bd\ln(ecx+dc)}{e^2} - \frac{b\ln\left(c^2d^2-2\,(ecx+dc)\,cd+(ecx+dc)^2+e^2\right)}{2\,ce} - \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctan(c*x))/(e*x+d),x)
```

```
[Out] a*x/e-a*d/e^2*ln(c*e*x+c*d)+b*x*arctan(c*x)/e-b*arctan(c*x)*d/e^2*ln(c*e*x+
c*d)-1/2/c*b/e*ln(c^2*d^2-2*(c*e*x+c*d)*c*d+(c*e*x+c*d)^2+e^2)-1/2*I*b/e^2*
d*ln(c*e*x+c*d)*ln((I*e-e*c*x)/(d*c+I*e))+1/2*I*b/e^2*d*ln(c*e*x+c*d)*ln((I
*e+e*c*x)/(I*e-d*c))-1/2*I*b/e^2*d*dilog((I*e-e*c*x)/(d*c+I*e))+1/2*I*b/e^2
*d*dilog((I*e+e*c*x)/(I*e-d*c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$a\left(\frac{x}{e} - \frac{d\log\left(ex + d\right)}{e^2}\right) + 2\,b\int \frac{x\arctan\left(cx\right)}{2\left(ex + d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))/(e*x+d),x, algorithm="maxima")
```

```
[Out] a*(x/e - d*log(e*x + d)/e^2) + 2*b*integrate(1/2*x*arctan(c*x)/(e*x + d), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{bx\arctan\left(cx\right) + ax}{ex + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b*x*arctan(c*x) + a*x)/(e*x + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(a + b\operatorname{atan}\left(cx\right)\right)}{d + ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atan(c*x))/(e*x+d),x)
```

```
[Out] Integral(x*(a + b*atan(c*x))/(d + e*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \arctan(cx) + a)x}{ex + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)*x/(e*x + d), x)
```

3.137 $\int \frac{a+b\tan^{-1}(cx)}{d+ex}\,dx$

Optimal. Leaf size=138

$$-\frac{ib\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(1-icx)(cd+ie)}\right)}{2e}+\frac{ib\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2e}+\frac{\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2c(d+ex)}{(1-icx)(cd+ie)}\right)}{e}-\frac{\log\left(\frac{2}{1-icx}\right)\left(a+b\right.}{e}$$

```
[Out] -(((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e) + ((a + b*ArcTan[c*x])*Log[(2
*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e + ((I/2)*b*PolyLog[2, 1 - 2/(1
- I*c*x)])/e - ((I/2)*b*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*
c*x))])/e
```

Rubi [A] time = 0.0745539, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4856, 2402, 2315, 2447}

$$-\frac{ib\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(1-icx)(cd+ie)}\right)}{2e}+\frac{ib\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2e}+\frac{\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2c(d+ex)}{(1-icx)(cd+ie)}\right)}{e}-\frac{\log\left(\frac{2}{1-icx}\right)\left(a+b\right.}{e}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])/(d + e*x),x]
```

```
[Out] -(((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e) + ((a + b*ArcTan[c*x])*Log[(2
*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e + ((I/2)*b*PolyLog[2, 1 - 2/(1
- I*c*x)])/e - ((I/2)*b*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*
c*x))])/e
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
```

```
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{a+b\tan^{-1}(cx)}{d+ex}\,dx = -\frac{\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2}{1-icx}\right)}{e}+\frac{\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{e}+\frac{(bc)\int\frac{\log\left(\frac{2}{1-icx}\right)}{1+c^2x^2}\,dx}{e}-\frac{(bc)}{}$$
$$= -\frac{\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2}{1-icx}\right)}{e}+\frac{\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{e}-\frac{ib\text{Li}_2\left(1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2e}+$$
$$= -\frac{\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2}{1-icx}\right)}{e}+\frac{\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{e}+\frac{ib\text{Li}_2\left(1-\frac{2}{1-icx}\right)}{2e}-\frac{ib\text{Li}_2}{}$$

Mathematica [A] time = 0.0559885, size = 138, normalized size = 1.

$$\frac{ib\text{PolyLog}\left(2,\frac{e(1-icx)}{e+icd}\right)-ib\text{PolyLog}\left(2,-\frac{e(cx-i)}{cd+ie}\right)+2a\log(d+ex)+ib\log(1-icx)\log\left(\frac{c(d+ex)}{cd-ie}\right)-ib\log(1+icx)\log\left(\frac{c(d+}{cd+}\right.}{2e}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c*x])/(d + e*x),x]
```

```
[Out] (2*a*Log[d + e*x] + I*b*Log[1 - I*c*x]*Log[(c*(d + e*x))/(c*d - I*e)] - I*b
*Log[1 + I*c*x]*Log[(c*(d + e*x))/(c*d + I*e)] + I*b*PolyLog[2, (e*(1 - I*c
*x))/(I*c*d + e)] - I*b*PolyLog[2, -((e*(-I + c*x))/(c*d + I*e))])/(2*e)
```

Maple [A] time = 0.038, size = 168, normalized size = 1.2

$$\frac{a\ln(ecx+dc)}{e}+\frac{b\ln(ecx+dc)\arctan(cx)}{e}+\frac{\frac{i}{2}b\ln(ecx+dc)}{e}\ln\left(\frac{ie-ecx}{dc+ie}\right)-\frac{\frac{i}{2}b\ln(ecx+dc)}{e}\ln\left(\frac{ie+ecx}{ie-dc}\right)+\frac{\frac{i}{2}b}{e}\mathrm{dilog}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/(e*x+d),x)
```

```
[Out] a*ln(c*e*x+c*d)/e+b*ln(c*e*x+c*d)/e*arctan(c*x)+1/2*I*b*ln(c*e*x+c*d)/e*ln(
(I*e-e*c*x)/(d*c+I*e))-1/2*I*b*ln(c*e*x+c*d)/e*ln((I*e+e*c*x)/(I*e-d*c))+1/
2*I*b/e*dilog((I*e-e*c*x)/(d*c+I*e))-1/2*I*b/e*dilog((I*e+e*c*x)/(I*e-d*c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$2b\int\frac{\arctan(cx)}{2(ex+d)}\,dx+\frac{a\log(ex+d)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/(e*x+d),x, algorithm="maxima")
```

```
[Out] 2*b*integrate(1/2*arctan(c*x)/(e*x + d), x) + a*log(e*x + d)/e
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b\arctan(cx)+a}{ex+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b*arctan(c*x) + a)/(e*x + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{a+b\,\mathrm{atan}(cx)}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/(e*x+d),x)
```

```
[Out] Integral((a + b*atan(c*x))/(d + e*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \arctan(cx) + a}{ex + d} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)/(e*x + d), x)
```

3.138 $\int \frac{a+b\tan^{-1}(cx)}{x(d+ex)}\,dx$

Optimal. Leaf size=181

$$\frac{ib\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(1-icx)(cd+ie)}\right)}{2d}+\frac{ib\text{PolyLog}(2,-icx)}{2d}-\frac{ib\text{PolyLog}(2,icx)}{2d}-\frac{ib\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2d}-\frac{\left(a+b\tan^{-1}(\right.}{}$$

```
[Out] (a*Log[x])/d + ((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/d - ((a + b*ArcTan[
c*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/d + ((I/2)*b*PolyLog[
2, (-I)*c*x])/d - ((I/2)*b*PolyLog[2, I*c*x])/d - ((I/2)*b*PolyLog[2, 1 - 2
/(1 - I*c*x)])/d + ((I/2)*b*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1
- I*c*x))])/d
```

Rubi [A] time = 0.186101, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {4876, 4848, 2391, 4856, 2402, 2315, 2447}

$$\frac{ib\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(1-icx)(cd+ie)}\right)}{2d}+\frac{ib\text{PolyLog}(2,-icx)}{2d}-\frac{ib\text{PolyLog}(2,icx)}{2d}-\frac{ib\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2d}-\frac{\left(a+b\tan^{-1}(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])/(x*(d + e*x)),x]
```


```
[Out] (a*Log[x])/d + ((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/d - ((a + b*ArcTan[
c*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/d + ((I/2)*b*PolyLog[
2, (-I)*c*x])/d - ((I/2)*b*PolyLog[2, I*c*x])/d - ((I/2)*b*PolyLog[2, 1 - 2
/(1 - I*c*x)])/d + ((I/2)*b*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1
- I*c*x))])/d
```


Rule 4876

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
```

```
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{a+b\tan^{-1}(cx)}{x(d+ex)}\,dx = \int \left(\frac{a+b\tan^{-1}(cx)}{dx} - \frac{e\left(a+b\tan^{-1}(cx)\right)}{d(d+ex)}\right) dx$$
$$= \frac{\int \frac{a+b\tan^{-1}(cx)}{x}\,dx}{d} - \frac{e\int \frac{a+b\tan^{-1}(cx)}{d+ex}\,dx}{d}$$
$$= \frac{a\log(x)}{d} + \frac{\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2}{1-icx}\right)}{d} - \frac{\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{d} + \frac{(ib)\int \frac{\log(1-icx}{x}}{2d}$$
$$= \frac{a\log(x)}{d} + \frac{\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2}{1-icx}\right)}{d} - \frac{\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{d} + \frac{ib\mathrm{Li}_2(-icx)}{2d} -$$
$$= \frac{a\log(x)}{d} + \frac{\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2}{1-icx}\right)}{d} - \frac{\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{d} + \frac{ib\mathrm{Li}_2(-icx)}{2d} -$$

Mathematica [A] time = 0.0950062, size = 169, normalized size = 0.93

$$\frac{-ib\text{PolyLog}\left(2,\frac{e(1-icx)}{e+icd}\right) + ib\text{PolyLog}\left(2,-\frac{e(cx-i)}{cd+ie}\right) + ib\text{PolyLog}(2,-icx) - ib\text{PolyLog}(2,icx) - 2a\log(d+ex) + 2a\log}{2d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c*x])/(x*(d + e*x)),x]
```

```
[Out] (2*a*Log[x] - 2*a*Log[d + e*x] - I*b*Log[1 - I*c*x]*Log[(c*(d + e*x))/(c*d
- I*e)] + I*b*Log[1 + I*c*x]*Log[(c*(d + e*x))/(c*d + I*e)] + I*b*PolyLog[2
, (-I)*c*x] - I*b*PolyLog[2, I*c*x] - I*b*PolyLog[2, (e*(1 - I*c*x))/(I*c*d
 + e)] + I*b*PolyLog[2, -((e*(-I + c*x))/(c*d + I*e))])/(2*d)
```

Maple [A] time = 0.051, size = 260, normalized size = 1.4

$$-\frac{a\ln(ecx+dc)}{d} + \frac{a\ln(cx)}{d} - \frac{b\arctan(cx)\ln(ecx+dc)}{d} + \frac{b\arctan(cx)\ln(cx)}{d} + \frac{\frac{i}{2}b\ln(cx)\ln(1+icx)}{d} - \frac{\frac{i}{2}b\ln(cx)\ln}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/x/(e*x+d),x)
```

```
[Out] -a/d*ln(c*e*x+c*d)+a/d*ln(c*x)-b*arctan(c*x)/d*ln(c*e*x+c*d)+b/d*arctan(c*x
)*ln(c*x)+1/2*I*b/d*ln(c*x)*ln(1+I*c*x)-1/2*I*b/d*ln(c*x)*ln(1-I*c*x)+1/2*I
*b/d*dilog(1+I*c*x)-1/2*I*b/d*dilog(1-I*c*x)-1/2*I*b/d*ln(c*e*x+c*d)*ln((I*
e-e*c*x)/(d*c+I*e))+1/2*I*b/d*ln(c*e*x+c*d)*ln((I*e+e*c*x)/(I*e-d*c))-1/2*I
*b/d*dilog((I*e-e*c*x)/(d*c+I*e))+1/2*I*b/d*dilog((I*e+e*c*x)/(I*e-d*c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-a\left(\frac{\log\left(ex+d\right)}{d}-\frac{\log\left(x\right)}{d}\right)+2\,b\int\frac{\arctan\left(cx\right)}{2\left(ex^2+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x/(e*x+d),x, algorithm="maxima")
```

```
[Out] -a*(log(e*x + d)/d - log(x)/d) + 2*b*integrate(1/2*arctan(c*x)/(e*x^2 + d*x
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b\arctan\left(cx\right)+a}{ex^2+dx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b*arctan(c*x) + a)/(e*x^2 + d*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/x/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \arctan(cx) + a}{(ex + d)x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)/((e*x + d)*x), x)
```

3.139 $\int \frac{a+b\tan^{-1}(cx)}{x^2(d+ex)}\,dx$

Optimal. Leaf size=232

$$-\frac{ibe\text{PolyLog}(2,-icx)}{2d^2}+\frac{ibe\text{PolyLog}(2,icx)}{2d^2}+\frac{ibe\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2d^2}-\frac{ibe\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(1-icx)(cd+ie)}\right)}{2d^2}-\frac{e\log\left(\frac{2}{1-ic}\right.}{}$$

```
[Out] -((a + b*ArcTan[c*x])/(d*x)) + (b*c*Log[x])/d - (a*e*Log[x])/d^2 - (e*(a +
b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/d^2 + (e*(a + b*ArcTan[c*x])*Log[(2*c*(d
 + e*x))/((c*d + I*e)*(1 - I*c*x))])/d^2 - (b*c*Log[1 + c^2*x^2])/(2*d) - (
(I/2)*b*e*PolyLog[2, (-I)*c*x])/d^2 + ((I/2)*b*e*PolyLog[2, I*c*x])/d^2 + (
(I/2)*b*e*PolyLog[2, 1 - 2/(1 - I*c*x)])/d^2 - ((I/2)*b*e*PolyLog[2, 1 - (2
*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/d^2
```

Rubi [A] time = 0.241335, antiderivative size = 232, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 12, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.632, Rules used = {4876, 4852, 266, 36, 29, 31, 4848, 2391, 4856, 2402, 2315, 2447}

$$-\frac{ibe\text{PolyLog}(2,-icx)}{2d^2}+\frac{ibe\text{PolyLog}(2,icx)}{2d^2}+\frac{ibe\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2d^2}-\frac{ibe\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(1-icx)(cd+ie)}\right)}{2d^2}-\frac{e\log\left(\frac{2}{1-ic}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])/(x^2*(d + e*x)),x]
```

```
[Out] -((a + b*ArcTan[c*x])/(d*x)) + (b*c*Log[x])/d - (a*e*Log[x])/d^2 - (e*(a +
b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/d^2 + (e*(a + b*ArcTan[c*x])*Log[(2*c*(d
 + e*x))/((c*d + I*e)*(1 - I*c*x))])/d^2 - (b*c*Log[1 + c^2*x^2])/(2*d) - (
(I/2)*b*e*PolyLog[2, (-I)*c*x])/d^2 + ((I/2)*b*e*PolyLog[2, I*c*x])/d^2 + (
(I/2)*b*e*PolyLog[2, 1 - 2/(1 - I*c*x)])/d^2 - ((I/2)*b*e*PolyLog[2, 1 - (2
*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/d^2
```

Rule 4876

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
```

```
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{a+b\tan^{-1}(cx)}{x^2(d+ex)}\,dx = \int \left(\frac{a+b\tan^{-1}(cx)}{dx^2} - \frac{e\left(a+b\tan^{-1}(cx)\right)}{d^2x} + \frac{e^2\left(a+b\tan^{-1}(cx)\right)}{d^2(d+ex)}\right) dx$$

$$= \frac{\int \frac{a+b\tan^{-1}(cx)}{x^2}\,dx}{d} - \frac{e\int \frac{a+b\tan^{-1}(cx)}{x}\,dx}{d^2} + \frac{e^2\int \frac{a+b\tan^{-1}(cx)}{d+ex}\,dx}{d^2}$$

$$= -\frac{a+b\tan^{-1}(cx)}{dx} - \frac{ae\log(x)}{d^2} - \frac{e\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2}{1-icx}\right)}{d^2} + \frac{e\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2c(d+}{(cd+ie)(1}\right.}{d^2}$$

$$= -\frac{a+b\tan^{-1}(cx)}{dx} - \frac{ae\log(x)}{d^2} - \frac{e\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2}{1-icx}\right)}{d^2} + \frac{e\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2c(d+}{(cd+ie)(1}\right.}{d^2}$$

$$= -\frac{a+b\tan^{-1}(cx)}{dx} - \frac{ae\log(x)}{d^2} - \frac{e\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2}{1-icx}\right)}{d^2} + \frac{e\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2c(d+}{(cd+ie)(1}\right.}{d^2}$$

$$= -\frac{a+b\tan^{-1}(cx)}{dx} + \frac{bc\log(x)}{d} - \frac{ae\log(x)}{d^2} - \frac{e\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2}{1-icx}\right)}{d^2} + \frac{e\left(a+b\tan^{-1}(cx)\right)}{d^2}$$

Mathematica [A] time = 0.141304, size = 223, normalized size = 0.96

$$-\frac{-ibex\text{PolyLog}\left(2,\frac{e(1-icx)}{e+icd}\right)+ibex\text{PolyLog}\left(2,-\frac{e(cx-i)}{cd+ie}\right)+ibex\text{PolyLog}(2,-icx)-ibex\text{PolyLog}(2,icx)-2aex\log(d+}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c*x])/(x^2*(d + e*x)),x]
```

```
[Out] -(2*a*d + 2*b*d*ArcTan[c*x] - 2*b*c*d*x*Log[x] + 2*a*e*x*Log[x] - 2*a*e*x*L
og[d + e*x] - I*b*e*x*Log[1 - I*c*x]*Log[(c*(d + e*x))/(c*d - I*e)] + I*b*e
*x*Log[1 + I*c*x]*Log[(c*(d + e*x))/(c*d + I*e)] + b*c*d*x*Log[1 + c^2*x^2]
 + I*b*e*x*PolyLog[2, (-I)*c*x] - I*b*e*x*PolyLog[2, I*c*x] - I*b*e*x*PolyL
og[2, (e*(1 - I*c*x))/(I*c*d + e)] + I*b*e*x*PolyLog[2, -((e*(-I + c*x))/(c
*d + I*e))])/(2*d^2*x)
```

Maple [A] time = 0.07, size = 321, normalized size = 1.4

$$\frac{ae\ln(ecx+dc)}{d^2}-\frac{a}{dx}-\frac{ae\ln(cx)}{d^2}+\frac{b\arctan(cx)e\ln(ecx+dc)}{d^2}-\frac{b\arctan(cx)}{dx}-\frac{b\arctan(cx)e\ln(cx)}{d^2}-\frac{\frac{i}{2}be\text{dilog}(1}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/x^2/(e*x+d),x)
```

```
[Out] a/d^2*e*ln(c*e*x+c*d)-a/d/x-a/d^2*e*ln(c*x)+b*arctan(c*x)/d^2*e*ln(c*e*x+c*
d)-b*arctan(c*x)/d/x-b*arctan(c*x)/d^2*e*ln(c*x)-1/2*I*b/d^2*e*dilog(1+I*c*
x)+1/2*I*b/d^2*e*dilog(1-I*c*x)+1/2*I*b/d^2*e*ln(c*x)*ln(1-I*c*x)-1/2*I*b/d
^2*e*dilog((I*e+e*c*x)/(I*e-d*c))-1/2*b*c*ln(c^2*x^2+1)/d+c*b/d*ln(c*x)+1/2
*I*b/d^2*e*ln(c*e*x+c*d)*ln((I*e-e*c*x)/(d*c+I*e))+1/2*I*b/d^2*e*dilog((I*e
-e*c*x)/(d*c+I*e))-1/2*I*b/d^2*e*ln(c*e*x+c*d)*ln((I*e+e*c*x)/(I*e-d*c))-1/
2*I*b/d^2*e*ln(c*x)*ln(1+I*c*x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$a\left(\frac{e\log(ex+d)}{d^2}-\frac{e\log(x)}{d^2}-\frac{1}{dx}\right)+2\,b\int\frac{\arctan(cx)}{2\left(ex^3+dx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^2/(e*x+d),x, algorithm="maxima")
```

```
[Out] a*(e*log(e*x + d)/d^2 - e*log(x)/d^2 - 1/(d*x)) + 2*b*integrate(1/2*arctan(
c*x)/(e*x^3 + d*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b\arctan(cx)+a}{ex^3+dx^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^2/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b*arctan(c*x) + a)/(e*x^3 + d*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/x**2/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\arctan(cx)+a}{(ex+d)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^2/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)/((e*x + d)*x^2), x)
```

3.140 $\int \frac{a+b\tan^{-1}(cx)}{x^3(d+ex)}\,dx$

Optimal. Leaf size=293

$$\frac{ibe^2\text{PolyLog}(2,-icx)}{2d^3}-\frac{ibe^2\text{PolyLog}(2,icx)}{2d^3}-\frac{ibe^2\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2d^3}+\frac{ibe^2\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(1-icx)(cd+ie)}\right)}{2d^3}+\frac{e^2\log}{}$$

```
[Out] -(b*c)/(2*d*x) - (b*c^2*ArcTan[c*x])/(2*d) - (a + b*ArcTan[c*x])/(2*d*x^2)
+ (e*(a + b*ArcTan[c*x]))/(d^2*x) - (b*c*e*Log[x])/d^2 + (a*e^2*Log[x])/d^3
 + (e^2*(a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/d^3 - (e^2*(a + b*ArcTan[c*
x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/d^3 + (b*c*e*Log[1 + c^
2*x^2])/(2*d^2) + ((I/2)*b*e^2*PolyLog[2, (-I)*c*x])/d^3 - ((I/2)*b*e^2*Pol
yLog[2, I*c*x])/d^3 - ((I/2)*b*e^2*PolyLog[2, 1 - 2/(1 - I*c*x)])/d^3 + ((I
/2)*b*e^2*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/d^3
```

Rubi [A] time = 0.284429, antiderivative size = 293, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 14, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.737, Rules used = {4876, 4852, 325, 203, 266, 36, 29, 31, 4848, 2391, 4856, 2402, 2315, 2447}

$$\frac{ibe^2\text{PolyLog}(2,-icx)}{2d^3}-\frac{ibe^2\text{PolyLog}(2,icx)}{2d^3}-\frac{ibe^2\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2d^3}+\frac{ibe^2\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(1-icx)(cd+ie)}\right)}{2d^3}+\frac{e^2\log}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])/(x^3*(d + e*x)),x]
```


```
[Out] -(b*c)/(2*d*x) - (b*c^2*ArcTan[c*x])/(2*d) - (a + b*ArcTan[c*x])/(2*d*x^2)
+ (e*(a + b*ArcTan[c*x]))/(d^2*x) - (b*c*e*Log[x])/d^2 + (a*e^2*Log[x])/d^3
 + (e^2*(a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/d^3 - (e^2*(a + b*ArcTan[c*
x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/d^3 + (b*c*e*Log[1 + c^
2*x^2])/(2*d^2) + ((I/2)*b*e^2*PolyLog[2, (-I)*c*x])/d^3 - ((I/2)*b*e^2*Pol
yLog[2, I*c*x])/d^3 - ((I/2)*b*e^2*PolyLog[2, 1 - 2/(1 - I*c*x)])/d^3 + ((I
/2)*b*e^2*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/d^3
```


Rule 4876

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
```

```
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{a+b\tan^{-1}(cx)}{x^3(d+ex)}\,dx = \int \left(\frac{a+b\tan^{-1}(cx)}{dx^3} - \frac{e\left(a+b\tan^{-1}(cx)\right)}{d^2x^2} + \frac{e^2\left(a+b\tan^{-1}(cx)\right)}{d^3x} - \frac{e^3\left(a+b\tan^{-1}(cx)\right)}{d^3(d+ex)}\right)dx$$

$$= \frac{\int \frac{a+b\tan^{-1}(cx)}{x^3}\,dx}{d} - \frac{e\int \frac{a+b\tan^{-1}(cx)}{x^2}\,dx}{d^2} + \frac{e^2\int \frac{a+b\tan^{-1}(cx)}{x}\,dx}{d^3} - \frac{e^3\int \frac{a+b\tan^{-1}(cx)}{d+ex}\,dx}{d^3}$$

$$= -\frac{a+b\tan^{-1}(cx)}{2dx^2} + \frac{e\left(a+b\tan^{-1}(cx)\right)}{d^2x} + \frac{ae^2\log(x)}{d^3} + \frac{e^2\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2}{1-icx}\right)}{d^3} - \frac{e^2\left(a+\right.}{}$$

$$= -\frac{bc}{2dx} - \frac{a+b\tan^{-1}(cx)}{2dx^2} + \frac{e\left(a+b\tan^{-1}(cx)\right)}{d^2x} + \frac{ae^2\log(x)}{d^3} + \frac{e^2\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2}{1-icx}\right)}{d^3} -$$

$$= -\frac{bc}{2dx} - \frac{bc^2\tan^{-1}(cx)}{2d} - \frac{a+b\tan^{-1}(cx)}{2dx^2} + \frac{e\left(a+b\tan^{-1}(cx)\right)}{d^2x} + \frac{ae^2\log(x)}{d^3} + \frac{e^2\left(a+b\tan^{-1}(c\right.}{d^3}$$

$$= -\frac{bc}{2dx} - \frac{bc^2\tan^{-1}(cx)}{2d} - \frac{a+b\tan^{-1}(cx)}{2dx^2} + \frac{e\left(a+b\tan^{-1}(cx)\right)}{d^2x} - \frac{bce\log(x)}{d^2} + \frac{ae^2\log(x)}{d^3} + \frac{e^2\left(a\right.}{}$$

Mathematica [C] time = 0.173295, size = 298, normalized size = 1.02

$$-\frac{bc\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-c^2x^2\right)}{2dx} + \frac{ibe^2\text{PolyLog}(2,-icx)}{2d^3} - \frac{ibe^2\text{PolyLog}(2,icx)}{2d^3} - \frac{ib\left(e^2\text{PolyLog}\left(2,\frac{e(1-icx)}{e+icd}\right)\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c*x])/(x^3*(d + e*x)),x]
```

```
[Out] -(a + b*ArcTan[c*x])/(2*d*x^2) + (e*(a + b*ArcTan[c*x]))/(d^2*x) - (b*c*Hyp
ergeometric2F1[-1/2, 1, 1/2, -(c^2*x^2)])/(2*d*x) + (a*e^2*Log[x])/d^3 - (a
*e^2*Log[d + e*x])/d^3 - (b*c*e*(2*Log[x] - Log[1 + c^2*x^2]))/(2*d^2) + ((
I/2)*b*e^2*PolyLog[2, (-I)*c*x])/d^3 - ((I/2)*b*e^2*PolyLog[2, I*c*x])/d^3
- ((I/2)*b*(e^2*Log[1 - I*c*x]*Log[(c*(d + e*x))/(c*d - I*e)] + e^2*PolyLog
[2, (e*(1 - I*c*x))/(I*c*d + e)]))/d^3 + ((I/2)*b*(e^2*Log[1 + I*c*x]*Log[(
c*(d + e*x))/(c*d + I*e)] + e^2*PolyLog[2, -((e*(1 + I*c*x))/(I*c*d - e))])
)/d^3
```

Maple [A] time = 0.061, size = 393, normalized size = 1.3

$$-\frac{e^2a\ln(ecx+dc)}{d^3} - \frac{a}{2\,dx^2} + \frac{e^2a\ln(cx)}{d^3} + \frac{ae}{d^2x} - \frac{b\arctan(cx)\,e^2\ln(ecx+dc)}{d^3} - \frac{b\arctan(cx)}{2\,dx^2} + \frac{b\arctan(cx)\,e^2\ln(cx)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/x^3/(e*x+d),x)
```

```
[Out] -a/d^3*e^2*ln(c*e*x+c*d)-1/2*a/d/x^2+a/d^3*e^2*ln(c*x)+a/d^2*e/x-b*arctan(c
*x)/d^3*e^2*ln(c*e*x+c*d)-1/2*b*arctan(c*x)/d/x^2+b*arctan(c*x)/d^3*e^2*ln(
c*x)+b*arctan(c*x)/d^2*e/x+1/2*b*c*e*ln(c^2*x^2+1)/d^2-1/2*b*c^2*arctan(c*x
)/d-c*b/d^2*e*ln(c*x)-1/2*b*c/d/x-1/2*I*b/d^3*e^2*ln(c*x)*ln(1-I*c*x)+1/2*I
*b/d^3*e^2*ln(c*x)*ln(1+I*c*x)-1/2*I*b/d^3*e^2*ln(c*e*x+c*d)*ln((I*e-e*c*x)
/(d*c+I*e))+1/2*I*b/d^3*e^2*dilog(1+I*c*x)+1/2*I*b/d^3*e^2*ln(c*e*x+c*d)*ln
((I*e+e*c*x)/(I*e-d*c))-1/2*I*b/d^3*e^2*dilog((I*e-e*c*x)/(d*c+I*e))+1/2*I*
b/d^3*e^2*dilog((I*e+e*c*x)/(I*e-d*c))-1/2*I*b/d^3*e^2*dilog(1-I*c*x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2}\,a\left(\frac{2\,e^2\log\left(ex+d\right)}{d^3}-\frac{2\,e^2\log\left(x\right)}{d^3}-\frac{2\,ex-d}{d^2x^2}\right)+2\,b\int\frac{\arctan\left(cx\right)}{2\left(ex^4+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^3/(e*x+d),x, algorithm="maxima")
```

```
[Out] -1/2*a*(2*e^2*log(e*x + d)/d^3 - 2*e^2*log(x)/d^3 - (2*e*x - d)/(d^2*x^2))
+ 2*b*integrate(1/2*arctan(c*x)/(e*x^4 + d*x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b\arctan\left(cx\right)+a}{ex^4+dx^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^3/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b*arctan(c*x) + a)/(e*x^4 + d*x^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/x**3/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \arctan(cx) + a}{(ex + d)x^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^3/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)/((e*x + d)*x^3), x)
```

3.141 $\int \frac{x^3\left(a+b\tan^{-1}(cx)\right)^2}{d+ex}\,dx$

Optimal. Leaf size=598

$$-\frac{ibd^3\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)\left(a+b\tan^{-1}(cx)\right)}{e^4}+\frac{ibd^3\left(a+b\tan^{-1}(cx)\right)\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(1-icx)(cd+ie)}\right)}{e^4}-\frac{ib^2\text{PolyLog}\left(2,\right.}{3c^3e}$$

```
[Out] (a*b*d*x)/(c*e^2) + (b^2*x)/(3*c^2*e) - (b^2*ArcTan[c*x])/(3*c^3*e) + (b^2*
d*x*ArcTan[c*x])/(c*e^2) - (b*x^2*(a + b*ArcTan[c*x]))/(3*c*e) + (I*d^2*(a
+ b*ArcTan[c*x])^2)/(c*e^3) - (d*(a + b*ArcTan[c*x])^2)/(2*c^2*e^2) - ((I/3
)*(a + b*ArcTan[c*x])^2)/(c^3*e) + (d^2*x*(a + b*ArcTan[c*x])^2)/e^3 - (d*x
^2*(a + b*ArcTan[c*x])^2)/(2*e^2) + (x^3*(a + b*ArcTan[c*x])^2)/(3*e) + (d^
3*(a + b*ArcTan[c*x])^2*Log[2/(1 - I*c*x)])/e^4 + (2*b*d^2*(a + b*ArcTan[c*
x])*Log[2/(1 + I*c*x)])/(c*e^3) - (2*b*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x
)])/(3*c^3*e) - (d^3*(a + b*ArcTan[c*x])^2*Log[(2*c*(d + e*x))/((c*d + I*e)
*(1 - I*c*x))])/e^4 - (b^2*d*Log[1 + c^2*x^2])/(2*c^2*e^2) - (I*b*d^3*(a +
b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 - I*c*x)])/e^4 + (I*b^2*d^2*PolyLog[2, 1
 - 2/(1 + I*c*x)])/(c*e^3) - ((I/3)*b^2*PolyLog[2, 1 - 2/(1 + I*c*x)])/(c^3
*e) + (I*b*d^3*(a + b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + I
*e)*(1 - I*c*x))])/e^4 + (b^2*d^3*PolyLog[3, 1 - 2/(1 - I*c*x)])/(2*e^4) -
(b^2*d^3*PolyLog[3, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/(2*e^4)
```

Rubi [A] time = 0.671122, antiderivative size = 598, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 13, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.619, Rules used = {4876, 4846, 4920, 4854, 2402, 2315, 4852, 4916, 260, 4884, 321, 203, 4858}

$$-\frac{ibd^3\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)\left(a+b\tan^{-1}(cx)\right)}{e^4}+\frac{ibd^3\left(a+b\tan^{-1}(cx)\right)\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(1-icx)(cd+ie)}\right)}{e^4}-\frac{ib^2\text{PolyLog}\left(2,\right.}{3c^3e}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcTan[c*x])^2)/(d + e*x),x]
```


```
[Out] (a*b*d*x)/(c*e^2) + (b^2*x)/(3*c^2*e) - (b^2*ArcTan[c*x])/(3*c^3*e) + (b^2*
d*x*ArcTan[c*x])/(c*e^2) - (b*x^2*(a + b*ArcTan[c*x]))/(3*c*e) + (I*d^2*(a
+ b*ArcTan[c*x])^2)/(c*e^3) - (d*(a + b*ArcTan[c*x])^2)/(2*c^2*e^2) - ((I/3
)*(a + b*ArcTan[c*x])^2)/(c^3*e) + (d^2*x*(a + b*ArcTan[c*x])^2)/e^3 - (d*x
^2*(a + b*ArcTan[c*x])^2)/(2*e^2) + (x^3*(a + b*ArcTan[c*x])^2)/(3*e) + (d^
3*(a + b*ArcTan[c*x])^2*Log[2/(1 - I*c*x)])/e^4 + (2*b*d^2*(a + b*ArcTan[c*
```

```
x])*Log[2/(1 + I*c*x)])/(c*e^3) - (2*b*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x
)])/(3*c^3*e) - (d^3*(a + b*ArcTan[c*x])^2*Log[(2*c*(d + e*x))/((c*d + I*e)
*(1 - I*c*x))])/e^4 - (b^2*d*Log[1 + c^2*x^2])/(2*c^2*e^2) - (I*b*d^3*(a +
b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 - I*c*x)])/e^4 + (I*b^2*d^2*PolyLog[2, 1
 - 2/(1 + I*c*x)])/(c*e^3) - ((I/3)*b^2*PolyLog[2, 1 - 2/(1 + I*c*x)])/(c^3
*e) + (I*b*d^3*(a + b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + I
*e)*(1 - I*c*x))])/e^4 + (b^2*d^3*PolyLog[3, 1 - 2/(1 - I*c*x)])/(2*e^4) -
(b^2*d^3*PolyLog[3, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/(2*e^4)
```

Rule 4876

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4858

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^2/((d_) + (e_.)*(x_)), x_Symbol] :>
-Simp[((a + b*ArcTan[c*x])^2*Log[2/(1 - I*c*x)])/e, x] + (Simp[((a + b*ArcT
an[c*x])^2*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x] + Simp[(I*
b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 - I*c*x)])/e, x] - Simp[(I*b*(a +
 b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/
e, x] - Simp[(b^2*PolyLog[3, 1 - 2/(1 - I*c*x)])/(2*e), x] + Simp[(b^2*Poly
Log[3, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/(2*e), x]) /; FreeQ[
{a, b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rubi steps

$$\int \frac{x^3\left(a+b\tan^{-1}(cx)\right)^2}{d+ex}\,dx = \int\left(\frac{d^2\left(a+b\tan^{-1}(cx)\right)^2}{e^3} - \frac{dx\left(a+b\tan^{-1}(cx)\right)^2}{e^2} + \frac{x^2\left(a+b\tan^{-1}(cx)\right)^2}{e} - \frac{d^3\left(a+b\tan\right.}{e^3(d+e}\right.$$

$$= \frac{d^2\int\left(a+b\tan^{-1}(cx)\right)^2dx}{e^3} - \frac{d^3\int\frac{\left(a+b\tan^{-1}(cx)\right)^2}{d+ex}dx}{e^3} - \frac{d\int x\left(a+b\tan^{-1}(cx)\right)^2dx}{e^2} + \frac{\int x^2\left(\right.}{}$$

$$= \frac{d^2x\left(a+b\tan^{-1}(cx)\right)^2}{e^3} - \frac{dx^2\left(a+b\tan^{-1}(cx)\right)^2}{2e^2} + \frac{x^3\left(a+b\tan^{-1}(cx)\right)^2}{3e} + \frac{d^3\left(a+b\tan^{-1}\right.}{}$$

$$= \frac{id^2\left(a+b\tan^{-1}(cx)\right)^2}{ce^3} + \frac{d^2x\left(a+b\tan^{-1}(cx)\right)^2}{e^3} - \frac{dx^2\left(a+b\tan^{-1}(cx)\right)^2}{2e^2} + \frac{x^3\left(a+b\tan^{-}\right.}{3e}$$

$$= \frac{abdx}{ce^2} - \frac{bx^2\left(a+b\tan^{-1}(cx)\right)}{3ce} + \frac{id^2\left(a+b\tan^{-1}(cx)\right)^2}{ce^3} - \frac{d\left(a+b\tan^{-1}(cx)\right)^2}{2c^2e^2} - \frac{i\left(a+b\tan\right.}{3c}$$

$$= \frac{abdx}{ce^2} + \frac{b^2x}{3c^2e} + \frac{b^2dx\tan^{-1}(cx)}{ce^2} - \frac{bx^2\left(a+b\tan^{-1}(cx)\right)}{3ce} + \frac{id^2\left(a+b\tan^{-1}(cx)\right)^2}{ce^3} - \frac{d\left(a+b\right.}{}$$

$$= \frac{abdx}{ce^2} + \frac{b^2x}{3c^2e} - \frac{b^2\tan^{-1}(cx)}{3c^3e} + \frac{b^2dx\tan^{-1}(cx)}{ce^2} - \frac{bx^2\left(a+b\tan^{-1}(cx)\right)}{3ce} + \frac{id^2\left(a+b\tan^{-1}\right.}{ce^3}$$

$$= \frac{abdx}{ce^2} + \frac{b^2x}{3c^2e} - \frac{b^2\tan^{-1}(cx)}{3c^3e} + \frac{b^2dx\tan^{-1}(cx)}{ce^2} - \frac{bx^2\left(a+b\tan^{-1}(cx)\right)}{3ce} + \frac{id^2\left(a+b\tan^{-1}\right.}{ce^3}$$

Mathematica [F] time = 180.002, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(x^3*(a + b*ArcTan[c*x])^2)/(d + e*x),x]
```

[Out] $Aborted

Maple [C] time = 14.474, size = 2136, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(a+b*arctan(c*x))^2/(e*x+d),x)

[Out] -c*b^2*d^4/e^4/(d*c-I*e)*arctan(c*x)^2*ln(1-(I*e-d*c)/(d*c+I*e)*(1+I*c*x)^2/(c^2*x^2+1))+I*b^2*d^3/e^3/(d*c-I*e)*arctan(c*x)^2*ln(1-(I*e-d*c)/(d*c+I*e)*(1+I*c*x)^2/(c^2*x^2+1))+I*a*b/e^4*d^3*ln(c*e*x+c*d)*ln((I*e+e*c*x)/(I*e-d*c))-1/2*I*b^2/e^4*d^3*Pi*csgn(I*(-I*(1+I*c*x)^2/(c^2*x^2+1)*e+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+d*c)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*arctan(c*x)^2-I*a*b/e^4*d^3*ln(c*e*x+c*d)*ln((I*e-e*c*x)/(d*c+I*e))+2/c*b^2/e^3*d^2*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-1/2*c*b^2*d^4/e^4/(d*c-I*e)*polylog(3,(I*e-d*c)/(d*c+I*e)*(1+I*c*x)^2/(c^2*x^2+1))-1/c*a*b/e^3*ln(c^2*d^2-2*(c*e*x+c*d)*c*d+(c*e*x+c*d)^2+e^2)*d^2-1/c^2*a*b/e^2*arctan(c*x)*d-I*b^2*d^3/e^4*arctan(c*x)*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))+1/2*I*b^2*d^3/e^3/(d*c-I*e)*polylog(3,(I*e-d*c)/(d*c+I*e)*(1+I*c*x)^2/(c^2*x^2+1))-I*a*b/e^4*d^3*dilog((I*e-e*c*x)/(d*c+I*e))-I/c^2*b^2*d*arctan(c*x)/e^2-2*I/c*b^2/e^3*d^2*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-I/c*b^2/e^3*d^2*arctan(c*x)^2-2*I/c*b^2/e^3*d^2*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+b^2*d^3/e^3/(d*c-I*e)*arctan(c*x)*polylog(2,(I*e-d*c)/(d*c+I*e)*(1+I*c*x)^2/(c^2*x^2+1))-a*b*arctan(c*x)/e^2*d*x^2+1/3*b^2*x/c^2/e-1/3*b^2*arctan(c*x)/c^3/e+I*c*b^2*d^4/e^4/(d*c-I*e)*arctan(c*x)*polylog(2,(I*e-d*c)/(d*c+I*e)*(1+I*c*x)^2/(c^2*x^2+1))+1/2*I*b^2/e^4*d^3*Pi*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*(-I*(1+I*c*x)^2/(c^2*x^2+1)*e+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+d*c)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2+1/2*I*b^2/e^4*d^3*Pi*csgn(I*(-I*(1+I*c*x)^2/(c^2*x^2+1)*e+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+d*c))*csgn(I*(-I*(1+I*c*x)^2/(c^2*x^2+1)*e+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+d*c)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2+a*b*d*x/c/e^2+b^2*d*x*arctan(c*x)/c/e^2-1/2*I*b^2/e^4*d^3*Pi*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*(-I*(1+I*c*x)^2/(c^2*x^2+1)*e+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+d*c))*csgn(I*(-I*(1+I*c*x)^2/(c^2*x^2+1)*e+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+d*c)/((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)^2+1/3*I/c^3*b^2/e+2*a*b*arctan(c*x)/e^3*d^2*x-2*a*b*arctan(c*x)*d^3/e^4*ln(c*e*x+c*d)+I*a*b/e^4*d^3*dilog((I*e+e*c*x)/(I*e-d*c))+2/c*b^2/e^3*d^2*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-1/3/c*a*b*x^2/e-1/2*b^2*arctan(c*x)^2/e^2*d*x^2+b^2*arctan(c*x)^2/e^3*x*d^2+2/3*a*b*arctan(c*x)/e*x^3+b^2*d^3/e^4*arctan(c*x)^2*ln(-I*(1+I*c*x)^2/(c^2*x^2+1)*e+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+d*c)-b^2*arctan(c*x)^2*d^3/e^4*ln(c*e*x+c*d)+1/c^2*b^2/e^2*d*ln

```
((1+I*c*x)^2/(c^2*x^2+1)+1)-1/2/c^2*b^2/e^2*d*arctan(c*x)^2+1/3/c^3*a*b/e*l
n(c^2*d^2-2*(c*e*x+c*d)*c*d+(c*e*x+c*d)^2+e^2)-2/3/c^3*b^2/e*arctan(c*x)*ln
(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-2/3/c^3*b^2/e*arctan(c*x)*ln(1-I*(1+I*c*x
)/(c^2*x^2+1)^(1/2))-1/3/c*b^2*arctan(c*x)/e*x^2+2/3*I/c^3*b^2/e*dilog(1-I*
(1+I*c*x)/(c^2*x^2+1)^(1/2))+1/3*I/c^3*b^2/e*arctan(c*x)^2+2/3*I/c^3*b^2/e*
dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+4/3/c*a*b/e^3*d^2+1/3*a^2/e*x^3+1/2*
b^2*d^3/e^4*polylog(3,-(1+I*c*x)^2/(c^2*x^2+1))-a^2*d^3/e^4*ln(c*e*x+c*d)+1
/3*b^2*arctan(c*x)^2/e*x^3+a^2/e^3*x*d^2-1/2*a^2/e^2*d*x^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{6}a^2\left(\frac{6d^3\log(ex+d)}{e^4}-\frac{2e^2x^3-3dex^2+6d^2x}{e^3}\right)+\frac{2e^3\int\frac{36\left(b^2c^2e^3x^5+b^2e^3x^3\right)\arctan(cx)^2+3\left(b^2c^2e^3x^5+b^2e^3x^3\right)\log\left(c^2x^2+1\right)^2+4\left(24abc^2\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))^2/(e*x+d),x, algorithm="maxima")
```

```
[Out] -1/6*a^2*(6*d^3*log(e*x + d)/e^4 - (2*e^2*x^3 - 3*d*e*x^2 + 6*d^2*x)/e^3) +
 1/96*(96*e^3*integrate(1/48*(36*(b^2*c^2*e^3*x^5 + b^2*e^3*x^3)*arctan(c*x
)^2 + 3*(b^2*c^2*e^3*x^5 + b^2*e^3*x^3)*log(c^2*x^2 + 1)^2 + 4*(24*a*b*c^2*
e^3*x^5 - 2*b^2*c*e^3*x^4 - 3*b^2*c*d^2*e*x^2 - 6*b^2*c*d^3*x + (b^2*c*d*e^
2 + 24*a*b*e^3)*x^3)*arctan(c*x) + 2*(2*b^2*c^2*e^3*x^5 - b^2*c^2*d*e^2*x^4
 + 3*b^2*c^2*d^2*e*x^3 + 6*b^2*c^2*d^3*x^2)*log(c^2*x^2 + 1))/(c^2*e^4*x^3
+ c^2*d*e^3*x^2 + e^4*x + d*e^3), x) + 4*(2*b^2*e^2*x^3 - 3*b^2*d*e*x^2 + 6
*b^2*d^2*x)*arctan(c*x)^2 - (2*b^2*e^2*x^3 - 3*b^2*d*e*x^2 + 6*b^2*d^2*x)*l
og(c^2*x^2 + 1)^2)/e^3
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b^2x^3\arctan(cx)^2+2\,abx^3\arctan(cx)+a^2x^3}{ex+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))^2/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^3*arctan(c*x)^2 + 2*a*b*x^3*arctan(c*x) + a^2*x^3)/(e*x + d
), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atan(c*x))**2/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\arctan(cx)+a)^2x^3}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))^2/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)^2*x^3/(e*x + d), x)
```

3.142 $\int \frac{x^2\left(a+b\tan^{-1}(cx)\right)^2}{d+ex}\,dx$

Optimal. Leaf size=430

$$\frac{ibd^2\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)\left(a+b\tan^{-1}(cx)\right)}{e^3}-\frac{ibd^2\left(a+b\tan^{-1}(cx)\right)\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(1-icx)(cd+ie)}\right)}{e^3}-\frac{b^2d^2\text{PolyLog}\left(3,1\right.}{2e^3}$$

```
[Out] -((a*b*x)/(c*e)) - (b^2*x*ArcTan[c*x])/(c*e) - (I*d*(a + b*ArcTan[c*x])^2)/
(c*e^2) + (a + b*ArcTan[c*x])^2/(2*c^2*e) - (d*x*(a + b*ArcTan[c*x])^2)/e^2
 + (x^2*(a + b*ArcTan[c*x])^2)/(2*e) - (d^2*(a + b*ArcTan[c*x])^2*Log[2/(1
- I*c*x)])/e^3 - (2*b*d*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(c*e^2) + (
d^2*(a + b*ArcTan[c*x])^2*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e
^3 + (b^2*Log[1 + c^2*x^2])/(2*c^2*e) + (I*b*d^2*(a + b*ArcTan[c*x])*PolyLo
g[2, 1 - 2/(1 - I*c*x)])/e^3 - (I*b^2*d*PolyLog[2, 1 - 2/(1 + I*c*x)])/(c*e
^2) - (I*b*d^2*(a + b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + I
*e)*(1 - I*c*x))])/e^3 - (b^2*d^2*PolyLog[3, 1 - 2/(1 - I*c*x)])/(2*e^3) +
(b^2*d^2*PolyLog[3, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/(2*e^3)
```

Rubi [A] time = 0.424568, antiderivative size = 430, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 11, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.524, Rules used = {4876, 4846, 4920, 4854, 2402, 2315, 4852, 4916, 260, 4884, 4858}

$$\frac{ibd^2\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)\left(a+b\tan^{-1}(cx)\right)}{e^3}-\frac{ibd^2\left(a+b\tan^{-1}(cx)\right)\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(1-icx)(cd+ie)}\right)}{e^3}-\frac{b^2d^2\text{PolyLog}\left(3,1\right.}{2e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcTan[c*x])^2)/(d + e*x),x]
```

```
[Out] -((a*b*x)/(c*e)) - (b^2*x*ArcTan[c*x])/(c*e) - (I*d*(a + b*ArcTan[c*x])^2)/
(c*e^2) + (a + b*ArcTan[c*x])^2/(2*c^2*e) - (d*x*(a + b*ArcTan[c*x])^2)/e^2
 + (x^2*(a + b*ArcTan[c*x])^2)/(2*e) - (d^2*(a + b*ArcTan[c*x])^2*Log[2/(1
- I*c*x)])/e^3 - (2*b*d*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(c*e^2) + (
d^2*(a + b*ArcTan[c*x])^2*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e
^3 + (b^2*Log[1 + c^2*x^2])/(2*c^2*e) + (I*b*d^2*(a + b*ArcTan[c*x])*PolyLo
g[2, 1 - 2/(1 - I*c*x)])/e^3 - (I*b^2*d*PolyLog[2, 1 - 2/(1 + I*c*x)])/(c*e
^2) - (I*b*d^2*(a + b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + I
*e)*(1 - I*c*x))])/e^3 - (b^2*d^2*PolyLog[3, 1 - 2/(1 - I*c*x)])/(2*e^3) +
(b^2*d^2*PolyLog[3, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/(2*e^3)
```

Rule 4876

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
```

```
erQ[m]) && NeQ[m, -1]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4858

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^2/((d_) + (e_.)*(x_)), x_Symbol] :>
-Simp[((a + b*ArcTan[c*x])^2*Log[2/(1 - I*c*x)])/e, x] + (Simp[((a + b*ArcT
an[c*x])^2*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x] + Simp[(I*
b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 - I*c*x)])/e, x] - Simp[(I*b*(a +
 b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/
e, x] - Simp[(b^2*PolyLog[3, 1 - 2/(1 - I*c*x)])/(2*e), x] + Simp[(b^2*Poly
Log[3, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/(2*e), x]) /; FreeQ[
{a, b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2\left(a+b\tan^{-1}(cx)\right)^2}{d+ex}\,dx &= \int \left(-\frac{d\left(a+b\tan^{-1}(cx)\right)^2}{e^2}+\frac{x\left(a+b\tan^{-1}(cx)\right)^2}{e}+\frac{d^2\left(a+b\tan^{-1}(cx)\right)^2}{e^2(d+ex)}\right)dx \\
&= -\frac{d\int \left(a+b\tan^{-1}(cx)\right)^2 dx}{e^2}+\frac{d^2\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{d+ex}dx}{e^2}+\frac{\int x\left(a+b\tan^{-1}(cx)\right)^2 dx}{e} \\
&= -\frac{dx\left(a+b\tan^{-1}(cx)\right)^2}{e^2}+\frac{x^2\left(a+b\tan^{-1}(cx)\right)^2}{2e}-\frac{d^2\left(a+b\tan^{-1}(cx)\right)^2\log\left(\frac{2}{1-icx}\right)}{e^3}+\cdots \\
&= -\frac{id\left(a+b\tan^{-1}(cx)\right)^2}{ce^2}-\frac{dx\left(a+b\tan^{-1}(cx)\right)^2}{e^2}+\frac{x^2\left(a+b\tan^{-1}(cx)\right)^2}{2e}-\cdots \\
&= -\frac{abx}{ce}-\frac{id\left(a+b\tan^{-1}(cx)\right)^2}{ce^2}+\frac{\left(a+b\tan^{-1}(cx)\right)^2}{2c^2e}-\frac{dx\left(a+b\tan^{-1}(cx)\right)^2}{e^2}+\cdots \\
&= -\frac{abx}{ce}-\frac{b^2x\tan^{-1}(cx)}{ce}-\frac{id\left(a+b\tan^{-1}(cx)\right)^2}{ce^2}+\frac{\left(a+b\tan^{-1}(cx)\right)^2}{2c^2e}-\cdots \\
&= -\frac{abx}{ce}-\frac{b^2x\tan^{-1}(cx)}{ce}-\frac{id\left(a+b\tan^{-1}(cx)\right)^2}{ce^2}+\frac{\left(a+b\tan^{-1}(cx)\right)^2}{2c^2e}-\cdots
\end{aligned}
$$

Mathematica [F] time = 122.316, size = 0, normalized size = 0.

$$\int \frac{x^2\left(a+b\tan^{-1}(cx)\right)^2}{d+ex}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^2*(a + b*ArcTan[c*x])^2)/(d + e*x),x]

[Out] Integrate[(x^2*(a + b*ArcTan[c*x])^2)/(d + e*x), x]
```

Maple [C] time = 8.652, size = 1784, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctan(c*x))^2/(e*x+d),x)
```

```
[Out] 1/2*I*b^2/e^3*d^2*Pi*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*(-I*(1+I*c*
x)^2/(c^2*x^2+1)*e+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+d*c))*csgn(I*(-I*(1+I*c*
x)^2/(c^2*x^2+1)*e+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+d*c)/((1+I*c*x)^2/(c^2*x
^2+1)+1))*arctan(c*x)^2-a*b*x/c/e-b^2*x*arctan(c*x)/c/e-1/2*I*b^2/e^3*d^2*P
i*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*(-I*(1+I*c*x)^2/(c^2*x^2+1)*e+
c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+d*c)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(
c*x)^2-1/2*I*b^2/e^3*d^2*Pi*csgn(I*(-I*(1+I*c*x)^2/(c^2*x^2+1)*e+c*d*(1+I*c
*x)^2/(c^2*x^2+1)+I*e+d*c))*csgn(I*(-I*(1+I*c*x)^2/(c^2*x^2+1)*e+c*d*(1+I*c
*x)^2/(c^2*x^2+1)+I*e+d*c)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2-I*c
*b^2*d^3/e^3/(d*c-I*e)*arctan(c*x)*polylog(2,(I*e-d*c)/(d*c+I*e)*(1+I*c*x)^
2/(c^2*x^2+1))-1/c*a*b*d/e^2-a^2*d/e^2*x+1/2*b^2*arctan(c*x)^2*x^2/e+a^2*d^
2/e^3*ln(c*e*x+c*d)-1/2*b^2*d^2/e^3*polylog(3,-(1+I*c*x)^2/(c^2*x^2+1))+1/2
/c^2*b^2*arctan(c*x)^2/e-1/c^2*b^2/e*ln((1+I*c*x)^2/(c^2*x^2+1)+1)+I*a*b/e^
3*d^2*ln(c*e*x+c*d)*ln((I*e-e*c*x)/(d*c+I*e))+c*b^2*d^3/e^3/(d*c-I*e)*arcta
n(c*x)^2*ln(1-(I*e-d*c)/(d*c+I*e)*(1+I*c*x)^2/(c^2*x^2+1))-I*b^2*d^2/e^2/(d
*c-I*e)*arctan(c*x)^2*ln(1-(I*e-d*c)/(d*c+I*e)*(1+I*c*x)^2/(c^2*x^2+1))+1/2
*I*b^2/e^3*d^2*Pi*csgn(I*(-I*(1+I*c*x)^2/(c^2*x^2+1)*e+c*d*(1+I*c*x)^2/(c^2
*x^2+1)+I*e+d*c)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*arctan(c*x)^2-I*a*b/e^3*d^2
*ln(c*e*x+c*d)*ln((I*e+e*c*x)/(I*e-d*c))+1/c*a*b/e^2*d*ln(c^2*d^2-2*(c*e*x+
c*d)*c*d+(c*e*x+c*d)^2+e^2)+1/2*c*b^2*d^3/e^3/(d*c-I*e)*polylog(3,(I*e-d*c)
/(d*c+I*e)*(1+I*c*x)^2/(c^2*x^2+1))+I/c*b^2/e^2*d*arctan(c*x)^2-2/c*b^2/e^2
*d*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-2/c*b^2/e^2*d*arctan(c*x
)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+I*a*b/e^3*d^2*dilog((I*e-e*c*x)/(d*c+
I*e))-2*a*b*arctan(c*x)*d/e^2*x-b^2*d^2/e^2/(d*c-I*e)*arctan(c*x)*polylog(2
,(I*e-d*c)/(d*c+I*e)*(1+I*c*x)^2/(c^2*x^2+1))+I*b^2*d^2/e^3*arctan(c*x)*pol
ylog(2,-(1+I*c*x)^2/(c^2*x^2+1))+2*a*b*arctan(c*x)*d^2/e^3*ln(c*e*x+c*d)+2*
I/c*b^2/e^2*d*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+2*I/c*b^2/e^2*d*dilog(
1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-1/2*I*b^2*d^2/e^2/(d*c-I*e)*polylog(3,(I*e
-d*c)/(d*c+I*e)*(1+I*c*x)^2/(c^2*x^2+1))-I*a*b/e^3*d^2*dilog((I*e+e*c*x)/(I
*e-d*c))+I/c^2*b^2*arctan(c*x)/e+1/c^2*a*b/e*arctan(c*x)+a*b*arctan(c*x)*x^
2/e-b^2*arctan(c*x)^2*d/e^2*x+b^2*arctan(c*x)^2*d^2/e^3*ln(c*e*x+c*d)-b^2*d
^2/e^3*arctan(c*x)^2*ln(-I*(1+I*c*x)^2/(c^2*x^2+1)*e+c*d*(1+I*c*x)^2/(c^2*x
^2+1)+I*e+d*c)+1/2*a^2*x^2/e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}\,a^2\left(\frac{2\,d^2\log\left(ex+d\right)}{e^3}+\frac{ex^2-2\,dx}{e^2}\right)+\frac{4\left(b^2ex^2-2\,b^2dx\right)\arctan\left(cx\right)^2+2\,e^2\int\frac{12\left(b^2c^2e^2x^4+b^2e^2x^2\right)\arctan(cx)^2+\left(b^2c^2e^2x^4+b^2e^2x^2\right)}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))^2/(e*x+d),x, algorithm="maxima")
```

```
[Out] 1/2*a^2*(2*d^2*log(e*x + d)/e^3 + (e*x^2 - 2*d*x)/e^2) + 1/32*(4*(b^2*e*x^2
 - 2*b^2*d*x)*arctan(c*x)^2 + 32*e^2*integrate(1/16*(12*(b^2*c^2*e^2*x^4 +
b^2*e^2*x^2)*arctan(c*x)^2 + (b^2*c^2*e^2*x^4 + b^2*e^2*x^2)*log(c^2*x^2 +
1)^2 + 4*(8*a*b*c^2*e^2*x^4 - b^2*c*e^2*x^3 + 2*b^2*c*d^2*x + (b^2*c*d*e +
8*a*b*e^2)*x^2)*arctan(c*x) + 2*(b^2*c^2*e^2*x^4 - b^2*c^2*d*e*x^3 - 2*b^2*
c^2*d^2*x^2)*log(c^2*x^2 + 1))/(c^2*e^3*x^3 + c^2*d*e^2*x^2 + e^3*x + d*e^2
), x) - (b^2*e*x^2 - 2*b^2*d*x)*log(c^2*x^2 + 1)^2)/e^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2x^2\arctan(cx)^2 + 2\,abx^2\arctan(cx) + a^2x^2}{ex+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))^2/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^2*arctan(c*x)^2 + 2*a*b*x^2*arctan(c*x) + a^2*x^2)/(e*x + d
), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atan(c*x))**2/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\arctan(cx) + a)^2x^2}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))^2/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)^2*x^2/(e*x + d), x)
```

3.143 $\int \frac{x\left(a+b\tan^{-1}(cx)\right)^2}{d+ex}\,dx$

Optimal. Leaf size=323

$$-\frac{ibd\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)\left(a+b\tan^{-1}(cx)\right)}{e^2}+\frac{ibd\left(a+b\tan^{-1}(cx)\right)\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(1-icx)(cd+ie)}\right)}{e^2}+\frac{b^2d\text{PolyLog}\left(3,1\right.}{2e^2}$$

```
[Out] (I*(a + b*ArcTan[c*x])^2)/(c*e) + (x*(a + b*ArcTan[c*x])^2)/e + (d*(a + b*A
rcTan[c*x])^2*Log[2/(1 - I*c*x)])/e^2 + (2*b*(a + b*ArcTan[c*x])*Log[2/(1 +
 I*c*x)])/(c*e) - (d*(a + b*ArcTan[c*x])^2*Log[(2*c*(d + e*x))/((c*d + I*e)
*(1 - I*c*x))])/e^2 - (I*b*d*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 - I*c*
x)])/e^2 + (I*b^2*PolyLog[2, 1 - 2/(1 + I*c*x)])/(c*e) + (I*b*d*(a + b*ArcT
an[c*x])*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e^2 + (
b^2*d*PolyLog[3, 1 - 2/(1 - I*c*x)])/(2*e^2) - (b^2*d*PolyLog[3, 1 - (2*c*(
d + e*x))/((c*d + I*e)*(1 - I*c*x))])/(2*e^2)
```

Rubi [A] time = 0.265561, antiderivative size = 323, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {4876, 4846, 4920, 4854, 2402, 2315, 4858}

$$-\frac{ibd\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)\left(a+b\tan^{-1}(cx)\right)}{e^2}+\frac{ibd\left(a+b\tan^{-1}(cx)\right)\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(1-icx)(cd+ie)}\right)}{e^2}+\frac{b^2d\text{PolyLog}\left(3,1\right.}{2e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcTan[c*x])^2)/(d + e*x),x]
```


```
[Out] (I*(a + b*ArcTan[c*x])^2)/(c*e) + (x*(a + b*ArcTan[c*x])^2)/e + (d*(a + b*A
rcTan[c*x])^2*Log[2/(1 - I*c*x)])/e^2 + (2*b*(a + b*ArcTan[c*x])*Log[2/(1 +
 I*c*x)])/(c*e) - (d*(a + b*ArcTan[c*x])^2*Log[(2*c*(d + e*x))/((c*d + I*e)
*(1 - I*c*x))])/e^2 - (I*b*d*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 - I*c*
x)])/e^2 + (I*b^2*PolyLog[2, 1 - 2/(1 + I*c*x)])/(c*e) + (I*b*d*(a + b*ArcT
an[c*x])*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e^2 + (
b^2*d*PolyLog[3, 1 - 2/(1 - I*c*x)])/(2*e^2) - (b^2*d*PolyLog[3, 1 - (2*c*(
d + e*x))/((c*d + I*e)*(1 - I*c*x))])/(2*e^2)
```


Rule 4876

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
```

```
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 4858

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^2/((d_) + (e_.)*(x_)), x_Symbol] :>
-Simp[((a + b*ArcTan[c*x])^2*Log[2/(1 - I*c*x)])/e, x] + (Simp[((a + b*ArcT
an[c*x])^2*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x] + Simp[(I*
b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 - I*c*x)])/e, x] - Simp[(I*b*(a +
 b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/
e, x] - Simp[(b^2*PolyLog[3, 1 - 2/(1 - I*c*x)])/(2*e), x] + Simp[(b^2*Poly
Log[3, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/(2*e), x]) /; FreeQ[
{a, b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rubi steps

$$\int \frac{x\left(a+b\tan^{-1}(cx)\right)^2}{d+ex}\,dx=\int \left(\frac{\left(a+b\tan^{-1}(cx)\right)^2}{e}-\frac{d\left(a+b\tan^{-1}(cx)\right)^2}{e(d+ex)}\right)dx$$

$$=\frac{\int \left(a+b\tan^{-1}(cx)\right)^2\,dx}{e}-\frac{d\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{d+ex}\,dx}{e}$$

$$=\frac{x\left(a+b\tan^{-1}(cx)\right)^2}{e}+\frac{d\left(a+b\tan^{-1}(cx)\right)^2\log\left(\frac{2}{1-icx}\right)}{e^2}-\frac{d\left(a+b\tan^{-1}(cx)\right)^2\log\left(\frac{2c(d+}{(cd+ie)(1}\right.}{e^2}$$

$$=\frac{i\left(a+b\tan^{-1}(cx)\right)^2}{ce}+\frac{x\left(a+b\tan^{-1}(cx)\right)^2}{e}+\frac{d\left(a+b\tan^{-1}(cx)\right)^2\log\left(\frac{2}{1-icx}\right)}{e^2}-\frac{d\left(a+b\,\mathrm{t}\right.}{}$$

$$=\frac{i\left(a+b\tan^{-1}(cx)\right)^2}{ce}+\frac{x\left(a+b\tan^{-1}(cx)\right)^2}{e}+\frac{d\left(a+b\tan^{-1}(cx)\right)^2\log\left(\frac{2}{1-icx}\right)}{e^2}+\frac{2b\left(a+b\right.}{}$$

$$=\frac{i\left(a+b\tan^{-1}(cx)\right)^2}{ce}+\frac{x\left(a+b\tan^{-1}(cx)\right)^2}{e}+\frac{d\left(a+b\tan^{-1}(cx)\right)^2\log\left(\frac{2}{1-icx}\right)}{e^2}+\frac{2b\left(a+b\right.}{}$$

$$=\frac{i\left(a+b\tan^{-1}(cx)\right)^2}{ce}+\frac{x\left(a+b\tan^{-1}(cx)\right)^2}{e}+\frac{d\left(a+b\tan^{-1}(cx)\right)^2\log\left(\frac{2}{1-icx}\right)}{e^2}+\frac{2b\left(a+b\right.}{}$$

Mathematica [F] time = 180.002, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(x*(a + b*ArcTan[c*x])^2)/(d + e*x),x]
```

```
[Out] $Aborted
```

Maple [C] time = 5.046, size = 16024, normalized size = 49.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctan(c*x))^2/(e*x+d),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$a^2\left(\frac{x}{e}-\frac{d\log\left(ex+d\right)}{e^2}\right)+\frac{4\,b^2x\arctan\left(cx\right)^2-b^2x\log\left(c^2x^2+1\right)^2+e\int\frac{12\left(b^2c^2ex^3+b^2ex\right)\arctan(cx)^2+\left(b^2c^2ex^3+b^2ex\right)\log\left(c^2x^2+1\right)^2+8}{c^2e^2}}{16\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))^2/(e*x+d),x, algorithm="maxima")
```

```
[Out] a^2*(x/e - d*log(e*x + d)/e^2) + 1/16*(4*b^2*x*arctan(c*x)^2 - b^2*x*log(c^
2*x^2 + 1)^2 + 16*e*integrate(1/16*(12*(b^2*c^2*e*x^3 + b^2*e*x)*arctan(c*x
)^2 + (b^2*c^2*e*x^3 + b^2*e*x)*log(c^2*x^2 + 1)^2 + 8*(4*a*b*c^2*e*x^3 - b
^2*c*e*x^2 - (b^2*c*d - 4*a*b*e)*x)*arctan(c*x) + 4*(b^2*c^2*e*x^3 + b^2*c^
2*d*x^2)*log(c^2*x^2 + 1))/(c^2*e^2*x^3 + c^2*d*e*x^2 + e^2*x + d*e), x))/e
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2x\arctan\left(cx\right)^2+2\,abx\arctan\left(cx\right)+a^2x}{ex+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))^2/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*x*arctan(c*x)^2 + 2*a*b*x*arctan(c*x) + a^2*x)/(e*x + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x\left(a+b\,\mathrm{atan}\left(cx\right)\right)^2}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atan(c*x))**2/(e*x+d),x)
```

```
[Out] Integral(x*(a + b*atan(c*x))**2/(d + e*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \arctan(cx) + a)^2 x}{ex + d} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))^2/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)^2*x/(e*x + d), x)
```

3.144 $\int \frac{(a+b\tan^{-1}(cx))^2}{d+ex}\,dx$

Optimal. Leaf size=223

$$-\frac{ib\left(a+b\tan^{-1}(cx)\right)\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(1-icx)(cd+ie)}\right)}{e}+\frac{ib\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)\left(a+b\tan^{-1}(cx)\right)}{e}+\frac{b^2\text{PolyLog}\left(3,1-\frac{}{(1-}\right.}{2e}$$

```
[Out] -(((a + b*ArcTan[c*x])^2*Log[2/(1 - I*c*x)])/e) + ((a + b*ArcTan[c*x])^2*Lo
g[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e + (I*b*(a + b*ArcTan[c*x])*
PolyLog[2, 1 - 2/(1 - I*c*x)])/e - (I*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 -
(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e - (b^2*PolyLog[3, 1 - 2/(1 -
I*c*x)])/(2*e) + (b^2*PolyLog[3, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*
x))])/(2*e)
```

Rubi [A] time = 0.0476133, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {4858}

$$-\frac{ib\left(a+b\tan^{-1}(cx)\right)\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(1-icx)(cd+ie)}\right)}{e}+\frac{ib\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)\left(a+b\tan^{-1}(cx)\right)}{e}+\frac{b^2\text{PolyLog}\left(3,1-\frac{}{(1-}\right.}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^2/(d + e*x),x]
```

```
[Out] -(((a + b*ArcTan[c*x])^2*Log[2/(1 - I*c*x)])/e) + ((a + b*ArcTan[c*x])^2*Lo
g[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e + (I*b*(a + b*ArcTan[c*x])*
PolyLog[2, 1 - 2/(1 - I*c*x)])/e - (I*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 -
(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e - (b^2*PolyLog[3, 1 - 2/(1 -
I*c*x)])/(2*e) + (b^2*PolyLog[3, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*
x))])/(2*e)
```

Rule 4858

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^2/((d_) + (e_.)*(x_)), x_Symbol] :>
-Simp[((a + b*ArcTan[c*x])^2*Log[2/(1 - I*c*x)])/e, x] + (Simp[((a + b*ArcT
an[c*x])^2*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x] + Simp[(I*
b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 - I*c*x)])/e, x] - Simp[(I*b*(a +
 b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/
e, x] - Simp[(b^2*PolyLog[3, 1 - 2/(1 - I*c*x)])/(2*e), x] + Simp[(b^2*Poly
```

```
Log[3, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/(2*e), x]) /; FreeQ[
{a, b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{d+ex}\,dx = -\frac{\left(a+b\tan^{-1}(cx)\right)^2\log\left(\frac{2}{1-icx}\right)}{e} + \frac{\left(a+b\tan^{-1}(cx)\right)^2\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{e} + \frac{ib\left(a+b\tan^{-1}(cx\right.}{e}$$

Mathematica [F] time = 0.154898, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{d+ex}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcTan[c*x])^2/(d + e*x),x]
```

```
[Out] Integrate[(a + b*ArcTan[c*x])^2/(d + e*x), x]
```

Maple [C] time = 0.26, size = 1297, normalized size = 5.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^2/(e*x+d),x)
```

```
[Out] a^2*ln(c*e*x+c*d)/e+b^2*ln(c*e*x+c*d)/e*arctan(c*x)^2-b^2/e*arctan(c*x)^2*l
n(-I*(1+I*c*x)^2/(c^2*x^2+1)*e+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+d*c)-I*a*b*l
n(c*e*x+c*d)/e*ln((I*e+e*c*x)/(I*e-d*c))-1/2*I*b^2/e*arctan(c*x)^2*Pi*csgn(
I*(-I*(1+I*c*x)^2/(c^2*x^2+1)*e+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+d*c)/((1+I*
c*x)^2/(c^2*x^2+1)+1))^2*csgn(I*(-I*(1+I*c*x)^2/(c^2*x^2+1)*e+c*d*(1+I*c*x)
^2/(c^2*x^2+1)+I*e+d*c))-1/2*I*b^2/e*arctan(c*x)^2*Pi*csgn(I/((1+I*c*x)^2/(
c^2*x^2+1)+1))*csgn(I*(-I*(1+I*c*x)^2/(c^2*x^2+1)*e+c*d*(1+I*c*x)^2/(c^2*x^
2+1)+I*e+d*c)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2+1/2*I*b^2/e*arctan(c*x)^2*Pi*c
sgn(I*(-I*(1+I*c*x)^2/(c^2*x^2+1)*e+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+d*c)/((
1+I*c*x)^2/(c^2*x^2+1)+1))^3+I*a*b*ln(c*e*x+c*d)/e*ln((I*e-e*c*x)/(d*c+I*e)
)-1/2*b^2/e*polylog(3,-(1+I*c*x)^2/(c^2*x^2+1))+c*b^2/e*d/(d*c-I*e)*arctan(
```

```
c*x)^2*ln(1-(I*e-d*c)/(d*c+I*e)*(1+I*c*x)^2/(c^2*x^2+1))-I*c*b^2/e*d/(d*c-I
*e)*arctan(c*x)*polylog(2,(I*e-d*c)/(d*c+I*e)*(1+I*c*x)^2/(c^2*x^2+1))+1/2*
c*b^2/e*d/(d*c-I*e)*polylog(3,(I*e-d*c)/(d*c+I*e)*(1+I*c*x)^2/(c^2*x^2+1))+
b^2*arctan(c*x)^2*ln(1-(I*e-d*c)/(d*c+I*e)*(1+I*c*x)^2/(c^2*x^2+1))/(e+I*d*
c)+I*b^2/e*arctan(c*x)*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))+1/2*b^2*polylog(
3,(I*e-d*c)/(d*c+I*e)*(1+I*c*x)^2/(c^2*x^2+1))/(e+I*d*c)+2*a*b*ln(c*e*x+c*d
)/e*arctan(c*x)-I*a*b/e*dilog((I*e+e*c*x)/(I*e-d*c))+1/2*I*b^2/e*arctan(c*x
)^2*Pi*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*(-I*(1+I*c*x)^2/(c^2*x^2+
1)*e+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+d*c)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn
(I*(-I*(1+I*c*x)^2/(c^2*x^2+1)*e+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+d*c))+I*a*
b/e*dilog((I*e-e*c*x)/(d*c+I*e))-I*b^2*arctan(c*x)*polylog(2,(I*e-d*c)/(d*c
+I*e)*(1+I*c*x)^2/(c^2*x^2+1))/(e+I*d*c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^2\log(ex+d)}{e}+\int\frac{12\,b^2\arctan(cx)^2+b^2\log\left(c^2x^2+1\right)^2+32\,ab\arctan(cx)}{16\,(ex+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/(e*x+d),x, algorithm="maxima")
```

```
[Out] a^2*log(e*x + d)/e + integrate(1/16*(12*b^2*arctan(c*x)^2 + b^2*log(c^2*x^2
 + 1)^2 + 32*a*b*arctan(c*x))/(e*x + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b^2\arctan(cx)^2+2\,ab\arctan(cx)+a^2}{ex+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*arctan(c*x)^2 + 2*a*b*arctan(c*x) + a^2)/(e*x + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b\,\mathrm{atan}\,(cx))^2}{d + ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**2/(e*x+d),x)
```

```
[Out] Integral((a + b*atan(c*x))**2/(d + e*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\arctan\,(cx) + a)^2}{ex + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)^2/(e*x + d), x)
```

3.145 $\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x(d+ex)}\,dx$

Optimal. Leaf size=369

$$\frac{ib\left(a+b\tan^{-1}(cx)\right)\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(1-icx)(cd+ie)}\right)}{d}-\frac{ib\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)\left(a+b\tan^{-1}(cx)\right)}{d}-\frac{ib\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{d}$$

```
[Out] (2*(a + b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1 + I*c*x)])/d + ((a + b*ArcTan[c*x
])^2*Log[2/(1 - I*c*x)])/d - ((a + b*ArcTan[c*x])^2*Log[(2*c*(d + e*x))/((c
*d + I*e)*(1 - I*c*x))])/d - (I*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 -
 I*c*x)])/d - (I*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)])/d + (
I*b*(a + b*ArcTan[c*x])*PolyLog[2, -1 + 2/(1 + I*c*x)])/d + (I*b*(a + b*Arc
Tan[c*x])*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/d + (b
^2*PolyLog[3, 1 - 2/(1 - I*c*x)])/(2*d) - (b^2*PolyLog[3, 1 - 2/(1 + I*c*x)
])/(2*d) + (b^2*PolyLog[3, -1 + 2/(1 + I*c*x)])/(2*d) - (b^2*PolyLog[3, 1 -
 (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/(2*d)
```

Rubi [A] time = 0.433021, antiderivative size = 369, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4876, 4850, 4988, 4884, 4994, 6610, 4858}

$$\frac{ib\left(a+b\tan^{-1}(cx)\right)\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(1-icx)(cd+ie)}\right)}{d}-\frac{ib\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)\left(a+b\tan^{-1}(cx)\right)}{d}-\frac{ib\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^2/(x*(d + e*x)),x]
```


```
[Out] (2*(a + b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1 + I*c*x)])/d + ((a + b*ArcTan[c*x
])^2*Log[2/(1 - I*c*x)])/d - ((a + b*ArcTan[c*x])^2*Log[(2*c*(d + e*x))/((c
*d + I*e)*(1 - I*c*x))])/d - (I*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 -
 I*c*x)])/d - (I*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)])/d + (
I*b*(a + b*ArcTan[c*x])*PolyLog[2, -1 + 2/(1 + I*c*x)])/d + (I*b*(a + b*Arc
Tan[c*x])*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/d + (b
^2*PolyLog[3, 1 - 2/(1 - I*c*x)])/(2*d) - (b^2*PolyLog[3, 1 - 2/(1 + I*c*x)
])/(2*d) + (b^2*PolyLog[3, -1 + 2/(1 + I*c*x)])/(2*d) - (b^2*PolyLog[3, 1 -
 (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/(2*d)
```


Rule 4876

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 4850

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTan[c*x])^(p - 1)*ArcTanh[1 - 2/(1 + I*c*x)])/(1 + c^2*x^2), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4988

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTan[c*x])^p)/(d + e
*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTan[c*x])^p)/(d + e*x^2
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 4858

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^2/((d_) + (e_.)*(x_)), x_Symbol] :>
-Simp[((a + b*ArcTan[c*x])^2*Log[2/(1 - I*c*x)])/e, x] + (Simp[((a + b*ArcT
an[c*x])^2*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x] + Simp[(I*
b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 - I*c*x)])/e, x] - Simp[(I*b*(a +
```

```
 b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/
e, x] - Simp[(b^2*PolyLog[3, 1 - 2/(1 - I*c*x)])/(2*e), x] + Simp[(b^2*Poly
Log[3, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/(2*e), x]) /; FreeQ[
{a, b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x(d+ex)}\,dx = \int \left(\frac{\left(a+b\tan^{-1}(cx)\right)^2}{dx} - \frac{e\left(a+b\tan^{-1}(cx)\right)^2}{d(d+ex)}\right) dx$$

$$= \frac{\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x}\,dx}{d} - \frac{e\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{d+ex}\,dx}{d}$$

$$= \frac{2\left(a+b\tan^{-1}(cx)\right)^2\tanh^{-1}\left(1-\frac{2}{1+icx}\right)}{d} + \frac{\left(a+b\tan^{-1}(cx)\right)^2\log\left(\frac{2}{1-icx}\right)}{d} - \frac{\left(a+b\tan^{-1}(cx)\right)^2}{d}$$

$$= \frac{2\left(a+b\tan^{-1}(cx)\right)^2\tanh^{-1}\left(1-\frac{2}{1+icx}\right)}{d} + \frac{\left(a+b\tan^{-1}(cx)\right)^2\log\left(\frac{2}{1-icx}\right)}{d} - \frac{\left(a+b\tan^{-1}(cx)\right)^2}{d}$$

$$= \frac{2\left(a+b\tan^{-1}(cx)\right)^2\tanh^{-1}\left(1-\frac{2}{1+icx}\right)}{d} + \frac{\left(a+b\tan^{-1}(cx)\right)^2\log\left(\frac{2}{1-icx}\right)}{d} - \frac{\left(a+b\tan^{-1}(cx)\right)^2}{d}$$

$$= \frac{2\left(a+b\tan^{-1}(cx)\right)^2\tanh^{-1}\left(1-\frac{2}{1+icx}\right)}{d} + \frac{\left(a+b\tan^{-1}(cx)\right)^2\log\left(\frac{2}{1-icx}\right)}{d} - \frac{\left(a+b\tan^{-1}(cx)\right)^2}{d}$$

Mathematica [F] time = 180.003, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcTan[c*x])^2/(x*(d + e*x)),x]
```

```
[Out] $Aborted
```

Maple [C] time = 0.664, size = 2363, normalized size = 6.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^2/x/(e*x+d),x)
```

```
[Out] I*a*b/d*ln(c*x)*ln(1+I*c*x)+I*a*b/d*ln(c*e*x+c*d)*ln((I*e+e*c*x)/(I*e-d*c))
-I*a*b/d*ln(c*x)*ln(1-I*c*x)-1/2*b^2*c/(d*c-I*e)*polylog(3,(I*e-d*c)/(d*c+I
*e)*(1+I*c*x)^2/(c^2*x^2+1))-a^2/d*ln(c*e*x+c*d)+a^2/d*ln(c*x)-2*I*b^2/d*ar
ctan(c*x)*polylog(2,-(1+I*c*x)/(c^2*x^2+1)^(1/2))+2*a*b*arctan(c*x)/d*ln(c*
x)-b^2*c/(d*c-I*e)*arctan(c*x)^2*ln(1-(I*e-d*c)/(d*c+I*e)*(1+I*c*x)^2/(c^2*
x^2+1))-1/2*b^2*e*polylog(3,(I*e-d*c)/(d*c+I*e)*(1+I*c*x)^2/(c^2*x^2+1))/d/
(e+I*d*c)-2*I*b^2/d*arctan(c*x)*polylog(2,(1+I*c*x)/(c^2*x^2+1)^(1/2))+1/2*
I*b^2/d*Pi*arctan(c*x)^2+I*a*b/d*dilog(1+I*c*x)+I*a*b/d*dilog((I*e+e*c*x)/(
I*e-d*c))-I*a*b/d*dilog(1-I*c*x)-I*a*b/d*dilog((I*e-e*c*x)/(d*c+I*e))-2*a*b
*arctan(c*x)/d*ln(c*e*x+c*d)-I*a*b/d*ln(c*e*x+c*d)*ln((I*e-e*c*x)/(d*c+I*e)
)-b^2*e*arctan(c*x)^2*ln(1-(I*e-d*c)/(d*c+I*e)*(1+I*c*x)^2/(c^2*x^2+1))/d/(
e+I*d*c)+I*b^2*c/(d*c-I*e)*arctan(c*x)*polylog(2,(I*e-d*c)/(d*c+I*e)*(1+I*c
*x)^2/(c^2*x^2+1))+1/2*I*b^2/d*Pi*arctan(c*x)^2*csgn(I*((1+I*c*x)^2/(c^2*x^
2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3+1/2*I*b^2/d*Pi*arctan(c*x)^2*csgn(((
1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3-1/2*I*b^2/d*Pi*arc
tan(c*x)^2*csgn(I*(-I*(1+I*c*x)^2/(c^2*x^2+1)*e+c*d*(1+I*c*x)^2/(c^2*x^2+1)
+I*e+d*c)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3-1/2*I*b^2/d*Pi*arctan(c*x)^2*csgn(
((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2+b^2*arctan(c*x)^
2/d*ln(-I*(1+I*c*x)^2/(c^2*x^2+1)*e+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+d*c)+b^
2/d*arctan(c*x)^2*ln(1-(1+I*c*x)/(c^2*x^2+1)^(1/2))-b^2*arctan(c*x)^2/d*ln(
c*e*x+c*d)+b^2*arctan(c*x)^2/d*ln(c*x)+b^2/d*arctan(c*x)^2*ln(1+(1+I*c*x)/(
c^2*x^2+1)^(1/2))-b^2*arctan(c*x)^2/d*ln((1+I*c*x)^2/(c^2*x^2+1)-1)+2*b^2/d
*polylog(3,(1+I*c*x)/(c^2*x^2+1)^(1/2))+2*b^2/d*polylog(3,-(1+I*c*x)/(c^2*x
^2+1)^(1/2))-1/2*I*b^2/d*Pi*arctan(c*x)^2*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1
))*csgn(I*(-I*(1+I*c*x)^2/(c^2*x^2+1)*e+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+d*c
))*csgn(I*(-I*(1+I*c*x)^2/(c^2*x^2+1)*e+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+d*c
)/((1+I*c*x)^2/(c^2*x^2+1)+1))+1/2*I*b^2/d*Pi*arctan(c*x)^2*csgn(I*((1+I*c*
x)^2/(c^2*x^2+1)-1))*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*((1+I*c*x)^
2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))+I*b^2*e*arctan(c*x)*polylog(2
,(I*e-d*c)/(d*c+I*e)*(1+I*c*x)^2/(c^2*x^2+1))/d/(e+I*d*c)-1/2*I*b^2/d*Pi*ar
ctan(c*x)^2*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*((1+I*c*x)^2/(c^2*x^
2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2+1/2*I*b^2/d*Pi*arctan(c*x)^2*csgn(I*
((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(((1+I*c*x)^2/
(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))-1/2*I*b^2/d*Pi*arctan(c*x)^2*cs
gn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(((1+I*c*
x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2+1/2*I*b^2/d*Pi*arctan(c*
x)^2*csgn(I*(-I*(1+I*c*x)^2/(c^2*x^2+1)*e+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+d
*c))*csgn(I*(-I*(1+I*c*x)^2/(c^2*x^2+1)*e+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+d
*c)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2+1/2*I*b^2/d*Pi*arctan(c*x)^2*csgn(I/((1+
I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*(-I*(1+I*c*x)^2/(c^2*x^2+1)*e+c*d*(1+I*c*x)
```

```
^2/(c^2*x^2+1)+I*e+d*c)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2-1/2*I*b^2/d*Pi*arcta
n(c*x)^2*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1
)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-a^2\left(\frac{\log(ex+d)}{d}-\frac{\log(x)}{d}\right)+\int\frac{12\,b^2\arctan(cx)^2+b^2\log\left(c^2x^2+1\right)^2+32\,ab\arctan(cx)}{16\left(ex^2+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x/(e*x+d),x, algorithm="maxima")
```

```
[Out] -a^2*(log(e*x + d)/d - log(x)/d) + integrate(1/16*(12*b^2*arctan(c*x)^2 + b
^2*log(c^2*x^2 + 1)^2 + 32*a*b*arctan(c*x))/(e*x^2 + d*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{b^2\arctan(cx)^2+2\,ab\arctan(cx)+a^2}{ex^2+dx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*arctan(c*x)^2 + 2*a*b*arctan(c*x) + a^2)/(e*x^2 + d*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**2/x/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\arctan(cx) + a)^2}{(ex + d)x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)^2/((e*x + d)*x), x)
```

3.146 $\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^2(d+ex)}\,dx$

Optimal. Leaf size=473

$$\frac{ibe\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)\left(a+b\tan^{-1}(cx)\right)}{d^2}+\frac{ibe\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{d^2}-\frac{ibe\text{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)}{d^2}$$

```
[Out] ((-I)*c*(a + b*ArcTan[c*x])^2)/d - (a + b*ArcTan[c*x])^2/(d*x) - (2*e*(a +
b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1 + I*c*x)])/d^2 - (e*(a + b*ArcTan[c*x])^2
*Log[2/(1 - I*c*x)])/d^2 + (e*(a + b*ArcTan[c*x])^2*Log[(2*c*(d + e*x))/((c
*d + I*e)*(1 - I*c*x))])/d^2 + (2*b*c*(a + b*ArcTan[c*x])*Log[2 - 2/(1 - I*
c*x)])/d + (I*b*e*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 - I*c*x)])/d^2 -
(I*b^2*c*PolyLog[2, -1 + 2/(1 - I*c*x)])/d + (I*b*e*(a + b*ArcTan[c*x])*Pol
yLog[2, 1 - 2/(1 + I*c*x)])/d^2 - (I*b*e*(a + b*ArcTan[c*x])*PolyLog[2, -1
+ 2/(1 + I*c*x)])/d^2 - (I*b*e*(a + b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(d +
 e*x))/((c*d + I*e)*(1 - I*c*x))])/d^2 - (b^2*e*PolyLog[3, 1 - 2/(1 - I*c*x
)])/(2*d^2) + (b^2*e*PolyLog[3, 1 - 2/(1 + I*c*x)])/(2*d^2) - (b^2*e*PolyLo
g[3, -1 + 2/(1 + I*c*x)])/(2*d^2) + (b^2*e*PolyLog[3, 1 - (2*c*(d + e*x))/(
(c*d + I*e)*(1 - I*c*x))])/(2*d^2)
```

Rubi [A] time = 0.603513, antiderivative size = 473, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 11, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.524, Rules used = {4876, 4852, 4924, 4868, 2447, 4850, 4988, 4884, 4994, 6610, 4858}

$$\frac{ibe\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)\left(a+b\tan^{-1}(cx)\right)}{d^2}+\frac{ibe\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{d^2}-\frac{ibe\text{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^2/(x^2*(d + e*x)),x]
```


```
[Out] ((-I)*c*(a + b*ArcTan[c*x])^2)/d - (a + b*ArcTan[c*x])^2/(d*x) - (2*e*(a +
b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1 + I*c*x)])/d^2 - (e*(a + b*ArcTan[c*x])^2
*Log[2/(1 - I*c*x)])/d^2 + (e*(a + b*ArcTan[c*x])^2*Log[(2*c*(d + e*x))/((c
*d + I*e)*(1 - I*c*x))])/d^2 + (2*b*c*(a + b*ArcTan[c*x])*Log[2 - 2/(1 - I*
c*x)])/d + (I*b*e*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 - I*c*x)])/d^2 -
(I*b^2*c*PolyLog[2, -1 + 2/(1 - I*c*x)])/d + (I*b*e*(a + b*ArcTan[c*x])*Pol
yLog[2, 1 - 2/(1 + I*c*x)])/d^2 - (I*b*e*(a + b*ArcTan[c*x])*PolyLog[2, -1
+ 2/(1 + I*c*x)])/d^2 - (I*b*e*(a + b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(d +
 e*x))/((c*d + I*e)*(1 - I*c*x))])/d^2 - (b^2*e*PolyLog[3, 1 - 2/(1 - I*c*x
```

```
)])/(2*d^2) + (b^2*e*PolyLog[3, 1 - 2/(1 + I*c*x)])/(2*d^2) - (b^2*e*PolyLo
g[3, -1 + 2/(1 + I*c*x)])/(2*d^2) + (b^2*e*PolyLog[3, 1 - (2*c*(d + e*x))/(
(c*d + I*e)*(1 - I*c*x))])/(2*d^2)
```

Rule 4876

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4850

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[((a + b
```

```
*ArcTan[c*x])^(p - 1)*ArcTanh[1 - 2/(1 + I*c*x)])/(1 + c^2*x^2), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4988

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTan[c*x])^p)/(d + e
*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTan[c*x])^p)/(d + e*x^2
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 4858

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^2/((d_) + (e_.)*(x_)), x_Symbol] :>
-Simp[((a + b*ArcTan[c*x])^2*Log[2/(1 - I*c*x)])/e, x] + (Simp[((a + b*ArcT
an[c*x])^2*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x] + Simp[(I*
b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 - I*c*x)])/e, x] - Simp[(I*b*(a +
 b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/
e, x] - Simp[(b^2*PolyLog[3, 1 - 2/(1 - I*c*x)])/(2*e), x] + Simp[(b^2*Poly
Log[3, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/(2*e), x]) /; FreeQ[
{a, b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^2(d+ex)}\,dx = \int \left(\frac{\left(a+b\tan^{-1}(cx)\right)^2}{dx^2} - \frac{e\left(a+b\tan^{-1}(cx)\right)^2}{d^2x} + \frac{e^2\left(a+b\tan^{-1}(cx)\right)^2}{d^2(d+ex)}\right)dx$$

$$= \frac{\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^2}\,dx}{d} - \frac{e\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x}\,dx}{d^2} + \frac{e^2\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{d+ex}\,dx}{d^2}$$

$$= -\frac{\left(a+b\tan^{-1}(cx)\right)^2}{dx} - \frac{2e\left(a+b\tan^{-1}(cx)\right)^2\tanh^{-1}\left(1-\frac{2}{1+icx}\right)}{d^2} - \frac{e\left(a+b\tan^{-1}(cx)\right)^2\log\left(\right.}{d^2}$$

$$= -\frac{ic\left(a+b\tan^{-1}(cx)\right)^2}{d} - \frac{\left(a+b\tan^{-1}(cx)\right)^2}{dx} - \frac{2e\left(a+b\tan^{-1}(cx)\right)^2\tanh^{-1}\left(1-\frac{2}{1+icx}\right)}{d^2} - \frac{e}{}$$

$$= -\frac{ic\left(a+b\tan^{-1}(cx)\right)^2}{d} - \frac{\left(a+b\tan^{-1}(cx)\right)^2}{dx} - \frac{2e\left(a+b\tan^{-1}(cx)\right)^2\tanh^{-1}\left(1-\frac{2}{1+icx}\right)}{d^2} - \frac{e}{}$$

$$= -\frac{ic\left(a+b\tan^{-1}(cx)\right)^2}{d} - \frac{\left(a+b\tan^{-1}(cx)\right)^2}{dx} - \frac{2e\left(a+b\tan^{-1}(cx)\right)^2\tanh^{-1}\left(1-\frac{2}{1+icx}\right)}{d^2} - \frac{e}{}$$

Mathematica [F] time = 120.59, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^2(d+ex)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcTan[c*x])^2/(x^2*(d + e*x)),x]
```

```
[Out] Integrate[(a + b*ArcTan[c*x])^2/(x^2*(d + e*x)), x]
```

Maple [C] time = 6.674, size = 40579, normalized size = 85.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^2/x^2/(e*x+d),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$a^2\left(\frac{e\log(ex+d)}{d^2} - \frac{e\log(x)}{d^2} - \frac{1}{dx}\right) - \frac{4\,b^2\arctan(cx)^2 - b^2\log\left(c^2x^2+1\right)^2 - dx\int \frac{12\left(b^2c^2dx^2+b^2d\right)\arctan(cx)^2+\left(b^2c^2dx^2+b^2d\right)\log}{}}{16\,dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^2/(e*x+d),x, algorithm="maxima")
```

```
[Out] a^2*(e*log(e*x + d)/d^2 - e*log(x)/d^2 - 1/(d*x)) - 1/16*(4*b^2*arctan(c*x)
^2 - b^2*log(c^2*x^2 + 1)^2 - 16*d*x*integrate(1/16*(12*(b^2*c^2*d*x^2 + b^
2*d)*arctan(c*x)^2 + (b^2*c^2*d*x^2 + b^2*d)*log(c^2*x^2 + 1)^2 + 8*(b^2*c*
d*x + 4*a*b*d + (4*a*b*c^2*d + b^2*c*e)*x^2)*arctan(c*x) - 4*(b^2*c^2*e*x^3
 + b^2*c^2*d*x^2)*log(c^2*x^2 + 1))/(c^2*d*e*x^5 + c^2*d^2*x^4 + d*e*x^3 +
d^2*x^2), x))/(d*x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2\arctan(cx)^2 + 2\,ab\arctan(cx) + a^2}{ex^3 + dx^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^2/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*arctan(c*x)^2 + 2*a*b*arctan(c*x) + a^2)/(e*x^3 + d*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**2/x**2/(e*x+d),x)
```

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \arctan(cx) + a)^2}{(ex + d)x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^2/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)^2/((e*x + d)*x^2), x)
```

3.147 $\int \frac{(a+b\tan^{-1}(cx))^2}{x^3(d+ex)}\,dx$

Optimal. Leaf size=591

$$-\frac{ibe^2\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)\left(a+b\tan^{-1}(cx)\right)}{d^3}-\frac{ibe^2\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{d^3}+\frac{ibe^2\text{PolyLog}\left(2,-1+\frac{}{1+}\right.}{d^3}$$

```
[Out] -((b*c*(a + b*ArcTan[c*x]))/(d*x)) - (c^2*(a + b*ArcTan[c*x])^2)/(2*d) + (I
*c*e*(a + b*ArcTan[c*x])^2)/d^2 - (a + b*ArcTan[c*x])^2/(2*d*x^2) + (e*(a +
 b*ArcTan[c*x])^2)/(d^2*x) + (2*e^2*(a + b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1
+ I*c*x)])/d^3 + (b^2*c^2*Log[x])/d + (e^2*(a + b*ArcTan[c*x])^2*Log[2/(1 -
 I*c*x)])/d^3 - (e^2*(a + b*ArcTan[c*x])^2*Log[(2*c*(d + e*x))/((c*d + I*e)
*(1 - I*c*x))])/d^3 - (b^2*c^2*Log[1 + c^2*x^2])/(2*d) - (2*b*c*e*(a + b*Ar
cTan[c*x])*Log[2 - 2/(1 - I*c*x)])/d^2 - (I*b*e^2*(a + b*ArcTan[c*x])*PolyL
og[2, 1 - 2/(1 - I*c*x)])/d^3 + (I*b^2*c*e*PolyLog[2, -1 + 2/(1 - I*c*x)])/
d^2 - (I*b*e^2*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)])/d^3 + (I*
b*e^2*(a + b*ArcTan[c*x])*PolyLog[2, -1 + 2/(1 + I*c*x)])/d^3 + (I*b*e^2*(a
 + b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))]
)/d^3 + (b^2*e^2*PolyLog[3, 1 - 2/(1 - I*c*x)])/(2*d^3) - (b^2*e^2*PolyLog[
3, 1 - 2/(1 + I*c*x)])/(2*d^3) + (b^2*e^2*PolyLog[3, -1 + 2/(1 + I*c*x)])/(
2*d^3) - (b^2*e^2*PolyLog[3, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))]
)/(2*d^3)
```

Rubi [A] time = 0.84186, antiderivative size = 591, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 16, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.762, Rules used = {4876, 4852, 4918, 266, 36, 29, 31, 4884, 4924, 4868, 2447, 4850, 4988, 4994, 6610, 4858}

$$-\frac{ibe^2\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)\left(a+b\tan^{-1}(cx)\right)}{d^3}-\frac{ibe^2\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{d^3}+\frac{ibe^2\text{PolyLog}\left(2,-1+\frac{}{1+}\right.}{d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^2/(x^3*(d + e*x)),x]
```

```
[Out] -((b*c*(a + b*ArcTan[c*x]))/(d*x)) - (c^2*(a + b*ArcTan[c*x])^2)/(2*d) + (I
*c*e*(a + b*ArcTan[c*x])^2)/d^2 - (a + b*ArcTan[c*x])^2/(2*d*x^2) + (e*(a +
 b*ArcTan[c*x])^2)/(d^2*x) + (2*e^2*(a + b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1
+ I*c*x)])/d^3 + (b^2*c^2*Log[x])/d + (e^2*(a + b*ArcTan[c*x])^2*Log[2/(1 -
 I*c*x)])/d^3 - (e^2*(a + b*ArcTan[c*x])^2*Log[(2*c*(d + e*x))/((c*d + I*e)
```

```
*(1 - I*c*x))])/d^3 - (b^2*c^2*Log[1 + c^2*x^2])/(2*d) - (2*b*c*e*(a + b*Ar
cTan[c*x])*Log[2 - 2/(1 - I*c*x)])/d^2 - (I*b*e^2*(a + b*ArcTan[c*x])*PolyL
og[2, 1 - 2/(1 - I*c*x)])/d^3 + (I*b^2*c*e*PolyLog[2, -1 + 2/(1 - I*c*x)])/
d^2 - (I*b*e^2*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)])/d^3 + (I*
b*e^2*(a + b*ArcTan[c*x])*PolyLog[2, -1 + 2/(1 + I*c*x)])/d^3 + (I*b*e^2*(a
 + b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))]
)/d^3 + (b^2*e^2*PolyLog[3, 1 - 2/(1 - I*c*x)])/(2*d^3) - (b^2*e^2*PolyLog[
3, 1 - 2/(1 + I*c*x)])/(2*d^3) + (b^2*e^2*PolyLog[3, -1 + 2/(1 + I*c*x)])/(
2*d^3) - (b^2*e^2*PolyLog[3, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))]
)/(2*d^3)
```

Rule 4876

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4918

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/(d + e*x^2),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4850

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTan[c*x])^(p - 1)*ArcTanh[1 - 2/(1 + I*c*x)])/(1 + c^2*x^2), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4988

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTan[c*x])^p)/(d + e
*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTan[c*x])^p)/(d + e*x^2
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 4858

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^2/((d_) + (e_.)*(x_)), x_Symbol] :>
-Simp[((a + b*ArcTan[c*x])^2*Log[2/(1 - I*c*x)])/e, x] + (Simp[((a + b*ArcT
an[c*x])^2*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x] + Simp[(I*
b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 - I*c*x)])/e, x] - Simp[(I*b*(a +
 b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/
e, x] - Simp[(b^2*PolyLog[3, 1 - 2/(1 - I*c*x)])/(2*e), x] + Simp[(b^2*Poly
Log[3, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/(2*e), x]) /; FreeQ[
{a, b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^3(d+ex)}\,dx = \int \left(\frac{\left(a+b\tan^{-1}(cx)\right)^2}{dx^3} - \frac{e\left(a+b\tan^{-1}(cx)\right)^2}{d^2x^2} + \frac{e^2\left(a+b\tan^{-1}(cx)\right)^2}{d^3x} - \frac{e^3\left(a+b\tan^{-1}(cx)\right)^2}{d^3(d+ex)}\right.$$

$$= \frac{\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^3}dx}{d} - \frac{e\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^2}dx}{d^2} + \frac{e^2\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x}dx}{d^3} - \frac{e^3\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{d+ex}dx}{d^3}$$

$$= -\frac{\left(a+b\tan^{-1}(cx)\right)^2}{2dx^2} + \frac{e\left(a+b\tan^{-1}(cx)\right)^2}{d^2x} + \frac{2e^2\left(a+b\tan^{-1}(cx)\right)^2\tanh^{-1}\left(1-\frac{2}{1+icx}\right)}{d^3} + \frac{e^2}{}$$

$$= \frac{ice\left(a+b\tan^{-1}(cx)\right)^2}{d^2} - \frac{\left(a+b\tan^{-1}(cx)\right)^2}{2dx^2} + \frac{e\left(a+b\tan^{-1}(cx)\right)^2}{d^2x} + \frac{2e^2\left(a+b\tan^{-1}(cx)\right)^2 \mathrm{t}}{d^3}$$

$$= -\frac{bc\left(a+b\tan^{-1}(cx)\right)}{dx} - \frac{c^2\left(a+b\tan^{-1}(cx)\right)^2}{2d} + \frac{ice\left(a+b\tan^{-1}(cx)\right)^2}{d^2} - \frac{\left(a+b\tan^{-1}(cx)\right)^2}{2dx^2}$$

$$= -\frac{bc\left(a+b\tan^{-1}(cx)\right)}{dx} - \frac{c^2\left(a+b\tan^{-1}(cx)\right)^2}{2d} + \frac{ice\left(a+b\tan^{-1}(cx)\right)^2}{d^2} - \frac{\left(a+b\tan^{-1}(cx)\right)^2}{2dx^2}$$

$$= -\frac{bc\left(a+b\tan^{-1}(cx)\right)}{dx} - \frac{c^2\left(a+b\tan^{-1}(cx)\right)^2}{2d} + \frac{ice\left(a+b\tan^{-1}(cx)\right)^2}{d^2} - \frac{\left(a+b\tan^{-1}(cx)\right)^2}{2dx^2}$$

$$= -\frac{bc\left(a+b\tan^{-1}(cx)\right)}{dx} - \frac{c^2\left(a+b\tan^{-1}(cx)\right)^2}{2d} + \frac{ice\left(a+b\tan^{-1}(cx)\right)^2}{d^2} - \frac{\left(a+b\tan^{-1}(cx)\right)^2}{2dx^2}$$

Mathematica [F] time = 180.003, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcTan[c*x])^2/(x^3*(d + e*x)),x]
```

```
[Out] $Aborted
```

Maple [C] time = 14.738, size = 2861, normalized size = 4.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arctan(c*x))^2/x^3/(e*x+d),x)

[Out] 1/2*I*b^2/d^3*e^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x
^2+1)+1))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*arc
tan(c*x)^2+1/2*I*b^2/d^3*e^2*Pi*csgn(I*(-I*(1+I*c*x)^2/(c^2*x^2+1)*e+c*d*(1
+I*c*x)^2/(c^2*x^2+1)+I*e+d*c))*csgn(I*(-I*(1+I*c*x)^2/(c^2*x^2+1)*e+c*d*(1
+I*c*x)^2/(c^2*x^2+1)+I*e+d*c)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2
-1/2*I*b^2/d^3*e^2*Pi*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*((1+I*c*x)
^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2-1/2*I*b^2/d^
3*e^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1
)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2+1/2*I*b^2/d^3*e^2*Pi*csgn
(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*(-I*(1+I*c*x)^2/(c^2*x^2+1)*e+c*d*(1
+I*c*x)^2/(c^2*x^2+1)+I*e+d*c)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2
-1/2*I*b^2/d^3*e^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*
x^2+1)+1))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*
arctan(c*x)^2+a^2/d^2*e/x-1/2*b^2*arctan(c*x)^2/d/x^2+a^2/d^3*e^2*ln(c*x)+2
*b^2/d^3*e^2*polylog(3,(1+I*c*x)/(c^2*x^2+1)^(1/2))+2*b^2/d^3*e^2*polylog(3
,-(1+I*c*x)/(c^2*x^2+1)^(1/2))-a^2/d^3*e^2*ln(c*e*x+c*d)+c^2*b^2/d*ln((1+I*
c*x)/(c^2*x^2+1)^(1/2)-1)-c*b^2*e^2*arctan(c*x)*polylog(2,(I*e-d*c)/(d*c+I*
e)*(1+I*c*x)^2/(c^2*x^2+1))/d^2/(e+I*d*c)+I*b^2*e^3*arctan(c*x)*polylog(2,(
I*e-d*c)/(d*c+I*e)*(1+I*c*x)^2/(c^2*x^2+1))/d^3/(e+I*d*c)+I*a*b/d^3*e^2*ln(
c*x)*ln(1+I*c*x)+I*a*b/d^3*e^2*ln(c*e*x+c*d)*ln((I*e+e*c*x)/(I*e-d*c))-1/2*
I*b^2/d^3*e^2*Pi*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+
1))^2*arctan(c*x)^2+1/2*I*b^2/d^3*e^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)
/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*arctan(c*x)^2-I*a*b/d^3*e^2*ln(c*x)*ln(1-I*
c*x)-I*a*b/d^3*e^2*ln(c*e*x+c*d)*ln((I*e-e*c*x)/(d*c+I*e))-1/2*I*c*b^2*e^2*
polylog(3,(I*e-d*c)/(d*c+I*e)*(1+I*c*x)^2/(c^2*x^2+1))/d^2/(e+I*d*c)+c^2*b^
2/d*ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))-1/2*c^2*b^2/d*arctan(c*x)^2-1/2*a^2/d
/x^2-2*a*b*arctan(c*x)/d^3*e^2*ln(c*e*x+c*d)+2*a*b*arctan(c*x)/d^3*e^2*ln(c
*x)+2*a*b*arctan(c*x)/d^2*e/x-2*c*b^2/d^2*ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))
*arctan(c*x)*e+c*a*b/d^2*e*ln(c^2*x^2+1)-2*c*a*b/d^2*e*ln(c*x)+I*c*b^2/d^2*
e*arctan(c*x)^2+I*a*b/d^3*e^2*dilog(1+I*c*x)+I*a*b/d^3*e^2*dilog((I*e+e*c*x
)/(I*e-d*c))-b^2*e^3*arctan(c*x)^2*ln(1-(I*e-d*c)/(d*c+I*e)*(1+I*c*x)^2/(c^
2*x^2+1))/d^3/(e+I*d*c)-2*I*b^2/d^3*e^2*arctan(c*x)*polylog(2,(1+I*c*x)/(c^
2*x^2+1)^(1/2))-2*I*b^2/d^3*e^2*arctan(c*x)*polylog(2,-(1+I*c*x)/(c^2*x^2+1
)^(1/2))+2*I*c*b^2/d^2*dilog(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))*e-2*I*c*b^2/d^2
*dilog((1+I*c*x)/(c^2*x^2+1)^(1/2))*e-I*a*b/d^3*e^2*dilog((I*e-e*c*x)/(d*c+
I*e))-I*a*b/d^3*e^2*dilog(1-I*c*x)+1/2*I*b^2/d^3*e^2*Pi*arctan(c*x)^2-c*a*b
/d/x-I*c*b^2*e^2*arctan(c*x)^2*ln(1-(I*e-d*c)/(d*c+I*e)*(1+I*c*x)^2/(c^2*x^
2+1))/d^2/(e+I*d*c)-1/2*I*b^2/d^3*e^2*Pi*csgn(I*(-I*(1+I*c*x)^2/(c^2*x^2+1)
*e+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+d*c)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*arct
an(c*x)^2+1/2*I*b^2/d^3*e^2*Pi*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^
2/(c^2*x^2+1)+1))^3*arctan(c*x)^2-c*b^2*arctan(c*x)/x/d-c^2*a*b/d*arctan(c*
x)-b^2/d^3*e^2*arctan(c*x)^2*ln((1+I*c*x)^2/(c^2*x^2+1)-1)+b^2*arctan(c*x)^

```
2/d^2*e/x-1/2*b^2*e^3*polylog(3,(I*e-d*c)/(d*c+I*e)*(1+I*c*x)^2/(c^2*x^2+1)
)/d^3/(e+I*d*c)-a*b*arctan(c*x)/d/x^2+b^2*arctan(c*x)^2/d^3*e^2*ln(c*x)-b^2
*arctan(c*x)^2/d^3*e^2*ln(c*e*x+c*d)+b^2/d^3*e^2*arctan(c*x)^2*ln(-I*(1+I*c
*x)^2/(c^2*x^2+1)*e+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+d*c)+b^2/d^3*e^2*arctan
(c*x)^2*ln(1-(1+I*c*x)/(c^2*x^2+1)^(1/2))+b^2/d^3*e^2*arctan(c*x)^2*ln(1+(1
+I*c*x)/(c^2*x^2+1)^(1/2))-I*c^2*b^2*arctan(c*x)/d-1/2*I*b^2/d^3*e^2*Pi*csg
n(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*(-I*(1+I*c*x)^2/(c^2*x^2+1)*e+c*d*(
1+I*c*x)^2/(c^2*x^2+1)+I*e+d*c))*csgn(I*(-I*(1+I*c*x)^2/(c^2*x^2+1)*e+c*d*(
1+I*c*x)^2/(c^2*x^2+1)+I*e+d*c)/((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)^2+
1/2*I*b^2/d^3*e^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I/((1+I*c*x)^
2/(c^2*x^2+1)+1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+
1)+1))*arctan(c*x)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2}a^2\left(\frac{2e^2\log(ex+d)}{d^3}-\frac{2e^2\log(x)}{d^3}-\frac{2ex-d}{d^2x^2}\right)+\frac{2d^2x^2\int\frac{12\left(b^2c^2d^2x^2+b^2d^2\right)\arctan(cx)^2+\left(b^2c^2d^2x^2+b^2d^2\right)\log\left(c^2x^2+1\right)^2-4\left(2b^2ce^2x^3-\right.}{c^2d^2e}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^3/(e*x+d),x, algorithm="maxima")
```

```
[Out] -1/2*a^2*(2*e^2*log(e*x + d)/d^3 - 2*e^2*log(x)/d^3 - (2*e*x - d)/(d^2*x^2)
) + 1/32*(32*d^2*x^2*integrate(1/16*(12*(b^2*c^2*d^2*x^2 + b^2*d^2)*arctan(
c*x)^2 + (b^2*c^2*d^2*x^2 + b^2*d^2)*log(c^2*x^2 + 1)^2 - 4*(2*b^2*c*e^2*x^
3 - b^2*c*d^2*x - 8*a*b*d^2 - (8*a*b*c^2*d^2 - b^2*c*d*e)*x^2)*arctan(c*x)
+ 2*(2*b^2*c^2*e^2*x^4 + b^2*c^2*d*e*x^3 - b^2*c^2*d^2*x^2)*log(c^2*x^2 + 1
))/(c^2*d^2*e*x^6 + c^2*d^3*x^5 + d^2*e*x^4 + d^3*x^3), x) + 4*(2*b^2*e*x -
 b^2*d)*arctan(c*x)^2 - (2*b^2*e*x - b^2*d)*log(c^2*x^2 + 1)^2)/(d^2*x^2)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2\arctan(cx)^2+2ab\arctan(cx)+a^2}{ex^4+dx^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^3/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*arctan(c*x)^2 + 2*a*b*arctan(c*x) + a^2)/(e*x^4 + d*x^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**2/x**3/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \arctan(cx) + a)^2}{(ex + d)x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^3/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)^2/((e*x + d)*x^3), x)
```

3.148 $\int \frac{1}{(d+ex)\left(a+b\tan^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left(\frac{1}{(d+ex)\left(a+b\tan^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable[1/((d + e*x)*(a + b*ArcTan[c*x])), x]
```

Rubi [A] time = 0.0313904, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{(d+ex)\left(a+b\tan^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((d + e*x)*(a + b*ArcTan[c*x])),x]
```

```
[Out] Defer[Int][1/((d + e*x)*(a + b*ArcTan[c*x])), x]
```

Rubi steps

$$\int \frac{1}{(d+ex)\left(a+b\tan^{-1}(cx)\right)}\,dx = \int \frac{1}{(d+ex)\left(a+b\tan^{-1}(cx)\right)}\,dx$$

Mathematica [A] time = 0.51471, size = 0, normalized size = 0.

$$\int \frac{1}{(d+ex)\left(a+b\tan^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((d + e*x)*(a + b*ArcTan[c*x])),x]
```

```
[Out] Integrate[1/((d + e*x)*(a + b*ArcTan[c*x])), x]
```

Maple [A] time = 0.622, size = 0, normalized size = 0.

$$\int \frac{1}{(ex+d)\,(a+b\arctan\,(cx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(a+b*arctan(c*x)),x)
```

```
[Out] int(1/(e*x+d)/(a+b*arctan(c*x)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(ex+d)(b\arctan\,(cx)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(1/((e*x + d)*(b*arctan(c*x) + a)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{aex+ad+(bex+bd)\arctan\,(cx)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] integral(1/(a*e*x + a*d + (b*e*x + b*d)*arctan(c*x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a + b\,\text{atan}\,(cx))\,(d + ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*atan(c*x)),x)
```

```
[Out] Integral(1/((a + b*atan(c*x))*(d + e*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(ex + d)(b\arctan\,(cx) + a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] integrate(1/((e*x + d)*(b*arctan(c*x) + a)), x)
```

3.149 $\int x^3 \left(c + a^2 c x^2\right) \tan^{-1}(ax) \, dx$

Optimal. Leaf size=69

$$\frac{1}{6}a^2cx^6\tan^{-1}(ax) + \frac{cx}{12a^3} - \frac{c\tan^{-1}(ax)}{12a^4} - \frac{1}{30}acx^5 - \frac{cx^3}{36a} + \frac{1}{4}cx^4\tan^{-1}(ax)$$

```
[Out] (c*x)/(12*a^3) - (c*x^3)/(36*a) - (a*c*x^5)/30 - (c*ArcTan[a*x])/(12*a^4) +
 (c*x^4*ArcTan[a*x])/4 + (a^2*c*x^6*ArcTan[a*x])/6
```

Rubi [A] time = 0.0855232, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4950, 4852, 302, 203}

$$\frac{1}{6}a^2cx^6\tan^{-1}(ax) + \frac{cx}{12a^3} - \frac{c\tan^{-1}(ax)}{12a^4} - \frac{1}{30}acx^5 - \frac{cx^3}{36a} + \frac{1}{4}cx^4\tan^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(c + a^2*c*x^2)*ArcTan[a*x],x]
```

```
[Out] (c*x)/(12*a^3) - (c*x^3)/(36*a) - (a*c*x^5)/30 - (c*ArcTan[a*x])/(12*a^4) +
 (c*x^4*ArcTan[a*x])/4 + (a^2*c*x^6*ArcTan[a*x])/6
```

Rule 4950

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int x^3\left(c+a^2cx^2\right)\tan^{-1}(ax)\,dx = c\int x^3\tan^{-1}(ax)\,dx+\left(a^2c\right)\int x^5\tan^{-1}(ax)\,dx$$

$$= \frac{1}{4}cx^4\tan^{-1}(ax)+\frac{1}{6}a^2cx^6\tan^{-1}(ax)-\frac{1}{4}(ac)\int\frac{x^4}{1+a^2x^2}\,dx-\frac{1}{6}\left(a^3c\right)\int\frac{x^6}{1+a^2x^2}\,dx$$

$$= \frac{1}{4}cx^4\tan^{-1}(ax)+\frac{1}{6}a^2cx^6\tan^{-1}(ax)-\frac{1}{4}(ac)\int\left(-\frac{1}{a^4}+\frac{x^2}{a^2}+\frac{1}{a^4\left(1+a^2x^2\right)}\right)dx-\frac{1}{6}\left(a^3c\right.$$

$$= \frac{cx}{12a^3}-\frac{cx^3}{36a}-\frac{1}{30}acx^5+\frac{1}{4}cx^4\tan^{-1}(ax)+\frac{1}{6}a^2cx^6\tan^{-1}(ax)+\frac{c\int\frac{1}{1+a^2x^2}\,dx}{6a^3}-\frac{c\int\frac{1}{1+a^2x}}{4a^3}$$

$$= \frac{cx}{12a^3}-\frac{cx^3}{36a}-\frac{1}{30}acx^5-\frac{c\tan^{-1}(ax)}{12a^4}+\frac{1}{4}cx^4\tan^{-1}(ax)+\frac{1}{6}a^2cx^6\tan^{-1}(ax)$$

Mathematica [A] time = 0.0054891, size = 69, normalized size = 1.

$$\frac{1}{6}a^2cx^6\tan^{-1}(ax)+\frac{cx}{12a^3}-\frac{c\tan^{-1}(ax)}{12a^4}-\frac{1}{30}acx^5-\frac{cx^3}{36a}+\frac{1}{4}cx^4\tan^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(c + a^2*c*x^2)*ArcTan[a*x],x]
```

```
[Out] (c*x)/(12*a^3) - (c*x^3)/(36*a) - (a*c*x^5)/30 - (c*ArcTan[a*x])/(12*a^4) +
 (c*x^4*ArcTan[a*x])/4 + (a^2*c*x^6*ArcTan[a*x])/6
```

Maple [A] time = 0.023, size = 58, normalized size = 0.8

$$\frac{cx}{12\,a^3}-\frac{cx^3}{36\,a}-\frac{acx^5}{30}-\frac{c\arctan(ax)}{12\,a^4}+\frac{cx^4\arctan(ax)}{4}+\frac{a^2cx^6\arctan(ax)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a^2*c*x^2+c)*arctan(a*x),x)
```

```
[Out] 1/12*c*x/a^3-1/36*c*x^3/a-1/30*a*c*x^5-1/12*c*arctan(a*x)/a^4+1/4*c*x^4*arc
tan(a*x)+1/6*a^2*c*x^6*arctan(a*x)
```

Maxima [A] time = 1.48318, size = 86, normalized size = 1.25

$$-\frac{1}{180}\,a\left(\frac{6\,a^4cx^5+5\,a^2cx^3-15\,cx}{a^4}+\frac{15\,c\arctan\left(ax\right)}{a^5}\right)+\frac{1}{12}\left(2\,a^2cx^6+3\,cx^4\right)\arctan\left(ax\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a^2*c*x^2+c)*arctan(a*x),x, algorithm="maxima")
```

```
[Out] -1/180*a*((6*a^4*c*x^5 + 5*a^2*c*x^3 - 15*c*x)/a^4 + 15*c*arctan(a*x)/a^5)
+ 1/12*(2*a^2*c*x^6 + 3*c*x^4)*arctan(a*x)
```

Fricas [A] time = 1.67906, size = 135, normalized size = 1.96

$$-\frac{6\,a^5cx^5+5\,a^3cx^3-15\,acx-15\left(2\,a^6cx^6+3\,a^4cx^4-c\right)\arctan\left(ax\right)}{180\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a^2*c*x^2+c)*arctan(a*x),x, algorithm="fricas")
```

```
[Out] -1/180*(6*a^5*c*x^5 + 5*a^3*c*x^3 - 15*a*c*x - 15*(2*a^6*c*x^6 + 3*a^4*c*x^
4 - c)*arctan(a*x))/a^4
```

Sympy [A] time = 1.85988, size = 65, normalized size = 0.94

$$\begin{cases}\frac{a^2cx^6\operatorname{atan}(ax)}{6}-\frac{acx^5}{30}+\frac{cx^4\operatorname{atan}(ax)}{4}-\frac{cx^3}{36a}+\frac{cx}{12a^3}-\frac{c\operatorname{atan}(ax)}{12a^4} & \text{for}\ a\neq 0\\ 0 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a**2*c*x**2+c)*atan(a*x),x)
```

```
[Out] Piecewise((a**2*c*x**6*atan(a*x)/6 - a*c*x**5/30 + c*x**4*atan(a*x)/4 - c*x
**3/(36*a) + c*x/(12*a**3) - c*atan(a*x)/(12*a**4), Ne(a, 0)), (0, True))
```

Giac [A] time = 1.12661, size = 86, normalized size = 1.25

$$\frac{1}{12}\left(2\,a^2cx^6+3\,cx^4\right)\arctan\left(ax\right)-\frac{c\arctan\left(ax\right)}{12\,a^4}-\frac{6\,a^{11}cx^5+5\,a^9cx^3-15\,a^7cx}{180\,a^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a^2*c*x^2+c)*arctan(a*x),x, algorithm="giac")
```

```
[Out] 1/12*(2*a^2*c*x^6 + 3*c*x^4)*arctan(a*x) - 1/12*c*arctan(a*x)/a^4 - 1/180*(
6*a^11*c*x^5 + 5*a^9*c*x^3 - 15*a^7*c*x)/a^10
```

3.150 $\int x^2 \left(c + a^2 c x^2\right) \tan^{-1}(ax)\, dx$

Optimal. Leaf size=66

$$\frac{c \log \left(a^2 x^2+1\right)}{15 a^3}+\frac{1}{5} a^2 c x^5 \tan^{-1}(a x)-\frac{1}{20} a c x^4-\frac{c x^2}{15 a}+\frac{1}{3} c x^3 \tan^{-1}(a x)$$

```
[Out] -(c*x^2)/(15*a) - (a*c*x^4)/20 + (c*x^3*ArcTan[a*x])/3 + (a^2*c*x^5*ArcTan[
a*x])/5 + (c*Log[1 + a^2*x^2])/(15*a^3)
```

Rubi [A] time = 0.0948406, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4950, 4852, 266, 43}

$$\frac{c \log \left(a^2 x^2+1\right)}{15 a^3}+\frac{1}{5} a^2 c x^5 \tan^{-1}(a x)-\frac{1}{20} a c x^4-\frac{c x^2}{15 a}+\frac{1}{3} c x^3 \tan^{-1}(a x)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(c + a^2*c*x^2)*ArcTan[a*x],x]
```

```
[Out] -(c*x^2)/(15*a) - (a*c*x^4)/20 + (c*x^3*ArcTan[a*x])/3 + (a^2*c*x^5*ArcTan[
a*x])/5 + (c*Log[1 + a^2*x^2])/(15*a^3)
```

Rule 4950

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int x^2\left(c+a^2cx^2\right)\tan^{-1}(ax)\,dx &= c\int x^2\tan^{-1}(ax)\,dx+\left(a^2c\right)\int x^4\tan^{-1}(ax)\,dx\\
&= \frac{1}{3}cx^3\tan^{-1}(ax)+\frac{1}{5}a^2cx^5\tan^{-1}(ax)-\frac{1}{3}(ac)\int\frac{x^3}{1+a^2x^2}\,dx-\frac{1}{5}\left(a^3c\right)\int\frac{x^5}{1+a^2x^2}\,dx\\
&= \frac{1}{3}cx^3\tan^{-1}(ax)+\frac{1}{5}a^2cx^5\tan^{-1}(ax)-\frac{1}{6}(ac)\,\text{Subst}\left(\int\frac{x}{1+a^2x}\,dx,x,x^2\right)-\frac{1}{10}\left(a^3c\right)\text{Sul}\\
&= \frac{1}{3}cx^3\tan^{-1}(ax)+\frac{1}{5}a^2cx^5\tan^{-1}(ax)-\frac{1}{6}(ac)\,\text{Subst}\left(\int\left(\frac{1}{a^2}-\frac{1}{a^2\left(1+a^2x\right)}\right)dx,x,x^2\right)-\\
&= -\frac{cx^2}{15a}-\frac{1}{20}acx^4+\frac{1}{3}cx^3\tan^{-1}(ax)+\frac{1}{5}a^2cx^5\tan^{-1}(ax)+\frac{c\log\left(1+a^2x^2\right)}{15a^3}
\end{aligned}$$

Mathematica [A] time = 0.0224866, size = 66, normalized size = 1.

$$\frac{c\log\left(a^2x^2+1\right)}{15a^3}+\frac{1}{5}a^2cx^5\tan^{-1}(ax)-\frac{1}{20}acx^4-\frac{cx^2}{15a}+\frac{1}{3}cx^3\tan^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(c + a^2*c*x^2)*ArcTan[a*x],x]
```

```
[Out] -(c*x^2)/(15*a) - (a*c*x^4)/20 + (c*x^3*ArcTan[a*x])/3 + (a^2*c*x^5*ArcTan[
a*x])/5 + (c*Log[1 + a^2*x^2])/(15*a^3)
```

Maple [A] time = 0.023, size = 57, normalized size = 0.9

$$-\frac{cx^2}{15\,a}-\frac{acx^4}{20}+\frac{cx^3\arctan\left(ax\right)}{3}+\frac{a^2cx^5\arctan\left(ax\right)}{5}+\frac{c\ln\left(a^2x^2+1\right)}{15\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a^2*c*x^2+c)*arctan(a*x),x)
```

```
[Out] -1/15*c*x^2/a-1/20*a*c*x^4+1/3*c*x^3*arctan(a*x)+1/5*a^2*c*x^5*arctan(a*x)+
1/15*c*ln(a^2*x^2+1)/a^3
```

Maxima [A] time = 0.987573, size = 85, normalized size = 1.29

$$-\frac{1}{60}\,a\left(\frac{3\,a^2cx^4+4\,cx^2}{a^2}-\frac{4\,c\log\left(a^2x^2+1\right)}{a^4}\right)+\frac{1}{15}\left(3\,a^2cx^5+5\,cx^3\right)\arctan\left(ax\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)*arctan(a*x),x, algorithm="maxima")
```

```
[Out] -1/60*a*((3*a^2*c*x^4 + 4*c*x^2)/a^2 - 4*c*log(a^2*x^2 + 1)/a^4) + 1/15*(3*
a^2*c*x^5 + 5*c*x^3)*arctan(a*x)
```

Fricas [A] time = 1.63626, size = 143, normalized size = 2.17

$$-\frac{3\,a^4cx^4+4\,a^2cx^2-4\left(3\,a^5cx^5+5\,a^3cx^3\right)\arctan\left(ax\right)-4\,c\log\left(a^2x^2+1\right)}{60\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)*arctan(a*x),x, algorithm="fricas")
```

```
[Out] -1/60*(3*a^4*c*x^4 + 4*a^2*c*x^2 - 4*(3*a^5*c*x^5 + 5*a^3*c*x^3)*arctan(a*x
) - 4*c*log(a^2*x^2 + 1))/a^3
```

Sympy [A] time = 1.36341, size = 61, normalized size = 0.92

$$\begin{cases} \frac{a^2cx^5\operatorname{atan}(ax)}{5} - \frac{acx^4}{20} + \frac{cx^3\operatorname{atan}(ax)}{3} - \frac{cx^2}{15a} + \frac{c\log\left(x^2+\frac{1}{a^2}\right)}{15a^3} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a**2*c*x**2+c)*atan(a*x),x)
```

```
[Out] Piecewise((a**2*c*x**5*atan(a*x)/5 - a*c*x**4/20 + c*x**3*atan(a*x)/3 - c*x
**2/(15*a) + c*log(x**2 + a**(-2))/(15*a**3), Ne(a, 0)), (0, True))
```

Giac [A] time = 1.15673, size = 85, normalized size = 1.29

$$\frac{1}{15}\left(3\,a^2cx^5 + 5\,cx^3\right)\arctan(ax) + \frac{c\log\left(a^2x^2+1\right)}{15\,a^3} - \frac{3\,a^5cx^4 + 4\,a^3cx^2}{60\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)*arctan(a*x),x, algorithm="giac")
```

```
[Out] 1/15*(3*a^2*c*x^5 + 5*c*x^3)*arctan(a*x) + 1/15*c*log(a^2*x^2 + 1)/a^3 - 1/
60*(3*a^5*c*x^4 + 4*a^3*c*x^2)/a^4
```

3.151 $\int x\left(c+a^2cx^2\right)\tan^{-1}(ax)\,dx$

Optimal. Leaf size=42

$$\frac{c\left(a^2x^2+1\right)^2\tan^{-1}(ax)}{4a^2}-\frac{1}{12}acx^3-\frac{cx}{4a}$$

```
[Out] -(c*x)/(4*a) - (a*c*x^3)/12 + (c*(1 + a^2*x^2)^2*ArcTan[a*x])/(4*a^2)
```

Rubi [A] time = 0.0249456, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {4930}

$$\frac{c\left(a^2x^2+1\right)^2\tan^{-1}(ax)}{4a^2}-\frac{1}{12}acx^3-\frac{cx}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[x*(c + a^2*c*x^2)*ArcTan[a*x],x]
```

```
[Out] -(c*x)/(4*a) - (a*c*x^3)/12 + (c*(1 + a^2*x^2)^2*ArcTan[a*x])/(4*a^2)
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rubi steps

$$\begin{aligned}\int x\left(c+a^2cx^2\right)\tan^{-1}(ax)\,dx&=\frac{c\left(1+a^2x^2\right)^2\tan^{-1}(ax)}{4a^2}-\frac{\int\left(c+a^2cx^2\right)\,dx}{4a}\\&=-\frac{cx}{4a}-\frac{1}{12}acx^3+\frac{c\left(1+a^2x^2\right)^2\tan^{-1}(ax)}{4a^2}\end{aligned}$$

Mathematica [A] time = 0.0042802, size = 58, normalized size = 1.38

$$\frac{1}{4}a^2cx^4\tan^{-1}(ax)+\frac{c\tan^{-1}(ax)}{4a^2}-\frac{1}{12}acx^3+\frac{1}{2}cx^2\tan^{-1}(ax)-\frac{cx}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(c + a^2*c*x^2)*ArcTan[a*x],x]
```

```
[Out] -(c*x)/(4*a) - (a*c*x^3)/12 + (c*ArcTan[a*x])/(4*a^2) + (c*x^2*ArcTan[a*x])
/2 + (a^2*c*x^4*ArcTan[a*x])/4
```

Maple [A] time = 0.021, size = 49, normalized size = 1.2

$$\frac{a^2c\arctan(ax)x^4}{4}+\frac{c\arctan(ax)x^2}{2}-\frac{acx^3}{12}-\frac{cx}{4a}+\frac{c\arctan(ax)}{4a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)*arctan(a*x),x)
```

```
[Out] 1/4*a^2*c*arctan(a*x)*x^4+1/2*c*arctan(a*x)*x^2-1/12*a*c*x^3-1/4*c*x/a+1/4/
a^2*c*arctan(a*x)
```

Maxima [A] time = 0.971041, size = 68, normalized size = 1.62

$$\frac{\left(a^2cx^2+c\right)^2\arctan(ax)}{4a^2c}-\frac{a^2c^2x^3+3c^2x}{12ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)*arctan(a*x),x, algorithm="maxima")
```

```
[Out] 1/4*(a^2*c*x^2 + c)^2*arctan(a*x)/(a^2*c) - 1/12*(a^2*c^2*x^3 + 3*c^2*x)/(a
*c)
```

Fricas [A] time = 1.55884, size = 107, normalized size = 2.55

$$-\frac{a^3cx^3 + 3\,acx - 3\left(a^4cx^4 + 2\,a^2cx^2 + c\right)\arctan\left(ax\right)}{12\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)*arctan(a*x),x, algorithm="fricas")
```

```
[Out] -1/12*(a^3*c*x^3 + 3*a*c*x - 3*(a^4*c*x^4 + 2*a^2*c*x^2 + c)*arctan(a*x))/a
^2
```

Sympy [A] time = 1.01982, size = 54, normalized size = 1.29

$$\begin{cases} \frac{a^2cx^4\operatorname{atan}\left(ax\right)}{4} - \frac{acx^3}{12} + \frac{cx^2\operatorname{atan}\left(ax\right)}{2} - \frac{cx}{4a} + \frac{c\operatorname{atan}\left(ax\right)}{4a^2} & \text{for}\: a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)*atan(a*x),x)
```

```
[Out] Piecewise((a**2*c*x**4*atan(a*x)/4 - a*c*x**3/12 + c*x**2*atan(a*x)/2 - c*x
/(4*a) + c*atan(a*x)/(4*a**2), Ne(a, 0)), (0, True))
```

Giac [A] time = 1.12867, size = 72, normalized size = 1.71

$$\frac{1}{4}\left(a^2cx^4 + 2\,cx^2\right)\arctan\left(ax\right) + \frac{c\arctan\left(ax\right)}{4\,a^2} - \frac{a^7cx^3 + 3\,a^5cx}{12\,a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)*arctan(a*x),x, algorithm="giac")
```

```
[Out] 1/4*(a^2*c*x^4 + 2*c*x^2)*arctan(a*x) + 1/4*c*arctan(a*x)/a^2 - 1/12*(a^7*c
*x^3 + 3*a^5*c*x)/a^6
```

3.152 $\int \left(c + a^2 c x^2\right) \tan^{-1}(ax)\,dx$

Optimal. Leaf size=50

$$-\frac{c\log\left(a^2x^2+1\right)}{3a}+\frac{1}{3}a^2cx^3\tan^{-1}(ax)-\frac{1}{6}acx^2+cx\tan^{-1}(ax)$$

```
[Out] -(a*c*x^2)/6 + c*x*ArcTan[a*x] + (a^2*c*x^3*ArcTan[a*x])/3 - (c*Log[1 + a^2
*x^2])/(3*a)
```

Rubi [A] time = 0.0234937, antiderivative size = 65, normalized size of antiderivative = 1.3, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4878, 4846, 260}

$$-\frac{c\left(a^2x^2+1\right)}{6a}-\frac{c\log\left(a^2x^2+1\right)}{3a}+\frac{1}{3}cx\left(a^2x^2+1\right)\tan^{-1}(ax)+\frac{2}{3}cx\tan^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[(c + a^2*c*x^2)*ArcTan[a*x],x]
```

```
[Out] -(c*(1 + a^2*x^2))/(6*a) + (2*c*x*ArcTan[a*x])/3 + (c*x*(1 + a^2*x^2)*ArcTa
n[a*x])/3 - (c*Log[1 + a^2*x^2])/(3*a)
```

Rule 4878

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symbo
l] :> -Simp[(b*(d + e*x^2)^q)/(2*c*q*(2*q + 1)), x] + (Dist[(2*d*q)/(2*q +
1), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x]), x], x] + Simp[(x*(d + e*x^
2)^q*(a + b*ArcTan[c*x]))/(2*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] && Eq
Q[e, c^2*d] && GtQ[q, 0]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int \left(c+a^2cx^2\right)\tan^{-1}(ax)\,dx &= -\frac{c\left(1+a^2x^2\right)}{6a}+\frac{1}{3}cx\left(1+a^2x^2\right)\tan^{-1}(ax)+\frac{1}{3}(2c)\int \tan^{-1}(ax)\,dx\\ &= -\frac{c\left(1+a^2x^2\right)}{6a}+\frac{2}{3}cx\tan^{-1}(ax)+\frac{1}{3}cx\left(1+a^2x^2\right)\tan^{-1}(ax)-\frac{1}{3}(2ac)\int \frac{x}{1+a^2x^2}\,dx\\ &= -\frac{c\left(1+a^2x^2\right)}{6a}+\frac{2}{3}cx\tan^{-1}(ax)+\frac{1}{3}cx\left(1+a^2x^2\right)\tan^{-1}(ax)-\frac{c\log\left(1+a^2x^2\right)}{3a}\end{aligned}$$

Mathematica [A] time = 0.0103881, size = 50, normalized size = 1.

$$-\frac{c\log\left(a^2x^2+1\right)}{3a}+\frac{1}{3}a^2cx^3\tan^{-1}(ax)-\frac{1}{6}acx^2+cx\tan^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + a^2*c*x^2)*ArcTan[a*x],x]
```

```
[Out] -(a*c*x^2)/6 + c*x*ArcTan[a*x] + (a^2*c*x^3*ArcTan[a*x])/3 - (c*Log[1 + a^2
*x^2])/(3*a)
```

Maple [A] time = 0.024, size = 45, normalized size = 0.9

$$-\frac{ax^2c}{6}+cx\arctan(ax)+\frac{a^2cx^3\arctan(ax)}{3}-\frac{c\ln\left(a^2x^2+1\right)}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)*arctan(a*x),x)
```

```
[Out] -1/6*a*x^2*c+c*x*arctan(a*x)+1/3*a^2*c*x^3*arctan(a*x)-1/3*c*ln(a^2*x^2+1)/
a
```

Maxima [A] time = 0.975009, size = 61, normalized size = 1.22

$$-\frac{1}{6}\left(cx^2+\frac{2c\log\left(a^2x^2+1\right)}{a^2}\right)a+\frac{1}{3}\left(a^2cx^3+3cx\right)\arctan(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x),x, algorithm="maxima")
```

```
[Out] -1/6*(c*x^2 + 2*c*log(a^2*x^2 + 1)/a^2)*a + 1/3*(a^2*c*x^3 + 3*c*x)*arctan(
a*x)
```

Fricas [A] time = 1.61202, size = 109, normalized size = 2.18

$$-\frac{a^2cx^2 - 2\left(a^3cx^3 + 3\,acx\right)\arctan\left(ax\right) + 2\,c\log\left(a^2x^2 + 1\right)}{6\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x),x, algorithm="fricas")
```

```
[Out] -1/6*(a^2*c*x^2 - 2*(a^3*c*x^3 + 3*a*c*x)*arctan(a*x) + 2*c*log(a^2*x^2 + 1
))/a
```

Sympy [A] time = 0.747634, size = 48, normalized size = 0.96

$$\begin{cases} \frac{a^2cx^3\operatorname{atan}\left(ax\right)}{3} - \frac{acx^2}{6} + cx\operatorname{atan}\left(ax\right) - \frac{c\log\left(x^2+\frac{1}{a^2}\right)}{3a} & \text{for}\ a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)*atan(a*x),x)
```

```
[Out] Piecewise((a**2*c*x**3*atan(a*x)/3 - a*c*x**2/6 + c*x*atan(a*x) - c*log(x**
2 + a**(-2))/(3*a), Ne(a, 0)), (0, True))
```

Giac [A] time = 1.12351, size = 58, normalized size = 1.16

$$-\frac{1}{6}\,acx^2 + \frac{1}{3}\left(a^2cx^3 + 3\,cx\right)\arctan\left(ax\right) - \frac{c\log\left(a^2x^2 + 1\right)}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x),x, algorithm="giac")
```

```
[Out] -1/6*a*c*x^2 + 1/3*(a^2*c*x^3 + 3*c*x)*arctan(a*x) - 1/3*c*log(a^2*x^2 + 1)
/a
```

3.153 $\int \frac{(c+a^2cx^2)\tan^{-1}(ax)}{x}\,dx$

Optimal. Leaf size=62

$$\frac{1}{2}ic\text{PolyLog}(2,-iax)-\frac{1}{2}ic\text{PolyLog}(2,iax)+\frac{1}{2}a^2cx^2\tan^{-1}(ax)-\frac{acx}{2}+\frac{1}{2}c\tan^{-1}(ax)$$

```
[Out] -(a*c*x)/2 + (c*ArcTan[a*x])/2 + (a^2*c*x^2*ArcTan[a*x])/2 + (I/2)*c*PolyLo
g[2, (-I)*a*x] - (I/2)*c*PolyLog[2, I*a*x]
```

Rubi [A] time = 0.0661995, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4950, 4848, 2391, 4852, 321, 203}

$$\frac{1}{2}ic\text{PolyLog}(2,-iax)-\frac{1}{2}ic\text{PolyLog}(2,iax)+\frac{1}{2}a^2cx^2\tan^{-1}(ax)-\frac{acx}{2}+\frac{1}{2}c\tan^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)*ArcTan[a*x])/x,x]
```

```
[Out] -(a*c*x)/2 + (c*ArcTan[a*x])/2 + (a^2*c*x^2*ArcTan[a*x])/2 + (I/2)*c*PolyLo
g[2, (-I)*a*x] - (I/2)*c*PolyLog[2, I*a*x]
```

Rule 4950

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(c+a^2cx^2\right)\tan^{-1}(ax)}{x}\,dx &= c\int \frac{\tan^{-1}(ax)}{x}\,dx+\left(a^2c\right)\int x\tan^{-1}(ax)\,dx\\
&= \frac{1}{2}a^2cx^2\tan^{-1}(ax)+\frac{1}{2}(ic)\int \frac{\log(1-iax)}{x}\,dx-\frac{1}{2}(ic)\int \frac{\log(1+iax)}{x}\,dx-\frac{1}{2}\left(a^3c\right)\int \frac{x}{1+\cdots}\\
&= -\frac{1}{2}acx+\frac{1}{2}a^2cx^2\tan^{-1}(ax)+\frac{1}{2}ic\text{Li}_2(-iax)-\frac{1}{2}ic\text{Li}_2(iax)+\frac{1}{2}(ac)\int \frac{1}{1+a^2x^2}\,dx\\
&= -\frac{1}{2}acx+\frac{1}{2}c\tan^{-1}(ax)+\frac{1}{2}a^2cx^2\tan^{-1}(ax)+\frac{1}{2}ic\text{Li}_2(-iax)-\frac{1}{2}ic\text{Li}_2(iax)
\end{aligned}$$

Mathematica [A] time = 0.0039463, size = 62, normalized size = 1.

$$\frac{1}{2}ic\text{PolyLog}(2,-iax)-\frac{1}{2}ic\text{PolyLog}(2,iax)+\frac{1}{2}a^2cx^2\tan^{-1}(ax)-\frac{acx}{2}+\frac{1}{2}c\tan^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + a^2*c*x^2)*ArcTan[a*x])/x,x]
```

```
[Out] -(a*c*x)/2 + (c*ArcTan[a*x])/2 + (a^2*c*x^2*ArcTan[a*x])/2 + (I/2)*c*PolyLo
g[2, (-I)*a*x] - (I/2)*c*PolyLog[2, I*a*x]
```

Maple [A] time = 0.037, size = 93, normalized size = 1.5

$$\frac{a^2cx^2\arctan(ax)}{2}+c\arctan(ax)\ln(ax)+\frac{i}{2}\ln(ax)\ln(1+iax)c-\frac{i}{2}\ln(ax)\ln(1-iax)c+\frac{i}{2}\mathrm{dilog}(1+iax)c-\frac{i}{2}\mathrm{dilog}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)*arctan(a*x)/x,x)
```

```
[Out] 1/2*a^2*c*x^2*arctan(a*x)+c*arctan(a*x)*ln(a*x)+1/2*I*ln(a*x)*ln(1+I*a*x)*c
-1/2*I*ln(a*x)*ln(1-I*a*x)*c+1/2*I*dilog(1+I*a*x)*c-1/2*I*dilog(1-I*a*x)*c-
1/2*a*c*x+1/2*c*arctan(a*x)
```

Maxima [A] time = 1.62166, size = 101, normalized size = 1.63

$$-\frac{1}{2}acx-\frac{1}{4}\pi c\log\left(a^2x^2+1\right)+c\arctan(ax)\log(x|a|)+\frac{1}{2}\left(a^2cx^2+c(2i\arctan(0,a)+1)\right)\arctan(ax)-\frac{1}{2}ic\mathrm{Li}_2(iax+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)/x,x, algorithm="maxima")
```

```
[Out] -1/2*a*c*x - 1/4*pi*c*log(a^2*x^2 + 1) + c*arctan(a*x)*log(x*abs(a)) + 1/2*
(a^2*c*x^2 + c*(2*I*arctan2(0, a) + 1))*arctan(a*x) - 1/2*I*c*dilog(I*a*x +
 1) + 1/2*I*c*dilog(-I*a*x + 1)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(a^2cx^2+c\right)\arctan(ax)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)/x,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)*arctan(a*x)/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$c\left(\int \frac{\operatorname{atan}(ax)}{x}\,dx + \int a^2 x \operatorname{atan}(ax)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)*atan(a*x)/x,x)
```

```
[Out] c*(Integral(atan(a*x)/x, x) + Integral(a**2*x*atan(a*x), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2 c x^2 + c\right) \arctan(ax)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)/x,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)*arctan(a*x)/x, x)
```

3.154 $\int \frac{\left(c+a^2cx^2\right)\tan^{-1}(ax)}{x^2}\,dx$

Optimal. Leaf size=40

$$-ac\log\left(a^2x^2+1\right)+a^2cx\tan^{-1}(ax)+ac\log(x)-\frac{c\tan^{-1}(ax)}{x}$$

```
[Out] -((c*ArcTan[a*x])/x) + a^2*c*x*ArcTan[a*x] + a*c*Log[x] - a*c*Log[1 + a^2*x
^2]
```

Rubi [A] time = 0.0548211, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {4950, 4852, 266, 36, 29, 31, 4846, 260}

$$-ac\log\left(a^2x^2+1\right)+a^2cx\tan^{-1}(ax)+ac\log(x)-\frac{c\tan^{-1}(ax)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)*ArcTan[a*x])/x^2,x]
```

```
[Out] -((c*ArcTan[a*x])/x) + a^2*c*x*ArcTan[a*x] + a*c*Log[x] - a*c*Log[1 + a^2*x
^2]
```

Rule 4950

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)\tan^{-1}(ax)}{x^2}\,dx = c\int \frac{\tan^{-1}(ax)}{x^2}\,dx + (a^2c)\int \tan^{-1}(ax)\,dx$$
$$= -\frac{c\tan^{-1}(ax)}{x} + a^2cx\tan^{-1}(ax) + (ac)\int \frac{1}{x(1+a^2x^2)}\,dx - (a^3c)\int \frac{x}{1+a^2x^2}\,dx$$
$$= -\frac{c\tan^{-1}(ax)}{x} + a^2cx\tan^{-1}(ax) - \frac{1}{2}ac\log(1+a^2x^2) + \frac{1}{2}(ac)\,\mathrm{Subst}\left(\int \frac{1}{x(1+a^2x)}\,dx, x, \right.$$
$$= -\frac{c\tan^{-1}(ax)}{x} + a^2cx\tan^{-1}(ax) - \frac{1}{2}ac\log(1+a^2x^2) + \frac{1}{2}(ac)\,\mathrm{Subst}\left(\int \frac{1}{x}\,dx, x, x^2\right) - \frac{1}{2}($$
$$= -\frac{c\tan^{-1}(ax)}{x} + a^2cx\tan^{-1}(ax) + ac\log(x) - ac\log(1+a^2x^2)$$

Mathematica [A] time = 0.0047011, size = 40, normalized size = 1.

$$-ac\log(a^2x^2+1) + a^2cx\tan^{-1}(ax) + ac\log(x) - \frac{c\tan^{-1}(ax)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + a^2*c*x^2)*ArcTan[a*x])/x^2,x]
```

```
[Out] -((c*ArcTan[a*x])/x) + a^2*c*x*ArcTan[a*x] + a*c*Log[x] - a*c*Log[1 + a^2*x
^2]
```

Maple [A] time = 0.029, size = 43, normalized size = 1.1

$$a^2cx\arctan(ax) - \frac{c\arctan(ax)}{x} + ac\ln(ax) - ac\ln(a^2x^2+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)*arctan(a*x)/x^2,x)
```

```
[Out] a^2*c*x*arctan(a*x)-c*arctan(a*x)/x+a*c*ln(a*x)-a*c*ln(a^2*x^2+1)
```

Maxima [A] time = 0.987712, size = 54, normalized size = 1.35

$$-\left(c\log\left(a^2x^2+1\right)-c\log\left(x\right)\right)a+\left(a^2cx-\frac{c}{x}\right)\arctan\left(ax\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)/x^2,x, algorithm="maxima")
```

```
[Out] -(c*log(a^2*x^2 + 1) - c*log(x))*a + (a^2*c*x - c/x)*arctan(a*x)
```

Fricas [A] time = 1.60348, size = 100, normalized size = 2.5

$$-\frac{acx\log\left(a^2x^2+1\right)-acx\log\left(x\right)-\left(a^2cx^2-c\right)\arctan\left(ax\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)/x^2,x, algorithm="fricas")
```

```
[Out] -(a*c*x*log(a^2*x^2 + 1) - a*c*x*log(x) - (a^2*c*x^2 - c)*arctan(a*x))/x
```

Sympy [A] time = 1.06056, size = 41, normalized size = 1.02

$$\begin{cases} a^2cx\operatorname{atan}\left(ax\right)+ac\log\left(x\right)-ac\log\left(x^2+\frac{1}{a^2}\right)-\frac{c\operatorname{atan}\left(ax\right)}{x} & \text{for } a\neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)*atan(a*x)/x**2,x)
```

```
[Out] Piecewise((a**2*c*x*atan(a*x) + a*c*log(x) - a*c*log(x**2 + a**(-2)) - c*at
an(a*x)/x, Ne(a, 0)), (0, True))
```

Giac [A] time = 1.15231, size = 55, normalized size = 1.38

$$-ac\log\left(a^2x^2+1\right)+\frac{1}{2}ac\log\left(x^2\right)+\left(a^2cx-\frac{c}{x}\right)\arctan\left(ax\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)/x^2,x, algorithm="giac")
```

```
[Out] -a*c*log(a^2*x^2 + 1) + 1/2*a*c*log(x^2) + (a^2*c*x - c/x)*arctan(a*x)
```

3.155 $\int \frac{(c+a^2cx^2)\tan^{-1}(ax)}{x^3}\,dx$

Optimal. Leaf size=70

$$\frac{1}{2}ia^2c\text{PolyLog}(2,-iax)-\frac{1}{2}ia^2c\text{PolyLog}(2,iax)-\frac{1}{2}a^2c\tan^{-1}(ax)-\frac{c\tan^{-1}(ax)}{2x^2}-\frac{ac}{2x}$$

```
[Out] -(a*c)/(2*x) - (a^2*c*ArcTan[a*x])/2 - (c*ArcTan[a*x])/(2*x^2) + (I/2)*a^2*
c*PolyLog[2, (-I)*a*x] - (I/2)*a^2*c*PolyLog[2, I*a*x]
```

Rubi [A] time = 0.0706943, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4950, 4852, 325, 203, 4848, 2391}

$$\frac{1}{2}ia^2c\text{PolyLog}(2,-iax)-\frac{1}{2}ia^2c\text{PolyLog}(2,iax)-\frac{1}{2}a^2c\tan^{-1}(ax)-\frac{c\tan^{-1}(ax)}{2x^2}-\frac{ac}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)*ArcTan[a*x])/x^3,x]
```

```
[Out] -(a*c)/(2*x) - (a^2*c*ArcTan[a*x])/2 - (c*ArcTan[a*x])/(2*x^2) + (I/2)*a^2*
c*PolyLog[2, (-I)*a*x] - (I/2)*a^2*c*PolyLog[2, I*a*x]
```

Rule 4950

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(c+a^2cx^2\right)\tan^{-1}(ax)}{x^3}\,dx &= c\int \frac{\tan^{-1}(ax)}{x^3}\,dx+\left(a^2c\right)\int \frac{\tan^{-1}(ax)}{x}\,dx\\ &= -\frac{c\tan^{-1}(ax)}{2x^2}+\frac{1}{2}(ac)\int \frac{1}{x^2\left(1+a^2x^2\right)}\,dx+\frac{1}{2}\left(ia^2c\right)\int \frac{\log(1-iax)}{x}\,dx-\frac{1}{2}\left(ia^2c\right)\int \frac{\log(\\ &= -\frac{ac}{2x}-\frac{c\tan^{-1}(ax)}{2x^2}+\frac{1}{2}ia^2c\text{Li}_2(-iax)-\frac{1}{2}ia^2c\text{Li}_2(iax)-\frac{1}{2}\left(a^3c\right)\int \frac{1}{1+a^2x^2}\,dx\\ &= -\frac{ac}{2x}-\frac{1}{2}a^2c\tan^{-1}(ax)-\frac{c\tan^{-1}(ax)}{2x^2}+\frac{1}{2}ia^2c\text{Li}_2(-iax)-\frac{1}{2}ia^2c\text{Li}_2(iax)\end{aligned}$$

Mathematica [C] time = 0.0045507, size = 74, normalized size = 1.06

$$-\frac{ac\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-a^2x^2\right)}{2x}+\frac{1}{2}ia^2c\text{PolyLog}(2,-iax)-\frac{1}{2}ia^2c\text{PolyLog}(2,iax)-\frac{c\tan^{-1}(ax)}{2x^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((c + a^2*c*x^2)*ArcTan[a*x])/x^3,x]
```

```
[Out] -(c*ArcTan[a*x])/(2*x^2) - (a*c*Hypergeometric2F1[-1/2, 1, 1/2, -(a^2*x^2)]
)/(2*x) + (I/2)*a^2*c*PolyLog[2, (-I)*a*x] - (I/2)*a^2*c*PolyLog[2, I*a*x]
```

Maple [A] time = 0.041, size = 110, normalized size = 1.6

$$-\frac{c\arctan(ax)}{2x^2}+a^2c\arctan(ax)\ln(ax)-\frac{a^2c\arctan(ax)}{2}-\frac{ac}{2x}+\frac{i}{2}a^2c\ln(ax)\ln(1+iax)-\frac{i}{2}a^2c\ln(ax)\ln(1-iax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)*arctan(a*x)/x^3,x)
```

```
[Out] -1/2*c*arctan(a*x)/x^2+a^2*c*arctan(a*x)*ln(a*x)-1/2*a^2*c*arctan(a*x)-1/2*
a*c/x+1/2*I*a^2*c*ln(a*x)*ln(1+I*a*x)-1/2*I*a^2*c*ln(a*x)*ln(1-I*a*x)+1/2*I
*a^2*c*dilog(1+I*a*x)-1/2*I*a^2*c*dilog(1-I*a*x)
```

Maxima [B] time = 1.63797, size = 142, normalized size = 2.03

$$-\frac{\pi a^2cx^2\log\left(a^2x^2+1\right)-4a^2cx^2\arctan(ax)\log(x|a|)+2ia^2cx^2\mathrm{Li}_2(iax+1)-2ia^2cx^2\mathrm{Li}_2(-iax+1)+2acx-2\left(a^2c\right.}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)/x^3,x, algorithm="maxima")
```

```
[Out] -1/4*(pi*a^2*c*x^2*log(a^2*x^2 + 1) - 4*a^2*c*x^2*arctan(a*x)*log(x*abs(a))
 + 2*I*a^2*c*x^2*dilog(I*a*x + 1) - 2*I*a^2*c*x^2*dilog(-I*a*x + 1) + 2*a*c
*x - 2*(a^2*c*x^2*(2*I*arctan2(0, a) - 1) - c)*arctan(a*x))/x^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(a^2cx^2+c\right)\arctan(ax)}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)/x^3,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)*arctan(a*x)/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$c\left(\int \frac{\operatorname{atan}(ax)}{x^3}\,dx + \int \frac{a^2\operatorname{atan}(ax)}{x}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)*atan(a*x)/x**3,x)
```

```
[Out] c*(Integral(atan(a*x)/x**3, x) + Integral(a**2*atan(a*x)/x, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a^2cx^2 + c)\arctan(ax)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)/x^3,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)*arctan(a*x)/x^3, x)
```

3.156 $\int \frac{(c+a^2cx^2)\tan^{-1}(ax)}{x^4}\,dx$

Optimal. Leaf size=63

$$-\frac{1}{3}a^3c\log\left(a^2x^2+1\right)+\frac{2}{3}a^3c\log(x)-\frac{a^2c\tan^{-1}(ax)}{x}-\frac{ac}{6x^2}-\frac{c\tan^{-1}(ax)}{3x^3}$$

```
[Out] -(a*c)/(6*x^2) - (c*ArcTan[a*x])/(3*x^3) - (a^2*c*ArcTan[a*x])/x + (2*a^3*c
*Log[x])/3 - (a^3*c*Log[1 + a^2*x^2])/3
```

Rubi [A] time = 0.0822095, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {4950, 4852, 266, 44, 36, 29, 31}

$$-\frac{1}{3}a^3c\log\left(a^2x^2+1\right)+\frac{2}{3}a^3c\log(x)-\frac{a^2c\tan^{-1}(ax)}{x}-\frac{ac}{6x^2}-\frac{c\tan^{-1}(ax)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)*ArcTan[a*x])/x^4,x]
```

```
[Out] -(a*c)/(6*x^2) - (c*ArcTan[a*x])/(3*x^3) - (a^2*c*ArcTan[a*x])/x + (2*a^3*c
*Log[x])/3 - (a^3*c*Log[1 + a^2*x^2])/3
```

Rule 4950

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{\left(c+a^2cx^2\right)\tan^{-1}(ax)}{x^4}\,dx = c\int \frac{\tan^{-1}(ax)}{x^4}\,dx + \left(a^2c\right)\int \frac{\tan^{-1}(ax)}{x^2}\,dx$$

$$= -\frac{c\tan^{-1}(ax)}{3x^3} - \frac{a^2c\tan^{-1}(ax)}{x} + \frac{1}{3}(ac)\int \frac{1}{x^3\left(1+a^2x^2\right)}\,dx + \left(a^3c\right)\int \frac{1}{x\left(1+a^2x^2\right)}\,dx$$

$$= -\frac{c\tan^{-1}(ax)}{3x^3} - \frac{a^2c\tan^{-1}(ax)}{x} + \frac{1}{6}(ac)\,\text{Subst}\left(\int \frac{1}{x^2\left(1+a^2x\right)}\,dx, x, x^2\right) + \frac{1}{2}\left(a^3c\right)\text{Subst}\left(\right.$$

$$= -\frac{c\tan^{-1}(ax)}{3x^3} - \frac{a^2c\tan^{-1}(ax)}{x} + \frac{1}{6}(ac)\,\text{Subst}\left(\int \left(\frac{1}{x^2} - \frac{a^2}{x} + \frac{a^4}{1+a^2x}\right)dx, x, x^2\right) + \frac{1}{2}\left(a^3c\right.$$

$$= -\frac{ac}{6x^2} - \frac{c\tan^{-1}(ax)}{3x^3} - \frac{a^2c\tan^{-1}(ax)}{x} + \frac{2}{3}a^3c\log(x) - \frac{1}{3}a^3c\log\left(1+a^2x^2\right)$$

Mathematica [A] time = 0.0200303, size = 58, normalized size = 0.92

$$\frac{c\left(ax\left(4a^2x^2\log(x)-2a^2x^2\log\left(a^2x^2+1\right)-1\right)-2\left(3a^2x^2+1\right)\tan^{-1}(ax)\right)}{6x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + a^2*c*x^2)*ArcTan[a*x])/x^4,x]
```

```
[Out] (c*(-2*(1 + 3*a^2*x^2)*ArcTan[a*x] + a*x*(-1 + 4*a^2*x^2*Log[x] - 2*a^2*x^2
*Log[1 + a^2*x^2])))/(6*x^3)
```

Maple [A] time = 0.033, size = 58, normalized size = 0.9

$$-\frac{a^2c\arctan(ax)}{x}-\frac{c\arctan(ax)}{3x^3}-\frac{a^3c\ln\left(a^2x^2+1\right)}{3}-\frac{ac}{6x^2}+\frac{2a^3c\ln(ax)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)*arctan(a*x)/x^4,x)
```

```
[Out] -a^2*c*arctan(a*x)/x-1/3*c*arctan(a*x)/x^3-1/3*a^3*c*ln(a^2*x^2+1)-1/6*a*c/
x^2+2/3*a^3*c*ln(a*x)
```

Maxima [A] time = 0.990587, size = 76, normalized size = 1.21

$$-\frac{1}{6}\left(2a^2c\log\left(a^2x^2+1\right)-2a^2c\log\left(x^2\right)+\frac{c}{x^2}\right)a-\frac{\left(3a^2cx^2+c\right)\arctan(ax)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)/x^4,x, algorithm="maxima")
```

```
[Out] -1/6*(2*a^2*c*log(a^2*x^2 + 1) - 2*a^2*c*log(x^2) + c/x^2)*a - 1/3*(3*a^2*c
*x^2 + c)*arctan(a*x)/x^3
```

Fricas [A] time = 1.59627, size = 140, normalized size = 2.22

$$-\frac{2\,a^3cx^3\log\left(a^2x^2+1\right)-4\,a^3cx^3\log\left(x\right)+acx+2\left(3\,a^2cx^2+c\right)\arctan\left(ax\right)}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)/x^4,x, algorithm="fricas")
```

```
[Out] -1/6*(2*a^3*c*x^3*log(a^2*x^2 + 1) - 4*a^3*c*x^3*log(x) + a*c*x + 2*(3*a^2*
c*x^2 + c)*arctan(a*x))/x^3
```

Sympy [A] time = 1.52307, size = 61, normalized size = 0.97

$$\begin{cases} \frac{2a^3c\log\left(x\right)}{3}-\frac{a^3c\log\left(x^2+\frac{1}{a^2}\right)}{3}-\frac{a^2c\,\mathrm{atan}\left(ax\right)}{x}-\frac{ac}{6x^2}-\frac{c\,\mathrm{atan}\left(ax\right)}{3x^3} & \text{for } a\neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)*atan(a*x)/x**4,x)
```

```
[Out] Piecewise((2*a**3*c*log(x)/3 - a**3*c*log(x**2 + a**(-2))/3 - a**2*c*atan(a
*x)/x - a*c/(6*x**2) - c*atan(a*x)/(3*x**3), Ne(a, 0)), (0, True))
```

Giac [A] time = 1.12177, size = 88, normalized size = 1.4

$$-\frac{1}{3}\,a^3c\log\left(a^2x^2+1\right)+\frac{1}{3}\,a^3c\log\left(x^2\right)-\frac{2\,a^3cx^2+ac}{6\,x^2}-\frac{\left(3\,a^2cx^2+c\right)\arctan\left(ax\right)}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)/x^4,x, algorithm="giac")
```

```
[Out] -1/3*a^3*c*log(a^2*x^2 + 1) + 1/3*a^3*c*log(x^2) - 1/6*(2*a^3*c*x^2 + a*c)/
x^2 - 1/3*(3*a^2*c*x^2 + c)*arctan(a*x)/x^3
```

3.157 $\int x^3 \left(c+a^2 c x^2\right)^2 \tan^{-1}(ax) \, dx$

Optimal. Leaf size=111

$$-\frac{1}{56}a^3c^2x^7+\frac{1}{8}a^4c^2x^8\tan^{-1}(ax)+\frac{1}{3}a^2c^2x^6\tan^{-1}(ax)+\frac{c^2x}{24a^3}-\frac{c^2\tan^{-1}(ax)}{24a^4}-\frac{1}{24}ac^2x^5-\frac{c^2x^3}{72a}+\frac{1}{4}c^2x^4\tan^{-1}(ax)$$

```
[Out] (c^2*x)/(24*a^3) - (c^2*x^3)/(72*a) - (a*c^2*x^5)/24 - (a^3*c^2*x^7)/56 - (
c^2*ArcTan[a*x])/(24*a^4) + (c^2*x^4*ArcTan[a*x])/4 + (a^2*c^2*x^6*ArcTan[a
*x])/3 + (a^4*c^2*x^8*ArcTan[a*x])/8
```

Rubi [A] time = 0.155648, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4948, 4852, 302, 203}

$$-\frac{1}{56}a^3c^2x^7+\frac{1}{8}a^4c^2x^8\tan^{-1}(ax)+\frac{1}{3}a^2c^2x^6\tan^{-1}(ax)+\frac{c^2x}{24a^3}-\frac{c^2\tan^{-1}(ax)}{24a^4}-\frac{1}{24}ac^2x^5-\frac{c^2x^3}{72a}+\frac{1}{4}c^2x^4\tan^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(c + a^2*c*x^2)^2*ArcTan[a*x],x]
```

```
[Out] (c^2*x)/(24*a^3) - (c^2*x^3)/(72*a) - (a*c^2*x^5)/24 - (a^3*c^2*x^7)/56 - (
c^2*ArcTan[a*x])/(24*a^4) + (c^2*x^4*ArcTan[a*x])/4 + (a^2*c^2*x^6*ArcTan[a
*x])/3 + (a^4*c^2*x^8*ArcTan[a*x])/8
```

Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int x^3 \left(c+a^2cx^2\right)^2 \tan^{-1}(ax)\,dx &= \int \left(c^2x^3\tan^{-1}(ax)+2a^2c^2x^5\tan^{-1}(ax)+a^4c^2x^7\tan^{-1}(ax)\right)dx\\
&= c^2\int x^3\tan^{-1}(ax)\,dx+\left(2a^2c^2\right)\int x^5\tan^{-1}(ax)\,dx+\left(a^4c^2\right)\int x^7\tan^{-1}(ax)\,dx\\
&= \frac{1}{4}c^2x^4\tan^{-1}(ax)+\frac{1}{3}a^2c^2x^6\tan^{-1}(ax)+\frac{1}{8}a^4c^2x^8\tan^{-1}(ax)-\frac{1}{4}\left(ac^2\right)\int\frac{x^4}{1+a^2x^2}\,dx-\\
&= \frac{1}{4}c^2x^4\tan^{-1}(ax)+\frac{1}{3}a^2c^2x^6\tan^{-1}(ax)+\frac{1}{8}a^4c^2x^8\tan^{-1}(ax)-\frac{1}{4}\left(ac^2\right)\int\left(-\frac{1}{a^4}+\frac{x^2}{a^2}+\right.\\
&= \frac{c^2x}{24a^3}-\frac{c^2x^3}{72a}-\frac{1}{24}ac^2x^5-\frac{1}{56}a^3c^2x^7+\frac{1}{4}c^2x^4\tan^{-1}(ax)+\frac{1}{3}a^2c^2x^6\tan^{-1}(ax)+\frac{1}{8}a^4c^2x\\
&= \frac{c^2x}{24a^3}-\frac{c^2x^3}{72a}-\frac{1}{24}ac^2x^5-\frac{1}{56}a^3c^2x^7-\frac{c^2\tan^{-1}(ax)}{24a^4}+\frac{1}{4}c^2x^4\tan^{-1}(ax)+\frac{1}{3}a^2c^2x^6\tan^{-1}
\end{aligned}$$

Mathematica [A] time = 0.0953528, size = 111, normalized size = 1.

$$-\frac{1}{56}a^3c^2x^7+\frac{1}{8}a^4c^2x^8\tan^{-1}(ax)+\frac{1}{3}a^2c^2x^6\tan^{-1}(ax)+\frac{c^2x}{24a^3}-\frac{c^2\tan^{-1}(ax)}{24a^4}-\frac{1}{24}ac^2x^5-\frac{c^2x^3}{72a}+\frac{1}{4}c^2x^4\tan^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(c + a^2*c*x^2)^2*ArcTan[a*x],x]
```

```
[Out] (c^2*x)/(24*a^3) - (c^2*x^3)/(72*a) - (a*c^2*x^5)/24 - (a^3*c^2*x^7)/56 - (
c^2*ArcTan[a*x])/(24*a^4) + (c^2*x^4*ArcTan[a*x])/4 + (a^2*c^2*x^6*ArcTan[a
*x])/3 + (a^4*c^2*x^8*ArcTan[a*x])/8
```

Maple [A] time = 0.025, size = 96, normalized size = 0.9

$$\frac{c^2x}{24\,a^3}-\frac{c^2x^3}{72\,a}-\frac{ac^2x^5}{24}-\frac{a^3c^2x^7}{56}-\frac{c^2\arctan{(ax)}}{24\,a^4}+\frac{c^2x^4\arctan{(ax)}}{4}+\frac{a^2c^2x^6\arctan{(ax)}}{3}+\frac{a^4c^2x^8\arctan{(ax)}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a^2*c*x^2+c)^2*arctan(a*x),x)
```

```
[Out] 1/24*c^2*x/a^3-1/72*c^2*x^3/a-1/24*a*c^2*x^5-1/56*a^3*c^2*x^7-1/24*c^2*arct
an(a*x)/a^4+1/4*c^2*x^4*arctan(a*x)+1/3*a^2*c^2*x^6*arctan(a*x)+1/8*a^4*c^2
*x^8*arctan(a*x)
```

Maxima [A] time = 1.44119, size = 132, normalized size = 1.19

$$-\frac{1}{504}\,a\left(\frac{21\,c^2\arctan{(ax)}}{a^5}+\frac{9\,a^6c^2x^7+21\,a^4c^2x^5+7\,a^2c^2x^3-21\,c^2x}{a^4}\right)+\frac{1}{24}\left(3\,a^4c^2x^8+8\,a^2c^2x^6+6\,c^2x^4\right)\arctan{(ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a^2*c*x^2+c)^2*arctan(a*x),x, algorithm="maxima")
```

```
[Out] -1/504*a*(21*c^2*arctan(a*x)/a^5 + (9*a^6*c^2*x^7 + 21*a^4*c^2*x^5 + 7*a^2*
c^2*x^3 - 21*c^2*x)/a^4) + 1/24*(3*a^4*c^2*x^8 + 8*a^2*c^2*x^6 + 6*c^2*x^4)
*arctan(a*x)
```

Fricas [A] time = 1.56244, size = 196, normalized size = 1.77

$$-\frac{9\,a^7c^2x^7+21\,a^5c^2x^5+7\,a^3c^2x^3-21\,ac^2x-21\left(3\,a^8c^2x^8+8\,a^6c^2x^6+6\,a^4c^2x^4-c^2\right)\arctan{(ax)}}{504\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a^2*c*x^2+c)^2*arctan(a*x),x, algorithm="fricas")
```

```
[Out] -1/504*(9*a^7*c^2*x^7 + 21*a^5*c^2*x^5 + 7*a^3*c^2*x^3 - 21*a*c^2*x - 21*(3
*a^8*c^2*x^8 + 8*a^6*c^2*x^6 + 6*a^4*c^2*x^4 - c^2)*arctan(a*x))/a^4
```

Sympy [A] time = 3.45755, size = 104, normalized size = 0.94

$$\begin{cases} \frac{a^4c^2x^8\operatorname{atan}(ax)}{8} - \frac{a^3c^2x^7}{56} + \frac{a^2c^2x^6\operatorname{atan}(ax)}{3} - \frac{ac^2x^5}{24} + \frac{c^2x^4\operatorname{atan}(ax)}{4} - \frac{c^2x^3}{72a} + \frac{c^2x}{24a^3} - \frac{c^2\operatorname{atan}(ax)}{24a^4} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a**2*c*x**2+c)**2*atan(a*x),x)
```

```
[Out] Piecewise((a**4*c**2*x**8*atan(a*x)/8 - a**3*c**2*x**7/56 + a**2*c**2*x**6*
atan(a*x)/3 - a*c**2*x**5/24 + c**2*x**4*atan(a*x)/4 - c**2*x**3/(72*a) + c
**2*x/(24*a**3) - c**2*atan(a*x)/(24*a**4), Ne(a, 0)), (0, True))
```

Giac [A] time = 1.14705, size = 132, normalized size = 1.19

$$\frac{1}{24}\left(3\,a^4c^2x^8 + 8\,a^2c^2x^6 + 6\,c^2x^4\right)\arctan(ax) - \frac{c^2\arctan(ax)}{24\,a^4} - \frac{9\,a^{17}c^2x^7 + 21\,a^{15}c^2x^5 + 7\,a^{13}c^2x^3 - 21\,a^{11}c^2x}{504\,a^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a^2*c*x^2+c)^2*arctan(a*x),x, algorithm="giac")
```

```
[Out] 1/24*(3*a^4*c^2*x^8 + 8*a^2*c^2*x^6 + 6*c^2*x^4)*arctan(a*x) - 1/24*c^2*arc
tan(a*x)/a^4 - 1/504*(9*a^17*c^2*x^7 + 21*a^15*c^2*x^5 + 7*a^13*c^2*x^3 - 2
1*a^11*c^2*x)/a^14
```

3.158 $\int x^2 \left(c+a^2 c x^2\right)^2 \tan^{-1}(a x) \, dx$

Optimal. Leaf size=106

$$-\frac{1}{42}a^3c^2x^6+\frac{4c^2\log\left(a^2x^2+1\right)}{105a^3}+\frac{1}{7}a^4c^2x^7\tan^{-1}(ax)+\frac{2}{5}a^2c^2x^5\tan^{-1}(ax)-\frac{9}{140}ac^2x^4-\frac{4c^2x^2}{105a}+\frac{1}{3}c^2x^3\tan^{-1}(ax)$$

```
[Out] (-4*c^2*x^2)/(105*a) - (9*a*c^2*x^4)/140 - (a^3*c^2*x^6)/42 + (c^2*x^3*ArcT
an[a*x])/3 + (2*a^2*c^2*x^5*ArcTan[a*x])/5 + (a^4*c^2*x^7*ArcTan[a*x])/7 +
(4*c^2*Log[1 + a^2*x^2])/(105*a^3)
```

Rubi [A] time = 0.171692, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4948, 4852, 266, 43}

$$-\frac{1}{42}a^3c^2x^6+\frac{4c^2\log\left(a^2x^2+1\right)}{105a^3}+\frac{1}{7}a^4c^2x^7\tan^{-1}(ax)+\frac{2}{5}a^2c^2x^5\tan^{-1}(ax)-\frac{9}{140}ac^2x^4-\frac{4c^2x^2}{105a}+\frac{1}{3}c^2x^3\tan^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(c + a^2*c*x^2)^2*ArcTan[a*x],x]
```

```
[Out] (-4*c^2*x^2)/(105*a) - (9*a*c^2*x^4)/140 - (a^3*c^2*x^6)/42 + (c^2*x^3*ArcT
an[a*x])/3 + (2*a^2*c^2*x^5*ArcTan[a*x])/5 + (a^4*c^2*x^7*ArcTan[a*x])/7 +
(4*c^2*Log[1 + a^2*x^2])/(105*a^3)
```

Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2\left(c+a^2cx^2\right)^2\tan^{-1}(ax)\,dx &= \int\left(c^2x^2\tan^{-1}(ax)+2a^2c^2x^4\tan^{-1}(ax)+a^4c^2x^6\tan^{-1}(ax)\right)dx\\ &= c^2\int x^2\tan^{-1}(ax)\,dx+\left(2a^2c^2\right)\int x^4\tan^{-1}(ax)\,dx+\left(a^4c^2\right)\int x^6\tan^{-1}(ax)\,dx\\ &= \frac{1}{3}c^2x^3\tan^{-1}(ax)+\frac{2}{5}a^2c^2x^5\tan^{-1}(ax)+\frac{1}{7}a^4c^2x^7\tan^{-1}(ax)-\frac{1}{3}\left(ac^2\right)\int\frac{x^3}{1+a^2x^2}\,dx-\\ &= \frac{1}{3}c^2x^3\tan^{-1}(ax)+\frac{2}{5}a^2c^2x^5\tan^{-1}(ax)+\frac{1}{7}a^4c^2x^7\tan^{-1}(ax)-\frac{1}{6}\left(ac^2\right)\text{Subst}\left(\int\frac{x}{1+a^2}\right.\\ &= \frac{1}{3}c^2x^3\tan^{-1}(ax)+\frac{2}{5}a^2c^2x^5\tan^{-1}(ax)+\frac{1}{7}a^4c^2x^7\tan^{-1}(ax)-\frac{1}{6}\left(ac^2\right)\text{Subst}\left(\int\left(\frac{1}{a^2}-\right.\right.\\ &= -\frac{4c^2x^2}{105a}-\frac{9}{140}ac^2x^4-\frac{1}{42}a^3c^2x^6+\frac{1}{3}c^2x^3\tan^{-1}(ax)+\frac{2}{5}a^2c^2x^5\tan^{-1}(ax)+\frac{1}{7}a^4c^2x^7\tan\end{aligned}$$

Mathematica [A] time = 0.063396, size = 106, normalized size = 1.

$$-\frac{1}{42}a^3c^2x^6+\frac{4c^2\log\left(a^2x^2+1\right)}{105a^3}+\frac{1}{7}a^4c^2x^7\tan^{-1}(ax)+\frac{2}{5}a^2c^2x^5\tan^{-1}(ax)-\frac{9}{140}ac^2x^4-\frac{4c^2x^2}{105a}+\frac{1}{3}c^2x^3\tan^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(c + a^2*c*x^2)^2*ArcTan[a*x],x]
```

```
[Out] (-4*c^2*x^2)/(105*a) - (9*a*c^2*x^4)/140 - (a^3*c^2*x^6)/42 + (c^2*x^3*ArcT
an[a*x])/3 + (2*a^2*c^2*x^5*ArcTan[a*x])/5 + (a^4*c^2*x^7*ArcTan[a*x])/7 +
(4*c^2*Log[1 + a^2*x^2])/(105*a^3)
```

Maple [A] time = 0.024, size = 93, normalized size = 0.9

$$-\frac{4c^2x^2}{105a}-\frac{9ac^2x^4}{140}-\frac{a^3c^2x^6}{42}+\frac{c^2x^3\arctan(ax)}{3}+\frac{2a^2c^2x^5\arctan(ax)}{5}+\frac{a^4c^2x^7\arctan(ax)}{7}+\frac{4c^2\ln\left(a^2x^2+1\right)}{105a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a^2*c*x^2+c)^2*arctan(a*x),x)
```

```
[Out] -4/105*c^2*x^2/a-9/140*a*c^2*x^4-1/42*a^3*c^2*x^6+1/3*c^2*x^3*arctan(a*x)+2
/5*a^2*c^2*x^5*arctan(a*x)+1/7*a^4*c^2*x^7*arctan(a*x)+4/105*c^2*ln(a^2*x^2
+1)/a^3
```

Maxima [A] time = 0.974115, size = 128, normalized size = 1.21

$$-\frac{1}{420}a\left(\frac{10a^4c^2x^6+27a^2c^2x^4+16c^2x^2}{a^2}-\frac{16c^2\log\left(a^2x^2+1\right)}{a^4}\right)+\frac{1}{105}\left(15a^4c^2x^7+42a^2c^2x^5+35c^2x^3\right)\arctan(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^2*arctan(a*x),x, algorithm="maxima")
```

```
[Out] -1/420*a*((10*a^4*c^2*x^6 + 27*a^2*c^2*x^4 + 16*c^2*x^2)/a^2 - 16*c^2*log(a
^2*x^2 + 1)/a^4) + 1/105*(15*a^4*c^2*x^7 + 42*a^2*c^2*x^5 + 35*c^2*x^3)*arc
tan(a*x)
```

Fricas [A] time = 1.6932, size = 211, normalized size = 1.99

$$-\frac{10a^6c^2x^6+27a^4c^2x^4+16a^2c^2x^2-16c^2\log\left(a^2x^2+1\right)-4\left(15a^7c^2x^7+42a^5c^2x^5+35a^3c^2x^3\right)\arctan(ax)}{420a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^2*arctan(a*x),x, algorithm="fricas")
```

```
[Out] -1/420*(10*a^6*c^2*x^6 + 27*a^4*c^2*x^4 + 16*a^2*c^2*x^2 - 16*c^2*log(a^2*x
^2 + 1) - 4*(15*a^7*c^2*x^7 + 42*a^5*c^2*x^5 + 35*a^3*c^2*x^3)*arctan(a*x))
```

```
/a^3
```

Sympy [A] time = 2.47349, size = 105, normalized size = 0.99

$$\begin{cases} \frac{a^4c^2x^7\operatorname{atan}(ax)}{7} - \frac{a^3c^2x^6}{42} + \frac{2a^2c^2x^5\operatorname{atan}(ax)}{5} - \frac{9ac^2x^4}{140} + \frac{c^2x^3\operatorname{atan}(ax)}{3} - \frac{4c^2x^2}{105a} + \frac{4c^2\log\left(x^2+\frac{1}{a^2}\right)}{105a^3} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a**2*c*x**2+c)**2*atan(a*x),x)
```

```
[Out] Piecewise((a**4*c**2*x**7*atan(a*x)/7 - a**3*c**2*x**6/42 + 2*a**2*c**2*x**
5*atan(a*x)/5 - 9*a*c**2*x**4/140 + c**2*x**3*atan(a*x)/3 - 4*c**2*x**2/(10
5*a) + 4*c**2*log(x**2 + a**(-2))/(105*a**3), Ne(a, 0)), (0, True))
```

Giac [A] time = 1.17053, size = 128, normalized size = 1.21

$$\frac{1}{105}\left(15\,a^4c^2x^7 + 42\,a^2c^2x^5 + 35\,c^2x^3\right)\arctan(ax) + \frac{4\,c^2\log\left(a^2x^2+1\right)}{105\,a^3} - \frac{10\,a^9c^2x^6 + 27\,a^7c^2x^4 + 16\,a^5c^2x^2}{420\,a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^2*arctan(a*x),x, algorithm="giac")
```

```
[Out] 1/105*(15*a^4*c^2*x^7 + 42*a^2*c^2*x^5 + 35*c^2*x^3)*arctan(a*x) + 4/105*c^
2*log(a^2*x^2 + 1)/a^3 - 1/420*(10*a^9*c^2*x^6 + 27*a^7*c^2*x^4 + 16*a^5*c^
2*x^2)/a^6
```

3.159 $\int x\left(c+a^2cx^2\right)^2\tan^{-1}(ax)\,dx$

Optimal. Leaf size=61

$$-\frac{1}{30}a^3c^2x^5+\frac{c^2\left(a^2x^2+1\right)^3\tan^{-1}(ax)}{6a^2}-\frac{1}{9}ac^2x^3-\frac{c^2x}{6a}$$

```
[Out] -(c^2*x)/(6*a) - (a*c^2*x^3)/9 - (a^3*c^2*x^5)/30 + (c^2*(1 + a^2*x^2)^3*Ar
cTan[a*x])/(6*a^2)
```

Rubi [A] time = 0.0425797, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4930, 194}

$$-\frac{1}{30}a^3c^2x^5+\frac{c^2\left(a^2x^2+1\right)^3\tan^{-1}(ax)}{6a^2}-\frac{1}{9}ac^2x^3-\frac{c^2x}{6a}$$

Antiderivative was successfully verified.

```
[In] Int[x*(c + a^2*c*x^2)^2*ArcTan[a*x],x]
```

```
[Out] -(c^2*x)/(6*a) - (a*c^2*x^3)/9 - (a^3*c^2*x^5)/30 + (c^2*(1 + a^2*x^2)^3*Ar
cTan[a*x])/(6*a^2)
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int x\left(c+a^2cx^2\right)^2\tan^{-1}(ax)\,dx = \frac{c^2\left(1+a^2x^2\right)^3\tan^{-1}(ax)}{6a^2} - \frac{\int \left(c+a^2cx^2\right)^2\,dx}{6a}$$
$$= \frac{c^2\left(1+a^2x^2\right)^3\tan^{-1}(ax)}{6a^2} - \frac{\int \left(c^2+2a^2c^2x^2+a^4c^2x^4\right)\,dx}{6a}$$
$$= -\frac{c^2x}{6a} - \frac{1}{9}ac^2x^3 - \frac{1}{30}a^3c^2x^5 + \frac{c^2\left(1+a^2x^2\right)^3\tan^{-1}(ax)}{6a^2}$$

Mathematica [A] time = 0.0483689, size = 98, normalized size = 1.61

$$-\frac{1}{30}a^3c^2x^5 + \frac{1}{6}a^4c^2x^6\tan^{-1}(ax) + \frac{1}{2}a^2c^2x^4\tan^{-1}(ax) + \frac{c^2\tan^{-1}(ax)}{6a^2} - \frac{1}{9}ac^2x^3 + \frac{1}{2}c^2x^2\tan^{-1}(ax) - \frac{c^2x}{6a}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(c + a^2*c*x^2)^2*ArcTan[a*x],x]
```

```
[Out] -(c^2*x)/(6*a) - (a*c^2*x^3)/9 - (a^3*c^2*x^5)/30 + (c^2*ArcTan[a*x])/(6*a^
2) + (c^2*x^2*ArcTan[a*x])/2 + (a^2*c^2*x^4*ArcTan[a*x])/2 + (a^4*c^2*x^6*A
rcTan[a*x])/6
```

Maple [A] time = 0.023, size = 85, normalized size = 1.4

$$\frac{a^4c^2\arctan(ax)\,x^6}{6} + \frac{a^2c^2\arctan(ax)\,x^4}{2} + \frac{c^2\arctan(ax)\,x^2}{2} - \frac{a^3c^2x^5}{30} - \frac{ac^2x^3}{9} - \frac{c^2x}{6\,a} + \frac{c^2\arctan(ax)}{6\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^2*arctan(a*x),x)
```

```
[Out] 1/6*a^4*c^2*arctan(a*x)*x^6+1/2*a^2*c^2*arctan(a*x)*x^4+1/2*c^2*arctan(a*x)
*x^2-1/30*a^3*c^2*x^5-1/9*a*c^2*x^3-1/6*c^2*x/a+1/6/a^2*c^2*arctan(a*x)
```

Maxima [A] time = 0.968101, size = 84, normalized size = 1.38

$$\frac{\left(a^2cx^2+c\right)^3\arctan(ax)}{6\,a^2c} - \frac{3\,a^4c^3x^5 + 10\,a^2c^3x^3 + 15\,c^3x}{90\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^2*arctan(a*x),x, algorithm="maxima")
```

```
[Out] 1/6*(a^2*c*x^2 + c)^3*arctan(a*x)/(a^2*c) - 1/90*(3*a^4*c^3*x^5 + 10*a^2*c^
3*x^3 + 15*c^3*x)/(a*c)
```

Fricas [A] time = 1.60719, size = 170, normalized size = 2.79

$$-\frac{3\,a^5c^2x^5+10\,a^3c^2x^3+15\,ac^2x-15\left(a^6c^2x^6+3\,a^4c^2x^4+3\,a^2c^2x^2+c^2\right)\arctan\left(ax\right)}{90\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^2*arctan(a*x),x, algorithm="fricas")
```

```
[Out] -1/90*(3*a^5*c^2*x^5 + 10*a^3*c^2*x^3 + 15*a*c^2*x - 15*(a^6*c^2*x^6 + 3*a^
4*c^2*x^4 + 3*a^2*c^2*x^2 + c^2)*arctan(a*x))/a^2
```

Sympy [A] time = 2.10347, size = 92, normalized size = 1.51

$$\begin{cases}\frac{a^4c^2x^6\operatorname{atan}\left(ax\right)}{6}-\frac{a^3c^2x^5}{30}+\frac{a^2c^2x^4\operatorname{atan}\left(ax\right)}{2}-\frac{ac^2x^3}{9}+\frac{c^2x^2\operatorname{atan}\left(ax\right)}{2}-\frac{c^2x}{6a}+\frac{c^2\operatorname{atan}\left(ax\right)}{6a^2} & \text{for}\ a\neq 0\\ 0 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**2*atan(a*x),x)
```

```
[Out] Piecewise((a**4*c**2*x**6*atan(a*x)/6 - a**3*c**2*x**5/30 + a**2*c**2*x**4*
atan(a*x)/2 - a*c**2*x**3/9 + c**2*x**2*atan(a*x)/2 - c**2*x/(6*a) + c**2*a
tan(a*x)/(6*a**2), Ne(a, 0)), (0, True))
```

Giac [A] time = 1.10864, size = 80, normalized size = 1.31

$$\frac{\left(a^2cx^2+c\right)^3\arctan\left(ax\right)}{6\,a^2c}-\frac{3\,a^4c^2x^5+10\,a^2c^2x^3+15\,c^2x}{90\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^2*arctan(a*x),x, algorithm="giac")
```

```
[Out] 1/6*(a^2*c*x^2 + c)^3*arctan(a*x)/(a^2*c) - 1/90*(3*a^4*c^2*x^5 + 10*a^2*c^
2*x^3 + 15*c^2*x)/a
```

3.160 $\int \left(c+a^2cx^2\right)^2 \tan^{-1}(ax)\,dx$

Optimal. Leaf size=117

$$-\frac{c^2\left(a^2x^2+1\right)^2}{20a}-\frac{2c^2\left(a^2x^2+1\right)}{15a}-\frac{4c^2\log\left(a^2x^2+1\right)}{15a}+\frac{1}{5}c^2x\left(a^2x^2+1\right)^2\tan^{-1}(ax)+\frac{4}{15}c^2x\left(a^2x^2+1\right)\tan^{-1}(ax)+\frac{8}{15}$$

```
[Out] (-2*c^2*(1 + a^2*x^2))/(15*a) - (c^2*(1 + a^2*x^2)^2)/(20*a) + (8*c^2*x*Arc
Tan[a*x])/15 + (4*c^2*x*(1 + a^2*x^2)*ArcTan[a*x])/15 + (c^2*x*(1 + a^2*x^2
)^2*ArcTan[a*x])/5 - (4*c^2*Log[1 + a^2*x^2])/(15*a)
```

Rubi [A] time = 0.0454285, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {4878, 4846, 260}

$$-\frac{c^2\left(a^2x^2+1\right)^2}{20a}-\frac{2c^2\left(a^2x^2+1\right)}{15a}-\frac{4c^2\log\left(a^2x^2+1\right)}{15a}+\frac{1}{5}c^2x\left(a^2x^2+1\right)^2\tan^{-1}(ax)+\frac{4}{15}c^2x\left(a^2x^2+1\right)\tan^{-1}(ax)+\frac{8}{15}$$

Antiderivative was successfully verified.

```
[In] Int[(c + a^2*c*x^2)^2*ArcTan[a*x],x]
```

```
[Out] (-2*c^2*(1 + a^2*x^2))/(15*a) - (c^2*(1 + a^2*x^2)^2)/(20*a) + (8*c^2*x*Arc
Tan[a*x])/15 + (4*c^2*x*(1 + a^2*x^2)*ArcTan[a*x])/15 + (c^2*x*(1 + a^2*x^2
)^2*ArcTan[a*x])/5 - (4*c^2*Log[1 + a^2*x^2])/(15*a)
```

Rule 4878

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symbo
l] :> -Simp[(b*(d + e*x^2)^q)/(2*c*q*(2*q + 1)), x] + (Dist[(2*d*q)/(2*q +
1), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x]), x], x] + Simp[(x*(d + e*x^
2)^q*(a + b*ArcTan[c*x]))/(2*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] && Eq
Q[e, c^2*d] && GtQ[q, 0]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \left(c+a^2cx^2\right)^2\tan^{-1}(ax)\,dx = -\frac{c^2\left(1+a^2x^2\right)^2}{20a}+\frac{1}{5}c^2x\left(1+a^2x^2\right)^2\tan^{-1}(ax)+\frac{1}{5}(4c)\int \left(c+a^2cx^2\right)\tan^{-1}(ax)\,dx$$
$$= -\frac{2c^2\left(1+a^2x^2\right)}{15a}-\frac{c^2\left(1+a^2x^2\right)^2}{20a}+\frac{4}{15}c^2x\left(1+a^2x^2\right)\tan^{-1}(ax)+\frac{1}{5}c^2x\left(1+a^2x^2\right)^2\tan^{-1}($$
$$= -\frac{2c^2\left(1+a^2x^2\right)}{15a}-\frac{c^2\left(1+a^2x^2\right)^2}{20a}+\frac{8}{15}c^2x\tan^{-1}(ax)+\frac{4}{15}c^2x\left(1+a^2x^2\right)\tan^{-1}(ax)+\frac{1}{5}c^2$$
$$= -\frac{2c^2\left(1+a^2x^2\right)}{15a}-\frac{c^2\left(1+a^2x^2\right)^2}{20a}+\frac{8}{15}c^2x\tan^{-1}(ax)+\frac{4}{15}c^2x\left(1+a^2x^2\right)\tan^{-1}(ax)+\frac{1}{5}c^2$$

Mathematica [A] time = 0.0558637, size = 65, normalized size = 0.56

$$\frac{c^2\left(-3a^4x^4-14a^2x^2-16\log\left(a^2x^2+1\right)+4ax\left(3a^4x^4+10a^2x^2+15\right)\tan^{-1}(ax)\right)}{60a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + a^2*c*x^2)^2*ArcTan[a*x],x]
```

```
[Out] (c^2*(-14*a^2*x^2 - 3*a^4*x^4 + 4*a*x*(15 + 10*a^2*x^2 + 3*a^4*x^4)*ArcTan[
a*x] - 16*Log[1 + a^2*x^2]))/(60*a)
```

Maple [A] time = 0.025, size = 79, normalized size = 0.7

$$\frac{a^4c^2\arctan(ax)\,x^5}{5}+\frac{2\,a^2c^2\arctan(ax)\,x^3}{3}+c^2x\arctan(ax)-\frac{a^3c^2x^4}{20}-\frac{7\,c^2x^2a}{30}-\frac{4\,c^2\ln\left(a^2x^2+1\right)}{15\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^2*arctan(a*x),x)
```

```
[Out] 1/5*a^4*c^2*arctan(a*x)*x^5+2/3*a^2*c^2*arctan(a*x)*x^3+c^2*x*arctan(a*x)-1
/20*a^3*c^2*x^4-7/30*c^2*x^2*a-4/15*c^2*ln(a^2*x^2+1)/a
```

Maxima [A] time = 0.980269, size = 104, normalized size = 0.89

$$-\frac{1}{60}\left(3\,a^2c^2x^4+14\,c^2x^2+\frac{16\,c^2\log\left(a^2x^2+1\right)}{a^2}\right)a+\frac{1}{15}\left(3\,a^4c^2x^5+10\,a^2c^2x^3+15\,c^2x\right)\arctan\left(ax\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x),x, algorithm="maxima")
```

```
[Out] -1/60*(3*a^2*c^2*x^4 + 14*c^2*x^2 + 16*c^2*log(a^2*x^2 + 1)/a^2)*a + 1/15*(
3*a^4*c^2*x^5 + 10*a^2*c^2*x^3 + 15*c^2*x)*arctan(a*x)
```

Fricas [A] time = 1.57001, size = 176, normalized size = 1.5

$$-\frac{3\,a^4c^2x^4+14\,a^2c^2x^2+16\,c^2\log\left(a^2x^2+1\right)-4\left(3\,a^5c^2x^5+10\,a^3c^2x^3+15\,ac^2x\right)\arctan\left(ax\right)}{60\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x),x, algorithm="fricas")
```

```
[Out] -1/60*(3*a^4*c^2*x^4 + 14*a^2*c^2*x^2 + 16*c^2*log(a^2*x^2 + 1) - 4*(3*a^5*
c^2*x^5 + 10*a^3*c^2*x^3 + 15*a*c^2*x)*arctan(a*x))/a
```

Sympy [A] time = 1.46037, size = 88, normalized size = 0.75

$$\begin{cases}\frac{a^4c^2x^5\operatorname{atan}(ax)}{5}-\frac{a^3c^2x^4}{20}+\frac{2a^2c^2x^3\operatorname{atan}(ax)}{3}-\frac{7ac^2x^2}{30}+c^2x\operatorname{atan}(ax)-\frac{4c^2\log\left(x^2+\frac{1}{a^2}\right)}{15a} & \text{for}\ a\neq 0\\ 0 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**2*atan(a*x),x)
```

```
[Out] Piecewise((a**4*c**2*x**5*atan(a*x)/5 - a**3*c**2*x**4/20 + 2*a**2*c**2*x**
3*atan(a*x)/3 - 7*a*c**2*x**2/30 + c**2*x*atan(a*x) - 4*c**2*log(x**2 + a**
(-2))/(15*a), Ne(a, 0)), (0, True))
```

Giac [A] time = 1.18454, size = 111, normalized size = 0.95

$$\frac{1}{15}\left(3\,a^4c^2x^5 + 10\,a^2c^2x^3 + 15\,c^2x\right)\arctan\left(ax\right) - \frac{4\,c^2\log\left(a^2x^2+1\right)}{15\,a} - \frac{3\,a^7c^2x^4 + 14\,a^5c^2x^2}{60\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x),x, algorithm="giac")
```

```
[Out] 1/15*(3*a^4*c^2*x^5 + 10*a^2*c^2*x^3 + 15*c^2*x)*arctan(a*x) - 4/15*c^2*log
(a^2*x^2 + 1)/a - 1/60*(3*a^7*c^2*x^4 + 14*a^5*c^2*x^2)/a^4
```

3.161 $\int \frac{\left(c+a^2cx^2\right)^2\tan^{-1}(ax)}{x}\,dx$

Optimal. Leaf size=99

$$\frac{1}{2}ic^2\text{PolyLog}(2,-iax)-\frac{1}{2}ic^2\text{PolyLog}(2,iax)-\frac{1}{12}a^3c^2x^3+\frac{1}{4}a^4c^2x^4\tan^{-1}(ax)+a^2c^2x^2\tan^{-1}(ax)-\frac{3}{4}ac^2x+\frac{3}{4}c^2\tan^{-1}(ax)$$

```
[Out] (-3*a*c^2*x)/4 - (a^3*c^2*x^3)/12 + (3*c^2*ArcTan[a*x])/4 + a^2*c^2*x^2*Arc
Tan[a*x] + (a^4*c^2*x^4*ArcTan[a*x])/4 + (I/2)*c^2*PolyLog[2, (-I)*a*x] - (
I/2)*c^2*PolyLog[2, I*a*x]
```

Rubi [A] time = 0.119193, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {4948, 4848, 2391, 4852, 321, 203, 302}

$$\frac{1}{2}ic^2\text{PolyLog}(2,-iax)-\frac{1}{2}ic^2\text{PolyLog}(2,iax)-\frac{1}{12}a^3c^2x^3+\frac{1}{4}a^4c^2x^4\tan^{-1}(ax)+a^2c^2x^2\tan^{-1}(ax)-\frac{3}{4}ac^2x+\frac{3}{4}c^2\tan^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)^2*ArcTan[a*x])/x,x]
```


```
[Out] (-3*a*c^2*x)/4 - (a^3*c^2*x^3)/12 + (3*c^2*ArcTan[a*x])/4 + a^2*c^2*x^2*Arc
Tan[a*x] + (a^4*c^2*x^4*ArcTan[a*x])/4 + (I/2)*c^2*PolyLog[2, (-I)*a*x] - (
I/2)*c^2*PolyLog[2, I*a*x]
```


Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\int \frac{\left(c+a^2cx^2\right)^2\tan^{-1}(ax)}{x}\,dx = \int \left(\frac{c^2\tan^{-1}(ax)}{x} + 2a^2c^2x\tan^{-1}(ax) + a^4c^2x^3\tan^{-1}(ax)\right)dx$$
$$= c^2\int \frac{\tan^{-1}(ax)}{x}\,dx + \left(2a^2c^2\right)\int x\tan^{-1}(ax)\,dx + \left(a^4c^2\right)\int x^3\tan^{-1}(ax)\,dx$$
$$= a^2c^2x^2\tan^{-1}(ax) + \frac{1}{4}a^4c^2x^4\tan^{-1}(ax) + \frac{1}{2}\left(ic^2\right)\int \frac{\log(1-iax)}{x}\,dx - \frac{1}{2}\left(ic^2\right)\int \frac{\log(1+}{x}$$
$$= -ac^2x + a^2c^2x^2\tan^{-1}(ax) + \frac{1}{4}a^4c^2x^4\tan^{-1}(ax) + \frac{1}{2}ic^2\mathrm{Li}_2(-iax) - \frac{1}{2}ic^2\mathrm{Li}_2(iax) + \left(ac^2\right)$$
$$= -\frac{3}{4}ac^2x - \frac{1}{12}a^3c^2x^3 + c^2\tan^{-1}(ax) + a^2c^2x^2\tan^{-1}(ax) + \frac{1}{4}a^4c^2x^4\tan^{-1}(ax) + \frac{1}{2}ic^2\mathrm{Li}_2($$
$$= -\frac{3}{4}ac^2x - \frac{1}{12}a^3c^2x^3 + \frac{3}{4}c^2\tan^{-1}(ax) + a^2c^2x^2\tan^{-1}(ax) + \frac{1}{4}a^4c^2x^4\tan^{-1}(ax) + \frac{1}{2}ic^2\mathrm{Li}$$

Mathematica [A] time = 0.0342253, size = 99, normalized size = 1.

$$\frac{1}{2}ic^2\mathrm{PolyLog}(2,-iax) - \frac{1}{2}ic^2\mathrm{PolyLog}(2,iax) - \frac{1}{12}a^3c^2x^3 + \frac{1}{4}a^4c^2x^4\tan^{-1}(ax) + a^2c^2x^2\tan^{-1}(ax) - \frac{3}{4}ac^2x + \frac{3}{4}c^2\tan^{-}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + a^2*c*x^2)^2*ArcTan[a*x])/x,x]
```

```
[Out] (-3*a*c^2*x)/4 - (a^3*c^2*x^3)/12 + (3*c^2*ArcTan[a*x])/4 + a^2*c^2*x^2*Arc
Tan[a*x] + (a^4*c^2*x^4*ArcTan[a*x])/4 + (I/2)*c^2*PolyLog[2, (-I)*a*x] - (
I/2)*c^2*PolyLog[2, I*a*x]
```

Maple [A] time = 0.036, size = 134, normalized size = 1.4

$$\frac{a^4c^2x^4\arctan(ax)}{4} + a^2c^2x^2\arctan(ax) + c^2\arctan(ax)\ln(ax) - \frac{a^3c^2x^3}{12} - \frac{3\,ac^2x}{4} + \frac{3\,c^2\arctan(ax)}{4} + \frac{i}{2}c^2\ln(ax)\ln$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^2*arctan(a*x)/x,x)
```

```
[Out] 1/4*a^4*c^2*x^4*arctan(a*x)+a^2*c^2*x^2*arctan(a*x)+c^2*arctan(a*x)*ln(a*x)
-1/12*a^3*c^2*x^3-3/4*a*c^2*x+3/4*c^2*arctan(a*x)+1/2*I*c^2*ln(a*x)*ln(1+I*
a*x)-1/2*I*c^2*ln(a*x)*ln(1-I*a*x)+1/2*I*c^2*dilog(1+I*a*x)-1/2*I*c^2*dilog
```

```
(1-I*a*x)
```

Maxima [A] time = 1.63901, size = 150, normalized size = 1.52

$$-\frac{1}{12}a^3c^2x^3 - \frac{3}{4}ac^2x - \frac{1}{4}\pi c^2\log\left(a^2x^2+1\right) + c^2\arctan\left(ax\right)\log\left(x|a|\right) - \frac{1}{2}ic^2\mathrm{Li}_2\left(iax+1\right) + \frac{1}{2}ic^2\mathrm{Li}_2\left(-iax+1\right) + \frac{1}{4}\left(a^4\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)/x,x, algorithm="maxima")
```

```
[Out] -1/12*a^3*c^2*x^3 - 3/4*a*c^2*x - 1/4*pi*c^2*log(a^2*x^2 + 1) + c^2*arctan(
a*x)*log(x*abs(a)) - 1/2*I*c^2*dilog(I*a*x + 1) + 1/2*I*c^2*dilog(-I*a*x +
1) + 1/4*(a^4*c^2*x^4 + 4*a^2*c^2*x^2 + c^2*(4*I*arctan2(0, a) + 3))*arctan
(a*x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(a^4c^2x^4 + 2\,a^2c^2x^2 + c^2\right)\arctan\left(ax\right)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)/x,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(a*x)/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$c^2\left(\int\frac{\operatorname{atan}\left(ax\right)}{x}\,dx + \int 2a^2x\operatorname{atan}\left(ax\right)dx + \int a^4x^3\operatorname{atan}\left(ax\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**2*atan(a*x)/x,x)
```

```
[Out] c**2*(Integral(atan(a*x)/x, x) + Integral(2*a**2*x*atan(a*x), x) + Integral
(a**4*x**3*atan(a*x), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^2 \arctan(ax)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)/x,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^2*arctan(a*x)/x, x)
```

3.162 $\int \frac{\left(c+a^2cx^2\right)^2 \tan^{-1}(ax)}{x^2} \, dx$

Optimal. Leaf size=81

$$-\frac{1}{6}a^3c^2x^2-\frac{4}{3}ac^2\log\left(a^2x^2+1\right)+\frac{1}{3}a^4c^2x^3\tan^{-1}(ax)+2a^2c^2x\tan^{-1}(ax)+ac^2\log(x)-\frac{c^2\tan^{-1}(ax)}{x}$$

```
[Out] -(a^3*c^2*x^2)/6 - (c^2*ArcTan[a*x])/x + 2*a^2*c^2*x*ArcTan[a*x] + (a^4*c^2
*x^3*ArcTan[a*x])/3 + a*c^2*Log[x] - (4*a*c^2*Log[1 + a^2*x^2])/3
```

Rubi [A] time = 0.116758, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.45, Rules used = {4948, 4846, 260, 4852, 266, 36, 29, 31, 43}

$$-\frac{1}{6}a^3c^2x^2-\frac{4}{3}ac^2\log\left(a^2x^2+1\right)+\frac{1}{3}a^4c^2x^3\tan^{-1}(ax)+2a^2c^2x\tan^{-1}(ax)+ac^2\log(x)-\frac{c^2\tan^{-1}(ax)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)^2*ArcTan[a*x])/x^2,x]
```

```
[Out] -(a^3*c^2*x^2)/6 - (c^2*ArcTan[a*x])/x + 2*a^2*c^2*x*ArcTan[a*x] + (a^4*c^2
*x^3*ArcTan[a*x])/3 + a*c^2*Log[x] - (4*a*c^2*Log[1 + a^2*x^2])/3
```

Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\left(c+a^2cx^2\right)^2 \tan^{-1}(ax)}{x^2}\,dx = \int \left(2a^2c^2\tan^{-1}(ax) + \frac{c^2\tan^{-1}(ax)}{x^2} + a^4c^2x^2\tan^{-1}(ax)\right)dx$$

$$= c^2\int \frac{\tan^{-1}(ax)}{x^2}\,dx + \left(2a^2c^2\right)\int \tan^{-1}(ax)\,dx + \left(a^4c^2\right)\int x^2\tan^{-1}(ax)\,dx$$

$$= -\frac{c^2\tan^{-1}(ax)}{x} + 2a^2c^2x\tan^{-1}(ax) + \frac{1}{3}a^4c^2x^3\tan^{-1}(ax) + \left(ac^2\right)\int \frac{1}{x\left(1+a^2x^2\right)}\,dx - \left(2a\right.$$

$$= -\frac{c^2\tan^{-1}(ax)}{x} + 2a^2c^2x\tan^{-1}(ax) + \frac{1}{3}a^4c^2x^3\tan^{-1}(ax) - ac^2\log\left(1+a^2x^2\right) + \frac{1}{2}\left(ac^2\right)\mathrm{Su}$$

$$= -\frac{c^2\tan^{-1}(ax)}{x} + 2a^2c^2x\tan^{-1}(ax) + \frac{1}{3}a^4c^2x^3\tan^{-1}(ax) - ac^2\log\left(1+a^2x^2\right) + \frac{1}{2}\left(ac^2\right)\mathrm{Su}$$

$$= -\frac{1}{6}a^3c^2x^2 - \frac{c^2\tan^{-1}(ax)}{x} + 2a^2c^2x\tan^{-1}(ax) + \frac{1}{3}a^4c^2x^3\tan^{-1}(ax) + ac^2\log(x) - \frac{4}{3}ac^2\,\mathrm{lo}$$

Mathematica [A] time = 0.0513391, size = 62, normalized size = 0.77

$$\frac{c^2\left(2\left(a^4x^4+6a^2x^2-3\right)\tan^{-1}(ax) - ax\left(a^2x^2+8\log\left(a^2x^2+1\right)-6\log(x)\right)\right)}{6x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + a^2*c*x^2)^2*ArcTan[a*x])/x^2,x]
```

```
[Out] (c^2*(2*(-3 + 6*a^2*x^2 + a^4*x^4)*ArcTan[a*x] - a*x*(a^2*x^2 - 6*Log[x] +
8*Log[1 + a^2*x^2])))/(6*x)
```

Maple [A] time = 0.03, size = 78, normalized size = 1.

$$\frac{a^4c^2x^3\arctan(ax)}{3} + 2a^2c^2x\arctan(ax) - \frac{c^2\arctan(ax)}{x} - \frac{c^2x^2a^3}{6} - \frac{4ac^2\ln\left(a^2x^2+1\right)}{3} + ac^2\ln(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^2*arctan(a*x)/x^2,x)
```

```
[Out] 1/3*a^4*c^2*x^3*arctan(a*x)+2*a^2*c^2*x*arctan(a*x)-c^2*arctan(a*x)/x-1/6*c
^2*x^2*a^3-4/3*a*c^2*ln(a^2*x^2+1)+a*c^2*ln(a*x)
```

Maxima [A] time = 0.968203, size = 96, normalized size = 1.19

$$-\frac{1}{6}\left(a^2c^2x^2+8\,c^2\log\left(a^2x^2+1\right)-6\,c^2\log\left(x\right)\right)a+\frac{1}{3}\left(a^4c^2x^3+6\,a^2c^2x-\frac{3\,c^2}{x}\right)\arctan\left(ax\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)/x^2,x, algorithm="maxima")
```

```
[Out] -1/6*(a^2*c^2*x^2 + 8*c^2*log(a^2*x^2 + 1) - 6*c^2*log(x))*a + 1/3*(a^4*c^2
*x^3 + 6*a^2*c^2*x - 3*c^2/x)*arctan(a*x)
```

Fricas [A] time = 1.70552, size = 167, normalized size = 2.06

$$-\frac{a^3c^2x^3+8\,ac^2x\log\left(a^2x^2+1\right)-6\,ac^2x\log\left(x\right)-2\left(a^4c^2x^4+6\,a^2c^2x^2-3\,c^2\right)\arctan\left(ax\right)}{6\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)/x^2,x, algorithm="fricas")
```

```
[Out] -1/6*(a^3*c^2*x^3 + 8*a*c^2*x*log(a^2*x^2 + 1) - 6*a*c^2*x*log(x) - 2*(a^4*
c^2*x^4 + 6*a^2*c^2*x^2 - 3*c^2)*arctan(a*x))/x
```

Sympy [A] time = 1.85078, size = 82, normalized size = 1.01

$$\begin{cases}\frac{a^4c^2x^3\operatorname{atan}\left(ax\right)}{3}-\frac{a^3c^2x^2}{6}+2a^2c^2x\operatorname{atan}\left(ax\right)+ac^2\log\left(x\right)-\frac{4ac^2\log\left(x^2+\frac{1}{a^2}\right)}{3}-\frac{c^2\operatorname{atan}\left(ax\right)}{x} & \text{for}\ a\neq 0\\ 0 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**2*atan(a*x)/x**2,x)
```

```
[Out] Piecewise((a**4*c**2*x**3*atan(a*x)/3 - a**3*c**2*x**2/6 + 2*a**2*c**2*x*at
an(a*x) + a*c**2*log(x) - 4*a*c**2*log(x**2 + a**(-2))/3 - c**2*atan(a*x)/x
, Ne(a, 0)), (0, True))
```

Giac [A] time = 1.10871, size = 97, normalized size = 1.2

$$-\frac{1}{6}a^{3}c^{2}x^{2}-\frac{4}{3}ac^{2}\log\left(a^{2}x^{2}+1\right)+\frac{1}{2}ac^{2}\log\left(x^{2}\right)+\frac{1}{3}\left(a^{4}c^{2}x^{3}+6a^{2}c^{2}x-\frac{3c^{2}}{x}\right)\arctan\left(ax\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)/x^2,x, algorithm="giac")
```

```
[Out] -1/6*a^3*c^2*x^2 - 4/3*a*c^2*log(a^2*x^2 + 1) + 1/2*a*c^2*log(x^2) + 1/3*(a
^4*c^2*x^3 + 6*a^2*c^2*x - 3*c^2/x)*arctan(a*x)
```

3.163 $\int \frac{\left(c+a^2cx^2\right)^2 \tan^{-1}(ax)}{x^3}\,dx$

Optimal. Leaf size=90

$$ia^2c^2\text{PolyLog}(2,-iax)-ia^2c^2\text{PolyLog}(2,iax)+\frac{1}{2}a^4c^2x^2\tan^{-1}(ax)-\frac{1}{2}a^3c^2x-\frac{c^2\tan^{-1}(ax)}{2x^2}-\frac{ac^2}{2x}$$

```
[Out] -(a*c^2)/(2*x) - (a^3*c^2*x)/2 - (c^2*ArcTan[a*x])/(2*x^2) + (a^4*c^2*x^2*A
rcTan[a*x])/2 + I*a^2*c^2*PolyLog[2, (-I)*a*x] - I*a^2*c^2*PolyLog[2, I*a*x
]
```

Rubi [A] time = 0.123163, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {4948, 4852, 325, 203, 4848, 2391, 321}

$$ia^2c^2\text{PolyLog}(2,-iax)-ia^2c^2\text{PolyLog}(2,iax)+\frac{1}{2}a^4c^2x^2\tan^{-1}(ax)-\frac{1}{2}a^3c^2x-\frac{c^2\tan^{-1}(ax)}{2x^2}-\frac{ac^2}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)^2*ArcTan[a*x])/x^3,x]
```

```
[Out] -(a*c^2)/(2*x) - (a^3*c^2*x)/2 - (c^2*ArcTan[a*x])/(2*x^2) + (a^4*c^2*x^2*A
rcTan[a*x])/2 + I*a^2*c^2*PolyLog[2, (-I)*a*x] - I*a^2*c^2*PolyLog[2, I*a*x
]
```

Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^2 \tan^{-1}(ax)}{x^3}\,dx = \int \left(\frac{c^2\tan^{-1}(ax)}{x^3} + \frac{2a^2c^2\tan^{-1}(ax)}{x} + a^4c^2x\tan^{-1}(ax)\right)dx$$
$$= c^2\int \frac{\tan^{-1}(ax)}{x^3}\,dx + (2a^2c^2)\int \frac{\tan^{-1}(ax)}{x}\,dx + (a^4c^2)\int x\tan^{-1}(ax)\,dx$$
$$= -\frac{c^2\tan^{-1}(ax)}{2x^2} + \frac{1}{2}a^4c^2x^2\tan^{-1}(ax) + \frac{1}{2}(ac^2)\int \frac{1}{x^2(1+a^2x^2)}\,dx + (ia^2c^2)\int \frac{\log(1-}{x}$$
$$= -\frac{ac^2}{2x} - \frac{1}{2}a^3c^2x - \frac{c^2\tan^{-1}(ax)}{2x^2} + \frac{1}{2}a^4c^2x^2\tan^{-1}(ax) + ia^2c^2\mathrm{Li}_2(-iax) - ia^2c^2\mathrm{Li}_2(iax)$$

Mathematica [C] time = 0.044859, size = 103, normalized size = 1.14

$$\frac{c^2\left(-ax\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-a^2x^2\right) + 2ia^2x^2\text{PolyLog}(2,-iax) - 2ia^2x^2\text{PolyLog}(2,iax) - a^3x^3 + a^4x^4\tan^{-}\right.}{2x^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((c + a^2*c*x^2)^2*ArcTan[a*x])/x^3,x]
```

```
[Out] (c^2*(-(a^3*x^3) - ArcTan[a*x] + a^2*x^2*ArcTan[a*x] + a^4*x^4*ArcTan[a*x]
- a*x*Hypergeometric2F1[-1/2, 1, 1/2, -(a^2*x^2)] + (2*I)*a^2*x^2*PolyLog[2
, (-I)*a*x] - (2*I)*a^2*x^2*PolyLog[2, I*a*x]))/(2*x^2)
```

Maple [A] time = 0.04, size = 139, normalized size = 1.5

$$\frac{a^4c^2x^2\arctan(ax)}{2} - \frac{c^2\arctan(ax)}{2x^2} + 2a^2c^2\arctan(ax)\ln(ax) - \frac{a^3c^2x}{2} - \frac{ac^2}{2x} + ia^2c^2\ln(ax)\ln(1+iax) - ia^2c^2\ln(a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^2*arctan(a*x)/x^3,x)
```

```
[Out] 1/2*a^4*c^2*x^2*arctan(a*x)-1/2*c^2*arctan(a*x)/x^2+2*a^2*c^2*arctan(a*x)*l
n(a*x)-1/2*a^3*c^2*x-1/2*a*c^2/x+I*a^2*c^2*ln(a*x)*ln(1+I*a*x)-I*a^2*c^2*ln
(a*x)*ln(1-I*a*x)+I*a^2*c^2*dilog(1+I*a*x)-I*a^2*c^2*dilog(1-I*a*x)
```

Maxima [A] time = 1.65607, size = 182, normalized size = 2.02

$$-\frac{a^3c^2x^3 + \pi a^2c^2x^2\log\left(a^2x^2+1\right) - 4\,a^2c^2x^2\arctan\left(ax\right)\log\left(x|a|\right) + 2i\,a^2c^2x^2\mathrm{Li}_2\left(i\,ax+1\right) - 2i\,a^2c^2x^2\mathrm{Li}_2\left(-i\,ax+1\right) + ac}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*(a^3*c^2*x^3 + pi*a^2*c^2*x^2*log(a^2*x^2 + 1) - 4*a^2*c^2*x^2*arctan(
a*x)*log(x*abs(a)) + 2*I*a^2*c^2*x^2*dilog(I*a*x + 1) - 2*I*a^2*c^2*x^2*dil
og(-I*a*x + 1) + a*c^2*x - (a^4*c^2*x^4 + 4*I*a^2*c^2*x^2*arctan2(0, a) - c
^2)*arctan(a*x))/x^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(a^4c^2x^4 + 2\,a^2c^2x^2 + c^2\right)\arctan\left(ax\right)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)/x^3,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(a*x)/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$c^2\left(\int\frac{\operatorname{atan}\left(ax\right)}{x^3}\,dx + \int\frac{2a^2\operatorname{atan}\left(ax\right)}{x}\,dx + \int a^4x\operatorname{atan}\left(ax\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**2*atan(a*x)/x**3,x)
```

```
[Out] c**2*(Integral(atan(a*x)/x**3, x) + Integral(2*a**2*atan(a*x)/x, x) + Integ
ral(a**4*x*atan(a*x), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^2 \arctan\left(ax\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)/x^3,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^2*arctan(a*x)/x^3, x)
```

3.164 $\int \frac{(c+a^2cx^2)^2 \tan^{-1}(ax)}{x^4} dx$

Optimal. Leaf size=85

$$-\frac{4}{3}a^3c^2 \log\left(a^2x^2+1\right)+\frac{5}{3}a^3c^2 \log(x)+a^4c^2x \tan^{-1}(ax)-\frac{2a^2c^2 \tan^{-1}(ax)}{x}-\frac{ac^2}{6x^2}-\frac{c^2 \tan^{-1}(ax)}{3x^3}$$

```
[Out] -(a*c^2)/(6*x^2) - (c^2*ArcTan[a*x])/(3*x^3) - (2*a^2*c^2*ArcTan[a*x])/x +
a^4*c^2*x*ArcTan[a*x] + (5*a^3*c^2*Log[x])/3 - (4*a^3*c^2*Log[1 + a^2*x^2])
/3
```

Rubi [A] time = 0.124028, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.45, Rules used = {4948, 4846, 260, 4852, 266, 44, 36, 29, 31}

$$-\frac{4}{3}a^3c^2 \log\left(a^2x^2+1\right)+\frac{5}{3}a^3c^2 \log(x)+a^4c^2x \tan^{-1}(ax)-\frac{2a^2c^2 \tan^{-1}(ax)}{x}-\frac{ac^2}{6x^2}-\frac{c^2 \tan^{-1}(ax)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)^2*ArcTan[a*x])/x^4,x]
```

```
[Out] -(a*c^2)/(6*x^2) - (c^2*ArcTan[a*x])/(3*x^3) - (2*a^2*c^2*ArcTan[a*x])/x +
a^4*c^2*x*ArcTan[a*x] + (5*a^3*c^2*Log[x])/3 - (4*a^3*c^2*Log[1 + a^2*x^2])
/3
```

Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{\left(c+a^2cx^2\right)^2 \tan^{-1}(ax)}{x^4}\,dx = \int \left(a^4c^2\tan^{-1}(ax) + \frac{c^2\tan^{-1}(ax)}{x^4} + \frac{2a^2c^2\tan^{-1}(ax)}{x^2}\right)dx$$

$$= c^2\int \frac{\tan^{-1}(ax)}{x^4}\,dx + \left(2a^2c^2\right)\int \frac{\tan^{-1}(ax)}{x^2}\,dx + \left(a^4c^2\right)\int \tan^{-1}(ax)\,dx$$

$$= -\frac{c^2\tan^{-1}(ax)}{3x^3} - \frac{2a^2c^2\tan^{-1}(ax)}{x} + a^4c^2x\tan^{-1}(ax) + \frac{1}{3}\left(ac^2\right)\int \frac{1}{x^3\left(1+a^2x^2\right)}\,dx + \left(2a\right.$$

$$= -\frac{c^2\tan^{-1}(ax)}{3x^3} - \frac{2a^2c^2\tan^{-1}(ax)}{x} + a^4c^2x\tan^{-1}(ax) - \frac{1}{2}a^3c^2\log\left(1+a^2x^2\right) + \frac{1}{6}\left(ac^2\right)\mathrm{Su}$$

$$= -\frac{c^2\tan^{-1}(ax)}{3x^3} - \frac{2a^2c^2\tan^{-1}(ax)}{x} + a^4c^2x\tan^{-1}(ax) - \frac{1}{2}a^3c^2\log\left(1+a^2x^2\right) + \frac{1}{6}\left(ac^2\right)\mathrm{Su}$$

$$= -\frac{ac^2}{6x^2} - \frac{c^2\tan^{-1}(ax)}{3x^3} - \frac{2a^2c^2\tan^{-1}(ax)}{x} + a^4c^2x\tan^{-1}(ax) + \frac{5}{3}a^3c^2\log(x) - \frac{4}{3}a^3c^2\log\left(1\right.$$

Mathematica [A] time = 0.0548929, size = 68, normalized size = 0.8

$$\frac{c^2\left(ax\left(10a^2x^2\log(x) - 8a^2x^2\log\left(a^2x^2+1\right)-1\right) + 2\left(3a^4x^4 - 6a^2x^2 - 1\right)\tan^{-1}(ax)\right)}{6x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + a^2*c*x^2)^2*ArcTan[a*x])/x^4,x]
```

```
[Out] (c^2*(2*(-1 - 6*a^2*x^2 + 3*a^4*x^4)*ArcTan[a*x] + a*x*(-1 + 10*a^2*x^2*Log
[x] - 8*a^2*x^2*Log[1 + a^2*x^2])))/(6*x^3)
```

Maple [A] time = 0.032, size = 80, normalized size = 0.9

$$a^4c^2x\arctan(ax) - 2\,\frac{a^2c^2\arctan(ax)}{x} - \frac{c^2\arctan(ax)}{3\,x^3} - \frac{4\,a^3c^2\ln\left(a^2x^2+1\right)}{3} - \frac{c^2a}{6\,x^2} + \frac{5\,a^3c^2\ln(ax)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^2*arctan(a*x)/x^4,x)
```

```
[Out] a^4*c^2*x*arctan(a*x)-2*a^2*c^2*arctan(a*x)/x-1/3*c^2*arctan(a*x)/x^3-4/3*a
^3*c^2*ln(a^2*x^2+1)-1/6*a*c^2/x^2+5/3*a^3*c^2*ln(a*x)
```

Maxima [A] time = 1.00723, size = 103, normalized size = 1.21

$$-\frac{1}{6}\left(8\,a^2c^2\log\left(a^2x^2+1\right)-10\,a^2c^2\log\left(x\right)+\frac{c^2}{x^2}\right)a+\frac{1}{3}\left(3\,a^4c^2x-\frac{6\,a^2c^2x^2+c^2}{x^3}\right)\arctan\left(ax\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)/x^4,x, algorithm="maxima")
```

```
[Out] -1/6*(8*a^2*c^2*log(a^2*x^2 + 1) - 10*a^2*c^2*log(x) + c^2/x^2)*a + 1/3*(3*
a^4*c^2*x - (6*a^2*c^2*x^2 + c^2)/x^3)*arctan(a*x)
```

Fricas [A] time = 1.72893, size = 177, normalized size = 2.08

$$-\frac{8\,a^3c^2x^3\log\left(a^2x^2+1\right)-10\,a^3c^2x^3\log\left(x\right)+ac^2x-2\left(3\,a^4c^2x^4-6\,a^2c^2x^2-c^2\right)\arctan\left(ax\right)}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)/x^4,x, algorithm="fricas")
```

```
[Out] -1/6*(8*a^3*c^2*x^3*log(a^2*x^2 + 1) - 10*a^3*c^2*x^3*log(x) + a*c^2*x - 2*
(3*a^4*c^2*x^4 - 6*a^2*c^2*x^2 - c^2)*arctan(a*x))/x^3
```

Sympy [A] time = 1.92768, size = 87, normalized size = 1.02

$$\begin{cases} a^4c^2x\operatorname{atan}\left(ax\right)+\frac{5a^3c^2\log\left(x\right)}{3}-\frac{4a^3c^2\log\left(x^2+\frac{1}{a^2}\right)}{3}-\frac{2a^2c^2\operatorname{atan}\left(ax\right)}{x}-\frac{ac^2}{6x^2}-\frac{c^2\operatorname{atan}\left(ax\right)}{3x^3} & \text{for } a\neq 0\\ 0 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**2*atan(a*x)/x**4,x)
```

```
[Out] Piecewise((a**4*c**2*x*atan(a*x) + 5*a**3*c**2*log(x)/3 - 4*a**3*c**2*log(x
**2 + a**(-2))/3 - 2*a**2*c**2*atan(a*x)/x - a*c**2/(6*x**2) - c**2*atan(a*
x)/(3*x**3), Ne(a, 0)), (0, True))
```

Giac [A] time = 1.12624, size = 120, normalized size = 1.41

$$-\frac{4}{3}\,a^{3}c^{2}\log\left(a^{2}x^{2}+1\right)+\frac{5}{6}\,a^{3}c^{2}\log\left(x^{2}\right)+\frac{1}{3}\left(3\,a^{4}c^{2}x-\frac{6\,a^{2}c^{2}x^{2}+c^{2}}{x^{3}}\right)\arctan\left(ax\right)-\frac{5\,a^{3}c^{2}x^{2}+ac^{2}}{6\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)/x^4,x, algorithm="giac")
```

```
[Out] -4/3*a^3*c^2*log(a^2*x^2 + 1) + 5/6*a^3*c^2*log(x^2) + 1/3*(3*a^4*c^2*x - (
6*a^2*c^2*x^2 + c^2)/x^3)*arctan(a*x) - 1/6*(5*a^3*c^2*x^2 + a*c^2)/x^2
```

3.165 $\int x^3 \left(c+a^2cx^2\right)^3 \tan^{-1}(ax)\,dx$

Optimal. Leaf size=141

$$-\frac{1}{90}a^5c^3x^9-\frac{11}{280}a^3c^3x^7+\frac{1}{10}a^6c^3x^{10}\tan^{-1}(ax)+\frac{3}{8}a^4c^3x^8\tan^{-1}(ax)+\frac{1}{2}a^2c^3x^6\tan^{-1}(ax)+\frac{c^3x}{40a^3}-\frac{c^3\tan^{-1}(ax)}{40a^4}-\frac{9}{20}$$

```
[Out] (c^3*x)/(40*a^3) - (c^3*x^3)/(120*a) - (9*a*c^3*x^5)/200 - (11*a^3*c^3*x^7)
/280 - (a^5*c^3*x^9)/90 - (c^3*ArcTan[a*x])/(40*a^4) + (c^3*x^4*ArcTan[a*x]
)/4 + (a^2*c^3*x^6*ArcTan[a*x])/2 + (3*a^4*c^3*x^8*ArcTan[a*x])/8 + (a^6*c^
3*x^10*ArcTan[a*x])/10
```

Rubi [A] time = 0.207256, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4948, 4852, 302, 203}

$$-\frac{1}{90}a^5c^3x^9-\frac{11}{280}a^3c^3x^7+\frac{1}{10}a^6c^3x^{10}\tan^{-1}(ax)+\frac{3}{8}a^4c^3x^8\tan^{-1}(ax)+\frac{1}{2}a^2c^3x^6\tan^{-1}(ax)+\frac{c^3x}{40a^3}-\frac{c^3\tan^{-1}(ax)}{40a^4}-\frac{9}{20}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(c + a^2*c*x^2)^3*ArcTan[a*x],x]
```


```
[Out] (c^3*x)/(40*a^3) - (c^3*x^3)/(120*a) - (9*a*c^3*x^5)/200 - (11*a^3*c^3*x^7)
/280 - (a^5*c^3*x^9)/90 - (c^3*ArcTan[a*x])/(40*a^4) + (c^3*x^4*ArcTan[a*x]
)/4 + (a^2*c^3*x^6*ArcTan[a*x])/2 + (3*a^4*c^3*x^8*ArcTan[a*x])/8 + (a^6*c^
3*x^10*ArcTan[a*x])/10
```


Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
```

```
erQ[m]) && NeQ[m, -1]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int x^3 \left(c+a^2 c x^2\right)^3 \tan^{-1}(ax)\,dx = \int \left(c^3 x^3 \tan^{-1}(ax) + 3a^2 c^3 x^5 \tan^{-1}(ax) + 3a^4 c^3 x^7 \tan^{-1}(ax) + a^6 c^3 x^9 \tan^{-1}(ax)\right)\,dx$$

$$= c^3 \int x^3 \tan^{-1}(ax)\,dx + \left(3a^2 c^3\right) \int x^5 \tan^{-1}(ax)\,dx + \left(3a^4 c^3\right) \int x^7 \tan^{-1}(ax)\,dx + \left(a^6 c^3\right.$$

$$= \frac{1}{4}c^3 x^4 \tan^{-1}(ax) + \frac{1}{2}a^2 c^3 x^6 \tan^{-1}(ax) + \frac{3}{8}a^4 c^3 x^8 \tan^{-1}(ax) + \frac{1}{10}a^6 c^3 x^{10} \tan^{-1}(ax) - \frac{1}{4}$$

$$= \frac{1}{4}c^3 x^4 \tan^{-1}(ax) + \frac{1}{2}a^2 c^3 x^6 \tan^{-1}(ax) + \frac{3}{8}a^4 c^3 x^8 \tan^{-1}(ax) + \frac{1}{10}a^6 c^3 x^{10} \tan^{-1}(ax) - \frac{1}{4}$$

$$= \frac{c^3 x}{40a^3} - \frac{c^3 x^3}{120a} - \frac{9}{200}a c^3 x^5 - \frac{11}{280}a^3 c^3 x^7 - \frac{1}{90}a^5 c^3 x^9 + \frac{1}{4}c^3 x^4 \tan^{-1}(ax) + \frac{1}{2}a^2 c^3 x^6 \tan^{-}$$

$$= \frac{c^3 x}{40a^3} - \frac{c^3 x^3}{120a} - \frac{9}{200}a c^3 x^5 - \frac{11}{280}a^3 c^3 x^7 - \frac{1}{90}a^5 c^3 x^9 - \frac{c^3 \tan^{-1}(ax)}{40a^4} + \frac{1}{4}c^3 x^4 \tan^{-1}(ax)$$

Mathematica [A] time = 0.147731, size = 141, normalized size = 1.

$$-\frac{1}{90}a^5 c^3 x^9 - \frac{11}{280}a^3 c^3 x^7 + \frac{1}{10}a^6 c^3 x^{10} \tan^{-1}(ax) + \frac{3}{8}a^4 c^3 x^8 \tan^{-1}(ax) + \frac{1}{2}a^2 c^3 x^6 \tan^{-1}(ax) + \frac{c^3 x}{40a^3} - \frac{c^3 \tan^{-1}(ax)}{40a^4} - \frac{9}{200}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(c + a^2*c*x^2)^3*ArcTan[a*x],x]
```

```
[Out] (c^3*x)/(40*a^3) - (c^3*x^3)/(120*a) - (9*a*c^3*x^5)/200 - (11*a^3*c^3*x^7)
/280 - (a^5*c^3*x^9)/90 - (c^3*ArcTan[a*x])/(40*a^4) + (c^3*x^4*ArcTan[a*x]
)/4 + (a^2*c^3*x^6*ArcTan[a*x])/2 + (3*a^4*c^3*x^8*ArcTan[a*x])/8 + (a^6*c^
```

```
3*x^10*ArcTan[a*x])/10
```

Maple [A] time = 0.023, size = 122, normalized size = 0.9

$$\frac{c^3x}{40\,a^3}-\frac{c^3x^3}{120\,a}-\frac{9\,ac^3x^5}{200}-\frac{11\,a^3c^3x^7}{280}-\frac{a^5c^3x^9}{90}-\frac{c^3\arctan(ax)}{40\,a^4}+\frac{c^3x^4\arctan(ax)}{4}+\frac{a^2c^3x^6\arctan(ax)}{2}+\frac{3\,a^4c^3x^8\,\mathrm{ar}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a^2*c*x^2+c)^3*arctan(a*x),x)
```

```
[Out] 1/40*c^3*x/a^3-1/120*c^3*x^3/a-9/200*a*c^3*x^5-11/280*a^3*c^3*x^7-1/90*a^5*
c^3*x^9-1/40*c^3*arctan(a*x)/a^4+1/4*c^3*x^4*arctan(a*x)+1/2*a^2*c^3*x^6*ar
ctan(a*x)+3/8*a^4*c^3*x^8*arctan(a*x)+1/10*a^6*c^3*x^10*arctan(a*x)
```

Maxima [A] time = 1.47741, size = 162, normalized size = 1.15

$$-\frac{1}{12600}\,a\left(\frac{315\,c^3\arctan(ax)}{a^5}+\frac{140\,a^8c^3x^9+495\,a^6c^3x^7+567\,a^4c^3x^5+105\,a^2c^3x^3-315\,c^3x}{a^4}\right)+\frac{1}{40}\left(4\,a^6c^3x^{10}+15\,a^4c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a^2*c*x^2+c)^3*arctan(a*x),x, algorithm="maxima")
```

```
[Out] -1/12600*a*(315*c^3*arctan(a*x)/a^5 + (140*a^8*c^3*x^9 + 495*a^6*c^3*x^7 +
567*a^4*c^3*x^5 + 105*a^2*c^3*x^3 - 315*c^3*x)/a^4) + 1/40*(4*a^6*c^3*x^10
+ 15*a^4*c^3*x^8 + 20*a^2*c^3*x^6 + 10*c^3*x^4)*arctan(a*x)
```

Fricas [A] time = 1.6332, size = 261, normalized size = 1.85

$$-\frac{140\,a^9c^3x^9+495\,a^7c^3x^7+567\,a^5c^3x^5+105\,a^3c^3x^3-315\,ac^3x-315\left(4\,a^{10}c^3x^{10}+15\,a^8c^3x^8+20\,a^6c^3x^6+10\,a^4c^3x^4-\right.}{12600\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a^2*c*x^2+c)^3*arctan(a*x),x, algorithm="fricas")
```

```
[Out] -1/12600*(140*a^9*c^3*x^9 + 495*a^7*c^3*x^7 + 567*a^5*c^3*x^5 + 105*a^3*c^3
*x^3 - 315*a*c^3*x - 315*(4*a^10*c^3*x^10 + 15*a^8*c^3*x^8 + 20*a^6*c^3*x^6
 + 10*a^4*c^3*x^4 - c^3)*arctan(a*x))/a^4
```

Sympy [A] time = 5.6534, size = 138, normalized size = 0.98

$$\begin{cases} \frac{a^6c^3x^{10}\operatorname{atan}(ax)}{10} - \frac{a^5c^3x^9}{90} + \frac{3a^4c^3x^8\operatorname{atan}(ax)}{8} - \frac{11a^3c^3x^7}{280} + \frac{a^2c^3x^6\operatorname{atan}(ax)}{2} - \frac{9ac^3x^5}{200} + \frac{c^3x^4\operatorname{atan}(ax)}{4} - \frac{c^3x^3}{120a} + \frac{c^3x}{40a^3} - \frac{c^3\operatorname{atan}(ax)}{40a^4} & \text{for } a \\ 0 & \text{othe} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a**2*c*x**2+c)**3*atan(a*x),x)
```

```
[Out] Piecewise((a**6*c**3*x**10*atan(a*x)/10 - a**5*c**3*x**9/90 + 3*a**4*c**3*x
**8*atan(a*x)/8 - 11*a**3*c**3*x**7/280 + a**2*c**3*x**6*atan(a*x)/2 - 9*a*
c**3*x**5/200 + c**3*x**4*atan(a*x)/4 - c**3*x**3/(120*a) + c**3*x/(40*a**3
) - c**3*atan(a*x)/(40*a**4), Ne(a, 0)), (0, True))
```

Giac [A] time = 1.13402, size = 162, normalized size = 1.15

$$\frac{1}{40}\left(4\,a^6c^3x^{10} + 15\,a^4c^3x^8 + 20\,a^2c^3x^6 + 10\,c^3x^4\right)\arctan(ax) - \frac{c^3\arctan(ax)}{40\,a^4} - \frac{140\,a^{23}c^3x^9 + 495\,a^{21}c^3x^7 + 567\,a^{19}c^3x^5}{12600\,a^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a^2*c*x^2+c)^3*arctan(a*x),x, algorithm="giac")
```

```
[Out] 1/40*(4*a^6*c^3*x^10 + 15*a^4*c^3*x^8 + 20*a^2*c^3*x^6 + 10*c^3*x^4)*arctan
(a*x) - 1/40*c^3*arctan(a*x)/a^4 - 1/12600*(140*a^23*c^3*x^9 + 495*a^21*c^3
*x^7 + 567*a^19*c^3*x^5 + 105*a^17*c^3*x^3 - 315*a^15*c^3*x)/a^18
```

3.166 $\int x^2 \left(c+a^2 c x^2\right)^3 \tan^{-1}(ax) \, dx$

Optimal. Leaf size=136

$$-\frac{1}{72}a^5c^3x^8-\frac{10}{189}a^3c^3x^6+\frac{8c^3\log\left(a^2x^2+1\right)}{315a^3}+\frac{1}{9}a^6c^3x^9\tan^{-1}(ax)+\frac{3}{7}a^4c^3x^7\tan^{-1}(ax)+\frac{3}{5}a^2c^3x^5\tan^{-1}(ax)-\frac{89ac^3x}{1260}$$

```
[Out] (-8*c^3*x^2)/(315*a) - (89*a*c^3*x^4)/1260 - (10*a^3*c^3*x^6)/189 - (a^5*c^
3*x^8)/72 + (c^3*x^3*ArcTan[a*x])/3 + (3*a^2*c^3*x^5*ArcTan[a*x])/5 + (3*a^
4*c^3*x^7*ArcTan[a*x])/7 + (a^6*c^3*x^9*ArcTan[a*x])/9 + (8*c^3*Log[1 + a^2
*x^2])/(315*a^3)
```

Rubi [A] time = 0.233357, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4948, 4852, 266, 43}

$$-\frac{1}{72}a^5c^3x^8-\frac{10}{189}a^3c^3x^6+\frac{8c^3\log\left(a^2x^2+1\right)}{315a^3}+\frac{1}{9}a^6c^3x^9\tan^{-1}(ax)+\frac{3}{7}a^4c^3x^7\tan^{-1}(ax)+\frac{3}{5}a^2c^3x^5\tan^{-1}(ax)-\frac{89ac^3x}{1260}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(c + a^2*c*x^2)^3*ArcTan[a*x],x]
```

```
[Out] (-8*c^3*x^2)/(315*a) - (89*a*c^3*x^4)/1260 - (10*a^3*c^3*x^6)/189 - (a^5*c^
3*x^8)/72 + (c^3*x^3*ArcTan[a*x])/3 + (3*a^2*c^3*x^5*ArcTan[a*x])/5 + (3*a^
4*c^3*x^7*ArcTan[a*x])/7 + (a^6*c^3*x^9*ArcTan[a*x])/9 + (8*c^3*Log[1 + a^2
*x^2])/(315*a^3)
```

Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
```

```
erQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x^2\left(c+a^2cx^2\right)^3\tan^{-1}(ax)\,dx=\int\left(c^3x^2\tan^{-1}(ax)+3a^2c^3x^4\tan^{-1}(ax)+3a^4c^3x^6\tan^{-1}(ax)+a^6c^3x^8\tan^{-1}(ax)\right)dx$$
$$=c^3\int x^2\tan^{-1}(ax)\,dx+\left(3a^2c^3\right)\int x^4\tan^{-1}(ax)\,dx+\left(3a^4c^3\right)\int x^6\tan^{-1}(ax)\,dx+\left(a^6c^3\right.$$
$$=\frac{1}{3}c^3x^3\tan^{-1}(ax)+\frac{3}{5}a^2c^3x^5\tan^{-1}(ax)+\frac{3}{7}a^4c^3x^7\tan^{-1}(ax)+\frac{1}{9}a^6c^3x^9\tan^{-1}(ax)-\frac{1}{3}\left(\right.$$
$$=\frac{1}{3}c^3x^3\tan^{-1}(ax)+\frac{3}{5}a^2c^3x^5\tan^{-1}(ax)+\frac{3}{7}a^4c^3x^7\tan^{-1}(ax)+\frac{1}{9}a^6c^3x^9\tan^{-1}(ax)-\frac{1}{6}\left(\right.$$
$$=\frac{1}{3}c^3x^3\tan^{-1}(ax)+\frac{3}{5}a^2c^3x^5\tan^{-1}(ax)+\frac{3}{7}a^4c^3x^7\tan^{-1}(ax)+\frac{1}{9}a^6c^3x^9\tan^{-1}(ax)-\frac{1}{6}\left(\right.$$
$$=-\frac{8c^3x^2}{315a}-\frac{89ac^3x^4}{1260}-\frac{10}{189}a^3c^3x^6-\frac{1}{72}a^5c^3x^8+\frac{1}{3}c^3x^3\tan^{-1}(ax)+\frac{3}{5}a^2c^3x^5\tan^{-1}(ax)+$$

Mathematica [A] time = 0.084463, size = 136, normalized size = 1.

$$-\frac{1}{72}a^5c^3x^8-\frac{10}{189}a^3c^3x^6+\frac{8c^3\log\left(a^2x^2+1\right)}{315a^3}+\frac{1}{9}a^6c^3x^9\tan^{-1}(ax)+\frac{3}{7}a^4c^3x^7\tan^{-1}(ax)+\frac{3}{5}a^2c^3x^5\tan^{-1}(ax)-\frac{89ac^3x^4}{1260}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(c + a^2*c*x^2)^3*ArcTan[a*x],x]
```

```
[Out] (-8*c^3*x^2)/(315*a) - (89*a*c^3*x^4)/1260 - (10*a^3*c^3*x^6)/189 - (a^5*c^
3*x^8)/72 + (c^3*x^3*ArcTan[a*x])/3 + (3*a^2*c^3*x^5*ArcTan[a*x])/5 + (3*a^
```

```
4*c^3*x^7*ArcTan[a*x])/7 + (a^6*c^3*x^9*ArcTan[a*x])/9 + (8*c^3*Log[1 + a^2
*x^2])/(315*a^3)
```

Maple [A] time = 0.026, size = 119, normalized size = 0.9

$$-\frac{8c^3x^2}{315a}-\frac{89ac^3x^4}{1260}-\frac{10a^3c^3x^6}{189}-\frac{a^5c^3x^8}{72}+\frac{c^3x^3\arctan(ax)}{3}+\frac{3a^2c^3x^5\arctan(ax)}{5}+\frac{3a^4c^3x^7\arctan(ax)}{7}+\frac{a^6c^3x^9 \text{a}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a^2*c*x^2+c)^3*arctan(a*x),x)
```

```
[Out] -8/315*c^3*x^2/a-89/1260*a*c^3*x^4-10/189*a^3*c^3*x^6-1/72*a^5*c^3*x^8+1/3*
c^3*x^3*arctan(a*x)+3/5*a^2*c^3*x^5*arctan(a*x)+3/7*a^4*c^3*x^7*arctan(a*x)
+1/9*a^6*c^3*x^9*arctan(a*x)+8/315*c^3*ln(a^2*x^2+1)/a^3
```

Maxima [A] time = 0.979308, size = 159, normalized size = 1.17

$$\frac{1}{7560}a\left(\frac{192c^3\log\left(a^2x^2+1\right)}{a^4}-\frac{105a^6c^3x^8+400a^4c^3x^6+534a^2c^3x^4+192c^3x^2}{a^2}\right)+\frac{1}{315}\left(35a^6c^3x^9+135a^4c^3x^7+189\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^3*arctan(a*x),x, algorithm="maxima")
```

```
[Out] 1/7560*a*(192*c^3*log(a^2*x^2 + 1)/a^4 - (105*a^6*c^3*x^8 + 400*a^4*c^3*x^6
 + 534*a^2*c^3*x^4 + 192*c^3*x^2)/a^2) + 1/315*(35*a^6*c^3*x^9 + 135*a^4*c^
3*x^7 + 189*a^2*c^3*x^5 + 105*c^3*x^3)*arctan(a*x)
```

Fricas [A] time = 1.72335, size = 270, normalized size = 1.99

$$-\frac{105a^8c^3x^8+400a^6c^3x^6+534a^4c^3x^4+192a^2c^3x^2-192c^3\log\left(a^2x^2+1\right)-24\left(35a^9c^3x^9+135a^7c^3x^7+189a^5c^3x^5\right.}{7560a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^3*arctan(a*x),x, algorithm="fricas")
```

```
[Out] -1/7560*(105*a^8*c^3*x^8 + 400*a^6*c^3*x^6 + 534*a^4*c^3*x^4 + 192*a^2*c^3*
x^2 - 192*c^3*log(a^2*x^2 + 1) - 24*(35*a^9*c^3*x^9 + 135*a^7*c^3*x^7 + 189
*a^5*c^3*x^5 + 105*a^3*c^3*x^3)*arctan(a*x))/a^3
```

Sympy [A] time = 4.32547, size = 138, normalized size = 1.01

$$\begin{cases} \frac{a^6c^3x^9\operatorname{atan}(ax)}{9} - \frac{a^5c^3x^8}{72} + \frac{3a^4c^3x^7\operatorname{atan}(ax)}{7} - \frac{10a^3c^3x^6}{189} + \frac{3a^2c^3x^5\operatorname{atan}(ax)}{5} - \frac{89ac^3x^4}{1260} + \frac{c^3x^3\operatorname{atan}(ax)}{3} - \frac{8c^3x^2}{315a} + \frac{8c^3\log\left(x^2+\frac{1}{a^2}\right)}{315a^3} & \text{for } a = \\ 0 & \text{other} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a**2*c*x**2+c)**3*atan(a*x),x)
```

```
[Out] Piecewise((a**6*c**3*x**9*atan(a*x)/9 - a**5*c**3*x**8/72 + 3*a**4*c**3*x**
7*atan(a*x)/7 - 10*a**3*c**3*x**6/189 + 3*a**2*c**3*x**5*atan(a*x)/5 - 89*a
*c**3*x**4/1260 + c**3*x**3*atan(a*x)/3 - 8*c**3*x**2/(315*a) + 8*c**3*log(
x**2 + a**(-2))/(315*a**3), Ne(a, 0)), (0, True))
```

Giac [A] time = 1.17765, size = 158, normalized size = 1.16

$$\frac{1}{315}\left(35\,a^6c^3x^9 + 135\,a^4c^3x^7 + 189\,a^2c^3x^5 + 105\,c^3x^3\right)\arctan(ax) + \frac{8\,c^3\log\left(a^2x^2+1\right)}{315\,a^3} - \frac{105\,a^{13}c^3x^8 + 400\,a^{11}c^3x^6 + 53}{7560\,a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^3*arctan(a*x),x, algorithm="giac")
```

```
[Out] 1/315*(35*a^6*c^3*x^9 + 135*a^4*c^3*x^7 + 189*a^2*c^3*x^5 + 105*c^3*x^3)*ar
ctan(a*x) + 8/315*c^3*log(a^2*x^2 + 1)/a^3 - 1/7560*(105*a^13*c^3*x^8 + 400
*a^11*c^3*x^6 + 534*a^9*c^3*x^4 + 192*a^7*c^3*x^2)/a^8
```

3.167 $\int x\left(c+a^2cx^2\right)^3\tan^{-1}(ax)\,dx$

Optimal. Leaf size=74

$$-\frac{1}{56}a^5c^3x^7-\frac{3}{40}a^3c^3x^5+\frac{c^3\left(a^2x^2+1\right)^4\tan^{-1}(ax)}{8a^2}-\frac{1}{8}ac^3x^3-\frac{c^3x}{8a}$$

```
[Out] -(c^3*x)/(8*a) - (a*c^3*x^3)/8 - (3*a^3*c^3*x^5)/40 - (a^5*c^3*x^7)/56 + (c
^3*(1 + a^2*x^2)^4*ArcTan[a*x])/(8*a^2)
```

Rubi [A] time = 0.050066, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4930, 194}

$$-\frac{1}{56}a^5c^3x^7-\frac{3}{40}a^3c^3x^5+\frac{c^3\left(a^2x^2+1\right)^4\tan^{-1}(ax)}{8a^2}-\frac{1}{8}ac^3x^3-\frac{c^3x}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[x*(c + a^2*c*x^2)^3*ArcTan[a*x],x]
```

```
[Out] -(c^3*x)/(8*a) - (a*c^3*x^3)/8 - (3*a^3*c^3*x^5)/40 - (a^5*c^3*x^7)/56 + (c
^3*(1 + a^2*x^2)^4*ArcTan[a*x])/(8*a^2)
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int x\left(c+a^2cx^2\right)^3\tan^{-1}(ax)\,dx=\frac{c^3\left(1+a^2x^2\right)^4\tan^{-1}(ax)}{8a^2}-\frac{\int\left(c+a^2cx^2\right)^3\,dx}{8a}$$
$$=\frac{c^3\left(1+a^2x^2\right)^4\tan^{-1}(ax)}{8a^2}-\frac{\int\left(c^3+3a^2c^3x^2+3a^4c^3x^4+a^6c^3x^6\right)\,dx}{8a}$$
$$=-\frac{c^3x}{8a}-\frac{1}{8}ac^3x^3-\frac{3}{40}a^3c^3x^5-\frac{1}{56}a^5c^3x^7+\frac{c^3\left(1+a^2x^2\right)^4\tan^{-1}(ax)}{8a^2}$$

Mathematica [A] time = 0.087915, size = 128, normalized size = 1.73

$$-\frac{1}{56}a^5c^3x^7-\frac{3}{40}a^3c^3x^5+\frac{1}{8}a^6c^3x^8\tan^{-1}(ax)+\frac{1}{2}a^4c^3x^6\tan^{-1}(ax)+\frac{3}{4}a^2c^3x^4\tan^{-1}(ax)+\frac{c^3\tan^{-1}(ax)}{8a^2}-\frac{1}{8}ac^3x^3+\frac{1}{2}c^3x$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(c + a^2*c*x^2)^3*ArcTan[a*x],x]
```

```
[Out] -(c^3*x)/(8*a) - (a*c^3*x^3)/8 - (3*a^3*c^3*x^5)/40 - (a^5*c^3*x^7)/56 + (c
^3*ArcTan[a*x])/(8*a^2) + (c^3*x^2*ArcTan[a*x])/2 + (3*a^2*c^3*x^4*ArcTan[a
*x])/4 + (a^4*c^3*x^6*ArcTan[a*x])/2 + (a^6*c^3*x^8*ArcTan[a*x])/8
```

Maple [A] time = 0.025, size = 111, normalized size = 1.5

$$\frac{a^6c^3\arctan(ax)x^8}{8}+\frac{a^4c^3\arctan(ax)x^6}{2}+\frac{3a^2c^3\arctan(ax)x^4}{4}+\frac{c^3\arctan(ax)x^2}{2}-\frac{a^5c^3x^7}{56}-\frac{3a^3c^3x^5}{40}-\frac{c^3x^3a}{8}-\frac{c^3}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^3*arctan(a*x),x)
```

```
[Out] 1/8*a^6*c^3*arctan(a*x)*x^8+1/2*a^4*c^3*arctan(a*x)*x^6+3/4*a^2*c^3*arctan(
a*x)*x^4+1/2*c^3*arctan(a*x)*x^2-1/56*a^5*c^3*x^7-3/40*a^3*c^3*x^5-1/8*c^3*
x^3*a-1/8*c^3*x/a+1/8/a^2*c^3*arctan(a*x)
```

Maxima [A] time = 0.972201, size = 99, normalized size = 1.34

$$\frac{\left(a^2cx^2+c\right)^4\arctan(ax)}{8\,a^2c}-\frac{5\,a^6c^4x^7+21\,a^4c^4x^5+35\,a^2c^4x^3+35\,c^4x}{280\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^3*arctan(a*x),x, algorithm="maxima")
```

```
[Out] 1/8*(a^2*c*x^2 + c)^4*arctan(a*x)/(a^2*c) - 1/280*(5*a^6*c^4*x^7 + 21*a^4*c
^4*x^5 + 35*a^2*c^4*x^3 + 35*c^4*x)/(a*c)
```

Fricas [A] time = 1.70776, size = 216, normalized size = 2.92

$$-\frac{5\,a^{7}c^{3}x^{7} + 21\,a^{5}c^{3}x^{5} + 35\,a^{3}c^{3}x^{3} + 35\,ac^{3}x - 35\left(a^{8}c^{3}x^{8} + 4\,a^{6}c^{3}x^{6} + 6\,a^{4}c^{3}x^{4} + 4\,a^{2}c^{3}x^{2} + c^{3}\right)\arctan\left(ax\right)}{280\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^3*arctan(a*x),x, algorithm="fricas")
```

```
[Out] -1/280*(5*a^7*c^3*x^7 + 21*a^5*c^3*x^5 + 35*a^3*c^3*x^3 + 35*a*c^3*x - 35*(
a^8*c^3*x^8 + 4*a^6*c^3*x^6 + 6*a^4*c^3*x^4 + 4*a^2*c^3*x^2 + c^3)*arctan(a
*x))/a^2
```

Sympy [A] time = 3.50299, size = 124, normalized size = 1.68

$$\begin{cases} \frac{a^{6}c^{3}x^{8}\operatorname{atan}{\left(ax\right)}}{8} - \frac{a^{5}c^{3}x^{7}}{56} + \frac{a^{4}c^{3}x^{6}\operatorname{atan}{\left(ax\right)}}{2} - \frac{3a^{3}c^{3}x^{5}}{40} + \frac{3a^{2}c^{3}x^{4}\operatorname{atan}{\left(ax\right)}}{4} - \frac{ac^{3}x^{3}}{8} + \frac{c^{3}x^{2}\operatorname{atan}{\left(ax\right)}}{2} - \frac{c^{3}x}{8a} + \frac{c^{3}\operatorname{atan}{\left(ax\right)}}{8a^{2}} & \text{for}\: a \neq 0 \\0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**3*atan(a*x),x)
```

```
[Out] Piecewise((a**6*c**3*x**8*atan(a*x)/8 - a**5*c**3*x**7/56 + a**4*c**3*x**6*
atan(a*x)/2 - 3*a**3*c**3*x**5/40 + 3*a**2*c**3*x**4*atan(a*x)/4 - a*c**3*x
**3/8 + c**3*x**2*atan(a*x)/2 - c**3*x/(8*a) + c**3*atan(a*x)/(8*a**2), Ne(
a, 0)), (0, True))
```

Giac [A] time = 1.12103, size = 95, normalized size = 1.28

$$\frac{\left(a^{2}cx^{2} + c\right)^{4}\arctan\left(ax\right)}{8\,a^{2}c} - \frac{5\,a^{6}c^{3}x^{7} + 21\,a^{4}c^{3}x^{5} + 35\,a^{2}c^{3}x^{3} + 35\,c^{3}x}{280\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^3*arctan(a*x),x, algorithm="giac")
```

```
[Out] 1/8*(a^2*c*x^2 + c)^4*arctan(a*x)/(a^2*c) - 1/280*(5*a^6*c^3*x^7 + 21*a^4*c
^3*x^5 + 35*a^2*c^3*x^3 + 35*c^3*x)/a
```

3.168 $\int \left(c + a^2cx^2\right)^3 \tan^{-1}(ax)\,dx$

Optimal. Leaf size=161

$$-\frac{c^3\left(a^2x^2+1\right)^3}{42a}-\frac{3c^3\left(a^2x^2+1\right)^2}{70a}-\frac{4c^3\left(a^2x^2+1\right)}{35a}-\frac{8c^3\log\left(a^2x^2+1\right)}{35a}+\frac{1}{7}c^3x\left(a^2x^2+1\right)^3\tan^{-1}(ax)+\frac{6}{35}c^3x\left(a^2x^2+\right.$$

```
[Out] (-4*c^3*(1 + a^2*x^2))/(35*a) - (3*c^3*(1 + a^2*x^2)^2)/(70*a) - (c^3*(1 +
a^2*x^2)^3)/(42*a) + (16*c^3*x*ArcTan[a*x])/35 + (8*c^3*x*(1 + a^2*x^2)*Arc
Tan[a*x])/35 + (6*c^3*x*(1 + a^2*x^2)^2*ArcTan[a*x])/35 + (c^3*x*(1 + a^2*x
^2)^3*ArcTan[a*x])/7 - (8*c^3*Log[1 + a^2*x^2])/(35*a)
```

Rubi [A] time = 0.0763048, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {4878, 4846, 260}

$$-\frac{c^3\left(a^2x^2+1\right)^3}{42a}-\frac{3c^3\left(a^2x^2+1\right)^2}{70a}-\frac{4c^3\left(a^2x^2+1\right)}{35a}-\frac{8c^3\log\left(a^2x^2+1\right)}{35a}+\frac{1}{7}c^3x\left(a^2x^2+1\right)^3\tan^{-1}(ax)+\frac{6}{35}c^3x\left(a^2x^2+\right.$$

Antiderivative was successfully verified.

```
[In] Int[(c + a^2*c*x^2)^3*ArcTan[a*x],x]
```


```
[Out] (-4*c^3*(1 + a^2*x^2))/(35*a) - (3*c^3*(1 + a^2*x^2)^2)/(70*a) - (c^3*(1 +
a^2*x^2)^3)/(42*a) + (16*c^3*x*ArcTan[a*x])/35 + (8*c^3*x*(1 + a^2*x^2)*Arc
Tan[a*x])/35 + (6*c^3*x*(1 + a^2*x^2)^2*ArcTan[a*x])/35 + (c^3*x*(1 + a^2*x
^2)^3*ArcTan[a*x])/7 - (8*c^3*Log[1 + a^2*x^2])/(35*a)
```


Rule 4878

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symbo
l] :> -Simp[(b*(d + e*x^2)^q)/(2*c*q*(2*q + 1)), x] + (Dist[(2*d*q)/(2*q +
1), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x]), x], x] + Simp[(x*(d + e*x^
2)^q*(a + b*ArcTan[c*x]))/(2*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] && Eq
Q[e, c^2*d] && GtQ[q, 0]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
```

```
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \left(c+a^2cx^2\right)^3\tan^{-1}(ax)\,dx = -\frac{c^3\left(1+a^2x^2\right)^3}{42a}+\frac{1}{7}c^3x\left(1+a^2x^2\right)^3\tan^{-1}(ax)+\frac{1}{7}(6c)\int \left(c+a^2cx^2\right)^2\tan^{-1}(ax)\,dx$$

$$= -\frac{3c^3\left(1+a^2x^2\right)^2}{70a}-\frac{c^3\left(1+a^2x^2\right)^3}{42a}+\frac{6}{35}c^3x\left(1+a^2x^2\right)^2\tan^{-1}(ax)+\frac{1}{7}c^3x\left(1+a^2x^2\right)^3\tan$$

$$= -\frac{4c^3\left(1+a^2x^2\right)}{35a}-\frac{3c^3\left(1+a^2x^2\right)^2}{70a}-\frac{c^3\left(1+a^2x^2\right)^3}{42a}+\frac{8}{35}c^3x\left(1+a^2x^2\right)\tan^{-1}(ax)+\frac{6}{35}c^3$$

$$= -\frac{4c^3\left(1+a^2x^2\right)}{35a}-\frac{3c^3\left(1+a^2x^2\right)^2}{70a}-\frac{c^3\left(1+a^2x^2\right)^3}{42a}+\frac{16}{35}c^3x\tan^{-1}(ax)+\frac{8}{35}c^3x\left(1+a^2x^2\right.$$

$$= -\frac{4c^3\left(1+a^2x^2\right)}{35a}-\frac{3c^3\left(1+a^2x^2\right)^2}{70a}-\frac{c^3\left(1+a^2x^2\right)^3}{42a}+\frac{16}{35}c^3x\tan^{-1}(ax)+\frac{8}{35}c^3x\left(1+a^2x^2\right.$$

Mathematica [A] time = 0.076559, size = 83, normalized size = 0.52

$$\frac{c^3\left(-a^2x^2\left(5a^4x^4+24a^2x^2+57\right)-48\log\left(a^2x^2+1\right)+6ax\left(5a^6x^6+21a^4x^4+35a^2x^2+35\right)\tan^{-1}(ax)\right)}{210a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + a^2*c*x^2)^3*ArcTan[a*x],x]
```

```
[Out] (c^3*(-(a^2*x^2*(57 + 24*a^2*x^2 + 5*a^4*x^4)) + 6*a*x*(35 + 35*a^2*x^2 + 2
1*a^4*x^4 + 5*a^6*x^6)*ArcTan[a*x] - 48*Log[1 + a^2*x^2]))/(210*a)
```

Maple [A] time = 0.026, size = 104, normalized size = 0.7

$$\frac{a^6c^3\arctan(ax)\,x^7}{7}+\frac{3\,a^4c^3\arctan(ax)\,x^5}{5}+a^2c^3\arctan(ax)\,x^3+c^3x\arctan(ax)-\frac{a^5c^3x^6}{42}-\frac{4\,a^3c^3x^4}{35}-\frac{19\,ac^3x^2}{70}-\frac{8}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^3*arctan(a*x),x)
```

```
[Out] 1/7*a^6*c^3*arctan(a*x)*x^7+3/5*a^4*c^3*arctan(a*x)*x^5+a^2*c^3*arctan(a*x)
*x^3+c^3*x*arctan(a*x)-1/42*a^5*c^3*x^6-4/35*a^3*c^3*x^4-19/70*a*c^3*x^2-8/
35*c^3*ln(a^2*x^2+1)/a
```

Maxima [A] time = 0.966886, size = 134, normalized size = 0.83

$$-\frac{1}{210}\left(5\,a^4c^3x^6+24\,a^2c^3x^4+57\,c^3x^2+\frac{48\,c^3\log\left(a^2x^2+1\right)}{a^2}\right)a+\frac{1}{35}\left(5\,a^6c^3x^7+21\,a^4c^3x^5+35\,a^2c^3x^3+35\,c^3x\right)\operatorname{arctan}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x),x, algorithm="maxima")
```

```
[Out] -1/210*(5*a^4*c^3*x^6 + 24*a^2*c^3*x^4 + 57*c^3*x^2 + 48*c^3*log(a^2*x^2 +
1)/a^2)*a + 1/35*(5*a^6*c^3*x^7 + 21*a^4*c^3*x^5 + 35*a^2*c^3*x^3 + 35*c^3*
x)*arctan(a*x)
```

Fricas [A] time = 1.65825, size = 223, normalized size = 1.39

$$-\frac{5\,a^6c^3x^6+24\,a^4c^3x^4+57\,a^2c^3x^2+48\,c^3\log\left(a^2x^2+1\right)-6\left(5\,a^7c^3x^7+21\,a^5c^3x^5+35\,a^3c^3x^3+35\,ac^3x\right)\arctan\left(ax\right)}{210\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x),x, algorithm="fricas")
```

```
[Out] -1/210*(5*a^6*c^3*x^6 + 24*a^4*c^3*x^4 + 57*a^2*c^3*x^2 + 48*c^3*log(a^2*x^
2 + 1) - 6*(5*a^7*c^3*x^7 + 21*a^5*c^3*x^5 + 35*a^3*c^3*x^3 + 35*a*c^3*x)*a
rctan(a*x))/a
```

Sympy [A] time = 2.76145, size = 117, normalized size = 0.73

$$\begin{cases}\frac{a^6c^3x^7\operatorname{atan}(ax)}{7}-\frac{a^5c^3x^6}{42}+\frac{3a^4c^3x^5\operatorname{atan}(ax)}{5}-\frac{4a^3c^3x^4}{35}+a^2c^3x^3\operatorname{atan}(ax)-\frac{19ac^3x^2}{70}+c^3x\operatorname{atan}(ax)-\frac{8c^3\log\left(x^2+\frac{1}{a^2}\right)}{35a} & \text{for } a\neq 0\\ 0 & \text{otherwi}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**3*atan(a*x),x)
```

```
[Out] Piecewise((a**6*c**3*x**7*atan(a*x)/7 - a**5*c**3*x**6/42 + 3*a**4*c**3*x**
5*atan(a*x)/5 - 4*a**3*c**3*x**4/35 + a**2*c**3*x**3*atan(a*x) - 19*a*c**3*
x**2/70 + c**3*x*atan(a*x) - 8*c**3*log(x**2 + a**(-2))/(35*a), Ne(a, 0)),
(0, True))
```

Giac [A] time = 1.13563, size = 140, normalized size = 0.87

$$-\frac{8\,c^3\log\left(a^2x^2+1\right)}{35\,a}+\frac{1}{35}\left(5\,a^6c^3x^7+21\,a^4c^3x^5+35\,a^2c^3x^3+35\,c^3x\right)\arctan\left(ax\right)-\frac{5\,a^{11}c^3x^6+24\,a^9c^3x^4+57\,a^7c^3x^2}{210\,a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x),x, algorithm="giac")
```

```
[Out] -8/35*c^3*log(a^2*x^2 + 1)/a + 1/35*(5*a^6*c^3*x^7 + 21*a^4*c^3*x^5 + 35*a^
2*c^3*x^3 + 35*c^3*x)*arctan(a*x) - 1/210*(5*a^11*c^3*x^6 + 24*a^9*c^3*x^4
+ 57*a^7*c^3*x^2)/a^6
```

3.169 $\int \frac{(c+a^2cx^2)^3 \tan^{-1}(ax)}{x} dx$

Optimal. Leaf size=132

$$\frac{1}{2}ic^3\text{PolyLog}(2,-iax)-\frac{1}{2}ic^3\text{PolyLog}(2,iax)-\frac{1}{30}a^5c^3x^5-\frac{7}{36}a^3c^3x^3+\frac{1}{6}a^6c^3x^6\tan^{-1}(ax)+\frac{3}{4}a^4c^3x^4\tan^{-1}(ax)+\frac{3}{2}a^2$$

```
[Out] (-11*a*c^3*x)/12 - (7*a^3*c^3*x^3)/36 - (a^5*c^3*x^5)/30 + (11*c^3*ArcTan[a
*x])/12 + (3*a^2*c^3*x^2*ArcTan[a*x])/2 + (3*a^4*c^3*x^4*ArcTan[a*x])/4 + (
a^6*c^3*x^6*ArcTan[a*x])/6 + (I/2)*c^3*PolyLog[2, (-I)*a*x] - (I/2)*c^3*Pol
yLog[2, I*a*x]
```

Rubi [A] time = 0.154044, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {4948, 4848, 2391, 4852, 321, 203, 302}

$$\frac{1}{2}ic^3\text{PolyLog}(2,-iax)-\frac{1}{2}ic^3\text{PolyLog}(2,iax)-\frac{1}{30}a^5c^3x^5-\frac{7}{36}a^3c^3x^3+\frac{1}{6}a^6c^3x^6\tan^{-1}(ax)+\frac{3}{4}a^4c^3x^4\tan^{-1}(ax)+\frac{3}{2}a^2$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)^3*ArcTan[a*x])/x,x]
```

```
[Out] (-11*a*c^3*x)/12 - (7*a^3*c^3*x^3)/36 - (a^5*c^3*x^5)/30 + (11*c^3*ArcTan[a
*x])/12 + (3*a^2*c^3*x^2*ArcTan[a*x])/2 + (3*a^4*c^3*x^4*ArcTan[a*x])/4 + (
a^6*c^3*x^6*ArcTan[a*x])/6 + (I/2)*c^3*PolyLog[2, (-I)*a*x] - (I/2)*c^3*Pol
yLog[2, I*a*x]
```

Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\int \frac{\left(c+a^2cx^2\right)^3\tan^{-1}(ax)}{x}\,dx = \int\left(\frac{c^3\tan^{-1}(ax)}{x}+3a^2c^3x\tan^{-1}(ax)+3a^4c^3x^3\tan^{-1}(ax)+a^6c^3x^5\tan^{-1}(ax)\right)dx$$
$$= c^3\int\frac{\tan^{-1}(ax)}{x}\,dx+\left(3a^2c^3\right)\int x\tan^{-1}(ax)\,dx+\left(3a^4c^3\right)\int x^3\tan^{-1}(ax)\,dx+\left(a^6c^3\right)\int$$
$$= \frac{3}{2}a^2c^3x^2\tan^{-1}(ax)+\frac{3}{4}a^4c^3x^4\tan^{-1}(ax)+\frac{1}{6}a^6c^3x^6\tan^{-1}(ax)+\frac{1}{2}\left(ic^3\right)\int\frac{\log(1-iax)}{x}$$
$$= -\frac{3}{2}ac^3x+\frac{3}{2}a^2c^3x^2\tan^{-1}(ax)+\frac{3}{4}a^4c^3x^4\tan^{-1}(ax)+\frac{1}{6}a^6c^3x^6\tan^{-1}(ax)+\frac{1}{2}ic^3\text{Li}_2(-ia$$
$$= -\frac{11}{12}ac^3x-\frac{7}{36}a^3c^3x^3-\frac{1}{30}a^5c^3x^5+\frac{3}{2}c^3\tan^{-1}(ax)+\frac{3}{2}a^2c^3x^2\tan^{-1}(ax)+\frac{3}{4}a^4c^3x^4\tan$$
$$= -\frac{11}{12}ac^3x-\frac{7}{36}a^3c^3x^3-\frac{1}{30}a^5c^3x^5+\frac{11}{12}c^3\tan^{-1}(ax)+\frac{3}{2}a^2c^3x^2\tan^{-1}(ax)+\frac{3}{4}a^4c^3x^4\tan$$

Mathematica [A] time = 0.0528136, size = 132, normalized size = 1.

$$\frac{1}{2}ic^3\text{PolyLog}(2,-iax)-\frac{1}{2}ic^3\text{PolyLog}(2,iax)-\frac{1}{30}a^5c^3x^5-\frac{7}{36}a^3c^3x^3+\frac{1}{6}a^6c^3x^6\tan^{-1}(ax)+\frac{3}{4}a^4c^3x^4\tan^{-1}(ax)+\frac{3}{2}a^2$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + a^2*c*x^2)^3*ArcTan[a*x])/x,x]
```

```
[Out] (-11*a*c^3*x)/12 - (7*a^3*c^3*x^3)/36 - (a^5*c^3*x^5)/30 + (11*c^3*ArcTan[a
*x])/12 + (3*a^2*c^3*x^2*ArcTan[a*x])/2 + (3*a^4*c^3*x^4*ArcTan[a*x])/4 + (
a^6*c^3*x^6*ArcTan[a*x])/6 + (I/2)*c^3*PolyLog[2, (-I)*a*x] - (I/2)*c^3*Pol
yLog[2, I*a*x]
```

Maple [A] time = 0.037, size = 161, normalized size = 1.2

$$\frac{a^6c^3x^6\arctan(ax)}{6}+\frac{3\,a^4c^3x^4\arctan(ax)}{4}+\frac{3\,a^2c^3x^2\arctan(ax)}{2}+c^3\arctan(ax)\ln(ax)-\frac{a^5c^3x^5}{30}-\frac{7\,c^3x^3a^3}{36}-\frac{11\,ac}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^3*arctan(a*x)/x,x)
```

```
[Out] 1/6*a^6*c^3*x^6*arctan(a*x)+3/4*a^4*c^3*x^4*arctan(a*x)+3/2*a^2*c^3*x^2*arc
tan(a*x)+c^3*arctan(a*x)*ln(a*x)-1/30*a^5*c^3*x^5-7/36*c^3*x^3*a^3-11/12*a*
```

```
c^3*x+11/12*c^3*arctan(a*x)+1/2*I*c^3*ln(a*x)*ln(1+I*a*x)-1/2*I*c^3*ln(a*x)
*ln(1-I*a*x)+1/2*I*c^3*dilog(1+I*a*x)-1/2*I*c^3*dilog(1-I*a*x)
```

Maxima [A] time = 1.63611, size = 181, normalized size = 1.37

$$-\frac{1}{30}a^5c^3x^5 - \frac{7}{36}a^3c^3x^3 - \frac{11}{12}ac^3x - \frac{1}{4}\pi c^3\log\left(a^2x^2+1\right) + c^3\arctan(ax)\log(x|a|) - \frac{1}{2}ic^3\text{Li}_2(i\,ax+1) + \frac{1}{2}ic^3\text{Li}_2(-i\,ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)/x,x, algorithm="maxima")
```

```
[Out] -1/30*a^5*c^3*x^5 - 7/36*a^3*c^3*x^3 - 11/12*a*c^3*x - 1/4*pi*c^3*log(a^2*x
^2 + 1) + c^3*arctan(a*x)*log(x*abs(a)) - 1/2*I*c^3*dilog(I*a*x + 1) + 1/2*
I*c^3*dilog(-I*a*x + 1) + 1/12*(2*a^6*c^3*x^6 + 9*a^4*c^3*x^4 + 18*a^2*c^3*
x^2 + c^3*(12*I*arctan2(0, a) + 11))*arctan(a*x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^6c^3x^6 + 3\,a^4c^3x^4 + 3\,a^2c^3x^2 + c^3\right)\arctan(ax)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)/x,x, algorithm="fricas")
```

```
[Out] integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*arctan(a*x)/x,
 x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$c^3\left(\int\frac{\operatorname{atan}(ax)}{x}\,dx + \int 3a^2x\operatorname{atan}(ax)\,dx + \int 3a^4x^3\operatorname{atan}(ax)\,dx + \int a^6x^5\operatorname{atan}(ax)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**3*atan(a*x)/x,x)
```

```
[Out] c**3*(Integral(atan(a*x)/x, x) + Integral(3*a**2*x*atan(a*x), x) + Integral
(3*a**4*x**3*atan(a*x), x) + Integral(a**6*x**5*atan(a*x), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^3 \arctan(ax)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)/x,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^3*arctan(a*x)/x, x)
```

3.170 $\int \frac{\left(c+a^2cx^2\right)^3 \tan^{-1}(ax)}{x^2}\,dx$

Optimal. Leaf size=108

$$-\frac{1}{20}a^5c^3x^4 - \frac{2}{5}a^3c^3x^2 - \frac{8}{5}ac^3\log\left(a^2x^2+1\right) + \frac{1}{5}a^6c^3x^5\tan^{-1}(ax) + a^4c^3x^3\tan^{-1}(ax) + 3a^2c^3x\tan^{-1}(ax) + ac^3\log(x) -$$

```
[Out] (-2*a^3*c^3*x^2)/5 - (a^5*c^3*x^4)/20 - (c^3*ArcTan[a*x])/x + 3*a^2*c^3*x*A
rcTan[a*x] + a^4*c^3*x^3*ArcTan[a*x] + (a^6*c^3*x^5*ArcTan[a*x])/5 + a*c^3*
Log[x] - (8*a*c^3*Log[1 + a^2*x^2])/5
```

Rubi [A] time = 0.155943, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.45, Rules used = {4948, 4846, 260, 4852, 266, 36, 29, 31, 43}

$$-\frac{1}{20}a^5c^3x^4 - \frac{2}{5}a^3c^3x^2 - \frac{8}{5}ac^3\log\left(a^2x^2+1\right) + \frac{1}{5}a^6c^3x^5\tan^{-1}(ax) + a^4c^3x^3\tan^{-1}(ax) + 3a^2c^3x\tan^{-1}(ax) + ac^3\log(x) -$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)^3*ArcTan[a*x])/x^2,x]
```

```
[Out] (-2*a^3*c^3*x^2)/5 - (a^5*c^3*x^4)/20 - (c^3*ArcTan[a*x])/x + 3*a^2*c^3*x*A
rcTan[a*x] + a^4*c^3*x^3*ArcTan[a*x] + (a^6*c^3*x^5*ArcTan[a*x])/5 + a*c^3*
Log[x] - (8*a*c^3*Log[1 + a^2*x^2])/5
```

Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^3 \tan^{-1}(ax)}{x^2} dx = \int \left(3a^2c^3 \tan^{-1}(ax) + \frac{c^3 \tan^{-1}(ax)}{x^2} + 3a^4c^3x^2 \tan^{-1}(ax) + a^6c^3x^4 \tan^{-1}(ax)\right) dx$$

$$= c^3 \int \frac{\tan^{-1}(ax)}{x^2} dx + (3a^2c^3) \int \tan^{-1}(ax)\, dx + (3a^4c^3) \int x^2 \tan^{-1}(ax)\, dx + (a^6c^3) \int x^4$$

$$= -\frac{c^3 \tan^{-1}(ax)}{x} + 3a^2c^3x \tan^{-1}(ax) + a^4c^3x^3 \tan^{-1}(ax) + \frac{1}{5}a^6c^3x^5 \tan^{-1}(ax) + (ac^3) \int \frac{}{x}$$

$$= -\frac{c^3 \tan^{-1}(ax)}{x} + 3a^2c^3x \tan^{-1}(ax) + a^4c^3x^3 \tan^{-1}(ax) + \frac{1}{5}a^6c^3x^5 \tan^{-1}(ax) - \frac{3}{2}ac^3 \log(1$$

$$= -\frac{c^3 \tan^{-1}(ax)}{x} + 3a^2c^3x \tan^{-1}(ax) + a^4c^3x^3 \tan^{-1}(ax) + \frac{1}{5}a^6c^3x^5 \tan^{-1}(ax) - \frac{3}{2}ac^3 \log(1$$

$$= -\frac{2}{5}a^3c^3x^2 - \frac{1}{20}a^5c^3x^4 - \frac{c^3 \tan^{-1}(ax)}{x} + 3a^2c^3x \tan^{-1}(ax) + a^4c^3x^3 \tan^{-1}(ax) + \frac{1}{5}a^6c^3x^5$$

Mathematica [A] time = 0.0702488, size = 78, normalized size = 0.72

$$\frac{c^3 \left(4 \left(a^6x^6 + 5a^4x^4 + 15a^2x^2 - 5\right) \tan^{-1}(ax) - ax \left(a^4x^4 + 8a^2x^2 + 32 \log\left(a^2x^2+1\right) - 20 \log(x)\right)\right)}{20x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + a^2*c*x^2)^3*ArcTan[a*x])/x^2,x]
```

```
[Out] (c^3*(4*(-5 + 15*a^2*x^2 + 5*a^4*x^4 + a^6*x^6)*ArcTan[a*x] - a*x*(8*a^2*x^
2 + a^4*x^4 - 20*Log[x] + 32*Log[1 + a^2*x^2])))/(20*x)
```

Maple [A] time = 0.031, size = 103, normalized size = 1.

$$\frac{a^6c^3x^5 \arctan(ax)}{5} + a^4c^3x^3 \arctan(ax) + 3\,a^2c^3x \arctan(ax) - \frac{c^3 \arctan(ax)}{x} - \frac{a^5c^3x^4}{20} - \frac{2\,a^3c^3x^2}{5} - \frac{8\,ac^3 \ln\left(a^2x^2+1\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^3*arctan(a*x)/x^2,x)
```

```
[Out] 1/5*a^6*c^3*x^5*arctan(a*x)+a^4*c^3*x^3*arctan(a*x)+3*a^2*c^3*x*arctan(a*x)
-c^3*arctan(a*x)/x-1/20*a^5*c^3*x^4-2/5*a^3*c^3*x^2-8/5*a*c^3*ln(a^2*x^2+1)
```

```
+a*c^3*ln(a*x)
```

Maxima [A] time = 0.971987, size = 126, normalized size = 1.17

$$-\frac{1}{20}\left(a^4c^3x^4+8\,a^2c^3x^2+32\,c^3\log\left(a^2x^2+1\right)-20\,c^3\log\left(x\right)\right)a+\frac{1}{5}\left(a^6c^3x^5+5\,a^4c^3x^3+15\,a^2c^3x-\frac{5\,c^3}{x}\right)\arctan\left(ax\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)/x^2,x, algorithm="maxima")
```

```
[Out] -1/20*(a^4*c^3*x^4 + 8*a^2*c^3*x^2 + 32*c^3*log(a^2*x^2 + 1) - 20*c^3*log(x
))*a + 1/5*(a^6*c^3*x^5 + 5*a^4*c^3*x^3 + 15*a^2*c^3*x - 5*c^3/x)*arctan(a*
x)
```

Fricas [A] time = 1.60866, size = 216, normalized size = 2.

$$-\frac{a^5c^3x^5+8\,a^3c^3x^3+32\,ac^3x\log\left(a^2x^2+1\right)-20\,ac^3x\log\left(x\right)-4\left(a^6c^3x^6+5\,a^4c^3x^4+15\,a^2c^3x^2-5\,c^3\right)\arctan\left(ax\right)}{20\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)/x^2,x, algorithm="fricas")
```

```
[Out] -1/20*(a^5*c^3*x^5 + 8*a^3*c^3*x^3 + 32*a*c^3*x*log(a^2*x^2 + 1) - 20*a*c^3
*x*log(x) - 4*(a^6*c^3*x^6 + 5*a^4*c^3*x^4 + 15*a^2*c^3*x^2 - 5*c^3)*arctan
(a*x))/x
```

Sympy [A] time = 3.10596, size = 110, normalized size = 1.02

$$\begin{cases}\frac{a^6c^3x^5\operatorname{atan}\left(ax\right)}{5}-\frac{a^5c^3x^4}{20}+a^4c^3x^3\operatorname{atan}\left(ax\right)-\frac{2a^3c^3x^2}{5}+3a^2c^3x\operatorname{atan}\left(ax\right)+ac^3\log\left(x\right)-\frac{8ac^3\log\left(x^2+\frac{1}{a^2}\right)}{5}-\frac{c^3\operatorname{atan}\left(ax\right)}{x} & \text{for } a \\ 0 & \text{othe}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**3*atan(a*x)/x**2,x)
```

```
[Out] Piecewise((a**6*c**3*x**5*atan(a*x)/5 - a**5*c**3*x**4/20 + a**4*c**3*x**3*
atan(a*x) - 2*a**3*c**3*x**2/5 + 3*a**2*c**3*x*atan(a*x) + a*c**3*log(x) -
8*a*c**3*log(x**2 + a**(-2))/5 - c**3*atan(a*x)/x, Ne(a, 0)), (0, True))
```

Giac [A] time = 1.16316, size = 134, normalized size = 1.24

$$-\frac{8}{5}\,ac^3\log\left(a^2x^2+1\right)+\frac{1}{2}\,ac^3\log\left(x^2\right)+\frac{1}{5}\left(a^6c^3x^5+5\,a^4c^3x^3+15\,a^2c^3x-\frac{5\,c^3}{x}\right)\arctan\left(ax\right)-\frac{a^9c^3x^4+8\,a^7c^3x^2}{20\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)/x^2,x, algorithm="giac")
```

```
[Out] -8/5*a*c^3*log(a^2*x^2 + 1) + 1/2*a*c^3*log(x^2) + 1/5*(a^6*c^3*x^5 + 5*a^4
*c^3*x^3 + 15*a^2*c^3*x - 5*c^3/x)*arctan(a*x) - 1/20*(a^9*c^3*x^4 + 8*a^7*
c^3*x^2)/a^4
```

3.171 $\int \frac{(c+a^2cx^2)^3 \tan^{-1}(ax)}{x^3} dx$

Optimal. Leaf size=138

$$\frac{3}{2}ia^2c^3\text{PolyLog}(2,-iax) - \frac{3}{2}ia^2c^3\text{PolyLog}(2,iax) - \frac{1}{12}a^5c^3x^3 + \frac{1}{4}a^6c^3x^4\tan^{-1}(ax) + \frac{3}{2}a^4c^3x^2\tan^{-1}(ax) - \frac{5}{4}a^3c^3x + \frac{3}{4}$$

```
[Out] -(a*c^3)/(2*x) - (5*a^3*c^3*x)/4 - (a^5*c^3*x^3)/12 + (3*a^2*c^3*ArcTan[a*x
])/4 - (c^3*ArcTan[a*x])/(2*x^2) + (3*a^4*c^3*x^2*ArcTan[a*x])/2 + (a^6*c^3
*x^4*ArcTan[a*x])/4 + ((3*I)/2)*a^2*c^3*PolyLog[2, (-I)*a*x] - ((3*I)/2)*a^
2*c^3*PolyLog[2, I*a*x]
```

Rubi [A] time = 0.152538, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {4948, 4852, 325, 203, 4848, 2391, 321, 302}

$$\frac{3}{2}ia^2c^3\text{PolyLog}(2,-iax) - \frac{3}{2}ia^2c^3\text{PolyLog}(2,iax) - \frac{1}{12}a^5c^3x^3 + \frac{1}{4}a^6c^3x^4\tan^{-1}(ax) + \frac{3}{2}a^4c^3x^2\tan^{-1}(ax) - \frac{5}{4}a^3c^3x + \frac{3}{4}$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)^3*ArcTan[a*x])/x^3,x]
```


```
[Out] -(a*c^3)/(2*x) - (5*a^3*c^3*x)/4 - (a^5*c^3*x^3)/12 + (3*a^2*c^3*ArcTan[a*x
])/4 - (c^3*ArcTan[a*x])/(2*x^2) + (3*a^4*c^3*x^2*ArcTan[a*x])/2 + (a^6*c^3
*x^4*ArcTan[a*x])/4 + ((3*I)/2)*a^2*c^3*PolyLog[2, (-I)*a*x] - ((3*I)/2)*a^
2*c^3*PolyLog[2, I*a*x]
```


Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
```

```
erQ[m]) && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\int \frac{\left(c+a^2cx^2\right)^3 \tan^{-1}(ax)}{x^3}\,dx = \int \left(\frac{c^3\tan^{-1}(ax)}{x^3} + \frac{3a^2c^3\tan^{-1}(ax)}{x} + 3a^4c^3x\tan^{-1}(ax) + a^6c^3x^3\tan^{-1}(ax)\right)dx$$
$$= c^3\int \frac{\tan^{-1}(ax)}{x^3}\,dx + \left(3a^2c^3\right)\int \frac{\tan^{-1}(ax)}{x}\,dx + \left(3a^4c^3\right)\int x\tan^{-1}(ax)\,dx + \left(a^6c^3\right)\int x$$
$$= -\frac{c^3\tan^{-1}(ax)}{2x^2} + \frac{3}{2}a^4c^3x^2\tan^{-1}(ax) + \frac{1}{4}a^6c^3x^4\tan^{-1}(ax) + \frac{1}{2}\left(ac^3\right)\int \frac{1}{x^2\left(1+a^2x^2\right)}\,dx$$
$$= -\frac{ac^3}{2x} - \frac{3}{2}a^3c^3x - \frac{c^3\tan^{-1}(ax)}{2x^2} + \frac{3}{2}a^4c^3x^2\tan^{-1}(ax) + \frac{1}{4}a^6c^3x^4\tan^{-1}(ax) + \frac{3}{2}ia^2c^3\mathrm{Li}_2($$
$$= -\frac{ac^3}{2x} - \frac{5}{4}a^3c^3x - \frac{1}{12}a^5c^3x^3 + a^2c^3\tan^{-1}(ax) - \frac{c^3\tan^{-1}(ax)}{2x^2} + \frac{3}{2}a^4c^3x^2\tan^{-1}(ax) + \frac{1}{4}a$$
$$= -\frac{ac^3}{2x} - \frac{5}{4}a^3c^3x - \frac{1}{12}a^5c^3x^3 + \frac{3}{4}a^2c^3\tan^{-1}(ax) - \frac{c^3\tan^{-1}(ax)}{2x^2} + \frac{3}{2}a^4c^3x^2\tan^{-1}(ax) + \frac{1}{4}$$

Mathematica [C] time = 0.0433007, size = 154, normalized size = 1.12

$$-\frac{ac^3\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-a^2x^2\right)}{2x} + \frac{3}{2}ia^2c^3\text{PolyLog}(2,-iax) - \frac{3}{2}ia^2c^3\text{PolyLog}(2,iax) - \frac{1}{12}a^5c^3x^3 + \frac{1}{4}a^6c^3$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((c + a^2*c*x^2)^3*ArcTan[a*x])/x^3,x]
```

```
[Out] (-5*a^3*c^3*x)/4 - (a^5*c^3*x^3)/12 + (5*a^2*c^3*ArcTan[a*x])/4 - (c^3*ArcT
an[a*x])/(2*x^2) + (3*a^4*c^3*x^2*ArcTan[a*x])/2 + (a^6*c^3*x^4*ArcTan[a*x]
)/4 - (a*c^3*Hypergeometric2F1[-1/2, 1, 1/2, -(a^2*x^2)])/(2*x) + ((3*I)/2)
*a^2*c^3*PolyLog[2, (-I)*a*x] - ((3*I)/2)*a^2*c^3*PolyLog[2, I*a*x]
```

Maple [A] time = 0.044, size = 177, normalized size = 1.3

$$\frac{a^6c^3x^4\arctan(ax)}{4} + \frac{3a^4c^3x^2\arctan(ax)}{2} - \frac{c^3\arctan(ax)}{2x^2} + 3a^2c^3\arctan(ax)\ln(ax) - \frac{a^5c^3x^3}{12} - \frac{5a^3c^3x}{4} + \frac{3a^2c^3\,\mathrm{ar}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^3*arctan(a*x)/x^3,x)
```

```
[Out] 1/4*a^6*c^3*x^4*arctan(a*x)+3/2*a^4*c^3*x^2*arctan(a*x)-1/2*c^3*arctan(a*x)
/x^2+3*a^2*c^3*arctan(a*x)*ln(a*x)-1/12*a^5*c^3*x^3-5/4*a^3*c^3*x+3/4*a^2*c
^3*arctan(a*x)-1/2*a*c^3/x+3/2*I*a^2*c^3*ln(a*x)*ln(1+I*a*x)-3/2*I*a^2*c^3*
ln(a*x)*ln(1-I*a*x)+3/2*I*a^2*c^3*dilog(1+I*a*x)-3/2*I*a^2*c^3*dilog(1-I*a*
x)
```

Maxima [A] time = 1.69919, size = 220, normalized size = 1.59

$$-\frac{a^5c^3x^5 + 15\,a^3c^3x^3 + 9\,\pi a^2c^3x^2\log\left(a^2x^2+1\right) - 36\,a^2c^3x^2\arctan\left(ax\right)\log\left(x|a|\right) + 18i\,a^2c^3x^2\mathrm{Li}_2\left(i\,ax+1\right) - 18i\,a^2c^3x^2\mathrm{L}}{12\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)/x^3,x, algorithm="maxima")
```

```
[Out] -1/12*(a^5*c^3*x^5 + 15*a^3*c^3*x^3 + 9*pi*a^2*c^3*x^2*log(a^2*x^2 + 1) - 3
6*a^2*c^3*x^2*arctan(a*x)*log(x*abs(a)) + 18*I*a^2*c^3*x^2*dilog(I*a*x + 1)
 - 18*I*a^2*c^3*x^2*dilog(-I*a*x + 1) + 6*a*c^3*x - 3*(a^6*c^3*x^6 + 6*a^4*
c^3*x^4 + 3*a^2*c^3*x^2*(4*I*arctan2(0, a) + 1) - 2*c^3)*arctan(a*x))/x^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(a^6c^3x^6 + 3\,a^4c^3x^4 + 3\,a^2c^3x^2 + c^3\right)\arctan\left(ax\right)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)/x^3,x, algorithm="fricas")
```

```
[Out] integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*arctan(a*x)/x^
3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$c^3\left(\int\frac{\operatorname{atan}\left(ax\right)}{x^3}\,dx + \int\frac{3a^2\operatorname{atan}\left(ax\right)}{x}\,dx + \int 3a^4x\operatorname{atan}\left(ax\right)dx + \int a^6x^3\operatorname{atan}\left(ax\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**3*atan(a*x)/x**3,x)
```

```
[Out] c**3*(Integral(atan(a*x)/x**3, x) + Integral(3*a**2*atan(a*x)/x, x) + Integ
ral(3*a**4*x*atan(a*x), x) + Integral(a**6*x**3*atan(a*x), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^3 \arctan(ax)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)/x^3,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^3*arctan(a*x)/x^3, x)
```

3.172 $\int \frac{\left(c+a^2cx^2\right)^3 \tan^{-1}(ax)}{x^4}\,dx$

Optimal. Leaf size=116

$$-\frac{1}{6}a^5c^3x^2-\frac{8}{3}a^3c^3\log\left(a^2x^2+1\right)+\frac{1}{3}a^6c^3x^3\tan^{-1}(ax)+\frac{8}{3}a^3c^3\log(x)+3a^4c^3x\tan^{-1}(ax)-\frac{3a^2c^3\tan^{-1}(ax)}{x}-\frac{ac^3}{6x^2}-\frac{c^3}{}$$

```
[Out] -(a*c^3)/(6*x^2) - (a^5*c^3*x^2)/6 - (c^3*ArcTan[a*x])/(3*x^3) - (3*a^2*c^3
*ArcTan[a*x])/x + 3*a^4*c^3*x*ArcTan[a*x] + (a^6*c^3*x^3*ArcTan[a*x])/3 + (
8*a^3*c^3*Log[x])/3 - (8*a^3*c^3*Log[1 + a^2*x^2])/3
```

Rubi [A] time = 0.157906, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4948, 4846, 260, 4852, 266, 44, 36, 29, 31, 43}

$$-\frac{1}{6}a^5c^3x^2-\frac{8}{3}a^3c^3\log\left(a^2x^2+1\right)+\frac{1}{3}a^6c^3x^3\tan^{-1}(ax)+\frac{8}{3}a^3c^3\log(x)+3a^4c^3x\tan^{-1}(ax)-\frac{3a^2c^3\tan^{-1}(ax)}{x}-\frac{ac^3}{6x^2}-\frac{c^3}{}$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)^3*ArcTan[a*x])/x^4,x]
```

```
[Out] -(a*c^3)/(6*x^2) - (a^5*c^3*x^2)/6 - (c^3*ArcTan[a*x])/(3*x^3) - (3*a^2*c^3
*ArcTan[a*x])/x + 3*a^4*c^3*x*ArcTan[a*x] + (a^6*c^3*x^3*ArcTan[a*x])/3 + (
8*a^3*c^3*Log[x])/3 - (8*a^3*c^3*Log[1 + a^2*x^2])/3
```

Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\left(c+a^2cx^2\right)^3 \tan^{-1}(ax)}{x^4}\,dx = \int \left(3a^4c^3\tan^{-1}(ax)+\frac{c^3\tan^{-1}(ax)}{x^4}+\frac{3a^2c^3\tan^{-1}(ax)}{x^2}+a^6c^3x^2\tan^{-1}(ax)\right)dx$$
$$= c^3\int\frac{\tan^{-1}(ax)}{x^4}\,dx+\left(3a^2c^3\right)\int\frac{\tan^{-1}(ax)}{x^2}\,dx+\left(3a^4c^3\right)\int\tan^{-1}(ax)\,dx+\left(a^6c^3\right)\int x^2\,\mathrm{ta}$$
$$= -\frac{c^3\tan^{-1}(ax)}{3x^3}-\frac{3a^2c^3\tan^{-1}(ax)}{x}+3a^4c^3x\tan^{-1}(ax)+\frac{1}{3}a^6c^3x^3\tan^{-1}(ax)+\frac{1}{3}\left(ac^3\right)\int$$
$$= -\frac{c^3\tan^{-1}(ax)}{3x^3}-\frac{3a^2c^3\tan^{-1}(ax)}{x}+3a^4c^3x\tan^{-1}(ax)+\frac{1}{3}a^6c^3x^3\tan^{-1}(ax)-\frac{3}{2}a^3c^3\log\Big($$
$$= -\frac{c^3\tan^{-1}(ax)}{3x^3}-\frac{3a^2c^3\tan^{-1}(ax)}{x}+3a^4c^3x\tan^{-1}(ax)+\frac{1}{3}a^6c^3x^3\tan^{-1}(ax)-\frac{3}{2}a^3c^3\log\Big($$
$$= -\frac{ac^3}{6x^2}-\frac{1}{6}a^5c^3x^2-\frac{c^3\tan^{-1}(ax)}{3x^3}-\frac{3a^2c^3\tan^{-1}(ax)}{x}+3a^4c^3x\tan^{-1}(ax)+\frac{1}{3}a^6c^3x^3\tan^{-1}($$

Mathematica [A] time = 0.0706868, size = 83, normalized size = 0.72

$$\frac{c^3\left(2\left(a^6x^6+9a^4x^4-9a^2x^2-1\right)\tan^{-1}(ax)-ax\left(a^4x^4-16a^2x^2\log(x)+16a^2x^2\log\left(a^2x^2+1\right)+1\right)\right)}{6x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + a^2*c*x^2)^3*ArcTan[a*x])/x^4,x]
```

```
[Out] (c^3*(2*(-1 - 9*a^2*x^2 + 9*a^4*x^4 + a^6*x^6)*ArcTan[a*x] - a*x*(1 + a^4*x
^4 - 16*a^2*x^2*Log[x] + 16*a^2*x^2*Log[1 + a^2*x^2])))/(6*x^3)
```

Maple [A] time = 0.033, size = 107, normalized size = 0.9

$$\frac{a^6c^3x^3\arctan(ax)}{3}+3\,a^4c^3x\arctan(ax)-3\,\frac{a^2c^3\arctan(ax)}{x}-\frac{c^3\arctan(ax)}{3\,x^3}-\frac{a^5c^3x^2}{6}-\frac{8\,a^3c^3\ln\left(a^2x^2+1\right)}{3}-\frac{ac^3}{6\,x^2}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^3*arctan(a*x)/x^4,x)
```

```
[Out] 1/3*a^6*c^3*x^3*arctan(a*x)+3*a^4*c^3*x*arctan(a*x)-3*a^2*c^3*arctan(a*x)/x
-1/3*c^3*arctan(a*x)/x^3-1/6*a^5*c^3*x^2-8/3*a^3*c^3*ln(a^2*x^2+1)-1/6*a*c^
3/x^2+8/3*a^3*c^3*ln(a*x)
```

Maxima [A] time = 1.12865, size = 130, normalized size = 1.12

$$-\frac{1}{6}\left(a^4c^3x^2+16\,a^2c^3\log\left(a^2x^2+1\right)-16\,a^2c^3\log\left(x\right)+\frac{c^3}{x^2}\right)a+\frac{1}{3}\left(a^6c^3x^3+9\,a^4c^3x-\frac{9\,a^2c^3x^2+c^3}{x^3}\right)\arctan\left(ax\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)/x^4,x, algorithm="maxima")
```

```
[Out] -1/6*(a^4*c^3*x^2 + 16*a^2*c^3*log(a^2*x^2 + 1) - 16*a^2*c^3*log(x) + c^3/x
^2)*a + 1/3*(a^6*c^3*x^3 + 9*a^4*c^3*x - (9*a^2*c^3*x^2 + c^3)/x^3)*arctan(
a*x)
```

Fricas [A] time = 1.74709, size = 216, normalized size = 1.86

$$-\frac{a^5c^3x^5+16\,a^3c^3x^3\log\left(a^2x^2+1\right)-16\,a^3c^3x^3\log\left(x\right)+ac^3x-2\left(a^6c^3x^6+9\,a^4c^3x^4-9\,a^2c^3x^2-c^3\right)\arctan\left(ax\right)}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)/x^4,x, algorithm="fricas")
```

```
[Out] -1/6*(a^5*c^3*x^5 + 16*a^3*c^3*x^3*log(a^2*x^2 + 1) - 16*a^3*c^3*x^3*log(x)
 + a*c^3*x - 2*(a^6*c^3*x^6 + 9*a^4*c^3*x^4 - 9*a^2*c^3*x^2 - c^3)*arctan(a
*x))/x^3
```

Sympy [A] time = 2.94068, size = 117, normalized size = 1.01

$$\begin{cases}\frac{a^6c^3x^3\operatorname{atan}(ax)}{3}-\frac{a^5c^3x^2}{6}+3a^4c^3x\operatorname{atan}(ax)+\frac{8a^3c^3\log(x)}{3}-\frac{8a^3c^3\log\left(x^2+\frac{1}{a^2}\right)}{3}-\frac{3a^2c^3\operatorname{atan}(ax)}{x}-\frac{ac^3}{6x^2}-\frac{c^3\operatorname{atan}(ax)}{3x^3} & \text{for}\ a\neq 0\\ 0 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**3*atan(a*x)/x**4,x)
```

```
[Out] Piecewise((a**6*c**3*x**3*atan(a*x)/3 - a**5*c**3*x**2/6 + 3*a**4*c**3*x*at
an(a*x) + 8*a**3*c**3*log(x)/3 - 8*a**3*c**3*log(x**2 + a**(-2))/3 - 3*a**2
*c**3*atan(a*x)/x - a*c**3/(6*x**2) - c**3*atan(a*x)/(3*x**3), Ne(a, 0)), (
0, True))
```

Giac [A] time = 1.18822, size = 149, normalized size = 1.28

$$-\frac{1}{6}a^5c^3x^2 - \frac{8}{3}a^3c^3\log\left(a^2x^2+1\right) + \frac{4}{3}a^3c^3\log\left(x^2\right) + \frac{1}{3}\left(a^6c^3x^3 + 9a^4c^3x - \frac{9a^2c^3x^2+c^3}{x^3}\right)\arctan(ax) - \frac{8a^3c^3x^2+ac^3}{6x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)/x^4,x, algorithm="giac")
```

```
[Out] -1/6*a^5*c^3*x^2 - 8/3*a^3*c^3*log(a^2*x^2 + 1) + 4/3*a^3*c^3*log(x^2) + 1/
3*(a^6*c^3*x^3 + 9*a^4*c^3*x - (9*a^2*c^3*x^2 + c^3)/x^3)*arctan(a*x) - 1/6
*(8*a^3*c^3*x^2 + a*c^3)/x^2
```

3.173 $\int \frac{x^4 \tan^{-1}(ax)}{c+a^2cx^2} dx$

Optimal. Leaf size=80

$$-\frac{x^2}{6a^3c}+\frac{2\log\left(a^2x^2+1\right)}{3a^5c}+\frac{x^3\tan^{-1}(ax)}{3a^2c}-\frac{x\tan^{-1}(ax)}{a^4c}+\frac{\tan^{-1}(ax)^2}{2a^5c}$$

```
[Out] -x^2/(6*a^3*c) - (x*ArcTan[a*x])/(a^4*c) + (x^3*ArcTan[a*x])/(3*a^2*c) + Ar
cTan[a*x]^2/(2*a^5*c) + (2*Log[1 + a^2*x^2])/(3*a^5*c)
```

Rubi [A] time = 0.154707, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {4916, 4852, 266, 43, 4846, 260, 4884}

$$-\frac{x^2}{6a^3c}+\frac{2\log\left(a^2x^2+1\right)}{3a^5c}+\frac{x^3\tan^{-1}(ax)}{3a^2c}-\frac{x\tan^{-1}(ax)}{a^4c}+\frac{\tan^{-1}(ax)^2}{2a^5c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*ArcTan[a*x])/(c + a^2*c*x^2),x]
```

```
[Out] -x^2/(6*a^3*c) - (x*ArcTan[a*x])/(a^4*c) + (x^3*ArcTan[a*x])/(3*a^2*c) + Ar
cTan[a*x]^2/(2*a^5*c) + (2*Log[1 + a^2*x^2])/(3*a^5*c)
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{x^4 \tan^{-1}(ax)}{c+a^2cx^2}\,dx = -\frac{\int \frac{x^2\tan^{-1}(ax)}{c+a^2cx^2}\,dx}{a^2} + \frac{\int x^2\tan^{-1}(ax)\,dx}{a^2c}$$
$$= \frac{x^3\tan^{-1}(ax)}{3a^2c} + \frac{\int \frac{\tan^{-1}(ax)}{c+a^2cx^2}\,dx}{a^4} - \frac{\int \tan^{-1}(ax)\,dx}{a^4c} - \frac{\int \frac{x^3}{1+a^2x^2}\,dx}{3ac}$$
$$= -\frac{x\tan^{-1}(ax)}{a^4c} + \frac{x^3\tan^{-1}(ax)}{3a^2c} + \frac{\tan^{-1}(ax)^2}{2a^5c} + \frac{\int \frac{x}{1+a^2x^2}\,dx}{a^3c} - \frac{\text{Subst}\left(\int \frac{x}{1+a^2x}\,dx, x, x^2\right)}{6ac}$$
$$= -\frac{x\tan^{-1}(ax)}{a^4c} + \frac{x^3\tan^{-1}(ax)}{3a^2c} + \frac{\tan^{-1}(ax)^2}{2a^5c} + \frac{\log\left(1+a^2x^2\right)}{2a^5c} - \frac{\text{Subst}\left(\int \left(\frac{1}{a^2} - \frac{1}{a^2(1+a^2x)}\right)dx, x, x^2\right)}{6ac}$$
$$= -\frac{x^2}{6a^3c} - \frac{x\tan^{-1}(ax)}{a^4c} + \frac{x^3\tan^{-1}(ax)}{3a^2c} + \frac{\tan^{-1}(ax)^2}{2a^5c} + \frac{2\log\left(1+a^2x^2\right)}{3a^5c}$$

Mathematica [A] time = 0.0557064, size = 56, normalized size = 0.7

$$\frac{-a^2x^2 + 4\log\left(a^2x^2+1\right) + 2ax\left(a^2x^2-3\right)\tan^{-1}(ax) + 3\tan^{-1}(ax)^2}{6a^5c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*ArcTan[a*x])/(c + a^2*c*x^2),x]
```

```
[Out] (-(a^2*x^2) + 2*a*x*(-3 + a^2*x^2)*ArcTan[a*x] + 3*ArcTan[a*x]^2 + 4*Log[1
+ a^2*x^2])/(6*a^5*c)
```

Maple [A] time = 0.033, size = 73, normalized size = 0.9

$$-\frac{x^2}{6\,a^3c} - \frac{x\arctan(ax)}{a^4c} + \frac{x^3\arctan(ax)}{3\,a^2c} + \frac{(\arctan(ax))^2}{2\,a^5c} + \frac{2\,\ln\left(a^2x^2+1\right)}{3\,a^5c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arctan(a*x)/(a^2*c*x^2+c),x)
```

```
[Out] -1/6*x^2/a^3/c-x*arctan(a*x)/a^4/c+1/3*x^3*arctan(a*x)/a^2/c+1/2*arctan(a*x
)^2/a^5/c+2/3*ln(a^2*x^2+1)/a^5/c
```

Maxima [A] time = 1.63382, size = 100, normalized size = 1.25

$$\frac{1}{3}\left(\frac{a^2x^3-3\,x}{a^4c}+\frac{3\,\arctan\left(ax\right)}{a^5c}\right)\arctan\left(ax\right)-\frac{a^2x^2+3\,\arctan\left(ax\right)^2-4\,\log\left(a^2x^2+1\right)}{6\,a^5c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctan(a*x)/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/3*((a^2*x^3 - 3*x)/(a^4*c) + 3*arctan(a*x)/(a^5*c))*arctan(a*x) - 1/6*(a^
2*x^2 + 3*arctan(a*x)^2 - 4*log(a^2*x^2 + 1))/(a^5*c)
```

Fricas [A] time = 1.77491, size = 131, normalized size = 1.64

$$-\frac{a^2x^2-2\left(a^3x^3-3\,ax\right)\arctan\left(ax\right)-3\,\arctan\left(ax\right)^2-4\,\log\left(a^2x^2+1\right)}{6\,a^5c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctan(a*x)/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] -1/6*(a^2*x^2 - 2*(a^3*x^3 - 3*a*x)*arctan(a*x) - 3*arctan(a*x)^2 - 4*log(a
^2*x^2 + 1))/(a^5*c)
```

Sympy [A] time = 2.59707, size = 110, normalized size = 1.38

$$\begin{cases} \frac{x^3\operatorname{atan}\left(ax\right)}{3a^2c}-\frac{x^2}{6a^3c}-\frac{x\operatorname{atan}\left(ax\right)}{a^4c}+\frac{2\log\left(x^2+\frac{1}{a^2}\right)}{3a^5c}+\frac{\operatorname{atan}^2\left(ax\right)}{2a^5c} & \text{for}\ c\neq 0 \\ \tilde{\infty}\left(\frac{x^5\operatorname{atan}\left(ax\right)}{5}-\frac{x^4}{20a}+\frac{x^2}{10a^3}-\frac{\log\left(a^2x^2+1\right)}{10a^5}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*atan(a*x)/(a**2*c*x**2+c),x)
```

```
[Out] Piecewise((x**3*atan(a*x)/(3*a**2*c) - x**2/(6*a**3*c) - x*atan(a*x)/(a**4*
c) + 2*log(x**2 + a**(-2))/(3*a**5*c) + atan(a*x)**2/(2*a**5*c), Ne(c, 0)),
```

```
 (zoo*(x**5*atan(a*x)/5 - x**4/(20*a) + x**2/(10*a**3) - log(a**2*x**2 + 1)
/(10*a**5)), True))
```

Giac [A] time = 1.28609, size = 77, normalized size = 0.96

$$\frac{2\,a^3x^3\arctan\left(ax\right)-a^2x^2-6\,ax\arctan\left(ax\right)+3\,\arctan\left(ax\right)^2+4\,\log\left(a^2x^2+1\right)}{6\,a^5c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctan(a*x)/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] 1/6*(2*a^3*x^3*arctan(a*x) - a^2*x^2 - 6*a*x*arctan(a*x) + 3*arctan(a*x)^2
+ 4*log(a^2*x^2 + 1))/(a^5*c)
```

3.174 $\int \frac{x^3 \tan^{-1}(ax)}{c+a^2cx^2} \, dx$

Optimal. Leaf size=113

$$\frac{i\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{2a^4c}+\frac{x^2\tan^{-1}(ax)}{2a^2c}-\frac{x}{2a^3c}+\frac{i\tan^{-1}(ax)^2}{2a^4c}+\frac{\tan^{-1}(ax)}{2a^4c}+\frac{\log\left(\frac{2}{1+iax}\right)\tan^{-1}(ax)}{a^4c}$$

```
[Out] -x/(2*a^3*c) + ArcTan[a*x]/(2*a^4*c) + (x^2*ArcTan[a*x])/(2*a^2*c) + ((I/2)
*ArcTan[a*x]^2)/(a^4*c) + (ArcTan[a*x]*Log[2/(1 + I*a*x)])/(a^4*c) + ((I/2)
*PolyLog[2, 1 - 2/(1 + I*a*x)])/(a^4*c)
```

Rubi [A] time = 0.142134, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {4916, 4852, 321, 203, 4920, 4854, 2402, 2315}

$$\frac{i\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{2a^4c}+\frac{x^2\tan^{-1}(ax)}{2a^2c}-\frac{x}{2a^3c}+\frac{i\tan^{-1}(ax)^2}{2a^4c}+\frac{\tan^{-1}(ax)}{2a^4c}+\frac{\log\left(\frac{2}{1+iax}\right)\tan^{-1}(ax)}{a^4c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcTan[a*x])/(c + a^2*c*x^2),x]
```


```
[Out] -x/(2*a^3*c) + ArcTan[a*x]/(2*a^4*c) + (x^2*ArcTan[a*x])/(2*a^2*c) + ((I/2)
*ArcTan[a*x]^2)/(a^4*c) + (ArcTan[a*x]*Log[2/(1 + I*a*x)])/(a^4*c) + ((I/2)
*PolyLog[2, 1 - 2/(1 + I*a*x)])/(a^4*c)
```


Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int \frac{x^3 \tan^{-1}(ax)}{c + a^2cx^2} dx = -\frac{\int \frac{x \tan^{-1}(ax)}{c+a^2cx^2} dx}{a^2} + \frac{\int x \tan^{-1}(ax) dx}{a^2c}$$
$$= \frac{x^2 \tan^{-1}(ax)}{2a^2c} + \frac{i \tan^{-1}(ax)^2}{2a^4c} + \frac{\int \frac{\tan^{-1}(ax)}{i-ax} dx}{a^3c} - \frac{\int \frac{x^2}{1+a^2x^2} dx}{2ac}$$
$$= -\frac{x}{2a^3c} + \frac{x^2 \tan^{-1}(ax)}{2a^2c} + \frac{i \tan^{-1}(ax)^2}{2a^4c} + \frac{\tan^{-1}(ax) \log\left(\frac{2}{1+iax}\right)}{a^4c} + \frac{\int \frac{1}{1+a^2x^2} dx}{2a^3c} - \frac{\int \frac{\log\left(\frac{2}{1+iax}\right)}{1+a^2x^2} dx}{a^3c}$$
$$= -\frac{x}{2a^3c} + \frac{\tan^{-1}(ax)}{2a^4c} + \frac{x^2 \tan^{-1}(ax)}{2a^2c} + \frac{i \tan^{-1}(ax)^2}{2a^4c} + \frac{\tan^{-1}(ax) \log\left(\frac{2}{1+iax}\right)}{a^4c} + \frac{i \operatorname{Subst}\left(\int \frac{\log(2x)}{1-2x} dx\right.}{a^4c}$$
$$= -\frac{x}{2a^3c} + \frac{\tan^{-1}(ax)}{2a^4c} + \frac{x^2 \tan^{-1}(ax)}{2a^2c} + \frac{i \tan^{-1}(ax)^2}{2a^4c} + \frac{\tan^{-1}(ax) \log\left(\frac{2}{1+iax}\right)}{a^4c} + \frac{i \operatorname{Li}_2\left(1 - \frac{2}{1+iax}\right)}{2a^4c}$$

Mathematica [A] time = 0.0338757, size = 120, normalized size = 1.06

$$\frac{i \operatorname{PolyLog}\left(2, -\frac{ax+i}{-ax+i}\right)}{2a^4c} + \frac{x^2 \tan^{-1}(ax)}{2a^2c} - \frac{x}{2a^3c} + \frac{i \tan^{-1}(ax)^2}{2a^4c} + \frac{\tan^{-1}(ax)}{2a^4c} + \frac{\log\left(\frac{2i}{-ax+i}\right) \tan^{-1}(ax)}{a^4c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*ArcTan[a*x])/(c + a^2*c*x^2),x]
```

```
[Out] -x/(2*a^3*c) + ArcTan[a*x]/(2*a^4*c) + (x^2*ArcTan[a*x])/(2*a^2*c) + ((I/2)
*ArcTan[a*x]^2)/(a^4*c) + (ArcTan[a*x]*Log[(2*I)/(I - a*x)])/(a^4*c) + ((I/
2)*PolyLog[2, -((I + a*x)/(I - a*x))])/(a^4*c)
```

Maple [B] time = 0.091, size = 238, normalized size = 2.1

$$\frac{x^2 \arctan(ax)}{2a^2c} - \frac{\arctan(ax) \ln\left(a^2x^2+1\right)}{2a^4c} - \frac{x}{2a^3c} + \frac{\arctan(ax)}{2a^4c} - \frac{\frac{i}{4} \ln\left(a^2x^2+1\right) \ln(ax-i)}{a^4c} + \frac{\frac{i}{8} (\ln(ax-i))^2}{a^4c} + \frac{\frac{i}{4} \ln(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctan(a*x)/(a^2*c*x^2+c),x)
```

```
[Out] 1/2*x^2*arctan(a*x)/a^2/c-1/2/a^4/c*arctan(a*x)*ln(a^2*x^2+1)-1/2*x/a^3/c+1
/2*arctan(a*x)/a^4/c-1/4*I/a^4/c*ln(a^2*x^2+1)*ln(a*x-I)+1/8*I/a^4/c*ln(a*x
```

```
-I)^2+1/4*I/a^4/c*ln(a*x-I)*ln(-1/2*I*(a*x+I))+1/4*I/a^4/c*dilog(-1/2*I*(a*
x+I))+1/4*I/a^4/c*ln(a^2*x^2+1)*ln(a*x+I)-1/8*I/a^4/c*ln(a*x+I)^2-1/4*I/a^4
/c*ln(a*x+I)*ln(1/2*I*(a*x-I))-1/4*I/a^4/c*dilog(1/2*I*(a*x-I))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \arctan(ax)}{a^2cx^2 + c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] integrate(x^3*arctan(a*x)/(a^2*c*x^2 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^3 \arctan(ax)}{a^2cx^2 + c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(x^3*arctan(a*x)/(a^2*c*x^2 + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^3 \operatorname{atan}(ax)}{a^2x^2+1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atan(a*x)/(a**2*c*x**2+c),x)
```

```
[Out] Integral(x**3*atan(a*x)/(a**2*x**2 + 1), x)/c
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \arctan(ax)}{a^2cx^2 + c} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(x^3*arctan(a*x)/(a^2*c*x^2 + c), x)
```

3.175 $\int \frac{x^2 \tan^{-1}(ax)}{c+a^2cx^2} \, dx$

Optimal. Leaf size=49

$$-\frac{\log\left(a^2x^2+1\right)}{2a^3c}-\frac{\tan^{-1}(ax)^2}{2a^3c}+\frac{x\tan^{-1}(ax)}{a^2c}$$

```
[Out] (x*ArcTan[a*x])/(a^2*c) - ArcTan[a*x]^2/(2*a^3*c) - Log[1 + a^2*x^2]/(2*a^3
*c)
```

Rubi [A] time = 0.072521, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4916, 4846, 260, 4884}

$$-\frac{\log\left(a^2x^2+1\right)}{2a^3c}-\frac{\tan^{-1}(ax)^2}{2a^3c}+\frac{x\tan^{-1}(ax)}{a^2c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcTan[a*x])/(c + a^2*c*x^2),x]
```

```
[Out] (x*ArcTan[a*x])/(a^2*c) - ArcTan[a*x]^2/(2*a^3*c) - Log[1 + a^2*x^2]/(2*a^3
*c)
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\tan^{-1}(ax)}{c+a^2cx^2}\,dx &= -\frac{\int \frac{\tan^{-1}(ax)}{c+a^2cx^2}\,dx}{a^2}+\frac{\int \tan^{-1}(ax)\,dx}{a^2c}\\ &= \frac{x\tan^{-1}(ax)}{a^2c}-\frac{\tan^{-1}(ax)^2}{2a^3c}-\frac{\int \frac{x}{1+a^2x^2}\,dx}{ac}\\ &= \frac{x\tan^{-1}(ax)}{a^2c}-\frac{\tan^{-1}(ax)^2}{2a^3c}-\frac{\log\left(1+a^2x^2\right)}{2a^3c}\end{aligned}$$

Mathematica [A] time = 0.0278609, size = 49, normalized size = 1.

$$-\frac{\log\left(a^2x^2+1\right)}{2a^3c}-\frac{\tan^{-1}(ax)^2}{2a^3c}+\frac{x\tan^{-1}(ax)}{a^2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*ArcTan[a*x])/(c + a^2*c*x^2),x]
```

```
[Out] (x*ArcTan[a*x])/(a^2*c) - ArcTan[a*x]^2/(2*a^3*c) - Log[1 + a^2*x^2]/(2*a^3
*c)
```

Maple [A] time = 0.03, size = 46, normalized size = 0.9

$$\frac{x\arctan(ax)}{a^2c}-\frac{(\arctan(ax))^2}{2\,a^3c}-\frac{\ln\left(a^2x^2+1\right)}{2\,a^3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(a*x)/(a^2*c*x^2+c),x)
```

```
[Out] x*arctan(a*x)/a^2/c-1/2*arctan(a*x)^2/a^3/c-1/2*ln(a^2*x^2+1)/a^3/c
```

Maxima [A] time = 1.67045, size = 73, normalized size = 1.49

$$\left(\frac{x}{a^2c} - \frac{\arctan(ax)}{a^3c}\right)\arctan(ax) + \frac{\arctan(ax)^2 - \log\left(a^2x^2 + 1\right)}{2\,a^3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] (x/(a^2*c) - arctan(a*x)/(a^3*c))*arctan(a*x) + 1/2*(arctan(a*x)^2 - log(a^
2*x^2 + 1))/(a^3*c)
```

Fricas [A] time = 1.70957, size = 92, normalized size = 1.88

$$\frac{2\,ax\arctan(ax) - \arctan(ax)^2 - \log\left(a^2x^2 + 1\right)}{2\,a^3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*x*arctan(a*x) - arctan(a*x)^2 - log(a^2*x^2 + 1))/(a^3*c)
```

Sympy [A] time = 1.41703, size = 75, normalized size = 1.53

$$\begin{cases} \frac{x\operatorname{atan}(ax)}{a^2c} - \frac{\log\left(x^2+\frac{1}{a^2}\right)}{2a^3c} - \frac{\operatorname{atan}^2(ax)}{2a^3c} & \text{for } c \neq 0 \\ \tilde{\infty}\left(\frac{x^3\operatorname{atan}(ax)}{3} - \frac{x^2}{6a} + \frac{\log\left(a^2x^2+1\right)}{6a^3}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(a*x)/(a**2*c*x**2+c),x)
```

```
[Out] Piecewise((x*atan(a*x)/(a**2*c) - log(x**2 + a**(-2))/(2*a**3*c) - atan(a*x
)**2/(2*a**3*c), Ne(c, 0)), (zoo*(x**3*atan(a*x)/3 - x**2/(6*a) + log(a**2*
x**2 + 1)/(6*a**3)), True))
```

Giac [A] time = 1.15817, size = 50, normalized size = 1.02

$$\frac{2\,ax\arctan\left(ax\right)-\arctan\left(ax\right)^{2}-\log\left(a^{2}x^{2}+1\right)}{2\,a^{3}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] 1/2*(2*a*x*arctan(a*x) - arctan(a*x)^2 - log(a^2*x^2 + 1))/(a^3*c)
```

3.176 $\int \frac{x \tan^{-1}(ax)}{c+a^2cx^2} dx$

Optimal. Leaf size=72

$$-\frac{i\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{2a^2c}-\frac{i\tan^{-1}(ax)^2}{2a^2c}-\frac{\log\left(\frac{2}{1+iax}\right)\tan^{-1}(ax)}{a^2c}$$

```
[Out] ((-I/2)*ArcTan[a*x]^2)/(a^2*c) - (ArcTan[a*x]*Log[2/(1 + I*a*x)])/(a^2*c) -
 ((I/2)*PolyLog[2, 1 - 2/(1 + I*a*x)])/(a^2*c)
```

Rubi [A] time = 0.0721384, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4920, 4854, 2402, 2315}

$$-\frac{i\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{2a^2c}-\frac{i\tan^{-1}(ax)^2}{2a^2c}-\frac{\log\left(\frac{2}{1+iax}\right)\tan^{-1}(ax)}{a^2c}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTan[a*x])/(c + a^2*c*x^2),x]
```


```
[Out] ((-I/2)*ArcTan[a*x]^2)/(a^2*c) - (ArcTan[a*x]*Log[2/(1 + I*a*x)])/(a^2*c) -
 ((I/2)*PolyLog[2, 1 - 2/(1 + I*a*x)])/(a^2*c)
```


Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x\tan^{-1}(ax)}{c+a^2cx^2}\,dx &= -\frac{i\tan^{-1}(ax)^2}{2a^2c}-\frac{\int \frac{\tan^{-1}(ax)}{i-ax}\,dx}{ac}\\
&= -\frac{i\tan^{-1}(ax)^2}{2a^2c}-\frac{\tan^{-1}(ax)\log\left(\frac{2}{1+iax}\right)}{a^2c}+\frac{\int \frac{\log\left(\frac{2}{1+iax}\right)}{1+a^2x^2}\,dx}{ac}\\
&= -\frac{i\tan^{-1}(ax)^2}{2a^2c}-\frac{\tan^{-1}(ax)\log\left(\frac{2}{1+iax}\right)}{a^2c}-\frac{i\,\text{Subst}\left(\int \frac{\log(2x)}{1-2x}\,dx,x,\frac{1}{1+iax}\right)}{a^2c}\\
&= -\frac{i\tan^{-1}(ax)^2}{2a^2c}-\frac{\tan^{-1}(ax)\log\left(\frac{2}{1+iax}\right)}{a^2c}-\frac{i\text{Li}_2\left(1-\frac{2}{1+iax}\right)}{2a^2c}
\end{aligned}
$$

Mathematica [A] time = 0.0052534, size = 77, normalized size = 1.07

$$
-\frac{i\text{PolyLog}\left(2,\frac{ax+i}{ax-i}\right)}{2a^2c}-\frac{i\tan^{-1}(ax)^2}{2a^2c}-\frac{\log\left(\frac{2i}{-ax+i}\right)\tan^{-1}(ax)}{a^2c}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcTan[a*x])/(c + a^2*c*x^2),x]
```

```
[Out] ((-I/2)*ArcTan[a*x]^2)/(a^2*c) - (ArcTan[a*x]*Log[(2*I)/(I - a*x)])/(a^2*c)
 - ((I/2)*PolyLog[2, (I + a*x)/(-I + a*x)])/(a^2*c)
```

Maple [B] time = 0.086, size = 202, normalized size = 2.8

$$
\frac{\arctan(ax)\ln\left(a^2x^2+1\right)}{2a^2c}-\frac{\frac{i}{8}\left(\ln(ax-i)\right)^2}{a^2c}+\frac{\frac{i}{4}\ln(ax-i)\ln\left(a^2x^2+1\right)}{a^2c}-\frac{\frac{i}{4}\ln(ax-i)\ln\left(-\frac{i}{2}(ax+i)\right)}{a^2c}-\frac{\frac{i}{4}\text{dilog}\left(-\frac{i}{2}(ax\right.}{a^2c}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(a*x)/(a^2*c*x^2+c),x)
```

```
[Out] 1/2/a^2/c*arctan(a*x)*ln(a^2*x^2+1)-1/8*I/a^2/c*ln(a*x-I)^2+1/4*I/a^2/c*ln(
a*x-I)*ln(a^2*x^2+1)-1/4*I/a^2/c*ln(a*x-I)*ln(-1/2*I*(a*x+I))-1/4*I/a^2/c*d
ilog(-1/2*I*(a*x+I))+1/8*I/a^2/c*ln(a*x+I)^2+1/4*I/a^2/c*ln(a*x+I)*ln(1/2*I
*(a*x-I))-1/4*I/a^2/c*ln(a*x+I)*ln(a^2*x^2+1)+1/4*I/a^2/c*dilog(1/2*I*(a*x-
I))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \arctan(ax)}{a^2cx^2 + c} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] integrate(x*arctan(a*x)/(a^2*c*x^2 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x \arctan(ax)}{a^2cx^2 + c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(x*arctan(a*x)/(a^2*c*x^2 + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x \operatorname{atan}(ax)}{a^2x^2+1} \, dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(a*x)/(a**2*c*x**2+c),x)
```

```
[Out] Integral(x*atan(a*x)/(a**2*x**2 + 1), x)/c
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \arctan(ax)}{a^2cx^2 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(x*arctan(a*x)/(a^2*c*x^2 + c), x)
```

3.177 $\int \frac{\tan^{-1}(ax)}{c+a^2cx^2}\,dx$

Optimal. Leaf size=16

$$\frac{\tan^{-1}(ax)^2}{2ac}$$

```
[Out] ArcTan[a*x]^2/(2*a*c)
```

Rubi [A] time = 0.0173921, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {4884}

$$\frac{\tan^{-1}(ax)^2}{2ac}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]/(c + a^2*c*x^2),x]
```

```
[Out] ArcTan[a*x]^2/(2*a*c)
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)}{c+a^2cx^2}\,dx=\frac{\tan^{-1}(ax)^2}{2ac}$$

Mathematica [A] time = 0.0027385, size = 16, normalized size = 1.

$$\frac{\tan^{-1}(ax)^2}{2ac}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]/(c + a^2*c*x^2),x]
```

```
[Out] ArcTan[a*x]^2/(2*a*c)
```

Maple [A] time = 0.028, size = 15, normalized size = 0.9

$$\frac{(\arctan(ax))^2}{2ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)/(a^2*c*x^2+c),x)
```

```
[Out] 1/2*arctan(a*x)^2/a/c
```

Maxima [A] time = 1.54783, size = 19, normalized size = 1.19

$$\frac{\arctan(ax)^2}{2ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/2*arctan(a*x)^2/(a*c)
```

Fricas [A] time = 1.6202, size = 34, normalized size = 2.12

$$\frac{\arctan(ax)^2}{2ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/2*arctan(a*x)^2/(a*c)
```

Sympy [A] time = 2.4263, size = 36, normalized size = 2.25

$$\begin{cases} 0 & \text{for } a = 0 \\ \tilde{\infty} \left(\begin{cases} 0 & \text{for } a = 0 \\ \frac{ax \operatorname{atan}(ax) - \frac{\log(a^2x^2+1)}{2}}{a} & \text{otherwise} \end{cases} \right) & \text{for } c = 0 \\ \frac{\operatorname{atan}^2(ax)}{2ac} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)/(a**2*c*x**2+c),x)
```

```
[Out] Piecewise((0, Eq(a, 0)), (zoo*Piecewise((0, Eq(a, 0)), ((a*x*atan(a*x) - lo
g(a**2*x**2 + 1)/2)/a, True)), Eq(c, 0)), (atan(a*x)**2/(2*a*c), True))
```

Giac [A] time = 1.11326, size = 19, normalized size = 1.19

$$\frac{\arctan(ax)^2}{2\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] 1/2*arctan(a*x)^2/(a*c)
```

3.178 $\int \frac{\tan^{-1}(ax)}{x(c+a^2cx^2)}\,dx$

Optimal. Leaf size=64

$$-\frac{i\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{2c}-\frac{i\tan^{-1}(ax)^2}{2c}+\frac{\log\left(2-\frac{2}{1-iax}\right)\tan^{-1}(ax)}{c}$$

```
[Out] ((-I/2)*ArcTan[a*x]^2)/c + (ArcTan[a*x]*Log[2 - 2/(1 - I*a*x)])/c - ((I/2)*
PolyLog[2, -1 + 2/(1 - I*a*x)])/c
```

Rubi [A] time = 0.0996789, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {4924, 4868, 2447}

$$-\frac{i\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{2c}-\frac{i\tan^{-1}(ax)^2}{2c}+\frac{\log\left(2-\frac{2}{1-iax}\right)\tan^{-1}(ax)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]/(x*(c + a^2*c*x^2)),x]
```



```
[Out] ((-I/2)*ArcTan[a*x]^2)/c + (ArcTan[a*x]*Log[2 - 2/(1 - I*a*x)])/c - ((I/2)*
PolyLog[2, -1 + 2/(1 - I*a*x)])/c
```


Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^{-1}(ax)}{x\left(c+a^2cx^2\right)}\,dx &= -\frac{i\tan^{-1}(ax)^2}{2c}+\frac{i\int \frac{\tan^{-1}(ax)}{x(i+ax)}\,dx}{c}\\ &= -\frac{i\tan^{-1}(ax)^2}{2c}+\frac{\tan^{-1}(ax)\log\left(2-\frac{2}{1-iax}\right)}{c}-\frac{a\int \frac{\log\left(2-\frac{2}{1-iax}\right)}{1+a^2x^2}\,dx}{c}\\ &= -\frac{i\tan^{-1}(ax)^2}{2c}+\frac{\tan^{-1}(ax)\log\left(2-\frac{2}{1-iax}\right)}{c}-\frac{i\mathrm{Li}_2\left(-1+\frac{2}{1-iax}\right)}{2c}\end{aligned}$$

Mathematica [A] time = 0.0243861, size = 103, normalized size = 1.61

$$\frac{i\mathrm{PolyLog}(2,-iax)}{2c}-\frac{i\mathrm{PolyLog}(2,iax)}{2c}+\frac{i\mathrm{PolyLog}\left(2,-\frac{ax+i}{-ax+i}\right)}{2c}+\frac{i\tan^{-1}(ax)^2}{2c}+\frac{\log\left(\frac{2i}{-ax+i}\right)\tan^{-1}(ax)}{c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTan[a*x]/(x*(c + a^2*c*x^2)),x]
```

```
[Out] ((I/2)*ArcTan[a*x]^2)/c + (ArcTan[a*x]*Log[(2*I)/(I - a*x)])/c + ((I/2)*Pol
yLog[2, (-I)*a*x])/c - ((I/2)*PolyLog[2, I*a*x])/c + ((I/2)*PolyLog[2, -((I
 + a*x)/(I - a*x))])/c
```

Maple [B] time = 0.092, size = 251, normalized size = 3.9

$$-\frac{\arctan(ax)\ln\left(a^2x^2+1\right)}{2c}+\frac{\arctan(ax)\ln(ax)}{c}+\frac{\frac{i}{2}\ln(ax)\ln(1+iax)}{c}-\frac{\frac{i}{2}\ln(ax)\ln(1-iax)}{c}+\frac{\frac{i}{2}\mathrm{dilog}(1+iax)}{c}-\frac{\frac{i}{2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)/x/(a^2*c*x^2+c),x)
```

```
[Out] -1/2/c*arctan(a*x)*ln(a^2*x^2+1)+1/c*arctan(a*x)*ln(a*x)+1/2*I/c*ln(a*x)*ln
(1+I*a*x)-1/2*I/c*ln(a*x)*ln(1-I*a*x)+1/2*I/c*dilog(1+I*a*x)-1/2*I/c*dilog(
1-I*a*x)-1/4*I/c*ln(a^2*x^2+1)*ln(a*x-I)+1/8*I/c*ln(a*x-I)^2+1/4*I/c*ln(a*x
-I)*ln(-1/2*I*(a*x+I))+1/4*I/c*dilog(-1/2*I*(a*x+I))+1/4*I/c*ln(a^2*x^2+1)*
ln(a*x+I)-1/8*I/c*ln(a*x+I)^2-1/4*I/c*ln(a*x+I)*ln(1/2*I*(a*x-I))-1/4*I/c*d
ilog(1/2*I*(a*x-I))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(ax)}{(a^2cx^2+c)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] integrate(arctan(a*x)/((a^2*c*x^2 + c)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\arctan(ax)}{a^2cx^3+cx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(arctan(a*x)/(a^2*c*x^3 + c*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\operatorname{atan}(ax)}{a^2x^3+x}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)/x/(a**2*c*x**2+c),x)
```

```
[Out] Integral(atan(a*x)/(a**2*x**3 + x), x)/c
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(ax)}{(a^2cx^2+c)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)/((a^2*c*x^2 + c)*x), x)
```

3.179 $\int \frac{\tan^{-1}(ax)}{x^2(c+a^2cx^2)}\,dx$

Optimal. Leaf size=52

$$-\frac{a\log\left(a^2x^2+1\right)}{2c}+\frac{a\log(x)}{c}-\frac{a\tan^{-1}(ax)^2}{2c}-\frac{\tan^{-1}(ax)}{cx}$$

```
[Out] -(ArcTan[a*x]/(c*x)) - (a*ArcTan[a*x]^2)/(2*c) + (a*Log[x])/c - (a*Log[1 +
a^2*x^2])/(2*c)
```

Rubi [A] time = 0.0865378, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {4918, 4852, 266, 36, 29, 31, 4884}

$$-\frac{a\log\left(a^2x^2+1\right)}{2c}+\frac{a\log(x)}{c}-\frac{a\tan^{-1}(ax)^2}{2c}-\frac{\tan^{-1}(ax)}{cx}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]/(x^2*(c + a^2*c*x^2)),x]
```

```
[Out] -(ArcTan[a*x]/(c*x)) - (a*ArcTan[a*x]^2)/(2*c) + (a*Log[x])/c - (a*Log[1 +
a^2*x^2])/(2*c)
```

Rule 4918

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/(d + e*x^2),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^{-1}(ax)}{x^2\left(c+a^2cx^2\right)}\,dx &= -\left(a^2\int \frac{\tan^{-1}(ax)}{c+a^2cx^2}\,dx\right)+\frac{\int \frac{\tan^{-1}(ax)}{x^2}\,dx}{c}\\
&= -\frac{\tan^{-1}(ax)}{cx}-\frac{a\tan^{-1}(ax)^2}{2c}+\frac{a\int \frac{1}{x\left(1+a^2x^2\right)}\,dx}{c}\\
&= -\frac{\tan^{-1}(ax)}{cx}-\frac{a\tan^{-1}(ax)^2}{2c}+\frac{a\,\mathrm{Subst}\left(\int \frac{1}{x\left(1+a^2x\right)}\,dx,x,x^2\right)}{2c}\\
&= -\frac{\tan^{-1}(ax)}{cx}-\frac{a\tan^{-1}(ax)^2}{2c}+\frac{a\,\mathrm{Subst}\left(\int \frac{1}{x}\,dx,x,x^2\right)}{2c}-\frac{a^3\,\mathrm{Subst}\left(\int \frac{1}{1+a^2x}\,dx,x,x^2\right)}{2c}\\
&= -\frac{\tan^{-1}(ax)}{cx}-\frac{a\tan^{-1}(ax)^2}{2c}+\frac{a\log(x)}{c}-\frac{a\log\left(1+a^2x^2\right)}{2c}
\end{aligned}$$

Mathematica [A] time = 0.0077087, size = 52, normalized size = 1.

$$-\frac{a\log\left(a^2x^2+1\right)}{2c}+\frac{a\log(x)}{c}-\frac{a\tan^{-1}(ax)^2}{2c}-\frac{\tan^{-1}(ax)}{cx}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]/(x^2*(c + a^2*c*x^2)),x]
```

```
[Out] -(ArcTan[a*x]/(c*x)) - (a*ArcTan[a*x]^2)/(2*c) + (a*Log[x])/c - (a*Log[1 +
a^2*x^2])/(2*c)
```

Maple [A] time = 0.036, size = 51, normalized size = 1.

$$-\frac{a\left(\arctan\left(ax\right)\right)^2}{2\,c}-\frac{\arctan\left(ax\right)}{cx}-\frac{a\ln\left(a^2x^2+1\right)}{2\,c}+\frac{a\ln\left(ax\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)/x^2/(a^2*c*x^2+c),x)
```

```
[Out] -1/2*a*arctan(a*x)^2/c-arctan(a*x)/c/x-1/2*a*ln(a^2*x^2+1)/c+a/c*ln(a*x)
```

Maxima [A] time = 1.60181, size = 72, normalized size = 1.38

$$-\left(\frac{a\arctan\left(ax\right)}{c}+\frac{1}{cx}\right)\arctan\left(ax\right)+\frac{\left(\arctan\left(ax\right)^2-\log\left(a^2x^2+1\right)+2\,\log\left(x\right)\right)a}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x^2/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] -(a*arctan(a*x)/c + 1/(c*x))*arctan(a*x) + 1/2*(arctan(a*x)^2 - log(a^2*x^2
 + 1) + 2*log(x))*a/c
```

Fricas [A] time = 1.69865, size = 116, normalized size = 2.23

$$-\frac{ax \arctan(ax)^2 + ax \log\left(a^2x^2+1\right) - 2\,ax \log(x) + 2\,\arctan(ax)}{2\,cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x^2/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] -1/2*(a*x*arctan(a*x)^2 + a*x*log(a^2*x^2 + 1) - 2*a*x*log(x) + 2*arctan(a*
x))/(c*x)
```

Sympy [A] time = 2.0772, size = 68, normalized size = 1.31

$$\begin{cases} \frac{a\log(x)}{c} - \frac{a\log\left(x^2+\frac{1}{a^2}\right)}{2c} - \frac{a\,\mathrm{atan}^2(ax)}{2c} - \frac{\mathrm{atan}(ax)}{cx} & \text{for}\ c \neq 0 \\ \tilde{\infty}\left(a\log(x) - \frac{a\log\left(a^2x^2+1\right)}{2} - \frac{\mathrm{atan}(ax)}{x}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)/x**2/(a**2*c*x**2+c),x)
```

```
[Out] Piecewise((a*log(x)/c - a*log(x**2 + a**(-2))/(2*c) - a*atan(a*x)**2/(2*c)
- atan(a*x)/(c*x), Ne(c, 0)), (zoo*(a*log(x) - a*log(a**2*x**2 + 1)/2 - ata
n(a*x)/x), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(ax)}{\left(a^2cx^2+c\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x^2/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)/((a^2*c*x^2 + c)*x^2), x)
```

3.180 $\int \frac{\tan^{-1}(ax)}{x^3(c+a^2cx^2)}\,dx$

Optimal. Leaf size=113

$$\frac{ia^2\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{2c}+\frac{ia^2\tan^{-1}(ax)^2}{2c}-\frac{a^2\tan^{-1}(ax)}{2c}-\frac{a^2\log\left(2-\frac{2}{1-iax}\right)\tan^{-1}(ax)}{c}-\frac{\tan^{-1}(ax)}{2cx^2}-\frac{a}{2cx}$$

```
[Out] -a/(2*c*x) - (a^2*ArcTan[a*x])/(2*c) - ArcTan[a*x]/(2*c*x^2) + ((I/2)*a^2*A
rcTan[a*x]^2)/c - (a^2*ArcTan[a*x]*Log[2 - 2/(1 - I*a*x)])/c + ((I/2)*a^2*P
olyLog[2, -1 + 2/(1 - I*a*x)])/c
```

Rubi [A] time = 0.162361, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {4918, 4852, 325, 203, 4924, 4868, 2447}

$$\frac{ia^2\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{2c}+\frac{ia^2\tan^{-1}(ax)^2}{2c}-\frac{a^2\tan^{-1}(ax)}{2c}-\frac{a^2\log\left(2-\frac{2}{1-iax}\right)\tan^{-1}(ax)}{c}-\frac{\tan^{-1}(ax)}{2cx^2}-\frac{a}{2cx}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]/(x^3*(c + a^2*c*x^2)),x]
```

```
[Out] -a/(2*c*x) - (a^2*ArcTan[a*x])/(2*c) - ArcTan[a*x]/(2*c*x^2) + ((I/2)*a^2*A
rcTan[a*x]^2)/c - (a^2*ArcTan[a*x]*Log[2 - 2/(1 - I*a*x)])/c + ((I/2)*a^2*P
olyLog[2, -1 + 2/(1 - I*a*x)])/c
```

Rule 4918

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/(d + e*x^2),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
```

```
erQ[m]) && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)}{x^3\left(c+a^2cx^2\right)}\,dx = -\left(a^2\int \frac{\tan^{-1}(ax)}{x\left(c+a^2cx^2\right)}\,dx\right)+\frac{\int \frac{\tan^{-1}(ax)}{x^3}\,dx}{c}$$

$$= -\frac{\tan^{-1}(ax)}{2cx^2}+\frac{ia^2\tan^{-1}(ax)^2}{2c}+\frac{a\int \frac{1}{x^2\left(1+a^2x^2\right)}\,dx}{2c}-\frac{\left(ia^2\right)\int \frac{\tan^{-1}(ax)}{x(i+ax)}\,dx}{c}$$

$$= -\frac{a}{2cx}-\frac{\tan^{-1}(ax)}{2cx^2}+\frac{ia^2\tan^{-1}(ax)^2}{2c}-\frac{a^2\tan^{-1}(ax)\log\left(2-\frac{2}{1-iax}\right)}{c}-\frac{a^3\int \frac{1}{1+a^2x^2}\,dx}{2c}+\frac{a^3\int \frac{\log(2-}{1+a}}{c}$$

$$= -\frac{a}{2cx}-\frac{a^2\tan^{-1}(ax)}{2c}-\frac{\tan^{-1}(ax)}{2cx^2}+\frac{ia^2\tan^{-1}(ax)^2}{2c}-\frac{a^2\tan^{-1}(ax)\log\left(2-\frac{2}{1-iax}\right)}{c}+\frac{ia^2\mathrm{Li}_2\left(-1-\right.}{2c}$$

Mathematica [C] time = 0.0606948, size = 142, normalized size = 1.26

$$-\frac{a\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-a^2x^2\right)}{2cx}-\frac{a^2\left(\frac{1}{2}i\text{PolyLog}(2,-iax)-\frac{1}{2}i\text{PolyLog}(2,iax)+\frac{1}{2}\left(i\text{PolyLog}\left(2,-\frac{ax+i}{-ax+i}\right)+\right.\right.}{c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTan[a*x]/(x^3*(c + a^2*c*x^2)),x]
```

```
[Out] -ArcTan[a*x]/(2*c*x^2) - (a*Hypergeometric2F1[-1/2, 1, 1/2, -(a^2*x^2)])/(2
*c*x) - (a^2*((I/2)*ArcTan[a*x]^2 + (I/2)*PolyLog[2, (-I)*a*x] - (I/2)*Poly
Log[2, I*a*x] + (2*ArcTan[a*x]*Log[(2*I)/(I - a*x)] + I*PolyLog[2, -((I + a
*x)/(I - a*x))])/2))/c
```

Maple [B] time = 0.095, size = 327, normalized size = 2.9

$$\frac{a^2\arctan(ax)\ln\left(a^2x^2+1\right)}{2c}-\frac{\arctan(ax)}{2cx^2}-\frac{a^2\arctan(ax)\ln(ax)}{c}-\frac{\frac{i}{4}a^2\ln\left(a^2x^2+1\right)\ln(ax+i)}{c}-\frac{\frac{i}{4}a^2\ln(ax-i)\ln\left(-\right.}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)/x^3/(a^2*c*x^2+c),x)
```

```
[Out] 1/2*a^2/c*arctan(a*x)*ln(a^2*x^2+1)-1/2*arctan(a*x)/c/x^2-a^2/c*arctan(a*x)
*ln(a*x)-1/4*I*a^2/c*ln(a^2*x^2+1)*ln(a*x+I)-1/4*I*a^2/c*ln(a*x-I)*ln(-1/2*
```

```
I*(a*x+I))+1/2*I*a^2/c*dilog(1-I*a*x)-1/2*I*a^2/c*ln(a*x)*ln(1+I*a*x)+1/4*I
*a^2/c*ln(a^2*x^2+1)*ln(a*x-I)-1/8*I*a^2/c*ln(a*x-I)^2-1/4*I*a^2/c*dilog(-1
/2*I*(a*x+I))+1/4*I*a^2/c*ln(a*x+I)*ln(1/2*I*(a*x-I))-1/2*a^2*arctan(a*x)/c
-1/2/c/x*a+1/2*I*a^2/c*ln(a*x)*ln(1-I*a*x)-1/2*I*a^2/c*dilog(1+I*a*x)+1/4*I
*a^2/c*dilog(1/2*I*(a*x-I))+1/8*I*a^2/c*ln(a*x+I)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(ax)}{(a^2cx^2+c)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x^3/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] integrate(arctan(a*x)/((a^2*c*x^2 + c)*x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\arctan(ax)}{a^2cx^5+cx^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x^3/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(arctan(a*x)/(a^2*c*x^5 + c*x^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\operatorname{atan}(ax)}{a^2x^5+x^3}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)/x**3/(a**2*c*x**2+c),x)
```

```
[Out] Integral(atan(a*x)/(a**2*x**5 + x**3), x)/c
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(ax)}{(a^2cx^2+c)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x^3/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)/((a^2*c*x^2 + c)*x^3), x)
```

3.181 $\int \frac{\tan^{-1}(ax)}{x^4(c+a^2cx^2)}\,dx$

Optimal. Leaf size=88

$$\frac{2a^3\log\left(a^2x^2+1\right)}{3c}-\frac{4a^3\log(x)}{3c}+\frac{a^3\tan^{-1}(ax)^2}{2c}+\frac{a^2\tan^{-1}(ax)}{cx}-\frac{a}{6cx^2}-\frac{\tan^{-1}(ax)}{3cx^3}$$

```
[Out] -a/(6*c*x^2) - ArcTan[a*x]/(3*c*x^3) + (a^2*ArcTan[a*x])/(c*x) + (a^3*ArcTa
n[a*x]^2)/(2*c) - (4*a^3*Log[x])/(3*c) + (2*a^3*Log[1 + a^2*x^2])/(3*c)
```

Rubi [A] time = 0.164881, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {4918, 4852, 266, 44, 36, 29, 31, 4884}

$$\frac{2a^3\log\left(a^2x^2+1\right)}{3c}-\frac{4a^3\log(x)}{3c}+\frac{a^3\tan^{-1}(ax)^2}{2c}+\frac{a^2\tan^{-1}(ax)}{cx}-\frac{a}{6cx^2}-\frac{\tan^{-1}(ax)}{3cx^3}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]/(x^4*(c + a^2*c*x^2)),x]
```

```
[Out] -a/(6*c*x^2) - ArcTan[a*x]/(3*c*x^3) + (a^2*ArcTan[a*x])/(c*x) + (a^3*ArcTa
n[a*x]^2)/(2*c) - (4*a^3*Log[x])/(3*c) + (2*a^3*Log[1 + a^2*x^2])/(3*c)
```

Rule 4918

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/(d + e*x^2),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)}{x^4\left(c+a^2cx^2\right)}\,dx = -\left(a^2\int \frac{\tan^{-1}(ax)}{x^2\left(c+a^2cx^2\right)}\,dx\right) + \frac{\int \frac{\tan^{-1}(ax)}{x^4}\,dx}{c}$$

$$= -\frac{\tan^{-1}(ax)}{3cx^3} + a^4\int \frac{\tan^{-1}(ax)}{c+a^2cx^2}\,dx + \frac{a\int \frac{1}{x^3\left(1+a^2x^2\right)}\,dx}{3c} - \frac{a^2\int \frac{\tan^{-1}(ax)}{x^2}\,dx}{c}$$

$$= -\frac{\tan^{-1}(ax)}{3cx^3} + \frac{a^2\tan^{-1}(ax)}{cx} + \frac{a^3\tan^{-1}(ax)^2}{2c} + \frac{a\,\mathrm{Subst}\left(\int \frac{1}{x^2\left(1+a^2x\right)}\,dx, x, x^2\right)}{6c} - \frac{a^3\int \frac{1}{x\left(1+a^2x^2\right)}\,d}{c}$$

$$= -\frac{\tan^{-1}(ax)}{3cx^3} + \frac{a^2\tan^{-1}(ax)}{cx} + \frac{a^3\tan^{-1}(ax)^2}{2c} + \frac{a\,\mathrm{Subst}\left(\int \left(\frac{1}{x^2} - \frac{a^2}{x} + \frac{a^4}{1+a^2x}\right)dx, x, x^2\right)}{6c} - \frac{a^3\,\mathrm{Su}}{}$$

$$= -\frac{a}{6cx^2} - \frac{\tan^{-1}(ax)}{3cx^3} + \frac{a^2\tan^{-1}(ax)}{cx} + \frac{a^3\tan^{-1}(ax)^2}{2c} - \frac{a^3\log(x)}{3c} + \frac{a^3\log\left(1+a^2x^2\right)}{6c} - \frac{a^3\,\mathrm{Subst}}{}$$

$$= -\frac{a}{6cx^2} - \frac{\tan^{-1}(ax)}{3cx^3} + \frac{a^2\tan^{-1}(ax)}{cx} + \frac{a^3\tan^{-1}(ax)^2}{2c} - \frac{4a^3\log(x)}{3c} + \frac{2a^3\log\left(1+a^2x^2\right)}{3c}$$

Mathematica [A] time = 0.0178617, size = 88, normalized size = 1.

$$\frac{2a^3\log\left(a^2x^2+1\right)}{3c} - \frac{4a^3\log(x)}{3c} + \frac{a^3\tan^{-1}(ax)^2}{2c} + \frac{a^2\tan^{-1}(ax)}{cx} - \frac{a}{6cx^2} - \frac{\tan^{-1}(ax)}{3cx^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]/(x^4*(c + a^2*c*x^2)),x]
```

```
[Out] -a/(6*c*x^2) - ArcTan[a*x]/(3*c*x^3) + (a^2*ArcTan[a*x])/(c*x) + (a^3*ArcTa
n[a*x]^2)/(2*c) - (4*a^3*Log[x])/(3*c) + (2*a^3*Log[1 + a^2*x^2])/(3*c)
```

Maple [A] time = 0.041, size = 81, normalized size = 0.9

$$\frac{a^3\left(\arctan(ax)\right)^2}{2c} - \frac{\arctan(ax)}{3cx^3} + \frac{a^2\arctan(ax)}{cx} + \frac{2a^3\ln\left(a^2x^2+1\right)}{3c} - \frac{a}{6cx^2} - \frac{4a^3\ln(ax)}{3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)/x^4/(a^2*c*x^2+c),x)
```

```
[Out] 1/2*a^3*arctan(a*x)^2/c-1/3*arctan(a*x)/c/x^3+a^2*arctan(a*x)/c/x+2/3*a^3*l
n(a^2*x^2+1)/c-1/6*a/c/x^2-4/3*a^3/c*ln(a*x)
```

Maxima [A] time = 1.65361, size = 122, normalized size = 1.39

$$\frac{1}{3}\left(\frac{3\,a^3\arctan\left(ax\right)}{c}+\frac{3\,a^2x^2-1}{cx^3}\right)\arctan\left(ax\right)-\frac{\left(3\,a^2x^2\arctan\left(ax\right)^2-4\,a^2x^2\log\left(a^2x^2+1\right)+8\,a^2x^2\log\left(x\right)+1\right)a}{6\,cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x^4/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/3*(3*a^3*arctan(a*x)/c + (3*a^2*x^2 - 1)/(c*x^3))*arctan(a*x) - 1/6*(3*a^
2*x^2*arctan(a*x)^2 - 4*a^2*x^2*log(a^2*x^2 + 1) + 8*a^2*x^2*log(x) + 1)*a/
(c*x^2)
```

Fricas [A] time = 1.63022, size = 169, normalized size = 1.92

$$\frac{3\,a^3x^3\arctan\left(ax\right)^2+4\,a^3x^3\log\left(a^2x^2+1\right)-8\,a^3x^3\log\left(x\right)-ax+2\left(3\,a^2x^2-1\right)\arctan\left(ax\right)}{6\,cx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x^4/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/6*(3*a^3*x^3*arctan(a*x)^2 + 4*a^3*x^3*log(a^2*x^2 + 1) - 8*a^3*x^3*log(x
) - a*x + 2*(3*a^2*x^2 - 1)*arctan(a*x))/(c*x^3)
```

Sympy [A] time = 3.48911, size = 117, normalized size = 1.33

$$\begin{cases} -\frac{4a^3\log\left(x\right)}{3c}+\frac{2a^3\log\left(x^2+\frac{1}{a^2}\right)}{3c}+\frac{a^3\operatorname{atan}^2\left(ax\right)}{2c}+\frac{a^2\operatorname{atan}\left(ax\right)}{cx}-\frac{a}{6cx^2}-\frac{\operatorname{atan}\left(ax\right)}{3cx^3} & \text{for}\ c\neq 0\\ \tilde{\infty}\left(-\frac{a^3\log\left(x\right)}{3}+\frac{a^3\log\left(a^2x^2+1\right)}{6}-\frac{a}{6x^2}-\frac{\operatorname{atan}\left(ax\right)}{3x^3}\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)/x**4/(a**2*c*x**2+c),x)
```

```
[Out] Piecewise((-4*a**3*log(x)/(3*c) + 2*a**3*log(x**2 + a**(-2))/(3*c) + a**3*a
tan(a*x)**2/(2*c) + a**2*atan(a*x)/(c*x) - a/(6*c*x**2) - atan(a*x)/(3*c*x*
*3), Ne(c, 0)), (zoo*(-a**3*log(x)/3 + a**3*log(a**2*x**2 + 1)/6 - a/(6*x**
2) - atan(a*x)/(3*x**3)), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(ax)}{(a^2cx^2+c)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x^4/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)/((a^2*c*x^2 + c)*x^4), x)
```

3.182 $\int \frac{x^5 \tan^{-1}(ax)}{(c+a^2cx^2)^2} dx$

Optimal. Leaf size=157

$$\frac{i\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{a^6c^2}+\frac{x}{4a^5c^2(a^2x^2+1)}+\frac{x^2\tan^{-1}(ax)}{2a^4c^2}-\frac{\tan^{-1}(ax)}{2a^6c^2(a^2x^2+1)}-\frac{x}{2a^5c^2}+\frac{i\tan^{-1}(ax)^2}{a^6c^2}+\frac{3\tan^{-1}(ax)}{4a^6c^2}+$$

```
[Out] -x/(2*a^5*c^2) + x/(4*a^5*c^2*(1 + a^2*x^2)) + (3*ArcTan[a*x])/(4*a^6*c^2)
+ (x^2*ArcTan[a*x])/(2*a^4*c^2) - ArcTan[a*x]/(2*a^6*c^2*(1 + a^2*x^2)) + (
I*ArcTan[a*x]^2)/(a^6*c^2) + (2*ArcTan[a*x]*Log[2/(1 + I*a*x)])/(a^6*c^2) +
 (I*PolyLog[2, 1 - 2/(1 + I*a*x)])/(a^6*c^2)
```

Rubi [A] time = 0.36319, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 12, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {4964, 4916, 4852, 321, 203, 4920, 4854, 2402, 2315, 4930, 199, 205}

$$\frac{i\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{a^6c^2}+\frac{x}{4a^5c^2(a^2x^2+1)}+\frac{x^2\tan^{-1}(ax)}{2a^4c^2}-\frac{\tan^{-1}(ax)}{2a^6c^2(a^2x^2+1)}-\frac{x}{2a^5c^2}+\frac{i\tan^{-1}(ax)^2}{a^6c^2}+\frac{3\tan^{-1}(ax)}{4a^6c^2}+$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*ArcTan[a*x])/(c + a^2*c*x^2)^2,x]
```


```
[Out] -x/(2*a^5*c^2) + x/(4*a^5*c^2*(1 + a^2*x^2)) + (3*ArcTan[a*x])/(4*a^6*c^2)
+ (x^2*ArcTan[a*x])/(2*a^4*c^2) - ArcTan[a*x]/(2*a^6*c^2*(1 + a^2*x^2)) + (
I*ArcTan[a*x]^2)/(a^6*c^2) + (2*ArcTan[a*x]*Log[2/(1 + I*a*x)])/(a^6*c^2) +
 (I*PolyLog[2, 1 - 2/(1 + I*a*x)])/(a^6*c^2)
```


Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Arc
Tan[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan[c
*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p
, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
```

```
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^5\tan^{-1}(ax)}{(c+a^2cx^2)^2}\,dx = -\frac{\int \frac{x^3\tan^{-1}(ax)}{(c+a^2cx^2)^2}\,dx}{a^2} + \frac{\int \frac{x^3\tan^{-1}(ax)}{c+a^2cx^2}\,dx}{a^2c}$$

$$= \frac{\int \frac{x\tan^{-1}(ax)}{(c+a^2cx^2)^2}\,dx}{a^4} + \frac{\int x\tan^{-1}(ax)\,dx}{a^4c^2} - 2\frac{\int \frac{x\tan^{-1}(ax)}{c+a^2cx^2}\,dx}{a^4c}$$

$$= \frac{x^2\tan^{-1}(ax)}{2a^4c^2} - \frac{\tan^{-1}(ax)}{2a^6c^2(1+a^2x^2)} + \frac{\int \frac{1}{(c+a^2cx^2)^2}\,dx}{2a^5} - 2\left(-\frac{i\tan^{-1}(ax)^2}{2a^6c^2} - \frac{\int \frac{\tan^{-1}(ax)}{i-ax}\,dx}{a^5c^2}\right) - \frac{\int \frac{x^2}{1+a^2x^2}}{2a^3c^2}$$

$$= -\frac{x}{2a^5c^2} + \frac{x}{4a^5c^2(1+a^2x^2)} + \frac{x^2\tan^{-1}(ax)}{2a^4c^2} - \frac{\tan^{-1}(ax)}{2a^6c^2(1+a^2x^2)} + \frac{\int \frac{1}{1+a^2x^2}\,dx}{2a^5c^2} - 2\left(-\frac{i\tan^{-1}(ax)^2}{2a^6c^2}\right.$$

$$= -\frac{x}{2a^5c^2} + \frac{x}{4a^5c^2(1+a^2x^2)} + \frac{3\tan^{-1}(ax)}{4a^6c^2} + \frac{x^2\tan^{-1}(ax)}{2a^4c^2} - \frac{\tan^{-1}(ax)}{2a^6c^2(1+a^2x^2)} - 2\left(-\frac{i\tan^{-1}(ax)^2}{2a^6c^2}\right.$$

$$= -\frac{x}{2a^5c^2} + \frac{x}{4a^5c^2(1+a^2x^2)} + \frac{3\tan^{-1}(ax)}{4a^6c^2} + \frac{x^2\tan^{-1}(ax)}{2a^4c^2} - \frac{\tan^{-1}(ax)}{2a^6c^2(1+a^2x^2)} - 2\left(-\frac{i\tan^{-1}(ax)^2}{2a^6c^2}\right.$$

Mathematica [A] time = 0.231975, size = 90, normalized size = 0.57

$$\frac{-8i\text{PolyLog}\left(2,-e^{2i\tan^{-1}(ax)}\right) + 2\tan^{-1}(ax)\left(2a^2x^2 + 8\log\left(1+e^{2i\tan^{-1}(ax)}\right) - \cos\left(2\tan^{-1}(ax)\right) + 2\right) - 4ax - 8i\tan^{-1}(}{8a^6c^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^5*ArcTan[a*x])/(c + a^2*c*x^2)^2,x]
```

```
[Out] (-4*a*x - (8*I)*ArcTan[a*x]^2 + 2*ArcTan[a*x]*(2 + 2*a^2*x^2 - Cos[2*ArcTan
[a*x]] + 8*Log[1 + E^((2*I)*ArcTan[a*x])]) - (8*I)*PolyLog[2, -E^((2*I)*Arc
Tan[a*x])] + Sin[2*ArcTan[a*x]])/(8*a^6*c^2)
```

Maple [A] time = 0.095, size = 281, normalized size = 1.8

$$\frac{x^2\arctan(ax)}{2a^4c^2} - \frac{\arctan(ax)\ln\left(a^2x^2+1\right)}{a^6c^2} - \frac{\arctan(ax)}{2a^6c^2\left(a^2x^2+1\right)} - \frac{x}{2a^5c^2} + \frac{x}{4a^5c^2\left(a^2x^2+1\right)} + \frac{3\arctan(ax)}{4a^6c^2} + \frac{\frac{i}{4}(\ln(a}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*arctan(a*x)/(a^2*c*x^2+c)^2,x)
```

```
[Out] 1/2*x^2*arctan(a*x)/a^4/c^2-1/a^6/c^2*arctan(a*x)*ln(a^2*x^2+1)-1/2*arctan(
a*x)/a^6/c^2/(a^2*x^2+1)-1/2*x/a^5/c^2+1/4*x/a^5/c^2/(a^2*x^2+1)+3/4*arctan
(a*x)/a^6/c^2+1/4*I/a^6/c^2*ln(a*x-I)^2+1/2*I/a^6/c^2*ln(a*x-I)*ln(-1/2*I*(
a*x+I))-1/2*I/a^6/c^2*ln(a*x-I)*ln(a^2*x^2+1)+1/2*I/a^6/c^2*dilog(-1/2*I*(a
*x+I))-1/4*I/a^6/c^2*ln(a*x+I)^2-1/2*I/a^6/c^2*ln(a*x+I)*ln(1/2*I*(a*x-I))+
1/2*I/a^6/c^2*ln(a*x+I)*ln(a^2*x^2+1)-1/2*I/a^6/c^2*dilog(1/2*I*(a*x-I))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5 \arctan(ax)}{\left(a^2cx^2 + c\right)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arctan(a*x)/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^5*arctan(a*x)/(a^2*c*x^2 + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^5 \arctan(ax)}{a^4c^2x^4 + 2\,a^2c^2x^2 + c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arctan(a*x)/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] integral(x^5*arctan(a*x)/(a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^5 \operatorname{atan}(ax)}{a^4x^4+2a^2x^2+1} \, dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*atan(a*x)/(a**2*c*x**2+c)**2,x)
```

```
[Out] Integral(x**5*atan(a*x)/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5 \arctan(ax)}{(a^2cx^2 + c)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arctan(a*x)/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(x^5*arctan(a*x)/(a^2*c*x^2 + c)^2, x)
```

3.183 $\int \frac{x^4 \tan^{-1}(ax)}{\left(c+a^2cx^2\right)^2} \, dx$

Optimal. Leaf size=96

$$\frac{1}{4a^5c^2\left(a^2x^2+1\right)} - \frac{\log\left(a^2x^2+1\right)}{2a^5c^2} + \frac{x\tan^{-1}(ax)}{2a^4c^2\left(a^2x^2+1\right)} - \frac{3\tan^{-1}(ax)^2}{4a^5c^2} + \frac{x\tan^{-1}(ax)}{a^4c^2}$$

```
[Out] 1/(4*a^5*c^2*(1 + a^2*x^2)) + (x*ArcTan[a*x])/(a^4*c^2) + (x*ArcTan[a*x])/(
2*a^4*c^2*(1 + a^2*x^2)) - (3*ArcTan[a*x]^2)/(4*a^5*c^2) - Log[1 + a^2*x^2]
/(2*a^5*c^2)
```

Rubi [A] time = 0.180487, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {4964, 4916, 4846, 260, 4884, 4934}

$$\frac{1}{4a^5c^2\left(a^2x^2+1\right)} - \frac{\log\left(a^2x^2+1\right)}{2a^5c^2} + \frac{x\tan^{-1}(ax)}{2a^4c^2\left(a^2x^2+1\right)} - \frac{3\tan^{-1}(ax)^2}{4a^5c^2} + \frac{x\tan^{-1}(ax)}{a^4c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*ArcTan[a*x])/(c + a^2*c*x^2)^2,x]
```

```
[Out] 1/(4*a^5*c^2*(1 + a^2*x^2)) + (x*ArcTan[a*x])/(a^4*c^2) + (x*ArcTan[a*x])/(
2*a^4*c^2*(1 + a^2*x^2)) - (3*ArcTan[a*x]^2)/(4*a^5*c^2) - Log[1 + a^2*x^2]
/(2*a^5*c^2)
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Arc
Tan[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan[c
*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p
, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
```

```
e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4934

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^2*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> -Simp[(b*(d + e*x^2)^(q + 1))/(4*c^3*d*(q + 1)^2), x] + (-Dist[
1/(2*c^2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]), x], x] +
Simp[(x*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]))/(2*c^2*d*(q + 1)), x]) /;
FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -5/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^2}\,dx &= -\frac{\int \frac{x^2\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^2}\,dx}{a^2}+\frac{\int \frac{x^2\tan^{-1}(ax)}{c+a^2cx^2}\,dx}{a^2c}\\
&= \frac{1}{4a^5c^2\left(1+a^2x^2\right)}+\frac{x\tan^{-1}(ax)}{2a^4c^2\left(1+a^2x^2\right)}+\frac{\int \tan^{-1}(ax)\,dx}{a^4c^2}-\frac{\int \frac{\tan^{-1}(ax)}{c+a^2cx^2}\,dx}{2a^4c}-\frac{\int \frac{\tan^{-1}(ax)}{c+a^2cx^2}\,dx}{a^4c}\\
&= \frac{1}{4a^5c^2\left(1+a^2x^2\right)}+\frac{x\tan^{-1}(ax)}{a^4c^2}+\frac{x\tan^{-1}(ax)}{2a^4c^2\left(1+a^2x^2\right)}-\frac{3\tan^{-1}(ax)^2}{4a^5c^2}-\frac{\int \frac{x}{1+a^2x^2}\,dx}{a^3c^2}\\
&= \frac{1}{4a^5c^2\left(1+a^2x^2\right)}+\frac{x\tan^{-1}(ax)}{a^4c^2}+\frac{x\tan^{-1}(ax)}{2a^4c^2\left(1+a^2x^2\right)}-\frac{3\tan^{-1}(ax)^2}{4a^5c^2}-\frac{\log\left(1+a^2x^2\right)}{2a^5c^2}
\end{aligned}$$

Mathematica [A] time = 0.0598482, size = 79, normalized size = 0.82

$$\frac{-2\left(a^2x^2+1\right)\log\left(a^2x^2+1\right)-3\left(a^2x^2+1\right)\tan^{-1}(ax)^2+\left(4a^3x^3+6ax\right)\tan^{-1}(ax)+1}{4a^5c^2\left(a^2x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*ArcTan[a*x])/(c + a^2*c*x^2)^2,x]
```

```
[Out] (1 + (6*a*x + 4*a^3*x^3)*ArcTan[a*x] - 3*(1 + a^2*x^2)*ArcTan[a*x]^2 - 2*(1
 + a^2*x^2)*Log[1 + a^2*x^2])/(4*a^5*c^2*(1 + a^2*x^2))
```

Maple [A] time = 0.044, size = 89, normalized size = 0.9

$$\frac{1}{4\,a^5c^2\left(a^2x^2+1\right)}+\frac{x\arctan(ax)}{a^4c^2}+\frac{x\arctan(ax)}{2\,a^4c^2\left(a^2x^2+1\right)}-\frac{3\,(\arctan(ax))^2}{4\,a^5c^2}-\frac{\ln\left(a^2x^2+1\right)}{2\,a^5c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arctan(a*x)/(a^2*c*x^2+c)^2,x)
```

```
[Out] 1/4/a^5/c^2/(a^2*x^2+1)+x*arctan(a*x)/a^4/c^2+1/2*x*arctan(a*x)/a^4/c^2/(a^
2*x^2+1)-3/4*arctan(a*x)^2/a^5/c^2-1/2*ln(a^2*x^2+1)/a^5/c^2
```

Maxima [A] time = 1.57983, size = 154, normalized size = 1.6

$$\frac{1}{2}\left(\frac{x}{a^6c^2x^2+a^4c^2}+\frac{2\,x}{a^4c^2}-\frac{3\,\arctan(ax)}{a^5c^2}\right)\arctan(ax)+\frac{\left(3\left(a^2x^2+1\right)\arctan(ax)^2-2\left(a^2x^2+1\right)\log\left(a^2x^2+1\right)+1\right)a}{4\left(a^8c^2x^2+a^6c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctan(a*x)/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(x/(a^6*c^2*x^2 + a^4*c^2) + 2*x/(a^4*c^2) - 3*arctan(a*x)/(a^5*c^2))*a
rctan(a*x) + 1/4*(3*(a^2*x^2 + 1)*arctan(a*x)^2 - 2*(a^2*x^2 + 1)*log(a^2*x
^2 + 1) + 1)*a/(a^8*c^2*x^2 + a^6*c^2)
```

Fricas [A] time = 1.63435, size = 185, normalized size = 1.93

$$-\frac{3\left(a^2x^2+1\right)\arctan\left(ax\right)^2-2\left(2\,a^3x^3+3\,ax\right)\arctan\left(ax\right)+2\left(a^2x^2+1\right)\log\left(a^2x^2+1\right)-1}{4\left(a^7c^2x^2+a^5c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctan(a*x)/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(3*(a^2*x^2 + 1)*arctan(a*x)^2 - 2*(2*a^3*x^3 + 3*a*x)*arctan(a*x) + 2
*(a^2*x^2 + 1)*log(a^2*x^2 + 1) - 1)/(a^7*c^2*x^2 + a^5*c^2)
```

Sympy [A] time = 3.1785, size = 291, normalized size = 3.03

$$\begin{cases} \frac{12a^3x^3\operatorname{atan}(ax)}{12a^7c^2x^2+12a^5c^2}-\frac{6a^2x^2\log\left(x^2+\frac{1}{a^2}\right)}{12a^7c^2x^2+12a^5c^2}-\frac{9a^2x^2\operatorname{atan}^2(ax)}{12a^7c^2x^2+12a^5c^2}-\frac{a^2x^2}{12a^7c^2x^2+12a^5c^2}+\frac{18ax\operatorname{atan}(ax)}{12a^7c^2x^2+12a^5c^2}-\frac{6\log\left(x^2+\frac{1}{a^2}\right)}{12a^7c^2x^2+12a^5c^2}-\frac{9\operatorname{atan}^2(ax)}{12a^7c^2x^2+12a^5c^2}+\frac{}{12a} \\ \tilde{\infty}\left(\frac{x^5\operatorname{atan}(ax)}{5}-\frac{x^4}{20a}+\frac{x^2}{10a^3}-\frac{\log\left(a^2x^2+1\right)}{10a^5}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*atan(a*x)/(a**2*c*x**2+c)**2,x)
```

```
[Out] Piecewise((12*a**3*x**3*atan(a*x)/(12*a**7*c**2*x**2 + 12*a**5*c**2) - 6*a*
*2*x**2*log(x**2 + a**(-2))/(12*a**7*c**2*x**2 + 12*a**5*c**2) - 9*a**2*x**
2*atan(a*x)**2/(12*a**7*c**2*x**2 + 12*a**5*c**2) - a**2*x**2/(12*a**7*c**2
*x**2 + 12*a**5*c**2) + 18*a*x*atan(a*x)/(12*a**7*c**2*x**2 + 12*a**5*c**2)
 - 6*log(x**2 + a**(-2))/(12*a**7*c**2*x**2 + 12*a**5*c**2) - 9*atan(a*x)**
2/(12*a**7*c**2*x**2 + 12*a**5*c**2) + 2/(12*a**7*c**2*x**2 + 12*a**5*c**2)
, Ne(c, 0)), (zoo*(x**5*atan(a*x)/5 - x**4/(20*a) + x**2/(10*a**3) - log(a*
*2*x**2 + 1)/(10*a**5)), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^4\arctan\left(ax\right)}{\left(a^2cx^2+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctan(a*x)/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(x^4*arctan(a*x)/(a^2*c*x^2 + c)^2, x)
```

3.184 $\int \frac{x^3 \tan^{-1}(ax)}{(c+a^2cx^2)^2} dx$

Optimal. Leaf size=133

$$-\frac{i\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{2a^4c^2}-\frac{x}{4a^3c^2(a^2x^2+1)}+\frac{\tan^{-1}(ax)}{2a^4c^2(a^2x^2+1)}-\frac{i\tan^{-1}(ax)^2}{2a^4c^2}-\frac{\tan^{-1}(ax)}{4a^4c^2}-\frac{\log\left(\frac{2}{1+iax}\right)\tan^{-1}(ax)}{a^4c^2}$$

```
[Out] -x/(4*a^3*c^2*(1 + a^2*x^2)) - ArcTan[a*x]/(4*a^4*c^2) + ArcTan[a*x]/(2*a^4
*c^2*(1 + a^2*x^2)) - ((I/2)*ArcTan[a*x]^2)/(a^4*c^2) - (ArcTan[a*x]*Log[2/
(1 + I*a*x)])/(a^4*c^2) - ((I/2)*PolyLog[2, 1 - 2/(1 + I*a*x)])/(a^4*c^2)
```

Rubi [A] time = 0.162915, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {4964, 4920, 4854, 2402, 2315, 4930, 199, 205}

$$-\frac{i\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{2a^4c^2}-\frac{x}{4a^3c^2(a^2x^2+1)}+\frac{\tan^{-1}(ax)}{2a^4c^2(a^2x^2+1)}-\frac{i\tan^{-1}(ax)^2}{2a^4c^2}-\frac{\tan^{-1}(ax)}{4a^4c^2}-\frac{\log\left(\frac{2}{1+iax}\right)\tan^{-1}(ax)}{a^4c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcTan[a*x])/(c + a^2*c*x^2)^2,x]
```


```
[Out] -x/(4*a^3*c^2*(1 + a^2*x^2)) - ArcTan[a*x]/(4*a^4*c^2) + ArcTan[a*x]/(2*a^4
*c^2*(1 + a^2*x^2)) - ((I/2)*ArcTan[a*x]^2)/(a^4*c^2) - (ArcTan[a*x]*Log[2/
(1 + I*a*x)])/(a^4*c^2) - ((I/2)*PolyLog[2, 1 - 2/(1 + I*a*x)])/(a^4*c^2)
```


Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Arc
Tan[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan[c
*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p
, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
```

```
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^3 \tan^{-1}(ax)}{\left(c+a^2cx^2\right)^2}\,dx = -\frac{\int \frac{x\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^2}\,dx}{a^2} + \frac{\int \frac{x\tan^{-1}(ax)}{c+a^2cx^2}\,dx}{a^2c}$$

$$= \frac{\tan^{-1}(ax)}{2a^4c^2\left(1+a^2x^2\right)} - \frac{i\tan^{-1}(ax)^2}{2a^4c^2} - \frac{\int \frac{1}{\left(c+a^2cx^2\right)^2}\,dx}{2a^3} - \frac{\int \frac{\tan^{-1}(ax)}{i-ax}\,dx}{a^3c^2}$$

$$= -\frac{x}{4a^3c^2\left(1+a^2x^2\right)} + \frac{\tan^{-1}(ax)}{2a^4c^2\left(1+a^2x^2\right)} - \frac{i\tan^{-1}(ax)^2}{2a^4c^2} - \frac{\tan^{-1}(ax)\log\left(\frac{2}{1+iax}\right)}{a^4c^2} + \frac{\int \frac{\log\left(\frac{2}{1+iax}\right)}{1+a^2x^2}\,dx}{a^3c^2} -$$

$$= -\frac{x}{4a^3c^2\left(1+a^2x^2\right)} - \frac{\tan^{-1}(ax)}{4a^4c^2} + \frac{\tan^{-1}(ax)}{2a^4c^2\left(1+a^2x^2\right)} - \frac{i\tan^{-1}(ax)^2}{2a^4c^2} - \frac{\tan^{-1}(ax)\log\left(\frac{2}{1+iax}\right)}{a^4c^2} - \frac{i\,\mathrm{Su}\ldots}{}$$

$$= -\frac{x}{4a^3c^2\left(1+a^2x^2\right)} - \frac{\tan^{-1}(ax)}{4a^4c^2} + \frac{\tan^{-1}(ax)}{2a^4c^2\left(1+a^2x^2\right)} - \frac{i\tan^{-1}(ax)^2}{2a^4c^2} - \frac{\tan^{-1}(ax)\log\left(\frac{2}{1+iax}\right)}{a^4c^2} - \frac{i\mathrm{Li}_2\ldots}{}$$

Mathematica [A] time = 0.121025, size = 77, normalized size = 0.58

$$\frac{4i\mathrm{PolyLog}\left(2,-e^{2i\tan^{-1}(ax)}\right) + 4i\tan^{-1}(ax)^2 - \sin\left(2\tan^{-1}(ax)\right) + 2\tan^{-1}(ax)\left(\cos\left(2\tan^{-1}(ax)\right) - 4\log\left(1+e^{2i\tan^{-1}(a\ldots}\right.\right.}{8a^4c^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*ArcTan[a*x])/(c + a^2*c*x^2)^2,x]
```

```
[Out] ((4*I)*ArcTan[a*x]^2 + 2*ArcTan[a*x]*(Cos[2*ArcTan[a*x]] - 4*Log[1 + E^((2*
I)*ArcTan[a*x])]) + (4*I)*PolyLog[2, -E^((2*I)*ArcTan[a*x])] - Sin[2*ArcTan
[a*x]])/(8*a^4*c^2)
```

Maple [B] time = 0.095, size = 257, normalized size = 1.9

$$\frac{\arctan(ax)\ln\left(a^2x^2+1\right)}{2\,a^4c^2} + \frac{\arctan(ax)}{2\,a^4c^2\left(a^2x^2+1\right)} - \frac{x}{4\,a^3c^2\left(a^2x^2+1\right)} - \frac{\arctan(ax)}{4\,a^4c^2} + \frac{\frac{i}{4}\ln\left(a^2x^2+1\right)\ln(ax-i)}{a^4c^2} - \frac{\frac{i}{8}(\ln(a\ldots}{a^4\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctan(a*x)/(a^2*c*x^2+c)^2,x)
```

```
[Out] 1/2/a^4/c^2*arctan(a*x)*ln(a^2*x^2+1)+1/2*arctan(a*x)/a^4/c^2/(a^2*x^2+1)-1
/4*x/a^3/c^2/(a^2*x^2+1)-1/4*arctan(a*x)/a^4/c^2+1/4*I/a^4/c^2*ln(a^2*x^2+1
)*ln(a*x-I)-1/8*I/a^4/c^2*ln(a*x-I)^2-1/4*I/a^4/c^2*ln(a*x-I)*ln(-1/2*I*(a*
x+I))-1/4*I/a^4/c^2*dilog(-1/2*I*(a*x+I))-1/4*I/a^4/c^2*ln(a^2*x^2+1)*ln(a*
x+I)+1/8*I/a^4/c^2*ln(a*x+I)^2+1/4*I/a^4/c^2*ln(a*x+I)*ln(1/2*I*(a*x-I))+1/
4*I/a^4/c^2*dilog(1/2*I*(a*x-I))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \arctan(ax)}{(a^2cx^2 + c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^3*arctan(a*x)/(a^2*c*x^2 + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x^3 \arctan(ax)}{a^4c^2x^4 + 2\,a^2c^2x^2 + c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] integral(x^3*arctan(a*x)/(a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^3 \operatorname{atan}(ax)}{a^4x^4+2a^2x^2+1}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atan(a*x)/(a**2*c*x**2+c)**2,x)
```

```
[Out] Integral(x**3*atan(a*x)/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \arctan (ax)}{\left(a^2cx^2 + c\right)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(x^3*arctan(a*x)/(a^2*c*x^2 + c)^2, x)
```

3.185 $\int \frac{x^2 \tan^{-1}(ax)}{\left(c+a^2cx^2\right)^2} dx$

Optimal. Leaf size=64

$$-\frac{1}{4a^3c^2\left(a^2x^2+1\right)}-\frac{x\tan^{-1}(ax)}{2a^2c^2\left(a^2x^2+1\right)}+\frac{\tan^{-1}(ax)^2}{4a^3c^2}$$

```
[Out] -1/(4*a^3*c^2*(1 + a^2*x^2)) - (x*ArcTan[a*x])/(2*a^2*c^2*(1 + a^2*x^2)) +
ArcTan[a*x]^2/(4*a^3*c^2)
```

Rubi [A] time = 0.0643417, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {4934, 4884}

$$-\frac{1}{4a^3c^2\left(a^2x^2+1\right)}-\frac{x\tan^{-1}(ax)}{2a^2c^2\left(a^2x^2+1\right)}+\frac{\tan^{-1}(ax)^2}{4a^3c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcTan[a*x])/(c + a^2*c*x^2)^2,x]
```

```
[Out] -1/(4*a^3*c^2*(1 + a^2*x^2)) - (x*ArcTan[a*x])/(2*a^2*c^2*(1 + a^2*x^2)) +
ArcTan[a*x]^2/(4*a^3*c^2)
```

Rule 4934

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^2*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> -Simp[(b*(d + e*x^2)^(q + 1))/(4*c^3*d*(q + 1)^2), x] + (-Dist[
1/(2*c^2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]), x], x] +
Simp[(x*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]))/(2*c^2*d*(q + 1)), x]) /;
FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -5/2]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{x^2 \tan^{-1}(ax)}{\left(c+a^2cx^2\right)^2}\,dx = -\frac{1}{4a^3c^2\left(1+a^2x^2\right)} - \frac{x\tan^{-1}(ax)}{2a^2c^2\left(1+a^2x^2\right)} + \frac{\int \frac{\tan^{-1}(ax)}{c+a^2cx^2}\,dx}{2a^2c}$$
$$= -\frac{1}{4a^3c^2\left(1+a^2x^2\right)} - \frac{x\tan^{-1}(ax)}{2a^2c^2\left(1+a^2x^2\right)} + \frac{\tan^{-1}(ax)^2}{4a^3c^2}$$

Mathematica [A] time = 0.0429549, size = 47, normalized size = 0.73

$$\frac{\left(a^2x^2+1\right)\tan^{-1}(ax)^2 - 2ax\tan^{-1}(ax) - 1}{4a^3c^2\left(a^2x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*ArcTan[a*x])/(c + a^2*c*x^2)^2,x]
```

```
[Out] (-1 - 2*a*x*ArcTan[a*x] + (1 + a^2*x^2)*ArcTan[a*x]^2)/(4*a^3*c^2*(1 + a^2*
x^2))
```

Maple [A] time = 0.036, size = 59, normalized size = 0.9

$$-\frac{1}{4\,a^3c^2\left(a^2x^2+1\right)} - \frac{x\arctan(ax)}{2\,a^2c^2\left(a^2x^2+1\right)} + \frac{(\arctan(ax))^2}{4\,a^3c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(a*x)/(a^2*c*x^2+c)^2,x)
```

```
[Out] -1/4/a^3/c^2/(a^2*x^2+1)-1/2*x*arctan(a*x)/a^2/c^2/(a^2*x^2+1)+1/4*arctan(a
*x)^2/a^3/c^2
```

Maxima [A] time = 1.61264, size = 112, normalized size = 1.75

$$-\frac{1}{2}\left(\frac{x}{a^4c^2x^2+a^2c^2} - \frac{\arctan(ax)}{a^3c^2}\right)\arctan(ax) - \frac{\left(\left(a^2x^2+1\right)\arctan(ax)^2+1\right)a}{4\left(a^6c^2x^2+a^4c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(x/(a^4*c^2*x^2 + a^2*c^2) - arctan(a*x)/(a^3*c^2))*arctan(a*x) - 1/4*
((a^2*x^2 + 1)*arctan(a*x)^2 + 1)*a/(a^6*c^2*x^2 + a^4*c^2)
```

Fricas [A] time = 1.61264, size = 113, normalized size = 1.77

$$-\frac{2\,ax\arctan\left(ax\right)-\left(a^{2}x^{2}+1\right)\arctan\left(ax\right)^{2}+1}{4\left(a^{5}c^{2}x^{2}+a^{3}c^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(2*a*x*arctan(a*x) - (a^2*x^2 + 1)*arctan(a*x)^2 + 1)/(a^5*c^2*x^2 + a
^3*c^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{x^{2}\operatorname{atan}\left(ax\right)}{a^{4}x^{4}+2a^{2}x^{2}+1}\,dx}{c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(a*x)/(a**2*c*x**2+c)**2,x)
```

```
[Out] Integral(x**2*atan(a*x)/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^{2}\arctan\left(ax\right)}{\left(a^{2}cx^{2}+c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)/(a^2*c*x^2+c)^2,x, algorithm="giac")

[Out] integrate(x^2*arctan(a*x)/(a^2*c*x^2 + c)^2, x)
```

3.186 $\int \frac{x\tan^{-1}(ax)}{(c+a^2cx^2)^2}\,dx$

Optimal. Leaf size=62

$$\frac{x}{4ac^2(a^2x^2+1)}-\frac{\tan^{-1}(ax)}{2a^2c^2(a^2x^2+1)}+\frac{\tan^{-1}(ax)}{4a^2c^2}$$

```
[Out] x/(4*a*c^2*(1 + a^2*x^2)) + ArcTan[a*x]/(4*a^2*c^2) - ArcTan[a*x]/(2*a^2*c^
2*(1 + a^2*x^2))
```

Rubi [A] time = 0.041147, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4930, 199, 205}

$$\frac{x}{4ac^2(a^2x^2+1)}-\frac{\tan^{-1}(ax)}{2a^2c^2(a^2x^2+1)}+\frac{\tan^{-1}(ax)}{4a^2c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTan[a*x])/(c + a^2*c*x^2)^2,x]
```

```
[Out] x/(4*a*c^2*(1 + a^2*x^2)) + ArcTan[a*x]/(4*a^2*c^2) - ArcTan[a*x]/(2*a^2*c^
2*(1 + a^2*x^2))
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{x\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^2}\,dx &= -\frac{\tan^{-1}(ax)}{2a^2c^2\left(1+a^2x^2\right)}+\frac{\int \frac{1}{\left(c+a^2cx^2\right)^2}\,dx}{2a}\\ &= \frac{x}{4ac^2\left(1+a^2x^2\right)}-\frac{\tan^{-1}(ax)}{2a^2c^2\left(1+a^2x^2\right)}+\frac{\int \frac{1}{c+a^2cx^2}\,dx}{4ac}\\ &= \frac{x}{4ac^2\left(1+a^2x^2\right)}+\frac{\tan^{-1}(ax)}{4a^2c^2}-\frac{\tan^{-1}(ax)}{2a^2c^2\left(1+a^2x^2\right)}\end{aligned}$$

Mathematica [A] time = 0.0283901, size = 39, normalized size = 0.63

$$\frac{\left(a^2x^2-1\right)\tan^{-1}(ax)+ax}{4a^2c^2\left(a^2x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcTan[a*x])/(c + a^2*c*x^2)^2,x]
```

```
[Out] (a*x + (-1 + a^2*x^2)*ArcTan[a*x])/(4*a^2*c^2*(1 + a^2*x^2))
```

Maple [A] time = 0.026, size = 57, normalized size = 0.9

$$\frac{x}{4\,ac^2\left(a^2x^2+1\right)}+\frac{\arctan(ax)}{4\,a^2c^2}-\frac{\arctan(ax)}{2\,a^2c^2\left(a^2x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(a*x)/(a^2*c*x^2+c)^2,x)
```

```
[Out] 1/4*x/a/c^2/(a^2*x^2+1)+1/4*arctan(a*x)/a^2/c^2-1/2*arctan(a*x)/a^2/c^2/(a^
2*x^2+1)
```

Maxima [A] time = 1.59456, size = 80, normalized size = 1.29

$$\frac{\frac{x}{a^2cx^2+c} + \frac{\arctan(ax)}{ac}}{4\,ac} - \frac{\arctan\left(ax\right)}{2\left(a^2cx^2+c\right)a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(x/(a^2*c*x^2 + c) + arctan(a*x)/(a*c))/(a*c) - 1/2*arctan(a*x)/((a^2*c
*x^2 + c)*a^2*c)
```

Fricas [A] time = 1.65179, size = 85, normalized size = 1.37

$$\frac{ax + \left(a^2x^2-1\right)\arctan\left(ax\right)}{4\left(a^4c^2x^2+a^2c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(a*x + (a^2*x^2 - 1)*arctan(a*x))/(a^4*c^2*x^2 + a^2*c^2)
```

Sympy [A] time = 2.04451, size = 107, normalized size = 1.73

$$\begin{cases} \frac{a^2x^2\operatorname{atan}\left(ax\right)}{4a^4c^2x^2+4a^2c^2} + \frac{ax}{4a^4c^2x^2+4a^2c^2} - \frac{\operatorname{atan}\left(ax\right)}{4a^4c^2x^2+4a^2c^2} & \text{for}\ c \neq 0 \\ \tilde{\infty}\left(\frac{x^2\operatorname{atan}\left(ax\right)}{2} - \frac{x}{2a} + \frac{\operatorname{atan}\left(ax\right)}{2a^2}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(a*x)/(a**2*c*x**2+c)**2,x)
```

```
[Out] Piecewise(((a**2*x**2*atan(a*x)/(4*a**4*c**2*x**2 + 4*a**2*c**2) + a*x/(4*a*
*4*c**2*x**2 + 4*a**2*c**2) - atan(a*x)/(4*a**4*c**2*x**2 + 4*a**2*c**2), N
e(c, 0)), (zoo*(x**2*atan(a*x)/2 - x/(2*a) + atan(a*x)/(2*a**2)), True))
```

Giac [A] time = 1.16595, size = 77, normalized size = 1.24

$$\frac{x}{4\left(a^{2}x^{2}+1\right)ac^{2}}+\frac{\arctan\left(ax\right)}{4\,a^{2}c^{2}}-\frac{\arctan\left(ax\right)}{2\left(a^{2}cx^{2}+c\right)a^{2}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/4*x/((a^2*x^2 + 1)*a*c^2) + 1/4*arctan(a*x)/(a^2*c^2) - 1/2*arctan(a*x)/(
(a^2*c*x^2 + c)*a^2*c)
```

3.187 $\int \frac{\tan^{-1}(ax)}{(c+a^2cx^2)^2}\,dx$

Optimal. Leaf size=61

$$\frac{1}{4ac^2(a^2x^2+1)}+\frac{x\tan^{-1}(ax)}{2c^2(a^2x^2+1)}+\frac{\tan^{-1}(ax)^2}{4ac^2}$$

```
[Out] 1/(4*a*c^2*(1 + a^2*x^2)) + (x*ArcTan[a*x])/(2*c^2*(1 + a^2*x^2)) + ArcTan[
a*x]^2/(4*a*c^2)
```

Rubi [A] time = 0.0258874, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {4892, 261}

$$\frac{1}{4ac^2(a^2x^2+1)}+\frac{x\tan^{-1}(ax)}{2c^2(a^2x^2+1)}+\frac{\tan^{-1}(ax)^2}{4ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]/(c + a^2*c*x^2)^2,x]
```

```
[Out] 1/(4*a*c^2*(1 + a^2*x^2)) + (x*ArcTan[a*x])/(2*c^2*(1 + a^2*x^2)) + ArcTan[
a*x]^2/(4*a*c^2)
```

Rule 4892

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[(x*(a + b*ArcTan[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*c*
p)/2, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(a +
 b*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^2}\,dx = \frac{x\tan^{-1}(ax)}{2c^2\left(1+a^2x^2\right)} + \frac{\tan^{-1}(ax)^2}{4ac^2} - \frac{1}{2}a\int \frac{x}{\left(c+a^2cx^2\right)^2}\,dx$$
$$= \frac{1}{4ac^2\left(1+a^2x^2\right)} + \frac{x\tan^{-1}(ax)}{2c^2\left(1+a^2x^2\right)} + \frac{\tan^{-1}(ax)^2}{4ac^2}$$

Mathematica [A] time = 0.0215124, size = 44, normalized size = 0.72

$$\frac{\left(a^2x^2+1\right)\tan^{-1}(ax)^2+2ax\tan^{-1}(ax)+1}{4c^2\left(a^3x^2+a\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]/(c + a^2*c*x^2)^2,x]
```

```
[Out] (1 + 2*a*x*ArcTan[a*x] + (1 + a^2*x^2)*ArcTan[a*x]^2)/(4*c^2*(a + a^3*x^2))
```

Maple [A] time = 0.034, size = 56, normalized size = 0.9

$$\frac{1}{4\,ac^2\left(a^2x^2+1\right)} + \frac{x\arctan(ax)}{2\,c^2\left(a^2x^2+1\right)} + \frac{(\arctan(ax))^2}{4\,ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)/(a^2*c*x^2+c)^2,x)
```

```
[Out] 1/4/a/c^2/(a^2*x^2+1)+1/2*x*arctan(a*x)/c^2/(a^2*x^2+1)+1/4*arctan(a*x)^2/a
/c^2
```

Maxima [A] time = 1.66636, size = 105, normalized size = 1.72

$$\frac{1}{2}\left(\frac{x}{a^2c^2x^2+c^2} + \frac{\arctan(ax)}{ac^2}\right)\arctan(ax) - \frac{\left(\left(a^2x^2+1\right)\arctan(ax)^2-1\right)a}{4\left(a^4c^2x^2+a^2c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(x/(a^2*c^2*x^2 + c^2) + arctan(a*x)/(a*c^2))*arctan(a*x) - 1/4*((a^2*x
^2 + 1)*arctan(a*x)^2 - 1)*a/(a^4*c^2*x^2 + a^2*c^2)
```

Fricas [A] time = 1.65389, size = 109, normalized size = 1.79

$$\frac{2\,ax\arctan\left(ax\right)+\left(a^{2}x^{2}+1\right)\arctan\left(ax\right)^{2}+1}{4\left(a^{3}c^{2}x^{2}+ac^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*a*x*arctan(a*x) + (a^2*x^2 + 1)*arctan(a*x)^2 + 1)/(a^3*c^2*x^2 + a*
c^2)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RecursionError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)/(a**2*c*x**2+c)**2,x)
```

```
[Out] Exception raised: RecursionError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\arctan\left(ax\right)}{\left(a^{2}cx^{2}+c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)/(a^2*c*x^2 + c)^2, x)
```

3.188 $\int \frac{\tan^{-1}(ax)}{x(c+a^2cx^2)^2}\,dx$

Optimal. Leaf size=117

$$-\frac{i\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{2c^2}-\frac{ax}{4c^2\left(a^2x^2+1\right)}+\frac{\tan^{-1}(ax)}{2c^2\left(a^2x^2+1\right)}-\frac{i\tan^{-1}(ax)^2}{2c^2}-\frac{\tan^{-1}(ax)}{4c^2}+\frac{\log\left(2-\frac{2}{1-iax}\right)\tan^{-1}(ax)}{c^2}$$

```
[Out] -(a*x)/(4*c^2*(1 + a^2*x^2)) - ArcTan[a*x]/(4*c^2) + ArcTan[a*x]/(2*c^2*(1
+ a^2*x^2)) - ((I/2)*ArcTan[a*x]^2)/c^2 + (ArcTan[a*x]*Log[2 - 2/(1 - I*a*x
)])/c^2 - ((I/2)*PolyLog[2, -1 + 2/(1 - I*a*x)])/c^2
```

Rubi [A] time = 0.18415, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {4966, 4924, 4868, 2447, 4930, 199, 205}

$$-\frac{i\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{2c^2}-\frac{ax}{4c^2\left(a^2x^2+1\right)}+\frac{\tan^{-1}(ax)}{2c^2\left(a^2x^2+1\right)}-\frac{i\tan^{-1}(ax)^2}{2c^2}-\frac{\tan^{-1}(ax)}{4c^2}+\frac{\log\left(2-\frac{2}{1-iax}\right)\tan^{-1}(ax)}{c^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]/(x*(c + a^2*c*x^2)^2),x]
```

```
[Out] -(a*x)/(4*c^2*(1 + a^2*x^2)) - ArcTan[a*x]/(4*c^2) + ArcTan[a*x]/(2*c^2*(1
+ a^2*x^2)) - ((I/2)*ArcTan[a*x]^2)/c^2 + (ArcTan[a*x]*Log[2 - 2/(1 - I*a*x
)])/c^2 - ((I/2)*PolyLog[2, -1 + 2/(1 - I*a*x)])/c^2
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*
x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^p
, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*q]
 && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
```

```
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)}{x\left(c+a^2cx^2\right)^2}\,dx = -\left(a^2\int \frac{x\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^2}\,dx\right) + \frac{\int \frac{\tan^{-1}(ax)}{x(c+a^2cx^2)}\,dx}{c}$$

$$= \frac{\tan^{-1}(ax)}{2c^2\left(1+a^2x^2\right)} - \frac{i\tan^{-1}(ax)^2}{2c^2} - \frac{1}{2}a\int \frac{1}{\left(c+a^2cx^2\right)^2}\,dx + \frac{i\int \frac{\tan^{-1}(ax)}{x(i+ax)}\,dx}{c^2}$$

$$= -\frac{ax}{4c^2\left(1+a^2x^2\right)} + \frac{\tan^{-1}(ax)}{2c^2\left(1+a^2x^2\right)} - \frac{i\tan^{-1}(ax)^2}{2c^2} + \frac{\tan^{-1}(ax)\log\left(2-\frac{2}{1-iax}\right)}{c^2} - \frac{a\int \frac{\log\left(2-\frac{2}{1-iax}\right)}{1+a^2x^2}\,d$$

$$= -\frac{ax}{4c^2\left(1+a^2x^2\right)} - \frac{\tan^{-1}(ax)}{4c^2} + \frac{\tan^{-1}(ax)}{2c^2\left(1+a^2x^2\right)} - \frac{i\tan^{-1}(ax)^2}{2c^2} + \frac{\tan^{-1}(ax)\log\left(2-\frac{2}{1-iax}\right)}{c^2} - \frac{i\mathrm{Li}$$

Mathematica [A] time = 0.157162, size = 72, normalized size = 0.62

$$-\frac{4i\mathrm{PolyLog}\left(2,e^{2i\tan^{-1}(ax)}\right) + 4i\tan^{-1}(ax)^2 + \sin\left(2\tan^{-1}(ax)\right) - 2\tan^{-1}(ax)\left(\cos\left(2\tan^{-1}(ax)\right) + 4\log\left(1-e^{2i\tan^{-1}(a}\right.\right.}{8c^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTan[a*x]/(x*(c + a^2*c*x^2)^2),x]
```

```
[Out] -((4*I)*ArcTan[a*x]^2 - 2*ArcTan[a*x]*(Cos[2*ArcTan[a*x]] + 4*Log[1 - E^((2
*I)*ArcTan[a*x])]) + (4*I)*PolyLog[2, E^((2*I)*ArcTan[a*x])] + Sin[2*ArcTan
[a*x]])/(8*c^2)
```

Maple [B] time = 0.049, size = 298, normalized size = 2.6

$$-\frac{\arctan(ax)\ln\left(a^2x^2+1\right)}{2c^2} + \frac{\arctan(ax)}{2c^2\left(a^2x^2+1\right)} + \frac{\arctan(ax)\ln(ax)}{c^2} - \frac{ax}{4c^2\left(a^2x^2+1\right)} - \frac{\arctan(ax)}{4c^2} + \frac{\frac{i}{4}\ln\left(a^2x^2+1\right)\ln}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)/x/(a^2*c*x^2+c)^2,x)
```

```
[Out] -1/2/c^2*arctan(a*x)*ln(a^2*x^2+1)+1/2*arctan(a*x)/c^2/(a^2*x^2+1)+1/c^2*ar
ctan(a*x)*ln(a*x)-1/4*a*x/c^2/(a^2*x^2+1)-1/4*arctan(a*x)/c^2+1/4*I/c^2*ln(
a^2*x^2+1)*ln(a*x+I)+1/2*I/c^2*ln(a*x)*ln(1+I*a*x)+1/2*I/c^2*dilog(1+I*a*x)
-1/2*I/c^2*dilog(1-I*a*x)-1/2*I/c^2*ln(a*x)*ln(1-I*a*x)+1/8*I/c^2*ln(a*x-I)
^2-1/8*I/c^2*ln(a*x+I)^2-1/4*I/c^2*ln(a*x+I)*ln(1/2*I*(a*x-I))+1/4*I/c^2*di
log(-1/2*I*(a*x+I))-1/4*I/c^2*dilog(1/2*I*(a*x-I))+1/4*I/c^2*ln(a*x-I)*ln(-
1/2*I*(a*x+I))-1/4*I/c^2*ln(a^2*x^2+1)*ln(a*x-I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(ax)}{\left(a^2cx^2+c\right)^2 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(arctan(a*x)/((a^2*c*x^2 + c)^2*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\arctan(ax)}{a^4c^2x^5+2\,a^2c^2x^3+c^2x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] integral(arctan(a*x)/(a^4*c^2*x^5 + 2*a^2*c^2*x^3 + c^2*x), x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RecursionError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)/x/(a**2*c*x**2+c)**2,x)
```

```
[Out] Exception raised: RecursionError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(ax)}{\left(a^2cx^2+c\right)^2x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)/((a^2*c*x^2 + c)^2*x), x)
```

3.189 $\int \frac{\tan^{-1}(ax)}{x^2(c+a^2cx^2)^2}\,dx$

Optimal. Leaf size=97

$$-\frac{a}{4c^2(a^2x^2+1)}-\frac{a\log(a^2x^2+1)}{2c^2}-\frac{a^2x\tan^{-1}(ax)}{2c^2(a^2x^2+1)}+\frac{a\log(x)}{c^2}-\frac{3a\tan^{-1}(ax)^2}{4c^2}-\frac{\tan^{-1}(ax)}{c^2x}$$

```
[Out] -a/(4*c^2*(1 + a^2*x^2)) - ArcTan[a*x]/(c^2*x) - (a^2*x*ArcTan[a*x])/(2*c^2
*(1 + a^2*x^2)) - (3*a*ArcTan[a*x]^2)/(4*c^2) + (a*Log[x])/c^2 - (a*Log[1 +
 a^2*x^2])/(2*c^2)
```

Rubi [A] time = 0.161934, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4966, 4918, 4852, 266, 36, 29, 31, 4884, 4892, 261}

$$-\frac{a}{4c^2(a^2x^2+1)}-\frac{a\log(a^2x^2+1)}{2c^2}-\frac{a^2x\tan^{-1}(ax)}{2c^2(a^2x^2+1)}+\frac{a\log(x)}{c^2}-\frac{3a\tan^{-1}(ax)^2}{4c^2}-\frac{\tan^{-1}(ax)}{c^2x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]/(x^2*(c + a^2*c*x^2)^2),x]
```



```
[Out] -a/(4*c^2*(1 + a^2*x^2)) - ArcTan[a*x]/(c^2*x) - (a^2*x*ArcTan[a*x])/(2*c^2
*(1 + a^2*x^2)) - (3*a*ArcTan[a*x]^2)/(4*c^2) + (a*Log[x])/c^2 - (a*Log[1 +
 a^2*x^2])/(2*c^2)
```


Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*
x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^p
, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*q]
 && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 4918

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/(d + e*x^2),
```

```
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4892

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[(x*(a + b*ArcTan[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*c*
p)/2, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(a +
 b*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)}{x^2\left(c+a^2cx^2\right)^2}\,dx = -\left(a^2\int \frac{\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^2}\,dx\right)+\frac{\int \frac{\tan^{-1}(ax)}{x^2\left(c+a^2cx^2\right)}\,dx}{c}$$

$$= -\frac{a^2x\tan^{-1}(ax)}{2c^2\left(1+a^2x^2\right)}-\frac{a\tan^{-1}(ax)^2}{4c^2}+\frac{1}{2}a^3\int \frac{x}{\left(c+a^2cx^2\right)^2}\,dx+\frac{\int \frac{\tan^{-1}(ax)}{x^2}\,dx}{c^2}-\frac{a^2\int \frac{\tan^{-1}(ax)}{c+a^2cx^2}\,dx}{c}$$

$$= -\frac{a}{4c^2\left(1+a^2x^2\right)}-\frac{\tan^{-1}(ax)}{c^2x}-\frac{a^2x\tan^{-1}(ax)}{2c^2\left(1+a^2x^2\right)}-\frac{3a\tan^{-1}(ax)^2}{4c^2}+\frac{a\int \frac{1}{x\left(1+a^2x^2\right)}\,dx}{c^2}$$

$$= -\frac{a}{4c^2\left(1+a^2x^2\right)}-\frac{\tan^{-1}(ax)}{c^2x}-\frac{a^2x\tan^{-1}(ax)}{2c^2\left(1+a^2x^2\right)}-\frac{3a\tan^{-1}(ax)^2}{4c^2}+\frac{a\,\mathrm{Subst}\left(\int \frac{1}{x\left(1+a^2x\right)}\,dx,x,x^2\right)}{2c^2}$$

$$= -\frac{a}{4c^2\left(1+a^2x^2\right)}-\frac{\tan^{-1}(ax)}{c^2x}-\frac{a^2x\tan^{-1}(ax)}{2c^2\left(1+a^2x^2\right)}-\frac{3a\tan^{-1}(ax)^2}{4c^2}+\frac{a\,\mathrm{Subst}\left(\int \frac{1}{x}\,dx,x,x^2\right)}{2c^2}-a^3\,\mathrm{S}$$

$$= -\frac{a}{4c^2\left(1+a^2x^2\right)}-\frac{\tan^{-1}(ax)}{c^2x}-\frac{a^2x\tan^{-1}(ax)}{2c^2\left(1+a^2x^2\right)}-\frac{3a\tan^{-1}(ax)^2}{4c^2}+\frac{a\log(x)}{c^2}-\frac{a\log\left(1+a^2x^2\right)}{2c^2}$$

Mathematica [A] time = 0.0732947, size = 94, normalized size = 0.97

$$-\frac{a}{4c^2\left(a^2x^2+1\right)}-\frac{a\log\left(a^2x^2+1\right)}{2c^2}-\frac{\left(3a^2x^2+2\right)\tan^{-1}(ax)}{2c^2x\left(a^2x^2+1\right)}+\frac{a\log(x)}{c^2}-\frac{3a\tan^{-1}(ax)^2}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]/(x^2*(c + a^2*c*x^2)^2),x]
```

```
[Out] -a/(4*c^2*(1 + a^2*x^2)) - ((2 + 3*a^2*x^2)*ArcTan[a*x])/(2*c^2*x*(1 + a^2*
x^2)) - (3*a*ArcTan[a*x]^2)/(4*c^2) + (a*Log[x])/c^2 - (a*Log[1 + a^2*x^2])
/(2*c^2)
```

Maple [A] time = 0.05, size = 92, normalized size = 1.

$$-\frac{a^{2} x \arctan\left(a x\right)}{2 c^{2} \left(a^{2} x^{2}+1\right)}-\frac{3 a \left(\arctan\left(a x\right)\right)^{2}}{4 c^{2}}-\frac{\arctan\left(a x\right)}{c^{2} x}-\frac{a \ln\left(a^{2} x^{2}+1\right)}{2 c^{2}}-\frac{a}{4 c^{2} \left(a^{2} x^{2}+1\right)}+\frac{a \ln\left(a x\right)}{c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)/x^2/(a^2*c*x^2+c)^2,x)
```

```
[Out] -1/2*a^2*x*arctan(a*x)/c^2/(a^2*x^2+1)-3/4*a*arctan(a*x)^2/c^2-arctan(a*x)/
c^2/x-1/2*a*ln(a^2*x^2+1)/c^2-1/4*a/c^2/(a^2*x^2+1)+a/c^2*ln(a*x)
```

Maxima [A] time = 1.59835, size = 161, normalized size = 1.66

$$-\frac{1}{2} \left(\frac{3 a^{2} x^{2}+2}{a^{2} c^{2} x^{3}+c^{2} x}+\frac{3 a \arctan\left(a x\right)}{c^{2}}\right) \arctan\left(a x\right)+\frac{\left(3 \left(a^{2} x^{2}+1\right) \arctan\left(a x\right)^{2}-2 \left(a^{2} x^{2}+1\right) \log\left(a^{2} x^{2}+1\right)+4 \left(a^{2} x^{2}+\right.\right.}{4 \left(a^{2} c^{2} x^{2}+c^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x^2/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((3*a^2*x^2 + 2)/(a^2*c^2*x^3 + c^2*x) + 3*a*arctan(a*x)/c^2)*arctan(a
*x) + 1/4*(3*(a^2*x^2 + 1)*arctan(a*x)^2 - 2*(a^2*x^2 + 1)*log(a^2*x^2 + 1)
 + 4*(a^2*x^2 + 1)*log(x) - 1)*a/(a^2*c^2*x^2 + c^2)
```

Fricas [A] time = 1.68617, size = 221, normalized size = 2.28

$$-\frac{3 \left(a^{3} x^{3}+a x\right) \arctan\left(a x\right)^{2}+a x+2 \left(3 a^{2} x^{2}+2\right) \arctan\left(a x\right)+2 \left(a^{3} x^{3}+a x\right) \log\left(a^{2} x^{2}+1\right)-4 \left(a^{3} x^{3}+a x\right) \log\left(x\right)}{4 \left(a^{2} c^{2} x^{3}+c^{2} x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x^2/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(3*(a^3*x^3 + a*x)*arctan(a*x)^2 + a*x + 2*(3*a^2*x^2 + 2)*arctan(a*x)
 + 2*(a^3*x^3 + a*x)*log(a^2*x^2 + 1) - 4*(a^3*x^3 + a*x)*log(x))/(a^2*c^2*
```

```
x^3 + c^2*x)
```

Sympy [B] time = 2.13428, size = 299, normalized size = 3.08

$$\frac{12a^3x^3\log(x)}{12a^2c^2x^3+12c^2x}-\frac{6a^3x^3\log\left(x^2+\frac{1}{a^2}\right)}{12a^2c^2x^3+12c^2x}-\frac{9a^3x^3\operatorname{atan}^2(ax)}{12a^2c^2x^3+12c^2x}+\frac{a^3x^3}{12a^2c^2x^3+12c^2x}-\frac{18a^2x^2\operatorname{atan}(ax)}{12a^2c^2x^3+12c^2x}+\frac{12ax\log(x)}{12a^2c^2x^3+12c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)/x**2/(a**2*c*x**2+c)**2,x)
```

```
[Out] 12*a**3*x**3*log(x)/(12*a**2*c**2*x**3 + 12*c**2*x) - 6*a**3*x**3*log(x**2
+ a**(-2))/(12*a**2*c**2*x**3 + 12*c**2*x) - 9*a**3*x**3*atan(a*x)**2/(12*a
**2*c**2*x**3 + 12*c**2*x) + a**3*x**3/(12*a**2*c**2*x**3 + 12*c**2*x) - 18
*a**2*x**2*atan(a*x)/(12*a**2*c**2*x**3 + 12*c**2*x) + 12*a*x*log(x)/(12*a*
*2*c**2*x**3 + 12*c**2*x) - 6*a*x*log(x**2 + a**(-2))/(12*a**2*c**2*x**3 +
12*c**2*x) - 9*a*x*atan(a*x)**2/(12*a**2*c**2*x**3 + 12*c**2*x) - 2*a*x/(12
*a**2*c**2*x**3 + 12*c**2*x) - 12*atan(a*x)/(12*a**2*c**2*x**3 + 12*c**2*x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(ax)}{\left(a^2cx^2+c\right)^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x^2/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)/((a^2*c*x^2 + c)^2*x^2), x)
```

3.190 $\int \frac{\tan^{-1}(ax)}{x^3(c+a^2cx^2)^2}\,dx$

Optimal. Leaf size=156

$$\frac{ia^2\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{c^2}+\frac{a^3x}{4c^2\left(a^2x^2+1\right)}-\frac{a^2\tan^{-1}(ax)}{2c^2\left(a^2x^2+1\right)}+\frac{ia^2\tan^{-1}(ax)^2}{c^2}-\frac{a^2\tan^{-1}(ax)}{4c^2}-\frac{2a^2\log\left(2-\frac{2}{1-iax}\right)\text{ta}}{c^2}$$

```
[Out] -a/(2*c^2*x) + (a^3*x)/(4*c^2*(1 + a^2*x^2)) - (a^2*ArcTan[a*x])/(4*c^2) -
ArcTan[a*x]/(2*c^2*x^2) - (a^2*ArcTan[a*x])/(2*c^2*(1 + a^2*x^2)) + (I*a^2*
ArcTan[a*x]^2)/c^2 - (2*a^2*ArcTan[a*x]*Log[2 - 2/(1 - I*a*x)])/c^2 + (I*a^
2*PolyLog[2, -1 + 2/(1 - I*a*x)])/c^2
```

Rubi [A] time = 0.405977, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 11, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.55, Rules used = {4966, 4918, 4852, 325, 203, 4924, 4868, 2447, 4930, 199, 205}

$$\frac{ia^2\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{c^2}+\frac{a^3x}{4c^2\left(a^2x^2+1\right)}-\frac{a^2\tan^{-1}(ax)}{2c^2\left(a^2x^2+1\right)}+\frac{ia^2\tan^{-1}(ax)^2}{c^2}-\frac{a^2\tan^{-1}(ax)}{4c^2}-\frac{2a^2\log\left(2-\frac{2}{1-iax}\right)\text{ta}}{c^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]/(x^3*(c + a^2*c*x^2)^2),x]
```


```
[Out] -a/(2*c^2*x) + (a^3*x)/(4*c^2*(1 + a^2*x^2)) - (a^2*ArcTan[a*x])/(4*c^2) -
ArcTan[a*x]/(2*c^2*x^2) - (a^2*ArcTan[a*x])/(2*c^2*(1 + a^2*x^2)) + (I*a^2*
ArcTan[a*x]^2)/c^2 - (2*a^2*ArcTan[a*x]*Log[2 - 2/(1 - I*a*x)])/c^2 + (I*a^
2*PolyLog[2, -1 + 2/(1 - I*a*x)])/c^2
```


Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*
x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^p
, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*q]
 && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 4918

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/(d + e*x^2),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)}{x^3(c+a^2cx^2)^2}\,dx = -\left(a^2\int \frac{\tan^{-1}(ax)}{x(c+a^2cx^2)^2}\,dx\right)+\frac{\int \frac{\tan^{-1}(ax)}{x^3(c+a^2cx^2)}\,dx}{c}$$

$$= a^4\int \frac{x\tan^{-1}(ax)}{(c+a^2cx^2)^2}\,dx+\frac{\int \frac{\tan^{-1}(ax)}{x^3}\,dx}{c^2}-2\frac{a^2\int \frac{\tan^{-1}(ax)}{x(c+a^2cx^2)}\,dx}{c}$$

$$= -\frac{\tan^{-1}(ax)}{2c^2x^2}-\frac{a^2\tan^{-1}(ax)}{2c^2(1+a^2x^2)}+\frac{1}{2}a^3\int \frac{1}{(c+a^2cx^2)^2}\,dx+\frac{a\int \frac{1}{x^2(1+a^2x^2)}\,dx}{2c^2}-2\left(-\frac{ia^2\tan^{-1}(ax)^2}{2c^2}\right.$$

$$= -\frac{a}{2c^2x}+\frac{a^3x}{4c^2(1+a^2x^2)}-\frac{\tan^{-1}(ax)}{2c^2x^2}-\frac{a^2\tan^{-1}(ax)}{2c^2(1+a^2x^2)}-\frac{a^3\int \frac{1}{1+a^2x^2}\,dx}{2c^2}-2\left(-\frac{ia^2\tan^{-1}(ax)^2}{2c^2}+\right.$$

$$= -\frac{a}{2c^2x}+\frac{a^3x}{4c^2(1+a^2x^2)}-\frac{a^2\tan^{-1}(ax)}{4c^2}-\frac{\tan^{-1}(ax)}{2c^2x^2}-\frac{a^2\tan^{-1}(ax)}{2c^2(1+a^2x^2)}-2\left(-\frac{ia^2\tan^{-1}(ax)^2}{2c^2}+\right.$$

Mathematica [A] time = 0.391501, size = 93, normalized size = 0.6

$$\frac{a^2\left(8i\text{PolyLog}\left(2,e^{2i\tan^{-1}(ax)}\right)+\tan^{-1}(ax)\left(-\frac{4}{a^2x^2}-16\log\left(1-e^{2i\tan^{-1}(ax)}\right)-2\cos\left(2\tan^{-1}(ax)\right)-4\right)-\frac{4}{ax}+8i\tan^{-1}(a\right.}{8c^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTan[a*x]/(x^3*(c + a^2*c*x^2)^2),x]
```

```
[Out] (a^2*(-4/(a*x) + (8*I)*ArcTan[a*x]^2 + ArcTan[a*x]*(-4 - 4/(a^2*x^2) - 2*Co
s[2*ArcTan[a*x]] - 16*Log[1 - E^((2*I)*ArcTan[a*x])]) + (8*I)*PolyLog[2, E^
((2*I)*ArcTan[a*x])] + Sin[2*ArcTan[a*x]]))/(8*c^2)
```

Maple [B] time = 0.108, size = 369, normalized size = 2.4

$$\frac{a^2\arctan(ax)\ln\left(a^2x^2+1\right)}{c^2}-\frac{a^2\arctan(ax)}{2c^2\left(a^2x^2+1\right)}-\frac{\arctan(ax)}{2c^2x^2}-2\frac{a^2\arctan(ax)\ln(ax)}{c^2}-\frac{\frac{i}{2}a^2\text{dilog}\left(-\frac{i}{2}(ax+i)\right)}{c^2}+\frac{ia^2\text{di}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)/x^3/(a^2*c*x^2+c)^2,x)
```

```
[Out] a^2/c^2*arctan(a*x)*ln(a^2*x^2+1)-1/2*a^2*arctan(a*x)/c^2/(a^2*x^2+1)-1/2*a
rctan(a*x)/c^2/x^2-2*a^2/c^2*arctan(a*x)*ln(a*x)-1/2*I*a^2/c^2*dilog(-1/2*I
*(a*x+I))+I*a^2/c^2*dilog(1-I*a*x)-1/2*I*a^2/c^2*ln(a*x+I)*ln(a^2*x^2+1)+I*
a^2/c^2*ln(a*x)*ln(1-I*a*x)-1/2*I*a^2/c^2*ln(a*x-I)*ln(-1/2*I*(a*x+I))+1/2*
I*a^2/c^2*dilog(1/2*I*(a*x-I))+1/2*I*a^2/c^2*ln(a*x+I)*ln(1/2*I*(a*x-I))+1/
2*I*a^2/c^2*ln(a*x-I)*ln(a^2*x^2+1)+1/4*I*a^2/c^2*ln(a*x+I)^2-I*a^2/c^2*ln(
a*x)*ln(1+I*a*x)-1/4*I*a^2/c^2*ln(a*x-I)^2-I*a^2/c^2*dilog(1+I*a*x)+1/4*a^3
*x/c^2/(a^2*x^2+1)-1/4*a^2*arctan(a*x)/c^2-1/2*a/c^2/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(ax)}{\left(a^2cx^2+c\right)^2x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x^3/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(arctan(a*x)/((a^2*c*x^2 + c)^2*x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\arctan(ax)}{a^4c^2x^7+2\,a^2c^2x^5+c^2x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x^3/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] integral(arctan(a*x)/(a^4*c^2*x^7 + 2*a^2*c^2*x^5 + c^2*x^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\operatorname{atan}(ax)}{a^4x^7+2a^2x^5+x^3}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)/x**3/(a**2*c*x**2+c)**2,x)
```

```
[Out] Integral(atan(a*x)/(a**4*x**7 + 2*a**2*x**5 + x**3), x)/c**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(ax)}{\left(a^2cx^2+c\right)^2 x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x^3/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)/((a^2*c*x^2 + c)^2*x^3), x)
```

3.191 $\int \frac{\tan^{-1}(ax)}{x^4(c+a^2cx^2)^2}\,dx$

Optimal. Leaf size=136

$$\frac{a^3}{4c^2\left(a^2x^2+1\right)}+\frac{7a^3\log\left(a^2x^2+1\right)}{6c^2}+\frac{a^4x\tan^{-1}(ax)}{2c^2\left(a^2x^2+1\right)}-\frac{7a^3\log(x)}{3c^2}+\frac{5a^3\tan^{-1}(ax)^2}{4c^2}+\frac{2a^2\tan^{-1}(ax)}{c^2x}-\frac{a}{6c^2x^2}-\frac{\tan^{-1}}{3c^2}$$

```
[Out] -a/(6*c^2*x^2) + a^3/(4*c^2*(1 + a^2*x^2)) - ArcTan[a*x]/(3*c^2*x^3) + (2*a
^2*ArcTan[a*x])/(c^2*x) + (a^4*x*ArcTan[a*x])/(2*c^2*(1 + a^2*x^2)) + (5*a^
3*ArcTan[a*x]^2)/(4*c^2) - (7*a^3*Log[x])/(3*c^2) + (7*a^3*Log[1 + a^2*x^2]
)/(6*c^2)
```

Rubi [A] time = 0.374903, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 11, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.55, Rules used = {4966, 4918, 4852, 266, 44, 36, 29, 31, 4884, 4892, 261}

$$\frac{a^3}{4c^2\left(a^2x^2+1\right)}+\frac{7a^3\log\left(a^2x^2+1\right)}{6c^2}+\frac{a^4x\tan^{-1}(ax)}{2c^2\left(a^2x^2+1\right)}-\frac{7a^3\log(x)}{3c^2}+\frac{5a^3\tan^{-1}(ax)^2}{4c^2}+\frac{2a^2\tan^{-1}(ax)}{c^2x}-\frac{a}{6c^2x^2}-\frac{\tan^{-1}}{3c^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]/(x^4*(c + a^2*c*x^2)^2),x]
```

```
[Out] -a/(6*c^2*x^2) + a^3/(4*c^2*(1 + a^2*x^2)) - ArcTan[a*x]/(3*c^2*x^3) + (2*a
^2*ArcTan[a*x])/(c^2*x) + (a^4*x*ArcTan[a*x])/(2*c^2*(1 + a^2*x^2)) + (5*a^
3*ArcTan[a*x]^2)/(4*c^2) - (7*a^3*Log[x])/(3*c^2) + (7*a^3*Log[1 + a^2*x^2]
)/(6*c^2)
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*
x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^p
, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*q]
 && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 4918

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
```

```
x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/(d + e*x^2),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4892

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[(x*(a + b*ArcTan[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*c*
p)/2, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(a +
 b*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)}{x^4\left(c+a^2cx^2\right)^2}\,dx = -\left(a^2\int \frac{\tan^{-1}(ax)}{x^2\left(c+a^2cx^2\right)^2}\,dx\right)+\frac{\int \frac{\tan^{-1}(ax)}{x^4\left(c+a^2cx^2\right)}\,dx}{c}$$

$$= a^4\int \frac{\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^2}\,dx+\frac{\int \frac{\tan^{-1}(ax)}{x^4}\,dx}{c^2}-2\frac{a^2\int \frac{\tan^{-1}(ax)}{x^2\left(c+a^2cx^2\right)}\,dx}{c}$$

$$= -\frac{\tan^{-1}(ax)}{3c^2x^3}+\frac{a^4x\tan^{-1}(ax)}{2c^2\left(1+a^2x^2\right)}+\frac{a^3\tan^{-1}(ax)^2}{4c^2}-\frac{1}{2}a^5\int \frac{x}{\left(c+a^2cx^2\right)^2}\,dx+\frac{a\int \frac{1}{x^3\left(1+a^2x^2\right)}\,dx}{3c^2}-2$$

$$= \frac{a^3}{4c^2\left(1+a^2x^2\right)}-\frac{\tan^{-1}(ax)}{3c^2x^3}+\frac{a^4x\tan^{-1}(ax)}{2c^2\left(1+a^2x^2\right)}+\frac{a^3\tan^{-1}(ax)^2}{4c^2}+\frac{a\,\mathrm{Subst}\left(\int \frac{1}{x^2\left(1+a^2x\right)}\,dx,x,x^2\right)}{6c^2}$$

$$= \frac{a^3}{4c^2\left(1+a^2x^2\right)}-\frac{\tan^{-1}(ax)}{3c^2x^3}+\frac{a^4x\tan^{-1}(ax)}{2c^2\left(1+a^2x^2\right)}+\frac{a^3\tan^{-1}(ax)^2}{4c^2}+\frac{a\,\mathrm{Subst}\left(\int \left(\frac{1}{x^2}-\frac{a^2}{x}+\frac{a^4}{1+a^2x}\right)\right.}{6c^2}$$

$$= -\frac{a}{6c^2x^2}+\frac{a^3}{4c^2\left(1+a^2x^2\right)}-\frac{\tan^{-1}(ax)}{3c^2x^3}+\frac{a^4x\tan^{-1}(ax)}{2c^2\left(1+a^2x^2\right)}+\frac{a^3\tan^{-1}(ax)^2}{4c^2}-\frac{a^3\log(x)}{3c^2}+\frac{a^3\log\left(\right.}{6}$$

$$= -\frac{a}{6c^2x^2}+\frac{a^3}{4c^2\left(1+a^2x^2\right)}-\frac{\tan^{-1}(ax)}{3c^2x^3}+\frac{a^4x\tan^{-1}(ax)}{2c^2\left(1+a^2x^2\right)}+\frac{a^3\tan^{-1}(ax)^2}{4c^2}-\frac{a^3\log(x)}{3c^2}+\frac{a^3\log\left(\right.}{6}$$

Mathematica [A] time = 0.0968163, size = 124, normalized size = 0.91

$$\frac{a^3}{4c^2\left(a^2x^2+1\right)}+\frac{7a^3\log\left(a^2x^2+1\right)}{6c^2}+\frac{\left(15a^4x^4+10a^2x^2-2\right)\tan^{-1}(ax)}{6c^2x^3\left(a^2x^2+1\right)}-\frac{7a^3\log(x)}{3c^2}+\frac{5a^3\tan^{-1}(ax)^2}{4c^2}-\frac{a}{6c^2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]/(x^4*(c + a^2*c*x^2)^2),x]
```

```
[Out] -a/(6*c^2*x^2) + a^3/(4*c^2*(1 + a^2*x^2)) + ((-2 + 10*a^2*x^2 + 15*a^4*x^4
)*ArcTan[a*x])/(6*c^2*x^3*(1 + a^2*x^2)) + (5*a^3*ArcTan[a*x]^2)/(4*c^2) -
(7*a^3*Log[x])/(3*c^2) + (7*a^3*Log[1 + a^2*x^2])/(6*c^2)
```

Maple [A] time = 0.05, size = 125, normalized size = 0.9

$$\frac{a^4x\arctan(ax)}{2c^2\left(a^2x^2+1\right)}+\frac{5a^3(\arctan(ax))^2}{4c^2}-\frac{\arctan(ax)}{3c^2x^3}+2\frac{a^2\arctan(ax)}{c^2x}+\frac{7a^3\ln\left(a^2x^2+1\right)}{6c^2}+\frac{a^3}{4c^2\left(a^2x^2+1\right)}-\frac{a}{6c^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)/x^4/(a^2*c*x^2+c)^2,x)
```

```
[Out] 1/2*a^4*x*arctan(a*x)/c^2/(a^2*x^2+1)+5/4*a^3*arctan(a*x)^2/c^2-1/3*arctan(
a*x)/c^2/x^3+2*a^2*arctan(a*x)/c^2/x+7/6*a^3*ln(a^2*x^2+1)/c^2+1/4*a^3/c^2/
(a^2*x^2+1)-1/6*a/c^2/x^2-7/3*a^3/c^2*ln(a*x)
```

Maxima [A] time = 1.63609, size = 216, normalized size = 1.59

$$\frac{1}{6}\left(\frac{15a^3\arctan(ax)}{c^2}+\frac{15a^4x^4+10a^2x^2-2}{a^2c^2x^5+c^2x^3}\right)\arctan(ax)+\frac{\left(a^2x^2-15\left(a^4x^4+a^2x^2\right)\arctan(ax)^2+14\left(a^4x^4+a^2x^2\right)\mathrm{lo}\right.}{12\left(a^2c^2x^4+c^2x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x^4/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/6*(15*a^3*arctan(a*x)/c^2 + (15*a^4*x^4 + 10*a^2*x^2 - 2)/(a^2*c^2*x^5 +
c^2*x^3))*arctan(a*x) + 1/12*(a^2*x^2 - 15*(a^4*x^4 + a^2*x^2)*arctan(a*x)^
```

```
2 + 14*(a^4*x^4 + a^2*x^2)*log(a^2*x^2 + 1) - 28*(a^4*x^4 + a^2*x^2)*log(x)
 - 2)*a/(a^2*c^2*x^4 + c^2*x^2)
```

Fricas [A] time = 1.75338, size = 279, normalized size = 2.05

$$\frac{a^3x^3 + 15\left(a^5x^5 + a^3x^3\right)\arctan\left(ax\right)^2 - 2\,ax + 2\left(15\,a^4x^4 + 10\,a^2x^2 - 2\right)\arctan\left(ax\right) + 14\left(a^5x^5 + a^3x^3\right)\log\left(a^2x^2 + 1\right) -}{12\left(a^2c^2x^5 + c^2x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x^4/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(a^3*x^3 + 15*(a^5*x^5 + a^3*x^3)*arctan(a*x)^2 - 2*a*x + 2*(15*a^4*x^
4 + 10*a^2*x^2 - 2)*arctan(a*x) + 14*(a^5*x^5 + a^3*x^3)*log(a^2*x^2 + 1) -
 28*(a^5*x^5 + a^3*x^3)*log(x))/(a^2*c^2*x^5 + c^2*x^3)
```

Sympy [B] time = 3.4458, size = 360, normalized size = 2.65

$$-\frac{28a^5x^5\log\left(x\right)}{12a^2c^2x^5 + 12c^2x^3} + \frac{14a^5x^5\log\left(x^2 + \frac{1}{a^2}\right)}{12a^2c^2x^5 + 12c^2x^3} + \frac{15a^5x^5\operatorname{atan}^2\left(ax\right)}{12a^2c^2x^5 + 12c^2x^3} + \frac{30a^4x^4\operatorname{atan}\left(ax\right)}{12a^2c^2x^5 + 12c^2x^3} - \frac{28a^3x^3\log\left(x\right)}{12a^2c^2x^5 + 12c^2x^3} + \frac{14a^3x^3 \mathrm{l}}{12a^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)/x**4/(a**2*c*x**2+c)**2,x)
```

```
[Out] -28*a**5*x**5*log(x)/(12*a**2*c**2*x**5 + 12*c**2*x**3) + 14*a**5*x**5*log(
x**2 + a**(-2))/(12*a**2*c**2*x**5 + 12*c**2*x**3) + 15*a**5*x**5*atan(a*x)
**2/(12*a**2*c**2*x**5 + 12*c**2*x**3) + 30*a**4*x**4*atan(a*x)/(12*a**2*c*
*2*x**5 + 12*c**2*x**3) - 28*a**3*x**3*log(x)/(12*a**2*c**2*x**5 + 12*c**2*
x**3) + 14*a**3*x**3*log(x**2 + a**(-2))/(12*a**2*c**2*x**5 + 12*c**2*x**3)
 + 15*a**3*x**3*atan(a*x)**2/(12*a**2*c**2*x**5 + 12*c**2*x**3) + a**3*x**3
/(12*a**2*c**2*x**5 + 12*c**2*x**3) + 20*a**2*x**2*atan(a*x)/(12*a**2*c**2*
x**5 + 12*c**2*x**3) - 2*a*x/(12*a**2*c**2*x**5 + 12*c**2*x**3) - 4*atan(a*
x)/(12*a**2*c**2*x**5 + 12*c**2*x**3)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(ax)}{(a^2cx^2 + c)^2 x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x^4/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)/((a^2*c*x^2 + c)^2*x^4), x)
```

3.192 $\int \frac{x^3 \tan^{-1}(ax)}{\left(c+a^2cx^2\right)^3} dx$

Optimal. Leaf size=86

$$\frac{x^3}{16ac^3\left(a^2x^2+1\right)^2}+\frac{3x}{32a^3c^3\left(a^2x^2+1\right)}+\frac{x^4\tan^{-1}(ax)}{4c^3\left(a^2x^2+1\right)^2}-\frac{3\tan^{-1}(ax)}{32a^4c^3}$$

```
[Out] x^3/(16*a*c^3*(1 + a^2*x^2)^2) + (3*x)/(32*a^3*c^3*(1 + a^2*x^2)) - (3*ArcT
an[a*x])/(32*a^4*c^3) + (x^4*ArcTan[a*x])/(4*c^3*(1 + a^2*x^2)^2)
```

Rubi [A] time = 0.0655447, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {4944, 288, 205}

$$\frac{x^3}{16ac^3\left(a^2x^2+1\right)^2}+\frac{3x}{32a^3c^3\left(a^2x^2+1\right)}+\frac{x^4\tan^{-1}(ax)}{4c^3\left(a^2x^2+1\right)^2}-\frac{3\tan^{-1}(ax)}{32a^4c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcTan[a*x])/(c + a^2*c*x^2)^3,x]
```

```
[Out] x^3/(16*a*c^3*(1 + a^2*x^2)^2) + (3*x)/(32*a^3*c^3*(1 + a^2*x^2)) - (3*ArcT
an[a*x])/(32*a^4*c^3) + (x^4*ArcTan[a*x])/(4*c^3*(1 + a^2*x^2)^2)
```

Rule 4944

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a +
 b*ArcTan[c*x])^p)/(d*f*(m + 1)), x] - Dist[(b*c*p)/(f*(m + 1)), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^3}\,dx &= \frac{x^4\tan^{-1}(ax)}{4c^3\left(1+a^2x^2\right)^2} - \frac{1}{4}a\int \frac{x^4}{\left(c+a^2cx^2\right)^3}\,dx \\
&= \frac{x^3}{16ac^3\left(1+a^2x^2\right)^2} + \frac{x^4\tan^{-1}(ax)}{4c^3\left(1+a^2x^2\right)^2} - \frac{3\int \frac{x^2}{\left(c+a^2cx^2\right)^2}\,dx}{16ac} \\
&= \frac{x^3}{16ac^3\left(1+a^2x^2\right)^2} + \frac{3x}{32a^3c^3\left(1+a^2x^2\right)} + \frac{x^4\tan^{-1}(ax)}{4c^3\left(1+a^2x^2\right)^2} - \frac{3\int \frac{1}{c+a^2cx^2}\,dx}{32a^3c^2} \\
&= \frac{x^3}{16ac^3\left(1+a^2x^2\right)^2} + \frac{3x}{32a^3c^3\left(1+a^2x^2\right)} - \frac{3\tan^{-1}(ax)}{32a^4c^3} + \frac{x^4\tan^{-1}(ax)}{4c^3\left(1+a^2x^2\right)^2}
\end{aligned}
$$

Mathematica [A] time = 0.141649, size = 58, normalized size = 0.67

$$
\frac{ax\left(5a^2x^2+3\right)+\left(5a^4x^4-6a^2x^2-3\right)\tan^{-1}(ax)}{32a^4c^3\left(a^2x^2+1\right)^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*ArcTan[a*x])/(c + a^2*c*x^2)^3,x]
```

```
[Out] (a*x*(3 + 5*a^2*x^2) + (-3 - 6*a^2*x^2 + 5*a^4*x^4)*ArcTan[a*x])/(32*a^4*c^
3*(1 + a^2*x^2)^2)
```

Maple [A] time = 0.036, size = 102, normalized size = 1.2

$$
\frac{\arctan(ax)}{4\,a^4c^3\left(a^2x^2+1\right)^2} - \frac{\arctan(ax)}{2\,a^4c^3\left(a^2x^2+1\right)} + \frac{5\,x^3}{32\,ac^3\left(a^2x^2+1\right)^2} + \frac{3\,x}{32\,a^3c^3\left(a^2x^2+1\right)^2} + \frac{5\,\arctan(ax)}{32\,a^4c^3}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctan(a*x)/(a^2*c*x^2+c)^3,x)
```

```
[Out] 1/4/a^4/c^3*arctan(a*x)/(a^2*x^2+1)^2-1/2*arctan(a*x)/a^4/c^3/(a^2*x^2+1)+5
/32*x^3/a/c^3/(a^2*x^2+1)^2+3/32/a^3/c^3/(a^2*x^2+1)^2*x+5/32*arctan(a*x)/a
^4/c^3
```

Maxima [A] time = 1.54169, size = 146, normalized size = 1.7

$$\frac{1}{32}\,a\left(\frac{5\,a^2x^3+3\,x}{a^8c^3x^4+2\,a^6c^3x^2+a^4c^3}+\frac{5\,\arctan\left(ax\right)}{a^5c^3}\right)-\frac{\left(2\,a^2x^2+1\right)\arctan\left(ax\right)}{4\left(a^8c^3x^4+2\,a^6c^3x^2+a^4c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/32*a*((5*a^2*x^3 + 3*x)/(a^8*c^3*x^4 + 2*a^6*c^3*x^2 + a^4*c^3) + 5*arcta
n(a*x)/(a^5*c^3)) - 1/4*(2*a^2*x^2 + 1)*arctan(a*x)/(a^8*c^3*x^4 + 2*a^6*c^
3*x^2 + a^4*c^3)
```

Fricas [A] time = 1.64915, size = 146, normalized size = 1.7

$$\frac{5\,a^3x^3+3\,ax+\left(5\,a^4x^4-6\,a^2x^2-3\right)\arctan\left(ax\right)}{32\left(a^8c^3x^4+2\,a^6c^3x^2+a^4c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 1/32*(5*a^3*x^3 + 3*a*x + (5*a^4*x^4 - 6*a^2*x^2 - 3)*arctan(a*x))/(a^8*c^3
*x^4 + 2*a^6*c^3*x^2 + a^4*c^3)
```

Sympy [A] time = 3.95857, size = 243, normalized size = 2.83

$$\begin{cases}\frac{5a^4x^4\operatorname{atan}(ax)}{32a^8c^3x^4+64a^6c^3x^2+32a^4c^3}+\frac{5a^3x^3}{32a^8c^3x^4+64a^6c^3x^2+32a^4c^3}-\frac{6a^2x^2\operatorname{atan}(ax)}{32a^8c^3x^4+64a^6c^3x^2+32a^4c^3}+\frac{3ax}{32a^8c^3x^4+64a^6c^3x^2+32a^4c^3}-\frac{3\operatorname{atan}(ax)}{32a^8c^3x^4+64a^6c^3x^2+32a^4c} \\ \tilde{\infty}\left(\frac{x^4\operatorname{atan}(ax)}{4}-\frac{x^3}{12a}+\frac{x}{4a^3}-\frac{\operatorname{atan}(ax)}{4a^4}\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atan(a*x)/(a**2*c*x**2+c)**3,x)
```

```
[Out] Piecewise((5*a**4*x**4*atan(a*x)/(32*a**8*c**3*x**4 + 64*a**6*c**3*x**2 + 3
2*a**4*c**3) + 5*a**3*x**3/(32*a**8*c**3*x**4 + 64*a**6*c**3*x**2 + 32*a**4
*c**3) - 6*a**2*x**2*atan(a*x)/(32*a**8*c**3*x**4 + 64*a**6*c**3*x**2 + 32*
a**4*c**3) + 3*a*x/(32*a**8*c**3*x**4 + 64*a**6*c**3*x**2 + 32*a**4*c**3) -
 3*atan(a*x)/(32*a**8*c**3*x**4 + 64*a**6*c**3*x**2 + 32*a**4*c**3), Ne(c,
0)), (zoo*(x**4*atan(a*x)/4 - x**3/(12*a) + x/(4*a**3) - atan(a*x)/(4*a**4)
), True))
```

Giac [A] time = 1.15508, size = 104, normalized size = 1.21

$$\frac{5 \arctan(ax)}{32 a^4 c^3} + \frac{5 a^2 x^3 + 3 x}{32 \left(a^2 x^2 + 1\right)^2 a^3 c^3} - \frac{\left(2 a^2 x^2 + 1\right) \arctan(ax)}{4 \left(a^2 x^2 + 1\right)^2 a^4 c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 5/32*arctan(a*x)/(a^4*c^3) + 1/32*(5*a^2*x^3 + 3*x)/((a^2*x^2 + 1)^2*a^3*c^
3) - 1/4*(2*a^2*x^2 + 1)*arctan(a*x)/((a^2*x^2 + 1)^2*a^4*c^3)
```

3.193 $\int \frac{x^2\tan^{-1}(ax)}{(c+a^2cx^2)^3}\,dx$

Optimal. Leaf size=111

$$\frac{1}{16a^3c^3\left(a^2x^2+1\right)}-\frac{1}{16a^3c^3\left(a^2x^2+1\right)^2}+\frac{x\tan^{-1}(ax)}{8a^2c^3\left(a^2x^2+1\right)}-\frac{x\tan^{-1}(ax)}{4a^2c^3\left(a^2x^2+1\right)^2}+\frac{\tan^{-1}(ax)^2}{16a^3c^3}$$

```
[Out] -1/(16*a^3*c^3*(1 + a^2*x^2)^2) + 1/(16*a^3*c^3*(1 + a^2*x^2)) - (x*ArcTan[
a*x])/(4*a^2*c^3*(1 + a^2*x^2)^2) + (x*ArcTan[a*x])/(8*a^2*c^3*(1 + a^2*x^2
)) + ArcTan[a*x]^2/(16*a^3*c^3)
```

Rubi [A] time = 0.0748615, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {4934, 4892, 261}

$$\frac{1}{16a^3c^3\left(a^2x^2+1\right)}-\frac{1}{16a^3c^3\left(a^2x^2+1\right)^2}+\frac{x\tan^{-1}(ax)}{8a^2c^3\left(a^2x^2+1\right)}-\frac{x\tan^{-1}(ax)}{4a^2c^3\left(a^2x^2+1\right)^2}+\frac{\tan^{-1}(ax)^2}{16a^3c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcTan[a*x])/(c + a^2*c*x^2)^3,x]
```

```
[Out] -1/(16*a^3*c^3*(1 + a^2*x^2)^2) + 1/(16*a^3*c^3*(1 + a^2*x^2)) - (x*ArcTan[
a*x])/(4*a^2*c^3*(1 + a^2*x^2)^2) + (x*ArcTan[a*x])/(8*a^2*c^3*(1 + a^2*x^2
)) + ArcTan[a*x]^2/(16*a^3*c^3)
```

Rule 4934

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^2*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> -Simp[(b*(d + e*x^2)^(q + 1))/(4*c^3*d*(q + 1)^2), x] + (-Dist[
1/(2*c^2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]), x], x] +
Simp[(x*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]))/(2*c^2*d*(q + 1)), x]) /;
FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -5/2]
```

Rule 4892

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[(x*(a + b*ArcTan[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*c*
p)/2, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(a +
```

```
b*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2 \tan^{-1}(ax)}{\left(c+a^2cx^2\right)^3}\,dx &= -\frac{1}{16a^3c^3\left(1+a^2x^2\right)^2}-\frac{x\tan^{-1}(ax)}{4a^2c^3\left(1+a^2x^2\right)^2}+\frac{\int \frac{\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^2}\,dx}{4a^2c}\\ &= -\frac{1}{16a^3c^3\left(1+a^2x^2\right)^2}-\frac{x\tan^{-1}(ax)}{4a^2c^3\left(1+a^2x^2\right)^2}+\frac{x\tan^{-1}(ax)}{8a^2c^3\left(1+a^2x^2\right)}+\frac{\tan^{-1}(ax)^2}{16a^3c^3}-\frac{\int \frac{x}{\left(c+a^2cx^2\right)^2}\,dx}{8ac}\\ &= -\frac{1}{16a^3c^3\left(1+a^2x^2\right)^2}+\frac{1}{16a^3c^3\left(1+a^2x^2\right)}-\frac{x\tan^{-1}(ax)}{4a^2c^3\left(1+a^2x^2\right)^2}+\frac{x\tan^{-1}(ax)}{8a^2c^3\left(1+a^2x^2\right)}+\frac{\tan^{-1}(ax)^2}{16a^3c^3}\end{aligned}$$

Mathematica [A] time = 0.0526421, size = 64, normalized size = 0.58

$$\frac{a^2x^2+2ax\left(a^2x^2-1\right)\tan^{-1}(ax)+\left(a^2x^2+1\right)^2\tan^{-1}(ax)^2}{16a^3c^3\left(a^2x^2+1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*ArcTan[a*x])/(c + a^2*c*x^2)^3,x]
```

```
[Out] (a^2*x^2 + 2*a*x*(-1 + a^2*x^2)*ArcTan[a*x] + (1 + a^2*x^2)^2*ArcTan[a*x]^2
)/(16*a^3*c^3*(1 + a^2*x^2)^2)
```

Maple [A] time = 0.042, size = 101, normalized size = 0.9

$$\frac{\arctan(ax)\,x^3}{8\,c^3\left(a^2x^2+1\right)^2}-\frac{x\arctan(ax)}{8\,a^2c^3\left(a^2x^2+1\right)^2}+\frac{(\arctan(ax))^2}{16\,a^3c^3}-\frac{1}{16\,a^3c^3\left(a^2x^2+1\right)^2}+\frac{1}{16\,a^3c^3\left(a^2x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(a*x)/(a^2*c*x^2+c)^3,x)
```

```
[Out] 1/8/c^3*arctan(a*x)/(a^2*x^2+1)^2*x^3-1/8*x*arctan(a*x)/a^2/c^3/(a^2*x^2+1)
^2+1/16*arctan(a*x)^2/a^3/c^3-1/16/a^3/c^3/(a^2*x^2+1)^2+1/16/a^3/c^3/(a^2*
x^2+1)
```

Maxima [A] time = 1.70585, size = 174, normalized size = 1.57

$$\frac{1}{8}\left(\frac{a^2x^3-x}{a^6c^3x^4+2\,a^4c^3x^2+a^2c^3}+\frac{\arctan(ax)}{a^3c^3}\right)\arctan(ax)+\frac{\left(a^2x^2-\left(a^4x^4+2\,a^2x^2+1\right)\arctan(ax)^2\right)a}{16\left(a^8c^3x^4+2\,a^6c^3x^2+a^4c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((a^2*x^3 - x)/(a^6*c^3*x^4 + 2*a^4*c^3*x^2 + a^2*c^3) + arctan(a*x)/(a
^3*c^3))*arctan(a*x) + 1/16*(a^2*x^2 - (a^4*x^4 + 2*a^2*x^2 + 1)*arctan(a*x
)^2)*a/(a^8*c^3*x^4 + 2*a^6*c^3*x^2 + a^4*c^3)
```

Fricas [A] time = 1.59638, size = 176, normalized size = 1.59

$$\frac{a^2x^2+\left(a^4x^4+2\,a^2x^2+1\right)\arctan(ax)^2+2\left(a^3x^3-ax\right)\arctan(ax)}{16\left(a^7c^3x^4+2\,a^5c^3x^2+a^3c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(a^2*x^2 + (a^4*x^4 + 2*a^2*x^2 + 1)*arctan(a*x)^2 + 2*(a^3*x^3 - a*x)
*arctan(a*x))/(a^7*c^3*x^4 + 2*a^5*c^3*x^2 + a^3*c^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^2\,\mathrm{atan}(ax)}{a^6x^6+3a^4x^4+3a^2x^2+1}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(a*x)/(a**2*c*x**2+c)**3,x)
```

```
[Out] Integral(x**2*atan(a*x)/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1), x)/c**
3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \arctan(ax)}{\left(a^2cx^2+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] integrate(x^2*arctan(a*x)/(a^2*c*x^2 + c)^3, x)
```

3.194 $\int \frac{x \tan^{-1}(ax)}{(c+a^2cx^2)^3} dx$

Optimal. Leaf size=84

$$\frac{3x}{32ac^3(a^2x^2+1)}+\frac{x}{16ac^3(a^2x^2+1)^2}-\frac{\tan^{-1}(ax)}{4a^2c^3(a^2x^2+1)^2}+\frac{3\tan^{-1}(ax)}{32a^2c^3}$$

```
[Out] x/(16*a*c^3*(1 + a^2*x^2)^2) + (3*x)/(32*a*c^3*(1 + a^2*x^2)) + (3*ArcTan[a
*x])/(32*a^2*c^3) - ArcTan[a*x]/(4*a^2*c^3*(1 + a^2*x^2)^2)
```

Rubi [A] time = 0.0496819, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4930, 199, 205}

$$\frac{3x}{32ac^3(a^2x^2+1)}+\frac{x}{16ac^3(a^2x^2+1)^2}-\frac{\tan^{-1}(ax)}{4a^2c^3(a^2x^2+1)^2}+\frac{3\tan^{-1}(ax)}{32a^2c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTan[a*x])/(c + a^2*c*x^2)^3,x]
```

```
[Out] x/(16*a*c^3*(1 + a^2*x^2)^2) + (3*x)/(32*a*c^3*(1 + a^2*x^2)) + (3*ArcTan[a
*x])/(32*a^2*c^3) - ArcTan[a*x]/(4*a^2*c^3*(1 + a^2*x^2)^2)
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^3}\,dx &= -\frac{\tan^{-1}(ax)}{4a^2c^3\left(1+a^2x^2\right)^2}+\frac{\int \frac{1}{\left(c+a^2cx^2\right)^3}\,dx}{4a}\\
&= \frac{x}{16ac^3\left(1+a^2x^2\right)^2}-\frac{\tan^{-1}(ax)}{4a^2c^3\left(1+a^2x^2\right)^2}+\frac{3\int \frac{1}{\left(c+a^2cx^2\right)^2}\,dx}{16ac}\\
&= \frac{x}{16ac^3\left(1+a^2x^2\right)^2}+\frac{3x}{32ac^3\left(1+a^2x^2\right)}-\frac{\tan^{-1}(ax)}{4a^2c^3\left(1+a^2x^2\right)^2}+\frac{3\int \frac{1}{c+a^2cx^2}\,dx}{32ac^2}\\
&= \frac{x}{16ac^3\left(1+a^2x^2\right)^2}+\frac{3x}{32ac^3\left(1+a^2x^2\right)}+\frac{3\tan^{-1}(ax)}{32a^2c^3}-\frac{\tan^{-1}(ax)}{4a^2c^3\left(1+a^2x^2\right)^2}
\end{aligned}$$

Mathematica [A] time = 0.0454185, size = 55, normalized size = 0.65

$$\frac{ax\left(3a^2x^2+5\right)+\left(3a^4x^4+6a^2x^2-5\right)\tan^{-1}(ax)}{32c^3\left(a^3x^2+a\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcTan[a*x])/(c + a^2*c*x^2)^3,x]
```

```
[Out] (a*x*(5 + 3*a^2*x^2) + (-5 + 6*a^2*x^2 + 3*a^4*x^4)*ArcTan[a*x])/(32*c^3*(a
 + a^3*x^2)^2)
```

Maple [A] time = 0.029, size = 77, normalized size = 0.9

$$\frac{x}{16\,ac^3\left(a^2x^2+1\right)^2}+\frac{3\,x}{32\,ac^3\left(a^2x^2+1\right)}+\frac{3\,\arctan(ax)}{32\,a^2c^3}-\frac{\arctan(ax)}{4\,a^2c^3\left(a^2x^2+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(a*x)/(a^2*c*x^2+c)^3,x)
```

```
[Out] 1/16*x/a/c^3/(a^2*x^2+1)^2+3/32*x/a/c^3/(a^2*x^2+1)+3/32*arctan(a*x)/a^2/c^
3-1/4*arctan(a*x)/a^2/c^3/(a^2*x^2+1)^2
```

Maxima [A] time = 1.48182, size = 116, normalized size = 1.38

$$\frac{\frac{3\,a^2x^3+5\,x}{a^4c^2x^4+2\,a^2c^2x^2+c^2}+\frac{3\,\arctan(ax)}{ac^2}}{32\,ac}-\frac{\arctan\left(ax\right)}{4\left(a^2cx^2+c\right)^2a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/32*((3*a^2*x^3 + 5*x)/(a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2) + 3*arctan(a*x)
/(a*c^2))/(a*c) - 1/4*arctan(a*x)/((a^2*c*x^2 + c)^2*a^2*c)
```

Fricas [A] time = 1.54218, size = 146, normalized size = 1.74

$$\frac{3\,a^3x^3+5\,ax+\left(3\,a^4x^4+6\,a^2x^2-5\right)\arctan\left(ax\right)}{32\left(a^6c^3x^4+2\,a^4c^3x^2+a^2c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 1/32*(3*a^3*x^3 + 5*a*x + (3*a^4*x^4 + 6*a^2*x^2 - 5)*arctan(a*x))/(a^6*c^3
*x^4 + 2*a^4*c^3*x^2 + a^2*c^3)
```

Sympy [A] time = 3.81606, size = 235, normalized size = 2.8

$$\begin{cases}\frac{3a^4x^4\operatorname{atan}(ax)}{32a^6c^3x^4+64a^4c^3x^2+32a^2c^3}+\frac{3a^3x^3}{32a^6c^3x^4+64a^4c^3x^2+32a^2c^3}+\frac{6a^2x^2\operatorname{atan}(ax)}{32a^6c^3x^4+64a^4c^3x^2+32a^2c^3}+\frac{5ax}{32a^6c^3x^4+64a^4c^3x^2+32a^2c^3}-\frac{5\operatorname{atan}(ax)}{32a^6c^3x^4+64a^4c^3x^2+32a^2c \\ \tilde{\infty}\left(\frac{x^2\operatorname{atan}(ax)}{2}-\frac{x}{2a}+\frac{\operatorname{atan}(ax)}{2a^2}\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(a*x)/(a**2*c*x**2+c)**3,x)
```

```
[Out] Piecewise((3*a**4*x**4*atan(a*x)/(32*a**6*c**3*x**4 + 64*a**4*c**3*x**2 + 3
2*a**2*c**3) + 3*a**3*x**3/(32*a**6*c**3*x**4 + 64*a**4*c**3*x**2 + 32*a**2
*c**3) + 6*a**2*x**2*atan(a*x)/(32*a**6*c**3*x**4 + 64*a**4*c**3*x**2 + 32*
a**2*c**3) + 5*a*x/(32*a**6*c**3*x**4 + 64*a**4*c**3*x**2 + 32*a**2*c**3) -
 5*atan(a*x)/(32*a**6*c**3*x**4 + 64*a**4*c**3*x**2 + 32*a**2*c**3), Ne(c,
0)), (zoo*(x**2*atan(a*x)/2 - x/(2*a) + atan(a*x)/(2*a**2)), True))
```

Giac [A] time = 1.18818, size = 92, normalized size = 1.1

$$\frac{3\arctan(ax)}{32\,a^2c^3} - \frac{\arctan(ax)}{4\left(a^2cx^2+c\right)^2a^2c} + \frac{3\,a^2x^3+5\,x}{32\left(a^2x^2+1\right)^2ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 3/32*arctan(a*x)/(a^2*c^3) - 1/4*arctan(a*x)/((a^2*c*x^2 + c)^2*a^2*c) + 1/
32*(3*a^2*x^3 + 5*x)/((a^2*x^2 + 1)^2*a*c^3)
```

3.195 $\int \frac{\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^3} \, dx$

Optimal. Leaf size=105

$$\frac{3}{16ac^3\left(a^2x^2+1\right)}+\frac{1}{16ac^3\left(a^2x^2+1\right)^2}+\frac{3x\tan^{-1}(ax)}{8c^3\left(a^2x^2+1\right)}+\frac{x\tan^{-1}(ax)}{4c^3\left(a^2x^2+1\right)^2}+\frac{3\tan^{-1}(ax)^2}{16ac^3}$$

```
[Out] 1/(16*a*c^3*(1 + a^2*x^2)^2) + 3/(16*a*c^3*(1 + a^2*x^2)) + (x*ArcTan[a*x])
/(4*c^3*(1 + a^2*x^2)^2) + (3*x*ArcTan[a*x])/(8*c^3*(1 + a^2*x^2)) + (3*Arc
Tan[a*x]^2)/(16*a*c^3)
```

Rubi [A] time = 0.0461152, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {4896, 4892, 261}

$$\frac{3}{16ac^3\left(a^2x^2+1\right)}+\frac{1}{16ac^3\left(a^2x^2+1\right)^2}+\frac{3x\tan^{-1}(ax)}{8c^3\left(a^2x^2+1\right)}+\frac{x\tan^{-1}(ax)}{4c^3\left(a^2x^2+1\right)^2}+\frac{3\tan^{-1}(ax)^2}{16ac^3}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]/(c + a^2*c*x^2)^3,x]
```

```
[Out] 1/(16*a*c^3*(1 + a^2*x^2)^2) + 3/(16*a*c^3*(1 + a^2*x^2)) + (x*ArcTan[a*x])
/(4*c^3*(1 + a^2*x^2)^2) + (3*x*ArcTan[a*x])/(8*c^3*(1 + a^2*x^2)) + (3*Arc
Tan[a*x]^2)/(16*a*c^3)
```

Rule 4896

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbol
] :> Simp[(b*(d + e*x^2)^(q + 1))/(4*c*d*(q + 1)^2), x] + (Dist[(2*q + 3)/(
2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]), x], x] - Simp[(x
*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]))/(2*d*(q + 1)), x]) /; FreeQ[{a, b
, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -3/2]
```

Rule 4892

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[(x*(a + b*ArcTan[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*c*
p)/2, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(a +
```

```
b*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
NeQ[p, -1]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^3}\,dx = \frac{1}{16ac^3\left(1+a^2x^2\right)^2}+\frac{x\tan^{-1}(ax)}{4c^3\left(1+a^2x^2\right)^2}+\frac{3\int \frac{\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^2}\,dx}{4c}$$
$$= \frac{1}{16ac^3\left(1+a^2x^2\right)^2}+\frac{x\tan^{-1}(ax)}{4c^3\left(1+a^2x^2\right)^2}+\frac{3x\tan^{-1}(ax)}{8c^3\left(1+a^2x^2\right)}+\frac{3\tan^{-1}(ax)^2}{16ac^3}-\frac{(3a)\int \frac{x}{\left(c+a^2cx^2\right)^2}\,dx}{8c}$$
$$= \frac{1}{16ac^3\left(1+a^2x^2\right)^2}+\frac{3}{16ac^3\left(1+a^2x^2\right)}+\frac{x\tan^{-1}(ax)}{4c^3\left(1+a^2x^2\right)^2}+\frac{3x\tan^{-1}(ax)}{8c^3\left(1+a^2x^2\right)}+\frac{3\tan^{-1}(ax)^2}{16ac^3}$$

Mathematica [A] time = 0.0256976, size = 68, normalized size = 0.65

$$\frac{3a^2x^2+2ax\left(3a^2x^2+5\right)\tan^{-1}(ax)+3\left(a^2x^2+1\right)^2\tan^{-1}(ax)^2+4}{16ac^3\left(a^2x^2+1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]/(c + a^2*c*x^2)^3,x]
```

```
[Out] (4 + 3*a^2*x^2 + 2*a*x*(5 + 3*a^2*x^2)*ArcTan[a*x] + 3*(1 + a^2*x^2)^2*ArcT
an[a*x]^2)/(16*a*c^3*(1 + a^2*x^2)^2)
```

Maple [A] time = 0.038, size = 96, normalized size = 0.9

$$\frac{1}{16\,ac^3\left(a^2x^2+1\right)^2}+\frac{3}{16\,ac^3\left(a^2x^2+1\right)}+\frac{x\arctan(ax)}{4\,c^3\left(a^2x^2+1\right)^2}+\frac{3\,x\arctan(ax)}{8\,c^3\left(a^2x^2+1\right)}+\frac{3\,(\arctan(ax))^2}{16\,ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)/(a^2*c*x^2+c)^3,x)
```

```
[Out] 1/16/a/c^3/(a^2*x^2+1)^2+3/16/a/c^3/(a^2*x^2+1)+1/4*x*arctan(a*x)/c^3/(a^2*
x^2+1)^2+3/8*x*arctan(a*x)/c^3/(a^2*x^2+1)+3/16*arctan(a*x)^2/a/c^3
```

Maxima [A] time = 1.6343, size = 174, normalized size = 1.66

$$\frac{1}{8}\left(\frac{3\,a^{2}x^{3}+5\,x}{a^{4}c^{3}x^{4}+2\,a^{2}c^{3}x^{2}+c^{3}}+\frac{3\,\arctan\left(ax\right)}{ac^{3}}\right)\arctan\left(ax\right)+\frac{\left(3\,a^{2}x^{2}-3\,\left(a^{4}x^{4}+2\,a^{2}x^{2}+1\right)\arctan\left(ax\right)^{2}+4\right)a}{16\,\left(a^{6}c^{3}x^{4}+2\,a^{4}c^{3}x^{2}+a^{2}c^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((3*a^2*x^3 + 5*x)/(a^4*c^3*x^4 + 2*a^2*c^3*x^2 + c^3) + 3*arctan(a*x)/
(a*c^3))*arctan(a*x) + 1/16*(3*a^2*x^2 - 3*(a^4*x^4 + 2*a^2*x^2 + 1)*arctan
(a*x)^2 + 4)*a/(a^6*c^3*x^4 + 2*a^4*c^3*x^2 + a^2*c^3)
```

Fricas [A] time = 1.63665, size = 189, normalized size = 1.8

$$\frac{3\,a^{2}x^{2}+3\,\left(a^{4}x^{4}+2\,a^{2}x^{2}+1\right)\arctan\left(ax\right)^{2}+2\,\left(3\,a^{3}x^{3}+5\,ax\right)\arctan\left(ax\right)+4}{16\,\left(a^{5}c^{3}x^{4}+2\,a^{3}c^{3}x^{2}+ac^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(3*a^2*x^2 + 3*(a^4*x^4 + 2*a^2*x^2 + 1)*arctan(a*x)^2 + 2*(3*a^3*x^3
+ 5*a*x)*arctan(a*x) + 4)/(a^5*c^3*x^4 + 2*a^3*c^3*x^2 + a*c^3)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RecursionError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)/(a**2*c*x**2+c)**3,x)
```

```
[Out] Exception raised: RecursionError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(ax)}{\left(a^2cx^2+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)/(a^2*c*x^2 + c)^3, x)
```

3.196 $\int \frac{\tan^{-1}(ax)}{x(c+a^2cx^2)^3}\,dx$

Optimal. Leaf size=159

$$-\frac{i\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{2c^3}-\frac{11ax}{32c^3\left(a^2x^2+1\right)}-\frac{ax}{16c^3\left(a^2x^2+1\right)^2}+\frac{\tan^{-1}(ax)}{2c^3\left(a^2x^2+1\right)}+\frac{\tan^{-1}(ax)}{4c^3\left(a^2x^2+1\right)^2}-\frac{i\tan^{-1}(ax)^2}{2c^3}-$$

```
[Out] -(a*x)/(16*c^3*(1 + a^2*x^2)^2) - (11*a*x)/(32*c^3*(1 + a^2*x^2)) - (11*Arc
Tan[a*x])/(32*c^3) + ArcTan[a*x]/(4*c^3*(1 + a^2*x^2)^2) + ArcTan[a*x]/(2*c
^3*(1 + a^2*x^2)) - ((I/2)*ArcTan[a*x]^2)/c^3 + (ArcTan[a*x]*Log[2 - 2/(1 -
 I*a*x)])/c^3 - ((I/2)*PolyLog[2, -1 + 2/(1 - I*a*x)])/c^3
```

Rubi [A] time = 0.283952, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {4966, 4924, 4868, 2447, 4930, 199, 205}

$$-\frac{i\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{2c^3}-\frac{11ax}{32c^3\left(a^2x^2+1\right)}-\frac{ax}{16c^3\left(a^2x^2+1\right)^2}+\frac{\tan^{-1}(ax)}{2c^3\left(a^2x^2+1\right)}+\frac{\tan^{-1}(ax)}{4c^3\left(a^2x^2+1\right)^2}-\frac{i\tan^{-1}(ax)^2}{2c^3}-1$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]/(x*(c + a^2*c*x^2)^3),x]
```


```
[Out] -(a*x)/(16*c^3*(1 + a^2*x^2)^2) - (11*a*x)/(32*c^3*(1 + a^2*x^2)) - (11*Arc
Tan[a*x])/(32*c^3) + ArcTan[a*x]/(4*c^3*(1 + a^2*x^2)^2) + ArcTan[a*x]/(2*c
^3*(1 + a^2*x^2)) - ((I/2)*ArcTan[a*x]^2)/c^3 + (ArcTan[a*x]*Log[2 - 2/(1 -
 I*a*x)])/c^3 - ((I/2)*PolyLog[2, -1 + 2/(1 - I*a*x)])/c^3
```


Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*
x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^p
, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*q]
 && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)}{x\left(c+a^2cx^2\right)^3}\,dx = -\left(a^2\int \frac{x\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^3}\,dx\right) + \frac{\int \frac{\tan^{-1}(ax)}{x\left(c+a^2cx^2\right)^2}\,dx}{c}$$

$$= \frac{\tan^{-1}(ax)}{4c^3\left(1+a^2x^2\right)^2} - \frac{1}{4}a\int \frac{1}{\left(c+a^2cx^2\right)^3}\,dx + \frac{\int \frac{\tan^{-1}(ax)}{x\left(c+a^2cx^2\right)}\,dx}{c^2} - \frac{a^2\int \frac{x\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^2}\,dx}{c}$$

$$= -\frac{ax}{16c^3\left(1+a^2x^2\right)^2} + \frac{\tan^{-1}(ax)}{4c^3\left(1+a^2x^2\right)^2} + \frac{\tan^{-1}(ax)}{2c^3\left(1+a^2x^2\right)} - \frac{i\tan^{-1}(ax)^2}{2c^3} + \frac{i\int \frac{\tan^{-1}(ax)}{x(i+ax)}\,dx}{c^3} - \frac{(3a)\int}{}$$

$$= -\frac{ax}{16c^3\left(1+a^2x^2\right)^2} - \frac{11ax}{32c^3\left(1+a^2x^2\right)} + \frac{\tan^{-1}(ax)}{4c^3\left(1+a^2x^2\right)^2} + \frac{\tan^{-1}(ax)}{2c^3\left(1+a^2x^2\right)} - \frac{i\tan^{-1}(ax)^2}{2c^3} + \frac{\tan^{-1}}{}$$

$$= -\frac{ax}{16c^3\left(1+a^2x^2\right)^2} - \frac{11ax}{32c^3\left(1+a^2x^2\right)} - \frac{11\tan^{-1}(ax)}{32c^3} + \frac{\tan^{-1}(ax)}{4c^3\left(1+a^2x^2\right)^2} + \frac{\tan^{-1}(ax)}{2c^3\left(1+a^2x^2\right)} - \frac{i\tan}{2}$$

Mathematica [A] time = 0.221693, size = 90, normalized size = 0.57

$$-\frac{64i\text{PolyLog}\left(2,e^{2i\tan^{-1}(ax)}\right) + 64i\tan^{-1}(ax)^2 + 24\sin\left(2\tan^{-1}(ax)\right) + \sin\left(4\tan^{-1}(ax)\right) - 4\tan^{-1}(ax)\left(32\log\left(1-e^{2i}\right.\right.}{128c^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTan[a*x]/(x*(c + a^2*c*x^2)^3),x]
```

```
[Out] -((64*I)*ArcTan[a*x]^2 - 4*ArcTan[a*x]*(12*Cos[2*ArcTan[a*x]] + Cos[4*ArcTa
n[a*x]] + 32*Log[1 - E^((2*I)*ArcTan[a*x])]) + (64*I)*PolyLog[2, E^((2*I)*A
rcTan[a*x])] + 24*Sin[2*ArcTan[a*x]] + Sin[4*ArcTan[a*x]])/(128*c^3)
```

Maple [B] time = 0.102, size = 340, normalized size = 2.1

$$\frac{\arctan(ax)}{4c^3\left(a^2x^2+1\right)^2} - \frac{\arctan(ax)\ln\left(a^2x^2+1\right)}{2c^3} + \frac{\arctan(ax)}{2c^3\left(a^2x^2+1\right)} + \frac{\arctan(ax)\ln(ax)}{c^3} - \frac{11a^3x^3}{32c^3\left(a^2x^2+1\right)^2} - \frac{13ax}{32c^3\left(a^2x^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)/x/(a^2*c*x^2+c)^3,x)
```

```
[Out] 1/4*arctan(a*x)/c^3/(a^2*x^2+1)^2-1/2/c^3*arctan(a*x)*ln(a^2*x^2+1)+1/2*arc
tan(a*x)/c^3/(a^2*x^2+1)+1/c^3*arctan(a*x)*ln(a*x)-11/32/c^3/(a^2*x^2+1)^2*
x^3*a^3-13/32*a*x/c^3/(a^2*x^2+1)^2-11/32*arctan(a*x)/c^3+1/8*I/c^3*ln(a*x-
I)^2-1/4*I/c^3*ln(a*x-I)*ln(a^2*x^2+1)-1/4*I/c^3*ln(a*x+I)*ln(1/2*I*(a*x-I)
)+1/4*I/c^3*ln(a*x+I)*ln(a^2*x^2+1)-1/8*I/c^3*ln(a*x+I)^2+1/4*I/c^3*ln(a*x-
I)*ln(-1/2*I*(a*x+I))+1/2*I/c^3*dilog(1+I*a*x)-1/2*I/c^3*ln(a*x)*ln(1-I*a*x
)-1/2*I/c^3*dilog(1-I*a*x)+1/2*I/c^3*ln(a*x)*ln(1+I*a*x)+1/4*I/c^3*dilog(-1
/2*I*(a*x+I))-1/4*I/c^3*dilog(1/2*I*(a*x-I))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(ax)}{\left(a^2cx^2+c\right)^3 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] integrate(arctan(a*x)/((a^2*c*x^2 + c)^3*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\arctan(ax)}{a^6c^3x^7+3\,a^4c^3x^5+3\,a^2c^3x^3+c^3x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] integral(arctan(a*x)/(a^6*c^3*x^7 + 3*a^4*c^3*x^5 + 3*a^2*c^3*x^3 + c^3*x),
 x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RecursionError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)/x/(a**2*c*x**2+c)**3,x)
```

```
[Out] Exception raised: RecursionError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(ax)}{(a^2cx^2+c)^3 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)/((a^2*c*x^2 + c)^3*x), x)
```

3.197 $\int \frac{\tan^{-1}(ax)}{x^2(c+a^2cx^2)^3}\,dx$

Optimal. Leaf size=142

$$-\frac{7a}{16c^3(a^2x^2+1)}-\frac{a}{16c^3(a^2x^2+1)^2}-\frac{a\log(a^2x^2+1)}{2c^3}-\frac{7a^2x\tan^{-1}(ax)}{8c^3(a^2x^2+1)}-\frac{a^2x\tan^{-1}(ax)}{4c^3(a^2x^2+1)^2}+\frac{a\log(x)}{c^3}-\frac{15a\tan^{-1}(ax)^2}{16c^3}$$

```
[Out] -a/(16*c^3*(1 + a^2*x^2)^2) - (7*a)/(16*c^3*(1 + a^2*x^2)) - ArcTan[a*x]/(c
^3*x) - (a^2*x*ArcTan[a*x])/(4*c^3*(1 + a^2*x^2)^2) - (7*a^2*x*ArcTan[a*x])
/(8*c^3*(1 + a^2*x^2)) - (15*a*ArcTan[a*x]^2)/(16*c^3) + (a*Log[x])/c^3 - (
a*Log[1 + a^2*x^2])/(2*c^3)
```

Rubi [A] time = 0.262566, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 11, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.55, Rules used = {4966, 4918, 4852, 266, 36, 29, 31, 4884, 4892, 261, 4896}

$$-\frac{7a}{16c^3(a^2x^2+1)}-\frac{a}{16c^3(a^2x^2+1)^2}-\frac{a\log(a^2x^2+1)}{2c^3}-\frac{7a^2x\tan^{-1}(ax)}{8c^3(a^2x^2+1)}-\frac{a^2x\tan^{-1}(ax)}{4c^3(a^2x^2+1)^2}+\frac{a\log(x)}{c^3}-\frac{15a\tan^{-1}(ax)^2}{16c^3}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]/(x^2*(c + a^2*c*x^2)^3),x]
```

```
[Out] -a/(16*c^3*(1 + a^2*x^2)^2) - (7*a)/(16*c^3*(1 + a^2*x^2)) - ArcTan[a*x]/(c
^3*x) - (a^2*x*ArcTan[a*x])/(4*c^3*(1 + a^2*x^2)^2) - (7*a^2*x*ArcTan[a*x])
/(8*c^3*(1 + a^2*x^2)) - (15*a*ArcTan[a*x]^2)/(16*c^3) + (a*Log[x])/c^3 - (
a*Log[1 + a^2*x^2])/(2*c^3)
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*
x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^p
, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*q]
 && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 4918

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
```

```
x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/(d + e*x^2),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4892

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[(x*(a + b*ArcTan[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*c*
p)/2, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(a +
 b*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d, e},
```

```
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 4896

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbol
] :> Simp[(b*(d + e*x^2)^(q + 1))/(4*c*d*(q + 1)^2), x] + (Dist[(2*q + 3)/(
2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]), x], x] - Simp[(x
*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]))/(2*d*(q + 1)), x]) /; FreeQ[{a, b
, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -3/2]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)}{x^2\left(c+a^2cx^2\right)^3}\,dx = -\left(a^2\int \frac{\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^3}\,dx\right)+\frac{\int \frac{\tan^{-1}(ax)}{x^2\left(c+a^2cx^2\right)^2}\,dx}{c}$$

$$= -\frac{a}{16c^3\left(1+a^2x^2\right)^2}-\frac{a^2x\tan^{-1}(ax)}{4c^3\left(1+a^2x^2\right)^2}+\frac{\int \frac{\tan^{-1}(ax)}{x^2\left(c+a^2cx^2\right)}\,dx}{c^2}-\frac{\left(3a^2\right)\int \frac{\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^2}\,dx}{4c}-\frac{a^2\int \frac{\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^2}\,dx}{c}$$

$$= -\frac{a}{16c^3\left(1+a^2x^2\right)^2}-\frac{a^2x\tan^{-1}(ax)}{4c^3\left(1+a^2x^2\right)^2}-\frac{7a^2x\tan^{-1}(ax)}{8c^3\left(1+a^2x^2\right)}-\frac{7a\tan^{-1}(ax)^2}{16c^3}+\frac{\int \frac{\tan^{-1}(ax)}{x^2}\,dx}{c^3}-$$

$$= -\frac{a}{16c^3\left(1+a^2x^2\right)^2}-\frac{7a}{16c^3\left(1+a^2x^2\right)}-\frac{\tan^{-1}(ax)}{c^3x}-\frac{a^2x\tan^{-1}(ax)}{4c^3\left(1+a^2x^2\right)^2}-\frac{7a^2x\tan^{-1}(ax)}{8c^3\left(1+a^2x^2\right)}-\frac{15a\tan}{16}$$

$$= -\frac{a}{16c^3\left(1+a^2x^2\right)^2}-\frac{7a}{16c^3\left(1+a^2x^2\right)}-\frac{\tan^{-1}(ax)}{c^3x}-\frac{a^2x\tan^{-1}(ax)}{4c^3\left(1+a^2x^2\right)^2}-\frac{7a^2x\tan^{-1}(ax)}{8c^3\left(1+a^2x^2\right)}-\frac{15a\tan}{16}$$

$$= -\frac{a}{16c^3\left(1+a^2x^2\right)^2}-\frac{7a}{16c^3\left(1+a^2x^2\right)}-\frac{\tan^{-1}(ax)}{c^3x}-\frac{a^2x\tan^{-1}(ax)}{4c^3\left(1+a^2x^2\right)^2}-\frac{7a^2x\tan^{-1}(ax)}{8c^3\left(1+a^2x^2\right)}-\frac{15a\tan}{16}$$

$$= -\frac{a}{16c^3\left(1+a^2x^2\right)^2}-\frac{7a}{16c^3\left(1+a^2x^2\right)}-\frac{\tan^{-1}(ax)}{c^3x}-\frac{a^2x\tan^{-1}(ax)}{4c^3\left(1+a^2x^2\right)^2}-\frac{7a^2x\tan^{-1}(ax)}{8c^3\left(1+a^2x^2\right)}-\frac{15a\tan}{16}$$

Mathematica [A] time = 0.0939301, size = 118, normalized size = 0.83

$$\frac{ax\left(-7a^2x^2+16\left(a^2x^2+1\right)^2\log(x)-8\left(a^2x^2+1\right)^2\log\left(a^2x^2+1\right)-8\right)-15ax\left(a^2x^2+1\right)^2\tan^{-1}(ax)^2-2\left(15a^4x^4+25a\right.}{16c^3x\left(a^2x^2+1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]/(x^2*(c + a^2*c*x^2)^3),x]
```

```
[Out] (-2*(8 + 25*a^2*x^2 + 15*a^4*x^4)*ArcTan[a*x] - 15*a*x*(1 + a^2*x^2)^2*ArcT
an[a*x]^2 + a*x*(-8 - 7*a^2*x^2 + 16*(1 + a^2*x^2)^2*Log[x] - 8*(1 + a^2*x^
2)^2*Log[1 + a^2*x^2]))/(16*c^3*x*(1 + a^2*x^2)^2)
```

Maple [A] time = 0.047, size = 135, normalized size = 1.

$$-\frac{7\arctan(ax)a^4x^3}{8c^3\left(a^2x^2+1\right)^2}-\frac{9a^2x\arctan(ax)}{8c^3\left(a^2x^2+1\right)^2}-\frac{15a(\arctan(ax))^2}{16c^3}-\frac{\arctan(ax)}{c^3x}-\frac{a}{16c^3\left(a^2x^2+1\right)^2}-\frac{a\ln\left(a^2x^2+1\right)}{2c^3}-\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)/x^2/(a^2*c*x^2+c)^3,x)
```

```
[Out] -7/8/c^3*arctan(a*x)/(a^2*x^2+1)^2*a^4*x^3-9/8*a^2*x*arctan(a*x)/c^3/(a^2*x
^2+1)^2-15/16*a*arctan(a*x)^2/c^3-arctan(a*x)/c^3/x-1/16*a/c^3/(a^2*x^2+1)^
2-1/2*a*ln(a^2*x^2+1)/c^3-7/16*a/c^3/(a^2*x^2+1)+a/c^3*ln(a*x)
```

Maxima [A] time = 1.62712, size = 244, normalized size = 1.72

$$-\frac{1}{8}\left(\frac{15a^4x^4+25a^2x^2+8}{a^4c^3x^5+2a^2c^3x^3+c^3x}+\frac{15a\arctan(ax)}{c^3}\right)\arctan(ax)-\frac{\left(7a^2x^2-15\left(a^4x^4+2a^2x^2+1\right)\arctan(ax)^2+8\left(a^4x^4\right.\right.}{16\left(a^4c^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x^2/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -1/8*((15*a^4*x^4 + 25*a^2*x^2 + 8)/(a^4*c^3*x^5 + 2*a^2*c^3*x^3 + c^3*x) +
 15*a*arctan(a*x)/c^3)*arctan(a*x) - 1/16*(7*a^2*x^2 - 15*(a^4*x^4 + 2*a^2*
```

```
x^2 + 1)*arctan(a*x)^2 + 8*(a^4*x^4 + 2*a^2*x^2 + 1)*log(a^2*x^2 + 1) - 16*
(a^4*x^4 + 2*a^2*x^2 + 1)*log(x) + 8)*a/(a^4*c^3*x^4 + 2*a^2*c^3*x^2 + c^3)
```

Fricas [A] time = 1.73396, size = 333, normalized size = 2.35

$$-\frac{7\,a^3x^3+15\left(a^5x^5+2\,a^3x^3+ax\right)\arctan\left(ax\right)^2+8\,ax+2\left(15\,a^4x^4+25\,a^2x^2+8\right)\arctan\left(ax\right)+8\left(a^5x^5+2\,a^3x^3+ax\right)\mathrm{lo}}{16\left(a^4c^3x^5+2\,a^2c^3x^3+c^3x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x^2/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(7*a^3*x^3 + 15*(a^5*x^5 + 2*a^3*x^3 + a*x)*arctan(a*x)^2 + 8*a*x + 2
*(15*a^4*x^4 + 25*a^2*x^2 + 8)*arctan(a*x) + 8*(a^5*x^5 + 2*a^3*x^3 + a*x)*
log(a^2*x^2 + 1) - 16*(a^5*x^5 + 2*a^3*x^3 + a*x)*log(x))/(a^4*c^3*x^5 + 2*
a^2*c^3*x^3 + c^3*x)
```

Sympy [B] time = 4.04583, size = 602, normalized size = 4.24

$$\frac{16a^5x^5\log\left(x\right)}{16a^4c^3x^5+32a^2c^3x^3+16c^3x}-\frac{8a^5x^5\log\left(x^2+\frac{1}{a^2}\right)}{16a^4c^3x^5+32a^2c^3x^3+16c^3x}-\frac{15a^5x^5\,\mathrm{atan}^2\left(ax\right)}{16a^4c^3x^5+32a^2c^3x^3+16c^3x}-\frac{30a^4x^4\,\mathrm{atan}\left(ax\right)}{16a^4c^3x^5+32a^2c^3x^3+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)/x**2/(a**2*c*x**2+c)**3,x)
```

```
[Out] 16*a**5*x**5*log(x)/(16*a**4*c**3*x**5 + 32*a**2*c**3*x**3 + 16*c**3*x) - 8
*a**5*x**5*log(x**2 + a**(-2))/(16*a**4*c**3*x**5 + 32*a**2*c**3*x**3 + 16*
c**3*x) - 15*a**5*x**5*atan(a*x)**2/(16*a**4*c**3*x**5 + 32*a**2*c**3*x**3
+ 16*c**3*x) - 30*a**4*x**4*atan(a*x)/(16*a**4*c**3*x**5 + 32*a**2*c**3*x**
3 + 16*c**3*x) + 32*a**3*x**3*log(x)/(16*a**4*c**3*x**5 + 32*a**2*c**3*x**3
 + 16*c**3*x) - 16*a**3*x**3*log(x**2 + a**(-2))/(16*a**4*c**3*x**5 + 32*a*
*2*c**3*x**3 + 16*c**3*x) - 30*a**3*x**3*atan(a*x)**2/(16*a**4*c**3*x**5 +
32*a**2*c**3*x**3 + 16*c**3*x) - 7*a**3*x**3/(16*a**4*c**3*x**5 + 32*a**2*c
**3*x**3 + 16*c**3*x) - 50*a**2*x**2*atan(a*x)/(16*a**4*c**3*x**5 + 32*a**2
*c**3*x**3 + 16*c**3*x) + 16*a*x*log(x)/(16*a**4*c**3*x**5 + 32*a**2*c**3*x
**3 + 16*c**3*x) - 8*a*x*log(x**2 + a**(-2))/(16*a**4*c**3*x**5 + 32*a**2*c
**3*x**3 + 16*c**3*x) - 15*a*x*atan(a*x)**2/(16*a**4*c**3*x**5 + 32*a**2*c*
*3*x**3 + 16*c**3*x) - 8*a*x/(16*a**4*c**3*x**5 + 32*a**2*c**3*x**3 + 16*c*
```

```
*3*x) - 16*atan(a*x)/(16*a**4*c**3*x**5 + 32*a**2*c**3*x**3 + 16*c**3*x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(ax)}{(a^2cx^2+c)^3 x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x^2/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)/((a^2*c*x^2 + c)^3*x^2), x)
```

3.198 $\int \frac{\tan^{-1}(ax)}{x^3(c+a^2cx^2)^3}\,dx$

Optimal. Leaf size=205

$$\frac{3ia^2\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{2c^3}+\frac{19a^3x}{32c^3\left(a^2x^2+1\right)}+\frac{a^3x}{16c^3\left(a^2x^2+1\right)^2}-\frac{a^2\tan^{-1}(ax)}{c^3\left(a^2x^2+1\right)}-\frac{a^2\tan^{-1}(ax)}{4c^3\left(a^2x^2+1\right)^2}+\frac{3ia^2\tan^{-1}(ax)^2}{2c^3}$$

```
[Out] -a/(2*c^3*x) + (a^3*x)/(16*c^3*(1 + a^2*x^2)^2) + (19*a^3*x)/(32*c^3*(1 + a
^2*x^2)) + (3*a^2*ArcTan[a*x])/(32*c^3) - ArcTan[a*x]/(2*c^3*x^2) - (a^2*Ar
cTan[a*x])/(4*c^3*(1 + a^2*x^2)^2) - (a^2*ArcTan[a*x])/(c^3*(1 + a^2*x^2))
+ (((3*I)/2)*a^2*ArcTan[a*x]^2)/c^3 - (3*a^2*ArcTan[a*x]*Log[2 - 2/(1 - I*a
*x)])/c^3 + (((3*I)/2)*a^2*PolyLog[2, -1 + 2/(1 - I*a*x)])/c^3
```

Rubi [A] time = 0.760318, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 28, number of rules used = 11, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.55, Rules used = {4966, 4918, 4852, 325, 203, 4924, 4868, 2447, 4930, 199, 205}

$$\frac{3ia^2\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{2c^3}+\frac{19a^3x}{32c^3\left(a^2x^2+1\right)}+\frac{a^3x}{16c^3\left(a^2x^2+1\right)^2}-\frac{a^2\tan^{-1}(ax)}{c^3\left(a^2x^2+1\right)}-\frac{a^2\tan^{-1}(ax)}{4c^3\left(a^2x^2+1\right)^2}+\frac{3ia^2\tan^{-1}(ax)^2}{2c^3}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]/(x^3*(c + a^2*c*x^2)^3),x]
```


```
[Out] -a/(2*c^3*x) + (a^3*x)/(16*c^3*(1 + a^2*x^2)^2) + (19*a^3*x)/(32*c^3*(1 + a
^2*x^2)) + (3*a^2*ArcTan[a*x])/(32*c^3) - ArcTan[a*x]/(2*c^3*x^2) - (a^2*Ar
cTan[a*x])/(4*c^3*(1 + a^2*x^2)^2) - (a^2*ArcTan[a*x])/(c^3*(1 + a^2*x^2))
+ (((3*I)/2)*a^2*ArcTan[a*x]^2)/c^3 - (3*a^2*ArcTan[a*x]*Log[2 - 2/(1 - I*a
*x)])/c^3 + (((3*I)/2)*a^2*PolyLog[2, -1 + 2/(1 - I*a*x)])/c^3
```


Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*
x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^p
, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*q]
 && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 4918

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/(d + e*x^2),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)}{x^3\left(c+a^2cx^2\right)^3}\,dx = -\left(a^2\int \frac{\tan^{-1}(ax)}{x\left(c+a^2cx^2\right)^3}\,dx\right) + \frac{\int \frac{\tan^{-1}(ax)}{x^3\left(c+a^2cx^2\right)^2}\,dx}{c}$$

$$= a^4\int \frac{x\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^3}\,dx + \frac{\int \frac{\tan^{-1}(ax)}{x^3\left(c+a^2cx^2\right)}\,dx}{c^2} - 2\frac{a^2\int \frac{\tan^{-1}(ax)}{x\left(c+a^2cx^2\right)^2}\,dx}{c}$$

$$= -\frac{a^2\tan^{-1}(ax)}{4c^3\left(1+a^2x^2\right)^2} + \frac{1}{4}a^3\int \frac{1}{\left(c+a^2cx^2\right)^3}\,dx + \frac{\int \frac{\tan^{-1}(ax)}{x^3}\,dx}{c^3} - \frac{a^2\int \frac{\tan^{-1}(ax)}{x\left(c+a^2cx^2\right)}\,dx}{c^2} - 2\left(\frac{a^2\int \frac{\tan^{-1}}{x(c+a^2}}{c^2}\right.$$

$$= \frac{a^3x}{16c^3\left(1+a^2x^2\right)^2} - \frac{\tan^{-1}(ax)}{2c^3x^2} - \frac{a^2\tan^{-1}(ax)}{4c^3\left(1+a^2x^2\right)^2} + \frac{ia^2\tan^{-1}(ax)^2}{2c^3} + \frac{a\int \frac{1}{x^2\left(1+a^2x^2\right)}\,dx}{2c^3} - \frac{\left(ia^2\right)\int}{}$$

$$= -\frac{a}{2c^3x} + \frac{a^3x}{16c^3\left(1+a^2x^2\right)^2} + \frac{3a^3x}{32c^3\left(1+a^2x^2\right)} - \frac{\tan^{-1}(ax)}{2c^3x^2} - \frac{a^2\tan^{-1}(ax)}{4c^3\left(1+a^2x^2\right)^2} + \frac{ia^2\tan^{-1}(ax)^2}{2c^3} -$$

$$= -\frac{a}{2c^3x} + \frac{a^3x}{16c^3\left(1+a^2x^2\right)^2} + \frac{3a^3x}{32c^3\left(1+a^2x^2\right)} - \frac{13a^2\tan^{-1}(ax)}{32c^3} - \frac{\tan^{-1}(ax)}{2c^3x^2} - \frac{a^2\tan^{-1}(ax)}{4c^3\left(1+a^2x^2\right)^2}$$

Mathematica [A] time = 0.592833, size = 111, normalized size = 0.54

$$\frac{a^2\left(192i\text{PolyLog}\left(2,e^{2i\tan^{-1}(ax)}\right)+\tan^{-1}(ax)\left(-\frac{64}{a^2x^2}-384\log\left(1-e^{2i\tan^{-1}(ax)}\right)-80\cos\left(2\tan^{-1}(ax)\right)-4\cos\left(4\tan^{-1}\right.\right.\right.}{128c^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTan[a*x]/(x^3*(c + a^2*c*x^2)^3),x]
```

```
[Out] (a^2*(-64/(a*x) + (192*I)*ArcTan[a*x]^2 + ArcTan[a*x]*(-64 - 64/(a^2*x^2) -
 80*Cos[2*ArcTan[a*x]] - 4*Cos[4*ArcTan[a*x]] - 384*Log[1 - E^((2*I)*ArcTan
[a*x])]) + (192*I)*PolyLog[2, E^((2*I)*ArcTan[a*x])] + 40*Sin[2*ArcTan[a*x]
] + Sin[4*ArcTan[a*x]]))/(128*c^3)
```

Maple [B] time = 0.103, size = 415, normalized size = 2.

$$-\frac{a^{2}\arctan\left(ax\right)}{4c^{3}\left(a^{2}x^{2}+1\right)^{2}}+\frac{3a^{2}\arctan\left(ax\right)\ln\left(a^{2}x^{2}+1\right)}{2c^{3}}-\frac{a^{2}\arctan\left(ax\right)}{c^{3}\left(a^{2}x^{2}+1\right)}-\frac{\arctan\left(ax\right)}{2c^{3}x^{2}}-3\frac{a^{2}\arctan\left(ax\right)\ln\left(ax\right)}{c^{3}}+\frac{\frac{3i}{4}a^{2}\mathrm{dilo}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)/x^3/(a^2*c*x^2+c)^3,x)
```

```
[Out] -1/4*a^2*arctan(a*x)/c^3/(a^2*x^2+1)^2+3/2*a^2/c^3*arctan(a*x)*ln(a^2*x^2+1
)-a^2*arctan(a*x)/c^3/(a^2*x^2+1)-1/2*arctan(a*x)/c^3/x^2-3*a^2/c^3*arctan(
a*x)*ln(a*x)+3/4*I*a^2/c^3*dilog(1/2*I*(a*x-I))-3/2*I*a^2/c^3*dilog(1+I*a*x
)-3/8*I*a^2/c^3*ln(a*x-I)^2-3/4*I*a^2/c^3*ln(a*x+I)*ln(a^2*x^2+1)+3/8*I*a^2
/c^3*ln(a*x+I)^2+3/2*I*a^2/c^3*ln(a*x)*ln(1-I*a*x)+3/2*I*a^2/c^3*dilog(1-I*
a*x)-3/2*I*a^2/c^3*ln(a*x)*ln(1+I*a*x)+3/4*I*a^2/c^3*ln(a*x+I)*ln(1/2*I*(a*
x-I))-3/4*I*a^2/c^3*ln(a*x-I)*ln(-1/2*I*(a*x+I))-3/4*I*a^2/c^3*dilog(-1/2*I
*(a*x+I))+3/4*I*a^2/c^3*ln(a*x-I)*ln(a^2*x^2+1)+19/32*a^5/c^3/(a^2*x^2+1)^2
*x^3+21/32*a^3*x/c^3/(a^2*x^2+1)^2+3/32*a^2*arctan(a*x)/c^3-1/2*a/c^3/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\arctan\left(ax\right)}{\left(a^{2}cx^{2}+c\right)^{3}x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x^3/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] integrate(arctan(a*x)/((a^2*c*x^2 + c)^3*x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\arctan\left(ax\right)}{a^{6}c^{3}x^{9}+3a^{4}c^{3}x^{7}+3a^{2}c^{3}x^{5}+c^{3}x^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x^3/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] integral(arctan(a*x)/(a^6*c^3*x^9 + 3*a^4*c^3*x^7 + 3*a^2*c^3*x^5 + c^3*x^3
), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\operatorname{atan}(ax)}{a^6x^9+3a^4x^7+3a^2x^5+x^3}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)/x**3/(a**2*c*x**2+c)**3,x)
```

```
[Out] Integral(atan(a*x)/(a**6*x**9 + 3*a**4*x**7 + 3*a**2*x**5 + x**3), x)/c**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(ax)}{\left(a^2cx^2+c\right)^3x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x^3/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)/((a^2*c*x^2 + c)^3*x^3), x)
```

3.199 $\int \frac{\tan^{-1}(ax)}{x^4(c+a^2cx^2)^3} dx$

Optimal. Leaf size=183

$$\frac{11a^3}{16c^3(a^2x^2+1)}+\frac{a^3}{16c^3(a^2x^2+1)^2}+\frac{5a^3\log(a^2x^2+1)}{3c^3}+\frac{11a^4x\tan^{-1}(ax)}{8c^3(a^2x^2+1)}+\frac{a^4x\tan^{-1}(ax)}{4c^3(a^2x^2+1)^2}-\frac{10a^3\log(x)}{3c^3}+\frac{35a^3\tan^{-1}}{16c}$$

```
[Out] -a/(6*c^3*x^2) + a^3/(16*c^3*(1 + a^2*x^2)^2) + (11*a^3)/(16*c^3*(1 + a^2*x
^2)) - ArcTan[a*x]/(3*c^3*x^3) + (3*a^2*ArcTan[a*x])/(c^3*x) + (a^4*x*ArcTa
n[a*x])/(4*c^3*(1 + a^2*x^2)^2) + (11*a^4*x*ArcTan[a*x])/(8*c^3*(1 + a^2*x^
2)) + (35*a^3*ArcTan[a*x]^2)/(16*c^3) - (10*a^3*Log[x])/(3*c^3) + (5*a^3*Lo
g[1 + a^2*x^2])/(3*c^3)
```

Rubi [A] time = 0.68997, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 38, number of rules used = 12, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {4966, 4918, 4852, 266, 44, 36, 29, 31, 4884, 4892, 261, 4896}

$$\frac{11a^3}{16c^3(a^2x^2+1)}+\frac{a^3}{16c^3(a^2x^2+1)^2}+\frac{5a^3\log(a^2x^2+1)}{3c^3}+\frac{11a^4x\tan^{-1}(ax)}{8c^3(a^2x^2+1)}+\frac{a^4x\tan^{-1}(ax)}{4c^3(a^2x^2+1)^2}-\frac{10a^3\log(x)}{3c^3}+\frac{35a^3\tan^{-1}}{16c}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]/(x^4*(c + a^2*c*x^2)^3),x]
```


```
[Out] -a/(6*c^3*x^2) + a^3/(16*c^3*(1 + a^2*x^2)^2) + (11*a^3)/(16*c^3*(1 + a^2*x
^2)) - ArcTan[a*x]/(3*c^3*x^3) + (3*a^2*ArcTan[a*x])/(c^3*x) + (a^4*x*ArcTa
n[a*x])/(4*c^3*(1 + a^2*x^2)^2) + (11*a^4*x*ArcTan[a*x])/(8*c^3*(1 + a^2*x^
2)) + (35*a^3*ArcTan[a*x]^2)/(16*c^3) - (10*a^3*Log[x])/(3*c^3) + (5*a^3*Lo
g[1 + a^2*x^2])/(3*c^3)
```


Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*
x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^p
, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*q]
 && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 4918

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/(d + e*x^2),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
```

```
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4892

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[(x*(a + b*ArcTan[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*c*
p)/2, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(a +
 b*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 4896

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbol
] :> Simp[(b*(d + e*x^2)^(q + 1))/(4*c*d*(q + 1)^2), x] + (Dist[(2*q + 3)/(
2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]), x], x] - Simp[(x
*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]))/(2*d*(q + 1)), x]) /; FreeQ[{a, b
, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -3/2]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)}{x^4\left(c+a^2cx^2\right)^3}\,dx = -\left(a^2\int \frac{\tan^{-1}(ax)}{x^2\left(c+a^2cx^2\right)^3}\,dx\right) + \frac{\int \frac{\tan^{-1}(ax)}{x^4\left(c+a^2cx^2\right)^2}\,dx}{c}$$

$$= a^4\int \frac{\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^3}\,dx + \frac{\int \frac{\tan^{-1}(ax)}{x^4\left(c+a^2cx^2\right)}\,dx}{c^2} - 2\frac{a^2\int \frac{\tan^{-1}(ax)}{x^2\left(c+a^2cx^2\right)^2}\,dx}{c}$$

$$= \frac{a^3}{16c^3\left(1+a^2x^2\right)^2} + \frac{a^4x\tan^{-1}(ax)}{4c^3\left(1+a^2x^2\right)^2} + \frac{\int \frac{\tan^{-1}(ax)}{x^4}\,dx}{c^3} - \frac{a^2\int \frac{\tan^{-1}(ax)}{x^2\left(c+a^2cx^2\right)}\,dx}{c^2} + \frac{\left(3a^4\right)\int \frac{\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^2}\,dx}{4c}$$

$$= \frac{a^3}{16c^3\left(1+a^2x^2\right)^2} - \frac{\tan^{-1}(ax)}{3c^3x^3} + \frac{a^4x\tan^{-1}(ax)}{4c^3\left(1+a^2x^2\right)^2} + \frac{3a^4x\tan^{-1}(ax)}{8c^3\left(1+a^2x^2\right)} + \frac{3a^3\tan^{-1}(ax)^2}{16c^3} + \frac{a\int \frac{}{x^3(1}}{3}$$

$$= \frac{a^3}{16c^3\left(1+a^2x^2\right)^2} + \frac{3a^3}{16c^3\left(1+a^2x^2\right)} - \frac{\tan^{-1}(ax)}{3c^3x^3} + \frac{a^2\tan^{-1}(ax)}{c^3x} + \frac{a^4x\tan^{-1}(ax)}{4c^3\left(1+a^2x^2\right)^2} + \frac{3a^4x\tan^{-1}}{8c^3\left(1+\right.}$$

$$= \frac{a^3}{16c^3\left(1+a^2x^2\right)^2} + \frac{3a^3}{16c^3\left(1+a^2x^2\right)} - \frac{\tan^{-1}(ax)}{3c^3x^3} + \frac{a^2\tan^{-1}(ax)}{c^3x} + \frac{a^4x\tan^{-1}(ax)}{4c^3\left(1+a^2x^2\right)^2} + \frac{3a^4x\tan^{-1}}{8c^3\left(1+\right.}$$

$$= -\frac{a}{6c^3x^2} + \frac{a^3}{16c^3\left(1+a^2x^2\right)^2} + \frac{3a^3}{16c^3\left(1+a^2x^2\right)} - \frac{\tan^{-1}(ax)}{3c^3x^3} + \frac{a^2\tan^{-1}(ax)}{c^3x} + \frac{a^4x\tan^{-1}(ax)}{4c^3\left(1+a^2x^2\right)^2} +$$

$$= -\frac{a}{6c^3x^2} + \frac{a^3}{16c^3\left(1+a^2x^2\right)^2} + \frac{3a^3}{16c^3\left(1+a^2x^2\right)} - \frac{\tan^{-1}(ax)}{3c^3x^3} + \frac{a^2\tan^{-1}(ax)}{c^3x} + \frac{a^4x\tan^{-1}(ax)}{4c^3\left(1+a^2x^2\right)^2} +$$

Mathematica [A] time = 0.129204, size = 142, normalized size = 0.78

$$\frac{ax\left(25a^4x^4+20a^2x^2-160\left(a^3x^3+ax\right)^2\log(x)+80\left(a^3x^3+ax\right)^2\log\left(a^2x^2+1\right)-8\right)+105a^3x^3\left(a^2x^2+1\right)^2\tan^{-1}(ax)^2}{48c^3x^3\left(a^2x^2+1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]/(x^4*(c + a^2*c*x^2)^3),x]
```

```
[Out] (2*(-8 + 56*a^2*x^2 + 175*a^4*x^4 + 105*a^6*x^6)*ArcTan[a*x] + 105*a^3*x^3*
(1 + a^2*x^2)^2*ArcTan[a*x]^2 + a*x*(-8 + 20*a^2*x^2 + 25*a^4*x^4 - 160*(a*
x + a^3*x^3)^2*Log[x] + 80*(a*x + a^3*x^3)^2*Log[1 + a^2*x^2]))/(48*c^3*x^3
*(1 + a^2*x^2)^2)
```

Maple [A] time = 0.051, size = 170, normalized size = 0.9

$$\frac{11\,a^6\arctan(ax)\,x^3}{8\,c^3\left(a^2x^2+1\right)^2}+\frac{13\,a^4x\arctan(ax)}{8\,c^3\left(a^2x^2+1\right)^2}+\frac{35\,a^3\left(\arctan(ax)\right)^2}{16\,c^3}-\frac{\arctan(ax)}{3\,c^3x^3}+3\,\frac{a^2\arctan(ax)}{c^3x}+\frac{a^3}{16\,c^3\left(a^2x^2+1\right)^2}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)/x^4/(a^2*c*x^2+c)^3,x)
```

```
[Out] 11/8*a^6/c^3*arctan(a*x)/(a^2*x^2+1)^2*x^3+13/8*a^4*x*arctan(a*x)/c^3/(a^2*
x^2+1)^2+35/16*a^3*arctan(a*x)^2/c^3-1/3*arctan(a*x)/c^3/x^3+3*a^2*arctan(a
*x)/c^3/x+1/16*a^3/c^3/(a^2*x^2+1)^2+5/3*a^3*ln(a^2*x^2+1)/c^3+11/16*a^3/c^
3/(a^2*x^2+1)-1/6*a/c^3/x^2-10/3*a^3/c^3*ln(a*x)
```

Maxima [A] time = 1.68526, size = 301, normalized size = 1.64

$$\frac{1}{24}\left(\frac{105\,a^3\arctan(ax)}{c^3}+\frac{105\,a^6x^6+175\,a^4x^4+56\,a^2x^2-8}{a^4c^3x^7+2\,a^2c^3x^5+c^3x^3}\right)\arctan(ax)+\frac{\left(25\,a^4x^4+20\,a^2x^2-105\left(a^6x^6+2\,a^4x^4+a^2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x^4/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/24*(105*a^3*arctan(a*x)/c^3 + (105*a^6*x^6 + 175*a^4*x^4 + 56*a^2*x^2 - 8
)/(a^4*c^3*x^7 + 2*a^2*c^3*x^5 + c^3*x^3))*arctan(a*x) + 1/48*(25*a^4*x^4 +
 20*a^2*x^2 - 105*(a^6*x^6 + 2*a^4*x^4 + a^2*x^2)*arctan(a*x)^2 + 80*(a^6*x
^6 + 2*a^4*x^4 + a^2*x^2)*log(a^2*x^2 + 1) - 160*(a^6*x^6 + 2*a^4*x^4 + a^2
*x^2)*log(x) - 8)*a/(a^4*c^3*x^6 + 2*a^2*c^3*x^4 + c^3*x^2)
```

Fricas [A] time = 1.78557, size = 394, normalized size = 2.15

$$\frac{25\,a^5x^5+20\,a^3x^3+105\left(a^7x^7+2\,a^5x^5+a^3x^3\right)\arctan(ax)^2-8\,ax+2\left(105\,a^6x^6+175\,a^4x^4+56\,a^2x^2-8\right)\arctan(ax)+}{48\left(a^4c^3x^7+2\,a^2c^3x^5+c^3x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x^4/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 1/48*(25*a^5*x^5 + 20*a^3*x^3 + 105*(a^7*x^7 + 2*a^5*x^5 + a^3*x^3)*arctan(
a*x)^2 - 8*a*x + 2*(105*a^6*x^6 + 175*a^4*x^4 + 56*a^2*x^2 - 8)*arctan(a*x)
 + 80*(a^7*x^7 + 2*a^5*x^5 + a^3*x^3)*log(a^2*x^2 + 1) - 160*(a^7*x^7 + 2*a
^5*x^5 + a^3*x^3)*log(x))/(a^4*c^3*x^7 + 2*a^2*c^3*x^5 + c^3*x^3)
```

Sympy [B] time = 6.95688, size = 763, normalized size = 4.17

$$-\frac{640a^7x^7\log(x)}{192a^4c^3x^7+384a^2c^3x^5+192c^3x^3}+\frac{320a^7x^7\log\left(x^2+\frac{1}{a^2}\right)}{192a^4c^3x^7+384a^2c^3x^5+192c^3x^3}+\frac{420a^7x^7\operatorname{atan}^2(ax)}{192a^4c^3x^7+384a^2c^3x^5+192c^3x^3}-\frac{}{192a^4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)/x**4/(a**2*c*x**2+c)**3,x)
```

```
[Out] -640*a**7*x**7*log(x)/(192*a**4*c**3*x**7 + 384*a**2*c**3*x**5 + 192*c**3*x
**3) + 320*a**7*x**7*log(x**2 + a**(-2))/(192*a**4*c**3*x**7 + 384*a**2*c**
3*x**5 + 192*c**3*x**3) + 420*a**7*x**7*atan(a*x)**2/(192*a**4*c**3*x**7 +
384*a**2*c**3*x**5 + 192*c**3*x**3) - 25*a**7*x**7/(192*a**4*c**3*x**7 + 38
4*a**2*c**3*x**5 + 192*c**3*x**3) + 840*a**6*x**6*atan(a*x)/(192*a**4*c**3*
x**7 + 384*a**2*c**3*x**5 + 192*c**3*x**3) - 1280*a**5*x**5*log(x)/(192*a**
4*c**3*x**7 + 384*a**2*c**3*x**5 + 192*c**3*x**3) + 640*a**5*x**5*log(x**2
+ a**(-2))/(192*a**4*c**3*x**7 + 384*a**2*c**3*x**5 + 192*c**3*x**3) + 840*
a**5*x**5*atan(a*x)**2/(192*a**4*c**3*x**7 + 384*a**2*c**3*x**5 + 192*c**3*
x**3) + 50*a**5*x**5/(192*a**4*c**3*x**7 + 384*a**2*c**3*x**5 + 192*c**3*x*
*3) + 1400*a**4*x**4*atan(a*x)/(192*a**4*c**3*x**7 + 384*a**2*c**3*x**5 + 1
92*c**3*x**3) - 640*a**3*x**3*log(x)/(192*a**4*c**3*x**7 + 384*a**2*c**3*x*
*5 + 192*c**3*x**3) + 320*a**3*x**3*log(x**2 + a**(-2))/(192*a**4*c**3*x**7
 + 384*a**2*c**3*x**5 + 192*c**3*x**3) + 420*a**3*x**3*atan(a*x)**2/(192*a*
*4*c**3*x**7 + 384*a**2*c**3*x**5 + 192*c**3*x**3) + 55*a**3*x**3/(192*a**4
*c**3*x**7 + 384*a**2*c**3*x**5 + 192*c**3*x**3) + 448*a**2*x**2*atan(a*x)/
(192*a**4*c**3*x**7 + 384*a**2*c**3*x**5 + 192*c**3*x**3) - 32*a*x/(192*a**
4*c**3*x**7 + 384*a**2*c**3*x**5 + 192*c**3*x**3) - 64*atan(a*x)/(192*a**4*
c**3*x**7 + 384*a**2*c**3*x**5 + 192*c**3*x**3)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(ax)}{\left(a^2cx^2+c\right)^3 x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x^4/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)/((a^2*c*x^2 + c)^3*x^4), x)
```

3.200 $\int x^3\sqrt{c+a^2cx^2}\tan^{-1}(ax)\,dx$

Optimal. Leaf size=160

$$-\frac{x^3\sqrt{a^2cx^2+c}}{20a}+\frac{x\sqrt{a^2cx^2+c}}{24a^3}+\frac{1}{5}x^4\sqrt{a^2cx^2+c}\tan^{-1}(ax)+\frac{x^2\sqrt{a^2cx^2+c}\tan^{-1}(ax)}{15a^2}-\frac{2\sqrt{a^2cx^2+c}\tan^{-1}(ax)}{15a^4}+\frac{11\sqrt{}}{}$$

```
[Out] (x*Sqrt[c + a^2*c*x^2])/(24*a^3) - (x^3*Sqrt[c + a^2*c*x^2])/(20*a) - (2*Sq
rt[c + a^2*c*x^2]*ArcTan[a*x])/(15*a^4) + (x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a
*x])/(15*a^2) + (x^4*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/5 + (11*Sqrt[c]*ArcTa
nh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]])/(120*a^4)
```

Rubi [A] time = 0.271102, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4946, 4952, 321, 217, 206, 4930}

$$-\frac{x^3\sqrt{a^2cx^2+c}}{20a}+\frac{x\sqrt{a^2cx^2+c}}{24a^3}+\frac{1}{5}x^4\sqrt{a^2cx^2+c}\tan^{-1}(ax)+\frac{x^2\sqrt{a^2cx^2+c}\tan^{-1}(ax)}{15a^2}-\frac{2\sqrt{a^2cx^2+c}\tan^{-1}(ax)}{15a^4}+\frac{11\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x],x]
```



```
[Out] (x*Sqrt[c + a^2*c*x^2])/(24*a^3) - (x^3*Sqrt[c + a^2*c*x^2])/(20*a) - (2*Sq
rt[c + a^2*c*x^2]*ArcTan[a*x])/(15*a^4) + (x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a
*x])/(15*a^2) + (x^4*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/5 + (11*Sqrt[c]*ArcTa
nh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]])/(120*a^4)
```


Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*ArcTan[c*x
]))/(f*(m + 2)), x] + (Dist[d/(m + 2), Int[((f*x)^m*(a + b*ArcTan[c*x]))/Sq
rt[d + e*x^2], x], x] - Dist[(b*c*d)/(f*(m + 2)), Int[(f*x)^(m + 1)/Sqrt[d
+ e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && Ne
Q[m, -2]
```

Rule 4952

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b*
```

```
ArcTan[c*x])^p)/(c^2*d*m), x] + (-Dist[(b*f*p)/(c*m), Int[((f*x)^(m - 1)*(a
 + b*ArcTan[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] - Dist[(f^2*(m - 1))/(c^
2*m), Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/Sqrt[d + e*x^2], x], x]) /;
 FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rubi steps

$$\int x^3\sqrt{c+a^2cx^2}\tan^{-1}(ax)\,dx = \frac{1}{5}x^4\sqrt{c+a^2cx^2}\tan^{-1}(ax) + \frac{1}{5}c\int \frac{x^3\tan^{-1}(ax)}{\sqrt{c+a^2cx^2}}\,dx - \frac{1}{5}(ac)\int \frac{x^4}{\sqrt{c+a^2cx^2}}\,dx$$

$$= -\frac{x^3\sqrt{c+a^2cx^2}}{20a} + \frac{x^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{15a^2} + \frac{1}{5}x^4\sqrt{c+a^2cx^2}\tan^{-1}(ax) - \frac{(2c)\int \frac{x\tan^{-1}}{\sqrt{c+a^2}}}{15a^2}$$

$$= \frac{x\sqrt{c+a^2cx^2}}{24a^3} - \frac{x^3\sqrt{c+a^2cx^2}}{20a} - \frac{2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{15a^4} + \frac{x^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{15a^2} + \frac{1}{5}$$

$$= \frac{x\sqrt{c+a^2cx^2}}{24a^3} - \frac{x^3\sqrt{c+a^2cx^2}}{20a} - \frac{2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{15a^4} + \frac{x^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{15a^2} + \frac{1}{5}$$

$$= \frac{x\sqrt{c+a^2cx^2}}{24a^3} - \frac{x^3\sqrt{c+a^2cx^2}}{20a} - \frac{2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{15a^4} + \frac{x^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{15a^2} + \frac{1}{5}$$

Mathematica [A] time = 0.123572, size = 105, normalized size = 0.66

$$\frac{ax\left(5-6a^2x^2\right)\sqrt{a^2cx^2+c}+11\sqrt{c}\log\left(\sqrt{c}\sqrt{a^2cx^2+c}+acx\right)+8\left(3a^4x^4+a^2x^2-2\right)\sqrt{a^2cx^2+c}\tan^{-1}(ax)}{120a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x],x]
```

```
[Out] (a*x*(5 - 6*a^2*x^2)*Sqrt[c + a^2*c*x^2] + 8*Sqrt[c + a^2*c*x^2]*(-2 + a^2*
x^2 + 3*a^4*x^4)*ArcTan[a*x] + 11*Sqrt[c]*Log[a*c*x + Sqrt[c]*Sqrt[c + a^2*
c*x^2]])/(120*a^4)
```

Maple [C] time = 0.776, size = 176, normalized size = 1.1

$$\frac{24\arctan(ax)x^4a^4-6a^3x^3+8\arctan(ax)a^2x^2+5ax-16\arctan(ax)}{120a^4}\sqrt{c(ax-i)(ax+i)} - \frac{11}{120a^4}\sqrt{c(ax-i)(ax+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctan(a*x)*(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] 1/120/a^4*(c*(a*x-I)*(a*x+I))^(1/2)*(24*arctan(a*x)*x^4*a^4-6*a^3*x^3+8*arc
tan(a*x)*a^2*x^2+5*a*x-16*arctan(a*x))-11/120/a^4*(c*(a*x-I)*(a*x+I))^(1/2)
```

```
*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)-I)/(a^2*x^2+1)^(1/2)+11/120/a^4*(c*(a*x-I)*
(a*x+I))^(1/2)*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)+I)/(a^2*x^2+1)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)*(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.74631, size = 228, normalized size = 1.42

$$-\frac{2\left(6\,a^{3}x^{3}-5\,ax-8\left(3\,a^{4}x^{4}+a^{2}x^{2}-2\right)\arctan\left(ax\right)\right)\sqrt{a^{2}cx^{2}+c}-11\,\sqrt{c}\log\left(-2\,a^{2}cx^{2}-2\,\sqrt{a^{2}cx^{2}+c}a\sqrt{c}x-c\right)}{240\,a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)*(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/240*(2*(6*a^3*x^3 - 5*a*x - 8*(3*a^4*x^4 + a^2*x^2 - 2)*arctan(a*x))*sqr
t(a^2*c*x^2 + c) - 11*sqrt(c)*log(-2*a^2*c*x^2 - 2*sqrt(a^2*c*x^2 + c)*a*sq
rt(c)*x - c))/a^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^{3}\sqrt{c\left(a^{2}x^{2}+1\right)}\operatorname{atan}\left(ax\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atan(a*x)*(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(c*(a**2*x**2 + 1))*atan(a*x), x)
```

Giac [A] time = 1.18046, size = 144, normalized size = 0.9

$$-\frac{\sqrt{a^2cx^2+c}\left(6\,a^2x^2-5\right)x+\frac{11\,\sqrt{c}\log\left(\left|-\sqrt{a^2c}x+\sqrt{a^2cx^2+c}\right|\right)}{|a|}}{120\,a^3}+\frac{\left(3\,\left(a^2cx^2+c\right)^{\frac{5}{2}}-5\,\left(a^2cx^2+c\right)^{\frac{3}{2}}c\right)\arctan\left(ax\right)}{15\,a^4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)*(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/120*(sqrt(a^2*c*x^2 + c)*(6*a^2*x^2 - 5)*x + 11*sqrt(c)*log(abs(-sqrt(a^
2*c)*x + sqrt(a^2*c*x^2 + c)))/abs(a))/a^3 + 1/15*(3*(a^2*c*x^2 + c)^(5/2)
- 5*(a^2*c*x^2 + c)^(3/2)*c)*arctan(a*x)/(a^4*c^2)
```

3.201 $\int x^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)\,dx$

Optimal. Leaf size=298

$$-\frac{ic\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{8a^3\sqrt{a^2cx^2+c}}+\frac{ic\sqrt{a^2x^2+1}\text{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{8a^3\sqrt{a^2cx^2+c}}-\frac{\left(a^2cx^2+c\right)^{3/2}}{12a^3c}+\frac{\sqrt{a^2cx^2+c}}{8a^3}+\frac{1}{4}x^3\sqrt{a^2cx^2+}$$

```
[Out] Sqrt[c + a^2*c*x^2]/(8*a^3) - (c + a^2*c*x^2)^(3/2)/(12*a^3*c) + (x*Sqrt[c
+ a^2*c*x^2]*ArcTan[a*x])/(8*a^2) + (x^3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/4
 + ((I/4)*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I
*a*x]])/(a^3*Sqrt[c + a^2*c*x^2]) - ((I/8)*c*Sqrt[1 + a^2*x^2]*PolyLog[2, (
(-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/(a^3*Sqrt[c + a^2*c*x^2]) + ((I/8)*
c*Sqrt[1 + a^2*x^2]*PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/(a^3*S
qrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.269668, antiderivative size = 298, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {4946, 4952, 261, 4890, 4886, 266, 43}

$$-\frac{ic\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{8a^3\sqrt{a^2cx^2+c}}+\frac{ic\sqrt{a^2x^2+1}\text{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{8a^3\sqrt{a^2cx^2+c}}-\frac{\left(a^2cx^2+c\right)^{3/2}}{12a^3c}+\frac{\sqrt{a^2cx^2+c}}{8a^3}+\frac{1}{4}x^3\sqrt{a^2cx^2+}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x],x]
```



```
[Out] Sqrt[c + a^2*c*x^2]/(8*a^3) - (c + a^2*c*x^2)^(3/2)/(12*a^3*c) + (x*Sqrt[c
+ a^2*c*x^2]*ArcTan[a*x])/(8*a^2) + (x^3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/4
 + ((I/4)*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I
*a*x]])/(a^3*Sqrt[c + a^2*c*x^2]) - ((I/8)*c*Sqrt[1 + a^2*x^2]*PolyLog[2, (
(-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/(a^3*Sqrt[c + a^2*c*x^2]) + ((I/8)*
c*Sqrt[1 + a^2*x^2]*PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/(a^3*S
qrt[c + a^2*c*x^2])
```


Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*ArcTan[c*x
]))/(f*(m + 2)), x] + (Dist[d/(m + 2), Int[((f*x)^m*(a + b*ArcTan[c*x]))/Sq
rt[d + e*x^2], x], x] - Dist[(b*c*d)/(f*(m + 2)), Int[(f*x)^(m + 1)/Sqrt[d
```

```
+ e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && Ne
Q[m, -2]
```

Rule 4952

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b*
ArcTan[c*x])^p)/(c^2*d*m), x] + (-Dist[(b*f*p)/(c*m), Int[((f*x)^(m - 1)*(a
 + b*ArcTan[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] - Dist[(f^2*(m - 1))/(c^
2*m), Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/Sqrt[d + e*x^2], x], x]) /;
 FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 4890

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4886

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[(-2*I*(a + b*ArcTan[c*x])*ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]])
/(c*Sqrt[d]), x] + (Simp[(I*b*PolyLog[2, -((I*Sqrt[1 + I*c*x])/Sqrt[1 - I*c
*x])])/(c*Sqrt[d]), x] - Simp[(I*b*PolyLog[2, (I*Sqrt[1 + I*c*x])/Sqrt[1 -
I*c*x]])/(c*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)\,dx = \frac{1}{4}x^3\sqrt{c+a^2cx^2}\tan^{-1}(ax)+\frac{1}{4}c\int\frac{x^2\tan^{-1}(ax)}{\sqrt{c+a^2cx^2}}\,dx-\frac{1}{4}(ac)\int\frac{x^3}{\sqrt{c+a^2cx^2}}\,dx$$

$$=\frac{x\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{8a^2}+\frac{1}{4}x^3\sqrt{c+a^2cx^2}\tan^{-1}(ax)-\frac{c\int\frac{\tan^{-1}(ax)}{\sqrt{c+a^2cx^2}}\,dx}{8a^2}-\frac{c\int\frac{x}{\sqrt{c+a^2cx^2}}\,dx}{8a}$$

$$=-\frac{\sqrt{c+a^2cx^2}}{8a^3}+\frac{x\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{8a^2}+\frac{1}{4}x^3\sqrt{c+a^2cx^2}\tan^{-1}(ax)-\frac{1}{8}(ac)\,\text{Subst}\Bigg(\int\Bigg($$

$$=\frac{\sqrt{c+a^2cx^2}}{8a^3}-\frac{\left(c+a^2cx^2\right)^{3/2}}{12a^3c}+\frac{x\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{8a^2}+\frac{1}{4}x^3\sqrt{c+a^2cx^2}\tan^{-1}(ax)+\frac{ic}{}$$

Mathematica [A] time = 2.83128, size = 278, normalized size = 0.93

$$\frac{\sqrt{c\left(a^2x^2+1\right)}\left(-6i\text{PolyLog}\left(2,-ie^{i\tan^{-1}(ax)}\right)+6i\text{PolyLog}\left(2,ie^{i\tan^{-1}(ax)}\right)-\frac{1}{4}\left(a^2x^2+1\right)^2\left(-\frac{2}{\sqrt{a^2x^2+1}}+3\tan^{-1}(ax)\left(-\frac{14}{\sqrt{a^2}}\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x],x]
```

```
[Out] (Sqrt[c*(1 + a^2*x^2)]*((-6*I)*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] + (6*I)*P
olyLog[2, I*E^(I*ArcTan[a*x])] - ((1 + a^2*x^2)^2*(-2/Sqrt[1 + a^2*x^2] - 6
*Cos[3*ArcTan[a*x]] + 3*ArcTan[a*x]*((-14*a*x)/Sqrt[1 + a^2*x^2] + 3*Log[1
- I*E^(I*ArcTan[a*x])] + 4*Cos[2*ArcTan[a*x]]*(Log[1 - I*E^(I*ArcTan[a*x])]
 - Log[1 + I*E^(I*ArcTan[a*x])]) + Cos[4*ArcTan[a*x]]*(Log[1 - I*E^(I*ArcTa
n[a*x])] - Log[1 + I*E^(I*ArcTan[a*x])]) - 3*Log[1 + I*E^(I*ArcTan[a*x])] +
 2*Sin[3*ArcTan[a*x]])))/4))/(48*a^3*Sqrt[1 + a^2*x^2])
```

Maple [A] time = 0.559, size = 199, normalized size = 0.7

$$\frac{6\arctan(ax)\,x^3a^3-2\,a^2x^2+3\,\arctan(ax)\,xa+1}{24\,a^3}\sqrt{c\,(ax-i)\,(ax+i)}+\frac{1}{8\,a^3}\sqrt{c\,(ax-i)\,(ax+i)}\Bigg(\arctan(ax)\ln\Bigg(1+i\,(1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(a*x)*(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] 1/24/a^3*(c*(a*x-I)*(a*x+I))^(1/2)*(6*arctan(a*x)*x^3*a^3-2*a^2*x^2+3*arcta
n(a*x)*x*a+1)+1/8*(c*(a*x-I)*(a*x+I))^(1/2)*(arctan(a*x)*ln(1+I*(1+I*a*x)/(
a^2*x^2+1)^(1/2))-arctan(a*x)*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*dilog(1
+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+I*dilog(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))/a
^3/(a^2*x^2+1)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)*(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{a^2cx^2 + c}x^2 \arctan(ax), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)*(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^2*arctan(a*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \sqrt{c\left(a^2x^2 + 1\right)} \operatorname{atan}(ax)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(a*x)*(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(c*(a**2*x**2 + 1))*atan(a*x), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)*(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.202 $\int x\sqrt{c+a^2cx^2}\tan^{-1}(ax)\,dx$

Optimal. Leaf size=86

$$-\frac{x\sqrt{a^2cx^2+c}}{6a}+\frac{\left(a^2cx^2+c\right)^{3/2}\tan^{-1}(ax)}{3a^2c}-\frac{\sqrt{c}\tanh^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{6a^2}$$

```
[Out] -(x*Sqrt[c + a^2*c*x^2])/(6*a) + ((c + a^2*c*x^2)^(3/2)*ArcTan[a*x])/(3*a^2
*c) - (Sqrt[c]*ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]])/(6*a^2)
```

Rubi [A] time = 0.0605705, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4930, 195, 217, 206}

$$-\frac{x\sqrt{a^2cx^2+c}}{6a}+\frac{\left(a^2cx^2+c\right)^{3/2}\tan^{-1}(ax)}{3a^2c}-\frac{\sqrt{c}\tanh^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{6a^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x],x]
```


```
[Out] -(x*Sqrt[c + a^2*c*x^2])/(6*a) + ((c + a^2*c*x^2)^(3/2)*ArcTan[a*x])/(3*a^2
*c) - (Sqrt[c]*ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]])/(6*a^2)
```


Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int x\sqrt{c+a^2cx^2}\tan^{-1}(ax)\,dx &= \frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}{3a^2c}-\frac{\int \sqrt{c+a^2cx^2}\,dx}{3a}\\ &= -\frac{x\sqrt{c+a^2cx^2}}{6a}+\frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}{3a^2c}-\frac{c\int \frac{1}{\sqrt{c+a^2cx^2}}\,dx}{6a}\\ &= -\frac{x\sqrt{c+a^2cx^2}}{6a}+\frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}{3a^2c}-\frac{c\,\mathrm{Subst}\left(\int \frac{1}{1-a^2cx^2}\,dx,x,\frac{x}{\sqrt{c+a^2cx^2}}\right)}{6a}\\ &= -\frac{x\sqrt{c+a^2cx^2}}{6a}+\frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}{3a^2c}-\frac{\sqrt{c}\tanh^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c+a^2cx^2}}\right)}{6a^2}\end{aligned}$$

Mathematica [A] time = 0.110979, size = 86, normalized size = 1.

$$-\frac{ax\sqrt{a^2cx^2+c}+\sqrt{c}\log\left(\sqrt{c}\sqrt{a^2cx^2+c}+acx\right)-2\left(a^2x^2+1\right)\sqrt{a^2cx^2+c}\tan^{-1}(ax)}{6a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x],x]
```

```
[Out] -(a*x*Sqrt[c + a^2*c*x^2] - 2*(1 + a^2*x^2)*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]
 + Sqrt[c]*Log[a*c*x + Sqrt[c]*Sqrt[c + a^2*c*x^2]])/(6*a^2)
```

Maple [C] time = 0.408, size = 156, normalized size = 1.8

$$\frac{2\arctan(ax)a^2x^2-ax+2\arctan(ax)}{6a^2}\sqrt{c(ax-i)(ax+i)}+\frac{1}{6a^2}\sqrt{c(ax-i)(ax+i)}\ln\left((1+iax)\frac{1}{\sqrt{a^2x^2+1}}-i\right)\frac{1}{\sqrt{a^2x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(a*x)*(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] 1/6/a^2*(c*(a*x-I)*(a*x+I))^(1/2)*(2*arctan(a*x)*a^2*x^2-a*x+2*arctan(a*x))
+1/6/a^2*(c*(a*x-I)*(a*x+I))^(1/2)*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)-I)/(a^2*x
^2+1)^(1/2)-1/6/a^2*(c*(a*x-I)*(a*x+I))^(1/2)*ln((1+I*a*x)/(a^2*x^2+1)^(1/2
)+I)/(a^2*x^2+1)^(1/2)
```

Maxima [B] time = 1.85979, size = 351, normalized size = 4.08

$$4\left(a^2x^2+1\right)^{\frac{3}{2}}\sqrt{c}\arctan\left(ax\right)-2\left(a^4x^4+10\,a^2x^2+9\right)^{\frac{1}{4}}\left(ax\cos\left(\frac{1}{2}\,\arctan\left(4\,ax,-a^2x^2+3\right)\right)+2\,\sin\left(\frac{1}{2}\,\arctan\left(4\,ax,-\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)*(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/12*(4*(a^2*x^2 + 1)^(3/2)*sqrt(c)*arctan(a*x) - 2*(a^4*x^4 + 10*a^2*x^2 +
 9)^(1/4)*(a*x*cos(1/2*arctan2(4*a*x, -a^2*x^2 + 3)) + 2*sin(1/2*arctan2(4*
a*x, -a^2*x^2 + 3)))*sqrt(c) + sqrt(c)*(arctan2((a^4*x^4 + 10*a^2*x^2 + 9)^
(1/4)*sin(1/2*arctan2(4*a*x, a^2*x^2 - 3)) + 2, a*x + (a^4*x^4 + 10*a^2*x^2
 + 9)^(1/4)*cos(1/2*arctan2(4*a*x, a^2*x^2 - 3))) + arctan2((a^4*x^4 + 10*a
^2*x^2 + 9)^(1/4)*sin(1/2*arctan2(4*a*x, a^2*x^2 - 3)) - 2, -a*x + (a^4*x^4
 + 10*a^2*x^2 + 9)^(1/4)*cos(1/2*arctan2(4*a*x, a^2*x^2 - 3)))))/a^2
```

Fricas [A] time = 1.73939, size = 188, normalized size = 2.19

$$-\frac{2\,\sqrt{a^2cx^2+c}\left(ax-2\left(a^2x^2+1\right)\arctan\left(ax\right)\right)-\sqrt{c}\log\left(-2\,a^2cx^2+2\,\sqrt{a^2cx^2+c}a\sqrt{c}x-c\right)}{12\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)*(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/12*(2*sqrt(a^2*c*x^2 + c)*(a*x - 2*(a^2*x^2 + 1)*arctan(a*x)) - sqrt(c)*
log(-2*a^2*c*x^2 + 2*sqrt(a^2*c*x^2 + c)*a*sqrt(c)*x - c))/a^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{c\left(a^2x^2+1\right)}\operatorname{atan}\left(ax\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(a*x)*(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x*sqrt(c*(a**2*x**2 + 1))*atan(a*x), x)
```

Giac [A] time = 1.15356, size = 107, normalized size = 1.24

$$-\frac{\sqrt{a^2cx^2+c}\,x-\frac{\sqrt{c}\log\left(\left|-\sqrt{a^2c}\,x+\sqrt{a^2cx^2+c}\right|\right)}{|a|}}{6\,a}+\frac{\left(a^2cx^2+c\right)^{\frac{3}{2}}\arctan\left(ax\right)}{3\,a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)*(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/6*(sqrt(a^2*c*x^2 + c)*x - sqrt(c)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x
^2 + c)))/abs(a))/a + 1/3*(a^2*c*x^2 + c)^(3/2)*arctan(a*x)/(a^2*c)
```

3.203 $\int \sqrt{c+a^2cx^2}\tan^{-1}(ax)\,dx$

Optimal. Leaf size=244

$$\frac{ic\sqrt{a^2x^2+1}\mathrm{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{2a\sqrt{a^2cx^2+c}}-\frac{ic\sqrt{a^2x^2+1}\mathrm{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{2a\sqrt{a^2cx^2+c}}-\frac{\sqrt{a^2cx^2+c}}{2a}-\frac{ic\sqrt{a^2x^2+1}\tan^{-1}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)\tan$$

```
[Out] -Sqrt[c + a^2*c*x^2]/(2*a) + (x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/2 - (I*c*S
qrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/(a*Sq
rt[c + a^2*c*x^2]) + ((I/2)*c*Sqrt[1 + a^2*x^2]*PolyLog[2, ((-I)*Sqrt[1 + I
*a*x])/Sqrt[1 - I*a*x]])/(a*Sqrt[c + a^2*c*x^2]) - ((I/2)*c*Sqrt[1 + a^2*x^
2]*PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/(a*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.0932314, antiderivative size = 244, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {4878, 4890, 4886}

$$\frac{ic\sqrt{a^2x^2+1}\mathrm{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{2a\sqrt{a^2cx^2+c}}-\frac{ic\sqrt{a^2x^2+1}\mathrm{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{2a\sqrt{a^2cx^2+c}}-\frac{\sqrt{a^2cx^2+c}}{2a}-\frac{ic\sqrt{a^2x^2+1}\tan^{-1}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)\tan$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + a^2*c*x^2]*ArcTan[a*x],x]
```


```
[Out] -Sqrt[c + a^2*c*x^2]/(2*a) + (x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/2 - (I*c*S
qrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/(a*Sq
rt[c + a^2*c*x^2]) + ((I/2)*c*Sqrt[1 + a^2*x^2]*PolyLog[2, ((-I)*Sqrt[1 + I
*a*x])/Sqrt[1 - I*a*x]])/(a*Sqrt[c + a^2*c*x^2]) - ((I/2)*c*Sqrt[1 + a^2*x^
2]*PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/(a*Sqrt[c + a^2*c*x^2])
```


Rule 4878

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symbo
l] :> -Simp[(b*(d + e*x^2)^q)/(2*c*q*(2*q + 1)), x] + (Dist[(2*d*q)/(2*q +
1), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x]), x], x] + Simp[(x*(d + e*x^
2)^q*(a + b*ArcTan[c*x]))/(2*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] && Eq
Q[e, c^2*d] && GtQ[q, 0]
```

Rule 4890

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4886

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[(-2*I*(a + b*ArcTan[c*x])*ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]])
/(c*Sqrt[d]), x] + (Simp[(I*b*PolyLog[2, -((I*Sqrt[1 + I*c*x])/Sqrt[1 - I*c
*x])])/(c*Sqrt[d]), x] - Simp[(I*b*PolyLog[2, (I*Sqrt[1 + I*c*x])/Sqrt[1 -
I*c*x]])/(c*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

Rubi steps

$$\int \sqrt{c+a^2cx^2}\tan^{-1}(ax)\,dx = -\frac{\sqrt{c+a^2cx^2}}{2a}+\frac{1}{2}x\sqrt{c+a^2cx^2}\tan^{-1}(ax)+\frac{1}{2}c\int \frac{\tan^{-1}(ax)}{\sqrt{c+a^2cx^2}}\,dx$$

$$= -\frac{\sqrt{c+a^2cx^2}}{2a}+\frac{1}{2}x\sqrt{c+a^2cx^2}\tan^{-1}(ax)+\frac{\left(c\sqrt{1+a^2x^2}\right)\int \frac{\tan^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx}{2\sqrt{c+a^2cx^2}}$$

$$= -\frac{\sqrt{c+a^2cx^2}}{2a}+\frac{1}{2}x\sqrt{c+a^2cx^2}\tan^{-1}(ax)-\frac{ic\sqrt{1+a^2x^2}\tan^{-1}(ax)\tan^{-1}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a\sqrt{c+a^2cx^2}}+\frac{ic\sqrt{1+}}{}$$

Mathematica [A] time = 0.571578, size = 141, normalized size = 0.58

$$\frac{\sqrt{c\left(a^2x^2+1\right)}\left(i\text{PolyLog}\left(2,-ie^{i\tan^{-1}(ax)}\right)-i\text{PolyLog}\left(2,ie^{i\tan^{-1}(ax)}\right)+\sqrt{a^2x^2+1}\left(ax\tan^{-1}(ax)-1\right)+\tan^{-1}(ax)\left(\log\left(\right.\right.\right.}{2a\sqrt{a^2x^2+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c + a^2*c*x^2]*ArcTan[a*x],x]
```

```
[Out] (Sqrt[c*(1 + a^2*x^2)]*(Sqrt[1 + a^2*x^2]*(-1 + a*x*ArcTan[a*x]) + ArcTan[a
*x]*(Log[1 - I*E^(I*ArcTan[a*x])] - Log[1 + I*E^(I*ArcTan[a*x])]) + I*PolyL
og[2, (-I)*E^(I*ArcTan[a*x])] - I*PolyLog[2, I*E^(I*ArcTan[a*x])]))/(2*a*Sq
rt[1 + a^2*x^2])
```

Maple [A] time = 0.38, size = 178, normalized size = 0.7

$$\frac{\arctan (ax)\, xa-1}{2a}\sqrt{c\,(ax-i)\,(ax+i)}-\frac{1}{2a}\sqrt{c\,(ax-i)\,(ax+i)}\left(\arctan (ax)\ln\left(1+i\,(1+iax)\,\frac{1}{\sqrt{a^2x^2+1}}\right)-\arctan (ax\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)*(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] 1/2/a*(c*(a*x-I)*(a*x+I))^(1/2)*(arctan(a*x)*x*a-1)-1/2*(c*(a*x-I)*(a*x+I))
^(1/2)*(arctan(a*x)*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-arctan(a*x)*ln(1-I*
(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*dilog(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+I*dil
og(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))/a/(a^2*x^2+1)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)*(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{a^2cx^2+c}\arctan (ax)\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)*(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*arctan(a*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{c\left(a^2x^2+1\right)}\operatorname{atan}(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)*(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(c*(a**2*x**2 + 1))*atan(a*x), x)
```

<u>**Giac**</u> **[F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)*(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.204 $\int \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{x}\,dx$

Optimal. Leaf size=229

$$\frac{ic\sqrt{a^2x^2+1}\,\text{PolyLog}\left(2,-\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{a^2cx^2+c}}-\frac{ic\sqrt{a^2x^2+1}\,\text{PolyLog}\left(2,\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{a^2cx^2+c}}+\sqrt{a^2cx^2+c}\tan^{-1}(ax)-\sqrt{c}\tanh^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+}}\right.$$

```
[Out] Sqrt[c + a^2*c*x^2]*ArcTan[a*x] - (2*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTan
h[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2] - Sqrt[c]*ArcTanh[(
a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]] + (I*c*Sqrt[1 + a^2*x^2]*PolyLog[2, -(Sqr
t[1 + I*a*x]/Sqrt[1 - I*a*x])])/Sqrt[c + a^2*c*x^2] - (I*c*Sqrt[1 + a^2*x^2
]*PolyLog[2, Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2]
```

Rubi [A] time = 0.220983, antiderivative size = 229, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {4946, 4958, 4954, 217, 206}

$$\frac{ic\sqrt{a^2x^2+1}\,\text{PolyLog}\left(2,-\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{a^2cx^2+c}}-\frac{ic\sqrt{a^2x^2+1}\,\text{PolyLog}\left(2,\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{a^2cx^2+c}}+\sqrt{a^2cx^2+c}\tan^{-1}(ax)-\sqrt{c}\tanh^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/x,x]
```

```
[Out] Sqrt[c + a^2*c*x^2]*ArcTan[a*x] - (2*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTan
h[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2] - Sqrt[c]*ArcTanh[(
a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]] + (I*c*Sqrt[1 + a^2*x^2]*PolyLog[2, -(Sqr
t[1 + I*a*x]/Sqrt[1 - I*a*x])])/Sqrt[c + a^2*c*x^2] - (I*c*Sqrt[1 + a^2*x^2
]*PolyLog[2, Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*ArcTan[c*x
]))/(f*(m + 2)), x] + (Dist[d/(m + 2), Int[((f*x)^m*(a + b*ArcTan[c*x]))/Sq
rt[d + e*x^2], x], x] - Dist[(b*c*d)/(f*(m + 2)), Int[(f*x)^(m + 1)/Sqrt[d
+ e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && Ne
Q[m, -2]
```

Rule 4958

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4954

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x_
Symbol] :> Simp[(-2*(a + b*ArcTan[c*x])*ArcTanh[Sqrt[1 + I*c*x]/Sqrt[1 - I*
c*x]])/Sqrt[d], x] + (Simp[(I*b*PolyLog[2, -(Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x
])])/Sqrt[d], x] - Simp[(I*b*PolyLog[2, Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]])/S
qrt[d], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{x}\,dx = \sqrt{c+a^2cx^2}\tan^{-1}(ax) + c\int \frac{\tan^{-1}(ax)}{x\sqrt{c+a^2cx^2}}\,dx - (ac)\int \frac{1}{\sqrt{c+a^2cx^2}}\,dx$$

$$= \sqrt{c+a^2cx^2}\tan^{-1}(ax) - (ac)\,\text{Subst}\left(\int \frac{1}{1-a^2cx^2}\,dx, x, \frac{x}{\sqrt{c+a^2cx^2}}\right) + \frac{\left(c\sqrt{1+a^2x^2}\right)\int \frac{\tan}{x\sqrt{}}}{\sqrt{c+a^2cx^2}}$$

$$= \sqrt{c+a^2cx^2}\tan^{-1}(ax) - \frac{2c\sqrt{1+a^2x^2}\tan^{-1}(ax)\tanh^{-1}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}} - \sqrt{c}\tanh^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c+a^2cx^2}}\right.$$

Mathematica [A] time = 0.180788, size = 164, normalized size = 0.72

$$\frac{\sqrt{a^2cx^2+c}\left(i\text{PolyLog}\left(2,-e^{i\tan^{-1}(ax)}\right) - i\text{PolyLog}\left(2,e^{i\tan^{-1}(ax)}\right) + \sqrt{a^2x^2+1}\tan^{-1}(ax) + \tan^{-1}(ax)\log\left(1-e^{i\tan^{-1}(ax)}\right)\right.}{\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/x,x]
```

```
[Out] (Sqrt[c + a^2*c*x^2]*(Sqrt[1 + a^2*x^2]*ArcTan[a*x] + ArcTan[a*x]*Log[1 - E
^(I*ArcTan[a*x])] - ArcTan[a*x]*Log[1 + E^(I*ArcTan[a*x])] + Log[Cos[ArcTan
[a*x]/2] - Sin[ArcTan[a*x]/2]] - Log[Cos[ArcTan[a*x]/2] + Sin[ArcTan[a*x]/2
]] + I*PolyLog[2, -E^(I*ArcTan[a*x])] - I*PolyLog[2, E^(I*ArcTan[a*x])]))/S
qrt[1 + a^2*x^2]
```

Maple [A] time = 0.412, size = 151, normalized size = 0.7

$$\sqrt{c\left(ax-i\right)\left(ax+i\right)}\arctan\left(ax\right)-\sqrt{c\left(ax-i\right)\left(ax+i\right)}\left(\arctan\left(ax\right)\ln\left(1+\left(1+iax\right)\frac{1}{\sqrt{a^{2}x^{2}+1}}\right)-2\,i\arctan\left(\left(1+iax\right)\frac{1}{\sqrt{}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)*(a^2*c*x^2+c)^(1/2)/x,x)
```

```
[Out] (c*(a*x-I)*(a*x+I))^(1/2)*arctan(a*x)-(c*(a*x-I)*(a*x+I))^(1/2)*(arctan(a*x
)*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*I*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2))
-I*dilog((1+I*a*x)/(a^2*x^2+1)^(1/2))-I*dilog(1+(1+I*a*x)/(a^2*x^2+1)^(1/2)
))/(a^2*x^2+1)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)*(a^2*c*x^2+c)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a^{2}cx^{2}+c}\arctan\left(ax\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)*(a^2*c*x^2+c)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c\left(a^2x^2+1\right)}\operatorname{atan}(ax)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)*(a**2*c*x**2+c)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(c*(a**2*x**2 + 1))*atan(a*x)/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)*(a^2*c*x^2+c)^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)*arctan(a*x)/x, x)
```

3.205 $\int \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{x^2}\,dx$

Optimal. Leaf size=242

$$\frac{iac\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{a^2cx^2+c}}-\frac{iac\sqrt{a^2x^2+1}\text{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{a^2cx^2+c}}-\frac{2iac\sqrt{a^2x^2+1}\tan^{-1}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)\tan^{-1}(ax)}{\sqrt{a^2cx^2+c}}-$$

```
[Out] -((Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/x) - ((2*I)*a*c*Sqrt[1 + a^2*x^2]*ArcTa
n[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2] - a*Sqr
t[c]*ArcTanh[Sqrt[c + a^2*c*x^2]/Sqrt[c]] + (I*a*c*Sqrt[1 + a^2*x^2]*PolyLo
g[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2] - (I*a*c*
Sqrt[1 + a^2*x^2]*PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/Sqrt[c +
 a^2*c*x^2]
```

Rubi [A] time = 0.226926, antiderivative size = 242, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {4950, 4944, 266, 63, 208, 4890, 4886}

$$\frac{iac\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{a^2cx^2+c}}-\frac{iac\sqrt{a^2x^2+1}\text{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{a^2cx^2+c}}-\frac{2iac\sqrt{a^2x^2+1}\tan^{-1}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)\tan^{-1}(ax)}{\sqrt{a^2cx^2+c}}-$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/x^2,x]
```


```
[Out] -((Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/x) - ((2*I)*a*c*Sqrt[1 + a^2*x^2]*ArcTa
n[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2] - a*Sqr
t[c]*ArcTanh[Sqrt[c + a^2*c*x^2]/Sqrt[c]] + (I*a*c*Sqrt[1 + a^2*x^2]*PolyLo
g[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2] - (I*a*c*
Sqrt[1 + a^2*x^2]*PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/Sqrt[c +
 a^2*c*x^2]
```


Rule 4950

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
```

```
IntegerQ[q]))
```

Rule 4944

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a +
 b*ArcTan[c*x])^p)/(d*f*(m + 1)), x] - Dist[(b*c*p)/(f*(m + 1)), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 4890

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4886

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[(-2*I*(a + b*ArcTan[c*x])*ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]])
/(c*Sqrt[d]), x] + (Simp[(I*b*PolyLog[2, -((I*Sqrt[1 + I*c*x])/Sqrt[1 - I*c
*x])])/(c*Sqrt[d]), x] - Simp[(I*b*PolyLog[2, (I*Sqrt[1 + I*c*x])/Sqrt[1 -
I*c*x]])/(c*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
```

GtQ[d, 0]

Rubi steps

$$\int \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{x^2}\,dx = c\int \frac{\tan^{-1}(ax)}{x^2\sqrt{c+a^2cx^2}}\,dx + \left(a^2c\right)\int \frac{\tan^{-1}(ax)}{\sqrt{c+a^2cx^2}}\,dx$$
$$= -\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{x} + (ac)\int \frac{1}{x\sqrt{c+a^2cx^2}}\,dx + \frac{\left(a^2c\sqrt{1+a^2x^2}\right)\int \frac{\tan^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx}{\sqrt{c+a^2cx^2}}$$
$$= -\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{x} - \frac{2iac\sqrt{1+a^2x^2}\tan^{-1}(ax)\tan^{-1}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}} + \frac{iac\sqrt{1+a^2x^2}\mathrm{Li}_2\left(\cdots\right.}{\sqrt{c+a^2cx^2}}$$
$$= -\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{x} - \frac{2iac\sqrt{1+a^2x^2}\tan^{-1}(ax)\tan^{-1}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}} + \frac{iac\sqrt{1+a^2x^2}\mathrm{Li}_2\left(\cdots\right.}{\sqrt{c+a^2cx^2}}$$
$$= -\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{x} - \frac{2iac\sqrt{1+a^2x^2}\tan^{-1}(ax)\tan^{-1}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}} - a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+\cdots}}{\cdots}\right.$$

Mathematica [A] time = 0.445629, size = 163, normalized size = 0.67

$$-\frac{a\sqrt{c\left(a^2x^2+1\right)}\left(-i\mathrm{PolyLog}\left(2,-ie^{i\tan^{-1}(ax)}\right)+i\mathrm{PolyLog}\left(2,ie^{i\tan^{-1}(ax)}\right)+\frac{\sqrt{a^2x^2+1}\tan^{-1}(ax)}{ax}+\tan^{-1}(ax)\left(-\log\left(1-ie^{i\cdots}\right.\right.\right.}{\sqrt{a^2x^2+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/x^2,x]
```

```
[Out] -((a*Sqrt[c*(1 + a^2*x^2)]*((Sqrt[1 + a^2*x^2]*ArcTan[a*x])/(a*x) - ArcTan[
a*x]*Log[1 - I*E^(I*ArcTan[a*x])] + ArcTan[a*x]*Log[1 + I*E^(I*ArcTan[a*x])
] + Log[Cos[ArcTan[a*x]/2]] - Log[Sin[ArcTan[a*x]/2]] - I*PolyLog[2, (-I)*E
^(I*ArcTan[a*x])] + I*PolyLog[2, I*E^(I*ArcTan[a*x])]))/Sqrt[1 + a^2*x^2])
```

Maple [A] time = 0.426, size = 221, normalized size = 0.9

$$-\frac{\arctan(ax)}{x}\sqrt{c(ax-i)(ax+i)}+ia\sqrt{c(ax-i)(ax+i)}\left(i\arctan(ax)\ln\left(1+i(1+iax)\frac{1}{\sqrt{a^2x^2+1}}\right)-i\arctan(ax)\ln\left(\cdots\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)*(a^2*c*x^2+c)^(1/2)/x^2,x)
```

```
[Out] -(c*(a*x-I)*(a*x+I))^(1/2)*arctan(a*x)/x+I*a*(c*(a*x-I)*(a*x+I))^(1/2)*(I*a
rctan(a*x)*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*arctan(a*x)*ln(1-I*(1+I*a*
x)/(a^2*x^2+1)^(1/2))+I*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*ln((1+I*a*x)/(a
^2*x^2+1)^(1/2)-1)-dilog(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+dilog(1+I*(1+I*a*
x)/(a^2*x^2+1)^(1/2)))/(a^2*x^2+1)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)*(a^2*c*x^2+c)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)*(a^2*c*x^2+c)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{c\left(a^2x^2+1\right)}\operatorname{atan}(ax)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)*(a**2*c*x**2+c)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(c*(a**2*x**2 + 1))*atan(a*x)/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a^2cx^2 + c}\arctan(ax)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)*(a^2*c*x^2+c)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)*arctan(a*x)/x^2, x)
```

3.206 $\int \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{x^3}\,dx$

Optimal. Leaf size=240

$$\frac{ia^2c\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{2\sqrt{a^2cx^2+c}}-\frac{ia^2c\sqrt{a^2x^2+1}\text{PolyLog}\left(2,\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{2\sqrt{a^2cx^2+c}}-\frac{a\sqrt{a^2cx^2+c}}{2x}-\frac{\sqrt{a^2cx^2+c}\tan^{-1}(ax)}{2x^2}-\frac{a^2c}{}$$

```
[Out] -(a*Sqrt[c + a^2*c*x^2])/(2*x) - (Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(2*x^2)
- (a^2*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTanh[Sqrt[1 + I*a*x]/Sqrt[1 - I*a
*x]])/Sqrt[c + a^2*c*x^2] + ((I/2)*a^2*c*Sqrt[1 + a^2*x^2]*PolyLog[2, -(Sqr
t[1 + I*a*x]/Sqrt[1 - I*a*x])])/Sqrt[c + a^2*c*x^2] - ((I/2)*a^2*c*Sqrt[1 +
 a^2*x^2]*PolyLog[2, Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2]
```

Rubi [A] time = 0.348808, antiderivative size = 240, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {4946, 4962, 264, 4958, 4954}

$$\frac{ia^2c\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{2\sqrt{a^2cx^2+c}}-\frac{ia^2c\sqrt{a^2x^2+1}\text{PolyLog}\left(2,\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{2\sqrt{a^2cx^2+c}}-\frac{a\sqrt{a^2cx^2+c}}{2x}-\frac{\sqrt{a^2cx^2+c}\tan^{-1}(ax)}{2x^2}-\frac{a^2c}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/x^3,x]
```


```
[Out] -(a*Sqrt[c + a^2*c*x^2])/(2*x) - (Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(2*x^2)
- (a^2*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTanh[Sqrt[1 + I*a*x]/Sqrt[1 - I*a
*x]])/Sqrt[c + a^2*c*x^2] + ((I/2)*a^2*c*Sqrt[1 + a^2*x^2]*PolyLog[2, -(Sqr
t[1 + I*a*x]/Sqrt[1 - I*a*x])])/Sqrt[c + a^2*c*x^2] - ((I/2)*a^2*c*Sqrt[1 +
 a^2*x^2]*PolyLog[2, Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2]
```


Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*ArcTan[c*x
]))/(f*(m + 2)), x] + (Dist[d/(m + 2), Int[((f*x)^m*(a + b*ArcTan[c*x]))/Sq
rt[d + e*x^2], x], x] - Dist[(b*c*d)/(f*(m + 2)), Int[(f*x)^(m + 1)/Sqrt[d
+ e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && Ne
Q[m, -2]
```

Rule 4962

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Ar
cTan[c*x])^p)/(d*f*(m + 1)), x] + (-Dist[(b*c*p)/(f*(m + 1)), Int[((f*x)^(m
 + 1)*(a + b*ArcTan[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] - Dist[(c^2*(m +
 2))/(f^2*(m + 1)), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/Sqrt[d + e*x^
2], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] &
& LtQ[m, -1] && NeQ[m, -2]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 4958

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4954

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x_
Symbol] :> Simp[(-2*(a + b*ArcTan[c*x])*ArcTanh[Sqrt[1 + I*c*x]/Sqrt[1 - I*
c*x]])/Sqrt[d], x] + (Simp[(I*b*PolyLog[2, -(Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x
])])/Sqrt[d], x] - Simp[(I*b*PolyLog[2, Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]])/S
qrt[d], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

Rubi steps

$$\int \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{x^3}\,dx = -\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{x^2} - c\int \frac{\tan^{-1}(ax)}{x^3\sqrt{c+a^2cx^2}}\,dx + (ac)\int \frac{1}{x^2\sqrt{c+a^2cx^2}}\,dx$$

$$= -\frac{a\sqrt{c+a^2cx^2}}{x} - \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{2x^2} - \frac{1}{2}(ac)\int \frac{1}{x^2\sqrt{c+a^2cx^2}}\,dx + \frac{1}{2}\left(a^2c\right)\int \frac{\tan^{-1}(a}{x\sqrt{c+a^2}}$$

$$= -\frac{a\sqrt{c+a^2cx^2}}{2x} - \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{2x^2} + \frac{\left(a^2c\sqrt{1+a^2x^2}\right)\int \frac{\tan^{-1}(ax)}{x\sqrt{1+a^2x^2}}\,dx}{2\sqrt{c+a^2cx^2}}$$

$$= -\frac{a\sqrt{c+a^2cx^2}}{2x} - \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{2x^2} - \frac{a^2c\sqrt{1+a^2x^2}\tan^{-1}(ax)\tanh^{-1}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}} + \frac{ia^2c}{}$$

Mathematica [A] time = 1.04801, size = 165, normalized size = 0.69

$$\frac{a^2\sqrt{c\left(a^2x^2+1\right)}\left(4i\text{PolyLog}\left(2,-e^{i\tan^{-1}(ax)}\right)-4i\text{PolyLog}\left(2,e^{i\tan^{-1}(ax)}\right)-2\tan\left(\frac{1}{2}\tan^{-1}(ax)\right)+4\tan^{-1}(ax)\log\left(1-e^{i}\right.\right.}{8}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/x^3,x]
```

```
[Out] (a^2*Sqrt[c*(1 + a^2*x^2)]*(-2*Cot[ArcTan[a*x]/2] - ArcTan[a*x]*Csc[ArcTan[
a*x]/2]^2 + 4*ArcTan[a*x]*Log[1 - E^(I*ArcTan[a*x])] - 4*ArcTan[a*x]*Log[1
+ E^(I*ArcTan[a*x])] + (4*I)*PolyLog[2, -E^(I*ArcTan[a*x])] - (4*I)*PolyLog
[2, E^(I*ArcTan[a*x])] + ArcTan[a*x]*Sec[ArcTan[a*x]/2]^2 - 2*Tan[ArcTan[a*
x]/2]))/(8*Sqrt[1 + a^2*x^2])
```

Maple [A] time = 0.418, size = 169, normalized size = 0.7

$$-\frac{ax+\arctan(ax)}{2x^2}\sqrt{c(ax-i)(ax+i)}+\frac{i}{2}a^2\sqrt{c(ax-i)(ax+i)}\left(i\arctan(ax)\ln\left(1+(1+iax)\frac{1}{\sqrt{a^2x^2+1}}\right)-i\arctan(ax\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)*(a^2*c*x^2+c)^(1/2)/x^3,x)
```

```
[Out] -1/2*(c*(a*x-I)*(a*x+I))^(1/2)*(a*x+arctan(a*x))/x^2+1/2*I*a^2*(c*(a*x-I)*(
a*x+I))^(1/2)*(I*arctan(a*x)*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*arctan(a*x
```

```
)*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))+polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))
-polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2)))/(a^2*x^2+1)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)*(a^2*c*x^2+c)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)*(a^2*c*x^2+c)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{c\left(a^2x^2+1\right)}\operatorname{atan}(ax)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)*(a**2*c*x**2+c)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(c*(a**2*x**2 + 1))*atan(a*x)/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)*(a^2*c*x^2+c)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)*arctan(a*x)/x^3, x)
```

3.207 $\int \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{x^4}\,dx$

Optimal. Leaf size=84

$$-\frac{a\sqrt{a^2cx^2+c}}{6x^2}-\frac{\left(a^2cx^2+c\right)^{3/2}\tan^{-1}(ax)}{3cx^3}-\frac{1}{6}a^3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)$$

```
[Out] -(a*Sqrt[c + a^2*c*x^2])/(6*x^2) - ((c + a^2*c*x^2)^(3/2)*ArcTan[a*x])/(3*c
*x^3) - (a^3*Sqrt[c]*ArcTanh[Sqrt[c + a^2*c*x^2]/Sqrt[c]])/6
```

Rubi [A] time = 0.101868, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {4944, 266, 47, 63, 208}

$$-\frac{a\sqrt{a^2cx^2+c}}{6x^2}-\frac{\left(a^2cx^2+c\right)^{3/2}\tan^{-1}(ax)}{3cx^3}-\frac{1}{6}a^3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/x^4,x]
```

```
[Out] -(a*Sqrt[c + a^2*c*x^2])/(6*x^2) - ((c + a^2*c*x^2)^(3/2)*ArcTan[a*x])/(3*c
*x^3) - (a^3*Sqrt[c]*ArcTanh[Sqrt[c + a^2*c*x^2]/Sqrt[c]])/6
```

Rule 4944

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a +
 b*ArcTan[c*x])^p)/(d*f*(m + 1)), x] - Dist[(b*c*p)/(f*(m + 1)), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{x^4}\,dx &= -\frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}{3cx^3}+\frac{1}{3}a\int \frac{\sqrt{c+a^2cx^2}}{x^3}\,dx\\
&= -\frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}{3cx^3}+\frac{1}{6}a\,\mathrm{Subst}\left(\int \frac{\sqrt{c+a^2cx}}{x^2}\,dx,x,x^2\right)\\
&= -\frac{a\sqrt{c+a^2cx^2}}{6x^2}-\frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}{3cx^3}+\frac{1}{12}\left(a^3c\right)\mathrm{Subst}\left(\int \frac{1}{x\sqrt{c+a^2cx}}\,dx,x,x^2\right)\\
&= -\frac{a\sqrt{c+a^2cx^2}}{6x^2}-\frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}{3cx^3}+\frac{1}{6}a\,\mathrm{Subst}\left(\int \frac{1}{-\frac{1}{a^2}+\frac{x^2}{a^2c}}\,dx,x,\sqrt{c+a^2cx^2}\right)\\
&= -\frac{a\sqrt{c+a^2cx^2}}{6x^2}-\frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}{3cx^3}-\frac{1}{6}a^3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+a^2cx^2}}{\sqrt{c}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.116101, size = 105, normalized size = 1.25

$$
\frac{a^3\sqrt{c}x^3\log(x)-ax\left(\sqrt{a^2cx^2+c}+a^2\sqrt{c}x^2\log\left(\sqrt{c}\sqrt{a^2cx^2+c}+c\right)\right)-2\left(a^2x^2+1\right)\sqrt{a^2cx^2+c}\tan^{-1}(ax)}{6x^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/x^4,x]
```

```
[Out] (-2*(1 + a^2*x^2)*Sqrt[c + a^2*c*x^2]*ArcTan[a*x] + a^3*Sqrt[c]*x^3*Log[x]
- a*x*(Sqrt[c + a^2*c*x^2] + a^2*Sqrt[c]*x^2*Log[c + Sqrt[c]*Sqrt[c + a^2*c
*x^2]]))/(6*x^3)
```

Maple [C] time = 0.552, size = 153, normalized size = 1.8

$$-\frac{2\arctan(ax)a^2x^2+ax+2\arctan(ax)}{6x^3}\sqrt{c(ax-i)(ax+i)}-\frac{a^3}{6}\sqrt{c(ax-i)(ax+i)}\ln\left(1+(1+iax)\frac{1}{\sqrt{a^2x^2+1}}\right)\frac{1}{\sqrt{a^2x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)*(a^2*c*x^2+c)^(1/2)/x^4,x)
```

```
[Out] -1/6*(c*(a*x-I)*(a*x+I))^(1/2)*(2*arctan(a*x)*a^2*x^2+a*x+2*arctan(a*x))/x^
3-1/6*a^3*(c*(a*x-I)*(a*x+I))^(1/2)*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*
x^2+1)^(1/2)+1/6*a^3*(c*(a*x-I)*(a*x+I))^(1/2)*ln((1+I*a*x)/(a^2*x^2+1)^(1/
2)-1)/(a^2*x^2+1)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)*(a^2*c*x^2+c)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.75676, size = 200, normalized size = 2.38

$$\frac{a^3\sqrt{c}x^3\log\left(-\frac{a^2cx^2-2\sqrt{a^2cx^2+c}\sqrt{c}+2c}{x^2}\right)-2\sqrt{a^2cx^2+c}\left(ax+2\left(a^2x^2+1\right)\arctan(ax)\right)}{12x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)*(a^2*c*x^2+c)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/12*(a^3*sqrt(c)*x^3*log(-(a^2*c*x^2 - 2*sqrt(a^2*c*x^2 + c)*sqrt(c) + 2*c
)/x^2) - 2*sqrt(a^2*c*x^2 + c)*(a*x + 2*(a^2*x^2 + 1)*arctan(a*x)))/x^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c\left(a^2x^2+1\right)}\,\mathrm{atan}\,(ax)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)*(a**2*c*x**2+c)**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt(c*(a**2*x**2 + 1))*atan(a*x)/x**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a^2cx^2+c}\,\arctan\,(ax)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)*(a^2*c*x^2+c)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)*arctan(a*x)/x^4, x)
```

3.208 $\int x^3 \left(c+a^2cx^2\right)^{3/2} \tan^{-1}(ax)\,dx$

Optimal. Leaf size=217

$$\frac{17c^{3/2}\tanh^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{560a^4}-\frac{1}{42}acx^5\sqrt{a^2cx^2+c}-\frac{23cx^3\sqrt{a^2cx^2+c}}{840a}+\frac{3cx\sqrt{a^2cx^2+c}}{112a^3}+\frac{1}{7}a^2cx^6\sqrt{a^2cx^2+c}\tan^{-1}(ax)+$$

```
[Out] (3*c*x*Sqrt[c + a^2*c*x^2])/(112*a^3) - (23*c*x^3*Sqrt[c + a^2*c*x^2])/(840
*a) - (a*c*x^5*Sqrt[c + a^2*c*x^2])/42 - (2*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*
x])/(35*a^4) + (c*x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(35*a^2) + (8*c*x^4*
Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/35 + (a^2*c*x^6*Sqrt[c + a^2*c*x^2]*ArcTan
[a*x])/7 + (17*c^(3/2)*ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]])/(560*a^4
)
```

Rubi [A] time = 0.764624, antiderivative size = 217, normalized size of antiderivative = 1., number of steps used = 31, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {4950, 4946, 4952, 321, 217, 206, 4930}

$$\frac{17c^{3/2}\tanh^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{560a^4}-\frac{1}{42}acx^5\sqrt{a^2cx^2+c}-\frac{23cx^3\sqrt{a^2cx^2+c}}{840a}+\frac{3cx\sqrt{a^2cx^2+c}}{112a^3}+\frac{1}{7}a^2cx^6\sqrt{a^2cx^2+c}\tan^{-1}(ax)+$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x],x]
```

```
[Out] (3*c*x*Sqrt[c + a^2*c*x^2])/(112*a^3) - (23*c*x^3*Sqrt[c + a^2*c*x^2])/(840
*a) - (a*c*x^5*Sqrt[c + a^2*c*x^2])/42 - (2*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*
x])/(35*a^4) + (c*x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(35*a^2) + (8*c*x^4*
Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/35 + (a^2*c*x^6*Sqrt[c + a^2*c*x^2]*ArcTan
[a*x])/7 + (17*c^(3/2)*ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]])/(560*a^4
)
```

Rule 4950

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*ArcTan[c*x
]))/(f*(m + 2)), x] + (Dist[d/(m + 2), Int[((f*x)^m*(a + b*ArcTan[c*x]))/Sq
rt[d + e*x^2], x], x] - Dist[(b*c*d)/(f*(m + 2)), Int[(f*x)^(m + 1)/Sqrt[d
+ e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && Ne
Q[m, -2]
```

Rule 4952

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b*
ArcTan[c*x])^p)/(c^2*d*m), x] + (-Dist[(b*f*p)/(c*m), Int[((f*x)^(m - 1)*(a
 + b*ArcTan[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] - Dist[(f^2*(m - 1))/(c^
2*m), Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/Sqrt[d + e*x^2], x], x]) /;
 FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rubi steps

$$\int x^3 \left(c+a^2cx^2\right)^{3/2} \tan^{-1}(ax)\,dx = c\int x^3\sqrt{c+a^2cx^2}\tan^{-1}(ax)\,dx + \left(a^2c\right)\int x^5\sqrt{c+a^2cx^2}\tan^{-1}(ax)\,dx$$
$$= \frac{1}{5}cx^4\sqrt{c+a^2cx^2}\tan^{-1}(ax) + \frac{1}{7}a^2cx^6\sqrt{c+a^2cx^2}\tan^{-1}(ax) + \frac{1}{5}c^2\int \frac{x^3\tan^{-1}(ax)}{\sqrt{c+a^2cx^2}}\,dx$$
$$= -\frac{cx^3\sqrt{c+a^2cx^2}}{20a} - \frac{1}{42}acx^5\sqrt{c+a^2cx^2} + \frac{cx^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{15a^2} + \frac{8}{35}cx^4\sqrt{c+a^2c}$$
$$= \frac{cx\sqrt{c+a^2cx^2}}{24a^3} - \frac{23cx^3\sqrt{c+a^2cx^2}}{840a} - \frac{1}{42}acx^5\sqrt{c+a^2cx^2} - \frac{2c\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{15a^4}$$
$$= \frac{3cx\sqrt{c+a^2cx^2}}{112a^3} - \frac{23cx^3\sqrt{c+a^2cx^2}}{840a} - \frac{1}{42}acx^5\sqrt{c+a^2cx^2} - \frac{2c\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{35a^4}$$
$$= \frac{3cx\sqrt{c+a^2cx^2}}{112a^3} - \frac{23cx^3\sqrt{c+a^2cx^2}}{840a} - \frac{1}{42}acx^5\sqrt{c+a^2cx^2} - \frac{2c\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{35a^4}$$
$$= \frac{3cx\sqrt{c+a^2cx^2}}{112a^3} - \frac{23cx^3\sqrt{c+a^2cx^2}}{840a} - \frac{1}{42}acx^5\sqrt{c+a^2cx^2} - \frac{2c\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{35a^4}$$

Mathematica [A] time = 0.165537, size = 119, normalized size = 0.55

$$\frac{51c^{3/2}\log\left(\sqrt{c}\sqrt{a^2cx^2+c}+acx\right)+acx\left(-40a^4x^4-46a^2x^2+45\right)\sqrt{a^2cx^2+c}+48c\left(5a^2x^2-2\right)\left(a^2x^2+1\right)^2\sqrt{a^2cx^2+c}\,\mathrm{t}}{1680a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x],x]
```

```
[Out] (a*c*x*Sqrt[c + a^2*c*x^2]*(45 - 46*a^2*x^2 - 40*a^4*x^4) + 48*c*(1 + a^2*x
^2)^2*(-2 + 5*a^2*x^2)*Sqrt[c + a^2*c*x^2]*ArcTan[a*x] + 51*c^(3/2)*Log[a*c
*x + Sqrt[c]*Sqrt[c + a^2*c*x^2]])/(1680*a^4)
```

Maple [C] time = 0.914, size = 199, normalized size = 0.9

$$\frac{c\left(240\,\arctan(ax)\,x^6a^6-40\,a^5x^5+384\,\arctan(ax)\,x^4a^4-46\,a^3x^3+48\,\arctan(ax)\,a^2x^2+45\,ax-96\,\arctan(ax)\right)}{1680\,a^4}\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a^2*c*x^2+c)^(3/2)*arctan(a*x),x)
```

```
[Out] 1/1680*c/a^4*(c*(a*x-I)*(a*x+I))^(1/2)*(240*arctan(a*x)*x^6*a^6-40*a^5*x^5+
384*arctan(a*x)*x^4*a^4-46*a^3*x^3+48*arctan(a*x)*a^2*x^2+45*a*x-96*arctan(
a*x))-17/560*c/a^4*(c*(a*x-I)*(a*x+I))^(1/2)*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)
-I)/(a^2*x^2+1)^(1/2)+17/560*c/a^4*(c*(a*x-I)*(a*x+I))^(1/2)*ln((1+I*a*x)/(
a^2*x^2+1)^(1/2)+I)/(a^2*x^2+1)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a^2*c*x^2+c)^(3/2)*arctan(a*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.86986, size = 285, normalized size = 1.31

$$\frac{51\,c^{\frac{3}{2}}\log\left(-2\,a^{2}cx^{2}-2\,\sqrt{a^{2}cx^{2}+c}a\sqrt{c}x-c\right)-2\left(40\,a^{5}cx^{5}+46\,a^{3}cx^{3}-45\,acx-48\left(5\,a^{6}cx^{6}+8\,a^{4}cx^{4}+a^{2}cx^{2}-2\,c\right)\mathrm{arc}\right.}{3360\,a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a^2*c*x^2+c)^(3/2)*arctan(a*x),x, algorithm="fricas")
```

```
[Out] 1/3360*(51*c^(3/2)*log(-2*a^2*c*x^2 - 2*sqrt(a^2*c*x^2 + c)*a*sqrt(c)*x - c
) - 2*(40*a^5*c*x^5 + 46*a^3*c*x^3 - 45*a*c*x - 48*(5*a^6*c*x^6 + 8*a^4*c*x
^4 + a^2*c*x^2 - 2*c)*arctan(a*x))*sqrt(a^2*c*x^2 + c))/a^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^{3}\left(c\left(a^{2}x^{2}+1\right)\right)^{\frac{3}{2}}\operatorname{atan}\left(ax\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a**2*c*x**2+c)**(3/2)*atan(a*x),x)
```

```
[Out] Integral(x**3*(c*(a**2*x**2 + 1))**(3/2)*atan(a*x), x)
```

Giac [A] time = 1.15812, size = 246, normalized size = 1.13

$$\frac{\left(\frac{7\left(3\left(a^2cx^2+c\right)^{\frac{5}{2}}-5\left(a^2cx^2+c\right)^{\frac{3}{2}}c\right)}{a^2c}+\frac{15\left(a^2cx^2+c\right)^{\frac{7}{2}}-42\left(a^2cx^2+c\right)^{\frac{5}{2}}c+35\left(a^2cx^2+c\right)^{\frac{3}{2}}c^2}{a^2c^2}\right)\arctan\left(ax\right)}{105\,a^2}-\frac{\sqrt{a^2cx^2+c}\left(2\left(20\,a^4cx^2+23\,a^2c\right)x^2\right.}{168}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a^2*c*x^2+c)^(3/2)*arctan(a*x),x, algorithm="giac")
```

```
[Out] 1/105*(7*(3*(a^2*c*x^2 + c)^(5/2) - 5*(a^2*c*x^2 + c)^(3/2)*c)/(a^2*c) + (1
5*(a^2*c*x^2 + c)^(7/2) - 42*(a^2*c*x^2 + c)^(5/2)*c + 35*(a^2*c*x^2 + c)^(
3/2)*c^2)/(a^2*c^2))*arctan(a*x)/a^2 - 1/1680*(sqrt(a^2*c*x^2 + c)*(2*(20*a
^4*c*x^2 + 23*a^2*c)*x^2 - 45*c)*x + 51*c^(3/2)*log(abs(-sqrt(a^2*c)*x + sq
rt(a^2*c*x^2 + c)))/abs(a))/a^3
```

3.209 $\int x^2 \left(c+a^2 c x^2\right)^{3/2} \tan^{-1}(ax)\, dx$

Optimal. Leaf size=357

$$-\frac{ic^2\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{16a^3\sqrt{a^2cx^2+c}}+\frac{ic^2\sqrt{a^2x^2+1}\text{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{16a^3\sqrt{a^2cx^2+c}}+\frac{ic^2\sqrt{a^2x^2+1}\tan^{-1}(ax)\tan^{-1}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{8a^3\sqrt{a^2cx^2+c}}-\frac{(}{}$$

```
[Out] (c*Sqrt[c + a^2*c*x^2])/(16*a^3) + (c + a^2*c*x^2)^(3/2)/(72*a^3) - (c + a^
2*c*x^2)^(5/2)/(30*a^3*c) + (c*x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(16*a^2)
+ (7*c*x^3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/24 + (a^2*c*x^5*Sqrt[c + a^2*c*
x^2]*ArcTan[a*x])/6 + ((I/8)*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTan[Sqrt[
1 + I*a*x]/Sqrt[1 - I*a*x]])/(a^3*Sqrt[c + a^2*c*x^2]) - ((I/16)*c^2*Sqrt[1
 + a^2*x^2]*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/(a^3*Sqrt[c
 + a^2*c*x^2]) + ((I/16)*c^2*Sqrt[1 + a^2*x^2]*PolyLog[2, (I*Sqrt[1 + I*a*x
])/Sqrt[1 - I*a*x]])/(a^3*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.782542, antiderivative size = 357, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {4950, 4946, 4952, 261, 4890, 4886, 266, 43}

$$-\frac{ic^2\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{16a^3\sqrt{a^2cx^2+c}}+\frac{ic^2\sqrt{a^2x^2+1}\text{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{16a^3\sqrt{a^2cx^2+c}}+\frac{ic^2\sqrt{a^2x^2+1}\tan^{-1}(ax)\tan^{-1}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{8a^3\sqrt{a^2cx^2+c}}-\frac{(}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x],x]
```


```
[Out] (c*Sqrt[c + a^2*c*x^2])/(16*a^3) + (c + a^2*c*x^2)^(3/2)/(72*a^3) - (c + a^
2*c*x^2)^(5/2)/(30*a^3*c) + (c*x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(16*a^2)
+ (7*c*x^3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/24 + (a^2*c*x^5*Sqrt[c + a^2*c*
x^2]*ArcTan[a*x])/6 + ((I/8)*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTan[Sqrt[
1 + I*a*x]/Sqrt[1 - I*a*x]])/(a^3*Sqrt[c + a^2*c*x^2]) - ((I/16)*c^2*Sqrt[1
 + a^2*x^2]*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/(a^3*Sqrt[c
 + a^2*c*x^2]) + ((I/16)*c^2*Sqrt[1 + a^2*x^2]*PolyLog[2, (I*Sqrt[1 + I*a*x
])/Sqrt[1 - I*a*x]])/(a^3*Sqrt[c + a^2*c*x^2])
```


Rule 4950

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
```

```
b*ArcTan[c*x])^p, x], x] + Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*ArcTan[c*x
]))/(f*(m + 2)), x] + (Dist[d/(m + 2), Int[((f*x)^m*(a + b*ArcTan[c*x]))/Sq
rt[d + e*x^2], x], x] - Dist[(b*c*d)/(f*(m + 2)), Int[(f*x)^(m + 1)/Sqrt[d
+ e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && Ne
Q[m, -2]
```

Rule 4952

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b*
ArcTan[c*x])^p)/(c^2*d*m), x] + (-Dist[(b*f*p)/(c*m), Int[((f*x)^(m - 1)*(a
 + b*ArcTan[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] - Dist[(f^2*(m - 1))/(c^
2*m), Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/Sqrt[d + e*x^2], x], x]) /;
 FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 4890

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4886

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[(-2*I*(a + b*ArcTan[c*x])*ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]])
/(c*Sqrt[d]), x] + (Simp[(I*b*PolyLog[2, -((I*Sqrt[1 + I*c*x])/Sqrt[1 - I*c
*x])])/(c*Sqrt[d]), x] - Simp[(I*b*PolyLog[2, (I*Sqrt[1 + I*c*x])/Sqrt[1 -
I*c*x]])/(c*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x^2\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)\,dx = c\int x^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)\,dx+\left(a^2c\right)\int x^4\sqrt{c+a^2cx^2}\tan^{-1}(ax)\,dx$$

$$= \frac{1}{4}cx^3\sqrt{c+a^2cx^2}\tan^{-1}(ax)+\frac{1}{6}a^2cx^5\sqrt{c+a^2cx^2}\tan^{-1}(ax)+\frac{1}{4}c^2\int\frac{x^2\tan^{-1}(ax)}{\sqrt{c+a^2cx^2}}\,dx$$

$$= \frac{cx\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{8a^2}+\frac{7}{24}cx^3\sqrt{c+a^2cx^2}\tan^{-1}(ax)+\frac{1}{6}a^2cx^5\sqrt{c+a^2cx^2}\tan^{-1}(ax$$

$$= -\frac{c\sqrt{c+a^2cx^2}}{8a^3}+\frac{cx\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{16a^2}+\frac{7}{24}cx^3\sqrt{c+a^2cx^2}\tan^{-1}(ax)+\frac{1}{6}a^2cx^5\sqrt{c$$

$$= \frac{c\sqrt{c+a^2cx^2}}{48a^3}+\frac{\left(c+a^2cx^2\right)^{3/2}}{36a^3}-\frac{\left(c+a^2cx^2\right)^{5/2}}{30a^3c}+\frac{cx\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{16a^2}+\frac{7}{24}cx^3\sqrt{c$$

$$= \frac{c\sqrt{c+a^2cx^2}}{16a^3}+\frac{\left(c+a^2cx^2\right)^{3/2}}{72a^3}-\frac{\left(c+a^2cx^2\right)^{5/2}}{30a^3c}+\frac{cx\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{16a^2}+\frac{7}{24}cx^3\sqrt{c$$

Mathematica [A] time = 5.94983, size = 576, normalized size = 1.61

$$c\sqrt{a^2cx^2+c}\left(-90i\text{PolyLog}\left(2,-ie^{i\tan^{-1}(ax)}\right)+90i\text{PolyLog}\left(2,ie^{i\tan^{-1}(ax)}\right)+\frac{3}{4}\left(a^2x^2+1\right)^{5/2}+\frac{55}{8}\left(a^2x^2+1\right)^3\cos\left(3\tan^{-1}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x],x]
```

```
[Out] (c*Sqrt[c + a^2*c*x^2]*((3*(1 + a^2*x^2)^(5/2))/4 + (55*(1 + a^2*x^2)^3*Cos
[3*ArcTan[a*x]])/8 - (45*(1 + a^2*x^2)^3*Cos[5*ArcTan[a*x]])/8 - (90*I)*Pol
yLog[2, (-I)*E^(I*ArcTan[a*x])] + (90*I)*PolyLog[2, I*E^(I*ArcTan[a*x])] -
(15*(1 + a^2*x^2)^2*(-2/Sqrt[1 + a^2*x^2] - 6*Cos[3*ArcTan[a*x]] + 3*ArcTan
[a*x]*((-14*a*x)/Sqrt[1 + a^2*x^2] + 3*Log[1 - I*E^(I*ArcTan[a*x])] + 4*Cos
[2*ArcTan[a*x]]*(Log[1 - I*E^(I*ArcTan[a*x])] - Log[1 + I*E^(I*ArcTan[a*x])
]) + Cos[4*ArcTan[a*x]]*(Log[1 - I*E^(I*ArcTan[a*x])] - Log[1 + I*E^(I*ArcT
an[a*x])]) - 3*Log[1 + I*E^(I*ArcTan[a*x])] + 2*Sin[3*ArcTan[a*x]])))/2 + (
15*(1 + a^2*x^2)^3*ArcTan[a*x]*((156*a*x)/Sqrt[1 + a^2*x^2] + 30*Log[1 - I*
E^(I*ArcTan[a*x])] + 3*Cos[6*ArcTan[a*x]]*Log[1 - I*E^(I*ArcTan[a*x])] + 45
*Cos[2*ArcTan[a*x]]*(Log[1 - I*E^(I*ArcTan[a*x])] - Log[1 + I*E^(I*ArcTan[a
*x])]) + 18*Cos[4*ArcTan[a*x]]*(Log[1 - I*E^(I*ArcTan[a*x])] - Log[1 + I*E^
(I*ArcTan[a*x])]) - 30*Log[1 + I*E^(I*ArcTan[a*x])] - 3*Cos[6*ArcTan[a*x]]*
Log[1 + I*E^(I*ArcTan[a*x])] - 94*Sin[3*ArcTan[a*x]] + 6*Sin[5*ArcTan[a*x]]
))/16))/(1440*a^3*Sqrt[1 + a^2*x^2])
```

Maple [A] time = 0.457, size = 221, normalized size = 0.6

$$\frac{c\left(120\arctan(ax)\,x^5a^5 - 24\,a^4x^4 + 210\arctan(ax)\,x^3a^3 - 38\,a^2x^2 + 45\arctan(ax)\,xa + 31\right)}{720\,a^3}\sqrt{c\,(ax-i)\,(ax+i)} + \frac{c}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x),x)
```

```
[Out] 1/720*c/a^3*(c*(a*x-I)*(a*x+I))^(1/2)*(120*arctan(a*x)*x^5*a^5-24*a^4*x^4+2
10*arctan(a*x)*x^3*a^3-38*a^2*x^2+45*arctan(a*x)*x*a+31)+1/16*c*(c*(a*x-I)*
(a*x+I))^(1/2)*(arctan(a*x)*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-arctan(a*x)
*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*dilog(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2
))+I*dilog(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))/a^3/(a^2*x^2+1)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^2cx^4 + cx^2\right)\sqrt{a^2cx^2 + c}\arctan\left(ax\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^4 + c*x^2)*sqrt(a^2*c*x^2 + c)*arctan(a*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\operatorname{atan}\left(ax\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a**2*c*x**2+c)**(3/2)*atan(a*x),x)
```

```
[Out] Integral(x**2*(c*(a**2*x**2 + 1))**(3/2)*atan(a*x), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.210 $\int x\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)\,dx$

Optimal. Leaf size=109

$$-\frac{3c^{3/2}\tanh^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{40a^2}-\frac{x\left(a^2cx^2+c\right)^{3/2}}{20a}-\frac{3cx\sqrt{a^2cx^2+c}}{40a}+\frac{\left(a^2cx^2+c\right)^{5/2}\tan^{-1}(ax)}{5a^2c}$$

```
[Out] (-3*c*x*Sqrt[c + a^2*c*x^2])/(40*a) - (x*(c + a^2*c*x^2)^(3/2))/(20*a) + ((
c + a^2*c*x^2)^(5/2)*ArcTan[a*x])/(5*a^2*c) - (3*c^(3/2)*ArcTanh[(a*Sqrt[c]
*x)/Sqrt[c + a^2*c*x^2]])/(40*a^2)
```

Rubi [A] time = 0.0746509, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4930, 195, 217, 206}

$$-\frac{3c^{3/2}\tanh^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{40a^2}-\frac{x\left(a^2cx^2+c\right)^{3/2}}{20a}-\frac{3cx\sqrt{a^2cx^2+c}}{40a}+\frac{\left(a^2cx^2+c\right)^{5/2}\tan^{-1}(ax)}{5a^2c}$$

Antiderivative was successfully verified.

```
[In] Int[x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x],x]
```


```
[Out] (-3*c*x*Sqrt[c + a^2*c*x^2])/(40*a) - (x*(c + a^2*c*x^2)^(3/2))/(20*a) + ((
c + a^2*c*x^2)^(5/2)*ArcTan[a*x])/(5*a^2*c) - (3*c^(3/2)*ArcTanh[(a*Sqrt[c]
*x)/Sqrt[c + a^2*c*x^2]])/(40*a^2)
```


Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
```

```
Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int x\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)\,dx &= \frac{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)}{5a^2c}-\frac{\int\left(c+a^2cx^2\right)^{3/2}\,dx}{5a}\\ &= -\frac{x\left(c+a^2cx^2\right)^{3/2}}{20a}+\frac{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)}{5a^2c}-\frac{(3c)\int\sqrt{c+a^2cx^2}\,dx}{20a}\\ &= -\frac{3cx\sqrt{c+a^2cx^2}}{40a}-\frac{x\left(c+a^2cx^2\right)^{3/2}}{20a}+\frac{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)}{5a^2c}-\frac{\left(3c^2\right)\int\frac{1}{\sqrt{c+a^2cx^2}}\,dx}{40a}\\ &= -\frac{3cx\sqrt{c+a^2cx^2}}{40a}-\frac{x\left(c+a^2cx^2\right)^{3/2}}{20a}+\frac{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)}{5a^2c}-\frac{\left(3c^2\right)\text{Subst}\left(\int\frac{1}{1-a^2c\ldots}\right.}{40a\ldots}\\ &= -\frac{3cx\sqrt{c+a^2cx^2}}{40a}-\frac{x\left(c+a^2cx^2\right)^{3/2}}{20a}+\frac{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)}{5a^2c}-\frac{3c^{3/2}\tanh^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c+a^2cx\ldots}}\right.}{40a^2}\end{aligned}$$

Mathematica [A] time = 0.162349, size = 101, normalized size = 0.93

$$-\frac{3c^{3/2}\log\left(\sqrt{c}\sqrt{a^2cx^2+c}+acx\right)+acx\left(2a^2x^2+5\right)\sqrt{a^2cx^2+c}-8c\left(a^2x^2+1\right)^2\sqrt{a^2cx^2+c}\tan^{-1}(ax)}{40a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x],x]
```

```
[Out] -(a*c*x*(5 + 2*a^2*x^2)*Sqrt[c + a^2*c*x^2] - 8*c*(1 + a^2*x^2)^2*Sqrt[c +
a^2*c*x^2]*ArcTan[a*x] + 3*c^(3/2)*Log[a*c*x + Sqrt[c]*Sqrt[c + a^2*c*x^2]]
```

```
)/(40*a^2)
```

Maple [C] time = 0.3, size = 179, normalized size = 1.6

$$\frac{c\left(8\arctan\left(ax\right)x^{4}a^{4}-2\,a^{3}x^{3}+16\arctan\left(ax\right)a^{2}x^{2}-5\,ax+8\arctan\left(ax\right)\right)}{40\,a^{2}}\sqrt{c\left(ax-i\right)\left(ax+i\right)}+\frac{3\,c}{40\,a^{2}}\sqrt{c\left(ax-i\right)\left(ax\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x),x)
```

```
[Out] 1/40*c/a^2*(c*(a*x-I)*(a*x+I))^(1/2)*(8*arctan(a*x)*x^4*a^4-2*a^3*x^3+16*ar
ctan(a*x)*a^2*x^2-5*a*x+8*arctan(a*x))+3/40*c/a^2*(c*(a*x-I)*(a*x+I))^(1/2)
*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)-I)/(a^2*x^2+1)^(1/2)-3/40*c/a^2*(c*(a*x-I)*
(a*x+I))^(1/2)*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)+I)/(a^2*x^2+1)^(1/2)
```

Maxima [B] time = 2.05227, size = 576, normalized size = 5.28

$$\frac{40\left(a^{2}cx^{2}+c\right)\sqrt{a^{2}x^{2}+1}\sqrt{c}\arctan\left(ax\right)-20\left(a^{4}x^{4}+10\,a^{2}x^{2}+9\right)^{\frac{1}{4}}\left(acx\cos\left(\frac{1}{2}\arctan\left(4\,ax,-a^{2}x^{2}+3\right)\right)+2\,c\sin\left(\frac{1}{2}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x),x, algorithm="maxima")
```

```
[Out] 1/120*(40*(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1)*sqrt(c)*arctan(a*x) - 20*(a^4*x
^4 + 10*a^2*x^2 + 9)^(1/4)*(a*c*x*cos(1/2*arctan2(4*a*x, -a^2*x^2 + 3)) + 2
*c*sin(1/2*arctan2(4*a*x, -a^2*x^2 + 3)))*sqrt(c) - ((a*(3*(2*(a^2*x^2 + 1)
^(3/2)*x/a^2 - sqrt(a^2*x^2 + 1)*x/a^2 - arcsinh(a^2*x/sqrt(a^2))/(sqrt(a^2
)*a^2))/a^2 - 8*(sqrt(a^2*x^2 + 1)*x + arcsinh(a^2*x/sqrt(a^2))/sqrt(a^2))/
a^4) - 8*(3*(a^2*x^2 + 1)^(3/2)*x^2/a^2 - 2*(a^2*x^2 + 1)^(3/2)/a^4)*arctan
(a*x))*a^4*c - 10*c*arctan2((a^4*x^4 + 10*a^2*x^2 + 9)^(1/4)*sin(1/2*arctan
2(4*a*x, a^2*x^2 - 3)) + 2, a*x + (a^4*x^4 + 10*a^2*x^2 + 9)^(1/4)*cos(1/2*
arctan2(4*a*x, a^2*x^2 - 3))) - 10*c*arctan2((a^4*x^4 + 10*a^2*x^2 + 9)^(1/
4)*sin(1/2*arctan2(4*a*x, a^2*x^2 - 3)) - 2, -a*x + (a^4*x^4 + 10*a^2*x^2 +
```

```
9)^(1/4)*cos(1/2*arctan2(4*a*x, a^2*x^2 - 3))))*sqrt(c))/a^2
```

Fricas [A] time = 1.76499, size = 235, normalized size = 2.16

$$\frac{3\,c^{\frac{3}{2}}\log\left(-2\,a^{2}cx^{2}+2\,\sqrt{a^{2}cx^{2}+c}a\sqrt{c}x-c\right)-2\left(2\,a^{3}cx^{3}+5\,acx-8\left(a^{4}cx^{4}+2\,a^{2}cx^{2}+c\right)\arctan\left(ax\right)\right)\sqrt{a^{2}cx^{2}+c}}{80\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x),x, algorithm="fricas")
```

```
[Out] 1/80*(3*c^(3/2)*log(-2*a^2*c*x^2 + 2*sqrt(a^2*c*x^2 + c)*a*sqrt(c)*x - c) -
 2*(2*a^3*c*x^3 + 5*a*c*x - 8*(a^4*c*x^4 + 2*a^2*c*x^2 + c)*arctan(a*x))*sq
rt(a^2*c*x^2 + c))/a^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\left(c\left(a^{2}x^{2}+1\right)\right)^{\frac{3}{2}}\operatorname{atan}\left(ax\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**(3/2)*atan(a*x),x)
```

```
[Out] Integral(x*(c*(a**2*x**2 + 1))**(3/2)*atan(a*x), x)
```

Giac [A] time = 1.28191, size = 170, normalized size = 1.56

$$-\frac{\left(2\,a^{2}cx^{2}+5\,c\right)\sqrt{a^{2}cx^{2}+c}x-\frac{3\,c^{\frac{3}{2}}\log\left(\left|-\sqrt{a^{2}c}x+\sqrt{a^{2}cx^{2}+c}\right|\right)}{|a|}}{40\,a}+\frac{\left(5\left(a^{2}cx^{2}+c\right)^{\frac{3}{2}}+\frac{3\left(a^{2}cx^{2}+c\right)^{\frac{5}{2}}-5\left(a^{2}cx^{2}+c\right)^{\frac{3}{2}}c}{c}\right)\arctan\left(ax\right)}{15\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x),x, algorithm="giac")
```

```
[Out] -1/40*((2*a^2*c*x^2 + 5*c)*sqrt(a^2*c*x^2 + c)*x - 3*c^(3/2)*log(abs(-sqrt(
a^2*c)*x + sqrt(a^2*c*x^2 + c)))/abs(a))/a + 1/15*(5*(a^2*c*x^2 + c)^(3/2)
+ (3*(a^2*c*x^2 + c)^(5/2) - 5*(a^2*c*x^2 + c)^(3/2)*c)/c)*arctan(a*x)/a^2
```

3.211 $\int \left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)\,dx$

Optimal. Leaf size=298

$$\frac{3ic^2\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{8a\sqrt{a^2cx^2+c}}-\frac{3ic^2\sqrt{a^2x^2+1}\text{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{8a\sqrt{a^2cx^2+c}}-\frac{3ic^2\sqrt{a^2x^2+1}\tan^{-1}(ax)\tan^{-1}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{4a\sqrt{a^2cx^2+c}}-$$

```
[Out] (-3*c*Sqrt[c + a^2*c*x^2])/(8*a) - (c + a^2*c*x^2)^(3/2)/(12*a) + (3*c*x*Sq
rt[c + a^2*c*x^2]*ArcTan[a*x])/8 + (x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x])/4
- (((3*I)/4)*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[
1 - I*a*x]])/(a*Sqrt[c + a^2*c*x^2]) + (((3*I)/8)*c^2*Sqrt[1 + a^2*x^2]*Pol
yLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/(a*Sqrt[c + a^2*c*x^2]) -
(((3*I)/8)*c^2*Sqrt[1 + a^2*x^2]*PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*
a*x]])/(a*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.138201, antiderivative size = 298, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {4878, 4890, 4886}

$$\frac{3ic^2\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{8a\sqrt{a^2cx^2+c}}-\frac{3ic^2\sqrt{a^2x^2+1}\text{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{8a\sqrt{a^2cx^2+c}}-\frac{3ic^2\sqrt{a^2x^2+1}\tan^{-1}(ax)\tan^{-1}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{4a\sqrt{a^2cx^2+c}}-$$

Antiderivative was successfully verified.

```
[In] Int[(c + a^2*c*x^2)^(3/2)*ArcTan[a*x],x]
```


```
[Out] (-3*c*Sqrt[c + a^2*c*x^2])/(8*a) - (c + a^2*c*x^2)^(3/2)/(12*a) + (3*c*x*Sq
rt[c + a^2*c*x^2]*ArcTan[a*x])/8 + (x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x])/4
- (((3*I)/4)*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[
1 - I*a*x]])/(a*Sqrt[c + a^2*c*x^2]) + (((3*I)/8)*c^2*Sqrt[1 + a^2*x^2]*Pol
yLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/(a*Sqrt[c + a^2*c*x^2]) -
(((3*I)/8)*c^2*Sqrt[1 + a^2*x^2]*PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*
a*x]])/(a*Sqrt[c + a^2*c*x^2])
```


Rule 4878

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symbo
l] :> -Simp[(b*(d + e*x^2)^q)/(2*c*q*(2*q + 1)), x] + (Dist[(2*d*q)/(2*q +
1), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x]), x], x] + Simp[(x*(d + e*x^
2)^q*(a + b*ArcTan[c*x]))/(2*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] && Eq
```

```
Q[e, c^2*d] && GtQ[q, 0]
```

Rule 4890

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4886

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[(-2*I*(a + b*ArcTan[c*x])*ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]])
/(c*Sqrt[d]), x] + (Simp[(I*b*PolyLog[2, -((I*Sqrt[1 + I*c*x])/Sqrt[1 - I*c
*x])])/(c*Sqrt[d]), x] - Simp[(I*b*PolyLog[2, (I*Sqrt[1 + I*c*x])/Sqrt[1 -
I*c*x]])/(c*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

Rubi steps

$$\int \left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)\,dx = -\frac{\left(c+a^2cx^2\right)^{3/2}}{12a}+\frac{1}{4}x\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)+\frac{1}{4}(3c)\int \sqrt{c+a^2cx^2}\tan^{-1}(ax)\,dx$$

$$= -\frac{3c\sqrt{c+a^2cx^2}}{8a}-\frac{\left(c+a^2cx^2\right)^{3/2}}{12a}+\frac{3}{8}cx\sqrt{c+a^2cx^2}\tan^{-1}(ax)+\frac{1}{4}x\left(c+a^2cx^2\right)^{3/2}\tan^{-1}$$

$$= -\frac{3c\sqrt{c+a^2cx^2}}{8a}-\frac{\left(c+a^2cx^2\right)^{3/2}}{12a}+\frac{3}{8}cx\sqrt{c+a^2cx^2}\tan^{-1}(ax)+\frac{1}{4}x\left(c+a^2cx^2\right)^{3/2}\tan^{-1}$$

$$= -\frac{3c\sqrt{c+a^2cx^2}}{8a}-\frac{\left(c+a^2cx^2\right)^{3/2}}{12a}+\frac{3}{8}cx\sqrt{c+a^2cx^2}\tan^{-1}(ax)+\frac{1}{4}x\left(c+a^2cx^2\right)^{3/2}\tan^{-1}$$

Mathematica [A] time = 2.58904, size = 351, normalized size = 1.18

$$\frac{c\sqrt{a^2cx^2+c}\left(72i\text{PolyLog}\left(2,-ie^{i\tan^{-1}(ax)}\right)-72i\text{PolyLog}\left(2,ie^{i\tan^{-1}(ax)}\right)+2\left(a^2x^2+1\right)^{3/2}+96\sqrt{a^2x^2+1}\left(ax\tan^{-1}(ax\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + a^2*c*x^2)^(3/2)*ArcTan[a*x],x]
```

```
[Out] (c*Sqrt[c + a^2*c*x^2]*(2*(1 + a^2*x^2)^(3/2) + 96*Sqrt[1 + a^2*x^2]*(-1 +
a*x*ArcTan[a*x]) + 6*(1 + a^2*x^2)^2*Cos[3*ArcTan[a*x]] + 96*ArcTan[a*x]*(L
og[1 - I*E^(I*ArcTan[a*x])] - Log[1 + I*E^(I*ArcTan[a*x])]) + (72*I)*PolyLo
g[2, (-I)*E^(I*ArcTan[a*x])] - (72*I)*PolyLog[2, I*E^(I*ArcTan[a*x])] - 3*(
1 + a^2*x^2)^2*ArcTan[a*x]*((-14*a*x)/Sqrt[1 + a^2*x^2] + 3*Log[1 - I*E^(I*
ArcTan[a*x])] + 4*Cos[2*ArcTan[a*x]]*(Log[1 - I*E^(I*ArcTan[a*x])] - Log[1
+ I*E^(I*ArcTan[a*x])]) + Cos[4*ArcTan[a*x]]*(Log[1 - I*E^(I*ArcTan[a*x])]
- Log[1 + I*E^(I*ArcTan[a*x])]) - 3*Log[1 + I*E^(I*ArcTan[a*x])] + 2*Sin[3*
ArcTan[a*x]])))/(192*a*Sqrt[1 + a^2*x^2])
```

Maple [A] time = 0.292, size = 201, normalized size = 0.7

$$\frac{c\left(6\arctan(ax)x^3a^3-2a^2x^2+15\arctan(ax)xa-11\right)}{24a}\sqrt{c(ax-i)(ax+i)}-\frac{3c}{8a}\sqrt{c(ax-i)(ax+i)}\left(\arctan(ax)\ln\left(1+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(3/2)*arctan(a*x),x)
```

```
[Out] 1/24*c/a*(c*(a*x-I)*(a*x+I))^(1/2)*(6*arctan(a*x)*x^3*a^3-2*a^2*x^2+15*arct
an(a*x)*x*a-11)-3/8*c*(c*(a*x-I)*(a*x+I))^(1/2)*(arctan(a*x)*ln(1+I*(1+I*a*
x)/(a^2*x^2+1)^(1/2))-arctan(a*x)*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*dil
og(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+I*dilog(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2)
))/a/(a^2*x^2+1)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^2cx^2+c\right)^{\frac{3}{2}}\arctan(ax),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)^(3/2)*arctan(a*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}} \operatorname{atan}(ax)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(3/2)*atan(a*x),x)
```

```
[Out] Integral((c*(a**2*x**2 + 1))**(3/2)*atan(a*x), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.212 $\int \frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}{x}\,dx$

Optimal. Leaf size=281

$$\frac{ic^2\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{a^2cx^2+c}}-\frac{ic^2\sqrt{a^2x^2+1}\text{PolyLog}\left(2,\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{a^2cx^2+c}}-\frac{7}{6}c^{3/2}\tanh^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)-\frac{2c^2\sqrt{a^2x^2+1}\tan$$

```
[Out] -(a*c*x*Sqrt[c + a^2*c*x^2])/6 + c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x] + ((c +
a^2*c*x^2)^(3/2)*ArcTan[a*x])/3 - (2*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcT
anh[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2] - (7*c^(3/2)*ArcT
anh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]])/6 + (I*c^2*Sqrt[1 + a^2*x^2]*PolyLo
g[2, -(Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x])])/Sqrt[c + a^2*c*x^2] - (I*c^2*Sqrt
[1 + a^2*x^2]*PolyLog[2, Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x
^2]
```

Rubi [A] time = 0.376018, antiderivative size = 281, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {4950, 4946, 4958, 4954, 217, 206, 4930, 195}

$$\frac{ic^2\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{a^2cx^2+c}}-\frac{ic^2\sqrt{a^2x^2+1}\text{PolyLog}\left(2,\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{a^2cx^2+c}}-\frac{7}{6}c^{3/2}\tanh^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)-\frac{2c^2\sqrt{a^2x^2+1}\tan$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x])/x,x]
```


```
[Out] -(a*c*x*Sqrt[c + a^2*c*x^2])/6 + c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x] + ((c +
a^2*c*x^2)^(3/2)*ArcTan[a*x])/3 - (2*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcT
anh[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2] - (7*c^(3/2)*ArcT
anh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]])/6 + (I*c^2*Sqrt[1 + a^2*x^2]*PolyLo
g[2, -(Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x])])/Sqrt[c + a^2*c*x^2] - (I*c^2*Sqrt
[1 + a^2*x^2]*PolyLog[2, Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x
^2]
```


Rule 4950

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2)^
```

```
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*ArcTan[c*x
]))/(f*(m + 2)), x] + (Dist[d/(m + 2), Int[((f*x)^m*(a + b*ArcTan[c*x]))/Sq
rt[d + e*x^2], x], x] - Dist[(b*c*d)/(f*(m + 2)), Int[(f*x)^(m + 1)/Sqrt[d
+ e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && Ne
Q[m, -2]
```

Rule 4958

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4954

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x_
Symbol] :> Simp[(-2*(a + b*ArcTan[c*x])*ArcTanh[Sqrt[1 + I*c*x]/Sqrt[1 - I*
c*x]])/Sqrt[d], x] + (Simp[(I*b*PolyLog[2, -(Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x
])])/Sqrt[d], x] - Simp[(I*b*PolyLog[2, Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]])/S
qrt[d], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
```

```
0] && NeQ[q, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rubi steps

$$\int \frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}{x}\,dx = c\int \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{x}\,dx + \left(a^2c\right)\int x\sqrt{c+a^2cx^2}\tan^{-1}(ax)\,dx$$

$$= c\sqrt{c+a^2cx^2}\tan^{-1}(ax) + \frac{1}{3}\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax) - \frac{1}{3}(ac)\int \sqrt{c+a^2cx^2}\,dx + c^2\int \frac{}{x}$$

$$= -\frac{1}{6}acx\sqrt{c+a^2cx^2} + c\sqrt{c+a^2cx^2}\tan^{-1}(ax) + \frac{1}{3}\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax) - \frac{1}{6}\left(ac^2\right)\int$$

$$= -\frac{1}{6}acx\sqrt{c+a^2cx^2} + c\sqrt{c+a^2cx^2}\tan^{-1}(ax) + \frac{1}{3}\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax) - \frac{2c^2\sqrt{1+a^2x}}{}$$

$$= -\frac{1}{6}acx\sqrt{c+a^2cx^2} + c\sqrt{c+a^2cx^2}\tan^{-1}(ax) + \frac{1}{3}\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax) - \frac{2c^2\sqrt{1+a^2x}}{}$$

Mathematica [A] time = 0.245586, size = 220, normalized size = 0.78

$$\frac{c\sqrt{a^2cx^2+c}\left(6i\text{PolyLog}\left(2,-e^{i\tan^{-1}(ax)}\right) - 6i\text{PolyLog}\left(2,e^{i\tan^{-1}(ax)}\right) - ax\sqrt{a^2x^2+1} + 2a^2x^2\sqrt{a^2x^2+1}\tan^{-1}(ax) + 8\sqrt{a}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x])/x,x]
```

```
[Out] (c*Sqrt[c + a^2*c*x^2]*(-(a*x*Sqrt[1 + a^2*x^2]) - ArcSinh[a*x] + 8*Sqrt[1
+ a^2*x^2]*ArcTan[a*x] + 2*a^2*x^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x] + 6*ArcTan
[a*x]*Log[1 - E^(I*ArcTan[a*x])] - 6*ArcTan[a*x]*Log[1 + E^(I*ArcTan[a*x])]
 + 6*Log[Cos[ArcTan[a*x]/2] - Sin[ArcTan[a*x]/2]] - 6*Log[Cos[ArcTan[a*x]/2
] + Sin[ArcTan[a*x]/2]] + (6*I)*PolyLog[2, -E^(I*ArcTan[a*x])] - (6*I)*Poly
```

```
Log[2, E^(I*ArcTan[a*x])]]))/(6*Sqrt[1 + a^2*x^2])
```

Maple [A] time = 0.312, size = 174, normalized size = 0.6

$$\frac{c\left(2\arctan\left(ax\right)a^2x^2-ax+8\arctan\left(ax\right)\right)}{6}\sqrt{c\left(ax-i\right)\left(ax+i\right)}+\frac{c}{3}\sqrt{c\left(ax-i\right)\left(ax+i\right)}\left(7i\arctan\left(\left(1+iax\right)\frac{1}{\sqrt{a^2x^2+1}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(3/2)*arctan(a*x)/x,x)
```

```
[Out] 1/6*c*(c*(a*x-I)*(a*x+I))^(1/2)*(2*arctan(a*x)*a^2*x^2-a*x+8*arctan(a*x))+1
/3*c*(c*(a*x-I)*(a*x+I))^(1/2)*(7*I*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2))+3*I
*dilog((1+I*a*x)/(a^2*x^2+1)^(1/2))+3*I*dilog(1+(1+I*a*x)/(a^2*x^2+1)^(1/2)
)-3*arctan(a*x)*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2)))/(a^2*x^2+1)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^2cx^2+c\right)^{\frac{3}{2}}\arctan\left(ax\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)/x,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)^(3/2)*arctan(a*x)/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\operatorname{atan}(ax)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(3/2)*atan(a*x)/x,x)
```

```
[Out] Integral((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^{\frac{3}{2}}\arctan(ax)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)/x,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)*arctan(a*x)/x, x)
```

3.213 $\int \frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}{x^2}\,dx$

Optimal. Leaf size=300

$$\frac{3iac^2\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{2\sqrt{a^2cx^2+c}}-\frac{3iac^2\sqrt{a^2x^2+1}\text{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{2\sqrt{a^2cx^2+c}}-\frac{3iac^2\sqrt{a^2x^2+1}\tan^{-1}(ax)\tan^{-1}\left(\frac{\sqrt{1+ia}}{\sqrt{1-ia}}\right.}{\sqrt{a^2cx^2+c}}$$

```
[Out] -(a*c*Sqrt[c + a^2*c*x^2])/2 - (c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/x + (a^2
*c*x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/2 - ((3*I)*a*c^2*Sqrt[1 + a^2*x^2]*Ar
cTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2] - a*
c^(3/2)*ArcTanh[Sqrt[c + a^2*c*x^2]/Sqrt[c]] + (((3*I)/2)*a*c^2*Sqrt[1 + a^
2*x^2]*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x
^2] - (((3*I)/2)*a*c^2*Sqrt[1 + a^2*x^2]*PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqr
t[1 - I*a*x]])/Sqrt[c + a^2*c*x^2]
```

Rubi [A] time = 0.422001, antiderivative size = 300, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {4950, 4944, 266, 63, 208, 4890, 4886, 4878}

$$\frac{3iac^2\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{2\sqrt{a^2cx^2+c}}-\frac{3iac^2\sqrt{a^2x^2+1}\text{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{2\sqrt{a^2cx^2+c}}-\frac{3iac^2\sqrt{a^2x^2+1}\tan^{-1}(ax)\tan^{-1}\left(\frac{\sqrt{1+ia}}{\sqrt{1-ia}}\right.}{\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x])/x^2,x]
```

```
[Out] -(a*c*Sqrt[c + a^2*c*x^2])/2 - (c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/x + (a^2
*c*x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/2 - ((3*I)*a*c^2*Sqrt[1 + a^2*x^2]*Ar
cTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2] - a*
c^(3/2)*ArcTanh[Sqrt[c + a^2*c*x^2]/Sqrt[c]] + (((3*I)/2)*a*c^2*Sqrt[1 + a^
2*x^2]*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x
^2] - (((3*I)/2)*a*c^2*Sqrt[1 + a^2*x^2]*PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqr
t[1 - I*a*x]])/Sqrt[c + a^2*c*x^2]
```

Rule 4950

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2)^
```

```
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 4944

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a +
 b*ArcTan[c*x])^p)/(d*f*(m + 1)), x] - Dist[(b*c*p)/(f*(m + 1)), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 4890

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4886

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[(-2*I*(a + b*ArcTan[c*x])*ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]])
/(c*Sqrt[d]), x] + (Simp[(I*b*PolyLog[2, -((I*Sqrt[1 + I*c*x])/Sqrt[1 - I*c
*x])])/(c*Sqrt[d]), x] - Simp[(I*b*PolyLog[2, (I*Sqrt[1 + I*c*x])/Sqrt[1 -
```

```
I*c*x]])/(c*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

Rule 4878

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symbo
l] :> -Simp[(b*(d + e*x^2)^q)/(2*c*q*(2*q + 1)), x] + (Dist[(2*d*q)/(2*q +
1), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x]), x], x] + Simp[(x*(d + e*x^
2)^q*(a + b*ArcTan[c*x]))/(2*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] && Eq
Q[e, c^2*d] && GtQ[q, 0]
```

Rubi steps

$$\int \frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}{x^2}\,dx = c\int \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{x^2}\,dx + \left(a^2c\right)\int \sqrt{c+a^2cx^2}\tan^{-1}(ax)\,dx$$

$$= -\frac{1}{2}ac\sqrt{c+a^2cx^2} + \frac{1}{2}a^2cx\sqrt{c+a^2cx^2}\tan^{-1}(ax) + c^2\int \frac{\tan^{-1}(ax)}{x^2\sqrt{c+a^2cx^2}}\,dx + \frac{1}{2}\left(a^2c^2\right)\int$$

$$= -\frac{1}{2}ac\sqrt{c+a^2cx^2} - \frac{c\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{x} + \frac{1}{2}a^2cx\sqrt{c+a^2cx^2}\tan^{-1}(ax) + \left(ac^2\right)\int \frac{}{x}$$

$$= -\frac{1}{2}ac\sqrt{c+a^2cx^2} - \frac{c\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{x} + \frac{1}{2}a^2cx\sqrt{c+a^2cx^2}\tan^{-1}(ax) - \frac{3iac^2\sqrt{1+}}{}$$

$$= -\frac{1}{2}ac\sqrt{c+a^2cx^2} - \frac{c\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{x} + \frac{1}{2}a^2cx\sqrt{c+a^2cx^2}\tan^{-1}(ax) - \frac{3iac^2\sqrt{1+}}{}$$

$$= -\frac{1}{2}ac\sqrt{c+a^2cx^2} - \frac{c\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{x} + \frac{1}{2}a^2cx\sqrt{c+a^2cx^2}\tan^{-1}(ax) - \frac{3iac^2\sqrt{1+}}{}$$

Mathematica [A] time = 0.897408, size = 218, normalized size = 0.73

$$\frac{c\sqrt{a^2cx^2+c}\left(3iax\text{PolyLog}\left(2,-ie^{i\tan^{-1}(ax)}\right) - 3iax\text{PolyLog}\left(2,ie^{i\tan^{-1}(ax)}\right) - ax\sqrt{a^2x^2+1} + a^2x^2\sqrt{a^2x^2+1}\tan^{-1}(ax)}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x])/x^2,x]
```

```
[Out] (c*Sqrt[c + a^2*c*x^2]*(-(a*x*Sqrt[1 + a^2*x^2]) - 2*Sqrt[1 + a^2*x^2]*ArcT
an[a*x] + a^2*x^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x] + 3*a*x*ArcTan[a*x]*Log[1 -
 I*E^(I*ArcTan[a*x])] - 3*a*x*ArcTan[a*x]*Log[1 + I*E^(I*ArcTan[a*x])] - 2*
a*x*Log[Cos[ArcTan[a*x]/2]] + 2*a*x*Log[Sin[ArcTan[a*x]/2]] + (3*I)*a*x*Pol
yLog[2, (-I)*E^(I*ArcTan[a*x])] - (3*I)*a*x*PolyLog[2, I*E^(I*ArcTan[a*x])]
))/(2*x*Sqrt[1 + a^2*x^2])
```

Maple [A] time = 0.319, size = 240, normalized size = 0.8

$$\frac{c\left(\arctan(ax)\,a^2x^2-ax-2\arctan(ax)\right)}{2x}\sqrt{c\,(ax-i)\,(ax+i)}+\frac{ac}{2}\sqrt{c\,(ax-i)\,(ax+i)}\left(3\arctan(ax)\ln\left(1-\frac{i\,(1+iax)}{\sqrt{a^2x^2+1}}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(3/2)*arctan(a*x)/x^2,x)
```

```
[Out] 1/2*c*(c*(a*x-I)*(a*x+I))^(1/2)*(arctan(a*x)*a^2*x^2-a*x-2*arctan(a*x))/x+1
/2*(c*(a*x-I)*(a*x+I))^(1/2)*(3*arctan(a*x)*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1
/2))-3*arctan(a*x)*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+3*I*dilog(1+I*(1+I*a
*x)/(a^2*x^2+1)^(1/2))-3*I*dilog(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*ln((1+I
*a*x)/(a^2*x^2+1)^(1/2)-1)-2*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2)))*a*c/(a^2*x^
2+1)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^2cx^2+c\right)^{\frac{3}{2}}\arctan(ax)}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)/x^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)^(3/2)*arctan(a*x)/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\operatorname{atan}(ax)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(3/2)*atan(a*x)/x**2,x)
```

```
[Out] Integral((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^{\frac{3}{2}}\arctan(ax)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)/x^2,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)*arctan(a*x)/x^2, x)
```

3.214 $\int \frac{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)}{x^3}\,dx$

Optimal. Leaf size=304

$$\frac{3ia^2c^2\sqrt{a^2x^2+1}\,\text{PolyLog}\left(2,-\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{2\sqrt{a^2cx^2+c}}-\frac{3ia^2c^2\sqrt{a^2x^2+1}\,\text{PolyLog}\left(2,\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{2\sqrt{a^2cx^2+c}}-a^2c^{3/2}\tanh^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)-\frac{3a^2c^2\sqrt{a}}{}$$

```
[Out] -(a*c*Sqrt[c + a^2*c*x^2])/(2*x) + a^2*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x] -
(c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(2*x^2) - (3*a^2*c^2*Sqrt[1 + a^2*x^2]*
ArcTan[a*x]*ArcTanh[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2] -
 a^2*c^(3/2)*ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]] + (((3*I)/2)*a^2*c^
2*Sqrt[1 + a^2*x^2]*PolyLog[2, -(Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x])])/Sqrt[c
+ a^2*c*x^2] - (((3*I)/2)*a^2*c^2*Sqrt[1 + a^2*x^2]*PolyLog[2, Sqrt[1 + I*a
*x]/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2]
```

Rubi [A] time = 0.641778, antiderivative size = 304, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {4950, 4946, 4962, 264, 4958, 4954, 217, 206}

$$\frac{3ia^2c^2\sqrt{a^2x^2+1}\,\text{PolyLog}\left(2,-\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{2\sqrt{a^2cx^2+c}}-\frac{3ia^2c^2\sqrt{a^2x^2+1}\,\text{PolyLog}\left(2,\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{2\sqrt{a^2cx^2+c}}-a^2c^{3/2}\tanh^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)-\frac{3a^2c^2\sqrt{a}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x])/x^3,x]
```


```
[Out] -(a*c*Sqrt[c + a^2*c*x^2])/(2*x) + a^2*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x] -
(c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(2*x^2) - (3*a^2*c^2*Sqrt[1 + a^2*x^2]*
ArcTan[a*x]*ArcTanh[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2] -
 a^2*c^(3/2)*ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]] + (((3*I)/2)*a^2*c^
2*Sqrt[1 + a^2*x^2]*PolyLog[2, -(Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x])])/Sqrt[c
+ a^2*c*x^2] - (((3*I)/2)*a^2*c^2*Sqrt[1 + a^2*x^2]*PolyLog[2, Sqrt[1 + I*a
*x]/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2]
```


Rule 4950

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2)^
```

```
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*ArcTan[c*x
]))/(f*(m + 2)), x] + (Dist[d/(m + 2), Int[((f*x)^m*(a + b*ArcTan[c*x]))/Sq
rt[d + e*x^2], x], x] - Dist[(b*c*d)/(f*(m + 2)), Int[(f*x)^(m + 1)/Sqrt[d
+ e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && Ne
Q[m, -2]
```

Rule 4962

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Ar
cTan[c*x])^p)/(d*f*(m + 1)), x] + (-Dist[(b*c*p)/(f*(m + 1)), Int[((f*x)^(m
 + 1)*(a + b*ArcTan[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] - Dist[(c^2*(m +
 2))/(f^2*(m + 1)), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/Sqrt[d + e*x^
2], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] &
& LtQ[m, -1] && NeQ[m, -2]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 4958

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4954

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x_
Symbol] :> Simp[(-2*(a + b*ArcTan[c*x])*ArcTanh[Sqrt[1 + I*c*x]/Sqrt[1 - I*
c*x]])/Sqrt[d], x] + (Simp[(I*b*PolyLog[2, -(Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x
])])/Sqrt[d], x] - Simp[(I*b*PolyLog[2, Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]])/S
qrt[d], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}{x^3}\,dx = c\int \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{x^3}\,dx + \left(a^2c\right)\int \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{x}\,dx$$

$$= a^2c\sqrt{c+a^2cx^2}\tan^{-1}(ax) - \frac{c\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{x^2} - c^2\int \frac{\tan^{-1}(ax)}{x^3\sqrt{c+a^2cx^2}}\,dx + \left(ac^2\right)\int$$

$$= -\frac{ac\sqrt{c+a^2cx^2}}{x} + a^2c\sqrt{c+a^2cx^2}\tan^{-1}(ax) - \frac{c\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{2x^2} - \frac{1}{2}\left(ac^2\right)\int \frac{}{x^2\sqrt{c}}$$

$$= -\frac{ac\sqrt{c+a^2cx^2}}{2x} + a^2c\sqrt{c+a^2cx^2}\tan^{-1}(ax) - \frac{c\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{2x^2} - \frac{2a^2c^2\sqrt{1+a^2x^2}}{}$$

$$= -\frac{ac\sqrt{c+a^2cx^2}}{2x} + a^2c\sqrt{c+a^2cx^2}\tan^{-1}(ax) - \frac{c\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{2x^2} - \frac{3a^2c^2\sqrt{1+a^2x^2}}{}$$

Mathematica [A] time = 1.60369, size = 301, normalized size = 0.99

$$\frac{a^2c\sqrt{a^2cx^2+c}\tan\left(\frac{1}{2}\tan^{-1}(ax)\right)\left(12i\cot\left(\frac{1}{2}\tan^{-1}(ax)\right)\text{PolyLog}\left(2,-e^{i\tan^{-1}(ax)}\right)-12i\cot\left(\frac{1}{2}\tan^{-1}(ax)\right)\text{PolyLog}\left(2,e^{i}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x])/x^3,x]
```

```
[Out] (a^2*c*Sqrt[c + a^2*c*x^2]*(-2 - 2*Cot[ArcTan[a*x]/2]^2 + 4*a*x*ArcTan[a*x]
*Csc[ArcTan[a*x]/2]^2 - ArcTan[a*x]*Cot[ArcTan[a*x]/2]*Csc[ArcTan[a*x]/2]^2
 + 12*ArcTan[a*x]*Cot[ArcTan[a*x]/2]*Log[1 - E^(I*ArcTan[a*x])] - 12*ArcTan
[a*x]*Cot[ArcTan[a*x]/2]*Log[1 + E^(I*ArcTan[a*x])] + 8*Cot[ArcTan[a*x]/2]*
```

```
Log[Cos[ArcTan[a*x]/2] - Sin[ArcTan[a*x]/2]] - 8*Cot[ArcTan[a*x]/2]*Log[Cos
[ArcTan[a*x]/2] + Sin[ArcTan[a*x]/2]] + (12*I)*Cot[ArcTan[a*x]/2]*PolyLog[2
, -E^(I*ArcTan[a*x])] - (12*I)*Cot[ArcTan[a*x]/2]*PolyLog[2, E^(I*ArcTan[a*
x])] + ArcTan[a*x]*Csc[ArcTan[a*x]/2]*Sec[ArcTan[a*x]/2])*Tan[ArcTan[a*x]/2
])/(8*Sqrt[1 + a^2*x^2])
```

Maple [A] time = 0.326, size = 180, normalized size = 0.6

$$\frac{c\left(2\arctan\left(ax\right)a^{2}x^{2}-ax-\arctan\left(ax\right)\right)}{2x^{2}}\sqrt{c\left(ax-i\right)\left(ax+i\right)}-\frac{a^{2}c}{2}\sqrt{c\left(ax-i\right)\left(ax+i\right)}\left(3\arctan\left(ax\right)\ln\left(1+\frac{1+iax}{\sqrt{a^{2}x^{2}+1}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(3/2)*arctan(a*x)/x^3,x)
```

```
[Out] 1/2*c*(c*(a*x-I)*(a*x+I))^(1/2)*(2*arctan(a*x)*a^2*x^2-a*x-arctan(a*x))/x^2
-1/2*a^2*c*(c*(a*x-I)*(a*x+I))^(1/2)*(3*arctan(a*x)*ln(1+(1+I*a*x)/(a^2*x^2
+1)^(1/2))-4*I*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2))-3*I*dilog((1+I*a*x)/(a^2
*x^2+1)^(1/2))-3*I*dilog(1+(1+I*a*x)/(a^2*x^2+1)^(1/2)))/(a^2*x^2+1)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^{2}cx^{2}+c\right)^{\frac{3}{2}}\arctan\left(ax\right)}{x^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)/x^3,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)^(3/2)*arctan(a*x)/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\operatorname{atan}\left(ax\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(3/2)*atan(a*x)/x**3,x)
```

```
[Out] Integral((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^{\frac{3}{2}}\arctan\left(ax\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)/x^3,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)*arctan(a*x)/x^3, x)
```

3.215 $\int \frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}{x^4}\,dx$

Optimal. Leaf size=310

$$\frac{ia^3c^2\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{a^2cx^2+c}}-\frac{ia^3c^2\sqrt{a^2x^2+1}\text{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{a^2cx^2+c}}-\frac{2ia^3c^2\sqrt{a^2x^2+1}\tan^{-1}(ax)\tan^{-1}\left(\frac{\sqrt{1+i}}{\sqrt{1-i}}\right.}{\sqrt{a^2cx^2+c}}$$

```
[Out] -(a*c*Sqrt[c + a^2*c*x^2])/(6*x^2) - (a^2*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]
)/x - ((c + a^2*c*x^2)^(3/2)*ArcTan[a*x])/(3*x^3) - ((2*I)*a^3*c^2*Sqrt[1 +
 a^2*x^2]*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/Sqrt[c + a^2
*c*x^2] - (7*a^3*c^(3/2)*ArcTanh[Sqrt[c + a^2*c*x^2]/Sqrt[c]])/6 + (I*a^3*c
^2*Sqrt[1 + a^2*x^2]*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/Sq
rt[c + a^2*c*x^2] - (I*a^3*c^2*Sqrt[1 + a^2*x^2]*PolyLog[2, (I*Sqrt[1 + I*a
*x])/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2]
```

Rubi [A] time = 0.434311, antiderivative size = 310, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {4950, 4944, 266, 47, 63, 208, 4890, 4886}

$$\frac{ia^3c^2\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{a^2cx^2+c}}-\frac{ia^3c^2\sqrt{a^2x^2+1}\text{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{a^2cx^2+c}}-\frac{2ia^3c^2\sqrt{a^2x^2+1}\tan^{-1}(ax)\tan^{-1}\left(\frac{\sqrt{1+i}}{\sqrt{1-i}}\right.}{\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x])/x^4,x]
```


```
[Out] -(a*c*Sqrt[c + a^2*c*x^2])/(6*x^2) - (a^2*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]
)/x - ((c + a^2*c*x^2)^(3/2)*ArcTan[a*x])/(3*x^3) - ((2*I)*a^3*c^2*Sqrt[1 +
 a^2*x^2]*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/Sqrt[c + a^2
*c*x^2] - (7*a^3*c^(3/2)*ArcTanh[Sqrt[c + a^2*c*x^2]/Sqrt[c]])/6 + (I*a^3*c
^2*Sqrt[1 + a^2*x^2]*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/Sq
rt[c + a^2*c*x^2] - (I*a^3*c^2*Sqrt[1 + a^2*x^2]*PolyLog[2, (I*Sqrt[1 + I*a
*x])/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2]
```


Rule 4950

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2)^
```

```
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 4944

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a +
 b*ArcTan[c*x])^p)/(d*f*(m + 1)), x] - Dist[(b*c*p)/(f*(m + 1)), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 4890

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
```

```
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4886

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[(-2*I*(a + b*ArcTan[c*x])*ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]])
/(c*Sqrt[d]), x] + (Simp[(I*b*PolyLog[2, -((I*Sqrt[1 + I*c*x])/Sqrt[1 - I*c
*x])])/(c*Sqrt[d]), x] - Simp[(I*b*PolyLog[2, (I*Sqrt[1 + I*c*x])/Sqrt[1 -
I*c*x]])/(c*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

Rubi steps

$$\int \frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}{x^4}\,dx = c\int \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{x^4}\,dx + \left(a^2c\right)\int \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{x^2}\,dx$$

$$= -\frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}{3x^3} + \frac{1}{3}(ac)\int \frac{\sqrt{c+a^2cx^2}}{x^3}\,dx + \left(a^2c^2\right)\int \frac{\tan^{-1}(ax)}{x^2\sqrt{c+a^2cx^2}}\,dx + \Big($$

$$= -\frac{a^2c\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{x} - \frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}{3x^3} + \frac{1}{6}(ac)\,\mathrm{Subst}\Bigg(\int \frac{\sqrt{c+a^2cx}}{x^2}$$

$$= -\frac{ac\sqrt{c+a^2cx^2}}{6x^2} - \frac{a^2c\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{x} - \frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}{3x^3} - \frac{2ia^3c^2\sqrt{1+}}{}$$

$$= -\frac{ac\sqrt{c+a^2cx^2}}{6x^2} - \frac{a^2c\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{x} - \frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}{3x^3} - \frac{2ia^3c^2\sqrt{1+}}{}$$

$$= -\frac{ac\sqrt{c+a^2cx^2}}{6x^2} - \frac{a^2c\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{x} - \frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}{3x^3} - \frac{2ia^3c^2\sqrt{1+}}{}$$

Mathematica [A] time = 0.475579, size = 263, normalized size = 0.85

$$-\frac{c\sqrt{a^2cx^2+c}\left(-6ia^3x^3\mathrm{PolyLog}\left(2,-ie^{i\tan^{-1}(ax)}\right)+6ia^3x^3\mathrm{PolyLog}\left(2,ie^{i\tan^{-1}(ax)}\right)+ax\sqrt{a^2x^2+1}+8a^2x^2\sqrt{a^2x^2+1}\,\mathrm{ta}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x])/x^4,x]
```

```
[Out] -(c*Sqrt[c + a^2*c*x^2]*(a*x*Sqrt[1 + a^2*x^2] + 2*Sqrt[1 + a^2*x^2]*ArcTan
[a*x] + 8*a^2*x^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x] + a^3*x^3*ArcTanh[Sqrt[1 +
a^2*x^2]] - 6*a^3*x^3*ArcTan[a*x]*Log[1 - I*E^(I*ArcTan[a*x])] + 6*a^3*x^3*
ArcTan[a*x]*Log[1 + I*E^(I*ArcTan[a*x])] + 6*a^3*x^3*Log[Cos[ArcTan[a*x]/2]
] - 6*a^3*x^3*Log[Sin[ArcTan[a*x]/2]] - (6*I)*a^3*x^3*PolyLog[2, (-I)*E^(I*
ArcTan[a*x])] + (6*I)*a^3*x^3*PolyLog[2, I*E^(I*ArcTan[a*x])]))/(6*x^3*Sqrt
[1 + a^2*x^2])
```

Maple [A] time = 0.456, size = 245, normalized size = 0.8

$$-\frac{c\left(8\arctan\left(ax\right)a^2x^2+ax+2\arctan\left(ax\right)\right)}{6x^3}\sqrt{c\left(ax-i\right)\left(ax+i\right)}-\frac{i}{6}a^3c\sqrt{c\left(ax-i\right)\left(ax+i\right)}\left(6i\arctan\left(ax\right)\ln\left(1-i(1+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(3/2)*arctan(a*x)/x^4,x)
```

```
[Out] -1/6*c*(c*(a*x-I)*(a*x+I))^(1/2)*(8*arctan(a*x)*a^2*x^2+a*x+2*arctan(a*x))/
x^3-1/6*I*a^3*c*(c*(a*x-I)*(a*x+I))^(1/2)*(6*I*arctan(a*x)*ln(1-I*(1+I*a*x)
/(a^2*x^2+1)^(1/2))-6*I*arctan(a*x)*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+7*I
*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)-1)-7*I*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*
dilog(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*dilog(1+I*(1+I*a*x)/(a^2*x^2+1)^(1
/2)))/(a^2*x^2+1)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^2cx^2+c\right)^{\frac{3}{2}}\arctan\left(ax\right)}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)/x^4,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)^(3/2)*arctan(a*x)/x^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\operatorname{atan}\left(ax\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(3/2)*atan(a*x)/x**4,x)
```

```
[Out] Integral((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)/x**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^{\frac{3}{2}}\arctan\left(ax\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)/x^4,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)*arctan(a*x)/x^4, x)
```

3.216 $\int x^3 \left(c + a^2 c x^2\right)^{5/2} \tan^{-1}(ax)\,dx$

Optimal. Leaf size=289

$$-\frac{1}{72}a^3c^2x^7\sqrt{a^2cx^2+c}-\frac{103ac^2x^5\sqrt{a^2cx^2+c}}{3024}-\frac{205c^2x^3\sqrt{a^2cx^2+c}}{12096a}+\frac{47c^2x\sqrt{a^2cx^2+c}}{2688a^3}+\frac{1}{9}a^4c^2x^8\sqrt{a^2cx^2+c}\tan^{-1}(ax)$$

```
[Out] (47*c^2*x*Sqrt[c + a^2*c*x^2])/(2688*a^3) - (205*c^2*x^3*Sqrt[c + a^2*c*x^2
])/(12096*a) - (103*a*c^2*x^5*Sqrt[c + a^2*c*x^2])/3024 - (a^3*c^2*x^7*Sqrt
[c + a^2*c*x^2])/72 - (2*c^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(63*a^4) + (c
^2*x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(63*a^2) + (5*c^2*x^4*Sqrt[c + a^2*
c*x^2]*ArcTan[a*x])/21 + (19*a^2*c^2*x^6*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/6
3 + (a^4*c^2*x^8*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/9 + (115*c^(5/2)*ArcTanh[
(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]])/(8064*a^4)
```

Rubi [A] time = 1.97681, antiderivative size = 289, normalized size of antiderivative = 1., number of steps used = 76, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {4950, 4946, 4952, 321, 217, 206, 4930}

$$-\frac{1}{72}a^3c^2x^7\sqrt{a^2cx^2+c}-\frac{103ac^2x^5\sqrt{a^2cx^2+c}}{3024}-\frac{205c^2x^3\sqrt{a^2cx^2+c}}{12096a}+\frac{47c^2x\sqrt{a^2cx^2+c}}{2688a^3}+\frac{1}{9}a^4c^2x^8\sqrt{a^2cx^2+c}\tan^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x],x]
```


```
[Out] (47*c^2*x*Sqrt[c + a^2*c*x^2])/(2688*a^3) - (205*c^2*x^3*Sqrt[c + a^2*c*x^2
])/(12096*a) - (103*a*c^2*x^5*Sqrt[c + a^2*c*x^2])/3024 - (a^3*c^2*x^7*Sqrt
[c + a^2*c*x^2])/72 - (2*c^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(63*a^4) + (c
^2*x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(63*a^2) + (5*c^2*x^4*Sqrt[c + a^2*
c*x^2]*ArcTan[a*x])/21 + (19*a^2*c^2*x^6*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/6
3 + (a^4*c^2*x^8*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/9 + (115*c^(5/2)*ArcTanh[
(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]])/(8064*a^4)
```


Rule 4950

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
```

```
EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
IntegerQ[q]))
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*ArcTan[c*x
]))/(f*(m + 2)), x] + (Dist[d/(m + 2), Int[((f*x)^m*(a + b*ArcTan[c*x]))/Sq
rt[d + e*x^2], x], x] - Dist[(b*c*d)/(f*(m + 2)), Int[(f*x)^(m + 1)/Sqrt[d
+ e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && Ne
Q[m, -2]
```

Rule 4952

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b*
ArcTan[c*x])^p)/(c^2*d*m), x] + (-Dist[(b*f*p)/(c*m), Int[((f*x)^(m - 1)*(a
 + b*ArcTan[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] - Dist[(f^2*(m - 1))/(c^
2*m), Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/Sqrt[d + e*x^2], x], x]) /;
 FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
```

```
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rubi steps

$$\int x^3 \left(c+a^2cx^2\right)^{5/2} \tan^{-1}(ax)\,dx = c\int x^3 \left(c+a^2cx^2\right)^{3/2} \tan^{-1}(ax)\,dx + \left(a^2c\right)\int x^5 \left(c+a^2cx^2\right)^{3/2} \tan^{-1}(ax)\,dx$$

$$= c^2\int x^3\sqrt{c+a^2cx^2}\tan^{-1}(ax)\,dx + 2\left(\left(a^2c^2\right)\int x^5\sqrt{c+a^2cx^2}\tan^{-1}(ax)\,dx\right) + \left(a^4c^2\right)$$

$$= \frac{1}{5}c^2x^4\sqrt{c+a^2cx^2}\tan^{-1}(ax) + \frac{1}{9}a^4c^2x^8\sqrt{c+a^2cx^2}\tan^{-1}(ax) + \frac{1}{5}c^3\int\frac{x^3\tan^{-1}(ax)}{\sqrt{c+a^2cx^2}}\,d$$

$$= -\frac{c^2x^3\sqrt{c+a^2cx^2}}{20a} - \frac{1}{72}a^3c^2x^7\sqrt{c+a^2cx^2} + \frac{c^2x^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{15a^2} + \frac{1}{5}c^2x^4\sqrt{c+a}$$

$$= \frac{c^2x\sqrt{c+a^2cx^2}}{24a^3} - \frac{c^2x^3\sqrt{c+a^2cx^2}}{20a} + \frac{41ac^2x^5\sqrt{c+a^2cx^2}}{3024} - \frac{1}{72}a^3c^2x^7\sqrt{c+a^2cx^2} - \frac{2c^2}{}$$

$$= \frac{c^2x\sqrt{c+a^2cx^2}}{24a^3} - \frac{3761c^2x^3\sqrt{c+a^2cx^2}}{60480a} + \frac{41ac^2x^5\sqrt{c+a^2cx^2}}{3024} - \frac{1}{72}a^3c^2x^7\sqrt{c+a^2cx^2} -$$

$$= \frac{127c^2x\sqrt{c+a^2cx^2}}{2688a^3} - \frac{3761c^2x^3\sqrt{c+a^2cx^2}}{60480a} + \frac{41ac^2x^5\sqrt{c+a^2cx^2}}{3024} - \frac{1}{72}a^3c^2x^7\sqrt{c+a^2c}$$

$$= \frac{127c^2x\sqrt{c+a^2cx^2}}{2688a^3} - \frac{3761c^2x^3\sqrt{c+a^2cx^2}}{60480a} + \frac{41ac^2x^5\sqrt{c+a^2cx^2}}{3024} - \frac{1}{72}a^3c^2x^7\sqrt{c+a^2c}$$

$$= \frac{127c^2x\sqrt{c+a^2cx^2}}{2688a^3} - \frac{3761c^2x^3\sqrt{c+a^2cx^2}}{60480a} + \frac{41ac^2x^5\sqrt{c+a^2cx^2}}{3024} - \frac{1}{72}a^3c^2x^7\sqrt{c+a^2c}$$

Mathematica [A] time = 0.238495, size = 129, normalized size = 0.45

$$\frac{c^2\left(-ax\left(336a^6x^6+824a^4x^4+410a^2x^2-423\right)\sqrt{a^2cx^2+c}+345\sqrt{c}\log\left(\sqrt{c}\sqrt{a^2cx^2+c}+acx\right)+384\left(7a^2x^2-2\right)\left(a^2x^2+\right.\right.}{24192a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x],x]
```

```
[Out] (c^2*(-(a*x*Sqrt[c + a^2*c*x^2]*(-423 + 410*a^2*x^2 + 824*a^4*x^4 + 336*a^6
*x^6)) + 384*(1 + a^2*x^2)^3*(-2 + 7*a^2*x^2)*Sqrt[c + a^2*c*x^2]*ArcTan[a*
x] + 345*Sqrt[c]*Log[a*c*x + Sqrt[c]*Sqrt[c + a^2*c*x^2]]))/(24192*a^4)
```

Maple [C] time = 0.934, size = 225, normalized size = 0.8

$$\frac{c^2\left(2688\arctan(ax)x^8a^8-336x^7a^7+7296\arctan(ax)x^6a^6-824a^5x^5+5760\arctan(ax)x^4a^4-410a^3x^3+384\,\mathrm{ar}\right.}{24192\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a^2*c*x^2+c)^(5/2)*arctan(a*x),x)
```

```
[Out] 1/24192*c^2/a^4*(c*(a*x-I)*(a*x+I))^(1/2)*(2688*arctan(a*x)*x^8*a^8-336*x^7
*a^7+7296*arctan(a*x)*x^6*a^6-824*a^5*x^5+5760*arctan(a*x)*x^4*a^4-410*a^3*
x^3+384*arctan(a*x)*a^2*x^2+423*a*x-768*arctan(a*x))-115/8064*c^2/a^4*(c*(a
*x-I)*(a*x+I))^(1/2)*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)-I)/(a^2*x^2+1)^(1/2)+11
5/8064*c^2/a^4*(c*(a*x-I)*(a*x+I))^(1/2)*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)+I)/
(a^2*x^2+1)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a^2*c*x^2+c)^(5/2)*arctan(a*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.2985, size = 360, normalized size = 1.25

$$\frac{345\,c^{\frac{5}{2}}\log\left(-2\,a^2cx^2-2\,\sqrt{a^2cx^2+c}a\sqrt{c}x-c\right)-2\left(336\,a^7c^2x^7+824\,a^5c^2x^5+410\,a^3c^2x^3-423\,ac^2x-384\left(7\,a^8c^2x^8+1\right.\right.}{48384\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a^2*c*x^2+c)^(5/2)*arctan(a*x),x, algorithm="fricas")
```

```
[Out] 1/48384*(345*c^(5/2)*log(-2*a^2*c*x^2 - 2*sqrt(a^2*c*x^2 + c)*a*sqrt(c)*x -
 c) - 2*(336*a^7*c^2*x^7 + 824*a^5*c^2*x^5 + 410*a^3*c^2*x^3 - 423*a*c^2*x
- 384*(7*a^8*c^2*x^8 + 19*a^6*c^2*x^6 + 15*a^4*c^2*x^4 + a^2*c^2*x^2 - 2*c^
2)*arctan(a*x))*sqrt(a^2*c*x^2 + c))/a^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a**2*c*x**2+c)**(5/2)*atan(a*x),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.22306, size = 366, normalized size = 1.27

$$\frac{\left(\frac{21\left(3\left(a^2cx^2+c\right)^{\frac{5}{2}}-5\left(a^2cx^2+c\right)^{\frac{3}{2}}c\right)}{a^2}+\frac{6\left(15\left(a^2cx^2+c\right)^{\frac{7}{2}}-42\left(a^2cx^2+c\right)^{\frac{5}{2}}c+35\left(a^2cx^2+c\right)^{\frac{3}{2}}c^2\right)}{a^2c}+\frac{35\left(a^2cx^2+c\right)^{\frac{9}{2}}-135\left(a^2cx^2+c\right)^{\frac{7}{2}}c+189\left(a^2cx^2+c\right)^{\frac{5}{2}}c^2-105\left(a^2\right.}{a^2c^2}}{315\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a^2*c*x^2+c)^(5/2)*arctan(a*x),x, algorithm="giac")
```

```
[Out] 1/315*(21*(3*(a^2*c*x^2 + c)^(5/2) - 5*(a^2*c*x^2 + c)^(3/2)*c)/a^2 + 6*(15
*(a^2*c*x^2 + c)^(7/2) - 42*(a^2*c*x^2 + c)^(5/2)*c + 35*(a^2*c*x^2 + c)^(3
/2)*c^2)/(a^2*c) + (35*(a^2*c*x^2 + c)^(9/2) - 135*(a^2*c*x^2 + c)^(7/2)*c
+ 189*(a^2*c*x^2 + c)^(5/2)*c^2 - 105*(a^2*c*x^2 + c)^(3/2)*c^3)/(a^2*c^2))
*arctan(a*x)/a^2 - 1/24192*(sqrt(a^2*c*x^2 + c)*(2*(205*a^2*c^2 + 4*(42*a^6
*c^2*x^2 + 103*a^4*c^2)*x^2)*x^2 - 423*c^2)*x + 345*c^(5/2)*log(abs(-sqrt(a
^2*c)*x + sqrt(a^2*c*x^2 + c)))/abs(a))/a^3
```

3.217 $\int x^2 \left(c+a^2 c x^2\right)^{5/2} \tan^{-1}(ax)\, dx$

Optimal. Leaf size=418

$$-\frac{5ic^3\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{128a^3\sqrt{a^2cx^2+c}}+\frac{5ic^3\sqrt{a^2x^2+1}\text{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{128a^3\sqrt{a^2cx^2+c}}+\frac{5c^2\sqrt{a^2cx^2+c}}{128a^3}+\frac{1}{8}a^4c^2x^7\sqrt{a^2cx^2+c}\,\text{ta}$$

```
[Out] (5*c^2*Sqrt[c + a^2*c*x^2])/(128*a^3) + (5*c*(c + a^2*c*x^2)^(3/2))/(576*a^
3) + (c + a^2*c*x^2)^(5/2)/(240*a^3) - (c + a^2*c*x^2)^(7/2)/(56*a^3*c) + (
5*c^2*x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(128*a^2) + (59*c^2*x^3*Sqrt[c + a
^2*c*x^2]*ArcTan[a*x])/192 + (17*a^2*c^2*x^5*Sqrt[c + a^2*c*x^2]*ArcTan[a*x
])/48 + (a^4*c^2*x^7*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/8 + (((5*I)/64)*c^3*S
qrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/(a^3*
Sqrt[c + a^2*c*x^2]) - (((5*I)/128)*c^3*Sqrt[1 + a^2*x^2]*PolyLog[2, ((-I)*
Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/(a^3*Sqrt[c + a^2*c*x^2]) + (((5*I)/128)
*c^3*Sqrt[1 + a^2*x^2]*PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/(a^
3*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 2.03111, antiderivative size = 418, normalized size of antiderivative = 1., number of steps used = 51, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {4950, 4946, 4952, 261, 4890, 4886, 266, 43}

$$-\frac{5ic^3\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{128a^3\sqrt{a^2cx^2+c}}+\frac{5ic^3\sqrt{a^2x^2+1}\text{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{128a^3\sqrt{a^2cx^2+c}}+\frac{5c^2\sqrt{a^2cx^2+c}}{128a^3}+\frac{1}{8}a^4c^2x^7\sqrt{a^2cx^2+c}\,\text{ta}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x],x]
```



```
[Out] (5*c^2*Sqrt[c + a^2*c*x^2])/(128*a^3) + (5*c*(c + a^2*c*x^2)^(3/2))/(576*a^
3) + (c + a^2*c*x^2)^(5/2)/(240*a^3) - (c + a^2*c*x^2)^(7/2)/(56*a^3*c) + (
5*c^2*x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(128*a^2) + (59*c^2*x^3*Sqrt[c + a
^2*c*x^2]*ArcTan[a*x])/192 + (17*a^2*c^2*x^5*Sqrt[c + a^2*c*x^2]*ArcTan[a*x
])/48 + (a^4*c^2*x^7*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/8 + (((5*I)/64)*c^3*S
qrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/(a^3*
Sqrt[c + a^2*c*x^2]) - (((5*I)/128)*c^3*Sqrt[1 + a^2*x^2]*PolyLog[2, ((-I)*
Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/(a^3*Sqrt[c + a^2*c*x^2]) + (((5*I)/128)
*c^3*Sqrt[1 + a^2*x^2]*PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/(a^
3*Sqrt[c + a^2*c*x^2])
```

Rule 4950

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*ArcTan[c*x
]))/(f*(m + 2)), x] + (Dist[d/(m + 2), Int[((f*x)^m*(a + b*ArcTan[c*x]))/Sq
rt[d + e*x^2], x], x] - Dist[(b*c*d)/(f*(m + 2)), Int[(f*x)^(m + 1)/Sqrt[d
+ e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && Ne
Q[m, -2]
```

Rule 4952

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b*
ArcTan[c*x])^p)/(c^2*d*m), x] + (-Dist[(b*f*p)/(c*m), Int[((f*x)^(m - 1)*(a
 + b*ArcTan[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] - Dist[(f^2*(m - 1))/(c^
2*m), Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/Sqrt[d + e*x^2], x], x]) /;
 FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 4890

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4886

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[(-2*I*(a + b*ArcTan[c*x])*ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]])
/(c*Sqrt[d]), x] + (Simp[(I*b*PolyLog[2, -((I*Sqrt[1 + I*c*x])/Sqrt[1 - I*c
```

```
*x])])/(c*Sqrt[d]), x] - Simp[(I*b*PolyLog[2, (I*Sqrt[1 + I*c*x])/Sqrt[1 -
I*c*x]])/(c*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x^2\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)\,dx = c\int x^2\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)\,dx+\left(a^2c\right)\int x^4\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)\,dx$$

$$= c^2\int x^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)\,dx+2\left(\left(a^2c^2\right)\int x^4\sqrt{c+a^2cx^2}\tan^{-1}(ax)\,dx\right)+\left(a^4c^2\right)$$

$$= \frac{1}{4}c^2x^3\sqrt{c+a^2cx^2}\tan^{-1}(ax)+\frac{1}{8}a^4c^2x^7\sqrt{c+a^2cx^2}\tan^{-1}(ax)+\frac{1}{4}c^3\int\frac{x^2\tan^{-1}(ax)}{\sqrt{c+a^2cx^2}}$$

$$= \frac{c^2x\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{8a^2}+\frac{1}{4}c^2x^3\sqrt{c+a^2cx^2}\tan^{-1}(ax)+\frac{1}{48}a^2c^2x^5\sqrt{c+a^2cx^2}\tan$$

$$= -\frac{c^2\sqrt{c+a^2cx^2}}{8a^3}+\frac{c^2x\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{8a^2}+\frac{43}{192}c^2x^3\sqrt{c+a^2cx^2}\tan^{-1}(ax)+\frac{1}{48}a$$

$$= \frac{c^2\sqrt{c+a^2cx^2}}{4a^3}-\frac{5c\left(c+a^2cx^2\right)^{3/2}}{24a^3}+\frac{3\left(c+a^2cx^2\right)^{5/2}}{40a^3}-\frac{\left(c+a^2cx^2\right)^{7/2}}{56a^3c}+\frac{21c^2x\sqrt{c+a}}{1}$$

$$= \frac{73c^2\sqrt{c+a^2cx^2}}{384a^3}-\frac{7c\left(c+a^2cx^2\right)^{3/2}}{36a^3}+\frac{17\left(c+a^2cx^2\right)^{5/2}}{240a^3}-\frac{\left(c+a^2cx^2\right)^{7/2}}{56a^3c}+\frac{21c^2x\sqrt{c}}{}$$

$$= \frac{21c^2\sqrt{c+a^2cx^2}}{128a^3}-\frac{107c\left(c+a^2cx^2\right)^{3/2}}{576a^3}+\frac{17\left(c+a^2cx^2\right)^{5/2}}{240a^3}-\frac{\left(c+a^2cx^2\right)^{7/2}}{56a^3c}+\frac{21c^2x}{}$$

Mathematica [B] time = 15.4626, size = 1059, normalized size = 2.53

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x],x]
```

```
[Out] (c^2*Sqrt[c*(1 + a^2*x^2)]*((-6*I)*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] + (6*
I)*PolyLog[2, I*E^(I*ArcTan[a*x])] - ((1 + a^2*x^2)^2*(-2/Sqrt[1 + a^2*x^2]
 - 6*Cos[3*ArcTan[a*x]] + 3*ArcTan[a*x]*((-14*a*x)/Sqrt[1 + a^2*x^2] + 3*Lo
g[1 - I*E^(I*ArcTan[a*x])] + 4*Cos[2*ArcTan[a*x]]*(Log[1 - I*E^(I*ArcTan[a*
x])] - Log[1 + I*E^(I*ArcTan[a*x])]) + Cos[4*ArcTan[a*x]]*(Log[1 - I*E^(I*A
rcTan[a*x])] - Log[1 + I*E^(I*ArcTan[a*x])]) - 3*Log[1 + I*E^(I*ArcTan[a*x]
)] + 2*Sin[3*ArcTan[a*x]])))/4))/(48*a^3*Sqrt[1 + a^2*x^2]) + (c^2*Sqrt[c*(
1 + a^2*x^2)]*((90*I)*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - (90*I)*PolyLog[2
, I*E^(I*ArcTan[a*x])] + ((1 + a^2*x^2)^3*(12/Sqrt[1 + a^2*x^2] + 110*Cos[3
*ArcTan[a*x]] - 90*Cos[5*ArcTan[a*x]] + 15*ArcTan[a*x]*((156*a*x)/Sqrt[1 +
a^2*x^2] + 30*Log[1 - I*E^(I*ArcTan[a*x])] + 3*Cos[6*ArcTan[a*x]]*Log[1 - I
*E^(I*ArcTan[a*x])] + 45*Cos[2*ArcTan[a*x]]*(Log[1 - I*E^(I*ArcTan[a*x])] -
 Log[1 + I*E^(I*ArcTan[a*x])]) + 18*Cos[4*ArcTan[a*x]]*(Log[1 - I*E^(I*ArcT
an[a*x])] - Log[1 + I*E^(I*ArcTan[a*x])]) - 30*Log[1 + I*E^(I*ArcTan[a*x])]
 - 3*Cos[6*ArcTan[a*x]]*Log[1 + I*E^(I*ArcTan[a*x])] - 94*Sin[3*ArcTan[a*x]
] + 6*Sin[5*ArcTan[a*x]])))/16))/(720*a^3*Sqrt[1 + a^2*x^2]) + (c^2*Sqrt[c*
(1 + a^2*x^2)]*((-3150*I)*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] + (3150*I)*Pol
yLog[2, I*E^(I*ArcTan[a*x])] - ((1 + a^2*x^2)^4*(38134/Sqrt[1 + a^2*x^2] +
7658*Cos[3*ArcTan[a*x]] + 35*(314*Cos[5*ArcTan[a*x]] - 90*Cos[7*ArcTan[a*x]
] + 3*ArcTan[a*x]*((-3530*a*x)/Sqrt[1 + a^2*x^2] + 525*Log[1 - I*E^(I*ArcTa
n[a*x])] + 120*Cos[6*ArcTan[a*x]]*Log[1 - I*E^(I*ArcTan[a*x])] + 15*Cos[8*A
rcTan[a*x]]*Log[1 - I*E^(I*ArcTan[a*x])] + 840*Cos[2*ArcTan[a*x]]*(Log[1 -
I*E^(I*ArcTan[a*x])] - Log[1 + I*E^(I*ArcTan[a*x])]) + 420*Cos[4*ArcTan[a*x
]]*(Log[1 - I*E^(I*ArcTan[a*x])] - Log[1 + I*E^(I*ArcTan[a*x])]) - 525*Log[
1 + I*E^(I*ArcTan[a*x])] - 120*Cos[6*ArcTan[a*x]]*Log[1 + I*E^(I*ArcTan[a*x
])] - 15*Cos[8*ArcTan[a*x]]*Log[1 + I*E^(I*ArcTan[a*x])] + 1790*Sin[3*ArcTa
n[a*x]] - 794*Sin[5*ArcTan[a*x]] + 30*Sin[7*ArcTan[a*x]]))))/64))/(80640*a^
3*Sqrt[1 + a^2*x^2])
```

Maple [A] time = 0.503, size = 245, normalized size = 0.6

$$\frac{c^2 \left(5040 \arctan(ax) x^7 a^7 - 720 x^6 a^6 + 14280 \arctan(ax) x^5 a^5 - 1992 a^4 x^4 + 12390 \arctan(ax) x^3 a^3 - 1474 a^2 x^2 + 157\right.}{40320 a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x),x)
```

```
[Out] 1/40320*c^2/a^3*(c*(a*x-I)*(a*x+I))^(1/2)*(5040*arctan(a*x)*x^7*a^7-720*x^6
*a^6+14280*arctan(a*x)*x^5*a^5-1992*a^4*x^4+12390*arctan(a*x)*x^3*a^3-1474*
a^2*x^2+1575*arctan(a*x)*x*a+1373)+5/128*c^2*(c*(a*x-I)*(a*x+I))^(1/2)*(arc
tan(a*x)*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-arctan(a*x)*ln(1-I*(1+I*a*x)/(
a^2*x^2+1)^(1/2))-I*dilog(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+I*dilog(1-I*(1+I
*a*x)/(a^2*x^2+1)^(1/2)))/a^3/(a^2*x^2+1)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^4c^2x^6+2\,a^2c^2x^4+c^2x^2\right)\sqrt{a^2cx^2+c}\arctan\left(ax\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^6 + 2*a^2*c^2*x^4 + c^2*x^2)*sqrt(a^2*c*x^2 + c)*arctan
(a*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a**2*c*x**2+c)**(5/2)*atan(a*x),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.218 $\int x\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)\,dx$

Optimal. Leaf size=134

$$-\frac{5c^2x\sqrt{a^2cx^2+c}}{112a}-\frac{5c^{5/2}\tanh^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{112a^2}-\frac{x\left(a^2cx^2+c\right)^{5/2}}{42a}-\frac{5cx\left(a^2cx^2+c\right)^{3/2}}{168a}+\frac{\left(a^2cx^2+c\right)^{7/2}\tan^{-1}(ax)}{7a^2c}$$

```
[Out] (-5*c^2*x*Sqrt[c + a^2*c*x^2])/(112*a) - (5*c*x*(c + a^2*c*x^2)^(3/2))/(168
*a) - (x*(c + a^2*c*x^2)^(5/2))/(42*a) + ((c + a^2*c*x^2)^(7/2)*ArcTan[a*x]
)/(7*a^2*c) - (5*c^(5/2)*ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]])/(112*a
^2)
```

Rubi [A] time = 0.0853006, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4930, 195, 217, 206}

$$-\frac{5c^2x\sqrt{a^2cx^2+c}}{112a}-\frac{5c^{5/2}\tanh^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{112a^2}-\frac{x\left(a^2cx^2+c\right)^{5/2}}{42a}-\frac{5cx\left(a^2cx^2+c\right)^{3/2}}{168a}+\frac{\left(a^2cx^2+c\right)^{7/2}\tan^{-1}(ax)}{7a^2c}$$

Antiderivative was successfully verified.

```
[In] Int[x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x],x]
```

```
[Out] (-5*c^2*x*Sqrt[c + a^2*c*x^2])/(112*a) - (5*c*x*(c + a^2*c*x^2)^(3/2))/(168
*a) - (x*(c + a^2*c*x^2)^(5/2))/(42*a) + ((c + a^2*c*x^2)^(7/2)*ArcTan[a*x]
)/(7*a^2*c) - (5*c^(5/2)*ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]])/(112*a
^2)
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
```

```
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)\,dx=\frac{\left(c+a^2cx^2\right)^{7/2}\tan^{-1}(ax)}{7a^2c}-\frac{\int\left(c+a^2cx^2\right)^{5/2}\,dx}{7a}$$

$$=-\frac{x\left(c+a^2cx^2\right)^{5/2}}{42a}+\frac{\left(c+a^2cx^2\right)^{7/2}\tan^{-1}(ax)}{7a^2c}-\frac{(5c)\int\left(c+a^2cx^2\right)^{3/2}\,dx}{42a}$$

$$=-\frac{5cx\left(c+a^2cx^2\right)^{3/2}}{168a}-\frac{x\left(c+a^2cx^2\right)^{5/2}}{42a}+\frac{\left(c+a^2cx^2\right)^{7/2}\tan^{-1}(ax)}{7a^2c}-\frac{\left(5c^2\right)\int\sqrt{c+a^2cx}}{56a}$$

$$=-\frac{5c^2x\sqrt{c+a^2cx^2}}{112a}-\frac{5cx\left(c+a^2cx^2\right)^{3/2}}{168a}-\frac{x\left(c+a^2cx^2\right)^{5/2}}{42a}+\frac{\left(c+a^2cx^2\right)^{7/2}\tan^{-1}(ax)}{7a^2c}-$$

$$=-\frac{5c^2x\sqrt{c+a^2cx^2}}{112a}-\frac{5cx\left(c+a^2cx^2\right)^{3/2}}{168a}-\frac{x\left(c+a^2cx^2\right)^{5/2}}{42a}+\frac{\left(c+a^2cx^2\right)^{7/2}\tan^{-1}(ax)}{7a^2c}-$$

$$=-\frac{5c^2x\sqrt{c+a^2cx^2}}{112a}-\frac{5cx\left(c+a^2cx^2\right)^{3/2}}{168a}-\frac{x\left(c+a^2cx^2\right)^{5/2}}{42a}+\frac{\left(c+a^2cx^2\right)^{7/2}\tan^{-1}(ax)}{7a^2c}-$$

Mathematica [A] time = 0.211469, size = 111, normalized size = 0.83

$$\frac{c^2\left(-ax\left(8a^4x^4+26a^2x^2+33\right)\sqrt{a^2cx^2+c}-15\sqrt{c}\log\left(\sqrt{c}\sqrt{a^2cx^2+c}+acx\right)+48\left(a^2x^2+1\right)^3\sqrt{a^2cx^2+c}\tan^{-1}(ax)\right)}{336a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x],x]
```

```
[Out] (c^2*(-(a*x*Sqrt[c + a^2*c*x^2]*(33 + 26*a^2*x^2 + 8*a^4*x^4)) + 48*(1 + a^
2*x^2)^3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x] - 15*Sqrt[c]*Log[a*c*x + Sqrt[c]*S
qrt[c + a^2*c*x^2]]))/(336*a^2)
```

Maple [C] time = 0.329, size = 205, normalized size = 1.5

$$\frac{c^2 \left(48 \arctan(ax) x^6 a^6 - 8 a^5 x^5 + 144 \arctan(ax) x^4 a^4 - 26 a^3 x^3 + 144 \arctan(ax) a^2 x^2 - 33 ax + 48 \arctan(ax)\right)}{336 a^2} \sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x),x)
```

```
[Out] 1/336*c^2/a^2*(c*(a*x-I)*(a*x+I))^(1/2)*(48*arctan(a*x)*x^6*a^6-8*a^5*x^5+1
44*arctan(a*x)*x^4*a^4-26*a^3*x^3+144*arctan(a*x)*a^2*x^2-33*a*x+48*arctan(
a*x))+5/112*c^2/a^2*(c*(a*x-I)*(a*x+I))^(1/2)*ln((1+I*a*x)/(a^2*x^2+1)^(1/2
)-I)/(a^2*x^2+1)^(1/2)-5/112*c^2/a^2*(c*(a*x-I)*(a*x+I))^(1/2)*ln((1+I*a*x)
/(a^2*x^2+1)^(1/2)+I)/(a^2*x^2+1)^(1/2)
```

Maxima [B] time = 2.35587, size = 933, normalized size = 6.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x),x, algorithm="maxima")
```

```
[Out] 1/1680*(560*(a^2*c^2*x^2 + c^2)*sqrt(a^2*x^2 + 1)*sqrt(c)*arctan(a*x) - 280
*(a^4*x^4 + 10*a^2*x^2 + 9)^(1/4)*(a*c^2*x*cos(1/2*arctan2(4*a*x, -a^2*x^2
+ 3)) + 2*c^2*sin(1/2*arctan2(4*a*x, -a^2*x^2 + 3)))*sqrt(c) - ((a*(5*(8*(a
^2*x^2 + 1)^(3/2)*x^3/a^2 - 6*(a^2*x^2 + 1)^(3/2)*x/a^4 + 3*sqrt(a^2*x^2 +
1)*x/a^4 + 3*arcsinh(a^2*x/sqrt(a^2))/(sqrt(a^2)*a^4))/a^2 - 24*(2*(a^2*x^2
 + 1)^(3/2)*x/a^2 - sqrt(a^2*x^2 + 1)*x/a^2 - arcsinh(a^2*x/sqrt(a^2))/(sqr
t(a^2)*a^2))/a^4 + 64*(sqrt(a^2*x^2 + 1)*x + arcsinh(a^2*x/sqrt(a^2))/sqrt(
a^2))/a^6) - 16*(15*(a^2*x^2 + 1)^(3/2)*x^4/a^2 - 12*(a^2*x^2 + 1)^(3/2)*x^
2/a^4 + 8*(a^2*x^2 + 1)^(3/2)/a^6)*arctan(a*x))*a^6*c^2 + 28*(a*(3*(2*(a^2*
x^2 + 1)^(3/2)*x/a^2 - sqrt(a^2*x^2 + 1)*x/a^2 - arcsinh(a^2*x/sqrt(a^2))/(
sqrt(a^2)*a^2))/a^2 - 8*(sqrt(a^2*x^2 + 1)*x + arcsinh(a^2*x/sqrt(a^2))/sqr
t(a^2))/a^4) - 8*(3*(a^2*x^2 + 1)^(3/2)*x^2/a^2 - 2*(a^2*x^2 + 1)^(3/2)/a^4
```

```
)*arctan(a*x))*a^4*c^2 - 140*c^2*arctan2((a^4*x^4 + 10*a^2*x^2 + 9)^(1/4)*s
in(1/2*arctan2(4*a*x, a^2*x^2 - 3)) + 2, a*x + (a^4*x^4 + 10*a^2*x^2 + 9)^(
1/4)*cos(1/2*arctan2(4*a*x, a^2*x^2 - 3))) - 140*c^2*arctan2((a^4*x^4 + 10*
a^2*x^2 + 9)^(1/4)*sin(1/2*arctan2(4*a*x, a^2*x^2 - 3)) - 2, -a*x + (a^4*x^
4 + 10*a^2*x^2 + 9)^(1/4)*cos(1/2*arctan2(4*a*x, a^2*x^2 - 3))))*sqrt(c))/a
^2
```

Fricas [A] time = 2.48669, size = 298, normalized size = 2.22

$$\frac{15\,c^{\frac{5}{2}}\log\left(-2\,a^2cx^2+2\,\sqrt{a^2cx^2+c}a\sqrt{c}x-c\right)-2\left(8\,a^5c^2x^5+26\,a^3c^2x^3+33\,ac^2x-48\left(a^6c^2x^6+3\,a^4c^2x^4+3\,a^2c^2x^2+c^2\right)\right.}{672\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x),x, algorithm="fricas")
```

```
[Out] 1/672*(15*c^(5/2)*log(-2*a^2*c*x^2 + 2*sqrt(a^2*c*x^2 + c)*a*sqrt(c)*x - c)
 - 2*(8*a^5*c^2*x^5 + 26*a^3*c^2*x^3 + 33*a*c^2*x - 48*(a^6*c^2*x^6 + 3*a^4
*c^2*x^4 + 3*a^2*c^2*x^2 + c^2)*arctan(a*x))*sqrt(a^2*c*x^2 + c))/a^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**(5/2)*atan(a*x),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19344, size = 238, normalized size = 1.78

$$-\frac{\sqrt{a^2cx^2+c}\left(2\left(4\,a^4c^2x^2+13\,a^2c^2\right)x^2+33\,c^2\right)x-\frac{15\,c^{\frac{5}{2}}\log\left(\left|-\sqrt{a^2c}x+\sqrt{a^2cx^2+c}\right|\right)}{|a|}}{336\,a}+\frac{\left(42\left(a^2cx^2+c\right)^{\frac{5}{2}}-35\left(a^2cx^2+c\right)^{\frac{3}{2}}c+\frac{15\left(a\right.}{}\right.}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x),x, algorithm="giac")

[Out] -1/336*(sqrt(a^2*c*x^2 + c)*(2*(4*a^4*c^2*x^2 + 13*a^2*c^2)*x^2 + 33*c^2)*x
 - 15*c^(5/2)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 + c)))/abs(a))/a + 1/
105*(42*(a^2*c*x^2 + c)^(5/2) - 35*(a^2*c*x^2 + c)^(3/2)*c + (15*(a^2*c*x^2
 + c)^(7/2) - 42*(a^2*c*x^2 + c)^(5/2)*c + 35*(a^2*c*x^2 + c)^(3/2)*c^2)/c)
*arctan(a*x)/a^2
```

3.219 $\int \left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)\,dx$

Optimal. Leaf size=348

$$\frac{5ic^3\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{16a\sqrt{a^2cx^2+c}}-\frac{5ic^3\sqrt{a^2x^2+1}\text{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{16a\sqrt{a^2cx^2+c}}-\frac{5c^2\sqrt{a^2cx^2+c}}{16a}-\frac{5ic^3\sqrt{a^2x^2+1}\tan^{-1}(ax)}{8a\sqrt{a^2cx^2+}}$$

```
[Out] (-5*c^2*Sqrt[c + a^2*c*x^2])/(16*a) - (5*c*(c + a^2*c*x^2)^(3/2))/(72*a) -
(c + a^2*c*x^2)^(5/2)/(30*a) + (5*c^2*x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/16
 + (5*c*x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x])/24 + (x*(c + a^2*c*x^2)^(5/2)*
ArcTan[a*x])/6 - (((5*I)/8)*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTan[Sqrt[1
 + I*a*x]/Sqrt[1 - I*a*x]])/(a*Sqrt[c + a^2*c*x^2]) + (((5*I)/16)*c^3*Sqrt[
1 + a^2*x^2]*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/(a*Sqrt[c
+ a^2*c*x^2]) - (((5*I)/16)*c^3*Sqrt[1 + a^2*x^2]*PolyLog[2, (I*Sqrt[1 + I*
a*x])/Sqrt[1 - I*a*x]])/(a*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.194221, antiderivative size = 348, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {4878, 4890, 4886}

$$\frac{5ic^3\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{16a\sqrt{a^2cx^2+c}}-\frac{5ic^3\sqrt{a^2x^2+1}\text{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{16a\sqrt{a^2cx^2+c}}-\frac{5c^2\sqrt{a^2cx^2+c}}{16a}-\frac{5ic^3\sqrt{a^2x^2+1}\tan^{-1}(ax)}{8a\sqrt{a^2cx^2+}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + a^2*c*x^2)^(5/2)*ArcTan[a*x],x]
```


```
[Out] (-5*c^2*Sqrt[c + a^2*c*x^2])/(16*a) - (5*c*(c + a^2*c*x^2)^(3/2))/(72*a) -
(c + a^2*c*x^2)^(5/2)/(30*a) + (5*c^2*x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/16
 + (5*c*x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x])/24 + (x*(c + a^2*c*x^2)^(5/2)*
ArcTan[a*x])/6 - (((5*I)/8)*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTan[Sqrt[1
 + I*a*x]/Sqrt[1 - I*a*x]])/(a*Sqrt[c + a^2*c*x^2]) + (((5*I)/16)*c^3*Sqrt[
1 + a^2*x^2]*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/(a*Sqrt[c
+ a^2*c*x^2]) - (((5*I)/16)*c^3*Sqrt[1 + a^2*x^2]*PolyLog[2, (I*Sqrt[1 + I*
a*x])/Sqrt[1 - I*a*x]])/(a*Sqrt[c + a^2*c*x^2])
```


Rule 4878

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symbo
l] :> -Simp[(b*(d + e*x^2)^q)/(2*c*q*(2*q + 1)), x] + (Dist[(2*d*q)/(2*q +
```

```
1), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x]), x], x] + Simp[(x*(d + e*x^
2)^q*(a + b*ArcTan[c*x]))/(2*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] && Eq
Q[e, c^2*d] && GtQ[q, 0]
```

Rule 4890

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4886

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[(-2*I*(a + b*ArcTan[c*x])*ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]])
/(c*Sqrt[d]), x] + (Simp[(I*b*PolyLog[2, -((I*Sqrt[1 + I*c*x])/Sqrt[1 - I*c
*x])])/(c*Sqrt[d]), x] - Simp[(I*b*PolyLog[2, (I*Sqrt[1 + I*c*x])/Sqrt[1 -
I*c*x]])/(c*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

Rubi steps

$$\begin{aligned}\int \left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)\,dx &= -\frac{\left(c+a^2cx^2\right)^{5/2}}{30a}+\frac{1}{6}x\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)+\frac{1}{6}(5c)\int \left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)\,dx\\
&= -\frac{5c\left(c+a^2cx^2\right)^{3/2}}{72a}-\frac{\left(c+a^2cx^2\right)^{5/2}}{30a}+\frac{5}{24}cx\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)+\frac{1}{6}x\left(c+a^2cx^2\right)^{5}\\
&= -\frac{5c^2\sqrt{c+a^2cx^2}}{16a}-\frac{5c\left(c+a^2cx^2\right)^{3/2}}{72a}-\frac{\left(c+a^2cx^2\right)^{5/2}}{30a}+\frac{5}{16}c^2x\sqrt{c+a^2cx^2}\tan^{-1}(ax)+\\
&= -\frac{5c^2\sqrt{c+a^2cx^2}}{16a}-\frac{5c\left(c+a^2cx^2\right)^{3/2}}{72a}-\frac{\left(c+a^2cx^2\right)^{5/2}}{30a}+\frac{5}{16}c^2x\sqrt{c+a^2cx^2}\tan^{-1}(ax)+\\
&= -\frac{5c^2\sqrt{c+a^2cx^2}}{16a}-\frac{5c\left(c+a^2cx^2\right)^{3/2}}{72a}-\frac{\left(c+a^2cx^2\right)^{5/2}}{30a}+\frac{5}{16}c^2x\sqrt{c+a^2cx^2}\tan^{-1}(ax)+\end{aligned}$$

Mathematica [A] time = 6.14915, size = 643, normalized size = 1.85

$$\frac{c^2\sqrt{a^2cx^2+c}\left(450i\text{PolyLog}\left(2,-ie^{i\tan^{-1}(ax)}\right)-450i\text{PolyLog}\left(2,ie^{i\tan^{-1}(ax)}\right)+\frac{3}{4}\left(a^2x^2+1\right)^{5/2}+720\sqrt{a^2x^2+1}\left(ax\tan\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + a^2*c*x^2)^(5/2)*ArcTan[a*x],x]
```

```
[Out] (c^2*Sqrt[c + a^2*c*x^2]*((3*(1 + a^2*x^2)^(5/2))/4 + 720*Sqrt[1 + a^2*x^2]
*(-1 + a*x*ArcTan[a*x]) + (55*(1 + a^2*x^2)^3*Cos[3*ArcTan[a*x]])/8 - (45*(
1 + a^2*x^2)^3*Cos[5*ArcTan[a*x]])/8 + 720*ArcTan[a*x]*(Log[1 - I*E^(I*ArcT
an[a*x])] - Log[1 + I*E^(I*ArcTan[a*x])]) + (450*I)*PolyLog[2, (-I)*E^(I*Ar
cTan[a*x])] - (450*I)*PolyLog[2, I*E^(I*ArcTan[a*x])] - 15*(1 + a^2*x^2)^2*
(-2/Sqrt[1 + a^2*x^2] - 6*Cos[3*ArcTan[a*x]] + 3*ArcTan[a*x]*((-14*a*x)/Sqr
t[1 + a^2*x^2] + 3*Log[1 - I*E^(I*ArcTan[a*x])] + 4*Cos[2*ArcTan[a*x]]*(Log
[1 - I*E^(I*ArcTan[a*x])] - Log[1 + I*E^(I*ArcTan[a*x])]) + Cos[4*ArcTan[a*
x]]*(Log[1 - I*E^(I*ArcTan[a*x])] - Log[1 + I*E^(I*ArcTan[a*x])]) - 3*Log[1
 + I*E^(I*ArcTan[a*x])] + 2*Sin[3*ArcTan[a*x]])) + (15*(1 + a^2*x^2)^3*ArcT
an[a*x]*((156*a*x)/Sqrt[1 + a^2*x^2] + 30*Log[1 - I*E^(I*ArcTan[a*x])] + 3*
Cos[6*ArcTan[a*x]]*Log[1 - I*E^(I*ArcTan[a*x])] + 45*Cos[2*ArcTan[a*x]]*(Lo
g[1 - I*E^(I*ArcTan[a*x])] - Log[1 + I*E^(I*ArcTan[a*x])]) + 18*Cos[4*ArcTa
n[a*x]]*(Log[1 - I*E^(I*ArcTan[a*x])] - Log[1 + I*E^(I*ArcTan[a*x])]) - 30*
Log[1 + I*E^(I*ArcTan[a*x])] - 3*Cos[6*ArcTan[a*x]]*Log[1 + I*E^(I*ArcTan[a
*x])] - 94*Sin[3*ArcTan[a*x]] + 6*Sin[5*ArcTan[a*x]]))/16))/(1440*a*Sqrt[1
+ a^2*x^2])
```

Maple [A] time = 0.315, size = 225, normalized size = 0.7

$$\frac{c^2\left(120\arctan(ax)x^5a^5 - 24a^4x^4 + 390\arctan(ax)x^3a^3 - 98a^2x^2 + 495\arctan(ax)xa - 299\right)}{720a}\sqrt{c(ax-i)(ax+i)} - \frac{5}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(5/2)*arctan(a*x),x)
```

```
[Out] 1/720*c^2/a*(c*(a*x-I)*(a*x+I))^(1/2)*(120*arctan(a*x)*x^5*a^5-24*a^4*x^4+3
90*arctan(a*x)*x^3*a^3-98*a^2*x^2+495*arctan(a*x)*x*a-299)-5/16*c^2*(c*(a*x
-I)*(a*x+I))^(1/2)*(arctan(a*x)*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-arctan(
a*x)*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*dilog(1+I*(1+I*a*x)/(a^2*x^2+1)^
(1/2))+I*dilog(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))/a/(a^2*x^2+1)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^4c^2x^4+2\,a^2c^2x^2+c^2\right)\sqrt{a^2cx^2+c}\arctan\left(ax\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*arctan(a*x
), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(5/2)*atan(a*x),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.220 $\int \frac{(c+a^2cx^2)^{5/2}\tan^{-1}(ax)}{x}\,dx$

Optimal. Leaf size=329

$$\frac{ic^3\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{a^2cx^2+c}}-\frac{ic^3\sqrt{a^2x^2+1}\text{PolyLog}\left(2,\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{a^2cx^2+c}}-\frac{29}{120}ac^2x\sqrt{a^2cx^2+c}+c^2\sqrt{a^2cx^2+c}\tan^{-1}(ax)$$

```
[Out] (-29*a*c^2*x*Sqrt[c + a^2*c*x^2])/120 - (a*c*x*(c + a^2*c*x^2)^(3/2))/20 +
c^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x] + (c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x])
/3 + ((c + a^2*c*x^2)^(5/2)*ArcTan[a*x])/5 - (2*c^3*Sqrt[1 + a^2*x^2]*ArcTa
n[a*x]*ArcTanh[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2] - (149
*c^(5/2)*ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]])/120 + (I*c^3*Sqrt[1 +
a^2*x^2]*PolyLog[2, -(Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x])])/Sqrt[c + a^2*c*x^2
] - (I*c^3*Sqrt[1 + a^2*x^2]*PolyLog[2, Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/S
qrt[c + a^2*c*x^2]
```

Rubi [A] time = 0.550225, antiderivative size = 329, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {4950, 4946, 4958, 4954, 217, 206, 4930, 195}

$$\frac{ic^3\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{a^2cx^2+c}}-\frac{ic^3\sqrt{a^2x^2+1}\text{PolyLog}\left(2,\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{a^2cx^2+c}}-\frac{29}{120}ac^2x\sqrt{a^2cx^2+c}+c^2\sqrt{a^2cx^2+c}\tan^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x])/x,x]
```


```
[Out] (-29*a*c^2*x*Sqrt[c + a^2*c*x^2])/120 - (a*c*x*(c + a^2*c*x^2)^(3/2))/20 +
c^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x] + (c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x])
/3 + ((c + a^2*c*x^2)^(5/2)*ArcTan[a*x])/5 - (2*c^3*Sqrt[1 + a^2*x^2]*ArcTa
n[a*x]*ArcTanh[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2] - (149
*c^(5/2)*ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]])/120 + (I*c^3*Sqrt[1 +
a^2*x^2]*PolyLog[2, -(Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x])])/Sqrt[c + a^2*c*x^2
] - (I*c^3*Sqrt[1 + a^2*x^2]*PolyLog[2, Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/S
qrt[c + a^2*c*x^2]
```


Rule 4950

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
```

```
b*ArcTan[c*x])^p, x], x] + Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*ArcTan[c*x
]))/(f*(m + 2)), x] + (Dist[d/(m + 2), Int[((f*x)^m*(a + b*ArcTan[c*x]))/Sq
rt[d + e*x^2], x], x] - Dist[(b*c*d)/(f*(m + 2)), Int[(f*x)^(m + 1)/Sqrt[d
+ e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && Ne
Q[m, -2]
```

Rule 4958

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4954

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x_
Symbol] :> Simp[(-2*(a + b*ArcTan[c*x])*ArcTanh[Sqrt[1 + I*c*x]/Sqrt[1 - I*
c*x]])/Sqrt[d], x] + (Simp[(I*b*PolyLog[2, -(Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x
])])/Sqrt[d], x] - Simp[(I*b*PolyLog[2, Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]])/S
qrt[d], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
```

```
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rubi steps

$$\int \frac{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)}{x}\,dx = c\int \frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}{x}\,dx + \left(a^2c\right)\int x\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)\,dx$$

$$= \frac{1}{5}\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax) - \frac{1}{5}(ac)\int \left(c+a^2cx^2\right)^{3/2}\,dx + c^2\int \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{x}\,dx$$

$$= -\frac{1}{20}acx\left(c+a^2cx^2\right)^{3/2} + c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax) + \frac{1}{3}c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax) + \frac{1}{5}\left(c+\right.$$

$$= -\frac{29}{120}ac^2x\sqrt{c+a^2cx^2} - \frac{1}{20}acx\left(c+a^2cx^2\right)^{3/2} + c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax) + \frac{1}{3}c\left(c+a^2cx^2\right.$$

$$= -\frac{29}{120}ac^2x\sqrt{c+a^2cx^2} - \frac{1}{20}acx\left(c+a^2cx^2\right)^{3/2} + c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax) + \frac{1}{3}c\left(c+a^2cx^2\right.$$

$$= -\frac{29}{120}ac^2x\sqrt{c+a^2cx^2} - \frac{1}{20}acx\left(c+a^2cx^2\right)^{3/2} + c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax) + \frac{1}{3}c\left(c+a^2cx^2\right.$$

Mathematica [A] time = 0.343376, size = 268, normalized size = 0.81

$$c^2\sqrt{a^2cx^2+c}\left(120i\text{PolyLog}\left(2,-e^{i\tan^{-1}(ax)}\right) - 120i\text{PolyLog}\left(2,e^{i\tan^{-1}(ax)}\right) - 6a^3x^3\sqrt{a^2x^2+1} - 35ax\sqrt{a^2x^2+1} + 24a^4x^4\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x])/x,x]
```

```
[Out] (c^2*Sqrt[c + a^2*c*x^2]*(-35*a*x*Sqrt[1 + a^2*x^2] - 6*a^3*x^3*Sqrt[1 + a^
2*x^2] - 29*ArcSinh[a*x] + 184*Sqrt[1 + a^2*x^2]*ArcTan[a*x] + 88*a^2*x^2*S
```

```
qrt[1 + a^2*x^2]*ArcTan[a*x] + 24*a^4*x^4*Sqrt[1 + a^2*x^2]*ArcTan[a*x] + 1
20*ArcTan[a*x]*Log[1 - E^(I*ArcTan[a*x])] - 120*ArcTan[a*x]*Log[1 + E^(I*Ar
cTan[a*x])] + 120*Log[Cos[ArcTan[a*x]/2] - Sin[ArcTan[a*x]/2]] - 120*Log[Co
s[ArcTan[a*x]/2] + Sin[ArcTan[a*x]/2]] + (120*I)*PolyLog[2, -E^(I*ArcTan[a*
x])] - (120*I)*PolyLog[2, E^(I*ArcTan[a*x])]))/(120*Sqrt[1 + a^2*x^2])
```

Maple [A] time = 0.342, size = 198, normalized size = 0.6

$$\frac{c^2\left(24\arctan(ax)x^4a^4-6a^3x^3+88\arctan(ax)a^2x^2-35ax+184\arctan(ax)\right)}{120}\sqrt{c(ax-i)(ax+i)}-\frac{c^2}{60}\sqrt{c(ax-i)(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(5/2)*arctan(a*x)/x,x)
```

```
[Out] 1/120*c^2*(c*(a*x-I)*(a*x+I))^(1/2)*(24*arctan(a*x)*x^4*a^4-6*a^3*x^3+88*ar
ctan(a*x)*a^2*x^2-35*a*x+184*arctan(a*x))-1/60*c^2*(c*(a*x-I)*(a*x+I))^(1/2
)*(60*arctan(a*x)*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-149*I*arctan((1+I*a*x)/
(a^2*x^2+1)^(1/2))-60*I*dilog((1+I*a*x)/(a^2*x^2+1)^(1/2))-60*I*dilog(1+(1+
I*a*x)/(a^2*x^2+1)^(1/2)))/(a^2*x^2+1)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(a^4c^2x^4+2\,a^2c^2x^2+c^2\right)\sqrt{a^2cx^2+c}\arctan(ax)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)/x,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*arctan(a*x
)/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(5/2)*atan(a*x)/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^{\frac{5}{2}}\arctan\left(ax\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)/x,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(5/2)*arctan(a*x)/x, x)
```

3.221 $\int \frac{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)}{x^2}\,dx$

Optimal. Leaf size=355

$$\frac{15iac^3\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{8\sqrt{a^2cx^2+c}}-\frac{15iac^3\sqrt{a^2x^2+1}\text{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{8\sqrt{a^2cx^2+c}}-\frac{7}{8}ac^2\sqrt{a^2cx^2+c}-\frac{15iac^3\sqrt{a^2x^2+1}\,t}{4\sqrt{}}$$

```
[Out] (-7*a*c^2*Sqrt[c + a^2*c*x^2])/8 - (a*c*(c + a^2*c*x^2)^(3/2))/12 - (c^2*Sq
rt[c + a^2*c*x^2]*ArcTan[a*x])/x + (7*a^2*c^2*x*Sqrt[c + a^2*c*x^2]*ArcTan[
a*x])/8 + (a^2*c*x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x])/4 - (((15*I)/4)*a*c^3
*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/Sqr
t[c + a^2*c*x^2] - a*c^(5/2)*ArcTanh[Sqrt[c + a^2*c*x^2]/Sqrt[c]] + (((15*I
)/8)*a*c^3*Sqrt[1 + a^2*x^2]*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a
*x]])/Sqrt[c + a^2*c*x^2] - (((15*I)/8)*a*c^3*Sqrt[1 + a^2*x^2]*PolyLog[2,
(I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2]
```

Rubi [A] time = 0.772699, antiderivative size = 355, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {4950, 4944, 266, 63, 208, 4890, 4886, 4878}

$$\frac{15iac^3\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{8\sqrt{a^2cx^2+c}}-\frac{15iac^3\sqrt{a^2x^2+1}\text{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{8\sqrt{a^2cx^2+c}}-\frac{7}{8}ac^2\sqrt{a^2cx^2+c}-\frac{15iac^3\sqrt{a^2x^2+1}\,t}{4\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x])/x^2,x]
```


```
[Out] (-7*a*c^2*Sqrt[c + a^2*c*x^2])/8 - (a*c*(c + a^2*c*x^2)^(3/2))/12 - (c^2*Sq
rt[c + a^2*c*x^2]*ArcTan[a*x])/x + (7*a^2*c^2*x*Sqrt[c + a^2*c*x^2]*ArcTan[
a*x])/8 + (a^2*c*x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x])/4 - (((15*I)/4)*a*c^3
*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/Sqr
t[c + a^2*c*x^2] - a*c^(5/2)*ArcTanh[Sqrt[c + a^2*c*x^2]/Sqrt[c]] + (((15*I
)/8)*a*c^3*Sqrt[1 + a^2*x^2]*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a
*x]])/Sqrt[c + a^2*c*x^2] - (((15*I)/8)*a*c^3*Sqrt[1 + a^2*x^2]*PolyLog[2,
(I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2]
```


Rule 4950

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
```

```
b*ArcTan[c*x])^p, x], x] + Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 4944

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a +
 b*ArcTan[c*x])^p)/(d*f*(m + 1)), x] - Dist[(b*c*p)/(f*(m + 1)), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 4890

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4886

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[(-2*I*(a + b*ArcTan[c*x])*ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]])
/(c*Sqrt[d]), x] + (Simp[(I*b*PolyLog[2, -((I*Sqrt[1 + I*c*x])/Sqrt[1 - I*c
```

```
*x])])/(c*Sqrt[d]), x] - Simp[(I*b*PolyLog[2, (I*Sqrt[1 + I*c*x])/Sqrt[1 -
I*c*x]])/(c*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

Rule 4878

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symbo
l] :> -Simp[(b*(d + e*x^2)^q)/(2*c*q*(2*q + 1)), x] + (Dist[(2*d*q)/(2*q +
1), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x]), x], x] + Simp[(x*(d + e*x^
2)^q*(a + b*ArcTan[c*x]))/(2*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] && Eq
Q[e, c^2*d] && GtQ[q, 0]
```

Rubi steps

$$\int \frac{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)}{x^2}\,dx = c\int \frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}{x^2}\,dx + \left(a^2c\right)\int \left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)\,dx$$

$$= -\frac{1}{12}ac\left(c+a^2cx^2\right)^{3/2} + \frac{1}{4}a^2cx\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax) + c^2\int \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{x^2}\,d$$

$$= -\frac{7}{8}ac^2\sqrt{c+a^2cx^2} - \frac{1}{12}ac\left(c+a^2cx^2\right)^{3/2} + \frac{7}{8}a^2c^2x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax) + \frac{1}{4}a^2cx\left(c+a\right.$$

$$= -\frac{7}{8}ac^2\sqrt{c+a^2cx^2} - \frac{1}{12}ac\left(c+a^2cx^2\right)^{3/2} - \frac{c^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{x} + \frac{7}{8}a^2c^2x\sqrt{c+a^2cx}$$

$$= -\frac{7}{8}ac^2\sqrt{c+a^2cx^2} - \frac{1}{12}ac\left(c+a^2cx^2\right)^{3/2} - \frac{c^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{x} + \frac{7}{8}a^2c^2x\sqrt{c+a^2cx}$$

$$= -\frac{7}{8}ac^2\sqrt{c+a^2cx^2} - \frac{1}{12}ac\left(c+a^2cx^2\right)^{3/2} - \frac{c^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{x} + \frac{7}{8}a^2c^2x\sqrt{c+a^2cx}$$

$$= -\frac{7}{8}ac^2\sqrt{c+a^2cx^2} - \frac{1}{12}ac\left(c+a^2cx^2\right)^{3/2} - \frac{c^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{x} + \frac{7}{8}a^2c^2x\sqrt{c+a^2cx}$$

Mathematica [A] time = 3.94454, size = 491, normalized size = 1.38

$$\frac{ac^2\sqrt{a^2cx^2+c}\left(-48\left(-i\text{PolyLog}\left(2,-ie^{i\tan^{-1}(ax)}\right)+i\text{PolyLog}\left(2,ie^{i\tan^{-1}(ax)}\right)+\frac{\sqrt{a^2x^2+1}\tan^{-1}(ax)}{ax}+\tan^{-1}(ax)\left(-\log\left(1-\right.\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x])/x^2,x]
```

```
[Out] (a*c^2*Sqrt[c + a^2*c*x^2]*((1 + a^2*x^2)^(3/2)/2 + 48*Sqrt[1 + a^2*x^2]*(-
1 + a*x*ArcTan[a*x]) + (3*(1 + a^2*x^2)^2*Cos[3*ArcTan[a*x]])/2 + 48*ArcTan
[a*x]*(Log[1 - I*E^(I*ArcTan[a*x])] - Log[1 + I*E^(I*ArcTan[a*x])]) + (42*I
)*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - 48*((Sqrt[1 + a^2*x^2]*ArcTan[a*x])/
(a*x) - ArcTan[a*x]*Log[1 - I*E^(I*ArcTan[a*x])] + ArcTan[a*x]*Log[1 + I*E^
(I*ArcTan[a*x])] + Log[Cos[ArcTan[a*x]/2]] - Log[Sin[ArcTan[a*x]/2]] - I*Po
lyLog[2, (-I)*E^(I*ArcTan[a*x])] + I*PolyLog[2, I*E^(I*ArcTan[a*x])]) - (42
*I)*PolyLog[2, I*E^(I*ArcTan[a*x])] - (3*(1 + a^2*x^2)^2*ArcTan[a*x]*((-14*
a*x)/Sqrt[1 + a^2*x^2] + 3*Log[1 - I*E^(I*ArcTan[a*x])] + 4*Cos[2*ArcTan[a*
x]]*(Log[1 - I*E^(I*ArcTan[a*x])] - Log[1 + I*E^(I*ArcTan[a*x])]) + Cos[4*A
rcTan[a*x]]*(Log[1 - I*E^(I*ArcTan[a*x])] - Log[1 + I*E^(I*ArcTan[a*x])]) -
 3*Log[1 + I*E^(I*ArcTan[a*x])] + 2*Sin[3*ArcTan[a*x]]))/4))/(48*Sqrt[1 + a
^2*x^2])
```

Maple [A] time = 0.353, size = 265, normalized size = 0.8

$$\frac{c^2\left(6\,\arctan(ax)\,x^4a^4-2\,a^3x^3+27\,\arctan(ax)\,a^2x^2-23\,ax-24\,\arctan(ax)\right)}{24\,x}\sqrt{c\,(ax-i)\,(ax+i)}-\frac{ac^2}{8}\sqrt{c\,(ax-i)\,(ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(5/2)*arctan(a*x)/x^2,x)
```

```
[Out] 1/24*c^2*(c*(a*x-I)*(a*x+I))^(1/2)*(6*arctan(a*x)*x^4*a^4-2*a^3*x^3+27*arct
an(a*x)*a^2*x^2-23*a*x-24*arctan(a*x))/x-1/8*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2
*x^2+1)^(1/2)*(15*arctan(a*x)*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-15*arctan
(a*x)*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-8*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)-
1)+8*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))+15*I*dilog(1-I*(1+I*a*x)/(a^2*x^2+1)
^(1/2))-15*I*dilog(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))*a*c^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)/x^2,x, algorithm="maxima")
```

[Out] Exception raised: ValueError

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^{4}c^{2}x^{4}+2\,a^{2}c^{2}x^{2}+c^{2}\right)\sqrt{a^{2}cx^{2}+c}\arctan\left(ax\right)}{x^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)/x^2,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*arctan(a*x
)/x^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(5/2)*atan(a*x)/x**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^{2}cx^{2}+c\right)^{\frac{5}{2}}\arctan\left(ax\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)/x^2,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(5/2)*arctan(a*x)/x^2, x)
```

3.222 $\int \frac{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)}{x^3}\,dx$

Optimal. Leaf size=364

$$\frac{5ia^2c^3\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{2\sqrt{a^2cx^2+c}}-\frac{5ia^2c^3\sqrt{a^2x^2+1}\text{PolyLog}\left(2,\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{2\sqrt{a^2cx^2+c}}-\frac{1}{6}a^3c^2x\sqrt{a^2cx^2+c}-\frac{ac^2\sqrt{a^2cx^2+c}}{2x}+2$$

```
[Out] -(a*c^2*Sqrt[c + a^2*c*x^2])/(2*x) - (a^3*c^2*x*Sqrt[c + a^2*c*x^2])/6 + 2*
a^2*c^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x] - (c^2*Sqrt[c + a^2*c*x^2]*ArcTan[a
*x])/(2*x^2) + (a^2*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x])/3 - (5*a^2*c^3*Sqr
t[1 + a^2*x^2]*ArcTan[a*x]*ArcTanh[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/Sqrt[c
 + a^2*c*x^2] - (13*a^2*c^(5/2)*ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]])
/6 + (((5*I)/2)*a^2*c^3*Sqrt[1 + a^2*x^2]*PolyLog[2, -(Sqrt[1 + I*a*x]/Sqrt
[1 - I*a*x])])/Sqrt[c + a^2*c*x^2] - (((5*I)/2)*a^2*c^3*Sqrt[1 + a^2*x^2]*P
olyLog[2, Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2]
```

Rubi [A] time = 1.14402, antiderivative size = 364, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {4950, 4946, 4962, 264, 4958, 4954, 217, 206, 4930, 195}

$$\frac{5ia^2c^3\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{2\sqrt{a^2cx^2+c}}-\frac{5ia^2c^3\sqrt{a^2x^2+1}\text{PolyLog}\left(2,\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{2\sqrt{a^2cx^2+c}}-\frac{1}{6}a^3c^2x\sqrt{a^2cx^2+c}-\frac{ac^2\sqrt{a^2cx^2+c}}{2x}+2$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x])/x^3,x]
```


```
[Out] -(a*c^2*Sqrt[c + a^2*c*x^2])/(2*x) - (a^3*c^2*x*Sqrt[c + a^2*c*x^2])/6 + 2*
a^2*c^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x] - (c^2*Sqrt[c + a^2*c*x^2]*ArcTan[a
*x])/(2*x^2) + (a^2*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x])/3 - (5*a^2*c^3*Sqr
t[1 + a^2*x^2]*ArcTan[a*x]*ArcTanh[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/Sqrt[c
 + a^2*c*x^2] - (13*a^2*c^(5/2)*ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]])
/6 + (((5*I)/2)*a^2*c^3*Sqrt[1 + a^2*x^2]*PolyLog[2, -(Sqrt[1 + I*a*x]/Sqrt
[1 - I*a*x])])/Sqrt[c + a^2*c*x^2] - (((5*I)/2)*a^2*c^3*Sqrt[1 + a^2*x^2]*P
olyLog[2, Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2]
```


Rule 4950

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
```

```
b*ArcTan[c*x])^p, x], x] + Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*ArcTan[c*x
]))/(f*(m + 2)), x] + (Dist[d/(m + 2), Int[((f*x)^m*(a + b*ArcTan[c*x]))/Sq
rt[d + e*x^2], x], x] - Dist[(b*c*d)/(f*(m + 2)), Int[(f*x)^(m + 1)/Sqrt[d
+ e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && Ne
Q[m, -2]
```

Rule 4962

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Ar
cTan[c*x])^p)/(d*f*(m + 1)), x] + (-Dist[(b*c*p)/(f*(m + 1)), Int[((f*x)^(m
 + 1)*(a + b*ArcTan[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] - Dist[(c^2*(m +
 2))/(f^2*(m + 1)), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/Sqrt[d + e*x^
2], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] &
& LtQ[m, -1] && NeQ[m, -2]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 4958

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4954

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x_
Symbol] :> Simp[(-2*(a + b*ArcTan[c*x])*ArcTanh[Sqrt[1 + I*c*x]/Sqrt[1 - I*
c*x]])/Sqrt[d], x] + (Simp[(I*b*PolyLog[2, -(Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x
])])/Sqrt[d], x] - Simp[(I*b*PolyLog[2, Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]])/S
qrt[d], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rubi steps

$$\int \frac{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)}{x^3}\,dx = c\int \frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}{x^3}\,dx + \left(a^2c\right)\int \frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}{x}\,dx$$

$$= c^2\int \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{x^3}\,dx + 2\left(\left(a^2c^2\right)\int \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{x}\,dx\right) + \left(a^4c^2\right)\int x$$

$$= -\frac{c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{x^2} + \frac{1}{3}a^2c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax) - \frac{1}{3}\left(a^3c^2\right)\int \sqrt{c+a^2cx^2}\,dx$$

$$= -\frac{ac^2\sqrt{c+a^2cx^2}}{x} - \frac{1}{6}a^3c^2x\sqrt{c+a^2cx^2} - \frac{c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{2x^2} + \frac{1}{3}a^2c\left(c+a^2cx^2\right)^{3/2}$$

$$= -\frac{ac^2\sqrt{c+a^2cx^2}}{2x} - \frac{1}{6}a^3c^2x\sqrt{c+a^2cx^2} - \frac{c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{2x^2} + \frac{1}{3}a^2c\left(c+a^2cx^2\right)^{3/2}$$

$$= -\frac{ac^2\sqrt{c+a^2cx^2}}{2x} - \frac{1}{6}a^3c^2x\sqrt{c+a^2cx^2} - \frac{c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{2x^2} + \frac{1}{3}a^2c\left(c+a^2cx^2\right)^{3/2}$$

Mathematica [A] time = 2.02946, size = 361, normalized size = 0.99

$$\frac{a^2c^2\sqrt{a^2cx^2+c}\tan\left(\frac{1}{2}\tan^{-1}(ax)\right)\left(60i\cot\left(\frac{1}{2}\tan^{-1}(ax)\right)\text{PolyLog}\left(2,-e^{i\tan^{-1}(ax)}\right)-60i\cot\left(\frac{1}{2}\tan^{-1}(ax)\right)\text{PolyLog}\left(2\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x])/x^3,x]
```

```
[Out] (a^2*c^2*Sqrt[c + a^2*c*x^2]*(-6 - 4*ArcSinh[a*x]*Cot[ArcTan[a*x]/2] - 6*Co
t[ArcTan[a*x]/2]^2 - 2*a^2*x^2*Csc[ArcTan[a*x]/2]^2 + 28*a*x*ArcTan[a*x]*Cs
c[ArcTan[a*x]/2]^2 + 4*a^3*x^3*ArcTan[a*x]*Csc[ArcTan[a*x]/2]^2 - 3*ArcTan[
a*x]*Cot[ArcTan[a*x]/2]*Csc[ArcTan[a*x]/2]^2 + 60*ArcTan[a*x]*Cot[ArcTan[a*
x]/2]*Log[1 - E^(I*ArcTan[a*x])] - 60*ArcTan[a*x]*Cot[ArcTan[a*x]/2]*Log[1
+ E^(I*ArcTan[a*x])] + 48*Cot[ArcTan[a*x]/2]*Log[Cos[ArcTan[a*x]/2] - Sin[A
rcTan[a*x]/2]] - 48*Cot[ArcTan[a*x]/2]*Log[Cos[ArcTan[a*x]/2] + Sin[ArcTan[
a*x]/2]] + (60*I)*Cot[ArcTan[a*x]/2]*PolyLog[2, -E^(I*ArcTan[a*x])] - (60*I
)*Cot[ArcTan[a*x]/2]*PolyLog[2, E^(I*ArcTan[a*x])] + 3*ArcTan[a*x]*Csc[ArcT
an[a*x]/2]*Sec[ArcTan[a*x]/2])*Tan[ArcTan[a*x]/2])/(24*Sqrt[1 + a^2*x^2])
```

Maple [A] time = 0.356, size = 204, normalized size = 0.6

$$\frac{c^2 \left(2 \arctan \left(a x\right) x^4 a^4-a^3 x^3+14 \arctan \left(a x\right) a^2 x^2-3 a x-3 \arctan \left(a x\right)\right)}{6 x^2} \sqrt{c \left(a x-i\right) \left(a x+i\right)}-\frac{a^2 c^2}{6} \sqrt{c \left(a x-i\right) \left(a x+i\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(5/2)*arctan(a*x)/x^3,x)
```

```
[Out] 1/6*c^2*(c*(a*x-I)*(a*x+I))^(1/2)*(2*arctan(a*x)*x^4*a^4-a^3*x^3+14*arctan(
a*x)*a^2*x^2-3*a*x-3*arctan(a*x))/x^2-1/6*a^2*c^2*(c*(a*x-I)*(a*x+I))^(1/2)
*(15*arctan(a*x)*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-26*I*arctan((1+I*a*x)/(a
^2*x^2+1)^(1/2))-15*I*dilog((1+I*a*x)/(a^2*x^2+1)^(1/2))-15*I*dilog(1+(1+I*
a*x)/(a^2*x^2+1)^(1/2)))/(a^2*x^2+1)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^4 c^2 x^4+2 a^2 c^2 x^2+c^2\right) \sqrt{a^2 c x^2+c} \arctan \left(a x\right)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)/x^3,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*arctan(a*x
)/x^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(5/2)*atan(a*x)/x**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^{\frac{5}{2}}\arctan\left(ax\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)/x^3,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(5/2)*arctan(a*x)/x^3, x)
```

3.223 $\int \frac{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)}{x^4}\,dx$

Optimal. Leaf size=372

$$\frac{5ia^3c^3\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{2\sqrt{a^2cx^2+c}}-\frac{5ia^3c^3\sqrt{a^2x^2+1}\text{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{2\sqrt{a^2cx^2+c}}-\frac{1}{2}a^3c^2\sqrt{a^2cx^2+c}-\frac{ac^2\sqrt{a^2cx^2+c}}{6x^2}+$$

```
[Out] -(a^3*c^2*Sqrt[c + a^2*c*x^2])/2 - (a*c^2*Sqrt[c + a^2*c*x^2])/(6*x^2) - (2
*a^2*c^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/x + (a^4*c^2*x*Sqrt[c + a^2*c*x^2
]*ArcTan[a*x])/2 - (c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x])/(3*x^3) - ((5*I)*a
^3*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]
])/Sqrt[c + a^2*c*x^2] - (13*a^3*c^(5/2)*ArcTanh[Sqrt[c + a^2*c*x^2]/Sqrt[c
]])/6 + (((5*I)/2)*a^3*c^3*Sqrt[1 + a^2*x^2]*PolyLog[2, ((-I)*Sqrt[1 + I*a*
x])/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2] - (((5*I)/2)*a^3*c^3*Sqrt[1 + a^2
*x^2]*PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2]
```

Rubi [A] time = 0.975109, antiderivative size = 372, normalized size of antiderivative = 1., number of steps used = 25, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {4950, 4944, 266, 47, 63, 208, 4890, 4886, 4878}

$$\frac{5ia^3c^3\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{2\sqrt{a^2cx^2+c}}-\frac{5ia^3c^3\sqrt{a^2x^2+1}\text{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{2\sqrt{a^2cx^2+c}}-\frac{1}{2}a^3c^2\sqrt{a^2cx^2+c}-\frac{ac^2\sqrt{a^2cx^2+c}}{6x^2}+$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x])/x^4,x]
```

```
[Out] -(a^3*c^2*Sqrt[c + a^2*c*x^2])/2 - (a*c^2*Sqrt[c + a^2*c*x^2])/(6*x^2) - (2
*a^2*c^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/x + (a^4*c^2*x*Sqrt[c + a^2*c*x^2
]*ArcTan[a*x])/2 - (c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x])/(3*x^3) - ((5*I)*a
^3*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]
])/Sqrt[c + a^2*c*x^2] - (13*a^3*c^(5/2)*ArcTanh[Sqrt[c + a^2*c*x^2]/Sqrt[c
]])/6 + (((5*I)/2)*a^3*c^3*Sqrt[1 + a^2*x^2]*PolyLog[2, ((-I)*Sqrt[1 + I*a*
x])/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2] - (((5*I)/2)*a^3*c^3*Sqrt[1 + a^2
*x^2]*PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2]
```

Rule 4950

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
```

```
b*ArcTan[c*x])^p, x], x] + Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 4944

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a +
 b*ArcTan[c*x])^p)/(d*f*(m + 1)), x] - Dist[(b*c*p)/(f*(m + 1)), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 4890

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4886

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[(-2*I*(a + b*ArcTan[c*x])*ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]])
/(c*Sqrt[d]), x] + (Simp[(I*b*PolyLog[2, -((I*Sqrt[1 + I*c*x])/Sqrt[1 - I*c
*x])])/(c*Sqrt[d]), x] - Simp[(I*b*PolyLog[2, (I*Sqrt[1 + I*c*x])/Sqrt[1 -
I*c*x]])/(c*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

Rule 4878

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symbo
l] :> -Simp[(b*(d + e*x^2)^q)/(2*c*q*(2*q + 1)), x] + (Dist[(2*d*q)/(2*q +
1), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x]), x], x] + Simp[(x*(d + e*x^
2)^q*(a + b*ArcTan[c*x]))/(2*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] && Eq
Q[e, c^2*d] && GtQ[q, 0]
```

Rubi steps

$$\int \frac{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)}{x^4}\,dx = c\int \frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}{x^4}\,dx + \left(a^2c\right)\int \frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}{x^2}\,dx$$

$$= c^2\int \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{x^4}\,dx + 2\left(\left(a^2c^2\right)\int \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{x^2}\,dx\right) + \left(a^4c^2\right)\int \sqrt{}$$

$$= -\frac{1}{2}a^3c^2\sqrt{c+a^2cx^2} + \frac{1}{2}a^4c^2x\sqrt{c+a^2cx^2}\tan^{-1}(ax) - \frac{c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}{3x^3} + \frac{1}{3}\left(a\right.$$

$$= -\frac{1}{2}a^3c^2\sqrt{c+a^2cx^2} + \frac{1}{2}a^4c^2x\sqrt{c+a^2cx^2}\tan^{-1}(ax) - \frac{c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}{3x^3} + \frac{1}{6}\left(a\right.$$

$$= -\frac{1}{2}a^3c^2\sqrt{c+a^2cx^2} - \frac{ac^2\sqrt{c+a^2cx^2}}{6x^2} + \frac{1}{2}a^4c^2x\sqrt{c+a^2cx^2}\tan^{-1}(ax) - \frac{c\left(c+a^2cx^2\right)^{3/2}}{3x^3}$$

$$= -\frac{1}{2}a^3c^2\sqrt{c+a^2cx^2} - \frac{ac^2\sqrt{c+a^2cx^2}}{6x^2} + \frac{1}{2}a^4c^2x\sqrt{c+a^2cx^2}\tan^{-1}(ax) - \frac{c\left(c+a^2cx^2\right)^{3/2}}{3x^3}$$

$$= -\frac{1}{2}a^3c^2\sqrt{c+a^2cx^2} - \frac{ac^2\sqrt{c+a^2cx^2}}{6x^2} + \frac{1}{2}a^4c^2x\sqrt{c+a^2cx^2}\tan^{-1}(ax) - \frac{c\left(c+a^2cx^2\right)^{3/2}}{3x^3}$$

Mathematica [A] time = 0.980372, size = 313, normalized size = 0.84

$$\frac{c^2\sqrt{a^2cx^2+c}\left(15ia^3x^3\text{PolyLog}\left(2,-ie^{i\tan^{-1}(ax)}\right) - 15ia^3x^3\text{PolyLog}\left(2,ie^{i\tan^{-1}(ax)}\right) - 3a^3x^3\sqrt{a^2x^2+1} - ax\sqrt{a^2x^2+1} + \right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x])/x^4,x]
```

```
[Out] (c^2*Sqrt[c + a^2*c*x^2]*(-(a*x*Sqrt[1 + a^2*x^2]) - 3*a^3*x^3*Sqrt[1 + a^2
*x^2] - 2*Sqrt[1 + a^2*x^2]*ArcTan[a*x] - 14*a^2*x^2*Sqrt[1 + a^2*x^2]*ArcT
an[a*x] + 3*a^4*x^4*Sqrt[1 + a^2*x^2]*ArcTan[a*x] - a^3*x^3*ArcTanh[Sqrt[1
+ a^2*x^2]] + 15*a^3*x^3*ArcTan[a*x]*Log[1 - I*E^(I*ArcTan[a*x])] - 15*a^3*
x^3*ArcTan[a*x]*Log[1 + I*E^(I*ArcTan[a*x])] - 12*a^3*x^3*Log[Cos[ArcTan[a*
x]/2]] + 12*a^3*x^3*Log[Sin[ArcTan[a*x]/2]] + (15*I)*a^3*x^3*PolyLog[2, (-I
)*E^(I*ArcTan[a*x])] - (15*I)*a^3*x^3*PolyLog[2, I*E^(I*ArcTan[a*x])]))/(6*
x^3*Sqrt[1 + a^2*x^2])
```

Maple [A] time = 0.487, size = 270, normalized size = 0.7

$$\frac{c^2 \left(3 \arctan (ax) x^4 a^4 - 3 a^3 x^3 - 14 \arctan (ax) a^2 x^2 - ax - 2 \arctan (ax)\right)}{6 x^3} \sqrt{c (ax - i) (ax + i)} + \frac{i}{6} a^3 c^2 \sqrt{c (ax - i) (ax +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(5/2)*arctan(a*x)/x^4,x)
```

```
[Out] 1/6*c^2*(c*(a*x-I)*(a*x+I))^(1/2)*(3*arctan(a*x)*x^4*a^4-3*a^3*x^3-14*arcta
n(a*x)*a^2*x^2-a*x-2*arctan(a*x))/x^3+1/6*I*a^3*c^2*(c*(a*x-I)*(a*x+I))^(1/
2)*(15*I*arctan(a*x)*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-15*I*arctan(a*x)*l
n(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-13*I*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)-1)+1
3*I*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-15*dilog(1-I*(1+I*a*x)/(a^2*x^2+1)^(1
/2))+15*dilog(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))/(a^2*x^2+1)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^4 c^2 x^4 + 2 a^2 c^2 x^2 + c^2\right) \sqrt{a^2 c x^2 + c} \arctan (ax)}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)/x^4,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*arctan(a*x
)/x^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(5/2)*atan(a*x)/x**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a^2cx^2 + c)^{\frac{5}{2}} \arctan(ax)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)/x^4,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(5/2)*arctan(a*x)/x^4, x)
```

3.224 $\int \frac{x^3 \tan^{-1}(ax)}{\sqrt{c+a^2cx^2}} dx$

Optimal. Leaf size=120

$$-\frac{x\sqrt{a^2cx^2+c}}{6a^3c}+\frac{x^2\sqrt{a^2cx^2+c}\tan^{-1}(ax)}{3a^2c}-\frac{2\sqrt{a^2cx^2+c}\tan^{-1}(ax)}{3a^4c}+\frac{5\tanh^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{6a^4\sqrt{c}}$$

```
[Out] -(x*Sqrt[c + a^2*c*x^2])/(6*a^3*c) - (2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(3
*a^4*c) + (x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(3*a^2*c) + (5*ArcTanh[(a*S
qrt[c]*x)/Sqrt[c + a^2*c*x^2]])/(6*a^4*Sqrt[c])
```

Rubi [A] time = 0.152662, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {4952, 321, 217, 206, 4930}

$$-\frac{x\sqrt{a^2cx^2+c}}{6a^3c}+\frac{x^2\sqrt{a^2cx^2+c}\tan^{-1}(ax)}{3a^2c}-\frac{2\sqrt{a^2cx^2+c}\tan^{-1}(ax)}{3a^4c}+\frac{5\tanh^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{6a^4\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcTan[a*x])/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] -(x*Sqrt[c + a^2*c*x^2])/(6*a^3*c) - (2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(3
*a^4*c) + (x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(3*a^2*c) + (5*ArcTanh[(a*S
qrt[c]*x)/Sqrt[c + a^2*c*x^2]])/(6*a^4*Sqrt[c])
```

Rule 4952

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b*
ArcTan[c*x])^p)/(c^2*d*m), x] + (-Dist[(b*f*p)/(c*m), Int[((f*x)^(m - 1)*(a
 + b*ArcTan[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] - Dist[(f^2*(m - 1))/(c^
2*m), Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/Sqrt[d + e*x^2], x], x]) /;
 FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
```

```
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rubi steps

$$\int \frac{x^3\tan^{-1}(ax)}{\sqrt{c+a^2cx^2}}\,dx = \frac{x^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{3a^2c} - \frac{2\int \frac{x\tan^{-1}(ax)}{\sqrt{c+a^2cx^2}}\,dx}{3a^2} - \frac{\int \frac{x^2}{\sqrt{c+a^2cx^2}}\,dx}{3a}$$

$$= -\frac{x\sqrt{c+a^2cx^2}}{6a^3c} - \frac{2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{3a^4c} + \frac{x^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{3a^2c} + \frac{\int \frac{1}{\sqrt{c+a^2cx^2}}\,dx}{6a^3} + \frac{2\int \frac{1}{\sqrt{c+a^2c}}}{3a^3}$$

$$= -\frac{x\sqrt{c+a^2cx^2}}{6a^3c} - \frac{2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{3a^4c} + \frac{x^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{3a^2c} + \frac{\text{Subst}\left(\int \frac{1}{1-a^2cx^2}\,dx, x, \frac{}{\sqrt{c+}}\right.}{6a^3}$$

$$= -\frac{x\sqrt{c+a^2cx^2}}{6a^3c} - \frac{2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{3a^4c} + \frac{x^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{3a^2c} + \frac{5\tanh^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c+a^2cx^2}}\right)}{6a^4\sqrt{c}}$$

Mathematica [A] time = 0.120434, size = 91, normalized size = 0.76

$$\frac{-ax\sqrt{a^2cx^2+c}+5\sqrt{c}\log\left(\sqrt{c}\sqrt{a^2cx^2+c}+acx\right)+2\left(a^2x^2-2\right)\sqrt{a^2cx^2+c}\tan^{-1}(ax)}{6a^4c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*ArcTan[a*x])/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] (-(a*x*Sqrt[c + a^2*c*x^2]) + 2*(-2 + a^2*x^2)*Sqrt[c + a^2*c*x^2]*ArcTan[a
*x] + 5*Sqrt[c]*Log[a*c*x + Sqrt[c]*Sqrt[c + a^2*c*x^2]])/(6*a^4*c)
```

Maple [C] time = 1.153, size = 165, normalized size = 1.4

$$\frac{2\arctan(ax)a^2x^2 - ax - 4\arctan(ax)}{6ca^4}\sqrt{c(ax-i)(ax+i)} + \frac{5}{6ca^4}\ln\left((1+iax)\frac{1}{\sqrt{a^2x^2+1}} + i\right)\sqrt{c(ax-i)(ax+i)}\frac{1}{\sqrt{a^2x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctan(a*x)/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] 1/6*(2*arctan(a*x)*a^2*x^2-a*x-4*arctan(a*x))*(c*(a*x-I)*(a*x+I))^(1/2)/c/a
^4+5/6*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)+I)*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2
+1)^(1/2)/a^4/c-5/6*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)-I)*(c*(a*x-I)*(a*x+I))^(
1/2)/(a^2*x^2+1)^(1/2)/a^4/c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.47118, size = 196, normalized size = 1.63

$$-\frac{2\sqrt{a^2cx^2+c}\left(ax-2\left(a^2x^2-2\right)\arctan(ax)\right) - 5\sqrt{c}\log\left(-2a^2cx^2 - 2\sqrt{a^2cx^2+c}a\sqrt{c}x - c\right)}{12a^4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/12*(2*sqrt(a^2*c*x^2 + c)*(a*x - 2*(a^2*x^2 - 2)*arctan(a*x)) - 5*sqrt(c
)*log(-2*a^2*c*x^2 - 2*sqrt(a^2*c*x^2 + c)*a*sqrt(c)*x - c))/(a^4*c)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \operatorname{atan}(ax)}{\sqrt{c(a^2x^2+1)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atan(a*x)/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**3*atan(a*x)/sqrt(c*(a**2*x**2 + 1)), x)
```

Giac [A] time = 1.22996, size = 134, normalized size = 1.12

$$-\frac{\sqrt{a^2cx^2+c}x}{6a^3c} - \frac{5\log\left(\left|-\sqrt{a^2c}x+\sqrt{a^2cx^2+c}\right|\right)}{6a^3\sqrt{c}|a|} + \frac{\left(\left(a^2cx^2+c\right)^{\frac{3}{2}} - 3\sqrt{a^2cx^2+c}c\right)\arctan(ax)}{3a^4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(a^2*c*x^2 + c)*x/(a^3*c) - 5/6*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*
c*x^2 + c)))/(a^3*sqrt(c)*abs(a)) + 1/3*((a^2*c*x^2 + c)^(3/2) - 3*sqrt(a^2
*c*x^2 + c)*c)*arctan(a*x)/(a^4*c^2)
```

3.225 $\int \frac{x^2 \tan^{-1}(ax)}{\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=250

$$-\frac{i\sqrt{a^2x^2+1}\,\text{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{2a^3\sqrt{a^2cx^2+c}}+\frac{i\sqrt{a^2x^2+1}\,\text{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{2a^3\sqrt{a^2cx^2+c}}-\frac{\sqrt{a^2cx^2+c}}{2a^3c}+\frac{i\sqrt{a^2x^2+1}\tan^{-1}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)\tan^{-1}(}{a^3\sqrt{a^2cx^2+c}}$$

```
[Out] -Sqrt[c + a^2*c*x^2]/(2*a^3*c) + (x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(2*a^2
*c) + (I*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*
x]])/(a^3*Sqrt[c + a^2*c*x^2]) - ((I/2)*Sqrt[1 + a^2*x^2]*PolyLog[2, ((-I)*
Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/(a^3*Sqrt[c + a^2*c*x^2]) + ((I/2)*Sqrt[
1 + a^2*x^2]*PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/(a^3*Sqrt[c +
 a^2*c*x^2])
```

Rubi [A] time = 0.146525, antiderivative size = 250, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4952, 261, 4890, 4886}

$$-\frac{i\sqrt{a^2x^2+1}\,\text{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{2a^3\sqrt{a^2cx^2+c}}+\frac{i\sqrt{a^2x^2+1}\,\text{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{2a^3\sqrt{a^2cx^2+c}}-\frac{\sqrt{a^2cx^2+c}}{2a^3c}+\frac{i\sqrt{a^2x^2+1}\tan^{-1}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)\tan^{-1}(}{a^3\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcTan[a*x])/Sqrt[c + a^2*c*x^2],x]
```


```
[Out] -Sqrt[c + a^2*c*x^2]/(2*a^3*c) + (x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(2*a^2
*c) + (I*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*
x]])/(a^3*Sqrt[c + a^2*c*x^2]) - ((I/2)*Sqrt[1 + a^2*x^2]*PolyLog[2, ((-I)*
Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/(a^3*Sqrt[c + a^2*c*x^2]) + ((I/2)*Sqrt[
1 + a^2*x^2]*PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/(a^3*Sqrt[c +
 a^2*c*x^2])
```


Rule 4952

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b*
ArcTan[c*x])^p)/(c^2*d*m), x] + (-Dist[(b*f*p)/(c*m), Int[((f*x)^(m - 1)*(a
 + b*ArcTan[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] - Dist[(f^2*(m - 1))/(c^
2*m), Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/Sqrt[d + e*x^2], x], x]) /;
```

```
FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 4890

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4886

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[(-2*I*(a + b*ArcTan[c*x])*ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]])
/(c*Sqrt[d]), x] + (Simp[(I*b*PolyLog[2, -((I*Sqrt[1 + I*c*x])/Sqrt[1 - I*c
*x])])/(c*Sqrt[d]), x] - Simp[(I*b*PolyLog[2, (I*Sqrt[1 + I*c*x])/Sqrt[1 -
I*c*x]])/(c*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

Rubi steps

$$\int \frac{x^2\tan^{-1}(ax)}{\sqrt{c+a^2cx^2}}\,dx = \frac{x\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{2a^2c} - \frac{\int \frac{\tan^{-1}(ax)}{\sqrt{c+a^2cx^2}}\,dx}{2a^2} - \frac{\int \frac{x}{\sqrt{c+a^2cx^2}}\,dx}{2a}$$

$$= -\frac{\sqrt{c+a^2cx^2}}{2a^3c} + \frac{x\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{2a^2c} - \frac{\sqrt{1+a^2x^2}\int \frac{\tan^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx}{2a^2\sqrt{c+a^2cx^2}}$$

$$= -\frac{\sqrt{c+a^2cx^2}}{2a^3c} + \frac{x\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{2a^2c} + \frac{i\sqrt{1+a^2x^2}\tan^{-1}(ax)\tan^{-1}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a^3\sqrt{c+a^2cx^2}} - \frac{i\sqrt{1+a^2x^2}\text{Li}_2\left(\right.}{2a^3\sqrt{c+a}}$$

Mathematica [A] time = 0.5769, size = 158, normalized size = 0.63

$$-\frac{\sqrt{c\left(a^2x^2+1\right)}\left(i\text{PolyLog}\left(2,-ie^{i\tan^{-1}(ax)}\right)-i\text{PolyLog}\left(2,ie^{i\tan^{-1}(ax)}\right)+\sqrt{a^2x^2+1}-ax\sqrt{a^2x^2+1}\tan^{-1}(ax)+\tan^{-1}(\right.}{2a^3c\sqrt{a^2x^2+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*ArcTan[a*x])/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] -(Sqrt[c*(1 + a^2*x^2)]*(Sqrt[1 + a^2*x^2] - a*x*Sqrt[1 + a^2*x^2]*ArcTan[a
*x] + ArcTan[a*x]*Log[1 - I*E^(I*ArcTan[a*x])] - ArcTan[a*x]*Log[1 + I*E^(I
*ArcTan[a*x])] + I*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - I*PolyLog[2, I*E^(I
*ArcTan[a*x])]))/(2*a^3*c*Sqrt[1 + a^2*x^2])
```

Maple [A] time = 0.895, size = 184, normalized size = 0.7

$$\frac{\arctan(ax)\,xa-1}{2\,ca^3}\sqrt{c\,(ax-i)\,(ax+i)}-\frac{\frac{i}{2}}{ca^3}\left(i\arctan(ax)\ln\left(1+i\,(1+iax)\,\frac{1}{\sqrt{a^2x^2+1}}\right)-i\arctan(ax)\ln\left(1-i\,(1+iax\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(a*x)/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] 1/2*(arctan(a*x)*x*a-1)*(c*(a*x-I)*(a*x+I))^(1/2)/c/a^3-1/2*I*(I*arctan(a*x
)*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*arctan(a*x)*ln(1-I*(1+I*a*x)/(a^2*x
^2+1)^(1/2))+dilog(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-dilog(1-I*(1+I*a*x)/(a^
2*x^2+1)^(1/2)))*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)^(1/2)/a^3/c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^2*arctan(a*x)/sqrt(a^2*c*x^2 + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \operatorname{atan}(ax)}{\sqrt{c(a^2x^2+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(a*x)/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**2*atan(a*x)/sqrt(c*(a**2*x**2 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \arctan(ax)}{\sqrt{a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctan(a*x)/sqrt(a^2*c*x^2 + c), x)
```

3.226 $\int \frac{x\tan^{-1}(ax)}{\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=59

$$\frac{\sqrt{a^2cx^2+c}\tan^{-1}(ax)}{a^2c}-\frac{\tanh^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a^2\sqrt{c}}$$

```
[Out] (Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(a^2*c) - ArcTanh[(a*Sqrt[c]*x)/Sqrt[c +
a^2*c*x^2]]/(a^2*Sqrt[c])
```

Rubi [A] time = 0.0576372, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {4930, 217, 206}

$$\frac{\sqrt{a^2cx^2+c}\tan^{-1}(ax)}{a^2c}-\frac{\tanh^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a^2\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTan[a*x])/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] (Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(a^2*c) - ArcTanh[(a*Sqrt[c]*x)/Sqrt[c +
a^2*c*x^2]]/(a^2*Sqrt[c])
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x\tan^{-1}(ax)}{\sqrt{c+a^2cx^2}}\,dx &= \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{a^2c}-\frac{\int \frac{1}{\sqrt{c+a^2cx^2}}\,dx}{a}\\ &= \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{a^2c}-\frac{\text{Subst}\left(\int \frac{1}{1-a^2cx^2}\,dx,x,\frac{x}{\sqrt{c+a^2cx^2}}\right)}{a}\\ &= \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{a^2c}-\frac{\tanh^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c+a^2cx^2}}\right)}{a^2\sqrt{c}}\end{aligned}$$

Mathematica [A] time = 0.0679329, size = 60, normalized size = 1.02

$$\frac{\sqrt{a^2cx^2+c}\tan^{-1}(ax)-\sqrt{c}\log\left(\sqrt{c}\sqrt{a^2cx^2+c}+acx\right)}{a^2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcTan[a*x])/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] (Sqrt[c + a^2*c*x^2]*ArcTan[a*x] - Sqrt[c]*Log[a*c*x + Sqrt[c]*Sqrt[c + a^2
*c*x^2]])/(a^2*c)
```

Maple [C] time = 0.431, size = 144, normalized size = 2.4

$$\frac{\arctan(ax)}{a^2c}\sqrt{c(ax-i)(ax+i)}-\frac{1}{a^2c}\ln\left((1+iax)\frac{1}{\sqrt{a^2x^2+1}}+i\right)\sqrt{c(ax-i)(ax+i)}\frac{1}{\sqrt{a^2x^2+1}}+\frac{1}{a^2c}\ln\left((1+iax)\frac{1}{\sqrt{a^2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(a*x)/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] arctan(a*x)*(c*(a*x-I)*(a*x+I))^(1/2)/a^2/c-ln((1+I*a*x)/(a^2*x^2+1)^(1/2)+
I)*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)^(1/2)/a^2/c+ln((1+I*a*x)/(a^2*x^2+
```

```
1)^(1/2)-I)*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)^(1/2)/a^2/c
```

Maxima [A] time = 1.77494, size = 82, normalized size = 1.39

$$\frac{2\sqrt{a^2x^2+1}\arctan(ax) - \log\left(ax + \sqrt{a^2x^2+1}\right) + \log\left(-ax + \sqrt{a^2x^2+1}\right)}{2a^2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(2*sqrt(a^2*x^2 + 1)*arctan(a*x) - log(a*x + sqrt(a^2*x^2 + 1)) + log(-
a*x + sqrt(a^2*x^2 + 1)))/(a^2*sqrt(c))
```

Fricas [A] time = 2.42572, size = 158, normalized size = 2.68

$$\frac{2\sqrt{a^2cx^2+c}\arctan(ax) + \sqrt{c}\log\left(-2a^2cx^2 + 2\sqrt{a^2cx^2+c}a\sqrt{c}x - c\right)}{2a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*sqrt(a^2*c*x^2 + c)*arctan(a*x) + sqrt(c)*log(-2*a^2*c*x^2 + 2*sqrt(
a^2*c*x^2 + c)*a*sqrt(c)*x - c))/(a^2*c)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(a*x)/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Exception raised: TypeError
```

Giac [A] time = 1.15659, size = 81, normalized size = 1.37

$$\frac{\sqrt{a^2cx^2+c}\arctan\left(ax\right)}{a^2c}+\frac{\log\left(\left|-\sqrt{a^2c}x+\sqrt{a^2cx^2+c}\right|\right)}{a\sqrt{c}|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(a^2*c*x^2 + c)*arctan(a*x)/(a^2*c) + log(abs(-sqrt(a^2*c)*x + sqrt(a^2
*c*x^2 + c)))/(a*sqrt(c)*abs(a))
```

3.227 $\int \frac{\tan^{-1}(ax)}{\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=193

$$\frac{i\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a\sqrt{a^2cx^2+c}}-\frac{i\sqrt{a^2x^2+1}\text{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a\sqrt{a^2cx^2+c}}-\frac{2i\sqrt{a^2x^2+1}\tan^{-1}(ax)\tan^{-1}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a\sqrt{a^2cx^2+c}}$$

```
[Out] ((-2*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x
]])/(a*Sqrt[c + a^2*c*x^2]) + (I*Sqrt[1 + a^2*x^2]*PolyLog[2, ((-I)*Sqrt[1
+ I*a*x])/Sqrt[1 - I*a*x]])/(a*Sqrt[c + a^2*c*x^2]) - (I*Sqrt[1 + a^2*x^2]*
PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/(a*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.0572581, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {4890, 4886}

$$\frac{i\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a\sqrt{a^2cx^2+c}}-\frac{i\sqrt{a^2x^2+1}\text{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a\sqrt{a^2cx^2+c}}-\frac{2i\sqrt{a^2x^2+1}\tan^{-1}(ax)\tan^{-1}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]/Sqrt[c + a^2*c*x^2],x]
```


```
[Out] ((-2*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x
]])/(a*Sqrt[c + a^2*c*x^2]) + (I*Sqrt[1 + a^2*x^2]*PolyLog[2, ((-I)*Sqrt[1
+ I*a*x])/Sqrt[1 - I*a*x]])/(a*Sqrt[c + a^2*c*x^2]) - (I*Sqrt[1 + a^2*x^2]*
PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/(a*Sqrt[c + a^2*c*x^2])
```


Rule 4890

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4886

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[(-2*I*(a + b*ArcTan[c*x])*ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]])
```

```
/(c*Sqrt[d]), x] + (Simp[(I*b*PolyLog[2, -((I*Sqrt[1 + I*c*x])/Sqrt[1 - I*c
*x])])/(c*Sqrt[d]), x] - Simp[(I*b*PolyLog[2, (I*Sqrt[1 + I*c*x])/Sqrt[1 -
I*c*x]])/(c*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)}{\sqrt{c+a^2cx^2}}\,dx = \frac{\sqrt{1+a^2x^2}\int \frac{\tan^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx}{\sqrt{c+a^2cx^2}}$$
$$= -\frac{2i\sqrt{1+a^2x^2}\tan^{-1}(ax)\tan^{-1}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a\sqrt{c+a^2cx^2}} + \frac{i\sqrt{1+a^2x^2}\mathrm{Li}_2\left(-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a\sqrt{c+a^2cx^2}} - \frac{i\sqrt{1+a^2x^2}\mathrm{Li}_2\left(\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 0.107153, size = 118, normalized size = 0.61

$$\frac{\sqrt{c\left(a^2x^2+1\right)}\left(i\mathrm{PolyLog}\left(2,-ie^{i\tan^{-1}(ax)}\right)-i\mathrm{PolyLog}\left(2,ie^{i\tan^{-1}(ax)}\right)+\tan^{-1}(ax)\left(\log\left(1-ie^{i\tan^{-1}(ax)}\right)-\log\left(1+ie^{i\,\mathrm{ta}}\right.\right.\right.}{ac\sqrt{a^2x^2+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTan[a*x]/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] (Sqrt[c*(1 + a^2*x^2)]*(ArcTan[a*x]*(Log[1 - I*E^(I*ArcTan[a*x])] - Log[1 +
 I*E^(I*ArcTan[a*x])]) + I*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - I*PolyLog[2
, I*E^(I*ArcTan[a*x])]))/(a*c*Sqrt[1 + a^2*x^2])
```

Maple [A] time = 0.352, size = 150, normalized size = 0.8

$$\frac{i}{ca}\left(i\arctan(ax)\ln\left(1+i(1+iax)\frac{1}{\sqrt{a^2x^2+1}}\right)-i\arctan(ax)\ln\left(1-i(1+iax)\frac{1}{\sqrt{a^2x^2+1}}\right)+\mathrm{dilog}\left(1+i(1+iax)\frac{}{\sqrt{a}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] I*(I*arctan(a*x)*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*arctan(a*x)*ln(1-I*(
1+I*a*x)/(a^2*x^2+1)^(1/2))+dilog(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-dilog(1-
```

```
I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)^(1/2)
/c/a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(arctan(a*x)/sqrt(a^2*c*x^2 + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atan}(ax)}{\sqrt{c(a^2x^2+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(atan(a*x)/sqrt(c*(a**2*x**2 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(ax)}{\sqrt{a^2cx^2 + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)/sqrt(a^2*c*x^2 + c), x)
```

3.228 $\int \frac{\tan^{-1}(ax)}{x\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=177

$$\frac{i\sqrt{a^2x^2+1}\,\text{PolyLog}\left(2,-\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{a^2cx^2+c}}-\frac{i\sqrt{a^2x^2+1}\,\text{PolyLog}\left(2,\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{a^2cx^2+c}}-\frac{2\sqrt{a^2x^2+1}\tan^{-1}(ax)\tanh^{-1}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{a^2cx^2+c}}$$

```
[Out] (-2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTanh[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])
/Sqrt[c + a^2*c*x^2] + (I*Sqrt[1 + a^2*x^2]*PolyLog[2, -(Sqrt[1 + I*a*x]/Sq
rt[1 - I*a*x])])/Sqrt[c + a^2*c*x^2] - (I*Sqrt[1 + a^2*x^2]*PolyLog[2, Sqrt
[1 + I*a*x]/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2]
```

Rubi [A] time = 0.134127, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4958, 4954}

$$\frac{i\sqrt{a^2x^2+1}\,\text{PolyLog}\left(2,-\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{a^2cx^2+c}}-\frac{i\sqrt{a^2x^2+1}\,\text{PolyLog}\left(2,\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{a^2cx^2+c}}-\frac{2\sqrt{a^2x^2+1}\tan^{-1}(ax)\tanh^{-1}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]/(x*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] (-2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTanh[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])
/Sqrt[c + a^2*c*x^2] + (I*Sqrt[1 + a^2*x^2]*PolyLog[2, -(Sqrt[1 + I*a*x]/Sq
rt[1 - I*a*x])])/Sqrt[c + a^2*c*x^2] - (I*Sqrt[1 + a^2*x^2]*PolyLog[2, Sqrt
[1 + I*a*x]/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2]
```

Rule 4958

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4954

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x_
Symbol] :> Simp[(-2*(a + b*ArcTan[c*x])*ArcTanh[Sqrt[1 + I*c*x]/Sqrt[1 - I*
```

```
c*x]])/Sqrt[d], x] + (Simp[(I*b*PolyLog[2, -(Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x
])])/Sqrt[d], x] - Simp[(I*b*PolyLog[2, Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]])/S
qrt[d], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)}{x\sqrt{c+a^2cx^2}}\,dx = \frac{\sqrt{1+a^2x^2}\int \frac{\tan^{-1}(ax)}{x\sqrt{1+a^2x^2}}\,dx}{\sqrt{c+a^2cx^2}}$$
$$= -\frac{2\sqrt{1+a^2x^2}\tan^{-1}(ax)\tanh^{-1}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}} + \frac{i\sqrt{1+a^2x^2}\text{Li}_2\left(-\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}} - \frac{i\sqrt{1+a^2x^2}\text{Li}_2\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 0.14381, size = 100, normalized size = 0.56

$$\frac{\sqrt{a^2x^2+1}\left(i\text{PolyLog}\left(2,-e^{i\tan^{-1}(ax)}\right)-i\text{PolyLog}\left(2,e^{i\tan^{-1}(ax)}\right)+\tan^{-1}(ax)\left(\log\left(1-e^{i\tan^{-1}(ax)}\right)-\log\left(1+e^{i\tan^{-1}(ax)}\right)\right.\right.}{\sqrt{c\left(a^2x^2+1\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTan[a*x]/(x*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] (Sqrt[1 + a^2*x^2]*(ArcTan[a*x]*(Log[1 - E^(I*ArcTan[a*x])] - Log[1 + E^(I*
ArcTan[a*x])]) + I*PolyLog[2, -E^(I*ArcTan[a*x])] - I*PolyLog[2, E^(I*ArcTa
n[a*x])]))/Sqrt[c*(1 + a^2*x^2)]
```

Maple [A] time = 0.393, size = 139, normalized size = 0.8

$$\frac{-i}{c}\left(i\arctan(ax)\ln\left(1-(1+iax)\frac{1}{\sqrt{a^2x^2+1}}\right)-i\arctan(ax)\ln\left(1+(1+iax)\frac{1}{\sqrt{a^2x^2+1}}\right)+\text{polylog}\left(2,(1+iax)\frac{1}{\sqrt{a^2x^2}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)/x/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] -I*(I*arctan(a*x)*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*arctan(a*x)*ln(1+(1+I
*a*x)/(a^2*x^2+1)^(1/2))+polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))-polylog(2,-
```

```
(1+I*a*x)/(a^2*x^2+1)^(1/2)))*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)^(1/2)/c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{a^2cx^3+cx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)/(a^2*c*x^3 + c*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{atan}(ax)}{x\sqrt{c\left(a^2x^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)/x/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(atan(a*x)/(x*sqrt(c*(a**2*x**2 + 1))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(ax)}{\sqrt{a^2cx^2 + cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)/(sqrt(a^2*c*x^2 + c)*x), x)
```

3.229 $\int \frac{\tan^{-1}(ax)}{x^2\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=56

$$-\frac{\sqrt{a^2cx^2+c}\tan^{-1}(ax)}{cx}-\frac{a\tanh^{-1}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}$$

```
[Out] -((Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(c*x)) - (a*ArcTanh[Sqrt[c + a^2*c*x^2]
/Sqrt[c]])/Sqrt[c]
```

Rubi [A] time = 0.0926184, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4944, 266, 63, 208}

$$-\frac{\sqrt{a^2cx^2+c}\tan^{-1}(ax)}{cx}-\frac{a\tanh^{-1}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]/(x^2*Sqrt[c + a^2*c*x^2]),x]
```


```
[Out] -((Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(c*x)) - (a*ArcTanh[Sqrt[c + a^2*c*x^2]
/Sqrt[c]])/Sqrt[c]
```


Rule 4944

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a +
 b*ArcTan[c*x])^p)/(d*f*(m + 1)), x] - Dist[(b*c*p)/(f*(m + 1)), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^{-1}(ax)}{x^2\sqrt{c+a^2cx^2}}\,dx &= -\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{cx} + a\int \frac{1}{x\sqrt{c+a^2cx^2}}\,dx \\
&= -\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{cx} + \frac{1}{2}a\,\text{Subst}\left(\int \frac{1}{x\sqrt{c+a^2cx}}\,dx, x, x^2\right) \\
&= -\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{cx} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{1}{a^2}+\frac{x^2}{a^2c}}\,dx, x, \sqrt{c+a^2cx^2}\right)}{ac} \\
&= -\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{cx} - \frac{a\tanh^{-1}\left(\frac{\sqrt{c+a^2cx^2}}{\sqrt{c}}\right)}{\sqrt{c}}
\end{aligned}$$

Mathematica [A] time = 0.0921398, size = 62, normalized size = 1.11

$$\frac{a\left(\log(x)-\log\left(\sqrt{c}\sqrt{a^2cx^2+c}+c\right)\right)}{\sqrt{c}} - \frac{\sqrt{a^2cx^2+c}\tan^{-1}(ax)}{cx}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]/(x^2*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] -((Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(c*x)) + (a*(Log[x] - Log[c + Sqrt[c]*S
qrt[c + a^2*c*x^2]]))/Sqrt[c]
```

Maple [C] time = 0.369, size = 139, normalized size = 2.5

$$-\frac{\arctan (ax)}{cx}\sqrt{c\,(ax-i)\,(ax+i)}-\frac{a}{c}\ln\left(1+(1+iax)\frac{1}{\sqrt{a^2x^2+1}}\right)\sqrt{c\,(ax-i)\,(ax+i)}\frac{1}{\sqrt{a^2x^2+1}}+\frac{a}{c}\ln\left((1+iax)\frac{1}{\sqrt{a^2x^2+}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)/x^2/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] -arctan(a*x)*(c*(a*x-I)*(a*x+I))^(1/2)/c/x-a*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/
2))*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)^(1/2)/c+a*ln((1+I*a*x)/(a^2*x^2+1
)^(1/2)-1)*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)^(1/2)/c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x^2/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.39954, size = 163, normalized size = 2.91

$$\frac{a\sqrt{c}x\log\left(-\frac{a^2cx^2-2\sqrt{a^2cx^2+c}\sqrt{c}+2\,c}{x^2}\right)-2\sqrt{a^2cx^2+c}\arctan (ax)}{2\,cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x^2/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(a*sqrt(c)*x*log(-(a^2*c*x^2 - 2*sqrt(a^2*c*x^2 + c)*sqrt(c) + 2*c)/x^2
) - 2*sqrt(a^2*c*x^2 + c)*arctan(a*x))/(c*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atan}(ax)}{x^2\sqrt{c\left(a^2x^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)/x**2/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(atan(a*x)/(x**2*sqrt(c*(a**2*x**2 + 1))), x)
```

Giac [B] time = 1.23342, size = 136, normalized size = 2.43

$$\left(\frac{a\arctan\left(x|a|\right)}{\sqrt{c}|a|}-\frac{2\arctan\left(-\frac{\sqrt{a^2c}x-\sqrt{a^2cx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}\right)|a|+\frac{2\sqrt{c}|a|\arctan\left(ax\right)}{\left(\sqrt{a^2c}x-\sqrt{a^2cx^2+c}\right)^2-c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x^2/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] (a*arctan(x*abs(a))/(sqrt(c)*abs(a)) - 2*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*
c*x^2 + c))/sqrt(-c))/sqrt(-c))*abs(a) + 2*sqrt(c)*abs(a)*arctan(a*x)/((sqr
t(a^2*c)*x - sqrt(a^2*c*x^2 + c))^2 - c)
```

3.230 $\int \frac{\tan^{-1}(ax)}{x^3\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=242

$$-\frac{ia^2\sqrt{a^2x^2+1}\,\text{PolyLog}\left(2,-\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{2\sqrt{a^2cx^2+c}}+\frac{ia^2\sqrt{a^2x^2+1}\,\text{PolyLog}\left(2,\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{2\sqrt{a^2cx^2+c}}-\frac{a\sqrt{a^2cx^2+c}}{2cx}-\frac{\sqrt{a^2cx^2+c}\,\tan^{-1}(ax)}{2cx^2}+\frac{a^2\sqrt{}}{}$$

```
[Out] -(a*Sqrt[c + a^2*c*x^2])/(2*c*x) - (Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(2*c*x
^2) + (a^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTanh[Sqrt[1 + I*a*x]/Sqrt[1 - I
*a*x]])/Sqrt[c + a^2*c*x^2] - ((I/2)*a^2*Sqrt[1 + a^2*x^2]*PolyLog[2, -(Sqr
t[1 + I*a*x]/Sqrt[1 - I*a*x])])/Sqrt[c + a^2*c*x^2] + ((I/2)*a^2*Sqrt[1 + a
^2*x^2]*PolyLog[2, Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2]
```

Rubi [A] time = 0.220612, antiderivative size = 242, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4962, 264, 4958, 4954}

$$-\frac{ia^2\sqrt{a^2x^2+1}\,\text{PolyLog}\left(2,-\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{2\sqrt{a^2cx^2+c}}+\frac{ia^2\sqrt{a^2x^2+1}\,\text{PolyLog}\left(2,\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{2\sqrt{a^2cx^2+c}}-\frac{a\sqrt{a^2cx^2+c}}{2cx}-\frac{\sqrt{a^2cx^2+c}\,\tan^{-1}(ax)}{2cx^2}+\frac{a^2\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]/(x^3*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] -(a*Sqrt[c + a^2*c*x^2])/(2*c*x) - (Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(2*c*x
^2) + (a^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTanh[Sqrt[1 + I*a*x]/Sqrt[1 - I
*a*x]])/Sqrt[c + a^2*c*x^2] - ((I/2)*a^2*Sqrt[1 + a^2*x^2]*PolyLog[2, -(Sqr
t[1 + I*a*x]/Sqrt[1 - I*a*x])])/Sqrt[c + a^2*c*x^2] + ((I/2)*a^2*Sqrt[1 + a
^2*x^2]*PolyLog[2, Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2]
```

Rule 4962

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Ar
cTan[c*x])^p)/(d*f*(m + 1)), x] + (-Dist[(b*c*p)/(f*(m + 1)), Int[((f*x)^(m
 + 1)*(a + b*ArcTan[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] - Dist[(c^2*(m +
 2))/(f^2*(m + 1)), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/Sqrt[d + e*x^
2], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] &
& LtQ[m, -1] && NeQ[m, -2]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 4958

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4954

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x_
Symbol] :> Simp[(-2*(a + b*ArcTan[c*x])*ArcTanh[Sqrt[1 + I*c*x]/Sqrt[1 - I*
c*x]])/Sqrt[d], x] + (Simp[(I*b*PolyLog[2, -(Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x
])])/Sqrt[d], x] - Simp[(I*b*PolyLog[2, Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]])/S
qrt[d], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^{-1}(ax)}{x^3\sqrt{c+a^2cx^2}}\,dx &= -\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{2cx^2}+\frac{1}{2}a\int \frac{1}{x^2\sqrt{c+a^2cx^2}}\,dx-\frac{1}{2}a^2\int \frac{\tan^{-1}(ax)}{x\sqrt{c+a^2cx^2}}\,dx\\ &= -\frac{a\sqrt{c+a^2cx^2}}{2cx}-\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{2cx^2}-\frac{\left(a^2\sqrt{1+a^2x^2}\right)\int \frac{\tan^{-1}(ax)}{x\sqrt{1+a^2x^2}}\,dx}{2\sqrt{c+a^2cx^2}}\\ &= -\frac{a\sqrt{c+a^2cx^2}}{2cx}-\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{2cx^2}+\frac{a^2\sqrt{1+a^2x^2}\tan^{-1}(ax)\tanh^{-1}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}}-\frac{ia^2\sqrt{1+a^2x}}{2\sqrt{c}}\end{aligned}$$

Mathematica [A] time = 0.688616, size = 165, normalized size = 0.68

$$\frac{a^2\sqrt{a^2x^2+1}\left(-4i\text{PolyLog}\left(2,-e^{i\tan^{-1}(ax)}\right)+4i\text{PolyLog}\left(2,e^{i\tan^{-1}(ax)}\right)-2\tan\left(\frac{1}{2}\tan^{-1}(ax)\right)-4\tan^{-1}(ax)\log\left(1-e^{i\tan}\right.\right.}{8\sqrt{}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTan[a*x]/(x^3*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] (a^2*Sqrt[1 + a^2*x^2]*(-2*Cot[ArcTan[a*x]/2] - ArcTan[a*x]*Csc[ArcTan[a*x]
/2]^2 - 4*ArcTan[a*x]*Log[1 - E^(I*ArcTan[a*x])] + 4*ArcTan[a*x]*Log[1 + E^
(I*ArcTan[a*x])] - (4*I)*PolyLog[2, -E^(I*ArcTan[a*x])] + (4*I)*PolyLog[2,
E^(I*ArcTan[a*x])] + ArcTan[a*x]*Sec[ArcTan[a*x]/2]^2 - 2*Tan[ArcTan[a*x]/2
]))/(8*Sqrt[c*(1 + a^2*x^2)])
```

Maple [A] time = 0.497, size = 175, normalized size = 0.7

$$-\frac{ax+\arctan(ax)}{2\,cx^2}\sqrt{c\,(ax-i)\,(ax+i)}+\frac{\frac{i}{2}a^2}{c}\left(i\arctan(ax)\ln\left(1-(1+iax)\frac{1}{\sqrt{a^2x^2+1}}\right)-i\arctan(ax)\ln\left(1+(1+iax)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)/x^3/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] -1/2*(a*x+arctan(a*x))*(c*(a*x-I)*(a*x+I))^(1/2)/x^2/c+1/2*I*a^2*(I*arctan(
a*x)*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*arctan(a*x)*ln(1+(1+I*a*x)/(a^2*x^
2+1)^(1/2))+polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))-polylog(2,-(1+I*a*x)/(a^
2*x^2+1)^(1/2)))/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(a*x+I))^(1/2)/c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x^3/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{a^2cx^5+cx^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x^3/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)/(a^2*c*x^5 + c*x^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atan}(ax)}{x^3\sqrt{c\left(a^2x^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)/x**3/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(atan(a*x)/(x**3*sqrt(c*(a**2*x**2 + 1))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x^3/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)/(sqrt(a^2*c*x^2 + c)*x^3), x)
```

3.231 $\int \frac{\tan^{-1}(ax)}{x^4\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=118

$$-\frac{a\sqrt{a^2cx^2+c}}{6cx^2}+\frac{2a^2\sqrt{a^2cx^2+c}\tan^{-1}(ax)}{3cx}-\frac{\sqrt{a^2cx^2+c}\tan^{-1}(ax)}{3cx^3}+\frac{5a^3\tanh^{-1}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{6\sqrt{c}}$$

```
[Out] -(a*Sqrt[c + a^2*c*x^2])/(6*c*x^2) - (Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(3*c
*x^3) + (2*a^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(3*c*x) + (5*a^3*ArcTanh[Sq
rt[c + a^2*c*x^2]/Sqrt[c]])/(6*Sqrt[c])
```

Rubi [A] time = 0.200355, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4962, 266, 51, 63, 208, 4944}

$$-\frac{a\sqrt{a^2cx^2+c}}{6cx^2}+\frac{2a^2\sqrt{a^2cx^2+c}\tan^{-1}(ax)}{3cx}-\frac{\sqrt{a^2cx^2+c}\tan^{-1}(ax)}{3cx^3}+\frac{5a^3\tanh^{-1}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{6\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]/(x^4*Sqrt[c + a^2*c*x^2]),x]
```


```
[Out] -(a*Sqrt[c + a^2*c*x^2])/(6*c*x^2) - (Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(3*c
*x^3) + (2*a^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(3*c*x) + (5*a^3*ArcTanh[Sq
rt[c + a^2*c*x^2]/Sqrt[c]])/(6*Sqrt[c])
```


Rule 4962

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Ar
cTan[c*x])^p)/(d*f*(m + 1)), x] + (-Dist[(b*c*p)/(f*(m + 1)), Int[((f*x)^(m
 + 1)*(a + b*ArcTan[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] - Dist[(c^2*(m +
 2))/(f^2*(m + 1)), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/Sqrt[d + e*x^
2], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] &
& LtQ[m, -1] && NeQ[m, -2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 4944

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a +
 b*ArcTan[c*x])^p)/(d*f*(m + 1)), x] - Dist[(b*c*p)/(f*(m + 1)), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)}{x^4\sqrt{c+a^2cx^2}}\,dx = -\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{3cx^3} + \frac{1}{3}a\int \frac{1}{x^3\sqrt{c+a^2cx^2}}\,dx - \frac{1}{3}\left(2a^2\right)\int \frac{\tan^{-1}(ax)}{x^2\sqrt{c+a^2cx^2}}\,dx$$

$$= -\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{3cx^3} + \frac{2a^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{3cx} + \frac{1}{6}a\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{c+a^2cx}}\,dx, x, x^2\right) - \frac{1}{3}(2$$

$$= -\frac{a\sqrt{c+a^2cx^2}}{6cx^2} - \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{3cx^3} + \frac{2a^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{3cx} - \frac{1}{12}a^3\,\text{Subst}\left(\int \frac{1}{x\sqrt{c+a^2cx}}\right.$$

$$= -\frac{a\sqrt{c+a^2cx^2}}{6cx^2} - \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{3cx^3} + \frac{2a^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{3cx} - \frac{a\,\text{Subst}\left(\int \frac{1}{-\frac{1}{a^2}+\frac{x^2}{a^2c}}\,dx, x, \sqrt{c}\right.}{6c}$$

$$= -\frac{a\sqrt{c+a^2cx^2}}{6cx^2} - \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{3cx^3} + \frac{2a^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{3cx} + \frac{5a^3\tanh^{-1}\left(\frac{\sqrt{c+a^2cx^2}}{\sqrt{c}}\right)}{6\sqrt{c}}$$

Mathematica [A] time = 0.118508, size = 110, normalized size = 0.93

$$\frac{-ax\sqrt{a^2cx^2+c} - 5a^3\sqrt{c}x^3\log(x) + 5a^3\sqrt{c}x^3\log\left(\sqrt{c}\sqrt{a^2cx^2+c}+c\right) + 2\left(2a^2x^2-1\right)\sqrt{a^2cx^2+c}\tan^{-1}(ax)}{6cx^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]/(x^4*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] (-(a*x*Sqrt[c + a^2*c*x^2]) + 2*(-1 + 2*a^2*x^2)*Sqrt[c + a^2*c*x^2]*ArcTan
[a*x] - 5*a^3*Sqrt[c]*x^3*Log[x] + 5*a^3*Sqrt[c]*x^3*Log[c + Sqrt[c]*Sqrt[c
 + a^2*c*x^2]])/(6*c*x^3)
```

Maple [C] time = 0.801, size = 163, normalized size = 1.4

$$\frac{4\arctan(ax)a^2x^2 - ax - 2\arctan(ax)}{6cx^3}\sqrt{c(ax-i)(ax+i)} - \frac{5a^3}{6c}\ln\left((1+iax)\frac{1}{\sqrt{a^2x^2+1}} - 1\right)\sqrt{c(ax-i)(ax+i)}\frac{1}{\sqrt{a^2x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)/x^4/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] 1/6*(4*arctan(a*x)*a^2*x^2-a*x-2*arctan(a*x))*(c*(a*x-I)*(a*x+I))^(1/2)/c/x
^3-5/6*a^3*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)-1)*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2
*x^2+1)^(1/2)/c+5/6*a^3*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))*(c*(a*x-I)*(a*x+I
))^(1/2)/(a^2*x^2+1)^(1/2)/c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x^4/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.44953, size = 211, normalized size = 1.79

$$\frac{5\,a^3\sqrt{c}x^3\log\left(-\frac{a^2cx^2+2\,\sqrt{a^2cx^2+c}\sqrt{c}+2\,c}{x^2}\right)-2\,\sqrt{a^2cx^2+c}\left(ax-2\left(2\,a^2x^2-1\right)\arctan\left(ax\right)\right)}{12\,cx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x^4/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*(5*a^3*sqrt(c)*x^3*log(-(a^2*c*x^2 + 2*sqrt(a^2*c*x^2 + c)*sqrt(c) + 2
*c)/x^2) - 2*sqrt(a^2*c*x^2 + c)*(a*x - 2*(2*a^2*x^2 - 1)*arctan(a*x)))/(c*
x^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{atan}\left(ax\right)}{x^4\sqrt{c\left(a^2x^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)/x**4/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(atan(a*x)/(x**4*sqrt(c*(a**2*x**2 + 1))), x)
```

Giac [B] time = 1.41381, size = 381, normalized size = 3.23

$$\frac{4\left(3\left(\sqrt{a^2c}x-\sqrt{a^2cx^2+c}\right)^2-c\right)a^2c^{\frac{3}{2}}|a|\arctan\left(ax\right)}{3\left(\left(\sqrt{a^2c}x-\sqrt{a^2cx^2+c}\right)^2-c\right)^3}-\frac{\left(\frac{8\,a^3\arctan(x|a|)}{|a|}-5\,a^2\log\left(\left|-x|a|+\sqrt{a^2x^2+1}-\frac{1}{x|a|-\sqrt{a^2x^2+1}}+2\right|\right)+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x^4/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 4/3*(3*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 + c))^2 - c)*a^2*c^(3/2)*abs(a)*arct
an(a*x)/((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 + c))^2 - c)^3 - 1/12*(8*a^3*arcta
n(x*abs(a))/abs(a) - 5*a^2*log(abs(-x*abs(a) + sqrt(a^2*x^2 + 1) - 1/(x*abs
(a) - sqrt(a^2*x^2 + 1)) + 2)) + 5*a^2*log(abs(-x*abs(a) + sqrt(a^2*x^2 + 1
) - 1/(x*abs(a) - sqrt(a^2*x^2 + 1)) - 2)) - 4*(x*abs(a) - sqrt(a^2*x^2 + 1
) + 1/(x*abs(a) - sqrt(a^2*x^2 + 1)))*a^2/((x*abs(a) - sqrt(a^2*x^2 + 1) +
1/(x*abs(a) - sqrt(a^2*x^2 + 1)))^2 - 4))*abs(a)/sqrt(c)
```

3.232 $\int \frac{x^3 \tan^{-1}(ax)}{(c+a^2cx^2)^{3/2}} \, dx$

Optimal. Leaf size=107

$$\frac{\sqrt{a^2cx^2+c}\tan^{-1}(ax)}{a^4c^2} - \frac{\tanh^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a^4c^{3/2}} - \frac{x}{a^3c\sqrt{a^2cx^2+c}} + \frac{\tan^{-1}(ax)}{a^4c\sqrt{a^2cx^2+c}}$$

```
[Out] -(x/(a^3*c*Sqrt[c + a^2*c*x^2])) + ArcTan[a*x]/(a^4*c*Sqrt[c + a^2*c*x^2])
+ (Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(a^4*c^2) - ArcTanh[(a*Sqrt[c]*x)/Sqrt[
c + a^2*c*x^2]]/(a^4*c^(3/2))
```

Rubi [A] time = 0.202065, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {4964, 4930, 217, 206, 191}

$$\frac{\sqrt{a^2cx^2+c}\tan^{-1}(ax)}{a^4c^2} - \frac{\tanh^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a^4c^{3/2}} - \frac{x}{a^3c\sqrt{a^2cx^2+c}} + \frac{\tan^{-1}(ax)}{a^4c\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcTan[a*x])/(c + a^2*c*x^2)^(3/2),x]
```


```
[Out] -(x/(a^3*c*Sqrt[c + a^2*c*x^2])) + ArcTan[a*x]/(a^4*c*Sqrt[c + a^2*c*x^2])
+ (Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(a^4*c^2) - ArcTanh[(a*Sqrt[c]*x)/Sqrt[
c + a^2*c*x^2]]/(a^4*c^(3/2))
```


Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Arc
Tan[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan[c
*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p
, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
```

```
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3 \tan^{-1}(ax)}{\left(c+a^2cx^2\right)^{3/2}}\,dx &= -\frac{\int \frac{x\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^{3/2}}\,dx}{a^2}+\frac{\int \frac{x\tan^{-1}(ax)}{\sqrt{c+a^2cx^2}}\,dx}{a^2c}\\
&= \frac{\tan^{-1}(ax)}{a^4c\sqrt{c+a^2cx^2}}+\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{a^4c^2}-\frac{\int \frac{1}{\left(c+a^2cx^2\right)^{3/2}}\,dx}{a^3}-\frac{\int \frac{1}{\sqrt{c+a^2cx^2}}\,dx}{a^3c}\\
&= -\frac{x}{a^3c\sqrt{c+a^2cx^2}}+\frac{\tan^{-1}(ax)}{a^4c\sqrt{c+a^2cx^2}}+\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{a^4c^2}-\frac{\text{Subst}\left(\int \frac{1}{1-a^2cx^2}\,dx,x,\frac{x}{\sqrt{c+a^2cx^2}}\right)}{a^3c}\\
&= -\frac{x}{a^3c\sqrt{c+a^2cx^2}}+\frac{\tan^{-1}(ax)}{a^4c\sqrt{c+a^2cx^2}}+\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{a^4c^2}-\frac{\tanh^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c+a^2cx^2}}\right)}{a^4c^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.126293, size = 107, normalized size = 1.

$$\frac{-ax\sqrt{a^2cx^2+c}-\sqrt{c}\left(a^2x^2+1\right)\log\left(\sqrt{c}\sqrt{a^2cx^2+c}+acx\right)+\left(a^2x^2+2\right)\sqrt{a^2cx^2+c}\tan^{-1}(ax)}{a^4c^2\left(a^2x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*ArcTan[a*x])/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] (-(a*x*Sqrt[c + a^2*c*x^2]) + (2 + a^2*x^2)*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]
 - Sqrt[c]*(1 + a^2*x^2)*Log[a*c*x + Sqrt[c]*Sqrt[c + a^2*c*x^2]])/(a^4*c^2
*(1 + a^2*x^2))
```

Maple [C] time = 1.029, size = 242, normalized size = 2.3

$$\frac{(\arctan (ax) + i)(1 + iax)}{\left(2\,a^2x^2 + 2\right)a^4c^2}\sqrt{c\,(ax - i)\,(ax + i)} - \frac{(-1 + iax)(\arctan (ax) - i)}{\left(2\,a^2x^2 + 2\right)a^4c^2}\sqrt{c\,(ax - i)\,(ax + i)} + \frac{\arctan (ax)}{a^4c^2}\sqrt{c\,(ax - i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctan(a*x)/(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] 1/2*(arctan(a*x)+I)*(1+I*a*x)*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)/a^4/c^2
-1/2*(c*(a*x-I)*(a*x+I))^(1/2)*(-1+I*a*x)*(arctan(a*x)-I)/(a^2*x^2+1)/a^4/c
^2+arctan(a*x)*(c*(a*x-I)*(a*x+I))^(1/2)/a^4/c^2+ln((1+I*a*x)/(a^2*x^2+1)^(
1/2)-I)/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(a*x+I))^(1/2)/a^4/c^2-ln((1+I*a*x)/(a
^2*x^2+1)^(1/2)+I)/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(a*x+I))^(1/2)/a^4/c^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.41378, size = 228, normalized size = 2.13

$$\frac{\left(a^2x^2 + 1\right)\sqrt{c}\log \left(-2\,a^2cx^2 + 2\,\sqrt{a^2cx^2 + c}a\sqrt{c}x - c\right) - 2\,\sqrt{a^2cx^2 + c}\left(ax - \left(a^2x^2 + 2\right)\arctan (ax)\right)}{2\left(a^6c^2x^2 + a^4c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*((a^2*x^2 + 1)*sqrt(c)*log(-2*a^2*c*x^2 + 2*sqrt(a^2*c*x^2 + c)*a*sqrt(
c)*x - c) - 2*sqrt(a^2*c*x^2 + c)*(a*x - (a^2*x^2 + 2)*arctan(a*x)))/(a^6*c
^2*x^2 + a^4*c^2)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atan(a*x)/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Exception raised: TypeError
```

Giac [A] time = 1.35594, size = 130, normalized size = 1.21

$$-\frac{x}{\sqrt{a^2cx^2+c}a^3c}+\frac{\left(\sqrt{a^2cx^2+c}+\frac{c}{\sqrt{a^2cx^2+c}}\right)\arctan\left(ax\right)}{a^4c^2}+\frac{\log\left(\left|-\sqrt{a^2c}x+\sqrt{a^2cx^2+c}\right|\right)}{a^3c^{\frac{3}{2}}|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] -x/(sqrt(a^2*c*x^2 + c)*a^3*c) + (sqrt(a^2*c*x^2 + c) + c/sqrt(a^2*c*x^2 +
c))*arctan(a*x)/(a^4*c^2) + log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 + c)))/
(a^3*c^(3/2)*abs(a))
```

3.233 $\int \frac{x^2 \tan^{-1}(ax)}{\left(c+a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=251

$$\frac{i\sqrt{a^2x^2+1}\mathrm{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a^3c\sqrt{a^2cx^2+c}}-\frac{i\sqrt{a^2x^2+1}\mathrm{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a^3c\sqrt{a^2cx^2+c}}-\frac{1}{a^3c\sqrt{a^2cx^2+c}}-\frac{2i\sqrt{a^2x^2+1}\tan^{-1}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)\mathrm{ta}}{a^3c\sqrt{a^2cx^2+c}}$$

```
[Out] -(1/(a^3*c*Sqrt[c + a^2*c*x^2])) - (x*ArcTan[a*x])/(a^2*c*Sqrt[c + a^2*c*x^
2]) - ((2*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 -
I*a*x]])/(a^3*c*Sqrt[c + a^2*c*x^2]) + (I*Sqrt[1 + a^2*x^2]*PolyLog[2, ((-I
)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/(a^3*c*Sqrt[c + a^2*c*x^2]) - (I*Sqrt[
1 + a^2*x^2]*PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/(a^3*c*Sqrt[c
 + a^2*c*x^2])
```

Rubi [A] time = 0.158616, antiderivative size = 251, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {4934, 4890, 4886}

$$\frac{i\sqrt{a^2x^2+1}\mathrm{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a^3c\sqrt{a^2cx^2+c}}-\frac{i\sqrt{a^2x^2+1}\mathrm{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a^3c\sqrt{a^2cx^2+c}}-\frac{1}{a^3c\sqrt{a^2cx^2+c}}-\frac{2i\sqrt{a^2x^2+1}\tan^{-1}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)\mathrm{ta}}{a^3c\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcTan[a*x])/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] -(1/(a^3*c*Sqrt[c + a^2*c*x^2])) - (x*ArcTan[a*x])/(a^2*c*Sqrt[c + a^2*c*x^
2]) - ((2*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 -
I*a*x]])/(a^3*c*Sqrt[c + a^2*c*x^2]) + (I*Sqrt[1 + a^2*x^2]*PolyLog[2, ((-I
)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/(a^3*c*Sqrt[c + a^2*c*x^2]) - (I*Sqrt[
1 + a^2*x^2]*PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/(a^3*c*Sqrt[c
 + a^2*c*x^2])
```

Rule 4934

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^2*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> -Simp[(b*(d + e*x^2)^(q + 1))/(4*c^3*d*(q + 1)^2), x] + (-Dist[
1/(2*c^2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]), x], x] +
Simp[(x*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]))/(2*c^2*d*(q + 1)), x]) /;
```

```
FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -5/2]
```

Rule 4890

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4886

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[(-2*I*(a + b*ArcTan[c*x])*ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]])
/(c*Sqrt[d]), x] + (Simp[(I*b*PolyLog[2, -((I*Sqrt[1 + I*c*x])/Sqrt[1 - I*c
*x])])/(c*Sqrt[d]), x] - Simp[(I*b*PolyLog[2, (I*Sqrt[1 + I*c*x])/Sqrt[1 -
I*c*x]])/(c*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

Rubi steps

$$\int \frac{x^2\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^{3/2}}\,dx = -\frac{1}{a^3c\sqrt{c+a^2cx^2}}-\frac{x\tan^{-1}(ax)}{a^2c\sqrt{c+a^2cx^2}}+\frac{\int \frac{\tan^{-1}(ax)}{\sqrt{c+a^2cx^2}}\,dx}{a^2c}$$

$$= -\frac{1}{a^3c\sqrt{c+a^2cx^2}}-\frac{x\tan^{-1}(ax)}{a^2c\sqrt{c+a^2cx^2}}+\frac{\sqrt{1+a^2x^2}\int \frac{\tan^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx}{a^2c\sqrt{c+a^2cx^2}}$$

$$= -\frac{1}{a^3c\sqrt{c+a^2cx^2}}-\frac{x\tan^{-1}(ax)}{a^2c\sqrt{c+a^2cx^2}}-\frac{2i\sqrt{1+a^2x^2}\tan^{-1}(ax)\tan^{-1}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a^3c\sqrt{c+a^2cx^2}}+\frac{i\sqrt{1+a^2x^2}\text{Li}_2\left(-\frac{i\sqrt{}}{\sqrt{}}\right.}{a^3c\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 0.239759, size = 155, normalized size = 0.62

$$-\frac{\sqrt{a^2x^2+1}\left(-i\text{PolyLog}\left(2,-ie^{i\tan^{-1}(ax)}\right)+i\text{PolyLog}\left(2,ie^{i\tan^{-1}(ax)}\right)+\frac{1}{\sqrt{a^2x^2+1}}+\frac{ax\tan^{-1}(ax)}{\sqrt{a^2x^2+1}}+\tan^{-1}(ax)\left(-\log\left(1-ie^{i\tan}\right.\right.\right.}{a^3c\sqrt{c\left(a^2x^2+1\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*ArcTan[a*x])/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] -((Sqrt[1 + a^2*x^2]*(1/Sqrt[1 + a^2*x^2] + (a*x*ArcTan[a*x])/Sqrt[1 + a^2*
x^2] - ArcTan[a*x]*Log[1 - I*E^(I*ArcTan[a*x])] + ArcTan[a*x]*Log[1 + I*E^(
I*ArcTan[a*x])] - I*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] + I*PolyLog[2, I*E^(
I*ArcTan[a*x])]))/(a^3*c*Sqrt[c*(1 + a^2*x^2)]))
```

Maple [A] time = 0.766, size = 247, normalized size = 1.

$$-\frac{(\arctan(ax)+i)(ax-i)}{(2a^2x^2+2)c^2a^3}\sqrt{c(ax-i)(ax+i)}-\frac{(ax+i)(\arctan(ax)-i)}{(2a^2x^2+2)c^2a^3}\sqrt{c(ax-i)(ax+i)}+\frac{i}{c^2a^3}\Big(i\arctan(ax)\ln\Big(1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(a*x)/(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] -1/2*(arctan(a*x)+I)*(a*x-I)*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)/c^2/a^3-
1/2*(c*(a*x-I)*(a*x+I))^(1/2)*(a*x+I)*(arctan(a*x)-I)/(a^2*x^2+1)/c^2/a^3+I
*(I*arctan(a*x)*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*arctan(a*x)*ln(1-I*(1
+I*a*x)/(a^2*x^2+1)^(1/2))+dilog(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-dilog(1-I
*(1+I*a*x)/(a^2*x^2+1)^(1/2)))*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)^(1/2)/
c^2/a^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a^2cx^2+c}\,x^2\arctan(ax)}{a^4c^2x^4+2\,a^2c^2x^2+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^2*arctan(a*x)/(a^4*c^2*x^4 + 2*a^2*c^2*x^2 +
 c^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \operatorname{atan}(ax)}{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(a*x)/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(x**2*atan(a*x)/(c*(a**2*x**2 + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \arctan(ax)}{\left(a^2cx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctan(a*x)/(a^2*c*x^2 + c)^(3/2), x)
```

3.234 $\int \frac{x \tan^{-1}(ax)}{(c+a^2cx^2)^{3/2}} \, dx$

Optimal. Leaf size=49

$$\frac{x}{ac\sqrt{a^2cx^2+c}} - \frac{\tan^{-1}(ax)}{a^2c\sqrt{a^2cx^2+c}}$$

```
[Out] x/(a*c*Sqrt[c + a^2*c*x^2]) - ArcTan[a*x]/(a^2*c*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.0553146, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {4930, 191}

$$\frac{x}{ac\sqrt{a^2cx^2+c}} - \frac{\tan^{-1}(ax)}{a^2c\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTan[a*x])/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] x/(a*c*Sqrt[c + a^2*c*x^2]) - ArcTan[a*x]/(a^2*c*Sqrt[c + a^2*c*x^2])
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{x\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^{3/2}}\,dx = -\frac{\tan^{-1}(ax)}{a^2c\sqrt{c+a^2cx^2}} + \frac{\int \frac{1}{\left(c+a^2cx^2\right)^{3/2}}\,dx}{a}$$
$$= \frac{x}{ac\sqrt{c+a^2cx^2}} - \frac{\tan^{-1}(ax)}{a^2c\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 0.0495469, size = 42, normalized size = 0.86

$$\frac{\sqrt{a^2cx^2+c}\left(ax-\tan^{-1}(ax)\right)}{a^2c^2\left(a^2x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcTan[a*x])/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] (Sqrt[c + a^2*c*x^2]*(a*x - ArcTan[a*x]))/(a^2*c^2*(1 + a^2*x^2))
```

Maple [C] time = 0.266, size = 100, normalized size = 2.

$$-\frac{(\arctan(ax)+i)(1+iax)}{\left(2a^2x^2+2\right)c^2a^2}\sqrt{c(ax-i)(ax+i)} + \frac{(-1+iax)(\arctan(ax)-i)}{\left(2a^2x^2+2\right)c^2a^2}\sqrt{c(ax-i)(ax+i)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(a*x)/(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] -1/2*(arctan(a*x)+I)*(1+I*a*x)*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)/c^2/a^
2+1/2*(c*(a*x-I)*(a*x+I))^(1/2)*(-1+I*a*x)*(arctan(a*x)-I)/(a^2*x^2+1)/c^2/
a^2
```

Maxima [A] time = 1.74233, size = 38, normalized size = 0.78

$$\frac{ax-\arctan(ax)}{\sqrt{a^2x^2+1}a^2c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] (a*x - arctan(a*x))/(sqrt(a^2*x^2 + 1)*a^2*c^(3/2))
```

Fricas [A] time = 2.2285, size = 88, normalized size = 1.8

$$\frac{\sqrt{a^2cx^2+c}(ax-\arctan(ax))}{a^4c^2x^2+a^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] sqrt(a^2*c*x^2 + c)*(a*x - arctan(a*x))/(a^4*c^2*x^2 + a^2*c^2)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(a*x)/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Exception raised: TypeError
```

Giac [A] time = 1.3689, size = 61, normalized size = 1.24

$$\frac{x}{\sqrt{a^2cx^2+c}ac}-\frac{\arctan(ax)}{\sqrt{a^2cx^2+c}a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] x/(sqrt(a^2*c*x^2 + c)*a*c) - arctan(a*x)/(sqrt(a^2*c*x^2 + c)*a^2*c)
```

3.235 $\int \frac{\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=45

$$\frac{1}{ac\sqrt{a^2cx^2+c}}+\frac{x\tan^{-1}(ax)}{c\sqrt{a^2cx^2+c}}$$

```
[Out] 1/(a*c*Sqrt[c + a^2*c*x^2]) + (x*ArcTan[a*x])/(c*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.0248004, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {4894}

$$\frac{1}{ac\sqrt{a^2cx^2+c}}+\frac{x\tan^{-1}(ax)}{c\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] 1/(a*c*Sqrt[c + a^2*c*x^2]) + (x*ArcTan[a*x])/(c*Sqrt[c + a^2*c*x^2])
```

Rule 4894

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[(x*(a + b*ArcTan[c*x]))/(d*Sq
rt[d + e*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^{3/2}}\,dx=\frac{1}{ac\sqrt{c+a^2cx^2}}+\frac{x\tan^{-1}(ax)}{c\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 0.0451827, size = 38, normalized size = 0.84

$$\frac{\sqrt{a^2cx^2+c}\left(ax\tan^{-1}(ax)+1\right)}{c^2\left(a^3x^2+a\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] (Sqrt[c + a^2*c*x^2]*(1 + a*x*ArcTan[a*x]))/(c^2*(a + a^3*x^2))
```

Maple [C] time = 0.231, size = 98, normalized size = 2.2

$$\frac{(\arctan(ax)+i)(ax-i)}{(2a^2x^2+2)c^2a}\sqrt{c(ax-i)(ax+i)}+\frac{(ax+i)(\arctan(ax)-i)}{(2a^2x^2+2)c^2a}\sqrt{c(ax-i)(ax+i)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)/(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] 1/2*(arctan(a*x)+I)*(a*x-I)*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)/c^2/a+1/2
*(c*(a*x-I)*(a*x+I))^(1/2)*(a*x+I)*(arctan(a*x)-I)/(a^2*x^2+1)/c^2/a
```

Maxima [A] time = 1.02519, size = 55, normalized size = 1.22

$$\frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}c}+\frac{1}{\sqrt{a^2cx^2+c}ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] x*arctan(a*x)/(sqrt(a^2*c*x^2 + c)*c) + 1/(sqrt(a^2*c*x^2 + c)*a*c)
```

Fricas [A] time = 2.31497, size = 88, normalized size = 1.96

$$\frac{\sqrt{a^2cx^2+c}(ax\arctan(ax)+1)}{a^3c^2x^2+ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] sqrt(a^2*c*x^2 + c)*(a*x*arctan(a*x) + 1)/(a^3*c^2*x^2 + a*c^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atan}(ax)}{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(atan(a*x)/(c*(a**2*x**2 + 1))**(3/2), x)
```

Giac [A] time = 1.38002, size = 55, normalized size = 1.22

$$\frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}c}+\frac{1}{\sqrt{a^2cx^2+c}ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] x*arctan(a*x)/(sqrt(a^2*c*x^2 + c)*c) + 1/(sqrt(a^2*c*x^2 + c)*a*c)
```

3.236 $\int \frac{\tan^{-1}(ax)}{x(c+a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=229

$$\frac{i\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{c\sqrt{a^2cx^2+c}}-\frac{i\sqrt{a^2x^2+1}\text{PolyLog}\left(2,\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{c\sqrt{a^2cx^2+c}}-\frac{ax}{c\sqrt{a^2cx^2+c}}+\frac{\tan^{-1}(ax)}{c\sqrt{a^2cx^2+c}}-\frac{2\sqrt{a^2x^2+1}\tan^{-1}}{c\sqrt{a}}$$

```
[Out] -((a*x)/(c*Sqrt[c + a^2*c*x^2])) + ArcTan[a*x]/(c*Sqrt[c + a^2*c*x^2]) - (2
*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTanh[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/(c
*Sqrt[c + a^2*c*x^2]) + (I*Sqrt[1 + a^2*x^2]*PolyLog[2, -(Sqrt[1 + I*a*x]/S
qrt[1 - I*a*x])])/(c*Sqrt[c + a^2*c*x^2]) - (I*Sqrt[1 + a^2*x^2]*PolyLog[2,
 Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/(c*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.279953, antiderivative size = 229, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {4966, 4958, 4954, 4930, 191}

$$\frac{i\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{c\sqrt{a^2cx^2+c}}-\frac{i\sqrt{a^2x^2+1}\text{PolyLog}\left(2,\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{c\sqrt{a^2cx^2+c}}-\frac{ax}{c\sqrt{a^2cx^2+c}}+\frac{\tan^{-1}(ax)}{c\sqrt{a^2cx^2+c}}-\frac{2\sqrt{a^2x^2+1}\tan^{-1}}{c\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]/(x*(c + a^2*c*x^2)^(3/2)),x]
```

```
[Out] -((a*x)/(c*Sqrt[c + a^2*c*x^2])) + ArcTan[a*x]/(c*Sqrt[c + a^2*c*x^2]) - (2
*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTanh[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/(c
*Sqrt[c + a^2*c*x^2]) + (I*Sqrt[1 + a^2*x^2]*PolyLog[2, -(Sqrt[1 + I*a*x]/S
qrt[1 - I*a*x])])/(c*Sqrt[c + a^2*c*x^2]) - (I*Sqrt[1 + a^2*x^2]*PolyLog[2,
 Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/(c*Sqrt[c + a^2*c*x^2])
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*
x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^p
, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*q]
 && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 4958

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4954

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x_
Symbol] :> Simp[(-2*(a + b*ArcTan[c*x])*ArcTanh[Sqrt[1 + I*c*x]/Sqrt[1 - I*
c*x]])/Sqrt[d], x] + (Simp[(I*b*PolyLog[2, -(Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x
])])/Sqrt[d], x] - Simp[(I*b*PolyLog[2, Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]])/S
qrt[d], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)}{x\left(c+a^2cx^2\right)^{3/2}}\,dx = -\left(a^2\int \frac{x\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^{3/2}}\,dx\right)+\frac{\int \frac{\tan^{-1}(ax)}{x\sqrt{c+a^2cx^2}}\,dx}{c}$$

$$= \frac{\tan^{-1}(ax)}{c\sqrt{c+a^2cx^2}} - a\int \frac{1}{\left(c+a^2cx^2\right)^{3/2}}\,dx + \frac{\sqrt{1+a^2x^2}\int \frac{\tan^{-1}(ax)}{x\sqrt{1+a^2x^2}}\,dx}{c\sqrt{c+a^2cx^2}}$$

$$= -\frac{ax}{c\sqrt{c+a^2cx^2}} + \frac{\tan^{-1}(ax)}{c\sqrt{c+a^2cx^2}} - \frac{2\sqrt{1+a^2x^2}\tan^{-1}(ax)\tanh^{-1}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{c\sqrt{c+a^2cx^2}} + \frac{i\sqrt{1+a^2x^2}\,\text{Li}_2\left(-\frac{\sqrt{1+i}}{\sqrt{1-i}}\right.}{c\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 0.218121, size = 141, normalized size = 0.62

$$\frac{\sqrt{a^2x^2+1}\left(i\text{PolyLog}\left(2,-e^{i\tan^{-1}(ax)}\right)-i\text{PolyLog}\left(2,e^{i\tan^{-1}(ax)}\right)-\frac{ax}{\sqrt{a^2x^2+1}}+\frac{\tan^{-1}(ax)}{\sqrt{a^2x^2+1}}+\tan^{-1}(ax)\log\left(1-e^{i\tan^{-1}(ax)}\right)-\right.}{c\sqrt{c\left(a^2x^2+1\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTan[a*x]/(x*(c + a^2*c*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[1 + a^2*x^2]*(-((a*x)/Sqrt[1 + a^2*x^2]) + ArcTan[a*x]/Sqrt[1 + a^2*x
^2] + ArcTan[a*x]*Log[1 - E^(I*ArcTan[a*x])] - ArcTan[a*x]*Log[1 + E^(I*Arc
Tan[a*x])] + I*PolyLog[2, -E^(I*ArcTan[a*x])] - I*PolyLog[2, E^(I*ArcTan[a*
x])]))/(c*Sqrt[c*(1 + a^2*x^2)])
```

Maple [A] time = 0.281, size = 232, normalized size = 1.

$$\frac{(\arctan(ax)+i)(1+iax)}{\left(2a^2x^2+2\right)c^2}\sqrt{c(ax-i)(ax+i)}-\frac{(-1+iax)(\arctan(ax)-i)}{\left(2a^2x^2+2\right)c^2}\sqrt{c(ax-i)(ax+i)}-\frac{i}{c^2}\left(i\arctan(ax)\ln\left(1\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)/x/(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] 1/2*(arctan(a*x)+I)*(1+I*a*x)*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)/c^2-1/2
*(c*(a*x-I)*(a*x+I))^(1/2)*(-1+I*a*x)*(arctan(a*x)-I)/(a^2*x^2+1)/c^2-I*(I*
arctan(a*x)*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*arctan(a*x)*ln(1+(1+I*a*x)/
(a^2*x^2+1)^(1/2))+polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))-polylog(2,-(1+I*a
*x)/(a^2*x^2+1)^(1/2)))*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)^(1/2)/c^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a^2cx^2+c}\arctan\left(ax\right)}{a^4c^2x^5+2\,a^2c^2x^3+c^2x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)/(a^4*c^2*x^5 + 2*a^2*c^2*x^3 + c^2
*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{atan}\left(ax\right)}{x\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)/x/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(atan(a*x)/(x*(c*(a**2*x**2 + 1))**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\arctan\left(ax\right)}{\left(a^2cx^2+c\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)/((a^2*c*x^2 + c)^(3/2)*x), x)
```

3.237 $\int \frac{\tan^{-1}(ax)}{x^2(c+a^2cx^2)^{3/2}} dx$

Optimal. Leaf size=103

$$-\frac{\sqrt{a^2cx^2+c}\tan^{-1}(ax)}{c^2x}-\frac{a\tanh^{-1}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{c^{3/2}}-\frac{a}{c\sqrt{a^2cx^2+c}}-\frac{a^2x\tan^{-1}(ax)}{c\sqrt{a^2cx^2+c}}$$

```
[Out] -(a/(c*Sqrt[c + a^2*c*x^2])) - (a^2*x*ArcTan[a*x])/(c*Sqrt[c + a^2*c*x^2])
- (Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(c^2*x) - (a*ArcTanh[Sqrt[c + a^2*c*x^2
]/Sqrt[c]])/c^(3/2)
```

Rubi [A] time = 0.20189, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4966, 4944, 266, 63, 208, 4894}

$$-\frac{\sqrt{a^2cx^2+c}\tan^{-1}(ax)}{c^2x}-\frac{a\tanh^{-1}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{c^{3/2}}-\frac{a}{c\sqrt{a^2cx^2+c}}-\frac{a^2x\tan^{-1}(ax)}{c\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]/(x^2*(c + a^2*c*x^2)^(3/2)),x]
```

```
[Out] -(a/(c*Sqrt[c + a^2*c*x^2])) - (a^2*x*ArcTan[a*x])/(c*Sqrt[c + a^2*c*x^2])
- (Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(c^2*x) - (a*ArcTanh[Sqrt[c + a^2*c*x^2
]/Sqrt[c]])/c^(3/2)
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*
x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^p
, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*q]
 && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 4944

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a +
```

```
b*ArcTan[c*x])^p)/(d*f*(m + 1)), x] - Dist[(b*c*p)/(f*(m + 1)), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 4894

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[(x*(a + b*ArcTan[c*x]))/(d*Sq
rt[d + e*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)}{x^2\left(c+a^2cx^2\right)^{3/2}}\,dx = -\left(a^2\int \frac{\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^{3/2}}\,dx\right)+\frac{\int \frac{\tan^{-1}(ax)}{x^2\sqrt{c+a^2cx^2}}\,dx}{c}$$

$$= -\frac{a}{c\sqrt{c+a^2cx^2}}-\frac{a^2x\tan^{-1}(ax)}{c\sqrt{c+a^2cx^2}}-\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{c^2x}+\frac{a\int \frac{1}{x\sqrt{c+a^2cx^2}}\,dx}{c}$$

$$= -\frac{a}{c\sqrt{c+a^2cx^2}}-\frac{a^2x\tan^{-1}(ax)}{c\sqrt{c+a^2cx^2}}-\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{c^2x}+\frac{a\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{c+a^2cx}}\,dx,x,x^2\right)}{2c}$$

$$= -\frac{a}{c\sqrt{c+a^2cx^2}}-\frac{a^2x\tan^{-1}(ax)}{c\sqrt{c+a^2cx^2}}-\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{c^2x}+\frac{\mathrm{Subst}\left(\int \frac{1}{-\frac{1}{a^2}+\frac{x^2}{a^2c}}\,dx,x,\sqrt{c+a^2cx^2}\right.}{ac^2}$$

$$= -\frac{a}{c\sqrt{c+a^2cx^2}}-\frac{a^2x\tan^{-1}(ax)}{c\sqrt{c+a^2cx^2}}-\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{c^2x}-\frac{a\tanh^{-1}\left(\frac{\sqrt{c+a^2cx^2}}{\sqrt{c}}\right)}{c^{3/2}}$$

<u>Mathematica</u> [A] time = 0.181418, size = 122, normalized size = 1.18

$$-\frac{a\sqrt{c\left(a^2x^2+1\right)}}{c^2\left(a^2x^2+1\right)}-\frac{a\log\left(\sqrt{c}\sqrt{c\left(a^2x^2+1\right)}+c\right)}{c^{3/2}}-\frac{\left(2a^2x^2+1\right)\sqrt{c\left(a^2x^2+1\right)}\tan^{-1}(ax)}{c^2x\left(a^2x^2+1\right)}+\frac{a\log(x)}{c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]/(x^2*(c + a^2*c*x^2)^(3/2)),x]
```

```
[Out] -((a*Sqrt[c*(1 + a^2*x^2)])/(c^2*(1 + a^2*x^2))) - (Sqrt[c*(1 + a^2*x^2)]*(
1 + 2*a^2*x^2)*ArcTan[a*x])/(c^2*x*(1 + a^2*x^2)) + (a*Log[x])/c^(3/2) - (a
*Log[c + Sqrt[c]*Sqrt[c*(1 + a^2*x^2)]])/c^(3/2)
```

<u>Maple</u> [C] time = 0.295, size = 231, normalized size = 2.2

$$-\frac{a\left(\arctan\left(ax\right)+i\right)\left(ax-i\right)}{\left(2\,a^2x^2+2\right)c^2}\sqrt{c\left(ax-i\right)\left(ax+i\right)}-\frac{\left(ax+i\right)\left(\arctan\left(ax\right)-i\right)a}{\left(2\,a^2x^2+2\right)c^2}\sqrt{c\left(ax-i\right)\left(ax+i\right)}-\frac{\arctan\left(ax\right)}{c^2x}\sqrt{c\left(ax-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)/x^2/(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] -1/2*a*(arctan(a*x)+I)*(a*x-I)*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)/c^2-1/
2*(c*(a*x-I)*(a*x+I))^(1/2)*(a*x+I)*(arctan(a*x)-I)*a/(a^2*x^2+1)/c^2-arcta
n(a*x)*(c*(a*x-I)*(a*x+I))^(1/2)/x/c^2-a*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))/
(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(a*x+I))^(1/2)/c^2+a*ln((1+I*a*x)/(a^2*x^2+1)^
(1/2)-1)/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(a*x+I))^(1/2)/c^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x^2/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.69173, size = 234, normalized size = 2.27

$$\frac{\left(a^{3}x^{3}+ax\right)\sqrt{c}\log\left(-\frac{a^{2}cx^{2}-2\sqrt{a^{2}cx^{2}+c}\sqrt{c}+2c}{x^{2}}\right)-2\sqrt{a^{2}cx^{2}+c}\left(ax+\left(2a^{2}x^{2}+1\right)\arctan\left(ax\right)\right)}{2\left(a^{2}c^{2}x^{3}+c^{2}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x^2/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*((a^3*x^3 + a*x)*sqrt(c)*log(-(a^2*c*x^2 - 2*sqrt(a^2*c*x^2 + c)*sqrt(c
) + 2*c)/x^2) - 2*sqrt(a^2*c*x^2 + c)*(a*x + (2*a^2*x^2 + 1)*arctan(a*x)))/
(a^2*c^2*x^3 + c^2*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{atan}\left(ax\right)}{x^{2}\left(c\left(a^{2}x^{2}+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)/x**2/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(atan(a*x)/(x**2*(c*(a**2*x**2 + 1))**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(ax)}{\left(a^2cx^2+c\right)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x^2/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)/((a^2*c*x^2 + c)^(3/2)*x^2), x)
```

3.238 $\int \frac{\tan^{-1}(ax)}{x^3(c+a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=300

$$-\frac{3ia^2\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{2c\sqrt{a^2cx^2+c}}+\frac{3ia^2\sqrt{a^2x^2+1}\text{PolyLog}\left(2,\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{2c\sqrt{a^2cx^2+c}}-\frac{a\sqrt{a^2cx^2+c}}{2c^2x}-\frac{\sqrt{a^2cx^2+c}\tan^{-1}(ax)}{2c^2x^2}+\dots$$

```
[Out] (a^3*x)/(c*Sqrt[c + a^2*c*x^2]) - (a*Sqrt[c + a^2*c*x^2])/(2*c^2*x) - (a^2*
ArcTan[a*x])/(c*Sqrt[c + a^2*c*x^2]) - (Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(2
*c^2*x^2) + (3*a^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTanh[Sqrt[1 + I*a*x]/Sq
rt[1 - I*a*x]])/(c*Sqrt[c + a^2*c*x^2]) - (((3*I)/2)*a^2*Sqrt[1 + a^2*x^2]*
PolyLog[2, -(Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x])])/(c*Sqrt[c + a^2*c*x^2]) + (
((3*I)/2)*a^2*Sqrt[1 + a^2*x^2]*PolyLog[2, Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]]
)/(c*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.614256, antiderivative size = 300, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {4966, 4962, 264, 4958, 4954, 4930, 191}

$$-\frac{3ia^2\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{2c\sqrt{a^2cx^2+c}}+\frac{3ia^2\sqrt{a^2x^2+1}\text{PolyLog}\left(2,\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{2c\sqrt{a^2cx^2+c}}-\frac{a\sqrt{a^2cx^2+c}}{2c^2x}-\frac{\sqrt{a^2cx^2+c}\tan^{-1}(ax)}{2c^2x^2}+\dots$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]/(x^3*(c + a^2*c*x^2)^(3/2)),x]
```


```
[Out] (a^3*x)/(c*Sqrt[c + a^2*c*x^2]) - (a*Sqrt[c + a^2*c*x^2])/(2*c^2*x) - (a^2*
ArcTan[a*x])/(c*Sqrt[c + a^2*c*x^2]) - (Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(2
*c^2*x^2) + (3*a^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTanh[Sqrt[1 + I*a*x]/Sq
rt[1 - I*a*x]])/(c*Sqrt[c + a^2*c*x^2]) - (((3*I)/2)*a^2*Sqrt[1 + a^2*x^2]*
PolyLog[2, -(Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x])])/(c*Sqrt[c + a^2*c*x^2]) + (
((3*I)/2)*a^2*Sqrt[1 + a^2*x^2]*PolyLog[2, Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]]
)/(c*Sqrt[c + a^2*c*x^2])
```


Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*
x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^p
```

```
, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*q]
 && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 4962

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Ar
cTan[c*x])^p)/(d*f*(m + 1)), x] + (-Dist[(b*c*p)/(f*(m + 1)), Int[((f*x)^(m
 + 1)*(a + b*ArcTan[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] - Dist[(c^2*(m +
 2))/(f^2*(m + 1)), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/Sqrt[d + e*x^
2], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] &
& LtQ[m, -1] && NeQ[m, -2]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 4958

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4954

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x_
Symbol] :> Simp[(-2*(a + b*ArcTan[c*x])*ArcTanh[Sqrt[1 + I*c*x]/Sqrt[1 - I*
c*x]])/Sqrt[d], x] + (Simp[(I*b*PolyLog[2, -(Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x
])])/Sqrt[d], x] - Simp[(I*b*PolyLog[2, Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]])/S
qrt[d], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)}{x^3\left(c+a^2cx^2\right)^{3/2}}\,dx = -\left(a^2\int \frac{\tan^{-1}(ax)}{x\left(c+a^2cx^2\right)^{3/2}}\,dx\right)+\frac{\int \frac{\tan^{-1}(ax)}{x^3\sqrt{c+a^2cx^2}}\,dx}{c}$$

$$= -\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{2c^2x^2}+a^4\int \frac{x\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^{3/2}}\,dx+\frac{a\int \frac{1}{x^2\sqrt{c+a^2cx^2}}\,dx}{2c}-\frac{a^2\int \frac{\tan^{-1}(ax)}{x\sqrt{c+a^2cx^2}}\,dx}{2c}-\frac{a^2\int$$

$$= -\frac{a\sqrt{c+a^2cx^2}}{2c^2x}-\frac{a^2\tan^{-1}(ax)}{c\sqrt{c+a^2cx^2}}-\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{2c^2x^2}+a^3\int \frac{1}{\left(c+a^2cx^2\right)^{3/2}}\,dx-\frac{\left(a^2\sqrt{1+a^2x\right.}}{2c\sqrt{}}$$

$$= \frac{a^3x}{c\sqrt{c+a^2cx^2}}-\frac{a\sqrt{c+a^2cx^2}}{2c^2x}-\frac{a^2\tan^{-1}(ax)}{c\sqrt{c+a^2cx^2}}-\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{2c^2x^2}+\frac{3a^2\sqrt{1+a^2x^2}\tan^{-1}(ax}{c\sqrt{c+a^2}}$$

Mathematica [A] time = 1.23818, size = 258, normalized size = 0.86

$$-\frac{a^2\left(12i\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-e^{i\tan^{-1}(ax)}\right)-12i\sqrt{a^2x^2+1}\text{PolyLog}\left(2,e^{i\tan^{-1}(ax)}\right)+2\sqrt{a^2x^2+1}\tan\left(\frac{1}{2}\tan^{-1}(ax)\right)+12\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTan[a*x]/(x^3*(c + a^2*c*x^2)^(3/2)),x]
```

```
[Out] -(a^2*(-8*a*x + 8*ArcTan[a*x] + a*x*Csc[ArcTan[a*x]/2]^2 + Sqrt[1 + a^2*x^2
]*ArcTan[a*x]*Csc[ArcTan[a*x]/2]^2 + 12*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*Log[1
 - E^(I*ArcTan[a*x])] - 12*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*Log[1 + E^(I*ArcTa
n[a*x])] + (12*I)*Sqrt[1 + a^2*x^2]*PolyLog[2, -E^(I*ArcTan[a*x])] - (12*I)
*Sqrt[1 + a^2*x^2]*PolyLog[2, E^(I*ArcTan[a*x])] - Sqrt[1 + a^2*x^2]*ArcTan
[a*x]*Sec[ArcTan[a*x]/2]^2 + 2*Sqrt[1 + a^2*x^2]*Tan[ArcTan[a*x]/2]))/(8*c*
Sqrt[c + a^2*c*x^2])
```

Maple [A] time = 0.367, size = 273, normalized size = 0.9

$$-\frac{a^2(\arctan(ax)+i)(1+iax)}{(2a^2x^2+2)c^2}\sqrt{c(ax-i)(ax+i)}+\frac{(-1+iax)(\arctan(ax)-i)a^2}{(2a^2x^2+2)c^2}\sqrt{c(ax-i)(ax+i)}-\frac{ax+\arctan(a}{2c^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)/x^3/(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] -1/2*a^2*(arctan(a*x)+I)*(1+I*a*x)*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)/c^
2+1/2*(c*(a*x-I)*(a*x+I))^(1/2)*(-1+I*a*x)*(arctan(a*x)-I)*a^2/(a^2*x^2+1)/
c^2-1/2*(a*x+arctan(a*x))*(c*(a*x-I)*(a*x+I))^(1/2)/c^2/x^2+3/2*I*a^2*(I*ar
ctan(a*x)*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*arctan(a*x)*ln(1+(1+I*a*x)/(a
^2*x^2+1)^(1/2))+polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))-polylog(2,-(1+I*a*x
)/(a^2*x^2+1)^(1/2)))/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(a*x+I))^(1/2)/c^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x^3/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{a^4c^2x^7+2a^2c^2x^5+c^2x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x^3/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)/(a^4*c^2*x^7 + 2*a^2*c^2*x^5 + c^2
*x^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atan}(ax)}{x^3 \left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)/x**3/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(atan(a*x)/(x**3*(c*(a**2*x**2 + 1))**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(ax)}{\left(a^2cx^2+c\right)^{\frac{3}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x^3/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)/((a^2*c*x^2 + c)^(3/2)*x^3), x)
```

3.239 $\int \frac{\tan^{-1}(ax)}{x^4\left(c+a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=165

$$-\frac{a\sqrt{a^2cx^2+c}}{6c^2x^2}+\frac{5a^2\sqrt{a^2cx^2+c}\,\tan^{-1}(ax)}{3c^2x}-\frac{\sqrt{a^2cx^2+c}\,\tan^{-1}(ax)}{3c^2x^3}+\frac{11a^3\tanh^{-1}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{6c^{3/2}}+\frac{a^3}{c\sqrt{a^2cx^2+c}}+\frac{a^4x\tan}{c\sqrt{a^2c}}$$

```
[Out] a^3/(c*Sqrt[c + a^2*c*x^2]) - (a*Sqrt[c + a^2*c*x^2])/(6*c^2*x^2) + (a^4*x*
ArcTan[a*x])/(c*Sqrt[c + a^2*c*x^2]) - (Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(3
*c^2*x^3) + (5*a^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(3*c^2*x) + (11*a^3*Arc
Tanh[Sqrt[c + a^2*c*x^2]/Sqrt[c]])/(6*c^(3/2))
```

Rubi [A] time = 0.494693, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {4966, 4962, 266, 51, 63, 208, 4944, 4894}

$$-\frac{a\sqrt{a^2cx^2+c}}{6c^2x^2}+\frac{5a^2\sqrt{a^2cx^2+c}\,\tan^{-1}(ax)}{3c^2x}-\frac{\sqrt{a^2cx^2+c}\,\tan^{-1}(ax)}{3c^2x^3}+\frac{11a^3\tanh^{-1}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{6c^{3/2}}+\frac{a^3}{c\sqrt{a^2cx^2+c}}+\frac{a^4x\tan}{c\sqrt{a^2c}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]/(x^4*(c + a^2*c*x^2)^(3/2)),x]
```

```
[Out] a^3/(c*Sqrt[c + a^2*c*x^2]) - (a*Sqrt[c + a^2*c*x^2])/(6*c^2*x^2) + (a^4*x*
ArcTan[a*x])/(c*Sqrt[c + a^2*c*x^2]) - (Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(3
*c^2*x^3) + (5*a^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(3*c^2*x) + (11*a^3*Arc
Tanh[Sqrt[c + a^2*c*x^2]/Sqrt[c]])/(6*c^(3/2))
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*
x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^p
, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*q]
 && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 4962

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Ar
cTan[c*x])^p)/(d*f*(m + 1)), x] + (-Dist[(b*c*p)/(f*(m + 1)), Int[((f*x)^(m
 + 1)*(a + b*ArcTan[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] - Dist[(c^2*(m +
 2))/(f^2*(m + 1)), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/Sqrt[d + e*x^
2], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] &
& LtQ[m, -1] && NeQ[m, -2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 4944

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a +
 b*ArcTan[c*x])^p)/(d*f*(m + 1)), x] - Dist[(b*c*p)/(f*(m + 1)), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rule 4894

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[(x*(a + b*ArcTan[c*x]))/(d*Sq
rt[d + e*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)}{x^4\left(c+a^2cx^2\right)^{3/2}}\,dx = -\left(a^2\int \frac{\tan^{-1}(ax)}{x^2\left(c+a^2cx^2\right)^{3/2}}\,dx\right)+\frac{\int \frac{\tan^{-1}(ax)}{x^4\sqrt{c+a^2cx^2}}\,dx}{c}$$

$$= -\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{3c^2x^3}+a^4\int \frac{\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^{3/2}}\,dx+\frac{a\int \frac{1}{x^3\sqrt{c+a^2cx^2}}\,dx}{3c}-\frac{\left(2a^2\right)\int \frac{\tan^{-1}(ax)}{x^2\sqrt{c+a^2cx^2}}\,dx}{3c}$$

$$= \frac{a^3}{c\sqrt{c+a^2cx^2}}+\frac{a^4x\tan^{-1}(ax)}{c\sqrt{c+a^2cx^2}}-\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{3c^2x^3}+\frac{5a^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{3c^2x}+\frac{a\,\mathrm{Subst}}{}$$

$$= \frac{a^3}{c\sqrt{c+a^2cx^2}}-\frac{a\sqrt{c+a^2cx^2}}{6c^2x^2}+\frac{a^4x\tan^{-1}(ax)}{c\sqrt{c+a^2cx^2}}-\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{3c^2x^3}+\frac{5a^2\sqrt{c+a^2cx^2}\tan^{-1}}{3c^2x}$$

$$= \frac{a^3}{c\sqrt{c+a^2cx^2}}-\frac{a\sqrt{c+a^2cx^2}}{6c^2x^2}+\frac{a^4x\tan^{-1}(ax)}{c\sqrt{c+a^2cx^2}}-\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{3c^2x^3}+\frac{5a^2\sqrt{c+a^2cx^2}\tan^{-1}}{3c^2x}$$

$$= \frac{a^3}{c\sqrt{c+a^2cx^2}}-\frac{a\sqrt{c+a^2cx^2}}{6c^2x^2}+\frac{a^4x\tan^{-1}(ax)}{c\sqrt{c+a^2cx^2}}-\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{3c^2x^3}+\frac{5a^2\sqrt{c+a^2cx^2}\tan^{-1}}{3c^2x}$$

Mathematica [A] time = 0.32687, size = 143, normalized size = 0.87

$$\frac{\frac{a\left(5a^2x^2-1\right)\sqrt{a^2cx^2+c}}{a^2x^4+x^2}+11a^3\sqrt{c}\log\left(\sqrt{c}\sqrt{a^2cx^2+c}+c\right)+\frac{2\left(8a^4x^4+4a^2x^2-1\right)\sqrt{a^2cx^2+c}\tan^{-1}(ax)}{a^2x^5+x^3}-11a^3\sqrt{c}\log(x)}{6c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]/(x^4*(c + a^2*c*x^2)^(3/2)),x]
```

```
[Out] ((a*(-1 + 5*a^2*x^2)*Sqrt[c + a^2*c*x^2])/(x^2 + a^2*x^4) + (2*Sqrt[c + a^2
*c*x^2]*(-1 + 4*a^2*x^2 + 8*a^4*x^4)*ArcTan[a*x])/(x^3 + a^2*x^5) - 11*a^3*
Sqrt[c]*Log[x] + 11*a^3*Sqrt[c]*Log[c + Sqrt[c]*Sqrt[c + a^2*c*x^2]])/(6*c^
```

2)

Maple [C] time = 0.655, size = 259, normalized size = 1.6

$$\frac{a^{3} \left(\arctan \left(a x\right)+i\right) \left(a x-i\right)}{\left(2 a^{2} x^{2}+2\right) c^{2}}\sqrt{c \left(a x-i\right) \left(a x+i\right)}+\frac{\left(a x+i\right) \left(\arctan \left(a x\right)-i\right) a^{3}}{\left(2 a^{2} x^{2}+2\right) c^{2}}\sqrt{c \left(a x-i\right) \left(a x+i\right)}+\frac{10 \arctan \left(a x\right) a^{2} x^{2}-}{6 c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)/x^4/(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] 1/2*a^3*(arctan(a*x)+I)*(a*x-I)*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)/c^2+1
/2*(c*(a*x-I)*(a*x+I))^(1/2)*(a*x+I)*(arctan(a*x)-I)*a^3/(a^2*x^2+1)/c^2+1/
6*(10*arctan(a*x)*a^2*x^2-a*x-2*arctan(a*x))*(c*(a*x-I)*(a*x+I))^(1/2)/x^3/
c^2-11/6*a^3*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)-1)/(a^2*x^2+1)^(1/2)*(c*(a*x-I)
*(a*x+I))^(1/2)/c^2+11/6*a^3*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*x^2+1)^
(1/2)*(c*(a*x-I)*(a*x+I))^(1/2)/c^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x^4/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.55897, size = 282, normalized size = 1.71

$$\frac{11 \left(a^{5} x^{5}+a^{3} x^{3}\right) \sqrt{c} \log \left(-\frac{a^{2} c x^{2}+2 \sqrt{a^{2} c x^{2}+c} \sqrt{c}+2 c}{x^{2}}\right)+2 \left(5 a^{3} x^{3}-a x+2 \left(8 a^{4} x^{4}+4 a^{2} x^{2}-1\right) \arctan \left(a x\right)\right) \sqrt{a^{2} c x^{2}+c}}{12 \left(a^{2} c^{2} x^{5}+c^{2} x^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x^4/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/12*(11*(a^5*x^5 + a^3*x^3)*sqrt(c)*log(-(a^2*c*x^2 + 2*sqrt(a^2*c*x^2 + c
)*sqrt(c) + 2*c)/x^2) + 2*(5*a^3*x^3 - a*x + 2*(8*a^4*x^4 + 4*a^2*x^2 - 1)*
arctan(a*x))*sqrt(a^2*c*x^2 + c))/(a^2*c^2*x^5 + c^2*x^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atan}(ax)}{x^4 \left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)/x**4/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(atan(a*x)/(x**4*(c*(a**2*x**2 + 1))**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(ax)}{\left(a^2cx^2+c\right)^{\frac{3}{2}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x^4/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)/((a^2*c*x^2 + c)^(3/2)*x^4), x)
```

3.240 $\int \frac{x^5 \tan^{-1}(ax)}{(c+a^2cx^2)^{5/2}} dx$

Optimal. Leaf size=170

$$-\frac{5x}{3a^5c^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{a^2cx^2+c}\tan^{-1}(ax)}{a^6c^3}+\frac{5\tan^{-1}(ax)}{3a^6c^2\sqrt{a^2cx^2+c}}-\frac{\tanh^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a^6c^{5/2}}-\frac{x^3}{9a^3c\left(a^2cx^2+c\right)^{3/2}}+\frac{x^2\tan^{-1}}{3a^4c\left(a^2cx^2\right.}$$

```
[Out] -x^3/(9*a^3*c*(c + a^2*c*x^2)^(3/2)) - (5*x)/(3*a^5*c^2*Sqrt[c + a^2*c*x^2]
) + (x^2*ArcTan[a*x])/(3*a^4*c*(c + a^2*c*x^2)^(3/2)) + (5*ArcTan[a*x])/(3*
a^6*c^2*Sqrt[c + a^2*c*x^2]) + (Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(a^6*c^3)
- ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]]/(a^6*c^(5/2))
```

Rubi [A] time = 0.433126, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4964, 4930, 217, 206, 191, 4938}

$$-\frac{5x}{3a^5c^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{a^2cx^2+c}\tan^{-1}(ax)}{a^6c^3}+\frac{5\tan^{-1}(ax)}{3a^6c^2\sqrt{a^2cx^2+c}}-\frac{\tanh^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a^6c^{5/2}}-\frac{x^3}{9a^3c\left(a^2cx^2+c\right)^{3/2}}+\frac{x^2\tan^{-1}}{3a^4c\left(a^2cx^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*ArcTan[a*x])/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] -x^3/(9*a^3*c*(c + a^2*c*x^2)^(3/2)) - (5*x)/(3*a^5*c^2*Sqrt[c + a^2*c*x^2]
) + (x^2*ArcTan[a*x])/(3*a^4*c*(c + a^2*c*x^2)^(3/2)) + (5*ArcTan[a*x])/(3*
a^6*c^2*Sqrt[c + a^2*c*x^2]) + (Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(a^6*c^3)
- ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]]/(a^6*c^(5/2))
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Arc
Tan[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan[c
*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p
, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 4938

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[(b*(f*x)^m*(d + e*x^2)^(q + 1))/(c*d*m^2), x] +
 (Dist[(f^2*(m - 1))/(c^2*d*m), Int[(f*x)^(m - 2)*(d + e*x^2)^(q + 1)*(a +
b*ArcTan[c*x]), x], x] - Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(q + 1)*(a + b*A
rcTan[c*x]))/(c^2*d*m), x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d
] && EqQ[m + 2*q + 2, 0] && LtQ[q, -1]
```

Rubi steps

$$\int \frac{x^5 \tan^{-1}(ax)}{\left(c+a^2cx^2\right)^{5/2}}\,dx = -\frac{\int \frac{x^3 \tan^{-1}(ax)}{\left(c+a^2cx^2\right)^{5/2}}\,dx}{a^2} + \frac{\int \frac{x^3 \tan^{-1}(ax)}{\left(c+a^2cx^2\right)^{3/2}}\,dx}{a^2c}$$

$$= -\frac{x^3}{9a^3c\left(c+a^2cx^2\right)^{3/2}} + \frac{x^2\tan^{-1}(ax)}{3a^4c\left(c+a^2cx^2\right)^{3/2}} + \frac{\int \frac{x\tan^{-1}(ax)}{\sqrt{c+a^2cx^2}}\,dx}{a^4c^2} - \frac{2\int \frac{x\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^{3/2}}\,dx}{3a^4c} - \frac{\int \frac{x\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^{3/2}}\,dx}{a^4c}$$

$$= -\frac{x^3}{9a^3c\left(c+a^2cx^2\right)^{3/2}} + \frac{x^2\tan^{-1}(ax)}{3a^4c\left(c+a^2cx^2\right)^{3/2}} + \frac{5\tan^{-1}(ax)}{3a^6c^2\sqrt{c+a^2cx^2}} + \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{a^6c^3} - \frac{\int \frac{1}{\sqrt{c+a^2}}}{a^5c}$$

$$= -\frac{x^3}{9a^3c\left(c+a^2cx^2\right)^{3/2}} - \frac{5x}{3a^5c^2\sqrt{c+a^2cx^2}} + \frac{x^2\tan^{-1}(ax)}{3a^4c\left(c+a^2cx^2\right)^{3/2}} + \frac{5\tan^{-1}(ax)}{3a^6c^2\sqrt{c+a^2cx^2}} + \frac{\sqrt{c+a^2cx^2}\,t}{a^6c^3}$$

$$= -\frac{x^3}{9a^3c\left(c+a^2cx^2\right)^{3/2}} - \frac{5x}{3a^5c^2\sqrt{c+a^2cx^2}} + \frac{x^2\tan^{-1}(ax)}{3a^4c\left(c+a^2cx^2\right)^{3/2}} + \frac{5\tan^{-1}(ax)}{3a^6c^2\sqrt{c+a^2cx^2}} + \frac{\sqrt{c+a^2cx^2}\,t}{a^6c^3}$$

Mathematica [A] time = 0.202126, size = 131, normalized size = 0.77

$$-\frac{ax\left(16a^2x^2+15\right)\sqrt{a^2cx^2+c}+9\sqrt{c}\left(a^2x^2+1\right)^2\log\left(\sqrt{c}\sqrt{a^2cx^2+c}+acx\right)-3\left(3a^4x^4+12a^2x^2+8\right)\sqrt{a^2cx^2+c}\tan^{-1}(a}{9a^6c^3\left(a^2x^2+1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*ArcTan[a*x])/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] -(a*x*(15 + 16*a^2*x^2)*Sqrt[c + a^2*c*x^2] - 3*Sqrt[c + a^2*c*x^2]*(8 + 12
*a^2*x^2 + 3*a^4*x^4)*ArcTan[a*x] + 9*Sqrt[c]*(1 + a^2*x^2)^2*Log[a*c*x + S
qrt[c]*Sqrt[c + a^2*c*x^2]])/(9*a^6*c^3*(1 + a^2*x^2)^2)
```

Maple [C] time = 1.507, size = 386, normalized size = 2.3

$$\frac{(i+3\arctan(ax))\left(ix^3a^3+3a^2x^2-3iax-1\right)}{72\left(a^2x^2+1\right)^2c^3a^6}\sqrt{c(ax-i)(ax+i)} + \frac{(7\arctan(ax)+7i)(1+iax)}{8c^3a^6\left(a^2x^2+1\right)}\sqrt{c(ax-i)(ax+i)} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*arctan(a*x)/(a^2*c*x^2+c)^(5/2),x)
```

```
[Out] 1/72*(I+3*arctan(a*x))*(I*x^3*a^3+3*a^2*x^2-3*I*a*x-1)*(c*(a*x-I)*(a*x+I))^
(1/2)/(a^2*x^2+1)^2/c^3/a^6+7/8*(arctan(a*x)+I)*(1+I*a*x)*(c*(a*x-I)*(a*x+I
))^(1/2)/a^6/c^3/(a^2*x^2+1)-7/8*(c*(a*x-I)*(a*x+I))^(1/2)*(-1+I*a*x)*(arct
an(a*x)-I)/a^6/c^3/(a^2*x^2+1)-1/72*(c*(a*x-I)*(a*x+I))^(1/2)*(I*x^3*a^3-3*
a^2*x^2-3*I*a*x+1)*(-I+3*arctan(a*x))/a^6/c^3/(a^4*x^4+2*a^2*x^2+1)+arctan(
a*x)*(c*(a*x-I)*(a*x+I))^(1/2)/c^3/a^6+ln((1+I*a*x)/(a^2*x^2+1)^(1/2)-I)/(a
^2*x^2+1)^(1/2)*(c*(a*x-I)*(a*x+I))^(1/2)/a^6/c^3-ln((1+I*a*x)/(a^2*x^2+1)^
(1/2)+I)/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(a*x+I))^(1/2)/a^6/c^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arctan(a*x)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.49651, size = 316, normalized size = 1.86

$$\frac{9\left(a^4x^4+2a^2x^2+1\right)\sqrt{c}\log\left(-2a^2cx^2+2\sqrt{a^2cx^2+ca}\sqrt{c}x-c\right)-2\left(16a^3x^3+15ax-3\left(3a^4x^4+12a^2x^2+8\right)\arctan(}{18\left(a^{10}c^3x^4+2a^8c^3x^2+a^6c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arctan(a*x)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/18*(9*(a^4*x^4 + 2*a^2*x^2 + 1)*sqrt(c)*log(-2*a^2*c*x^2 + 2*sqrt(a^2*c*x
^2 + c)*a*sqrt(c)*x - c) - 2*(16*a^3*x^3 + 15*a*x - 3*(3*a^4*x^4 + 12*a^2*x
^2 + 8)*arctan(a*x))*sqrt(a^2*c*x^2 + c))/(a^10*c^3*x^4 + 2*a^8*c^3*x^2 + a
^6*c^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5 \operatorname{atan}(ax)}{\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*atan(a*x)/(a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral(x**5*atan(a*x)/(c*(a**2*x**2 + 1))**(5/2), x)
```

Giac [A] time = 1.24713, size = 177, normalized size = 1.04

$$-\frac{x\left(\frac{16\,x^2}{a^3c}+\frac{15}{a^5c}\right)}{9\left(a^2cx^2+c\right)^{\frac{3}{2}}}+\frac{\left(3\sqrt{a^2cx^2+c}+\frac{6\left(a^2cx^2+c\right)c-c^2}{\left(a^2cx^2+c\right)^{\frac{3}{2}}}\right)\arctan(ax)}{3\,a^6c^3}+\frac{\log\left(\left|-\sqrt{a^2c}x+\sqrt{a^2cx^2+c}\right|\right)}{a^5c^{\frac{5}{2}}|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arctan(a*x)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] -1/9*x*(16*x^2/(a^3*c) + 15/(a^5*c))/(a^2*c*x^2 + c)^(3/2) + 1/3*(3*sqrt(a^
2*c*x^2 + c) + (6*(a^2*c*x^2 + c)*c - c^2)/(a^2*c*x^2 + c)^(3/2))*arctan(a*
x)/(a^6*c^3) + log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 + c)))/(a^5*c^(5/2)*
abs(a))
```

3.241 $\int \frac{x^4 \tan^{-1}(ax)}{\left(c+a^2cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=308

$$\frac{i\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a^5c^2\sqrt{a^2cx^2+c}}-\frac{i\sqrt{a^2x^2+1}\text{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a^5c^2\sqrt{a^2cx^2+c}}-\frac{4}{3a^5c^2\sqrt{a^2cx^2+c}}-\frac{x\tan^{-1}(ax)}{a^4c^2\sqrt{a^2cx^2+c}}-2i\sqrt{a^2x^2}$$

```
[Out] 1/(9*a^5*c*(c + a^2*c*x^2)^(3/2)) - 4/(3*a^5*c^2*Sqrt[c + a^2*c*x^2]) - (x^
3*ArcTan[a*x])/(3*a^2*c*(c + a^2*c*x^2)^(3/2)) - (x*ArcTan[a*x])/(a^4*c^2*S
qrt[c + a^2*c*x^2]) - ((2*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTan[Sqrt[1 +
I*a*x]/Sqrt[1 - I*a*x]])/(a^5*c^2*Sqrt[c + a^2*c*x^2]) + (I*Sqrt[1 + a^2*x^
2]*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/(a^5*c^2*Sqrt[c + a^
2*c*x^2]) - (I*Sqrt[1 + a^2*x^2]*PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*
a*x]])/(a^5*c^2*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.36996, antiderivative size = 308, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {4964, 4934, 4890, 4886, 4944, 266, 43}

$$\frac{i\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a^5c^2\sqrt{a^2cx^2+c}}-\frac{i\sqrt{a^2x^2+1}\text{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a^5c^2\sqrt{a^2cx^2+c}}-\frac{4}{3a^5c^2\sqrt{a^2cx^2+c}}-\frac{x\tan^{-1}(ax)}{a^4c^2\sqrt{a^2cx^2+c}}-2i\sqrt{a^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*ArcTan[a*x])/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] 1/(9*a^5*c*(c + a^2*c*x^2)^(3/2)) - 4/(3*a^5*c^2*Sqrt[c + a^2*c*x^2]) - (x^
3*ArcTan[a*x])/(3*a^2*c*(c + a^2*c*x^2)^(3/2)) - (x*ArcTan[a*x])/(a^4*c^2*S
qrt[c + a^2*c*x^2]) - ((2*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTan[Sqrt[1 +
I*a*x]/Sqrt[1 - I*a*x]])/(a^5*c^2*Sqrt[c + a^2*c*x^2]) + (I*Sqrt[1 + a^2*x^
2]*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/(a^5*c^2*Sqrt[c + a^
2*c*x^2]) - (I*Sqrt[1 + a^2*x^2]*PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*
a*x]])/(a^5*c^2*Sqrt[c + a^2*c*x^2])
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Arc
```

```
Tan[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan[c
*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p
, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rule 4934

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^2*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> -Simp[(b*(d + e*x^2)^(q + 1))/(4*c^3*d*(q + 1)^2), x] + (-Dist[
1/(2*c^2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]), x], x] +
Simp[(x*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]))/(2*c^2*d*(q + 1)), x]) /;
FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -5/2]
```

Rule 4890

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4886

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[(-2*I*(a + b*ArcTan[c*x])*ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]])
/(c*Sqrt[d]), x] + (Simp[(I*b*PolyLog[2, -((I*Sqrt[1 + I*c*x])/Sqrt[1 - I*c
*x])])/(c*Sqrt[d]), x] - Simp[(I*b*PolyLog[2, (I*Sqrt[1 + I*c*x])/Sqrt[1 -
I*c*x]])/(c*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

Rule 4944

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a +
 b*ArcTan[c*x])^p)/(d*f*(m + 1)), x] - Dist[(b*c*p)/(f*(m + 1)), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^4 \tan^{-1}(ax)}{\left(c+a^2cx^2\right)^{5/2}}\,dx = -\frac{\int \frac{x^2\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^{5/2}}\,dx}{a^2}+\frac{\int \frac{x^2\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^{3/2}}\,dx}{a^2c}$$

$$= -\frac{1}{a^5c^2\sqrt{c+a^2cx^2}}-\frac{x^3\tan^{-1}(ax)}{3a^2c\left(c+a^2cx^2\right)^{3/2}}-\frac{x\tan^{-1}(ax)}{a^4c^2\sqrt{c+a^2cx^2}}+\frac{\int \frac{x^3}{\left(c+a^2cx^2\right)^{5/2}}\,dx}{3a}+\frac{\int \frac{\tan^{-1}(ax)}{\sqrt{c+a^2cx^2}}\,dx}{a^4c^2}$$

$$= -\frac{1}{a^5c^2\sqrt{c+a^2cx^2}}-\frac{x^3\tan^{-1}(ax)}{3a^2c\left(c+a^2cx^2\right)^{3/2}}-\frac{x\tan^{-1}(ax)}{a^4c^2\sqrt{c+a^2cx^2}}+\frac{\text{Subst}\left(\int \frac{x}{\left(c+a^2cx\right)^{5/2}}\,dx,x,x^2\right)}{6a}+ \frac{\sqrt{1}\ldots}{\ldots}$$

$$= -\frac{1}{a^5c^2\sqrt{c+a^2cx^2}}-\frac{x^3\tan^{-1}(ax)}{3a^2c\left(c+a^2cx^2\right)^{3/2}}-\frac{x\tan^{-1}(ax)}{a^4c^2\sqrt{c+a^2cx^2}}-\frac{2i\sqrt{1+a^2x^2}\tan^{-1}(ax)\tan^{-1}\left(\frac{\sqrt{1+ia\ldots}}{\sqrt{1-ia\ldots}}\right.\ldots}{a^5c^2\sqrt{c+a^2cx^2}}$$

$$= \frac{1}{9a^5c\left(c+a^2cx^2\right)^{3/2}}-\frac{4}{3a^5c^2\sqrt{c+a^2cx^2}}-\frac{x^3\tan^{-1}(ax)}{3a^2c\left(c+a^2cx^2\right)^{3/2}}-\frac{x\tan^{-1}(ax)}{a^4c^2\sqrt{c+a^2cx^2}}-\frac{2i\sqrt{1+a^2x^2}\,\text{ta}\ldots}{a^5c\ldots}$$

Mathematica [A] time = 0.386175, size = 177, normalized size = 0.57

$$\frac{\sqrt{c\left(a^2x^2+1\right)}\left(36i\left(\text{PolyLog}\left(2,-ie^{i\tan^{-1}(ax)}\right)-\text{PolyLog}\left(2,ie^{i\tan^{-1}(ax)}\right)\right)-\frac{45}{\sqrt{a^2x^2+1}}-\frac{45ax\tan^{-1}(ax)}{\sqrt{a^2x^2+1}}+36\tan^{-1}(ax)\left(\log\ldots\right.\right.}{36a^5c^3\sqrt{a^2x^2+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*ArcTan[a*x])/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] (Sqrt[c*(1 + a^2*x^2)]*(-45/Sqrt[1 + a^2*x^2] - (45*a*x*ArcTan[a*x])/Sqrt[1
 + a^2*x^2] + Cos[3*ArcTan[a*x]] + 36*ArcTan[a*x]*(Log[1 - I*E^(I*ArcTan[a*
x])] - Log[1 + I*E^(I*ArcTan[a*x])]) + (36*I)*(PolyLog[2, (-I)*E^(I*ArcTan[
a*x])] - PolyLog[2, I*E^(I*ArcTan[a*x])]) + 3*ArcTan[a*x]*Sin[3*ArcTan[a*x]
```

```
]))/(36*a^5*c^3*Sqrt[1 + a^2*x^2])
```

Maple [A] time = 0.756, size = 389, normalized size = 1.3

$$-\frac{(i+3\arctan(ax))\left(a^3x^3-3\,ia^2x^2-3\,ax+i\right)}{72\left(a^2x^2+1\right)^2c^3a^5}\sqrt{c\,(ax-i)\,(ax+i)}-\frac{(5\arctan(ax)+5\,i)\,(ax-i)}{8\,c^3a^5\left(a^2x^2+1\right)}\sqrt{c\,(ax-i)\,(ax+i)}-\frac{(5}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arctan(a*x)/(a^2*c*x^2+c)^(5/2),x)
```

```
[Out] -1/72*(I+3*arctan(a*x))*(a^3*x^3-3*I*a^2*x^2-3*a*x+I)*(c*(a*x-I)*(a*x+I))^(
1/2)/(a^2*x^2+1)^2/c^3/a^5-5/8*(arctan(a*x)+I)*(a*x-I)*(c*(a*x-I)*(a*x+I))^
(1/2)/a^5/c^3/(a^2*x^2+1)-5/8*(c*(a*x-I)*(a*x+I))^(1/2)*(a*x+I)*(arctan(a*x
)-I)/a^5/c^3/(a^2*x^2+1)-1/72*(-I+3*arctan(a*x))*(c*(a*x-I)*(a*x+I))^(1/2)*
(a^3*x^3+3*I*a^2*x^2-3*a*x-I)/(a^4*x^4+2*a^2*x^2+1)/c^3/a^5+I*(I*arctan(a*x
)*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*arctan(a*x)*ln(1-I*(1+I*a*x)/(a^2*x
^2+1)^(1/2))+dilog(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-dilog(1-I*(1+I*a*x)/(a^
2*x^2+1)^(1/2)))*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)^(1/2)/a^5/c^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctan(a*x)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a^2cx^2+c}\,x^4\arctan(ax)}{a^6c^3x^6+3\,a^4c^3x^4+3\,a^2c^3x^2+c^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctan(a*x)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^4*arctan(a*x)/(a^6*c^3*x^6 + 3*a^4*c^3*x^4 +
 3*a^2*c^3*x^2 + c^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4 \operatorname{atan}(ax)}{\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*atan(a*x)/(a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral(x**4*atan(a*x)/(c*(a**2*x**2 + 1))**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4 \arctan(ax)}{\left(a^2cx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctan(a*x)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x^4*arctan(a*x)/(a^2*c*x^2 + c)^(5/2), x)
```

3.242 $\int \frac{x^3 \tan^{-1}(ax)}{(c+a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=112

$$\frac{2x}{3a^3c^2\sqrt{a^2cx^2+c}}-\frac{2\tan^{-1}(ax)}{3a^4c^2\sqrt{a^2cx^2+c}}+\frac{x^3}{9ac\left(a^2cx^2+c\right)^{3/2}}-\frac{x^2\tan^{-1}(ax)}{3a^2c\left(a^2cx^2+c\right)^{3/2}}$$

```
[Out] x^3/(9*a*c*(c + a^2*c*x^2)^(3/2)) + (2*x)/(3*a^3*c^2*Sqrt[c + a^2*c*x^2]) -
 (x^2*ArcTan[a*x])/(3*a^2*c*(c + a^2*c*x^2)^(3/2)) - (2*ArcTan[a*x])/(3*a^4
*c^2*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.137188, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {4938, 4930, 191}

$$\frac{2x}{3a^3c^2\sqrt{a^2cx^2+c}}-\frac{2\tan^{-1}(ax)}{3a^4c^2\sqrt{a^2cx^2+c}}+\frac{x^3}{9ac\left(a^2cx^2+c\right)^{3/2}}-\frac{x^2\tan^{-1}(ax)}{3a^2c\left(a^2cx^2+c\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcTan[a*x])/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] x^3/(9*a*c*(c + a^2*c*x^2)^(3/2)) + (2*x)/(3*a^3*c^2*Sqrt[c + a^2*c*x^2]) -
 (x^2*ArcTan[a*x])/(3*a^2*c*(c + a^2*c*x^2)^(3/2)) - (2*ArcTan[a*x])/(3*a^4
*c^2*Sqrt[c + a^2*c*x^2])
```

Rule 4938

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[(b*(f*x)^m*(d + e*x^2)^(q + 1))/(c*d*m^2), x] +
 (Dist[(f^2*(m - 1))/(c^2*d*m), Int[(f*x)^(m - 2)*(d + e*x^2)^(q + 1)*(a +
b*ArcTan[c*x]), x], x] - Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(q + 1)*(a + b*A
rcTan[c*x]))/(c^2*d*m), x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d
] && EqQ[m + 2*q + 2, 0] && LtQ[q, -1]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
```

```
1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^{5/2}}\,dx &= \frac{x^3}{9ac\left(c+a^2cx^2\right)^{3/2}}-\frac{x^2\tan^{-1}(ax)}{3a^2c\left(c+a^2cx^2\right)^{3/2}}+\frac{2\int \frac{x\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^{3/2}}\,dx}{3a^2c}\\
&= \frac{x^3}{9ac\left(c+a^2cx^2\right)^{3/2}}-\frac{x^2\tan^{-1}(ax)}{3a^2c\left(c+a^2cx^2\right)^{3/2}}-\frac{2\tan^{-1}(ax)}{3a^4c^2\sqrt{c+a^2cx^2}}+\frac{2\int \frac{1}{\left(c+a^2cx^2\right)^{3/2}}\,dx}{3a^3c}\\
&= \frac{x^3}{9ac\left(c+a^2cx^2\right)^{3/2}}+\frac{2x}{3a^3c^2\sqrt{c+a^2cx^2}}-\frac{x^2\tan^{-1}(ax)}{3a^2c\left(c+a^2cx^2\right)^{3/2}}-\frac{2\tan^{-1}(ax)}{3a^4c^2\sqrt{c+a^2cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.0792966, size = 65, normalized size = 0.58

$$\frac{\sqrt{a^2cx^2+c}\left(ax\left(7a^2x^2+6\right)-3\left(3a^2x^2+2\right)\tan^{-1}(ax)\right)}{9a^4c^3\left(a^2x^2+1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*ArcTan[a*x])/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] (Sqrt[c + a^2*c*x^2]*(a*x*(6 + 7*a^2*x^2) - 3*(2 + 3*a^2*x^2)*ArcTan[a*x]))
/(9*a^4*c^3*(1 + a^2*x^2)^2)
```

Maple [C] time = 0.971, size = 244, normalized size = 2.2

$$-\frac{(i+3\arctan(ax))\left(ix^3a^3+3a^2x^2-3iax-1\right)}{72\left(a^2x^2+1\right)^2c^3a^4}\sqrt{c(ax-i)(ax+i)}-\frac{(3\arctan(ax)+3i)(1+iax)}{8c^3a^4\left(a^2x^2+1\right)}\sqrt{c(ax-i)(ax+i)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctan(a*x)/(a^2*c*x^2+c)^(5/2),x)
```

```
[Out] -1/72*(I+3*arctan(a*x))*(I*x^3*a^3+3*a^2*x^2-3*I*a*x-1)*(c*(a*x-I)*(a*x+I))
^(1/2)/(a^2*x^2+1)^2/c^3/a^4-3/8*(arctan(a*x)+I)*(1+I*a*x)*(c*(a*x-I)*(a*x+
I))^(1/2)/a^4/c^3/(a^2*x^2+1)+3/8*(c*(a*x-I)*(a*x+I))^(1/2)*(-1+I*a*x)*(arc
tan(a*x)-I)/a^4/c^3/(a^2*x^2+1)+1/72*(c*(a*x-I)*(a*x+I))^(1/2)*(I*x^3*a^3-3
*a^2*x^2-3*I*a*x+1)*(-I+3*arctan(a*x))/a^4/c^3/(a^4*x^4+2*a^2*x^2+1)
```

Maxima [A] time = 1.39263, size = 88, normalized size = 0.79

$$\frac{7\,a^3x^3 + 6\,ax - 3\left(3\,a^2x^2 + 2\right)\arctan\left(ax\right)}{9\left(a^6c^2x^2 + a^4c^2\right)\sqrt{a^2x^2 + 1}\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/9*(7*a^3*x^3 + 6*a*x - 3*(3*a^2*x^2 + 2)*arctan(a*x))/((a^6*c^2*x^2 + a^4
*c^2)*sqrt(a^2*x^2 + 1)*sqrt(c))
```

Fricas [A] time = 2.66274, size = 158, normalized size = 1.41

$$\frac{\left(7\,a^3x^3 + 6\,ax - 3\left(3\,a^2x^2 + 2\right)\arctan\left(ax\right)\right)\sqrt{a^2cx^2 + c}}{9\left(a^8c^3x^4 + 2\,a^6c^3x^2 + a^4c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/9*(7*a^3*x^3 + 6*a*x - 3*(3*a^2*x^2 + 2)*arctan(a*x))*sqrt(a^2*c*x^2 + c)
/(a^8*c^3*x^4 + 2*a^6*c^3*x^2 + a^4*c^3)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atan(a*x)/(a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Exception raised: TypeError
```

Giac [A] time = 1.21678, size = 99, normalized size = 0.88

$$\frac{x\left(\frac{7x^2}{ac}+\frac{6}{a^3c}\right)}{9\left(a^2cx^2+c\right)^{\frac{3}{2}}}-\frac{\left(3a^2cx^2+2c\right)\arctan\left(ax\right)}{3\left(a^2cx^2+c\right)^{\frac{3}{2}}a^4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/9*x*(7*x^2/(a*c) + 6/(a^3*c))/(a^2*c*x^2 + c)^(3/2) - 1/3*(3*a^2*c*x^2 +
2*c)*arctan(a*x)/((a^2*c*x^2 + c)^(3/2)*a^4*c^2)
```

3.243 $\int \frac{x^2 \tan^{-1}(ax)}{\left(c+a^2cx^2\right)^{5/2}} dx$

Optimal. Leaf size=77

$$\frac{1}{3a^3c^2\sqrt{a^2cx^2+c}} - \frac{1}{9a^3c\left(a^2cx^2+c\right)^{3/2}} + \frac{x^3\tan^{-1}(ax)}{3c\left(a^2cx^2+c\right)^{3/2}}$$

```
[Out] -1/(9*a^3*c*(c + a^2*c*x^2)^(3/2)) + 1/(3*a^3*c^2*Sqrt[c + a^2*c*x^2]) + (x
^3*ArcTan[a*x])/(3*c*(c + a^2*c*x^2)^(3/2))
```

Rubi [A] time = 0.112883, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {4944, 266, 43}

$$\frac{1}{3a^3c^2\sqrt{a^2cx^2+c}} - \frac{1}{9a^3c\left(a^2cx^2+c\right)^{3/2}} + \frac{x^3\tan^{-1}(ax)}{3c\left(a^2cx^2+c\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcTan[a*x])/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] -1/(9*a^3*c*(c + a^2*c*x^2)^(3/2)) + 1/(3*a^3*c^2*Sqrt[c + a^2*c*x^2]) + (x
^3*ArcTan[a*x])/(3*c*(c + a^2*c*x^2)^(3/2))
```

Rule 4944

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a +
 b*ArcTan[c*x])^p)/(d*f*(m + 1)), x] - Dist[(b*c*p)/(f*(m + 1)), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^{5/2}}\,dx &= \frac{x^3\tan^{-1}(ax)}{3c\left(c+a^2cx^2\right)^{3/2}}-\frac{1}{3}a\int \frac{x^3}{\left(c+a^2cx^2\right)^{5/2}}\,dx\\ &= \frac{x^3\tan^{-1}(ax)}{3c\left(c+a^2cx^2\right)^{3/2}}-\frac{1}{6}a\,\text{Subst}\left(\int \frac{x}{\left(c+a^2cx\right)^{5/2}}\,dx,x,x^2\right)\\ &= \frac{x^3\tan^{-1}(ax)}{3c\left(c+a^2cx^2\right)^{3/2}}-\frac{1}{6}a\,\text{Subst}\left(\int \left(-\frac{1}{a^2\left(c+a^2cx\right)^{5/2}}+\frac{1}{a^2c\left(c+a^2cx\right)^{3/2}}\right)dx,x,x^2\right)\\ &= -\frac{1}{9a^3c\left(c+a^2cx^2\right)^{3/2}}+\frac{1}{3a^3c^2\sqrt{c+a^2cx^2}}+\frac{x^3\tan^{-1}(ax)}{3c\left(c+a^2cx^2\right)^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.0618613, size = 57, normalized size = 0.74

$$\frac{\sqrt{a^2cx^2+c}\left(3a^2x^2+3a^3x^3\tan^{-1}(ax)+2\right)}{9a^3c^3\left(a^2x^2+1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*ArcTan[a*x])/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] (Sqrt[c + a^2*c*x^2]*(2 + 3*a^2*x^2 + 3*a^3*x^3*ArcTan[a*x]))/(9*a^3*c^3*(1
 + a^2*x^2)^2)
```

Maple [C] time = 0.713, size = 240, normalized size = 3.1

$$\frac{(i+3\arctan(ax))\left(a^3x^3-3ia^2x^2-3ax+i\right)}{72\left(a^2x^2+1\right)^2c^3a^3}\sqrt{c(ax-i)(ax+i)}+\frac{(\arctan(ax)+i)(ax-i)}{8c^3a^3\left(a^2x^2+1\right)}\sqrt{c(ax-i)(ax+i)}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(a*x)/(a^2*c*x^2+c)^(5/2),x)
```

```
[Out] 1/72*(I+3*arctan(a*x))*(a^3*x^3-3*I*a^2*x^2-3*a*x+I)*(c*(a*x-I)*(a*x+I))^(1
/2)/(a^2*x^2+1)^2/c^3/a^3+1/8*(arctan(a*x)+I)*(a*x-I)*(c*(a*x-I)*(a*x+I))^(
1/2)/a^3/c^3/(a^2*x^2+1)+1/8*(c*(a*x-I)*(a*x+I))^(1/2)*(a*x+I)*(arctan(a*x)
-I)/a^3/c^3/(a^2*x^2+1)+1/72*(-I+3*arctan(a*x))*(c*(a*x-I)*(a*x+I))^(1/2)*(
a^3*x^3+3*I*a^2*x^2-3*a*x-I)/(a^4*x^4+2*a^2*x^2+1)/c^3/a^3
```

Maxima [A] time = 1.03397, size = 126, normalized size = 1.64

$$\frac{1}{9}a\left(\frac{3}{\sqrt{a^2cx^2+c}a^4c^2}-\frac{1}{\left(a^2cx^2+c\right)^{\frac{3}{2}}a^4c}\right)+\frac{1}{3}\left(\frac{x}{\sqrt{a^2cx^2+c}a^2c^2}-\frac{x}{\left(a^2cx^2+c\right)^{\frac{3}{2}}a^2c}\right)\arctan(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/9*a*(3/(sqrt(a^2*c*x^2 + c)*a^4*c^2) - 1/((a^2*c*x^2 + c)^(3/2)*a^4*c)) +
 1/3*(x/(sqrt(a^2*c*x^2 + c)*a^2*c^2) - x/((a^2*c*x^2 + c)^(3/2)*a^2*c))*ar
ctan(a*x)
```

Fricas [A] time = 2.55572, size = 142, normalized size = 1.84

$$\frac{\left(3\,a^3x^3\arctan(ax)+3\,a^2x^2+2\right)\sqrt{a^2cx^2+c}}{9\left(a^7c^3x^4+2\,a^5c^3x^2+a^3c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/9*(3*a^3*x^3*arctan(a*x) + 3*a^2*x^2 + 2)*sqrt(a^2*c*x^2 + c)/(a^7*c^3*x^
4 + 2*a^5*c^3*x^2 + a^3*c^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \operatorname{atan}(ax)}{\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(a*x)/(a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral(x**2*atan(a*x)/(c*(a**2*x**2 + 1))**(5/2), x)
```

Giac [A] time = 1.24118, size = 78, normalized size = 1.01

$$\frac{x^3 \arctan(ax)}{3\left(a^2cx^2+c\right)^{\frac{3}{2}}c}+\frac{3\,a^2cx^2+2\,c}{9\left(a^2cx^2+c\right)^{\frac{3}{2}}a^3c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*x^3*arctan(a*x)/((a^2*c*x^2 + c)^(3/2)*c) + 1/9*(3*a^2*c*x^2 + 2*c)/((a
^2*c*x^2 + c)^(3/2)*a^3*c^2)
```

3.244 $\int \frac{x\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=79

$$\frac{2x}{9ac^2\sqrt{a^2cx^2+c}}+\frac{x}{9ac\left(a^2cx^2+c\right)^{3/2}}-\frac{\tan^{-1}(ax)}{3a^2c\left(a^2cx^2+c\right)^{3/2}}$$

```
[Out] x/(9*a*c*(c + a^2*c*x^2)^(3/2)) + (2*x)/(9*a*c^2*Sqrt[c + a^2*c*x^2]) - Arc
Tan[a*x]/(3*a^2*c*(c + a^2*c*x^2)^(3/2))
```

Rubi [A] time = 0.0606544, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {4930, 192, 191}

$$\frac{2x}{9ac^2\sqrt{a^2cx^2+c}}+\frac{x}{9ac\left(a^2cx^2+c\right)^{3/2}}-\frac{\tan^{-1}(ax)}{3a^2c\left(a^2cx^2+c\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTan[a*x])/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] x/(9*a*c*(c + a^2*c*x^2)^(3/2)) + (2*x)/(9*a*c^2*Sqrt[c + a^2*c*x^2]) - Arc
Tan[a*x]/(3*a^2*c*(c + a^2*c*x^2)^(3/2))
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^{5/2}}\,dx &= -\frac{\tan^{-1}(ax)}{3a^2c\left(c+a^2cx^2\right)^{3/2}}+\frac{\int \frac{1}{\left(c+a^2cx^2\right)^{5/2}}\,dx}{3a}\\ &= \frac{x}{9ac\left(c+a^2cx^2\right)^{3/2}}-\frac{\tan^{-1}(ax)}{3a^2c\left(c+a^2cx^2\right)^{3/2}}+\frac{2\int \frac{1}{\left(c+a^2cx^2\right)^{3/2}}\,dx}{9ac}\\ &= \frac{x}{9ac\left(c+a^2cx^2\right)^{3/2}}+\frac{2x}{9ac^2\sqrt{c+a^2cx^2}}-\frac{\tan^{-1}(ax)}{3a^2c\left(c+a^2cx^2\right)^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.0543117, size = 51, normalized size = 0.65

$$\frac{\sqrt{a^2cx^2+c}\left(2a^3x^3+3ax-3\tan^{-1}(ax)\right)}{9c^3\left(a^3x^2+a\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcTan[a*x])/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] (Sqrt[c + a^2*c*x^2]*(3*a*x + 2*a^3*x^3 - 3*ArcTan[a*x]))/(9*c^3*(a + a^3*x
^2)^2)
```

Maple [C] time = 0.303, size = 244, normalized size = 3.1

$$\frac{(i+3\arctan(ax))\left(ix^3a^3+3a^2x^2-3iax-1\right)}{72\left(a^2x^2+1\right)^2c^3a^2}\sqrt{c(ax-i)(ax+i)}-\frac{(\arctan(ax)+i)(1+iax)}{8c^3a^2\left(a^2x^2+1\right)}\sqrt{c(ax-i)(ax+i)}+\frac{(-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(a*x)/(a^2*c*x^2+c)^(5/2),x)
```

```
[Out] 1/72*(I+3*arctan(a*x))*(I*x^3*a^3+3*a^2*x^2-3*I*a*x-1)*(c*(a*x-I)*(a*x+I))^
(1/2)/(a^2*x^2+1)^2/c^3/a^2-1/8*(arctan(a*x)+I)*(1+I*a*x)*(c*(a*x-I)*(a*x+I
))^(1/2)/a^2/c^3/(a^2*x^2+1)+1/8*(c*(a*x-I)*(a*x+I))^(1/2)*(-1+I*a*x)*(arct
an(a*x)-I)/a^2/c^3/(a^2*x^2+1)-1/72*(c*(a*x-I)*(a*x+I))^(1/2)*(I*x^3*a^3-3*
a^2*x^2-3*I*a*x+1)*(-I+3*arctan(a*x))/a^2/c^3/(a^4*x^4+2*a^2*x^2+1)
```

Maxima [A] time = 1.30878, size = 89, normalized size = 1.13

$$\frac{\left(2\,a^3x^3+3\,ax-3\,\arctan\left(ax\right)\right)\sqrt{a^2x^2+1}\sqrt{c}}{9\left(a^6c^3x^4+2\,a^4c^3x^2+a^2c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/9*(2*a^3*x^3 + 3*a*x - 3*arctan(a*x))*sqrt(a^2*x^2 + 1)*sqrt(c)/(a^6*c^3*
x^4 + 2*a^4*c^3*x^2 + a^2*c^3)
```

Fricas [A] time = 2.54219, size = 136, normalized size = 1.72

$$\frac{\left(2\,a^3x^3+3\,ax-3\,\arctan\left(ax\right)\right)\sqrt{a^2cx^2+c}}{9\left(a^6c^3x^4+2\,a^4c^3x^2+a^2c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/9*(2*a^3*x^3 + 3*a*x - 3*arctan(a*x))*sqrt(a^2*c*x^2 + c)/(a^6*c^3*x^4 +
2*a^4*c^3*x^2 + a^2*c^3)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(a*x)/(a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Exception raised: TypeError
```

Giac [A] time = 1.19164, size = 78, normalized size = 0.99

$$\frac{\left(\frac{2\,ax^2}{c}+\frac{3}{ac}\right)x}{9\left(a^2cx^2+c\right)^{\frac{3}{2}}}-\frac{\arctan\left(ax\right)}{3\left(a^2cx^2+c\right)^{\frac{3}{2}}a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/9*(2*a*x^2/c + 3/(a*c))*x/(a^2*c*x^2 + c)^(3/2) - 1/3*arctan(a*x)/((a^2*c
*x^2 + c)^(3/2)*a^2*c)
```

3.245 $\int \frac{\tan^{-1}(ax)}{(c+a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=101

$$\frac{2}{3ac^2\sqrt{a^2cx^2+c}}+\frac{2x\tan^{-1}(ax)}{3c^2\sqrt{a^2cx^2+c}}+\frac{1}{9ac\left(a^2cx^2+c\right)^{3/2}}+\frac{x\tan^{-1}(ax)}{3c\left(a^2cx^2+c\right)^{3/2}}$$

```
[Out] 1/(9*a*c*(c + a^2*c*x^2)^(3/2)) + 2/(3*a*c^2*Sqrt[c + a^2*c*x^2]) + (x*ArcT
an[a*x])/(3*c*(c + a^2*c*x^2)^(3/2)) + (2*x*ArcTan[a*x])/(3*c^2*Sqrt[c + a^
2*c*x^2])
```

Rubi [A] time = 0.0539432, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {4896, 4894}

$$\frac{2}{3ac^2\sqrt{a^2cx^2+c}}+\frac{2x\tan^{-1}(ax)}{3c^2\sqrt{a^2cx^2+c}}+\frac{1}{9ac\left(a^2cx^2+c\right)^{3/2}}+\frac{x\tan^{-1}(ax)}{3c\left(a^2cx^2+c\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]/(c + a^2*c*x^2)^(5/2),x]
```


```
[Out] 1/(9*a*c*(c + a^2*c*x^2)^(3/2)) + 2/(3*a*c^2*Sqrt[c + a^2*c*x^2]) + (x*ArcT
an[a*x])/(3*c*(c + a^2*c*x^2)^(3/2)) + (2*x*ArcTan[a*x])/(3*c^2*Sqrt[c + a^
2*c*x^2])
```


Rule 4896

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbol
] :> Simp[(b*(d + e*x^2)^(q + 1))/(4*c*d*(q + 1)^2), x] + (Dist[(2*q + 3)/(
2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]), x], x] - Simp[(x
*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]))/(2*d*(q + 1)), x]) /; FreeQ[{a, b
, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -3/2]
```

Rule 4894

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[(x*(a + b*ArcTan[c*x]))/(d*Sq
rt[d + e*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^{5/2}}\,dx = \frac{1}{9ac\left(c+a^2cx^2\right)^{3/2}}+\frac{x\tan^{-1}(ax)}{3c\left(c+a^2cx^2\right)^{3/2}}+\frac{2\int \frac{\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^{3/2}}\,dx}{3c}$$

$$= \frac{1}{9ac\left(c+a^2cx^2\right)^{3/2}}+\frac{2}{3ac^2\sqrt{c+a^2cx^2}}+\frac{x\tan^{-1}(ax)}{3c\left(c+a^2cx^2\right)^{3/2}}+\frac{2x\tan^{-1}(ax)}{3c^2\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 0.0520958, size = 63, normalized size = 0.62

$$\frac{\sqrt{a^2cx^2+c}\left(6a^2x^2+\left(6a^3x^3+9ax\right)\tan^{-1}(ax)+7\right)}{9ac^3\left(a^2x^2+1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] (Sqrt[c + a^2*c*x^2]*(7 + 6*a^2*x^2 + (9*a*x + 6*a^3*x^3)*ArcTan[a*x]))/(9*
a*c^3*(1 + a^2*x^2)^2)
```

Maple [C] time = 0.267, size = 240, normalized size = 2.4

$$-\frac{(i+3\arctan(ax))\left(a^3x^3-3ia^2x^2-3ax+i\right)}{72\left(a^2x^2+1\right)^2ac^3}\sqrt{c(ax-i)(ax+i)}+\frac{(3\arctan(ax)+3i)(ax-i)}{8ac^3\left(a^2x^2+1\right)}\sqrt{c(ax-i)(ax+i)}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)/(a^2*c*x^2+c)^(5/2),x)
```

```
[Out] -1/72*(I+3*arctan(a*x))*(a^3*x^3-3*I*a^2*x^2-3*a*x+I)*(c*(a*x-I)*(a*x+I))^(
1/2)/(a^2*x^2+1)^2/a/c^3+3/8*(arctan(a*x)+I)*(a*x-I)*(c*(a*x-I)*(a*x+I))^(1
/2)/c^3/a/(a^2*x^2+1)+3/8*(c*(a*x-I)*(a*x+I))^(1/2)*(a*x+I)*(arctan(a*x)-I)
/c^3/a/(a^2*x^2+1)-1/72*(-I+3*arctan(a*x))*(c*(a*x-I)*(a*x+I))^(1/2)*(a^3*x
^3+3*I*a^2*x^2-3*a*x-I)/(a^4*x^4+2*a^2*x^2+1)/a/c^3
```

Maxima [A] time = 1.06625, size = 116, normalized size = 1.15

$$\frac{1}{9}\,a\left(\frac{6}{\sqrt{a^2cx^2+c}a^2c^2}+\frac{1}{\left(a^2cx^2+c\right)^{\frac{3}{2}}a^2c}\right)+\frac{1}{3}\left(\frac{2\,x}{\sqrt{a^2cx^2+c}c^2}+\frac{x}{\left(a^2cx^2+c\right)^{\frac{3}{2}}c}\right)\arctan\left(ax\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/9*a*(6/(sqrt(a^2*c*x^2 + c)*a^2*c^2) + 1/((a^2*c*x^2 + c)^(3/2)*a^2*c)) +
 1/3*(2*x/(sqrt(a^2*c*x^2 + c)*c^2) + x/((a^2*c*x^2 + c)^(3/2)*c))*arctan(a
*x)
```

Fricas [A] time = 2.25932, size = 155, normalized size = 1.53

$$\frac{\sqrt{a^2cx^2+c}\left(6\,a^2x^2+3\left(2\,a^3x^3+3\,ax\right)\arctan\left(ax\right)+7\right)}{9\left(a^5c^3x^4+2\,a^3c^3x^2+ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/9*sqrt(a^2*c*x^2 + c)*(6*a^2*x^2 + 3*(2*a^3*x^3 + 3*a*x)*arctan(a*x) + 7)
/(a^5*c^3*x^4 + 2*a^3*c^3*x^2 + a*c^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atan}\left(ax\right)}{\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)/(a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral(atan(a*x)/(c*(a**2*x**2 + 1))**(5/2), x)
```

Giac [A] time = 1.21544, size = 95, normalized size = 0.94

$$\frac{\left(\frac{2\,a^2x^2}{c}+\frac{3}{c}\right)x\arctan\left(ax\right)}{3\left(a^2cx^2+c\right)^{\frac{3}{2}}}+\frac{6\,a^2cx^2+7\,c}{9\left(a^2cx^2+c\right)^{\frac{3}{2}}ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(2*a^2*x^2/c + 3/c)*x*arctan(a*x)/(a^2*c*x^2 + c)^(3/2) + 1/9*(6*a^2*c*
x^2 + 7*c)/((a^2*c*x^2 + c)^(3/2)*a*c^2)
```

3.246 $\int \frac{\tan^{-1}(ax)}{x\left(c+a^2cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=279

$$\frac{i\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{c^2\sqrt{a^2cx^2+c}}-\frac{i\sqrt{a^2x^2+1}\text{PolyLog}\left(2,\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{c^2\sqrt{a^2cx^2+c}}-\frac{11ax}{9c^2\sqrt{a^2cx^2+c}}+\frac{\tan^{-1}(ax)}{c^2\sqrt{a^2cx^2+c}}-\frac{2\sqrt{a^2x^2+1}\tan}{c^2\surd}$$

```
[Out] -(a*x)/(9*c*(c + a^2*c*x^2)^(3/2)) - (11*a*x)/(9*c^2*Sqrt[c + a^2*c*x^2]) +
 ArcTan[a*x]/(3*c*(c + a^2*c*x^2)^(3/2)) + ArcTan[a*x]/(c^2*Sqrt[c + a^2*c*
x^2]) - (2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTanh[Sqrt[1 + I*a*x]/Sqrt[1 - I
*a*x]])/(c^2*Sqrt[c + a^2*c*x^2]) + (I*Sqrt[1 + a^2*x^2]*PolyLog[2, -(Sqrt[
1 + I*a*x]/Sqrt[1 - I*a*x])])/(c^2*Sqrt[c + a^2*c*x^2]) - (I*Sqrt[1 + a^2*x
^2]*PolyLog[2, Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/(c^2*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.430169, antiderivative size = 279, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4966, 4958, 4954, 4930, 191, 192}

$$\frac{i\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{c^2\sqrt{a^2cx^2+c}}-\frac{i\sqrt{a^2x^2+1}\text{PolyLog}\left(2,\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{c^2\sqrt{a^2cx^2+c}}-\frac{11ax}{9c^2\sqrt{a^2cx^2+c}}+\frac{\tan^{-1}(ax)}{c^2\sqrt{a^2cx^2+c}}-\frac{2\sqrt{a^2x^2+1}\tan}{c^2\surd}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]/(x*(c + a^2*c*x^2)^(5/2)),x]
```

```
[Out] -(a*x)/(9*c*(c + a^2*c*x^2)^(3/2)) - (11*a*x)/(9*c^2*Sqrt[c + a^2*c*x^2]) +
 ArcTan[a*x]/(3*c*(c + a^2*c*x^2)^(3/2)) + ArcTan[a*x]/(c^2*Sqrt[c + a^2*c*
x^2]) - (2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTanh[Sqrt[1 + I*a*x]/Sqrt[1 - I
*a*x]])/(c^2*Sqrt[c + a^2*c*x^2]) + (I*Sqrt[1 + a^2*x^2]*PolyLog[2, -(Sqrt[
1 + I*a*x]/Sqrt[1 - I*a*x])])/(c^2*Sqrt[c + a^2*c*x^2]) - (I*Sqrt[1 + a^2*x
^2]*PolyLog[2, Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/(c^2*Sqrt[c + a^2*c*x^2])
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*
x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^p
, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*q]
```

```
 && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 4958

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4954

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x_
Symbol] :> Simp[(-2*(a + b*ArcTan[c*x])*ArcTanh[Sqrt[1 + I*c*x]/Sqrt[1 - I*
c*x]])/Sqrt[d], x] + (Simp[(I*b*PolyLog[2, -(Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x
])])/Sqrt[d], x] - Simp[(I*b*PolyLog[2, Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]])/S
qrt[d], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)}{x\left(c+a^2cx^2\right)^{5/2}}\,dx = -\left(a^2\int \frac{x\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^{5/2}}\,dx\right)+\frac{\int \frac{\tan^{-1}(ax)}{x\left(c+a^2cx^2\right)^{3/2}}\,dx}{c}$$

$$= \frac{\tan^{-1}(ax)}{3c\left(c+a^2cx^2\right)^{3/2}}-\frac{1}{3}a\int \frac{1}{\left(c+a^2cx^2\right)^{5/2}}\,dx+\frac{\int \frac{\tan^{-1}(ax)}{x\sqrt{c+a^2cx^2}}\,dx}{c^2}-\frac{a^2\int \frac{x\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^{3/2}}\,dx}{c}$$

$$= -\frac{ax}{9c\left(c+a^2cx^2\right)^{3/2}}+\frac{\tan^{-1}(ax)}{3c\left(c+a^2cx^2\right)^{3/2}}+\frac{\tan^{-1}(ax)}{c^2\sqrt{c+a^2cx^2}}-\frac{(2a)\int \frac{1}{\left(c+a^2cx^2\right)^{3/2}}\,dx}{9c}-\frac{a\int \frac{1}{\left(c+a^2cx^2\right)^{3/2}}\,d}{c}$$

$$= -\frac{ax}{9c\left(c+a^2cx^2\right)^{3/2}}-\frac{11ax}{9c^2\sqrt{c+a^2cx^2}}+\frac{\tan^{-1}(ax)}{3c\left(c+a^2cx^2\right)^{3/2}}+\frac{\tan^{-1}(ax)}{c^2\sqrt{c+a^2cx^2}}-\frac{2\sqrt{1+a^2x^2}\tan^{-1}(ax}{c^2\sqrt{c+a}}$$

Mathematica [A] time = 0.366334, size = 168, normalized size = 0.6

$$\frac{\left(a^2x^2+1\right)^{3/2}\left(36i\text{PolyLog}\left(2,-e^{i\tan^{-1}(ax)}\right)-36i\text{PolyLog}\left(2,e^{i\tan^{-1}(ax)}\right)-\frac{45ax}{\sqrt{a^2x^2+1}}+\frac{45\tan^{-1}(ax)}{\sqrt{a^2x^2+1}}+36\tan^{-1}(ax)\log\left(1-e\right.\right.}{36c\left(c\left(a^2x^2+1\right)\right)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTan[a*x]/(x*(c + a^2*c*x^2)^(5/2)),x]
```

```
[Out] ((1 + a^2*x^2)^(3/2)*((-45*a*x)/Sqrt[1 + a^2*x^2] + (45*ArcTan[a*x])/Sqrt[1
 + a^2*x^2] + 3*ArcTan[a*x]*Cos[3*ArcTan[a*x]] + 36*ArcTan[a*x]*Log[1 - E^(
I*ArcTan[a*x])] - 36*ArcTan[a*x]*Log[1 + E^(I*ArcTan[a*x])] + (36*I)*PolyLo
g[2, -E^(I*ArcTan[a*x])] - (36*I)*PolyLog[2, E^(I*ArcTan[a*x])] - Sin[3*Arc
Tan[a*x]]))/(36*c*(c*(1 + a^2*x^2))^(3/2))
```

Maple [A] time = 0.323, size = 370, normalized size = 1.3

$$-\frac{(i+3\arctan(ax))\left(ix^3a^3+3a^2x^2-3iax-1\right)}{72\left(a^2x^2+1\right)^2c^3}\sqrt{c(ax-i)(ax+i)}+\frac{(5\arctan(ax)+5i)(1+iax)}{8c^3\left(a^2x^2+1\right)}\sqrt{c(ax-i)(ax+i)}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)/x/(a^2*c*x^2+c)^(5/2),x)
```

```
[Out] -1/72*(I+3*arctan(a*x))*(I*x^3*a^3+3*a^2*x^2-3*I*a*x-1)*(c*(a*x-I)*(a*x+I))
^(1/2)/(a^2*x^2+1)^2/c^3+5/8*(arctan(a*x)+I)*(1+I*a*x)*(c*(a*x-I)*(a*x+I))^
(1/2)/c^3/(a^2*x^2+1)-5/8*(c*(a*x-I)*(a*x+I))^(1/2)*(-1+I*a*x)*(arctan(a*x)
-I)/c^3/(a^2*x^2+1)+1/72*(c*(a*x-I)*(a*x+I))^(1/2)*(I*x^3*a^3-3*a^2*x^2-3*I
*a*x+1)*(-I+3*arctan(a*x))/c^3/(a^4*x^4+2*a^2*x^2+1)-I*(I*arctan(a*x)*ln(1-
(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*arctan(a*x)*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2)
)+polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))-polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(
1/2)))*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)^(1/2)/c^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{a^6c^3x^7+3\,a^4c^3x^5+3\,a^2c^3x^3+c^3x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)/(a^6*c^3*x^7 + 3*a^4*c^3*x^5 + 3*a
^2*c^3*x^3 + c^3*x), x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)/x/(a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(ax)}{(a^2cx^2+c)^{\frac{5}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)/((a^2*c*x^2 + c)^(5/2)*x), x)
```

3.247 $\int \frac{\tan^{-1}(ax)}{x^2\left(c+a^2cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=158

$$-\frac{5a}{3c^2\sqrt{a^2cx^2+c}}-\frac{5a^2x\tan^{-1}(ax)}{3c^2\sqrt{a^2cx^2+c}}-\frac{\sqrt{a^2cx^2+c}\tan^{-1}(ax)}{c^3x}-\frac{a\tanh^{-1}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{c^{5/2}}-\frac{a}{9c\left(a^2cx^2+c\right)^{3/2}}-\frac{a^2x\tan^{-1}(ax)}{3c\left(a^2cx^2+c\right)^{3}}$$

```
[Out] -a/(9*c*(c + a^2*c*x^2)^(3/2)) - (5*a)/(3*c^2*Sqrt[c + a^2*c*x^2]) - (a^2*x
*ArcTan[a*x])/(3*c*(c + a^2*c*x^2)^(3/2)) - (5*a^2*x*ArcTan[a*x])/(3*c^2*Sq
rt[c + a^2*c*x^2]) - (Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(c^3*x) - (a*ArcTanh
[Sqrt[c + a^2*c*x^2]/Sqrt[c]])/c^(5/2)
```

Rubi [A] time = 0.33748, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {4966, 4944, 266, 63, 208, 4894, 4896}

$$-\frac{5a}{3c^2\sqrt{a^2cx^2+c}}-\frac{5a^2x\tan^{-1}(ax)}{3c^2\sqrt{a^2cx^2+c}}-\frac{\sqrt{a^2cx^2+c}\tan^{-1}(ax)}{c^3x}-\frac{a\tanh^{-1}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{c^{5/2}}-\frac{a}{9c\left(a^2cx^2+c\right)^{3/2}}-\frac{a^2x\tan^{-1}(ax)}{3c\left(a^2cx^2+c\right)^{3}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]/(x^2*(c + a^2*c*x^2)^(5/2)),x]
```

```
[Out] -a/(9*c*(c + a^2*c*x^2)^(3/2)) - (5*a)/(3*c^2*Sqrt[c + a^2*c*x^2]) - (a^2*x
*ArcTan[a*x])/(3*c*(c + a^2*c*x^2)^(3/2)) - (5*a^2*x*ArcTan[a*x])/(3*c^2*Sq
rt[c + a^2*c*x^2]) - (Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(c^3*x) - (a*ArcTanh
[Sqrt[c + a^2*c*x^2]/Sqrt[c]])/c^(5/2)
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*
x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^p
, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*q]
 && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 4944

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a +
 b*ArcTan[c*x])^p)/(d*f*(m + 1)), x] - Dist[(b*c*p)/(f*(m + 1)), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 4894

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[(x*(a + b*ArcTan[c*x]))/(d*Sq
rt[d + e*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

Rule 4896

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbol
] :> Simp[(b*(d + e*x^2)^(q + 1))/(4*c*d*(q + 1)^2), x] + (Dist[(2*q + 3)/(
2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]), x], x] - Simp[(x
*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]))/(2*d*(q + 1)), x]) /; FreeQ[{a, b
, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -3/2]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)}{x^2\left(c+a^2cx^2\right)^{5/2}}\,dx = -\left(a^2\int \frac{\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^{5/2}}\,dx\right) + \frac{\int \frac{\tan^{-1}(ax)}{x^2\left(c+a^2cx^2\right)^{3/2}}\,dx}{c}$$

$$= -\frac{a}{9c\left(c+a^2cx^2\right)^{3/2}} - \frac{a^2x\tan^{-1}(ax)}{3c\left(c+a^2cx^2\right)^{3/2}} + \frac{\int \frac{\tan^{-1}(ax)}{x^2\sqrt{c+a^2cx^2}}\,dx}{c^2} - \frac{\left(2a^2\right)\int \frac{\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^{3/2}}\,dx}{3c} - \frac{a^2\int \frac{\tan^{-1}}{\left(c+a^2c\right.}}{c}$$

$$= -\frac{a}{9c\left(c+a^2cx^2\right)^{3/2}} - \frac{5a}{3c^2\sqrt{c+a^2cx^2}} - \frac{a^2x\tan^{-1}(ax)}{3c\left(c+a^2cx^2\right)^{3/2}} - \frac{5a^2x\tan^{-1}(ax)}{3c^2\sqrt{c+a^2cx^2}} - \frac{\sqrt{c+a^2cx^2}\tan^{-}}{c^3x}$$

$$= -\frac{a}{9c\left(c+a^2cx^2\right)^{3/2}} - \frac{5a}{3c^2\sqrt{c+a^2cx^2}} - \frac{a^2x\tan^{-1}(ax)}{3c\left(c+a^2cx^2\right)^{3/2}} - \frac{5a^2x\tan^{-1}(ax)}{3c^2\sqrt{c+a^2cx^2}} - \frac{\sqrt{c+a^2cx^2}\tan^{-}}{c^3x}$$

$$= -\frac{a}{9c\left(c+a^2cx^2\right)^{3/2}} - \frac{5a}{3c^2\sqrt{c+a^2cx^2}} - \frac{a^2x\tan^{-1}(ax)}{3c\left(c+a^2cx^2\right)^{3/2}} - \frac{5a^2x\tan^{-1}(ax)}{3c^2\sqrt{c+a^2cx^2}} - \frac{\sqrt{c+a^2cx^2}\tan^{-}}{c^3x}$$

$$= -\frac{a}{9c\left(c+a^2cx^2\right)^{3/2}} - \frac{5a}{3c^2\sqrt{c+a^2cx^2}} - \frac{a^2x\tan^{-1}(ax)}{3c\left(c+a^2cx^2\right)^{3/2}} - \frac{5a^2x\tan^{-1}(ax)}{3c^2\sqrt{c+a^2cx^2}} - \frac{\sqrt{c+a^2cx^2}\tan^{-}}{c^3x}$$

<u>Mathematica</u> [A] time = 0.24921, size = 151, normalized size = 0.96

$$\frac{ax\left(-\left(15a^2x^2+16\right)\sqrt{a^2cx^2+c}+9\sqrt{c}\left(a^2x^2+1\right)^2\log(x)-9\sqrt{c}\left(a^2x^2+1\right)^2\log\left(\sqrt{c}\sqrt{a^2cx^2+c}+c\right)\right)-3\left(8a^4x^4+12a^2\right.}{9c^3x\left(a^2x^2+1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]/(x^2*(c + a^2*c*x^2)^(5/2)),x]
```

```
[Out] (-3*Sqrt[c + a^2*c*x^2]*(3 + 12*a^2*x^2 + 8*a^4*x^4)*ArcTan[a*x] + a*x*(-((
16 + 15*a^2*x^2)*Sqrt[c + a^2*c*x^2]) + 9*Sqrt[c]*(1 + a^2*x^2)^2*Log[x] -
9*Sqrt[c]*(1 + a^2*x^2)^2*Log[c + Sqrt[c]*Sqrt[c + a^2*c*x^2]]))/(9*c^3*x*(
1 + a^2*x^2)^2)
```

Maple [C] time = 0.337, size = 369, normalized size = 2.3

$$\frac{a\left(i+3\arctan\left(ax\right)\right)\left(a^{3}x^{3}-3\,i a^{2}x^{2}-3\,ax+i\right)}{72\left(a^{2}x^{2}+1\right)^{2}c^{3}}\sqrt{c\left(ax-i\right)\left(ax+i\right)}-\frac{7\,a\left(\arctan\left(ax\right)+i\right)\left(ax-i\right)}{8\,c^{3}\left(a^{2}x^{2}+1\right)}\sqrt{c\left(ax-i\right)\left(ax+i\right)}-\frac{(7}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)/x^2/(a^2*c*x^2+c)^(5/2),x)
```

```
[Out] 1/72*a*(I+3*arctan(a*x))*(a^3*x^3-3*I*a^2*x^2-3*a*x+I)*(c*(a*x-I)*(a*x+I))^
(1/2)/(a^2*x^2+1)^2/c^3-7/8*a*(arctan(a*x)+I)*(a*x-I)*(c*(a*x-I)*(a*x+I))^(
1/2)/c^3/(a^2*x^2+1)-7/8*(c*(a*x-I)*(a*x+I))^(1/2)*(a*x+I)*(arctan(a*x)-I)*
a/c^3/(a^2*x^2+1)+1/72*(c*(a*x-I)*(a*x+I))^(1/2)*(a^3*x^3+3*I*a^2*x^2-3*a*x
-I)*(-I+3*arctan(a*x))*a/c^3/(a^4*x^4+2*a^2*x^2+1)-arctan(a*x)*(c*(a*x-I)*(
a*x+I))^(1/2)/x/c^3-a*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*x^2+1)^(1/2)*(
c*(a*x-I)*(a*x+I))^(1/2)/c^3+a*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)-1)/(a^2*x^2+1
)^(1/2)*(c*(a*x-I)*(a*x+I))^(1/2)/c^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x^2/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.78061, size = 317, normalized size = 2.01

$$\frac{9\left(a^{5}x^{5}+2\,a^{3}x^{3}+ax\right)\sqrt{c}\log\left(-\frac{a^{2}cx^{2}-2\,\sqrt{a^{2}cx^{2}+c}\sqrt{c}+2\,c}{x^{2}}\right)-2\left(15\,a^{3}x^{3}+16\,ax+3\left(8\,a^{4}x^{4}+12\,a^{2}x^{2}+3\right)\arctan\left(ax\right)\right)\sqrt{a^{2}cx^{2}}}{18\left(a^{4}c^{3}x^{5}+2\,a^{2}c^{3}x^{3}+c^{3}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x^2/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/18*(9*(a^5*x^5 + 2*a^3*x^3 + a*x)*sqrt(c)*log(-(a^2*c*x^2 - 2*sqrt(a^2*c*
x^2 + c)*sqrt(c) + 2*c)/x^2) - 2*(15*a^3*x^3 + 16*a*x + 3*(8*a^4*x^4 + 12*a
^2*x^2 + 3)*arctan(a*x))*sqrt(a^2*c*x^2 + c))/(a^4*c^3*x^5 + 2*a^2*c^3*x^3
+ c^3*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atan}(ax)}{x^2\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)/x**2/(a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral(atan(a*x)/(x**2*(c*(a**2*x**2 + 1))**(5/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(ax)}{\left(a^2cx^2+c\right)^{\frac{5}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x^2/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)/((a^2*c*x^2 + c)^(5/2)*x^2), x)
```

3.248 $\int x^m \left(c+a^2cx^2\right)^3 \tan^{-1}(ax)\,dx$

Optimal. Leaf size=270

$$-\frac{ac^3x^{m+2}\text{Hypergeometric2F1}\left(1,\frac{m+2}{2},\frac{m+4}{2},-a^2x^2\right)}{m^2+3m+2}-\frac{3a^3c^3x^{m+4}\text{Hypergeometric2F1}\left(1,\frac{m+4}{2},\frac{m+6}{2},-a^2x^2\right)}{m^2+7m+12}-\frac{3a^5c^3x^{m}}{}$$

```
[Out] (c^3*x^(1 + m)*ArcTan[a*x])/(1 + m) + (3*a^2*c^3*x^(3 + m)*ArcTan[a*x])/(3
+ m) + (3*a^4*c^3*x^(5 + m)*ArcTan[a*x])/(5 + m) + (a^6*c^3*x^(7 + m)*ArcTa
n[a*x])/(7 + m) - (a*c^3*x^(2 + m)*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/
2, -(a^2*x^2)])/(2 + 3*m + m^2) - (3*a^3*c^3*x^(4 + m)*Hypergeometric2F1[1,
 (4 + m)/2, (6 + m)/2, -(a^2*x^2)])/(12 + 7*m + m^2) - (3*a^5*c^3*x^(6 + m)
*Hypergeometric2F1[1, (6 + m)/2, (8 + m)/2, -(a^2*x^2)])/((5 + m)*(6 + m))
- (a^7*c^3*x^(8 + m)*Hypergeometric2F1[1, (8 + m)/2, (10 + m)/2, -(a^2*x^2)
])/((7 + m)*(8 + m))
```

Rubi [A] time = 0.225201, antiderivative size = 270, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {4948, 4852, 364}

$$-\frac{ac^3x^{m+2}\,{}_2F_1\left(1,\frac{m+2}{2};\frac{m+4}{2};-a^2x^2\right)}{m^2+3m+2}-\frac{3a^3c^3x^{m+4}\,{}_2F_1\left(1,\frac{m+4}{2};\frac{m+6}{2};-a^2x^2\right)}{m^2+7m+12}-\frac{3a^5c^3x^{m+6}\,{}_2F_1\left(1,\frac{m+6}{2};\frac{m+8}{2};-a^2x^2\right)}{(m+5)(m+6)}-\frac{a^7c^3x^{m}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(c + a^2*c*x^2)^3*ArcTan[a*x],x]
```

```
[Out] (c^3*x^(1 + m)*ArcTan[a*x])/(1 + m) + (3*a^2*c^3*x^(3 + m)*ArcTan[a*x])/(3
+ m) + (3*a^4*c^3*x^(5 + m)*ArcTan[a*x])/(5 + m) + (a^6*c^3*x^(7 + m)*ArcTa
n[a*x])/(7 + m) - (a*c^3*x^(2 + m)*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/
2, -(a^2*x^2)])/(2 + 3*m + m^2) - (3*a^3*c^3*x^(4 + m)*Hypergeometric2F1[1,
 (4 + m)/2, (6 + m)/2, -(a^2*x^2)])/(12 + 7*m + m^2) - (3*a^5*c^3*x^(6 + m)
*Hypergeometric2F1[1, (6 + m)/2, (8 + m)/2, -(a^2*x^2)])/((5 + m)*(6 + m))
- (a^7*c^3*x^(8 + m)*Hypergeometric2F1[1, (8 + m)/2, (10 + m)/2, -(a^2*x^2)
])/((7 + m)*(8 + m))
```

Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
```

```
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int x^m \left(c+a^2cx^2\right)^3 \tan^{-1}(ax)\,dx = \int \left(c^3x^m \tan^{-1}(ax) + 3a^2c^3x^{2+m}\tan^{-1}(ax) + 3a^4c^3x^{4+m}\tan^{-1}(ax) + a^6c^3x^{6+m}\tan^{-1}\right.$$

$$= c^3\int x^m \tan^{-1}(ax)\,dx + \left(3a^2c^3\right)\int x^{2+m}\tan^{-1}(ax)\,dx + \left(3a^4c^3\right)\int x^{4+m}\tan^{-1}(ax)\,dx$$

$$= \frac{c^3x^{1+m}\tan^{-1}(ax)}{1+m} + \frac{3a^2c^3x^{3+m}\tan^{-1}(ax)}{3+m} + \frac{3a^4c^3x^{5+m}\tan^{-1}(ax)}{5+m} + \frac{a^6c^3x^{7+m}\tan^{-1}}{7+m}$$

$$= \frac{c^3x^{1+m}\tan^{-1}(ax)}{1+m} + \frac{3a^2c^3x^{3+m}\tan^{-1}(ax)}{3+m} + \frac{3a^4c^3x^{5+m}\tan^{-1}(ax)}{5+m} + \frac{a^6c^3x^{7+m}\tan^{-1}}{7+m}$$

Mathematica [A] time = 0.338003, size = 234, normalized size = 0.87

$$c^3x^{m+1}\left(-\frac{3a^3x^3\text{Hypergeometric2F1}\left(1,\frac{m+4}{2},\frac{m+6}{2},-a^2x^2\right)}{m^2+7m+12} - \frac{ax\text{Hypergeometric2F1}\left(1,\frac{m+2}{2},\frac{m+4}{2},-a^2x^2\right)}{m^2+3m+2} - \frac{a^7x^7\text{Hy}}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(c + a^2*c*x^2)^3*ArcTan[a*x],x]
```

```
[Out] c^3*x^(1 + m)*(ArcTan[a*x]/(1 + m) + (3*a^2*x^2*ArcTan[a*x])/(3 + m) + (3*a
^4*x^4*ArcTan[a*x])/(5 + m) + (a^6*x^6*ArcTan[a*x])/(7 + m) - (a^7*x^7*Hype
```

```
rgeometric2F1[1, 4 + m/2, 5 + m/2, -(a^2*x^2)])/((7 + m)*(8 + m)) - (a*x*Hy
pergeometric2F1[1, (2 + m)/2, (4 + m)/2, -(a^2*x^2)])/(2 + 3*m + m^2) - (3*
a^3*x^3*Hypergeometric2F1[1, (4 + m)/2, (6 + m)/2, -(a^2*x^2)])/(12 + 7*m +
 m^2) - (3*a^5*x^5*Hypergeometric2F1[1, (6 + m)/2, (8 + m)/2, -(a^2*x^2)])/
((5 + m)*(6 + m)))
```

Maple [C] time = 0.747, size = 600, normalized size = 2.2

$$\frac{a^{-1-m}c^3}{4}\left(-4\,\frac{x^m a^m\left(a^6m^3x^6+6\,a^6m^2x^6+8\,mx^6a^6-a^4m^3x^4-8\,a^4m^2x^4-12\,mx^4a^4+a^2m^3x^2+10\,a^2m^2x^2+24\,mx^2a^2-\right.}{(7+m)\,m\,(2+m)\,(4+m)\,(6+m)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)^3*arctan(a*x),x)
```

```
[Out] 1/4*a^(-1-m)*c^3*(-4*x^m*a^m*(a^6*m^3*x^6+6*a^6*m^2*x^6+8*a^6*m*x^6-a^4*m^3
*x^4-8*a^4*m^2*x^4-12*a^4*m*x^4+a^2*m^3*x^2+10*a^2*m^2*x^2+24*a^2*m*x^2-m^3
-12*m^2-44*m-48)/(7+m)/m/(2+m)/(4+m)/(6+m)+8*x^(8+m)*a^(8+m)/(14+2*m)/(a^2*
x^2)^(1/2)*arctan((a^2*x^2)^(1/2))+2/(8+m)*x^m*a^m*(-8-m)/(7+m)*LerchPhi(-a
^2*x^2,1,1/2*m))+3/4*a^(-1-m)*c^3*(-4*x^m*a^m*(a^4*m^2*x^4+2*a^4*m*x^4-a^2*
m^2*x^2-4*a^2*m*x^2+m^2+6*m+8)/(5+m)/m/(2+m)/(4+m)+8*x^(6+m)*a^(6+m)/(10+2*
m)/(a^2*x^2)^(1/2)*arctan((a^2*x^2)^(1/2))+2*x^m*a^m/(5+m)*LerchPhi(-a^2*x^
2,1,1/2*m))+3/4*a^(-1-m)*c^3*(-4*x^m*a^m*(a^2*m*x^2-m-2)/(3+m)/m/(2+m)+8*x^
(4+m)*a^(4+m)/(6+2*m)/(a^2*x^2)^(1/2)*arctan((a^2*x^2)^(1/2))+2/(4+m)*x^m*a
^m*(-4-m)/(3+m)*LerchPhi(-a^2*x^2,1,1/2*m))+1/4*a^(-1-m)*c^3*(4/(2+m)*x^m*a
^m*(-m-2)/(1+m)/m+8*x^(2+m)*a^(2+m)/(2+2*m)/(a^2*x^2)^(1/2)*arctan((a^2*x^2
)^(1/2))+2*x^m*a^m/(1+m)*LerchPhi(-a^2*x^2,1,1/2*m))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^3*arctan(a*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^{6}c^{3}x^{6}+3\,a^{4}c^{3}x^{4}+3\,a^{2}c^{3}x^{2}+c^{3}\right)x^{m}\arctan\left(ax\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^3*arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*x^m*arctan(a*x
), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$c^{3}\left(\int x^{m}\operatorname{atan}\left(ax\right)dx+\int 3a^{2}x^{2}x^{m}\operatorname{atan}\left(ax\right)dx+\int 3a^{4}x^{4}x^{m}\operatorname{atan}\left(ax\right)dx+\int a^{6}x^{6}x^{m}\operatorname{atan}\left(ax\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**3*atan(a*x),x)
```

```
[Out] c**3*(Integral(x**m*atan(a*x), x) + Integral(3*a**2*x**2*x**m*atan(a*x), x)
 + Integral(3*a**4*x**4*x**m*atan(a*x), x) + Integral(a**6*x**6*x**m*atan(a
*x), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a^{2}cx^{2}+c\right)^{3}x^{m}\arctan\left(ax\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^3*arctan(a*x),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^3*x^m*arctan(a*x), x)
```

3.249 $\int x^m \left(c+a^2cx^2\right)^2 \tan^{-1}(ax)\,dx$

Optimal. Leaf size=201

$$-\frac{ac^2x^{m+2}\text{Hypergeometric2F1}\left(1,\frac{m+2}{2},\frac{m+4}{2},-a^2x^2\right)}{m^2+3m+2}-\frac{2a^3c^2x^{m+4}\text{Hypergeometric2F1}\left(1,\frac{m+4}{2},\frac{m+6}{2},-a^2x^2\right)}{m^2+7m+12}-\frac{a^5c^2x^{m\cdots}}{\cdots}$$

```
[Out] (c^2*x^(1 + m)*ArcTan[a*x])/(1 + m) + (2*a^2*c^2*x^(3 + m)*ArcTan[a*x])/(3
+ m) + (a^4*c^2*x^(5 + m)*ArcTan[a*x])/(5 + m) - (a*c^2*x^(2 + m)*Hypergeom
etric2F1[1, (2 + m)/2, (4 + m)/2, -(a^2*x^2)])/(2 + 3*m + m^2) - (2*a^3*c^2
*x^(4 + m)*Hypergeometric2F1[1, (4 + m)/2, (6 + m)/2, -(a^2*x^2)])/(12 + 7*
m + m^2) - (a^5*c^2*x^(6 + m)*Hypergeometric2F1[1, (6 + m)/2, (8 + m)/2, -(
a^2*x^2)])/((5 + m)*(6 + m))
```

Rubi [A] time = 0.156996, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {4948, 4852, 364}

$$-\frac{ac^2x^{m+2}\,{}_2F_1\left(1,\frac{m+2}{2};\frac{m+4}{2};-a^2x^2\right)}{m^2+3m+2}-\frac{2a^3c^2x^{m+4}\,{}_2F_1\left(1,\frac{m+4}{2};\frac{m+6}{2};-a^2x^2\right)}{m^2+7m+12}-\frac{a^5c^2x^{m+6}\,{}_2F_1\left(1,\frac{m+6}{2};\frac{m+8}{2};-a^2x^2\right)}{(m+5)(m+6)}+\frac{2a^2c^2x^{m\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(c + a^2*c*x^2)^2*ArcTan[a*x],x]
```



```
[Out] (c^2*x^(1 + m)*ArcTan[a*x])/(1 + m) + (2*a^2*c^2*x^(3 + m)*ArcTan[a*x])/(3
+ m) + (a^4*c^2*x^(5 + m)*ArcTan[a*x])/(5 + m) - (a*c^2*x^(2 + m)*Hypergeom
etric2F1[1, (2 + m)/2, (4 + m)/2, -(a^2*x^2)])/(2 + 3*m + m^2) - (2*a^3*c^2
*x^(4 + m)*Hypergeometric2F1[1, (4 + m)/2, (6 + m)/2, -(a^2*x^2)])/(12 + 7*
m + m^2) - (a^5*c^2*x^(6 + m)*Hypergeometric2F1[1, (6 + m)/2, (8 + m)/2, -(
a^2*x^2)])/((5 + m)*(6 + m))
```


Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int x^m \left(c+a^2cx^2\right)^2 \tan^{-1}(ax)\,dx = \int \left(c^2x^m \tan^{-1}(ax)+2a^2c^2x^{2+m}\tan^{-1}(ax)+a^4c^2x^{4+m}\tan^{-1}(ax)\right)\,dx$$

$$= c^2\int x^m \tan^{-1}(ax)\,dx+\left(2a^2c^2\right)\int x^{2+m}\tan^{-1}(ax)\,dx+\left(a^4c^2\right)\int x^{4+m}\tan^{-1}(ax)\,dx$$

$$= \frac{c^2x^{1+m}\tan^{-1}(ax)}{1+m}+\frac{2a^2c^2x^{3+m}\tan^{-1}(ax)}{3+m}+\frac{a^4c^2x^{5+m}\tan^{-1}(ax)}{5+m}-\frac{\left(ac^2\right)\int \frac{x^{1+m}}{1+a^2x^2}\,dx}{1+m}$$

$$= \frac{c^2x^{1+m}\tan^{-1}(ax)}{1+m}+\frac{2a^2c^2x^{3+m}\tan^{-1}(ax)}{3+m}+\frac{a^4c^2x^{5+m}\tan^{-1}(ax)}{5+m}-\frac{ac^2x^{2+m}\,{}_2F_1\left(1,\frac{2}{}\right.}{2+3m}$$

Mathematica [A] time = 0.138472, size = 175, normalized size = 0.87

$$c^2x^{m+1}\left(-\frac{2a^3x^3\text{Hypergeometric2F1}\left(1,\frac{m+4}{2},\frac{m+6}{2},-a^2x^2\right)}{m^2+7m+12}-\frac{ax\text{Hypergeometric2F1}\left(1,\frac{m+2}{2},\frac{m+4}{2},-a^2x^2\right)}{m^2+3m+2}-\frac{a^5x^5\text{Hy}}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(c + a^2*c*x^2)^2*ArcTan[a*x],x]
```

```
[Out] c^2*x^(1 + m)*(ArcTan[a*x]/(1 + m) + (2*a^2*x^2*ArcTan[a*x])/(3 + m) + (a^4
*x^4*ArcTan[a*x])/(5 + m) - (a*x*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2,
 -(a^2*x^2)])/(2 + 3*m + m^2) - (2*a^3*x^3*Hypergeometric2F1[1, (4 + m)/2,
(6 + m)/2, -(a^2*x^2)])/(12 + 7*m + m^2) - (a^5*x^5*Hypergeometric2F1[1, (6
 + m)/2, (8 + m)/2, -(a^2*x^2)])/((5 + m)*(6 + m)))
```

Maple [C] time = 0.577, size = 376, normalized size = 1.9

$$\frac{a^{-1-m}c^2}{4}\left(-4\,\frac{x^m a^m\left(a^4m^2x^4+2\,mx^4a^4-a^2m^2x^2-4\,mx^2a^2+m^2+6\,m+8\right)}{(5+m)\,m\,(2+m)\,(4+m)}+8\,\frac{x^{6+m}a^{6+m}\arctan\left(\sqrt{a^2x^2}\right)}{(10+2\,m)\sqrt{a^2x^2}}+2\,\frac{x^m a^m\mathrm{Le}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)^2*arctan(a*x),x)
```

```
[Out] 1/4*a^(-1-m)*c^2*(-4*x^m*a^m*(a^4*m^2*x^4+2*a^4*m*x^4-a^2*m^2*x^2-4*a^2*m*x
^2+m^2+6*m+8)/(5+m)/m/(2+m)/(4+m)+8*x^(6+m)*a^(6+m)/(10+2*m)/(a^2*x^2)^(1/2
)*arctan((a^2*x^2)^(1/2))+2*x^m*a^m/(5+m)*LerchPhi(-a^2*x^2,1,1/2*m))+1/2*a
^(-1-m)*c^2*(-4*x^m*a^m*(a^2*m*x^2-m-2)/(3+m)/m/(2+m)+8*x^(4+m)*a^(4+m)/(6+
2*m)/(a^2*x^2)^(1/2)*arctan((a^2*x^2)^(1/2))+2/(4+m)*x^m*a^m*(-4-m)/(3+m)*L
erchPhi(-a^2*x^2,1,1/2*m))+1/4*a^(-1-m)*c^2*(4/(2+m)*x^m*a^m*(-m-2)/(1+m)/m
+8*x^(2+m)*a^(2+m)/(2+2*m)/(a^2*x^2)^(1/2)*arctan((a^2*x^2)^(1/2))+2*x^m*a^
m/(1+m)*LerchPhi(-a^2*x^2,1,1/2*m))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^2*arctan(a*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a^4c^2x^4+2\,a^2c^2x^2+c^2\right)x^m\arctan\left(ax\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^2*arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*x^m*arctan(a*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$c^2\left(\int x^m \operatorname{atan}(ax)\,dx + \int 2a^2x^2x^m \operatorname{atan}(ax)\,dx + \int a^4x^4x^m \operatorname{atan}(ax)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**2*atan(a*x),x)
```

```
[Out] c**2*(Integral(x**m*atan(a*x), x) + Integral(2*a**2*x**2*x**m*atan(a*x), x)
 + Integral(a**4*x**4*x**m*atan(a*x), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2+c\right)^2 x^m \arctan(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^2*arctan(a*x),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^2*x^m*arctan(a*x), x)
```

3.250 $\int x^m \left(c+a^2cx^2\right) \tan^{-1}(ax) \, dx$

Optimal. Leaf size=124

$$-\frac{acx^{m+2}\text{Hypergeometric2F1}\left(1,\frac{m+2}{2},\frac{m+4}{2},-a^2x^2\right)}{m^2+3m+2}-\frac{a^3cx^{m+4}\text{Hypergeometric2F1}\left(1,\frac{m+4}{2},\frac{m+6}{2},-a^2x^2\right)}{m^2+7m+12}+\frac{a^2cx^{m+3}\,\text{ta}}{m+}$$

```
[Out] (c*x^(1 + m)*ArcTan[a*x])/(1 + m) + (a^2*c*x^(3 + m)*ArcTan[a*x])/(3 + m) -
 (a*c*x^(2 + m)*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, -(a^2*x^2)])/(2
+ 3*m + m^2) - (a^3*c*x^(4 + m)*Hypergeometric2F1[1, (4 + m)/2, (6 + m)/2,
-(a^2*x^2)])/(12 + 7*m + m^2)
```

Rubi [A] time = 0.0758609, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4950, 4852, 364}

$$-\frac{acx^{m+2}\,{}_2F_1\left(1,\frac{m+2}{2};\frac{m+4}{2};-a^2x^2\right)}{m^2+3m+2}-\frac{a^3cx^{m+4}\,{}_2F_1\left(1,\frac{m+4}{2};\frac{m+6}{2};-a^2x^2\right)}{m^2+7m+12}+\frac{a^2cx^{m+3}\tan^{-1}(ax)}{m+3}+\frac{cx^{m+1}\tan^{-1}(ax)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(c + a^2*c*x^2)*ArcTan[a*x],x]
```

```
[Out] (c*x^(1 + m)*ArcTan[a*x])/(1 + m) + (a^2*c*x^(3 + m)*ArcTan[a*x])/(3 + m) -
 (a*c*x^(2 + m)*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, -(a^2*x^2)])/(2
+ 3*m + m^2) - (a^3*c*x^(4 + m)*Hypergeometric2F1[1, (4 + m)/2, (6 + m)/2,
-(a^2*x^2)])/(12 + 7*m + m^2)
```

Rule 4950

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
```

```
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int x^m\left(c+a^2cx^2\right)\tan^{-1}(ax)\,dx = c\int x^m\tan^{-1}(ax)\,dx+\left(a^2c\right)\int x^{2+m}\tan^{-1}(ax)\,dx$$
$$=\frac{cx^{1+m}\tan^{-1}(ax)}{1+m}+\frac{a^2cx^{3+m}\tan^{-1}(ax)}{3+m}-\frac{(ac)\int\frac{x^{1+m}}{1+a^2x^2}\,dx}{1+m}-\frac{\left(a^3c\right)\int\frac{x^{3+m}}{1+a^2x^2}\,dx}{3+m}$$
$$=\frac{cx^{1+m}\tan^{-1}(ax)}{1+m}+\frac{a^2cx^{3+m}\tan^{-1}(ax)}{3+m}-\frac{acx^{2+m}\,{}_2F_1\left(1,\frac{2+m}{2};\frac{4+m}{2};-a^2x^2\right)}{2+3m+m^2}-\frac{a^3cx^{4+m}\,{}_2}{}$$

Mathematica [A] time = 0.110708, size = 111, normalized size = 0.9

$$cx^{m+1}\left(-\frac{a^3x^3\text{Hypergeometric2F1}\left(1,\frac{m+4}{2},\frac{m+6}{2},-a^2x^2\right)}{m^2+7m+12}-\frac{ax\text{Hypergeometric2F1}\left(1,\frac{m+2}{2},\frac{m+4}{2},-a^2x^2\right)}{m^2+3m+2}+\left(\frac{a^2x^2}{m+3}+\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(c + a^2*c*x^2)*ArcTan[a*x],x]
```

```
[Out] c*x^(1 + m)*(((1 + m)^(-1) + (a^2*x^2)/(3 + m))*ArcTan[a*x] - (a*x*Hypergeo
metric2F1[1, (2 + m)/2, (4 + m)/2, -(a^2*x^2)])/(2 + 3*m + m^2) - (a^3*x^3*
Hypergeometric2F1[1, (4 + m)/2, (6 + m)/2, -(a^2*x^2)])/(12 + 7*m + m^2))
```

Maple [C] time = 0.454, size = 222, normalized size = 1.8

$$\frac{a^{-1-m}c}{4}\left(-4\frac{x^ma^m\left(mx^2a^2-m-2\right)}{(3+m)\,m\,(2+m)}+8\frac{x^{4+m}a^{4+m}\arctan\left(\sqrt{a^2x^2}\right)}{(6+2\,m)\sqrt{a^2x^2}}+2\frac{x^ma^m\,(-4-m)\,\text{LerchPhi}\left(-a^2x^2,1,m/2\right)}{(4+m)\,(3+m)}\right)+\frac{a^{-}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)*arctan(a*x),x)
```

```
[Out] 1/4*a^(-1-m)*c*(-4*x^m*a^m*(a^2*m*x^2-m-2)/(3+m)/m/(2+m)+8*x^(4+m)*a^(4+m)/
(6+2*m)/(a^2*x^2)^(1/2)*arctan((a^2*x^2)^(1/2))+2/(4+m)*x^m*a^m*(-4-m)/(3+m
)*LerchPhi(-a^2*x^2,1,1/2*m))+1/4*a^(-1-m)*c*(4/(2+m)*x^m*a^m*(-m-2)/(1+m)/
m+8*x^(2+m)*a^(2+m)/(2+2*m)/(a^2*x^2)^(1/2)*arctan((a^2*x^2)^(1/2))+2*x^m*a
^m/(1+m)*LerchPhi(-a^2*x^2,1,1/2*m))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)*arctan(a*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^2cx^2+c\right)x^m\arctan\left(ax\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)*arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)*x^m*arctan(a*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$c\left(\int x^m\,\mathrm{atan}\left(ax\right)dx+\int a^2x^2x^m\,\mathrm{atan}\left(ax\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)*atan(a*x),x)
```

```
[Out] c*(Integral(x**m*atan(a*x), x) + Integral(a**2*x**2*x**m*atan(a*x), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2+c\right)x^m \arctan(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)*arctan(a*x),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)*x^m*arctan(a*x), x)
```

3.251 $\int \frac{x^m \tan^{-1}(ax)}{c+a^2cx^2} \, dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{x^m \tan^{-1}(ax)}{a^2cx^2+c},x\right)$$

```
[Out] Unintegrable[(x^m*ArcTan[a*x])/(c + a^2*c*x^2), x]
```

Rubi [A] time = 0.0479666, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m \tan^{-1}(ax)}{c+a^2cx^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*ArcTan[a*x])/(c + a^2*c*x^2),x]
```

```
[Out] Defer[Int][(x^m*ArcTan[a*x])/(c + a^2*c*x^2), x]
```

Rubi steps

$$\int \frac{x^m \tan^{-1}(ax)}{c+a^2cx^2} \, dx = \int \frac{x^m \tan^{-1}(ax)}{c+a^2cx^2} \, dx$$

Mathematica [A] time = 0.851674, size = 0, normalized size = 0.

$$\int \frac{x^m \tan^{-1}(ax)}{c+a^2cx^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*ArcTan[a*x])/(c + a^2*c*x^2),x]
```

```
[Out] Integrate[(x^m*ArcTan[a*x])/(c + a^2*c*x^2), x]
```

Maple [A] time = 0.499, size = 0, normalized size = 0.

$$\int \frac{x^m \arctan(ax)}{a^2 c x^2 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arctan(a*x)/(a^2*c*x^2+c),x)
```

```
[Out] int(x^m*arctan(a*x)/(a^2*c*x^2+c),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m \arctan(ax)}{a^2 c x^2 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] integrate(x^m*arctan(a*x)/(a^2*c*x^2 + c), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^m \arctan(ax)}{a^2 c x^2 + c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(x^m*arctan(a*x)/(a^2*c*x^2 + c), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^m \operatorname{atan}(ax)}{a^2 x^2 + 1} dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*atan(a*x)/(a**2*c*x**2+c),x)
```

```
[Out] Integral(x**m*atan(a*x)/(a**2*x**2 + 1), x)/c
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m \arctan(ax)}{a^2cx^2 + c} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(x^m*arctan(a*x)/(a^2*c*x^2 + c), x)
```

3.252 $\int \frac{x^m \tan^{-1}(ax)}{\left(c+a^2cx^2\right)^2} dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{x^m \tan^{-1}(ax)}{\left(a^2cx^2+c\right)^2}, x\right)$$

```
[Out] Unintegrable[(x^m*ArcTan[a*x])/(c + a^2*c*x^2)^2, x]
```

Rubi [A] time = 0.0498953, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m \tan^{-1}(ax)}{\left(c+a^2cx^2\right)^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*ArcTan[a*x])/(c + a^2*c*x^2)^2,x]
```

```
[Out] Defer[Int][(x^m*ArcTan[a*x])/(c + a^2*c*x^2)^2, x]
```

Rubi steps

$$\int \frac{x^m \tan^{-1}(ax)}{\left(c+a^2cx^2\right)^2} dx = \int \frac{x^m \tan^{-1}(ax)}{\left(c+a^2cx^2\right)^2} dx$$

Mathematica [A] time = 0.604582, size = 0, normalized size = 0.

$$\int \frac{x^m \tan^{-1}(ax)}{\left(c+a^2cx^2\right)^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*ArcTan[a*x])/(c + a^2*c*x^2)^2,x]
```

```
[Out] Integrate[(x^m*ArcTan[a*x])/(c + a^2*c*x^2)^2, x]
```

Maple [A] time = 1.122, size = 0, normalized size = 0.

$$\int \frac{x^m \arctan(ax)}{\left(a^2cx^2+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arctan(a*x)/(a^2*c*x^2+c)^2,x)
```

```
[Out] int(x^m*arctan(a*x)/(a^2*c*x^2+c)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m \arctan(ax)}{\left(a^2cx^2+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^m*arctan(a*x)/(a^2*c*x^2 + c)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^m \arctan(ax)}{a^4c^2x^4+2\,a^2c^2x^2+c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] integral(x^m*arctan(a*x)/(a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^m \operatorname{atan}(ax)}{a^4x^4+2a^2x^2+1}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*atan(a*x)/(a**2*c*x**2+c)**2,x)
```

```
[Out] Integral(x**m*atan(a*x)/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m \arctan(ax)}{\left(a^2cx^2+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(x^m*arctan(a*x)/(a^2*c*x^2 + c)^2, x)
```

3.253 $\int x^m \left(c+a^2 c x^2\right)^{5/2} \tan^{-1}(ax) \, dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(x^m \left(a^2 c x^2+c\right)^{5/2} \tan^{-1}(ax),x\right)$$

```
[Out] Unintegrable[x^m*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x], x]
```

Rubi [A] time = 0.0855001, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^m \left(c+a^2 c x^2\right)^{5/2} \tan^{-1}(ax) \, dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x],x]
```

```
[Out] Defer[Int][x^m*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x], x]
```

Rubi steps

$$\int x^m \left(c+a^2 c x^2\right)^{5/2} \tan^{-1}(ax) \, dx=\int x^m \left(c+a^2 c x^2\right)^{5/2} \tan^{-1}(ax) \, dx$$

Mathematica [A] time = 0.91851, size = 0, normalized size = 0.

$$\int x^m \left(c+a^2 c x^2\right)^{5/2} \tan^{-1}(ax) \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x],x]
```

```
[Out] Integrate[x^m*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x], x]
```

Maple [A] time = 0.53, size = 0, normalized size = 0.

$$\int x^{m} \left(a^{2} c \,x^{2}+c \right)^{\frac{5}{2}} \arctan \left(a x \right)d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)^(5/2)*arctan(a*x),x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)^(5/2)*arctan(a*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(a^{2} c x^{2}+c\right)^{\frac{5}{2}} x^{m} \arctan \left(a x\right)\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(5/2)*arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(5/2)*x^m*arctan(a*x), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$${\rm integral}\left({\left(a^{4} c^{2} x^{4} + 2 \, a^{2} c^{2} x^{2} + c^{2}\right)} \sqrt{a^{2} c x^{2} + c} x^{m} \arctan \left(a x\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(5/2)*arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*x^m*arctan
(a*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**(5/2)*atan(a*x),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(5/2)*arctan(a*x),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.254 $\int x^m \left(c+a^2 c x^2\right)^{3/2} \tan^{-1}(ax) \, dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(x^m \left(a^2 c x^2+c\right)^{3/2} \tan^{-1}(ax),x\right)$$

```
[Out] Unintegrable[x^m*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x], x]
```

Rubi [A] time = 0.0812673, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int x^m \left(c+a^2 c x^2\right)^{3/2} \tan^{-1}(ax) \, dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x],x]
```

```
[Out] Defer[Int][x^m*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x], x]
```

Rubi steps

$$\int x^m \left(c+a^2 c x^2\right)^{3/2} \tan^{-1}(ax) \, dx=\int x^m \left(c+a^2 c x^2\right)^{3/2} \tan^{-1}(ax) \, dx$$

Mathematica [A] time = 0.494558, size = 0, normalized size = 0.

$$\int x^m \left(c+a^2 c x^2\right)^{3/2} \tan^{-1}(ax) \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x],x]
```

```
[Out] Integrate[x^m*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x], x]
```

Maple [A] time = 0.482, size = 0, normalized size = 0.

$$\int x^{m} \left(a^{2} c \,x^{2}+c \right)^{\frac{3}{2}} \arctan \left(a x \right)d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x),x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(a^{2} c x^{2} + c\right)^{\frac{3}{2}} x^{m} \arctan\left(a x\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)*x^m*arctan(a*x), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left({\left(a^{2} c x^{2} + c\right)}^{\frac{3}{2}} x^{m} \arctan\left(a x\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)^(3/2)*x^m*arctan(a*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**(3/2)*atan(a*x),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.255 $\int x^m \sqrt{c+a^2 c x^2} \tan^{-1}(ax) \, dx$

Optimal. Leaf size=112

$$-\frac{a x^{m+2} \sqrt{a^2 c x^2+c}\,\text{Hypergeometric2F1}\left(1,\frac{m+3}{2},\frac{m+4}{2},-a^2 x^2\right)}{(m+2)^2}+\frac{c\,\text{Unintegrable}\left(\frac{x^m \tan^{-1}(ax)}{\sqrt{a^2 c x^2+c}},x\right)}{m+2}+\frac{x^{m+1} \sqrt{a^2 c x^2+c}\,\tan^{-1}(ax)}{m+2}$$

```
[Out] (x^(1 + m)*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(2 + m) - (a*x^(2 + m)*Sqrt[c +
 a^2*c*x^2]*Hypergeometric2F1[1, (3 + m)/2, (4 + m)/2, -(a^2*x^2)])/(2 + m)
^2 + (c*Unintegrable[(x^m*ArcTan[a*x])/Sqrt[c + a^2*c*x^2], x])/(2 + m)
```

Rubi [A] time = 0.17032, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^m \sqrt{c+a^2 c x^2} \tan^{-1}(ax) \, dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*Sqrt[c + a^2*c*x^2]*ArcTan[a*x],x]
```

```
[Out] (x^(1 + m)*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(2 + m) - (a*c*x^(2 + m)*Sqrt[1
 + a^2*x^2]*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, -(a^2*x^2)])/((2 +
 m)^2*Sqrt[c + a^2*c*x^2]) + (c*Defer[Int][(x^m*ArcTan[a*x])/Sqrt[c + a^2*c
*x^2], x])/(2 + m)
```

Rubi steps

$$\begin{aligned}
\int x^m \sqrt{c+a^2 c x^2} \tan^{-1}(ax) \, dx &= \frac{x^{1+m} \sqrt{c+a^2 c x^2} \tan^{-1}(ax)}{2+m}+\frac{c \int \frac{x^m \tan^{-1}(ax)}{\sqrt{c+a^2 c x^2}} \, dx}{2+m}-\frac{(ac) \int \frac{x^{1+m}}{\sqrt{c+a^2 c x^2}} \, dx}{2+m}\\
&= \frac{x^{1+m} \sqrt{c+a^2 c x^2} \tan^{-1}(ax)}{2+m}+\frac{c \int \frac{x^m \tan^{-1}(ax)}{\sqrt{c+a^2 c x^2}} \, dx}{2+m}-\frac{\left(ac\sqrt{1+a^2 x^2}\right) \int \frac{x^{1+m}}{\sqrt{1+a^2 x^2}} \, dx}{(2+m)\sqrt{c+a^2 c x^2}}\\
&= \frac{x^{1+m} \sqrt{c+a^2 c x^2} \tan^{-1}(ax)}{2+m}-\frac{a c x^{2+m} \sqrt{1+a^2 x^2}\, {}_2F_1\left(\frac{1}{2},\frac{2+m}{2};\frac{4+m}{2};-a^2 x^2\right)}{(2+m)^2 \sqrt{c+a^2 c x^2}}+\frac{c \int \frac{x^m \tan^{-1}(ax)}{\sqrt{c+a^2 c x^2}} \, dx}{2+m}
\end{aligned}$$

Mathematica [A] time = 0.10003, size = 0, normalized size = 0.

$$\int x^m \sqrt{c + a^2 c x^2} \tan^{-1}(ax)\, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*Sqrt[c + a^2*c*x^2]*ArcTan[a*x],x]
```

```
[Out] Integrate[x^m*Sqrt[c + a^2*c*x^2]*ArcTan[a*x], x]
```

Maple [A] time = 0.596, size = 0, normalized size = 0.

$$\int x^m \sqrt{a^2 c x^2 + c} \arctan(ax)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)^(1/2)*arctan(a*x),x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)^(1/2)*arctan(a*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a^2 c x^2 + c} x^m \arctan(ax)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(1/2)*arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)*x^m*arctan(a*x), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{a^2 c x^2 + c} x^m \arctan(ax), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(1/2)*arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^m*arctan(a*x), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int x^m\sqrt{c\left(a^2x^2+1\right)}\operatorname{atan}(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**(1/2)*atan(a*x),x)
```

```
[Out] Integral(x**m*sqrt(c*(a**2*x**2 + 1))*atan(a*x), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(1/2)*arctan(a*x),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.256 $\int \frac{x^m \tan^{-1}(ax)}{\sqrt{c+a^2cx^2}} \, dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{x^m \tan^{-1}(ax)}{\sqrt{a^2cx^2+c}}, x\right)$$

```
[Out] Unintegrable[(x^m*ArcTan[a*x])/Sqrt[c + a^2*c*x^2], x]
```

Rubi [A] time = 0.0726201, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m \tan^{-1}(ax)}{\sqrt{c+a^2cx^2}} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*ArcTan[a*x])/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] Defer[Int][(x^m*ArcTan[a*x])/Sqrt[c + a^2*c*x^2], x]
```

Rubi steps

$$\int \frac{x^m \tan^{-1}(ax)}{\sqrt{c+a^2cx^2}} \, dx = \int \frac{x^m \tan^{-1}(ax)}{\sqrt{c+a^2cx^2}} \, dx$$

Mathematica [A] time = 0.443544, size = 0, normalized size = 0.

$$\int \frac{x^m \tan^{-1}(ax)}{\sqrt{c+a^2cx^2}} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*ArcTan[a*x])/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] Integrate[(x^m*ArcTan[a*x])/Sqrt[c + a^2*c*x^2], x]
```

Maple [A] time = 0.961, size = 0, normalized size = 0.

$$\int x^{m} \arctan(ax)\frac{1}{\sqrt{a^{2}c x^{2}+c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arctan(a*x)/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(x^m*arctan(a*x)/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{m} \arctan(ax)}{\sqrt{a^{2}c x^{2}+c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m*arctan(a*x)/sqrt(a^2*c*x^2 + c), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^{m} \arctan(ax)}{\sqrt{a^{2}c x^{2}+c}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^m*arctan(a*x)/sqrt(a^2*c*x^2 + c), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{m} \operatorname{atan}(ax)}{\sqrt{c\left(a^{2}x^{2}+1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*atan(a*x)/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**m*atan(a*x)/sqrt(c*(a**2*x**2 + 1)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m \arctan(ax)}{\sqrt{a^2cx^2 + c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^m*arctan(a*x)/sqrt(a^2*c*x^2 + c), x)
```

3.257 $\int \frac{x^m \tan^{-1}(ax)}{\left(c+a^2cx^2\right)^{3/2}} dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{x^m \tan^{-1}(ax)}{\left(a^2cx^2+c\right)^{3/2}},x\right)$$

```
[Out] Unintegrable[(x^m*ArcTan[a*x])/(c + a^2*c*x^2)^(3/2), x]
```

Rubi [A] time = 0.0843549, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m \tan^{-1}(ax)}{\left(c+a^2cx^2\right)^{3/2}} dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*ArcTan[a*x])/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] Defer[Int][(x^m*ArcTan[a*x])/(c + a^2*c*x^2)^(3/2), x]
```

Rubi steps

$$\int \frac{x^m \tan^{-1}(ax)}{\left(c+a^2cx^2\right)^{3/2}} dx = \int \frac{x^m \tan^{-1}(ax)}{\left(c+a^2cx^2\right)^{3/2}} dx$$

Mathematica [A] time = 0.509031, size = 0, normalized size = 0.

$$\int \frac{x^m \tan^{-1}(ax)}{\left(c+a^2cx^2\right)^{3/2}} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*ArcTan[a*x])/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] Integrate[(x^m*ArcTan[a*x])/(c + a^2*c*x^2)^(3/2), x]
```

Maple [A] time = 1.092, size = 0, normalized size = 0.

$$\int x^{m}\arctan\left(ax\right)\left(a^{2}cx^{2}+c\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arctan(a*x)/(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] int(x^m*arctan(a*x)/(a^2*c*x^2+c)^(3/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{m}\arctan\left(ax\right)}{\left(a^{2}cx^{2}+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m*arctan(a*x)/(a^2*c*x^2 + c)^(3/2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{a^{2}cx^{2}+c}\,x^{m}\arctan\left(ax\right)}{a^{4}c^{2}x^{4}+2\,a^{2}c^{2}x^{2}+c^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^m*arctan(a*x)/(a^4*c^2*x^4 + 2*a^2*c^2*x^2 +
 c^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*atan(a*x)/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m \arctan(ax)}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^m*arctan(a*x)/(a^2*c*x^2 + c)^(3/2), x)
```

3.258 $\int x^3 \left(c + a^2 c x^2\right) \tan^{-1}(ax)^2 \, dx$

Optimal. Leaf size=124

$$-\frac{cx^2}{180a^2} - \frac{7c \log\left(a^2x^2+1\right)}{90a^4} + \frac{1}{6}a^2cx^6 \tan^{-1}(ax)^2 + \frac{cx \tan^{-1}(ax)}{6a^3} - \frac{c \tan^{-1}(ax)^2}{12a^4} - \frac{1}{15}acx^5 \tan^{-1}(ax) + \frac{1}{4}cx^4 \tan^{-1}(ax)$$

```
[Out] -(c*x^2)/(180*a^2) + (c*x^4)/60 + (c*x*ArcTan[a*x])/(6*a^3) - (c*x^3*ArcTan
[a*x])/(18*a) - (a*c*x^5*ArcTan[a*x])/15 - (c*ArcTan[a*x]^2)/(12*a^4) + (c*
x^4*ArcTan[a*x]^2)/4 + (a^2*c*x^6*ArcTan[a*x]^2)/6 - (7*c*Log[1 + a^2*x^2])
/(90*a^4)
```

Rubi [A] time = 0.425179, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 26, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {4950, 4852, 4916, 266, 43, 4846, 260, 4884}

$$-\frac{cx^2}{180a^2} - \frac{7c \log\left(a^2x^2+1\right)}{90a^4} + \frac{1}{6}a^2cx^6 \tan^{-1}(ax)^2 + \frac{cx \tan^{-1}(ax)}{6a^3} - \frac{c \tan^{-1}(ax)^2}{12a^4} - \frac{1}{15}acx^5 \tan^{-1}(ax) + \frac{1}{4}cx^4 \tan^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(c + a^2*c*x^2)*ArcTan[a*x]^2,x]
```

```
[Out] -(c*x^2)/(180*a^2) + (c*x^4)/60 + (c*x*ArcTan[a*x])/(6*a^3) - (c*x^3*ArcTan
[a*x])/(18*a) - (a*c*x^5*ArcTan[a*x])/15 - (c*ArcTan[a*x]^2)/(12*a^4) + (c*
x^4*ArcTan[a*x]^2)/4 + (a^2*c*x^6*ArcTan[a*x]^2)/6 - (7*c*Log[1 + a^2*x^2])
/(90*a^4)
```

Rule 4950

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
```

```
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rubi steps

$$\int x^3\left(c+a^2cx^2\right)\tan^{-1}(ax)^2\,dx = c\int x^3\tan^{-1}(ax)^2\,dx + \left(a^2c\right)\int x^5\tan^{-1}(ax)^2\,dx$$

$$= \frac{1}{4}cx^4\tan^{-1}(ax)^2 + \frac{1}{6}a^2cx^6\tan^{-1}(ax)^2 - \frac{1}{2}(ac)\int\frac{x^4\tan^{-1}(ax)}{1+a^2x^2}\,dx - \frac{1}{3}\left(a^3c\right)\int\frac{x^6\tan}{1+}$$

$$= \frac{1}{4}cx^4\tan^{-1}(ax)^2 + \frac{1}{6}a^2cx^6\tan^{-1}(ax)^2 - \frac{c\int x^2\tan^{-1}(ax)\,dx}{2a} + \frac{c\int\frac{x^2\tan^{-1}(ax)}{1+a^2x^2}\,dx}{2a} - \frac{1}{3}$$

$$= -\frac{cx^3\tan^{-1}(ax)}{6a} - \frac{1}{15}acx^5\tan^{-1}(ax) + \frac{1}{4}cx^4\tan^{-1}(ax)^2 + \frac{1}{6}a^2cx^6\tan^{-1}(ax)^2 + \frac{1}{6}c\int$$

$$= \frac{cx\tan^{-1}(ax)}{2a^3} - \frac{cx^3\tan^{-1}(ax)}{18a} - \frac{1}{15}acx^5\tan^{-1}(ax) - \frac{c\tan^{-1}(ax)^2}{4a^4} + \frac{1}{4}cx^4\tan^{-1}(ax)^2$$

$$= \frac{cx\tan^{-1}(ax)}{6a^3} - \frac{cx^3\tan^{-1}(ax)}{18a} - \frac{1}{15}acx^5\tan^{-1}(ax) - \frac{c\tan^{-1}(ax)^2}{12a^4} + \frac{1}{4}cx^4\tan^{-1}(ax)^2$$

$$= \frac{cx^2}{20a^2} + \frac{cx^4}{60} + \frac{cx\tan^{-1}(ax)}{6a^3} - \frac{cx^3\tan^{-1}(ax)}{18a} - \frac{1}{15}acx^5\tan^{-1}(ax) - \frac{c\tan^{-1}(ax)^2}{12a^4} + \frac{1}{4}$$

$$= -\frac{cx^2}{180a^2} + \frac{cx^4}{60} + \frac{cx\tan^{-1}(ax)}{6a^3} - \frac{cx^3\tan^{-1}(ax)}{18a} - \frac{1}{15}acx^5\tan^{-1}(ax) - \frac{c\tan^{-1}(ax)^2}{12a^4} +$$

Mathematica [A] time = 0.0382577, size = 89, normalized size = 0.72

$$\frac{c\left(3a^4x^4 - a^2x^2 - 14\log\left(a^2x^2+1\right) - 2ax\left(6a^4x^4+5a^2x^2-15\right)\tan^{-1}(ax) + 15\left(2a^6x^6+3a^4x^4-1\right)\tan^{-1}(ax)^2\right)}{180a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(c + a^2*c*x^2)*ArcTan[a*x]^2,x]
```

```
[Out] (c*(-(a^2*x^2) + 3*a^4*x^4 - 2*a*x*(-15 + 5*a^2*x^2 + 6*a^4*x^4)*ArcTan[a*x
] + 15*(-1 + 3*a^4*x^4 + 2*a^6*x^6)*ArcTan[a*x]^2 - 14*Log[1 + a^2*x^2]))/(
180*a^4)
```

Maple [A] time = 0.033, size = 107, normalized size = 0.9

$$-\frac{cx^2}{180\,a^2} + \frac{cx^4}{60} + \frac{cx\arctan(ax)}{6\,a^3} - \frac{cx^3\arctan(ax)}{18\,a} - \frac{acx^5\arctan(ax)}{15} - \frac{c(\arctan(ax))^2}{12\,a^4} + \frac{cx^4(\arctan(ax))^2}{4} + \frac{a^2cx}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a^2*c*x^2+c)*arctan(a*x)^2,x)
```

```
[Out] -1/180*c*x^2/a^2+1/60*c*x^4+1/6*c*x*arctan(a*x)/a^3-1/18*c*x^3*arctan(a*x)/
a-1/15*a*c*x^5*arctan(a*x)-1/12*c*arctan(a*x)^2/a^4+1/4*c*x^4*arctan(a*x)^2
+1/6*a^2*c*x^6*arctan(a*x)^2-7/90*c*ln(a^2*x^2+1)/a^4
```

Maxima [A] time = 1.52954, size = 157, normalized size = 1.27

$$-\frac{1}{90}\,a\left(\frac{6\,a^4cx^5+5\,a^2cx^3-15\,cx}{a^4}+\frac{15\,c\arctan(ax)}{a^5}\right)\arctan(ax)+\frac{1}{12}\left(2\,a^2cx^6+3\,cx^4\right)\arctan(ax)^2+\frac{3\,a^4cx^4-a^2cx^2+\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a^2*c*x^2+c)*arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] -1/90*a*((6*a^4*c*x^5 + 5*a^2*c*x^3 - 15*c*x)/a^4 + 15*c*arctan(a*x)/a^5)*a
rctan(a*x) + 1/12*(2*a^2*c*x^6 + 3*c*x^4)*arctan(a*x)^2 + 1/180*(3*a^4*c*x^
4 - a^2*c*x^2 + 15*c*arctan(a*x)^2 - 14*c*log(a^2*x^2 + 1))/a^4
```

Fricas [A] time = 2.2027, size = 225, normalized size = 1.81

$$\frac{3\,a^4cx^4-a^2cx^2+15\left(2\,a^6cx^6+3\,a^4cx^4-c\right)\arctan(ax)^2-2\left(6\,a^5cx^5+5\,a^3cx^3-15\,acx\right)\arctan(ax)-14\,c\log\left(a^2x^2+1\right.}{180\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a^2*c*x^2+c)*arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/180*(3*a^4*c*x^4 - a^2*c*x^2 + 15*(2*a^6*c*x^6 + 3*a^4*c*x^4 - c)*arctan(
a*x)^2 - 2*(6*a^5*c*x^5 + 5*a^3*c*x^3 - 15*a*c*x)*arctan(a*x) - 14*c*log(a^
2*x^2 + 1))/a^4
```

Sympy [A] time = 2.808, size = 121, normalized size = 0.98

$$\begin{cases}\frac{a^2cx^6\operatorname{atan}^2(ax)}{6}-\frac{acx^5\operatorname{atan}(ax)}{15}+\frac{cx^4\operatorname{atan}^2(ax)}{4}+\frac{cx^4}{60}-\frac{cx^3\operatorname{atan}(ax)}{18a}-\frac{cx^2}{180a^2}+\frac{cx\operatorname{atan}(ax)}{6a^3}-\frac{7c\log\left(x^2+\frac{1}{a^2}\right)}{90a^4}-\frac{c\operatorname{atan}^2(ax)}{12a^4} & \text{for}\ a\neq 0\\ 0 & \text{otherwis}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a**2*c*x**2+c)*atan(a*x)**2,x)
```

```
[Out] Piecewise((a**2*c*x**6*atan(a*x)**2/6 - a*c*x**5*atan(a*x)/15 + c*x**4*atan
(a*x)**2/4 + c*x**4/60 - c*x**3*atan(a*x)/(18*a) - c*x**2/(180*a**2) + c*x*
atan(a*x)/(6*a**3) - 7*c*log(x**2 + a**(-2))/(90*a**4) - c*atan(a*x)**2/(12
*a**4), Ne(a, 0)), (0, True))
```

Giac [A] time = 1.15108, size = 142, normalized size = 1.15

$$\frac{1}{12}\left(2\,a^2cx^6+3\,cx^4\right)\arctan\left(ax\right)^2-\frac{12\,a^5cx^5\arctan\left(ax\right)-3\,a^4cx^4+10\,a^3cx^3\arctan\left(ax\right)+a^2cx^2-30\,acx\arctan\left(ax\right)}{180\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a^2*c*x^2+c)*arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] 1/12*(2*a^2*c*x^6 + 3*c*x^4)*arctan(a*x)^2 - 1/180*(12*a^5*c*x^5*arctan(a*x
) - 3*a^4*c*x^4 + 10*a^3*c*x^3*arctan(a*x) + a^2*c*x^2 - 30*a*c*x*arctan(a*
x) + 15*c*arctan(a*x)^2 + 14*c*log(a^2*x^2 + 1))/a^4
```

3.259 $\int x^2 \left(c+a^2 c x^2\right) \tan^{-1}(ax)^2 \, dx$

Optimal. Leaf size=156

$$-\frac{2ic\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{15a^3}+\frac{1}{5}a^2cx^5\tan^{-1}(ax)^2+\frac{cx}{30a^2}-\frac{2ic\tan^{-1}(ax)^2}{15a^3}-\frac{c\tan^{-1}(ax)}{30a^3}-\frac{4c\log\left(\frac{2}{1+iax}\right)\tan^{-1}(ax)}{15a^3}-\frac{1}{10}$$

```
[Out] (c*x)/(30*a^2) + (c*x^3)/30 - (c*ArcTan[a*x])/(30*a^3) - (2*c*x^2*ArcTan[a*
x])/(15*a) - (a*c*x^4*ArcTan[a*x])/10 - (((2*I)/15)*c*ArcTan[a*x]^2)/a^3 +
(c*x^3*ArcTan[a*x]^2)/3 + (a^2*c*x^5*ArcTan[a*x]^2)/5 - (4*c*ArcTan[a*x]*Lo
g[2/(1 + I*a*x)])/(15*a^3) - (((2*I)/15)*c*PolyLog[2, 1 - 2/(1 + I*a*x)])/a
^3
```

Rubi [A] time = 0.40942, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4950, 4852, 4916, 321, 203, 4920, 4854, 2402, 2315, 302}

$$-\frac{2ic\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{15a^3}+\frac{1}{5}a^2cx^5\tan^{-1}(ax)^2+\frac{cx}{30a^2}-\frac{2ic\tan^{-1}(ax)^2}{15a^3}-\frac{c\tan^{-1}(ax)}{30a^3}-\frac{4c\log\left(\frac{2}{1+iax}\right)\tan^{-1}(ax)}{15a^3}-\frac{1}{10}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(c + a^2*c*x^2)*ArcTan[a*x]^2,x]
```


```
[Out] (c*x)/(30*a^2) + (c*x^3)/30 - (c*ArcTan[a*x])/(30*a^3) - (2*c*x^2*ArcTan[a*
x])/(15*a) - (a*c*x^4*ArcTan[a*x])/10 - (((2*I)/15)*c*ArcTan[a*x]^2)/a^3 +
(c*x^3*ArcTan[a*x]^2)/3 + (a^2*c*x^5*ArcTan[a*x]^2)/5 - (4*c*ArcTan[a*x]*Lo
g[2/(1 + I*a*x)])/(15*a^3) - (((2*I)/15)*c*PolyLog[2, 1 - 2/(1 + I*a*x)])/a
^3
```


Rule 4950

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
```

```
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\int x^2\left(c+a^2cx^2\right)\tan^{-1}(ax)^2\,dx = c\int x^2\tan^{-1}(ax)^2\,dx + \left(a^2c\right)\int x^4\tan^{-1}(ax)^2\,dx$$

$$= \frac{1}{3}cx^3\tan^{-1}(ax)^2 + \frac{1}{5}a^2cx^5\tan^{-1}(ax)^2 - \frac{1}{3}(2ac)\int\frac{x^3\tan^{-1}(ax)}{1+a^2x^2}\,dx - \frac{1}{5}\left(2a^3c\right)\int\frac{x^5\tan}{1+}$$

$$= \frac{1}{3}cx^3\tan^{-1}(ax)^2 + \frac{1}{5}a^2cx^5\tan^{-1}(ax)^2 - \frac{(2c)\int x\tan^{-1}(ax)\,dx}{3a} + \frac{(2c)\int\frac{x\tan^{-1}(ax)}{1+a^2x^2}\,dx}{3a} -$$

$$= -\frac{cx^2\tan^{-1}(ax)}{3a} - \frac{1}{10}acx^4\tan^{-1}(ax) - \frac{ic\tan^{-1}(ax)^2}{3a^3} + \frac{1}{3}cx^3\tan^{-1}(ax)^2 + \frac{1}{5}a^2cx^5\tan$$

$$= \frac{cx}{3a^2} - \frac{2cx^2\tan^{-1}(ax)}{15a} - \frac{1}{10}acx^4\tan^{-1}(ax) - \frac{2ic\tan^{-1}(ax)^2}{15a^3} + \frac{1}{3}cx^3\tan^{-1}(ax)^2 + \frac{1}{5}a^2c$$

$$= \frac{cx}{30a^2} + \frac{cx^3}{30} - \frac{c\tan^{-1}(ax)}{3a^3} - \frac{2cx^2\tan^{-1}(ax)}{15a} - \frac{1}{10}acx^4\tan^{-1}(ax) - \frac{2ic\tan^{-1}(ax)^2}{15a^3} + \frac{1}{3}$$

$$= \frac{cx}{30a^2} + \frac{cx^3}{30} - \frac{c\tan^{-1}(ax)}{30a^3} - \frac{2cx^2\tan^{-1}(ax)}{15a} - \frac{1}{10}acx^4\tan^{-1}(ax) - \frac{2ic\tan^{-1}(ax)^2}{15a^3} + \frac{1}{3}$$

$$= \frac{cx}{30a^2} + \frac{cx^3}{30} - \frac{c\tan^{-1}(ax)}{30a^3} - \frac{2cx^2\tan^{-1}(ax)}{15a} - \frac{1}{10}acx^4\tan^{-1}(ax) - \frac{2ic\tan^{-1}(ax)^2}{15a^3} + \frac{1}{3}$$

Mathematica [A] time = 0.607208, size = 104, normalized size = 0.67

$$\frac{c\left(4i\text{PolyLog}\left(2,-e^{2i\tan^{-1}(ax)}\right)+a^3x^3+2\left(3a^5x^5+5a^3x^3+2i\right)\tan^{-1}(ax)^2-\tan^{-1}(ax)\left(3a^4x^4+4a^2x^2+8\log\left(1+e^{2i\tan}\right.\right.\right.}{30a^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(c + a^2*c*x^2)*ArcTan[a*x]^2,x]
```

```
[Out] (c*(a*x + a^3*x^3 + 2*(2*I + 5*a^3*x^3 + 3*a^5*x^5)*ArcTan[a*x]^2 - ArcTan[
a*x]*(1 + 4*a^2*x^2 + 3*a^4*x^4 + 8*Log[1 + E^((2*I)*ArcTan[a*x])]) + (4*I)
*PolyLog[2, -E^((2*I)*ArcTan[a*x])]))/(30*a^3)
```

Maple [A] time = 0.095, size = 258, normalized size = 1.7

$$\frac{a^2cx^5 (\arctan (ax))^2}{5} + \frac{cx^3 (\arctan (ax))^2}{3} - \frac{acx^4 \arctan (ax)}{10} - \frac{2\,cx^2 \arctan (ax)}{15\,a} + \frac{2\,c \arctan (ax) \ln \left(a^2x^2+1\right)}{15\,a^3} + \frac{cx}{30}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a^2*c*x^2+c)*arctan(a*x)^2,x)
```

```
[Out] 1/5*a^2*c*x^5*arctan(a*x)^2+1/3*c*x^3*arctan(a*x)^2-1/10*a*c*x^4*arctan(a*x
)-2/15*c*x^2*arctan(a*x)/a+2/15/a^3*c*arctan(a*x)*ln(a^2*x^2+1)+1/30*c*x^3+
1/30*c*x/a^2-1/30*c*arctan(a*x)/a^3+1/15*I/a^3*c*ln(a*x+I)*ln(1/2*I*(a*x-I)
)+1/15*I/a^3*c*dilog(1/2*I*(a*x-I))-1/15*I/a^3*c*ln(a*x-I)*ln(-1/2*I*(a*x+I
))-1/15*I/a^3*c*dilog(-1/2*I*(a*x+I))+1/30*I/a^3*c*ln(a*x+I)^2+1/15*I/a^3*c
*ln(a^2*x^2+1)*ln(a*x-I)-1/30*I/a^3*c*ln(a*x-I)^2-1/15*I/a^3*c*ln(a^2*x^2+1
)*ln(a*x+I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{60}\left(3\,a^2cx^5+5\,cx^3\right)\arctan (ax)^2 - \frac{1}{240}\left(3\,a^2cx^5+5\,cx^3\right)\log\left(a^2x^2+1\right)^2 + \int \frac{180\left(a^4cx^6+2\,a^2cx^4+cx^2\right)\arctan (ax)^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)*arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/60*(3*a^2*c*x^5 + 5*c*x^3)*arctan(a*x)^2 - 1/240*(3*a^2*c*x^5 + 5*c*x^3)*
log(a^2*x^2 + 1)^2 + integrate(1/240*(180*(a^4*c*x^6 + 2*a^2*c*x^4 + c*x^2)
*arctan(a*x)^2 + 15*(a^4*c*x^6 + 2*a^2*c*x^4 + c*x^2)*log(a^2*x^2 + 1)^2 -
8*(3*a^3*c*x^5 + 5*a*c*x^3)*arctan(a*x) + 4*(3*a^4*c*x^6 + 5*a^2*c*x^4)*log
```

```
(a^2*x^2 + 1))/(a^2*x^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^2cx^4 + cx^2\right)\arctan\left(ax\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)*arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^4 + c*x^2)*arctan(a*x)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$c\left(\int x^2\,\text{atan}^2\left(ax\right)dx + \int a^2x^4\,\text{atan}^2\left(ax\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a**2*c*x**2+c)*atan(a*x)**2,x)
```

```
[Out] c*(Integral(x**2*atan(a*x)**2, x) + Integral(a**2*x**4*atan(a*x)**2, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2 + c\right)x^2\arctan\left(ax\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)*arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)*x^2*arctan(a*x)^2, x)
```

3.260 $\int x\left(c+a^2cx^2\right)\tan^{-1}(ax)^2\,dx$

Optimal. Leaf size=96

$$\frac{c\left(a^2x^2+1\right)}{12a^2}+\frac{c\log\left(a^2x^2+1\right)}{6a^2}+\frac{c\left(a^2x^2+1\right)^2\tan^{-1}(ax)^2}{4a^2}-\frac{cx\left(a^2x^2+1\right)\tan^{-1}(ax)}{6a}-\frac{cx\tan^{-1}(ax)}{3a}$$

```
[Out] (c*(1 + a^2*x^2))/(12*a^2) - (c*x*ArcTan[a*x])/(3*a) - (c*x*(1 + a^2*x^2)*A
rcTan[a*x])/(6*a) + (c*(1 + a^2*x^2)^2*ArcTan[a*x]^2)/(4*a^2) + (c*Log[1 +
a^2*x^2])/(6*a^2)
```

Rubi [A] time = 0.0525261, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4930, 4878, 4846, 260}

$$\frac{c\left(a^2x^2+1\right)}{12a^2}+\frac{c\log\left(a^2x^2+1\right)}{6a^2}+\frac{c\left(a^2x^2+1\right)^2\tan^{-1}(ax)^2}{4a^2}-\frac{cx\left(a^2x^2+1\right)\tan^{-1}(ax)}{6a}-\frac{cx\tan^{-1}(ax)}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[x*(c + a^2*c*x^2)*ArcTan[a*x]^2,x]
```

```
[Out] (c*(1 + a^2*x^2))/(12*a^2) - (c*x*ArcTan[a*x])/(3*a) - (c*x*(1 + a^2*x^2)*A
rcTan[a*x])/(6*a) + (c*(1 + a^2*x^2)^2*ArcTan[a*x]^2)/(4*a^2) + (c*Log[1 +
a^2*x^2])/(6*a^2)
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 4878

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symbo
l] :> -Simp[(b*(d + e*x^2)^q)/(2*c*q*(2*q + 1)), x] + (Dist[(2*d*q)/(2*q +
1), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x]), x], x] + Simp[(x*(d + e*x^
2)^q*(a + b*ArcTan[c*x]))/(2*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] && Eq
Q[e, c^2*d] && GtQ[q, 0]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int x\left(c+a^2cx^2\right)\tan^{-1}(ax)^2\,dx &= \frac{c\left(1+a^2x^2\right)^2\tan^{-1}(ax)^2}{4a^2}-\frac{\int\left(c+a^2cx^2\right)\tan^{-1}(ax)\,dx}{2a}\\ &= \frac{c\left(1+a^2x^2\right)}{12a^2}-\frac{cx\left(1+a^2x^2\right)\tan^{-1}(ax)}{6a}+\frac{c\left(1+a^2x^2\right)^2\tan^{-1}(ax)^2}{4a^2}-\frac{c\int\tan^{-1}(ax)\,dx}{3a}\\ &= \frac{c\left(1+a^2x^2\right)}{12a^2}-\frac{cx\tan^{-1}(ax)}{3a}-\frac{cx\left(1+a^2x^2\right)\tan^{-1}(ax)}{6a}+\frac{c\left(1+a^2x^2\right)^2\tan^{-1}(ax)^2}{4a^2}+\frac{1}{3}c\\ &= \frac{c\left(1+a^2x^2\right)}{12a^2}-\frac{cx\tan^{-1}(ax)}{3a}-\frac{cx\left(1+a^2x^2\right)\tan^{-1}(ax)}{6a}+\frac{c\left(1+a^2x^2\right)^2\tan^{-1}(ax)^2}{4a^2}+\frac{c\,\mathrm{l}}{}\end{aligned}$$

Mathematica [A] time = 0.0296227, size = 64, normalized size = 0.67

$$\frac{c\left(a^2x^2+2\log\left(a^2x^2+1\right)-2ax\left(a^2x^2+3\right)\tan^{-1}(ax)+3\left(a^2x^2+1\right)^2\tan^{-1}(ax)^2\right)}{12a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(c + a^2*c*x^2)*ArcTan[a*x]^2,x]
```

```
[Out] (c*(a^2*x^2 - 2*a*x*(3 + a^2*x^2)*ArcTan[a*x] + 3*(1 + a^2*x^2)^2*ArcTan[a*
x]^2 + 2*Log[1 + a^2*x^2]))/(12*a^2)
```

Maple [A] time = 0.033, size = 85, normalized size = 0.9

$$\frac{a^2c\left(\arctan(ax)\right)^2x^4}{4}+\frac{c\left(\arctan(ax)\right)^2x^2}{2}-\frac{ac\arctan(ax)x^3}{6}-\frac{cx\arctan(ax)}{2a}+\frac{c\left(\arctan(ax)\right)^2}{4a^2}+\frac{cx^2}{12}+\frac{c\ln\left(a^2x^2+\right.}{6a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)*arctan(a*x)^2,x)
```

```
[Out] 1/4*a^2*c*arctan(a*x)^2*x^4+1/2*c*arctan(a*x)^2*x^2-1/6*a*c*arctan(a*x)*x^3
-1/2*c*x*arctan(a*x)/a+1/4/a^2*c*arctan(a*x)^2+1/12*c*x^2+1/6*c*ln(a^2*x^2+
1)/a^2
```

Maxima [A] time = 1.00575, size = 117, normalized size = 1.22

$$\frac{\left(a^2cx^2+c\right)^2\arctan\left(ax\right)^2}{4\,a^2c}+\frac{\left(c^2x^2+\frac{2\,c^2\log\left(a^2x^2+1\right)}{a^2}\right)a-2\left(a^2c^2x^3+3\,c^2x\right)\arctan\left(ax\right)}{12\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)*arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(a^2*c*x^2 + c)^2*arctan(a*x)^2/(a^2*c) + 1/12*((c^2*x^2 + 2*c^2*log(a^
2*x^2 + 1)/a^2)*a - 2*(a^2*c^2*x^3 + 3*c^2*x)*arctan(a*x))/(a*c)
```

Fricas [A] time = 2.20166, size = 177, normalized size = 1.84

$$\frac{a^2cx^2+3\left(a^4cx^4+2\,a^2cx^2+c\right)\arctan\left(ax\right)^2-2\left(a^3cx^3+3\,acx\right)\arctan\left(ax\right)+2\,c\log\left(a^2x^2+1\right)}{12\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)*arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(a^2*c*x^2 + 3*(a^4*c*x^4 + 2*a^2*c*x^2 + c)*arctan(a*x)^2 - 2*(a^3*c*
x^3 + 3*a*c*x)*arctan(a*x) + 2*c*log(a^2*x^2 + 1))/a^2
```

Sympy [A] time = 1.49577, size = 94, normalized size = 0.98

$$\begin{cases} \frac{a^2cx^4\operatorname{atan}^2\left(ax\right)}{4}-\frac{acx^3\operatorname{atan}\left(ax\right)}{6}+\frac{cx^2\operatorname{atan}^2\left(ax\right)}{2}+\frac{cx^2}{12}-\frac{cx\operatorname{atan}\left(ax\right)}{2a}+\frac{c\log\left(x^2+\frac{1}{a^2}\right)}{6a^2}+\frac{c\operatorname{atan}^2\left(ax\right)}{4a^2} & \text{for}\: a\neq 0\\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)*atan(a*x)**2,x)
```

```
[Out] Piecewise((a**2*c*x**4*atan(a*x)**2/4 - a*c*x**3*atan(a*x)/6 + c*x**2*atan(
a*x)**2/2 + c*x**2/12 - c*x*atan(a*x)/(2*a) + c*log(x**2 + a**(-2))/(6*a**2
) + c*atan(a*x)**2/(4*a**2), Ne(a, 0)), (0, True))
```

Giac [A] time = 1.14065, size = 112, normalized size = 1.17

$$\frac{1}{4}\left(a^{2}cx^{4}+2\,cx^{2}\right)\arctan\left(ax\right)^{2}-\frac{2\,a^{3}cx^{3}\arctan\left(ax\right)-a^{2}cx^{2}+6\,acx\arctan\left(ax\right)-3\,c\arctan\left(ax\right)^{2}-2\,c\log\left(a^{2}x^{2}+1\right)}{12\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)*arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] 1/4*(a^2*c*x^4 + 2*c*x^2)*arctan(a*x)^2 - 1/12*(2*a^3*c*x^3*arctan(a*x) - a
^2*c*x^2 + 6*a*c*x*arctan(a*x) - 3*c*arctan(a*x)^2 - 2*c*log(a^2*x^2 + 1))/
a^2
```

3.261 $\int \left(c+a^2cx^2\right) \tan^{-1}(ax)^2 \, dx$

Optimal. Leaf size=128

$$\frac{2ic\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{3a}+\frac{1}{3}cx\left(a^2x^2+1\right)\tan^{-1}(ax)^2-\frac{c\left(a^2x^2+1\right)\tan^{-1}(ax)}{3a}+\frac{2ic\tan^{-1}(ax)^2}{3a}+\frac{2}{3}cx\tan^{-1}(ax)^2+$$

```
[Out] (c*x)/3 - (c*(1 + a^2*x^2)*ArcTan[a*x])/(3*a) + (((2*I)/3)*c*ArcTan[a*x]^2)
/a + (2*c*x*ArcTan[a*x]^2)/3 + (c*x*(1 + a^2*x^2)*ArcTan[a*x]^2)/3 + (4*c*A
rcTan[a*x]*Log[2/(1 + I*a*x)])/(3*a) + (((2*I)/3)*c*PolyLog[2, 1 - 2/(1 + I
*a*x)])/a
```

Rubi [A] time = 0.0960557, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {4880, 4846, 4920, 4854, 2402, 2315, 8}

$$\frac{2ic\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{3a}+\frac{1}{3}cx\left(a^2x^2+1\right)\tan^{-1}(ax)^2-\frac{c\left(a^2x^2+1\right)\tan^{-1}(ax)}{3a}+\frac{2ic\tan^{-1}(ax)^2}{3a}+\frac{2}{3}cx\tan^{-1}(ax)^2+$$

Antiderivative was successfully verified.

```
[In] Int[(c + a^2*c*x^2)*ArcTan[a*x]^2,x]
```

```
[Out] (c*x)/3 - (c*(1 + a^2*x^2)*ArcTan[a*x])/(3*a) + (((2*I)/3)*c*ArcTan[a*x]^2)
/a + (2*c*x*ArcTan[a*x]^2)/3 + (c*x*(1 + a^2*x^2)*ArcTan[a*x]^2)/3 + (4*c*A
rcTan[a*x]*Log[2/(1 + I*a*x)])/(3*a) + (((2*I)/3)*c*PolyLog[2, 1 - 2/(1 + I
*a*x)])/a
```

Rule 4880

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> -Simp[(b*p*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1))/(2*c*q*(2*
q + 1)), x] + (Dist[(2*d*q)/(2*q + 1), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTa
n[c*x])^p, x], x] + Dist[(b^2*d*p*(p - 1))/(2*q*(2*q + 1)), Int[(d + e*x^2)
^(q - 1)*(a + b*ArcTan[c*x])^(p - 2), x], x] + Simp[(x*(d + e*x^2)^q*(a + b
*ArcTan[c*x])^p)/(2*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[q, 0] && GtQ[p, 1]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \left(c+a^2cx^2\right)\tan^{-1}(ax)^2\,dx = -\frac{c\left(1+a^2x^2\right)\tan^{-1}(ax)}{3a}+\frac{1}{3}cx\left(1+a^2x^2\right)\tan^{-1}(ax)^2+\frac{1}{3}c\int 1\,dx+\frac{1}{3}(2c)\int \tan^{-1}(ax)$$

$$=\frac{cx}{3}-\frac{c\left(1+a^2x^2\right)\tan^{-1}(ax)}{3a}+\frac{2}{3}cx\tan^{-1}(ax)^2+\frac{1}{3}cx\left(1+a^2x^2\right)\tan^{-1}(ax)^2-\frac{1}{3}(4ac)\int$$

$$=\frac{cx}{3}-\frac{c\left(1+a^2x^2\right)\tan^{-1}(ax)}{3a}+\frac{2ic\tan^{-1}(ax)^2}{3a}+\frac{2}{3}cx\tan^{-1}(ax)^2+\frac{1}{3}cx\left(1+a^2x^2\right)\tan^{-1}$$

$$=\frac{cx}{3}-\frac{c\left(1+a^2x^2\right)\tan^{-1}(ax)}{3a}+\frac{2ic\tan^{-1}(ax)^2}{3a}+\frac{2}{3}cx\tan^{-1}(ax)^2+\frac{1}{3}cx\left(1+a^2x^2\right)\tan^{-1}$$

$$=\frac{cx}{3}-\frac{c\left(1+a^2x^2\right)\tan^{-1}(ax)}{3a}+\frac{2ic\tan^{-1}(ax)^2}{3a}+\frac{2}{3}cx\tan^{-1}(ax)^2+\frac{1}{3}cx\left(1+a^2x^2\right)\tan^{-1}$$

$$=\frac{cx}{3}-\frac{c\left(1+a^2x^2\right)\tan^{-1}(ax)}{3a}+\frac{2ic\tan^{-1}(ax)^2}{3a}+\frac{2}{3}cx\tan^{-1}(ax)^2+\frac{1}{3}cx\left(1+a^2x^2\right)\tan^{-1}$$

Mathematica [A] time = 0.0504754, size = 82, normalized size = 0.64

$$\frac{c\left(-2i\text{PolyLog}\left(2,-e^{2i\tan^{-1}(ax)}\right)+\left(a^3x^3+3ax-2i\right)\tan^{-1}(ax)^2-\tan^{-1}(ax)\left(a^2x^2-4\log\left(1+e^{2i\tan^{-1}(ax)}\right)+1\right)+ax\right)}{3a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + a^2*c*x^2)*ArcTan[a*x]^2,x]
```

```
[Out] (c*(a*x + (-2*I + 3*a*x + a^3*x^3)*ArcTan[a*x]^2 - ArcTan[a*x]*(1 + a^2*x^2
 - 4*Log[1 + E^((2*I)*ArcTan[a*x])]) - (2*I)*PolyLog[2, -E^((2*I)*ArcTan[a*
x])]))/(3*a)
```

Maple [B] time = 0.086, size = 233, normalized size = 1.8

$$\frac{a^2c\left(\arctan(ax)\right)^2x^3}{3}+cx\left(\arctan(ax)\right)^2-\frac{ac\arctan(ax)\,x^2}{3}-\frac{2\,c\arctan(ax)\ln\left(a^2x^2+1\right)}{3\,a}+\frac{cx}{3}-\frac{c\arctan(ax)}{3\,a}-\frac{i}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)*arctan(a*x)^2,x)
```

```
[Out] 1/3*a^2*c*arctan(a*x)^2*x^3+c*x*arctan(a*x)^2-1/3*a*c*arctan(a*x)*x^2-2/3/a
*c*arctan(a*x)*ln(a^2*x^2+1)+1/3*c*x-1/3/a*c*arctan(a*x)-1/3*I/a*c*ln(a^2*x
^2+1)*ln(a*x-I)+1/6*I/a*c*ln(a*x-I)^2+1/3*I/a*c*ln(a*x-I)*ln(-1/2*I*(a*x+I)
)+1/3*I/a*c*dilog(-1/2*I*(a*x+I))+1/3*I/a*c*ln(a^2*x^2+1)*ln(a*x+I)-1/6*I/a
*c*ln(a*x+I)^2-1/3*I/a*c*ln(a*x+I)*ln(1/2*I*(a*x-I))-1/3*I/a*c*dilog(1/2*I*
(a*x-I))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$36\,a^4c\int\frac{x^4\arctan(ax)^2}{48\left(a^2x^2+1\right)}\,dx+3\,a^4c\int\frac{x^4\log\left(a^2x^2+1\right)^2}{48\left(a^2x^2+1\right)}\,dx+4\,a^4c\int\frac{x^4\log\left(a^2x^2+1\right)}{48\left(a^2x^2+1\right)}\,dx-8\,a^3c\int\frac{x^3\arctan(ax)}{48\left(a^2x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] 36*a^4*c*integrate(1/48*x^4*arctan(a*x)^2/(a^2*x^2 + 1), x) + 3*a^4*c*integ
rate(1/48*x^4*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x) + 4*a^4*c*integrate(1/48
*x^4*log(a^2*x^2 + 1)/(a^2*x^2 + 1), x) - 8*a^3*c*integrate(1/48*x^3*arctan
(a*x)/(a^2*x^2 + 1), x) + 72*a^2*c*integrate(1/48*x^2*arctan(a*x)^2/(a^2*x^
2 + 1), x) + 6*a^2*c*integrate(1/48*x^2*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x
) + 12*a^2*c*integrate(1/48*x^2*log(a^2*x^2 + 1)/(a^2*x^2 + 1), x) + 1/12*(
a^2*c*x^3 + 3*c*x)*arctan(a*x)^2 + 1/4*c*arctan(a*x)^3/a - 24*a*c*integrate
(1/48*x*arctan(a*x)/(a^2*x^2 + 1), x) - 1/48*(a^2*c*x^3 + 3*c*x)*log(a^2*x^
2 + 1)^2 + 3*c*integrate(1/48*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a^2cx^2+c\right)\arctan(ax)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)*arctan(a*x)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$c\left(\int a^2x^2\,\mathrm{atan}^2\,(ax)\,dx+\int \mathrm{atan}^2\,(ax)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)*atan(a*x)**2,x)
```

```
[Out] c*(Integral(a**2*x**2*atan(a*x)**2, x) + Integral(atan(a*x)**2, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2+c\right)\arctan\,(ax)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)*arctan(a*x)^2, x)
```

3.262 $\int \frac{(c+a^2cx^2)\tan^{-1}(ax)^2}{x} dx$

Optimal. Leaf size=169

$$-\frac{1}{2}c\text{PolyLog}\left(3,1-\frac{2}{1+iax}\right)+\frac{1}{2}c\text{PolyLog}\left(3,-1+\frac{2}{1+iax}\right)-ic\tan^{-1}(ax)\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)+ic\tan^{-1}(ax)\text{Pol}$$

```
[Out] -(a*c*x*ArcTan[a*x]) + (c*ArcTan[a*x]^2)/2 + (a^2*c*x^2*ArcTan[a*x]^2)/2 +
2*c*ArcTan[a*x]^2*ArcTanh[1 - 2/(1 + I*a*x)] + (c*Log[1 + a^2*x^2])/2 - I*c
*ArcTan[a*x]*PolyLog[2, 1 - 2/(1 + I*a*x)] + I*c*ArcTan[a*x]*PolyLog[2, -1
+ 2/(1 + I*a*x)] - (c*PolyLog[3, 1 - 2/(1 + I*a*x)])/2 + (c*PolyLog[3, -1 +
 2/(1 + I*a*x)])/2
```

Rubi [A] time = 0.311247, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4950, 4850, 4988, 4884, 4994, 6610, 4852, 4916, 4846, 260}

$$-\frac{1}{2}c\text{PolyLog}\left(3,1-\frac{2}{1+iax}\right)+\frac{1}{2}c\text{PolyLog}\left(3,-1+\frac{2}{1+iax}\right)-ic\tan^{-1}(ax)\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)+ic\tan^{-1}(ax)\text{Pol}$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)*ArcTan[a*x]^2)/x,x]
```

```
[Out] -(a*c*x*ArcTan[a*x]) + (c*ArcTan[a*x]^2)/2 + (a^2*c*x^2*ArcTan[a*x]^2)/2 +
2*c*ArcTan[a*x]^2*ArcTanh[1 - 2/(1 + I*a*x)] + (c*Log[1 + a^2*x^2])/2 - I*c
*ArcTan[a*x]*PolyLog[2, 1 - 2/(1 + I*a*x)] + I*c*ArcTan[a*x]*PolyLog[2, -1
+ 2/(1 + I*a*x)] - (c*PolyLog[3, 1 - 2/(1 + I*a*x)])/2 + (c*PolyLog[3, -1 +
 2/(1 + I*a*x)])/2
```

Rule 4950

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 4850

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTan[c*x])^(p - 1)*ArcTanh[1 - 2/(1 + I*c*x)])/(1 + c^2*x^2), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4988

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTan[c*x])^p)/(d + e
*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTan[c*x])^p)/(d + e*x^2
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
```

```
e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{\left(c+a^2cx^2\right)\tan^{-1}(ax)^2}{x}\,dx = c\int \frac{\tan^{-1}(ax)^2}{x}\,dx + \left(a^2c\right)\int x\tan^{-1}(ax)^2\,dx$$

$$= \frac{1}{2}a^2cx^2\tan^{-1}(ax)^2 + 2c\tan^{-1}(ax)^2\tanh^{-1}\left(1-\frac{2}{1+iax}\right) - (4ac)\int \frac{\tan^{-1}(ax)\tanh^{-1}\left(1\right.}{1+a^2x^2}$$

$$= \frac{1}{2}a^2cx^2\tan^{-1}(ax)^2 + 2c\tan^{-1}(ax)^2\tanh^{-1}\left(1-\frac{2}{1+iax}\right) - (ac)\int \tan^{-1}(ax)\,dx + (ac)\int$$

$$= -acx\tan^{-1}(ax) + \frac{1}{2}c\tan^{-1}(ax)^2 + \frac{1}{2}a^2cx^2\tan^{-1}(ax)^2 + 2c\tan^{-1}(ax)^2\tanh^{-1}\left(1-\frac{2}{1+ia\right.}$$

$$= -acx\tan^{-1}(ax) + \frac{1}{2}c\tan^{-1}(ax)^2 + \frac{1}{2}a^2cx^2\tan^{-1}(ax)^2 + 2c\tan^{-1}(ax)^2\tanh^{-1}\left(1-\frac{2}{1+ia\right.}$$

Mathematica [A] time = 0.0439985, size = 177, normalized size = 1.05

$$\frac{1}{2}c\text{PolyLog}\left(3,\frac{-ax-i}{ax-i}\right) - \frac{1}{2}c\text{PolyLog}\left(3,\frac{ax+i}{ax-i}\right) + ic\tan^{-1}(ax)\text{PolyLog}\left(2,\frac{-ax-i}{ax-i}\right) - ic\tan^{-1}(ax)\text{PolyLog}\left(2,\frac{ax+}{ax-}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + a^2*c*x^2)*ArcTan[a*x]^2)/x,x]
```

```
[Out] -(a*c*x*ArcTan[a*x]) + (c*(1 + a^2*x^2)*ArcTan[a*x]^2)/2 + 2*c*ArcTan[a*x]^
2*ArcTanh[1 - (2*I)/(I - a*x)] + (c*Log[1 + a^2*x^2])/2 + I*c*ArcTan[a*x]*P
olyLog[2, (-I - a*x)/(-I + a*x)] - I*c*ArcTan[a*x]*PolyLog[2, (I + a*x)/(-I
```

```
 + a*x)] + (c*PolyLog[3, (-I - a*x)/(-I + a*x)])/2 - (c*PolyLog[3, (I + a*x
)/(-I + a*x)])/2
```

Maple [C] time = 1.577, size = 1078, normalized size = 6.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)*arctan(a*x)^2/x,x)
```

```
[Out] 1/2*a^2*c*x^2*arctan(a*x)^2+c*arctan(a*x)^2*ln(a*x)-c*arctan(a*x)^2*ln((1+I
*a*x)^2/(a^2*x^2+1)-1)+c*arctan(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-1/
2*I*c*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1
)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2*arctan(a*x)^2+2*c*polylog(3,(1+I*a*x)/(
a^2*x^2+1)^(1/2))+c*arctan(a*x)^2*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))+I*c*arc
tan(a*x)*polylog(2,-(1+I*a*x)^2/(a^2*x^2+1))+2*c*polylog(3,-(1+I*a*x)/(a^2*
x^2+1)^(1/2))-1/2*I*c*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a
^2*x^2+1)+1))*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))
^2*arctan(a*x)^2-1/2*c*polylog(3,-(1+I*a*x)^2/(a^2*x^2+1))+1/2*I*c*Pi*csgn(
I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^3*arctan(a*x)^2-
2*I*c*arctan(a*x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+1/2*c*arctan(a*x)^
2-1/2*I*c*Pi*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^
2*arctan(a*x)^2-c*ln((1+I*a*x)^2/(a^2*x^2+1)+1)-a*c*x*arctan(a*x)+1/2*I*c*P
i*arctan(a*x)^2+1/2*I*c*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/
(a^2*x^2+1)+1))*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1
))*arctan(a*x)^2+1/2*I*c*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1))*csgn(I/((1+
I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a
^2*x^2+1)+1))*arctan(a*x)^2+I*c*arctan(a*x)-1/2*I*c*Pi*csgn(I*((1+I*a*x)^2/
(a^2*x^2+1)-1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)
+1))^2*arctan(a*x)^2+1/2*I*c*Pi*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)
^2/(a^2*x^2+1)+1))^3*arctan(a*x)^2-2*I*c*arctan(a*x)*polylog(2,-(1+I*a*x)/(
a^2*x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{8}\,a^2cx^2\arctan\left(ax\right)^2-\frac{1}{32}\,a^2cx^2\log\left(a^2x^2+1\right)^2+12\,a^4c\int\frac{x^4\arctan\left(ax\right)^2}{16\left(a^2x^3+x\right)}\,dx+a^4c\int\frac{x^4\log\left(a^2x^2+1\right)^2}{16\left(a^2x^3+x\right)}\,dx+2\,a^4c\int$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^2/x,x, algorithm="maxima")
```

```
[Out] 1/8*a^2*c*x^2*arctan(a*x)^2 - 1/32*a^2*c*x^2*log(a^2*x^2 + 1)^2 + 12*a^4*c*
integrate(1/16*x^4*arctan(a*x)^2/(a^2*x^3 + x), x) + a^4*c*integrate(1/16*x
^4*log(a^2*x^2 + 1)^2/(a^2*x^3 + x), x) + 2*a^4*c*integrate(1/16*x^4*log(a^
2*x^2 + 1)/(a^2*x^3 + x), x) - 4*a^3*c*integrate(1/16*x^3*arctan(a*x)/(a^2*
x^3 + x), x) + 24*a^2*c*integrate(1/16*x^2*arctan(a*x)^2/(a^2*x^3 + x), x)
+ 1/48*c*log(a^2*x^2 + 1)^3 + 12*c*integrate(1/16*arctan(a*x)^2/(a^2*x^3 +
x), x) + c*integrate(1/16*log(a^2*x^2 + 1)^2/(a^2*x^3 + x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^2cx^2+c\right)\arctan\left(ax\right)^2}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^2/x,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)*arctan(a*x)^2/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$c\left(\int\frac{\operatorname{atan}^2\left(ax\right)}{x}\,dx+\int a^2x\operatorname{atan}^2\left(ax\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)*atan(a*x)**2/x,x)
```

```
[Out] c*(Integral(atan(a*x)**2/x, x) + Integral(a**2*x*atan(a*x)**2, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a^2cx^2+c\right)\arctan\left(ax\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^2/x,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)*arctan(a*x)^2/x, x)
```

3.263 $\int \frac{(c+a^2cx^2)\tan^{-1}(ax)^2}{x^2}\,dx$

Optimal. Leaf size=113

$$-iac\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)+iac\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)+a^2cx\tan^{-1}(ax)^2-\frac{c\tan^{-1}(ax)^2}{x}+2ac\log\left(\frac{2}{1+iax}\right)\tan$$

```
[Out] -((c*ArcTan[a*x]^2)/x) + a^2*c*x*ArcTan[a*x]^2 + 2*a*c*ArcTan[a*x]*Log[2/(1
 + I*a*x)] + 2*a*c*ArcTan[a*x]*Log[2 - 2/(1 - I*a*x)] - I*a*c*PolyLog[2, -1
 + 2/(1 - I*a*x)] + I*a*c*PolyLog[2, 1 - 2/(1 + I*a*x)]
```

Rubi [A] time = 0.222704, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4950, 4852, 4924, 4868, 2447, 4846, 4920, 4854, 2402, 2315}

$$-iac\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)+iac\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)+a^2cx\tan^{-1}(ax)^2-\frac{c\tan^{-1}(ax)^2}{x}+2ac\log\left(\frac{2}{1+iax}\right)\tan$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)*ArcTan[a*x]^2)/x^2,x]
```


```
[Out] -((c*ArcTan[a*x]^2)/x) + a^2*c*x*ArcTan[a*x]^2 + 2*a*c*ArcTan[a*x]*Log[2/(1
 + I*a*x)] + 2*a*c*ArcTan[a*x]*Log[2 - 2/(1 - I*a*x)] - I*a*c*PolyLog[2, -1
 + 2/(1 - I*a*x)] + I*a*c*PolyLog[2, 1 - 2/(1 + I*a*x)]
```


Rule 4950

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
```

```
erQ[m]) && NeQ[m, -1]
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(c+a^2cx^2\right)\tan^{-1}(ax)^2}{x^2}\,dx &= c\int \frac{\tan^{-1}(ax)^2}{x^2}\,dx+\left(a^2c\right)\int \tan^{-1}(ax)^2\,dx\\ &= -\frac{c\tan^{-1}(ax)^2}{x}+a^2cx\tan^{-1}(ax)^2+(2ac)\int \frac{\tan^{-1}(ax)}{x\left(1+a^2x^2\right)}\,dx-\left(2a^3c\right)\int \frac{x\tan^{-1}(ax)}{1+a^2x^2}\,dx\\ &= -\frac{c\tan^{-1}(ax)^2}{x}+a^2cx\tan^{-1}(ax)^2+(2iac)\int \frac{\tan^{-1}(ax)}{x(i+ax)}\,dx+\left(2a^2c\right)\int \frac{\tan^{-1}(ax)}{i-ax}\,dx\\ &= -\frac{c\tan^{-1}(ax)^2}{x}+a^2cx\tan^{-1}(ax)^2+2ac\tan^{-1}(ax)\log\left(\frac{2}{1+iax}\right)+2ac\tan^{-1}(ax)\log\left(2-\right.\\ &= -\frac{c\tan^{-1}(ax)^2}{x}+a^2cx\tan^{-1}(ax)^2+2ac\tan^{-1}(ax)\log\left(\frac{2}{1+iax}\right)+2ac\tan^{-1}(ax)\log\left(2-\right.\\ &= -\frac{c\tan^{-1}(ax)^2}{x}+a^2cx\tan^{-1}(ax)^2+2ac\tan^{-1}(ax)\log\left(\frac{2}{1+iax}\right)+2ac\tan^{-1}(ax)\log\left(2-\right.\end{aligned}$$

Mathematica [A] time = 0.148467, size = 123, normalized size = 1.09

$$ac\left(-i\text{PolyLog}\left(2,-e^{2i\tan^{-1}(ax)}\right)+ax\tan^{-1}(ax)^2-i\tan^{-1}(ax)^2+2\tan^{-1}(ax)\log\left(1+e^{2i\tan^{-1}(ax)}\right)\right)+ac\left(-i\left(\tan^{-1}(ax)^2\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((c + a^2*c*x^2)*ArcTan[a*x]^2)/x^2,x]
```

```
[Out] a*c*((-I)*ArcTan[a*x]^2 + a*x*ArcTan[a*x]^2 + 2*ArcTan[a*x]*Log[1 + E^((2*I
)*ArcTan[a*x])] - I*PolyLog[2, -E^((2*I)*ArcTan[a*x])]) + a*c*(-(ArcTan[a*x
]^2/(a*x)) + 2*ArcTan[a*x]*Log[1 - E^((2*I)*ArcTan[a*x])] - I*(ArcTan[a*x]^
2 + PolyLog[2, E^((2*I)*ArcTan[a*x])]))
```

Maple [B] time = 0.092, size = 262, normalized size = 2.3

$$a^2cx(\arctan(ax))^2-\frac{c(\arctan(ax))^2}{x}-2ac\arctan(ax)\ln\left(a^2x^2+1\right)+2ac\arctan(ax)\ln(ax)-iac\ln\left(a^2x^2+1\right)\ln($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)*arctan(a*x)^2/x^2,x)
```

```
[Out] a^2*c*x*arctan(a*x)^2-c*arctan(a*x)^2/x-2*a*c*arctan(a*x)*ln(a^2*x^2+1)+2*a
*c*arctan(a*x)*ln(a*x)-I*a*c*ln(a^2*x^2+1)*ln(a*x-I)-I*a*c*ln(a*x)*ln(1-I*a
*x)-I*a*c*dilog(1/2*I*(a*x-I))+I*a*c*ln(a*x)*ln(1+I*a*x)+I*a*c*dilog(1+I*a*
x)+1/2*I*a*c*ln(a*x-I)^2+I*a*c*ln(a^2*x^2+1)*ln(a*x+I)-I*a*c*dilog(1-I*a*x)
+I*a*c*ln(a*x-I)*ln(-1/2*I*(a*x+I))-I*a*c*ln(a*x+I)*ln(1/2*I*(a*x-I))-1/2*I
*a*c*ln(a*x+I)^2+I*a*c*dilog(-1/2*I*(a*x+I))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^2/x^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^2cx^2+c\right)\arctan(ax)^2}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^2/x^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)*arctan(a*x)^2/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$c\left(\int a^2\,\mathrm{atan}^2\left(ax\right)dx+\int\frac{\mathrm{atan}^2\left(ax\right)}{x^2}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)*atan(a*x)**2/x**2,x)
```

```
[Out] c*(Integral(a**2*atan(a*x)**2, x) + Integral(atan(a*x)**2/x**2, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a^2cx^2+c\right)\arctan\left(ax\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^2/x^2,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)*arctan(a*x)^2/x^2, x)
```

3.264 $\int \frac{(c+a^2cx^2)\tan^{-1}(ax)^2}{x^3}\,dx$

Optimal. Leaf size=196

$$-\frac{1}{2}a^2c\text{PolyLog}\left(3,1-\frac{2}{1+iax}\right)+\frac{1}{2}a^2c\text{PolyLog}\left(3,-1+\frac{2}{1+iax}\right)-ia^2c\tan^{-1}(ax)\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)+ia^2c\tan$$

```
[Out] -((a*c*ArcTan[a*x])/x) - (a^2*c*ArcTan[a*x]^2)/2 - (c*ArcTan[a*x]^2)/(2*x^2
) + 2*a^2*c*ArcTan[a*x]^2*ArcTanh[1 - 2/(1 + I*a*x)] + a^2*c*Log[x] - (a^2*
c*Log[1 + a^2*x^2])/2 - I*a^2*c*ArcTan[a*x]*PolyLog[2, 1 - 2/(1 + I*a*x)] +
 I*a^2*c*ArcTan[a*x]*PolyLog[2, -1 + 2/(1 + I*a*x)] - (a^2*c*PolyLog[3, 1 -
 2/(1 + I*a*x)])/2 + (a^2*c*PolyLog[3, -1 + 2/(1 + I*a*x)])/2
```

Rubi [A] time = 0.325356, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 12, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {4950, 4852, 4918, 266, 36, 29, 31, 4884, 4850, 4988, 4994, 6610}

$$-\frac{1}{2}a^2c\text{PolyLog}\left(3,1-\frac{2}{1+iax}\right)+\frac{1}{2}a^2c\text{PolyLog}\left(3,-1+\frac{2}{1+iax}\right)-ia^2c\tan^{-1}(ax)\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)+ia^2c\tan$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)*ArcTan[a*x]^2)/x^3,x]
```


```
[Out] -((a*c*ArcTan[a*x])/x) - (a^2*c*ArcTan[a*x]^2)/2 - (c*ArcTan[a*x]^2)/(2*x^2
) + 2*a^2*c*ArcTan[a*x]^2*ArcTanh[1 - 2/(1 + I*a*x)] + a^2*c*Log[x] - (a^2*
c*Log[1 + a^2*x^2])/2 - I*a^2*c*ArcTan[a*x]*PolyLog[2, 1 - 2/(1 + I*a*x)] +
 I*a^2*c*ArcTan[a*x]*PolyLog[2, -1 + 2/(1 + I*a*x)] - (a^2*c*PolyLog[3, 1 -
 2/(1 + I*a*x)])/2 + (a^2*c*PolyLog[3, -1 + 2/(1 + I*a*x)])/2
```


Rule 4950

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4918

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/(d + e*x^2),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4850

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTan[c*x])^(p - 1)*ArcTanh[1 - 2/(1 + I*c*x)])/(1 + c^2*x^2), x], x] /;
```

```
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4988

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTan[c*x])^p)/(d + e
*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTan[c*x])^p)/(d + e*x^2
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{(c + a^2cx^2)\tan^{-1}(ax)^2}{x^3}\,dx = c\int \frac{\tan^{-1}(ax)^2}{x^3}\,dx + (a^2c)\int \frac{\tan^{-1}(ax)^2}{x}\,dx$$

$$= -\frac{c\tan^{-1}(ax)^2}{2x^2} + 2a^2c\tan^{-1}(ax)^2\tanh^{-1}\left(1 - \frac{2}{1+iax}\right) + (ac)\int \frac{\tan^{-1}(ax)}{x^2(1+a^2x^2)}\,dx - (4a^3c$$

$$= -\frac{c\tan^{-1}(ax)^2}{2x^2} + 2a^2c\tan^{-1}(ax)^2\tanh^{-1}\left(1 - \frac{2}{1+iax}\right) + (ac)\int \frac{\tan^{-1}(ax)}{x^2}\,dx - (a^3c)\int$$

$$= -\frac{ac\tan^{-1}(ax)}{x} - \frac{1}{2}a^2c\tan^{-1}(ax)^2 - \frac{c\tan^{-1}(ax)^2}{2x^2} + 2a^2c\tan^{-1}(ax)^2\tanh^{-1}\left(1 - \frac{2}{1+iax}\right)$$

$$= -\frac{ac\tan^{-1}(ax)}{x} - \frac{1}{2}a^2c\tan^{-1}(ax)^2 - \frac{c\tan^{-1}(ax)^2}{2x^2} + 2a^2c\tan^{-1}(ax)^2\tanh^{-1}\left(1 - \frac{2}{1+iax}\right)$$

$$= -\frac{ac\tan^{-1}(ax)}{x} - \frac{1}{2}a^2c\tan^{-1}(ax)^2 - \frac{c\tan^{-1}(ax)^2}{2x^2} + 2a^2c\tan^{-1}(ax)^2\tanh^{-1}\left(1 - \frac{2}{1+iax}\right)$$

$$= -\frac{ac\tan^{-1}(ax)}{x} - \frac{1}{2}a^2c\tan^{-1}(ax)^2 - \frac{c\tan^{-1}(ax)^2}{2x^2} + 2a^2c\tan^{-1}(ax)^2\tanh^{-1}\left(1 - \frac{2}{1+iax}\right)$$

Mathematica [A] time = 0.0874597, size = 208, normalized size = 1.06

$$\frac{1}{2}a^2c\text{PolyLog}\left(3, \frac{-ax-i}{ax-i}\right) - \frac{1}{2}a^2c\text{PolyLog}\left(3, \frac{ax+i}{ax-i}\right) + ia^2c\tan^{-1}(ax)\text{PolyLog}\left(2, \frac{-ax-i}{ax-i}\right) - ia^2c\tan^{-1}(ax)\text{PolyLog}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + a^2*c*x^2)*ArcTan[a*x]^2)/x^3,x]
```

```
[Out] -((a*c*ArcTan[a*x])/x) + (c*(-1 - a^2*x^2)*ArcTan[a*x]^2)/(2*x^2) + 2*a^2*c
*ArcTan[a*x]^2*ArcTanh[1 - (2*I)/(I - a*x)] + a^2*c*Log[x] - (a^2*c*Log[1 +
 a^2*x^2])/2 + I*a^2*c*ArcTan[a*x]*PolyLog[2, (-I - a*x)/(-I + a*x)] - I*a^
2*c*ArcTan[a*x]*PolyLog[2, (I + a*x)/(-I + a*x)] + (a^2*c*PolyLog[3, (-I -
a*x)/(-I + a*x)])/2 - (a^2*c*PolyLog[3, (I + a*x)/(-I + a*x)])/2
```

Maple [C] time = 3.306, size = 1167, normalized size = 6.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)*arctan(a*x)^2/x^3,x)
```

```
[Out] 1/2*I*a^2*c*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1))*csgn(I/((1+I*a*x)^2/(a^2
*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))
*arctan(a*x)^2-1/2*a^2*c*arctan(a*x)^2-1/2*c*arctan(a*x)^2/x^2-a*c*arctan(a
*x)/x+2*a^2*c*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*a^2*c*polylog(3,-(1+
I*a*x)/(a^2*x^2+1)^(1/2))-1/2*a^2*c*polylog(3,-(1+I*a*x)^2/(a^2*x^2+1))+a^2
*c*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))+a^2*c*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)-1
)+I*a^2*c*arctan(a*x)*polylog(2,-(1+I*a*x)^2/(a^2*x^2+1))+1/2*I*a^2*c*Pi*ar
ctan(a*x)^2-2*I*a^2*c*arctan(a*x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*
I*a^2*c*arctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+1/2*I*a^2*c*Pi*
csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^3*arctan(a*x)
^2-1/2*I*a^2*c*Pi*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)
+1))^2*arctan(a*x)^2+1/2*I*a^2*c*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+
I*a*x)^2/(a^2*x^2+1)+1))^3*arctan(a*x)^2+1/2*I*a^2*c*Pi*csgn(I*((1+I*a*x)^2
/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(((1+I*a*x)^2/(a^2*x^2+1)-
1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*arctan(a*x)^2-1/2*I*a^2*c*Pi*csgn(I*((1+I*a
*x)^2/(a^2*x^2+1)-1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*
x^2+1)+1))^2*arctan(a*x)^2-1/2*I*a^2*c*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1
))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2*arctan
(a*x)^2-1/2*I*a^2*c*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2
*x^2+1)+1))*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2
*arctan(a*x)^2-a^2*c*arctan(a*x)^2*ln((1+I*a*x)^2/(a^2*x^2+1)-1)+a^2*c*arct
an(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))+a^2*c*arctan(a*x)^2*ln(1+(1+I*a
*x)/(a^2*x^2+1)^(1/2))+a^2*c*arctan(a*x)^2*ln(a*x)-I*a^2*c*arctan(a*x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(72\,a^4c\int\frac{x^4\arctan(ax)^2}{a^2x^5+x^3}\,dx+a^2c\log\left(a^2x^2+1\right)^3-3\left(a^2\left(\frac{\log\left(a^2x^2+1\right)^2}{a^2}-\frac{2\left(2\,\log\left(a^2x^2+1\right)\log(x)+\mathrm{Li}_2\left(-a^2x^2\right)\right)}{a^2}\right)-2\left(\log\left(a^2x^2+1\right)\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^2/x^3,x, algorithm="maxima")
```

```
[Out] 1/96*((1152*a^4*c*integrate(1/16*x^4*arctan(a*x)^2/(a^2*x^5 + x^3), x) + a^
2*c*log(a^2*x^2 + 1)^3 + 2304*a^2*c*integrate(1/16*x^2*arctan(a*x)^2/(a^2*x
^5 + x^3), x) + 192*a^2*c*integrate(1/16*x^2*log(a^2*x^2 + 1)^2/(a^2*x^5 +
x^3), x) - 192*a^2*c*integrate(1/16*x^2*log(a^2*x^2 + 1)/(a^2*x^5 + x^3), x
) + 384*a*c*integrate(1/16*x*arctan(a*x)/(a^2*x^5 + x^3), x) + 1152*c*integ
rate(1/16*arctan(a*x)^2/(a^2*x^5 + x^3), x) + 96*c*integrate(1/16*log(a^2*x
```

```
^2 + 1)^2/(a^2*x^5 + x^3), x))*x^2 - 12*c*arctan(a*x)^2 + 3*c*log(a^2*x^2 +
 1)^2)/x^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^2cx^2+c\right)\arctan\left(ax\right)^2}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^2/x^3,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)*arctan(a*x)^2/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$c\left(\int\frac{\operatorname{atan}^2\left(ax\right)}{x^3}\,dx+\int\frac{a^2\operatorname{atan}^2\left(ax\right)}{x}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)*atan(a*x)**2/x**3,x)
```

```
[Out] c*(Integral(atan(a*x)**2/x**3, x) + Integral(a**2*atan(a*x)**2/x, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a^2cx^2+c\right)\arctan\left(ax\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^2/x^3,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)*arctan(a*x)^2/x^3, x)
```

3.265 $\int \frac{(c+a^2cx^2)\tan^{-1}(ax)^2}{x^4}\,dx$

Optimal. Leaf size=135

$$-\frac{2}{3}ia^3c\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)-\frac{a^2c}{3x}-\frac{2}{3}ia^3c\tan^{-1}(ax)^2-\frac{1}{3}a^3c\tan^{-1}(ax)-\frac{a^2c\tan^{-1}(ax)^2}{x}+\frac{4}{3}a^3c\log\left(2-\frac{2}{1-iax}\right)$$

```
[Out] -(a^2*c)/(3*x) - (a^3*c*ArcTan[a*x])/3 - (a*c*ArcTan[a*x])/(3*x^2) - ((2*I)
/3)*a^3*c*ArcTan[a*x]^2 - (c*ArcTan[a*x]^2)/(3*x^3) - (a^2*c*ArcTan[a*x]^2)
/x + (4*a^3*c*ArcTan[a*x]*Log[2 - 2/(1 - I*a*x)])/3 - ((2*I)/3)*a^3*c*PolyL
og[2, -1 + 2/(1 - I*a*x)]
```

Rubi [A] time = 0.312138, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {4950, 4852, 4918, 325, 203, 4924, 4868, 2447}

$$-\frac{2}{3}ia^3c\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)-\frac{a^2c}{3x}-\frac{2}{3}ia^3c\tan^{-1}(ax)^2-\frac{1}{3}a^3c\tan^{-1}(ax)-\frac{a^2c\tan^{-1}(ax)^2}{x}+\frac{4}{3}a^3c\log\left(2-\frac{2}{1-iax}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)*ArcTan[a*x]^2)/x^4,x]
```

```
[Out] -(a^2*c)/(3*x) - (a^3*c*ArcTan[a*x])/3 - (a*c*ArcTan[a*x])/(3*x^2) - ((2*I)
/3)*a^3*c*ArcTan[a*x]^2 - (c*ArcTan[a*x]^2)/(3*x^3) - (a^2*c*ArcTan[a*x]^2)
/x + (4*a^3*c*ArcTan[a*x]*Log[2 - 2/(1 - I*a*x)])/3 - ((2*I)/3)*a^3*c*PolyL
og[2, -1 + 2/(1 - I*a*x)]
```

Rule 4950

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
```

```
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4918

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/(d + e*x^2),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
```

```
x]][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{\left(c+a^2cx^2\right)\tan^{-1}(ax)^2}{x^4}\,dx = c\int \frac{\tan^{-1}(ax)^2}{x^4}\,dx + \left(a^2c\right)\int \frac{\tan^{-1}(ax)^2}{x^2}\,dx$$
$$= -\frac{c\tan^{-1}(ax)^2}{3x^3} - \frac{a^2c\tan^{-1}(ax)^2}{x} + \frac{1}{3}(2ac)\int \frac{\tan^{-1}(ax)}{x^3\left(1+a^2x^2\right)}\,dx + \left(2a^3c\right)\int \frac{\tan^{-1}(ax)}{x\left(1+a^2x^2\right)}$$
$$= -ia^3c\tan^{-1}(ax)^2 - \frac{c\tan^{-1}(ax)^2}{3x^3} - \frac{a^2c\tan^{-1}(ax)^2}{x} + \frac{1}{3}(2ac)\int \frac{\tan^{-1}(ax)}{x^3}\,dx + \left(2ia^3c\right)$$
$$= -\frac{ac\tan^{-1}(ax)}{3x^2} - \frac{2}{3}ia^3c\tan^{-1}(ax)^2 - \frac{c\tan^{-1}(ax)^2}{3x^3} - \frac{a^2c\tan^{-1}(ax)^2}{x} + 2a^3c\tan^{-1}(ax)\,\mathrm{lo}$$
$$= -\frac{a^2c}{3x} - \frac{ac\tan^{-1}(ax)}{3x^2} - \frac{2}{3}ia^3c\tan^{-1}(ax)^2 - \frac{c\tan^{-1}(ax)^2}{3x^3} - \frac{a^2c\tan^{-1}(ax)^2}{x} + \frac{4}{3}a^3c\tan^{-}$$
$$= -\frac{a^2c}{3x} - \frac{1}{3}a^3c\tan^{-1}(ax) - \frac{ac\tan^{-1}(ax)}{3x^2} - \frac{2}{3}ia^3c\tan^{-1}(ax)^2 - \frac{c\tan^{-1}(ax)^2}{3x^3} - \frac{a^2c\tan^{-1}(}{x}$$

Mathematica [A] time = 0.590859, size = 103, normalized size = 0.76

$$\frac{c\left(-2ia^3x^3\text{PolyLog}\left(2,e^{2i\tan^{-1}(ax)}\right) - a^2x^2 + ax\tan^{-1}(ax)\left(-a^2x^2 + 4a^2x^2\log\left(1-e^{2i\tan^{-1}(ax)}\right) - 1\right) + (1-2iax)(ax-i)^{}\right.}{3x^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((c + a^2*c*x^2)*ArcTan[a*x]^2)/x^4,x]
```

```
[Out] (c*(-(a^2*x^2) + (1 - (2*I)*a*x)*(-I + a*x)^2*ArcTan[a*x]^2 + a*x*ArcTan[a*
x]*(-1 - a^2*x^2 + 4*a^2*x^2*Log[1 - E^((2*I)*ArcTan[a*x])]) - (2*I)*a^3*x^
3*PolyLog[2, E^((2*I)*ArcTan[a*x])]))/(3*x^3)
```

Maple [B] time = 0.096, size = 323, normalized size = 2.4

$$-\frac{a^2c\left(\arctan(ax)\right)^2}{x} - \frac{c\left(\arctan(ax)\right)^2}{3x^3} - \frac{2a^3c\arctan(ax)\ln\left(a^2x^2+1\right)}{3} - \frac{ac\arctan(ax)}{3x^2} + \frac{4a^3c\arctan(ax)\ln(ax)}{3} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)*arctan(a*x)^2/x^4,x)
```

```
[Out] -a^2*c*arctan(a*x)^2/x-1/3*c*arctan(a*x)^2/x^3-2/3*a^3*c*arctan(a*x)*ln(a^2
*x^2+1)-1/3*a*c*arctan(a*x)/x^2+4/3*a^3*c*arctan(a*x)*ln(a*x)-1/3*a^3*c*arc
tan(a*x)-1/3*a^2*c/x-1/3*I*a^3*c*ln(a*x+I)*ln(1/2*I*(a*x-I))+1/3*I*a^3*c*di
log(-1/2*I*(a*x+I))-1/3*I*a^3*c*ln(a^2*x^2+1)*ln(a*x-I)-1/3*I*a^3*c*dilog(1
/2*I*(a*x-I))-2/3*I*a^3*c*dilog(1-I*a*x)+2/3*I*a^3*c*dilog(1+I*a*x)+2/3*I*a
^3*c*ln(a*x)*ln(1+I*a*x)-2/3*I*a^3*c*ln(a*x)*ln(1-I*a*x)+1/3*I*a^3*c*ln(a*x
-I)*ln(-1/2*I*(a*x+I))+1/6*I*a^3*c*ln(a*x-I)^2+1/3*I*a^3*c*ln(a^2*x^2+1)*ln
(a*x+I)-1/6*I*a^3*c*ln(a*x+I)^2
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^2/x^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^2cx^2+c\right)\arctan\left(ax\right)^2}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^2/x^4,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)*arctan(a*x)^2/x^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$c\left(\int\frac{\operatorname{atan}^2\left(ax\right)}{x^4}\,dx+\int\frac{a^2\operatorname{atan}^2\left(ax\right)}{x^2}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)*atan(a*x)**2/x**4,x)
```

```
[Out] c*(Integral(atan(a*x)**2/x**4, x) + Integral(a**2*atan(a*x)**2/x**2, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a^2cx^2+c)\arctan(ax)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^2/x^4,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)*arctan(a*x)^2/x^4, x)
```

3.266 $\int x^3 \left(c+a^2 c x^2\right)^2 \tan^{-1}(ax)^2 \, dx$

Optimal. Leaf size=191

$$\frac{1}{168}a^2c^2x^6-\frac{5c^2x^2}{504a^2}-\frac{2c^2\log\left(a^2x^2+1\right)}{63a^4}+\frac{1}{8}a^4c^2x^8\tan^{-1}(ax)^2-\frac{1}{28}a^3c^2x^7\tan^{-1}(ax)+\frac{1}{3}a^2c^2x^6\tan^{-1}(ax)^2+\frac{c^2x\tan^{-1}}{12a^3}$$

```
[Out] (-5*c^2*x^2)/(504*a^2) + (c^2*x^4)/84 + (a^2*c^2*x^6)/168 + (c^2*x*ArcTan[a
*x])/(12*a^3) - (c^2*x^3*ArcTan[a*x])/(36*a) - (a*c^2*x^5*ArcTan[a*x])/12 -
 (a^3*c^2*x^7*ArcTan[a*x])/28 - (c^2*ArcTan[a*x]^2)/(24*a^4) + (c^2*x^4*Arc
Tan[a*x]^2)/4 + (a^2*c^2*x^6*ArcTan[a*x]^2)/3 + (a^4*c^2*x^8*ArcTan[a*x]^2)
/8 - (2*c^2*Log[1 + a^2*x^2])/(63*a^4)
```

Rubi [A] time = 0.788643, antiderivative size = 191, normalized size of antiderivative = 1., number of steps used = 47, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {4948, 4852, 4916, 266, 43, 4846, 260, 4884}

$$\frac{1}{168}a^2c^2x^6-\frac{5c^2x^2}{504a^2}-\frac{2c^2\log\left(a^2x^2+1\right)}{63a^4}+\frac{1}{8}a^4c^2x^8\tan^{-1}(ax)^2-\frac{1}{28}a^3c^2x^7\tan^{-1}(ax)+\frac{1}{3}a^2c^2x^6\tan^{-1}(ax)^2+\frac{c^2x\tan^{-1}}{12a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(c + a^2*c*x^2)^2*ArcTan[a*x]^2,x]
```

```
[Out] (-5*c^2*x^2)/(504*a^2) + (c^2*x^4)/84 + (a^2*c^2*x^6)/168 + (c^2*x*ArcTan[a
*x])/(12*a^3) - (c^2*x^3*ArcTan[a*x])/(36*a) - (a*c^2*x^5*ArcTan[a*x])/12 -
 (a^3*c^2*x^7*ArcTan[a*x])/28 - (c^2*ArcTan[a*x]^2)/(24*a^4) + (c^2*x^4*Arc
Tan[a*x]^2)/4 + (a^2*c^2*x^6*ArcTan[a*x]^2)/3 + (a^4*c^2*x^8*ArcTan[a*x]^2)
/8 - (2*c^2*Log[1 + a^2*x^2])/(63*a^4)
```

Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
```

```
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rubi steps

$$\int x^3\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^2\,dx=\int\left(c^2x^3\tan^{-1}(ax)^2+2a^2c^2x^5\tan^{-1}(ax)^2+a^4c^2x^7\tan^{-1}(ax)^2\right)dx$$
$$=c^2\int x^3\tan^{-1}(ax)^2\,dx+\left(2a^2c^2\right)\int x^5\tan^{-1}(ax)^2\,dx+\left(a^4c^2\right)\int x^7\tan^{-1}(ax)^2\,dx$$
$$=\frac{1}{4}c^2x^4\tan^{-1}(ax)^2+\frac{1}{3}a^2c^2x^6\tan^{-1}(ax)^2+\frac{1}{8}a^4c^2x^8\tan^{-1}(ax)^2-\frac{1}{2}\left(ac^2\right)\int\frac{x^4\tan^{-1}(}{1+a^2x}$$
$$=\frac{1}{4}c^2x^4\tan^{-1}(ax)^2+\frac{1}{3}a^2c^2x^6\tan^{-1}(ax)^2+\frac{1}{8}a^4c^2x^8\tan^{-1}(ax)^2-\frac{c^2\int x^2\tan^{-1}(ax)\,dx}{2a}$$
$$=-\frac{c^2x^3\tan^{-1}(ax)}{6a}-\frac{2}{15}ac^2x^5\tan^{-1}(ax)-\frac{1}{28}a^3c^2x^7\tan^{-1}(ax)+\frac{1}{4}c^2x^4\tan^{-1}(ax)^2+\frac{1}{3}$$
$$=\frac{c^2x\tan^{-1}(ax)}{2a^3}+\frac{c^2x^3\tan^{-1}(ax)}{18a}-\frac{1}{12}ac^2x^5\tan^{-1}(ax)-\frac{1}{28}a^3c^2x^7\tan^{-1}(ax)-\frac{c^2\tan}{4}$$
$$=-\frac{c^2x\tan^{-1}(ax)}{6a^3}-\frac{c^2x^3\tan^{-1}(ax)}{36a}-\frac{1}{12}ac^2x^5\tan^{-1}(ax)-\frac{1}{28}a^3c^2x^7\tan^{-1}(ax)+\frac{c^2\tan}{1}$$
$$=\frac{29c^2x^2}{840a^2}+\frac{41c^2x^4}{1680}+\frac{1}{168}a^2c^2x^6+\frac{c^2x\tan^{-1}(ax)}{12a^3}-\frac{c^2x^3\tan^{-1}(ax)}{36a}-\frac{1}{12}ac^2x^5\tan^{-1}(ax$$
$$=-\frac{13c^2x^2}{252a^2}+\frac{c^2x^4}{84}+\frac{1}{168}a^2c^2x^6+\frac{c^2x\tan^{-1}(ax)}{12a^3}-\frac{c^2x^3\tan^{-1}(ax)}{36a}-\frac{1}{12}ac^2x^5\tan^{-1}(ax)$$
$$=-\frac{5c^2x^2}{504a^2}+\frac{c^2x^4}{84}+\frac{1}{168}a^2c^2x^6+\frac{c^2x\tan^{-1}(ax)}{12a^3}-\frac{c^2x^3\tan^{-1}(ax)}{36a}-\frac{1}{12}ac^2x^5\tan^{-1}(ax)$$

Mathematica [A] time = 0.0805569, size = 110, normalized size = 0.58

$$\frac{c^2\left(3a^6x^6+6a^4x^4-5a^2x^2-16\log\left(a^2x^2+1\right)-2ax\left(9a^6x^6+21a^4x^4+7a^2x^2-21\right)\tan^{-1}(ax)+21\left(a^2x^2+1\right)^3\left(3a^2x^2-1\right.\right.}{504a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(c + a^2*c*x^2)^2*ArcTan[a*x]^2,x]
```

```
[Out] (c^2*(-5*a^2*x^2 + 6*a^4*x^4 + 3*a^6*x^6 - 2*a*x*(-21 + 7*a^2*x^2 + 21*a^4*
x^4 + 9*a^6*x^6)*ArcTan[a*x] + 21*(1 + a^2*x^2)^3*(-1 + 3*a^2*x^2)*ArcTan[a
*x]^2 - 16*Log[1 + a^2*x^2]))/(504*a^4)
```

Maple [A] time = 0.034, size = 168, normalized size = 0.9

$$-\frac{5\,c^2x^2}{504\,a^2}+\frac{c^2x^4}{84}+\frac{a^2c^2x^6}{168}+\frac{c^2x\arctan(ax)}{12\,a^3}-\frac{c^2x^3\arctan(ax)}{36\,a}-\frac{ac^2x^5\arctan(ax)}{12}-\frac{a^3c^2x^7\arctan(ax)}{28}-\frac{c^2\,(\text{arctar}}{24\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a^2*c*x^2+c)^2*arctan(a*x)^2,x)
```

```
[Out] -5/504*c^2*x^2/a^2+1/84*c^2*x^4+1/168*a^2*c^2*x^6+1/12*c^2*x*arctan(a*x)/a^
3-1/36*c^2*x^3*arctan(a*x)/a-1/12*a*c^2*x^5*arctan(a*x)-1/28*a^3*c^2*x^7*ar
ctan(a*x)-1/24*c^2*arctan(a*x)^2/a^4+1/4*c^2*x^4*arctan(a*x)^2+1/3*a^2*c^2*
x^6*arctan(a*x)^2+1/8*a^4*c^2*x^8*arctan(a*x)^2-2/63*c^2*ln(a^2*x^2+1)/a^4
```

Maxima [A] time = 1.56325, size = 228, normalized size = 1.19

$$-\frac{1}{252}\,a\left(\frac{21\,c^2\arctan(ax)}{a^5}+\frac{9\,a^6c^2x^7+21\,a^4c^2x^5+7\,a^2c^2x^3-21\,c^2x}{a^4}\right)\arctan(ax)+\frac{1}{24}\left(3\,a^4c^2x^8+8\,a^2c^2x^6+6\,c^2x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a^2*c*x^2+c)^2*arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] -1/252*a*(21*c^2*arctan(a*x)/a^5 + (9*a^6*c^2*x^7 + 21*a^4*c^2*x^5 + 7*a^2*
c^2*x^3 - 21*c^2*x)/a^4)*arctan(a*x) + 1/24*(3*a^4*c^2*x^8 + 8*a^2*c^2*x^6
+ 6*c^2*x^4)*arctan(a*x)^2 + 1/504*(3*a^6*c^2*x^6 + 6*a^4*c^2*x^4 - 5*a^2*c
^2*x^2 + 21*c^2*arctan(a*x)^2 - 16*c^2*log(a^2*x^2 + 1))/a^4
```

Fricas [A] time = 2.17902, size = 319, normalized size = 1.67

$$\frac{3\,a^6c^2x^6+6\,a^4c^2x^4-5\,a^2c^2x^2+21\left(3\,a^8c^2x^8+8\,a^6c^2x^6+6\,a^4c^2x^4-c^2\right)\arctan(ax)^2-16\,c^2\log\left(a^2x^2+1\right)-2\left(9\,a^7c^2x\right.}{504\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a^2*c*x^2+c)^2*arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/504*(3*a^6*c^2*x^6 + 6*a^4*c^2*x^4 - 5*a^2*c^2*x^2 + 21*(3*a^8*c^2*x^8 +
8*a^6*c^2*x^6 + 6*a^4*c^2*x^4 - c^2)*arctan(a*x)^2 - 16*c^2*log(a^2*x^2 + 1
```

```
) - 2*(9*a^7*c^2*x^7 + 21*a^5*c^2*x^5 + 7*a^3*c^2*x^3 - 21*a*c^2*x)*arctan(
a*x))/a^4
```

Sympy [A] time = 4.81681, size = 185, normalized size = 0.97

$$\begin{cases} \frac{a^4c^2x^8\operatorname{atan}^2(ax)}{8} - \frac{a^3c^2x^7\operatorname{atan}(ax)}{28} + \frac{a^2c^2x^6\operatorname{atan}^2(ax)}{3} + \frac{a^2c^2x^6}{168} - \frac{ac^2x^5\operatorname{atan}(ax)}{12} + \frac{c^2x^4\operatorname{atan}^2(ax)}{4} + \frac{c^2x^4}{84} - \frac{c^2x^3\operatorname{atan}(ax)}{36a} - \frac{5c^2x^2}{504a^2} + \frac{c^2x\,\mathrm{at}}{1} \\ 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a**2*c*x**2+c)**2*atan(a*x)**2,x)
```

```
[Out] Piecewise((a**4*c**2*x**8*atan(a*x)**2/8 - a**3*c**2*x**7*atan(a*x)/28 + a*
*2*c**2*x**6*atan(a*x)**2/3 + a**2*c**2*x**6/168 - a*c**2*x**5*atan(a*x)/12
 + c**2*x**4*atan(a*x)**2/4 + c**2*x**4/84 - c**2*x**3*atan(a*x)/(36*a) - 5
*c**2*x**2/(504*a**2) + c**2*x*atan(a*x)/(12*a**3) - 2*c**2*log(x**2 + a**(
-2))/(63*a**4) - c**2*atan(a*x)**2/(24*a**4), Ne(a, 0)), (0, True))
```

Giac [A] time = 1.18099, size = 217, normalized size = 1.14

$$\frac{1}{24}\left(3\,a^4c^2x^8 + 8\,a^2c^2x^6 + 6\,c^2x^4\right)\arctan(ax)^2 - \frac{18\,a^7c^2x^7\arctan(ax) - 3\,a^6c^2x^6 + 42\,a^5c^2x^5\arctan(ax) - 6\,a^4c^2x^4 + 14}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a^2*c*x^2+c)^2*arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] 1/24*(3*a^4*c^2*x^8 + 8*a^2*c^2*x^6 + 6*c^2*x^4)*arctan(a*x)^2 - 1/504*(18*
a^7*c^2*x^7*arctan(a*x) - 3*a^6*c^2*x^6 + 42*a^5*c^2*x^5*arctan(a*x) - 6*a^
4*c^2*x^4 + 14*a^3*c^2*x^3*arctan(a*x) + 5*a^2*c^2*x^2 - 42*a*c^2*x*arctan(
a*x) + 21*c^2*arctan(a*x)^2 + 16*c^2*log(a^2*x^2 + 1))/a^4
```

3.267 $\int x^2 \left(c+a^2 c x^2\right)^2 \tan^{-1}(ax)^2\,dx$

Optimal. Leaf size=225

$$-\frac{8ic^2\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{105a^3}+\frac{1}{105}a^2c^2x^5+\frac{1}{7}a^4c^2x^7\tan^{-1}(ax)^2-\frac{1}{21}a^3c^2x^6\tan^{-1}(ax)+\frac{2}{5}a^2c^2x^5\tan^{-1}(ax)^2-\frac{c^2x}{210a^2}$$

```
[Out] -(c^2*x)/(210*a^2) + (17*c^2*x^3)/630 + (a^2*c^2*x^5)/105 + (c^2*ArcTan[a*x
])/(210*a^3) - (8*c^2*x^2*ArcTan[a*x])/(105*a) - (9*a*c^2*x^4*ArcTan[a*x])/
70 - (a^3*c^2*x^6*ArcTan[a*x])/21 - (((8*I)/105)*c^2*ArcTan[a*x]^2)/a^3 + (
c^2*x^3*ArcTan[a*x]^2)/3 + (2*a^2*c^2*x^5*ArcTan[a*x]^2)/5 + (a^4*c^2*x^7*A
rcTan[a*x]^2)/7 - (16*c^2*ArcTan[a*x]*Log[2/(1 + I*a*x)])/(105*a^3) - (((8*
I)/105)*c^2*PolyLog[2, 1 - 2/(1 + I*a*x)])/a^3
```

Rubi [A] time = 0.752474, antiderivative size = 225, normalized size of antiderivative = 1., number of steps used = 44, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {4948, 4852, 4916, 321, 203, 4920, 4854, 2402, 2315, 302}

$$-\frac{8ic^2\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{105a^3}+\frac{1}{105}a^2c^2x^5+\frac{1}{7}a^4c^2x^7\tan^{-1}(ax)^2-\frac{1}{21}a^3c^2x^6\tan^{-1}(ax)+\frac{2}{5}a^2c^2x^5\tan^{-1}(ax)^2-\frac{c^2x}{210a^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]^2,x]
```

```
[Out] -(c^2*x)/(210*a^2) + (17*c^2*x^3)/630 + (a^2*c^2*x^5)/105 + (c^2*ArcTan[a*x
])/(210*a^3) - (8*c^2*x^2*ArcTan[a*x])/(105*a) - (9*a*c^2*x^4*ArcTan[a*x])/
70 - (a^3*c^2*x^6*ArcTan[a*x])/21 - (((8*I)/105)*c^2*ArcTan[a*x]^2)/a^3 + (
c^2*x^3*ArcTan[a*x]^2)/3 + (2*a^2*c^2*x^5*ArcTan[a*x]^2)/5 + (a^4*c^2*x^7*A
rcTan[a*x]^2)/7 - (16*c^2*ArcTan[a*x]*Log[2/(1 + I*a*x)])/(105*a^3) - (((8*
I)/105)*c^2*PolyLog[2, 1 - 2/(1 + I*a*x)])/a^3
```

Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
```

```
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\int x^2 \left(c+a^2cx^2\right)^2 \tan^{-1}(ax)^2\,dx = \int \left(c^2x^2\tan^{-1}(ax)^2+2a^2c^2x^4\tan^{-1}(ax)^2+a^4c^2x^6\tan^{-1}(ax)^2\right)dx$$

$$= c^2\int x^2\tan^{-1}(ax)^2\,dx+\left(2a^2c^2\right)\int x^4\tan^{-1}(ax)^2\,dx+\left(a^4c^2\right)\int x^6\tan^{-1}(ax)^2\,dx$$

$$= \frac{1}{3}c^2x^3\tan^{-1}(ax)^2+\frac{2}{5}a^2c^2x^5\tan^{-1}(ax)^2+\frac{1}{7}a^4c^2x^7\tan^{-1}(ax)^2-\frac{1}{3}\left(2ac^2\right)\int \frac{x^3\tan^{-1}}{1+a}$$

$$= \frac{1}{3}c^2x^3\tan^{-1}(ax)^2+\frac{2}{5}a^2c^2x^5\tan^{-1}(ax)^2+\frac{1}{7}a^4c^2x^7\tan^{-1}(ax)^2-\frac{\left(2c^2\right)\int x\tan^{-1}(ax}{3a}$$

$$= -\frac{c^2x^2\tan^{-1}(ax)}{3a}-\frac{1}{5}ac^2x^4\tan^{-1}(ax)-\frac{1}{21}a^3c^2x^6\tan^{-1}(ax)-\frac{ic^2\tan^{-1}(ax)^2}{3a^3}+\frac{1}{3}c^2x$$

$$= \frac{c^2x}{3a^2}+\frac{c^2x^2\tan^{-1}(ax)}{15a}-\frac{9}{70}ac^2x^4\tan^{-1}(ax)-\frac{1}{21}a^3c^2x^6\tan^{-1}(ax)+\frac{ic^2\tan^{-1}(ax)^2}{15a^3}$$

$$= -\frac{23c^2x}{105a^2}+\frac{16c^2x^3}{315}+\frac{1}{105}a^2c^2x^5-\frac{c^2\tan^{-1}(ax)}{3a^3}-\frac{8c^2x^2\tan^{-1}(ax)}{105a}-\frac{9}{70}ac^2x^4\tan^{-1}($$

$$= -\frac{c^2x}{210a^2}+\frac{17c^2x^3}{630}+\frac{1}{105}a^2c^2x^5+\frac{23c^2\tan^{-1}(ax)}{105a^3}-\frac{8c^2x^2\tan^{-1}(ax)}{105a}-\frac{9}{70}ac^2x^4\tan^{-1}$$

$$= -\frac{c^2x}{210a^2}+\frac{17c^2x^3}{630}+\frac{1}{105}a^2c^2x^5+\frac{c^2\tan^{-1}(ax)}{210a^3}-\frac{8c^2x^2\tan^{-1}(ax)}{105a}-\frac{9}{70}ac^2x^4\tan^{-1}($$

$$= -\frac{c^2x}{210a^2}+\frac{17c^2x^3}{630}+\frac{1}{105}a^2c^2x^5+\frac{c^2\tan^{-1}(ax)}{210a^3}-\frac{8c^2x^2\tan^{-1}(ax)}{105a}-\frac{9}{70}ac^2x^4\tan^{-1}($$

Mathematica [A] time = 1.24216, size = 133, normalized size = 0.59

$$\frac{c^2\left(48i\text{PolyLog}\left(2,-e^{2i\tan^{-1}(ax)}\right)+ax\left(6a^4x^4+17a^2x^2-3\right)+6\left(15a^7x^7+42a^5x^5+35a^3x^3+8i\right)\tan^{-1}(ax)^2-3\tan^{-1}(a$$
$$630a^3$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]^2,x]
```

```
[Out] (c^2*(a*x*(-3 + 17*a^2*x^2 + 6*a^4*x^4) + 6*(8*I + 35*a^3*x^3 + 42*a^5*x^5
+ 15*a^7*x^7)*ArcTan[a*x]^2 - 3*ArcTan[a*x]*(-1 + 16*a^2*x^2 + 27*a^4*x^4 +
 10*a^6*x^6 + 32*Log[1 + E^((2*I)*ArcTan[a*x])]) + (48*I)*PolyLog[2, -E^((2
*I)*ArcTan[a*x])]))/(630*a^3)
```

Maple [A] time = 0.092, size = 333, normalized size = 1.5

$$\frac{a^4c^2x^7(\arctan(ax))^2}{7}+\frac{2a^2c^2x^5(\arctan(ax))^2}{5}+\frac{c^2x^3(\arctan(ax))^2}{3}-\frac{a^3c^2x^6\arctan(ax)}{21}-\frac{9ac^2x^4\arctan(ax)}{70}-\frac{8c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a^2*c*x^2+c)^2*arctan(a*x)^2,x)
```

```
[Out] 1/7*a^4*c^2*x^7*arctan(a*x)^2+2/5*a^2*c^2*x^5*arctan(a*x)^2+1/3*c^2*x^3*arc
tan(a*x)^2-1/21*a^3*c^2*x^6*arctan(a*x)-9/70*a*c^2*x^4*arctan(a*x)-8/105*c^
2*x^2*arctan(a*x)/a+8/105/a^3*c^2*arctan(a*x)*ln(a^2*x^2+1)+1/105*a^2*c^2*x
^5+17/630*c^2*x^3-1/210*c^2*x/a^2+1/210*c^2*arctan(a*x)/a^3+4/105*I/a^3*c^2
*ln(a^2*x^2+1)*ln(a*x-I)+4/105*I/a^3*c^2*dilog(1/2*I*(a*x-I))+4/105*I/a^3*c
^2*ln(a*x+I)*ln(1/2*I*(a*x-I))-2/105*I/a^3*c^2*ln(a*x-I)^2+2/105*I/a^3*c^2*
ln(a*x+I)^2-4/105*I/a^3*c^2*ln(a^2*x^2+1)*ln(a*x+I)-4/105*I/a^3*c^2*dilog(-
1/2*I*(a*x+I))-4/105*I/a^3*c^2*ln(a*x-I)*ln(-1/2*I*(a*x+I))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{420}\left(15a^4c^2x^7+42a^2c^2x^5+35c^2x^3\right)\arctan(ax)^2-\frac{1}{1680}\left(15a^4c^2x^7+42a^2c^2x^5+35c^2x^3\right)\log\left(a^2x^2+1\right)^2+\int\frac{1260(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^2*arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/420*(15*a^4*c^2*x^7 + 42*a^2*c^2*x^5 + 35*c^2*x^3)*arctan(a*x)^2 - 1/1680
*(15*a^4*c^2*x^7 + 42*a^2*c^2*x^5 + 35*c^2*x^3)*log(a^2*x^2 + 1)^2 + integr
ate(1/1680*(1260*(a^6*c^2*x^8 + 3*a^4*c^2*x^6 + 3*a^2*c^2*x^4 + c^2*x^2)*ar
ctan(a*x)^2 + 105*(a^6*c^2*x^8 + 3*a^4*c^2*x^6 + 3*a^2*c^2*x^4 + c^2*x^2)*l
og(a^2*x^2 + 1)^2 - 8*(15*a^5*c^2*x^7 + 42*a^3*c^2*x^5 + 35*a*c^2*x^3)*arct
an(a*x) + 4*(15*a^6*c^2*x^8 + 42*a^4*c^2*x^6 + 35*a^2*c^2*x^4)*log(a^2*x^2
+ 1))/(a^2*x^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^4c^2x^6 + 2\,a^2c^2x^4 + c^2x^2\right)\arctan\left(ax\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^2*arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^6 + 2*a^2*c^2*x^4 + c^2*x^2)*arctan(a*x)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$c^2\left(\int x^2\,\text{atan}^2\left(ax\right)dx + \int 2a^2x^4\,\text{atan}^2\left(ax\right)dx + \int a^4x^6\,\text{atan}^2\left(ax\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a**2*c*x**2+c)**2*atan(a*x)**2,x)
```

```
[Out] c**2*(Integral(x**2*atan(a*x)**2, x) + Integral(2*a**2*x**4*atan(a*x)**2, x
) + Integral(a**4*x**6*atan(a*x)**2, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2 + c\right)^2 x^2 \arctan\left(ax\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^2*arctan(a*x)^2,x, algorithm="giac")

[Out] integrate((a^2*c*x^2 + c)^2*x^2*arctan(a*x)^2, x)
```

3.268 $\int x \left(c + a^2 c x^2\right)^2 \tan^{-1}(ax)^2 \, dx$

Optimal. Leaf size=153

$$\frac{c^2 \left(a^2x^2+1\right)^2}{60a^2}+\frac{2c^2 \left(a^2x^2+1\right)}{45a^2}+\frac{4c^2 \log \left(a^2x^2+1\right)}{45a^2}+\frac{c^2 \left(a^2x^2+1\right)^3 \tan^{-1}(ax)^2}{6a^2}-\frac{c^2 x \left(a^2x^2+1\right)^2 \tan^{-1}(ax)}{15a}-\frac{4c^2 x \left(a\right.}{}$$

```
[Out] (2*c^2*(1 + a^2*x^2))/(45*a^2) + (c^2*(1 + a^2*x^2)^2)/(60*a^2) - (8*c^2*x*
ArcTan[a*x])/(45*a) - (4*c^2*x*(1 + a^2*x^2)*ArcTan[a*x])/(45*a) - (c^2*x*(
1 + a^2*x^2)^2*ArcTan[a*x])/(15*a) + (c^2*(1 + a^2*x^2)^3*ArcTan[a*x]^2)/(6
*a^2) + (4*c^2*Log[1 + a^2*x^2])/(45*a^2)
```

Rubi [A] time = 0.0930782, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4930, 4878, 4846, 260}

$$\frac{c^2 \left(a^2x^2+1\right)^2}{60a^2}+\frac{2c^2 \left(a^2x^2+1\right)}{45a^2}+\frac{4c^2 \log \left(a^2x^2+1\right)}{45a^2}+\frac{c^2 \left(a^2x^2+1\right)^3 \tan^{-1}(ax)^2}{6a^2}-\frac{c^2 x \left(a^2x^2+1\right)^2 \tan^{-1}(ax)}{15a}-\frac{4c^2 x \left(a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x*(c + a^2*c*x^2)^2*ArcTan[a*x]^2,x]
```


```
[Out] (2*c^2*(1 + a^2*x^2))/(45*a^2) + (c^2*(1 + a^2*x^2)^2)/(60*a^2) - (8*c^2*x*
ArcTan[a*x])/(45*a) - (4*c^2*x*(1 + a^2*x^2)*ArcTan[a*x])/(45*a) - (c^2*x*(
1 + a^2*x^2)^2*ArcTan[a*x])/(15*a) + (c^2*(1 + a^2*x^2)^3*ArcTan[a*x]^2)/(6
*a^2) + (4*c^2*Log[1 + a^2*x^2])/(45*a^2)
```


Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 4878

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symbo
l] :> -Simp[(b*(d + e*x^2)^q)/(2*c*q*(2*q + 1)), x] + (Dist[(2*d*q)/(2*q +
1), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x]), x], x] + Simp[(x*(d + e*x^
```

```
2)^q*(a + b*ArcTan[c*x]))/(2*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] && Eq
Q[e, c^2*d] && GtQ[q, 0]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int x\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^2\,dx = \frac{c^2\left(1+a^2x^2\right)^3\tan^{-1}(ax)^2}{6a^2}-\frac{\int\left(c+a^2cx^2\right)^2\tan^{-1}(ax)\,dx}{3a}$$
$$= \frac{c^2\left(1+a^2x^2\right)^2}{60a^2}-\frac{c^2x\left(1+a^2x^2\right)^2\tan^{-1}(ax)}{15a}+\frac{c^2\left(1+a^2x^2\right)^3\tan^{-1}(ax)^2}{6a^2}-\frac{(4c)\int\left(c+a\right.}{}$$
$$= \frac{2c^2\left(1+a^2x^2\right)}{45a^2}+\frac{c^2\left(1+a^2x^2\right)^2}{60a^2}-\frac{4c^2x\left(1+a^2x^2\right)\tan^{-1}(ax)}{45a}-\frac{c^2x\left(1+a^2x^2\right)^2\tan^{-1}(a}{15a}$$
$$= \frac{2c^2\left(1+a^2x^2\right)}{45a^2}+\frac{c^2\left(1+a^2x^2\right)^2}{60a^2}-\frac{8c^2x\tan^{-1}(ax)}{45a}-\frac{4c^2x\left(1+a^2x^2\right)\tan^{-1}(ax)}{45a}-\frac{c^2x\left(1\right.}{}$$
$$= \frac{2c^2\left(1+a^2x^2\right)}{45a^2}+\frac{c^2\left(1+a^2x^2\right)^2}{60a^2}-\frac{8c^2x\tan^{-1}(ax)}{45a}-\frac{4c^2x\left(1+a^2x^2\right)\tan^{-1}(ax)}{45a}-\frac{c^2x\left(1\right.}{}$$

Mathematica [A] time = 0.0649818, size = 84, normalized size = 0.55

$$\frac{c^2\left(3a^4x^4+14a^2x^2+16\log\left(a^2x^2+1\right)-4ax\left(3a^4x^4+10a^2x^2+15\right)\tan^{-1}(ax)+30\left(a^2x^2+1\right)^3\tan^{-1}(ax)^2\right)}{180a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(c + a^2*c*x^2)^2*ArcTan[a*x]^2,x]
```

```
[Out] (c^2*(14*a^2*x^2 + 3*a^4*x^4 - 4*a*x*(15 + 10*a^2*x^2 + 3*a^4*x^4)*ArcTan[a
*x] + 30*(1 + a^2*x^2)^3*ArcTan[a*x]^2 + 16*Log[1 + a^2*x^2]))/(180*a^2)
```

Maple [A] time = 0.031, size = 142, normalized size = 0.9

$$\frac{a^4c^2 (\arctan(ax))^2 x^6}{6} + \frac{a^2c^2 (\arctan(ax))^2 x^4}{2} + \frac{c^2 (\arctan(ax))^2 x^2}{2} - \frac{a^3c^2 \arctan(ax) x^5}{15} - \frac{2ac^2 \arctan(ax) x^3}{9} - \frac{c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^2*arctan(a*x)^2,x)
```

```
[Out] 1/6*a^4*c^2*arctan(a*x)^2*x^6+1/2*a^2*c^2*arctan(a*x)^2*x^4+1/2*c^2*arctan(
a*x)^2*x^2-1/15*a^3*c^2*arctan(a*x)*x^5-2/9*a*c^2*arctan(a*x)*x^3-1/3*c^2*x
*arctan(a*x)/a+1/6/a^2*c^2*arctan(a*x)^2+1/60*a^2*c^2*x^4+7/90*c^2*x^2+4/45
*c^2*ln(a^2*x^2+1)/a^2
```

Maxima [A] time = 1.00444, size = 150, normalized size = 0.98

$$\frac{\left(a^2cx^2+c\right)^3 \arctan(ax)^2}{6a^2c} + \frac{\left(3a^2c^3x^4 + 14c^3x^2 + \frac{16c^3\log\left(a^2x^2+1\right)}{a^2}\right)a - 4\left(3a^4c^3x^5 + 10a^2c^3x^3 + 15c^3x\right)\arctan(ax)}{180ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^2*arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/6*(a^2*c*x^2 + c)^3*arctan(a*x)^2/(a^2*c) + 1/180*((3*a^2*c^3*x^4 + 14*c^
3*x^2 + 16*c^3*log(a^2*x^2 + 1)/a^2)*a - 4*(3*a^4*c^3*x^5 + 10*a^2*c^3*x^3
+ 15*c^3*x)*arctan(a*x))/(a*c)
```

Fricas [A] time = 2.21317, size = 274, normalized size = 1.79

$$\frac{3a^4c^2x^4 + 14a^2c^2x^2 + 30\left(a^6c^2x^6 + 3a^4c^2x^4 + 3a^2c^2x^2 + c^2\right)\arctan(ax)^2 + 16c^2\log\left(a^2x^2+1\right) - 4\left(3a^5c^2x^5 + 10a^3c^2\right.}{180a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^2*arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/180*(3*a^4*c^2*x^4 + 14*a^2*c^2*x^2 + 30*(a^6*c^2*x^6 + 3*a^4*c^2*x^4 + 3
*a^2*c^2*x^2 + c^2)*arctan(a*x)^2 + 16*c^2*log(a^2*x^2 + 1) - 4*(3*a^5*c^2*
x^5 + 10*a^3*c^2*x^3 + 15*a*c^2*x)*arctan(a*x))/a^2
```

Sympy [A] time = 2.78782, size = 158, normalized size = 1.03

$$\begin{cases} \frac{a^4c^2x^6\operatorname{atan}^2(ax)}{6} - \frac{a^3c^2x^5\operatorname{atan}(ax)}{15} + \frac{a^2c^2x^4\operatorname{atan}^2(ax)}{2} + \frac{a^2c^2x^4}{60} - \frac{2ac^2x^3\operatorname{atan}(ax)}{9} + \frac{c^2x^2\operatorname{atan}^2(ax)}{2} + \frac{7c^2x^2}{90} - \frac{c^2x\operatorname{atan}(ax)}{3a} + \frac{4c^2\log\left(x^2+\frac{1}{a^2}\right)}{45a^2} \\ 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**2*atan(a*x)**2,x)
```

```
[Out] Piecewise((a**4*c**2*x**6*atan(a*x)**2/6 - a**3*c**2*x**5*atan(a*x)/15 + a*
*2*c**2*x**4*atan(a*x)**2/2 + a**2*c**2*x**4/60 - 2*a*c**2*x**3*atan(a*x)/9
 + c**2*x**2*atan(a*x)**2/2 + 7*c**2*x**2/90 - c**2*x*atan(a*x)/(3*a) + 4*c
**2*log(x**2 + a**(-2))/(45*a**2) + c**2*atan(a*x)**2/(6*a**2), Ne(a, 0)),
(0, True))
```

Giac [A] time = 1.16454, size = 216, normalized size = 1.41

$$\frac{\left(a^2cx^2+c\right)^3\arctan(ax)^2}{6\,a^2c} - \frac{3\left(4\,x^5\arctan(ax) - a\left(\frac{a^2x^4-2\,x^2}{a^4} + \frac{2\,\log\left(a^2x^2+1\right)}{a^6}\right)\right)a^4c^2 + 20\left(2\,x^3\arctan(ax) - a\left(\frac{x^2}{a^2} - \frac{\log\left(a^2x^2\right.}{a^4}\right.\right.}{180\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^2*arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] 1/6*(a^2*c*x^2 + c)^3*arctan(a*x)^2/(a^2*c) - 1/180*(3*(4*x^5*arctan(a*x) -
 a*((a^2*x^4 - 2*x^2)/a^4 + 2*log(a^2*x^2 + 1)/a^6))*a^4*c^2 + 20*(2*x^3*ar
ctan(a*x) - a*(x^2/a^2 - log(a^2*x^2 + 1)/a^4))*a^2*c^2 + 30*(2*a*x*arctan(
a*x) - log(a^2*x^2 + 1))*c^2/a)/a
```

3.269 $\int \left(c+a^2 c x^2\right)^2 \tan^{-1}(ax)^2 \, dx$

Optimal. Leaf size=205

$$\frac{8ic^2\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{15a}+\frac{1}{30}a^2c^2x^3+\frac{1}{5}c^2x\left(a^2x^2+1\right)^2\tan^{-1}(ax)^2+\frac{4}{15}c^2x\left(a^2x^2+1\right)\tan^{-1}(ax)^2-\frac{c^2\left(a^2x^2+1\right)^2 \text{t}}{10a}$$

```
[Out] (11*c^2*x)/30 + (a^2*c^2*x^3)/30 - (4*c^2*(1 + a^2*x^2)*ArcTan[a*x])/(15*a)
 - (c^2*(1 + a^2*x^2)^2*ArcTan[a*x])/(10*a) + (((8*I)/15)*c^2*ArcTan[a*x]^2
)/a + (8*c^2*x*ArcTan[a*x]^2)/15 + (4*c^2*x*(1 + a^2*x^2)*ArcTan[a*x]^2)/15
 + (c^2*x*(1 + a^2*x^2)^2*ArcTan[a*x]^2)/5 + (16*c^2*ArcTan[a*x]*Log[2/(1 +
 I*a*x)])/(15*a) + (((8*I)/15)*c^2*PolyLog[2, 1 - 2/(1 + I*a*x)])/a
```

Rubi [A] time = 0.138938, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {4880, 4846, 4920, 4854, 2402, 2315, 8}

$$\frac{8ic^2\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{15a}+\frac{1}{30}a^2c^2x^3+\frac{1}{5}c^2x\left(a^2x^2+1\right)^2\tan^{-1}(ax)^2+\frac{4}{15}c^2x\left(a^2x^2+1\right)\tan^{-1}(ax)^2-\frac{c^2\left(a^2x^2+1\right)^2 \text{t}}{10a}$$

Antiderivative was successfully verified.

```
[In] Int[(c + a^2*c*x^2)^2*ArcTan[a*x]^2,x]
```

```
[Out] (11*c^2*x)/30 + (a^2*c^2*x^3)/30 - (4*c^2*(1 + a^2*x^2)*ArcTan[a*x])/(15*a)
 - (c^2*(1 + a^2*x^2)^2*ArcTan[a*x])/(10*a) + (((8*I)/15)*c^2*ArcTan[a*x]^2
)/a + (8*c^2*x*ArcTan[a*x]^2)/15 + (4*c^2*x*(1 + a^2*x^2)*ArcTan[a*x]^2)/15
 + (c^2*x*(1 + a^2*x^2)^2*ArcTan[a*x]^2)/5 + (16*c^2*ArcTan[a*x]*Log[2/(1 +
 I*a*x)])/(15*a) + (((8*I)/15)*c^2*PolyLog[2, 1 - 2/(1 + I*a*x)])/a
```

Rule 4880

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> -Simp[(b*p*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1))/(2*c*q*(2*
q + 1)), x] + (Dist[(2*d*q)/(2*q + 1), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTa
n[c*x])^p, x], x] + Dist[(b^2*d*p*(p - 1))/(2*q*(2*q + 1)), Int[(d + e*x^2)
^(q - 1)*(a + b*ArcTan[c*x])^(p - 2), x], x] + Simp[(x*(d + e*x^2)^q*(a + b
*ArcTan[c*x])^p)/(2*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[q, 0] && GtQ[p, 1]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \left(c+a^2cx^2\right)^2 \tan^{-1}(ax)^2\,dx = -\frac{c^2\left(1+a^2x^2\right)^2\tan^{-1}(ax)}{10a} + \frac{1}{5}c^2x\left(1+a^2x^2\right)^2\tan^{-1}(ax)^2 + \frac{1}{10}c\int\left(c+a^2cx^2\right)\,dx + \frac{1}{5}$$

$$= \frac{c^2x}{10} + \frac{1}{30}a^2c^2x^3 - \frac{4c^2\left(1+a^2x^2\right)\tan^{-1}(ax)}{15a} - \frac{c^2\left(1+a^2x^2\right)^2\tan^{-1}(ax)}{10a} + \frac{4}{15}c^2x\left(1+a\right.$$

$$= \frac{11c^2x}{30} + \frac{1}{30}a^2c^2x^3 - \frac{4c^2\left(1+a^2x^2\right)\tan^{-1}(ax)}{15a} - \frac{c^2\left(1+a^2x^2\right)^2\tan^{-1}(ax)}{10a} + \frac{8}{15}c^2x\tan$$

$$= \frac{11c^2x}{30} + \frac{1}{30}a^2c^2x^3 - \frac{4c^2\left(1+a^2x^2\right)\tan^{-1}(ax)}{15a} - \frac{c^2\left(1+a^2x^2\right)^2\tan^{-1}(ax)}{10a} + \frac{8ic^2\tan^{-1}}{15a}$$

$$= \frac{11c^2x}{30} + \frac{1}{30}a^2c^2x^3 - \frac{4c^2\left(1+a^2x^2\right)\tan^{-1}(ax)}{15a} - \frac{c^2\left(1+a^2x^2\right)^2\tan^{-1}(ax)}{10a} + \frac{8ic^2\tan^{-1}}{15a}$$

$$= \frac{11c^2x}{30} + \frac{1}{30}a^2c^2x^3 - \frac{4c^2\left(1+a^2x^2\right)\tan^{-1}(ax)}{15a} - \frac{c^2\left(1+a^2x^2\right)^2\tan^{-1}(ax)}{10a} + \frac{8ic^2\tan^{-1}}{15a}$$

$$= \frac{11c^2x}{30} + \frac{1}{30}a^2c^2x^3 - \frac{4c^2\left(1+a^2x^2\right)\tan^{-1}(ax)}{15a} - \frac{c^2\left(1+a^2x^2\right)^2\tan^{-1}(ax)}{10a} + \frac{8ic^2\tan^{-1}}{15a}$$

Mathematica [A] time = 0.654809, size = 112, normalized size = 0.55

$$\frac{c^2\left(-16i\text{PolyLog}\left(2,-e^{2i\tan^{-1}(ax)}\right)+ax\left(a^2x^2+11\right)+2\left(3a^5x^5+10a^3x^3+15ax-8i\right)\tan^{-1}(ax)^2-\tan^{-1}(ax)\left(3a^4x^4+\right.\right.}{30a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + a^2*c*x^2)^2*ArcTan[a*x]^2,x]
```

```
[Out] (c^2*(a*x*(11 + a^2*x^2) + 2*(-8*I + 15*a*x + 10*a^3*x^3 + 3*a^5*x^5)*ArcTa
n[a*x]^2 - ArcTan[a*x]*(11 + 14*a^2*x^2 + 3*a^4*x^4 - 32*Log[1 + E^((2*I)*A
rcTan[a*x])]) - (16*I)*PolyLog[2, -E^((2*I)*ArcTan[a*x])]))/(30*a)
```

Maple [A] time = 0.085, size = 304, normalized size = 1.5

$$\frac{a^4c^2\left(\arctan(ax)\right)^2x^5}{5} + \frac{2a^2c^2\left(\arctan(ax)\right)^2x^3}{3} + c^2x\left(\arctan(ax)\right)^2 - \frac{a^3c^2\arctan(ax)\,x^4}{10} - \frac{7ac^2\arctan(ax)\,x^2}{15} - \frac{8}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^2*arctan(a*x)^2,x)
```

```
[Out] 1/5*a^4*c^2*arctan(a*x)^2*x^5+2/3*a^2*c^2*arctan(a*x)^2*x^3+c^2*x*arctan(a*
x)^2-1/10*a^3*c^2*arctan(a*x)*x^4-7/15*a*c^2*arctan(a*x)*x^2-8/15/a*c^2*arc
tan(a*x)*ln(a^2*x^2+1)+1/30*a^2*c^2*x^3+11/30*c^2*x-11/30/a*c^2*arctan(a*x)
+4/15*I/a*c^2*ln(a^2*x^2+1)*ln(a*x+I)+4/15*I/a*c^2*ln(a*x-I)*ln(-1/2*I*(a*x
+I))+4/15*I/a*c^2*dilog(-1/2*I*(a*x+I))-4/15*I/a*c^2*dilog(1/2*I*(a*x-I))-2
/15*I/a*c^2*ln(a*x+I)^2+2/15*I/a*c^2*ln(a*x-I)^2-4/15*I/a*c^2*ln(a*x+I)*ln(
1/2*I*(a*x-I))-4/15*I/a*c^2*ln(a^2*x^2+1)*ln(a*x-I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$180\,a^6c^2\int\frac{x^6\arctan(ax)^2}{240\left(a^2x^2+1\right)}\,dx+15\,a^6c^2\int\frac{x^6\log\left(a^2x^2+1\right)^2}{240\left(a^2x^2+1\right)}\,dx+12\,a^6c^2\int\frac{x^6\log\left(a^2x^2+1\right)}{240\left(a^2x^2+1\right)}\,dx-24\,a^5c^2\int\frac{x^5\arctan}{240\left(a^2x^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] 180*a^6*c^2*integrate(1/240*x^6*arctan(a*x)^2/(a^2*x^2 + 1), x) + 15*a^6*c^
2*integrate(1/240*x^6*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x) + 12*a^6*c^2*int
egrate(1/240*x^6*log(a^2*x^2 + 1)/(a^2*x^2 + 1), x) - 24*a^5*c^2*integrate(
1/240*x^5*arctan(a*x)/(a^2*x^2 + 1), x) + 540*a^4*c^2*integrate(1/240*x^4*a
rctan(a*x)^2/(a^2*x^2 + 1), x) + 45*a^4*c^2*integrate(1/240*x^4*log(a^2*x^2
 + 1)^2/(a^2*x^2 + 1), x) + 40*a^4*c^2*integrate(1/240*x^4*log(a^2*x^2 + 1)
/(a^2*x^2 + 1), x) - 80*a^3*c^2*integrate(1/240*x^3*arctan(a*x)/(a^2*x^2 +
1), x) + 540*a^2*c^2*integrate(1/240*x^2*arctan(a*x)^2/(a^2*x^2 + 1), x) +
45*a^2*c^2*integrate(1/240*x^2*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x) + 60*a^
2*c^2*integrate(1/240*x^2*log(a^2*x^2 + 1)/(a^2*x^2 + 1), x) + 1/4*c^2*arct
an(a*x)^3/a - 120*a*c^2*integrate(1/240*x*arctan(a*x)/(a^2*x^2 + 1), x) + 1
/60*(3*a^4*c^2*x^5 + 10*a^2*c^2*x^3 + 15*c^2*x)*arctan(a*x)^2 + 15*c^2*inte
grate(1/240*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x) - 1/240*(3*a^4*c^2*x^5 + 1
0*a^2*c^2*x^3 + 15*c^2*x)*log(a^2*x^2 + 1)^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^4c^2x^4+2\,a^2c^2x^2+c^2\right)\arctan(ax)^2\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(a*x)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$c^2 \left(\int 2a^2x^2 \operatorname{atan}^2(ax)\,dx + \int a^4x^4 \operatorname{atan}^2(ax)\,dx + \int \operatorname{atan}^2(ax)\,dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**2*atan(a*x)**2,x)
```

```
[Out] c**2*(Integral(2*a**2*x**2*atan(a*x)**2, x) + Integral(a**4*x**4*atan(a*x)*
*2, x) + Integral(atan(a*x)**2, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2 + c\right)^2 \arctan(ax)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^2*arctan(a*x)^2, x)
```

3.270 $\int \frac{(c+a^2cx^2)^2 \tan^{-1}(ax)^2}{x} dx$

Optimal. Leaf size=235

$$-\frac{1}{2}c^2\text{PolyLog}\left(3,1-\frac{2}{1+iax}\right)+\frac{1}{2}c^2\text{PolyLog}\left(3,-1+\frac{2}{1+iax}\right)-ic^2\tan^{-1}(ax)\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)+ic^2\tan^{-1}(ax$$

```
[Out] (a^2*c^2*x^2)/12 - (3*a*c^2*x*ArcTan[a*x])/2 - (a^3*c^2*x^3*ArcTan[a*x])/6
+ (3*c^2*ArcTan[a*x]^2)/4 + a^2*c^2*x^2*ArcTan[a*x]^2 + (a^4*c^2*x^4*ArcTan
[a*x]^2)/4 + 2*c^2*ArcTan[a*x]^2*ArcTanh[1 - 2/(1 + I*a*x)] + (2*c^2*Log[1
+ a^2*x^2])/3 - I*c^2*ArcTan[a*x]*PolyLog[2, 1 - 2/(1 + I*a*x)] + I*c^2*Arc
Tan[a*x]*PolyLog[2, -1 + 2/(1 + I*a*x)] - (c^2*PolyLog[3, 1 - 2/(1 + I*a*x)
])/2 + (c^2*PolyLog[3, -1 + 2/(1 + I*a*x)])/2
```

Rubi [A] time = 0.514568, antiderivative size = 235, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 12, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {4948, 4850, 4988, 4884, 4994, 6610, 4852, 4916, 4846, 260, 266, 43}

$$-\frac{1}{2}c^2\text{PolyLog}\left(3,1-\frac{2}{1+iax}\right)+\frac{1}{2}c^2\text{PolyLog}\left(3,-1+\frac{2}{1+iax}\right)-ic^2\tan^{-1}(ax)\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)+ic^2\tan^{-1}(ax$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)^2*ArcTan[a*x]^2)/x,x]
```


```
[Out] (a^2*c^2*x^2)/12 - (3*a*c^2*x*ArcTan[a*x])/2 - (a^3*c^2*x^3*ArcTan[a*x])/6
+ (3*c^2*ArcTan[a*x]^2)/4 + a^2*c^2*x^2*ArcTan[a*x]^2 + (a^4*c^2*x^4*ArcTan
[a*x]^2)/4 + 2*c^2*ArcTan[a*x]^2*ArcTanh[1 - 2/(1 + I*a*x)] + (2*c^2*Log[1
+ a^2*x^2])/3 - I*c^2*ArcTan[a*x]*PolyLog[2, 1 - 2/(1 + I*a*x)] + I*c^2*Arc
Tan[a*x]*PolyLog[2, -1 + 2/(1 + I*a*x)] - (c^2*PolyLog[3, 1 - 2/(1 + I*a*x)
])/2 + (c^2*PolyLog[3, -1 + 2/(1 + I*a*x)])/2
```


Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

Rule 4850

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTan[c*x])^(p - 1)*ArcTanh[1 - 2/(1 + I*c*x)])/(1 + c^2*x^2), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4988

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTan[c*x])^p)/(d + e
*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTan[c*x])^p)/(d + e*x^2
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
```

```
e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^2}{x}\,dx = \int \left(\frac{c^2\tan^{-1}(ax)^2}{x} + 2a^2c^2x\tan^{-1}(ax)^2 + a^4c^2x^3\tan^{-1}(ax)^2\right)dx$$

$$= c^2\int \frac{\tan^{-1}(ax)^2}{x}\,dx + \left(2a^2c^2\right)\int x\tan^{-1}(ax)^2\,dx + \left(a^4c^2\right)\int x^3\tan^{-1}(ax)^2\,dx$$

$$= a^2c^2x^2\tan^{-1}(ax)^2 + \frac{1}{4}a^4c^2x^4\tan^{-1}(ax)^2 + 2c^2\tan^{-1}(ax)^2\tanh^{-1}\left(1-\frac{2}{1+iax}\right) - \left(4ac\right.$$

$$= a^2c^2x^2\tan^{-1}(ax)^2 + \frac{1}{4}a^4c^2x^4\tan^{-1}(ax)^2 + 2c^2\tan^{-1}(ax)^2\tanh^{-1}\left(1-\frac{2}{1+iax}\right) - \left(2ac\right.$$

$$= -2ac^2x\tan^{-1}(ax) - \frac{1}{6}a^3c^2x^3\tan^{-1}(ax) + c^2\tan^{-1}(ax)^2 + a^2c^2x^2\tan^{-1}(ax)^2 + \frac{1}{4}a^4c^2x$$

$$= -\frac{3}{2}ac^2x\tan^{-1}(ax) - \frac{1}{6}a^3c^2x^3\tan^{-1}(ax) + \frac{3}{4}c^2\tan^{-1}(ax)^2 + a^2c^2x^2\tan^{-1}(ax)^2 + \frac{1}{4}a^4c$$

$$= -\frac{3}{2}ac^2x\tan^{-1}(ax) - \frac{1}{6}a^3c^2x^3\tan^{-1}(ax) + \frac{3}{4}c^2\tan^{-1}(ax)^2 + a^2c^2x^2\tan^{-1}(ax)^2 + \frac{1}{4}a^4c$$

$$= \frac{1}{12}a^2c^2x^2 - \frac{3}{2}ac^2x\tan^{-1}(ax) - \frac{1}{6}a^3c^2x^3\tan^{-1}(ax) + \frac{3}{4}c^2\tan^{-1}(ax)^2 + a^2c^2x^2\tan^{-1}(ax$$

Mathematica [A] time = 0.315548, size = 218, normalized size = 0.93

$$\frac{1}{24}c^2\left(24i\tan^{-1}(ax)\text{PolyLog}\left(2,e^{-2i\tan^{-1}(ax)}\right) + 24i\tan^{-1}(ax)\text{PolyLog}\left(2,-e^{2i\tan^{-1}(ax)}\right) + 12\text{PolyLog}\left(3,e^{-2i\tan^{-1}(ax)}\right)\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((c + a^2*c*x^2)^2*ArcTan[a*x]^2)/x,x]
```

```
[Out] (c^2*(2 - I*Pi^3 + 2*a^2*x^2 - 36*a*x*ArcTan[a*x] - 4*a^3*x^3*ArcTan[a*x] +
 18*ArcTan[a*x]^2 + 24*a^2*x^2*ArcTan[a*x]^2 + 6*a^4*x^4*ArcTan[a*x]^2 + (1
6*I)*ArcTan[a*x]^3 + 24*ArcTan[a*x]^2*Log[1 - E^((-2*I)*ArcTan[a*x])] - 24*
ArcTan[a*x]^2*Log[1 + E^((2*I)*ArcTan[a*x])] + 16*Log[1 + a^2*x^2] + (24*I)
*ArcTan[a*x]*PolyLog[2, E^((-2*I)*ArcTan[a*x])] + (24*I)*ArcTan[a*x]*PolyLo
g[2, -E^((2*I)*ArcTan[a*x])] + 12*PolyLog[3, E^((-2*I)*ArcTan[a*x])] - 12*P
olyLog[3, -E^((2*I)*ArcTan[a*x])]))/24
```

Maple [C] time = 3.003, size = 1173, normalized size = 5.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^2*arctan(a*x)^2/x,x)
```

```
[Out] 1/12*c^2-3/2*a*c^2*x*arctan(a*x)-1/6*a^3*c^2*x^3*arctan(a*x)+1/4*a^4*c^2*x^
4*arctan(a*x)^2+1/12*a^2*c^2*x^2+3/4*c^2*arctan(a*x)^2+a^2*c^2*x^2*arctan(a
*x)^2+4/3*I*c^2*arctan(a*x)+c^2*arctan(a*x)^2*ln(a*x)-1/2*I*c^2*Pi*csgn(I*(
(1+I*a*x)^2/(a^2*x^2+1)-1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2
/(a^2*x^2+1)+1))^2*arctan(a*x)^2-1/2*I*c^2*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+
1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a
*x)^2/(a^2*x^2+1)+1))^2*arctan(a*x)^2+1/2*I*c^2*Pi*csgn(I*((1+I*a*x)^2/(a^2
*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((
1+I*a*x)^2/(a^2*x^2+1)+1))*arctan(a*x)^2-1/2*I*c^2*Pi*csgn(I/((1+I*a*x)^2/(
a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+
1))^2*arctan(a*x)^2+1/2*I*c^2*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a
*x)^2/(a^2*x^2+1)+1))^3*arctan(a*x)^2+1/2*I*c^2*Pi*csgn(I*((1+I*a*x)^2/(a^2
*x^2+1)-1))*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^
2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*arctan(a*x)^2-c^2*arctan(a*x)^2*ln((1+
I*a*x)^2/(a^2*x^2+1)-1)+c^2*arctan(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))
+c^2*arctan(a*x)^2*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-1/2*I*c^2*Pi*csgn(((1+
I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2*arctan(a*x)^2+1/2*I*
c^2*Pi*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^3*arct
an(a*x)^2-4/3*c^2*ln((1+I*a*x)^2/(a^2*x^2+1)+1)-1/2*c^2*polylog(3,-(1+I*a*x
)^2/(a^2*x^2+1))+2*c^2*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*c^2*polylo
g(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))+I*c^2*arctan(a*x)*polylog(2,-(1+I*a*x)^2/(
a^2*x^2+1))+1/2*I*c^2*Pi*arctan(a*x)^2-2*I*c^2*arctan(a*x)*polylog(2,-(1+I*
a*x)/(a^2*x^2+1)^(1/2))-2*I*c^2*arctan(a*x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)
^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$12\,a^6c^2\int\frac{x^6\arctan\left(ax\right)^2}{16\left(a^2x^3+x\right)}\,dx+a^6c^2\int\frac{x^6\log\left(a^2x^2+1\right)^2}{16\left(a^2x^3+x\right)}\,dx+a^6c^2\int\frac{x^6\log\left(a^2x^2+1\right)}{16\left(a^2x^3+x\right)}\,dx-2\,a^5c^2\int\frac{x^5\arctan\left(ax\right)}{16\left(a^2x^3+x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^2/x,x, algorithm="maxima")
```

```
[Out] 12*a^6*c^2*integrate(1/16*x^6*arctan(a*x)^2/(a^2*x^3 + x), x) + a^6*c^2*int
egrate(1/16*x^6*log(a^2*x^2 + 1)^2/(a^2*x^3 + x), x) + a^6*c^2*integrate(1/
16*x^6*log(a^2*x^2 + 1)/(a^2*x^3 + x), x) - 2*a^5*c^2*integrate(1/16*x^5*ar
ctan(a*x)/(a^2*x^3 + x), x) + 36*a^4*c^2*integrate(1/16*x^4*arctan(a*x)^2/(
a^2*x^3 + x), x) + 3*a^4*c^2*integrate(1/16*x^4*log(a^2*x^2 + 1)^2/(a^2*x^3
 + x), x) + 4*a^4*c^2*integrate(1/16*x^4*log(a^2*x^2 + 1)/(a^2*x^3 + x), x)
 - 8*a^3*c^2*integrate(1/16*x^3*arctan(a*x)/(a^2*x^3 + x), x) + 36*a^2*c^2*
integrate(1/16*x^2*arctan(a*x)^2/(a^2*x^3 + x), x) + 1/32*c^2*log(a^2*x^2 +
 1)^3 + 1/16*(a^4*c^2*x^4 + 4*a^2*c^2*x^2)*arctan(a*x)^2 + 12*c^2*integrate
(1/16*arctan(a*x)^2/(a^2*x^3 + x), x) + c^2*integrate(1/16*log(a^2*x^2 + 1)
^2/(a^2*x^3 + x), x) - 1/64*(a^4*c^2*x^4 + 4*a^2*c^2*x^2)*log(a^2*x^2 + 1)^
2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^{4}c^{2}x^{4}+2\,a^{2}c^{2}x^{2}+c^{2}\right)\arctan\left(ax\right)^{2}}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^2/x,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(a*x)^2/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$c^{2}\left(\int\frac{\operatorname{atan}^{2}\left(ax\right)}{x}\,dx+\int 2a^{2}x\operatorname{atan}^{2}\left(ax\right)dx+\int a^{4}x^{3}\operatorname{atan}^{2}\left(ax\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**2*atan(a*x)**2/x,x)
```

```
[Out] c**2*(Integral(atan(a*x)**2/x, x) + Integral(2*a**2*x*atan(a*x)**2, x) + In
tegral(a**4*x**3*atan(a*x)**2, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^2 \arctan (ax)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^2/x,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^2*arctan(a*x)^2/x, x)
```

3.271 $\int \frac{(c+a^2cx^2)^2 \tan^{-1}(ax)^2}{x^2} dx$

Optimal. Leaf size=205

$$-iac^2\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)+\frac{5}{3}iac^2\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)+\frac{1}{3}a^4c^2x^3\tan^{-1}(ax)^2-\frac{1}{3}a^3c^2x^2\tan^{-1}(ax)+\frac{1}{3}a^2c^2$$

```
[Out] (a^2*c^2*x)/3 - (a*c^2*ArcTan[a*x])/3 - (a^3*c^2*x^2*ArcTan[a*x])/3 + ((2*I
)/3)*a*c^2*ArcTan[a*x]^2 - (c^2*ArcTan[a*x]^2)/x + 2*a^2*c^2*x*ArcTan[a*x]^
2 + (a^4*c^2*x^3*ArcTan[a*x]^2)/3 + (10*a*c^2*ArcTan[a*x]*Log[2/(1 + I*a*x)
])/3 + 2*a*c^2*ArcTan[a*x]*Log[2 - 2/(1 - I*a*x)] - I*a*c^2*PolyLog[2, -1 +
 2/(1 - I*a*x)] + ((5*I)/3)*a*c^2*PolyLog[2, 1 - 2/(1 + I*a*x)]
```

Rubi [A] time = 0.4231, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 13, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.591, Rules used = {4948, 4846, 4920, 4854, 2402, 2315, 4852, 4924, 4868, 2447, 4916, 321, 203}

$$-iac^2\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)+\frac{5}{3}iac^2\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)+\frac{1}{3}a^4c^2x^3\tan^{-1}(ax)^2-\frac{1}{3}a^3c^2x^2\tan^{-1}(ax)+\frac{1}{3}a^2c^2$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)^2*ArcTan[a*x]^2)/x^2,x]
```


```
[Out] (a^2*c^2*x)/3 - (a*c^2*ArcTan[a*x])/3 - (a^3*c^2*x^2*ArcTan[a*x])/3 + ((2*I
)/3)*a*c^2*ArcTan[a*x]^2 - (c^2*ArcTan[a*x]^2)/x + 2*a^2*c^2*x*ArcTan[a*x]^
2 + (a^4*c^2*x^3*ArcTan[a*x]^2)/3 + (10*a*c^2*ArcTan[a*x]*Log[2/(1 + I*a*x)
])/3 + 2*a*c^2*ArcTan[a*x]*Log[2 - 2/(1 - I*a*x)] - I*a*c^2*PolyLog[2, -1 +
 2/(1 - I*a*x)] + ((5*I)/3)*a*c^2*PolyLog[2, 1 - 2/(1 + I*a*x)]
```


Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^2\tan^{-1}(ax)^2}{x^2}\,dx = \int \left(2a^2c^2\tan^{-1}(ax)^2 + \frac{c^2\tan^{-1}(ax)^2}{x^2} + a^4c^2x^2\tan^{-1}(ax)^2\right)dx$$

$$= c^2\int \frac{\tan^{-1}(ax)^2}{x^2}\,dx + (2a^2c^2)\int \tan^{-1}(ax)^2\,dx + (a^4c^2)\int x^2\tan^{-1}(ax)^2\,dx$$

$$= -\frac{c^2\tan^{-1}(ax)^2}{x} + 2a^2c^2x\tan^{-1}(ax)^2 + \frac{1}{3}a^4c^2x^3\tan^{-1}(ax)^2 + (2ac^2)\int \frac{\tan^{-1}(ax)}{x(1+a^2x^2)}\,dx$$

$$= iac^2\tan^{-1}(ax)^2 - \frac{c^2\tan^{-1}(ax)^2}{x} + 2a^2c^2x\tan^{-1}(ax)^2 + \frac{1}{3}a^4c^2x^3\tan^{-1}(ax)^2 + (2iac^2)\int$$

$$= -\frac{1}{3}a^3c^2x^2\tan^{-1}(ax) + \frac{2}{3}iac^2\tan^{-1}(ax)^2 - \frac{c^2\tan^{-1}(ax)^2}{x} + 2a^2c^2x\tan^{-1}(ax)^2 + \frac{1}{3}a^4c^2x$$

$$= \frac{1}{3}a^2c^2x - \frac{1}{3}a^3c^2x^2\tan^{-1}(ax) + \frac{2}{3}iac^2\tan^{-1}(ax)^2 - \frac{c^2\tan^{-1}(ax)^2}{x} + 2a^2c^2x\tan^{-1}(ax)^2 +$$

$$= \frac{1}{3}a^2c^2x - \frac{1}{3}ac^2\tan^{-1}(ax) - \frac{1}{3}a^3c^2x^2\tan^{-1}(ax) + \frac{2}{3}iac^2\tan^{-1}(ax)^2 - \frac{c^2\tan^{-1}(ax)^2}{x} + 2a$$

$$= \frac{1}{3}a^2c^2x - \frac{1}{3}ac^2\tan^{-1}(ax) - \frac{1}{3}a^3c^2x^2\tan^{-1}(ax) + \frac{2}{3}iac^2\tan^{-1}(ax)^2 - \frac{c^2\tan^{-1}(ax)^2}{x} + 2a$$

Mathematica [A] time = 0.353405, size = 167, normalized size = 0.81

$$\frac{c^2\left(-5iax\text{PolyLog}\left(2,-e^{2i\tan^{-1}(ax)}\right) - 3iax\text{PolyLog}\left(2,e^{2i\tan^{-1}(ax)}\right) + a^2x^2 + a^4x^4\tan^{-1}(ax)^2 - a^3x^3\tan^{-1}(ax) + 6a^2x^2\tan\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((c + a^2*c*x^2)^2*ArcTan[a*x]^2)/x^2,x]
```

```
[Out] (c^2*(a^2*x^2 - a*x*ArcTan[a*x] - a^3*x^3*ArcTan[a*x] - 3*ArcTan[a*x]^2 - (
8*I)*a*x*ArcTan[a*x]^2 + 6*a^2*x^2*ArcTan[a*x]^2 + a^4*x^4*ArcTan[a*x]^2 +
6*a*x*ArcTan[a*x]*Log[1 - E^((2*I)*ArcTan[a*x])] + 10*a*x*ArcTan[a*x]*Log[1
 + E^((2*I)*ArcTan[a*x])] - (5*I)*a*x*PolyLog[2, -E^((2*I)*ArcTan[a*x])] -
(3*I)*a*x*PolyLog[2, E^((2*I)*ArcTan[a*x])]))/(3*x)
```

Maple [A] time = 0.096, size = 346, normalized size = 1.7

$$\frac{a^4c^2x^3(\arctan(ax))^2}{3} + 2a^2c^2x(\arctan(ax))^2 - \frac{c^2(\arctan(ax))^2}{x} - \frac{a^3c^2x^2\arctan(ax)}{3} - \frac{8ac^2\arctan(ax)\ln(a^2x^2+1}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^2*arctan(a*x)^2/x^2,x)
```

```
[Out] 1/3*a^4*c^2*x^3*arctan(a*x)^2+2*a^2*c^2*x*arctan(a*x)^2-c^2*arctan(a*x)^2/x
-1/3*a^3*c^2*x^2*arctan(a*x)-8/3*a*c^2*arctan(a*x)*ln(a^2*x^2+1)+2*a*c^2*ar
ctan(a*x)*ln(a*x)+1/3*a^2*c^2*x-1/3*a*c^2*arctan(a*x)+I*a*c^2*ln(a*x)*ln(1+
I*a*x)+4/3*I*a*c^2*ln(a*x-I)*ln(-1/2*I*(a*x+I))-I*a*c^2*ln(a*x)*ln(1-I*a*x)
-I*a*c^2*dilog(1-I*a*x)+I*a*c^2*dilog(1+I*a*x)+4/3*I*a*c^2*ln(a^2*x^2+1)*ln
(a*x+I)-2/3*I*a*c^2*ln(a*x+I)^2-4/3*I*a*c^2*ln(a^2*x^2+1)*ln(a*x-I)+2/3*I*a
*c^2*ln(a*x-I)^2-4/3*I*a*c^2*dilog(1/2*I*(a*x-I))+4/3*I*a*c^2*dilog(-1/2*I*
(a*x+I))-4/3*I*a*c^2*ln(a*x+I)*ln(1/2*I*(a*x-I))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^2/x^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^4c^2x^4 + 2\,a^2c^2x^2 + c^2\right)\arctan\left(ax\right)^2}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^2/x^2,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(a*x)^2/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$c^2\left(\int 2a^2\operatorname{atan}^2\left(ax\right)dx + \int\frac{\operatorname{atan}^2\left(ax\right)}{x^2}\,dx + \int a^4x^2\operatorname{atan}^2\left(ax\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**2*atan(a*x)**2/x**2,x)
```

```
[Out] c**2*(Integral(2*a**2*atan(a*x)**2, x) + Integral(atan(a*x)**2/x**2, x) + I
ntegral(a**4*x**2*atan(a*x)**2, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^2 \arctan(ax)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^2/x^2,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^2*arctan(a*x)^2/x^2, x)
```

3.272 $\int \frac{(c+a^2cx^2)^2 \tan^{-1}(ax)^2}{x^3} dx$

Optimal. Leaf size=207

$$-a^2c^2\text{PolyLog}\left(3,1-\frac{2}{1+iax}\right)+a^2c^2\text{PolyLog}\left(3,-1+\frac{2}{1+iax}\right)-2ia^2c^2\tan^{-1}(ax)\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)+2ia^2c^2$$

```
[Out] -((a*c^2*ArcTan[a*x])/x) - a^3*c^2*x*ArcTan[a*x] - (c^2*ArcTan[a*x]^2)/(2*x
^2) + (a^4*c^2*x^2*ArcTan[a*x]^2)/2 + 4*a^2*c^2*ArcTan[a*x]^2*ArcTanh[1 - 2
/(1 + I*a*x)] + a^2*c^2*Log[x] - (2*I)*a^2*c^2*ArcTan[a*x]*PolyLog[2, 1 - 2
/(1 + I*a*x)] + (2*I)*a^2*c^2*ArcTan[a*x]*PolyLog[2, -1 + 2/(1 + I*a*x)] -
a^2*c^2*PolyLog[3, 1 - 2/(1 + I*a*x)] + a^2*c^2*PolyLog[3, -1 + 2/(1 + I*a*
x)]
```

Rubi [A] time = 0.452362, antiderivative size = 207, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 15, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.682, Rules used = {4948, 4852, 4918, 266, 36, 29, 31, 4884, 4850, 4988, 4994, 6610, 4916, 4846, 260}

$$-a^2c^2\text{PolyLog}\left(3,1-\frac{2}{1+iax}\right)+a^2c^2\text{PolyLog}\left(3,-1+\frac{2}{1+iax}\right)-2ia^2c^2\tan^{-1}(ax)\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)+2ia^2c^2$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)^2*ArcTan[a*x]^2)/x^3,x]
```


```
[Out] -((a*c^2*ArcTan[a*x])/x) - a^3*c^2*x*ArcTan[a*x] - (c^2*ArcTan[a*x]^2)/(2*x
^2) + (a^4*c^2*x^2*ArcTan[a*x]^2)/2 + 4*a^2*c^2*ArcTan[a*x]^2*ArcTanh[1 - 2
/(1 + I*a*x)] + a^2*c^2*Log[x] - (2*I)*a^2*c^2*ArcTan[a*x]*PolyLog[2, 1 - 2
/(1 + I*a*x)] + (2*I)*a^2*c^2*ArcTan[a*x]*PolyLog[2, -1 + 2/(1 + I*a*x)] -
a^2*c^2*PolyLog[3, 1 - 2/(1 + I*a*x)] + a^2*c^2*PolyLog[3, -1 + 2/(1 + I*a*
x)]
```


Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4918

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/(d + e*x^2),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4850

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTan[c*x])^(p - 1)*ArcTanh[1 - 2/(1 + I*c*x)])/(1 + c^2*x^2), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4988

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTan[c*x])^p)/(d + e
*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTan[c*x])^p)/(d + e*x^2
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{\left(c+a^2cx^2\right)^2 \tan^{-1}(ax)^2}{x^3}\,dx = \int \left(\frac{c^2\tan^{-1}(ax)^2}{x^3} + \frac{2a^2c^2\tan^{-1}(ax)^2}{x} + a^4c^2x\tan^{-1}(ax)^2\right)dx$$
$$= c^2\int \frac{\tan^{-1}(ax)^2}{x^3}\,dx + \left(2a^2c^2\right)\int \frac{\tan^{-1}(ax)^2}{x}\,dx + \left(a^4c^2\right)\int x\tan^{-1}(ax)^2\,dx$$
$$= -\frac{c^2\tan^{-1}(ax)^2}{2x^2} + \frac{1}{2}a^4c^2x^2\tan^{-1}(ax)^2 + 4a^2c^2\tan^{-1}(ax)^2\tanh^{-1}\left(1-\frac{2}{1+iax}\right) + \left(ac^2\right)$$
$$= -\frac{c^2\tan^{-1}(ax)^2}{2x^2} + \frac{1}{2}a^4c^2x^2\tan^{-1}(ax)^2 + 4a^2c^2\tan^{-1}(ax)^2\tanh^{-1}\left(1-\frac{2}{1+iax}\right) + \left(ac^2\right)$$
$$= -\frac{ac^2\tan^{-1}(ax)}{x} - a^3c^2x\tan^{-1}(ax) - \frac{c^2\tan^{-1}(ax)^2}{2x^2} + \frac{1}{2}a^4c^2x^2\tan^{-1}(ax)^2 + 4a^2c^2\tan^{-1}$$
$$= -\frac{ac^2\tan^{-1}(ax)}{x} - a^3c^2x\tan^{-1}(ax) - \frac{c^2\tan^{-1}(ax)^2}{2x^2} + \frac{1}{2}a^4c^2x^2\tan^{-1}(ax)^2 + 4a^2c^2\tan^{-1}$$
$$= -\frac{ac^2\tan^{-1}(ax)}{x} - a^3c^2x\tan^{-1}(ax) - \frac{c^2\tan^{-1}(ax)^2}{2x^2} + \frac{1}{2}a^4c^2x^2\tan^{-1}(ax)^2 + 4a^2c^2\tan^{-1}$$
$$= -\frac{ac^2\tan^{-1}(ax)}{x} - a^3c^2x\tan^{-1}(ax) - \frac{c^2\tan^{-1}(ax)^2}{2x^2} + \frac{1}{2}a^4c^2x^2\tan^{-1}(ax)^2 + 4a^2c^2\tan^{-1}$$

Mathematica [A] time = 0.307287, size = 226, normalized size = 1.09

$$a^2c^2\left(2i\tan^{-1}(ax)\text{PolyLog}\left(2,e^{-2i\tan^{-1}(ax)}\right) + 2i\tan^{-1}(ax)\text{PolyLog}\left(2,-e^{2i\tan^{-1}(ax)}\right) + \text{PolyLog}\left(3,e^{-2i\tan^{-1}(ax)}\right) - \text{Poly}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((c + a^2*c*x^2)^2*ArcTan[a*x]^2)/x^3,x]
```

```
[Out] a^2*c^2*((-I/12)*Pi^3 - ArcTan[a*x]/(a*x) - a*x*ArcTan[a*x] - ArcTan[a*x]^2
/(2*a^2*x^2) + (a^2*x^2*ArcTan[a*x]^2)/2 + ((4*I)/3)*ArcTan[a*x]^3 + 2*ArcT
an[a*x]^2*Log[1 - E^((-2*I)*ArcTan[a*x])] - 2*ArcTan[a*x]^2*Log[1 + E^((2*I
)*ArcTan[a*x])] + Log[(a*x)/Sqrt[1 + a^2*x^2]] + Log[1 + a^2*x^2]/2 + (2*I)
*ArcTan[a*x]*PolyLog[2, E^((-2*I)*ArcTan[a*x])] + (2*I)*ArcTan[a*x]*PolyLog
[2, -E^((2*I)*ArcTan[a*x])] + PolyLog[3, E^((-2*I)*ArcTan[a*x])] - PolyLog[
3, -E^((2*I)*ArcTan[a*x])])
```

Maple [C] time = 3.27, size = 1255, normalized size = 6.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^2*arctan(a*x)^2/x^3,x)
```

```
[Out] 1/2*a^4*c^2*x^2*arctan(a*x)^2-I*a^2*c^2*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-
1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)
^2/(a^2*x^2+1)+1))^2*arctan(a*x)^2+I*a^2*c^2*Pi*arctan(a*x)^2-4*I*a^2*c^2*a
rctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-4*I*a^2*c^2*arctan(a*x)*
polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*I*a^2*c^2*arctan(a*x)*polylog(2,-(
1+I*a*x)^2/(a^2*x^2+1))-1/2*c^2*arctan(a*x)^2/x^2-a*c^2*arctan(a*x)/x-a^3*c
^2*x*arctan(a*x)+I*a^2*c^2*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)
^2/(a^2*x^2+1)+1))^3*arctan(a*x)^2+I*a^2*c^2*Pi*csgn(((1+I*a*x)^2/(a^2*x^2+
1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^3*arctan(a*x)^2-I*a^2*c^2*Pi*csgn(((1+I*
a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2*arctan(a*x)^2+I*a^2*c^
2*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1))*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)
)*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*arctan(a*
x)^2-I*a^2*c^2*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(
a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2*arctan(a*x)^2-I*a^2*c^2*Pi*csg
n(I*((1+I*a*x)^2/(a^2*x^2+1)-1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a
*x)^2/(a^2*x^2+1)+1))^2*arctan(a*x)^2-2*a^2*c^2*arctan(a*x)^2*ln((1+I*a*x)^
2/(a^2*x^2+1)-1)+2*a^2*c^2*arctan(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))+
2*a^2*c^2*arctan(a*x)^2*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*a^2*c^2*arctan(
a*x)^2*ln(a*x)+I*a^2*c^2*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2
/(a^2*x^2+1)+1))*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+
1))*arctan(a*x)^2-a^2*c^2*polylog(3,-(1+I*a*x)^2/(a^2*x^2+1))-a^2*c^2*ln((1
+I*a*x)^2/(a^2*x^2+1)+1)+a^2*c^2*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))+a^2*c^2*
ln((1+I*a*x)/(a^2*x^2+1)^(1/2)-1)+4*a^2*c^2*polylog(3,(1+I*a*x)/(a^2*x^2+1)
^(1/2))+4*a^2*c^2*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^2/x^3,x, algorithm="maxima")
```

[Out] Timed out

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^4c^2x^4 + 2\,a^2c^2x^2 + c^2\right)\arctan\left(ax\right)^2}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^2/x^3,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(a*x)^2/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$c^2\left(\int \frac{\operatorname{atan}^2\left(ax\right)}{x^3}\,dx + \int \frac{2a^2\operatorname{atan}^2\left(ax\right)}{x}\,dx + \int a^4x\operatorname{atan}^2\left(ax\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**2*atan(a*x)**2/x**3,x)
```

```
[Out] c**2*(Integral(atan(a*x)**2/x**3, x) + Integral(2*a**2*atan(a*x)**2/x, x) +
 Integral(a**4*x*atan(a*x)**2, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2 + c\right)^2\arctan\left(ax\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^2/x^3,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^2*arctan(a*x)^2/x^3, x)
```

3.273 $\int \frac{(c+a^2cx^2)^2 \tan^{-1}(ax)^2}{x^4} dx$

Optimal. Leaf size=216

$$-\frac{5}{3}ia^3c^2\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)+ia^3c^2\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)-\frac{a^2c^2}{3x}+a^4c^2x\tan^{-1}(ax)^2-\frac{2}{3}ia^3c^2\tan^{-1}(ax)^2-$$

```
[Out] -(a^2*c^2)/(3*x) - (a^3*c^2*ArcTan[a*x])/3 - (a*c^2*ArcTan[a*x])/(3*x^2) -
((2*I)/3)*a^3*c^2*ArcTan[a*x]^2 - (c^2*ArcTan[a*x]^2)/(3*x^3) - (2*a^2*c^2*
ArcTan[a*x]^2)/x + a^4*c^2*x*ArcTan[a*x]^2 + 2*a^3*c^2*ArcTan[a*x]*Log[2/(1
 + I*a*x)] + (10*a^3*c^2*ArcTan[a*x]*Log[2 - 2/(1 - I*a*x)])/3 - ((5*I)/3)*
a^3*c^2*PolyLog[2, -1 + 2/(1 - I*a*x)] + I*a^3*c^2*PolyLog[2, 1 - 2/(1 + I*
a*x)]
```

Rubi [A] time = 0.439726, antiderivative size = 216, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 13, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.591, Rules used = {4948, 4846, 4920, 4854, 2402, 2315, 4852, 4918, 325, 203, 4924, 4868, 2447}

$$-\frac{5}{3}ia^3c^2\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)+ia^3c^2\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)-\frac{a^2c^2}{3x}+a^4c^2x\tan^{-1}(ax)^2-\frac{2}{3}ia^3c^2\tan^{-1}(ax)^2-$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)^2*ArcTan[a*x]^2)/x^4,x]
```


```
[Out] -(a^2*c^2)/(3*x) - (a^3*c^2*ArcTan[a*x])/3 - (a*c^2*ArcTan[a*x])/(3*x^2) -
((2*I)/3)*a^3*c^2*ArcTan[a*x]^2 - (c^2*ArcTan[a*x]^2)/(3*x^3) - (2*a^2*c^2*
ArcTan[a*x]^2)/x + a^4*c^2*x*ArcTan[a*x]^2 + 2*a^3*c^2*ArcTan[a*x]*Log[2/(1
 + I*a*x)] + (10*a^3*c^2*ArcTan[a*x]*Log[2 - 2/(1 - I*a*x)])/3 - ((5*I)/3)*
a^3*c^2*PolyLog[2, -1 + 2/(1 - I*a*x)] + I*a^3*c^2*PolyLog[2, 1 - 2/(1 + I*
a*x)]
```


Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4918

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/(d + e*x^2),
```

```
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^2 \tan^{-1}(ax)^2}{x^4}\,dx = \int \left(a^4c^2\tan^{-1}(ax)^2 + \frac{c^2\tan^{-1}(ax)^2}{x^4} + \frac{2a^2c^2\tan^{-1}(ax)^2}{x^2}\right)dx$$

$$= c^2\int \frac{\tan^{-1}(ax)^2}{x^4}\,dx + (2a^2c^2)\int \frac{\tan^{-1}(ax)^2}{x^2}\,dx + (a^4c^2)\int \tan^{-1}(ax)^2\,dx$$

$$= -\frac{c^2\tan^{-1}(ax)^2}{3x^3} - \frac{2a^2c^2\tan^{-1}(ax)^2}{x} + a^4c^2x\tan^{-1}(ax)^2 + \frac{1}{3}(2ac^2)\int \frac{\tan^{-1}(ax)}{x^3(1+a^2x^2)}\,dx +$$

$$= -ia^3c^2\tan^{-1}(ax)^2 - \frac{c^2\tan^{-1}(ax)^2}{3x^3} - \frac{2a^2c^2\tan^{-1}(ax)^2}{x} + a^4c^2x\tan^{-1}(ax)^2 + \frac{1}{3}(2ac^2)\int$$

$$= -\frac{ac^2\tan^{-1}(ax)}{3x^2} - \frac{2}{3}ia^3c^2\tan^{-1}(ax)^2 - \frac{c^2\tan^{-1}(ax)^2}{3x^3} - \frac{2a^2c^2\tan^{-1}(ax)^2}{x} + a^4c^2x\tan^{-1}$$

$$= -\frac{a^2c^2}{3x} - \frac{ac^2\tan^{-1}(ax)}{3x^2} - \frac{2}{3}ia^3c^2\tan^{-1}(ax)^2 - \frac{c^2\tan^{-1}(ax)^2}{3x^3} - \frac{2a^2c^2\tan^{-1}(ax)^2}{x} + a^4c^2$$

$$= -\frac{a^2c^2}{3x} - \frac{1}{3}a^3c^2\tan^{-1}(ax) - \frac{ac^2\tan^{-1}(ax)}{3x^2} - \frac{2}{3}ia^3c^2\tan^{-1}(ax)^2 - \frac{c^2\tan^{-1}(ax)^2}{3x^3} - \frac{2a^2c^2}{}$$

Mathematica [A] time = 0.381343, size = 189, normalized size = 0.88

$$\frac{c^2\left(-3ia^3x^3\text{PolyLog}\left(2,-e^{2i\tan^{-1}(ax)}\right) - 5ia^3x^3\text{PolyLog}\left(2,e^{2i\tan^{-1}(ax)}\right) - a^2x^2 + 3a^4x^4\tan^{-1}(ax)^2 - 8ia^3x^3\tan^{-1}(ax)^2 -}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((c + a^2*c*x^2)^2*ArcTan[a*x]^2)/x^4,x]
```

```
[Out] (c^2*(-(a^2*x^2) - a*x*ArcTan[a*x] - a^3*x^3*ArcTan[a*x] - ArcTan[a*x]^2 -
6*a^2*x^2*ArcTan[a*x]^2 - (8*I)*a^3*x^3*ArcTan[a*x]^2 + 3*a^4*x^4*ArcTan[a*
x]^2 + 10*a^3*x^3*ArcTan[a*x]*Log[1 - E^((2*I)*ArcTan[a*x])] + 6*a^3*x^3*Ar
cTan[a*x]*Log[1 + E^((2*I)*ArcTan[a*x])] - (3*I)*a^3*x^3*PolyLog[2, -E^((2*
I)*ArcTan[a*x])] - (5*I)*a^3*x^3*PolyLog[2, E^((2*I)*ArcTan[a*x])]))/(3*x^3
)
```

Maple [A] time = 0.102, size = 375, normalized size = 1.7

$$a^4c^2x(\arctan(ax))^2 - 2\frac{a^2c^2(\arctan(ax))^2}{x} - \frac{c^2(\arctan(ax))^2}{3x^3} - \frac{8a^3c^2\arctan(ax)\ln(a^2x^2+1)}{3} - \frac{ac^2\arctan(ax)}{3x^2} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^2*arctan(a*x)^2/x^4,x)
```

```
[Out] a^4*c^2*x*arctan(a*x)^2-2*a^2*c^2*arctan(a*x)^2/x-1/3*c^2*arctan(a*x)^2/x^3
-8/3*a^3*c^2*arctan(a*x)*ln(a^2*x^2+1)-1/3*a*c^2*arctan(a*x)/x^2+10/3*a^3*c
^2*arctan(a*x)*ln(a*x)-1/3*a^3*c^2*arctan(a*x)-1/3*a^2*c^2/x-4/3*I*a^3*c^2*
dilog(1/2*I*(a*x-I))+4/3*I*a^3*c^2*ln(a*x-I)*ln(-1/2*I*(a*x+I))-5/3*I*a^3*c
^2*ln(a*x)*ln(1-I*a*x)-4/3*I*a^3*c^2*ln(a*x+I)*ln(1/2*I*(a*x-I))-4/3*I*a^3*
c^2*ln(a^2*x^2+1)*ln(a*x-I)-2/3*I*a^3*c^2*ln(a*x+I)^2+4/3*I*a^3*c^2*dilog(-
1/2*I*(a*x+I))+5/3*I*a^3*c^2*ln(a*x)*ln(1+I*a*x)+4/3*I*a^3*c^2*ln(a^2*x^2+1
)*ln(a*x+I)+5/3*I*a^3*c^2*dilog(1+I*a*x)-5/3*I*a^3*c^2*dilog(1-I*a*x)+2/3*I
*a^3*c^2*ln(a*x-I)^2
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^2/x^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(a^4c^2x^4+2\,a^2c^2x^2+c^2\right)\arctan\left(ax\right)^2}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^2/x^4,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(a*x)^2/x^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$c^{2}\left(\int a^{4}\operatorname{atan}^{2}(ax)\,dx+\int\frac{\operatorname{atan}^{2}(ax)}{x^{4}}\,dx+\int\frac{2a^{2}\operatorname{atan}^{2}(ax)}{x^{2}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**2*atan(a*x)**2/x**4,x)
```

```
[Out] c**2*(Integral(a**4*atan(a*x)**2, x) + Integral(atan(a*x)**2/x**4, x) + Int
egral(2*a**2*atan(a*x)**2/x**2, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(a^{2}cx^{2}+c)^{2}\arctan(ax)^{2}}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^2/x^4,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^2*arctan(a*x)^2/x^4, x)
```

3.274 $\int x^3 \left(c + a^2cx^2\right)^3 \tan^{-1}(ax)^2 \, dx$

Optimal. Leaf size=240

$$\frac{1}{360}a^4c^3x^8 + \frac{71a^2c^3x^6}{7560} - \frac{107c^3x^2}{12600a^2} - \frac{26c^3\log\left(a^2x^2+1\right)}{1575a^4} + \frac{1}{10}a^6c^3x^{10}\tan^{-1}(ax)^2 - \frac{1}{45}a^5c^3x^9\tan^{-1}(ax) + \frac{3}{8}a^4c^3x^8\tan$$

```
[Out] (-107*c^3*x^2)/(12600*a^2) + (53*c^3*x^4)/6300 + (71*a^2*c^3*x^6)/7560 + (a
^4*c^3*x^8)/360 + (c^3*x*ArcTan[a*x])/(20*a^3) - (c^3*x^3*ArcTan[a*x])/(60*
a) - (9*a*c^3*x^5*ArcTan[a*x])/100 - (11*a^3*c^3*x^7*ArcTan[a*x])/140 - (a^
5*c^3*x^9*ArcTan[a*x])/45 - (c^3*ArcTan[a*x]^2)/(40*a^4) + (c^3*x^4*ArcTan[
a*x]^2)/4 + (a^2*c^3*x^6*ArcTan[a*x]^2)/2 + (3*a^4*c^3*x^8*ArcTan[a*x]^2)/8
 + (a^6*c^3*x^10*ArcTan[a*x]^2)/10 - (26*c^3*Log[1 + a^2*x^2])/(1575*a^4)
```

Rubi [A] time = 1.22695, antiderivative size = 240, normalized size of antiderivative = 1., number of steps used = 72, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {4948, 4852, 4916, 266, 43, 4846, 260, 4884}

$$\frac{1}{360}a^4c^3x^8 + \frac{71a^2c^3x^6}{7560} - \frac{107c^3x^2}{12600a^2} - \frac{26c^3\log\left(a^2x^2+1\right)}{1575a^4} + \frac{1}{10}a^6c^3x^{10}\tan^{-1}(ax)^2 - \frac{1}{45}a^5c^3x^9\tan^{-1}(ax) + \frac{3}{8}a^4c^3x^8\tan$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(c + a^2*c*x^2)^3*ArcTan[a*x]^2,x]
```

```
[Out] (-107*c^3*x^2)/(12600*a^2) + (53*c^3*x^4)/6300 + (71*a^2*c^3*x^6)/7560 + (a
^4*c^3*x^8)/360 + (c^3*x*ArcTan[a*x])/(20*a^3) - (c^3*x^3*ArcTan[a*x])/(60*
a) - (9*a*c^3*x^5*ArcTan[a*x])/100 - (11*a^3*c^3*x^7*ArcTan[a*x])/140 - (a^
5*c^3*x^9*ArcTan[a*x])/45 - (c^3*ArcTan[a*x]^2)/(40*a^4) + (c^3*x^4*ArcTan[
a*x]^2)/4 + (a^2*c^3*x^6*ArcTan[a*x]^2)/2 + (3*a^4*c^3*x^8*ArcTan[a*x]^2)/8
 + (a^6*c^3*x^10*ArcTan[a*x]^2)/10 - (26*c^3*Log[1 + a^2*x^2])/(1575*a^4)
```

Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rubi steps

$$\int x^3 \left(c+a^2cx^2\right)^3 \tan^{-1}(ax)^2\,dx = \int \left(c^3x^3\tan^{-1}(ax)^2 + 3a^2c^3x^5\tan^{-1}(ax)^2 + 3a^4c^3x^7\tan^{-1}(ax)^2 + a^6c^3x^9\tan^{-1}(ax)^2\right.$$

$$= c^3\int x^3\tan^{-1}(ax)^2\,dx + \left(3a^2c^3\right)\int x^5\tan^{-1}(ax)^2\,dx + \left(3a^4c^3\right)\int x^7\tan^{-1}(ax)^2\,dx +$$

$$= \frac{1}{4}c^3x^4\tan^{-1}(ax)^2 + \frac{1}{2}a^2c^3x^6\tan^{-1}(ax)^2 + \frac{3}{8}a^4c^3x^8\tan^{-1}(ax)^2 + \frac{1}{10}a^6c^3x^{10}\tan^{-1}(a$$

$$= \frac{1}{4}c^3x^4\tan^{-1}(ax)^2 + \frac{1}{2}a^2c^3x^6\tan^{-1}(ax)^2 + \frac{3}{8}a^4c^3x^8\tan^{-1}(ax)^2 + \frac{1}{10}a^6c^3x^{10}\tan^{-1}(a$$

$$= -\frac{c^3x^3\tan^{-1}(ax)}{6a} - \frac{1}{5}ac^3x^5\tan^{-1}(ax) - \frac{3}{28}a^3c^3x^7\tan^{-1}(ax) - \frac{1}{45}a^5c^3x^9\tan^{-1}(ax) +$$

$$= \frac{c^3x\tan^{-1}(ax)}{2a^3} + \frac{c^3x^3\tan^{-1}(ax)}{6a} - \frac{1}{20}ac^3x^5\tan^{-1}(ax) - \frac{11}{140}a^3c^3x^7\tan^{-1}(ax) - \frac{1}{45}a$$

$$= -\frac{c^3x\tan^{-1}(ax)}{2a^3} - \frac{c^3x^3\tan^{-1}(ax)}{12a} - \frac{9}{100}ac^3x^5\tan^{-1}(ax) - \frac{11}{140}a^3c^3x^7\tan^{-1}(ax) - \frac{1}{4}$$

$$= \frac{13c^3x^2}{504a^2} + \frac{29c^3x^4}{1008} + \frac{107a^2c^3x^6}{7560} + \frac{1}{360}a^4c^3x^8 + \frac{c^3x\tan^{-1}(ax)}{4a^3} - \frac{c^3x^3\tan^{-1}(ax)}{60a} - \frac{9}{10}$$

$$= -\frac{101c^3x^2}{1260a^2} - \frac{c^3x^4}{630} + \frac{71a^2c^3x^6}{7560} + \frac{1}{360}a^4c^3x^8 + \frac{c^3x\tan^{-1}(ax)}{20a^3} - \frac{c^3x^3\tan^{-1}(ax)}{60a} - \frac{9}{100}$$

$$= \frac{313c^3x^2}{12600a^2} + \frac{53c^3x^4}{6300} + \frac{71a^2c^3x^6}{7560} + \frac{1}{360}a^4c^3x^8 + \frac{c^3x\tan^{-1}(ax)}{20a^3} - \frac{c^3x^3\tan^{-1}(ax)}{60a} - \frac{9}{10}$$

$$= -\frac{107c^3x^2}{12600a^2} + \frac{53c^3x^4}{6300} + \frac{71a^2c^3x^6}{7560} + \frac{1}{360}a^4c^3x^8 + \frac{c^3x\tan^{-1}(ax)}{20a^3} - \frac{c^3x^3\tan^{-1}(ax)}{60a} -$$

Mathematica [A] time = 0.0938761, size = 126, normalized size = 0.52

$$\frac{c^3\left(105a^8x^8+355a^6x^6+318a^4x^4-321a^2x^2-624\log\left(a^2x^2+1\right)-6ax\left(140a^8x^8+495a^6x^6+567a^4x^4+105a^2x^2-315\right.\right.}{37800a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(c + a^2*c*x^2)^3*ArcTan[a*x]^2,x]
```

```
[Out] (c^3*(-321*a^2*x^2 + 318*a^4*x^4 + 355*a^6*x^6 + 105*a^8*x^8 - 6*a*x*(-315
+ 105*a^2*x^2 + 567*a^4*x^4 + 495*a^6*x^6 + 140*a^8*x^8)*ArcTan[a*x] + 945*
(1 + a^2*x^2)^4*(-1 + 4*a^2*x^2)*ArcTan[a*x]^2 - 624*Log[1 + a^2*x^2]))/(37
```

```
800*a^4)
```

Maple [A] time = 0.033, size = 211, normalized size = 0.9

$$-\frac{107\,c^3x^2}{12600\,a^2}+\frac{53\,c^3x^4}{6300}+\frac{71\,a^2c^3x^6}{7560}+\frac{a^4c^3x^8}{360}+\frac{c^3x\arctan(ax)}{20\,a^3}-\frac{c^3x^3\arctan(ax)}{60\,a}-\frac{9\,ac^3x^5\arctan(ax)}{100}-\frac{11\,a^3c^3x^7\,\mathrm{arct}}{140}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a^2*c*x^2+c)^3*arctan(a*x)^2,x)
```

```
[Out] -107/12600*c^3*x^2/a^2+53/6300*c^3*x^4+71/7560*a^2*c^3*x^6+1/360*a^4*c^3*x^
8+1/20*c^3*x*arctan(a*x)/a^3-1/60*c^3*x^3*arctan(a*x)/a-9/100*a*c^3*x^5*arc
tan(a*x)-11/140*a^3*c^3*x^7*arctan(a*x)-1/45*a^5*c^3*x^9*arctan(a*x)-1/40*c
^3*arctan(a*x)^2/a^4+1/4*c^3*x^4*arctan(a*x)^2+1/2*a^2*c^3*x^6*arctan(a*x)^
2+3/8*a^4*c^3*x^8*arctan(a*x)^2+1/10*a^6*c^3*x^10*arctan(a*x)^2-26/1575*c^3
*ln(a^2*x^2+1)/a^4
```

Maxima [A] time = 1.52462, size = 273, normalized size = 1.14

$$-\frac{1}{6300}\,a\left(\frac{315\,c^3\arctan(ax)}{a^5}+\frac{140\,a^8c^3x^9+495\,a^6c^3x^7+567\,a^4c^3x^5+105\,a^2c^3x^3-315\,c^3x}{a^4}\right)\arctan(ax)+\frac{1}{40}\left(4\,a^6c^3x^1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a^2*c*x^2+c)^3*arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] -1/6300*a*(315*c^3*arctan(a*x)/a^5 + (140*a^8*c^3*x^9 + 495*a^6*c^3*x^7 + 5
67*a^4*c^3*x^5 + 105*a^2*c^3*x^3 - 315*c^3*x)/a^4)*arctan(a*x) + 1/40*(4*a^
6*c^3*x^10 + 15*a^4*c^3*x^8 + 20*a^2*c^3*x^6 + 10*c^3*x^4)*arctan(a*x)^2 +
1/37800*(105*a^8*c^3*x^8 + 355*a^6*c^3*x^6 + 318*a^4*c^3*x^4 - 321*a^2*c^3*
x^2 + 945*c^3*arctan(a*x)^2 - 624*c^3*log(a^2*x^2 + 1))/a^4
```

Fricas [A] time = 2.22803, size = 417, normalized size = 1.74

$$\frac{105\,a^8c^3x^8+355\,a^6c^3x^6+318\,a^4c^3x^4-321\,a^2c^3x^2-624\,c^3\log\left(a^2x^2+1\right)+945\left(4\,a^{10}c^3x^{10}+15\,a^8c^3x^8+20\,a^6c^3x^6+10\right.}{37800\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a^2*c*x^2+c)^3*arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/37800*(105*a^8*c^3*x^8 + 355*a^6*c^3*x^6 + 318*a^4*c^3*x^4 - 321*a^2*c^3*
x^2 - 624*c^3*log(a^2*x^2 + 1) + 945*(4*a^10*c^3*x^10 + 15*a^8*c^3*x^8 + 20
*a^6*c^3*x^6 + 10*a^4*c^3*x^4 - c^3)*arctan(a*x)^2 - 6*(140*a^9*c^3*x^9 + 4
95*a^7*c^3*x^7 + 567*a^5*c^3*x^5 + 105*a^3*c^3*x^3 - 315*a*c^3*x)*arctan(a*
x))/a^4
```

Sympy [A] time = 7.61995, size = 241, normalized size = 1.

$$\begin{cases} \frac{a^6c^3x^{10}\operatorname{atan}^2(ax)}{10} - \frac{a^5c^3x^9\operatorname{atan}(ax)}{45} + \frac{3a^4c^3x^8\operatorname{atan}^2(ax)}{8} + \frac{a^4c^3x^8}{360} - \frac{11a^3c^3x^7\operatorname{atan}(ax)}{140} + \frac{a^2c^3x^6\operatorname{atan}^2(ax)}{2} + \frac{71a^2c^3x^6}{7560} - \frac{9ac^3x^5\operatorname{atan}(ax)}{100} + \\ 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a**2*c*x**2+c)**3*atan(a*x)**2,x)
```

```
[Out] Piecewise((a**6*c**3*x**10*atan(a*x)**2/10 - a**5*c**3*x**9*atan(a*x)/45 +
3*a**4*c**3*x**8*atan(a*x)**2/8 + a**4*c**3*x**8/360 - 11*a**3*c**3*x**7*at
an(a*x)/140 + a**2*c**3*x**6*atan(a*x)**2/2 + 71*a**2*c**3*x**6/7560 - 9*a*
c**3*x**5*atan(a*x)/100 + c**3*x**4*atan(a*x)**2/4 + 53*c**3*x**4/6300 - c*
*3*x**3*atan(a*x)/(60*a) - 107*c**3*x**2/(12600*a**2) + c**3*x*atan(a*x)/(2
0*a**3) - 26*c**3*log(x**2 + a**(-2))/(1575*a**4) - c**3*atan(a*x)**2/(40*a
**4), Ne(a, 0)), (0, True))
```

Giac [A] time = 1.17823, size = 267, normalized size = 1.11

$$\frac{1}{40}\left(4\,a^6c^3x^{10} + 15\,a^4c^3x^8 + 20\,a^2c^3x^6 + 10\,c^3x^4\right)\arctan(ax)^2 - \frac{840\,a^9c^3x^9\arctan(ax) - 105\,a^8c^3x^8 + 2970\,a^7c^3x^7\operatorname{arc}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a^2*c*x^2+c)^3*arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] 1/40*(4*a^6*c^3*x^10 + 15*a^4*c^3*x^8 + 20*a^2*c^3*x^6 + 10*c^3*x^4)*arctan
(a*x)^2 - 1/37800*(840*a^9*c^3*x^9*arctan(a*x) - 105*a^8*c^3*x^8 + 2970*a^7
```

```
*c^3*x^7*arctan(a*x) - 355*a^6*c^3*x^6 + 3402*a^5*c^3*x^5*arctan(a*x) - 318
*a^4*c^3*x^4 + 630*a^3*c^3*x^3*arctan(a*x) + 321*a^2*c^3*x^2 - 1890*a*c^3*x
*arctan(a*x) + 945*c^3*arctan(a*x)^2 + 624*c^3*log(a^2*x^2 + 1))/a^4
```

3.275 $\int x^2 \left(c+a^2cx^2\right)^3 \tan^{-1}(ax)^2 \, dx$

Optimal. Leaf size=274

$$-\frac{16ic^3\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{315a^3}+\frac{1}{252}a^4c^3x^7+\frac{59a^2c^3x^5}{3780}+\frac{1}{9}a^6c^3x^9\tan^{-1}(ax)^2-\frac{1}{36}a^5c^3x^8\tan^{-1}(ax)+\frac{3}{7}a^4c^3x^7\tan^{-1}($$

```
[Out] (-47*c^3*x)/(3780*a^2) + (239*c^3*x^3)/11340 + (59*a^2*c^3*x^5)/3780 + (a^4
*c^3*x^7)/252 + (47*c^3*ArcTan[a*x])/(3780*a^3) - (16*c^3*x^2*ArcTan[a*x])/
(315*a) - (89*a*c^3*x^4*ArcTan[a*x])/630 - (20*a^3*c^3*x^6*ArcTan[a*x])/189
 - (a^5*c^3*x^8*ArcTan[a*x])/36 - (((16*I)/315)*c^3*ArcTan[a*x]^2)/a^3 + (c
^3*x^3*ArcTan[a*x]^2)/3 + (3*a^2*c^3*x^5*ArcTan[a*x]^2)/5 + (3*a^4*c^3*x^7*
ArcTan[a*x]^2)/7 + (a^6*c^3*x^9*ArcTan[a*x]^2)/9 - (32*c^3*ArcTan[a*x]*Log[
2/(1 + I*a*x)])/(315*a^3) - (((16*I)/315)*c^3*PolyLog[2, 1 - 2/(1 + I*a*x)]
)/a^3
```

Rubi [A] time = 1.154, antiderivative size = 274, normalized size of antiderivative = 1., number of steps used = 68, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {4948, 4852, 4916, 321, 203, 4920, 4854, 2402, 2315, 302}

$$-\frac{16ic^3\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{315a^3}+\frac{1}{252}a^4c^3x^7+\frac{59a^2c^3x^5}{3780}+\frac{1}{9}a^6c^3x^9\tan^{-1}(ax)^2-\frac{1}{36}a^5c^3x^8\tan^{-1}(ax)+\frac{3}{7}a^4c^3x^7\tan^{-1}($$

Antiderivative was successfully verified.

```
[In] Int[x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]^2,x]
```


```
[Out] (-47*c^3*x)/(3780*a^2) + (239*c^3*x^3)/11340 + (59*a^2*c^3*x^5)/3780 + (a^4
*c^3*x^7)/252 + (47*c^3*ArcTan[a*x])/(3780*a^3) - (16*c^3*x^2*ArcTan[a*x])/
(315*a) - (89*a*c^3*x^4*ArcTan[a*x])/630 - (20*a^3*c^3*x^6*ArcTan[a*x])/189
 - (a^5*c^3*x^8*ArcTan[a*x])/36 - (((16*I)/315)*c^3*ArcTan[a*x]^2)/a^3 + (c
^3*x^3*ArcTan[a*x]^2)/3 + (3*a^2*c^3*x^5*ArcTan[a*x]^2)/5 + (3*a^4*c^3*x^7*
ArcTan[a*x]^2)/7 + (a^6*c^3*x^9*ArcTan[a*x]^2)/9 - (32*c^3*ArcTan[a*x]*Log[
2/(1 + I*a*x)])/(315*a^3) - (((16*I)/315)*c^3*PolyLog[2, 1 - 2/(1 + I*a*x)]
)/a^3
```


Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
```

```
b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
```

```
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\int x^2\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^2\,dx = \int\left(c^3x^2\tan^{-1}(ax)^2+3a^2c^3x^4\tan^{-1}(ax)^2+3a^4c^3x^6\tan^{-1}(ax)^2+a^6c^3x^8\tan^{-1}(ax)^2\right)$$

$$= c^3\int x^2\tan^{-1}(ax)^2\,dx+\left(3a^2c^3\right)\int x^4\tan^{-1}(ax)^2\,dx+\left(3a^4c^3\right)\int x^6\tan^{-1}(ax)^2\,dx+($$

$$= \frac{1}{3}c^3x^3\tan^{-1}(ax)^2+\frac{3}{5}a^2c^3x^5\tan^{-1}(ax)^2+\frac{3}{7}a^4c^3x^7\tan^{-1}(ax)^2+\frac{1}{9}a^6c^3x^9\tan^{-1}(ax)^2$$

$$= \frac{1}{3}c^3x^3\tan^{-1}(ax)^2+\frac{3}{5}a^2c^3x^5\tan^{-1}(ax)^2+\frac{3}{7}a^4c^3x^7\tan^{-1}(ax)^2+\frac{1}{9}a^6c^3x^9\tan^{-1}(ax)^2$$

$$= -\frac{c^3x^2\tan^{-1}(ax)}{3a}-\frac{3}{10}ac^3x^4\tan^{-1}(ax)-\frac{1}{7}a^3c^3x^6\tan^{-1}(ax)-\frac{1}{36}a^5c^3x^8\tan^{-1}(ax)-$$

$$= \frac{c^3x}{3a^2}+\frac{4c^3x^2\tan^{-1}(ax)}{15a}-\frac{3}{35}ac^3x^4\tan^{-1}(ax)-\frac{20}{189}a^3c^3x^6\tan^{-1}(ax)-\frac{1}{36}a^5c^3x^8\tan^{-1}$$

$$= -\frac{569c^3x}{1260a^2}+\frac{233c^3x^3}{3780}+\frac{29a^2c^3x^5}{1260}+\frac{1}{252}a^4c^3x^7-\frac{c^3\tan^{-1}(ax)}{3a^3}-\frac{17c^3x^2\tan^{-1}(ax)}{105a}-$$

$$= \frac{583c^3x}{3780a^2}+\frac{29c^3x^3}{11340}+\frac{59a^2c^3x^5}{3780}+\frac{1}{252}a^4c^3x^7+\frac{569c^3\tan^{-1}(ax)}{1260a^3}-\frac{16c^3x^2\tan^{-1}(ax)}{315a}-$$

$$= -\frac{47c^3x}{3780a^2}+\frac{239c^3x^3}{11340}+\frac{59a^2c^3x^5}{3780}+\frac{1}{252}a^4c^3x^7-\frac{583c^3\tan^{-1}(ax)}{3780a^3}-\frac{16c^3x^2\tan^{-1}(ax)}{315a}$$

$$= -\frac{47c^3x}{3780a^2}+\frac{239c^3x^3}{11340}+\frac{59a^2c^3x^5}{3780}+\frac{1}{252}a^4c^3x^7+\frac{47c^3\tan^{-1}(ax)}{3780a^3}-\frac{16c^3x^2\tan^{-1}(ax)}{315a}$$

$$= -\frac{47c^3x}{3780a^2}+\frac{239c^3x^3}{11340}+\frac{59a^2c^3x^5}{3780}+\frac{1}{252}a^4c^3x^7+\frac{47c^3\tan^{-1}(ax)}{3780a^3}-\frac{16c^3x^2\tan^{-1}(ax)}{315a}$$

Mathematica [A] time = 2.22103, size = 157, normalized size = 0.57

$$\frac{c^3\left(576i\text{PolyLog}\left(2,-e^{2i\tan^{-1}(ax)}\right)+ax\left(45a^6x^6+177a^4x^4+239a^2x^2-141\right)+36\left(35a^9x^9+135a^7x^7+189a^5x^5+105a^3\right.}{11340a^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]^2,x]
```

```
[Out] (c^3*(a*x*(-141 + 239*a^2*x^2 + 177*a^4*x^4 + 45*a^6*x^6) + 36*(16*I + 105*
a^3*x^3 + 189*a^5*x^5 + 135*a^7*x^7 + 35*a^9*x^9)*ArcTan[a*x]^2 - 3*ArcTan[
```

```
a*x]*(-47 + 192*a^2*x^2 + 534*a^4*x^4 + 400*a^6*x^6 + 105*a^8*x^8 + 384*Log
[1 + E^((2*I)*ArcTan[a*x])]) + (576*I)*PolyLog[2, -E^((2*I)*ArcTan[a*x])]))
/(11340*a^3)
```

Maple [A] time = 0.089, size = 376, normalized size = 1.4

$$\frac{a^6c^3x^9 (\arctan (ax))^2}{9} + \frac{3\,a^4c^3x^7 (\arctan (ax))^2}{7} + \frac{3\,a^2c^3x^5 (\arctan (ax))^2}{5} + \frac{c^3x^3 (\arctan (ax))^2}{3} - \frac{a^5c^3x^8 \arctan (ax)}{36}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a^2*c*x^2+c)^3*arctan(a*x)^2,x)
```

```
[Out] 1/9*a^6*c^3*x^9*arctan(a*x)^2+3/7*a^4*c^3*x^7*arctan(a*x)^2+3/5*a^2*c^3*x^5
*arctan(a*x)^2+1/3*c^3*x^3*arctan(a*x)^2-1/36*a^5*c^3*x^8*arctan(a*x)-20/18
9*a^3*c^3*x^6*arctan(a*x)-89/630*a*c^3*x^4*arctan(a*x)-16/315*c^3*x^2*arcta
n(a*x)/a+16/315/a^3*c^3*arctan(a*x)*ln(a^2*x^2+1)+1/252*a^4*c^3*x^7+59/3780
*a^2*c^3*x^5+239/11340*c^3*x^3-47/3780*c^3*x/a^2+47/3780*c^3*arctan(a*x)/a^
3-8/315*I/a^3*c^3*ln(a*x-I)*ln(-1/2*I*(a*x+I))+8/315*I/a^3*c^3*ln(a*x-I)*ln
(a^2*x^2+1)-8/315*I/a^3*c^3*ln(a^2*x^2+1)*ln(a*x+I)-8/315*I/a^3*c^3*dilog(-
1/2*I*(a*x+I))-4/315*I/a^3*c^3*ln(a*x-I)^2+8/315*I/a^3*c^3*dilog(1/2*I*(a*x
-I))+4/315*I/a^3*c^3*ln(a*x+I)^2+8/315*I/a^3*c^3*ln(a*x+I)*ln(1/2*I*(a*x-I)
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{1260}\left(35\,a^6c^3x^9 + 135\,a^4c^3x^7 + 189\,a^2c^3x^5 + 105\,c^3x^3\right)\arctan (ax)^2 - \frac{1}{5040}\left(35\,a^6c^3x^9 + 135\,a^4c^3x^7 + 189\,a^2c^3x^5 + 105\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^3*arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/1260*(35*a^6*c^3*x^9 + 135*a^4*c^3*x^7 + 189*a^2*c^3*x^5 + 105*c^3*x^3)*a
rctan(a*x)^2 - 1/5040*(35*a^6*c^3*x^9 + 135*a^4*c^3*x^7 + 189*a^2*c^3*x^5 +
 105*c^3*x^3)*log(a^2*x^2 + 1)^2 + integrate(1/5040*(3780*(a^8*c^3*x^10 + 4
*a^6*c^3*x^8 + 6*a^4*c^3*x^6 + 4*a^2*c^3*x^4 + c^3*x^2)*arctan(a*x)^2 + 315
*(a^8*c^3*x^10 + 4*a^6*c^3*x^8 + 6*a^4*c^3*x^6 + 4*a^2*c^3*x^4 + c^3*x^2)*l
```

```
og(a^2*x^2 + 1)^2 - 8*(35*a^7*c^3*x^9 + 135*a^5*c^3*x^7 + 189*a^3*c^3*x^5 +
 105*a*c^3*x^3)*arctan(a*x) + 4*(35*a^8*c^3*x^10 + 135*a^6*c^3*x^8 + 189*a^
4*c^3*x^6 + 105*a^2*c^3*x^4)*log(a^2*x^2 + 1))/(a^2*x^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^6c^3x^8 + 3\,a^4c^3x^6 + 3\,a^2c^3x^4 + c^3x^2\right)\arctan\left(ax\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^3*arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral((a^6*c^3*x^8 + 3*a^4*c^3*x^6 + 3*a^2*c^3*x^4 + c^3*x^2)*arctan(a*x
)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$c^3\left(\int x^2\,\mathrm{atan}^2\left(ax\right)dx + \int 3a^2x^4\,\mathrm{atan}^2\left(ax\right)dx + \int 3a^4x^6\,\mathrm{atan}^2\left(ax\right)dx + \int a^6x^8\,\mathrm{atan}^2\left(ax\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a**2*c*x**2+c)**3*atan(a*x)**2,x)
```

```
[Out] c**3*(Integral(x**2*atan(a*x)**2, x) + Integral(3*a**2*x**4*atan(a*x)**2, x
) + Integral(3*a**4*x**6*atan(a*x)**2, x) + Integral(a**6*x**8*atan(a*x)**2
, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2 + c\right)^3x^2\arctan\left(ax\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^3*arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^3*x^2*arctan(a*x)^2, x)
```

3.276 $\int x\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^2\,dx$

Optimal. Leaf size=200

$$\frac{c^3\left(a^2x^2+1\right)^3}{168a^2}+\frac{3c^3\left(a^2x^2+1\right)^2}{280a^2}+\frac{c^3\left(a^2x^2+1\right)}{35a^2}+\frac{2c^3\log\left(a^2x^2+1\right)}{35a^2}+\frac{c^3\left(a^2x^2+1\right)^4\tan^{-1}(ax)^2}{8a^2}-\frac{c^3x\left(a^2x^2+1\right)^3\tan}{28a}$$

```
[Out] (c^3*(1 + a^2*x^2))/(35*a^2) + (3*c^3*(1 + a^2*x^2)^2)/(280*a^2) + (c^3*(1
+ a^2*x^2)^3)/(168*a^2) - (4*c^3*x*ArcTan[a*x])/(35*a) - (2*c^3*x*(1 + a^2*
x^2)*ArcTan[a*x])/(35*a) - (3*c^3*x*(1 + a^2*x^2)^2*ArcTan[a*x])/(70*a) - (
c^3*x*(1 + a^2*x^2)^3*ArcTan[a*x])/(28*a) + (c^3*(1 + a^2*x^2)^4*ArcTan[a*x
]^2)/(8*a^2) + (2*c^3*Log[1 + a^2*x^2])/(35*a^2)
```

Rubi [A] time = 0.121294, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4930, 4878, 4846, 260}

$$\frac{c^3\left(a^2x^2+1\right)^3}{168a^2}+\frac{3c^3\left(a^2x^2+1\right)^2}{280a^2}+\frac{c^3\left(a^2x^2+1\right)}{35a^2}+\frac{2c^3\log\left(a^2x^2+1\right)}{35a^2}+\frac{c^3\left(a^2x^2+1\right)^4\tan^{-1}(ax)^2}{8a^2}-\frac{c^3x\left(a^2x^2+1\right)^3\tan}{28a}$$

Antiderivative was successfully verified.

```
[In] Int[x*(c + a^2*c*x^2)^3*ArcTan[a*x]^2,x]
```


```
[Out] (c^3*(1 + a^2*x^2))/(35*a^2) + (3*c^3*(1 + a^2*x^2)^2)/(280*a^2) + (c^3*(1
+ a^2*x^2)^3)/(168*a^2) - (4*c^3*x*ArcTan[a*x])/(35*a) - (2*c^3*x*(1 + a^2*
x^2)*ArcTan[a*x])/(35*a) - (3*c^3*x*(1 + a^2*x^2)^2*ArcTan[a*x])/(70*a) - (
c^3*x*(1 + a^2*x^2)^3*ArcTan[a*x])/(28*a) + (c^3*(1 + a^2*x^2)^4*ArcTan[a*x
]^2)/(8*a^2) + (2*c^3*Log[1 + a^2*x^2])/(35*a^2)
```


Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 4878

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symbo
l] :> -Simp[(b*(d + e*x^2)^q)/(2*c*q*(2*q + 1)), x] + (Dist[(2*d*q)/(2*q +
1), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x]), x], x] + Simp[(x*(d + e*x^
2)^q*(a + b*ArcTan[c*x]))/(2*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] && Eq
Q[e, c^2*d] && GtQ[q, 0]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int x\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^2\,dx = \frac{c^3\left(1+a^2x^2\right)^4\tan^{-1}(ax)^2}{8a^2} - \frac{\int\left(c+a^2cx^2\right)^3\tan^{-1}(ax)\,dx}{4a}$$

$$= \frac{c^3\left(1+a^2x^2\right)^3}{168a^2} - \frac{c^3x\left(1+a^2x^2\right)^3\tan^{-1}(ax)}{28a} + \frac{c^3\left(1+a^2x^2\right)^4\tan^{-1}(ax)^2}{8a^2} - (3c)\int\left(c+a\right.$$

$$= \frac{3c^3\left(1+a^2x^2\right)^2}{280a^2} + \frac{c^3\left(1+a^2x^2\right)^3}{168a^2} - \frac{3c^3x\left(1+a^2x^2\right)^2\tan^{-1}(ax)}{70a} - \frac{c^3x\left(1+a^2x^2\right)^3\tan^{-1}}{28a}$$

$$= \frac{c^3\left(1+a^2x^2\right)}{35a^2} + \frac{3c^3\left(1+a^2x^2\right)^2}{280a^2} + \frac{c^3\left(1+a^2x^2\right)^3}{168a^2} - \frac{2c^3x\left(1+a^2x^2\right)\tan^{-1}(ax)}{35a} - \frac{3c^3x\left(\right.}{}$$

$$= \frac{c^3\left(1+a^2x^2\right)}{35a^2} + \frac{3c^3\left(1+a^2x^2\right)^2}{280a^2} + \frac{c^3\left(1+a^2x^2\right)^3}{168a^2} - \frac{4c^3x\tan^{-1}(ax)}{35a} - \frac{2c^3x\left(1+a^2x^2\right)t}{35a}$$

$$= \frac{c^3\left(1+a^2x^2\right)}{35a^2} + \frac{3c^3\left(1+a^2x^2\right)^2}{280a^2} + \frac{c^3\left(1+a^2x^2\right)^3}{168a^2} - \frac{4c^3x\tan^{-1}(ax)}{35a} - \frac{2c^3x\left(1+a^2x^2\right)t}{35a}$$

Mathematica [A] time = 0.0801937, size = 100, normalized size = 0.5

$$\frac{c^3\left(5a^6x^6+24a^4x^4+57a^2x^2+48\log\left(a^2x^2+1\right)-6ax\left(5a^6x^6+21a^4x^4+35a^2x^2+35\right)\tan^{-1}(ax)+105\left(a^2x^2+1\right)^4\tan\right.}{840a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(c + a^2*c*x^2)^3*ArcTan[a*x]^2,x]
```

```
[Out] (c^3*(57*a^2*x^2 + 24*a^4*x^4 + 5*a^6*x^6 - 6*a*x*(35 + 35*a^2*x^2 + 21*a^4
*x^4 + 5*a^6*x^6)*ArcTan[a*x] + 105*(1 + a^2*x^2)^4*ArcTan[a*x]^2 + 48*Log[
1 + a^2*x^2]))/(840*a^2)
```

Maple [A] time = 0.034, size = 185, normalized size = 0.9

$$\frac{a^6c^3 (\arctan (ax))^2 x^8}{8}+\frac{a^4c^3 (\arctan (ax))^2 x^6}{2}+\frac{3\,a^2c^3 (\arctan (ax))^2 x^4}{4}+\frac{c^3 (\arctan (ax))^2 x^2}{2}-\frac{a^5c^3 \arctan (ax)\, x^7}{28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^3*arctan(a*x)^2,x)
```

```
[Out] 1/8*a^6*c^3*arctan(a*x)^2*x^8+1/2*a^4*c^3*arctan(a*x)^2*x^6+3/4*a^2*c^3*arc
tan(a*x)^2*x^4+1/2*c^3*arctan(a*x)^2*x^2-1/28*a^5*c^3*arctan(a*x)*x^7-3/20*
a^3*c^3*arctan(a*x)*x^5-1/4*a*c^3*arctan(a*x)*x^3-1/4*c^3*x*arctan(a*x)/a+1
/8/a^2*c^3*arctan(a*x)^2+1/168*a^4*c^3*x^6+1/35*a^2*x^4*c^3+19/280*x^2*c^3+
2/35*c^3*ln(a^2*x^2+1)/a^2
```

Maxima [A] time = 0.995014, size = 180, normalized size = 0.9

$$\frac{\left(a^2cx^2+c\right)^4 \arctan (ax)^2}{8\,a^2c}+\frac{\left(5\,a^4c^4x^6+24\,a^2c^4x^4+57\,c^4x^2+\frac{48\,c^4\log\left(a^2x^2+1\right)}{a^2}\right)a-6\left(5\,a^6c^4x^7+21\,a^4c^4x^5+35\,a^2c^4x^3+\right.}{840\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^3*arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/8*(a^2*c*x^2 + c)^4*arctan(a*x)^2/(a^2*c) + 1/840*((5*a^4*c^4*x^6 + 24*a^
2*c^4*x^4 + 57*c^4*x^2 + 48*c^4*log(a^2*x^2 + 1)/a^2)*a - 6*(5*a^6*c^4*x^7
+ 21*a^4*c^4*x^5 + 35*a^2*c^4*x^3 + 35*c^4*x)*arctan(a*x))/(a*c)
```

Fricas [A] time = 2.23156, size = 343, normalized size = 1.72

$$\frac{5\,a^6c^3x^6+24\,a^4c^3x^4+57\,a^2c^3x^2+48\,c^3\log\left(a^2x^2+1\right)+105\left(a^8c^3x^8+4\,a^6c^3x^6+6\,a^4c^3x^4+4\,a^2c^3x^2+c^3\right)\arctan (ax}{840\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^3*arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/840*(5*a^6*c^3*x^6 + 24*a^4*c^3*x^4 + 57*a^2*c^3*x^2 + 48*c^3*log(a^2*x^2
 + 1) + 105*(a^8*c^3*x^8 + 4*a^6*c^3*x^6 + 6*a^4*c^3*x^4 + 4*a^2*c^3*x^2 +
c^3)*arctan(a*x)^2 - 6*(5*a^7*c^3*x^7 + 21*a^5*c^3*x^5 + 35*a^3*c^3*x^3 + 3
5*a*c^3*x)*arctan(a*x))/a^2
```

Sympy [A] time = 4.81666, size = 207, normalized size = 1.03

$$\begin{cases} \frac{a^6c^3x^8\operatorname{atan}^2(ax)}{8} - \frac{a^5c^3x^7\operatorname{atan}(ax)}{28} + \frac{a^4c^3x^6\operatorname{atan}^2(ax)}{2} + \frac{a^4c^3x^6}{168} - \frac{3a^3c^3x^5\operatorname{atan}(ax)}{20} + \frac{3a^2c^3x^4\operatorname{atan}^2(ax)}{4} + \frac{a^2c^3x^4}{35} - \frac{ac^3x^3\operatorname{atan}(ax)}{4} + \frac{c^3x^2 a\ldots \\ 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**3*atan(a*x)**2,x)
```

```
[Out] Piecewise((a**6*c**3*x**8*atan(a*x)**2/8 - a**5*c**3*x**7*atan(a*x)/28 + a*
*4*c**3*x**6*atan(a*x)**2/2 + a**4*c**3*x**6/168 - 3*a**3*c**3*x**5*atan(a*
x)/20 + 3*a**2*c**3*x**4*atan(a*x)**2/4 + a**2*c**3*x**4/35 - a*c**3*x**3*a
tan(a*x)/4 + c**3*x**2*atan(a*x)**2/2 + 19*c**3*x**2/280 - c**3*x*atan(a*x)
/(4*a) + 2*c**3*log(x**2 + a**(-2))/(35*a**2) + c**3*atan(a*x)**2/(8*a**2),
 Ne(a, 0)), (0, True))
```

Giac [A] time = 1.16338, size = 301, normalized size = 1.5

$$\frac{\left(a^2cx^2+c\right)^4\arctan(ax)^2}{8\,a^2c} - \frac{5\left(12\,x^7\arctan(ax) - a\left(\frac{2\,a^4x^6-3\,a^2x^4+6\,x^2}{a^6} - \frac{6\,\log\left(a^2x^2+1\right)}{a^8}\right)\right)a^6c^3 + 63\left(4\,x^5\arctan(ax) - a\left(\frac{a^2x^4}{\ldots}\right.\right.\ldots}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^3*arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] 1/8*(a^2*c*x^2 + c)^4*arctan(a*x)^2/(a^2*c) - 1/1680*(5*(12*x^7*arctan(a*x)
 - a*((2*a^4*x^6 - 3*a^2*x^4 + 6*x^2)/a^6 - 6*log(a^2*x^2 + 1)/a^8))*a^6*c^
3 + 63*(4*x^5*arctan(a*x) - a*((a^2*x^4 - 2*x^2)/a^4 + 2*log(a^2*x^2 + 1)/a
```

```
^6))*a^4*c^3 + 210*(2*x^3*arctan(a*x) - a*(x^2/a^2 - log(a^2*x^2 + 1)/a^4))
*a^2*c^3 + 210*(2*a*x*arctan(a*x) - log(a^2*x^2 + 1))*c^3/a)/a
```

3.277 $\int \left(c+a^2 c x^2\right)^3 \tan^{-1}(ax)^2 \, dx$

Optimal. Leaf size=268

$$\frac{16ic^3 \text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{35a}+\frac{1}{105}a^4c^3x^5+\frac{19}{315}a^2c^3x^3+\frac{1}{7}c^3x\left(a^2x^2+1\right)^3\tan^{-1}(ax)^2+\frac{6}{35}c^3x\left(a^2x^2+1\right)^2\tan^{-1}(ax)^2+$$

```
[Out] (38*c^3*x)/105 + (19*a^2*c^3*x^3)/315 + (a^4*c^3*x^5)/105 - (8*c^3*(1 + a^2
*x^2)*ArcTan[a*x])/(35*a) - (3*c^3*(1 + a^2*x^2)^2*ArcTan[a*x])/(35*a) - (c
^3*(1 + a^2*x^2)^3*ArcTan[a*x])/(21*a) + (((16*I)/35)*c^3*ArcTan[a*x]^2)/a
+ (16*c^3*x*ArcTan[a*x]^2)/35 + (8*c^3*x*(1 + a^2*x^2)*ArcTan[a*x]^2)/35 +
(6*c^3*x*(1 + a^2*x^2)^2*ArcTan[a*x]^2)/35 + (c^3*x*(1 + a^2*x^2)^3*ArcTan[
a*x]^2)/7 + (32*c^3*ArcTan[a*x]*Log[2/(1 + I*a*x)])/(35*a) + (((16*I)/35)*c
^3*PolyLog[2, 1 - 2/(1 + I*a*x)])/a
```

Rubi [A] time = 0.184195, antiderivative size = 268, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {4880, 4846, 4920, 4854, 2402, 2315, 8, 194}

$$\frac{16ic^3 \text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{35a}+\frac{1}{105}a^4c^3x^5+\frac{19}{315}a^2c^3x^3+\frac{1}{7}c^3x\left(a^2x^2+1\right)^3\tan^{-1}(ax)^2+\frac{6}{35}c^3x\left(a^2x^2+1\right)^2\tan^{-1}(ax)^2+$$

Antiderivative was successfully verified.

```
[In] Int[(c + a^2*c*x^2)^3*ArcTan[a*x]^2,x]
```

```
[Out] (38*c^3*x)/105 + (19*a^2*c^3*x^3)/315 + (a^4*c^3*x^5)/105 - (8*c^3*(1 + a^2
*x^2)*ArcTan[a*x])/(35*a) - (3*c^3*(1 + a^2*x^2)^2*ArcTan[a*x])/(35*a) - (c
^3*(1 + a^2*x^2)^3*ArcTan[a*x])/(21*a) + (((16*I)/35)*c^3*ArcTan[a*x]^2)/a
+ (16*c^3*x*ArcTan[a*x]^2)/35 + (8*c^3*x*(1 + a^2*x^2)*ArcTan[a*x]^2)/35 +
(6*c^3*x*(1 + a^2*x^2)^2*ArcTan[a*x]^2)/35 + (c^3*x*(1 + a^2*x^2)^3*ArcTan[
a*x]^2)/7 + (32*c^3*ArcTan[a*x]*Log[2/(1 + I*a*x)])/(35*a) + (((16*I)/35)*c
^3*PolyLog[2, 1 - 2/(1 + I*a*x)])/a
```

Rule 4880

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> -Simp[(b*p*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1))/(2*c*q*(2*
q + 1)), x] + (Dist[(2*d*q)/(2*q + 1), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTa
n[c*x])^p, x], x] + Dist[(b^2*d*p*(p - 1))/(2*q*(2*q + 1)), Int[(d + e*x^2)
```

```
^(q - 1)*(a + b*ArcTan[c*x])^(p - 2), x], x] + Simp[(x*(d + e*x^2)^q*(a + b
*ArcTan[c*x])^p)/(2*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[q, 0] && GtQ[p, 1]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \left(c+a^2cx^2\right)^3 \tan^{-1}(ax)^2\,dx = -\frac{c^3\left(1+a^2x^2\right)^3\tan^{-1}(ax)}{21a} + \frac{1}{7}c^3x\left(1+a^2x^2\right)^3\tan^{-1}(ax)^2 + \frac{1}{21}c\int \left(c+a^2cx^2\right)^2\,dx + \frac{1}{7}($$

$$= -\frac{3c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)}{35a} - \frac{c^3\left(1+a^2x^2\right)^3\tan^{-1}(ax)}{21a} + \frac{6}{35}c^3x\left(1+a^2x^2\right)^2\tan^{-1}(ax)^2$$

$$= \frac{2c^3x}{15} + \frac{19}{315}a^2c^3x^3 + \frac{1}{105}a^4c^3x^5 - \frac{8c^3\left(1+a^2x^2\right)\tan^{-1}(ax)}{35a} - \frac{3c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)}{35a}$$

$$= \frac{38c^3x}{105} + \frac{19}{315}a^2c^3x^3 + \frac{1}{105}a^4c^3x^5 - \frac{8c^3\left(1+a^2x^2\right)\tan^{-1}(ax)}{35a} - \frac{3c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax}{35a}$$

$$= \frac{38c^3x}{105} + \frac{19}{315}a^2c^3x^3 + \frac{1}{105}a^4c^3x^5 - \frac{8c^3\left(1+a^2x^2\right)\tan^{-1}(ax)}{35a} - \frac{3c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax}{35a}$$

$$= \frac{38c^3x}{105} + \frac{19}{315}a^2c^3x^3 + \frac{1}{105}a^4c^3x^5 - \frac{8c^3\left(1+a^2x^2\right)\tan^{-1}(ax)}{35a} - \frac{3c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax}{35a}$$

$$= \frac{38c^3x}{105} + \frac{19}{315}a^2c^3x^3 + \frac{1}{105}a^4c^3x^5 - \frac{8c^3\left(1+a^2x^2\right)\tan^{-1}(ax)}{35a} - \frac{3c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax}{35a}$$

$$= \frac{38c^3x}{105} + \frac{19}{315}a^2c^3x^3 + \frac{1}{105}a^4c^3x^5 - \frac{8c^3\left(1+a^2x^2\right)\tan^{-1}(ax)}{35a} - \frac{3c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax}{35a}$$

Mathematica [A] time = 1.16528, size = 137, normalized size = 0.51

$$\frac{c^3\left(-144i\text{PolyLog}\left(2,-e^{2i\tan^{-1}(ax)}\right) + ax\left(3a^4x^4+19a^2x^2+114\right) + 9\left(5a^7x^7+21a^5x^5+35a^3x^3+35ax-16i\right)\tan^{-1}(ax)^2}{315a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + a^2*c*x^2)^3*ArcTan[a*x]^2,x]
```

```
[Out] (c^3*(a*x*(114 + 19*a^2*x^2 + 3*a^4*x^4) + 9*(-16*I + 35*a*x + 35*a^3*x^3 +
 21*a^5*x^5 + 5*a^7*x^7)*ArcTan[a*x]^2 - 3*ArcTan[a*x]*(38 + 57*a^2*x^2 + 2
4*a^4*x^4 + 5*a^6*x^6 - 96*Log[1 + E^((2*I)*ArcTan[a*x])]) - (144*I)*PolyLo
g[2, -E^((2*I)*ArcTan[a*x])]))/(315*a)
```

Maple [A] time = 0.069, size = 346, normalized size = 1.3

$$\frac{a^6c^3 (\arctan(ax))^2 x^7}{7}+\frac{3\,a^4c^3 (\arctan(ax))^2 x^5}{5}+a^2c^3 (\arctan(ax))^2 x^3+c^3x (\arctan(ax))^2-\frac{a^5c^3\arctan(ax)\,x^6}{21}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^3*arctan(a*x)^2,x)
```

```
[Out] 1/7*a^6*c^3*arctan(a*x)^2*x^7+3/5*a^4*c^3*arctan(a*x)^2*x^5+a^2*c^3*arctan(
a*x)^2*x^3+c^3*x*arctan(a*x)^2-1/21*a^5*c^3*arctan(a*x)*x^6-8/35*a^3*c^3*ar
ctan(a*x)*x^4-19/35*a*c^3*arctan(a*x)*x^2-16/35/a*c^3*arctan(a*x)*ln(a^2*x^
2+1)+1/105*a^4*c^3*x^5+19/315*a^2*c^3*x^3+38/105*c^3*x-38/105/a*c^3*arctan(
a*x)-4/35*I/a*c^3*ln(a*x+I)^2+8/35*I/a*c^3*ln(a^2*x^2+1)*ln(a*x+I)+8/35*I/a
*c^3*ln(a*x-I)*ln(-1/2*I*(a*x+I))+8/35*I/a*c^3*dilog(-1/2*I*(a*x+I))-8/35*I
/a*c^3*ln(a^2*x^2+1)*ln(a*x-I)-8/35*I/a*c^3*dilog(1/2*I*(a*x-I))-8/35*I/a*c
^3*ln(a*x+I)*ln(1/2*I*(a*x-I))+4/35*I/a*c^3*ln(a*x-I)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] 420*a^8*c^3*integrate(1/560*x^8*arctan(a*x)^2/(a^2*x^2 + 1), x) + 35*a^8*c^
3*integrate(1/560*x^8*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x) + 20*a^8*c^3*int
egrate(1/560*x^8*log(a^2*x^2 + 1)/(a^2*x^2 + 1), x) - 40*a^7*c^3*integrate(
1/560*x^7*arctan(a*x)/(a^2*x^2 + 1), x) + 1680*a^6*c^3*integrate(1/560*x^6*
arctan(a*x)^2/(a^2*x^2 + 1), x) + 140*a^6*c^3*integrate(1/560*x^6*log(a^2*x
^2 + 1)^2/(a^2*x^2 + 1), x) + 84*a^6*c^3*integrate(1/560*x^6*log(a^2*x^2 +
1)/(a^2*x^2 + 1), x) - 168*a^5*c^3*integrate(1/560*x^5*arctan(a*x)/(a^2*x^2
 + 1), x) + 2520*a^4*c^3*integrate(1/560*x^4*arctan(a*x)^2/(a^2*x^2 + 1), x
) + 210*a^4*c^3*integrate(1/560*x^4*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x) +
140*a^4*c^3*integrate(1/560*x^4*log(a^2*x^2 + 1)/(a^2*x^2 + 1), x) - 280*a^
3*c^3*integrate(1/560*x^3*arctan(a*x)/(a^2*x^2 + 1), x) + 1680*a^2*c^3*inte
grate(1/560*x^2*arctan(a*x)^2/(a^2*x^2 + 1), x) + 140*a^2*c^3*integrate(1/5
60*x^2*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x) + 140*a^2*c^3*integrate(1/560*x
^2*log(a^2*x^2 + 1)/(a^2*x^2 + 1), x) + 1/4*c^3*arctan(a*x)^3/a - 280*a*c^3
*integrate(1/560*x*arctan(a*x)/(a^2*x^2 + 1), x) + 35*c^3*integrate(1/560*l
```

```
og(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x) + 1/140*(5*a^6*c^3*x^7 + 21*a^4*c^3*x^5
 + 35*a^2*c^3*x^3 + 35*c^3*x)*arctan(a*x)^2 - 1/560*(5*a^6*c^3*x^7 + 21*a^4
*c^3*x^5 + 35*a^2*c^3*x^3 + 35*c^3*x)*log(a^2*x^2 + 1)^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^6c^3x^6 + 3\,a^4c^3x^4 + 3\,a^2c^3x^2 + c^3\right)\arctan\left(ax\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*arctan(a*x)^2,
 x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$c^3\left(\int 3a^2x^2\,\mathrm{atan}^2\left(ax\right)dx + \int 3a^4x^4\,\mathrm{atan}^2\left(ax\right)dx + \int a^6x^6\,\mathrm{atan}^2\left(ax\right)dx + \int \mathrm{atan}^2\left(ax\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**3*atan(a*x)**2,x)
```

```
[Out] c**3*(Integral(3*a**2*x**2*atan(a*x)**2, x) + Integral(3*a**4*x**4*atan(a*x
)**2, x) + Integral(a**6*x**6*atan(a*x)**2, x) + Integral(atan(a*x)**2, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2 + c\right)^3\arctan\left(ax\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^3*arctan(a*x)^2, x)
```

3.278 $\int \frac{\left(c+a^2cx^2\right)^3 \tan^{-1}(ax)^2}{x} dx$

Optimal. Leaf size=287

$$-\frac{1}{2}c^3\text{PolyLog}\left(3,1-\frac{2}{1+iax}\right)+\frac{1}{2}c^3\text{PolyLog}\left(3,-1+\frac{2}{1+iax}\right)-ic^3\tan^{-1}(ax)\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)+ic^3\tan^{-1}(a$$

```
[Out] (29*a^2*c^3*x^2)/180 + (a^4*c^3*x^4)/60 - (11*a*c^3*x*ArcTan[a*x])/6 - (7*a
^3*c^3*x^3*ArcTan[a*x])/18 - (a^5*c^3*x^5*ArcTan[a*x])/15 + (11*c^3*ArcTan[
a*x]^2)/12 + (3*a^2*c^3*x^2*ArcTan[a*x]^2)/2 + (3*a^4*c^3*x^4*ArcTan[a*x]^2
)/4 + (a^6*c^3*x^6*ArcTan[a*x]^2)/6 + 2*c^3*ArcTan[a*x]^2*ArcTanh[1 - 2/(1
+ I*a*x)] + (34*c^3*Log[1 + a^2*x^2])/45 - I*c^3*ArcTan[a*x]*PolyLog[2, 1 -
 2/(1 + I*a*x)] + I*c^3*ArcTan[a*x]*PolyLog[2, -1 + 2/(1 + I*a*x)] - (c^3*P
olyLog[3, 1 - 2/(1 + I*a*x)])/2 + (c^3*PolyLog[3, -1 + 2/(1 + I*a*x)])/2
```

Rubi [A] time = 0.743414, antiderivative size = 287, normalized size of antiderivative = 1., number of steps used = 38, number of rules used = 12, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {4948, 4850, 4988, 4884, 4994, 6610, 4852, 4916, 4846, 260, 266, 43}

$$-\frac{1}{2}c^3\text{PolyLog}\left(3,1-\frac{2}{1+iax}\right)+\frac{1}{2}c^3\text{PolyLog}\left(3,-1+\frac{2}{1+iax}\right)-ic^3\tan^{-1}(ax)\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)+ic^3\tan^{-1}(a$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)^3*ArcTan[a*x]^2)/x,x]
```


```
[Out] (29*a^2*c^3*x^2)/180 + (a^4*c^3*x^4)/60 - (11*a*c^3*x*ArcTan[a*x])/6 - (7*a
^3*c^3*x^3*ArcTan[a*x])/18 - (a^5*c^3*x^5*ArcTan[a*x])/15 + (11*c^3*ArcTan[
a*x]^2)/12 + (3*a^2*c^3*x^2*ArcTan[a*x]^2)/2 + (3*a^4*c^3*x^4*ArcTan[a*x]^2
)/4 + (a^6*c^3*x^6*ArcTan[a*x]^2)/6 + 2*c^3*ArcTan[a*x]^2*ArcTanh[1 - 2/(1
+ I*a*x)] + (34*c^3*Log[1 + a^2*x^2])/45 - I*c^3*ArcTan[a*x]*PolyLog[2, 1 -
 2/(1 + I*a*x)] + I*c^3*ArcTan[a*x]*PolyLog[2, -1 + 2/(1 + I*a*x)] - (c^3*P
olyLog[3, 1 - 2/(1 + I*a*x)])/2 + (c^3*PolyLog[3, -1 + 2/(1 + I*a*x)])/2
```


Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

Rule 4850

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTan[c*x])^(p - 1)*ArcTanh[1 - 2/(1 + I*c*x)])/(1 + c^2*x^2), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4988

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTan[c*x])^p)/(d + e
*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTan[c*x])^p)/(d + e*x^2
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
```

```
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\left(c+a^2cx^2\right)^3 \tan^{-1}(ax)^2}{x}\,dx = \int \left(\frac{c^3\tan^{-1}(ax)^2}{x} + 3a^2c^3x\tan^{-1}(ax)^2 + 3a^4c^3x^3\tan^{-1}(ax)^2 + a^6c^3x^5\tan^{-1}(ax)^2\right)dx$$

$$= c^3\int \frac{\tan^{-1}(ax)^2}{x}\,dx + \left(3a^2c^3\right)\int x\tan^{-1}(ax)^2\,dx + \left(3a^4c^3\right)\int x^3\tan^{-1}(ax)^2\,dx + \left(a^6c^3\right)$$

$$= \frac{3}{2}a^2c^3x^2\tan^{-1}(ax)^2 + \frac{3}{4}a^4c^3x^4\tan^{-1}(ax)^2 + \frac{1}{6}a^6c^3x^6\tan^{-1}(ax)^2 + 2c^3\tan^{-1}(ax)^2\tanh$$

$$= \frac{3}{2}a^2c^3x^2\tan^{-1}(ax)^2 + \frac{3}{4}a^4c^3x^4\tan^{-1}(ax)^2 + \frac{1}{6}a^6c^3x^6\tan^{-1}(ax)^2 + 2c^3\tan^{-1}(ax)^2\tanh$$

$$= -3ac^3x\tan^{-1}(ax) - \frac{1}{2}a^3c^3x^3\tan^{-1}(ax) - \frac{1}{15}a^5c^3x^5\tan^{-1}(ax) + \frac{3}{2}c^3\tan^{-1}(ax)^2 + \frac{3}{2}a^2c^3$$

$$= -\frac{3}{2}ac^3x\tan^{-1}(ax) - \frac{7}{18}a^3c^3x^3\tan^{-1}(ax) - \frac{1}{15}a^5c^3x^5\tan^{-1}(ax) + \frac{3}{4}c^3\tan^{-1}(ax)^2 + \frac{3}{2}a^2$$

$$= -\frac{11}{6}ac^3x\tan^{-1}(ax) - \frac{7}{18}a^3c^3x^3\tan^{-1}(ax) - \frac{1}{15}a^5c^3x^5\tan^{-1}(ax) + \frac{11}{12}c^3\tan^{-1}(ax)^2 + \frac{3}{2}$$

$$= \frac{13}{60}a^2c^3x^2 + \frac{1}{60}a^4c^3x^4 - \frac{11}{6}ac^3x\tan^{-1}(ax) - \frac{7}{18}a^3c^3x^3\tan^{-1}(ax) - \frac{1}{15}a^5c^3x^5\tan^{-1}(ax)$$

$$= \frac{29}{180}a^2c^3x^2 + \frac{1}{60}a^4c^3x^4 - \frac{11}{6}ac^3x\tan^{-1}(ax) - \frac{7}{18}a^3c^3x^3\tan^{-1}(ax) - \frac{1}{15}a^5c^3x^5\tan^{-1}(ax)$$

Mathematica [A] time = 0.528906, size = 252, normalized size = 0.88

$$\frac{1}{360}c^3\left(360i\tan^{-1}(ax)\text{PolyLog}\left(2,e^{-2i\tan^{-1}(ax)}\right) + 360i\tan^{-1}(ax)\text{PolyLog}\left(2,-e^{2i\tan^{-1}(ax)}\right) + 180\text{PolyLog}\left(3,e^{-2i\tan^{-1}(a}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((c + a^2*c*x^2)^3*ArcTan[a*x]^2)/x,x]
```

```
[Out] (c^3*(52 - (15*I)*Pi^3 + 58*a^2*x^2 + 6*a^4*x^4 - 660*a*x*ArcTan[a*x] - 140
*a^3*x^3*ArcTan[a*x] - 24*a^5*x^5*ArcTan[a*x] + 330*ArcTan[a*x]^2 + 540*a^2
*x^2*ArcTan[a*x]^2 + 270*a^4*x^4*ArcTan[a*x]^2 + 60*a^6*x^6*ArcTan[a*x]^2 +
 (240*I)*ArcTan[a*x]^3 + 360*ArcTan[a*x]^2*Log[1 - E^((-2*I)*ArcTan[a*x])]
- 360*ArcTan[a*x]^2*Log[1 + E^((2*I)*ArcTan[a*x])] + 272*Log[1 + a^2*x^2] +
 (360*I)*ArcTan[a*x]*PolyLog[2, E^((-2*I)*ArcTan[a*x])] + (360*I)*ArcTan[a*
x]*PolyLog[2, -E^((2*I)*ArcTan[a*x])] + 180*PolyLog[3, E^((-2*I)*ArcTan[a*x
])] - 180*PolyLog[3, -E^((2*I)*ArcTan[a*x])]))/360
```

Maple [C] time = 3.927, size = 1217, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^3*arctan(a*x)^2/x,x)
```

```
[Out] 13/90*c^3-11/6*a*c^3*x*arctan(a*x)-7/18*a^3*c^3*x^3*arctan(a*x)-1/15*a^5*c^
3*x^5*arctan(a*x)+3/2*a^2*c^3*x^2*arctan(a*x)^2+3/4*a^4*c^3*x^4*arctan(a*x)
^2+1/6*a^6*c^3*x^6*arctan(a*x)^2+29/180*a^2*c^3*x^2+1/60*a^4*c^3*x^4+11/12*
c^3*arctan(a*x)^2+2*c^3*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))-1/2*c^3*poly
log(3,-(1+I*a*x)^2/(a^2*x^2+1))-68/45*c^3*ln((1+I*a*x)^2/(a^2*x^2+1)+1)+2*c
^3*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-1/2*I*c^3*Pi*csgn(((1+I*a*x)^2/(
a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2*arctan(a*x)^2+1/2*I*c^3*Pi*csg
n(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^3*arctan(a*x)^2+
I*c^3*arctan(a*x)*polylog(2,-(1+I*a*x)^2/(a^2*x^2+1))+1/2*I*c^3*Pi*arctan(a
*x)^2-2*I*c^3*arctan(a*x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*I*c^3*ar
ctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+1/2*I*c^3*Pi*csgn(I*((1+I
*a*x)^2/(a^2*x^2+1)-1))*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*
x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*arctan(a*x)^2+1/2*I*c^3*Pi
*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^3*arctan(a
*x)^2+c^3*arctan(a*x)^2*ln(a*x)+c^3*arctan(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1
)^(1/2))+c^3*arctan(a*x)^2*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))+68/45*I*c^3*ar
ctan(a*x)-1/2*I*c^3*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1))*csgn(I*((1+I*a*x
)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2*arctan(a*x)^2-1/2*I*c^3*P
i*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(((1+
I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2*arctan(a*x)^2+1/2*I*
c^3*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn
(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*arctan(a*x)^2-1/2
*I*c^3*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+
1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2*arctan(a*x)^2-c^3*arctan(a*x)^2*ln((1+
I*a*x)^2/(a^2*x^2+1)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^2/x,x, algorithm="maxima")
```

```
[Out] 36*a^8*c^3*integrate(1/48*x^8*arctan(a*x)^2/(a^2*x^3 + x), x) + 3*a^8*c^3*i
ntegrate(1/48*x^8*log(a^2*x^2 + 1)^2/(a^2*x^3 + x), x) + 2*a^8*c^3*integrat
e(1/48*x^8*log(a^2*x^2 + 1)/(a^2*x^3 + x), x) - 4*a^7*c^3*integrate(1/48*x^
7*arctan(a*x)/(a^2*x^3 + x), x) + 144*a^6*c^3*integrate(1/48*x^6*arctan(a*x
)^2/(a^2*x^3 + x), x) + 12*a^6*c^3*integrate(1/48*x^6*log(a^2*x^2 + 1)^2/(a
^2*x^3 + x), x) + 9*a^6*c^3*integrate(1/48*x^6*log(a^2*x^2 + 1)/(a^2*x^3 +
x), x) - 18*a^5*c^3*integrate(1/48*x^5*arctan(a*x)/(a^2*x^3 + x), x) + 216*
a^4*c^3*integrate(1/48*x^4*arctan(a*x)^2/(a^2*x^3 + x), x) + 18*a^4*c^3*int
egrate(1/48*x^4*log(a^2*x^2 + 1)^2/(a^2*x^3 + x), x) + 18*a^4*c^3*integrate
(1/48*x^4*log(a^2*x^2 + 1)/(a^2*x^3 + x), x) - 36*a^3*c^3*integrate(1/48*x^
3*arctan(a*x)/(a^2*x^3 + x), x) + 144*a^2*c^3*integrate(1/48*x^2*arctan(a*x
)^2/(a^2*x^3 + x), x) + 1/24*c^3*log(a^2*x^2 + 1)^3 + 36*c^3*integrate(1/48
*arctan(a*x)^2/(a^2*x^3 + x), x) + 3*c^3*integrate(1/48*log(a^2*x^2 + 1)^2/
(a^2*x^3 + x), x) + 1/48*(2*a^6*c^3*x^6 + 9*a^4*c^3*x^4 + 18*a^2*c^3*x^2)*a
rctan(a*x)^2 - 1/192*(2*a^6*c^3*x^6 + 9*a^4*c^3*x^4 + 18*a^2*c^3*x^2)*log(a
^2*x^2 + 1)^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^{6}c^{3}x^{6}+3\,a^{4}c^{3}x^{4}+3\,a^{2}c^{3}x^{2}+c^{3}\right)\arctan\left(ax\right)^{2}}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^2/x,x, algorithm="fricas")
```

```
[Out] integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*arctan(a*x)^2/
x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$c^{3}\left(\int\frac{\operatorname{atan}^{2}(ax)}{x}\,dx+\int 3a^{2}x\operatorname{atan}^{2}(ax)\,dx+\int 3a^{4}x^{3}\operatorname{atan}^{2}(ax)\,dx+\int a^{6}x^{5}\operatorname{atan}^{2}(ax)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**3*atan(a*x)**2/x,x)
```

```
[Out] c**3*(Integral(atan(a*x)**2/x, x) + Integral(3*a**2*x*atan(a*x)**2, x) + In
tegral(3*a**4*x**3*atan(a*x)**2, x) + Integral(a**6*x**5*atan(a*x)**2, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^3 \arctan\left(ax\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^2/x,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^3*arctan(a*x)^2/x, x)
```

3.279 $\int \frac{(c+a^2cx^2)^3 \tan^{-1}(ax)^2}{x^2} dx$

Optimal. Leaf size=251

$$-iac^3\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)+\frac{11}{5}iac^3\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)+\frac{1}{30}a^4c^3x^3+\frac{1}{5}a^6c^3x^5\tan^{-1}(ax)^2-\frac{1}{10}a^5c^3x^4\tan^{-1}($$

```
[Out] (7*a^2*c^3*x)/10 + (a^4*c^3*x^3)/30 - (7*a*c^3*ArcTan[a*x])/10 - (4*a^3*c^3
*x^2*ArcTan[a*x])/5 - (a^5*c^3*x^4*ArcTan[a*x])/10 + ((6*I)/5)*a*c^3*ArcTan
[a*x]^2 - (c^3*ArcTan[a*x]^2)/x + 3*a^2*c^3*x*ArcTan[a*x]^2 + a^4*c^3*x^3*A
rcTan[a*x]^2 + (a^6*c^3*x^5*ArcTan[a*x]^2)/5 + (22*a*c^3*ArcTan[a*x]*Log[2/
(1 + I*a*x)])/5 + 2*a*c^3*ArcTan[a*x]*Log[2 - 2/(1 - I*a*x)] - I*a*c^3*Poly
Log[2, -1 + 2/(1 - I*a*x)] + ((11*I)/5)*a*c^3*PolyLog[2, 1 - 2/(1 + I*a*x)]
```

Rubi [A] time = 0.64533, antiderivative size = 251, normalized size of antiderivative = 1., number of steps used = 34, number of rules used = 14, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {4948, 4846, 4920, 4854, 2402, 2315, 4852, 4924, 4868, 2447, 4916, 321, 203, 302}

$$-iac^3\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)+\frac{11}{5}iac^3\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)+\frac{1}{30}a^4c^3x^3+\frac{1}{5}a^6c^3x^5\tan^{-1}(ax)^2-\frac{1}{10}a^5c^3x^4\tan^{-1}($$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)^3*ArcTan[a*x]^2)/x^2,x]
```


```
[Out] (7*a^2*c^3*x)/10 + (a^4*c^3*x^3)/30 - (7*a*c^3*ArcTan[a*x])/10 - (4*a^3*c^3
*x^2*ArcTan[a*x])/5 - (a^5*c^3*x^4*ArcTan[a*x])/10 + ((6*I)/5)*a*c^3*ArcTan
[a*x]^2 - (c^3*ArcTan[a*x]^2)/x + 3*a^2*c^3*x*ArcTan[a*x]^2 + a^4*c^3*x^3*A
rcTan[a*x]^2 + (a^6*c^3*x^5*ArcTan[a*x]^2)/5 + (22*a*c^3*ArcTan[a*x]*Log[2/
(1 + I*a*x)])/5 + 2*a*c^3*ArcTan[a*x]*Log[2 - 2/(1 - I*a*x)] - I*a*c^3*Poly
Log[2, -1 + 2/(1 - I*a*x)] + ((11*I)/5)*a*c^3*PolyLog[2, 1 - 2/(1 + I*a*x)]
```


Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\int \frac{\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^2}{x^2}\,dx = \int \left(3a^2c^3\tan^{-1}(ax)^2 + \frac{c^3\tan^{-1}(ax)^2}{x^2} + 3a^4c^3x^2\tan^{-1}(ax)^2 + a^6c^3x^4\tan^{-1}(ax)^2\right)dx$$

$$= c^3\int \frac{\tan^{-1}(ax)^2}{x^2}\,dx + \left(3a^2c^3\right)\int \tan^{-1}(ax)^2\,dx + \left(3a^4c^3\right)\int x^2\tan^{-1}(ax)^2\,dx + \left(a^6c^3\right)$$

$$= -\frac{c^3\tan^{-1}(ax)^2}{x} + 3a^2c^3x\tan^{-1}(ax)^2 + a^4c^3x^3\tan^{-1}(ax)^2 + \frac{1}{5}a^6c^3x^5\tan^{-1}(ax)^2 + \left(2ac\right.$$

$$= 2iac^3\tan^{-1}(ax)^2 - \frac{c^3\tan^{-1}(ax)^2}{x} + 3a^2c^3x\tan^{-1}(ax)^2 + a^4c^3x^3\tan^{-1}(ax)^2 + \frac{1}{5}a^6c^3x^5$$

$$= -a^3c^3x^2\tan^{-1}(ax) - \frac{1}{10}a^5c^3x^4\tan^{-1}(ax) + iac^3\tan^{-1}(ax)^2 - \frac{c^3\tan^{-1}(ax)^2}{x} + 3a^2c^3x\,t$$

$$= a^2c^3x - \frac{4}{5}a^3c^3x^2\tan^{-1}(ax) - \frac{1}{10}a^5c^3x^4\tan^{-1}(ax) + \frac{6}{5}iac^3\tan^{-1}(ax)^2 - \frac{c^3\tan^{-1}(ax)^2}{x}$$

$$= \frac{7}{10}a^2c^3x + \frac{1}{30}a^4c^3x^3 - ac^3\tan^{-1}(ax) - \frac{4}{5}a^3c^3x^2\tan^{-1}(ax) - \frac{1}{10}a^5c^3x^4\tan^{-1}(ax) + \frac{6}{5}i$$

$$= \frac{7}{10}a^2c^3x + \frac{1}{30}a^4c^3x^3 - \frac{7}{10}ac^3\tan^{-1}(ax) - \frac{4}{5}a^3c^3x^2\tan^{-1}(ax) - \frac{1}{10}a^5c^3x^4\tan^{-1}(ax) +$$

$$= \frac{7}{10}a^2c^3x + \frac{1}{30}a^4c^3x^3 - \frac{7}{10}ac^3\tan^{-1}(ax) - \frac{4}{5}a^3c^3x^2\tan^{-1}(ax) - \frac{1}{10}a^5c^3x^4\tan^{-1}(ax) +$$

Mathematica [A] time = 0.705295, size = 202, normalized size = 0.8

$$c^3\left(-66iax\text{PolyLog}\left(2,-e^{2i\tan^{-1}(ax)}\right) - 30iax\text{PolyLog}\left(2,e^{2i\tan^{-1}(ax)}\right) + a^4x^4 + 21a^2x^2 + 6a^6x^6\tan^{-1}(ax)^2 - 3a^5x^5\tan$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((c + a^2*c*x^2)^3*ArcTan[a*x]^2)/x^2,x]
```

```
[Out] (c^3*(21*a^2*x^2 + a^4*x^4 - 21*a*x*ArcTan[a*x] - 24*a^3*x^3*ArcTan[a*x] -
3*a^5*x^5*ArcTan[a*x] - 30*ArcTan[a*x]^2 - (96*I)*a*x*ArcTan[a*x]^2 + 90*a^
2*x^2*ArcTan[a*x]^2 + 30*a^4*x^4*ArcTan[a*x]^2 + 6*a^6*x^6*ArcTan[a*x]^2 +
60*a*x*ArcTan[a*x]*Log[1 - E^((2*I)*ArcTan[a*x])] + 132*a*x*ArcTan[a*x]*Log
[1 + E^((2*I)*ArcTan[a*x])] - (66*I)*a*x*PolyLog[2, -E^((2*I)*ArcTan[a*x])]
 - (30*I)*a*x*PolyLog[2, E^((2*I)*ArcTan[a*x])]))/(30*x)
```

Maple [A] time = 0.097, size = 388, normalized size = 1.6

$$\frac{a^6c^3x^5(\arctan(ax))^2}{5}+a^4c^3x^3(\arctan(ax))^2+3a^2c^3x(\arctan(ax))^2-\frac{c^3(\arctan(ax))^2}{x}-\frac{a^5c^3x^4\arctan(ax)}{10}-\frac{4a^3c^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^3*arctan(a*x)^2/x^2,x)
```

```
[Out] 1/5*a^6*c^3*x^5*arctan(a*x)^2+a^4*c^3*x^3*arctan(a*x)^2+3*a^2*c^3*x*arctan(
a*x)^2-c^3*arctan(a*x)^2/x-1/10*a^5*c^3*x^4*arctan(a*x)-4/5*a^3*c^3*x^2*arc
tan(a*x)-16/5*a*c^3*arctan(a*x)*ln(a^2*x^2+1)+2*a*c^3*arctan(a*x)*ln(a*x)+1
/30*a^4*c^3*x^3+7/10*a^2*c^3*x-7/10*a*c^3*arctan(a*x)+I*a*c^3*dilog(1+I*a*x
)+I*a*c^3*ln(a*x)*ln(1+I*a*x)-8/5*I*a*c^3*ln(a^2*x^2+1)*ln(a*x-I)-4/5*I*a*c
^3*ln(a*x+I)^2-8/5*I*a*c^3*ln(a*x+I)*ln(1/2*I*(a*x-I))-I*a*c^3*dilog(1-I*a*
x)+4/5*I*a*c^3*ln(a*x-I)^2-8/5*I*a*c^3*dilog(1/2*I*(a*x-I))+8/5*I*a*c^3*dil
og(-1/2*I*(a*x+I))+8/5*I*a*c^3*ln(a^2*x^2+1)*ln(a*x+I)+8/5*I*a*c^3*ln(a*x-I
)*ln(-1/2*I*(a*x+I))-I*a*c^3*ln(a*x)*ln(1-I*a*x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^2/x^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^6c^3x^6+3a^4c^3x^4+3a^2c^3x^2+c^3\right)\arctan(ax)^2}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^2/x^2,x, algorithm="fricas")
```

```
[Out] integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*arctan(a*x)^2/
x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$c^3\left(\int 3a^2\,\mathrm{atan}^2\,(ax)\,dx + \int \frac{\mathrm{atan}^2\,(ax)}{x^2}\,dx + \int 3a^4x^2\,\mathrm{atan}^2\,(ax)\,dx + \int a^6x^4\,\mathrm{atan}^2\,(ax)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**3*atan(a*x)**2/x**2,x)
```

```
[Out] c**3*(Integral(3*a**2*atan(a*x)**2, x) + Integral(atan(a*x)**2/x**2, x) + I
ntegral(3*a**4*x**2*atan(a*x)**2, x) + Integral(a**6*x**4*atan(a*x)**2, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2 + c\right)^3 \arctan\,(ax)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^2/x^2,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^3*arctan(a*x)^2/x^2, x)
```

3.280 $\int \frac{(c+a^2cx^2)^3 \tan^{-1}(ax)^2}{x^3} dx$

Optimal. Leaf size=299

$$-\frac{3}{2}a^2c^3\text{PolyLog}\left(3,1-\frac{2}{1+iax}\right)+\frac{3}{2}a^2c^3\text{PolyLog}\left(3,-1+\frac{2}{1+iax}\right)-3ia^2c^3\tan^{-1}(ax)\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)+3ia^2c$$

```
[Out] (a^4*c^3*x^2)/12 - (a*c^3*ArcTan[a*x])/x - (5*a^3*c^3*x*ArcTan[a*x])/2 - (a
^5*c^3*x^3*ArcTan[a*x])/6 + (3*a^2*c^3*ArcTan[a*x]^2)/4 - (c^3*ArcTan[a*x]^
2)/(2*x^2) + (3*a^4*c^3*x^2*ArcTan[a*x]^2)/2 + (a^6*c^3*x^4*ArcTan[a*x]^2)/
4 + 6*a^2*c^3*ArcTan[a*x]^2*ArcTanh[1 - 2/(1 + I*a*x)] + a^2*c^3*Log[x] + (
2*a^2*c^3*Log[1 + a^2*x^2])/3 - (3*I)*a^2*c^3*ArcTan[a*x]*PolyLog[2, 1 - 2/
(1 + I*a*x)] + (3*I)*a^2*c^3*ArcTan[a*x]*PolyLog[2, -1 + 2/(1 + I*a*x)] - (
3*a^2*c^3*PolyLog[3, 1 - 2/(1 + I*a*x)])/2 + (3*a^2*c^3*PolyLog[3, -1 + 2/(
1 + I*a*x)])/2
```

Rubi [A] time = 0.603183, antiderivative size = 299, normalized size of antiderivative = 1., number of steps used = 31, number of rules used = 16, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.727, Rules used = {4948, 4852, 4918, 266, 36, 29, 31, 4884, 4850, 4988, 4994, 6610, 4916, 4846, 260, 43}

$$-\frac{3}{2}a^2c^3\text{PolyLog}\left(3,1-\frac{2}{1+iax}\right)+\frac{3}{2}a^2c^3\text{PolyLog}\left(3,-1+\frac{2}{1+iax}\right)-3ia^2c^3\tan^{-1}(ax)\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)+3ia^2c$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)^3*ArcTan[a*x]^2)/x^3,x]
```


```
[Out] (a^4*c^3*x^2)/12 - (a*c^3*ArcTan[a*x])/x - (5*a^3*c^3*x*ArcTan[a*x])/2 - (a
^5*c^3*x^3*ArcTan[a*x])/6 + (3*a^2*c^3*ArcTan[a*x]^2)/4 - (c^3*ArcTan[a*x]^
2)/(2*x^2) + (3*a^4*c^3*x^2*ArcTan[a*x]^2)/2 + (a^6*c^3*x^4*ArcTan[a*x]^2)/
4 + 6*a^2*c^3*ArcTan[a*x]^2*ArcTanh[1 - 2/(1 + I*a*x)] + a^2*c^3*Log[x] + (
2*a^2*c^3*Log[1 + a^2*x^2])/3 - (3*I)*a^2*c^3*ArcTan[a*x]*PolyLog[2, 1 - 2/
(1 + I*a*x)] + (3*I)*a^2*c^3*ArcTan[a*x]*PolyLog[2, -1 + 2/(1 + I*a*x)] - (
3*a^2*c^3*PolyLog[3, 1 - 2/(1 + I*a*x)])/2 + (3*a^2*c^3*PolyLog[3, -1 + 2/(
1 + I*a*x)])/2
```


Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
```

```
b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4918

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/(d + e*x^2),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4850

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTan[c*x])^(p - 1)*ArcTanh[1 - 2/(1 + I*c*x)])/(1 + c^2*x^2), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4988

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTan[c*x])^p)/(d + e
*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTan[c*x])^p)/(d + e*x^2
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^2}{x^3}\,dx = \int \left(\frac{c^3\tan^{-1}(ax)^2}{x^3}+\frac{3a^2c^3\tan^{-1}(ax)^2}{x}+3a^4c^3x\tan^{-1}(ax)^2+a^6c^3x^3\tan^{-1}(ax)^2\right)dx$$

$$= c^3\int \frac{\tan^{-1}(ax)^2}{x^3}\,dx+\left(3a^2c^3\right)\int \frac{\tan^{-1}(ax)^2}{x}\,dx+\left(3a^4c^3\right)\int x\tan^{-1}(ax)^2\,dx+\left(a^6c^3\right)$$

$$= -\frac{c^3\tan^{-1}(ax)^2}{2x^2}+\frac{3}{2}a^4c^3x^2\tan^{-1}(ax)^2+\frac{1}{4}a^6c^3x^4\tan^{-1}(ax)^2+6a^2c^3\tan^{-1}(ax)^2\tanh$$

$$= -\frac{c^3\tan^{-1}(ax)^2}{2x^2}+\frac{3}{2}a^4c^3x^2\tan^{-1}(ax)^2+\frac{1}{4}a^6c^3x^4\tan^{-1}(ax)^2+6a^2c^3\tan^{-1}(ax)^2\tanh$$

$$= -\frac{ac^3\tan^{-1}(ax)}{x}-3a^3c^3x\tan^{-1}(ax)-\frac{1}{6}a^5c^3x^3\tan^{-1}(ax)+a^2c^3\tan^{-1}(ax)^2-\frac{c^3\tan^{-1}}{2x^2}$$

$$= -\frac{ac^3\tan^{-1}(ax)}{x}-\frac{5}{2}a^3c^3x\tan^{-1}(ax)-\frac{1}{6}a^5c^3x^3\tan^{-1}(ax)+\frac{3}{4}a^2c^3\tan^{-1}(ax)^2-\frac{c^3\tan}{2}$$

$$= -\frac{ac^3\tan^{-1}(ax)}{x}-\frac{5}{2}a^3c^3x\tan^{-1}(ax)-\frac{1}{6}a^5c^3x^3\tan^{-1}(ax)+\frac{3}{4}a^2c^3\tan^{-1}(ax)^2-\frac{c^3\tan}{2}$$

$$= \frac{1}{12}a^4c^3x^2-\frac{ac^3\tan^{-1}(ax)}{x}-\frac{5}{2}a^3c^3x\tan^{-1}(ax)-\frac{1}{6}a^5c^3x^3\tan^{-1}(ax)+\frac{3}{4}a^2c^3\tan^{-1}(ax$$

Mathematica [A] time = 0.386366, size = 333, normalized size = 1.11

$$\frac{c^3\left(72ia^2x^2\tan^{-1}(ax)\text{PolyLog}\left(2,e^{-2i\tan^{-1}(ax)}\right)+72ia^2x^2\tan^{-1}(ax)\text{PolyLog}\left(2,-e^{2i\tan^{-1}(ax)}\right)+36a^2x^2\text{PolyLog}\left(3,e^{-}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((c + a^2*c*x^2)^3*ArcTan[a*x]^2)/x^3,x]
```

```
[Out] (c^3*(2*a^2*x^2 - (3*I)*a^2*Pi^3*x^2 + 2*a^4*x^4 - 24*a*x*ArcTan[a*x] - 60*
a^3*x^3*ArcTan[a*x] - 4*a^5*x^5*ArcTan[a*x] - 12*ArcTan[a*x]^2 + 18*a^2*x^2
*ArcTan[a*x]^2 + 36*a^4*x^4*ArcTan[a*x]^2 + 6*a^6*x^6*ArcTan[a*x]^2 + (48*I
)*a^2*x^2*ArcTan[a*x]^3 + 72*a^2*x^2*ArcTan[a*x]^2*Log[1 - E^((-2*I)*ArcTan
[a*x])] - 72*a^2*x^2*ArcTan[a*x]^2*Log[1 + E^((2*I)*ArcTan[a*x])] + 24*a^2*
x^2*Log[(a*x)/Sqrt[1 + a^2*x^2]] + 28*a^2*x^2*Log[1 + a^2*x^2] + (72*I)*a^2
*x^2*ArcTan[a*x]*PolyLog[2, E^((-2*I)*ArcTan[a*x])] + (72*I)*a^2*x^2*ArcTan
[a*x]*PolyLog[2, -E^((2*I)*ArcTan[a*x])] + 36*a^2*x^2*PolyLog[3, E^((-2*I)*
ArcTan[a*x])] - 36*a^2*x^2*PolyLog[3, -E^((2*I)*ArcTan[a*x])]))/(24*x^2)
```

Maple [C] time = 4.543, size = 1333, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^3*arctan(a*x)^2/x^3,x)
```

```
[Out] -5/2*a^3*c^3*x*arctan(a*x)-1/6*a^5*c^3*x^3*arctan(a*x)+3/2*a^4*c^3*x^2*arct
an(a*x)^2+1/4*a^6*c^3*x^4*arctan(a*x)^2+3*I*a^2*c^3*arctan(a*x)*polylog(2,-
(1+I*a*x)^2/(a^2*x^2+1))-6*I*a^2*c^3*arctan(a*x)*polylog(2,(1+I*a*x)/(a^2*x
^2+1)^(1/2))-6*I*a^2*c^3*arctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2)
)+3/2*I*a^2*c^3*Pi*arctan(a*x)^2+3/4*a^2*c^3*arctan(a*x)^2-1/2*c^3*arctan(a
*x)^2/x^2+1/12*a^4*c^3*x^2-a*c^3*arctan(a*x)/x+1/12*c^3*a^2-7/3*a^2*c^3*ln(
(1+I*a*x)^2/(a^2*x^2+1)+1)+6*a^2*c^3*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2)
)+6*a^2*c^3*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))-3/2*a^2*c^3*polylog(3,-(
1+I*a*x)^2/(a^2*x^2+1))+a^2*c^3*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)-1)+a^2*c^3*l
n(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-3*a^2*c^3*arctan(a*x)^2*ln((1+I*a*x)^2/(a^
2*x^2+1)-1)+3*a^2*c^3*arctan(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))+3*a^2
*c^3*arctan(a*x)^2*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))+3*a^2*c^3*arctan(a*x)^
2*ln(a*x)+4/3*I*a^2*c^3*arctan(a*x)-3/2*I*a^2*c^3*Pi*csgn(((1+I*a*x)^2/(a^2
*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2*arctan(a*x)^2+3/2*I*a^2*c^3*Pi*cs
gn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^3*arctan(a*x)^2
+3/2*I*a^2*c^3*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+
1)+1))^3*arctan(a*x)^2+3/2*I*a^2*c^3*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1))
*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1
+I*a*x)^2/(a^2*x^2+1)+1))*arctan(a*x)^2-3/2*I*a^2*c^3*Pi*csgn(I*((1+I*a*x)^
2/(a^2*x^2+1)-1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+
1)+1))^2*arctan(a*x)^2-3/2*I*a^2*c^3*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/
((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/
(a^2*x^2+1)+1))^2*arctan(a*x)^2+3/2*I*a^2*c^3*Pi*csgn(I*((1+I*a*x)^2/(a^2*x
^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+
```

```
I*a*x)^2/(a^2*x^2+1)+1))*arctan(a*x)^2-3/2*I*a^2*c^3*Pi*csgn(I/((1+I*a*x)^2
/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1
)+1))^2*arctan(a*x)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^2/x^3,x, algorithm="maxima")
```

```
[Out] 1/64*(4*(192*a^8*c^3*integrate(1/16*x^8*arctan(a*x)^2/(a^2*x^5 + x^3), x) +
 16*a^8*c^3*integrate(1/16*x^8*log(a^2*x^2 + 1)^2/(a^2*x^5 + x^3), x) + 16*
a^8*c^3*integrate(1/16*x^8*log(a^2*x^2 + 1)/(a^2*x^5 + x^3), x) - 32*a^7*c^
3*integrate(1/16*x^7*arctan(a*x)/(a^2*x^5 + x^3), x) + 768*a^6*c^3*integrat
e(1/16*x^6*arctan(a*x)^2/(a^2*x^5 + x^3), x) + 64*a^6*c^3*integrate(1/16*x^
6*log(a^2*x^2 + 1)^2/(a^2*x^5 + x^3), x) + 96*a^6*c^3*integrate(1/16*x^6*lo
g(a^2*x^2 + 1)/(a^2*x^5 + x^3), x) - 192*a^5*c^3*integrate(1/16*x^5*arctan(
a*x)/(a^2*x^5 + x^3), x) + 1152*a^4*c^3*integrate(1/16*x^4*arctan(a*x)^2/(a
^2*x^5 + x^3), x) + a^2*c^3*log(a^2*x^2 + 1)^3 + 768*a^2*c^3*integrate(1/16
*x^2*arctan(a*x)^2/(a^2*x^5 + x^3), x) + 64*a^2*c^3*integrate(1/16*x^2*log(
a^2*x^2 + 1)^2/(a^2*x^5 + x^3), x) - 32*a^2*c^3*integrate(1/16*x^2*log(a^2*
x^2 + 1)/(a^2*x^5 + x^3), x) + 64*a*c^3*integrate(1/16*x*arctan(a*x)/(a^2*x
^5 + x^3), x) + 192*c^3*integrate(1/16*arctan(a*x)^2/(a^2*x^5 + x^3), x) +
16*c^3*integrate(1/16*log(a^2*x^2 + 1)^2/(a^2*x^5 + x^3), x))*x^2 + 4*(a^6*
c^3*x^6 + 6*a^4*c^3*x^4 - 2*c^3)*arctan(a*x)^2 - (a^6*c^3*x^6 + 6*a^4*c^3*x
^4 - 2*c^3)*log(a^2*x^2 + 1)^2)/x^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^{6}c^{3}x^{6}+3\,a^{4}c^{3}x^{4}+3\,a^{2}c^{3}x^{2}+c^{3}\right)\arctan\left(ax\right)^{2}}{x^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^2/x^3,x, algorithm="fricas")
```

```
[Out] integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*arctan(a*x)^2/
x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$c^{3}\left(\int \frac{\operatorname{atan}^{2}(ax)}{x^{3}}\,dx + \int \frac{3a^{2}\operatorname{atan}^{2}(ax)}{x}\,dx + \int 3a^{4}x\operatorname{atan}^{2}(ax)\,dx + \int a^{6}x^{3}\operatorname{atan}^{2}(ax)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**3*atan(a*x)**2/x**3,x)
```

```
[Out] c**3*(Integral(atan(a*x)**2/x**3, x) + Integral(3*a**2*atan(a*x)**2/x, x) +
 Integral(3*a**4*x*atan(a*x)**2, x) + Integral(a**6*x**3*atan(a*x)**2, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^{2}cx^{2}+c\right)^{3}\arctan(ax)^{2}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^2/x^3,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^3*arctan(a*x)^2/x^3, x)
```

3.281 $\int \frac{\left(c+a^2cx^2\right)^3 \tan^{-1}(ax)^2}{x^4}\,dx$

Optimal. Leaf size=250

$$-\frac{8}{3}ia^3c^3\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)+\frac{8}{3}ia^3c^3\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)+\frac{1}{3}a^6c^3x^3\tan^{-1}(ax)^2-\frac{1}{3}a^5c^3x^2\tan^{-1}(ax)+\frac{1}{3}a$$

```
[Out] -(a^2*c^3)/(3*x) + (a^4*c^3*x)/3 - (2*a^3*c^3*ArcTan[a*x])/3 - (a*c^3*ArcTa
n[a*x])/(3*x^2) - (a^5*c^3*x^2*ArcTan[a*x])/3 - (c^3*ArcTan[a*x]^2)/(3*x^3)
 - (3*a^2*c^3*ArcTan[a*x]^2)/x + 3*a^4*c^3*x*ArcTan[a*x]^2 + (a^6*c^3*x^3*A
rcTan[a*x]^2)/3 + (16*a^3*c^3*ArcTan[a*x]*Log[2/(1 + I*a*x)])/3 + (16*a^3*c
^3*ArcTan[a*x]*Log[2 - 2/(1 - I*a*x)])/3 - ((8*I)/3)*a^3*c^3*PolyLog[2, -1
+ 2/(1 - I*a*x)] + ((8*I)/3)*a^3*c^3*PolyLog[2, 1 - 2/(1 + I*a*x)]
```

Rubi [A] time = 0.608375, antiderivative size = 250, normalized size of antiderivative = 1., number of steps used = 28, number of rules used = 15, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.682, Rules used = {4948, 4846, 4920, 4854, 2402, 2315, 4852, 4918, 325, 203, 4924, 4868, 2447, 4916, 321}

$$-\frac{8}{3}ia^3c^3\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)+\frac{8}{3}ia^3c^3\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)+\frac{1}{3}a^6c^3x^3\tan^{-1}(ax)^2-\frac{1}{3}a^5c^3x^2\tan^{-1}(ax)+\frac{1}{3}a$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)^3*ArcTan[a*x]^2)/x^4,x]
```

```
[Out] -(a^2*c^3)/(3*x) + (a^4*c^3*x)/3 - (2*a^3*c^3*ArcTan[a*x])/3 - (a*c^3*ArcTa
n[a*x])/(3*x^2) - (a^5*c^3*x^2*ArcTan[a*x])/3 - (c^3*ArcTan[a*x]^2)/(3*x^3)
 - (3*a^2*c^3*ArcTan[a*x]^2)/x + 3*a^4*c^3*x*ArcTan[a*x]^2 + (a^6*c^3*x^3*A
rcTan[a*x]^2)/3 + (16*a^3*c^3*ArcTan[a*x]*Log[2/(1 + I*a*x)])/3 + (16*a^3*c
^3*ArcTan[a*x]*Log[2 - 2/(1 - I*a*x)])/3 - ((8*I)/3)*a^3*c^3*PolyLog[2, -1
+ 2/(1 - I*a*x)] + ((8*I)/3)*a^3*c^3*PolyLog[2, 1 - 2/(1 + I*a*x)]
```

Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4918

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/(d + e*x^2),
```

```
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^2}{x^4}\,dx = \int \left(3a^4c^3\tan^{-1}(ax)^2+\frac{c^3\tan^{-1}(ax)^2}{x^4}+\frac{3a^2c^3\tan^{-1}(ax)^2}{x^2}+a^6c^3x^2\tan^{-1}(ax)^2\right)dx$$

$$= c^3\int \frac{\tan^{-1}(ax)^2}{x^4}\,dx+\left(3a^2c^3\right)\int \frac{\tan^{-1}(ax)^2}{x^2}\,dx+\left(3a^4c^3\right)\int \tan^{-1}(ax)^2\,dx+\left(a^6c^3\right)\int$$

$$= -\frac{c^3\tan^{-1}(ax)^2}{3x^3}-\frac{3a^2c^3\tan^{-1}(ax)^2}{x}+3a^4c^3x\tan^{-1}(ax)^2+\frac{1}{3}a^6c^3x^3\tan^{-1}(ax)^2+\frac{1}{3}\left(2ac\right.$$

$$= -\frac{c^3\tan^{-1}(ax)^2}{3x^3}-\frac{3a^2c^3\tan^{-1}(ax)^2}{x}+3a^4c^3x\tan^{-1}(ax)^2+\frac{1}{3}a^6c^3x^3\tan^{-1}(ax)^2+\frac{1}{3}\left(2ac\right.$$

$$= -\frac{ac^3\tan^{-1}(ax)}{3x^2}-\frac{1}{3}a^5c^3x^2\tan^{-1}(ax)-\frac{c^3\tan^{-1}(ax)^2}{3x^3}-\frac{3a^2c^3\tan^{-1}(ax)^2}{x}+3a^4c^3x\tan$$

$$= -\frac{a^2c^3}{3x}+\frac{1}{3}a^4c^3x-\frac{ac^3\tan^{-1}(ax)}{3x^2}-\frac{1}{3}a^5c^3x^2\tan^{-1}(ax)-\frac{c^3\tan^{-1}(ax)^2}{3x^3}-\frac{3a^2c^3\tan^{-1}(a}{x}$$

$$= -\frac{a^2c^3}{3x}+\frac{1}{3}a^4c^3x-\frac{2}{3}a^3c^3\tan^{-1}(ax)-\frac{ac^3\tan^{-1}(ax)}{3x^2}-\frac{1}{3}a^5c^3x^2\tan^{-1}(ax)-\frac{c^3\tan^{-1}(ax}{3x^3}$$

$$= -\frac{a^2c^3}{3x}+\frac{1}{3}a^4c^3x-\frac{2}{3}a^3c^3\tan^{-1}(ax)-\frac{ac^3\tan^{-1}(ax)}{3x^2}-\frac{1}{3}a^5c^3x^2\tan^{-1}(ax)-\frac{c^3\tan^{-1}(ax}{3x^3}$$

Mathematica [A] time = 0.573921, size = 221, normalized size = 0.88

$$c^3\left(-8ia^3x^3\text{PolyLog}\left(2,-e^{2i\tan^{-1}(ax)}\right)-8ia^3x^3\text{PolyLog}\left(2,e^{2i\tan^{-1}(ax)}\right)+a^4x^4-a^2x^2+a^6x^6\tan^{-1}(ax)^2-a^5x^5\tan^{-1}(ax\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((c + a^2*c*x^2)^3*ArcTan[a*x]^2)/x^4,x]
```

```
[Out] (c^3*(-(a^2*x^2) + a^4*x^4 - a*x*ArcTan[a*x] - 2*a^3*x^3*ArcTan[a*x] - a^5*
x^5*ArcTan[a*x] - ArcTan[a*x]^2 - 9*a^2*x^2*ArcTan[a*x]^2 - (16*I)*a^3*x^3*
ArcTan[a*x]^2 + 9*a^4*x^4*ArcTan[a*x]^2 + a^6*x^6*ArcTan[a*x]^2 + 16*a^3*x^
3*ArcTan[a*x]*Log[1 - E^((2*I)*ArcTan[a*x])] + 16*a^3*x^3*ArcTan[a*x]*Log[1
 + E^((2*I)*ArcTan[a*x])] - (8*I)*a^3*x^3*PolyLog[2, -E^((2*I)*ArcTan[a*x])
] - (8*I)*a^3*x^3*PolyLog[2, E^((2*I)*ArcTan[a*x])]))/(3*x^3)
```

Maple [A] time = 0.106, size = 417, normalized size = 1.7

$$\frac{a^6c^3x^3\left(\arctan\left(ax\right)\right)^2}{3}+3\,a^4c^3x\left(\arctan\left(ax\right)\right)^2-3\,\frac{a^2c^3\left(\arctan\left(ax\right)\right)^2}{x}-\frac{c^3\left(\arctan\left(ax\right)\right)^2}{3\,x^3}-\frac{a^5c^3x^2\arctan\left(ax\right)}{3}-\frac{16\,a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^3*arctan(a*x)^2/x^4,x)
```

```
[Out] 1/3*a^6*c^3*x^3*arctan(a*x)^2+3*a^4*c^3*x*arctan(a*x)^2-3*a^2*c^3*arctan(a*
x)^2/x-1/3*c^3*arctan(a*x)^2/x^3-1/3*a^5*c^3*x^2*arctan(a*x)-16/3*a^3*c^3*a
rctan(a*x)*ln(a^2*x^2+1)-1/3*a*c^3*arctan(a*x)/x^2+16/3*a^3*c^3*arctan(a*x)
*ln(a*x)+1/3*a^4*c^3*x-2/3*a^3*c^3*arctan(a*x)-1/3*a^2*c^3/x+8/3*I*a^3*c^3*
dilog(1+I*a*x)+8/3*I*a^3*c^3*ln(a*x-I)*ln(-1/2*I*(a*x+I))+8/3*I*a^3*c^3*dil
og(-1/2*I*(a*x+I))-8/3*I*a^3*c^3*ln(a*x)*ln(1-I*a*x)+8/3*I*a^3*c^3*ln(a^2*x
^2+1)*ln(a*x+I)-8/3*I*a^3*c^3*ln(a*x+I)*ln(1/2*I*(a*x-I))+8/3*I*a^3*c^3*ln(
a*x)*ln(1+I*a*x)+4/3*I*a^3*c^3*ln(a*x-I)^2-4/3*I*a^3*c^3*ln(a*x+I)^2-8/3*I*
a^3*c^3*dilog(1-I*a*x)-8/3*I*a^3*c^3*ln(a^2*x^2+1)*ln(a*x-I)-8/3*I*a^3*c^3*
dilog(1/2*I*(a*x-I))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^2/x^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^6c^3x^6 + 3\,a^4c^3x^4 + 3\,a^2c^3x^2 + c^3\right)\arctan\left(ax\right)^2}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^2/x^4,x, algorithm="fricas")
```

```
[Out] integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*arctan(a*x)^2/
x^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$c^3\left(\int 3a^4\,\mathrm{atan}^2\left(ax\right)dx + \int \frac{\mathrm{atan}^2\left(ax\right)}{x^4}\,dx + \int \frac{3a^2\,\mathrm{atan}^2\left(ax\right)}{x^2}\,dx + \int a^6x^2\,\mathrm{atan}^2\left(ax\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**3*atan(a*x)**2/x**4,x)
```

```
[Out] c**3*(Integral(3*a**4*atan(a*x)**2, x) + Integral(atan(a*x)**2/x**4, x) + I
ntegral(3*a**2*atan(a*x)**2/x**2, x) + Integral(a**6*x**2*atan(a*x)**2, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2 + c\right)^3\arctan\left(ax\right)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^2/x^4,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^3*arctan(a*x)^2/x^4, x)
```

3.282 $\int \frac{x^4 \tan^{-1}(ax)^2}{c+a^2cx^2} dx$

Optimal. Leaf size=166

$$-\frac{4i\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{3a^5c}+\frac{x^3\tan^{-1}(ax)^2}{3a^2c}-\frac{x^2\tan^{-1}(ax)}{3a^3c}+\frac{x}{3a^4c}-\frac{x\tan^{-1}(ax)^2}{a^4c}+\frac{\tan^{-1}(ax)^3}{3a^5c}-\frac{4i\tan^{-1}(ax)^2}{3a^5c}-\frac{\text{ta}}{}$$

```
[Out] x/(3*a^4*c) - ArcTan[a*x]/(3*a^5*c) - (x^2*ArcTan[a*x])/(3*a^3*c) - (((4*I)
/3)*ArcTan[a*x]^2)/(a^5*c) - (x*ArcTan[a*x]^2)/(a^4*c) + (x^3*ArcTan[a*x]^2
)/(3*a^2*c) + ArcTan[a*x]^3/(3*a^5*c) - (8*ArcTan[a*x]*Log[2/(1 + I*a*x)])/
(3*a^5*c) - (((4*I)/3)*PolyLog[2, 1 - 2/(1 + I*a*x)])/(a^5*c)
```

Rubi [A] time = 0.387175, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {4916, 4852, 321, 203, 4920, 4854, 2402, 2315, 4846, 4884}

$$-\frac{4i\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{3a^5c}+\frac{x^3\tan^{-1}(ax)^2}{3a^2c}-\frac{x^2\tan^{-1}(ax)}{3a^3c}+\frac{x}{3a^4c}-\frac{x\tan^{-1}(ax)^2}{a^4c}+\frac{\tan^{-1}(ax)^3}{3a^5c}-\frac{4i\tan^{-1}(ax)^2}{3a^5c}-\frac{\text{ta}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*ArcTan[a*x]^2)/(c + a^2*c*x^2),x]
```


```
[Out] x/(3*a^4*c) - ArcTan[a*x]/(3*a^5*c) - (x^2*ArcTan[a*x])/(3*a^3*c) - (((4*I)
/3)*ArcTan[a*x]^2)/(a^5*c) - (x*ArcTan[a*x]^2)/(a^4*c) + (x^3*ArcTan[a*x]^2
)/(3*a^2*c) + ArcTan[a*x]^3/(3*a^5*c) - (8*ArcTan[a*x]*Log[2/(1 + I*a*x)])/
(3*a^5*c) - (((4*I)/3)*PolyLog[2, 1 - 2/(1 + I*a*x)])/(a^5*c)
```


Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
```

```
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{x^4 \tan^{-1}(ax)^2}{c+a^2cx^2}\,dx = -\frac{\int \frac{x^2\tan^{-1}(ax)^2}{c+a^2cx^2}\,dx}{a^2} + \frac{\int x^2\tan^{-1}(ax)^2\,dx}{a^2c}$$

$$= \frac{x^3\tan^{-1}(ax)^2}{3a^2c} + \frac{\int \frac{\tan^{-1}(ax)^2}{c+a^2cx^2}\,dx}{a^4} - \frac{\int \tan^{-1}(ax)^2\,dx}{a^4c} - \frac{2\int \frac{x^3\tan^{-1}(ax)}{1+a^2x^2}\,dx}{3ac}$$

$$= -\frac{x\tan^{-1}(ax)^2}{a^4c} + \frac{x^3\tan^{-1}(ax)^2}{3a^2c} + \frac{\tan^{-1}(ax)^3}{3a^5c} - \frac{2\int x\tan^{-1}(ax)\,dx}{3a^3c} + \frac{2\int \frac{x\tan^{-1}(ax)}{1+a^2x^2}\,dx}{3a^3c} + \ldots$$

$$= -\frac{x^2\tan^{-1}(ax)}{3a^3c} - \frac{4i\tan^{-1}(ax)^2}{3a^5c} - \frac{x\tan^{-1}(ax)^2}{a^4c} + \frac{x^3\tan^{-1}(ax)^2}{3a^2c} + \frac{\tan^{-1}(ax)^3}{3a^5c} - \frac{2\int \frac{\tan^{-1}(ax)}{i-ax}\,dx}{3a^4c} \ldots$$

$$= \frac{x}{3a^4c} - \frac{x^2\tan^{-1}(ax)}{3a^3c} - \frac{4i\tan^{-1}(ax)^2}{3a^5c} - \frac{x\tan^{-1}(ax)^2}{a^4c} + \frac{x^3\tan^{-1}(ax)^2}{3a^2c} + \frac{\tan^{-1}(ax)^3}{3a^5c} - \frac{8\tan^{-1}(\ldots}{\ldots}$$

$$= \frac{x}{3a^4c} - \frac{\tan^{-1}(ax)}{3a^5c} - \frac{x^2\tan^{-1}(ax)}{3a^3c} - \frac{4i\tan^{-1}(ax)^2}{3a^5c} - \frac{x\tan^{-1}(ax)^2}{a^4c} + \frac{x^3\tan^{-1}(ax)^2}{3a^2c} + \frac{\tan^{-1}(ax)\ldots}{3a^5c}$$

$$= \frac{x}{3a^4c} - \frac{\tan^{-1}(ax)}{3a^5c} - \frac{x^2\tan^{-1}(ax)}{3a^3c} - \frac{4i\tan^{-1}(ax)^2}{3a^5c} - \frac{x\tan^{-1}(ax)^2}{a^4c} + \frac{x^3\tan^{-1}(ax)^2}{3a^2c} + \frac{\tan^{-1}(ax)\ldots}{3a^5c}$$

Mathematica [A] time = 0.280561, size = 90, normalized size = 0.54

$$\frac{4i\text{PolyLog}\left(2,-e^{2i\tan^{-1}(ax)}\right) + \left(a^3x^3-3ax+4i\right)\tan^{-1}(ax)^2 - \tan^{-1}(ax)\left(a^2x^2+8\log\left(1+e^{2i\tan^{-1}(ax)}\right)+1\right) + ax + \tan\ldots}{3a^5c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*ArcTan[a*x]^2)/(c + a^2*c*x^2),x]
```

```
[Out] (a*x + (4*I - 3*a*x + a^3*x^3)*ArcTan[a*x]^2 + ArcTan[a*x]^3 - ArcTan[a*x]*
(1 + a^2*x^2 + 8*Log[1 + E^((2*I)*ArcTan[a*x])]) + (4*I)*PolyLog[2, -E^((2*
I)*ArcTan[a*x])])/(3*a^5*c)
```

Maple [A] time = 0.088, size = 284, normalized size = 1.7

$$\frac{x^3 (\arctan(ax))^2}{3a^2c} - \frac{x(\arctan(ax))^2}{a^4c} + \frac{(\arctan(ax))^3}{3a^5c} - \frac{x^2\arctan(ax)}{3a^3c} + \frac{4\arctan(ax)\ln(a^2x^2+1)}{3a^5c} + \frac{x}{3a^4c} - \frac{\arctan}{3a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arctan(a*x)^2/(a^2*c*x^2+c),x)
```

```
[Out] 1/3*x^3*arctan(a*x)^2/a^2/c-x*arctan(a*x)^2/a^4/c+1/3*arctan(a*x)^3/a^5/c-1
/3*x^2*arctan(a*x)/a^3/c+4/3/a^5/c*arctan(a*x)*ln(a^2*x^2+1)+1/3*x/a^4/c-1/
3*arctan(a*x)/a^5/c-1/3*I/a^5/c*ln(a*x-I)^2-2/3*I/a^5/c*ln(a*x+I)*ln(a^2*x^
2+1)+2/3*I/a^5/c*ln(1/2*I*(a*x-I))*ln(a*x+I)+2/3*I/a^5/c*dilog(1/2*I*(a*x-I
))-2/3*I/a^5/c*ln(a*x-I)*ln(-1/2*I*(a*x+I))+1/3*I/a^5/c*ln(a*x+I)^2-2/3*I/a
^5/c*dilog(-1/2*I*(a*x+I))+2/3*I/a^5/c*ln(a*x-I)*ln(a^2*x^2+1)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctan(a*x)^2/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^4\arctan(ax)^2}{a^2cx^2+c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctan(a*x)^2/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(x^4*arctan(a*x)^2/(a^2*c*x^2 + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^4 \operatorname{atan}^2(ax)}{a^2x^2+1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*atan(a*x)**2/(a**2*c*x**2+c),x)
```

```
[Out] Integral(x**4*atan(a*x)**2/(a**2*x**2 + 1), x)/c
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4 \arctan(ax)^2}{a^2cx^2 + c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctan(a*x)^2/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(x^4*arctan(a*x)^2/(a^2*c*x^2 + c), x)
```

3.283 $\int \frac{x^3 \tan^{-1}(ax)^2}{c+a^2cx^2} dx$

Optimal. Leaf size=169

$$\frac{\text{PolyLog}\left(3,1-\frac{2}{1+iax}\right)}{2a^4c}+\frac{i\tan^{-1}(ax)\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{a^4c}+\frac{\log\left(a^2x^2+1\right)}{2a^4c}+\frac{x^2\tan^{-1}(ax)^2}{2a^2c}+\frac{i\tan^{-1}(ax)^3}{3a^4c}+\frac{\tan^{-1}(a}{2a^4c}$$

```
[Out] -((x*ArcTan[a*x])/(a^3*c)) + ArcTan[a*x]^2/(2*a^4*c) + (x^2*ArcTan[a*x]^2)/
(2*a^2*c) + ((I/3)*ArcTan[a*x]^3)/(a^4*c) + (ArcTan[a*x]^2*Log[2/(1 + I*a*x
)])/(a^4*c) + Log[1 + a^2*x^2]/(2*a^4*c) + (I*ArcTan[a*x]*PolyLog[2, 1 - 2/
(1 + I*a*x)])/(a^4*c) + PolyLog[3, 1 - 2/(1 + I*a*x)]/(2*a^4*c)
```

Rubi [A] time = 0.293706, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {4916, 4852, 4846, 260, 4884, 4920, 4854, 4994, 6610}

$$\frac{\text{PolyLog}\left(3,1-\frac{2}{1+iax}\right)}{2a^4c}+\frac{i\tan^{-1}(ax)\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{a^4c}+\frac{\log\left(a^2x^2+1\right)}{2a^4c}+\frac{x^2\tan^{-1}(ax)^2}{2a^2c}+\frac{i\tan^{-1}(ax)^3}{3a^4c}+\frac{\tan^{-1}(a}{2a^4c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcTan[a*x]^2)/(c + a^2*c*x^2),x]
```



```
[Out] -((x*ArcTan[a*x])/(a^3*c)) + ArcTan[a*x]^2/(2*a^4*c) + (x^2*ArcTan[a*x]^2)/
(2*a^2*c) + ((I/3)*ArcTan[a*x]^3)/(a^4*c) + (ArcTan[a*x]^2*Log[2/(1 + I*a*x
)])/(a^4*c) + Log[1 + a^2*x^2]/(2*a^4*c) + (I*ArcTan[a*x]*PolyLog[2, 1 - 2/
(1 + I*a*x)])/(a^4*c) + PolyLog[3, 1 - 2/(1 + I*a*x)]/(2*a^4*c)
```


Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
```

```
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{x^3\tan^{-1}(ax)^2}{c+a^2cx^2}\,dx = -\frac{\int \frac{x\tan^{-1}(ax)^2}{c+a^2cx^2}\,dx}{a^2}+\frac{\int x\tan^{-1}(ax)^2\,dx}{a^2c}$$
$$= \frac{x^2\tan^{-1}(ax)^2}{2a^2c}+\frac{i\tan^{-1}(ax)^3}{3a^4c}+\frac{\int \frac{\tan^{-1}(ax)^2}{i-ax}\,dx}{a^3c}-\frac{\int \frac{x^2\tan^{-1}(ax)}{1+a^2x^2}\,dx}{ac}$$
$$= \frac{x^2\tan^{-1}(ax)^2}{2a^2c}+\frac{i\tan^{-1}(ax)^3}{3a^4c}+\frac{\tan^{-1}(ax)^2\log\left(\frac{2}{1+iax}\right)}{a^4c}-\frac{\int \tan^{-1}(ax)\,dx}{a^3c}+\frac{\int \frac{\tan^{-1}(ax)}{1+a^2x^2}\,dx}{a^3c}-\frac{2\int}{}$$
$$= -\frac{x\tan^{-1}(ax)}{a^3c}+\frac{\tan^{-1}(ax)^2}{2a^4c}+\frac{x^2\tan^{-1}(ax)^2}{2a^2c}+\frac{i\tan^{-1}(ax)^3}{3a^4c}+\frac{\tan^{-1}(ax)^2\log\left(\frac{2}{1+iax}\right)}{a^4c}+\frac{i\tan^{-1}(a}{}$$
$$= -\frac{x\tan^{-1}(ax)}{a^3c}+\frac{\tan^{-1}(ax)^2}{2a^4c}+\frac{x^2\tan^{-1}(ax)^2}{2a^2c}+\frac{i\tan^{-1}(ax)^3}{3a^4c}+\frac{\tan^{-1}(ax)^2\log\left(\frac{2}{1+iax}\right)}{a^4c}+\frac{\log(1+}{2a^4c}$$

Mathematica [A] time = 0.112094, size = 123, normalized size = 0.73

$$\frac{-i\tan^{-1}(ax)\text{PolyLog}\left(2,-e^{2i\tan^{-1}(ax)}\right)+\frac{1}{2}\text{PolyLog}\left(3,-e^{2i\tan^{-1}(ax)}\right)-\log\left(\frac{1}{\sqrt{a^2x^2+1}}\right)+\frac{1}{2}\left(a^2x^2+1\right)\tan^{-1}(ax)^2-\frac{1}{3}i\tan}{a^4c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*ArcTan[a*x]^2)/(c + a^2*c*x^2),x]
```

```
[Out] (-(a*x*ArcTan[a*x]) + ((1 + a^2*x^2)*ArcTan[a*x]^2)/2 - (I/3)*ArcTan[a*x]^3
 + ArcTan[a*x]^2*Log[1 + E^((2*I)*ArcTan[a*x])] - Log[1/Sqrt[1 + a^2*x^2]]
- I*ArcTan[a*x]*PolyLog[2, -E^((2*I)*ArcTan[a*x])] + PolyLog[3, -E^((2*I)*A
rcTan[a*x])]/2)/(a^4*c)
```

Maple [C] time = 0.727, size = 1695, normalized size = 10.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctan(a*x)^2/(a^2*c*x^2+c),x)
```

```
[Out] 1/a^4/c*arctan(a*x)^2*ln((1+I*a*x)/(a^2*x^2+1)^(1/2))-1/a^4/c*ln((1+I*a*x)^
2/(a^2*x^2+1)+1)+1/2/a^4/c*polylog(3,-(1+I*a*x)^2/(a^2*x^2+1))-1/4*I/a^4/c*
arctan(a*x)^2*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(
a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)+1
/2*arctan(a*x)^2/a^4/c+1/8/a^3/c*arctan(a*x)^2*Pi*csgn(I*((1+I*a*x)^2/(a^2*
x^2+1)+1))^2*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*x-1/4/a^3/c*arctan(a*x)^
2*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)
^2)^2*x-x*arctan(a*x)/a^3/c+1/2*x^2*arctan(a*x)^2/a^2/c-1/8/a^3/c*arctan(a*
x)^2*Pi*csgn(I*(1+I*a*x)^4/(a^2*x^2+1)^2+2*I*(1+I*a*x)^2/(a^2*x^2+1)+I)^3*x
+1/8/a^3/c*arctan(a*x)^2*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3*x+1/8*I
/a^4/c*arctan(a*x)^2*Pi*csgn(I*(1+I*a*x)^4/(a^2*x^2+1)^2+2*I*(1+I*a*x)^2/(a
^2*x^2+1)+I)^3+1/8*I/a^4/c*arctan(a*x)^2*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)
+1)^2)^3-1/4*I/a^4/c*arctan(a*x)^2*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^3-1/4
*I/a^4/c*arctan(a*x)^2*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*
x^2+1)+1)^2)^3-1/4*I/a^4/c*arctan(a*x)^2*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))
*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))^2+1/4*I/a^4/c*arctan(a*x)^2*Pi*csgn(I*
(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x
^2+1)+1)^2)^2+1/8*I/a^4/c*arctan(a*x)^2*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+
1))^2*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)-1/4*I/a^4/c*arctan(a*x)^2*Pi*cs
gn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2-I
/a^4/c*arctan(a*x)*polylog(2,-(1+I*a*x)^2/(a^2*x^2+1))-1/2/a^4/c*arctan(a*x
)^2*ln(a^2*x^2+1)+1/a^4/c*arctan(a*x)^2*ln(2)+I/a^4/c*arctan(a*x)-1/3*I/a^4
/c*arctan(a*x)^3+1/4/a^3/c*arctan(a*x)^2*Pi*csgn(I*(1+I*a*x)^4/(a^2*x^2+1)^
2+2*I*(1+I*a*x)^2/(a^2*x^2+1)+I)^2*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)+I)*x-1/4*
I/a^4/c*arctan(a*x)^2*Pi*csgn(I*(1+I*a*x)^4/(a^2*x^2+1)^2+2*I*(1+I*a*x)^2/(
a^2*x^2+1)+I)^2*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)+I)+1/8*I/a^4/c*arctan(a*x)^2
*Pi*csgn(I*(1+I*a*x)^4/(a^2*x^2+1)^2+2*I*(1+I*a*x)^2/(a^2*x^2+1)+I)*csgn(I*
(1+I*a*x)^2/(a^2*x^2+1)+I)^2-1/8/a^3/c*arctan(a*x)^2*Pi*csgn(I*(1+I*a*x)^4/
(a^2*x^2+1)^2+2*I*(1+I*a*x)^2/(a^2*x^2+1)+I)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)
+I)^2*x+1/4*I/a^4/c*arctan(a*x)^2*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*
csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2+1/2*I/a^4/c
*arctan(a*x)^2*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^2*csgn(I*(1+I*a*x)/(a^2*x
^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \arctan(ax)^2}{a^2cx^2 + c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^2/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] integrate(x^3*arctan(a*x)^2/(a^2*c*x^2 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^3 \arctan{(ax)}^2}{a^2cx^2 + c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^2/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(x^3*arctan(a*x)^2/(a^2*c*x^2 + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^3 \operatorname{atan}^2{(ax)}}{a^2x^2+1}\, dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atan(a*x)**2/(a**2*c*x**2+c),x)
```

```
[Out] Integral(x**3*atan(a*x)**2/(a**2*x**2 + 1), x)/c
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \arctan{(ax)}^2}{a^2cx^2 + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^2/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(x^3*arctan(a*x)^2/(a^2*c*x^2 + c), x)
```

3.284 $\int \frac{x^2 \tan^{-1}(ax)^2}{c+a^2cx^2} dx$

Optimal. Leaf size=98

$$\frac{i\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{a^3c}-\frac{\tan^{-1}(ax)^3}{3a^3c}+\frac{x\tan^{-1}(ax)^2}{a^2c}+\frac{i\tan^{-1}(ax)^2}{a^3c}+\frac{2\log\left(\frac{2}{1+iax}\right)\tan^{-1}(ax)}{a^3c}$$

```
[Out] (I*ArcTan[a*x]^2)/(a^3*c) + (x*ArcTan[a*x]^2)/(a^2*c) - ArcTan[a*x]^3/(3*a^
3*c) + (2*ArcTan[a*x]*Log[2/(1 + I*a*x)])/(a^3*c) + (I*PolyLog[2, 1 - 2/(1
+ I*a*x)])/(a^3*c)
```

Rubi [A] time = 0.166925, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {4916, 4846, 4920, 4854, 2402, 2315, 4884}

$$\frac{i\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{a^3c}-\frac{\tan^{-1}(ax)^3}{3a^3c}+\frac{x\tan^{-1}(ax)^2}{a^2c}+\frac{i\tan^{-1}(ax)^2}{a^3c}+\frac{2\log\left(\frac{2}{1+iax}\right)\tan^{-1}(ax)}{a^3c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcTan[a*x]^2)/(c + a^2*c*x^2),x]
```


```
[Out] (I*ArcTan[a*x]^2)/(a^3*c) + (x*ArcTan[a*x]^2)/(a^2*c) - ArcTan[a*x]^3/(3*a^
3*c) + (2*ArcTan[a*x]*Log[2/(1 + I*a*x)])/(a^3*c) + (I*PolyLog[2, 1 - 2/(1
+ I*a*x)])/(a^3*c)
```


Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{x^2 \tan^{-1}(ax)^2}{c+a^2cx^2}\,dx = -\frac{\int \frac{\tan^{-1}(ax)^2}{c+a^2cx^2}\,dx}{a^2} + \frac{\int \tan^{-1}(ax)^2\,dx}{a^2c}$$
$$= \frac{x\tan^{-1}(ax)^2}{a^2c} - \frac{\tan^{-1}(ax)^3}{3a^3c} - \frac{2\int \frac{x\tan^{-1}(ax)}{1+a^2x^2}\,dx}{ac}$$
$$= \frac{i\tan^{-1}(ax)^2}{a^3c} + \frac{x\tan^{-1}(ax)^2}{a^2c} - \frac{\tan^{-1}(ax)^3}{3a^3c} + \frac{2\int \frac{\tan^{-1}(ax)}{i-ax}\,dx}{a^2c}$$
$$= \frac{i\tan^{-1}(ax)^2}{a^3c} + \frac{x\tan^{-1}(ax)^2}{a^2c} - \frac{\tan^{-1}(ax)^3}{3a^3c} + \frac{2\tan^{-1}(ax)\log\left(\frac{2}{1+iax}\right)}{a^3c} - \frac{2\int \frac{\log\left(\frac{2}{1+iax}\right)}{1+a^2x^2}\,dx}{a^2c}$$
$$= \frac{i\tan^{-1}(ax)^2}{a^3c} + \frac{x\tan^{-1}(ax)^2}{a^2c} - \frac{\tan^{-1}(ax)^3}{3a^3c} + \frac{2\tan^{-1}(ax)\log\left(\frac{2}{1+iax}\right)}{a^3c} + \frac{(2i)\,\text{Subst}\left(\int \frac{\log(2x)}{1-2x}\,dx,\right.}{a^3c}$$
$$= \frac{i\tan^{-1}(ax)^2}{a^3c} + \frac{x\tan^{-1}(ax)^2}{a^2c} - \frac{\tan^{-1}(ax)^3}{3a^3c} + \frac{2\tan^{-1}(ax)\log\left(\frac{2}{1+iax}\right)}{a^3c} + \frac{i\text{Li}_2\left(1-\frac{2}{1+iax}\right)}{a^3c}$$

Mathematica [A] time = 0.172592, size = 69, normalized size = 0.7

$$\frac{-i\text{PolyLog}\left(2,-e^{2i\tan^{-1}(ax)}\right) - \frac{1}{3}\tan^{-1}(ax)\left(\tan^{-1}(ax)^2 + (-3ax+3i)\tan^{-1}(ax) - 6\log\left(1+e^{2i\tan^{-1}(ax)}\right)\right)}{a^3c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*ArcTan[a*x]^2)/(c + a^2*c*x^2),x]
```

```
[Out] (-(ArcTan[a*x]*((3*I - 3*a*x)*ArcTan[a*x] + ArcTan[a*x]^2 - 6*Log[1 + E^((2
*I)*ArcTan[a*x])]))/3 - I*PolyLog[2, -E^((2*I)*ArcTan[a*x])])/(a^3*c)
```

Maple [B] time = 0.093, size = 230, normalized size = 2.4

$$\frac{x(\arctan(ax))^2}{a^2c} - \frac{(\arctan(ax))^3}{3a^3c} - \frac{\arctan(ax)\ln\left(a^2x^2+1\right)}{a^3c} - \frac{\frac{i}{2}\ln(ax-i)\ln\left(a^2x^2+1\right)}{a^3c} + \frac{\frac{i}{4}(\ln(ax-i))^2}{a^3c} + \frac{\frac{i}{2}\ln(ax}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(a*x)^2/(a^2*c*x^2+c),x)
```

```
[Out] x*arctan(a*x)^2/a^2/c-1/3*arctan(a*x)^3/a^3/c-1/a^3/c*arctan(a*x)*ln(a^2*x^
2+1)-1/2*I/a^3/c*ln(a*x-I)*ln(a^2*x^2+1)+1/4*I/a^3/c*ln(a*x-I)^2+1/2*I/a^3/
c*ln(a*x-I)*ln(-1/2*I*(a*x+I))+1/2*I/a^3/c*dilog(-1/2*I*(a*x+I))+1/2*I/a^3/
c*ln(a*x+I)*ln(a^2*x^2+1)-1/4*I/a^3/c*ln(a*x+I)^2-1/2*I/a^3/c*ln(1/2*I*(a*x
-I))*ln(a*x+I)-1/2*I/a^3/c*dilog(1/2*I*(a*x-I))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^2/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^2 \arctan(ax)^2}{a^2cx^2 + c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^2/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(x^2*arctan(a*x)^2/(a^2*c*x^2 + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^2 \operatorname{atan}^2(ax)}{a^2x^2+1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(a*x)**2/(a**2*c*x**2+c),x)
```

```
[Out] Integral(x**2*atan(a*x)**2/(a**2*x**2 + 1), x)/c
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \arctan{(ax)}^2}{a^2cx^2 + c} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^2/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctan(a*x)^2/(a^2*c*x^2 + c), x)
```

3.285 $\int \frac{x\tan^{-1}(ax)^2}{c+a^2cx^2}\,dx$

Optimal. Leaf size=102

$$-\frac{\text{PolyLog}\left(3,1-\frac{2}{1+iax}\right)}{2a^2c}-\frac{i\tan^{-1}(ax)\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{a^2c}-\frac{i\tan^{-1}(ax)^3}{3a^2c}-\frac{\log\left(\frac{2}{1+iax}\right)\tan^{-1}(ax)^2}{a^2c}$$

```
[Out] ((-I/3)*ArcTan[a*x]^3)/(a^2*c) - (ArcTan[a*x]^2*Log[2/(1 + I*a*x)])/(a^2*c)
 - (I*ArcTan[a*x]*PolyLog[2, 1 - 2/(1 + I*a*x)])/(a^2*c) - PolyLog[3, 1 - 2
/(1 + I*a*x)]/(2*a^2*c)
```

Rubi [A] time = 0.146067, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4920, 4854, 4884, 4994, 6610}

$$-\frac{\text{PolyLog}\left(3,1-\frac{2}{1+iax}\right)}{2a^2c}-\frac{i\tan^{-1}(ax)\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{a^2c}-\frac{i\tan^{-1}(ax)^3}{3a^2c}-\frac{\log\left(\frac{2}{1+iax}\right)\tan^{-1}(ax)^2}{a^2c}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTan[a*x]^2)/(c + a^2*c*x^2),x]
```


```
[Out] ((-I/3)*ArcTan[a*x]^3)/(a^2*c) - (ArcTan[a*x]^2*Log[2/(1 + I*a*x)])/(a^2*c)
 - (I*ArcTan[a*x]*PolyLog[2, 1 - 2/(1 + I*a*x)])/(a^2*c) - PolyLog[3, 1 - 2
/(1 + I*a*x)]/(2*a^2*c)
```


Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x\tan^{-1}(ax)^2}{c+a^2cx^2}\,dx &= -\frac{i\tan^{-1}(ax)^3}{3a^2c}-\frac{\int \frac{\tan^{-1}(ax)^2}{i-ax}\,dx}{ac}\\
&= -\frac{i\tan^{-1}(ax)^3}{3a^2c}-\frac{\tan^{-1}(ax)^2\log\left(\frac{2}{1+iax}\right)}{a^2c}+\frac{2\int \frac{\tan^{-1}(ax)\log\left(\frac{2}{1+iax}\right)}{1+a^2x^2}\,dx}{ac}\\
&= -\frac{i\tan^{-1}(ax)^3}{3a^2c}-\frac{\tan^{-1}(ax)^2\log\left(\frac{2}{1+iax}\right)}{a^2c}-\frac{i\tan^{-1}(ax)\text{Li}_2\left(1-\frac{2}{1+iax}\right)}{a^2c}+\frac{i\int \frac{\text{Li}_2\left(1-\frac{2}{1+iax}\right)}{1+a^2x^2}\,dx}{ac}\\
&= -\frac{i\tan^{-1}(ax)^3}{3a^2c}-\frac{\tan^{-1}(ax)^2\log\left(\frac{2}{1+iax}\right)}{a^2c}-\frac{i\tan^{-1}(ax)\text{Li}_2\left(1-\frac{2}{1+iax}\right)}{a^2c}-\frac{\text{Li}_3\left(1-\frac{2}{1+iax}\right)}{2a^2c}
\end{aligned}
$$

Mathematica [A] time = 0.0098974, size = 110, normalized size = 1.08

$$
-\frac{\text{PolyLog}\left(3,\frac{ax+i}{ax-i}\right)}{2a^2c}-\frac{i\tan^{-1}(ax)\text{PolyLog}\left(2,\frac{ax+i}{ax-i}\right)}{a^2c}-\frac{i\tan^{-1}(ax)^3}{3a^2c}-\frac{\log\left(\frac{2i}{-ax+i}\right)\tan^{-1}(ax)^2}{a^2c}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcTan[a*x]^2)/(c + a^2*c*x^2),x]
```

```
[Out] ((-I/3)*ArcTan[a*x]^3)/(a^2*c) - (ArcTan[a*x]^2*Log[(2*I)/(I - a*x)])/(a^2*
c) - (I*ArcTan[a*x]*PolyLog[2, (I + a*x)/(-I + a*x)])/(a^2*c) - PolyLog[3,
(I + a*x)/(-I + a*x)]/(2*a^2*c)
```

Maple [C] time = 0.346, size = 897, normalized size = 8.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(a*x)^2/(a^2*c*x^2+c),x)
```

```
[Out] 1/2/a^2/c*arctan(a*x)^2*ln(a^2*x^2+1)-1/a^2/c*arctan(a*x)^2*ln((1+I*a*x)/(a
^2*x^2+1)^(1/2))+1/3*I/a^2/c*arctan(a*x)^3+1/2*I/a^2/c*arctan(a*x)^2*Pi*csg
n(I*((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2+1/
4*I/a^2/c*arctan(a*x)^2*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^3+1/4*I/a^2/c*ar
ctan(a*x)^2*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^
2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)+1/4
*I/a^2/c*arctan(a*x)^2*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*
x^2+1)+1)^2)^3-1/4*I/a^2/c*arctan(a*x)^2*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)
+1))^2*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)+1/4*I/a^2/c*arctan(a*x)^2*Pi*c
sgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))^2-1/4*I/
a^2/c*arctan(a*x)^2*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(
a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2-1/4*I/a^2/c*arctan(a*x)^2*Pi*cs
gn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*
x)^2/(a^2*x^2+1)+1)^2)^2-1/2*I/a^2/c*arctan(a*x)^2*Pi*csgn(I*(1+I*a*x)^2/(a
^2*x^2+1))^2*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-1/a^2/c*arctan(a*x)^2*ln(2
)-1/4*I/a^2/c*arctan(a*x)^2*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3-1/2/
a^2/c*polylog(3,-(1+I*a*x)^2/(a^2*x^2+1))+I/a^2/c*arctan(a*x)*polylog(2,-(1
+I*a*x)^2/(a^2*x^2+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \arctan (ax)^2}{a^2cx^2 + c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^2/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] integrate(x*arctan(a*x)^2/(a^2*c*x^2 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x\arctan\left(ax\right)^{2}}{a^{2}cx^{2}+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^2/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(x*arctan(a*x)^2/(a^2*c*x^2 + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x\operatorname{atan}^{2}\left(ax\right)}{a^{2}x^{2}+1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(a*x)**2/(a**2*c*x**2+c),x)
```

```
[Out] Integral(x*atan(a*x)**2/(a**2*x**2 + 1), x)/c
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\arctan\left(ax\right)^{2}}{a^{2}cx^{2}+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^2/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(x*arctan(a*x)^2/(a^2*c*x^2 + c), x)
```

3.286 $\int \frac{\tan^{-1}(ax)^2}{c+a^2cx^2}\,dx$

Optimal. Leaf size=16

$$\frac{\tan^{-1}(ax)^3}{3ac}$$

```
[Out] ArcTan[a*x]^3/(3*a*c)
```

Rubi [A] time = 0.0235709, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {4884}

$$\frac{\tan^{-1}(ax)^3}{3ac}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^2/(c + a^2*c*x^2),x]
```

```
[Out] ArcTan[a*x]^3/(3*a*c)
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^2}{c+a^2cx^2}\,dx = \frac{\tan^{-1}(ax)^3}{3ac}$$

Mathematica [A] time = 0.0033675, size = 16, normalized size = 1.

$$\frac{\tan^{-1}(ax)^3}{3ac}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]^2/(c + a^2*c*x^2),x]
```

```
[Out] ArcTan[a*x]^3/(3*a*c)
```

Maple [A] time = 0.024, size = 15, normalized size = 0.9

$$\frac{(\arctan(ax))^3}{3\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^2/(a^2*c*x^2+c),x)
```

```
[Out] 1/3*arctan(a*x)^3/a/c
```

Maxima [A] time = 1.52937, size = 19, normalized size = 1.19

$$\frac{\arctan(ax)^3}{3\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/3*arctan(a*x)^3/(a*c)
```

Fricas [A] time = 2.12695, size = 34, normalized size = 2.12

$$\frac{\arctan(ax)^3}{3\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/3*arctan(a*x)^3/(a*c)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\operatorname{atan}^2(ax)}{a^2x^2+1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**2/(a**2*c*x**2+c),x)
```

```
[Out] Integral(atan(a*x)**2/(a**2*x**2 + 1), x)/c
```

Giac [A] time = 1.14008, size = 19, normalized size = 1.19

$$\frac{\arctan(ax)^3}{3\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] 1/3*arctan(a*x)^3/(a*c)
```

3.287 $\int \frac{\tan^{-1}(ax)^2}{x(c+a^2cx^2)}\,dx$

Optimal. Leaf size=91

$$\frac{\text{PolyLog}\left(3,-1+\frac{2}{1-iax}\right)}{2c}-\frac{i\tan^{-1}(ax)\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{c}-\frac{i\tan^{-1}(ax)^3}{3c}+\frac{\log\left(2-\frac{2}{1-iax}\right)\tan^{-1}(ax)^2}{c}$$

```
[Out] ((-I/3)*ArcTan[a*x]^3)/c + (ArcTan[a*x]^2*Log[2 - 2/(1 - I*a*x)])/c - (I*Ar
cTan[a*x]*PolyLog[2, -1 + 2/(1 - I*a*x)])/c + PolyLog[3, -1 + 2/(1 - I*a*x)
]/(2*c)
```

Rubi [A] time = 0.179331, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {4924, 4868, 4884, 4992, 6610}

$$\frac{\text{PolyLog}\left(3,-1+\frac{2}{1-iax}\right)}{2c}-\frac{i\tan^{-1}(ax)\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{c}-\frac{i\tan^{-1}(ax)^3}{3c}+\frac{\log\left(2-\frac{2}{1-iax}\right)\tan^{-1}(ax)^2}{c}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^2/(x*(c + a^2*c*x^2)),x]
```

```
[Out] ((-I/3)*ArcTan[a*x]^3)/c + (ArcTan[a*x]^2*Log[2 - 2/(1 - I*a*x)])/c - (I*Ar
cTan[a*x]*PolyLog[2, -1 + 2/(1 - I*a*x)])/c + PolyLog[3, -1 + 2/(1 - I*a*x)
]/(2*c)
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
```

```
^2 + e^2, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4992

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d
] && EqQ[(1 - u)^2 - (1 - (2*I)/(I + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^{-1}(ax)^2}{x\left(c+a^2cx^2\right)}\,dx &= -\frac{i\tan^{-1}(ax)^3}{3c}+\frac{i\int \frac{\tan^{-1}(ax)^2}{x(i+ax)}\,dx}{c}\\ &= -\frac{i\tan^{-1}(ax)^3}{3c}+\frac{\tan^{-1}(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{c}-\frac{(2a)\int \frac{\tan^{-1}(ax)\log\left(2-\frac{2}{1-iax}\right)}{1+a^2x^2}\,dx}{c}\\ &= -\frac{i\tan^{-1}(ax)^3}{3c}+\frac{\tan^{-1}(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{c}-\frac{i\tan^{-1}(ax)\mathrm{Li}_2\left(-1+\frac{2}{1-iax}\right)}{c}+\frac{(ia)\int \frac{\mathrm{Li}_2\left(-1+\frac{2}{1-iax}\right)}{1+a^2x^2}\,dx}{c}\\ &= -\frac{i\tan^{-1}(ax)^3}{3c}+\frac{\tan^{-1}(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{c}-\frac{i\tan^{-1}(ax)\mathrm{Li}_2\left(-1+\frac{2}{1-iax}\right)}{c}+\frac{\mathrm{Li}_3\left(-1+\frac{2}{1-iax}\right)}{2c}\end{aligned}$$

Mathematica [B] time = 0.0496027, size = 243, normalized size = 2.67

$$\frac{\mathrm{PolyLog}\left(3,\frac{-ax-i}{ax-i}\right)}{2c}+\frac{\mathrm{PolyLog}\left(3,-\frac{ax+i}{-ax+i}\right)}{2c}-\frac{\mathrm{PolyLog}\left(3,\frac{ax+i}{ax-i}\right)}{2c}+\frac{i\tan^{-1}(ax)\mathrm{PolyLog}\left(2,\frac{-ax-i}{ax-i}\right)}{c}+\frac{i\tan^{-1}(ax)\mathrm{PolyLo}}{c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTan[a*x]^2/(x*(c + a^2*c*x^2)),x]
```

```
[Out] ((I/3)*ArcTan[a*x]^3)/c + (2*ArcTan[a*x]^2*ArcTanh[1 - (2*I)/(I - a*x)])/c
+ (ArcTan[a*x]^2*Log[(2*I)/(I - a*x)])/c + (I*ArcTan[a*x]*PolyLog[2, (-I -
a*x)/(-I + a*x)])/c + (I*ArcTan[a*x]*PolyLog[2, -((I + a*x)/(I - a*x))])/c
- (I*ArcTan[a*x]*PolyLog[2, (I + a*x)/(-I + a*x)])/c + PolyLog[3, (-I - a*x
)/(-I + a*x)]/(2*c) + PolyLog[3, -((I + a*x)/(I - a*x))]/(2*c) - PolyLog[3,
 (I + a*x)/(-I + a*x)]/(2*c)
```

Maple [C] time = 0.283, size = 1767, normalized size = 19.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^2/x/(a^2*c*x^2+c),x)
```

```
[Out] 1/c*arctan(a*x)^2*ln(2)-1/c*arctan(a*x)^2*ln((1+I*a*x)^2/(a^2*x^2+1)-1)+1/c
*arctan(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))+1/c*arctan(a*x)^2*ln(1+(1+
I*a*x)/(a^2*x^2+1)^(1/2))+1/c*arctan(a*x)^2*ln((1+I*a*x)/(a^2*x^2+1)^(1/2))
-1/2/c*arctan(a*x)^2*ln(a^2*x^2+1)+1/c*arctan(a*x)^2*ln(a*x)+1/2*I/c*Pi*csg
n(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^3*arctan(a*x)^2-
1/4*I/c*arctan(a*x)^2*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x
^2+1)+1)^2)^3+1/2*I/c*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a
^2*x^2+1)+1))^3*arctan(a*x)^2+1/4*I/c*arctan(a*x)^2*Pi*csgn(I*((1+I*a*x)^2/
(a^2*x^2+1)+1)^2)^3+1/2*I/c*arctan(a*x)^2*Pi*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(
1/2))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^2+1/4*I/c*arctan(a*x)^2*Pi*csgn(I*((1
+I*a*x)^2/(a^2*x^2+1)+1))^2*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)-1/2*I/c*P
i*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((
1+I*a*x)^2/(a^2*x^2+1)+1))^2*arctan(a*x)^2-1/2*I/c*arctan(a*x)^2*Pi*csgn(I*
((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2+1/4*I/
c*arctan(a*x)^2*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2
/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2-1/4*I/c*arctan(a*x)^2*Pi*csgn
(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))^2*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))-1/2*I/c*P
i*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(((1+
I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2*arctan(a*x)^2+1/4*I/
c*arctan(a*x)^2*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*
x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2-1/2*I/c*Pi*csgn(I/((1+I*a*x)^2/(a^2
*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))
^2*arctan(a*x)^2+1/2*I/c*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2
/(a^2*x^2+1)+1))*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+
1))*arctan(a*x)^2-1/4*I/c*arctan(a*x)^2*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^
```

```
3-1/2*I/c*Pi*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^
2*arctan(a*x)^2-1/3*I*arctan(a*x)^3/c+1/2*I/c*Pi*csgn(I*((1+I*a*x)^2/(a^2*x
^2+1)-1))*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+
1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*arctan(a*x)^2-1/4*I/c*arctan(a*x)^2*Pi*c
sgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I
*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)+1/2*I/c*Pi*arctan(a
*x)^2-2*I/c*arctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*I/c*arcta
n(a*x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+2/c*polylog(3,-(1+I*a*x)/(a^2
*x^2+1)^(1/2))+2/c*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(ax)^2}{(a^2cx^2+c)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] integrate(arctan(a*x)^2/((a^2*c*x^2 + c)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\arctan(ax)^2}{a^2cx^3+cx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(arctan(a*x)^2/(a^2*c*x^3 + c*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\operatorname{atan}^2(ax)}{a^2x^3+x}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**2/x/(a**2*c*x**2+c),x)
```

```
[Out] Integral(atan(a*x)**2/(a**2*x**3 + x), x)/c
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(ax)^2}{(a^2cx^2+c)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)^2/((a^2*c*x^2 + c)*x), x)
```

3.288 $\int \frac{\tan^{-1}(ax)^2}{x^2(c+a^2cx^2)}\,dx$

Optimal. Leaf size=92

$$-\frac{ia\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{c}-\frac{a\tan^{-1}(ax)^3}{3c}-\frac{ia\tan^{-1}(ax)^2}{c}-\frac{\tan^{-1}(ax)^2}{cx}+\frac{2a\log\left(2-\frac{2}{1-iax}\right)\tan^{-1}(ax)}{c}$$

```
[Out] ((-I)*a*ArcTan[a*x]^2)/c - ArcTan[a*x]^2/(c*x) - (a*ArcTan[a*x]^3)/(3*c) +
(2*a*ArcTan[a*x]*Log[2 - 2/(1 - I*a*x)])/c - (I*a*PolyLog[2, -1 + 2/(1 - I*
a*x)])/c
```

Rubi [A] time = 0.195493, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4918, 4852, 4924, 4868, 2447, 4884}

$$-\frac{ia\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{c}-\frac{a\tan^{-1}(ax)^3}{3c}-\frac{ia\tan^{-1}(ax)^2}{c}-\frac{\tan^{-1}(ax)^2}{cx}+\frac{2a\log\left(2-\frac{2}{1-iax}\right)\tan^{-1}(ax)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^2/(x^2*(c + a^2*c*x^2)),x]
```

```
[Out] ((-I)*a*ArcTan[a*x]^2)/c - ArcTan[a*x]^2/(c*x) - (a*ArcTan[a*x]^3)/(3*c) +
(2*a*ArcTan[a*x]*Log[2 - 2/(1 - I*a*x)])/c - (I*a*PolyLog[2, -1 + 2/(1 - I*
a*x)])/c
```

Rule 4918

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/(d + e*x^2),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
```

```
erQ[m]) && NeQ[m, -1]
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^2}{x^2\left(c+a^2cx^2\right)}\,dx = -\left(a^2\int \frac{\tan^{-1}(ax)^2}{c+a^2cx^2}\,dx\right) + \frac{\int \frac{\tan^{-1}(ax)^2}{x^2}\,dx}{c}$$
$$= -\frac{\tan^{-1}(ax)^2}{cx} - \frac{a\tan^{-1}(ax)^3}{3c} + \frac{(2a)\int \frac{\tan^{-1}(ax)}{x\left(1+a^2x^2\right)}\,dx}{c}$$
$$= -\frac{ia\tan^{-1}(ax)^2}{c} - \frac{\tan^{-1}(ax)^2}{cx} - \frac{a\tan^{-1}(ax)^3}{3c} + \frac{(2ia)\int \frac{\tan^{-1}(ax)}{x(i+ax)}\,dx}{c}$$
$$= -\frac{ia\tan^{-1}(ax)^2}{c} - \frac{\tan^{-1}(ax)^2}{cx} - \frac{a\tan^{-1}(ax)^3}{3c} + \frac{2a\tan^{-1}(ax)\log\left(2-\frac{2}{1-iax}\right)}{c} - \frac{\left(2a^2\right)\int \frac{\log\left(2-\frac{2}{1-iax}\right)}{1+a^2x^2}$$
$$= -\frac{ia\tan^{-1}(ax)^2}{c} - \frac{\tan^{-1}(ax)^2}{cx} - \frac{a\tan^{-1}(ax)^3}{3c} + \frac{2a\tan^{-1}(ax)\log\left(2-\frac{2}{1-iax}\right)}{c} - \frac{ia\mathrm{Li}_2\left(-1+\frac{2}{1-iax}\right)}{c}$$

Mathematica [A] time = 0.179234, size = 73, normalized size = 0.79

$$\frac{a\left(-i\mathrm{PolyLog}\left(2,e^{2i\tan^{-1}(ax)}\right)-\frac{1}{3}\tan^{-1}(ax)\left(\left(\tan^{-1}(ax)+3i\right)\tan^{-1}(ax)+\frac{3\tan^{-1}(ax)}{ax}-6\log\left(1-e^{2i\tan^{-1}(ax)}\right)\right)\right)}{c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTan[a*x]^2/(x^2*(c + a^2*c*x^2)),x]
```

```
[Out] (a*(-(ArcTan[a*x]*((3*ArcTan[a*x])/(a*x) + ArcTan[a*x]*(3*I + ArcTan[a*x])
- 6*Log[1 - E^((2*I)*ArcTan[a*x])]))/3 - I*PolyLog[2, E^((2*I)*ArcTan[a*x])
]))/c
```

Maple [B] time = 0.099, size = 292, normalized size = 3.2

$$-\frac{a\left(\arctan(ax)\right)^3}{3c} - \frac{\left(\arctan(ax)\right)^2}{cx} - \frac{a\arctan(ax)\ln\left(a^2x^2+1\right)}{c} + 2\frac{a\arctan(ax)\ln(ax)}{c} + \frac{\frac{i}{2}a\mathrm{dilog}\left(-\frac{i}{2}(ax+i)\right)}{c} + \frac{ia}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^2/x^2/(a^2*c*x^2+c),x)
```

```
[Out] -1/3*a*arctan(a*x)^3/c-arctan(a*x)^2/c/x-a/c*arctan(a*x)*ln(a^2*x^2+1)+2*a/
c*arctan(a*x)*ln(a*x)+1/2*I*a/c*dilog(-1/2*I*(a*x+I))+I*a/c*dilog(1+I*a*x)-
1/2*I*a/c*dilog(1/2*I*(a*x-I))-1/4*I*a/c*ln(a*x+I)^2+1/2*I*a/c*ln(a*x+I)*ln
(a^2*x^2+1)-1/2*I*a/c*ln(1/2*I*(a*x-I))*ln(a*x+I)-1/2*I*a/c*ln(a*x-I)*ln(a^
2*x^2+1)+I*a/c*ln(a*x)*ln(1+I*a*x)+1/4*I*a/c*ln(a*x-I)^2-I*a/c*dilog(1-I*a*
x)+1/2*I*a/c*ln(a*x-I)*ln(-1/2*I*(a*x+I))-I*a/c*ln(a*x)*ln(1-I*a*x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x^2/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\arctan(ax)^2}{a^2cx^4+cx^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x^2/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(arctan(a*x)^2/(a^2*c*x^4 + c*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\operatorname{atan}^2(ax)}{a^2x^4+x^2}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**2/x**2/(a**2*c*x**2+c),x)
```

```
[Out] Integral(atan(a*x)**2/(a**2*x**4 + x**2), x)/c
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(ax)^2}{(a^2cx^2+c)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x^2/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)^2/((a^2*c*x^2 + c)*x^2), x)
```

3.289 $\int \frac{\tan^{-1}(ax)^2}{x^3(c+a^2cx^2)}\,dx$

Optimal. Leaf size=178

$$-\frac{a^2\text{PolyLog}\left(3,-1+\frac{2}{1-iax}\right)}{2c}+\frac{ia^2\tan^{-1}(ax)\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{c}-\frac{a^2\log\left(a^2x^2+1\right)}{2c}+\frac{a^2\log(x)}{c}+\frac{ia^2\tan^{-1}(ax)^3}{3c}$$

```
[Out] -((a*ArcTan[a*x])/(c*x)) - (a^2*ArcTan[a*x]^2)/(2*c) - ArcTan[a*x]^2/(2*c*x
^2) + ((I/3)*a^2*ArcTan[a*x]^3)/c + (a^2*Log[x])/c - (a^2*Log[1 + a^2*x^2])
/(2*c) - (a^2*ArcTan[a*x]^2*Log[2 - 2/(1 - I*a*x)])/c + (I*a^2*ArcTan[a*x]*
PolyLog[2, -1 + 2/(1 - I*a*x)])/c - (a^2*PolyLog[3, -1 + 2/(1 - I*a*x)])/(2
*c)
```

Rubi [A] time = 0.335515, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 11, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4918, 4852, 266, 36, 29, 31, 4884, 4924, 4868, 4992, 6610}

$$-\frac{a^2\text{PolyLog}\left(3,-1+\frac{2}{1-iax}\right)}{2c}+\frac{ia^2\tan^{-1}(ax)\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{c}-\frac{a^2\log\left(a^2x^2+1\right)}{2c}+\frac{a^2\log(x)}{c}+\frac{ia^2\tan^{-1}(ax)^3}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^2/(x^3*(c + a^2*c*x^2)),x]
```


```
[Out] -((a*ArcTan[a*x])/(c*x)) - (a^2*ArcTan[a*x]^2)/(2*c) - ArcTan[a*x]^2/(2*c*x
^2) + ((I/3)*a^2*ArcTan[a*x]^3)/c + (a^2*Log[x])/c - (a^2*Log[1 + a^2*x^2])
/(2*c) - (a^2*ArcTan[a*x]^2*Log[2 - 2/(1 - I*a*x)])/c + (I*a^2*ArcTan[a*x]*
PolyLog[2, -1 + 2/(1 - I*a*x)])/c - (a^2*PolyLog[3, -1 + 2/(1 - I*a*x)])/(2
*c)
```


Rule 4918

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/(d + e*x^2),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
```

```
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 4992

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d
] && EqQ[(1 - u)^2 - (1 - (2*I)/(I + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^2}{x^3\left(c+a^2cx^2\right)}\,dx = -\left(a^2\int \frac{\tan^{-1}(ax)^2}{x\left(c+a^2cx^2\right)}\,dx\right)+\frac{\int \frac{\tan^{-1}(ax)^2}{x^3}\,dx}{c}$$

$$= -\frac{\tan^{-1}(ax)^2}{2cx^2}+\frac{ia^2\tan^{-1}(ax)^3}{3c}+\frac{a\int \frac{\tan^{-1}(ax)}{x^2\left(1+a^2x^2\right)}\,dx}{c}-\frac{\left(ia^2\right)\int \frac{\tan^{-1}(ax)^2}{x(i+ax)}\,dx}{c}$$

$$= -\frac{\tan^{-1}(ax)^2}{2cx^2}+\frac{ia^2\tan^{-1}(ax)^3}{3c}-\frac{a^2\tan^{-1}(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{c}+\frac{a\int \frac{\tan^{-1}(ax)}{x^2}\,dx}{c}-\frac{a^3\int \frac{\tan^{-1}(ax)}{1+a^2x^2}}{c}$$

$$= -\frac{a\tan^{-1}(ax)}{cx}-\frac{a^2\tan^{-1}(ax)^2}{2c}-\frac{\tan^{-1}(ax)^2}{2cx^2}+\frac{ia^2\tan^{-1}(ax)^3}{3c}-\frac{a^2\tan^{-1}(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{c}+$$

$$= -\frac{a\tan^{-1}(ax)}{cx}-\frac{a^2\tan^{-1}(ax)^2}{2c}-\frac{\tan^{-1}(ax)^2}{2cx^2}+\frac{ia^2\tan^{-1}(ax)^3}{3c}-\frac{a^2\tan^{-1}(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{c}+$$

$$= -\frac{a\tan^{-1}(ax)}{cx}-\frac{a^2\tan^{-1}(ax)^2}{2c}-\frac{\tan^{-1}(ax)^2}{2cx^2}+\frac{ia^2\tan^{-1}(ax)^3}{3c}-\frac{a^2\tan^{-1}(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{c}+$$

$$= -\frac{a\tan^{-1}(ax)}{cx}-\frac{a^2\tan^{-1}(ax)^2}{2c}-\frac{\tan^{-1}(ax)^2}{2cx^2}+\frac{ia^2\tan^{-1}(ax)^3}{3c}+\frac{a^2\log(x)}{c}-\frac{a^2\log\left(1+a^2x^2\right)}{2c}-$$

Mathematica [A] time = 0.305383, size = 142, normalized size = 0.8

$$\frac{a^2\left(-i\tan^{-1}(ax)\mathrm{PolyLog}\left(2,e^{-2i\tan^{-1}(ax)}\right)-\frac{1}{2}\mathrm{PolyLog}\left(3,e^{-2i\tan^{-1}(ax)}\right)+\log\left(\frac{ax}{\sqrt{a^2x^2+1}}\right)-\frac{\left(a^2x^2+1\right)\tan^{-1}(ax)^2}{2a^2x^2}-\frac{1}{3}i\tan^{-1}(ax$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTan[a*x]^2/(x^3*(c + a^2*c*x^2)),x]
```

```
[Out] (a^2*((I/24)*Pi^3 - ArcTan[a*x]/(a*x) - ((1 + a^2*x^2)*ArcTan[a*x]^2)/(2*a^
2*x^2) - (I/3)*ArcTan[a*x]^3 - ArcTan[a*x]^2*Log[1 - E^((-2*I)*ArcTan[a*x])
] + Log[(a*x)/Sqrt[1 + a^2*x^2]] - I*ArcTan[a*x]*PolyLog[2, E^((-2*I)*ArcTa
n[a*x])] - PolyLog[3, E^((-2*I)*ArcTan[a*x])]/2))/c
```

Maple [C] time = 0.715, size = 5491, normalized size = 30.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^2/x^3/(a^2*c*x^2+c),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(ax)^2}{\left(a^2cx^2+c\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x^3/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] integrate(arctan(a*x)^2/((a^2*c*x^2 + c)*x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\arctan(ax)^2}{a^2cx^5 + cx^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x^3/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(arctan(a*x)^2/(a^2*c*x^5 + c*x^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\operatorname{atan}^2(ax)}{a^2x^5+x^3}\, dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**2/x**3/(a**2*c*x**2+c),x)
```

```
[Out] Integral(atan(a*x)**2/(a**2*x**5 + x**3), x)/c
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(ax)^2}{(a^2cx^2 + c)x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x^3/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)^2/((a^2*c*x^2 + c)*x^3), x)
```

3.290 $\int \frac{\tan^{-1}(ax)^2}{x^4(c+a^2cx^2)}\,dx$

Optimal. Leaf size=166

$$\frac{4ia^3\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{3c}-\frac{a^2}{3cx}+\frac{a^3\tan^{-1}(ax)^3}{3c}+\frac{4ia^3\tan^{-1}(ax)^2}{3c}-\frac{a^3\tan^{-1}(ax)}{3c}+\frac{a^2\tan^{-1}(ax)^2}{cx}-\frac{8a^3\log\left(2-\frac{}{1}\right.}{3}$$

```
[Out] -a^2/(3*c*x) - (a^3*ArcTan[a*x])/(3*c) - (a*ArcTan[a*x])/(3*c*x^2) + (((4*I
)/3)*a^3*ArcTan[a*x]^2)/c - ArcTan[a*x]^2/(3*c*x^3) + (a^2*ArcTan[a*x]^2)/(
c*x) + (a^3*ArcTan[a*x]^3)/(3*c) - (8*a^3*ArcTan[a*x]*Log[2 - 2/(1 - I*a*x)
])/(3*c) + (((4*I)/3)*a^3*PolyLog[2, -1 + 2/(1 - I*a*x)])/c
```

Rubi [A] time = 0.435952, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {4918, 4852, 325, 203, 4924, 4868, 2447, 4884}

$$\frac{4ia^3\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{3c}-\frac{a^2}{3cx}+\frac{a^3\tan^{-1}(ax)^3}{3c}+\frac{4ia^3\tan^{-1}(ax)^2}{3c}-\frac{a^3\tan^{-1}(ax)}{3c}+\frac{a^2\tan^{-1}(ax)^2}{cx}-\frac{8a^3\log\left(2-\frac{}{1}\right.}{3}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^2/(x^4*(c + a^2*c*x^2)),x]
```


```
[Out] -a^2/(3*c*x) - (a^3*ArcTan[a*x])/(3*c) - (a*ArcTan[a*x])/(3*c*x^2) + (((4*I
)/3)*a^3*ArcTan[a*x]^2)/c - ArcTan[a*x]^2/(3*c*x^3) + (a^2*ArcTan[a*x]^2)/(
c*x) + (a^3*ArcTan[a*x]^3)/(3*c) - (8*a^3*ArcTan[a*x]*Log[2 - 2/(1 - I*a*x)
])/(3*c) + (((4*I)/3)*a^3*PolyLog[2, -1 + 2/(1 - I*a*x)])/c
```


Rule 4918

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/(d + e*x^2),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
```

```
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^2}{x^4\left(c+a^2cx^2\right)}\,dx = -\left(a^2\int \frac{\tan^{-1}(ax)^2}{x^2\left(c+a^2cx^2\right)}\,dx\right)+\frac{\int \frac{\tan^{-1}(ax)^2}{x^4}\,dx}{c}$$

$$= -\frac{\tan^{-1}(ax)^2}{3cx^3}+a^4\int \frac{\tan^{-1}(ax)^2}{c+a^2cx^2}\,dx+\frac{(2a)\int \frac{\tan^{-1}(ax)}{x^3\left(1+a^2x^2\right)}\,dx}{3c}-\frac{a^2\int \frac{\tan^{-1}(ax)^2}{x^2}\,dx}{c}$$

$$= -\frac{\tan^{-1}(ax)^2}{3cx^3}+\frac{a^2\tan^{-1}(ax)^2}{cx}+\frac{a^3\tan^{-1}(ax)^3}{3c}+\frac{(2a)\int \frac{\tan^{-1}(ax)}{x^3}\,dx}{3c}-\frac{\left(2a^3\right)\int \frac{\tan^{-1}(ax)}{x\left(1+a^2x^2\right)}\,dx}{3c}-\frac{(2a^3}{}$$

$$= -\frac{a\tan^{-1}(ax)}{3cx^2}+\frac{4ia^3\tan^{-1}(ax)^2}{3c}-\frac{\tan^{-1}(ax)^2}{3cx^3}+\frac{a^2\tan^{-1}(ax)^2}{cx}+\frac{a^3\tan^{-1}(ax)^3}{3c}+\frac{a^2\int \frac{1}{x^2\left(1+a^2x^2\right)}}{3c}$$

$$= -\frac{a^2}{3cx}-\frac{a\tan^{-1}(ax)}{3cx^2}+\frac{4ia^3\tan^{-1}(ax)^2}{3c}-\frac{\tan^{-1}(ax)^2}{3cx^3}+\frac{a^2\tan^{-1}(ax)^2}{cx}+\frac{a^3\tan^{-1}(ax)^3}{3c}-\frac{8a^3\tan}{}$$

$$= -\frac{a^2}{3cx}-\frac{a^3\tan^{-1}(ax)}{3c}-\frac{a\tan^{-1}(ax)}{3cx^2}+\frac{4ia^3\tan^{-1}(ax)^2}{3c}-\frac{\tan^{-1}(ax)^2}{3cx^3}+\frac{a^2\tan^{-1}(ax)^2}{cx}+\frac{a^3\tan^{-1}}{3c}$$

Mathematica [A] time = 0.345505, size = 120, normalized size = 0.72

$$\frac{a^3\left(4i\text{PolyLog}\left(2,e^{2i\tan^{-1}(ax)}\right)-\frac{\frac{\left(a^2x^2+1\right)\tan^{-1}(ax)^2}{a^2x^2}-4\tan^{-1}(ax)^2+1}{ax}+\tan^{-1}(ax)\left(-\frac{a^2x^2+1}{a^2x^2}+\tan^{-1}(ax)\left(\tan^{-1}(ax)+4i\right)-8\log\right.\right.}{3c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTan[a*x]^2/(x^4*(c + a^2*c*x^2)),x]
```

```
[Out] (a^3*(-((1 - 4*ArcTan[a*x]^2 + ((1 + a^2*x^2)*ArcTan[a*x]^2)/(a^2*x^2))/(a*
x)) + ArcTan[a*x]*(-((1 + a^2*x^2)/(a^2*x^2)) + ArcTan[a*x]*(4*I + ArcTan[a
*x]) - 8*Log[1 - E^((2*I)*ArcTan[a*x])]) + (4*I)*PolyLog[2, E^((2*I)*ArcTan
[a*x])]))/(3*c)
```

Maple [B] time = 0.108, size = 374, normalized size = 2.3

$$\frac{a^3(\arctan(ax))^3}{3c}-\frac{(\arctan(ax))^2}{3cx^3}+\frac{a^2(\arctan(ax))^2}{cx}+\frac{4a^3\arctan(ax)\ln\left(a^2x^2+1\right)}{3c}-\frac{a\arctan(ax)}{3cx^2}-\frac{8a^3\arctan(}{3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^2/x^4/(a^2*c*x^2+c),x)
```

```
[Out] 1/3*a^3*arctan(a*x)^3/c-1/3*arctan(a*x)^2/c/x^3+a^2*arctan(a*x)^2/c/x+4/3*a
^3/c*arctan(a*x)*ln(a^2*x^2+1)-1/3*a*arctan(a*x)/c/x^2-8/3*a^3/c*arctan(a*x
)*ln(a*x)-1/3*a^3*arctan(a*x)/c-1/3*a^2/c/x+4/3*I*a^3/c*dilog(1-I*a*x)-2/3*
I*a^3/c*ln(a*x+I)*ln(a^2*x^2+1)+2/3*I*a^3/c*ln(1/2*I*(a*x-I))*ln(a*x+I)-1/3
*I*a^3/c*ln(a*x-I)^2-4/3*I*a^3/c*dilog(1+I*a*x)-4/3*I*a^3/c*ln(a*x)*ln(1+I*
a*x)+2/3*I*a^3/c*ln(a*x-I)*ln(a^2*x^2+1)-2/3*I*a^3/c*ln(a*x-I)*ln(-1/2*I*(a
*x+I))+4/3*I*a^3/c*ln(a*x)*ln(1-I*a*x)-2/3*I*a^3/c*dilog(-1/2*I*(a*x+I))+2/
3*I*a^3/c*dilog(1/2*I*(a*x-I))+1/3*I*a^3/c*ln(a*x+I)^2
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x^4/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\arctan\left(ax\right)^{2}}{a^{2}cx^{6}+cx^{4}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x^4/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(arctan(a*x)^2/(a^2*c*x^6 + c*x^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\operatorname{atan}^{2}\left(ax\right)}{a^{2}x^{6}+x^{4}}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**2/x**4/(a**2*c*x**2+c),x)
```

```
[Out] Integral(atan(a*x)**2/(a**2*x**6 + x**4), x)/c
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan (ax)^2}{(a^2cx^2 + c)x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x^4/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)^2/((a^2*c*x^2 + c)*x^4), x)
```

3.291 $\int \frac{x^3 \tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^2} dx$

Optimal. Leaf size=192

$$-\frac{\text{PolyLog}\left(3,1-\frac{2}{1+iax}\right)}{2a^4c^2}-\frac{i\tan^{-1}(ax)\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{a^4c^2}-\frac{1}{4a^4c^2\left(a^2x^2+1\right)}+\frac{\tan^{-1}(ax)^2}{2a^4c^2\left(a^2x^2+1\right)}-\frac{x\tan^{-1}(ax)}{2a^3c^2\left(a^2x^2+1\right)}$$

```
[Out] -1/(4*a^4*c^2*(1 + a^2*x^2)) - (x*ArcTan[a*x])/(2*a^3*c^2*(1 + a^2*x^2)) -
ArcTan[a*x]^2/(4*a^4*c^2) + ArcTan[a*x]^2/(2*a^4*c^2*(1 + a^2*x^2)) - ((I/3
)*ArcTan[a*x]^3)/(a^4*c^2) - (ArcTan[a*x]^2*Log[2/(1 + I*a*x)])/(a^4*c^2) -
 (I*ArcTan[a*x]*PolyLog[2, 1 - 2/(1 + I*a*x)])/(a^4*c^2) - PolyLog[3, 1 - 2
/(1 + I*a*x)]/(2*a^4*c^2)
```

Rubi [A] time = 0.289703, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {4964, 4920, 4854, 4884, 4994, 6610, 4930, 4892, 261}

$$-\frac{\text{PolyLog}\left(3,1-\frac{2}{1+iax}\right)}{2a^4c^2}-\frac{i\tan^{-1}(ax)\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{a^4c^2}-\frac{1}{4a^4c^2\left(a^2x^2+1\right)}+\frac{\tan^{-1}(ax)^2}{2a^4c^2\left(a^2x^2+1\right)}-\frac{x\tan^{-1}(ax)}{2a^3c^2\left(a^2x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcTan[a*x]^2)/(c + a^2*c*x^2)^2,x]
```

```
[Out] -1/(4*a^4*c^2*(1 + a^2*x^2)) - (x*ArcTan[a*x])/(2*a^3*c^2*(1 + a^2*x^2)) -
ArcTan[a*x]^2/(4*a^4*c^2) + ArcTan[a*x]^2/(2*a^4*c^2*(1 + a^2*x^2)) - ((I/3
)*ArcTan[a*x]^3)/(a^4*c^2) - (ArcTan[a*x]^2*Log[2/(1 + I*a*x)])/(a^4*c^2) -
 (I*ArcTan[a*x]*PolyLog[2, 1 - 2/(1 + I*a*x)])/(a^4*c^2) - PolyLog[3, 1 - 2
/(1 + I*a*x)]/(2*a^4*c^2)
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Arc
Tan[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan[c
*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p
, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 4892

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[(x*(a + b*ArcTan[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*c*
```

```
p)/2, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(a +
 b*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x^3\tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^2}\,dx = -\frac{\int \frac{x\tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^2}\,dx}{a^2}+\frac{\int \frac{x\tan^{-1}(ax)^2}{c+a^2cx^2}\,dx}{a^2c}$$

$$= \frac{\tan^{-1}(ax)^2}{2a^4c^2\left(1+a^2x^2\right)}-\frac{i\tan^{-1}(ax)^3}{3a^4c^2}-\frac{\int \frac{\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^2}\,dx}{a^3}-\frac{\int \frac{\tan^{-1}(ax)^2}{i-ax}\,dx}{a^3c^2}$$

$$= -\frac{x\tan^{-1}(ax)}{2a^3c^2\left(1+a^2x^2\right)}-\frac{\tan^{-1}(ax)^2}{4a^4c^2}+\frac{\tan^{-1}(ax)^2}{2a^4c^2\left(1+a^2x^2\right)}-\frac{i\tan^{-1}(ax)^3}{3a^4c^2}-\frac{\tan^{-1}(ax)^2\log\left(\frac{2}{1+iax}\right)}{a^4c^2}+\cdots$$

$$= -\frac{1}{4a^4c^2\left(1+a^2x^2\right)}-\frac{x\tan^{-1}(ax)}{2a^3c^2\left(1+a^2x^2\right)}-\frac{\tan^{-1}(ax)^2}{4a^4c^2}+\frac{\tan^{-1}(ax)^2}{2a^4c^2\left(1+a^2x^2\right)}-\frac{i\tan^{-1}(ax)^3}{3a^4c^2}-\frac{\tan^{-1}(\cdots}{\cdots}$$

$$= -\frac{1}{4a^4c^2\left(1+a^2x^2\right)}-\frac{x\tan^{-1}(ax)}{2a^3c^2\left(1+a^2x^2\right)}-\frac{\tan^{-1}(ax)^2}{4a^4c^2}+\frac{\tan^{-1}(ax)^2}{2a^4c^2\left(1+a^2x^2\right)}-\frac{i\tan^{-1}(ax)^3}{3a^4c^2}-\frac{\tan^{-1}(\cdots}{\cdots}$$

Mathematica [A] time = 0.178734, size = 117, normalized size = 0.61

$$\frac{i\tan^{-1}(ax)\text{PolyLog}\left(2,-e^{2i\tan^{-1}(ax)}\right)-\frac{1}{2}\text{PolyLog}\left(3,-e^{2i\tan^{-1}(ax)}\right)+\frac{1}{3}i\tan^{-1}(ax)^3-\tan^{-1}(ax)^2\log\left(1+e^{2i\tan^{-1}(ax)}\right)\cdots}{a^4c^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*ArcTan[a*x]^2)/(c + a^2*c*x^2)^2,x]
```

```
[Out] ((I/3)*ArcTan[a*x]^3 + ((-1 + 2*ArcTan[a*x]^2)*Cos[2*ArcTan[a*x]])/8 - ArcT
an[a*x]^2*Log[1 + E^((2*I)*ArcTan[a*x])] + I*ArcTan[a*x]*PolyLog[2, -E^((2*
```

```
I)*ArcTan[a*x])] - PolyLog[3, -E^((2*I)*ArcTan[a*x])]/2 - (ArcTan[a*x]*Sin[
2*ArcTan[a*x]])/4)/(a^4*c^2)
```

Maple [C] time = 0.423, size = 1092, normalized size = 5.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctan(a*x)^2/(a^2*c*x^2+c)^2,x)
```

```
[Out] 1/4*I/a^4/c^2*arctan(a*x)^2*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*
a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I/((1+I*a*x)^2/(a^2*
x^2+1)+1)^2)+I/a^3/c^2*arctan(a*x)/(8*a*x-8*I)*x+1/4*I/a^4/c^2*arctan(a*x)^
2*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^3+1/4*I/a^4/c^2*arctan(a*x)^2*Pi*csgn(
I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3-1/4*I/a^4/c^2*ar
ctan(a*x)^2*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3-I/a^3/c^2*arctan(a*x
)/(8*a*x+8*I)*x+1/16/a^3/c^2/(a*x+I)*x+1/3*I/a^4/c^2*arctan(a*x)^3+1/16*I/a
^4/c^2/(a*x-I)-1/16*I/a^4/c^2/(a*x+I)-1/2/a^4/c^2*polylog(3,-(1+I*a*x)^2/(a
^2*x^2+1))-1/4*arctan(a*x)^2/a^4/c^2-1/a^4/c^2*arctan(a*x)^2*ln(2)+1/16/a^3
/c^2/(a*x-I)*x+1/2/a^4/c^2*arctan(a*x)^2*ln(a^2*x^2+1)-1/a^4/c^2*arctan(a*x
)/(8*a*x-8*I)+1/2*I/a^4/c^2*arctan(a*x)^2*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1
)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2-1/2*I/a^4/c^2*arctan(a*x)^2*P
i*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^2*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+1/2
*arctan(a*x)^2/a^4/c^2/(a^2*x^2+1)-1/4*I/a^4/c^2*arctan(a*x)^2*Pi*csgn(I*(1
+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2
+1)+1)^2)^2+1/4*I/a^4/c^2*arctan(a*x)^2*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*
csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))^2-1/4*I/a^4/c^2*arctan(a*x)^2*Pi*csgn(I
*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2*csgn(I/((1+I*a*x)
^2/(a^2*x^2+1)+1)^2)-1/4*I/a^4/c^2*arctan(a*x)^2*Pi*csgn(I*((1+I*a*x)^2/(a^
2*x^2+1)+1))^2*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)-1/a^4/c^2*arctan(a*x)^
2*ln((1+I*a*x)/(a^2*x^2+1)^(1/2))+I/a^4/c^2*arctan(a*x)*polylog(2,-(1+I*a*x
)^2/(a^2*x^2+1))-1/a^4/c^2*arctan(a*x)/(8*a*x+8*I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \arctan(ax)^2}{\left(a^2cx^2 + c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^2/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^3*arctan(a*x)^2/(a^2*c*x^2 + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^3 \arctan\left(ax\right)^2}{a^4c^2x^4 + 2\,a^2c^2x^2 + c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^2/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] integral(x^3*arctan(a*x)^2/(a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^3 \operatorname{atan}^2\left(ax\right)}{a^4x^4+2a^2x^2+1}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atan(a*x)**2/(a**2*c*x**2+c)**2,x)
```

```
[Out] Integral(x**3*atan(a*x)**2/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \arctan\left(ax\right)^2}{\left(a^2cx^2 + c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^2/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(x^3*arctan(a*x)^2/(a^2*c*x^2 + c)^2, x)
```

3.292 $\int \frac{x^2 \tan^{-1}(ax)^2}{(c+a^2cx^2)^2} dx$

Optimal. Leaf size=106

$$\frac{x}{4a^2c^2(a^2x^2+1)} - \frac{x\tan^{-1}(ax)^2}{2a^2c^2(a^2x^2+1)} - \frac{\tan^{-1}(ax)}{2a^3c^2(a^2x^2+1)} + \frac{\tan^{-1}(ax)^3}{6a^3c^2} + \frac{\tan^{-1}(ax)}{4a^3c^2}$$

```
[Out] x/(4*a^2*c^2*(1 + a^2*x^2)) + ArcTan[a*x]/(4*a^3*c^2) - ArcTan[a*x]/(2*a^3*
c^2*(1 + a^2*x^2)) - (x*ArcTan[a*x]^2)/(2*a^2*c^2*(1 + a^2*x^2)) + ArcTan[a
*x]^3/(6*a^3*c^2)
```

Rubi [A] time = 0.110012, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4936, 4930, 199, 205}

$$\frac{x}{4a^2c^2(a^2x^2+1)} - \frac{x\tan^{-1}(ax)^2}{2a^2c^2(a^2x^2+1)} - \frac{\tan^{-1}(ax)}{2a^3c^2(a^2x^2+1)} + \frac{\tan^{-1}(ax)^3}{6a^3c^2} + \frac{\tan^{-1}(ax)}{4a^3c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcTan[a*x]^2)/(c + a^2*c*x^2)^2,x]
```


```
[Out] x/(4*a^2*c^2*(1 + a^2*x^2)) + ArcTan[a*x]/(4*a^3*c^2) - ArcTan[a*x]/(2*a^3*
c^2*(1 + a^2*x^2)) - (x*ArcTan[a*x]^2)/(2*a^2*c^2*(1 + a^2*x^2)) + ArcTan[a
*x]^3/(6*a^3*c^2)
```


Rule 4936

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^2)/((d_) + (e_.)*(x_)^2)
^2, x_Symbol] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(2*b*c^3*d^2*(p + 1)), x]
 + (Dist[(b*p)/(2*c), Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(d + e*x^2)^2, x]
, x] - Simp[(x*(a + b*ArcTan[c*x])^p)/(2*c^2*d*(d + e*x^2)), x]) /; FreeQ[{
a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
```

```
0] && NeQ[q, -1]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^2}\,dx &= -\frac{x\tan^{-1}(ax)^2}{2a^2c^2\left(1+a^2x^2\right)}+\frac{\tan^{-1}(ax)^3}{6a^3c^2}+\frac{\int \frac{x\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^2}\,dx}{a}\\ &= -\frac{\tan^{-1}(ax)}{2a^3c^2\left(1+a^2x^2\right)}-\frac{x\tan^{-1}(ax)^2}{2a^2c^2\left(1+a^2x^2\right)}+\frac{\tan^{-1}(ax)^3}{6a^3c^2}+\frac{\int \frac{1}{\left(c+a^2cx^2\right)^2}\,dx}{2a^2}\\ &= \frac{x}{4a^2c^2\left(1+a^2x^2\right)}-\frac{\tan^{-1}(ax)}{2a^3c^2\left(1+a^2x^2\right)}-\frac{x\tan^{-1}(ax)^2}{2a^2c^2\left(1+a^2x^2\right)}+\frac{\tan^{-1}(ax)^3}{6a^3c^2}+\frac{\int \frac{1}{c+a^2cx^2}\,dx}{4a^2c}\\ &= \frac{x}{4a^2c^2\left(1+a^2x^2\right)}+\frac{\tan^{-1}(ax)}{4a^3c^2}-\frac{\tan^{-1}(ax)}{2a^3c^2\left(1+a^2x^2\right)}-\frac{x\tan^{-1}(ax)^2}{2a^2c^2\left(1+a^2x^2\right)}+\frac{\tan^{-1}(ax)^3}{6a^3c^2}\end{aligned}$$

Mathematica [A] time = 0.0904656, size = 68, normalized size = 0.64

$$\frac{2\left(a^2x^2+1\right)\tan^{-1}(ax)^3+3\left(a^2x^2-1\right)\tan^{-1}(ax)+3ax-6ax\tan^{-1}(ax)^2}{12a^3c^2\left(a^2x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*ArcTan[a*x]^2)/(c + a^2*c*x^2)^2,x]
```

```
[Out] (3*a*x + 3*(-1 + a^2*x^2)*ArcTan[a*x] - 6*a*x*ArcTan[a*x]^2 + 2*(1 + a^2*x^
2)*ArcTan[a*x]^3)/(12*a^3*c^2*(1 + a^2*x^2))
```

Maple [A] time = 0.035, size = 97, normalized size = 0.9

$$\frac{x}{4\,a^2c^2\left(a^2x^2+1\right)}+\frac{\arctan\left(ax\right)}{4\,a^3c^2}-\frac{\arctan\left(ax\right)}{2\,a^3c^2\left(a^2x^2+1\right)}-\frac{x\left(\arctan\left(ax\right)\right)^2}{2\,a^2c^2\left(a^2x^2+1\right)}+\frac{\left(\arctan\left(ax\right)\right)^3}{6\,a^3c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(a*x)^2/(a^2*c*x^2+c)^2,x)
```

```
[Out] 1/4*x/a^2/c^2/(a^2*x^2+1)+1/4*arctan(a*x)/a^3/c^2-1/2*arctan(a*x)/a^3/c^2/(
a^2*x^2+1)-1/2*x*arctan(a*x)^2/a^2/c^2/(a^2*x^2+1)+1/6*arctan(a*x)^3/a^3/c^
2
```

Maxima [A] time = 1.63624, size = 204, normalized size = 1.92

$$-\frac{1}{2}\left(\frac{x}{a^4c^2x^2+a^2c^2}-\frac{\arctan\left(ax\right)}{a^3c^2}\right)\arctan\left(ax\right)^2+\frac{\left(2\left(a^2x^2+1\right)\arctan\left(ax\right)^3+3\,ax+3\left(a^2x^2+1\right)\arctan\left(ax\right)\right)a^2}{12\left(a^7c^2x^2+a^5c^2\right)}-\frac{\left(\left(a\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^2/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(x/(a^4*c^2*x^2 + a^2*c^2) - arctan(a*x)/(a^3*c^2))*arctan(a*x)^2 + 1/
12*(2*(a^2*x^2 + 1)*arctan(a*x)^3 + 3*a*x + 3*(a^2*x^2 + 1)*arctan(a*x))*a^
2/(a^7*c^2*x^2 + a^5*c^2) - 1/2*((a^2*x^2 + 1)*arctan(a*x)^2 + 1)*a*arctan(
a*x)/(a^6*c^2*x^2 + a^4*c^2)
```

Fricas [A] time = 2.12255, size = 166, normalized size = 1.57

$$-\frac{6\,ax\arctan\left(ax\right)^2-2\left(a^2x^2+1\right)\arctan\left(ax\right)^3-3\,ax-3\left(a^2x^2-1\right)\arctan\left(ax\right)}{12\left(a^5c^2x^2+a^3c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^2/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] -1/12*(6*a*x*arctan(a*x)^2 - 2*(a^2*x^2 + 1)*arctan(a*x)^3 - 3*a*x - 3*(a^2
*x^2 - 1)*arctan(a*x))/(a^5*c^2*x^2 + a^3*c^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^2 \operatorname{atan}^2(ax)}{a^4x^4+2a^2x^2+1}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(a*x)**2/(a**2*c*x**2+c)**2,x)
```

```
[Out] Integral(x**2*atan(a*x)**2/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \arctan(ax)^2}{(a^2cx^2+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^2/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*arctan(a*x)^2/(a^2*c*x^2 + c)^2, x)
```

3.293 $\int \frac{x \tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^2} dx$

Optimal. Leaf size=91

$$\frac{1}{4a^2c^2\left(a^2x^2+1\right)}-\frac{\tan^{-1}(ax)^2}{2a^2c^2\left(a^2x^2+1\right)}+\frac{x\tan^{-1}(ax)}{2ac^2\left(a^2x^2+1\right)}+\frac{\tan^{-1}(ax)^2}{4a^2c^2}$$

```
[Out] 1/(4*a^2*c^2*(1 + a^2*x^2)) + (x*ArcTan[a*x])/(2*a*c^2*(1 + a^2*x^2)) + Arc
Tan[a*x]^2/(4*a^2*c^2) - ArcTan[a*x]^2/(2*a^2*c^2*(1 + a^2*x^2))
```

Rubi [A] time = 0.069906, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {4930, 4892, 261}

$$\frac{1}{4a^2c^2\left(a^2x^2+1\right)}-\frac{\tan^{-1}(ax)^2}{2a^2c^2\left(a^2x^2+1\right)}+\frac{x\tan^{-1}(ax)}{2ac^2\left(a^2x^2+1\right)}+\frac{\tan^{-1}(ax)^2}{4a^2c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTan[a*x]^2)/(c + a^2*c*x^2)^2,x]
```

```
[Out] 1/(4*a^2*c^2*(1 + a^2*x^2)) + (x*ArcTan[a*x])/(2*a*c^2*(1 + a^2*x^2)) + Arc
Tan[a*x]^2/(4*a^2*c^2) - ArcTan[a*x]^2/(2*a^2*c^2*(1 + a^2*x^2))
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 4892

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[(x*(a + b*ArcTan[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*c*
p)/2, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(a +
 b*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x\tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^2}\,dx = -\frac{\tan^{-1}(ax)^2}{2a^2c^2\left(1+a^2x^2\right)} + \frac{\int \frac{\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^2}\,dx}{a}$$
$$= \frac{x\tan^{-1}(ax)}{2ac^2\left(1+a^2x^2\right)} + \frac{\tan^{-1}(ax)^2}{4a^2c^2} - \frac{\tan^{-1}(ax)^2}{2a^2c^2\left(1+a^2x^2\right)} - \frac{1}{2}\int \frac{x}{\left(c+a^2cx^2\right)^2}\,dx$$
$$= \frac{1}{4a^2c^2\left(1+a^2x^2\right)} + \frac{x\tan^{-1}(ax)}{2ac^2\left(1+a^2x^2\right)} + \frac{\tan^{-1}(ax)^2}{4a^2c^2} - \frac{\tan^{-1}(ax)^2}{2a^2c^2\left(1+a^2x^2\right)}$$

Mathematica [A] time = 0.0314355, size = 47, normalized size = 0.52

$$\frac{\left(a^2x^2-1\right)\tan^{-1}(ax)^2+2ax\tan^{-1}(ax)+1}{4a^2c^2\left(a^2x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcTan[a*x]^2)/(c + a^2*c*x^2)^2,x]
```

```
[Out] (1 + 2*a*x*ArcTan[a*x] + (-1 + a^2*x^2)*ArcTan[a*x]^2)/(4*a^2*c^2*(1 + a^2*
x^2))
```

Maple [A] time = 0.032, size = 84, normalized size = 0.9

$$\frac{1}{4\,a^2c^2\left(a^2x^2+1\right)}+\frac{x\arctan(ax)}{2\,ac^2\left(a^2x^2+1\right)}+\frac{(\arctan(ax))^2}{4\,a^2c^2}-\frac{(\arctan(ax))^2}{2\,a^2c^2\left(a^2x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(a*x)^2/(a^2*c*x^2+c)^2,x)
```

```
[Out] 1/4/a^2/c^2/(a^2*x^2+1)+1/2*x*arctan(a*x)/a/c^2/(a^2*x^2+1)+1/4*arctan(a*x)
^2/a^2/c^2-1/2*arctan(a*x)^2/a^2/c^2/(a^2*x^2+1)
```

Maxima [A] time = 1.52726, size = 140, normalized size = 1.54

$$\frac{\left(\frac{x}{a^2cx^2+c}+\frac{\arctan(ax)}{ac}\right)\arctan\left(ax\right)}{2\,ac}-\frac{\left(a^2x^2+1\right)\arctan\left(ax\right)^2-1}{4\left(a^4cx^2+a^2c\right)c}-\frac{\arctan\left(ax\right)^2}{2\left(a^2cx^2+c\right)a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^2/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(x/(a^2*c*x^2 + c) + arctan(a*x)/(a*c))*arctan(a*x)/(a*c) - 1/4*((a^2*x
^2 + 1)*arctan(a*x)^2 - 1)/((a^4*c*x^2 + a^2*c)*c) - 1/2*arctan(a*x)^2/((a^
2*c*x^2 + c)*a^2*c)
```

Fricas [A] time = 2.17174, size = 112, normalized size = 1.23

$$\frac{2\,ax\arctan\left(ax\right)+\left(a^2x^2-1\right)\arctan\left(ax\right)^2+1}{4\left(a^4c^2x^2+a^2c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^2/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*a*x*arctan(a*x) + (a^2*x^2 - 1)*arctan(a*x)^2 + 1)/(a^4*c^2*x^2 + a^
2*c^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{x\,\mathrm{atan}^2\left(ax\right)}{a^4x^4+2a^2x^2+1}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(a*x)**2/(a**2*c*x**2+c)**2,x)
```

```
[Out] Integral(x*atan(a*x)**2/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \arctan (ax)^2}{\left(a^2 c x^2+c\right)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^2/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(x*arctan(a*x)^2/(a^2*c*x^2 + c)^2, x)
```

3.294 $\int \frac{\tan^{-1}(ax)^2}{(c+a^2cx^2)^2} dx$

Optimal. Leaf size=100

$$-\frac{x}{4c^2(a^2x^2+1)}+\frac{x\tan^{-1}(ax)^2}{2c^2(a^2x^2+1)}+\frac{\tan^{-1}(ax)}{2ac^2(a^2x^2+1)}+\frac{\tan^{-1}(ax)^3}{6ac^2}-\frac{\tan^{-1}(ax)}{4ac^2}$$

```
[Out] -x/(4*c^2*(1 + a^2*x^2)) - ArcTan[a*x]/(4*a*c^2) + ArcTan[a*x]/(2*a*c^2*(1
+ a^2*x^2)) + (x*ArcTan[a*x]^2)/(2*c^2*(1 + a^2*x^2)) + ArcTan[a*x]^3/(6*a*
c^2)
```

Rubi [A] time = 0.0688284, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {4892, 4930, 199, 205}

$$-\frac{x}{4c^2(a^2x^2+1)}+\frac{x\tan^{-1}(ax)^2}{2c^2(a^2x^2+1)}+\frac{\tan^{-1}(ax)}{2ac^2(a^2x^2+1)}+\frac{\tan^{-1}(ax)^3}{6ac^2}-\frac{\tan^{-1}(ax)}{4ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^2/(c + a^2*c*x^2)^2,x]
```

```
[Out] -x/(4*c^2*(1 + a^2*x^2)) - ArcTan[a*x]/(4*a*c^2) + ArcTan[a*x]/(2*a*c^2*(1
+ a^2*x^2)) + (x*ArcTan[a*x]^2)/(2*c^2*(1 + a^2*x^2)) + ArcTan[a*x]^3/(6*a*
c^2)
```

Rule 4892

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[(x*(a + b*ArcTan[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*c*
p)/2, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(a +
 b*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
```

```
0] && NeQ[q, -1]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^2}\,dx &= \frac{x\tan^{-1}(ax)^2}{2c^2\left(1+a^2x^2\right)}+\frac{\tan^{-1}(ax)^3}{6ac^2}-a\int \frac{x\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^2}\,dx \\
&= \frac{\tan^{-1}(ax)}{2ac^2\left(1+a^2x^2\right)}+\frac{x\tan^{-1}(ax)^2}{2c^2\left(1+a^2x^2\right)}+\frac{\tan^{-1}(ax)^3}{6ac^2}-\frac{1}{2}\int \frac{1}{\left(c+a^2cx^2\right)^2}\,dx \\
&= -\frac{x}{4c^2\left(1+a^2x^2\right)}+\frac{\tan^{-1}(ax)}{2ac^2\left(1+a^2x^2\right)}+\frac{x\tan^{-1}(ax)^2}{2c^2\left(1+a^2x^2\right)}+\frac{\tan^{-1}(ax)^3}{6ac^2}-\frac{\int \frac{1}{c+a^2cx^2}\,dx}{4c} \\
&= -\frac{x}{4c^2\left(1+a^2x^2\right)}-\frac{\tan^{-1}(ax)}{4ac^2}+\frac{\tan^{-1}(ax)}{2ac^2\left(1+a^2x^2\right)}+\frac{x\tan^{-1}(ax)^2}{2c^2\left(1+a^2x^2\right)}+\frac{\tan^{-1}(ax)^3}{6ac^2}
\end{aligned}
$$

Mathematica [A] time = 0.0400243, size = 65, normalized size = 0.65

$$
\frac{2\left(a^2x^2+1\right)\tan^{-1}(ax)^3+\left(3-3a^2x^2\right)\tan^{-1}(ax)-3ax+6ax\tan^{-1}(ax)^2}{12c^2\left(a^3x^2+a\right)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]^2/(c + a^2*c*x^2)^2,x]
```

```
[Out] (-3*a*x + (3 - 3*a^2*x^2)*ArcTan[a*x] + 6*a*x*ArcTan[a*x]^2 + 2*(1 + a^2*x^
2)*ArcTan[a*x]^3)/(12*c^2*(a + a^3*x^2))
```

Maple [A] time = 0.029, size = 91, normalized size = 0.9

$$-\frac{x}{4c^{2}\left(a^{2}x^{2}+1\right)}-\frac{\arctan\left(ax\right)}{4ac^{2}}+\frac{\arctan\left(ax\right)}{2ac^{2}\left(a^{2}x^{2}+1\right)}+\frac{x\left(\arctan\left(ax\right)\right)^{2}}{2c^{2}\left(a^{2}x^{2}+1\right)}+\frac{\left(\arctan\left(ax\right)\right)^{3}}{6ac^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^2/(a^2*c*x^2+c)^2,x)
```

```
[Out] -1/4*x/c^2/(a^2*x^2+1)-1/4*arctan(a*x)/a/c^2+1/2*arctan(a*x)/a/c^2/(a^2*x^2
+1)+1/2*x*arctan(a*x)^2/c^2/(a^2*x^2+1)+1/6*arctan(a*x)^3/a/c^2
```

Maxima [A] time = 1.60479, size = 197, normalized size = 1.97

$$\frac{1}{2}\left(\frac{x}{a^{2}c^{2}x^{2}+c^{2}}+\frac{\arctan\left(ax\right)}{ac^{2}}\right)\arctan\left(ax\right)^{2}+\frac{\left(2\left(a^{2}x^{2}+1\right)\arctan\left(ax\right)^{3}-3ax-3\left(a^{2}x^{2}+1\right)\arctan\left(ax\right)\right)a^{2}}{12\left(a^{5}c^{2}x^{2}+a^{3}c^{2}\right)}-\frac{\left(\left(a^{2}x^{2}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(x/(a^2*c^2*x^2 + c^2) + arctan(a*x)/(a*c^2))*arctan(a*x)^2 + 1/12*(2*(
a^2*x^2 + 1)*arctan(a*x)^3 - 3*a*x - 3*(a^2*x^2 + 1)*arctan(a*x))*a^2/(a^5*
c^2*x^2 + a^3*c^2) - 1/2*((a^2*x^2 + 1)*arctan(a*x)^2 - 1)*a*arctan(a*x)/(a
^4*c^2*x^2 + a^2*c^2)
```

Fricas [A] time = 2.12475, size = 162, normalized size = 1.62

$$\frac{6ax\arctan\left(ax\right)^{2}+2\left(a^{2}x^{2}+1\right)\arctan\left(ax\right)^{3}-3ax-3\left(a^{2}x^{2}-1\right)\arctan\left(ax\right)}{12\left(a^{3}c^{2}x^{2}+ac^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(6*a*x*arctan(a*x)^2 + 2*(a^2*x^2 + 1)*arctan(a*x)^3 - 3*a*x - 3*(a^2*
x^2 - 1)*arctan(a*x))/(a^3*c^2*x^2 + a*c^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\operatorname{atan}^2(ax)}{a^4x^4+2a^2x^2+1}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**2/(a**2*c*x**2+c)**2,x)
```

```
[Out] Integral(atan(a*x)**2/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(ax)^2}{\left(a^2cx^2+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)^2/(a^2*c*x^2 + c)^2, x)
```

3.295 $\int \frac{\tan^{-1}(ax)^2}{x\left(c+a^2cx^2\right)^2}\,dx$

Optimal. Leaf size=170

$$\frac{\text{PolyLog}\left(3,-1+\frac{2}{1-iax}\right)}{2c^2}-\frac{i\tan^{-1}(ax)\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{c^2}-\frac{1}{4c^2\left(a^2x^2+1\right)}+\frac{\tan^{-1}(ax)^2}{2c^2\left(a^2x^2+1\right)}-\frac{ax\tan^{-1}(ax)}{2c^2\left(a^2x^2+1\right)}-$$

```
[Out] -1/(4*c^2*(1 + a^2*x^2)) - (a*x*ArcTan[a*x])/(2*c^2*(1 + a^2*x^2)) - ArcTan
[a*x]^2/(4*c^2) + ArcTan[a*x]^2/(2*c^2*(1 + a^2*x^2)) - ((I/3)*ArcTan[a*x]^
3)/c^2 + (ArcTan[a*x]^2*Log[2 - 2/(1 - I*a*x)])/c^2 - (I*ArcTan[a*x]*PolyLo
g[2, -1 + 2/(1 - I*a*x)])/c^2 + PolyLog[3, -1 + 2/(1 - I*a*x)]/(2*c^2)
```

Rubi [A] time = 0.311218, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {4966, 4924, 4868, 4884, 4992, 6610, 4930, 4892, 261}

$$\frac{\text{PolyLog}\left(3,-1+\frac{2}{1-iax}\right)}{2c^2}-\frac{i\tan^{-1}(ax)\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{c^2}-\frac{1}{4c^2\left(a^2x^2+1\right)}+\frac{\tan^{-1}(ax)^2}{2c^2\left(a^2x^2+1\right)}-\frac{ax\tan^{-1}(ax)}{2c^2\left(a^2x^2+1\right)}-$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^2/(x*(c + a^2*c*x^2)^2),x]
```


```
[Out] -1/(4*c^2*(1 + a^2*x^2)) - (a*x*ArcTan[a*x])/(2*c^2*(1 + a^2*x^2)) - ArcTan
[a*x]^2/(4*c^2) + ArcTan[a*x]^2/(2*c^2*(1 + a^2*x^2)) - ((I/3)*ArcTan[a*x]^
3)/c^2 + (ArcTan[a*x]^2*Log[2 - 2/(1 - I*a*x)])/c^2 - (I*ArcTan[a*x]*PolyLo
g[2, -1 + 2/(1 - I*a*x)])/c^2 + PolyLog[3, -1 + 2/(1 - I*a*x)]/(2*c^2)
```


Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*
x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^p
, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*q]
 && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4992

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d
] && EqQ[(1 - u)^2 - (1 - (2*I)/(I + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 4892

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[(x*(a + b*ArcTan[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*c*
p)/2, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(a +
```

```
b*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^2}{x\left(c+a^2cx^2\right)^2}\,dx = -\left(a^2\int \frac{x\tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^2}\,dx\right)+\frac{\int \frac{\tan^{-1}(ax)^2}{x\left(c+a^2cx^2\right)}\,dx}{c}$$

$$= \frac{\tan^{-1}(ax)^2}{2c^2\left(1+a^2x^2\right)}-\frac{i\tan^{-1}(ax)^3}{3c^2}-a\int \frac{\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^2}\,dx+\frac{i\int \frac{\tan^{-1}(ax)^2}{x(i+ax)}\,dx}{c^2}$$

$$= -\frac{ax\tan^{-1}(ax)}{2c^2\left(1+a^2x^2\right)}-\frac{\tan^{-1}(ax)^2}{4c^2}+\frac{\tan^{-1}(ax)^2}{2c^2\left(1+a^2x^2\right)}-\frac{i\tan^{-1}(ax)^3}{3c^2}+\frac{\tan^{-1}(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{c^2}+\frac{1}{2}a$$

$$= -\frac{1}{4c^2\left(1+a^2x^2\right)}-\frac{ax\tan^{-1}(ax)}{2c^2\left(1+a^2x^2\right)}-\frac{\tan^{-1}(ax)^2}{4c^2}+\frac{\tan^{-1}(ax)^2}{2c^2\left(1+a^2x^2\right)}-\frac{i\tan^{-1}(ax)^3}{3c^2}+\frac{\tan^{-1}(ax)^2\log}{c^2}$$

$$= -\frac{1}{4c^2\left(1+a^2x^2\right)}-\frac{ax\tan^{-1}(ax)}{2c^2\left(1+a^2x^2\right)}-\frac{\tan^{-1}(ax)^2}{4c^2}+\frac{\tan^{-1}(ax)^2}{2c^2\left(1+a^2x^2\right)}-\frac{i\tan^{-1}(ax)^3}{3c^2}+\frac{\tan^{-1}(ax)^2\log}{c^2}$$

Mathematica [A] time = 0.198638, size = 119, normalized size = 0.7

$$\frac{24i\tan^{-1}(ax)\text{PolyLog}\left(2,e^{-2i\tan^{-1}(ax)}\right)+12\text{PolyLog}\left(3,e^{-2i\tan^{-1}(ax)}\right)+8i\tan^{-1}(ax)^3+24\tan^{-1}(ax)^2\log\left(1-e^{-2i\tan^{-1}(}\right.}{24c^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTan[a*x]^2/(x*(c + a^2*c*x^2)^2),x]
```

```
[Out] ((-I)*Pi^3 + (8*I)*ArcTan[a*x]^3 - 3*Cos[2*ArcTan[a*x]] + 6*ArcTan[a*x]^2*C
os[2*ArcTan[a*x]] + 24*ArcTan[a*x]^2*Log[1 - E^((-2*I)*ArcTan[a*x])] + (24*
I)*ArcTan[a*x]*PolyLog[2, E^((-2*I)*ArcTan[a*x])] + 12*PolyLog[3, E^((-2*I)
```

*ArcTan[a*x])] - 6*ArcTan[a*x]*Sin[2*ArcTan[a*x]])/(24*c^2)

Maple [C] time = 0.53, size = 1936, normalized size = 11.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arctan(a*x)^2/x/(a^2*c*x^2+c)^2,x)

[Out] -1/2/c^2*arctan(a*x)^2*ln(a^2*x^2+1)+1/c^2*arctan(a*x)^2*ln(2)-1/c^2*arctan(a*x)^2*ln((1+I*a*x)^2/(a^2*x^2+1)-1)+1/c^2*arctan(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))+1/2*I/c^2*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1))*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*arctan(a*x)^2+1/2*arctan(a*x)^2/c^2/(a^2*x^2+1)-1/3*I*arctan(a*x)^3/c^2+1/c^2*arctan(a*x)^2*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))+1/c^2*arctan(a*x)^2*ln((1+I*a*x)/(a^2*x^2+1)^(1/2))-1/c^2*arctan(a*x)/(8*a*x-8*I)-1/c^2*arctan(a*x)/(8*a*x+8*I)+1/16*I/c^2/(a*x-I)-1/16*I/c^2/(a*x+I)+1/c^2*arctan(a*x)^2*ln(a*x)+I/c^2*arctan(a*x)/(8*a*x-8*I)*a*x-1/4*I/c^2*arctan(a*x)^2*Pi*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))^2*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))-1/2*I/c^2*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2*arctan(a*x)^2+1/4*I/c^2*arctan(a*x)^2*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2-1/2*I/c^2*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2*arctan(a*x)^2+1/2*I/c^2*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*arctan(a*x)^2+1/2*I/c^2*arctan(a*x)^2*Pi*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^2+1/4*I/c^2*arctan(a*x)^2*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))^2*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)-1/2*I/c^2*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2*arctan(a*x)^2-1/2*I/c^2*arctan(a*x)^2*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2+1/4*I/c^2*arctan(a*x)^2*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2-I/c^2*arctan(a*x)/(8*a*x+8*I)*a*x+2/c^2*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+2/c^2*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))-1/4*arctan(a*x)^2/c^2-1/4*I/c^2*arctan(a*x)^2*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)+1/16/c^2/(a*x+I)*a*x-2*I/c^2*arctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*I/c^2*arctan(a*x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+1/2*I/c^2*Pi*arctan(a*x)^2+1/16/c^2/(a*x-I)*a*x-1/4*I/c^2*arctan(a*x)^2*Pi*csgn(I*

```
(1+I*a*x)^2/(a^2*x^2+1))^3-1/2*I/c^2*Pi*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((
1+I*a*x)^2/(a^2*x^2+1)+1))^2*arctan(a*x)^2+1/4*I/c^2*arctan(a*x)^2*Pi*csgn(
I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3+1/2*I/c^2*Pi*csgn(((1+I*a*x)^2/(a^2*x^2+
1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^3*arctan(a*x)^2-1/4*I/c^2*arctan(a*x)^2*
Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3+1/2*I/c^
2*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^3*arct
an(a*x)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(ax)^2}{(a^2cx^2+c)^2 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(arctan(a*x)^2/((a^2*c*x^2 + c)^2*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\arctan(ax)^2}{a^4c^2x^5+2\,a^2c^2x^3+c^2x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] integral(arctan(a*x)^2/(a^4*c^2*x^5 + 2*a^2*c^2*x^3 + c^2*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\operatorname{atan}^2(ax)}{a^4x^5+2a^2x^3+x}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**2/x/(a**2*c*x**2+c)**2,x)
```

```
[Out] Integral(atan(a*x)**2/(a**4*x**5 + 2*a**2*x**3 + x), x)/c**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(ax)^2}{(a^2cx^2+c)^2 x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)^2/((a^2*c*x^2 + c)^2*x), x)
```

3.296 $\int \frac{\tan^{-1}(ax)^2}{x^2(c+a^2cx^2)^2}\,dx$

Optimal. Leaf size=177

$$-\frac{ia\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{c^2}+\frac{a^2x}{4c^2\left(a^2x^2+1\right)}-\frac{a^2x\tan^{-1}(ax)^2}{2c^2\left(a^2x^2+1\right)}-\frac{a\tan^{-1}(ax)}{2c^2\left(a^2x^2+1\right)}-\frac{a\tan^{-1}(ax)^3}{2c^2}-\frac{\tan^{-1}(ax)^2}{c^2x}-\frac{ia\tan^{-}}{c^2}$$

```
[Out] (a^2*x)/(4*c^2*(1 + a^2*x^2)) + (a*ArcTan[a*x])/(4*c^2) - (a*ArcTan[a*x])/(
2*c^2*(1 + a^2*x^2)) - (I*a*ArcTan[a*x]^2)/c^2 - ArcTan[a*x]^2/(c^2*x) - (a
^2*x*ArcTan[a*x]^2)/(2*c^2*(1 + a^2*x^2)) - (a*ArcTan[a*x]^3)/(2*c^2) + (2*
a*ArcTan[a*x]*Log[2 - 2/(1 - I*a*x)])/c^2 - (I*a*PolyLog[2, -1 + 2/(1 - I*a
*x)])/c^2
```

Rubi [A] time = 0.339919, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 11, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4966, 4918, 4852, 4924, 4868, 2447, 4884, 4892, 4930, 199, 205}

$$-\frac{ia\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{c^2}+\frac{a^2x}{4c^2\left(a^2x^2+1\right)}-\frac{a^2x\tan^{-1}(ax)^2}{2c^2\left(a^2x^2+1\right)}-\frac{a\tan^{-1}(ax)}{2c^2\left(a^2x^2+1\right)}-\frac{a\tan^{-1}(ax)^3}{2c^2}-\frac{\tan^{-1}(ax)^2}{c^2x}-\frac{ia\tan^{-}}{c^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^2/(x^2*(c + a^2*c*x^2)^2),x]
```


```
[Out] (a^2*x)/(4*c^2*(1 + a^2*x^2)) + (a*ArcTan[a*x])/(4*c^2) - (a*ArcTan[a*x])/(
2*c^2*(1 + a^2*x^2)) - (I*a*ArcTan[a*x]^2)/c^2 - ArcTan[a*x]^2/(c^2*x) - (a
^2*x*ArcTan[a*x]^2)/(2*c^2*(1 + a^2*x^2)) - (a*ArcTan[a*x]^3)/(2*c^2) + (2*
a*ArcTan[a*x]*Log[2 - 2/(1 - I*a*x)])/c^2 - (I*a*PolyLog[2, -1 + 2/(1 - I*a
*x)])/c^2
```


Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*
x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^p
, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*q]
 && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 4918

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/(d + e*x^2),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4892

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[(x*(a + b*ArcTan[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*c*
p)/2, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(a +
 b*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^2}{x^2\left(c+a^2cx^2\right)^2}\,dx = -\left(a^2\int \frac{\tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^2}\,dx\right)+\frac{\int \frac{\tan^{-1}(ax)^2}{x^2\left(c+a^2cx^2\right)}\,dx}{c}$$

$$= -\frac{a^2x\tan^{-1}(ax)^2}{2c^2\left(1+a^2x^2\right)}-\frac{a\tan^{-1}(ax)^3}{6c^2}+a^3\int \frac{x\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^2}\,dx+\frac{\int \frac{\tan^{-1}(ax)^2}{x^2}\,dx}{c^2}-\frac{a^2\int \frac{\tan^{-1}(ax)^2}{c+a^2cx^2}\,dx}{c}$$

$$= -\frac{a\tan^{-1}(ax)}{2c^2\left(1+a^2x^2\right)}-\frac{\tan^{-1}(ax)^2}{c^2x}-\frac{a^2x\tan^{-1}(ax)^2}{2c^2\left(1+a^2x^2\right)}-\frac{a\tan^{-1}(ax)^3}{2c^2}+\frac{1}{2}a^2\int \frac{1}{\left(c+a^2cx^2\right)^2}\,dx+ \frac{(2}{}$$

$$= \frac{a^2x}{4c^2\left(1+a^2x^2\right)}-\frac{a\tan^{-1}(ax)}{2c^2\left(1+a^2x^2\right)}-\frac{ia\tan^{-1}(ax)^2}{c^2}-\frac{\tan^{-1}(ax)^2}{c^2x}-\frac{a^2x\tan^{-1}(ax)^2}{2c^2\left(1+a^2x^2\right)}-\frac{a\tan^{-1}(ax)}{2c^2}$$

$$= \frac{a^2x}{4c^2\left(1+a^2x^2\right)}+\frac{a\tan^{-1}(ax)}{4c^2}-\frac{a\tan^{-1}(ax)}{2c^2\left(1+a^2x^2\right)}-\frac{ia\tan^{-1}(ax)^2}{c^2}-\frac{\tan^{-1}(ax)^2}{c^2x}-\frac{a^2x\tan^{-1}(ax)^2}{2c^2\left(1+a^2x^2\right)}$$

$$= \frac{a^2x}{4c^2\left(1+a^2x^2\right)}+\frac{a\tan^{-1}(ax)}{4c^2}-\frac{a\tan^{-1}(ax)}{2c^2\left(1+a^2x^2\right)}-\frac{ia\tan^{-1}(ax)^2}{c^2}-\frac{\tan^{-1}(ax)^2}{c^2x}-\frac{a^2x\tan^{-1}(ax)^2}{2c^2\left(1+a^2x^2\right)}$$

Mathematica [A] time = 0.325082, size = 109, normalized size = 0.62

$$-\frac{8iax\text{PolyLog}\left(2,e^{2i\tan^{-1}(ax)}\right)+4ax\tan^{-1}(ax)^3+2\tan^{-1}(ax)^2\left(4iax+ax\sin\left(2\tan^{-1}(ax)\right)+4\right)-ax\sin\left(2\tan^{-1}(ax\right.}{8c^2x}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTan[a*x]^2/(x^2*(c + a^2*c*x^2)^2),x]
```

```
[Out] -(4*a*x*ArcTan[a*x]^3 + 2*a*x*ArcTan[a*x]*(Cos[2*ArcTan[a*x]] - 8*Log[1 - E
^((2*I)*ArcTan[a*x])]) + (8*I)*a*x*PolyLog[2, E^((2*I)*ArcTan[a*x])] - a*x*
Sin[2*ArcTan[a*x]] + 2*ArcTan[a*x]^2*(4 + (4*I)*a*x + a*x*Sin[2*ArcTan[a*x]
]))/(8*c^2*x)
```

Maple [B] time = 0.11, size = 369, normalized size = 2.1

$$-\frac{a^2x\left(\arctan(ax)\right)^2}{2c^2\left(a^2x^2+1\right)}-\frac{a\left(\arctan(ax)\right)^3}{2c^2}-\frac{\left(\arctan(ax)\right)^2}{c^2x}-\frac{a\arctan(ax)\ln\left(a^2x^2+1\right)}{c^2}-\frac{a\arctan(ax)}{2c^2\left(a^2x^2+1\right)}+2\frac{a\arctan(}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^2/x^2/(a^2*c*x^2+c)^2,x)
```

```
[Out] -1/2*a^2*x*arctan(a*x)^2/c^2/(a^2*x^2+1)-1/2*a*arctan(a*x)^3/c^2-arctan(a*x
)^2/c^2/x-a/c^2*arctan(a*x)*ln(a^2*x^2+1)-1/2*a*arctan(a*x)/c^2/(a^2*x^2+1)
+2*a/c^2*arctan(a*x)*ln(a*x)+1/4*a^2*x/c^2/(a^2*x^2+1)+1/4*a*arctan(a*x)/c^
2-1/2*I*a/c^2*ln(a^2*x^2+1)*ln(a*x-I)-1/4*I*a/c^2*ln(a*x+I)^2+I*a/c^2*ln(a*
x)*ln(1+I*a*x)+1/2*I*a/c^2*ln(a^2*x^2+1)*ln(a*x+I)-I*a/c^2*dilog(1-I*a*x)+1
/4*I*a/c^2*ln(a*x-I)^2-I*a/c^2*ln(a*x)*ln(1-I*a*x)+I*a/c^2*dilog(1+I*a*x)-1
/2*I*a/c^2*dilog(1/2*I*(a*x-I))+1/2*I*a/c^2*ln(a*x-I)*ln(-1/2*I*(a*x+I))-1/
2*I*a/c^2*ln(a*x+I)*ln(1/2*I*(a*x-I))+1/2*I*a/c^2*dilog(-1/2*I*(a*x+I))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x^2/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\arctan\left(ax\right)^{2}}{a^{4}c^{2}x^{6}+2\,a^{2}c^{2}x^{4}+c^{2}x^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x^2/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] integral(arctan(a*x)^2/(a^4*c^2*x^6 + 2*a^2*c^2*x^4 + c^2*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{\operatorname{atan}^{2}\left(ax\right)}{a^{4}x^{6}+2a^{2}x^{4}+x^{2}}\,dx}{c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**2/x**2/(a**2*c*x**2+c)**2,x)
```

```
[Out] Integral(atan(a*x)**2/(a**4*x**6 + 2*a**2*x**4 + x**2), x)/c**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan{(ax)}^2}{\left(a^2cx^2+c\right)^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x^2/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)^2/((a^2*c*x^2 + c)^2*x^2), x)
```

3.297 $\int \frac{\tan^{-1}(ax)^2}{x^3\left(c+a^2cx^2\right)^2}\,dx$

Optimal. Leaf size=250

$$-\frac{a^2\text{PolyLog}\left(3,-1+\frac{2}{1-iax}\right)}{c^2}+\frac{2ia^2\tan^{-1}(ax)\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{c^2}+\frac{a^2}{4c^2\left(a^2x^2+1\right)}-\frac{a^2\log\left(a^2x^2+1\right)}{2c^2}+\frac{a^3x\tan^{-1}}{2c^2\left(a^2x^2\right.}$$

```
[Out] a^2/(4*c^2*(1 + a^2*x^2)) - (a*ArcTan[a*x])/(c^2*x) + (a^3*x*ArcTan[a*x])/(
2*c^2*(1 + a^2*x^2)) - (a^2*ArcTan[a*x]^2)/(4*c^2) - ArcTan[a*x]^2/(2*c^2*x
^2) - (a^2*ArcTan[a*x]^2)/(2*c^2*(1 + a^2*x^2)) + (((2*I)/3)*a^2*ArcTan[a*x
]^3)/c^2 + (a^2*Log[x])/c^2 - (a^2*Log[1 + a^2*x^2])/(2*c^2) - (2*a^2*ArcTa
n[a*x]^2*Log[2 - 2/(1 - I*a*x)])/c^2 + ((2*I)*a^2*ArcTan[a*x]*PolyLog[2, -1
 + 2/(1 - I*a*x)])/c^2 - (a^2*PolyLog[3, -1 + 2/(1 - I*a*x)])/c^2
```

Rubi [A] time = 0.74027, antiderivative size = 250, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 15, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.682, Rules used = {4966, 4918, 4852, 266, 36, 29, 31, 4884, 4924, 4868, 4992, 6610, 4930, 4892, 261}

$$-\frac{a^2\text{PolyLog}\left(3,-1+\frac{2}{1-iax}\right)}{c^2}+\frac{2ia^2\tan^{-1}(ax)\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{c^2}+\frac{a^2}{4c^2\left(a^2x^2+1\right)}-\frac{a^2\log\left(a^2x^2+1\right)}{2c^2}+\frac{a^3x\tan^{-1}}{2c^2\left(a^2x^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^2/(x^3*(c + a^2*c*x^2)^2),x]
```


```
[Out] a^2/(4*c^2*(1 + a^2*x^2)) - (a*ArcTan[a*x])/(c^2*x) + (a^3*x*ArcTan[a*x])/(
2*c^2*(1 + a^2*x^2)) - (a^2*ArcTan[a*x]^2)/(4*c^2) - ArcTan[a*x]^2/(2*c^2*x
^2) - (a^2*ArcTan[a*x]^2)/(2*c^2*(1 + a^2*x^2)) + (((2*I)/3)*a^2*ArcTan[a*x
]^3)/c^2 + (a^2*Log[x])/c^2 - (a^2*Log[1 + a^2*x^2])/(2*c^2) - (2*a^2*ArcTa
n[a*x]^2*Log[2 - 2/(1 - I*a*x)])/c^2 + ((2*I)*a^2*ArcTan[a*x]*PolyLog[2, -1
 + 2/(1 - I*a*x)])/c^2 - (a^2*PolyLog[3, -1 + 2/(1 - I*a*x)])/c^2
```


Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*
x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^p
```

```
, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*q]
 && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 4918

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/(d + e*x^2),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 4992

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d
] && EqQ[(1 - u)^2 - (1 - (2*I)/(I + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 4892

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[(x*(a + b*ArcTan[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*c*
p)/2, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(a +
 b*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^2}{x^3\left(c+a^2cx^2\right)^2}\,dx = -\left(a^2\int \frac{\tan^{-1}(ax)^2}{x\left(c+a^2cx^2\right)^2}\,dx\right)+\frac{\int \frac{\tan^{-1}(ax)^2}{x^3\left(c+a^2cx^2\right)}\,dx}{c}$$

$$= a^4\int \frac{x\tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^2}\,dx+\frac{\int \frac{\tan^{-1}(ax)^2}{x^3}\,dx}{c^2}-2\frac{a^2\int \frac{\tan^{-1}(ax)^2}{x\left(c+a^2cx^2\right)}\,dx}{c}$$

$$= -\frac{\tan^{-1}(ax)^2}{2c^2x^2}-\frac{a^2\tan^{-1}(ax)^2}{2c^2\left(1+a^2x^2\right)}+a^3\int \frac{\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^2}\,dx+\frac{a\int \frac{\tan^{-1}(ax)}{x^2\left(1+a^2x^2\right)}\,dx}{c^2}-2\left(-\frac{ia^2\tan^{-1}(ax)^3}{3c^2}\right.$$

$$= \frac{a^3x\tan^{-1}(ax)}{2c^2\left(1+a^2x^2\right)}+\frac{a^2\tan^{-1}(ax)^2}{4c^2}-\frac{\tan^{-1}(ax)^2}{2c^2x^2}-\frac{a^2\tan^{-1}(ax)^2}{2c^2\left(1+a^2x^2\right)}-\frac{1}{2}a^4\int \frac{x}{\left(c+a^2cx^2\right)^2}\,dx+\frac{a\int}{}$$

$$= \frac{a^2}{4c^2\left(1+a^2x^2\right)}-\frac{a\tan^{-1}(ax)}{c^2x}+\frac{a^3x\tan^{-1}(ax)}{2c^2\left(1+a^2x^2\right)}-\frac{a^2\tan^{-1}(ax)^2}{4c^2}-\frac{\tan^{-1}(ax)^2}{2c^2x^2}-\frac{a^2\tan^{-1}(ax)^2}{2c^2\left(1+a^2x^2\right)}$$

$$= \frac{a^2}{4c^2\left(1+a^2x^2\right)}-\frac{a\tan^{-1}(ax)}{c^2x}+\frac{a^3x\tan^{-1}(ax)}{2c^2\left(1+a^2x^2\right)}-\frac{a^2\tan^{-1}(ax)^2}{4c^2}-\frac{\tan^{-1}(ax)^2}{2c^2x^2}-\frac{a^2\tan^{-1}(ax)^2}{2c^2\left(1+a^2x^2\right)}$$

$$= \frac{a^2}{4c^2\left(1+a^2x^2\right)}-\frac{a\tan^{-1}(ax)}{c^2x}+\frac{a^3x\tan^{-1}(ax)}{2c^2\left(1+a^2x^2\right)}-\frac{a^2\tan^{-1}(ax)^2}{4c^2}-\frac{\tan^{-1}(ax)^2}{2c^2x^2}-\frac{a^2\tan^{-1}(ax)^2}{2c^2\left(1+a^2x^2\right)}$$

$$= \frac{a^2}{4c^2\left(1+a^2x^2\right)}-\frac{a\tan^{-1}(ax)}{c^2x}+\frac{a^3x\tan^{-1}(ax)}{2c^2\left(1+a^2x^2\right)}-\frac{a^2\tan^{-1}(ax)^2}{4c^2}-\frac{\tan^{-1}(ax)^2}{2c^2x^2}-\frac{a^2\tan^{-1}(ax)^2}{2c^2\left(1+a^2x^2\right)}$$

Mathematica [A] time = 0.618, size = 183, normalized size = 0.73

$$a^2\left(-2i\tan^{-1}(ax)\text{PolyLog}\left(2,e^{-2i\tan^{-1}(ax)}\right)-\text{PolyLog}\left(3,e^{-2i\tan^{-1}(ax)}\right)+\log\left(\frac{ax}{\sqrt{a^2x^2+1}}\right)-\frac{\left(a^2x^2+1\right)\tan^{-1}(ax)^2}{2a^2x^2}-\frac{2}{3}i\tan^{-1}(\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTan[a*x]^2/(x^3*(c + a^2*c*x^2)^2),x]
```

```
[Out] (a^2*((I/12)*Pi^3 - ArcTan[a*x]/(a*x) - ((1 + a^2*x^2)*ArcTan[a*x]^2)/(2*a^
2*x^2) - ((2*I)/3)*ArcTan[a*x]^3 + Cos[2*ArcTan[a*x]]/8 - (ArcTan[a*x]^2*Co
s[2*ArcTan[a*x]])/4 - 2*ArcTan[a*x]^2*Log[1 - E^((-2*I)*ArcTan[a*x])] + Log
[(a*x)/Sqrt[1 + a^2*x^2]] - (2*I)*ArcTan[a*x]*PolyLog[2, E^((-2*I)*ArcTan[a
*x])] - PolyLog[3, E^((-2*I)*ArcTan[a*x])] + (ArcTan[a*x]*Sin[2*ArcTan[a*x]
])/4))/c^2
```

Maple [C] time = 3.573, size = 5115, normalized size = 20.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^2/x^3/(a^2*c*x^2+c)^2,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan\left(ax\right)^{2}}{\left(a^{2}cx^{2}+c\right)^{2}x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x^3/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(arctan(a*x)^2/((a^2*c*x^2 + c)^2*x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\arctan\left(ax\right)^{2}}{a^{4}c^{2}x^{7}+2\,a^{2}c^{2}x^{5}+c^{2}x^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x^3/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] integral(arctan(a*x)^2/(a^4*c^2*x^7 + 2*a^2*c^2*x^5 + c^2*x^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\operatorname{atan}^2(ax)}{a^4x^7+2a^2x^5+x^3}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**2/x**3/(a**2*c*x**2+c)**2,x)
```

```
[Out] Integral(atan(a*x)**2/(a**4*x**7 + 2*a**2*x**5 + x**3), x)/c**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(ax)^2}{\left(a^2cx^2+c\right)^2x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x^3/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)^2/((a^2*c*x^2 + c)^2*x^3), x)
```

3.298 $\int \frac{\tan^{-1}(ax)^2}{x^4\left(c+a^2cx^2\right)^2}\,dx$

Optimal. Leaf size=242

$$\frac{7ia^3\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{3c^2}-\frac{a^4x}{4c^2\left(a^2x^2+1\right)}+\frac{a^4x\tan^{-1}(ax)^2}{2c^2\left(a^2x^2+1\right)}+\frac{a^3\tan^{-1}(ax)}{2c^2\left(a^2x^2+1\right)}-\frac{a^2}{3c^2x}+\frac{5a^3\tan^{-1}(ax)^3}{6c^2}+\frac{7ia^3\tan^{-1}}{3c^2}$$

```
[Out] -a^2/(3*c^2*x) - (a^4*x)/(4*c^2*(1 + a^2*x^2)) - (7*a^3*ArcTan[a*x])/(12*c^
2) - (a*ArcTan[a*x])/(3*c^2*x^2) + (a^3*ArcTan[a*x])/(2*c^2*(1 + a^2*x^2))
+ (((7*I)/3)*a^3*ArcTan[a*x]^2)/c^2 - ArcTan[a*x]^2/(3*c^2*x^3) + (2*a^2*Ar
cTan[a*x]^2)/(c^2*x) + (a^4*x*ArcTan[a*x]^2)/(2*c^2*(1 + a^2*x^2)) + (5*a^3
*ArcTan[a*x]^3)/(6*c^2) - (14*a^3*ArcTan[a*x]*Log[2 - 2/(1 - I*a*x)])/(3*c^
2) + (((7*I)/3)*a^3*PolyLog[2, -1 + 2/(1 - I*a*x)])/c^2
```

Rubi [A] time = 0.87263, antiderivative size = 242, normalized size of antiderivative = 1., number of steps used = 27, number of rules used = 13, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.591, Rules used = {4966, 4918, 4852, 325, 203, 4924, 4868, 2447, 4884, 4892, 4930, 199, 205}

$$\frac{7ia^3\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{3c^2}-\frac{a^4x}{4c^2\left(a^2x^2+1\right)}+\frac{a^4x\tan^{-1}(ax)^2}{2c^2\left(a^2x^2+1\right)}+\frac{a^3\tan^{-1}(ax)}{2c^2\left(a^2x^2+1\right)}-\frac{a^2}{3c^2x}+\frac{5a^3\tan^{-1}(ax)^3}{6c^2}+\frac{7ia^3\tan^{-1}}{3c^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^2/(x^4*(c + a^2*c*x^2)^2),x]
```

```
[Out] -a^2/(3*c^2*x) - (a^4*x)/(4*c^2*(1 + a^2*x^2)) - (7*a^3*ArcTan[a*x])/(12*c^
2) - (a*ArcTan[a*x])/(3*c^2*x^2) + (a^3*ArcTan[a*x])/(2*c^2*(1 + a^2*x^2))
+ (((7*I)/3)*a^3*ArcTan[a*x]^2)/c^2 - ArcTan[a*x]^2/(3*c^2*x^3) + (2*a^2*Ar
cTan[a*x]^2)/(c^2*x) + (a^4*x*ArcTan[a*x]^2)/(2*c^2*(1 + a^2*x^2)) + (5*a^3
*ArcTan[a*x]^3)/(6*c^2) - (14*a^3*ArcTan[a*x]*Log[2 - 2/(1 - I*a*x)])/(3*c^
2) + (((7*I)/3)*a^3*PolyLog[2, -1 + 2/(1 - I*a*x)])/c^2
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*
x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^p
```

```
, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*q]
 && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 4918

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/(d + e*x^2),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
```

```
^2 + e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4892

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[(x*(a + b*ArcTan[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*c*
p)/2, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(a +
 b*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^2}{x^4\left(c+a^2cx^2\right)^2}\,dx = -\left(a^2\int \frac{\tan^{-1}(ax)^2}{x^2\left(c+a^2cx^2\right)^2}\,dx\right) + \frac{\int \frac{\tan^{-1}(ax)^2}{x^4\left(c+a^2cx^2\right)}\,dx}{c}$$

$$= a^4\int \frac{\tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^2}\,dx + \frac{\int \frac{\tan^{-1}(ax)^2}{x^4}\,dx}{c^2} - 2\frac{a^2\int \frac{\tan^{-1}(ax)^2}{x^2\left(c+a^2cx^2\right)}\,dx}{c}$$

$$= -\frac{\tan^{-1}(ax)^2}{3c^2x^3} + \frac{a^4x\tan^{-1}(ax)^2}{2c^2\left(1+a^2x^2\right)} + \frac{a^3\tan^{-1}(ax)^3}{6c^2} - a^5\int \frac{x\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^2}\,dx + \frac{(2a)\int \frac{\tan^{-1}(ax)}{x^3\left(1+a^2x^2\right)}\,dx}{3c^2}$$

$$= \frac{a^3\tan^{-1}(ax)}{2c^2\left(1+a^2x^2\right)} - \frac{\tan^{-1}(ax)^2}{3c^2x^3} + \frac{a^4x\tan^{-1}(ax)^2}{2c^2\left(1+a^2x^2\right)} + \frac{a^3\tan^{-1}(ax)^3}{6c^2} - \frac{1}{2}a^4\int \frac{1}{\left(c+a^2cx^2\right)^2}\,dx + \frac{(2a}{}$$

$$= -\frac{a^4x}{4c^2\left(1+a^2x^2\right)} - \frac{a\tan^{-1}(ax)}{3c^2x^2} + \frac{a^3\tan^{-1}(ax)}{2c^2\left(1+a^2x^2\right)} + \frac{ia^3\tan^{-1}(ax)^2}{3c^2} - \frac{\tan^{-1}(ax)^2}{3c^2x^3} + \frac{a^4x\tan^{-1}(a}{2c^2\left(1+a^2x\right.}$$

$$= -\frac{a^2}{3c^2x} - \frac{a^4x}{4c^2\left(1+a^2x^2\right)} - \frac{a^3\tan^{-1}(ax)}{4c^2} - \frac{a\tan^{-1}(ax)}{3c^2x^2} + \frac{a^3\tan^{-1}(ax)}{2c^2\left(1+a^2x^2\right)} + \frac{ia^3\tan^{-1}(ax)^2}{3c^2} - \text{tan}$$

$$= -\frac{a^2}{3c^2x} - \frac{a^4x}{4c^2\left(1+a^2x^2\right)} - \frac{7a^3\tan^{-1}(ax)}{12c^2} - \frac{a\tan^{-1}(ax)}{3c^2x^2} + \frac{a^3\tan^{-1}(ax)}{2c^2\left(1+a^2x^2\right)} + \frac{ia^3\tan^{-1}(ax)^2}{3c^2} - \text{ta}$$

Mathematica [A] time = 0.419227, size = 166, normalized size = 0.69

$$\frac{56ia^3x^3\text{PolyLog}\left(2,e^{2i\tan^{-1}(ax)}\right) + 20a^3x^3\tan^{-1}(ax)^3 - a^2x^2\left(3ax\sin\left(2\tan^{-1}(ax)\right)+8\right) + \tan^{-1}(ax)^2\left(56ia^3x^3+48a^2x\right.}{24c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTan[a*x]^2/(x^4*(c + a^2*c*x^2)^2),x]
```

```
[Out] (20*a^3*x^3*ArcTan[a*x]^3 + 2*a*x*ArcTan[a*x]*(-4 - 4*a^2*x^2 + 3*a^2*x^2*C
os[2*ArcTan[a*x]] - 56*a^2*x^2*Log[1 - E^((2*I)*ArcTan[a*x])]) + (56*I)*a^3
*x^3*PolyLog[2, E^((2*I)*ArcTan[a*x])] - a^2*x^2*(8 + 3*a*x*Sin[2*ArcTan[a*
x]]) + ArcTan[a*x]^2*(-8 + 48*a^2*x^2 + (56*I)*a^3*x^3 + 6*a^3*x^3*Sin[2*Ar
```

```
cTan[a*x]]))/(24*c^2*x^3)
```

Maple [B] time = 0.12, size = 444, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^2/x^4/(a^2*c*x^2+c)^2,x)
```

```
[Out] 1/2*a^4*x*arctan(a*x)^2/c^2/(a^2*x^2+1)+5/6*a^3*arctan(a*x)^3/c^2-1/3*arcta
n(a*x)^2/c^2/x^3+2*a^2*arctan(a*x)^2/c^2/x+7/3*a^3/c^2*arctan(a*x)*ln(a^2*x
^2+1)+1/2*a^3*arctan(a*x)/c^2/(a^2*x^2+1)-1/3*a*arctan(a*x)/c^2/x^2-14/3*a^
3/c^2*arctan(a*x)*ln(a*x)-7/12*I*a^3/c^2*ln(a*x-I)^2-7/3*I*a^3/c^2*ln(a*x)*
ln(1+I*a*x)+7/6*I*a^3/c^2*dilog(1/2*I*(a*x-I))+7/3*I*a^3/c^2*ln(a*x)*ln(1-I
*a*x)-7/3*I*a^3/c^2*dilog(1+I*a*x)+7/3*I*a^3/c^2*dilog(1-I*a*x)+7/6*I*a^3/c
^2*ln(a*x-I)*ln(a^2*x^2+1)-7/6*I*a^3/c^2*ln(a*x+I)*ln(a^2*x^2+1)+7/12*I*a^3
/c^2*ln(a*x+I)^2-7/6*I*a^3/c^2*dilog(-1/2*I*(a*x+I))-7/6*I*a^3/c^2*ln(a*x-I
)*ln(-1/2*I*(a*x+I))+7/6*I*a^3/c^2*ln(a*x+I)*ln(1/2*I*(a*x-I))-1/4*a^4*x/c^
2/(a^2*x^2+1)-7/12*a^3*arctan(a*x)/c^2-1/3*a^2/c^2/x
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x^4/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\arctan\left(ax\right)^{2}}{a^{4}c^{2}x^{8}+2\,a^{2}c^{2}x^{6}+c^{2}x^{4}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x^4/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] integral(arctan(a*x)^2/(a^4*c^2*x^8 + 2*a^2*c^2*x^6 + c^2*x^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\operatorname{atan}^{2}(ax)}{a^{4}x^{8}+2a^{2}x^{6}+x^{4}}\,dx}{c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**2/x**4/(a**2*c*x**2+c)**2,x)
```

```
[Out] Integral(atan(a*x)**2/(a**4*x**8 + 2*a**2*x**6 + x**4), x)/c**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan\left(ax\right)^{2}}{\left(a^{2}cx^{2}+c\right)^{2}x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x^4/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)^2/((a^2*c*x^2 + c)^2*x^4), x)
```

3.299 $\int \frac{x^3 \tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^3} dx$

Optimal. Leaf size=140

$$-\frac{x^4}{32c^3\left(a^2x^2+1\right)^2}+\frac{3}{32a^4c^3\left(a^2x^2+1\right)}+\frac{x^4\tan^{-1}(ax)^2}{4c^3\left(a^2x^2+1\right)^2}+\frac{x^3\tan^{-1}(ax)}{8ac^3\left(a^2x^2+1\right)^2}+\frac{3x\tan^{-1}(ax)}{16a^3c^3\left(a^2x^2+1\right)}-\frac{3\tan^{-1}(ax)^2}{32a^4c^3}$$

```
[Out] -x^4/(32*c^3*(1 + a^2*x^2)^2) + 3/(32*a^4*c^3*(1 + a^2*x^2)) + (x^3*ArcTan[
a*x])/(8*a*c^3*(1 + a^2*x^2)^2) + (3*x*ArcTan[a*x])/(16*a^3*c^3*(1 + a^2*x^
2)) - (3*ArcTan[a*x]^2)/(32*a^4*c^3) + (x^4*ArcTan[a*x]^2)/(4*c^3*(1 + a^2*
x^2)^2)
```

Rubi [A] time = 0.185193, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4944, 4938, 4934, 4884}

$$-\frac{x^4}{32c^3\left(a^2x^2+1\right)^2}+\frac{3}{32a^4c^3\left(a^2x^2+1\right)}+\frac{x^4\tan^{-1}(ax)^2}{4c^3\left(a^2x^2+1\right)^2}+\frac{x^3\tan^{-1}(ax)}{8ac^3\left(a^2x^2+1\right)^2}+\frac{3x\tan^{-1}(ax)}{16a^3c^3\left(a^2x^2+1\right)}-\frac{3\tan^{-1}(ax)^2}{32a^4c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcTan[a*x]^2)/(c + a^2*c*x^2)^3,x]
```

```
[Out] -x^4/(32*c^3*(1 + a^2*x^2)^2) + 3/(32*a^4*c^3*(1 + a^2*x^2)) + (x^3*ArcTan[
a*x])/(8*a*c^3*(1 + a^2*x^2)^2) + (3*x*ArcTan[a*x])/(16*a^3*c^3*(1 + a^2*x^
2)) - (3*ArcTan[a*x]^2)/(32*a^4*c^3) + (x^4*ArcTan[a*x]^2)/(4*c^3*(1 + a^2*
x^2)^2)
```

Rule 4944

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a +
 b*ArcTan[c*x])^p)/(d*f*(m + 1)), x] - Dist[(b*c*p)/(f*(m + 1)), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rule 4938

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[(b*(f*x)^m*(d + e*x^2)^(q + 1))/(c*d*m^2), x] +
 (Dist[(f^2*(m - 1))/(c^2*d*m), Int[(f*x)^(m - 2)*(d + e*x^2)^(q + 1)*(a +
b*ArcTan[c*x]), x], x] - Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(q + 1)*(a + b*A
rcTan[c*x]))/(c^2*d*m), x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d
] && EqQ[m + 2*q + 2, 0] && LtQ[q, -1]
```

Rule 4934

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^2*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> -Simp[(b*(d + e*x^2)^(q + 1))/(4*c^3*d*(q + 1)^2), x] + (-Dist[
1/(2*c^2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]), x], x] +
Simp[(x*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]))/(2*c^2*d*(q + 1)), x]) /;
FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -5/2]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3\tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^3}\,dx &= \frac{x^4\tan^{-1}(ax)^2}{4c^3\left(1+a^2x^2\right)^2}-\frac{1}{2}a\int \frac{x^4\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^3}\,dx\\ &= -\frac{x^4}{32c^3\left(1+a^2x^2\right)^2}+\frac{x^3\tan^{-1}(ax)}{8ac^3\left(1+a^2x^2\right)^2}+\frac{x^4\tan^{-1}(ax)^2}{4c^3\left(1+a^2x^2\right)^2}-\frac{3\int \frac{x^2\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^2}\,dx}{8ac}\\ &= -\frac{x^4}{32c^3\left(1+a^2x^2\right)^2}+\frac{3}{32a^4c^3\left(1+a^2x^2\right)}+\frac{x^3\tan^{-1}(ax)}{8ac^3\left(1+a^2x^2\right)^2}+\frac{3x\tan^{-1}(ax)}{16a^3c^3\left(1+a^2x^2\right)}+\frac{x^4\tan^{-1}(ax)^2}{4c^3\left(1+a^2x^2\right)}\\ &= -\frac{x^4}{32c^3\left(1+a^2x^2\right)^2}+\frac{3}{32a^4c^3\left(1+a^2x^2\right)}+\frac{x^3\tan^{-1}(ax)}{8ac^3\left(1+a^2x^2\right)^2}+\frac{3x\tan^{-1}(ax)}{16a^3c^3\left(1+a^2x^2\right)}-\frac{3\tan^{-1}(ax)^2}{32a^4c^3}\end{aligned}$$

Mathematica [A] time = 0.0856415, size = 74, normalized size = 0.53

$$\frac{5a^2x^2+2ax\left(5a^2x^2+3\right)\tan^{-1}(ax)+\left(5a^4x^4-6a^2x^2-3\right)\tan^{-1}(ax)^2+4}{32a^4c^3\left(a^2x^2+1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*ArcTan[a*x]^2)/(c + a^2*c*x^2)^3,x]
```

```
[Out] (4 + 5*a^2*x^2 + 2*a*x*(3 + 5*a^2*x^2)*ArcTan[a*x] + (-3 - 6*a^2*x^2 + 5*a^
4*x^4)*ArcTan[a*x]^2)/(32*a^4*c^3*(1 + a^2*x^2)^2)
```

Maple [A] time = 0.047, size = 154, normalized size = 1.1

$$\frac{(\arctan(ax))^2}{4c^3a^4(a^2x^2+1)^2} - \frac{(\arctan(ax))^2}{2c^3a^4(a^2x^2+1)} + \frac{5x^3\arctan(ax)}{16ac^3(a^2x^2+1)^2} + \frac{3\arctan(ax)x}{16c^3a^3(a^2x^2+1)^2} + \frac{5(\arctan(ax))^2}{32c^3a^4} - \frac{1}{32c^3a^4(a^2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctan(a*x)^2/(a^2*c*x^2+c)^3,x)
```

```
[Out] 1/4/a^4/c^3*arctan(a*x)^2/(a^2*x^2+1)^2-1/2/a^4/c^3*arctan(a*x)^2/(a^2*x^2+
1)+5/16*x^3*arctan(a*x)/a/c^3/(a^2*x^2+1)^2+3/16/a^3/c^3*arctan(a*x)*x/(a^2
*x^2+1)^2+5/32*arctan(a*x)^2/a^4/c^3-1/32/a^4/c^3/(a^2*x^2+1)^2+5/32/a^4/c^
3/(a^2*x^2+1)
```

Maxima [A] time = 1.58772, size = 250, normalized size = 1.79

$$\frac{1}{16}a\left(\frac{5a^2x^3+3x}{a^8c^3x^4+2a^6c^3x^2+a^4c^3} + \frac{5\arctan(ax)}{a^5c^3}\right)\arctan(ax) + \frac{\left(5a^2x^2-5\left(a^4x^4+2a^2x^2+1\right)\arctan(ax)^2+4\right)a^2}{32\left(a^{10}c^3x^4+2a^8c^3x^2+a^6c^3\right)} - \frac{(}{4(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^2/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/16*a*((5*a^2*x^3 + 3*x)/(a^8*c^3*x^4 + 2*a^6*c^3*x^2 + a^4*c^3) + 5*arcta
n(a*x)/(a^5*c^3))*arctan(a*x) + 1/32*(5*a^2*x^2 - 5*(a^4*x^4 + 2*a^2*x^2 +
1)*arctan(a*x)^2 + 4)*a^2/(a^10*c^3*x^4 + 2*a^8*c^3*x^2 + a^6*c^3) - 1/4*(2
*a^2*x^2 + 1)*arctan(a*x)^2/(a^8*c^3*x^4 + 2*a^6*c^3*x^2 + a^4*c^3)
```

Fricas [A] time = 2.18483, size = 192, normalized size = 1.37

$$\frac{5\,a^2x^2 + \left(5\,a^4x^4 - 6\,a^2x^2 - 3\right)\arctan\left(ax\right)^2 + 2\left(5\,a^3x^3 + 3\,ax\right)\arctan\left(ax\right) + 4}{32\left(a^8c^3x^4 + 2\,a^6c^3x^2 + a^4c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^2/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 1/32*(5*a^2*x^2 + (5*a^4*x^4 - 6*a^2*x^2 - 3)*arctan(a*x)^2 + 2*(5*a^3*x^3
+ 3*a*x)*arctan(a*x) + 4)/(a^8*c^3*x^4 + 2*a^6*c^3*x^2 + a^4*c^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^3\,\mathrm{atan}^2\left(ax\right)}{a^6x^6+3a^4x^4+3a^2x^2+1}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atan(a*x)**2/(a**2*c*x**2+c)**3,x)
```

```
[Out] Integral(x**3*atan(a*x)**2/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1), x)/
c**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3\arctan\left(ax\right)^2}{\left(a^2cx^2 + c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^2/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] integrate(x^3*arctan(a*x)^2/(a^2*c*x^2 + c)^3, x)
```

3.300 $\int \frac{x^2\tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^3}\,dx$

Optimal. Leaf size=181

$$-\frac{x}{64a^2c^3\left(a^2x^2+1\right)}+\frac{x}{32a^2c^3\left(a^2x^2+1\right)^2}+\frac{x\tan^{-1}(ax)^2}{8a^2c^3\left(a^2x^2+1\right)}-\frac{x\tan^{-1}(ax)^2}{4a^2c^3\left(a^2x^2+1\right)^2}+\frac{\tan^{-1}(ax)}{8a^3c^3\left(a^2x^2+1\right)}-\frac{\tan^{-1}(ax)}{8a^3c^3\left(a^2x^2+1\right)^2}$$

```
[Out] x/(32*a^2*c^3*(1 + a^2*x^2)^2) - x/(64*a^2*c^3*(1 + a^2*x^2)) - ArcTan[a*x]
/(64*a^3*c^3) - ArcTan[a*x]/(8*a^3*c^3*(1 + a^2*x^2)^2) + ArcTan[a*x]/(8*a^
3*c^3*(1 + a^2*x^2)) - (x*ArcTan[a*x]^2)/(4*a^2*c^3*(1 + a^2*x^2)^2) + (x*A
rcTan[a*x]^2)/(8*a^2*c^3*(1 + a^2*x^2)) + ArcTan[a*x]^3/(24*a^3*c^3)
```

Rubi [A] time = 0.266443, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4964, 4892, 4930, 199, 205, 4900}

$$-\frac{x}{64a^2c^3\left(a^2x^2+1\right)}+\frac{x}{32a^2c^3\left(a^2x^2+1\right)^2}+\frac{x\tan^{-1}(ax)^2}{8a^2c^3\left(a^2x^2+1\right)}-\frac{x\tan^{-1}(ax)^2}{4a^2c^3\left(a^2x^2+1\right)^2}+\frac{\tan^{-1}(ax)}{8a^3c^3\left(a^2x^2+1\right)}-\frac{\tan^{-1}(ax)}{8a^3c^3\left(a^2x^2+1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcTan[a*x]^2)/(c + a^2*c*x^2)^3,x]
```

```
[Out] x/(32*a^2*c^3*(1 + a^2*x^2)^2) - x/(64*a^2*c^3*(1 + a^2*x^2)) - ArcTan[a*x]
/(64*a^3*c^3) - ArcTan[a*x]/(8*a^3*c^3*(1 + a^2*x^2)^2) + ArcTan[a*x]/(8*a^
3*c^3*(1 + a^2*x^2)) - (x*ArcTan[a*x]^2)/(4*a^2*c^3*(1 + a^2*x^2)^2) + (x*A
rcTan[a*x]^2)/(8*a^2*c^3*(1 + a^2*x^2)) + ArcTan[a*x]^3/(24*a^3*c^3)
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Arc
Tan[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan[c
*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p
, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rule 4892

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[(x*(a + b*ArcTan[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*c*
```

```
p)/2, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(a +
 b*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 4900

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(b*p*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^(p - 1))/(4*c*d
*(q + 1)^2), x] + (Dist[(2*q + 3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a
 + b*ArcTan[c*x])^p, x], x] - Dist[(b^2*p*(p - 1))/(4*(q + 1)^2), Int[(d +
e*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x] - Simp[(x*(d + e*x^2)^(q + 1)*
(a + b*ArcTan[c*x])^p)/(2*d*(q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] && E
qQ[e, c^2*d] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

Rubi steps

$$\int \frac{x^2 \tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^3}\,dx = -\frac{\int \frac{\tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^3}\,dx}{a^2} + \frac{\int \frac{\tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^2}\,dx}{a^2c}$$

$$= -\frac{\tan^{-1}(ax)}{8a^3c^3\left(1+a^2x^2\right)^2} - \frac{x\tan^{-1}(ax)^2}{4a^2c^3\left(1+a^2x^2\right)^2} + \frac{x\tan^{-1}(ax)^2}{2a^2c^3\left(1+a^2x^2\right)} + \frac{\tan^{-1}(ax)^3}{6a^3c^3} + \frac{\int \frac{1}{\left(c+a^2cx^2\right)^3}\,dx}{8a^2} - \frac{3\int}{}$$

$$= \frac{x}{32a^2c^3\left(1+a^2x^2\right)^2} - \frac{\tan^{-1}(ax)}{8a^3c^3\left(1+a^2x^2\right)^2} + \frac{\tan^{-1}(ax)}{2a^3c^3\left(1+a^2x^2\right)} - \frac{x\tan^{-1}(ax)^2}{4a^2c^3\left(1+a^2x^2\right)^2} + \frac{x\tan^{-1}(ax)^2}{8a^2c^3\left(1+a^2x^2\right.}$$

$$= \frac{x}{32a^2c^3\left(1+a^2x^2\right)^2} - \frac{13x}{64a^2c^3\left(1+a^2x^2\right)} - \frac{\tan^{-1}(ax)}{8a^3c^3\left(1+a^2x^2\right)^2} + \frac{\tan^{-1}(ax)}{8a^3c^3\left(1+a^2x^2\right)} - \frac{x\tan^{-1}(ax)^2}{4a^2c^3\left(1+a^2x^2\right.}$$

$$= \frac{x}{32a^2c^3\left(1+a^2x^2\right)^2} - \frac{x}{64a^2c^3\left(1+a^2x^2\right)} - \frac{13\tan^{-1}(ax)}{64a^3c^3} - \frac{\tan^{-1}(ax)}{8a^3c^3\left(1+a^2x^2\right)^2} + \frac{\tan^{-1}(ax)}{8a^3c^3\left(1+a^2x^2\right)} -$$

$$= \frac{x}{32a^2c^3\left(1+a^2x^2\right)^2} - \frac{x}{64a^2c^3\left(1+a^2x^2\right)} - \frac{\tan^{-1}(ax)}{64a^3c^3} - \frac{\tan^{-1}(ax)}{8a^3c^3\left(1+a^2x^2\right)^2} + \frac{\tan^{-1}(ax)}{8a^3c^3\left(1+a^2x^2\right)} - \frac{}{4a}$$

Mathematica [A] time = 0.108138, size = 95, normalized size = 0.52

$$\frac{-3a^3x^3 + 24ax\left(a^2x^2-1\right)\tan^{-1}(ax)^2 + 8\left(a^2x^2+1\right)^2\tan^{-1}(ax)^3 - 3\left(a^4x^4-6a^2x^2+1\right)\tan^{-1}(ax) + 3ax}{192a^3c^3\left(a^2x^2+1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*ArcTan[a*x]^2)/(c + a^2*c*x^2)^3,x]
```

```
[Out] (3*a*x - 3*a^3*x^3 - 3*(1 - 6*a^2*x^2 + a^4*x^4)*ArcTan[a*x] + 24*a*x*(-1 +
 a^2*x^2)*ArcTan[a*x]^2 + 8*(1 + a^2*x^2)^2*ArcTan[a*x]^3)/(192*a^3*c^3*(1
+ a^2*x^2)^2)
```

Maple [A] time = 0.041, size = 164, normalized size = 0.9

$$\frac{(\arctan(ax))^2 x^3}{8c^3\left(a^2x^2+1\right)^2} - \frac{x(\arctan(ax))^2}{8c^3a^2\left(a^2x^2+1\right)^2} + \frac{(\arctan(ax))^3}{24c^3a^3} - \frac{\arctan(ax)}{8c^3a^3\left(a^2x^2+1\right)^2} + \frac{\arctan(ax)}{8c^3a^3\left(a^2x^2+1\right)} - \frac{x^3}{64c^3\left(a^2x^2+1\right)^2} + \frac{}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(a*x)^2/(a^2*c*x^2+c)^3,x)
```

```
[Out] 1/8/c^3*arctan(a*x)^2*x^3/(a^2*x^2+1)^2-1/8*x*arctan(a*x)^2/a^2/c^3/(a^2*x^
2+1)^2+1/24*arctan(a*x)^3/a^3/c^3-1/8*arctan(a*x)/a^3/c^3/(a^2*x^2+1)^2+1/8
*arctan(a*x)/a^3/c^3/(a^2*x^2+1)-1/64/c^3/(a^2*x^2+1)^2*x^3+1/64*x/a^2/c^3/
(a^2*x^2+1)^2-1/64*arctan(a*x)/a^3/c^3
```

Maxima [A] time = 1.66396, size = 313, normalized size = 1.73

$$\frac{1}{8}\left(\frac{a^2x^3 - x}{a^6c^3x^4 + 2\,a^4c^3x^2 + a^2c^3} + \frac{\arctan\left(ax\right)}{a^3c^3}\right)\arctan\left(ax\right)^2 - \frac{\left(3\,a^3x^3 - 8\left(a^4x^4 + 2\,a^2x^2 + 1\right)\arctan\left(ax\right)^3 - 3\,ax + 3\left(a^4x\right.\right.}{192\left(a^9c^3x^4 + 2\,a^7c^3x^2 + a^5c^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^2/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((a^2*x^3 - x)/(a^6*c^3*x^4 + 2*a^4*c^3*x^2 + a^2*c^3) + arctan(a*x)/(a
^3*c^3))*arctan(a*x)^2 - 1/192*(3*a^3*x^3 - 8*(a^4*x^4 + 2*a^2*x^2 + 1)*arc
tan(a*x)^3 - 3*a*x + 3*(a^4*x^4 + 2*a^2*x^2 + 1)*arctan(a*x))*a^2/(a^9*c^3*
x^4 + 2*a^7*c^3*x^2 + a^5*c^3) + 1/8*(a^2*x^2 - (a^4*x^4 + 2*a^2*x^2 + 1)*a
rctan(a*x)^2)*a*arctan(a*x)/(a^8*c^3*x^4 + 2*a^6*c^3*x^2 + a^4*c^3)
```

Fricas [A] time = 2.14738, size = 255, normalized size = 1.41

$$-\frac{3\,a^3x^3 - 8\left(a^4x^4 + 2\,a^2x^2 + 1\right)\arctan\left(ax\right)^3 - 24\left(a^3x^3 - ax\right)\arctan\left(ax\right)^2 - 3\,ax + 3\left(a^4x^4 - 6\,a^2x^2 + 1\right)\arctan\left(ax\right)}{192\left(a^7c^3x^4 + 2\,a^5c^3x^2 + a^3c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^2/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] -1/192*(3*a^3*x^3 - 8*(a^4*x^4 + 2*a^2*x^2 + 1)*arctan(a*x)^3 - 24*(a^3*x^3
 - a*x)*arctan(a*x)^2 - 3*a*x + 3*(a^4*x^4 - 6*a^2*x^2 + 1)*arctan(a*x))/(a
^7*c^3*x^4 + 2*a^5*c^3*x^2 + a^3*c^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^2 \operatorname{atan}^2(ax)}{a^6x^6+3a^4x^4+3a^2x^2+1}\, dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(a*x)**2/(a**2*c*x**2+c)**3,x)
```

```
[Out] Integral(x**2*atan(a*x)**2/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1), x)/
c**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \arctan(ax)^2}{(a^2cx^2+c)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^2/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] integrate(x^2*arctan(a*x)^2/(a^2*c*x^2 + c)^3, x)
```

3.301 $\int \frac{x\tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^3}\,dx$

Optimal. Leaf size=138

$$\frac{3}{32a^2c^3\left(a^2x^2+1\right)}+\frac{1}{32a^2c^3\left(a^2x^2+1\right)^2}-\frac{\tan^{-1}(ax)^2}{4a^2c^3\left(a^2x^2+1\right)^2}+\frac{3x\tan^{-1}(ax)}{16ac^3\left(a^2x^2+1\right)}+\frac{x\tan^{-1}(ax)}{8ac^3\left(a^2x^2+1\right)^2}+\frac{3\tan^{-1}(ax)^2}{32a^2c^3}$$

```
[Out] 1/(32*a^2*c^3*(1 + a^2*x^2)^2) + 3/(32*a^2*c^3*(1 + a^2*x^2)) + (x*ArcTan[a
*x])/(8*a*c^3*(1 + a^2*x^2)^2) + (3*x*ArcTan[a*x])/(16*a*c^3*(1 + a^2*x^2))
 + (3*ArcTan[a*x]^2)/(32*a^2*c^3) - ArcTan[a*x]^2/(4*a^2*c^3*(1 + a^2*x^2)^
2)
```

Rubi [A] time = 0.0964198, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4930, 4896, 4892, 261}

$$\frac{3}{32a^2c^3\left(a^2x^2+1\right)}+\frac{1}{32a^2c^3\left(a^2x^2+1\right)^2}-\frac{\tan^{-1}(ax)^2}{4a^2c^3\left(a^2x^2+1\right)^2}+\frac{3x\tan^{-1}(ax)}{16ac^3\left(a^2x^2+1\right)}+\frac{x\tan^{-1}(ax)}{8ac^3\left(a^2x^2+1\right)^2}+\frac{3\tan^{-1}(ax)^2}{32a^2c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTan[a*x]^2)/(c + a^2*c*x^2)^3,x]
```

```
[Out] 1/(32*a^2*c^3*(1 + a^2*x^2)^2) + 3/(32*a^2*c^3*(1 + a^2*x^2)) + (x*ArcTan[a
*x])/(8*a*c^3*(1 + a^2*x^2)^2) + (3*x*ArcTan[a*x])/(16*a*c^3*(1 + a^2*x^2))
 + (3*ArcTan[a*x]^2)/(32*a^2*c^3) - ArcTan[a*x]^2/(4*a^2*c^3*(1 + a^2*x^2)^
2)
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 4896

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbol
] :> Simp[(b*(d + e*x^2)^(q + 1))/(4*c*d*(q + 1)^2), x] + (Dist[(2*q + 3)/(
```

```
2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]), x], x] - Simp[(x
*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]))/(2*d*(q + 1)), x]) /; FreeQ[{a, b
, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -3/2]
```

Rule 4892

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[(x*(a + b*ArcTan[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*c*
p)/2, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(a +
 b*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x\tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^3}\,dx = -\frac{\tan^{-1}(ax)^2}{4a^2c^3\left(1+a^2x^2\right)^2}+\frac{\int \frac{\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^3}\,dx}{2a}$$

$$= \frac{1}{32a^2c^3\left(1+a^2x^2\right)^2}+\frac{x\tan^{-1}(ax)}{8ac^3\left(1+a^2x^2\right)^2}-\frac{\tan^{-1}(ax)^2}{4a^2c^3\left(1+a^2x^2\right)^2}+\frac{3\int \frac{\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^2}\,dx}{8ac}$$

$$= \frac{1}{32a^2c^3\left(1+a^2x^2\right)^2}+\frac{x\tan^{-1}(ax)}{8ac^3\left(1+a^2x^2\right)^2}+\frac{3x\tan^{-1}(ax)}{16ac^3\left(1+a^2x^2\right)}+\frac{3\tan^{-1}(ax)^2}{32a^2c^3}-\frac{\tan^{-1}(ax)^2}{4a^2c^3\left(1+a^2x^2\right)^2}-$$

$$= \frac{1}{32a^2c^3\left(1+a^2x^2\right)^2}+\frac{3}{32a^2c^3\left(1+a^2x^2\right)}+\frac{x\tan^{-1}(ax)}{8ac^3\left(1+a^2x^2\right)^2}+\frac{3x\tan^{-1}(ax)}{16ac^3\left(1+a^2x^2\right)}+\frac{3\tan^{-1}(ax)^2}{32a^2c^3}-$$

Mathematica [A] time = 0.0385074, size = 71, normalized size = 0.51

$$\frac{3a^2x^2+2ax\left(3a^2x^2+5\right)\tan^{-1}(ax)+\left(3a^4x^4+6a^2x^2-5\right)\tan^{-1}(ax)^2+4}{32c^3\left(a^3x^2+a\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcTan[a*x]^2)/(c + a^2*c*x^2)^3,x]
```

```
[Out] (4 + 3*a^2*x^2 + 2*a*x*(5 + 3*a^2*x^2)*ArcTan[a*x] + (-5 + 6*a^2*x^2 + 3*a^
4*x^4)*ArcTan[a*x]^2)/(32*c^3*(a + a^3*x^2)^2)
```

Maple [A] time = 0.041, size = 127, normalized size = 0.9

$$\frac{1}{32\,c^3a^2\left(a^2x^2+1\right)^2}+\frac{3}{32\,c^3a^2\left(a^2x^2+1\right)}+\frac{x\arctan\left(ax\right)}{8\,ac^3\left(a^2x^2+1\right)^2}+\frac{3\,x\arctan\left(ax\right)}{16\,ac^3\left(a^2x^2+1\right)}+\frac{3\,\left(\arctan\left(ax\right)\right)^2}{32\,c^3a^2}-\frac{\left(\arctan\left(ax\right)\right)^2}{4\,c^3a^2\left(a^2x^2+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(a*x)^2/(a^2*c*x^2+c)^3,x)
```

```
[Out] 1/32/a^2/c^3/(a^2*x^2+1)^2+3/32/a^2/c^3/(a^2*x^2+1)+1/8*x*arctan(a*x)/a/c^3
/(a^2*x^2+1)^2+3/16*x*arctan(a*x)/a/c^3/(a^2*x^2+1)+3/32*arctan(a*x)^2/a^2/
c^3-1/4*arctan(a*x)^2/a^2/c^3/(a^2*x^2+1)^2
```

Maxima [A] time = 1.58952, size = 220, normalized size = 1.59

$$\frac{\left(\frac{3\,a^2x^3+5\,x}{a^4c^2x^4+2\,a^2c^2x^2+c^2}+\frac{3\,\arctan(ax)}{ac^2}\right)\arctan\left(ax\right)}{16\,ac}+\frac{3\,a^2x^2-3\left(a^4x^4+2\,a^2x^2+1\right)\arctan\left(ax\right)^2+4}{32\left(a^6c^2x^4+2\,a^4c^2x^2+a^2c^2\right)c}-\frac{\arctan\left(ax\right)^2}{4\left(a^2cx^2+c\right)^2a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^2/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/16*((3*a^2*x^3 + 5*x)/(a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2) + 3*arctan(a*x)
/(a*c^2))*arctan(a*x)/(a*c) + 1/32*(3*a^2*x^2 - 3*(a^4*x^4 + 2*a^2*x^2 + 1)
*arctan(a*x)^2 + 4)/((a^6*c^2*x^4 + 2*a^4*c^2*x^2 + a^2*c^2)*c) - 1/4*arcta
n(a*x)^2/((a^2*c*x^2 + c)^2*a^2*c)
```

Fricas [A] time = 2.12659, size = 192, normalized size = 1.39

$$\frac{3\,a^2x^2+\left(3\,a^4x^4+6\,a^2x^2-5\right)\arctan\left(ax\right)^2+2\left(3\,a^3x^3+5\,ax\right)\arctan\left(ax\right)+4}{32\left(a^6c^3x^4+2\,a^4c^3x^2+a^2c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^2/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 1/32*(3*a^2*x^2 + (3*a^4*x^4 + 6*a^2*x^2 - 5)*arctan(a*x)^2 + 2*(3*a^3*x^3
+ 5*a*x)*arctan(a*x) + 4)/(a^6*c^3*x^4 + 2*a^4*c^3*x^2 + a^2*c^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x \operatorname{atan}^2(ax)}{a^6x^6+3a^4x^4+3a^2x^2+1}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(a*x)**2/(a**2*c*x**2+c)**3,x)
```

```
[Out] Integral(x*atan(a*x)**2/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1), x)/c**
3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \arctan(ax)^2}{\left(a^2cx^2+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^2/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] integrate(x*arctan(a*x)^2/(a^2*c*x^2 + c)^3, x)
```

3.302 $\int \frac{\tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^3}\,dx$

Optimal. Leaf size=169

$$-\frac{15x}{64c^3\left(a^2x^2+1\right)}-\frac{x}{32c^3\left(a^2x^2+1\right)^2}+\frac{3x\tan^{-1}(ax)^2}{8c^3\left(a^2x^2+1\right)}+\frac{x\tan^{-1}(ax)^2}{4c^3\left(a^2x^2+1\right)^2}+\frac{3\tan^{-1}(ax)}{8ac^3\left(a^2x^2+1\right)}+\frac{\tan^{-1}(ax)}{8ac^3\left(a^2x^2+1\right)^2}+\frac{\tan^{-1}}{8a}$$

```
[Out] -x/(32*c^3*(1 + a^2*x^2)^2) - (15*x)/(64*c^3*(1 + a^2*x^2)) - (15*ArcTan[a*
x])/(64*a*c^3) + ArcTan[a*x]/(8*a*c^3*(1 + a^2*x^2)^2) + (3*ArcTan[a*x])/(8
*a*c^3*(1 + a^2*x^2)) + (x*ArcTan[a*x]^2)/(4*c^3*(1 + a^2*x^2)^2) + (3*x*Ar
cTan[a*x]^2)/(8*c^3*(1 + a^2*x^2)) + ArcTan[a*x]^3/(8*a*c^3)
```

Rubi [A] time = 0.118644, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {4900, 4892, 4930, 199, 205}

$$-\frac{15x}{64c^3\left(a^2x^2+1\right)}-\frac{x}{32c^3\left(a^2x^2+1\right)^2}+\frac{3x\tan^{-1}(ax)^2}{8c^3\left(a^2x^2+1\right)}+\frac{x\tan^{-1}(ax)^2}{4c^3\left(a^2x^2+1\right)^2}+\frac{3\tan^{-1}(ax)}{8ac^3\left(a^2x^2+1\right)}+\frac{\tan^{-1}(ax)}{8ac^3\left(a^2x^2+1\right)^2}+\frac{\tan^{-1}}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^2/(c + a^2*c*x^2)^3,x]
```

```
[Out] -x/(32*c^3*(1 + a^2*x^2)^2) - (15*x)/(64*c^3*(1 + a^2*x^2)) - (15*ArcTan[a*
x])/(64*a*c^3) + ArcTan[a*x]/(8*a*c^3*(1 + a^2*x^2)^2) + (3*ArcTan[a*x])/(8
*a*c^3*(1 + a^2*x^2)) + (x*ArcTan[a*x]^2)/(4*c^3*(1 + a^2*x^2)^2) + (3*x*Ar
cTan[a*x]^2)/(8*c^3*(1 + a^2*x^2)) + ArcTan[a*x]^3/(8*a*c^3)
```

Rule 4900

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(b*p*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^(p - 1))/(4*c*d
*(q + 1)^2), x] + (Dist[(2*q + 3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a
 + b*ArcTan[c*x])^p, x], x] - Dist[(b^2*p*(p - 1))/(4*(q + 1)^2), Int[(d +
e*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x] - Simp[(x*(d + e*x^2)^(q + 1)*
(a + b*ArcTan[c*x])^p)/(2*d*(q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] && E
qQ[e, c^2*d] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

Rule 4892

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[(x*(a + b*ArcTan[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*c*
p)/2, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(a +
 b*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^3}\,dx = \frac{\tan^{-1}(ax)}{8ac^3\left(1+a^2x^2\right)^2} + \frac{x\tan^{-1}(ax)^2}{4c^3\left(1+a^2x^2\right)^2} - \frac{1}{8}\int \frac{1}{\left(c+a^2cx^2\right)^3}\,dx + \frac{3\int \frac{\tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^2}\,dx}{4c}$$

$$= -\frac{x}{32c^3\left(1+a^2x^2\right)^2} + \frac{\tan^{-1}(ax)}{8ac^3\left(1+a^2x^2\right)^2} + \frac{x\tan^{-1}(ax)^2}{4c^3\left(1+a^2x^2\right)^2} + \frac{3x\tan^{-1}(ax)^2}{8c^3\left(1+a^2x^2\right)} + \frac{\tan^{-1}(ax)^3}{8ac^3} - \frac{3\int \cdots}{\cdots}$$

$$= -\frac{x}{32c^3\left(1+a^2x^2\right)^2} - \frac{3x}{64c^3\left(1+a^2x^2\right)} + \frac{\tan^{-1}(ax)}{8ac^3\left(1+a^2x^2\right)^2} + \frac{3\tan^{-1}(ax)}{8ac^3\left(1+a^2x^2\right)} + \frac{x\tan^{-1}(ax)^2}{4c^3\left(1+a^2x^2\right)^2} + \cdots$$

$$= -\frac{x}{32c^3\left(1+a^2x^2\right)^2} - \frac{15x}{64c^3\left(1+a^2x^2\right)} - \frac{3\tan^{-1}(ax)}{64ac^3} + \frac{\tan^{-1}(ax)}{8ac^3\left(1+a^2x^2\right)^2} + \frac{3\tan^{-1}(ax)}{8ac^3\left(1+a^2x^2\right)} + \cdots$$

$$= -\frac{x}{32c^3\left(1+a^2x^2\right)^2} - \frac{15x}{64c^3\left(1+a^2x^2\right)} - \frac{15\tan^{-1}(ax)}{64ac^3} + \frac{\tan^{-1}(ax)}{8ac^3\left(1+a^2x^2\right)^2} + \frac{3\tan^{-1}(ax)}{8ac^3\left(1+a^2x^2\right)} + \cdots$$

Mathematica [A] time = 0.0447453, size = 98, normalized size = 0.58

$$\frac{-ax\left(15a^2x^2+17\right)+8\left(a^2x^2+1\right)^2\tan^{-1}(ax)^3+8ax\left(3a^2x^2+5\right)\tan^{-1}(ax)^2+\left(-15a^4x^4-6a^2x^2+17\right)\tan^{-1}(ax)}{64ac^3\left(a^2x^2+1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]^2/(c + a^2*c*x^2)^3,x]
```

```
[Out] (-(a*x*(17 + 15*a^2*x^2)) + (17 - 6*a^2*x^2 - 15*a^4*x^4)*ArcTan[a*x] + 8*a
*x*(5 + 3*a^2*x^2)*ArcTan[a*x]^2 + 8*(1 + a^2*x^2)^2*ArcTan[a*x]^3)/(64*a*c
^3*(1 + a^2*x^2)^2)
```

Maple [A] time = 0.041, size = 159, normalized size = 0.9

$$\frac{x(\arctan(ax))^2}{4c^3\left(a^2x^2+1\right)^2} + \frac{3x(\arctan(ax))^2}{8c^3\left(a^2x^2+1\right)} + \frac{(\arctan(ax))^3}{8ac^3} + \frac{\arctan(ax)}{8ac^3\left(a^2x^2+1\right)^2} + \frac{3\arctan(ax)}{8ac^3\left(a^2x^2+1\right)} - \frac{15a^2x^3}{64c^3\left(a^2x^2+1\right)^2} - \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^2/(a^2*c*x^2+c)^3,x)
```

```
[Out] 1/4*x*arctan(a*x)^2/c^3/(a^2*x^2+1)^2+3/8*x*arctan(a*x)^2/c^3/(a^2*x^2+1)+1
/8*arctan(a*x)^3/a/c^3+1/8*arctan(a*x)/a/c^3/(a^2*x^2+1)^2+3/8*arctan(a*x)/
a/c^3/(a^2*x^2+1)-15/64*a^2/c^3/(a^2*x^2+1)^2*x^3-17/64*x/c^3/(a^2*x^2+1)^2
-15/64*arctan(a*x)/a/c^3
```

Maxima [A] time = 1.67147, size = 313, normalized size = 1.85

$$\frac{1}{8}\left(\frac{3\,a^2x^3+5\,x}{a^4c^3x^4+2\,a^2c^3x^2+c^3}+\frac{3\,\arctan\left(ax\right)}{ac^3}\right)\arctan\left(ax\right)^2-\frac{\left(15\,a^3x^3-8\left(a^4x^4+2\,a^2x^2+1\right)\arctan\left(ax\right)^3+17\,ax+15\left(a\right.\right.}{64\left(a^7c^3x^4+2\,a^5c^3x^2+a^3c^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((3*a^2*x^3 + 5*x)/(a^4*c^3*x^4 + 2*a^2*c^3*x^2 + c^3) + 3*arctan(a*x)/
(a*c^3))*arctan(a*x)^2 - 1/64*(15*a^3*x^3 - 8*(a^4*x^4 + 2*a^2*x^2 + 1)*arc
tan(a*x)^3 + 17*a*x + 15*(a^4*x^4 + 2*a^2*x^2 + 1)*arctan(a*x))*a^2/(a^7*c^
3*x^4 + 2*a^5*c^3*x^2 + a^3*c^3) + 1/8*(3*a^2*x^2 - 3*(a^4*x^4 + 2*a^2*x^2
+ 1)*arctan(a*x)^2 + 4)*a*arctan(a*x)/(a^6*c^3*x^4 + 2*a^4*c^3*x^2 + a^2*c^
3)
```

Fricas [A] time = 2.15604, size = 261, normalized size = 1.54

$$-\frac{15\,a^3x^3-8\left(a^4x^4+2\,a^2x^2+1\right)\arctan\left(ax\right)^3-8\left(3\,a^3x^3+5\,ax\right)\arctan\left(ax\right)^2+17\,ax+\left(15\,a^4x^4+6\,a^2x^2-17\right)\arctan\left(\right.}{64\left(a^5c^3x^4+2\,a^3c^3x^2+ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] -1/64*(15*a^3*x^3 - 8*(a^4*x^4 + 2*a^2*x^2 + 1)*arctan(a*x)^3 - 8*(3*a^3*x^
3 + 5*a*x)*arctan(a*x)^2 + 17*a*x + (15*a^4*x^4 + 6*a^2*x^2 - 17)*arctan(a*
x))/(a^5*c^3*x^4 + 2*a^3*c^3*x^2 + a*c^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\operatorname{atan}^2(ax)}{a^6x^6+3a^4x^4+3a^2x^2+1}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**2/(a**2*c*x**2+c)**3,x)
```

```
[Out] Integral(atan(a*x)**2/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1), x)/c**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(ax)^2}{\left(a^2cx^2+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)^2/(a^2*c*x^2 + c)^3, x)
```

3.303 $\int \frac{\tan^{-1}(ax)^2}{x(c+a^2cx^2)^3}\,dx$

Optimal. Leaf size=236

$$\frac{\text{PolyLog}\left(3,-1+\frac{2}{1-iax}\right)}{2c^3}-\frac{i\tan^{-1}(ax)\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{c^3}-\frac{11}{32c^3\left(a^2x^2+1\right)}-\frac{1}{32c^3\left(a^2x^2+1\right)^2}+\frac{\tan^{-1}(ax)^2}{2c^3\left(a^2x^2+1\right)}+$$

```
[Out] -1/(32*c^3*(1 + a^2*x^2)^2) - 11/(32*c^3*(1 + a^2*x^2)) - (a*x*ArcTan[a*x])
/(8*c^3*(1 + a^2*x^2)^2) - (11*a*x*ArcTan[a*x])/(16*c^3*(1 + a^2*x^2)) - (1
1*ArcTan[a*x]^2)/(32*c^3) + ArcTan[a*x]^2/(4*c^3*(1 + a^2*x^2)^2) + ArcTan[
a*x]^2/(2*c^3*(1 + a^2*x^2)) - ((I/3)*ArcTan[a*x]^3)/c^3 + (ArcTan[a*x]^2*L
og[2 - 2/(1 - I*a*x)])/c^3 - (I*ArcTan[a*x]*PolyLog[2, -1 + 2/(1 - I*a*x)])
/c^3 + PolyLog[3, -1 + 2/(1 - I*a*x)]/(2*c^3)
```

Rubi [A] time = 0.484914, antiderivative size = 236, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {4966, 4924, 4868, 4884, 4992, 6610, 4930, 4892, 261, 4896}

$$\frac{\text{PolyLog}\left(3,-1+\frac{2}{1-iax}\right)}{2c^3}-\frac{i\tan^{-1}(ax)\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{c^3}-\frac{11}{32c^3\left(a^2x^2+1\right)}-\frac{1}{32c^3\left(a^2x^2+1\right)^2}+\frac{\tan^{-1}(ax)^2}{2c^3\left(a^2x^2+1\right)}+$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^2/(x*(c + a^2*c*x^2)^3),x]
```


```
[Out] -1/(32*c^3*(1 + a^2*x^2)^2) - 11/(32*c^3*(1 + a^2*x^2)) - (a*x*ArcTan[a*x])
/(8*c^3*(1 + a^2*x^2)^2) - (11*a*x*ArcTan[a*x])/(16*c^3*(1 + a^2*x^2)) - (1
1*ArcTan[a*x]^2)/(32*c^3) + ArcTan[a*x]^2/(4*c^3*(1 + a^2*x^2)^2) + ArcTan[
a*x]^2/(2*c^3*(1 + a^2*x^2)) - ((I/3)*ArcTan[a*x]^3)/c^3 + (ArcTan[a*x]^2*L
og[2 - 2/(1 - I*a*x)])/c^3 - (I*ArcTan[a*x]*PolyLog[2, -1 + 2/(1 - I*a*x)])
/c^3 + PolyLog[3, -1 + 2/(1 - I*a*x)]/(2*c^3)
```


Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*
x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^p
, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*q]
```

```
&& LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4992

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d
] && EqQ[(1 - u)^2 - (1 - (2*I)/(I + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 4892

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[(x*(a + b*ArcTan[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*c*
p)/2, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(a +
 b*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 4896

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbol
] :> Simp[(b*(d + e*x^2)^(q + 1))/(4*c*d*(q + 1)^2), x] + (Dist[(2*q + 3)/(
2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]), x], x] - Simp[(x
*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]))/(2*d*(q + 1)), x]) /; FreeQ[{a, b
, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -3/2]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^2}{x\left(c+a^2cx^2\right)^3}\,dx = -\left(a^2\int \frac{x\tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^3}\,dx\right) + \frac{\int \frac{\tan^{-1}(ax)^2}{x\left(c+a^2cx^2\right)^2}\,dx}{c}$$

$$= \frac{\tan^{-1}(ax)^2}{4c^3\left(1+a^2x^2\right)^2} - \frac{1}{2}a\int \frac{\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^3}\,dx + \frac{\int \frac{\tan^{-1}(ax)^2}{x\left(c+a^2cx^2\right)}\,dx}{c^2} - \frac{a^2\int \frac{x\tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^2}\,dx}{c}$$

$$= -\frac{1}{32c^3\left(1+a^2x^2\right)^2} - \frac{ax\tan^{-1}(ax)}{8c^3\left(1+a^2x^2\right)^2} + \frac{\tan^{-1}(ax)^2}{4c^3\left(1+a^2x^2\right)^2} + \frac{\tan^{-1}(ax)^2}{2c^3\left(1+a^2x^2\right)} - \frac{i\tan^{-1}(ax)^3}{3c^3} + \frac{i\int \frac{\text{ta}}{x}}{}$$

$$= -\frac{1}{32c^3\left(1+a^2x^2\right)^2} - \frac{ax\tan^{-1}(ax)}{8c^3\left(1+a^2x^2\right)^2} - \frac{11ax\tan^{-1}(ax)}{16c^3\left(1+a^2x^2\right)} - \frac{11\tan^{-1}(ax)^2}{32c^3} + \frac{\tan^{-1}(ax)^2}{4c^3\left(1+a^2x^2\right)^2} + \frac{\text{ta}}{2c^3}$$

$$= -\frac{1}{32c^3\left(1+a^2x^2\right)^2} - \frac{11}{32c^3\left(1+a^2x^2\right)} - \frac{ax\tan^{-1}(ax)}{8c^3\left(1+a^2x^2\right)^2} - \frac{11ax\tan^{-1}(ax)}{16c^3\left(1+a^2x^2\right)} - \frac{11\tan^{-1}(ax)^2}{32c^3} + \frac{\text{t}}{4c^3}$$

$$= -\frac{1}{32c^3\left(1+a^2x^2\right)^2} - \frac{11}{32c^3\left(1+a^2x^2\right)} - \frac{ax\tan^{-1}(ax)}{8c^3\left(1+a^2x^2\right)^2} - \frac{11ax\tan^{-1}(ax)}{16c^3\left(1+a^2x^2\right)} - \frac{11\tan^{-1}(ax)^2}{32c^3} + \frac{\text{t}}{4c^3}$$

Mathematica [A] time = 0.255242, size = 156, normalized size = 0.66

$$\frac{768i\tan^{-1}(ax)\text{PolyLog}\left(2,e^{-2i\tan^{-1}(ax)}\right) + 384\text{PolyLog}\left(3,e^{-2i\tan^{-1}(ax)}\right) + 256i\tan^{-1}(ax)^3 + 768\tan^{-1}(ax)^2\log\left(1-e\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTan[a*x]^2/(x*(c + a^2*c*x^2)^3),x]
```

```
[Out] ((-32*I)*Pi^3 + (256*I)*ArcTan[a*x]^3 - 144*Cos[2*ArcTan[a*x]] + 288*ArcTan
[a*x]^2*Cos[2*ArcTan[a*x]] - 3*Cos[4*ArcTan[a*x]] + 24*ArcTan[a*x]^2*Cos[4*
ArcTan[a*x]] + 768*ArcTan[a*x]^2*Log[1 - E^((-2*I)*ArcTan[a*x])] + (768*I)*
ArcTan[a*x]*PolyLog[2, E^((-2*I)*ArcTan[a*x])] + 384*PolyLog[3, E^((-2*I)*A
rcTan[a*x])] - 288*ArcTan[a*x]*Sin[2*ArcTan[a*x]] - 12*ArcTan[a*x]*Sin[4*Ar
cTan[a*x]])/(768*c^3)
```

Maple [C] time = 0.559, size = 2176, normalized size = 9.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^2/x/(a^2*c*x^2+c)^3,x)
```

```
[Out] 1/2*I/c^3*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1))*csgn(I/((1+I*a*x)^2/(a^2*x
^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*a
rctan(a*x)^2-3/32*I/c^3/(a*x+I)+3/32*I/c^3/(a*x-I)-1/512/c^3/(a*x+I)^2*a^2*
x^2-1/512/c^3/(a*x-I)^2*a^2*x^2+3/32/c^3/(a*x+I)*a*x+3/32/c^3/(a*x-I)*a*x-1
/128*I/c^3*arctan(a*x)/(a*x+I)^2+1/128*I/c^3*arctan(a*x)/(a*x-I)^2-2*I/c^3*
arctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+1/2*I/c^3*Pi*arctan(a*x
)^2-2*I/c^3*arctan(a*x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+1/4*I/c^3*Pi
*arctan(a*x)^2*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))^2*csgn(I*((1+I*a*x)^2/(a
^2*x^2+1)+1)^2)+1/4*I/c^3*Pi*arctan(a*x)^2*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+
1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2+1/2*I
/c^3*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*csg
n(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*arctan(a*x)^2+1/
128*I/c^3*arctan(a*x)/(a*x+I)^2*a^2*x^2-3/2*I/c^3*arctan(a*x)/(8*a*x+8*I)*a
*x+3/2*I/c^3*arctan(a*x)/(8*a*x-8*I)*a*x-1/128*I/c^3*arctan(a*x)/(a*x-I)^2*
a^2*x^2-1/3*I*arctan(a*x)^3/c^3+1/4*arctan(a*x)^2/c^3/(a^2*x^2+1)^2+1/2*arc
tan(a*x)^2/c^3/(a^2*x^2+1)-1/2*I/c^3*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1))
*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2*arctan(a
*x)^2-1/2*I/c^3*Pi*arctan(a*x)^2*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I
*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2-1/2*I/c^3*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2
+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2*a
rctan(a*x)^2+1/2*I/c^3*Pi*arctan(a*x)^2*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))
*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^2-1/4*I/c^3*Pi*arctan(a*x)^2*csgn(I*(1+I*a
*x)/(a^2*x^2+1)^(1/2))^2*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))+1/4*I/c^3*Pi*arcta
n(a*x)^2*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1
+I*a*x)^2/(a^2*x^2+1)+1)^2)^2-1/2*I/c^3*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-
1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)
^2/(a^2*x^2+1)+1))^2*arctan(a*x)^2-1/4*I/c^3*Pi*arctan(a*x)^2*csgn(I/((1+I*
a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2
/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)+1/c^3*arctan(a*x)^2*ln(a*x)-1/2
/c^3*arctan(a*x)^2*ln(a^2*x^2+1)+2/c^3*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/
2))+2/c^3*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))+1/512/c^3/(a*x+I)^2+1/512/
c^3/(a*x-I)^2+1/c^3*arctan(a*x)^2*ln(2)-1/c^3*arctan(a*x)^2*ln((1+I*a*x)^2/
(a^2*x^2+1)-1)-11/32*arctan(a*x)^2/c^3-3/2/c^3*arctan(a*x)/(8*a*x+8*I)-3/2/
c^3*arctan(a*x)/(8*a*x-8*I)+1/c^3*arctan(a*x)^2*ln(1+(1+I*a*x)/(a^2*x^2+1)^
(1/2))+1/c^3*arctan(a*x)^2*ln((1+I*a*x)/(a^2*x^2+1)^(1/2))+1/2*I/c^3*Pi*csg
n(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^3*arctan(a*x)^2+
```

```
1/2*I/c^3*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1)
)^3*arctan(a*x)^2-1/4*I/c^3*Pi*arctan(a*x)^2*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)
)^3-1/4*I/c^3*Pi*arctan(a*x)^2*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/
(a^2*x^2+1)+1)^2)^3+1/4*I/c^3*Pi*arctan(a*x)^2*csgn(I*((1+I*a*x)^2/(a^2*x^2
+1)+1)^2)^3+1/256*I/c^3/(a*x+I)^2*a*x-1/256*I/c^3/(a*x-I)^2*a*x-1/2*I/c^3*P
i*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2*arctan(a*
x)^2+1/64/c^3*arctan(a*x)/(a*x+I)^2*a*x+1/64/c^3*arctan(a*x)/(a*x-I)^2*a*x+
1/c^3*arctan(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(ax)^2}{\left(a^2cx^2+c\right)^3x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] integrate(arctan(a*x)^2/((a^2*c*x^2 + c)^3*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\arctan(ax)^2}{a^6c^3x^7+3\,a^4c^3x^5+3\,a^2c^3x^3+c^3x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] integral(arctan(a*x)^2/(a^6*c^3*x^7 + 3*a^4*c^3*x^5 + 3*a^2*c^3*x^3 + c^3*x
), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\operatorname{atan}^2(ax)}{a^6x^7+3a^4x^5+3a^2x^3+x}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**2/x/(a**2*c*x**2+c)**3,x)
```

```
[Out] Integral(atan(a*x)**2/(a**6*x**7 + 3*a**4*x**5 + 3*a**2*x**3 + x), x)/c**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(ax)^2}{\left(a^2cx^2+c\right)^3 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)^2/((a^2*c*x^2 + c)^3*x), x)
```

3.304 $\int \frac{\tan^{-1}(ax)^2}{x^2\left(c+a^2cx^2\right)^3}\,dx$

Optimal. Leaf size=250

$$-\frac{ia\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{c^3}+\frac{31a^2x}{64c^3\left(a^2x^2+1\right)}+\frac{a^2x}{32c^3\left(a^2x^2+1\right)^2}-\frac{7a^2x\tan^{-1}(ax)^2}{8c^3\left(a^2x^2+1\right)}-\frac{a^2x\tan^{-1}(ax)^2}{4c^3\left(a^2x^2+1\right)^2}-\frac{7a\tan^{-1}(ax}{8c^3(a^2x^2+1}$$

```
[Out] (a^2*x)/(32*c^3*(1 + a^2*x^2)^2) + (31*a^2*x)/(64*c^3*(1 + a^2*x^2)) + (31*
a*ArcTan[a*x])/(64*c^3) - (a*ArcTan[a*x])/(8*c^3*(1 + a^2*x^2)^2) - (7*a*Ar
cTan[a*x])/(8*c^3*(1 + a^2*x^2)) - (I*a*ArcTan[a*x]^2)/c^3 - ArcTan[a*x]^2/
(c^3*x) - (a^2*x*ArcTan[a*x]^2)/(4*c^3*(1 + a^2*x^2)^2) - (7*a^2*x*ArcTan[a
*x]^2)/(8*c^3*(1 + a^2*x^2)) - (5*a*ArcTan[a*x]^3)/(8*c^3) + (2*a*ArcTan[a*
x]*Log[2 - 2/(1 - I*a*x)])/c^3 - (I*a*PolyLog[2, -1 + 2/(1 - I*a*x)])/c^3
```

Rubi [A] time = 0.551444, antiderivative size = 250, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 12, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {4966, 4918, 4852, 4924, 4868, 2447, 4884, 4892, 4930, 199, 205, 4900}

$$-\frac{ia\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{c^3}+\frac{31a^2x}{64c^3\left(a^2x^2+1\right)}+\frac{a^2x}{32c^3\left(a^2x^2+1\right)^2}-\frac{7a^2x\tan^{-1}(ax)^2}{8c^3\left(a^2x^2+1\right)}-\frac{a^2x\tan^{-1}(ax)^2}{4c^3\left(a^2x^2+1\right)^2}-\frac{7a\tan^{-1}(ax}{8c^3(a^2x^2+1}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^2/(x^2*(c + a^2*c*x^2)^3),x]
```

```
[Out] (a^2*x)/(32*c^3*(1 + a^2*x^2)^2) + (31*a^2*x)/(64*c^3*(1 + a^2*x^2)) + (31*
a*ArcTan[a*x])/(64*c^3) - (a*ArcTan[a*x])/(8*c^3*(1 + a^2*x^2)^2) - (7*a*Ar
cTan[a*x])/(8*c^3*(1 + a^2*x^2)) - (I*a*ArcTan[a*x]^2)/c^3 - ArcTan[a*x]^2/
(c^3*x) - (a^2*x*ArcTan[a*x]^2)/(4*c^3*(1 + a^2*x^2)^2) - (7*a^2*x*ArcTan[a
*x]^2)/(8*c^3*(1 + a^2*x^2)) - (5*a*ArcTan[a*x]^3)/(8*c^3) + (2*a*ArcTan[a*
x]*Log[2 - 2/(1 - I*a*x)])/c^3 - (I*a*PolyLog[2, -1 + 2/(1 - I*a*x)])/c^3
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*
x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^p
, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*q]
```

```
&& LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 4918

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/(d + e*x^2),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4892

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[(x*(a + b*ArcTan[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*c*
p)/2, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(a +
 b*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 4900

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(b*p*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^(p - 1))/(4*c*d
*(q + 1)^2), x] + (Dist[(2*q + 3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a
 + b*ArcTan[c*x])^p, x], x] - Dist[(b^2*p*(p - 1))/(4*(q + 1)^2), Int[(d +
e*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x] - Simp[(x*(d + e*x^2)^(q + 1)*
(a + b*ArcTan[c*x])^p)/(2*d*(q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] && E
qQ[e, c^2*d] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^2}{x^2\left(c+a^2cx^2\right)^3}\,dx = -\left(a^2\int \frac{\tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^3}\,dx\right) + \frac{\int \frac{\tan^{-1}(ax)^2}{x^2\left(c+a^2cx^2\right)^2}\,dx}{c}$$

$$= -\frac{a\tan^{-1}(ax)}{8c^3\left(1+a^2x^2\right)^2} - \frac{a^2x\tan^{-1}(ax)^2}{4c^3\left(1+a^2x^2\right)^2} + \frac{1}{8}a^2\int \frac{1}{\left(c+a^2cx^2\right)^3}\,dx + \frac{\int \frac{\tan^{-1}(ax)^2}{x^2\left(c+a^2cx^2\right)}\,dx}{c^2} - \frac{\left(3a^2\right)\int \frac{\tan^{-1}(a}{(c+a^2cx}}{4c}$$

$$= \frac{a^2x}{32c^3\left(1+a^2x^2\right)^2} - \frac{a\tan^{-1}(ax)}{8c^3\left(1+a^2x^2\right)^2} - \frac{a^2x\tan^{-1}(ax)^2}{4c^3\left(1+a^2x^2\right)^2} - \frac{7a^2x\tan^{-1}(ax)^2}{8c^3\left(1+a^2x^2\right)} - \frac{7a\tan^{-1}(ax)^3}{24c^3} + \frac{\int \frac{\tan}{}}{}$$

$$= \frac{a^2x}{32c^3\left(1+a^2x^2\right)^2} + \frac{3a^2x}{64c^3\left(1+a^2x^2\right)} - \frac{a\tan^{-1}(ax)}{8c^3\left(1+a^2x^2\right)^2} - \frac{7a\tan^{-1}(ax)}{8c^3\left(1+a^2x^2\right)} - \frac{\tan^{-1}(ax)^2}{c^3x} - \frac{a^2x\tan^{-}}{4c^3\left(1+\right.}$$

$$= \frac{a^2x}{32c^3\left(1+a^2x^2\right)^2} + \frac{31a^2x}{64c^3\left(1+a^2x^2\right)} + \frac{3a\tan^{-1}(ax)}{64c^3} - \frac{a\tan^{-1}(ax)}{8c^3\left(1+a^2x^2\right)^2} - \frac{7a\tan^{-1}(ax)}{8c^3\left(1+a^2x^2\right)} - \frac{ia\tan^{-}}{c}$$

$$= \frac{a^2x}{32c^3\left(1+a^2x^2\right)^2} + \frac{31a^2x}{64c^3\left(1+a^2x^2\right)} + \frac{31a\tan^{-1}(ax)}{64c^3} - \frac{a\tan^{-1}(ax)}{8c^3\left(1+a^2x^2\right)^2} - \frac{7a\tan^{-1}(ax)}{8c^3\left(1+a^2x^2\right)} - \frac{ia\tan}{}$$

$$= \frac{a^2x}{32c^3\left(1+a^2x^2\right)^2} + \frac{31a^2x}{64c^3\left(1+a^2x^2\right)} + \frac{31a\tan^{-1}(ax)}{64c^3} - \frac{a\tan^{-1}(ax)}{8c^3\left(1+a^2x^2\right)^2} - \frac{7a\tan^{-1}(ax)}{8c^3\left(1+a^2x^2\right)} - \frac{ia\tan}{}$$

Mathematica [A] time = 0.400255, size = 139, normalized size = 0.56

$$-\frac{256iax\text{PolyLog}\left(2,e^{2i\tan^{-1}(ax)}\right) + 160ax\tan^{-1}(ax)^3 + 8\tan^{-1}(ax)^2\left(32iax + 16ax\sin\left(2\tan^{-1}(ax)\right) + ax\sin\left(4\tan^{-1}(a\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTan[a*x]^2/(x^2*(c + a^2*c*x^2)^3),x]
```

```
[Out] -(160*a*x*ArcTan[a*x]^3 + 4*a*x*ArcTan[a*x]*(32*Cos[2*ArcTan[a*x]] + Cos[4*
ArcTan[a*x]] - 128*Log[1 - E^((2*I)*ArcTan[a*x])]) + (256*I)*a*x*PolyLog[2,
 E^((2*I)*ArcTan[a*x])] - a*x*(64*Sin[2*ArcTan[a*x]] + Sin[4*ArcTan[a*x]])
+ 8*ArcTan[a*x]^2*(32 + (32*I)*a*x + 16*a*x*Sin[2*ArcTan[a*x]] + a*x*Sin[4*
ArcTan[a*x]]))/(256*c^3*x)
```

Maple [A] time = 0.115, size = 440, normalized size = 1.8

$$-\frac{7\left(\arctan\left(ax\right)\right)^{2}a^{4}x^{3}}{8c^{3}\left(a^{2}x^{2}+1\right)^{2}}-\frac{9a^{2}x\left(\arctan\left(ax\right)\right)^{2}}{8c^{3}\left(a^{2}x^{2}+1\right)^{2}}-\frac{5a\left(\arctan\left(ax\right)\right)^{3}}{8c^{3}}-\frac{\left(\arctan\left(ax\right)\right)^{2}}{c^{3}x}-\frac{a\arctan\left(ax\right)\ln\left(a^{2}x^{2}+1\right)}{c^{3}}-\frac{}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^2/x^2/(a^2*c*x^2+c)^3,x)
```

```
[Out] -7/8/c^3*arctan(a*x)^2*a^4*x^3/(a^2*x^2+1)^2-9/8*a^2*x*arctan(a*x)^2/c^3/(a
^2*x^2+1)^2-5/8*a*arctan(a*x)^3/c^3-arctan(a*x)^2/c^3/x-a/c^3*arctan(a*x)*l
n(a^2*x^2+1)-1/8*a*arctan(a*x)/c^3/(a^2*x^2+1)^2-7/8*a*arctan(a*x)/c^3/(a^2
*x^2+1)+2*a/c^3*arctan(a*x)*ln(a*x)-1/4*I*a/c^3*ln(a*x+I)^2+1/4*I*a/c^3*ln(
a*x-I)^2+1/2*I*a/c^3*ln(a*x+I)*ln(a^2*x^2+1)+I*a/c^3*ln(a*x)*ln(1+I*a*x)-1/
2*I*a/c^3*ln(a*x+I)*ln(1/2*I*(a*x-I))-I*a/c^3*ln(a*x)*ln(1-I*a*x)-1/2*I*a/c
^3*dilog(1/2*I*(a*x-I))-I*a/c^3*dilog(1-I*a*x)+1/2*I*a/c^3*ln(a*x-I)*ln(-1/
2*I*(a*x+I))+I*a/c^3*dilog(1+I*a*x)-1/2*I*a/c^3*ln(a*x-I)*ln(a^2*x^2+1)+1/2
*I*a/c^3*dilog(-1/2*I*(a*x+I))+31/64/c^3/(a^2*x^2+1)^2*x^3*a^4+33/64*a^2*x/
c^3/(a^2*x^2+1)^2+31/64*a*arctan(a*x)/c^3
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x^2/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\arctan\left(ax\right)^{2}}{a^{6}c^{3}x^{8}+3\,a^{4}c^{3}x^{6}+3\,a^{2}c^{3}x^{4}+c^{3}x^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x^2/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] integral(arctan(a*x)^2/(a^6*c^3*x^8 + 3*a^4*c^3*x^6 + 3*a^2*c^3*x^4 + c^3*x
^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\operatorname{atan}^{2}(ax)}{a^{6}x^{8}+3a^{4}x^{6}+3a^{2}x^{4}+x^{2}}\,dx}{c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**2/x**2/(a**2*c*x**2+c)**3,x)
```

```
[Out] Integral(atan(a*x)**2/(a**6*x**8 + 3*a**4*x**6 + 3*a**2*x**4 + x**2), x)/c*
*3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(ax)^{2}}{\left(a^{2}cx^{2}+c\right)^{3}x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x^2/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)^2/((a^2*c*x^2 + c)^3*x^2), x)
```

3.305 $\int \frac{\tan^{-1}(ax)^2}{x^3\left(c+a^2cx^2\right)^3}\,dx$

Optimal. Leaf size=322

$$-\frac{3a^2\text{PolyLog}\left(3,-1+\frac{2}{1-iax}\right)}{2c^3}+\frac{3ia^2\tan^{-1}(ax)\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{c^3}+\frac{19a^2}{32c^3\left(a^2x^2+1\right)}+\frac{a^2}{32c^3\left(a^2x^2+1\right)^2}-\frac{a^2\log}{}$$

```
[Out] a^2/(32*c^3*(1 + a^2*x^2)^2) + (19*a^2)/(32*c^3*(1 + a^2*x^2)) - (a*ArcTan[
a*x])/(c^3*x) + (a^3*x*ArcTan[a*x])/(8*c^3*(1 + a^2*x^2)^2) + (19*a^3*x*Arc
Tan[a*x])/(16*c^3*(1 + a^2*x^2)) + (3*a^2*ArcTan[a*x]^2)/(32*c^3) - ArcTan[
a*x]^2/(2*c^3*x^2) - (a^2*ArcTan[a*x]^2)/(4*c^3*(1 + a^2*x^2)^2) - (a^2*Arc
Tan[a*x]^2)/(c^3*(1 + a^2*x^2)) + (I*a^2*ArcTan[a*x]^3)/c^3 + (a^2*Log[x])/
c^3 - (a^2*Log[1 + a^2*x^2])/(2*c^3) - (3*a^2*ArcTan[a*x]^2*Log[2 - 2/(1 -
I*a*x)])/c^3 + ((3*I)*a^2*ArcTan[a*x]*PolyLog[2, -1 + 2/(1 - I*a*x)])/c^3 -
 (3*a^2*PolyLog[3, -1 + 2/(1 - I*a*x)])/(2*c^3)
```

Rubi [A] time = 1.33344, antiderivative size = 322, normalized size of antiderivative = 1., number of steps used = 36, number of rules used = 16, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.727, Rules used = {4966, 4918, 4852, 266, 36, 29, 31, 4884, 4924, 4868, 4992, 6610, 4930, 4892, 261, 4896}

$$-\frac{3a^2\text{PolyLog}\left(3,-1+\frac{2}{1-iax}\right)}{2c^3}+\frac{3ia^2\tan^{-1}(ax)\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{c^3}+\frac{19a^2}{32c^3\left(a^2x^2+1\right)}+\frac{a^2}{32c^3\left(a^2x^2+1\right)^2}-\frac{a^2\log}{}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^2/(x^3*(c + a^2*c*x^2)^3),x]
```


```
[Out] a^2/(32*c^3*(1 + a^2*x^2)^2) + (19*a^2)/(32*c^3*(1 + a^2*x^2)) - (a*ArcTan[
a*x])/(c^3*x) + (a^3*x*ArcTan[a*x])/(8*c^3*(1 + a^2*x^2)^2) + (19*a^3*x*Arc
Tan[a*x])/(16*c^3*(1 + a^2*x^2)) + (3*a^2*ArcTan[a*x]^2)/(32*c^3) - ArcTan[
a*x]^2/(2*c^3*x^2) - (a^2*ArcTan[a*x]^2)/(4*c^3*(1 + a^2*x^2)^2) - (a^2*Arc
Tan[a*x]^2)/(c^3*(1 + a^2*x^2)) + (I*a^2*ArcTan[a*x]^3)/c^3 + (a^2*Log[x])/
c^3 - (a^2*Log[1 + a^2*x^2])/(2*c^3) - (3*a^2*ArcTan[a*x]^2*Log[2 - 2/(1 -
I*a*x)])/c^3 + ((3*I)*a^2*ArcTan[a*x]*PolyLog[2, -1 + 2/(1 - I*a*x)])/c^3 -
 (3*a^2*PolyLog[3, -1 + 2/(1 - I*a*x)])/(2*c^3)
```


Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*
x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^p
, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*q]
 && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 4918

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/(d + e*x^2),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 4992

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d
] && EqQ[(1 - u)^2 - (1 - (2*I)/(I + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 4892

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[(x*(a + b*ArcTan[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*c*
p)/2, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(a +
```

```
 b*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 4896

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbol
] :> Simp[(b*(d + e*x^2)^(q + 1))/(4*c*d*(q + 1)^2), x] + (Dist[(2*q + 3)/(
2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]), x], x] - Simp[(x
*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]))/(2*d*(q + 1)), x]) /; FreeQ[{a, b
, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -3/2]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^2}{x^3\left(c+a^2cx^2\right)^3}\,dx = -\left(a^2\int \frac{\tan^{-1}(ax)^2}{x\left(c+a^2cx^2\right)^3}\,dx\right) + \frac{\int \frac{\tan^{-1}(ax)^2}{x^3\left(c+a^2cx^2\right)^2}\,dx}{c}$$

$$= a^4\int \frac{x\tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^3}\,dx + \frac{\int \frac{\tan^{-1}(ax)^2}{x^3\left(c+a^2cx^2\right)}\,dx}{c^2} - 2\frac{a^2\int \frac{\tan^{-1}(ax)^2}{x\left(c+a^2cx^2\right)^2}\,dx}{c}$$

$$= -\frac{a^2\tan^{-1}(ax)^2}{4c^3\left(1+a^2x^2\right)^2} + \frac{1}{2}a^3\int \frac{\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^3}\,dx + \frac{\int \frac{\tan^{-1}(ax)^2}{x^3}\,dx}{c^3} - \frac{a^2\int \frac{\tan^{-1}(ax)^2}{x\left(c+a^2cx^2\right)}\,dx}{c^2} - 2\left(\frac{a^2\int \frac{\tan^{-1}}{x(c+a}}{c^2}\right.$$

$$= \frac{a^2}{32c^3\left(1+a^2x^2\right)^2} + \frac{a^3x\tan^{-1}(ax)}{8c^3\left(1+a^2x^2\right)^2} - \frac{\tan^{-1}(ax)^2}{2c^3x^2} - \frac{a^2\tan^{-1}(ax)^2}{4c^3\left(1+a^2x^2\right)^2} + \frac{ia^2\tan^{-1}(ax)^3}{3c^3} + \frac{a\int \frac{\tan}{x^2(}}{}$$

$$= \frac{a^2}{32c^3\left(1+a^2x^2\right)^2} + \frac{a^3x\tan^{-1}(ax)}{8c^3\left(1+a^2x^2\right)^2} + \frac{3a^3x\tan^{-1}(ax)}{16c^3\left(1+a^2x^2\right)} + \frac{3a^2\tan^{-1}(ax)^2}{32c^3} - \frac{\tan^{-1}(ax)^2}{2c^3x^2} - \frac{a^2\tan}{4c^3(1}$$

$$= \frac{a^2}{32c^3\left(1+a^2x^2\right)^2} + \frac{3a^2}{32c^3\left(1+a^2x^2\right)} - \frac{a\tan^{-1}(ax)}{c^3x} + \frac{a^3x\tan^{-1}(ax)}{8c^3\left(1+a^2x^2\right)^2} + \frac{3a^3x\tan^{-1}(ax)}{16c^3\left(1+a^2x^2\right)} - \frac{13a^2}{}$$

$$= \frac{a^2}{32c^3\left(1+a^2x^2\right)^2} + \frac{3a^2}{32c^3\left(1+a^2x^2\right)} - \frac{a\tan^{-1}(ax)}{c^3x} + \frac{a^3x\tan^{-1}(ax)}{8c^3\left(1+a^2x^2\right)^2} + \frac{3a^3x\tan^{-1}(ax)}{16c^3\left(1+a^2x^2\right)} - \frac{13a^2}{}$$

$$= \frac{a^2}{32c^3\left(1+a^2x^2\right)^2} + \frac{3a^2}{32c^3\left(1+a^2x^2\right)} - \frac{a\tan^{-1}(ax)}{c^3x} + \frac{a^3x\tan^{-1}(ax)}{8c^3\left(1+a^2x^2\right)^2} + \frac{3a^3x\tan^{-1}(ax)}{16c^3\left(1+a^2x^2\right)} - \frac{13a^2}{}$$

$$= \frac{a^2}{32c^3\left(1+a^2x^2\right)^2} + \frac{3a^2}{32c^3\left(1+a^2x^2\right)} - \frac{a\tan^{-1}(ax)}{c^3x} + \frac{a^3x\tan^{-1}(ax)}{8c^3\left(1+a^2x^2\right)^2} + \frac{3a^3x\tan^{-1}(ax)}{16c^3\left(1+a^2x^2\right)} - \frac{13a^2}{}$$

Mathematica [A] time = 0.771168, size = 226, normalized size = 0.7

$$a^2\left(-3i\tan^{-1}(ax)\text{PolyLog}\left(2,e^{-2i\tan^{-1}(ax)}\right) - \frac{3}{2}\text{PolyLog}\left(3,e^{-2i\tan^{-1}(ax)}\right) + \log\left(\frac{ax}{\sqrt{a^2x^2+1}}\right) - \frac{\left(a^2x^2+1\right)\tan^{-1}(ax)^2}{2a^2x^2} - i\tan^{-1}(a\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTan[a*x]^2/(x^3*(c + a^2*c*x^2)^3),x]
```

```
[Out] (a^2*((I/8)*Pi^3 - ArcTan[a*x]/(a*x) - ((1 + a^2*x^2)*ArcTan[a*x]^2)/(2*a^2
*x^2) - I*ArcTan[a*x]^3 + (5*Cos[2*ArcTan[a*x]])/16 - (5*ArcTan[a*x]^2*Cos[
2*ArcTan[a*x]])/8 + Cos[4*ArcTan[a*x]]/256 - (ArcTan[a*x]^2*Cos[4*ArcTan[a*
x]])/32 - 3*ArcTan[a*x]^2*Log[1 - E^((-2*I)*ArcTan[a*x])] + Log[(a*x)/Sqrt[
1 + a^2*x^2]] - (3*I)*ArcTan[a*x]*PolyLog[2, E^((-2*I)*ArcTan[a*x])] - (3*P
olyLog[3, E^((-2*I)*ArcTan[a*x])])/2 + (5*ArcTan[a*x]*Sin[2*ArcTan[a*x]])/8
 + (ArcTan[a*x]*Sin[4*ArcTan[a*x]])/64))/c^3
```

Maple [C] time = 2.632, size = 2421, normalized size = 7.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^2/x^3/(a^2*c*x^2+c)^3,x)
```

```
[Out] -6*a^2/c^3*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*a^2/c^3*polylog(3,(1+I
*a*x)/(a^2*x^2+1)^(1/2))+a^2/c^3*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)-1)+a^2/c^3*
ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-1/512*a^2/c^3/(a*x+I)^2-1/512*a^2/c^3/(a*
x-I)^2-a*arctan(a*x)/c^3/x-1/4*a^2*arctan(a*x)^2/c^3/(a^2*x^2+1)^2-1/2*arct
an(a*x)^2/c^3/x^2+3/32*a^2*arctan(a*x)^2/c^3+I*a^2*arctan(a*x)^3/c^3-3/2*I*
a^2/c^3*Pi*arctan(a*x)^2*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1))*csgn(I/((1+I*a
*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*
x^2+1)+1))+3/4*I*a^2/c^3*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1
+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2
+1)+1)^2)*arctan(a*x)^2-3/2*I*a^2/c^3*Pi*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2)
)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^2*arctan(a*x)^2-a^2*arctan(a*x)^2/c^3/(a^
2*x^2+1)-3*a^2/c^3*arctan(a*x)^2*ln(2)-5/32*a^3/c^3/(a*x+I)*x-5/32*a^3/c^3/
(a*x-I)*x+1/512*a^4/c^3/(a*x+I)^2*x^2+1/512*a^4/c^3/(a*x-I)^2*x^2-3*a^2/c^3
*arctan(a*x)^2*ln(a*x)+3/2*a^2/c^3*arctan(a*x)^2*ln(a^2*x^2+1)+5/2*a^2/c^3*
arctan(a*x)/(8*a*x+8*I)+5/2*a^2/c^3*arctan(a*x)/(8*a*x-8*I)+3*a^2/c^3*arcta
n(a*x)^2*ln((1+I*a*x)^2/(a^2*x^2+1)-1)-3*a^2/c^3*arctan(a*x)^2*ln(1-(1+I*a*
x)/(a^2*x^2+1)^(1/2))-3*a^2/c^3*arctan(a*x)^2*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1
/2))-3*a^2/c^3*arctan(a*x)^2*ln((1+I*a*x)/(a^2*x^2+1)^(1/2))-I*a^2/c^3*arct
an(a*x)+5/32*I*a^2/c^3/(a*x+I)-5/32*I*a^2/c^3/(a*x-I)-1/64*a^3/c^3*arctan(a
*x)/(a*x-I)^2*x-1/64*a^3/c^3*arctan(a*x)/(a*x+I)^2*x-1/256*I*a^3/c^3/(a*x+I
)^2*x+1/256*I*a^3/c^3/(a*x-I)^2*x+1/128*I*a^2/c^3*arctan(a*x)/(a*x+I)^2-1/1
28*I*a^2/c^3*arctan(a*x)/(a*x-I)^2+6*I*a^2/c^3*arctan(a*x)*polylog(2,-(1+I*
```

```
a*x)/(a^2*x^2+1)^(1/2))+6*I*a^2/c^3*arctan(a*x)*polylog(2,(1+I*a*x)/(a^2*x^
2+1)^(1/2))-3/2*I*a^2/c^3*Pi*arctan(a*x)^2-3/4*I*a^2/c^3*Pi*arctan(a*x)^2*c
sgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a
*x)^2/(a^2*x^2+1)+1)^2)^2-3/2*I*a^2/c^3*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-
1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)
^2/(a^2*x^2+1)+1))*arctan(a*x)^2+3/2*I*a^2/c^3*Pi*csgn(I*((1+I*a*x)^2/(a^2*
x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1
+I*a*x)^2/(a^2*x^2+1)+1))^2*arctan(a*x)^2+3/4*I*a^2/c^3*Pi*csgn(I*(1+I*a*x)
/(a^2*x^2+1)^(1/2))^2*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*arctan(a*x)^2+3/2*I*a
^2/c^3*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+
1)+1)^2)^2*arctan(a*x)^2-3/4*I*a^2/c^3*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1
))^2*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*arctan(a*x)^2+3/2*I*a^2/c^3*Pi*c
sgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I
*a*x)^2/(a^2*x^2+1)+1))^2*arctan(a*x)^2-3/4*I*a^2/c^3*Pi*csgn(I*(1+I*a*x)^2
/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)
^2*arctan(a*x)^2+3/2*I*a^2/c^3*Pi*arctan(a*x)^2*csgn(I*((1+I*a*x)^2/(a^2*x^
2+1)-1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2+
3/4*I*a^2/c^3*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)
^2)^3*arctan(a*x)^2-3/4*I*a^2/c^3*Pi*arctan(a*x)^2*csgn(I*((1+I*a*x)^2/(a^2
*x^2+1)+1)^2)^3-3/2*I*a^2/c^3*Pi*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x
)^2/(a^2*x^2+1)+1))^3*arctan(a*x)^2-3/2*I*a^2/c^3*Pi*csgn(I*((1+I*a*x)^2/(a
^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^3*arctan(a*x)^2+3/4*I*a^2/c^3*Pi*
csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^3*arctan(a*x)^2+3/2*I*a^2/c^3*Pi*csgn(((1+I
*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2*arctan(a*x)^2+5/2*I*a
^3/c^3*arctan(a*x)/(8*a*x+8*I)*x-1/128*I*a^4/c^3*arctan(a*x)/(a*x+I)^2*x^2+
1/128*I*a^4/c^3*arctan(a*x)/(a*x-I)^2*x^2-5/2*I*a^3/c^3*arctan(a*x)/(8*a*x-
8*I)*x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(ax)^2}{\left(a^2cx^2+c\right)^3x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x^3/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] integrate(arctan(a*x)^2/((a^2*c*x^2 + c)^3*x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\arctan(ax)^2}{a^6c^3x^9 + 3\,a^4c^3x^7 + 3\,a^2c^3x^5 + c^3x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x^3/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] integral(arctan(a*x)^2/(a^6*c^3*x^9 + 3*a^4*c^3*x^7 + 3*a^2*c^3*x^5 + c^3*x
^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\operatorname{atan}^2(ax)}{a^6x^9+3a^4x^7+3a^2x^5+x^3}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**2/x**3/(a**2*c*x**2+c)**3,x)
```

```
[Out] Integral(atan(a*x)**2/(a**6*x**9 + 3*a**4*x**7 + 3*a**2*x**5 + x**3), x)/c*
*3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(ax)^2}{\left(a^2cx^2 + c\right)^3 x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x^3/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)^2/((a^2*c*x^2 + c)^3*x^3), x)
```

3.306 $\int \frac{\tan^{-1}(ax)^2}{x^4(c+a^2cx^2)^3}\,dx$

Optimal. Leaf size=317

$$\frac{10ia^3\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{3c^3}-\frac{47a^4x}{64c^3\left(a^2x^2+1\right)}-\frac{a^4x}{32c^3\left(a^2x^2+1\right)^2}+\frac{11a^4x\tan^{-1}(ax)^2}{8c^3\left(a^2x^2+1\right)}+\frac{a^4x\tan^{-1}(ax)^2}{4c^3\left(a^2x^2+1\right)^2}+\frac{11a^3\tan^{-1}}{8c^3\left(a^2x^2\right.}$$

```
[Out] -a^2/(3*c^3*x) - (a^4*x)/(32*c^3*(1 + a^2*x^2)^2) - (47*a^4*x)/(64*c^3*(1 +
 a^2*x^2)) - (205*a^3*ArcTan[a*x])/(192*c^3) - (a*ArcTan[a*x])/(3*c^3*x^2)
+ (a^3*ArcTan[a*x])/(8*c^3*(1 + a^2*x^2)^2) + (11*a^3*ArcTan[a*x])/(8*c^3*(
1 + a^2*x^2)) + (((10*I)/3)*a^3*ArcTan[a*x]^2)/c^3 - ArcTan[a*x]^2/(3*c^3*x
^3) + (3*a^2*ArcTan[a*x]^2)/(c^3*x) + (a^4*x*ArcTan[a*x]^2)/(4*c^3*(1 + a^2
*x^2)^2) + (11*a^4*x*ArcTan[a*x]^2)/(8*c^3*(1 + a^2*x^2)) + (35*a^3*ArcTan[
a*x]^3)/(24*c^3) - (20*a^3*ArcTan[a*x]*Log[2 - 2/(1 - I*a*x)])/(3*c^3) + ((
(10*I)/3)*a^3*PolyLog[2, -1 + 2/(1 - I*a*x)])/c^3
```

Rubi [A] time = 1.52575, antiderivative size = 317, normalized size of antiderivative = 1., number of steps used = 48, number of rules used = 14, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {4966, 4918, 4852, 325, 203, 4924, 4868, 2447, 4884, 4892, 4930, 199, 205, 4900}

$$\frac{10ia^3\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{3c^3}-\frac{47a^4x}{64c^3\left(a^2x^2+1\right)}-\frac{a^4x}{32c^3\left(a^2x^2+1\right)^2}+\frac{11a^4x\tan^{-1}(ax)^2}{8c^3\left(a^2x^2+1\right)}+\frac{a^4x\tan^{-1}(ax)^2}{4c^3\left(a^2x^2+1\right)^2}+\frac{11a^3\tan^{-1}}{8c^3\left(a^2x^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^2/(x^4*(c + a^2*c*x^2)^3),x]
```


```
[Out] -a^2/(3*c^3*x) - (a^4*x)/(32*c^3*(1 + a^2*x^2)^2) - (47*a^4*x)/(64*c^3*(1 +
 a^2*x^2)) - (205*a^3*ArcTan[a*x])/(192*c^3) - (a*ArcTan[a*x])/(3*c^3*x^2)
+ (a^3*ArcTan[a*x])/(8*c^3*(1 + a^2*x^2)^2) + (11*a^3*ArcTan[a*x])/(8*c^3*(
1 + a^2*x^2)) + (((10*I)/3)*a^3*ArcTan[a*x]^2)/c^3 - ArcTan[a*x]^2/(3*c^3*x
^3) + (3*a^2*ArcTan[a*x]^2)/(c^3*x) + (a^4*x*ArcTan[a*x]^2)/(4*c^3*(1 + a^2
*x^2)^2) + (11*a^4*x*ArcTan[a*x]^2)/(8*c^3*(1 + a^2*x^2)) + (35*a^3*ArcTan[
a*x]^3)/(24*c^3) - (20*a^3*ArcTan[a*x]*Log[2 - 2/(1 - I*a*x)])/(3*c^3) + ((
(10*I)/3)*a^3*PolyLog[2, -1 + 2/(1 - I*a*x)])/c^3
```


Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*
x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^p
, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*q]
 && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 4918

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/(d + e*x^2),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4892

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[(x*(a + b*ArcTan[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*c*
p)/2, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(a +
 b*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 4900

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(b*p*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^(p - 1))/(4*c*d
*(q + 1)^2), x] + (Dist[(2*q + 3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a
 + b*ArcTan[c*x])^p, x], x] - Dist[(b^2*p*(p - 1))/(4*(q + 1)^2), Int[(d +
e*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x] - Simp[(x*(d + e*x^2)^(q + 1)*
(a + b*ArcTan[c*x])^p)/(2*d*(q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] && E
qQ[e, c^2*d] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^2}{x^4\left(c+a^2cx^2\right)^3}\,dx = -\left(a^2\int \frac{\tan^{-1}(ax)^2}{x^2\left(c+a^2cx^2\right)^3}\,dx\right) + \frac{\int \frac{\tan^{-1}(ax)^2}{x^4\left(c+a^2cx^2\right)^2}\,dx}{c}$$

$$= a^4\int \frac{\tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^3}\,dx + \frac{\int \frac{\tan^{-1}(ax)^2}{x^4\left(c+a^2cx^2\right)}\,dx}{c^2} - 2\frac{a^2\int \frac{\tan^{-1}(ax)^2}{x^2\left(c+a^2cx^2\right)^2}\,dx}{c}$$

$$= \frac{a^3\tan^{-1}(ax)}{8c^3\left(1+a^2x^2\right)^2} + \frac{a^4x\tan^{-1}(ax)^2}{4c^3\left(1+a^2x^2\right)^2} - \frac{1}{8}a^4\int \frac{1}{\left(c+a^2cx^2\right)^3}\,dx + \frac{\int \frac{\tan^{-1}(ax)^2}{x^4}\,dx}{c^3} - \frac{a^2\int \frac{\tan^{-1}(ax)^2}{x^2\left(c+a^2cx^2\right)}}{c^2}$$

$$= -\frac{a^4x}{32c^3\left(1+a^2x^2\right)^2} + \frac{a^3\tan^{-1}(ax)}{8c^3\left(1+a^2x^2\right)^2} - \frac{\tan^{-1}(ax)^2}{3c^3x^3} + \frac{a^4x\tan^{-1}(ax)^2}{4c^3\left(1+a^2x^2\right)^2} + \frac{3a^4x\tan^{-1}(ax)^2}{8c^3\left(1+a^2x^2\right)} + \frac{a^3\,\mathrm{t}}{}$$

$$= -\frac{a^4x}{32c^3\left(1+a^2x^2\right)^2} - \frac{3a^4x}{64c^3\left(1+a^2x^2\right)} + \frac{a^3\tan^{-1}(ax)}{8c^3\left(1+a^2x^2\right)^2} + \frac{3a^3\tan^{-1}(ax)}{8c^3\left(1+a^2x^2\right)} - \frac{\tan^{-1}(ax)^2}{3c^3x^3} + \frac{a^2\tan}{}$$

$$= -\frac{a^4x}{32c^3\left(1+a^2x^2\right)^2} - \frac{15a^4x}{64c^3\left(1+a^2x^2\right)} - \frac{3a^3\tan^{-1}(ax)}{64c^3} - \frac{a\tan^{-1}(ax)}{3c^3x^2} + \frac{a^3\tan^{-1}(ax)}{8c^3\left(1+a^2x^2\right)^2} + \frac{3a^3\,\mathrm{t}}{8c^3(}$$

$$= -\frac{a^2}{3c^3x} - \frac{a^4x}{32c^3\left(1+a^2x^2\right)^2} - \frac{15a^4x}{64c^3\left(1+a^2x^2\right)} - \frac{15a^3\tan^{-1}(ax)}{64c^3} - \frac{a\tan^{-1}(ax)}{3c^3x^2} + \frac{a^3\tan^{-1}(ax)}{8c^3\left(1+a^2x^2\right)^2}$$

$$= -\frac{a^2}{3c^3x} - \frac{a^4x}{32c^3\left(1+a^2x^2\right)^2} - \frac{15a^4x}{64c^3\left(1+a^2x^2\right)} - \frac{109a^3\tan^{-1}(ax)}{192c^3} - \frac{a\tan^{-1}(ax)}{3c^3x^2} + \frac{a^3\tan^{-1}(ax)}{8c^3\left(1+a^2x^2\right)}$$

Mathematica [A] time = 0.763991, size = 189, normalized size = 0.6

$$a^3\left(2560i\left(\tan^{-1}(ax)^2+\mathrm{PolyLog}\left(2,e^{2i\tan^{-1}(ax)}\right)\right) - \frac{256\left(a^2x^2+1\right)\tan^{-1}(ax)^2}{a^3x^3} - \frac{256\left(a^2x^2+1\right)\tan^{-1}(ax)}{a^2x^2} + 1120\tan^{-1}(ax)^3 + \frac{256(1}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTan[a*x]^2/(x^4*(c + a^2*c*x^2)^3),x]
```

```
[Out] (a^3*((-256*(1 + a^2*x^2)*ArcTan[a*x])/(a^2*x^2) - (256*(1 + a^2*x^2)*ArcTa
n[a*x]^2)/(a^3*x^3) + 1120*ArcTan[a*x]^3 + (256*(-1 + 10*ArcTan[a*x]^2))/(a
*x) + 576*ArcTan[a*x]*Cos[2*ArcTan[a*x]] + 12*ArcTan[a*x]*Cos[4*ArcTan[a*x]
] - 5120*ArcTan[a*x]*Log[1 - E^((2*I)*ArcTan[a*x])] + (2560*I)*(ArcTan[a*x]
^2 + PolyLog[2, E^((2*I)*ArcTan[a*x])]) + 288*(-1 + 2*ArcTan[a*x]^2)*Sin[2*
ArcTan[a*x]] + 3*(-1 + 8*ArcTan[a*x]^2)*Sin[4*ArcTan[a*x]]))/(768*c^3)
```

Maple [A] time = 0.116, size = 517, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^2/x^4/(a^2*c*x^2+c)^3,x)
```

```
[Out] 5/3*I*a^3/c^3*dilog(1/2*I*(a*x-I))-10/3*I*a^3/c^3*dilog(1+I*a*x)+10/3*I*a^3
/c^3*dilog(1-I*a*x)-5/3*I*a^3/c^3*dilog(-1/2*I*(a*x+I))-47/64*a^6/c^3/(a^2*
x^2+1)^2*x^3-20/3*a^3/c^3*arctan(a*x)*ln(a*x)+10/3*a^3/c^3*arctan(a*x)*ln(a
^2*x^2+1)-5/6*I*a^3/c^3*ln(a*x-I)^2+5/6*I*a^3/c^3*ln(a*x+I)^2+11/8*a^6/c^3*
arctan(a*x)^2/(a^2*x^2+1)^2*x^3+5/3*I*a^3/c^3*ln(a*x+I)*ln(1/2*I*(a*x-I))+1
0/3*I*a^3/c^3*ln(a*x)*ln(1-I*a*x)-10/3*I*a^3/c^3*ln(a*x)*ln(1+I*a*x)+5/3*I*
a^3/c^3*ln(a^2*x^2+1)*ln(a*x-I)-5/3*I*a^3/c^3*ln(a*x-I)*ln(-1/2*I*(a*x+I))-
5/3*I*a^3/c^3*ln(a*x+I)*ln(a^2*x^2+1)-49/64*a^4*x/c^3/(a^2*x^2+1)^2-1/3*a*a
rctan(a*x)/c^3/x^2+1/8*a^3*arctan(a*x)/c^3/(a^2*x^2+1)^2+11/8*a^3*arctan(a*
x)/c^3/(a^2*x^2+1)+3*a^2*arctan(a*x)^2/c^3/x+13/8*a^4*x*arctan(a*x)^2/c^3/(
a^2*x^2+1)^2-1/3*a^2/c^3/x-205/192*a^3*arctan(a*x)/c^3-1/3*arctan(a*x)^2/c^
3/x^3+35/24*a^3*arctan(a*x)^3/c^3
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x^4/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\arctan(ax)^2}{a^6c^3x^{10}+3\,a^4c^3x^8+3\,a^2c^3x^6+c^3x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x^4/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] integral(arctan(a*x)^2/(a^6*c^3*x^10 + 3*a^4*c^3*x^8 + 3*a^2*c^3*x^6 + c^3*
x^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\operatorname{atan}^2(ax)}{a^6x^{10}+3a^4x^8+3a^2x^6+x^4}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**2/x**4/(a**2*c*x**2+c)**3,x)
```

```
[Out] Integral(atan(a*x)**2/(a**6*x**10 + 3*a**4*x**8 + 3*a**2*x**6 + x**4), x)/c
**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(ax)^2}{\left(a^2cx^2+c\right)^3x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x^4/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)^2/((a^2*c*x^2 + c)^3*x^4), x)
```

3.307 $\int x^3\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2\,dx$

Optimal. Leaf size=385

$$\frac{11ic\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{60a^4\sqrt{a^2cx^2+c}}-\frac{11ic\sqrt{a^2x^2+1}\text{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{60a^4\sqrt{a^2cx^2+c}}+\frac{\left(a^2cx^2+c\right)^{3/2}}{30a^4c}-\frac{11\sqrt{a^2cx^2+c}}{60a^4}+\frac{1}{5}x^4\sqrt{a^2c}$$

```
[Out] (-11*Sqrt[c + a^2*c*x^2])/(60*a^4) + (c + a^2*c*x^2)^(3/2)/(30*a^4*c) + (x*
Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(12*a^3) - (x^3*Sqrt[c + a^2*c*x^2]*ArcTan
[a*x])/(10*a) - (2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(15*a^4) + (x^2*Sqrt[
c + a^2*c*x^2]*ArcTan[a*x]^2)/(15*a^2) + (x^4*Sqrt[c + a^2*c*x^2]*ArcTan[a*
x]^2)/5 - (((11*I)/30)*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*
x]/Sqrt[1 - I*a*x]])/(a^4*Sqrt[c + a^2*c*x^2]) + (((11*I)/60)*c*Sqrt[1 + a^
2*x^2]*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/(a^4*Sqrt[c + a^
2*c*x^2]) - (((11*I)/60)*c*Sqrt[1 + a^2*x^2]*PolyLog[2, (I*Sqrt[1 + I*a*x])
/Sqrt[1 - I*a*x]])/(a^4*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 1.42525, antiderivative size = 385, normalized size of antiderivative = 1., number of steps used = 26, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4950, 4952, 261, 4890, 4886, 4930, 266, 43}

$$\frac{11ic\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{60a^4\sqrt{a^2cx^2+c}}-\frac{11ic\sqrt{a^2x^2+1}\text{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{60a^4\sqrt{a^2cx^2+c}}+\frac{\left(a^2cx^2+c\right)^{3/2}}{30a^4c}-\frac{11\sqrt{a^2cx^2+c}}{60a^4}+\frac{1}{5}x^4\sqrt{a^2c}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2,x]
```


```
[Out] (-11*Sqrt[c + a^2*c*x^2])/(60*a^4) + (c + a^2*c*x^2)^(3/2)/(30*a^4*c) + (x*
Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(12*a^3) - (x^3*Sqrt[c + a^2*c*x^2]*ArcTan
[a*x])/(10*a) - (2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(15*a^4) + (x^2*Sqrt[
c + a^2*c*x^2]*ArcTan[a*x]^2)/(15*a^2) + (x^4*Sqrt[c + a^2*c*x^2]*ArcTan[a*
x]^2)/5 - (((11*I)/30)*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*
x]/Sqrt[1 - I*a*x]])/(a^4*Sqrt[c + a^2*c*x^2]) + (((11*I)/60)*c*Sqrt[1 + a^
2*x^2]*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/(a^4*Sqrt[c + a^
2*c*x^2]) - (((11*I)/60)*c*Sqrt[1 + a^2*x^2]*PolyLog[2, (I*Sqrt[1 + I*a*x])
/Sqrt[1 - I*a*x]])/(a^4*Sqrt[c + a^2*c*x^2])
```


Rule 4950

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 4952

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b*
ArcTan[c*x])^p)/(c^2*d*m), x] + (-Dist[(b*f*p)/(c*m), Int[((f*x)^(m - 1)*(a
 + b*ArcTan[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] - Dist[(f^2*(m - 1))/(c^
2*m), Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/Sqrt[d + e*x^2], x], x]) /;
 FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 4890

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4886

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[(-2*I*(a + b*ArcTan[c*x])*ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]])
/(c*Sqrt[d]), x] + (Simp[(I*b*PolyLog[2, -((I*Sqrt[1 + I*c*x])/Sqrt[1 - I*c
*x])])/(c*Sqrt[d]), x] - Simp[(I*b*PolyLog[2, (I*Sqrt[1 + I*c*x])/Sqrt[1 -
I*c*x]])/(c*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
```

```
0] && NeQ[q, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x^3\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2\,dx = c\int \frac{x^3\tan^{-1}(ax)^2}{\sqrt{c+a^2cx^2}}\,dx + \left(a^2c\right)\int \frac{x^5\tan^{-1}(ax)^2}{\sqrt{c+a^2cx^2}}\,dx$$

$$= \frac{x^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{3a^2} + \frac{1}{5}x^4\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2 - \frac{1}{5}(4c)\int \frac{x^3\tan^{-1}(ax)^2}{\sqrt{c+a^2cx^2}}\,dx - \cdots$$

$$= -\frac{x\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{3a^3} - \frac{x^3\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{10a} - \frac{2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{3a^4} + \frac{x^2\sqrt{c}\cdots}{\cdots}$$

$$= \frac{\sqrt{c+a^2cx^2}}{3a^4} + \frac{x\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{12a^3} - \frac{x^3\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{10a} - \frac{2\sqrt{c+a^2cx^2}\tan^{-1}(\cdots}{15a^4}$$

$$= -\frac{\sqrt{c+a^2cx^2}}{12a^4} + \frac{x\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{12a^3} - \frac{x^3\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{10a} - \frac{2\sqrt{c+a^2cx^2}\tan^{-1}\cdots}{15a^4}$$

$$= -\frac{11\sqrt{c+a^2cx^2}}{60a^4} + \frac{\left(c+a^2cx^2\right)^{3/2}}{30a^4c} + \frac{x\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{12a^3} - \frac{x^3\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{10a} - \cdots$$

Mathematica [A] time = 1.14219, size = 360, normalized size = 0.94

$$-\frac{\left(a^2x^2+1\right)^2\sqrt{c\left(a^2x^2+1\right)}\left(-\frac{176i\text{PolyLog}\left(2,-ie^{i\tan^{-1}(ax)}\right)}{\left(a^2x^2+1\right)^{5/2}} + \frac{176i\text{PolyLog}\left(2,ie^{i\tan^{-1}(ax)}\right)}{\left(a^2x^2+1\right)^{5/2}} - \frac{110\tan^{-1}(ax)\log\left(1-ie^{i\tan^{-1}(ax)}\right)}{\sqrt{a^2x^2+1}} + \frac{110\tan^{-1}(ax\cdots}{\cdots}\cdots\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2,x]
```

```
[Out] -((1 + a^2*x^2)^2*Sqrt[c*(1 + a^2*x^2)]*(50 - 32*ArcTan[a*x]^2 + 72*Cos[2*A
rcTan[a*x]] + 160*ArcTan[a*x]^2*Cos[2*ArcTan[a*x]] + 22*Cos[4*ArcTan[a*x]]
- (110*ArcTan[a*x]*Log[1 - I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - 55*Arc
Tan[a*x]*Cos[3*ArcTan[a*x]]*Log[1 - I*E^(I*ArcTan[a*x])] - 11*ArcTan[a*x]*C
os[5*ArcTan[a*x]]*Log[1 - I*E^(I*ArcTan[a*x])] + (110*ArcTan[a*x]*Log[1 + I
*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + 55*ArcTan[a*x]*Cos[3*ArcTan[a*x]]*
Log[1 + I*E^(I*ArcTan[a*x])] + 11*ArcTan[a*x]*Cos[5*ArcTan[a*x]]*Log[1 + I*
E^(I*ArcTan[a*x])] - ((176*I)*PolyLog[2, (-I)*E^(I*ArcTan[a*x])])/(1 + a^2*
x^2)^(5/2) + ((176*I)*PolyLog[2, I*E^(I*ArcTan[a*x])])/(1 + a^2*x^2)^(5/2)
+ 4*ArcTan[a*x]*Sin[2*ArcTan[a*x]] - 22*ArcTan[a*x]*Sin[4*ArcTan[a*x]]))/(9
60*a^4)
```

Maple [A] time = 0.999, size = 235, normalized size = 0.6

$$\frac{12\,(\arctan(ax))^2 x^4a^4 - 6\,\arctan(ax)\,x^3a^3 + 4\,(\arctan(ax))^2 x^2a^2 + 2\,a^2x^2 + 5\,\arctan(ax)\,xa - 8\,(\arctan(ax))^2 - 9}{60\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] 1/60/a^4*(c*(a*x-I)*(a*x+I))^(1/2)*(12*arctan(a*x)^2*x^4*a^4-6*arctan(a*x)*
x^3*a^3+4*arctan(a*x)^2*x^2*a^2+2*a^2*x^2+5*arctan(a*x)*x*a-8*arctan(a*x)^2
-9)-11/60*(c*(a*x-I)*(a*x+I))^(1/2)*(arctan(a*x)*ln(1+I*(1+I*a*x)/(a^2*x^2+
1)^(1/2))-arctan(a*x)*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*dilog(1+I*(1+I*
a*x)/(a^2*x^2+1)^(1/2))+I*dilog(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))/a^4/(a^2*
x^2+1)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{a^2cx^2+c}x^3\arctan(ax)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^3*arctan(a*x)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\sqrt{c\left(a^2x^2+1\right)}\,\text{atan}^2(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atan(a*x)**2*(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(c*(a**2*x**2 + 1))*atan(a*x)**2, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.308 $\int x^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2\,dx$

Optimal. Leaf size=436

$$-\frac{ic\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,-ie^{i\tan^{-1}(ax)}\right)}{4a^3\sqrt{a^2cx^2+c}}+\frac{ic\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,ie^{i\tan^{-1}(ax)}\right)}{4a^3\sqrt{a^2cx^2+c}}+\frac{c\sqrt{a^2x^2+1}\text{Poly}}{4a^3\sqrt{}}$$

```
[Out] (x*Sqrt[c + a^2*c*x^2])/(12*a^2) + (Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(12*a^
3) - (x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(6*a) + (x*Sqrt[c + a^2*c*x^2]*A
rcTan[a*x]^2)/(8*a^2) + (x^3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/4 + ((I/4)*
c*Sqrt[1 + a^2*x^2]*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2)/(a^3*Sqrt[c +
a^2*c*x^2]) - (Sqrt[c]*ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]])/(6*a^3)
- ((I/4)*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])]
)/(a^3*Sqrt[c + a^2*c*x^2]) + ((I/4)*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLo
g[2, I*E^(I*ArcTan[a*x])])/(a^3*Sqrt[c + a^2*c*x^2]) + (c*Sqrt[1 + a^2*x^2]
*PolyLog[3, (-I)*E^(I*ArcTan[a*x])])/(4*a^3*Sqrt[c + a^2*c*x^2]) - (c*Sqrt[
1 + a^2*x^2]*PolyLog[3, I*E^(I*ArcTan[a*x])])/(4*a^3*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 1.13451, antiderivative size = 436, normalized size of antiderivative = 1., number of steps used = 35, number of rules used = 12, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4950, 4952, 4930, 217, 206, 4890, 4888, 4181, 2531, 2282, 6589, 321}

$$-\frac{ic\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,-ie^{i\tan^{-1}(ax)}\right)}{4a^3\sqrt{a^2cx^2+c}}+\frac{ic\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,ie^{i\tan^{-1}(ax)}\right)}{4a^3\sqrt{a^2cx^2+c}}+\frac{c\sqrt{a^2x^2+1}\text{Poly}}{4a^3\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2,x]
```

```
[Out] (x*Sqrt[c + a^2*c*x^2])/(12*a^2) + (Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(12*a^
3) - (x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(6*a) + (x*Sqrt[c + a^2*c*x^2]*A
rcTan[a*x]^2)/(8*a^2) + (x^3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/4 + ((I/4)*
c*Sqrt[1 + a^2*x^2]*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2)/(a^3*Sqrt[c +
a^2*c*x^2]) - (Sqrt[c]*ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]])/(6*a^3)
- ((I/4)*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])]
)/(a^3*Sqrt[c + a^2*c*x^2]) + ((I/4)*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLo
g[2, I*E^(I*ArcTan[a*x])])/(a^3*Sqrt[c + a^2*c*x^2]) + (c*Sqrt[1 + a^2*x^2]
*PolyLog[3, (-I)*E^(I*ArcTan[a*x])])/(4*a^3*Sqrt[c + a^2*c*x^2]) - (c*Sqrt[
1 + a^2*x^2]*PolyLog[3, I*E^(I*ArcTan[a*x])])/(4*a^3*Sqrt[c + a^2*c*x^2])
```

Rule 4950

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 4952

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b*
ArcTan[c*x])^p)/(c^2*d*m), x] + (-Dist[(b*f*p)/(c*m), Int[((f*x)^(m - 1)*(a
 + b*ArcTan[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] - Dist[(f^2*(m - 1))/(c^
2*m), Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/Sqrt[d + e*x^2], x], x]) /;
 FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 4890

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4888

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[c
*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && GtQ
[d, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int x^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2\,dx = c\int \frac{x^2\tan^{-1}(ax)^2}{\sqrt{c+a^2cx^2}}\,dx + \left(a^2c\right)\int \frac{x^4\tan^{-1}(ax)^2}{\sqrt{c+a^2cx^2}}\,dx$$

$$= \frac{x\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2a^2} + \frac{1}{4}x^3\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2 - \frac{1}{4}(3c)\int \frac{x^2\tan^{-1}(ax)^2}{\sqrt{c+a^2cx^2}}\,dx - \frac{c}{}$$

$$= -\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{a^3} - \frac{x^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{6a} + \frac{x\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{8a^2} + \frac{1}{4}x^3\sqrt{c}$$

$$= \frac{x\sqrt{c+a^2cx^2}}{12a^2} + \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{12a^3} - \frac{x^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{6a} + \frac{x\sqrt{c+a^2cx^2}\tan^{-1}(}{8a^2}$$

$$= \frac{x\sqrt{c+a^2cx^2}}{12a^2} + \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{12a^3} - \frac{x^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{6a} + \frac{x\sqrt{c+a^2cx^2}\tan^{-1}(}{8a^2}$$

$$= \frac{x\sqrt{c+a^2cx^2}}{12a^2} + \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{12a^3} - \frac{x^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{6a} + \frac{x\sqrt{c+a^2cx^2}\tan^{-1}(}{8a^2}$$

$$= \frac{x\sqrt{c+a^2cx^2}}{12a^2} + \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{12a^3} - \frac{x^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{6a} + \frac{x\sqrt{c+a^2cx^2}\tan^{-1}(}{8a^2}$$

$$= \frac{x\sqrt{c+a^2cx^2}}{12a^2} + \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{12a^3} - \frac{x^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{6a} + \frac{x\sqrt{c+a^2cx^2}\tan^{-1}(}{8a^2}$$

$$= \frac{x\sqrt{c+a^2cx^2}}{12a^2} + \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{12a^3} - \frac{x^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{6a} + \frac{x\sqrt{c+a^2cx^2}\tan^{-1}(}{8a^2}$$

Mathematica [A] time = 1.21586, size = 267, normalized size = 0.61

$$\frac{\sqrt{a^2cx^2+c}\left(\left(a^2x^2+1\right)^{3/2}\left(-3\tan^{-1}(ax)^2\left(\sqrt{a^2x^2+1}\sin\left(3\tan^{-1}(ax)\right)-7ax\right)+2\left(\sqrt{a^2x^2+1}\sin\left(3\tan^{-1}(ax)\right)+ax\right)+\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2,x]
```

```
[Out] (Sqrt[c + a^2*c*x^2]*(8*((3*I)*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2 - 2*
ArcTanh[(a*x)/Sqrt[1 + a^2*x^2]] - (3*I)*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*A
rcTan[a*x])] + (3*I)*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])] + 3*PolyLo
g[3, (-I)*E^(I*ArcTan[a*x])] - 3*PolyLog[3, I*E^(I*ArcTan[a*x])]) + (1 + a^
```

```
2*x^2)^(3/2)*(ArcTan[a*x]*(2 + 6*Sqrt[1 + a^2*x^2]*Cos[3*ArcTan[a*x]]) - 3*
ArcTan[a*x]^2*(-7*a*x + Sqrt[1 + a^2*x^2]*Sin[3*ArcTan[a*x]]) + 2*(a*x + Sq
rt[1 + a^2*x^2]*Sin[3*ArcTan[a*x]]))))/(96*a^3*Sqrt[1 + a^2*x^2])
```

Maple [A] time = 0.59, size = 302, normalized size = 0.7

$$\frac{6\left(\arctan\left(ax\right)\right)^{2}x^{3}a^{3}-4\arctan\left(ax\right)a^{2}x^{2}+3\left(\arctan\left(ax\right)\right)^{2}xa+2\,ax+2\arctan\left(ax\right)}{24\,a^{3}}\sqrt{c\left(ax-i\right)\left(ax+i\right)}-\frac{\frac{i}{24}}{a^{3}}\sqrt{c\left(ax\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] 1/24/a^3*(c*(a*x-I)*(a*x+I))^(1/2)*(6*arctan(a*x)^2*x^3*a^3-4*arctan(a*x)*a
^2*x^2+3*arctan(a*x)^2*x*a+2*a*x+2*arctan(a*x))-1/24*I*(c*(a*x-I)*(a*x+I))^
(1/2)*(3*I*arctan(a*x)^2*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-3*I*arctan(a*x
)^2*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*arctan(a*x)*polylog(2,-I*(1+I*a*x
)/(a^2*x^2+1)^(1/2))-6*arctan(a*x)*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))
+6*I*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*I*polylog(3,I*(1+I*a*x)/(a
^2*x^2+1)^(1/2))-8*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2)))/a^3/(a^2*x^2+1)^(1/
2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{a^{2}cx^{2}+c}x^{2}\arctan\left(ax\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^2*arctan(a*x)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\sqrt{c\left(a^2x^2+1\right)}\,\mathrm{atan}^2\left(ax\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(a*x)**2*(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(c*(a**2*x**2 + 1))*atan(a*x)**2, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.309 $\int x\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2\,dx$

Optimal. Leaf size=279

$$-\frac{ic\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{3a^2\sqrt{a^2cx^2+c}}+\frac{ic\sqrt{a^2x^2+1}\text{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{3a^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{a^2cx^2+c}}{3a^2}+\frac{\left(a^2cx^2+c\right)^{3/2}\tan^{-1}(ax)^2}{3a^2c}+\cdots$$

```
[Out] Sqrt[c + a^2*c*x^2]/(3*a^2) - (x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(3*a) + (
(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2)/(3*a^2*c) + (((2*I)/3)*c*Sqrt[1 + a^2*
x^2]*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/(a^2*Sqrt[c + a^2
*c*x^2]) - ((I/3)*c*Sqrt[1 + a^2*x^2]*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqr
t[1 - I*a*x]])/(a^2*Sqrt[c + a^2*c*x^2]) + ((I/3)*c*Sqrt[1 + a^2*x^2]*PolyL
og[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/(a^2*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.175936, antiderivative size = 279, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4930, 4878, 4890, 4886}

$$-\frac{ic\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{3a^2\sqrt{a^2cx^2+c}}+\frac{ic\sqrt{a^2x^2+1}\text{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{3a^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{a^2cx^2+c}}{3a^2}+\frac{\left(a^2cx^2+c\right)^{3/2}\tan^{-1}(ax)^2}{3a^2c}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2,x]
```


```
[Out] Sqrt[c + a^2*c*x^2]/(3*a^2) - (x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(3*a) + (
(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2)/(3*a^2*c) + (((2*I)/3)*c*Sqrt[1 + a^2*
x^2]*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/(a^2*Sqrt[c + a^2
*c*x^2]) - ((I/3)*c*Sqrt[1 + a^2*x^2]*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqr
t[1 - I*a*x]])/(a^2*Sqrt[c + a^2*c*x^2]) + ((I/3)*c*Sqrt[1 + a^2*x^2]*PolyL
og[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/(a^2*Sqrt[c + a^2*c*x^2])
```


Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 4878

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symbo
l] :> -Simp[(b*(d + e*x^2)^q)/(2*c*q*(2*q + 1)), x] + (Dist[(2*d*q)/(2*q +
1), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x]), x], x] + Simp[(x*(d + e*x^
2)^q*(a + b*ArcTan[c*x]))/(2*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] && Eq
Q[e, c^2*d] && GtQ[q, 0]
```

Rule 4890

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4886

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[(-2*I*(a + b*ArcTan[c*x])*ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]])
/(c*Sqrt[d]), x] + (Simp[(I*b*PolyLog[2, -((I*Sqrt[1 + I*c*x])/Sqrt[1 - I*c
*x])])/(c*Sqrt[d]), x] - Simp[(I*b*PolyLog[2, (I*Sqrt[1 + I*c*x])/Sqrt[1 -
I*c*x]])/(c*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

Rubi steps

$$\begin{aligned}\int x\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2\,dx &= \frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2}{3a^2c}-\frac{2\int\sqrt{c+a^2cx^2}\tan^{-1}(ax)\,dx}{3a}\\ &= \frac{\sqrt{c+a^2cx^2}}{3a^2}-\frac{x\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{3a}+\frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2}{3a^2c}-\frac{c\int\frac{\tan^{-1}(ax)}{\sqrt{c+a^2cx^2}}\,dx}{3a}\\ &= \frac{\sqrt{c+a^2cx^2}}{3a^2}-\frac{x\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{3a}+\frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2}{3a^2c}-\frac{\left(c\sqrt{1+a^2x^2}\right)\int\frac{\tan}{\sqrt{1\cdot}}}{3a\sqrt{c+a^2cx}}\\ &= \frac{\sqrt{c+a^2cx^2}}{3a^2}-\frac{x\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{3a}+\frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2}{3a^2c}+\frac{2ic\sqrt{1+a^2x^2}\tan^{-1}}{3a^2\sqrt{c}}\end{aligned}$$

Mathematica [A] time = 0.580792, size = 260, normalized size = 0.93

$$\left(a^2x^2+1\right)\sqrt{c\left(a^2x^2+1\right)}\left(-\frac{4i\text{PolyLog}\left(2,-ie^{i\tan^{-1}(ax)}\right)}{\left(a^2x^2+1\right)^{3/2}}+\frac{4i\text{PolyLog}\left(2,ie^{i\tan^{-1}(ax)}\right)}{\left(a^2x^2+1\right)^{3/2}}-\frac{3\tan^{-1}(ax)\log\left(1-ie^{i\tan^{-1}(ax)}\right)}{\sqrt{a^2x^2+1}}+\frac{3\tan^{-1}(ax)\log\left(1+ie^{i\tan}\right.}{\sqrt{a^2x^2+1}}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2,x]
```

```
[Out] ((1 + a^2*x^2)*Sqrt[c*(1 + a^2*x^2)]*(2 + 4*ArcTan[a*x]^2 + 2*Cos[2*ArcTan[
a*x]] - (3*ArcTan[a*x]*Log[1 - I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - Ar
cTan[a*x]*Cos[3*ArcTan[a*x]]*Log[1 - I*E^(I*ArcTan[a*x])] + (3*ArcTan[a*x]*
Log[1 + I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + ArcTan[a*x]*Cos[3*ArcTan[
a*x]]*Log[1 + I*E^(I*ArcTan[a*x])] - ((4*I)*PolyLog[2, (-I)*E^(I*ArcTan[a*x
])])/(1 + a^2*x^2)^(3/2) + ((4*I)*PolyLog[2, I*E^(I*ArcTan[a*x])])/(1 + a^2
*x^2)^(3/2) - 2*ArcTan[a*x]*Sin[2*ArcTan[a*x]]))/(12*a^2)
```

Maple [A] time = 0.404, size = 198, normalized size = 0.7

$$\frac{(\arctan(ax))^2 x^2a^2 - \arctan(ax)\,xa + (\arctan(ax))^2 + 1}{3\,a^2}\sqrt{c\,(ax-i)\,(ax+i)} + \frac{1}{3\,a^2}\sqrt{c\,(ax-i)\,(ax+i)}\left(\arctan(ax)\ln\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] 1/3/a^2*(c*(a*x-I)*(a*x+I))^(1/2)*(arctan(a*x)^2*x^2*a^2-arctan(a*x)*x*a+ar
ctan(a*x)^2+1)+1/3*(c*(a*x-I)*(a*x+I))^(1/2)*(arctan(a*x)*ln(1+I*(1+I*a*x)/
(a^2*x^2+1)^(1/2))-arctan(a*x)*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*dilog(
1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+I*dilog(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))/
a^2/(a^2*x^2+1)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a^2cx^2+c}x\arctan(ax)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)*x*arctan(a*x)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{a^2cx^2+c}x\arctan\left(ax\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x*arctan(a*x)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{c\left(a^2x^2+1\right)}\,\text{atan}^2\left(ax\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(a*x)**2*(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x*sqrt(c*(a**2*x**2 + 1))*atan(a*x)**2, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.310 $\int \sqrt{c+a^2cx^2}\tan^{-1}(ax)^2\,dx$

Optimal. Leaf size=340

$$\frac{ic\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,-ie^{i\tan^{-1}(ax)}\right)}{a\sqrt{a^2cx^2+c}}-\frac{ic\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,ie^{i\tan^{-1}(ax)}\right)}{a\sqrt{a^2cx^2+c}}-\frac{c\sqrt{a^2x^2+1}\text{PolyL}}{a\sqrt{a^2}}$$

```
[Out] -((Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/a) + (x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]
^2)/2 - (I*c*Sqrt[1 + a^2*x^2]*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2)/(a*
Sqrt[c + a^2*c*x^2]) + (Sqrt[c]*ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]])
/a + (I*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])])
/(a*Sqrt[c + a^2*c*x^2]) - (I*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, I*
E^(I*ArcTan[a*x])])/(a*Sqrt[c + a^2*c*x^2]) - (c*Sqrt[1 + a^2*x^2]*PolyLog[
3, (-I)*E^(I*ArcTan[a*x])])/(a*Sqrt[c + a^2*c*x^2]) + (c*Sqrt[1 + a^2*x^2]*
PolyLog[3, I*E^(I*ArcTan[a*x])])/(a*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.215215, antiderivative size = 340, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4880, 4890, 4888, 4181, 2531, 2282, 6589, 217, 206}

$$\frac{ic\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,-ie^{i\tan^{-1}(ax)}\right)}{a\sqrt{a^2cx^2+c}}-\frac{ic\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,ie^{i\tan^{-1}(ax)}\right)}{a\sqrt{a^2cx^2+c}}-\frac{c\sqrt{a^2x^2+1}\text{PolyL}}{a\sqrt{a^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2,x]
```

```
[Out] -((Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/a) + (x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]
^2)/2 - (I*c*Sqrt[1 + a^2*x^2]*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2)/(a*
Sqrt[c + a^2*c*x^2]) + (Sqrt[c]*ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]])
/a + (I*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])])
/(a*Sqrt[c + a^2*c*x^2]) - (I*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, I*
E^(I*ArcTan[a*x])])/(a*Sqrt[c + a^2*c*x^2]) - (c*Sqrt[1 + a^2*x^2]*PolyLog[
3, (-I)*E^(I*ArcTan[a*x])])/(a*Sqrt[c + a^2*c*x^2]) + (c*Sqrt[1 + a^2*x^2]*
PolyLog[3, I*E^(I*ArcTan[a*x])])/(a*Sqrt[c + a^2*c*x^2])
```

Rule 4880

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> -Simp[(b*p*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1))/(2*c*q*(2*
```

```
q + 1)), x] + (Dist[(2*d*q)/(2*q + 1), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTa
n[c*x])^p, x], x] + Dist[(b^2*d*p*(p - 1))/(2*q*(2*q + 1)), Int[(d + e*x^2)
^(q - 1)*(a + b*ArcTan[c*x])^(p - 2), x], x] + Simp[(x*(d + e*x^2)^q*(a + b
*ArcTan[c*x])^p)/(2*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[q, 0] && GtQ[p, 1]
```

Rule 4890

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4888

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[c
*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && GtQ
[d, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sqrt{c+a^2cx^2}\tan^{-1}(ax)^2\,dx = -\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{a}+\frac{1}{2}x\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2+\frac{1}{2}c\int \frac{\tan^{-1}(ax)^2}{\sqrt{c+a^2cx^2}}\,dx+c\int \frac{}{\sqrt{c+}}$$

$$= -\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{a}+\frac{1}{2}x\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2+c\,\mathrm{Subst}\left(\int \frac{1}{1-a^2cx^2}\,dx,x,\frac{}{\sqrt{c+}}\right.$$

$$= -\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{a}+\frac{1}{2}x\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2+\frac{\sqrt{c}\tanh^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c+a^2cx^2}}\right)}{a}+\frac{\left(c\sqrt{1+}\right.}{}$$

$$= -\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{a}+\frac{1}{2}x\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2-\frac{ic\sqrt{1+a^2x^2}\tan^{-1}\left(e^{i\tan^{-1}(ax)}\right)\mathrm{ta}}{a\sqrt{c+a^2cx^2}}$$

$$= -\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{a}+\frac{1}{2}x\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2-\frac{ic\sqrt{1+a^2x^2}\tan^{-1}\left(e^{i\tan^{-1}(ax)}\right)\mathrm{ta}}{a\sqrt{c+a^2cx^2}}$$

$$= -\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{a}+\frac{1}{2}x\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2-\frac{ic\sqrt{1+a^2x^2}\tan^{-1}\left(e^{i\tan^{-1}(ax)}\right)\mathrm{ta}}{a\sqrt{c+a^2cx^2}}$$

$$= -\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{a}+\frac{1}{2}x\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2-\frac{ic\sqrt{1+a^2x^2}\tan^{-1}\left(e^{i\tan^{-1}(ax)}\right)\mathrm{ta}}{a\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 0.381306, size = 201, normalized size = 0.59

$$\frac{\sqrt{c\left(a^2x^2+1\right)}\left(2i\tan^{-1}(ax)\text{PolyLog}\left(2,-ie^{i\tan^{-1}(ax)}\right)-2i\tan^{-1}(ax)\text{PolyLog}\left(2,ie^{i\tan^{-1}(ax)}\right)-2\text{PolyLog}\left(3,-ie^{i\tan^{-1}(ax)}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2,x]
```

```
[Out] (Sqrt[c*(1 + a^2*x^2)]*(-2*Sqrt[1 + a^2*x^2]*ArcTan[a*x] + a*x*Sqrt[1 + a^2
*x^2]*ArcTan[a*x]^2 - (2*I)*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2 + 2*Arc
Tanh[(a*x)/Sqrt[1 + a^2*x^2]] + (2*I)*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcT
an[a*x])] - (2*I)*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])] - 2*PolyLog[3
, (-I)*E^(I*ArcTan[a*x])] + 2*PolyLog[3, I*E^(I*ArcTan[a*x])]))/(2*a*Sqrt[1
 + a^2*x^2])
```

Maple [A] time = 0.405, size = 268, normalized size = 0.8

$$\frac{\arctan(ax)(\arctan(ax)xa-2)}{2a}\sqrt{c(ax-i)(ax+i)}+\frac{\frac{i}{2}}{a}\sqrt{c(ax-i)(ax+i)}\left(i(\arctan(ax))^2\ln\left(1+i(1+iax)\frac{1}{\sqrt{a^2x^2+1}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^2*(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] 1/2/a*(c*(a*x-I)*(a*x+I))^(1/2)*arctan(a*x)*(arctan(a*x)*x*a-2)+1/2*I*(c*(a
*x-I)*(a*x+I))^(1/2)*(I*arctan(a*x)^2*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-I
*arctan(a*x)^2*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*arctan(a*x)*polylog(2,
-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*arctan(a*x)*polylog(2,I*(1+I*a*x)/(a^2*x^
2+1)^(1/2))+2*I*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*I*polylog(3,I*(
1+I*a*x)/(a^2*x^2+1)^(1/2))-4*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2)))/a/(a^2*x
^2+1)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2*(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{a^2cx^2+c}\arctan(ax)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2*(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{c\left(a^2x^2+1\right)}\,\text{atan}^2(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**2*(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(c*(a**2*x**2 + 1))*atan(a*x)**2, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2*(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.311 $\int \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{x}\,dx$

Optimal. Leaf size=439

$$-\frac{2ic\sqrt{a^2x^2+1}\mathrm{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{a^2cx^2+c}}+\frac{2ic\sqrt{a^2x^2+1}\mathrm{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{a^2cx^2+c}}+\frac{2ic\sqrt{a^2x^2+1}\tan^{-1}(ax)\mathrm{PolyLog}\left(2,-e^{i\tan^{-1}(ax)}\right.}{\sqrt{a^2cx^2+c}}$$

```
[Out] Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2 + ((4*I)*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*
ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2] - (2*c*Sqrt[1
+ a^2*x^2]*ArcTan[a*x]^2*ArcTanh[E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] +
((2*I)*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])])/Sqrt
[c + a^2*c*x^2] - ((2*I)*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, E^(I*Ar
cTan[a*x])])/Sqrt[c + a^2*c*x^2] - ((2*I)*c*Sqrt[1 + a^2*x^2]*PolyLog[2, ((
-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2] + ((2*I)*c*Sqrt[
1 + a^2*x^2]*PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*
c*x^2] - (2*c*Sqrt[1 + a^2*x^2]*PolyLog[3, -E^(I*ArcTan[a*x])])/Sqrt[c + a^
2*c*x^2] + (2*c*Sqrt[1 + a^2*x^2]*PolyLog[3, E^(I*ArcTan[a*x])])/Sqrt[c + a
^2*c*x^2]
```

Rubi [A] time = 0.513494, antiderivative size = 439, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {4950, 4958, 4956, 4183, 2531, 2282, 6589, 4930, 4890, 4886}

$$-\frac{2ic\sqrt{a^2x^2+1}\mathrm{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{a^2cx^2+c}}+\frac{2ic\sqrt{a^2x^2+1}\mathrm{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{a^2cx^2+c}}+\frac{2ic\sqrt{a^2x^2+1}\tan^{-1}(ax)\mathrm{PolyLog}\left(2,-e^{i\tan^{-1}(ax)}\right.}{\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/x,x]
```


```
[Out] Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2 + ((4*I)*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*
ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2] - (2*c*Sqrt[1
+ a^2*x^2]*ArcTan[a*x]^2*ArcTanh[E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] +
((2*I)*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])])/Sqrt
[c + a^2*c*x^2] - ((2*I)*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, E^(I*Ar
cTan[a*x])])/Sqrt[c + a^2*c*x^2] - ((2*I)*c*Sqrt[1 + a^2*x^2]*PolyLog[2, ((
-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2] + ((2*I)*c*Sqrt[
1 + a^2*x^2]*PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*
c*x^2] - (2*c*Sqrt[1 + a^2*x^2]*PolyLog[3, -E^(I*ArcTan[a*x])])/Sqrt[c + a^
2*c*x^2] + (2*c*Sqrt[1 + a^2*x^2]*PolyLog[3, E^(I*ArcTan[a*x])])/Sqrt[c + a
```

```
^2*c*x^2]
```

Rule 4950

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 4958

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4956

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Dist[1/Sqrt[d], Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTan
[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && G
tQ[d, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
```

```
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 4890

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4886

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[(-2*I*(a + b*ArcTan[c*x])*ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]])
/(c*Sqrt[d]), x] + (Simp[(I*b*PolyLog[2, -((I*Sqrt[1 + I*c*x])/Sqrt[1 - I*c
*x])])/(c*Sqrt[d]), x] - Simp[(I*b*PolyLog[2, (I*Sqrt[1 + I*c*x])/Sqrt[1 -
I*c*x]])/(c*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

Rubi steps

$$\int \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{x}\,dx = c\int \frac{\tan^{-1}(ax)^2}{x\sqrt{c+a^2cx^2}}\,dx + \left(a^2c\right)\int \frac{x\tan^{-1}(ax)^2}{\sqrt{c+a^2cx^2}}\,dx$$

$$= \sqrt{c+a^2cx^2}\tan^{-1}(ax)^2 - (2ac)\int \frac{\tan^{-1}(ax)}{\sqrt{c+a^2cx^2}}\,dx + \frac{\left(c\sqrt{1+a^2x^2}\right)\int \frac{\tan^{-1}(ax)^2}{x\sqrt{1+a^2x^2}}\,dx}{\sqrt{c+a^2cx^2}}$$

$$= \sqrt{c+a^2cx^2}\tan^{-1}(ax)^2 + \frac{\left(c\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int x^2\csc(x)\,dx, x, \tan^{-1}(ax)\right)}{\sqrt{c+a^2cx^2}} - \frac{\left(2ac\sqrt{1\right.}$$

$$= \sqrt{c+a^2cx^2}\tan^{-1}(ax)^2 + \frac{4ic\sqrt{1+a^2x^2}\tan^{-1}(ax)\tan^{-1}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}} - \frac{2c\sqrt{1+a^2x^2}\tan^{-1}(}{\sqrt{c}}$$

$$= \sqrt{c+a^2cx^2}\tan^{-1}(ax)^2 + \frac{4ic\sqrt{1+a^2x^2}\tan^{-1}(ax)\tan^{-1}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}} - \frac{2c\sqrt{1+a^2x^2}\tan^{-1}(}{\sqrt{c}}$$

$$= \sqrt{c+a^2cx^2}\tan^{-1}(ax)^2 + \frac{4ic\sqrt{1+a^2x^2}\tan^{-1}(ax)\tan^{-1}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}} - \frac{2c\sqrt{1+a^2x^2}\tan^{-1}(}{\sqrt{c}}$$

$$= \sqrt{c+a^2cx^2}\tan^{-1}(ax)^2 + \frac{4ic\sqrt{1+a^2x^2}\tan^{-1}(ax)\tan^{-1}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}} - \frac{2c\sqrt{1+a^2x^2}\tan^{-1}(}{\sqrt{c}}$$

Mathematica [A] time = 0.255023, size = 250, normalized size = 0.57

$$\frac{\sqrt{a^2cx^2+c}\left(2i\tan^{-1}(ax)\text{PolyLog}\left(2,-e^{i\tan^{-1}(ax)}\right) - 2i\tan^{-1}(ax)\text{PolyLog}\left(2,e^{i\tan^{-1}(ax)}\right) - 2i\text{PolyLog}\left(2,-ie^{i\tan^{-1}(ax)}\right)\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/x,x]
```

```
[Out] (Sqrt[c + a^2*c*x^2]*(Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2 + ArcTan[a*x]^2*Log[1
 - E^(I*ArcTan[a*x])] - 2*ArcTan[a*x]*Log[1 - I*E^(I*ArcTan[a*x])] + 2*ArcT
an[a*x]*Log[1 + I*E^(I*ArcTan[a*x])] - ArcTan[a*x]^2*Log[1 + E^(I*ArcTan[a*
x])] + (2*I)*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])] - (2*I)*PolyLog[2,
(-I)*E^(I*ArcTan[a*x])] + (2*I)*PolyLog[2, I*E^(I*ArcTan[a*x])] - (2*I)*Arc
Tan[a*x]*PolyLog[2, E^(I*ArcTan[a*x])] - 2*PolyLog[3, -E^(I*ArcTan[a*x])] +
 2*PolyLog[3, E^(I*ArcTan[a*x])]))/Sqrt[1 + a^2*x^2]
```

Maple [A] time = 0.454, size = 337, normalized size = 0.8

$$\sqrt{c\,(ax-i)\,(ax+i)}\,(\arctan\,(ax))^2+i\sqrt{c\,(ax-i)\,(ax+i)}\left(i\,(\arctan\,(ax))^2\ln\left(1+(1+iax)\frac{1}{\sqrt{a^2x^2+1}}\right)-i\,(\arctan\,(ax))^2\,\mathrm{l}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/x,x)
```

```
[Out] (c*(a*x-I)*(a*x+I))^(1/2)*arctan(a*x)^2+I*(c*(a*x-I)*(a*x+I))^(1/2)*(I*arct
an(a*x)^2*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*arctan(a*x)^2*ln(1-(1+I*a*x)/
(a^2*x^2+1)^(1/2))-2*I*arctan(a*x)*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*I*
arctan(a*x)*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*arctan(a*x)*polylog(2,-(1
+I*a*x)/(a^2*x^2+1)^(1/2))-2*arctan(a*x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1
/2))+2*I*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*I*polylog(3,(1+I*a*x)/(a
^2*x^2+1)^(1/2))-2*dilog(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*dilog(1-I*(1+I*
a*x)/(a^2*x^2+1)^(1/2)))/(a^2*x^2+1)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{a^2cx^2+c}\arctan\,(ax)^2}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c\left(a^2x^2+1\right)}\,\mathrm{atan}^2\left(ax\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**2*(a**2*c*x**2+c)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(c*(a**2*x**2 + 1))*atan(a*x)**2/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a^2cx^2+c}\,\arctan\left(ax\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2/x, x)
```

3.312 $\int \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{x^2}\,dx$

Optimal. Leaf size=458

$$\frac{2iac\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{a^2cx^2+c}}-\frac{2iac\sqrt{a^2x^2+1}\text{PolyLog}\left(2,\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{a^2cx^2+c}}+\frac{2iac\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,-ie^{i\tan^{-1}(ax)}\right)}{\sqrt{a^2cx^2+c}}$$

```
[Out] -((Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/x) - ((2*I)*a*c*Sqrt[1 + a^2*x^2]*Arc
Tan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2)/Sqrt[c + a^2*c*x^2] - (4*a*c*Sqrt[1 +
 a^2*x^2]*ArcTan[a*x]*ArcTanh[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/Sqrt[c + a^
2*c*x^2] + ((2*I)*a*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*Ar
cTan[a*x])])/Sqrt[c + a^2*c*x^2] - ((2*I)*a*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]
*PolyLog[2, I*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + ((2*I)*a*c*Sqrt[1 +
 a^2*x^2]*PolyLog[2, -(Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x])])/Sqrt[c + a^2*c*x^
2] - ((2*I)*a*c*Sqrt[1 + a^2*x^2]*PolyLog[2, Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x
]])/Sqrt[c + a^2*c*x^2] - (2*a*c*Sqrt[1 + a^2*x^2]*PolyLog[3, (-I)*E^(I*Arc
Tan[a*x])])/Sqrt[c + a^2*c*x^2] + (2*a*c*Sqrt[1 + a^2*x^2]*PolyLog[3, I*E^(
I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2]
```

Rubi [A] time = 0.529323, antiderivative size = 458, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {4950, 4944, 4958, 4954, 4890, 4888, 4181, 2531, 2282, 6589}

$$\frac{2iac\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{a^2cx^2+c}}-\frac{2iac\sqrt{a^2x^2+1}\text{PolyLog}\left(2,\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{a^2cx^2+c}}+\frac{2iac\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,-ie^{i\tan^{-1}(ax)}\right)}{\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/x^2,x]
```


```
[Out] -((Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/x) - ((2*I)*a*c*Sqrt[1 + a^2*x^2]*Arc
Tan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2)/Sqrt[c + a^2*c*x^2] - (4*a*c*Sqrt[1 +
 a^2*x^2]*ArcTan[a*x]*ArcTanh[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/Sqrt[c + a^
2*c*x^2] + ((2*I)*a*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*Ar
cTan[a*x])])/Sqrt[c + a^2*c*x^2] - ((2*I)*a*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]
*PolyLog[2, I*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + ((2*I)*a*c*Sqrt[1 +
 a^2*x^2]*PolyLog[2, -(Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x])])/Sqrt[c + a^2*c*x^
2] - ((2*I)*a*c*Sqrt[1 + a^2*x^2]*PolyLog[2, Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x
]])/Sqrt[c + a^2*c*x^2] - (2*a*c*Sqrt[1 + a^2*x^2]*PolyLog[3, (-I)*E^(I*Arc
Tan[a*x])])/Sqrt[c + a^2*c*x^2] + (2*a*c*Sqrt[1 + a^2*x^2]*PolyLog[3, I*E^(
```

```
I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2]
```

Rule 4950

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 4944

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a +
 b*ArcTan[c*x])^p)/(d*f*(m + 1)), x] - Dist[(b*c*p)/(f*(m + 1)), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rule 4958

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4954

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x_
Symbol] :> Simp[(-2*(a + b*ArcTan[c*x])*ArcTanh[Sqrt[1 + I*c*x]/Sqrt[1 - I*
c*x]])/Sqrt[d], x] + (Simp[(I*b*PolyLog[2, -(Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x
])])/Sqrt[d], x] - Simp[(I*b*PolyLog[2, Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]])/S
qrt[d], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

Rule 4890

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4888

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[c
*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && GtQ
[d, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{x^2}\,dx = c\int \frac{\tan^{-1}(ax)^2}{x^2\sqrt{c+a^2cx^2}}\,dx + \left(a^2c\right)\int \frac{\tan^{-1}(ax)^2}{\sqrt{c+a^2cx^2}}\,dx$$

$$= -\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{x} + (2ac)\int \frac{\tan^{-1}(ax)}{x\sqrt{c+a^2cx^2}}\,dx + \frac{\left(a^2c\sqrt{1+a^2x^2}\right)\int \frac{\tan^{-1}(ax)^2}{\sqrt{1+a^2x^2}}\,dx}{\sqrt{c+a^2cx^2}}$$

$$= -\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{x} + \frac{\left(ac\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int x^2\sec(x)\,dx, x, \tan^{-1}(ax)\right)}{\sqrt{c+a^2cx^2}} + \frac{(2ac}{}$$

$$= -\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{x} - \frac{2iac\sqrt{1+a^2x^2}\tan^{-1}\left(e^{i\tan^{-1}(ax)}\right)\tan^{-1}(ax)^2}{\sqrt{c+a^2cx^2}} - \frac{4ac\sqrt{1+a^2x}}{}$$

$$= -\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{x} - \frac{2iac\sqrt{1+a^2x^2}\tan^{-1}\left(e^{i\tan^{-1}(ax)}\right)\tan^{-1}(ax)^2}{\sqrt{c+a^2cx^2}} - \frac{4ac\sqrt{1+a^2x}}{}$$

$$= -\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{x} - \frac{2iac\sqrt{1+a^2x^2}\tan^{-1}\left(e^{i\tan^{-1}(ax)}\right)\tan^{-1}(ax)^2}{\sqrt{c+a^2cx^2}} - \frac{4ac\sqrt{1+a^2x}}{}$$

$$= -\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{x} - \frac{2iac\sqrt{1+a^2x^2}\tan^{-1}\left(e^{i\tan^{-1}(ax)}\right)\tan^{-1}(ax)^2}{\sqrt{c+a^2cx^2}} - \frac{4ac\sqrt{1+a^2x}}{}$$

Mathematica [A] time = 0.74865, size = 265, normalized size = 0.58

$$-\frac{a\sqrt{c\left(a^2x^2+1\right)}\left(-2i\tan^{-1}(ax)\text{PolyLog}\left(2,-ie^{i\tan^{-1}(ax)}\right)+2i\tan^{-1}(ax)\text{PolyLog}\left(2,ie^{i\tan^{-1}(ax)}\right)-2i\text{PolyLog}\left(2,-e^{i\,\text{t}}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/x^2,x]
```

```
[Out] -((a*Sqrt[c*(1 + a^2*x^2)]*((Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2)/(a*x) - 2*Arc
Tan[a*x]*Log[1 - E^(I*ArcTan[a*x])] - ArcTan[a*x]^2*Log[1 - I*E^(I*ArcTan[a
*x])] + ArcTan[a*x]^2*Log[1 + I*E^(I*ArcTan[a*x])] + 2*ArcTan[a*x]*Log[1 +
E^(I*ArcTan[a*x])] - (2*I)*PolyLog[2, -E^(I*ArcTan[a*x])] - (2*I)*ArcTan[a*
x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] + (2*I)*ArcTan[a*x]*PolyLog[2, I*E^(I
*ArcTan[a*x])] + (2*I)*PolyLog[2, E^(I*ArcTan[a*x])] + 2*PolyLog[3, (-I)*E^
(I*ArcTan[a*x])] - 2*PolyLog[3, I*E^(I*ArcTan[a*x])]))/Sqrt[1 + a^2*x^2])
```

Maple [A] time = 0.453, size = 309, normalized size = 0.7

$$-\frac{(\arctan(ax))^2}{x}\sqrt{c(ax-i)(ax+i)}-a\sqrt{c(ax-i)(ax+i)}\left((\arctan(ax))^2\ln\left(1+i(1+iax)\frac{1}{\sqrt{a^2x^2+1}}\right)-(\arctan(ax))^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/x^2,x)
```

```
[Out] -(c*(a*x-I)*(a*x+I))^(1/2)*arctan(a*x)^2/x-a*(c*(a*x-I)*(a*x+I))^(1/2)*(arc
tan(a*x)^2*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-arctan(a*x)^2*ln(1-I*(1+I*a*
x)/(a^2*x^2+1)^(1/2))-2*I*arctan(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1
/2))+2*I*arctan(a*x)*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*arctan(a*x)
*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*I*dilog((1+I*a*x)/(a^2*x^2+1)^(1/2))-2
*I*dilog(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1
)^(1/2))-2*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))/(a^2*x^2+1)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c\left(a^2x^2+1\right)}\,\mathrm{atan}^2\left(ax\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**2*(a**2*c*x**2+c)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(c*(a**2*x**2 + 1))*atan(a*x)**2/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a^2cx^2+c}\,\arctan\left(ax\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2/x^2, x)
```

3.313 $\int \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{x^3}\,dx$

Optimal. Leaf size=328

$$\frac{ia^2c\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,-e^{i\tan^{-1}(ax)}\right)}{\sqrt{a^2cx^2+c}}-\frac{ia^2c\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,e^{i\tan^{-1}(ax)}\right)}{\sqrt{a^2cx^2+c}}-\frac{a^2c\sqrt{a^2x^2+1}\text{Po}}{\sqrt{a}}$$

```
[Out] -((a*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/x) - (Sqrt[c + a^2*c*x^2]*ArcTan[a*x]
^2)/(2*x^2) - (a^2*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*ArcTanh[E^(I*ArcTan[a*
x])])/Sqrt[c + a^2*c*x^2] - a^2*Sqrt[c]*ArcTanh[Sqrt[c + a^2*c*x^2]/Sqrt[c]
] + (I*a^2*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])])/
Sqrt[c + a^2*c*x^2] - (I*a^2*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, E^(
I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - (a^2*c*Sqrt[1 + a^2*x^2]*PolyLog[3,
-E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + (a^2*c*Sqrt[1 + a^2*x^2]*PolyLog
[3, E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2]
```

Rubi [A] time = 0.856418, antiderivative size = 328, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 12, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4950, 4962, 4944, 266, 63, 208, 4958, 4956, 4183, 2531, 2282, 6589}

$$\frac{ia^2c\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,-e^{i\tan^{-1}(ax)}\right)}{\sqrt{a^2cx^2+c}}-\frac{ia^2c\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,e^{i\tan^{-1}(ax)}\right)}{\sqrt{a^2cx^2+c}}-\frac{a^2c\sqrt{a^2x^2+1}\text{Po}}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/x^3,x]
```


```
[Out] -((a*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/x) - (Sqrt[c + a^2*c*x^2]*ArcTan[a*x]
^2)/(2*x^2) - (a^2*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*ArcTanh[E^(I*ArcTan[a*
x])])/Sqrt[c + a^2*c*x^2] - a^2*Sqrt[c]*ArcTanh[Sqrt[c + a^2*c*x^2]/Sqrt[c]
] + (I*a^2*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])])/
Sqrt[c + a^2*c*x^2] - (I*a^2*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, E^(
I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - (a^2*c*Sqrt[1 + a^2*x^2]*PolyLog[3,
-E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + (a^2*c*Sqrt[1 + a^2*x^2]*PolyLog
[3, E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2]
```


Rule 4950

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
```

```
b*ArcTan[c*x])^p, x], x] + Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 4962

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Ar
cTan[c*x])^p)/(d*f*(m + 1)), x] + (-Dist[(b*c*p)/(f*(m + 1)), Int[((f*x)^(m
 + 1)*(a + b*ArcTan[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] - Dist[(c^2*(m +
 2))/(f^2*(m + 1)), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/Sqrt[d + e*x^
2], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] &
& LtQ[m, -1] && NeQ[m, -2]
```

Rule 4944

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a +
 b*ArcTan[c*x])^p)/(d*f*(m + 1)), x] - Dist[(b*c*p)/(f*(m + 1)), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 4958

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4956

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Dist[1/Sqrt[d], Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTan
[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && G
tQ[d, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{x^3}\,dx = c\int \frac{\tan^{-1}(ax)^2}{x^3\sqrt{c+a^2cx^2}}\,dx + \left(a^2c\right)\int \frac{\tan^{-1}(ax)^2}{x\sqrt{c+a^2cx^2}}\,dx$$

$$= -\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2x^2} + (ac)\int \frac{\tan^{-1}(ax)}{x^2\sqrt{c+a^2cx^2}}\,dx - \frac{1}{2}\left(a^2c\right)\int \frac{\tan^{-1}(ax)^2}{x\sqrt{c+a^2cx^2}}\,dx + \frac{\left(a^2\right.}{}$$

$$= -\frac{a\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{x} - \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2x^2} + \left(a^2c\right)\int \frac{1}{x\sqrt{c+a^2cx^2}}\,dx - \frac{\left(a^2c\sqrt{1\right.}{}$$

$$= -\frac{a\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{x} - \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2x^2} - \frac{2a^2c\sqrt{1+a^2x^2}\tan^{-1}(ax)^2\tanh^{-1}}{\sqrt{c+a^2cx^2}}$$

$$= -\frac{a\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{x} - \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2x^2} - \frac{a^2c\sqrt{1+a^2x^2}\tan^{-1}(ax)^2\tanh^{-1}\left(\right.}{\sqrt{c+a^2cx^2}}$$

$$= -\frac{a\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{x} - \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2x^2} - \frac{a^2c\sqrt{1+a^2x^2}\tan^{-1}(ax)^2\tanh^{-1}\left(\right.}{\sqrt{c+a^2cx^2}}$$

$$= -\frac{a\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{x} - \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2x^2} - \frac{a^2c\sqrt{1+a^2x^2}\tan^{-1}(ax)^2\tanh^{-1}\left(\right.}{\sqrt{c+a^2cx^2}}$$

$$= -\frac{a\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{x} - \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2x^2} - \frac{a^2c\sqrt{1+a^2x^2}\tan^{-1}(ax)^2\tanh^{-1}\left(\right.}{\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 1.79175, size = 222, normalized size = 0.68

$$\frac{a^2\sqrt{c\left(a^2x^2+1\right)}\left(8i\tan^{-1}(ax)\left(\text{PolyLog}\left(2,-e^{i\tan^{-1}(ax)}\right)-\text{PolyLog}\left(2,e^{i\tan^{-1}(ax)}\right)\right)+8\left(\text{PolyLog}\left(3,e^{i\tan^{-1}(ax)}\right)-\text{Pol}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/x^3,x]
```

```
[Out] (a^2*Sqrt[c*(1 + a^2*x^2)]*(-4*ArcTan[a*x]*Cot[ArcTan[a*x]/2] - ArcTan[a*x]
^2*Csc[ArcTan[a*x]/2]^2 + 4*ArcTan[a*x]^2*(Log[1 - E^(I*ArcTan[a*x])] - Log
[1 + E^(I*ArcTan[a*x])]) + 8*Log[Tan[ArcTan[a*x]/2]] + (8*I)*ArcTan[a*x]*(P
olyLog[2, -E^(I*ArcTan[a*x])] - PolyLog[2, E^(I*ArcTan[a*x])]) + 8*(-PolyLo
g[3, -E^(I*ArcTan[a*x])] + PolyLog[3, E^(I*ArcTan[a*x])]) + ArcTan[a*x]^2*S
ec[ArcTan[a*x]/2]^2 - 4*ArcTan[a*x]*Tan[ArcTan[a*x]/2]))/(8*Sqrt[1 + a^2*x^
```

```
2])
```

Maple [A] time = 0.414, size = 255, normalized size = 0.8

$$-\frac{\arctan(ax)(2ax+\arctan(ax))}{2x^2}\sqrt{c(ax-i)(ax+i)}-\frac{a^2}{2}\sqrt{c(ax-i)(ax+i)}\left((\arctan(ax))^2\ln\left(1+(1+iax)\frac{1}{\sqrt{a^2x^2+1}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/x^3,x)
```

```
[Out] -1/2*(c*(a*x-I)*(a*x+I))^(1/2)*arctan(a*x)*(2*a*x+arctan(a*x))/x^2-1/2*a^2*
(c*(a*x-I)*(a*x+I))^(1/2)*(arctan(a*x)^2*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-
arctan(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*I*arctan(a*x)*polylog(2,-
(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*I*arctan(a*x)*polylog(2,(1+I*a*x)/(a^2*x^2+1
)^(1/2))+2*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*polylog(3,(1+I*a*x)/(a
^2*x^2+1)^(1/2))+4*arctanh((1+I*a*x)/(a^2*x^2+1)^(1/2)))/(a^2*x^2+1)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c\left(a^2x^2+1\right)}\operatorname{atan}^2\left(ax\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**2*(a**2*c*x**2+c)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(c*(a**2*x**2 + 1))*atan(a*x)**2/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a^2cx^2+c}\arctan\left(ax\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2/x^3, x)
```

3.314 $\int \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{x^4}\,dx$

Optimal. Leaf size=275

$$\frac{ia^3c\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{3\sqrt{a^2cx^2+c}}-\frac{ia^3c\sqrt{a^2x^2+1}\text{PolyLog}\left(2,\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{3\sqrt{a^2cx^2+c}}-\frac{a^2\sqrt{a^2cx^2+c}}{3x}-\frac{a\sqrt{a^2cx^2+c}\tan^{-1}(ax)}{3x^2}-($$

```
[Out] -(a^2*Sqrt[c + a^2*c*x^2])/(3*x) - (a*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(3*x
^2) - ((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2)/(3*c*x^3) - (2*a^3*c*Sqrt[1 + a
^2*x^2]*ArcTan[a*x]*ArcTanh[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/(3*Sqrt[c + a
^2*c*x^2]) + ((I/3)*a^3*c*Sqrt[1 + a^2*x^2]*PolyLog[2, -(Sqrt[1 + I*a*x]/Sq
rt[1 - I*a*x])])/Sqrt[c + a^2*c*x^2] - ((I/3)*a^3*c*Sqrt[1 + a^2*x^2]*PolyL
og[2, Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2]
```

Rubi [A] time = 0.426936, antiderivative size = 275, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4944, 4946, 4962, 264, 4958, 4954}

$$\frac{ia^3c\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{3\sqrt{a^2cx^2+c}}-\frac{ia^3c\sqrt{a^2x^2+1}\text{PolyLog}\left(2,\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{3\sqrt{a^2cx^2+c}}-\frac{a^2\sqrt{a^2cx^2+c}}{3x}-\frac{a\sqrt{a^2cx^2+c}\tan^{-1}(ax)}{3x^2}-($$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/x^4,x]
```

```
[Out] -(a^2*Sqrt[c + a^2*c*x^2])/(3*x) - (a*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(3*x
^2) - ((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2)/(3*c*x^3) - (2*a^3*c*Sqrt[1 + a
^2*x^2]*ArcTan[a*x]*ArcTanh[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/(3*Sqrt[c + a
^2*c*x^2]) + ((I/3)*a^3*c*Sqrt[1 + a^2*x^2]*PolyLog[2, -(Sqrt[1 + I*a*x]/Sq
rt[1 - I*a*x])])/Sqrt[c + a^2*c*x^2] - ((I/3)*a^3*c*Sqrt[1 + a^2*x^2]*PolyL
og[2, Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2]
```

Rule 4944

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a +
 b*ArcTan[c*x])^p)/(d*f*(m + 1)), x] - Dist[(b*c*p)/(f*(m + 1)), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
```

```
& NeQ[m, -1]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*ArcTan[c*x
]))/(f*(m + 2)), x] + (Dist[d/(m + 2), Int[((f*x)^m*(a + b*ArcTan[c*x]))/Sq
rt[d + e*x^2], x], x] - Dist[(b*c*d)/(f*(m + 2)), Int[(f*x)^(m + 1)/Sqrt[d
+ e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && Ne
Q[m, -2]
```

Rule 4962

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Ar
cTan[c*x])^p)/(d*f*(m + 1)), x] + (-Dist[(b*c*p)/(f*(m + 1)), Int[((f*x)^(m
 + 1)*(a + b*ArcTan[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] - Dist[(c^2*(m +
 2))/(f^2*(m + 1)), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/Sqrt[d + e*x^
2], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] &
& LtQ[m, -1] && NeQ[m, -2]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 4958

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4954

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x_
Symbol] :> Simp[(-2*(a + b*ArcTan[c*x])*ArcTanh[Sqrt[1 + I*c*x]/Sqrt[1 - I*
c*x]])/Sqrt[d], x] + (Simp[(I*b*PolyLog[2, -(Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x
])])/Sqrt[d], x] - Simp[(I*b*PolyLog[2, Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]])/S
qrt[d], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

Rubi steps

$$\int \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{x^4}\,dx = -\frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2}{3cx^3} + \frac{1}{3}(2a)\int \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{x^3}\,dx$$

$$= -\frac{2a\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{3x^2} - \frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2}{3cx^3} - \frac{1}{3}(2ac)\int \frac{\tan^{-1}(ax)}{x^3\sqrt{c+a^2cx^2}}\,dx + \frac{1}{3}$$

$$= -\frac{2a^2\sqrt{c+a^2cx^2}}{3x} - \frac{a\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{3x^2} - \frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2}{3cx^3} - \frac{1}{3}\left(a^2c\right)\int$$

$$= -\frac{a^2\sqrt{c+a^2cx^2}}{3x} - \frac{a\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{3x^2} - \frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2}{3cx^3} + \frac{\left(a^3c\sqrt{1+a^2x^2}\right)}{3\sqrt{c+a}}$$

$$= -\frac{a^2\sqrt{c+a^2cx^2}}{3x} - \frac{a\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{3x^2} - \frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2}{3cx^3} - \frac{2a^3c\sqrt{1+a^2x^2}}{3}$$

Mathematica [A] time = 1.66286, size = 239, normalized size = 0.87

$$-\frac{c\sqrt{a^2x^2+1}\left(-4ia^3x^3\text{PolyLog}\left(2,-e^{i\tan^{-1}(ax)}\right)+4ia^3x^3\text{PolyLog}\left(2,e^{i\tan^{-1}(ax)}\right)+\sqrt{a^2x^2+1}\left(4a^2x^2+4\left(a^2x^2+1\right)\tan^{-1}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/x^4,x]
```

```
[Out] -(c*Sqrt[1 + a^2*x^2]*((-4*I)*a^3*x^3*PolyLog[2, -E^(I*ArcTan[a*x])] + (4*I
)*a^3*x^3*PolyLog[2, E^(I*ArcTan[a*x])] + Sqrt[1 + a^2*x^2]*(4*a^2*x^2 + 4*
(1 + a^2*x^2)*ArcTan[a*x]^2 + ArcTan[a*x]*(a*x*(4 - 3*Sqrt[1 + a^2*x^2]*Log
[1 - E^(I*ArcTan[a*x])] + 3*Sqrt[1 + a^2*x^2]*Log[1 + E^(I*ArcTan[a*x])]) +
 (1 + a^2*x^2)*(Log[1 - E^(I*ArcTan[a*x])] - Log[1 + E^(I*ArcTan[a*x])])*Si
n[3*ArcTan[a*x]]))))/(12*x^3*Sqrt[c + a^2*c*x^2])
```

Maple [A] time = 0.549, size = 195, normalized size = 0.7

$$-\frac{(\arctan(ax))^2x^2a^2+a^2x^2+\arctan(ax)xa+(\arctan(ax))^2}{3x^3}\sqrt{c(ax-i)(ax+i)}+\frac{i}{3}a^3\sqrt{c(ax-i)(ax+i)}\left(i\arctan(ax\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/x^4,x)
```

```
[Out] -1/3*(c*(a*x-I)*(a*x+I))^(1/2)*(arctan(a*x)^2*x^2*a^2+a^2*x^2+arctan(a*x)*x
*a+arctan(a*x)^2)/x^3+1/3*I*a^3*(c*(a*x-I)*(a*x+I))^(1/2)*(I*arctan(a*x)*ln
(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*arctan(a*x)*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1
/2))+polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-polylog(2,(1+I*a*x)/(a^2*x^2+1
)^(1/2)))/(a^2*x^2+1)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2/x^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c\left(a^2x^2+1\right)}\,\text{atan}^2\,(ax)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**2*(a**2*c*x**2+c)**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt(c*(a**2*x**2 + 1))*atan(a*x)**2/x**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a^2cx^2+c}\arctan{(ax)}^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2/x^4, x)
```

3.315 $\int x^3 \left(c+a^2 c x^2\right)^{3/2} \tan^{-1}(ax)^2 \, dx$

Optimal. Leaf size=476

$$\frac{17ic^2\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{280a^4\sqrt{a^2cx^2+c}}-\frac{17ic^2\sqrt{a^2x^2+1}\text{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{280a^4\sqrt{a^2cx^2+c}}-\frac{17ic^2\sqrt{a^2x^2+1}\tan^{-1}(ax)\tan^{-1}\left(\frac{\sqrt{1+ia}}{\sqrt{1-ia}}\right.}{140a^4\sqrt{a^2cx^2+c}}$$

```
[Out] (-17*c*Sqrt[c + a^2*c*x^2])/(280*a^4) - (17*(c + a^2*c*x^2)^(3/2))/(1260*a^
4) + (c + a^2*c*x^2)^(5/2)/(105*a^4*c) + (3*c*x*Sqrt[c + a^2*c*x^2]*ArcTan[
a*x])/(56*a^3) - (23*c*x^3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(420*a) - (a*c*
x^5*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/21 - (2*c*Sqrt[c + a^2*c*x^2]*ArcTan[a
*x]^2)/(35*a^4) + (c*x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(35*a^2) + (8*c
*x^4*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/35 + (a^2*c*x^6*Sqrt[c + a^2*c*x^2]
*ArcTan[a*x]^2)/7 - (((17*I)/140)*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTan[
Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/(a^4*Sqrt[c + a^2*c*x^2]) + (((17*I)/280)
*c^2*Sqrt[1 + a^2*x^2]*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/
(a^4*Sqrt[c + a^2*c*x^2]) - (((17*I)/280)*c^2*Sqrt[1 + a^2*x^2]*PolyLog[2,
(I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/(a^4*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 4.07276, antiderivative size = 476, normalized size of antiderivative = 1., number of steps used = 75, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4950, 4952, 261, 4890, 4886, 4930, 266, 43}

$$\frac{17ic^2\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{280a^4\sqrt{a^2cx^2+c}}-\frac{17ic^2\sqrt{a^2x^2+1}\text{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{280a^4\sqrt{a^2cx^2+c}}-\frac{17ic^2\sqrt{a^2x^2+1}\tan^{-1}(ax)\tan^{-1}\left(\frac{\sqrt{1+ia}}{\sqrt{1-ia}}\right.}{140a^4\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2,x]
```


```
[Out] (-17*c*Sqrt[c + a^2*c*x^2])/(280*a^4) - (17*(c + a^2*c*x^2)^(3/2))/(1260*a^
4) + (c + a^2*c*x^2)^(5/2)/(105*a^4*c) + (3*c*x*Sqrt[c + a^2*c*x^2]*ArcTan[
a*x])/(56*a^3) - (23*c*x^3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(420*a) - (a*c*
x^5*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/21 - (2*c*Sqrt[c + a^2*c*x^2]*ArcTan[a
*x]^2)/(35*a^4) + (c*x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(35*a^2) + (8*c
*x^4*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/35 + (a^2*c*x^6*Sqrt[c + a^2*c*x^2]
*ArcTan[a*x]^2)/7 - (((17*I)/140)*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTan[
Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/(a^4*Sqrt[c + a^2*c*x^2]) + (((17*I)/280)
*c^2*Sqrt[1 + a^2*x^2]*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/
(a^4*Sqrt[c + a^2*c*x^2]) - (((17*I)/280)*c^2*Sqrt[1 + a^2*x^2]*PolyLog[2,
```

```
(I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/(a^4*Sqrt[c + a^2*c*x^2])
```

Rule 4950

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 4952

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b*
ArcTan[c*x])^p)/(c^2*d*m), x] + (-Dist[(b*f*p)/(c*m), Int[((f*x)^(m - 1)*(a
 + b*ArcTan[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] - Dist[(f^2*(m - 1))/(c^
2*m), Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/Sqrt[d + e*x^2], x], x]) /;
 FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 4890

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4886

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[(-2*I*(a + b*ArcTan[c*x])*ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]])
/(c*Sqrt[d]), x] + (Simp[(I*b*PolyLog[2, -((I*Sqrt[1 + I*c*x])/Sqrt[1 - I*c
*x])])/(c*Sqrt[d]), x] - Simp[(I*b*PolyLog[2, (I*Sqrt[1 + I*c*x])/Sqrt[1 -
I*c*x]])/(c*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x^3\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2\,dx = c\int x^3\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2\,dx+\left(a^2c\right)\int x^5\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2\,dx$$

$$= c^2\int\frac{x^3\tan^{-1}(ax)^2}{\sqrt{c+a^2cx^2}}\,dx+2\left(\left(a^2c^2\right)\int\frac{x^5\tan^{-1}(ax)^2}{\sqrt{c+a^2cx^2}}\,dx\right)+\left(a^4c^2\right)\int\frac{x^7\tan^{-1}(ax)^2}{\sqrt{c+a^2cx^2}}\,d$$

$$= \frac{cx^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{3a^2}+\frac{1}{7}a^2cx^6\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2-\frac{\left(2c^2\right)\int\frac{x\tan^{-1}(ax)^2}{\sqrt{c+a^2cx^2}}\,dx}{3a^2}$$

$$= -\frac{cx\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{3a^3}-\frac{1}{21}acx^5\sqrt{c+a^2cx^2}\tan^{-1}(ax)-\frac{2c\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{3a^4}$$

$$= \frac{c\sqrt{c+a^2cx^2}}{3a^4}-\frac{cx\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{3a^3}+\frac{61cx^3\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{420a}-\frac{1}{21}acx^5\sqrt{c}$$

$$= \frac{c\sqrt{c+a^2cx^2}}{3a^4}-\frac{131cx\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{168a^3}+\frac{61cx^3\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{420a}-\frac{1}{21}acx^5$$

$$= \frac{139c\sqrt{c+a^2cx^2}}{168a^4}-\frac{2\left(c+a^2cx^2\right)^{3/2}}{63a^4}+\frac{\left(c+a^2cx^2\right)^{5/2}}{105a^4c}-\frac{131cx\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{168a^3}+$$

$$= \frac{817c\sqrt{c+a^2cx^2}}{840a^4}-\frac{101\left(c+a^2cx^2\right)^{3/2}}{1260a^4}+\frac{\left(c+a^2cx^2\right)^{5/2}}{105a^4c}-\frac{131cx\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{168a^3}$$

Mathematica [A] time = 4.56689, size = 797, normalized size = 1.67

$$\frac{c\left(a^2x^2+1\right)^2\sqrt{a^2cx^2+c}\left(\left(a^2x^2+1\right)\left(-5376\cos\left(2\tan^{-1}(ax)\right)\tan^{-1}(ax)^2+6720\cos\left(4\tan^{-1}(ax)\right)\tan^{-1}(ax)^2+10944\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2,x]
```

```
[Out] (c*(1 + a^2*x^2)^2*Sqrt[c + a^2*c*x^2]*(-168*(50 - 32*ArcTan[a*x]^2 + 72*Co
s[2*ArcTan[a*x]] + 160*ArcTan[a*x]^2*Cos[2*ArcTan[a*x]] + 22*Cos[4*ArcTan[a
*x]] - (110*ArcTan[a*x]*Log[1 - I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - 5
5*ArcTan[a*x]*Cos[3*ArcTan[a*x]]*Log[1 - I*E^(I*ArcTan[a*x])] - 11*ArcTan[a
*x]*Cos[5*ArcTan[a*x]]*Log[1 - I*E^(I*ArcTan[a*x])] + (110*ArcTan[a*x]*Log[
```

```
1 + I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + 55*ArcTan[a*x]*Cos[3*ArcTan[a
*x]]*Log[1 + I*E^(I*ArcTan[a*x])] + 11*ArcTan[a*x]*Cos[5*ArcTan[a*x]]*Log[1
 + I*E^(I*ArcTan[a*x])] - ((176*I)*PolyLog[2, (-I)*E^(I*ArcTan[a*x])])/(1 +
 a^2*x^2)^(5/2) + ((176*I)*PolyLog[2, I*E^(I*ArcTan[a*x])])/(1 + a^2*x^2)^(
5/2) + 4*ArcTan[a*x]*Sin[2*ArcTan[a*x]] - 22*ArcTan[a*x]*Sin[4*ArcTan[a*x]]
) + (1 + a^2*x^2)*(4116 + 10944*ArcTan[a*x]^2 + 6262*Cos[2*ArcTan[a*x]] - 5
376*ArcTan[a*x]^2*Cos[2*ArcTan[a*x]] + 2764*Cos[4*ArcTan[a*x]] + 6720*ArcTa
n[a*x]^2*Cos[4*ArcTan[a*x]] + 618*Cos[6*ArcTan[a*x]] - (10815*ArcTan[a*x]*L
og[1 - I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - 6489*ArcTan[a*x]*Cos[3*Arc
Tan[a*x]]*Log[1 - I*E^(I*ArcTan[a*x])] - 2163*ArcTan[a*x]*Cos[5*ArcTan[a*x]
]*Log[1 - I*E^(I*ArcTan[a*x])] - 309*ArcTan[a*x]*Cos[7*ArcTan[a*x]]*Log[1 -
 I*E^(I*ArcTan[a*x])] + (10815*ArcTan[a*x]*Log[1 + I*E^(I*ArcTan[a*x])])/Sq
rt[1 + a^2*x^2] + 6489*ArcTan[a*x]*Cos[3*ArcTan[a*x]]*Log[1 + I*E^(I*ArcTan
[a*x])] + 2163*ArcTan[a*x]*Cos[5*ArcTan[a*x]]*Log[1 + I*E^(I*ArcTan[a*x])]
+ 309*ArcTan[a*x]*Cos[7*ArcTan[a*x]]*Log[1 + I*E^(I*ArcTan[a*x])] - ((19776
*I)*PolyLog[2, (-I)*E^(I*ArcTan[a*x])])/(1 + a^2*x^2)^(7/2) + ((19776*I)*Po
lyLog[2, I*E^(I*ArcTan[a*x])])/(1 + a^2*x^2)^(7/2) - 1266*ArcTan[a*x]*Sin[2
*ArcTan[a*x]] + 360*ArcTan[a*x]*Sin[4*ArcTan[a*x]] - 618*ArcTan[a*x]*Sin[6*
ArcTan[a*x]])))/(161280*a^4)
```

Maple [A] time = 0.909, size = 271, normalized size = 0.6

$$\frac{c\left(360\,(\arctan(ax))^2\,x^6a^6-120\,\arctan(ax)\,x^5a^5+576\,(\arctan(ax))^2\,x^4a^4+24\,a^4x^4-138\,\arctan(ax)\,x^3a^3+72\,(a\right.}{2520\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^2,x)
```

```
[Out] 1/2520*c/a^4*(c*(a*x-I)*(a*x+I))^(1/2)*(360*arctan(a*x)^2*x^6*a^6-120*arcta
n(a*x)*x^5*a^5+576*arctan(a*x)^2*x^4*a^4+24*a^4*x^4-138*arctan(a*x)*x^3*a^3
+72*arctan(a*x)^2*x^2*a^2+14*a^2*x^2+135*arctan(a*x)*x*a-144*arctan(a*x)^2-
163)-17/280*c*(c*(a*x-I)*(a*x+I))^(1/2)*(arctan(a*x)*ln(1+I*(1+I*a*x)/(a^2*
x^2+1)^(1/2))-arctan(a*x)*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*dilog(1+I*(
1+I*a*x)/(a^2*x^2+1)^(1/2))+I*dilog(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))/a^4/(
a^2*x^2+1)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^2cx^5 + cx^3\right)\sqrt{a^2cx^2 + c}\arctan\left(ax\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^5 + c*x^3)*sqrt(a^2*c*x^2 + c)*arctan(a*x)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a**2*c*x**2+c)**(3/2)*atan(a*x)**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.316 $\int x^2 \left(c+a^2 c x^2\right)^{3/2} \tan^{-1}(ax)^2 \, dx$

Optimal. Leaf size=531

$$-\frac{ic^2\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,-ie^{i\tan^{-1}(ax)}\right)}{8a^3\sqrt{a^2cx^2+c}}+\frac{ic^2\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,ie^{i\tan^{-1}(ax)}\right)}{8a^3\sqrt{a^2cx^2+c}}+\frac{c^2\sqrt{a^2x^2+1}\text{P}}{8a}$$

```
[Out] (c*x*Sqrt[c + a^2*c*x^2])/(36*a^2) + (c*x^3*Sqrt[c + a^2*c*x^2])/60 + (31*c
*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(360*a^3) - (19*c*x^2*Sqrt[c + a^2*c*x^2]
*ArcTan[a*x])/(180*a) - (a*c*x^4*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/15 + (c*x
*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(16*a^2) + (7*c*x^3*Sqrt[c + a^2*c*x^2]
*ArcTan[a*x]^2)/24 + (a^2*c*x^5*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/6 + ((I/
8)*c^2*Sqrt[1 + a^2*x^2]*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2)/(a^3*Sqrt
[c + a^2*c*x^2]) - (41*c^(3/2)*ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]])/
(360*a^3) - ((I/8)*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*A
rcTan[a*x])])/(a^3*Sqrt[c + a^2*c*x^2]) + ((I/8)*c^2*Sqrt[1 + a^2*x^2]*ArcT
an[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])])/(a^3*Sqrt[c + a^2*c*x^2]) + (c^2*S
qrt[1 + a^2*x^2]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])])/(8*a^3*Sqrt[c + a^2*c*
x^2]) - (c^2*Sqrt[1 + a^2*x^2]*PolyLog[3, I*E^(I*ArcTan[a*x])])/(8*a^3*Sqrt
[c + a^2*c*x^2])
```

Rubi [A] time = 3.18631, antiderivative size = 531, normalized size of antiderivative = 1., number of steps used = 92, number of rules used = 12, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4950, 4952, 4930, 217, 206, 4890, 4888, 4181, 2531, 2282, 6589, 321}

$$-\frac{ic^2\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,-ie^{i\tan^{-1}(ax)}\right)}{8a^3\sqrt{a^2cx^2+c}}+\frac{ic^2\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,ie^{i\tan^{-1}(ax)}\right)}{8a^3\sqrt{a^2cx^2+c}}+\frac{c^2\sqrt{a^2x^2+1}\text{P}}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2,x]
```


```
[Out] (c*x*Sqrt[c + a^2*c*x^2])/(36*a^2) + (c*x^3*Sqrt[c + a^2*c*x^2])/60 + (31*c
*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(360*a^3) - (19*c*x^2*Sqrt[c + a^2*c*x^2]
*ArcTan[a*x])/(180*a) - (a*c*x^4*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/15 + (c*x
*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(16*a^2) + (7*c*x^3*Sqrt[c + a^2*c*x^2]
*ArcTan[a*x]^2)/24 + (a^2*c*x^5*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/6 + ((I/
8)*c^2*Sqrt[1 + a^2*x^2]*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2)/(a^3*Sqrt
[c + a^2*c*x^2]) - (41*c^(3/2)*ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]])/
(360*a^3) - ((I/8)*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*A
```

```
rcTan[a*x])])/(a^3*Sqrt[c + a^2*c*x^2]) + ((I/8)*c^2*Sqrt[1 + a^2*x^2]*ArcT
an[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])])/(a^3*Sqrt[c + a^2*c*x^2]) + (c^2*S
qrt[1 + a^2*x^2]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])])/(8*a^3*Sqrt[c + a^2*c*
x^2]) - (c^2*Sqrt[1 + a^2*x^2]*PolyLog[3, I*E^(I*ArcTan[a*x])])/(8*a^3*Sqrt
[c + a^2*c*x^2])
```

Rule 4950

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 4952

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b*
ArcTan[c*x])^p)/(c^2*d*m), x] + (-Dist[(b*f*p)/(c*m), Int[((f*x)^(m - 1)*(a
 + b*ArcTan[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] - Dist[(f^2*(m - 1))/(c^
2*m), Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/Sqrt[d + e*x^2], x], x]) /;
 FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 4890

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4888

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[c
*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && GtQ
[d, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int x^2\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2\,dx = c\int x^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2\,dx+\left(a^2c\right)\int x^4\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2\,dx$$

$$= c^2\int\frac{x^2\tan^{-1}(ax)^2}{\sqrt{c+a^2cx^2}}\,dx+2\left(\left(a^2c^2\right)\int\frac{x^4\tan^{-1}(ax)^2}{\sqrt{c+a^2cx^2}}\,dx\right)+\left(a^4c^2\right)\int\frac{x^6\tan^{-1}(ax)^2}{\sqrt{c+a^2cx^2}}$$

$$= \frac{cx\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2a^2}+\frac{1}{6}a^2cx^5\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2-\frac{c^2\int\frac{\tan^{-1}(ax)^2}{\sqrt{c+a^2cx^2}}\,dx}{2a^2}-\frac{c^2}{}$$

$$= -\frac{c\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{a^3}-\frac{1}{15}acx^4\sqrt{c+a^2cx^2}\tan^{-1}(ax)+\frac{cx\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{2a^2}$$

$$= \frac{1}{60}cx^3\sqrt{c+a^2cx^2}-\frac{c\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{a^3}+\frac{41cx^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{180a}-\frac{1}{15}acx$$

$$= -\frac{5cx\sqrt{c+a^2cx^2}}{36a^2}+\frac{1}{60}cx^3\sqrt{c+a^2cx^2}-\frac{749c\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{360a^3}+\frac{41cx^2\sqrt{c+a^2}}{18}$$

$$= -\frac{5cx\sqrt{c+a^2cx^2}}{36a^2}+\frac{1}{60}cx^3\sqrt{c+a^2cx^2}-\frac{749c\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{360a^3}+\frac{41cx^2\sqrt{c+a^2}}{18}$$

$$= -\frac{5cx\sqrt{c+a^2cx^2}}{36a^2}+\frac{1}{60}cx^3\sqrt{c+a^2cx^2}-\frac{749c\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{360a^3}+\frac{41cx^2\sqrt{c+a^2}}{18}$$

$$= -\frac{5cx\sqrt{c+a^2cx^2}}{36a^2}+\frac{1}{60}cx^3\sqrt{c+a^2cx^2}-\frac{749c\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{360a^3}+\frac{41cx^2\sqrt{c+a^2}}{18}$$

$$= -\frac{5cx\sqrt{c+a^2cx^2}}{36a^2}+\frac{1}{60}cx^3\sqrt{c+a^2cx^2}-\frac{749c\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{360a^3}+\frac{41cx^2\sqrt{c+a^2}}{18}$$

$$= -\frac{5cx\sqrt{c+a^2cx^2}}{36a^2}+\frac{1}{60}cx^3\sqrt{c+a^2cx^2}-\frac{749c\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{360a^3}+\frac{41cx^2\sqrt{c+a^2}}{18}$$

Mathematica [A] time = 3.46325, size = 527, normalized size = 0.99

$$\frac{c\sqrt{a^2cx^2+c}\left(960\left(-3i\tan^{-1}(ax)\text{PolyLog}\left(2,-ie^{i\tan^{-1}(ax)}\right)+3i\tan^{-1}(ax)\text{PolyLog}\left(2,ie^{i\tan^{-1}(ax)}\right)+3\text{PolyLog}\left(3,-ie^{i}\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2,x]
```

```
[Out] (c*Sqrt[c + a^2*c*x^2]*(960*((3*I)*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2
- 2*ArcTanh[(a*x)/Sqrt[1 + a^2*x^2]] - (3*I)*ArcTan[a*x]*PolyLog[2, (-I)*E^
(I*ArcTan[a*x])] + (3*I)*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])] + 3*Po
lyLog[3, (-I)*E^(I*ArcTan[a*x])] - 3*PolyLog[3, I*E^(I*ArcTan[a*x])]) + 32*
((-45*I)*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2 + 19*ArcTanh[(a*x)/Sqrt[1
+ a^2*x^2]] + (45*I)*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - (45*I
)*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])] - 45*PolyLog[3, (-I)*E^(I*Arc
Tan[a*x])] + 45*PolyLog[3, I*E^(I*ArcTan[a*x])]) + 120*(1 + a^2*x^2)^(3/2)*
(ArcTan[a*x]*(2 + 6*Sqrt[1 + a^2*x^2]*Cos[3*ArcTan[a*x]]) - 3*ArcTan[a*x]^2
*(-7*a*x + Sqrt[1 + a^2*x^2]*Sin[3*ArcTan[a*x]]) + 2*(a*x + Sqrt[1 + a^2*x^
2]*Sin[3*ArcTan[a*x]])) + (1 + a^2*x^2)^3*((-56*a*x)/Sqrt[1 + a^2*x^2] + Ar
cTan[a*x]*(12/Sqrt[1 + a^2*x^2] + 110*Cos[3*ArcTan[a*x]] - 90*Cos[5*ArcTan[
a*x]]) - 108*Sin[3*ArcTan[a*x]] - 52*Sin[5*ArcTan[a*x]] + 15*ArcTan[a*x]^2*
((78*a*x)/Sqrt[1 + a^2*x^2] - 47*Sin[3*ArcTan[a*x]] + 3*Sin[5*ArcTan[a*x]])
)))/(11520*a^3*Sqrt[1 + a^2*x^2])
```

Maple [A] time = 0.487, size = 338, normalized size = 0.6

$$\frac{c\left(120\,(\arctan(ax))^2 x^5a^5 - 48\,\arctan(ax)\,x^4a^4 + 210\,(\arctan(ax))^2 x^3a^3 + 12\,a^3x^3 - 76\,\arctan(ax)\,a^2x^2 + 45\,(\mathrm{arcta}\right.}{720\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^2,x)
```

```
[Out] 1/720*c/a^3*(c*(a*x-I)*(a*x+I))^(1/2)*(120*arctan(a*x)^2*x^5*a^5-48*arctan(
a*x)*x^4*a^4+210*arctan(a*x)^2*x^3*a^3+12*a^3*x^3-76*arctan(a*x)*a^2*x^2+45
*arctan(a*x)^2*x*a+20*a*x+62*arctan(a*x))-1/720*I*c*(c*(a*x-I)*(a*x+I))^(1/
2)*(45*I*arctan(a*x)^2*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-45*I*arctan(a*x)
^2*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+90*arctan(a*x)*polylog(2,-I*(1+I*a*x
)/(a^2*x^2+1)^(1/2))-90*arctan(a*x)*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2)
)+90*I*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-90*I*polylog(3,I*(1+I*a*x)
/(a^2*x^2+1)^(1/2))-164*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2)))/a^3/(a^2*x^2+1
)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^2cx^4 + cx^2\right)\sqrt{a^2cx^2 + c}\arctan\left(ax\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^4 + c*x^2)*sqrt(a^2*c*x^2 + c)*arctan(a*x)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\operatorname{atan}^2\left(ax\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a**2*c*x**2+c)**(3/2)*atan(a*x)**2,x)
```

```
[Out] Integral(x**2*(c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**2, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.317 $\int x\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2\,dx$

Optimal. Leaf size=334

$$-\frac{3ic^2\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{20a^2\sqrt{a^2cx^2+c}}+\frac{3ic^2\sqrt{a^2x^2+1}\text{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{20a^2\sqrt{a^2cx^2+c}}+\frac{3ic^2\sqrt{a^2x^2+1}\tan^{-1}(ax)\tan^{-1}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{10a^2\sqrt{a^2cx^2+c}}$$

```
[Out] (3*c*Sqrt[c + a^2*c*x^2])/(20*a^2) + (c + a^2*c*x^2)^(3/2)/(30*a^2) - (3*c*
x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(20*a) - (x*(c + a^2*c*x^2)^(3/2)*ArcTan
[a*x])/(10*a) + ((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2)/(5*a^2*c) + (((3*I)/1
0)*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]
])/(a^2*Sqrt[c + a^2*c*x^2]) - (((3*I)/20)*c^2*Sqrt[1 + a^2*x^2]*PolyLog[2,
 ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/(a^2*Sqrt[c + a^2*c*x^2]) + (((3*
I)/20)*c^2*Sqrt[1 + a^2*x^2]*PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]
])/(a^2*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.232196, antiderivative size = 334, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4930, 4878, 4890, 4886}

$$-\frac{3ic^2\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{20a^2\sqrt{a^2cx^2+c}}+\frac{3ic^2\sqrt{a^2x^2+1}\text{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{20a^2\sqrt{a^2cx^2+c}}+\frac{3ic^2\sqrt{a^2x^2+1}\tan^{-1}(ax)\tan^{-1}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{10a^2\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2,x]
```

```
[Out] (3*c*Sqrt[c + a^2*c*x^2])/(20*a^2) + (c + a^2*c*x^2)^(3/2)/(30*a^2) - (3*c*
x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(20*a) - (x*(c + a^2*c*x^2)^(3/2)*ArcTan
[a*x])/(10*a) + ((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2)/(5*a^2*c) + (((3*I)/1
0)*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]
])/(a^2*Sqrt[c + a^2*c*x^2]) - (((3*I)/20)*c^2*Sqrt[1 + a^2*x^2]*PolyLog[2,
 ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/(a^2*Sqrt[c + a^2*c*x^2]) + (((3*
I)/20)*c^2*Sqrt[1 + a^2*x^2]*PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]
])/(a^2*Sqrt[c + a^2*c*x^2])
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
```

```
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 4878

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symbo
l] :> -Simp[(b*(d + e*x^2)^q)/(2*c*q*(2*q + 1)), x] + (Dist[(2*d*q)/(2*q +
1), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x]), x], x] + Simp[(x*(d + e*x^
2)^q*(a + b*ArcTan[c*x]))/(2*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] && Eq
Q[e, c^2*d] && GtQ[q, 0]
```

Rule 4890

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4886

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[(-2*I*(a + b*ArcTan[c*x])*ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]])
/(c*Sqrt[d]), x] + (Simp[(I*b*PolyLog[2, -((I*Sqrt[1 + I*c*x])/Sqrt[1 - I*c
*x])])/(c*Sqrt[d]), x] - Simp[(I*b*PolyLog[2, (I*Sqrt[1 + I*c*x])/Sqrt[1 -
I*c*x]])/(c*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

Rubi steps

$$\int x\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2\,dx = \frac{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^2}{5a^2c} - \frac{2\int\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)\,dx}{5a}$$
$$= \frac{\left(c+a^2cx^2\right)^{3/2}}{30a^2} - \frac{x\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}{10a} + \frac{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^2}{5a^2c} - \frac{(3c)\int\sqrt{c}}{}$$
$$= \frac{3c\sqrt{c+a^2cx^2}}{20a^2} + \frac{\left(c+a^2cx^2\right)^{3/2}}{30a^2} - \frac{3cx\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{20a} - \frac{x\left(c+a^2cx^2\right)^{3/2}\tan^{-1}}{10a}$$
$$= \frac{3c\sqrt{c+a^2cx^2}}{20a^2} + \frac{\left(c+a^2cx^2\right)^{3/2}}{30a^2} - \frac{3cx\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{20a} - \frac{x\left(c+a^2cx^2\right)^{3/2}\tan^{-1}}{10a}$$
$$= \frac{3c\sqrt{c+a^2cx^2}}{20a^2} + \frac{\left(c+a^2cx^2\right)^{3/2}}{30a^2} - \frac{3cx\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{20a} - \frac{x\left(c+a^2cx^2\right)^{3/2}\tan^{-1}}{10a}$$

Mathematica [A] time = 4.04465, size = 601, normalized size = 1.8

$$\frac{c\left(a^2x^2+1\right)\sqrt{a^2cx^2+c}\left(80\left(-\frac{4i\mathrm{PolyLog}\left(2,-ie^{i\tan^{-1}(ax)}\right)}{\left(a^2x^2+1\right)^{3/2}} + \frac{4i\mathrm{PolyLog}\left(2,ie^{i\tan^{-1}(ax)}\right)}{\left(a^2x^2+1\right)^{3/2}} - \frac{3\tan^{-1}(ax)\log\left(1-ie^{i\tan^{-1}(ax)}\right)}{\sqrt{a^2x^2+1}} + \frac{3\tan^{-1}(ax)\log\left(1\right.}{\sqrt{a^2x^2}}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2,x]
```

```
[Out] (c*(1 + a^2*x^2)*Sqrt[c + a^2*c*x^2]*(80*(2 + 4*ArcTan[a*x]^2 + 2*Cos[2*Arc
Tan[a*x]] - (3*ArcTan[a*x]*Log[1 - I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2]
- ArcTan[a*x]*Cos[3*ArcTan[a*x]]*Log[1 - I*E^(I*ArcTan[a*x])] + (3*ArcTan[a
*x]*Log[1 + I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + ArcTan[a*x]*Cos[3*Arc
Tan[a*x]]*Log[1 + I*E^(I*ArcTan[a*x])] - ((4*I)*PolyLog[2, (-I)*E^(I*ArcTan
[a*x])])/(1 + a^2*x^2)^(3/2) + ((4*I)*PolyLog[2, I*E^(I*ArcTan[a*x])])/(1 +
 a^2*x^2)^(3/2) - 2*ArcTan[a*x]*Sin[2*ArcTan[a*x]]) - (1 + a^2*x^2)*(50 - 3
2*ArcTan[a*x]^2 + 72*Cos[2*ArcTan[a*x]] + 160*ArcTan[a*x]^2*Cos[2*ArcTan[a*
x]] + 22*Cos[4*ArcTan[a*x]] - (110*ArcTan[a*x]*Log[1 - I*E^(I*ArcTan[a*x])]
)/Sqrt[1 + a^2*x^2] - 55*ArcTan[a*x]*Cos[3*ArcTan[a*x]]*Log[1 - I*E^(I*ArcT
an[a*x])] - 11*ArcTan[a*x]*Cos[5*ArcTan[a*x]]*Log[1 - I*E^(I*ArcTan[a*x])]
+ (110*ArcTan[a*x]*Log[1 + I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + 55*Arc
Tan[a*x]*Cos[3*ArcTan[a*x]]*Log[1 + I*E^(I*ArcTan[a*x])] + 11*ArcTan[a*x]*C
os[5*ArcTan[a*x]]*Log[1 + I*E^(I*ArcTan[a*x])] - ((176*I)*PolyLog[2, (-I)*E
^(I*ArcTan[a*x])])/(1 + a^2*x^2)^(5/2) + ((176*I)*PolyLog[2, I*E^(I*ArcTan[
a*x])])/(1 + a^2*x^2)^(5/2) + 4*ArcTan[a*x]*Sin[2*ArcTan[a*x]] - 22*ArcTan[
```

```
a*x]*Sin[4*ArcTan[a*x]])))/(960*a^2)
```

Maple [A] time = 0.309, size = 237, normalized size = 0.7

$$\frac{c\left(12\left(\arctan\left(ax\right)\right)^{2}x^{4}a^{4}-6\arctan\left(ax\right)x^{3}a^{3}+24\left(\arctan\left(ax\right)\right)^{2}x^{2}a^{2}+2a^{2}x^{2}-15\arctan\left(ax\right)xa+12\left(\arctan\left(ax\right)\right)^{2}}{60a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^2,x)
```

```
[Out] 1/60*c/a^2*(c*(a*x-I)*(a*x+I))^(1/2)*(12*arctan(a*x)^2*x^4*a^4-6*arctan(a*x
)*x^3*a^3+24*arctan(a*x)^2*x^2*a^2+2*a^2*x^2-15*arctan(a*x)*x*a+12*arctan(a
*x)^2+11)+3/20*c*(c*(a*x-I)*(a*x+I))^(1/2)*(arctan(a*x)*ln(1+I*(1+I*a*x)/(a
^2*x^2+1)^(1/2))-arctan(a*x)*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*dilog(1+
I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+I*dilog(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))/a^
2/(a^2*x^2+1)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a^{2}cx^{2}+c\right)^{\frac{3}{2}}x\arctan\left(ax\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)*x*arctan(a*x)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a^{2}cx^{3}+cx\right)\sqrt{a^{2}cx^{2}+c}\arctan\left(ax\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^2,x, algorithm="fricas")
```

[Out] integral((a^2*c*x^3 + c*x)*sqrt(a^2*c*x^2 + c)*arctan(a*x)^2, x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\operatorname{atan}^2(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a**2*c*x**2+c)**(3/2)*atan(a*x)**2,x)

[Out] Integral(x*(c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**2, x)

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^2,x, algorithm="giac")

[Out] Exception raised: TypeError

3.318 $\int \left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2\,dx$

Optimal. Leaf size=438

$$\frac{3ic^2\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,-ie^{i\tan^{-1}(ax)}\right)}{4a\sqrt{a^2cx^2+c}}-\frac{3ic^2\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,ie^{i\tan^{-1}(ax)}\right)}{4a\sqrt{a^2cx^2+c}}-\frac{3c^2\sqrt{a^2x^2+1}\text{P}}{4a}$$

```
[Out] (c*x*Sqrt[c + a^2*c*x^2])/12 - (3*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(4*a)
- ((c + a^2*c*x^2)^(3/2)*ArcTan[a*x])/(6*a) + (3*c*x*Sqrt[c + a^2*c*x^2]*Ar
cTan[a*x]^2)/8 + (x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2)/4 - (((3*I)/4)*c^2
*Sqrt[1 + a^2*x^2]*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2)/(a*Sqrt[c + a^2
*c*x^2]) + (5*c^(3/2)*ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]])/(6*a) + (
((3*I)/4)*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x
])])/(a*Sqrt[c + a^2*c*x^2]) - (((3*I)/4)*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]
*PolyLog[2, I*E^(I*ArcTan[a*x])])/(a*Sqrt[c + a^2*c*x^2]) - (3*c^2*Sqrt[1 +
 a^2*x^2]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])])/(4*a*Sqrt[c + a^2*c*x^2]) + (
3*c^2*Sqrt[1 + a^2*x^2]*PolyLog[3, I*E^(I*ArcTan[a*x])])/(4*a*Sqrt[c + a^2*
c*x^2])
```

Rubi [A] time = 0.311359, antiderivative size = 438, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {4880, 4890, 4888, 4181, 2531, 2282, 6589, 217, 206, 195}

$$\frac{3ic^2\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,-ie^{i\tan^{-1}(ax)}\right)}{4a\sqrt{a^2cx^2+c}}-\frac{3ic^2\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,ie^{i\tan^{-1}(ax)}\right)}{4a\sqrt{a^2cx^2+c}}-\frac{3c^2\sqrt{a^2x^2+1}\text{P}}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2,x]
```


```
[Out] (c*x*Sqrt[c + a^2*c*x^2])/12 - (3*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(4*a)
- ((c + a^2*c*x^2)^(3/2)*ArcTan[a*x])/(6*a) + (3*c*x*Sqrt[c + a^2*c*x^2]*Ar
cTan[a*x]^2)/8 + (x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2)/4 - (((3*I)/4)*c^2
*Sqrt[1 + a^2*x^2]*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2)/(a*Sqrt[c + a^2
*c*x^2]) + (5*c^(3/2)*ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]])/(6*a) + (
((3*I)/4)*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x
])])/(a*Sqrt[c + a^2*c*x^2]) - (((3*I)/4)*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]
*PolyLog[2, I*E^(I*ArcTan[a*x])])/(a*Sqrt[c + a^2*c*x^2]) - (3*c^2*Sqrt[1 +
 a^2*x^2]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])])/(4*a*Sqrt[c + a^2*c*x^2]) + (
3*c^2*Sqrt[1 + a^2*x^2]*PolyLog[3, I*E^(I*ArcTan[a*x])])/(4*a*Sqrt[c + a^2*
```

```
c*x^2])
```

Rule 4880

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> -Simp[(b*p*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1))/(2*c*q*(2*
q + 1)), x] + (Dist[(2*d*q)/(2*q + 1), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTa
n[c*x])^p, x], x] + Dist[(b^2*d*p*(p - 1))/(2*q*(2*q + 1)), Int[(d + e*x^2)
^(q - 1)*(a + b*ArcTan[c*x])^(p - 2), x], x] + Simp[(x*(d + e*x^2)^q*(a + b
*ArcTan[c*x])^p)/(2*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[q, 0] && GtQ[p, 1]
```

Rule 4890

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4888

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[c
*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && GtQ
[d, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rubi steps

$$\int \left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2\,dx = -\frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}{6a} + \frac{1}{4}x\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2 + \frac{1}{6}c\int\sqrt{c+a^2cx^2}\,dx + \frac{1}{4}$$

$$= \frac{1}{12}cx\sqrt{c+a^2cx^2} - \frac{3c\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{4a} - \frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}{6a} + \frac{3}{8}cx\sqrt{c+a^2}$$

$$= \frac{1}{12}cx\sqrt{c+a^2cx^2} - \frac{3c\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{4a} - \frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}{6a} + \frac{3}{8}cx\sqrt{c+a^2}$$

$$= \frac{1}{12}cx\sqrt{c+a^2cx^2} - \frac{3c\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{4a} - \frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}{6a} + \frac{3}{8}cx\sqrt{c+a^2}$$

$$= \frac{1}{12}cx\sqrt{c+a^2cx^2} - \frac{3c\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{4a} - \frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}{6a} + \frac{3}{8}cx\sqrt{c+a^2}$$

$$= \frac{1}{12}cx\sqrt{c+a^2cx^2} - \frac{3c\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{4a} - \frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}{6a} + \frac{3}{8}cx\sqrt{c+a^2}$$

$$= \frac{1}{12}cx\sqrt{c+a^2cx^2} - \frac{3c\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{4a} - \frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}{6a} + \frac{3}{8}cx\sqrt{c+a^2}$$

$$= \frac{1}{12}cx\sqrt{c+a^2cx^2} - \frac{3c\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{4a} - \frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}{6a} + \frac{3}{8}cx\sqrt{c+a^2}$$

Mathematica [A] time = 0.935191, size = 439, normalized size = 1.

$$\frac{c\sqrt{a^2cx^2+c}\left(72i\tan^{-1}(ax)\mathrm{PolyLog}\left(2,-ie^{i\tan^{-1}(ax)}\right)-72i\tan^{-1}(ax)\mathrm{PolyLog}\left(2,ie^{i\tan^{-1}(ax)}\right)-72\mathrm{PolyLog}\left(3,-ie^{i\tan\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2,x]
```

```
[Out] (c*Sqrt[c + a^2*c*x^2]*(2*a*x*Sqrt[1 + a^2*x^2] + 2*a^3*x^3*Sqrt[1 + a^2*x^
2] - 94*Sqrt[1 + a^2*x^2]*ArcTan[a*x] + 2*a^2*x^2*Sqrt[1 + a^2*x^2]*ArcTan[
a*x] + 69*a*x*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2 + 21*a^3*x^3*Sqrt[1 + a^2*x^2
]*ArcTan[a*x]^2 - (72*I)*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2 + 80*ArcTa
nh[(a*x)/Sqrt[1 + a^2*x^2]] + 6*ArcTan[a*x]*Cos[3*ArcTan[a*x]] + 12*a^2*x^2
*ArcTan[a*x]*Cos[3*ArcTan[a*x]] + 6*a^4*x^4*ArcTan[a*x]*Cos[3*ArcTan[a*x]]
+ (72*I)*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - (72*I)*ArcTan[a*x
```

```
]*PolyLog[2, I*E^(I*ArcTan[a*x])] - 72*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] +
 72*PolyLog[3, I*E^(I*ArcTan[a*x])] + 2*Sin[3*ArcTan[a*x]] + 4*a^2*x^2*Sin[
3*ArcTan[a*x]] + 2*a^4*x^4*Sin[3*ArcTan[a*x]] - 3*ArcTan[a*x]^2*Sin[3*ArcTa
n[a*x]] - 6*a^2*x^2*ArcTan[a*x]^2*Sin[3*ArcTan[a*x]] - 3*a^4*x^4*ArcTan[a*x
]^2*Sin[3*ArcTan[a*x]]))/(96*a*Sqrt[1 + a^2*x^2])
```

Maple [A] time = 0.317, size = 304, normalized size = 0.7

$$\frac{c\left(6\,(\arctan(ax))^2x^3a^3-4\arctan(ax)\,a^2x^2+15\,(\arctan(ax))^2xa+2\,ax-22\arctan(ax)\right)}{24\,a}\sqrt{c\,(ax-i)\,(ax+i)}+\frac{\frac{i}{24}c}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(3/2)*arctan(a*x)^2,x)
```

```
[Out] 1/24*c/a*(c*(a*x-I)*(a*x+I))^(1/2)*(6*arctan(a*x)^2*x^3*a^3-4*arctan(a*x)*a
^2*x^2+15*arctan(a*x)^2*x*a+2*a*x-22*arctan(a*x))+1/24*I*c*(c*(a*x-I)*(a*x+
I))^(1/2)*(9*I*arctan(a*x)^2*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-9*I*arctan
(a*x)^2*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+18*arctan(a*x)*polylog(2,-I*(1+
I*a*x)/(a^2*x^2+1)^(1/2))-18*arctan(a*x)*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^
(1/2))+18*I*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-18*I*polylog(3,I*(1+I
*a*x)/(a^2*x^2+1)^(1/2))-40*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2)))/a/(a^2*x^2
+1)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a^2cx^2+c\right)^{\frac{3}{2}}\arctan(ax)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}} \operatorname{atan}^2(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(3/2)*atan(a*x)**2,x)
```

```
[Out] Integral((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**2, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.319 $\int \frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2}{x}\,dx$

Optimal. Leaf size=530

$$-\frac{7ic^2\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{3\sqrt{a^2cx^2+c}}+\frac{7ic^2\sqrt{a^2x^2+1}\text{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{3\sqrt{a^2cx^2+c}}+\frac{2ic^2\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,-e^{i}\right.}{\sqrt{a^2cx^2+c}}$$

```
[Out] (c*Sqrt[c + a^2*c*x^2])/3 - (a*c*x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/3 + c*S
qrt[c + a^2*c*x^2]*ArcTan[a*x]^2 + ((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2)/3
+ (((14*I)/3)*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt
[1 - I*a*x]])/Sqrt[c + a^2*c*x^2] - (2*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*
ArcTanh[E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + ((2*I)*c^2*Sqrt[1 + a^2*x
^2]*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - ((2*I
)*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, E^(I*ArcTan[a*x])])/Sqrt[c +
 a^2*c*x^2] - (((7*I)/3)*c^2*Sqrt[1 + a^2*x^2]*PolyLog[2, ((-I)*Sqrt[1 + I*
a*x])/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2] + (((7*I)/3)*c^2*Sqrt[1 + a^2*x
^2]*PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2] -
(2*c^2*Sqrt[1 + a^2*x^2]*PolyLog[3, -E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2
] + (2*c^2*Sqrt[1 + a^2*x^2]*PolyLog[3, E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*
x^2]
```

Rubi [A] time = 0.882195, antiderivative size = 530, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {4950, 4958, 4956, 4183, 2531, 2282, 6589, 4930, 4890, 4886, 4878}

$$-\frac{7ic^2\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{3\sqrt{a^2cx^2+c}}+\frac{7ic^2\sqrt{a^2x^2+1}\text{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{3\sqrt{a^2cx^2+c}}+\frac{2ic^2\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,-e^{i}\right.}{\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2)/x,x]
```


```
[Out] (c*Sqrt[c + a^2*c*x^2])/3 - (a*c*x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/3 + c*S
qrt[c + a^2*c*x^2]*ArcTan[a*x]^2 + ((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2)/3
+ (((14*I)/3)*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt
[1 - I*a*x]])/Sqrt[c + a^2*c*x^2] - (2*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*
ArcTanh[E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + ((2*I)*c^2*Sqrt[1 + a^2*x
^2]*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - ((2*I
)*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, E^(I*ArcTan[a*x])])/Sqrt[c +
```

```
 a^2*c*x^2] - (((7*I)/3)*c^2*Sqrt[1 + a^2*x^2]*PolyLog[2, ((-I)*Sqrt[1 + I*
a*x])/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2] + (((7*I)/3)*c^2*Sqrt[1 + a^2*x
^2]*PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2] -
(2*c^2*Sqrt[1 + a^2*x^2]*PolyLog[3, -E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2
] + (2*c^2*Sqrt[1 + a^2*x^2]*PolyLog[3, E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*
x^2]
```

Rule 4950

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 4958

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4956

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Dist[1/Sqrt[d], Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTan
[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && G
tQ[d, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
```

```
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 4890

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4886

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[(-2*I*(a + b*ArcTan[c*x])*ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]])
/(c*Sqrt[d]), x] + (Simp[(I*b*PolyLog[2, -((I*Sqrt[1 + I*c*x])/Sqrt[1 - I*c
*x])])/(c*Sqrt[d]), x] - Simp[(I*b*PolyLog[2, (I*Sqrt[1 + I*c*x])/Sqrt[1 -
I*c*x]])/(c*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

Rule 4878

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symbo
l] :> -Simp[(b*(d + e*x^2)^q)/(2*c*q*(2*q + 1)), x] + (Dist[(2*d*q)/(2*q +
1), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x]), x], x] + Simp[(x*(d + e*x^
```

```
2)^q*(a + b*ArcTan[c*x]))/(2*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] && Eq
Q[e, c^2*d] && GtQ[q, 0]
```

Rubi steps

$$\int \frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2}{x}\,dx = c\int \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{x}\,dx + \left(a^2c\right)\int x\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2\,dx$$

$$= \frac{1}{3}\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2 - \frac{1}{3}(2ac)\int \sqrt{c+a^2cx^2}\tan^{-1}(ax)\,dx + c^2\int \frac{\tan^{-1}(ax)^2}{x\sqrt{c+a^2cx^2}}$$

$$= \frac{1}{3}c\sqrt{c+a^2cx^2} - \frac{1}{3}acx\sqrt{c+a^2cx^2}\tan^{-1}(ax) + c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2 + \frac{1}{3}\left(c+a^2cx^2\right.$$

$$= \frac{1}{3}c\sqrt{c+a^2cx^2} - \frac{1}{3}acx\sqrt{c+a^2cx^2}\tan^{-1}(ax) + c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2 + \frac{1}{3}\left(c+a^2cx^2\right.$$

$$= \frac{1}{3}c\sqrt{c+a^2cx^2} - \frac{1}{3}acx\sqrt{c+a^2cx^2}\tan^{-1}(ax) + c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2 + \frac{1}{3}\left(c+a^2cx^2\right.$$

$$= \frac{1}{3}c\sqrt{c+a^2cx^2} - \frac{1}{3}acx\sqrt{c+a^2cx^2}\tan^{-1}(ax) + c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2 + \frac{1}{3}\left(c+a^2cx^2\right.$$

$$= \frac{1}{3}c\sqrt{c+a^2cx^2} - \frac{1}{3}acx\sqrt{c+a^2cx^2}\tan^{-1}(ax) + c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2 + \frac{1}{3}\left(c+a^2cx^2\right.$$

$$= \frac{1}{3}c\sqrt{c+a^2cx^2} - \frac{1}{3}acx\sqrt{c+a^2cx^2}\tan^{-1}(ax) + c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2 + \frac{1}{3}\left(c+a^2cx^2\right.$$

Mathematica [A] time = 3.17827, size = 496, normalized size = 0.94

$$\frac{1}{12}c\sqrt{a^2cx^2+c}\left(\frac{12\left(2i\tan^{-1}(ax)\text{PolyLog}\left(2,-e^{i\tan^{-1}(ax)}\right)-2i\tan^{-1}(ax)\text{PolyLog}\left(2,e^{i\tan^{-1}(ax)}\right)-2i\text{PolyLog}\left(2,-ie^{i\tan^{-1}(ax)}\right.\right.}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2)/x,x]
```

```
[Out] (c*Sqrt[c + a^2*c*x^2]*((12*(Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2 + ArcTan[a*x]^
2*Log[1 - E^(I*ArcTan[a*x])] - 2*ArcTan[a*x]*Log[1 - I*E^(I*ArcTan[a*x])] +
```

```
 2*ArcTan[a*x]*Log[1 + I*E^(I*ArcTan[a*x])] - ArcTan[a*x]^2*Log[1 + E^(I*Ar
cTan[a*x])] + (2*I)*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])] - (2*I)*Poly
Log[2, (-I)*E^(I*ArcTan[a*x])] + (2*I)*PolyLog[2, I*E^(I*ArcTan[a*x])] - (2
*I)*ArcTan[a*x]*PolyLog[2, E^(I*ArcTan[a*x])] - 2*PolyLog[3, -E^(I*ArcTan[a
*x])] + 2*PolyLog[3, E^(I*ArcTan[a*x])]))/Sqrt[1 + a^2*x^2] + (1 + a^2*x^2)
*(2 + 4*ArcTan[a*x]^2 + 2*Cos[2*ArcTan[a*x]] - (3*ArcTan[a*x]*Log[1 - I*E^(
I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - ArcTan[a*x]*Cos[3*ArcTan[a*x]]*Log[1 -
 I*E^(I*ArcTan[a*x])] + (3*ArcTan[a*x]*Log[1 + I*E^(I*ArcTan[a*x])])/Sqrt[1
 + a^2*x^2] + ArcTan[a*x]*Cos[3*ArcTan[a*x]]*Log[1 + I*E^(I*ArcTan[a*x])] -
 ((4*I)*PolyLog[2, (-I)*E^(I*ArcTan[a*x])])/(1 + a^2*x^2)^(3/2) + ((4*I)*Po
lyLog[2, I*E^(I*ArcTan[a*x])])/(1 + a^2*x^2)^(3/2) - 2*ArcTan[a*x]*Sin[2*Ar
cTan[a*x]])))/12
```

Maple [A] time = 0.371, size = 365, normalized size = 0.7

$$\frac{c\left(\left(\arctan\left(ax\right)\right)^{2}x^{2}a^{2}-\arctan\left(ax\right)xa+4\left(\arctan\left(ax\right)\right)^{2}+1\right)}{3}\sqrt{c\left(ax-i\right)\left(ax+i\right)}+\frac{i}{3}c\sqrt{c\left(ax-i\right)\left(ax+i\right)}\left(3i\left(\arctan\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(3/2)*arctan(a*x)^2/x,x)
```

```
[Out] 1/3*c*(c*(a*x-I)*(a*x+I))^(1/2)*(arctan(a*x)^2*x^2*a^2-arctan(a*x)*x*a+4*ar
ctan(a*x)^2+1)+1/3*I*c*(c*(a*x-I)*(a*x+I))^(1/2)*(3*I*arctan(a*x)^2*ln(1+(1
+I*a*x)/(a^2*x^2+1)^(1/2))-3*I*arctan(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/
2))-7*I*arctan(a*x)*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+7*I*arctan(a*x)*ln(
1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*arctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^
2+1)^(1/2))-6*arctan(a*x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I*polylo
g(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*I*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2)
)-7*dilog(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+7*dilog(1-I*(1+I*a*x)/(a^2*x^2+1
)^(1/2)))/(a^2*x^2+1)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^2/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^2cx^2+c\right)^{\frac{3}{2}}\arctan\left(ax\right)^2}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^2/x,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^2/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\operatorname{atan}^2\left(ax\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(3/2)*atan(a*x)**2/x,x)
```

```
[Out] Integral((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**2/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a^2cx^2+c\right)^{\frac{3}{2}}\arctan\left(ax\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^2/x,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^2/x, x)
```

3.320 $\int \frac{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^2}{x^2}\,dx$

Optimal. Leaf size=556

$$\frac{2iac^2\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{a^2cx^2+c}}-\frac{2iac^2\sqrt{a^2x^2+1}\text{PolyLog}\left(2,\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{a^2cx^2+c}}+\frac{3iac^2\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,-i\right.}{\sqrt{a^2cx^2+c}}$$

```
[Out] -(a*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]) - (c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]
^2)/x + (a^2*c*x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/2 - ((3*I)*a*c^2*Sqrt[1
 + a^2*x^2]*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2)/Sqrt[c + a^2*c*x^2] -
(4*a*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTanh[Sqrt[1 + I*a*x]/Sqrt[1 - I*a
*x]])/Sqrt[c + a^2*c*x^2] + a*c^(3/2)*ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*
x^2]] + ((3*I)*a*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*Arc
Tan[a*x])])/Sqrt[c + a^2*c*x^2] - ((3*I)*a*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x
]*PolyLog[2, I*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + ((2*I)*a*c^2*Sqrt[
1 + a^2*x^2]*PolyLog[2, -(Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x])])/Sqrt[c + a^2*c
*x^2] - ((2*I)*a*c^2*Sqrt[1 + a^2*x^2]*PolyLog[2, Sqrt[1 + I*a*x]/Sqrt[1 -
I*a*x]])/Sqrt[c + a^2*c*x^2] - (3*a*c^2*Sqrt[1 + a^2*x^2]*PolyLog[3, (-I)*E
^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + (3*a*c^2*Sqrt[1 + a^2*x^2]*PolyLog
[3, I*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2]
```

Rubi [A] time = 0.967633, antiderivative size = 556, normalized size of antiderivative = 1., number of steps used = 26, number of rules used = 13, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.542, Rules used = {4950, 4944, 4958, 4954, 4890, 4888, 4181, 2531, 2282, 6589, 4880, 217, 206}

$$\frac{2iac^2\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{a^2cx^2+c}}-\frac{2iac^2\sqrt{a^2x^2+1}\text{PolyLog}\left(2,\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{a^2cx^2+c}}+\frac{3iac^2\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,-i\right.}{\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2)/x^2,x]
```


```
[Out] -(a*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]) - (c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]
^2)/x + (a^2*c*x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/2 - ((3*I)*a*c^2*Sqrt[1
 + a^2*x^2]*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2)/Sqrt[c + a^2*c*x^2] -
(4*a*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTanh[Sqrt[1 + I*a*x]/Sqrt[1 - I*a
*x]])/Sqrt[c + a^2*c*x^2] + a*c^(3/2)*ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*
x^2]] + ((3*I)*a*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*Arc
```

```
Tan[a*x])])/Sqrt[c + a^2*c*x^2] - ((3*I)*a*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x
]*PolyLog[2, I*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + ((2*I)*a*c^2*Sqrt[
1 + a^2*x^2]*PolyLog[2, -(Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x])])/Sqrt[c + a^2*c
*x^2] - ((2*I)*a*c^2*Sqrt[1 + a^2*x^2]*PolyLog[2, Sqrt[1 + I*a*x]/Sqrt[1 -
I*a*x]])/Sqrt[c + a^2*c*x^2] - (3*a*c^2*Sqrt[1 + a^2*x^2]*PolyLog[3, (-I)*E
^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + (3*a*c^2*Sqrt[1 + a^2*x^2]*PolyLog
[3, I*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2]
```

Rule 4950

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 4944

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a +
 b*ArcTan[c*x])^p)/(d*f*(m + 1)), x] - Dist[(b*c*p)/(f*(m + 1)), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rule 4958

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4954

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x_
Symbol] :> Simp[(-2*(a + b*ArcTan[c*x])*ArcTanh[Sqrt[1 + I*c*x]/Sqrt[1 - I*
c*x]])/Sqrt[d], x] + (Simp[(I*b*PolyLog[2, -(Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x
])])/Sqrt[d], x] - Simp[(I*b*PolyLog[2, Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]])/S
qrt[d], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

Rule 4890

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
```

```
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4888

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[c
*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && GtQ
[d, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4880

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> -Simp[(b*p*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1))/(2*c*q*(2*
q + 1)), x] + (Dist[(2*d*q)/(2*q + 1), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTa
```

```
n[c*x])^p, x], x] + Dist[(b^2*d*p*(p - 1))/(2*q*(2*q + 1)), Int[(d + e*x^2)
^(q - 1)*(a + b*ArcTan[c*x])^(p - 2), x], x] + Simp[(x*(d + e*x^2)^q*(a + b
*ArcTan[c*x])^p)/(2*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[q, 0] && GtQ[p, 1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2}{x^2}\,dx = c\int \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{x^2}\,dx+\left(a^2c\right)\int \sqrt{c+a^2cx^2}\tan^{-1}(ax)^2\,dx$$

$$= -ac\sqrt{c+a^2cx^2}\tan^{-1}(ax)+\frac{1}{2}a^2cx\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2+c^2\int \frac{\tan^{-1}(ax)^2}{x^2\sqrt{c+a^2cx^2}}\,dx+$$

$$= -ac\sqrt{c+a^2cx^2}\tan^{-1}(ax)-\frac{c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{x}+\frac{1}{2}a^2cx\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2$$

$$= -ac\sqrt{c+a^2cx^2}\tan^{-1}(ax)-\frac{c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{x}+\frac{1}{2}a^2cx\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2$$

$$= -ac\sqrt{c+a^2cx^2}\tan^{-1}(ax)-\frac{c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{x}+\frac{1}{2}a^2cx\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2$$

$$= -ac\sqrt{c+a^2cx^2}\tan^{-1}(ax)-\frac{c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{x}+\frac{1}{2}a^2cx\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2$$

$$= -ac\sqrt{c+a^2cx^2}\tan^{-1}(ax)-\frac{c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{x}+\frac{1}{2}a^2cx\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2$$

$$= -ac\sqrt{c+a^2cx^2}\tan^{-1}(ax)-\frac{c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{x}+\frac{1}{2}a^2cx\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2$$

Mathematica [A] time = 1.02912, size = 376, normalized size = 0.68

$$\frac{c\sqrt{a^2cx^2+c}\left(6iax\tan^{-1}(ax)\text{PolyLog}\left(2,-ie^{i\tan^{-1}(ax)}\right)-6iax\tan^{-1}(ax)\text{PolyLog}\left(2,ie^{i\tan^{-1}(ax)}\right)+4iax\text{PolyLog}\left(2,-e^{i\tan^{-1}(ax)}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2)/x^2,x]
```

```
[Out] (c*Sqrt[c + a^2*c*x^2]*(-2*a*x*Sqrt[1 + a^2*x^2]*ArcTan[a*x] - 2*Sqrt[1 + a
^2*x^2]*ArcTan[a*x]^2 + a^2*x^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2 - (2*I)*a*x
*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2 + 2*a*x*ArcTanh[(a*x)/Sqrt[1 + a^2
*x^2]] + 4*a*x*ArcTan[a*x]*Log[1 - E^(I*ArcTan[a*x])] + 2*a*x*ArcTan[a*x]^2
*Log[1 - I*E^(I*ArcTan[a*x])] - 2*a*x*ArcTan[a*x]^2*Log[1 + I*E^(I*ArcTan[a
*x])] - 4*a*x*ArcTan[a*x]*Log[1 + E^(I*ArcTan[a*x])] + (4*I)*a*x*PolyLog[2,
 -E^(I*ArcTan[a*x])] + (6*I)*a*x*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*
x])] - (6*I)*a*x*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])] - (4*I)*a*x*Po
lyLog[2, E^(I*ArcTan[a*x])] - 6*a*x*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] + 6*
a*x*PolyLog[3, I*E^(I*ArcTan[a*x])]))/(2*x*Sqrt[1 + a^2*x^2])
```

Maple [A] time = 0.383, size = 356, normalized size = 0.6

$$\frac{c\arctan(ax)\left(\arctan(ax)a^2x^2-2ax-2\arctan(ax)\right)}{2x}\sqrt{c(ax-i)(ax+i)}+\frac{i}{2}ac\sqrt{c(ax-i)(ax+i)}\left(3i(\arctan(ax))^2\ln\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(3/2)*arctan(a*x)^2/x^2,x)
```

```
[Out] 1/2*c*(c*(a*x-I)*(a*x+I))^(1/2)*arctan(a*x)*(arctan(a*x)*a^2*x^2-2*a*x-2*ar
ctan(a*x))/x+1/2*I*a*c*(c*(a*x-I)*(a*x+I))^(1/2)*(3*I*arctan(a*x)^2*ln(1+I*
(1+I*a*x)/(a^2*x^2+1)^(1/2))-3*I*arctan(a*x)^2*ln(1-I*(1+I*a*x)/(a^2*x^2+1)
^(1/2))+4*I*arctan(a*x)*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*arctan(a*x)*pol
ylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*arctan(a*x)*polylog(2,I*(1+I*a*x)/
(a^2*x^2+1)^(1/2))+6*I*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*I*polylo
g(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+4*dilog((1+I*a*x)/(a^2*x^2+1)^(1/2))+4*d
ilog(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-4*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2)))/
(a^2*x^2+1)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^2/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^2cx^2+c\right)^{\frac{3}{2}}\arctan\left(ax\right)^2}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^2/x^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^2/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\operatorname{atan}^2\left(ax\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(3/2)*atan(a*x)**2/x**2,x)
```

```
[Out] Integral((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**2/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^{\frac{3}{2}}\arctan\left(ax\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^2/x^2,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^2/x^2, x)
```

3.321 $\int \frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2}{x^3}\,dx$

Optimal. Leaf size=567

$$-\frac{2ia^2c^2\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{a^2cx^2+c}}+\frac{2ia^2c^2\sqrt{a^2x^2+1}\text{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{a^2cx^2+c}}+\frac{3ia^2c^2\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLo}}{\sqrt{a^2cx^2+c}}$$

```
[Out] -((a*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/x) + a^2*c*Sqrt[c + a^2*c*x^2]*ArcT
an[a*x]^2 - (c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(2*x^2) + ((4*I)*a^2*c^2*
Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/Sqrt
[c + a^2*c*x^2] - (3*a^2*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*ArcTanh[E^(I*A
rcTan[a*x])])/Sqrt[c + a^2*c*x^2] - a^2*c^(3/2)*ArcTanh[Sqrt[c + a^2*c*x^2]
/Sqrt[c]] + ((3*I)*a^2*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, -E^(I*A
rcTan[a*x])])/Sqrt[c + a^2*c*x^2] - ((3*I)*a^2*c^2*Sqrt[1 + a^2*x^2]*ArcTan
[a*x]*PolyLog[2, E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - ((2*I)*a^2*c^2*S
qrt[1 + a^2*x^2]*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/Sqrt[c
 + a^2*c*x^2] + ((2*I)*a^2*c^2*Sqrt[1 + a^2*x^2]*PolyLog[2, (I*Sqrt[1 + I*a
*x])/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2] - (3*a^2*c^2*Sqrt[1 + a^2*x^2]*P
olyLog[3, -E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + (3*a^2*c^2*Sqrt[1 + a^
2*x^2]*PolyLog[3, E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2]
```

Rubi [A] time = 1.65459, antiderivative size = 567, normalized size of antiderivative = 1., number of steps used = 38, number of rules used = 15, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {4950, 4962, 4944, 266, 63, 208, 4958, 4956, 4183, 2531, 2282, 6589, 4930, 4890, 4886}

$$-\frac{2ia^2c^2\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{a^2cx^2+c}}+\frac{2ia^2c^2\sqrt{a^2x^2+1}\text{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{a^2cx^2+c}}+\frac{3ia^2c^2\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLo}}{\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2)/x^3,x]
```

```
[Out] -((a*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/x) + a^2*c*Sqrt[c + a^2*c*x^2]*ArcT
an[a*x]^2 - (c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(2*x^2) + ((4*I)*a^2*c^2*
Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/Sqrt
[c + a^2*c*x^2] - (3*a^2*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*ArcTanh[E^(I*A
rcTan[a*x])])/Sqrt[c + a^2*c*x^2] - a^2*c^(3/2)*ArcTanh[Sqrt[c + a^2*c*x^2]
/Sqrt[c]] + ((3*I)*a^2*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, -E^(I*A
```

```
rcTan[a*x])])/Sqrt[c + a^2*c*x^2] - ((3*I)*a^2*c^2*Sqrt[1 + a^2*x^2]*ArcTan
[a*x]*PolyLog[2, E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - ((2*I)*a^2*c^2*S
qrt[1 + a^2*x^2]*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/Sqrt[c
 + a^2*c*x^2] + ((2*I)*a^2*c^2*Sqrt[1 + a^2*x^2]*PolyLog[2, (I*Sqrt[1 + I*a
*x])/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2] - (3*a^2*c^2*Sqrt[1 + a^2*x^2]*P
olyLog[3, -E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + (3*a^2*c^2*Sqrt[1 + a^
2*x^2]*PolyLog[3, E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2]
```

Rule 4950

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 4962

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Ar
cTan[c*x])^p)/(d*f*(m + 1)), x] + (-Dist[(b*c*p)/(f*(m + 1)), Int[((f*x)^(m
 + 1)*(a + b*ArcTan[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] - Dist[(c^2*(m +
 2))/(f^2*(m + 1)), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/Sqrt[d + e*x^
2], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] &
& LtQ[m, -1] && NeQ[m, -2]
```

Rule 4944

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a +
 b*ArcTan[c*x])^p)/(d*f*(m + 1)), x] - Dist[(b*c*p)/(f*(m + 1)), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
```

```
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 4958

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4956

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Dist[1/Sqrt[d], Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTan
[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && G
tQ[d, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 4890

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4886

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[(-2*I*(a + b*ArcTan[c*x])*ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]])
/(c*Sqrt[d]), x] + (Simp[(I*b*PolyLog[2, -((I*Sqrt[1 + I*c*x])/Sqrt[1 - I*c
*x])])/(c*Sqrt[d]), x] - Simp[(I*b*PolyLog[2, (I*Sqrt[1 + I*c*x])/Sqrt[1 -
I*c*x]])/(c*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

Rubi steps

$$\int \frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2}{x^3}\,dx = c\int \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{x^3}\,dx + \left(a^2c\right)\int \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{x}\,dx$$

$$= c^2\int \frac{\tan^{-1}(ax)^2}{x^3\sqrt{c+a^2cx^2}}\,dx + 2\left(\left(a^2c^2\right)\int \frac{\tan^{-1}(ax)^2}{x\sqrt{c+a^2cx^2}}\,dx\right) + \left(a^4c^2\right)\int \frac{x\tan^{-1}(ax)^2}{\sqrt{c+a^2cx^2}}\,dx$$

$$= a^2c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2 - \frac{c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2x^2} + \left(ac^2\right)\int \frac{\tan^{-1}(ax)}{x^2\sqrt{c+a^2cx^2}}\,dx - \frac{1}{2}$$

$$= -\frac{ac\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{x} + a^2c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2 - \frac{c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2x^2} + ($$

$$= -\frac{ac\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{x} + a^2c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2 - \frac{c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2x^2} + \frac{4}{}$$

$$= -\frac{ac\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{x} + a^2c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2 - \frac{c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2x^2} + \frac{4}{}$$

$$= -\frac{ac\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{x} + a^2c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2 - \frac{c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2x^2} + \frac{4}{}$$

$$= -\frac{ac\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{x} + a^2c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2 - \frac{c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2x^2} + \frac{4}{}$$

$$= -\frac{ac\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{x} + a^2c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2 - \frac{c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2x^2} + \frac{4}{}$$

Mathematica [A] time = 2.9497, size = 455, normalized size = 0.8

$$\frac{a^2c\sqrt{a^2cx^2+c}\tan\left(\frac{1}{2}\tan^{-1}(ax)\right)\left(24i\tan^{-1}(ax)\cot\left(\frac{1}{2}\tan^{-1}(ax)\right)\text{PolyLog}\left(2,-e^{i\tan^{-1}(ax)}\right) - 24i\tan^{-1}(ax)\cot\left(\frac{1}{2}\tan\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2)/x^3,x]
```

```
[Out] (a^2*c*Sqrt[c + a^2*c*x^2]*(-4*ArcTan[a*x] - 4*ArcTan[a*x]*Cot[ArcTan[a*x]/
2]^2 + 4*a*x*ArcTan[a*x]^2*Csc[ArcTan[a*x]/2]^2 - ArcTan[a*x]^2*Cot[ArcTan[
```

```
a*x]/2]*Csc[ArcTan[a*x]/2]^2 + 12*ArcTan[a*x]^2*Cot[ArcTan[a*x]/2]*Log[1 -
E^(I*ArcTan[a*x])] - 16*ArcTan[a*x]*Cot[ArcTan[a*x]/2]*Log[1 - I*E^(I*ArcTa
n[a*x])] + 16*ArcTan[a*x]*Cot[ArcTan[a*x]/2]*Log[1 + I*E^(I*ArcTan[a*x])] -
 12*ArcTan[a*x]^2*Cot[ArcTan[a*x]/2]*Log[1 + E^(I*ArcTan[a*x])] + 8*Cot[Arc
Tan[a*x]/2]*Log[Tan[ArcTan[a*x]/2]] + (24*I)*ArcTan[a*x]*Cot[ArcTan[a*x]/2]
*PolyLog[2, -E^(I*ArcTan[a*x])] - (16*I)*Cot[ArcTan[a*x]/2]*PolyLog[2, (-I)
*E^(I*ArcTan[a*x])] + (16*I)*Cot[ArcTan[a*x]/2]*PolyLog[2, I*E^(I*ArcTan[a*
x])] - (24*I)*ArcTan[a*x]*Cot[ArcTan[a*x]/2]*PolyLog[2, E^(I*ArcTan[a*x])]
- 24*Cot[ArcTan[a*x]/2]*PolyLog[3, -E^(I*ArcTan[a*x])] + 24*Cot[ArcTan[a*x]
/2]*PolyLog[3, E^(I*ArcTan[a*x])] + ArcTan[a*x]^2*Csc[ArcTan[a*x]/2]*Sec[Ar
cTan[a*x]/2])*Tan[ArcTan[a*x]/2])/(8*Sqrt[1 + a^2*x^2])
```

Maple [A] time = 0.385, size = 412, normalized size = 0.7

$$\frac{c\arctan(ax)\left(2\arctan(ax)a^2x^2-2ax-\arctan(ax)\right)}{2x^2}\sqrt{c(ax-i)(ax+i)}-\frac{a^2c}{2}\sqrt{c(ax-i)(ax+i)}\left(3(\arctan(ax))^2\ln\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(3/2)*arctan(a*x)^2/x^3,x)
```

```
[Out] 1/2*c*(c*(a*x-I)*(a*x+I))^(1/2)*arctan(a*x)*(2*arctan(a*x)*a^2*x^2-2*a*x-ar
ctan(a*x))/x^2-1/2*a^2*c*(c*(a*x-I)*(a*x+I))^(1/2)*(3*arctan(a*x)^2*ln(1+(1
+I*a*x)/(a^2*x^2+1)^(1/2))-3*arctan(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2)
)-6*I*arctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I*arctan(a*x)*p
olylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))-4*arctan(a*x)*ln(1+I*(1+I*a*x)/(a^2*x
^2+1)^(1/2))+4*arctan(a*x)*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+4*I*dilog(1+
I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-4*I*dilog(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+2
*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2)
)-6*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)
-1))/(a^2*x^2+1)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^2/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^2cx^2+c\right)^{\frac{3}{2}}\arctan\left(ax\right)^2}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^2/x^3,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^2/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\operatorname{atan}^2\left(ax\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(3/2)*atan(a*x)**2/x**3,x)
```

```
[Out] Integral((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**2/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a^2cx^2+c\right)^{\frac{3}{2}}\arctan\left(ax\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^2/x^3,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^2/x^3, x)
```

3.322 $\int \frac{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^2}{x^4}\,dx$

Optimal. Leaf size=579

$$\frac{7ia^3c^2\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{3\sqrt{a^2cx^2+c}}-\frac{7ia^3c^2\sqrt{a^2x^2+1}\text{PolyLog}\left(2,\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{3\sqrt{a^2cx^2+c}}+\frac{2ia^3c^2\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2\right.}{\sqrt{a^2cx^2+c}}$$

```
[Out] -(a^2*c*Sqrt[c + a^2*c*x^2])/(3*x) - (a*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/
(3*x^2) - (a^2*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/x - ((c + a^2*c*x^2)^(3
/2)*ArcTan[a*x]^2)/(3*x^3) - ((2*I)*a^3*c^2*Sqrt[1 + a^2*x^2]*ArcTan[E^(I*A
rcTan[a*x])]*ArcTan[a*x]^2)/Sqrt[c + a^2*c*x^2] - (14*a^3*c^2*Sqrt[1 + a^2*
x^2]*ArcTan[a*x]*ArcTanh[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/(3*Sqrt[c + a^2*
c*x^2]) + ((2*I)*a^3*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, (-I)*E^(I
*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - ((2*I)*a^3*c^2*Sqrt[1 + a^2*x^2]*ArcT
an[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + (((7*I)/3)*a
^3*c^2*Sqrt[1 + a^2*x^2]*PolyLog[2, -(Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x])])/Sq
rt[c + a^2*c*x^2] - (((7*I)/3)*a^3*c^2*Sqrt[1 + a^2*x^2]*PolyLog[2, Sqrt[1
+ I*a*x]/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2] - (2*a^3*c^2*Sqrt[1 + a^2*x^
2]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + (2*a^3*c^2*Sqr
t[1 + a^2*x^2]*PolyLog[3, I*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2]
```

Rubi [A] time = 1.14642, antiderivative size = 579, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 13, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.542, Rules used = {4950, 4944, 4946, 4962, 264, 4958, 4954, 4890, 4888, 4181, 2531, 2282, 6589}

$$\frac{7ia^3c^2\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{3\sqrt{a^2cx^2+c}}-\frac{7ia^3c^2\sqrt{a^2x^2+1}\text{PolyLog}\left(2,\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{3\sqrt{a^2cx^2+c}}+\frac{2ia^3c^2\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2\right.}{\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2)/x^4,x]
```


```
[Out] -(a^2*c*Sqrt[c + a^2*c*x^2])/(3*x) - (a*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/
(3*x^2) - (a^2*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/x - ((c + a^2*c*x^2)^(3
/2)*ArcTan[a*x]^2)/(3*x^3) - ((2*I)*a^3*c^2*Sqrt[1 + a^2*x^2]*ArcTan[E^(I*A
rcTan[a*x])]*ArcTan[a*x]^2)/Sqrt[c + a^2*c*x^2] - (14*a^3*c^2*Sqrt[1 + a^2*
x^2]*ArcTan[a*x]*ArcTanh[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/(3*Sqrt[c + a^2*
c*x^2]) + ((2*I)*a^3*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, (-I)*E^(I
```

```
*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - ((2*I)*a^3*c^2*Sqrt[1 + a^2*x^2]*ArcT
an[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + (((7*I)/3)*a
^3*c^2*Sqrt[1 + a^2*x^2]*PolyLog[2, -(Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x])])/Sq
rt[c + a^2*c*x^2] - (((7*I)/3)*a^3*c^2*Sqrt[1 + a^2*x^2]*PolyLog[2, Sqrt[1
+ I*a*x]/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2] - (2*a^3*c^2*Sqrt[1 + a^2*x^
2]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + (2*a^3*c^2*Sqr
t[1 + a^2*x^2]*PolyLog[3, I*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2]
```

Rule 4950

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 4944

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a +
 b*ArcTan[c*x])^p)/(d*f*(m + 1)), x] - Dist[(b*c*p)/(f*(m + 1)), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*ArcTan[c*x
]))/(f*(m + 2)), x] + (Dist[d/(m + 2), Int[((f*x)^m*(a + b*ArcTan[c*x]))/Sq
rt[d + e*x^2], x], x] - Dist[(b*c*d)/(f*(m + 2)), Int[(f*x)^(m + 1)/Sqrt[d
+ e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && Ne
Q[m, -2]
```

Rule 4962

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Ar
cTan[c*x])^p)/(d*f*(m + 1)), x] + (-Dist[(b*c*p)/(f*(m + 1)), Int[((f*x)^(m
 + 1)*(a + b*ArcTan[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] - Dist[(c^2*(m +
 2))/(f^2*(m + 1)), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/Sqrt[d + e*x^
2], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] &
& LtQ[m, -1] && NeQ[m, -2]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 4958

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4954

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x_
Symbol] :> Simp[(-2*(a + b*ArcTan[c*x])*ArcTanh[Sqrt[1 + I*c*x]/Sqrt[1 - I*
c*x]])/Sqrt[d], x] + (Simp[(I*b*PolyLog[2, -(Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x
])])/Sqrt[d], x] - Simp[(I*b*PolyLog[2, Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]])/S
qrt[d], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

Rule 4890

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4888

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[c
*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && GtQ
[d, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2}{x^4}\,dx = c\int \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{x^4}\,dx + \left(a^2c\right)\int \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{x^2}\,dx$$

$$= -\frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2}{3x^3} + \frac{1}{3}(2ac)\int \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{x^3}\,dx + \left(a^2c^2\right)\int \frac{\tan^{-1}(a}{x^2\sqrt{c+a}}$$

$$= -\frac{2ac\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{3x^2} - \frac{a^2c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{x} - \frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2}{3x^3} -$$

$$= -\frac{2a^2c\sqrt{c+a^2cx^2}}{3x} - \frac{ac\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{3x^2} - \frac{a^2c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{x} - \frac{\left(c+a^2cx^2\right.}{}$$

$$= -\frac{a^2c\sqrt{c+a^2cx^2}}{3x} - \frac{ac\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{3x^2} - \frac{a^2c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{x} - \frac{\left(c+a^2cx^2\right)}{3}$$

$$= -\frac{a^2c\sqrt{c+a^2cx^2}}{3x} - \frac{ac\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{3x^2} - \frac{a^2c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{x} - \frac{\left(c+a^2cx^2\right)}{3}$$

$$= -\frac{a^2c\sqrt{c+a^2cx^2}}{3x} - \frac{ac\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{3x^2} - \frac{a^2c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{x} - \frac{\left(c+a^2cx^2\right)}{3}$$

$$= -\frac{a^2c\sqrt{c+a^2cx^2}}{3x} - \frac{ac\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{3x^2} - \frac{a^2c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{x} - \frac{\left(c+a^2cx^2\right)}{3}$$

Mathematica [A] time = 7.29432, size = 537, normalized size = 0.93

$$\frac{a^3c^2\sqrt{a^2x^2+1}\left(8i\text{PolyLog}\left(2,-e^{i\tan^{-1}(ax)}\right) - \frac{2\left(a^2x^2+1\right)^{3/2}\left(\frac{4ia^3x^3\text{PolyLog}\left(2,e^{i\tan^{-1}(ax)}\right)}{\left(a^2x^2+1\right)^{3/2}} - \frac{3ax\tan^{-1}(ax)\log\left(1-e^{i\tan^{-1}(ax)}\right)}{\sqrt{a^2x^2+1}} + \frac{3ax\tan^{-1}(ax)\log\left(1+e^{i\tan}\right.}{\sqrt{a^2x^2+1}}\right.}{}\right.}{24}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2)/x^4,x]
```

```
[Out] -((a^3*c*Sqrt[c*(1 + a^2*x^2)]*((Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2)/(a*x) - 2
*ArcTan[a*x]*Log[1 - E^(I*ArcTan[a*x])] - ArcTan[a*x]^2*Log[1 - I*E^(I*ArcT
an[a*x])] + ArcTan[a*x]^2*Log[1 + I*E^(I*ArcTan[a*x])] + 2*ArcTan[a*x]*Log[
```

```
1 + E^(I*ArcTan[a*x])] - (2*I)*PolyLog[2, -E^(I*ArcTan[a*x])] - (2*I)*ArcTa
n[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] + (2*I)*ArcTan[a*x]*PolyLog[2, I*
E^(I*ArcTan[a*x])] + (2*I)*PolyLog[2, E^(I*ArcTan[a*x])] + 2*PolyLog[3, (-I
)*E^(I*ArcTan[a*x])] - 2*PolyLog[3, I*E^(I*ArcTan[a*x])]))/Sqrt[1 + a^2*x^2
]) + (a^3*c^2*Sqrt[1 + a^2*x^2]*((8*I)*PolyLog[2, -E^(I*ArcTan[a*x])] - (2*
(1 + a^2*x^2)^(3/2)*(2 + 4*ArcTan[a*x]^2 - 2*Cos[2*ArcTan[a*x]] - (3*a*x*Ar
cTan[a*x]*Log[1 - E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + (3*a*x*ArcTan[a*x
]*Log[1 + E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + ((4*I)*a^3*x^3*PolyLog[2,
 E^(I*ArcTan[a*x])])/(1 + a^2*x^2)^(3/2) + 2*ArcTan[a*x]*Sin[2*ArcTan[a*x]]
 + ArcTan[a*x]*Log[1 - E^(I*ArcTan[a*x])]*Sin[3*ArcTan[a*x]] - ArcTan[a*x]*
Log[1 + E^(I*ArcTan[a*x])]*Sin[3*ArcTan[a*x]]))/(a^3*x^3)))/(24*Sqrt[c*(1 +
 a^2*x^2)])
```

Maple [A] time = 0.5, size = 343, normalized size = 0.6

$$-\frac{c\left(4\left(\arctan\left(ax\right)\right)^{2}x^{2}a^{2}+a^{2}x^{2}+\arctan\left(ax\right)xa+\left(\arctan\left(ax\right)\right)^{2}\right)}{3x^{3}}\sqrt{c\left(ax-i\right)\left(ax+i\right)}-\frac{a^{3}c}{3}\sqrt{c\left(ax-i\right)\left(ax+i\right)}\left(3\left(\mathrm{ar}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(3/2)*arctan(a*x)^2/x^4,x)
```

```
[Out] -1/3*c*(c*(a*x-I)*(a*x+I))^(1/2)*(4*arctan(a*x)^2*x^2*a^2+a^2*x^2+arctan(a*
x)*x*a+arctan(a*x)^2)/x^3-1/3*a^3*c*(c*(a*x-I)*(a*x+I))^(1/2)*(3*arctan(a*x
)^2*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-3*arctan(a*x)^2*ln(1-I*(1+I*a*x)/(a
^2*x^2+1)^(1/2))-6*I*arctan(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+
6*I*arctan(a*x)*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+7*arctan(a*x)*ln(1
+(1+I*a*x)/(a^2*x^2+1)^(1/2))-7*I*dilog((1+I*a*x)/(a^2*x^2+1)^(1/2))-7*I*di
log(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/
2))-6*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))/(a^2*x^2+1)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^2/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^2cx^2+c\right)^{\frac{3}{2}}\arctan\left(ax\right)^2}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^2/x^4,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^2/x^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\operatorname{atan}^2\left(ax\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(3/2)*atan(a*x)**2/x**4,x)
```

```
[Out] Integral((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**2/x**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a^2cx^2+c\right)^{\frac{3}{2}}\arctan\left(ax\right)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^2/x^4,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^2/x^4, x)
```

3.323 $\int x^3 \left(c+a^2 c x^2\right)^{5/2} \tan^{-1}(ax)^2 \, dx$

Optimal. Leaf size=578

$$\frac{115ic^3\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{4032a^4\sqrt{a^2cx^2+c}}-\frac{115ic^3\sqrt{a^2x^2+1}\text{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{4032a^4\sqrt{a^2cx^2+c}}-\frac{115c^2\sqrt{a^2cx^2+c}}{4032a^4}+\frac{1}{9}a^4c^2x^8\sqrt{a^2cx^2+}$$

```
[Out] (-115*c^2*Sqrt[c + a^2*c*x^2])/(4032*a^4) - (115*c*(c + a^2*c*x^2)^(3/2))/(
18144*a^4) - (23*(c + a^2*c*x^2)^(5/2))/(7560*a^4) + (c + a^2*c*x^2)^(7/2)/
(252*a^4*c) + (47*c^2*x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(1344*a^3) - (205*
c^2*x^3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(6048*a) - (103*a*c^2*x^5*Sqrt[c +
 a^2*c*x^2]*ArcTan[a*x])/1512 - (a^3*c^2*x^7*Sqrt[c + a^2*c*x^2]*ArcTan[a*x
])/36 - (2*c^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(63*a^4) + (c^2*x^2*Sqrt[
c + a^2*c*x^2]*ArcTan[a*x]^2)/(63*a^2) + (5*c^2*x^4*Sqrt[c + a^2*c*x^2]*Arc
Tan[a*x]^2)/21 + (19*a^2*c^2*x^6*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/63 + (a
^4*c^2*x^8*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/9 - (((115*I)/2016)*c^3*Sqrt[
1 + a^2*x^2]*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/(a^4*Sqrt
[c + a^2*c*x^2]) + (((115*I)/4032)*c^3*Sqrt[1 + a^2*x^2]*PolyLog[2, ((-I)*S
qrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/(a^4*Sqrt[c + a^2*c*x^2]) - (((115*I)/403
2)*c^3*Sqrt[1 + a^2*x^2]*PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/(
a^4*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 10.7013, antiderivative size = 578, normalized size of antiderivative = 1., number of steps used = 203, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4950, 4952, 261, 4890, 4886, 4930, 266, 43}

$$\frac{115ic^3\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{4032a^4\sqrt{a^2cx^2+c}}-\frac{115ic^3\sqrt{a^2x^2+1}\text{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{4032a^4\sqrt{a^2cx^2+c}}-\frac{115c^2\sqrt{a^2cx^2+c}}{4032a^4}+\frac{1}{9}a^4c^2x^8\sqrt{a^2cx^2+}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2,x]
```

```
[Out] (-115*c^2*Sqrt[c + a^2*c*x^2])/(4032*a^4) - (115*c*(c + a^2*c*x^2)^(3/2))/(
18144*a^4) - (23*(c + a^2*c*x^2)^(5/2))/(7560*a^4) + (c + a^2*c*x^2)^(7/2)/
(252*a^4*c) + (47*c^2*x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(1344*a^3) - (205*
c^2*x^3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(6048*a) - (103*a*c^2*x^5*Sqrt[c +
 a^2*c*x^2]*ArcTan[a*x])/1512 - (a^3*c^2*x^7*Sqrt[c + a^2*c*x^2]*ArcTan[a*x
])/36 - (2*c^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(63*a^4) + (c^2*x^2*Sqrt[
c + a^2*c*x^2]*ArcTan[a*x]^2)/(63*a^2) + (5*c^2*x^4*Sqrt[c + a^2*c*x^2]*Arc
```

```
Tan[a*x]^2)/21 + (19*a^2*c^2*x^6*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/63 + (a
^4*c^2*x^8*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/9 - (((115*I)/2016)*c^3*Sqrt[
1 + a^2*x^2]*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/(a^4*Sqrt
[c + a^2*c*x^2]) + (((115*I)/4032)*c^3*Sqrt[1 + a^2*x^2]*PolyLog[2, ((-I)*S
qrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/(a^4*Sqrt[c + a^2*c*x^2]) - (((115*I)/403
2)*c^3*Sqrt[1 + a^2*x^2]*PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/(
a^4*Sqrt[c + a^2*c*x^2])
```

Rule 4950

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 4952

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b*
ArcTan[c*x])^p)/(c^2*d*m), x] + (-Dist[(b*f*p)/(c*m), Int[((f*x)^(m - 1)*(a
 + b*ArcTan[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] - Dist[(f^2*(m - 1))/(c^
2*m), Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/Sqrt[d + e*x^2], x], x]) /;
 FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 4890

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4886

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[(-2*I*(a + b*ArcTan[c*x])*ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]])
/(c*Sqrt[d]), x] + (Simp[(I*b*PolyLog[2, -((I*Sqrt[1 + I*c*x])/Sqrt[1 - I*c
*x])])/(c*Sqrt[d]), x] - Simp[(I*b*PolyLog[2, (I*Sqrt[1 + I*c*x])/Sqrt[1 -
```

```
I*c*x]])/(c*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x^3\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^2\,dx = c\int x^3\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2\,dx + \left(a^2c\right)\int x^5\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2\,dx$$

$$= c^2\int x^3\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2\,dx + 2\left(\left(a^2c^2\right)\int x^5\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2\,dx\right) + \left(a^4c\right.$$

$$= c^3\int \frac{x^3\tan^{-1}(ax)^2}{\sqrt{c+a^2cx^2}}\,dx + \left(a^2c^3\right)\int \frac{x^5\tan^{-1}(ax)^2}{\sqrt{c+a^2cx^2}}\,dx + \left(a^4c^3\right)\int \frac{x^7\tan^{-1}(ax)^2}{\sqrt{c+a^2cx^2}}\,dx +$$

$$= \frac{c^2x^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{3a^2} + \frac{1}{5}c^2x^4\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2 + \frac{1}{7}a^2c^2x^6\sqrt{c+a^2cx^2}\tan$$

$$= -\frac{c^2x\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{3a^3} - \frac{c^2x^3\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{10a} - \frac{1}{21}ac^2x^5\sqrt{c+a^2cx^2}\tan^{-1}$$

$$= \frac{c^2\sqrt{c+a^2cx^2}}{3a^4} + \frac{c^2x\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{12a^3} + \frac{19c^2x^3\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{420a} + \frac{41ac^2x^5}{}$$

$$= -\frac{c^2\sqrt{c+a^2cx^2}}{12a^4} - \frac{61c^2x\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{168a^3} - \frac{3761c^2x^3\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{30240a} + \frac{41}{}$$

$$= \frac{713c^2\sqrt{c+a^2cx^2}}{2520a^4} + \frac{37c\left(c+a^2cx^2\right)^{3/2}}{1260a^4} - \frac{\left(c+a^2cx^2\right)^{5/2}}{140a^4} + \frac{\left(c+a^2cx^2\right)^{7/2}}{252a^4c} + \frac{127c^2x\sqrt{}}{}$$

$$= -\frac{6299c^2\sqrt{c+a^2cx^2}}{60480a^4} + \frac{349c\left(c+a^2cx^2\right)^{3/2}}{11340a^4} - \frac{167\left(c+a^2cx^2\right)^{5/2}}{7560a^4} + \frac{\left(c+a^2cx^2\right)^{7/2}}{252a^4c} + \frac{1}{}$$

$$= -\frac{5519c^2\sqrt{c+a^2cx^2}}{20160a^4} + \frac{7921c\left(c+a^2cx^2\right)^{3/2}}{90720a^4} - \frac{167\left(c+a^2cx^2\right)^{5/2}}{7560a^4} + \frac{\left(c+a^2cx^2\right)^{7/2}}{252a^4c} +$$

Mathematica [B] time = 8.25716, size = 1320, normalized size = 2.28

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2,x]
```

```
[Out] ((c + a^2*c*x^2)^(5/2)*(-48384*(50 - 32*ArcTan[a*x]^2 + 72*Cos[2*ArcTan[a*x
]] + 160*ArcTan[a*x]^2*Cos[2*ArcTan[a*x]] + 22*Cos[4*ArcTan[a*x]] - (110*Ar
```

cTan[a*x]*Log[1 - I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - 55*ArcTan[a*x]*
Cos[3*ArcTan[a*x]]*Log[1 - I*E^(I*ArcTan[a*x])] - 11*ArcTan[a*x]*Cos[5*ArcT
an[a*x]]*Log[1 - I*E^(I*ArcTan[a*x])] + (110*ArcTan[a*x]*Log[1 + I*E^(I*Arc
Tan[a*x])])/Sqrt[1 + a^2*x^2] + 55*ArcTan[a*x]*Cos[3*ArcTan[a*x]]*Log[1 + I
*E^(I*ArcTan[a*x])] + 11*ArcTan[a*x]*Cos[5*ArcTan[a*x]]*Log[1 + I*E^(I*ArcT
an[a*x])] - ((176*I)*PolyLog[2, (-I)*E^(I*ArcTan[a*x])])/(1 + a^2*x^2)^(5/2
) + ((176*I)*PolyLog[2, I*E^(I*ArcTan[a*x])])/(1 + a^2*x^2)^(5/2) + 4*ArcTa
n[a*x]*Sin[2*ArcTan[a*x]] - 22*ArcTan[a*x]*Sin[4*ArcTan[a*x]]) + 576*(1 + a
^2*x^2)*(4116 + 10944*ArcTan[a*x]^2 + 6262*Cos[2*ArcTan[a*x]] - 5376*ArcTan
[a*x]^2*Cos[2*ArcTan[a*x]] + 2764*Cos[4*ArcTan[a*x]] + 6720*ArcTan[a*x]^2*C
os[4*ArcTan[a*x]] + 618*Cos[6*ArcTan[a*x]] - (10815*ArcTan[a*x]*Log[1 - I*E
^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - 6489*ArcTan[a*x]*Cos[3*ArcTan[a*x]]*
Log[1 - I*E^(I*ArcTan[a*x])] - 2163*ArcTan[a*x]*Cos[5*ArcTan[a*x]]*Log[1 -
I*E^(I*ArcTan[a*x])] - 309*ArcTan[a*x]*Cos[7*ArcTan[a*x]]*Log[1 - I*E^(I*Ar
cTan[a*x])] + (10815*ArcTan[a*x]*Log[1 + I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2
*x^2] + 6489*ArcTan[a*x]*Cos[3*ArcTan[a*x]]*Log[1 + I*E^(I*ArcTan[a*x])] +
2163*ArcTan[a*x]*Cos[5*ArcTan[a*x]]*Log[1 + I*E^(I*ArcTan[a*x])] + 309*ArcT
an[a*x]*Cos[7*ArcTan[a*x]]*Log[1 + I*E^(I*ArcTan[a*x])] - ((19776*I)*PolyLo
g[2, (-I)*E^(I*ArcTan[a*x])])/(1 + a^2*x^2)^(7/2) + ((19776*I)*PolyLog[2, I
*E^(I*ArcTan[a*x])])/(1 + a^2*x^2)^(7/2) - 1266*ArcTan[a*x]*Sin[2*ArcTan[a*
x]] + 360*ArcTan[a*x]*Sin[4*ArcTan[a*x]] - 618*ArcTan[a*x]*Sin[6*ArcTan[a*x
]]) - (1 + a^2*x^2)^2*(657578 - 820224*ArcTan[a*x]^2 + 1083168*Cos[2*ArcTan
[a*x]] + 3276288*ArcTan[a*x]^2*Cos[2*ArcTan[a*x]] + 576936*Cos[4*ArcTan[a*x
]] - 580608*ArcTan[a*x]^2*Cos[4*ArcTan[a*x]] + 184160*Cos[6*ArcTan[a*x]] +
483840*ArcTan[a*x]^2*Cos[6*ArcTan[a*x]] + 32814*Cos[8*ArcTan[a*x]] - (20672
82*ArcTan[a*x]*Log[1 - I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - 1378188*Ar
cTan[a*x]*Cos[3*ArcTan[a*x]]*Log[1 - I*E^(I*ArcTan[a*x])] - 590652*ArcTan[a
*x]*Cos[5*ArcTan[a*x]]*Log[1 - I*E^(I*ArcTan[a*x])] - 147663*ArcTan[a*x]*Co
s[7*ArcTan[a*x]]*Log[1 - I*E^(I*ArcTan[a*x])] - 16407*ArcTan[a*x]*Cos[9*Arc
Tan[a*x]]*Log[1 - I*E^(I*ArcTan[a*x])] + (2067282*ArcTan[a*x]*Log[1 + I*E^(
I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + 1378188*ArcTan[a*x]*Cos[3*ArcTan[a*x]]
*Log[1 + I*E^(I*ArcTan[a*x])] + 590652*ArcTan[a*x]*Cos[5*ArcTan[a*x]]*Log[1
 + I*E^(I*ArcTan[a*x])] + 147663*ArcTan[a*x]*Cos[7*ArcTan[a*x]]*Log[1 + I*E
^(I*ArcTan[a*x])] + 16407*ArcTan[a*x]*Cos[9*ArcTan[a*x]]*Log[1 + I*E^(I*Arc
Tan[a*x])] - ((4200192*I)*PolyLog[2, (-I)*E^(I*ArcTan[a*x])])/(1 + a^2*x^2)
^(9/2) + ((4200192*I)*PolyLog[2, I*E^(I*ArcTan[a*x])])/(1 + a^2*x^2)^(9/2)
+ 78444*ArcTan[a*x]*Sin[2*ArcTan[a*x]] - 160452*ArcTan[a*x]*Sin[4*ArcTan[a*
x]] + 38172*ArcTan[a*x]*Sin[6*ArcTan[a*x]] - 32814*ArcTan[a*x]*Sin[8*ArcTan
[a*x]])))/(46448640*a^4)

Maple [A] time = 0.957, size = 309, normalized size = 0.5

$$\frac{c^2 \left(20160 (\arctan(ax))^2 x^8 a^8 - 5040 \arctan(ax) x^7 a^7 + 54720 (\arctan(ax))^2 x^6 a^6 + 720 x^6 a^6 - 12360 \arctan(ax) x^5 a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^2,x)
```

```
[Out] 1/181440*c^2/a^4*(c*(a*x-I)*(a*x+I))^(1/2)*(20160*arctan(a*x)^2*x^8*a^8-504
0*arctan(a*x)*x^7*a^7+54720*arctan(a*x)^2*x^6*a^6+720*x^6*a^6-12360*arctan(
a*x)*x^5*a^5+43200*arctan(a*x)^2*x^4*a^4+1608*a^4*x^4-6150*arctan(a*x)*x^3*
a^3+2880*arctan(a*x)^2*x^2*a^2-94*a^2*x^2+6345*arctan(a*x)*x*a-5760*arctan(
a*x)^2-6157)+115/4032*c^2*(c*(a*x-I)*(a*x+I))^(1/2)*(arctan(a*x)*ln(1-I*(1+
I*a*x)/(a^2*x^2+1)^(1/2))-arctan(a*x)*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+I
*dilog(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*dilog(1-I*(1+I*a*x)/(a^2*x^2+1)^(
1/2)))/a^4/(a^2*x^2+1)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^4c^2x^7 + 2\,a^2c^2x^5 + c^2x^3\right)\sqrt{a^2cx^2 + c}\arctan(ax)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^7 + 2*a^2*c^2*x^5 + c^2*x^3)*sqrt(a^2*c*x^2 + c)*arctan
(a*x)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a**2*c*x**2+c)**(5/2)*atan(a*x)**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.324 $\int x^2 \left(c+a^2 c x^2\right)^{5/2} \tan^{-1}(ax)^2 \, dx$

Optimal. Leaf size=638

$$-\frac{5ic^3\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,-ie^{i\tan^{-1}(ax)}\right)}{64a^3\sqrt{a^2cx^2+c}}+\frac{5ic^3\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,ie^{i\tan^{-1}(ax)}\right)}{64a^3\sqrt{a^2cx^2+c}}+\frac{5c^3\sqrt{a^2x^2+1}}{6}$$

```
[Out] (43*c^2*x*Sqrt[c + a^2*c*x^2])/(4032*a^2) + (29*c^2*x^3*Sqrt[c + a^2*c*x^2]
)/1680 + (a^2*c^2*x^5*Sqrt[c + a^2*c*x^2])/168 + (1373*c^2*Sqrt[c + a^2*c*x
^2]*ArcTan[a*x])/(20160*a^3) - (737*c^2*x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]
)/(10080*a) - (83*a*c^2*x^4*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/840 - (a^3*c^2
*x^6*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/28 + (5*c^2*x*Sqrt[c + a^2*c*x^2]*Arc
Tan[a*x]^2)/(128*a^2) + (59*c^2*x^3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/192
+ (17*a^2*c^2*x^5*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/48 + (a^4*c^2*x^7*Sqrt
[c + a^2*c*x^2]*ArcTan[a*x]^2)/8 + (((5*I)/64)*c^3*Sqrt[1 + a^2*x^2]*ArcTan
[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2)/(a^3*Sqrt[c + a^2*c*x^2]) - (397*c^(5/2)
*ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]])/(5040*a^3) - (((5*I)/64)*c^3*S
qrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])])/(a^3*Sqrt[
c + a^2*c*x^2]) + (((5*I)/64)*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2,
I*E^(I*ArcTan[a*x])])/(a^3*Sqrt[c + a^2*c*x^2]) + (5*c^3*Sqrt[1 + a^2*x^2]*
PolyLog[3, (-I)*E^(I*ArcTan[a*x])])/(64*a^3*Sqrt[c + a^2*c*x^2]) - (5*c^3*S
qrt[1 + a^2*x^2]*PolyLog[3, I*E^(I*ArcTan[a*x])])/(64*a^3*Sqrt[c + a^2*c*x^
2])
```

Rubi [A] time = 8.34799, antiderivative size = 638, normalized size of antiderivative = 1., number of steps used = 238, number of rules used = 12, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4950, 4952, 4930, 217, 206, 4890, 4888, 4181, 2531, 2282, 6589, 321}

$$-\frac{5ic^3\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,-ie^{i\tan^{-1}(ax)}\right)}{64a^3\sqrt{a^2cx^2+c}}+\frac{5ic^3\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,ie^{i\tan^{-1}(ax)}\right)}{64a^3\sqrt{a^2cx^2+c}}+\frac{5c^3\sqrt{a^2x^2+1}}{6}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2,x]
```


```
[Out] (43*c^2*x*Sqrt[c + a^2*c*x^2])/(4032*a^2) + (29*c^2*x^3*Sqrt[c + a^2*c*x^2]
)/1680 + (a^2*c^2*x^5*Sqrt[c + a^2*c*x^2])/168 + (1373*c^2*Sqrt[c + a^2*c*x
^2]*ArcTan[a*x])/(20160*a^3) - (737*c^2*x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]
)/(10080*a) - (83*a*c^2*x^4*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/840 - (a^3*c^2
*x^6*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/28 + (5*c^2*x*Sqrt[c + a^2*c*x^2]*Arc
```

```
Tan[a*x]^2)/(128*a^2) + (59*c^2*x^3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/192
+ (17*a^2*c^2*x^5*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/48 + (a^4*c^2*x^7*Sqrt
[c + a^2*c*x^2]*ArcTan[a*x]^2)/8 + (((5*I)/64)*c^3*Sqrt[1 + a^2*x^2]*ArcTan
[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2)/(a^3*Sqrt[c + a^2*c*x^2]) - (397*c^(5/2)
*ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]])/(5040*a^3) - (((5*I)/64)*c^3*S
qrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])])/(a^3*Sqrt[
c + a^2*c*x^2]) + (((5*I)/64)*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2,
I*E^(I*ArcTan[a*x])])/(a^3*Sqrt[c + a^2*c*x^2]) + (5*c^3*Sqrt[1 + a^2*x^2]*
PolyLog[3, (-I)*E^(I*ArcTan[a*x])])/(64*a^3*Sqrt[c + a^2*c*x^2]) - (5*c^3*S
qrt[1 + a^2*x^2]*PolyLog[3, I*E^(I*ArcTan[a*x])])/(64*a^3*Sqrt[c + a^2*c*x^
2])
```

Rule 4950

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 4952

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b*
ArcTan[c*x])^p)/(c^2*d*m), x] + (-Dist[(b*f*p)/(c*m), Int[((f*x)^(m - 1)*(a
 + b*ArcTan[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] - Dist[(f^2*(m - 1))/(c^
2*m), Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/Sqrt[d + e*x^2], x], x]) /;
 FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 4890

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4888

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[c
*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && GtQ
[d, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int x^2\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^2\,dx = c\int x^2\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2\,dx + \left(a^2c\right)\int x^4\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2\,dx$$

$$= c^2\int x^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2\,dx + 2\left(\left(a^2c^2\right)\int x^4\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2\,dx\right) + \left(a^4c\right.$$

$$= c^3\int\frac{x^2\tan^{-1}(ax)^2}{\sqrt{c+a^2cx^2}}\,dx + \left(a^2c^3\right)\int\frac{x^4\tan^{-1}(ax)^2}{\sqrt{c+a^2cx^2}}\,dx + \left(a^4c^3\right)\int\frac{x^6\tan^{-1}(ax)^2}{\sqrt{c+a^2cx^2}}\,dx +$$

$$= \frac{c^2x\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2a^2} + \frac{1}{4}c^2x^3\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2 + \frac{1}{6}a^2c^2x^5\sqrt{c+a^2cx^2}\tan$$

$$= -\frac{c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{a^3} - \frac{c^2x^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{6a} - \frac{1}{15}ac^2x^4\sqrt{c+a^2cx^2}\tan^{-1}($$

$$= \frac{c^2x\sqrt{c+a^2cx^2}}{12a^2} + \frac{1}{60}c^2x^3\sqrt{c+a^2cx^2} + \frac{1}{168}a^2c^2x^5\sqrt{c+a^2cx^2} + \frac{c^2\sqrt{c+a^2cx^2}\tan^{-1}(}{12a^3}$$

$$= -\frac{c^2x\sqrt{c+a^2cx^2}}{18a^2} - \frac{9}{560}c^2x^3\sqrt{c+a^2cx^2} + \frac{1}{168}a^2c^2x^5\sqrt{c+a^2cx^2} - \frac{359c^2\sqrt{c+a^2cx^2}\,\mathrm{t}}{360a^3}$$

$$= \frac{491c^2x\sqrt{c+a^2cx^2}}{4032a^2} - \frac{9}{560}c^2x^3\sqrt{c+a^2cx^2} + \frac{1}{168}a^2c^2x^5\sqrt{c+a^2cx^2} + \frac{1261c^2\sqrt{c+a^2c}}{2016}$$

$$= \frac{491c^2x\sqrt{c+a^2cx^2}}{4032a^2} - \frac{9}{560}c^2x^3\sqrt{c+a^2cx^2} + \frac{1}{168}a^2c^2x^5\sqrt{c+a^2cx^2} + \frac{1261c^2\sqrt{c+a^2c}}{2016}$$

$$= \frac{491c^2x\sqrt{c+a^2cx^2}}{4032a^2} - \frac{9}{560}c^2x^3\sqrt{c+a^2cx^2} + \frac{1}{168}a^2c^2x^5\sqrt{c+a^2cx^2} + \frac{1261c^2\sqrt{c+a^2c}}{2016}$$

$$= \frac{491c^2x\sqrt{c+a^2cx^2}}{4032a^2} - \frac{9}{560}c^2x^3\sqrt{c+a^2cx^2} + \frac{1}{168}a^2c^2x^5\sqrt{c+a^2cx^2} + \frac{1261c^2\sqrt{c+a^2c}}{2016}$$

$$= \frac{491c^2x\sqrt{c+a^2cx^2}}{4032a^2} - \frac{9}{560}c^2x^3\sqrt{c+a^2cx^2} + \frac{1}{168}a^2c^2x^5\sqrt{c+a^2cx^2} + \frac{1261c^2\sqrt{c+a^2c}}{2016}$$

$$= \frac{491c^2x\sqrt{c+a^2cx^2}}{4032a^2} - \frac{9}{560}c^2x^3\sqrt{c+a^2cx^2} + \frac{1}{168}a^2c^2x^5\sqrt{c+a^2cx^2} + \frac{1261c^2\sqrt{c+a^2c}}{2016}$$

Mathematica [A] time = 4.83101, size = 759, normalized size = 1.19

$$\frac{c^2\sqrt{a^2cx^2+c}\left(-201600i\tan^{-1}(ax)\text{PolyLog}\left(2,-ie^{i\tan^{-1}(ax)}\right)+201600i\tan^{-1}(ax)\text{PolyLog}\left(2,ie^{i\tan^{-1}(ax)}\right)+201600\text{Po}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2,x]
```

```
[Out] (c^2*Sqrt[c + a^2*c*x^2]*(53760*a*x*(1 + a^2*x^2)^(3/2) - 25088*a*x*(1 + a^
2*x^2)^(5/2) + 7006*a*x*(1 + a^2*x^2)^(7/2) + 53760*(1 + a^2*x^2)^(3/2)*Arc
Tan[a*x] + 5376*(1 + a^2*x^2)^(5/2)*ArcTan[a*x] - 38134*(1 + a^2*x^2)^(7/2)
*ArcTan[a*x] + 564480*a*x*(1 + a^2*x^2)^(3/2)*ArcTan[a*x]^2 + 524160*a*x*(1
 + a^2*x^2)^(5/2)*ArcTan[a*x]^2 + 185325*a*x*(1 + a^2*x^2)^(7/2)*ArcTan[a*x
]^2 + (201600*I)*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2 - 203264*ArcTanh[(
a*x)/Sqrt[1 + a^2*x^2]] + 161280*(1 + a^2*x^2)^2*ArcTan[a*x]*Cos[3*ArcTan[a
*x]] + 49280*(1 + a^2*x^2)^3*ArcTan[a*x]*Cos[3*ArcTan[a*x]] - 7658*(1 + a^2
*x^2)^4*ArcTan[a*x]*Cos[3*ArcTan[a*x]] - 40320*(1 + a^2*x^2)^3*ArcTan[a*x]*
Cos[5*ArcTan[a*x]] - 10990*(1 + a^2*x^2)^4*ArcTan[a*x]*Cos[5*ArcTan[a*x]] +
 3150*(1 + a^2*x^2)^4*ArcTan[a*x]*Cos[7*ArcTan[a*x]] - (201600*I)*ArcTan[a*
x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] + (201600*I)*ArcTan[a*x]*PolyLog[2, I
*E^(I*ArcTan[a*x])] + 201600*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] - 201600*Po
lyLog[3, I*E^(I*ArcTan[a*x])] + 53760*(1 + a^2*x^2)^2*Sin[3*ArcTan[a*x]] -
48384*(1 + a^2*x^2)^3*Sin[3*ArcTan[a*x]] + 12246*(1 + a^2*x^2)^4*Sin[3*ArcT
an[a*x]] - 80640*(1 + a^2*x^2)^2*ArcTan[a*x]^2*Sin[3*ArcTan[a*x]] - 315840*
(1 + a^2*x^2)^3*ArcTan[a*x]^2*Sin[3*ArcTan[a*x]] - 93975*(1 + a^2*x^2)^4*Ar
cTan[a*x]^2*Sin[3*ArcTan[a*x]] - 23296*(1 + a^2*x^2)^3*Sin[5*ArcTan[a*x]] +
 7678*(1 + a^2*x^2)^4*Sin[5*ArcTan[a*x]] + 20160*(1 + a^2*x^2)^3*ArcTan[a*x
]^2*Sin[5*ArcTan[a*x]] + 41685*(1 + a^2*x^2)^4*ArcTan[a*x]^2*Sin[5*ArcTan[a
*x]] + 2438*(1 + a^2*x^2)^4*Sin[7*ArcTan[a*x]] - 1575*(1 + a^2*x^2)^4*ArcTa
n[a*x]^2*Sin[7*ArcTan[a*x]]))/(2580480*a^3*Sqrt[1 + a^2*x^2])
```

Maple [A] time = 0.569, size = 376, normalized size = 0.6

$$\frac{c^2\left(5040\,(\arctan(ax))^2x^7a^7-1440\,\arctan(ax)\,x^6a^6+14280\,(\arctan(ax))^2x^5a^5+240\,a^5x^5-3984\,\arctan(ax)\,x^4a^4\right.}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^2,x)
```

```
[Out] 1/40320*c^2/a^3*(c*(a*x-I)*(a*x+I))^(1/2)*(5040*arctan(a*x)^2*x^7*a^7-1440*
arctan(a*x)*x^6*a^6+14280*arctan(a*x)^2*x^5*a^5+240*a^5*x^5-3984*arctan(a*x
)*x^4*a^4+12390*arctan(a*x)^2*x^3*a^3+696*a^3*x^3-2948*arctan(a*x)*a^2*x^2+
1575*arctan(a*x)^2*x*a+430*a*x+2746*arctan(a*x))-1/40320*I*c^2*(c*(a*x-I)*(
a*x+I))^(1/2)*(1575*I*arctan(a*x)^2*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-157
5*I*arctan(a*x)^2*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+3150*arctan(a*x)*poly
log(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-3150*arctan(a*x)*polylog(2,I*(1+I*a*x
)/(a^2*x^2+1)^(1/2))+3150*I*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-3150*
I*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-6352*arctan((1+I*a*x)/(a^2*x^2+1
)^(1/2)))/a^3/(a^2*x^2+1)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^4c^2x^6 + 2\,a^2c^2x^4 + c^2x^2\right)\sqrt{a^2cx^2 + c}\,\arctan\left(ax\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^6 + 2*a^2*c^2*x^4 + c^2*x^2)*sqrt(a^2*c*x^2 + c)*arctan
(a*x)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a**2*c*x**2+c)**(5/2)*atan(a*x)**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.325 $\int x\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^2\,dx$

Optimal. Leaf size=387

$$-\frac{5ic^3\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{56a^2\sqrt{a^2cx^2+c}}+\frac{5ic^3\sqrt{a^2x^2+1}\text{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{56a^2\sqrt{a^2cx^2+c}}+\frac{5c^2\sqrt{a^2cx^2+c}}{56a^2}-\frac{5c^2x\sqrt{a^2cx^2+c}\tan^{-1}(}{56a}$$

```
[Out] (5*c^2*Sqrt[c + a^2*c*x^2])/(56*a^2) + (5*c*(c + a^2*c*x^2)^(3/2))/(252*a^2
) + (c + a^2*c*x^2)^(5/2)/(105*a^2) - (5*c^2*x*Sqrt[c + a^2*c*x^2]*ArcTan[a
*x])/(56*a) - (5*c*x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x])/(84*a) - (x*(c + a^
2*c*x^2)^(5/2)*ArcTan[a*x])/(21*a) + ((c + a^2*c*x^2)^(7/2)*ArcTan[a*x]^2)/
(7*a^2*c) + (((5*I)/28)*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTan[Sqrt[1 + I
*a*x]/Sqrt[1 - I*a*x]])/(a^2*Sqrt[c + a^2*c*x^2]) - (((5*I)/56)*c^3*Sqrt[1
+ a^2*x^2]*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/(a^2*Sqrt[c
+ a^2*c*x^2]) + (((5*I)/56)*c^3*Sqrt[1 + a^2*x^2]*PolyLog[2, (I*Sqrt[1 + I*
a*x])/Sqrt[1 - I*a*x]])/(a^2*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.280812, antiderivative size = 387, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4930, 4878, 4890, 4886}

$$-\frac{5ic^3\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{56a^2\sqrt{a^2cx^2+c}}+\frac{5ic^3\sqrt{a^2x^2+1}\text{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{56a^2\sqrt{a^2cx^2+c}}+\frac{5c^2\sqrt{a^2cx^2+c}}{56a^2}-\frac{5c^2x\sqrt{a^2cx^2+c}\tan^{-1}(}{56a}$$

Antiderivative was successfully verified.

```
[In] Int[x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2,x]
```


```
[Out] (5*c^2*Sqrt[c + a^2*c*x^2])/(56*a^2) + (5*c*(c + a^2*c*x^2)^(3/2))/(252*a^2
) + (c + a^2*c*x^2)^(5/2)/(105*a^2) - (5*c^2*x*Sqrt[c + a^2*c*x^2]*ArcTan[a
*x])/(56*a) - (5*c*x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x])/(84*a) - (x*(c + a^
2*c*x^2)^(5/2)*ArcTan[a*x])/(21*a) + ((c + a^2*c*x^2)^(7/2)*ArcTan[a*x]^2)/
(7*a^2*c) + (((5*I)/28)*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTan[Sqrt[1 + I
*a*x]/Sqrt[1 - I*a*x]])/(a^2*Sqrt[c + a^2*c*x^2]) - (((5*I)/56)*c^3*Sqrt[1
+ a^2*x^2]*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/(a^2*Sqrt[c
+ a^2*c*x^2]) + (((5*I)/56)*c^3*Sqrt[1 + a^2*x^2]*PolyLog[2, (I*Sqrt[1 + I*
a*x])/Sqrt[1 - I*a*x]])/(a^2*Sqrt[c + a^2*c*x^2])
```


Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 4878

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symbo
l] :> -Simp[(b*(d + e*x^2)^q)/(2*c*q*(2*q + 1)), x] + (Dist[(2*d*q)/(2*q +
1), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x]), x], x] + Simp[(x*(d + e*x^
2)^q*(a + b*ArcTan[c*x]))/(2*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] && Eq
Q[e, c^2*d] && GtQ[q, 0]
```

Rule 4890

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4886

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[(-2*I*(a + b*ArcTan[c*x])*ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]])
/(c*Sqrt[d]), x] + (Simp[(I*b*PolyLog[2, -((I*Sqrt[1 + I*c*x])/Sqrt[1 - I*c
*x])])/(c*Sqrt[d]), x] - Simp[(I*b*PolyLog[2, (I*Sqrt[1 + I*c*x])/Sqrt[1 -
I*c*x]])/(c*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

Rubi steps

$$\int x\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^2\,dx = \frac{\left(c+a^2cx^2\right)^{7/2}\tan^{-1}(ax)^2}{7a^2c} - \frac{2\int\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)\,dx}{7a}$$

$$= \frac{\left(c+a^2cx^2\right)^{5/2}}{105a^2} - \frac{x\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)}{21a} + \frac{\left(c+a^2cx^2\right)^{7/2}\tan^{-1}(ax)^2}{7a^2c} - (5c)\int\left(c+\right.$$

$$= \frac{5c\left(c+a^2cx^2\right)^{3/2}}{252a^2} + \frac{\left(c+a^2cx^2\right)^{5/2}}{105a^2} - \frac{5cx\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}{84a} - \frac{x\left(c+a^2cx^2\right)^{5/2}\,\mathrm{ta}}{21a}$$

$$= \frac{5c^2\sqrt{c+a^2cx^2}}{56a^2} + \frac{5c\left(c+a^2cx^2\right)^{3/2}}{252a^2} + \frac{\left(c+a^2cx^2\right)^{5/2}}{105a^2} - \frac{5c^2x\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{56a} - 5$$

$$= \frac{5c^2\sqrt{c+a^2cx^2}}{56a^2} + \frac{5c\left(c+a^2cx^2\right)^{3/2}}{252a^2} + \frac{\left(c+a^2cx^2\right)^{5/2}}{105a^2} - \frac{5c^2x\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{56a} - 5$$

$$= \frac{5c^2\sqrt{c+a^2cx^2}}{56a^2} + \frac{5c\left(c+a^2cx^2\right)^{3/2}}{252a^2} + \frac{\left(c+a^2cx^2\right)^{5/2}}{105a^2} - \frac{5c^2x\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{56a} - 5$$

Mathematica [B] time = 7.78639, size = 1087, normalized size = 2.81

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2,x]
```

```
[Out] (c^2*(1 + a^2*x^2)*Sqrt[c*(1 + a^2*x^2)]*(2 + 4*ArcTan[a*x]^2 + 2*Cos[2*Arc
Tan[a*x]] - (3*ArcTan[a*x]*Log[1 - I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2]
- ArcTan[a*x]*Cos[3*ArcTan[a*x]]*Log[1 - I*E^(I*ArcTan[a*x])] + (3*ArcTan[a
*x]*Log[1 + I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + ArcTan[a*x]*Cos[3*Arc
Tan[a*x]]*Log[1 + I*E^(I*ArcTan[a*x])] - ((4*I)*PolyLog[2, (-I)*E^(I*ArcTan
[a*x])])/(1 + a^2*x^2)^(3/2) + ((4*I)*PolyLog[2, I*E^(I*ArcTan[a*x])])/(1 +
 a^2*x^2)^(3/2) - 2*ArcTan[a*x]*Sin[2*ArcTan[a*x]]))/(12*a^2) - (c^2*(1 + a
^2*x^2)^2*Sqrt[c*(1 + a^2*x^2)]*(50 - 32*ArcTan[a*x]^2 + 72*Cos[2*ArcTan[a*
x]] + 160*ArcTan[a*x]^2*Cos[2*ArcTan[a*x]] + 22*Cos[4*ArcTan[a*x]] - (110*A
rcTan[a*x]*Log[1 - I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - 55*ArcTan[a*x]
*Cos[3*ArcTan[a*x]]*Log[1 - I*E^(I*ArcTan[a*x])] - 11*ArcTan[a*x]*Cos[5*Arc
Tan[a*x]]*Log[1 - I*E^(I*ArcTan[a*x])] + (110*ArcTan[a*x]*Log[1 + I*E^(I*Ar
cTan[a*x])])/Sqrt[1 + a^2*x^2] + 55*ArcTan[a*x]*Cos[3*ArcTan[a*x]]*Log[1 +
I*E^(I*ArcTan[a*x])] + 11*ArcTan[a*x]*Cos[5*ArcTan[a*x]]*Log[1 + I*E^(I*Arc
Tan[a*x])] - ((176*I)*PolyLog[2, (-I)*E^(I*ArcTan[a*x])])/(1 + a^2*x^2)^(5/
2) + ((176*I)*PolyLog[2, I*E^(I*ArcTan[a*x])])/(1 + a^2*x^2)^(5/2) + 4*ArcT
```

```
an[a*x]*Sin[2*ArcTan[a*x]] - 22*ArcTan[a*x]*Sin[4*ArcTan[a*x]]))/(480*a^2)
+ (c^2*(1 + a^2*x^2)^3*Sqrt[c*(1 + a^2*x^2)]*(4116 + 10944*ArcTan[a*x]^2 +
6262*Cos[2*ArcTan[a*x]] - 5376*ArcTan[a*x]^2*Cos[2*ArcTan[a*x]] + 2764*Cos[
4*ArcTan[a*x]] + 6720*ArcTan[a*x]^2*Cos[4*ArcTan[a*x]] + 618*Cos[6*ArcTan[a
*x]] - (10815*ArcTan[a*x]*Log[1 - I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] -
 6489*ArcTan[a*x]*Cos[3*ArcTan[a*x]]*Log[1 - I*E^(I*ArcTan[a*x])] - 2163*Ar
cTan[a*x]*Cos[5*ArcTan[a*x]]*Log[1 - I*E^(I*ArcTan[a*x])] - 309*ArcTan[a*x]
*Cos[7*ArcTan[a*x]]*Log[1 - I*E^(I*ArcTan[a*x])] + (10815*ArcTan[a*x]*Log[1
 + I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + 6489*ArcTan[a*x]*Cos[3*ArcTan[
a*x]]*Log[1 + I*E^(I*ArcTan[a*x])] + 2163*ArcTan[a*x]*Cos[5*ArcTan[a*x]]*Lo
g[1 + I*E^(I*ArcTan[a*x])] + 309*ArcTan[a*x]*Cos[7*ArcTan[a*x]]*Log[1 + I*E
^(I*ArcTan[a*x])] - ((19776*I)*PolyLog[2, (-I)*E^(I*ArcTan[a*x])])/(1 + a^2
*x^2)^(7/2) + ((19776*I)*PolyLog[2, I*E^(I*ArcTan[a*x])])/(1 + a^2*x^2)^(7/
2) - 1266*ArcTan[a*x]*Sin[2*ArcTan[a*x]] + 360*ArcTan[a*x]*Sin[4*ArcTan[a*x
]] - 618*ArcTan[a*x]*Sin[6*ArcTan[a*x]]))/(161280*a^2)
```

Maple [A] time = 0.351, size = 275, normalized size = 0.7

$$\frac{c^2 \left(360 \left(\arctan (ax)\right)^2 x^6 a^6 - 120 \arctan (ax) x^5 a^5 + 1080 \left(\arctan (ax)\right)^2 x^4 a^4 + 24 a^4 x^4 - 390 \arctan (ax) x^3 a^3 + 108\right.}{2520 a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^2,x)
```

```
[Out] 1/2520*c^2/a^2*(c*(a*x-I)*(a*x+I))^(1/2)*(360*arctan(a*x)^2*x^6*a^6-120*arc
tan(a*x)*x^5*a^5+1080*arctan(a*x)^2*x^4*a^4+24*a^4*x^4-390*arctan(a*x)*x^3*
a^3+1080*arctan(a*x)^2*x^2*a^2+98*a^2*x^2-495*arctan(a*x)*x*a+360*arctan(a*
x)^2+299)+5/56*c^2*(c*(a*x-I)*(a*x+I))^(1/2)*(arctan(a*x)*ln(1+I*(1+I*a*x)/
(a^2*x^2+1)^(1/2))-arctan(a*x)*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*dilog(
1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+I*dilog(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))/
a^2/(a^2*x^2+1)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2 c x^2 + c\right)^{\frac{5}{2}} x \arctan (ax)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(5/2)*x*arctan(a*x)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^4c^2x^5 + 2\,a^2c^2x^3 + c^2x\right)\sqrt{a^2cx^2 + c}\arctan\left(ax\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^5 + 2*a^2*c^2*x^3 + c^2*x)*sqrt(a^2*c*x^2 + c)*arctan(a
*x)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**(5/2)*atan(a*x)**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.326 $\int \left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^2\,dx$

Optimal. Leaf size=516

$$\frac{5ic^3\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,-ie^{i\tan^{-1}(ax)}\right)}{8a\sqrt{a^2cx^2+c}}-\frac{5ic^3\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,ie^{i\tan^{-1}(ax)}\right)}{8a\sqrt{a^2cx^2+c}}-\frac{5c^3\sqrt{a^2x^2+1}}{8}$$

```
[Out] (17*c^2*x*Sqrt[c + a^2*c*x^2])/180 + (c*x*(c + a^2*c*x^2)^(3/2))/60 - (5*c^
2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(8*a) - (5*c*(c + a^2*c*x^2)^(3/2)*ArcTa
n[a*x])/(36*a) - ((c + a^2*c*x^2)^(5/2)*ArcTan[a*x])/(15*a) + (5*c^2*x*Sqrt
[c + a^2*c*x^2]*ArcTan[a*x]^2)/16 + (5*c*x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x
]^2)/24 + (x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2)/6 - (((5*I)/8)*c^3*Sqrt[1
 + a^2*x^2]*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2)/(a*Sqrt[c + a^2*c*x^2]
) + (259*c^(5/2)*ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]])/(360*a) + (((5
*I)/8)*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])]
)/(a*Sqrt[c + a^2*c*x^2]) - (((5*I)/8)*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*Po
lyLog[2, I*E^(I*ArcTan[a*x])])/(a*Sqrt[c + a^2*c*x^2]) - (5*c^3*Sqrt[1 + a^
2*x^2]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])])/(8*a*Sqrt[c + a^2*c*x^2]) + (5*c
^3*Sqrt[1 + a^2*x^2]*PolyLog[3, I*E^(I*ArcTan[a*x])])/(8*a*Sqrt[c + a^2*c*x
^2])
```

Rubi [A] time = 0.39048, antiderivative size = 516, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {4880, 4890, 4888, 4181, 2531, 2282, 6589, 217, 206, 195}

$$\frac{5ic^3\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,-ie^{i\tan^{-1}(ax)}\right)}{8a\sqrt{a^2cx^2+c}}-\frac{5ic^3\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,ie^{i\tan^{-1}(ax)}\right)}{8a\sqrt{a^2cx^2+c}}-\frac{5c^3\sqrt{a^2x^2+1}}{8}$$

Antiderivative was successfully verified.

```
[In] Int[(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2,x]
```


```
[Out] (17*c^2*x*Sqrt[c + a^2*c*x^2])/180 + (c*x*(c + a^2*c*x^2)^(3/2))/60 - (5*c^
2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(8*a) - (5*c*(c + a^2*c*x^2)^(3/2)*ArcTa
n[a*x])/(36*a) - ((c + a^2*c*x^2)^(5/2)*ArcTan[a*x])/(15*a) + (5*c^2*x*Sqrt
[c + a^2*c*x^2]*ArcTan[a*x]^2)/16 + (5*c*x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x
]^2)/24 + (x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2)/6 - (((5*I)/8)*c^3*Sqrt[1
 + a^2*x^2]*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2)/(a*Sqrt[c + a^2*c*x^2]
) + (259*c^(5/2)*ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]])/(360*a) + (((5
*I)/8)*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])]
```

```
)/(a*Sqrt[c + a^2*c*x^2]) - (((5*I)/8)*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*Po
lyLog[2, I*E^(I*ArcTan[a*x])])/(a*Sqrt[c + a^2*c*x^2]) - (5*c^3*Sqrt[1 + a^
2*x^2]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])])/(8*a*Sqrt[c + a^2*c*x^2]) + (5*c
^3*Sqrt[1 + a^2*x^2]*PolyLog[3, I*E^(I*ArcTan[a*x])])/(8*a*Sqrt[c + a^2*c*x
^2])
```

Rule 4880

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> -Simp[(b*p*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1))/(2*c*q*(2*
q + 1)), x] + (Dist[(2*d*q)/(2*q + 1), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTa
n[c*x])^p, x], x] + Dist[(b^2*d*p*(p - 1))/(2*q*(2*q + 1)), Int[(d + e*x^2)
^(q - 1)*(a + b*ArcTan[c*x])^(p - 2), x], x] + Simp[(x*(d + e*x^2)^q*(a + b
*ArcTan[c*x])^p)/(2*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[q, 0] && GtQ[p, 1]
```

Rule 4890

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4888

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[c
*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && GtQ
[d, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
```

```
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rubi steps

$$\int \left(c+a^2cx^2\right)^{5/2} \tan^{-1}(ax)^2\,dx = -\frac{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)}{15a} + \frac{1}{6}x\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^2 + \frac{1}{15}c\int \left(c+a^2cx^2\right)^{3/2}\,dx +$$

$$= \frac{1}{60}cx\left(c+a^2cx^2\right)^{3/2} - \frac{5c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}{36a} - \frac{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)}{15a} + \frac{5}{24}cx\,(c$$

$$= \frac{17}{180}c^2x\sqrt{c+a^2cx^2} + \frac{1}{60}cx\left(c+a^2cx^2\right)^{3/2} - \frac{5c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{8a} - \frac{5c\left(c+a^2cx^2\right)^{3/}}{36a}$$

$$= \frac{17}{180}c^2x\sqrt{c+a^2cx^2} + \frac{1}{60}cx\left(c+a^2cx^2\right)^{3/2} - \frac{5c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{8a} - \frac{5c\left(c+a^2cx^2\right)^{3/}}{36a}$$

$$= \frac{17}{180}c^2x\sqrt{c+a^2cx^2} + \frac{1}{60}cx\left(c+a^2cx^2\right)^{3/2} - \frac{5c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{8a} - \frac{5c\left(c+a^2cx^2\right)^{3/}}{36a}$$

$$= \frac{17}{180}c^2x\sqrt{c+a^2cx^2} + \frac{1}{60}cx\left(c+a^2cx^2\right)^{3/2} - \frac{5c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{8a} - \frac{5c\left(c+a^2cx^2\right)^{3/}}{36a}$$

$$= \frac{17}{180}c^2x\sqrt{c+a^2cx^2} + \frac{1}{60}cx\left(c+a^2cx^2\right)^{3/2} - \frac{5c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{8a} - \frac{5c\left(c+a^2cx^2\right)^{3/}}{36a}$$

$$= \frac{17}{180}c^2x\sqrt{c+a^2cx^2} + \frac{1}{60}cx\left(c+a^2cx^2\right)^{3/2} - \frac{5c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{8a} - \frac{5c\left(c+a^2cx^2\right)^{3/}}{36a}$$

$$= \frac{17}{180}c^2x\sqrt{c+a^2cx^2} + \frac{1}{60}cx\left(c+a^2cx^2\right)^{3/2} - \frac{5c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{8a} - \frac{5c\left(c+a^2cx^2\right)^{3/}}{36a}$$

Mathematica [A] time = 1.57735, size = 771, normalized size = 1.49

$$\frac{c^2\sqrt{a^2cx^2+c}\left(7200i\tan^{-1}(ax)\text{PolyLog}\left(2,-ie^{i\tan^{-1}(ax)}\right) - 7200i\tan^{-1}(ax)\text{PolyLog}\left(2,ie^{i\tan^{-1}(ax)}\right) - 7200\text{PolyLog}\left(3,\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2,x]
```

```
[Out] (c^2*Sqrt[c + a^2*c*x^2]*(424*a*x*Sqrt[1 + a^2*x^2] + 368*a^3*x^3*Sqrt[1 +
a^2*x^2] - 56*a^5*x^5*Sqrt[1 + a^2*x^2] - 11028*Sqrt[1 + a^2*x^2]*ArcTan[a*
x] + 504*a^2*x^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x] + 12*a^4*x^4*Sqrt[1 + a^2*x^
2]*ArcTan[a*x] + 11970*a*x*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2 + 7380*a^3*x^3*S
qrt[1 + a^2*x^2]*ArcTan[a*x]^2 + 1170*a^5*x^5*Sqrt[1 + a^2*x^2]*ArcTan[a*x]
```

```
^2 - (7200*I)*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2 + 8288*ArcTanh[(a*x)/
Sqrt[1 + a^2*x^2]] + 1550*ArcTan[a*x]*Cos[3*ArcTan[a*x]] + 3210*a^2*x^2*Arc
Tan[a*x]*Cos[3*ArcTan[a*x]] + 1770*a^4*x^4*ArcTan[a*x]*Cos[3*ArcTan[a*x]] +
 110*a^6*x^6*ArcTan[a*x]*Cos[3*ArcTan[a*x]] - 90*ArcTan[a*x]*Cos[5*ArcTan[a
*x]] - 270*a^2*x^2*ArcTan[a*x]*Cos[5*ArcTan[a*x]] - 270*a^4*x^4*ArcTan[a*x]
*Cos[5*ArcTan[a*x]] - 90*a^6*x^6*ArcTan[a*x]*Cos[5*ArcTan[a*x]] + (7200*I)*
ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - (7200*I)*ArcTan[a*x]*PolyL
og[2, I*E^(I*ArcTan[a*x])] - 7200*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] + 7200
*PolyLog[3, I*E^(I*ArcTan[a*x])] + 372*Sin[3*ArcTan[a*x]] + 636*a^2*x^2*Sin
[3*ArcTan[a*x]] + 156*a^4*x^4*Sin[3*ArcTan[a*x]] - 108*a^6*x^6*Sin[3*ArcTan
[a*x]] - 1425*ArcTan[a*x]^2*Sin[3*ArcTan[a*x]] - 3555*a^2*x^2*ArcTan[a*x]^2
*Sin[3*ArcTan[a*x]] - 2835*a^4*x^4*ArcTan[a*x]^2*Sin[3*ArcTan[a*x]] - 705*a
^6*x^6*ArcTan[a*x]^2*Sin[3*ArcTan[a*x]] - 52*Sin[5*ArcTan[a*x]] - 156*a^2*x
^2*Sin[5*ArcTan[a*x]] - 156*a^4*x^4*Sin[5*ArcTan[a*x]] - 52*a^6*x^6*Sin[5*A
rcTan[a*x]] + 45*ArcTan[a*x]^2*Sin[5*ArcTan[a*x]] + 135*a^2*x^2*ArcTan[a*x]
^2*Sin[5*ArcTan[a*x]] + 135*a^4*x^4*ArcTan[a*x]^2*Sin[5*ArcTan[a*x]] + 45*a
^6*x^6*ArcTan[a*x]^2*Sin[5*ArcTan[a*x]]))/(11520*a*Sqrt[1 + a^2*x^2])
```

Maple [A] time = 0.364, size = 342, normalized size = 0.7

$$\frac{c^2\left(120\,(\arctan(ax))^2 x^5a^5 - 48\,\arctan(ax)\,x^4a^4 + 390\,(\arctan(ax))^2 x^3a^3 + 12\,a^3x^3 - 196\,\arctan(ax)\,a^2x^2 + 495\,(a\right.}{720\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(5/2)*arctan(a*x)^2,x)
```

```
[Out] 1/720*c^2/a*(c*(a*x-I)*(a*x+I))^(1/2)*(120*arctan(a*x)^2*x^5*a^5-48*arctan(
a*x)*x^4*a^4+390*arctan(a*x)^2*x^3*a^3+12*a^3*x^3-196*arctan(a*x)*a^2*x^2+4
95*arctan(a*x)^2*x*a+80*a*x-598*arctan(a*x))+1/720*I*c^2*(c*(a*x-I)*(a*x+I)
)^(1/2)*(225*I*arctan(a*x)^2*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-225*I*arct
an(a*x)^2*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+450*arctan(a*x)*polylog(2,-I*
(1+I*a*x)/(a^2*x^2+1)^(1/2))-450*arctan(a*x)*polylog(2,I*(1+I*a*x)/(a^2*x^2
+1)^(1/2))+450*I*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-450*I*polylog(3,
I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-1036*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2)))/a/
(a^2*x^2+1)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^{4}c^{2}x^{4}+2\,a^{2}c^{2}x^{2}+c^{2}\right)\sqrt{a^{2}cx^{2}+c}\,\arctan\left(ax\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*arctan(a*x
)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(5/2)*atan(a*x)**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.327 $\int \frac{\left(c+a^2cx^2\right)^{5/2} \tan^{-1}(ax)^2}{x} \, dx$

Optimal. Leaf size=605

$$-\frac{149ic^3\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{60\sqrt{a^2cx^2+c}}+\frac{149ic^3\sqrt{a^2x^2+1}\text{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{60\sqrt{a^2cx^2+c}}+\frac{2ic^3\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}}{\sqrt{a^2cx^2+c}}$$

```
[Out] (29*c^2*Sqrt[c + a^2*c*x^2])/60 + (c*(c + a^2*c*x^2)^(3/2))/30 - (29*a*c^2*
x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/60 - (a*c*x*(c + a^2*c*x^2)^(3/2)*ArcTan
[a*x])/10 + c^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2 + (c*(c + a^2*c*x^2)^(3/2
)*ArcTan[a*x]^2)/3 + ((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2)/5 + (((149*I)/30
)*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]]
)/Sqrt[c + a^2*c*x^2] - (2*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*ArcTanh[E^(I
*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + ((2*I)*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a
*x]*PolyLog[2, -E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - ((2*I)*c^3*Sqrt[1
 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2]
- (((149*I)/60)*c^3*Sqrt[1 + a^2*x^2]*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqr
t[1 - I*a*x]])/Sqrt[c + a^2*c*x^2] + (((149*I)/60)*c^3*Sqrt[1 + a^2*x^2]*Po
lyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2] - (2*c^3
*Sqrt[1 + a^2*x^2]*PolyLog[3, -E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + (2
*c^3*Sqrt[1 + a^2*x^2]*PolyLog[3, E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2]
```

Rubi [A] time = 1.263, antiderivative size = 605, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {4950, 4958, 4956, 4183, 2531, 2282, 6589, 4930, 4890, 4886, 4878}

$$-\frac{149ic^3\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{60\sqrt{a^2cx^2+c}}+\frac{149ic^3\sqrt{a^2x^2+1}\text{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{60\sqrt{a^2cx^2+c}}+\frac{2ic^3\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}}{\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2)/x,x]
```


```
[Out] (29*c^2*Sqrt[c + a^2*c*x^2])/60 + (c*(c + a^2*c*x^2)^(3/2))/30 - (29*a*c^2*
x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/60 - (a*c*x*(c + a^2*c*x^2)^(3/2)*ArcTan
[a*x])/10 + c^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2 + (c*(c + a^2*c*x^2)^(3/2
)*ArcTan[a*x]^2)/3 + ((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2)/5 + (((149*I)/30
)*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]]
)/Sqrt[c + a^2*c*x^2] - (2*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*ArcTanh[E^(I
```

```
*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + ((2*I)*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a
*x]*PolyLog[2, -E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - ((2*I)*c^3*Sqrt[1
 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2]
- (((149*I)/60)*c^3*Sqrt[1 + a^2*x^2]*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqr
t[1 - I*a*x]])/Sqrt[c + a^2*c*x^2] + (((149*I)/60)*c^3*Sqrt[1 + a^2*x^2]*Po
lyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2] - (2*c^3
*Sqrt[1 + a^2*x^2]*PolyLog[3, -E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + (2
*c^3*Sqrt[1 + a^2*x^2]*PolyLog[3, E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2]
```

Rule 4950

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 4958

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4956

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Dist[1/Sqrt[d], Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTan
[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && G
tQ[d, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
```

```
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 4890

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4886

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[(-2*I*(a + b*ArcTan[c*x])*ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]])
/(c*Sqrt[d]), x] + (Simp[(I*b*PolyLog[2, -((I*Sqrt[1 + I*c*x])/Sqrt[1 - I*c
*x])])/(c*Sqrt[d]), x] - Simp[(I*b*PolyLog[2, (I*Sqrt[1 + I*c*x])/Sqrt[1 -
I*c*x]])/(c*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

Rule 4878

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symbo
l] :> -Simp[(b*(d + e*x^2)^q)/(2*c*q*(2*q + 1)), x] + (Dist[(2*d*q)/(2*q +
```

```
1), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x]), x], x] + Simp[(x*(d + e*x^
2)^q*(a + b*ArcTan[c*x]))/(2*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] && Eq
Q[e, c^2*d] && GtQ[q, 0]
```

Rubi steps

$$\int \frac{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^2}{x}\,dx = c\int \frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2}{x}\,dx + \left(a^2c\right)\int x\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2\,dx$$

$$= \frac{1}{5}\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^2 - \frac{1}{5}(2ac)\int \left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)\,dx + c^2\int \frac{\sqrt{c+a^2cx^2}}{}$$

$$= \frac{1}{30}c\left(c+a^2cx^2\right)^{3/2} - \frac{1}{10}acx\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax) + \frac{1}{3}c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2 + \frac{1}{5}$$

$$= \frac{29}{60}c^2\sqrt{c+a^2cx^2} + \frac{1}{30}c\left(c+a^2cx^2\right)^{3/2} - \frac{29}{60}ac^2x\sqrt{c+a^2cx^2}\tan^{-1}(ax) - \frac{1}{10}acx\left(c+a^2c\right.$$

$$= \frac{29}{60}c^2\sqrt{c+a^2cx^2} + \frac{1}{30}c\left(c+a^2cx^2\right)^{3/2} - \frac{29}{60}ac^2x\sqrt{c+a^2cx^2}\tan^{-1}(ax) - \frac{1}{10}acx\left(c+a^2c\right.$$

$$= \frac{29}{60}c^2\sqrt{c+a^2cx^2} + \frac{1}{30}c\left(c+a^2cx^2\right)^{3/2} - \frac{29}{60}ac^2x\sqrt{c+a^2cx^2}\tan^{-1}(ax) - \frac{1}{10}acx\left(c+a^2c\right.$$

$$= \frac{29}{60}c^2\sqrt{c+a^2cx^2} + \frac{1}{30}c\left(c+a^2cx^2\right)^{3/2} - \frac{29}{60}ac^2x\sqrt{c+a^2cx^2}\tan^{-1}(ax) - \frac{1}{10}acx\left(c+a^2c\right.$$

$$= \frac{29}{60}c^2\sqrt{c+a^2cx^2} + \frac{1}{30}c\left(c+a^2cx^2\right)^{3/2} - \frac{29}{60}ac^2x\sqrt{c+a^2cx^2}\tan^{-1}(ax) - \frac{1}{10}acx\left(c+a^2c\right.$$

$$= \frac{29}{60}c^2\sqrt{c+a^2cx^2} + \frac{1}{30}c\left(c+a^2cx^2\right)^{3/2} - \frac{29}{60}ac^2x\sqrt{c+a^2cx^2}\tan^{-1}(ax) - \frac{1}{10}acx\left(c+a^2c\right.$$

Mathematica [A] time = 7.09301, size = 889, normalized size = 1.47

$$\sqrt{c\left(a^2x^2+1\right)}\left(\frac{\left(\log\left(1-e^{i\tan^{-1}(ax)}\right)-\log\left(1+e^{i\tan^{-1}(ax)}\right)\right)\tan^{-1}(ax)^2}{\sqrt{a^2x^2+1}}+\tan^{-1}(ax)^2+\frac{2i\left(\text{PolyLog}\left(2,-e^{i\tan^{-1}(ax)}\right)-\text{Po}\right.}{\sqrt{a^2x}}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2)/x,x]
```

[Out] c^2*Sqrt[c*(1 + a^2*x^2)]*(ArcTan[a*x]^2 + (ArcTan[a*x]^2*(Log[1 - E^(I*ArcTan[a*x])] - Log[1 + E^(I*ArcTan[a*x])]))/Sqrt[1 + a^2*x^2] - (2*(ArcTan[a*x]*(Log[1 - I*E^(I*ArcTan[a*x])] - Log[1 + I*E^(I*ArcTan[a*x])]) + I*(PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - PolyLog[2, I*E^(I*ArcTan[a*x])])))/Sqrt[1 + a^2*x^2] + ((2*I)*ArcTan[a*x]*(PolyLog[2, -E^(I*ArcTan[a*x])] - PolyLog[2, E^(I*ArcTan[a*x])]))/Sqrt[1 + a^2*x^2] + (2*(-PolyLog[3, -E^(I*ArcTan[a*x])] + PolyLog[3, E^(I*ArcTan[a*x])]))/Sqrt[1 + a^2*x^2]) + (c^2*(1 + a^2*x^2)*Sqrt[c*(1 + a^2*x^2)]*(2 + 4*ArcTan[a*x]^2 + 2*Cos[2*ArcTan[a*x]] - (3*ArcTan[a*x]*Log[1 - I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - ArcTan[a*x]*Cos[3*ArcTan[a*x]]*Log[1 - I*E^(I*ArcTan[a*x])] + (3*ArcTan[a*x]*Log[1 + I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + ArcTan[a*x]*Cos[3*ArcTan[a*x]]*Log[1 + I*E^(I*ArcTan[a*x])] - ((4*I)*PolyLog[2, (-I)*E^(I*ArcTan[a*x])])/(1 + a^2*x^2)^(3/2) + ((4*I)*PolyLog[2, I*E^(I*ArcTan[a*x])])/(1 + a^2*x^2)^(3/2) - 2*ArcTan[a*x]*Sin[2*ArcTan[a*x]]))/6 - (c^2*(1 + a^2*x^2)^2*Sqrt[c*(1 + a^2*x^2)]*(50 - 32*ArcTan[a*x]^2 + 72*Cos[2*ArcTan[a*x]] + 160*ArcTan[a*x]^2*Cos[2*ArcTan[a*x]] + 22*Cos[4*ArcTan[a*x]] - (110*ArcTan[a*x]*Log[1 - I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - 55*ArcTan[a*x]*Cos[3*ArcTan[a*x]]*Log[1 - I*E^(I*ArcTan[a*x])] - 11*ArcTan[a*x]*Cos[5*ArcTan[a*x]]*Log[1 - I*E^(I*ArcTan[a*x])] + (110*ArcTan[a*x]*Log[1 + I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + 55*ArcTan[a*x]*Cos[3*ArcTan[a*x]]*Log[1 + I*E^(I*ArcTan[a*x])] + 11*ArcTan[a*x]*Cos[5*ArcTan[a*x]]*Log[1 + I*E^(I*ArcTan[a*x])] - ((176*I)*PolyLog[2, (-I)*E^(I*ArcTan[a*x])])/(1 + a^2*x^2)^(5/2) + ((176*I)*PolyLog[2, I*E^(I*ArcTan[a*x])])/(1 + a^2*x^2)^(5/2) + 4*ArcTan[a*x]*Sin[2*ArcTan[a*x]] - 22*ArcTan[a*x]*Sin[4*ArcTan[a*x]]))/960

Maple [A] time = 0.422, size = 404, normalized size = 0.7

$$\frac{c^2\left(12\,(\arctan(ax))^2 x^4a^4 - 6\arctan(ax)\,x^3a^3 + 44\,(\arctan(ax))^2 x^2a^2 + 2\,a^2x^2 - 35\arctan(ax)\,xa + 92\,(\arctan(a\right.}{60}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^2*c*x^2+c)^(5/2)*arctan(a*x)^2/x,x)

[Out] 1/60*c^2*(c*(a*x-I)*(a*x+I))^(1/2)*(12*arctan(a*x)^2*x^4*a^4-6*arctan(a*x)*x^3*a^3+44*arctan(a*x)^2*x^2*a^2+2*a^2*x^2-35*arctan(a*x)*x*a+92*arctan(a*x)^2+31)-1/60*I*c^2*(c*(a*x-I)*(a*x+I))^(1/2)*(60*I*arctan(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-60*I*arctan(a*x)^2*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))+149*I*arctan(a*x)*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-149*I*arctan(a*x)*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+120*arctan(a*x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))-120*arctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+120*I*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))-120*I*polylog(3,-(1+I*a*x)/(a^2*x^

```
2+1)^(1/2))+149*dilog(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-149*dilog(1-I*(1+I*a
*x)/(a^2*x^2+1)^(1/2)))/(a^2*x^2+1)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^2/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^{4}c^{2}x^{4}+2\,a^{2}c^{2}x^{2}+c^{2}\right)\sqrt{a^{2}cx^{2}+c}\,\arctan\left(ax\right)^{2}}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^2/x,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*arctan(a*x
)^2/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(5/2)*atan(a*x)**2/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^{\frac{5}{2}}\arctan\left(ax\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^2/x,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^2/x, x)
```

3.328 $\int \frac{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^2}{x^2}\,dx$

Optimal. Leaf size=655

$$\frac{2iac^3\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{a^2cx^2+c}}-\frac{2iac^3\sqrt{a^2x^2+1}\text{PolyLog}\left(2,\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{a^2cx^2+c}}+\frac{15iac^3\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,-\right.}{4\sqrt{a^2cx^2+c}}$$

```
[Out] (a^2*c^2*x*Sqrt[c + a^2*c*x^2])/12 - (7*a*c^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*
x])/4 - (a*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x])/6 - (c^2*Sqrt[c + a^2*c*x^2
]*ArcTan[a*x]^2)/x + (7*a^2*c^2*x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/8 + (a
^2*c*x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2)/4 - (((15*I)/4)*a*c^3*Sqrt[1 +
a^2*x^2]*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2)/Sqrt[c + a^2*c*x^2] - (4*
a*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTanh[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]
])/Sqrt[c + a^2*c*x^2] + (11*a*c^(5/2)*ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c
*x^2]])/6 + (((15*I)/4)*a*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, (-I)
*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - (((15*I)/4)*a*c^3*Sqrt[1 + a^2*x
^2]*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + ((2*
I)*a*c^3*Sqrt[1 + a^2*x^2]*PolyLog[2, -(Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x])])/
Sqrt[c + a^2*c*x^2] - ((2*I)*a*c^3*Sqrt[1 + a^2*x^2]*PolyLog[2, Sqrt[1 + I*
a*x]/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2] - (15*a*c^3*Sqrt[1 + a^2*x^2]*Po
lyLog[3, (-I)*E^(I*ArcTan[a*x])])/(4*Sqrt[c + a^2*c*x^2]) + (15*a*c^3*Sqrt[
1 + a^2*x^2]*PolyLog[3, I*E^(I*ArcTan[a*x])])/(4*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 1.4075, antiderivative size = 655, normalized size of antiderivative = 1., number of steps used = 43, number of rules used = 14, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {4950, 4944, 4958, 4954, 4890, 4888, 4181, 2531, 2282, 6589, 4880, 217, 206, 195}

$$\frac{2iac^3\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{a^2cx^2+c}}-\frac{2iac^3\sqrt{a^2x^2+1}\text{PolyLog}\left(2,\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{a^2cx^2+c}}+\frac{15iac^3\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,-\right.}{4\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2)/x^2,x]
```

```
[Out] (a^2*c^2*x*Sqrt[c + a^2*c*x^2])/12 - (7*a*c^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*
x])/4 - (a*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x])/6 - (c^2*Sqrt[c + a^2*c*x^2
]*ArcTan[a*x]^2)/x + (7*a^2*c^2*x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/8 + (a
^2*c*x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2)/4 - (((15*I)/4)*a*c^3*Sqrt[1 +
```

```
a^2*x^2]*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2)/Sqrt[c + a^2*c*x^2] - (4*
a*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTanh[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]
])/Sqrt[c + a^2*c*x^2] + (11*a*c^(5/2)*ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c
*x^2]])/6 + (((15*I)/4)*a*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, (-I)
*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - (((15*I)/4)*a*c^3*Sqrt[1 + a^2*x
^2]*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + ((2*
I)*a*c^3*Sqrt[1 + a^2*x^2]*PolyLog[2, -(Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x])])/
Sqrt[c + a^2*c*x^2] - ((2*I)*a*c^3*Sqrt[1 + a^2*x^2]*PolyLog[2, Sqrt[1 + I*
a*x]/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2] - (15*a*c^3*Sqrt[1 + a^2*x^2]*Po
lyLog[3, (-I)*E^(I*ArcTan[a*x])])/(4*Sqrt[c + a^2*c*x^2]) + (15*a*c^3*Sqrt[
1 + a^2*x^2]*PolyLog[3, I*E^(I*ArcTan[a*x])])/(4*Sqrt[c + a^2*c*x^2])
```

Rule 4950

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 4944

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a +
 b*ArcTan[c*x])^p)/(d*f*(m + 1)), x] - Dist[(b*c*p)/(f*(m + 1)), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rule 4958

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4954

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x_
Symbol] :> Simp[(-2*(a + b*ArcTan[c*x])*ArcTanh[Sqrt[1 + I*c*x]/Sqrt[1 - I*
c*x]])/Sqrt[d], x] + (Simp[(I*b*PolyLog[2, -(Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x
])])/Sqrt[d], x] - Simp[(I*b*PolyLog[2, Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]])/S
qrt[d], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

Rule 4890

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4888

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[c
*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && GtQ
[d, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4880

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> -Simp[(b*p*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1))/(2*c*q*(2*
q + 1)), x] + (Dist[(2*d*q)/(2*q + 1), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTa
n[c*x])^p, x], x] + Dist[(b^2*d*p*(p - 1))/(2*q*(2*q + 1)), Int[(d + e*x^2)
^(q - 1)*(a + b*ArcTan[c*x])^(p - 2), x], x] + Simp[(x*(d + e*x^2)^q*(a + b
*ArcTan[c*x])^p)/(2*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[q, 0] && GtQ[p, 1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rubi steps

$$\int \frac{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^2}{x^2}\,dx = c\int \frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2}{x^2}\,dx + \left(a^2c\right)\int \left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2\,dx$$

$$= -\frac{1}{6}ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax) + \frac{1}{4}a^2cx\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2 + c^2\int \frac{\sqrt{c+a^2cx^2}\,\mathrm{ta}}{x^2}$$

$$= \frac{1}{12}a^2c^2x\sqrt{c+a^2cx^2} - \frac{7}{4}ac^2\sqrt{c+a^2cx^2}\tan^{-1}(ax) - \frac{1}{6}ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax) + \frac{7}{8}a^2$$

$$= \frac{1}{12}a^2c^2x\sqrt{c+a^2cx^2} - \frac{7}{4}ac^2\sqrt{c+a^2cx^2}\tan^{-1}(ax) - \frac{1}{6}ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax) - \frac{c^2\sqrt{}}{}$$

$$= \frac{1}{12}a^2c^2x\sqrt{c+a^2cx^2} - \frac{7}{4}ac^2\sqrt{c+a^2cx^2}\tan^{-1}(ax) - \frac{1}{6}ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax) - \frac{c^2\sqrt{}}{}$$

$$= \frac{1}{12}a^2c^2x\sqrt{c+a^2cx^2} - \frac{7}{4}ac^2\sqrt{c+a^2cx^2}\tan^{-1}(ax) - \frac{1}{6}ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax) - \frac{c^2\sqrt{}}{}$$

$$= \frac{1}{12}a^2c^2x\sqrt{c+a^2cx^2} - \frac{7}{4}ac^2\sqrt{c+a^2cx^2}\tan^{-1}(ax) - \frac{1}{6}ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax) - \frac{c^2\sqrt{}}{}$$

$$= \frac{1}{12}a^2c^2x\sqrt{c+a^2cx^2} - \frac{7}{4}ac^2\sqrt{c+a^2cx^2}\tan^{-1}(ax) - \frac{1}{6}ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax) - \frac{c^2\sqrt{}}{}$$

$$= \frac{1}{12}a^2c^2x\sqrt{c+a^2cx^2} - \frac{7}{4}ac^2\sqrt{c+a^2cx^2}\tan^{-1}(ax) - \frac{1}{6}ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax) - \frac{c^2\sqrt{}}{}$$

Mathematica [A] time = 1.72348, size = 626, normalized size = 0.96

$$\frac{c^2\sqrt{a^2cx^2+c}\left(192iax\text{PolyLog}\left(2,-e^{i\tan^{-1}(ax)}\right)+360iax\tan^{-1}(ax)\text{PolyLog}\left(2,-ie^{i\tan^{-1}(ax)}\right)-360iax\tan^{-1}(ax)\text{PolyLog}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2)/x^2,x]
```

```
[Out] (c^2*Sqrt[c + a^2*c*x^2]*(2*a^2*x^2*Sqrt[1 + a^2*x^2] + 2*a^4*x^4*Sqrt[1 +
a^2*x^2] - 190*a*x*Sqrt[1 + a^2*x^2]*ArcTan[a*x] + 2*a^3*x^3*Sqrt[1 + a^2*x
^2]*ArcTan[a*x] - 96*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2 + 117*a^2*x^2*Sqrt[1 +
 a^2*x^2]*ArcTan[a*x]^2 + 21*a^4*x^4*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2 - (168
*I)*a*x*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2 + 176*a*x*ArcTanh[(a*x)/Sqr
```

```
t[1 + a^2*x^2]] + 6*a*x*ArcTan[a*x]*Cos[3*ArcTan[a*x]] + 12*a^3*x^3*ArcTan[
a*x]*Cos[3*ArcTan[a*x]] + 6*a^5*x^5*ArcTan[a*x]*Cos[3*ArcTan[a*x]] + 192*a*
x*ArcTan[a*x]*Log[1 - E^(I*ArcTan[a*x])] + 96*a*x*ArcTan[a*x]^2*Log[1 - I*E
^(I*ArcTan[a*x])] - 96*a*x*ArcTan[a*x]^2*Log[1 + I*E^(I*ArcTan[a*x])] - 192
*a*x*ArcTan[a*x]*Log[1 + E^(I*ArcTan[a*x])] + (192*I)*a*x*PolyLog[2, -E^(I*
ArcTan[a*x])] + (360*I)*a*x*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])]
- (360*I)*a*x*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])] - (192*I)*a*x*Pol
yLog[2, E^(I*ArcTan[a*x])] - 360*a*x*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] + 3
60*a*x*PolyLog[3, I*E^(I*ArcTan[a*x])] + 2*a*x*Sin[3*ArcTan[a*x]] + 4*a^3*x
^3*Sin[3*ArcTan[a*x]] + 2*a^5*x^5*Sin[3*ArcTan[a*x]] - 3*a*x*ArcTan[a*x]^2*
Sin[3*ArcTan[a*x]] - 6*a^3*x^3*ArcTan[a*x]^2*Sin[3*ArcTan[a*x]] - 3*a^5*x^5
*ArcTan[a*x]^2*Sin[3*ArcTan[a*x]]))/(96*x*Sqrt[1 + a^2*x^2])
```

Maple [A] time = 0.426, size = 399, normalized size = 0.6

$$\frac{c^2\left(6\,(\arctan(ax))^2\,x^4a^4-4\,\arctan(ax)\,x^3a^3+27\,(\arctan(ax))^2\,x^2a^2+2\,a^2x^2-46\,\arctan(ax)\,xa-24\,(\arctan(ax$$
$$24\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(5/2)*arctan(a*x)^2/x^2,x)
```

```
[Out] 1/24*c^2*(c*(a*x-I)*(a*x+I))^(1/2)*(6*arctan(a*x)^2*x^4*a^4-4*arctan(a*x)*x
^3*a^3+27*arctan(a*x)^2*x^2*a^2+2*a^2*x^2-46*arctan(a*x)*x*a-24*arctan(a*x)
^2)/x+1/24*I*a*c^2*(c*(a*x-I)*(a*x+I))^(1/2)*(45*I*arctan(a*x)^2*ln(1+I*(1+
I*a*x)/(a^2*x^2+1)^(1/2))-45*I*arctan(a*x)^2*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(
1/2))+48*I*arctan(a*x)*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))+90*arctan(a*x)*pol
ylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-90*arctan(a*x)*polylog(2,I*(1+I*a*x)
/(a^2*x^2+1)^(1/2))+90*I*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-90*I*pol
ylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+48*dilog((1+I*a*x)/(a^2*x^2+1)^(1/2))
+48*dilog(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-88*arctan((1+I*a*x)/(a^2*x^2+1)^(1
/2)))/(a^2*x^2+1)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^2/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^4c^2x^4+2\,a^2c^2x^2+c^2\right)\sqrt{a^2cx^2+c}\arctan\left(ax\right)^2}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^2/x^2,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*arctan(a*x
)^2/x^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(5/2)*atan(a*x)**2/x**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^{\frac{5}{2}}\arctan\left(ax\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^2/x^2,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^2/x^2, x)
```

3.329 $\int \frac{(c+a^2cx^2)^{5/2}\tan^{-1}(ax)^2}{x^3}\,dx$

Optimal. Leaf size=661

$$-\frac{13ia^2c^3\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{3\sqrt{a^2cx^2+c}}+\frac{13ia^2c^3\sqrt{a^2x^2+1}\text{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{3\sqrt{a^2cx^2+c}}+\frac{5ia^2c^3\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyL}}{\sqrt{a^2cx^2+c}}$$

```
[Out] (a^2*c^2*Sqrt[c + a^2*c*x^2])/3 - (a*c^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/x
 - (a^3*c^2*x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/3 + 2*a^2*c^2*Sqrt[c + a^2*c
*x^2]*ArcTan[a*x]^2 - (c^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(2*x^2) + (a^
2*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2)/3 + (((26*I)/3)*a^2*c^3*Sqrt[1 + a
^2*x^2]*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c
*x^2] - (5*a^2*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*ArcTanh[E^(I*ArcTan[a*x]
)])/Sqrt[c + a^2*c*x^2] - a^2*c^(5/2)*ArcTanh[Sqrt[c + a^2*c*x^2]/Sqrt[c]]
+ ((5*I)*a^2*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x]
)])/Sqrt[c + a^2*c*x^2] - ((5*I)*a^2*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*Poly
Log[2, E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - (((13*I)/3)*a^2*c^3*Sqrt[1
 + a^2*x^2]*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/Sqrt[c + a^
2*c*x^2] + (((13*I)/3)*a^2*c^3*Sqrt[1 + a^2*x^2]*PolyLog[2, (I*Sqrt[1 + I*a
*x])/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2] - (5*a^2*c^3*Sqrt[1 + a^2*x^2]*P
olyLog[3, -E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + (5*a^2*c^3*Sqrt[1 + a^
2*x^2]*PolyLog[3, E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2]
```

Rubi [A] time = 2.6211, antiderivative size = 661, normalized size of antiderivative = 1., number of steps used = 57, number of rules used = 16, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {4950, 4962, 4944, 266, 63, 208, 4958, 4956, 4183, 2531, 2282, 6589, 4930, 4890, 4886, 4878}

$$-\frac{13ia^2c^3\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{3\sqrt{a^2cx^2+c}}+\frac{13ia^2c^3\sqrt{a^2x^2+1}\text{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{3\sqrt{a^2cx^2+c}}+\frac{5ia^2c^3\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyL}}{\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2)/x^3,x]
```

```
[Out] (a^2*c^2*Sqrt[c + a^2*c*x^2])/3 - (a*c^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/x
 - (a^3*c^2*x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/3 + 2*a^2*c^2*Sqrt[c + a^2*c
*x^2]*ArcTan[a*x]^2 - (c^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(2*x^2) + (a^
2*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2)/3 + (((26*I)/3)*a^2*c^3*Sqrt[1 + a
```

```
^2*x^2]*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c
*x^2] - (5*a^2*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*ArcTanh[E^(I*ArcTan[a*x]
)])/Sqrt[c + a^2*c*x^2] - a^2*c^(5/2)*ArcTanh[Sqrt[c + a^2*c*x^2]/Sqrt[c]]
+ ((5*I)*a^2*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x]
)])/Sqrt[c + a^2*c*x^2] - ((5*I)*a^2*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*Poly
Log[2, E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - (((13*I)/3)*a^2*c^3*Sqrt[1
 + a^2*x^2]*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/Sqrt[c + a^
2*c*x^2] + (((13*I)/3)*a^2*c^3*Sqrt[1 + a^2*x^2]*PolyLog[2, (I*Sqrt[1 + I*a
*x])/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2] - (5*a^2*c^3*Sqrt[1 + a^2*x^2]*P
olyLog[3, -E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + (5*a^2*c^3*Sqrt[1 + a^
2*x^2]*PolyLog[3, E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2]
```

Rule 4950

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 4962

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Ar
cTan[c*x])^p)/(d*f*(m + 1)), x] + (-Dist[(b*c*p)/(f*(m + 1)), Int[((f*x)^(m
 + 1)*(a + b*ArcTan[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] - Dist[(c^2*(m +
 2))/(f^2*(m + 1)), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/Sqrt[d + e*x^
2], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] &
& LtQ[m, -1] && NeQ[m, -2]
```

Rule 4944

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a +
 b*ArcTan[c*x])^p)/(d*f*(m + 1)), x] - Dist[(b*c*p)/(f*(m + 1)), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 4958

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4956

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Dist[1/Sqrt[d], Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTan
[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && G
tQ[d, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 4890

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4886

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[(-2*I*(a + b*ArcTan[c*x])*ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]])
/(c*Sqrt[d]), x] + (Simp[(I*b*PolyLog[2, -((I*Sqrt[1 + I*c*x])/Sqrt[1 - I*c
*x])])/(c*Sqrt[d]), x] - Simp[(I*b*PolyLog[2, (I*Sqrt[1 + I*c*x])/Sqrt[1 -
I*c*x]])/(c*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

Rule 4878

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symbo
l] :> -Simp[(b*(d + e*x^2)^q)/(2*c*q*(2*q + 1)), x] + (Dist[(2*d*q)/(2*q +
1), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x]), x], x] + Simp[(x*(d + e*x^
2)^q*(a + b*ArcTan[c*x]))/(2*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] && Eq
```

```
Q[e, c^2*d] && GtQ[q, 0]
```

Rubi steps

$$\int \frac{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^2}{x^3}\,dx = c\int \frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2}{x^3}\,dx + \left(a^2c\right)\int \frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2}{x}\,dx$$

$$= c^2\int \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{x^3}\,dx + 2\left(\left(a^2c^2\right)\int \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{x}\,dx\right) + \left(a^4c^2\right)\int$$

$$= \frac{1}{3}a^2c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2 - \frac{1}{3}\left(2a^3c^2\right)\int \sqrt{c+a^2cx^2}\tan^{-1}(ax)\,dx + c^3\int \frac{\tan^{-}}{x^3\sqrt{c}}$$

$$= \frac{1}{3}a^2c^2\sqrt{c+a^2cx^2} - \frac{1}{3}a^3c^2x\sqrt{c+a^2cx^2}\tan^{-1}(ax) - \frac{c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2x^2} + \frac{1}{3}a^2c$$

$$= \frac{1}{3}a^2c^2\sqrt{c+a^2cx^2} - \frac{ac^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{x} - \frac{1}{3}a^3c^2x\sqrt{c+a^2cx^2}\tan^{-1}(ax) - \frac{c^2\sqrt{c}}{}$$

$$= \frac{1}{3}a^2c^2\sqrt{c+a^2cx^2} - \frac{ac^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{x} - \frac{1}{3}a^3c^2x\sqrt{c+a^2cx^2}\tan^{-1}(ax) - \frac{c^2\sqrt{c}}{}$$

$$= \frac{1}{3}a^2c^2\sqrt{c+a^2cx^2} - \frac{ac^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{x} - \frac{1}{3}a^3c^2x\sqrt{c+a^2cx^2}\tan^{-1}(ax) - \frac{c^2\sqrt{c}}{}$$

$$= \frac{1}{3}a^2c^2\sqrt{c+a^2cx^2} - \frac{ac^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{x} - \frac{1}{3}a^3c^2x\sqrt{c+a^2cx^2}\tan^{-1}(ax) - \frac{c^2\sqrt{c}}{}$$

$$= \frac{1}{3}a^2c^2\sqrt{c+a^2cx^2} - \frac{ac^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{x} - \frac{1}{3}a^3c^2x\sqrt{c+a^2cx^2}\tan^{-1}(ax) - \frac{c^2\sqrt{c}}{}$$

$$= \frac{1}{3}a^2c^2\sqrt{c+a^2cx^2} - \frac{ac^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{x} - \frac{1}{3}a^3c^2x\sqrt{c+a^2cx^2}\tan^{-1}(ax) - \frac{c^2\sqrt{c}}{}$$

Mathematica [A] time = 7.72253, size = 761, normalized size = 1.15

$$2a^2c^2\sqrt{c\left(a^2x^2+1\right)}\left(\frac{2i\tan^{-1}(ax)\left(\text{PolyLog}\left(2,-e^{i\tan^{-1}(ax)}\right)-\text{PolyLog}\left(2,e^{i\tan^{-1}(ax)}\right)\right)}{\sqrt{a^2x^2+1}} + \frac{2\left(\text{PolyLog}\left(3,e^{i\tan^{-1}(ax)}\right)-\right.}{\sqrt{a^2x^2}}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2)/x^3,x]
```

```
[Out] 2*a^2*c^2*Sqrt[c*(1 + a^2*x^2)]*(ArcTan[a*x]^2 + (ArcTan[a*x]^2*(Log[1 - E^
(I*ArcTan[a*x])] - Log[1 + E^(I*ArcTan[a*x])]))/Sqrt[1 + a^2*x^2] - (2*(Arc
Tan[a*x]*(Log[1 - I*E^(I*ArcTan[a*x])] - Log[1 + I*E^(I*ArcTan[a*x])]) + I*
(PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - PolyLog[2, I*E^(I*ArcTan[a*x])])))/Sq
rt[1 + a^2*x^2] + ((2*I)*ArcTan[a*x]*(PolyLog[2, -E^(I*ArcTan[a*x])] - Poly
Log[2, E^(I*ArcTan[a*x])]))/Sqrt[1 + a^2*x^2] + (2*(-PolyLog[3, -E^(I*ArcTa
n[a*x])] + PolyLog[3, E^(I*ArcTan[a*x])]))/Sqrt[1 + a^2*x^2]) + (a^2*c^2*(1
 + a^2*x^2)*Sqrt[c*(1 + a^2*x^2)]*(2 + 4*ArcTan[a*x]^2 + 2*Cos[2*ArcTan[a*x
]] - (3*ArcTan[a*x]*Log[1 - I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - ArcTa
n[a*x]*Cos[3*ArcTan[a*x]]*Log[1 - I*E^(I*ArcTan[a*x])] + (3*ArcTan[a*x]*Log
[1 + I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + ArcTan[a*x]*Cos[3*ArcTan[a*x
]]*Log[1 + I*E^(I*ArcTan[a*x])] - ((4*I)*PolyLog[2, (-I)*E^(I*ArcTan[a*x])]
)/(1 + a^2*x^2)^(3/2) + ((4*I)*PolyLog[2, I*E^(I*ArcTan[a*x])])/(1 + a^2*x^
2)^(3/2) - 2*ArcTan[a*x]*Sin[2*ArcTan[a*x]]))/12 + (a^2*c^2*Sqrt[c*(1 + a^2
*x^2)]*(-4*ArcTan[a*x]*Cot[ArcTan[a*x]/2] - ArcTan[a*x]^2*Csc[ArcTan[a*x]/2
]^2 + 4*ArcTan[a*x]^2*(Log[1 - E^(I*ArcTan[a*x])] - Log[1 + E^(I*ArcTan[a*x
])]) + 8*Log[Tan[ArcTan[a*x]/2]] + (8*I)*ArcTan[a*x]*(PolyLog[2, -E^(I*ArcT
an[a*x])] - PolyLog[2, E^(I*ArcTan[a*x])]) + 8*(-PolyLog[3, -E^(I*ArcTan[a*
x])] + PolyLog[3, E^(I*ArcTan[a*x])]) + ArcTan[a*x]^2*Sec[ArcTan[a*x]/2]^2
- 4*ArcTan[a*x]*Tan[ArcTan[a*x]/2]))/(8*Sqrt[1 + a^2*x^2])
```

Maple [A] time = 0.425, size = 454, normalized size = 0.7

$$\frac{c^2\left(2\,(\arctan(ax))^2x^4a^4-2\arctan(ax)\,x^3a^3+14\,(\arctan(ax))^2x^2a^2+2\,a^2x^2-6\arctan(ax)\,xa-3\,(\arctan(ax))^2\right)}{6\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(5/2)*arctan(a*x)^2/x^3,x)
```

```
[Out] 1/6*c^2*(c*(a*x-I)*(a*x+I))^(1/2)*(2*arctan(a*x)^2*x^4*a^4-2*arctan(a*x)*x^
3*a^3+14*arctan(a*x)^2*x^2*a^2+2*a^2*x^2-6*arctan(a*x)*x*a-3*arctan(a*x)^2)
/x^2-1/6*a^2*c^2*(c*(a*x-I)*(a*x+I))^(1/2)*(15*arctan(a*x)^2*ln(1+(1+I*a*x)
/(a^2*x^2+1)^(1/2))-15*arctan(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-30*I
*arctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+30*I*arctan(a*x)*polyl
og(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))-26*arctan(a*x)*ln(1+I*(1+I*a*x)/(a^2*x^2+
1)^(1/2))+26*arctan(a*x)*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+26*I*dilog(1+I
```

```
*(1+I*a*x)/(a^2*x^2+1)^(1/2))-26*I*dilog(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+6
*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))+30*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2
))-30*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*ln((1+I*a*x)/(a^2*x^2+1)^(1/
2)-1))/(a^2*x^2+1)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^2/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^4c^2x^4 + 2\,a^2c^2x^2 + c^2\right)\sqrt{a^2cx^2 + c}\,\arctan\left(ax\right)^2}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^2/x^3,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*arctan(a*x
)^2/x^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(5/2)*atan(a*x)**2/x**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^{\frac{5}{2}}\arctan\left(ax\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^2/x^3,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^2/x^3, x)
```

3.330 $\int \frac{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^2}{x^4}\,dx$

Optimal. Leaf size=675

$$\frac{13ia^3c^3\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{3\sqrt{a^2cx^2+c}}-\frac{13ia^3c^3\sqrt{a^2x^2+1}\text{PolyLog}\left(2,\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{3\sqrt{a^2cx^2+c}}+\frac{5ia^3c^3\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLo}}{\sqrt{a^2cx^2+c}}$$

```
[Out] -(a^2*c^2*Sqrt[c + a^2*c*x^2])/(3*x) - a^3*c^2*Sqrt[c + a^2*c*x^2]*ArcTan[a
*x] - (a*c^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(3*x^2) - (2*a^2*c^2*Sqrt[c +
 a^2*c*x^2]*ArcTan[a*x]^2)/x + (a^4*c^2*x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2
)/2 - (c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2)/(3*x^3) - ((5*I)*a^3*c^3*Sqrt
[1 + a^2*x^2]*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2)/Sqrt[c + a^2*c*x^2]
- (26*a^3*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTanh[Sqrt[1 + I*a*x]/Sqrt[1
- I*a*x]])/(3*Sqrt[c + a^2*c*x^2]) + a^3*c^(5/2)*ArcTanh[(a*Sqrt[c]*x)/Sqrt
[c + a^2*c*x^2]] + ((5*I)*a^3*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2,
(-I)*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - ((5*I)*a^3*c^3*Sqrt[1 + a^2*
x^2]*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + (((
13*I)/3)*a^3*c^3*Sqrt[1 + a^2*x^2]*PolyLog[2, -(Sqrt[1 + I*a*x]/Sqrt[1 - I*
a*x])])/Sqrt[c + a^2*c*x^2] - (((13*I)/3)*a^3*c^3*Sqrt[1 + a^2*x^2]*PolyLog
[2, Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2] - (5*a^3*c^3*Sqrt
[1 + a^2*x^2]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + (5*
a^3*c^3*Sqrt[1 + a^2*x^2]*PolyLog[3, I*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x
^2]
```

Rubi [A] time = 2.30888, antiderivative size = 675, normalized size of antiderivative = 1., number of steps used = 48, number of rules used = 16, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {4950, 4944, 4946, 4962, 264, 4958, 4954, 4890, 4888, 4181, 2531, 2282, 6589, 4880, 217, 206}

$$\frac{13ia^3c^3\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{3\sqrt{a^2cx^2+c}}-\frac{13ia^3c^3\sqrt{a^2x^2+1}\text{PolyLog}\left(2,\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{3\sqrt{a^2cx^2+c}}+\frac{5ia^3c^3\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLo}}{\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2)/x^4,x]
```

```
[Out] -(a^2*c^2*Sqrt[c + a^2*c*x^2])/(3*x) - a^3*c^2*Sqrt[c + a^2*c*x^2]*ArcTan[a
*x] - (a*c^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(3*x^2) - (2*a^2*c^2*Sqrt[c +
 a^2*c*x^2]*ArcTan[a*x]^2)/x + (a^4*c^2*x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2
```

```
)/2 - (c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2)/(3*x^3) - ((5*I)*a^3*c^3*Sqrt
[1 + a^2*x^2]*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2)/Sqrt[c + a^2*c*x^2]
- (26*a^3*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTanh[Sqrt[1 + I*a*x]/Sqrt[1
- I*a*x]])/(3*Sqrt[c + a^2*c*x^2]) + a^3*c^(5/2)*ArcTanh[(a*Sqrt[c]*x)/Sqrt
[c + a^2*c*x^2]] + ((5*I)*a^3*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2,
(-I)*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - ((5*I)*a^3*c^3*Sqrt[1 + a^2*
x^2]*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + (((
13*I)/3)*a^3*c^3*Sqrt[1 + a^2*x^2]*PolyLog[2, -(Sqrt[1 + I*a*x]/Sqrt[1 - I*
a*x])])/Sqrt[c + a^2*c*x^2] - (((13*I)/3)*a^3*c^3*Sqrt[1 + a^2*x^2]*PolyLog
[2, Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2] - (5*a^3*c^3*Sqrt
[1 + a^2*x^2]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + (5*
a^3*c^3*Sqrt[1 + a^2*x^2]*PolyLog[3, I*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x
^2]
```

Rule 4950

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 4944

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a +
 b*ArcTan[c*x])^p)/(d*f*(m + 1)), x] - Dist[(b*c*p)/(f*(m + 1)), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*ArcTan[c*x
]))/(f*(m + 2)), x] + (Dist[d/(m + 2), Int[((f*x)^m*(a + b*ArcTan[c*x]))/Sq
rt[d + e*x^2], x], x] - Dist[(b*c*d)/(f*(m + 2)), Int[(f*x)^(m + 1)/Sqrt[d
+ e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && Ne
Q[m, -2]
```

Rule 4962

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Ar
```

```
cTan[c*x])^p)/(d*f*(m + 1)), x] + (-Dist[(b*c*p)/(f*(m + 1)), Int[((f*x)^(m
 + 1)*(a + b*ArcTan[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] - Dist[(c^2*(m +
 2))/(f^2*(m + 1)), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/Sqrt[d + e*x^
2], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] &
& LtQ[m, -1] && NeQ[m, -2]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 4958

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4954

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x_
Symbol] :> Simp[(-2*(a + b*ArcTan[c*x])*ArcTanh[Sqrt[1 + I*c*x]/Sqrt[1 - I*
c*x]])/Sqrt[d], x] + (Simp[(I*b*PolyLog[2, -(Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x
])])/Sqrt[d], x] - Simp[(I*b*PolyLog[2, Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]])/S
qrt[d], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

Rule 4890

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4888

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[c
*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && GtQ
[d, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
```

```
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4880

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> -Simp[(b*p*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1))/(2*c*q*(2*
q + 1)), x] + (Dist[(2*d*q)/(2*q + 1), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTa
n[c*x])^p, x], x] + Dist[(b^2*d*p*(p - 1))/(2*q*(2*q + 1)), Int[(d + e*x^2)
^(q - 1)*(a + b*ArcTan[c*x])^(p - 2), x], x] + Simp[(x*(d + e*x^2)^q*(a + b
*ArcTan[c*x])^p)/(2*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[q, 0] && GtQ[p, 1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^2}{x^4}\,dx = c\int \frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2}{x^4}\,dx + \left(a^2c\right)\int \frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2}{x^2}\,dx$$

$$= c^2\int \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{x^4}\,dx + 2\left(\left(a^2c^2\right)\int \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{x^2}\,dx\right) + \left(a^4c^2\right)\int$$

$$= -a^3c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax) + \frac{1}{2}a^4c^2x\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2 - \frac{c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(}{3x^3}$$

$$= -a^3c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax) - \frac{2ac^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{3x^2} + \frac{1}{2}a^4c^2x\sqrt{c+a^2cx^2}\tan^{-1}($$

$$= -\frac{2a^2c^2\sqrt{c+a^2cx^2}}{3x} - a^3c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax) - \frac{ac^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{3x^2} + \frac{1}{2}a^4c^2x$$

$$= -\frac{a^2c^2\sqrt{c+a^2cx^2}}{3x} - a^3c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax) - \frac{ac^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{3x^2} + \frac{1}{2}a^4c^2x$$

$$= -\frac{a^2c^2\sqrt{c+a^2cx^2}}{3x} - a^3c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax) - \frac{ac^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{3x^2} + \frac{1}{2}a^4c^2x$$

$$= -\frac{a^2c^2\sqrt{c+a^2cx^2}}{3x} - a^3c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax) - \frac{ac^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{3x^2} + \frac{1}{2}a^4c^2x$$

$$= -\frac{a^2c^2\sqrt{c+a^2cx^2}}{3x} - a^3c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax) - \frac{ac^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{3x^2} + \frac{1}{2}a^4c^2x$$

Mathematica [A] time = 4.56544, size = 644, normalized size = 0.95

$$-\frac{c^3\sqrt{a^2x^2+1}\left(-52ia^3x^3\text{PolyLog}\left(2,-e^{i\tan^{-1}(ax)}\right) - 60ia^3x^3\tan^{-1}(ax)\text{PolyLog}\left(2,-ie^{i\tan^{-1}(ax)}\right) + 60ia^3x^3\tan^{-1}(ax)\text{Po}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2)/x^4,x]
```

```
[Out] -(c^3*Sqrt[1 + a^2*x^2]*(2*(1 + a^2*x^2)^(3/2) + 12*a^3*x^3*Sqrt[1 + a^2*x^
2]*ArcTan[a*x] + 24*a^2*x^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2 - 6*a^4*x^4*Sqr
t[1 + a^2*x^2]*ArcTan[a*x]^2 + 4*(1 + a^2*x^2)^(3/2)*ArcTan[a*x]^2 + (12*I)
*a^3*x^3*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2 - 12*a^3*x^3*ArcTanh[(a*x)
/Sqrt[1 + a^2*x^2]] - 2*(1 + a^2*x^2)^(3/2)*Cos[2*ArcTan[a*x]] - 3*a*x*ArcT
an[a*x]*Log[1 - E^(I*ArcTan[a*x])] - 51*a^3*x^3*ArcTan[a*x]*Log[1 - E^(I*Ar
cTan[a*x])] - 24*a^3*x^3*ArcTan[a*x]^2*Log[1 - I*E^(I*ArcTan[a*x])] + 24*a^
3*x^3*ArcTan[a*x]^2*Log[1 + I*E^(I*ArcTan[a*x])] + 3*a*x*ArcTan[a*x]*Log[1
+ E^(I*ArcTan[a*x])] + 51*a^3*x^3*ArcTan[a*x]*Log[1 + E^(I*ArcTan[a*x])] -
(52*I)*a^3*x^3*PolyLog[2, -E^(I*ArcTan[a*x])] - (60*I)*a^3*x^3*ArcTan[a*x]*
PolyLog[2, (-I)*E^(I*ArcTan[a*x])] + (60*I)*a^3*x^3*ArcTan[a*x]*PolyLog[2,
I*E^(I*ArcTan[a*x])] + (52*I)*a^3*x^3*PolyLog[2, E^(I*ArcTan[a*x])] + 60*a^
3*x^3*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] - 60*a^3*x^3*PolyLog[3, I*E^(I*Arc
Tan[a*x])] + 2*(1 + a^2*x^2)^(3/2)*ArcTan[a*x]*Sin[2*ArcTan[a*x]] + (1 + a^
2*x^2)^(3/2)*ArcTan[a*x]*Log[1 - E^(I*ArcTan[a*x])]*Sin[3*ArcTan[a*x]] - (1
 + a^2*x^2)^(3/2)*ArcTan[a*x]*Log[1 + E^(I*ArcTan[a*x])]*Sin[3*ArcTan[a*x]]
))/(12*x^3*Sqrt[c + a^2*c*x^2])
```

Maple [A] time = 0.556, size = 401, normalized size = 0.6

$$\frac{c^2\left(3\,(\arctan(ax))^2x^4a^4-6\arctan(ax)\,x^3a^3-14\,(\arctan(ax))^2x^2a^2-2\,a^2x^2-2\arctan(ax)\,xa-2\,(\arctan(ax))^2\right)}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(5/2)*arctan(a*x)^2/x^4,x)
```

```
[Out] 1/6*c^2*(c*(a*x-I)*(a*x+I))^(1/2)*(3*arctan(a*x)^2*x^4*a^4-6*arctan(a*x)*x^
3*a^3-14*arctan(a*x)^2*x^2*a^2-2*a^2*x^2-2*arctan(a*x)*x*a-2*arctan(a*x)^2)
/x^3+1/6*I*a^3*c^2*(c*(a*x-I)*(a*x+I))^(1/2)*(15*I*arctan(a*x)^2*ln(1+I*(1+
I*a*x)/(a^2*x^2+1)^(1/2))-15*I*arctan(a*x)^2*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(
1/2))+26*I*arctan(a*x)*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))+30*arctan(a*x)*pol
ylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-30*arctan(a*x)*polylog(2,I*(1+I*a*x)
/(a^2*x^2+1)^(1/2))+30*I*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-30*I*pol
ylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+26*dilog((1+I*a*x)/(a^2*x^2+1)^(1/2))
+26*dilog(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-12*arctan((1+I*a*x)/(a^2*x^2+1)^(1
/2)))/(a^2*x^2+1)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^2/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^4c^2x^4+2\,a^2c^2x^2+c^2\right)\sqrt{a^2cx^2+c}\,\arctan\left(ax\right)^2}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^2/x^4,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*arctan(a*x
)^2/x^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(5/2)*atan(a*x)**2/x**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a^2cx^2+c\right)^{\frac{5}{2}}\arctan\left(ax\right)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^2/x^4,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^2/x^4, x)
```

3.331 $\int \frac{x^3\tan^{-1}(ax)^2}{\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=315

$$\frac{5i\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{3a^4\sqrt{a^2cx^2+c}}-\frac{5i\sqrt{a^2x^2+1}\text{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{3a^4\sqrt{a^2cx^2+c}}+\frac{\sqrt{a^2cx^2+c}}{3a^4c}+\frac{x^2\sqrt{a^2cx^2+c}\tan^{-1}(ax)^2}{3a^2c}-2\sqrt{\ldots}$$

```
[Out] Sqrt[c + a^2*c*x^2]/(3*a^4*c) - (x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(3*a^3*
c) - (2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(3*a^4*c) + (x^2*Sqrt[c + a^2*c*
x^2]*ArcTan[a*x]^2)/(3*a^2*c) - (((10*I)/3)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*A
rcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/(a^4*Sqrt[c + a^2*c*x^2]) + (((5*I)
/3)*Sqrt[1 + a^2*x^2]*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/(
a^4*Sqrt[c + a^2*c*x^2]) - (((5*I)/3)*Sqrt[1 + a^2*x^2]*PolyLog[2, (I*Sqrt[
1 + I*a*x])/Sqrt[1 - I*a*x]])/(a^4*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.425524, antiderivative size = 315, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {4952, 261, 4890, 4886, 4930}

$$\frac{5i\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{3a^4\sqrt{a^2cx^2+c}}-\frac{5i\sqrt{a^2x^2+1}\text{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{3a^4\sqrt{a^2cx^2+c}}+\frac{\sqrt{a^2cx^2+c}}{3a^4c}+\frac{x^2\sqrt{a^2cx^2+c}\tan^{-1}(ax)^2}{3a^2c}-2\sqrt{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcTan[a*x]^2)/Sqrt[c + a^2*c*x^2],x]
```


```
[Out] Sqrt[c + a^2*c*x^2]/(3*a^4*c) - (x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(3*a^3*
c) - (2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(3*a^4*c) + (x^2*Sqrt[c + a^2*c*
x^2]*ArcTan[a*x]^2)/(3*a^2*c) - (((10*I)/3)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*A
rcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/(a^4*Sqrt[c + a^2*c*x^2]) + (((5*I)
/3)*Sqrt[1 + a^2*x^2]*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/(
a^4*Sqrt[c + a^2*c*x^2]) - (((5*I)/3)*Sqrt[1 + a^2*x^2]*PolyLog[2, (I*Sqrt[
1 + I*a*x])/Sqrt[1 - I*a*x]])/(a^4*Sqrt[c + a^2*c*x^2])
```


Rule 4952

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b*
ArcTan[c*x])^p)/(c^2*d*m), x] + (-Dist[(b*f*p)/(c*m), Int[((f*x)^(m - 1)*(a
```

```
 + b*ArcTan[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] - Dist[(f^2*(m - 1))/(c^
2*m), Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/Sqrt[d + e*x^2], x], x]) /;
 FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 4890

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4886

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[(-2*I*(a + b*ArcTan[c*x])*ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]])
/(c*Sqrt[d]), x] + (Simp[(I*b*PolyLog[2, -((I*Sqrt[1 + I*c*x])/Sqrt[1 - I*c
*x])])/(c*Sqrt[d]), x] - Simp[(I*b*PolyLog[2, (I*Sqrt[1 + I*c*x])/Sqrt[1 -
I*c*x]])/(c*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rubi steps

$$\int \frac{x^3 \tan^{-1}(ax)^2}{\sqrt{c+a^2cx^2}}\,dx = \frac{x^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{3a^2c} - \frac{2\int \frac{x\tan^{-1}(ax)^2}{\sqrt{c+a^2cx^2}}\,dx}{3a^2} - \frac{2\int \frac{x^2\tan^{-1}(ax)}{\sqrt{c+a^2cx^2}}\,dx}{3a}$$

$$= -\frac{x\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{3a^3c} - \frac{2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{3a^4c} + \frac{x^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{3a^2c} + \frac{\int \frac{\tan^{-1}(ax)}{\sqrt{c+a^2cx^2}}\,dx}{3a^3}$$

$$= \frac{\sqrt{c+a^2cx^2}}{3a^4c} - \frac{x\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{3a^3c} - \frac{2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{3a^4c} + \frac{x^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{3a^2c} + \cdots$$

$$= \frac{\sqrt{c+a^2cx^2}}{3a^4c} - \frac{x\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{3a^3c} - \frac{2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{3a^4c} + \frac{x^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{3a^2c} - \cdots$$

Mathematica [A] time = 0.675842, size = 279, normalized size = 0.89

$$\frac{\left(a^2x^2+1\right)\sqrt{c\left(a^2x^2+1\right)}\left(\frac{20i\text{PolyLog}\left(2,-ie^{i\tan^{-1}(ax)}\right)}{\left(a^2x^2+1\right)^{3/2}} - \frac{20i\text{PolyLog}\left(2,ie^{i\tan^{-1}(ax)}\right)}{\left(a^2x^2+1\right)^{3/2}} + \frac{15\tan^{-1}(ax)\log\left(1-ie^{i\tan^{-1}(ax)}\right)}{\sqrt{a^2x^2+1}} - \frac{15\tan^{-1}(ax)\log\left(1+\cdots\right.}{\sqrt{a^2x^2+\cdots}}\cdots\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*ArcTan[a*x]^2)/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] ((1 + a^2*x^2)*Sqrt[c*(1 + a^2*x^2)]*(2 - 2*ArcTan[a*x]^2 + 2*Cos[2*ArcTan[
a*x]] - 6*ArcTan[a*x]^2*Cos[2*ArcTan[a*x]] + (15*ArcTan[a*x]*Log[1 - I*E^(I
*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + 5*ArcTan[a*x]*Cos[3*ArcTan[a*x]]*Log[1
- I*E^(I*ArcTan[a*x])] - (15*ArcTan[a*x]*Log[1 + I*E^(I*ArcTan[a*x])])/Sqrt
[1 + a^2*x^2] - 5*ArcTan[a*x]*Cos[3*ArcTan[a*x]]*Log[1 + I*E^(I*ArcTan[a*x]
)] + ((20*I)*PolyLog[2, (-I)*E^(I*ArcTan[a*x])])/(1 + a^2*x^2)^(3/2) - ((20
*I)*PolyLog[2, I*E^(I*ArcTan[a*x])])/(1 + a^2*x^2)^(3/2) - 2*ArcTan[a*x]*Si
n[2*ArcTan[a*x]]))/(12*a^4*c)
```

Maple [A] time = 1.143, size = 206, normalized size = 0.7

$$\frac{(\arctan(ax))^2x^2a^2 - \arctan(ax)xa - 2(\arctan(ax))^2 + 1}{3ca^4}\sqrt{c(ax-i)(ax+i)} + \frac{\frac{5i}{3}}{ca^4}\left(i\arctan(ax)\ln\left(1+i(1+iax)\cdots\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] 1/3*(arctan(a*x)^2*x^2*a^2-arctan(a*x)*x*a-2*arctan(a*x)^2+1)*(c*(a*x-I)*(a
*x+I))^(1/2)/c/a^4+5/3*I*(I*arctan(a*x)*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))
-I*arctan(a*x)*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+dilog(1+I*(1+I*a*x)/(a^2
*x^2+1)^(1/2))-dilog(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))*(c*(a*x-I)*(a*x+I))^
(1/2)/(a^2*x^2+1)^(1/2)/a^4/c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^3\arctan\left(ax\right)^2}{\sqrt{a^2cx^2+c}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^3*arctan(a*x)^2/sqrt(a^2*c*x^2 + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^3\operatorname{atan}^2\left(ax\right)}{\sqrt{c\left(a^2x^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atan(a*x)**2/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**3*atan(a*x)**2/sqrt(c*(a**2*x**2 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \arctan{(ax)}^2}{\sqrt{a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^3*arctan(a*x)^2/sqrt(a^2*c*x^2 + c), x)
```

3.332 $\int \frac{x^2 \tan^{-1}(ax)^2}{\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=344

$$-\frac{i\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,-ie^{i\tan^{-1}(ax)}\right)}{a^3\sqrt{a^2cx^2+c}}+\frac{i\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,ie^{i\tan^{-1}(ax)}\right)}{a^3\sqrt{a^2cx^2+c}}+\frac{\sqrt{a^2x^2+1}\text{PolyLog}}{a^3\sqrt{a^2cx}}$$

```
[Out] -((Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(a^3*c)) + (x*Sqrt[c + a^2*c*x^2]*ArcTa
n[a*x]^2)/(2*a^2*c) + (I*Sqrt[1 + a^2*x^2]*ArcTan[E^(I*ArcTan[a*x])]*ArcTan
[a*x]^2)/(a^3*Sqrt[c + a^2*c*x^2]) + ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x
^2]]/(a^3*Sqrt[c]) - (I*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*
ArcTan[a*x])])/(a^3*Sqrt[c + a^2*c*x^2]) + (I*Sqrt[1 + a^2*x^2]*ArcTan[a*x]
*PolyLog[2, I*E^(I*ArcTan[a*x])])/(a^3*Sqrt[c + a^2*c*x^2]) + (Sqrt[1 + a^2
*x^2]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])])/(a^3*Sqrt[c + a^2*c*x^2]) - (Sqrt
[1 + a^2*x^2]*PolyLog[3, I*E^(I*ArcTan[a*x])])/(a^3*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.333879, antiderivative size = 344, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {4952, 4930, 217, 206, 4890, 4888, 4181, 2531, 2282, 6589}

$$-\frac{i\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,-ie^{i\tan^{-1}(ax)}\right)}{a^3\sqrt{a^2cx^2+c}}+\frac{i\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,ie^{i\tan^{-1}(ax)}\right)}{a^3\sqrt{a^2cx^2+c}}+\frac{\sqrt{a^2x^2+1}\text{PolyLog}}{a^3\sqrt{a^2cx}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcTan[a*x]^2)/Sqrt[c + a^2*c*x^2],x]
```


```
[Out] -((Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(a^3*c)) + (x*Sqrt[c + a^2*c*x^2]*ArcTa
n[a*x]^2)/(2*a^2*c) + (I*Sqrt[1 + a^2*x^2]*ArcTan[E^(I*ArcTan[a*x])]*ArcTan
[a*x]^2)/(a^3*Sqrt[c + a^2*c*x^2]) + ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x
^2]]/(a^3*Sqrt[c]) - (I*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*
ArcTan[a*x])])/(a^3*Sqrt[c + a^2*c*x^2]) + (I*Sqrt[1 + a^2*x^2]*ArcTan[a*x]
*PolyLog[2, I*E^(I*ArcTan[a*x])])/(a^3*Sqrt[c + a^2*c*x^2]) + (Sqrt[1 + a^2
*x^2]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])])/(a^3*Sqrt[c + a^2*c*x^2]) - (Sqrt
[1 + a^2*x^2]*PolyLog[3, I*E^(I*ArcTan[a*x])])/(a^3*Sqrt[c + a^2*c*x^2])
```


Rule 4952

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b*
```

```
ArcTan[c*x])^p)/(c^2*d*m), x] + (-Dist[(b*f*p)/(c*m), Int[((f*x)^(m - 1)*(a
 + b*ArcTan[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] - Dist[(f^2*(m - 1))/(c^
2*m), Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/Sqrt[d + e*x^2], x], x]) /;
 FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 4890

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4888

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[c
*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && GtQ
[d, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
```

```
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^2 \tan^{-1}(ax)^2}{\sqrt{c+a^2cx^2}}\,dx = \frac{x\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2a^2c} - \frac{\int \frac{\tan^{-1}(ax)^2}{\sqrt{c+a^2cx^2}}\,dx}{2a^2} - \frac{\int \frac{x\tan^{-1}(ax)}{\sqrt{c+a^2cx^2}}\,dx}{a}$$

$$= -\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{a^3c} + \frac{x\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2a^2c} + \frac{\int \frac{1}{\sqrt{c+a^2cx^2}}\,dx}{a^2} - \frac{\sqrt{1+a^2x^2}\int \frac{\tan^{-1}(ax)^2}{\sqrt{1+a^2x^2}}\,dx}{2a^2\sqrt{c+a^2cx^2}}$$

$$= -\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{a^3c} + \frac{x\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2a^2c} + \frac{\text{Subst}\left(\int \frac{1}{1-a^2cx^2}\,dx, x, \frac{x}{\sqrt{c+a^2cx^2}}\right)}{a^2} - \sqrt{1+a}$$

$$= -\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{a^3c} + \frac{x\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2a^2c} + \frac{i\sqrt{1+a^2x^2}\tan^{-1}\left(e^{i\tan^{-1}(ax)}\right)\tan^{-1}(ax)^2}{a^3\sqrt{c+a^2cx^2}} +$$

$$= -\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{a^3c} + \frac{x\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2a^2c} + \frac{i\sqrt{1+a^2x^2}\tan^{-1}\left(e^{i\tan^{-1}(ax)}\right)\tan^{-1}(ax)^2}{a^3\sqrt{c+a^2cx^2}} +$$

$$= -\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{a^3c} + \frac{x\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2a^2c} + \frac{i\sqrt{1+a^2x^2}\tan^{-1}\left(e^{i\tan^{-1}(ax)}\right)\tan^{-1}(ax)^2}{a^3\sqrt{c+a^2cx^2}} +$$

$$= -\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{a^3c} + \frac{x\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2a^2c} + \frac{i\sqrt{1+a^2x^2}\tan^{-1}\left(e^{i\tan^{-1}(ax)}\right)\tan^{-1}(ax)^2}{a^3\sqrt{c+a^2cx^2}} +$$

Mathematica [A] time = 0.343681, size = 175, normalized size = 0.51

$$\frac{\sqrt{a^2cx^2+c}\left(\tan^{-1}(ax)\left(ax\tan^{-1}(ax)-2\right) + \frac{2\left(-i\tan^{-1}(ax)\text{PolyLog}\left(2,-ie^{i\tan^{-1}(ax)}\right)+i\tan^{-1}(ax)\text{PolyLog}\left(2,ie^{i\tan^{-1}(ax)}\right)+\text{PolyLog}\left(3,-ie^{i\tan}\right.\right.}{\sqrt{a^2x^2+1}}\right.}{2a^3c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*ArcTan[a*x]^2)/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] (Sqrt[c + a^2*c*x^2]*(ArcTan[a*x]*(-2 + a*x*ArcTan[a*x]) + (2*(I*ArcTan[E^(
I*ArcTan[a*x])]*ArcTan[a*x]^2 + ArcTanh[(a*x)/Sqrt[1 + a^2*x^2]] - I*ArcTan
[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] + I*ArcTan[a*x]*PolyLog[2, I*E^(I*
ArcTan[a*x])] + PolyLog[3, (-I)*E^(I*ArcTan[a*x])] - PolyLog[3, I*E^(I*ArcT
an[a*x])]))/Sqrt[1 + a^2*x^2]))/(2*a^3*c)
```

Maple [A] time = 0.898, size = 271, normalized size = 0.8

$$\frac{(\arctan (ax)\,xa-2)\arctan (ax)}{2\,ca^3}\sqrt{c\,(ax-i)\,(ax+i)}+\frac{1}{2\,ca^3}\left((\arctan (ax))^2\ln \left(1+i\,(1+iax)\,\frac{1}{\sqrt{a^2x^2+1}}\right)-(\arctan (ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] 1/2*(arctan(a*x)*x*a-2)*arctan(a*x)*(c*(a*x-I)*(a*x+I))^(1/2)/c/a^3+1/2*(ar
ctan(a*x)^2*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-arctan(a*x)^2*ln(1-I*(1+I*a
*x)/(a^2*x^2+1)^(1/2))-2*I*arctan(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(
1/2))+2*I*arctan(a*x)*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-4*I*arctan((
1+I*a*x)/(a^2*x^2+1)^(1/2))+2*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*p
olylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2
+1)^(1/2)/a^3/c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^2\arctan (ax)^2}{\sqrt{a^2cx^2+c}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^2*arctan(a*x)^2/sqrt(a^2*c*x^2 + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \operatorname{atan}^2(ax)}{\sqrt{c(a^2x^2+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(a*x)**2/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**2*atan(a*x)**2/sqrt(c*(a**2*x**2 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \arctan(ax)^2}{\sqrt{a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctan(a*x)^2/sqrt(a^2*c*x^2 + c), x)
```

3.333 $\int \frac{x\tan^{-1}(ax)^2}{\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=220

$$-\frac{2i\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a^2\sqrt{a^2cx^2+c}}+\frac{2i\sqrt{a^2x^2+1}\text{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{a^2cx^2+c}\tan^{-1}(ax)^2}{a^2c}+\frac{4i\sqrt{a^2x^2+1}\tan^{-1}}{a^2\sqrt{a^2}}$$

```
[Out] (Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(a^2*c) + ((4*I)*Sqrt[1 + a^2*x^2]*ArcT
an[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/(a^2*Sqrt[c + a^2*c*x^2])
- ((2*I)*Sqrt[1 + a^2*x^2]*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x
]])/(a^2*Sqrt[c + a^2*c*x^2]) + ((2*I)*Sqrt[1 + a^2*x^2]*PolyLog[2, (I*Sqrt
[1 + I*a*x])/Sqrt[1 - I*a*x]])/(a^2*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.143122, antiderivative size = 220, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {4930, 4890, 4886}

$$-\frac{2i\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a^2\sqrt{a^2cx^2+c}}+\frac{2i\sqrt{a^2x^2+1}\text{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{a^2cx^2+c}\tan^{-1}(ax)^2}{a^2c}+\frac{4i\sqrt{a^2x^2+1}\tan^{-1}}{a^2\sqrt{a^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTan[a*x]^2)/Sqrt[c + a^2*c*x^2],x]
```



```
[Out] (Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(a^2*c) + ((4*I)*Sqrt[1 + a^2*x^2]*ArcT
an[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/(a^2*Sqrt[c + a^2*c*x^2])
- ((2*I)*Sqrt[1 + a^2*x^2]*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x
]])/(a^2*Sqrt[c + a^2*c*x^2]) + ((2*I)*Sqrt[1 + a^2*x^2]*PolyLog[2, (I*Sqrt
[1 + I*a*x])/Sqrt[1 - I*a*x]])/(a^2*Sqrt[c + a^2*c*x^2])
```


Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 4890

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4886

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[(-2*I*(a + b*ArcTan[c*x])*ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]])
/(c*Sqrt[d]), x] + (Simp[(I*b*PolyLog[2, -((I*Sqrt[1 + I*c*x])/Sqrt[1 - I*c
*x])])/(c*Sqrt[d]), x] - Simp[(I*b*PolyLog[2, (I*Sqrt[1 + I*c*x])/Sqrt[1 -
I*c*x]])/(c*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

Rubi steps

$$\int \frac{x\tan^{-1}(ax)^2}{\sqrt{c+a^2cx^2}}\,dx = \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{a^2c} - \frac{2\int \frac{\tan^{-1}(ax)}{\sqrt{c+a^2cx^2}}\,dx}{a}$$
$$= \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{a^2c} - \frac{\left(2\sqrt{1+a^2x^2}\right)\int \frac{\tan^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx}{a\sqrt{c+a^2cx^2}}$$
$$= \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{a^2c} + \frac{4i\sqrt{1+a^2x^2}\tan^{-1}(ax)\tan^{-1}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a^2\sqrt{c+a^2cx^2}} - \frac{2i\sqrt{1+a^2x^2}\,\mathrm{Li}_2\left(-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a^2\sqrt{c+a^2cx^2}} + $$

Mathematica [A] time = 0.23889, size = 126, normalized size = 0.57

$$\frac{\sqrt{c\left(a^2x^2+1\right)}\left(\tan^{-1}(ax)^2 - \frac{2\left(i\left(\mathrm{PolyLog}\left(2,-ie^{i\tan^{-1}(ax)}\right)-\mathrm{PolyLog}\left(2,ie^{i\tan^{-1}(ax)}\right)\right)+\tan^{-1}(ax)\left(\log\left(1-ie^{i\tan^{-1}(ax)}\right)-\log\left(1+ie^{i\tan^{-1}(ax)}\right)\right)\right)}{\sqrt{a^2x^2+1}}\right)}{a^2c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*ArcTan[a*x]^2)/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] (Sqrt[c*(1 + a^2*x^2)]*(ArcTan[a*x]^2 - (2*(ArcTan[a*x]*(Log[1 - I*E^(I*Arc
Tan[a*x])] - Log[1 + I*E^(I*ArcTan[a*x])]) + I*(PolyLog[2, (-I)*E^(I*ArcTan
[a*x])] - PolyLog[2, I*E^(I*ArcTan[a*x])])))/Sqrt[1 + a^2*x^2]))/(a^2*c)
```

Maple [A] time = 0.43, size = 180, normalized size = 0.8

$$\frac{(\arctan(ax))^2}{a^2c}\sqrt{c(ax-i)(ax+i)}-\frac{2i}{a^2c}\left(i\arctan(ax)\ln\left(1+i(1+iax)\frac{1}{\sqrt{a^2x^2+1}}\right)-i\arctan(ax)\ln\left(1-i(1+iax)\frac{}{\sqrt{a}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] arctan(a*x)^2*(c*(a*x-I)*(a*x+I))^(1/2)/a^2/c-2*I*(I*arctan(a*x)*ln(1+I*(1+
I*a*x)/(a^2*x^2+1)^(1/2))-I*arctan(a*x)*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))
+dilog(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-dilog(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/
2)))*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)^(1/2)/a^2/c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x*arctan(a*x)^2/sqrt(a^2*c*x^2 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x*arctan(a*x)^2/sqrt(a^2*c*x^2 + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \operatorname{atan}^2(ax)}{\sqrt{c(a^2x^2+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(a*x)**2/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x*atan(a*x)**2/sqrt(c*(a**2*x**2 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \arctan(ax)^2}{\sqrt{a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x*arctan(a*x)^2/sqrt(a^2*c*x^2 + c), x)
```

3.334 $\int \frac{\tan^{-1}(ax)^2}{\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=256

$$\frac{2i\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,-ie^{i\tan^{-1}(ax)}\right)}{a\sqrt{a^2cx^2+c}}-\frac{2i\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,ie^{i\tan^{-1}(ax)}\right)}{a\sqrt{a^2cx^2+c}}-\frac{2\sqrt{a^2x^2+1}\text{PolyLo}}{a\sqrt{a^2c}}$$

```
[Out] ((-2*I)*Sqrt[1 + a^2*x^2]*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2)/(a*Sqrt[
c + a^2*c*x^2]) + ((2*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, (-I)*E^(I
*ArcTan[a*x])])/(a*Sqrt[c + a^2*c*x^2]) - ((2*I)*Sqrt[1 + a^2*x^2]*ArcTan[a
*x]*PolyLog[2, I*E^(I*ArcTan[a*x])])/(a*Sqrt[c + a^2*c*x^2]) - (2*Sqrt[1 +
a^2*x^2]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])])/(a*Sqrt[c + a^2*c*x^2]) + (2*S
qrt[1 + a^2*x^2]*PolyLog[3, I*E^(I*ArcTan[a*x])])/(a*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.153852, antiderivative size = 256, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4890, 4888, 4181, 2531, 2282, 6589}

$$\frac{2i\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,-ie^{i\tan^{-1}(ax)}\right)}{a\sqrt{a^2cx^2+c}}-\frac{2i\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,ie^{i\tan^{-1}(ax)}\right)}{a\sqrt{a^2cx^2+c}}-\frac{2\sqrt{a^2x^2+1}\text{PolyLo}}{a\sqrt{a^2c}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^2/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] ((-2*I)*Sqrt[1 + a^2*x^2]*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2)/(a*Sqrt[
c + a^2*c*x^2]) + ((2*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, (-I)*E^(I
*ArcTan[a*x])])/(a*Sqrt[c + a^2*c*x^2]) - ((2*I)*Sqrt[1 + a^2*x^2]*ArcTan[a
*x]*PolyLog[2, I*E^(I*ArcTan[a*x])])/(a*Sqrt[c + a^2*c*x^2]) - (2*Sqrt[1 +
a^2*x^2]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])])/(a*Sqrt[c + a^2*c*x^2]) + (2*S
qrt[1 + a^2*x^2]*PolyLog[3, I*E^(I*ArcTan[a*x])])/(a*Sqrt[c + a^2*c*x^2])
```

Rule 4890

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4888

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[c
*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && GtQ
[d, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^2}{\sqrt{c+a^2cx^2}}\,dx = \frac{\sqrt{1+a^2x^2}\int \frac{\tan^{-1}(ax)^2}{\sqrt{1+a^2x^2}}\,dx}{\sqrt{c+a^2cx^2}}$$
$$= \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int x^2\sec(x)\,dx, x, \tan^{-1}(ax)\right)}{a\sqrt{c+a^2cx^2}}$$
$$= -\frac{2i\sqrt{1+a^2x^2}\tan^{-1}\left(e^{i\tan^{-1}(ax)}\right)\tan^{-1}(ax)^2}{a\sqrt{c+a^2cx^2}} - \frac{\left(2\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int x\log\left(1-ie^{ix}\right)\,dx, x, \tan^{-1}(a$$
$$= -\frac{2i\sqrt{1+a^2x^2}\tan^{-1}\left(e^{i\tan^{-1}(ax)}\right)\tan^{-1}(ax)^2}{a\sqrt{c+a^2cx^2}} + \frac{2i\sqrt{1+a^2x^2}\tan^{-1}(ax)\text{Li}_2\left(-ie^{i\tan^{-1}(ax)}\right)}{a\sqrt{c+a^2cx^2}} - \frac{2i\sqrt{1+a^2}}{}$$
$$= -\frac{2i\sqrt{1+a^2x^2}\tan^{-1}\left(e^{i\tan^{-1}(ax)}\right)\tan^{-1}(ax)^2}{a\sqrt{c+a^2cx^2}} + \frac{2i\sqrt{1+a^2x^2}\tan^{-1}(ax)\text{Li}_2\left(-ie^{i\tan^{-1}(ax)}\right)}{a\sqrt{c+a^2cx^2}} - \frac{2i\sqrt{1+a^2}}{}$$
$$= -\frac{2i\sqrt{1+a^2x^2}\tan^{-1}\left(e^{i\tan^{-1}(ax)}\right)\tan^{-1}(ax)^2}{a\sqrt{c+a^2cx^2}} + \frac{2i\sqrt{1+a^2x^2}\tan^{-1}(ax)\text{Li}_2\left(-ie^{i\tan^{-1}(ax)}\right)}{a\sqrt{c+a^2cx^2}} - \frac{2i\sqrt{1+a^2}}{}$$

Mathematica [A] time = 0.115355, size = 140, normalized size = 0.55

$$\frac{2\sqrt{c\left(a^2x^2+1\right)}\left(i\tan^{-1}(ax)\text{PolyLog}\left(2,-ie^{i\tan^{-1}(ax)}\right)-i\tan^{-1}(ax)\text{PolyLog}\left(2,ie^{i\tan^{-1}(ax)}\right)-\text{PolyLog}\left(3,-ie^{i\tan^{-1}(ax)}\right)}{ac\sqrt{a^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]^2/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] (2*Sqrt[c*(1 + a^2*x^2)]*((-I)*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2 + I*
ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - I*ArcTan[a*x]*PolyLog[2, I
*E^(I*ArcTan[a*x])] - PolyLog[3, (-I)*E^(I*ArcTan[a*x])] + PolyLog[3, I*E^(
I*ArcTan[a*x])]))/(a*c*Sqrt[1 + a^2*x^2])
```

Maple [F] time = 0.45, size = 0, normalized size = 0.

$$\int (\arctan(ax))^2 \frac{1}{\sqrt{a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\arctan\left(ax\right)^{2}}{\sqrt{a^{2}cx^{2}+c}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(arctan(a*x)^2/sqrt(a^2*c*x^2 + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atan}^{2}\left(ax\right)}{\sqrt{c\left(a^{2}x^{2}+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**2/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(atan(a*x)**2/sqrt(c*(a**2*x**2 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)^2/sqrt(a^2*c*x^2 + c), x)
```

3.335 $\int \frac{\tan^{-1}(ax)^2}{x\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=227

$$\frac{2i\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,-e^{i\tan^{-1}(ax)}\right)}{\sqrt{a^2cx^2+c}}-\frac{2i\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,e^{i\tan^{-1}(ax)}\right)}{\sqrt{a^2cx^2+c}}-\frac{2\sqrt{a^2x^2+1}\text{PolyLo}}{\sqrt{a^2cx}}$$

```
[Out] (-2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*ArcTanh[E^(I*ArcTan[a*x])])/Sqrt[c + a^
2*c*x^2] + ((2*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x
])])/Sqrt[c + a^2*c*x^2] - ((2*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2,
E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - (2*Sqrt[1 + a^2*x^2]*PolyLog[3, -
E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + (2*Sqrt[1 + a^2*x^2]*PolyLog[3, E
^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2]
```

Rubi [A] time = 0.252572, antiderivative size = 227, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4958, 4956, 4183, 2531, 2282, 6589}

$$\frac{2i\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,-e^{i\tan^{-1}(ax)}\right)}{\sqrt{a^2cx^2+c}}-\frac{2i\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,e^{i\tan^{-1}(ax)}\right)}{\sqrt{a^2cx^2+c}}-\frac{2\sqrt{a^2x^2+1}\text{PolyLo}}{\sqrt{a^2cx}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^2/(x*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] (-2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*ArcTanh[E^(I*ArcTan[a*x])])/Sqrt[c + a^
2*c*x^2] + ((2*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x
])])/Sqrt[c + a^2*c*x^2] - ((2*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2,
E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - (2*Sqrt[1 + a^2*x^2]*PolyLog[3, -
E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + (2*Sqrt[1 + a^2*x^2]*PolyLog[3, E
^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2]
```

Rule 4958

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4956

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Dist[1/Sqrt[d], Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTan
[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && G
tQ[d, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^2}{x\sqrt{c+a^2cx^2}}\,dx = \frac{\sqrt{1+a^2x^2}\int \frac{\tan^{-1}(ax)^2}{x\sqrt{1+a^2x^2}}\,dx}{\sqrt{c+a^2cx^2}}$$
$$= \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int x^2\csc(x)\,dx, x, \tan^{-1}(ax)\right)}{\sqrt{c+a^2cx^2}}$$
$$= -\frac{2\sqrt{1+a^2x^2}\tan^{-1}(ax)^2\tanh^{-1}\left(e^{i\tan^{-1}(ax)}\right)}{\sqrt{c+a^2cx^2}} - \frac{\left(2\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int x\log\left(1-e^{ix}\right)dx, x, \tan^{-1}\right.}{\sqrt{c+a^2cx^2}}$$
$$= -\frac{2\sqrt{1+a^2x^2}\tan^{-1}(ax)^2\tanh^{-1}\left(e^{i\tan^{-1}(ax)}\right)}{\sqrt{c+a^2cx^2}} + \frac{2i\sqrt{1+a^2x^2}\tan^{-1}(ax)\text{Li}_2\left(-e^{i\tan^{-1}(ax)}\right)}{\sqrt{c+a^2cx^2}} - \frac{2i\sqrt{1+}}{}$$
$$= -\frac{2\sqrt{1+a^2x^2}\tan^{-1}(ax)^2\tanh^{-1}\left(e^{i\tan^{-1}(ax)}\right)}{\sqrt{c+a^2cx^2}} + \frac{2i\sqrt{1+a^2x^2}\tan^{-1}(ax)\text{Li}_2\left(-e^{i\tan^{-1}(ax)}\right)}{\sqrt{c+a^2cx^2}} - \frac{2i\sqrt{1+}}{}$$
$$= -\frac{2\sqrt{1+a^2x^2}\tan^{-1}(ax)^2\tanh^{-1}\left(e^{i\tan^{-1}(ax)}\right)}{\sqrt{c+a^2cx^2}} + \frac{2i\sqrt{1+a^2x^2}\tan^{-1}(ax)\text{Li}_2\left(-e^{i\tan^{-1}(ax)}\right)}{\sqrt{c+a^2cx^2}} - \frac{2i\sqrt{1+}}{}$$

Mathematica [A] time = 0.165126, size = 145, normalized size = 0.64

$$\frac{\sqrt{a^2x^2+1}\left(2i\tan^{-1}(ax)\text{PolyLog}\left(2,-e^{i\tan^{-1}(ax)}\right)-2i\tan^{-1}(ax)\text{PolyLog}\left(2,e^{i\tan^{-1}(ax)}\right)-2\text{PolyLog}\left(3,-e^{i\tan^{-1}(ax)}\right)+\right.}{\sqrt{c\left(a^2x^2+1\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTan[a*x]^2/(x*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] (Sqrt[1 + a^2*x^2]*(ArcTan[a*x]^2*Log[1 - E^(I*ArcTan[a*x])] - ArcTan[a*x]^
2*Log[1 + E^(I*ArcTan[a*x])] + (2*I)*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*
x])] - (2*I)*ArcTan[a*x]*PolyLog[2, E^(I*ArcTan[a*x])] - 2*PolyLog[3, -E^(I
*ArcTan[a*x])] + 2*PolyLog[3, E^(I*ArcTan[a*x])]))/Sqrt[c*(1 + a^2*x^2)]
```

Maple [A] time = 0.393, size = 198, normalized size = 0.9

$$-\frac{1}{c}\left((\arctan(ax))^2\ln\left(1+(1+iax)\frac{1}{\sqrt{a^2x^2+1}}\right)-(\arctan(ax))^2\ln\left(1-(1+iax)\frac{1}{\sqrt{a^2x^2+1}}\right)-2\,i\arctan(ax)\,\text{polylog}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^2/x/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] -(arctan(a*x)^2*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-arctan(a*x)^2*ln(1-(1+I*a
*x)/(a^2*x^2+1)^(1/2))-2*I*arctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/
2))+2*I*arctan(a*x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*polylog(3,-(1+
I*a*x)/(a^2*x^2+1)^(1/2))-2*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2)))*(c*(a*x
-I)*(a*x+I))^(1/2)/(a^2*x^2+1)^(1/2)/c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a^2cx^2+c}\arctan\left(ax\right)^2}{a^2cx^3+cx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2/(a^2*c*x^3 + c*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{atan}^2\left(ax\right)}{x\sqrt{c\left(a^2x^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**2/x/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(atan(a*x)**2/(x*sqrt(c*(a**2*x**2 + 1))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan (ax)^2}{\sqrt{a^2cx^2+c}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)^2/(sqrt(a^2*c*x^2 + c)*x), x)
```

3.336 $\int \frac{\tan^{-1}(ax)^2}{x^2\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=208

$$\frac{2ia\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{a^2cx^2+c}}-\frac{2ia\sqrt{a^2x^2+1}\text{PolyLog}\left(2,\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{a^2cx^2+c}}-\frac{\sqrt{a^2cx^2+c}\tan^{-1}(ax)^2}{cx}-\frac{4a\sqrt{a^2x^2+1}\tan^{-1}(}{\sqrt{a^2c}}$$

```
[Out] -((Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(c*x)) - (4*a*Sqrt[1 + a^2*x^2]*ArcTa
n[a*x]*ArcTanh[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2] + ((2*
I)*a*Sqrt[1 + a^2*x^2]*PolyLog[2, -(Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x])])/Sqrt
[c + a^2*c*x^2] - ((2*I)*a*Sqrt[1 + a^2*x^2]*PolyLog[2, Sqrt[1 + I*a*x]/Sqr
t[1 - I*a*x]])/Sqrt[c + a^2*c*x^2]
```

Rubi [A] time = 0.251814, antiderivative size = 208, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4944, 4958, 4954}

$$\frac{2ia\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{a^2cx^2+c}}-\frac{2ia\sqrt{a^2x^2+1}\text{PolyLog}\left(2,\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{a^2cx^2+c}}-\frac{\sqrt{a^2cx^2+c}\tan^{-1}(ax)^2}{cx}-\frac{4a\sqrt{a^2x^2+1}\tan^{-1}(}{\sqrt{a^2c}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^2/(x^2*Sqrt[c + a^2*c*x^2]),x]
```


```
[Out] -((Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(c*x)) - (4*a*Sqrt[1 + a^2*x^2]*ArcTa
n[a*x]*ArcTanh[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2] + ((2*
I)*a*Sqrt[1 + a^2*x^2]*PolyLog[2, -(Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x])])/Sqrt
[c + a^2*c*x^2] - ((2*I)*a*Sqrt[1 + a^2*x^2]*PolyLog[2, Sqrt[1 + I*a*x]/Sqr
t[1 - I*a*x]])/Sqrt[c + a^2*c*x^2]
```


Rule 4944

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a +
 b*ArcTan[c*x])^p)/(d*f*(m + 1)), x] - Dist[(b*c*p)/(f*(m + 1)), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rule 4958

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4954

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x_
Symbol] :> Simp[(-2*(a + b*ArcTan[c*x])*ArcTanh[Sqrt[1 + I*c*x]/Sqrt[1 - I*
c*x]])/Sqrt[d], x] + (Simp[(I*b*PolyLog[2, -(Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x
])])/Sqrt[d], x] - Simp[(I*b*PolyLog[2, Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]])/S
qrt[d], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^2}{x^2\sqrt{c+a^2cx^2}}\,dx = -\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{cx}+(2a)\int \frac{\tan^{-1}(ax)}{x\sqrt{c+a^2cx^2}}\,dx$$
$$= -\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{cx}+\frac{\left(2a\sqrt{1+a^2x^2}\right)\int \frac{\tan^{-1}(ax)}{x\sqrt{1+a^2x^2}}\,dx}{\sqrt{c+a^2cx^2}}$$
$$= -\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{cx}-\frac{4a\sqrt{1+a^2x^2}\tan^{-1}(ax)\tanh^{-1}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}}+\frac{2ia\sqrt{1+a^2x^2}\text{Li}_2\left(-\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right.}{\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 0.419303, size = 128, normalized size = 0.62

$$-\frac{a\sqrt{a^2x^2+1}\left(-2i\text{PolyLog}\left(2,-e^{i\tan^{-1}(ax)}\right)+2i\text{PolyLog}\left(2,e^{i\tan^{-1}(ax)}\right)+\tan^{-1}(ax)\left(\frac{\sqrt{a^2x^2+1}\tan^{-1}(ax)}{ax}-2\log\left(1-e^{i\tan^{-1}}\right.\right.\right.}{\sqrt{c\left(a^2x^2+1\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTan[a*x]^2/(x^2*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] -((a*Sqrt[1 + a^2*x^2]*(ArcTan[a*x]*((Sqrt[1 + a^2*x^2]*ArcTan[a*x])/(a*x)
- 2*Log[1 - E^(I*ArcTan[a*x])] + 2*Log[1 + E^(I*ArcTan[a*x])]) - (2*I)*Poly
Log[2, -E^(I*ArcTan[a*x])] + (2*I)*PolyLog[2, E^(I*ArcTan[a*x])]))/Sqrt[c*(
1 + a^2*x^2)])
```

Maple [A] time = 0.369, size = 171, normalized size = 0.8

$$-\frac{(\arctan(ax))^2}{cx}\sqrt{c(ax-i)(ax+i)}-\frac{2ia}{c}\left(i\arctan(ax)\ln\left(1-(1+iax)\frac{1}{\sqrt{a^2x^2+1}}\right)-i\arctan(ax)\ln\left(1+(1+iax)\frac{1}{\sqrt{a^2\dots}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^2/x^2/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] -arctan(a*x)^2*(c*(a*x-I)*(a*x+I))^(1/2)/c/x-2*I*a*(I*arctan(a*x)*ln(1-(1+I
*a*x)/(a^2*x^2+1)^(1/2))-I*arctan(a*x)*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))+po
lylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))-polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2)
))/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(a*x+I))^(1/2)/c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x^2/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{a^2cx^4+cx^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x^2/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2/(a^2*c*x^4 + c*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atan}^2(ax)}{x^2\sqrt{c\left(a^2x^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**2/x**2/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(atan(a*x)**2/(x**2*sqrt(c*(a**2*x**2 + 1))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x^2/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)^2/(sqrt(a^2*c*x^2 + c)*x^2), x)
```

3.337 $\int \frac{\tan^{-1}(ax)^2}{x^3\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=328

$$-\frac{ia^2\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,-e^{i\tan^{-1}(ax)}\right)}{\sqrt{a^2cx^2+c}}+\frac{ia^2\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,e^{i\tan^{-1}(ax)}\right)}{\sqrt{a^2cx^2+c}}+\frac{a^2\sqrt{a^2x^2+1}\text{Poly}}{\sqrt{a^2}}$$

```
[Out] -((a*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(c*x)) - (Sqrt[c + a^2*c*x^2]*ArcTan[
a*x]^2)/(2*c*x^2) + (a^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*ArcTanh[E^(I*ArcTa
n[a*x])])/Sqrt[c + a^2*c*x^2] - (a^2*ArcTanh[Sqrt[c + a^2*c*x^2]/Sqrt[c]])/
Sqrt[c] - (I*a^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x]
)])/Sqrt[c + a^2*c*x^2] + (I*a^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, E
^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + (a^2*Sqrt[1 + a^2*x^2]*PolyLog[3,
-E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - (a^2*Sqrt[1 + a^2*x^2]*PolyLog[3
, E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2]
```

Rubi [A] time = 0.475394, antiderivative size = 328, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {4962, 4944, 266, 63, 208, 4958, 4956, 4183, 2531, 2282, 6589}

$$-\frac{ia^2\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,-e^{i\tan^{-1}(ax)}\right)}{\sqrt{a^2cx^2+c}}+\frac{ia^2\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,e^{i\tan^{-1}(ax)}\right)}{\sqrt{a^2cx^2+c}}+\frac{a^2\sqrt{a^2x^2+1}\text{Poly}}{\sqrt{a^2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^2/(x^3*Sqrt[c + a^2*c*x^2]),x]
```


```
[Out] -((a*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(c*x)) - (Sqrt[c + a^2*c*x^2]*ArcTan[
a*x]^2)/(2*c*x^2) + (a^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*ArcTanh[E^(I*ArcTa
n[a*x])])/Sqrt[c + a^2*c*x^2] - (a^2*ArcTanh[Sqrt[c + a^2*c*x^2]/Sqrt[c]])/
Sqrt[c] - (I*a^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x]
)])/Sqrt[c + a^2*c*x^2] + (I*a^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, E
^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + (a^2*Sqrt[1 + a^2*x^2]*PolyLog[3,
-E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - (a^2*Sqrt[1 + a^2*x^2]*PolyLog[3
, E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2]
```


Rule 4962

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Ar
cTan[c*x])^p)/(d*f*(m + 1)), x] + (-Dist[(b*c*p)/(f*(m + 1)), Int[((f*x)^(m
 + 1)*(a + b*ArcTan[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] - Dist[(c^2*(m +
 2))/(f^2*(m + 1)), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/Sqrt[d + e*x^
2], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] &
& LtQ[m, -1] && NeQ[m, -2]
```

Rule 4944

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a +
 b*ArcTan[c*x])^p)/(d*f*(m + 1)), x] - Dist[(b*c*p)/(f*(m + 1)), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 4958

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4956

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Dist[1/Sqrt[d], Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTan
[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && G
tQ[d, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^2}{x^3\sqrt{c+a^2cx^2}}\,dx = -\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2cx^2} + a\int \frac{\tan^{-1}(ax)}{x^2\sqrt{c+a^2cx^2}}\,dx - \frac{1}{2}a^2\int \frac{\tan^{-1}(ax)^2}{x\sqrt{c+a^2cx^2}}\,dx$$

$$= -\frac{a\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{cx} - \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2cx^2} + a^2\int \frac{1}{x\sqrt{c+a^2cx^2}}\,dx - \frac{\left(a^2\sqrt{1+a^2x^2}\right)\int \frac{\tan}{x}}{2\sqrt{c+a^2c}}$$

$$= -\frac{a\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{cx} - \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2cx^2} + \frac{1}{2}a^2\,\text{Subst}\left(\int \frac{1}{x\sqrt{c+a^2cx}}\,dx, x, x^2\right) - \frac{\left(a^2\right.}{}$$

$$= -\frac{a\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{cx} - \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2cx^2} + \frac{a^2\sqrt{1+a^2x^2}\tan^{-1}(ax)^2\tanh^{-1}\left(e^{i\tan^{-1}(ax)}\right)}{\sqrt{c+a^2cx^2}}$$

$$= -\frac{a\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{cx} - \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2cx^2} + \frac{a^2\sqrt{1+a^2x^2}\tan^{-1}(ax)^2\tanh^{-1}\left(e^{i\tan^{-1}(ax)}\right)}{\sqrt{c+a^2cx^2}}$$

$$= -\frac{a\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{cx} - \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2cx^2} + \frac{a^2\sqrt{1+a^2x^2}\tan^{-1}(ax)^2\tanh^{-1}\left(e^{i\tan^{-1}(ax)}\right)}{\sqrt{c+a^2cx^2}}$$

$$= -\frac{a\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{cx} - \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2cx^2} + \frac{a^2\sqrt{1+a^2x^2}\tan^{-1}(ax)^2\tanh^{-1}\left(e^{i\tan^{-1}(ax)}\right)}{\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 1.16839, size = 231, normalized size = 0.7

$$\frac{a^2\sqrt{a^2x^2+1}\left(-8i\tan^{-1}(ax)\text{PolyLog}\left(2,-e^{i\tan^{-1}(ax)}\right)+8i\tan^{-1}(ax)\text{PolyLog}\left(2,e^{i\tan^{-1}(ax)}\right)+8\text{PolyLog}\left(3,-e^{i\tan^{-1}(ax)}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTan[a*x]^2/(x^3*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] (a^2*Sqrt[1 + a^2*x^2]*(-4*ArcTan[a*x]*Cot[ArcTan[a*x]/2] - ArcTan[a*x]^2*C
sc[ArcTan[a*x]/2]^2 - 4*ArcTan[a*x]^2*Log[1 - E^(I*ArcTan[a*x])] + 4*ArcTan
[a*x]^2*Log[1 + E^(I*ArcTan[a*x])] + 8*Log[Tan[ArcTan[a*x]/2]] - (8*I)*ArcT
an[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])] + (8*I)*ArcTan[a*x]*PolyLog[2, E^(I*
ArcTan[a*x])] + 8*PolyLog[3, -E^(I*ArcTan[a*x])] - 8*PolyLog[3, E^(I*ArcTan
[a*x])] + ArcTan[a*x]^2*Sec[ArcTan[a*x]/2]^2 - 4*ArcTan[a*x]*Tan[ArcTan[a*x
]/2]))/(8*Sqrt[c*(1 + a^2*x^2)])
```

Maple [A] time = 0.493, size = 261, normalized size = 0.8

$$-\frac{(2ax+\arctan(ax))\arctan(ax)}{2\,cx^2}\sqrt{c\,(ax-i)\,(ax+i)}+\frac{a^2}{2\,c}\left((\arctan(ax))^2\ln\left(1+(1+iax)\frac{1}{\sqrt{a^2x^2+1}}\right)-(\arctan(ax))^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^2/x^3/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] -1/2*(2*a*x+arctan(a*x))*arctan(a*x)*(c*(a*x-I)*(a*x+I))^(1/2)/x^2/c+1/2*a^
2*(arctan(a*x)^2*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-arctan(a*x)^2*ln(1-(1+I*
a*x)/(a^2*x^2+1)^(1/2))-2*I*arctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1
/2))+2*I*arctan(a*x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*polylog(3,-(1
+I*a*x)/(a^2*x^2+1)^(1/2))-2*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))-4*arcta
nh((1+I*a*x)/(a^2*x^2+1)^(1/2)))/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(a*x+I))^(1/2
)/c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x^3/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{a^2cx^5+cx^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x^3/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2/(a^2*c*x^5 + c*x^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atan}^2(ax)}{x^3\sqrt{c(a^2x^2+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**2/x**3/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(atan(a*x)**2/(x**3*sqrt(c*(a**2*x**2 + 1))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x^3/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)^2/(sqrt(a^2*c*x^2 + c)*x^3), x)
```

3.338 $\int \frac{\tan^{-1}(ax)^2}{x^4\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=311

$$-\frac{5ia^3\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{3\sqrt{a^2cx^2+c}}+\frac{5ia^3\sqrt{a^2x^2+1}\text{PolyLog}\left(2,\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{3\sqrt{a^2cx^2+c}}-\frac{a^2\sqrt{a^2cx^2+c}}{3cx}+\frac{2a^2\sqrt{a^2cx^2+c}\tan^{-1}(ax)}{3cx}$$

```
[Out] -(a^2*Sqrt[c + a^2*c*x^2])/(3*c*x) - (a*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(3
*c*x^2) - (Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(3*c*x^3) + (2*a^2*Sqrt[c + a
^2*c*x^2]*ArcTan[a*x]^2)/(3*c*x) + (10*a^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*Ar
cTanh[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/(3*Sqrt[c + a^2*c*x^2]) - (((5*I)/3
)*a^3*Sqrt[1 + a^2*x^2]*PolyLog[2, -(Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x])])/Sqr
t[c + a^2*c*x^2] + (((5*I)/3)*a^3*Sqrt[1 + a^2*x^2]*PolyLog[2, Sqrt[1 + I*a
*x]/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2]
```

Rubi [A] time = 0.627991, antiderivative size = 311, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {4962, 264, 4958, 4954, 4944}

$$-\frac{5ia^3\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{3\sqrt{a^2cx^2+c}}+\frac{5ia^3\sqrt{a^2x^2+1}\text{PolyLog}\left(2,\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{3\sqrt{a^2cx^2+c}}-\frac{a^2\sqrt{a^2cx^2+c}}{3cx}+\frac{2a^2\sqrt{a^2cx^2+c}\tan^{-1}(ax)}{3cx}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^2/(x^4*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] -(a^2*Sqrt[c + a^2*c*x^2])/(3*c*x) - (a*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(3
*c*x^2) - (Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(3*c*x^3) + (2*a^2*Sqrt[c + a
^2*c*x^2]*ArcTan[a*x]^2)/(3*c*x) + (10*a^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*Ar
cTanh[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/(3*Sqrt[c + a^2*c*x^2]) - (((5*I)/3
)*a^3*Sqrt[1 + a^2*x^2]*PolyLog[2, -(Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x])])/Sqr
t[c + a^2*c*x^2] + (((5*I)/3)*a^3*Sqrt[1 + a^2*x^2]*PolyLog[2, Sqrt[1 + I*a
*x]/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2]
```

Rule 4962

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Ar
cTan[c*x])^p)/(d*f*(m + 1)), x] + (-Dist[(b*c*p)/(f*(m + 1)), Int[((f*x)^(m
```

```
 + 1)*(a + b*ArcTan[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] - Dist[(c^2*(m +
 2))/(f^2*(m + 1)), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/Sqrt[d + e*x^
2], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] &
& LtQ[m, -1] && NeQ[m, -2]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 4958

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4954

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x_
Symbol] :> Simp[(-2*(a + b*ArcTan[c*x])*ArcTanh[Sqrt[1 + I*c*x]/Sqrt[1 - I*
c*x]])/Sqrt[d], x] + (Simp[(I*b*PolyLog[2, -(Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x
])])/Sqrt[d], x] - Simp[(I*b*PolyLog[2, Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]])/S
qrt[d], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

Rule 4944

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a +
 b*ArcTan[c*x])^p)/(d*f*(m + 1)), x] - Dist[(b*c*p)/(f*(m + 1)), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^2}{x^4\sqrt{c+a^2cx^2}}\,dx = -\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{3cx^3} + \frac{1}{3}(2a)\int \frac{\tan^{-1}(ax)}{x^3\sqrt{c+a^2cx^2}}\,dx - \frac{1}{3}\left(2a^2\right)\int \frac{\tan^{-1}(ax)^2}{x^2\sqrt{c+a^2cx^2}}\,dx$$

$$= -\frac{a\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{3cx^2} - \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{3cx^3} + \frac{2a^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{3cx} + \frac{1}{3}a^2\int \frac{1}{x^2\sqrt{c+}}$$

$$= -\frac{a^2\sqrt{c+a^2cx^2}}{3cx} - \frac{a\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{3cx^2} - \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{3cx^3} + \frac{2a^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{3cx} -$$

$$= -\frac{a^2\sqrt{c+a^2cx^2}}{3cx} - \frac{a\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{3cx^2} - \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{3cx^3} + \frac{2a^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{3cx} +$$

Mathematica [A] time = 2.60985, size = 228, normalized size = 0.73

$$\frac{a^3\sqrt{a^2cx^2+c}\left(\frac{\left(a^2x^2+1\right)^{3/2}\left(\frac{20ia^3x^3\text{PolyLog}\left(2,e^{i\tan^{-1}(ax)}\right)}{\left(a^2x^2+1\right)^{3/2}}+\tan^{-1}(ax)\left(-2\sin\left(2\tan^{-1}(ax)\right)+\frac{5\left(\log\left(1-e^{i\tan^{-1}(ax)}\right)-\log\left(1+e^{i\tan^{-1}(ax)}\right)\right)\left(\sqrt{a^2x^2+1}\sin\left(3\tan^{-1}(ax)\right)\right.}{\sqrt{a^2x^2+1}}\right.\right.}{a^3x^3}\right.}{12c\sqrt{a^2x^2+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTan[a*x]^2/(x^4*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] (a^3*Sqrt[c + a^2*c*x^2]*((-20*I)*PolyLog[2, -E^(I*ArcTan[a*x])] + ((1 + a^
2*x^2)^(3/2)*(ArcTan[a*x]^2*(2 - 6*Cos[2*ArcTan[a*x]]) + 2*(-1 + Cos[2*ArcT
an[a*x]]) + ((20*I)*a^3*x^3*PolyLog[2, E^(I*ArcTan[a*x])])/(1 + a^2*x^2)^(3
/2) + ArcTan[a*x]*(-2*Sin[2*ArcTan[a*x]] + (5*(Log[1 - E^(I*ArcTan[a*x])] -
 Log[1 + E^(I*ArcTan[a*x])])*(-3*a*x + Sqrt[1 + a^2*x^2]*Sin[3*ArcTan[a*x]]
))/Sqrt[1 + a^2*x^2])))/(a^3*x^3)))/(12*c*Sqrt[1 + a^2*x^2])
```

Maple [A] time = 0.798, size = 206, normalized size = 0.7

$$\frac{2\left(\arctan(ax)\right)^2x^2a^2 - a^2x^2 - \arctan(ax)\,xa - \left(\arctan(ax)\right)^2}{3\,cx^3}\sqrt{c\,(ax-i)\,(ax+i)} + \frac{\frac{5\,i}{3}a^3}{c}\left(i\arctan(ax)\ln\left(1-(1+iax)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^2/x^4/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] 1/3*(2*arctan(a*x)^2*x^2*a^2-a^2*x^2-arctan(a*x)*x*a-arctan(a*x)^2)*(c*(a*x
-I)*(a*x+I))^(1/2)/c/x^3+5/3*I*a^3*(I*arctan(a*x)*ln(1-(1+I*a*x)/(a^2*x^2+1
)^(1/2))-I*arctan(a*x)*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))+polylog(2,(1+I*a*x
)/(a^2*x^2+1)^(1/2))-polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2)))/(a^2*x^2+1)^(
1/2)*(c*(a*x-I)*(a*x+I))^(1/2)/c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x^4/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a^2cx^2+c}\arctan\left(ax\right)^2}{a^2cx^6+cx^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x^4/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2/(a^2*c*x^6 + c*x^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{atan}^2\left(ax\right)}{x^4\sqrt{c\left(a^2x^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**2/x**4/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(atan(a*x)**2/(x**4*sqrt(c*(a**2*x**2 + 1))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan (ax)^2}{\sqrt{a^2cx^2 + cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x^4/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)^2/(sqrt(a^2*c*x^2 + c)*x^4), x)
```

3.339 $\int \frac{x^3 \tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^{3/2}} \, dx$

Optimal. Leaf size=305

$$-\frac{2i\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a^4c\sqrt{a^2cx^2+c}}+\frac{2i\sqrt{a^2x^2+1}\text{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a^4c\sqrt{a^2cx^2+c}}+\frac{\sqrt{a^2cx^2+c}\tan^{-1}(ax)^2}{a^4c^2}-\frac{2}{a^4c\sqrt{a^2cx^2+c}}+$$

```
[Out] -2/(a^4*c*Sqrt[c + a^2*c*x^2]) - (2*x*ArcTan[a*x])/(a^3*c*Sqrt[c + a^2*c*x^
2]) + ArcTan[a*x]^2/(a^4*c*Sqrt[c + a^2*c*x^2]) + (Sqrt[c + a^2*c*x^2]*ArcT
an[a*x]^2)/(a^4*c^2) + ((4*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTan[Sqrt[1 +
 I*a*x]/Sqrt[1 - I*a*x]])/(a^4*c*Sqrt[c + a^2*c*x^2]) - ((2*I)*Sqrt[1 + a^2
*x^2]*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/(a^4*c*Sqrt[c + a
^2*c*x^2]) + ((2*I)*Sqrt[1 + a^2*x^2]*PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1
 - I*a*x]])/(a^4*c*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.396028, antiderivative size = 305, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {4964, 4930, 4890, 4886, 4894}

$$-\frac{2i\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a^4c\sqrt{a^2cx^2+c}}+\frac{2i\sqrt{a^2x^2+1}\text{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a^4c\sqrt{a^2cx^2+c}}+\frac{\sqrt{a^2cx^2+c}\tan^{-1}(ax)^2}{a^4c^2}-\frac{2}{a^4c\sqrt{a^2cx^2+c}}+$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcTan[a*x]^2)/(c + a^2*c*x^2)^(3/2),x]
```


```
[Out] -2/(a^4*c*Sqrt[c + a^2*c*x^2]) - (2*x*ArcTan[a*x])/(a^3*c*Sqrt[c + a^2*c*x^
2]) + ArcTan[a*x]^2/(a^4*c*Sqrt[c + a^2*c*x^2]) + (Sqrt[c + a^2*c*x^2]*ArcT
an[a*x]^2)/(a^4*c^2) + ((4*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTan[Sqrt[1 +
 I*a*x]/Sqrt[1 - I*a*x]])/(a^4*c*Sqrt[c + a^2*c*x^2]) - ((2*I)*Sqrt[1 + a^2
*x^2]*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/(a^4*c*Sqrt[c + a
^2*c*x^2]) + ((2*I)*Sqrt[1 + a^2*x^2]*PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1
 - I*a*x]])/(a^4*c*Sqrt[c + a^2*c*x^2])
```


Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Arc
Tan[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan[c
```

```
*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p
, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 4890

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4886

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[(-2*I*(a + b*ArcTan[c*x])*ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]])
/(c*Sqrt[d]), x] + (Simp[(I*b*PolyLog[2, -((I*Sqrt[1 + I*c*x])/Sqrt[1 - I*c
*x])])/(c*Sqrt[d]), x] - Simp[(I*b*PolyLog[2, (I*Sqrt[1 + I*c*x])/Sqrt[1 -
I*c*x]])/(c*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

Rule 4894

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[(x*(a + b*ArcTan[c*x]))/(d*Sq
rt[d + e*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

Rubi steps

$$\int \frac{x^3 \tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^{3/2}}\,dx = -\frac{\int \frac{x\tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^{3/2}}\,dx}{a^2} + \frac{\int \frac{x\tan^{-1}(ax)^2}{\sqrt{c+a^2cx^2}}\,dx}{a^2c}$$

$$= \frac{\tan^{-1}(ax)^2}{a^4c\sqrt{c+a^2cx^2}} + \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{a^4c^2} - \frac{2\int \frac{\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^{3/2}}\,dx}{a^3} - \frac{2\int \frac{\tan^{-1}(ax)}{\sqrt{c+a^2cx^2}}\,dx}{a^3c}$$

$$= -\frac{2}{a^4c\sqrt{c+a^2cx^2}} - \frac{2x\tan^{-1}(ax)}{a^3c\sqrt{c+a^2cx^2}} + \frac{\tan^{-1}(ax)^2}{a^4c\sqrt{c+a^2cx^2}} + \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{a^4c^2} - \frac{\left(2\sqrt{1+a^2x^2}\right)\int}{a^3c\sqrt{c+}}$$

$$= -\frac{2}{a^4c\sqrt{c+a^2cx^2}} - \frac{2x\tan^{-1}(ax)}{a^3c\sqrt{c+a^2cx^2}} + \frac{\tan^{-1}(ax)^2}{a^4c\sqrt{c+a^2cx^2}} + \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{a^4c^2} + \frac{4i\sqrt{1+a^2x^2}\tan}{a^4c}$$

Mathematica [A] time = 0.841571, size = 209, normalized size = 0.69

$$\frac{\sqrt{c\left(a^2x^2+1\right)}\left(-\frac{4i\text{PolyLog}\left(2,-ie^{i\tan^{-1}(ax)}\right)}{\sqrt{a^2x^2+1}} + \frac{4i\text{PolyLog}\left(2,ie^{i\tan^{-1}(ax)}\right)}{\sqrt{a^2x^2+1}} - \frac{4\tan^{-1}(ax)\log\left(1-ie^{i\tan^{-1}(ax)}\right)}{\sqrt{a^2x^2+1}} + \frac{4\tan^{-1}(ax)\log\left(1+ie^{i\tan^{-1}(ax)}\right)}{\sqrt{a^2x^2+1}} + \right.}{2a^4c^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*ArcTan[a*x]^2)/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] (Sqrt[c*(1 + a^2*x^2)]*(-2 + 3*ArcTan[a*x]^2 - 2*Cos[2*ArcTan[a*x]] + ArcTa
n[a*x]^2*Cos[2*ArcTan[a*x]] - (4*ArcTan[a*x]*Log[1 - I*E^(I*ArcTan[a*x])])/
Sqrt[1 + a^2*x^2] + (4*ArcTan[a*x]*Log[1 + I*E^(I*ArcTan[a*x])])/Sqrt[1 + a
^2*x^2] - ((4*I)*PolyLog[2, (-I)*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + ((
4*I)*PolyLog[2, I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - 2*ArcTan[a*x]*Sin
[2*ArcTan[a*x]]))/(2*a^4*c^2)
```

Maple [A] time = 1., size = 294, normalized size = 1.

$$\frac{\left((\arctan(ax))^2 - 2 + 2i\arctan(ax)\right)(1+iax)}{\left(2a^2x^2+2\right)a^4c^2}\sqrt{c(ax-i)(ax+i)} - \frac{(-1+iax)\left((\arctan(ax))^2 - 2 - 2i\arctan(ax)\right)}{\left(2a^2x^2+2\right)a^4c^2}\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctan(a*x)^2/(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] 1/2*(arctan(a*x)^2-2+2*I*arctan(a*x))*(1+I*a*x)*(c*(a*x-I)*(a*x+I))^(1/2)/(
a^2*x^2+1)/a^4/c^2-1/2*(c*(a*x-I)*(a*x+I))^(1/2)*(-1+I*a*x)*(arctan(a*x)^2-
2-2*I*arctan(a*x))/(a^2*x^2+1)/a^4/c^2+arctan(a*x)^2*(c*(a*x-I)*(a*x+I))^(1
/2)/a^4/c^2-2*I*(I*arctan(a*x)*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*arctan
(a*x)*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+dilog(1+I*(1+I*a*x)/(a^2*x^2+1)^(
1/2))-dilog(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))*(c*(a*x-I)*(a*x+I))^(1/2)/(a^
2*x^2+1)^(1/2)/a^4/c^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^2/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a^2cx^2+c}\,x^3\arctan\left(ax\right)^2}{a^4c^2x^4+2\,a^2c^2x^2+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^2/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^3*arctan(a*x)^2/(a^4*c^2*x^4 + 2*a^2*c^2*x^2
 + c^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3\,\operatorname{atan}^2\left(ax\right)}{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atan(a*x)**2/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(x**3*atan(a*x)**2/(c*(a**2*x**2 + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \arctan(ax)^2}{\left(a^2cx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^2/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^3*arctan(a*x)^2/(a^2*c*x^2 + c)^(3/2), x)
```

3.340 $\int \frac{x^2 \tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=349

$$\frac{2i\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,-ie^{i\tan^{-1}(ax)}\right)}{a^3c\sqrt{a^2cx^2+c}}-\frac{2i\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,ie^{i\tan^{-1}(ax)}\right)}{a^3c\sqrt{a^2cx^2+c}}-\frac{2\sqrt{a^2x^2+1}\text{PolyLo}}{a^3c\sqrt{a^2}}$$

```
[Out] (2*x)/(a^2*c*Sqrt[c + a^2*c*x^2]) - (2*ArcTan[a*x])/(a^3*c*Sqrt[c + a^2*c*x
^2]) - (x*ArcTan[a*x]^2)/(a^2*c*Sqrt[c + a^2*c*x^2]) - ((2*I)*Sqrt[1 + a^2*
x^2]*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2)/(a^3*c*Sqrt[c + a^2*c*x^2]) +
 ((2*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])])/(
a^3*c*Sqrt[c + a^2*c*x^2]) - ((2*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2
, I*E^(I*ArcTan[a*x])])/(a^3*c*Sqrt[c + a^2*c*x^2]) - (2*Sqrt[1 + a^2*x^2]*
PolyLog[3, (-I)*E^(I*ArcTan[a*x])])/(a^3*c*Sqrt[c + a^2*c*x^2]) + (2*Sqrt[1
 + a^2*x^2]*PolyLog[3, I*E^(I*ArcTan[a*x])])/(a^3*c*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.341122, antiderivative size = 349, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4964, 4890, 4888, 4181, 2531, 2282, 6589, 4898, 191}

$$\frac{2i\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,-ie^{i\tan^{-1}(ax)}\right)}{a^3c\sqrt{a^2cx^2+c}}-\frac{2i\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,ie^{i\tan^{-1}(ax)}\right)}{a^3c\sqrt{a^2cx^2+c}}-\frac{2\sqrt{a^2x^2+1}\text{PolyLo}}{a^3c\sqrt{a^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcTan[a*x]^2)/(c + a^2*c*x^2)^(3/2),x]
```


```
[Out] (2*x)/(a^2*c*Sqrt[c + a^2*c*x^2]) - (2*ArcTan[a*x])/(a^3*c*Sqrt[c + a^2*c*x
^2]) - (x*ArcTan[a*x]^2)/(a^2*c*Sqrt[c + a^2*c*x^2]) - ((2*I)*Sqrt[1 + a^2*
x^2]*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2)/(a^3*c*Sqrt[c + a^2*c*x^2]) +
 ((2*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])])/(
a^3*c*Sqrt[c + a^2*c*x^2]) - ((2*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2
, I*E^(I*ArcTan[a*x])])/(a^3*c*Sqrt[c + a^2*c*x^2]) - (2*Sqrt[1 + a^2*x^2]*
PolyLog[3, (-I)*E^(I*ArcTan[a*x])])/(a^3*c*Sqrt[c + a^2*c*x^2]) + (2*Sqrt[1
 + a^2*x^2]*PolyLog[3, I*E^(I*ArcTan[a*x])])/(a^3*c*Sqrt[c + a^2*c*x^2])
```


Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Arc
```

```
Tan[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan[c
*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p
, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rule 4890

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4888

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[c
*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && GtQ
[d, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4898

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[(b*p*(a + b*ArcTan[c*x])^(p - 1))/(c*d*Sqrt[d + e*x^2]), x]
 + (-Dist[b^2*p*(p - 1), Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2),
 x], x] + Simp[(x*(a + b*ArcTan[c*x])^p)/(d*Sqrt[d + e*x^2]), x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{x^2 \tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^{3/2}}\,dx = -\frac{\int \frac{\tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^{3/2}}\,dx}{a^2} + \frac{\int \frac{\tan^{-1}(ax)^2}{\sqrt{c+a^2cx^2}}\,dx}{a^2c}$$

$$= -\frac{2\tan^{-1}(ax)}{a^3c\sqrt{c+a^2cx^2}} - \frac{x\tan^{-1}(ax)^2}{a^2c\sqrt{c+a^2cx^2}} + \frac{2\int \frac{1}{\left(c+a^2cx^2\right)^{3/2}}\,dx}{a^2} + \frac{\sqrt{1+a^2x^2}\int \frac{\tan^{-1}(ax)^2}{\sqrt{1+a^2x^2}}\,dx}{a^2c\sqrt{c+a^2cx^2}}$$

$$= \frac{2x}{a^2c\sqrt{c+a^2cx^2}} - \frac{2\tan^{-1}(ax)}{a^3c\sqrt{c+a^2cx^2}} - \frac{x\tan^{-1}(ax)^2}{a^2c\sqrt{c+a^2cx^2}} + \frac{\sqrt{1+a^2x^2}\,\mathrm{Subst}\left(\int x^2\sec(x)\,dx, x, \tan^{-1}(ax\right.}{a^3c\sqrt{c+a^2cx^2}}$$

$$= \frac{2x}{a^2c\sqrt{c+a^2cx^2}} - \frac{2\tan^{-1}(ax)}{a^3c\sqrt{c+a^2cx^2}} - \frac{x\tan^{-1}(ax)^2}{a^2c\sqrt{c+a^2cx^2}} - \frac{2i\sqrt{1+a^2x^2}\tan^{-1}\left(e^{i\tan^{-1}(ax)}\right)\tan^{-1}(ax)^2}{a^3c\sqrt{c+a^2cx^2}} -$$

$$= \frac{2x}{a^2c\sqrt{c+a^2cx^2}} - \frac{2\tan^{-1}(ax)}{a^3c\sqrt{c+a^2cx^2}} - \frac{x\tan^{-1}(ax)^2}{a^2c\sqrt{c+a^2cx^2}} - \frac{2i\sqrt{1+a^2x^2}\tan^{-1}\left(e^{i\tan^{-1}(ax)}\right)\tan^{-1}(ax)^2}{a^3c\sqrt{c+a^2cx^2}} +$$

$$= \frac{2x}{a^2c\sqrt{c+a^2cx^2}} - \frac{2\tan^{-1}(ax)}{a^3c\sqrt{c+a^2cx^2}} - \frac{x\tan^{-1}(ax)^2}{a^2c\sqrt{c+a^2cx^2}} - \frac{2i\sqrt{1+a^2x^2}\tan^{-1}\left(e^{i\tan^{-1}(ax)}\right)\tan^{-1}(ax)^2}{a^3c\sqrt{c+a^2cx^2}} +$$

$$= \frac{2x}{a^2c\sqrt{c+a^2cx^2}} - \frac{2\tan^{-1}(ax)}{a^3c\sqrt{c+a^2cx^2}} - \frac{x\tan^{-1}(ax)^2}{a^2c\sqrt{c+a^2cx^2}} - \frac{2i\sqrt{1+a^2x^2}\tan^{-1}\left(e^{i\tan^{-1}(ax)}\right)\tan^{-1}(ax)^2}{a^3c\sqrt{c+a^2cx^2}} +$$

Mathematica [A] time = 0.347808, size = 228, normalized size = 0.65

$$-\frac{\sqrt{a^2x^2+1}\left(-2i\tan^{-1}(ax)\text{PolyLog}\left(2,-ie^{i\tan^{-1}(ax)}\right)+2i\tan^{-1}(ax)\text{PolyLog}\left(2,ie^{i\tan^{-1}(ax)}\right)+2\text{PolyLog}\left(3,-ie^{i\tan^{-1}(ax)}\right)\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*ArcTan[a*x]^2)/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] -((Sqrt[1 + a^2*x^2]*((-2*a*x)/Sqrt[1 + a^2*x^2] + (2*ArcTan[a*x])/Sqrt[1 +
 a^2*x^2] + (a*x*ArcTan[a*x]^2)/Sqrt[1 + a^2*x^2] - ArcTan[a*x]^2*Log[1 - I
*E^(I*ArcTan[a*x])] + ArcTan[a*x]^2*Log[1 + I*E^(I*ArcTan[a*x])] - (2*I)*Ar
cTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] + (2*I)*ArcTan[a*x]*PolyLog[2,
 I*E^(I*ArcTan[a*x])] + 2*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] - 2*PolyLog[3,
 I*E^(I*ArcTan[a*x])]))/(a^3*c*Sqrt[c*(1 + a^2*x^2)]))
```

Maple [F] time = 0.873, size = 0, normalized size = 0.

$$\int x^2(\arctan(ax))^2\left(a^2cx^2+c\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(a*x)^2/(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] int(x^2*arctan(a*x)^2/(a^2*c*x^2+c)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^2/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a^2cx^2+c}\,x^2\arctan\left(ax\right)^2}{a^4c^2x^4+2\,a^2c^2x^2+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^2/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^2*arctan(a*x)^2/(a^4*c^2*x^4 + 2*a^2*c^2*x^2
 + c^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\,\mathrm{atan}^2\left(ax\right)}{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(a*x)**2/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(x**2*atan(a*x)**2/(c*(a**2*x**2 + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\arctan\left(ax\right)^2}{\left(a^2cx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^2/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctan(a*x)^2/(a^2*c*x^2 + c)^(3/2), x)
```

3.341 $\int \frac{x \tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^{3/2}} dx$

Optimal. Leaf size=78

$$\frac{2}{a^2c\sqrt{a^2cx^2+c}}-\frac{\tan^{-1}(ax)^2}{a^2c\sqrt{a^2cx^2+c}}+\frac{2x\tan^{-1}(ax)}{ac\sqrt{a^2cx^2+c}}$$

```
[Out] 2/(a^2*c*Sqrt[c + a^2*c*x^2]) + (2*x*ArcTan[a*x])/(a*c*Sqrt[c + a^2*c*x^2])
 - ArcTan[a*x]^2/(a^2*c*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.110577, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4930, 4894}

$$\frac{2}{a^2c\sqrt{a^2cx^2+c}}-\frac{\tan^{-1}(ax)^2}{a^2c\sqrt{a^2cx^2+c}}+\frac{2x\tan^{-1}(ax)}{ac\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTan[a*x]^2)/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] 2/(a^2*c*Sqrt[c + a^2*c*x^2]) + (2*x*ArcTan[a*x])/(a*c*Sqrt[c + a^2*c*x^2])
 - ArcTan[a*x]^2/(a^2*c*Sqrt[c + a^2*c*x^2])
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 4894

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[(x*(a + b*ArcTan[c*x]))/(d*Sq
rt[d + e*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

Rubi steps

$$\int \frac{x\tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^{3/2}}\,dx = -\frac{\tan^{-1}(ax)^2}{a^2c\sqrt{c+a^2cx^2}} + \frac{2\int \frac{\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^{3/2}}\,dx}{a}$$

$$= \frac{2}{a^2c\sqrt{c+a^2cx^2}} + \frac{2x\tan^{-1}(ax)}{ac\sqrt{c+a^2cx^2}} - \frac{\tan^{-1}(ax)^2}{a^2c\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 0.0748785, size = 50, normalized size = 0.64

$$\frac{\sqrt{a^2cx^2+c}\left(-\tan^{-1}(ax)^2+2ax\tan^{-1}(ax)+2\right)}{a^2c^2\left(a^2x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcTan[a*x]^2)/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] (Sqrt[c + a^2*c*x^2]*(2 + 2*a*x*ArcTan[a*x] - ArcTan[a*x]^2))/(a^2*c^2*(1 +
 a^2*x^2))
```

Maple [C] time = 0.265, size = 116, normalized size = 1.5

$$-\frac{\left((\arctan(ax))^2-2+2\,i\arctan(ax)\right)(1+iax)}{\left(2\,a^2x^2+2\right)c^2a^2}\sqrt{c\,(ax-i)\,(ax+i)} + \frac{(-1+iax)\left((\arctan(ax))^2-2-2\,i\arctan(ax)\right)}{\left(2\,a^2x^2+2\right)c^2a^2}\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(a*x)^2/(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] -1/2*(arctan(a*x)^2-2+2*I*arctan(a*x))*(1+I*a*x)*(c*(a*x-I)*(a*x+I))^(1/2)/
(a^2*x^2+1)/c^2/a^2+1/2*(c*(a*x-I)*(a*x+I))^(1/2)*(-1+I*a*x)*(arctan(a*x)^2
-2-2*I*arctan(a*x))/(a^2*x^2+1)/c^2/a^2
```

Maxima [A] time = 2.53735, size = 99, normalized size = 1.27

$$\sqrt{c}\left(\frac{2\,x\arctan(ax)}{\sqrt{a^2x^2+1}ac^2} - \frac{\arctan(ax)^2}{\sqrt{a^2x^2+1}a^2c^2} + \frac{2}{\sqrt{a^2x^2+1}a^2c^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^2/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] sqrt(c)*(2*x*arctan(a*x)/(sqrt(a^2*x^2 + 1)*a*c^2) - arctan(a*x)^2/(sqrt(a^
2*x^2 + 1)*a^2*c^2) + 2/(sqrt(a^2*x^2 + 1)*a^2*c^2))
```

Fricas [A] time = 2.24516, size = 115, normalized size = 1.47

$$\frac{\sqrt{a^2cx^2+c}\left(2\,ax\arctan\left(ax\right)-\arctan\left(ax\right)^2+2\right)}{a^4c^2x^2+a^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^2/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] sqrt(a^2*c*x^2 + c)*(2*a*x*arctan(a*x) - arctan(a*x)^2 + 2)/(a^4*c^2*x^2 +
a^2*c^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x\,\mathrm{atan}^2\left(ax\right)}{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(a*x)**2/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(x*atan(a*x)**2/(c*(a**2*x**2 + 1))**(3/2), x)
```

Giac [A] time = 1.22588, size = 97, normalized size = 1.24

$$\frac{2\,x\arctan\left(ax\right)}{\sqrt{a^2cx^2+c}ac}-\frac{\arctan\left(ax\right)^2}{\sqrt{a^2cx^2+c}a^2c}+\frac{2}{\sqrt{a^2cx^2+c}a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^2/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] 2*x*arctan(a*x)/(sqrt(a^2*c*x^2 + c)*a*c) - arctan(a*x)^2/(sqrt(a^2*c*x^2 +
 c)*a^2*c) + 2/(sqrt(a^2*c*x^2 + c)*a^2*c)
```

3.342 $\int \frac{\tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=72

$$-\frac{2x}{c\sqrt{a^2cx^2+c}}+\frac{x\tan^{-1}(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\tan^{-1}(ax)}{ac\sqrt{a^2cx^2+c}}$$

```
[Out] (-2*x)/(c*Sqrt[c + a^2*c*x^2]) + (2*ArcTan[a*x])/(a*c*Sqrt[c + a^2*c*x^2])
+ (x*ArcTan[a*x]^2)/(c*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.0455449, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4898, 191}

$$-\frac{2x}{c\sqrt{a^2cx^2+c}}+\frac{x\tan^{-1}(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\tan^{-1}(ax)}{ac\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^2/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] (-2*x)/(c*Sqrt[c + a^2*c*x^2]) + (2*ArcTan[a*x])/(a*c*Sqrt[c + a^2*c*x^2])
+ (x*ArcTan[a*x]^2)/(c*Sqrt[c + a^2*c*x^2])
```

Rule 4898

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[(b*p*(a + b*ArcTan[c*x])^(p - 1))/(c*d*Sqrt[d + e*x^2]), x]
 + (-Dist[b^2*p*(p - 1), Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2),
 x], x] + Simp[(x*(a + b*ArcTan[c*x])^p)/(d*Sqrt[d + e*x^2]), x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^{3/2}}\,dx = \frac{2\tan^{-1}(ax)}{ac\sqrt{c+a^2cx^2}} + \frac{x\tan^{-1}(ax)^2}{c\sqrt{c+a^2cx^2}} - 2\int \frac{1}{\left(c+a^2cx^2\right)^{3/2}}\,dx$$
$$= -\frac{2x}{c\sqrt{c+a^2cx^2}} + \frac{2\tan^{-1}(ax)}{ac\sqrt{c+a^2cx^2}} + \frac{x\tan^{-1}(ax)^2}{c\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 0.062047, size = 49, normalized size = 0.68

$$\frac{\sqrt{a^2cx^2+c}\left(-2ax+ax\tan^{-1}(ax)^2+2\tan^{-1}(ax)\right)}{c^2\left(a^3x^2+a\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]^2/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] (Sqrt[c + a^2*c*x^2]*(-2*a*x + 2*ArcTan[a*x] + a*x*ArcTan[a*x]^2))/(c^2*(a
+ a^3*x^2))
```

Maple [C] time = 0.223, size = 114, normalized size = 1.6

$$\frac{\left((\arctan(ax))^2-2+2\,i\arctan(ax)\right)(ax-i)}{\left(2\,a^2x^2+2\right)c^2a}\sqrt{c\,(ax-i)\,(ax+i)}+\frac{(ax+i)\left((\arctan(ax))^2-2-2\,i\arctan(ax)\right)}{\left(2\,a^2x^2+2\right)c^2a}\sqrt{c\,(ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^2/(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] 1/2*(arctan(a*x)^2-2+2*I*arctan(a*x))*(a*x-I)*(c*(a*x-I)*(a*x+I))^(1/2)/(a^
2*x^2+1)/c^2/a+1/2*(c*(a*x-I)*(a*x+I))^(1/2)*(a*x+I)*(arctan(a*x)^2-2-2*I*a
rctan(a*x))/(a^2*x^2+1)/c^2/a
```

Maxima [A] time = 1.76498, size = 72, normalized size = 1.

$$\frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}\,c} - \frac{2\,(ax-\arctan(ax))}{\sqrt{a^2x^2+1}\,ac^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] x*arctan(a*x)^2/(sqrt(a^2*c*x^2 + c)*c) - 2*(a*x - arctan(a*x))/(sqrt(a^2*x
^2 + 1)*a*c^(3/2))
```

Fricas [A] time = 2.29909, size = 117, normalized size = 1.62

$$\frac{\sqrt{a^2cx^2+c}\left(ax\arctan\left(ax\right)^2-2\,ax+2\,\arctan\left(ax\right)\right)}{a^3c^2x^2+ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] sqrt(a^2*c*x^2 + c)*(a*x*arctan(a*x)^2 - 2*a*x + 2*arctan(a*x))/(a^3*c^2*x^
2 + a*c^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{atan}^2\left(ax\right)}{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**2/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(atan(a*x)**2/(c*(a**2*x**2 + 1))**(3/2), x)
```

Giac [A] time = 1.2316, size = 97, normalized size = 1.35

$$-2\,a\left(\frac{x}{\sqrt{a^2cx^2+c}ac}-\frac{\arctan\left(ax\right)}{\sqrt{a^2cx^2+c}a^2c}\right)+\frac{x\arctan\left(ax\right)^2}{\sqrt{a^2cx^2+c}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] -2*a*(x/(sqrt(a^2*c*x^2 + c)*a*c) - arctan(a*x)/(sqrt(a^2*c*x^2 + c)*a^2*c)
) + x*arctan(a*x)^2/(sqrt(a^2*c*x^2 + c)*c)
```

3.343 $\int \frac{\tan^{-1}(ax)^2}{x\left(c+a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=310

$$\frac{2i\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,-e^{i\tan^{-1}(ax)}\right)}{c\sqrt{a^2cx^2+c}}-\frac{2i\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,e^{i\tan^{-1}(ax)}\right)}{c\sqrt{a^2cx^2+c}}-\frac{2\sqrt{a^2x^2+1}\text{PolyLo}}{c\sqrt{a^2c}}$$

```
[Out] -2/(c*Sqrt[c + a^2*c*x^2]) - (2*a*x*ArcTan[a*x])/(c*Sqrt[c + a^2*c*x^2]) +
ArcTan[a*x]^2/(c*Sqrt[c + a^2*c*x^2]) - (2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*
ArcTanh[E^(I*ArcTan[a*x])])/(c*Sqrt[c + a^2*c*x^2]) + ((2*I)*Sqrt[1 + a^2*x
^2]*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])])/(c*Sqrt[c + a^2*c*x^2]) - (
(2*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, E^(I*ArcTan[a*x])])/(c*Sqrt[
c + a^2*c*x^2]) - (2*Sqrt[1 + a^2*x^2]*PolyLog[3, -E^(I*ArcTan[a*x])])/(c*S
qrt[c + a^2*c*x^2]) + (2*Sqrt[1 + a^2*x^2]*PolyLog[3, E^(I*ArcTan[a*x])])/(
c*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.50629, antiderivative size = 310, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4966, 4958, 4956, 4183, 2531, 2282, 6589, 4930, 4894}

$$\frac{2i\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,-e^{i\tan^{-1}(ax)}\right)}{c\sqrt{a^2cx^2+c}}-\frac{2i\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,e^{i\tan^{-1}(ax)}\right)}{c\sqrt{a^2cx^2+c}}-\frac{2\sqrt{a^2x^2+1}\text{PolyLo}}{c\sqrt{a^2c}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^2/(x*(c + a^2*c*x^2)^(3/2)),x]
```


```
[Out] -2/(c*Sqrt[c + a^2*c*x^2]) - (2*a*x*ArcTan[a*x])/(c*Sqrt[c + a^2*c*x^2]) +
ArcTan[a*x]^2/(c*Sqrt[c + a^2*c*x^2]) - (2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*
ArcTanh[E^(I*ArcTan[a*x])])/(c*Sqrt[c + a^2*c*x^2]) + ((2*I)*Sqrt[1 + a^2*x
^2]*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])])/(c*Sqrt[c + a^2*c*x^2]) - (
(2*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, E^(I*ArcTan[a*x])])/(c*Sqrt[
c + a^2*c*x^2]) - (2*Sqrt[1 + a^2*x^2]*PolyLog[3, -E^(I*ArcTan[a*x])])/(c*S
qrt[c + a^2*c*x^2]) + (2*Sqrt[1 + a^2*x^2]*PolyLog[3, E^(I*ArcTan[a*x])])/(
c*Sqrt[c + a^2*c*x^2])
```


Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*
```

```
x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^p
, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*q]
 && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 4958

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4956

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Dist[1/Sqrt[d], Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTan
[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && G
tQ[d, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 4894

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[(x*(a + b*ArcTan[c*x]))/(d*Sq
rt[d + e*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^2}{x\left(c+a^2cx^2\right)^{3/2}}\,dx = -\left(a^2\int \frac{x\tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^{3/2}}\,dx\right)+\frac{\int \frac{\tan^{-1}(ax)^2}{x\sqrt{c+a^2cx^2}}\,dx}{c}$$

$$= \frac{\tan^{-1}(ax)^2}{c\sqrt{c+a^2cx^2}}-(2a)\int \frac{\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^{3/2}}\,dx+\frac{\sqrt{1+a^2x^2}\int \frac{\tan^{-1}(ax)^2}{x\sqrt{1+a^2x^2}}\,dx}{c\sqrt{c+a^2cx^2}}$$

$$= -\frac{2}{c\sqrt{c+a^2cx^2}}-\frac{2ax\tan^{-1}(ax)}{c\sqrt{c+a^2cx^2}}+\frac{\tan^{-1}(ax)^2}{c\sqrt{c+a^2cx^2}}+\frac{\sqrt{1+a^2x^2}\,\mathrm{Subst}\left(\int x^2\csc(x)\,dx,x,\tan^{-1}(ax\right.}{c\sqrt{c+a^2cx^2}}$$

$$= -\frac{2}{c\sqrt{c+a^2cx^2}}-\frac{2ax\tan^{-1}(ax)}{c\sqrt{c+a^2cx^2}}+\frac{\tan^{-1}(ax)^2}{c\sqrt{c+a^2cx^2}}-\frac{2\sqrt{1+a^2x^2}\tan^{-1}(ax)^2\tanh^{-1}\left(e^{i\tan^{-1}(ax)}\right)}{c\sqrt{c+a^2cx^2}}-$$

$$= -\frac{2}{c\sqrt{c+a^2cx^2}}-\frac{2ax\tan^{-1}(ax)}{c\sqrt{c+a^2cx^2}}+\frac{\tan^{-1}(ax)^2}{c\sqrt{c+a^2cx^2}}-\frac{2\sqrt{1+a^2x^2}\tan^{-1}(ax)^2\tanh^{-1}\left(e^{i\tan^{-1}(ax)}\right)}{c\sqrt{c+a^2cx^2}}+$$

$$= -\frac{2}{c\sqrt{c+a^2cx^2}}-\frac{2ax\tan^{-1}(ax)}{c\sqrt{c+a^2cx^2}}+\frac{\tan^{-1}(ax)^2}{c\sqrt{c+a^2cx^2}}-\frac{2\sqrt{1+a^2x^2}\tan^{-1}(ax)^2\tanh^{-1}\left(e^{i\tan^{-1}(ax)}\right)}{c\sqrt{c+a^2cx^2}}+$$

$$= -\frac{2}{c\sqrt{c+a^2cx^2}}-\frac{2ax\tan^{-1}(ax)}{c\sqrt{c+a^2cx^2}}+\frac{\tan^{-1}(ax)^2}{c\sqrt{c+a^2cx^2}}-\frac{2\sqrt{1+a^2x^2}\tan^{-1}(ax)^2\tanh^{-1}\left(e^{i\tan^{-1}(ax)}\right)}{c\sqrt{c+a^2cx^2}}+$$

Mathematica [A] time = 0.308159, size = 204, normalized size = 0.66

$$\frac{\sqrt{a^2x^2+1}\left(2i\tan^{-1}(ax)\text{PolyLog}\left(2,-e^{i\tan^{-1}(ax)}\right)-2i\tan^{-1}(ax)\text{PolyLog}\left(2,e^{i\tan^{-1}(ax)}\right)-2\text{PolyLog}\left(3,-e^{i\tan^{-1}(ax)}\right)+2\right.}{c\sqrt{c}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTan[a*x]^2/(x*(c + a^2*c*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[1 + a^2*x^2]*(-2/Sqrt[1 + a^2*x^2] - (2*a*x*ArcTan[a*x])/Sqrt[1 + a^2
*x^2] + ArcTan[a*x]^2/Sqrt[1 + a^2*x^2] + ArcTan[a*x]^2*Log[1 - E^(I*ArcTan
[a*x])] - ArcTan[a*x]^2*Log[1 + E^(I*ArcTan[a*x])] + (2*I)*ArcTan[a*x]*Poly
Log[2, -E^(I*ArcTan[a*x])] - (2*I)*ArcTan[a*x]*PolyLog[2, E^(I*ArcTan[a*x])
] - 2*PolyLog[3, -E^(I*ArcTan[a*x])] + 2*PolyLog[3, E^(I*ArcTan[a*x])]))/(c
*Sqrt[c*(1 + a^2*x^2)])
```

Maple [A] time = 0.297, size = 307, normalized size = 1.

$$\frac{\left((\arctan(ax))^2-2+2\,i\arctan(ax)\right)(1+iax)}{\left(2\,a^2x^2+2\right)c^2}\sqrt{c\,(ax-i)\,(ax+i)}-\frac{(-1+iax)\left((\arctan(ax))^2-2-2\,i\arctan(ax)\right)}{\left(2\,a^2x^2+2\right)c^2}\sqrt{c\,(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^2/x/(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] 1/2*(arctan(a*x)^2-2+2*I*arctan(a*x))*(1+I*a*x)*(c*(a*x-I)*(a*x+I))^(1/2)/(
a^2*x^2+1)/c^2-1/2*(c*(a*x-I)*(a*x+I))^(1/2)*(-1+I*a*x)*(arctan(a*x)^2-2-2*
I*arctan(a*x))/(a^2*x^2+1)/c^2-(arctan(a*x)^2*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1
/2))-arctan(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*I*arctan(a*x)*polylo
g(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*I*arctan(a*x)*polylog(2,(1+I*a*x)/(a^2*
x^2+1)^(1/2))+2*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*polylog(3,(1+I*a*
x)/(a^2*x^2+1)^(1/2)))*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)^(1/2)/c^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a^2cx^2+c}\arctan\left(ax\right)^2}{a^4c^2x^5+2\,a^2c^2x^3+c^2x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2/(a^4*c^2*x^5 + 2*a^2*c^2*x^3 + c
^2*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atan}^2\left(ax\right)}{x\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**2/x/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(atan(a*x)**2/(x*(c*(a**2*x**2 + 1))**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan\left(ax\right)^2}{\left(a^2cx^2+c\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)^2/((a^2*c*x^2 + c)^(3/2)*x), x)
```

3.344 $\int \frac{\tan^{-1}(ax)^2}{x^2\left(c+a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=293

$$\frac{2ia\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{c\sqrt{a^2cx^2+c}}-\frac{2ia\sqrt{a^2x^2+1}\text{PolyLog}\left(2,\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{c\sqrt{a^2cx^2+c}}-\frac{\sqrt{a^2cx^2+c}\tan^{-1}(ax)^2}{c^2x}+\frac{2a^2x}{c\sqrt{a^2cx^2+c}}-\frac{a^2}{c}$$

```
[Out] (2*a^2*x)/(c*Sqrt[c + a^2*c*x^2]) - (2*a*ArcTan[a*x])/(c*Sqrt[c + a^2*c*x^2
]) - (a^2*x*ArcTan[a*x]^2)/(c*Sqrt[c + a^2*c*x^2]) - (Sqrt[c + a^2*c*x^2]*A
rcTan[a*x]^2)/(c^2*x) - (4*a*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTanh[Sqrt[1 +
 I*a*x]/Sqrt[1 - I*a*x]])/(c*Sqrt[c + a^2*c*x^2]) + ((2*I)*a*Sqrt[1 + a^2*x
^2]*PolyLog[2, -(Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x])])/(c*Sqrt[c + a^2*c*x^2])
 - ((2*I)*a*Sqrt[1 + a^2*x^2]*PolyLog[2, Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/
(c*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.431192, antiderivative size = 293, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4966, 4944, 4958, 4954, 4898, 191}

$$\frac{2ia\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{c\sqrt{a^2cx^2+c}}-\frac{2ia\sqrt{a^2x^2+1}\text{PolyLog}\left(2,\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{c\sqrt{a^2cx^2+c}}-\frac{\sqrt{a^2cx^2+c}\tan^{-1}(ax)^2}{c^2x}+\frac{2a^2x}{c\sqrt{a^2cx^2+c}}-\frac{a^2}{c}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^2/(x^2*(c + a^2*c*x^2)^(3/2)),x]
```


```
[Out] (2*a^2*x)/(c*Sqrt[c + a^2*c*x^2]) - (2*a*ArcTan[a*x])/(c*Sqrt[c + a^2*c*x^2
]) - (a^2*x*ArcTan[a*x]^2)/(c*Sqrt[c + a^2*c*x^2]) - (Sqrt[c + a^2*c*x^2]*A
rcTan[a*x]^2)/(c^2*x) - (4*a*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTanh[Sqrt[1 +
 I*a*x]/Sqrt[1 - I*a*x]])/(c*Sqrt[c + a^2*c*x^2]) + ((2*I)*a*Sqrt[1 + a^2*x
^2]*PolyLog[2, -(Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x])])/(c*Sqrt[c + a^2*c*x^2])
 - ((2*I)*a*Sqrt[1 + a^2*x^2]*PolyLog[2, Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/
(c*Sqrt[c + a^2*c*x^2])
```


Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*
x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^p
```

```
, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*q]
 && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 4944

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a +
 b*ArcTan[c*x])^p)/(d*f*(m + 1)), x] - Dist[(b*c*p)/(f*(m + 1)), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rule 4958

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4954

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x_
Symbol] :> Simp[(-2*(a + b*ArcTan[c*x])*ArcTanh[Sqrt[1 + I*c*x]/Sqrt[1 - I*
c*x]])/Sqrt[d], x] + (Simp[(I*b*PolyLog[2, -(Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x
])])/Sqrt[d], x] - Simp[(I*b*PolyLog[2, Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]])/S
qrt[d], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

Rule 4898

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[(b*p*(a + b*ArcTan[c*x])^(p - 1))/(c*d*Sqrt[d + e*x^2]), x]
 + (-Dist[b^2*p*(p - 1), Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2),
 x], x] + Simp[(x*(a + b*ArcTan[c*x])^p)/(d*Sqrt[d + e*x^2]), x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^2}{x^2\left(c+a^2cx^2\right)^{3/2}}\,dx = -\left(a^2\int \frac{\tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^{3/2}}\,dx\right) + \frac{\int \frac{\tan^{-1}(ax)^2}{x^2\sqrt{c+a^2cx^2}}\,dx}{c}$$

$$= -\frac{2a\tan^{-1}(ax)}{c\sqrt{c+a^2cx^2}} - \frac{a^2x\tan^{-1}(ax)^2}{c\sqrt{c+a^2cx^2}} - \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{c^2x} + \left(2a^2\right)\int \frac{1}{\left(c+a^2cx^2\right)^{3/2}}\,dx + \frac{(2a}{}$$

$$= \frac{2a^2x}{c\sqrt{c+a^2cx^2}} - \frac{2a\tan^{-1}(ax)}{c\sqrt{c+a^2cx^2}} - \frac{a^2x\tan^{-1}(ax)^2}{c\sqrt{c+a^2cx^2}} - \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{c^2x} + \frac{\left(2a\sqrt{1+a^2x^2}\right)\int}{c\sqrt{c+a^2c}}$$

$$= \frac{2a^2x}{c\sqrt{c+a^2cx^2}} - \frac{2a\tan^{-1}(ax)}{c\sqrt{c+a^2cx^2}} - \frac{a^2x\tan^{-1}(ax)^2}{c\sqrt{c+a^2cx^2}} - \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{c^2x} - \frac{4a\sqrt{1+a^2x^2}\tan^{-}}{c\sqrt{c}}$$

Mathematica [A] time = 1.04784, size = 226, normalized size = 0.77

$$\frac{a\left(4i\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-e^{i\tan^{-1}(ax)}\right) - 4i\sqrt{a^2x^2+1}\text{PolyLog}\left(2,e^{i\tan^{-1}(ax)}\right) + 4\sqrt{a^2x^2+1}\tan^{-1}(ax)\log\left(1-e^{i\tan^{-1}(}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTan[a*x]^2/(x^2*(c + a^2*c*x^2)^(3/2)),x]
```

```
[Out] (a*(4*a*x - 4*ArcTan[a*x] - 2*a*x*ArcTan[a*x]^2 - (a*x*ArcTan[a*x]^2*Csc[Ar
cTan[a*x]/2]^2)/2 + 4*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*Log[1 - E^(I*ArcTan[a*x
])] - 4*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*Log[1 + E^(I*ArcTan[a*x])] + (4*I)*Sq
rt[1 + a^2*x^2]*PolyLog[2, -E^(I*ArcTan[a*x])] - (4*I)*Sqrt[1 + a^2*x^2]*Po
lyLog[2, E^(I*ArcTan[a*x])] - (2*(1 + a^2*x^2)*ArcTan[a*x]^2*Sin[ArcTan[a*x
]/2]^2)/(a*x)))/(2*c*Sqrt[c + a^2*c*x^2])
```

Maple [A] time = 0.306, size = 279, normalized size = 1.

$$-\frac{a\left((\arctan(ax))^2-2+2i\arctan(ax)\right)(ax-i)}{\left(2a^2x^2+2\right)c^2}\sqrt{c(ax-i)(ax+i)} - \frac{(ax+i)\left((\arctan(ax))^2-2-2i\arctan(ax)\right)a}{\left(2a^2x^2+2\right)c^2}\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^2/x^2/(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] -1/2*a*(arctan(a*x)^2-2+2*I*arctan(a*x))*(a*x-I)*(c*(a*x-I)*(a*x+I))^(1/2)/
(a^2*x^2+1)/c^2-1/2*(c*(a*x-I)*(a*x+I))^(1/2)*(a*x+I)*(arctan(a*x)^2-2-2*I*
arctan(a*x))*a/(a^2*x^2+1)/c^2-arctan(a*x)^2*(c*(a*x-I)*(a*x+I))^(1/2)/x/c^
2-2*I*a*(I*arctan(a*x)*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*arctan(a*x)*ln(1
+(1+I*a*x)/(a^2*x^2+1)^(1/2))+polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))-polylo
g(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2)))/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(a*x+I))^(1
/2)/c^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x^2/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a^2cx^2+c}\arctan\left(ax\right)^2}{a^4c^2x^6+2\,a^2c^2x^4+c^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x^2/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2/(a^4*c^2*x^6 + 2*a^2*c^2*x^4 + c
^2*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{atan}^2\left(ax\right)}{x^2\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**2/x**2/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(atan(a*x)**2/(x**2*(c*(a**2*x**2 + 1))**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan (ax)^2}{\left(a^2cx^2+c\right)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x^2/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)^2/((a^2*c*x^2 + c)^(3/2)*x^2), x)
```

3.345 $\int \frac{\tan^{-1}(ax)^2}{x^3\left(c+a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=422

$$-\frac{3ia^2\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,-e^{i\tan^{-1}(ax)}\right)}{c\sqrt{a^2cx^2+c}}+\frac{3ia^2\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,e^{i\tan^{-1}(ax)}\right)}{c\sqrt{a^2cx^2+c}}+\frac{3a^2\sqrt{a^2x^2+1}}{c}$$

```
[Out] (2*a^2)/(c*Sqrt[c + a^2*c*x^2]) + (2*a^3*x*ArcTan[a*x])/(c*Sqrt[c + a^2*c*x
^2]) - (a*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(c^2*x) - (a^2*ArcTan[a*x]^2)/(c
*Sqrt[c + a^2*c*x^2]) - (Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(2*c^2*x^2) + (
3*a^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*ArcTanh[E^(I*ArcTan[a*x])])/(c*Sqrt[c
 + a^2*c*x^2]) - (a^2*ArcTanh[Sqrt[c + a^2*c*x^2]/Sqrt[c]])/c^(3/2) - ((3*I
)*a^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])])/(c*Sqrt
[c + a^2*c*x^2]) + ((3*I)*a^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, E^(I
*ArcTan[a*x])])/(c*Sqrt[c + a^2*c*x^2]) + (3*a^2*Sqrt[1 + a^2*x^2]*PolyLog[
3, -E^(I*ArcTan[a*x])])/(c*Sqrt[c + a^2*c*x^2]) - (3*a^2*Sqrt[1 + a^2*x^2]*
PolyLog[3, E^(I*ArcTan[a*x])])/(c*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 1.1284, antiderivative size = 422, normalized size of antiderivative = 1., number of steps used = 27, number of rules used = 14, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {4966, 4962, 4944, 266, 63, 208, 4958, 4956, 4183, 2531, 2282, 6589, 4930, 4894}

$$-\frac{3ia^2\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,-e^{i\tan^{-1}(ax)}\right)}{c\sqrt{a^2cx^2+c}}+\frac{3ia^2\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,e^{i\tan^{-1}(ax)}\right)}{c\sqrt{a^2cx^2+c}}+\frac{3a^2\sqrt{a^2x^2+1}}{c}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^2/(x^3*(c + a^2*c*x^2)^(3/2)),x]
```



```
[Out] (2*a^2)/(c*Sqrt[c + a^2*c*x^2]) + (2*a^3*x*ArcTan[a*x])/(c*Sqrt[c + a^2*c*x
^2]) - (a*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(c^2*x) - (a^2*ArcTan[a*x]^2)/(c
*Sqrt[c + a^2*c*x^2]) - (Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(2*c^2*x^2) + (
3*a^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*ArcTanh[E^(I*ArcTan[a*x])])/(c*Sqrt[c
 + a^2*c*x^2]) - (a^2*ArcTanh[Sqrt[c + a^2*c*x^2]/Sqrt[c]])/c^(3/2) - ((3*I
)*a^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])])/(c*Sqrt
[c + a^2*c*x^2]) + ((3*I)*a^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, E^(I
*ArcTan[a*x])])/(c*Sqrt[c + a^2*c*x^2]) + (3*a^2*Sqrt[1 + a^2*x^2]*PolyLog[
```

```
3, -E^(I*ArcTan[a*x])])/(c*Sqrt[c + a^2*c*x^2]) - (3*a^2*Sqrt[1 + a^2*x^2]*
PolyLog[3, E^(I*ArcTan[a*x])])/(c*Sqrt[c + a^2*c*x^2])
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*
x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^p
, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*q]
 && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 4962

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Ar
cTan[c*x])^p)/(d*f*(m + 1)), x] + (-Dist[(b*c*p)/(f*(m + 1)), Int[((f*x)^(m
 + 1)*(a + b*ArcTan[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] - Dist[(c^2*(m +
 2))/(f^2*(m + 1)), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/Sqrt[d + e*x^
2], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] &
& LtQ[m, -1] && NeQ[m, -2]
```

Rule 4944

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a +
 b*ArcTan[c*x])^p)/(d*f*(m + 1)), x] - Dist[(b*c*p)/(f*(m + 1)), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 4958

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4956

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Dist[1/Sqrt[d], Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTan
[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && G
tQ[d, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 4894

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[(x*(a + b*ArcTan[c*x]))/(d*Sq
rt[d + e*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^2}{x^3\left(c+a^2cx^2\right)^{3/2}}\,dx = -\left(a^2\int \frac{\tan^{-1}(ax)^2}{x\left(c+a^2cx^2\right)^{3/2}}\,dx\right) + \frac{\int \frac{\tan^{-1}(ax)^2}{x^3\sqrt{c+a^2cx^2}}\,dx}{c}$$

$$= -\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2c^2x^2} + a^4\int \frac{x\tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^{3/2}}\,dx + \frac{a\int \frac{\tan^{-1}(ax)}{x^2\sqrt{c+a^2cx^2}}\,dx}{c} - \frac{a^2\int \frac{\tan^{-1}(ax)^2}{x\sqrt{c+a^2cx^2}}\,dx}{2c} - \frac{a^2}{}$$

$$= -\frac{a\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{c^2x} - \frac{a^2\tan^{-1}(ax)^2}{c\sqrt{c+a^2cx^2}} - \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2c^2x^2} + \left(2a^3\right)\int \frac{\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^{3/2}}\,d$$

$$= \frac{2a^2}{c\sqrt{c+a^2cx^2}} + \frac{2a^3x\tan^{-1}(ax)}{c\sqrt{c+a^2cx^2}} - \frac{a\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{c^2x} - \frac{a^2\tan^{-1}(ax)^2}{c\sqrt{c+a^2cx^2}} - \frac{\sqrt{c+a^2cx^2}\tan^{-1}(}{2c^2x^2}$$

$$= \frac{2a^2}{c\sqrt{c+a^2cx^2}} + \frac{2a^3x\tan^{-1}(ax)}{c\sqrt{c+a^2cx^2}} - \frac{a\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{c^2x} - \frac{a^2\tan^{-1}(ax)^2}{c\sqrt{c+a^2cx^2}} - \frac{\sqrt{c+a^2cx^2}\tan^{-1}(}{2c^2x^2}$$

$$= \frac{2a^2}{c\sqrt{c+a^2cx^2}} + \frac{2a^3x\tan^{-1}(ax)}{c\sqrt{c+a^2cx^2}} - \frac{a\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{c^2x} - \frac{a^2\tan^{-1}(ax)^2}{c\sqrt{c+a^2cx^2}} - \frac{\sqrt{c+a^2cx^2}\tan^{-1}(}{2c^2x^2}$$

$$= \frac{2a^2}{c\sqrt{c+a^2cx^2}} + \frac{2a^3x\tan^{-1}(ax)}{c\sqrt{c+a^2cx^2}} - \frac{a\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{c^2x} - \frac{a^2\tan^{-1}(ax)^2}{c\sqrt{c+a^2cx^2}} - \frac{\sqrt{c+a^2cx^2}\tan^{-1}(}{2c^2x^2}$$

$$= \frac{2a^2}{c\sqrt{c+a^2cx^2}} + \frac{2a^3x\tan^{-1}(ax)}{c\sqrt{c+a^2cx^2}} - \frac{a\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{c^2x} - \frac{a^2\tan^{-1}(ax)^2}{c\sqrt{c+a^2cx^2}} - \frac{\sqrt{c+a^2cx^2}\tan^{-1}(}{2c^2x^2}$$

<u>Mathematica</u> [A] time = 2.23918, size = 371, normalized size = 0.88

$$\frac{a^2\left(-24i\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,-e^{i\tan^{-1}(ax)}\right)+24i\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,e^{i\tan^{-1}(ax)}\right)+24\sqrt{a^2x^2+1}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTan[a*x]^2/(x^3*(c + a^2*c*x^2)^(3/2)),x]
```

```
[Out] (a^2*(16 + 16*a*x*ArcTan[a*x] - 8*ArcTan[a*x]^2 - 2*a*x*ArcTan[a*x]*Csc[Arc
Tan[a*x]/2]^2 - Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*Csc[ArcTan[a*x]/2]^2 - 12*S
qrt[1 + a^2*x^2]*ArcTan[a*x]^2*Log[1 - E^(I*ArcTan[a*x])] + 12*Sqrt[1 + a^2
```

```
*x^2]*ArcTan[a*x]^2*Log[1 + E^(I*ArcTan[a*x])] + 8*Sqrt[1 + a^2*x^2]*Log[Ta
n[ArcTan[a*x]/2]] - (24*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, -E^(I*A
rcTan[a*x])] + (24*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, E^(I*ArcTan[
a*x])] + 24*Sqrt[1 + a^2*x^2]*PolyLog[3, -E^(I*ArcTan[a*x])] - 24*Sqrt[1 +
a^2*x^2]*PolyLog[3, E^(I*ArcTan[a*x])] + Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*Se
c[ArcTan[a*x]/2]^2 - 4*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*Tan[ArcTan[a*x]/2]))/(
8*c*Sqrt[c + a^2*c*x^2])
```

Maple [A] time = 0.393, size = 376, normalized size = 0.9

$$-\frac{a^2\left((\arctan(ax))^2-2+2i\arctan(ax)\right)(1+iax)}{\left(2a^2x^2+2\right)c^2}\sqrt{c(ax-i)(ax+i)}+\frac{(-1+iax)\left((\arctan(ax))^2-2-2i\arctan(ax\right.}{\left(2a^2x^2+2\right)c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^2/x^3/(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] -1/2*a^2*(arctan(a*x)^2-2+2*I*arctan(a*x))*(1+I*a*x)*(c*(a*x-I)*(a*x+I))^(1
/2)/(a^2*x^2+1)/c^2+1/2*(c*(a*x-I)*(a*x+I))^(1/2)*(-1+I*a*x)*(arctan(a*x)^2
-2-2*I*arctan(a*x))*a^2/(a^2*x^2+1)/c^2-1/2*(2*a*x+arctan(a*x))*arctan(a*x)
*(c*(a*x-I)*(a*x+I))^(1/2)/c^2/x^2+1/2*a^2*(3*arctan(a*x)^2*ln(1+(1+I*a*x)/
(a^2*x^2+1)^(1/2))-3*arctan(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*I*ar
ctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I*arctan(a*x)*polylog(2
,(1+I*a*x)/(a^2*x^2+1)^(1/2))-4*arctanh((1+I*a*x)/(a^2*x^2+1)^(1/2))+6*poly
log(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2)
))*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)^(1/2)/c^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x^3/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{a^4c^2x^7+2\,a^2c^2x^5+c^2x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x^3/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2/(a^4*c^2*x^7 + 2*a^2*c^2*x^5 + c
^2*x^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atan}^2(ax)}{x^3\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**2/x**3/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(atan(a*x)**2/(x**3*(c*(a**2*x**2 + 1))**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(ax)^2}{\left(a^2cx^2+c\right)^{\frac{3}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x^3/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)^2/((a^2*c*x^2 + c)^(3/2)*x^3), x)
```

3.346 $\int \frac{\tan^{-1}(ax)^2}{x^4(c+a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=397

$$-\frac{11ia^3\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{3c\sqrt{a^2cx^2+c}}+\frac{11ia^3\sqrt{a^2x^2+1}\text{PolyLog}\left(2,\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{3c\sqrt{a^2cx^2+c}}-\frac{a^2\sqrt{a^2cx^2+c}}{3c^2x}+\frac{5a^2\sqrt{a^2cx^2+c}\tan^{-1}}{3c^2x}$$

```
[Out] (-2*a^4*x)/(c*Sqrt[c + a^2*c*x^2]) - (a^2*Sqrt[c + a^2*c*x^2])/(3*c^2*x) +
(2*a^3*ArcTan[a*x])/(c*Sqrt[c + a^2*c*x^2]) - (a*Sqrt[c + a^2*c*x^2]*ArcTan
[a*x])/(3*c^2*x^2) + (a^4*x*ArcTan[a*x]^2)/(c*Sqrt[c + a^2*c*x^2]) - (Sqrt[
c + a^2*c*x^2]*ArcTan[a*x]^2)/(3*c^2*x^3) + (5*a^2*Sqrt[c + a^2*c*x^2]*ArcT
an[a*x]^2)/(3*c^2*x) + (22*a^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTanh[Sqrt[1
 + I*a*x]/Sqrt[1 - I*a*x]])/(3*c*Sqrt[c + a^2*c*x^2]) - (((11*I)/3)*a^3*Sqr
t[1 + a^2*x^2]*PolyLog[2, -(Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x])])/(c*Sqrt[c +
a^2*c*x^2]) + (((11*I)/3)*a^3*Sqrt[1 + a^2*x^2]*PolyLog[2, Sqrt[1 + I*a*x]/
Sqrt[1 - I*a*x]])/(c*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 1.20162, antiderivative size = 397, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4966, 4962, 264, 4958, 4954, 4944, 4898, 191}

$$-\frac{11ia^3\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{3c\sqrt{a^2cx^2+c}}+\frac{11ia^3\sqrt{a^2x^2+1}\text{PolyLog}\left(2,\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{3c\sqrt{a^2cx^2+c}}-\frac{a^2\sqrt{a^2cx^2+c}}{3c^2x}+\frac{5a^2\sqrt{a^2cx^2+c}\tan^{-1}}{3c^2x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^2/(x^4*(c + a^2*c*x^2)^(3/2)),x]
```


```
[Out] (-2*a^4*x)/(c*Sqrt[c + a^2*c*x^2]) - (a^2*Sqrt[c + a^2*c*x^2])/(3*c^2*x) +
(2*a^3*ArcTan[a*x])/(c*Sqrt[c + a^2*c*x^2]) - (a*Sqrt[c + a^2*c*x^2]*ArcTan
[a*x])/(3*c^2*x^2) + (a^4*x*ArcTan[a*x]^2)/(c*Sqrt[c + a^2*c*x^2]) - (Sqrt[
c + a^2*c*x^2]*ArcTan[a*x]^2)/(3*c^2*x^3) + (5*a^2*Sqrt[c + a^2*c*x^2]*ArcT
an[a*x]^2)/(3*c^2*x) + (22*a^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTanh[Sqrt[1
 + I*a*x]/Sqrt[1 - I*a*x]])/(3*c*Sqrt[c + a^2*c*x^2]) - (((11*I)/3)*a^3*Sqr
t[1 + a^2*x^2]*PolyLog[2, -(Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x])])/(c*Sqrt[c +
a^2*c*x^2]) + (((11*I)/3)*a^3*Sqrt[1 + a^2*x^2]*PolyLog[2, Sqrt[1 + I*a*x]/
Sqrt[1 - I*a*x]])/(c*Sqrt[c + a^2*c*x^2])
```


Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*
x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^p
, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*q]
 && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 4962

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Ar
cTan[c*x])^p)/(d*f*(m + 1)), x] + (-Dist[(b*c*p)/(f*(m + 1)), Int[((f*x)^(m
 + 1)*(a + b*ArcTan[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] - Dist[(c^2*(m +
 2))/(f^2*(m + 1)), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/Sqrt[d + e*x^
2], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] &
& LtQ[m, -1] && NeQ[m, -2]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 4958

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4954

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x_
Symbol] :> Simp[(-2*(a + b*ArcTan[c*x])*ArcTanh[Sqrt[1 + I*c*x]/Sqrt[1 - I*
c*x]])/Sqrt[d], x] + (Simp[(I*b*PolyLog[2, -(Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x
])])/Sqrt[d], x] - Simp[(I*b*PolyLog[2, Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]])/S
qrt[d], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

Rule 4944

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a +
 b*ArcTan[c*x])^p)/(d*f*(m + 1)), x] - Dist[(b*c*p)/(f*(m + 1)), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
```

```
& NeQ[m, -1]
```

Rule 4898

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[(b*p*(a + b*ArcTan[c*x])^(p - 1))/(c*d*Sqrt[d + e*x^2]), x]
 + (-Dist[b^2*p*(p - 1), Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2),
 x], x] + Simp[(x*(a + b*ArcTan[c*x])^p)/(d*Sqrt[d + e*x^2]), x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^2}{x^4\left(c+a^2cx^2\right)^{3/2}}\,dx = -\left(a^2\int \frac{\tan^{-1}(ax)^2}{x^2\left(c+a^2cx^2\right)^{3/2}}\,dx\right)+\frac{\int \frac{\tan^{-1}(ax)^2}{x^4\sqrt{c+a^2cx^2}}\,dx}{c}$$

$$= -\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{3c^2x^3}+a^4\int \frac{\tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^{3/2}}\,dx+\frac{(2a)\int \frac{\tan^{-1}(ax)}{x^3\sqrt{c+a^2cx^2}}\,dx}{3c}-\frac{\left(2a^2\right)\int \frac{\tan^{-1}(ax)^2}{x^2\sqrt{c+a^2cx^2}}}{3c}$$

$$= \frac{2a^3\tan^{-1}(ax)}{c\sqrt{c+a^2cx^2}}-\frac{a\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{3c^2x^2}+\frac{a^4x\tan^{-1}(ax)^2}{c\sqrt{c+a^2cx^2}}-\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{3c^2x^3}+\frac{5a^2\sqrt{c}}{}$$

$$= -\frac{2a^4x}{c\sqrt{c+a^2cx^2}}-\frac{a^2\sqrt{c+a^2cx^2}}{3c^2x}+\frac{2a^3\tan^{-1}(ax)}{c\sqrt{c+a^2cx^2}}-\frac{a\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{3c^2x^2}+\frac{a^4x\tan^{-1}(ax)^2}{c\sqrt{c+a^2cx^2}}-$$

$$= -\frac{2a^4x}{c\sqrt{c+a^2cx^2}}-\frac{a^2\sqrt{c+a^2cx^2}}{3c^2x}+\frac{2a^3\tan^{-1}(ax)}{c\sqrt{c+a^2cx^2}}-\frac{a\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{3c^2x^2}+\frac{a^4x\tan^{-1}(ax)^2}{c\sqrt{c+a^2cx^2}}-$$

Mathematica [A] time = 3.41547, size = 270, normalized size = 0.68

$$a^3\sqrt{a^2x^2+1}\left(\frac{\left(a^2x^2+1\right)^{3/2}\left(\frac{88ia^3x^3\text{PolyLog}\left(2,e^{i\tan^{-1}(ax)}\right)}{\left(a^2x^2+1\right)^{3/2}}+\tan^{-1}(ax)\left(\frac{66ax\left(\log\left(1+e^{i\tan^{-1}(ax)}\right)-\log\left(1-e^{i\tan^{-1}(ax)}\right)\right)}{\sqrt{a^2x^2+1}}+8\sin\left(2\tan^{-1}(ax)\right)-6\sin\left(4\tan^{-1}(ax)\right)+\right.\right.}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTan[a*x]^2/(x^4*(c + a^2*c*x^2)^(3/2)),x]
```

```
[Out] (a^3*Sqrt[1 + a^2*x^2]*((-88*I)*PolyLog[2, -E^(I*ArcTan[a*x])] + ((1 + a^2*
x^2)^(3/2)*(-22 + 28*Cos[2*ArcTan[a*x]] - 6*Cos[4*ArcTan[a*x]] + ArcTan[a*x
]^2*(25 - 36*Cos[2*ArcTan[a*x]] + 3*Cos[4*ArcTan[a*x]]) + ((88*I)*a^3*x^3*P
olyLog[2, E^(I*ArcTan[a*x])])/(1 + a^2*x^2)^(3/2) + ArcTan[a*x]*((66*a*x*(-
Log[1 - E^(I*ArcTan[a*x])] + Log[1 + E^(I*ArcTan[a*x])]))/Sqrt[1 + a^2*x^2]
 + 8*Sin[2*ArcTan[a*x]] + 22*(Log[1 - E^(I*ArcTan[a*x])] - Log[1 + E^(I*Arc
Tan[a*x])])*Sin[3*ArcTan[a*x]] - 6*Sin[4*ArcTan[a*x]])))/(a^3*x^3)))/(24*c*
Sqrt[c + a^2*c*x^2])
```

Maple [A] time = 0.683, size = 318, normalized size = 0.8

$$\frac{a^3\left((\arctan(ax))^2-2+2i\arctan(ax)\right)(ax-i)}{\left(2a^2x^2+2\right)c^2}\sqrt{c(ax-i)(ax+i)}+\frac{(ax+i)\left((\arctan(ax))^2-2-2i\arctan(ax)\right)a^3}{\left(2a^2x^2+2\right)c^2}\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^2/x^4/(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] 1/2*a^3*(arctan(a*x)^2-2+2*I*arctan(a*x))*(a*x-I)*(c*(a*x-I)*(a*x+I))^(1/2)
/(a^2*x^2+1)/c^2+1/2*(c*(a*x-I)*(a*x+I))^(1/2)*(a*x+I)*(arctan(a*x)^2-2-2*I
*arctan(a*x))*a^3/(a^2*x^2+1)/c^2+1/3*(5*arctan(a*x)^2*x^2*a^2-a^2*x^2-arct
an(a*x)*x*a-arctan(a*x)^2)*(c*(a*x-I)*(a*x+I))^(1/2)/x^3/c^2+11/3*I*a^3*(I*
arctan(a*x)*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*arctan(a*x)*ln(1+(1+I*a*x)/
(a^2*x^2+1)^(1/2))+polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))-polylog(2,-(1+I*a
*x)/(a^2*x^2+1)^(1/2)))/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(a*x+I))^(1/2)/c^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x^4/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{a^4c^2x^8+2\,a^2c^2x^6+c^2x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x^4/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2/(a^4*c^2*x^8 + 2*a^2*c^2*x^6 + c
^2*x^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{atan}^2(ax)}{x^4\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**2/x**4/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(atan(a*x)**2/(x**4*(c*(a**2*x**2 + 1))**(3/2)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x^4/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.347 $\int \frac{x^5 \tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^{5/2}} \, dx$

Optimal. Leaf size=400

$$-\frac{2i\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a^6c^2\sqrt{a^2cx^2+c}}+\frac{2i\sqrt{a^2x^2+1}\text{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a^6c^2\sqrt{a^2cx^2+c}}-\frac{32}{9a^6c^2\sqrt{a^2cx^2+c}}-\frac{10x\tan^{-1}(ax)}{3a^5c^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{a^2c}}{}$$

```
[Out] 2/(27*a^6*c*(c + a^2*c*x^2)^(3/2)) - 32/(9*a^6*c^2*Sqrt[c + a^2*c*x^2]) - (
2*x^3*ArcTan[a*x])/(9*a^3*c*(c + a^2*c*x^2)^(3/2)) - (10*x*ArcTan[a*x])/(3*
a^5*c^2*Sqrt[c + a^2*c*x^2]) + (x^2*ArcTan[a*x]^2)/(3*a^4*c*(c + a^2*c*x^2)
^(3/2)) + (5*ArcTan[a*x]^2)/(3*a^6*c^2*Sqrt[c + a^2*c*x^2]) + (Sqrt[c + a^2
*c*x^2]*ArcTan[a*x]^2)/(a^6*c^3) + ((4*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*Arc
Tan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/(a^6*c^2*Sqrt[c + a^2*c*x^2]) - ((2*I
)*Sqrt[1 + a^2*x^2]*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/(a^
6*c^2*Sqrt[c + a^2*c*x^2]) + ((2*I)*Sqrt[1 + a^2*x^2]*PolyLog[2, (I*Sqrt[1
+ I*a*x])/Sqrt[1 - I*a*x]])/(a^6*c^2*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.81751, antiderivative size = 400, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4964, 4930, 4890, 4886, 4894, 4940, 266, 43}

$$-\frac{2i\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a^6c^2\sqrt{a^2cx^2+c}}+\frac{2i\sqrt{a^2x^2+1}\text{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a^6c^2\sqrt{a^2cx^2+c}}-\frac{32}{9a^6c^2\sqrt{a^2cx^2+c}}-\frac{10x\tan^{-1}(ax)}{3a^5c^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{a^2c}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*ArcTan[a*x]^2)/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] 2/(27*a^6*c*(c + a^2*c*x^2)^(3/2)) - 32/(9*a^6*c^2*Sqrt[c + a^2*c*x^2]) - (
2*x^3*ArcTan[a*x])/(9*a^3*c*(c + a^2*c*x^2)^(3/2)) - (10*x*ArcTan[a*x])/(3*
a^5*c^2*Sqrt[c + a^2*c*x^2]) + (x^2*ArcTan[a*x]^2)/(3*a^4*c*(c + a^2*c*x^2)
^(3/2)) + (5*ArcTan[a*x]^2)/(3*a^6*c^2*Sqrt[c + a^2*c*x^2]) + (Sqrt[c + a^2
*c*x^2]*ArcTan[a*x]^2)/(a^6*c^3) + ((4*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*Arc
Tan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/(a^6*c^2*Sqrt[c + a^2*c*x^2]) - ((2*I
)*Sqrt[1 + a^2*x^2]*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/(a^
6*c^2*Sqrt[c + a^2*c*x^2]) + ((2*I)*Sqrt[1 + a^2*x^2]*PolyLog[2, (I*Sqrt[1
+ I*a*x])/Sqrt[1 - I*a*x]])/(a^6*c^2*Sqrt[c + a^2*c*x^2])
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Arc
Tan[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan[c
*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p
, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 4890

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4886

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[(-2*I*(a + b*ArcTan[c*x])*ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]])
/(c*Sqrt[d]), x] + (Simp[(I*b*PolyLog[2, -((I*Sqrt[1 + I*c*x])/Sqrt[1 - I*c
*x])])/(c*Sqrt[d]), x] - Simp[(I*b*PolyLog[2, (I*Sqrt[1 + I*c*x])/Sqrt[1 -
I*c*x]])/(c*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

Rule 4894

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[(x*(a + b*ArcTan[c*x]))/(d*Sq
rt[d + e*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_)*((d_) + (e_.)
*(x_)^2)^(q_), x_Symbol] :> Simp[(b*p*(f*x)^m*(d + e*x^2)^(q + 1)*(a + b*Ar
cTan[c*x])^(p - 1))/(c*d*m^2), x] + (Dist[(f^2*(m - 1))/(c^2*d*m), Int[(f*x
)^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p, x], x] - Dist[(b^2*p*(
p - 1))/m^2, Int[(f*x)^m*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x]
```

```
- Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(c^2*d*m
), x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q +
 2, 0] && LtQ[q, -1] && GtQ[p, 1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^5 \tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^{5/2}}\,dx = -\frac{\int \frac{x^3 \tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^{5/2}}\,dx}{a^2} + \frac{\int \frac{x^3 \tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^{3/2}}\,dx}{a^2 c}$$

$$= -\frac{2x^3 \tan^{-1}(ax)}{9a^3c\left(c+a^2cx^2\right)^{3/2}} + \frac{x^2 \tan^{-1}(ax)^2}{3a^4c\left(c+a^2cx^2\right)^{3/2}} + \frac{2\int \frac{x^3}{\left(c+a^2cx^2\right)^{5/2}}\,dx}{9a^2} + \frac{\int \frac{x \tan^{-1}(ax)^2}{\sqrt{c+a^2cx^2}}\,dx}{a^4c^2} - \frac{2\int \frac{x \tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^{3/2}}}{3a^4c}$$

$$= -\frac{2x^3 \tan^{-1}(ax)}{9a^3c\left(c+a^2cx^2\right)^{3/2}} + \frac{x^2 \tan^{-1}(ax)^2}{3a^4c\left(c+a^2cx^2\right)^{3/2}} + \frac{5\tan^{-1}(ax)^2}{3a^6c^2\sqrt{c+a^2cx^2}} + \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{a^6c^3} + \frac{\text{Subst}}{}$$

$$= -\frac{10}{3a^6c^2\sqrt{c+a^2cx^2}} - \frac{2x^3 \tan^{-1}(ax)}{9a^3c\left(c+a^2cx^2\right)^{3/2}} - \frac{10x \tan^{-1}(ax)}{3a^5c^2\sqrt{c+a^2cx^2}} + \frac{x^2 \tan^{-1}(ax)^2}{3a^4c\left(c+a^2cx^2\right)^{3/2}} + \frac{5\tan^{-1}(ax}{3a^6c^2\sqrt{c+a^2}}$$

$$= \frac{2}{27a^6c\left(c+a^2cx^2\right)^{3/2}} - \frac{32}{9a^6c^2\sqrt{c+a^2cx^2}} - \frac{2x^3 \tan^{-1}(ax)}{9a^3c\left(c+a^2cx^2\right)^{3/2}} - \frac{10x \tan^{-1}(ax)}{3a^5c^2\sqrt{c+a^2cx^2}} + \frac{x^2 \tan^{-1}(a}{3a^4c\left(c+a^2c\right.}$$

Mathematica [A] time = 1.31527, size = 229, normalized size = 0.57

$$\frac{-432i\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-ie^{i\tan^{-1}(ax)}\right)+432i\sqrt{a^2x^2+1}\text{PolyLog}\left(2,ie^{i\tan^{-1}(ax)}\right)-9\left(a^2x^2+1\right)\tan^{-1}(ax)^2\left(-20\cos\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^5*ArcTan[a*x]^2)/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] (8*(-95 + Cos[2*ArcTan[a*x]]) - 9*(1 + a^2*x^2)*ArcTan[a*x]^2*(-45 - 20*Cos
[2*ArcTan[a*x]] + Cos[4*ArcTan[a*x]]) - (432*I)*Sqrt[1 + a^2*x^2]*PolyLog[2
, (-I)*E^(I*ArcTan[a*x])] + (432*I)*Sqrt[1 + a^2*x^2]*PolyLog[2, I*E^(I*Arc
Tan[a*x])] + 6*ArcTan[a*x]*(-124*a*x - 72*Sqrt[1 + a^2*x^2]*Log[1 - I*E^(I*
ArcTan[a*x])] + 72*Sqrt[1 + a^2*x^2]*Log[1 + I*E^(I*ArcTan[a*x])] + (1 + a^
2*x^2)*Sin[4*ArcTan[a*x]]))/(216*a^6*c^2*Sqrt[c + a^2*c*x^2])
```

Maple [A] time = 1.515, size = 454, normalized size = 1.1

$$\frac{\left(6\,i\arctan(ax)+9\,(\arctan(ax))^2-2\right)\left(ix^3a^3+3\,a^2x^2-3\,iax-1\right)}{216\,\left(a^2x^2+1\right)^2c^3a^6}\sqrt{c\,(ax-i)\,(ax+i)}+\frac{\left(7\,(\arctan(ax))^2-14+14\,i\right.}{8\,c^3a^6\left(a^2x^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*arctan(a*x)^2/(a^2*c*x^2+c)^(5/2),x)
```

```
[Out] 1/216*(6*I*arctan(a*x)+9*arctan(a*x)^2-2)*(I*x^3*a^3+3*a^2*x^2-3*I*a*x-1)*(
c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)^2/c^3/a^6+7/8*(arctan(a*x)^2-2+2*I*arc
tan(a*x))*(1+I*a*x)*(c*(a*x-I)*(a*x+I))^(1/2)/a^6/c^3/(a^2*x^2+1)-7/8*(c*(a
*x-I)*(a*x+I))^(1/2)*(-1+I*a*x)*(arctan(a*x)^2-2-2*I*arctan(a*x))/a^6/c^3/(
a^2*x^2+1)-1/216*(c*(a*x-I)*(a*x+I))^(1/2)*(I*x^3*a^3-3*a^2*x^2-3*I*a*x+1)*
(-6*I*arctan(a*x)+9*arctan(a*x)^2-2)/a^6/c^3/(a^4*x^4+2*a^2*x^2+1)+arctan(a
*x)^2*(c*(a*x-I)*(a*x+I))^(1/2)/c^3/a^6-2*I*(I*arctan(a*x)*ln(1+I*(1+I*a*x)
/(a^2*x^2+1)^(1/2))-I*arctan(a*x)*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+dilog
(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-dilog(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))*(
c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)^(1/2)/a^6/c^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arctan(a*x)^2/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a^2cx^2 + c}x^5 \arctan(ax)^2}{a^6c^3x^6 + 3a^4c^3x^4 + 3a^2c^3x^2 + c^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arctan(a*x)^2/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^5*arctan(a*x)^2/(a^6*c^3*x^6 + 3*a^4*c^3*x^4
 + 3*a^2*c^3*x^2 + c^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5 \operatorname{atan}^2(ax)}{\left(c\left(a^2x^2 + 1\right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*atan(a*x)**2/(a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral(x**5*atan(a*x)**2/(c*(a**2*x**2 + 1))**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5 \arctan(ax)^2}{\left(a^2cx^2 + c\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arctan(a*x)^2/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x^5*arctan(a*x)^2/(a^2*c*x^2 + c)^(5/2), x)
```

3.348 $\int \frac{x^4 \tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=444

$$\frac{2i\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,-ie^{i\tan^{-1}(ax)}\right)}{a^5c^2\sqrt{a^2cx^2+c}}-\frac{2i\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,ie^{i\tan^{-1}(ax)}\right)}{a^5c^2\sqrt{a^2cx^2+c}}-\frac{2\sqrt{a^2x^2+1}\text{PolyLo}}{a^5c^2\sqrt{a^2}}$$

```
[Out] (2*x^3)/(27*a^2*c*(c + a^2*c*x^2)^(3/2)) + (22*x)/(9*a^4*c^2*Sqrt[c + a^2*c
*x^2]) - (2*x^2*ArcTan[a*x])/(9*a^3*c*(c + a^2*c*x^2)^(3/2)) - (22*ArcTan[a
*x])/(9*a^5*c^2*Sqrt[c + a^2*c*x^2]) - (x^3*ArcTan[a*x]^2)/(3*a^2*c*(c + a^
2*c*x^2)^(3/2)) - (x*ArcTan[a*x]^2)/(a^4*c^2*Sqrt[c + a^2*c*x^2]) - ((2*I)*
Sqrt[1 + a^2*x^2]*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2)/(a^5*c^2*Sqrt[c
+ a^2*c*x^2]) + ((2*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*A
rcTan[a*x])])/(a^5*c^2*Sqrt[c + a^2*c*x^2]) - ((2*I)*Sqrt[1 + a^2*x^2]*ArcT
an[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])])/(a^5*c^2*Sqrt[c + a^2*c*x^2]) - (2
*Sqrt[1 + a^2*x^2]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])])/(a^5*c^2*Sqrt[c + a^
2*c*x^2]) + (2*Sqrt[1 + a^2*x^2]*PolyLog[3, I*E^(I*ArcTan[a*x])])/(a^5*c^2*
Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.769034, antiderivative size = 444, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 12, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4964, 4890, 4888, 4181, 2531, 2282, 6589, 4898, 191, 4944, 4938, 4930}

$$\frac{2i\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,-ie^{i\tan^{-1}(ax)}\right)}{a^5c^2\sqrt{a^2cx^2+c}}-\frac{2i\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,ie^{i\tan^{-1}(ax)}\right)}{a^5c^2\sqrt{a^2cx^2+c}}-\frac{2\sqrt{a^2x^2+1}\text{PolyLo}}{a^5c^2\sqrt{a^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*ArcTan[a*x]^2)/(c + a^2*c*x^2)^(5/2),x]
```


```
[Out] (2*x^3)/(27*a^2*c*(c + a^2*c*x^2)^(3/2)) + (22*x)/(9*a^4*c^2*Sqrt[c + a^2*c
*x^2]) - (2*x^2*ArcTan[a*x])/(9*a^3*c*(c + a^2*c*x^2)^(3/2)) - (22*ArcTan[a
*x])/(9*a^5*c^2*Sqrt[c + a^2*c*x^2]) - (x^3*ArcTan[a*x]^2)/(3*a^2*c*(c + a^
2*c*x^2)^(3/2)) - (x*ArcTan[a*x]^2)/(a^4*c^2*Sqrt[c + a^2*c*x^2]) - ((2*I)*
Sqrt[1 + a^2*x^2]*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2)/(a^5*c^2*Sqrt[c
+ a^2*c*x^2]) + ((2*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*A
rcTan[a*x])])/(a^5*c^2*Sqrt[c + a^2*c*x^2]) - ((2*I)*Sqrt[1 + a^2*x^2]*ArcT
an[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])])/(a^5*c^2*Sqrt[c + a^2*c*x^2]) - (2
```

```
*Sqrt[1 + a^2*x^2]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])])/(a^5*c^2*Sqrt[c + a^
2*c*x^2]) + (2*Sqrt[1 + a^2*x^2]*PolyLog[3, I*E^(I*ArcTan[a*x])])/(a^5*c^2*
Sqrt[c + a^2*c*x^2])
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Arc
Tan[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan[c
*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p
, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rule 4890

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4888

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[c
*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && GtQ
[d, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4898

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[(b*p*(a + b*ArcTan[c*x])^(p - 1))/(c*d*Sqrt[d + e*x^2]), x]
 + (-Dist[b^2*p*(p - 1), Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2),
 x], x] + Simp[(x*(a + b*ArcTan[c*x])^p)/(d*Sqrt[d + e*x^2]), x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 4944

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a +
 b*ArcTan[c*x])^p)/(d*f*(m + 1)), x] - Dist[(b*c*p)/(f*(m + 1)), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rule 4938

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[(b*(f*x)^m*(d + e*x^2)^(q + 1))/(c*d*m^2), x] +
 (Dist[(f^2*(m - 1))/(c^2*d*m), Int[(f*x)^(m - 2)*(d + e*x^2)^(q + 1)*(a +
b*ArcTan[c*x]), x], x] - Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(q + 1)*(a + b*A
rcTan[c*x]))/(c^2*d*m), x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d
] && EqQ[m + 2*q + 2, 0] && LtQ[q, -1]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rubi steps

$$\int \frac{x^4 \tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^{5/2}}\,dx = -\frac{\int \frac{x^2\tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^{5/2}}\,dx}{a^2} + \frac{\int \frac{x^2\tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^{3/2}}\,dx}{a^2c}$$

$$= -\frac{x^3\tan^{-1}(ax)^2}{3a^2c\left(c+a^2cx^2\right)^{3/2}} + \frac{2\int \frac{x^3\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^{5/2}}\,dx}{3a} + \frac{\int \frac{\tan^{-1}(ax)^2}{\sqrt{c+a^2cx^2}}\,dx}{a^4c^2} - \frac{\int \frac{\tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^{3/2}}\,dx}{a^4c}$$

$$= \frac{2x^3}{27a^2c\left(c+a^2cx^2\right)^{3/2}} - \frac{2x^2\tan^{-1}(ax)}{9a^3c\left(c+a^2cx^2\right)^{3/2}} - \frac{2\tan^{-1}(ax)}{a^5c^2\sqrt{c+a^2cx^2}} - \frac{x^3\tan^{-1}(ax)^2}{3a^2c\left(c+a^2cx^2\right)^{3/2}} - \frac{x\tan^{-1}(a}{a^4c^2\sqrt{c+a}$$

$$= \frac{2x^3}{27a^2c\left(c+a^2cx^2\right)^{3/2}} + \frac{2x}{a^4c^2\sqrt{c+a^2cx^2}} - \frac{2x^2\tan^{-1}(ax)}{9a^3c\left(c+a^2cx^2\right)^{3/2}} - \frac{22\tan^{-1}(ax)}{9a^5c^2\sqrt{c+a^2cx^2}} - \frac{x^3\tan^{-1}(a}{3a^2c\left(c+a^2c\right.}$$

$$= \frac{2x^3}{27a^2c\left(c+a^2cx^2\right)^{3/2}} + \frac{22x}{9a^4c^2\sqrt{c+a^2cx^2}} - \frac{2x^2\tan^{-1}(ax)}{9a^3c\left(c+a^2cx^2\right)^{3/2}} - \frac{22\tan^{-1}(ax)}{9a^5c^2\sqrt{c+a^2cx^2}} - \frac{x^3\tan^{-1}(}{3a^2c\left(c+a^2\right.}$$

$$= \frac{2x^3}{27a^2c\left(c+a^2cx^2\right)^{3/2}} + \frac{22x}{9a^4c^2\sqrt{c+a^2cx^2}} - \frac{2x^2\tan^{-1}(ax)}{9a^3c\left(c+a^2cx^2\right)^{3/2}} - \frac{22\tan^{-1}(ax)}{9a^5c^2\sqrt{c+a^2cx^2}} - \frac{x^3\tan^{-1}(}{3a^2c\left(c+a^2\right.}$$

$$= \frac{2x^3}{27a^2c\left(c+a^2cx^2\right)^{3/2}} + \frac{22x}{9a^4c^2\sqrt{c+a^2cx^2}} - \frac{2x^2\tan^{-1}(ax)}{9a^3c\left(c+a^2cx^2\right)^{3/2}} - \frac{22\tan^{-1}(ax)}{9a^5c^2\sqrt{c+a^2cx^2}} - \frac{x^3\tan^{-1}(}{3a^2c\left(c+a^2\right.}$$

$$= \frac{2x^3}{27a^2c\left(c+a^2cx^2\right)^{3/2}} + \frac{22x}{9a^4c^2\sqrt{c+a^2cx^2}} - \frac{2x^2\tan^{-1}(ax)}{9a^3c\left(c+a^2cx^2\right)^{3/2}} - \frac{22\tan^{-1}(ax)}{9a^5c^2\sqrt{c+a^2cx^2}} - \frac{x^3\tan^{-1}(}{3a^2c\left(c+a^2\right.}$$

Mathematica [A] time = 0.574489, size = 239, normalized size = 0.54

$$\frac{\sqrt{c\left(a^2x^2+1\right)}\left(216i\tan^{-1}(ax)\left(\text{PolyLog}\left(2,-ie^{i\tan^{-1}(ax)}\right)-\text{PolyLog}\left(2,ie^{i\tan^{-1}(ax)}\right)\right)-216\left(\text{PolyLog}\left(3,-ie^{i\tan^{-1}(ax)}\right)\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*ArcTan[a*x]^2)/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] (Sqrt[c*(1 + a^2*x^2)]*((-270*ArcTan[a*x])/Sqrt[1 + a^2*x^2] - (135*a*x*(-2
 + ArcTan[a*x]^2))/Sqrt[1 + a^2*x^2] + 6*ArcTan[a*x]*Cos[3*ArcTan[a*x]] + 1
08*ArcTan[a*x]^2*(Log[1 - I*E^(I*ArcTan[a*x])] - Log[1 + I*E^(I*ArcTan[a*x]
)]) + (216*I)*ArcTan[a*x]*(PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - PolyLog[2,
I*E^(I*ArcTan[a*x])]) - 216*(PolyLog[3, (-I)*E^(I*ArcTan[a*x])] - PolyLog[3
, I*E^(I*ArcTan[a*x])]) + (-2 + 9*ArcTan[a*x]^2)*Sin[3*ArcTan[a*x]]))/(108*
a^5*c^3*Sqrt[1 + a^2*x^2])
```

Maple [F] time = 0.904, size = 0, normalized size = 0.

$$\int x^{4} \left(\arctan \left(a x\right)\right)^{2} \left(a^{2} c \,x^{2}+c\right)^{-\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arctan(a*x)^2/(a^2*c*x^2+c)^(5/2),x)
```

```
[Out] int(x^4*arctan(a*x)^2/(a^2*c*x^2+c)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctan(a*x)^2/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{a^{2} c x^{2} + c} x^{4} \arctan\left(a x\right)^{2}}{a^{6} c^{3} x^{6} + 3 \, a^{4} c^{3} x^{4} + 3 \, a^{2} c^{3} x^{2} + c^{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctan(a*x)^2/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^4*arctan(a*x)^2/(a^6*c^3*x^6 + 3*a^4*c^3*x^4
 + 3*a^2*c^3*x^2 + c^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4 \operatorname{atan}^2(ax)}{\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*atan(a*x)**2/(a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral(x**4*atan(a*x)**2/(c*(a**2*x**2 + 1))**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4 \arctan(ax)^2}{\left(a^2cx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctan(a*x)^2/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x^4*arctan(a*x)^2/(a^2*c*x^2 + c)^(5/2), x)
```

3.349 $\int \frac{x^3 \tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=172

$$\frac{14}{9a^4c^2\sqrt{a^2cx^2+c}}+\frac{4x\tan^{-1}(ax)}{3a^3c^2\sqrt{a^2cx^2+c}}-\frac{2\tan^{-1}(ax)^2}{3a^4c^2\sqrt{a^2cx^2+c}}-\frac{2}{27a^4c\left(a^2cx^2+c\right)^{3/2}}+\frac{2x^3\tan^{-1}(ax)}{9ac\left(a^2cx^2+c\right)^{3/2}}-\frac{x^2\tan^{-1}(ax)^2}{3a^2c\left(a^2cx^2+c\right)^{3/2}}$$

```
[Out] -2/(27*a^4*c*(c + a^2*c*x^2)^(3/2)) + 14/(9*a^4*c^2*Sqrt[c + a^2*c*x^2]) +
(2*x^3*ArcTan[a*x])/(9*a*c*(c + a^2*c*x^2)^(3/2)) + (4*x*ArcTan[a*x])/(3*a^
3*c^2*Sqrt[c + a^2*c*x^2]) - (x^2*ArcTan[a*x]^2)/(3*a^2*c*(c + a^2*c*x^2)^(
3/2)) - (2*ArcTan[a*x]^2)/(3*a^4*c^2*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.282745, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {4940, 4930, 4894, 266, 43}

$$\frac{14}{9a^4c^2\sqrt{a^2cx^2+c}}+\frac{4x\tan^{-1}(ax)}{3a^3c^2\sqrt{a^2cx^2+c}}-\frac{2\tan^{-1}(ax)^2}{3a^4c^2\sqrt{a^2cx^2+c}}-\frac{2}{27a^4c\left(a^2cx^2+c\right)^{3/2}}+\frac{2x^3\tan^{-1}(ax)}{9ac\left(a^2cx^2+c\right)^{3/2}}-\frac{x^2\tan^{-1}(ax)^2}{3a^2c\left(a^2cx^2+c\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcTan[a*x]^2)/(c + a^2*c*x^2)^(5/2),x]
```


```
[Out] -2/(27*a^4*c*(c + a^2*c*x^2)^(3/2)) + 14/(9*a^4*c^2*Sqrt[c + a^2*c*x^2]) +
(2*x^3*ArcTan[a*x])/(9*a*c*(c + a^2*c*x^2)^(3/2)) + (4*x*ArcTan[a*x])/(3*a^
3*c^2*Sqrt[c + a^2*c*x^2]) - (x^2*ArcTan[a*x]^2)/(3*a^2*c*(c + a^2*c*x^2)^(
3/2)) - (2*ArcTan[a*x]^2)/(3*a^4*c^2*Sqrt[c + a^2*c*x^2])
```


Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_)*((d_) + (e_.)
*(x_)^2)^(q_), x_Symbol] :> Simp[(b*p*(f*x)^m*(d + e*x^2)^(q + 1)*(a + b*Ar
cTan[c*x])^(p - 1))/(c*d*m^2), x] + (Dist[(f^2*(m - 1))/(c^2*d*m), Int[(f*x
)^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p, x], x] - Dist[(b^2*p*(
p - 1))/m^2, Int[(f*x)^m*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x]
- Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(c^2*d*m
), x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q +
 2, 0] && LtQ[q, -1] && GtQ[p, 1]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 4894

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[(x*(a + b*ArcTan[c*x]))/(d*Sq
rt[d + e*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^3\tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^{5/2}}\,dx = \frac{2x^3\tan^{-1}(ax)}{9ac\left(c+a^2cx^2\right)^{3/2}}-\frac{x^2\tan^{-1}(ax)^2}{3a^2c\left(c+a^2cx^2\right)^{3/2}}-\frac{2}{9}\int\frac{x^3}{\left(c+a^2cx^2\right)^{5/2}}\,dx+\frac{2\int\frac{x\tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^{3/2}}\,dx}{3a^2c}$$

$$= \frac{2x^3\tan^{-1}(ax)}{9ac\left(c+a^2cx^2\right)^{3/2}}-\frac{x^2\tan^{-1}(ax)^2}{3a^2c\left(c+a^2cx^2\right)^{3/2}}-\frac{2\tan^{-1}(ax)^2}{3a^4c^2\sqrt{c+a^2cx^2}}-\frac{1}{9}\operatorname{Subst}\left(\int\frac{x}{\left(c+a^2cx\right)^{5/2}}\,dx,x,x^{\cdots}\right.$$

$$= \frac{4}{3a^4c^2\sqrt{c+a^2cx^2}}+\frac{2x^3\tan^{-1}(ax)}{9ac\left(c+a^2cx^2\right)^{3/2}}+\frac{4x\tan^{-1}(ax)}{3a^3c^2\sqrt{c+a^2cx^2}}-\frac{x^2\tan^{-1}(ax)^2}{3a^2c\left(c+a^2cx^2\right)^{3/2}}-\frac{2\tan^{-1}(ax)}{3a^4c^2\sqrt{c+a^2\cdots}}$$

$$= -\frac{2}{27a^4c\left(c+a^2cx^2\right)^{3/2}}+\frac{14}{9a^4c^2\sqrt{c+a^2cx^2}}+\frac{2x^3\tan^{-1}(ax)}{9ac\left(c+a^2cx^2\right)^{3/2}}+\frac{4x\tan^{-1}(ax)}{3a^3c^2\sqrt{c+a^2cx^2}}-\frac{x^2\tan^{-1}\cdots}{3a^2c\left(c+a\cdots\right.}$$

Mathematica [A] time = 0.113212, size = 81, normalized size = 0.47

$$\frac{\sqrt{a^2cx^2+c}\left(42a^2x^2+6ax\left(7a^2x^2+6\right)\tan^{-1}(ax)-9\left(3a^2x^2+2\right)\tan^{-1}(ax)^2+40\right)}{27a^4c^3\left(a^2x^2+1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*ArcTan[a*x]^2)/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] (Sqrt[c + a^2*c*x^2]*(40 + 42*a^2*x^2 + 6*a*x*(6 + 7*a^2*x^2)*ArcTan[a*x] -
 9*(2 + 3*a^2*x^2)*ArcTan[a*x]^2))/(27*a^4*c^3*(1 + a^2*x^2)^2)
```

Maple [C] time = 0.982, size = 276, normalized size = 1.6

$$-\frac{\left(6\,i\arctan\left(ax\right)+9\,\left(\arctan\left(ax\right)\right)^2-2\right)\left(ix^3a^3+3\,a^2x^2-3\,iax-1\right)}{216\,\left(a^2x^2+1\right)^2c^3a^4}\sqrt{c\left(ax-i\right)\left(ax+i\right)}-\frac{\left(3\,\left(\arctan\left(ax\right)\right)^2-6+6\,i\arc}{8\,c^3a^4\left(a^2x^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctan(a*x)^2/(a^2*c*x^2+c)^(5/2),x)
```

```
[Out] -1/216*(6*I*arctan(a*x)+9*arctan(a*x)^2-2)*(I*x^3*a^3+3*a^2*x^2-3*I*a*x-1)*
(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)^2/c^3/a^4-3/8*(arctan(a*x)^2-2+2*I*ar
ctan(a*x))*(1+I*a*x)*(c*(a*x-I)*(a*x+I))^(1/2)/a^4/c^3/(a^2*x^2+1)+3/8*(c*(
a*x-I)*(a*x+I))^(1/2)*(-1+I*a*x)*(arctan(a*x)^2-2-2*I*arctan(a*x))/a^4/c^3/
(a^2*x^2+1)+1/216*(c*(a*x-I)*(a*x+I))^(1/2)*(I*x^3*a^3-3*a^2*x^2-3*I*a*x+1)
*(-6*I*arctan(a*x)+9*arctan(a*x)^2-2)/a^4/c^3/(a^4*x^4+2*a^2*x^2+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3\arctan\left(ax\right)^2}{\left(a^2cx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^2/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3*arctan(a*x)^2/(a^2*c*x^2 + c)^(5/2), x)
```

Fricas [A] time = 2.2528, size = 208, normalized size = 1.21

$$\frac{\sqrt{a^2cx^2+c}\left(42\,a^2x^2-9\left(3\,a^2x^2+2\right)\arctan\left(ax\right)^2+6\left(7\,a^3x^3+6\,ax\right)\arctan\left(ax\right)+40\right)}{27\left(a^8c^3x^4+2\,a^6c^3x^2+a^4c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^2/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/27*sqrt(a^2*c*x^2 + c)*(42*a^2*x^2 - 9*(3*a^2*x^2 + 2)*arctan(a*x)^2 + 6*
(7*a^3*x^3 + 6*a*x)*arctan(a*x) + 40)/(a^8*c^3*x^4 + 2*a^6*c^3*x^2 + a^4*c^
3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3\operatorname{atan}^2\left(ax\right)}{\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atan(a*x)**2/(a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral(x**3*atan(a*x)**2/(c*(a**2*x**2 + 1))**(5/2), x)
```

Giac [A] time = 1.26272, size = 151, normalized size = 0.88

$$\frac{2\,x\left(\frac{7\,x^2}{ac}+\frac{6}{a^3c}\right)\arctan\left(ax\right)}{9\left(a^2cx^2+c\right)^{\frac{3}{2}}}-\frac{\left(3\,a^2cx^2+2\,c\right)\arctan\left(ax\right)^2}{3\left(a^2cx^2+c\right)^{\frac{3}{2}}a^4c^2}+\frac{2\left(21\,a^2cx^2+20\,c\right)}{27\left(a^2cx^2+c\right)^{\frac{3}{2}}a^4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^2/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] 2/9*x*(7*x^2/(a*c) + 6/(a^3*c))*arctan(a*x)/(a^2*c*x^2 + c)^(3/2) - 1/3*(3*
a^2*c*x^2 + 2*c)*arctan(a*x)^2/((a^2*c*x^2 + c)^(3/2)*a^4*c^2) + 2/27*(21*a
^2*c*x^2 + 20*c)/((a^2*c*x^2 + c)^(3/2)*a^4*c^2)
```

3.350 $\int \frac{x^2 \tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=139

$$-\frac{4x}{9a^2c^2\sqrt{a^2cx^2+c}}+\frac{4\tan^{-1}(ax)}{9a^3c^2\sqrt{a^2cx^2+c}}-\frac{2x^3}{27c\left(a^2cx^2+c\right)^{3/2}}+\frac{x^3\tan^{-1}(ax)^2}{3c\left(a^2cx^2+c\right)^{3/2}}+\frac{2x^2\tan^{-1}(ax)}{9ac\left(a^2cx^2+c\right)^{3/2}}$$

```
[Out] (-2*x^3)/(27*c*(c + a^2*c*x^2)^(3/2)) - (4*x)/(9*a^2*c^2*Sqrt[c + a^2*c*x^2
]) + (2*x^2*ArcTan[a*x])/(9*a*c*(c + a^2*c*x^2)^(3/2)) + (4*ArcTan[a*x])/(9
*a^3*c^2*Sqrt[c + a^2*c*x^2]) + (x^3*ArcTan[a*x]^2)/(3*c*(c + a^2*c*x^2)^(3
/2))
```

Rubi [A] time = 0.267383, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4944, 4938, 4930, 191}

$$-\frac{4x}{9a^2c^2\sqrt{a^2cx^2+c}}+\frac{4\tan^{-1}(ax)}{9a^3c^2\sqrt{a^2cx^2+c}}-\frac{2x^3}{27c\left(a^2cx^2+c\right)^{3/2}}+\frac{x^3\tan^{-1}(ax)^2}{3c\left(a^2cx^2+c\right)^{3/2}}+\frac{2x^2\tan^{-1}(ax)}{9ac\left(a^2cx^2+c\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcTan[a*x]^2)/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] (-2*x^3)/(27*c*(c + a^2*c*x^2)^(3/2)) - (4*x)/(9*a^2*c^2*Sqrt[c + a^2*c*x^2
]) + (2*x^2*ArcTan[a*x])/(9*a*c*(c + a^2*c*x^2)^(3/2)) + (4*ArcTan[a*x])/(9
*a^3*c^2*Sqrt[c + a^2*c*x^2]) + (x^3*ArcTan[a*x]^2)/(3*c*(c + a^2*c*x^2)^(3
/2))
```

Rule 4944

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a +
 b*ArcTan[c*x])^p)/(d*f*(m + 1)), x] - Dist[(b*c*p)/(f*(m + 1)), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rule 4938

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[(b*(f*x)^m*(d + e*x^2)^(q + 1))/(c*d*m^2), x] +
 (Dist[(f^2*(m - 1))/(c^2*d*m), Int[(f*x)^(m - 2)*(d + e*x^2)^(q + 1)*(a +
b*ArcTan[c*x]), x], x] - Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(q + 1)*(a + b*A
rcTan[c*x]))/(c^2*d*m), x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d
] && EqQ[m + 2*q + 2, 0] && LtQ[q, -1]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{x^2\tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^{5/2}}\,dx = \frac{x^3\tan^{-1}(ax)^2}{3c\left(c+a^2cx^2\right)^{3/2}}-\frac{1}{3}(2a)\int \frac{x^3\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^{5/2}}\,dx$$

$$= -\frac{2x^3}{27c\left(c+a^2cx^2\right)^{3/2}}+\frac{2x^2\tan^{-1}(ax)}{9ac\left(c+a^2cx^2\right)^{3/2}}+\frac{x^3\tan^{-1}(ax)^2}{3c\left(c+a^2cx^2\right)^{3/2}}-\frac{4\int \frac{x\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^{3/2}}\,dx}{9ac}$$

$$= -\frac{2x^3}{27c\left(c+a^2cx^2\right)^{3/2}}+\frac{2x^2\tan^{-1}(ax)}{9ac\left(c+a^2cx^2\right)^{3/2}}+\frac{4\tan^{-1}(ax)}{9a^3c^2\sqrt{c+a^2cx^2}}+\frac{x^3\tan^{-1}(ax)^2}{3c\left(c+a^2cx^2\right)^{3/2}}-\frac{4\int \frac{1}{\left(c+a^2cx^2\right)^{3/2}}\,d}{9a^2c}$$

$$= -\frac{2x^3}{27c\left(c+a^2cx^2\right)^{3/2}}-\frac{4x}{9a^2c^2\sqrt{c+a^2cx^2}}+\frac{2x^2\tan^{-1}(ax)}{9ac\left(c+a^2cx^2\right)^{3/2}}+\frac{4\tan^{-1}(ax)}{9a^3c^2\sqrt{c+a^2cx^2}}+\frac{x^3\tan^{-1}(ax)^2}{3c\left(c+a^2cx^2\right)^{3}}$$

Mathematica [A] time = 0.0866473, size = 80, normalized size = 0.58

$$\frac{\sqrt{a^2cx^2+c}\left(-2ax\left(7a^2x^2+6\right)+9a^3x^3\tan^{-1}(ax)^2+6\left(3a^2x^2+2\right)\tan^{-1}(ax)\right)}{27a^3c^3\left(a^2x^2+1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*ArcTan[a*x]^2)/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] (Sqrt[c + a^2*c*x^2]*(-2*a*x*(6 + 7*a^2*x^2) + 6*(2 + 3*a^2*x^2)*ArcTan[a*x
] + 9*a^3*x^3*ArcTan[a*x]^2))/(27*a^3*c^3*(1 + a^2*x^2)^2)
```

Maple [C] time = 0.733, size = 272, normalized size = 2.

$$\frac{\left(6\,i\arctan\left(ax\right)+9\,\left(\arctan\left(ax\right)\right)^{2}-2\right)\left(a^{3}x^{3}-3\,ia^{2}x^{2}-3\,ax+i\right)}{216\,\left(a^{2}x^{2}+1\right)^{2}c^{3}a^{3}}\sqrt{c\left(ax-i\right)\left(ax+i\right)}+\frac{\left(\left(\arctan\left(ax\right)\right)^{2}-2+2\,i\arcta\right.}{8\,c^{3}a^{3}\left(a^{2}x^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(a*x)^2/(a^2*c*x^2+c)^(5/2),x)
```

```
[Out] 1/216*(6*I*arctan(a*x)+9*arctan(a*x)^2-2)*(a^3*x^3-3*I*a^2*x^2-3*a*x+I)*(c*
(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)^2/c^3/a^3+1/8*(arctan(a*x)^2-2+2*I*arcta
n(a*x))*(a*x-I)*(c*(a*x-I)*(a*x+I))^(1/2)/a^3/c^3/(a^2*x^2+1)+1/8*(c*(a*x-I
)*(a*x+I))^(1/2)*(a*x+I)*(arctan(a*x)^2-2-2*I*arctan(a*x))/a^3/c^3/(a^2*x^2
+1)+1/216*(-6*I*arctan(a*x)+9*arctan(a*x)^2-2)*(c*(a*x-I)*(a*x+I))^(1/2)*(a
^3*x^3+3*I*a^2*x^2-3*a*x-I)/(a^4*x^4+2*a^2*x^2+1)/c^3/a^3
```

Maxima [A] time = 1.39334, size = 158, normalized size = 1.14

$$\frac{1}{3}\left(\frac{x}{\sqrt{a^{2}cx^{2}+c}a^{2}c^{2}}-\frac{x}{\left(a^{2}cx^{2}+c\right)^{\frac{3}{2}}a^{2}c}\right)\arctan\left(ax\right)^{2}-\frac{2\left(7\,a^{3}x^{3}+6\,ax-3\left(3\,a^{2}x^{2}+2\right)\arctan\left(ax\right)\right)a}{27\left(a^{6}c^{2}x^{2}+a^{4}c^{2}\right)\sqrt{a^{2}x^{2}+1}\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^2/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*(x/(sqrt(a^2*c*x^2 + c)*a^2*c^2) - x/((a^2*c*x^2 + c)^(3/2)*a^2*c))*arc
tan(a*x)^2 - 2/27*(7*a^3*x^3 + 6*a*x - 3*(3*a^2*x^2 + 2)*arctan(a*x))*a/((a
^6*c^2*x^2 + a^4*c^2)*sqrt(a^2*x^2 + 1)*sqrt(c))
```

Fricas [A] time = 2.32938, size = 197, normalized size = 1.42

$$\frac{\left(9\,a^{3}x^{3}\arctan\left(ax\right)^{2}-14\,a^{3}x^{3}-12\,ax+6\left(3\,a^{2}x^{2}+2\right)\arctan\left(ax\right)\right)\sqrt{a^{2}cx^{2}+c}}{27\left(a^{7}c^{3}x^{4}+2\,a^{5}c^{3}x^{2}+a^{3}c^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^2/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/27*(9*a^3*x^3*arctan(a*x)^2 - 14*a^3*x^3 - 12*a*x + 6*(3*a^2*x^2 + 2)*arc
tan(a*x))*sqrt(a^2*c*x^2 + c)/(a^7*c^3*x^4 + 2*a^5*c^3*x^2 + a^3*c^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^{2}\operatorname{atan}^{2}\left(ax\right)}{\left(c\left(a^{2}x^{2}+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(a*x)**2/(a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral(x**2*atan(a*x)**2/(c*(a**2*x**2 + 1))**(5/2), x)
```

Giac [A] time = 1.24564, size = 138, normalized size = 0.99

$$\frac{x^{3}\arctan\left(ax\right)^{2}}{3\left(a^{2}cx^{2}+c\right)^{\frac{3}{2}}c}-\frac{2}{27}\,a\left(\frac{x\left(\frac{7\,x^{2}}{ac}+\frac{6}{a^{3}c}\right)}{\left(a^{2}cx^{2}+c\right)^{\frac{3}{2}}}-\frac{3\left(3\,a^{2}cx^{2}+2\,c\right)\arctan\left(ax\right)}{\left(a^{2}cx^{2}+c\right)^{\frac{3}{2}}a^{4}c^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^2/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*x^3*arctan(a*x)^2/((a^2*c*x^2 + c)^(3/2)*c) - 2/27*a*(x*(7*x^2/(a*c) +
6/(a^3*c))/(a^2*c*x^2 + c)^(3/2) - 3*(3*a^2*c*x^2 + 2*c)*arctan(a*x)/((a^2*
c*x^2 + c)^(3/2)*a^4*c^2))
```

3.351 $\int \frac{x \tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^{5/2}} \, dx$

Optimal. Leaf size=137

$$\frac{4}{9a^2c^2\sqrt{a^2cx^2+c}}+\frac{4x\tan^{-1}(ax)}{9ac^2\sqrt{a^2cx^2+c}}+\frac{2}{27a^2c\left(a^2cx^2+c\right)^{3/2}}-\frac{\tan^{-1}(ax)^2}{3a^2c\left(a^2cx^2+c\right)^{3/2}}+\frac{2x\tan^{-1}(ax)}{9ac\left(a^2cx^2+c\right)^{3/2}}$$

```
[Out] 2/(27*a^2*c*(c + a^2*c*x^2)^(3/2)) + 4/(9*a^2*c^2*Sqrt[c + a^2*c*x^2]) + (2
*x*ArcTan[a*x])/(9*a*c*(c + a^2*c*x^2)^(3/2)) + (4*x*ArcTan[a*x])/(9*a*c^2*
Sqrt[c + a^2*c*x^2]) - ArcTan[a*x]^2/(3*a^2*c*(c + a^2*c*x^2)^(3/2))
```

Rubi [A] time = 0.141851, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {4930, 4896, 4894}

$$\frac{4}{9a^2c^2\sqrt{a^2cx^2+c}}+\frac{4x\tan^{-1}(ax)}{9ac^2\sqrt{a^2cx^2+c}}+\frac{2}{27a^2c\left(a^2cx^2+c\right)^{3/2}}-\frac{\tan^{-1}(ax)^2}{3a^2c\left(a^2cx^2+c\right)^{3/2}}+\frac{2x\tan^{-1}(ax)}{9ac\left(a^2cx^2+c\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTan[a*x]^2)/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] 2/(27*a^2*c*(c + a^2*c*x^2)^(3/2)) + 4/(9*a^2*c^2*Sqrt[c + a^2*c*x^2]) + (2
*x*ArcTan[a*x])/(9*a*c*(c + a^2*c*x^2)^(3/2)) + (4*x*ArcTan[a*x])/(9*a*c^2*
Sqrt[c + a^2*c*x^2]) - ArcTan[a*x]^2/(3*a^2*c*(c + a^2*c*x^2)^(3/2))
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 4896

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbol
] :> Simp[(b*(d + e*x^2)^(q + 1))/(4*c*d*(q + 1)^2), x] + (Dist[(2*q + 3)/(
2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]), x], x] - Simp[(x
```

```
*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]))/(2*d*(q + 1)), x]) /; FreeQ[{a, b
, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -3/2]
```

Rule 4894

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[(x*(a + b*ArcTan[c*x]))/(d*Sq
rt[d + e*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

Rubi steps

$$\int \frac{x\tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^{5/2}}\,dx = -\frac{\tan^{-1}(ax)^2}{3a^2c\left(c+a^2cx^2\right)^{3/2}}+\frac{2\int \frac{\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^{5/2}}\,dx}{3a}$$

$$= \frac{2}{27a^2c\left(c+a^2cx^2\right)^{3/2}}+\frac{2x\tan^{-1}(ax)}{9ac\left(c+a^2cx^2\right)^{3/2}}-\frac{\tan^{-1}(ax)^2}{3a^2c\left(c+a^2cx^2\right)^{3/2}}+\frac{4\int \frac{\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^{3/2}}\,dx}{9ac}$$

$$= \frac{2}{27a^2c\left(c+a^2cx^2\right)^{3/2}}+\frac{4}{9a^2c^2\sqrt{c+a^2cx^2}}+\frac{2x\tan^{-1}(ax)}{9ac\left(c+a^2cx^2\right)^{3/2}}+\frac{4x\tan^{-1}(ax)}{9ac^2\sqrt{c+a^2cx^2}}-\frac{\tan^{-1}(ax)^2}{3a^2c\left(c+a^2cx^2\right.}$$

Mathematica [A] time = 0.0815654, size = 71, normalized size = 0.52

$$\frac{\sqrt{a^2cx^2+c}\left(2\left(6a^2x^2+7\right)+6ax\left(2a^2x^2+3\right)\tan^{-1}(ax)-9\tan^{-1}(ax)^2\right)}{27c^3\left(a^3x^2+a\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcTan[a*x]^2)/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] (Sqrt[c + a^2*c*x^2]*(2*(7 + 6*a^2*x^2) + 6*a*x*(3 + 2*a^2*x^2)*ArcTan[a*x]
 - 9*ArcTan[a*x]^2))/(27*c^3*(a + a^3*x^2)^2)
```

Maple [C] time = 0.294, size = 276, normalized size = 2.

$$\frac{\left(6\,i\arctan(ax)+9\,(\arctan(ax))^2-2\right)\left(ix^3a^3+3\,a^2x^2-3\,iax-1\right)}{216\,\left(a^2x^2+1\right)^2c^3a^2}\sqrt{c\,(ax-i)\,(ax+i)}-\frac{\left((\arctan(ax))^2-2+2\,i\arctan\right.}{8\,c^3a^2\left(a^2x^2+1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(a*x)^2/(a^2*c*x^2+c)^(5/2),x)
```

```
[Out] 1/216*(6*I*arctan(a*x)+9*arctan(a*x)^2-2)*(I*x^3*a^3+3*a^2*x^2-3*I*a*x-1)*(
c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)^2/c^3/a^2-1/8*(arctan(a*x)^2-2+2*I*arc
tan(a*x))*(1+I*a*x)*(c*(a*x-I)*(a*x+I))^(1/2)/a^2/c^3/(a^2*x^2+1)+1/8*(c*(a
*x-I)*(a*x+I))^(1/2)*(-1+I*a*x)*(arctan(a*x)^2-2-2*I*arctan(a*x))/a^2/c^3/(
a^2*x^2+1)-1/216*(c*(a*x-I)*(a*x+I))^(1/2)*(I*x^3*a^3-3*a^2*x^2-3*I*a*x+1)*
(-6*I*arctan(a*x)+9*arctan(a*x)^2-2)/a^2/c^3/(a^4*x^4+2*a^2*x^2+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\arctan(ax)^2}{\left(a^2cx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^2/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x*arctan(a*x)^2/(a^2*c*x^2 + c)^(5/2), x)
```

Fricas [A] time = 2.29556, size = 186, normalized size = 1.36

$$\frac{\sqrt{a^2cx^2+c}\left(12\,a^2x^2+6\left(2\,a^3x^3+3\,ax\right)\arctan(ax)-9\,\arctan(ax)^2+14\right)}{27\left(a^6c^3x^4+2\,a^4c^3x^2+a^2c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^2/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/27*sqrt(a^2*c*x^2 + c)*(12*a^2*x^2 + 6*(2*a^3*x^3 + 3*a*x)*arctan(a*x) -
9*arctan(a*x)^2 + 14)/(a^6*c^3*x^4 + 2*a^4*c^3*x^2 + a^2*c^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\,\mathrm{atan}^2(ax)}{\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(a*x)**2/(a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral(x*atan(a*x)**2/(c*(a**2*x**2 + 1))**(5/2), x)
```

Giac [A] time = 1.25566, size = 131, normalized size = 0.96

$$\frac{2\left(\frac{2\,ax^2}{c}+\frac{3}{ac}\right)x\arctan(ax)}{9\left(a^2cx^2+c\right)^{\frac{3}{2}}}-\frac{\arctan(ax)^2}{3\left(a^2cx^2+c\right)^{\frac{3}{2}}a^2c}+\frac{2\left(6\,a^2cx^2+7\,c\right)}{27\left(a^2cx^2+c\right)^{\frac{3}{2}}a^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^2/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] 2/9*(2*a*x^2/c + 3/(a*c))*x*arctan(a*x)/(a^2*c*x^2 + c)^(3/2) - 1/3*arctan(
a*x)^2/((a^2*c*x^2 + c)^(3/2)*a^2*c) + 2/27*(6*a^2*c*x^2 + 7*c)/((a^2*c*x^2
 + c)^(3/2)*a^2*c^2)
```

3.352 $\int \frac{\tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=157

$$-\frac{40x}{27c^2\sqrt{a^2cx^2+c}}+\frac{2x\tan^{-1}(ax)^2}{3c^2\sqrt{a^2cx^2+c}}+\frac{4\tan^{-1}(ax)}{3ac^2\sqrt{a^2cx^2+c}}-\frac{2x}{27c\left(a^2cx^2+c\right)^{3/2}}+\frac{x\tan^{-1}(ax)^2}{3c\left(a^2cx^2+c\right)^{3/2}}+\frac{2\tan^{-1}(ax)}{9ac\left(a^2cx^2+c\right)^{3/2}}$$

```
[Out] (-2*x)/(27*c*(c + a^2*c*x^2)^(3/2)) - (40*x)/(27*c^2*Sqrt[c + a^2*c*x^2]) +
 (2*ArcTan[a*x])/(9*a*c*(c + a^2*c*x^2)^(3/2)) + (4*ArcTan[a*x])/(3*a*c^2*S
qrt[c + a^2*c*x^2]) + (x*ArcTan[a*x]^2)/(3*c*(c + a^2*c*x^2)^(3/2)) + (2*x*
ArcTan[a*x]^2)/(3*c^2*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.102493, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {4900, 4898, 191, 192}

$$-\frac{40x}{27c^2\sqrt{a^2cx^2+c}}+\frac{2x\tan^{-1}(ax)^2}{3c^2\sqrt{a^2cx^2+c}}+\frac{4\tan^{-1}(ax)}{3ac^2\sqrt{a^2cx^2+c}}-\frac{2x}{27c\left(a^2cx^2+c\right)^{3/2}}+\frac{x\tan^{-1}(ax)^2}{3c\left(a^2cx^2+c\right)^{3/2}}+\frac{2\tan^{-1}(ax)}{9ac\left(a^2cx^2+c\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^2/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] (-2*x)/(27*c*(c + a^2*c*x^2)^(3/2)) - (40*x)/(27*c^2*Sqrt[c + a^2*c*x^2]) +
 (2*ArcTan[a*x])/(9*a*c*(c + a^2*c*x^2)^(3/2)) + (4*ArcTan[a*x])/(3*a*c^2*S
qrt[c + a^2*c*x^2]) + (x*ArcTan[a*x]^2)/(3*c*(c + a^2*c*x^2)^(3/2)) + (2*x*
ArcTan[a*x]^2)/(3*c^2*Sqrt[c + a^2*c*x^2])
```

Rule 4900

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(b*p*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^(p - 1))/(4*c*d
*(q + 1)^2), x] + (Dist[(2*q + 3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a
 + b*ArcTan[c*x])^p, x], x] - Dist[(b^2*p*(p - 1))/(4*(q + 1)^2), Int[(d +
e*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x] - Simp[(x*(d + e*x^2)^(q + 1)*
(a + b*ArcTan[c*x])^p)/(2*d*(q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] && E
qQ[e, c^2*d] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

Rule 4898

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[(b*p*(a + b*ArcTan[c*x])^(p - 1))/(c*d*Sqrt[d + e*x^2]), x]
 + (-Dist[b^2*p*(p - 1), Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2),
 x], x] + Simp[(x*(a + b*ArcTan[c*x])^p)/(d*Sqrt[d + e*x^2]), x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^{5/2}}\,dx = \frac{2\tan^{-1}(ax)}{9ac\left(c+a^2cx^2\right)^{3/2}} + \frac{x\tan^{-1}(ax)^2}{3c\left(c+a^2cx^2\right)^{3/2}} - \frac{2}{9}\int \frac{1}{\left(c+a^2cx^2\right)^{5/2}}\,dx + \frac{2\int \frac{\tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^{3/2}}\,dx}{3c}$$

$$= -\frac{2x}{27c\left(c+a^2cx^2\right)^{3/2}} + \frac{2\tan^{-1}(ax)}{9ac\left(c+a^2cx^2\right)^{3/2}} + \frac{4\tan^{-1}(ax)}{3ac^2\sqrt{c+a^2cx^2}} + \frac{x\tan^{-1}(ax)^2}{3c\left(c+a^2cx^2\right)^{3/2}} + \frac{2x\tan^{-1}(ax)^2}{3c^2\sqrt{c+a^2cx^2}}$$

$$= -\frac{2x}{27c\left(c+a^2cx^2\right)^{3/2}} - \frac{40x}{27c^2\sqrt{c+a^2cx^2}} + \frac{2\tan^{-1}(ax)}{9ac\left(c+a^2cx^2\right)^{3/2}} + \frac{4\tan^{-1}(ax)}{3ac^2\sqrt{c+a^2cx^2}} + \frac{x\tan^{-1}(ax)^2}{3c\left(c+a^2cx^2\right)^{3/2}}$$

Mathematica [A] time = 0.0701325, size = 86, normalized size = 0.55

$$\frac{\sqrt{a^2cx^2+c}\left(-2ax\left(20a^2x^2+21\right)+9ax\left(2a^2x^2+3\right)\tan^{-1}(ax)^2+6\left(6a^2x^2+7\right)\tan^{-1}(ax)\right)}{27ac^3\left(a^2x^2+1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]^2/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] (Sqrt[c + a^2*c*x^2]*(-2*a*x*(21 + 20*a^2*x^2) + 6*(7 + 6*a^2*x^2)*ArcTan[a
*x] + 9*a*x*(3 + 2*a^2*x^2)*ArcTan[a*x]^2))/(27*a*c^3*(1 + a^2*x^2)^2)
```

Maple [C] time = 0.271, size = 272, normalized size = 1.7

$$-\frac{\left(6 i \arctan (ax)+9 (\arctan (ax))^2-2\right)\left(a^3x^3-3 i a^2x^2-3 ax+i\right)}{216 \left(a^2x^2+1\right)^2 ac^3}\sqrt{c(ax-i)(ax+i)}+\frac{\left(3 (\arctan (ax))^2-6+6 i \text{ar}\right.}{8 ac^3 \left(a^2x^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^2/(a^2*c*x^2+c)^(5/2),x)
```

```
[Out] -1/216*(6*I*arctan(a*x)+9*arctan(a*x)^2-2)*(a^3*x^3-3*I*a^2*x^2-3*a*x+I)*(c
*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)^2/a/c^3+3/8*(arctan(a*x)^2-2+2*I*arctan
(a*x))*(a*x-I)*(c*(a*x-I)*(a*x+I))^(1/2)/c^3/a/(a^2*x^2+1)+3/8*(c*(a*x-I)*(
a*x+I))^(1/2)*(a*x+I)*(arctan(a*x)^2-2-2*I*arctan(a*x))/c^3/a/(a^2*x^2+1)-1
/216*(-6*I*arctan(a*x)+9*arctan(a*x)^2-2)*(c*(a*x-I)*(a*x+I))^(1/2)*(a^3*x^
3+3*I*a^2*x^2-3*a*x-I)/(a^4*x^4+2*a^2*x^2+1)/a/c^3
```

Maxima [A] time = 1.4638, size = 150, normalized size = 0.96

$$\frac{1}{3}\left(\frac{2x}{\sqrt{a^2cx^2+c}c^2}+\frac{x}{\left(a^2cx^2+c\right)^{\frac{3}{2}}c}\right)\arctan (ax)^2-\frac{2\left(20 a^3x^3+21 ax-3\left(6 a^2x^2+7\right)\arctan (ax)\right)a}{27\left(a^4c^2x^2+a^2c^2\right)\sqrt{a^2x^2+1}\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*(2*x/(sqrt(a^2*c*x^2 + c)*c^2) + x/((a^2*c*x^2 + c)^(3/2)*c))*arctan(a*
x)^2 - 2/27*(20*a^3*x^3 + 21*a*x - 3*(6*a^2*x^2 + 7)*arctan(a*x))*a/((a^4*c
^2*x^2 + a^2*c^2)*sqrt(a^2*x^2 + 1)*sqrt(c))
```

Fricas [A] time = 2.24433, size = 212, normalized size = 1.35

$$-\frac{\left(40 a^3x^3-9\left(2 a^3x^3+3 ax\right)\arctan (ax)^2+42 ax-6\left(6 a^2x^2+7\right)\arctan (ax)\right)\sqrt{a^2cx^2+c}}{27\left(a^5c^3x^4+2 a^3c^3x^2+ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/27*(40*a^3*x^3 - 9*(2*a^3*x^3 + 3*a*x)*arctan(a*x)^2 + 42*a*x - 6*(6*a^2
*x^2 + 7)*arctan(a*x))*sqrt(a^2*c*x^2 + c)/(a^5*c^3*x^4 + 2*a^3*c^3*x^2 + a
*c^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atan}^2(ax)}{\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**2/(a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral(atan(a*x)**2/(c*(a**2*x**2 + 1))**(5/2), x)
```

Giac [A] time = 1.25547, size = 146, normalized size = 0.93

$$\frac{\left(\frac{2\,a^2x^2}{c}+\frac{3}{c}\right)x\arctan(ax)^2}{3\left(a^2cx^2+c\right)^{\frac{3}{2}}}-\frac{2\left(\frac{20\,a^2x^2}{c}+\frac{21}{c}\right)x}{27\left(a^2cx^2+c\right)^{\frac{3}{2}}}+\frac{2\left(6\,a^2cx^2+7\,c\right)\arctan(ax)}{9\left(a^2cx^2+c\right)^{\frac{3}{2}}ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(2*a^2*x^2/c + 3/c)*x*arctan(a*x)^2/(a^2*c*x^2 + c)^(3/2) - 2/27*(20*a^
2*x^2/c + 21/c)*x/(a^2*c*x^2 + c)^(3/2) + 2/9*(6*a^2*c*x^2 + 7*c)*arctan(a*
x)/((a^2*c*x^2 + c)^(3/2)*a*c^2)
```

3.353 $\int \frac{\tan^{-1}(ax)^2}{x\left(c+a^2cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=389

$$\frac{2i\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,-e^{i\tan^{-1}(ax)}\right)}{c^2\sqrt{a^2cx^2+c}}-\frac{2i\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,e^{i\tan^{-1}(ax)}\right)}{c^2\sqrt{a^2cx^2+c}}-\frac{2\sqrt{a^2x^2+1}\text{PolyLo}}{c^2\sqrt{a^2c}}$$

```
[Out] -2/(27*c*(c + a^2*c*x^2)^(3/2)) - 22/(9*c^2*Sqrt[c + a^2*c*x^2]) - (2*a*x*A
rcTan[a*x])/(9*c*(c + a^2*c*x^2)^(3/2)) - (22*a*x*ArcTan[a*x])/(9*c^2*Sqrt[
c + a^2*c*x^2]) + ArcTan[a*x]^2/(3*c*(c + a^2*c*x^2)^(3/2)) + ArcTan[a*x]^2
/(c^2*Sqrt[c + a^2*c*x^2]) - (2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*ArcTanh[E^(
I*ArcTan[a*x])])/(c^2*Sqrt[c + a^2*c*x^2]) + ((2*I)*Sqrt[1 + a^2*x^2]*ArcTa
n[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])])/(c^2*Sqrt[c + a^2*c*x^2]) - ((2*I)*S
qrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, E^(I*ArcTan[a*x])])/(c^2*Sqrt[c + a
^2*c*x^2]) - (2*Sqrt[1 + a^2*x^2]*PolyLog[3, -E^(I*ArcTan[a*x])])/(c^2*Sqrt
[c + a^2*c*x^2]) + (2*Sqrt[1 + a^2*x^2]*PolyLog[3, E^(I*ArcTan[a*x])])/(c^2
*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.7834, antiderivative size = 389, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {4966, 4958, 4956, 4183, 2531, 2282, 6589, 4930, 4894, 4896}

$$\frac{2i\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,-e^{i\tan^{-1}(ax)}\right)}{c^2\sqrt{a^2cx^2+c}}-\frac{2i\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,e^{i\tan^{-1}(ax)}\right)}{c^2\sqrt{a^2cx^2+c}}-\frac{2\sqrt{a^2x^2+1}\text{PolyLo}}{c^2\sqrt{a^2c}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^2/(x*(c + a^2*c*x^2)^(5/2)),x]
```


```
[Out] -2/(27*c*(c + a^2*c*x^2)^(3/2)) - 22/(9*c^2*Sqrt[c + a^2*c*x^2]) - (2*a*x*A
rcTan[a*x])/(9*c*(c + a^2*c*x^2)^(3/2)) - (22*a*x*ArcTan[a*x])/(9*c^2*Sqrt[
c + a^2*c*x^2]) + ArcTan[a*x]^2/(3*c*(c + a^2*c*x^2)^(3/2)) + ArcTan[a*x]^2
/(c^2*Sqrt[c + a^2*c*x^2]) - (2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*ArcTanh[E^(
I*ArcTan[a*x])])/(c^2*Sqrt[c + a^2*c*x^2]) + ((2*I)*Sqrt[1 + a^2*x^2]*ArcTa
n[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])])/(c^2*Sqrt[c + a^2*c*x^2]) - ((2*I)*S
qrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, E^(I*ArcTan[a*x])])/(c^2*Sqrt[c + a
^2*c*x^2]) - (2*Sqrt[1 + a^2*x^2]*PolyLog[3, -E^(I*ArcTan[a*x])])/(c^2*Sqrt
[c + a^2*c*x^2]) + (2*Sqrt[1 + a^2*x^2]*PolyLog[3, E^(I*ArcTan[a*x])])/(c^2
```

```
*Sqrt[c + a^2*c*x^2])
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*
x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^p
, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*q]
 && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 4958

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4956

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Dist[1/Sqrt[d], Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTan
[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && G
tQ[d, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 4894

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[(x*(a + b*ArcTan[c*x]))/(d*Sq
rt[d + e*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

Rule 4896

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbol
] :> Simp[(b*(d + e*x^2)^(q + 1))/(4*c*d*(q + 1)^2), x] + (Dist[(2*q + 3)/(
2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]), x], x] - Simp[(x
*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]))/(2*d*(q + 1)), x]) /; FreeQ[{a, b
, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -3/2]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^2}{x\left(c+a^2cx^2\right)^{5/2}}\,dx = -\left(a^2\int \frac{x\tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^{5/2}}\,dx\right) + \frac{\int \frac{\tan^{-1}(ax)^2}{x\left(c+a^2cx^2\right)^{3/2}}\,dx}{c}$$

$$= \frac{\tan^{-1}(ax)^2}{3c\left(c+a^2cx^2\right)^{3/2}} - \frac{1}{3}(2a)\int \frac{\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^{5/2}}\,dx + \frac{\int \frac{\tan^{-1}(ax)^2}{x\sqrt{c+a^2cx^2}}\,dx}{c^2} - \frac{a^2\int \frac{x\tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^{3/2}}\,dx}{c}$$

$$= -\frac{2}{27c\left(c+a^2cx^2\right)^{3/2}} - \frac{2ax\tan^{-1}(ax)}{9c\left(c+a^2cx^2\right)^{3/2}} + \frac{\tan^{-1}(ax)^2}{3c\left(c+a^2cx^2\right)^{3/2}} + \frac{\tan^{-1}(ax)^2}{c^2\sqrt{c+a^2cx^2}} - \frac{(4a)\int \frac{\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^{3/2}}\,d}{9c}$$

$$= -\frac{2}{27c\left(c+a^2cx^2\right)^{3/2}} - \frac{22}{9c^2\sqrt{c+a^2cx^2}} - \frac{2ax\tan^{-1}(ax)}{9c\left(c+a^2cx^2\right)^{3/2}} - \frac{22ax\tan^{-1}(ax)}{9c^2\sqrt{c+a^2cx^2}} + \frac{\tan^{-1}(ax)^2}{3c\left(c+a^2cx^2\right)^{3/2}}$$

$$= -\frac{2}{27c\left(c+a^2cx^2\right)^{3/2}} - \frac{22}{9c^2\sqrt{c+a^2cx^2}} - \frac{2ax\tan^{-1}(ax)}{9c\left(c+a^2cx^2\right)^{3/2}} - \frac{22ax\tan^{-1}(ax)}{9c^2\sqrt{c+a^2cx^2}} + \frac{\tan^{-1}(ax)^2}{3c\left(c+a^2cx^2\right)^{3/2}}$$

$$= -\frac{2}{27c\left(c+a^2cx^2\right)^{3/2}} - \frac{22}{9c^2\sqrt{c+a^2cx^2}} - \frac{2ax\tan^{-1}(ax)}{9c\left(c+a^2cx^2\right)^{3/2}} - \frac{22ax\tan^{-1}(ax)}{9c^2\sqrt{c+a^2cx^2}} + \frac{\tan^{-1}(ax)^2}{3c\left(c+a^2cx^2\right)^{3/2}}$$

$$= -\frac{2}{27c\left(c+a^2cx^2\right)^{3/2}} - \frac{22}{9c^2\sqrt{c+a^2cx^2}} - \frac{2ax\tan^{-1}(ax)}{9c\left(c+a^2cx^2\right)^{3/2}} - \frac{22ax\tan^{-1}(ax)}{9c^2\sqrt{c+a^2cx^2}} + \frac{\tan^{-1}(ax)^2}{3c\left(c+a^2cx^2\right)^{3/2}}$$

$$= -\frac{2}{27c\left(c+a^2cx^2\right)^{3/2}} - \frac{22}{9c^2\sqrt{c+a^2cx^2}} - \frac{2ax\tan^{-1}(ax)}{9c\left(c+a^2cx^2\right)^{3/2}} - \frac{22ax\tan^{-1}(ax)}{9c^2\sqrt{c+a^2cx^2}} + \frac{\tan^{-1}(ax)^2}{3c\left(c+a^2cx^2\right)^{3/2}}$$

Mathematica [A] time = 0.50098, size = 246, normalized size = 0.63

$$\left(a^2x^2+1\right)^{3/2}\left(216i\tan^{-1}(ax)\mathrm{PolyLog}\left(2,-e^{i\tan^{-1}(ax)}\right) - 216i\tan^{-1}(ax)\mathrm{PolyLog}\left(2,e^{i\tan^{-1}(ax)}\right) - 216\mathrm{PolyLog}\left(3,-e^{i\tan}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTan[a*x]^2/(x*(c + a^2*c*x^2)^(5/2)),x]
```

```
[Out] ((1 + a^2*x^2)^(3/2)*(-270/Sqrt[1 + a^2*x^2] - (270*a*x*ArcTan[a*x])/Sqrt[1
 + a^2*x^2] + (135*ArcTan[a*x]^2)/Sqrt[1 + a^2*x^2] - 2*Cos[3*ArcTan[a*x]]
```

```
+ 9*ArcTan[a*x]^2*Cos[3*ArcTan[a*x]] + 108*ArcTan[a*x]^2*Log[1 - E^(I*ArcTa
n[a*x])] - 108*ArcTan[a*x]^2*Log[1 + E^(I*ArcTan[a*x])] + (216*I)*ArcTan[a*
x]*PolyLog[2, -E^(I*ArcTan[a*x])] - (216*I)*ArcTan[a*x]*PolyLog[2, E^(I*Arc
Tan[a*x])] - 216*PolyLog[3, -E^(I*ArcTan[a*x])] + 216*PolyLog[3, E^(I*ArcTa
n[a*x])] - 6*ArcTan[a*x]*Sin[3*ArcTan[a*x]]))/(108*c*(c*(1 + a^2*x^2))^(3/2
))
```

Maple [A] time = 0.345, size = 461, normalized size = 1.2

$$-\frac{\left(6\,i\arctan\left(ax\right)+9\,\left(\arctan\left(ax\right)\right)^{2}-2\right)\left(ix^{3}a^{3}+3\,a^{2}x^{2}-3\,iax-1\right)}{216\,\left(a^{2}x^{2}+1\right)^{2}c^{3}}\sqrt{c\left(ax-i\right)\left(ax+i\right)}+\frac{\left(5\,\left(\arctan\left(ax\right)\right)^{2}-10+10}{8\,c^{3}\left(a^{2}x^{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^2/x/(a^2*c*x^2+c)^(5/2),x)
```

```
[Out] -1/216*(6*I*arctan(a*x)+9*arctan(a*x)^2-2)*(I*x^3*a^3+3*a^2*x^2-3*I*a*x-1)*
(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)^2/c^3+5/8*(arctan(a*x)^2-2+2*I*arctan
(a*x))*(1+I*a*x)*(c*(a*x-I)*(a*x+I))^(1/2)/c^3/(a^2*x^2+1)-5/8*(c*(a*x-I)*(
a*x+I))^(1/2)*(-1+I*a*x)*(arctan(a*x)^2-2-2*I*arctan(a*x))/c^3/(a^2*x^2+1)+
1/216*(c*(a*x-I)*(a*x+I))^(1/2)*(I*x^3*a^3-3*a^2*x^2-3*I*a*x+1)*(-6*I*arcta
n(a*x)+9*arctan(a*x)^2-2)/c^3/(a^4*x^4+2*a^2*x^2+1)-(arctan(a*x)^2*ln(1+(1+
I*a*x)/(a^2*x^2+1)^(1/2))-arctan(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-2
*I*arctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*I*arctan(a*x)*poly
log(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2)
)-2*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2)))*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*
x^2+1)^(1/2)/c^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{a^6c^3x^7+3\,a^4c^3x^5+3\,a^2c^3x^3+c^3x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2/(a^6*c^3*x^7 + 3*a^4*c^3*x^5 + 3
*a^2*c^3*x^3 + c^3*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{atan}^2(ax)}{x\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**2/x/(a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral(atan(a*x)**2/(x*(c*(a**2*x**2 + 1))**(5/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\arctan(ax)^2}{\left(a^2cx^2+c\right)^{\frac{5}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)^2/((a^2*c*x^2 + c)^(5/2)*x), x)
```

3.354 $\int \frac{\tan^{-1}(ax)^2}{x^2\left(c+a^2cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=381

$$\frac{2ia\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{c^2\sqrt{a^2cx^2+c}}-\frac{2ia\sqrt{a^2x^2+1}\text{PolyLog}\left(2,\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{c^2\sqrt{a^2cx^2+c}}+\frac{94a^2x}{27c^2\sqrt{a^2cx^2+c}}-\frac{5a^2x\tan^{-1}(ax)^2}{3c^2\sqrt{a^2cx^2+c}}-\frac{10a\,\mathrm{t}}{3c^2\sqrt{}}$$

```
[Out] (2*a^2*x)/(27*c*(c + a^2*c*x^2)^(3/2)) + (94*a^2*x)/(27*c^2*Sqrt[c + a^2*c*
x^2]) - (2*a*ArcTan[a*x])/(9*c*(c + a^2*c*x^2)^(3/2)) - (10*a*ArcTan[a*x])/
(3*c^2*Sqrt[c + a^2*c*x^2]) - (a^2*x*ArcTan[a*x]^2)/(3*c*(c + a^2*c*x^2)^(3
/2)) - (5*a^2*x*ArcTan[a*x]^2)/(3*c^2*Sqrt[c + a^2*c*x^2]) - (Sqrt[c + a^2*
c*x^2]*ArcTan[a*x]^2)/(c^3*x) - (4*a*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTanh[
Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/(c^2*Sqrt[c + a^2*c*x^2]) + ((2*I)*a*Sqrt
[1 + a^2*x^2]*PolyLog[2, -(Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x])])/(c^2*Sqrt[c +
 a^2*c*x^2]) - ((2*I)*a*Sqrt[1 + a^2*x^2]*PolyLog[2, Sqrt[1 + I*a*x]/Sqrt[1
 - I*a*x]])/(c^2*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.685573, antiderivative size = 381, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4966, 4944, 4958, 4954, 4898, 191, 4900, 192}

$$\frac{2ia\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{c^2\sqrt{a^2cx^2+c}}-\frac{2ia\sqrt{a^2x^2+1}\text{PolyLog}\left(2,\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{c^2\sqrt{a^2cx^2+c}}+\frac{94a^2x}{27c^2\sqrt{a^2cx^2+c}}-\frac{5a^2x\tan^{-1}(ax)^2}{3c^2\sqrt{a^2cx^2+c}}-\frac{10a\,\mathrm{t}}{3c^2\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^2/(x^2*(c + a^2*c*x^2)^(5/2)),x]
```

```
[Out] (2*a^2*x)/(27*c*(c + a^2*c*x^2)^(3/2)) + (94*a^2*x)/(27*c^2*Sqrt[c + a^2*c*
x^2]) - (2*a*ArcTan[a*x])/(9*c*(c + a^2*c*x^2)^(3/2)) - (10*a*ArcTan[a*x])/
(3*c^2*Sqrt[c + a^2*c*x^2]) - (a^2*x*ArcTan[a*x]^2)/(3*c*(c + a^2*c*x^2)^(3
/2)) - (5*a^2*x*ArcTan[a*x]^2)/(3*c^2*Sqrt[c + a^2*c*x^2]) - (Sqrt[c + a^2*
c*x^2]*ArcTan[a*x]^2)/(c^3*x) - (4*a*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTanh[
Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/(c^2*Sqrt[c + a^2*c*x^2]) + ((2*I)*a*Sqrt
[1 + a^2*x^2]*PolyLog[2, -(Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x])])/(c^2*Sqrt[c +
 a^2*c*x^2]) - ((2*I)*a*Sqrt[1 + a^2*x^2]*PolyLog[2, Sqrt[1 + I*a*x]/Sqrt[1
 - I*a*x]])/(c^2*Sqrt[c + a^2*c*x^2])
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*
x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^p
, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*q]
 && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 4944

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a +
 b*ArcTan[c*x])^p)/(d*f*(m + 1)), x] - Dist[(b*c*p)/(f*(m + 1)), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rule 4958

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4954

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x_
Symbol] :> Simp[(-2*(a + b*ArcTan[c*x])*ArcTanh[Sqrt[1 + I*c*x]/Sqrt[1 - I*
c*x]])/Sqrt[d], x] + (Simp[(I*b*PolyLog[2, -(Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x
])])/Sqrt[d], x] - Simp[(I*b*PolyLog[2, Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]])/S
qrt[d], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

Rule 4898

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[(b*p*(a + b*ArcTan[c*x])^(p - 1))/(c*d*Sqrt[d + e*x^2]), x]
 + (-Dist[b^2*p*(p - 1), Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2),
 x], x] + Simp[(x*(a + b*ArcTan[c*x])^p)/(d*Sqrt[d + e*x^2]), x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 4900

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(b*p*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^(p - 1))/(4*c*d
*(q + 1)^2), x] + (Dist[(2*q + 3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a
 + b*ArcTan[c*x])^p, x], x] - Dist[(b^2*p*(p - 1))/(4*(q + 1)^2), Int[(d +
e*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x] - Simp[(x*(d + e*x^2)^(q + 1)*
(a + b*ArcTan[c*x])^p)/(2*d*(q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] && E
qQ[e, c^2*d] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^2}{x^2\left(c+a^2cx^2\right)^{5/2}}\,dx = -\left(a^2\int \frac{\tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^{5/2}}\,dx\right)+\frac{\int \frac{\tan^{-1}(ax)^2}{x^2\left(c+a^2cx^2\right)^{3/2}}\,dx}{c}$$

$$= -\frac{2a\tan^{-1}(ax)}{9c\left(c+a^2cx^2\right)^{3/2}}-\frac{a^2x\tan^{-1}(ax)^2}{3c\left(c+a^2cx^2\right)^{3/2}}+\frac{1}{9}\left(2a^2\right)\int \frac{1}{\left(c+a^2cx^2\right)^{5/2}}\,dx+\frac{\int \frac{\tan^{-1}(ax)^2}{x^2\sqrt{c+a^2cx^2}}\,dx}{c^2}-\frac{(2a}{}$$

$$= \frac{2a^2x}{27c\left(c+a^2cx^2\right)^{3/2}}-\frac{2a\tan^{-1}(ax)}{9c\left(c+a^2cx^2\right)^{3/2}}-\frac{10a\tan^{-1}(ax)}{3c^2\sqrt{c+a^2cx^2}}-\frac{a^2x\tan^{-1}(ax)^2}{3c\left(c+a^2cx^2\right)^{3/2}}-\frac{5a^2x\tan^{-1}(ax)^2}{3c^2\sqrt{c+a^2cx^2}}$$

$$= \frac{2a^2x}{27c\left(c+a^2cx^2\right)^{3/2}}+\frac{94a^2x}{27c^2\sqrt{c+a^2cx^2}}-\frac{2a\tan^{-1}(ax)}{9c\left(c+a^2cx^2\right)^{3/2}}-\frac{10a\tan^{-1}(ax)}{3c^2\sqrt{c+a^2cx^2}}-\frac{a^2x\tan^{-1}(ax)^2}{3c\left(c+a^2cx^2\right)^{3/2}}$$

$$= \frac{2a^2x}{27c\left(c+a^2cx^2\right)^{3/2}}+\frac{94a^2x}{27c^2\sqrt{c+a^2cx^2}}-\frac{2a\tan^{-1}(ax)}{9c\left(c+a^2cx^2\right)^{3/2}}-\frac{10a\tan^{-1}(ax)}{3c^2\sqrt{c+a^2cx^2}}-\frac{a^2x\tan^{-1}(ax)^2}{3c\left(c+a^2cx^2\right)^{3/2}}$$

Mathematica [A] time = 1.60847, size = 296, normalized size = 0.78

$$-\frac{a\left(-216i\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-e^{i\tan^{-1}(ax)}\right)+216i\sqrt{a^2x^2+1}\text{PolyLog}\left(2,e^{i\tan^{-1}(ax)}\right)+54\sqrt{a^2x^2+1}\tan\left(\frac{1}{2}\tan^{-1}(ax)\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTan[a*x]^2/(x^2*(c + a^2*c*x^2)^(5/2)),x]
```

```
[Out] -(a*(-378*a*x + 378*ArcTan[a*x] + 189*a*x*ArcTan[a*x]^2 + 6*Sqrt[1 + a^2*x^
2]*ArcTan[a*x]*Cos[3*ArcTan[a*x]] + 27*a*x*ArcTan[a*x]^2*Csc[ArcTan[a*x]/2]
^2 - 216*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*Log[1 - E^(I*ArcTan[a*x])] + 216*Sqr
t[1 + a^2*x^2]*ArcTan[a*x]*Log[1 + E^(I*ArcTan[a*x])] - (216*I)*Sqrt[1 + a^
2*x^2]*PolyLog[2, -E^(I*ArcTan[a*x])] + (216*I)*Sqrt[1 + a^2*x^2]*PolyLog[2
, E^(I*ArcTan[a*x])] - 2*Sqrt[1 + a^2*x^2]*Sin[3*ArcTan[a*x]] + 9*Sqrt[1 +
a^2*x^2]*ArcTan[a*x]^2*Sin[3*ArcTan[a*x]] + 54*Sqrt[1 + a^2*x^2]*ArcTan[a*x
]^2*Tan[ArcTan[a*x]/2]))/(108*c^2*Sqrt[c + a^2*c*x^2])
```

Maple [A] time = 0.345, size = 433, normalized size = 1.1

$$\frac{a\left(6\,i\arctan\left(ax\right)+9\,\left(\arctan\left(ax\right)\right)^{2}-2\right)\left(a^{3}x^{3}-3\,ia^{2}x^{2}-3\,ax+i\right)}{216\,\left(a^{2}x^{2}+1\right)^{2}c^{3}}\sqrt{c\left(ax-i\right)\left(ax+i\right)}-\frac{7\,a\left(\left(\arctan\left(ax\right)\right)^{2}-2+2\,i\arc}{8\,c^{3}\left(a^{2}x^{2}+1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^2/x^2/(a^2*c*x^2+c)^(5/2),x)
```

```
[Out] 1/216*a*(6*I*arctan(a*x)+9*arctan(a*x)^2-2)*(a^3*x^3-3*I*a^2*x^2-3*a*x+I)*(
c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)^2/c^3-7/8*a*(arctan(a*x)^2-2+2*I*arcta
n(a*x))*(a*x-I)*(c*(a*x-I)*(a*x+I))^(1/2)/c^3/(a^2*x^2+1)-7/8*(c*(a*x-I)*(a
*x+I))^(1/2)*(a*x+I)*(arctan(a*x)^2-2-2*I*arctan(a*x))*a/c^3/(a^2*x^2+1)+1/
216*(c*(a*x-I)*(a*x+I))^(1/2)*(a^3*x^3+3*I*a^2*x^2-3*a*x-I)*(-6*I*arctan(a*
x)+9*arctan(a*x)^2-2)*a/c^3/(a^4*x^4+2*a^2*x^2+1)-arctan(a*x)^2*(c*(a*x-I)*
(a*x+I))^(1/2)/x/c^3-2*I*a*(I*arctan(a*x)*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))
-I*arctan(a*x)*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))+polylog(2,(1+I*a*x)/(a^2*x
^2+1)^(1/2))-polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2)))/(a^2*x^2+1)^(1/2)*(c*
(a*x-I)*(a*x+I))^(1/2)/c^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x^2/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a^2cx^2+c}\arctan\left(ax\right)^2}{a^6c^3x^8+3\,a^4c^3x^6+3\,a^2c^3x^4+c^3x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x^2/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2/(a^6*c^3*x^8 + 3*a^4*c^3*x^6 + 3
*a^2*c^3*x^4 + c^3*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{atan}^2\left(ax\right)}{x^2\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**2/x**2/(a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral(atan(a*x)**2/(x**2*(c*(a**2*x**2 + 1))**(5/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\arctan\left(ax\right)^2}{\left(a^2cx^2+c\right)^{\frac{5}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x^2/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)^2/((a^2*c*x^2 + c)^(5/2)*x^2), x)
```

3.355 $\int x^m \left(c+a^2 c x^2\right)^2 \tan^{-1}(a x)^2 \, dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(x^m \left(a^2 c x^2+c\right)^2 \tan^{-1}(a x)^2,x\right)$$

```
[Out] Unintegrable[x^m*(c + a^2*c*x^2)^2*ArcTan[a*x]^2, x]
```

Rubi [A] time = 0.0541744, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^m \left(c+a^2 c x^2\right)^2 \tan^{-1}(a x)^2 \, dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*(c + a^2*c*x^2)^2*ArcTan[a*x]^2,x]
```

```
[Out] Defer[Int][x^m*(c + a^2*c*x^2)^2*ArcTan[a*x]^2, x]
```

Rubi steps

$$\int x^m \left(c+a^2 c x^2\right)^2 \tan^{-1}(a x)^2 \, dx=\int x^m \left(c+a^2 c x^2\right)^2 \tan^{-1}(a x)^2 \, dx$$

Mathematica [A] time = 1.76929, size = 0, normalized size = 0.

$$\int x^m \left(c+a^2 c x^2\right)^2 \tan^{-1}(a x)^2 \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*(c + a^2*c*x^2)^2*ArcTan[a*x]^2,x]
```

```
[Out] Integrate[x^m*(c + a^2*c*x^2)^2*ArcTan[a*x]^2, x]
```

Maple [A] time = 0.588, size = 0, normalized size = 0.

$$\int x^{m} \left(a^{2} c x^{2}+c \right)^{2} \left(\arctan \left(a x \right)\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^2,x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a^{4} c^{2} x^{4}+2\, a^{2} c^{2} x^{2}+c^{2}\right) x^{m} \arctan \left(a x\right)^{2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*x^m*arctan(a*x)^2, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$c^{2} \left(\int x^{m} \operatorname{atan}^{2}{\left(a x \right)}\, dx + \int 2 a^{2} x^{2} x^{m} \operatorname{atan}^{2}{\left(a x \right)}\, dx + \int a^{4} x^{4} x^{m} \operatorname{atan}^{2}{\left(a x \right)}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**2*atan(a*x)**2,x)
```

```
[Out] c**2*(Integral(x**m*atan(a*x)**2, x) + Integral(2*a**2*x**2*x**m*atan(a*x)*
*2, x) + Integral(a**4*x**4*x**m*atan(a*x)**2, x))
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2+c\right)^2 x^m \arctan(ax)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^2*x^m*arctan(a*x)^2, x)
```

3.356 $\int x^m \left(c+a^2 c x^2\right) \tan^{-1}(a x)^2 \, dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(x^m \left(a^2 c x^2+c\right) \tan^{-1}(a x)^2,x\right)$$

```
[Out] Unintegrable[x^m*(c + a^2*c*x^2)*ArcTan[a*x]^2, x]
```

Rubi [A] time = 0.0353108, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^m \left(c+a^2 c x^2\right) \tan^{-1}(a x)^2 \, dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*(c + a^2*c*x^2)*ArcTan[a*x]^2,x]
```

```
[Out] Defer[Int][x^m*(c + a^2*c*x^2)*ArcTan[a*x]^2, x]
```

Rubi steps

$$\int x^m \left(c+a^2 c x^2\right) \tan^{-1}(a x)^2 \, dx=\int x^m \left(c+a^2 c x^2\right) \tan^{-1}(a x)^2 \, dx$$

Mathematica [A] time = 0.876566, size = 0, normalized size = 0.

$$\int x^m \left(c+a^2 c x^2\right) \tan^{-1}(a x)^2 \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*(c + a^2*c*x^2)*ArcTan[a*x]^2,x]
```

```
[Out] Integrate[x^m*(c + a^2*c*x^2)*ArcTan[a*x]^2, x]
```

Maple [A] time = 0.462, size = 0, normalized size = 0.

$$\int x^{m} \left(a^{2} c \,x^{2}+c \right) \left(\arctan \left(a x \right)\right)^{2}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)*arctan(a*x)^2,x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)*arctan(a*x)^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)*arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left({\left(a^{2} c x^{2} + c\right)} x^{m} \arctan\left(a x\right)^{2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)*arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)*x^m*arctan(a*x)^2, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$c \left(\int x^{m} \operatorname{atan}^{2}{\left(a x \right)}\, dx + \int a^{2} x^{2} x^{m} \operatorname{atan}^{2}{\left(a x \right)}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)*atan(a*x)**2,x)
```

```
[Out] c*(Integral(x**m*atan(a*x)**2, x) + Integral(a**2*x**2*x**m*atan(a*x)**2, x
))
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2+c\right)x^m \arctan (ax)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)*arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)*x^m*arctan(a*x)^2, x)
```

3.357 $\int \frac{x^m \tan^{-1}(ax)^2}{c+a^2cx^2} \, dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{x^m \tan^{-1}(ax)^2}{a^2cx^2+c},x\right)$$

```
[Out] Unintegrable[(x^m*ArcTan[a*x]^2)/(c + a^2*c*x^2), x]
```

Rubi [A] time = 0.0621514, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m \tan^{-1}(ax)^2}{c+a^2cx^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*ArcTan[a*x]^2)/(c + a^2*c*x^2),x]
```

```
[Out] Defer[Int][(x^m*ArcTan[a*x]^2)/(c + a^2*c*x^2), x]
```

Rubi steps

$$\int \frac{x^m \tan^{-1}(ax)^2}{c+a^2cx^2} \, dx = \int \frac{x^m \tan^{-1}(ax)^2}{c+a^2cx^2} \, dx$$

Mathematica [A] time = 0.85548, size = 0, normalized size = 0.

$$\int \frac{x^m \tan^{-1}(ax)^2}{c+a^2cx^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*ArcTan[a*x]^2)/(c + a^2*c*x^2),x]
```

```
[Out] Integrate[(x^m*ArcTan[a*x]^2)/(c + a^2*c*x^2), x]
```

Maple [A] time = 0.464, size = 0, normalized size = 0.

$$\int \frac{x^m (\arctan (ax))^2}{a^2 c x^2 + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arctan(a*x)^2/(a^2*c*x^2+c),x)
```

```
[Out] int(x^m*arctan(a*x)^2/(a^2*c*x^2+c),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m \arctan (ax)^2}{a^2 c x^2 + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^2/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] integrate(x^m*arctan(a*x)^2/(a^2*c*x^2 + c), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^m \arctan (ax)^2}{a^2 c x^2 + c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^2/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(x^m*arctan(a*x)^2/(a^2*c*x^2 + c), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^m \operatorname{atan}^2 (ax)}{a^2 x^2 + 1}\, dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*atan(a*x)**2/(a**2*c*x**2+c),x)
```

```
[Out] Integral(x**m*atan(a*x)**2/(a**2*x**2 + 1), x)/c
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m \arctan(ax)^2}{a^2cx^2 + c} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^2/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(x^m*arctan(a*x)^2/(a^2*c*x^2 + c), x)
```

3.358 $\int \frac{x^m \tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^2}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{x^m \tan^{-1}(ax)^2}{\left(a^2cx^2+c\right)^2},x\right)$$

```
[Out] Unintegrable[(x^m*ArcTan[a*x]^2)/(c + a^2*c*x^2)^2, x]
```

Rubi [A] time = 0.0625476, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m \tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*ArcTan[a*x]^2)/(c + a^2*c*x^2)^2,x]
```

```
[Out] Defer[Int][(x^m*ArcTan[a*x]^2)/(c + a^2*c*x^2)^2, x]
```

Rubi steps

$$\int \frac{x^m \tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^2}\,dx=\int \frac{x^m \tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^2}\,dx$$

Mathematica [A] time = 0.675697, size = 0, normalized size = 0.

$$\int \frac{x^m \tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*ArcTan[a*x]^2)/(c + a^2*c*x^2)^2,x]
```

```
[Out] Integrate[(x^m*ArcTan[a*x]^2)/(c + a^2*c*x^2)^2, x]
```

Maple [A] time = 1.135, size = 0, normalized size = 0.

$$\int \frac{x^{m} \left(\arctan \left(ax\right)\right)^{2}}{\left(a^{2} c x^{2}+c\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arctan(a*x)^2/(a^2*c*x^2+c)^2,x)
```

```
[Out] int(x^m*arctan(a*x)^2/(a^2*c*x^2+c)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{m} \arctan \left(ax\right)^{2}}{\left(a^{2} c x^{2}+c\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^2/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^m*arctan(a*x)^2/(a^2*c*x^2 + c)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x^{m} \arctan \left(ax\right)^{2}}{a^{4} c^{2} x^{4}+2\, a^{2} c^{2} x^{2}+c^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^2/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] integral(x^m*arctan(a*x)^2/(a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^m \operatorname{atan}^2(ax)}{a^4x^4+2a^2x^2+1}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*atan(a*x)**2/(a**2*c*x**2+c)**2,x)
```

```
[Out] Integral(x**m*atan(a*x)**2/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m \arctan(ax)^2}{\left(a^2cx^2+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^2/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(x^m*arctan(a*x)^2/(a^2*c*x^2 + c)^2, x)
```

3.359 $\int x^m \left(c+a^2cx^2\right)^{3/2} \tan^{-1}(ax)^2 \, dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(x^m \left(a^2cx^2+c\right)^{3/2} \tan^{-1}(ax)^2,x\right)$$

```
[Out] Unintegrable[x^m*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2, x]
```

Rubi [A] time = 0.109411, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^m \left(c+a^2cx^2\right)^{3/2} \tan^{-1}(ax)^2 \, dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2,x]
```

```
[Out] Defer[Int][x^m*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2, x]
```

Rubi steps

$$\int x^m \left(c+a^2cx^2\right)^{3/2} \tan^{-1}(ax)^2 \, dx = \int x^m \left(c+a^2cx^2\right)^{3/2} \tan^{-1}(ax)^2 \, dx$$

Mathematica [A] time = 0.951496, size = 0, normalized size = 0.

$$\int x^m \left(c+a^2cx^2\right)^{3/2} \tan^{-1}(ax)^2 \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2,x]
```

```
[Out] Integrate[x^m*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2, x]
```

Maple [A] time = 0.49, size = 0, normalized size = 0.

$$\int x^{m} \left(a^{2} c x^{2}+c\right)^{\frac{3}{2}} \left(\arctan \left(a x\right)\right)^{2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^2,x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(a^{2} c x^{2}+c\right)^{\frac{3}{2}} x^{m} \arctan \left(a x\right)^{2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)*x^m*arctan(a*x)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^{2} c x^{2}+c\right)^{\frac{3}{2}} x^{m} \arctan \left(a x\right)^{2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)^(3/2)*x^m*arctan(a*x)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**(3/2)*atan(a*x)**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.360 $\int x^m \sqrt{c+a^2 c x^2} \tan^{-1}(a x)^2 \, dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(x^m \sqrt{a^2 c x^2+c} \tan^{-1}(a x)^2,x\right)$$

```
[Out] Unintegrable[x^m*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2, x]
```

Rubi [A] time = 0.0977815, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^m \sqrt{c+a^2 c x^2} \tan^{-1}(a x)^2 \, dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2,x]
```

```
[Out] Defer[Int][x^m*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2, x]
```

Rubi steps

$$\int x^m \sqrt{c+a^2 c x^2} \tan^{-1}(a x)^2 \, dx=\int x^m \sqrt{c+a^2 c x^2} \tan^{-1}(a x)^2 \, dx$$

Mathematica [A] time = 0.155392, size = 0, normalized size = 0.

$$\int x^m \sqrt{c+a^2 c x^2} \tan^{-1}(a x)^2 \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2,x]
```

```
[Out] Integrate[x^m*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2, x]
```

Maple [A] time = 0.616, size = 0, normalized size = 0.

$$\int x^{m}\sqrt{a^{2}c\,x^{2}+c}\,\left(\arctan\left(a x\right)\right)^{2}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^2,x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a^{2}cx^{2}+c}\,x^{m}\arctan\left(ax\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)*x^m*arctan(a*x)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{a^{2}cx^{2}+c}\,x^{m}\arctan\left(ax\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^m*arctan(a*x)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**(1/2)*atan(a*x)**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.361 $\int \frac{x^m \tan^{-1}(ax)^2}{\sqrt{c+a^2cx^2}} \, dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{x^m \tan^{-1}(ax)^2}{\sqrt{a^2cx^2+c}},x\right)$$

```
[Out] Unintegrable[(x^m*ArcTan[a*x]^2)/Sqrt[c + a^2*c*x^2], x]
```

Rubi [A] time = 0.101626, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m \tan^{-1}(ax)^2}{\sqrt{c+a^2cx^2}} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*ArcTan[a*x]^2)/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] Defer[Int][(x^m*ArcTan[a*x]^2)/Sqrt[c + a^2*c*x^2], x]
```

Rubi steps

$$\int \frac{x^m \tan^{-1}(ax)^2}{\sqrt{c+a^2cx^2}} \, dx = \int \frac{x^m \tan^{-1}(ax)^2}{\sqrt{c+a^2cx^2}} \, dx$$

Mathematica [A] time = 0.502206, size = 0, normalized size = 0.

$$\int \frac{x^m \tan^{-1}(ax)^2}{\sqrt{c+a^2cx^2}} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*ArcTan[a*x]^2)/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] Integrate[(x^m*ArcTan[a*x]^2)/Sqrt[c + a^2*c*x^2], x]
```

Maple [A] time = 0.925, size = 0, normalized size = 0.

$$\int x^{m} \left(\arctan \left(a x\right)\right)^{2} \frac{1}{\sqrt{a^{2} c \,x^{2}+c}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(x^m*arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{m} \arctan \left(a x\right)^{2}}{\sqrt{a^{2} c x^{2} + c}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m*arctan(a*x)^2/sqrt(a^2*c*x^2 + c), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$${\rm integral}\left(\frac{x^{m} \arctan \left(a x\right)^{2}}{\sqrt{a^{2} c x^{2} + c}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^m*arctan(a*x)^2/sqrt(a^2*c*x^2 + c), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m \operatorname{atan}^2(ax)}{\sqrt{c\left(a^2x^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*atan(a*x)**2/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**m*atan(a*x)**2/sqrt(c*(a**2*x**2 + 1)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m \arctan(ax)^2}{\sqrt{a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^m*arctan(a*x)^2/sqrt(a^2*c*x^2 + c), x)
```

3.362 $\int \frac{x^m \tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^{3/2}} \, dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{x^m \tan^{-1}(ax)^2}{\left(a^2cx^2+c\right)^{3/2}},x\right)$$

```
[Out] Unintegrable[(x^m*ArcTan[a*x]^2)/(c + a^2*c*x^2)^(3/2), x]
```

Rubi [A] time = 0.114937, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m \tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^{3/2}} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*ArcTan[a*x]^2)/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] Defer[Int][(x^m*ArcTan[a*x]^2)/(c + a^2*c*x^2)^(3/2), x]
```

Rubi steps

$$\int \frac{x^m \tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^{3/2}} \, dx = \int \frac{x^m \tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^{3/2}} \, dx$$

Mathematica [A] time = 0.629828, size = 0, normalized size = 0.

$$\int \frac{x^m \tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^{3/2}} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*ArcTan[a*x]^2)/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] Integrate[(x^m*ArcTan[a*x]^2)/(c + a^2*c*x^2)^(3/2), x]
```

Maple [A] time = 1.092, size = 0, normalized size = 0.

$$\int x^{m} (\arctan(ax))^{2} \left(a^{2}c x^{2}+c\right)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arctan(a*x)^2/(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] int(x^m*arctan(a*x)^2/(a^2*c*x^2+c)^(3/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{m} \arctan(ax)^{2}}{\left(a^{2}c x^{2}+c\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^2/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m*arctan(a*x)^2/(a^2*c*x^2 + c)^(3/2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{a^{2}c x^{2}+c}\, x^{m} \arctan(ax)^{2}}{a^{4}c^{2}x^{4}+2\,a^{2}c^{2}x^{2}+c^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^2/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^m*arctan(a*x)^2/(a^4*c^2*x^4 + 2*a^2*c^2*x^2
 + c^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*atan(a*x)**2/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m \arctan(ax)^2}{(a^2cx^2 + c)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^2/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^m*arctan(a*x)^2/(a^2*c*x^2 + c)^(3/2), x)
```

3.363 $\int x^3\left(c+a^2cx^2\right)\tan^{-1}(ax)^3\,dx$

Optimal. Leaf size=219

$$\frac{7ic\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{30a^4}+\frac{1}{6}a^2cx^6\tan^{-1}(ax)^3-\frac{cx^2\tan^{-1}(ax)}{60a^2}+\frac{cx}{15a^3}+\frac{cx\tan^{-1}(ax)^2}{4a^3}-\frac{c\tan^{-1}(ax)^3}{12a^4}+\frac{7ic\tan^{-1}(a}{30a^4}$$

```
[Out] (c*x)/(15*a^3) - (c*x^3)/(60*a) - (c*ArcTan[a*x])/(15*a^4) - (c*x^2*ArcTan[
a*x])/(60*a^2) + (c*x^4*ArcTan[a*x])/20 + (((7*I)/30)*c*ArcTan[a*x]^2)/a^4
+ (c*x*ArcTan[a*x]^2)/(4*a^3) - (c*x^3*ArcTan[a*x]^2)/(12*a) - (a*c*x^5*Arc
Tan[a*x]^2)/10 - (c*ArcTan[a*x]^3)/(12*a^4) + (c*x^4*ArcTan[a*x]^3)/4 + (a^
2*c*x^6*ArcTan[a*x]^3)/6 + (7*c*ArcTan[a*x]*Log[2/(1 + I*a*x)])/(15*a^4) +
(((7*I)/30)*c*PolyLog[2, 1 - 2/(1 + I*a*x)])/a^4
```

Rubi [A] time = 1.11374, antiderivative size = 219, normalized size of antiderivative = 1., number of steps used = 52, number of rules used = 12, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {4950, 4852, 4916, 321, 203, 4920, 4854, 2402, 2315, 4846, 4884, 302}

$$\frac{7ic\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{30a^4}+\frac{1}{6}a^2cx^6\tan^{-1}(ax)^3-\frac{cx^2\tan^{-1}(ax)}{60a^2}+\frac{cx}{15a^3}+\frac{cx\tan^{-1}(ax)^2}{4a^3}-\frac{c\tan^{-1}(ax)^3}{12a^4}+\frac{7ic\tan^{-1}(a}{30a^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(c + a^2*c*x^2)*ArcTan[a*x]^3,x]
```


```
[Out] (c*x)/(15*a^3) - (c*x^3)/(60*a) - (c*ArcTan[a*x])/(15*a^4) - (c*x^2*ArcTan[
a*x])/(60*a^2) + (c*x^4*ArcTan[a*x])/20 + (((7*I)/30)*c*ArcTan[a*x]^2)/a^4
+ (c*x*ArcTan[a*x]^2)/(4*a^3) - (c*x^3*ArcTan[a*x]^2)/(12*a) - (a*c*x^5*Arc
Tan[a*x]^2)/10 - (c*ArcTan[a*x]^3)/(12*a^4) + (c*x^4*ArcTan[a*x]^3)/4 + (a^
2*c*x^6*ArcTan[a*x]^3)/6 + (7*c*ArcTan[a*x]*Log[2/(1 + I*a*x)])/(15*a^4) +
(((7*I)/30)*c*PolyLog[2, 1 - 2/(1 + I*a*x)])/a^4
```


Rule 4950

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\int x^3\left(c+a^2cx^2\right)\tan^{-1}(ax)^3\,dx = c\int x^3\tan^{-1}(ax)^3\,dx+\left(a^2c\right)\int x^5\tan^{-1}(ax)^3\,dx$$

$$=\frac{1}{4}cx^4\tan^{-1}(ax)^3+\frac{1}{6}a^2cx^6\tan^{-1}(ax)^3-\frac{1}{4}(3ac)\int\frac{x^4\tan^{-1}(ax)^2}{1+a^2x^2}\,dx-\frac{1}{2}\left(a^3c\right)\int\frac{x^6\tan}{1+}$$

$$=\frac{1}{4}cx^4\tan^{-1}(ax)^3+\frac{1}{6}a^2cx^6\tan^{-1}(ax)^3-\frac{(3c)\int x^2\tan^{-1}(ax)^2\,dx}{4a}+\frac{(3c)\int\frac{x^2\tan^{-1}(ax)^2}{1+a^2x^2}\,d}{4a}$$

$$=-\frac{cx^3\tan^{-1}(ax)^2}{4a}-\frac{1}{10}acx^5\tan^{-1}(ax)^2+\frac{1}{4}cx^4\tan^{-1}(ax)^3+\frac{1}{6}a^2cx^6\tan^{-1}(ax)^3+\frac{1}{2}c\int$$

$$=\frac{3cx\tan^{-1}(ax)^2}{4a^3}-\frac{cx^3\tan^{-1}(ax)^2}{12a}-\frac{1}{10}acx^5\tan^{-1}(ax)^2-\frac{c\tan^{-1}(ax)^3}{4a^4}+\frac{1}{4}cx^4\tan^{-1}(a$$

$$=\frac{cx^2\tan^{-1}(ax)}{4a^2}+\frac{1}{20}cx^4\tan^{-1}(ax)+\frac{ic\tan^{-1}(ax)^2}{a^4}+\frac{cx\tan^{-1}(ax)^2}{4a^3}-\frac{cx^3\tan^{-1}(ax)^2}{12a}-$$

$$=-\frac{cx}{4a^3}-\frac{cx^2\tan^{-1}(ax)}{60a^2}+\frac{1}{20}cx^4\tan^{-1}(ax)+\frac{7ic\tan^{-1}(ax)^2}{30a^4}+\frac{cx\tan^{-1}(ax)^2}{4a^3}-\frac{cx^3\tan}{12}$$

$$=\frac{cx}{15a^3}-\frac{cx^3}{60a}+\frac{c\tan^{-1}(ax)}{4a^4}-\frac{cx^2\tan^{-1}(ax)}{60a^2}+\frac{1}{20}cx^4\tan^{-1}(ax)+\frac{7ic\tan^{-1}(ax)^2}{30a^4}+\frac{cx}{}$$

$$=\frac{cx}{15a^3}-\frac{cx^3}{60a}-\frac{c\tan^{-1}(ax)}{15a^4}-\frac{cx^2\tan^{-1}(ax)}{60a^2}+\frac{1}{20}cx^4\tan^{-1}(ax)+\frac{7ic\tan^{-1}(ax)^2}{30a^4}+\frac{cx}{}$$

$$=\frac{cx}{15a^3}-\frac{cx^3}{60a}-\frac{c\tan^{-1}(ax)}{15a^4}-\frac{cx^2\tan^{-1}(ax)}{60a^2}+\frac{1}{20}cx^4\tan^{-1}(ax)+\frac{7ic\tan^{-1}(ax)^2}{30a^4}+\frac{cx}{}$$

Mathematica [A] time = 0.612491, size = 135, normalized size = 0.62

$$\frac{c\left(-14i\text{PolyLog}\left(2,-e^{2i\tan^{-1}(ax)}\right)-a^3x^3+5\left(2a^6x^6+3a^4x^4-1\right)\tan^{-1}(ax)^3-\left(6a^5x^5+5a^3x^3-15ax+14i\right)\tan^{-1}(ax)^2-}{60a^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*(c + a^2*c*x^2)*ArcTan[a*x]^3,x]
```

```
[Out] (c*(4*a*x - a^3*x^3 - (14*I - 15*a*x + 5*a^3*x^3 + 6*a^5*x^5)*ArcTan[a*x]^2
 + 5*(-1 + 3*a^4*x^4 + 2*a^6*x^6)*ArcTan[a*x]^3 + ArcTan[a*x]*(-4 - a^2*x^2
 + 3*a^4*x^4 + 28*Log[1 + E^((2*I)*ArcTan[a*x])]) - (14*I)*PolyLog[2, -E^((
2*I)*ArcTan[a*x])]))/(60*a^4)
```

Maple [A] time = 0.097, size = 313, normalized size = 1.4

$$\frac{a^2cx^6 (\arctan(ax))^3}{6} + \frac{cx^4 (\arctan(ax))^3}{4} - \frac{acx^5 (\arctan(ax))^2}{10} - \frac{cx^3 (\arctan(ax))^2}{12\,a} + \frac{cx (\arctan(ax))^2}{4\,a^3} - \frac{c (\arctan}{12\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a^2*c*x^2+c)*arctan(a*x)^3,x)
```

```
[Out] 1/6*a^2*c*x^6*arctan(a*x)^3+1/4*c*x^4*arctan(a*x)^3-1/10*a*c*x^5*arctan(a*x
)^2-1/12*c*x^3*arctan(a*x)^2/a+1/4*c*x*arctan(a*x)^2/a^3-1/12*c*arctan(a*x)
^3/a^4+1/20*c*x^4*arctan(a*x)-1/60*c*x^2*arctan(a*x)/a^2-7/30/a^4*c*arctan(
a*x)*ln(a^2*x^2+1)-1/60*c*x^3/a+1/15*c*x/a^3-1/15*c*arctan(a*x)/a^4-7/60*I/
a^4*c*ln(a*x-I)*ln(a^2*x^2+1)+7/60*I/a^4*c*ln(a*x-I)*ln(-1/2*I*(a*x+I))+7/1
20*I/a^4*c*ln(a*x-I)^2-7/60*I/a^4*c*ln(a*x+I)*ln(1/2*I*(a*x-I))+7/60*I/a^4*
c*ln(a*x+I)*ln(a^2*x^2+1)-7/120*I/a^4*c*ln(a*x+I)^2+7/60*I/a^4*c*dilog(-1/2
*I*(a*x+I))-7/60*I/a^4*c*dilog(1/2*I*(a*x-I))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a^2*c*x^2+c)*arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^2cx^5 + cx^3\right)\arctan(ax)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a^2*c*x^2+c)*arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^5 + c*x^3)*arctan(a*x)^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$c\left(\int x^{3}\operatorname{atan}^{3}(ax)\,dx+\int a^{2}x^{5}\operatorname{atan}^{3}(ax)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a**2*c*x**2+c)*atan(a*x)**3,x)
```

```
[Out] c*(Integral(x**3*atan(a*x)**3, x) + Integral(a**2*x**5*atan(a*x)**3, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^{2}cx^{2}+c\right)x^{3}\arctan\left(ax\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a^2*c*x^2+c)*arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)*x^3*arctan(a*x)^3, x)
```

3.364 $\int x^2 \left(c+a^2 c x^2\right) \tan^{-1}(ax)^3 \, dx$

Optimal. Leaf size=211

$$-\frac{c\text{PolyLog}\left(3,1-\frac{2}{1+iax}\right)}{5a^3}-\frac{2ic\tan^{-1}(ax)\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{5a^3}+\frac{1}{5}a^2cx^5\tan^{-1}(ax)^3+\frac{cx\tan^{-1}(ax)}{10a^2}-\frac{2ic\tan^{-1}(ax)^3}{15a^3}$$

```
[Out] -(c*x^2)/(20*a) + (c*x*ArcTan[a*x])/(10*a^2) + (c*x^3*ArcTan[a*x])/10 - (c*
ArcTan[a*x]^2)/(20*a^3) - (c*x^2*ArcTan[a*x]^2)/(5*a) - (3*a*c*x^4*ArcTan[a
*x]^2)/20 - (((2*I)/15)*c*ArcTan[a*x]^3)/a^3 + (c*x^3*ArcTan[a*x]^3)/3 + (a
^2*c*x^5*ArcTan[a*x]^3)/5 - (2*c*ArcTan[a*x]^2*Log[2/(1 + I*a*x)])/(5*a^3)
- (((2*I)/5)*c*ArcTan[a*x]*PolyLog[2, 1 - 2/(1 + I*a*x)])/a^3 - (c*PolyLog[
3, 1 - 2/(1 + I*a*x)])/(5*a^3)
```

Rubi [A] time = 0.881975, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 34, number of rules used = 12, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {4950, 4852, 4916, 4846, 260, 4884, 4920, 4854, 4994, 6610, 266, 43}

$$-\frac{c\text{PolyLog}\left(3,1-\frac{2}{1+iax}\right)}{5a^3}-\frac{2ic\tan^{-1}(ax)\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{5a^3}+\frac{1}{5}a^2cx^5\tan^{-1}(ax)^3+\frac{cx\tan^{-1}(ax)}{10a^2}-\frac{2ic\tan^{-1}(ax)^3}{15a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(c + a^2*c*x^2)*ArcTan[a*x]^3,x]
```


```
[Out] -(c*x^2)/(20*a) + (c*x*ArcTan[a*x])/(10*a^2) + (c*x^3*ArcTan[a*x])/10 - (c*
ArcTan[a*x]^2)/(20*a^3) - (c*x^2*ArcTan[a*x]^2)/(5*a) - (3*a*c*x^4*ArcTan[a
*x]^2)/20 - (((2*I)/15)*c*ArcTan[a*x]^3)/a^3 + (c*x^3*ArcTan[a*x]^3)/3 + (a
^2*c*x^5*ArcTan[a*x]^3)/5 - (2*c*ArcTan[a*x]^2*Log[2/(1 + I*a*x)])/(5*a^3)
- (((2*I)/5)*c*ArcTan[a*x]*PolyLog[2, 1 - 2/(1 + I*a*x)])/a^3 - (c*PolyLog[
3, 1 - 2/(1 + I*a*x)])/(5*a^3)
```


Rule 4950

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
```

```
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x^2\left(c+a^2cx^2\right)\tan^{-1}(ax)^3\,dx = c\int x^2\tan^{-1}(ax)^3\,dx+\left(a^2c\right)\int x^4\tan^{-1}(ax)^3\,dx$$

$$= \frac{1}{3}cx^3\tan^{-1}(ax)^3+\frac{1}{5}a^2cx^5\tan^{-1}(ax)^3-(ac)\int\frac{x^3\tan^{-1}(ax)^2}{1+a^2x^2}\,dx-\frac{1}{5}\left(3a^3c\right)\int\frac{x^5\tan}{1+}$$

$$= \frac{1}{3}cx^3\tan^{-1}(ax)^3+\frac{1}{5}a^2cx^5\tan^{-1}(ax)^3-\frac{c\int x\tan^{-1}(ax)^2\,dx}{a}+\frac{c\int\frac{x\tan^{-1}(ax)^2}{1+a^2x^2}\,dx}{a}-\frac{1}{5}(3$$

$$= -\frac{cx^2\tan^{-1}(ax)^2}{2a}-\frac{3}{20}acx^4\tan^{-1}(ax)^2-\frac{ic\tan^{-1}(ax)^3}{3a^3}+\frac{1}{3}cx^3\tan^{-1}(ax)^3+\frac{1}{5}a^2cx^5\,\mathrm{ta}$$

$$= -\frac{cx^2\tan^{-1}(ax)^2}{5a}-\frac{3}{20}acx^4\tan^{-1}(ax)^2-\frac{2ic\tan^{-1}(ax)^3}{15a^3}+\frac{1}{3}cx^3\tan^{-1}(ax)^3+\frac{1}{5}a^2cx^5\,\mathrm{t}$$

$$= \frac{cx\tan^{-1}(ax)}{a^2}+\frac{1}{10}cx^3\tan^{-1}(ax)-\frac{c\tan^{-1}(ax)^2}{2a^3}-\frac{cx^2\tan^{-1}(ax)^2}{5a}-\frac{3}{20}acx^4\tan^{-1}(ax)$$

$$= \frac{cx\tan^{-1}(ax)}{10a^2}+\frac{1}{10}cx^3\tan^{-1}(ax)-\frac{c\tan^{-1}(ax)^2}{20a^3}-\frac{cx^2\tan^{-1}(ax)^2}{5a}-\frac{3}{20}acx^4\tan^{-1}(ax)$$

$$= \frac{cx\tan^{-1}(ax)}{10a^2}+\frac{1}{10}cx^3\tan^{-1}(ax)-\frac{c\tan^{-1}(ax)^2}{20a^3}-\frac{cx^2\tan^{-1}(ax)^2}{5a}-\frac{3}{20}acx^4\tan^{-1}(ax)$$

$$= -\frac{cx^2}{20a}+\frac{cx\tan^{-1}(ax)}{10a^2}+\frac{1}{10}cx^3\tan^{-1}(ax)-\frac{c\tan^{-1}(ax)^2}{20a^3}-\frac{cx^2\tan^{-1}(ax)^2}{5a}-\frac{3}{20}acx^4\,\mathrm{ta}$$

Mathematica [A] time = 0.531563, size = 171, normalized size = 0.81

$$c\left(24i\tan^{-1}(ax)\mathrm{PolyLog}\left(2,-e^{2i\tan^{-1}(ax)}\right)-12\mathrm{PolyLog}\left(3,-e^{2i\tan^{-1}(ax)}\right)-3a^2x^2+12a^5x^5\tan^{-1}(ax)^3-9a^4x^4\tan^{-1}(ax$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(c + a^2*c*x^2)*ArcTan[a*x]^3,x]
```

```
[Out] (c*(-3 - 3*a^2*x^2 + 6*a*x*ArcTan[a*x] + 6*a^3*x^3*ArcTan[a*x] - 3*ArcTan[a
*x]^2 - 12*a^2*x^2*ArcTan[a*x]^2 - 9*a^4*x^4*ArcTan[a*x]^2 + (8*I)*ArcTan[a
*x]^3 + 20*a^3*x^3*ArcTan[a*x]^3 + 12*a^5*x^5*ArcTan[a*x]^3 - 24*ArcTan[a*x
]^2*Log[1 + E^((2*I)*ArcTan[a*x])] + (24*I)*ArcTan[a*x]*PolyLog[2, -E^((2*I
)*ArcTan[a*x])] - 12*PolyLog[3, -E^((2*I)*ArcTan[a*x])]))/(60*a^3)
```

Maple [C] time = 2.629, size = 2555, normalized size = 12.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a^2*c*x^2+c)*arctan(a*x)^3,x)
```

```
[Out] 1/3*c*x^3*arctan(a*x)^3-2/5/a^3*c*arctan(a*x)^2*ln((1+I*a*x)/(a^2*x^2+1)^(1
/2))-2/5/a^3*c*arctan(a*x)^2*ln(2)+1/5/a^3*c*arctan(a*x)^2*ln(a^2*x^2+1)+2/
15*I/a^3*c*arctan(a*x)^3-1/20*c*x^2/a+1/10*c*x^3*arctan(a*x)-1/20*c*arctan(
a*x)^2/a^3-3/40*I/a*c*arctan(a*x)^2*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2
)^2*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))*x^2+3/80*I/a*c*arctan(a*x)^2*Pi*csg
n(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))^2*x^
2+3/40*I/a*c*arctan(a*x)^2*Pi*csgn(I*(1+I*a*x)^4/(a^2*x^2+1)^2+2*I*(1+I*a*x
)^2/(a^2*x^2+1)+I)^2*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)+I)*x^2-3/80*I/a*c*arcta
n(a*x)^2*Pi*csgn(I*(1+I*a*x)^4/(a^2*x^2+1)^2+2*I*(1+I*a*x)^2/(a^2*x^2+1)+I)
*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)+I)^2*x^2+1/10*I/a^3*c*arctan(a*x)^2*Pi*csgn
(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1
+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)+1/10*c*x*arctan(a*x)/a
^2-1/5*c*x^2*arctan(a*x)^2/a-3/20*a*c*x^4*arctan(a*x)^2+1/5*a^2*c*x^5*arcta
n(a*x)^3-3/80/a^2*c*arctan(a*x)^2*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))^2*
csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*x+3/40/a^2*c*arctan(a*x)^2*Pi*csgn(I*
((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2*x-1/5/
a^3*c*polylog(3,-(1+I*a*x)^2/(a^2*x^2+1))-1/80*c*arctan(a*x)^2*Pi*csgn(I*(1
+I*a*x)^4/(a^2*x^2+1)^2+2*I*(1+I*a*x)^2/(a^2*x^2+1)+I)^3*x^3+2/5*I/a^3*c*ar
ctan(a*x)*polylog(2,-(1+I*a*x)^2/(a^2*x^2+1))-3/80/a^2*c*arctan(a*x)^2*Pi*c
sgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3*x+3/80/a^2*c*arctan(a*x)^2*Pi*csgn(I
*(1+I*a*x)^4/(a^2*x^2+1)^2+2*I*(1+I*a*x)^2/(a^2*x^2+1)+I)^3*x-1/20/a^3*c+7/
40*I/a^3*c*arctan(a*x)^2*Pi*csgn(I*(1+I*a*x)^4/(a^2*x^2+1)^2+2*I*(1+I*a*x)^
2/(a^2*x^2+1)+I)^2*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)+I)-7/80*I/a^3*c*arctan(a*
x)^2*Pi*csgn(I*(1+I*a*x)^4/(a^2*x^2+1)^2+2*I*(1+I*a*x)^2/(a^2*x^2+1)+I)*csg
n(I*(1+I*a*x)^2/(a^2*x^2+1)+I)^2-1/5*I/a^3*c*arctan(a*x)^2*Pi*csgn(I*(1+I*a
*x)^2/(a^2*x^2+1))^2*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-1/10*I/a^3*c*arcta
n(a*x)^2*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/
((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2+1/10*I/a^3*c*arctan(a*x)^2*Pi*csgn(I*(1+I*
a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))^2-1/10*I/a^3*c*arct
an(a*x)^2*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*
x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2+3/80*I/a*c*arctan(a*x)^2*Pi*csgn(I*
((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3*x^2-3/80*I/a*c*arctan(a*x)^2*Pi*csgn(I*(1+
I*a*x)^4/(a^2*x^2+1)^2+2*I*(1+I*a*x)^2/(a^2*x^2+1)+I)^3*x^2+3/80/a^2*c*arct
an(a*x)^2*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)+I)^2*csgn(I*(1+I*a*x)^4/(a^2*x^
2+1)^2+2*I*(1+I*a*x)^2/(a^2*x^2+1)+I)*x-3/40/a^2*c*arctan(a*x)^2*Pi*csgn(I*
(1+I*a*x)^2/(a^2*x^2+1)+I)*csgn(I*(1+I*a*x)^4/(a^2*x^2+1)^2+2*I*(1+I*a*x)^2
```

```
/(a^2*x^2+1)+I)^2*x+1/40*I/a^3*c*arctan(a*x)^2*Pi*csgn(I*((1+I*a*x)^2/(a^2*
x^2+1)+1)^2)^2*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))-1/80*I/a^3*c*arctan(a*x)
^2*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)
+1))^2+1/80*c*arctan(a*x)^2*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3*x^3+
1/80*c*arctan(a*x)^2*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))^2*csgn(I*((1+I*
a*x)^2/(a^2*x^2+1)+1)^2)*x^3-1/40*c*arctan(a*x)^2*Pi*csgn(I*((1+I*a*x)^2/(a
^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2*x^3-1/80*c*arctan(a*x
)^2*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)+I)^2*csgn(I*(1+I*a*x)^4/(a^2*x^2+1)^2
+2*I*(1+I*a*x)^2/(a^2*x^2+1)+I)*x^3+1/40*c*arctan(a*x)^2*Pi*csgn(I*(1+I*a*x
)^2/(a^2*x^2+1)+I)*csgn(I*(1+I*a*x)^4/(a^2*x^2+1)^2+2*I*(1+I*a*x)^2/(a^2*x^
2+1)+I)^2*x^3-1/80*I/a^3*c*arctan(a*x)^2*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)
+1)^2)^3-7/80*I/a^3*c*arctan(a*x)^2*Pi*csgn(I*(1+I*a*x)^4/(a^2*x^2+1)^2+2*I
*(1+I*a*x)^2/(a^2*x^2+1)+I)^3+1/10*I/a^3*c*arctan(a*x)^2*Pi*csgn(I*(1+I*a*x
)^2/(a^2*x^2+1))^3+1/10*I/a^3*c*arctan(a*x)^2*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^
2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{120}\left(3\,a^2cx^5+5\,cx^3\right)\arctan\left(ax\right)^3-\frac{1}{160}\left(3\,a^2cx^5+5\,cx^3\right)\arctan\left(ax\right)\log\left(a^2x^2+1\right)^2+\int\frac{140\left(a^4cx^6+2\,a^2cx^4+cx^2\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)*arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/120*(3*a^2*c*x^5 + 5*c*x^3)*arctan(a*x)^3 - 1/160*(3*a^2*c*x^5 + 5*c*x^3)
*arctan(a*x)*log(a^2*x^2 + 1)^2 + integrate(1/160*(140*(a^4*c*x^6 + 2*a^2*c
*x^4 + c*x^2)*arctan(a*x)^3 - 4*(3*a^3*c*x^5 + 5*a*c*x^3)*arctan(a*x)^2 + 4
*(3*a^4*c*x^6 + 5*a^2*c*x^4)*arctan(a*x)*log(a^2*x^2 + 1) + (3*a^3*c*x^5 +
5*a*c*x^3 + 15*(a^4*c*x^6 + 2*a^2*c*x^4 + c*x^2)*arctan(a*x))*log(a^2*x^2 +
 1)^2)/(a^2*x^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^2cx^4+cx^2\right)\arctan\left(ax\right)^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)*arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^4 + c*x^2)*arctan(a*x)^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$c\left(\int x^2 \operatorname{atan}^3(ax)\,dx + \int a^2x^4 \operatorname{atan}^3(ax)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a**2*c*x**2+c)*atan(a*x)**3,x)
```

```
[Out] c*(Integral(x**2*atan(a*x)**3, x) + Integral(a**2*x**4*atan(a*x)**3, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2 + c\right)x^2 \arctan(ax)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)*arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)*x^2*arctan(a*x)^3, x)
```

3.365 $\int x\left(c+a^2cx^2\right)\tan^{-1}(ax)^3\,dx$

Optimal. Leaf size=160

$$-\frac{ic\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{2a^2}+\frac{c\left(a^2x^2+1\right)^2\tan^{-1}(ax)^3}{4a^2}-\frac{cx\left(a^2x^2+1\right)\tan^{-1}(ax)^2}{4a}+\frac{c\left(a^2x^2+1\right)\tan^{-1}(ax)}{4a^2}-\frac{ic\tan^{-1}(ax}{2a^2}$$

```
[Out] -(c*x)/(4*a) + (c*(1 + a^2*x^2)*ArcTan[a*x])/(4*a^2) - ((I/2)*c*ArcTan[a*x]
^2)/a^2 - (c*x*ArcTan[a*x]^2)/(2*a) - (c*x*(1 + a^2*x^2)*ArcTan[a*x]^2)/(4*
a) + (c*(1 + a^2*x^2)^2*ArcTan[a*x]^3)/(4*a^2) - (c*ArcTan[a*x]*Log[2/(1 +
I*a*x)])/a^2 - ((I/2)*c*PolyLog[2, 1 - 2/(1 + I*a*x)])/a^2
```

Rubi [A] time = 0.129083, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {4930, 4880, 4846, 4920, 4854, 2402, 2315, 8}

$$-\frac{ic\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{2a^2}+\frac{c\left(a^2x^2+1\right)^2\tan^{-1}(ax)^3}{4a^2}-\frac{cx\left(a^2x^2+1\right)\tan^{-1}(ax)^2}{4a}+\frac{c\left(a^2x^2+1\right)\tan^{-1}(ax)}{4a^2}-\frac{ic\tan^{-1}(ax}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(c + a^2*c*x^2)*ArcTan[a*x]^3,x]
```

```
[Out] -(c*x)/(4*a) + (c*(1 + a^2*x^2)*ArcTan[a*x])/(4*a^2) - ((I/2)*c*ArcTan[a*x]
^2)/a^2 - (c*x*ArcTan[a*x]^2)/(2*a) - (c*x*(1 + a^2*x^2)*ArcTan[a*x]^2)/(4*
a) + (c*(1 + a^2*x^2)^2*ArcTan[a*x]^3)/(4*a^2) - (c*ArcTan[a*x]*Log[2/(1 +
I*a*x)])/a^2 - ((I/2)*c*PolyLog[2, 1 - 2/(1 + I*a*x)])/a^2
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 4880

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> -Simp[(b*p*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1))/(2*c*q*(2*
q + 1)), x] + (Dist[(2*d*q)/(2*q + 1), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTa
```

```
n[c*x])^p, x], x] + Dist[(b^2*d*p*(p - 1))/(2*q*(2*q + 1)), Int[(d + e*x^2)
^(q - 1)*(a + b*ArcTan[c*x])^(p - 2), x], x] + Simp[(x*(d + e*x^2)^q*(a + b
*ArcTan[c*x])^p)/(2*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[q, 0] && GtQ[p, 1]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int x\left(c+a^2cx^2\right)\tan^{-1}(ax)^3\,dx = \frac{c\left(1+a^2x^2\right)^2\tan^{-1}(ax)^3}{4a^2} - \frac{3\int\left(c+a^2cx^2\right)\tan^{-1}(ax)^2\,dx}{4a}$$

$$= \frac{c\left(1+a^2x^2\right)\tan^{-1}(ax)}{4a^2} - \frac{cx\left(1+a^2x^2\right)\tan^{-1}(ax)^2}{4a} + \frac{c\left(1+a^2x^2\right)^2\tan^{-1}(ax)^3}{4a^2} - \frac{c\int 1\,dx}{4a}$$

$$= -\frac{cx}{4a} + \frac{c\left(1+a^2x^2\right)\tan^{-1}(ax)}{4a^2} - \frac{cx\tan^{-1}(ax)^2}{2a} - \frac{cx\left(1+a^2x^2\right)\tan^{-1}(ax)^2}{4a} + \frac{c\left(1+a^2x^2\right)}{}$$

$$= -\frac{cx}{4a} + \frac{c\left(1+a^2x^2\right)\tan^{-1}(ax)}{4a^2} - \frac{ic\tan^{-1}(ax)^2}{2a^2} - \frac{cx\tan^{-1}(ax)^2}{2a} - \frac{cx\left(1+a^2x^2\right)\tan^{-1}(a}{4a}$$

$$= -\frac{cx}{4a} + \frac{c\left(1+a^2x^2\right)\tan^{-1}(ax)}{4a^2} - \frac{ic\tan^{-1}(ax)^2}{2a^2} - \frac{cx\tan^{-1}(ax)^2}{2a} - \frac{cx\left(1+a^2x^2\right)\tan^{-1}(a}{4a}$$

$$= -\frac{cx}{4a} + \frac{c\left(1+a^2x^2\right)\tan^{-1}(ax)}{4a^2} - \frac{ic\tan^{-1}(ax)^2}{2a^2} - \frac{cx\tan^{-1}(ax)^2}{2a} - \frac{cx\left(1+a^2x^2\right)\tan^{-1}(a}{4a}$$

$$= -\frac{cx}{4a} + \frac{c\left(1+a^2x^2\right)\tan^{-1}(ax)}{4a^2} - \frac{ic\tan^{-1}(ax)^2}{2a^2} - \frac{cx\tan^{-1}(ax)^2}{2a} - \frac{cx\left(1+a^2x^2\right)\tan^{-1}(a}{4a}$$

Mathematica [A] time = 0.0674966, size = 101, normalized size = 0.63

$$\frac{c\left(2i\text{PolyLog}\left(2,-e^{2i\tan^{-1}(ax)}\right)+\left(a^2x^2+1\right)^2\tan^{-1}(ax)^3-\left(a^3x^3+3ax-2i\right)\tan^{-1}(ax)^2+\tan^{-1}(ax)\left(a^2x^2-4\log\left(1+e^{2}\right.\right.\right.}{4a^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*(c + a^2*c*x^2)*ArcTan[a*x]^3,x]
```

```
[Out] (c*(-(a*x) - (-2*I + 3*a*x + a^3*x^3)*ArcTan[a*x]^2 + (1 + a^2*x^2)^2*ArcTa
n[a*x]^3 + ArcTan[a*x]*(1 + a^2*x^2 - 4*Log[1 + E^((2*I)*ArcTan[a*x])]) + (
2*I)*PolyLog[2, -E^((2*I)*ArcTan[a*x])]))/(4*a^2)
```

Maple [A] time = 0.097, size = 276, normalized size = 1.7

$$\frac{a^2c\left(\arctan(ax)\right)^3x^4}{4} + \frac{c\left(\arctan(ax)\right)^3x^2}{2} - \frac{ac\left(\arctan(ax)\right)^2x^3}{4} - \frac{3\,cx\left(\arctan(ax)\right)^2}{4\,a} + \frac{c\left(\arctan(ax)\right)^3}{4\,a^2} + \frac{c\arctan(a}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)*arctan(a*x)^3,x)
```

```
[Out] 1/4*a^2*c*arctan(a*x)^3*x^4+1/2*c*arctan(a*x)^3*x^2-1/4*a*c*arctan(a*x)^2*x
^3-3/4*c*x*arctan(a*x)^2/a+1/4/a^2*c*arctan(a*x)^3+1/4*c*arctan(a*x)*x^2+1/
2/a^2*c*arctan(a*x)*ln(a^2*x^2+1)-1/4*c*x/a+1/4/a^2*c*arctan(a*x)+1/8*I/a^2
*c*ln(a*x+I)^2+1/4*I/a^2*c*ln(1/2*I*(a*x-I))*ln(a*x+I)-1/4*I/a^2*c*ln(a*x+I
)*ln(a^2*x^2+1)+1/4*I/a^2*c*dilog(1/2*I*(a*x-I))-1/8*I/a^2*c*ln(a*x-I)^2-1/
4*I/a^2*c*ln(a*x-I)*ln(-1/2*I*(a*x+I))-1/4*I/a^2*c*dilog(-1/2*I*(a*x+I))+1/
4*I/a^2*c*ln(a*x-I)*ln(a^2*x^2+1)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)*arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^2cx^3+cx\right)\arctan\left(ax\right)^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)*arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^3 + c*x)*arctan(a*x)^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$c\left(\int x\operatorname{atan}^3\left(ax\right)dx+\int a^2x^3\operatorname{atan}^3\left(ax\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)*atan(a*x)**3,x)
```

```
[Out] c*(Integral(x*atan(a*x)**3, x) + Integral(a**2*x**3*atan(a*x)**3, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a^2cx^2 + c)x \arctan(ax)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)*arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)*x*arctan(a*x)^3, x)
```

3.366 $\int \left(c + a^2cx^2\right) \tan^{-1}(ax)^3 \, dx$

Optimal. Leaf size=172

$$\frac{c\text{PolyLog}\left(3,1-\frac{2}{1+iax}\right)}{a}+\frac{2ic\tan^{-1}(ax)\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{a}-\frac{c\log\left(a^2x^2+1\right)}{2a}+\frac{1}{3}cx\left(a^2x^2+1\right)\tan^{-1}(ax)^3-\frac{c\left(a^2\right.}{}$$

```
[Out] c*x*ArcTan[a*x] - (c*(1 + a^2*x^2)*ArcTan[a*x]^2)/(2*a) + (((2*I)/3)*c*ArcT
an[a*x]^3)/a + (2*c*x*ArcTan[a*x]^3)/3 + (c*x*(1 + a^2*x^2)*ArcTan[a*x]^3)/
3 + (2*c*ArcTan[a*x]^2*Log[2/(1 + I*a*x)])/a - (c*Log[1 + a^2*x^2])/(2*a) +
 ((2*I)*c*ArcTan[a*x]*PolyLog[2, 1 - 2/(1 + I*a*x)])/a + (c*PolyLog[3, 1 -
2/(1 + I*a*x)])/a
```

Rubi [A] time = 0.182432, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {4880, 4846, 4920, 4854, 4884, 4994, 6610, 260}

$$\frac{c\text{PolyLog}\left(3,1-\frac{2}{1+iax}\right)}{a}+\frac{2ic\tan^{-1}(ax)\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{a}-\frac{c\log\left(a^2x^2+1\right)}{2a}+\frac{1}{3}cx\left(a^2x^2+1\right)\tan^{-1}(ax)^3-\frac{c\left(a^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + a^2*c*x^2)*ArcTan[a*x]^3,x]
```

```
[Out] c*x*ArcTan[a*x] - (c*(1 + a^2*x^2)*ArcTan[a*x]^2)/(2*a) + (((2*I)/3)*c*ArcT
an[a*x]^3)/a + (2*c*x*ArcTan[a*x]^3)/3 + (c*x*(1 + a^2*x^2)*ArcTan[a*x]^3)/
3 + (2*c*ArcTan[a*x]^2*Log[2/(1 + I*a*x)])/a - (c*Log[1 + a^2*x^2])/(2*a) +
 ((2*I)*c*ArcTan[a*x]*PolyLog[2, 1 - 2/(1 + I*a*x)])/a + (c*PolyLog[3, 1 -
2/(1 + I*a*x)])/a
```

Rule 4880

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> -Simp[(b*p*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1))/(2*c*q*(2*
q + 1)), x] + (Dist[(2*d*q)/(2*q + 1), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTa
n[c*x])^p, x], x] + Dist[(b^2*d*p*(p - 1))/(2*q*(2*q + 1)), Int[(d + e*x^2)
^(q - 1)*(a + b*ArcTan[c*x])^(p - 2), x], x] + Simp[(x*(d + e*x^2)^q*(a + b
*ArcTan[c*x])^p)/(2*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[q, 0] && GtQ[p, 1]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \left(c+a^2cx^2\right)\tan^{-1}(ax)^3\,dx = -\frac{c\left(1+a^2x^2\right)\tan^{-1}(ax)^2}{2a}+\frac{1}{3}cx\left(1+a^2x^2\right)\tan^{-1}(ax)^3+\frac{1}{3}(2c)\int \tan^{-1}(ax)^3\,dx+c\int \text{t}$$

$$= cx\tan^{-1}(ax)-\frac{c\left(1+a^2x^2\right)\tan^{-1}(ax)^2}{2a}+\frac{2}{3}cx\tan^{-1}(ax)^3+\frac{1}{3}cx\left(1+a^2x^2\right)\tan^{-1}(ax)^3 -$$

$$= cx\tan^{-1}(ax)-\frac{c\left(1+a^2x^2\right)\tan^{-1}(ax)^2}{2a}+\frac{2ic\tan^{-1}(ax)^3}{3a}+\frac{2}{3}cx\tan^{-1}(ax)^3+\frac{1}{3}cx\left(1+a\right.$$

$$= cx\tan^{-1}(ax)-\frac{c\left(1+a^2x^2\right)\tan^{-1}(ax)^2}{2a}+\frac{2ic\tan^{-1}(ax)^3}{3a}+\frac{2}{3}cx\tan^{-1}(ax)^3+\frac{1}{3}cx\left(1+a\right.$$

$$= cx\tan^{-1}(ax)-\frac{c\left(1+a^2x^2\right)\tan^{-1}(ax)^2}{2a}+\frac{2ic\tan^{-1}(ax)^3}{3a}+\frac{2}{3}cx\tan^{-1}(ax)^3+\frac{1}{3}cx\left(1+a\right.$$

$$= cx\tan^{-1}(ax)-\frac{c\left(1+a^2x^2\right)\tan^{-1}(ax)^2}{2a}+\frac{2ic\tan^{-1}(ax)^3}{3a}+\frac{2}{3}cx\tan^{-1}(ax)^3+\frac{1}{3}cx\left(1+a\right.$$

Mathematica [A] time = 0.0487358, size = 144, normalized size = 0.84

$$\frac{c\left(-12i\tan^{-1}(ax)\text{PolyLog}\left(2,-e^{2i\tan^{-1}(ax)}\right)+6\text{PolyLog}\left(3,-e^{2i\tan^{-1}(ax)}\right)-3\log\left(a^2x^2+1\right)+2a^3x^3\tan^{-1}(ax)^3-3a^2x\right.}{6a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + a^2*c*x^2)*ArcTan[a*x]^3,x]
```

```
[Out] (c*(6*a*x*ArcTan[a*x] - 3*ArcTan[a*x]^2 - 3*a^2*x^2*ArcTan[a*x]^2 - (4*I)*A
rcTan[a*x]^3 + 6*a*x*ArcTan[a*x]^3 + 2*a^3*x^3*ArcTan[a*x]^3 + 12*ArcTan[a*
x]^2*Log[1 + E^((2*I)*ArcTan[a*x])] - 3*Log[1 + a^2*x^2] - (12*I)*ArcTan[a*
x]*PolyLog[2, -E^((2*I)*ArcTan[a*x])] + 6*PolyLog[3, -E^((2*I)*ArcTan[a*x])
]))/(6*a)
```

Maple [C] time = 1.237, size = 1635, normalized size = 9.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)*arctan(a*x)^3,x)
```

```
[Out] 1/a*c*ln((1+I*a*x)^2/(a^2*x^2+1)+1)+1/a*c*polylog(3,-(1+I*a*x)^2/(a^2*x^2+1
))-1/2/a*c*arctan(a*x)^2-1/2*a*c*arctan(a*x)^2*x^2+1/3*a^2*c*arctan(a*x)^3*
x^3-1/a*c*arctan(a*x)^2*ln(a^2*x^2+1)+2/a*c*arctan(a*x)^2*ln(2)+2/a*c*arcta
n(a*x)^2*ln((1+I*a*x)/(a^2*x^2+1)^(1/2))-I/a*c*arctan(a*x)-2/3*I/a*c*arctan
(a*x)^3-1/2*I/a*c*arctan(a*x)^2*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I/(
(1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(
a^2*x^2+1)+1)^2)-1/4*c*arctan(a*x)^2*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)+I)^2
*csgn(I*(1+I*a*x)^4/(a^2*x^2+1)^2+2*I*(1+I*a*x)^2/(a^2*x^2+1)+I)*x+1/2*c*ar
ctan(a*x)^2*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)+I)*csgn(I*(1+I*a*x)^4/(a^2*x^
2+1)^2+2*I*(1+I*a*x)^2/(a^2*x^2+1)+I)^2*x+1/4*I/a*c*arctan(a*x)^2*Pi*csgn(I
*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3+1/4*I/a*c*arctan(a*x)^2*Pi*csgn(I*(1+I*a*
x)^4/(a^2*x^2+1)^2+2*I*(1+I*a*x)^2/(a^2*x^2+1)+I)^3-1/2*I/a*c*arctan(a*x)^2
*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^3-1/2*I/a*c*arctan(a*x)^2*Pi*csgn(I*(1+
I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3+c*x*arctan(a*x)^3-1/2
*I/a*c*arctan(a*x)^2*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)+I)*csgn(I*(1+I*a*x)^
4/(a^2*x^2+1)^2+2*I*(1+I*a*x)^2/(a^2*x^2+1)+I)^2-1/2*I/a*c*arctan(a*x)^2*Pi
*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))^2*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))+1/2*
I/a*c*arctan(a*x)^2*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(
a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2+1/2*I/a*c*arctan(a*x)^2*Pi*csgn
(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)
^2/(a^2*x^2+1)+1)^2)^2+c*x*arctan(a*x)+1/4*c*arctan(a*x)^2*Pi*csgn(I*((1+I*
a*x)^2/(a^2*x^2+1)+1))^2*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*x-1/2*c*arct
an(a*x)^2*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x
^2+1)+1)^2)^2*x+I/a*c*arctan(a*x)^2*Pi*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*
csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^2+1/4*I/a*c*arctan(a*x)^2*Pi*csgn(I*((1+I*a
*x)^2/(a^2*x^2+1)+1))^2*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)-1/2*I/a*c*arc
tan(a*x)^2*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*
x^2+1)+1)^2)^2+1/4*I/a*c*arctan(a*x)^2*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)+I)
^2*csgn(I*(1+I*a*x)^4/(a^2*x^2+1)^2+2*I*(1+I*a*x)^2/(a^2*x^2+1)+I)+1/4*c*ar
ctan(a*x)^2*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3*x-1/4*c*arctan(a*x)^
2*Pi*csgn(I*(1+I*a*x)^4/(a^2*x^2+1)^2+2*I*(1+I*a*x)^2/(a^2*x^2+1)+I)^3*x-2*
I/a*c*arctan(a*x)*polylog(2,-(1+I*a*x)^2/(a^2*x^2+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$28\,a^4c\int\frac{x^4\arctan(ax)^3}{32\left(a^2x^2+1\right)}\,dx+3\,a^4c\int\frac{x^4\arctan(ax)\log\left(a^2x^2+1\right)^2}{32\left(a^2x^2+1\right)}\,dx+4\,a^4c\int\frac{x^4\arctan(ax)\log\left(a^2x^2+1\right)}{32\left(a^2x^2+1\right)}\,dx-4\,a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] 28*a^4*c*integrate(1/32*x^4*arctan(a*x)^3/(a^2*x^2 + 1), x) + 3*a^4*c*integ
rate(1/32*x^4*arctan(a*x)*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x) + 4*a^4*c*in
tegrate(1/32*x^4*arctan(a*x)*log(a^2*x^2 + 1)/(a^2*x^2 + 1), x) - 4*a^3*c*i
ntegrate(1/32*x^3*arctan(a*x)^2/(a^2*x^2 + 1), x) + a^3*c*integrate(1/32*x^
3*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x) + 1/24*(a^2*c*x^3 + 3*c*x)*arctan(a*
x)^3 + 7/32*c*arctan(a*x)^4/a + 56*a^2*c*integrate(1/32*x^2*arctan(a*x)^3/(
a^2*x^2 + 1), x) + 6*a^2*c*integrate(1/32*x^2*arctan(a*x)*log(a^2*x^2 + 1)^
2/(a^2*x^2 + 1), x) + 12*a^2*c*integrate(1/32*x^2*arctan(a*x)*log(a^2*x^2 +
 1)/(a^2*x^2 + 1), x) - 1/32*(a^2*c*x^3 + 3*c*x)*arctan(a*x)*log(a^2*x^2 +
1)^2 - 12*a*c*integrate(1/32*x*arctan(a*x)^2/(a^2*x^2 + 1), x) + 3*a*c*inte
grate(1/32*x*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x) + 3*c*integrate(1/32*arct
an(a*x)*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^2cx^2+c\right)\arctan\left(ax\right)^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)*arctan(a*x)^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$c\left(\int a^2x^2\,\text{atan}^3\left(ax\right)dx+\int \text{atan}^3\left(ax\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)*atan(a*x)**3,x)
```

```
[Out] c*(Integral(a**2*x**2*atan(a*x)**3, x) + Integral(atan(a*x)**3, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2+c\right)\arctan\left(ax\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)*arctan(a*x)^3, x)
```

3.367 $\int \frac{(c+a^2cx^2)\tan^{-1}(ax)^3}{x}\,dx$

Optimal. Leaf size=276

$$-\frac{3}{2}ic\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)+\frac{3}{4}ic\text{PolyLog}\left(4,1-\frac{2}{1+iax}\right)-\frac{3}{4}ic\text{PolyLog}\left(4,-1+\frac{2}{1+iax}\right)-\frac{3}{2}ic\tan^{-1}(ax)^2\text{PolyL}$$

```
[Out] ((-3*I)/2)*c*ArcTan[a*x]^2 - (3*a*c*x*ArcTan[a*x]^2)/2 + (c*ArcTan[a*x]^3)/
2 + (a^2*c*x^2*ArcTan[a*x]^3)/2 + 2*c*ArcTan[a*x]^3*ArcTanh[1 - 2/(1 + I*a*
x)] - 3*c*ArcTan[a*x]*Log[2/(1 + I*a*x)] - ((3*I)/2)*c*PolyLog[2, 1 - 2/(1
+ I*a*x)] - ((3*I)/2)*c*ArcTan[a*x]^2*PolyLog[2, 1 - 2/(1 + I*a*x)] + ((3*I
)/2)*c*ArcTan[a*x]^2*PolyLog[2, -1 + 2/(1 + I*a*x)] - (3*c*ArcTan[a*x]*Poly
Log[3, 1 - 2/(1 + I*a*x)])/2 + (3*c*ArcTan[a*x]*PolyLog[3, -1 + 2/(1 + I*a*
x)])/2 + ((3*I)/4)*c*PolyLog[4, 1 - 2/(1 + I*a*x)] - ((3*I)/4)*c*PolyLog[4,
 -1 + 2/(1 + I*a*x)]
```

Rubi [A] time = 0.522815, antiderivative size = 276, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 14, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.7, Rules used = {4950, 4850, 4988, 4884, 4994, 4998, 6610, 4852, 4916, 4846, 4920, 4854, 2402, 2315}

$$-\frac{3}{2}ic\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)+\frac{3}{4}ic\text{PolyLog}\left(4,1-\frac{2}{1+iax}\right)-\frac{3}{4}ic\text{PolyLog}\left(4,-1+\frac{2}{1+iax}\right)-\frac{3}{2}ic\tan^{-1}(ax)^2\text{PolyL}$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)*ArcTan[a*x]^3)/x,x]
```


```
[Out] ((-3*I)/2)*c*ArcTan[a*x]^2 - (3*a*c*x*ArcTan[a*x]^2)/2 + (c*ArcTan[a*x]^3)/
2 + (a^2*c*x^2*ArcTan[a*x]^3)/2 + 2*c*ArcTan[a*x]^3*ArcTanh[1 - 2/(1 + I*a*
x)] - 3*c*ArcTan[a*x]*Log[2/(1 + I*a*x)] - ((3*I)/2)*c*PolyLog[2, 1 - 2/(1
+ I*a*x)] - ((3*I)/2)*c*ArcTan[a*x]^2*PolyLog[2, 1 - 2/(1 + I*a*x)] + ((3*I
)/2)*c*ArcTan[a*x]^2*PolyLog[2, -1 + 2/(1 + I*a*x)] - (3*c*ArcTan[a*x]*Poly
Log[3, 1 - 2/(1 + I*a*x)])/2 + (3*c*ArcTan[a*x]*PolyLog[3, -1 + 2/(1 + I*a*
x)])/2 + ((3*I)/4)*c*PolyLog[4, 1 - 2/(1 + I*a*x)] - ((3*I)/4)*c*PolyLog[4,
 -1 + 2/(1 + I*a*x)]
```


Rule 4950

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
```

```
b*ArcTan[c*x])^p, x], x] + Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 4850

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTan[c*x])^(p - 1)*ArcTanh[1 - 2/(1 + I*c*x)])/(1 + c^2*x^2), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4988

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTan[c*x])^p)/(d + e
*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTan[c*x])^p)/(d + e*x^2
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 4998

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[k + 1, u])/(2
*c*d), x] - Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[k + 1,
 u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
qQ[e, c^2*d] && EqQ[u^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int \frac{\left(c+a^2cx^2\right)\tan^{-1}(ax)^3}{x}\,dx = c\int \frac{\tan^{-1}(ax)^3}{x}\,dx + \left(a^2c\right)\int x\tan^{-1}(ax)^3\,dx$$

$$= \frac{1}{2}a^2cx^2\tan^{-1}(ax)^3 + 2c\tan^{-1}(ax)^3\tanh^{-1}\left(1-\frac{2}{1+iax}\right) - (6ac)\int \frac{\tan^{-1}(ax)^2\tanh^{-1}\left(\right.}{1+a^2x^2}$$

$$= \frac{1}{2}a^2cx^2\tan^{-1}(ax)^3 + 2c\tan^{-1}(ax)^3\tanh^{-1}\left(1-\frac{2}{1+iax}\right) - \frac{1}{2}(3ac)\int \tan^{-1}(ax)^2\,dx + \frac{1}{2}($$

$$= -\frac{3}{2}acx\tan^{-1}(ax)^2 + \frac{1}{2}c\tan^{-1}(ax)^3 + \frac{1}{2}a^2cx^2\tan^{-1}(ax)^3 + 2c\tan^{-1}(ax)^3\tanh^{-1}\left(1-\frac{}{1+}\right.$$

$$= -\frac{3}{2}ic\tan^{-1}(ax)^2 - \frac{3}{2}acx\tan^{-1}(ax)^2 + \frac{1}{2}c\tan^{-1}(ax)^3 + \frac{1}{2}a^2cx^2\tan^{-1}(ax)^3 + 2c\tan^{-1}(ax$$

$$= -\frac{3}{2}ic\tan^{-1}(ax)^2 - \frac{3}{2}acx\tan^{-1}(ax)^2 + \frac{1}{2}c\tan^{-1}(ax)^3 + \frac{1}{2}a^2cx^2\tan^{-1}(ax)^3 + 2c\tan^{-1}(ax$$

$$= -\frac{3}{2}ic\tan^{-1}(ax)^2 - \frac{3}{2}acx\tan^{-1}(ax)^2 + \frac{1}{2}c\tan^{-1}(ax)^3 + \frac{1}{2}a^2cx^2\tan^{-1}(ax)^3 + 2c\tan^{-1}(ax$$

$$= -\frac{3}{2}ic\tan^{-1}(ax)^2 - \frac{3}{2}acx\tan^{-1}(ax)^2 + \frac{1}{2}c\tan^{-1}(ax)^3 + \frac{1}{2}a^2cx^2\tan^{-1}(ax)^3 + 2c\tan^{-1}(ax$$

Mathematica [A] time = 0.0709174, size = 284, normalized size = 1.03

$$-\frac{3}{4}ic\text{PolyLog}\left(4,\frac{-ax-i}{ax-i}\right) + \frac{3}{4}ic\text{PolyLog}\left(4,\frac{ax+i}{ax-i}\right) + \frac{3}{2}ic\tan^{-1}(ax)^2\text{PolyLog}\left(2,\frac{-ax-i}{ax-i}\right) - \frac{3}{2}ic\tan^{-1}(ax)^2\text{PolyLog}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((c + a^2*c*x^2)*ArcTan[a*x]^3)/x,x]
```

```
[Out] (c*(1 + a^2*x^2)*ArcTan[a*x]^3)/2 + 2*c*ArcTan[a*x]^3*ArcTanh[1 - (2*I)/(I
- a*x)] - (3*c*((-I)*ArcTan[a*x]^2 + a*x*ArcTan[a*x]^2 + 2*ArcTan[a*x]*Log[
1 + E^((2*I)*ArcTan[a*x])] - I*PolyLog[2, -E^((2*I)*ArcTan[a*x])]))/2 + ((3
```

```
*I)/2)*c*ArcTan[a*x]^2*PolyLog[2, (-I - a*x)/(-I + a*x)] - ((3*I)/2)*c*ArcT
an[a*x]^2*PolyLog[2, (I + a*x)/(-I + a*x)] + (3*c*ArcTan[a*x]*PolyLog[3, (-
I - a*x)/(-I + a*x)])/2 - (3*c*ArcTan[a*x]*PolyLog[3, (I + a*x)/(-I + a*x)]
)/2 - ((3*I)/4)*c*PolyLog[4, (-I - a*x)/(-I + a*x)] + ((3*I)/4)*c*PolyLog[4
, (I + a*x)/(-I + a*x)]
```

Maple [A] time = 1.325, size = 460, normalized size = 1.7

$$\frac{c\left(\arctan\left(ax\right)\right)^{2}\left(-i\arctan\left(ax\right)+\arctan\left(ax\right)xa-3\right)\left(ax+i\right)}{2}+6\,ic\mathrm{polylog}\left(4,\left(1+iax\right)\frac{1}{\sqrt{a^{2}x^{2}+1}}\right)-3\,c\arctan\left(ax\right)\mathrm{l}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)*arctan(a*x)^3/x,x)
```

```
[Out] 1/2*c*arctan(a*x)^2*(-I*arctan(a*x)+arctan(a*x)*x*a-3)*(a*x+I)+6*I*c*polylo
g(4,(1+I*a*x)/(a^2*x^2+1)^(1/2))-3*c*arctan(a*x)*ln((1+I*a*x)^2/(a^2*x^2+1)
+1)+3/2*I*c*polylog(2,-(1+I*a*x)^2/(a^2*x^2+1))+c*arctan(a*x)^3*ln(1-(1+I*a
*x)/(a^2*x^2+1)^(1/2))+3*I*c*arctan(a*x)^2+6*c*arctan(a*x)*polylog(3,(1+I*a
*x)/(a^2*x^2+1)^(1/2))+6*I*c*polylog(4,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+c*arct
an(a*x)^3*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-3*I*c*arctan(a*x)^2*polylog(2,(
1+I*a*x)/(a^2*x^2+1)^(1/2))+6*c*arctan(a*x)*polylog(3,-(1+I*a*x)/(a^2*x^2+1
)^(1/2))-3*I*c*arctan(a*x)^2*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-c*arct
an(a*x)^3*ln((1+I*a*x)^2/(a^2*x^2+1)+1)+3/2*I*c*arctan(a*x)^2*polylog(2,-(1
+I*a*x)^2/(a^2*x^2+1))-3/2*c*arctan(a*x)*polylog(3,-(1+I*a*x)^2/(a^2*x^2+1)
)-3/4*I*c*polylog(4,-(1+I*a*x)^2/(a^2*x^2+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{16}\,a^{2}cx^{2}\arctan\left(ax\right)^{3}-\frac{3}{64}\,a^{2}cx^{2}\arctan\left(ax\right)\log\left(a^{2}x^{2}+1\right)^{2}+\int\frac{12\,a^{4}cx^{4}\arctan\left(ax\right)\log\left(a^{2}x^{2}+1\right)-12\,a^{3}cx^{3}\,\mathrm{arcta}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^3/x,x, algorithm="maxima")
```

```
[Out] 1/16*a^2*c*x^2*arctan(a*x)^3 - 3/64*a^2*c*x^2*arctan(a*x)*log(a^2*x^2 + 1)^
2 + integrate(1/64*(12*a^4*c*x^4*arctan(a*x)*log(a^2*x^2 + 1) - 12*a^3*c*x^
3*arctan(a*x)^2 + 56*(a^4*c*x^4 + 2*a^2*c*x^2 + c)*arctan(a*x)^3 + 3*(a^3*c
```

```
*x^3 + 2*(a^4*c*x^4 + 2*a^2*c*x^2 + c)*arctan(a*x))*log(a^2*x^2 + 1)^2)/(a^
2*x^3 + x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^2cx^2+c\right)\arctan\left(ax\right)^3}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^3/x,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)*arctan(a*x)^3/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$c\left(\int\frac{\operatorname{atan}^3\left(ax\right)}{x}\,dx+\int a^2x\operatorname{atan}^3\left(ax\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)*atan(a*x)**3/x,x)
```

```
[Out] c*(Integral(atan(a*x)**3/x, x) + Integral(a**2*x*atan(a*x)**3, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a^2cx^2+c\right)\arctan\left(ax\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^3/x,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)*arctan(a*x)^3/x, x)
```

3.368 $\int \frac{(c+a^2cx^2)\tan^{-1}(ax)^3}{x^2}\,dx$

Optimal. Leaf size=169

$$\frac{3}{2}ac\text{PolyLog}\left(3,-1+\frac{2}{1-iax}\right)+\frac{3}{2}ac\text{PolyLog}\left(3,1-\frac{2}{1+iax}\right)-3iac\tan^{-1}(ax)\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)+3iac\tan^{-1}$$

```
[Out] -((c*ArcTan[a*x]^3)/x) + a^2*c*x*ArcTan[a*x]^3 + 3*a*c*ArcTan[a*x]^2*Log[2/
(1 + I*a*x)] + 3*a*c*ArcTan[a*x]^2*Log[2 - 2/(1 - I*a*x)] - (3*I)*a*c*ArcTa
n[a*x]*PolyLog[2, -1 + 2/(1 - I*a*x)] + (3*I)*a*c*ArcTan[a*x]*PolyLog[2, 1
- 2/(1 + I*a*x)] + (3*a*c*PolyLog[3, -1 + 2/(1 - I*a*x)])/2 + (3*a*c*PolyLo
g[3, 1 - 2/(1 + I*a*x)])/2
```

Rubi [A] time = 0.39471, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 11, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.55, Rules used = {4950, 4852, 4924, 4868, 4884, 4992, 6610, 4846, 4920, 4854, 4994}

$$\frac{3}{2}ac\text{PolyLog}\left(3,-1+\frac{2}{1-iax}\right)+\frac{3}{2}ac\text{PolyLog}\left(3,1-\frac{2}{1+iax}\right)-3iac\tan^{-1}(ax)\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)+3iac\tan^{-1}$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)*ArcTan[a*x]^3)/x^2,x]
```


```
[Out] -((c*ArcTan[a*x]^3)/x) + a^2*c*x*ArcTan[a*x]^3 + 3*a*c*ArcTan[a*x]^2*Log[2/
(1 + I*a*x)] + 3*a*c*ArcTan[a*x]^2*Log[2 - 2/(1 - I*a*x)] - (3*I)*a*c*ArcTa
n[a*x]*PolyLog[2, -1 + 2/(1 - I*a*x)] + (3*I)*a*c*ArcTan[a*x]*PolyLog[2, 1
- 2/(1 + I*a*x)] + (3*a*c*PolyLog[3, -1 + 2/(1 - I*a*x)])/2 + (3*a*c*PolyLo
g[3, 1 - 2/(1 + I*a*x)])/2
```


Rule 4950

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4992

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d
] && EqQ[(1 - u)^2 - (1 - (2*I)/(I + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
```

```
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rubi steps

$$\int \frac{\left(c+a^2cx^2\right)\tan^{-1}(ax)^3}{x^2}\,dx = c\int \frac{\tan^{-1}(ax)^3}{x^2}\,dx+\left(a^2c\right)\int \tan^{-1}(ax)^3\,dx$$

$$= -\frac{c\tan^{-1}(ax)^3}{x}+a^2cx\tan^{-1}(ax)^3+(3ac)\int \frac{\tan^{-1}(ax)^2}{x\left(1+a^2x^2\right)}\,dx-\left(3a^3c\right)\int \frac{x\tan^{-1}(ax)^2}{1+a^2x^2}\,d$$

$$= -\frac{c\tan^{-1}(ax)^3}{x}+a^2cx\tan^{-1}(ax)^3+(3iac)\int \frac{\tan^{-1}(ax)^2}{x(i+ax)}\,dx+\left(3a^2c\right)\int \frac{\tan^{-1}(ax)^2}{i-ax}\,dx$$

$$= -\frac{c\tan^{-1}(ax)^3}{x}+a^2cx\tan^{-1}(ax)^3+3ac\tan^{-1}(ax)^2\log\left(\frac{2}{1+iax}\right)+3ac\tan^{-1}(ax)^2\log\Big($$

$$= -\frac{c\tan^{-1}(ax)^3}{x}+a^2cx\tan^{-1}(ax)^3+3ac\tan^{-1}(ax)^2\log\left(\frac{2}{1+iax}\right)+3ac\tan^{-1}(ax)^2\log\Big($$

$$= -\frac{c\tan^{-1}(ax)^3}{x}+a^2cx\tan^{-1}(ax)^3+3ac\tan^{-1}(ax)^2\log\left(\frac{2}{1+iax}\right)+3ac\tan^{-1}(ax)^2\log\Big($$

Mathematica [A] time = 0.175158, size = 181, normalized size = 1.07

$$ac\left(3i\tan^{-1}(ax)\text{PolyLog}\left(2,e^{-2i\tan^{-1}(ax)}\right)+\frac{3}{2}\text{PolyLog}\left(3,e^{-2i\tan^{-1}(ax)}\right)-\frac{\tan^{-1}(ax)^3}{ax}+i\tan^{-1}(ax)^3+3\tan^{-1}(ax)^2\log\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((c + a^2*c*x^2)*ArcTan[a*x]^3)/x^2,x]
```

```
[Out] a*c*((-I/8)*Pi^3 + I*ArcTan[a*x]^3 - ArcTan[a*x]^3/(a*x) + 3*ArcTan[a*x]^2*
Log[1 - E^((-2*I)*ArcTan[a*x])] + (3*I)*ArcTan[a*x]*PolyLog[2, E^((-2*I)*Ar
cTan[a*x])] + (3*PolyLog[3, E^((-2*I)*ArcTan[a*x])])/2) + a*c*((-I)*ArcTan[
a*x]^3 + a*x*ArcTan[a*x]^3 + 3*ArcTan[a*x]^2*Log[1 + E^((2*I)*ArcTan[a*x])]
 - (3*I)*ArcTan[a*x]*PolyLog[2, -E^((2*I)*ArcTan[a*x])] + (3*PolyLog[3, -E^
((2*I)*ArcTan[a*x])])/2)
```

Maple [C] time = 0.431, size = 1826, normalized size = 10.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)*arctan(a*x)^3/x^2,x)
```

```
[Out] 3/2*I*a*c*Pi*arctan(a*x)^2*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1))*csgn(I/((1+I
*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^
2*x^2+1)+1))-3/2*I*a*c*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I
*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1
)+1)^2)*arctan(a*x)^2+3/2*a*c*polylog(3,-(1+I*a*x)^2/(a^2*x^2+1))+6*a*c*pol
ylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*a*c*polylog(3,(1+I*a*x)/(a^2*x^2+1)^
(1/2))+3*I*a*c*Pi*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*csgn(I*(1+I*a*x)^2/(a
^2*x^2+1))^2*arctan(a*x)^2+3/2*I*a*c*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^
2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2*arctan(a
*x)^2-3/2*I*a*c*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/
(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2*arctan(a*x)^2-3/2*I*a*c*Pi*cs
gn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(((1+I*a*
x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2*arctan(a*x)^2-c*arctan(a
*x)^3/x+a^2*c*x*arctan(a*x)^3+6*a*c*arctan(a*x)^2*ln(2)-3*a*c*arctan(a*x)^2
*ln((1+I*a*x)^2/(a^2*x^2+1)-1)-3*a*c*arctan(a*x)^2*ln(a^2*x^2+1)+3*a*c*arct
an(a*x)^2*ln(a*x)+6*a*c*arctan(a*x)^2*ln((1+I*a*x)/(a^2*x^2+1)^(1/2))+3*a*c
*arctan(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))+3*a*c*arctan(a*x)^2*ln(1+(
```

```
1+I*a*x)/(a^2*x^2+1)^(1/2))-2*I*a*c*arctan(a*x)^3-3/2*I*a*c*Pi*csgn(I*((1+I
*a*x)^2/(a^2*x^2+1)-1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^
2*x^2+1)+1))^2*arctan(a*x)^2-3/2*I*a*c*Pi*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2
))^2*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*arctan(a*x)^2-3*I*a*c*Pi*csgn(I*((1+I*
a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2*arctan(a*x)^
2+3/2*I*a*c*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+
1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2*arctan(a*x)^2+3/2*I*a*c*Pi*csgn(I*((1+I
*a*x)^2/(a^2*x^2+1)+1))^2*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*arctan(a*x)
^2+3/2*I*a*c*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)
+1))*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*arctan(a
*x)^2+3/2*I*a*c*Pi*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1
)+1))^3*arctan(a*x)^2+3/2*I*a*c*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I
*a*x)^2/(a^2*x^2+1)+1))^3*arctan(a*x)^2-3/2*I*a*c*Pi*csgn(I*(1+I*a*x)^2/(a^
2*x^2+1))^3*arctan(a*x)^2-3/2*I*a*c*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I
*a*x)^2/(a^2*x^2+1)+1)^2)^3*arctan(a*x)^2+3/2*I*a*c*Pi*csgn(I*((1+I*a*x)^2/
(a^2*x^2+1)+1)^2)^3*arctan(a*x)^2-3/2*I*a*c*Pi*csgn(((1+I*a*x)^2/(a^2*x^2+1
)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2*arctan(a*x)^2-3*I*a*c*arctan(a*x)*polyl
og(2,-(1+I*a*x)^2/(a^2*x^2+1))-6*I*a*c*arctan(a*x)*polylog(2,(1+I*a*x)/(a^2
*x^2+1)^(1/2))-6*I*a*c*arctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+
3/2*I*a*c*Pi*arctan(a*x)^2
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^3/x^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^2cx^2+c\right)\arctan\left(ax\right)^3}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^3/x^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)*arctan(a*x)^3/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$c\left(\int a^2\operatorname{atan}^3(ax)\,dx+\int\frac{\operatorname{atan}^3(ax)}{x^2}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)*atan(a*x)**3/x**2,x)
```

```
[Out] c*(Integral(a**2*atan(a*x)**3, x) + Integral(atan(a*x)**3/x**2, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a^2cx^2+c\right)\arctan(ax)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^3/x^2,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)*arctan(a*x)^3/x^2, x)
```

3.369 $\int \frac{(c+a^2cx^2)\tan^{-1}(ax)^3}{x^3}\,dx$

Optimal. Leaf size=310

$$-\frac{3}{2}ia^2c\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)+\frac{3}{4}ia^2c\text{PolyLog}\left(4,1-\frac{2}{1+iax}\right)-\frac{3}{4}ia^2c\text{PolyLog}\left(4,-1+\frac{2}{1+iax}\right)-\frac{3}{2}ia^2c\tan^{-1}($$

```
[Out] ((-3*I)/2)*a^2*c*ArcTan[a*x]^2 - (3*a*c*ArcTan[a*x]^2)/(2*x) - (a^2*c*ArcTa
n[a*x]^3)/2 - (c*ArcTan[a*x]^3)/(2*x^2) + 2*a^2*c*ArcTan[a*x]^3*ArcTanh[1 -
 2/(1 + I*a*x)] + 3*a^2*c*ArcTan[a*x]*Log[2 - 2/(1 - I*a*x)] - ((3*I)/2)*a^
2*c*PolyLog[2, -1 + 2/(1 - I*a*x)] - ((3*I)/2)*a^2*c*ArcTan[a*x]^2*PolyLog[
2, 1 - 2/(1 + I*a*x)] + ((3*I)/2)*a^2*c*ArcTan[a*x]^2*PolyLog[2, -1 + 2/(1
+ I*a*x)] - (3*a^2*c*ArcTan[a*x]*PolyLog[3, 1 - 2/(1 + I*a*x)])/2 + (3*a^2*
c*ArcTan[a*x]*PolyLog[3, -1 + 2/(1 + I*a*x)])/2 + ((3*I)/4)*a^2*c*PolyLog[4
, 1 - 2/(1 + I*a*x)] - ((3*I)/4)*a^2*c*PolyLog[4, -1 + 2/(1 + I*a*x)]
```

Rubi [A] time = 0.556854, antiderivative size = 310, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 12, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {4950, 4852, 4918, 4924, 4868, 2447, 4884, 4850, 4988, 4994, 4998, 6610}

$$-\frac{3}{2}ia^2c\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)+\frac{3}{4}ia^2c\text{PolyLog}\left(4,1-\frac{2}{1+iax}\right)-\frac{3}{4}ia^2c\text{PolyLog}\left(4,-1+\frac{2}{1+iax}\right)-\frac{3}{2}ia^2c\tan^{-1}($$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)*ArcTan[a*x]^3)/x^3,x]
```


```
[Out] ((-3*I)/2)*a^2*c*ArcTan[a*x]^2 - (3*a*c*ArcTan[a*x]^2)/(2*x) - (a^2*c*ArcTa
n[a*x]^3)/2 - (c*ArcTan[a*x]^3)/(2*x^2) + 2*a^2*c*ArcTan[a*x]^3*ArcTanh[1 -
 2/(1 + I*a*x)] + 3*a^2*c*ArcTan[a*x]*Log[2 - 2/(1 - I*a*x)] - ((3*I)/2)*a^
2*c*PolyLog[2, -1 + 2/(1 - I*a*x)] - ((3*I)/2)*a^2*c*ArcTan[a*x]^2*PolyLog[
2, 1 - 2/(1 + I*a*x)] + ((3*I)/2)*a^2*c*ArcTan[a*x]^2*PolyLog[2, -1 + 2/(1
+ I*a*x)] - (3*a^2*c*ArcTan[a*x]*PolyLog[3, 1 - 2/(1 + I*a*x)])/2 + (3*a^2*
c*ArcTan[a*x]*PolyLog[3, -1 + 2/(1 + I*a*x)])/2 + ((3*I)/4)*a^2*c*PolyLog[4
, 1 - 2/(1 + I*a*x)] - ((3*I)/4)*a^2*c*PolyLog[4, -1 + 2/(1 + I*a*x)]
```


Rule 4950

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2)^
```

```
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4918

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/(d + e*x^2),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
```

```
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4850

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTan[c*x])^(p - 1)*ArcTanh[1 - 2/(1 + I*c*x)])/(1 + c^2*x^2), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4988

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTan[c*x])^p)/(d + e
*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTan[c*x])^p)/(d + e*x^2
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 4998

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[k + 1, u])/(2
*c*d), x] - Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[k + 1,
 u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
qQ[e, c^2*d] && EqQ[u^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)\tan^{-1}(ax)^3}{x^3}\,dx = c\int \frac{\tan^{-1}(ax)^3}{x^3}\,dx + (a^2c)\int \frac{\tan^{-1}(ax)^3}{x}\,dx$$

$$= -\frac{c\tan^{-1}(ax)^3}{2x^2} + 2a^2c\tan^{-1}(ax)^3\tanh^{-1}\left(1-\frac{2}{1+iax}\right) + \frac{1}{2}(3ac)\int \frac{\tan^{-1}(ax)^2}{x^2(1+a^2x^2)}\,dx - (6$$

$$= -\frac{c\tan^{-1}(ax)^3}{2x^2} + 2a^2c\tan^{-1}(ax)^3\tanh^{-1}\left(1-\frac{2}{1+iax}\right) + \frac{1}{2}(3ac)\int \frac{\tan^{-1}(ax)^2}{x^2}\,dx - \frac{1}{2}(3$$

$$= -\frac{3ac\tan^{-1}(ax)^2}{2x} - \frac{1}{2}a^2c\tan^{-1}(ax)^3 - \frac{c\tan^{-1}(ax)^3}{2x^2} + 2a^2c\tan^{-1}(ax)^3\tanh^{-1}\left(1-\frac{2}{1+ia\right.$$

$$= -\frac{3}{2}ia^2c\tan^{-1}(ax)^2 - \frac{3ac\tan^{-1}(ax)^2}{2x} - \frac{1}{2}a^2c\tan^{-1}(ax)^3 - \frac{c\tan^{-1}(ax)^3}{2x^2} + 2a^2c\tan^{-1}(ax)$$

$$= -\frac{3}{2}ia^2c\tan^{-1}(ax)^2 - \frac{3ac\tan^{-1}(ax)^2}{2x} - \frac{1}{2}a^2c\tan^{-1}(ax)^3 - \frac{c\tan^{-1}(ax)^3}{2x^2} + 2a^2c\tan^{-1}(ax)$$

$$= -\frac{3}{2}ia^2c\tan^{-1}(ax)^2 - \frac{3ac\tan^{-1}(ax)^2}{2x} - \frac{1}{2}a^2c\tan^{-1}(ax)^3 - \frac{c\tan^{-1}(ax)^3}{2x^2} + 2a^2c\tan^{-1}(ax)$$

Mathematica [A] time = 0.240957, size = 337, normalized size = 1.09

$$-\frac{3}{4}ia^2c\text{PolyLog}\left(4,\frac{-ax-i}{ax-i}\right) + \frac{3}{4}ia^2c\text{PolyLog}\left(4,\frac{ax+i}{ax-i}\right) + \frac{3}{2}ia^2c\tan^{-1}(ax)^2\text{PolyLog}\left(2,\frac{-ax-i}{ax-i}\right) - \frac{3}{2}ia^2c\tan^{-1}(ax)^2\text{P}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((c + a^2*c*x^2)*ArcTan[a*x]^3)/x^3,x]
```

```
[Out] (a^2*c*ArcTan[a*x]^3)/2 + (c*(-1 - a^2*x^2)*ArcTan[a*x]^3)/(2*x^2) + 2*a^2*
c*ArcTan[a*x]^3*ArcTanh[1 - (2*I)/(I - a*x)] + (3*a^2*c*(-(ArcTan[a*x]*((3*
ArcTan[a*x])/(a*x) + ArcTan[a*x]*(3*I + ArcTan[a*x]) - 6*Log[1 - E^((2*I)*A
rcTan[a*x])]))/3 - I*PolyLog[2, E^((2*I)*ArcTan[a*x])]))/2 + ((3*I)/2)*a^2*
c*ArcTan[a*x]^2*PolyLog[2, (-I - a*x)/(-I + a*x)] - ((3*I)/2)*a^2*c*ArcTan[
a*x]^2*PolyLog[2, (I + a*x)/(-I + a*x)] + (3*a^2*c*ArcTan[a*x]*PolyLog[3, (
-I - a*x)/(-I + a*x)])/2 - (3*a^2*c*ArcTan[a*x]*PolyLog[3, (I + a*x)/(-I +
a*x)])/2 - ((3*I)/4)*a^2*c*PolyLog[4, (-I - a*x)/(-I + a*x)] + ((3*I)/4)*a^
2*c*PolyLog[4, (I + a*x)/(-I + a*x)]
```

Maple [B] time = 2.037, size = 568, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)*arctan(a*x)^3/x^3,x)
```

```
[Out] -1/2*a^2*c*arctan(a*x)^3-3/4*I*a^2*c*polylog(4,-(1+I*a*x)^2/(a^2*x^2+1))-3/
2*a*c*arctan(a*x)^2/x-1/2*c*arctan(a*x)^3/x^2+6*I*a^2*c*polylog(4,-(1+I*a*x
)/(a^2*x^2+1)^(1/2))+3*a^2*c*arctan(a*x)*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-
3/2*I*a^2*c*arctan(a*x)^2+a^2*c*arctan(a*x)^3*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1
/2))+6*I*a^2*c*polylog(4,(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*a^2*c*arctan(a*x)*p
olylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+a^2*c*arctan(a*x)^3*ln(1-(1+I*a*x)/(
a^2*x^2+1)^(1/2))-3*I*a^2*c*arctan(a*x)^2*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^
(1/2))-3/2*a^2*c*arctan(a*x)*polylog(3,-(1+I*a*x)^2/(a^2*x^2+1))+3/2*I*a^2*
c*arctan(a*x)^2*polylog(2,-(1+I*a*x)^2/(a^2*x^2+1))-a^2*c*arctan(a*x)^3*ln(
(1+I*a*x)^2/(a^2*x^2+1)+1)-3*I*a^2*c*arctan(a*x)^2*polylog(2,(1+I*a*x)/(a^2
*x^2+1)^(1/2))+6*a^2*c*arctan(a*x)*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))-3
*I*a^2*c*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+3*a^2*c*arctan(a*x)*ln(1-(1
+I*a*x)/(a^2*x^2+1)^(1/2))-3*I*a^2*c*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2)
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{4\,c\arctan\left(ax\right)^{3} - 3\,c\arctan\left(ax\right)\log\left(a^{2}x^{2}+1\right)^{2} + x^{2}\int \frac{12\,a^{2}cx^{2}\arctan(ax)\log\left(a^{2}x^{2}+1\right) - 12\,acx\arctan(ax)^{2} - 56\left(a^{4}cx^{4}+2\,a^{2}cx^{2}+c\right)\mathrm{arc}}{a^{2}x^{5}+x^{3}}}{64\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^3/x^3,x, algorithm="maxima")
```

```
[Out] -1/64*(4*c*arctan(a*x)^3 - 3*c*arctan(a*x)*log(a^2*x^2 + 1)^2 - 64*x^2*inte
grate(-1/64*(12*a^2*c*x^2*arctan(a*x)*log(a^2*x^2 + 1) - 12*a*c*x*arctan(a*
x)^2 - 56*(a^4*c*x^4 + 2*a^2*c*x^2 + c)*arctan(a*x)^3 + 3*(a*c*x - 2*(a^4*c
*x^4 + 2*a^2*c*x^2 + c)*arctan(a*x))*log(a^2*x^2 + 1)^2)/(a^2*x^5 + x^3), x
))/x^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^2cx^2+c\right)\arctan\left(ax\right)^3}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^3/x^3,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)*arctan(a*x)^3/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$c\left(\int\frac{\operatorname{atan}^3\left(ax\right)}{x^3}\,dx+\int\frac{a^2\operatorname{atan}^3\left(ax\right)}{x}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)*atan(a*x)**3/x**3,x)
```

```
[Out] c*(Integral(atan(a*x)**3/x**3, x) + Integral(a**2*atan(a*x)**3/x, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a^2cx^2+c\right)\arctan\left(ax\right)^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^3/x^3,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)*arctan(a*x)^3/x^3, x)
```

3.370 $\int \frac{(c+a^2cx^2)\tan^{-1}(ax)^3}{x^4}\,dx$

Optimal. Leaf size=189

$$a^3c\text{PolyLog}\left(3,-1+\frac{2}{1-iax}\right)-2ia^3c\tan^{-1}(ax)\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)-\frac{1}{2}a^3c\log\left(a^2x^2+1\right)+a^3c\log(x)-\frac{2}{3}ia^3c$$

```
[Out] -((a^2*c*ArcTan[a*x])/x) - (a^3*c*ArcTan[a*x]^2)/2 - (a*c*ArcTan[a*x]^2)/(2
*x^2) - ((2*I)/3)*a^3*c*ArcTan[a*x]^3 - (c*ArcTan[a*x]^3)/(3*x^3) - (a^2*c*
ArcTan[a*x]^3)/x + a^3*c*Log[x] - (a^3*c*Log[1 + a^2*x^2])/2 + 2*a^3*c*ArcT
an[a*x]^2*Log[2 - 2/(1 - I*a*x)] - (2*I)*a^3*c*ArcTan[a*x]*PolyLog[2, -1 +
2/(1 - I*a*x)] + a^3*c*PolyLog[3, -1 + 2/(1 - I*a*x)]
```

Rubi [A] time = 0.58492, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 12, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {4950, 4852, 4918, 266, 36, 29, 31, 4884, 4924, 4868, 4992, 6610}

$$a^3c\text{PolyLog}\left(3,-1+\frac{2}{1-iax}\right)-2ia^3c\tan^{-1}(ax)\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)-\frac{1}{2}a^3c\log\left(a^2x^2+1\right)+a^3c\log(x)-\frac{2}{3}ia^3c$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)*ArcTan[a*x]^3)/x^4,x]
```



```
[Out] -((a^2*c*ArcTan[a*x])/x) - (a^3*c*ArcTan[a*x]^2)/2 - (a*c*ArcTan[a*x]^2)/(2
*x^2) - ((2*I)/3)*a^3*c*ArcTan[a*x]^3 - (c*ArcTan[a*x]^3)/(3*x^3) - (a^2*c*
ArcTan[a*x]^3)/x + a^3*c*Log[x] - (a^3*c*Log[1 + a^2*x^2])/2 + 2*a^3*c*ArcT
an[a*x]^2*Log[2 - 2/(1 - I*a*x)] - (2*I)*a^3*c*ArcTan[a*x]*PolyLog[2, -1 +
2/(1 - I*a*x)] + a^3*c*PolyLog[3, -1 + 2/(1 - I*a*x)]
```


Rule 4950

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4918

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/(d + e*x^2),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 4992

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d
] && EqQ[(1 - u)^2 - (1 - (2*I)/(I + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(c+a^2cx^2\right)\tan^{-1}(ax)^3}{x^4}\,dx = c\int \frac{\tan^{-1}(ax)^3}{x^4}\,dx + \left(a^2c\right)\int \frac{\tan^{-1}(ax)^3}{x^2}\,dx$$

$$= -\frac{c\tan^{-1}(ax)^3}{3x^3} - \frac{a^2c\tan^{-1}(ax)^3}{x} + (ac)\int \frac{\tan^{-1}(ax)^2}{x^3\left(1+a^2x^2\right)}\,dx + \left(3a^3c\right)\int \frac{\tan^{-1}(ax)^2}{x\left(1+a^2x^2\right)}\,dx$$

$$= -ia^3c\tan^{-1}(ax)^3 - \frac{c\tan^{-1}(ax)^3}{3x^3} - \frac{a^2c\tan^{-1}(ax)^3}{x} + (ac)\int \frac{\tan^{-1}(ax)^2}{x^3}\,dx + \left(3ia^3c\right)\int$$

$$= -\frac{ac\tan^{-1}(ax)^2}{2x^2} - \frac{2}{3}ia^3c\tan^{-1}(ax)^3 - \frac{c\tan^{-1}(ax)^3}{3x^3} - \frac{a^2c\tan^{-1}(ax)^3}{x} + 3a^3c\tan^{-1}(ax)^2\,\mathrm{l}$$

$$= -\frac{ac\tan^{-1}(ax)^2}{2x^2} - \frac{2}{3}ia^3c\tan^{-1}(ax)^3 - \frac{c\tan^{-1}(ax)^3}{3x^3} - \frac{a^2c\tan^{-1}(ax)^3}{x} + 2a^3c\tan^{-1}(ax)^2\,\mathrm{l}$$

$$= -\frac{a^2c\tan^{-1}(ax)}{x} - \frac{1}{2}a^3c\tan^{-1}(ax)^2 - \frac{ac\tan^{-1}(ax)^2}{2x^2} - \frac{2}{3}ia^3c\tan^{-1}(ax)^3 - \frac{c\tan^{-1}(ax)^3}{3x^3} -$$

$$= -\frac{a^2c\tan^{-1}(ax)}{x} - \frac{1}{2}a^3c\tan^{-1}(ax)^2 - \frac{ac\tan^{-1}(ax)^2}{2x^2} - \frac{2}{3}ia^3c\tan^{-1}(ax)^3 - \frac{c\tan^{-1}(ax)^3}{3x^3} -$$

$$= -\frac{a^2c\tan^{-1}(ax)}{x} - \frac{1}{2}a^3c\tan^{-1}(ax)^2 - \frac{ac\tan^{-1}(ax)^2}{2x^2} - \frac{2}{3}ia^3c\tan^{-1}(ax)^3 - \frac{c\tan^{-1}(ax)^3}{3x^3} -$$

$$= -\frac{a^2c\tan^{-1}(ax)}{x} - \frac{1}{2}a^3c\tan^{-1}(ax)^2 - \frac{ac\tan^{-1}(ax)^2}{2x^2} - \frac{2}{3}ia^3c\tan^{-1}(ax)^3 - \frac{c\tan^{-1}(ax)^3}{3x^3} -$$

Mathematica [A] time = 0.384004, size = 177, normalized size = 0.94

$$\frac{1}{12}c\left(24ia^3\tan^{-1}(ax)\mathrm{PolyLog}\left(2,e^{-2i\tan^{-1}(ax)}\right)+12a^3\mathrm{PolyLog}\left(3,e^{-2i\tan^{-1}(ax)}\right)+12a^3\log\left(\frac{ax}{\sqrt{a^2x^2+1}}\right)+8ia^3\tan^{-1}(ax\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((c + a^2*c*x^2)*ArcTan[a*x]^3)/x^4,x]
```

```
[Out] (c*((-I)*a^3*Pi^3 - (12*a^2*ArcTan[a*x])/x - 6*a^3*ArcTan[a*x]^2 - (6*a*Arc
Tan[a*x]^2)/x^2 + (8*I)*a^3*ArcTan[a*x]^3 - (4*ArcTan[a*x]^3)/x^3 - (12*a^2
*ArcTan[a*x]^3)/x + 24*a^3*ArcTan[a*x]^2*Log[1 - E^((-2*I)*ArcTan[a*x])] +
12*a^3*Log[(a*x)/Sqrt[1 + a^2*x^2]] + (24*I)*a^3*ArcTan[a*x]*PolyLog[2, E^(
(-2*I)*ArcTan[a*x])] + 12*a^3*PolyLog[3, E^((-2*I)*ArcTan[a*x])]))/12
```

Maple [C] time = 1.928, size = 5426, normalized size = 28.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)*arctan(a*x)^3/x^4,x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^3/x^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^2cx^2+c\right)\arctan\left(ax\right)^3}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^3/x^4,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)*arctan(a*x)^3/x^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$c\left(\int\frac{\operatorname{atan}^3\left(ax\right)}{x^4}\,dx+\int\frac{a^2\operatorname{atan}^3\left(ax\right)}{x^2}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)*atan(a*x)**3/x**4,x)
```

```
[Out] c*(Integral(atan(a*x)**3/x**4, x) + Integral(a**2*atan(a*x)**3/x**2, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a^2cx^2 + c)\arctan(ax)^3}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^3/x^4,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)*arctan(a*x)^3/x^4, x)
```

3.371 $\int x^3 \left(c+a^2 c x^2\right)^2 \tan^{-1}(ax)^3 \, dx$

Optimal. Leaf size=313

$$\frac{2ic^2 \text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{21a^4}+\frac{1}{8}a^4c^2x^8\tan^{-1}(ax)^3-\frac{3}{56}a^3c^2x^7\tan^{-1}(ax)^2+\frac{1}{3}a^2c^2x^6\tan^{-1}(ax)^3+\frac{1}{56}a^2c^2x^6\tan^{-1}(ax)$$

```
[Out] (c^2*x)/(21*a^3) - (c^2*x^3)/(168*a) - (a*c^2*x^5)/280 - (c^2*ArcTan[a*x])/
(21*a^4) - (5*c^2*x^2*ArcTan[a*x])/(168*a^2) + (c^2*x^4*ArcTan[a*x])/28 + (
a^2*c^2*x^6*ArcTan[a*x])/56 + (((2*I)/21)*c^2*ArcTan[a*x]^2)/a^4 + (c^2*x*A
rcTan[a*x]^2)/(8*a^3) - (c^2*x^3*ArcTan[a*x]^2)/(24*a) - (a*c^2*x^5*ArcTan[
a*x]^2)/8 - (3*a^3*c^2*x^7*ArcTan[a*x]^2)/56 - (c^2*ArcTan[a*x]^3)/(24*a^4)
 + (c^2*x^4*ArcTan[a*x]^3)/4 + (a^2*c^2*x^6*ArcTan[a*x]^3)/3 + (a^4*c^2*x^8
*ArcTan[a*x]^3)/8 + (4*c^2*ArcTan[a*x]*Log[2/(1 + I*a*x)])/(21*a^4) + (((2*
I)/21)*c^2*PolyLog[2, 1 - 2/(1 + I*a*x)])/a^4
```

Rubi [A] time = 2.28292, antiderivative size = 313, normalized size of antiderivative = 1., number of steps used = 106, number of rules used = 12, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {4948, 4852, 4916, 321, 203, 4920, 4854, 2402, 2315, 4846, 4884, 302}

$$\frac{2ic^2 \text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{21a^4}+\frac{1}{8}a^4c^2x^8\tan^{-1}(ax)^3-\frac{3}{56}a^3c^2x^7\tan^{-1}(ax)^2+\frac{1}{3}a^2c^2x^6\tan^{-1}(ax)^3+\frac{1}{56}a^2c^2x^6\tan^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(c + a^2*c*x^2)^2*ArcTan[a*x]^3,x]
```


```
[Out] (c^2*x)/(21*a^3) - (c^2*x^3)/(168*a) - (a*c^2*x^5)/280 - (c^2*ArcTan[a*x])/
(21*a^4) - (5*c^2*x^2*ArcTan[a*x])/(168*a^2) + (c^2*x^4*ArcTan[a*x])/28 + (
a^2*c^2*x^6*ArcTan[a*x])/56 + (((2*I)/21)*c^2*ArcTan[a*x]^2)/a^4 + (c^2*x*A
rcTan[a*x]^2)/(8*a^3) - (c^2*x^3*ArcTan[a*x]^2)/(24*a) - (a*c^2*x^5*ArcTan[
a*x]^2)/8 - (3*a^3*c^2*x^7*ArcTan[a*x]^2)/56 - (c^2*ArcTan[a*x]^3)/(24*a^4)
 + (c^2*x^4*ArcTan[a*x]^3)/4 + (a^2*c^2*x^6*ArcTan[a*x]^3)/3 + (a^4*c^2*x^8
*ArcTan[a*x]^3)/8 + (4*c^2*ArcTan[a*x]*Log[2/(1 + I*a*x)])/(21*a^4) + (((2*
I)/21)*c^2*PolyLog[2, 1 - 2/(1 + I*a*x)])/a^4
```


Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
```

```
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
```

```
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\int x^3 \left(c+a^2cx^2\right)^2 \tan^{-1}(ax)^3\,dx = \int \left(c^2x^3\tan^{-1}(ax)^3 + 2a^2c^2x^5\tan^{-1}(ax)^3 + a^4c^2x^7\tan^{-1}(ax)^3\right)dx$$
$$= c^2\int x^3\tan^{-1}(ax)^3\,dx + \left(2a^2c^2\right)\int x^5\tan^{-1}(ax)^3\,dx + \left(a^4c^2\right)\int x^7\tan^{-1}(ax)^3\,dx$$
$$= \frac{1}{4}c^2x^4\tan^{-1}(ax)^3 + \frac{1}{3}a^2c^2x^6\tan^{-1}(ax)^3 + \frac{1}{8}a^4c^2x^8\tan^{-1}(ax)^3 - \frac{1}{4}\left(3ac^2\right)\int \frac{x^4\tan^{-1}}{1+a^2}$$
$$= \frac{1}{4}c^2x^4\tan^{-1}(ax)^3 + \frac{1}{3}a^2c^2x^6\tan^{-1}(ax)^3 + \frac{1}{8}a^4c^2x^8\tan^{-1}(ax)^3 - \frac{\left(3c^2\right)\int x^2\tan^{-1}(ax)}{4a}$$
$$= -\frac{c^2x^3\tan^{-1}(ax)^2}{4a} - \frac{1}{5}ac^2x^5\tan^{-1}(ax)^2 - \frac{3}{56}a^3c^2x^7\tan^{-1}(ax)^2 + \frac{1}{4}c^2x^4\tan^{-1}(ax)^3 +$$
$$= \frac{3c^2x\tan^{-1}(ax)^2}{4a^3} + \frac{c^2x^3\tan^{-1}(ax)^2}{12a} - \frac{1}{8}ac^2x^5\tan^{-1}(ax)^2 - \frac{3}{56}a^3c^2x^7\tan^{-1}(ax)^2 - \frac{c^2}{}$$
$$= \frac{c^2x^2\tan^{-1}(ax)}{4a^2} + \frac{1}{10}c^2x^4\tan^{-1}(ax) + \frac{1}{56}a^2c^2x^6\tan^{-1}(ax) + \frac{ic^2\tan^{-1}(ax)^2}{a^4} - \frac{c^2x\tan}{4}$$
$$= -\frac{c^2x}{4a^3} - \frac{17c^2x^2\tan^{-1}(ax)}{60a^2} + \frac{1}{28}c^2x^4\tan^{-1}(ax) + \frac{1}{56}a^2c^2x^6\tan^{-1}(ax) - \frac{8ic^2\tan^{-1}(ax)}{15a^4}$$
$$= \frac{307c^2x}{840a^3} - \frac{23c^2x^3}{840a} - \frac{1}{280}ac^2x^5 + \frac{c^2\tan^{-1}(ax)}{4a^4} - \frac{5c^2x^2\tan^{-1}(ax)}{168a^2} + \frac{1}{28}c^2x^4\tan^{-1}(ax)$$
$$= \frac{c^2x}{21a^3} - \frac{c^2x^3}{168a} - \frac{1}{280}ac^2x^5 - \frac{307c^2\tan^{-1}(ax)}{840a^4} - \frac{5c^2x^2\tan^{-1}(ax)}{168a^2} + \frac{1}{28}c^2x^4\tan^{-1}(ax) -$$
$$= \frac{c^2x}{21a^3} - \frac{c^2x^3}{168a} - \frac{1}{280}ac^2x^5 - \frac{c^2\tan^{-1}(ax)}{21a^4} - \frac{5c^2x^2\tan^{-1}(ax)}{168a^2} + \frac{1}{28}c^2x^4\tan^{-1}(ax) + \frac{1}{56}$$
$$= \frac{c^2x}{21a^3} - \frac{c^2x^3}{168a} - \frac{1}{280}ac^2x^5 - \frac{c^2\tan^{-1}(ax)}{21a^4} - \frac{5c^2x^2\tan^{-1}(ax)}{168a^2} + \frac{1}{28}c^2x^4\tan^{-1}(ax) + \frac{1}{56}$$

Mathematica [A] time = 1.22391, size = 165, normalized size = 0.53

$$\frac{c^2\left(-80i\text{PolyLog}\left(2,-e^{2i\tan^{-1}(ax)}\right) - 3a^5x^5 - 5a^3x^3 + 35\left(a^2x^2+1\right)^3\left(3a^2x^2-1\right)\tan^{-1}(ax)^3 - 5\left(9a^7x^7 + 21a^5x^5 + 7a^3x^3\right.}{840a^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*(c + a^2*c*x^2)^2*ArcTan[a*x]^3,x]
```

```
[Out] (c^2*(40*a*x - 5*a^3*x^3 - 3*a^5*x^5 - 5*(16*I - 21*a*x + 7*a^3*x^3 + 21*a^
5*x^5 + 9*a^7*x^7)*ArcTan[a*x]^2 + 35*(1 + a^2*x^2)^3*(-1 + 3*a^2*x^2)*ArcT
an[a*x]^3 + 5*ArcTan[a*x]*(-8 - 5*a^2*x^2 + 6*a^4*x^4 + 3*a^6*x^6 + 32*Log[
1 + E^((2*I)*ArcTan[a*x])]) - (80*I)*PolyLog[2, -E^((2*I)*ArcTan[a*x])]))/(
840*a^4)
```

Maple [A] time = 0.099, size = 411, normalized size = 1.3

$$\frac{a^4c^2x^8(\arctan(ax))^3}{8}+\frac{a^2c^2x^6(\arctan(ax))^3}{3}+\frac{c^2x^4(\arctan(ax))^3}{4}-\frac{3\,a^3c^2x^7(\arctan(ax))^2}{56}-\frac{ac^2x^5(\arctan(ax))^2}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a^2*c*x^2+c)^2*arctan(a*x)^3,x)
```

```
[Out] 1/8*a^4*c^2*x^8*arctan(a*x)^3+1/3*a^2*c^2*x^6*arctan(a*x)^3+1/4*c^2*x^4*arc
tan(a*x)^3-3/56*a^3*c^2*x^7*arctan(a*x)^2-1/8*a*c^2*x^5*arctan(a*x)^2-1/24*
c^2*x^3*arctan(a*x)^2/a+1/8*c^2*x*arctan(a*x)^2/a^3-1/24*c^2*arctan(a*x)^3/
a^4+1/56*a^2*c^2*x^6*arctan(a*x)+1/28*c^2*x^4*arctan(a*x)-5/168*c^2*x^2*arc
tan(a*x)/a^2-2/21/a^4*c^2*arctan(a*x)*ln(a^2*x^2+1)-1/280*a*c^2*x^5-1/168*c
^2*x^3/a+1/21*c^2*x/a^3-1/21*c^2*arctan(a*x)/a^4-1/21*I/a^4*c^2*ln(a*x-I)*l
n(a^2*x^2+1)-1/21*I/a^4*c^2*ln(a*x+I)*ln(1/2*I*(a*x-I))-1/21*I/a^4*c^2*dilo
g(1/2*I*(a*x-I))+1/21*I/a^4*c^2*dilog(-1/2*I*(a*x+I))-1/42*I/a^4*c^2*ln(a*x
+I)^2+1/42*I/a^4*c^2*ln(a*x-I)^2+1/21*I/a^4*c^2*ln(a*x+I)*ln(a^2*x^2+1)+1/2
1*I/a^4*c^2*ln(a*x-I)*ln(-1/2*I*(a*x+I))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a^2*c*x^2+c)^2*arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^4c^2x^7 + 2\,a^2c^2x^5 + c^2x^3\right)\arctan\left(ax\right)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a^2*c*x^2+c)^2*arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^7 + 2*a^2*c^2*x^5 + c^2*x^3)*arctan(a*x)^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$c^2\left(\int x^3\,\text{atan}^3\left(ax\right)dx + \int 2a^2x^5\,\text{atan}^3\left(ax\right)dx + \int a^4x^7\,\text{atan}^3\left(ax\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a**2*c*x**2+c)**2*atan(a*x)**3,x)
```

```
[Out] c**2*(Integral(x**3*atan(a*x)**3, x) + Integral(2*a**2*x**5*atan(a*x)**3, x
) + Integral(a**4*x**7*atan(a*x)**3, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2 + c\right)^2 x^3 \arctan\left(ax\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a^2*c*x^2+c)^2*arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^2*x^3*arctan(a*x)^3, x)
```

3.372 $\int x^2 \left(c+a^2cx^2\right)^2 \tan^{-1}(ax)^3\,dx$

Optimal. Leaf size=321

$$-\frac{4c^2\text{PolyLog}\left(3,1-\frac{2}{1+iax}\right)}{35a^3}-\frac{8ic^2\tan^{-1}(ax)\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{35a^3}+\frac{c^2\log\left(a^2x^2+1\right)}{30a^3}+\frac{1}{7}a^4c^2x^7\tan^{-1}(ax)^3-\frac{1}{14}a^3c$$

```
[Out] (-11*c^2*x^2)/(420*a) - (a*c^2*x^4)/140 - (c^2*x*ArcTan[a*x])/(70*a^2) + (1
7*c^2*x^3*ArcTan[a*x])/210 + (a^2*c^2*x^5*ArcTan[a*x])/35 + (c^2*ArcTan[a*x
]^2)/(140*a^3) - (4*c^2*x^2*ArcTan[a*x]^2)/(35*a) - (27*a*c^2*x^4*ArcTan[a*
x]^2)/140 - (a^3*c^2*x^6*ArcTan[a*x]^2)/14 - (((8*I)/105)*c^2*ArcTan[a*x]^3
)/a^3 + (c^2*x^3*ArcTan[a*x]^3)/3 + (2*a^2*c^2*x^5*ArcTan[a*x]^3)/5 + (a^4*
c^2*x^7*ArcTan[a*x]^3)/7 - (8*c^2*ArcTan[a*x]^2*Log[2/(1 + I*a*x)])/(35*a^3
) + (c^2*Log[1 + a^2*x^2])/(30*a^3) - (((8*I)/35)*c^2*ArcTan[a*x]*PolyLog[2
, 1 - 2/(1 + I*a*x)])/a^3 - (4*c^2*PolyLog[3, 1 - 2/(1 + I*a*x)])/(35*a^3)
```

Rubi [A] time = 1.79533, antiderivative size = 321, normalized size of antiderivative = 1., number of steps used = 73, number of rules used = 12, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {4948, 4852, 4916, 4846, 260, 4884, 4920, 4854, 4994, 6610, 266, 43}

$$-\frac{4c^2\text{PolyLog}\left(3,1-\frac{2}{1+iax}\right)}{35a^3}-\frac{8ic^2\tan^{-1}(ax)\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{35a^3}+\frac{c^2\log\left(a^2x^2+1\right)}{30a^3}+\frac{1}{7}a^4c^2x^7\tan^{-1}(ax)^3-\frac{1}{14}a^3c$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]^3,x]
```


```
[Out] (-11*c^2*x^2)/(420*a) - (a*c^2*x^4)/140 - (c^2*x*ArcTan[a*x])/(70*a^2) + (1
7*c^2*x^3*ArcTan[a*x])/210 + (a^2*c^2*x^5*ArcTan[a*x])/35 + (c^2*ArcTan[a*x
]^2)/(140*a^3) - (4*c^2*x^2*ArcTan[a*x]^2)/(35*a) - (27*a*c^2*x^4*ArcTan[a*
x]^2)/140 - (a^3*c^2*x^6*ArcTan[a*x]^2)/14 - (((8*I)/105)*c^2*ArcTan[a*x]^3
)/a^3 + (c^2*x^3*ArcTan[a*x]^3)/3 + (2*a^2*c^2*x^5*ArcTan[a*x]^3)/5 + (a^4*
c^2*x^7*ArcTan[a*x]^3)/7 - (8*c^2*ArcTan[a*x]^2*Log[2/(1 + I*a*x)])/(35*a^3
) + (c^2*Log[1 + a^2*x^2])/(30*a^3) - (((8*I)/35)*c^2*ArcTan[a*x]*PolyLog[2
, 1 - 2/(1 + I*a*x)])/a^3 - (4*c^2*PolyLog[3, 1 - 2/(1 + I*a*x)])/(35*a^3)
```


Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
```

```
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x^2 \left(c+a^2cx^2\right)^2 \tan^{-1}(ax)^3 \, dx = \int \left(c^2x^2 \tan^{-1}(ax)^3 + 2a^2c^2x^4 \tan^{-1}(ax)^3 + a^4c^2x^6 \tan^{-1}(ax)^3\right) dx$$

$$= c^2 \int x^2 \tan^{-1}(ax)^3 \, dx + \left(2a^2c^2\right) \int x^4 \tan^{-1}(ax)^3 \, dx + \left(a^4c^2\right) \int x^6 \tan^{-1}(ax)^3 \, dx$$

$$= \frac{1}{3}c^2x^3 \tan^{-1}(ax)^3 + \frac{2}{5}a^2c^2x^5 \tan^{-1}(ax)^3 + \frac{1}{7}a^4c^2x^7 \tan^{-1}(ax)^3 - \left(ac^2\right) \int \frac{x^3 \tan^{-1}(ax}{1+a^2x^2}$$

$$= \frac{1}{3}c^2x^3 \tan^{-1}(ax)^3 + \frac{2}{5}a^2c^2x^5 \tan^{-1}(ax)^3 + \frac{1}{7}a^4c^2x^7 \tan^{-1}(ax)^3 - \frac{c^2 \int x \tan^{-1}(ax)^2 \, dx}{a}$$

$$= -\frac{c^2x^2 \tan^{-1}(ax)^2}{2a} - \frac{3}{10}ac^2x^4 \tan^{-1}(ax)^2 - \frac{1}{14}a^3c^2x^6 \tan^{-1}(ax)^2 - \frac{ic^2 \tan^{-1}(ax)^3}{3a^3} + \frac{1}{3}$$

$$= \frac{c^2x^2 \tan^{-1}(ax)^2}{10a} - \frac{27}{140}ac^2x^4 \tan^{-1}(ax)^2 - \frac{1}{14}a^3c^2x^6 \tan^{-1}(ax)^2 + \frac{ic^2 \tan^{-1}(ax)^3}{15a^3} + \frac{1}{3}$$

$$= \frac{c^2x \tan^{-1}(ax)}{a^2} + \frac{1}{5}c^2x^3 \tan^{-1}(ax) + \frac{1}{35}a^2c^2x^5 \tan^{-1}(ax) - \frac{c^2 \tan^{-1}(ax)^2}{2a^3} - \frac{4c^2x^2 \tan}{35a}$$

$$= -\frac{4c^2x \tan^{-1}(ax)}{5a^2} + \frac{17}{210}c^2x^3 \tan^{-1}(ax) + \frac{1}{35}a^2c^2x^5 \tan^{-1}(ax) + \frac{2c^2 \tan^{-1}(ax)^2}{5a^3} - \frac{4c^2}{}$$

$$= -\frac{c^2x \tan^{-1}(ax)}{70a^2} + \frac{17}{210}c^2x^3 \tan^{-1}(ax) + \frac{1}{35}a^2c^2x^5 \tan^{-1}(ax) + \frac{c^2 \tan^{-1}(ax)^2}{140a^3} - \frac{4c^2x^2}{}$$

$$= -\frac{3c^2x^2}{35a} - \frac{1}{140}ac^2x^4 - \frac{c^2x \tan^{-1}(ax)}{70a^2} + \frac{17}{210}c^2x^3 \tan^{-1}(ax) + \frac{1}{35}a^2c^2x^5 \tan^{-1}(ax) + \frac{c^2}{}$$

$$= -\frac{11c^2x^2}{420a} - \frac{1}{140}ac^2x^4 - \frac{c^2x \tan^{-1}(ax)}{70a^2} + \frac{17}{210}c^2x^3 \tan^{-1}(ax) + \frac{1}{35}a^2c^2x^5 \tan^{-1}(ax) + \frac{c}{}$$

Mathematica [A] time = 1.08935, size = 233, normalized size = 0.73

$$c^2 \left(96i \tan^{-1}(ax)\text{PolyLog}\left(2, -e^{2i\tan^{-1}(ax)}\right) - 48\text{PolyLog}\left(3, -e^{2i\tan^{-1}(ax)}\right) - 3a^4x^4 - 11a^2x^2 + 14\log\left(a^2x^2+1\right) + 60a^7x^7$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]^3,x]
```

```
[Out] (c^2*(-8 - 11*a^2*x^2 - 3*a^4*x^4 - 6*a*x*ArcTan[a*x] + 34*a^3*x^3*ArcTan[a
*x] + 12*a^5*x^5*ArcTan[a*x] + 3*ArcTan[a*x]^2 - 48*a^2*x^2*ArcTan[a*x]^2 -
```

```
 81*a^4*x^4*ArcTan[a*x]^2 - 30*a^6*x^6*ArcTan[a*x]^2 + (32*I)*ArcTan[a*x]^3
 + 140*a^3*x^3*ArcTan[a*x]^3 + 168*a^5*x^5*ArcTan[a*x]^3 + 60*a^7*x^7*ArcTa
n[a*x]^3 - 96*ArcTan[a*x]^2*Log[1 + E^((2*I)*ArcTan[a*x])] + 14*Log[1 + a^2
*x^2] + (96*I)*ArcTan[a*x]*PolyLog[2, -E^((2*I)*ArcTan[a*x])] - 48*PolyLog[
3, -E^((2*I)*ArcTan[a*x])]))/(420*a^3)
```

Maple [C] time = 1.699, size = 1121, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a^2*c*x^2+c)^2*arctan(a*x)^3,x)

[Out] -1/140*a*c^2*x^4+1/35*a^2*c^2*x^5*arctan(a*x)-11/420*c^2*x^2/a+17/210*c^2*x
^3*arctan(a*x)+1/3*c^2*x^3*arctan(a*x)^3+1/140*c^2*arctan(a*x)^2/a^3+2/35*I
/a^3*c^2*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x
^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*arctan
(a*x)^2-1/70*c^2*x*arctan(a*x)/a^2-4/35*c^2*x^2*arctan(a*x)^2/a-27/140*a*c^
2*x^4*arctan(a*x)^2-1/14*a^3*c^2*x^6*arctan(a*x)^2+2/5*a^2*c^2*x^5*arctan(a
*x)^3+1/7*a^4*c^2*x^7*arctan(a*x)^3-4/35/a^3*c^2*polylog(3,-(1+I*a*x)^2/(a^
2*x^2+1))-1/15/a^3*c^2*ln((1+I*a*x)^2/(a^2*x^2+1)+1)+2/35*I/a^3*c^2*Pi*csgn
(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3*arctan(a*x)^2-2
/35*I/a^3*c^2*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3*arctan(a*x)^2+2/35
*I/a^3*c^2*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^3*arctan(a*x)^2+4/35/a^3*c^2*
arctan(a*x)^2*ln(a^2*x^2+1)-8/35/a^3*c^2*arctan(a*x)^2*ln((1+I*a*x)/(a^2*x^
2+1)^(1/2))-8/35/a^3*c^2*arctan(a*x)^2*ln(2)+1/15*I/a^3*c^2*arctan(a*x)+8/1
05*I/a^3*c^2*arctan(a*x)^3-2/105/a^3*c^2-4/35*I/a^3*c^2*Pi*csgn(I*(1+I*a*x)
/(a^2*x^2+1)^(1/2))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^2*arctan(a*x)^2+4/35*I/
a^3*c^2*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2
+1)+1)^2)^2*arctan(a*x)^2+2/35*I/a^3*c^2*Pi*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1
/2))^2*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*arctan(a*x)^2-2/35*I/a^3*c^2*Pi*csgn
(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^
2*x^2+1)+1)^2)^2*arctan(a*x)^2-2/35*I/a^3*c^2*Pi*csgn(I/((1+I*a*x)^2/(a^2*x
^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2
*arctan(a*x)^2-2/35*I/a^3*c^2*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))^2*csgn
(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*arctan(a*x)^2+8/35*I/a^3*c^2*arctan(a*x)*
polylog(2,-(1+I*a*x)^2/(a^2*x^2+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{840}\left(15\,a^4c^2x^7+42\,a^2c^2x^5+35\,c^2x^3\right)\arctan\left(ax\right)^3-\frac{1}{1120}\left(15\,a^4c^2x^7+42\,a^2c^2x^5+35\,c^2x^3\right)\arctan\left(ax\right)\log\left(a^2x^2+1\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^2*arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/840*(15*a^4*c^2*x^7 + 42*a^2*c^2*x^5 + 35*c^2*x^3)*arctan(a*x)^3 - 1/1120
*(15*a^4*c^2*x^7 + 42*a^2*c^2*x^5 + 35*c^2*x^3)*arctan(a*x)*log(a^2*x^2 + 1
)^2 + integrate(1/1120*(980*(a^6*c^2*x^8 + 3*a^4*c^2*x^6 + 3*a^2*c^2*x^4 +
c^2*x^2)*arctan(a*x)^3 - 4*(15*a^5*c^2*x^7 + 42*a^3*c^2*x^5 + 35*a*c^2*x^3)
*arctan(a*x)^2 + 4*(15*a^6*c^2*x^8 + 42*a^4*c^2*x^6 + 35*a^2*c^2*x^4)*arcta
n(a*x)*log(a^2*x^2 + 1) + (15*a^5*c^2*x^7 + 42*a^3*c^2*x^5 + 35*a*c^2*x^3 +
 105*(a^6*c^2*x^8 + 3*a^4*c^2*x^6 + 3*a^2*c^2*x^4 + c^2*x^2)*arctan(a*x))*l
og(a^2*x^2 + 1)^2)/(a^2*x^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^4c^2x^6+2\,a^2c^2x^4+c^2x^2\right)\arctan\left(ax\right)^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^2*arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^6 + 2*a^2*c^2*x^4 + c^2*x^2)*arctan(a*x)^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$c^2\left(\int x^2\,\mathrm{atan}^3\left(ax\right)dx+\int 2a^2x^4\,\mathrm{atan}^3\left(ax\right)dx+\int a^4x^6\,\mathrm{atan}^3\left(ax\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a**2*c*x**2+c)**2*atan(a*x)**3,x)
```

```
[Out] c**2*(Integral(x**2*atan(a*x)**3, x) + Integral(2*a**2*x**4*atan(a*x)**3, x
) + Integral(a**4*x**6*atan(a*x)**3, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2+c\right)^2 x^2 \arctan(ax)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^2*arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^2*x^2*arctan(a*x)^3, x)
```

3.373 $\int x\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^3\,dx$

Optimal. Leaf size=242

$$-\frac{4ic^2\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{15a^2}-\frac{c^2x\left(a^2x^2+1\right)^2\tan^{-1}(ax)^2}{10a}-\frac{2c^2x\left(a^2x^2+1\right)\tan^{-1}(ax)^2}{15a}+\frac{c^2\left(a^2x^2+1\right)^3\tan^{-1}(ax)^3}{6a^2}+\frac{c}{-}$$

```
[Out] (-11*c^2*x)/(60*a) - (a*c^2*x^3)/60 + (2*c^2*(1 + a^2*x^2)*ArcTan[a*x])/(15
*a^2) + (c^2*(1 + a^2*x^2)^2*ArcTan[a*x])/(20*a^2) - (((4*I)/15)*c^2*ArcTan
[a*x]^2)/a^2 - (4*c^2*x*ArcTan[a*x]^2)/(15*a) - (2*c^2*x*(1 + a^2*x^2)*ArcT
an[a*x]^2)/(15*a) - (c^2*x*(1 + a^2*x^2)^2*ArcTan[a*x]^2)/(10*a) + (c^2*(1
+ a^2*x^2)^3*ArcTan[a*x]^3)/(6*a^2) - (8*c^2*ArcTan[a*x]*Log[2/(1 + I*a*x)]
)/(15*a^2) - (((4*I)/15)*c^2*PolyLog[2, 1 - 2/(1 + I*a*x)])/a^2
```

Rubi [A] time = 0.188007, antiderivative size = 242, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {4930, 4880, 4846, 4920, 4854, 2402, 2315, 8}

$$-\frac{4ic^2\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{15a^2}-\frac{c^2x\left(a^2x^2+1\right)^2\tan^{-1}(ax)^2}{10a}-\frac{2c^2x\left(a^2x^2+1\right)\tan^{-1}(ax)^2}{15a}+\frac{c^2\left(a^2x^2+1\right)^3\tan^{-1}(ax)^3}{6a^2}+\frac{c}{-}$$

Antiderivative was successfully verified.

```
[In] Int[x*(c + a^2*c*x^2)^2*ArcTan[a*x]^3,x]
```


```
[Out] (-11*c^2*x)/(60*a) - (a*c^2*x^3)/60 + (2*c^2*(1 + a^2*x^2)*ArcTan[a*x])/(15
*a^2) + (c^2*(1 + a^2*x^2)^2*ArcTan[a*x])/(20*a^2) - (((4*I)/15)*c^2*ArcTan
[a*x]^2)/a^2 - (4*c^2*x*ArcTan[a*x]^2)/(15*a) - (2*c^2*x*(1 + a^2*x^2)*ArcT
an[a*x]^2)/(15*a) - (c^2*x*(1 + a^2*x^2)^2*ArcTan[a*x]^2)/(10*a) + (c^2*(1
+ a^2*x^2)^3*ArcTan[a*x]^3)/(6*a^2) - (8*c^2*ArcTan[a*x]*Log[2/(1 + I*a*x)]
)/(15*a^2) - (((4*I)/15)*c^2*PolyLog[2, 1 - 2/(1 + I*a*x)])/a^2
```


Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 4880

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> -Simp[(b*p*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1))/(2*c*q*(2*
q + 1)), x] + (Dist[(2*d*q)/(2*q + 1), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTa
n[c*x])^p, x], x] + Dist[(b^2*d*p*(p - 1))/(2*q*(2*q + 1)), Int[(d + e*x^2)
^(q - 1)*(a + b*ArcTan[c*x])^(p - 2), x], x] + Simp[(x*(d + e*x^2)^q*(a + b
*ArcTan[c*x])^p)/(2*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[q, 0] && GtQ[p, 1]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int x\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^3\,dx=\frac{c^2\left(1+a^2x^2\right)^3\tan^{-1}(ax)^3}{6a^2}-\frac{\int\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^2\,dx}{2a}$$

$$=\frac{c^2\left(1+a^2x^2\right)^2\tan^{-1}(ax)}{20a^2}-\frac{c^2x\left(1+a^2x^2\right)^2\tan^{-1}(ax)^2}{10a}+\frac{c^2\left(1+a^2x^2\right)^3\tan^{-1}(ax)^3}{6a^2}-$$

$$=-\frac{c^2x}{20a}-\frac{1}{60}ac^2x^3+\frac{2c^2\left(1+a^2x^2\right)\tan^{-1}(ax)}{15a^2}+\frac{c^2\left(1+a^2x^2\right)^2\tan^{-1}(ax)}{20a^2}-\frac{2c^2x\left(1+a^2\right.}{}$$

$$=-\frac{11c^2x}{60a}-\frac{1}{60}ac^2x^3+\frac{2c^2\left(1+a^2x^2\right)\tan^{-1}(ax)}{15a^2}+\frac{c^2\left(1+a^2x^2\right)^2\tan^{-1}(ax)}{20a^2}-\frac{4c^2x\tan^{-}}{15a}$$

$$=-\frac{11c^2x}{60a}-\frac{1}{60}ac^2x^3+\frac{2c^2\left(1+a^2x^2\right)\tan^{-1}(ax)}{15a^2}+\frac{c^2\left(1+a^2x^2\right)^2\tan^{-1}(ax)}{20a^2}-\frac{4ic^2\tan^{-1}}{15a^2}$$

$$=-\frac{11c^2x}{60a}-\frac{1}{60}ac^2x^3+\frac{2c^2\left(1+a^2x^2\right)\tan^{-1}(ax)}{15a^2}+\frac{c^2\left(1+a^2x^2\right)^2\tan^{-1}(ax)}{20a^2}-\frac{4ic^2\tan^{-1}}{15a^2}$$

$$=-\frac{11c^2x}{60a}-\frac{1}{60}ac^2x^3+\frac{2c^2\left(1+a^2x^2\right)\tan^{-1}(ax)}{15a^2}+\frac{c^2\left(1+a^2x^2\right)^2\tan^{-1}(ax)}{20a^2}-\frac{4ic^2\tan^{-1}}{15a^2}$$

$$=-\frac{11c^2x}{60a}-\frac{1}{60}ac^2x^3+\frac{2c^2\left(1+a^2x^2\right)\tan^{-1}(ax)}{15a^2}+\frac{c^2\left(1+a^2x^2\right)^2\tan^{-1}(ax)}{20a^2}-\frac{4ic^2\tan^{-1}}{15a^2}$$

Mathematica [A] time = 0.743275, size = 131, normalized size = 0.54

$$\frac{c^2\left(16i\text{PolyLog}\left(2,-e^{2i\tan^{-1}(ax)}\right)-ax\left(a^2x^2+11\right)+10\left(a^2x^2+1\right)^3\tan^{-1}(ax)^3-2\left(3a^5x^5+10a^3x^3+15ax-8i\right)\tan^{-1}(a\right.}{60a^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*(c + a^2*c*x^2)^2*ArcTan[a*x]^3,x]
```

```
[Out] (c^2*(-(a*x*(11 + a^2*x^2)) - 2*(-8*I + 15*a*x + 10*a^3*x^3 + 3*a^5*x^5)*Ar
cTan[a*x]^2 + 10*(1 + a^2*x^2)^3*ArcTan[a*x]^3 + ArcTan[a*x]*(11 + 14*a^2*x
^2 + 3*a^4*x^4 - 32*Log[1 + E^((2*I)*ArcTan[a*x])]) + (16*I)*PolyLog[2, -E^
((2*I)*ArcTan[a*x])]))/(60*a^2)
```

Maple [A] time = 0.11, size = 368, normalized size = 1.5

$$\frac{a^4c^2(\arctan(ax))^3x^6}{6}+\frac{a^2c^2(\arctan(ax))^3x^4}{2}+\frac{c^2(\arctan(ax))^3x^2}{2}-\frac{a^3c^2(\arctan(ax))^2x^5}{10}-\frac{ac^2(\arctan(ax))^2x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^2*arctan(a*x)^3,x)
```

```
[Out] 1/6*a^4*c^2*arctan(a*x)^3*x^6+1/2*a^2*c^2*arctan(a*x)^3*x^4+1/2*c^2*arctan(
a*x)^3*x^2-1/10*a^3*c^2*arctan(a*x)^2*x^5-1/3*a*c^2*arctan(a*x)^2*x^3-1/2*c
^2*x*arctan(a*x)^2/a+1/6/a^2*c^2*arctan(a*x)^3+1/20*a^2*c^2*arctan(a*x)*x^4
+7/30*c^2*arctan(a*x)*x^2+4/15/a^2*c^2*arctan(a*x)*ln(a^2*x^2+1)-1/60*a*c^2
*x^3-11/60*c^2*x/a+11/60/a^2*c^2*arctan(a*x)-1/15*I/a^2*c^2*ln(a*x-I)^2+1/1
5*I/a^2*c^2*ln(a*x+I)^2+2/15*I/a^2*c^2*ln(a*x+I)*ln(1/2*I*(a*x-I))-2/15*I/a
^2*c^2*dilog(-1/2*I*(a*x+I))+2/15*I/a^2*c^2*dilog(1/2*I*(a*x-I))+2/15*I/a^2
*c^2*ln(a*x-I)*ln(a^2*x^2+1)-2/15*I/a^2*c^2*ln(a*x-I)*ln(-1/2*I*(a*x+I))-2/
15*I/a^2*c^2*ln(a*x+I)*ln(a^2*x^2+1)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^2*arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^4c^2x^5+2\,a^2c^2x^3+c^2x\right)\arctan(ax)^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^2*arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^5 + 2*a^2*c^2*x^3 + c^2*x)*arctan(a*x)^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$c^2\left(\int x\operatorname{atan}^3(ax)\,dx+\int 2a^2x^3\operatorname{atan}^3(ax)\,dx+\int a^4x^5\operatorname{atan}^3(ax)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**2*atan(a*x)**3,x)
```

```
[Out] c**2*(Integral(x*atan(a*x)**3, x) + Integral(2*a**2*x**3*atan(a*x)**3, x) +
 Integral(a**4*x**5*atan(a*x)**3, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2+c\right)^2 x\arctan(ax)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^2*arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^2*x*arctan(a*x)^3, x)
```

3.374 $\int \left(c+a^2cx^2\right)^2 \tan^{-1}(ax)^3\,dx$

Optimal. Leaf size=289

$$\frac{4c^2\text{PolyLog}\left(3,1-\frac{2}{1+iax}\right)}{5a}+\frac{8ic^2\tan^{-1}(ax)\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{5a}-\frac{c^2\left(a^2x^2+1\right)}{20a}-\frac{c^2\log\left(a^2x^2+1\right)}{2a}+\frac{1}{5}c^2x\left(a^2x^2+\right.$$

```
[Out] -(c^2*(1 + a^2*x^2))/(20*a) + c^2*x*ArcTan[a*x] + (c^2*x*(1 + a^2*x^2)*ArcT
an[a*x])/10 - (2*c^2*(1 + a^2*x^2)*ArcTan[a*x]^2)/(5*a) - (3*c^2*(1 + a^2*x
^2)^2*ArcTan[a*x]^2)/(20*a) + (((8*I)/15)*c^2*ArcTan[a*x]^3)/a + (8*c^2*x*A
rcTan[a*x]^3)/15 + (4*c^2*x*(1 + a^2*x^2)*ArcTan[a*x]^3)/15 + (c^2*x*(1 + a
^2*x^2)^2*ArcTan[a*x]^3)/5 + (8*c^2*ArcTan[a*x]^2*Log[2/(1 + I*a*x)])/(5*a)
 - (c^2*Log[1 + a^2*x^2])/(2*a) + (((8*I)/5)*c^2*ArcTan[a*x]*PolyLog[2, 1 -
 2/(1 + I*a*x)])/a + (4*c^2*PolyLog[3, 1 - 2/(1 + I*a*x)])/(5*a)
```

Rubi [A] time = 0.245234, antiderivative size = 289, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {4880, 4846, 4920, 4854, 4884, 4994, 6610, 260, 4878}

$$\frac{4c^2\text{PolyLog}\left(3,1-\frac{2}{1+iax}\right)}{5a}+\frac{8ic^2\tan^{-1}(ax)\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{5a}-\frac{c^2\left(a^2x^2+1\right)}{20a}-\frac{c^2\log\left(a^2x^2+1\right)}{2a}+\frac{1}{5}c^2x\left(a^2x^2+\right.$$

Antiderivative was successfully verified.

```
[In] Int[(c + a^2*c*x^2)^2*ArcTan[a*x]^3,x]
```


```
[Out] -(c^2*(1 + a^2*x^2))/(20*a) + c^2*x*ArcTan[a*x] + (c^2*x*(1 + a^2*x^2)*ArcT
an[a*x])/10 - (2*c^2*(1 + a^2*x^2)*ArcTan[a*x]^2)/(5*a) - (3*c^2*(1 + a^2*x
^2)^2*ArcTan[a*x]^2)/(20*a) + (((8*I)/15)*c^2*ArcTan[a*x]^3)/a + (8*c^2*x*A
rcTan[a*x]^3)/15 + (4*c^2*x*(1 + a^2*x^2)*ArcTan[a*x]^3)/15 + (c^2*x*(1 + a
^2*x^2)^2*ArcTan[a*x]^3)/5 + (8*c^2*ArcTan[a*x]^2*Log[2/(1 + I*a*x)])/(5*a)
 - (c^2*Log[1 + a^2*x^2])/(2*a) + (((8*I)/5)*c^2*ArcTan[a*x]*PolyLog[2, 1 -
 2/(1 + I*a*x)])/a + (4*c^2*PolyLog[3, 1 - 2/(1 + I*a*x)])/(5*a)
```


Rule 4880

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> -Simp[(b*p*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1))/(2*c*q*(2*
q + 1)), x] + (Dist[(2*d*q)/(2*q + 1), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTa
n[c*x])^p, x], x] + Dist[(b^2*d*p*(p - 1))/(2*q*(2*q + 1)), Int[(d + e*x^2)
```

```
^(q - 1)*(a + b*ArcTan[c*x])^(p - 2), x], x] + Simp[(x*(d + e*x^2)^q*(a + b
*ArcTan[c*x])^p)/(2*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[q, 0] && GtQ[p, 1]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4878

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symbo
l] :> -Simp[(b*(d + e*x^2)^q)/(2*c*q*(2*q + 1)), x] + (Dist[(2*d*q)/(2*q +
1), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x]), x], x] + Simp[(x*(d + e*x^
2)^q*(a + b*ArcTan[c*x]))/(2*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] && Eq
Q[e, c^2*d] && GtQ[q, 0]
```

Rubi steps

$$\int \left(c+a^2cx^2\right)^2 \tan^{-1}(ax)^3\,dx = -\frac{3c^2\left(1+a^2x^2\right)^2\tan^{-1}(ax)^2}{20a} + \frac{1}{5}c^2x\left(1+a^2x^2\right)^2\tan^{-1}(ax)^3 + \frac{1}{10}(3c)\int \left(c+a^2cx^2\right) \tan$$

$$= -\frac{c^2\left(1+a^2x^2\right)}{20a} + \frac{1}{10}c^2x\left(1+a^2x^2\right)\tan^{-1}(ax) - \frac{2c^2\left(1+a^2x^2\right)\tan^{-1}(ax)^2}{5a} - \frac{3c^2\left(1+a^2\right.}{}$$

$$= -\frac{c^2\left(1+a^2x^2\right)}{20a} + c^2x\tan^{-1}(ax) + \frac{1}{10}c^2x\left(1+a^2x^2\right)\tan^{-1}(ax) - \frac{2c^2\left(1+a^2x^2\right)\tan^{-1}(a}{5a}$$

$$= -\frac{c^2\left(1+a^2x^2\right)}{20a} + c^2x\tan^{-1}(ax) + \frac{1}{10}c^2x\left(1+a^2x^2\right)\tan^{-1}(ax) - \frac{2c^2\left(1+a^2x^2\right)\tan^{-1}(a}{5a}$$

$$= -\frac{c^2\left(1+a^2x^2\right)}{20a} + c^2x\tan^{-1}(ax) + \frac{1}{10}c^2x\left(1+a^2x^2\right)\tan^{-1}(ax) - \frac{2c^2\left(1+a^2x^2\right)\tan^{-1}(a}{5a}$$

$$= -\frac{c^2\left(1+a^2x^2\right)}{20a} + c^2x\tan^{-1}(ax) + \frac{1}{10}c^2x\left(1+a^2x^2\right)\tan^{-1}(ax) - \frac{2c^2\left(1+a^2x^2\right)\tan^{-1}(a}{5a}$$

$$= -\frac{c^2\left(1+a^2x^2\right)}{20a} + c^2x\tan^{-1}(ax) + \frac{1}{10}c^2x\left(1+a^2x^2\right)\tan^{-1}(ax) - \frac{2c^2\left(1+a^2x^2\right)\tan^{-1}(a}{5a}$$

Mathematica [A] time = 0.572706, size = 195, normalized size = 0.67

$$c^2\left(-96i\tan^{-1}(ax)\text{PolyLog}\left(2,-e^{2i\tan^{-1}(ax)}\right) + 48\text{PolyLog}\left(3,-e^{2i\tan^{-1}(ax)}\right) - 3a^2x^2 - 30\log\left(a^2x^2+1\right) + 12a^5x^5\tan^{-}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + a^2*c*x^2)^2*ArcTan[a*x]^3,x]
```

```
[Out] (c^2*(-3 - 3*a^2*x^2 + 66*a*x*ArcTan[a*x] + 6*a^3*x^3*ArcTan[a*x] - 33*ArcT
an[a*x]^2 - 42*a^2*x^2*ArcTan[a*x]^2 - 9*a^4*x^4*ArcTan[a*x]^2 - (32*I)*Arc
Tan[a*x]^3 + 60*a*x*ArcTan[a*x]^3 + 40*a^3*x^3*ArcTan[a*x]^3 + 12*a^5*x^5*A
rcTan[a*x]^3 + 96*ArcTan[a*x]^2*Log[1 + E^((2*I)*ArcTan[a*x])] - 30*Log[1 +
 a^2*x^2] - (96*I)*ArcTan[a*x]*PolyLog[2, -E^((2*I)*ArcTan[a*x])] + 48*Poly
Log[3, -E^((2*I)*ArcTan[a*x])]))/(60*a)
```

Maple [C] time = 2.737, size = 2691, normalized size = 9.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^2*arctan(a*x)^3,x)
```

```
[Out] 1/a*c^2*ln((1+I*a*x)^2/(a^2*x^2+1)+1)+4/5/a*c^2*polylog(3,-(1+I*a*x)^2/(a^2
*x^2+1))-11/20/a*c^2*arctan(a*x)^2-1/20*c^2*x^2*a+11/10*c^2*x*arctan(a*x)+c
^2*x*arctan(a*x)^3+3/20*I*a*c^2*arctan(a*x)^2*Pi*csgn(I*(1+I*a*x)^4/(a^2*x^
2+1)^2+2*I*(1+I*a*x)^2/(a^2*x^2+1)+I)^3*x^2-3/20*I*a*c^2*arctan(a*x)^2*Pi*c
sgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3*x^2-7/10*I/a*c^2*arctan(a*x)^2*Pi*cs
gn(I*(1+I*a*x)^4/(a^2*x^2+1)^2+2*I*(1+I*a*x)^2/(a^2*x^2+1)+I)^2*csgn(I*(1+I
*a*x)^2/(a^2*x^2+1)+I)+7/20*I/a*c^2*arctan(a*x)^2*Pi*csgn(I*(1+I*a*x)^4/(a^
2*x^2+1)^2+2*I*(1+I*a*x)^2/(a^2*x^2+1)+I)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)+I)
^2-3/10*I*a*c^2*arctan(a*x)^2*Pi*csgn(I*(1+I*a*x)^4/(a^2*x^2+1)^2+2*I*(1+I*
a*x)^2/(a^2*x^2+1)+I)^2*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)+I)*x^2+3/20*I*a*c^2*
arctan(a*x)^2*Pi*csgn(I*(1+I*a*x)^4/(a^2*x^2+1)^2+2*I*(1+I*a*x)^2/(a^2*x^2+
1)+I)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)+I)^2*x^2-3/20*I*a*c^2*arctan(a*x)^2*Pi
*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))^2*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2
)*x^2+3/10*I*a*c^2*arctan(a*x)^2*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))*csg
n(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2*x^2-2/5*I/a*c^2*arctan(a*x)^2*Pi*csgn(
I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2
*x^2+1)+1)^2)*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)+4/5*I/a*c^2*arctan(a*x)
^2*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^2*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))
+2/5*I/a*c^2*arctan(a*x)^2*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a
*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2-2/5*I/a*c^2*arctan(a*x)^
2*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))^2+
2/5*I/a*c^2*arctan(a*x)^2*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a
^2*x^2+1)+1)^2)^2*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)+1/20*I/a*c^2*arctan
(a*x)^2*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))^2*csgn(I*((1+I*a*x)^2/(a^2*x
^2+1)+1)^2)-1/10*I/a*c^2*arctan(a*x)^2*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1
))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2-1/20/a*c^2-7/10*a*c^2*arctan(a*x
)^2*x^2+1/10*a^2*c^2*arctan(a*x)*x^3-3/20*a^3*c^2*arctan(a*x)^2*x^4+1/5*a^4
```

```
*c^2*arctan(a*x)^3*x^5+2/3*a^2*c^2*arctan(a*x)^3*x^3-4/5/a*c^2*arctan(a*x)^
2*ln(a^2*x^2+1)+8/5/a*c^2*arctan(a*x)^2*ln((1+I*a*x)/(a^2*x^2+1)^(1/2))+8/5
/a*c^2*arctan(a*x)^2*ln(2)-8/15*I/a*c^2*arctan(a*x)^3-I/a*c^2*arctan(a*x)+1
/10*a^2*c^2*arctan(a*x)^2*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1
+I*a*x)^2/(a^2*x^2+1)+1)^2)^2*x^3+1/20*a^2*c^2*arctan(a*x)^2*Pi*csgn(I*(1+I
*a*x)^2/(a^2*x^2+1)+I)^2*csgn(I*(1+I*a*x)^4/(a^2*x^2+1)^2+2*I*(1+I*a*x)^2/(
a^2*x^2+1)+I)*x^3-1/10*a^2*c^2*arctan(a*x)^2*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2
+1)+I)*csgn(I*(1+I*a*x)^4/(a^2*x^2+1)^2+2*I*(1+I*a*x)^2/(a^2*x^2+1)+I)^2*x^
3-1/20*a^2*c^2*arctan(a*x)^2*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))^2*csgn(
I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*x^3+3/20*c^2*arctan(a*x)^2*Pi*csgn(I*((1+I
*a*x)^2/(a^2*x^2+1)+1))^2*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*x-3/20*c^2*
arctan(a*x)^2*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)+I)^2*csgn(I*(1+I*a*x)^4/(a^
2*x^2+1)^2+2*I*(1+I*a*x)^2/(a^2*x^2+1)+I)*x+3/10*c^2*arctan(a*x)^2*Pi*csgn(
I*(1+I*a*x)^2/(a^2*x^2+1)+I)*csgn(I*(1+I*a*x)^4/(a^2*x^2+1)^2+2*I*(1+I*a*x)
^2/(a^2*x^2+1)+I)^2*x-3/10*c^2*arctan(a*x)^2*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^
2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2*x-1/20*a^2*c^2*arctan(a*x)
^2*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3*x^3+1/20*a^2*c^2*arctan(a*x)^
2*Pi*csgn(I*(1+I*a*x)^4/(a^2*x^2+1)^2+2*I*(1+I*a*x)^2/(a^2*x^2+1)+I)^3*x^3+
7/20*I/a*c^2*arctan(a*x)^2*Pi*csgn(I*(1+I*a*x)^4/(a^2*x^2+1)^2+2*I*(1+I*a*x
)^2/(a^2*x^2+1)+I)^3-2/5*I/a*c^2*arctan(a*x)^2*Pi*csgn(I*(1+I*a*x)^2/(a^2*x
^2+1))^3-2/5*I/a*c^2*arctan(a*x)^2*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*
a*x)^2/(a^2*x^2+1)+1)^2)^3+1/20*I/a*c^2*arctan(a*x)^2*Pi*csgn(I*((1+I*a*x)^
2/(a^2*x^2+1)+1)^2)^3-3/20*c^2*arctan(a*x)^2*Pi*csgn(I*(1+I*a*x)^4/(a^2*x^2
+1)^2+2*I*(1+I*a*x)^2/(a^2*x^2+1)+I)^3*x+3/20*c^2*arctan(a*x)^2*Pi*csgn(I*(
(1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3*x-8/5*I/a*c^2*arctan(a*x)*polylog(2,-(1+I*a
*x)^2/(a^2*x^2+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] 140*a^6*c^2*integrate(1/160*x^6*arctan(a*x)^3/(a^2*x^2 + 1), x) + 15*a^6*c^
2*integrate(1/160*x^6*arctan(a*x)*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x) + 12
*a^6*c^2*integrate(1/160*x^6*arctan(a*x)*log(a^2*x^2 + 1)/(a^2*x^2 + 1), x)
 - 12*a^5*c^2*integrate(1/160*x^5*arctan(a*x)^2/(a^2*x^2 + 1), x) + 3*a^5*c
^2*integrate(1/160*x^5*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x) + 420*a^4*c^2*i
ntegrate(1/160*x^4*arctan(a*x)^3/(a^2*x^2 + 1), x) + 45*a^4*c^2*integrate(1
/160*x^4*arctan(a*x)*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x) + 40*a^4*c^2*inte
```

```
grate(1/160*x^4*arctan(a*x)*log(a^2*x^2 + 1)/(a^2*x^2 + 1), x) - 40*a^3*c^2
*integrate(1/160*x^3*arctan(a*x)^2/(a^2*x^2 + 1), x) + 10*a^3*c^2*integrate
(1/160*x^3*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x) + 7/32*c^2*arctan(a*x)^4/a
+ 420*a^2*c^2*integrate(1/160*x^2*arctan(a*x)^3/(a^2*x^2 + 1), x) + 45*a^2*
c^2*integrate(1/160*x^2*arctan(a*x)*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x) +
60*a^2*c^2*integrate(1/160*x^2*arctan(a*x)*log(a^2*x^2 + 1)/(a^2*x^2 + 1),
x) + 1/120*(3*a^4*c^2*x^5 + 10*a^2*c^2*x^3 + 15*c^2*x)*arctan(a*x)^3 - 60*a
*c^2*integrate(1/160*x*arctan(a*x)^2/(a^2*x^2 + 1), x) + 15*a*c^2*integrate
(1/160*x*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x) - 1/160*(3*a^4*c^2*x^5 + 10*a
^2*c^2*x^3 + 15*c^2*x)*arctan(a*x)*log(a^2*x^2 + 1)^2 + 15*c^2*integrate(1/
160*arctan(a*x)*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^4c^2x^4 + 2\,a^2c^2x^2 + c^2\right)\arctan\left(ax\right)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(a*x)^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$c^2\left(\int 2a^2x^2\,\text{atan}^3\left(ax\right)dx + \int a^4x^4\,\text{atan}^3\left(ax\right)dx + \int \text{atan}^3\left(ax\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**2*atan(a*x)**3,x)
```

```
[Out] c**2*(Integral(2*a**2*x**2*atan(a*x)**3, x) + Integral(a**4*x**4*atan(a*x)*
*3, x) + Integral(atan(a*x)**3, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2 + c\right)^2\arctan\left(ax\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^2*arctan(a*x)^3, x)
```

3.375 $\int \frac{(c+a^2cx^2)^2 \tan^{-1}(ax)^3}{x} \, dx$

Optimal. Leaf size=370

$$-2ic^2\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)+\frac{3}{4}ic^2\text{PolyLog}\left(4,1-\frac{2}{1+iax}\right)-\frac{3}{4}ic^2\text{PolyLog}\left(4,-1+\frac{2}{1+iax}\right)-\frac{3}{2}ic^2\tan^{-1}(ax)^2\text{Poly}$$

```
[Out] -(a*c^2*x)/4 + (c^2*ArcTan[a*x])/4 + (a^2*c^2*x^2*ArcTan[a*x])/4 - (2*I)*c^
2*ArcTan[a*x]^2 - (9*a*c^2*x*ArcTan[a*x]^2)/4 - (a^3*c^2*x^3*ArcTan[a*x]^2)
/4 + (3*c^2*ArcTan[a*x]^3)/4 + a^2*c^2*x^2*ArcTan[a*x]^3 + (a^4*c^2*x^4*Arc
Tan[a*x]^3)/4 + 2*c^2*ArcTan[a*x]^3*ArcTanh[1 - 2/(1 + I*a*x)] - 4*c^2*ArcT
an[a*x]*Log[2/(1 + I*a*x)] - (2*I)*c^2*PolyLog[2, 1 - 2/(1 + I*a*x)] - ((3*
I)/2)*c^2*ArcTan[a*x]^2*PolyLog[2, 1 - 2/(1 + I*a*x)] + ((3*I)/2)*c^2*ArcTa
n[a*x]^2*PolyLog[2, -1 + 2/(1 + I*a*x)] - (3*c^2*ArcTan[a*x]*PolyLog[3, 1 -
 2/(1 + I*a*x)])/2 + (3*c^2*ArcTan[a*x]*PolyLog[3, -1 + 2/(1 + I*a*x)])/2 +
 ((3*I)/4)*c^2*PolyLog[4, 1 - 2/(1 + I*a*x)] - ((3*I)/4)*c^2*PolyLog[4, -1
+ 2/(1 + I*a*x)]
```

Rubi [A] time = 0.970399, antiderivative size = 370, normalized size of antiderivative = 1., number of steps used = 36, number of rules used = 16, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.727, Rules used = {4948, 4850, 4988, 4884, 4994, 4998, 6610, 4852, 4916, 4846, 4920, 4854, 2402, 2315, 321, 203}

$$-2ic^2\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)+\frac{3}{4}ic^2\text{PolyLog}\left(4,1-\frac{2}{1+iax}\right)-\frac{3}{4}ic^2\text{PolyLog}\left(4,-1+\frac{2}{1+iax}\right)-\frac{3}{2}ic^2\tan^{-1}(ax)^2\text{Poly}$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)^2*ArcTan[a*x]^3)/x,x]
```


```
[Out] -(a*c^2*x)/4 + (c^2*ArcTan[a*x])/4 + (a^2*c^2*x^2*ArcTan[a*x])/4 - (2*I)*c^
2*ArcTan[a*x]^2 - (9*a*c^2*x*ArcTan[a*x]^2)/4 - (a^3*c^2*x^3*ArcTan[a*x]^2)
/4 + (3*c^2*ArcTan[a*x]^3)/4 + a^2*c^2*x^2*ArcTan[a*x]^3 + (a^4*c^2*x^4*Arc
Tan[a*x]^3)/4 + 2*c^2*ArcTan[a*x]^3*ArcTanh[1 - 2/(1 + I*a*x)] - 4*c^2*ArcT
an[a*x]*Log[2/(1 + I*a*x)] - (2*I)*c^2*PolyLog[2, 1 - 2/(1 + I*a*x)] - ((3*
I)/2)*c^2*ArcTan[a*x]^2*PolyLog[2, 1 - 2/(1 + I*a*x)] + ((3*I)/2)*c^2*ArcTa
n[a*x]^2*PolyLog[2, -1 + 2/(1 + I*a*x)] - (3*c^2*ArcTan[a*x]*PolyLog[3, 1 -
 2/(1 + I*a*x)])/2 + (3*c^2*ArcTan[a*x]*PolyLog[3, -1 + 2/(1 + I*a*x)])/2 +
 ((3*I)/4)*c^2*PolyLog[4, 1 - 2/(1 + I*a*x)] - ((3*I)/4)*c^2*PolyLog[4, -1
+ 2/(1 + I*a*x)]
```

Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

Rule 4850

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTan[c*x])^(p - 1)*ArcTanh[1 - 2/(1 + I*c*x)])/(1 + c^2*x^2), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4988

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTan[c*x])^p)/(d + e
*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTan[c*x])^p)/(d + e*x^2
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 4998

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[k + 1, u])/(2
*c*d), x] - Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[k + 1,
 u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
qQ[e, c^2*d] && EqQ[u^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^2\tan^{-1}(ax)^3}{x}\,dx = \int\left(\frac{c^2\tan^{-1}(ax)^3}{x} + 2a^2c^2x\tan^{-1}(ax)^3 + a^4c^2x^3\tan^{-1}(ax)^3\right)dx$$

$$= c^2\int\frac{\tan^{-1}(ax)^3}{x}\,dx + (2a^2c^2)\int x\tan^{-1}(ax)^3\,dx + (a^4c^2)\int x^3\tan^{-1}(ax)^3\,dx$$

$$= a^2c^2x^2\tan^{-1}(ax)^3 + \frac{1}{4}a^4c^2x^4\tan^{-1}(ax)^3 + 2c^2\tan^{-1}(ax)^3\tanh^{-1}\left(1-\frac{2}{1+iax}\right) - (6ac^2$$

$$= a^2c^2x^2\tan^{-1}(ax)^3 + \frac{1}{4}a^4c^2x^4\tan^{-1}(ax)^3 + 2c^2\tan^{-1}(ax)^3\tanh^{-1}\left(1-\frac{2}{1+iax}\right) - (3ac^2$$

$$= -3ac^2x\tan^{-1}(ax)^2 - \frac{1}{4}a^3c^2x^3\tan^{-1}(ax)^2 + c^2\tan^{-1}(ax)^3 + a^2c^2x^2\tan^{-1}(ax)^3 + \frac{1}{4}a^4c^2x$$

$$= -3ic^2\tan^{-1}(ax)^2 - \frac{9}{4}ac^2x\tan^{-1}(ax)^2 - \frac{1}{4}a^3c^2x^3\tan^{-1}(ax)^2 + \frac{3}{4}c^2\tan^{-1}(ax)^3 + a^2c^2x^2$$

$$= \frac{1}{4}a^2c^2x^2\tan^{-1}(ax) - 2ic^2\tan^{-1}(ax)^2 - \frac{9}{4}ac^2x\tan^{-1}(ax)^2 - \frac{1}{4}a^3c^2x^3\tan^{-1}(ax)^2 + \frac{3}{4}c^2$$

$$= -\frac{1}{4}ac^2x + \frac{1}{4}a^2c^2x^2\tan^{-1}(ax) - 2ic^2\tan^{-1}(ax)^2 - \frac{9}{4}ac^2x\tan^{-1}(ax)^2 - \frac{1}{4}a^3c^2x^3\tan^{-1}(ax$$

$$= -\frac{1}{4}ac^2x + \frac{1}{4}c^2\tan^{-1}(ax) + \frac{1}{4}a^2c^2x^2\tan^{-1}(ax) - 2ic^2\tan^{-1}(ax)^2 - \frac{9}{4}ac^2x\tan^{-1}(ax)^2 -$$

$$= -\frac{1}{4}ac^2x + \frac{1}{4}c^2\tan^{-1}(ax) + \frac{1}{4}a^2c^2x^2\tan^{-1}(ax) - 2ic^2\tan^{-1}(ax)^2 - \frac{9}{4}ac^2x\tan^{-1}(ax)^2 -$$

Mathematica [A] time = 0.558051, size = 302, normalized size = 0.82

$$\frac{1}{64}c^2\left(96i\tan^{-1}(ax)^2\text{PolyLog}\left(2,e^{-2i\tan^{-1}(ax)}\right) + 32i\left(3\tan^{-1}(ax)^2+4\right)\text{PolyLog}\left(2,-e^{2i\tan^{-1}(ax)}\right) + 96\tan^{-1}(ax)\text{PolyLo}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((c + a^2*c*x^2)^2*ArcTan[a*x]^3)/x,x]
```

```
[Out] (c^2*((-I)*Pi^4 - 16*a*x + 16*ArcTan[a*x] + 16*a^2*x^2*ArcTan[a*x] + (128*I
)*ArcTan[a*x]^2 - 144*a*x*ArcTan[a*x]^2 - 16*a^3*x^3*ArcTan[a*x]^2 + 48*Arc
Tan[a*x]^3 + 64*a^2*x^2*ArcTan[a*x]^3 + 16*a^4*x^4*ArcTan[a*x]^3 + (32*I)*A
rcTan[a*x]^4 + 64*ArcTan[a*x]^3*Log[1 - E^((-2*I)*ArcTan[a*x])] - 256*ArcTa
n[a*x]*Log[1 + E^((2*I)*ArcTan[a*x])] - 64*ArcTan[a*x]^3*Log[1 + E^((2*I)*A
```

```
rcTan[a*x])] + (96*I)*ArcTan[a*x]^2*PolyLog[2, E^((-2*I)*ArcTan[a*x])] + (3
2*I)*(4 + 3*ArcTan[a*x]^2)*PolyLog[2, -E^((2*I)*ArcTan[a*x])] + 96*ArcTan[a
*x]*PolyLog[3, E^((-2*I)*ArcTan[a*x])] - 96*ArcTan[a*x]*PolyLog[3, -E^((2*I
)*ArcTan[a*x])] - (48*I)*PolyLog[4, E^((-2*I)*ArcTan[a*x])] - (48*I)*PolyLo
g[4, -E^((2*I)*ArcTan[a*x])]))/64
```

Maple [A] time = 1.497, size = 566, normalized size = 1.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^2*arctan(a*x)^3/x,x)
```

```
[Out] 1/4*c^2*(-3*I*arctan(a*x)^3+3*arctan(a*x)^3*a*x-I*arctan(a*x)^3*a^2*x^2+arc
tan(a*x)^3*a^3*x^3-8*arctan(a*x)^2+I*arctan(a*x)^2*a*x-arctan(a*x)^2*x^2*a^
2-I*arctan(a*x)+arctan(a*x)*x*a-1)*(a*x+I)-3*I*c^2*arctan(a*x)^2*polylog(2,
(1+I*a*x)/(a^2*x^2+1)^(1/2))-3/2*c^2*arctan(a*x)*polylog(3,-(1+I*a*x)^2/(a^
2*x^2+1))+6*I*c^2*polylog(4,(1+I*a*x)/(a^2*x^2+1)^(1/2))+c^2*arctan(a*x)^3*
ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-3*I*c^2*arctan(a*x)^2*polylog(2,-(1+I*a*x
)/(a^2*x^2+1)^(1/2))+6*c^2*arctan(a*x)*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2
))+c^2*arctan(a*x)^3*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I*c^2*polylog(4,-(
1+I*a*x)/(a^2*x^2+1)^(1/2))+6*c^2*arctan(a*x)*polylog(3,-(1+I*a*x)/(a^2*x^2
+1)^(1/2))-c^2*arctan(a*x)^3*ln((1+I*a*x)^2/(a^2*x^2+1)+1)+2*I*c^2*polylog(
2,-(1+I*a*x)^2/(a^2*x^2+1))-4*c^2*arctan(a*x)*ln((1+I*a*x)^2/(a^2*x^2+1)+1)
+3/2*I*c^2*arctan(a*x)^2*polylog(2,-(1+I*a*x)^2/(a^2*x^2+1))+4*I*c^2*arctan
(a*x)^2-3/4*I*c^2*polylog(4,-(1+I*a*x)^2/(a^2*x^2+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{32}\left(a^4c^2x^4+4\,a^2c^2x^2\right)\arctan\left(ax\right)^3-\frac{3}{128}\left(a^4c^2x^4+4\,a^2c^2x^2\right)\arctan\left(ax\right)\log\left(a^2x^2+1\right)^2+\int\frac{112\left(a^6c^2x^6+3\,a^4c^2x^4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^3/x,x, algorithm="maxima")
```

```
[Out] 1/32*(a^4*c^2*x^4 + 4*a^2*c^2*x^2)*arctan(a*x)^3 - 3/128*(a^4*c^2*x^4 + 4*a
^2*c^2*x^2)*arctan(a*x)*log(a^2*x^2 + 1)^2 + integrate(1/128*(112*(a^6*c^2*
```

```
x^6 + 3*a^4*c^2*x^4 + 3*a^2*c^2*x^2 + c^2)*arctan(a*x)^3 - 12*(a^5*c^2*x^5
+ 4*a^3*c^2*x^3)*arctan(a*x)^2 + 12*(a^6*c^2*x^6 + 4*a^4*c^2*x^4)*arctan(a*
x)*log(a^2*x^2 + 1) + 3*(a^5*c^2*x^5 + 4*a^3*c^2*x^3 + 4*(a^6*c^2*x^6 + 3*a
^4*c^2*x^4 + 3*a^2*c^2*x^2 + c^2)*arctan(a*x))*log(a^2*x^2 + 1)^2)/(a^2*x^3
 + x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^4c^2x^4 + 2\,a^2c^2x^2 + c^2\right)\arctan\left(ax\right)^3}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^3/x,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(a*x)^3/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$c^2\left(\int \frac{\operatorname{atan}^3\left(ax\right)}{x}\,dx + \int 2a^2x\operatorname{atan}^3\left(ax\right)dx + \int a^4x^3\operatorname{atan}^3\left(ax\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**2*atan(a*x)**3/x,x)
```

```
[Out] c**2*(Integral(atan(a*x)**3/x, x) + Integral(2*a**2*x*atan(a*x)**3, x) + In
tegral(a**4*x**3*atan(a*x)**3, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2 + c\right)^2\arctan\left(ax\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^3/x,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^2*arctan(a*x)^3/x, x)
```

3.376 $\int \frac{\left(c+a^2cx^2\right)^2 \tan^{-1}(ax)^3}{x^2} dx$

Optimal. Leaf size=284

$$\frac{3}{2}ac^2\text{PolyLog}\left(3,-1+\frac{2}{1-iax}\right)+\frac{5}{2}ac^2\text{PolyLog}\left(3,1-\frac{2}{1+iax}\right)-3iac^2\tan^{-1}(ax)\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)+5iac^2\text{ta}$$

```
[Out] a^2*c^2*x*ArcTan[a*x] - (a*c^2*ArcTan[a*x]^2)/2 - (a^3*c^2*x^2*ArcTan[a*x]^
2)/2 + ((2*I)/3)*a*c^2*ArcTan[a*x]^3 - (c^2*ArcTan[a*x]^3)/x + 2*a^2*c^2*x*
ArcTan[a*x]^3 + (a^4*c^2*x^3*ArcTan[a*x]^3)/3 + 5*a*c^2*ArcTan[a*x]^2*Log[2
/(1 + I*a*x)] - (a*c^2*Log[1 + a^2*x^2])/2 + 3*a*c^2*ArcTan[a*x]^2*Log[2 -
2/(1 - I*a*x)] - (3*I)*a*c^2*ArcTan[a*x]*PolyLog[2, -1 + 2/(1 - I*a*x)] + (
5*I)*a*c^2*ArcTan[a*x]*PolyLog[2, 1 - 2/(1 + I*a*x)] + (3*a*c^2*PolyLog[3,
-1 + 2/(1 - I*a*x)])/2 + (5*a*c^2*PolyLog[3, 1 - 2/(1 + I*a*x)])/2
```

Rubi [A] time = 0.751286, antiderivative size = 284, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 13, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.591, Rules used = {4948, 4846, 4920, 4854, 4884, 4994, 6610, 4852, 4924, 4868, 4992, 4916, 260}

$$\frac{3}{2}ac^2\text{PolyLog}\left(3,-1+\frac{2}{1-iax}\right)+\frac{5}{2}ac^2\text{PolyLog}\left(3,1-\frac{2}{1+iax}\right)-3iac^2\tan^{-1}(ax)\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)+5iac^2\text{ta}$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)^2*ArcTan[a*x]^3)/x^2,x]
```


```
[Out] a^2*c^2*x*ArcTan[a*x] - (a*c^2*ArcTan[a*x]^2)/2 - (a^3*c^2*x^2*ArcTan[a*x]^
2)/2 + ((2*I)/3)*a*c^2*ArcTan[a*x]^3 - (c^2*ArcTan[a*x]^3)/x + 2*a^2*c^2*x*
ArcTan[a*x]^3 + (a^4*c^2*x^3*ArcTan[a*x]^3)/3 + 5*a*c^2*ArcTan[a*x]^2*Log[2
/(1 + I*a*x)] - (a*c^2*Log[1 + a^2*x^2])/2 + 3*a*c^2*ArcTan[a*x]^2*Log[2 -
2/(1 - I*a*x)] - (3*I)*a*c^2*ArcTan[a*x]*PolyLog[2, -1 + 2/(1 - I*a*x)] + (
5*I)*a*c^2*ArcTan[a*x]*PolyLog[2, 1 - 2/(1 + I*a*x)] + (3*a*c^2*PolyLog[3,
-1 + 2/(1 - I*a*x)])/2 + (5*a*c^2*PolyLog[3, 1 - 2/(1 + I*a*x)])/2
```


Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
```

```
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
```

```
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 4992

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d
] && EqQ[(1 - u)^2 - (1 - (2*I)/(I + c*x))^2, 0]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^2 \tan^{-1}(ax)^3}{x^2}\,dx = \int \left(2a^2c^2\tan^{-1}(ax)^3 + \frac{c^2\tan^{-1}(ax)^3}{x^2} + a^4c^2x^2\tan^{-1}(ax)^3\right)dx$$

$$= c^2\int \frac{\tan^{-1}(ax)^3}{x^2}\,dx + (2a^2c^2)\int \tan^{-1}(ax)^3\,dx + (a^4c^2)\int x^2\tan^{-1}(ax)^3\,dx$$

$$= -\frac{c^2\tan^{-1}(ax)^3}{x} + 2a^2c^2x\tan^{-1}(ax)^3 + \frac{1}{3}a^4c^2x^3\tan^{-1}(ax)^3 + (3ac^2)\int \frac{\tan^{-1}(ax)^2}{x(1+a^2x^2)}\,dx$$

$$= iac^2\tan^{-1}(ax)^3 - \frac{c^2\tan^{-1}(ax)^3}{x} + 2a^2c^2x\tan^{-1}(ax)^3 + \frac{1}{3}a^4c^2x^3\tan^{-1}(ax)^3 + (3iac^2)$$

$$= -\frac{1}{2}a^3c^2x^2\tan^{-1}(ax)^2 + \frac{2}{3}iac^2\tan^{-1}(ax)^3 - \frac{c^2\tan^{-1}(ax)^3}{x} + 2a^2c^2x\tan^{-1}(ax)^3 + \frac{1}{3}a^4c$$

$$= -\frac{1}{2}a^3c^2x^2\tan^{-1}(ax)^2 + \frac{2}{3}iac^2\tan^{-1}(ax)^3 - \frac{c^2\tan^{-1}(ax)^3}{x} + 2a^2c^2x\tan^{-1}(ax)^3 + \frac{1}{3}a^4c$$

$$= a^2c^2x\tan^{-1}(ax) - \frac{1}{2}ac^2\tan^{-1}(ax)^2 - \frac{1}{2}a^3c^2x^2\tan^{-1}(ax)^2 + \frac{2}{3}iac^2\tan^{-1}(ax)^3 - \frac{c^2\tan}{}$$

$$= a^2c^2x\tan^{-1}(ax) - \frac{1}{2}ac^2\tan^{-1}(ax)^2 - \frac{1}{2}a^3c^2x^2\tan^{-1}(ax)^2 + \frac{2}{3}iac^2\tan^{-1}(ax)^3 - \frac{c^2\tan}{}$$

Mathematica [A] time = 0.388447, size = 246, normalized size = 0.87

$$\frac{c^2\left(72iax\tan^{-1}(ax)\text{PolyLog}\left(2,e^{-2i\tan^{-1}(ax)}\right) - 120iax\tan^{-1}(ax)\text{PolyLog}\left(2,-e^{2i\tan^{-1}(ax)}\right) + 36ax\text{PolyLog}\left(3,e^{-2i\tan}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((c + a^2*c*x^2)^2*ArcTan[a*x]^3)/x^2,x]
```

```
[Out] (c^2*((-3*I)*a*Pi^3*x + 24*a^2*x^2*ArcTan[a*x] - 12*a*x*ArcTan[a*x]^2 - 12*
a^3*x^3*ArcTan[a*x]^2 - 24*ArcTan[a*x]^3 - (16*I)*a*x*ArcTan[a*x]^3 + 48*a^
2*x^2*ArcTan[a*x]^3 + 8*a^4*x^4*ArcTan[a*x]^3 + 72*a*x*ArcTan[a*x]^2*Log[1
- E^((-2*I)*ArcTan[a*x])] + 120*a*x*ArcTan[a*x]^2*Log[1 + E^((2*I)*ArcTan[a
*x])] - 12*a*x*Log[1 + a^2*x^2] + (72*I)*a*x*ArcTan[a*x]*PolyLog[2, E^((-2*
I)*ArcTan[a*x])] - (120*I)*a*x*ArcTan[a*x]*PolyLog[2, -E^((2*I)*ArcTan[a*x]
)] + 36*a*x*PolyLog[3, E^((-2*I)*ArcTan[a*x])] + 60*a*x*PolyLog[3, -E^((2*I
)*ArcTan[a*x])]))/(24*x)
```

Maple [C] time = 2.39, size = 5486, normalized size = 19.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^2*arctan(a*x)^3/x^2,x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^3/x^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^{4} c^{2} x^{4} + 2 \, a^{2} c^{2} x^{2} + c^{2}\right) \arctan\left(a x\right)^{3}}{x^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^3/x^2,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(a*x)^3/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$c^{2} \left(\int 2 a^{2} \operatorname{atan}^{3}{\left(a x \right)}\, dx + \int \frac{\operatorname{atan}^{3}{\left(a x \right)}}{x^{2}}\, dx + \int a^{4} x^{2} \operatorname{atan}^{3}{\left(a x \right)}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**2*atan(a*x)**3/x**2,x)
```

```
[Out] c**2*(Integral(2*a**2*atan(a*x)**3, x) + Integral(atan(a*x)**3/x**2, x) + I
ntegral(a**4*x**2*atan(a*x)**3, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^2 \arctan\left(ax\right)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^3/x^2,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^2*arctan(a*x)^3/x^2, x)
```

3.377 $\int \frac{(c+a^2cx^2)^2 \tan^{-1}(ax)^3}{x^3} dx$

Optimal. Leaf size=399

$$-\frac{3}{2}ia^2c^2\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)-\frac{3}{2}ia^2c^2\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)+\frac{3}{2}ia^2c^2\text{PolyLog}\left(4,1-\frac{2}{1+iax}\right)-\frac{3}{2}ia^2c^2\text{PolyL}$$

```
[Out] (-3*I)*a^2*c^2*ArcTan[a*x]^2 - (3*a*c^2*ArcTan[a*x]^2)/(2*x) - (3*a^3*c^2*x
*ArcTan[a*x]^2)/2 - (c^2*ArcTan[a*x]^3)/(2*x^2) + (a^4*c^2*x^2*ArcTan[a*x]^
3)/2 + 4*a^2*c^2*ArcTan[a*x]^3*ArcTanh[1 - 2/(1 + I*a*x)] - 3*a^2*c^2*ArcTa
n[a*x]*Log[2/(1 + I*a*x)] + 3*a^2*c^2*ArcTan[a*x]*Log[2 - 2/(1 - I*a*x)] -
((3*I)/2)*a^2*c^2*PolyLog[2, -1 + 2/(1 - I*a*x)] - ((3*I)/2)*a^2*c^2*PolyLo
g[2, 1 - 2/(1 + I*a*x)] - (3*I)*a^2*c^2*ArcTan[a*x]^2*PolyLog[2, 1 - 2/(1 +
 I*a*x)] + (3*I)*a^2*c^2*ArcTan[a*x]^2*PolyLog[2, -1 + 2/(1 + I*a*x)] - 3*a
^2*c^2*ArcTan[a*x]*PolyLog[3, 1 - 2/(1 + I*a*x)] + 3*a^2*c^2*ArcTan[a*x]*Po
lyLog[3, -1 + 2/(1 + I*a*x)] + ((3*I)/2)*a^2*c^2*PolyLog[4, 1 - 2/(1 + I*a*
x)] - ((3*I)/2)*a^2*c^2*PolyLog[4, -1 + 2/(1 + I*a*x)]
```

Rubi [A] time = 0.796296, antiderivative size = 399, normalized size of antiderivative = 1., number of steps used = 25, number of rules used = 18, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.818, Rules used = {4948, 4852, 4918, 4924, 4868, 2447, 4884, 4850, 4988, 4994, 4998, 6610, 4916, 4846, 4920, 4854, 2402, 2315}

$$-\frac{3}{2}ia^2c^2\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)-\frac{3}{2}ia^2c^2\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)+\frac{3}{2}ia^2c^2\text{PolyLog}\left(4,1-\frac{2}{1+iax}\right)-\frac{3}{2}ia^2c^2\text{PolyL}$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)^2*ArcTan[a*x]^3)/x^3,x]
```


```
[Out] (-3*I)*a^2*c^2*ArcTan[a*x]^2 - (3*a*c^2*ArcTan[a*x]^2)/(2*x) - (3*a^3*c^2*x
*ArcTan[a*x]^2)/2 - (c^2*ArcTan[a*x]^3)/(2*x^2) + (a^4*c^2*x^2*ArcTan[a*x]^
3)/2 + 4*a^2*c^2*ArcTan[a*x]^3*ArcTanh[1 - 2/(1 + I*a*x)] - 3*a^2*c^2*ArcTa
n[a*x]*Log[2/(1 + I*a*x)] + 3*a^2*c^2*ArcTan[a*x]*Log[2 - 2/(1 - I*a*x)] -
((3*I)/2)*a^2*c^2*PolyLog[2, -1 + 2/(1 - I*a*x)] - ((3*I)/2)*a^2*c^2*PolyLo
g[2, 1 - 2/(1 + I*a*x)] - (3*I)*a^2*c^2*ArcTan[a*x]^2*PolyLog[2, 1 - 2/(1 +
 I*a*x)] + (3*I)*a^2*c^2*ArcTan[a*x]^2*PolyLog[2, -1 + 2/(1 + I*a*x)] - 3*a
^2*c^2*ArcTan[a*x]*PolyLog[3, 1 - 2/(1 + I*a*x)] + 3*a^2*c^2*ArcTan[a*x]*Po
lyLog[3, -1 + 2/(1 + I*a*x)] + ((3*I)/2)*a^2*c^2*PolyLog[4, 1 - 2/(1 + I*a*
x)] - ((3*I)/2)*a^2*c^2*PolyLog[4, -1 + 2/(1 + I*a*x)]
```

Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4918

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/(d + e*x^2),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4850

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTan[c*x])^(p - 1)*ArcTanh[1 - 2/(1 + I*c*x)])/(1 + c^2*x^2), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4988

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTan[c*x])^p)/(d + e
*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTan[c*x])^p)/(d + e*x^2
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 4998

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[k + 1, u])/(2
*c*d), x] - Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[k + 1,
 u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
qQ[e, c^2*d] && EqQ[u^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
```

```
e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^2 \tan^{-1}(ax)^3}{x^3}\,dx = \int \left(\frac{c^2\tan^{-1}(ax)^3}{x^3} + \frac{2a^2c^2\tan^{-1}(ax)^3}{x} + a^4c^2x\tan^{-1}(ax)^3\right) dx$$

$$= c^2\int \frac{\tan^{-1}(ax)^3}{x^3}\,dx + (2a^2c^2)\int \frac{\tan^{-1}(ax)^3}{x}\,dx + (a^4c^2)\int x\tan^{-1}(ax)^3\,dx$$

$$= -\frac{c^2\tan^{-1}(ax)^3}{2x^2} + \frac{1}{2}a^4c^2x^2\tan^{-1}(ax)^3 + 4a^2c^2\tan^{-1}(ax)^3\tanh^{-1}\left(1-\frac{2}{1+iax}\right) + \frac{1}{2}(3a$$

$$= -\frac{c^2\tan^{-1}(ax)^3}{2x^2} + \frac{1}{2}a^4c^2x^2\tan^{-1}(ax)^3 + 4a^2c^2\tan^{-1}(ax)^3\tanh^{-1}\left(1-\frac{2}{1+iax}\right) + \frac{1}{2}(3a$$

$$= -\frac{3ac^2\tan^{-1}(ax)^2}{2x} - \frac{3}{2}a^3c^2x\tan^{-1}(ax)^2 - \frac{c^2\tan^{-1}(ax)^3}{2x^2} + \frac{1}{2}a^4c^2x^2\tan^{-1}(ax)^3 + 4a^2c^2\,\mathrm{t}$$

$$= -3ia^2c^2\tan^{-1}(ax)^2 - \frac{3ac^2\tan^{-1}(ax)^2}{2x} - \frac{3}{2}a^3c^2x\tan^{-1}(ax)^2 - \frac{c^2\tan^{-1}(ax)^3}{2x^2} + \frac{1}{2}a^4c^2x^2$$

$$= -3ia^2c^2\tan^{-1}(ax)^2 - \frac{3ac^2\tan^{-1}(ax)^2}{2x} - \frac{3}{2}a^3c^2x\tan^{-1}(ax)^2 - \frac{c^2\tan^{-1}(ax)^3}{2x^2} + \frac{1}{2}a^4c^2x^2$$

$$= -3ia^2c^2\tan^{-1}(ax)^2 - \frac{3ac^2\tan^{-1}(ax)^2}{2x} - \frac{3}{2}a^3c^2x\tan^{-1}(ax)^2 - \frac{c^2\tan^{-1}(ax)^3}{2x^2} + \frac{1}{2}a^4c^2x^2$$

$$= -3ia^2c^2\tan^{-1}(ax)^2 - \frac{3ac^2\tan^{-1}(ax)^2}{2x} - \frac{3}{2}a^3c^2x\tan^{-1}(ax)^2 - \frac{c^2\tan^{-1}(ax)^3}{2x^2} + \frac{1}{2}a^4c^2x^2$$

Mathematica [A] time = 0.601002, size = 302, normalized size = 0.76

$$\frac{1}{32}a^2c^2\left(96i\tan^{-1}(ax)^2\text{PolyLog}\left(2,e^{-2i\tan^{-1}(ax)}\right) + 96\tan^{-1}(ax)\text{PolyLog}\left(3,e^{-2i\tan^{-1}(ax)}\right) - 96\tan^{-1}(ax)\text{PolyLog}\left(3,-\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((c + a^2*c*x^2)^2*ArcTan[a*x]^3)/x^3,x]
```

```
[Out] (a^2*c^2*((-I)*Pi^4 - (48*ArcTan[a*x]^2)/(a*x) - 48*a*x*ArcTan[a*x]^2 - (16
*ArcTan[a*x]^3)/(a^2*x^2) + 16*a^2*x^2*ArcTan[a*x]^3 + (32*I)*ArcTan[a*x]^4
 + 64*ArcTan[a*x]^3*Log[1 - E^((-2*I)*ArcTan[a*x])] + 96*ArcTan[a*x]*Log[1
- E^((2*I)*ArcTan[a*x])] - 96*ArcTan[a*x]*Log[1 + E^((2*I)*ArcTan[a*x])] -
64*ArcTan[a*x]^3*Log[1 + E^((2*I)*ArcTan[a*x])] + (96*I)*ArcTan[a*x]^2*Poly
Log[2, E^((-2*I)*ArcTan[a*x])] + (48*I)*(1 + 2*ArcTan[a*x]^2)*PolyLog[2, -E
^((2*I)*ArcTan[a*x])] - (48*I)*PolyLog[2, E^((2*I)*ArcTan[a*x])] + 96*ArcTa
```

```
n[a*x]*PolyLog[3, E^((-2*I)*ArcTan[a*x])] - 96*ArcTan[a*x]*PolyLog[3, -E^((
2*I)*ArcTan[a*x])] - (48*I)*PolyLog[4, E^((-2*I)*ArcTan[a*x])] - (48*I)*Pol
yLog[4, -E^((2*I)*ArcTan[a*x])]))/32
```

Maple [A] time = 2.365, size = 682, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^2*arctan(a*x)^3/x^3,x)
```

```
[Out] 1/2*a^4*c^2*x^2*arctan(a*x)^3-3/2*a^3*c^2*x*arctan(a*x)^2-3/2*a*c^2*arctan(
a*x)^2/x-1/2*c^2*arctan(a*x)^3/x^2+3*a^2*c^2*arctan(a*x)*ln(1+(1+I*a*x)/(a^
2*x^2+1)^(1/2))+3/2*I*a^2*c^2*polylog(2,-(1+I*a*x)^2/(a^2*x^2+1))+2*a^2*c^2
*arctan(a*x)^3*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-3*I*a^2*c^2*polylog(2,(1+I
*a*x)/(a^2*x^2+1)^(1/2))+12*a^2*c^2*arctan(a*x)*polylog(3,-(1+I*a*x)/(a^2*x
^2+1)^(1/2))+12*I*a^2*c^2*polylog(4,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*a^2*c^2
*arctan(a*x)^3*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*I*a^2*c^2*arctan(a*x)^2*
polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))-3*a^2*c^2*arctan(a*x)*polylog(3,-(1+
I*a*x)^2/(a^2*x^2+1))-3*I*a^2*c^2*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-3
*a^2*c^2*arctan(a*x)*ln((1+I*a*x)^2/(a^2*x^2+1)+1)-6*I*a^2*c^2*arctan(a*x)^
2*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*a^2*c^2*arctan(a*x)^3*ln((1+I*a
*x)^2/(a^2*x^2+1)+1)+3*I*a^2*c^2*arctan(a*x)^2*polylog(2,-(1+I*a*x)^2/(a^2*
x^2+1))+12*a^2*c^2*arctan(a*x)*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))-3/2*I
*a^2*c^2*polylog(4,-(1+I*a*x)^2/(a^2*x^2+1))+3*a^2*c^2*arctan(a*x)*ln(1-(1+
I*a*x)/(a^2*x^2+1)^(1/2))+12*I*a^2*c^2*polylog(4,(1+I*a*x)/(a^2*x^2+1)^(1/2
))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^3/x^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^4c^2x^4 + 2\,a^2c^2x^2 + c^2\right)\arctan\left(ax\right)^3}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^3/x^3,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(a*x)^3/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$c^2\left(\int\frac{\operatorname{atan}^3\left(ax\right)}{x^3}\,dx + \int\frac{2a^2\operatorname{atan}^3\left(ax\right)}{x}\,dx + \int a^4x\operatorname{atan}^3\left(ax\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**2*atan(a*x)**3/x**3,x)
```

```
[Out] c**2*(Integral(atan(a*x)**3/x**3, x) + Integral(2*a**2*atan(a*x)**3/x, x) +
 Integral(a**4*x*atan(a*x)**3, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a^2cx^2 + c\right)^2\arctan\left(ax\right)^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^3/x^3,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^2*arctan(a*x)^3/x^3, x)
```

3.378 $\int \frac{(c+a^2cx^2)^2 \tan^{-1}(ax)^3}{x^4} dx$

Optimal. Leaf size=311

$$\frac{5}{2}a^3c^2\text{PolyLog}\left(3,-1+\frac{2}{1-iax}\right)+\frac{3}{2}a^3c^2\text{PolyLog}\left(3,1-\frac{2}{1+iax}\right)-5ia^3c^2\tan^{-1}(ax)\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)+3ia$$

```
[Out] -((a^2*c^2*ArcTan[a*x])/x) - (a^3*c^2*ArcTan[a*x]^2)/2 - (a*c^2*ArcTan[a*x]
^2)/(2*x^2) - ((2*I)/3)*a^3*c^2*ArcTan[a*x]^3 - (c^2*ArcTan[a*x]^3)/(3*x^3)
 - (2*a^2*c^2*ArcTan[a*x]^3)/x + a^4*c^2*x*ArcTan[a*x]^3 + a^3*c^2*Log[x] +
 3*a^3*c^2*ArcTan[a*x]^2*Log[2/(1 + I*a*x)] - (a^3*c^2*Log[1 + a^2*x^2])/2
+ 5*a^3*c^2*ArcTan[a*x]^2*Log[2 - 2/(1 - I*a*x)] - (5*I)*a^3*c^2*ArcTan[a*x
]*PolyLog[2, -1 + 2/(1 - I*a*x)] + (3*I)*a^3*c^2*ArcTan[a*x]*PolyLog[2, 1 -
 2/(1 + I*a*x)] + (5*a^3*c^2*PolyLog[3, -1 + 2/(1 - I*a*x)])/2 + (3*a^3*c^2
*PolyLog[3, 1 - 2/(1 + I*a*x)])/2
```

Rubi [A] time = 0.789185, antiderivative size = 311, normalized size of antiderivative = 1., number of steps used = 26, number of rules used = 16, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.727, Rules used = {4948, 4846, 4920, 4854, 4884, 4994, 6610, 4852, 4918, 266, 36, 29, 31, 4924, 4868, 4992}

$$\frac{5}{2}a^3c^2\text{PolyLog}\left(3,-1+\frac{2}{1-iax}\right)+\frac{3}{2}a^3c^2\text{PolyLog}\left(3,1-\frac{2}{1+iax}\right)-5ia^3c^2\tan^{-1}(ax)\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)+3ia$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)^2*ArcTan[a*x]^3)/x^4,x]
```


```
[Out] -((a^2*c^2*ArcTan[a*x])/x) - (a^3*c^2*ArcTan[a*x]^2)/2 - (a*c^2*ArcTan[a*x]
^2)/(2*x^2) - ((2*I)/3)*a^3*c^2*ArcTan[a*x]^3 - (c^2*ArcTan[a*x]^3)/(3*x^3)
 - (2*a^2*c^2*ArcTan[a*x]^3)/x + a^4*c^2*x*ArcTan[a*x]^3 + a^3*c^2*Log[x] +
 3*a^3*c^2*ArcTan[a*x]^2*Log[2/(1 + I*a*x)] - (a^3*c^2*Log[1 + a^2*x^2])/2
+ 5*a^3*c^2*ArcTan[a*x]^2*Log[2 - 2/(1 - I*a*x)] - (5*I)*a^3*c^2*ArcTan[a*x
]*PolyLog[2, -1 + 2/(1 - I*a*x)] + (3*I)*a^3*c^2*ArcTan[a*x]*PolyLog[2, 1 -
 2/(1 + I*a*x)] + (5*a^3*c^2*PolyLog[3, -1 + 2/(1 - I*a*x)])/2 + (3*a^3*c^2
*PolyLog[3, 1 - 2/(1 + I*a*x)])/2
```


Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
```

```
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4918

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/(d + e*x^2),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
```

```
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 4992

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d
] && EqQ[(1 - u)^2 - (1 - (2*I)/(I + c*x))^2, 0]
```

Rubi steps

$$\int \frac{\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^3}{x^4}\,dx = \int \left(a^4c^2\tan^{-1}(ax)^3+\frac{c^2\tan^{-1}(ax)^3}{x^4}+\frac{2a^2c^2\tan^{-1}(ax)^3}{x^2}\right)dx$$

$$= c^2\int \frac{\tan^{-1}(ax)^3}{x^4}\,dx+\left(2a^2c^2\right)\int \frac{\tan^{-1}(ax)^3}{x^2}\,dx+\left(a^4c^2\right)\int \tan^{-1}(ax)^3\,dx$$

$$= -\frac{c^2\tan^{-1}(ax)^3}{3x^3}-\frac{2a^2c^2\tan^{-1}(ax)^3}{x}+a^4c^2x\tan^{-1}(ax)^3+\left(ac^2\right)\int \frac{\tan^{-1}(ax)^2}{x^3\left(1+a^2x^2\right)}\,dx+\big(6$$

$$= -ia^3c^2\tan^{-1}(ax)^3-\frac{c^2\tan^{-1}(ax)^3}{3x^3}-\frac{2a^2c^2\tan^{-1}(ax)^3}{x}+a^4c^2x\tan^{-1}(ax)^3+\left(ac^2\right)\int \frac{\text{ta}}{}$$

$$= -\frac{ac^2\tan^{-1}(ax)^2}{2x^2}-\frac{2}{3}ia^3c^2\tan^{-1}(ax)^3-\frac{c^2\tan^{-1}(ax)^3}{3x^3}-\frac{2a^2c^2\tan^{-1}(ax)^3}{x}+a^4c^2x\tan^{-}$$

$$= -\frac{ac^2\tan^{-1}(ax)^2}{2x^2}-\frac{2}{3}ia^3c^2\tan^{-1}(ax)^3-\frac{c^2\tan^{-1}(ax)^3}{3x^3}-\frac{2a^2c^2\tan^{-1}(ax)^3}{x}+a^4c^2x\tan^{-}$$

$$= -\frac{a^2c^2\tan^{-1}(ax)}{x}-\frac{1}{2}a^3c^2\tan^{-1}(ax)^2-\frac{ac^2\tan^{-1}(ax)^2}{2x^2}-\frac{2}{3}ia^3c^2\tan^{-1}(ax)^3-\frac{c^2\tan^{-1}(a}{3x^3}$$

$$= -\frac{a^2c^2\tan^{-1}(ax)}{x}-\frac{1}{2}a^3c^2\tan^{-1}(ax)^2-\frac{ac^2\tan^{-1}(ax)^2}{2x^2}-\frac{2}{3}ia^3c^2\tan^{-1}(ax)^3-\frac{c^2\tan^{-1}(a}{3x^3}$$

$$= -\frac{a^2c^2\tan^{-1}(ax)}{x}-\frac{1}{2}a^3c^2\tan^{-1}(ax)^2-\frac{ac^2\tan^{-1}(ax)^2}{2x^2}-\frac{2}{3}ia^3c^2\tan^{-1}(ax)^3-\frac{c^2\tan^{-1}(a}{3x^3}$$

$$= -\frac{a^2c^2\tan^{-1}(ax)}{x}-\frac{1}{2}a^3c^2\tan^{-1}(ax)^2-\frac{ac^2\tan^{-1}(ax)^2}{2x^2}-\frac{2}{3}ia^3c^2\tan^{-1}(ax)^3-\frac{c^2\tan^{-1}(a}{3x^3}$$

Mathematica [A] time = 0.502387, size = 289, normalized size = 0.93

$$\frac{c^2\left(120ia^3x^3\tan^{-1}(ax)\text{PolyLog}\left(2,e^{-2i\tan^{-1}(ax)}\right)-72ia^3x^3\tan^{-1}(ax)\text{PolyLog}\left(2,-e^{2i\tan^{-1}(ax)}\right)+60a^3x^3\text{PolyLog}\left(3,e\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((c + a^2*c*x^2)^2*ArcTan[a*x]^3)/x^4,x]
```

```
[Out] (c^2*((-5*I)*a^3*Pi^3*x^3 - 24*a^2*x^2*ArcTan[a*x] - 12*a*x*ArcTan[a*x]^2 -
 12*a^3*x^3*ArcTan[a*x]^2 - 8*ArcTan[a*x]^3 - 48*a^2*x^2*ArcTan[a*x]^3 + (1
6*I)*a^3*x^3*ArcTan[a*x]^3 + 24*a^4*x^4*ArcTan[a*x]^3 + 120*a^3*x^3*ArcTan[
a*x]^2*Log[1 - E^((-2*I)*ArcTan[a*x])] + 72*a^3*x^3*ArcTan[a*x]^2*Log[1 + E
^((2*I)*ArcTan[a*x])] + 24*a^3*x^3*Log[(a*x)/Sqrt[1 + a^2*x^2]] + (120*I)*a
^3*x^3*ArcTan[a*x]*PolyLog[2, E^((-2*I)*ArcTan[a*x])] - (72*I)*a^3*x^3*ArcT
an[a*x]*PolyLog[2, -E^((2*I)*ArcTan[a*x])] + 60*a^3*x^3*PolyLog[3, E^((-2*I
)*ArcTan[a*x])] + 36*a^3*x^3*PolyLog[3, -E^((2*I)*ArcTan[a*x])]))/(24*x^3)
```

Maple [C] time = 2.881, size = 5651, normalized size = 18.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^2*arctan(a*x)^3/x^4,x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^3/x^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^4c^2x^4 + 2\,a^2c^2x^2 + c^2\right)\arctan\left(ax\right)^3}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^3/x^4,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(a*x)^3/x^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$c^2\left(\int a^4\,\text{atan}^3\left(ax\right)dx + \int \frac{\text{atan}^3\left(ax\right)}{x^4}\,dx + \int \frac{2a^2\,\text{atan}^3\left(ax\right)}{x^2}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**2*atan(a*x)**3/x**4,x)
```

```
[Out] c**2*(Integral(a**4*atan(a*x)**3, x) + Integral(atan(a*x)**3/x**4, x) + Int
egral(2*a**2*atan(a*x)**3/x**2, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2 + c\right)^2\arctan\left(ax\right)^3}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^3/x^4,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^2*arctan(a*x)^3/x^4, x)
```

3.379 $\int x^3 \left(c + a^2 c x^2\right)^3 \tan^{-1}(ax)^3 \, dx$

Optimal. Leaf size=381

$$\frac{26ic^3\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{525a^4} - \frac{1}{840}a^3c^3x^7 + \frac{1}{10}a^6c^3x^{10}\tan^{-1}(ax)^3 - \frac{1}{30}a^5c^3x^9\tan^{-1}(ax)^2 + \frac{3}{8}a^4c^3x^8\tan^{-1}(ax)^3 + \frac{1}{120}$$

```
[Out] (389*c^3*x)/(12600*a^3) - (17*c^3*x^3)/(9450*a) - (a*c^3*x^5)/252 - (a^3*c^
3*x^7)/840 - (389*c^3*ArcTan[a*x])/(12600*a^4) - (107*c^3*x^2*ArcTan[a*x])/
(4200*a^2) + (53*c^3*x^4*ArcTan[a*x])/2100 + (71*a^2*c^3*x^6*ArcTan[a*x])/2
520 + (a^4*c^3*x^8*ArcTan[a*x])/120 + (((26*I)/525)*c^3*ArcTan[a*x]^2)/a^4
+ (3*c^3*x*ArcTan[a*x]^2)/(40*a^3) - (c^3*x^3*ArcTan[a*x]^2)/(40*a) - (27*a
*c^3*x^5*ArcTan[a*x]^2)/200 - (33*a^3*c^3*x^7*ArcTan[a*x]^2)/280 - (a^5*c^3
*x^9*ArcTan[a*x]^2)/30 - (c^3*ArcTan[a*x]^3)/(40*a^4) + (c^3*x^4*ArcTan[a*x
]^3)/4 + (a^2*c^3*x^6*ArcTan[a*x]^3)/2 + (3*a^4*c^3*x^8*ArcTan[a*x]^3)/8 +
(a^6*c^3*x^10*ArcTan[a*x]^3)/10 + (52*c^3*ArcTan[a*x]*Log[2/(1 + I*a*x)])/(
525*a^4) + (((26*I)/525)*c^3*PolyLog[2, 1 - 2/(1 + I*a*x)])/a^4
```

Rubi [A] time = 3.72532, antiderivative size = 381, normalized size of antiderivative = 1., number of steps used = 184, number of rules used = 12, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {4948, 4852, 4916, 321, 203, 4920, 4854, 2402, 2315, 4846, 4884, 302}

$$\frac{26ic^3\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{525a^4} - \frac{1}{840}a^3c^3x^7 + \frac{1}{10}a^6c^3x^{10}\tan^{-1}(ax)^3 - \frac{1}{30}a^5c^3x^9\tan^{-1}(ax)^2 + \frac{3}{8}a^4c^3x^8\tan^{-1}(ax)^3 + \frac{1}{120}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(c + a^2*c*x^2)^3*ArcTan[a*x]^3,x]
```


```
[Out] (389*c^3*x)/(12600*a^3) - (17*c^3*x^3)/(9450*a) - (a*c^3*x^5)/252 - (a^3*c^
3*x^7)/840 - (389*c^3*ArcTan[a*x])/(12600*a^4) - (107*c^3*x^2*ArcTan[a*x])/
(4200*a^2) + (53*c^3*x^4*ArcTan[a*x])/2100 + (71*a^2*c^3*x^6*ArcTan[a*x])/2
520 + (a^4*c^3*x^8*ArcTan[a*x])/120 + (((26*I)/525)*c^3*ArcTan[a*x]^2)/a^4
+ (3*c^3*x*ArcTan[a*x]^2)/(40*a^3) - (c^3*x^3*ArcTan[a*x]^2)/(40*a) - (27*a
*c^3*x^5*ArcTan[a*x]^2)/200 - (33*a^3*c^3*x^7*ArcTan[a*x]^2)/280 - (a^5*c^3
*x^9*ArcTan[a*x]^2)/30 - (c^3*ArcTan[a*x]^3)/(40*a^4) + (c^3*x^4*ArcTan[a*x
]^3)/4 + (a^2*c^3*x^6*ArcTan[a*x]^3)/2 + (3*a^4*c^3*x^8*ArcTan[a*x]^3)/8 +
(a^6*c^3*x^10*ArcTan[a*x]^3)/10 + (52*c^3*ArcTan[a*x]*Log[2/(1 + I*a*x)])/(
525*a^4) + (((26*I)/525)*c^3*PolyLog[2, 1 - 2/(1 + I*a*x)])/a^4
```

Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\int x^3 \left(c+a^2cx^2\right)^3 \tan^{-1}(ax)^3\,dx = \int \left(c^3x^3\tan^{-1}(ax)^3 + 3a^2c^3x^5\tan^{-1}(ax)^3 + 3a^4c^3x^7\tan^{-1}(ax)^3 + a^6c^3x^9\tan^{-1}(ax)^3\right)$$

$$= c^3\int x^3\tan^{-1}(ax)^3\,dx + \left(3a^2c^3\right)\int x^5\tan^{-1}(ax)^3\,dx + \left(3a^4c^3\right)\int x^7\tan^{-1}(ax)^3\,dx + \left(\right.$$

$$= \frac{1}{4}c^3x^4\tan^{-1}(ax)^3 + \frac{1}{2}a^2c^3x^6\tan^{-1}(ax)^3 + \frac{3}{8}a^4c^3x^8\tan^{-1}(ax)^3 + \frac{1}{10}a^6c^3x^{10}\tan^{-1}(ax$$

$$= \frac{1}{4}c^3x^4\tan^{-1}(ax)^3 + \frac{1}{2}a^2c^3x^6\tan^{-1}(ax)^3 + \frac{3}{8}a^4c^3x^8\tan^{-1}(ax)^3 + \frac{1}{10}a^6c^3x^{10}\tan^{-1}(ax$$

$$= -\frac{c^3x^3\tan^{-1}(ax)^2}{4a} - \frac{3}{10}ac^3x^5\tan^{-1}(ax)^2 - \frac{9}{56}a^3c^3x^7\tan^{-1}(ax)^2 - \frac{1}{30}a^5c^3x^9\tan^{-1}(ax$$

$$= \frac{3c^3x\tan^{-1}(ax)^2}{4a^3} + \frac{c^3x^3\tan^{-1}(ax)^2}{4a} - \frac{3}{40}ac^3x^5\tan^{-1}(ax)^2 - \frac{33}{280}a^3c^3x^7\tan^{-1}(ax)^2 -$$

$$= \frac{c^3x^2\tan^{-1}(ax)}{4a^2} + \frac{3}{20}c^3x^4\tan^{-1}(ax) + \frac{3}{56}a^2c^3x^6\tan^{-1}(ax) + \frac{1}{120}a^4c^3x^8\tan^{-1}(ax) +$$

$$= -\frac{c^3x}{4a^3} - \frac{11c^3x^2\tan^{-1}(ax)}{20a^2} - \frac{3}{70}c^3x^4\tan^{-1}(ax) + \frac{71a^2c^3x^6\tan^{-1}(ax)}{2520} + \frac{1}{120}a^4c^3x^8\tan$$

$$= \frac{55c^3x}{84a^3} - \frac{11c^3x^3}{315a} - \frac{19ac^3x^5}{2100} - \frac{1}{840}a^3c^3x^7 + \frac{c^3\tan^{-1}(ax)}{4a^4} + \frac{59c^3x^2\tan^{-1}(ax)}{280a^2} + \frac{53c^3x}{}$$

$$= -\frac{689c^3x}{2520a^3} + \frac{79c^3x^3}{3780a} - \frac{1}{252}ac^3x^5 - \frac{1}{840}a^3c^3x^7 - \frac{55c^3\tan^{-1}(ax)}{84a^4} - \frac{107c^3x^2\tan^{-1}(ax)}{4200a^2}$$

$$= \frac{389c^3x}{12600a^3} - \frac{17c^3x^3}{9450a} - \frac{1}{252}ac^3x^5 - \frac{1}{840}a^3c^3x^7 + \frac{689c^3\tan^{-1}(ax)}{2520a^4} - \frac{107c^3x^2\tan^{-1}(ax)}{4200a^2}$$

$$= \frac{389c^3x}{12600a^3} - \frac{17c^3x^3}{9450a} - \frac{1}{252}ac^3x^5 - \frac{1}{840}a^3c^3x^7 - \frac{389c^3\tan^{-1}(ax)}{12600a^4} - \frac{107c^3x^2\tan^{-1}(ax)}{4200a^2}$$

$$= \frac{389c^3x}{12600a^3} - \frac{17c^3x^3}{9450a} - \frac{1}{252}ac^3x^5 - \frac{1}{840}a^3c^3x^7 - \frac{389c^3\tan^{-1}(ax)}{12600a^4} - \frac{107c^3x^2\tan^{-1}(ax)}{4200a^2}$$

Mathematica [A] time = 1.9998, size = 191, normalized size = 0.5

$$c^3\left(-1872i\text{PolyLog}\left(2,-e^{2i\tan^{-1}(ax)}\right) - ax\left(45a^6x^6+150a^4x^4+68a^2x^2-1167\right) + 945\left(4a^2x^2-1\right)\left(a^2x^2+1\right)^4\tan^{-1}(ax)^3\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*(c + a^2*c*x^2)^3*ArcTan[a*x]^3,x]
```

```
[Out] (c^3*(-(a*x*(-1167 + 68*a^2*x^2 + 150*a^4*x^4 + 45*a^6*x^6)) - 9*(208*I - 3
15*a*x + 105*a^3*x^3 + 567*a^5*x^5 + 495*a^7*x^7 + 140*a^9*x^9)*ArcTan[a*x]
^2 + 945*(1 + a^2*x^2)^4*(-1 + 4*a^2*x^2)*ArcTan[a*x]^3 + 3*ArcTan[a*x]*(-3
89 - 321*a^2*x^2 + 318*a^4*x^4 + 355*a^6*x^6 + 105*a^8*x^8 + 1248*Log[1 + E
^((2*I)*ArcTan[a*x])]) - (1872*I)*PolyLog[2, -E^((2*I)*ArcTan[a*x])]))/(378
00*a^4)
```

Maple [A] time = 0.099, size = 471, normalized size = 1.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a^2*c*x^2+c)^3*arctan(a*x)^3,x)
```

```
[Out] 71/2520*a^2*c^3*x^6*arctan(a*x)+1/120*a^4*c^3*x^8*arctan(a*x)-13/525*I/a^4*
c^3*ln(a*x+I)*ln(1/2*I*(a*x-I))-13/525*I/a^4*c^3*ln(a*x-I)*ln(a^2*x^2+1)+13
/525*I/a^4*c^3*ln(a*x-I)*ln(-1/2*I*(a*x+I))+13/525*I/a^4*c^3*ln(a*x+I)*ln(a
^2*x^2+1)+13/1050*I/a^4*c^3*ln(a*x-I)^2-13/525*I/a^4*c^3*dilog(1/2*I*(a*x-I
))+13/525*I/a^4*c^3*dilog(-1/2*I*(a*x+I))-26/525/a^4*c^3*arctan(a*x)*ln(a^2
*x^2+1)-13/1050*I/a^4*c^3*ln(a*x+I)^2+389/12600*c^3*x/a^3-17/9450*c^3*x^3/a
-1/252*a*c^3*x^5-1/840*a^3*c^3*x^7-389/12600*c^3*arctan(a*x)/a^4+53/2100*c^
3*x^4*arctan(a*x)-1/40*c^3*arctan(a*x)^3/a^4+1/4*c^3*x^4*arctan(a*x)^3-107/
4200*c^3*x^2*arctan(a*x)/a^2+3/40*c^3*x*arctan(a*x)^2/a^3-1/40*c^3*x^3*arct
an(a*x)^2/a-27/200*a*c^3*x^5*arctan(a*x)^2-33/280*a^3*c^3*x^7*arctan(a*x)^2
-1/30*a^5*c^3*x^9*arctan(a*x)^2+1/2*a^2*c^3*x^6*arctan(a*x)^3+3/8*a^4*c^3*x
^8*arctan(a*x)^3+1/10*a^6*c^3*x^10*arctan(a*x)^3
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a^2*c*x^2+c)^3*arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^6c^3x^9+3\,a^4c^3x^7+3\,a^2c^3x^5+c^3x^3\right)\arctan\left(ax\right)^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a^2*c*x^2+c)^3*arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral((a^6*c^3*x^9 + 3*a^4*c^3*x^7 + 3*a^2*c^3*x^5 + c^3*x^3)*arctan(a*x
)^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$c^3\left(\int x^3\operatorname{atan}^3\left(ax\right)dx+\int 3a^2x^5\operatorname{atan}^3\left(ax\right)dx+\int 3a^4x^7\operatorname{atan}^3\left(ax\right)dx+\int a^6x^9\operatorname{atan}^3\left(ax\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a**2*c*x**2+c)**3*atan(a*x)**3,x)
```

```
[Out] c**3*(Integral(x**3*atan(a*x)**3, x) + Integral(3*a**2*x**5*atan(a*x)**3, x
) + Integral(3*a**4*x**7*atan(a*x)**3, x) + Integral(a**6*x**9*atan(a*x)**3
, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a^2cx^2+c\right)^3x^3\arctan\left(ax\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a^2*c*x^2+c)^3*arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^3*x^3*arctan(a*x)^3, x)
```

3.380 $\int x^2 \left(c+a^2cx^2\right)^3 \tan^{-1}(ax)^3\,dx$

Optimal. Leaf size=389

$$-\frac{8c^3\text{PolyLog}\left(3,1-\frac{2}{1+iax}\right)}{105a^3}-\frac{16ic^3\tan^{-1}(ax)\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{105a^3}-\frac{1}{504}a^3c^3x^6+\frac{31c^3\log\left(a^2x^2+1\right)}{945a^3}+\frac{1}{9}a^6c^3x^9\,\text{ta}$$

```
[Out] (-107*c^3*x^2)/(7560*a) - (11*a*c^3*x^4)/1260 - (a^3*c^3*x^6)/504 - (47*c^3
*x*ArcTan[a*x])/(1260*a^2) + (239*c^3*x^3*ArcTan[a*x])/3780 + (59*a^2*c^3*x
^5*ArcTan[a*x])/1260 + (a^4*c^3*x^7*ArcTan[a*x])/84 + (47*c^3*ArcTan[a*x]^2
)/(2520*a^3) - (8*c^3*x^2*ArcTan[a*x]^2)/(105*a) - (89*a*c^3*x^4*ArcTan[a*x
]^2)/420 - (10*a^3*c^3*x^6*ArcTan[a*x]^2)/63 - (a^5*c^3*x^8*ArcTan[a*x]^2)/
24 - (((16*I)/315)*c^3*ArcTan[a*x]^3)/a^3 + (c^3*x^3*ArcTan[a*x]^3)/3 + (3*
a^2*c^3*x^5*ArcTan[a*x]^3)/5 + (3*a^4*c^3*x^7*ArcTan[a*x]^3)/7 + (a^6*c^3*x
^9*ArcTan[a*x]^3)/9 - (16*c^3*ArcTan[a*x]^2*Log[2/(1 + I*a*x)])/(105*a^3) +
 (31*c^3*Log[1 + a^2*x^2])/(945*a^3) - (((16*I)/105)*c^3*ArcTan[a*x]*PolyLo
g[2, 1 - 2/(1 + I*a*x)])/a^3 - (8*c^3*PolyLog[3, 1 - 2/(1 + I*a*x)])/(105*a
^3)
```

Rubi [A] time = 3.03923, antiderivative size = 389, normalized size of antiderivative = 1., number of steps used = 132, number of rules used = 12, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {4948, 4852, 4916, 4846, 260, 4884, 4920, 4854, 4994, 6610, 266, 43}

$$-\frac{8c^3\text{PolyLog}\left(3,1-\frac{2}{1+iax}\right)}{105a^3}-\frac{16ic^3\tan^{-1}(ax)\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{105a^3}-\frac{1}{504}a^3c^3x^6+\frac{31c^3\log\left(a^2x^2+1\right)}{945a^3}+\frac{1}{9}a^6c^3x^9\,\text{ta}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]^3,x]
```

```
[Out] (-107*c^3*x^2)/(7560*a) - (11*a*c^3*x^4)/1260 - (a^3*c^3*x^6)/504 - (47*c^3
*x*ArcTan[a*x])/(1260*a^2) + (239*c^3*x^3*ArcTan[a*x])/3780 + (59*a^2*c^3*x
^5*ArcTan[a*x])/1260 + (a^4*c^3*x^7*ArcTan[a*x])/84 + (47*c^3*ArcTan[a*x]^2
)/(2520*a^3) - (8*c^3*x^2*ArcTan[a*x]^2)/(105*a) - (89*a*c^3*x^4*ArcTan[a*x
]^2)/420 - (10*a^3*c^3*x^6*ArcTan[a*x]^2)/63 - (a^5*c^3*x^8*ArcTan[a*x]^2)/
24 - (((16*I)/315)*c^3*ArcTan[a*x]^3)/a^3 + (c^3*x^3*ArcTan[a*x]^3)/3 + (3*
a^2*c^3*x^5*ArcTan[a*x]^3)/5 + (3*a^4*c^3*x^7*ArcTan[a*x]^3)/7 + (a^6*c^3*x
^9*ArcTan[a*x]^3)/9 - (16*c^3*ArcTan[a*x]^2*Log[2/(1 + I*a*x)])/(105*a^3) +
 (31*c^3*Log[1 + a^2*x^2])/(945*a^3) - (((16*I)/105)*c^3*ArcTan[a*x]*PolyLo
g[2, 1 - 2/(1 + I*a*x)])/a^3 - (8*c^3*PolyLog[3, 1 - 2/(1 + I*a*x)])/(105*a
```

```
^3)
```

Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
```

```
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x^2\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^3\,dx = \int\left(c^3x^2\tan^{-1}(ax)^3+3a^2c^3x^4\tan^{-1}(ax)^3+3a^4c^3x^6\tan^{-1}(ax)^3+a^6c^3x^8\tan^{-1}(ax)^3\right)$$
$$= c^3\int x^2\tan^{-1}(ax)^3\,dx+\left(3a^2c^3\right)\int x^4\tan^{-1}(ax)^3\,dx+\left(3a^4c^3\right)\int x^6\tan^{-1}(ax)^3\,dx+($$
$$= \frac{1}{3}c^3x^3\tan^{-1}(ax)^3+\frac{3}{5}a^2c^3x^5\tan^{-1}(ax)^3+\frac{3}{7}a^4c^3x^7\tan^{-1}(ax)^3+\frac{1}{9}a^6c^3x^9\tan^{-1}(ax)^3$$
$$= \frac{1}{3}c^3x^3\tan^{-1}(ax)^3+\frac{3}{5}a^2c^3x^5\tan^{-1}(ax)^3+\frac{3}{7}a^4c^3x^7\tan^{-1}(ax)^3+\frac{1}{9}a^6c^3x^9\tan^{-1}(ax)^3$$
$$= -\frac{c^3x^2\tan^{-1}(ax)^2}{2a}-\frac{9}{20}ac^3x^4\tan^{-1}(ax)^2-\frac{3}{14}a^3c^3x^6\tan^{-1}(ax)^2-\frac{1}{24}a^5c^3x^8\tan^{-1}(ax$$
$$= \frac{2c^3x^2\tan^{-1}(ax)^2}{5a}-\frac{9}{70}ac^3x^4\tan^{-1}(ax)^2-\frac{10}{63}a^3c^3x^6\tan^{-1}(ax)^2-\frac{1}{24}a^5c^3x^8\tan^{-1}(ax$$
$$= \frac{c^3x\tan^{-1}(ax)}{a^2}+\frac{3}{10}c^3x^3\tan^{-1}(ax)+\frac{3}{35}a^2c^3x^5\tan^{-1}(ax)+\frac{1}{84}a^4c^3x^7\tan^{-1}(ax)-$$
$$= -\frac{17c^3x\tan^{-1}(ax)}{10a^2}-\frac{2}{35}c^3x^3\tan^{-1}(ax)+\frac{59a^2c^3x^5\tan^{-1}(ax)}{1260}+\frac{1}{84}a^4c^3x^7\tan^{-1}(ax)+$$
$$= \frac{23c^3x\tan^{-1}(ax)}{35a^2}+\frac{239c^3x^3\tan^{-1}(ax)}{3780}+\frac{59a^2c^3x^5\tan^{-1}(ax)}{1260}+\frac{1}{84}a^4c^3x^7\tan^{-1}(ax)-$$
$$= -\frac{19c^3x^2}{168a}-\frac{31ac^3x^4}{1680}-\frac{1}{504}a^3c^3x^6-\frac{47c^3x\tan^{-1}(ax)}{1260a^2}+\frac{239c^3x^3\tan^{-1}(ax)}{3780}+$$
$$= \frac{29c^3x^2}{630a}-\frac{11ac^3x^4}{1260}-\frac{1}{504}a^3c^3x^6-\frac{47c^3x\tan^{-1}(ax)}{1260a^2}+\frac{239c^3x^3\tan^{-1}(ax)}{3780}+$$
$$= -\frac{107c^3x^2}{7560a}-\frac{11ac^3x^4}{1260}-\frac{1}{504}a^3c^3x^6-\frac{47c^3x\tan^{-1}(ax)}{1260a^2}+\frac{239c^3x^3\tan^{-1}(ax)}{3780}+$$

Mathematica [A] time = 1.86585, size = 281, normalized size = 0.72

$$c^3\left(1152i\tan^{-1}(ax)\text{PolyLog}\left(2,-e^{2i\tan^{-1}(ax)}\right)-576\text{PolyLog}\left(3,-e^{2i\tan^{-1}(ax)}\right)-15a^6x^6-66a^4x^4-107a^2x^2+248\log\left(a\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]^3,x]
```

```
[Out] (c^3*(-56 - 107*a^2*x^2 - 66*a^4*x^4 - 15*a^6*x^6 - 282*a*x*ArcTan[a*x] + 4
78*a^3*x^3*ArcTan[a*x] + 354*a^5*x^5*ArcTan[a*x] + 90*a^7*x^7*ArcTan[a*x] +
 141*ArcTan[a*x]^2 - 576*a^2*x^2*ArcTan[a*x]^2 - 1602*a^4*x^4*ArcTan[a*x]^2
 - 1200*a^6*x^6*ArcTan[a*x]^2 - 315*a^8*x^8*ArcTan[a*x]^2 + (384*I)*ArcTan[
a*x]^3 + 2520*a^3*x^3*ArcTan[a*x]^3 + 4536*a^5*x^5*ArcTan[a*x]^3 + 3240*a^7
*x^7*ArcTan[a*x]^3 + 840*a^9*x^9*ArcTan[a*x]^3 - 1152*ArcTan[a*x]^2*Log[1 +
 E^((2*I)*ArcTan[a*x])] + 248*Log[1 + a^2*x^2] + (1152*I)*ArcTan[a*x]*PolyL
og[2, -E^((2*I)*ArcTan[a*x])] - 576*PolyLog[3, -E^((2*I)*ArcTan[a*x])]))/(7
560*a^3)
```

Maple [C] time = 8.589, size = 1181, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a^2*c*x^2+c)^3*arctan(a*x)^3,x)
```

```
[Out] 59/1260*a^2*c^3*x^5*arctan(a*x)+1/84*a^4*c^3*x^7*arctan(a*x)-107/7560*c^3*x
^2/a-11/1260*a*c^3*x^4-1/504*a^3*c^3*x^6+239/3780*c^3*x^3*arctan(a*x)+1/3*c
^3*x^3*arctan(a*x)^3+47/2520*c^3*arctan(a*x)^2/a^3-62/945/a^3*c^3*ln((1+I*a
*x)^2/(a^2*x^2+1)+1)-8/105/a^3*c^3*polylog(3,-(1+I*a*x)^2/(a^2*x^2+1))-16/1
05/a^3*c^3*arctan(a*x)^2*ln(2)+8/105/a^3*c^3*arctan(a*x)^2*ln(a^2*x^2+1)-16
/105/a^3*c^3*arctan(a*x)^2*ln((1+I*a*x)/(a^2*x^2+1)^(1/2))+62/945*I/a^3*c^3
*arctan(a*x)+16/315*I/a^3*c^3*arctan(a*x)^3+4/105*I/a^3*c^3*Pi*csgn(I/((1+I
*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^
2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*arctan(a*x)^2-47/1260*c^3*x*ar
ctan(a*x)/a^2-8/105*c^3*x^2*arctan(a*x)^2/a-89/420*a*c^3*x^4*arctan(a*x)^2-
10/63*a^3*c^3*x^6*arctan(a*x)^2-1/24*a^5*c^3*x^8*arctan(a*x)^2+3/5*a^2*c^3*
x^5*arctan(a*x)^3+3/7*a^4*c^3*x^7*arctan(a*x)^3+1/9*a^6*c^3*x^9*arctan(a*x)
^3+16/105*I/a^3*c^3*arctan(a*x)*polylog(2,-(1+I*a*x)^2/(a^2*x^2+1))-4/105*I
/a^3*c^3*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))^2*csgn(I*((1+I*a*x)^2/(a^2*
x^2+1)+1)^2)*arctan(a*x)^2-8/105*I/a^3*c^3*Pi*csgn(I*(1+I*a*x)/(a^2*x^2+1)^
(1/2))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^2*arctan(a*x)^2-4/105*I/a^3*c^3*Pi*c
sgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a
*x)^2/(a^2*x^2+1)+1)^2)^2*arctan(a*x)^2+4/105*I/a^3*c^3*Pi*csgn(I*(1+I*a*x)
/(a^2*x^2+1)^(1/2))^2*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*arctan(a*x)^2-4/105*I
/a^3*c^3*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/
((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2*arctan(a*x)^2+8/105*I/a^3*c^3*Pi*csgn(I*((
1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2*arctan(a
*x)^2+4/105*I/a^3*c^3*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^3*arctan(a*x)^2+4/
105*I/a^3*c^3*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)
```

```
^2)^3*arctan(a*x)^2-4/105*I/a^3*c^3*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2
)^3*arctan(a*x)^2-1/135/a^3*c^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2520}\left(35\,a^{6}c^{3}x^{9}+135\,a^{4}c^{3}x^{7}+189\,a^{2}c^{3}x^{5}+105\,c^{3}x^{3}\right)\arctan\left(ax\right)^{3}-\frac{1}{3360}\left(35\,a^{6}c^{3}x^{9}+135\,a^{4}c^{3}x^{7}+189\,a^{2}c^{3}x^{5}+105\,c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^3*arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/2520*(35*a^6*c^3*x^9 + 135*a^4*c^3*x^7 + 189*a^2*c^3*x^5 + 105*c^3*x^3)*a
rctan(a*x)^3 - 1/3360*(35*a^6*c^3*x^9 + 135*a^4*c^3*x^7 + 189*a^2*c^3*x^5 +
 105*c^3*x^3)*arctan(a*x)*log(a^2*x^2 + 1)^2 + integrate(1/3360*(2940*(a^8*
c^3*x^10 + 4*a^6*c^3*x^8 + 6*a^4*c^3*x^6 + 4*a^2*c^3*x^4 + c^3*x^2)*arctan(
a*x)^3 - 4*(35*a^7*c^3*x^9 + 135*a^5*c^3*x^7 + 189*a^3*c^3*x^5 + 105*a*c^3*
x^3)*arctan(a*x)^2 + 4*(35*a^8*c^3*x^10 + 135*a^6*c^3*x^8 + 189*a^4*c^3*x^6
 + 105*a^2*c^3*x^4)*arctan(a*x)*log(a^2*x^2 + 1) + (35*a^7*c^3*x^9 + 135*a^
5*c^3*x^7 + 189*a^3*c^3*x^5 + 105*a*c^3*x^3 + 315*(a^8*c^3*x^10 + 4*a^6*c^3
*x^8 + 6*a^4*c^3*x^6 + 4*a^2*c^3*x^4 + c^3*x^2)*arctan(a*x))*log(a^2*x^2 +
1)^2)/(a^2*x^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a^{6}c^{3}x^{8}+3\,a^{4}c^{3}x^{6}+3\,a^{2}c^{3}x^{4}+c^{3}x^{2}\right)\arctan\left(ax\right)^{3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^3*arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral((a^6*c^3*x^8 + 3*a^4*c^3*x^6 + 3*a^2*c^3*x^4 + c^3*x^2)*arctan(a*x
)^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$c^{3}\left(\int x^{2}\,\mathrm{atan}^{3}\left(ax\right)dx+\int 3a^{2}x^{4}\,\mathrm{atan}^{3}\left(ax\right)dx+\int 3a^{4}x^{6}\,\mathrm{atan}^{3}\left(ax\right)dx+\int a^{6}x^{8}\,\mathrm{atan}^{3}\left(ax\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a**2*c*x**2+c)**3*atan(a*x)**3,x)
```

```
[Out] c**3*(Integral(x**2*atan(a*x)**3, x) + Integral(3*a**2*x**4*atan(a*x)**3, x
) + Integral(3*a**4*x**6*atan(a*x)**3, x) + Integral(a**6*x**8*atan(a*x)**3
, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2+c\right)^3 x^2 \arctan(ax)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^3*arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^3*x^2*arctan(a*x)^3, x)
```

3.381 $\int x\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^3\,dx$

Optimal. Leaf size=308

$$-\frac{6ic^3\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{35a^2}-\frac{1}{280}a^3c^3x^5-\frac{3c^3x\left(a^2x^2+1\right)^3\tan^{-1}(ax)^2}{56a}-\frac{9c^3x\left(a^2x^2+1\right)^2\tan^{-1}(ax)^2}{140a}-\frac{3c^3x\left(a^2x^2+1\right.}{35}$$

```
[Out] (-19*c^3*x)/(140*a) - (19*a*c^3*x^3)/840 - (a^3*c^3*x^5)/280 + (3*c^3*(1 +
a^2*x^2)*ArcTan[a*x])/(35*a^2) + (9*c^3*(1 + a^2*x^2)^2*ArcTan[a*x])/(280*a
^2) + (c^3*(1 + a^2*x^2)^3*ArcTan[a*x])/(56*a^2) - (((6*I)/35)*c^3*ArcTan[a
*x]^2)/a^2 - (6*c^3*x*ArcTan[a*x]^2)/(35*a) - (3*c^3*x*(1 + a^2*x^2)*ArcTan
[a*x]^2)/(35*a) - (9*c^3*x*(1 + a^2*x^2)^2*ArcTan[a*x]^2)/(140*a) - (3*c^3*
x*(1 + a^2*x^2)^3*ArcTan[a*x]^2)/(56*a) + (c^3*(1 + a^2*x^2)^4*ArcTan[a*x]^
3)/(8*a^2) - (12*c^3*ArcTan[a*x]*Log[2/(1 + I*a*x)])/(35*a^2) - (((6*I)/35)
*c^3*PolyLog[2, 1 - 2/(1 + I*a*x)])/a^2
```

Rubi [A] time = 0.254217, antiderivative size = 308, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.45, Rules used = {4930, 4880, 4846, 4920, 4854, 2402, 2315, 8, 194}

$$-\frac{6ic^3\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{35a^2}-\frac{1}{280}a^3c^3x^5-\frac{3c^3x\left(a^2x^2+1\right)^3\tan^{-1}(ax)^2}{56a}-\frac{9c^3x\left(a^2x^2+1\right)^2\tan^{-1}(ax)^2}{140a}-\frac{3c^3x\left(a^2x^2+1\right.}{35}$$

Antiderivative was successfully verified.

```
[In] Int[x*(c + a^2*c*x^2)^3*ArcTan[a*x]^3,x]
```



```
[Out] (-19*c^3*x)/(140*a) - (19*a*c^3*x^3)/840 - (a^3*c^3*x^5)/280 + (3*c^3*(1 +
a^2*x^2)*ArcTan[a*x])/(35*a^2) + (9*c^3*(1 + a^2*x^2)^2*ArcTan[a*x])/(280*a
^2) + (c^3*(1 + a^2*x^2)^3*ArcTan[a*x])/(56*a^2) - (((6*I)/35)*c^3*ArcTan[a
*x]^2)/a^2 - (6*c^3*x*ArcTan[a*x]^2)/(35*a) - (3*c^3*x*(1 + a^2*x^2)*ArcTan
[a*x]^2)/(35*a) - (9*c^3*x*(1 + a^2*x^2)^2*ArcTan[a*x]^2)/(140*a) - (3*c^3*
x*(1 + a^2*x^2)^3*ArcTan[a*x]^2)/(56*a) + (c^3*(1 + a^2*x^2)^4*ArcTan[a*x]^
3)/(8*a^2) - (12*c^3*ArcTan[a*x]*Log[2/(1 + I*a*x)])/(35*a^2) - (((6*I)/35)
*c^3*PolyLog[2, 1 - 2/(1 + I*a*x)])/a^2
```


Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
```

```
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 4880

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> -Simp[(b*p*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1))/(2*c*q*(2*
q + 1)), x] + (Dist[(2*d*q)/(2*q + 1), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTa
n[c*x])^p, x], x] + Dist[(b^2*d*p*(p - 1))/(2*q*(2*q + 1)), Int[(d + e*x^2)
^(q - 1)*(a + b*ArcTan[c*x])^(p - 2), x], x] + Simp[(x*(d + e*x^2)^q*(a + b
*ArcTan[c*x])^p)/(2*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[q, 0] && GtQ[p, 1]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int x\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^3\,dx = \frac{c^3\left(1+a^2x^2\right)^4\tan^{-1}(ax)^3}{8a^2}-\frac{3\int\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^2\,dx}{8a}$$
$$=\frac{c^3\left(1+a^2x^2\right)^3\tan^{-1}(ax)}{56a^2}-\frac{3c^3x\left(1+a^2x^2\right)^3\tan^{-1}(ax)^2}{56a}+\frac{c^3\left(1+a^2x^2\right)^4\tan^{-1}(ax)^3}{8a^2}-$$
$$=\frac{9c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)}{280a^2}+\frac{c^3\left(1+a^2x^2\right)^3\tan^{-1}(ax)}{56a^2}-\frac{9c^3x\left(1+a^2x^2\right)^2\tan^{-1}(ax)^2}{140a}-$$
$$=-\frac{c^3x}{20a}-\frac{19}{840}ac^3x^3-\frac{1}{280}a^3c^3x^5+\frac{3c^3\left(1+a^2x^2\right)\tan^{-1}(ax)}{35a^2}+\frac{9c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax}{280a^2}$$
$$=-\frac{19c^3x}{140a}-\frac{19}{840}ac^3x^3-\frac{1}{280}a^3c^3x^5+\frac{3c^3\left(1+a^2x^2\right)\tan^{-1}(ax)}{35a^2}+\frac{9c^3\left(1+a^2x^2\right)^2\tan^{-1}(}{280a^2}$$
$$=-\frac{19c^3x}{140a}-\frac{19}{840}ac^3x^3-\frac{1}{280}a^3c^3x^5+\frac{3c^3\left(1+a^2x^2\right)\tan^{-1}(ax)}{35a^2}+\frac{9c^3\left(1+a^2x^2\right)^2\tan^{-1}(}{280a^2}$$
$$=-\frac{19c^3x}{140a}-\frac{19}{840}ac^3x^3-\frac{1}{280}a^3c^3x^5+\frac{3c^3\left(1+a^2x^2\right)\tan^{-1}(ax)}{35a^2}+\frac{9c^3\left(1+a^2x^2\right)^2\tan^{-1}(}{280a^2}$$
$$=-\frac{19c^3x}{140a}-\frac{19}{840}ac^3x^3-\frac{1}{280}a^3c^3x^5+\frac{3c^3\left(1+a^2x^2\right)\tan^{-1}(ax)}{35a^2}+\frac{9c^3\left(1+a^2x^2\right)^2\tan^{-1}(}{280a^2}$$
$$=-\frac{19c^3x}{140a}-\frac{19}{840}ac^3x^3-\frac{1}{280}a^3c^3x^5+\frac{3c^3\left(1+a^2x^2\right)\tan^{-1}(ax)}{35a^2}+\frac{9c^3\left(1+a^2x^2\right)^2\tan^{-1}(}{280a^2}$$

Mathematica [A] time = 1.31684, size = 157, normalized size = 0.51

$$\frac{c^3\left(144i\text{PolyLog}\left(2,-e^{2i\tan^{-1}(ax)}\right)-ax\left(3a^4x^4+19a^2x^2+114\right)+105\left(a^2x^2+1\right)^4\tan^{-1}(ax)^3-9\left(5a^7x^7+21a^5x^5+35a^3\right.}{840a^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*(c + a^2*c*x^2)^3*ArcTan[a*x]^3,x]
```

```
[Out] (c^3*(-(a*x*(114 + 19*a^2*x^2 + 3*a^4*x^4)) - 9*(-16*I + 35*a*x + 35*a^3*x^
3 + 21*a^5*x^5 + 5*a^7*x^7)*ArcTan[a*x]^2 + 105*(1 + a^2*x^2)^4*ArcTan[a*x]
^3 + 3*ArcTan[a*x]*(38 + 57*a^2*x^2 + 24*a^4*x^4 + 5*a^6*x^6 - 96*Log[1 + E
^((2*I)*ArcTan[a*x])]) + (144*I)*PolyLog[2, -E^((2*I)*ArcTan[a*x])]))/(840*
a^2)
```

Maple [A] time = 0.096, size = 428, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^3*arctan(a*x)^3,x)
```

```
[Out] -19/840*c^3*x^3*a+1/2*a^4*c^3*arctan(a*x)^3*x^6+3/4*a^2*c^3*arctan(a*x)^3*x
^4+1/56*a^4*c^3*arctan(a*x)*x^6+3/35*a^2*c^3*arctan(a*x)*x^4-3/56*a^5*c^3*a
rctan(a*x)^2*x^7-9/40*a^3*c^3*arctan(a*x)^2*x^5-3/8*a*c^3*arctan(a*x)^2*x^3
+3/35*I/a^2*c^3*dilog(1/2*I*(a*x-I))+3/70*I/a^2*c^3*ln(a*x+I)^2-3/35*I/a^2*
c^3*dilog(-1/2*I*(a*x+I))-3/70*I/a^2*c^3*ln(a*x-I)^2+6/35/a^2*c^3*arctan(a*
x)*ln(a^2*x^2+1)+1/8*a^6*c^3*arctan(a*x)^3*x^8-3/35*I/a^2*c^3*ln(a*x-I)*ln(
-1/2*I*(a*x+I))+3/35*I/a^2*c^3*ln(a*x-I)*ln(a^2*x^2+1)-3/35*I/a^2*c^3*ln(a*
x+I)*ln(a^2*x^2+1)+3/35*I/a^2*c^3*ln(a*x+I)*ln(1/2*I*(a*x-I))+1/2*c^3*arcta
n(a*x)^3*x^2+57/280*c^3*arctan(a*x)*x^2+1/8/a^2*c^3*arctan(a*x)^3+19/140/a^
2*c^3*arctan(a*x)-19/140*c^3*x/a-1/280*a^3*c^3*x^5-3/8*c^3*x*arctan(a*x)^2/
a
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^3*arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^6c^3x^7 + 3\,a^4c^3x^5 + 3\,a^2c^3x^3 + c^3x\right)\arctan\left(ax\right)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^3*arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral((a^6*c^3*x^7 + 3*a^4*c^3*x^5 + 3*a^2*c^3*x^3 + c^3*x)*arctan(a*x)^
3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$c^3\left(\int x\,\text{atan}^3\left(ax\right)dx + \int 3a^2x^3\,\text{atan}^3\left(ax\right)dx + \int 3a^4x^5\,\text{atan}^3\left(ax\right)dx + \int a^6x^7\,\text{atan}^3\left(ax\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**3*atan(a*x)**3,x)
```

```
[Out] c**3*(Integral(x*atan(a*x)**3, x) + Integral(3*a**2*x**3*atan(a*x)**3, x) +
 Integral(3*a**4*x**5*atan(a*x)**3, x) + Integral(a**6*x**7*atan(a*x)**3, x
))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a^2cx^2 + c\right)^3 x\arctan\left(ax\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^3*arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^3*x*arctan(a*x)^3, x)
```

3.382 $\int \left(c+a^2cx^2\right)^3 \tan^{-1}(ax)^3\,dx$

Optimal. Leaf size=388

$$\frac{24c^3\text{PolyLog}\left(3,1-\frac{2}{1+iax}\right)}{35a}+\frac{48ic^3\tan^{-1}(ax)\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{35a}-\frac{c^3\left(a^2x^2+1\right)^2}{140a}-\frac{13c^3\left(a^2x^2+1\right)}{210a}-\frac{7c^3\log\left(a^2x\right.}{15a}$$

```
[Out] (-13*c^3*(1 + a^2*x^2))/(210*a) - (c^3*(1 + a^2*x^2)^2)/(140*a) + (14*c^3*x
*ArcTan[a*x])/15 + (13*c^3*x*(1 + a^2*x^2)*ArcTan[a*x])/105 + (c^3*x*(1 + a
^2*x^2)^2*ArcTan[a*x])/35 - (12*c^3*(1 + a^2*x^2)*ArcTan[a*x]^2)/(35*a) - (
9*c^3*(1 + a^2*x^2)^2*ArcTan[a*x]^2)/(70*a) - (c^3*(1 + a^2*x^2)^3*ArcTan[a
*x]^2)/(14*a) + (((16*I)/35)*c^3*ArcTan[a*x]^3)/a + (16*c^3*x*ArcTan[a*x]^3
)/35 + (8*c^3*x*(1 + a^2*x^2)*ArcTan[a*x]^3)/35 + (6*c^3*x*(1 + a^2*x^2)^2*
ArcTan[a*x]^3)/35 + (c^3*x*(1 + a^2*x^2)^3*ArcTan[a*x]^3)/7 + (48*c^3*ArcTa
n[a*x]^2*Log[2/(1 + I*a*x)])/(35*a) - (7*c^3*Log[1 + a^2*x^2])/(15*a) + (((
48*I)/35)*c^3*ArcTan[a*x]*PolyLog[2, 1 - 2/(1 + I*a*x)])/a + (24*c^3*PolyLo
g[3, 1 - 2/(1 + I*a*x)])/(35*a)
```

Rubi [A] time = 0.340402, antiderivative size = 388, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {4880, 4846, 4920, 4854, 4884, 4994, 6610, 260, 4878}

$$\frac{24c^3\text{PolyLog}\left(3,1-\frac{2}{1+iax}\right)}{35a}+\frac{48ic^3\tan^{-1}(ax)\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{35a}-\frac{c^3\left(a^2x^2+1\right)^2}{140a}-\frac{13c^3\left(a^2x^2+1\right)}{210a}-\frac{7c^3\log\left(a^2x\right.}{15a}$$

Antiderivative was successfully verified.

```
[In] Int[(c + a^2*c*x^2)^3*ArcTan[a*x]^3,x]
```


```
[Out] (-13*c^3*(1 + a^2*x^2))/(210*a) - (c^3*(1 + a^2*x^2)^2)/(140*a) + (14*c^3*x
*ArcTan[a*x])/15 + (13*c^3*x*(1 + a^2*x^2)*ArcTan[a*x])/105 + (c^3*x*(1 + a
^2*x^2)^2*ArcTan[a*x])/35 - (12*c^3*(1 + a^2*x^2)*ArcTan[a*x]^2)/(35*a) - (
9*c^3*(1 + a^2*x^2)^2*ArcTan[a*x]^2)/(70*a) - (c^3*(1 + a^2*x^2)^3*ArcTan[a
*x]^2)/(14*a) + (((16*I)/35)*c^3*ArcTan[a*x]^3)/a + (16*c^3*x*ArcTan[a*x]^3
)/35 + (8*c^3*x*(1 + a^2*x^2)*ArcTan[a*x]^3)/35 + (6*c^3*x*(1 + a^2*x^2)^2*
ArcTan[a*x]^3)/35 + (c^3*x*(1 + a^2*x^2)^3*ArcTan[a*x]^3)/7 + (48*c^3*ArcTa
n[a*x]^2*Log[2/(1 + I*a*x)])/(35*a) - (7*c^3*Log[1 + a^2*x^2])/(15*a) + (((
48*I)/35)*c^3*ArcTan[a*x]*PolyLog[2, 1 - 2/(1 + I*a*x)])/a + (24*c^3*PolyLo
g[3, 1 - 2/(1 + I*a*x)])/(35*a)
```

Rule 4880

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> -Simp[(b*p*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1))/(2*c*q*(2*
q + 1)), x] + (Dist[(2*d*q)/(2*q + 1), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTa
n[c*x])^p, x], x] + Dist[(b^2*d*p*(p - 1))/(2*q*(2*q + 1)), Int[(d + e*x^2)
^(q - 1)*(a + b*ArcTan[c*x])^(p - 2), x], x] + Simp[(x*(d + e*x^2)^q*(a + b
*ArcTan[c*x])^p)/(2*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[q, 0] && GtQ[p, 1]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4878

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symbo
l] :> -Simp[(b*(d + e*x^2)^q)/(2*c*q*(2*q + 1)), x] + (Dist[(2*d*q)/(2*q +
1), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x]), x], x] + Simp[(x*(d + e*x^
2)^q*(a + b*ArcTan[c*x]))/(2*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] && Eq
Q[e, c^2*d] && GtQ[q, 0]
```

Rubi steps

$$\int \left(c+a^2cx^2\right)^3\tan^{-1}(ax)^3\,dx = -\frac{c^3\left(1+a^2x^2\right)^3\tan^{-1}(ax)^2}{14a}+\frac{1}{7}c^3x\left(1+a^2x^2\right)^3\tan^{-1}(ax)^3+\frac{1}{7}c\int\left(c+a^2cx^2\right)^2\tan^{-1}($$

$$= -\frac{c^3\left(1+a^2x^2\right)^2}{140a}+\frac{1}{35}c^3x\left(1+a^2x^2\right)^2\tan^{-1}(ax)-\frac{9c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)^2}{70a}-\frac{c^3\left(1+\right.}{}$$

$$= -\frac{13c^3\left(1+a^2x^2\right)}{210a}-\frac{c^3\left(1+a^2x^2\right)^2}{140a}+\frac{13}{105}c^3x\left(1+a^2x^2\right)\tan^{-1}(ax)+\frac{1}{35}c^3x\left(1+a^2x^2\right)^2$$

$$= -\frac{13c^3\left(1+a^2x^2\right)}{210a}-\frac{c^3\left(1+a^2x^2\right)^2}{140a}+\frac{14}{15}c^3x\tan^{-1}(ax)+\frac{13}{105}c^3x\left(1+a^2x^2\right)\tan^{-1}(ax)+$$

$$= -\frac{13c^3\left(1+a^2x^2\right)}{210a}-\frac{c^3\left(1+a^2x^2\right)^2}{140a}+\frac{14}{15}c^3x\tan^{-1}(ax)+\frac{13}{105}c^3x\left(1+a^2x^2\right)\tan^{-1}(ax)+$$

$$= -\frac{13c^3\left(1+a^2x^2\right)}{210a}-\frac{c^3\left(1+a^2x^2\right)^2}{140a}+\frac{14}{15}c^3x\tan^{-1}(ax)+\frac{13}{105}c^3x\left(1+a^2x^2\right)\tan^{-1}(ax)+$$

$$= -\frac{13c^3\left(1+a^2x^2\right)}{210a}-\frac{c^3\left(1+a^2x^2\right)^2}{140a}+\frac{14}{15}c^3x\tan^{-1}(ax)+\frac{13}{105}c^3x\left(1+a^2x^2\right)\tan^{-1}(ax)+$$

$$= -\frac{13c^3\left(1+a^2x^2\right)}{210a}-\frac{c^3\left(1+a^2x^2\right)^2}{140a}+\frac{14}{15}c^3x\tan^{-1}(ax)+\frac{13}{105}c^3x\left(1+a^2x^2\right)\tan^{-1}(ax)+$$

Mathematica [A] time = 1.12658, size = 243, normalized size = 0.63

$$\frac{c^3 \left(-576 i \tan^{-1}(ax)\text{PolyLog}\left(2,-e^{2 i \tan^{-1}(ax)}\right)+288\text{PolyLog}\left(3,-e^{2 i \tan^{-1}(ax)}\right)-3 a^4 x^4-32 a^2 x^2-196 \log \left(a^2 x^2+1\right)+6\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + a^2*c*x^2)^3*ArcTan[a*x]^3,x]
```

```
[Out] (c^3*(-29 - 32*a^2*x^2 - 3*a^4*x^4 + 456*a*x*ArcTan[a*x] + 76*a^3*x^3*ArcTa
n[a*x] + 12*a^5*x^5*ArcTan[a*x] - 228*ArcTan[a*x]^2 - 342*a^2*x^2*ArcTan[a*
x]^2 - 144*a^4*x^4*ArcTan[a*x]^2 - 30*a^6*x^6*ArcTan[a*x]^2 - (192*I)*ArcTa
n[a*x]^3 + 420*a*x*ArcTan[a*x]^3 + 420*a^3*x^3*ArcTan[a*x]^3 + 252*a^5*x^5*
ArcTan[a*x]^3 + 60*a^7*x^7*ArcTan[a*x]^3 + 576*ArcTan[a*x]^2*Log[1 + E^((2*
I)*ArcTan[a*x])] - 196*Log[1 + a^2*x^2] - (576*I)*ArcTan[a*x]*PolyLog[2, -E
^((2*I)*ArcTan[a*x])] + 288*PolyLog[3, -E^((2*I)*ArcTan[a*x])]))/(420*a)
```

Maple [C] time = 2.551, size = 1134, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^3*arctan(a*x)^3,x)
```

```
[Out] -12/35*I/a*c^3*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1
)^2)^3*arctan(a*x)^2+12/35*I/a*c^3*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)
^3*arctan(a*x)^2-12/35*I/a*c^3*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^3*arctan(
a*x)^2+c^3*x*arctan(a*x)^3+38/35*c^3*x*arctan(a*x)-29/420/a*c^3-12/35*I/a*c
^3*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))
*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*arctan(a*x)^
2-1/140*a^3*x^4*c^3-8/105*a*c^3*x^2-19/35/a*c^3*arctan(a*x)^2+14/15/a*c^3*l
n((1+I*a*x)^2/(a^2*x^2+1)+1)+24/35/a*c^3*polylog(3,-(1+I*a*x)^2/(a^2*x^2+1)
)+19/105*a^2*c^3*arctan(a*x)*x^3-57/70*a*c^3*arctan(a*x)^2*x^2+1/7*a^6*c^3*
arctan(a*x)^3*x^7+3/5*a^4*c^3*arctan(a*x)^3*x^5+a^2*c^3*arctan(a*x)^3*x^3-2
4/35/a*c^3*arctan(a*x)^2*ln(a^2*x^2+1)+48/35/a*c^3*arctan(a*x)^2*ln((1+I*a*
x)/(a^2*x^2+1)^(1/2))+48/35/a*c^3*arctan(a*x)^2*ln(2)-1/14*a^5*c^3*arctan(a
*x)^2*x^6-12/35*a^3*c^3*arctan(a*x)^2*x^4+1/35*a^4*c^3*arctan(a*x)*x^5-14/1
5*I/a*c^3*arctan(a*x)-16/35*I/a*c^3*arctan(a*x)^3+12/35*I/a*c^3*Pi*csgn(I*(
(1+I*a*x)^2/(a^2*x^2+1)+1))^2*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*arctan(
a*x)^2+24/35*I/a*c^3*Pi*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*csgn(I*(1+I*a*x
```

```
)^2/(a^2*x^2+1))^2*arctan(a*x)^2+12/35*I/a*c^3*Pi*csgn(I/((1+I*a*x)^2/(a^2*
x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^
2*arctan(a*x)^2-12/35*I/a*c^3*Pi*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))^2*csgn
(I*(1+I*a*x)^2/(a^2*x^2+1))*arctan(a*x)^2+12/35*I/a*c^3*Pi*csgn(I*(1+I*a*x)
^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^
2)^2*arctan(a*x)^2-24/35*I/a*c^3*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))*csg
n(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2*arctan(a*x)^2-48/35*I/a*c^3*arctan(a*x
)*polylog(2,-(1+I*a*x)^2/(a^2*x^2+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] 980*a^8*c^3*integrate(1/1120*x^8*arctan(a*x)^3/(a^2*x^2 + 1), x) + 105*a^8*
c^3*integrate(1/1120*x^8*arctan(a*x)*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x) +
 60*a^8*c^3*integrate(1/1120*x^8*arctan(a*x)*log(a^2*x^2 + 1)/(a^2*x^2 + 1)
, x) - 60*a^7*c^3*integrate(1/1120*x^7*arctan(a*x)^2/(a^2*x^2 + 1), x) + 15
*a^7*c^3*integrate(1/1120*x^7*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x) + 3920*a
^6*c^3*integrate(1/1120*x^6*arctan(a*x)^3/(a^2*x^2 + 1), x) + 420*a^6*c^3*i
ntegrate(1/1120*x^6*arctan(a*x)*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x) + 252*
a^6*c^3*integrate(1/1120*x^6*arctan(a*x)*log(a^2*x^2 + 1)/(a^2*x^2 + 1), x)
 - 252*a^5*c^3*integrate(1/1120*x^5*arctan(a*x)^2/(a^2*x^2 + 1), x) + 63*a^
5*c^3*integrate(1/1120*x^5*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x) + 5880*a^4*
c^3*integrate(1/1120*x^4*arctan(a*x)^3/(a^2*x^2 + 1), x) + 630*a^4*c^3*inte
grate(1/1120*x^4*arctan(a*x)*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x) + 420*a^4
*c^3*integrate(1/1120*x^4*arctan(a*x)*log(a^2*x^2 + 1)/(a^2*x^2 + 1), x) -
420*a^3*c^3*integrate(1/1120*x^3*arctan(a*x)^2/(a^2*x^2 + 1), x) + 105*a^3*
c^3*integrate(1/1120*x^3*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x) + 7/32*c^3*ar
ctan(a*x)^4/a + 3920*a^2*c^3*integrate(1/1120*x^2*arctan(a*x)^3/(a^2*x^2 +
1), x) + 420*a^2*c^3*integrate(1/1120*x^2*arctan(a*x)*log(a^2*x^2 + 1)^2/(a
^2*x^2 + 1), x) + 420*a^2*c^3*integrate(1/1120*x^2*arctan(a*x)*log(a^2*x^2
+ 1)/(a^2*x^2 + 1), x) - 420*a*c^3*integrate(1/1120*x*arctan(a*x)^2/(a^2*x^
2 + 1), x) + 105*a*c^3*integrate(1/1120*x*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1),
 x) + 1/280*(5*a^6*c^3*x^7 + 21*a^4*c^3*x^5 + 35*a^2*c^3*x^3 + 35*c^3*x)*ar
ctan(a*x)^3 + 105*c^3*integrate(1/1120*arctan(a*x)*log(a^2*x^2 + 1)^2/(a^2*
x^2 + 1), x) - 3/1120*(5*a^6*c^3*x^7 + 21*a^4*c^3*x^5 + 35*a^2*c^3*x^3 + 35
*c^3*x)*arctan(a*x)*log(a^2*x^2 + 1)^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^6c^3x^6+3\,a^4c^3x^4+3\,a^2c^3x^2+c^3\right)\arctan\left(ax\right)^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*arctan(a*x)^3,
 x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$c^3\left(\int 3a^2x^2\operatorname{atan}^3\left(ax\right)dx+\int 3a^4x^4\operatorname{atan}^3\left(ax\right)dx+\int a^6x^6\operatorname{atan}^3\left(ax\right)dx+\int\operatorname{atan}^3\left(ax\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**3*atan(a*x)**3,x)
```

```
[Out] c**3*(Integral(3*a**2*x**2*atan(a*x)**3, x) + Integral(3*a**4*x**4*atan(a*x
)**3, x) + Integral(a**6*x**6*atan(a*x)**3, x) + Integral(atan(a*x)**3, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a^2cx^2+c\right)^3\arctan\left(ax\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^3*arctan(a*x)^3, x)
```

3.383 $\int \frac{\left(c+a^2cx^2\right)^3 \tan^{-1}(ax)^3}{x} dx$

Optimal. Leaf size=447

$$-\frac{34}{15}ic^3\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)+\frac{3}{4}ic^3\text{PolyLog}\left(4,1-\frac{2}{1+iax}\right)-\frac{3}{4}ic^3\text{PolyLog}\left(4,-1+\frac{2}{1+iax}\right)-\frac{3}{2}ic^3\tan^{-1}(ax)^2\text{F}$$

```
[Out] (-13*a*c^3*x)/30 - (a^3*c^3*x^3)/60 + (13*c^3*ArcTan[a*x])/30 + (29*a^2*c^3
*x^2*ArcTan[a*x])/60 + (a^4*c^3*x^4*ArcTan[a*x])/20 - ((34*I)/15)*c^3*ArcTa
n[a*x]^2 - (11*a*c^3*x*ArcTan[a*x]^2)/4 - (7*a^3*c^3*x^3*ArcTan[a*x]^2)/12
- (a^5*c^3*x^5*ArcTan[a*x]^2)/10 + (11*c^3*ArcTan[a*x]^3)/12 + (3*a^2*c^3*x
^2*ArcTan[a*x]^3)/2 + (3*a^4*c^3*x^4*ArcTan[a*x]^3)/4 + (a^6*c^3*x^6*ArcTan
[a*x]^3)/6 + 2*c^3*ArcTan[a*x]^3*ArcTanh[1 - 2/(1 + I*a*x)] - (68*c^3*ArcTa
n[a*x]*Log[2/(1 + I*a*x)])/15 - ((34*I)/15)*c^3*PolyLog[2, 1 - 2/(1 + I*a*x
)] - ((3*I)/2)*c^3*ArcTan[a*x]^2*PolyLog[2, 1 - 2/(1 + I*a*x)] + ((3*I)/2)*
c^3*ArcTan[a*x]^2*PolyLog[2, -1 + 2/(1 + I*a*x)] - (3*c^3*ArcTan[a*x]*PolyL
og[3, 1 - 2/(1 + I*a*x)])/2 + (3*c^3*ArcTan[a*x]*PolyLog[3, -1 + 2/(1 + I*a
*x)])/2 + ((3*I)/4)*c^3*PolyLog[4, 1 - 2/(1 + I*a*x)] - ((3*I)/4)*c^3*PolyL
og[4, -1 + 2/(1 + I*a*x)]
```

Rubi [A] time = 1.65542, antiderivative size = 447, normalized size of antiderivative = 1., number of steps used = 69, number of rules used = 17, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.773, Rules used = {4948, 4850, 4988, 4884, 4994, 4998, 6610, 4852, 4916, 4846, 4920, 4854, 2402, 2315, 321, 203, 302}

$$-\frac{34}{15}ic^3\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)+\frac{3}{4}ic^3\text{PolyLog}\left(4,1-\frac{2}{1+iax}\right)-\frac{3}{4}ic^3\text{PolyLog}\left(4,-1+\frac{2}{1+iax}\right)-\frac{3}{2}ic^3\tan^{-1}(ax)^2\text{F}$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)^3*ArcTan[a*x]^3)/x,x]
```


```
[Out] (-13*a*c^3*x)/30 - (a^3*c^3*x^3)/60 + (13*c^3*ArcTan[a*x])/30 + (29*a^2*c^3
*x^2*ArcTan[a*x])/60 + (a^4*c^3*x^4*ArcTan[a*x])/20 - ((34*I)/15)*c^3*ArcTa
n[a*x]^2 - (11*a*c^3*x*ArcTan[a*x]^2)/4 - (7*a^3*c^3*x^3*ArcTan[a*x]^2)/12
- (a^5*c^3*x^5*ArcTan[a*x]^2)/10 + (11*c^3*ArcTan[a*x]^3)/12 + (3*a^2*c^3*x
^2*ArcTan[a*x]^3)/2 + (3*a^4*c^3*x^4*ArcTan[a*x]^3)/4 + (a^6*c^3*x^6*ArcTan
[a*x]^3)/6 + 2*c^3*ArcTan[a*x]^3*ArcTanh[1 - 2/(1 + I*a*x)] - (68*c^3*ArcTa
n[a*x]*Log[2/(1 + I*a*x)])/15 - ((34*I)/15)*c^3*PolyLog[2, 1 - 2/(1 + I*a*x
)] - ((3*I)/2)*c^3*ArcTan[a*x]^2*PolyLog[2, 1 - 2/(1 + I*a*x)] + ((3*I)/2)*
c^3*ArcTan[a*x]^2*PolyLog[2, -1 + 2/(1 + I*a*x)] - (3*c^3*ArcTan[a*x]*PolyL
```

```
og[3, 1 - 2/(1 + I*a*x)])/2 + (3*c^3*ArcTan[a*x]*PolyLog[3, -1 + 2/(1 + I*a
*x)])/2 + ((3*I)/4)*c^3*PolyLog[4, 1 - 2/(1 + I*a*x)] - ((3*I)/4)*c^3*PolyL
og[4, -1 + 2/(1 + I*a*x)]
```

Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

Rule 4850

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTan[c*x])^(p - 1)*ArcTanh[1 - 2/(1 + I*c*x)])/(1 + c^2*x^2), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4988

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTan[c*x])^p)/(d + e
*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTan[c*x])^p)/(d + e*x^2
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 4998

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[k + 1, u])/(2
*c*d), x] - Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[k + 1,
```

```
 u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
qQ[e, c^2*d] && EqQ[u^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\int \frac{\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^3}{x}\,dx = \int\left(\frac{c^3\tan^{-1}(ax)^3}{x}+3a^2c^3x\tan^{-1}(ax)^3+3a^4c^3x^3\tan^{-1}(ax)^3+a^6c^3x^5\tan^{-1}(ax)^3\right)dx$$

$$= c^3\int\frac{\tan^{-1}(ax)^3}{x}\,dx+\left(3a^2c^3\right)\int x\tan^{-1}(ax)^3\,dx+\left(3a^4c^3\right)\int x^3\tan^{-1}(ax)^3\,dx+\left(a^6c\right.$$

$$= \frac{3}{2}a^2c^3x^2\tan^{-1}(ax)^3+\frac{3}{4}a^4c^3x^4\tan^{-1}(ax)^3+\frac{1}{6}a^6c^3x^6\tan^{-1}(ax)^3+2c^3\tan^{-1}(ax)^3\tan$$

$$= \frac{3}{2}a^2c^3x^2\tan^{-1}(ax)^3+\frac{3}{4}a^4c^3x^4\tan^{-1}(ax)^3+\frac{1}{6}a^6c^3x^6\tan^{-1}(ax)^3+2c^3\tan^{-1}(ax)^3\tan$$

$$= -\frac{9}{2}ac^3x\tan^{-1}(ax)^2-\frac{3}{4}a^3c^3x^3\tan^{-1}(ax)^2-\frac{1}{10}a^5c^3x^5\tan^{-1}(ax)^2+\frac{3}{2}c^3\tan^{-1}(ax)^3+$$

$$= -\frac{9}{2}ic^3\tan^{-1}(ax)^2-\frac{9}{4}ac^3x\tan^{-1}(ax)^2-\frac{7}{12}a^3c^3x^3\tan^{-1}(ax)^2-\frac{1}{10}a^5c^3x^5\tan^{-1}(ax)^2+$$

$$= \frac{3}{4}a^2c^3x^2\tan^{-1}(ax)+\frac{1}{20}a^4c^3x^4\tan^{-1}(ax)-\frac{3}{2}ic^3\tan^{-1}(ax)^2-\frac{11}{4}ac^3x\tan^{-1}(ax)^2-\frac{7}{12}$$

$$= -\frac{3}{4}ac^3x+\frac{29}{60}a^2c^3x^2\tan^{-1}(ax)+\frac{1}{20}a^4c^3x^4\tan^{-1}(ax)-\frac{34}{15}ic^3\tan^{-1}(ax)^2-\frac{11}{4}ac^3x\tan$$

$$= -\frac{13}{30}ac^3x-\frac{1}{60}a^3c^3x^3+\frac{3}{4}c^3\tan^{-1}(ax)+\frac{29}{60}a^2c^3x^2\tan^{-1}(ax)+\frac{1}{20}a^4c^3x^4\tan^{-1}(ax)-$$

$$= -\frac{13}{30}ac^3x-\frac{1}{60}a^3c^3x^3+\frac{13}{30}c^3\tan^{-1}(ax)+\frac{29}{60}a^2c^3x^2\tan^{-1}(ax)+\frac{1}{20}a^4c^3x^4\tan^{-1}(ax)-$$

$$= -\frac{13}{30}ac^3x-\frac{1}{60}a^3c^3x^3+\frac{13}{30}c^3\tan^{-1}(ax)+\frac{29}{60}a^2c^3x^2\tan^{-1}(ax)+\frac{1}{20}a^4c^3x^4\tan^{-1}(ax)-$$

Mathematica [A] time = 1.00972, size = 350, normalized size = 0.78

$$\frac{1}{960}c^3\left(1440i\tan^{-1}(ax)^2\text{PolyLog}\left(2,e^{-2i\tan^{-1}(ax)}\right)+32i\left(45\tan^{-1}(ax)^2+68\right)\text{PolyLog}\left(2,-e^{2i\tan^{-1}(ax)}\right)+1440\tan^{-1}(\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((c + a^2*c*x^2)^3*ArcTan[a*x]^3)/x,x]
```

```
[Out] (c^3*((-15*I)*Pi^4 - 416*a*x - 16*a^3*x^3 + 416*ArcTan[a*x] + 464*a^2*x^2*A
rcTan[a*x] + 48*a^4*x^4*ArcTan[a*x] + (2176*I)*ArcTan[a*x]^2 - 2640*a*x*Arc
```

```
Tan[a*x]^2 - 560*a^3*x^3*ArcTan[a*x]^2 - 96*a^5*x^5*ArcTan[a*x]^2 + 880*Arc
Tan[a*x]^3 + 1440*a^2*x^2*ArcTan[a*x]^3 + 720*a^4*x^4*ArcTan[a*x]^3 + 160*a
^6*x^6*ArcTan[a*x]^3 + (480*I)*ArcTan[a*x]^4 + 960*ArcTan[a*x]^3*Log[1 - E^
((-2*I)*ArcTan[a*x])] - 4352*ArcTan[a*x]*Log[1 + E^((2*I)*ArcTan[a*x])] - 9
60*ArcTan[a*x]^3*Log[1 + E^((2*I)*ArcTan[a*x])] + (1440*I)*ArcTan[a*x]^2*Po
lyLog[2, E^((-2*I)*ArcTan[a*x])] + (32*I)*(68 + 45*ArcTan[a*x]^2)*PolyLog[2
, -E^((2*I)*ArcTan[a*x])] + 1440*ArcTan[a*x]*PolyLog[3, E^((-2*I)*ArcTan[a*
x])] - 1440*ArcTan[a*x]*PolyLog[3, -E^((2*I)*ArcTan[a*x])] - (720*I)*PolyLo
g[4, E^((-2*I)*ArcTan[a*x])] - (720*I)*PolyLog[4, -E^((2*I)*ArcTan[a*x])]))
/960
```

Maple [A] time = 1.26, size = 664, normalized size = 1.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^3*arctan(a*x)^3/x,x)
```

```
[Out] 1/60*c^3*(-3*I*arctan(a*x)*a^2*x^2+55*arctan(a*x)^3*a*x+29*I*arctan(a*x)^2*
a*x+35*arctan(a*x)^3*a^3*x^3-35*I*arctan(a*x)^3*a^2*x^2+10*arctan(a*x)^3*a^
5*x^5-136*arctan(a*x)^2+I*a*x-29*arctan(a*x)^2*x^2*a^2-55*I*arctan(a*x)^3-6
*arctan(a*x)^2*x^4*a^4+6*I*arctan(a*x)^2*a^3*x^3+26*arctan(a*x)*x*a-10*I*ar
ctan(a*x)^3*a^4*x^4+3*arctan(a*x)*x^3*a^3-25-26*I*arctan(a*x)-a^2*x^2)*(a*x
+I)+6*I*c^3*polylog(4,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-68/15*c^3*arctan(a*x)*l
n((1+I*a*x)^2/(a^2*x^2+1)+1)+3/2*I*c^3*arctan(a*x)^2*polylog(2,-(1+I*a*x)^2
/(a^2*x^2+1))+c^3*arctan(a*x)^3*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-3*I*c^3*a
rctan(a*x)^2*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*c^3*arctan(a*x)*poly
log(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I*c^3*polylog(4,(1+I*a*x)/(a^2*x^2+1)^
(1/2))+c^3*arctan(a*x)^3*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))+34/15*I*c^3*poly
log(2,-(1+I*a*x)^2/(a^2*x^2+1))+6*c^3*arctan(a*x)*polylog(3,-(1+I*a*x)/(a^2
*x^2+1)^(1/2))+68/15*I*c^3*arctan(a*x)^2-c^3*arctan(a*x)^3*ln((1+I*a*x)^2/(
a^2*x^2+1)+1)-3/4*I*c^3*polylog(4,-(1+I*a*x)^2/(a^2*x^2+1))-3/2*c^3*arctan(
a*x)*polylog(3,-(1+I*a*x)^2/(a^2*x^2+1))-3*I*c^3*arctan(a*x)^2*polylog(2,(1
+I*a*x)/(a^2*x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{96}\left(2\,a^6c^3x^6+9\,a^4c^3x^4+18\,a^2c^3x^2\right)\arctan\left(ax\right)^3-\frac{1}{128}\left(2\,a^6c^3x^6+9\,a^4c^3x^4+18\,a^2c^3x^2\right)\arctan\left(ax\right)\log\left(a^2x^2+1\right)^2+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^3/x,x, algorithm="maxima")
```

```
[Out] 1/96*(2*a^6*c^3*x^6 + 9*a^4*c^3*x^4 + 18*a^2*c^3*x^2)*arctan(a*x)^3 - 1/128
*(2*a^6*c^3*x^6 + 9*a^4*c^3*x^4 + 18*a^2*c^3*x^2)*arctan(a*x)*log(a^2*x^2 +
 1)^2 + integrate(1/128*(112*(a^8*c^3*x^8 + 4*a^6*c^3*x^6 + 6*a^4*c^3*x^4 +
 4*a^2*c^3*x^2 + c^3)*arctan(a*x)^3 - 4*(2*a^7*c^3*x^7 + 9*a^5*c^3*x^5 + 18
*a^3*c^3*x^3)*arctan(a*x)^2 + 4*(2*a^8*c^3*x^8 + 9*a^6*c^3*x^6 + 18*a^4*c^3
*x^4)*arctan(a*x)*log(a^2*x^2 + 1) + (2*a^7*c^3*x^7 + 9*a^5*c^3*x^5 + 18*a^
3*c^3*x^3 + 12*(a^8*c^3*x^8 + 4*a^6*c^3*x^6 + 6*a^4*c^3*x^4 + 4*a^2*c^3*x^2
 + c^3)*arctan(a*x))*log(a^2*x^2 + 1)^2)/(a^2*x^3 + x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^{6}c^{3}x^{6}+3\,a^{4}c^{3}x^{4}+3\,a^{2}c^{3}x^{2}+c^{3}\right)\arctan\left(ax\right)^{3}}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^3/x,x, algorithm="fricas")
```

```
[Out] integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*arctan(a*x)^3/
x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$c^{3}\left(\int\frac{\operatorname{atan}^{3}\left(ax\right)}{x}\,dx+\int 3a^{2}x\operatorname{atan}^{3}\left(ax\right)dx+\int 3a^{4}x^{3}\operatorname{atan}^{3}\left(ax\right)dx+\int a^{6}x^{5}\operatorname{atan}^{3}\left(ax\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**3*atan(a*x)**3/x,x)
```

```
[Out] c**3*(Integral(atan(a*x)**3/x, x) + Integral(3*a**2*x*atan(a*x)**3, x) + In
tegral(3*a**4*x**3*atan(a*x)**3, x) + Integral(a**6*x**5*atan(a*x)**3, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^3 \arctan(ax)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^3/x,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^3*arctan(a*x)^3/x, x)
```

3.384 $\int \frac{(c+a^2cx^2)^3 \tan^{-1}(ax)^3}{x^2} dx$

Optimal. Leaf size=354

$$\frac{3}{2}ac^3\text{PolyLog}\left(3,-1+\frac{2}{1-iax}\right)+\frac{33}{10}ac^3\text{PolyLog}\left(3,1-\frac{2}{1+iax}\right)-3iac^3\tan^{-1}(ax)\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)+\frac{33}{5}ia$$

```
[Out] -(a^3*c^3*x^2)/20 + (21*a^2*c^3*x*ArcTan[a*x])/10 + (a^4*c^3*x^3*ArcTan[a*x
])/10 - (21*a*c^3*ArcTan[a*x]^2)/20 - (6*a^3*c^3*x^2*ArcTan[a*x]^2)/5 - (3*
a^5*c^3*x^4*ArcTan[a*x]^2)/20 + ((6*I)/5)*a*c^3*ArcTan[a*x]^3 - (c^3*ArcTan
[a*x]^3)/x + 3*a^2*c^3*x*ArcTan[a*x]^3 + a^4*c^3*x^3*ArcTan[a*x]^3 + (a^6*c
^3*x^5*ArcTan[a*x]^3)/5 + (33*a*c^3*ArcTan[a*x]^2*Log[2/(1 + I*a*x)])/5 - a
*c^3*Log[1 + a^2*x^2] + 3*a*c^3*ArcTan[a*x]^2*Log[2 - 2/(1 - I*a*x)] - (3*I
)*a*c^3*ArcTan[a*x]*PolyLog[2, -1 + 2/(1 - I*a*x)] + ((33*I)/5)*a*c^3*ArcTa
n[a*x]*PolyLog[2, 1 - 2/(1 + I*a*x)] + (3*a*c^3*PolyLog[3, -1 + 2/(1 - I*a*
x)])/2 + (33*a*c^3*PolyLog[3, 1 - 2/(1 + I*a*x)])/10
```

Rubi [A] time = 1.28021, antiderivative size = 354, normalized size of antiderivative = 1., number of steps used = 45, number of rules used = 15, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.682, Rules used = {4948, 4846, 4920, 4854, 4884, 4994, 6610, 4852, 4924, 4868, 4992, 4916, 260, 266, 43}

$$\frac{3}{2}ac^3\text{PolyLog}\left(3,-1+\frac{2}{1-iax}\right)+\frac{33}{10}ac^3\text{PolyLog}\left(3,1-\frac{2}{1+iax}\right)-3iac^3\tan^{-1}(ax)\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)+\frac{33}{5}ia$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)^3*ArcTan[a*x]^3)/x^2,x]
```


```
[Out] -(a^3*c^3*x^2)/20 + (21*a^2*c^3*x*ArcTan[a*x])/10 + (a^4*c^3*x^3*ArcTan[a*x
])/10 - (21*a*c^3*ArcTan[a*x]^2)/20 - (6*a^3*c^3*x^2*ArcTan[a*x]^2)/5 - (3*
a^5*c^3*x^4*ArcTan[a*x]^2)/20 + ((6*I)/5)*a*c^3*ArcTan[a*x]^3 - (c^3*ArcTan
[a*x]^3)/x + 3*a^2*c^3*x*ArcTan[a*x]^3 + a^4*c^3*x^3*ArcTan[a*x]^3 + (a^6*c
^3*x^5*ArcTan[a*x]^3)/5 + (33*a*c^3*ArcTan[a*x]^2*Log[2/(1 + I*a*x)])/5 - a
*c^3*Log[1 + a^2*x^2] + 3*a*c^3*ArcTan[a*x]^2*Log[2 - 2/(1 - I*a*x)] - (3*I
)*a*c^3*ArcTan[a*x]*PolyLog[2, -1 + 2/(1 - I*a*x)] + ((33*I)/5)*a*c^3*ArcTa
n[a*x]*PolyLog[2, 1 - 2/(1 + I*a*x)] + (3*a*c^3*PolyLog[3, -1 + 2/(1 - I*a*
x)])/2 + (33*a*c^3*PolyLog[3, 1 - 2/(1 + I*a*x)])/10
```


Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 4992

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d
] && EqQ[(1 - u)^2 - (1 - (2*I)/(I + c*x))^2, 0]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^3}{x^2}\,dx = \int \left(3a^2c^3\tan^{-1}(ax)^3+\frac{c^3\tan^{-1}(ax)^3}{x^2}+3a^4c^3x^2\tan^{-1}(ax)^3+a^6c^3x^4\tan^{-1}(ax)^3\right)dx$$

$$= c^3\int \frac{\tan^{-1}(ax)^3}{x^2}\,dx+\left(3a^2c^3\right)\int \tan^{-1}(ax)^3\,dx+\left(3a^4c^3\right)\int x^2\tan^{-1}(ax)^3\,dx+\left(a^6c^3\right)$$

$$= -\frac{c^3\tan^{-1}(ax)^3}{x}+3a^2c^3x\tan^{-1}(ax)^3+a^4c^3x^3\tan^{-1}(ax)^3+\frac{1}{5}a^6c^3x^5\tan^{-1}(ax)^3+\left(3ac^3\right.$$

$$= 2iac^3\tan^{-1}(ax)^3-\frac{c^3\tan^{-1}(ax)^3}{x}+3a^2c^3x\tan^{-1}(ax)^3+a^4c^3x^3\tan^{-1}(ax)^3+\frac{1}{5}a^6c^3x^5$$

$$= -\frac{3}{2}a^3c^3x^2\tan^{-1}(ax)^2-\frac{3}{20}a^5c^3x^4\tan^{-1}(ax)^2+iac^3\tan^{-1}(ax)^3-\frac{c^3\tan^{-1}(ax)^3}{x}+3a^2c^3$$

$$= -\frac{6}{5}a^3c^3x^2\tan^{-1}(ax)^2-\frac{3}{20}a^5c^3x^4\tan^{-1}(ax)^2+\frac{6}{5}iac^3\tan^{-1}(ax)^3-\frac{c^3\tan^{-1}(ax)^3}{x}+3a^2$$

$$= 3a^2c^3x\tan^{-1}(ax)+\frac{1}{10}a^4c^3x^3\tan^{-1}(ax)-\frac{3}{2}ac^3\tan^{-1}(ax)^2-\frac{6}{5}a^3c^3x^2\tan^{-1}(ax)^2-\frac{3}{20}a$$

$$= \frac{21}{10}a^2c^3x\tan^{-1}(ax)+\frac{1}{10}a^4c^3x^3\tan^{-1}(ax)-\frac{21}{20}ac^3\tan^{-1}(ax)^2-\frac{6}{5}a^3c^3x^2\tan^{-1}(ax)^2-$$

$$= \frac{21}{10}a^2c^3x\tan^{-1}(ax)+\frac{1}{10}a^4c^3x^3\tan^{-1}(ax)-\frac{21}{20}ac^3\tan^{-1}(ax)^2-\frac{6}{5}a^3c^3x^2\tan^{-1}(ax)^2-$$

$$= -\frac{1}{20}a^3c^3x^2+\frac{21}{10}a^2c^3x\tan^{-1}(ax)+\frac{1}{10}a^4c^3x^3\tan^{-1}(ax)-\frac{21}{20}ac^3\tan^{-1}(ax)^2-\frac{6}{5}a^3c^3x^2$$

Mathematica [A] time = 0.72241, size = 298, normalized size = 0.84

$$\frac{c^3\left(120iax\tan^{-1}(ax)\text{PolyLog}\left(2,e^{-2i\tan^{-1}(ax)}\right)-264iax\tan^{-1}(ax)\text{PolyLog}\left(2,-e^{2i\tan^{-1}(ax)}\right)+60ax\text{PolyLog}\left(3,e^{-2i\tan\ldots}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((c + a^2*c*x^2)^3*ArcTan[a*x]^3)/x^2,x]
```

```
[Out] (c^3*(-2*a*x - (5*I)*a*Pi^3*x - 2*a^3*x^3 + 84*a^2*x^2*ArcTan[a*x] + 4*a^4*
x^4*ArcTan[a*x] - 42*a*x*ArcTan[a*x]^2 - 48*a^3*x^3*ArcTan[a*x]^2 - 6*a^5*x
^5*ArcTan[a*x]^2 - 40*ArcTan[a*x]^3 - (48*I)*a*x*ArcTan[a*x]^3 + 120*a^2*x^
2*ArcTan[a*x]^3 + 40*a^4*x^4*ArcTan[a*x]^3 + 8*a^6*x^6*ArcTan[a*x]^3 + 120*
a*x*ArcTan[a*x]^2*Log[1 - E^((-2*I)*ArcTan[a*x])] + 264*a*x*ArcTan[a*x]^2*L
og[1 + E^((2*I)*ArcTan[a*x])] - 40*a*x*Log[1 + a^2*x^2] + (120*I)*a*x*ArcTa
n[a*x]*PolyLog[2, E^((-2*I)*ArcTan[a*x])] - (264*I)*a*x*ArcTan[a*x]*PolyLog
[2, -E^((2*I)*ArcTan[a*x])] + 60*a*x*PolyLog[3, E^((-2*I)*ArcTan[a*x])] + 1
32*a*x*PolyLog[3, -E^((2*I)*ArcTan[a*x])]))/(40*x)
```

Maple [C] time = 10.748, size = 10139, normalized size = 28.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^3*arctan(a*x)^3/x^2,x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^3/x^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^{6} c^{3} x^{6} + 3 \, a^{4} c^{3} x^{4} + 3 \, a^{2} c^{3} x^{2} + c^{3}\right) \arctan\left(a x\right)^{3}}{x^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^3/x^2,x, algorithm="fricas")
```

```
[Out] integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*arctan(a*x)^3/
x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$c^{3} \left(\int 3 a^{2} \operatorname{atan}^{3}{\left(a x \right)}\, dx + \int \frac{\operatorname{atan}^{3}{\left(a x \right)}}{x^{2}}\, dx + \int 3 a^{4} x^{2} \operatorname{atan}^{3}{\left(a x \right)}\, dx + \int a^{6} x^{4} \operatorname{atan}^{3}{\left(a x \right)}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**3*atan(a*x)**3/x**2,x)
```

```
[Out] c**3*(Integral(3*a**2*atan(a*x)**3, x) + Integral(atan(a*x)**3/x**2, x) + I
ntegral(3*a**4*x**2*atan(a*x)**3, x) + Integral(a**6*x**4*atan(a*x)**3, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{{\left(a^{2} c x^{2} + c\right)}^{3} \arctan\left(a x\right)^{3}}{x^{2}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^3/x^2,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^3*arctan(a*x)^3/x^2, x)
```

3.385 $\int \frac{(c+a^2cx^2)^3\tan^{-1}(ax)^3}{x^3}\,dx$

Optimal. Leaf size=503

$$-\frac{3}{2}ia^2c^3\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)-\frac{7}{2}ia^2c^3\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)+\frac{9}{4}ia^2c^3\text{PolyLog}\left(4,1-\frac{2}{1+iax}\right)-\frac{9}{4}ia^2c^3\text{Poly}$$

```
[Out] -(a^3*c^3*x)/4 + (a^2*c^3*ArcTan[a*x])/4 + (a^4*c^3*x^2*ArcTan[a*x])/4 - (5
*I)*a^2*c^3*ArcTan[a*x]^2 - (3*a*c^3*ArcTan[a*x]^2)/(2*x) - (15*a^3*c^3*x*A
rcTan[a*x]^2)/4 - (a^5*c^3*x^3*ArcTan[a*x]^2)/4 + (3*a^2*c^3*ArcTan[a*x]^3)
/4 - (c^3*ArcTan[a*x]^3)/(2*x^2) + (3*a^4*c^3*x^2*ArcTan[a*x]^3)/2 + (a^6*c
^3*x^4*ArcTan[a*x]^3)/4 + 6*a^2*c^3*ArcTan[a*x]^3*ArcTanh[1 - 2/(1 + I*a*x)
] - 7*a^2*c^3*ArcTan[a*x]*Log[2/(1 + I*a*x)] + 3*a^2*c^3*ArcTan[a*x]*Log[2
- 2/(1 - I*a*x)] - ((3*I)/2)*a^2*c^3*PolyLog[2, -1 + 2/(1 - I*a*x)] - ((7*I
)/2)*a^2*c^3*PolyLog[2, 1 - 2/(1 + I*a*x)] - ((9*I)/2)*a^2*c^3*ArcTan[a*x]^
2*PolyLog[2, 1 - 2/(1 + I*a*x)] + ((9*I)/2)*a^2*c^3*ArcTan[a*x]^2*PolyLog[2
, -1 + 2/(1 + I*a*x)] - (9*a^2*c^3*ArcTan[a*x]*PolyLog[3, 1 - 2/(1 + I*a*x)
])/2 + (9*a^2*c^3*ArcTan[a*x]*PolyLog[3, -1 + 2/(1 + I*a*x)])/2 + ((9*I)/4)
*a^2*c^3*PolyLog[4, 1 - 2/(1 + I*a*x)] - ((9*I)/4)*a^2*c^3*PolyLog[4, -1 +
2/(1 + I*a*x)]
```

Rubi [A] time = 1.19198, antiderivative size = 503, normalized size of antiderivative = 1., number of steps used = 43, number of rules used = 20, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.909, Rules used = {4948, 4852, 4918, 4924, 4868, 2447, 4884, 4850, 4988, 4994, 4998, 6610, 4916, 4846, 4920, 4854, 2402, 2315, 321, 203}

$$-\frac{3}{2}ia^2c^3\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)-\frac{7}{2}ia^2c^3\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)+\frac{9}{4}ia^2c^3\text{PolyLog}\left(4,1-\frac{2}{1+iax}\right)-\frac{9}{4}ia^2c^3\text{Poly}$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)^3*ArcTan[a*x]^3)/x^3,x]
```


```
[Out] -(a^3*c^3*x)/4 + (a^2*c^3*ArcTan[a*x])/4 + (a^4*c^3*x^2*ArcTan[a*x])/4 - (5
*I)*a^2*c^3*ArcTan[a*x]^2 - (3*a*c^3*ArcTan[a*x]^2)/(2*x) - (15*a^3*c^3*x*A
rcTan[a*x]^2)/4 - (a^5*c^3*x^3*ArcTan[a*x]^2)/4 + (3*a^2*c^3*ArcTan[a*x]^3)
/4 - (c^3*ArcTan[a*x]^3)/(2*x^2) + (3*a^4*c^3*x^2*ArcTan[a*x]^3)/2 + (a^6*c
^3*x^4*ArcTan[a*x]^3)/4 + 6*a^2*c^3*ArcTan[a*x]^3*ArcTanh[1 - 2/(1 + I*a*x)
] - 7*a^2*c^3*ArcTan[a*x]*Log[2/(1 + I*a*x)] + 3*a^2*c^3*ArcTan[a*x]*Log[2
- 2/(1 - I*a*x)] - ((3*I)/2)*a^2*c^3*PolyLog[2, -1 + 2/(1 - I*a*x)] - ((7*I
)/2)*a^2*c^3*PolyLog[2, 1 - 2/(1 + I*a*x)] - ((9*I)/2)*a^2*c^3*ArcTan[a*x]^
```

```
2*PolyLog[2, 1 - 2/(1 + I*a*x)] + ((9*I)/2)*a^2*c^3*ArcTan[a*x]^2*PolyLog[2
, -1 + 2/(1 + I*a*x)] - (9*a^2*c^3*ArcTan[a*x]*PolyLog[3, 1 - 2/(1 + I*a*x)
])/2 + (9*a^2*c^3*ArcTan[a*x]*PolyLog[3, -1 + 2/(1 + I*a*x)])/2 + ((9*I)/4)
*a^2*c^3*PolyLog[4, 1 - 2/(1 + I*a*x)] - ((9*I)/4)*a^2*c^3*PolyLog[4, -1 +
2/(1 + I*a*x)]
```

Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4918

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/(d + e*x^2),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4850

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTan[c*x])^(p - 1)*ArcTanh[1 - 2/(1 + I*c*x)])/(1 + c^2*x^2), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4988

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTan[c*x])^p)/(d + e
*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTan[c*x])^p)/(d + e*x^2
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 4998

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[k + 1, u])/(2
*c*d), x] - Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[k + 1,
 u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
qQ[e, c^2*d] && EqQ[u^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
```

```
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^3}{x^3}\,dx = \int \left(\frac{c^3\tan^{-1}(ax)^3}{x^3}+\frac{3a^2c^3\tan^{-1}(ax)^3}{x}+3a^4c^3x\tan^{-1}(ax)^3+a^6c^3x^3\tan^{-1}(ax)^3\right)dx$$

$$= c^3\int \frac{\tan^{-1}(ax)^3}{x^3}\,dx+\left(3a^2c^3\right)\int \frac{\tan^{-1}(ax)^3}{x}\,dx+\left(3a^4c^3\right)\int x\tan^{-1}(ax)^3\,dx+\left(a^6c^3\right)$$

$$= -\frac{c^3\tan^{-1}(ax)^3}{2x^2}+\frac{3}{2}a^4c^3x^2\tan^{-1}(ax)^3+\frac{1}{4}a^6c^3x^4\tan^{-1}(ax)^3+6a^2c^3\tan^{-1}(ax)^3\tanh$$

$$= -\frac{c^3\tan^{-1}(ax)^3}{2x^2}+\frac{3}{2}a^4c^3x^2\tan^{-1}(ax)^3+\frac{1}{4}a^6c^3x^4\tan^{-1}(ax)^3+6a^2c^3\tan^{-1}(ax)^3\tanh$$

$$= -\frac{3ac^3\tan^{-1}(ax)^2}{2x}-\frac{9}{2}a^3c^3x\tan^{-1}(ax)^2-\frac{1}{4}a^5c^3x^3\tan^{-1}(ax)^2+a^2c^3\tan^{-1}(ax)^3-\frac{c^3}{}$$

$$= -6ia^2c^3\tan^{-1}(ax)^2-\frac{3ac^3\tan^{-1}(ax)^2}{2x}-\frac{15}{4}a^3c^3x\tan^{-1}(ax)^2-\frac{1}{4}a^5c^3x^3\tan^{-1}(ax)^2+$$

$$= \frac{1}{4}a^4c^3x^2\tan^{-1}(ax)-5ia^2c^3\tan^{-1}(ax)^2-\frac{3ac^3\tan^{-1}(ax)^2}{2x}-\frac{15}{4}a^3c^3x\tan^{-1}(ax)^2-\frac{1}{4}a$$

$$= -\frac{1}{4}a^3c^3x+\frac{1}{4}a^4c^3x^2\tan^{-1}(ax)-5ia^2c^3\tan^{-1}(ax)^2-\frac{3ac^3\tan^{-1}(ax)^2}{2x}-\frac{15}{4}a^3c^3x\tan^{-1}$$

$$= -\frac{1}{4}a^3c^3x+\frac{1}{4}a^2c^3\tan^{-1}(ax)+\frac{1}{4}a^4c^3x^2\tan^{-1}(ax)-5ia^2c^3\tan^{-1}(ax)^2-\frac{3ac^3\tan^{-1}(ax}{2x}$$

$$= -\frac{1}{4}a^3c^3x+\frac{1}{4}a^2c^3\tan^{-1}(ax)+\frac{1}{4}a^4c^3x^2\tan^{-1}(ax)-5ia^2c^3\tan^{-1}(ax)^2-\frac{3ac^3\tan^{-1}(ax}{2x}$$

Mathematica [A] time = 0.749681, size = 464, normalized size = 0.92

$$\frac{c^3\left(288ia^2x^2\tan^{-1}(ax)^2\text{PolyLog}\left(2,e^{-2i\tan^{-1}(ax)}\right)+32ia^2x^2\left(9\tan^{-1}(ax)^2+7\right)\text{PolyLog}\left(2,-e^{2i\tan^{-1}(ax)}\right)-96ia^2x^2\text{Poly}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((c + a^2*c*x^2)^3*ArcTan[a*x]^3)/x^3,x]
```

```
[Out] (c^3*((-3*I)*a^2*Pi^4*x^2 - 16*a^3*x^3 + 16*a^2*x^2*ArcTan[a*x] + 16*a^4*x^
4*ArcTan[a*x] - 96*a*x*ArcTan[a*x]^2 + (128*I)*a^2*x^2*ArcTan[a*x]^2 - 240*
a^3*x^3*ArcTan[a*x]^2 - 16*a^5*x^5*ArcTan[a*x]^2 - 32*ArcTan[a*x]^3 + 48*a^
2*x^2*ArcTan[a*x]^3 + 96*a^4*x^4*ArcTan[a*x]^3 + 16*a^6*x^6*ArcTan[a*x]^3 +
 (96*I)*a^2*x^2*ArcTan[a*x]^4 + 192*a^2*x^2*ArcTan[a*x]^3*Log[1 - E^((-2*I)
*ArcTan[a*x])] + 192*a^2*x^2*ArcTan[a*x]*Log[1 - E^((2*I)*ArcTan[a*x])] - 4
48*a^2*x^2*ArcTan[a*x]*Log[1 + E^((2*I)*ArcTan[a*x])] - 192*a^2*x^2*ArcTan[
a*x]^3*Log[1 + E^((2*I)*ArcTan[a*x])] + (288*I)*a^2*x^2*ArcTan[a*x]^2*PolyL
og[2, E^((-2*I)*ArcTan[a*x])] + (32*I)*a^2*x^2*(7 + 9*ArcTan[a*x]^2)*PolyLo
g[2, -E^((2*I)*ArcTan[a*x])] - (96*I)*a^2*x^2*PolyLog[2, E^((2*I)*ArcTan[a*
x])] + 288*a^2*x^2*ArcTan[a*x]*PolyLog[3, E^((-2*I)*ArcTan[a*x])] - 288*a^2
*x^2*ArcTan[a*x]*PolyLog[3, -E^((2*I)*ArcTan[a*x])] - (144*I)*a^2*x^2*PolyL
og[4, E^((-2*I)*ArcTan[a*x])] - (144*I)*a^2*x^2*PolyLog[4, -E^((2*I)*ArcTan
[a*x])]))/(64*x^2)
```

Maple [A] time = 6.651, size = 790, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^3*arctan(a*x)^3/x^3,x)
```

```
[Out] 3/4*a^2*c^3*arctan(a*x)^3-1/2*c^3*arctan(a*x)^3/x^2-1/4*a^3*c^3*x+1/4*a^2*c
^3*arctan(a*x)-3/2*a*c^3*arctan(a*x)^2/x-15/4*a^3*c^3*x*arctan(a*x)^2-1/4*a
^5*c^3*x^3*arctan(a*x)^2+3/2*a^4*c^3*x^2*arctan(a*x)^3+1/4*a^6*c^3*x^4*arct
an(a*x)^3-3*I*a^2*c^3*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+18*I*a^2*c^3*
polylog(4,(1+I*a*x)/(a^2*x^2+1)^(1/2))+7/2*I*a^2*c^3*polylog(2,-(1+I*a*x)^2
/(a^2*x^2+1))-3*I*a^2*c^3*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+3*a^2*c^3*
arctan(a*x)*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))+3*a^2*c^3*arctan(a*x)^3*ln(1+
(1+I*a*x)/(a^2*x^2+1)^(1/2))+18*a^2*c^3*arctan(a*x)*polylog(3,-(1+I*a*x)/(a
^2*x^2+1)^(1/2))+3*a^2*c^3*arctan(a*x)^3*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-
```

```
9/2*a^2*c^3*arctan(a*x)*polylog(3,-(1+I*a*x)^2/(a^2*x^2+1))-7*a^2*c^3*arcta
n(a*x)*ln((1+I*a*x)^2/(a^2*x^2+1)+1)-3*a^2*c^3*arctan(a*x)^3*ln((1+I*a*x)^2
/(a^2*x^2+1)+1)+18*a^2*c^3*arctan(a*x)*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2
))+3*a^2*c^3*arctan(a*x)*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))+18*I*a^2*c^3*pol
ylog(4,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*I*a^2*c^3*arctan(a*x)^2-9/4*I*a^2*c^
3*polylog(4,-(1+I*a*x)^2/(a^2*x^2+1))+9/2*I*a^2*c^3*arctan(a*x)^2*polylog(2
,-(1+I*a*x)^2/(a^2*x^2+1))-9*I*a^2*c^3*arctan(a*x)^2*polylog(2,-(1+I*a*x)/(
a^2*x^2+1)^(1/2))+1/4*a^4*c^3*x^2*arctan(a*x)-1/4*I*a^2*c^3-9*I*a^2*c^3*arc
tan(a*x)^2*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{4\left(a^6c^3x^6+6\,a^4c^3x^4-2\,c^3\right)\arctan\left(ax\right)^3-3\left(a^6c^3x^6+6\,a^4c^3x^4-2\,c^3\right)\arctan\left(ax\right)\log\left(a^2x^2+1\right)^2+x^2\int\frac{112\left(a^8c^3x^8+4\,a^6\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^3/x^3,x, algorithm="maxima")
```

```
[Out] 1/128*(4*(a^6*c^3*x^6 + 6*a^4*c^3*x^4 - 2*c^3)*arctan(a*x)^3 - 3*(a^6*c^3*x
^6 + 6*a^4*c^3*x^4 - 2*c^3)*arctan(a*x)*log(a^2*x^2 + 1)^2 + 128*x^2*integr
ate(1/128*(112*(a^8*c^3*x^8 + 4*a^6*c^3*x^6 + 6*a^4*c^3*x^4 + 4*a^2*c^3*x^2
 + c^3)*arctan(a*x)^3 - 12*(a^7*c^3*x^7 + 6*a^5*c^3*x^5 - 2*a*c^3*x)*arctan
(a*x)^2 + 12*(a^8*c^3*x^8 + 6*a^6*c^3*x^6 - 2*a^2*c^3*x^2)*arctan(a*x)*log(
a^2*x^2 + 1) + 3*(a^7*c^3*x^7 + 6*a^5*c^3*x^5 - 2*a*c^3*x + 4*(a^8*c^3*x^8
+ 4*a^6*c^3*x^6 + 6*a^4*c^3*x^4 + 4*a^2*c^3*x^2 + c^3)*arctan(a*x))*log(a^2
*x^2 + 1)^2)/(a^2*x^5 + x^3), x))/x^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^6c^3x^6+3\,a^4c^3x^4+3\,a^2c^3x^2+c^3\right)\arctan\left(ax\right)^3}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^3/x^3,x, algorithm="fricas")
```

```
[Out] integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*arctan(a*x)^3/
x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$c^{3}\left(\int\frac{\operatorname{atan}^{3}(ax)}{x^{3}}\,dx+\int\frac{3a^{2}\operatorname{atan}^{3}(ax)}{x}\,dx+\int 3a^{4}x\operatorname{atan}^{3}(ax)\,dx+\int a^{6}x^{3}\operatorname{atan}^{3}(ax)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**3*atan(a*x)**3/x**3,x)
```

```
[Out] c**3*(Integral(atan(a*x)**3/x**3, x) + Integral(3*a**2*atan(a*x)**3/x, x) +
 Integral(3*a**4*x*atan(a*x)**3, x) + Integral(a**6*x**3*atan(a*x)**3, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a^{2}cx^{2}+c\right)^{3}\arctan(ax)^{3}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^3/x^3,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^3*arctan(a*x)^3/x^3, x)
```

3.386 $\int \frac{(c+a^2cx^2)^3 \tan^{-1}(ax)^3}{x^4} dx$

Optimal. Leaf size=336

$$4a^3c^3\text{PolyLog}\left(3,-1+\frac{2}{1-iax}\right)+4a^3c^3\text{PolyLog}\left(3,1-\frac{2}{1+iax}\right)-8ia^3c^3\tan^{-1}(ax)\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)+8ia^3$$

```
[Out] -((a^2*c^3*ArcTan[a*x])/x) + a^4*c^3*x*ArcTan[a*x] - a^3*c^3*ArcTan[a*x]^2
- (a*c^3*ArcTan[a*x]^2)/(2*x^2) - (a^5*c^3*x^2*ArcTan[a*x]^2)/2 - (c^3*ArcT
an[a*x]^3)/(3*x^3) - (3*a^2*c^3*ArcTan[a*x]^3)/x + 3*a^4*c^3*x*ArcTan[a*x]^
3 + (a^6*c^3*x^3*ArcTan[a*x]^3)/3 + a^3*c^3*Log[x] + 8*a^3*c^3*ArcTan[a*x]^
2*Log[2/(1 + I*a*x)] - a^3*c^3*Log[1 + a^2*x^2] + 8*a^3*c^3*ArcTan[a*x]^2*L
og[2 - 2/(1 - I*a*x)] - (8*I)*a^3*c^3*ArcTan[a*x]*PolyLog[2, -1 + 2/(1 - I*
a*x)] + (8*I)*a^3*c^3*ArcTan[a*x]*PolyLog[2, 1 - 2/(1 + I*a*x)] + 4*a^3*c^3
*PolyLog[3, -1 + 2/(1 - I*a*x)] + 4*a^3*c^3*PolyLog[3, 1 - 2/(1 + I*a*x)]
```

Rubi [A] time = 1.10857, antiderivative size = 336, normalized size of antiderivative = 1., number of steps used = 37, number of rules used = 18, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.818, Rules used = {4948, 4846, 4920, 4854, 4884, 4994, 6610, 4852, 4918, 266, 36, 29, 31, 4924, 4868, 4992, 4916, 260}

$$4a^3c^3\text{PolyLog}\left(3,-1+\frac{2}{1-iax}\right)+4a^3c^3\text{PolyLog}\left(3,1-\frac{2}{1+iax}\right)-8ia^3c^3\tan^{-1}(ax)\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)+8ia^3$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)^3*ArcTan[a*x]^3)/x^4,x]
```


```
[Out] -((a^2*c^3*ArcTan[a*x])/x) + a^4*c^3*x*ArcTan[a*x] - a^3*c^3*ArcTan[a*x]^2
- (a*c^3*ArcTan[a*x]^2)/(2*x^2) - (a^5*c^3*x^2*ArcTan[a*x]^2)/2 - (c^3*ArcT
an[a*x]^3)/(3*x^3) - (3*a^2*c^3*ArcTan[a*x]^3)/x + 3*a^4*c^3*x*ArcTan[a*x]^
3 + (a^6*c^3*x^3*ArcTan[a*x]^3)/3 + a^3*c^3*Log[x] + 8*a^3*c^3*ArcTan[a*x]^
2*Log[2/(1 + I*a*x)] - a^3*c^3*Log[1 + a^2*x^2] + 8*a^3*c^3*ArcTan[a*x]^2*L
og[2 - 2/(1 - I*a*x)] - (8*I)*a^3*c^3*ArcTan[a*x]*PolyLog[2, -1 + 2/(1 - I*
a*x)] + (8*I)*a^3*c^3*ArcTan[a*x]*PolyLog[2, 1 - 2/(1 + I*a*x)] + 4*a^3*c^3
*PolyLog[3, -1 + 2/(1 - I*a*x)] + 4*a^3*c^3*PolyLog[3, 1 - 2/(1 + I*a*x)]
```


Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
```

```
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4918

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/(d + e*x^2),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
```

```
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 4992

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d
] && EqQ[(1 - u)^2 - (1 - (2*I)/(I + c*x))^2, 0]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^3 \tan^{-1}(ax)^3}{x^4} dx = \int \left(3a^4c^3 \tan^{-1}(ax)^3 + \frac{c^3 \tan^{-1}(ax)^3}{x^4} + \frac{3a^2c^3 \tan^{-1}(ax)^3}{x^2} + a^6c^3x^2 \tan^{-1}(ax)^3\right) dx$$
$$= c^3 \int \frac{\tan^{-1}(ax)^3}{x^4} dx + (3a^2c^3) \int \frac{\tan^{-1}(ax)^3}{x^2} dx + (3a^4c^3) \int \tan^{-1}(ax)^3 dx + (a^6c^3) \int$$
$$= -\frac{c^3 \tan^{-1}(ax)^3}{3x^3} - \frac{3a^2c^3 \tan^{-1}(ax)^3}{x} + 3a^4c^3x \tan^{-1}(ax)^3 + \frac{1}{3}a^6c^3x^3 \tan^{-1}(ax)^3 + (ac^3)$$
$$= -\frac{c^3 \tan^{-1}(ax)^3}{3x^3} - \frac{3a^2c^3 \tan^{-1}(ax)^3}{x} + 3a^4c^3x \tan^{-1}(ax)^3 + \frac{1}{3}a^6c^3x^3 \tan^{-1}(ax)^3 + (ac^3)$$
$$= -\frac{ac^3 \tan^{-1}(ax)^2}{2x^2} - \frac{1}{2}a^5c^3x^2 \tan^{-1}(ax)^2 - \frac{c^3 \tan^{-1}(ax)^3}{3x^3} - \frac{3a^2c^3 \tan^{-1}(ax)^3}{x} + 3a^4c^3x \mathrm{t}$$
$$= -\frac{ac^3 \tan^{-1}(ax)^2}{2x^2} - \frac{1}{2}a^5c^3x^2 \tan^{-1}(ax)^2 - \frac{c^3 \tan^{-1}(ax)^3}{3x^3} - \frac{3a^2c^3 \tan^{-1}(ax)^3}{x} + 3a^4c^3x \mathrm{t}$$
$$= -\frac{a^2c^3 \tan^{-1}(ax)}{x} + a^4c^3x \tan^{-1}(ax) - a^3c^3 \tan^{-1}(ax)^2 - \frac{ac^3 \tan^{-1}(ax)^2}{2x^2} - \frac{1}{2}a^5c^3x^2 \mathrm{tan}$$
$$= -\frac{a^2c^3 \tan^{-1}(ax)}{x} + a^4c^3x \tan^{-1}(ax) - a^3c^3 \tan^{-1}(ax)^2 - \frac{ac^3 \tan^{-1}(ax)^2}{2x^2} - \frac{1}{2}a^5c^3x^2 \mathrm{tan}$$
$$= -\frac{a^2c^3 \tan^{-1}(ax)}{x} + a^4c^3x \tan^{-1}(ax) - a^3c^3 \tan^{-1}(ax)^2 - \frac{ac^3 \tan^{-1}(ax)^2}{2x^2} - \frac{1}{2}a^5c^3x^2 \mathrm{tan}$$
$$= -\frac{a^2c^3 \tan^{-1}(ax)}{x} + a^4c^3x \tan^{-1}(ax) - a^3c^3 \tan^{-1}(ax)^2 - \frac{ac^3 \tan^{-1}(ax)^2}{2x^2} - \frac{1}{2}a^5c^3x^2 \mathrm{tan}$$

Mathematica [A] time = 0.706, size = 331, normalized size = 0.99

$$\frac{c^3 \left(48ia^3x^3 \tan^{-1}(ax) \mathrm{PolyLog}\left(2, e^{-2i \tan^{-1}(ax)}\right) - 48ia^3x^3 \tan^{-1}(ax) \mathrm{PolyLog}\left(2, -e^{2i \tan^{-1}(ax)}\right) + 24a^3x^3 \mathrm{PolyLog}\left(3, e^{-}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((c + a^2*c*x^2)^3*ArcTan[a*x]^3)/x^4,x]
```

```
[Out] (c^3*((-2*I)*a^3*Pi^3*x^3 - 6*a^2*x^2*ArcTan[a*x] + 6*a^4*x^4*ArcTan[a*x] -
 3*a*x*ArcTan[a*x]^2 - 6*a^3*x^3*ArcTan[a*x]^2 - 3*a^5*x^5*ArcTan[a*x]^2 -
2*ArcTan[a*x]^3 - 18*a^2*x^2*ArcTan[a*x]^3 + 18*a^4*x^4*ArcTan[a*x]^3 + 2*a
^6*x^6*ArcTan[a*x]^3 + 48*a^3*x^3*ArcTan[a*x]^2*Log[1 - E^((-2*I)*ArcTan[a*
```

```
x])] + 48*a^3*x^3*ArcTan[a*x]^2*Log[1 + E^((2*I)*ArcTan[a*x])] + 6*a^3*x^3*
Log[(a*x)/Sqrt[1 + a^2*x^2]] - 3*a^3*x^3*Log[1 + a^2*x^2] + (48*I)*a^3*x^3*
ArcTan[a*x]*PolyLog[2, E^((-2*I)*ArcTan[a*x])] - (48*I)*a^3*x^3*ArcTan[a*x]
*PolyLog[2, -E^((2*I)*ArcTan[a*x])] + 24*a^3*x^3*PolyLog[3, E^((-2*I)*ArcTa
n[a*x])] + 24*a^3*x^3*PolyLog[3, -E^((2*I)*ArcTan[a*x])]))/(6*x^3)
```

Maple [C] time = 18.202, size = 7948, normalized size = 23.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^3*arctan(a*x)^3/x^4,x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^3/x^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^{6} c^{3} x^{6} + 3 \, a^{4} c^{3} x^{4} + 3 \, a^{2} c^{3} x^{2} + c^{3}\right) \arctan\left(a x\right)^{3}}{x^{4}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^3/x^4,x, algorithm="fricas")
```

```
[Out] integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*arctan(a*x)^3/
x^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$c^3\left(\int 3a^4\operatorname{atan}^3(ax)\,dx+\int\frac{\operatorname{atan}^3(ax)}{x^4}\,dx+\int\frac{3a^2\operatorname{atan}^3(ax)}{x^2}\,dx+\int a^6x^2\operatorname{atan}^3(ax)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**3*atan(a*x)**3/x**4,x)
```

```
[Out] c**3*(Integral(3*a**4*atan(a*x)**3, x) + Integral(atan(a*x)**3/x**4, x) + I
ntegral(3*a**2*atan(a*x)**3/x**2, x) + Integral(a**6*x**2*atan(a*x)**3, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a^2cx^2+c\right)^3\arctan(ax)^3}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^3/x^4,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^3*arctan(a*x)^3/x^4, x)
```

3.387 $\int \frac{x^4 \tan^{-1}(ax)^3}{c+a^2cx^2} dx$

Optimal. Leaf size=217

$$-\frac{2\text{PolyLog}\left(3,1-\frac{2}{1+iax}\right)}{a^5c}-\frac{4i\tan^{-1}(ax)\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{a^5c}-\frac{\log\left(a^2x^2+1\right)}{2a^5c}+\frac{x^3\tan^{-1}(ax)^3}{3a^2c}-\frac{x^2\tan^{-1}(ax)^2}{2a^3c}+\frac{\tan}{}$$

```
[Out] (x*ArcTan[a*x])/(a^4*c) - ArcTan[a*x]^2/(2*a^5*c) - (x^2*ArcTan[a*x]^2)/(2*
a^3*c) - (((4*I)/3)*ArcTan[a*x]^3)/(a^5*c) - (x*ArcTan[a*x]^3)/(a^4*c) + (x
^3*ArcTan[a*x]^3)/(3*a^2*c) + ArcTan[a*x]^4/(4*a^5*c) - (4*ArcTan[a*x]^2*Lo
g[2/(1 + I*a*x)])/(a^5*c) - Log[1 + a^2*x^2]/(2*a^5*c) - ((4*I)*ArcTan[a*x]
*PolyLog[2, 1 - 2/(1 + I*a*x)])/(a^5*c) - (2*PolyLog[3, 1 - 2/(1 + I*a*x)])
/(a^5*c)
```

Rubi [A] time = 0.626298, antiderivative size = 217, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {4916, 4852, 4846, 260, 4884, 4920, 4854, 4994, 6610}

$$-\frac{2\text{PolyLog}\left(3,1-\frac{2}{1+iax}\right)}{a^5c}-\frac{4i\tan^{-1}(ax)\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{a^5c}-\frac{\log\left(a^2x^2+1\right)}{2a^5c}+\frac{x^3\tan^{-1}(ax)^3}{3a^2c}-\frac{x^2\tan^{-1}(ax)^2}{2a^3c}+\frac{\tan}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*ArcTan[a*x]^3)/(c + a^2*c*x^2),x]
```

```
[Out] (x*ArcTan[a*x])/(a^4*c) - ArcTan[a*x]^2/(2*a^5*c) - (x^2*ArcTan[a*x]^2)/(2*
a^3*c) - (((4*I)/3)*ArcTan[a*x]^3)/(a^5*c) - (x*ArcTan[a*x]^3)/(a^4*c) + (x
^3*ArcTan[a*x]^3)/(3*a^2*c) + ArcTan[a*x]^4/(4*a^5*c) - (4*ArcTan[a*x]^2*Lo
g[2/(1 + I*a*x)])/(a^5*c) - Log[1 + a^2*x^2]/(2*a^5*c) - ((4*I)*ArcTan[a*x]
*PolyLog[2, 1 - 2/(1 + I*a*x)])/(a^5*c) - (2*PolyLog[3, 1 - 2/(1 + I*a*x)])
/(a^5*c)
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{x^4 \tan^{-1}(ax)^3}{c+a^2cx^2}\,dx = -\frac{\int \frac{x^2 \tan^{-1}(ax)^3}{c+a^2cx^2}\,dx}{a^2} + \frac{\int x^2 \tan^{-1}(ax)^3\,dx}{a^2c}$$
$$= \frac{x^3 \tan^{-1}(ax)^3}{3a^2c} + \frac{\int \frac{\tan^{-1}(ax)^3}{c+a^2cx^2}\,dx}{a^4} - \frac{\int \tan^{-1}(ax)^3\,dx}{a^4c} - \frac{\int \frac{x^3 \tan^{-1}(ax)^2}{1+a^2x^2}\,dx}{ac}$$
$$= -\frac{x \tan^{-1}(ax)^3}{a^4c} + \frac{x^3 \tan^{-1}(ax)^3}{3a^2c} + \frac{\tan^{-1}(ax)^4}{4a^5c} - \frac{\int x \tan^{-1}(ax)^2\,dx}{a^3c} + \frac{\int \frac{x \tan^{-1}(ax)^2}{1+a^2x^2}\,dx}{a^3c} + \frac{3\int \frac{x \tan^{-1}}{1+a^{}}}{a^{3}}$$
$$= -\frac{x^2 \tan^{-1}(ax)^2}{2a^3c} - \frac{4i \tan^{-1}(ax)^3}{3a^5c} - \frac{x \tan^{-1}(ax)^3}{a^4c} + \frac{x^3 \tan^{-1}(ax)^3}{3a^2c} + \frac{\tan^{-1}(ax)^4}{4a^5c} - \frac{\int \frac{\tan^{-1}(ax)^2}{i-ax}\,dx}{a^4c}$$
$$= -\frac{x^2 \tan^{-1}(ax)^2}{2a^3c} - \frac{4i \tan^{-1}(ax)^3}{3a^5c} - \frac{x \tan^{-1}(ax)^3}{a^4c} + \frac{x^3 \tan^{-1}(ax)^3}{3a^2c} + \frac{\tan^{-1}(ax)^4}{4a^5c} - \frac{4 \tan^{-1}(ax)^2 \log}{a^5c}$$
$$= \frac{x \tan^{-1}(ax)}{a^4c} - \frac{\tan^{-1}(ax)^2}{2a^5c} - \frac{x^2 \tan^{-1}(ax)^2}{2a^3c} - \frac{4i \tan^{-1}(ax)^3}{3a^5c} - \frac{x \tan^{-1}(ax)^3}{a^4c} + \frac{x^3 \tan^{-1}(ax)^3}{3a^2c} + \mathrm{ta}$$
$$= \frac{x \tan^{-1}(ax)}{a^4c} - \frac{\tan^{-1}(ax)^2}{2a^5c} - \frac{x^2 \tan^{-1}(ax)^2}{2a^3c} - \frac{4i \tan^{-1}(ax)^3}{3a^5c} - \frac{x \tan^{-1}(ax)^3}{a^4c} + \frac{x^3 \tan^{-1}(ax)^3}{3a^2c} + \mathrm{ta}$$

Mathematica [A] time = 0.246642, size = 154, normalized size = 0.71

$$48i \tan^{-1}(ax)\mathrm{PolyLog}\left(2,-e^{2i \tan^{-1}(ax)}\right) - 24\mathrm{PolyLog}\left(3,-e^{2i \tan^{-1}(ax)}\right) - 6 \log\left(a^2x^2+1\right) + 4a^3x^3 \tan^{-1}(ax)^3 - 6a^2x^2 \tan$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*ArcTan[a*x]^3)/(c + a^2*c*x^2),x]
```

```
[Out] (12*a*x*ArcTan[a*x] - 6*ArcTan[a*x]^2 - 6*a^2*x^2*ArcTan[a*x]^2 + (16*I)*Ar
cTan[a*x]^3 - 12*a*x*ArcTan[a*x]^3 + 4*a^3*x^3*ArcTan[a*x]^3 + 3*ArcTan[a*x
]^4 - 48*ArcTan[a*x]^2*Log[1 + E^((2*I)*ArcTan[a*x])] - 6*Log[1 + a^2*x^2]
+ (48*I)*ArcTan[a*x]*PolyLog[2, -E^((2*I)*ArcTan[a*x])] - 24*PolyLog[3, -E^
((2*I)*ArcTan[a*x])])/(12*a^5*c)
```

Maple [C] time = 3.701, size = 1740, normalized size = 8.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arctan(a*x)^3/(a^2*c*x^2+c),x)
```

```
[Out] 1/a^5/c*ln((1+I*a*x)^2/(a^2*x^2+1)+1)-2/a^5/c*polylog(3,-(1+I*a*x)^2/(a^2*x
^2+1))-1/2/a^4/c*arctan(a*x)^2*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3*x
+I/a^5/c*arctan(a*x)^2*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^3+I/a^5/c*arctan(
a*x)^2*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3-1
/2*I/a^5/c*arctan(a*x)^2*Pi*csgn(I*(1+I*a*x)^4/(a^2*x^2+1)^2+2*I*(1+I*a*x)^
2/(a^2*x^2+1)+I)^3-1/2*I/a^5/c*arctan(a*x)^2*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^
2+1)+1)^2)^3+1/2/a^4/c*arctan(a*x)^2*Pi*csgn(I*(1+I*a*x)^4/(a^2*x^2+1)^2+2*
I*(1+I*a*x)^2/(a^2*x^2+1)+I)^3*x-4/a^5/c*arctan(a*x)^2*ln((1+I*a*x)/(a^2*x^
2+1)^(1/2))+1/4*arctan(a*x)^4/a^5/c-1/2*x^2*arctan(a*x)^2/a^3/c-x*arctan(a*
x)^3/a^4/c+1/3*x^3*arctan(a*x)^3/a^2/c+I/a^5/c*arctan(a*x)^2*Pi*csgn(I*(1+I
*a*x)^2/(a^2*x^2+1))*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)
^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)-1/2*arctan(a*x)^2/a^5/c+x*arc
tan(a*x)/a^4/c-I/a^5/c*arctan(a*x)+4/3*I/a^5/c*arctan(a*x)^3-4/a^5/c*arctan
(a*x)^2*ln(2)+2/a^5/c*arctan(a*x)^2*ln(a^2*x^2+1)+I/a^5/c*arctan(a*x)^2*Pi*
csgn(I*(1+I*a*x)^2/(a^2*x^2+1)+I)*csgn(I*(1+I*a*x)^4/(a^2*x^2+1)^2+2*I*(1+I
*a*x)^2/(a^2*x^2+1)+I)^2+I/a^5/c*arctan(a*x)^2*Pi*csgn(I*((1+I*a*x)^2/(a^2*
x^2+1)+1)^2)^2*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))+I/a^5/c*arctan(a*x)^2*Pi
*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))^2*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))+4*I/
a^5/c*arctan(a*x)*polylog(2,-(1+I*a*x)^2/(a^2*x^2+1))+1/2/a^4/c*arctan(a*x)
^2*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)+I)^2*csgn(I*(1+I*a*x)^4/(a^2*x^2+1)^2+
2*I*(1+I*a*x)^2/(a^2*x^2+1)+I)*x-1/a^4/c*arctan(a*x)^2*Pi*csgn(I*(1+I*a*x)^
2/(a^2*x^2+1)+I)*csgn(I*(1+I*a*x)^4/(a^2*x^2+1)^2+2*I*(1+I*a*x)^2/(a^2*x^2+
1)+I)^2*x+1/a^4/c*arctan(a*x)^2*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2*
csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))*x-1/2/a^4/c*arctan(a*x)^2*Pi*csgn(I*((1
+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))^2*x-1/2*I/a
^5/c*arctan(a*x)^2*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)+I)^2*csgn(I*(1+I*a*x)^
4/(a^2*x^2+1)^2+2*I*(1+I*a*x)^2/(a^2*x^2+1)+I)-1/2*I/a^5/c*arctan(a*x)^2*Pi
*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))^
2-2*I/a^5/c*arctan(a*x)^2*Pi*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*csgn(I*(1+
I*a*x)^2/(a^2*x^2+1))^2-I/a^5/c*arctan(a*x)^2*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^
2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2-I/a^5
/c*arctan(a*x)^2*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^
2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctan(a*x)^3/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^4 \arctan(ax)^3}{a^2cx^2 + c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctan(a*x)^3/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(x^4*arctan(a*x)^3/(a^2*c*x^2 + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^4 \operatorname{atan}^3(ax)}{a^2x^2+1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*atan(a*x)**3/(a**2*c*x**2+c),x)
```

```
[Out] Integral(x**4*atan(a*x)**3/(a**2*x**2 + 1), x)/c
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4 \arctan(ax)^3}{a^2cx^2 + c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctan(a*x)^3/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(x^4*arctan(a*x)^3/(a^2*c*x^2 + c), x)
```

3.388 $\int \frac{x^3 \tan^{-1}(ax)^3}{c+a^2cx^2}\,dx$

Optimal. Leaf size=260

$$-\frac{3i\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{2a^4c}-\frac{3i\text{PolyLog}\left(4,1-\frac{2}{1+iax}\right)}{4a^4c}+\frac{3i\tan^{-1}(ax)^2\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{2a^4c}+\frac{3\tan^{-1}(ax)\text{PolyLog}\left(3\right.}{2a^4c}$$

```
[Out] (((-3*I)/2)*ArcTan[a*x]^2)/(a^4*c) - (3*x*ArcTan[a*x]^2)/(2*a^3*c) + ArcTan
[a*x]^3/(2*a^4*c) + (x^2*ArcTan[a*x]^3)/(2*a^2*c) + ((I/4)*ArcTan[a*x]^4)/(
a^4*c) - (3*ArcTan[a*x]*Log[2/(1 + I*a*x)])/(a^4*c) + (ArcTan[a*x]^3*Log[2/
(1 + I*a*x)])/(a^4*c) - (((3*I)/2)*PolyLog[2, 1 - 2/(1 + I*a*x)])/(a^4*c) +
 (((3*I)/2)*ArcTan[a*x]^2*PolyLog[2, 1 - 2/(1 + I*a*x)])/(a^4*c) + (3*ArcTa
n[a*x]*PolyLog[3, 1 - 2/(1 + I*a*x)])/(2*a^4*c) - (((3*I)/4)*PolyLog[4, 1 -
 2/(1 + I*a*x)])/(a^4*c)
```

Rubi [A] time = 0.446751, antiderivative size = 260, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 11, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4916, 4852, 4846, 4920, 4854, 2402, 2315, 4884, 4994, 4998, 6610}

$$-\frac{3i\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{2a^4c}-\frac{3i\text{PolyLog}\left(4,1-\frac{2}{1+iax}\right)}{4a^4c}+\frac{3i\tan^{-1}(ax)^2\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{2a^4c}+\frac{3\tan^{-1}(ax)\text{PolyLog}\left(3\right.}{2a^4c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcTan[a*x]^3)/(c + a^2*c*x^2),x]
```


```
[Out] (((-3*I)/2)*ArcTan[a*x]^2)/(a^4*c) - (3*x*ArcTan[a*x]^2)/(2*a^3*c) + ArcTan
[a*x]^3/(2*a^4*c) + (x^2*ArcTan[a*x]^3)/(2*a^2*c) + ((I/4)*ArcTan[a*x]^4)/(
a^4*c) - (3*ArcTan[a*x]*Log[2/(1 + I*a*x)])/(a^4*c) + (ArcTan[a*x]^3*Log[2/
(1 + I*a*x)])/(a^4*c) - (((3*I)/2)*PolyLog[2, 1 - 2/(1 + I*a*x)])/(a^4*c) +
 (((3*I)/2)*ArcTan[a*x]^2*PolyLog[2, 1 - 2/(1 + I*a*x)])/(a^4*c) + (3*ArcTa
n[a*x]*PolyLog[3, 1 - 2/(1 + I*a*x)])/(2*a^4*c) - (((3*I)/4)*PolyLog[4, 1 -
 2/(1 + I*a*x)])/(a^4*c)
```


Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 4998

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[k + 1, u])/(2
*c*d), x] - Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[k + 1,
 u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
qQ[e, c^2*d] && EqQ[u^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{x^3 \tan^{-1}(ax)^3}{c+a^2cx^2}\,dx = -\frac{\int \frac{x\tan^{-1}(ax)^3}{c+a^2cx^2}dx}{a^2} + \frac{\int x\tan^{-1}(ax)^3\,dx}{a^2c}$$
$$= \frac{x^2\tan^{-1}(ax)^3}{2a^2c} + \frac{i\tan^{-1}(ax)^4}{4a^4c} + \frac{\int \frac{\tan^{-1}(ax)^3}{i-ax}dx}{a^3c} - \frac{3\int \frac{x^2\tan^{-1}(ax)^2}{1+a^2x^2}dx}{2ac}$$
$$= \frac{x^2\tan^{-1}(ax)^3}{2a^2c} + \frac{i\tan^{-1}(ax)^4}{4a^4c} + \frac{\tan^{-1}(ax)^3\log\left(\frac{2}{1+iax}\right)}{a^4c} - \frac{3\int \tan^{-1}(ax)^2\,dx}{2a^3c} + \frac{3\int \frac{\tan^{-1}(ax)^2}{1+a^2x^2}dx}{2a^3c}$$
$$= -\frac{3x\tan^{-1}(ax)^2}{2a^3c} + \frac{\tan^{-1}(ax)^3}{2a^4c} + \frac{x^2\tan^{-1}(ax)^3}{2a^2c} + \frac{i\tan^{-1}(ax)^4}{4a^4c} + \frac{\tan^{-1}(ax)^3\log\left(\frac{2}{1+iax}\right)}{a^4c} + \frac{3i\tan}{}$$
$$= -\frac{3i\tan^{-1}(ax)^2}{2a^4c} - \frac{3x\tan^{-1}(ax)^2}{2a^3c} + \frac{\tan^{-1}(ax)^3}{2a^4c} + \frac{x^2\tan^{-1}(ax)^3}{2a^2c} + \frac{i\tan^{-1}(ax)^4}{4a^4c} + \frac{\tan^{-1}(ax)^3\log}{a^4c}$$
$$= -\frac{3i\tan^{-1}(ax)^2}{2a^4c} - \frac{3x\tan^{-1}(ax)^2}{2a^3c} + \frac{\tan^{-1}(ax)^3}{2a^4c} + \frac{x^2\tan^{-1}(ax)^3}{2a^2c} + \frac{i\tan^{-1}(ax)^4}{4a^4c} - \frac{3\tan^{-1}(ax)\log}{a^4c}$$
$$= -\frac{3i\tan^{-1}(ax)^2}{2a^4c} - \frac{3x\tan^{-1}(ax)^2}{2a^3c} + \frac{\tan^{-1}(ax)^3}{2a^4c} + \frac{x^2\tan^{-1}(ax)^3}{2a^2c} + \frac{i\tan^{-1}(ax)^4}{4a^4c} - \frac{3\tan^{-1}(ax)\log}{a^4c}$$
$$= -\frac{3i\tan^{-1}(ax)^2}{2a^4c} - \frac{3x\tan^{-1}(ax)^2}{2a^3c} + \frac{\tan^{-1}(ax)^3}{2a^4c} + \frac{x^2\tan^{-1}(ax)^3}{2a^2c} + \frac{i\tan^{-1}(ax)^4}{4a^4c} - \frac{3\tan^{-1}(ax)\log}{a^4c}$$

Mathematica [A] time = 0.289916, size = 162, normalized size = 0.62

$$\frac{6\tan^{-1}(ax)\text{PolyLog}\left(3,-e^{2i\tan^{-1}(ax)}\right) - 6i\left(\tan^{-1}(ax)^2-1\right)\text{PolyLog}\left(2,-e^{2i\tan^{-1}(ax)}\right) + 3i\text{PolyLog}\left(4,-e^{2i\tan^{-1}(ax)}\right) + }{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*ArcTan[a*x]^3)/(c + a^2*c*x^2),x]
```

```
[Out] ((6*I)*ArcTan[a*x]^2 - 6*a*x*ArcTan[a*x]^2 + 2*(1 + a^2*x^2)*ArcTan[a*x]^3
- I*ArcTan[a*x]^4 - 12*ArcTan[a*x]*Log[1 + E^((2*I)*ArcTan[a*x])] + 4*ArcTa
n[a*x]^3*Log[1 + E^((2*I)*ArcTan[a*x])] - (6*I)*(-1 + ArcTan[a*x]^2)*PolyLo
g[2, -E^((2*I)*ArcTan[a*x])] + 6*ArcTan[a*x]*PolyLog[3, -E^((2*I)*ArcTan[a*
x])] + (3*I)*PolyLog[4, -E^((2*I)*ArcTan[a*x])])/(4*a^4*c)
```

Maple [A] time = 5.585, size = 292, normalized size = 1.1

$$\frac{-\frac{i}{4}(\arctan(ax))^4}{a^4c}+\frac{x^2(\arctan(ax))^3}{2a^2c}+\frac{(\arctan(ax))^3}{2a^4c}-\frac{3x(\arctan(ax))^2}{2a^3c}+\frac{\frac{3i}{2}(\arctan(ax))^2}{a^4c}+\frac{(\arctan(ax))^3}{a^4c}\ln\Bigg(\Bigg($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctan(a*x)^3/(a^2*c*x^2+c),x)
```

```
[Out] -1/4*I/a^4/c*arctan(a*x)^4+1/2*x^2*arctan(a*x)^3/a^2/c+1/2*arctan(a*x)^3/a^
4/c-3/2*x*arctan(a*x)^2/a^3/c+3/2*I/a^4/c*arctan(a*x)^2+1/a^4/c*arctan(a*x)
^3*ln((1+I*a*x)^2/(a^2*x^2+1)+1)-3/2*I/a^4/c*arctan(a*x)^2*polylog(2,-(1+I*
a*x)^2/(a^2*x^2+1))+3/2/a^4/c*arctan(a*x)*polylog(3,-(1+I*a*x)^2/(a^2*x^2+1
))+3/4*I/a^4/c*polylog(4,-(1+I*a*x)^2/(a^2*x^2+1))-3/a^4/c*arctan(a*x)*ln((
1+I*a*x)^2/(a^2*x^2+1)+1)+3/2*I/a^4/c*polylog(2,-(1+I*a*x)^2/(a^2*x^2+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^3\arctan(ax)^3}{a^2cx^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^3/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] integrate(x^3*arctan(a*x)^3/(a^2*c*x^2 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^3\arctan(ax)^3}{a^2cx^2+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^3/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(x^3*arctan(a*x)^3/(a^2*c*x^2 + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^3 \operatorname{atan}^3(ax)}{a^2x^2+1}\, dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atan(a*x)**3/(a**2*c*x**2+c),x)
```

```
[Out] Integral(x**3*atan(a*x)**3/(a**2*x**2 + 1), x)/c
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \arctan(ax)^3}{a^2cx^2 + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^3/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(x^3*arctan(a*x)^3/(a^2*c*x^2 + c), x)
```

3.389 $\int \frac{x^2 \tan^{-1}(ax)^3}{c+a^2cx^2} dx$

Optimal. Leaf size=130

$$\frac{3\text{PolyLog}\left(3,1-\frac{2}{1+iax}\right)}{2a^3c}+\frac{3i\tan^{-1}(ax)\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{a^3c}-\frac{\tan^{-1}(ax)^4}{4a^3c}+\frac{x\tan^{-1}(ax)^3}{a^2c}+\frac{i\tan^{-1}(ax)^3}{a^3c}+\frac{3\log\left(\frac{2}{1+i}\right.}{}$$

```
[Out] (I*ArcTan[a*x]^3)/(a^3*c) + (x*ArcTan[a*x]^3)/(a^2*c) - ArcTan[a*x]^4/(4*a^
3*c) + (3*ArcTan[a*x]^2*Log[2/(1 + I*a*x)])/(a^3*c) + ((3*I)*ArcTan[a*x]*Po
lyLog[2, 1 - 2/(1 + I*a*x)])/(a^3*c) + (3*PolyLog[3, 1 - 2/(1 + I*a*x)])/(2
*a^3*c)
```

Rubi [A] time = 0.245269, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {4916, 4846, 4920, 4854, 4884, 4994, 6610}

$$\frac{3\text{PolyLog}\left(3,1-\frac{2}{1+iax}\right)}{2a^3c}+\frac{3i\tan^{-1}(ax)\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{a^3c}-\frac{\tan^{-1}(ax)^4}{4a^3c}+\frac{x\tan^{-1}(ax)^3}{a^2c}+\frac{i\tan^{-1}(ax)^3}{a^3c}+\frac{3\log\left(\frac{2}{1+i}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcTan[a*x]^3)/(c + a^2*c*x^2),x]
```


```
[Out] (I*ArcTan[a*x]^3)/(a^3*c) + (x*ArcTan[a*x]^3)/(a^2*c) - ArcTan[a*x]^4/(4*a^
3*c) + (3*ArcTan[a*x]^2*Log[2/(1 + I*a*x)])/(a^3*c) + ((3*I)*ArcTan[a*x]*Po
lyLog[2, 1 - 2/(1 + I*a*x)])/(a^3*c) + (3*PolyLog[3, 1 - 2/(1 + I*a*x)])/(2
*a^3*c)
```


Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{x^2 \tan^{-1}(ax)^3}{c+a^2cx^2}\,dx = -\frac{\int \frac{\tan^{-1}(ax)^3}{c+a^2cx^2}\,dx}{a^2} + \frac{\int \tan^{-1}(ax)^3\,dx}{a^2c}$$
$$= \frac{x\tan^{-1}(ax)^3}{a^2c} - \frac{\tan^{-1}(ax)^4}{4a^3c} - \frac{3\int \frac{x\tan^{-1}(ax)^2}{1+a^2x^2}\,dx}{ac}$$
$$= \frac{i\tan^{-1}(ax)^3}{a^3c} + \frac{x\tan^{-1}(ax)^3}{a^2c} - \frac{\tan^{-1}(ax)^4}{4a^3c} + \frac{3\int \frac{\tan^{-1}(ax)^2}{i-ax}\,dx}{a^2c}$$
$$= \frac{i\tan^{-1}(ax)^3}{a^3c} + \frac{x\tan^{-1}(ax)^3}{a^2c} - \frac{\tan^{-1}(ax)^4}{4a^3c} + \frac{3\tan^{-1}(ax)^2\log\left(\frac{2}{1+iax}\right)}{a^3c} - \frac{6\int \frac{\tan^{-1}(ax)\log\left(\frac{2}{1+iax}\right)}{1+a^2x^2}\,dx}{a^2c}$$
$$= \frac{i\tan^{-1}(ax)^3}{a^3c} + \frac{x\tan^{-1}(ax)^3}{a^2c} - \frac{\tan^{-1}(ax)^4}{4a^3c} + \frac{3\tan^{-1}(ax)^2\log\left(\frac{2}{1+iax}\right)}{a^3c} + \frac{3i\tan^{-1}(ax)\mathrm{Li}_2\left(1-\frac{2}{1+ia\ldots}\right.}{a^3c}$$
$$= \frac{i\tan^{-1}(ax)^3}{a^3c} + \frac{x\tan^{-1}(ax)^3}{a^2c} - \frac{\tan^{-1}(ax)^4}{4a^3c} + \frac{3\tan^{-1}(ax)^2\log\left(\frac{2}{1+iax}\right)}{a^3c} + \frac{3i\tan^{-1}(ax)\mathrm{Li}_2\left(1-\frac{2}{1+ia\ldots}\right.}{a^3c}$$

Mathematica [A] time = 0.209472, size = 93, normalized size = 0.72

$$\frac{-3i\tan^{-1}(ax)\mathrm{PolyLog}\left(2,-e^{2i\tan^{-1}(ax)}\right) + \frac{3}{2}\mathrm{PolyLog}\left(3,-e^{2i\tan^{-1}(ax)}\right) - \frac{1}{4}\tan^{-1}(ax)^2\left(\tan^{-1}(ax)^2 + (-4ax+4i)\tan^{-1}(a\ldots\right.}{a^3c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*ArcTan[a*x]^3)/(c + a^2*c*x^2),x]
```

```
[Out] (-(ArcTan[a*x]^2*((4*I - 4*a*x)*ArcTan[a*x] + ArcTan[a*x]^2 - 12*Log[1 + E^
((2*I)*ArcTan[a*x])]))/4 - (3*I)*ArcTan[a*x]*PolyLog[2, -E^((2*I)*ArcTan[a*
x])] + (3*PolyLog[3, -E^((2*I)*ArcTan[a*x])])/2)/(a^3*c)
```

Maple [C] time = 1.033, size = 925, normalized size = 7.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(a*x)^3/(a^2*c*x^2+c),x)
```

```
[Out] x*arctan(a*x)^3/a^2/c-1/4*arctan(a*x)^4/a^3/c-3/2/a^3/c*arctan(a*x)^2*ln(a^
2*x^2+1)+3/a^3/c*arctan(a*x)^2*ln((1+I*a*x)/(a^2*x^2+1)^(1/2))+3/4*I/a^3/c*
arctan(a*x)^2*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(
a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2-3/2*I/a^3/c*arctan(a*x)^2*Pi*cs
gn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2-3
/4*I/a^3/c*arctan(a*x)^2*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^3+3/4*I/a^3/c*a
rctan(a*x)^2*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2
+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2+3/4*I/a^3/c*arctan(a*x)^2*Pi*csgn(I*((
1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3+3/4*I/a^3/c*arctan(a*x)^2*Pi*csgn(I*((1+I*a*
x)^2/(a^2*x^2+1)+1))^2*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)-3/4*I/a^3/c*ar
ctan(a*x)^2*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)/(a^2*x^2+1)
^(1/2))^2-3/4*I/a^3/c*arctan(a*x)^2*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I
*a*x)^2/(a^2*x^2+1)+1)^2)^3-3/4*I/a^3/c*arctan(a*x)^2*Pi*csgn(I/((1+I*a*x)^
2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2
*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)-I/a^3/c*arctan(a*x)^3+3/a^3/c*arctan
(a*x)^2*ln(2)+3/2*I/a^3/c*arctan(a*x)^2*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^
2*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+3/2/a^3/c*polylog(3,-(1+I*a*x)^2/(a^2
*x^2+1))-3*I/a^3/c*arctan(a*x)*polylog(2,-(1+I*a*x)^2/(a^2*x^2+1))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^3/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^2\arctan(ax)^3}{a^2cx^2+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^3/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(x^2*arctan(a*x)^3/(a^2*c*x^2 + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^2 \operatorname{atan}^3(ax)}{a^2x^2+1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(a*x)**3/(a**2*c*x**2+c),x)
```

```
[Out] Integral(x**2*atan(a*x)**3/(a**2*x**2 + 1), x)/c
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \arctan(ax)^3}{a^2cx^2 + c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^3/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctan(a*x)^3/(a^2*c*x^2 + c), x)
```

3.390 $\int \frac{x \tan^{-1}(ax)^3}{c+a^2cx^2} dx$

Optimal. Leaf size=138

$$\frac{3i\text{PolyLog}\left(4,1-\frac{2}{1+iax}\right)}{4a^2c}-\frac{3i\tan^{-1}(ax)^2\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{2a^2c}-\frac{3\tan^{-1}(ax)\text{PolyLog}\left(3,1-\frac{2}{1+iax}\right)}{2a^2c}-\frac{i\tan^{-1}(ax)^4}{4a^2c}-$$

```
[Out] ((-I/4)*ArcTan[a*x]^4)/(a^2*c) - (ArcTan[a*x]^3*Log[2/(1 + I*a*x)])/(a^2*c)
 - (((3*I)/2)*ArcTan[a*x]^2*PolyLog[2, 1 - 2/(1 + I*a*x)])/(a^2*c) - (3*Arc
Tan[a*x]*PolyLog[3, 1 - 2/(1 + I*a*x)])/(2*a^2*c) + (((3*I)/4)*PolyLog[4, 1
 - 2/(1 + I*a*x)])/(a^2*c)
```

Rubi [A] time = 0.217725, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {4920, 4854, 4884, 4994, 4998, 6610}

$$\frac{3i\text{PolyLog}\left(4,1-\frac{2}{1+iax}\right)}{4a^2c}-\frac{3i\tan^{-1}(ax)^2\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{2a^2c}-\frac{3\tan^{-1}(ax)\text{PolyLog}\left(3,1-\frac{2}{1+iax}\right)}{2a^2c}-\frac{i\tan^{-1}(ax)^4}{4a^2c}-$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTan[a*x]^3)/(c + a^2*c*x^2),x]
```

```
[Out] ((-I/4)*ArcTan[a*x]^4)/(a^2*c) - (ArcTan[a*x]^3*Log[2/(1 + I*a*x)])/(a^2*c)
 - (((3*I)/2)*ArcTan[a*x]^2*PolyLog[2, 1 - 2/(1 + I*a*x)])/(a^2*c) - (3*Arc
Tan[a*x]*PolyLog[3, 1 - 2/(1 + I*a*x)])/(2*a^2*c) + (((3*I)/4)*PolyLog[4, 1
 - 2/(1 + I*a*x)])/(a^2*c)
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
```

```
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 4998

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[k + 1, u])/(2
*c*d), x] - Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[k + 1,
 u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
qQ[e, c^2*d] && EqQ[u^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{x\tan^{-1}(ax)^3}{c+a^2cx^2}\,dx = -\frac{i\tan^{-1}(ax)^4}{4a^2c} - \frac{\int \frac{\tan^{-1}(ax)^3}{i-ax}\,dx}{ac}$$

$$= -\frac{i\tan^{-1}(ax)^4}{4a^2c} - \frac{\tan^{-1}(ax)^3\log\left(\frac{2}{1+iax}\right)}{a^2c} + \frac{3\int \frac{\tan^{-1}(ax)^2\log\left(\frac{2}{1+iax}\right)}{1+a^2x^2}\,dx}{ac}$$

$$= -\frac{i\tan^{-1}(ax)^4}{4a^2c} - \frac{\tan^{-1}(ax)^3\log\left(\frac{2}{1+iax}\right)}{a^2c} - \frac{3i\tan^{-1}(ax)^2\text{Li}_2\left(1-\frac{2}{1+iax}\right)}{2a^2c} + \frac{(3i)\int \frac{\tan^{-1}(ax)\text{Li}_2\left(1-\frac{2}{1+iax}\right.}{1+a^2x^2}}{ac}$$

$$= -\frac{i\tan^{-1}(ax)^4}{4a^2c} - \frac{\tan^{-1}(ax)^3\log\left(\frac{2}{1+iax}\right)}{a^2c} - \frac{3i\tan^{-1}(ax)^2\text{Li}_2\left(1-\frac{2}{1+iax}\right)}{2a^2c} - \frac{3\tan^{-1}(ax)\text{Li}_3\left(1-\frac{2}{1+ia}\right.}{2a^2c}$$

$$= -\frac{i\tan^{-1}(ax)^4}{4a^2c} - \frac{\tan^{-1}(ax)^3\log\left(\frac{2}{1+iax}\right)}{a^2c} - \frac{3i\tan^{-1}(ax)^2\text{Li}_2\left(1-\frac{2}{1+iax}\right)}{2a^2c} - \frac{3\tan^{-1}(ax)\text{Li}_3\left(1-\frac{2}{1+ia}\right.}{2a^2c}$$

Mathematica [A] time = 0.0118383, size = 149, normalized size = 1.08

$$\frac{3i\text{PolyLog}\left(4,\frac{ax+i}{ax-i}\right)}{4a^2c} - \frac{3i\tan^{-1}(ax)^2\text{PolyLog}\left(2,\frac{ax+i}{ax-i}\right)}{2a^2c} - \frac{3\tan^{-1}(ax)\text{PolyLog}\left(3,\frac{ax+i}{ax-i}\right)}{2a^2c} - \frac{i\tan^{-1}(ax)^4}{4a^2c} - \frac{\log\left(\frac{2i}{-ax+i}\right)\text{t}}{a^2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcTan[a*x]^3)/(c + a^2*c*x^2),x]
```

```
[Out] ((-I/4)*ArcTan[a*x]^4)/(a^2*c) - (ArcTan[a*x]^3*Log[(2*I)/(I - a*x)])/(a^2*
c) - (((3*I)/2)*ArcTan[a*x]^2*PolyLog[2, (I + a*x)/(-I + a*x)])/(a^2*c) - (
3*ArcTan[a*x]*PolyLog[3, (I + a*x)/(-I + a*x)])/(2*a^2*c) + (((3*I)/4)*Poly
Log[4, (I + a*x)/(-I + a*x)])/(a^2*c)
```

Maple [C] time = 1.132, size = 936, normalized size = 6.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(a*x)^3/(a^2*c*x^2+c),x)
```

```
[Out] 1/2/a^2/c*ln(a^2*x^2+1)*arctan(a*x)^3-1/a^2/c*arctan(a*x)^3*ln((1+I*a*x)/(a
^2*x^2+1)^(1/2))+1/4*I/a^2/c*arctan(a*x)^3*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1
```

```
)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3-3/4*I/a^2/c*polylog(4,-(1+I*a*x)^2/(a^2*
x^2+1))+1/4*I/a^2/c*arctan(a*x)^4+3/2*I/a^2/c*arctan(a*x)^2*polylog(2,-(1+I
*a*x)^2/(a^2*x^2+1))+1/4*I/a^2/c*arctan(a*x)^3*Pi*csgn(I*(1+I*a*x)^2/(a^2*x
^2+1))*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))^2+1/2*I/a^2/c*arctan(a*x)^3*Pi*c
sgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2-
1/4*I/a^2/c*arctan(a*x)^3*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*
x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2+1/4*I/a^2/c*arctan(a*x)^3
*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^3-1/2*I/a^2/c*arctan(a*x)^3*Pi*csgn(I*(
1+I*a*x)^2/(a^2*x^2+1))^2*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-1/4*I/a^2/c*a
rctan(a*x)^3*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a
^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2-1/a^2/c*arctan(a*x)^3*ln(2)-1/4*
I/a^2/c*arctan(a*x)^3*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3-3/2/a^2/c*
arctan(a*x)*polylog(3,-(1+I*a*x)^2/(a^2*x^2+1))-1/4*I/a^2/c*arctan(a*x)^3*P
i*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))^2*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^
2)+1/4*I/a^2/c*arctan(a*x)^3*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(
I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2
*x^2+1)+1)^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \arctan{(ax)}^3}{a^2cx^2 + c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^3/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] integrate(x*arctan(a*x)^3/(a^2*c*x^2 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x \arctan{(ax)}^3}{a^2cx^2 + c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^3/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(x*arctan(a*x)^3/(a^2*c*x^2 + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x \operatorname{atan}^{3}(ax)}{a^{2}x^{2}+1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(a*x)**3/(a**2*c*x**2+c),x)
```

```
[Out] Integral(x*atan(a*x)**3/(a**2*x**2 + 1), x)/c
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \arctan(ax)^{3}}{a^{2}cx^{2}+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^3/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(x*arctan(a*x)^3/(a^2*c*x^2 + c), x)
```

3.391 $\int \frac{\tan^{-1}(ax)^3}{c+a^2cx^2}\,dx$

Optimal. Leaf size=16

$$\frac{\tan^{-1}(ax)^4}{4ac}$$

```
[Out] ArcTan[a*x]^4/(4*a*c)
```

Rubi [A] time = 0.0242376, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {4884}

$$\frac{\tan^{-1}(ax)^4}{4ac}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^3/(c + a^2*c*x^2),x]
```

```
[Out] ArcTan[a*x]^4/(4*a*c)
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^3}{c+a^2cx^2}\,dx = \frac{\tan^{-1}(ax)^4}{4ac}$$

Mathematica [A] time = 0.0035491, size = 16, normalized size = 1.

$$\frac{\tan^{-1}(ax)^4}{4ac}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]^3/(c + a^2*c*x^2),x]
```

```
[Out] ArcTan[a*x]^4/(4*a*c)
```

Maple [A] time = 0.078, size = 15, normalized size = 0.9

$$\frac{(\arctan(ax))^4}{4ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^3/(a^2*c*x^2+c),x)
```

```
[Out] 1/4*arctan(a*x)^4/a/c
```

Maxima [A] time = 1.49116, size = 19, normalized size = 1.19

$$\frac{\arctan(ax)^4}{4ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/4*arctan(a*x)^4/(a*c)
```

Fricas [A] time = 1.78814, size = 34, normalized size = 2.12

$$\frac{\arctan(ax)^4}{4ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/4*arctan(a*x)^4/(a*c)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\operatorname{atan}^3(ax)}{a^2x^2+1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**3/(a**2*c*x**2+c),x)
```

```
[Out] Integral(atan(a*x)**3/(a**2*x**2 + 1), x)/c
```

Giac [A] time = 1.17405, size = 19, normalized size = 1.19

$$\frac{\arctan(ax)^4}{4\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] 1/4*arctan(a*x)^4/(a*c)
```

3.392 $\int \frac{\tan^{-1}(ax)^3}{x(c+a^2cx^2)}\,dx$

Optimal. Leaf size=124

$$\frac{3i\text{PolyLog}\left(4,-1+\frac{2}{1-iax}\right)}{4c}-\frac{3i\tan^{-1}(ax)^2\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{2c}+\frac{3\tan^{-1}(ax)\text{PolyLog}\left(3,-1+\frac{2}{1-iax}\right)}{2c}-\frac{i\tan^{-1}(a}{4c}$$

```
[Out] ((-I/4)*ArcTan[a*x]^4)/c + (ArcTan[a*x]^3*Log[2 - 2/(1 - I*a*x)])/c - (((3*
I)/2)*ArcTan[a*x]^2*PolyLog[2, -1 + 2/(1 - I*a*x)])/c + (3*ArcTan[a*x]*Poly
Log[3, -1 + 2/(1 - I*a*x)])/(2*c) + (((3*I)/4)*PolyLog[4, -1 + 2/(1 - I*a*x
)])/c
```

Rubi [A] time = 0.230754, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4924, 4868, 4884, 4992, 4996, 6610}

$$\frac{3i\text{PolyLog}\left(4,-1+\frac{2}{1-iax}\right)}{4c}-\frac{3i\tan^{-1}(ax)^2\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{2c}+\frac{3\tan^{-1}(ax)\text{PolyLog}\left(3,-1+\frac{2}{1-iax}\right)}{2c}-\frac{i\tan^{-1}(a}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^3/(x*(c + a^2*c*x^2)),x]
```

```
[Out] ((-I/4)*ArcTan[a*x]^4)/c + (ArcTan[a*x]^3*Log[2 - 2/(1 - I*a*x)])/c - (((3*
I)/2)*ArcTan[a*x]^2*PolyLog[2, -1 + 2/(1 - I*a*x)])/c + (3*ArcTan[a*x]*Poly
Log[3, -1 + 2/(1 - I*a*x)])/(2*c) + (((3*I)/4)*PolyLog[4, -1 + 2/(1 - I*a*x
)])/c
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
```

```
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4992

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d
] && EqQ[(1 - u)^2 - (1 - (2*I)/(I + c*x))^2, 0]
```

Rule 4996

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[k + 1, u])/(
2*c*d), x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[k + 1
, u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] &&
EqQ[e, c^2*d] && EqQ[u^2 - (1 - (2*I)/(I + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^3}{x\left(c+a^2cx^2\right)}\,dx = -\frac{i\tan^{-1}(ax)^4}{4c} + \frac{i\int \frac{\tan^{-1}(ax)^3}{x(i+ax)}\,dx}{c}$$

$$= -\frac{i\tan^{-1}(ax)^4}{4c} + \frac{\tan^{-1}(ax)^3\log\left(2-\frac{2}{1-iax}\right)}{c} - \frac{(3a)\int \frac{\tan^{-1}(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{1+a^2x^2}\,dx}{c}$$

$$= -\frac{i\tan^{-1}(ax)^4}{4c} + \frac{\tan^{-1}(ax)^3\log\left(2-\frac{2}{1-iax}\right)}{c} - \frac{3i\tan^{-1}(ax)^2\mathrm{Li}_2\left(-1+\frac{2}{1-iax}\right)}{2c} + \frac{(3ia)\int \frac{\tan^{-1}(ax)\mathrm{Li}_2}{1+a^2}}{c}$$

$$= -\frac{i\tan^{-1}(ax)^4}{4c} + \frac{\tan^{-1}(ax)^3\log\left(2-\frac{2}{1-iax}\right)}{c} - \frac{3i\tan^{-1}(ax)^2\mathrm{Li}_2\left(-1+\frac{2}{1-iax}\right)}{2c} + \frac{3\tan^{-1}(ax)\mathrm{Li}_3\left(-\right.}{2c}$$

$$= -\frac{i\tan^{-1}(ax)^4}{4c} + \frac{\tan^{-1}(ax)^3\log\left(2-\frac{2}{1-iax}\right)}{c} - \frac{3i\tan^{-1}(ax)^2\mathrm{Li}_2\left(-1+\frac{2}{1-iax}\right)}{2c} + \frac{3\tan^{-1}(ax)\mathrm{Li}_3\left(-\right.}{2c}$$

Mathematica [B] time = 0.0573479, size = 354, normalized size = 2.85

$$-\frac{3i\mathrm{PolyLog}\left(4,\frac{-ax-i}{ax-i}\right)}{4c} - \frac{3i\mathrm{PolyLog}\left(4,-\frac{ax+i}{-ax+i}\right)}{4c} + \frac{3i\mathrm{PolyLog}\left(4,\frac{ax+i}{ax-i}\right)}{4c} + \frac{3i\tan^{-1}(ax)^2\mathrm{PolyLog}\left(2,\frac{-ax-i}{ax-i}\right)}{2c} + \frac{3i\tan^{-1}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTan[a*x]^3/(x*(c + a^2*c*x^2)),x]
```

```
[Out] ((I/4)*ArcTan[a*x]^4)/c + (2*ArcTan[a*x]^3*ArcTanh[1 - (2*I)/(I - a*x)])/c
+ (ArcTan[a*x]^3*Log[(2*I)/(I - a*x)])/c + (((3*I)/2)*ArcTan[a*x]^2*PolyLog
[2, (-I - a*x)/(-I + a*x)])/c + (((3*I)/2)*ArcTan[a*x]^2*PolyLog[2, -((I +
a*x)/(I - a*x))])/c - (((3*I)/2)*ArcTan[a*x]^2*PolyLog[2, (I + a*x)/(-I + a
*x)])/c + (3*ArcTan[a*x]*PolyLog[3, (-I - a*x)/(-I + a*x)])/(2*c) + (3*ArcT
an[a*x]*PolyLog[3, -((I + a*x)/(I - a*x))])/(2*c) - (3*ArcTan[a*x]*PolyLog[
3, (I + a*x)/(-I + a*x)])/(2*c) - (((3*I)/4)*PolyLog[4, (-I - a*x)/(-I + a*
x)])/c - (((3*I)/4)*PolyLog[4, -((I + a*x)/(I - a*x))])/c + (((3*I)/4)*Poly
Log[4, (I + a*x)/(-I + a*x)])/c
```

Maple [C] time = 1.253, size = 1834, normalized size = 14.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^3/x/(a^2*c*x^2+c),x)
```

```
[Out] -1/2*I/c*arctan(a*x)^3*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(
a^2*x^2+1)+1))*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1)
)^2-1/2*I/c*arctan(a*x)^3*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1))*csgn(I*((1
+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2+1/4*I/c*arctan(a*x)
^3*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*
a*x)^2/(a^2*x^2+1)+1)^2)^2-1/4*I/c*arctan(a*x)^3*Pi*csgn(I*(1+I*a*x)/(a^2*x
^2+1)^(1/2))^2*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))+1/4*I/c*arctan(a*x)^3*Pi*csg
n(I*((1+I*a*x)^2/(a^2*x^2+1)+1))^2*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)+1/
2*I/c*arctan(a*x)^3*Pi*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*csgn(I*(1+I*a*x)
^2/(a^2*x^2+1))^2+1/4*I/c*arctan(a*x)^3*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+
1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2-1/2*I
/c*arctan(a*x)^3*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2
/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2+1/2*I/c*arctan(a*x)^3*Pi*csg
n(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(((1+I*a*x
)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))-1/2*I/c*arctan(a*x)^3*Pi*cs
gn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2-1
/4*I/c*arctan(a*x)^3*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a
*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+
1)^2)+1/2*I/c*arctan(a*x)^3*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1))*csgn(I/(
(1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2
/(a^2*x^2+1)+1))+1/2*I/c*arctan(a*x)^3*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1
)/((1+I*a*x)^2/(a^2*x^2+1)+1))^3+1/2*I/c*arctan(a*x)^3*Pi*csgn(((1+I*a*x)^2
/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^3+1/4*I/c*arctan(a*x)^3*Pi*csg
n(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3-1/4*I/c*arctan(a*x)^3*Pi*csgn(I*(1+I*a
*x)^2/(a^2*x^2+1))^3-1/2*I/c*arctan(a*x)^3*Pi*csgn(((1+I*a*x)^2/(a^2*x^2+1)
-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2-1/4*I/c*arctan(a*x)^3*Pi*csgn(I*(1+I*a*x
)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3-1/4*I*arctan(a*x)^4/c+1/c*
arctan(a*x)^3*ln(2)+1/c*arctan(a*x)^3*ln((1+I*a*x)/(a^2*x^2+1)^(1/2))-1/c*a
rctan(a*x)^3*ln((1+I*a*x)^2/(a^2*x^2+1)-1)+1/c*arctan(a*x)^3*ln(1-(1+I*a*x)
/(a^2*x^2+1)^(1/2))+6/c*arctan(a*x)*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))+
1/c*arctan(a*x)^3*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))+6/c*arctan(a*x)*polylog
(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I/c*polylog(4,(1+I*a*x)/(a^2*x^2+1)^(1/2
))+6*I/c*polylog(4,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-1/2/c*ln(a^2*x^2+1)*arctan
(a*x)^3+1/c*arctan(a*x)^3*ln(a*x)+1/2*I/c*arctan(a*x)^3*Pi-3*I/c*arctan(a*x
)^2*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))-3*I/c*arctan(a*x)^2*polylog(2,-(
1+I*a*x)/(a^2*x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(ax)^3}{(a^2cx^2+c)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] integrate(arctan(a*x)^3/((a^2*c*x^2 + c)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\arctan(ax)^3}{a^2cx^3+cx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(arctan(a*x)^3/(a^2*c*x^3 + c*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\operatorname{atan}^3(ax)}{a^2x^3+x}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**3/x/(a**2*c*x**2+c),x)
```

```
[Out] Integral(atan(a*x)**3/(a**2*x**3 + x), x)/c
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(ax)^3}{(a^2cx^2+c)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)^3/((a^2*c*x^2 + c)*x), x)
```

3.393 $\int \frac{\tan^{-1}(ax)^3}{x^2(c+a^2cx^2)}\,dx$

Optimal. Leaf size=122

$$\frac{3a\text{PolyLog}\left(3,-1+\frac{2}{1-iax}\right)}{2c}-\frac{3ia\tan^{-1}(ax)\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{c}-\frac{a\tan^{-1}(ax)^4}{4c}-\frac{ia\tan^{-1}(ax)^3}{c}-\frac{\tan^{-1}(ax)^3}{cx}+\frac{3}{}$$

```
[Out] ((-I)*a*ArcTan[a*x]^3)/c - ArcTan[a*x]^3/(c*x) - (a*ArcTan[a*x]^4)/(4*c) +
(3*a*ArcTan[a*x]^2*Log[2 - 2/(1 - I*a*x)])/c - ((3*I)*a*ArcTan[a*x]*PolyLog
[2, -1 + 2/(1 - I*a*x)])/c + (3*a*PolyLog[3, -1 + 2/(1 - I*a*x)])/(2*c)
```

Rubi [A] time = 0.285911, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {4918, 4852, 4924, 4868, 4884, 4992, 6610}

$$\frac{3a\text{PolyLog}\left(3,-1+\frac{2}{1-iax}\right)}{2c}-\frac{3ia\tan^{-1}(ax)\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{c}-\frac{a\tan^{-1}(ax)^4}{4c}-\frac{ia\tan^{-1}(ax)^3}{c}-\frac{\tan^{-1}(ax)^3}{cx}+\frac{3}{}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^3/(x^2*(c + a^2*c*x^2)),x]
```

```
[Out] ((-I)*a*ArcTan[a*x]^3)/c - ArcTan[a*x]^3/(c*x) - (a*ArcTan[a*x]^4)/(4*c) +
(3*a*ArcTan[a*x]^2*Log[2 - 2/(1 - I*a*x)])/c - ((3*I)*a*ArcTan[a*x]*PolyLog
[2, -1 + 2/(1 - I*a*x)])/c + (3*a*PolyLog[3, -1 + 2/(1 - I*a*x)])/(2*c)
```

Rule 4918

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/(d + e*x^2),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
```

```
erQ[m]) && NeQ[m, -1]
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4992

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d
] && EqQ[(1 - u)^2 - (1 - (2*I)/(I + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^3}{x^2\left(c+a^2cx^2\right)}\,dx = -\left(a^2\int \frac{\tan^{-1}(ax)^3}{c+a^2cx^2}\,dx\right)+\frac{\int \frac{\tan^{-1}(ax)^3}{x^2}\,dx}{c}$$
$$= -\frac{\tan^{-1}(ax)^3}{cx}-\frac{a\tan^{-1}(ax)^4}{4c}+\frac{(3a)\int \frac{\tan^{-1}(ax)^2}{x\left(1+a^2x^2\right)}\,dx}{c}$$
$$= -\frac{ia\tan^{-1}(ax)^3}{c}-\frac{\tan^{-1}(ax)^3}{cx}-\frac{a\tan^{-1}(ax)^4}{4c}+\frac{(3ia)\int \frac{\tan^{-1}(ax)^2}{x(i+ax)}\,dx}{c}$$
$$= -\frac{ia\tan^{-1}(ax)^3}{c}-\frac{\tan^{-1}(ax)^3}{cx}-\frac{a\tan^{-1}(ax)^4}{4c}+\frac{3a\tan^{-1}(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{c}-\frac{\left(6a^2\right)\int \frac{\tan^{-1}(ax}{1}}{c}$$
$$= -\frac{ia\tan^{-1}(ax)^3}{c}-\frac{\tan^{-1}(ax)^3}{cx}-\frac{a\tan^{-1}(ax)^4}{4c}+\frac{3a\tan^{-1}(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{c}-\frac{3ia\tan^{-1}(ax)\mathrm{Li}}{c}$$
$$= -\frac{ia\tan^{-1}(ax)^3}{c}-\frac{\tan^{-1}(ax)^3}{cx}-\frac{a\tan^{-1}(ax)^4}{4c}+\frac{3a\tan^{-1}(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{c}-\frac{3ia\tan^{-1}(ax)\mathrm{Li}}{c}$$

Mathematica [A] time = 0.163603, size = 108, normalized size = 0.89

$$\frac{a\left(3i\tan^{-1}(ax)\text{PolyLog}\left(2,e^{-2i\tan^{-1}(ax)}\right)+\frac{3}{2}\text{PolyLog}\left(3,e^{-2i\tan^{-1}(ax)}\right)-\frac{1}{4}\tan^{-1}(ax)^4-\frac{\tan^{-1}(ax)^3}{ax}+i\tan^{-1}(ax)^3+3\,\mathrm{ta}\right.}{c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTan[a*x]^3/(x^2*(c + a^2*c*x^2)),x]
```

```
[Out] (a*((-I/8)*Pi^3 + I*ArcTan[a*x]^3 - ArcTan[a*x]^3/(a*x) - ArcTan[a*x]^4/4 +
 3*ArcTan[a*x]^2*Log[1 - E^((-2*I)*ArcTan[a*x])] + (3*I)*ArcTan[a*x]*PolyLo
g[2, E^((-2*I)*ArcTan[a*x])] + (3*PolyLog[3, E^((-2*I)*ArcTan[a*x])])/2))/c
```

Maple [C] time = 0.959, size = 1829, normalized size = 15.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^3/x^2/(a^2*c*x^2+c),x)
```

```
[Out] 3/2*I*a/c*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1))*csgn(I/((1+I*a*x)^2/(a^2*x
^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*a
rctan(a*x)^2-3/4*I*a/c*arctan(a*x)^2*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^
2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x
)^2/(a^2*x^2+1)+1)^2)+6*a/c*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*a/c*p
olylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*I*a/c*arctan(a*x)*polylog(2,-(1+I*a
*x)/(a^2*x^2+1)^(1/2))-6*I*a/c*arctan(a*x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^
(1/2))+3/2*I*a/c*Pi*arctan(a*x)^2-arctan(a*x)^3/c/x-1/4*a*arctan(a*x)^4/c-3
/2*I*a/c*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1))*csgn(I*((1+I*a*x)^2/(a^2*x^
2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2*arctan(a*x)^2-3/4*I*a/c*arctan(a*x)^
2*Pi*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))^2*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))+
3/4*I*a/c*arctan(a*x)^2*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+
I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2+3/2*I*a/c*arctan(a*x)
^2*Pi*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^2
-3/2*I*a/c*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*
x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2*arctan(a*x)^2+3/2*I*a/c*Pi*csgn(I*
((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(((1+I*a*x)^2/
(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*arctan(a*x)^2+3/4*I*a/c*arctan(
a*x)^2*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))^2*csgn(I*((1+I*a*x)^2/(a^2*x^
2+1)+1)^2)+3/4*I*a/c*arctan(a*x)^2*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(
I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2-3/2*I*a/c*arctan
(a*x)^2*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2
+1)+1)^2)^2-3/2*I*a/c*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a
^2*x^2+1)+1))*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))
^2*arctan(a*x)^2-3/4*I*a/c*arctan(a*x)^2*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/
((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3+3/4*I*a/c*arctan(a*x)^2*Pi*csgn(I*((1+I*a*
x)^2/(a^2*x^2+1)+1)^2)^3+3/2*I*a/c*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((
1+I*a*x)^2/(a^2*x^2+1)+1))^3*arctan(a*x)^2-3/4*I*a/c*arctan(a*x)^2*Pi*csgn(
I*(1+I*a*x)^2/(a^2*x^2+1))^3-3/2*I*a/c*Pi*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/
((1+I*a*x)^2/(a^2*x^2+1)+1))^2*arctan(a*x)^2+3/2*I*a/c*Pi*csgn(((1+I*a*x)^2
/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^3*arctan(a*x)^2+3*a/c*arctan(a
*x)^2*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))+3*a/c*arctan(a*x)^2*ln(2)-3/2*a/c*a
rctan(a*x)^2*ln(a^2*x^2+1)+3*a/c*arctan(a*x)^2*ln(a*x)+3*a/c*arctan(a*x)^2*
ln((1+I*a*x)/(a^2*x^2+1)^(1/2))-3*a/c*arctan(a*x)^2*ln((1+I*a*x)^2/(a^2*x^2
+1)-1)+3*a/c*arctan(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*a*arctan(a*x
)^3/c
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x^2/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\arctan(ax)^3}{a^2cx^4+cx^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x^2/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(arctan(a*x)^3/(a^2*c*x^4 + c*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\operatorname{atan}^3(ax)}{a^2x^4+x^2}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**3/x**2/(a**2*c*x**2+c),x)
```

```
[Out] Integral(atan(a*x)**3/(a**2*x**4 + x**2), x)/c
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(ax)^3}{(a^2cx^2+c)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x^2/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)^3/((a^2*c*x^2 + c)*x^2), x)
```

3.394 $\int \frac{\tan^{-1}(ax)^3}{x^3(c+a^2cx^2)}\,dx$

Optimal. Leaf size=262

$$-\frac{3ia^2\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{2c}-\frac{3ia^2\text{PolyLog}\left(4,-1+\frac{2}{1-iax}\right)}{4c}+\frac{3ia^2\tan^{-1}(ax)^2\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{2c}-3a^2\tan^{-1}(ax$$

```
[Out] (((-3*I)/2)*a^2*ArcTan[a*x]^2)/c - (3*a*ArcTan[a*x]^2)/(2*c*x) - (a^2*ArcTa
n[a*x]^3)/(2*c) - ArcTan[a*x]^3/(2*c*x^2) + ((I/4)*a^2*ArcTan[a*x]^4)/c + (
3*a^2*ArcTan[a*x]*Log[2 - 2/(1 - I*a*x)])/c - (a^2*ArcTan[a*x]^3*Log[2 - 2/
(1 - I*a*x)])/c - (((3*I)/2)*a^2*PolyLog[2, -1 + 2/(1 - I*a*x)])/c + (((3*I
)/2)*a^2*ArcTan[a*x]^2*PolyLog[2, -1 + 2/(1 - I*a*x)])/c - (3*a^2*ArcTan[a*
x]*PolyLog[3, -1 + 2/(1 - I*a*x)])/(2*c) - (((3*I)/4)*a^2*PolyLog[4, -1 + 2
/(1 - I*a*x)])/c
```

Rubi [A] time = 0.510552, antiderivative size = 262, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {4918, 4852, 4924, 4868, 2447, 4884, 4992, 4996, 6610}

$$-\frac{3ia^2\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{2c}-\frac{3ia^2\text{PolyLog}\left(4,-1+\frac{2}{1-iax}\right)}{4c}+\frac{3ia^2\tan^{-1}(ax)^2\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{2c}-3a^2\tan^{-1}(ax$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^3/(x^3*(c + a^2*c*x^2)),x]
```

```
[Out] (((-3*I)/2)*a^2*ArcTan[a*x]^2)/c - (3*a*ArcTan[a*x]^2)/(2*c*x) - (a^2*ArcTa
n[a*x]^3)/(2*c) - ArcTan[a*x]^3/(2*c*x^2) + ((I/4)*a^2*ArcTan[a*x]^4)/c + (
3*a^2*ArcTan[a*x]*Log[2 - 2/(1 - I*a*x)])/c - (a^2*ArcTan[a*x]^3*Log[2 - 2/
(1 - I*a*x)])/c - (((3*I)/2)*a^2*PolyLog[2, -1 + 2/(1 - I*a*x)])/c + (((3*I
)/2)*a^2*ArcTan[a*x]^2*PolyLog[2, -1 + 2/(1 - I*a*x)])/c - (3*a^2*ArcTan[a*
x]*PolyLog[3, -1 + 2/(1 - I*a*x)])/(2*c) - (((3*I)/4)*a^2*PolyLog[4, -1 + 2
/(1 - I*a*x)])/c
```

Rule 4918

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/(d + e*x^2),
```

```
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4992

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d
```

```
] && EqQ[(1 - u)^2 - (1 - (2*I)/(I + c*x))^2, 0]
```

Rule 4996

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[k + 1, u])/(
2*c*d), x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[k + 1
, u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] &&
EqQ[e, c^2*d] && EqQ[u^2 - (1 - (2*I)/(I + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^3}{x^3\left(c+a^2cx^2\right)}\,dx = -\left(a^2\int \frac{\tan^{-1}(ax)^3}{x\left(c+a^2cx^2\right)}\,dx\right)+\frac{\int \frac{\tan^{-1}(ax)^3}{x^3}\,dx}{c}$$

$$= -\frac{\tan^{-1}(ax)^3}{2cx^2}+\frac{ia^2\tan^{-1}(ax)^4}{4c}+\frac{(3a)\int \frac{\tan^{-1}(ax)^2}{x^2\left(1+a^2x^2\right)}\,dx}{2c}-\frac{\left(ia^2\right)\int \frac{\tan^{-1}(ax)^3}{x(i+ax)}\,dx}{c}$$

$$= -\frac{\tan^{-1}(ax)^3}{2cx^2}+\frac{ia^2\tan^{-1}(ax)^4}{4c}-\frac{a^2\tan^{-1}(ax)^3\log\left(2-\frac{2}{1-iax}\right)}{c}+\frac{(3a)\int \frac{\tan^{-1}(ax)^2}{x^2}\,dx}{2c}-\frac{\left(3a^3\right)\int \frac{\tan}{1}}{2c}$$

$$= -\frac{3a\tan^{-1}(ax)^2}{2cx}-\frac{a^2\tan^{-1}(ax)^3}{2c}-\frac{\tan^{-1}(ax)^3}{2cx^2}+\frac{ia^2\tan^{-1}(ax)^4}{4c}-\frac{a^2\tan^{-1}(ax)^3\log\left(2-\frac{2}{1-iax}\right)}{c}$$

$$= -\frac{3ia^2\tan^{-1}(ax)^2}{2c}-\frac{3a\tan^{-1}(ax)^2}{2cx}-\frac{a^2\tan^{-1}(ax)^3}{2c}-\frac{\tan^{-1}(ax)^3}{2cx^2}+\frac{ia^2\tan^{-1}(ax)^4}{4c}-\frac{a^2\tan^{-1}(ax}{}$$

$$= -\frac{3ia^2\tan^{-1}(ax)^2}{2c}-\frac{3a\tan^{-1}(ax)^2}{2cx}-\frac{a^2\tan^{-1}(ax)^3}{2c}-\frac{\tan^{-1}(ax)^3}{2cx^2}+\frac{ia^2\tan^{-1}(ax)^4}{4c}+\frac{3a^2\tan^{-1}(a}{}$$

$$= -\frac{3ia^2\tan^{-1}(ax)^2}{2c}-\frac{3a\tan^{-1}(ax)^2}{2cx}-\frac{a^2\tan^{-1}(ax)^3}{2c}-\frac{\tan^{-1}(ax)^3}{2cx^2}+\frac{ia^2\tan^{-1}(ax)^4}{4c}+\frac{3a^2\tan^{-1}(a}{}$$

Mathematica [A] time = 0.397151, size = 189, normalized size = 0.72

$$\frac{ia^2\left(-96\tan^{-1}(ax)^2\text{PolyLog}\left(2,e^{-2i\tan^{-1}(ax)}\right)+96i\tan^{-1}(ax)\text{PolyLog}\left(3,e^{-2i\tan^{-1}(ax)}\right)-96\text{PolyLog}\left(2,e^{2i\tan^{-1}(ax)}\right)+\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTan[a*x]^3/(x^3*(c + a^2*c*x^2)),x]
```

```
[Out] ((I/64)*a^2*(Pi^4 - 96*ArcTan[a*x]^2 + ((96*I)*ArcTan[a*x]^2)/(a*x) + ((32*
I)*(1 + a^2*x^2)*ArcTan[a*x]^3)/(a^2*x^2) - 16*ArcTan[a*x]^4 + (64*I)*ArcTa
n[a*x]^3*Log[1 - E^((-2*I)*ArcTan[a*x])] - (192*I)*ArcTan[a*x]*Log[1 - E^((
2*I)*ArcTan[a*x])] - 96*ArcTan[a*x]^2*PolyLog[2, E^((-2*I)*ArcTan[a*x])] -
96*PolyLog[2, E^((2*I)*ArcTan[a*x])] + (96*I)*ArcTan[a*x]*PolyLog[3, E^((-2
*I)*ArcTan[a*x])] + 48*PolyLog[4, E^((-2*I)*ArcTan[a*x])]))/c
```

Maple [B] time = 7.375, size = 479, normalized size = 1.8

$$\frac{-\frac{3i}{2}a^2(\arctan(ax))^2}{c}-\frac{a^2(\arctan(ax))^3}{2c}-\frac{6ia^2}{c}\text{polylog}\left(4,(1+iax)\frac{1}{\sqrt{a^2x^2+1}}\right)-\frac{3a(\arctan(ax))^2}{2cx}-\frac{(\arctan(ax))^3}{2cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^3/x^3/(a^2*c*x^2+c),x)
```

```
[Out] -3/2*I*a^2*arctan(a*x)^2/c-1/2*a^2*arctan(a*x)^3/c-6*I*a^2/c*polylog(4,(1+I
*a*x)/(a^2*x^2+1)^(1/2))-3/2*a*arctan(a*x)^2/c/x-1/2*arctan(a*x)^3/c/x^2-3*
I*a^2/c*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))-a^2/c*arctan(a*x)^3*ln(1-(1+
I*a*x)/(a^2*x^2+1)^(1/2))+1/4*I*a^2*arctan(a*x)^4/c-6*a^2/c*arctan(a*x)*pol
ylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))+3*I*a^2/c*arctan(a*x)^2*polylog(2,-(1+I
*a*x)/(a^2*x^2+1)^(1/2))-a^2/c*arctan(a*x)^3*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/
2))-3*I*a^2/c*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*a^2/c*arctan(a*x)*p
olylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*I*a^2/c*polylog(4,-(1+I*a*x)/(a^2*
x^2+1)^(1/2))+3*I*a^2/c*arctan(a*x)^2*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2)
)+3*a^2/c*arctan(a*x)*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))+3*a^2/c*arctan(a*x)
*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(ax)^3}{(a^2cx^2+c)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x^3/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] integrate(arctan(a*x)^3/((a^2*c*x^2 + c)*x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\arctan(ax)^3}{a^2cx^5+cx^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x^3/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(arctan(a*x)^3/(a^2*c*x^5 + c*x^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\operatorname{atan}^3(ax)}{a^2x^5+x^3}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**3/x**3/(a**2*c*x**2+c),x)
```

```
[Out] Integral(atan(a*x)**3/(a**2*x**5 + x**3), x)/c
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(ax)^3}{(a^2cx^2+c)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x^3/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)^3/((a^2*c*x^2 + c)*x^3), x)
```

3.395 $\int \frac{\tan^{-1}(ax)^3}{x^4(c+a^2cx^2)}\,dx$

Optimal. Leaf size=227

$$-\frac{2a^3\text{PolyLog}\left(3,-1+\frac{2}{1-iax}\right)}{c}+\frac{4ia^3\tan^{-1}(ax)\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{c}-\frac{a^3\log\left(a^2x^2+1\right)}{2c}+\frac{a^3\log(x)}{c}+\frac{a^3\tan^{-1}(ax)^4}{4c}$$

```
[Out] -((a^2*ArcTan[a*x])/(c*x)) - (a^3*ArcTan[a*x]^2)/(2*c) - (a*ArcTan[a*x]^2)/
(2*c*x^2) + (((4*I)/3)*a^3*ArcTan[a*x]^3)/c - ArcTan[a*x]^3/(3*c*x^3) + (a^
2*ArcTan[a*x]^3)/(c*x) + (a^3*ArcTan[a*x]^4)/(4*c) + (a^3*Log[x])/c - (a^3*
Log[1 + a^2*x^2])/(2*c) - (4*a^3*ArcTan[a*x]^2*Log[2 - 2/(1 - I*a*x)])/c +
((4*I)*a^3*ArcTan[a*x]*PolyLog[2, -1 + 2/(1 - I*a*x)])/c - (2*a^3*PolyLog[3
, -1 + 2/(1 - I*a*x)])/c
```

Rubi [A] time = 0.722126, antiderivative size = 227, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 11, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4918, 4852, 266, 36, 29, 31, 4884, 4924, 4868, 4992, 6610}

$$-\frac{2a^3\text{PolyLog}\left(3,-1+\frac{2}{1-iax}\right)}{c}+\frac{4ia^3\tan^{-1}(ax)\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{c}-\frac{a^3\log\left(a^2x^2+1\right)}{2c}+\frac{a^3\log(x)}{c}+\frac{a^3\tan^{-1}(ax)^4}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^3/(x^4*(c + a^2*c*x^2)),x]
```



```
[Out] -((a^2*ArcTan[a*x])/(c*x)) - (a^3*ArcTan[a*x]^2)/(2*c) - (a*ArcTan[a*x]^2)/
(2*c*x^2) + (((4*I)/3)*a^3*ArcTan[a*x]^3)/c - ArcTan[a*x]^3/(3*c*x^3) + (a^
2*ArcTan[a*x]^3)/(c*x) + (a^3*ArcTan[a*x]^4)/(4*c) + (a^3*Log[x])/c - (a^3*
Log[1 + a^2*x^2])/(2*c) - (4*a^3*ArcTan[a*x]^2*Log[2 - 2/(1 - I*a*x)])/c +
((4*I)*a^3*ArcTan[a*x]*PolyLog[2, -1 + 2/(1 - I*a*x)])/c - (2*a^3*PolyLog[3
, -1 + 2/(1 - I*a*x)])/c
```


Rule 4918

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/(d + e*x^2),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 4992

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d
] && EqQ[(1 - u)^2 - (1 - (2*I)/(I + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^3}{x^4(c+a^2cx^2)}\,dx = -\left(a^2\int \frac{\tan^{-1}(ax)^3}{x^2(c+a^2cx^2)}\,dx\right) + \frac{\int \frac{\tan^{-1}(ax)^3}{x^4}\,dx}{c}$$

$$= -\frac{\tan^{-1}(ax)^3}{3cx^3} + a^4\int \frac{\tan^{-1}(ax)^3}{c+a^2cx^2}\,dx + \frac{a\int \frac{\tan^{-1}(ax)^2}{x^3(1+a^2x^2)}\,dx}{c} - \frac{a^2\int \frac{\tan^{-1}(ax)^3}{x^2}\,dx}{c}$$

$$= -\frac{\tan^{-1}(ax)^3}{3cx^3} + \frac{a^2\tan^{-1}(ax)^3}{cx} + \frac{a^3\tan^{-1}(ax)^4}{4c} + \frac{a\int \frac{\tan^{-1}(ax)^2}{x^3}\,dx}{c} - \frac{a^3\int \frac{\tan^{-1}(ax)^2}{x(1+a^2x^2)}\,dx}{c} - \frac{(3a^3)\int}{}$$

$$= -\frac{a\tan^{-1}(ax)^2}{2cx^2} + \frac{4ia^3\tan^{-1}(ax)^3}{3c} - \frac{\tan^{-1}(ax)^3}{3cx^3} + \frac{a^2\tan^{-1}(ax)^3}{cx} + \frac{a^3\tan^{-1}(ax)^4}{4c} + \frac{a^2\int \frac{\tan^{-1}(}{x^2(1+a^2}}{c}$$

$$= -\frac{a\tan^{-1}(ax)^2}{2cx^2} + \frac{4ia^3\tan^{-1}(ax)^3}{3c} - \frac{\tan^{-1}(ax)^3}{3cx^3} + \frac{a^2\tan^{-1}(ax)^3}{cx} + \frac{a^3\tan^{-1}(ax)^4}{4c} - \frac{4a^3\tan^{-1}(}{}$$

$$= -\frac{a^2\tan^{-1}(ax)}{cx} - \frac{a^3\tan^{-1}(ax)^2}{2c} - \frac{a\tan^{-1}(ax)^2}{2cx^2} + \frac{4ia^3\tan^{-1}(ax)^3}{3c} - \frac{\tan^{-1}(ax)^3}{3cx^3} + \frac{a^2\tan^{-1}(ax)}{cx}$$

$$= -\frac{a^2\tan^{-1}(ax)}{cx} - \frac{a^3\tan^{-1}(ax)^2}{2c} - \frac{a\tan^{-1}(ax)^2}{2cx^2} + \frac{4ia^3\tan^{-1}(ax)^3}{3c} - \frac{\tan^{-1}(ax)^3}{3cx^3} + \frac{a^2\tan^{-1}(ax)}{cx}$$

$$= -\frac{a^2\tan^{-1}(ax)}{cx} - \frac{a^3\tan^{-1}(ax)^2}{2c} - \frac{a\tan^{-1}(ax)^2}{2cx^2} + \frac{4ia^3\tan^{-1}(ax)^3}{3c} - \frac{\tan^{-1}(ax)^3}{3cx^3} + \frac{a^2\tan^{-1}(ax)}{cx}$$

$$= -\frac{a^2\tan^{-1}(ax)}{cx} - \frac{a^3\tan^{-1}(ax)^2}{2c} - \frac{a\tan^{-1}(ax)^2}{2cx^2} + \frac{4ia^3\tan^{-1}(ax)^3}{3c} - \frac{\tan^{-1}(ax)^3}{3cx^3} + \frac{a^2\tan^{-1}(ax)}{cx}$$

Mathematica [A] time = 0.483151, size = 180, normalized size = 0.79

$$\frac{a^3\left(-4i\tan^{-1}(ax)\text{PolyLog}\left(2,e^{-2i\tan^{-1}(ax)}\right) - 2\text{PolyLog}\left(3,e^{-2i\tan^{-1}(ax)}\right) + \log\left(\frac{ax}{\sqrt{a^2x^2+1}}\right) - \frac{\tan^{-1}(ax)^3}{3a^3x^3} - \frac{\tan^{-1}(ax)^2}{2a^2x^2} + \frac{1}{4}\text{ta}}{c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTan[a*x]^3/(x^4*(c + a^2*c*x^2)),x]
```

```
[Out] (a^3*((I/6)*Pi^3 - ArcTan[a*x]/(a*x) - ArcTan[a*x]^2/2 - ArcTan[a*x]^2/(2*a
^2*x^2) - ((4*I)/3)*ArcTan[a*x]^3 - ArcTan[a*x]^3/(3*a^3*x^3) + ArcTan[a*x]
^3/(a*x) + ArcTan[a*x]^4/4 - 4*ArcTan[a*x]^2*Log[1 - E^((-2*I)*ArcTan[a*x])
] + Log[(a*x)/Sqrt[1 + a^2*x^2]] - (4*I)*ArcTan[a*x]*PolyLog[2, E^((-2*I)*A
```

```
rcTan[a*x])] - 2*PolyLog[3, E^((-2*I)*ArcTan[a*x])]))/c
```

Maple [C] time = 7.244, size = 5574, normalized size = 24.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^3/x^4/(a^2*c*x^2+c),x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x^4/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\arctan\left(ax\right)^{3}}{a^{2}cx^{6}+cx^{4}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x^4/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(arctan(a*x)^3/(a^2*c*x^6 + c*x^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\operatorname{atan}^3(ax)}{a^2x^6+x^4}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**3/x**4/(a**2*c*x**2+c),x)
```

```
[Out] Integral(atan(a*x)**3/(a**2*x**6 + x**4), x)/c
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(ax)^3}{(a^2cx^2+c)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x^4/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)^3/((a^2*c*x^2 + c)*x^4), x)
```

3.396 $\int \frac{x^3 \tan^{-1}(ax)^3}{(c+a^2cx^2)^2} dx$

Optimal. Leaf size=270

$$\frac{3i\text{PolyLog}\left(4,1-\frac{2}{1+iax}\right)}{4a^4c^2}-\frac{3i\tan^{-1}(ax)^2\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{2a^4c^2}-\frac{3\tan^{-1}(ax)\text{PolyLog}\left(3,1-\frac{2}{1+iax}\right)}{2a^4c^2}+\frac{3x}{8a^3c^2\left(a^2x^2+1\right)}$$

```
[Out] (3*x)/(8*a^3*c^2*(1 + a^2*x^2)) + (3*ArcTan[a*x])/(8*a^4*c^2) - (3*ArcTan[a
*x])/(4*a^4*c^2*(1 + a^2*x^2)) - (3*x*ArcTan[a*x]^2)/(4*a^3*c^2*(1 + a^2*x^
2)) - ArcTan[a*x]^3/(4*a^4*c^2) + ArcTan[a*x]^3/(2*a^4*c^2*(1 + a^2*x^2)) -
 ((I/4)*ArcTan[a*x]^4)/(a^4*c^2) - (ArcTan[a*x]^3*Log[2/(1 + I*a*x)])/(a^4*
c^2) - (((3*I)/2)*ArcTan[a*x]^2*PolyLog[2, 1 - 2/(1 + I*a*x)])/(a^4*c^2) -
(3*ArcTan[a*x]*PolyLog[3, 1 - 2/(1 + I*a*x)])/(2*a^4*c^2) + (((3*I)/4)*Poly
Log[4, 1 - 2/(1 + I*a*x)])/(a^4*c^2)
```

Rubi [A] time = 0.412732, antiderivative size = 270, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 11, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4964, 4920, 4854, 4884, 4994, 4998, 6610, 4930, 4892, 199, 205}

$$\frac{3i\text{PolyLog}\left(4,1-\frac{2}{1+iax}\right)}{4a^4c^2}-\frac{3i\tan^{-1}(ax)^2\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{2a^4c^2}-\frac{3\tan^{-1}(ax)\text{PolyLog}\left(3,1-\frac{2}{1+iax}\right)}{2a^4c^2}+\frac{3x}{8a^3c^2\left(a^2x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcTan[a*x]^3)/(c + a^2*c*x^2)^2,x]
```


```
[Out] (3*x)/(8*a^3*c^2*(1 + a^2*x^2)) + (3*ArcTan[a*x])/(8*a^4*c^2) - (3*ArcTan[a
*x])/(4*a^4*c^2*(1 + a^2*x^2)) - (3*x*ArcTan[a*x]^2)/(4*a^3*c^2*(1 + a^2*x^
2)) - ArcTan[a*x]^3/(4*a^4*c^2) + ArcTan[a*x]^3/(2*a^4*c^2*(1 + a^2*x^2)) -
 ((I/4)*ArcTan[a*x]^4)/(a^4*c^2) - (ArcTan[a*x]^3*Log[2/(1 + I*a*x)])/(a^4*
c^2) - (((3*I)/2)*ArcTan[a*x]^2*PolyLog[2, 1 - 2/(1 + I*a*x)])/(a^4*c^2) -
(3*ArcTan[a*x]*PolyLog[3, 1 - 2/(1 + I*a*x)])/(2*a^4*c^2) + (((3*I)/4)*Poly
Log[4, 1 - 2/(1 + I*a*x)])/(a^4*c^2)
```


Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Arc
```

```
Tan[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan[c
*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p
, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 4998

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[k + 1, u])/(2
*c*d), x] - Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[k + 1,
 u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
qQ[e, c^2*d] && EqQ[u^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 4892

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[(x*(a + b*ArcTan[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*c*
p)/2, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(a +
 b*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^3 \tan^{-1}(ax)^3}{\left(c+a^2cx^2\right)^2}\,dx = -\frac{\int \frac{x\tan^{-1}(ax)^3}{\left(c+a^2cx^2\right)^2}\,dx}{a^2} + \frac{\int \frac{x\tan^{-1}(ax)^3}{c+a^2cx^2}\,dx}{a^2c}$$

$$= \frac{\tan^{-1}(ax)^3}{2a^4c^2\left(1+a^2x^2\right)} - \frac{i\tan^{-1}(ax)^4}{4a^4c^2} - \frac{3\int \frac{\tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^2}\,dx}{2a^3} - \frac{\int \frac{\tan^{-1}(ax)^3}{i-ax}\,dx}{a^3c^2}$$

$$= -\frac{3x\tan^{-1}(ax)^2}{4a^3c^2\left(1+a^2x^2\right)} - \frac{\tan^{-1}(ax)^3}{4a^4c^2} + \frac{\tan^{-1}(ax)^3}{2a^4c^2\left(1+a^2x^2\right)} - \frac{i\tan^{-1}(ax)^4}{4a^4c^2} - \frac{\tan^{-1}(ax)^3\log\left(\frac{2}{1+iax}\right)}{a^4c^2} + \cdots$$

$$= -\frac{3\tan^{-1}(ax)}{4a^4c^2\left(1+a^2x^2\right)} - \frac{3x\tan^{-1}(ax)^2}{4a^3c^2\left(1+a^2x^2\right)} - \frac{\tan^{-1}(ax)^3}{4a^4c^2} + \frac{\tan^{-1}(ax)^3}{2a^4c^2\left(1+a^2x^2\right)} - \frac{i\tan^{-1}(ax)^4}{4a^4c^2} - \cdots$$

$$= \frac{3x}{8a^3c^2\left(1+a^2x^2\right)} - \frac{3\tan^{-1}(ax)}{4a^4c^2\left(1+a^2x^2\right)} - \frac{3x\tan^{-1}(ax)^2}{4a^3c^2\left(1+a^2x^2\right)} - \frac{\tan^{-1}(ax)^3}{4a^4c^2} + \frac{\tan^{-1}(ax)^3}{2a^4c^2\left(1+a^2x^2\right)} - \cdots$$

$$= \frac{3x}{8a^3c^2\left(1+a^2x^2\right)} + \frac{3\tan^{-1}(ax)}{8a^4c^2} - \frac{3\tan^{-1}(ax)}{4a^4c^2\left(1+a^2x^2\right)} - \frac{3x\tan^{-1}(ax)^2}{4a^3c^2\left(1+a^2x^2\right)} - \frac{\tan^{-1}(ax)^3}{4a^4c^2} + \cdots$$

Mathematica [A] time = 0.188492, size = 156, normalized size = 0.58

$$24i\tan^{-1}(ax)^2\text{PolyLog}\left(2,-e^{2i\tan^{-1}(ax)}\right) - 24\tan^{-1}(ax)\text{PolyLog}\left(3,-e^{2i\tan^{-1}(ax)}\right) - 12i\text{PolyLog}\left(4,-e^{2i\tan^{-1}(ax)}\right) + 4i \cdots$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*ArcTan[a*x]^3)/(c + a^2*c*x^2)^2,x]
```

```
[Out] ((4*I)*ArcTan[a*x]^4 - 6*ArcTan[a*x]*Cos[2*ArcTan[a*x]] + 4*ArcTan[a*x]^3*C
os[2*ArcTan[a*x]] - 16*ArcTan[a*x]^3*Log[1 + E^((2*I)*ArcTan[a*x])] + (24*I
)*ArcTan[a*x]^2*PolyLog[2, -E^((2*I)*ArcTan[a*x])] - 24*ArcTan[a*x]*PolyLog
[3, -E^((2*I)*ArcTan[a*x])] - (12*I)*PolyLog[4, -E^((2*I)*ArcTan[a*x])] + 3
*Sin[2*ArcTan[a*x]] - 6*ArcTan[a*x]^2*Sin[2*ArcTan[a*x]])/(16*a^4*c^2)
```

Maple [C] time = 1.636, size = 1227, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctan(a*x)^3/(a^2*c*x^2+c)^2,x)
```

```
[Out] -3/2*I/a^3/c^2*arctan(a*x)^2/(8*a*x+8*I)*x+1/4*I/a^4/c^2*arctan(a*x)^3*Pi*c
sgn(I*(1+I*a*x)^2/(a^2*x^2+1))^3+1/4*I/a^4/c^2*arctan(a*x)^3*Pi*csgn(I*(1+I
*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3-1/4*I/a^4/c^2*arctan(a
*x)^3*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3+1/4*I/a^4/c^2*arctan(a*x)^
3*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a
*x)^2/(a^2*x^2+1)+1)^2)*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)+3/2*I/a^3/c^2
*arctan(a*x)^2/(8*a*x-8*I)*x+3/2/a^4/c^2/(16*a*x+16*I)+3/2/a^4/c^2/(16*a*x-
16*I)+1/2*I/a^4/c^2*arctan(a*x)^3*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))*cs
gn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2-1/2*I/a^4/c^2*arctan(a*x)^3*Pi*csgn(I
*(1+I*a*x)^2/(a^2*x^2+1))^2*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-1/4*arctan(
a*x)^3/a^4/c^2-3/2/a^4/c^2*arctan(a*x)^2/(8*a*x-8*I)-3/2/a^4/c^2*arctan(a*x
)^2/(8*a*x+8*I)-1/a^4/c^2*arctan(a*x)^3*ln(2)-1/a^4/c^2*arctan(a*x)^3*ln((1
+I*a*x)/(a^2*x^2+1)^(1/2))-3/2/a^4/c^2*arctan(a*x)*polylog(3,-(1+I*a*x)^2/(
a^2*x^2+1))+1/4*I/a^4/c^2*arctan(a*x)^4-3/4*I/a^4/c^2*polylog(4,-(1+I*a*x)^
2/(a^2*x^2+1))+1/2/a^4/c^2*arctan(a*x)^3*ln(a^2*x^2+1)+1/4*I/a^4/c^2*arctan
(a*x)^3*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/
2))^2-1/4*I/a^4/c^2*arctan(a*x)^3*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I
*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2-1/4*I/a^4/c^2*arc
tan(a*x)^3*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)
^2*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)-1/4*I/a^4/c^2*arctan(a*x)^3*Pi*csg
n(I*((1+I*a*x)^2/(a^2*x^2+1)+1))^2*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)+3/
16/a^3/c^2*arctan(a*x)/(a*x-I)*x+3/16/a^3/c^2*arctan(a*x)/(a*x+I)*x-3/2*I/a
^3/c^2/(16*a*x-16*I)*x+3/2*I/a^3/c^2/(16*a*x+16*I)*x+3/2*I/a^4/c^2*arctan(a
*x)^2*polylog(2,-(1+I*a*x)^2/(a^2*x^2+1))+3/16*I/a^4/c^2*arctan(a*x)/(a*x-I
)-3/16*I/a^4/c^2*arctan(a*x)/(a*x+I)+1/2*arctan(a*x)^3/a^4/c^2/(a^2*x^2+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \arctan(ax)^3}{\left(a^2cx^2 + c\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^3/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^3*arctan(a*x)^3/(a^2*c*x^2 + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^3\arctan(ax)^3}{a^4c^2x^4 + 2\,a^2c^2x^2 + c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^3/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] integral(x^3*arctan(a*x)^3/(a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^3\operatorname{atan}^3(ax)}{a^4x^4+2a^2x^2+1}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atan(a*x)**3/(a**2*c*x**2+c)**2,x)
```

```
[Out] Integral(x**3*atan(a*x)**3/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3\arctan(ax)^3}{\left(a^2cx^2 + c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^3/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(x^3*arctan(a*x)^3/(a^2*c*x^2 + c)^2, x)
```

3.397 $\int \frac{x^2 \tan^{-1}(ax)^3}{\left(c+a^2cx^2\right)^2} dx$

Optimal. Leaf size=135

$$\frac{3}{8a^3c^2\left(a^2x^2+1\right)}-\frac{x\tan^{-1}(ax)^3}{2a^2c^2\left(a^2x^2+1\right)}-\frac{3\tan^{-1}(ax)^2}{4a^3c^2\left(a^2x^2+1\right)}+\frac{3x\tan^{-1}(ax)}{4a^2c^2\left(a^2x^2+1\right)}+\frac{\tan^{-1}(ax)^4}{8a^3c^2}+\frac{3\tan^{-1}(ax)^2}{8a^3c^2}$$

```
[Out] 3/(8*a^3*c^2*(1 + a^2*x^2)) + (3*x*ArcTan[a*x])/(4*a^2*c^2*(1 + a^2*x^2)) +
 (3*ArcTan[a*x]^2)/(8*a^3*c^2) - (3*ArcTan[a*x]^2)/(4*a^3*c^2*(1 + a^2*x^2)
) - (x*ArcTan[a*x]^3)/(2*a^2*c^2*(1 + a^2*x^2)) + ArcTan[a*x]^4/(8*a^3*c^2)
```

Rubi [A] time = 0.143749, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4936, 4930, 4892, 261}

$$\frac{3}{8a^3c^2\left(a^2x^2+1\right)}-\frac{x\tan^{-1}(ax)^3}{2a^2c^2\left(a^2x^2+1\right)}-\frac{3\tan^{-1}(ax)^2}{4a^3c^2\left(a^2x^2+1\right)}+\frac{3x\tan^{-1}(ax)}{4a^2c^2\left(a^2x^2+1\right)}+\frac{\tan^{-1}(ax)^4}{8a^3c^2}+\frac{3\tan^{-1}(ax)^2}{8a^3c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcTan[a*x]^3)/(c + a^2*c*x^2)^2,x]
```


```
[Out] 3/(8*a^3*c^2*(1 + a^2*x^2)) + (3*x*ArcTan[a*x])/(4*a^2*c^2*(1 + a^2*x^2)) +
 (3*ArcTan[a*x]^2)/(8*a^3*c^2) - (3*ArcTan[a*x]^2)/(4*a^3*c^2*(1 + a^2*x^2)
) - (x*ArcTan[a*x]^3)/(2*a^2*c^2*(1 + a^2*x^2)) + ArcTan[a*x]^4/(8*a^3*c^2)
```


Rule 4936

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^2)/((d_) + (e_.)*(x_)^2)
^2, x_Symbol] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(2*b*c^3*d^2*(p + 1)), x]
 + (Dist[(b*p)/(2*c), Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(d + e*x^2)^2, x]
, x] - Simp[(x*(a + b*ArcTan[c*x])^p)/(2*c^2*d*(d + e*x^2)), x]) /; FreeQ[{
a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
```

```
0] && NeQ[q, -1]
```

Rule 4892

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[(x*(a + b*ArcTan[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*c*
p)/2, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(a +
 b*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2 \tan^{-1}(ax)^3}{\left(c+a^2cx^2\right)^2}\,dx &= -\frac{x\tan^{-1}(ax)^3}{2a^2c^2\left(1+a^2x^2\right)}+\frac{\tan^{-1}(ax)^4}{8a^3c^2}+\frac{3\int \frac{x\tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^2}\,dx}{2a}\\ &= -\frac{3\tan^{-1}(ax)^2}{4a^3c^2\left(1+a^2x^2\right)}-\frac{x\tan^{-1}(ax)^3}{2a^2c^2\left(1+a^2x^2\right)}+\frac{\tan^{-1}(ax)^4}{8a^3c^2}+\frac{3\int \frac{\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^2}\,dx}{2a^2}\\ &= \frac{3x\tan^{-1}(ax)}{4a^2c^2\left(1+a^2x^2\right)}+\frac{3\tan^{-1}(ax)^2}{8a^3c^2}-\frac{3\tan^{-1}(ax)^2}{4a^3c^2\left(1+a^2x^2\right)}-\frac{x\tan^{-1}(ax)^3}{2a^2c^2\left(1+a^2x^2\right)}+\frac{\tan^{-1}(ax)^4}{8a^3c^2}-\frac{3\int \frac{}{(c+a}}{4}\\ &= \frac{3}{8a^3c^2\left(1+a^2x^2\right)}+\frac{3x\tan^{-1}(ax)}{4a^2c^2\left(1+a^2x^2\right)}+\frac{3\tan^{-1}(ax)^2}{8a^3c^2}-\frac{3\tan^{-1}(ax)^2}{4a^3c^2\left(1+a^2x^2\right)}-\frac{x\tan^{-1}(ax)^3}{2a^2c^2\left(1+a^2x^2\right)}+\frac{\mathrm{ta}}{}\end{aligned}$$

Mathematica [A] time = 0.0596125, size = 74, normalized size = 0.55

$$\frac{\left(a^2x^2+1\right)\tan^{-1}(ax)^4+3\left(a^2x^2-1\right)\tan^{-1}(ax)^2-4ax\tan^{-1}(ax)^3+6ax\tan^{-1}(ax)+3}{8a^3c^2\left(a^2x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*ArcTan[a*x]^3)/(c + a^2*c*x^2)^2,x]
```

```
[Out] (3 + 6*a*x*ArcTan[a*x] + 3*(-1 + a^2*x^2)*ArcTan[a*x]^2 - 4*a*x*ArcTan[a*x]
^3 + (1 + a^2*x^2)*ArcTan[a*x]^4)/(8*a^3*c^2*(1 + a^2*x^2))
```

Maple [A] time = 0.158, size = 124, normalized size = 0.9

$$\frac{3}{8\,a^{3}c^{2}\left(a^{2}x^{2}+1\right)}+\frac{3\,x\arctan\left(ax\right)}{4\,a^{2}c^{2}\left(a^{2}x^{2}+1\right)}+\frac{3\,\left(\arctan\left(ax\right)\right)^{2}}{8\,a^{3}c^{2}}-\frac{3\,\left(\arctan\left(ax\right)\right)^{2}}{4\,a^{3}c^{2}\left(a^{2}x^{2}+1\right)}-\frac{x\left(\arctan\left(ax\right)\right)^{3}}{2\,a^{2}c^{2}\left(a^{2}x^{2}+1\right)}+\frac{\left(\arctan\left(ax\right)\right)^{4}}{8\,a^{3}c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(a*x)^3/(a^2*c*x^2+c)^2,x)
```

```
[Out] 3/8/a^3/c^2/(a^2*x^2+1)+3/4*x*arctan(a*x)/a^2/c^2/(a^2*x^2+1)+3/8*arctan(a*
x)^2/a^3/c^2-3/4*arctan(a*x)^2/a^3/c^2/(a^2*x^2+1)-1/2*x*arctan(a*x)^3/a^2/
c^2/(a^2*x^2+1)+1/8*arctan(a*x)^4/a^3/c^2
```

Maxima [A] time = 1.73454, size = 294, normalized size = 2.18

$$-\frac{1}{2}\left(\frac{x}{a^{4}c^{2}x^{2}+a^{2}c^{2}}-\frac{\arctan\left(ax\right)}{a^{3}c^{2}}\right)\arctan\left(ax\right)^{3}-\frac{3\left(\left(a^{2}x^{2}+1\right)\arctan\left(ax\right)^{2}+1\right)a\arctan\left(ax\right)^{2}}{4\left(a^{6}c^{2}x^{2}+a^{4}c^{2}\right)}-\frac{1}{8}\left(\frac{\left(\left(a^{2}x^{2}+1\right)\arctan\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^3/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(x/(a^4*c^2*x^2 + a^2*c^2) - arctan(a*x)/(a^3*c^2))*arctan(a*x)^3 - 3/
4*((a^2*x^2 + 1)*arctan(a*x)^2 + 1)*a*arctan(a*x)^2/(a^6*c^2*x^2 + a^4*c^2)
 - 1/8*(((a^2*x^2 + 1)*arctan(a*x)^4 + 3*(a^2*x^2 + 1)*arctan(a*x)^2 - 3)*a
^2/(a^8*c^2*x^2 + a^6*c^2) - 2*(2*(a^2*x^2 + 1)*arctan(a*x)^3 + 3*a*x + 3*(
a^2*x^2 + 1)*arctan(a*x))*a*arctan(a*x)/(a^7*c^2*x^2 + a^5*c^2))*a
```

Fricas [A] time = 1.92814, size = 186, normalized size = 1.38

$$-\frac{4\,ax\arctan\left(ax\right)^{3}-\left(a^{2}x^{2}+1\right)\arctan\left(ax\right)^{4}-6\,ax\arctan\left(ax\right)-3\left(a^{2}x^{2}-1\right)\arctan\left(ax\right)^{2}-3}{8\left(a^{5}c^{2}x^{2}+a^{3}c^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^3/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(4*a*x*arctan(a*x)^3 - (a^2*x^2 + 1)*arctan(a*x)^4 - 6*a*x*arctan(a*x)
 - 3*(a^2*x^2 - 1)*arctan(a*x)^2 - 3)/(a^5*c^2*x^2 + a^3*c^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^2 \operatorname{atan}^3(ax)}{a^4x^4+2a^2x^2+1}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(a*x)**3/(a**2*c*x**2+c)**2,x)
```

```
[Out] Integral(x**2*atan(a*x)**3/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \arctan(ax)^3}{\left(a^2cx^2+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^3/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*arctan(a*x)^3/(a^2*c*x^2 + c)^2, x)
```

3.398 $\int \frac{x\tan^{-1}(ax)^3}{\left(c+a^2cx^2\right)^2}\,dx$

Optimal. Leaf size=133

$$-\frac{3x}{8ac^2\left(a^2x^2+1\right)}-\frac{\tan^{-1}(ax)^3}{2a^2c^2\left(a^2x^2+1\right)}+\frac{3x\tan^{-1}(ax)^2}{4ac^2\left(a^2x^2+1\right)}+\frac{3\tan^{-1}(ax)}{4a^2c^2\left(a^2x^2+1\right)}+\frac{\tan^{-1}(ax)^3}{4a^2c^2}-\frac{3\tan^{-1}(ax)}{8a^2c^2}$$

```
[Out] (-3*x)/(8*a*c^2*(1 + a^2*x^2)) - (3*ArcTan[a*x])/(8*a^2*c^2) + (3*ArcTan[a*
x])/(4*a^2*c^2*(1 + a^2*x^2)) + (3*x*ArcTan[a*x]^2)/(4*a*c^2*(1 + a^2*x^2))
 + ArcTan[a*x]^3/(4*a^2*c^2) - ArcTan[a*x]^3/(2*a^2*c^2*(1 + a^2*x^2))
```

Rubi [A] time = 0.12161, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4930, 4892, 199, 205}

$$-\frac{3x}{8ac^2\left(a^2x^2+1\right)}-\frac{\tan^{-1}(ax)^3}{2a^2c^2\left(a^2x^2+1\right)}+\frac{3x\tan^{-1}(ax)^2}{4ac^2\left(a^2x^2+1\right)}+\frac{3\tan^{-1}(ax)}{4a^2c^2\left(a^2x^2+1\right)}+\frac{\tan^{-1}(ax)^3}{4a^2c^2}-\frac{3\tan^{-1}(ax)}{8a^2c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTan[a*x]^3)/(c + a^2*c*x^2)^2,x]
```

```
[Out] (-3*x)/(8*a*c^2*(1 + a^2*x^2)) - (3*ArcTan[a*x])/(8*a^2*c^2) + (3*ArcTan[a*
x])/(4*a^2*c^2*(1 + a^2*x^2)) + (3*x*ArcTan[a*x]^2)/(4*a*c^2*(1 + a^2*x^2))
 + ArcTan[a*x]^3/(4*a^2*c^2) - ArcTan[a*x]^3/(2*a^2*c^2*(1 + a^2*x^2))
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 4892

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[(x*(a + b*ArcTan[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*c*
p)/2, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(a +
 b*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d, e},
```

```
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x\tan^{-1}(ax)^3}{\left(c+a^2cx^2\right)^2}\,dx &= -\frac{\tan^{-1}(ax)^3}{2a^2c^2\left(1+a^2x^2\right)}+\frac{3\int \frac{\tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^2}\,dx}{2a}\\
&= \frac{3x\tan^{-1}(ax)^2}{4ac^2\left(1+a^2x^2\right)}+\frac{\tan^{-1}(ax)^3}{4a^2c^2}-\frac{\tan^{-1}(ax)^3}{2a^2c^2\left(1+a^2x^2\right)}-\frac{3}{2}\int \frac{x\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^2}\,dx\\
&= \frac{3\tan^{-1}(ax)}{4a^2c^2\left(1+a^2x^2\right)}+\frac{3x\tan^{-1}(ax)^2}{4ac^2\left(1+a^2x^2\right)}+\frac{\tan^{-1}(ax)^3}{4a^2c^2}-\frac{\tan^{-1}(ax)^3}{2a^2c^2\left(1+a^2x^2\right)}-\frac{3\int \frac{1}{\left(c+a^2cx^2\right)^2}\,dx}{4a}\\
&= -\frac{3x}{8ac^2\left(1+a^2x^2\right)}+\frac{3\tan^{-1}(ax)}{4a^2c^2\left(1+a^2x^2\right)}+\frac{3x\tan^{-1}(ax)^2}{4ac^2\left(1+a^2x^2\right)}+\frac{\tan^{-1}(ax)^3}{4a^2c^2}-\frac{\tan^{-1}(ax)^3}{2a^2c^2\left(1+a^2x^2\right)}-\frac{3\int \frac{}{c}}{8}\\
&= -\frac{3x}{8ac^2\left(1+a^2x^2\right)}-\frac{3\tan^{-1}(ax)}{8a^2c^2}+\frac{3\tan^{-1}(ax)}{4a^2c^2\left(1+a^2x^2\right)}+\frac{3x\tan^{-1}(ax)^2}{4ac^2\left(1+a^2x^2\right)}+\frac{\tan^{-1}(ax)^3}{4a^2c^2}-\frac{\tan^{-1}(a}{2a^2c^2\left(1+\right.}
\end{aligned}$$

Mathematica [A] time = 0.0416157, size = 68, normalized size = 0.51

$$\frac{2\left(a^2x^2-1\right)\tan^{-1}(ax)^3+\left(3-3a^2x^2\right)\tan^{-1}(ax)-3ax+6ax\tan^{-1}(ax)^2}{8a^2c^2\left(a^2x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcTan[a*x]^3)/(c + a^2*c*x^2)^2,x]
```

```
[Out] (-3*a*x + (3 - 3*a^2*x^2)*ArcTan[a*x] + 6*a*x*ArcTan[a*x]^2 + 2*(-1 + a^2*x
^2)*ArcTan[a*x]^3)/(8*a^2*c^2*(1 + a^2*x^2))
```

Maple [A] time = 0.126, size = 122, normalized size = 0.9

$$-\frac{3x}{8ac^2\left(a^2x^2+1\right)}-\frac{3\arctan\left(ax\right)}{8a^2c^2}+\frac{3\arctan\left(ax\right)}{4a^2c^2\left(a^2x^2+1\right)}+\frac{3x\left(\arctan\left(ax\right)\right)^2}{4ac^2\left(a^2x^2+1\right)}+\frac{\left(\arctan\left(ax\right)\right)^3}{4a^2c^2}-\frac{\left(\arctan\left(ax\right)\right)^3}{2a^2c^2\left(a^2x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(a*x)^3/(a^2*c*x^2+c)^2,x)
```

```
[Out] -3/8*x/a/c^2/(a^2*x^2+1)-3/8*arctan(a*x)/a^2/c^2+3/4*arctan(a*x)/a^2/c^2/(a
^2*x^2+1)+3/4*x*arctan(a*x)^2/a/c^2/(a^2*x^2+1)+1/4*arctan(a*x)^3/a^2/c^2-1
/2*arctan(a*x)^3/a^2/c^2/(a^2*x^2+1)
```

Maxima [A] time = 1.63228, size = 235, normalized size = 1.77

$$\frac{3\left(\frac{x}{a^2cx^2+c}+\frac{\arctan(ax)}{ac}\right)\arctan\left(ax\right)^2}{4ac}+\frac{\frac{\left(2\left(a^2x^2+1\right)\arctan(ax)^3-3ax-3\left(a^2x^2+1\right)\arctan(ax)\right)a^2}{a^5cx^2+a^3c}-\frac{6\left(\left(a^2x^2+1\right)\arctan(ax)^2-1\right)a\arctan(ax)}{a^4cx^2+a^2c}}{8ac}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^3/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 3/4*(x/(a^2*c*x^2 + c) + arctan(a*x)/(a*c))*arctan(a*x)^2/(a*c) + 1/8*((2*(
a^2*x^2 + 1)*arctan(a*x)^3 - 3*a*x - 3*(a^2*x^2 + 1)*arctan(a*x))*a^2/(a^5*
c*x^2 + a^3*c) - 6*((a^2*x^2 + 1)*arctan(a*x)^2 - 1)*a*arctan(a*x)/(a^4*c*x
^2 + a^2*c))/(a*c) - 1/2*arctan(a*x)^3/((a^2*c*x^2 + c)*a^2*c)
```

Fricas [A] time = 1.99894, size = 163, normalized size = 1.23

$$\frac{6ax\arctan\left(ax\right)^2+2\left(a^2x^2-1\right)\arctan\left(ax\right)^3-3ax-3\left(a^2x^2-1\right)\arctan\left(ax\right)}{8\left(a^4c^2x^2+a^2c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^3/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(6*a*x*arctan(a*x)^2 + 2*(a^2*x^2 - 1)*arctan(a*x)^3 - 3*a*x - 3*(a^2*x
^2 - 1)*arctan(a*x))/(a^4*c^2*x^2 + a^2*c^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x \operatorname{atan}^3(ax)}{a^4x^4+2a^2x^2+1}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(a*x)**3/(a**2*c*x**2+c)**2,x)
```

```
[Out] Integral(x*atan(a*x)**3/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \arctan(ax)^3}{(a^2cx^2+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^3/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(x*arctan(a*x)^3/(a^2*c*x^2 + c)^2, x)
```

3.399 $\int \frac{\tan^{-1}(ax)^3}{\left(c+a^2cx^2\right)^2} dx$

Optimal. Leaf size=129

$$-\frac{3}{8ac^2\left(a^2x^2+1\right)}+\frac{x\tan^{-1}(ax)^3}{2c^2\left(a^2x^2+1\right)}+\frac{3\tan^{-1}(ax)^2}{4ac^2\left(a^2x^2+1\right)}-\frac{3x\tan^{-1}(ax)}{4c^2\left(a^2x^2+1\right)}+\frac{\tan^{-1}(ax)^4}{8ac^2}-\frac{3\tan^{-1}(ax)^2}{8ac^2}$$

```
[Out] -3/(8*a*c^2*(1 + a^2*x^2)) - (3*x*ArcTan[a*x])/(4*c^2*(1 + a^2*x^2)) - (3*A
rcTan[a*x]^2)/(8*a*c^2) + (3*ArcTan[a*x]^2)/(4*a*c^2*(1 + a^2*x^2)) + (x*Ar
cTan[a*x]^3)/(2*c^2*(1 + a^2*x^2)) + ArcTan[a*x]^4/(8*a*c^2)
```

Rubi [A] time = 0.104387, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {4892, 4930, 261}

$$-\frac{3}{8ac^2\left(a^2x^2+1\right)}+\frac{x\tan^{-1}(ax)^3}{2c^2\left(a^2x^2+1\right)}+\frac{3\tan^{-1}(ax)^2}{4ac^2\left(a^2x^2+1\right)}-\frac{3x\tan^{-1}(ax)}{4c^2\left(a^2x^2+1\right)}+\frac{\tan^{-1}(ax)^4}{8ac^2}-\frac{3\tan^{-1}(ax)^2}{8ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^3/(c + a^2*c*x^2)^2,x]
```

```
[Out] -3/(8*a*c^2*(1 + a^2*x^2)) - (3*x*ArcTan[a*x])/(4*c^2*(1 + a^2*x^2)) - (3*A
rcTan[a*x]^2)/(8*a*c^2) + (3*ArcTan[a*x]^2)/(4*a*c^2*(1 + a^2*x^2)) + (x*Ar
cTan[a*x]^3)/(2*c^2*(1 + a^2*x^2)) + ArcTan[a*x]^4/(8*a*c^2)
```

Rule 4892

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[(x*(a + b*ArcTan[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*c*
p)/2, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(a +
 b*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
```

```
0] && NeQ[q, -1]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^{-1}(ax)^3}{\left(c+a^2cx^2\right)^2}\,dx &= \frac{x\tan^{-1}(ax)^3}{2c^2\left(1+a^2x^2\right)}+\frac{\tan^{-1}(ax)^4}{8ac^2}-\frac{1}{2}(3a)\int \frac{x\tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^2}\,dx\\
&= \frac{3\tan^{-1}(ax)^2}{4ac^2\left(1+a^2x^2\right)}+\frac{x\tan^{-1}(ax)^3}{2c^2\left(1+a^2x^2\right)}+\frac{\tan^{-1}(ax)^4}{8ac^2}-\frac{3}{2}\int \frac{\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^2}\,dx\\
&= -\frac{3x\tan^{-1}(ax)}{4c^2\left(1+a^2x^2\right)}-\frac{3\tan^{-1}(ax)^2}{8ac^2}+\frac{3\tan^{-1}(ax)^2}{4ac^2\left(1+a^2x^2\right)}+\frac{x\tan^{-1}(ax)^3}{2c^2\left(1+a^2x^2\right)}+\frac{\tan^{-1}(ax)^4}{8ac^2}+\frac{1}{4}(3a)\int \frac{}{(c-}\\
&= -\frac{3}{8ac^2\left(1+a^2x^2\right)}-\frac{3x\tan^{-1}(ax)}{4c^2\left(1+a^2x^2\right)}-\frac{3\tan^{-1}(ax)^2}{8ac^2}+\frac{3\tan^{-1}(ax)^2}{4ac^2\left(1+a^2x^2\right)}+\frac{x\tan^{-1}(ax)^3}{2c^2\left(1+a^2x^2\right)}+\frac{\tan^{-1}(a}{8ac^2}
\end{aligned}$$

Mathematica [A] time = 0.0307542, size = 71, normalized size = 0.55

$$\frac{\left(a^2x^2+1\right)\tan^{-1}(ax)^4+\left(3-3a^2x^2\right)\tan^{-1}(ax)^2+4ax\tan^{-1}(ax)^3-6ax\tan^{-1}(ax)-3}{8c^2\left(a^3x^2+a\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]^3/(c + a^2*c*x^2)^2,x]
```

```
[Out] (-3 - 6*a*x*ArcTan[a*x] + (3 - 3*a^2*x^2)*ArcTan[a*x]^2 + 4*a*x*ArcTan[a*x]
^3 + (1 + a^2*x^2)*ArcTan[a*x]^4)/(8*c^2*(a + a^3*x^2))
```

Maple [A] time = 0.144, size = 118, normalized size = 0.9

$$-\frac{3}{8\,ac^2\left(a^2x^2+1\right)}-\frac{3\,x\arctan(ax)}{4\,c^2\left(a^2x^2+1\right)}-\frac{3\,(\arctan(ax))^2}{8\,ac^2}+\frac{3\,(\arctan(ax))^2}{4\,ac^2\left(a^2x^2+1\right)}+\frac{x\,(\arctan(ax))^3}{2\,c^2\left(a^2x^2+1\right)}+\frac{(\arctan(ax))^4}{8\,ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^3/(a^2*c*x^2+c)^2,x)
```

```
[Out] -3/8/a/c^2/(a^2*x^2+1)-3/4*x*arctan(a*x)/c^2/(a^2*x^2+1)-3/8*arctan(a*x)^2/
a/c^2+3/4*arctan(a*x)^2/a/c^2/(a^2*x^2+1)+1/2*x*arctan(a*x)^3/c^2/(a^2*x^2+
1)+1/8*arctan(a*x)^4/a/c^2
```

Maximina [A] time = 1.76755, size = 288, normalized size = 2.23

$$\frac{1}{2}\left(\frac{x}{a^2c^2x^2+c^2}+\frac{\arctan(ax)}{ac^2}\right)\arctan(ax)^3-\frac{3\left(\left(a^2x^2+1\right)\arctan(ax)^2-1\right)a\arctan(ax)^2}{4\left(a^4c^2x^2+a^2c^2\right)}-\frac{1}{8}\left(\frac{\left(\left(a^2x^2+1\right)\arctan(a\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(x/(a^2*c^2*x^2 + c^2) + arctan(a*x)/(a*c^2))*arctan(a*x)^3 - 3/4*((a^2
*x^2 + 1)*arctan(a*x)^2 - 1)*a*arctan(a*x)^2/(a^4*c^2*x^2 + a^2*c^2) - 1/8*
(((a^2*x^2 + 1)*arctan(a*x)^4 - 3*(a^2*x^2 + 1)*arctan(a*x)^2 + 3)*a^2/(a^6
*c^2*x^2 + a^4*c^2) - 2*(2*(a^2*x^2 + 1)*arctan(a*x)^3 - 3*a*x - 3*(a^2*x^2
 + 1)*arctan(a*x))*a*arctan(a*x)/(a^5*c^2*x^2 + a^3*c^2))*a
```

Fricas [A] time = 1.85608, size = 182, normalized size = 1.41

$$\frac{4\,ax\arctan(ax)^3+\left(a^2x^2+1\right)\arctan(ax)^4-6\,ax\arctan(ax)-3\left(a^2x^2-1\right)\arctan(ax)^2-3}{8\left(a^3c^2x^2+ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*a*x*arctan(a*x)^3 + (a^2*x^2 + 1)*arctan(a*x)^4 - 6*a*x*arctan(a*x)
- 3*(a^2*x^2 - 1)*arctan(a*x)^2 - 3)/(a^3*c^2*x^2 + a*c^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\operatorname{atan}^3(ax)}{a^4x^4+2a^2x^2+1}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**3/(a**2*c*x**2+c)**2,x)
```

```
[Out] Integral(atan(a*x)**3/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(ax)^3}{\left(a^2cx^2+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)^3/(a^2*c*x^2 + c)^2, x)
```

3.400 $\int \frac{\tan^{-1}(ax)^3}{x\left(c+a^2cx^2\right)^2}\,dx$

Optimal. Leaf size=240

$$\frac{3i\text{PolyLog}\left(4,-1+\frac{2}{1-iax}\right)}{4c^2}-\frac{3i\tan^{-1}(ax)^2\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{2c^2}+\frac{3\tan^{-1}(ax)\text{PolyLog}\left(3,-1+\frac{2}{1-iax}\right)}{2c^2}+\frac{3ax}{8c^2\left(a^2x^2+\right.}$$

```
[Out] (3*a*x)/(8*c^2*(1 + a^2*x^2)) + (3*ArcTan[a*x])/(8*c^2) - (3*ArcTan[a*x])/(
4*c^2*(1 + a^2*x^2)) - (3*a*x*ArcTan[a*x]^2)/(4*c^2*(1 + a^2*x^2)) - ArcTan
[a*x]^3/(4*c^2) + ArcTan[a*x]^3/(2*c^2*(1 + a^2*x^2)) - ((I/4)*ArcTan[a*x]^
4)/c^2 + (ArcTan[a*x]^3*Log[2 - 2/(1 - I*a*x)])/c^2 - (((3*I)/2)*ArcTan[a*x
]^2*PolyLog[2, -1 + 2/(1 - I*a*x)])/c^2 + (3*ArcTan[a*x]*PolyLog[3, -1 + 2/
(1 - I*a*x)])/(2*c^2) + (((3*I)/4)*PolyLog[4, -1 + 2/(1 - I*a*x)])/c^2
```

Rubi [A] time = 0.428387, antiderivative size = 240, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 11, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4966, 4924, 4868, 4884, 4992, 4996, 6610, 4930, 4892, 199, 205}

$$\frac{3i\text{PolyLog}\left(4,-1+\frac{2}{1-iax}\right)}{4c^2}-\frac{3i\tan^{-1}(ax)^2\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{2c^2}+\frac{3\tan^{-1}(ax)\text{PolyLog}\left(3,-1+\frac{2}{1-iax}\right)}{2c^2}+\frac{3ax}{8c^2\left(a^2x^2+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^3/(x*(c + a^2*c*x^2)^2),x]
```


```
[Out] (3*a*x)/(8*c^2*(1 + a^2*x^2)) + (3*ArcTan[a*x])/(8*c^2) - (3*ArcTan[a*x])/(
4*c^2*(1 + a^2*x^2)) - (3*a*x*ArcTan[a*x]^2)/(4*c^2*(1 + a^2*x^2)) - ArcTan
[a*x]^3/(4*c^2) + ArcTan[a*x]^3/(2*c^2*(1 + a^2*x^2)) - ((I/4)*ArcTan[a*x]^
4)/c^2 + (ArcTan[a*x]^3*Log[2 - 2/(1 - I*a*x)])/c^2 - (((3*I)/2)*ArcTan[a*x
]^2*PolyLog[2, -1 + 2/(1 - I*a*x)])/c^2 + (3*ArcTan[a*x]*PolyLog[3, -1 + 2/
(1 - I*a*x)])/(2*c^2) + (((3*I)/4)*PolyLog[4, -1 + 2/(1 - I*a*x)])/c^2
```


Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*
x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^p
, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*q]
```

```
&& LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4992

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d
] && EqQ[(1 - u)^2 - (1 - (2*I)/(I + c*x))^2, 0]
```

Rule 4996

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[k + 1, u])/(
2*c*d), x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[k + 1
, u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] &&
EqQ[e, c^2*d] && EqQ[u^2 - (1 - (2*I)/(I + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 4892

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[(x*(a + b*ArcTan[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*c*
p)/2, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(a +
 b*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^3}{x\left(c+a^2cx^2\right)^2}\,dx = -\left(a^2\int \frac{x\tan^{-1}(ax)^3}{\left(c+a^2cx^2\right)^2}\,dx\right) + \frac{\int \frac{\tan^{-1}(ax)^3}{x\left(c+a^2cx^2\right)}\,dx}{c}$$

$$= \frac{\tan^{-1}(ax)^3}{2c^2\left(1+a^2x^2\right)} - \frac{i\tan^{-1}(ax)^4}{4c^2} - \frac{1}{2}(3a)\int \frac{\tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^2}\,dx + \frac{i\int \frac{\tan^{-1}(ax)^3}{x(i+ax)}\,dx}{c^2}$$

$$= -\frac{3ax\tan^{-1}(ax)^2}{4c^2\left(1+a^2x^2\right)} - \frac{\tan^{-1}(ax)^3}{4c^2} + \frac{\tan^{-1}(ax)^3}{2c^2\left(1+a^2x^2\right)} - \frac{i\tan^{-1}(ax)^4}{4c^2} + \frac{\tan^{-1}(ax)^3\log\left(2-\frac{2}{1-iax}\right)}{c^2} +$$

$$= -\frac{3\tan^{-1}(ax)}{4c^2\left(1+a^2x^2\right)} - \frac{3ax\tan^{-1}(ax)^2}{4c^2\left(1+a^2x^2\right)} - \frac{\tan^{-1}(ax)^3}{4c^2} + \frac{\tan^{-1}(ax)^3}{2c^2\left(1+a^2x^2\right)} - \frac{i\tan^{-1}(ax)^4}{4c^2} + \frac{\tan^{-1}(ax)^3\,\mathrm{l}}{}$$

$$= \frac{3ax}{8c^2\left(1+a^2x^2\right)} - \frac{3\tan^{-1}(ax)}{4c^2\left(1+a^2x^2\right)} - \frac{3ax\tan^{-1}(ax)^2}{4c^2\left(1+a^2x^2\right)} - \frac{\tan^{-1}(ax)^3}{4c^2} + \frac{\tan^{-1}(ax)^3}{2c^2\left(1+a^2x^2\right)} - \frac{i\tan^{-1}(ax)^4}{4c^2}$$

$$= \frac{3ax}{8c^2\left(1+a^2x^2\right)} + \frac{3\tan^{-1}(ax)}{8c^2} - \frac{3\tan^{-1}(ax)}{4c^2\left(1+a^2x^2\right)} - \frac{3ax\tan^{-1}(ax)^2}{4c^2\left(1+a^2x^2\right)} - \frac{\tan^{-1}(ax)^3}{4c^2} + \frac{\tan^{-1}(ax)^3}{2c^2\left(1+a^2x^2\right)}$$

Mathematica [A] time = 0.22227, size = 156, normalized size = 0.65

$$\frac{96i\tan^{-1}(ax)^2\mathrm{PolyLog}\left(2,e^{-2i\tan^{-1}(ax)}\right) + 96\tan^{-1}(ax)\mathrm{PolyLog}\left(3,e^{-2i\tan^{-1}(ax)}\right) - 48i\mathrm{PolyLog}\left(4,e^{-2i\tan^{-1}(ax)}\right) + 16i}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTan[a*x]^3/(x*(c + a^2*c*x^2)^2),x]
```

```
[Out] ((-I)*Pi^4 + (16*I)*ArcTan[a*x]^4 - 24*ArcTan[a*x]*Cos[2*ArcTan[a*x]] + 16*
ArcTan[a*x]^3*Cos[2*ArcTan[a*x]] + 64*ArcTan[a*x]^3*Log[1 - E^((-2*I)*ArcTa
n[a*x])] + (96*I)*ArcTan[a*x]^2*PolyLog[2, E^((-2*I)*ArcTan[a*x])] + 96*Arc
Tan[a*x]*PolyLog[3, E^((-2*I)*ArcTan[a*x])] - (48*I)*PolyLog[4, E^((-2*I)*A
rcTan[a*x])] + 12*Sin[2*ArcTan[a*x]] - 24*ArcTan[a*x]^2*Sin[2*ArcTan[a*x]])
/(64*c^2)
```

Maple [C] time = 1.752, size = 2089, normalized size = 8.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^3/x/(a^2*c*x^2+c)^2,x)
```

```
[Out] -1/4*arctan(a*x)^3/c^2+3/2/c^2/(16*a*x+16*I)+1/2*I/c^2*arctan(a*x)^3*Pi*csg
n(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^3+3/16/c^2*arc
tan(a*x)/(a*x-I)*a*x-1/4*I/c^2*arctan(a*x)^3*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2
+1))^3+3/2*I/c^2/(16*a*x+16*I)*a*x-1/4*I/c^2*arctan(a*x)^3*Pi*csgn(I*(1+I*a
*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3+1/c^2*arctan(a*x)^3*ln(2
)+6/c^2*arctan(a*x)*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+1/c^2*arctan(a*
x)^3*ln((1+I*a*x)/(a^2*x^2+1)^(1/2))-1/c^2*arctan(a*x)^3*ln((1+I*a*x)^2/(a^
2*x^2+1)-1)+1/c^2*arctan(a*x)^3*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))+6/c^2*arc
tan(a*x)*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I/c^2*polylog(4,(1+I*a*x)
/(a^2*x^2+1)^(1/2))+6*I/c^2*polylog(4,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-3/2/c^2
*arctan(a*x)^2/(8*a*x+8*I)-3/2/c^2*arctan(a*x)^2/(8*a*x-8*I)+1/c^2*arctan(a
*x)^3*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))+3/16/c^2*arctan(a*x)/(a*x+I)*a*x+3/
2/c^2/(16*a*x-16*I)+1/2*arctan(a*x)^3/c^2/(a^2*x^2+1)-1/4*I*arctan(a*x)^4/c
^2-1/2/c^2*arctan(a*x)^3*ln(a^2*x^2+1)+1/c^2*arctan(a*x)^3*ln(a*x)+1/4*I/c^
2*arctan(a*x)^3*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2
/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2-1/4*I/c^2*arctan(a*x)^3*Pi*cs
gn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))^2*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))-1/2*I/c
^2*arctan(a*x)^3*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^
2+1)+1))*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2+1/
4*I/c^2*arctan(a*x)^3*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))^2*csgn(I*((1+I
*a*x)^2/(a^2*x^2+1)+1)^2)+1/2*I/c^2*arctan(a*x)^3*Pi*csgn(I*(1+I*a*x)/(a^2*
x^2+1)^(1/2))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^2+1/4*I/c^2*arctan(a*x)^3*Pi*
csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2
/(a^2*x^2+1)+1)^2)^2-1/2*I/c^2*arctan(a*x)^3*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^
2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2-
1/2*I/c^2*arctan(a*x)^3*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1))*csgn(I*((1+I
*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2+1/2*I/c^2*arctan(a*x)
^3*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(
((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))-1/2*I/c^2*arctan(a
*x)^3*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1
)+1)^2)^2-3/2*I/c^2*arctan(a*x)^2/(8*a*x+8*I)*a*x+3/2*I/c^2*arctan(a*x)^2/(
8*a*x-8*I)*a*x-1/2*I/c^2*arctan(a*x)^3*Pi*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/
((1+I*a*x)^2/(a^2*x^2+1)+1))^2+1/2*I/c^2*arctan(a*x)^3*Pi*csgn(((1+I*a*x)^2
/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^3+1/4*I/c^2*arctan(a*x)^3*Pi*c
sgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3-3/2*I/c^2/(16*a*x-16*I)*a*x-3*I/c^2*
arctan(a*x)^2*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+3/16*I/c^2*arctan(a*x
```

```
)/(a*x-I)-3/16*I/c^2*arctan(a*x)/(a*x+I)+1/2*I/c^2*arctan(a*x)^3*Pi-3*I/c^2
*arctan(a*x)^2*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))-1/4*I/c^2*arctan(a*x)
^3*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))
*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)+1/2*I/c^2*ar
ctan(a*x)^3*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1))*csgn(I/((1+I*a*x)^2/(a^2
*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(ax)^3}{\left(a^2cx^2+c\right)^2 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(arctan(a*x)^3/((a^2*c*x^2 + c)^2*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\arctan(ax)^3}{a^4c^2x^5+2\,a^2c^2x^3+c^2x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] integral(arctan(a*x)^3/(a^4*c^2*x^5 + 2*a^2*c^2*x^3 + c^2*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\operatorname{atan}^3(ax)}{a^4x^5+2a^2x^3+x}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**3/x/(a**2*c*x**2+c)**2,x)
```

```
[Out] Integral(atan(a*x)**3/(a**4*x**5 + 2*a**2*x**3 + x), x)/c**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(ax)^3}{(a^2cx^2 + c)^2 x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)^3/((a^2*c*x^2 + c)^2*x), x)
```

3.401 $\int \frac{\tan^{-1}(ax)^3}{x^2(c+a^2cx^2)^2}\,dx$

Optimal. Leaf size=234

$$\frac{3a\text{PolyLog}\left(3,-1+\frac{2}{1-iax}\right)}{2c^2}-\frac{3ia\tan^{-1}(ax)\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{c^2}+\frac{3a}{8c^2\left(a^2x^2+1\right)}-\frac{a^2x\tan^{-1}(ax)^3}{2c^2\left(a^2x^2+1\right)}-\frac{3a\tan^{-1}(ax)}{4c^2\left(a^2x^2+1\right)}$$

```
[Out] (3*a)/(8*c^2*(1 + a^2*x^2)) + (3*a^2*x*ArcTan[a*x])/(4*c^2*(1 + a^2*x^2)) +
 (3*a*ArcTan[a*x]^2)/(8*c^2) - (3*a*ArcTan[a*x]^2)/(4*c^2*(1 + a^2*x^2)) -
(I*a*ArcTan[a*x]^3)/c^2 - ArcTan[a*x]^3/(c^2*x) - (a^2*x*ArcTan[a*x]^3)/(2*
c^2*(1 + a^2*x^2)) - (3*a*ArcTan[a*x]^4)/(8*c^2) + (3*a*ArcTan[a*x]^2*Log[2
 - 2/(1 - I*a*x)])/c^2 - ((3*I)*a*ArcTan[a*x]*PolyLog[2, -1 + 2/(1 - I*a*x)
])/c^2 + (3*a*PolyLog[3, -1 + 2/(1 - I*a*x)])/(2*c^2)
```

Rubi [A] time = 0.468147, antiderivative size = 234, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 11, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4966, 4918, 4852, 4924, 4868, 4884, 4992, 6610, 4892, 4930, 261}

$$\frac{3a\text{PolyLog}\left(3,-1+\frac{2}{1-iax}\right)}{2c^2}-\frac{3ia\tan^{-1}(ax)\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{c^2}+\frac{3a}{8c^2\left(a^2x^2+1\right)}-\frac{a^2x\tan^{-1}(ax)^3}{2c^2\left(a^2x^2+1\right)}-\frac{3a\tan^{-1}(ax)}{4c^2\left(a^2x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^3/(x^2*(c + a^2*c*x^2)^2),x]
```

```
[Out] (3*a)/(8*c^2*(1 + a^2*x^2)) + (3*a^2*x*ArcTan[a*x])/(4*c^2*(1 + a^2*x^2)) +
 (3*a*ArcTan[a*x]^2)/(8*c^2) - (3*a*ArcTan[a*x]^2)/(4*c^2*(1 + a^2*x^2)) -
(I*a*ArcTan[a*x]^3)/c^2 - ArcTan[a*x]^3/(c^2*x) - (a^2*x*ArcTan[a*x]^3)/(2*
c^2*(1 + a^2*x^2)) - (3*a*ArcTan[a*x]^4)/(8*c^2) + (3*a*ArcTan[a*x]^2*Log[2
 - 2/(1 - I*a*x)])/c^2 - ((3*I)*a*ArcTan[a*x]*PolyLog[2, -1 + 2/(1 - I*a*x)
])/c^2 + (3*a*PolyLog[3, -1 + 2/(1 - I*a*x)])/(2*c^2)
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*
x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^p
, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*q]
```

```
 && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 4918

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/(d + e*x^2),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4992

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d
```

```
] && EqQ[(1 - u)^2 - (1 - (2*I)/(I + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 4892

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[(x*(a + b*ArcTan[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*c*
p)/2, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(a +
 b*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^3}{x^2\left(c+a^2cx^2\right)^2}\,dx = -\left(a^2\int \frac{\tan^{-1}(ax)^3}{\left(c+a^2cx^2\right)^2}\,dx\right) + \frac{\int \frac{\tan^{-1}(ax)^3}{x^2\left(c+a^2cx^2\right)}\,dx}{c}$$

$$= -\frac{a^2x\tan^{-1}(ax)^3}{2c^2\left(1+a^2x^2\right)} - \frac{a\tan^{-1}(ax)^4}{8c^2} + \frac{1}{2}\left(3a^3\right)\int \frac{x\tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^2}\,dx + \frac{\int \frac{\tan^{-1}(ax)^3}{x^2}\,dx}{c^2} - \frac{a^2\int \frac{\tan^{-1}(ax)^3}{c+a^2cx^2}}{c}$$

$$= -\frac{3a\tan^{-1}(ax)^2}{4c^2\left(1+a^2x^2\right)} - \frac{\tan^{-1}(ax)^3}{c^2x} - \frac{a^2x\tan^{-1}(ax)^3}{2c^2\left(1+a^2x^2\right)} - \frac{3a\tan^{-1}(ax)^4}{8c^2} + \frac{1}{2}\left(3a^2\right)\int \frac{\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^2}\,dx +$$

$$= \frac{3a^2x\tan^{-1}(ax)}{4c^2\left(1+a^2x^2\right)} + \frac{3a\tan^{-1}(ax)^2}{8c^2} - \frac{3a\tan^{-1}(ax)^2}{4c^2\left(1+a^2x^2\right)} - \frac{ia\tan^{-1}(ax)^3}{c^2} - \frac{\tan^{-1}(ax)^3}{c^2x} - \frac{a^2x\tan^{-1}(ax}{2c^2\left(1+a^2x^2\right.}$$

$$= \frac{3a}{8c^2\left(1+a^2x^2\right)} + \frac{3a^2x\tan^{-1}(ax)}{4c^2\left(1+a^2x^2\right)} + \frac{3a\tan^{-1}(ax)^2}{8c^2} - \frac{3a\tan^{-1}(ax)^2}{4c^2\left(1+a^2x^2\right)} - \frac{ia\tan^{-1}(ax)^3}{c^2} - \frac{\tan^{-1}(ax)}{c^2x}$$

$$= \frac{3a}{8c^2\left(1+a^2x^2\right)} + \frac{3a^2x\tan^{-1}(ax)}{4c^2\left(1+a^2x^2\right)} + \frac{3a\tan^{-1}(ax)^2}{8c^2} - \frac{3a\tan^{-1}(ax)^2}{4c^2\left(1+a^2x^2\right)} - \frac{ia\tan^{-1}(ax)^3}{c^2} - \frac{\tan^{-1}(ax)}{c^2x}$$

$$= \frac{3a}{8c^2\left(1+a^2x^2\right)} + \frac{3a^2x\tan^{-1}(ax)}{4c^2\left(1+a^2x^2\right)} + \frac{3a\tan^{-1}(ax)^2}{8c^2} - \frac{3a\tan^{-1}(ax)^2}{4c^2\left(1+a^2x^2\right)} - \frac{ia\tan^{-1}(ax)^3}{c^2} - \frac{\tan^{-1}(ax)}{c^2x}$$

Mathematica [A] time = 0.340198, size = 157, normalized size = 0.67

$$a\left(48i\tan^{-1}(ax)\text{PolyLog}\left(2,e^{-2i\tan^{-1}(ax)}\right) + 24\text{PolyLog}\left(3,e^{-2i\tan^{-1}(ax)}\right) - 6\tan^{-1}(ax)^4 - \frac{16\tan^{-1}(ax)^3}{ax} + 16i\tan^{-1}(ax)^3 +$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTan[a*x]^3/(x^2*(c + a^2*c*x^2)^2),x]
```

```
[Out] (a*((-2*I)*Pi^3 + (16*I)*ArcTan[a*x]^3 - (16*ArcTan[a*x]^3)/(a*x) - 6*ArcTa
n[a*x]^4 + 3*Cos[2*ArcTan[a*x]] - 6*ArcTan[a*x]^2*Cos[2*ArcTan[a*x]] + 48*A
rcTan[a*x]^2*Log[1 - E^((-2*I)*ArcTan[a*x])] + (48*I)*ArcTan[a*x]*PolyLog[2
, E^((-2*I)*ArcTan[a*x])] + 24*PolyLog[3, E^((-2*I)*ArcTan[a*x])] + 6*ArcTa
n[a*x]*Sin[2*ArcTan[a*x]] - 4*ArcTan[a*x]^3*Sin[2*ArcTan[a*x]]))/(16*c^2)
```

Maple [C] time = 1.958, size = 2038, normalized size = 8.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^3/x^2/(a^2*c*x^2+c)^2,x)
```

```
[Out] 3/2*I*a/c^2*Pi*arctan(a*x)^2*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*csgn(I*(1+
I*a*x)^2/(a^2*x^2+1))^2+3/2*I*a/c^2*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/(
(1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(
a^2*x^2+1)+1))*arctan(a*x)^2+3/4*I*a/c^2*Pi*arctan(a*x)^2*csgn(I*(1+I*a*x)^
2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2
)^2+3/2*I*a/c^2*Pi*arctan(a*x)^2-6*I*a/c^2*arctan(a*x)*polylog(2,-(1+I*a*x)
/(a^2*x^2+1)^(1/2))-6*I*a/c^2*arctan(a*x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(
1/2))-3/4*I*a/c^2*Pi*arctan(a*x)^2*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*cs
gn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(
a^2*x^2+1)+1)^2)+3/2*I*a/c^2*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1))*csgn(I/
((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^
2/(a^2*x^2+1)+1))*arctan(a*x)^2-3/2*I*a/c^2*Pi*arctan(a*x)^2*csgn(I*((1+I*a
*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2-arctan(a*x)^3
/c^2/x-3/8*a*arctan(a*x)^4/c^2-3/2*I*a/c^2*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+
1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2*ar
ctan(a*x)^2-3/2*I*a/c^2*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/
(a^2*x^2+1)+1))*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1
))^2*arctan(a*x)^2-3/2*I*a/c^2*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1))*csgn(
I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2*arctan(a*x)^2+
3/8*a*arctan(a*x)^2/c^2-1/2*a^2*x*arctan(a*x)^3/c^2/(a^2*x^2+1)+6*a/c^2*pol
ylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*a/c^2*polylog(3,(1+I*a*x)/(a^2*x^2+1
)^(1/2))-3/4*a*arctan(a*x)^2/c^2/(a^2*x^2+1)-I*a*arctan(a*x)^3/c^2-3*a/c^2*
arctan(a*x)^2*ln((1+I*a*x)^2/(a^2*x^2+1)-1)+3*a/c^2*arctan(a*x)^2*ln(a*x)-3
/2*a/c^2*arctan(a*x)^2*ln(a^2*x^2+1)+3*a/c^2*arctan(a*x)^2*ln((1+I*a*x)/(a^
2*x^2+1)^(1/2))+3*a/c^2*arctan(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))+3*a
/c^2*arctan(a*x)^2*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-3/4*I*a/c^2*Pi*arctan(
a*x)^2*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))^2*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)
)+3/4*I*a/c^2*Pi*arctan(a*x)^2*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I
*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2+3/4*I*a/c^2*Pi*ar
ctan(a*x)^2*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))^2*csgn(I*((1+I*a*x)^2/(a^2*
x^2+1)+1)^2)+3/32*I*a/c^2/(a*x+I)-3/32*I*a/c^2/(a*x-I)+3/2*a/c^2*arctan(a*x
)/(8*a*x-8*I)-3/32/c^2/(a*x+I)*a^2*x-3/32/c^2/(a*x-I)*a^2*x+3/2*a/c^2*arcta
n(a*x)/(8*a*x+8*I)+3*a/c^2*arctan(a*x)^2*ln(2)-3/2*I/c^2*arctan(a*x)/(8*a*x
-8*I)*a^2*x-3/2*I*a/c^2*Pi*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a
^2*x^2+1)+1))^2*arctan(a*x)^2-3/4*I*a/c^2*Pi*arctan(a*x)^2*csgn(I*(1+I*a*x)
^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3+3/2*I/c^2*arctan(a*x)/(8*a*
```

```
x+8*I)*a^2*x+3/2*I*a/c^2*Pi*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(
a^2*x^2+1)+1))^3*arctan(a*x)^2+3/2*I*a/c^2*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+
1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^3*arctan(a*x)^2-3/4*I*a/c^2*Pi*arctan(a*
x)^2*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^3+3/4*I*a/c^2*Pi*arctan(a*x)^2*csgn(I*
((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x^2/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\arctan(ax)^3}{a^4c^2x^6 + 2\,a^2c^2x^4 + c^2x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x^2/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] integral(arctan(a*x)^3/(a^4*c^2*x^6 + 2*a^2*c^2*x^4 + c^2*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\operatorname{atan}^3(ax)}{a^4x^6+2a^2x^4+x^2}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**3/x**2/(a**2*c*x**2+c)**2,x)
```

```
[Out] Integral(atan(a*x)**3/(a**4*x**6 + 2*a**2*x**4 + x**2), x)/c**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(ax)^3}{\left(a^2cx^2+c\right)^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x^2/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)^3/((a^2*c*x^2 + c)^2*x^2), x)
```

3.402 $\int \frac{\tan^{-1}(ax)^3}{x^3(c+a^2cx^2)^2}\,dx$

Optimal. Leaf size=374

$$-\frac{3ia^2\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{2c^2}-\frac{3ia^2\text{PolyLog}\left(4,-1+\frac{2}{1-iax}\right)}{2c^2}+\frac{3ia^2\tan^{-1}(ax)^2\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{c^2}-\frac{3a^2\tan^{-1}(ax\cdots}{\cdots}$$

```
[Out] (-3*a^3*x)/(8*c^2*(1 + a^2*x^2)) - (3*a^2*ArcTan[a*x])/(8*c^2) + (3*a^2*Arc
Tan[a*x])/(4*c^2*(1 + a^2*x^2)) - (((3*I)/2)*a^2*ArcTan[a*x]^2)/c^2 - (3*a*
ArcTan[a*x]^2)/(2*c^2*x) + (3*a^3*x*ArcTan[a*x]^2)/(4*c^2*(1 + a^2*x^2)) -
(a^2*ArcTan[a*x]^3)/(4*c^2) - ArcTan[a*x]^3/(2*c^2*x^2) - (a^2*ArcTan[a*x]^
3)/(2*c^2*(1 + a^2*x^2)) + ((I/2)*a^2*ArcTan[a*x]^4)/c^2 + (3*a^2*ArcTan[a*
x]*Log[2 - 2/(1 - I*a*x)])/c^2 - (2*a^2*ArcTan[a*x]^3*Log[2 - 2/(1 - I*a*x)
])/c^2 - (((3*I)/2)*a^2*PolyLog[2, -1 + 2/(1 - I*a*x)])/c^2 + ((3*I)*a^2*Ar
cTan[a*x]^2*PolyLog[2, -1 + 2/(1 - I*a*x)])/c^2 - (3*a^2*ArcTan[a*x]*PolyLo
g[3, -1 + 2/(1 - I*a*x)])/c^2 - (((3*I)/2)*a^2*PolyLog[4, -1 + 2/(1 - I*a*x
)])/c^2
```

Rubi [A] time = 1.02629, antiderivative size = 374, normalized size of antiderivative = 1., number of steps used = 25, number of rules used = 14, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {4966, 4918, 4852, 4924, 4868, 2447, 4884, 4992, 4996, 6610, 4930, 4892, 199, 205}

$$-\frac{3ia^2\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{2c^2}-\frac{3ia^2\text{PolyLog}\left(4,-1+\frac{2}{1-iax}\right)}{2c^2}+\frac{3ia^2\tan^{-1}(ax)^2\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{c^2}-\frac{3a^2\tan^{-1}(ax\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^3/(x^3*(c + a^2*c*x^2)^2),x]
```


```
[Out] (-3*a^3*x)/(8*c^2*(1 + a^2*x^2)) - (3*a^2*ArcTan[a*x])/(8*c^2) + (3*a^2*Arc
Tan[a*x])/(4*c^2*(1 + a^2*x^2)) - (((3*I)/2)*a^2*ArcTan[a*x]^2)/c^2 - (3*a*
ArcTan[a*x]^2)/(2*c^2*x) + (3*a^3*x*ArcTan[a*x]^2)/(4*c^2*(1 + a^2*x^2)) -
(a^2*ArcTan[a*x]^3)/(4*c^2) - ArcTan[a*x]^3/(2*c^2*x^2) - (a^2*ArcTan[a*x]^
3)/(2*c^2*(1 + a^2*x^2)) + ((I/2)*a^2*ArcTan[a*x]^4)/c^2 + (3*a^2*ArcTan[a*
x]*Log[2 - 2/(1 - I*a*x)])/c^2 - (2*a^2*ArcTan[a*x]^3*Log[2 - 2/(1 - I*a*x)
])/c^2 - (((3*I)/2)*a^2*PolyLog[2, -1 + 2/(1 - I*a*x)])/c^2 + ((3*I)*a^2*Ar
cTan[a*x]^2*PolyLog[2, -1 + 2/(1 - I*a*x)])/c^2 - (3*a^2*ArcTan[a*x]*PolyLo
```

```
g[3, -1 + 2/(1 - I*a*x)])/c^2 - (((3*I)/2)*a^2*PolyLog[4, -1 + 2/(1 - I*a*x
)])/c^2
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*
x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^p
, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*q]
 && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 4918

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/(d + e*x^2),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
```

```
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4992

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d
] && EqQ[(1 - u)^2 - (1 - (2*I)/(I + c*x))^2, 0]
```

Rule 4996

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[k + 1, u])/(
2*c*d), x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[k + 1
, u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] &&
EqQ[e, c^2*d] && EqQ[u^2 - (1 - (2*I)/(I + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 4892

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[(x*(a + b*ArcTan[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*c*
p)/2, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(a +
 b*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d, e},
```

```
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^3}{x^3\left(c+a^2cx^2\right)^2}\,dx = -\left(a^2\int \frac{\tan^{-1}(ax)^3}{x\left(c+a^2cx^2\right)^2}\,dx\right)+\frac{\int \frac{\tan^{-1}(ax)^3}{x^3\left(c+a^2cx^2\right)}\,dx}{c}$$

$$= a^4\int \frac{x\tan^{-1}(ax)^3}{\left(c+a^2cx^2\right)^2}\,dx+\frac{\int \frac{\tan^{-1}(ax)^3}{x^3}\,dx}{c^2}-2\frac{a^2\int \frac{\tan^{-1}(ax)^3}{x\left(c+a^2cx^2\right)}\,dx}{c}$$

$$= -\frac{\tan^{-1}(ax)^3}{2c^2x^2}-\frac{a^2\tan^{-1}(ax)^3}{2c^2\left(1+a^2x^2\right)}+\frac{1}{2}\left(3a^3\right)\int \frac{\tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^2}\,dx+\frac{(3a)\int \frac{\tan^{-1}(ax)^2}{x^2\left(1+a^2x^2\right)}\,dx}{2c^2}-2\left(-\frac{ia^2\tan^{-1}}{4c}\right.$$

$$= \frac{3a^3x\tan^{-1}(ax)^2}{4c^2\left(1+a^2x^2\right)}+\frac{a^2\tan^{-1}(ax)^3}{4c^2}-\frac{\tan^{-1}(ax)^3}{2c^2x^2}-\frac{a^2\tan^{-1}(ax)^3}{2c^2\left(1+a^2x^2\right)}-\frac{1}{2}\left(3a^4\right)\int \frac{x\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^2}\,dx+$$

$$= \frac{3a^2\tan^{-1}(ax)}{4c^2\left(1+a^2x^2\right)}-\frac{3a\tan^{-1}(ax)^2}{2c^2x}+\frac{3a^3x\tan^{-1}(ax)^2}{4c^2\left(1+a^2x^2\right)}-\frac{a^2\tan^{-1}(ax)^3}{4c^2}-\frac{\tan^{-1}(ax)^3}{2c^2x^2}-\frac{a^2\tan^{-1}(ax}{2c^2\left(1+a^2x\right.}$$

$$= -\frac{3a^3x}{8c^2\left(1+a^2x^2\right)}+\frac{3a^2\tan^{-1}(ax)}{4c^2\left(1+a^2x^2\right)}-\frac{3ia^2\tan^{-1}(ax)^2}{2c^2}-\frac{3a\tan^{-1}(ax)^2}{2c^2x}+\frac{3a^3x\tan^{-1}(ax)^2}{4c^2\left(1+a^2x^2\right)}-\frac{a^2\tan}{}$$

$$= -\frac{3a^3x}{8c^2\left(1+a^2x^2\right)}-\frac{3a^2\tan^{-1}(ax)}{8c^2}+\frac{3a^2\tan^{-1}(ax)}{4c^2\left(1+a^2x^2\right)}-\frac{3ia^2\tan^{-1}(ax)^2}{2c^2}-\frac{3a\tan^{-1}(ax)^2}{2c^2x}+\frac{3a^3x\tan}{4c^2\left(1\right.}$$

$$= -\frac{3a^3x}{8c^2\left(1+a^2x^2\right)}-\frac{3a^2\tan^{-1}(ax)}{8c^2}+\frac{3a^2\tan^{-1}(ax)}{4c^2\left(1+a^2x^2\right)}-\frac{3ia^2\tan^{-1}(ax)^2}{2c^2}-\frac{3a\tan^{-1}(ax)^2}{2c^2x}+\frac{3a^3x\tan}{4c^2\left(1\right.}$$

Mathematica [A] time = 0.7366, size = 243, normalized size = 0.65

$$\frac{a^2\left(-96i\tan^{-1}(ax)^2\text{PolyLog}\left(2,e^{-2i\tan^{-1}(ax)}\right)-96\tan^{-1}(ax)\text{PolyLog}\left(3,e^{-2i\tan^{-1}(ax)}\right)-48i\text{PolyLog}\left(2,e^{2i\tan^{-1}(ax)}\right)+4\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTan[a*x]^3/(x^3*(c + a^2*c*x^2)^2),x]
```

```
[Out] (a^2*(I*Pi^4 - (48*I)*ArcTan[a*x]^2 - (48*ArcTan[a*x]^2)/(a*x) - (16*(1 + a
^2*x^2)*ArcTan[a*x]^3)/(a^2*x^2) - (16*I)*ArcTan[a*x]^4 + 12*ArcTan[a*x]*Co
s[2*ArcTan[a*x]] - 8*ArcTan[a*x]^3*Cos[2*ArcTan[a*x]] - 64*ArcTan[a*x]^3*Lo
g[1 - E^((-2*I)*ArcTan[a*x])] + 96*ArcTan[a*x]*Log[1 - E^((2*I)*ArcTan[a*x]
)] - (96*I)*ArcTan[a*x]^2*PolyLog[2, E^((-2*I)*ArcTan[a*x])] - (48*I)*PolyL
og[2, E^((2*I)*ArcTan[a*x])] - 96*ArcTan[a*x]*PolyLog[3, E^((-2*I)*ArcTan[a
*x])] + (48*I)*PolyLog[4, E^((-2*I)*ArcTan[a*x])] - 6*Sin[2*ArcTan[a*x]] +
12*ArcTan[a*x]^2*Sin[2*ArcTan[a*x]]))/(32*c^2)
```

Maple [B] time = 7.276, size = 815, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^3/x^3/(a^2*c*x^2+c)^2,x)
```

```
[Out] -1/2*arctan(a*x)^3/c^2/x^2-1/2*a^2*arctan(a*x)^3/c^2+3/16*a^2/c^2/(a*x+I)*a
rctan(a*x)^2+3/16*a^2/c^2/(a*x-I)*arctan(a*x)^2-3*I*a^2/c^2*polylog(2,-(1+I
*a*x)/(a^2*x^2+1)^(1/2))-12*I*a^2/c^2*polylog(4,-(1+I*a*x)/(a^2*x^2+1)^(1/2
))-3*I*a^2/c^2*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+1/2*I*a^2*arctan(a*x)
^4/c^2-3/2*a*arctan(a*x)^2/c^2/x-3/2*I*a^2*arctan(a*x)^2/c^2-3/32*a^2/c^2/(
a*x+I)-3/32*a^2/c^2/(a*x-I)+3/16*I*a^3/c^2/(a*x+I)*arctan(a*x)^2*x-3/16*I*a
^3/c^2/(a*x-I)*arctan(a*x)^2*x-12*I*a^2/c^2*polylog(4,(1+I*a*x)/(a^2*x^2+1)
^(1/2))-12*a^2/c^2*arctan(a*x)*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*a^2
/c^2*arctan(a*x)^3*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-12*a^2/c^2*arctan(a*x)
*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+3*a^2/c^2*arctan(a*x)*ln(1-(1+I*a*
x)/(a^2*x^2+1)^(1/2))+3*a^2/c^2*arctan(a*x)*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2
))-2*a^2/c^2*arctan(a*x)^3*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))+1/8*a^3/c^2/(a
*x+I)*arctan(a*x)^3*x-3/16*a^3/c^2/(a*x+I)*arctan(a*x)*x+1/8*a^3/c^2/(a*x-I
)*arctan(a*x)^3*x-3/16*a^3/c^2/(a*x-I)*arctan(a*x)*x+6*I*a^2/c^2*arctan(a*x
)^2*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I*a^2/c^2*arctan(a*x)^2*polyl
og(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))-1/8*I*a^2/c^2/(a*x+I)*arctan(a*x)^3+3/16*
I*a^2/c^2/(a*x+I)*arctan(a*x)+1/8*I*a^2/c^2/(a*x-I)*arctan(a*x)^3-3/16*I*a^
2/c^2/(a*x-I)*arctan(a*x)-3/32*I*a^3/c^2/(a*x+I)*x+3/32*I*a^3/c^2/(a*x-I)*x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(ax)^3}{\left(a^2cx^2+c\right)^2x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x^3/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(arctan(a*x)^3/((a^2*c*x^2 + c)^2*x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\arctan\left(ax\right)^{3}}{a^{4}c^{2}x^{7}+2\,a^{2}c^{2}x^{5}+c^{2}x^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x^3/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] integral(arctan(a*x)^3/(a^4*c^2*x^7 + 2*a^2*c^2*x^5 + c^2*x^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\operatorname{atan}^{3}\left(ax\right)}{a^{4}x^{7}+2a^{2}x^{5}+x^{3}}\,dx}{c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**3/x**3/(a**2*c*x**2+c)**2,x)
```

```
[Out] Integral(atan(a*x)**3/(a**4*x**7 + 2*a**2*x**5 + x**3), x)/c**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan\left(ax\right)^{3}}{\left(a^{2}cx^{2}+c\right)^{2}x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x^3/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)^3/((a^2*c*x^2 + c)^2*x^3), x)
```

3.403 $\int \frac{\tan^{-1}(ax)^3}{x^4\left(c+a^2cx^2\right)^2}\,dx$

Optimal. Leaf size=332

$$-\frac{7a^3\text{PolyLog}\left(3,-1+\frac{2}{1-iax}\right)}{2c^2}+\frac{7ia^3\tan^{-1}(ax)\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{c^2}-\frac{3a^3}{8c^2\left(a^2x^2+1\right)}-\frac{a^3\log\left(a^2x^2+1\right)}{2c^2}+\frac{a^4x\tan^{-}}{2c^2\left(a^2x\right.}$$

```
[Out] (-3*a^3)/(8*c^2*(1 + a^2*x^2)) - (a^2*ArcTan[a*x])/(c^2*x) - (3*a^4*x*ArcTa
n[a*x])/(4*c^2*(1 + a^2*x^2)) - (7*a^3*ArcTan[a*x]^2)/(8*c^2) - (a*ArcTan[a
*x]^2)/(2*c^2*x^2) + (3*a^3*ArcTan[a*x]^2)/(4*c^2*(1 + a^2*x^2)) + (((7*I)/
3)*a^3*ArcTan[a*x]^3)/c^2 - ArcTan[a*x]^3/(3*c^2*x^3) + (2*a^2*ArcTan[a*x]^
3)/(c^2*x) + (a^4*x*ArcTan[a*x]^3)/(2*c^2*(1 + a^2*x^2)) + (5*a^3*ArcTan[a*
x]^4)/(8*c^2) + (a^3*Log[x])/c^2 - (a^3*Log[1 + a^2*x^2])/(2*c^2) - (7*a^3*
ArcTan[a*x]^2*Log[2 - 2/(1 - I*a*x)])/c^2 + ((7*I)*a^3*ArcTan[a*x]*PolyLog[
2, -1 + 2/(1 - I*a*x)])/c^2 - (7*a^3*PolyLog[3, -1 + 2/(1 - I*a*x)])/(2*c^2
)
```

Rubi [A] time = 1.29004, antiderivative size = 332, normalized size of antiderivative = 1., number of steps used = 35, number of rules used = 15, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.682, Rules used = {4966, 4918, 4852, 266, 36, 29, 31, 4884, 4924, 4868, 4992, 6610, 4892, 4930, 261}

$$-\frac{7a^3\text{PolyLog}\left(3,-1+\frac{2}{1-iax}\right)}{2c^2}+\frac{7ia^3\tan^{-1}(ax)\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{c^2}-\frac{3a^3}{8c^2\left(a^2x^2+1\right)}-\frac{a^3\log\left(a^2x^2+1\right)}{2c^2}+\frac{a^4x\tan^{-}}{2c^2\left(a^2x\right.}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^3/(x^4*(c + a^2*c*x^2)^2),x]
```

```
[Out] (-3*a^3)/(8*c^2*(1 + a^2*x^2)) - (a^2*ArcTan[a*x])/(c^2*x) - (3*a^4*x*ArcTa
n[a*x])/(4*c^2*(1 + a^2*x^2)) - (7*a^3*ArcTan[a*x]^2)/(8*c^2) - (a*ArcTan[a
*x]^2)/(2*c^2*x^2) + (3*a^3*ArcTan[a*x]^2)/(4*c^2*(1 + a^2*x^2)) + (((7*I)/
3)*a^3*ArcTan[a*x]^3)/c^2 - ArcTan[a*x]^3/(3*c^2*x^3) + (2*a^2*ArcTan[a*x]^
3)/(c^2*x) + (a^4*x*ArcTan[a*x]^3)/(2*c^2*(1 + a^2*x^2)) + (5*a^3*ArcTan[a*
x]^4)/(8*c^2) + (a^3*Log[x])/c^2 - (a^3*Log[1 + a^2*x^2])/(2*c^2) - (7*a^3*
ArcTan[a*x]^2*Log[2 - 2/(1 - I*a*x)])/c^2 + ((7*I)*a^3*ArcTan[a*x]*PolyLog[
2, -1 + 2/(1 - I*a*x)])/c^2 - (7*a^3*PolyLog[3, -1 + 2/(1 - I*a*x)])/(2*c^2
)
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*
x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^p
, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*q]
 && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 4918

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/(d + e*x^2),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 4992

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d
] && EqQ[(1 - u)^2 - (1 - (2*I)/(I + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 4892

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[(x*(a + b*ArcTan[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*c*
p)/2, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(a +
 b*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
```

```
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^3}{x^4\left(c+a^2cx^2\right)^2}\,dx = -\left(a^2\int \frac{\tan^{-1}(ax)^3}{x^2\left(c+a^2cx^2\right)^2}\,dx\right)+\frac{\int \frac{\tan^{-1}(ax)^3}{x^4\left(c+a^2cx^2\right)}\,dx}{c}$$

$$= a^4\int \frac{\tan^{-1}(ax)^3}{\left(c+a^2cx^2\right)^2}\,dx+\frac{\int \frac{\tan^{-1}(ax)^3}{x^4}\,dx}{c^2}-2\frac{a^2\int \frac{\tan^{-1}(ax)^3}{x^2\left(c+a^2cx^2\right)}\,dx}{c}$$

$$= -\frac{\tan^{-1}(ax)^3}{3c^2x^3}+\frac{a^4x\tan^{-1}(ax)^3}{2c^2\left(1+a^2x^2\right)}+\frac{a^3\tan^{-1}(ax)^4}{8c^2}-\frac{1}{2}\left(3a^5\right)\int \frac{x\tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^2}\,dx+\frac{a\int \frac{\tan^{-1}(ax)^2}{x^3\left(1+a^2x^2\right)}\,dx}{c^2}$$

$$= \frac{3a^3\tan^{-1}(ax)^2}{4c^2\left(1+a^2x^2\right)}-\frac{\tan^{-1}(ax)^3}{3c^2x^3}+\frac{a^4x\tan^{-1}(ax)^3}{2c^2\left(1+a^2x^2\right)}+\frac{a^3\tan^{-1}(ax)^4}{8c^2}-\frac{1}{2}\left(3a^4\right)\int \frac{\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^2}\,dx+$$

$$= -\frac{3a^4x\tan^{-1}(ax)}{4c^2\left(1+a^2x^2\right)}-\frac{3a^3\tan^{-1}(ax)^2}{8c^2}-\frac{a\tan^{-1}(ax)^2}{2c^2x^2}+\frac{3a^3\tan^{-1}(ax)^2}{4c^2\left(1+a^2x^2\right)}+\frac{ia^3\tan^{-1}(ax)^3}{3c^2}-\frac{\tan^{-1}(}{3c^2x}$$

$$= -\frac{3a^3}{8c^2\left(1+a^2x^2\right)}-\frac{3a^4x\tan^{-1}(ax)}{4c^2\left(1+a^2x^2\right)}-\frac{3a^3\tan^{-1}(ax)^2}{8c^2}-\frac{a\tan^{-1}(ax)^2}{2c^2x^2}+\frac{3a^3\tan^{-1}(ax)^2}{4c^2\left(1+a^2x^2\right)}+\frac{ia^3\tan}{3c}$$

$$= -\frac{3a^3}{8c^2\left(1+a^2x^2\right)}-\frac{a^2\tan^{-1}(ax)}{c^2x}-\frac{3a^4x\tan^{-1}(ax)}{4c^2\left(1+a^2x^2\right)}-\frac{7a^3\tan^{-1}(ax)^2}{8c^2}-\frac{a\tan^{-1}(ax)^2}{2c^2x^2}+\frac{3a^3\tan^{-1}}{4c^2\left(1+a\right.}$$

$$= -\frac{3a^3}{8c^2\left(1+a^2x^2\right)}-\frac{a^2\tan^{-1}(ax)}{c^2x}-\frac{3a^4x\tan^{-1}(ax)}{4c^2\left(1+a^2x^2\right)}-\frac{7a^3\tan^{-1}(ax)^2}{8c^2}-\frac{a\tan^{-1}(ax)^2}{2c^2x^2}+\frac{3a^3\tan^{-1}}{4c^2\left(1+a\right.}$$

$$= -\frac{3a^3}{8c^2\left(1+a^2x^2\right)}-\frac{a^2\tan^{-1}(ax)}{c^2x}-\frac{3a^4x\tan^{-1}(ax)}{4c^2\left(1+a^2x^2\right)}-\frac{7a^3\tan^{-1}(ax)^2}{8c^2}-\frac{a\tan^{-1}(ax)^2}{2c^2x^2}+\frac{3a^3\tan^{-1}}{4c^2\left(1+a\right.}$$

$$= -\frac{3a^3}{8c^2\left(1+a^2x^2\right)}-\frac{a^2\tan^{-1}(ax)}{c^2x}-\frac{3a^4x\tan^{-1}(ax)}{4c^2\left(1+a^2x^2\right)}-\frac{7a^3\tan^{-1}(ax)^2}{8c^2}-\frac{a\tan^{-1}(ax)^2}{2c^2x^2}+\frac{3a^3\tan^{-1}}{4c^2\left(1+a\right.}$$

Mathematica [A] time = 0.941906, size = 243, normalized size = 0.73

$$a^3\left(-7i\tan^{-1}(ax)\mathrm{PolyLog}\left(2,e^{-2i\tan^{-1}(ax)}\right)-\frac{7}{2}\mathrm{PolyLog}\left(3,e^{-2i\tan^{-1}(ax)}\right)+\log\left(\frac{ax}{\sqrt{a^2x^2+1}}\right)-\frac{\tan^{-1}(ax)^3}{3a^3x^3}-\frac{\tan^{-1}(ax)^2}{2a^2x^2}+\frac{5}{8}\tan\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTan[a*x]^3/(x^4*(c + a^2*c*x^2)^2),x]
```

```
[Out] (a^3*(((7*I)/24)*Pi^3 - ArcTan[a*x]/(a*x) - ArcTan[a*x]^2/2 - ArcTan[a*x]^2
/(2*a^2*x^2) - ((7*I)/3)*ArcTan[a*x]^3 - ArcTan[a*x]^3/(3*a^3*x^3) + (2*Arc
Tan[a*x]^3)/(a*x) + (5*ArcTan[a*x]^4)/8 - (3*Cos[2*ArcTan[a*x]])/16 + (3*Ar
cTan[a*x]^2*Cos[2*ArcTan[a*x]])/8 - 7*ArcTan[a*x]^2*Log[1 - E^((-2*I)*ArcTa
n[a*x])] + Log[(a*x)/Sqrt[1 + a^2*x^2]] - (7*I)*ArcTan[a*x]*PolyLog[2, E^((
-2*I)*ArcTan[a*x])] - (7*PolyLog[3, E^((-2*I)*ArcTan[a*x])])/2 - (3*ArcTan[
a*x]*Sin[2*ArcTan[a*x]])/8 + (ArcTan[a*x]^3*Sin[2*ArcTan[a*x]])/4))/c^2
```

Maple [C] time = 8.694, size = 5190, normalized size = 15.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^3/x^4/(a^2*c*x^2+c)^2,x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x^4/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\arctan\left(ax\right)^{3}}{a^{4}c^{2}x^{8}+2\,a^{2}c^{2}x^{6}+c^{2}x^{4}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x^4/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] integral(arctan(a*x)^3/(a^4*c^2*x^8 + 2*a^2*c^2*x^6 + c^2*x^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\operatorname{atan}^{3}(ax)}{a^{4}x^{8}+2a^{2}x^{6}+x^{4}}\,dx}{c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**3/x**4/(a**2*c*x**2+c)**2,x)
```

```
[Out] Integral(atan(a*x)**3/(a**4*x**8 + 2*a**2*x**6 + x**4), x)/c**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(ax)^{3}}{\left(a^{2}cx^{2}+c\right)^{2}x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x^4/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)^3/((a^2*c*x^2 + c)^2*x^4), x)
```

3.404 $\int \frac{x^3 \tan^{-1}(ax)^3}{(c+a^2cx^2)^3}\,dx$

Optimal. Leaf size=212

$$-\frac{3x^3}{128ac^3\left(a^2x^2+1\right)^2}-\frac{45x}{256a^3c^3\left(a^2x^2+1\right)}+\frac{x^4\tan^{-1}(ax)^3}{4c^3\left(a^2x^2+1\right)^2}-\frac{3x^4\tan^{-1}(ax)}{32c^3\left(a^2x^2+1\right)^2}+\frac{3x^3\tan^{-1}(ax)^2}{16ac^3\left(a^2x^2+1\right)^2}+\frac{9x\tan^{-1}(ax)^2}{32a^3c^3\left(a^2x^2+\right.}$$

```
[Out] (-3*x^3)/(128*a*c^3*(1 + a^2*x^2)^2) - (45*x)/(256*a^3*c^3*(1 + a^2*x^2)) -
 (27*ArcTan[a*x])/(256*a^4*c^3) - (3*x^4*ArcTan[a*x])/(32*c^3*(1 + a^2*x^2)
^2) + (9*ArcTan[a*x])/(32*a^4*c^3*(1 + a^2*x^2)) + (3*x^3*ArcTan[a*x]^2)/(1
6*a*c^3*(1 + a^2*x^2)^2) + (9*x*ArcTan[a*x]^2)/(32*a^3*c^3*(1 + a^2*x^2)) -
 (3*ArcTan[a*x]^3)/(32*a^4*c^3) + (x^4*ArcTan[a*x]^3)/(4*c^3*(1 + a^2*x^2)^
2)
```

Rubi [A] time = 0.293902, antiderivative size = 212, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {4944, 4940, 4936, 4930, 199, 205, 288}

$$-\frac{3x^3}{128ac^3\left(a^2x^2+1\right)^2}-\frac{45x}{256a^3c^3\left(a^2x^2+1\right)}+\frac{x^4\tan^{-1}(ax)^3}{4c^3\left(a^2x^2+1\right)^2}-\frac{3x^4\tan^{-1}(ax)}{32c^3\left(a^2x^2+1\right)^2}+\frac{3x^3\tan^{-1}(ax)^2}{16ac^3\left(a^2x^2+1\right)^2}+\frac{9x\tan^{-1}(ax)^2}{32a^3c^3\left(a^2x^2+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcTan[a*x]^3)/(c + a^2*c*x^2)^3,x]
```


```
[Out] (-3*x^3)/(128*a*c^3*(1 + a^2*x^2)^2) - (45*x)/(256*a^3*c^3*(1 + a^2*x^2)) -
 (27*ArcTan[a*x])/(256*a^4*c^3) - (3*x^4*ArcTan[a*x])/(32*c^3*(1 + a^2*x^2)
^2) + (9*ArcTan[a*x])/(32*a^4*c^3*(1 + a^2*x^2)) + (3*x^3*ArcTan[a*x]^2)/(1
6*a*c^3*(1 + a^2*x^2)^2) + (9*x*ArcTan[a*x]^2)/(32*a^3*c^3*(1 + a^2*x^2)) -
 (3*ArcTan[a*x]^3)/(32*a^4*c^3) + (x^4*ArcTan[a*x]^3)/(4*c^3*(1 + a^2*x^2)^
2)
```


Rule 4944

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a +
 b*ArcTan[c*x])^p)/(d*f*(m + 1)), x] - Dist[(b*c*p)/(f*(m + 1)), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
```

```
& NeQ[m, -1]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_)*((d_) + (e_.)
*(x_)^2)^(q_), x_Symbol] :> Simp[(b*p*(f*x)^m*(d + e*x^2)^(q + 1)*(a + b*Ar
cTan[c*x])^(p - 1))/(c*d*m^2), x] + (Dist[(f^2*(m - 1))/(c^2*d*m), Int[(f*x
)^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p, x], x] - Dist[(b^2*p*(
p - 1))/m^2, Int[(f*x)^m*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x]
- Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(c^2*d*m
), x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q +
 2, 0] && LtQ[q, -1] && GtQ[p, 1]
```

Rule 4936

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^2)/((d_) + (e_.)*(x_)^2)
^2, x_Symbol] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(2*b*c^3*d^2*(p + 1)), x]
 + (Dist[(b*p)/(2*c), Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(d + e*x^2)^2, x]
, x] - Simp[(x*(a + b*ArcTan[c*x])^p)/(2*c^2*d*(d + e*x^2)), x]) /; FreeQ[{
a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^3\tan^{-1}(ax)^3}{\left(c+a^2cx^2\right)^3}\,dx = \frac{x^4\tan^{-1}(ax)^3}{4c^3\left(1+a^2x^2\right)^2}-\frac{1}{4}(3a)\int \frac{x^4\tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^3}\,dx$$

$$= -\frac{3x^4\tan^{-1}(ax)}{32c^3\left(1+a^2x^2\right)^2}+\frac{3x^3\tan^{-1}(ax)^2}{16ac^3\left(1+a^2x^2\right)^2}+\frac{x^4\tan^{-1}(ax)^3}{4c^3\left(1+a^2x^2\right)^2}+\frac{1}{32}(3a)\int \frac{x^4}{\left(c+a^2cx^2\right)^3}\,dx-\frac{9\int \frac{x^2\,\mathrm{t}}{(c}}{1}$$

$$= -\frac{3x^3}{128ac^3\left(1+a^2x^2\right)^2}-\frac{3x^4\tan^{-1}(ax)}{32c^3\left(1+a^2x^2\right)^2}+\frac{3x^3\tan^{-1}(ax)^2}{16ac^3\left(1+a^2x^2\right)^2}+\frac{9x\tan^{-1}(ax)^2}{32a^3c^3\left(1+a^2x^2\right)}-\frac{3\tan^{-1}(ax)}{32a^4c^3}$$

$$= -\frac{3x^3}{128ac^3\left(1+a^2x^2\right)^2}-\frac{9x}{256a^3c^3\left(1+a^2x^2\right)}-\frac{3x^4\tan^{-1}(ax)}{32c^3\left(1+a^2x^2\right)^2}+\frac{9\tan^{-1}(ax)}{32a^4c^3\left(1+a^2x^2\right)}+\frac{3x^3\tan^{-1}}{16ac^3\left(1+\right.}$$

$$= -\frac{3x^3}{128ac^3\left(1+a^2x^2\right)^2}-\frac{45x}{256a^3c^3\left(1+a^2x^2\right)}+\frac{9\tan^{-1}(ax)}{256a^4c^3}-\frac{3x^4\tan^{-1}(ax)}{32c^3\left(1+a^2x^2\right)^2}+\frac{9\tan^{-1}(ax)}{32a^4c^3\left(1+a^2x^2\right.}$$

$$= -\frac{3x^3}{128ac^3\left(1+a^2x^2\right)^2}-\frac{45x}{256a^3c^3\left(1+a^2x^2\right)}-\frac{27\tan^{-1}(ax)}{256a^4c^3}-\frac{3x^4\tan^{-1}(ax)}{32c^3\left(1+a^2x^2\right)^2}+\frac{9\tan^{-1}(ax)}{32a^4c^3\left(1+a^2x\right.}$$

Mathematica [A] time = 0.242411, size = 105, normalized size = 0.5

$$\frac{-3ax\left(17a^2x^2+15\right)+8\left(5a^4x^4-6a^2x^2-3\right)\tan^{-1}(ax)^3+24ax\left(5a^2x^2+3\right)\tan^{-1}(ax)^2+\left(-51a^4x^4+18a^2x^2+45\right)\tan^{-1}}{256a^4c^3\left(a^2x^2+1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*ArcTan[a*x]^3)/(c + a^2*c*x^2)^3,x]
```

```
[Out] (-3*a*x*(15 + 17*a^2*x^2) + (45 + 18*a^2*x^2 - 51*a^4*x^4)*ArcTan[a*x] + 24
*a*x*(3 + 5*a^2*x^2)*ArcTan[a*x]^2 + 8*(-3 - 6*a^2*x^2 + 5*a^4*x^4)*ArcTan[
```

```
a*x]^3)/(256*a^4*c^3*(1 + a^2*x^2)^2)
```

Maple [A] time = 0.261, size = 220, normalized size = 1.

$$\frac{(\arctan(ax))^3}{4c^3a^4(a^2x^2+1)^2}-\frac{(\arctan(ax))^3}{2c^3a^4(a^2x^2+1)}+\frac{15x^3(\arctan(ax))^2}{32ac^3(a^2x^2+1)^2}+\frac{9x(\arctan(ax))^2}{32c^3a^3(a^2x^2+1)^2}+\frac{5(\arctan(ax))^3}{32c^3a^4}-\frac{3\arctan(a}{32c^3a^4(a^2x^2+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctan(a*x)^3/(a^2*c*x^2+c)^3,x)
```

```
[Out] 1/4/a^4/c^3*arctan(a*x)^3/(a^2*x^2+1)^2-1/2/a^4/c^3*arctan(a*x)^3/(a^2*x^2+
1)+15/32*x^3*arctan(a*x)^2/a/c^3/(a^2*x^2+1)^2+9/32/a^3/c^3*x/(a^2*x^2+1)^2
*arctan(a*x)^2+5/32*arctan(a*x)^3/a^4/c^3-3/32/a^4/c^3/(a^2*x^2+1)^2*arctan
(a*x)+15/32*arctan(a*x)/a^4/c^3/(a^2*x^2+1)-51/256*x^3/a/c^3/(a^2*x^2+1)^2-
45/256/a^3/c^3/(a^2*x^2+1)^2*x-51/256*arctan(a*x)/a^4/c^3
```

Maxima [A] time = 1.6801, size = 390, normalized size = 1.84

$$\frac{3}{32}a\left(\frac{5a^2x^3+3x}{a^8c^3x^4+2a^6c^3x^2+a^4c^3}+\frac{5\arctan(ax)}{a^5c^3}\right)\arctan(ax)^2-\frac{(2a^2x^2+1)\arctan(ax)^3}{4(a^8c^3x^4+2a^6c^3x^2+a^4c^3)}-\frac{1}{256}\left(\frac{(51a^3x^3-40(a^4x^4}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^3/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 3/32*a*((5*a^2*x^3 + 3*x)/(a^8*c^3*x^4 + 2*a^6*c^3*x^2 + a^4*c^3) + 5*arcta
n(a*x)/(a^5*c^3))*arctan(a*x)^2 - 1/4*(2*a^2*x^2 + 1)*arctan(a*x)^3/(a^8*c^
3*x^4 + 2*a^6*c^3*x^2 + a^4*c^3) - 1/256*((51*a^3*x^3 - 40*(a^4*x^4 + 2*a^2
*x^2 + 1)*arctan(a*x)^3 + 45*a*x + 51*(a^4*x^4 + 2*a^2*x^2 + 1)*arctan(a*x)
)*a^2/(a^11*c^3*x^4 + 2*a^9*c^3*x^2 + a^7*c^3) - 24*(5*a^2*x^2 - 5*(a^4*x^4
 + 2*a^2*x^2 + 1)*arctan(a*x)^2 + 4)*a*arctan(a*x)/(a^10*c^3*x^4 + 2*a^8*c^
3*x^2 + a^6*c^3))*a
```

Fricas [A] time = 1.8075, size = 271, normalized size = 1.28

$$-\frac{51a^3x^3-8(5a^4x^4-6a^2x^2-3)\arctan(ax)^3-24(5a^3x^3+3ax)\arctan(ax)^2+45ax+3(17a^4x^4-6a^2x^2-15)\mathrm{arct}}{256(a^8c^3x^4+2a^6c^3x^2+a^4c^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^3/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] -1/256*(51*a^3*x^3 - 8*(5*a^4*x^4 - 6*a^2*x^2 - 3)*arctan(a*x)^3 - 24*(5*a^
3*x^3 + 3*a*x)*arctan(a*x)^2 + 45*a*x + 3*(17*a^4*x^4 - 6*a^2*x^2 - 15)*arc
tan(a*x))/(a^8*c^3*x^4 + 2*a^6*c^3*x^2 + a^4*c^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^3 \operatorname{atan}^3(ax)}{a^6x^6+3a^4x^4+3a^2x^2+1}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atan(a*x)**3/(a**2*c*x**2+c)**3,x)
```

```
[Out] Integral(x**3*atan(a*x)**3/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1), x)/
c**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \arctan(ax)^3}{\left(a^2cx^2+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^3/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] integrate(x^3*arctan(a*x)^3/(a^2*c*x^2 + c)^3, x)
```

3.405 $\int \frac{x^2 \tan^{-1}(ax)^3}{(c+a^2cx^2)^3} dx$

Optimal. Leaf size=237

$$-\frac{3}{128a^3c^3(a^2x^2+1)}+\frac{3}{128a^3c^3(a^2x^2+1)^2}+\frac{x\tan^{-1}(ax)^3}{8a^2c^3(a^2x^2+1)}-\frac{x\tan^{-1}(ax)^3}{4a^2c^3(a^2x^2+1)^2}+\frac{3\tan^{-1}(ax)^2}{16a^3c^3(a^2x^2+1)}-\frac{3\tan^{-1}(ax}{16a^3c^3(a^2x^2+}$$

```
[Out] 3/(128*a^3*c^3*(1 + a^2*x^2)^2) - 3/(128*a^3*c^3*(1 + a^2*x^2)) + (3*x*ArcT
an[a*x])/(32*a^2*c^3*(1 + a^2*x^2)^2) - (3*x*ArcTan[a*x])/(64*a^2*c^3*(1 +
a^2*x^2)) - (3*ArcTan[a*x]^2)/(128*a^3*c^3) - (3*ArcTan[a*x]^2)/(16*a^3*c^3
*(1 + a^2*x^2)^2) + (3*ArcTan[a*x]^2)/(16*a^3*c^3*(1 + a^2*x^2)) - (x*ArcTa
n[a*x]^3)/(4*a^2*c^3*(1 + a^2*x^2)^2) + (x*ArcTan[a*x]^3)/(8*a^2*c^3*(1 + a
^2*x^2)) + ArcTan[a*x]^4/(32*a^3*c^3)
```

Rubi [A] time = 0.388572, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4964, 4892, 4930, 261, 4900, 4896}

$$-\frac{3}{128a^3c^3(a^2x^2+1)}+\frac{3}{128a^3c^3(a^2x^2+1)^2}+\frac{x\tan^{-1}(ax)^3}{8a^2c^3(a^2x^2+1)}-\frac{x\tan^{-1}(ax)^3}{4a^2c^3(a^2x^2+1)^2}+\frac{3\tan^{-1}(ax)^2}{16a^3c^3(a^2x^2+1)}-\frac{3\tan^{-1}(ax}{16a^3c^3(a^2x^2+}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcTan[a*x]^3)/(c + a^2*c*x^2)^3,x]
```


```
[Out] 3/(128*a^3*c^3*(1 + a^2*x^2)^2) - 3/(128*a^3*c^3*(1 + a^2*x^2)) + (3*x*ArcT
an[a*x])/(32*a^2*c^3*(1 + a^2*x^2)^2) - (3*x*ArcTan[a*x])/(64*a^2*c^3*(1 +
a^2*x^2)) - (3*ArcTan[a*x]^2)/(128*a^3*c^3) - (3*ArcTan[a*x]^2)/(16*a^3*c^3
*(1 + a^2*x^2)^2) + (3*ArcTan[a*x]^2)/(16*a^3*c^3*(1 + a^2*x^2)) - (x*ArcTa
n[a*x]^3)/(4*a^2*c^3*(1 + a^2*x^2)^2) + (x*ArcTan[a*x]^3)/(8*a^2*c^3*(1 + a
^2*x^2)) + ArcTan[a*x]^4/(32*a^3*c^3)
```


Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Arc
Tan[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan[c
*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p
, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rule 4892

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[(x*(a + b*ArcTan[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*c*
p)/2, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(a +
 b*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 4900

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(b*p*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^(p - 1))/(4*c*d
*(q + 1)^2), x] + (Dist[(2*q + 3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a
 + b*ArcTan[c*x])^p, x], x] - Dist[(b^2*p*(p - 1))/(4*(q + 1)^2), Int[(d +
e*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x] - Simp[(x*(d + e*x^2)^(q + 1)*
(a + b*ArcTan[c*x])^p)/(2*d*(q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] && E
qQ[e, c^2*d] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

Rule 4896

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbol
] :> Simp[(b*(d + e*x^2)^(q + 1))/(4*c*d*(q + 1)^2), x] + (Dist[(2*q + 3)/(
2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]), x], x] - Simp[(x
*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]))/(2*d*(q + 1)), x]) /; FreeQ[{a, b
, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -3/2]
```

Rubi steps

$$\int \frac{x^2 \tan^{-1}(ax)^3}{\left(c+a^2cx^2\right)^3} dx = -\frac{\int \frac{\tan^{-1}(ax)^3}{\left(c+a^2cx^2\right)^3} dx}{a^2} + \frac{\int \frac{\tan^{-1}(ax)^3}{\left(c+a^2cx^2\right)^2} dx}{a^2c}$$

$$= -\frac{3\tan^{-1}(ax)^2}{16a^3c^3\left(1+a^2x^2\right)^2} - \frac{x\tan^{-1}(ax)^3}{4a^2c^3\left(1+a^2x^2\right)^2} + \frac{x\tan^{-1}(ax)^3}{2a^2c^3\left(1+a^2x^2\right)} + \frac{\tan^{-1}(ax)^4}{8a^3c^3} + \frac{3\int \frac{\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^3} dx}{8a^2} - - 3$$

$$= \frac{3}{128a^3c^3\left(1+a^2x^2\right)^2} + \frac{3x\tan^{-1}(ax)}{32a^2c^3\left(1+a^2x^2\right)^2} - \frac{3\tan^{-1}(ax)^2}{16a^3c^3\left(1+a^2x^2\right)^2} + \frac{3\tan^{-1}(ax)^2}{4a^3c^3\left(1+a^2x^2\right)} - \frac{x\tan^{-1}(a}{4a^2c^3\left(1+a\right.}$$

$$= \frac{3}{128a^3c^3\left(1+a^2x^2\right)^2} + \frac{3x\tan^{-1}(ax)}{32a^2c^3\left(1+a^2x^2\right)^2} - \frac{39x\tan^{-1}(ax)}{64a^2c^3\left(1+a^2x^2\right)} - \frac{39\tan^{-1}(ax)^2}{128a^3c^3} - \frac{3\tan^{-1}(ax)^2}{16a^3c^3\left(1+a^2x\right.}$$

$$= \frac{3}{128a^3c^3\left(1+a^2x^2\right)^2} - \frac{39}{128a^3c^3\left(1+a^2x^2\right)} + \frac{3x\tan^{-1}(ax)}{32a^2c^3\left(1+a^2x^2\right)^2} - \frac{3x\tan^{-1}(ax)}{64a^2c^3\left(1+a^2x^2\right)} - \frac{3\tan^{-1}(ax}{128a^3c^3}$$

$$= \frac{3}{128a^3c^3\left(1+a^2x^2\right)^2} - \frac{3}{128a^3c^3\left(1+a^2x^2\right)} + \frac{3x\tan^{-1}(ax)}{32a^2c^3\left(1+a^2x^2\right)^2} - \frac{3x\tan^{-1}(ax)}{64a^2c^3\left(1+a^2x^2\right)} - \frac{3\tan^{-1}(ax}{128a^3c^3}$$

Mathematica [A] time = 0.0727406, size = 111, normalized size = 0.47

$$\frac{-3a^2x^2 + 4\left(a^2x^2+1\right)^2\tan^{-1}(ax)^4 + 16ax\left(a^2x^2-1\right)\tan^{-1}(ax)^3 - 3\left(a^4x^4-6a^2x^2+1\right)\tan^{-1}(ax)^2 + \left(6ax-6a^3x^3\right)\tan^{-}}{128a^3c^3\left(a^2x^2+1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*ArcTan[a*x]^3)/(c + a^2*c*x^2)^3,x]
```

```
[Out] (-3*a^2*x^2 + (6*a*x - 6*a^3*x^3)*ArcTan[a*x] - 3*(1 - 6*a^2*x^2 + a^4*x^4)
*ArcTan[a*x]^2 + 16*a*x*(-1 + a^2*x^2)*ArcTan[a*x]^3 + 4*(1 + a^2*x^2)^2*Ar
cTan[a*x]^4)/(128*a^3*c^3*(1 + a^2*x^2)^2)
```

Maple [A] time = 0.144, size = 216, normalized size = 0.9

$$\frac{(\arctan(ax))^3 x^3}{8c^3\left(a^2x^2+1\right)^2} - \frac{x(\arctan(ax))^3}{8c^3a^2\left(a^2x^2+1\right)^2} + \frac{(\arctan(ax))^4}{32c^3a^3} - \frac{3(\arctan(ax))^2}{16c^3a^3\left(a^2x^2+1\right)^2} + \frac{3(\arctan(ax))^2}{16c^3a^3\left(a^2x^2+1\right)} - \frac{3\arctan(ax)x^3}{64c^3\left(a^2x^2+1\right)^2} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(a*x)^3/(a^2*c*x^2+c)^3,x)
```

```
[Out] 1/8/c^3*arctan(a*x)^3*x^3/(a^2*x^2+1)^2-1/8*x*arctan(a*x)^3/a^2/c^3/(a^2*x^
2+1)^2+1/32*arctan(a*x)^4/a^3/c^3-3/16*arctan(a*x)^2/a^3/c^3/(a^2*x^2+1)^2+
3/16*arctan(a*x)^2/a^3/c^3/(a^2*x^2+1)-3/64/c^3*arctan(a*x)*x^3/(a^2*x^2+1)
^2+3/64*x*arctan(a*x)/a^2/c^3/(a^2*x^2+1)^2-3/128*arctan(a*x)^2/a^3/c^3+3/1
28/a^3/c^3/(a^2*x^2+1)^2-3/128/a^3/c^3/(a^2*x^2+1)
```

Maxima [A] time = 1.866, size = 451, normalized size = 1.9

$$\frac{1}{8}\left(\frac{a^2x^3 - x}{a^6c^3x^4 + 2\,a^4c^3x^2 + a^2c^3} + \frac{\arctan\left(ax\right)}{a^3c^3}\right)\arctan\left(ax\right)^3 + \frac{3\left(a^2x^2 - \left(a^4x^4 + 2\,a^2x^2 + 1\right)\arctan\left(ax\right)^2\right)a\arctan\left(ax\right)^2}{16\left(a^8c^3x^4 + 2\,a^6c^3x^2 + a^4c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^3/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((a^2*x^3 - x)/(a^6*c^3*x^4 + 2*a^4*c^3*x^2 + a^2*c^3) + arctan(a*x)/(a
^3*c^3))*arctan(a*x)^3 + 3/16*(a^2*x^2 - (a^4*x^4 + 2*a^2*x^2 + 1)*arctan(a
*x)^2)*a*arctan(a*x)^2/(a^8*c^3*x^4 + 2*a^6*c^3*x^2 + a^4*c^3) - 1/128*((4*
(a^4*x^4 + 2*a^2*x^2 + 1)*arctan(a*x)^4 + 3*a^2*x^2 - 3*(a^4*x^4 + 2*a^2*x^
2 + 1)*arctan(a*x)^2)*a^2/(a^10*c^3*x^4 + 2*a^8*c^3*x^2 + a^6*c^3) + 2*(3*a
^3*x^3 - 8*(a^4*x^4 + 2*a^2*x^2 + 1)*arctan(a*x)^3 - 3*a*x + 3*(a^4*x^4 + 2
*a^2*x^2 + 1)*arctan(a*x))*a*arctan(a*x)/(a^9*c^3*x^4 + 2*a^7*c^3*x^2 + a^5
*c^3))*a
```

Fricas [A] time = 1.96763, size = 289, normalized size = 1.22

$$\frac{4\left(a^4x^4 + 2\,a^2x^2 + 1\right)\arctan\left(ax\right)^4 - 3\,a^2x^2 + 16\left(a^3x^3 - ax\right)\arctan\left(ax\right)^3 - 3\left(a^4x^4 - 6\,a^2x^2 + 1\right)\arctan\left(ax\right)^2 - 6\left(a^3x^3\right.}{128\left(a^7c^3x^4 + 2\,a^5c^3x^2 + a^3c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^3/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 1/128*(4*(a^4*x^4 + 2*a^2*x^2 + 1)*arctan(a*x)^4 - 3*a^2*x^2 + 16*(a^3*x^3
- a*x)*arctan(a*x)^3 - 3*(a^4*x^4 - 6*a^2*x^2 + 1)*arctan(a*x)^2 - 6*(a^3*x
```

```
^3 - a*x)*arctan(a*x))/(a^7*c^3*x^4 + 2*a^5*c^3*x^2 + a^3*c^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^2 \operatorname{atan}^3(ax)}{a^6x^6+3a^4x^4+3a^2x^2+1}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(a*x)**3/(a**2*c*x**2+c)**3,x)
```

```
[Out] Integral(x**2*atan(a*x)**3/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1), x)/
c**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \arctan(ax)^3}{\left(a^2cx^2+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^3/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] integrate(x^2*arctan(a*x)^3/(a^2*c*x^2 + c)^3, x)
```

3.406 $\int \frac{x\tan^{-1}(ax)^3}{\left(c+a^2cx^2\right)^3}\,dx$

Optimal. Leaf size=208

$$-\frac{45x}{256ac^3\left(a^2x^2+1\right)}-\frac{3x}{128ac^3\left(a^2x^2+1\right)^2}-\frac{\tan^{-1}(ax)^3}{4a^2c^3\left(a^2x^2+1\right)^2}+\frac{9x\tan^{-1}(ax)^2}{32ac^3\left(a^2x^2+1\right)}+\frac{3x\tan^{-1}(ax)^2}{16ac^3\left(a^2x^2+1\right)^2}+\frac{9\tan^{-1}(ax)}{32a^2c^3\left(a^2x^2+\right.}$$

```
[Out] (-3*x)/(128*a*c^3*(1 + a^2*x^2)^2) - (45*x)/(256*a*c^3*(1 + a^2*x^2)) - (45
*ArcTan[a*x])/(256*a^2*c^3) + (3*ArcTan[a*x])/(32*a^2*c^3*(1 + a^2*x^2)^2)
+ (9*ArcTan[a*x])/(32*a^2*c^3*(1 + a^2*x^2)) + (3*x*ArcTan[a*x]^2)/(16*a*c^
3*(1 + a^2*x^2)^2) + (9*x*ArcTan[a*x]^2)/(32*a*c^3*(1 + a^2*x^2)) + (3*ArcT
an[a*x]^3)/(32*a^2*c^3) - ArcTan[a*x]^3/(4*a^2*c^3*(1 + a^2*x^2)^2)
```

Rubi [A] time = 0.177407, antiderivative size = 208, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4930, 4900, 4892, 199, 205}

$$-\frac{45x}{256ac^3\left(a^2x^2+1\right)}-\frac{3x}{128ac^3\left(a^2x^2+1\right)^2}-\frac{\tan^{-1}(ax)^3}{4a^2c^3\left(a^2x^2+1\right)^2}+\frac{9x\tan^{-1}(ax)^2}{32ac^3\left(a^2x^2+1\right)}+\frac{3x\tan^{-1}(ax)^2}{16ac^3\left(a^2x^2+1\right)^2}+\frac{9\tan^{-1}(ax)}{32a^2c^3\left(a^2x^2+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTan[a*x]^3)/(c + a^2*c*x^2)^3,x]
```



```
[Out] (-3*x)/(128*a*c^3*(1 + a^2*x^2)^2) - (45*x)/(256*a*c^3*(1 + a^2*x^2)) - (45
*ArcTan[a*x])/(256*a^2*c^3) + (3*ArcTan[a*x])/(32*a^2*c^3*(1 + a^2*x^2)^2)
+ (9*ArcTan[a*x])/(32*a^2*c^3*(1 + a^2*x^2)) + (3*x*ArcTan[a*x]^2)/(16*a*c^
3*(1 + a^2*x^2)^2) + (9*x*ArcTan[a*x]^2)/(32*a*c^3*(1 + a^2*x^2)) + (3*ArcT
an[a*x]^3)/(32*a^2*c^3) - ArcTan[a*x]^3/(4*a^2*c^3*(1 + a^2*x^2)^2)
```


Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 4900

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(b*p*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^(p - 1))/(4*c*d
*(q + 1)^2), x] + (Dist[(2*q + 3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a
 + b*ArcTan[c*x])^p, x], x] - Dist[(b^2*p*(p - 1))/(4*(q + 1)^2), Int[(d +
e*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x] - Simp[(x*(d + e*x^2)^(q + 1)*
(a + b*ArcTan[c*x])^p)/(2*d*(q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] && E
qQ[e, c^2*d] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

Rule 4892

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[(x*(a + b*ArcTan[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*c*
p)/2, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(a +
 b*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x \tan^{-1}(ax)^3}{(c+a^2cx^2)^3} dx = -\frac{\tan^{-1}(ax)^3}{4a^2c^3(1+a^2x^2)^2} + \frac{3\int \frac{\tan^{-1}(ax)^2}{(c+a^2cx^2)^3} dx}{4a}$$

$$= \frac{3\tan^{-1}(ax)}{32a^2c^3(1+a^2x^2)^2} + \frac{3x\tan^{-1}(ax)^2}{16ac^3(1+a^2x^2)^2} - \frac{\tan^{-1}(ax)^3}{4a^2c^3(1+a^2x^2)^2} - \frac{3\int \frac{1}{(c+a^2cx^2)^3} dx}{32a} + \frac{9\int \frac{\tan^{-1}(ax)^2}{(c+a^2cx^2)^2} dx}{16ac}$$

$$= -\frac{3x}{128ac^3(1+a^2x^2)^2} + \frac{3\tan^{-1}(ax)}{32a^2c^3(1+a^2x^2)^2} + \frac{3x\tan^{-1}(ax)^2}{16ac^3(1+a^2x^2)^2} + \frac{9x\tan^{-1}(ax)^2}{32ac^3(1+a^2x^2)} + \frac{3\tan^{-1}(ax)}{32a^2c^3}$$

$$= -\frac{3x}{128ac^3(1+a^2x^2)^2} - \frac{9x}{256ac^3(1+a^2x^2)} + \frac{3\tan^{-1}(ax)}{32a^2c^3(1+a^2x^2)^2} + \frac{9\tan^{-1}(ax)}{32a^2c^3(1+a^2x^2)} + \frac{3x\tan^{-1}}{16ac^3(1+}$$

$$= -\frac{3x}{128ac^3(1+a^2x^2)^2} - \frac{45x}{256ac^3(1+a^2x^2)} - \frac{9\tan^{-1}(ax)}{256a^2c^3} + \frac{3\tan^{-1}(ax)}{32a^2c^3(1+a^2x^2)^2} + \frac{9\tan^{-1}(ax)}{32a^2c^3(1+a^2x^2}$$

$$= -\frac{3x}{128ac^3(1+a^2x^2)^2} - \frac{45x}{256ac^3(1+a^2x^2)} - \frac{45\tan^{-1}(ax)}{256a^2c^3} + \frac{3\tan^{-1}(ax)}{32a^2c^3(1+a^2x^2)^2} + \frac{9\tan^{-1}(ax)}{32a^2c^3(1+a^2x}$$

Mathematica [A] time = 0.0777072, size = 103, normalized size = 0.5

$$\frac{-3ax(15a^2x^2+17)+8(3a^4x^4+6a^2x^2-5)\tan^{-1}(ax)^3+24ax(3a^2x^2+5)\tan^{-1}(ax)^2-3(15a^4x^4+6a^2x^2-17)\tan^{-1}}{256c^3(a^3x^2+a)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcTan[a*x]^3)/(c + a^2*c*x^2)^3,x]
```

```
[Out] (-3*a*x*(17 + 15*a^2*x^2) - 3*(-17 + 6*a^2*x^2 + 15*a^4*x^4)*ArcTan[a*x] +
24*a*x*(5 + 3*a^2*x^2)*ArcTan[a*x]^2 + 8*(-5 + 6*a^2*x^2 + 3*a^4*x^4)*ArcTa
n[a*x]^3)/(256*c^3*(a + a^3*x^2)^2)
```

Maple [A] time = 0.133, size = 191, normalized size = 0.9

$$-\frac{(\arctan(ax))^3}{4c^3a^2(a^2x^2+1)^2} + \frac{3x(\arctan(ax))^2}{16ac^3(a^2x^2+1)^2} + \frac{9x(\arctan(ax))^2}{32ac^3(a^2x^2+1)} + \frac{3(\arctan(ax))^3}{32c^3a^2} + \frac{3\arctan(ax)}{32c^3a^2(a^2x^2+1)^2} + \frac{9\arctan(}{32c^3a^2(a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(a*x)^3/(a^2*c*x^2+c)^3,x)
```

```
[Out] -1/4*arctan(a*x)^3/a^2/c^3/(a^2*x^2+1)^2+3/16*x*arctan(a*x)^2/a/c^3/(a^2*x^
2+1)^2+9/32*x*arctan(a*x)^2/a/c^3/(a^2*x^2+1)+3/32*arctan(a*x)^3/a^2/c^3+3/
32*arctan(a*x)/a^2/c^3/(a^2*x^2+1)^2+9/32*arctan(a*x)/a^2/c^3/(a^2*x^2+1)-4
5/256*a/c^3/(a^2*x^2+1)^2*x^3-51/256*x/a/c^3/(a^2*x^2+1)^2-45/256*arctan(a*
x)/a^2/c^3
```

Maxima [A] time = 1.69194, size = 367, normalized size = 1.76

$$\frac{3\left(\frac{3\,a^2x^3+5\,x}{a^4c^2x^4+2\,a^2c^2x^2+c^2}+\frac{3\,\arctan(ax)}{ac^2}\right)\arctan\left(ax\right)^2}{32\,ac}-\frac{3\left(\frac{\left(15\,a^3x^3-8\left(a^4x^4+2\,a^2x^2+1\right)\arctan(ax)^3+17\,ax+15\left(a^4x^4+2\,a^2x^2+1\right)\arctan(ax)\right)a^2}{a^7c^2x^4+2\,a^5c^2x^2+a^3c^2}-\frac{8\cdots}{\cdots}\right.}{256\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^3/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 3/32*((3*a^2*x^3 + 5*x)/(a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2) + 3*arctan(a*x)
/(a*c^2))*arctan(a*x)^2/(a*c) - 3/256*((15*a^3*x^3 - 8*(a^4*x^4 + 2*a^2*x^2
 + 1)*arctan(a*x)^3 + 17*a*x + 15*(a^4*x^4 + 2*a^2*x^2 + 1)*arctan(a*x))*a^
2/(a^7*c^2*x^4 + 2*a^5*c^2*x^2 + a^3*c^2) - 8*(3*a^2*x^2 - 3*(a^4*x^4 + 2*a
^2*x^2 + 1)*arctan(a*x)^2 + 4)*a*arctan(a*x)/(a^6*c^2*x^4 + 2*a^4*c^2*x^2 +
 a^2*c^2))/(a*c) - 1/4*arctan(a*x)^3/((a^2*c*x^2 + c)^2*a^2*c)
```

Fricas [A] time = 1.88186, size = 271, normalized size = 1.3

$$-\frac{45\,a^3x^3-8\left(3\,a^4x^4+6\,a^2x^2-5\right)\arctan\left(ax\right)^3-24\left(3\,a^3x^3+5\,ax\right)\arctan\left(ax\right)^2+51\,ax+3\left(15\,a^4x^4+6\,a^2x^2-17\right)\mathrm{arc}\cdots}{256\left(a^6c^3x^4+2\,a^4c^3x^2+a^2c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^3/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] -1/256*(45*a^3*x^3 - 8*(3*a^4*x^4 + 6*a^2*x^2 - 5)*arctan(a*x)^3 - 24*(3*a^
3*x^3 + 5*a*x)*arctan(a*x)^2 + 51*a*x + 3*(15*a^4*x^4 + 6*a^2*x^2 - 17)*arc
```

```
tan(a*x))/(a^6*c^3*x^4 + 2*a^4*c^3*x^2 + a^2*c^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x \operatorname{atan}^3(ax)}{a^6x^6+3a^4x^4+3a^2x^2+1}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(a*x)**3/(a**2*c*x**2+c)**3,x)
```

```
[Out] Integral(x*atan(a*x)**3/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1), x)/c**
3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \arctan(ax)^3}{(a^2cx^2+c)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^3/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] integrate(x*arctan(a*x)^3/(a^2*c*x^2 + c)^3, x)
```

3.407 $\int \frac{\tan^{-1}(ax)^3}{(c+a^2cx^2)^3}\,dx$

Optimal. Leaf size=225

$$-\frac{45}{128ac^3\left(a^2x^2+1\right)}-\frac{3}{128ac^3\left(a^2x^2+1\right)^2}+\frac{3x\tan^{-1}(ax)^3}{8c^3\left(a^2x^2+1\right)}+\frac{x\tan^{-1}(ax)^3}{4c^3\left(a^2x^2+1\right)^2}+\frac{9\tan^{-1}(ax)^2}{16ac^3\left(a^2x^2+1\right)}+\frac{3\tan^{-1}(ax)^2}{16ac^3\left(a^2x^2+1\right)^2}-$$

```
[Out] -3/(128*a*c^3*(1 + a^2*x^2)^2) - 45/(128*a*c^3*(1 + a^2*x^2)) - (3*x*ArcTan
[a*x])/(32*c^3*(1 + a^2*x^2)^2) - (45*x*ArcTan[a*x])/(64*c^3*(1 + a^2*x^2))
 - (45*ArcTan[a*x]^2)/(128*a*c^3) + (3*ArcTan[a*x]^2)/(16*a*c^3*(1 + a^2*x^
2)^2) + (9*ArcTan[a*x]^2)/(16*a*c^3*(1 + a^2*x^2)) + (x*ArcTan[a*x]^3)/(4*c
^3*(1 + a^2*x^2)^2) + (3*x*ArcTan[a*x]^3)/(8*c^3*(1 + a^2*x^2)) + (3*ArcTan
[a*x]^4)/(32*a*c^3)
```

Rubi [A] time = 0.196444, antiderivative size = 225, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {4900, 4892, 4930, 261, 4896}

$$-\frac{45}{128ac^3\left(a^2x^2+1\right)}-\frac{3}{128ac^3\left(a^2x^2+1\right)^2}+\frac{3x\tan^{-1}(ax)^3}{8c^3\left(a^2x^2+1\right)}+\frac{x\tan^{-1}(ax)^3}{4c^3\left(a^2x^2+1\right)^2}+\frac{9\tan^{-1}(ax)^2}{16ac^3\left(a^2x^2+1\right)}+\frac{3\tan^{-1}(ax)^2}{16ac^3\left(a^2x^2+1\right)^2}-$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^3/(c + a^2*c*x^2)^3,x]
```


```
[Out] -3/(128*a*c^3*(1 + a^2*x^2)^2) - 45/(128*a*c^3*(1 + a^2*x^2)) - (3*x*ArcTan
[a*x])/(32*c^3*(1 + a^2*x^2)^2) - (45*x*ArcTan[a*x])/(64*c^3*(1 + a^2*x^2))
 - (45*ArcTan[a*x]^2)/(128*a*c^3) + (3*ArcTan[a*x]^2)/(16*a*c^3*(1 + a^2*x^
2)^2) + (9*ArcTan[a*x]^2)/(16*a*c^3*(1 + a^2*x^2)) + (x*ArcTan[a*x]^3)/(4*c
^3*(1 + a^2*x^2)^2) + (3*x*ArcTan[a*x]^3)/(8*c^3*(1 + a^2*x^2)) + (3*ArcTan
[a*x]^4)/(32*a*c^3)
```


Rule 4900

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(b*p*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^(p - 1))/(4*c*d
*(q + 1)^2), x] + (Dist[(2*q + 3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a
 + b*ArcTan[c*x])^p, x], x] - Dist[(b^2*p*(p - 1))/(4*(q + 1)^2), Int[(d +
e*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x] - Simp[(x*(d + e*x^2)^(q + 1)*
```

```
(a + b*ArcTan[c*x])^p)/(2*d*(q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] && E
qQ[e, c^2*d] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

Rule 4892

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[(x*(a + b*ArcTan[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*c*
p)/2, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(a +
 b*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 4896

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbol
] :> Simp[(b*(d + e*x^2)^(q + 1))/(4*c*d*(q + 1)^2), x] + (Dist[(2*q + 3)/(
2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]), x], x] - Simp[(x
*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]))/(2*d*(q + 1)), x]) /; FreeQ[{a, b
, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -3/2]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^3}{(c+a^2cx^2)^3}\,dx = \frac{3\tan^{-1}(ax)^2}{16ac^3(1+a^2x^2)^2} + \frac{x\tan^{-1}(ax)^3}{4c^3(1+a^2x^2)^2} - \frac{3}{8}\int \frac{\tan^{-1}(ax)}{(c+a^2cx^2)^3}\,dx + \frac{3\int \frac{\tan^{-1}(ax)^3}{(c+a^2cx^2)^2}\,dx}{4c}$$

$$= -\frac{3}{128ac^3(1+a^2x^2)^2} - \frac{3x\tan^{-1}(ax)}{32c^3(1+a^2x^2)^2} + \frac{3\tan^{-1}(ax)^2}{16ac^3(1+a^2x^2)^2} + \frac{x\tan^{-1}(ax)^3}{4c^3(1+a^2x^2)^2} + \frac{3x\tan^{-1}(ax)^3}{8c^3(1+a^2x^2)} +$$

$$= -\frac{3}{128ac^3(1+a^2x^2)^2} - \frac{3x\tan^{-1}(ax)}{32c^3(1+a^2x^2)^2} - \frac{9x\tan^{-1}(ax)}{64c^3(1+a^2x^2)} - \frac{9\tan^{-1}(ax)^2}{128ac^3} + \frac{3\tan^{-1}(ax)^2}{16ac^3(1+a^2x^2)^2} + -$$

$$= -\frac{3}{128ac^3(1+a^2x^2)^2} - \frac{9}{128ac^3(1+a^2x^2)} - \frac{3x\tan^{-1}(ax)}{32c^3(1+a^2x^2)^2} - \frac{45x\tan^{-1}(ax)}{64c^3(1+a^2x^2)} - \frac{45\tan^{-1}(ax)^2}{128ac^3} +$$

$$= -\frac{3}{128ac^3(1+a^2x^2)^2} - \frac{45}{128ac^3(1+a^2x^2)} - \frac{3x\tan^{-1}(ax)}{32c^3(1+a^2x^2)^2} - \frac{45x\tan^{-1}(ax)}{64c^3(1+a^2x^2)} - \frac{45\tan^{-1}(ax)^2}{128ac^3} +$$

Mathematica [A] time = 0.0564496, size = 114, normalized size = 0.51

$$-\frac{45a^2x^2 - 12(a^2x^2+1)^2\tan^{-1}(ax)^4 - 16ax(3a^2x^2+5)\tan^{-1}(ax)^3 + 3(15a^4x^4+6a^2x^2-17)\tan^{-1}(ax)^2 + 6ax(15a^2x^2}{128ac^3(a^2x^2+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]^3/(c + a^2*c*x^2)^3,x]
```

```
[Out] -(48 + 45*a^2*x^2 + 6*a*x*(17 + 15*a^2*x^2)*ArcTan[a*x] + 3*(-17 + 6*a^2*x^
2 + 15*a^4*x^4)*ArcTan[a*x]^2 - 16*a*x*(5 + 3*a^2*x^2)*ArcTan[a*x]^3 - 12*(
1 + a^2*x^2)^2*ArcTan[a*x]^4)/(128*a*c^3*(1 + a^2*x^2)^2)
```

Maple [A] time = 0.302, size = 211, normalized size = 0.9

$$\frac{x(\arctan(ax))^3}{4c^3(a^2x^2+1)^2} + \frac{3x(\arctan(ax))^3}{8c^3(a^2x^2+1)} + \frac{3(\arctan(ax))^4}{32ac^3} + \frac{3(\arctan(ax))^2}{16ac^3(a^2x^2+1)^2} + \frac{9(\arctan(ax))^2}{16ac^3(a^2x^2+1)} - \frac{45a^2\arctan(ax)x^3}{64c^3(a^2x^2+1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^3/(a^2*c*x^2+c)^3,x)
```

```
[Out] 1/4*x*arctan(a*x)^3/c^3/(a^2*x^2+1)^2+3/8*x*arctan(a*x)^3/c^3/(a^2*x^2+1)+3
/32*arctan(a*x)^4/a/c^3+3/16*arctan(a*x)^2/a/c^3/(a^2*x^2+1)^2+9/16*arctan(
a*x)^2/a/c^3/(a^2*x^2+1)-45/64*a^2/c^3*arctan(a*x)*x^3/(a^2*x^2+1)^2-51/64*
x*arctan(a*x)/c^3/(a^2*x^2+1)^2-45/128*arctan(a*x)^2/a/c^3-3/128/a/c^3/(a^2
*x^2+1)^2-45/128/a/c^3/(a^2*x^2+1)
```

Maxima [A] time = 1.88486, size = 452, normalized size = 2.01

$$\frac{1}{8}\left(\frac{3\,a^2x^3+5\,x}{a^4c^3x^4+2\,a^2c^3x^2+c^3}+\frac{3\,\arctan\left(ax\right)}{ac^3}\right)\arctan\left(ax\right)^3+\frac{3\left(3\,a^2x^2-3\left(a^4x^4+2\,a^2x^2+1\right)\arctan\left(ax\right)^2+4\right)a\,\mathrm{arctan}}{16\left(a^6c^3x^4+2\,a^4c^3x^2+a^2c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((3*a^2*x^3 + 5*x)/(a^4*c^3*x^4 + 2*a^2*c^3*x^2 + c^3) + 3*arctan(a*x)/
(a*c^3))*arctan(a*x)^3 + 3/16*(3*a^2*x^2 - 3*(a^4*x^4 + 2*a^2*x^2 + 1)*arct
an(a*x)^2 + 4)*a*arctan(a*x)^2/(a^6*c^3*x^4 + 2*a^4*c^3*x^2 + a^2*c^3) - 3/
128*((4*(a^4*x^4 + 2*a^2*x^2 + 1)*arctan(a*x)^4 + 15*a^2*x^2 - 15*(a^4*x^4
+ 2*a^2*x^2 + 1)*arctan(a*x)^2 + 16)*a^2/(a^8*c^3*x^4 + 2*a^6*c^3*x^2 + a^4
*c^3) + 2*(15*a^3*x^3 - 8*(a^4*x^4 + 2*a^2*x^2 + 1)*arctan(a*x)^3 + 17*a*x
+ 15*(a^4*x^4 + 2*a^2*x^2 + 1)*arctan(a*x))*a*arctan(a*x)/(a^7*c^3*x^4 + 2*
a^5*c^3*x^2 + a^3*c^3))*a
```

Fricas [A] time = 2.0007, size = 315, normalized size = 1.4

$$\frac{12\left(a^4x^4+2\,a^2x^2+1\right)\arctan\left(ax\right)^4-45\,a^2x^2+16\left(3\,a^3x^3+5\,ax\right)\arctan\left(ax\right)^3-3\left(15\,a^4x^4+6\,a^2x^2-17\right)\arctan\left(ax\right)^2}{128\left(a^5c^3x^4+2\,a^3c^3x^2+ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 1/128*(12*(a^4*x^4 + 2*a^2*x^2 + 1)*arctan(a*x)^4 - 45*a^2*x^2 + 16*(3*a^3*
x^3 + 5*a*x)*arctan(a*x)^3 - 3*(15*a^4*x^4 + 6*a^2*x^2 - 17)*arctan(a*x)^2
- 6*(15*a^3*x^3 + 17*a*x)*arctan(a*x) - 48)/(a^5*c^3*x^4 + 2*a^3*c^3*x^2 +
```

```
a*c^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\operatorname{atan}^3(ax)}{a^6x^6+3a^4x^4+3a^2x^2+1}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**3/(a**2*c*x**2+c)**3,x)
```

```
[Out] Integral(atan(a*x)**3/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1), x)/c**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(ax)^3}{(a^2cx^2+c)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)^3/(a^2*c*x^2 + c)^3, x)
```

3.408 $\int \frac{\tan^{-1}(ax)^3}{x(c+a^2cx^2)^3}\,dx$

Optimal. Leaf size=332

$$\frac{3i\text{PolyLog}\left(4,-1+\frac{2}{1-iax}\right)}{4c^3}-\frac{3i\tan^{-1}(ax)^2\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{2c^3}+\frac{3\tan^{-1}(ax)\text{PolyLog}\left(3,-1+\frac{2}{1-iax}\right)}{2c^3}+\frac{141a}{256c^3\left(a^2\right.}$$

```
[Out] (3*a*x)/(128*c^3*(1 + a^2*x^2)^2) + (141*a*x)/(256*c^3*(1 + a^2*x^2)) + (14
1*ArcTan[a*x])/(256*c^3) - (3*ArcTan[a*x])/(32*c^3*(1 + a^2*x^2)^2) - (33*A
rcTan[a*x])/(32*c^3*(1 + a^2*x^2)) - (3*a*x*ArcTan[a*x]^2)/(16*c^3*(1 + a^2
*x^2)^2) - (33*a*x*ArcTan[a*x]^2)/(32*c^3*(1 + a^2*x^2)) - (11*ArcTan[a*x]^
3)/(32*c^3) + ArcTan[a*x]^3/(4*c^3*(1 + a^2*x^2)^2) + ArcTan[a*x]^3/(2*c^3*
(1 + a^2*x^2)) - ((I/4)*ArcTan[a*x]^4)/c^3 + (ArcTan[a*x]^3*Log[2 - 2/(1 -
I*a*x)])/c^3 - (((3*I)/2)*ArcTan[a*x]^2*PolyLog[2, -1 + 2/(1 - I*a*x)])/c^3
 + (3*ArcTan[a*x]*PolyLog[3, -1 + 2/(1 - I*a*x)])/(2*c^3) + (((3*I)/4)*Poly
Log[4, -1 + 2/(1 - I*a*x)])/c^3
```

Rubi [A] time = 0.709367, antiderivative size = 332, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 12, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {4966, 4924, 4868, 4884, 4992, 4996, 6610, 4930, 4892, 199, 205, 4900}

$$\frac{3i\text{PolyLog}\left(4,-1+\frac{2}{1-iax}\right)}{4c^3}-\frac{3i\tan^{-1}(ax)^2\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{2c^3}+\frac{3\tan^{-1}(ax)\text{PolyLog}\left(3,-1+\frac{2}{1-iax}\right)}{2c^3}+\frac{141a}{256c^3\left(a^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^3/(x*(c + a^2*c*x^2)^3),x]
```



```
[Out] (3*a*x)/(128*c^3*(1 + a^2*x^2)^2) + (141*a*x)/(256*c^3*(1 + a^2*x^2)) + (14
1*ArcTan[a*x])/(256*c^3) - (3*ArcTan[a*x])/(32*c^3*(1 + a^2*x^2)^2) - (33*A
rcTan[a*x])/(32*c^3*(1 + a^2*x^2)) - (3*a*x*ArcTan[a*x]^2)/(16*c^3*(1 + a^2
*x^2)^2) - (33*a*x*ArcTan[a*x]^2)/(32*c^3*(1 + a^2*x^2)) - (11*ArcTan[a*x]^
3)/(32*c^3) + ArcTan[a*x]^3/(4*c^3*(1 + a^2*x^2)^2) + ArcTan[a*x]^3/(2*c^3*
(1 + a^2*x^2)) - ((I/4)*ArcTan[a*x]^4)/c^3 + (ArcTan[a*x]^3*Log[2 - 2/(1 -
I*a*x)])/c^3 - (((3*I)/2)*ArcTan[a*x]^2*PolyLog[2, -1 + 2/(1 - I*a*x)])/c^3
 + (3*ArcTan[a*x]*PolyLog[3, -1 + 2/(1 - I*a*x)])/(2*c^3) + (((3*I)/4)*Poly
Log[4, -1 + 2/(1 - I*a*x)])/c^3
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*
x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^p
, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*q]
 && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4992

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d
] && EqQ[(1 - u)^2 - (1 - (2*I)/(I + c*x))^2, 0]
```

Rule 4996

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[k + 1, u])/(
2*c*d), x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[k + 1
, u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] &&
EqQ[e, c^2*d] && EqQ[u^2 - (1 - (2*I)/(I + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 4892

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[(x*(a + b*ArcTan[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*c*
p)/2, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(a +
 b*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 4900

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(b*p*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^(p - 1))/(4*c*d
*(q + 1)^2), x] + (Dist[(2*q + 3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a
 + b*ArcTan[c*x])^p, x], x] - Dist[(b^2*p*(p - 1))/(4*(q + 1)^2), Int[(d +
e*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x] - Simp[(x*(d + e*x^2)^(q + 1)*
(a + b*ArcTan[c*x])^p)/(2*d*(q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] && E
qQ[e, c^2*d] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^3}{x\left(c+a^2cx^2\right)^3}\,dx = -\left(a^2\int \frac{x\tan^{-1}(ax)^3}{\left(c+a^2cx^2\right)^3}\,dx\right)+\frac{\int \frac{\tan^{-1}(ax)^3}{x\left(c+a^2cx^2\right)^2}\,dx}{c}$$

$$= \frac{\tan^{-1}(ax)^3}{4c^3\left(1+a^2x^2\right)^2}-\frac{1}{4}(3a)\int \frac{\tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^3}\,dx+\frac{\int \frac{\tan^{-1}(ax)^3}{x\left(c+a^2cx^2\right)}\,dx}{c^2}-\frac{a^2\int \frac{x\tan^{-1}(ax)^3}{\left(c+a^2cx^2\right)^2}\,dx}{c}$$

$$= -\frac{3\tan^{-1}(ax)}{32c^3\left(1+a^2x^2\right)^2}-\frac{3ax\tan^{-1}(ax)^2}{16c^3\left(1+a^2x^2\right)^2}+\frac{\tan^{-1}(ax)^3}{4c^3\left(1+a^2x^2\right)^2}+\frac{\tan^{-1}(ax)^3}{2c^3\left(1+a^2x^2\right)}-\frac{i\tan^{-1}(ax)^4}{4c^3}+\frac{1}{32}(3a$$

$$= \frac{3ax}{128c^3\left(1+a^2x^2\right)^2}-\frac{3\tan^{-1}(ax)}{32c^3\left(1+a^2x^2\right)^2}-\frac{3ax\tan^{-1}(ax)^2}{16c^3\left(1+a^2x^2\right)^2}-\frac{33ax\tan^{-1}(ax)^2}{32c^3\left(1+a^2x^2\right)}-\frac{11\tan^{-1}(ax)^3}{32c^3}+ -$$

$$= \frac{3ax}{128c^3\left(1+a^2x^2\right)^2}+\frac{9ax}{256c^3\left(1+a^2x^2\right)}-\frac{3\tan^{-1}(ax)}{32c^3\left(1+a^2x^2\right)^2}-\frac{33\tan^{-1}(ax)}{32c^3\left(1+a^2x^2\right)}-\frac{3ax\tan^{-1}(ax)^2}{16c^3\left(1+a^2x^2\right)^2}-$$

$$= \frac{3ax}{128c^3\left(1+a^2x^2\right)^2}+\frac{141ax}{256c^3\left(1+a^2x^2\right)}+\frac{9\tan^{-1}(ax)}{256c^3}-\frac{3\tan^{-1}(ax)}{32c^3\left(1+a^2x^2\right)^2}-\frac{33\tan^{-1}(ax)}{32c^3\left(1+a^2x^2\right)}-\frac{3ax}{16c^3}$$

$$= \frac{3ax}{128c^3\left(1+a^2x^2\right)^2}+\frac{141ax}{256c^3\left(1+a^2x^2\right)}+\frac{141\tan^{-1}(ax)}{256c^3}-\frac{3\tan^{-1}(ax)}{32c^3\left(1+a^2x^2\right)^2}-\frac{33\tan^{-1}(ax)}{32c^3\left(1+a^2x^2\right)}-\frac{3}{16}$$

Mathematica [A] time = 0.313079, size = 208, normalized size = 0.63

$$\frac{1536i\tan^{-1}(ax)^2\text{PolyLog}\left(2,e^{-2i\tan^{-1}(ax)}\right)+1536\tan^{-1}(ax)\text{PolyLog}\left(3,e^{-2i\tan^{-1}(ax)}\right)-768i\text{PolyLog}\left(4,e^{-2i\tan^{-1}(ax)}\right)+}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTan[a*x]^3/(x*(c + a^2*c*x^2)^3),x]
```

```
[Out] ((-16*I)*Pi^4 + (256*I)*ArcTan[a*x]^4 - 576*ArcTan[a*x]*Cos[2*ArcTan[a*x]]
+ 384*ArcTan[a*x]^3*Cos[2*ArcTan[a*x]] - 12*ArcTan[a*x]*Cos[4*ArcTan[a*x]]
+ 32*ArcTan[a*x]^3*Cos[4*ArcTan[a*x]] + 1024*ArcTan[a*x]^3*Log[1 - E^((-2*I
```

```
)*ArcTan[a*x])] + (1536*I)*ArcTan[a*x]^2*PolyLog[2, E^((-2*I)*ArcTan[a*x])]
 + 1536*ArcTan[a*x]*PolyLog[3, E^((-2*I)*ArcTan[a*x])] - (768*I)*PolyLog[4,
 E^((-2*I)*ArcTan[a*x])] + 288*Sin[2*ArcTan[a*x]] - 576*ArcTan[a*x]^2*Sin[2
*ArcTan[a*x]] + 3*Sin[4*ArcTan[a*x]] - 24*ArcTan[a*x]^2*Sin[4*ArcTan[a*x]])
/(1024*c^3)
```

Maple [C] time = 3.697, size = 2463, normalized size = 7.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^3/x/(a^2*c*x^2+c)^3,x)
```

```
[Out] -3/1024/c^3/(a*x+I)^2*a*x-3/1024/c^3/(a*x-I)^2*a*x-9/32*I/c^3*arctan(a*x)/(
a*x+I)+9/32*I/c^3*arctan(a*x)/(a*x-I)-11/32*arctan(a*x)^3/c^3+9/4/c^3/(16*a
*x+16*I)-1/2/c^3*arctan(a*x)^3*ln(a^2*x^2+1)+1/c^3*arctan(a*x)^3*ln(a*x)+1/
c^3*arctan(a*x)^3*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))+6/c^3*arctan(a*x)*polyl
og(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+3/512/c^3*arctan(a*x)/(a*x-I)^2+3/512/c^
3*arctan(a*x)/(a*x+I)^2-9/4/c^3*arctan(a*x)^2/(8*a*x+8*I)-9/4/c^3*arctan(a*
x)^2/(8*a*x-8*I)+6*I/c^3*polylog(4,(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I/c^3*pol
ylog(4,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+3/2048*I/c^3/(a*x+I)^2-3/2048*I/c^3/(a
*x-I)^2+1/c^3*arctan(a*x)^3*ln(2)+1/c^3*arctan(a*x)^3*ln((1+I*a*x)/(a^2*x^2
+1)^(1/2))-1/c^3*arctan(a*x)^3*ln((1+I*a*x)^2/(a^2*x^2+1)-1)+1/c^3*arctan(a
*x)^3*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))+6/c^3*arctan(a*x)*polylog(3,(1+I*a*
x)/(a^2*x^2+1)^(1/2))+1/2*I/c^3*Pi*arctan(a*x)^3*csgn(I*((1+I*a*x)^2/(a^2*x
^2+1)-1))*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+
1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))+1/4*I/c^3*arctan(a*x)^3*Pi*csgn(I*(1+I*a
*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+
1)^2)^2+1/4*I/c^3*Pi*arctan(a*x)^3*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))^2*cs
gn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)-1/4*I/c^3*Pi*arctan(a*x)^3*csgn(I*(1+I*
a*x)/(a^2*x^2+1)^(1/2))^2*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))-1/2*I/c^3*Pi*arct
an(a*x)^3*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+
1)+1)^2)^2-1/4*I/c^3*Pi*arctan(a*x)^3*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)
*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^
2/(a^2*x^2+1)+1)^2)+1/4*arctan(a*x)^3/c^3/(a^2*x^2+1)^2+1/2*arctan(a*x)^3/c
^3/(a^2*x^2+1)-1/4*I*arctan(a*x)^4/c^3-3/256*I/c^3*arctan(a*x)^2/(a*x+I)^2+
3/256*I/c^3*arctan(a*x)^2/(a*x-I)^2-3*I/c^3*arctan(a*x)^2*polylog(2,(1+I*a*
x)/(a^2*x^2+1)^(1/2))-3*I/c^3*arctan(a*x)^2*polylog(2,-(1+I*a*x)/(a^2*x^2+1
)^(1/2))+1/2*I/c^3*arctan(a*x)^3*Pi+1/2*I/c^3*Pi*arctan(a*x)^3*csgn(I*((1+I
*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(((1+I*a*x)^2/(a^2*
x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))-1/2*I/c^3*Pi*arctan(a*x)^3*csgn(I*((
```

```
1+I*a*x)^2/(a^2*x^2+1)-1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/
(a^2*x^2+1)+1))^2-1/2*I/c^3*Pi*arctan(a*x)^3*csgn(I*((1+I*a*x)^2/(a^2*x^2+1
)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*
x)^2/(a^2*x^2+1)+1))^2+1/4*I/c^3*Pi*arctan(a*x)^3*csgn(I/((1+I*a*x)^2/(a^2*
x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^
2+1/2*I/c^3*Pi*arctan(a*x)^3*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*csgn(I*(1+
I*a*x)^2/(a^2*x^2+1))^2-1/2*I/c^3*Pi*arctan(a*x)^3*csgn(I/((1+I*a*x)^2/(a^2
*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))
^2-3/256*I/c^3*arctan(a*x)/(a*x-I)^2*a*x+3/256*I/c^3*arctan(a*x)/(a*x+I)^2*
a*x-9/4*I/c^3*arctan(a*x)^2/(8*a*x+8*I)*a*x+3/256*I/c^3*arctan(a*x)^2/(a*x+
I)^2*a^2*x^2-3/256*I/c^3*arctan(a*x)^2/(a*x-I)^2*a^2*x^2+9/4*I/c^3*arctan(a
*x)^2/(8*a*x-8*I)*a*x+9/4/c^3/(16*a*x-16*I)+3/128/c^3*arctan(a*x)^2/(a*x+I)
^2*a*x+3/128/c^3*arctan(a*x)^2/(a*x-I)^2*a*x-3/512/c^3*arctan(a*x)/(a*x-I)^
2*a^2*x^2+9/32/c^3*arctan(a*x)/(a*x+I)*a*x-3/512/c^3*arctan(a*x)/(a*x+I)^2*
a^2*x^2+9/32/c^3*arctan(a*x)/(a*x-I)*a*x+1/2*I/c^3*Pi*arctan(a*x)^3*csgn(I*
((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^3-1/2*I/c^3*Pi*arc
tan(a*x)^3*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2+
1/2*I/c^3*Pi*arctan(a*x)^3*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a
^2*x^2+1)+1))^3+1/4*I/c^3*Pi*arctan(a*x)^3*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+
1)^2)^3-1/4*I/c^3*Pi*arctan(a*x)^3*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x
)^2/(a^2*x^2+1)+1)^2)^3-1/4*I/c^3*arctan(a*x)^3*Pi*csgn(I*(1+I*a*x)^2/(a^2*
x^2+1))^3+9/4*I/c^3/(16*a*x+16*I)*a*x-3/2048*I/c^3/(a*x+I)^2*a^2*x^2+3/2048
*I/c^3/(a*x-I)^2*a^2*x^2-9/4*I/c^3/(16*a*x-16*I)*a*x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(ax)^3}{\left(a^2cx^2+c\right)^3x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] integrate(arctan(a*x)^3/((a^2*c*x^2 + c)^3*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\arctan(ax)^3}{a^6c^3x^7+3\,a^4c^3x^5+3\,a^2c^3x^3+c^3x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] integral(arctan(a*x)^3/(a^6*c^3*x^7 + 3*a^4*c^3*x^5 + 3*a^2*c^3*x^3 + c^3*x
), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\operatorname{atan}^3(ax)}{a^6x^7+3a^4x^5+3a^2x^3+x}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**3/x/(a**2*c*x**2+c)**3,x)
```

```
[Out] Integral(atan(a*x)**3/(a**6*x**7 + 3*a**4*x**5 + 3*a**2*x**3 + x), x)/c**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(ax)^3}{\left(a^2cx^2+c\right)^3 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)^3/((a^2*c*x^2 + c)^3*x), x)
```

3.409 $\int \frac{\tan^{-1}(ax)^3}{x^2(c+a^2cx^2)^3}\,dx$

Optimal. Leaf size=332

$$\frac{3a\text{PolyLog}\left(3,-1+\frac{2}{1-iax}\right)}{2c^3}-\frac{3ia\tan^{-1}(ax)\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{c^3}+\frac{93a}{128c^3\left(a^2x^2+1\right)}+\frac{3a}{128c^3\left(a^2x^2+1\right)^2}-\frac{7a^2x\tan^{-1}}{8c^3\left(a^2x\right.}$$

```
[Out] (3*a)/(128*c^3*(1 + a^2*x^2)^2) + (93*a)/(128*c^3*(1 + a^2*x^2)) + (3*a^2*x
*ArcTan[a*x])/(32*c^3*(1 + a^2*x^2)^2) + (93*a^2*x*ArcTan[a*x])/(64*c^3*(1
+ a^2*x^2)) + (93*a*ArcTan[a*x]^2)/(128*c^3) - (3*a*ArcTan[a*x]^2)/(16*c^3*
(1 + a^2*x^2)^2) - (21*a*ArcTan[a*x]^2)/(16*c^3*(1 + a^2*x^2)) - (I*a*ArcTa
n[a*x]^3)/c^3 - ArcTan[a*x]^3/(c^3*x) - (a^2*x*ArcTan[a*x]^3)/(4*c^3*(1 + a
^2*x^2)^2) - (7*a^2*x*ArcTan[a*x]^3)/(8*c^3*(1 + a^2*x^2)) - (15*a*ArcTan[a
*x]^4)/(32*c^3) + (3*a*ArcTan[a*x]^2*Log[2 - 2/(1 - I*a*x)])/c^3 - ((3*I)*a
*ArcTan[a*x]*PolyLog[2, -1 + 2/(1 - I*a*x)])/c^3 + (3*a*PolyLog[3, -1 + 2/(
1 - I*a*x)])/(2*c^3)
```

Rubi [A] time = 0.75429, antiderivative size = 332, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 13, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.591, Rules used = {4966, 4918, 4852, 4924, 4868, 4884, 4992, 6610, 4892, 4930, 261, 4900, 4896}

$$\frac{3a\text{PolyLog}\left(3,-1+\frac{2}{1-iax}\right)}{2c^3}-\frac{3ia\tan^{-1}(ax)\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{c^3}+\frac{93a}{128c^3\left(a^2x^2+1\right)}+\frac{3a}{128c^3\left(a^2x^2+1\right)^2}-\frac{7a^2x\tan^{-1}}{8c^3\left(a^2x\right.}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^3/(x^2*(c + a^2*c*x^2)^3),x]
```


```
[Out] (3*a)/(128*c^3*(1 + a^2*x^2)^2) + (93*a)/(128*c^3*(1 + a^2*x^2)) + (3*a^2*x
*ArcTan[a*x])/(32*c^3*(1 + a^2*x^2)^2) + (93*a^2*x*ArcTan[a*x])/(64*c^3*(1
+ a^2*x^2)) + (93*a*ArcTan[a*x]^2)/(128*c^3) - (3*a*ArcTan[a*x]^2)/(16*c^3*
(1 + a^2*x^2)^2) - (21*a*ArcTan[a*x]^2)/(16*c^3*(1 + a^2*x^2)) - (I*a*ArcTa
n[a*x]^3)/c^3 - ArcTan[a*x]^3/(c^3*x) - (a^2*x*ArcTan[a*x]^3)/(4*c^3*(1 + a
^2*x^2)^2) - (7*a^2*x*ArcTan[a*x]^3)/(8*c^3*(1 + a^2*x^2)) - (15*a*ArcTan[a
*x]^4)/(32*c^3) + (3*a*ArcTan[a*x]^2*Log[2 - 2/(1 - I*a*x)])/c^3 - ((3*I)*a
*ArcTan[a*x]*PolyLog[2, -1 + 2/(1 - I*a*x)])/c^3 + (3*a*PolyLog[3, -1 + 2/(
1 - I*a*x)])/(2*c^3)
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*
x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^p
, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*q]
 && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 4918

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/(d + e*x^2),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4992

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d
] && EqQ[(1 - u)^2 - (1 - (2*I)/(I + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 4892

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[(x*(a + b*ArcTan[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*c*
p)/2, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(a +
 b*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 4900

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(b*p*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^(p - 1))/(4*c*d
*(q + 1)^2), x] + (Dist[(2*q + 3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a
 + b*ArcTan[c*x])^p, x], x] - Dist[(b^2*p*(p - 1))/(4*(q + 1)^2), Int[(d +
e*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x] - Simp[(x*(d + e*x^2)^(q + 1)*
(a + b*ArcTan[c*x])^p)/(2*d*(q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] && E
qQ[e, c^2*d] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

Rule 4896

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbol
] :> Simp[(b*(d + e*x^2)^(q + 1))/(4*c*d*(q + 1)^2), x] + (Dist[(2*q + 3)/(
2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]), x], x] - Simp[(x
*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]))/(2*d*(q + 1)), x]) /; FreeQ[{a, b
, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -3/2]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^3}{x^2\left(c+a^2cx^2\right)^3}\,dx = -\left(a^2\int \frac{\tan^{-1}(ax)^3}{\left(c+a^2cx^2\right)^3}\,dx\right)+\frac{\int \frac{\tan^{-1}(ax)^3}{x^2\left(c+a^2cx^2\right)^2}\,dx}{c}$$

$$= -\frac{3a\tan^{-1}(ax)^2}{16c^3\left(1+a^2x^2\right)^2}-\frac{a^2x\tan^{-1}(ax)^3}{4c^3\left(1+a^2x^2\right)^2}+\frac{1}{8}\left(3a^2\right)\int \frac{\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^3}\,dx+\frac{\int \frac{\tan^{-1}(ax)^3}{x^2\left(c+a^2cx^2\right)}\,dx}{c^2}-\frac{\left(3a^2\right)\int}{}$$

$$= \frac{3a}{128c^3\left(1+a^2x^2\right)^2}+\frac{3a^2x\tan^{-1}(ax)}{32c^3\left(1+a^2x^2\right)^2}-\frac{3a\tan^{-1}(ax)^2}{16c^3\left(1+a^2x^2\right)^2}-\frac{a^2x\tan^{-1}(ax)^3}{4c^3\left(1+a^2x^2\right)^2}-\frac{7a^2x\tan^{-1}(ax)^3}{8c^3\left(1+a^2x^2\right)}$$

$$= \frac{3a}{128c^3\left(1+a^2x^2\right)^2}+\frac{3a^2x\tan^{-1}(ax)}{32c^3\left(1+a^2x^2\right)^2}+\frac{9a^2x\tan^{-1}(ax)}{64c^3\left(1+a^2x^2\right)}+\frac{9a\tan^{-1}(ax)^2}{128c^3}-\frac{3a\tan^{-1}(ax)^2}{16c^3\left(1+a^2x^2\right)^2}-$$

$$= \frac{3a}{128c^3\left(1+a^2x^2\right)^2}+\frac{9a}{128c^3\left(1+a^2x^2\right)}+\frac{3a^2x\tan^{-1}(ax)}{32c^3\left(1+a^2x^2\right)^2}+\frac{93a^2x\tan^{-1}(ax)}{64c^3\left(1+a^2x^2\right)}+\frac{93a\tan^{-1}(ax)^2}{128c^3}$$

$$= \frac{3a}{128c^3\left(1+a^2x^2\right)^2}+\frac{93a}{128c^3\left(1+a^2x^2\right)}+\frac{3a^2x\tan^{-1}(ax)}{32c^3\left(1+a^2x^2\right)^2}+\frac{93a^2x\tan^{-1}(ax)}{64c^3\left(1+a^2x^2\right)}+\frac{93a\tan^{-1}(ax)^2}{128c^3}$$

$$= \frac{3a}{128c^3\left(1+a^2x^2\right)^2}+\frac{93a}{128c^3\left(1+a^2x^2\right)}+\frac{3a^2x\tan^{-1}(ax)}{32c^3\left(1+a^2x^2\right)^2}+\frac{93a^2x\tan^{-1}(ax)}{64c^3\left(1+a^2x^2\right)}+\frac{93a\tan^{-1}(ax)^2}{128c^3}$$

$$= \frac{3a}{128c^3\left(1+a^2x^2\right)^2}+\frac{93a}{128c^3\left(1+a^2x^2\right)}+\frac{3a^2x\tan^{-1}(ax)}{32c^3\left(1+a^2x^2\right)^2}+\frac{93a^2x\tan^{-1}(ax)}{64c^3\left(1+a^2x^2\right)}+\frac{93a\tan^{-1}(ax)^2}{128c^3}$$

Mathematica [A] time = 0.546762, size = 232, normalized size = 0.7

$$a\left(3i\tan^{-1}(ax)\text{PolyLog}\left(2,e^{-2i\tan^{-1}(ax)}\right)+\frac{3}{2}\text{PolyLog}\left(3,e^{-2i\tan^{-1}(ax)}\right)-\frac{ax\tan^{-1}(ax)^3}{a^2x^2+1}-\frac{15}{32}\tan^{-1}(ax)^4-\frac{\tan^{-1}(ax)^3}{ax}+i\tan\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTan[a*x]^3/(x^2*(c + a^2*c*x^2)^3),x]
```

```
[Out] (a*((-I/8)*Pi^3 + I*ArcTan[a*x]^3 - ArcTan[a*x]^3/(a*x) - (a*x*ArcTan[a*x]^
3)/(1 + a^2*x^2) - (15*ArcTan[a*x]^4)/32 + (3*Cos[2*ArcTan[a*x]])/8 - (3*Ar
cTan[a*x]^2*Cos[2*ArcTan[a*x]])/4 + (3*Cos[4*ArcTan[a*x]])/1024 - (3*ArcTan
[a*x]^2*Cos[4*ArcTan[a*x]])/128 + 3*ArcTan[a*x]^2*Log[1 - E^((-2*I)*ArcTan[
a*x])] + (3*I)*ArcTan[a*x]*PolyLog[2, E^((-2*I)*ArcTan[a*x])] + (3*PolyLog[
3, E^((-2*I)*ArcTan[a*x])])/2 + (3*ArcTan[a*x]*Sin[2*ArcTan[a*x]])/4 + (3*A
rcTan[a*x]*Sin[4*ArcTan[a*x]])/256 - (ArcTan[a*x]^3*Sin[4*ArcTan[a*x]])/32)
)/c^3
```

Maple [C] time = 3.306, size = 2315, normalized size = 7.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^3/x^2/(a^2*c*x^2+c)^3,x)
```

```
[Out] 3/4*I*a/c^3*Pi*arctan(a*x)^2*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*
x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2-3/2*I*a/c^3*Pi*arctan(a*x
)^2*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^
2)^2+3/2*I*a/c^3*Pi*arctan(a*x)^2*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*
a*x)^2/(a^2*x^2+1)+1))*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x
^2+1)+1))+3/4*I*a/c^3*Pi*arctan(a*x)^2*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))^
2*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)+3/512*I/c^3*arctan(a*x)/(a*x-I)^2*a
^3*x^2+3/8*I/c^3*arctan(a*x)/(a*x+I)*a^2*x-3/512*I/c^3*arctan(a*x)/(a*x+I)^
2*a^3*x^2-3/8*I/c^3*arctan(a*x)/(a*x-I)*a^2*x+3/2*I*a/c^3*Pi*arctan(a*x)^2*
csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^3-3/4*I*a/c^3
*Pi*arctan(a*x)^2*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1
)^2)^3+3/4*I*a/c^3*Pi*arctan(a*x)^2*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3
-3/2*I*a/c^3*Pi*arctan(a*x)^2*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2
/(a^2*x^2+1)+1))^2-3/4*I*a/c^3*Pi*arctan(a*x)^2*csgn(I*(1+I*a*x)^2/(a^2*x^2
+1))^3+3/2*I*a/c^3*Pi*arctan(a*x)^2*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+
I*a*x)^2/(a^2*x^2+1)+1))^3+6*a/c^3*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))+6
*a/c^3*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-3/2048*a/c^3/(a*x+I)^2-3/204
8*a/c^3/(a*x-I)^2-9/8*a^2*x*arctan(a*x)^3/c^3/(a^2*x^2+1)^2+3*a/c^3*arctan(
a*x)^2*ln(2)+3*a/c^3*arctan(a*x)^2*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))+3*a/c^
3*arctan(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))+3*a/c^3*arctan(a*x)^2*ln(
a*x)-3/2*a/c^3*arctan(a*x)^2*ln(a^2*x^2+1)-3/16*I*a/c^3/(a*x-I)+3/16*I*a/c^
```

```
3/(a*x+I)+3*a/c^3*arctan(a*x)^2*ln((1+I*a*x)/(a^2*x^2+1)^(1/2))-3*a/c^3*arc
tan(a*x)^2*ln((1+I*a*x)^2/(a^2*x^2+1)-1)-3/16/c^3/(a*x-I)*a^2*x+3/2048/c^3/
(a*x+I)^2*a^3*x^2+3/2048/c^3/(a*x-I)^2*a^3*x^2-3/16/c^3/(a*x+I)*a^2*x-arcta
n(a*x)^3/c^3/x-15/32*a*arctan(a*x)^4/c^3+93/128*a*arctan(a*x)^2/c^3+3/8*a/c
^3*arctan(a*x)/(a*x+I)+3/8*a/c^3*arctan(a*x)/(a*x-I)-3/4*I*a/c^3*Pi*arctan(
a*x)^2*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)
)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)+3/2*I*a/c^3
*Pi*arctan(a*x)^2*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1))*csgn(I/((1+I*a*x)^2/(
a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+
1))-3/16*a*arctan(a*x)^2/c^3/(a^2*x^2+1)^2-21/16*a*arctan(a*x)^2/c^3/(a^2*x
^2+1)-I*a*arctan(a*x)^3/c^3-6*I*a/c^3*arctan(a*x)*polylog(2,-(1+I*a*x)/(a^2
*x^2+1)^(1/2))+3/2*I*a/c^3*Pi*arctan(a*x)^2-3/512*I*a/c^3*arctan(a*x)/(a*x-
I)^2+3/512*I*a/c^3*arctan(a*x)/(a*x+I)^2-3/1024*I/c^3/(a*x+I)^2*a^2*x+3/102
4*I/c^3/(a*x-I)^2*a^2*x-3/256/c^3*arctan(a*x)/(a*x-I)^2*a^2*x-3/256/c^3*arc
tan(a*x)/(a*x+I)^2*a^2*x-7/8/c^3*arctan(a*x)^3*a^4*x^3/(a^2*x^2+1)^2-6*I*a/
c^3*arctan(a*x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))-3/2*I*a/c^3*Pi*arcta
n(a*x)^2*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1
)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2-3/2*I*a/c^3*Pi*arctan(a*x)^2*csgn(I*((1
+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(((1+I*a*x)^2/(a^
2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2+3/4*I*a/c^3*Pi*arctan(a*x)^2*csg
n(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x
)^2/(a^2*x^2+1)+1)^2)^2-3/4*I*a/c^3*Pi*arctan(a*x)^2*csgn(I*(1+I*a*x)/(a^2*
x^2+1)^(1/2))^2*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))+3/2*I*a/c^3*Pi*arctan(a*x)^
2*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^2-3/2
*I*a/c^3*Pi*arctan(a*x)^2*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*
a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x^2/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\arctan\left(ax\right)^{3}}{a^{6}c^{3}x^{8}+3\,a^{4}c^{3}x^{6}+3\,a^{2}c^{3}x^{4}+c^{3}x^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x^2/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] integral(arctan(a*x)^3/(a^6*c^3*x^8 + 3*a^4*c^3*x^6 + 3*a^2*c^3*x^4 + c^3*x
^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\operatorname{atan}^{3}(ax)}{a^{6}x^{8}+3a^{4}x^{6}+3a^{2}x^{4}+x^{2}}\,dx}{c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**3/x**2/(a**2*c*x**2+c)**3,x)
```

```
[Out] Integral(atan(a*x)**3/(a**6*x**8 + 3*a**4*x**6 + 3*a**2*x**4 + x**2), x)/c*
*3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(ax)^{3}}{\left(a^{2}cx^{2}+c\right)^{3}x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x^2/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)^3/((a^2*c*x^2 + c)^3*x^2), x)
```

3.410 $\int \frac{\tan^{-1}(ax)^3}{x^3\left(c+a^2cx^2\right)^3}\,dx$

Optimal. Leaf size=478

$$-\frac{3ia^2\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{2c^3}-\frac{9ia^2\text{PolyLog}\left(4,-1+\frac{2}{1-iax}\right)}{4c^3}+\frac{9ia^2\tan^{-1}(ax)^2\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{2c^3}-\frac{9a^2\tan^{-1}(\cdots}{\cdots}$$

```
[Out] (-3*a^3*x)/(128*c^3*(1 + a^2*x^2)^2) - (237*a^3*x)/(256*c^3*(1 + a^2*x^2))
- (237*a^2*ArcTan[a*x])/(256*c^3) + (3*a^2*ArcTan[a*x])/(32*c^3*(1 + a^2*x^
2)^2) + (57*a^2*ArcTan[a*x])/(32*c^3*(1 + a^2*x^2)) - (((3*I)/2)*a^2*ArcTan
[a*x]^2)/c^3 - (3*a*ArcTan[a*x]^2)/(2*c^3*x) + (3*a^3*x*ArcTan[a*x]^2)/(16*
c^3*(1 + a^2*x^2)^2) + (57*a^3*x*ArcTan[a*x]^2)/(32*c^3*(1 + a^2*x^2)) + (3
*a^2*ArcTan[a*x]^3)/(32*c^3) - ArcTan[a*x]^3/(2*c^3*x^2) - (a^2*ArcTan[a*x]
^3)/(4*c^3*(1 + a^2*x^2)^2) - (a^2*ArcTan[a*x]^3)/(c^3*(1 + a^2*x^2)) + (((
3*I)/4)*a^2*ArcTan[a*x]^4)/c^3 + (3*a^2*ArcTan[a*x]*Log[2 - 2/(1 - I*a*x)])
/c^3 - (3*a^2*ArcTan[a*x]^3*Log[2 - 2/(1 - I*a*x)])/c^3 - (((3*I)/2)*a^2*Po
lyLog[2, -1 + 2/(1 - I*a*x)])/c^3 + (((9*I)/2)*a^2*ArcTan[a*x]^2*PolyLog[2,
 -1 + 2/(1 - I*a*x)])/c^3 - (9*a^2*ArcTan[a*x]*PolyLog[3, -1 + 2/(1 - I*a*x
)])/(2*c^3) - (((9*I)/4)*a^2*PolyLog[4, -1 + 2/(1 - I*a*x)])/c^3
```

Rubi [A] time = 1.83844, antiderivative size = 478, normalized size of antiderivative = 1., number of steps used = 47, number of rules used = 15, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.682, Rules used = {4966, 4918, 4852, 4924, 4868, 2447, 4884, 4992, 4996, 6610, 4930, 4892, 199, 205, 4900}

$$-\frac{3ia^2\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{2c^3}-\frac{9ia^2\text{PolyLog}\left(4,-1+\frac{2}{1-iax}\right)}{4c^3}+\frac{9ia^2\tan^{-1}(ax)^2\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{2c^3}-\frac{9a^2\tan^{-1}(\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^3/(x^3*(c + a^2*c*x^2)^3),x]
```


```
[Out] (-3*a^3*x)/(128*c^3*(1 + a^2*x^2)^2) - (237*a^3*x)/(256*c^3*(1 + a^2*x^2))
- (237*a^2*ArcTan[a*x])/(256*c^3) + (3*a^2*ArcTan[a*x])/(32*c^3*(1 + a^2*x^
2)^2) + (57*a^2*ArcTan[a*x])/(32*c^3*(1 + a^2*x^2)) - (((3*I)/2)*a^2*ArcTan
[a*x]^2)/c^3 - (3*a*ArcTan[a*x]^2)/(2*c^3*x) + (3*a^3*x*ArcTan[a*x]^2)/(16*
c^3*(1 + a^2*x^2)^2) + (57*a^3*x*ArcTan[a*x]^2)/(32*c^3*(1 + a^2*x^2)) + (3
*a^2*ArcTan[a*x]^3)/(32*c^3) - ArcTan[a*x]^3/(2*c^3*x^2) - (a^2*ArcTan[a*x]
```

```
^3)/(4*c^3*(1 + a^2*x^2)^2) - (a^2*ArcTan[a*x]^3)/(c^3*(1 + a^2*x^2)) + (((
3*I)/4)*a^2*ArcTan[a*x]^4)/c^3 + (3*a^2*ArcTan[a*x]*Log[2 - 2/(1 - I*a*x)])
/c^3 - (3*a^2*ArcTan[a*x]^3*Log[2 - 2/(1 - I*a*x)])/c^3 - (((3*I)/2)*a^2*Po
lyLog[2, -1 + 2/(1 - I*a*x)])/c^3 + (((9*I)/2)*a^2*ArcTan[a*x]^2*PolyLog[2,
 -1 + 2/(1 - I*a*x)])/c^3 - (9*a^2*ArcTan[a*x]*PolyLog[3, -1 + 2/(1 - I*a*x
)])/(2*c^3) - (((9*I)/4)*a^2*PolyLog[4, -1 + 2/(1 - I*a*x)])/c^3
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*
x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^p
, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*q]
 && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 4918

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/(d + e*x^2),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4992

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d
] && EqQ[(1 - u)^2 - (1 - (2*I)/(I + c*x))^2, 0]
```

Rule 4996

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[k + 1, u])/(
2*c*d), x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[k + 1
, u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] &&
EqQ[e, c^2*d] && EqQ[u^2 - (1 - (2*I)/(I + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 4892

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
```

```
bol] :> Simp[(x*(a + b*ArcTan[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*c*
p)/2, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(a +
 b*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 4900

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(b*p*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^(p - 1))/(4*c*d
*(q + 1)^2), x] + (Dist[(2*q + 3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a
 + b*ArcTan[c*x])^p, x], x] - Dist[(b^2*p*(p - 1))/(4*(q + 1)^2), Int[(d +
e*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x] - Simp[(x*(d + e*x^2)^(q + 1)*
(a + b*ArcTan[c*x])^p)/(2*d*(q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] && E
qQ[e, c^2*d] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^3}{x^3\left(c+a^2cx^2\right)^3}\,dx = -\left(a^2\int \frac{\tan^{-1}(ax)^3}{x\left(c+a^2cx^2\right)^3}\,dx\right) + \frac{\int \frac{\tan^{-1}(ax)^3}{x^3\left(c+a^2cx^2\right)^2}\,dx}{c}$$

$$= a^4\int \frac{x\tan^{-1}(ax)^3}{\left(c+a^2cx^2\right)^3}\,dx + \frac{\int \frac{\tan^{-1}(ax)^3}{x^3\left(c+a^2cx^2\right)}\,dx}{c^2} - 2\frac{a^2\int \frac{\tan^{-1}(ax)^3}{x\left(c+a^2cx^2\right)^2}\,dx}{c}$$

$$= -\frac{a^2\tan^{-1}(ax)^3}{4c^3\left(1+a^2x^2\right)^2} + \frac{1}{4}\left(3a^3\right)\int \frac{\tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^3}\,dx + \frac{\int \frac{\tan^{-1}(ax)^3}{x^3}\,dx}{c^3} - \frac{a^2\int \frac{\tan^{-1}(ax)^3}{x\left(c+a^2cx^2\right)}\,dx}{c^2} - 2\left(\frac{a^2\int \ldots}{\ldots}\right.$$

$$= \frac{3a^2\tan^{-1}(ax)}{32c^3\left(1+a^2x^2\right)^2} + \frac{3a^3x\tan^{-1}(ax)^2}{16c^3\left(1+a^2x^2\right)^2} - \frac{\tan^{-1}(ax)^3}{2c^3x^2} - \frac{a^2\tan^{-1}(ax)^3}{4c^3\left(1+a^2x^2\right)^2} + \frac{ia^2\tan^{-1}(ax)^4}{4c^3} - \frac{1}{32}\left(3\ldots\right.$$

$$= -\frac{3a^3x}{128c^3\left(1+a^2x^2\right)^2} + \frac{3a^2\tan^{-1}(ax)}{32c^3\left(1+a^2x^2\right)^2} + \frac{3a^3x\tan^{-1}(ax)^2}{16c^3\left(1+a^2x^2\right)^2} + \frac{9a^3x\tan^{-1}(ax)^2}{32c^3\left(1+a^2x^2\right)} + \frac{3a^2\tan^{-1}(ax)\ldots}{32c^3}$$

$$= -\frac{3a^3x}{128c^3\left(1+a^2x^2\right)^2} - \frac{9a^3x}{256c^3\left(1+a^2x^2\right)} + \frac{3a^2\tan^{-1}(ax)}{32c^3\left(1+a^2x^2\right)^2} + \frac{9a^2\tan^{-1}(ax)}{32c^3\left(1+a^2x^2\right)} - \frac{3a\tan^{-1}(ax)^2}{2c^3x}$$

$$= -\frac{3a^3x}{128c^3\left(1+a^2x^2\right)^2} - \frac{45a^3x}{256c^3\left(1+a^2x^2\right)} - \frac{9a^2\tan^{-1}(ax)}{256c^3} + \frac{3a^2\tan^{-1}(ax)}{32c^3\left(1+a^2x^2\right)^2} + \frac{9a^2\tan^{-1}(ax)}{32c^3\left(1+a^2x^2\right)}$$

$$= -\frac{3a^3x}{128c^3\left(1+a^2x^2\right)^2} - \frac{45a^3x}{256c^3\left(1+a^2x^2\right)} - \frac{45a^2\tan^{-1}(ax)}{256c^3} + \frac{3a^2\tan^{-1}(ax)}{32c^3\left(1+a^2x^2\right)^2} + \frac{9a^2\tan^{-1}(ax)}{32c^3\left(1+a^2x^2\right)}$$

$$= -\frac{3a^3x}{128c^3\left(1+a^2x^2\right)^2} - \frac{45a^3x}{256c^3\left(1+a^2x^2\right)} - \frac{45a^2\tan^{-1}(ax)}{256c^3} + \frac{3a^2\tan^{-1}(ax)}{32c^3\left(1+a^2x^2\right)^2} + \frac{9a^2\tan^{-1}(ax)}{32c^3\left(1+a^2x^2\right)}$$

Mathematica [A] time = 1.0469, size = 295, normalized size = 0.62

$$a^2\left(-4608i\tan^{-1}(ax)^2\text{PolyLog}\left(2,e^{-2i\tan^{-1}(ax)}\right) - 4608\tan^{-1}(ax)\text{PolyLog}\left(3,e^{-2i\tan^{-1}(ax)}\right) - 1536i\text{PolyLog}\left(2,e^{2i\tan\ldots}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTan[a*x]^3/(x^3*(c + a^2*c*x^2)^3),x]
```

```
[Out] (a^2*((48*I)*Pi^4 - (1536*I)*ArcTan[a*x]^2 - (1536*ArcTan[a*x]^2)/(a*x) - (
512*(1 + a^2*x^2)*ArcTan[a*x]^3)/(a^2*x^2) - (768*I)*ArcTan[a*x]^4 + 960*Ar
cTan[a*x]*Cos[2*ArcTan[a*x]] - 640*ArcTan[a*x]^3*Cos[2*ArcTan[a*x]] + 12*Ar
cTan[a*x]*Cos[4*ArcTan[a*x]] - 32*ArcTan[a*x]^3*Cos[4*ArcTan[a*x]] - 3072*A
rcTan[a*x]^3*Log[1 - E^((-2*I)*ArcTan[a*x])] + 3072*ArcTan[a*x]*Log[1 - E^(
(2*I)*ArcTan[a*x])] - (4608*I)*ArcTan[a*x]^2*PolyLog[2, E^((-2*I)*ArcTan[a*
x])] - (1536*I)*PolyLog[2, E^((2*I)*ArcTan[a*x])] - 4608*ArcTan[a*x]*PolyLo
g[3, E^((-2*I)*ArcTan[a*x])] + (2304*I)*PolyLog[4, E^((-2*I)*ArcTan[a*x])]
- 480*Sin[2*ArcTan[a*x]] + 960*ArcTan[a*x]^2*Sin[2*ArcTan[a*x]] - 3*Sin[4*A
rcTan[a*x]] + 24*ArcTan[a*x]^2*Sin[4*ArcTan[a*x]]))/(1024*c^3)
```

Maple [B] time = 13.75, size = 1339, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^3/x^3/(a^2*c*x^2+c)^3,x)
```

```
[Out] 3/4*I*a^2*arctan(a*x)^4/c^3-3/2*I*a^2*arctan(a*x)^2/c^3-1/2*a^2*arctan(a*x)
^3/c^3-1/2*arctan(a*x)^3/c^3/x^2-3/2*a*arctan(a*x)^2/c^3/x-3/256*I*a^4/c^3/
(a*x+I)^2*arctan(a*x)^2*x^2+1/32*I*a^3/c^3/(a*x+I)^2*arctan(a*x)^3*x-3/256*
I*a^3/c^3/(a*x+I)^2*arctan(a*x)*x+15/32*I*a^3/c^3/(a*x+I)*arctan(a*x)^2*x-1
5/32*I*a^3/c^3/(a*x-I)*arctan(a*x)^2*x+3/256*I*a^4/c^3/(a*x-I)^2*arctan(a*x
)^2*x^2-1/32*I*a^3/c^3/(a*x-I)^2*arctan(a*x)^3*x+3/256*I*a^3/c^3/(a*x-I)^2*
arctan(a*x)*x-3*a^2/c^3*arctan(a*x)^3*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-18*
a^2/c^3*arctan(a*x)*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))-3*a^2/c^3*arctan
(a*x)^3*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-18*a^2/c^3*arctan(a*x)*polylog(3,
-(1+I*a*x)/(a^2*x^2+1)^(1/2))+3*a^2/c^3*arctan(a*x)*ln(1-(1+I*a*x)/(a^2*x^2
+1)^(1/2))+3*a^2/c^3*arctan(a*x)*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))+1/64*a^2
/c^3/(a*x+I)^2*arctan(a*x)^3-3/512*a^2/c^3/(a*x+I)^2*arctan(a*x)+15/32*a^2/
c^3/(a*x+I)*arctan(a*x)^2+15/32*a^2/c^3/(a*x-I)*arctan(a*x)^2+1/64*a^2/c^3/
(a*x-I)^2*arctan(a*x)^3-3/512*a^2/c^3/(a*x-I)^2*arctan(a*x)+3/1024*a^3/c^3/
(a*x+I)^2*x+3/1024*a^3/c^3/(a*x-I)^2*x-3/2048*I*a^2/c^3/(a*x+I)^2+3/2048*I*
a^2/c^3/(a*x-I)^2-3*I*a^2/c^3*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-18*I*
a^2/c^3*polylog(4,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-18*I*a^2/c^3*polylog(4,(1+I
*a*x)/(a^2*x^2+1)^(1/2))-3*I*a^2/c^3*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))
-15/64*a^2/c^3/(a*x+I)-15/64*a^2/c^3/(a*x-I)-15/32*a^3/c^3/(a*x-I)*arctan(a
```

```
*x)*x-1/64*a^4/c^3/(a*x-I)^2*arctan(a*x)^3*x^2+3/512*a^4/c^3/(a*x-I)^2*arct
an(a*x)*x^2-3/128*a^3/c^3/(a*x+I)^2*arctan(a*x)^2*x-3/128*a^3/c^3/(a*x-I)^2
*arctan(a*x)^2*x+9*I*a^2/c^3*arctan(a*x)^2*polylog(2,(1+I*a*x)/(a^2*x^2+1)^
(1/2))+9*I*a^2/c^3*arctan(a*x)^2*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+3/
256*I*a^2/c^3/(a*x+I)^2*arctan(a*x)^2+5/16*a^3/c^3/(a*x-I)*arctan(a*x)^3*x-
5/16*I*a^2/c^3/(a*x+I)*arctan(a*x)^3+15/32*I*a^2/c^3/(a*x+I)*arctan(a*x)+5/
16*I*a^2/c^3/(a*x-I)*arctan(a*x)^3-15/32*I*a^2/c^3/(a*x-I)*arctan(a*x)-3/25
6*I*a^2/c^3/(a*x-I)^2*arctan(a*x)^2+3/2048*I*a^4/c^3/(a*x+I)^2*x^2-15/64*I*
a^3/c^3/(a*x+I)*x+15/64*I*a^3/c^3/(a*x-I)*x-3/2048*I*a^4/c^3/(a*x-I)^2*x^2-
1/64*a^4/c^3/(a*x+I)^2*arctan(a*x)^3*x^2+3/512*a^4/c^3/(a*x+I)^2*arctan(a*x
)*x^2+5/16*a^3/c^3/(a*x+I)*arctan(a*x)^3*x-15/32*a^3/c^3/(a*x+I)*arctan(a*x
)*x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(ax)^3}{(a^2cx^2+c)^3 x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x^3/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] integrate(arctan(a*x)^3/((a^2*c*x^2 + c)^3*x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\arctan(ax)^3}{a^6c^3x^9+3\,a^4c^3x^7+3\,a^2c^3x^5+c^3x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x^3/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] integral(arctan(a*x)^3/(a^6*c^3*x^9 + 3*a^4*c^3*x^7 + 3*a^2*c^3*x^5 + c^3*x
^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\operatorname{atan}^{3}(ax)}{a^{6}x^{9}+3a^{4}x^{7}+3a^{2}x^{5}+x^{3}}\,dx}{c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**3/x**3/(a**2*c*x**2+c)**3,x)
```

```
[Out] Integral(atan(a*x)**3/(a**6*x**9 + 3*a**4*x**7 + 3*a**2*x**5 + x**3), x)/c*
*3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan (ax)^{3}}{\left(a^{2}cx^{2}+c\right)^{3}x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x^3/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)^3/((a^2*c*x^2 + c)^3*x^3), x)
```

3.411 $\int \frac{\tan^{-1}(ax)^3}{x^4\left(c+a^2cx^2\right)^3}\,dx$

Optimal. Leaf size=432

$$-\frac{5a^3\text{PolyLog}\left(3,-1+\frac{2}{1-iax}\right)}{c^3}+\frac{10ia^3\tan^{-1}(ax)\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{c^3}-\frac{141a^3}{128c^3\left(a^2x^2+1\right)}-\frac{3a^3}{128c^3\left(a^2x^2+1\right)^2}-\frac{a^3 \text{I}}{}$$

```
[Out] (-3*a^3)/(128*c^3*(1 + a^2*x^2)^2) - (141*a^3)/(128*c^3*(1 + a^2*x^2)) - (a
^2*ArcTan[a*x])/(c^3*x) - (3*a^4*x*ArcTan[a*x])/(32*c^3*(1 + a^2*x^2)^2) -
(141*a^4*x*ArcTan[a*x])/(64*c^3*(1 + a^2*x^2)) - (205*a^3*ArcTan[a*x]^2)/(1
28*c^3) - (a*ArcTan[a*x]^2)/(2*c^3*x^2) + (3*a^3*ArcTan[a*x]^2)/(16*c^3*(1
+ a^2*x^2)^2) + (33*a^3*ArcTan[a*x]^2)/(16*c^3*(1 + a^2*x^2)) + (((10*I)/3)
*a^3*ArcTan[a*x]^3)/c^3 - ArcTan[a*x]^3/(3*c^3*x^3) + (3*a^2*ArcTan[a*x]^3)
/(c^3*x) + (a^4*x*ArcTan[a*x]^3)/(4*c^3*(1 + a^2*x^2)^2) + (11*a^4*x*ArcTan
[a*x]^3)/(8*c^3*(1 + a^2*x^2)) + (35*a^3*ArcTan[a*x]^4)/(32*c^3) + (a^3*Log
[x])/c^3 - (a^3*Log[1 + a^2*x^2])/(2*c^3) - (10*a^3*ArcTan[a*x]^2*Log[2 - 2
/(1 - I*a*x)])/c^3 + ((10*I)*a^3*ArcTan[a*x]*PolyLog[2, -1 + 2/(1 - I*a*x)]
)/c^3 - (5*a^3*PolyLog[3, -1 + 2/(1 - I*a*x)])/c^3
```

Rubi [A] time = 2.15952, antiderivative size = 432, normalized size of antiderivative = 1., number of steps used = 57, number of rules used = 17, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.773, Rules used = {4966, 4918, 4852, 266, 36, 29, 31, 4884, 4924, 4868, 4992, 6610, 4892, 4930, 261, 4900, 4896}

$$-\frac{5a^3\text{PolyLog}\left(3,-1+\frac{2}{1-iax}\right)}{c^3}+\frac{10ia^3\tan^{-1}(ax)\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{c^3}-\frac{141a^3}{128c^3\left(a^2x^2+1\right)}-\frac{3a^3}{128c^3\left(a^2x^2+1\right)^2}-\frac{a^3 \text{I}}{}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^3/(x^4*(c + a^2*c*x^2)^3),x]
```


```
[Out] (-3*a^3)/(128*c^3*(1 + a^2*x^2)^2) - (141*a^3)/(128*c^3*(1 + a^2*x^2)) - (a
^2*ArcTan[a*x])/(c^3*x) - (3*a^4*x*ArcTan[a*x])/(32*c^3*(1 + a^2*x^2)^2) -
(141*a^4*x*ArcTan[a*x])/(64*c^3*(1 + a^2*x^2)) - (205*a^3*ArcTan[a*x]^2)/(1
28*c^3) - (a*ArcTan[a*x]^2)/(2*c^3*x^2) + (3*a^3*ArcTan[a*x]^2)/(16*c^3*(1
+ a^2*x^2)^2) + (33*a^3*ArcTan[a*x]^2)/(16*c^3*(1 + a^2*x^2)) + (((10*I)/3)
*a^3*ArcTan[a*x]^3)/c^3 - ArcTan[a*x]^3/(3*c^3*x^3) + (3*a^2*ArcTan[a*x]^3)
/(c^3*x) + (a^4*x*ArcTan[a*x]^3)/(4*c^3*(1 + a^2*x^2)^2) + (11*a^4*x*ArcTan
```

```
[a*x]^3)/(8*c^3*(1 + a^2*x^2)) + (35*a^3*ArcTan[a*x]^4)/(32*c^3) + (a^3*Log
[x])/c^3 - (a^3*Log[1 + a^2*x^2])/(2*c^3) - (10*a^3*ArcTan[a*x]^2*Log[2 - 2
/(1 - I*a*x)])/c^3 + ((10*I)*a^3*ArcTan[a*x]*PolyLog[2, -1 + 2/(1 - I*a*x)]
)/c^3 - (5*a^3*PolyLog[3, -1 + 2/(1 - I*a*x)])/c^3
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*
x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^p
, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*q]
 && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 4918

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/(d + e*x^2),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 4992

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d
] && EqQ[(1 - u)^2 - (1 - (2*I)/(I + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 4892

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[(x*(a + b*ArcTan[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*c*
p)/2, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(a +
 b*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d, e},
```

```
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 4900

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(b*p*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^(p - 1))/(4*c*d
*(q + 1)^2), x] + (Dist[(2*q + 3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a
 + b*ArcTan[c*x])^p, x], x] - Dist[(b^2*p*(p - 1))/(4*(q + 1)^2), Int[(d +
e*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x] - Simp[(x*(d + e*x^2)^(q + 1)*
(a + b*ArcTan[c*x])^p)/(2*d*(q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] && E
qQ[e, c^2*d] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

Rule 4896

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbol
] :> Simp[(b*(d + e*x^2)^(q + 1))/(4*c*d*(q + 1)^2), x] + (Dist[(2*q + 3)/(
2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]), x], x] - Simp[(x
*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]))/(2*d*(q + 1)), x]) /; FreeQ[{a, b
, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -3/2]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^3}{x^4\left(c+a^2cx^2\right)^3}\,dx = -\left(a^2\int \frac{\tan^{-1}(ax)^3}{x^2\left(c+a^2cx^2\right)^3}\,dx\right) + \frac{\int \frac{\tan^{-1}(ax)^3}{x^4\left(c+a^2cx^2\right)^2}\,dx}{c}$$

$$= a^4\int \frac{\tan^{-1}(ax)^3}{\left(c+a^2cx^2\right)^3}\,dx + \frac{\int \frac{\tan^{-1}(ax)^3}{x^4\left(c+a^2cx^2\right)}\,dx}{c^2} - 2\frac{a^2\int \frac{\tan^{-1}(ax)^3}{x^2\left(c+a^2cx^2\right)^2}\,dx}{c}$$

$$= \frac{3a^3\tan^{-1}(ax)^2}{16c^3\left(1+a^2x^2\right)^2} + \frac{a^4x\tan^{-1}(ax)^3}{4c^3\left(1+a^2x^2\right)^2} - \frac{1}{8}\left(3a^4\right)\int \frac{\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^3}\,dx + \frac{\int \frac{\tan^{-1}(ax)^3}{x^4}\,dx}{c^3} - \frac{a^2\int \frac{\tan^{-1}(}{x^2(c+a}}{c^2}$$

$$= -\frac{3a^3}{128c^3\left(1+a^2x^2\right)^2} - \frac{3a^4x\tan^{-1}(ax)}{32c^3\left(1+a^2x^2\right)^2} + \frac{3a^3\tan^{-1}(ax)^2}{16c^3\left(1+a^2x^2\right)^2} - \frac{\tan^{-1}(ax)^3}{3c^3x^3} + \frac{a^4x\tan^{-1}(ax)^3}{4c^3\left(1+a^2x^2\right)^2} + 3$$

$$= -\frac{3a^3}{128c^3\left(1+a^2x^2\right)^2} - \frac{3a^4x\tan^{-1}(ax)}{32c^3\left(1+a^2x^2\right)^2} - \frac{9a^4x\tan^{-1}(ax)}{64c^3\left(1+a^2x^2\right)} - \frac{9a^3\tan^{-1}(ax)^2}{128c^3} + \frac{3a^3\tan^{-1}(ax)^2}{16c^3\left(1+a^2x^2\right)^2}$$

$$= -\frac{3a^3}{128c^3\left(1+a^2x^2\right)^2} - \frac{9a^3}{128c^3\left(1+a^2x^2\right)} - \frac{3a^4x\tan^{-1}(ax)}{32c^3\left(1+a^2x^2\right)^2} - \frac{45a^4x\tan^{-1}(ax)}{64c^3\left(1+a^2x^2\right)} - \frac{45a^3\tan^{-1}(ax}{128c^3}$$

$$= -\frac{3a^3}{128c^3\left(1+a^2x^2\right)^2} - \frac{45a^3}{128c^3\left(1+a^2x^2\right)} - \frac{3a^4x\tan^{-1}(ax)}{32c^3\left(1+a^2x^2\right)^2} - \frac{45a^4x\tan^{-1}(ax)}{64c^3\left(1+a^2x^2\right)} - \frac{45a^3\tan^{-1}(ax}{128c^3}$$

$$= -\frac{3a^3}{128c^3\left(1+a^2x^2\right)^2} - \frac{45a^3}{128c^3\left(1+a^2x^2\right)} - \frac{a^2\tan^{-1}(ax)}{c^3x} - \frac{3a^4x\tan^{-1}(ax)}{32c^3\left(1+a^2x^2\right)^2} - \frac{45a^4x\tan^{-1}(ax)}{64c^3\left(1+a^2x^2\right)} -$$

$$= -\frac{3a^3}{128c^3\left(1+a^2x^2\right)^2} - \frac{45a^3}{128c^3\left(1+a^2x^2\right)} - \frac{a^2\tan^{-1}(ax)}{c^3x} - \frac{3a^4x\tan^{-1}(ax)}{32c^3\left(1+a^2x^2\right)^2} - \frac{45a^4x\tan^{-1}(ax)}{64c^3\left(1+a^2x^2\right)} -$$

$$= -\frac{3a^3}{128c^3\left(1+a^2x^2\right)^2} - \frac{45a^3}{128c^3\left(1+a^2x^2\right)} - \frac{a^2\tan^{-1}(ax)}{c^3x} - \frac{3a^4x\tan^{-1}(ax)}{32c^3\left(1+a^2x^2\right)^2} - \frac{45a^4x\tan^{-1}(ax)}{64c^3\left(1+a^2x^2\right)} -$$

$$= -\frac{3a^3}{128c^3\left(1+a^2x^2\right)^2} - \frac{45a^3}{128c^3\left(1+a^2x^2\right)} - \frac{a^2\tan^{-1}(ax)}{c^3x} - \frac{3a^4x\tan^{-1}(ax)}{32c^3\left(1+a^2x^2\right)^2} - \frac{45a^4x\tan^{-1}(ax)}{64c^3\left(1+a^2x^2\right)} -$$

Mathematica [A] time = 1.16333, size = 301, normalized size = 0.7

$$a^3\left(-10i\tan^{-1}(ax)\text{PolyLog}\left(2,e^{-2i\tan^{-1}(ax)}\right)-5\text{PolyLog}\left(3,e^{-2i\tan^{-1}(ax)}\right)+\log\left(\frac{ax}{\sqrt{a^2x^2+1}}\right)-\frac{\tan^{-1}(ax)^3}{3a^3x^3}-\frac{\tan^{-1}(ax)^2}{2a^2x^2}+\frac{35}{32}\text{ta}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTan[a*x]^3/(x^4*(c + a^2*c*x^2)^3),x]
```

```
[Out] (a^3*(((5*I)/12)*Pi^3 - ArcTan[a*x]/(a*x) - ArcTan[a*x]^2/2 - ArcTan[a*x]^2
/(2*a^2*x^2) - ((10*I)/3)*ArcTan[a*x]^3 - ArcTan[a*x]^3/(3*a^3*x^3) + (3*Ar
cTan[a*x]^3)/(a*x) + (35*ArcTan[a*x]^4)/32 - (9*Cos[2*ArcTan[a*x]])/16 + (9
*ArcTan[a*x]^2*Cos[2*ArcTan[a*x]])/8 - (3*Cos[4*ArcTan[a*x]])/1024 + (3*Arc
Tan[a*x]^2*Cos[4*ArcTan[a*x]])/128 - 10*ArcTan[a*x]^2*Log[1 - E^((-2*I)*Arc
Tan[a*x])] + Log[(a*x)/Sqrt[1 + a^2*x^2]] - (10*I)*ArcTan[a*x]*PolyLog[2, E
^((-2*I)*ArcTan[a*x])] - 5*PolyLog[3, E^((-2*I)*ArcTan[a*x])] - (9*ArcTan[a
*x]*Sin[2*ArcTan[a*x]])/8 + (3*ArcTan[a*x]^3*Sin[2*ArcTan[a*x]])/4 - (3*Arc
Tan[a*x]*Sin[4*ArcTan[a*x]])/256 + (ArcTan[a*x]^3*Sin[4*ArcTan[a*x]])/32))/
c^3
```

Maple [C] time = 11.602, size = 2523, normalized size = 5.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^3/x^4/(a^2*c*x^2+c)^3,x)
```

```
[Out] -a^2*arctan(a*x)/c^3/x+3/256*a^4/c^3*arctan(a*x)/(a*x+I)^2*x+3/256*a^4/c^3*
arctan(a*x)/(a*x-I)^2*x+11/8*a^6/c^3*arctan(a*x)^3*x^3/(a^2*x^2+1)^2+3/1024
*I*a^4/c^3/(a*x+I)^2*x-3/1024*I*a^4/c^3/(a*x-I)^2*x-3/512*I*a^3/c^3*arctan(
a*x)/(a*x+I)^2+3/512*I*a^3/c^3*arctan(a*x)/(a*x-I)^2+20*I*a^3/c^3*arctan(a*
x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+20*I*a^3/c^3*arctan(a*x)*polylog
(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))-5*I*a^3/c^3*Pi*arctan(a*x)^2-1/2*a*arctan(a
*x)^2/c^3/x^2+3/16*a^3*arctan(a*x)^2/c^3/(a^2*x^2+1)^2+33/16*a^3*arctan(a*x
)^2/c^3/(a^2*x^2+1)+3*a^2*arctan(a*x)^3/c^3/x-1/3*arctan(a*x)^3/c^3/x^3+35/
32*a^3*arctan(a*x)^4/c^3-205/128*a^3*arctan(a*x)^2/c^3+13/8*a^4*x*arctan(a*
x)^3/c^3/(a^2*x^2+1)^2-5*I*a^3/c^3*Pi*arctan(a*x)^2*csgn(I*((1+I*a*x)^2/(a^
2*x^2+1)-1))*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x
^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))+5/2*I*a^3/c^3*Pi*csgn(I/((1+I*a*x)^2/
(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x
```

```
^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*arctan(a*x)^2+5*I*a^3/c^3*Pi*arctan(a*
x)^2*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)
/((1+I*a*x)^2/(a^2*x^2+1)+1))^2+10/3*I*a^3*arctan(a*x)^3/c^3+5*I*a^3/c^3*Pi
*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1
+I*a*x)^2/(a^2*x^2+1)+1))^2*arctan(a*x)^2-5/2*I*a^3/c^3*Pi*csgn(I/((1+I*a*x
)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+
1)+1)^2)^2*arctan(a*x)^2+5*I*a^3/c^3*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/
((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/
(a^2*x^2+1)+1))^2*arctan(a*x)^2-5/2*I*a^3/c^3*Pi*csgn(I*((1+I*a*x)^2/(a^2*x
^2+1)+1))^2*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*arctan(a*x)^2-5*I*a^3/c^3
*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(((
1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*arctan(a*x)^2+5*I*a^
3/c^3*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1
)+1)^2)^2*arctan(a*x)^2-5*I*a^3/c^3*Pi*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*
csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^2*arctan(a*x)^2+a^3/c^3*ln((1+I*a*x)/(a^2*x
^2+1)^(1/2)-1)-20*a^3/c^3*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-20*a^3/c^
3*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))+a^3/c^3*ln(1+(1+I*a*x)/(a^2*x^2+1)
^(1/2))+3/2048*a^3/c^3/(a*x+I)^2+3/2048*a^3/c^3/(a*x-I)^2+5/2*I*a^3/c^3*Pi*
arctan(a*x)^2*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)
^3-5*I*a^3/c^3*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+
1)+1))^3*arctan(a*x)^2-5*I*a^3/c^3*Pi*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+
I*a*x)^2/(a^2*x^2+1)+1))^3*arctan(a*x)^2-5/2*I*a^3/c^3*Pi*csgn(I*((1+I*a*x)
^2/(a^2*x^2+1)+1)^2)^3*arctan(a*x)^2+5/2*I*a^3/c^3*Pi*csgn(I*(1+I*a*x)^2/(a
^2*x^2+1))^3*arctan(a*x)^2-9/8*I*a^4/c^3*arctan(a*x)/(2*a*x+2*I)*x+9/8*I*a^
4/c^3*arctan(a*x)/(2*a*x-2*I)*x+3/512*I*a^5/c^3*arctan(a*x)/(a*x+I)^2*x^2-3
/512*I*a^5/c^3*arctan(a*x)/(a*x-I)^2*x^2+5*I*a^3/c^3*Pi*csgn(((1+I*a*x)^2/(
a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2*arctan(a*x)^2+5/2*I*a^3/c^3*Pi
*arctan(a*x)^2*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))^2*csgn(I*(1+I*a*x)^2/(a^
2*x^2+1))-5/2*I*a^3/c^3*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)
^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2*arctan(a*x)^2+9/32*a^4/c^3/
(a*x-I)*x-3/2048*a^5/c^3/(a*x+I)^2*x^2-3/2048*a^5/c^3/(a*x-I)^2*x^2+9/32*a^
4/c^3/(a*x+I)*x-10*a^3/c^3*arctan(a*x)^2*ln(2)-10*a^3/c^3*arctan(a*x)^2*ln(
a*x)+5*a^3/c^3*arctan(a*x)^2*ln(a^2*x^2+1)-9/8*a^3/c^3*arctan(a*x)/(2*a*x+2
*I)-9/8*a^3/c^3*arctan(a*x)/(2*a*x-2*I)-10*a^3/c^3*arctan(a*x)^2*ln((1+I*a*
x)/(a^2*x^2+1)^(1/2))+10*a^3/c^3*arctan(a*x)^2*ln((1+I*a*x)^2/(a^2*x^2+1)-1
)-10*a^3/c^3*arctan(a*x)^2*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-10*a^3/c^3*arc
tan(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))+9/32*I*a^3/c^3/(a*x-I)-9/32*I*
a^3/c^3/(a*x+I)-I*a^3/c^3*arctan(a*x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x^4/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\arctan(ax)^3}{a^6c^3x^{10} + 3\,a^4c^3x^8 + 3\,a^2c^3x^6 + c^3x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x^4/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] integral(arctan(a*x)^3/(a^6*c^3*x^10 + 3*a^4*c^3*x^8 + 3*a^2*c^3*x^6 + c^3*
x^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\operatorname{atan}^3(ax)}{a^6x^{10}+3a^4x^8+3a^2x^6+x^4}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**3/x**4/(a**2*c*x**2+c)**3,x)
```

```
[Out] Integral(atan(a*x)**3/(a**6*x**10 + 3*a**4*x**8 + 3*a**2*x**6 + x**4), x)/c
**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(ax)^3}{\left(a^2cx^2 + c\right)^3x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x^4/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)^3/((a^2*c*x^2 + c)^3*x^4), x)
```

3.412 $\int x^3\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3\,dx$

Optimal. Leaf size=523

$$\frac{11ic\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,-ie^{i\tan^{-1}(ax)}\right)}{20a^4\sqrt{a^2cx^2+c}}-\frac{11ic\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,ie^{i\tan^{-1}(ax)}\right)}{20a^4\sqrt{a^2cx^2+c}}-\frac{11c\sqrt{a^2x^2+1}\text{P}}{20a}$$

```
[Out] -(x*Sqrt[c + a^2*c*x^2])/(20*a^3) - (9*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(20
*a^4) + (x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(10*a^2) + (x*Sqrt[c + a^2*c*
x^2]*ArcTan[a*x]^2)/(8*a^3) - (3*x^3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(20
*a) - (((11*I)/20)*c*Sqrt[1 + a^2*x^2]*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x
]^2)/(a^4*Sqrt[c + a^2*c*x^2]) - (2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/(15*
a^4) + (x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/(15*a^2) + (x^4*Sqrt[c + a^2
*c*x^2]*ArcTan[a*x]^3)/5 + (Sqrt[c]*ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^
2]])/(2*a^4) + (((11*I)/20)*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, (-I)
*E^(I*ArcTan[a*x])])/(a^4*Sqrt[c + a^2*c*x^2]) - (((11*I)/20)*c*Sqrt[1 + a^
2*x^2]*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])])/(a^4*Sqrt[c + a^2*c*x^2
]) - (11*c*Sqrt[1 + a^2*x^2]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])])/(20*a^4*Sq
rt[c + a^2*c*x^2]) + (11*c*Sqrt[1 + a^2*x^2]*PolyLog[3, I*E^(I*ArcTan[a*x])
])/(20*a^4*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 2.4372, antiderivative size = 523, normalized size of antiderivative = 1., number of steps used = 71, number of rules used = 12, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4950, 4952, 4930, 217, 206, 4890, 4888, 4181, 2531, 2282, 6589, 321}

$$\frac{11ic\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,-ie^{i\tan^{-1}(ax)}\right)}{20a^4\sqrt{a^2cx^2+c}}-\frac{11ic\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,ie^{i\tan^{-1}(ax)}\right)}{20a^4\sqrt{a^2cx^2+c}}-\frac{11c\sqrt{a^2x^2+1}\text{P}}{20a}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3,x]
```


```
[Out] -(x*Sqrt[c + a^2*c*x^2])/(20*a^3) - (9*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(20
*a^4) + (x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(10*a^2) + (x*Sqrt[c + a^2*c*
x^2]*ArcTan[a*x]^2)/(8*a^3) - (3*x^3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(20
*a) - (((11*I)/20)*c*Sqrt[1 + a^2*x^2]*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x
]^2)/(a^4*Sqrt[c + a^2*c*x^2]) - (2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/(15*
a^4) + (x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/(15*a^2) + (x^4*Sqrt[c + a^2
*c*x^2]*ArcTan[a*x]^3)/5 + (Sqrt[c]*ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^
2]])/(2*a^4) + (((11*I)/20)*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, (-I)
```

```
*E^(I*ArcTan[a*x])])/(a^4*Sqrt[c + a^2*c*x^2]) - (((11*I)/20)*c*Sqrt[1 + a^
2*x^2]*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])])/(a^4*Sqrt[c + a^2*c*x^2
]) - (11*c*Sqrt[1 + a^2*x^2]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])])/(20*a^4*Sq
rt[c + a^2*c*x^2]) + (11*c*Sqrt[1 + a^2*x^2]*PolyLog[3, I*E^(I*ArcTan[a*x])
])/(20*a^4*Sqrt[c + a^2*c*x^2])
```

Rule 4950

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 4952

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b*
ArcTan[c*x])^p)/(c^2*d*m), x] + (-Dist[(b*f*p)/(c*m), Int[((f*x)^(m - 1)*(a
 + b*ArcTan[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] - Dist[(f^2*(m - 1))/(c^
2*m), Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/Sqrt[d + e*x^2], x], x]) /;
 FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 4890

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4888

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[c
*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && GtQ
[d, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int x^3\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3\,dx = c\int \frac{x^3\tan^{-1}(ax)^3}{\sqrt{c+a^2cx^2}}\,dx + \left(a^2c\right)\int \frac{x^5\tan^{-1}(ax)^3}{\sqrt{c+a^2cx^2}}\,dx$$

$$= \frac{x^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{3a^2} + \frac{1}{5}x^4\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3 - \frac{1}{5}(4c)\int \frac{x^3\tan^{-1}(ax)^3}{\sqrt{c+a^2cx^2}}\,dx -$$

$$= -\frac{x\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2a^3} - \frac{3x^3\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{20a} - \frac{2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{3a^4} +$$

$$= \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{a^4} + \frac{x^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{10a^2} + \frac{x\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{8a^3} -$$

$$= -\frac{x\sqrt{c+a^2cx^2}}{20a^3} - \frac{9\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{20a^4} + \frac{x^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{10a^2} + \frac{x\sqrt{c+a^2cx^2}\,\mathrm{ta}}{8a^3}$$

$$= -\frac{x\sqrt{c+a^2cx^2}}{20a^3} - \frac{9\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{20a^4} + \frac{x^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{10a^2} + \frac{x\sqrt{c+a^2cx^2}\,\mathrm{ta}}{8a^3}$$

$$= -\frac{x\sqrt{c+a^2cx^2}}{20a^3} - \frac{9\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{20a^4} + \frac{x^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{10a^2} + \frac{x\sqrt{c+a^2cx^2}\,\mathrm{ta}}{8a^3}$$

$$= -\frac{x\sqrt{c+a^2cx^2}}{20a^3} - \frac{9\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{20a^4} + \frac{x^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{10a^2} + \frac{x\sqrt{c+a^2cx^2}\,\mathrm{ta}}{8a^3}$$

$$= -\frac{x\sqrt{c+a^2cx^2}}{20a^3} - \frac{9\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{20a^4} + \frac{x^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{10a^2} + \frac{x\sqrt{c+a^2cx^2}\,\mathrm{ta}}{8a^3}$$

$$= -\frac{x\sqrt{c+a^2cx^2}}{20a^3} - \frac{9\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{20a^4} + \frac{x^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{10a^2} + \frac{x\sqrt{c+a^2cx^2}\,\mathrm{ta}}{8a^3}$$

Mathematica [A] time = 1.16379, size = 262, normalized size = 0.5

$$\frac{\sqrt{a^2cx^2+c}\left(-\left(a^2x^2+1\right)^2\left(\frac{48ax}{\left(a^2x^2+1\right)^2}+\tan^{-1}(ax)^2\left(6\sin\left(2\tan^{-1}(ax)\right)-33\sin\left(4\tan^{-1}(ax)\right)\right)+32\tan^{-1}(ax)^3\left(5\cos\left(2\,\mathrm{t}\right.\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3,x]
```

```
[Out] (Sqrt[c + a^2*c*x^2]*((48*((-11*I)*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2
+ 10*ArcTanh[(a*x)/Sqrt[1 + a^2*x^2]] + (11*I)*ArcTan[a*x]*PolyLog[2, (-I)*
E^(I*ArcTan[a*x])] - (11*I)*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])] - 1
1*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] + 11*PolyLog[3, I*E^(I*ArcTan[a*x])]))
/Sqrt[1 + a^2*x^2] - (1 + a^2*x^2)^2*((48*a*x)/(1 + a^2*x^2)^2 + 32*ArcTan[
a*x]^3*(-1 + 5*Cos[2*ArcTan[a*x]]) + 6*ArcTan[a*x]*(25 + 36*Cos[2*ArcTan[a*
x]] + 11*Cos[4*ArcTan[a*x]]) + ArcTan[a*x]^2*(6*Sin[2*ArcTan[a*x]] - 33*Sin
[4*ArcTan[a*x]]))))/(960*a^4)
```

Maple [A] time = 4.634, size = 417, normalized size = 0.8

$$\frac{24\,(\arctan(ax))^3x^4a^4-18\,(\arctan(ax))^2x^3a^3+8\,(\arctan(ax))^3x^2a^2+12\arctan(ax)\,a^2x^2+15\,(\arctan(ax))^2xa-}{120\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctan(a*x)^3*(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] 1/120/a^4*(c*(a*x-I)*(a*x+I))^(1/2)*(24*arctan(a*x)^3*x^4*a^4-18*arctan(a*x
)^2*x^3*a^3+8*arctan(a*x)^3*x^2*a^2+12*arctan(a*x)*a^2*x^2+15*arctan(a*x)^2
*x*a-16*arctan(a*x)^3-6*a*x-54*arctan(a*x))+11/120*(c*(a*x-I)*(a*x+I))^(1/2
)*(I*arctan(a*x)^3-3*arctan(a*x)^2*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I*
arctan(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*polylog(3,-I*(1+I*a
*x)/(a^2*x^2+1)^(1/2)))/a^4/(a^2*x^2+1)^(1/2)-11/120*(c*(a*x-I)*(a*x+I))^(1
/2)*(I*arctan(a*x)^3+6*I*arctan(a*x)*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2
))-3*arctan(a*x)^2*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*polylog(3,I*(1+I*a
*x)/(a^2*x^2+1)^(1/2)))/a^4/(a^2*x^2+1)^(1/2)-I/a^4*(c*(a*x-I)*(a*x+I))^(1/
2)*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*x^2+1)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^3*(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{a^2cx^2 + c}x^3 \arctan(ax)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^3*(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^3*arctan(a*x)^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\sqrt{c\left(a^2x^2 + 1\right)}\,\text{atan}^3\,(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atan(a*x)**3*(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(c*(a**2*x**2 + 1))*atan(a*x)**3, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^3*(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.413 $\int x^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3\,dx$

Optimal. Leaf size=747

$$-\frac{ic\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{2a^3\sqrt{a^2cx^2+c}}+\frac{ic\sqrt{a^2x^2+1}\text{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{2a^3\sqrt{a^2cx^2+c}}-\frac{3ic\sqrt{a^2x^2+1}\tan^{-1}(ax)^2\text{PolyLog}\left(2,-ie^{i\tan\right.}}{8a^3\sqrt{a^2cx^2+c}}$$

```
[Out] -Sqrt[c + a^2*c*x^2]/(4*a^3) + (x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(4*a^2)
+ (Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(8*a^3) - (x^2*Sqrt[c + a^2*c*x^2]*Ar
cTan[a*x]^2)/(4*a) + (x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/(8*a^2) + (x^3*S
qrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/4 + ((I/4)*c*Sqrt[1 + a^2*x^2]*ArcTan[E^(
I*ArcTan[a*x])]*ArcTan[a*x]^3)/(a^3*Sqrt[c + a^2*c*x^2]) + (I*c*Sqrt[1 + a^
2*x^2]*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/(a^3*Sqrt[c + a
^2*c*x^2]) - (((3*I)/8)*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog[2, (-I)*E
^(I*ArcTan[a*x])])/(a^3*Sqrt[c + a^2*c*x^2]) + (((3*I)/8)*c*Sqrt[1 + a^2*x^
2]*ArcTan[a*x]^2*PolyLog[2, I*E^(I*ArcTan[a*x])])/(a^3*Sqrt[c + a^2*c*x^2])
 - ((I/2)*c*Sqrt[1 + a^2*x^2]*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*
a*x]])/(a^3*Sqrt[c + a^2*c*x^2]) + ((I/2)*c*Sqrt[1 + a^2*x^2]*PolyLog[2, (I
*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/(a^3*Sqrt[c + a^2*c*x^2]) + (3*c*Sqrt[1
 + a^2*x^2]*ArcTan[a*x]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])])/(4*a^3*Sqrt[c +
 a^2*c*x^2]) - (3*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[3, I*E^(I*ArcTan[
a*x])])/(4*a^3*Sqrt[c + a^2*c*x^2]) + (((3*I)/4)*c*Sqrt[1 + a^2*x^2]*PolyLo
g[4, (-I)*E^(I*ArcTan[a*x])])/(a^3*Sqrt[c + a^2*c*x^2]) - (((3*I)/4)*c*Sqrt
[1 + a^2*x^2]*PolyLog[4, I*E^(I*ArcTan[a*x])])/(a^3*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 1.84627, antiderivative size = 747, normalized size of antiderivative = 1., number of steps used = 40, number of rules used = 12, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4950, 4952, 4930, 4890, 4886, 4888, 4181, 2531, 6609, 2282, 6589, 261}

$$-\frac{ic\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{2a^3\sqrt{a^2cx^2+c}}+\frac{ic\sqrt{a^2x^2+1}\text{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{2a^3\sqrt{a^2cx^2+c}}-\frac{3ic\sqrt{a^2x^2+1}\tan^{-1}(ax)^2\text{PolyLog}\left(2,-ie^{i\tan\right.}}{8a^3\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3,x]
```

```
[Out] -Sqrt[c + a^2*c*x^2]/(4*a^3) + (x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(4*a^2)
+ (Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(8*a^3) - (x^2*Sqrt[c + a^2*c*x^2]*Ar
cTan[a*x]^2)/(4*a) + (x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/(8*a^2) + (x^3*S
qrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/4 + ((I/4)*c*Sqrt[1 + a^2*x^2]*ArcTan[E^(
```

```
I*ArcTan[a*x])]*ArcTan[a*x]^3)/(a^3*Sqrt[c + a^2*c*x^2]) + (I*c*Sqrt[1 + a^
2*x^2]*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/(a^3*Sqrt[c + a
^2*c*x^2]) - (((3*I)/8)*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog[2, (-I)*E
^(I*ArcTan[a*x])])/(a^3*Sqrt[c + a^2*c*x^2]) + (((3*I)/8)*c*Sqrt[1 + a^2*x^
2]*ArcTan[a*x]^2*PolyLog[2, I*E^(I*ArcTan[a*x])])/(a^3*Sqrt[c + a^2*c*x^2])
 - ((I/2)*c*Sqrt[1 + a^2*x^2]*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*
a*x]])/(a^3*Sqrt[c + a^2*c*x^2]) + ((I/2)*c*Sqrt[1 + a^2*x^2]*PolyLog[2, (I
*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/(a^3*Sqrt[c + a^2*c*x^2]) + (3*c*Sqrt[1
 + a^2*x^2]*ArcTan[a*x]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])])/(4*a^3*Sqrt[c +
 a^2*c*x^2]) - (3*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[3, I*E^(I*ArcTan[
a*x])])/(4*a^3*Sqrt[c + a^2*c*x^2]) + (((3*I)/4)*c*Sqrt[1 + a^2*x^2]*PolyLo
g[4, (-I)*E^(I*ArcTan[a*x])])/(a^3*Sqrt[c + a^2*c*x^2]) - (((3*I)/4)*c*Sqrt
[1 + a^2*x^2]*PolyLog[4, I*E^(I*ArcTan[a*x])])/(a^3*Sqrt[c + a^2*c*x^2])
```

Rule 4950

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 4952

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b*
ArcTan[c*x])^p)/(c^2*d*m), x] + (-Dist[(b*f*p)/(c*m), Int[((f*x)^(m - 1)*(a
 + b*ArcTan[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] - Dist[(f^2*(m - 1))/(c^
2*m), Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/Sqrt[d + e*x^2], x], x]) /;
 FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 4890

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
```

```
IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4886

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[(-2*I*(a + b*ArcTan[c*x])*ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]])
/(c*Sqrt[d]), x] + (Simp[(I*b*PolyLog[2, -((I*Sqrt[1 + I*c*x])/Sqrt[1 - I*c
*x])])/(c*Sqrt[d]), x] - Simp[(I*b*PolyLog[2, (I*Sqrt[1 + I*c*x])/Sqrt[1 -
I*c*x]])/(c*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

Rule 4888

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[c
*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && GtQ
[d, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3\,dx = c\int \frac{x^2\tan^{-1}(ax)^3}{\sqrt{c+a^2cx^2}}\,dx + \left(a^2c\right)\int \frac{x^4\tan^{-1}(ax)^3}{\sqrt{c+a^2cx^2}}\,dx$$

$$= \frac{x\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{2a^2} + \frac{1}{4}x^3\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3 - \frac{1}{4}(3c)\int \frac{x^2\tan^{-1}(ax)^3}{\sqrt{c+a^2cx^2}}\,dx -$$

$$= -\frac{3\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2a^3} - \frac{x^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{4a} + \frac{x\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{8a^2} + \frac{1}{4}$$

$$= \frac{x\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{4a^2} + \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{8a^3} - \frac{x^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{4a} + \frac{x\sqrt{c}}{}$$

$$= -\frac{\sqrt{c+a^2cx^2}}{4a^3} + \frac{x\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{4a^2} + \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{8a^3} - \frac{x^2\sqrt{c+a^2cx^2}\tan}{4a}$$

$$= -\frac{\sqrt{c+a^2cx^2}}{4a^3} + \frac{x\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{4a^2} + \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{8a^3} - \frac{x^2\sqrt{c+a^2cx^2}\tan}{4a}$$

$$= -\frac{\sqrt{c+a^2cx^2}}{4a^3} + \frac{x\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{4a^2} + \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{8a^3} - \frac{x^2\sqrt{c+a^2cx^2}\tan}{4a}$$

$$= -\frac{\sqrt{c+a^2cx^2}}{4a^3} + \frac{x\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{4a^2} + \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{8a^3} - \frac{x^2\sqrt{c+a^2cx^2}\tan}{4a}$$

$$= -\frac{\sqrt{c+a^2cx^2}}{4a^3} + \frac{x\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{4a^2} + \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{8a^3} - \frac{x^2\sqrt{c+a^2cx^2}\tan}{4a}$$

$$= -\frac{\sqrt{c+a^2cx^2}}{4a^3} + \frac{x\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{4a^2} + \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{8a^3} - \frac{x^2\sqrt{c+a^2cx^2}\tan}{4a}$$

Mathematica [B] time = 12.1331, size = 1844, normalized size = 2.47

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3,x]
```

```
[Out] ((Sqrt[c*(1 + a^2*x^2)]*(-1 + ArcTan[a*x]^2))/(4*Sqrt[1 + a^2*x^2]) + (Sqrt
[c*(1 + a^2*x^2)]*(-(ArcTan[a*x]*(Log[1 - I*E^(I*ArcTan[a*x])] - Log[1 + I*
E^(I*ArcTan[a*x])])) - I*(PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - PolyLog[2, I
```

```
*E^(I*ArcTan[a*x])]])))/(2*Sqrt[1 + a^2*x^2]) + (Sqrt[c*(1 + a^2*x^2)]*(-(Pi
^3*Log[Cot[(Pi/2 - ArcTan[a*x])/2]])/8 - (3*Pi^2*((Pi/2 - ArcTan[a*x])*(Log
[1 - E^(I*(Pi/2 - ArcTan[a*x]))] - Log[1 + E^(I*(Pi/2 - ArcTan[a*x]))]) + I
*(PolyLog[2, -E^(I*(Pi/2 - ArcTan[a*x]))] - PolyLog[2, E^(I*(Pi/2 - ArcTan[
a*x]))])))/4 + (3*Pi*((Pi/2 - ArcTan[a*x])^2*(Log[1 - E^(I*(Pi/2 - ArcTan[a
*x]))] - Log[1 + E^(I*(Pi/2 - ArcTan[a*x]))]) + (2*I)*(Pi/2 - ArcTan[a*x])*
(PolyLog[2, -E^(I*(Pi/2 - ArcTan[a*x]))] - PolyLog[2, E^(I*(Pi/2 - ArcTan[a
*x]))]) + 2*(-PolyLog[3, -E^(I*(Pi/2 - ArcTan[a*x]))] + PolyLog[3, E^(I*(Pi
/2 - ArcTan[a*x]))])))/2 - 8*((I/64)*(Pi/2 - ArcTan[a*x])^4 + (I/4)*(Pi/2 +
 (-Pi/2 + ArcTan[a*x])/2)^4 - ((Pi/2 - ArcTan[a*x])^3*Log[1 + E^(I*(Pi/2 -
ArcTan[a*x]))])/8 - (Pi^3*(I*(Pi/2 + (-Pi/2 + ArcTan[a*x])/2) - Log[1 + E^(
(2*I)*(Pi/2 + (-Pi/2 + ArcTan[a*x])/2))]))/8 - (Pi/2 + (-Pi/2 + ArcTan[a*x]
)/2)^3*Log[1 + E^((2*I)*(Pi/2 + (-Pi/2 + ArcTan[a*x])/2))] + ((3*I)/8)*(Pi/
2 - ArcTan[a*x])^2*PolyLog[2, -E^(I*(Pi/2 - ArcTan[a*x]))] + (3*Pi^2*((I/2)
*(Pi/2 + (-Pi/2 + ArcTan[a*x])/2)^2 - (Pi/2 + (-Pi/2 + ArcTan[a*x])/2)*Log[
1 + E^((2*I)*(Pi/2 + (-Pi/2 + ArcTan[a*x])/2))] + (I/2)*PolyLog[2, -E^((2*I
)*(Pi/2 + (-Pi/2 + ArcTan[a*x])/2))]))/4 + ((3*I)/2)*(Pi/2 + (-Pi/2 + ArcTa
n[a*x])/2)^2*PolyLog[2, -E^((2*I)*(Pi/2 + (-Pi/2 + ArcTan[a*x])/2))] - (3*(
Pi/2 - ArcTan[a*x])*PolyLog[3, -E^(I*(Pi/2 - ArcTan[a*x]))])/4 - (3*Pi*((I/
3)*(Pi/2 + (-Pi/2 + ArcTan[a*x])/2)^3 - (Pi/2 + (-Pi/2 + ArcTan[a*x])/2)^2*
Log[1 + E^((2*I)*(Pi/2 + (-Pi/2 + ArcTan[a*x])/2))] + I*(Pi/2 + (-Pi/2 + Ar
cTan[a*x])/2)*PolyLog[2, -E^((2*I)*(Pi/2 + (-Pi/2 + ArcTan[a*x])/2))] - Pol
yLog[3, -E^((2*I)*(Pi/2 + (-Pi/2 + ArcTan[a*x])/2))]/2))/2 - (3*(Pi/2 + (-P
i/2 + ArcTan[a*x])/2)*PolyLog[3, -E^((2*I)*(Pi/2 + (-Pi/2 + ArcTan[a*x])/2)
)])/2 - ((3*I)/4)*PolyLog[4, -E^(I*(Pi/2 - ArcTan[a*x]))] - ((3*I)/4)*PolyL
og[4, -E^((2*I)*(Pi/2 + (-Pi/2 + ArcTan[a*x])/2))])))/(8*Sqrt[1 + a^2*x^2])
 + (Sqrt[c*(1 + a^2*x^2)]*ArcTan[a*x]^3)/(16*Sqrt[1 + a^2*x^2]*(Cos[ArcTan[
a*x]/2] - Sin[ArcTan[a*x]/2])^4) + (Sqrt[c*(1 + a^2*x^2)]*(2*ArcTan[a*x] -
ArcTan[a*x]^2 - ArcTan[a*x]^3))/(16*Sqrt[1 + a^2*x^2]*(Cos[ArcTan[a*x]/2] -
 Sin[ArcTan[a*x]/2])^2) - (Sqrt[c*(1 + a^2*x^2)]*ArcTan[a*x]^2*Sin[ArcTan[a
*x]/2])/(8*Sqrt[1 + a^2*x^2]*(Cos[ArcTan[a*x]/2] - Sin[ArcTan[a*x]/2])^3) -
 (Sqrt[c*(1 + a^2*x^2)]*ArcTan[a*x]^3)/(16*Sqrt[1 + a^2*x^2]*(Cos[ArcTan[a*
x]/2] + Sin[ArcTan[a*x]/2])^4) + (Sqrt[c*(1 + a^2*x^2)]*ArcTan[a*x]^2*Sin[A
rcTan[a*x]/2])/(8*Sqrt[1 + a^2*x^2]*(Cos[ArcTan[a*x]/2] + Sin[ArcTan[a*x]/2
])^3) + (Sqrt[c*(1 + a^2*x^2)]*(-2*ArcTan[a*x] - ArcTan[a*x]^2 + ArcTan[a*x
]^3))/(16*Sqrt[1 + a^2*x^2]*(Cos[ArcTan[a*x]/2] + Sin[ArcTan[a*x]/2])^2) +
(Sqrt[c*(1 + a^2*x^2)]*(Sin[ArcTan[a*x]/2] - ArcTan[a*x]^2*Sin[ArcTan[a*x]/
2]))/(4*Sqrt[1 + a^2*x^2]*(Cos[ArcTan[a*x]/2] + Sin[ArcTan[a*x]/2])) + (Sqr
t[c*(1 + a^2*x^2)]*(-Sin[ArcTan[a*x]/2] + ArcTan[a*x]^2*Sin[ArcTan[a*x]/2])
)/(4*Sqrt[1 + a^2*x^2]*(Cos[ArcTan[a*x]/2] - Sin[ArcTan[a*x]/2])))/a^3
```

Maple [A] time = 2.878, size = 460, normalized size = 0.6

$$\frac{2 (\arctan (ax))^3 a^3x^3 - 2 (\arctan (ax))^2 x^2a^2 + (\arctan (ax))^3 ax + 2 \arctan (ax) xa + (\arctan (ax))^2 - 2}{8 a^3}\sqrt{c (ax - i) (a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(a*x)^3*(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] 1/8/a^3*(c*(a*x-I)*(a*x+I))^(1/2)*(2*arctan(a*x)^3*a^3*x^3-2*arctan(a*x)^2*
x^2*a^2+arctan(a*x)^3*a*x+2*arctan(a*x)*x*a+arctan(a*x)^2-2)+1/8*(c*(a*x-I)
*(a*x+I))^(1/2)*(arctan(a*x)^3*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-arctan(a
*x)^3*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-3*I*arctan(a*x)^2*polylog(2,-I*(1
+I*a*x)/(a^2*x^2+1)^(1/2))+3*I*arctan(a*x)^2*polylog(2,I*(1+I*a*x)/(a^2*x^2
+1)^(1/2))+4*arctan(a*x)*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*arctan(a*x)*
polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-4*arctan(a*x)*ln(1-I*(1+I*a*x)/(a
^2*x^2+1)^(1/2))-6*arctan(a*x)*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I
*polylog(4,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-4*I*dilog(1+I*(1+I*a*x)/(a^2*x^2
+1)^(1/2))+4*I*dilog(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*I*polylog(4,I*(1+I*
a*x)/(a^2*x^2+1)^(1/2)))/a^3/(a^2*x^2+1)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^3*(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{a^2cx^2 + c}x^2 \arctan (ax)^3 , x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^3*(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^2*arctan(a*x)^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \sqrt{c \left(a^2 x^2 + 1\right)} \operatorname{atan}^3(ax) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(a*x)**3*(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(c*(a**2*x**2 + 1))*atan(a*x)**3, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^3*(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.414 $\int x\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3\,dx$

Optimal. Leaf size=373

$$-\frac{ic\sqrt{a^2x^2+1}\tan^{-1}(ax)\mathrm{PolyLog}\left(2,-ie^{i\tan^{-1}(ax)}\right)}{a^2\sqrt{a^2cx^2+c}}+\frac{ic\sqrt{a^2x^2+1}\tan^{-1}(ax)\mathrm{PolyLog}\left(2,ie^{i\tan^{-1}(ax)}\right)}{a^2\sqrt{a^2cx^2+c}}+\frac{c\sqrt{a^2x^2+1}\mathrm{Poly}\cdots}{a^2\sqrt{\cdots}}$$

```
[Out] (Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/a^2 - (x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^
2)/(2*a) + (I*c*Sqrt[1 + a^2*x^2]*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2)/
(a^2*Sqrt[c + a^2*c*x^2]) + ((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3)/(3*a^2*c)
 - (Sqrt[c]*ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]])/a^2 - (I*c*Sqrt[1 +
 a^2*x^2]*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])])/(a^2*Sqrt[c + a^2
*c*x^2]) + (I*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x]
)])/(a^2*Sqrt[c + a^2*c*x^2]) + (c*Sqrt[1 + a^2*x^2]*PolyLog[3, (-I)*E^(I*A
rcTan[a*x])])/(a^2*Sqrt[c + a^2*c*x^2]) - (c*Sqrt[1 + a^2*x^2]*PolyLog[3, I
*E^(I*ArcTan[a*x])])/(a^2*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.355473, antiderivative size = 373, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {4930, 4880, 4890, 4888, 4181, 2531, 2282, 6589, 217, 206}

$$-\frac{ic\sqrt{a^2x^2+1}\tan^{-1}(ax)\mathrm{PolyLog}\left(2,-ie^{i\tan^{-1}(ax)}\right)}{a^2\sqrt{a^2cx^2+c}}+\frac{ic\sqrt{a^2x^2+1}\tan^{-1}(ax)\mathrm{PolyLog}\left(2,ie^{i\tan^{-1}(ax)}\right)}{a^2\sqrt{a^2cx^2+c}}+\frac{c\sqrt{a^2x^2+1}\mathrm{Poly}\cdots}{a^2\sqrt{\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3,x]
```

```
[Out] (Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/a^2 - (x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^
2)/(2*a) + (I*c*Sqrt[1 + a^2*x^2]*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2)/
(a^2*Sqrt[c + a^2*c*x^2]) + ((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3)/(3*a^2*c)
 - (Sqrt[c]*ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]])/a^2 - (I*c*Sqrt[1 +
 a^2*x^2]*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])])/(a^2*Sqrt[c + a^2
*c*x^2]) + (I*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x]
)])/(a^2*Sqrt[c + a^2*c*x^2]) + (c*Sqrt[1 + a^2*x^2]*PolyLog[3, (-I)*E^(I*A
rcTan[a*x])])/(a^2*Sqrt[c + a^2*c*x^2]) - (c*Sqrt[1 + a^2*x^2]*PolyLog[3, I
*E^(I*ArcTan[a*x])])/(a^2*Sqrt[c + a^2*c*x^2])
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 4880

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> -Simp[(b*p*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1))/(2*c*q*(2*
q + 1)), x] + (Dist[(2*d*q)/(2*q + 1), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTa
n[c*x])^p, x], x] + Dist[(b^2*d*p*(p - 1))/(2*q*(2*q + 1)), Int[(d + e*x^2)
^(q - 1)*(a + b*ArcTan[c*x])^(p - 2), x], x] + Simp[(x*(d + e*x^2)^q*(a + b
*ArcTan[c*x])^p)/(2*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[q, 0] && GtQ[p, 1]
```

Rule 4890

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4888

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[c
*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && GtQ
[d, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
```

```
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3\,dx = \frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^3}{3a^2c} - \frac{\int \sqrt{c+a^2cx^2}\tan^{-1}(ax)^2\,dx}{a}$$

$$= \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{a^2} - \frac{x\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2a} + \frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^3}{3a^2c} - \frac{c\int \frac{\tan^{-}}{\sqrt{c+}}}{2}$$

$$= \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{a^2} - \frac{x\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2a} + \frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^3}{3a^2c} - \frac{c\,\mathrm{Subst}}{}$$

$$= \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{a^2} - \frac{x\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2a} + \frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^3}{3a^2c} - \frac{\sqrt{c}\tanh}{}$$

$$= \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{a^2} - \frac{x\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2a} + \frac{ic\sqrt{1+a^2x^2}\tan^{-1}\left(e^{i\tan^{-1}(ax)}\right)\tan^{-}}{a^2\sqrt{c+a^2cx^2}}$$

$$= \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{a^2} - \frac{x\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2a} + \frac{ic\sqrt{1+a^2x^2}\tan^{-1}\left(e^{i\tan^{-1}(ax)}\right)\tan^{-}}{a^2\sqrt{c+a^2cx^2}}$$

$$= \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{a^2} - \frac{x\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2a} + \frac{ic\sqrt{1+a^2x^2}\tan^{-1}\left(e^{i\tan^{-1}(ax)}\right)\tan^{-}}{a^2\sqrt{c+a^2cx^2}}$$

$$= \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{a^2} - \frac{x\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2a} + \frac{ic\sqrt{1+a^2x^2}\tan^{-1}\left(e^{i\tan^{-1}(ax)}\right)\tan^{-}}{a^2\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 0.581, size = 206, normalized size = 0.55

$$\frac{\sqrt{a^2cx^2+c}\left(\left(a^2x^2+1\right)\tan^{-1}(ax)\left(4\tan^{-1}(ax)^2-3\tan^{-1}(ax)\sin\left(2\tan^{-1}(ax)\right)+6\cos\left(2\tan^{-1}(ax)\right)+6\right)+\frac{12\left(-i\tan^{-1}(a\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3,x]
```

```
[Out] (Sqrt[c + a^2*c*x^2]*((12*(I*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2 - ArcT
anh[(a*x)/Sqrt[1 + a^2*x^2]] - I*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*
x])] + I*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])] + PolyLog[3, (-I)*E^(I
*ArcTan[a*x])] - PolyLog[3, I*E^(I*ArcTan[a*x])]))/Sqrt[1 + a^2*x^2] + (1 +
 a^2*x^2)*ArcTan[a*x]*(6 + 4*ArcTan[a*x]^2 + 6*Cos[2*ArcTan[a*x]] - 3*ArcTa
n[a*x]*Sin[2*ArcTan[a*x]])))/(12*a^2)
```

Maple [A] time = 1.884, size = 370, normalized size = 1.

$$\frac{\arctan(ax)\left(2\left(\arctan(ax)\right)^2x^2a^2-3\arctan(ax)xa+2\left(\arctan(ax)\right)^2+6\right)}{6a^2}\sqrt{c(ax-i)(ax+i)}-\frac{1}{6a^2}\sqrt{c(ax-i)(ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(a*x)^3*(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] 1/6/a^2*(c*(a*x-I)*(a*x+I))^(1/2)*arctan(a*x)*(2*arctan(a*x)^2*x^2*a^2-3*ar
ctan(a*x)*x*a+2*arctan(a*x)^2+6)-1/6*(c*(a*x-I)*(a*x+I))^(1/2)*(I*arctan(a*
x)^3-3*arctan(a*x)^2*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I*arctan(a*x)*po
lylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1
)^(1/2)))/a^2/(a^2*x^2+1)^(1/2)+1/6*(c*(a*x-I)*(a*x+I))^(1/2)*(I*arctan(a*x
)^3+6*I*arctan(a*x)*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-3*arctan(a*x)^
2*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(
1/2)))/a^2/(a^2*x^2+1)^(1/2)+2*I/a^2*(c*(a*x-I)*(a*x+I))^(1/2)*arctan((1+I*
a*x)/(a^2*x^2+1)^(1/2))/(a^2*x^2+1)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a^2cx^2+c}\,x\arctan(ax)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^3*(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)*x*arctan(a*x)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{a^2cx^2+c}\,x\arctan(ax)^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^3*(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x*arctan(a*x)^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{c\left(a^{2}x^{2}+1\right)}\operatorname{atan}^{3}\left(ax\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(a*x)**3*(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x*sqrt(c*(a**2*x**2 + 1))*atan(a*x)**3, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^3*(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.415 $\int \sqrt{c+a^2cx^2}\tan^{-1}(ax)^3\,dx$

Optimal. Leaf size=626

$$\frac{3ic\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a\sqrt{a^2cx^2+c}}-\frac{3ic\sqrt{a^2x^2+1}\text{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a\sqrt{a^2cx^2+c}}+\frac{3ic\sqrt{a^2x^2+1}\tan^{-1}(ax)^2\text{PolyLog}\left(2,-ie^{i\tan^{-1}(ax)}\right)}{2a\sqrt{a^2cx^2+c}}$$

```
[Out] (-3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(2*a) + (x*Sqrt[c + a^2*c*x^2]*ArcTa
n[a*x]^3)/2 - (I*c*Sqrt[1 + a^2*x^2]*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^
3)/(a*Sqrt[c + a^2*c*x^2]) - ((6*I)*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTan[
Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/(a*Sqrt[c + a^2*c*x^2]) + (((3*I)/2)*c*Sq
rt[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog[2, (-I)*E^(I*ArcTan[a*x])])/(a*Sqrt[c
 + a^2*c*x^2]) - (((3*I)/2)*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog[2, I*
E^(I*ArcTan[a*x])])/(a*Sqrt[c + a^2*c*x^2]) + ((3*I)*c*Sqrt[1 + a^2*x^2]*Po
lyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/(a*Sqrt[c + a^2*c*x^2]) -
 ((3*I)*c*Sqrt[1 + a^2*x^2]*PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]]
)/(a*Sqrt[c + a^2*c*x^2]) - (3*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[3, (
-I)*E^(I*ArcTan[a*x])])/(a*Sqrt[c + a^2*c*x^2]) + (3*c*Sqrt[1 + a^2*x^2]*Ar
cTan[a*x]*PolyLog[3, I*E^(I*ArcTan[a*x])])/(a*Sqrt[c + a^2*c*x^2]) - ((3*I)
*c*Sqrt[1 + a^2*x^2]*PolyLog[4, (-I)*E^(I*ArcTan[a*x])])/(a*Sqrt[c + a^2*c*
x^2]) + ((3*I)*c*Sqrt[1 + a^2*x^2]*PolyLog[4, I*E^(I*ArcTan[a*x])])/(a*Sqrt
[c + a^2*c*x^2])
```

Rubi [A] time = 0.361453, antiderivative size = 626, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4880, 4890, 4888, 4181, 2531, 6609, 2282, 6589, 4886}

$$\frac{3ic\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a\sqrt{a^2cx^2+c}}-\frac{3ic\sqrt{a^2x^2+1}\text{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a\sqrt{a^2cx^2+c}}+\frac{3ic\sqrt{a^2x^2+1}\tan^{-1}(ax)^2\text{PolyLog}\left(2,-ie^{i\tan^{-1}(ax)}\right)}{2a\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3,x]
```


```
[Out] (-3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(2*a) + (x*Sqrt[c + a^2*c*x^2]*ArcTa
n[a*x]^3)/2 - (I*c*Sqrt[1 + a^2*x^2]*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^
3)/(a*Sqrt[c + a^2*c*x^2]) - ((6*I)*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTan[
Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/(a*Sqrt[c + a^2*c*x^2]) + (((3*I)/2)*c*Sq
rt[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog[2, (-I)*E^(I*ArcTan[a*x])])/(a*Sqrt[c
 + a^2*c*x^2]) - (((3*I)/2)*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog[2, I*
```

```
E^(I*ArcTan[a*x])])/(a*Sqrt[c + a^2*c*x^2]) + ((3*I)*c*Sqrt[1 + a^2*x^2]*Po
lyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/(a*Sqrt[c + a^2*c*x^2]) -
 ((3*I)*c*Sqrt[1 + a^2*x^2]*PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]]
)/(a*Sqrt[c + a^2*c*x^2]) - (3*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[3, (
-I)*E^(I*ArcTan[a*x])])/(a*Sqrt[c + a^2*c*x^2]) + (3*c*Sqrt[1 + a^2*x^2]*Ar
cTan[a*x]*PolyLog[3, I*E^(I*ArcTan[a*x])])/(a*Sqrt[c + a^2*c*x^2]) - ((3*I)
*c*Sqrt[1 + a^2*x^2]*PolyLog[4, (-I)*E^(I*ArcTan[a*x])])/(a*Sqrt[c + a^2*c*
x^2]) + ((3*I)*c*Sqrt[1 + a^2*x^2]*PolyLog[4, I*E^(I*ArcTan[a*x])])/(a*Sqrt
[c + a^2*c*x^2])
```

Rule 4880

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> -Simp[(b*p*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1))/(2*c*q*(2*
q + 1)), x] + (Dist[(2*d*q)/(2*q + 1), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTa
n[c*x])^p, x], x] + Dist[(b^2*d*p*(p - 1))/(2*q*(2*q + 1)), Int[(d + e*x^2)
^(q - 1)*(a + b*ArcTan[c*x])^(p - 2), x], x] + Simp[(x*(d + e*x^2)^q*(a + b
*ArcTan[c*x])^p)/(2*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[q, 0] && GtQ[p, 1]
```

Rule 4890

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4888

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[c
*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && GtQ
[d, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
```

```
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4886

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[(-2*I*(a + b*ArcTan[c*x])*ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]])
/(c*Sqrt[d]), x] + (Simp[(I*b*PolyLog[2, -((I*Sqrt[1 + I*c*x])/Sqrt[1 - I*c
*x])])/(c*Sqrt[d]), x] - Simp[(I*b*PolyLog[2, (I*Sqrt[1 + I*c*x])/Sqrt[1 -
I*c*x]])/(c*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

Rubi steps

$$\int \sqrt{c+a^2cx^2}\tan^{-1}(ax)^3\,dx = -\frac{3\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2a} + \frac{1}{2}x\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3 + \frac{1}{2}c\int \frac{\tan^{-1}(ax)^3}{\sqrt{c+a^2cx^2}}\,dx + (3c)\int \frac{\mathrm{t}}{\sqrt{}}$$

$$= -\frac{3\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2a} + \frac{1}{2}x\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3 + \frac{\left(c\sqrt{1+a^2x^2}\right)\int \frac{\tan^{-1}(ax)^3}{\sqrt{1+a^2x^2}}\,dx}{2\sqrt{c+a^2cx^2}} + \frac{(}{}$$

$$= -\frac{3\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2a} + \frac{1}{2}x\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3 - \frac{6ic\sqrt{1+a^2x^2}\tan^{-1}(ax)\tan^{-1}\left(\right.}{a\sqrt{c+a^2cx^2}}$$

$$= -\frac{3\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2a} + \frac{1}{2}x\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3 - \frac{ic\sqrt{1+a^2x^2}\tan^{-1}\left(e^{i\tan^{-1}(ax)}\right)\mathrm{ta}}{a\sqrt{c+a^2cx^2}}$$

$$= -\frac{3\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2a} + \frac{1}{2}x\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3 - \frac{ic\sqrt{1+a^2x^2}\tan^{-1}\left(e^{i\tan^{-1}(ax)}\right)\mathrm{ta}}{a\sqrt{c+a^2cx^2}}$$

$$= -\frac{3\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2a} + \frac{1}{2}x\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3 - \frac{ic\sqrt{1+a^2x^2}\tan^{-1}\left(e^{i\tan^{-1}(ax)}\right)\mathrm{ta}}{a\sqrt{c+a^2cx^2}}$$

$$= -\frac{3\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2a} + \frac{1}{2}x\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3 - \frac{ic\sqrt{1+a^2x^2}\tan^{-1}\left(e^{i\tan^{-1}(ax)}\right)\mathrm{ta}}{a\sqrt{c+a^2cx^2}}$$

$$= -\frac{3\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2a} + \frac{1}{2}x\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3 - \frac{ic\sqrt{1+a^2x^2}\tan^{-1}\left(e^{i\tan^{-1}(ax)}\right)\mathrm{ta}}{a\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 0.798037, size = 258, normalized size = 0.41

$$-\frac{i\sqrt{c\left(a^2x^2+1\right)}\left(-6i\tan^{-1}(ax)\mathrm{PolyLog}\left(3,-ie^{i\tan^{-1}(ax)}\right)+6i\tan^{-1}(ax)\mathrm{PolyLog}\left(3,ie^{i\tan^{-1}(ax)}\right)-3\left(\tan^{-1}(ax)^2+2\right)\mathrm{Po}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3,x]
```

```
[Out] ((-I/2)*Sqrt[c*(1 + a^2*x^2)]*(12*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x] - (
3*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2 + I*a*x*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^
3 + 2*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^3 - 3*(2 + ArcTan[a*x]^2)*PolyL
og[2, (-I)*E^(I*ArcTan[a*x])] + 3*(2 + ArcTan[a*x]^2)*PolyLog[2, I*E^(I*Arc
Tan[a*x])] - (6*I)*ArcTan[a*x]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] + (6*I)*A
rcTan[a*x]*PolyLog[3, I*E^(I*ArcTan[a*x])] + 6*PolyLog[4, (-I)*E^(I*ArcTan[
```

```
a*x])] - 6*PolyLog[4, I*E^(I*ArcTan[a*x])]))/(a*Sqrt[1 + a^2*x^2])
```

Maple [A] time = 2.117, size = 422, normalized size = 0.7

$$\frac{(\arctan(ax))^2(\arctan(ax)xa-3)}{2a}\sqrt{c(ax-i)(ax+i)}-\frac{1}{2a}\sqrt{c(ax-i)(ax+i)}\left((\arctan(ax))^3\ln\left(1+i(1+iax)\frac{1}{\sqrt{a^2x^2}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^3*(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] 1/2/a*(c*(a*x-I)*(a*x+I))^(1/2)*arctan(a*x)^2*(arctan(a*x)*x*a-3)-1/2*(c*(a
*x-I)*(a*x+I))^(1/2)*(arctan(a*x)^3*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-arc
tan(a*x)^3*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-3*I*arctan(a*x)^2*polylog(2,
-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+3*I*arctan(a*x)^2*polylog(2,I*(1+I*a*x)/(a^
2*x^2+1)^(1/2))+6*arctan(a*x)*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*arctan(
a*x)*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*arctan(a*x)*ln(1-I*(1+I*a*
x)/(a^2*x^2+1)^(1/2))-6*arctan(a*x)*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2)
)+6*I*polylog(4,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*I*polylog(4,I*(1+I*a*x)/(
a^2*x^2+1)^(1/2))-6*I*dilog(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I*dilog(1-I*
(1+I*a*x)/(a^2*x^2+1)^(1/2)))/a/(a^2*x^2+1)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3*(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{a^2cx^2+c}\arctan(ax)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3*(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c\left(a^{2}x^{2}+1\right)}\,\mathrm{atan}^{3}\left(ax\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**3*(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(c*(a**2*x**2 + 1))*atan(a*x)**3, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3*(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.416 $\int \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{x}\,dx$

Optimal. Leaf size=600

$$\frac{3ic\sqrt{a^2x^2+1}\tan^{-1}(ax)^2\text{PolyLog}\left(2,-e^{i\tan^{-1}(ax)}\right)}{\sqrt{a^2cx^2+c}}-\frac{3ic\sqrt{a^2x^2+1}\tan^{-1}(ax)^2\text{PolyLog}\left(2,e^{i\tan^{-1}(ax)}\right)}{\sqrt{a^2cx^2+c}}-\frac{6ic\sqrt{a^2x^2+1}\,\text{t}}{}$$

```
[Out] ((6*I)*c*Sqrt[1 + a^2*x^2]*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2)/Sqrt[c
+ a^2*c*x^2] + Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3 - (2*c*Sqrt[1 + a^2*x^2]*A
rcTan[a*x]^3*ArcTanh[E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + ((3*I)*c*Sqr
t[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog[2, -E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c
*x^2] - ((6*I)*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[
a*x])])/Sqrt[c + a^2*c*x^2] + ((6*I)*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLo
g[2, I*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - ((3*I)*c*Sqrt[1 + a^2*x^2]
*ArcTan[a*x]^2*PolyLog[2, E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - (6*c*Sq
rt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[3, -E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*
x^2] + (6*c*Sqrt[1 + a^2*x^2]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])])/Sqrt[c +
a^2*c*x^2] - (6*c*Sqrt[1 + a^2*x^2]*PolyLog[3, I*E^(I*ArcTan[a*x])])/Sqrt[c
 + a^2*c*x^2] + (6*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[3, E^(I*ArcTan[a
*x])])/Sqrt[c + a^2*c*x^2] - ((6*I)*c*Sqrt[1 + a^2*x^2]*PolyLog[4, -E^(I*Ar
cTan[a*x])])/Sqrt[c + a^2*c*x^2] + ((6*I)*c*Sqrt[1 + a^2*x^2]*PolyLog[4, E^
(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2]
```

Rubi [A] time = 0.727169, antiderivative size = 600, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 12, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4950, 4958, 4956, 4183, 2531, 6609, 2282, 6589, 4930, 4890, 4888, 4181}

$$\frac{3ic\sqrt{a^2x^2+1}\tan^{-1}(ax)^2\text{PolyLog}\left(2,-e^{i\tan^{-1}(ax)}\right)}{\sqrt{a^2cx^2+c}}-\frac{3ic\sqrt{a^2x^2+1}\tan^{-1}(ax)^2\text{PolyLog}\left(2,e^{i\tan^{-1}(ax)}\right)}{\sqrt{a^2cx^2+c}}-\frac{6ic\sqrt{a^2x^2+1}\,\text{t}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/x,x]
```

```
[Out] ((6*I)*c*Sqrt[1 + a^2*x^2]*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2)/Sqrt[c
+ a^2*c*x^2] + Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3 - (2*c*Sqrt[1 + a^2*x^2]*A
rcTan[a*x]^3*ArcTanh[E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + ((3*I)*c*Sqr
t[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog[2, -E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c
*x^2] - ((6*I)*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[
a*x])])/Sqrt[c + a^2*c*x^2] + ((6*I)*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLo
```

```
g[2, I*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - ((3*I)*c*Sqrt[1 + a^2*x^2]
*ArcTan[a*x]^2*PolyLog[2, E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - (6*c*Sq
rt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[3, -E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*
x^2] + (6*c*Sqrt[1 + a^2*x^2]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])])/Sqrt[c +
a^2*c*x^2] - (6*c*Sqrt[1 + a^2*x^2]*PolyLog[3, I*E^(I*ArcTan[a*x])])/Sqrt[c
 + a^2*c*x^2] + (6*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[3, E^(I*ArcTan[a
*x])])/Sqrt[c + a^2*c*x^2] - ((6*I)*c*Sqrt[1 + a^2*x^2]*PolyLog[4, -E^(I*Ar
cTan[a*x])])/Sqrt[c + a^2*c*x^2] + ((6*I)*c*Sqrt[1 + a^2*x^2]*PolyLog[4, E^
(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2]
```

Rule 4950

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 4958

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4956

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Dist[1/Sqrt[d], Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTan
[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && G
tQ[d, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
```

```
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 4890

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4888

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[c
```

```
*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && GtQ
[d, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{x}\,dx = c\int \frac{\tan^{-1}(ax)^3}{x\sqrt{c+a^2cx^2}}\,dx + \left(a^2c\right)\int \frac{x\tan^{-1}(ax)^3}{\sqrt{c+a^2cx^2}}\,dx$$

$$= \sqrt{c+a^2cx^2}\tan^{-1}(ax)^3 - (3ac)\int \frac{\tan^{-1}(ax)^2}{\sqrt{c+a^2cx^2}}\,dx + \frac{\left(c\sqrt{1+a^2x^2}\right)\int \frac{\tan^{-1}(ax)^3}{x\sqrt{1+a^2x^2}}\,dx}{\sqrt{c+a^2cx^2}}$$

$$= \sqrt{c+a^2cx^2}\tan^{-1}(ax)^3 + \frac{\left(c\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int x^3\csc(x)\,dx, x, \tan^{-1}(ax)\right)}{\sqrt{c+a^2cx^2}} - \frac{\left(3ac\sqrt{1+\right.}{}$$

$$= \sqrt{c+a^2cx^2}\tan^{-1}(ax)^3 - \frac{2c\sqrt{1+a^2x^2}\tan^{-1}(ax)^3\tanh^{-1}\left(e^{i\tan^{-1}(ax)}\right)}{\sqrt{c+a^2cx^2}} - \frac{\left(3c\sqrt{1+a^2x^2}\right)\text{S}}{}$$

$$= \frac{6ic\sqrt{1+a^2x^2}\tan^{-1}\left(e^{i\tan^{-1}(ax)}\right)\tan^{-1}(ax)^2}{\sqrt{c+a^2cx^2}} + \sqrt{c+a^2cx^2}\tan^{-1}(ax)^3 - \frac{2c\sqrt{1+a^2x^2}\tan}{}$$

$$= \frac{6ic\sqrt{1+a^2x^2}\tan^{-1}\left(e^{i\tan^{-1}(ax)}\right)\tan^{-1}(ax)^2}{\sqrt{c+a^2cx^2}} + \sqrt{c+a^2cx^2}\tan^{-1}(ax)^3 - \frac{2c\sqrt{1+a^2x^2}\tan}{}$$

$$= \frac{6ic\sqrt{1+a^2x^2}\tan^{-1}\left(e^{i\tan^{-1}(ax)}\right)\tan^{-1}(ax)^2}{\sqrt{c+a^2cx^2}} + \sqrt{c+a^2cx^2}\tan^{-1}(ax)^3 - \frac{2c\sqrt{1+a^2x^2}\tan}{}$$

$$= \frac{6ic\sqrt{1+a^2x^2}\tan^{-1}\left(e^{i\tan^{-1}(ax)}\right)\tan^{-1}(ax)^2}{\sqrt{c+a^2cx^2}} + \sqrt{c+a^2cx^2}\tan^{-1}(ax)^3 - \frac{2c\sqrt{1+a^2x^2}\tan}{}$$

Mathematica [A] time = 0.585957, size = 366, normalized size = 0.61

$$\frac{\sqrt{a^2cx^2+c}\left(24i\tan^{-1}(ax)^2\text{PolyLog}\left(2,e^{-i\tan^{-1}(ax)}\right)+24i\tan^{-1}(ax)^2\text{PolyLog}\left(2,-e^{i\tan^{-1}(ax)}\right)-48i\tan^{-1}(ax)\text{PolyLog}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/x,x]
```

```
[Out] (Sqrt[c + a^2*c*x^2]*((-I)*Pi^4 + 8*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^3 + (2*I)
*ArcTan[a*x]^4 + 8*ArcTan[a*x]^3*Log[1 - E^((-I)*ArcTan[a*x])] - 24*ArcTan[
a*x]^2*Log[1 - I*E^(I*ArcTan[a*x])] + 24*ArcTan[a*x]^2*Log[1 + I*E^(I*ArcTa
n[a*x])] - 8*ArcTan[a*x]^3*Log[1 + E^(I*ArcTan[a*x])] + (24*I)*ArcTan[a*x]^
2*PolyLog[2, E^((-I)*ArcTan[a*x])] + (24*I)*ArcTan[a*x]^2*PolyLog[2, -E^(I*
ArcTan[a*x])] - (48*I)*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] + (48
*I)*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])] + 48*ArcTan[a*x]*PolyLog[3,
 E^((-I)*ArcTan[a*x])] - 48*ArcTan[a*x]*PolyLog[3, -E^(I*ArcTan[a*x])] + 48
*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] - 48*PolyLog[3, I*E^(I*ArcTan[a*x])] -
(48*I)*PolyLog[4, E^((-I)*ArcTan[a*x])] - (48*I)*PolyLog[4, -E^(I*ArcTan[a*
x])]))/(8*Sqrt[1 + a^2*x^2])
```

Maple [A] time = 2.316, size = 453, normalized size = 0.8

$$\sqrt{c\,(ax-i)\,(ax+i)}\,(\arctan(ax))^3+\sqrt{c\,(ax-i)\,(ax+i)}\left((\arctan(ax))^3\ln\left(1-(1+iax)\frac{1}{\sqrt{a^2x^2+1}}\right)-(\arctan(ax))^3\ln\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)/x,x)
```

```
[Out] (c*(a*x-I)*(a*x+I))^(1/2)*arctan(a*x)^3+(c*(a*x-I)*(a*x+I))^(1/2)*(arctan(a
*x)^3*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-arctan(a*x)^3*ln(1+(1+I*a*x)/(a^2*x
^2+1)^(1/2))-3*I*arctan(a*x)^2*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+3*I*a
rctan(a*x)^2*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+3*arctan(a*x)^2*ln(1+I
*(1+I*a*x)/(a^2*x^2+1)^(1/2))-3*arctan(a*x)^2*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^
(1/2))-6*I*arctan(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I*arctan
(a*x)*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*arctan(a*x)*polylog(3,(1+I
*a*x)/(a^2*x^2+1)^(1/2))-6*arctan(a*x)*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/
2))+6*I*polylog(4,(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*I*polylog(4,-(1+I*a*x)/(a^
2*x^2+1)^(1/2))+6*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*polylog(3,I*(
1+I*a*x)/(a^2*x^2+1)^(1/2)))/(a^2*x^2+1)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a^2cx^2+c}\arctan\left(ax\right)^3}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^3/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{c\left(a^2x^2+1\right)}\operatorname{atan}^3\left(ax\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**3*(a**2*c*x**2+c)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(c*(a**2*x**2 + 1))*atan(a*x)**3/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{a^2cx^2+c}\arctan\left(ax\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)*arctan(a*x)^3/x, x)
```

3.417 $\int \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{x^2}\,dx$

Optimal. Leaf size=622

$$\frac{3iac\sqrt{a^2x^2+1}\tan^{-1}(ax)^2\text{PolyLog}\left(2,-ie^{i\tan^{-1}(ax)}\right)}{\sqrt{a^2cx^2+c}}-\frac{3iac\sqrt{a^2x^2+1}\tan^{-1}(ax)^2\text{PolyLog}\left(2,ie^{i\tan^{-1}(ax)}\right)}{\sqrt{a^2cx^2+c}}+\frac{6iac\sqrt{a^2x^2+}}{}$$

```
[Out] -((Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/x) - ((2*I)*a*c*Sqrt[1 + a^2*x^2]*Arc
Tan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^3)/Sqrt[c + a^2*c*x^2] - (6*a*c*Sqrt[1 +
 a^2*x^2]*ArcTan[a*x]^2*ArcTanh[E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + (
(6*I)*a*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])])/Sqr
t[c + a^2*c*x^2] + ((3*I)*a*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog[2, (-
I)*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - ((3*I)*a*c*Sqrt[1 + a^2*x^2]*A
rcTan[a*x]^2*PolyLog[2, I*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - ((6*I)*
a*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, E^(I*ArcTan[a*x])])/Sqrt[c + a
^2*c*x^2] - (6*a*c*Sqrt[1 + a^2*x^2]*PolyLog[3, -E^(I*ArcTan[a*x])])/Sqrt[c
 + a^2*c*x^2] - (6*a*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[3, (-I)*E^(I*A
rcTan[a*x])])/Sqrt[c + a^2*c*x^2] + (6*a*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*Po
lyLog[3, I*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + (6*a*c*Sqrt[1 + a^2*x^
2]*PolyLog[3, E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - ((6*I)*a*c*Sqrt[1 +
 a^2*x^2]*PolyLog[4, (-I)*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + ((6*I)*
a*c*Sqrt[1 + a^2*x^2]*PolyLog[4, I*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2]
```

Rubi [A] time = 0.774342, antiderivative size = 622, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 12, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4950, 4944, 4958, 4956, 4183, 2531, 2282, 6589, 4890, 4888, 4181, 6609}

$$\frac{3iac\sqrt{a^2x^2+1}\tan^{-1}(ax)^2\text{PolyLog}\left(2,-ie^{i\tan^{-1}(ax)}\right)}{\sqrt{a^2cx^2+c}}-\frac{3iac\sqrt{a^2x^2+1}\tan^{-1}(ax)^2\text{PolyLog}\left(2,ie^{i\tan^{-1}(ax)}\right)}{\sqrt{a^2cx^2+c}}+\frac{6iac\sqrt{a^2x^2+}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/x^2,x]
```

```
[Out] -((Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/x) - ((2*I)*a*c*Sqrt[1 + a^2*x^2]*Arc
Tan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^3)/Sqrt[c + a^2*c*x^2] - (6*a*c*Sqrt[1 +
 a^2*x^2]*ArcTan[a*x]^2*ArcTanh[E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + (
(6*I)*a*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])])/Sqr
t[c + a^2*c*x^2] + ((3*I)*a*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog[2, (-
I)*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - ((3*I)*a*c*Sqrt[1 + a^2*x^2]*A
```

```
rcTan[a*x]^2*PolyLog[2, I*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - ((6*I)*
a*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, E^(I*ArcTan[a*x])])/Sqrt[c + a
^2*c*x^2] - (6*a*c*Sqrt[1 + a^2*x^2]*PolyLog[3, -E^(I*ArcTan[a*x])])/Sqrt[c
 + a^2*c*x^2] - (6*a*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[3, (-I)*E^(I*A
rcTan[a*x])])/Sqrt[c + a^2*c*x^2] + (6*a*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*Po
lyLog[3, I*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + (6*a*c*Sqrt[1 + a^2*x^
2]*PolyLog[3, E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - ((6*I)*a*c*Sqrt[1 +
 a^2*x^2]*PolyLog[4, (-I)*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + ((6*I)*
a*c*Sqrt[1 + a^2*x^2]*PolyLog[4, I*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2]
```

Rule 4950

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 4944

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a +
 b*ArcTan[c*x])^p)/(d*f*(m + 1)), x] - Dist[(b*c*p)/(f*(m + 1)), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rule 4958

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4956

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Dist[1/Sqrt[d], Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTan
[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && G
tQ[d, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
```

```
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4890

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4888

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[c
*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && GtQ
[d, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
```

```
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{x^2}\,dx = c\int \frac{\tan^{-1}(ax)^3}{x^2\sqrt{c+a^2cx^2}}\,dx + \left(a^2c\right)\int \frac{\tan^{-1}(ax)^3}{\sqrt{c+a^2cx^2}}\,dx$$

$$= -\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{x} + (3ac)\int \frac{\tan^{-1}(ax)^2}{x\sqrt{c+a^2cx^2}}\,dx + \frac{\left(a^2c\sqrt{1+a^2x^2}\right)\int \frac{\tan^{-1}(ax)^3}{\sqrt{1+a^2x^2}}\,dx}{\sqrt{c+a^2cx^2}}$$

$$= -\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{x} + \frac{\left(ac\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int x^3\sec(x)\,dx, x, \tan^{-1}(ax)\right)}{\sqrt{c+a^2cx^2}} + \frac{\left(3ac\right.}{}$$

$$= -\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{x} - \frac{2iac\sqrt{1+a^2x^2}\tan^{-1}\left(e^{i\tan^{-1}(ax)}\right)\tan^{-1}(ax)^3}{\sqrt{c+a^2cx^2}} + \frac{\left(3ac\sqrt{1+a^2\right.}{}$$

$$= -\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{x} - \frac{2iac\sqrt{1+a^2x^2}\tan^{-1}\left(e^{i\tan^{-1}(ax)}\right)\tan^{-1}(ax)^3}{\sqrt{c+a^2cx^2}} - \frac{6ac\sqrt{1+a^2x}}{}$$

$$= -\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{x} - \frac{2iac\sqrt{1+a^2x^2}\tan^{-1}\left(e^{i\tan^{-1}(ax)}\right)\tan^{-1}(ax)^3}{\sqrt{c+a^2cx^2}} - \frac{6ac\sqrt{1+a^2x}}{}$$

$$= -\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{x} - \frac{2iac\sqrt{1+a^2x^2}\tan^{-1}\left(e^{i\tan^{-1}(ax)}\right)\tan^{-1}(ax)^3}{\sqrt{c+a^2cx^2}} - \frac{6ac\sqrt{1+a^2x}}{}$$

$$= -\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{x} - \frac{2iac\sqrt{1+a^2x^2}\tan^{-1}\left(e^{i\tan^{-1}(ax)}\right)\tan^{-1}(ax)^3}{\sqrt{c+a^2cx^2}} - \frac{6ac\sqrt{1+a^2x}}{}$$

Mathematica [A] time = 3.1749, size = 768, normalized size = 1.23

$$\frac{a\sqrt{a^2cx^2+c}\csc\left(\frac{1}{2}\tan^{-1}(ax)\right)\sec\left(\frac{1}{2}\tan^{-1}(ax)\right)\left(192iax\tan^{-1}(ax)^2\text{PolyLog}\left(2,-ie^{-i\tan^{-1}(ax)}\right)+192iax\tan^{-1}(ax)^2\text{Po}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/x^2,x]
```

```
[Out] (a*Sqrt[c + a^2*c*x^2]*Csc[ArcTan[a*x]/2]*((-7*I)*a*Pi^4*x - (8*I)*a*Pi^3*x
*ArcTan[a*x] + (24*I)*a*Pi^2*x*ArcTan[a*x]^2 - (32*I)*a*Pi*x*ArcTan[a*x]^3
- 64*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^3 + (16*I)*a*x*ArcTan[a*x]^4 + 48*a*Pi^2
*x*ArcTan[a*x]*Log[1 - I/E^(I*ArcTan[a*x])] - 96*a*Pi*x*ArcTan[a*x]^2*Log[1
 - I/E^(I*ArcTan[a*x])] - 8*a*Pi^3*x*Log[1 + I/E^(I*ArcTan[a*x])] + 64*a*x*
ArcTan[a*x]^3*Log[1 + I/E^(I*ArcTan[a*x])] + 192*a*x*ArcTan[a*x]^2*Log[1 -
E^(I*ArcTan[a*x])] + 8*a*Pi^3*x*Log[1 + I*E^(I*ArcTan[a*x])] - 48*a*Pi^2*x*
ArcTan[a*x]*Log[1 + I*E^(I*ArcTan[a*x])] + 96*a*Pi*x*ArcTan[a*x]^2*Log[1 +
I*E^(I*ArcTan[a*x])] - 64*a*x*ArcTan[a*x]^3*Log[1 + I*E^(I*ArcTan[a*x])] -
192*a*x*ArcTan[a*x]^2*Log[1 + E^(I*ArcTan[a*x])] + 8*a*Pi^3*x*Log[Tan[(Pi +
 2*ArcTan[a*x])/4]] + (192*I)*a*x*ArcTan[a*x]^2*PolyLog[2, (-I)/E^(I*ArcTan
[a*x])] + (48*I)*a*Pi*x*(Pi - 4*ArcTan[a*x])*PolyLog[2, I/E^(I*ArcTan[a*x])
] + (384*I)*a*x*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])] + (48*I)*a*Pi^2*
x*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - (192*I)*a*Pi*x*ArcTan[a*x]*PolyLog[2
, (-I)*E^(I*ArcTan[a*x])] + (192*I)*a*x*ArcTan[a*x]^2*PolyLog[2, (-I)*E^(I*
ArcTan[a*x])] - (384*I)*a*x*ArcTan[a*x]*PolyLog[2, E^(I*ArcTan[a*x])] + 384
*a*x*ArcTan[a*x]*PolyLog[3, (-I)/E^(I*ArcTan[a*x])] - 192*a*Pi*x*PolyLog[3,
 I/E^(I*ArcTan[a*x])] - 384*a*x*PolyLog[3, -E^(I*ArcTan[a*x])] + 192*a*Pi*x
*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] - 384*a*x*ArcTan[a*x]*PolyLog[3, (-I)*E
^(I*ArcTan[a*x])] + 384*a*x*PolyLog[3, E^(I*ArcTan[a*x])] - (384*I)*a*x*Pol
yLog[4, (-I)/E^(I*ArcTan[a*x])] - (384*I)*a*x*PolyLog[4, (-I)*E^(I*ArcTan[a
*x])])*Sec[ArcTan[a*x]/2])/(128*(1 + a^2*x^2))
```

Maple [A] time = 2.294, size = 466, normalized size = 0.8

$$-\frac{(\arctan(ax))^3}{x}\sqrt{c(ax-i)(ax+i)}+ia\sqrt{c(ax-i)(ax+i)}\left(i(\arctan(ax))^3\ln\left(1+i(1+iax)\frac{1}{\sqrt{a^2x^2+1}}\right)-i(\arctan(ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)/x^2,x)
```

```
[Out] -(c*(a*x-I)*(a*x+I))^(1/2)*arctan(a*x)^3/x+I*a*(c*(a*x-I)*(a*x+I))^(1/2)*(I
*arctan(a*x)^3*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*arctan(a*x)^3*ln(1-I*(
1+I*a*x)/(a^2*x^2+1)^(1/2))-3*I*arctan(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1
/2))+3*I*arctan(a*x)^2*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))+3*arctan(a*x)^2*po
lylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-3*arctan(a*x)^2*polylog(2,I*(1+I*a*
```

```
x)/(a^2*x^2+1)^(1/2))+6*I*arctan(a*x)*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1
/2))-6*I*arctan(a*x)*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*arctan(a*x)
*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*arctan(a*x)*polylog(2,-(1+I*a*x)/
(a^2*x^2+1)^(1/2))-6*I*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I*polylog(3
,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*polylog(4,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+
6*polylog(4,I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))/(a^2*x^2+1)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a^2cx^2+c}\,\arctan(ax)^3}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^3/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c\left(a^2x^2+1\right)}\,\operatorname{atan}^3(ax)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**3*(a**2*c*x**2+c)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(c*(a**2*x**2 + 1))*atan(a*x)**3/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)*arctan(a*x)^3/x^2, x)
```

3.418 $\int \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{x^3}\,dx$

Optimal. Leaf size=602

$$\frac{3ia^2c\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{a^2cx^2+c}}-\frac{3ia^2c\sqrt{a^2x^2+1}\text{PolyLog}\left(2,\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{a^2cx^2+c}}+\frac{3ia^2c\sqrt{a^2x^2+1}\tan^{-1}(ax)^2\text{PolyLog}\left(2,\right.}{2\sqrt{a^2cx^2+c}}$$

```
[Out] (-3*a*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(2*x) - (Sqrt[c + a^2*c*x^2]*ArcTa
n[a*x]^3)/(2*x^2) - (a^2*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^3*ArcTanh[E^(I*Arc
Tan[a*x])])/Sqrt[c + a^2*c*x^2] - (6*a^2*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*Ar
cTanh[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2] + (((3*I)/2)*a^
2*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog[2, -E^(I*ArcTan[a*x])])/Sqrt[c
+ a^2*c*x^2] - (((3*I)/2)*a^2*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog[2,
E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + ((3*I)*a^2*c*Sqrt[1 + a^2*x^2]*Po
lyLog[2, -(Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x])])/Sqrt[c + a^2*c*x^2] - ((3*I)*
a^2*c*Sqrt[1 + a^2*x^2]*PolyLog[2, Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/Sqrt[c
 + a^2*c*x^2] - (3*a^2*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[3, -E^(I*Arc
Tan[a*x])])/Sqrt[c + a^2*c*x^2] + (3*a^2*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*Po
lyLog[3, E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - ((3*I)*a^2*c*Sqrt[1 + a^
2*x^2]*PolyLog[4, -E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + ((3*I)*a^2*c*S
qrt[1 + a^2*x^2]*PolyLog[4, E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2]
```

Rubi [A] time = 1.23769, antiderivative size = 602, normalized size of antiderivative = 1., number of steps used = 27, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {4950, 4962, 4944, 4958, 4954, 4956, 4183, 2531, 6609, 2282, 6589}

$$\frac{3ia^2c\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{a^2cx^2+c}}-\frac{3ia^2c\sqrt{a^2x^2+1}\text{PolyLog}\left(2,\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{a^2cx^2+c}}+\frac{3ia^2c\sqrt{a^2x^2+1}\tan^{-1}(ax)^2\text{PolyLog}\left(2,\right.}{2\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/x^3,x]
```



```
[Out] (-3*a*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(2*x) - (Sqrt[c + a^2*c*x^2]*ArcTa
n[a*x]^3)/(2*x^2) - (a^2*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^3*ArcTanh[E^(I*Arc
Tan[a*x])])/Sqrt[c + a^2*c*x^2] - (6*a^2*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*Ar
cTanh[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2] + (((3*I)/2)*a^
2*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog[2, -E^(I*ArcTan[a*x])])/Sqrt[c
+ a^2*c*x^2] - (((3*I)/2)*a^2*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog[2,
E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + ((3*I)*a^2*c*Sqrt[1 + a^2*x^2]*Po
```

```
lyLog[2, -(Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x])])/Sqrt[c + a^2*c*x^2] - ((3*I)*
a^2*c*Sqrt[1 + a^2*x^2]*PolyLog[2, Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/Sqrt[c
 + a^2*c*x^2] - (3*a^2*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[3, -E^(I*Arc
Tan[a*x])])/Sqrt[c + a^2*c*x^2] + (3*a^2*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*Po
lyLog[3, E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - ((3*I)*a^2*c*Sqrt[1 + a^
2*x^2]*PolyLog[4, -E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + ((3*I)*a^2*c*S
qrt[1 + a^2*x^2]*PolyLog[4, E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2]
```

Rule 4950

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 4962

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Ar
cTan[c*x])^p)/(d*f*(m + 1)), x] + (-Dist[(b*c*p)/(f*(m + 1)), Int[((f*x)^(m
 + 1)*(a + b*ArcTan[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] - Dist[(c^2*(m +
 2))/(f^2*(m + 1)), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/Sqrt[d + e*x^
2], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] &
& LtQ[m, -1] && NeQ[m, -2]
```

Rule 4944

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a +
 b*ArcTan[c*x])^p)/(d*f*(m + 1)), x] - Dist[(b*c*p)/(f*(m + 1)), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rule 4958

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4954

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x_
Symbol] :> Simp[(-2*(a + b*ArcTan[c*x])*ArcTanh[Sqrt[1 + I*c*x]/Sqrt[1 - I*
c*x]])/Sqrt[d], x] + (Simp[(I*b*PolyLog[2, -(Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x
])])/Sqrt[d], x] - Simp[(I*b*PolyLog[2, Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]])/S
qrt[d], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

Rule 4956

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Dist[1/Sqrt[d], Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTan
[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && G
tQ[d, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{x^3}\,dx = c\int \frac{\tan^{-1}(ax)^3}{x^3\sqrt{c+a^2cx^2}}\,dx + \left(a^2c\right)\int \frac{\tan^{-1}(ax)^3}{x\sqrt{c+a^2cx^2}}\,dx$$

$$= -\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{2x^2} + \frac{1}{2}(3ac)\int \frac{\tan^{-1}(ax)^2}{x^2\sqrt{c+a^2cx^2}}\,dx - \frac{1}{2}\left(a^2c\right)\int \frac{\tan^{-1}(ax)^3}{x\sqrt{c+a^2cx^2}}\,dx + \frac{(a}{}$$

$$= -\frac{3a\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2x} - \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{2x^2} + \left(3a^2c\right)\int \frac{\tan^{-1}(ax)}{x\sqrt{c+a^2cx^2}}\,dx - \frac{(a^2c}{}$$

$$= -\frac{3a\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2x} - \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{2x^2} - \frac{2a^2c\sqrt{1+a^2x^2}\tan^{-1}(ax)^3\tanh^{-}}{\sqrt{c+a^2cx^2}}$$

$$= -\frac{3a\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2x} - \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{2x^2} - \frac{a^2c\sqrt{1+a^2x^2}\tan^{-1}(ax)^3\tanh^{-1}}{\sqrt{c+a^2cx^2}}$$

$$= -\frac{3a\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2x} - \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{2x^2} - \frac{a^2c\sqrt{1+a^2x^2}\tan^{-1}(ax)^3\tanh^{-1}}{\sqrt{c+a^2cx^2}}$$

$$= -\frac{3a\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2x} - \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{2x^2} - \frac{a^2c\sqrt{1+a^2x^2}\tan^{-1}(ax)^3\tanh^{-1}}{\sqrt{c+a^2cx^2}}$$

$$= -\frac{3a\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2x} - \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{2x^2} - \frac{a^2c\sqrt{1+a^2x^2}\tan^{-1}(ax)^3\tanh^{-1}}{\sqrt{c+a^2cx^2}}$$

$$= -\frac{3a\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2x} - \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{2x^2} - \frac{a^2c\sqrt{1+a^2x^2}\tan^{-1}(ax)^3\tanh^{-1}}{\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 5.51688, size = 345, normalized size = 0.57

$$\frac{a^2\sqrt{c\left(a^2x^2+1\right)}\left(24i\tan^{-1}(ax)^2\text{PolyLog}\left(2,e^{-i\tan^{-1}(ax)}\right)+48\tan^{-1}(ax)\text{PolyLog}\left(3,e^{-i\tan^{-1}(ax)}\right)-48\tan^{-1}(ax)\text{PolyLo}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/x^3,x]
```

```
[Out] (a^2*Sqrt[c*(1 + a^2*x^2)]*((-I)*Pi^4 + (2*I)*ArcTan[a*x]^4 - 12*ArcTan[a*x
]^2*Cot[ArcTan[a*x]/2] - 2*ArcTan[a*x]^3*Csc[ArcTan[a*x]/2]^2 + 8*ArcTan[a*
x]^3*Log[1 - E^((-I)*ArcTan[a*x])] + 48*ArcTan[a*x]*Log[1 - E^(I*ArcTan[a*x
])] - 48*ArcTan[a*x]*Log[1 + E^(I*ArcTan[a*x])] - 8*ArcTan[a*x]^3*Log[1 + E
^(I*ArcTan[a*x])] + (24*I)*ArcTan[a*x]^2*PolyLog[2, E^((-I)*ArcTan[a*x])] +
 (24*I)*(2 + ArcTan[a*x]^2)*PolyLog[2, -E^(I*ArcTan[a*x])] - (48*I)*PolyLog
[2, E^(I*ArcTan[a*x])] + 48*ArcTan[a*x]*PolyLog[3, E^((-I)*ArcTan[a*x])] -
48*ArcTan[a*x]*PolyLog[3, -E^(I*ArcTan[a*x])] - (48*I)*PolyLog[4, E^((-I)*A
rcTan[a*x])] - (48*I)*PolyLog[4, -E^(I*ArcTan[a*x])] + 2*ArcTan[a*x]^3*Sec[
ArcTan[a*x]/2]^2 - 12*ArcTan[a*x]^2*Tan[ArcTan[a*x]/2]))/(16*Sqrt[1 + a^2*x
^2])
```

Maple [A] time = 1.722, size = 404, normalized size = 0.7

$$-\frac{(\arctan(ax))^2(3ax+\arctan(ax))}{2x^2}\sqrt{c(ax-i)(ax+i)}+\frac{i}{2}a^2\sqrt{c(ax-i)(ax+i)}\left(i(\arctan(ax))^3\ln\left(1+(1+iax)\frac{}{\sqrt{a^2}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)/x^3,x)
```

```
[Out] -1/2*(c*(a*x-I)*(a*x+I))^(1/2)*arctan(a*x)^2*(3*a*x+arctan(a*x))/x^2+1/2*I*
a^2*(c*(a*x-I)*(a*x+I))^(1/2)*(I*arctan(a*x)^3*ln(1+(1+I*a*x)/(a^2*x^2+1)^(
1/2))-I*arctan(a*x)^3*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))+3*arctan(a*x)^2*pol
ylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-3*arctan(a*x)^2*polylog(2,(1+I*a*x)/(a
^2*x^2+1)^(1/2))+6*I*arctan(a*x)*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I*arct
an(a*x)*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*I*arctan(a*x)*ln(1-(1+I*a
*x)/(a^2*x^2+1)^(1/2))-6*I*arctan(a*x)*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2
))+6*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*polylog(4,-(1+I*a*x)/(a^2*x^
2+1)^(1/2))-6*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*polylog(4,(1+I*a*x)/
(a^2*x^2+1)^(1/2)))/(a^2*x^2+1)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a^2cx^2+c}\arctan{(ax)}^3}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^3/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{c\left(a^2x^2+1\right)}\operatorname{atan}^3{(ax)}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**3*(a**2*c*x**2+c)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(c*(a**2*x**2 + 1))*atan(a*x)**3/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{a^2cx^2+c}\arctan{(ax)}^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)*arctan(a*x)^3/x^3, x)
```

3.419 $\int \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{x^4}\,dx$

Optimal. Leaf size=361

$$\frac{ia^3c\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,-e^{i\tan^{-1}(ax)}\right)}{\sqrt{a^2cx^2+c}}-\frac{ia^3c\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,e^{i\tan^{-1}(ax)}\right)}{\sqrt{a^2cx^2+c}}-\frac{a^3c\sqrt{a^2x^2+1}\text{P}}{}$$

```
[Out] -((a^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/x) - (a*Sqrt[c + a^2*c*x^2]*ArcTan[
a*x]^2)/(2*x^2) - ((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3)/(3*c*x^3) - (a^3*c*
Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*ArcTanh[E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*
x^2] - a^3*Sqrt[c]*ArcTanh[Sqrt[c + a^2*c*x^2]/Sqrt[c]] + (I*a^3*c*Sqrt[1 +
 a^2*x^2]*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] -
 (I*a^3*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, E^(I*ArcTan[a*x])])/Sqrt
[c + a^2*c*x^2] - (a^3*c*Sqrt[1 + a^2*x^2]*PolyLog[3, -E^(I*ArcTan[a*x])])/
Sqrt[c + a^2*c*x^2] + (a^3*c*Sqrt[1 + a^2*x^2]*PolyLog[3, E^(I*ArcTan[a*x])
])/Sqrt[c + a^2*c*x^2]
```

Rubi [A] time = 1.01622, antiderivative size = 361, normalized size of antiderivative = 1., number of steps used = 25, number of rules used = 12, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4944, 4950, 4962, 266, 63, 208, 4958, 4956, 4183, 2531, 2282, 6589}

$$\frac{ia^3c\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,-e^{i\tan^{-1}(ax)}\right)}{\sqrt{a^2cx^2+c}}-\frac{ia^3c\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,e^{i\tan^{-1}(ax)}\right)}{\sqrt{a^2cx^2+c}}-\frac{a^3c\sqrt{a^2x^2+1}\text{P}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/x^4,x]
```


```
[Out] -((a^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/x) - (a*Sqrt[c + a^2*c*x^2]*ArcTan[
a*x]^2)/(2*x^2) - ((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3)/(3*c*x^3) - (a^3*c*
Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*ArcTanh[E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*
x^2] - a^3*Sqrt[c]*ArcTanh[Sqrt[c + a^2*c*x^2]/Sqrt[c]] + (I*a^3*c*Sqrt[1 +
 a^2*x^2]*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] -
 (I*a^3*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, E^(I*ArcTan[a*x])])/Sqrt
[c + a^2*c*x^2] - (a^3*c*Sqrt[1 + a^2*x^2]*PolyLog[3, -E^(I*ArcTan[a*x])])/
Sqrt[c + a^2*c*x^2] + (a^3*c*Sqrt[1 + a^2*x^2]*PolyLog[3, E^(I*ArcTan[a*x])
])/Sqrt[c + a^2*c*x^2]
```


Rule 4944

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a +
 b*ArcTan[c*x])^p)/(d*f*(m + 1)), x] - Dist[(b*c*p)/(f*(m + 1)), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rule 4950

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 4962

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Ar
cTan[c*x])^p)/(d*f*(m + 1)), x] + (-Dist[(b*c*p)/(f*(m + 1)), Int[((f*x)^(m
 + 1)*(a + b*ArcTan[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] - Dist[(c^2*(m +
 2))/(f^2*(m + 1)), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/Sqrt[d + e*x^
2], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] &
& LtQ[m, -1] && NeQ[m, -2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 4958

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4956

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Dist[1/Sqrt[d], Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTan
[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && G
tQ[d, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{x^4}\,dx = -\frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^3}{3cx^3} + a\int \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{x^3}\,dx$$

$$= -\frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^3}{3cx^3} + (ac)\int \frac{\tan^{-1}(ax)^2}{x^3\sqrt{c+a^2cx^2}}\,dx + \left(a^3c\right)\int \frac{\tan^{-1}(ax)^2}{x\sqrt{c+a^2cx^2}}\,dx$$

$$= -\frac{a\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2x^2} - \frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^3}{3cx^3} + \left(a^2c\right)\int \frac{\tan^{-1}(ax)}{x^2\sqrt{c+a^2cx^2}}\,dx - \frac{1}{2}\left(a\right.$$

$$= -\frac{a^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{x} - \frac{a\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2x^2} - \frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^3}{3cx^3} + \left(a^3c\right)$$

$$= -\frac{a^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{x} - \frac{a\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2x^2} - \frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^3}{3cx^3} - \frac{2a^3c}{}$$

$$= -\frac{a^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{x} - \frac{a\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2x^2} - \frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^3}{3cx^3} - \frac{a^3c\sqrt{}}{}$$

$$= -\frac{a^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{x} - \frac{a\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2x^2} - \frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^3}{3cx^3} - \frac{a^3c\sqrt{}}{}$$

$$= -\frac{a^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{x} - \frac{a\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2x^2} - \frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^3}{3cx^3} - \frac{a^3c\sqrt{}}{}$$

$$= -\frac{a^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{x} - \frac{a\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2x^2} - \frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^3}{3cx^3} - \frac{a^3c\sqrt{}}{}$$

Mathematica [A] time = 3.49065, size = 341, normalized size = 0.94

$$\frac{a^3c\sqrt{a^2x^2+1}\left(24i\tan^{-1}(ax)\text{PolyLog}\left(2,-e^{i\tan^{-1}(ax)}\right) - 24i\tan^{-1}(ax)\text{PolyLog}\left(2,e^{i\tan^{-1}(ax)}\right) - 24\text{PolyLog}\left(3,-e^{i\tan^{-1}(a\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/x^4,x]
```

```
[Out] (a^3*c*Sqrt[1 + a^2*x^2]*(-12*ArcTan[a*x]*Cot[ArcTan[a*x]/2] - 2*ArcTan[a*x
]^3*Cot[ArcTan[a*x]/2] - 3*ArcTan[a*x]^2*Csc[ArcTan[a*x]/2]^2 - (a*x*ArcTan
[a*x]^3*Csc[ArcTan[a*x]/2]^4)/(2*Sqrt[1 + a^2*x^2]) + 12*ArcTan[a*x]^2*Log[
1 - E^(I*ArcTan[a*x])] - 12*ArcTan[a*x]^2*Log[1 + E^(I*ArcTan[a*x])] + 24*L
og[Tan[ArcTan[a*x]/2]] + (24*I)*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])]
- (24*I)*ArcTan[a*x]*PolyLog[2, E^(I*ArcTan[a*x])] - 24*PolyLog[3, -E^(I*Ar
cTan[a*x])] + 24*PolyLog[3, E^(I*ArcTan[a*x])] + 3*ArcTan[a*x]^2*Sec[ArcTan
[a*x]/2]^2 - (8*(1 + a^2*x^2)^(3/2)*ArcTan[a*x]^3*Sin[ArcTan[a*x]/2]^4)/(a^
3*x^3) - 12*ArcTan[a*x]*Tan[ArcTan[a*x]/2] - 2*ArcTan[a*x]^3*Tan[ArcTan[a*x
]/2]))/(24*Sqrt[c*(1 + a^2*x^2)])
```

Maple [A] time = 2.786, size = 462, normalized size = 1.3

$$-\frac{\arctan(ax)\left(2\,(\arctan(ax))^2 x^2a^2+6\,a^2x^2+3\,\arctan(ax)\,xa+2\,(\arctan(ax))^2\right)}{6\,x^3}\sqrt{c\,(ax-i)\,(ax+i)}+\frac{a^3\,(\arctan(}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)/x^4,x)
```

```
[Out] -1/6*(c*(a*x-I)*(a*x+I))^(1/2)*arctan(a*x)*(2*arctan(a*x)^2*x^2*a^2+6*a^2*x
^2+3*arctan(a*x)*x*a+2*arctan(a*x)^2)/x^3+1/2*a^3*(c*(a*x-I)*(a*x+I))^(1/2)
*arctan(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*x^2+1)^(1/2)-I*a^3*(c
*(a*x-I)*(a*x+I))^(1/2)*arctan(a*x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))/
(a^2*x^2+1)^(1/2)+a^3*(c*(a*x-I)*(a*x+I))^(1/2)*polylog(3,(1+I*a*x)/(a^2*x^
2+1)^(1/2))/(a^2*x^2+1)^(1/2)-1/2*a^3*(c*(a*x-I)*(a*x+I))^(1/2)*arctan(a*x)
^2*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*x^2+1)^(1/2)+I*a^3*(c*(a*x-I)*(a*
x+I))^(1/2)*arctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*x^2+1)
^(1/2)-a^3*(c*(a*x-I)*(a*x+I))^(1/2)*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2)
)/(a^2*x^2+1)^(1/2)-2*a^3*(c*(a*x-I)*(a*x+I))^(1/2)*arctanh((1+I*a*x)/(a^2*
x^2+1)^(1/2))/(a^2*x^2+1)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a^2cx^2 + c}\arctan(ax)^3}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^3/x^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c\left(a^2x^2 + 1\right)}\operatorname{atan}^3(ax)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**3*(a**2*c*x**2+c)**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt(c*(a**2*x**2 + 1))*atan(a*x)**3/x**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a^2cx^2 + c}\arctan(ax)^3}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)*arctan(a*x)^3/x^4, x)
```

3.420 $\int x^3 \left(c+a^2 c x^2\right)^{3/2} \tan^{-1}(ax)^3 \, dx$

Optimal. Leaf size=652

$$\frac{51 i c^2 \sqrt{a^2 x^2+1} \tan^{-1}(ax) \text{PolyLog}\left(2,-i e^{i \tan^{-1}(ax)}\right)}{280 a^4 \sqrt{a^2 c x^2+c}}-\frac{51 i c^2 \sqrt{a^2 x^2+1} \tan^{-1}(ax) \text{PolyLog}\left(2,i e^{i \tan^{-1}(ax)}\right)}{280 a^4 \sqrt{a^2 c x^2+c}}-\frac{51 c^2 \sqrt{a^2 x^2}}{}$$

```
[Out] (c*x*Sqrt[c + a^2*c*x^2])/(420*a^3) - (c*x^3*Sqrt[c + a^2*c*x^2])/(140*a) -
 (163*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(840*a^4) + (c*x^2*Sqrt[c + a^2*c*
x^2]*ArcTan[a*x])/(60*a^2) + (c*x^4*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/35 + (
9*c*x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(112*a^3) - (23*c*x^3*Sqrt[c + a^2
*c*x^2]*ArcTan[a*x]^2)/(280*a) - (a*c*x^5*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2
)/14 - (((51*I)/280)*c^2*Sqrt[1 + a^2*x^2]*ArcTan[E^(I*ArcTan[a*x])]*ArcTan
[a*x]^2)/(a^4*Sqrt[c + a^2*c*x^2]) - (2*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3
)/(35*a^4) + (c*x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/(35*a^2) + (8*c*x^4*
Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/35 + (a^2*c*x^6*Sqrt[c + a^2*c*x^2]*ArcT
an[a*x]^3)/7 + (23*c^(3/2)*ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]])/(120
*a^4) + (((51*I)/280)*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, (-I)*E^(
I*ArcTan[a*x])])/(a^4*Sqrt[c + a^2*c*x^2]) - (((51*I)/280)*c^2*Sqrt[1 + a^2
*x^2]*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])])/(a^4*Sqrt[c + a^2*c*x^2]
) - (51*c^2*Sqrt[1 + a^2*x^2]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])])/(280*a^4*
Sqrt[c + a^2*c*x^2]) + (51*c^2*Sqrt[1 + a^2*x^2]*PolyLog[3, I*E^(I*ArcTan[a
*x])])/(280*a^4*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 7.37059, antiderivative size = 652, normalized size of antiderivative = 1., number of steps used = 200, number of rules used = 12, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4950, 4952, 4930, 217, 206, 4890, 4888, 4181, 2531, 2282, 6589, 321}

$$\frac{51 i c^2 \sqrt{a^2 x^2+1} \tan^{-1}(ax) \text{PolyLog}\left(2,-i e^{i \tan^{-1}(ax)}\right)}{280 a^4 \sqrt{a^2 c x^2+c}}-\frac{51 i c^2 \sqrt{a^2 x^2+1} \tan^{-1}(ax) \text{PolyLog}\left(2,i e^{i \tan^{-1}(ax)}\right)}{280 a^4 \sqrt{a^2 c x^2+c}}-\frac{51 c^2 \sqrt{a^2 x^2}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3,x]
```



```
[Out] (c*x*Sqrt[c + a^2*c*x^2])/(420*a^3) - (c*x^3*Sqrt[c + a^2*c*x^2])/(140*a) -
 (163*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(840*a^4) + (c*x^2*Sqrt[c + a^2*c*
x^2]*ArcTan[a*x])/(60*a^2) + (c*x^4*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/35 + (
9*c*x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(112*a^3) - (23*c*x^3*Sqrt[c + a^2
*c*x^2]*ArcTan[a*x]^2)/(280*a) - (a*c*x^5*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2
```

```
)/14 - (((51*I)/280)*c^2*Sqrt[1 + a^2*x^2]*ArcTan[E^(I*ArcTan[a*x])]*ArcTan
[a*x]^2)/(a^4*Sqrt[c + a^2*c*x^2]) - (2*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3
)/(35*a^4) + (c*x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/(35*a^2) + (8*c*x^4*
Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/35 + (a^2*c*x^6*Sqrt[c + a^2*c*x^2]*ArcT
an[a*x]^3)/7 + (23*c^(3/2)*ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]])/(120
*a^4) + (((51*I)/280)*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, (-I)*E^(
I*ArcTan[a*x])])/(a^4*Sqrt[c + a^2*c*x^2]) - (((51*I)/280)*c^2*Sqrt[1 + a^2
*x^2]*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])])/(a^4*Sqrt[c + a^2*c*x^2]
) - (51*c^2*Sqrt[1 + a^2*x^2]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])])/(280*a^4*
Sqrt[c + a^2*c*x^2]) + (51*c^2*Sqrt[1 + a^2*x^2]*PolyLog[3, I*E^(I*ArcTan[a
*x])])/(280*a^4*Sqrt[c + a^2*c*x^2])
```

Rule 4950

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 4952

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b*
ArcTan[c*x])^p)/(c^2*d*m), x] + (-Dist[(b*f*p)/(c*m), Int[((f*x)^(m - 1)*(a
 + b*ArcTan[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] - Dist[(f^2*(m - 1))/(c^
2*m), Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/Sqrt[d + e*x^2], x], x]) /;
 FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 4890

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4888

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[c
*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && GtQ
[d, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int x^3 \left(c+a^2cx^2\right)^{3/2} \tan^{-1}(ax)^3\,dx = c\int x^3\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3\,dx + \left(a^2c\right)\int x^5\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3\,dx$$

$$= c^2\int \frac{x^3\tan^{-1}(ax)^3}{\sqrt{c+a^2cx^2}}\,dx + 2\left(\left(a^2c^2\right)\int \frac{x^5\tan^{-1}(ax)^3}{\sqrt{c+a^2cx^2}}\,dx\right) + \left(a^4c^2\right)\int \frac{x^7\tan^{-1}(ax)^3}{\sqrt{c+a^2cx^2}}$$

$$= \frac{cx^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{3a^2} + \frac{1}{7}a^2cx^6\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3 - \frac{\left(2c^2\right)\int \frac{x\tan^{-1}(ax)^3}{\sqrt{c+a^2cx^2}}\,dx}{3a^2}$$

$$= -\frac{cx\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2a^3} - \frac{1}{14}acx^5\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2 - \frac{2c\sqrt{c+a^2cx^2}\tan^{-1}(}{3a^4}$$

$$= \frac{c\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{a^4} + \frac{1}{35}cx^4\sqrt{c+a^2cx^2}\tan^{-1}(ax) - \frac{cx\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2a^3}$$

$$= -\frac{cx^3\sqrt{c+a^2cx^2}}{140a} + \frac{c\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{a^4} - \frac{11cx^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{60a^2} + \frac{1}{35}cx^4$$

$$= \frac{43cx\sqrt{c+a^2cx^2}}{420a^3} - \frac{cx^3\sqrt{c+a^2cx^2}}{140a} + \frac{2273c\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{840a^4} - \frac{11cx^2\sqrt{c+a^2c}}{60a}$$

$$= \frac{43cx\sqrt{c+a^2cx^2}}{420a^3} - \frac{cx^3\sqrt{c+a^2cx^2}}{140a} + \frac{2273c\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{840a^4} - \frac{11cx^2\sqrt{c+a^2c}}{60a}$$

$$= \frac{43cx\sqrt{c+a^2cx^2}}{420a^3} - \frac{cx^3\sqrt{c+a^2cx^2}}{140a} + \frac{2273c\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{840a^4} - \frac{11cx^2\sqrt{c+a^2c}}{60a}$$

$$= \frac{43cx\sqrt{c+a^2cx^2}}{420a^3} - \frac{cx^3\sqrt{c+a^2cx^2}}{140a} + \frac{2273c\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{840a^4} - \frac{11cx^2\sqrt{c+a^2c}}{60a}$$

$$= \frac{43cx\sqrt{c+a^2cx^2}}{420a^3} - \frac{cx^3\sqrt{c+a^2cx^2}}{140a} + \frac{2273c\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{840a^4} - \frac{11cx^2\sqrt{c+a^2c}}{60a}$$

$$= \frac{43cx\sqrt{c+a^2cx^2}}{420a^3} - \frac{cx^3\sqrt{c+a^2cx^2}}{140a} + \frac{2273c\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{840a^4} - \frac{11cx^2\sqrt{c+a^2c}}{60a}$$

Mathematica [A] time = 3.26548, size = 538, normalized size = 0.83

$$\frac{c\sqrt{a^2cx^2+c}\left(64\left(-309i\tan^{-1}(ax)\text{PolyLog}\left(2,-ie^{i\tan^{-1}(ax)}\right)+309i\tan^{-1}(ax)\text{PolyLog}\left(2,ie^{i\tan^{-1}(ax)}\right)+309\text{PolyLog}\left(3,\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3,x]
```

```
[Out] (c*Sqrt[c + a^2*c*x^2]*(64*((309*I)*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2
 - 259*ArcTanh[(a*x)/Sqrt[1 + a^2*x^2]] - (309*I)*ArcTan[a*x]*PolyLog[2, (-
I)*E^(I*ArcTan[a*x])] + (309*I)*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])]
 + 309*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] - 309*PolyLog[3, I*E^(I*ArcTan[a*
x])]) + 2688*((-11*I)*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2 + 10*ArcTanh[
(a*x)/Sqrt[1 + a^2*x^2]] + (11*I)*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a
*x])] - (11*I)*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])] - 11*PolyLog[3,
(-I)*E^(I*ArcTan[a*x])] + 11*PolyLog[3, I*E^(I*ArcTan[a*x])]) - 56*(1 + a^2
*x^2)^(5/2)*((48*a*x)/(1 + a^2*x^2)^2 + 32*ArcTan[a*x]^3*(-1 + 5*Cos[2*ArcT
an[a*x]]) + 6*ArcTan[a*x]*(25 + 36*Cos[2*ArcTan[a*x]] + 11*Cos[4*ArcTan[a*x
]]) + ArcTan[a*x]^2*(6*Sin[2*ArcTan[a*x]] - 33*Sin[4*ArcTan[a*x]])) + (1 +
a^2*x^2)^(7/2)*(64*ArcTan[a*x]^3*(57 - 28*Cos[2*ArcTan[a*x]] + 35*Cos[4*Arc
Tan[a*x]]) + (8*ArcTan[a*x]*(647 + 764*Cos[2*ArcTan[a*x]] + 309*Cos[4*ArcTa
n[a*x]]))/(1 + a^2*x^2) + 4*(101*Sin[2*ArcTan[a*x]] + 88*Sin[4*ArcTan[a*x]]
 + 25*Sin[6*ArcTan[a*x]]) - 3*ArcTan[a*x]^2*(211*Sin[2*ArcTan[a*x]] - 60*Si
n[4*ArcTan[a*x]] + 103*Sin[6*ArcTan[a*x]]))))/(53760*a^4*Sqrt[1 + a^2*x^2])
```

Maple [A] time = 4.163, size = 469, normalized size = 0.7

$$\frac{c\left(240\,(\arctan(ax))^3x^6a^6-120\,(\arctan(ax))^2x^5a^5+384\,(\arctan(ax))^3x^4a^4+48\arctan(ax)\,x^4a^4-138\,(\arctan(ax\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^3,x)
```

```
[Out] 1/1680*c/a^4*(c*(a*x-I)*(a*x+I))^(1/2)*(240*arctan(a*x)^3*x^6*a^6-120*arcta
n(a*x)^2*x^5*a^5+384*arctan(a*x)^3*x^4*a^4+48*arctan(a*x)*x^4*a^4-138*arcta
n(a*x)^2*x^3*a^3+48*arctan(a*x)^3*x^2*a^2-12*a^3*x^3+28*arctan(a*x)*a^2*x^2
+135*arctan(a*x)^2*x*a-96*arctan(a*x)^3+4*a*x-326*arctan(a*x))+17/560*c*(c*
(a*x-I)*(a*x+I))^(1/2)*(I*arctan(a*x)^3-3*arctan(a*x)^2*ln(1+I*(1+I*a*x)/(a
```

```
^2*x^2+1)^(1/2))+6*I*arctan(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-
6*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))/a^4/(a^2*x^2+1)^(1/2)-17/560*c
*(c*(a*x-I)*(a*x+I))^(1/2)*(I*arctan(a*x)^3+6*I*arctan(a*x)*polylog(2,I*(1+
I*a*x)/(a^2*x^2+1)^(1/2))-3*arctan(a*x)^2*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2
))-6*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))/a^4/(a^2*x^2+1)^(1/2)-23/60*
I*c/a^4*(c*(a*x-I)*(a*x+I))^(1/2)*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*
x^2+1)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^2cx^5 + cx^3\right)\sqrt{a^2cx^2 + c}\arctan\left(ax\right)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^5 + c*x^3)*sqrt(a^2*c*x^2 + c)*arctan(a*x)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a**2*c*x**2+c)**(3/2)*atan(a*x)**3,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.421 $\int x^2 \left(c+a^2cx^2\right)^{3/2} \tan^{-1}(ax)^3 \, dx$

Optimal. Leaf size=882

$$\frac{1}{6}a^2c\sqrt{a^2cx^2+c}\tan^{-1}(ax)^3x^5-\frac{1}{10}ac\sqrt{a^2cx^2+c}\tan^{-1}(ax)^2x^4+\frac{7}{24}c\sqrt{a^2cx^2+c}\tan^{-1}(ax)^3x^3+\frac{1}{20}c\sqrt{a^2cx^2+c}\tan^{-1}($$

```
[Out] -(c*Sqrt[c + a^2*c*x^2])/(30*a^3) - (c + a^2*c*x^2)^(3/2)/(60*a^3) + (c*x*S
qrt[c + a^2*c*x^2]*ArcTan[a*x])/(12*a^2) + (c*x^3*Sqrt[c + a^2*c*x^2]*ArcTa
n[a*x])/20 + (31*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(240*a^3) - (19*c*x^2
*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(120*a) - (a*c*x^4*Sqrt[c + a^2*c*x^2]*
ArcTan[a*x]^2)/10 + (c*x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/(16*a^2) + (7*c
*x^3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/24 + (a^2*c*x^5*Sqrt[c + a^2*c*x^2]
*ArcTan[a*x]^3)/6 + ((I/8)*c^2*Sqrt[1 + a^2*x^2]*ArcTan[E^(I*ArcTan[a*x])]*
ArcTan[a*x]^3)/(a^3*Sqrt[c + a^2*c*x^2]) + (((41*I)/60)*c^2*Sqrt[1 + a^2*x^
2]*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/(a^3*Sqrt[c + a^2*c
*x^2]) - (((3*I)/16)*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog[2, (-I)*E^
(I*ArcTan[a*x])])/(a^3*Sqrt[c + a^2*c*x^2]) + (((3*I)/16)*c^2*Sqrt[1 + a^2*
x^2]*ArcTan[a*x]^2*PolyLog[2, I*E^(I*ArcTan[a*x])])/(a^3*Sqrt[c + a^2*c*x^2
]) - (((41*I)/120)*c^2*Sqrt[1 + a^2*x^2]*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/
Sqrt[1 - I*a*x]])/(a^3*Sqrt[c + a^2*c*x^2]) + (((41*I)/120)*c^2*Sqrt[1 + a^
2*x^2]*PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/(a^3*Sqrt[c + a^2*c
*x^2]) + (3*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[3, (-I)*E^(I*ArcTan[a
*x])])/(8*a^3*Sqrt[c + a^2*c*x^2]) - (3*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*P
olyLog[3, I*E^(I*ArcTan[a*x])])/(8*a^3*Sqrt[c + a^2*c*x^2]) + (((3*I)/8)*c^
2*Sqrt[1 + a^2*x^2]*PolyLog[4, (-I)*E^(I*ArcTan[a*x])])/(a^3*Sqrt[c + a^2*c
*x^2]) - (((3*I)/8)*c^2*Sqrt[1 + a^2*x^2]*PolyLog[4, I*E^(I*ArcTan[a*x])])/
(a^3*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 5.46991, antiderivative size = 882, normalized size of antiderivative = 1., number of steps used = 108, number of rules used = 14, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {4950, 4952, 4930, 4890, 4886, 4888, 4181, 2531, 6609, 2282, 6589, 261, 266, 43}

$$\frac{1}{6}a^2c\sqrt{a^2cx^2+c}\tan^{-1}(ax)^3x^5-\frac{1}{10}ac\sqrt{a^2cx^2+c}\tan^{-1}(ax)^2x^4+\frac{7}{24}c\sqrt{a^2cx^2+c}\tan^{-1}(ax)^3x^3+\frac{1}{20}c\sqrt{a^2cx^2+c}\tan^{-1}($$

Antiderivative was successfully verified.

```
[In] Int[x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3,x]
```

```
[Out] -(c*Sqrt[c + a^2*c*x^2])/(30*a^3) - (c + a^2*c*x^2)^(3/2)/(60*a^3) + (c*x*S
qrt[c + a^2*c*x^2]*ArcTan[a*x])/(12*a^2) + (c*x^3*Sqrt[c + a^2*c*x^2]*ArcTa
n[a*x])/20 + (31*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(240*a^3) - (19*c*x^2
*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(120*a) - (a*c*x^4*Sqrt[c + a^2*c*x^2]*
ArcTan[a*x]^2)/10 + (c*x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/(16*a^2) + (7*c
*x^3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/24 + (a^2*c*x^5*Sqrt[c + a^2*c*x^2]
*ArcTan[a*x]^3)/6 + ((I/8)*c^2*Sqrt[1 + a^2*x^2]*ArcTan[E^(I*ArcTan[a*x])]*
ArcTan[a*x]^3)/(a^3*Sqrt[c + a^2*c*x^2]) + (((41*I)/60)*c^2*Sqrt[1 + a^2*x^
2]*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/(a^3*Sqrt[c + a^2*c
*x^2]) - (((3*I)/16)*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog[2, (-I)*E^
(I*ArcTan[a*x])])/(a^3*Sqrt[c + a^2*c*x^2]) + (((3*I)/16)*c^2*Sqrt[1 + a^2*
x^2]*ArcTan[a*x]^2*PolyLog[2, I*E^(I*ArcTan[a*x])])/(a^3*Sqrt[c + a^2*c*x^2
]) - (((41*I)/120)*c^2*Sqrt[1 + a^2*x^2]*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/
Sqrt[1 - I*a*x]])/(a^3*Sqrt[c + a^2*c*x^2]) + (((41*I)/120)*c^2*Sqrt[1 + a^
2*x^2]*PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/(a^3*Sqrt[c + a^2*c
*x^2]) + (3*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[3, (-I)*E^(I*ArcTan[a
*x])])/(8*a^3*Sqrt[c + a^2*c*x^2]) - (3*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*P
olyLog[3, I*E^(I*ArcTan[a*x])])/(8*a^3*Sqrt[c + a^2*c*x^2]) + (((3*I)/8)*c^
2*Sqrt[1 + a^2*x^2]*PolyLog[4, (-I)*E^(I*ArcTan[a*x])])/(a^3*Sqrt[c + a^2*c
*x^2]) - (((3*I)/8)*c^2*Sqrt[1 + a^2*x^2]*PolyLog[4, I*E^(I*ArcTan[a*x])])/
(a^3*Sqrt[c + a^2*c*x^2])
```

Rule 4950

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 4952

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b*
ArcTan[c*x])^p)/(c^2*d*m), x] + (-Dist[(b*f*p)/(c*m), Int[((f*x)^(m - 1)*(a
 + b*ArcTan[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] - Dist[(f^2*(m - 1))/(c^
2*m), Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/Sqrt[d + e*x^2], x], x]) /;
 FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
```

```
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 4890

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4886

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[(-2*I*(a + b*ArcTan[c*x])*ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]])
/(c*Sqrt[d]), x] + (Simp[(I*b*PolyLog[2, -((I*Sqrt[1 + I*c*x])/Sqrt[1 - I*c
*x])])/(c*Sqrt[d]), x] - Simp[(I*b*PolyLog[2, (I*Sqrt[1 + I*c*x])/Sqrt[1 -
I*c*x]])/(c*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

Rule 4888

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[c
*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && GtQ
[d, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x^2 \left(c+a^2cx^2\right)^{3/2} \tan^{-1}(ax)^3\,dx = c\int x^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3\,dx + \left(a^2c\right)\int x^4\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3\,dx$$

$$= c^2\int \frac{x^2\tan^{-1}(ax)^3}{\sqrt{c+a^2cx^2}}\,dx + 2\left(\left(a^2c^2\right)\int \frac{x^4\tan^{-1}(ax)^3}{\sqrt{c+a^2cx^2}}\,dx\right) + \left(a^4c^2\right)\int \frac{x^6\tan^{-1}(ax)^3}{\sqrt{c+a^2cx^2}}$$

$$= \frac{cx\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{2a^2} + \frac{1}{6}a^2cx^5\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3 - \frac{c^2\int \frac{\tan^{-1}(ax)^3}{\sqrt{c+a^2cx^2}}\,dx}{2a^2} - \frac{(3}{}$$

$$= -\frac{3c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2a^3} - \frac{1}{10}acx^4\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2 + \frac{cx\sqrt{c+a^2cx^2}\tan^{-1}(}{2a^2}$$

$$= \frac{1}{20}cx^3\sqrt{c+a^2cx^2}\tan^{-1}(ax) - \frac{3c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2a^3} + \frac{41cx^2\sqrt{c+a^2cx^2}\tan^{-1}(}{120a}$$

$$= -\frac{5cx\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{12a^2} + \frac{1}{20}cx^3\sqrt{c+a^2cx^2}\tan^{-1}(ax) - \frac{749c\sqrt{c+a^2cx^2}\tan^{-1}}{240a^3}$$

$$= \frac{5c\sqrt{c+a^2cx^2}}{12a^3} - \frac{5cx\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{12a^2} + \frac{1}{20}cx^3\sqrt{c+a^2cx^2}\tan^{-1}(ax) - \frac{749c\sqrt{}}{}$$

$$= \frac{7c\sqrt{c+a^2cx^2}}{15a^3} - \frac{\left(c+a^2cx^2\right)^{3/2}}{60a^3} - \frac{5cx\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{12a^2} + \frac{1}{20}cx^3\sqrt{c+a^2cx^2}\tan$$

$$= \frac{7c\sqrt{c+a^2cx^2}}{15a^3} - \frac{\left(c+a^2cx^2\right)^{3/2}}{60a^3} - \frac{5cx\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{12a^2} + \frac{1}{20}cx^3\sqrt{c+a^2cx^2}\tan$$

$$= \frac{7c\sqrt{c+a^2cx^2}}{15a^3} - \frac{\left(c+a^2cx^2\right)^{3/2}}{60a^3} - \frac{5cx\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{12a^2} + \frac{1}{20}cx^3\sqrt{c+a^2cx^2}\tan$$

$$= \frac{7c\sqrt{c+a^2cx^2}}{15a^3} - \frac{\left(c+a^2cx^2\right)^{3/2}}{60a^3} - \frac{5cx\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{12a^2} + \frac{1}{20}cx^3\sqrt{c+a^2cx^2}\tan$$

$$= \frac{7c\sqrt{c+a^2cx^2}}{15a^3} - \frac{\left(c+a^2cx^2\right)^{3/2}}{60a^3} - \frac{5cx\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{12a^2} + \frac{1}{20}cx^3\sqrt{c+a^2cx^2}\tan$$

Mathematica [B] time = 18.2558, size = 4015, normalized size = 4.55

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3,x]
```

```
[Out] (c*((Sqrt[c*(1 + a^2*x^2)]*(-1 + ArcTan[a*x]^2))/(4*Sqrt[1 + a^2*x^2]) + (S
qrt[c*(1 + a^2*x^2)]*(-(ArcTan[a*x]*(Log[1 - I*E^(I*ArcTan[a*x])] - Log[1 +
 I*E^(I*ArcTan[a*x])])) - I*(PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - PolyLog[2
, I*E^(I*ArcTan[a*x])])))/(2*Sqrt[1 + a^2*x^2]) + (Sqrt[c*(1 + a^2*x^2)]*(-
(Pi^3*Log[Cot[(Pi/2 - ArcTan[a*x])/2]])/8 - (3*Pi^2*((Pi/2 - ArcTan[a*x])*(
Log[1 - E^(I*(Pi/2 - ArcTan[a*x]))] - Log[1 + E^(I*(Pi/2 - ArcTan[a*x]))])
+ I*(PolyLog[2, -E^(I*(Pi/2 - ArcTan[a*x]))] - PolyLog[2, E^(I*(Pi/2 - ArcT
an[a*x]))])))/4 + (3*Pi*((Pi/2 - ArcTan[a*x])^2*(Log[1 - E^(I*(Pi/2 - ArcTa
n[a*x]))] - Log[1 + E^(I*(Pi/2 - ArcTan[a*x]))]) + (2*I)*(Pi/2 - ArcTan[a*x
])*(PolyLog[2, -E^(I*(Pi/2 - ArcTan[a*x]))] - PolyLog[2, E^(I*(Pi/2 - ArcTa
n[a*x]))]) + 2*(-PolyLog[3, -E^(I*(Pi/2 - ArcTan[a*x]))] + PolyLog[3, E^(I*
(Pi/2 - ArcTan[a*x]))])))/2 - 8*((I/64)*(Pi/2 - ArcTan[a*x])^4 + (I/4)*(Pi/
2 + (-Pi/2 + ArcTan[a*x])/2)^4 - ((Pi/2 - ArcTan[a*x])^3*Log[1 + E^(I*(Pi/2
 - ArcTan[a*x]))])/8 - (Pi^3*(I*(Pi/2 + (-Pi/2 + ArcTan[a*x])/2) - Log[1 +
E^((2*I)*(Pi/2 + (-Pi/2 + ArcTan[a*x])/2))]))/8 - (Pi/2 + (-Pi/2 + ArcTan[a
*x])/2)^3*Log[1 + E^((2*I)*(Pi/2 + (-Pi/2 + ArcTan[a*x])/2))] + ((3*I)/8)*(
Pi/2 - ArcTan[a*x])^2*PolyLog[2, -E^(I*(Pi/2 - ArcTan[a*x]))] + (3*Pi^2*((I
/2)*(Pi/2 + (-Pi/2 + ArcTan[a*x])/2)^2 - (Pi/2 + (-Pi/2 + ArcTan[a*x])/2)*L
og[1 + E^((2*I)*(Pi/2 + (-Pi/2 + ArcTan[a*x])/2))] + (I/2)*PolyLog[2, -E^((
2*I)*(Pi/2 + (-Pi/2 + ArcTan[a*x])/2))]))/4 + ((3*I)/2)*(Pi/2 + (-Pi/2 + Ar
cTan[a*x])/2)^2*PolyLog[2, -E^((2*I)*(Pi/2 + (-Pi/2 + ArcTan[a*x])/2))] - (
3*(Pi/2 - ArcTan[a*x])*PolyLog[3, -E^(I*(Pi/2 - ArcTan[a*x]))])/4 - (3*Pi*(
(I/3)*(Pi/2 + (-Pi/2 + ArcTan[a*x])/2)^3 - (Pi/2 + (-Pi/2 + ArcTan[a*x])/2)
^2*Log[1 + E^((2*I)*(Pi/2 + (-Pi/2 + ArcTan[a*x])/2))] + I*(Pi/2 + (-Pi/2 +
 ArcTan[a*x])/2)*PolyLog[2, -E^((2*I)*(Pi/2 + (-Pi/2 + ArcTan[a*x])/2))] -
PolyLog[3, -E^((2*I)*(Pi/2 + (-Pi/2 + ArcTan[a*x])/2))]/2))/2 - (3*(Pi/2 +
(-Pi/2 + ArcTan[a*x])/2)*PolyLog[3, -E^((2*I)*(Pi/2 + (-Pi/2 + ArcTan[a*x])
/2))])/2 - ((3*I)/4)*PolyLog[4, -E^(I*(Pi/2 - ArcTan[a*x]))] - ((3*I)/4)*Po
lyLog[4, -E^((2*I)*(Pi/2 + (-Pi/2 + ArcTan[a*x])/2))])))/(8*Sqrt[1 + a^2*x^
2]) + (Sqrt[c*(1 + a^2*x^2)]*ArcTan[a*x]^3)/(16*Sqrt[1 + a^2*x^2]*(Cos[ArcT
an[a*x]/2] - Sin[ArcTan[a*x]/2])^4) + (Sqrt[c*(1 + a^2*x^2)]*(2*ArcTan[a*x]
 - ArcTan[a*x]^2 - ArcTan[a*x]^3))/(16*Sqrt[1 + a^2*x^2]*(Cos[ArcTan[a*x]/2
] - Sin[ArcTan[a*x]/2])^2) - (Sqrt[c*(1 + a^2*x^2)]*ArcTan[a*x]^2*Sin[ArcTa
n[a*x]/2])/(8*Sqrt[1 + a^2*x^2]*(Cos[ArcTan[a*x]/2] - Sin[ArcTan[a*x]/2])^3
) - (Sqrt[c*(1 + a^2*x^2)]*ArcTan[a*x]^3)/(16*Sqrt[1 + a^2*x^2]*(Cos[ArcTan
[a*x]/2] + Sin[ArcTan[a*x]/2])^4) + (Sqrt[c*(1 + a^2*x^2)]*ArcTan[a*x]^2*Si
```

n[ArcTan[a*x]/2])/(8*Sqrt[1 + a^2*x^2]*(Cos[ArcTan[a*x]/2] + Sin[ArcTan[a*x
]/2])^3) + (Sqrt[c*(1 + a^2*x^2)]*(-2*ArcTan[a*x] - ArcTan[a*x]^2 + ArcTan[
a*x]^3))/(16*Sqrt[1 + a^2*x^2]*(Cos[ArcTan[a*x]/2] + Sin[ArcTan[a*x]/2])^2)
 + (Sqrt[c*(1 + a^2*x^2)]*(Sin[ArcTan[a*x]/2] - ArcTan[a*x]^2*Sin[ArcTan[a*
x]/2]))/(4*Sqrt[1 + a^2*x^2]*(Cos[ArcTan[a*x]/2] + Sin[ArcTan[a*x]/2])) + (
Sqrt[c*(1 + a^2*x^2)]*(-Sin[ArcTan[a*x]/2] + ArcTan[a*x]^2*Sin[ArcTan[a*x]/
2]))/(4*Sqrt[1 + a^2*x^2]*(Cos[ArcTan[a*x]/2] - Sin[ArcTan[a*x]/2]))))/a^3
 + (c*((Sqrt[c*(1 + a^2*x^2)]*(50 - 19*ArcTan[a*x]^2))/(240*Sqrt[1 + a^2*x^2
]) + (19*Sqrt[c*(1 + a^2*x^2)]*(ArcTan[a*x]*(Log[1 - I*E^(I*ArcTan[a*x])] -
 Log[1 + I*E^(I*ArcTan[a*x])]) + I*(PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - Po
lyLog[2, I*E^(I*ArcTan[a*x])])))/(120*Sqrt[1 + a^2*x^2]) + (Sqrt[c*(1 + a^2
x^2)]((Pi^3*Log[Cot[(Pi/2 - ArcTan[a*x])/2]])/8 + (3*Pi^2*((Pi/2 - ArcTan
[a*x])*(Log[1 - E^(I*(Pi/2 - ArcTan[a*x]))] - Log[1 + E^(I*(Pi/2 - ArcTan[a
x]))]) + I(PolyLog[2, -E^(I*(Pi/2 - ArcTan[a*x]))] - PolyLog[2, E^(I*(Pi/
2 - ArcTan[a*x]))])))/4 - (3*Pi*((Pi/2 - ArcTan[a*x])^2*(Log[1 - E^(I*(Pi/2
 - ArcTan[a*x]))] - Log[1 + E^(I*(Pi/2 - ArcTan[a*x]))]) + (2*I)*(Pi/2 - Ar
cTan[a*x])*(PolyLog[2, -E^(I*(Pi/2 - ArcTan[a*x]))] - PolyLog[2, E^(I*(Pi/2
 - ArcTan[a*x]))]) + 2*(-PolyLog[3, -E^(I*(Pi/2 - ArcTan[a*x]))] + PolyLog[
3, E^(I*(Pi/2 - ArcTan[a*x]))])))/2 + 8*((I/64)*(Pi/2 - ArcTan[a*x])^4 + (I
/4)*(Pi/2 + (-Pi/2 + ArcTan[a*x])/2)^4 - ((Pi/2 - ArcTan[a*x])^3*Log[1 + E^
(I*(Pi/2 - ArcTan[a*x]))])/8 - (Pi^3*(I*(Pi/2 + (-Pi/2 + ArcTan[a*x])/2) -
Log[1 + E^((2*I)*(Pi/2 + (-Pi/2 + ArcTan[a*x])/2))]))/8 - (Pi/2 + (-Pi/2 +
ArcTan[a*x])/2)^3*Log[1 + E^((2*I)*(Pi/2 + (-Pi/2 + ArcTan[a*x])/2))] + ((3
I)/8)(Pi/2 - ArcTan[a*x])^2*PolyLog[2, -E^(I*(Pi/2 - ArcTan[a*x]))] + (3*
Pi^2*((I/2)*(Pi/2 + (-Pi/2 + ArcTan[a*x])/2)^2 - (Pi/2 + (-Pi/2 + ArcTan[a*
x])/2)*Log[1 + E^((2*I)*(Pi/2 + (-Pi/2 + ArcTan[a*x])/2))] + (I/2)*PolyLog[
2, -E^((2*I)*(Pi/2 + (-Pi/2 + ArcTan[a*x])/2))]))/4 + ((3*I)/2)*(Pi/2 + (-P
i/2 + ArcTan[a*x])/2)^2*PolyLog[2, -E^((2*I)*(Pi/2 + (-Pi/2 + ArcTan[a*x])/
2))] - (3*(Pi/2 - ArcTan[a*x])*PolyLog[3, -E^(I*(Pi/2 - ArcTan[a*x]))])/4 -
 (3*Pi*((I/3)*(Pi/2 + (-Pi/2 + ArcTan[a*x])/2)^3 - (Pi/2 + (-Pi/2 + ArcTan[
a*x])/2)^2*Log[1 + E^((2*I)*(Pi/2 + (-Pi/2 + ArcTan[a*x])/2))] + I*(Pi/2 +
(-Pi/2 + ArcTan[a*x])/2)*PolyLog[2, -E^((2*I)*(Pi/2 + (-Pi/2 + ArcTan[a*x])
/2))] - PolyLog[3, -E^((2*I)*(Pi/2 + (-Pi/2 + ArcTan[a*x])/2))]/2))/2 - (3*
(Pi/2 + (-Pi/2 + ArcTan[a*x])/2)*PolyLog[3, -E^((2*I)*(Pi/2 + (-Pi/2 + ArcT
an[a*x])/2))])/2 - ((3*I)/4)*PolyLog[4, -E^(I*(Pi/2 - ArcTan[a*x]))] - ((3*
I)/4)*PolyLog[4, -E^((2*I)*(Pi/2 + (-Pi/2 + ArcTan[a*x])/2))])))/(16*Sqrt[1
 + a^2*x^2]) + (Sqrt[c*(1 + a^2*x^2)]*ArcTan[a*x]^3)/(48*Sqrt[1 + a^2*x^2]*
(Cos[ArcTan[a*x]/2] - Sin[ArcTan[a*x]/2])^6) + (Sqrt[c*(1 + a^2*x^2)]*(ArcT
an[a*x] - ArcTan[a*x]^2 - 5*ArcTan[a*x]^3))/(80*Sqrt[1 + a^2*x^2]*(Cos[ArcT
an[a*x]/2] - Sin[ArcTan[a*x]/2])^4) + (Sqrt[c*(1 + a^2*x^2)]*(-2 - 52*ArcTa
n[a*x] + 26*ArcTan[a*x]^2 + 15*ArcTan[a*x]^3))/(480*Sqrt[1 + a^2*x^2]*(Cos[
ArcTan[a*x]/2] - Sin[ArcTan[a*x]/2])^2) - (Sqrt[c*(1 + a^2*x^2)]*ArcTan[a*x
]^2*Sin[ArcTan[a*x]/2])/(40*Sqrt[1 + a^2*x^2]*(Cos[ArcTan[a*x]/2] - Sin[Arc
Tan[a*x]/2])^5) - (Sqrt[c*(1 + a^2*x^2)]*ArcTan[a*x]^3)/(48*Sqrt[1 + a^2*x^
2]*(Cos[ArcTan[a*x]/2] + Sin[ArcTan[a*x]/2])^6) + (Sqrt[c*(1 + a^2*x^2)]*Ar

```
cTan[a*x]^2*Sin[ArcTan[a*x]/2])/(40*Sqrt[1 + a^2*x^2]*(Cos[ArcTan[a*x]/2] +
 Sin[ArcTan[a*x]/2])^5) + (Sqrt[c*(1 + a^2*x^2)]*(-ArcTan[a*x] - ArcTan[a*x
]^2 + 5*ArcTan[a*x]^3))/(80*Sqrt[1 + a^2*x^2]*(Cos[ArcTan[a*x]/2] + Sin[Arc
Tan[a*x]/2])^4) + (Sqrt[c*(1 + a^2*x^2)]*(-2 + 52*ArcTan[a*x] + 26*ArcTan[a
*x]^2 - 15*ArcTan[a*x]^3))/(480*Sqrt[1 + a^2*x^2]*(Cos[ArcTan[a*x]/2] + Sin
[ArcTan[a*x]/2])^2) + (Sqrt[c*(1 + a^2*x^2)]*(50*Sin[ArcTan[a*x]/2] - 19*Ar
cTan[a*x]^2*Sin[ArcTan[a*x]/2]))/(240*Sqrt[1 + a^2*x^2]*(Cos[ArcTan[a*x]/2]
 - Sin[ArcTan[a*x]/2])) + (Sqrt[c*(1 + a^2*x^2)]*(Sin[ArcTan[a*x]/2] - 13*A
rcTan[a*x]^2*Sin[ArcTan[a*x]/2]))/(120*Sqrt[1 + a^2*x^2]*(Cos[ArcTan[a*x]/2
] + Sin[ArcTan[a*x]/2])^3) + (Sqrt[c*(1 + a^2*x^2)]*(-Sin[ArcTan[a*x]/2] +
13*ArcTan[a*x]^2*Sin[ArcTan[a*x]/2]))/(120*Sqrt[1 + a^2*x^2]*(Cos[ArcTan[a*
x]/2] - Sin[ArcTan[a*x]/2])^3) + (Sqrt[c*(1 + a^2*x^2)]*(-50*Sin[ArcTan[a*x
]/2] + 19*ArcTan[a*x]^2*Sin[ArcTan[a*x]/2]))/(240*Sqrt[1 + a^2*x^2]*(Cos[Ar
cTan[a*x]/2] + Sin[ArcTan[a*x]/2]))))/a^3
```

Maple [A] time = 2.363, size = 514, normalized size = 0.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^3,x)
```

```
[Out] 1/240*c/a^3*(c*(a*x-I)*(a*x+I))^(1/2)*(40*arctan(a*x)^3*a^5*x^5-24*arctan(a
*x)^2*x^4*a^4+70*arctan(a*x)^3*a^3*x^3+12*arctan(a*x)*x^3*a^3-38*arctan(a*x
)^2*x^2*a^2+15*arctan(a*x)^3*a*x-4*a^2*x^2+20*arctan(a*x)*x*a+31*arctan(a*x
)^2-12)+1/240*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)^(1/2)/a^3*(15*arctan(a*
x)^3*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-15*arctan(a*x)^3*ln(1-I*(1+I*a*x)/
(a^2*x^2+1)^(1/2))-45*I*arctan(a*x)^2*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1
/2))+45*I*arctan(a*x)^2*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+82*arctan(
a*x)*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+90*arctan(a*x)*polylog(3,-I*(1+I*a
*x)/(a^2*x^2+1)^(1/2))-82*arctan(a*x)*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-9
0*arctan(a*x)*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+90*I*polylog(4,-I*(1
+I*a*x)/(a^2*x^2+1)^(1/2))-90*I*polylog(4,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-82
*I*dilog(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+82*I*dilog(1-I*(1+I*a*x)/(a^2*x^2
+1)^(1/2)))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a^2cx^4 + cx^2\right)\sqrt{a^2cx^2 + c}\arctan\left(ax\right)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^4 + c*x^2)*sqrt(a^2*c*x^2 + c)*arctan(a*x)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a**2*c*x**2+c)**(3/2)*atan(a*x)**3,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.422 $\int x\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^3\,dx$

Optimal. Leaf size=477

$$-\frac{9ic^2\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,-ie^{i\tan^{-1}(ax)}\right)}{20a^2\sqrt{a^2cx^2+c}}+\frac{9ic^2\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,ie^{i\tan^{-1}(ax)}\right)}{20a^2\sqrt{a^2cx^2+c}}+\frac{9c^2\sqrt{a^2x^2+1}}{2}$$

```
[Out] -(c*x*Sqrt[c + a^2*c*x^2])/(20*a) + (9*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(
20*a^2) + ((c + a^2*c*x^2)^(3/2)*ArcTan[a*x])/(10*a^2) - (9*c*x*Sqrt[c + a^
2*c*x^2]*ArcTan[a*x]^2)/(40*a) - (3*x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2)/
(20*a) + (((9*I)/20)*c^2*Sqrt[1 + a^2*x^2]*ArcTan[E^(I*ArcTan[a*x])]*ArcTan
[a*x]^2)/(a^2*Sqrt[c + a^2*c*x^2]) + ((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3)/
(5*a^2*c) - (c^(3/2)*ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]])/(2*a^2) -
(((9*I)/20)*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a
*x])])/(a^2*Sqrt[c + a^2*c*x^2]) + (((9*I)/20)*c^2*Sqrt[1 + a^2*x^2]*ArcTan
[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])])/(a^2*Sqrt[c + a^2*c*x^2]) + (9*c^2*S
qrt[1 + a^2*x^2]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])])/(20*a^2*Sqrt[c + a^2*c
*x^2]) - (9*c^2*Sqrt[1 + a^2*x^2]*PolyLog[3, I*E^(I*ArcTan[a*x])])/(20*a^2*
Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.416141, antiderivative size = 477, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 11, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4930, 4880, 4890, 4888, 4181, 2531, 2282, 6589, 217, 206, 195}

$$-\frac{9ic^2\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,-ie^{i\tan^{-1}(ax)}\right)}{20a^2\sqrt{a^2cx^2+c}}+\frac{9ic^2\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,ie^{i\tan^{-1}(ax)}\right)}{20a^2\sqrt{a^2cx^2+c}}+\frac{9c^2\sqrt{a^2x^2+1}}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3,x]
```


```
[Out] -(c*x*Sqrt[c + a^2*c*x^2])/(20*a) + (9*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(
20*a^2) + ((c + a^2*c*x^2)^(3/2)*ArcTan[a*x])/(10*a^2) - (9*c*x*Sqrt[c + a^
2*c*x^2]*ArcTan[a*x]^2)/(40*a) - (3*x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2)/
(20*a) + (((9*I)/20)*c^2*Sqrt[1 + a^2*x^2]*ArcTan[E^(I*ArcTan[a*x])]*ArcTan
[a*x]^2)/(a^2*Sqrt[c + a^2*c*x^2]) + ((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3)/
(5*a^2*c) - (c^(3/2)*ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]])/(2*a^2) -
(((9*I)/20)*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a
*x])])/(a^2*Sqrt[c + a^2*c*x^2]) + (((9*I)/20)*c^2*Sqrt[1 + a^2*x^2]*ArcTan
[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])])/(a^2*Sqrt[c + a^2*c*x^2]) + (9*c^2*S
```

```
qrt[1 + a^2*x^2]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])])/(20*a^2*Sqrt[c + a^2*c
*x^2]) - (9*c^2*Sqrt[1 + a^2*x^2]*PolyLog[3, I*E^(I*ArcTan[a*x])])/(20*a^2*
Sqrt[c + a^2*c*x^2])
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 4880

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> -Simp[(b*p*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1))/(2*c*q*(2*
q + 1)), x] + (Dist[(2*d*q)/(2*q + 1), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTa
n[c*x])^p, x], x] + Dist[(b^2*d*p*(p - 1))/(2*q*(2*q + 1)), Int[(d + e*x^2)
^(q - 1)*(a + b*ArcTan[c*x])^(p - 2), x], x] + Simp[(x*(d + e*x^2)^q*(a + b
*ArcTan[c*x])^p)/(2*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[q, 0] && GtQ[p, 1]
```

Rule 4890

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4888

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[c
*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && GtQ
[d, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rubi steps

$$\int x\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^3\,dx = \frac{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^3}{5a^2c} - \frac{3\int\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2\,dx}{5a}$$

$$= \frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}{10a^2} - \frac{3x\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2}{20a} + \frac{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^3}{5a^2c}$$

$$= -\frac{cx\sqrt{c+a^2cx^2}}{20a} + \frac{9c\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{20a^2} + \frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}{10a^2} - \frac{9cx\sqrt{c+a^2}}{}$$

$$= -\frac{cx\sqrt{c+a^2cx^2}}{20a} + \frac{9c\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{20a^2} + \frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}{10a^2} - \frac{9cx\sqrt{c+a^2}}{}$$

$$= -\frac{cx\sqrt{c+a^2cx^2}}{20a} + \frac{9c\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{20a^2} + \frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}{10a^2} - \frac{9cx\sqrt{c+a^2}}{}$$

$$= -\frac{cx\sqrt{c+a^2cx^2}}{20a} + \frac{9c\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{20a^2} + \frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}{10a^2} - \frac{9cx\sqrt{c+a^2}}{}$$

$$= -\frac{cx\sqrt{c+a^2cx^2}}{20a} + \frac{9c\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{20a^2} + \frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}{10a^2} - \frac{9cx\sqrt{c+a^2}}{}$$

$$= -\frac{cx\sqrt{c+a^2cx^2}}{20a} + \frac{9c\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{20a^2} + \frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}{10a^2} - \frac{9cx\sqrt{c+a^2}}{}$$

$$= -\frac{cx\sqrt{c+a^2cx^2}}{20a} + \frac{9c\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{20a^2} + \frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}{10a^2} - \frac{9cx\sqrt{c+a^2}}{}$$

Mathematica [A] time = 3.85049, size = 441, normalized size = 0.92

$$\frac{c\sqrt{a^2cx^2+c}\left(960\left(-i\tan^{-1}(ax)\text{PolyLog}\left(2,-ie^{i\tan^{-1}(ax)}\right)+i\tan^{-1}(ax)\text{PolyLog}\left(2,ie^{i\tan^{-1}(ax)}\right)+\text{PolyLog}\left(3,-ie^{i\tan^{-1}}\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3,x]
```

```
[Out] (c*Sqrt[c + a^2*c*x^2]*(960*(I*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2 - Ar
cTanh[(a*x)/Sqrt[1 + a^2*x^2]] - I*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[
a*x])] + I*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])] + PolyLog[3, (-I)*E^
(I*ArcTan[a*x])] - PolyLog[3, I*E^(I*ArcTan[a*x])]) + 48*((-11*I)*ArcTan[E^
```

```
(I*ArcTan[a*x])]*ArcTan[a*x]^2 + 10*ArcTanh[(a*x)/Sqrt[1 + a^2*x^2]] + (11*
I)*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - (11*I)*ArcTan[a*x]*Poly
Log[2, I*E^(I*ArcTan[a*x])] - 11*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] + 11*Po
lyLog[3, I*E^(I*ArcTan[a*x])]) + 80*(1 + a^2*x^2)^(3/2)*ArcTan[a*x]*(6 + 4*
ArcTan[a*x]^2 + 6*Cos[2*ArcTan[a*x]] - 3*ArcTan[a*x]*Sin[2*ArcTan[a*x]]) -
(1 + a^2*x^2)^(5/2)*((48*a*x)/(1 + a^2*x^2)^2 + 32*ArcTan[a*x]^3*(-1 + 5*Co
s[2*ArcTan[a*x]]) + 6*ArcTan[a*x]*(25 + 36*Cos[2*ArcTan[a*x]] + 11*Cos[4*Ar
cTan[a*x]]) + ArcTan[a*x]^2*(6*Sin[2*ArcTan[a*x]] - 33*Sin[4*ArcTan[a*x]]))
))/(960*a^2*Sqrt[1 + a^2*x^2])
```

Maple [A] time = 1.543, size = 421, normalized size = 0.9

$$\frac{c\left(8\ (\arctan(ax))^3\ x^4a^4-6\ (\arctan(ax))^2\ x^3a^3+16\ (\arctan(ax))^3\ x^2a^2+4\ \arctan(ax)\ a^2x^2-15\ (\arctan(ax))^2\ xa+\right.}{40\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^3,x)
```

```
[Out] 1/40*c/a^2*(c*(a*x-I)*(a*x+I))^(1/2)*(8*arctan(a*x)^3*x^4*a^4-6*arctan(a*x)
^2*x^3*a^3+16*arctan(a*x)^3*x^2*a^2+4*arctan(a*x)*a^2*x^2-15*arctan(a*x)^2*
x*a+8*arctan(a*x)^3-2*a*x+22*arctan(a*x))-3/40*c*(c*(a*x-I)*(a*x+I))^(1/2)*
(I*arctan(a*x)^3-3*arctan(a*x)^2*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I*ar
ctan(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*polylog(3,-I*(1+I*a*x
)/(a^2*x^2+1)^(1/2)))/a^2/(a^2*x^2+1)^(1/2)+3/40*c*(c*(a*x-I)*(a*x+I))^(1/2
)*(I*arctan(a*x)^3+6*I*arctan(a*x)*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))
-3*arctan(a*x)^2*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*polylog(3,I*(1+I*a*x
)/(a^2*x^2+1)^(1/2)))/a^2/(a^2*x^2+1)^(1/2)+I*c/a^2*(c*(a*x-I)*(a*x+I))^(1/
2)*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*x^2+1)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2+c\right)^{\frac{3}{2}} x \arctan(ax)^3\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)*x*arctan(a*x)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^2cx^3+cx\right)\sqrt{a^2cx^2+c}\arctan\left(ax\right)^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^3 + c*x)*sqrt(a^2*c*x^2 + c)*arctan(a*x)^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\operatorname{atan}^3\left(ax\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**(3/2)*atan(a*x)**3,x)
```

```
[Out] Integral(x*(c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**3, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.423 $\int \left(c+a^2cx^2\right)^{3/2} \tan^{-1}(ax)^3\,dx$

Optimal. Leaf size=760

$$\frac{5ic^2\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{2a\sqrt{a^2cx^2+c}}-\frac{5ic^2\sqrt{a^2x^2+1}\text{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{2a\sqrt{a^2cx^2+c}}+\frac{9ic^2\sqrt{a^2x^2+1}\tan^{-1}(ax)^2\text{PolyLog}\left(2,-ie^{i}\right.}{8a\sqrt{a^2cx^2+c}}$$

```
[Out] -(c*Sqrt[c + a^2*c*x^2])/(4*a) + (c*x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/4 -
(9*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(8*a) - ((c + a^2*c*x^2)^(3/2)*ArcT
an[a*x]^2)/(4*a) + (3*c*x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/8 + (x*(c + a^
2*c*x^2)^(3/2)*ArcTan[a*x]^3)/4 - (((3*I)/4)*c^2*Sqrt[1 + a^2*x^2]*ArcTan[E
^(I*ArcTan[a*x])]*ArcTan[a*x]^3)/(a*Sqrt[c + a^2*c*x^2]) - ((5*I)*c^2*Sqrt[
1 + a^2*x^2]*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/(a*Sqrt[c
 + a^2*c*x^2]) + (((9*I)/8)*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog[2,
(-I)*E^(I*ArcTan[a*x])])/(a*Sqrt[c + a^2*c*x^2]) - (((9*I)/8)*c^2*Sqrt[1 +
a^2*x^2]*ArcTan[a*x]^2*PolyLog[2, I*E^(I*ArcTan[a*x])])/(a*Sqrt[c + a^2*c*x
^2]) + (((5*I)/2)*c^2*Sqrt[1 + a^2*x^2]*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/S
qrt[1 - I*a*x]])/(a*Sqrt[c + a^2*c*x^2]) - (((5*I)/2)*c^2*Sqrt[1 + a^2*x^2]
*PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/(a*Sqrt[c + a^2*c*x^2]) -
 (9*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])])/(
4*a*Sqrt[c + a^2*c*x^2]) + (9*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[3,
I*E^(I*ArcTan[a*x])])/(4*a*Sqrt[c + a^2*c*x^2]) - (((9*I)/4)*c^2*Sqrt[1 + a
^2*x^2]*PolyLog[4, (-I)*E^(I*ArcTan[a*x])])/(a*Sqrt[c + a^2*c*x^2]) + (((9*
I)/4)*c^2*Sqrt[1 + a^2*x^2]*PolyLog[4, I*E^(I*ArcTan[a*x])])/(a*Sqrt[c + a^
2*c*x^2])
```

Rubi [A] time = 0.524236, antiderivative size = 760, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {4880, 4890, 4888, 4181, 2531, 6609, 2282, 6589, 4886, 4878}

$$\frac{5ic^2\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{2a\sqrt{a^2cx^2+c}}-\frac{5ic^2\sqrt{a^2x^2+1}\text{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{2a\sqrt{a^2cx^2+c}}+\frac{9ic^2\sqrt{a^2x^2+1}\tan^{-1}(ax)^2\text{PolyLog}\left(2,-ie^{i}\right.}{8a\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3,x]
```

```
[Out] -(c*Sqrt[c + a^2*c*x^2])/(4*a) + (c*x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/4 -
(9*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(8*a) - ((c + a^2*c*x^2)^(3/2)*ArcT
an[a*x]^2)/(4*a) + (3*c*x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/8 + (x*(c + a^
```

```
2*c*x^2)^(3/2)*ArcTan[a*x]^3)/4 - (((3*I)/4)*c^2*Sqrt[1 + a^2*x^2]*ArcTan[E
^(I*ArcTan[a*x])]*ArcTan[a*x]^3)/(a*Sqrt[c + a^2*c*x^2]) - ((5*I)*c^2*Sqrt[
1 + a^2*x^2]*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/(a*Sqrt[c
 + a^2*c*x^2]) + (((9*I)/8)*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog[2,
(-I)*E^(I*ArcTan[a*x])])/(a*Sqrt[c + a^2*c*x^2]) - (((9*I)/8)*c^2*Sqrt[1 +
a^2*x^2]*ArcTan[a*x]^2*PolyLog[2, I*E^(I*ArcTan[a*x])])/(a*Sqrt[c + a^2*c*x
^2]) + (((5*I)/2)*c^2*Sqrt[1 + a^2*x^2]*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/S
qrt[1 - I*a*x]])/(a*Sqrt[c + a^2*c*x^2]) - (((5*I)/2)*c^2*Sqrt[1 + a^2*x^2]
*PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/(a*Sqrt[c + a^2*c*x^2]) -
 (9*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])])/(
4*a*Sqrt[c + a^2*c*x^2]) + (9*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[3,
I*E^(I*ArcTan[a*x])])/(4*a*Sqrt[c + a^2*c*x^2]) - (((9*I)/4)*c^2*Sqrt[1 + a
^2*x^2]*PolyLog[4, (-I)*E^(I*ArcTan[a*x])])/(a*Sqrt[c + a^2*c*x^2]) + (((9*
I)/4)*c^2*Sqrt[1 + a^2*x^2]*PolyLog[4, I*E^(I*ArcTan[a*x])])/(a*Sqrt[c + a^
2*c*x^2])
```

Rule 4880

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> -Simp[(b*p*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1))/(2*c*q*(2*
q + 1)), x] + (Dist[(2*d*q)/(2*q + 1), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTa
n[c*x])^p, x], x] + Dist[(b^2*d*p*(p - 1))/(2*q*(2*q + 1)), Int[(d + e*x^2)
^(q - 1)*(a + b*ArcTan[c*x])^(p - 2), x], x] + Simp[(x*(d + e*x^2)^q*(a + b
*ArcTan[c*x])^p)/(2*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[q, 0] && GtQ[p, 1]
```

Rule 4890

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4888

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[c
*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && GtQ
[d, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
```

```
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4886

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[(-2*I*(a + b*ArcTan[c*x])*ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]])
/(c*Sqrt[d]), x] + (Simp[(I*b*PolyLog[2, -((I*Sqrt[1 + I*c*x])/Sqrt[1 - I*c
*x])])/(c*Sqrt[d]), x] - Simp[(I*b*PolyLog[2, (I*Sqrt[1 + I*c*x])/Sqrt[1 -
I*c*x]])/(c*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

Rule 4878

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symbo
l] :> -Simp[(b*(d + e*x^2)^q)/(2*c*q*(2*q + 1)), x] + (Dist[(2*d*q)/(2*q +
1), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x]), x], x] + Simp[(x*(d + e*x^
2)^q*(a + b*ArcTan[c*x]))/(2*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] && Eq
Q[e, c^2*d] && GtQ[q, 0]
```

Rubi steps

$$\int \left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^3\,dx=-\frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2}{4a}+\frac{1}{4}x\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^3+\frac{1}{2}c\int\sqrt{c+a^2cx^2}\tan^{-1}($$

$$=-\frac{c\sqrt{c+a^2cx^2}}{4a}+\frac{1}{4}cx\sqrt{c+a^2cx^2}\tan^{-1}(ax)-\frac{9c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{8a}-\frac{\left(c+a^2cx^2\right)^{3}}{4}$$

$$=-\frac{c\sqrt{c+a^2cx^2}}{4a}+\frac{1}{4}cx\sqrt{c+a^2cx^2}\tan^{-1}(ax)-\frac{9c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{8a}-\frac{\left(c+a^2cx^2\right)^{3}}{4}$$

$$=-\frac{c\sqrt{c+a^2cx^2}}{4a}+\frac{1}{4}cx\sqrt{c+a^2cx^2}\tan^{-1}(ax)-\frac{9c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{8a}-\frac{\left(c+a^2cx^2\right)^{3}}{4}$$

$$=-\frac{c\sqrt{c+a^2cx^2}}{4a}+\frac{1}{4}cx\sqrt{c+a^2cx^2}\tan^{-1}(ax)-\frac{9c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{8a}-\frac{\left(c+a^2cx^2\right)^{3}}{4}$$

$$=-\frac{c\sqrt{c+a^2cx^2}}{4a}+\frac{1}{4}cx\sqrt{c+a^2cx^2}\tan^{-1}(ax)-\frac{9c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{8a}-\frac{\left(c+a^2cx^2\right)^{3}}{4}$$

$$=-\frac{c\sqrt{c+a^2cx^2}}{4a}+\frac{1}{4}cx\sqrt{c+a^2cx^2}\tan^{-1}(ax)-\frac{9c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{8a}-\frac{\left(c+a^2cx^2\right)^{3}}{4}$$

$$=-\frac{c\sqrt{c+a^2cx^2}}{4a}+\frac{1}{4}cx\sqrt{c+a^2cx^2}\tan^{-1}(ax)-\frac{9c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{8a}-\frac{\left(c+a^2cx^2\right)^{3}}{4}$$

$$=-\frac{c\sqrt{c+a^2cx^2}}{4a}+\frac{1}{4}cx\sqrt{c+a^2cx^2}\tan^{-1}(ax)-\frac{9c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{8a}-\frac{\left(c+a^2cx^2\right)^{3}}{4}$$

Mathematica [B] time = 12.9469, size = 2105, normalized size = 2.77

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3,x]

[Out] ((-I/2)*c*Sqrt[c*(1 + a^2*x^2)]*(12*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x] -
 (3*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2 + I*a*x*Sqrt[1 + a^2*x^2]*ArcTan[a*x
]^3 + 2*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^3 - 3*(2 + ArcTan[a*x]^2)*Pol
yLog[2, (-I)*E^(I*ArcTan[a*x])] + 3*(2 + ArcTan[a*x]^2)*PolyLog[2, I*E^(I*A
rcTan[a*x])] - (6*I)*ArcTan[a*x]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] + (6*I)
*ArcTan[a*x]*PolyLog[3, I*E^(I*ArcTan[a*x])] + 6*PolyLog[4, (-I)*E^(I*ArcTa
n[a*x])] - 6*PolyLog[4, I*E^(I*ArcTan[a*x])]))/(a*Sqrt[1 + a^2*x^2]) + (c*(
(Sqrt[c*(1 + a^2*x^2)]*(-1 + ArcTan[a*x]^2))/(4*Sqrt[1 + a^2*x^2]) + (Sqrt[
c*(1 + a^2*x^2)]*(-(ArcTan[a*x]*(Log[1 - I*E^(I*ArcTan[a*x])] - Log[1 + I*E
^(I*ArcTan[a*x])])) - I*(PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - PolyLog[2, I*
E^(I*ArcTan[a*x])])))/(2*Sqrt[1 + a^2*x^2]) + (Sqrt[c*(1 + a^2*x^2)]*(-(Pi^
3*Log[Cot[(Pi/2 - ArcTan[a*x])/2]])/8 - (3*Pi^2*((Pi/2 - ArcTan[a*x])*(Log[
1 - E^(I*(Pi/2 - ArcTan[a*x]))] - Log[1 + E^(I*(Pi/2 - ArcTan[a*x]))]) + I*
(PolyLog[2, -E^(I*(Pi/2 - ArcTan[a*x]))] - PolyLog[2, E^(I*(Pi/2 - ArcTan[a
*x]))])))/4 + (3*Pi*((Pi/2 - ArcTan[a*x])^2*(Log[1 - E^(I*(Pi/2 - ArcTan[a*
x]))] - Log[1 + E^(I*(Pi/2 - ArcTan[a*x]))]) + (2*I)*(Pi/2 - ArcTan[a*x])*(
PolyLog[2, -E^(I*(Pi/2 - ArcTan[a*x]))] - PolyLog[2, E^(I*(Pi/2 - ArcTan[a*
x]))]) + 2*(-PolyLog[3, -E^(I*(Pi/2 - ArcTan[a*x]))] + PolyLog[3, E^(I*(Pi/
2 - ArcTan[a*x]))])))/2 - 8*((I/64)*(Pi/2 - ArcTan[a*x])^4 + (I/4)*(Pi/2 +
(-Pi/2 + ArcTan[a*x])/2)^4 - ((Pi/2 - ArcTan[a*x])^3*Log[1 + E^(I*(Pi/2 - A
rcTan[a*x]))])/8 - (Pi^3*(I*(Pi/2 + (-Pi/2 + ArcTan[a*x])/2) - Log[1 + E^((
2*I)*(Pi/2 + (-Pi/2 + ArcTan[a*x])/2))]))/8 - (Pi/2 + (-Pi/2 + ArcTan[a*x])
/2)^3*Log[1 + E^((2*I)*(Pi/2 + (-Pi/2 + ArcTan[a*x])/2))] + ((3*I)/8)*(Pi/2
 - ArcTan[a*x])^2*PolyLog[2, -E^(I*(Pi/2 - ArcTan[a*x]))] + (3*Pi^2*((I/2)*
(Pi/2 + (-Pi/2 + ArcTan[a*x])/2)^2 - (Pi/2 + (-Pi/2 + ArcTan[a*x])/2)*Log[1
 + E^((2*I)*(Pi/2 + (-Pi/2 + ArcTan[a*x])/2))] + (I/2)*PolyLog[2, -E^((2*I)
*(Pi/2 + (-Pi/2 + ArcTan[a*x])/2))]))/4 + ((3*I)/2)*(Pi/2 + (-Pi/2 + ArcTan
[a*x])/2)^2*PolyLog[2, -E^((2*I)*(Pi/2 + (-Pi/2 + ArcTan[a*x])/2))] - (3*(P
i/2 - ArcTan[a*x])*PolyLog[3, -E^(I*(Pi/2 - ArcTan[a*x]))])/4 - (3*Pi*((I/3
)*(Pi/2 + (-Pi/2 + ArcTan[a*x])/2)^3 - (Pi/2 + (-Pi/2 + ArcTan[a*x])/2)^2*L
og[1 + E^((2*I)*(Pi/2 + (-Pi/2 + ArcTan[a*x])/2))] + I*(Pi/2 + (-Pi/2 + Arc
Tan[a*x])/2)*PolyLog[2, -E^((2*I)*(Pi/2 + (-Pi/2 + ArcTan[a*x])/2))] - Poly
Log[3, -E^((2*I)*(Pi/2 + (-Pi/2 + ArcTan[a*x])/2))]/2))/2 - (3*(Pi/2 + (-Pi
/2 + ArcTan[a*x])/2)*PolyLog[3, -E^((2*I)*(Pi/2 + (-Pi/2 + ArcTan[a*x])/2))
])/2 - ((3*I)/4)*PolyLog[4, -E^(I*(Pi/2 - ArcTan[a*x]))] - ((3*I)/4)*PolyLo
g[4, -E^((2*I)*(Pi/2 + (-Pi/2 + ArcTan[a*x])/2))])))/(8*Sqrt[1 + a^2*x^2])
+ (Sqrt[c*(1 + a^2*x^2)]*ArcTan[a*x]^3)/(16*Sqrt[1 + a^2*x^2]*(Cos[ArcTan[a
*x]/2] - Sin[ArcTan[a*x]/2])^4) + (Sqrt[c*(1 + a^2*x^2)]*(2*ArcTan[a*x] - A
rcTan[a*x]^2 - ArcTan[a*x]^3))/(16*Sqrt[1 + a^2*x^2]*(Cos[ArcTan[a*x]/2] -
Sin[ArcTan[a*x]/2])^2) - (Sqrt[c*(1 + a^2*x^2)]*ArcTan[a*x]^2*Sin[ArcTan[a*
x]/2])/(8*Sqrt[1 + a^2*x^2]*(Cos[ArcTan[a*x]/2] - Sin[ArcTan[a*x]/2])^3) -
(Sqrt[c*(1 + a^2*x^2)]*ArcTan[a*x]^3)/(16*Sqrt[1 + a^2*x^2]*(Cos[ArcTan[a*x
```

```
]/2] + Sin[ArcTan[a*x]/2])^4) + (Sqrt[c*(1 + a^2*x^2)]*ArcTan[a*x]^2*Sin[Ar
cTan[a*x]/2])/(8*Sqrt[1 + a^2*x^2]*(Cos[ArcTan[a*x]/2] + Sin[ArcTan[a*x]/2]
)^3) + (Sqrt[c*(1 + a^2*x^2)]*(-2*ArcTan[a*x] - ArcTan[a*x]^2 + ArcTan[a*x]
^3))/(16*Sqrt[1 + a^2*x^2]*(Cos[ArcTan[a*x]/2] + Sin[ArcTan[a*x]/2])^2) + (
Sqrt[c*(1 + a^2*x^2)]*(Sin[ArcTan[a*x]/2] - ArcTan[a*x]^2*Sin[ArcTan[a*x]/2
]))/(4*Sqrt[1 + a^2*x^2]*(Cos[ArcTan[a*x]/2] + Sin[ArcTan[a*x]/2])) + (Sqrt
[c*(1 + a^2*x^2)]*(-Sin[ArcTan[a*x]/2] + ArcTan[a*x]^2*Sin[ArcTan[a*x]/2]))
/(4*Sqrt[1 + a^2*x^2]*(Cos[ArcTan[a*x]/2] - Sin[ArcTan[a*x]/2]))))/a
```

Maple [A] time = 1.434, size = 466, normalized size = 0.6

$$\frac{c\left(2\,(\arctan(ax))^3 a^3x^3 - 2\,(\arctan(ax))^2 x^2a^2 + 5\,(\arctan(ax))^3 ax + 2\arctan(ax)\,xa - 11\,(\arctan(ax))^2 - 2\right)}{8a}\sqrt{c(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(3/2)*arctan(a*x)^3,x)
```

```
[Out] 1/8*c/a*(c*(a*x-I)*(a*x+I))^(1/2)*(2*arctan(a*x)^3*a^3*x^3-2*arctan(a*x)^2*
x^2*a^2+5*arctan(a*x)^3*a*x+2*arctan(a*x)*x*a-11*arctan(a*x)^2-2)-1/8*c*(c*
(a*x-I)*(a*x+I))^(1/2)*(3*arctan(a*x)^3*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))
-3*arctan(a*x)^3*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-9*I*arctan(a*x)^2*poly
log(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+9*I*arctan(a*x)^2*polylog(2,I*(1+I*a*
x)/(a^2*x^2+1)^(1/2))+20*arctan(a*x)*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+18
*arctan(a*x)*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-20*arctan(a*x)*ln(1-
I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-18*arctan(a*x)*polylog(3,I*(1+I*a*x)/(a^2*x^
2+1)^(1/2))+18*I*polylog(4,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-18*I*polylog(4,I
*(1+I*a*x)/(a^2*x^2+1)^(1/2))-20*I*dilog(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+2
0*I*dilog(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))/a/(a^2*x^2+1)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^2cx^2+c\right)^{\frac{3}{2}}\arctan\left(ax\right)^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\operatorname{atan}^3\left(ax\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(3/2)*atan(a*x)**3,x)
```

```
[Out] Integral((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**3, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.424 $\int \frac{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^3}{x}\,dx$

Optimal. Leaf size=726

$$\frac{3ic^2\sqrt{a^2x^2+1}\tan^{-1}(ax)^2\text{PolyLog}\left(2,-e^{i\tan^{-1}(ax)}\right)}{\sqrt{a^2cx^2+c}}-\frac{3ic^2\sqrt{a^2x^2+1}\tan^{-1}(ax)^2\text{PolyLog}\left(2,e^{i\tan^{-1}(ax)}\right)}{\sqrt{a^2cx^2+c}}-\frac{7ic^2\sqrt{a^2x^2+}}{}$$

```
[Out] c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x] - (a*c*x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^
2)/2 + ((7*I)*c^2*Sqrt[1 + a^2*x^2]*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2
)/Sqrt[c + a^2*c*x^2] + c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3 + ((c + a^2*c*x
^2)^(3/2)*ArcTan[a*x]^3)/3 - (2*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^3*ArcTanh
[E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - c^(3/2)*ArcTanh[(a*Sqrt[c]*x)/Sq
rt[c + a^2*c*x^2]] + ((3*I)*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog[2,
-E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - ((7*I)*c^2*Sqrt[1 + a^2*x^2]*Arc
Tan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + ((7*I)*c
^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])])/Sqrt[c +
a^2*c*x^2] - ((3*I)*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog[2, E^(I*Arc
Tan[a*x])])/Sqrt[c + a^2*c*x^2] - (6*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*Poly
Log[3, -E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + (7*c^2*Sqrt[1 + a^2*x^2]*
PolyLog[3, (-I)*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - (7*c^2*Sqrt[1 + a
^2*x^2]*PolyLog[3, I*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + (6*c^2*Sqrt[
1 + a^2*x^2]*ArcTan[a*x]*PolyLog[3, E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2]
 - ((6*I)*c^2*Sqrt[1 + a^2*x^2]*PolyLog[4, -E^(I*ArcTan[a*x])])/Sqrt[c + a^
2*c*x^2] + ((6*I)*c^2*Sqrt[1 + a^2*x^2]*PolyLog[4, E^(I*ArcTan[a*x])])/Sqrt
[c + a^2*c*x^2]
```

Rubi [A] time = 1.14392, antiderivative size = 726, normalized size of antiderivative = 1., number of steps used = 36, number of rules used = 15, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {4950, 4958, 4956, 4183, 2531, 6609, 2282, 6589, 4930, 4890, 4888, 4181, 4880, 217, 206}

$$\frac{3ic^2\sqrt{a^2x^2+1}\tan^{-1}(ax)^2\text{PolyLog}\left(2,-e^{i\tan^{-1}(ax)}\right)}{\sqrt{a^2cx^2+c}}-\frac{3ic^2\sqrt{a^2x^2+1}\tan^{-1}(ax)^2\text{PolyLog}\left(2,e^{i\tan^{-1}(ax)}\right)}{\sqrt{a^2cx^2+c}}-\frac{7ic^2\sqrt{a^2x^2+}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3)/x,x]
```

```
[Out] c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x] - (a*c*x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^
2)/2 + ((7*I)*c^2*Sqrt[1 + a^2*x^2]*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2
```

```
)/Sqrt[c + a^2*c*x^2] + c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3 + ((c + a^2*c*x
^2)^(3/2)*ArcTan[a*x]^3)/3 - (2*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^3*ArcTanh
[E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - c^(3/2)*ArcTanh[(a*Sqrt[c]*x)/Sq
rt[c + a^2*c*x^2]] + ((3*I)*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog[2,
-E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - ((7*I)*c^2*Sqrt[1 + a^2*x^2]*Arc
Tan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + ((7*I)*c
^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])])/Sqrt[c +
a^2*c*x^2] - ((3*I)*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog[2, E^(I*Arc
Tan[a*x])])/Sqrt[c + a^2*c*x^2] - (6*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*Poly
Log[3, -E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + (7*c^2*Sqrt[1 + a^2*x^2]*
PolyLog[3, (-I)*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - (7*c^2*Sqrt[1 + a
^2*x^2]*PolyLog[3, I*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + (6*c^2*Sqrt[
1 + a^2*x^2]*ArcTan[a*x]*PolyLog[3, E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2]
 - ((6*I)*c^2*Sqrt[1 + a^2*x^2]*PolyLog[4, -E^(I*ArcTan[a*x])])/Sqrt[c + a^
2*c*x^2] + ((6*I)*c^2*Sqrt[1 + a^2*x^2]*PolyLog[4, E^(I*ArcTan[a*x])])/Sqrt
[c + a^2*c*x^2]
```

Rule 4950

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 4958

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4956

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Dist[1/Sqrt[d], Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTan
[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && G
tQ[d, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
```

```
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 4890

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
```

```
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4888

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[c
*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && GtQ
[d, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4880

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> -Simp[(b*p*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1))/(2*c*q*(2*
q + 1)), x] + (Dist[(2*d*q)/(2*q + 1), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTa
n[c*x])^p, x], x] + Dist[(b^2*d*p*(p - 1))/(2*q*(2*q + 1)), Int[(d + e*x^2)
^(q - 1)*(a + b*ArcTan[c*x])^(p - 2), x], x] + Simp[(x*(d + e*x^2)^q*(a + b
*ArcTan[c*x])^p)/(2*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[q, 0] && GtQ[p, 1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^3}{x}\,dx = c\int \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{x}\,dx + \left(a^2c\right)\int x\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3\,dx$$
$$= \frac{1}{3}\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^3 - (ac)\int \sqrt{c+a^2cx^2}\tan^{-1}(ax)^2\,dx + c^2\int \frac{\tan^{-1}(ax)^3}{x\sqrt{c+a^2cx^2}}\,d$$
$$= c\sqrt{c+a^2cx^2}\tan^{-1}(ax) - \frac{1}{2}acx\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2 + c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3 + \frac{1}{3}($$
$$= c\sqrt{c+a^2cx^2}\tan^{-1}(ax) - \frac{1}{2}acx\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2 + c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3 + \frac{1}{3}($$
$$= c\sqrt{c+a^2cx^2}\tan^{-1}(ax) - \frac{1}{2}acx\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2 + c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3 + \frac{1}{3}($$
$$= c\sqrt{c+a^2cx^2}\tan^{-1}(ax) - \frac{1}{2}acx\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2 + \frac{7ic^2\sqrt{1+a^2x^2}\tan^{-1}\left(e^{i\tan^{-1}(}\right.}{\sqrt{c+a^2cx^2}}$$
$$= c\sqrt{c+a^2cx^2}\tan^{-1}(ax) - \frac{1}{2}acx\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2 + \frac{7ic^2\sqrt{1+a^2x^2}\tan^{-1}\left(e^{i\tan^{-1}(}\right.}{\sqrt{c+a^2cx^2}}$$
$$= c\sqrt{c+a^2cx^2}\tan^{-1}(ax) - \frac{1}{2}acx\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2 + \frac{7ic^2\sqrt{1+a^2x^2}\tan^{-1}\left(e^{i\tan^{-1}(}\right.}{\sqrt{c+a^2cx^2}}$$
$$= c\sqrt{c+a^2cx^2}\tan^{-1}(ax) - \frac{1}{2}acx\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2 + \frac{7ic^2\sqrt{1+a^2x^2}\tan^{-1}\left(e^{i\tan^{-1}(}\right.}{\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 1.3687, size = 555, normalized size = 0.76

$$\frac{c\sqrt{a^2cx^2+c}\left(72i\tan^{-1}(ax)^2\text{PolyLog}\left(2,e^{-i\tan^{-1}(ax)}\right)+72i\tan^{-1}(ax)^2\text{PolyLog}\left(2,-e^{i\tan^{-1}(ax)}\right)-168i\tan^{-1}(ax)\text{PolyL}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3)/x,x]
```

```
[Out] (c*Sqrt[c + a^2*c*x^2]*((-3*I)*Pi^4 + (24*ArcTan[a*x])/Sqrt[1 + a^2*x^2] +
(24*a^2*x^2*ArcTan[a*x])/Sqrt[1 + a^2*x^2] - (12*a*x*ArcTan[a*x]^2)/Sqrt[1
+ a^2*x^2] - (12*a^3*x^3*ArcTan[a*x]^2)/Sqrt[1 + a^2*x^2] + (24*I)*ArcTan[E
^(I*ArcTan[a*x])]*ArcTan[a*x]^2 + (32*ArcTan[a*x]^3)/Sqrt[1 + a^2*x^2] + (4
0*a^2*x^2*ArcTan[a*x]^3)/Sqrt[1 + a^2*x^2] + (8*a^4*x^4*ArcTan[a*x]^3)/Sqrt
```

```
[1 + a^2*x^2] + (6*I)*ArcTan[a*x]^4 - 24*ArcTanh[(a*x)/Sqrt[1 + a^2*x^2]] +
 24*ArcTan[a*x]^3*Log[1 - E^((-I)*ArcTan[a*x])] - 72*ArcTan[a*x]^2*Log[1 -
I*E^(I*ArcTan[a*x])] + 72*ArcTan[a*x]^2*Log[1 + I*E^(I*ArcTan[a*x])] - 24*A
rcTan[a*x]^3*Log[1 + E^(I*ArcTan[a*x])] + (72*I)*ArcTan[a*x]^2*PolyLog[2, E
^((-I)*ArcTan[a*x])] + (72*I)*ArcTan[a*x]^2*PolyLog[2, -E^(I*ArcTan[a*x])]
- (168*I)*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] + (168*I)*ArcTan[a
*x]*PolyLog[2, I*E^(I*ArcTan[a*x])] + 144*ArcTan[a*x]*PolyLog[3, E^((-I)*Ar
cTan[a*x])] - 144*ArcTan[a*x]*PolyLog[3, -E^(I*ArcTan[a*x])] + 168*PolyLog[
3, (-I)*E^(I*ArcTan[a*x])] - 168*PolyLog[3, I*E^(I*ArcTan[a*x])] - (144*I)*
PolyLog[4, E^((-I)*ArcTan[a*x])] - (144*I)*PolyLog[4, -E^(I*ArcTan[a*x])]))
/(24*Sqrt[1 + a^2*x^2])
```

Maple [A] time = 1.52, size = 511, normalized size = 0.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(3/2)*arctan(a*x)^3/x,x)
```

```
[Out] 1/6*c*(c*(a*x-I)*(a*x+I))^(1/2)*arctan(a*x)*(2*arctan(a*x)^2*x^2*a^2-3*arct
an(a*x)*x*a+8*arctan(a*x)^2+6)+1/2*c*(c*(a*x-I)*(a*x+I))^(1/2)*(2*arctan(a*
x)^3*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*arctan(a*x)^3*ln(1+(1+I*a*x)/(a^2*
x^2+1)^(1/2))-6*I*arctan(a*x)^2*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I*
arctan(a*x)^2*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+7*arctan(a*x)^2*ln(1+
I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-7*arctan(a*x)^2*ln(1-I*(1+I*a*x)/(a^2*x^2+1)
^(1/2))-14*I*arctan(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+14*I*arc
tan(a*x)*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+12*arctan(a*x)*polylog(3,
(1+I*a*x)/(a^2*x^2+1)^(1/2))-12*arctan(a*x)*polylog(3,-(1+I*a*x)/(a^2*x^2+1
)^(1/2))+12*I*polylog(4,(1+I*a*x)/(a^2*x^2+1)^(1/2))-12*I*polylog(4,-(1+I*a
*x)/(a^2*x^2+1)^(1/2))+4*I*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2))+14*polylog(3
,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-14*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2)
))/(a^2*x^2+1)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^3/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^2cx^2+c\right)^{\frac{3}{2}}\arctan\left(ax\right)^3}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^3/x,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^3/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\operatorname{atan}^3\left(ax\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(3/2)*atan(a*x)**3/x,x)
```

```
[Out] Integral((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**3/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^{\frac{3}{2}}\arctan\left(ax\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^3/x,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^3/x, x)
```

3.425 $\int \frac{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^3}{x^2}\,dx$

Optimal. Leaf size=901

$$-\frac{3iac^2\sqrt{a^2x^2+1}\tan^{-1}\left(e^{i\tan^{-1}(ax)}\right)\tan^{-1}(ax)^3}{\sqrt{a^2cx^2+c}}+\frac{1}{2}a^2cx\sqrt{a^2cx^2+c}\tan^{-1}(ax)^3-\frac{c\sqrt{a^2cx^2+c}\tan^{-1}(ax)^3}{x}-\frac{6ac^2\sqrt{a^2x^2}}{}$$

```
[Out] (-3*a*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/2 - (c*Sqrt[c + a^2*c*x^2]*ArcTa
n[a*x]^3)/x + (a^2*c*x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/2 - ((3*I)*a*c^2*
Sqrt[1 + a^2*x^2]*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^3)/Sqrt[c + a^2*c*x
^2] - ((6*I)*a*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqr
t[1 - I*a*x]])/Sqrt[c + a^2*c*x^2] - (6*a*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]
^2*ArcTanh[E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + ((6*I)*a*c^2*Sqrt[1 +
a^2*x^2]*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] +
(((9*I)/2)*a*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog[2, (-I)*E^(I*ArcTa
n[a*x])])/Sqrt[c + a^2*c*x^2] - (((9*I)/2)*a*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a
*x]^2*PolyLog[2, I*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - ((6*I)*a*c^2*S
qrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*
x^2] + ((3*I)*a*c^2*Sqrt[1 + a^2*x^2]*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqr
t[1 - I*a*x]])/Sqrt[c + a^2*c*x^2] - ((3*I)*a*c^2*Sqrt[1 + a^2*x^2]*PolyLog
[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2] - (6*a*c^2*Sq
rt[1 + a^2*x^2]*PolyLog[3, -E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - (9*a*
c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])])/Sqrt[
c + a^2*c*x^2] + (9*a*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[3, I*E^(I*A
rcTan[a*x])])/Sqrt[c + a^2*c*x^2] + (6*a*c^2*Sqrt[1 + a^2*x^2]*PolyLog[3, E
^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - ((9*I)*a*c^2*Sqrt[1 + a^2*x^2]*Pol
yLog[4, (-I)*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + ((9*I)*a*c^2*Sqrt[1
+ a^2*x^2]*PolyLog[4, I*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2]
```

Rubi [A] time = 1.23727, antiderivative size = 901, normalized size of antiderivative = 1., number of steps used = 37, number of rules used = 14, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {4950, 4944, 4958, 4956, 4183, 2531, 2282, 6589, 4890, 4888, 4181, 6609, 4880, 4886}

$$-\frac{3iac^2\sqrt{a^2x^2+1}\tan^{-1}\left(e^{i\tan^{-1}(ax)}\right)\tan^{-1}(ax)^3}{\sqrt{a^2cx^2+c}}+\frac{1}{2}a^2cx\sqrt{a^2cx^2+c}\tan^{-1}(ax)^3-\frac{c\sqrt{a^2cx^2+c}\tan^{-1}(ax)^3}{x}-\frac{6ac^2\sqrt{a^2x^2}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3)/x^2,x]
```

```
[Out] (-3*a*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/2 - (c*Sqrt[c + a^2*c*x^2]*ArcTa
n[a*x]^3)/x + (a^2*c*x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/2 - ((3*I)*a*c^2*
Sqrt[1 + a^2*x^2]*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^3)/Sqrt[c + a^2*c*x
^2] - ((6*I)*a*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqr
t[1 - I*a*x]])/Sqrt[c + a^2*c*x^2] - (6*a*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]
^2*ArcTanh[E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + ((6*I)*a*c^2*Sqrt[1 +
a^2*x^2]*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] +
(((9*I)/2)*a*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog[2, (-I)*E^(I*ArcTa
n[a*x])])/Sqrt[c + a^2*c*x^2] - (((9*I)/2)*a*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a
*x]^2*PolyLog[2, I*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - ((6*I)*a*c^2*S
qrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*
x^2] + ((3*I)*a*c^2*Sqrt[1 + a^2*x^2]*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqr
t[1 - I*a*x]])/Sqrt[c + a^2*c*x^2] - ((3*I)*a*c^2*Sqrt[1 + a^2*x^2]*PolyLog
[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2] - (6*a*c^2*Sq
rt[1 + a^2*x^2]*PolyLog[3, -E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - (9*a*
c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])])/Sqrt[
c + a^2*c*x^2] + (9*a*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[3, I*E^(I*A
rcTan[a*x])])/Sqrt[c + a^2*c*x^2] + (6*a*c^2*Sqrt[1 + a^2*x^2]*PolyLog[3, E
^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - ((9*I)*a*c^2*Sqrt[1 + a^2*x^2]*Pol
yLog[4, (-I)*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + ((9*I)*a*c^2*Sqrt[1
+ a^2*x^2]*PolyLog[4, I*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2]
```

Rule 4950

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 4944

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a +
 b*ArcTan[c*x])^p)/(d*f*(m + 1)), x] - Dist[(b*c*p)/(f*(m + 1)), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rule 4958

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4956

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Dist[1/Sqrt[d], Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTan
[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && G
tQ[d, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4890

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4888

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[c
*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && GtQ
[d, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 4880

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> -Simp[(b*p*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1))/(2*c*q*(2*
q + 1)), x] + (Dist[(2*d*q)/(2*q + 1), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTa
n[c*x])^p, x], x] + Dist[(b^2*d*p*(p - 1))/(2*q*(2*q + 1)), Int[(d + e*x^2)
^(q - 1)*(a + b*ArcTan[c*x])^(p - 2), x], x] + Simp[(x*(d + e*x^2)^q*(a + b
*ArcTan[c*x])^p)/(2*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[q, 0] && GtQ[p, 1]
```

Rule 4886

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[(-2*I*(a + b*ArcTan[c*x])*ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]])
/(c*Sqrt[d]), x] + (Simp[(I*b*PolyLog[2, -((I*Sqrt[1 + I*c*x])/Sqrt[1 - I*c
*x])])/(c*Sqrt[d]), x] - Simp[(I*b*PolyLog[2, (I*Sqrt[1 + I*c*x])/Sqrt[1 -
I*c*x]])/(c*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
```

GtQ[d, 0]

Rubi steps

$$\int \frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^3}{x^2}\,dx = c\int \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{x^2}\,dx + \left(a^2c\right)\int \sqrt{c+a^2cx^2}\tan^{-1}(ax)^3\,dx$$
$$= -\frac{3}{2}ac\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2 + \frac{1}{2}a^2cx\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3 + c^2\int \frac{\tan^{-1}(ax)^3}{x^2\sqrt{c+a^2cx^2}}\,dx$$
$$= -\frac{3}{2}ac\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2 - \frac{c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{x} + \frac{1}{2}a^2cx\sqrt{c+a^2cx^2}\tan^{-1}(ax$$
$$= -\frac{3}{2}ac\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2 - \frac{c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{x} + \frac{1}{2}a^2cx\sqrt{c+a^2cx^2}\tan^{-1}(ax$$
$$= -\frac{3}{2}ac\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2 - \frac{c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{x} + \frac{1}{2}a^2cx\sqrt{c+a^2cx^2}\tan^{-1}(ax$$
$$= -\frac{3}{2}ac\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2 - \frac{c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{x} + \frac{1}{2}a^2cx\sqrt{c+a^2cx^2}\tan^{-1}(ax$$
$$= -\frac{3}{2}ac\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2 - \frac{c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{x} + \frac{1}{2}a^2cx\sqrt{c+a^2cx^2}\tan^{-1}(ax$$
$$= -\frac{3}{2}ac\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2 - \frac{c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{x} + \frac{1}{2}a^2cx\sqrt{c+a^2cx^2}\tan^{-1}(ax$$
$$= -\frac{3}{2}ac\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2 - \frac{c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{x} + \frac{1}{2}a^2cx\sqrt{c+a^2cx^2}\tan^{-1}(ax$$

Mathematica [A] time = 6.50192, size = 1387, normalized size = 1.54

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3)/x^2,x]
```

```
[Out] (a*c*Sqrt[c + a^2*c*x^2]*((-7*I)*Pi^4*Sqrt[1 + a^2*x^2] - (8*I)*Pi^3*Sqrt[1
 + a^2*x^2]*ArcTan[a*x] - (384*I)*Sqrt[1 + a^2*x^2]*ArcTan[E^(I*ArcTan[a*x]
```

```
)]*ArcTan[a*x] - 96*ArcTan[a*x]^2 - 96*a^2*x^2*ArcTan[a*x]^2 + (24*I)*Pi^2*
Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2 - (64*ArcTan[a*x]^3)/(a*x) - 32*a*x*ArcTan[
a*x]^3 + 32*a^3*x^3*ArcTan[a*x]^3 - (32*I)*Pi*Sqrt[1 + a^2*x^2]*ArcTan[a*x]
^3 - (64*I)*Sqrt[1 + a^2*x^2]*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^3 + (16
*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^4 + 48*Pi^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]
*Log[1 - I/E^(I*ArcTan[a*x])] - 96*Pi*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*Log[1
 - I/E^(I*ArcTan[a*x])] - 8*Pi^3*Sqrt[1 + a^2*x^2]*Log[1 + I/E^(I*ArcTan[a*
x])] + 64*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^3*Log[1 + I/E^(I*ArcTan[a*x])] + 19
2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*Log[1 - E^(I*ArcTan[a*x])] + 8*Pi^3*Sqrt[
1 + a^2*x^2]*Log[1 + I*E^(I*ArcTan[a*x])] - 48*Pi^2*Sqrt[1 + a^2*x^2]*ArcTa
n[a*x]*Log[1 + I*E^(I*ArcTan[a*x])] + 96*Pi*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2
*Log[1 + I*E^(I*ArcTan[a*x])] - 64*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^3*Log[1 +
I*E^(I*ArcTan[a*x])] - 192*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*Log[1 + E^(I*Arc
Tan[a*x])] + 8*Pi^3*Sqrt[1 + a^2*x^2]*Log[2*Sqrt[1 + a^2*x^2]*Sin[(Pi + 2*A
rcTan[a*x])/4]^2] + (192*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog[2, (-I)
/E^(I*ArcTan[a*x])] + (48*I)*Pi*Sqrt[1 + a^2*x^2]*(Pi - 4*ArcTan[a*x])*Poly
Log[2, I/E^(I*ArcTan[a*x])] + (384*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog
[2, -E^(I*ArcTan[a*x])] + (192*I)*Sqrt[1 + a^2*x^2]*PolyLog[2, (-I)*E^(I*Ar
cTan[a*x])] + (48*I)*Pi^2*Sqrt[1 + a^2*x^2]*PolyLog[2, (-I)*E^(I*ArcTan[a*x
])] - (192*I)*Pi*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[
a*x])] + (288*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog[2, (-I)*E^(I*ArcTa
n[a*x])] - (192*I)*Sqrt[1 + a^2*x^2]*PolyLog[2, I*E^(I*ArcTan[a*x])] - (96*
I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog[2, I*E^(I*ArcTan[a*x])] - (384*I
)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, E^(I*ArcTan[a*x])] + 384*Sqrt[1
+ a^2*x^2]*ArcTan[a*x]*PolyLog[3, (-I)/E^(I*ArcTan[a*x])] - 192*Pi*Sqrt[1 +
 a^2*x^2]*PolyLog[3, I/E^(I*ArcTan[a*x])] - 384*Sqrt[1 + a^2*x^2]*PolyLog[3
, -E^(I*ArcTan[a*x])] + 192*Pi*Sqrt[1 + a^2*x^2]*PolyLog[3, (-I)*E^(I*ArcTa
n[a*x])] - 576*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[3, (-I)*E^(I*ArcTan[a*
x])] + 192*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[3, I*E^(I*ArcTan[a*x])] +
384*Sqrt[1 + a^2*x^2]*PolyLog[3, E^(I*ArcTan[a*x])] - (384*I)*Sqrt[1 + a^2*
x^2]*PolyLog[4, (-I)/E^(I*ArcTan[a*x])] - (576*I)*Sqrt[1 + a^2*x^2]*PolyLog
[4, (-I)*E^(I*ArcTan[a*x])] + (192*I)*Sqrt[1 + a^2*x^2]*PolyLog[4, I*E^(I*A
rcTan[a*x])]))/(64*(1 + a^2*x^2))
```

Maple [A] time = 2.009, size = 602, normalized size = 0.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(3/2)*arctan(a*x)^3/x^2,x)
```

```
[Out] 1/2*c*(c*(a*x-I)*(a*x+I))^(1/2)*arctan(a*x)^2*(arctan(a*x)*a^2*x^2-3*a*x-2*
arctan(a*x))/x-3/2*I*a*c*(c*(a*x-I)*(a*x+I))^(1/2)*(-4*I*polylog(3,-(1+I*a*
x)/(a^2*x^2+1)^(1/2))+2*I*arctan(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))+2
*I*arctan(a*x)*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*I*arctan(a*x)*ln(1+I*(
1+I*a*x)/(a^2*x^2+1)^(1/2))-6*I*arctan(a*x)*polylog(3,-I*(1+I*a*x)/(a^2*x^2
+1)^(1/2))+6*I*arctan(a*x)*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+I*arcta
n(a*x)^3*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*I*arctan(a*x)^2*ln(1+(1+I*a*
x)/(a^2*x^2+1)^(1/2))-3*arctan(a*x)^2*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1
/2))+3*arctan(a*x)^2*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*arctan(a*x)
^3*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+4*I*polylog(3,(1+I*a*x)/(a^2*x^2+1)^
(1/2))+4*arctan(a*x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))-4*arctan(a*x)*p
olylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)
^(1/2))+6*polylog(4,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*polylog(2,I*(1+I*a*x)
/(a^2*x^2+1)^(1/2))-6*polylog(4,I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))/(a^2*x^2+1)
^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^3/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^2cx^2+c\right)^{\frac{3}{2}}\arctan\left(ax\right)^3}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^3/x^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^3/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c\left(a^{2}x^{2}+1\right)\right)^{\frac{3}{2}} \operatorname{atan}^{3}\left(ax\right)}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(3/2)*atan(a*x)**3/x**2,x)
```

```
[Out] Integral((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**3/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^{2}cx^{2}+c\right)^{\frac{3}{2}} \arctan\left(ax\right)^{3}}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^3/x^2,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^3/x^2, x)
```

3.426 $\int \frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^3}{x^3}\,dx$

Optimal. Leaf size=919

$$-\frac{3a^2c^2\sqrt{a^2x^2+1}\tanh^{-1}\left(e^{i\tan^{-1}(ax)}\right)\tan^{-1}(ax)^3}{\sqrt{a^2cx^2+c}}+a^2c\sqrt{a^2cx^2+c}\tan^{-1}(ax)^3-\frac{c\sqrt{a^2cx^2+c}\tan^{-1}(ax)^3}{2x^2}+\frac{6ia^2c^2\sqrt{a^2x^2}}{}$$

```
[Out] (-3*a*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(2*x) + ((6*I)*a^2*c^2*Sqrt[1 +
a^2*x^2]*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2)/Sqrt[c + a^2*c*x^2] + a^2
*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3 - (c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3
)/(2*x^2) - (3*a^2*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^3*ArcTanh[E^(I*ArcTan[
a*x])])/Sqrt[c + a^2*c*x^2] - (6*a^2*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcT
anh[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2] + (((9*I)/2)*a^2*
c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog[2, -E^(I*ArcTan[a*x])])/Sqrt[c
+ a^2*c*x^2] - ((6*I)*a^2*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, (-I)
*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + ((6*I)*a^2*c^2*Sqrt[1 + a^2*x^2]
*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - (((9*I)
/2)*a^2*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog[2, E^(I*ArcTan[a*x])])/
Sqrt[c + a^2*c*x^2] + ((3*I)*a^2*c^2*Sqrt[1 + a^2*x^2]*PolyLog[2, -(Sqrt[1
+ I*a*x]/Sqrt[1 - I*a*x])])/Sqrt[c + a^2*c*x^2] - ((3*I)*a^2*c^2*Sqrt[1 + a
^2*x^2]*PolyLog[2, Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2] -
(9*a^2*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[3, -E^(I*ArcTan[a*x])])/Sq
rt[c + a^2*c*x^2] + (6*a^2*c^2*Sqrt[1 + a^2*x^2]*PolyLog[3, (-I)*E^(I*ArcTa
n[a*x])])/Sqrt[c + a^2*c*x^2] - (6*a^2*c^2*Sqrt[1 + a^2*x^2]*PolyLog[3, I*E
^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + (9*a^2*c^2*Sqrt[1 + a^2*x^2]*ArcTa
n[a*x]*PolyLog[3, E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - ((9*I)*a^2*c^2*
Sqrt[1 + a^2*x^2]*PolyLog[4, -E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + ((9
*I)*a^2*c^2*Sqrt[1 + a^2*x^2]*PolyLog[4, E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c
*x^2]
```

Rubi [A] time = 2.01579, antiderivative size = 919, normalized size of antiderivative = 1., number of steps used = 50, number of rules used = 15, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {4950, 4962, 4944, 4958, 4954, 4956, 4183, 2531, 6609, 2282, 6589, 4930, 4890, 4888, 4181}

$$-\frac{3a^2c^2\sqrt{a^2x^2+1}\tanh^{-1}\left(e^{i\tan^{-1}(ax)}\right)\tan^{-1}(ax)^3}{\sqrt{a^2cx^2+c}}+a^2c\sqrt{a^2cx^2+c}\tan^{-1}(ax)^3-\frac{c\sqrt{a^2cx^2+c}\tan^{-1}(ax)^3}{2x^2}+\frac{6ia^2c^2\sqrt{a^2x^2}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3)/x^3,x]
```

```
[Out] (-3*a*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(2*x) + ((6*I)*a^2*c^2*Sqrt[1 +
a^2*x^2]*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2)/Sqrt[c + a^2*c*x^2] + a^2
*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3 - (c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3
)/(2*x^2) - (3*a^2*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^3*ArcTanh[E^(I*ArcTan[
a*x])])/Sqrt[c + a^2*c*x^2] - (6*a^2*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcT
anh[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2] + (((9*I)/2)*a^2*
c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog[2, -E^(I*ArcTan[a*x])])/Sqrt[c
+ a^2*c*x^2] - ((6*I)*a^2*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, (-I)
*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + ((6*I)*a^2*c^2*Sqrt[1 + a^2*x^2]
*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - (((9*I)
/2)*a^2*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog[2, E^(I*ArcTan[a*x])])/
Sqrt[c + a^2*c*x^2] + ((3*I)*a^2*c^2*Sqrt[1 + a^2*x^2]*PolyLog[2, -(Sqrt[1
+ I*a*x]/Sqrt[1 - I*a*x])])/Sqrt[c + a^2*c*x^2] - ((3*I)*a^2*c^2*Sqrt[1 + a
^2*x^2]*PolyLog[2, Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2] -
(9*a^2*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[3, -E^(I*ArcTan[a*x])])/Sq
rt[c + a^2*c*x^2] + (6*a^2*c^2*Sqrt[1 + a^2*x^2]*PolyLog[3, (-I)*E^(I*ArcTa
n[a*x])])/Sqrt[c + a^2*c*x^2] - (6*a^2*c^2*Sqrt[1 + a^2*x^2]*PolyLog[3, I*E
^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + (9*a^2*c^2*Sqrt[1 + a^2*x^2]*ArcTa
n[a*x]*PolyLog[3, E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - ((9*I)*a^2*c^2*
Sqrt[1 + a^2*x^2]*PolyLog[4, -E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + ((9
*I)*a^2*c^2*Sqrt[1 + a^2*x^2]*PolyLog[4, E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c
*x^2]
```

<u>Rule 4950</u>

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

<u>Rule 4962</u>

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Ar
cTan[c*x])^p)/(d*f*(m + 1)), x] + (-Dist[(b*c*p)/(f*(m + 1)), Int[((f*x)^(m
 + 1)*(a + b*ArcTan[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] - Dist[(c^2*(m +
 2))/(f^2*(m + 1)), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/Sqrt[d + e*x^
2], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] &
& LtQ[m, -1] && NeQ[m, -2]
```

Rule 4944

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a +
 b*ArcTan[c*x])^p)/(d*f*(m + 1)), x] - Dist[(b*c*p)/(f*(m + 1)), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rule 4958

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4954

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x_
Symbol] :> Simp[(-2*(a + b*ArcTan[c*x])*ArcTanh[Sqrt[1 + I*c*x]/Sqrt[1 - I*
c*x]])/Sqrt[d], x] + (Simp[(I*b*PolyLog[2, -(Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x
])])/Sqrt[d], x] - Simp[(I*b*PolyLog[2, Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]])/S
qrt[d], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

Rule 4956

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Dist[1/Sqrt[d], Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTan
[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && G
tQ[d, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
```

```
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 4890

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4888

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[c
*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && GtQ
```

```
[d, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^3}{x^3}\,dx = c\int \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{x^3}\,dx + (a^2c)\int \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{x}\,dx$$

$$= c^2\int \frac{\tan^{-1}(ax)^3}{x^3\sqrt{c+a^2cx^2}}\,dx + 2\left((a^2c^2)\int \frac{\tan^{-1}(ax)^3}{x\sqrt{c+a^2cx^2}}\,dx\right) + (a^4c^2)\int \frac{x\tan^{-1}(ax)^3}{\sqrt{c+a^2cx^2}}\,dx$$

$$= a^2c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3 - \frac{c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{2x^2} + \frac{1}{2}(3ac^2)\int \frac{\tan^{-1}(ax)^2}{x^2\sqrt{c+a^2cx^2}}\,dx -$$

$$= -\frac{3ac\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2x} + a^2c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3 - \frac{c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{2x^2} +$$

$$= -\frac{3ac\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2x} + a^2c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3 - \frac{c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{2x^2} -$$

$$= -\frac{3ac\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2x} + \frac{6ia^2c^2\sqrt{1+a^2x^2}\tan^{-1}\left(e^{i\tan^{-1}(ax)}\right)\tan^{-1}(ax)^2}{\sqrt{c+a^2cx^2}} + a^2c\sqrt{c}$$

$$= -\frac{3ac\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2x} + \frac{6ia^2c^2\sqrt{1+a^2x^2}\tan^{-1}\left(e^{i\tan^{-1}(ax)}\right)\tan^{-1}(ax)^2}{\sqrt{c+a^2cx^2}} + a^2c\sqrt{c}$$

$$= -\frac{3ac\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2x} + \frac{6ia^2c^2\sqrt{1+a^2x^2}\tan^{-1}\left(e^{i\tan^{-1}(ax)}\right)\tan^{-1}(ax)^2}{\sqrt{c+a^2cx^2}} + a^2c\sqrt{c}$$

$$= -\frac{3ac\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2x} + \frac{6ia^2c^2\sqrt{1+a^2x^2}\tan^{-1}\left(e^{i\tan^{-1}(ax)}\right)\tan^{-1}(ax)^2}{\sqrt{c+a^2cx^2}} + a^2c\sqrt{c}$$

$$= -\frac{3ac\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2x} + \frac{6ia^2c^2\sqrt{1+a^2x^2}\tan^{-1}\left(e^{i\tan^{-1}(ax)}\right)\tan^{-1}(ax)^2}{\sqrt{c+a^2cx^2}} + a^2c\sqrt{c}$$

Mathematica [A] time = 9.57279, size = 691, normalized size = 0.75

$$\frac{a^2c\sqrt{a^2cx^2+c}\tan\left(\frac{1}{2}\tan^{-1}(ax)\right)\left(72i\tan^{-1}(ax)^2\cot\left(\frac{1}{2}\tan^{-1}(ax)\right)\text{PolyLog}\left(2,e^{-i\tan^{-1}(ax)}\right) - 96i\tan^{-1}(ax)\cot\left(\frac{1}{2}\tan^{-1}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3)/x^3,x]
```

```
[Out] (a^2*c*Sqrt[c + a^2*c*x^2]*(-12*ArcTan[a*x]^2 - (3*I)*Pi^4*Cot[ArcTan[a*x]/
2] + (6*I)*ArcTan[a*x]^4*Cot[ArcTan[a*x]/2] - 12*ArcTan[a*x]^2*Cot[ArcTan[a
*x]/2]^2 + 8*a*x*ArcTan[a*x]^3*Csc[ArcTan[a*x]/2]^2 - 2*ArcTan[a*x]^3*Cot[A
rcTan[a*x]/2]*Csc[ArcTan[a*x]/2]^2 + 24*ArcTan[a*x]^3*Cot[ArcTan[a*x]/2]*Lo
g[1 - E^((-I)*ArcTan[a*x])] + 48*ArcTan[a*x]*Cot[ArcTan[a*x]/2]*Log[1 - E^(
I*ArcTan[a*x])] - 48*ArcTan[a*x]^2*Cot[ArcTan[a*x]/2]*Log[1 - I*E^(I*ArcTan
[a*x])] + 48*ArcTan[a*x]^2*Cot[ArcTan[a*x]/2]*Log[1 + I*E^(I*ArcTan[a*x])]
- 48*ArcTan[a*x]*Cot[ArcTan[a*x]/2]*Log[1 + E^(I*ArcTan[a*x])] - 24*ArcTan[
a*x]^3*Cot[ArcTan[a*x]/2]*Log[1 + E^(I*ArcTan[a*x])] + (72*I)*ArcTan[a*x]^2
*Cot[ArcTan[a*x]/2]*PolyLog[2, E^((-I)*ArcTan[a*x])] + (24*I)*(2 + 3*ArcTan
[a*x]^2)*Cot[ArcTan[a*x]/2]*PolyLog[2, -E^(I*ArcTan[a*x])] - (96*I)*ArcTan[
a*x]*Cot[ArcTan[a*x]/2]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] + (96*I)*ArcTan[
a*x]*Cot[ArcTan[a*x]/2]*PolyLog[2, I*E^(I*ArcTan[a*x])] - (48*I)*Cot[ArcTan
[a*x]/2]*PolyLog[2, E^(I*ArcTan[a*x])] + 144*ArcTan[a*x]*Cot[ArcTan[a*x]/2]
*PolyLog[3, E^((-I)*ArcTan[a*x])] - 144*ArcTan[a*x]*Cot[ArcTan[a*x]/2]*Poly
Log[3, -E^(I*ArcTan[a*x])] + 96*Cot[ArcTan[a*x]/2]*PolyLog[3, (-I)*E^(I*Arc
Tan[a*x])] - 96*Cot[ArcTan[a*x]/2]*PolyLog[3, I*E^(I*ArcTan[a*x])] - (144*I
)*Cot[ArcTan[a*x]/2]*PolyLog[4, E^((-I)*ArcTan[a*x])] - (144*I)*Cot[ArcTan[
a*x]/2]*PolyLog[4, -E^(I*ArcTan[a*x])] + 2*ArcTan[a*x]^3*Csc[ArcTan[a*x]/2]
*Sec[ArcTan[a*x]/2])*Tan[ArcTan[a*x]/2])/(16*Sqrt[1 + a^2*x^2])
```

Maple [A] time = 1.964, size = 592, normalized size = 0.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(3/2)*arctan(a*x)^3/x^3,x)
```

```
[Out] 1/2*c*(c*(a*x-I)*(a*x+I))^(1/2)*arctan(a*x)^2*(2*arctan(a*x)*a^2*x^2-3*a*x-
arctan(a*x))/x^2+3/2*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)^(1/2)*(arctan(a*
x)^3*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-arctan(a*x)^3*ln(1+(1+I*a*x)/(a^2*x^
2+1)^(1/2))-6*I*polylog(4,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-3*I*arctan(a*x)^2*p
olylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*arctan(a*x)^2*ln(1+I*(1+I*a*x)/(a^2
*x^2+1)^(1/2))-2*arctan(a*x)^2*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*I*poly
log(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-4*I*arctan(a*x)*polylog(2,-I*(1+I*a*x)/
(a^2*x^2+1)^(1/2))+2*arctan(a*x)*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*arctan
(a*x)*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*arctan(a*x)*ln(1+(1+I*a*x)/(
a^2*x^2+1)^(1/2))-6*arctan(a*x)*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+4*I
*arctan(a*x)*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*I*polylog(2,(1+I*a*
x)/(a^2*x^2+1)^(1/2))+3*I*arctan(a*x)^2*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1
/2))+6*I*polylog(4,(1+I*a*x)/(a^2*x^2+1)^(1/2))+4*polylog(3,-I*(1+I*a*x)/(a
```

```
^2*x^2+1)^(1/2))-4*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))*a^2*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^3/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^2cx^2+c\right)^{\frac{3}{2}}\arctan\left(ax\right)^3}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^3/x^3,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^3/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\operatorname{atan}^3\left(ax\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(3/2)*atan(a*x)**3/x**3,x)
```

```
[Out] Integral((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**3/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^{\frac{3}{2}} \arctan\left(ax\right)^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^3/x^3,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^3/x^3, x)
```

3.427 $\int \frac{(c+a^2cx^2)^{3/2} \tan^{-1}(ax)^3}{x^4} \, dx$

Optimal. Leaf size=788

$$\frac{7ia^3c^2\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,-e^{i\tan^{-1}(ax)}\right)}{\sqrt{a^2cx^2+c}}+\frac{3ia^3c^2\sqrt{a^2x^2+1}\tan^{-1}(ax)^2\text{PolyLog}\left(2,-ie^{i\tan^{-1}(ax)}\right)}{\sqrt{a^2cx^2+c}}-\frac{3ia^3c^2\sqrt{}}{}$$

```
[Out] -((a^2*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/x) - (a*c*Sqrt[c + a^2*c*x^2]*Arc
Tan[a*x]^2)/(2*x^2) - (a^2*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/x - ((c + a
^2*c*x^2)^(3/2)*ArcTan[a*x]^3)/(3*x^3) - ((2*I)*a^3*c^2*Sqrt[1 + a^2*x^2]*A
rcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^3)/Sqrt[c + a^2*c*x^2] - (7*a^3*c^2*Sq
rt[1 + a^2*x^2]*ArcTan[a*x]^2*ArcTanh[E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^
2] - a^3*c^(3/2)*ArcTanh[Sqrt[c + a^2*c*x^2]/Sqrt[c]] + ((7*I)*a^3*c^2*Sqrt
[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^
2] + ((3*I)*a^3*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog[2, (-I)*E^(I*Ar
cTan[a*x])])/Sqrt[c + a^2*c*x^2] - ((3*I)*a^3*c^2*Sqrt[1 + a^2*x^2]*ArcTan[
a*x]^2*PolyLog[2, I*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - ((7*I)*a^3*c^
2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, E^(I*ArcTan[a*x])])/Sqrt[c + a^2
*c*x^2] - (7*a^3*c^2*Sqrt[1 + a^2*x^2]*PolyLog[3, -E^(I*ArcTan[a*x])])/Sqrt
[c + a^2*c*x^2] - (6*a^3*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[3, (-I)*
E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + (6*a^3*c^2*Sqrt[1 + a^2*x^2]*ArcT
an[a*x]*PolyLog[3, I*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + (7*a^3*c^2*S
qrt[1 + a^2*x^2]*PolyLog[3, E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - ((6*I
)*a^3*c^2*Sqrt[1 + a^2*x^2]*PolyLog[4, (-I)*E^(I*ArcTan[a*x])])/Sqrt[c + a^
2*c*x^2] + ((6*I)*a^3*c^2*Sqrt[1 + a^2*x^2]*PolyLog[4, I*E^(I*ArcTan[a*x])]
)/Sqrt[c + a^2*c*x^2]
```

Rubi [A] time = 1.87913, antiderivative size = 788, normalized size of antiderivative = 1., number of steps used = 48, number of rules used = 16, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {4950, 4944, 4962, 266, 63, 208, 4958, 4956, 4183, 2531, 2282, 6589, 4890, 4888, 4181, 6609}

$$\frac{7ia^3c^2\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,-e^{i\tan^{-1}(ax)}\right)}{\sqrt{a^2cx^2+c}}+\frac{3ia^3c^2\sqrt{a^2x^2+1}\tan^{-1}(ax)^2\text{PolyLog}\left(2,-ie^{i\tan^{-1}(ax)}\right)}{\sqrt{a^2cx^2+c}}-\frac{3ia^3c^2\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3)/x^4,x]
```

```
[Out] -((a^2*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/x) - (a*c*Sqrt[c + a^2*c*x^2]*Arc
Tan[a*x]^2)/(2*x^2) - (a^2*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/x - ((c + a
^2*c*x^2)^(3/2)*ArcTan[a*x]^3)/(3*x^3) - ((2*I)*a^3*c^2*Sqrt[1 + a^2*x^2]*A
rcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^3)/Sqrt[c + a^2*c*x^2] - (7*a^3*c^2*Sq
rt[1 + a^2*x^2]*ArcTan[a*x]^2*ArcTanh[E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^
2] - a^3*c^(3/2)*ArcTanh[Sqrt[c + a^2*c*x^2]/Sqrt[c]] + ((7*I)*a^3*c^2*Sqrt
[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^
2] + ((3*I)*a^3*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog[2, (-I)*E^(I*Ar
cTan[a*x])])/Sqrt[c + a^2*c*x^2] - ((3*I)*a^3*c^2*Sqrt[1 + a^2*x^2]*ArcTan[
a*x]^2*PolyLog[2, I*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - ((7*I)*a^3*c^
2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, E^(I*ArcTan[a*x])])/Sqrt[c + a^2
*c*x^2] - (7*a^3*c^2*Sqrt[1 + a^2*x^2]*PolyLog[3, -E^(I*ArcTan[a*x])])/Sqrt
[c + a^2*c*x^2] - (6*a^3*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[3, (-I)*
E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + (6*a^3*c^2*Sqrt[1 + a^2*x^2]*ArcT
an[a*x]*PolyLog[3, I*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + (7*a^3*c^2*S
qrt[1 + a^2*x^2]*PolyLog[3, E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - ((6*I
)*a^3*c^2*Sqrt[1 + a^2*x^2]*PolyLog[4, (-I)*E^(I*ArcTan[a*x])])/Sqrt[c + a^
2*c*x^2] + ((6*I)*a^3*c^2*Sqrt[1 + a^2*x^2]*PolyLog[4, I*E^(I*ArcTan[a*x])]
)/Sqrt[c + a^2*c*x^2]
```

Rule 4950

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 4944

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a +
 b*ArcTan[c*x])^p)/(d*f*(m + 1)), x] - Dist[(b*c*p)/(f*(m + 1)), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rule 4962

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Ar
cTan[c*x])^p)/(d*f*(m + 1)), x] + (-Dist[(b*c*p)/(f*(m + 1)), Int[((f*x)^(m
 + 1)*(a + b*ArcTan[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] - Dist[(c^2*(m +
 2))/(f^2*(m + 1)), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/Sqrt[d + e*x^
```

```
2], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] &
& LtQ[m, -1] && NeQ[m, -2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 4958

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4956

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Dist[1/Sqrt[d], Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTan
[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && G
tQ[d, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4890

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4888

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[c
*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && GtQ
[d, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^3}{x^4}\,dx = c\int \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{x^4}\,dx+\left(a^2c\right)\int \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{x^2}\,dx$$

$$= -\frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^3}{3x^3}+(ac)\int \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{x^3}\,dx+\left(a^2c^2\right)\int \frac{\tan^{-1}(ax}{x^2\sqrt{c+a^2c}}$$

$$= -\frac{a^2c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{x}-\frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^3}{3x^3}+\left(ac^2\right)\int \frac{\tan^{-1}(ax)^2}{x^3\sqrt{c+a^2cx^2}}\,dx+$$

$$= -\frac{ac\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2x^2}-\frac{a^2c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{x}-\frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^3}{3x^3}+$$

$$= -\frac{a^2c\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{x}-\frac{ac\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2x^2}-\frac{a^2c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{x}-$$

$$= -\frac{a^2c\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{x}-\frac{ac\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2x^2}-\frac{a^2c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{x}-$$

$$= -\frac{a^2c\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{x}-\frac{ac\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2x^2}-\frac{a^2c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{x}-$$

$$= -\frac{a^2c\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{x}-\frac{ac\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2x^2}-\frac{a^2c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{x}-$$

$$= -\frac{a^2c\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{x}-\frac{ac\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2x^2}-\frac{a^2c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{x}-$$

$$= -\frac{a^2c\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{x}-\frac{ac\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2x^2}-\frac{a^2c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{x}-$$

Mathematica [A] time = 10.0864, size = 1508, normalized size = 1.91

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3)/x^4,x]
```

```
[Out] (a^3*c*Sqrt[c*(1 + a^2*x^2)]*Csc[ArcTan[a*x]/2]*(((-7*I)*a*Pi^4*x)/Sqrt[1 +
 a^2*x^2] - ((8*I)*a*Pi^3*x*ArcTan[a*x])/Sqrt[1 + a^2*x^2] + ((24*I)*a*Pi^2
*x*ArcTan[a*x]^2)/Sqrt[1 + a^2*x^2] - 64*ArcTan[a*x]^3 - ((32*I)*a*Pi*x*Arc
Tan[a*x]^3)/Sqrt[1 + a^2*x^2] + ((16*I)*a*x*ArcTan[a*x]^4)/Sqrt[1 + a^2*x^2
] + (48*a*Pi^2*x*ArcTan[a*x]*Log[1 - I/E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2
] - (96*a*Pi*x*ArcTan[a*x]^2*Log[1 - I/E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2
] - (8*a*Pi^3*x*Log[1 + I/E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + (64*a*x*A
rcTan[a*x]^3*Log[1 + I/E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + (192*a*x*Arc
Tan[a*x]^2*Log[1 - E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + (8*a*Pi^3*x*Log[
1 + I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - (48*a*Pi^2*x*ArcTan[a*x]*Log[
1 + I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + (96*a*Pi*x*ArcTan[a*x]^2*Log[
1 + I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - (64*a*x*ArcTan[a*x]^3*Log[1 +
 I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - (192*a*x*ArcTan[a*x]^2*Log[1 + E
^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + (8*a*Pi^3*x*Log[Tan[(Pi + 2*ArcTan[a
*x])/4]])/Sqrt[1 + a^2*x^2] + ((192*I)*a*x*ArcTan[a*x]^2*PolyLog[2, (-I)/E^
(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + ((48*I)*a*Pi*x*(Pi - 4*ArcTan[a*x])*P
olyLog[2, I/E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + ((384*I)*a*x*ArcTan[a*x
]*PolyLog[2, -E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + ((48*I)*a*Pi^2*x*Poly
Log[2, (-I)*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - ((192*I)*a*Pi*x*ArcTan[
a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + ((192*I)*a*x*A
rcTan[a*x]^2*PolyLog[2, (-I)*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - ((384*
I)*a*x*ArcTan[a*x]*PolyLog[2, E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + (384*
a*x*ArcTan[a*x]*PolyLog[3, (-I)/E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - (19
2*a*Pi*x*PolyLog[3, I/E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - (384*a*x*Poly
Log[3, -E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + (192*a*Pi*x*PolyLog[3, (-I)
*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - (384*a*x*ArcTan[a*x]*PolyLog[3, (-
I)*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + (384*a*x*PolyLog[3, E^(I*ArcTan[
a*x])])/Sqrt[1 + a^2*x^2] - ((384*I)*a*x*PolyLog[4, (-I)/E^(I*ArcTan[a*x])]
)/Sqrt[1 + a^2*x^2] - ((384*I)*a*x*PolyLog[4, (-I)*E^(I*ArcTan[a*x])])/Sqrt
[1 + a^2*x^2])*Sec[ArcTan[a*x]/2])/(128*Sqrt[1 + a^2*x^2]) + (a^3*c^2*Sqrt[
1 + a^2*x^2]*(-12*ArcTan[a*x]*Cot[ArcTan[a*x]/2] - 2*ArcTan[a*x]^3*Cot[ArcT
an[a*x]/2] - 3*ArcTan[a*x]^2*Csc[ArcTan[a*x]/2]^2 - (a*x*ArcTan[a*x]^3*Csc[
ArcTan[a*x]/2]^4)/(2*Sqrt[1 + a^2*x^2]) + 12*ArcTan[a*x]^2*Log[1 - E^(I*Arc
Tan[a*x])] - 12*ArcTan[a*x]^2*Log[1 + E^(I*ArcTan[a*x])] + 24*Log[Tan[ArcTa
n[a*x]/2]] + (24*I)*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])] - (24*I)*Arc
Tan[a*x]*PolyLog[2, E^(I*ArcTan[a*x])] - 24*PolyLog[3, -E^(I*ArcTan[a*x])]
+ 24*PolyLog[3, E^(I*ArcTan[a*x])] + 3*ArcTan[a*x]^2*Sec[ArcTan[a*x]/2]^2 -
 (8*(1 + a^2*x^2)^(3/2)*ArcTan[a*x]^3*Sin[ArcTan[a*x]/2]^4)/(a^3*x^3) - 12*
ArcTan[a*x]*Tan[ArcTan[a*x]/2] - 2*ArcTan[a*x]^3*Tan[ArcTan[a*x]/2]))/(24*S
qrt[c*(1 + a^2*x^2)])
```

Maple [A] time = 2.326, size = 557, normalized size = 0.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(3/2)*arctan(a*x)^3/x^4,x)
```

```
[Out] -1/6*c*(c*(a*x-I)*(a*x+I))^(1/2)*arctan(a*x)*(8*arctan(a*x)^2*x^2*a^2+6*a^2
*x^2+3*arctan(a*x)*x*a+2*arctan(a*x)^2)/x^3+1/2*I*a^3*c*(c*(a*x-I)*(a*x+I))
^(1/2)*(14*I*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-12*I*arctan(a*x)*polyl
og(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*I*arctan(a*x)^3*ln(1-I*(1+I*a*x)/(a^2
*x^2+1)^(1/2))-7*I*arctan(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*I*ln((
1+I*a*x)/(a^2*x^2+1)^(1/2)-1)+2*I*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*arcta
n(a*x)^2*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*arctan(a*x)^2*polylog(
2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*I*arctan(a*x)^3*ln(1+I*(1+I*a*x)/(a^2*x^
2+1)^(1/2))+12*I*arctan(a*x)*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+7*I*
arctan(a*x)^2*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-14*I*polylog(3,(1+I*a*x)/(a
^2*x^2+1)^(1/2))-14*arctan(a*x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+14*a
rctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-12*polylog(4,-I*(1+I*a*x
)/(a^2*x^2+1)^(1/2))+12*polylog(4,I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))/(a^2*x^2+
1)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^3/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^2cx^2+c\right)^{\frac{3}{2}}\arctan\left(ax\right)^3}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^3/x^4,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^3/x^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\operatorname{atan}^3(ax)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(3/2)*atan(a*x)**3/x**4,x)
```

```
[Out] Integral((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**3/x**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^{\frac{3}{2}}\arctan(ax)^3}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^3/x^4,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^3/x^4, x)
```

3.428 $\int x^3 \left(c+a^2cx^2\right)^{5/2} \tan^{-1}(ax)^3 \, dx$

Optimal. Leaf size=798

$$\frac{1}{9}a^4c^2\sqrt{a^2cx^2+c}\tan^{-1}(ax)^3x^8 - \frac{1}{24}a^3c^2\sqrt{a^2cx^2+c}\tan^{-1}(ax)^2x^7 + \frac{19}{63}a^2c^2\sqrt{a^2cx^2+c}\tan^{-1}(ax)^3x^6 + \frac{1}{84}a^2c^2\sqrt{a^2cx^2+c}$$

```
[Out] (85*c^2*x*Sqrt[c + a^2*c*x^2])/(12096*a^3) - (c^2*x^3*Sqrt[c + a^2*c*x^2])/
(240*a) - (a*c^2*x^5*Sqrt[c + a^2*c*x^2])/504 - (6157*c^2*Sqrt[c + a^2*c*x^
2]*ArcTan[a*x])/(60480*a^4) - (47*c^2*x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/
(30240*a^2) + (67*c^2*x^4*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/2520 + (a^2*c^2*
x^6*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/84 + (47*c^2*x*Sqrt[c + a^2*c*x^2]*Arc
Tan[a*x]^2)/(896*a^3) - (205*c^2*x^3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(40
32*a) - (103*a*c^2*x^5*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/1008 - (a^3*c^2*x
^7*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/24 - (((115*I)/1344)*c^3*Sqrt[1 + a^2
*x^2]*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2)/(a^4*Sqrt[c + a^2*c*x^2]) -
(2*c^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/(63*a^4) + (c^2*x^2*Sqrt[c + a^2*
c*x^2]*ArcTan[a*x]^3)/(63*a^2) + (5*c^2*x^4*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]
^3)/21 + (19*a^2*c^2*x^6*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/63 + (a^4*c^2*x
^8*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/9 + (1433*c^(5/2)*ArcTanh[(a*Sqrt[c]*
x)/Sqrt[c + a^2*c*x^2]])/(15120*a^4) + (((115*I)/1344)*c^3*Sqrt[1 + a^2*x^2
]*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])])/(a^4*Sqrt[c + a^2*c*x^2])
 - (((115*I)/1344)*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcT
an[a*x])])/(a^4*Sqrt[c + a^2*c*x^2]) - (115*c^3*Sqrt[1 + a^2*x^2]*PolyLog[3
, (-I)*E^(I*ArcTan[a*x])])/(1344*a^4*Sqrt[c + a^2*c*x^2]) + (115*c^3*Sqrt[1
 + a^2*x^2]*PolyLog[3, I*E^(I*ArcTan[a*x])])/(1344*a^4*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 19.6636, antiderivative size = 798, normalized size of antiderivative = 1., number of steps used = 547, number of rules used = 12, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4950, 4952, 4930, 217, 206, 4890, 4888, 4181, 2531, 2282, 6589, 321}

$$\frac{1}{9}a^4c^2\sqrt{a^2cx^2+c}\tan^{-1}(ax)^3x^8 - \frac{1}{24}a^3c^2\sqrt{a^2cx^2+c}\tan^{-1}(ax)^2x^7 + \frac{19}{63}a^2c^2\sqrt{a^2cx^2+c}\tan^{-1}(ax)^3x^6 + \frac{1}{84}a^2c^2\sqrt{a^2cx^2+c}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3,x]
```

```
[Out] (85*c^2*x*Sqrt[c + a^2*c*x^2])/(12096*a^3) - (c^2*x^3*Sqrt[c + a^2*c*x^2])/
(240*a) - (a*c^2*x^5*Sqrt[c + a^2*c*x^2])/504 - (6157*c^2*Sqrt[c + a^2*c*x^
```

```
2]*ArcTan[a*x])/(60480*a^4) - (47*c^2*x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/
(30240*a^2) + (67*c^2*x^4*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/2520 + (a^2*c^2*
x^6*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/84 + (47*c^2*x*Sqrt[c + a^2*c*x^2]*Arc
Tan[a*x]^2)/(896*a^3) - (205*c^2*x^3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(40
32*a) - (103*a*c^2*x^5*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/1008 - (a^3*c^2*x
^7*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/24 - (((115*I)/1344)*c^3*Sqrt[1 + a^2
*x^2]*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2)/(a^4*Sqrt[c + a^2*c*x^2]) -
(2*c^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/(63*a^4) + (c^2*x^2*Sqrt[c + a^2*
c*x^2]*ArcTan[a*x]^3)/(63*a^2) + (5*c^2*x^4*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]
^3)/21 + (19*a^2*c^2*x^6*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/63 + (a^4*c^2*x
^8*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/9 + (1433*c^(5/2)*ArcTanh[(a*Sqrt[c]*
x)/Sqrt[c + a^2*c*x^2]])/(15120*a^4) + (((115*I)/1344)*c^3*Sqrt[1 + a^2*x^2
]*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])])/(a^4*Sqrt[c + a^2*c*x^2])
 - (((115*I)/1344)*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcT
an[a*x])])/(a^4*Sqrt[c + a^2*c*x^2]) - (115*c^3*Sqrt[1 + a^2*x^2]*PolyLog[3
, (-I)*E^(I*ArcTan[a*x])])/(1344*a^4*Sqrt[c + a^2*c*x^2]) + (115*c^3*Sqrt[1
 + a^2*x^2]*PolyLog[3, I*E^(I*ArcTan[a*x])])/(1344*a^4*Sqrt[c + a^2*c*x^2])
```

Rule 4950

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 4952

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b*
ArcTan[c*x])^p)/(c^2*d*m), x] + (-Dist[(b*f*p)/(c*m), Int[((f*x)^(m - 1)*(a
 + b*ArcTan[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] - Dist[(f^2*(m - 1))/(c^
2*m), Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/Sqrt[d + e*x^2], x], x]) /;
 FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 4890

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4888

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[c
*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && GtQ
[d, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
```

```
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int x^3 \left(c+a^2cx^2\right)^{5/2} \tan^{-1}(ax)^3 \, dx = c \int x^3 \left(c+a^2cx^2\right)^{3/2} \tan^{-1}(ax)^3 \, dx + \left(a^2c\right) \int x^5 \left(c+a^2cx^2\right)^{3/2} \tan^{-1}(ax)^3 \, dx$$

Mathematica [A] time = 6.47453, size = 850, normalized size = 1.07

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3,x]
```

```
[Out] (c^2*Sqrt[c + a^2*c*x^2]*(774144*((-11*I)*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[
a*x]^2 + 10*ArcTanh[(a*x)/Sqrt[1 + a^2*x^2]] + (11*I)*ArcTan[a*x]*PolyLog[2
, (-I)*E^(I*ArcTan[a*x])] - (11*I)*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x
])] - 11*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] + 11*PolyLog[3, I*E^(I*ArcTan[a
*x])]) + 256*((-16407*I)*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2 + 12788*Ar
cTanh[(a*x)/Sqrt[1 + a^2*x^2]] + (16407*I)*ArcTan[a*x]*PolyLog[2, (-I)*E^(I
*ArcTan[a*x])] - (16407*I)*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])] - 16
407*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] + 16407*PolyLog[3, I*E^(I*ArcTan[a*x
])]) - 16128*(1 + a^2*x^2)^(5/2)*((48*a*x)/(1 + a^2*x^2)^2 + 32*ArcTan[a*x]
^3*(-1 + 5*Cos[2*ArcTan[a*x]]) + 6*ArcTan[a*x]*(25 + 36*Cos[2*ArcTan[a*x]]
+ 11*Cos[4*ArcTan[a*x]]) + ArcTan[a*x]^2*(6*Sin[2*ArcTan[a*x]] - 33*Sin[4*A
```

```
rcTan[a*x]])) + 576*(64*((309*I)*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2 -
259*ArcTanh[(a*x)/Sqrt[1 + a^2*x^2]] - (309*I)*ArcTan[a*x]*PolyLog[2, (-I)*
E^(I*ArcTan[a*x])] + (309*I)*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])] +
309*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] - 309*PolyLog[3, I*E^(I*ArcTan[a*x])
]) + (1 + a^2*x^2)^(7/2)*(64*ArcTan[a*x]^3*(57 - 28*Cos[2*ArcTan[a*x]] + 35
*Cos[4*ArcTan[a*x]]) + (8*ArcTan[a*x]*(647 + 764*Cos[2*ArcTan[a*x]] + 309*C
os[4*ArcTan[a*x]]))/(1 + a^2*x^2) + 4*(101*Sin[2*ArcTan[a*x]] + 88*Sin[4*Ar
cTan[a*x]] + 25*Sin[6*ArcTan[a*x]]) - 3*ArcTan[a*x]^2*(211*Sin[2*ArcTan[a*x
]] - 60*Sin[4*ArcTan[a*x]] + 103*Sin[6*ArcTan[a*x]]))) - (1 + a^2*x^2)^(9/2
)*(1536*ArcTan[a*x]^3*(-178 + 711*Cos[2*ArcTan[a*x]] - 126*Cos[4*ArcTan[a*x
]] + 105*Cos[6*ArcTan[a*x]]) + (8*ArcTan[a*x]*(87630 + 153529*Cos[2*ArcTan[
a*x]] + 59266*Cos[4*ArcTan[a*x]] + 16407*Cos[6*ArcTan[a*x]]))/(1 + a^2*x^2)
 + 74932*Sin[2*ArcTan[a*x]] + 77908*Sin[4*ArcTan[a*x]] + 36612*Sin[6*ArcTan
[a*x]] + 3*ArcTan[a*x]^2*(13074*Sin[2*ArcTan[a*x]] - 26742*Sin[4*ArcTan[a*x
]] + 6362*Sin[6*ArcTan[a*x]] - 5469*Sin[8*ArcTan[a*x]]) + 7238*Sin[8*ArcTan
[a*x]])))/(15482880*a^4*Sqrt[1 + a^2*x^2])
```

Maple [A] time = 4.576, size = 525, normalized size = 0.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^3,x)
```

```
[Out] 1/120960*c^2/a^4*(c*(a*x-I)*(a*x+I))^(1/2)*(13440*arctan(a*x)^3*x^8*a^8-504
0*arctan(a*x)^2*x^7*a^7+36480*arctan(a*x)^3*x^6*a^6+1440*arctan(a*x)*x^6*a^
6-12360*arctan(a*x)^2*x^5*a^5+28800*arctan(a*x)^3*x^4*a^4-240*a^5*x^5+3216*
arctan(a*x)*x^4*a^4-6150*arctan(a*x)^2*x^3*a^3+1920*arctan(a*x)^3*x^2*a^2-5
04*a^3*x^3-188*arctan(a*x)*a^2*x^2+6345*arctan(a*x)^2*x*a-3840*arctan(a*x)^
3+850*a*x-12314*arctan(a*x))+115/8064*c^2*(c*(a*x-I)*(a*x+I))^(1/2)*(I*arct
an(a*x)^3-3*arctan(a*x)^2*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I*arctan(a*
x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*polylog(3,-I*(1+I*a*x)/(a^2*
x^2+1)^(1/2)))/a^4/(a^2*x^2+1)^(1/2)-115/8064*c^2*(c*(a*x-I)*(a*x+I))^(1/2)
*(I*arctan(a*x)^3+6*I*arctan(a*x)*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-
3*arctan(a*x)^2*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*polylog(3,I*(1+I*a*x)
/(a^2*x^2+1)^(1/2)))/a^4/(a^2*x^2+1)^(1/2)-1433/7560*I*c^2/a^4*(c*(a*x-I)*(
a*x+I))^(1/2)*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*x^2+1)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^4c^2x^7 + 2\,a^2c^2x^5 + c^2x^3\right)\sqrt{a^2cx^2 + c}\,\arctan\left(ax\right)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^7 + 2*a^2*c^2*x^5 + c^2*x^3)*sqrt(a^2*c*x^2 + c)*arctan
(a*x)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a**2*c*x**2+c)**(5/2)*atan(a*x)**3,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.429 $\int x^2 \left(c+a^2cx^2\right)^{5/2} \tan^{-1}(ax)^3 \, dx$

Optimal. Leaf size=1019

result too large to display

```
[Out] (13*c^2*Sqrt[c + a^2*c*x^2])/(6720*a^3) - (3*c*(c + a^2*c*x^2)^(3/2))/(560*
a^3) - (c + a^2*c*x^2)^(5/2)/(280*a^3) + (43*c^2*x*Sqrt[c + a^2*c*x^2]*ArcT
an[a*x])/(1344*a^2) + (29*c^2*x^3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/560 + (a
^2*c^2*x^5*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/56 + (1373*c^2*Sqrt[c + a^2*c*x
^2]*ArcTan[a*x]^2)/(13440*a^3) - (737*c^2*x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*
x]^2)/(6720*a) - (83*a*c^2*x^4*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/560 - (3*
a^3*c^2*x^6*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/56 + (5*c^2*x*Sqrt[c + a^2*c
*x^2]*ArcTan[a*x]^3)/(128*a^2) + (59*c^2*x^3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x
]^3)/192 + (17*a^2*c^2*x^5*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/48 + (a^4*c^2
*x^7*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/8 + (((5*I)/64)*c^3*Sqrt[1 + a^2*x^
2]*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^3)/(a^3*Sqrt[c + a^2*c*x^2]) + (((
397*I)/840)*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1
 - I*a*x]])/(a^3*Sqrt[c + a^2*c*x^2]) - (((15*I)/128)*c^3*Sqrt[1 + a^2*x^2]
*ArcTan[a*x]^2*PolyLog[2, (-I)*E^(I*ArcTan[a*x])])/(a^3*Sqrt[c + a^2*c*x^2]
) + (((15*I)/128)*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog[2, I*E^(I*Arc
Tan[a*x])])/(a^3*Sqrt[c + a^2*c*x^2]) - (((397*I)/1680)*c^3*Sqrt[1 + a^2*x^
2]*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/(a^3*Sqrt[c + a^2*c*
x^2]) + (((397*I)/1680)*c^3*Sqrt[1 + a^2*x^2]*PolyLog[2, (I*Sqrt[1 + I*a*x]
)/Sqrt[1 - I*a*x]])/(a^3*Sqrt[c + a^2*c*x^2]) + (15*c^3*Sqrt[1 + a^2*x^2]*A
rcTan[a*x]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])])/(64*a^3*Sqrt[c + a^2*c*x^2])
 - (15*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[3, I*E^(I*ArcTan[a*x])])/(
64*a^3*Sqrt[c + a^2*c*x^2]) + (((15*I)/64)*c^3*Sqrt[1 + a^2*x^2]*PolyLog[4,
 (-I)*E^(I*ArcTan[a*x])])/(a^3*Sqrt[c + a^2*c*x^2]) - (((15*I)/64)*c^3*Sqrt
[1 + a^2*x^2]*PolyLog[4, I*E^(I*ArcTan[a*x])])/(a^3*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 15.4173, antiderivative size = 1019, normalized size of antiderivative = 1., number of steps used = 293, number of rules used = 14, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {4950, 4952, 4930, 4890, 4886, 4888, 4181, 2531, 6609, 2282, 6589, 261, 266, 43}

$$\frac{1}{8}a^4c^2\sqrt{a^2cx^2+c}\tan^{-1}(ax)^3x^7 - \frac{3}{56}a^3c^2\sqrt{a^2cx^2+c}\tan^{-1}(ax)^2x^6 + \frac{17}{48}a^2c^2\sqrt{a^2cx^2+c}\tan^{-1}(ax)^3x^5 + \frac{1}{56}a^2c^2\sqrt{a^2cx^2+}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3,x]
```

```
[Out] (13*c^2*Sqrt[c + a^2*c*x^2])/(6720*a^3) - (3*c*(c + a^2*c*x^2)^(3/2))/(560*
a^3) - (c + a^2*c*x^2)^(5/2)/(280*a^3) + (43*c^2*x*Sqrt[c + a^2*c*x^2]*ArcT
an[a*x])/(1344*a^2) + (29*c^2*x^3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/560 + (a
^2*c^2*x^5*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/56 + (1373*c^2*Sqrt[c + a^2*c*x
^2]*ArcTan[a*x]^2)/(13440*a^3) - (737*c^2*x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*
x]^2)/(6720*a) - (83*a*c^2*x^4*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/560 - (3*
a^3*c^2*x^6*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/56 + (5*c^2*x*Sqrt[c + a^2*c
*x^2]*ArcTan[a*x]^3)/(128*a^2) + (59*c^2*x^3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x
]^3)/192 + (17*a^2*c^2*x^5*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/48 + (a^4*c^2
*x^7*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/8 + (((5*I)/64)*c^3*Sqrt[1 + a^2*x^
2]*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^3)/(a^3*Sqrt[c + a^2*c*x^2]) + (((
397*I)/840)*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1
 - I*a*x]])/(a^3*Sqrt[c + a^2*c*x^2]) - (((15*I)/128)*c^3*Sqrt[1 + a^2*x^2]
*ArcTan[a*x]^2*PolyLog[2, (-I)*E^(I*ArcTan[a*x])])/(a^3*Sqrt[c + a^2*c*x^2]
) + (((15*I)/128)*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog[2, I*E^(I*Arc
Tan[a*x])])/(a^3*Sqrt[c + a^2*c*x^2]) - (((397*I)/1680)*c^3*Sqrt[1 + a^2*x^
2]*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/(a^3*Sqrt[c + a^2*c*
x^2]) + (((397*I)/1680)*c^3*Sqrt[1 + a^2*x^2]*PolyLog[2, (I*Sqrt[1 + I*a*x]
)/Sqrt[1 - I*a*x]])/(a^3*Sqrt[c + a^2*c*x^2]) + (15*c^3*Sqrt[1 + a^2*x^2]*A
rcTan[a*x]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])])/(64*a^3*Sqrt[c + a^2*c*x^2])
 - (15*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[3, I*E^(I*ArcTan[a*x])])/(
64*a^3*Sqrt[c + a^2*c*x^2]) + (((15*I)/64)*c^3*Sqrt[1 + a^2*x^2]*PolyLog[4,
 (-I)*E^(I*ArcTan[a*x])])/(a^3*Sqrt[c + a^2*c*x^2]) - (((15*I)/64)*c^3*Sqrt
[1 + a^2*x^2]*PolyLog[4, I*E^(I*ArcTan[a*x])])/(a^3*Sqrt[c + a^2*c*x^2])
```

Rule 4950

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 4952

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b*
ArcTan[c*x])^p)/(c^2*d*m), x] + (-Dist[(b*f*p)/(c*m), Int[((f*x)^(m - 1)*(a
 + b*ArcTan[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] - Dist[(f^2*(m - 1))/(c^
2*m), Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/Sqrt[d + e*x^2], x], x]) /;
 FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 4890

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4886

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[(-2*I*(a + b*ArcTan[c*x])*ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]])
/(c*Sqrt[d]), x] + (Simp[(I*b*PolyLog[2, -((I*Sqrt[1 + I*c*x])/Sqrt[1 - I*c
*x])])/(c*Sqrt[d]), x] - Simp[(I*b*PolyLog[2, (I*Sqrt[1 + I*c*x])/Sqrt[1 -
I*c*x]])/(c*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

Rule 4888

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[c
*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && GtQ
[d, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
```

```
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x^2 \left(c+a^2cx^2\right)^{5/2} \tan^{-1}(ax)^3 \, dx = c \int x^2 \left(c+a^2cx^2\right)^{3/2} \tan^{-1}(ax)^3 \, dx + \left(a^2c\right) \int x^4 \left(c+a^2cx^2\right)^{3/2} \tan^{-1}(ax)^3 \, dx$$

$$= c^2 \int x^2 \sqrt{c+a^2cx^2} \tan^{-1}(ax)^3 \, dx + 2\left(\left(a^2c^2\right) \int x^4 \sqrt{c+a^2cx^2} \tan^{-1}(ax)^3 \, dx\right) + \left(a^4c\right.$$

$$= c^3 \int \frac{x^2 \tan^{-1}(ax)^3}{\sqrt{c+a^2cx^2}} \, dx + \left(a^2c^3\right) \int \frac{x^4 \tan^{-1}(ax)^3}{\sqrt{c+a^2cx^2}} \, dx + \left(a^4c^3\right) \int \frac{x^6 \tan^{-1}(ax)^3}{\sqrt{c+a^2cx^2}} \, dx +$$

$$= \frac{c^2 x\sqrt{c+a^2cx^2} \tan^{-1}(ax)^3}{2a^2} + \frac{1}{4}c^2x^3\sqrt{c+a^2cx^2} \tan^{-1}(ax)^3 + \frac{1}{6}a^2c^2x^5\sqrt{c+a^2cx^2} \tan$$

$$= -\frac{3c^2\sqrt{c+a^2cx^2} \tan^{-1}(ax)^2}{2a^3} - \frac{c^2x^2\sqrt{c+a^2cx^2} \tan^{-1}(ax)^2}{4a} - \frac{1}{10}ac^2x^4\sqrt{c+a^2cx^2} \tan$$

$$= \frac{c^2 x\sqrt{c+a^2cx^2} \tan^{-1}(ax)}{4a^2} + \frac{1}{20}c^2x^3\sqrt{c+a^2cx^2} \tan^{-1}(ax) + \frac{1}{56}a^2c^2x^5\sqrt{c+a^2cx^2} \tan$$

$$= -\frac{c^2\sqrt{c+a^2cx^2}}{4a^3} - \frac{c^2x\sqrt{c+a^2cx^2} \tan^{-1}(ax)}{6a^2} - \frac{27}{560}c^2x^3\sqrt{c+a^2cx^2} \tan^{-1}(ax) + \frac{1}{56}a^2$$

$$= \frac{c^2\sqrt{c+a^2cx^2}}{6a^3} + \frac{491c^2x\sqrt{c+a^2cx^2} \tan^{-1}(ax)}{1344a^2} - \frac{27}{560}c^2x^3\sqrt{c+a^2cx^2} \tan^{-1}(ax) + \frac{1}{56}$$

$$= -\frac{2239c^2\sqrt{c+a^2cx^2}}{6720a^3} - \frac{c\left(c+a^2cx^2\right)^{3/2}}{210a^3} - \frac{\left(c+a^2cx^2\right)^{5/2}}{280a^3} + \frac{491c^2x\sqrt{c+a^2cx^2} \tan^{-1}(}{1344a^2}$$

$$= -\frac{2899c^2\sqrt{c+a^2cx^2}}{6720a^3} + \frac{47c\left(c+a^2cx^2\right)^{3/2}}{1680a^3} - \frac{\left(c+a^2cx^2\right)^{5/2}}{280a^3} + \frac{491c^2x\sqrt{c+a^2cx^2} \tan^{-}}{1344a^2}$$

$$= -\frac{2899c^2\sqrt{c+a^2cx^2}}{6720a^3} + \frac{47c\left(c+a^2cx^2\right)^{3/2}}{1680a^3} - \frac{\left(c+a^2cx^2\right)^{5/2}}{280a^3} + \frac{491c^2x\sqrt{c+a^2cx^2} \tan^{-}}{1344a^2}$$

$$= -\frac{2899c^2\sqrt{c+a^2cx^2}}{6720a^3} + \frac{47c\left(c+a^2cx^2\right)^{3/2}}{1680a^3} - \frac{\left(c+a^2cx^2\right)^{5/2}}{280a^3} + \frac{491c^2x\sqrt{c+a^2cx^2} \tan^{-}}{1344a^2}$$

$$= -\frac{2899c^2\sqrt{c+a^2cx^2}}{6720a^3} + \frac{47c\left(c+a^2cx^2\right)^{3/2}}{1680a^3} - \frac{\left(c+a^2cx^2\right)^{5/2}}{280a^3} + \frac{491c^2x\sqrt{c+a^2cx^2} \tan^{-}}{1344a^2}$$

$$= -\frac{2899c^2\sqrt{c+a^2cx^2}}{6720a^3} + \frac{47c\left(c+a^2cx^2\right)^{3/2}}{1680a^3} - \frac{\left(c+a^2cx^2\right)^{5/2}}{280a^3} + \frac{491c^2x\sqrt{c+a^2cx^2} \tan^{-}}{1344a^2}$$

Mathematica [B] time = 24.4021, size = 6517, normalized size = 6.4

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3,x]
```

```
[Out] Result too large to show
```

Maple [A] time = 2.773, size = 566, normalized size = 0.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^3,x)
```

```
[Out] 1/13440*c^2/a^3*(c*(a*x-I)*(a*x+I))^(1/2)*(1680*arctan(a*x)^3*x^7*a^7-720*a
rctan(a*x)^2*x^6*a^6+4760*arctan(a*x)^3*a^5*x^5+240*arctan(a*x)*x^5*a^5-199
2*arctan(a*x)^2*x^4*a^4+4130*arctan(a*x)^3*a^3*x^3-48*a^4*x^4+696*arctan(a*
x)*x^3*a^3-1474*arctan(a*x)^2*x^2*a^2+525*arctan(a*x)^3*a*x-168*a^2*x^2+430
*arctan(a*x)*x*a+1373*arctan(a*x)^2-94)+1/13440*(c*(a*x-I)*(a*x+I))^(1/2)/(
a^2*x^2+1)^(1/2)/a^3*(525*arctan(a*x)^3*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))
-525*arctan(a*x)^3*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-1575*I*arctan(a*x)^2
*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+1575*I*arctan(a*x)^2*polylog(2,I
*(1+I*a*x)/(a^2*x^2+1)^(1/2))+3176*arctan(a*x)*ln(1+I*(1+I*a*x)/(a^2*x^2+1)
^(1/2))+3150*arctan(a*x)*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-3176*arc
tan(a*x)*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-3150*arctan(a*x)*polylog(3,I*(
1+I*a*x)/(a^2*x^2+1)^(1/2))+3150*I*polylog(4,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2)
)-3150*I*polylog(4,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-3176*I*dilog(1+I*(1+I*a*x
)/(a^2*x^2+1)^(1/2))+3176*I*dilog(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))*c^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^4c^2x^6+2\,a^2c^2x^4+c^2x^2\right)\sqrt{a^2cx^2+c}\,\arctan\left(ax\right)^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^6 + 2*a^2*c^2*x^4 + c^2*x^2)*sqrt(a^2*c*x^2 + c)*arctan
(a*x)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a**2*c*x**2+c)**(5/2)*atan(a*x)**3,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.430 $\int x\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^3\,dx$

Optimal. Leaf size=561

$$-\frac{15ic^3\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,-ie^{i\tan^{-1}(ax)}\right)}{56a^2\sqrt{a^2cx^2+c}}+\frac{15ic^3\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,ie^{i\tan^{-1}(ax)}\right)}{56a^2\sqrt{a^2cx^2+c}}+\frac{15c^3\sqrt{a^2x}}{}$$

```
[Out] (-17*c^2*x*Sqrt[c + a^2*c*x^2])/(420*a) - (c*x*(c + a^2*c*x^2)^(3/2))/(140*
a) + (15*c^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(56*a^2) + (5*c*(c + a^2*c*x^
2)^(3/2)*ArcTan[a*x])/(84*a^2) + ((c + a^2*c*x^2)^(5/2)*ArcTan[a*x])/(35*a^
2) - (15*c^2*x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(112*a) - (5*c*x*(c + a^2
*c*x^2)^(3/2)*ArcTan[a*x]^2)/(56*a) - (x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^
2)/(14*a) + (((15*I)/56)*c^3*Sqrt[1 + a^2*x^2]*ArcTan[E^(I*ArcTan[a*x])]*Ar
cTan[a*x]^2)/(a^2*Sqrt[c + a^2*c*x^2]) + ((c + a^2*c*x^2)^(7/2)*ArcTan[a*x]
^3)/(7*a^2*c) - (37*c^(5/2)*ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]])/(12
0*a^2) - (((15*I)/56)*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, (-I)*E^(
I*ArcTan[a*x])])/(a^2*Sqrt[c + a^2*c*x^2]) + (((15*I)/56)*c^3*Sqrt[1 + a^2*
x^2]*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])])/(a^2*Sqrt[c + a^2*c*x^2])
 + (15*c^3*Sqrt[1 + a^2*x^2]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])])/(56*a^2*Sq
rt[c + a^2*c*x^2]) - (15*c^3*Sqrt[1 + a^2*x^2]*PolyLog[3, I*E^(I*ArcTan[a*x
])])/(56*a^2*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.531382, antiderivative size = 561, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 11, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4930, 4880, 4890, 4888, 4181, 2531, 2282, 6589, 217, 206, 195}

$$-\frac{15ic^3\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,-ie^{i\tan^{-1}(ax)}\right)}{56a^2\sqrt{a^2cx^2+c}}+\frac{15ic^3\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,ie^{i\tan^{-1}(ax)}\right)}{56a^2\sqrt{a^2cx^2+c}}+\frac{15c^3\sqrt{a^2x}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3,x]
```


```
[Out] (-17*c^2*x*Sqrt[c + a^2*c*x^2])/(420*a) - (c*x*(c + a^2*c*x^2)^(3/2))/(140*
a) + (15*c^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(56*a^2) + (5*c*(c + a^2*c*x^
2)^(3/2)*ArcTan[a*x])/(84*a^2) + ((c + a^2*c*x^2)^(5/2)*ArcTan[a*x])/(35*a^
2) - (15*c^2*x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(112*a) - (5*c*x*(c + a^2
*c*x^2)^(3/2)*ArcTan[a*x]^2)/(56*a) - (x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^
2)/(14*a) + (((15*I)/56)*c^3*Sqrt[1 + a^2*x^2]*ArcTan[E^(I*ArcTan[a*x])]*Ar
cTan[a*x]^2)/(a^2*Sqrt[c + a^2*c*x^2]) + ((c + a^2*c*x^2)^(7/2)*ArcTan[a*x]
```

```
^3)/(7*a^2*c) - (37*c^(5/2)*ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]])/(12
0*a^2) - (((15*I)/56)*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, (-I)*E^(
I*ArcTan[a*x])])/(a^2*Sqrt[c + a^2*c*x^2]) + (((15*I)/56)*c^3*Sqrt[1 + a^2*
x^2]*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])])/(a^2*Sqrt[c + a^2*c*x^2])
 + (15*c^3*Sqrt[1 + a^2*x^2]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])])/(56*a^2*Sq
rt[c + a^2*c*x^2]) - (15*c^3*Sqrt[1 + a^2*x^2]*PolyLog[3, I*E^(I*ArcTan[a*x
])])/(56*a^2*Sqrt[c + a^2*c*x^2])
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 4880

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> -Simp[(b*p*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1))/(2*c*q*(2*
q + 1)), x] + (Dist[(2*d*q)/(2*q + 1), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTa
n[c*x])^p, x], x] + Dist[(b^2*d*p*(p - 1))/(2*q*(2*q + 1)), Int[(d + e*x^2)
^(q - 1)*(a + b*ArcTan[c*x])^(p - 2), x], x] + Simp[(x*(d + e*x^2)^q*(a + b
*ArcTan[c*x])^p)/(2*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[q, 0] && GtQ[p, 1]
```

Rule 4890

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4888

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[c
*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && GtQ
[d, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
```

```
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rubi steps

$$\int x\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^3\,dx=\frac{\left(c+a^2cx^2\right)^{7/2}\tan^{-1}(ax)^3}{7a^2c}-\frac{3\int\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^2\,dx}{7a}$$
$$=\frac{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)}{35a^2}-\frac{x\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^2}{14a}+\frac{\left(c+a^2cx^2\right)^{7/2}\tan^{-1}(ax)^3}{7a^2c}-$$
$$=-\frac{cx\left(c+a^2cx^2\right)^{3/2}}{140a}+\frac{5c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}{84a^2}+\frac{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)}{35a^2}-\frac{5cx\left(c\right.}{}$$
$$=-\frac{17c^2x\sqrt{c+a^2cx^2}}{420a}-\frac{cx\left(c+a^2cx^2\right)^{3/2}}{140a}+\frac{15c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{56a^2}+\frac{5c\left(c+a^2cx^2\right)^3}{84a}$$
$$=-\frac{17c^2x\sqrt{c+a^2cx^2}}{420a}-\frac{cx\left(c+a^2cx^2\right)^{3/2}}{140a}+\frac{15c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{56a^2}+\frac{5c\left(c+a^2cx^2\right)^3}{84a}$$
$$=-\frac{17c^2x\sqrt{c+a^2cx^2}}{420a}-\frac{cx\left(c+a^2cx^2\right)^{3/2}}{140a}+\frac{15c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{56a^2}+\frac{5c\left(c+a^2cx^2\right)^3}{84a}$$
$$=-\frac{17c^2x\sqrt{c+a^2cx^2}}{420a}-\frac{cx\left(c+a^2cx^2\right)^{3/2}}{140a}+\frac{15c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{56a^2}+\frac{5c\left(c+a^2cx^2\right)^3}{84a}$$
$$=-\frac{17c^2x\sqrt{c+a^2cx^2}}{420a}-\frac{cx\left(c+a^2cx^2\right)^{3/2}}{140a}+\frac{15c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{56a^2}+\frac{5c\left(c+a^2cx^2\right)^3}{84a}$$
$$=-\frac{17c^2x\sqrt{c+a^2cx^2}}{420a}-\frac{cx\left(c+a^2cx^2\right)^{3/2}}{140a}+\frac{15c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{56a^2}+\frac{5c\left(c+a^2cx^2\right)^3}{84a}$$
$$=-\frac{17c^2x\sqrt{c+a^2cx^2}}{420a}-\frac{cx\left(c+a^2cx^2\right)^{3/2}}{140a}+\frac{15c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{56a^2}+\frac{5c\left(c+a^2cx^2\right)^3}{84a}$$

Mathematica [A] time = 5.54493, size = 718, normalized size = 1.28

$$\frac{c^2\sqrt{a^2cx^2+c}\left(64\left(-309i\tan^{-1}(ax)\text{PolyLog}\left(2,-ie^{i\tan^{-1}(ax)}\right)+309i\tan^{-1}(ax)\text{PolyLog}\left(2,ie^{i\tan^{-1}(ax)}\right)+309\text{PolyLog}\left(3\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3,x]
```

```
[Out] (c^2*Sqrt[c + a^2*c*x^2]*(64*((309*I)*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]
^2 - 259*ArcTanh[(a*x)/Sqrt[1 + a^2*x^2]] - (309*I)*ArcTan[a*x]*PolyLog[2,
(-I)*E^(I*ArcTan[a*x])] + (309*I)*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x]
)] + 309*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] - 309*PolyLog[3, I*E^(I*ArcTan[
a*x])]) + 53760*(I*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2 - ArcTanh[(a*x)/
Sqrt[1 + a^2*x^2]] - I*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] + I*A
rcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])] + PolyLog[3, (-I)*E^(I*ArcTan[a*
x])] - PolyLog[3, I*E^(I*ArcTan[a*x])]) + 4480*(1 + a^2*x^2)^(3/2)*ArcTan[a
*x]*(6 + 4*ArcTan[a*x]^2 + 6*Cos[2*ArcTan[a*x]] - 3*ArcTan[a*x]*Sin[2*ArcTa
n[a*x]]) - 112*(48*((11*I)*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2 - 10*Arc
Tanh[(a*x)/Sqrt[1 + a^2*x^2]] - (11*I)*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*Arc
Tan[a*x])] + (11*I)*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])] + 11*PolyLo
g[3, (-I)*E^(I*ArcTan[a*x])] - 11*PolyLog[3, I*E^(I*ArcTan[a*x])]) + (1 + a
^2*x^2)^(5/2)*((48*a*x)/(1 + a^2*x^2)^2 + 32*ArcTan[a*x]^3*(-1 + 5*Cos[2*Ar
cTan[a*x]]) + 6*ArcTan[a*x]*(25 + 36*Cos[2*ArcTan[a*x]] + 11*Cos[4*ArcTan[a
*x]]) + ArcTan[a*x]^2*(6*Sin[2*ArcTan[a*x]] - 33*Sin[4*ArcTan[a*x]]))) + (1
 + a^2*x^2)^(7/2)*(64*ArcTan[a*x]^3*(57 - 28*Cos[2*ArcTan[a*x]] + 35*Cos[4*
ArcTan[a*x]]) + (8*ArcTan[a*x]*(647 + 764*Cos[2*ArcTan[a*x]] + 309*Cos[4*Ar
cTan[a*x]]))/(1 + a^2*x^2) + 4*(101*Sin[2*ArcTan[a*x]] + 88*Sin[4*ArcTan[a*
x]] + 25*Sin[6*ArcTan[a*x]]) - 3*ArcTan[a*x]^2*(211*Sin[2*ArcTan[a*x]] - 60
*Sin[4*ArcTan[a*x]] + 103*Sin[6*ArcTan[a*x]]))))/(53760*a^2*Sqrt[1 + a^2*x^
2])
```

Maple [A] time = 1.486, size = 477, normalized size = 0.9

$$\frac{c^2 \left(240 (\arctan(ax))^3 x^6a^6 - 120 (\arctan(ax))^2 x^5a^5 + 720 (\arctan(ax))^3 x^4a^4 + 48 \arctan(ax) x^4a^4 - 390 (\arctan\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^3,x)
```

```
[Out] 1/1680*c^2/a^2*(c*(a*x-I)*(a*x+I))^(1/2)*(240*arctan(a*x)^3*x^6*a^6-120*arc
tan(a*x)^2*x^5*a^5+720*arctan(a*x)^3*x^4*a^4+48*arctan(a*x)*x^4*a^4-390*arc
tan(a*x)^2*x^3*a^3+720*arctan(a*x)^3*x^2*a^2-12*a^3*x^3+196*arctan(a*x)*a^2
*x^2-495*arctan(a*x)^2*x*a+240*arctan(a*x)^3-80*a*x+598*arctan(a*x))-5/112*
c^2*(c*(a*x-I)*(a*x+I))^(1/2)*(I*arctan(a*x)^3-3*arctan(a*x)^2*ln(1+I*(1+I*
a*x)/(a^2*x^2+1)^(1/2))+6*I*arctan(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^
(1/2))-6*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))/a^2/(a^2*x^2+1)^(1/2)+5
/112*c^2*(c*(a*x-I)*(a*x+I))^(1/2)*(I*arctan(a*x)^3+6*I*arctan(a*x)*polylog
(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-3*arctan(a*x)^2*ln(1-I*(1+I*a*x)/(a^2*x^2
+1)^(1/2))-6*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))/a^2/(a^2*x^2+1)^(1/2
```

```
)+37/60*I*c^2/a^2*(c*(a*x-I)*(a*x+I))^(1/2)*arctan((1+I*a*x)/(a^2*x^2+1)^(1
/2))/(a^2*x^2+1)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2+c\right)^{\frac{5}{2}}x\arctan\left(ax\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(5/2)*x*arctan(a*x)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^4c^2x^5+2\,a^2c^2x^3+c^2x\right)\sqrt{a^2cx^2+c}\arctan\left(ax\right)^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^5 + 2*a^2*c^2*x^3 + c^2*x)*sqrt(a^2*c*x^2 + c)*arctan(a
*x)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**(5/2)*atan(a*x)**3,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.431 $\int \left(c+a^2cx^2\right)^{5/2} \tan^{-1}(ax)^3 \, dx$

Optimal. Leaf size=870

$$-\frac{5i\sqrt{a^2x^2+1}\tan^{-1}\left(e^{i\tan^{-1}(ax)}\right)\tan^{-1}(ax)^3c^3}{8a\sqrt{a^2cx^2+c}}-\frac{259i\sqrt{a^2x^2+1}\tan^{-1}(ax)\tan^{-1}\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)c^3}{60a\sqrt{a^2cx^2+c}}+\frac{15i\sqrt{a^2x^2+1}\tan^{-1}(ax)^2\mathrm{P}}{16a\sqrt{a}}$$

```
[Out] (-17*c^2*Sqrt[c + a^2*c*x^2])/(60*a) - (c*(c + a^2*c*x^2)^(3/2))/(60*a) + (
17*c^2*x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/60 + (c*x*(c + a^2*c*x^2)^(3/2)*A
rcTan[a*x])/20 - (15*c^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(16*a) - (5*c*(
c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2)/(24*a) - ((c + a^2*c*x^2)^(5/2)*ArcTan[
a*x]^2)/(10*a) + (5*c^2*x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/16 + (5*c*x*(c
 + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3)/24 + (x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x
]^3)/6 - (((5*I)/8)*c^3*Sqrt[1 + a^2*x^2]*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[
a*x]^3)/(a*Sqrt[c + a^2*c*x^2]) - (((259*I)/60)*c^3*Sqrt[1 + a^2*x^2]*ArcTa
n[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/(a*Sqrt[c + a^2*c*x^2]) + (
((15*I)/16)*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog[2, (-I)*E^(I*ArcTan
[a*x])])/(a*Sqrt[c + a^2*c*x^2]) - (((15*I)/16)*c^3*Sqrt[1 + a^2*x^2]*ArcTa
n[a*x]^2*PolyLog[2, I*E^(I*ArcTan[a*x])])/(a*Sqrt[c + a^2*c*x^2]) + (((259*
I)/120)*c^3*Sqrt[1 + a^2*x^2]*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*
a*x]])/(a*Sqrt[c + a^2*c*x^2]) - (((259*I)/120)*c^3*Sqrt[1 + a^2*x^2]*PolyL
og[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/(a*Sqrt[c + a^2*c*x^2]) - (15*c
^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])])/(8*a*S
qrt[c + a^2*c*x^2]) + (15*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[3, I*E^
(I*ArcTan[a*x])])/(8*a*Sqrt[c + a^2*c*x^2]) - (((15*I)/8)*c^3*Sqrt[1 + a^2*
x^2]*PolyLog[4, (-I)*E^(I*ArcTan[a*x])])/(a*Sqrt[c + a^2*c*x^2]) + (((15*I)
/8)*c^3*Sqrt[1 + a^2*x^2]*PolyLog[4, I*E^(I*ArcTan[a*x])])/(a*Sqrt[c + a^2*
c*x^2])
```

Rubi [A] time = 0.792093, antiderivative size = 870, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {4880, 4890, 4888, 4181, 2531, 6609, 2282, 6589, 4886, 4878}

$$-\frac{5i\sqrt{a^2x^2+1}\tan^{-1}\left(e^{i\tan^{-1}(ax)}\right)\tan^{-1}(ax)^3c^3}{8a\sqrt{a^2cx^2+c}}-\frac{259i\sqrt{a^2x^2+1}\tan^{-1}(ax)\tan^{-1}\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)c^3}{60a\sqrt{a^2cx^2+c}}+\frac{15i\sqrt{a^2x^2+1}\tan^{-1}(ax)^2\mathrm{P}}{16a\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3,x]
```

```
[Out] (-17*c^2*Sqrt[c + a^2*c*x^2])/(60*a) - (c*(c + a^2*c*x^2)^(3/2))/(60*a) + (
17*c^2*x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/60 + (c*x*(c + a^2*c*x^2)^(3/2)*A
rcTan[a*x])/20 - (15*c^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(16*a) - (5*c*(
c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2)/(24*a) - ((c + a^2*c*x^2)^(5/2)*ArcTan[
a*x]^2)/(10*a) + (5*c^2*x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/16 + (5*c*x*(c
 + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3)/24 + (x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x
]^3)/6 - (((5*I)/8)*c^3*Sqrt[1 + a^2*x^2]*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[
a*x]^3)/(a*Sqrt[c + a^2*c*x^2]) - (((259*I)/60)*c^3*Sqrt[1 + a^2*x^2]*ArcTa
n[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/(a*Sqrt[c + a^2*c*x^2]) + (
((15*I)/16)*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog[2, (-I)*E^(I*ArcTan
[a*x])])/(a*Sqrt[c + a^2*c*x^2]) - (((15*I)/16)*c^3*Sqrt[1 + a^2*x^2]*ArcTa
n[a*x]^2*PolyLog[2, I*E^(I*ArcTan[a*x])])/(a*Sqrt[c + a^2*c*x^2]) + (((259*
I)/120)*c^3*Sqrt[1 + a^2*x^2]*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*
a*x]])/(a*Sqrt[c + a^2*c*x^2]) - (((259*I)/120)*c^3*Sqrt[1 + a^2*x^2]*PolyL
og[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/(a*Sqrt[c + a^2*c*x^2]) - (15*c
^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])])/(8*a*S
qrt[c + a^2*c*x^2]) + (15*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[3, I*E^
(I*ArcTan[a*x])])/(8*a*Sqrt[c + a^2*c*x^2]) - (((15*I)/8)*c^3*Sqrt[1 + a^2*
x^2]*PolyLog[4, (-I)*E^(I*ArcTan[a*x])])/(a*Sqrt[c + a^2*c*x^2]) + (((15*I)
/8)*c^3*Sqrt[1 + a^2*x^2]*PolyLog[4, I*E^(I*ArcTan[a*x])])/(a*Sqrt[c + a^2*
c*x^2])
```

Rule 4880

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> -Simp[(b*p*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1))/(2*c*q*(2*
q + 1)), x] + (Dist[(2*d*q)/(2*q + 1), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTa
n[c*x])^p, x], x] + Dist[(b^2*d*p*(p - 1))/(2*q*(2*q + 1)), Int[(d + e*x^2)
^(q - 1)*(a + b*ArcTan[c*x])^(p - 2), x], x] + Simp[(x*(d + e*x^2)^q*(a + b
*ArcTan[c*x])^p)/(2*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[q, 0] && GtQ[p, 1]
```

Rule 4890

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4888

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[c
*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && GtQ
[d, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4886

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[(-2*I*(a + b*ArcTan[c*x])*ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]])
/(c*Sqrt[d]), x] + (Simp[(I*b*PolyLog[2, -((I*Sqrt[1 + I*c*x])/Sqrt[1 - I*c
*x])])/(c*Sqrt[d]), x] - Simp[(I*b*PolyLog[2, (I*Sqrt[1 + I*c*x])/Sqrt[1 -
```

```
I*c*x]])/(c*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

Rule 4878

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symbo
l] :> -Simp[(b*(d + e*x^2)^q)/(2*c*q*(2*q + 1)), x] + (Dist[(2*d*q)/(2*q +
1), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x]), x], x] + Simp[(x*(d + e*x^
2)^q*(a + b*ArcTan[c*x]))/(2*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] && Eq
Q[e, c^2*d] && GtQ[q, 0]
```

Rubi steps

$$\int \left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^3\,dx = -\frac{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^2}{10a}+\frac{1}{6}x\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^3+\frac{1}{5}c\int\left(c+a^2cx^2\right)^{3/2}\tan$$

$$= -\frac{c\left(c+a^2cx^2\right)^{3/2}}{60a}+\frac{1}{20}cx\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)-\frac{5c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2}{24a}-\frac{(c}{}$$

$$= -\frac{17c^2\sqrt{c+a^2cx^2}}{60a}-\frac{c\left(c+a^2cx^2\right)^{3/2}}{60a}+\frac{17}{60}c^2x\sqrt{c+a^2cx^2}\tan^{-1}(ax)+\frac{1}{20}cx\left(c+a^2cx^2\right.$$

$$= -\frac{17c^2\sqrt{c+a^2cx^2}}{60a}-\frac{c\left(c+a^2cx^2\right)^{3/2}}{60a}+\frac{17}{60}c^2x\sqrt{c+a^2cx^2}\tan^{-1}(ax)+\frac{1}{20}cx\left(c+a^2cx^2\right.$$

$$= -\frac{17c^2\sqrt{c+a^2cx^2}}{60a}-\frac{c\left(c+a^2cx^2\right)^{3/2}}{60a}+\frac{17}{60}c^2x\sqrt{c+a^2cx^2}\tan^{-1}(ax)+\frac{1}{20}cx\left(c+a^2cx^2\right.$$

$$= -\frac{17c^2\sqrt{c+a^2cx^2}}{60a}-\frac{c\left(c+a^2cx^2\right)^{3/2}}{60a}+\frac{17}{60}c^2x\sqrt{c+a^2cx^2}\tan^{-1}(ax)+\frac{1}{20}cx\left(c+a^2cx^2\right.$$

$$= -\frac{17c^2\sqrt{c+a^2cx^2}}{60a}-\frac{c\left(c+a^2cx^2\right)^{3/2}}{60a}+\frac{17}{60}c^2x\sqrt{c+a^2cx^2}\tan^{-1}(ax)+\frac{1}{20}cx\left(c+a^2cx^2\right.$$

$$= -\frac{17c^2\sqrt{c+a^2cx^2}}{60a}-\frac{c\left(c+a^2cx^2\right)^{3/2}}{60a}+\frac{17}{60}c^2x\sqrt{c+a^2cx^2}\tan^{-1}(ax)+\frac{1}{20}cx\left(c+a^2cx^2\right.$$

$$= -\frac{17c^2\sqrt{c+a^2cx^2}}{60a}-\frac{c\left(c+a^2cx^2\right)^{3/2}}{60a}+\frac{17}{60}c^2x\sqrt{c+a^2cx^2}\tan^{-1}(ax)+\frac{1}{20}cx\left(c+a^2cx^2\right.$$

$$= -\frac{17c^2\sqrt{c+a^2cx^2}}{60a}-\frac{c\left(c+a^2cx^2\right)^{3/2}}{60a}+\frac{17}{60}c^2x\sqrt{c+a^2cx^2}\tan^{-1}(ax)+\frac{1}{20}cx\left(c+a^2cx^2\right.$$

Mathematica [B] time = 18.9039, size = 4281, normalized size = 4.92

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3,x]
```

```
[Out] ((-I/2)*c^2*Sqrt[c*(1 + a^2*x^2)]*(12*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]
 - (3*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2 + I*a*x*Sqrt[1 + a^2*x^2]*ArcTan[a
*x]^3 + 2*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^3 - 3*(2 + ArcTan[a*x]^2)*P
olyLog[2, (-I)*E^(I*ArcTan[a*x])] + 3*(2 + ArcTan[a*x]^2)*PolyLog[2, I*E^(I
*ArcTan[a*x])] - (6*I)*ArcTan[a*x]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] + (6*
I)*ArcTan[a*x]*PolyLog[3, I*E^(I*ArcTan[a*x])] + 6*PolyLog[4, (-I)*E^(I*Arc
Tan[a*x])] - 6*PolyLog[4, I*E^(I*ArcTan[a*x])]))/(a*Sqrt[1 + a^2*x^2]) + (2
*c^2*((Sqrt[c*(1 + a^2*x^2)]*(-1 + ArcTan[a*x]^2))/(4*Sqrt[1 + a^2*x^2]) +
(Sqrt[c*(1 + a^2*x^2)]*(-(ArcTan[a*x]*(Log[1 - I*E^(I*ArcTan[a*x])] - Log[1
 + I*E^(I*ArcTan[a*x])])) - I*(PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - PolyLog
[2, I*E^(I*ArcTan[a*x])])))/(2*Sqrt[1 + a^2*x^2]) + (Sqrt[c*(1 + a^2*x^2)]*
(-(Pi^3*Log[Cot[(Pi/2 - ArcTan[a*x])/2]])/8 - (3*Pi^2*((Pi/2 - ArcTan[a*x])
*(Log[1 - E^(I*(Pi/2 - ArcTan[a*x]))] - Log[1 + E^(I*(Pi/2 - ArcTan[a*x]))]
) + I*(PolyLog[2, -E^(I*(Pi/2 - ArcTan[a*x]))] - PolyLog[2, E^(I*(Pi/2 - Ar
cTan[a*x]))])))/4 + (3*Pi*((Pi/2 - ArcTan[a*x])^2*(Log[1 - E^(I*(Pi/2 - Arc
Tan[a*x]))] - Log[1 + E^(I*(Pi/2 - ArcTan[a*x]))]) + (2*I)*(Pi/2 - ArcTan[a
*x])*(PolyLog[2, -E^(I*(Pi/2 - ArcTan[a*x]))] - PolyLog[2, E^(I*(Pi/2 - Arc
Tan[a*x]))]) + 2*(-PolyLog[3, -E^(I*(Pi/2 - ArcTan[a*x]))] + PolyLog[3, E^(
I*(Pi/2 - ArcTan[a*x]))])))/2 - 8*((I/64)*(Pi/2 - ArcTan[a*x])^4 + (I/4)*(P
i/2 + (-Pi/2 + ArcTan[a*x])/2)^4 - ((Pi/2 - ArcTan[a*x])^3*Log[1 + E^(I*(Pi
/2 - ArcTan[a*x]))])/8 - (Pi^3*(I*(Pi/2 + (-Pi/2 + ArcTan[a*x])/2) - Log[1
+ E^((2*I)*(Pi/2 + (-Pi/2 + ArcTan[a*x])/2))]))/8 - (Pi/2 + (-Pi/2 + ArcTan
[a*x])/2)^3*Log[1 + E^((2*I)*(Pi/2 + (-Pi/2 + ArcTan[a*x])/2))] + ((3*I)/8)
*(Pi/2 - ArcTan[a*x])^2*PolyLog[2, -E^(I*(Pi/2 - ArcTan[a*x]))] + (3*Pi^2*(
(I/2)*(Pi/2 + (-Pi/2 + ArcTan[a*x])/2)^2 - (Pi/2 + (-Pi/2 + ArcTan[a*x])/2)
*Log[1 + E^((2*I)*(Pi/2 + (-Pi/2 + ArcTan[a*x])/2))] + (I/2)*PolyLog[2, -E^
((2*I)*(Pi/2 + (-Pi/2 + ArcTan[a*x])/2))]))/4 + ((3*I)/2)*(Pi/2 + (-Pi/2 +
ArcTan[a*x])/2)^2*PolyLog[2, -E^((2*I)*(Pi/2 + (-Pi/2 + ArcTan[a*x])/2))] -
 (3*(Pi/2 - ArcTan[a*x])*PolyLog[3, -E^(I*(Pi/2 - ArcTan[a*x]))])/4 - (3*Pi
*((I/3)*(Pi/2 + (-Pi/2 + ArcTan[a*x])/2)^3 - (Pi/2 + (-Pi/2 + ArcTan[a*x])/
2)^2*Log[1 + E^((2*I)*(Pi/2 + (-Pi/2 + ArcTan[a*x])/2))] + I*(Pi/2 + (-Pi/2
 + ArcTan[a*x])/2)*PolyLog[2, -E^((2*I)*(Pi/2 + (-Pi/2 + ArcTan[a*x])/2))]
- PolyLog[3, -E^((2*I)*(Pi/2 + (-Pi/2 + ArcTan[a*x])/2))]/2))/2 - (3*(Pi/2
+ (-Pi/2 + ArcTan[a*x])/2)*PolyLog[3, -E^((2*I)*(Pi/2 + (-Pi/2 + ArcTan[a*x
])/2))])/2 - ((3*I)/4)*PolyLog[4, -E^(I*(Pi/2 - ArcTan[a*x]))] - ((3*I)/4)*
PolyLog[4, -E^((2*I)*(Pi/2 + (-Pi/2 + ArcTan[a*x])/2))])))/(8*Sqrt[1 + a^2*
```

x^2]) + (Sqrt[c*(1 + a^2*x^2)]*ArcTan[a*x]^3)/(16*Sqrt[1 + a^2*x^2]*(Cos[ArcTan[a*x]/2] - Sin[ArcTan[a*x]/2])^4) + (Sqrt[c*(1 + a^2*x^2)]*(2*ArcTan[a*x] - ArcTan[a*x]^2 - ArcTan[a*x]^3))/(16*Sqrt[1 + a^2*x^2]*(Cos[ArcTan[a*x]/2] - Sin[ArcTan[a*x]/2])^2) - (Sqrt[c*(1 + a^2*x^2)]*ArcTan[a*x]^2*Sin[ArcTan[a*x]/2])/(8*Sqrt[1 + a^2*x^2]*(Cos[ArcTan[a*x]/2] - Sin[ArcTan[a*x]/2])^3) - (Sqrt[c*(1 + a^2*x^2)]*ArcTan[a*x]^3)/(16*Sqrt[1 + a^2*x^2]*(Cos[ArcTan[a*x]/2] + Sin[ArcTan[a*x]/2])^4) + (Sqrt[c*(1 + a^2*x^2)]*ArcTan[a*x]^2*Sin[ArcTan[a*x]/2])/(8*Sqrt[1 + a^2*x^2]*(Cos[ArcTan[a*x]/2] + Sin[ArcTan[a*x]/2])^3) + (Sqrt[c*(1 + a^2*x^2)]*(-2*ArcTan[a*x] - ArcTan[a*x]^2 + ArcTan[a*x]^3))/(16*Sqrt[1 + a^2*x^2]*(Cos[ArcTan[a*x]/2] + Sin[ArcTan[a*x]/2])^2) + (Sqrt[c*(1 + a^2*x^2)]*(Sin[ArcTan[a*x]/2] - ArcTan[a*x]^2*Sin[ArcTan[a*x]/2]))/(4*Sqrt[1 + a^2*x^2]*(Cos[ArcTan[a*x]/2] + Sin[ArcTan[a*x]/2])) + (Sqrt[c*(1 + a^2*x^2)]*(-Sin[ArcTan[a*x]/2] + ArcTan[a*x]^2*Sin[ArcTan[a*x]/2]))/(4*Sqrt[1 + a^2*x^2]*(Cos[ArcTan[a*x]/2] - Sin[ArcTan[a*x]/2]))))/a + (c^2*((Sqrt[c*(1 + a^2*x^2)]*(50 - 19*ArcTan[a*x]^2))/(240*Sqrt[1 + a^2*x^2]) + (19*Sqrt[c*(1 + a^2*x^2)]*(ArcTan[a*x]*(Log[1 - I*E^(I*ArcTan[a*x])] - Log[1 + I*E^(I*ArcTan[a*x])]) + I*(PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - PolyLog[2, I*E^(I*ArcTan[a*x])])))/(120*Sqrt[1 + a^2*x^2]) + (Sqrt[c*(1 + a^2*x^2)]*((Pi^3*Log[Cot[(Pi/2 - ArcTan[a*x])/2]])/8 + (3*Pi^2*((Pi/2 - ArcTan[a*x])*(Log[1 - E^(I*(Pi/2 - ArcTan[a*x]))] - Log[1 + E^(I*(Pi/2 - ArcTan[a*x]))]) + I*(PolyLog[2, -E^(I*(Pi/2 - ArcTan[a*x]))] - PolyLog[2, E^(I*(Pi/2 - ArcTan[a*x]))])))/4 - (3*Pi*((Pi/2 - ArcTan[a*x])^2*(Log[1 - E^(I*(Pi/2 - ArcTan[a*x]))] - Log[1 + E^(I*(Pi/2 - ArcTan[a*x]))]) + (2*I)*(Pi/2 - ArcTan[a*x])*(PolyLog[2, -E^(I*(Pi/2 - ArcTan[a*x]))] - PolyLog[2, E^(I*(Pi/2 - ArcTan[a*x]))]) + 2*(-PolyLog[3, -E^(I*(Pi/2 - ArcTan[a*x]))] + PolyLog[3, E^(I*(Pi/2 - ArcTan[a*x]))])))/2 + 8*((I/64)*(Pi/2 - ArcTan[a*x])^4 + (I/4)*(Pi/2 + (-Pi/2 + ArcTan[a*x])/2)^4 - ((Pi/2 - ArcTan[a*x])^3*Log[1 + E^(I*(Pi/2 - ArcTan[a*x]))])/8 - (Pi^3*(I*(Pi/2 + (-Pi/2 + ArcTan[a*x])/2) - Log[1 + E^((2*I)*(Pi/2 + (-Pi/2 + ArcTan[a*x])/2))]))/8 - (Pi/2 + (-Pi/2 + ArcTan[a*x])/2)^3*Log[1 + E^((2*I)*(Pi/2 + (-Pi/2 + ArcTan[a*x])/2))] + ((3*I)/8)*(Pi/2 - ArcTan[a*x])^2*PolyLog[2, -E^(I*(Pi/2 - ArcTan[a*x]))] + (3*Pi^2*((I/2)*(Pi/2 + (-Pi/2 + ArcTan[a*x])/2)^2 - (Pi/2 + (-Pi/2 + ArcTan[a*x])/2)*Log[1 + E^((2*I)*(Pi/2 + (-Pi/2 + ArcTan[a*x])/2))] + (I/2)*PolyLog[2, -E^((2*I)*(Pi/2 + (-Pi/2 + ArcTan[a*x])/2))]))/4 + ((3*I)/2)*(Pi/2 + (-Pi/2 + ArcTan[a*x])/2)^2*PolyLog[2, -E^((2*I)*(Pi/2 + (-Pi/2 + ArcTan[a*x])/2))] - (3*(Pi/2 - ArcTan[a*x])*PolyLog[3, -E^(I*(Pi/2 - ArcTan[a*x]))])/4 - (3*Pi*((I/3)*(Pi/2 + (-Pi/2 + ArcTan[a*x])/2)^3 - (Pi/2 + (-Pi/2 + ArcTan[a*x])/2)^2*Log[1 + E^((2*I)*(Pi/2 + (-Pi/2 + ArcTan[a*x])/2))] + I*(Pi/2 + (-Pi/2 + ArcTan[a*x])/2)*PolyLog[2, -E^((2*I)*(Pi/2 + (-Pi/2 + ArcTan[a*x])/2))] - PolyLog[3, -E^((2*I)*(Pi/2 + (-Pi/2 + ArcTan[a*x])/2))]/2))/2 - (3*(Pi/2 + (-Pi/2 + ArcTan[a*x])/2)*PolyLog[3, -E^((2*I)*(Pi/2 + (-Pi/2 + ArcTan[a*x])/2))])/2 - ((3*I)/4)*PolyLog[4, -E^(I*(Pi/2 - ArcTan[a*x]))] - ((3*I)/4)*PolyLog[4, -E^((2*I)*(Pi/2 + (-Pi/2 + ArcTan[a*x])/2))])))/(16*Sqrt[1 + a^2*x^2]) + (Sqrt[c*(1 + a^2*x^2)]*ArcTan[a*x]^3)/(48*Sqrt[1 + a^2*x^2]*(Cos[ArcTan[a*x]/2] - Sin[ArcTan[a*x]/2])^6) + (Sqrt[c*(1 + a^2*x^2)]*(Ar

```
cTan[a*x] - ArcTan[a*x]^2 - 5*ArcTan[a*x]^3))/(80*Sqrt[1 + a^2*x^2]*(Cos[Ar
cTan[a*x]/2] - Sin[ArcTan[a*x]/2])^4) + (Sqrt[c*(1 + a^2*x^2)]*(-2 - 52*Arc
Tan[a*x] + 26*ArcTan[a*x]^2 + 15*ArcTan[a*x]^3))/(480*Sqrt[1 + a^2*x^2]*(Co
s[ArcTan[a*x]/2] - Sin[ArcTan[a*x]/2])^2) - (Sqrt[c*(1 + a^2*x^2)]*ArcTan[a
*x]^2*Sin[ArcTan[a*x]/2])/(40*Sqrt[1 + a^2*x^2]*(Cos[ArcTan[a*x]/2] - Sin[A
rcTan[a*x]/2])^5) - (Sqrt[c*(1 + a^2*x^2)]*ArcTan[a*x]^3)/(48*Sqrt[1 + a^2*
x^2]*(Cos[ArcTan[a*x]/2] + Sin[ArcTan[a*x]/2])^6) + (Sqrt[c*(1 + a^2*x^2)]*
ArcTan[a*x]^2*Sin[ArcTan[a*x]/2])/(40*Sqrt[1 + a^2*x^2]*(Cos[ArcTan[a*x]/2]
 + Sin[ArcTan[a*x]/2])^5) + (Sqrt[c*(1 + a^2*x^2)]*(-ArcTan[a*x] - ArcTan[a
*x]^2 + 5*ArcTan[a*x]^3))/(80*Sqrt[1 + a^2*x^2]*(Cos[ArcTan[a*x]/2] + Sin[A
rcTan[a*x]/2])^4) + (Sqrt[c*(1 + a^2*x^2)]*(-2 + 52*ArcTan[a*x] + 26*ArcTan
[a*x]^2 - 15*ArcTan[a*x]^3))/(480*Sqrt[1 + a^2*x^2]*(Cos[ArcTan[a*x]/2] + S
in[ArcTan[a*x]/2])^2) + (Sqrt[c*(1 + a^2*x^2)]*(50*Sin[ArcTan[a*x]/2] - 19*
ArcTan[a*x]^2*Sin[ArcTan[a*x]/2]))/(240*Sqrt[1 + a^2*x^2]*(Cos[ArcTan[a*x]/
2] - Sin[ArcTan[a*x]/2])) + (Sqrt[c*(1 + a^2*x^2)]*(Sin[ArcTan[a*x]/2] - 13
*ArcTan[a*x]^2*Sin[ArcTan[a*x]/2]))/(120*Sqrt[1 + a^2*x^2]*(Cos[ArcTan[a*x]
/2] + Sin[ArcTan[a*x]/2])^3) + (Sqrt[c*(1 + a^2*x^2)]*(-Sin[ArcTan[a*x]/2]
+ 13*ArcTan[a*x]^2*Sin[ArcTan[a*x]/2]))/(120*Sqrt[1 + a^2*x^2]*(Cos[ArcTan[
a*x]/2] - Sin[ArcTan[a*x]/2])^3) + (Sqrt[c*(1 + a^2*x^2)]*(-50*Sin[ArcTan[a
*x]/2] + 19*ArcTan[a*x]^2*Sin[ArcTan[a*x]/2]))/(240*Sqrt[1 + a^2*x^2]*(Cos[
ArcTan[a*x]/2] + Sin[ArcTan[a*x]/2]))))/a
```

Maple [A] time = 2.278, size = 518, normalized size = 0.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(5/2)*arctan(a*x)^3,x)
```

```
[Out] 1/240*c^2/a*(c*(a*x-I)*(a*x+I))^(1/2)*(40*arctan(a*x)^3*a^5*x^5-24*arctan(a
*x)^2*x^4*a^4+130*arctan(a*x)^3*a^3*x^3+12*arctan(a*x)*x^3*a^3-98*arctan(a*
x)^2*x^2*a^2+165*arctan(a*x)^3*a*x-4*a^2*x^2+80*arctan(a*x)*x*a-299*arctan(
a*x)^2-72)-1/240*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)^(1/2)/a*(75*arctan(a
*x)^3*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-75*arctan(a*x)^3*ln(1-I*(1+I*a*x)
/(a^2*x^2+1)^(1/2))-225*I*arctan(a*x)^2*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^
(1/2))+225*I*arctan(a*x)^2*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+518*arc
tan(a*x)*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+450*arctan(a*x)*polylog(3,-I*(
1+I*a*x)/(a^2*x^2+1)^(1/2))-518*arctan(a*x)*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1
/2))-450*arctan(a*x)*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+450*I*polylog
(4,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-450*I*polylog(4,I*(1+I*a*x)/(a^2*x^2+1)^
(1/2))-518*I*dilog(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+518*I*dilog(1-I*(1+I*a*
```

```
x)/(a^2*x^2+1)^(1/2)))*c^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^4c^2x^4 + 2\,a^2c^2x^2 + c^2\right)\sqrt{a^2cx^2 + c}\arctan\left(ax\right)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*arctan(a*x
)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(5/2)*atan(a*x)**3,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.432 $\int \frac{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^3}{x}\,dx$

Optimal. Leaf size=845

$$\frac{149i\sqrt{a^2x^2+1}\tan^{-1}\left(e^{i\tan^{-1}(ax)}\right)\tan^{-1}(ax)^2c^3}{20\sqrt{a^2cx^2+c}}-\frac{2\sqrt{a^2x^2+1}\tan^{-1}(ax)^3\tanh^{-1}\left(e^{i\tan^{-1}(ax)}\right)c^3}{\sqrt{a^2cx^2+c}}+\frac{3i\sqrt{a^2x^2+1}\tan^{-1}(a}{\sqrt{}}$$

```
[Out] -(a*c^2*x*Sqrt[c + a^2*c*x^2])/20 + (29*c^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]
)/20 + (c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x])/10 - (29*a*c^2*x*Sqrt[c + a^2*
c*x^2]*ArcTan[a*x]^2)/40 - (3*a*c*x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2)/20
 + (((149*I)/20)*c^3*Sqrt[1 + a^2*x^2]*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x
]^2)/Sqrt[c + a^2*c*x^2] + c^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3 + (c*(c +
a^2*c*x^2)^(3/2)*ArcTan[a*x]^3)/3 + ((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3)/5
 - (2*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^3*ArcTanh[E^(I*ArcTan[a*x])])/Sqrt[
c + a^2*c*x^2] - (3*c^(5/2)*ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]])/2 +
 ((3*I)*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog[2, -E^(I*ArcTan[a*x])])
/Sqrt[c + a^2*c*x^2] - (((149*I)/20)*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*Poly
Log[2, (-I)*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + (((149*I)/20)*c^3*Sqr
t[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*
x^2] - ((3*I)*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog[2, E^(I*ArcTan[a*
x])])/Sqrt[c + a^2*c*x^2] - (6*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[3,
 -E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + (149*c^3*Sqrt[1 + a^2*x^2]*Poly
Log[3, (-I)*E^(I*ArcTan[a*x])])/(20*Sqrt[c + a^2*c*x^2]) - (149*c^3*Sqrt[1
+ a^2*x^2]*PolyLog[3, I*E^(I*ArcTan[a*x])])/(20*Sqrt[c + a^2*c*x^2]) + (6*c
^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[3, E^(I*ArcTan[a*x])])/Sqrt[c + a^
2*c*x^2] - ((6*I)*c^3*Sqrt[1 + a^2*x^2]*PolyLog[4, -E^(I*ArcTan[a*x])])/Sqr
t[c + a^2*c*x^2] + ((6*I)*c^3*Sqrt[1 + a^2*x^2]*PolyLog[4, E^(I*ArcTan[a*x]
)])/Sqrt[c + a^2*c*x^2]
```

Rubi [A] time = 1.78395, antiderivative size = 845, normalized size of antiderivative = 1., number of steps used = 54, number of rules used = 16, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {4950, 4958, 4956, 4183, 2531, 6609, 2282, 6589, 4930, 4890, 4888, 4181, 4880, 217, 206, 195}

$$\frac{149i\sqrt{a^2x^2+1}\tan^{-1}\left(e^{i\tan^{-1}(ax)}\right)\tan^{-1}(ax)^2c^3}{20\sqrt{a^2cx^2+c}}-\frac{2\sqrt{a^2x^2+1}\tan^{-1}(ax)^3\tanh^{-1}\left(e^{i\tan^{-1}(ax)}\right)c^3}{\sqrt{a^2cx^2+c}}+\frac{3i\sqrt{a^2x^2+1}\tan^{-1}(a}{\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3)/x,x]
```

```
[Out] -(a*c^2*x*Sqrt[c + a^2*c*x^2])/20 + (29*c^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]
)/20 + (c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x])/10 - (29*a*c^2*x*Sqrt[c + a^2*
c*x^2]*ArcTan[a*x]^2)/40 - (3*a*c*x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2)/20
 + (((149*I)/20)*c^3*Sqrt[1 + a^2*x^2]*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x
]^2)/Sqrt[c + a^2*c*x^2] + c^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3 + (c*(c +
a^2*c*x^2)^(3/2)*ArcTan[a*x]^3)/3 + ((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3)/5
 - (2*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^3*ArcTanh[E^(I*ArcTan[a*x])])/Sqrt[
c + a^2*c*x^2] - (3*c^(5/2)*ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]])/2 +
 ((3*I)*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog[2, -E^(I*ArcTan[a*x])])
/Sqrt[c + a^2*c*x^2] - (((149*I)/20)*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*Poly
Log[2, (-I)*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + (((149*I)/20)*c^3*Sqr
t[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*
x^2] - ((3*I)*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog[2, E^(I*ArcTan[a*
x])])/Sqrt[c + a^2*c*x^2] - (6*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[3,
 -E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + (149*c^3*Sqrt[1 + a^2*x^2]*Poly
Log[3, (-I)*E^(I*ArcTan[a*x])])/(20*Sqrt[c + a^2*c*x^2]) - (149*c^3*Sqrt[1
+ a^2*x^2]*PolyLog[3, I*E^(I*ArcTan[a*x])])/(20*Sqrt[c + a^2*c*x^2]) + (6*c
^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[3, E^(I*ArcTan[a*x])])/Sqrt[c + a^
2*c*x^2] - ((6*I)*c^3*Sqrt[1 + a^2*x^2]*PolyLog[4, -E^(I*ArcTan[a*x])])/Sqr
t[c + a^2*c*x^2] + ((6*I)*c^3*Sqrt[1 + a^2*x^2]*PolyLog[4, E^(I*ArcTan[a*x]
)])/Sqrt[c + a^2*c*x^2]
```

Rule 4950

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 4958

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4956

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Dist[1/Sqrt[d], Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTan
[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && G
tQ[d, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
```

```
0] && NeQ[q, -1]
```

Rule 4890

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4888

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[c
*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && GtQ
[d, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4880

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> -Simp[(b*p*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1))/(2*c*q*(2*
q + 1)), x] + (Dist[(2*d*q)/(2*q + 1), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTa
n[c*x])^p, x], x] + Dist[(b^2*d*p*(p - 1))/(2*q*(2*q + 1)), Int[(d + e*x^2)
^(q - 1)*(a + b*ArcTan[c*x])^(p - 2), x], x] + Simp[(x*(d + e*x^2)^q*(a + b
*ArcTan[c*x])^p)/(2*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[q, 0] && GtQ[p, 1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rubi steps

$$\int \frac{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^3}{x}\,dx = c\int \frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^3}{x}\,dx + \left(a^2c\right)\int x\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^3\,dx$$

$$= \frac{1}{5}\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^3 - \frac{1}{5}(3ac)\int \left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2\,dx + c^2\int \frac{\sqrt{c+a^2}}{}$$

$$= \frac{1}{10}c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax) - \frac{3}{20}acx\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2 + \frac{1}{3}c\left(c+a^2cx^2\right)^{3/2}\text{ta}$$

$$= -\frac{1}{20}ac^2x\sqrt{c+a^2cx^2} + \frac{29}{20}c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax) + \frac{1}{10}c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax) - \frac{2}{4}$$

$$= -\frac{1}{20}ac^2x\sqrt{c+a^2cx^2} + \frac{29}{20}c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax) + \frac{1}{10}c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax) - \frac{2}{4}$$

$$= -\frac{1}{20}ac^2x\sqrt{c+a^2cx^2} + \frac{29}{20}c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax) + \frac{1}{10}c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax) - \frac{2}{4}$$

$$= -\frac{1}{20}ac^2x\sqrt{c+a^2cx^2} + \frac{29}{20}c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax) + \frac{1}{10}c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax) - \frac{2}{4}$$

$$= -\frac{1}{20}ac^2x\sqrt{c+a^2cx^2} + \frac{29}{20}c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax) + \frac{1}{10}c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax) - \frac{2}{4}$$

$$= -\frac{1}{20}ac^2x\sqrt{c+a^2cx^2} + \frac{29}{20}c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax) + \frac{1}{10}c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax) - \frac{2}{4}$$

$$= -\frac{1}{20}ac^2x\sqrt{c+a^2cx^2} + \frac{29}{20}c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax) + \frac{1}{10}c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax) - \frac{2}{4}$$

Mathematica [A] time = 6.59863, size = 723, normalized size = 0.86

$$\frac{c^2\sqrt{a^2cx^2+c}\left(2880i\tan^{-1}(ax)^2\text{PolyLog}\left(2,e^{-i\tan^{-1}(ax)}\right)+2880i\tan^{-1}(ax)^2\text{PolyLog}\left(2,-e^{i\tan^{-1}(ax)}\right)-7152i\tan^{-1}(ax\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3)/x,x]
```

```
[Out] (c^2*Sqrt[c + a^2*c*x^2]*((-120*I)*Pi^4 + 960*(1 + a^2*x^2)^(3/2)*ArcTan[a*
x] - 150*(1 + a^2*x^2)^(5/2)*ArcTan[a*x] + (1392*I)*ArcTan[E^(I*ArcTan[a*x]
)]*ArcTan[a*x]^2 + 960*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^3 + 640*(1 + a^2*x^2)^
(3/2)*ArcTan[a*x]^3 + 32*(1 + a^2*x^2)^(5/2)*ArcTan[a*x]^3 + (240*I)*ArcTan
[a*x]^4 - 1440*ArcTanh[(a*x)/Sqrt[1 + a^2*x^2]] + 960*(1 + a^2*x^2)^(3/2)*A
rcTan[a*x]*Cos[2*ArcTan[a*x]] - 216*(1 + a^2*x^2)^(5/2)*ArcTan[a*x]*Cos[2*A
rcTan[a*x]] - 160*(1 + a^2*x^2)^(5/2)*ArcTan[a*x]^3*Cos[2*ArcTan[a*x]] - 66
*(1 + a^2*x^2)^(5/2)*ArcTan[a*x]*Cos[4*ArcTan[a*x]] + 960*ArcTan[a*x]^3*Log
[1 - E^((-I)*ArcTan[a*x])] - 2880*ArcTan[a*x]^2*Log[1 - I*E^(I*ArcTan[a*x])
] + 2880*ArcTan[a*x]^2*Log[1 + I*E^(I*ArcTan[a*x])] - 960*ArcTan[a*x]^3*Log
[1 + E^(I*ArcTan[a*x])] + (2880*I)*ArcTan[a*x]^2*PolyLog[2, E^((-I)*ArcTan[
a*x])] + (2880*I)*ArcTan[a*x]^2*PolyLog[2, -E^(I*ArcTan[a*x])] - (7152*I)*A
rcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] + (7152*I)*ArcTan[a*x]*PolyLo
g[2, I*E^(I*ArcTan[a*x])] + 5760*ArcTan[a*x]*PolyLog[3, E^((-I)*ArcTan[a*x]
)] - 5760*ArcTan[a*x]*PolyLog[3, -E^(I*ArcTan[a*x])] + 7152*PolyLog[3, (-I)
*E^(I*ArcTan[a*x])] - 7152*PolyLog[3, I*E^(I*ArcTan[a*x])] - (5760*I)*PolyL
og[4, E^((-I)*ArcTan[a*x])] - (5760*I)*PolyLog[4, -E^(I*ArcTan[a*x])] - 12*
(1 + a^2*x^2)^(5/2)*Sin[2*ArcTan[a*x]] - 480*(1 + a^2*x^2)^(3/2)*ArcTan[a*x
]^2*Sin[2*ArcTan[a*x]] - 6*(1 + a^2*x^2)^(5/2)*ArcTan[a*x]^2*Sin[2*ArcTan[a
*x]] - 6*(1 + a^2*x^2)^(5/2)*Sin[4*ArcTan[a*x]] + 33*(1 + a^2*x^2)^(5/2)*Ar
cTan[a*x]^2*Sin[4*ArcTan[a*x]]))/(960*Sqrt[1 + a^2*x^2])
```

Maple [A] time = 1.787, size = 562, normalized size = 0.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(5/2)*arctan(a*x)^3/x,x)
```

```
[Out] 1/120*c^2*(c*(a*x-I)*(a*x+I))^(1/2)*(24*arctan(a*x)^3*x^4*a^4-18*arctan(a*x
)^2*x^3*a^3+88*arctan(a*x)^3*x^2*a^2+12*arctan(a*x)*a^2*x^2-105*arctan(a*x)
^2*x*a+184*arctan(a*x)^3-6*a*x+186*arctan(a*x))+1/40*(c*(a*x-I)*(a*x+I))^(1
/2)/(a^2*x^2+1)^(1/2)*(40*arctan(a*x)^3*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-4
0*arctan(a*x)^3*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-120*I*arctan(a*x)^2*polyl
og(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+120*I*arctan(a*x)^2*polylog(2,-(1+I*a*x)/
(a^2*x^2+1)^(1/2))+149*arctan(a*x)^2*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-14
9*arctan(a*x)^2*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-298*I*arctan(a*x)*polyl
```

```
og(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+298*I*arctan(a*x)*polylog(2,I*(1+I*a*x
)/(a^2*x^2+1)^(1/2))+240*arctan(a*x)*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))
-240*arctan(a*x)*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+240*I*polylog(4,(1
+I*a*x)/(a^2*x^2+1)^(1/2))-240*I*polylog(4,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+12
0*I*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2))+298*polylog(3,-I*(1+I*a*x)/(a^2*x^2
+1)^(1/2))-298*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))*c^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^3/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(a^4c^2x^4+2\,a^2c^2x^2+c^2\right)\sqrt{a^2cx^2+c}\,\arctan\left(ax\right)^3}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^3/x,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*arctan(a*x
)^3/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(5/2)*atan(a*x)**3/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^{\frac{5}{2}}\arctan\left(ax\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^3/x,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^3/x, x)
```

3.433 $\int \frac{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^3}{x^2}\,dx$

Optimal. Leaf size=1027

result too large to display

```
[Out] -(a*c^2*Sqrt[c + a^2*c*x^2])/4 + (a^2*c^2*x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]
)/4 - (21*a*c^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/8 - (a*c*(c + a^2*c*x^2)
^(3/2)*ArcTan[a*x]^2)/4 - (c^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/x + (7*a^
2*c^2*x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/8 + (a^2*c*x*(c + a^2*c*x^2)^(3/
2)*ArcTan[a*x]^3)/4 - (((15*I)/4)*a*c^3*Sqrt[1 + a^2*x^2]*ArcTan[E^(I*ArcTa
n[a*x])]*ArcTan[a*x]^3)/Sqrt[c + a^2*c*x^2] - ((11*I)*a*c^3*Sqrt[1 + a^2*x^
2]*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2]
 - (6*a*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*ArcTanh[E^(I*ArcTan[a*x])])/Sqr
t[c + a^2*c*x^2] + ((6*I)*a*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, -E
^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + (((45*I)/8)*a*c^3*Sqrt[1 + a^2*x^2
]*ArcTan[a*x]^2*PolyLog[2, (-I)*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - (
((45*I)/8)*a*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog[2, I*E^(I*ArcTan[a
*x])])/Sqrt[c + a^2*c*x^2] - ((6*I)*a*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*Pol
yLog[2, E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + (((11*I)/2)*a*c^3*Sqrt[1
+ a^2*x^2]*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/Sqrt[c + a^2
*c*x^2] - (((11*I)/2)*a*c^3*Sqrt[1 + a^2*x^2]*PolyLog[2, (I*Sqrt[1 + I*a*x]
)/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2] - (6*a*c^3*Sqrt[1 + a^2*x^2]*PolyLo
g[3, -E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - (45*a*c^3*Sqrt[1 + a^2*x^2]
*ArcTan[a*x]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])])/(4*Sqrt[c + a^2*c*x^2]) +
(45*a*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[3, I*E^(I*ArcTan[a*x])])/(4
*Sqrt[c + a^2*c*x^2]) + (6*a*c^3*Sqrt[1 + a^2*x^2]*PolyLog[3, E^(I*ArcTan[a
*x])])/Sqrt[c + a^2*c*x^2] - (((45*I)/4)*a*c^3*Sqrt[1 + a^2*x^2]*PolyLog[4,
 (-I)*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + (((45*I)/4)*a*c^3*Sqrt[1 +
a^2*x^2]*PolyLog[4, I*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2]
```

Rubi [A] time = 2.11121, antiderivative size = 1027, normalized size of antiderivative = 1., number of steps used = 56, number of rules used = 15, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {4950, 4944, 4958, 4956, 4183, 2531, 2282, 6589, 4890, 4888, 4181, 6609, 4880, 4886, 4878}

$$-\frac{15ia\sqrt{a^2x^2+1}\tan^{-1}\left(e^{i\tan^{-1}(ax)}\right)\tan^{-1}(ax)^3c^3}{4\sqrt{a^2cx^2+c}}-\frac{11ia\sqrt{a^2x^2+1}\tan^{-1}(ax)\tan^{-1}\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)c^3}{\sqrt{a^2cx^2+c}}-\frac{6a\sqrt{a^2x^2+1}\tan^{-1}(ax}{\sqrt{a^2c}}$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3)/x^2,x]
```

```
[Out] -(a*c^2*Sqrt[c + a^2*c*x^2])/4 + (a^2*c^2*x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]
)/4 - (21*a*c^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/8 - (a*c*(c + a^2*c*x^2)
^(3/2)*ArcTan[a*x]^2)/4 - (c^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/x + (7*a^
2*c^2*x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/8 + (a^2*c*x*(c + a^2*c*x^2)^(3/
2)*ArcTan[a*x]^3)/4 - (((15*I)/4)*a*c^3*Sqrt[1 + a^2*x^2]*ArcTan[E^(I*ArcTa
n[a*x])]*ArcTan[a*x]^3)/Sqrt[c + a^2*c*x^2] - ((11*I)*a*c^3*Sqrt[1 + a^2*x^
2]*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2]
 - (6*a*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*ArcTanh[E^(I*ArcTan[a*x])])/Sqr
t[c + a^2*c*x^2] + ((6*I)*a*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, -E
^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + (((45*I)/8)*a*c^3*Sqrt[1 + a^2*x^2
]*ArcTan[a*x]^2*PolyLog[2, (-I)*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - (
((45*I)/8)*a*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog[2, I*E^(I*ArcTan[a
*x])])/Sqrt[c + a^2*c*x^2] - ((6*I)*a*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*Pol
yLog[2, E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + (((11*I)/2)*a*c^3*Sqrt[1
+ a^2*x^2]*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/Sqrt[c + a^2
*c*x^2] - (((11*I)/2)*a*c^3*Sqrt[1 + a^2*x^2]*PolyLog[2, (I*Sqrt[1 + I*a*x]
)/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2] - (6*a*c^3*Sqrt[1 + a^2*x^2]*PolyLo
g[3, -E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - (45*a*c^3*Sqrt[1 + a^2*x^2]
*ArcTan[a*x]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])])/(4*Sqrt[c + a^2*c*x^2]) +
(45*a*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[3, I*E^(I*ArcTan[a*x])])/(4
*Sqrt[c + a^2*c*x^2]) + (6*a*c^3*Sqrt[1 + a^2*x^2]*PolyLog[3, E^(I*ArcTan[a
*x])])/Sqrt[c + a^2*c*x^2] - (((45*I)/4)*a*c^3*Sqrt[1 + a^2*x^2]*PolyLog[4,
 (-I)*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + (((45*I)/4)*a*c^3*Sqrt[1 +
a^2*x^2]*PolyLog[4, I*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2]
```

Rule 4950

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 4944

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a +
 b*ArcTan[c*x])^p)/(d*f*(m + 1)), x] - Dist[(b*c*p)/(f*(m + 1)), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rule 4958

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4956

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Dist[1/Sqrt[d], Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTan
[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && G
tQ[d, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4890

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4888

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[c
*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && GtQ
[d, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 4880

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> -Simp[(b*p*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1))/(2*c*q*(2*
q + 1)), x] + (Dist[(2*d*q)/(2*q + 1), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTa
n[c*x])^p, x], x] + Dist[(b^2*d*p*(p - 1))/(2*q*(2*q + 1)), Int[(d + e*x^2)
^(q - 1)*(a + b*ArcTan[c*x])^(p - 2), x], x] + Simp[(x*(d + e*x^2)^q*(a + b
*ArcTan[c*x])^p)/(2*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[q, 0] && GtQ[p, 1]
```

Rule 4886

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[(-2*I*(a + b*ArcTan[c*x])*ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]])
/(c*Sqrt[d]), x] + (Simp[(I*b*PolyLog[2, -((I*Sqrt[1 + I*c*x])/Sqrt[1 - I*c
```

```
*x])])/(c*Sqrt[d]), x] - Simp[(I*b*PolyLog[2, (I*Sqrt[1 + I*c*x])/Sqrt[1 -
I*c*x]])/(c*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

Rule 4878

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symbo
l] :> -Simp[(b*(d + e*x^2)^q)/(2*c*q*(2*q + 1)), x] + (Dist[(2*d*q)/(2*q +
1), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x]), x], x] + Simp[(x*(d + e*x^
2)^q*(a + b*ArcTan[c*x]))/(2*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] && Eq
Q[e, c^2*d] && GtQ[q, 0]
```

Rubi steps

$$\int \frac{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^3}{x^2}\,dx = c\int \frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^3}{x^2}\,dx + \left(a^2c\right)\int \left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^3\,dx$$

$$= -\frac{1}{4}ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2 + \frac{1}{4}a^2cx\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^3 + c^2\int \frac{\sqrt{c+a^2cx^2}}{x}$$

$$= -\frac{1}{4}ac^2\sqrt{c+a^2cx^2} + \frac{1}{4}a^2c^2x\sqrt{c+a^2cx^2}\tan^{-1}(ax) - \frac{21}{8}ac^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2 - \frac{1}{4}$$

$$= -\frac{1}{4}ac^2\sqrt{c+a^2cx^2} + \frac{1}{4}a^2c^2x\sqrt{c+a^2cx^2}\tan^{-1}(ax) - \frac{21}{8}ac^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2 - \frac{1}{4}$$

$$= -\frac{1}{4}ac^2\sqrt{c+a^2cx^2} + \frac{1}{4}a^2c^2x\sqrt{c+a^2cx^2}\tan^{-1}(ax) - \frac{21}{8}ac^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2 - \frac{1}{4}$$

$$= -\frac{1}{4}ac^2\sqrt{c+a^2cx^2} + \frac{1}{4}a^2c^2x\sqrt{c+a^2cx^2}\tan^{-1}(ax) - \frac{21}{8}ac^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2 - \frac{1}{4}$$

$$= -\frac{1}{4}ac^2\sqrt{c+a^2cx^2} + \frac{1}{4}a^2c^2x\sqrt{c+a^2cx^2}\tan^{-1}(ax) - \frac{21}{8}ac^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2 - \frac{1}{4}$$

$$= -\frac{1}{4}ac^2\sqrt{c+a^2cx^2} + \frac{1}{4}a^2c^2x\sqrt{c+a^2cx^2}\tan^{-1}(ax) - \frac{21}{8}ac^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2 - \frac{1}{4}$$

$$= -\frac{1}{4}ac^2\sqrt{c+a^2cx^2} + \frac{1}{4}a^2c^2x\sqrt{c+a^2cx^2}\tan^{-1}(ax) - \frac{21}{8}ac^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2 - \frac{1}{4}$$

$$= -\frac{1}{4}ac^2\sqrt{c+a^2cx^2} + \frac{1}{4}a^2c^2x\sqrt{c+a^2cx^2}\tan^{-1}(ax) - \frac{21}{8}ac^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2 - \frac{1}{4}$$

Mathematica [B] time = 15.9356, size = 3267, normalized size = 3.18

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3)/x^2,x]
```

```
[Out] ((-I)*a*c^2*Sqrt[c*(1 + a^2*x^2)]*(12*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]
 - (3*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2 + I*a*x*Sqrt[1 + a^2*x^2]*ArcTan[a
*x]^3 + 2*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^3 - 3*(2 + ArcTan[a*x]^2)*P
olyLog[2, (-I)*E^(I*ArcTan[a*x])] + 3*(2 + ArcTan[a*x]^2)*PolyLog[2, I*E^(I
*ArcTan[a*x])] - (6*I)*ArcTan[a*x]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] + (6*
I)*ArcTan[a*x]*PolyLog[3, I*E^(I*ArcTan[a*x])] + 6*PolyLog[4, (-I)*E^(I*Arc
Tan[a*x])] - 6*PolyLog[4, I*E^(I*ArcTan[a*x])]))/Sqrt[1 + a^2*x^2] + (a*c^2
*Sqrt[c*(1 + a^2*x^2)]*Csc[ArcTan[a*x]/2]*(((-7*I)*a*Pi^4*x)/Sqrt[1 + a^2*x
^2] - ((8*I)*a*Pi^3*x*ArcTan[a*x])/Sqrt[1 + a^2*x^2] + ((24*I)*a*Pi^2*x*Arc
Tan[a*x]^2)/Sqrt[1 + a^2*x^2] - 64*ArcTan[a*x]^3 - ((32*I)*a*Pi*x*ArcTan[a*
x]^3)/Sqrt[1 + a^2*x^2] + ((16*I)*a*x*ArcTan[a*x]^4)/Sqrt[1 + a^2*x^2] + (4
8*a*Pi^2*x*ArcTan[a*x]*Log[1 - I/E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - (9
6*a*Pi*x*ArcTan[a*x]^2*Log[1 - I/E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - (8
*a*Pi^3*x*Log[1 + I/E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + (64*a*x*ArcTan[
a*x]^3*Log[1 + I/E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + (192*a*x*ArcTan[a*
x]^2*Log[1 - E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + (8*a*Pi^3*x*Log[1 + I*
E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - (48*a*Pi^2*x*ArcTan[a*x]*Log[1 + I*
E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + (96*a*Pi*x*ArcTan[a*x]^2*Log[1 + I*
E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - (64*a*x*ArcTan[a*x]^3*Log[1 + I*E^(
I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - (192*a*x*ArcTan[a*x]^2*Log[1 + E^(I*Ar
cTan[a*x])])/Sqrt[1 + a^2*x^2] + (8*a*Pi^3*x*Log[Tan[(Pi + 2*ArcTan[a*x])/4
]])/Sqrt[1 + a^2*x^2] + ((192*I)*a*x*ArcTan[a*x]^2*PolyLog[2, (-I)/E^(I*Arc
Tan[a*x])])/Sqrt[1 + a^2*x^2] + ((48*I)*a*Pi*x*(Pi - 4*ArcTan[a*x])*PolyLog
[2, I/E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + ((384*I)*a*x*ArcTan[a*x]*Poly
Log[2, -E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + ((48*I)*a*Pi^2*x*PolyLog[2,
 (-I)*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - ((192*I)*a*Pi*x*ArcTan[a*x]*P
olyLog[2, (-I)*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + ((192*I)*a*x*ArcTan[
a*x]^2*PolyLog[2, (-I)*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - ((384*I)*a*x
*ArcTan[a*x]*PolyLog[2, E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + (384*a*x*Ar
cTan[a*x]*PolyLog[3, (-I)/E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - (192*a*Pi
*x*PolyLog[3, I/E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - (384*a*x*PolyLog[3,
 -E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + (192*a*Pi*x*PolyLog[3, (-I)*E^(I*
ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - (384*a*x*ArcTan[a*x]*PolyLog[3, (-I)*E^(
I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + (384*a*x*PolyLog[3, E^(I*ArcTan[a*x])]
)/Sqrt[1 + a^2*x^2] - ((384*I)*a*x*PolyLog[4, (-I)/E^(I*ArcTan[a*x])])/Sqrt
[1 + a^2*x^2] - ((384*I)*a*x*PolyLog[4, (-I)*E^(I*ArcTan[a*x])])/Sqrt[1 + a
```

```
^2*x^2])*Sec[ArcTan[a*x]/2])/(128*Sqrt[1 + a^2*x^2]) + a*c^2*((Sqrt[c*(1 +
a^2*x^2)]*(-1 + ArcTan[a*x]^2))/(4*Sqrt[1 + a^2*x^2]) + (Sqrt[c*(1 + a^2*x^
2)]*(-(ArcTan[a*x]*(Log[1 - I*E^(I*ArcTan[a*x])] - Log[1 + I*E^(I*ArcTan[a*
x])])) - I*(PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - PolyLog[2, I*E^(I*ArcTan[a
*x])])))/(2*Sqrt[1 + a^2*x^2]) + (Sqrt[c*(1 + a^2*x^2)]*(-(Pi^3*Log[Cot[(Pi
/2 - ArcTan[a*x])/2]])/8 - (3*Pi^2*((Pi/2 - ArcTan[a*x])*(Log[1 - E^(I*(Pi/
2 - ArcTan[a*x]))] - Log[1 + E^(I*(Pi/2 - ArcTan[a*x]))]) + I*(PolyLog[2, -
E^(I*(Pi/2 - ArcTan[a*x]))] - PolyLog[2, E^(I*(Pi/2 - ArcTan[a*x]))])))/4 +
 (3*Pi*((Pi/2 - ArcTan[a*x])^2*(Log[1 - E^(I*(Pi/2 - ArcTan[a*x]))] - Log[1
 + E^(I*(Pi/2 - ArcTan[a*x]))]) + (2*I)*(Pi/2 - ArcTan[a*x])*(PolyLog[2, -E
^(I*(Pi/2 - ArcTan[a*x]))] - PolyLog[2, E^(I*(Pi/2 - ArcTan[a*x]))]) + 2*(-
PolyLog[3, -E^(I*(Pi/2 - ArcTan[a*x]))] + PolyLog[3, E^(I*(Pi/2 - ArcTan[a*
x]))])))/2 - 8*((I/64)*(Pi/2 - ArcTan[a*x])^4 + (I/4)*(Pi/2 + (-Pi/2 + ArcT
an[a*x])/2)^4 - ((Pi/2 - ArcTan[a*x])^3*Log[1 + E^(I*(Pi/2 - ArcTan[a*x]))]
)/8 - (Pi^3*(I*(Pi/2 + (-Pi/2 + ArcTan[a*x])/2) - Log[1 + E^((2*I)*(Pi/2 +
(-Pi/2 + ArcTan[a*x])/2))]))/8 - (Pi/2 + (-Pi/2 + ArcTan[a*x])/2)^3*Log[1 +
 E^((2*I)*(Pi/2 + (-Pi/2 + ArcTan[a*x])/2))] + ((3*I)/8)*(Pi/2 - ArcTan[a*x
])^2*PolyLog[2, -E^(I*(Pi/2 - ArcTan[a*x]))] + (3*Pi^2*((I/2)*(Pi/2 + (-Pi/
2 + ArcTan[a*x])/2)^2 - (Pi/2 + (-Pi/2 + ArcTan[a*x])/2)*Log[1 + E^((2*I)*(
Pi/2 + (-Pi/2 + ArcTan[a*x])/2))] + (I/2)*PolyLog[2, -E^((2*I)*(Pi/2 + (-Pi
/2 + ArcTan[a*x])/2))]))/4 + ((3*I)/2)*(Pi/2 + (-Pi/2 + ArcTan[a*x])/2)^2*P
olyLog[2, -E^((2*I)*(Pi/2 + (-Pi/2 + ArcTan[a*x])/2))] - (3*(Pi/2 - ArcTan[
a*x])*PolyLog[3, -E^(I*(Pi/2 - ArcTan[a*x]))])/4 - (3*Pi*((I/3)*(Pi/2 + (-P
i/2 + ArcTan[a*x])/2)^3 - (Pi/2 + (-Pi/2 + ArcTan[a*x])/2)^2*Log[1 + E^((2*
I)*(Pi/2 + (-Pi/2 + ArcTan[a*x])/2))] + I*(Pi/2 + (-Pi/2 + ArcTan[a*x])/2)*
PolyLog[2, -E^((2*I)*(Pi/2 + (-Pi/2 + ArcTan[a*x])/2))] - PolyLog[3, -E^((2
*I)*(Pi/2 + (-Pi/2 + ArcTan[a*x])/2))]/2))/2 - (3*(Pi/2 + (-Pi/2 + ArcTan[a
*x])/2)*PolyLog[3, -E^((2*I)*(Pi/2 + (-Pi/2 + ArcTan[a*x])/2))])/2 - ((3*I)
/4)*PolyLog[4, -E^(I*(Pi/2 - ArcTan[a*x]))] - ((3*I)/4)*PolyLog[4, -E^((2*I
)*(Pi/2 + (-Pi/2 + ArcTan[a*x])/2))])))/(8*Sqrt[1 + a^2*x^2]) + (Sqrt[c*(1
+ a^2*x^2)]*ArcTan[a*x]^3)/(16*Sqrt[1 + a^2*x^2]*(Cos[ArcTan[a*x]/2] - Sin[
ArcTan[a*x]/2])^4) + (Sqrt[c*(1 + a^2*x^2)]*(2*ArcTan[a*x] - ArcTan[a*x]^2
- ArcTan[a*x]^3))/(16*Sqrt[1 + a^2*x^2]*(Cos[ArcTan[a*x]/2] - Sin[ArcTan[a*
x]/2])^2) - (Sqrt[c*(1 + a^2*x^2)]*ArcTan[a*x]^2*Sin[ArcTan[a*x]/2])/(8*Sqr
t[1 + a^2*x^2]*(Cos[ArcTan[a*x]/2] - Sin[ArcTan[a*x]/2])^3) - (Sqrt[c*(1 +
a^2*x^2)]*ArcTan[a*x]^3)/(16*Sqrt[1 + a^2*x^2]*(Cos[ArcTan[a*x]/2] + Sin[Ar
cTan[a*x]/2])^4) + (Sqrt[c*(1 + a^2*x^2)]*ArcTan[a*x]^2*Sin[ArcTan[a*x]/2])
/(8*Sqrt[1 + a^2*x^2]*(Cos[ArcTan[a*x]/2] + Sin[ArcTan[a*x]/2])^3) + (Sqrt[
c*(1 + a^2*x^2)]*(-2*ArcTan[a*x] - ArcTan[a*x]^2 + ArcTan[a*x]^3))/(16*Sqrt
[1 + a^2*x^2]*(Cos[ArcTan[a*x]/2] + Sin[ArcTan[a*x]/2])^2) + (Sqrt[c*(1 + a
^2*x^2)]*(Sin[ArcTan[a*x]/2] - ArcTan[a*x]^2*Sin[ArcTan[a*x]/2]))/(4*Sqrt[1
 + a^2*x^2]*(Cos[ArcTan[a*x]/2] + Sin[ArcTan[a*x]/2])) + (Sqrt[c*(1 + a^2*x
^2)]*(-Sin[ArcTan[a*x]/2] + ArcTan[a*x]^2*Sin[ArcTan[a*x]/2]))/(4*Sqrt[1 +
a^2*x^2]*(Cos[ArcTan[a*x]/2] - Sin[ArcTan[a*x]/2])))
```

Maple [A] time = 1.996, size = 655, normalized size = 0.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(5/2)*arctan(a*x)^3/x^2,x)
```

```
[Out] 1/8*c^2*(c*(a*x-I)*(a*x+I))^(1/2)*(2*arctan(a*x)^3*x^4*a^4-2*arctan(a*x)^2*
x^3*a^3+9*arctan(a*x)^3*x^2*a^2+2*arctan(a*x)*a^2*x^2-23*arctan(a*x)^2*x*a-
8*arctan(a*x)^3-2*a*x)/x-1/8*I*a*c^2*(c*(a*x-I)*(a*x+I))^(1/2)*(-48*I*polyl
og(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+44*I*arctan(a*x)*ln(1-I*(1+I*a*x)/(a^2*x
^2+1)^(1/2))-44*I*arctan(a*x)*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-90*I*arct
an(a*x)*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+90*I*arctan(a*x)*polylog(
3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-24*I*arctan(a*x)^2*ln(1+(1+I*a*x)/(a^2*x^2
+1)^(1/2))+15*I*arctan(a*x)^3*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-15*I*arct
an(a*x)^3*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-45*arctan(a*x)^2*polylog(2,-I
*(1+I*a*x)/(a^2*x^2+1)^(1/2))+45*arctan(a*x)^2*polylog(2,I*(1+I*a*x)/(a^2*x
^2+1)^(1/2))+48*I*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))+24*I*arctan(a*x)^2
*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))+48*arctan(a*x)*polylog(2,(1+I*a*x)/(a^2*
x^2+1)^(1/2))-48*arctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-44*pol
ylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+90*polylog(4,-I*(1+I*a*x)/(a^2*x^2+1
)^(1/2))+44*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-90*polylog(4,I*(1+I*a*
x)/(a^2*x^2+1)^(1/2)))/(a^2*x^2+1)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^3/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^4c^2x^4 + 2\,a^2c^2x^2 + c^2\right)\sqrt{a^2cx^2 + c}\,\arctan\left(ax\right)^3}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^3/x^2,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*arctan(a*x
)^3/x^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(5/2)*atan(a*x)**3/x**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^{\frac{5}{2}}\arctan\left(ax\right)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^3/x^2,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^3/x^2, x)
```

3.434 $\int \frac{(c+a^2cx^2)^{5/2}\tan^{-1}(ax)^3}{x^3}\,dx$

Optimal. Leaf size=1043

result too large to display

```
[Out] a^2*c^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x] - (3*a*c^2*Sqrt[c + a^2*c*x^2]*ArcT
an[a*x]^2)/(2*x) - (a^3*c^2*x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/2 + ((13*I
)*a^2*c^3*Sqrt[1 + a^2*x^2]*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2)/Sqrt[c
 + a^2*c*x^2] + 2*a^2*c^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3 - (c^2*Sqrt[c +
 a^2*c*x^2]*ArcTan[a*x]^3)/(2*x^2) + (a^2*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*
x]^3)/3 - (5*a^2*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^3*ArcTanh[E^(I*ArcTan[a*
x])])/Sqrt[c + a^2*c*x^2] - (6*a^2*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTan
h[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2] - a^2*c^(5/2)*ArcTa
nh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]] + (((15*I)/2)*a^2*c^3*Sqrt[1 + a^2*x^
2]*ArcTan[a*x]^2*PolyLog[2, -E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - ((13
*I)*a^2*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])
])/Sqrt[c + a^2*c*x^2] + ((13*I)*a^2*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*Poly
Log[2, I*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - (((15*I)/2)*a^2*c^3*Sqrt
[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog[2, E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x
^2] + ((3*I)*a^2*c^3*Sqrt[1 + a^2*x^2]*PolyLog[2, -(Sqrt[1 + I*a*x]/Sqrt[1
- I*a*x])])/Sqrt[c + a^2*c*x^2] - ((3*I)*a^2*c^3*Sqrt[1 + a^2*x^2]*PolyLog[
2, Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2] - (15*a^2*c^3*Sqrt
[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[3, -E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^
2] + (13*a^2*c^3*Sqrt[1 + a^2*x^2]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])])/Sqrt
[c + a^2*c*x^2] - (13*a^2*c^3*Sqrt[1 + a^2*x^2]*PolyLog[3, I*E^(I*ArcTan[a*
x])])/Sqrt[c + a^2*c*x^2] + (15*a^2*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyL
og[3, E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - ((15*I)*a^2*c^3*Sqrt[1 + a^
2*x^2]*PolyLog[4, -E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + ((15*I)*a^2*c^
3*Sqrt[1 + a^2*x^2]*PolyLog[4, E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2]
```

Rubi [A] time = 3.53672, antiderivative size = 1043, normalized size of antiderivative = 1., number of steps used = 87, number of rules used = 18, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75, Rules used = {4950, 4962, 4944, 4958, 4954, 4956, 4183, 2531, 6609, 2282, 6589, 4930, 4890, 4888, 4181, 4880, 217, 206}

$$-\frac{1}{2}c^2x\sqrt{a^2cx^2+c}\tan^{-1}(ax)^2a^3+\frac{1}{3}c\left(a^2cx^2+c\right)^{3/2}\tan^{-1}(ax)^3a^2+2c^2\sqrt{a^2cx^2+c}\tan^{-1}(ax)^3a^2+\frac{13ic^3\sqrt{a^2x^2+1}\tan^{-1}(}{\sqrt{a^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3)/x^3,x]
```

```
[Out] a^2*c^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x] - (3*a*c^2*Sqrt[c + a^2*c*x^2]*ArcT
an[a*x]^2)/(2*x) - (a^3*c^2*x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/2 + ((13*I
)*a^2*c^3*Sqrt[1 + a^2*x^2]*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2)/Sqrt[c
 + a^2*c*x^2] + 2*a^2*c^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3 - (c^2*Sqrt[c +
 a^2*c*x^2]*ArcTan[a*x]^3)/(2*x^2) + (a^2*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*
x]^3)/3 - (5*a^2*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^3*ArcTanh[E^(I*ArcTan[a*
x])])/Sqrt[c + a^2*c*x^2] - (6*a^2*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTan
h[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2] - a^2*c^(5/2)*ArcTa
nh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]] + (((15*I)/2)*a^2*c^3*Sqrt[1 + a^2*x^
2]*ArcTan[a*x]^2*PolyLog[2, -E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - ((13
*I)*a^2*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])
])/Sqrt[c + a^2*c*x^2] + ((13*I)*a^2*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*Poly
Log[2, I*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - (((15*I)/2)*a^2*c^3*Sqrt
[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog[2, E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x
^2] + ((3*I)*a^2*c^3*Sqrt[1 + a^2*x^2]*PolyLog[2, -(Sqrt[1 + I*a*x]/Sqrt[1
- I*a*x])])/Sqrt[c + a^2*c*x^2] - ((3*I)*a^2*c^3*Sqrt[1 + a^2*x^2]*PolyLog[
2, Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2] - (15*a^2*c^3*Sqrt
[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[3, -E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^
2] + (13*a^2*c^3*Sqrt[1 + a^2*x^2]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])])/Sqrt
[c + a^2*c*x^2] - (13*a^2*c^3*Sqrt[1 + a^2*x^2]*PolyLog[3, I*E^(I*ArcTan[a*
x])])/Sqrt[c + a^2*c*x^2] + (15*a^2*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyL
og[3, E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - ((15*I)*a^2*c^3*Sqrt[1 + a^
2*x^2]*PolyLog[4, -E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + ((15*I)*a^2*c^
3*Sqrt[1 + a^2*x^2]*PolyLog[4, E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2]
```

Rule 4950

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 4962

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Ar
cTan[c*x])^p)/(d*f*(m + 1)), x] + (-Dist[(b*c*p)/(f*(m + 1)), Int[((f*x)^(m
 + 1)*(a + b*ArcTan[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] - Dist[(c^2*(m +
 2))/(f^2*(m + 1)), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/Sqrt[d + e*x^
2], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] &
```

& LtQ[m, -1] && NeQ[m, -2]

Rule 4944

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a +
 b*ArcTan[c*x])^p)/(d*f*(m + 1)), x] - Dist[(b*c*p)/(f*(m + 1)), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rule 4958

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4954

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x_
Symbol] :> Simp[(-2*(a + b*ArcTan[c*x])*ArcTanh[Sqrt[1 + I*c*x]/Sqrt[1 - I*
c*x]])/Sqrt[d], x] + (Simp[(I*b*PolyLog[2, -(Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x
])])/Sqrt[d], x] - Simp[(I*b*PolyLog[2, Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]])/S
qrt[d], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

Rule 4956

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Dist[1/Sqrt[d], Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTan
[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && G
tQ[d, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
```

```
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 4890

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4888

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[c
```

```
*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && GtQ
[d, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4880

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> -Simp[(b*p*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1))/(2*c*q*(2*
q + 1)), x] + (Dist[(2*d*q)/(2*q + 1), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTa
n[c*x])^p, x], x] + Dist[(b^2*d*p*(p - 1))/(2*q*(2*q + 1)), Int[(d + e*x^2)
^(q - 1)*(a + b*ArcTan[c*x])^(p - 2), x], x] + Simp[(x*(d + e*x^2)^q*(a + b
*ArcTan[c*x])^p)/(2*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[q, 0] && GtQ[p, 1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^3}{x^3}\,dx = c\int \frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^3}{x^3}\,dx + \left(a^2c\right)\int \frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^3}{x}\,dx$$

$$= c^2\int \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{x^3}\,dx + 2\left(\left(a^2c^2\right)\int \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{x}\,dx\right) + \left(a^4c^2\right)\int$$

$$= \frac{1}{3}a^2c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^3 - \left(a^3c^2\right)\int \sqrt{c+a^2cx^2}\tan^{-1}(ax)^2\,dx + c^3\int \frac{\tan^{-1}(}{x^3\sqrt{c+}}$$

$$= a^2c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax) - \frac{1}{2}a^3c^2x\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2 - \frac{c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{2x^2}$$

$$= a^2c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax) - \frac{3ac^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2x} - \frac{1}{2}a^3c^2x\sqrt{c+a^2cx^2}\tan^{-1}($$

$$= a^2c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax) - \frac{3ac^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2x} - \frac{1}{2}a^3c^2x\sqrt{c+a^2cx^2}\tan^{-1}($$

$$= a^2c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax) - \frac{3ac^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2x} - \frac{1}{2}a^3c^2x\sqrt{c+a^2cx^2}\tan^{-1}($$

$$= a^2c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax) - \frac{3ac^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2x} - \frac{1}{2}a^3c^2x\sqrt{c+a^2cx^2}\tan^{-1}($$

$$= a^2c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax) - \frac{3ac^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2x} - \frac{1}{2}a^3c^2x\sqrt{c+a^2cx^2}\tan^{-1}($$

$$= a^2c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax) - \frac{3ac^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2x} - \frac{1}{2}a^3c^2x\sqrt{c+a^2cx^2}\tan^{-1}($$

$$= a^2c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax) - \frac{3ac^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2x} - \frac{1}{2}a^3c^2x\sqrt{c+a^2cx^2}\tan^{-1}($$

Mathematica [A] time = 10.2065, size = 1128, normalized size = 1.08

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3)/x^3,x]
```

```
[Out] (a^2*c^2*Sqrt[c*(1 + a^2*x^2)]*(((-I)*Pi^4)/Sqrt[1 + a^2*x^2] + 8*ArcTan[a*
x]^3 + ((2*I)*ArcTan[a*x]^4)/Sqrt[1 + a^2*x^2] + (8*ArcTan[a*x]^3*Log[1 - E
^((-I)*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - (24*ArcTan[a*x]^2*Log[1 - I*E^(I*
ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + (24*ArcTan[a*x]^2*Log[1 + I*E^(I*ArcTan[
a*x])])/Sqrt[1 + a^2*x^2] - (8*ArcTan[a*x]^3*Log[1 + E^(I*ArcTan[a*x])])/Sq
rt[1 + a^2*x^2] + ((24*I)*ArcTan[a*x]^2*PolyLog[2, E^((-I)*ArcTan[a*x])])/S
qrt[1 + a^2*x^2] + ((24*I)*ArcTan[a*x]^2*PolyLog[2, -E^(I*ArcTan[a*x])])/Sq
rt[1 + a^2*x^2] - ((48*I)*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])])/S
qrt[1 + a^2*x^2] + ((48*I)*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])])/Sqr
t[1 + a^2*x^2] + (48*ArcTan[a*x]*PolyLog[3, E^((-I)*ArcTan[a*x])])/Sqrt[1 +
 a^2*x^2] - (48*ArcTan[a*x]*PolyLog[3, -E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^
2] + (48*PolyLog[3, (-I)*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - (48*PolyLo
g[3, I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - ((48*I)*PolyLog[4, E^((-I)*A
rcTan[a*x])])/Sqrt[1 + a^2*x^2] - ((48*I)*PolyLog[4, -E^(I*ArcTan[a*x])])/S
qrt[1 + a^2*x^2]))/4 + a^2*c^2*((Sqrt[c*(1 + a^2*x^2)]*(I*ArcTan[E^(I*ArcTa
n[a*x])]*ArcTan[a*x]^2 - ArcTanh[(a*x)/Sqrt[1 + a^2*x^2]] - I*ArcTan[a*x]*P
olyLog[2, (-I)*E^(I*ArcTan[a*x])] + I*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[
a*x])] + PolyLog[3, (-I)*E^(I*ArcTan[a*x])] - PolyLog[3, I*E^(I*ArcTan[a*x]
)]))/Sqrt[1 + a^2*x^2] + ((1 + a^2*x^2)*Sqrt[c*(1 + a^2*x^2)]*ArcTan[a*x]*(
6 + 4*ArcTan[a*x]^2 + 6*Cos[2*ArcTan[a*x]] - 3*ArcTan[a*x]*Sin[2*ArcTan[a*x
]]))/12) + (a^2*c^2*Sqrt[c*(1 + a^2*x^2)]*((-I)*Pi^4 + (2*I)*ArcTan[a*x]^4
- 12*ArcTan[a*x]^2*Cot[ArcTan[a*x]/2] - 2*ArcTan[a*x]^3*Csc[ArcTan[a*x]/2]^
2 + 8*ArcTan[a*x]^3*Log[1 - E^((-I)*ArcTan[a*x])] + 48*ArcTan[a*x]*Log[1 -
E^(I*ArcTan[a*x])] - 48*ArcTan[a*x]*Log[1 + E^(I*ArcTan[a*x])] - 8*ArcTan[a
*x]^3*Log[1 + E^(I*ArcTan[a*x])] + (24*I)*ArcTan[a*x]^2*PolyLog[2, E^((-I)*
ArcTan[a*x])] + (24*I)*(2 + ArcTan[a*x]^2)*PolyLog[2, -E^(I*ArcTan[a*x])] -
 (48*I)*PolyLog[2, E^(I*ArcTan[a*x])] + 48*ArcTan[a*x]*PolyLog[3, E^((-I)*A
rcTan[a*x])] - 48*ArcTan[a*x]*PolyLog[3, -E^(I*ArcTan[a*x])] - (48*I)*PolyL
og[4, E^((-I)*ArcTan[a*x])] - (48*I)*PolyLog[4, -E^(I*ArcTan[a*x])] + 2*Arc
Tan[a*x]^3*Sec[ArcTan[a*x]/2]^2 - 12*ArcTan[a*x]^2*Tan[ArcTan[a*x]/2]))/(16
*Sqrt[1 + a^2*x^2])
```

Maple [A] time = 1.978, size = 660, normalized size = 0.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(5/2)*arctan(a*x)^3/x^3,x)
```

```
[Out] 1/6*c^2*(c*(a*x-I)*(a*x+I))^(1/2)*arctan(a*x)*(2*arctan(a*x)^2*x^4*a^4-3*ar
ctan(a*x)*x^3*a^3+14*arctan(a*x)^2*x^2*a^2+6*a^2*x^2-9*arctan(a*x)*x*a-3*ar
ctan(a*x)^2)/x^2+1/2*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)^(1/2)*(5*arctan(
a*x)^3*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-5*arctan(a*x)^3*ln(1+(1+I*a*x)/(a^
2*x^2+1)^(1/2))+4*I*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2))-30*I*polylog(4,-(1+
I*a*x)/(a^2*x^2+1)^(1/2))+13*arctan(a*x)^2*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/
2))-13*arctan(a*x)^2*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+26*I*arctan(a*x)*p
olylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-15*I*arctan(a*x)^2*polylog(2,(1+I*a
*x)/(a^2*x^2+1)^(1/2))+6*arctan(a*x)*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))+30*a
rctan(a*x)*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*arctan(a*x)*ln(1+(1+I*a
*x)/(a^2*x^2+1)^(1/2))-30*arctan(a*x)*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2
))-26*I*arctan(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I*polylog(2
,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*I*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+1
5*I*arctan(a*x)^2*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+30*I*polylog(4,(1
+I*a*x)/(a^2*x^2+1)^(1/2))+26*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-26*
polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))*a^2*c^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^3/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^4c^2x^4 + 2\,a^2c^2x^2 + c^2\right)\sqrt{a^2cx^2 + c}\,\arctan\left(ax\right)^3}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^3/x^3,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*arctan(a*x
)^3/x^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(5/2)*atan(a*x)**3/x**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^{\frac{5}{2}}\arctan\left(ax\right)^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^3/x^3,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^3/x^3, x)
```

3.435 $\int \frac{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^3}{x^4}\,dx$

Optimal. Leaf size=1061

result too large to display

```
[Out] -((a^2*c^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/x) - (3*a^3*c^2*Sqrt[c + a^2*c*
x^2]*ArcTan[a*x]^2)/2 - (a*c^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(2*x^2) -
 (2*a^2*c^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/x + (a^4*c^2*x*Sqrt[c + a^2*
c*x^2]*ArcTan[a*x]^3)/2 - (c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3)/(3*x^3) -
 ((5*I)*a^3*c^3*Sqrt[1 + a^2*x^2]*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^3)/
Sqrt[c + a^2*c*x^2] - ((6*I)*a^3*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTan[S
qrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2] - (13*a^3*c^3*Sqrt[1 +
 a^2*x^2]*ArcTan[a*x]^2*ArcTanh[E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - a
^3*c^(5/2)*ArcTanh[Sqrt[c + a^2*c*x^2]/Sqrt[c]] + ((13*I)*a^3*c^3*Sqrt[1 +
a^2*x^2]*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] +
(((15*I)/2)*a^3*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog[2, (-I)*E^(I*Ar
cTan[a*x])])/Sqrt[c + a^2*c*x^2] - (((15*I)/2)*a^3*c^3*Sqrt[1 + a^2*x^2]*Ar
cTan[a*x]^2*PolyLog[2, I*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - ((13*I)*
a^3*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, E^(I*ArcTan[a*x])])/Sqrt[c
 + a^2*c*x^2] + ((3*I)*a^3*c^3*Sqrt[1 + a^2*x^2]*PolyLog[2, ((-I)*Sqrt[1 +
I*a*x])/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2] - ((3*I)*a^3*c^3*Sqrt[1 + a^2
*x^2]*PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2]
- (13*a^3*c^3*Sqrt[1 + a^2*x^2]*PolyLog[3, -E^(I*ArcTan[a*x])])/Sqrt[c + a^
2*c*x^2] - (15*a^3*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[3, (-I)*E^(I*A
rcTan[a*x])])/Sqrt[c + a^2*c*x^2] + (15*a^3*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*
x]*PolyLog[3, I*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + (13*a^3*c^3*Sqrt[
1 + a^2*x^2]*PolyLog[3, E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - ((15*I)*a
^3*c^3*Sqrt[1 + a^2*x^2]*PolyLog[4, (-I)*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c
*x^2] + ((15*I)*a^3*c^3*Sqrt[1 + a^2*x^2]*PolyLog[4, I*E^(I*ArcTan[a*x])])/
Sqrt[c + a^2*c*x^2]
```

Rubi [A] time = 3.37545, antiderivative size = 1061, normalized size of antiderivative = 1., number of steps used = 86, number of rules used = 18, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75, Rules used = {4950, 4944, 4962, 266, 63, 208, 4958, 4956, 4183, 2531, 2282, 6589, 4890, 4888, 4181, 6609, 4880, 4886}

$$\frac{1}{2}c^2x\sqrt{a^2cx^2+c}\tan^{-1}(ax)^3a^4-\frac{5ic^3\sqrt{a^2x^2+1}\tan^{-1}\left(e^{i\tan^{-1}(ax)}\right)\tan^{-1}(ax)^3a^3}{\sqrt{a^2cx^2+c}}-\frac{3}{2}c^2\sqrt{a^2cx^2+c}\tan^{-1}(ax)^2a^3-\frac{6ic^3\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3)/x^4,x]
```

```
[Out] -((a^2*c^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/x) - (3*a^3*c^2*Sqrt[c + a^2*c*
x^2]*ArcTan[a*x]^2)/2 - (a*c^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(2*x^2) -
 (2*a^2*c^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/x + (a^4*c^2*x*Sqrt[c + a^2*
c*x^2]*ArcTan[a*x]^3)/2 - (c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3)/(3*x^3) -
 ((5*I)*a^3*c^3*Sqrt[1 + a^2*x^2]*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^3)/
Sqrt[c + a^2*c*x^2] - ((6*I)*a^3*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTan[S
qrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2] - (13*a^3*c^3*Sqrt[1 +
 a^2*x^2]*ArcTan[a*x]^2*ArcTanh[E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - a
^3*c^(5/2)*ArcTanh[Sqrt[c + a^2*c*x^2]/Sqrt[c]] + ((13*I)*a^3*c^3*Sqrt[1 +
a^2*x^2]*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] +
(((15*I)/2)*a^3*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog[2, (-I)*E^(I*Ar
cTan[a*x])])/Sqrt[c + a^2*c*x^2] - (((15*I)/2)*a^3*c^3*Sqrt[1 + a^2*x^2]*Ar
cTan[a*x]^2*PolyLog[2, I*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - ((13*I)*
a^3*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, E^(I*ArcTan[a*x])])/Sqrt[c
 + a^2*c*x^2] + ((3*I)*a^3*c^3*Sqrt[1 + a^2*x^2]*PolyLog[2, ((-I)*Sqrt[1 +
I*a*x])/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2] - ((3*I)*a^3*c^3*Sqrt[1 + a^2
*x^2]*PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2]
- (13*a^3*c^3*Sqrt[1 + a^2*x^2]*PolyLog[3, -E^(I*ArcTan[a*x])])/Sqrt[c + a^
2*c*x^2] - (15*a^3*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[3, (-I)*E^(I*A
rcTan[a*x])])/Sqrt[c + a^2*c*x^2] + (15*a^3*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*
x]*PolyLog[3, I*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + (13*a^3*c^3*Sqrt[
1 + a^2*x^2]*PolyLog[3, E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - ((15*I)*a
^3*c^3*Sqrt[1 + a^2*x^2]*PolyLog[4, (-I)*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c
*x^2] + ((15*I)*a^3*c^3*Sqrt[1 + a^2*x^2]*PolyLog[4, I*E^(I*ArcTan[a*x])])/
Sqrt[c + a^2*c*x^2]
```

Rule 4950

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 4944

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a +
 b*ArcTan[c*x])^p)/(d*f*(m + 1)), x] - Dist[(b*c*p)/(f*(m + 1)), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
```

```
& NeQ[m, -1]
```

Rule 4962

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Ar
cTan[c*x])^p)/(d*f*(m + 1)), x] + (-Dist[(b*c*p)/(f*(m + 1)), Int[((f*x)^(m
 + 1)*(a + b*ArcTan[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] - Dist[(c^2*(m +
 2))/(f^2*(m + 1)), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/Sqrt[d + e*x^
2], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] &
& LtQ[m, -1] && NeQ[m, -2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 4958

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4956

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Dist[1/Sqrt[d], Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTan
[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && G
tQ[d, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4890

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4888

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[c
*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && GtQ
[d, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 4880

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> -Simp[(b*p*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1))/(2*c*q*(2*
q + 1)), x] + (Dist[(2*d*q)/(2*q + 1), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTa
n[c*x])^p, x], x] + Dist[(b^2*d*p*(p - 1))/(2*q*(2*q + 1)), Int[(d + e*x^2)
^(q - 1)*(a + b*ArcTan[c*x])^(p - 2), x], x] + Simp[(x*(d + e*x^2)^q*(a + b
*ArcTan[c*x])^p)/(2*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[q, 0] && GtQ[p, 1]
```

Rule 4886

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[(-2*I*(a + b*ArcTan[c*x])*ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]])
/(c*Sqrt[d]), x] + (Simp[(I*b*PolyLog[2, -((I*Sqrt[1 + I*c*x])/Sqrt[1 - I*c
*x])])/(c*Sqrt[d]), x] - Simp[(I*b*PolyLog[2, (I*Sqrt[1 + I*c*x])/Sqrt[1 -
I*c*x]])/(c*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

Rubi steps

$$\int \frac{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^3}{x^4}\,dx = c\int \frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^3}{x^4}\,dx + \left(a^2c\right)\int \frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^3}{x^2}\,dx$$

$$= c^2\int \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{x^4}\,dx + 2\left(\left(a^2c^2\right)\int \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{x^2}\,dx\right) + \left(a^4c^2\right)\int$$

$$= -\frac{3}{2}a^3c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2 + \frac{1}{2}a^4c^2x\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3 - \frac{c\left(c+a^2cx^2\right)^{3/2}\tan^{-}}{3x^3}$$

$$= -\frac{3}{2}a^3c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2 + \frac{1}{2}a^4c^2x\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3 - \frac{c\left(c+a^2cx^2\right)^{3/2}\tan^{-}}{3x^3}$$

$$= -\frac{3}{2}a^3c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2 - \frac{ac^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2x^2} + \frac{1}{2}a^4c^2x\sqrt{c+a^2cx^2}\tan^{-1}$$

$$= -\frac{a^2c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{x} - \frac{3}{2}a^3c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2 - \frac{ac^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{2x^2}$$

$$= -\frac{a^2c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{x} - \frac{3}{2}a^3c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2 - \frac{ac^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{2x^2}$$

$$= -\frac{a^2c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{x} - \frac{3}{2}a^3c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2 - \frac{ac^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{2x^2}$$

$$= -\frac{a^2c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{x} - \frac{3}{2}a^3c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2 - \frac{ac^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{2x^2}$$

$$= -\frac{a^2c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{x} - \frac{3}{2}a^3c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2 - \frac{ac^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{2x^2}$$

$$= -\frac{a^2c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{x} - \frac{3}{2}a^3c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2 - \frac{ac^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{2x^2}$$

Mathematica [A] time = 11.4207, size = 1771, normalized size = 1.67

result too large to display

Warning: Unable to verify antiderivative.

[In] Integrate[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3)/x^4,x]

[Out] ((-I/2)*a^3*c^2*Sqrt[c*(1 + a^2*x^2)]*(12*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x] - (3*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2 + I*a*x*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^3 + 2*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^3 - 3*(2 + ArcTan[a*x]^2)*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] + 3*(2 + ArcTan[a*x]^2)*PolyLog[2, I*E^(I*ArcTan[a*x])] - (6*I)*ArcTan[a*x]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] + (6*I)*ArcTan[a*x]*PolyLog[3, I*E^(I*ArcTan[a*x])] + 6*PolyLog[4, (-I)*E^(I*ArcTan[a*x])] - 6*PolyLog[4, I*E^(I*ArcTan[a*x])]))/Sqrt[1 + a^2*x^2] + (a^3*c^2*Sqrt[c*(1 + a^2*x^2)]*Csc[ArcTan[a*x]/2]*(((-7*I)*a*Pi^4*x)/Sqrt[1 + a^2*x^2] - ((8*I)*a*Pi^3*x*ArcTan[a*x])/Sqrt[1 + a^2*x^2] + ((24*I)*a*Pi^2*x*ArcTan[a*x]^2)/Sqrt[1 + a^2*x^2] - 64*ArcTan[a*x]^3 - ((32*I)*a*Pi*x*ArcTan[a*x]^3)/Sqrt[1 + a^2*x^2] + ((16*I)*a*x*ArcTan[a*x]^4)/Sqrt[1 + a^2*x^2] + (48*a*Pi^2*x*ArcTan[a*x]*Log[1 - I/E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - (96*a*Pi*x*ArcTan[a*x]^2*Log[1 - I/E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - (8*a*Pi^3*x*Log[1 + I/E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + (64*a*x*ArcTan[a*x]^3*Log[1 + I/E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + (192*a*x*ArcTan[a*x]^2*Log[1 - E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + (8*a*Pi^3*x*Log[1 + I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - (48*a*Pi^2*x*ArcTan[a*x]*Log[1 + I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + (96*a*Pi*x*ArcTan[a*x]^2*Log[1 + I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - (64*a*x*ArcTan[a*x]^3*Log[1 + I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - (192*a*x*ArcTan[a*x]^2*Log[1 + E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + (8*a*Pi^3*x*Log[Tan[(Pi + 2*ArcTan[a*x])/4]])/Sqrt[1 + a^2*x^2] + ((192*I)*a*x*ArcTan[a*x]^2*PolyLog[2, (-I)/E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + ((48*I)*a*Pi*x*(Pi - 4*ArcTan[a*x])*PolyLog[2, I/E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + ((384*I)*a*x*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + ((48*I)*a*Pi^2*x*PolyLog[2, (-I)*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - ((192*I)*a*Pi*x*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + ((192*I)*a*x*ArcTan[a*x]^2*PolyLog[2, (-I)*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - ((384*I)*a*x*ArcTan[a*x]*PolyLog[2, E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + (384*a*x*ArcTan[a*x]*PolyLog[3, (-I)/E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - (192*a*Pi*x*PolyLog[3, I/E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - (384*a*x*PolyLog[3, -E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + (192*a*Pi*x*PolyLog[3, (-I)*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - (384*a*x*ArcTan[a*x]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + (384*a*x*PolyLog[3, E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - ((384*I)*a*x*PolyLog[4, (-I)/E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - ((384*I)*a*x*PolyLog[4, (-I)*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2])*Sec[ArcTan[a*x]/2])/(64*Sqrt[1 + a^2*x^2]) + (a^3*c^3*Sqrt[1 + a^2*x^2]*(-12*ArcTan[a*x]*Cot[ArcTan[a*x]/2] - 2*ArcTan[a*x]^3*Cot[ArcTan[a*x]/2] - 3*ArcTan[a*x]^2*Csc[ArcTan[a*x]/2]^2 - (a*x*ArcTan[a*x]^3*Csc[ArcTan[a*x]/2]^4)/(2*Sqrt[1 + a^2*x^2]) + 12*ArcTan[a*x]^2*Log[1 - E^(I*ArcTan[a*x])] - 12*ArcTan[a*x]^2*Log[1 + E^(I*ArcTan[a*x])] + 24*Log[Tan[ArcTan[a*x]/2]] + (24*I)*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])] - (24*I)*ArcT

```
an[a*x]*PolyLog[2, E^(I*ArcTan[a*x])] - 24*PolyLog[3, -E^(I*ArcTan[a*x])] +
 24*PolyLog[3, E^(I*ArcTan[a*x])] + 3*ArcTan[a*x]^2*Sec[ArcTan[a*x]/2]^2 -
(8*(1 + a^2*x^2)^(3/2)*ArcTan[a*x]^3*Sin[ArcTan[a*x]/2]^4)/(a^3*x^3) - 12*A
rcTan[a*x]*Tan[ArcTan[a*x]/2] - 2*ArcTan[a*x]^3*Tan[ArcTan[a*x]/2]))/(24*Sq
rt[c*(1 + a^2*x^2)])
```

Maple [A] time = 2.281, size = 694, normalized size = 0.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(5/2)*arctan(a*x)^3/x^4,x)
```

```
[Out] 1/6*c^2*(c*(a*x-I)*(a*x+I))^(1/2)*arctan(a*x)*(3*arctan(a*x)^2*x^4*a^4-9*ar
ctan(a*x)*x^3*a^3-14*arctan(a*x)^2*x^2*a^2-6*a^2*x^2-3*arctan(a*x)*x*a-2*ar
ctan(a*x)^2)/x^3+1/2*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)^(1/2)*(5*arctan(
a*x)^3*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-5*arctan(a*x)^3*ln(1+I*(1+I*a*x)
/(a^2*x^2+1)^(1/2))-13*arctan(a*x)^2*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))+13*a
rctan(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-15*I*arctan(a*x)^2*polylog(2
,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+15*I*arctan(a*x)^2*polylog(2,-I*(1+I*a*x)/(
a^2*x^2+1)^(1/2))+6*arctan(a*x)*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+30*arct
an(a*x)*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-30*arctan(a*x)*polylog(3,-
I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*arctan(a*x)*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(
1/2))+30*I*polylog(4,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*I*polylog(2,I*(1+I*a*
x)/(a^2*x^2+1)^(1/2))+2*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)-1)-2*ln(1+(1+I*a*x)/
(a^2*x^2+1)^(1/2))+26*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))-26*polylog(3,-
(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-
26*I*arctan(a*x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+26*I*arctan(a*x)*po
lylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-30*I*polylog(4,-I*(1+I*a*x)/(a^2*x^2+
1)^(1/2)))*a^3*c^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^3/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^4c^2x^4+2\,a^2c^2x^2+c^2\right)\sqrt{a^2cx^2+c}\,\arctan\left(ax\right)^3}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^3/x^4,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*arctan(a*x
)^3/x^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(5/2)*atan(a*x)**3/x**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a^2cx^2+c\right)^{\frac{5}{2}}\arctan\left(ax\right)^3}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^3/x^4,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^3/x^4, x)
```

3.436 $\int \frac{x^3 \tan^{-1}(ax)^3}{\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=408

$$\frac{5i\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,-ie^{i\tan^{-1}(ax)}\right)}{a^4\sqrt{a^2cx^2+c}}-\frac{5i\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,ie^{i\tan^{-1}(ax)}\right)}{a^4\sqrt{a^2cx^2+c}}-\frac{5\sqrt{a^2x^2+1}\text{PolyLo}}{a^4\sqrt{a^2c}}$$

```
[Out] (Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(a^4*c) - (x*Sqrt[c + a^2*c*x^2]*ArcTan[a
*x]^2)/(2*a^3*c) - ((5*I)*Sqrt[1 + a^2*x^2]*ArcTan[E^(I*ArcTan[a*x])]*ArcTa
n[a*x]^2)/(a^4*Sqrt[c + a^2*c*x^2]) - (2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)
/(3*a^4*c) + (x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/(3*a^2*c) - ArcTanh[(a
*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]]/(a^4*Sqrt[c]) + ((5*I)*Sqrt[1 + a^2*x^2]*A
rcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])])/(a^4*Sqrt[c + a^2*c*x^2]) -
((5*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])])/(a^4*
Sqrt[c + a^2*c*x^2]) - (5*Sqrt[1 + a^2*x^2]*PolyLog[3, (-I)*E^(I*ArcTan[a*x
])])/(a^4*Sqrt[c + a^2*c*x^2]) + (5*Sqrt[1 + a^2*x^2]*PolyLog[3, I*E^(I*Arc
Tan[a*x])])/(a^4*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.728413, antiderivative size = 408, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {4952, 4930, 217, 206, 4890, 4888, 4181, 2531, 2282, 6589}

$$\frac{5i\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,-ie^{i\tan^{-1}(ax)}\right)}{a^4\sqrt{a^2cx^2+c}}-\frac{5i\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,ie^{i\tan^{-1}(ax)}\right)}{a^4\sqrt{a^2cx^2+c}}-\frac{5\sqrt{a^2x^2+1}\text{PolyLo}}{a^4\sqrt{a^2c}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcTan[a*x]^3)/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] (Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(a^4*c) - (x*Sqrt[c + a^2*c*x^2]*ArcTan[a
*x]^2)/(2*a^3*c) - ((5*I)*Sqrt[1 + a^2*x^2]*ArcTan[E^(I*ArcTan[a*x])]*ArcTa
n[a*x]^2)/(a^4*Sqrt[c + a^2*c*x^2]) - (2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)
/(3*a^4*c) + (x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/(3*a^2*c) - ArcTanh[(a
*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]]/(a^4*Sqrt[c]) + ((5*I)*Sqrt[1 + a^2*x^2]*A
rcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])])/(a^4*Sqrt[c + a^2*c*x^2]) -
((5*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])])/(a^4*
Sqrt[c + a^2*c*x^2]) - (5*Sqrt[1 + a^2*x^2]*PolyLog[3, (-I)*E^(I*ArcTan[a*x
])])/(a^4*Sqrt[c + a^2*c*x^2]) + (5*Sqrt[1 + a^2*x^2]*PolyLog[3, I*E^(I*Arc
Tan[a*x])])/(a^4*Sqrt[c + a^2*c*x^2])
```

Rule 4952

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b*
ArcTan[c*x])^p)/(c^2*d*m), x] + (-Dist[(b*f*p)/(c*m), Int[((f*x)^(m - 1)*(a
 + b*ArcTan[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] - Dist[(f^2*(m - 1))/(c^
2*m), Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/Sqrt[d + e*x^2], x], x]) /;
 FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 4890

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4888

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[c
*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && GtQ
[d, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
```

```
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^3 \tan^{-1}(ax)^3}{\sqrt{c+a^2cx^2}}\,dx = \frac{x^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{3a^2c} - \frac{2\int \frac{x\tan^{-1}(ax)^3}{\sqrt{c+a^2cx^2}}\,dx}{3a^2} - \frac{\int \frac{x^2\tan^{-1}(ax)^2}{\sqrt{c+a^2cx^2}}\,dx}{a}$$

$$= -\frac{x\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2a^3c} - \frac{2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{3a^4c} + \frac{x^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{3a^2c} + \frac{\int \frac{\tan^{-1}(ax)^2}{\sqrt{c+a^2cx^2}}\,d}{2a^3}$$

$$= \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{a^4c} - \frac{x\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2a^3c} - \frac{2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{3a^4c} + \frac{x^2\sqrt{c+a^2cx^2}\tan}{3a^2c}$$

$$= \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{a^4c} - \frac{x\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2a^3c} - \frac{2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{3a^4c} + \frac{x^2\sqrt{c+a^2cx^2}\tan}{3a^2c}$$

$$= \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{a^4c} - \frac{x\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2a^3c} - \frac{5i\sqrt{1+a^2x^2}\tan^{-1}\left(e^{i\tan^{-1}(ax)}\right)\tan^{-1}(ax)^2}{a^4\sqrt{c+a^2cx^2}} -$$

$$= \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{a^4c} - \frac{x\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2a^3c} - \frac{5i\sqrt{1+a^2x^2}\tan^{-1}\left(e^{i\tan^{-1}(ax)}\right)\tan^{-1}(ax)^2}{a^4\sqrt{c+a^2cx^2}} -$$

$$= \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{a^4c} - \frac{x\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2a^3c} - \frac{5i\sqrt{1+a^2x^2}\tan^{-1}\left(e^{i\tan^{-1}(ax)}\right)\tan^{-1}(ax)^2}{a^4\sqrt{c+a^2cx^2}} -$$

$$= \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{a^4c} - \frac{x\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2a^3c} - \frac{5i\sqrt{1+a^2x^2}\tan^{-1}\left(e^{i\tan^{-1}(ax)}\right)\tan^{-1}(ax)^2}{a^4\sqrt{c+a^2cx^2}} -$$

Mathematica [A] time = 0.79027, size = 220, normalized size = 0.54

$$\frac{\sqrt{a^2cx^2+c}\left(-\left(a^2x^2+1\right)\tan^{-1}(ax)\left(2\tan^{-1}(ax)^2+3\tan^{-1}(ax)\sin\left(2\tan^{-1}(ax)\right)+6\left(\tan^{-1}(ax)^2-1\right)\cos\left(2\tan^{-1}(ax\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*ArcTan[a*x]^3)/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] (Sqrt[c + a^2*c*x^2]*((12*((-5*I)*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2 -
 ArcTanh[(a*x)/Sqrt[1 + a^2*x^2]] + (5*I)*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*
ArcTan[a*x])] - (5*I)*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])] - 5*PolyL
og[3, (-I)*E^(I*ArcTan[a*x])] + 5*PolyLog[3, I*E^(I*ArcTan[a*x])]))/Sqrt[1
+ a^2*x^2] - (1 + a^2*x^2)*ArcTan[a*x]*(-6 + 2*ArcTan[a*x]^2 + 6*(-1 + ArcT
an[a*x]^2)*Cos[2*ArcTan[a*x]] + 3*ArcTan[a*x]*Sin[2*ArcTan[a*x]])))/(12*a^4
```

```
*c)
```

Maple [A] time = 4.271, size = 380, normalized size = 0.9

$$\frac{\left(2\left(\arctan\left(ax\right)\right)^{2}x^{2}a^{2}-3\arctan\left(ax\right)xa-4\left(\arctan\left(ax\right)\right)^{2}+6\right)\arctan\left(ax\right)}{6\,c a^{4}}\sqrt{c\left(ax-i\right)\left(ax+i\right)}+\frac{\frac{5i}{6}}{c a^{4}}\left(3i\left(\arctan\left(ax\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] 1/6*(2*arctan(a*x)^2*x^2*a^2-3*arctan(a*x)*x*a-4*arctan(a*x)^2+6)*arctan(a*
x)*(c*(a*x-I)*(a*x+I))^(1/2)/c/a^4+5/6*I*(3*I*arctan(a*x)^2*ln(1+I*(1+I*a*x
)/(a^2*x^2+1)^(1/2))+arctan(a*x)^3+6*arctan(a*x)*polylog(2,-I*(1+I*a*x)/(a^
2*x^2+1)^(1/2))+6*I*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))*(c*(a*x-I)*(
a*x+I))^(1/2)/(a^2*x^2+1)^(1/2)/a^4/c-5/6*I*(3*I*arctan(a*x)^2*ln(1-I*(1+I*
a*x)/(a^2*x^2+1)^(1/2))+arctan(a*x)^3+6*arctan(a*x)*polylog(2,I*(1+I*a*x)/(
a^2*x^2+1)^(1/2))+6*I*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))*(c*(a*x-I)*
(a*x+I))^(1/2)/(a^2*x^2+1)^(1/2)/a^4/c+2*I*arctan((1+I*a*x)/(a^2*x^2+1)^(1/
2))*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)^(1/2)/a^4/c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^{3}\arctan\left(ax\right)^{3}}{\sqrt{a^{2}cx^{2}+c}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^3*arctan(a*x)^3/sqrt(a^2*c*x^2 + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \operatorname{atan}^3 (ax)}{\sqrt{c \left(a^2 x^2 + 1\right)}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atan(a*x)**3/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**3*atan(a*x)**3/sqrt(c*(a**2*x**2 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \arctan (ax)^3}{\sqrt{a^2 c x^2 + c}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^3*arctan(a*x)^3/sqrt(a^2*c*x^2 + c), x)
```

3.437 $\int \frac{x^2 \tan^{-1}(ax)^3}{\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=625

$$\frac{3i\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a^3\sqrt{a^2cx^2+c}}-\frac{3i\sqrt{a^2x^2+1}\text{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a^3\sqrt{a^2cx^2+c}}-\frac{3i\sqrt{a^2x^2+1}\tan^{-1}(ax)^2\text{PolyLog}\left(2,-ie^{i\tan^{-1}(a}\right.}{2a^3\sqrt{a^2cx^2+c}}$$

```
[Out] (-3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(2*a^3*c) + (x*Sqrt[c + a^2*c*x^2]*A
rcTan[a*x]^3)/(2*a^2*c) + (I*Sqrt[1 + a^2*x^2]*ArcTan[E^(I*ArcTan[a*x])]*Ar
cTan[a*x]^3)/(a^3*Sqrt[c + a^2*c*x^2]) - ((6*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*
x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/(a^3*Sqrt[c + a^2*c*x^2]) - (((
3*I)/2)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog[2, (-I)*E^(I*ArcTan[a*x])])
/(a^3*Sqrt[c + a^2*c*x^2]) + (((3*I)/2)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*Pol
yLog[2, I*E^(I*ArcTan[a*x])])/(a^3*Sqrt[c + a^2*c*x^2]) + ((3*I)*Sqrt[1 + a
^2*x^2]*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/(a^3*Sqrt[c + a
^2*c*x^2]) - ((3*I)*Sqrt[1 + a^2*x^2]*PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1
 - I*a*x]])/(a^3*Sqrt[c + a^2*c*x^2]) + (3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*Po
lyLog[3, (-I)*E^(I*ArcTan[a*x])])/(a^3*Sqrt[c + a^2*c*x^2]) - (3*Sqrt[1 + a
^2*x^2]*ArcTan[a*x]*PolyLog[3, I*E^(I*ArcTan[a*x])])/(a^3*Sqrt[c + a^2*c*x^
2]) + ((3*I)*Sqrt[1 + a^2*x^2]*PolyLog[4, (-I)*E^(I*ArcTan[a*x])])/(a^3*Sqr
t[c + a^2*c*x^2]) - ((3*I)*Sqrt[1 + a^2*x^2]*PolyLog[4, I*E^(I*ArcTan[a*x])
])/(a^3*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.489111, antiderivative size = 625, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {4952, 4930, 4890, 4886, 4888, 4181, 2531, 6609, 2282, 6589}

$$\frac{3i\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a^3\sqrt{a^2cx^2+c}}-\frac{3i\sqrt{a^2x^2+1}\text{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a^3\sqrt{a^2cx^2+c}}-\frac{3i\sqrt{a^2x^2+1}\tan^{-1}(ax)^2\text{PolyLog}\left(2,-ie^{i\tan^{-1}(a}\right.}{2a^3\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcTan[a*x]^3)/Sqrt[c + a^2*c*x^2],x]
```


```
[Out] (-3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(2*a^3*c) + (x*Sqrt[c + a^2*c*x^2]*A
rcTan[a*x]^3)/(2*a^2*c) + (I*Sqrt[1 + a^2*x^2]*ArcTan[E^(I*ArcTan[a*x])]*Ar
cTan[a*x]^3)/(a^3*Sqrt[c + a^2*c*x^2]) - ((6*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*
x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/(a^3*Sqrt[c + a^2*c*x^2]) - (((
3*I)/2)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog[2, (-I)*E^(I*ArcTan[a*x])])
```

```
/(a^3*Sqrt[c + a^2*c*x^2]) + (((3*I)/2)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*Pol
yLog[2, I*E^(I*ArcTan[a*x])])/(a^3*Sqrt[c + a^2*c*x^2]) + ((3*I)*Sqrt[1 + a
^2*x^2]*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/(a^3*Sqrt[c + a
^2*c*x^2]) - ((3*I)*Sqrt[1 + a^2*x^2]*PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1
 - I*a*x]])/(a^3*Sqrt[c + a^2*c*x^2]) + (3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*Po
lyLog[3, (-I)*E^(I*ArcTan[a*x])])/(a^3*Sqrt[c + a^2*c*x^2]) - (3*Sqrt[1 + a
^2*x^2]*ArcTan[a*x]*PolyLog[3, I*E^(I*ArcTan[a*x])])/(a^3*Sqrt[c + a^2*c*x^
2]) + ((3*I)*Sqrt[1 + a^2*x^2]*PolyLog[4, (-I)*E^(I*ArcTan[a*x])])/(a^3*Sqr
t[c + a^2*c*x^2]) - ((3*I)*Sqrt[1 + a^2*x^2]*PolyLog[4, I*E^(I*ArcTan[a*x])
])/(a^3*Sqrt[c + a^2*c*x^2])
```

Rule 4952

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b*
ArcTan[c*x])^p)/(c^2*d*m), x] + (-Dist[(b*f*p)/(c*m), Int[((f*x)^(m - 1)*(a
 + b*ArcTan[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] - Dist[(f^2*(m - 1))/(c^
2*m), Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/Sqrt[d + e*x^2], x], x]) /;
 FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 4890

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4886

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[(-2*I*(a + b*ArcTan[c*x])*ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]])
/(c*Sqrt[d]), x] + (Simp[(I*b*PolyLog[2, -((I*Sqrt[1 + I*c*x])/Sqrt[1 - I*c
*x])])/(c*Sqrt[d]), x] - Simp[(I*b*PolyLog[2, (I*Sqrt[1 + I*c*x])/Sqrt[1 -
I*c*x]])/(c*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

Rule 4888

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[c
*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && GtQ
[d, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^2 \tan^{-1}(ax)^3}{\sqrt{c+a^2cx^2}}\,dx = \frac{x\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{2a^2c} - \frac{\int \frac{\tan^{-1}(ax)^3}{\sqrt{c+a^2cx^2}}\,dx}{2a^2} - \frac{3\int \frac{x\tan^{-1}(ax)^2}{\sqrt{c+a^2cx^2}}\,dx}{2a}$$

$$= -\frac{3\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2a^3c} + \frac{x\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{2a^2c} + \frac{3\int \frac{\tan^{-1}(ax)}{\sqrt{c+a^2cx^2}}\,dx}{a^2} - \frac{\sqrt{1+a^2x^2}\int \frac{\tan^{-1}(ax)^3}{\sqrt{1+a^2x^2}}}{2a^2\sqrt{c+a^2cx^2}}$$

$$= -\frac{3\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2a^3c} + \frac{x\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{2a^2c} - \frac{\sqrt{1+a^2x^2}\,\mathrm{Subst}\left(\int x^3\sec(x)\,dx, x, \tan^{-}\right.}{2a^3\sqrt{c+a^2cx^2}}$$

$$= -\frac{3\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2a^3c} + \frac{x\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{2a^2c} + \frac{i\sqrt{1+a^2x^2}\tan^{-1}\left(e^{i\tan^{-1}(ax)}\right)\tan^{-1}(ax)^3}{a^3\sqrt{c+a^2cx^2}}$$

$$= -\frac{3\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2a^3c} + \frac{x\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{2a^2c} + \frac{i\sqrt{1+a^2x^2}\tan^{-1}\left(e^{i\tan^{-1}(ax)}\right)\tan^{-1}(ax)^3}{a^3\sqrt{c+a^2cx^2}}$$

$$= -\frac{3\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2a^3c} + \frac{x\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{2a^2c} + \frac{i\sqrt{1+a^2x^2}\tan^{-1}\left(e^{i\tan^{-1}(ax)}\right)\tan^{-1}(ax)^3}{a^3\sqrt{c+a^2cx^2}}$$

$$= -\frac{3\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2a^3c} + \frac{x\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{2a^2c} + \frac{i\sqrt{1+a^2x^2}\tan^{-1}\left(e^{i\tan^{-1}(ax)}\right)\tan^{-1}(ax)^3}{a^3\sqrt{c+a^2cx^2}}$$

$$= -\frac{3\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2a^3c} + \frac{x\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{2a^2c} + \frac{i\sqrt{1+a^2x^2}\tan^{-1}\left(e^{i\tan^{-1}(ax)}\right)\tan^{-1}(ax)^3}{a^3\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 6.00671, size = 812, normalized size = 1.3

$$\frac{\sqrt{c\left(a^2x^2+1\right)}\left(-\frac{1}{2}i\tan^{-1}(ax)^4 - 2\log\left(1+ie^{-i\tan^{-1}(ax)}\right)\tan^{-1}(ax)^3 + 2\log\left(1+ie^{i\tan^{-1}(ax)}\right)\tan^{-1}(ax)^3 + \frac{\tan}{\left(\cos\left(\frac{1}{2}\tan^{-1}(ax\right.\right.}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*ArcTan[a*x]^3)/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] (Sqrt[c*(1 + a^2*x^2)]*(((7*I)/32)*Pi^4 + (I/4)*Pi^3*ArcTan[a*x] - 6*ArcTan
[a*x]^2 - ((3*I)/4)*Pi^2*ArcTan[a*x]^2 + I*Pi*ArcTan[a*x]^3 - (I/2)*ArcTan[
a*x]^4 - (3*Pi^2*ArcTan[a*x]*Log[1 - I/E^(I*ArcTan[a*x])])/2 + 3*Pi*ArcTan[
a*x]^2*Log[1 - I/E^(I*ArcTan[a*x])] + (Pi^3*Log[1 + I/E^(I*ArcTan[a*x])])/4
```

```
 - 2*ArcTan[a*x]^3*Log[1 + I/E^(I*ArcTan[a*x])] + 12*ArcTan[a*x]*Log[1 - I*
E^(I*ArcTan[a*x])] - (Pi^3*Log[1 + I*E^(I*ArcTan[a*x])])/4 - 12*ArcTan[a*x]
*Log[1 + I*E^(I*ArcTan[a*x])] + (3*Pi^2*ArcTan[a*x]*Log[1 + I*E^(I*ArcTan[a
*x])])/2 - 3*Pi*ArcTan[a*x]^2*Log[1 + I*E^(I*ArcTan[a*x])] + 2*ArcTan[a*x]^
3*Log[1 + I*E^(I*ArcTan[a*x])] - (Pi^3*Log[Tan[(Pi + 2*ArcTan[a*x])/4]])/4
- (6*I)*ArcTan[a*x]^2*PolyLog[2, (-I)/E^(I*ArcTan[a*x])] - ((3*I)/2)*Pi*(Pi
 - 4*ArcTan[a*x])*PolyLog[2, I/E^(I*ArcTan[a*x])] + (12*I)*PolyLog[2, (-I)*
E^(I*ArcTan[a*x])] - ((3*I)/2)*Pi^2*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] + (6
*I)*Pi*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - (6*I)*ArcTan[a*x]^2
*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - (12*I)*PolyLog[2, I*E^(I*ArcTan[a*x])
] - 12*ArcTan[a*x]*PolyLog[3, (-I)/E^(I*ArcTan[a*x])] + 6*Pi*PolyLog[3, I/E
^(I*ArcTan[a*x])] - 6*Pi*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] + 12*ArcTan[a*x
]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] + (12*I)*PolyLog[4, (-I)/E^(I*ArcTan[a
*x])] + (12*I)*PolyLog[4, (-I)*E^(I*ArcTan[a*x])] + ArcTan[a*x]^3/(Cos[ArcT
an[a*x]/2] - Sin[ArcTan[a*x]/2])^2 - (6*ArcTan[a*x]^2*Sin[ArcTan[a*x]/2])/(
Cos[ArcTan[a*x]/2] - Sin[ArcTan[a*x]/2]) - ArcTan[a*x]^3/(Cos[ArcTan[a*x]/2
] + Sin[ArcTan[a*x]/2])^2 + (6*ArcTan[a*x]^2*Sin[ArcTan[a*x]/2])/(Cos[ArcTa
n[a*x]/2] + Sin[ArcTan[a*x]/2])))/(4*a^3*c*Sqrt[1 + a^2*x^2])
```

Maple [A] time = 3.78, size = 430, normalized size = 0.7

$$\frac{(\arctan(ax)\,xa-3)(\arctan(ax))^2}{2\,ca^3}\sqrt{c\,(ax-i)(ax+i)}-\frac{\frac{i}{2}}{ca^3}\left(i\,(\arctan(ax))^3\ln\left(1+i\,(1+iax)\frac{1}{\sqrt{a^2x^2+1}}\right)-i\,(\arctan($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] 1/2*(arctan(a*x)*x*a-3)*arctan(a*x)^2*(c*(a*x-I)*(a*x+I))^(1/2)/c/a^3-1/2*I
*(I*arctan(a*x)^3*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*arctan(a*x)^3*ln(1-
I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+3*arctan(a*x)^2*polylog(2,-I*(1+I*a*x)/(a^2*
x^2+1)^(1/2))-3*arctan(a*x)^2*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*I*
arctan(a*x)*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I*arctan(a*x)*polylog(3,-
I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I*arctan(a*x)*ln(1-I*(1+I*a*x)/(a^2*x^2+1)
^(1/2))-6*I*arctan(a*x)*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*polylog(
4,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*polylog(4,I*(1+I*a*x)/(a^2*x^2+1)^(1/2)
)-6*dilog(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*dilog(1-I*(1+I*a*x)/(a^2*x^2+1
)^(1/2)))*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)^(1/2)/a^3/c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^2 \arctan(ax)^3}{\sqrt{a^2cx^2+c}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^2*arctan(a*x)^3/sqrt(a^2*c*x^2 + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \operatorname{atan}^3(ax)}{\sqrt{c(a^2x^2+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(a*x)**3/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**2*atan(a*x)**3/sqrt(c*(a**2*x**2 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \arctan(ax)^3}{\sqrt{a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctan(a*x)^3/sqrt(a^2*c*x^2 + c), x)
```

3.438 $\int \frac{x\tan^{-1}(ax)^3}{\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=283

$$-\frac{6i\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,-ie^{i\tan^{-1}(ax)}\right)}{a^2\sqrt{a^2cx^2+c}}+\frac{6i\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,ie^{i\tan^{-1}(ax)}\right)}{a^2\sqrt{a^2cx^2+c}}+\frac{6\sqrt{a^2x^2+1}\text{Poly}}{a^2\sqrt{}}$$

```
[Out] ((6*I)*Sqrt[1 + a^2*x^2]*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2)/(a^2*Sqrt
[c + a^2*c*x^2]) + (Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/(a^2*c) - ((6*I)*Sqr
t[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])])/(a^2*Sqrt[c
+ a^2*c*x^2]) + ((6*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcT
an[a*x])])/(a^2*Sqrt[c + a^2*c*x^2]) + (6*Sqrt[1 + a^2*x^2]*PolyLog[3, (-I)
*E^(I*ArcTan[a*x])])/(a^2*Sqrt[c + a^2*c*x^2]) - (6*Sqrt[1 + a^2*x^2]*PolyL
og[3, I*E^(I*ArcTan[a*x])])/(a^2*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.233566, antiderivative size = 283, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {4930, 4890, 4888, 4181, 2531, 2282, 6589}

$$-\frac{6i\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,-ie^{i\tan^{-1}(ax)}\right)}{a^2\sqrt{a^2cx^2+c}}+\frac{6i\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,ie^{i\tan^{-1}(ax)}\right)}{a^2\sqrt{a^2cx^2+c}}+\frac{6\sqrt{a^2x^2+1}\text{Poly}}{a^2\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTan[a*x]^3)/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] ((6*I)*Sqrt[1 + a^2*x^2]*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2)/(a^2*Sqrt
[c + a^2*c*x^2]) + (Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/(a^2*c) - ((6*I)*Sqr
t[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])])/(a^2*Sqrt[c
+ a^2*c*x^2]) + ((6*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcT
an[a*x])])/(a^2*Sqrt[c + a^2*c*x^2]) + (6*Sqrt[1 + a^2*x^2]*PolyLog[3, (-I)
*E^(I*ArcTan[a*x])])/(a^2*Sqrt[c + a^2*c*x^2]) - (6*Sqrt[1 + a^2*x^2]*PolyL
og[3, I*E^(I*ArcTan[a*x])])/(a^2*Sqrt[c + a^2*c*x^2])
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
```

```
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 4890

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4888

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[c
*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && GtQ
[d, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x\tan^{-1}(ax)^3}{\sqrt{c+a^2cx^2}}\,dx = \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{a^2c} - \frac{3\int \frac{\tan^{-1}(ax)^2}{\sqrt{c+a^2cx^2}}\,dx}{a}$$

$$= \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{a^2c} - \frac{\left(3\sqrt{1+a^2x^2}\right)\int \frac{\tan^{-1}(ax)^2}{\sqrt{1+a^2x^2}}\,dx}{a\sqrt{c+a^2cx^2}}$$

$$= \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{a^2c} - \frac{\left(3\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int x^2\sec(x)\,dx, x, \tan^{-1}(ax)\right)}{a^2\sqrt{c+a^2cx^2}}$$

$$= \frac{6i\sqrt{1+a^2x^2}\tan^{-1}\left(e^{i\tan^{-1}(ax)}\right)\tan^{-1}(ax)^2}{a^2\sqrt{c+a^2cx^2}} + \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{a^2c} + \frac{\left(6\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int x\right.}{a^2\sqrt{c}}$$

$$= \frac{6i\sqrt{1+a^2x^2}\tan^{-1}\left(e^{i\tan^{-1}(ax)}\right)\tan^{-1}(ax)^2}{a^2\sqrt{c+a^2cx^2}} + \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{a^2c} - \frac{6i\sqrt{1+a^2x^2}\tan^{-1}(ax)\text{Li}_2}{a^2\sqrt{c+a^2cx}}$$

$$= \frac{6i\sqrt{1+a^2x^2}\tan^{-1}\left(e^{i\tan^{-1}(ax)}\right)\tan^{-1}(ax)^2}{a^2\sqrt{c+a^2cx^2}} + \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{a^2c} - \frac{6i\sqrt{1+a^2x^2}\tan^{-1}(ax)\text{Li}_2}{a^2\sqrt{c+a^2cx}}$$

$$= \frac{6i\sqrt{1+a^2x^2}\tan^{-1}\left(e^{i\tan^{-1}(ax)}\right)\tan^{-1}(ax)^2}{a^2\sqrt{c+a^2cx^2}} + \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{a^2c} - \frac{6i\sqrt{1+a^2x^2}\tan^{-1}(ax)\text{Li}_2}{a^2\sqrt{c+a^2cx}}$$

Mathematica [A] time = 0.275091, size = 168, normalized size = 0.59

$$\frac{\sqrt{c\left(a^2x^2+1\right)}\left(\tan^{-1}(ax)^3 - \frac{3\left(2i\tan^{-1}(ax)\left(\text{PolyLog}\left(2,-ie^{i\tan^{-1}(ax)}\right)-\text{PolyLog}\left(2,ie^{i\tan^{-1}(ax)}\right)\right)-2\text{PolyLog}\left(3,-ie^{i\tan^{-1}(ax)}\right)+2\text{PolyLog}\left(3,ie^{i\tan^{-1}}\right.\right.}{\sqrt{a^2x^2+1}}}{a^2c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*ArcTan[a*x]^3)/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] (Sqrt[c*(1 + a^2*x^2)]*(ArcTan[a*x]^3 - (3*(ArcTan[a*x]^2*(Log[1 - I*E^(I*A
rcTan[a*x])] - Log[1 + I*E^(I*ArcTan[a*x])]) + (2*I)*ArcTan[a*x]*(PolyLog[2
, (-I)*E^(I*ArcTan[a*x])] - PolyLog[2, I*E^(I*ArcTan[a*x])]) - 2*PolyLog[3,
 (-I)*E^(I*ArcTan[a*x])] + 2*PolyLog[3, I*E^(I*ArcTan[a*x])]))/Sqrt[1 + a^2
*x^2]))/(a^2*c)
```

Maple [F] time = 1.898, size = 0, normalized size = 0.

$$\int x\left(\arctan\left(ax\right)\right)^{3}\frac{1}{\sqrt{a^{2}c\,x^{2}+c}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(x*arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\arctan\left(ax\right)^{3}}{\sqrt{a^{2}cx^{2}+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x*arctan(a*x)^3/sqrt(a^2*c*x^2 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x\arctan\left(ax\right)^{3}}{\sqrt{a^{2}cx^{2}+c}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x*arctan(a*x)^3/sqrt(a^2*c*x^2 + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \operatorname{atan}^3(ax)}{\sqrt{c(a^2x^2+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(a*x)**3/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x*atan(a*x)**3/sqrt(c*(a**2*x**2 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \arctan(ax)^3}{\sqrt{a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x*arctan(a*x)^3/sqrt(a^2*c*x^2 + c), x)
```

3.439 $\int \frac{\tan^{-1}(ax)^3}{\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=368

$$\frac{3i\sqrt{a^2x^2+1}\tan^{-1}(ax)^2\text{PolyLog}\left(2,-ie^{i\tan^{-1}(ax)}\right)}{a\sqrt{a^2cx^2+c}}-\frac{3i\sqrt{a^2x^2+1}\tan^{-1}(ax)^2\text{PolyLog}\left(2,ie^{i\tan^{-1}(ax)}\right)}{a\sqrt{a^2cx^2+c}}-\frac{6\sqrt{a^2x^2+1}\tan^{-}}{}$$

```
[Out] ((-2*I)*Sqrt[1 + a^2*x^2]*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^3)/(a*Sqrt[
c + a^2*c*x^2]) + ((3*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog[2, (-I)*E^
(I*ArcTan[a*x])])/(a*Sqrt[c + a^2*c*x^2]) - ((3*I)*Sqrt[1 + a^2*x^2]*ArcTan
[a*x]^2*PolyLog[2, I*E^(I*ArcTan[a*x])])/(a*Sqrt[c + a^2*c*x^2]) - (6*Sqrt[
1 + a^2*x^2]*ArcTan[a*x]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])])/(a*Sqrt[c + a^
2*c*x^2]) + (6*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[3, I*E^(I*ArcTan[a*x])
])/(a*Sqrt[c + a^2*c*x^2]) - ((6*I)*Sqrt[1 + a^2*x^2]*PolyLog[4, (-I)*E^(I*
ArcTan[a*x])])/(a*Sqrt[c + a^2*c*x^2]) + ((6*I)*Sqrt[1 + a^2*x^2]*PolyLog[4
, I*E^(I*ArcTan[a*x])])/(a*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.185535, antiderivative size = 368, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4890, 4888, 4181, 2531, 6609, 2282, 6589}

$$\frac{3i\sqrt{a^2x^2+1}\tan^{-1}(ax)^2\text{PolyLog}\left(2,-ie^{i\tan^{-1}(ax)}\right)}{a\sqrt{a^2cx^2+c}}-\frac{3i\sqrt{a^2x^2+1}\tan^{-1}(ax)^2\text{PolyLog}\left(2,ie^{i\tan^{-1}(ax)}\right)}{a\sqrt{a^2cx^2+c}}-\frac{6\sqrt{a^2x^2+1}\tan^{-}}{}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^3/Sqrt[c + a^2*c*x^2],x]
```


```
[Out] ((-2*I)*Sqrt[1 + a^2*x^2]*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^3)/(a*Sqrt[
c + a^2*c*x^2]) + ((3*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog[2, (-I)*E^
(I*ArcTan[a*x])])/(a*Sqrt[c + a^2*c*x^2]) - ((3*I)*Sqrt[1 + a^2*x^2]*ArcTan
[a*x]^2*PolyLog[2, I*E^(I*ArcTan[a*x])])/(a*Sqrt[c + a^2*c*x^2]) - (6*Sqrt[
1 + a^2*x^2]*ArcTan[a*x]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])])/(a*Sqrt[c + a^
2*c*x^2]) + (6*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[3, I*E^(I*ArcTan[a*x])
])/(a*Sqrt[c + a^2*c*x^2]) - ((6*I)*Sqrt[1 + a^2*x^2]*PolyLog[4, (-I)*E^(I*
ArcTan[a*x])])/(a*Sqrt[c + a^2*c*x^2]) + ((6*I)*Sqrt[1 + a^2*x^2]*PolyLog[4
, I*E^(I*ArcTan[a*x])])/(a*Sqrt[c + a^2*c*x^2])
```


Rule 4890

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4888

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[c
*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && GtQ
[d, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^3}{\sqrt{c+a^2cx^2}}\,dx = \frac{\sqrt{1+a^2x^2}\int \frac{\tan^{-1}(ax)^3}{\sqrt{1+a^2x^2}}\,dx}{\sqrt{c+a^2cx^2}}$$
$$= \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int x^3\sec(x)\,dx, x, \tan^{-1}(ax)\right)}{a\sqrt{c+a^2cx^2}}$$
$$= -\frac{2i\sqrt{1+a^2x^2}\tan^{-1}\left(e^{i\tan^{-1}(ax)}\right)\tan^{-1}(ax)^3}{a\sqrt{c+a^2cx^2}} - \frac{\left(3\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int x^2\log\left(1-ie^{ix}\right)dx, x, \tan^{-1}(a\right.}{a\sqrt{c+a^2cx^2}}$$
$$= -\frac{2i\sqrt{1+a^2x^2}\tan^{-1}\left(e^{i\tan^{-1}(ax)}\right)\tan^{-1}(ax)^3}{a\sqrt{c+a^2cx^2}} + \frac{3i\sqrt{1+a^2x^2}\tan^{-1}(ax)^2\text{Li}_2\left(-ie^{i\tan^{-1}(ax)}\right)}{a\sqrt{c+a^2cx^2}} - \frac{3i\sqrt{1+a}}{}$$
$$= -\frac{2i\sqrt{1+a^2x^2}\tan^{-1}\left(e^{i\tan^{-1}(ax)}\right)\tan^{-1}(ax)^3}{a\sqrt{c+a^2cx^2}} + \frac{3i\sqrt{1+a^2x^2}\tan^{-1}(ax)^2\text{Li}_2\left(-ie^{i\tan^{-1}(ax)}\right)}{a\sqrt{c+a^2cx^2}} - \frac{3i\sqrt{1+a}}{}$$
$$= -\frac{2i\sqrt{1+a^2x^2}\tan^{-1}\left(e^{i\tan^{-1}(ax)}\right)\tan^{-1}(ax)^3}{a\sqrt{c+a^2cx^2}} + \frac{3i\sqrt{1+a^2x^2}\tan^{-1}(ax)^2\text{Li}_2\left(-ie^{i\tan^{-1}(ax)}\right)}{a\sqrt{c+a^2cx^2}} - \frac{3i\sqrt{1+a}}{}$$
$$= -\frac{2i\sqrt{1+a^2x^2}\tan^{-1}\left(e^{i\tan^{-1}(ax)}\right)\tan^{-1}(ax)^3}{a\sqrt{c+a^2cx^2}} + \frac{3i\sqrt{1+a^2x^2}\tan^{-1}(ax)^2\text{Li}_2\left(-ie^{i\tan^{-1}(ax)}\right)}{a\sqrt{c+a^2cx^2}} - \frac{3i\sqrt{1+a}}{}$$

Mathematica [A] time = 0.138806, size = 190, normalized size = 0.52

$$-\frac{i\sqrt{c\left(a^2x^2+1\right)}\left(-3\tan^{-1}(ax)^2\text{PolyLog}\left(2,-ie^{i\tan^{-1}(ax)}\right)+3\tan^{-1}(ax)^2\text{PolyLog}\left(2,ie^{i\tan^{-1}(ax)}\right)-6i\tan^{-1}(ax)\text{PolyLo}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]^3/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] ((-I)*Sqrt[c*(1 + a^2*x^2)]*(2*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^3 - 3*
ArcTan[a*x]^2*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] + 3*ArcTan[a*x]^2*PolyLog[
```

```
2, I*E^(I*ArcTan[a*x])] - (6*I)*ArcTan[a*x]*PolyLog[3, (-I)*E^(I*ArcTan[a*x
])] + (6*I)*ArcTan[a*x]*PolyLog[3, I*E^(I*ArcTan[a*x])] + 6*PolyLog[4, (-I)
*E^(I*ArcTan[a*x])] - 6*PolyLog[4, I*E^(I*ArcTan[a*x])]))/(a*c*Sqrt[1 + a^2
*x^2])
```

Maple [F] time = 1.368, size = 0, normalized size = 0.

$$\int (\arctan(ax))^3 \frac{1}{\sqrt{a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\arctan(ax)^3}{\sqrt{a^2cx^2+c}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(arctan(a*x)^3/sqrt(a^2*c*x^2 + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atan}^{3}\left(ax\right)}{\sqrt{c\left(a^{2}x^{2}+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**3/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(atan(a*x)**3/sqrt(c*(a**2*x**2 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan\left(ax\right)^{3}}{\sqrt{a^{2}cx^{2}+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)^3/sqrt(a^2*c*x^2 + c), x)
```

3.440 $\int \frac{\tan^{-1}(ax)^3}{x\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=327

$$\frac{3i\sqrt{a^2x^2+1}\tan^{-1}(ax)^2\text{PolyLog}\left(2,-e^{i\tan^{-1}(ax)}\right)}{\sqrt{a^2cx^2+c}}-\frac{3i\sqrt{a^2x^2+1}\tan^{-1}(ax)^2\text{PolyLog}\left(2,e^{i\tan^{-1}(ax)}\right)}{\sqrt{a^2cx^2+c}}-\frac{6\sqrt{a^2x^2+1}\tan^{-}}{}$$

```
[Out] (-2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^3*ArcTanh[E^(I*ArcTan[a*x])])/Sqrt[c + a^
2*c*x^2] + ((3*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog[2, -E^(I*ArcTan[a
*x])])/Sqrt[c + a^2*c*x^2] - ((3*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog
[2, E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - (6*Sqrt[1 + a^2*x^2]*ArcTan[a
*x]*PolyLog[3, -E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + (6*Sqrt[1 + a^2*x
^2]*ArcTan[a*x]*PolyLog[3, E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - ((6*I)
*Sqrt[1 + a^2*x^2]*PolyLog[4, -E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + ((
6*I)*Sqrt[1 + a^2*x^2]*PolyLog[4, E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2]
```

Rubi [A] time = 0.280881, antiderivative size = 327, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {4958, 4956, 4183, 2531, 6609, 2282, 6589}

$$\frac{3i\sqrt{a^2x^2+1}\tan^{-1}(ax)^2\text{PolyLog}\left(2,-e^{i\tan^{-1}(ax)}\right)}{\sqrt{a^2cx^2+c}}-\frac{3i\sqrt{a^2x^2+1}\tan^{-1}(ax)^2\text{PolyLog}\left(2,e^{i\tan^{-1}(ax)}\right)}{\sqrt{a^2cx^2+c}}-\frac{6\sqrt{a^2x^2+1}\tan^{-}}{}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^3/(x*Sqrt[c + a^2*c*x^2]),x]
```


```
[Out] (-2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^3*ArcTanh[E^(I*ArcTan[a*x])])/Sqrt[c + a^
2*c*x^2] + ((3*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog[2, -E^(I*ArcTan[a
*x])])/Sqrt[c + a^2*c*x^2] - ((3*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog
[2, E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - (6*Sqrt[1 + a^2*x^2]*ArcTan[a
*x]*PolyLog[3, -E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + (6*Sqrt[1 + a^2*x
^2]*ArcTan[a*x]*PolyLog[3, E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - ((6*I)
*Sqrt[1 + a^2*x^2]*PolyLog[4, -E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + ((
6*I)*Sqrt[1 + a^2*x^2]*PolyLog[4, E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2]
```


Rule 4958

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
```

```
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4956

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Dist[1/Sqrt[d], Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTan
[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && G
tQ[d, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^3}{x\sqrt{c+a^2cx^2}}\,dx = \frac{\sqrt{1+a^2x^2}\int \frac{\tan^{-1}(ax)^3}{x\sqrt{1+a^2x^2}}\,dx}{\sqrt{c+a^2cx^2}}$$
$$= \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int x^3\csc(x)\,dx, x, \tan^{-1}(ax)\right)}{\sqrt{c+a^2cx^2}}$$
$$= -\frac{2\sqrt{1+a^2x^2}\tan^{-1}(ax)^3\tanh^{-1}\left(e^{i\tan^{-1}(ax)}\right)}{\sqrt{c+a^2cx^2}} - \frac{\left(3\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int x^2\log\left(1-e^{ix}\right)\,dx, x, \tan^{\cdot}\right.}{\sqrt{c+a^2cx^2}}$$
$$= -\frac{2\sqrt{1+a^2x^2}\tan^{-1}(ax)^3\tanh^{-1}\left(e^{i\tan^{-1}(ax)}\right)}{\sqrt{c+a^2cx^2}} + \frac{3i\sqrt{1+a^2x^2}\tan^{-1}(ax)^2\text{Li}_2\left(-e^{i\tan^{-1}(ax)}\right)}{\sqrt{c+a^2cx^2}} - \frac{3i\sqrt{1}\cdots}{}$$
$$= -\frac{2\sqrt{1+a^2x^2}\tan^{-1}(ax)^3\tanh^{-1}\left(e^{i\tan^{-1}(ax)}\right)}{\sqrt{c+a^2cx^2}} + \frac{3i\sqrt{1+a^2x^2}\tan^{-1}(ax)^2\text{Li}_2\left(-e^{i\tan^{-1}(ax)}\right)}{\sqrt{c+a^2cx^2}} - \frac{3i\sqrt{1}\cdots}{}$$
$$= -\frac{2\sqrt{1+a^2x^2}\tan^{-1}(ax)^3\tanh^{-1}\left(e^{i\tan^{-1}(ax)}\right)}{\sqrt{c+a^2cx^2}} + \frac{3i\sqrt{1+a^2x^2}\tan^{-1}(ax)^2\text{Li}_2\left(-e^{i\tan^{-1}(ax)}\right)}{\sqrt{c+a^2cx^2}} - \frac{3i\sqrt{1}\cdots}{}$$
$$= -\frac{2\sqrt{1+a^2x^2}\tan^{-1}(ax)^3\tanh^{-1}\left(e^{i\tan^{-1}(ax)}\right)}{\sqrt{c+a^2cx^2}} + \frac{3i\sqrt{1+a^2x^2}\tan^{-1}(ax)^2\text{Li}_2\left(-e^{i\tan^{-1}(ax)}\right)}{\sqrt{c+a^2cx^2}} - \frac{3i\sqrt{1}\cdots}{}$$

Mathematica [A] time = 0.217309, size = 208, normalized size = 0.64

$$-\frac{i\sqrt{a^2x^2+1}\left(-24\tan^{-1}(ax)^2\text{PolyLog}\left(2,e^{-i\tan^{-1}(ax)}\right)-24\tan^{-1}(ax)^2\text{PolyLog}\left(2,-e^{i\tan^{-1}(ax)}\right)+48i\tan^{-1}(ax)\text{PolyL}\cdots\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTan[a*x]^3/(x*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] ((-I/8)*Sqrt[1 + a^2*x^2]*(Pi^4 - 2*ArcTan[a*x]^4 + (8*I)*ArcTan[a*x]^3*Log
[1 - E^((-I)*ArcTan[a*x])] - (8*I)*ArcTan[a*x]^3*Log[1 + E^(I*ArcTan[a*x])]
 - 24*ArcTan[a*x]^2*PolyLog[2, E^((-I)*ArcTan[a*x])] - 24*ArcTan[a*x]^2*Pol
```

```
yLog[2, -E^(I*ArcTan[a*x])] + (48*I)*ArcTan[a*x]*PolyLog[3, E^((-I)*ArcTan[
a*x])] - (48*I)*ArcTan[a*x]*PolyLog[3, -E^(I*ArcTan[a*x])] + 48*PolyLog[4,
E^((-I)*ArcTan[a*x])] + 48*PolyLog[4, -E^(I*ArcTan[a*x])]))/Sqrt[c*(1 + a^2
*x^2)]
```

Maple [A] time = 0.404, size = 261, normalized size = 0.8

$$\frac{i}{c}\left(i\left(\arctan\left(ax\right)\right)^{3}\ln\left(1+\left(1+iax\right)\frac{1}{\sqrt{a^{2}x^{2}+1}}\right)-i\left(\arctan\left(ax\right)\right)^{3}\ln\left(1-\left(1+iax\right)\frac{1}{\sqrt{a^{2}x^{2}+1}}\right)+3\left(\arctan\left(ax\right)\right)^{2}\operatorname{polylog}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^3/x/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] I*(I*arctan(a*x)^3*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*arctan(a*x)^3*ln(1-(
1+I*a*x)/(a^2*x^2+1)^(1/2))+3*arctan(a*x)^2*polylog(2,-(1+I*a*x)/(a^2*x^2+1
)^(1/2))-3*arctan(a*x)^2*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I*arctan(
a*x)*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*I*arctan(a*x)*polylog(3,(1+I
*a*x)/(a^2*x^2+1)^(1/2))-6*polylog(4,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*polylo
g(4,(1+I*a*x)/(a^2*x^2+1)^(1/2)))*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)^(1/
2)/c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a^{2}cx^{2}+c}\arctan\left(ax\right)^{3}}{a^{2}cx^{3}+cx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^3/(a^2*c*x^3 + c*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atan}^3(ax)}{x\sqrt{c(a^2x^2+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**3/x/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(atan(a*x)**3/(x*sqrt(c*(a**2*x**2 + 1))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(ax)^3}{\sqrt{a^2cx^2+c}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)^3/(sqrt(a^2*c*x^2 + c)*x), x)
```

3.441 $\int \frac{\tan^{-1}(ax)^3}{x^2\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=260

$$\frac{6ia\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,-e^{i\tan^{-1}(ax)}\right)}{\sqrt{a^2cx^2+c}}-\frac{6ia\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,e^{i\tan^{-1}(ax)}\right)}{\sqrt{a^2cx^2+c}}-\frac{6a\sqrt{a^2x^2+1}\text{PolyL}}{\sqrt{a^2c}}$$

```
[Out] -((Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/(c*x)) - (6*a*Sqrt[1 + a^2*x^2]*ArcTa
n[a*x]^2*ArcTanh[E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + ((6*I)*a*Sqrt[1
+ a^2*x^2]*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2]
- ((6*I)*a*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, E^(I*ArcTan[a*x])])/Sqr
t[c + a^2*c*x^2] - (6*a*Sqrt[1 + a^2*x^2]*PolyLog[3, -E^(I*ArcTan[a*x])])/S
qrt[c + a^2*c*x^2] + (6*a*Sqrt[1 + a^2*x^2]*PolyLog[3, E^(I*ArcTan[a*x])])/
Sqrt[c + a^2*c*x^2]
```

Rubi [A] time = 0.372423, antiderivative size = 260, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {4944, 4958, 4956, 4183, 2531, 2282, 6589}

$$\frac{6ia\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,-e^{i\tan^{-1}(ax)}\right)}{\sqrt{a^2cx^2+c}}-\frac{6ia\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,e^{i\tan^{-1}(ax)}\right)}{\sqrt{a^2cx^2+c}}-\frac{6a\sqrt{a^2x^2+1}\text{PolyL}}{\sqrt{a^2c}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^3/(x^2*Sqrt[c + a^2*c*x^2]),x]
```


```
[Out] -((Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/(c*x)) - (6*a*Sqrt[1 + a^2*x^2]*ArcTa
n[a*x]^2*ArcTanh[E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + ((6*I)*a*Sqrt[1
+ a^2*x^2]*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2]
- ((6*I)*a*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, E^(I*ArcTan[a*x])])/Sqr
t[c + a^2*c*x^2] - (6*a*Sqrt[1 + a^2*x^2]*PolyLog[3, -E^(I*ArcTan[a*x])])/S
qrt[c + a^2*c*x^2] + (6*a*Sqrt[1 + a^2*x^2]*PolyLog[3, E^(I*ArcTan[a*x])])/
Sqrt[c + a^2*c*x^2]
```


Rule 4944

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a +
 b*ArcTan[c*x])^p)/(d*f*(m + 1)), x] - Dist[(b*c*p)/(f*(m + 1)), Int[(f*x)^
```

(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] && NeQ[m, -1]

Rule 4958

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4956

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x_Symbol] :> Dist[1/Sqrt[d], Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && GtQ[d, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m - 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; FunctionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^3}{x^2\sqrt{c+a^2cx^2}}\,dx = -\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{cx} + (3a)\int \frac{\tan^{-1}(ax)^2}{x\sqrt{c+a^2cx^2}}\,dx$$

$$= -\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{cx} + \frac{\left(3a\sqrt{1+a^2x^2}\right)\int \frac{\tan^{-1}(ax)^2}{x\sqrt{1+a^2x^2}}\,dx}{\sqrt{c+a^2cx^2}}$$

$$= -\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{cx} + \frac{\left(3a\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int x^2\csc(x)\,dx, x, \tan^{-1}(ax)\right)}{\sqrt{c+a^2cx^2}}$$

$$= -\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{cx} - \frac{6a\sqrt{1+a^2x^2}\tan^{-1}(ax)^2\tanh^{-1}\left(e^{i\tan^{-1}(ax)}\right)}{\sqrt{c+a^2cx^2}} - \frac{\left(6a\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int\right.}{\sqrt{}}$$

$$= -\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{cx} - \frac{6a\sqrt{1+a^2x^2}\tan^{-1}(ax)^2\tanh^{-1}\left(e^{i\tan^{-1}(ax)}\right)}{\sqrt{c+a^2cx^2}} + \frac{6ia\sqrt{1+a^2x^2}\tan^{-1}(ax}{\sqrt{c+a^2c}}$$

$$= -\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{cx} - \frac{6a\sqrt{1+a^2x^2}\tan^{-1}(ax)^2\tanh^{-1}\left(e^{i\tan^{-1}(ax)}\right)}{\sqrt{c+a^2cx^2}} + \frac{6ia\sqrt{1+a^2x^2}\tan^{-1}(ax}{\sqrt{c+a^2c}}$$

$$= -\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{cx} - \frac{6a\sqrt{1+a^2x^2}\tan^{-1}(ax)^2\tanh^{-1}\left(e^{i\tan^{-1}(ax)}\right)}{\sqrt{c+a^2cx^2}} + \frac{6ia\sqrt{1+a^2x^2}\tan^{-1}(ax}{\sqrt{c+a^2c}}$$

Mathematica [A] time = 0.325929, size = 174, normalized size = 0.67

$$-\frac{a\sqrt{a^2x^2+1}\left(-6i\tan^{-1}(ax)\text{PolyLog}\left(2,-e^{i\tan^{-1}(ax)}\right)+6i\tan^{-1}(ax)\text{PolyLog}\left(2,e^{i\tan^{-1}(ax)}\right)+6\text{PolyLog}\left(3,-e^{i\tan^{-1}(ax)}\right)\right.}{\sqrt{c\left(a^2x\right.}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTan[a*x]^3/(x^2*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] -((a*Sqrt[1 + a^2*x^2]*((Sqrt[1 + a^2*x^2]*ArcTan[a*x]^3)/(a*x) - 3*ArcTan[
a*x]^2*Log[1 - E^(I*ArcTan[a*x])] + 3*ArcTan[a*x]^2*Log[1 + E^(I*ArcTan[a*x
])] - (6*I)*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])] + (6*I)*ArcTan[a*x]*
```

```
PolyLog[2, E^(I*ArcTan[a*x])] + 6*PolyLog[3, -E^(I*ArcTan[a*x])] - 6*PolyLo
g[3, E^(I*ArcTan[a*x])]))/Sqrt[c*(1 + a^2*x^2)])
```

Maple [A] time = 0.38, size = 230, normalized size = 0.9

$$-\frac{(\arctan(ax))^3}{cx}\sqrt{c(ax-i)(ax+i)}-3\frac{a\sqrt{c(ax-i)(ax+i)}}{\sqrt{a^2x^2+1}c}\left((\arctan(ax))^2\ln\left(1+\frac{1+iax}{\sqrt{a^2x^2+1}}\right)-(\arctan(ax))^2\ln\left(1-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^3/x^2/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] -arctan(a*x)^3*(c*(a*x-I)*(a*x+I))^(1/2)/c/x-3*a*(arctan(a*x)^2*ln(1+(1+I*a
*x)/(a^2*x^2+1)^(1/2))-arctan(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*I*
arctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*I*arctan(a*x)*polylog
(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-2
*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2)))*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2
+1)^(1/2)/c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x^2/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{a^2cx^4+cx^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x^2/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^3/(a^2*c*x^4 + c*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atan}^3(ax)}{x^2\sqrt{c\left(a^2x^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**3/x**2/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(atan(a*x)**3/(x**2*sqrt(c*(a**2*x**2 + 1))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(ax)^3}{\sqrt{a^2cx^2+c}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x^2/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)^3/(sqrt(a^2*c*x^2 + c)*x^2), x)
```

3.442 $\int \frac{\tan^{-1}(ax)^3}{x^3\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=597

$$\frac{3ia^2\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{a^2cx^2+c}}-\frac{3ia^2\sqrt{a^2x^2+1}\text{PolyLog}\left(2,\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{a^2cx^2+c}}-\frac{3ia^2\sqrt{a^2x^2+1}\tan^{-1}(ax)^2\text{PolyLog}\left(2,-e\right.}{2\sqrt{a^2cx^2+c}}$$

```
[Out] (-3*a*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(2*c*x) - (Sqrt[c + a^2*c*x^2]*Arc
Tan[a*x]^3)/(2*c*x^2) + (a^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^3*ArcTanh[E^(I*A
rcTan[a*x])])/Sqrt[c + a^2*c*x^2] - (6*a^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*Ar
cTanh[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2] - (((3*I)/2)*a^
2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog[2, -E^(I*ArcTan[a*x])])/Sqrt[c +
a^2*c*x^2] + (((3*I)/2)*a^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog[2, E^(I
*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + ((3*I)*a^2*Sqrt[1 + a^2*x^2]*PolyLog[
2, -(Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x])])/Sqrt[c + a^2*c*x^2] - ((3*I)*a^2*Sq
rt[1 + a^2*x^2]*PolyLog[2, Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c
*x^2] + (3*a^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[3, -E^(I*ArcTan[a*x])]
)/Sqrt[c + a^2*c*x^2] - (3*a^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[3, E^(
I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + ((3*I)*a^2*Sqrt[1 + a^2*x^2]*PolyLog
[4, -E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - ((3*I)*a^2*Sqrt[1 + a^2*x^2]
*PolyLog[4, E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2]
```

Rubi [A] time = 0.684633, antiderivative size = 597, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {4962, 4944, 4958, 4954, 4956, 4183, 2531, 6609, 2282, 6589}

$$\frac{3ia^2\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{a^2cx^2+c}}-\frac{3ia^2\sqrt{a^2x^2+1}\text{PolyLog}\left(2,\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{a^2cx^2+c}}-\frac{3ia^2\sqrt{a^2x^2+1}\tan^{-1}(ax)^2\text{PolyLog}\left(2,-e\right.}{2\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^3/(x^3*Sqrt[c + a^2*c*x^2]),x]
```


```
[Out] (-3*a*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(2*c*x) - (Sqrt[c + a^2*c*x^2]*Arc
Tan[a*x]^3)/(2*c*x^2) + (a^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^3*ArcTanh[E^(I*A
rcTan[a*x])])/Sqrt[c + a^2*c*x^2] - (6*a^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*Ar
cTanh[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2] - (((3*I)/2)*a^
2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog[2, -E^(I*ArcTan[a*x])])/Sqrt[c +
a^2*c*x^2] + (((3*I)/2)*a^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog[2, E^(I
```

```
*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + ((3*I)*a^2*Sqrt[1 + a^2*x^2]*PolyLog[
2, -(Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x])])/Sqrt[c + a^2*c*x^2] - ((3*I)*a^2*Sq
rt[1 + a^2*x^2]*PolyLog[2, Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c
*x^2] + (3*a^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[3, -E^(I*ArcTan[a*x])]
)/Sqrt[c + a^2*c*x^2] - (3*a^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[3, E^(
I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + ((3*I)*a^2*Sqrt[1 + a^2*x^2]*PolyLog
[4, -E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - ((3*I)*a^2*Sqrt[1 + a^2*x^2]
*PolyLog[4, E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2]
```

Rule 4962

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Ar
cTan[c*x])^p)/(d*f*(m + 1)), x] + (-Dist[(b*c*p)/(f*(m + 1)), Int[((f*x)^(m
 + 1)*(a + b*ArcTan[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] - Dist[(c^2*(m +
 2))/(f^2*(m + 1)), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/Sqrt[d + e*x^
2], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] &
& LtQ[m, -1] && NeQ[m, -2]
```

Rule 4944

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a +
 b*ArcTan[c*x])^p)/(d*f*(m + 1)), x] - Dist[(b*c*p)/(f*(m + 1)), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rule 4958

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4954

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x_
Symbol] :> Simp[(-2*(a + b*ArcTan[c*x])*ArcTanh[Sqrt[1 + I*c*x]/Sqrt[1 - I*
c*x]])/Sqrt[d], x] + (Simp[(I*b*PolyLog[2, -(Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x
])])/Sqrt[d], x] - Simp[(I*b*PolyLog[2, Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]])/S
qrt[d], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

Rule 4956

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Dist[1/Sqrt[d], Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTan
[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && G
tQ[d, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^3}{x^3\sqrt{c+a^2cx^2}}\,dx = -\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{2cx^2} + \frac{1}{2}(3a)\int \frac{\tan^{-1}(ax)^2}{x^2\sqrt{c+a^2cx^2}}\,dx - \frac{1}{2}a^2\int \frac{\tan^{-1}(ax)^3}{x\sqrt{c+a^2cx^2}}\,dx$$

$$= -\frac{3a\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2cx} - \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{2cx^2} + \left(3a^2\right)\int \frac{\tan^{-1}(ax)}{x\sqrt{c+a^2cx^2}}\,dx - \frac{\left(a^2\sqrt{1+a^2x^2}\right)}{2\sqrt{c+}}$$

$$= -\frac{3a\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2cx} - \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{2cx^2} - \frac{\left(a^2\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int x^3\csc(x)\,dx, x, \text{ta}\right.}{2\sqrt{c+a^2cx^2}}$$

$$= -\frac{3a\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2cx} - \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{2cx^2} + \frac{a^2\sqrt{1+a^2x^2}\tan^{-1}(ax)^3\tanh^{-1}\left(e^{i\tan^{-1}(ax}\right.}{\sqrt{c+a^2cx^2}}$$

$$= -\frac{3a\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2cx} - \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{2cx^2} + \frac{a^2\sqrt{1+a^2x^2}\tan^{-1}(ax)^3\tanh^{-1}\left(e^{i\tan^{-1}(ax}\right.}{\sqrt{c+a^2cx^2}}$$

$$= -\frac{3a\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2cx} - \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{2cx^2} + \frac{a^2\sqrt{1+a^2x^2}\tan^{-1}(ax)^3\tanh^{-1}\left(e^{i\tan^{-1}(ax}\right.}{\sqrt{c+a^2cx^2}}$$

$$= -\frac{3a\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2cx} - \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{2cx^2} + \frac{a^2\sqrt{1+a^2x^2}\tan^{-1}(ax)^3\tanh^{-1}\left(e^{i\tan^{-1}(ax}\right.}{\sqrt{c+a^2cx^2}}$$

$$= -\frac{3a\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2cx} - \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{2cx^2} + \frac{a^2\sqrt{1+a^2x^2}\tan^{-1}(ax)^3\tanh^{-1}\left(e^{i\tan^{-1}(ax}\right.}{\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 3.78778, size = 345, normalized size = 0.58

$$\frac{a^2\sqrt{a^2x^2+1}\left(-24i\tan^{-1}(ax)^2\text{PolyLog}\left(2,e^{-i\tan^{-1}(ax)}\right) - 48\tan^{-1}(ax)\text{PolyLog}\left(3,e^{-i\tan^{-1}(ax)}\right) + 48\tan^{-1}(ax)\text{PolyLog}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTan[a*x]^3/(x^3*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] (a^2*Sqrt[1 + a^2*x^2]*(I*Pi^4 - (2*I)*ArcTan[a*x]^4 - 12*ArcTan[a*x]^2*Cot
[ArcTan[a*x]/2] - 2*ArcTan[a*x]^3*Csc[ArcTan[a*x]/2]^2 - 8*ArcTan[a*x]^3*Lo
g[1 - E^((-I)*ArcTan[a*x])] + 48*ArcTan[a*x]*Log[1 - E^(I*ArcTan[a*x])] - 4
8*ArcTan[a*x]*Log[1 + E^(I*ArcTan[a*x])] + 8*ArcTan[a*x]^3*Log[1 + E^(I*Arc
Tan[a*x])] - (24*I)*ArcTan[a*x]^2*PolyLog[2, E^((-I)*ArcTan[a*x])] - (24*I)
*(-2 + ArcTan[a*x]^2)*PolyLog[2, -E^(I*ArcTan[a*x])] - (48*I)*PolyLog[2, E^
```

```
(I*ArcTan[a*x])] - 48*ArcTan[a*x]*PolyLog[3, E^((-I)*ArcTan[a*x])] + 48*Arc
Tan[a*x]*PolyLog[3, -E^(I*ArcTan[a*x])] + (48*I)*PolyLog[4, E^((-I)*ArcTan[
a*x])] + (48*I)*PolyLog[4, -E^(I*ArcTan[a*x])] + 2*ArcTan[a*x]^3*Sec[ArcTan
[a*x]/2]^2 - 12*ArcTan[a*x]^2*Tan[ArcTan[a*x]/2]))/(16*Sqrt[c*(1 + a^2*x^2)
])
```

Maple [A] time = 0.508, size = 410, normalized size = 0.7

$$-\frac{(3\,ax+\arctan(ax))\,(\arctan(ax))^2}{2\,cx^2}\sqrt{c\,(ax-i)\,(ax+i)}+\frac{\frac{i}{2}a^2}{c}\left(i\,(\arctan(ax))^3\ln\left(1-(1+iax)\frac{1}{\sqrt{a^2x^2+1}}\right)-i\,(\arctan$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^3/x^3/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] -1/2*(3*a*x+arctan(a*x))*arctan(a*x)^2*(c*(a*x-I)*(a*x+I))^(1/2)/x^2/c+1/2*
I*a^2*(I*arctan(a*x)^3*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*arctan(a*x)^3*ln
(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))+3*arctan(a*x)^2*polylog(2,(1+I*a*x)/(a^2*x^
2+1)^(1/2))-3*arctan(a*x)^2*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*I*arc
tan(a*x)*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I*arctan(a*x)*polylog(3,(1+I*a
*x)/(a^2*x^2+1)^(1/2))+6*I*arctan(a*x)*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*
I*arctan(a*x)*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*polylog(2,(1+I*a*x)
/(a^2*x^2+1)^(1/2))-6*polylog(4,(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*polylog(2,-(
1+I*a*x)/(a^2*x^2+1)^(1/2))+6*polylog(4,-(1+I*a*x)/(a^2*x^2+1)^(1/2)))*(c*(
a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)^(1/2)/c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x^3/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{a^2cx^5+cx^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x^3/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^3/(a^2*c*x^5 + c*x^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atan}^3(ax)}{x^3\sqrt{c(a^2x^2+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**3/x**3/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(atan(a*x)**3/(x**3*sqrt(c*(a**2*x**2 + 1))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(ax)^3}{\sqrt{a^2cx^2+c}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x^3/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)^3/(sqrt(a^2*c*x^2 + c)*x^3), x)
```

3.443 $\int \frac{\tan^{-1}(ax)^3}{x^4\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=396

$$-\frac{5ia^3\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,-e^{i\tan^{-1}(ax)}\right)}{\sqrt{a^2cx^2+c}}+\frac{5ia^3\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,e^{i\tan^{-1}(ax)}\right)}{\sqrt{a^2cx^2+c}}+\frac{5a^3\sqrt{a^2x^2+}}{}$$

```
[Out] -((a^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(c*x)) - (a*Sqrt[c + a^2*c*x^2]*Arc
Tan[a*x]^2)/(2*c*x^2) - (Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/(3*c*x^3) + (2*
a^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/(3*c*x) + (5*a^3*Sqrt[1 + a^2*x^2]*A
rcTan[a*x]^2*ArcTanh[E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - (a^3*ArcTanh
[Sqrt[c + a^2*c*x^2]/Sqrt[c]])/Sqrt[c] - ((5*I)*a^3*Sqrt[1 + a^2*x^2]*ArcTa
n[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + ((5*I)*a^3*Sqr
t[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^
2] + (5*a^3*Sqrt[1 + a^2*x^2]*PolyLog[3, -E^(I*ArcTan[a*x])])/Sqrt[c + a^2*
c*x^2] - (5*a^3*Sqrt[1 + a^2*x^2]*PolyLog[3, E^(I*ArcTan[a*x])])/Sqrt[c + a
^2*c*x^2]
```

Rubi [A] time = 0.986335, antiderivative size = 396, normalized size of antiderivative = 1., number of steps used = 25, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {4962, 4944, 266, 63, 208, 4958, 4956, 4183, 2531, 2282, 6589}

$$-\frac{5ia^3\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,-e^{i\tan^{-1}(ax)}\right)}{\sqrt{a^2cx^2+c}}+\frac{5ia^3\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,e^{i\tan^{-1}(ax)}\right)}{\sqrt{a^2cx^2+c}}+\frac{5a^3\sqrt{a^2x^2+}}{}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^3/(x^4*Sqrt[c + a^2*c*x^2]),x]
```


```
[Out] -((a^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(c*x)) - (a*Sqrt[c + a^2*c*x^2]*Arc
Tan[a*x]^2)/(2*c*x^2) - (Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/(3*c*x^3) + (2*
a^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/(3*c*x) + (5*a^3*Sqrt[1 + a^2*x^2]*A
rcTan[a*x]^2*ArcTanh[E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - (a^3*ArcTanh
[Sqrt[c + a^2*c*x^2]/Sqrt[c]])/Sqrt[c] - ((5*I)*a^3*Sqrt[1 + a^2*x^2]*ArcTa
n[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + ((5*I)*a^3*Sqr
t[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^
2] + (5*a^3*Sqrt[1 + a^2*x^2]*PolyLog[3, -E^(I*ArcTan[a*x])])/Sqrt[c + a^2*
c*x^2] - (5*a^3*Sqrt[1 + a^2*x^2]*PolyLog[3, E^(I*ArcTan[a*x])])/Sqrt[c + a
```

```
^2*c*x^2]
```

Rule 4962

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Ar
cTan[c*x])^p)/(d*f*(m + 1)), x] + (-Dist[(b*c*p)/(f*(m + 1)), Int[((f*x)^(m
 + 1)*(a + b*ArcTan[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] - Dist[(c^2*(m +
 2))/(f^2*(m + 1)), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/Sqrt[d + e*x^
2], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] &
& LtQ[m, -1] && NeQ[m, -2]
```

Rule 4944

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a +
 b*ArcTan[c*x])^p)/(d*f*(m + 1)), x] - Dist[(b*c*p)/(f*(m + 1)), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 4958

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
```

```
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4956

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Dist[1/Sqrt[d], Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTan
[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && G
tQ[d, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^3}{x^4\sqrt{c+a^2cx^2}}\,dx = -\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{3cx^3} + a\int \frac{\tan^{-1}(ax)^2}{x^3\sqrt{c+a^2cx^2}}\,dx - \frac{1}{3}\left(2a^2\right)\int \frac{\tan^{-1}(ax)^3}{x^2\sqrt{c+a^2cx^2}}\,dx$$

$$= -\frac{a\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2cx^2} - \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{3cx^3} + \frac{2a^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{3cx} + a^2\int \frac{\tan^{-1}(}{x^2\sqrt{c+a}}$$

$$= -\frac{a^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{cx} - \frac{a\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2cx^2} - \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{3cx^3} + \frac{2a^2\sqrt{c+a^2cx^2}\,\mathrm{ta}}{3cx}$$

$$= -\frac{a^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{cx} - \frac{a\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2cx^2} - \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{3cx^3} + \frac{2a^2\sqrt{c+a^2cx^2}\,\mathrm{ta}}{3cx}$$

$$= -\frac{a^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{cx} - \frac{a\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2cx^2} - \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{3cx^3} + \frac{2a^2\sqrt{c+a^2cx^2}\,\mathrm{ta}}{3cx}$$

$$= -\frac{a^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{cx} - \frac{a\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2cx^2} - \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{3cx^3} + \frac{2a^2\sqrt{c+a^2cx^2}\,\mathrm{ta}}{3cx}$$

$$= -\frac{a^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{cx} - \frac{a\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2cx^2} - \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{3cx^3} + \frac{2a^2\sqrt{c+a^2cx^2}\,\mathrm{ta}}{3cx}$$

$$= -\frac{a^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{cx} - \frac{a\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2cx^2} - \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{3cx^3} + \frac{2a^2\sqrt{c+a^2cx^2}\,\mathrm{ta}}{3cx}$$

Mathematica [A] time = 5.65628, size = 343, normalized size = 0.87

$$\frac{a^3\sqrt{c\left(a^2x^2+1\right)}\left(-120i\tan^{-1}(ax)\mathrm{PolyLog}\left(2,-e^{i\tan^{-1}(ax)}\right)+120i\tan^{-1}(ax)\mathrm{PolyLog}\left(2,e^{i\tan^{-1}(ax)}\right)+120\mathrm{PolyLog}\left(3,-\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTan[a*x]^3/(x^4*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] (a^3*Sqrt[c*(1 + a^2*x^2)]*(-12*ArcTan[a*x]*Cot[ArcTan[a*x]/2] + 10*ArcTan[
a*x]^3*Cot[ArcTan[a*x]/2] - 3*ArcTan[a*x]^2*Csc[ArcTan[a*x]/2]^2 - (a*x*Arc
Tan[a*x]^3*Csc[ArcTan[a*x]/2]^4)/(2*Sqrt[1 + a^2*x^2]) - 60*ArcTan[a*x]^2*L
og[1 - E^(I*ArcTan[a*x])] + 60*ArcTan[a*x]^2*Log[1 + E^(I*ArcTan[a*x])] + 2
4*Log[Tan[ArcTan[a*x]/2]] - (120*I)*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x
```

```
])] + (120*I)*ArcTan[a*x]*PolyLog[2, E^(I*ArcTan[a*x])] + 120*PolyLog[3, -E
^(I*ArcTan[a*x])] - 120*PolyLog[3, E^(I*ArcTan[a*x])] + 3*ArcTan[a*x]^2*Sec
[ArcTan[a*x]/2]^2 - (8*(1 + a^2*x^2)^(3/2)*ArcTan[a*x]^3*Sin[ArcTan[a*x]/2]
^4)/(a^3*x^3) - 12*ArcTan[a*x]*Tan[ArcTan[a*x]/2] + 10*ArcTan[a*x]^3*Tan[Ar
cTan[a*x]/2]))/(24*c*Sqrt[1 + a^2*x^2])
```

Maple [A] time = 0.842, size = 487, normalized size = 1.2

$$\frac{\left(4\,(\arctan(ax))^2x^2a^2-6\,a^2x^2-3\,\arctan(ax)\,xa-2\,(\arctan(ax))^2\right)\arctan(ax)}{6\,cx^3}\sqrt{c\,(ax-i)\,(ax+i)}-2\,\frac{a^3\sqrt{c\,(ax-i)}}{\sqrt{a^2x^2+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^3/x^4/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] 1/6*(4*arctan(a*x)^2*x^2*a^2-6*a^2*x^2-3*arctan(a*x)*x*a-2*arctan(a*x)^2)*a
rctan(a*x)*(c*(a*x-I)*(a*x+I))^(1/2)/c/x^3-2*a^3*arctanh((1+I*a*x)/(a^2*x^2
+1)^(1/2))*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)^(1/2)/c-5/2*a^3*ln(1-(1+I*
a*x)/(a^2*x^2+1)^(1/2))*arctan(a*x)^2*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)
^(1/2)/c+5*I*a^3*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))*arctan(a*x)*(c*(a*x
-I)*(a*x+I))^(1/2)/(a^2*x^2+1)^(1/2)/c-5*a^3*polylog(3,(1+I*a*x)/(a^2*x^2+1
)^(1/2))*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)^(1/2)/c+5/2*a^3*ln(1+(1+I*a*
x)/(a^2*x^2+1)^(1/2))*arctan(a*x)^2*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)^(
1/2)/c-5*I*a^3*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))*arctan(a*x)*(c*(a*x-
I)*(a*x+I))^(1/2)/(a^2*x^2+1)^(1/2)/c+5*a^3*polylog(3,-(1+I*a*x)/(a^2*x^2+1
)^(1/2))*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)^(1/2)/c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x^4/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{a^2cx^6+cx^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x^4/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^3/(a^2*c*x^6 + c*x^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{atan}^3(ax)}{x^4\sqrt{c\left(a^2x^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**3/x**4/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(atan(a*x)**3/(x**4*sqrt(c*(a**2*x**2 + 1))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\arctan(ax)^3}{\sqrt{a^2cx^2+c}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x^4/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)^3/(sqrt(a^2*c*x^2 + c)*x^4), x)
```

3.444 $\int \frac{x^3 \tan^{-1}(ax)^3}{\left(c+a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=403

$$-\frac{6i\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,-ie^{i\tan^{-1}(ax)}\right)}{a^4c\sqrt{a^2cx^2+c}}+\frac{6i\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,ie^{i\tan^{-1}(ax)}\right)}{a^4c\sqrt{a^2cx^2+c}}+\frac{6\sqrt{a^2x^2+1}\text{Poly}}{a^4c}$$

```
[Out] (6*x)/(a^3*c*Sqrt[c + a^2*c*x^2]) - (6*ArcTan[a*x])/(a^4*c*Sqrt[c + a^2*c*x
^2]) - (3*x*ArcTan[a*x]^2)/(a^3*c*Sqrt[c + a^2*c*x^2]) + ((6*I)*Sqrt[1 + a^
2*x^2]*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2)/(a^4*c*Sqrt[c + a^2*c*x^2])
 + ArcTan[a*x]^3/(a^4*c*Sqrt[c + a^2*c*x^2]) + (Sqrt[c + a^2*c*x^2]*ArcTan[
a*x]^3)/(a^4*c^2) - ((6*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, (-I)*E^
(I*ArcTan[a*x])])/(a^4*c*Sqrt[c + a^2*c*x^2]) + ((6*I)*Sqrt[1 + a^2*x^2]*Ar
cTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])])/(a^4*c*Sqrt[c + a^2*c*x^2]) + (6
*Sqrt[1 + a^2*x^2]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])])/(a^4*c*Sqrt[c + a^2*
c*x^2]) - (6*Sqrt[1 + a^2*x^2]*PolyLog[3, I*E^(I*ArcTan[a*x])])/(a^4*c*Sqrt
[c + a^2*c*x^2])
```

Rubi [A] time = 0.515933, antiderivative size = 403, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {4964, 4930, 4890, 4888, 4181, 2531, 2282, 6589, 4898, 191}

$$-\frac{6i\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,-ie^{i\tan^{-1}(ax)}\right)}{a^4c\sqrt{a^2cx^2+c}}+\frac{6i\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,ie^{i\tan^{-1}(ax)}\right)}{a^4c\sqrt{a^2cx^2+c}}+\frac{6\sqrt{a^2x^2+1}\text{Poly}}{a^4c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcTan[a*x]^3)/(c + a^2*c*x^2)^(3/2),x]
```


```
[Out] (6*x)/(a^3*c*Sqrt[c + a^2*c*x^2]) - (6*ArcTan[a*x])/(a^4*c*Sqrt[c + a^2*c*x
^2]) - (3*x*ArcTan[a*x]^2)/(a^3*c*Sqrt[c + a^2*c*x^2]) + ((6*I)*Sqrt[1 + a^
2*x^2]*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2)/(a^4*c*Sqrt[c + a^2*c*x^2])
 + ArcTan[a*x]^3/(a^4*c*Sqrt[c + a^2*c*x^2]) + (Sqrt[c + a^2*c*x^2]*ArcTan[
a*x]^3)/(a^4*c^2) - ((6*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, (-I)*E^
(I*ArcTan[a*x])])/(a^4*c*Sqrt[c + a^2*c*x^2]) + ((6*I)*Sqrt[1 + a^2*x^2]*Ar
cTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])])/(a^4*c*Sqrt[c + a^2*c*x^2]) + (6
*Sqrt[1 + a^2*x^2]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])])/(a^4*c*Sqrt[c + a^2*
c*x^2]) - (6*Sqrt[1 + a^2*x^2]*PolyLog[3, I*E^(I*ArcTan[a*x])])/(a^4*c*Sqrt
```

```
[c + a^2*c*x^2])
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Arc
Tan[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan[c
*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p
, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 4890

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4888

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[c
*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && GtQ
[d, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
```

```
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4898

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[(b*p*(a + b*ArcTan[c*x])^(p - 1))/(c*d*Sqrt[d + e*x^2]), x]
 + (-Dist[b^2*p*(p - 1), Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2),
 x], x] + Simp[(x*(a + b*ArcTan[c*x])^p)/(d*Sqrt[d + e*x^2]), x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{x^3 \tan^{-1}(ax)^3}{\left(c+a^2cx^2\right)^{3/2}}\,dx = -\frac{\int \frac{x\tan^{-1}(ax)^3}{\left(c+a^2cx^2\right)^{3/2}}\,dx}{a^2} + \frac{\int \frac{x\tan^{-1}(ax)^3}{\sqrt{c+a^2cx^2}}\,dx}{a^2c}$$

$$= \frac{\tan^{-1}(ax)^3}{a^4c\sqrt{c+a^2cx^2}} + \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{a^4c^2} - \frac{3\int \frac{\tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^{3/2}}\,dx}{a^3} - \frac{3\int \frac{\tan^{-1}(ax)^2}{\sqrt{c+a^2cx^2}}\,dx}{a^3c}$$

$$= -\frac{6\tan^{-1}(ax)}{a^4c\sqrt{c+a^2cx^2}} - \frac{3x\tan^{-1}(ax)^2}{a^3c\sqrt{c+a^2cx^2}} + \frac{\tan^{-1}(ax)^3}{a^4c\sqrt{c+a^2cx^2}} + \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{a^4c^2} + \frac{6\int \frac{1}{\left(c+a^2cx^2\right)^{3/2}}\,dx}{a^3}$$

$$= \frac{6x}{a^3c\sqrt{c+a^2cx^2}} - \frac{6\tan^{-1}(ax)}{a^4c\sqrt{c+a^2cx^2}} - \frac{3x\tan^{-1}(ax)^2}{a^3c\sqrt{c+a^2cx^2}} + \frac{\tan^{-1}(ax)^3}{a^4c\sqrt{c+a^2cx^2}} + \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{a^4c^2} -$$

$$= \frac{6x}{a^3c\sqrt{c+a^2cx^2}} - \frac{6\tan^{-1}(ax)}{a^4c\sqrt{c+a^2cx^2}} - \frac{3x\tan^{-1}(ax)^2}{a^3c\sqrt{c+a^2cx^2}} + \frac{6i\sqrt{1+a^2x^2}\tan^{-1}\left(e^{i\tan^{-1}(ax)}\right)\tan^{-1}(ax)^2}{a^4c\sqrt{c+a^2cx^2}} +$$

$$= \frac{6x}{a^3c\sqrt{c+a^2cx^2}} - \frac{6\tan^{-1}(ax)}{a^4c\sqrt{c+a^2cx^2}} - \frac{3x\tan^{-1}(ax)^2}{a^3c\sqrt{c+a^2cx^2}} + \frac{6i\sqrt{1+a^2x^2}\tan^{-1}\left(e^{i\tan^{-1}(ax)}\right)\tan^{-1}(ax)^2}{a^4c\sqrt{c+a^2cx^2}} +$$

$$= \frac{6x}{a^3c\sqrt{c+a^2cx^2}} - \frac{6\tan^{-1}(ax)}{a^4c\sqrt{c+a^2cx^2}} - \frac{3x\tan^{-1}(ax)^2}{a^3c\sqrt{c+a^2cx^2}} + \frac{6i\sqrt{1+a^2x^2}\tan^{-1}\left(e^{i\tan^{-1}(ax)}\right)\tan^{-1}(ax)^2}{a^4c\sqrt{c+a^2cx^2}} +$$

$$= \frac{6x}{a^3c\sqrt{c+a^2cx^2}} - \frac{6\tan^{-1}(ax)}{a^4c\sqrt{c+a^2cx^2}} - \frac{3x\tan^{-1}(ax)^2}{a^3c\sqrt{c+a^2cx^2}} + \frac{6i\sqrt{1+a^2x^2}\tan^{-1}\left(e^{i\tan^{-1}(ax)}\right)\tan^{-1}(ax)^2}{a^4c\sqrt{c+a^2cx^2}} +$$

Mathematica [A] time = 0.795253, size = 308, normalized size = 0.76

$$\frac{\sqrt{a^2x^2+1}\left(-6i\tan^{-1}(ax)\text{PolyLog}\left(2,-ie^{i\tan^{-1}(ax)}\right)+6i\tan^{-1}(ax)\text{PolyLog}\left(2,ie^{i\tan^{-1}(ax)}\right)+6\text{PolyLog}\left(3,-ie^{i\tan^{-1}(ax)}\right)\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*ArcTan[a*x]^3)/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] (Sqrt[1 + a^2*x^2]*((6*a*x)/Sqrt[1 + a^2*x^2] - 3*Sqrt[1 + a^2*x^2]*ArcTan[
a*x] - (3*a*x*ArcTan[a*x]^2)/Sqrt[1 + a^2*x^2] + (3*Sqrt[1 + a^2*x^2]*ArcTa
n[a*x]^3)/2 - 3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*Cos[2*ArcTan[a*x]] + (Sqrt[1
+ a^2*x^2]*ArcTan[a*x]^3*Cos[2*ArcTan[a*x]])/2 - 3*ArcTan[a*x]^2*Log[1 - I*
E^(I*ArcTan[a*x])] + 3*ArcTan[a*x]^2*Log[1 + I*E^(I*ArcTan[a*x])] - (6*I)*A
```

```
rcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] + (6*I)*ArcTan[a*x]*PolyLog[2
, I*E^(I*ArcTan[a*x])] + 6*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] - 6*PolyLog[3
, I*E^(I*ArcTan[a*x])]))/(a^4*c*Sqrt[c*(1 + a^2*x^2)])
```

Maple [F] time = 1.137, size = 0, normalized size = 0.

$$\int x^3 (\arctan(ax))^3 \left(a^2cx^2+c\right)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctan(a*x)^3/(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] int(x^3*arctan(a*x)^3/(a^2*c*x^2+c)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^3/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{a^2cx^2+c}\,x^3\arctan(ax)^3}{a^4c^2x^4+2\,a^2c^2x^2+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^3/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^3*arctan(a*x)^3/(a^4*c^2*x^4 + 2*a^2*c^2*x^2
 + c^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \operatorname{atan}^3(ax)}{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atan(a*x)**3/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(x**3*atan(a*x)**3/(c*(a**2*x**2 + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \arctan(ax)^3}{\left(a^2cx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^3/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^3*arctan(a*x)^3/(a^2*c*x^2 + c)^(3/2), x)
```

3.445 $\int \frac{x^2 \tan^{-1}(ax)^3}{\left(c+a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=495

$$\frac{3i\sqrt{a^2x^2+1}\tan^{-1}(ax)^2\text{PolyLog}\left(2,-ie^{i\tan^{-1}(ax)}\right)}{a^3c\sqrt{a^2cx^2+c}}-\frac{3i\sqrt{a^2x^2+1}\tan^{-1}(ax)^2\text{PolyLog}\left(2,ie^{i\tan^{-1}(ax)}\right)}{a^3c\sqrt{a^2cx^2+c}}-\frac{6\sqrt{a^2x^2+1}\tan}{}$$

```
[Out] 6/(a^3*c*Sqrt[c + a^2*c*x^2]) + (6*x*ArcTan[a*x])/(a^2*c*Sqrt[c + a^2*c*x^2
]) - (3*ArcTan[a*x]^2)/(a^3*c*Sqrt[c + a^2*c*x^2]) - (x*ArcTan[a*x]^3)/(a^2
*c*Sqrt[c + a^2*c*x^2]) - ((2*I)*Sqrt[1 + a^2*x^2]*ArcTan[E^(I*ArcTan[a*x])
]*ArcTan[a*x]^3)/(a^3*c*Sqrt[c + a^2*c*x^2]) + ((3*I)*Sqrt[1 + a^2*x^2]*Arc
Tan[a*x]^2*PolyLog[2, (-I)*E^(I*ArcTan[a*x])])/(a^3*c*Sqrt[c + a^2*c*x^2])
- ((3*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog[2, I*E^(I*ArcTan[a*x])])/(
a^3*c*Sqrt[c + a^2*c*x^2]) - (6*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[3, (-
I)*E^(I*ArcTan[a*x])])/(a^3*c*Sqrt[c + a^2*c*x^2]) + (6*Sqrt[1 + a^2*x^2]*A
rcTan[a*x]*PolyLog[3, I*E^(I*ArcTan[a*x])])/(a^3*c*Sqrt[c + a^2*c*x^2]) - (
(6*I)*Sqrt[1 + a^2*x^2]*PolyLog[4, (-I)*E^(I*ArcTan[a*x])])/(a^3*c*Sqrt[c +
 a^2*c*x^2]) + ((6*I)*Sqrt[1 + a^2*x^2]*PolyLog[4, I*E^(I*ArcTan[a*x])])/(a
^3*c*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.411755, antiderivative size = 495, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {4964, 4890, 4888, 4181, 2531, 6609, 2282, 6589, 4898, 4894}

$$\frac{3i\sqrt{a^2x^2+1}\tan^{-1}(ax)^2\text{PolyLog}\left(2,-ie^{i\tan^{-1}(ax)}\right)}{a^3c\sqrt{a^2cx^2+c}}-\frac{3i\sqrt{a^2x^2+1}\tan^{-1}(ax)^2\text{PolyLog}\left(2,ie^{i\tan^{-1}(ax)}\right)}{a^3c\sqrt{a^2cx^2+c}}-\frac{6\sqrt{a^2x^2+1}\tan}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcTan[a*x]^3)/(c + a^2*c*x^2)^(3/2),x]
```


```
[Out] 6/(a^3*c*Sqrt[c + a^2*c*x^2]) + (6*x*ArcTan[a*x])/(a^2*c*Sqrt[c + a^2*c*x^2
]) - (3*ArcTan[a*x]^2)/(a^3*c*Sqrt[c + a^2*c*x^2]) - (x*ArcTan[a*x]^3)/(a^2
*c*Sqrt[c + a^2*c*x^2]) - ((2*I)*Sqrt[1 + a^2*x^2]*ArcTan[E^(I*ArcTan[a*x])
]*ArcTan[a*x]^3)/(a^3*c*Sqrt[c + a^2*c*x^2]) + ((3*I)*Sqrt[1 + a^2*x^2]*Arc
Tan[a*x]^2*PolyLog[2, (-I)*E^(I*ArcTan[a*x])])/(a^3*c*Sqrt[c + a^2*c*x^2])
- ((3*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog[2, I*E^(I*ArcTan[a*x])])/(
a^3*c*Sqrt[c + a^2*c*x^2]) - (6*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[3, (-
I)*E^(I*ArcTan[a*x])])/(a^3*c*Sqrt[c + a^2*c*x^2]) + (6*Sqrt[1 + a^2*x^2]*A
```

rcTan[a*x]*PolyLog[3, I*E^(I*ArcTan[a*x])])/(a^3*c*Sqrt[c + a^2*c*x^2]) - ((6*I)*Sqrt[1 + a^2*x^2]*PolyLog[4, (-I)*E^(I*ArcTan[a*x])])/(a^3*c*Sqrt[c + a^2*c*x^2]) + ((6*I)*Sqrt[1 + a^2*x^2]*PolyLog[4, I*E^(I*ArcTan[a*x])])/(a^3*c*Sqrt[c + a^2*c*x^2])

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rule 4890

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4888

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && GtQ[d, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m - 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4898

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[(b*p*(a + b*ArcTan[c*x])^(p - 1))/(c*d*Sqrt[d + e*x^2]), x]
 + (-Dist[b^2*p*(p - 1), Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2),
 x], x] + Simp[(x*(a + b*ArcTan[c*x])^p)/(d*Sqrt[d + e*x^2]), x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

Rule 4894

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[(x*(a + b*ArcTan[c*x]))/(d*Sq
rt[d + e*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

Rubi steps

$$\int \frac{x^2 \tan^{-1}(ax)^3}{\left(c+a^2cx^2\right)^{3/2}}\,dx = -\frac{\int \frac{\tan^{-1}(ax)^3}{\left(c+a^2cx^2\right)^{3/2}}\,dx}{a^2} + \frac{\int \frac{\tan^{-1}(ax)^3}{\sqrt{c+a^2cx^2}}\,dx}{a^2c}$$

$$= -\frac{3\tan^{-1}(ax)^2}{a^3c\sqrt{c+a^2cx^2}} - \frac{x\tan^{-1}(ax)^3}{a^2c\sqrt{c+a^2cx^2}} + \frac{6\int \frac{\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^{3/2}}\,dx}{a^2} + \frac{\sqrt{1+a^2x^2}\int \frac{\tan^{-1}(ax)^3}{\sqrt{1+a^2x^2}}\,dx}{a^2c\sqrt{c+a^2cx^2}}$$

$$= \frac{6}{a^3c\sqrt{c+a^2cx^2}} + \frac{6x\tan^{-1}(ax)}{a^2c\sqrt{c+a^2cx^2}} - \frac{3\tan^{-1}(ax)^2}{a^3c\sqrt{c+a^2cx^2}} - \frac{x\tan^{-1}(ax)^3}{a^2c\sqrt{c+a^2cx^2}} + \frac{\sqrt{1+a^2x^2}\,\mathrm{Subst}\left(\int x^3 \sec(\right.}{a^3c\sqrt{c+a^2}}$$

$$= \frac{6}{a^3c\sqrt{c+a^2cx^2}} + \frac{6x\tan^{-1}(ax)}{a^2c\sqrt{c+a^2cx^2}} - \frac{3\tan^{-1}(ax)^2}{a^3c\sqrt{c+a^2cx^2}} - \frac{x\tan^{-1}(ax)^3}{a^2c\sqrt{c+a^2cx^2}} - \frac{2i\sqrt{1+a^2x^2}\tan^{-1}\left(e^{i\tan^{-1}(}\right.}{a^3c\sqrt{c+a^2cx}}$$

$$= \frac{6}{a^3c\sqrt{c+a^2cx^2}} + \frac{6x\tan^{-1}(ax)}{a^2c\sqrt{c+a^2cx^2}} - \frac{3\tan^{-1}(ax)^2}{a^3c\sqrt{c+a^2cx^2}} - \frac{x\tan^{-1}(ax)^3}{a^2c\sqrt{c+a^2cx^2}} - \frac{2i\sqrt{1+a^2x^2}\tan^{-1}\left(e^{i\tan^{-1}(}\right.}{a^3c\sqrt{c+a^2cx}}$$

$$= \frac{6}{a^3c\sqrt{c+a^2cx^2}} + \frac{6x\tan^{-1}(ax)}{a^2c\sqrt{c+a^2cx^2}} - \frac{3\tan^{-1}(ax)^2}{a^3c\sqrt{c+a^2cx^2}} - \frac{x\tan^{-1}(ax)^3}{a^2c\sqrt{c+a^2cx^2}} - \frac{2i\sqrt{1+a^2x^2}\tan^{-1}\left(e^{i\tan^{-1}(}\right.}{a^3c\sqrt{c+a^2cx}}$$

$$= \frac{6}{a^3c\sqrt{c+a^2cx^2}} + \frac{6x\tan^{-1}(ax)}{a^2c\sqrt{c+a^2cx^2}} - \frac{3\tan^{-1}(ax)^2}{a^3c\sqrt{c+a^2cx^2}} - \frac{x\tan^{-1}(ax)^3}{a^2c\sqrt{c+a^2cx^2}} - \frac{2i\sqrt{1+a^2x^2}\tan^{-1}\left(e^{i\tan^{-1}(}\right.}{a^3c\sqrt{c+a^2cx}}$$

$$= \frac{6}{a^3c\sqrt{c+a^2cx^2}} + \frac{6x\tan^{-1}(ax)}{a^2c\sqrt{c+a^2cx^2}} - \frac{3\tan^{-1}(ax)^2}{a^3c\sqrt{c+a^2cx^2}} - \frac{x\tan^{-1}(ax)^3}{a^2c\sqrt{c+a^2cx^2}} - \frac{2i\sqrt{1+a^2x^2}\tan^{-1}\left(e^{i\tan^{-1}(}\right.}{a^3c\sqrt{c+a^2cx}}$$

Mathematica [A] time = 1.82668, size = 639, normalized size = 1.29

$$-\frac{\sqrt{a^2x^2+1}\left(-192i\tan^{-1}(ax)^2\mathrm{PolyLog}\left(2,-ie^{-i\tan^{-1}(ax)}\right)-192i\tan^{-1}(ax)^2\mathrm{PolyLog}\left(2,-ie^{i\tan^{-1}(ax)}\right)+192i\pi\tan^{-1}(ax)\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*ArcTan[a*x]^3)/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] -(Sqrt[1 + a^2*x^2]*((7*I)*Pi^4 - 384/Sqrt[1 + a^2*x^2] + (8*I)*Pi^3*ArcTan
[a*x] - (384*a*x*ArcTan[a*x])/Sqrt[1 + a^2*x^2] - (24*I)*Pi^2*ArcTan[a*x]^2
 + (192*ArcTan[a*x]^2)/Sqrt[1 + a^2*x^2] + (32*I)*Pi*ArcTan[a*x]^3 + (64*a*
x*ArcTan[a*x]^3)/Sqrt[1 + a^2*x^2] - (16*I)*ArcTan[a*x]^4 - 48*Pi^2*ArcTan[
a*x]*Log[1 - I/E^(I*ArcTan[a*x])] + 96*Pi*ArcTan[a*x]^2*Log[1 - I/E^(I*ArcT
```

```
an[a*x])] + 8*Pi^3*Log[1 + I/E^(I*ArcTan[a*x])] - 64*ArcTan[a*x]^3*Log[1 +
I/E^(I*ArcTan[a*x])] - 8*Pi^3*Log[1 + I*E^(I*ArcTan[a*x])] + 48*Pi^2*ArcTan
[a*x]*Log[1 + I*E^(I*ArcTan[a*x])] - 96*Pi*ArcTan[a*x]^2*Log[1 + I*E^(I*Arc
Tan[a*x])] + 64*ArcTan[a*x]^3*Log[1 + I*E^(I*ArcTan[a*x])] - 8*Pi^3*Log[Tan
[(Pi + 2*ArcTan[a*x])/4]] - (192*I)*ArcTan[a*x]^2*PolyLog[2, (-I)/E^(I*ArcT
an[a*x])] - (48*I)*Pi*(Pi - 4*ArcTan[a*x])*PolyLog[2, I/E^(I*ArcTan[a*x])]
- (48*I)*Pi^2*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] + (192*I)*Pi*ArcTan[a*x]*P
olyLog[2, (-I)*E^(I*ArcTan[a*x])] - (192*I)*ArcTan[a*x]^2*PolyLog[2, (-I)*E
^(I*ArcTan[a*x])] - 384*ArcTan[a*x]*PolyLog[3, (-I)/E^(I*ArcTan[a*x])] + 19
2*Pi*PolyLog[3, I/E^(I*ArcTan[a*x])] - 192*Pi*PolyLog[3, (-I)*E^(I*ArcTan[a
*x])] + 384*ArcTan[a*x]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] + (384*I)*PolyLo
g[4, (-I)/E^(I*ArcTan[a*x])] + (384*I)*PolyLog[4, (-I)*E^(I*ArcTan[a*x])]))
/(64*a^3*c*Sqrt[c*(1 + a^2*x^2)])
```

Maple [F] time = 0.885, size = 0, normalized size = 0.

$$\int x^{2} \left(\arctan \left(a x\right)\right)^{3} \left(a^{2} c \,x^{2}+c \right)^{-\frac{3}{2}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(a*x)^3/(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] int(x^2*arctan(a*x)^3/(a^2*c*x^2+c)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^3/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{a^{2} c x^{2} + c} x^{2} \arctan\left(a x\right)^{3}}{a^{4} c^{2} x^{4} + 2 \, a^{2} c^{2} x^{2} + c^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^3/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^2*arctan(a*x)^3/(a^4*c^2*x^4 + 2*a^2*c^2*x^2
 + c^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \operatorname{atan}^3(ax)}{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(a*x)**3/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(x**2*atan(a*x)**3/(c*(a**2*x**2 + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \arctan(ax)^3}{\left(a^2cx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^3/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctan(a*x)^3/(a^2*c*x^2 + c)^(3/2), x)
```

3.446 $\int \frac{x \tan^{-1}(ax)^3}{\left(c+a^2cx^2\right)^{3/2}} dx$

Optimal. Leaf size=107

$$-\frac{6x}{ac\sqrt{a^2cx^2+c}}-\frac{\tan^{-1}(ax)^3}{a^2c\sqrt{a^2cx^2+c}}+\frac{3x\tan^{-1}(ax)^2}{ac\sqrt{a^2cx^2+c}}+\frac{6\tan^{-1}(ax)}{a^2c\sqrt{a^2cx^2+c}}$$

```
[Out] (-6*x)/(a*c*Sqrt[c + a^2*c*x^2]) + (6*ArcTan[a*x])/(a^2*c*Sqrt[c + a^2*c*x^
2]) + (3*x*ArcTan[a*x]^2)/(a*c*Sqrt[c + a^2*c*x^2]) - ArcTan[a*x]^3/(a^2*c*
Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.13112, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {4930, 4898, 191}

$$-\frac{6x}{ac\sqrt{a^2cx^2+c}}-\frac{\tan^{-1}(ax)^3}{a^2c\sqrt{a^2cx^2+c}}+\frac{3x\tan^{-1}(ax)^2}{ac\sqrt{a^2cx^2+c}}+\frac{6\tan^{-1}(ax)}{a^2c\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTan[a*x]^3)/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] (-6*x)/(a*c*Sqrt[c + a^2*c*x^2]) + (6*ArcTan[a*x])/(a^2*c*Sqrt[c + a^2*c*x^
2]) + (3*x*ArcTan[a*x]^2)/(a*c*Sqrt[c + a^2*c*x^2]) - ArcTan[a*x]^3/(a^2*c*
Sqrt[c + a^2*c*x^2])
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 4898

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[(b*p*(a + b*ArcTan[c*x])^(p - 1))/(c*d*Sqrt[d + e*x^2]), x]
 + (-Dist[b^2*p*(p - 1), Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2),
 x], x] + Simp[(x*(a + b*ArcTan[c*x])^p)/(d*Sqrt[d + e*x^2]), x]) /; FreeQ[
```

```
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x\tan^{-1}(ax)^3}{\left(c+a^2cx^2\right)^{3/2}}\,dx &= -\frac{\tan^{-1}(ax)^3}{a^2c\sqrt{c+a^2cx^2}}+\frac{3\int \frac{\tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^{3/2}}\,dx}{a}\\ &= \frac{6\tan^{-1}(ax)}{a^2c\sqrt{c+a^2cx^2}}+\frac{3x\tan^{-1}(ax)^2}{ac\sqrt{c+a^2cx^2}}-\frac{\tan^{-1}(ax)^3}{a^2c\sqrt{c+a^2cx^2}}-\frac{6\int \frac{1}{\left(c+a^2cx^2\right)^{3/2}}\,dx}{a}\\ &= -\frac{6x}{ac\sqrt{c+a^2cx^2}}+\frac{6\tan^{-1}(ax)}{a^2c\sqrt{c+a^2cx^2}}+\frac{3x\tan^{-1}(ax)^2}{ac\sqrt{c+a^2cx^2}}-\frac{\tan^{-1}(ax)^3}{a^2c\sqrt{c+a^2cx^2}}\end{aligned}$$

Mathematica [A] time = 0.0934493, size = 61, normalized size = 0.57

$$\frac{\sqrt{a^2cx^2+c}\left(-6ax-\tan^{-1}(ax)^3+3ax\tan^{-1}(ax)^2+6\tan^{-1}(ax)\right)}{a^2c^2\left(a^2x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcTan[a*x]^3)/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] (Sqrt[c + a^2*c*x^2]*(-6*a*x + 6*ArcTan[a*x] + 3*a*x*ArcTan[a*x]^2 - ArcTan
[a*x]^3))/(a^2*c^2*(1 + a^2*x^2))
```

Maple [C] time = 0.275, size = 134, normalized size = 1.3

$$-\frac{\left((\arctan(ax))^3-6\arctan(ax)+3i(\arctan(ax))^2-6i\right)(1+iax)}{\left(2a^2x^2+2\right)c^2a^2}\sqrt{c(ax-i)(ax+i)}+\frac{(-1+iax)\left((\arctan(ax))^3-6\right.}{\left(2a^{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(a*x)^3/(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] -1/2*(arctan(a*x)^3-6*arctan(a*x)+3*I*arctan(a*x)^2-6*I)*(1+I*a*x)*(c*(a*x-
I)*(a*x+I))^(1/2)/(a^2*x^2+1)/c^2/a^2+1/2*(c*(a*x-I)*(a*x+I))^(1/2)*(-1+I*a
*x)*(arctan(a*x)^3-6*arctan(a*x)-3*I*arctan(a*x)^2+6*I)/(a^2*x^2+1)/c^2/a^2
```

Maxima [A] time = 3.28509, size = 132, normalized size = 1.23

$$\sqrt{c}\left(\frac{3\,x\arctan\left(ax\right)^{2}}{\sqrt{a^{2}x^{2}+1}ac^{2}}-\frac{\arctan\left(ax\right)^{3}}{\sqrt{a^{2}x^{2}+1}a^{2}c^{2}}-\frac{6\left(\frac{x}{\sqrt{a^{2}x^{2}+1}}-\frac{\arctan(ax)}{\sqrt{a^{2}x^{2}+1}a}\right)}{ac^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^3/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] sqrt(c)*(3*x*arctan(a*x)^2/(sqrt(a^2*x^2 + 1)*a*c^2) - arctan(a*x)^3/(sqrt(
a^2*x^2 + 1)*a^2*c^2) - 6*(x/sqrt(a^2*x^2 + 1) - arctan(a*x)/(sqrt(a^2*x^2
+ 1)*a))/(a*c^2))
```

Fricas [A] time = 1.75106, size = 144, normalized size = 1.35

$$\frac{\sqrt{a^{2}cx^{2}+c}\left(3\,ax\arctan\left(ax\right)^{2}-\arctan\left(ax\right)^{3}-6\,ax+6\,\arctan\left(ax\right)\right)}{a^{4}c^{2}x^{2}+a^{2}c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^3/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] sqrt(a^2*c*x^2 + c)*(3*a*x*arctan(a*x)^2 - arctan(a*x)^3 - 6*a*x + 6*arctan
(a*x))/(a^4*c^2*x^2 + a^2*c^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x\,\mathrm{atan}^{3}\left(ax\right)}{\left(c\left(a^{2}x^{2}+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(a*x)**3/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(x*atan(a*x)**3/(c*(a**2*x**2 + 1))**(3/2), x)
```

Giac [A] time = 1.29754, size = 134, normalized size = 1.25

$$\frac{3\,x\arctan\left(ax\right)^{2}}{\sqrt{a^{2}cx^{2}+c}ac}-\frac{\arctan\left(ax\right)^{3}}{\sqrt{a^{2}cx^{2}+c}a^{2}c}-\frac{6\,x}{\sqrt{a^{2}cx^{2}+c}ac}+\frac{6\,\arctan\left(ax\right)}{\sqrt{a^{2}cx^{2}+c}a^{2}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^3/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] 3*x*arctan(a*x)^2/(sqrt(a^2*c*x^2 + c)*a*c) - arctan(a*x)^3/(sqrt(a^2*c*x^2
 + c)*a^2*c) - 6*x/(sqrt(a^2*c*x^2 + c)*a*c) + 6*arctan(a*x)/(sqrt(a^2*c*x^
2 + c)*a^2*c)
```

3.447 $\int \frac{\tan^{-1}(ax)^3}{\left(c+a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=100

$$-\frac{6}{ac\sqrt{a^2cx^2+c}}+\frac{x\tan^{-1}(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3\tan^{-1}(ax)^2}{ac\sqrt{a^2cx^2+c}}-\frac{6x\tan^{-1}(ax)}{c\sqrt{a^2cx^2+c}}$$

```
[Out] -6/(a*c*Sqrt[c + a^2*c*x^2]) - (6*x*ArcTan[a*x])/(c*Sqrt[c + a^2*c*x^2]) +
(3*ArcTan[a*x]^2)/(a*c*Sqrt[c + a^2*c*x^2]) + (x*ArcTan[a*x]^3)/(c*Sqrt[c +
 a^2*c*x^2])
```

Rubi [A] time = 0.0690659, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4898, 4894}

$$-\frac{6}{ac\sqrt{a^2cx^2+c}}+\frac{x\tan^{-1}(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3\tan^{-1}(ax)^2}{ac\sqrt{a^2cx^2+c}}-\frac{6x\tan^{-1}(ax)}{c\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^3/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] -6/(a*c*Sqrt[c + a^2*c*x^2]) - (6*x*ArcTan[a*x])/(c*Sqrt[c + a^2*c*x^2]) +
(3*ArcTan[a*x]^2)/(a*c*Sqrt[c + a^2*c*x^2]) + (x*ArcTan[a*x]^3)/(c*Sqrt[c +
 a^2*c*x^2])
```

Rule 4898

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[(b*p*(a + b*ArcTan[c*x])^(p - 1))/(c*d*Sqrt[d + e*x^2]), x]
 + (-Dist[b^2*p*(p - 1), Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2),
 x], x] + Simp[(x*(a + b*ArcTan[c*x])^p)/(d*Sqrt[d + e*x^2]), x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

Rule 4894

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[(x*(a + b*ArcTan[c*x]))/(d*Sq
rt[d + e*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^3}{\left(c+a^2cx^2\right)^{3/2}}\,dx = \frac{3\tan^{-1}(ax)^2}{ac\sqrt{c+a^2cx^2}}+\frac{x\tan^{-1}(ax)^3}{c\sqrt{c+a^2cx^2}}-6\int \frac{\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^{3/2}}\,dx$$
$$= -\frac{6}{ac\sqrt{c+a^2cx^2}}-\frac{6x\tan^{-1}(ax)}{c\sqrt{c+a^2cx^2}}+\frac{3\tan^{-1}(ax)^2}{ac\sqrt{c+a^2cx^2}}+\frac{x\tan^{-1}(ax)^3}{c\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 0.0674057, size = 56, normalized size = 0.56

$$\frac{\sqrt{a^2cx^2+c}\left(ax\tan^{-1}(ax)^3+3\tan^{-1}(ax)^2-6ax\tan^{-1}(ax)-6\right)}{c^2\left(a^3x^2+a\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]^3/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] (Sqrt[c + a^2*c*x^2]*(-6 - 6*a*x*ArcTan[a*x] + 3*ArcTan[a*x]^2 + a*x*ArcTan
[a*x]^3))/(c^2*(a + a^3*x^2))
```

Maple [C] time = 0.236, size = 132, normalized size = 1.3

$$\frac{\left((\arctan(ax))^3-6\arctan(ax)+3i(\arctan(ax))^2-6i\right)(ax-i)}{\left(2a^2x^2+2\right)c^2a}\sqrt{c(ax-i)(ax+i)}+\frac{(ax+i)\left((\arctan(ax))^3-6\,\mathrm{arct}\right.}{\left(2a^2x^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^3/(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] 1/2*(arctan(a*x)^3-6*arctan(a*x)+3*I*arctan(a*x)^2-6*I)*(a*x-I)*(c*(a*x-I)*
(a*x+I))^(1/2)/(a^2*x^2+1)/c^2/a+1/2*(c*(a*x-I)*(a*x+I))^(1/2)*(a*x+I)*(arc
tan(a*x)^3-6*arctan(a*x)-3*I*arctan(a*x)^2+6*I)/(a^2*x^2+1)/c^2/a
```

Maxima [A] time = 2.55137, size = 134, normalized size = 1.34

$$\frac{x\arctan(ax)^3}{\sqrt{a^2cx^2+cc}}-\frac{3a\left(\frac{2x\arctan(ax)}{\sqrt{a^2x^2+1}ac}-\frac{\arctan(ax)^2}{\sqrt{a^2x^2+1}a^2c}+\frac{2}{\sqrt{a^2x^2+1}a^2c}\right)}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] x*arctan(a*x)^3/(sqrt(a^2*c*x^2 + c)*c) - 3*a*(2*x*arctan(a*x)/(sqrt(a^2*x^
2 + 1)*a*c) - arctan(a*x)^2/(sqrt(a^2*x^2 + 1)*a^2*c) + 2/(sqrt(a^2*x^2 + 1
)*a^2*c))/sqrt(c)
```

Fricas [A] time = 1.78762, size = 142, normalized size = 1.42

$$\frac{\sqrt{a^2cx^2+c}\left(ax\arctan(ax)^3-6\,ax\arctan(ax)+3\,\arctan(ax)^2-6\right)}{a^3c^2x^2+ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] sqrt(a^2*c*x^2 + c)*(a*x*arctan(a*x)^3 - 6*a*x*arctan(a*x) + 3*arctan(a*x)^
2 - 6)/(a^3*c^2*x^2 + a*c^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{atan}^3(ax)}{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**3/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(atan(a*x)**3/(c*(a**2*x**2 + 1))**(3/2), x)
```

Giac [A] time = 1.27381, size = 134, normalized size = 1.34

$$\frac{x\arctan(ax)^3}{\sqrt{a^2cx^2+c}c}-3\,a\left(\frac{2\,x\arctan(ax)}{\sqrt{a^2cx^2+c}ac}-\frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}a^2c}+\frac{2}{\sqrt{a^2cx^2+c}a^2c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] x*arctan(a*x)^3/(sqrt(a^2*c*x^2 + c)*c) - 3*a*(2*x*arctan(a*x)/(sqrt(a^2*c*
x^2 + c)*a*c) - arctan(a*x)^2/(sqrt(a^2*c*x^2 + c)*a^2*c) + 2/(sqrt(a^2*c*x
^2 + c)*a^2*c))
```

3.448 $\int \frac{\tan^{-1}(ax)^3}{x\left(c+a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=443

$$\frac{3i\sqrt{a^2x^2+1}\tan^{-1}(ax)^2\text{PolyLog}\left(2,-e^{i\tan^{-1}(ax)}\right)}{c\sqrt{a^2cx^2+c}}-\frac{3i\sqrt{a^2x^2+1}\tan^{-1}(ax)^2\text{PolyLog}\left(2,e^{i\tan^{-1}(ax)}\right)}{c\sqrt{a^2cx^2+c}}-\frac{6\sqrt{a^2x^2+1}\tan^{-1}}{}$$

```
[Out] (6*a*x)/(c*Sqrt[c + a^2*c*x^2]) - (6*ArcTan[a*x])/(c*Sqrt[c + a^2*c*x^2]) -
 (3*a*x*ArcTan[a*x]^2)/(c*Sqrt[c + a^2*c*x^2]) + ArcTan[a*x]^3/(c*Sqrt[c +
a^2*c*x^2]) - (2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^3*ArcTanh[E^(I*ArcTan[a*x])]
)/(c*Sqrt[c + a^2*c*x^2]) + ((3*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog[
2, -E^(I*ArcTan[a*x])])/(c*Sqrt[c + a^2*c*x^2]) - ((3*I)*Sqrt[1 + a^2*x^2]*
ArcTan[a*x]^2*PolyLog[2, E^(I*ArcTan[a*x])])/(c*Sqrt[c + a^2*c*x^2]) - (6*S
qrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[3, -E^(I*ArcTan[a*x])])/(c*Sqrt[c + a^
2*c*x^2]) + (6*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[3, E^(I*ArcTan[a*x])])
/(c*Sqrt[c + a^2*c*x^2]) - ((6*I)*Sqrt[1 + a^2*x^2]*PolyLog[4, -E^(I*ArcTan
[a*x])])/(c*Sqrt[c + a^2*c*x^2]) + ((6*I)*Sqrt[1 + a^2*x^2]*PolyLog[4, E^(I
*ArcTan[a*x])])/(c*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.557865, antiderivative size = 443, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {4966, 4958, 4956, 4183, 2531, 6609, 2282, 6589, 4930, 4898, 191}

$$\frac{3i\sqrt{a^2x^2+1}\tan^{-1}(ax)^2\text{PolyLog}\left(2,-e^{i\tan^{-1}(ax)}\right)}{c\sqrt{a^2cx^2+c}}-\frac{3i\sqrt{a^2x^2+1}\tan^{-1}(ax)^2\text{PolyLog}\left(2,e^{i\tan^{-1}(ax)}\right)}{c\sqrt{a^2cx^2+c}}-\frac{6\sqrt{a^2x^2+1}\tan^{-1}}{}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^3/(x*(c + a^2*c*x^2)^(3/2)),x]
```


```
[Out] (6*a*x)/(c*Sqrt[c + a^2*c*x^2]) - (6*ArcTan[a*x])/(c*Sqrt[c + a^2*c*x^2]) -
 (3*a*x*ArcTan[a*x]^2)/(c*Sqrt[c + a^2*c*x^2]) + ArcTan[a*x]^3/(c*Sqrt[c +
a^2*c*x^2]) - (2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^3*ArcTanh[E^(I*ArcTan[a*x])]
)/(c*Sqrt[c + a^2*c*x^2]) + ((3*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog[
2, -E^(I*ArcTan[a*x])])/(c*Sqrt[c + a^2*c*x^2]) - ((3*I)*Sqrt[1 + a^2*x^2]*
ArcTan[a*x]^2*PolyLog[2, E^(I*ArcTan[a*x])])/(c*Sqrt[c + a^2*c*x^2]) - (6*S
qrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[3, -E^(I*ArcTan[a*x])])/(c*Sqrt[c + a^
2*c*x^2]) + (6*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[3, E^(I*ArcTan[a*x])])
/(c*Sqrt[c + a^2*c*x^2]) - ((6*I)*Sqrt[1 + a^2*x^2]*PolyLog[4, -E^(I*ArcTan
```

```
[a*x])])/(c*Sqrt[c + a^2*c*x^2]) + ((6*I)*Sqrt[1 + a^2*x^2]*PolyLog[4, E^(I
*ArcTan[a*x])])/(c*Sqrt[c + a^2*c*x^2])
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*
x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^p
, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*q]
 && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 4958

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4956

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Dist[1/Sqrt[d], Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTan
[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && G
tQ[d, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
```

```
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 4898

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[(b*p*(a + b*ArcTan[c*x])^(p - 1))/(c*d*Sqrt[d + e*x^2]), x]
 + (-Dist[b^2*p*(p - 1), Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2),
 x], x] + Simp[(x*(a + b*ArcTan[c*x])^p)/(d*Sqrt[d + e*x^2]), x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^3}{x\left(c+a^2cx^2\right)^{3/2}}\,dx = -\left(a^2\int \frac{x\tan^{-1}(ax)^3}{\left(c+a^2cx^2\right)^{3/2}}\,dx\right)+\frac{\int \frac{\tan^{-1}(ax)^3}{x\sqrt{c+a^2cx^2}}\,dx}{c}$$

$$= \frac{\tan^{-1}(ax)^3}{c\sqrt{c+a^2cx^2}}-(3a)\int \frac{\tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^{3/2}}\,dx+\frac{\sqrt{1+a^2x^2}\int \frac{\tan^{-1}(ax)^3}{x\sqrt{1+a^2x^2}}\,dx}{c\sqrt{c+a^2cx^2}}$$

$$= -\frac{6\tan^{-1}(ax)}{c\sqrt{c+a^2cx^2}}-\frac{3ax\tan^{-1}(ax)^2}{c\sqrt{c+a^2cx^2}}+\frac{\tan^{-1}(ax)^3}{c\sqrt{c+a^2cx^2}}+(6a)\int \frac{1}{\left(c+a^2cx^2\right)^{3/2}}\,dx+\frac{\sqrt{1+a^2x^2}\,\mathrm{Subst}\,(}{c}$$

$$= \frac{6ax}{c\sqrt{c+a^2cx^2}}-\frac{6\tan^{-1}(ax)}{c\sqrt{c+a^2cx^2}}-\frac{3ax\tan^{-1}(ax)^2}{c\sqrt{c+a^2cx^2}}+\frac{\tan^{-1}(ax)^3}{c\sqrt{c+a^2cx^2}}-\frac{2\sqrt{1+a^2x^2}\tan^{-1}(ax)^3\tanh^{-1}(}{c\sqrt{c+a^2cx^2}}$$

$$= \frac{6ax}{c\sqrt{c+a^2cx^2}}-\frac{6\tan^{-1}(ax)}{c\sqrt{c+a^2cx^2}}-\frac{3ax\tan^{-1}(ax)^2}{c\sqrt{c+a^2cx^2}}+\frac{\tan^{-1}(ax)^3}{c\sqrt{c+a^2cx^2}}-\frac{2\sqrt{1+a^2x^2}\tan^{-1}(ax)^3\tanh^{-1}(}{c\sqrt{c+a^2cx^2}}$$

$$= \frac{6ax}{c\sqrt{c+a^2cx^2}}-\frac{6\tan^{-1}(ax)}{c\sqrt{c+a^2cx^2}}-\frac{3ax\tan^{-1}(ax)^2}{c\sqrt{c+a^2cx^2}}+\frac{\tan^{-1}(ax)^3}{c\sqrt{c+a^2cx^2}}-\frac{2\sqrt{1+a^2x^2}\tan^{-1}(ax)^3\tanh^{-1}(}{c\sqrt{c+a^2cx^2}}$$

$$= \frac{6ax}{c\sqrt{c+a^2cx^2}}-\frac{6\tan^{-1}(ax)}{c\sqrt{c+a^2cx^2}}-\frac{3ax\tan^{-1}(ax)^2}{c\sqrt{c+a^2cx^2}}+\frac{\tan^{-1}(ax)^3}{c\sqrt{c+a^2cx^2}}-\frac{2\sqrt{1+a^2x^2}\tan^{-1}(ax)^3\tanh^{-1}(}{c\sqrt{c+a^2cx^2}}$$

$$= \frac{6ax}{c\sqrt{c+a^2cx^2}}-\frac{6\tan^{-1}(ax)}{c\sqrt{c+a^2cx^2}}-\frac{3ax\tan^{-1}(ax)^2}{c\sqrt{c+a^2cx^2}}+\frac{\tan^{-1}(ax)^3}{c\sqrt{c+a^2cx^2}}-\frac{2\sqrt{1+a^2x^2}\tan^{-1}(ax)^3\tanh^{-1}(}{c\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 0.423663, size = 295, normalized size = 0.67

$$\frac{\sqrt{a^2x^2+1}\left(24i\tan^{-1}(ax)^2\mathrm{PolyLog}\left(2,e^{-i\tan^{-1}(ax)}\right)+24i\tan^{-1}(ax)^2\mathrm{PolyLog}\left(2,-e^{i\tan^{-1}(ax)}\right)+48\tan^{-1}(ax)\mathrm{PolyLog}\left(3\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTan[a*x]^3/(x*(c + a^2*c*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[1 + a^2*x^2]*((-I)*Pi^4 + (48*a*x)/Sqrt[1 + a^2*x^2] - (48*ArcTan[a*x
])/Sqrt[1 + a^2*x^2] - (24*a*x*ArcTan[a*x]^2)/Sqrt[1 + a^2*x^2] + (8*ArcTan
[a*x]^3)/Sqrt[1 + a^2*x^2] + (2*I)*ArcTan[a*x]^4 + 8*ArcTan[a*x]^3*Log[1 -
E^((-I)*ArcTan[a*x])] - 8*ArcTan[a*x]^3*Log[1 + E^(I*ArcTan[a*x])] + (24*I)
*ArcTan[a*x]^2*PolyLog[2, E^((-I)*ArcTan[a*x])] + (24*I)*ArcTan[a*x]^2*Poly
```

```
Log[2, -E^(I*ArcTan[a*x])] + 48*ArcTan[a*x]*PolyLog[3, E^((-I)*ArcTan[a*x])
] - 48*ArcTan[a*x]*PolyLog[3, -E^(I*ArcTan[a*x])] - (48*I)*PolyLog[4, E^((-
I)*ArcTan[a*x])] - (48*I)*PolyLog[4, -E^(I*ArcTan[a*x])]))/(8*c*Sqrt[c*(1 +
 a^2*x^2)])
```

Maple [A] time = 0.309, size = 388, normalized size = 0.9

$$\frac{\left((\arctan(ax))^3 - 6\arctan(ax) + 3i(\arctan(ax))^2 - 6i\right)(1+iax)}{\left(2a^2x^2+2\right)c^2}\sqrt{c(ax-i)(ax+i)} - \frac{(-1+iax)\left((\arctan(ax))^3 - 6\right.}{\left(2a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^3/x/(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] 1/2*(arctan(a*x)^3-6*arctan(a*x)+3*I*arctan(a*x)^2-6*I)*(1+I*a*x)*(c*(a*x-I
)*(a*x+I))^(1/2)/(a^2*x^2+1)/c^2-1/2*(c*(a*x-I)*(a*x+I))^(1/2)*(-1+I*a*x)*(
arctan(a*x)^3-6*arctan(a*x)-3*I*arctan(a*x)^2+6*I)/(a^2*x^2+1)/c^2+I*(I*arc
tan(a*x)^3*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*arctan(a*x)^3*ln(1-(1+I*a*x)
/(a^2*x^2+1)^(1/2))+3*arctan(a*x)^2*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))
-3*arctan(a*x)^2*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I*arctan(a*x)*pol
ylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*I*arctan(a*x)*polylog(3,(1+I*a*x)/(a
^2*x^2+1)^(1/2))-6*polylog(4,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*polylog(4,(1+I
*a*x)/(a^2*x^2+1)^(1/2)))*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)^(1/2)/c^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{a^4c^2x^5+2a^2c^2x^3+c^2x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^3/(a^4*c^2*x^5 + 2*a^2*c^2*x^3 + c
^2*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atan}^3(ax)}{x\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**3/x/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(atan(a*x)**3/(x*(c*(a**2*x**2 + 1))**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(ax)^3}{\left(a^2cx^2+c\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)^3/((a^2*c*x^2 + c)^(3/2)*x), x)
```

3.449 $\int \frac{\tan^{-1}(ax)^3}{x^2\left(c+a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=377

$$\frac{6ia\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,-e^{i\tan^{-1}(ax)}\right)}{c\sqrt{a^2cx^2+c}}-\frac{6ia\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,e^{i\tan^{-1}(ax)}\right)}{c\sqrt{a^2cx^2+c}}-\frac{6a\sqrt{a^2x^2+1}\text{Pol}}{c\sqrt{a}}$$

```
[Out] (6*a)/(c*Sqrt[c + a^2*c*x^2]) + (6*a^2*x*ArcTan[a*x])/(c*Sqrt[c + a^2*c*x^2
]) - (3*a*ArcTan[a*x]^2)/(c*Sqrt[c + a^2*c*x^2]) - (a^2*x*ArcTan[a*x]^3)/(c
*Sqrt[c + a^2*c*x^2]) - (Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/(c^2*x) - (6*a*
Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*ArcTanh[E^(I*ArcTan[a*x])])/(c*Sqrt[c + a^2
*c*x^2]) + ((6*I)*a*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a
*x])])/(c*Sqrt[c + a^2*c*x^2]) - ((6*I)*a*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*Pol
yLog[2, E^(I*ArcTan[a*x])])/(c*Sqrt[c + a^2*c*x^2]) - (6*a*Sqrt[1 + a^2*x^2
]*PolyLog[3, -E^(I*ArcTan[a*x])])/(c*Sqrt[c + a^2*c*x^2]) + (6*a*Sqrt[1 + a
^2*x^2]*PolyLog[3, E^(I*ArcTan[a*x])])/(c*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.584324, antiderivative size = 377, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {4966, 4944, 4958, 4956, 4183, 2531, 2282, 6589, 4898, 4894}

$$\frac{6ia\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,-e^{i\tan^{-1}(ax)}\right)}{c\sqrt{a^2cx^2+c}}-\frac{6ia\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,e^{i\tan^{-1}(ax)}\right)}{c\sqrt{a^2cx^2+c}}-\frac{6a\sqrt{a^2x^2+1}\text{Pol}}{c\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^3/(x^2*(c + a^2*c*x^2)^(3/2)),x]
```


```
[Out] (6*a)/(c*Sqrt[c + a^2*c*x^2]) + (6*a^2*x*ArcTan[a*x])/(c*Sqrt[c + a^2*c*x^2
]) - (3*a*ArcTan[a*x]^2)/(c*Sqrt[c + a^2*c*x^2]) - (a^2*x*ArcTan[a*x]^3)/(c
*Sqrt[c + a^2*c*x^2]) - (Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/(c^2*x) - (6*a*
Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*ArcTanh[E^(I*ArcTan[a*x])])/(c*Sqrt[c + a^2
*c*x^2]) + ((6*I)*a*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a
*x])])/(c*Sqrt[c + a^2*c*x^2]) - ((6*I)*a*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*Pol
yLog[2, E^(I*ArcTan[a*x])])/(c*Sqrt[c + a^2*c*x^2]) - (6*a*Sqrt[1 + a^2*x^2
]*PolyLog[3, -E^(I*ArcTan[a*x])])/(c*Sqrt[c + a^2*c*x^2]) + (6*a*Sqrt[1 + a
^2*x^2]*PolyLog[3, E^(I*ArcTan[a*x])])/(c*Sqrt[c + a^2*c*x^2])
```


Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*
x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^p
, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*q]
 && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 4944

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a +
 b*ArcTan[c*x])^p)/(d*f*(m + 1)), x] - Dist[(b*c*p)/(f*(m + 1)), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rule 4958

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4956

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Dist[1/Sqrt[d], Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTan
[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && G
tQ[d, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4898

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[(b*p*(a + b*ArcTan[c*x])^(p - 1))/(c*d*Sqrt[d + e*x^2]), x]
 + (-Dist[b^2*p*(p - 1), Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2),
 x], x] + Simp[(x*(a + b*ArcTan[c*x])^p)/(d*Sqrt[d + e*x^2]), x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

Rule 4894

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[(x*(a + b*ArcTan[c*x]))/(d*Sq
rt[d + e*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^3}{x^2\left(c+a^2cx^2\right)^{3/2}}\,dx = -\left(a^2\int \frac{\tan^{-1}(ax)^3}{\left(c+a^2cx^2\right)^{3/2}}\,dx\right) + \frac{\int \frac{\tan^{-1}(ax)^3}{x^2\sqrt{c+a^2cx^2}}\,dx}{c}$$

$$= -\frac{3a\tan^{-1}(ax)^2}{c\sqrt{c+a^2cx^2}} - \frac{a^2x\tan^{-1}(ax)^3}{c\sqrt{c+a^2cx^2}} - \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{c^2x} + \left(6a^2\right)\int \frac{\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^{3/2}}\,dx + \frac{(3a)}{}$$

$$= \frac{6a}{c\sqrt{c+a^2cx^2}} + \frac{6a^2x\tan^{-1}(ax)}{c\sqrt{c+a^2cx^2}} - \frac{3a\tan^{-1}(ax)^2}{c\sqrt{c+a^2cx^2}} - \frac{a^2x\tan^{-1}(ax)^3}{c\sqrt{c+a^2cx^2}} - \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{c^2x} +$$

$$= \frac{6a}{c\sqrt{c+a^2cx^2}} + \frac{6a^2x\tan^{-1}(ax)}{c\sqrt{c+a^2cx^2}} - \frac{3a\tan^{-1}(ax)^2}{c\sqrt{c+a^2cx^2}} - \frac{a^2x\tan^{-1}(ax)^3}{c\sqrt{c+a^2cx^2}} - \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{c^2x} +$$

$$= \frac{6a}{c\sqrt{c+a^2cx^2}} + \frac{6a^2x\tan^{-1}(ax)}{c\sqrt{c+a^2cx^2}} - \frac{3a\tan^{-1}(ax)^2}{c\sqrt{c+a^2cx^2}} - \frac{a^2x\tan^{-1}(ax)^3}{c\sqrt{c+a^2cx^2}} - \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{c^2x} -$$

$$= \frac{6a}{c\sqrt{c+a^2cx^2}} + \frac{6a^2x\tan^{-1}(ax)}{c\sqrt{c+a^2cx^2}} - \frac{3a\tan^{-1}(ax)^2}{c\sqrt{c+a^2cx^2}} - \frac{a^2x\tan^{-1}(ax)^3}{c\sqrt{c+a^2cx^2}} - \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{c^2x} -$$

$$= \frac{6a}{c\sqrt{c+a^2cx^2}} + \frac{6a^2x\tan^{-1}(ax)}{c\sqrt{c+a^2cx^2}} - \frac{3a\tan^{-1}(ax)^2}{c\sqrt{c+a^2cx^2}} - \frac{a^2x\tan^{-1}(ax)^3}{c\sqrt{c+a^2cx^2}} - \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{c^2x} -$$

$$= \frac{6a}{c\sqrt{c+a^2cx^2}} + \frac{6a^2x\tan^{-1}(ax)}{c\sqrt{c+a^2cx^2}} - \frac{3a\tan^{-1}(ax)^2}{c\sqrt{c+a^2cx^2}} - \frac{a^2x\tan^{-1}(ax)^3}{c\sqrt{c+a^2cx^2}} - \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{c^2x} -$$

Mathematica [A] time = 1.47256, size = 301, normalized size = 0.8

$$\frac{a\left(12i\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,-e^{i\tan^{-1}(ax)}\right) - 12i\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,e^{i\tan^{-1}(ax)}\right) - 12\sqrt{a^2x^2+1}\text{Pol}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTan[a*x]^3/(x^2*(c + a^2*c*x^2)^(3/2)),x]
```

```
[Out] (a*(12 + 12*a*x*ArcTan[a*x] - 6*ArcTan[a*x]^2 - 2*a*x*ArcTan[a*x]^3 - (a*x*
ArcTan[a*x]^3*Csc[ArcTan[a*x]/2]^2)/2 + 6*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*L
og[1 - E^(I*ArcTan[a*x])] - 6*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*Log[1 + E^(I*
ArcTan[a*x])] + (12*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTa
n[a*x])] - (12*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, E^(I*ArcTan[a*x]
```

```
)] - 12*Sqrt[1 + a^2*x^2]*PolyLog[3, -E^(I*ArcTan[a*x])] + 12*Sqrt[1 + a^2*
x^2]*PolyLog[3, E^(I*ArcTan[a*x])] - (2*(1 + a^2*x^2)*ArcTan[a*x]^3*Sin[Arc
Tan[a*x]/2]^2)/(a*x)))/(2*c*Sqrt[c + a^2*c*x^2])
```

Maple [A] time = 0.319, size = 356, normalized size = 0.9

$$-\frac{a\left((\arctan(ax))^3-6\arctan(ax)+3i(\arctan(ax))^2-6i\right)(ax-i)}{\left(2a^2x^2+2\right)c^2}\sqrt{c(ax-i)(ax+i)}-\frac{(ax+i)\left((\arctan(ax))^3-6\right.}{\left(2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^3/x^2/(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] -1/2*a*(arctan(a*x)^3-6*arctan(a*x)+3*I*arctan(a*x)^2-6*I)*(a*x-I)*(c*(a*x-
I)*(a*x+I))^(1/2)/(a^2*x^2+1)/c^2-1/2*(c*(a*x-I)*(a*x+I))^(1/2)*(a*x+I)*(ar
ctan(a*x)^3-6*arctan(a*x)-3*I*arctan(a*x)^2+6*I)*a/(a^2*x^2+1)/c^2-arctan(a
*x)^3*(c*(a*x-I)*(a*x+I))^(1/2)/x/c^2-3*a*(arctan(a*x)^2*ln(1+(1+I*a*x)/(a^
2*x^2+1)^(1/2))-arctan(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*I*arctan(
a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*I*arctan(a*x)*polylog(2,(1+I
*a*x)/(a^2*x^2+1)^(1/2))+2*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*polylo
g(3,(1+I*a*x)/(a^2*x^2+1)^(1/2)))*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)^(1/
2)/c^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x^2/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{a^4c^2x^6+2a^2c^2x^4+c^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x^2/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^3/(a^4*c^2*x^6 + 2*a^2*c^2*x^4 + c
^2*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atan}^3(ax)}{x^2\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**3/x**2/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(atan(a*x)**3/(x**2*(c*(a**2*x**2 + 1))**(3/2)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x^2/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.450 $\int \frac{x^5 \tan^{-1}(ax)^3}{\left(c+a^2cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=534

$$-\frac{6i\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,-ie^{i\tan^{-1}(ax)}\right)}{a^6c^2\sqrt{a^2cx^2+c}}+\frac{6i\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,ie^{i\tan^{-1}(ax)}\right)}{a^6c^2\sqrt{a^2cx^2+c}}+\frac{6\sqrt{a^2x^2+1}\text{Poly}}{a^6c^2}$$

```
[Out] (2*x^3)/(27*a^3*c*(c + a^2*c*x^2)^(3/2)) + (94*x)/(9*a^5*c^2*Sqrt[c + a^2*c
*x^2]) - (2*x^2*ArcTan[a*x])/(9*a^4*c*(c + a^2*c*x^2)^(3/2)) - (94*ArcTan[a
*x])/(9*a^6*c^2*Sqrt[c + a^2*c*x^2]) - (x^3*ArcTan[a*x]^2)/(3*a^3*c*(c + a^
2*c*x^2)^(3/2)) - (5*x*ArcTan[a*x]^2)/(a^5*c^2*Sqrt[c + a^2*c*x^2]) + ((6*I
)*Sqrt[1 + a^2*x^2]*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2)/(a^6*c^2*Sqrt[
c + a^2*c*x^2]) + (x^2*ArcTan[a*x]^3)/(3*a^4*c*(c + a^2*c*x^2)^(3/2)) + (5*
ArcTan[a*x]^3)/(3*a^6*c^2*Sqrt[c + a^2*c*x^2]) + (Sqrt[c + a^2*c*x^2]*ArcTa
n[a*x]^3)/(a^6*c^3) - ((6*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, (-I)*
E^(I*ArcTan[a*x])])/(a^6*c^2*Sqrt[c + a^2*c*x^2]) + ((6*I)*Sqrt[1 + a^2*x^2
]*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])])/(a^6*c^2*Sqrt[c + a^2*c*x^2]
) + (6*Sqrt[1 + a^2*x^2]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])])/(a^6*c^2*Sqrt[
c + a^2*c*x^2]) - (6*Sqrt[1 + a^2*x^2]*PolyLog[3, I*E^(I*ArcTan[a*x])])/(a^
6*c^2*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 1.11098, antiderivative size = 534, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 12, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4964, 4930, 4890, 4888, 4181, 2531, 2282, 6589, 4898, 191, 4940, 4938}

$$-\frac{6i\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,-ie^{i\tan^{-1}(ax)}\right)}{a^6c^2\sqrt{a^2cx^2+c}}+\frac{6i\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,ie^{i\tan^{-1}(ax)}\right)}{a^6c^2\sqrt{a^2cx^2+c}}+\frac{6\sqrt{a^2x^2+1}\text{Poly}}{a^6c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*ArcTan[a*x]^3)/(c + a^2*c*x^2)^(5/2),x]
```


```
[Out] (2*x^3)/(27*a^3*c*(c + a^2*c*x^2)^(3/2)) + (94*x)/(9*a^5*c^2*Sqrt[c + a^2*c
*x^2]) - (2*x^2*ArcTan[a*x])/(9*a^4*c*(c + a^2*c*x^2)^(3/2)) - (94*ArcTan[a
*x])/(9*a^6*c^2*Sqrt[c + a^2*c*x^2]) - (x^3*ArcTan[a*x]^2)/(3*a^3*c*(c + a^
2*c*x^2)^(3/2)) - (5*x*ArcTan[a*x]^2)/(a^5*c^2*Sqrt[c + a^2*c*x^2]) + ((6*I
)*Sqrt[1 + a^2*x^2]*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2)/(a^6*c^2*Sqrt[
c + a^2*c*x^2]) + (x^2*ArcTan[a*x]^3)/(3*a^4*c*(c + a^2*c*x^2)^(3/2)) + (5*
```

```
ArcTan[a*x]^3)/(3*a^6*c^2*Sqrt[c + a^2*c*x^2]) + (Sqrt[c + a^2*c*x^2]*ArcTa
n[a*x]^3)/(a^6*c^3) - ((6*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, (-I)*
E^(I*ArcTan[a*x])])/(a^6*c^2*Sqrt[c + a^2*c*x^2]) + ((6*I)*Sqrt[1 + a^2*x^2
]*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])])/(a^6*c^2*Sqrt[c + a^2*c*x^2]
) + (6*Sqrt[1 + a^2*x^2]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])])/(a^6*c^2*Sqrt[
c + a^2*c*x^2]) - (6*Sqrt[1 + a^2*x^2]*PolyLog[3, I*E^(I*ArcTan[a*x])])/(a^
6*c^2*Sqrt[c + a^2*c*x^2])
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Arc
Tan[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan[c
*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p
, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 4890

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4888

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[c
*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && GtQ
[d, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
```

```
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4898

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[(b*p*(a + b*ArcTan[c*x])^(p - 1))/(c*d*Sqrt[d + e*x^2]), x]
 + (-Dist[b^2*p*(p - 1), Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2),
 x], x] + Simp[(x*(a + b*ArcTan[c*x])^p)/(d*Sqrt[d + e*x^2]), x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_)*((d_) + (e_.)
*(x_)^2)^(q_), x_Symbol] :> Simp[(b*p*(f*x)^m*(d + e*x^2)^(q + 1)*(a + b*Ar
cTan[c*x])^(p - 1))/(c*d*m^2), x] + (Dist[(f^2*(m - 1))/(c^2*d*m), Int[(f*x
)^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p, x], x] - Dist[(b^2*p*(
p - 1))/m^2, Int[(f*x)^m*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x]
- Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(c^2*d*m
```

```
), x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q +
 2, 0] && LtQ[q, -1] && GtQ[p, 1]
```

Rule 4938

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[(b*(f*x)^m*(d + e*x^2)^(q + 1))/(c*d*m^2), x] +
 (Dist[(f^2*(m - 1))/(c^2*d*m), Int[(f*x)^(m - 2)*(d + e*x^2)^(q + 1)*(a +
b*ArcTan[c*x]), x], x] - Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(q + 1)*(a + b*A
rcTan[c*x]))/(c^2*d*m), x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d
] && EqQ[m + 2*q + 2, 0] && LtQ[q, -1]
```

Rubi steps

$$\int \frac{x^5 \tan^{-1}(ax)^3}{\left(c+a^2cx^2\right)^{5/2}}\,dx = -\frac{\int \frac{x^3 \tan^{-1}(ax)^3}{\left(c+a^2cx^2\right)^{5/2}}\,dx}{a^2} + \frac{\int \frac{x^3 \tan^{-1}(ax)^3}{\left(c+a^2cx^2\right)^{3/2}}\,dx}{a^2c}$$

$$= -\frac{x^3 \tan^{-1}(ax)^2}{3a^3c\left(c+a^2cx^2\right)^{3/2}} + \frac{x^2 \tan^{-1}(ax)^3}{3a^4c\left(c+a^2cx^2\right)^{3/2}} + \frac{2\int \frac{x^3 \tan^{-1}(ax)}{\left(c+a^2cx^2\right)^{5/2}}\,dx}{3a^2} + \frac{\int \frac{x \tan^{-1}(ax)^3}{\sqrt{c+a^2cx^2}}\,dx}{a^4c^2} - \frac{2\int \frac{x \tan^{-1}(a}{\left(c+a^2cx^2\right)}}{3a^4c}$$

$$= \frac{2x^3}{27a^3c\left(c+a^2cx^2\right)^{3/2}} - \frac{2x^2 \tan^{-1}(ax)}{9a^4c\left(c+a^2cx^2\right)^{3/2}} - \frac{x^3 \tan^{-1}(ax)^2}{3a^3c\left(c+a^2cx^2\right)^{3/2}} + \frac{x^2 \tan^{-1}(ax)^3}{3a^4c\left(c+a^2cx^2\right)^{3/2}} + \frac{5\tan}{3a^6c^2\sqrt{}}$$

$$= \frac{2x^3}{27a^3c\left(c+a^2cx^2\right)^{3/2}} - \frac{2x^2 \tan^{-1}(ax)}{9a^4c\left(c+a^2cx^2\right)^{3/2}} - \frac{94\tan^{-1}(ax)}{9a^6c^2\sqrt{c+a^2cx^2}} - \frac{x^3 \tan^{-1}(ax)^2}{3a^3c\left(c+a^2cx^2\right)^{3/2}} - \frac{5x\tan^{-1}}{a^5c^2\sqrt{c+}}$$

$$= \frac{2x^3}{27a^3c\left(c+a^2cx^2\right)^{3/2}} + \frac{94x}{9a^5c^2\sqrt{c+a^2cx^2}} - \frac{2x^2 \tan^{-1}(ax)}{9a^4c\left(c+a^2cx^2\right)^{3/2}} - \frac{94\tan^{-1}(ax)}{9a^6c^2\sqrt{c+a^2cx^2}} - \frac{x^3 \tan^{-1}(}{3a^3c\left(c+a^2\right.}$$

$$= \frac{2x^3}{27a^3c\left(c+a^2cx^2\right)^{3/2}} + \frac{94x}{9a^5c^2\sqrt{c+a^2cx^2}} - \frac{2x^2 \tan^{-1}(ax)}{9a^4c\left(c+a^2cx^2\right)^{3/2}} - \frac{94\tan^{-1}(ax)}{9a^6c^2\sqrt{c+a^2cx^2}} - \frac{x^3 \tan^{-1}(}{3a^3c\left(c+a^2\right.}$$

$$= \frac{2x^3}{27a^3c\left(c+a^2cx^2\right)^{3/2}} + \frac{94x}{9a^5c^2\sqrt{c+a^2cx^2}} - \frac{2x^2 \tan^{-1}(ax)}{9a^4c\left(c+a^2cx^2\right)^{3/2}} - \frac{94\tan^{-1}(ax)}{9a^6c^2\sqrt{c+a^2cx^2}} - \frac{x^3 \tan^{-1}(}{3a^3c\left(c+a^2\right.}$$

$$= \frac{2x^3}{27a^3c\left(c+a^2cx^2\right)^{3/2}} + \frac{94x}{9a^5c^2\sqrt{c+a^2cx^2}} - \frac{2x^2 \tan^{-1}(ax)}{9a^4c\left(c+a^2cx^2\right)^{3/2}} - \frac{94\tan^{-1}(ax)}{9a^6c^2\sqrt{c+a^2cx^2}} - \frac{x^3 \tan^{-1}(}{3a^3c\left(c+a^2\right.}$$

$$= \frac{2x^3}{27a^3c\left(c+a^2cx^2\right)^{3/2}} + \frac{94x}{9a^5c^2\sqrt{c+a^2cx^2}} - \frac{2x^2 \tan^{-1}(ax)}{9a^4c\left(c+a^2cx^2\right)^{3/2}} - \frac{94\tan^{-1}(ax)}{9a^6c^2\sqrt{c+a^2cx^2}} - \frac{x^3 \tan^{-1}(}{3a^3c\left(c+a^2\right.}$$

Mathematica [A] time = 2.56383, size = 367, normalized size = 0.69

$$-\frac{\left(a^2x^2+1\right)^2\left(\frac{1296i\tan^{-1}(ax)\text{PolyLog}\left(2,-ie^{i\tan^{-1}(ax)}\right)}{\sqrt{a^2x^2+1}} - \frac{1296i\tan^{-1}(ax)\text{PolyLog}\left(2,ie^{i\tan^{-1}(ax)}\right)}{\sqrt{a^2x^2+1}} - \frac{1296\text{PolyLog}\left(3,-ie^{i\tan^{-1}(ax)}\right)}{\sqrt{a^2x^2+1}} + \frac{1296\text{PolyLo}}{\sqrt{a}}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^5*ArcTan[a*x]^3)/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] -((1 + a^2*x^2)^2*(1134*ArcTan[a*x] - 405*ArcTan[a*x]^3 + 1128*ArcTan[a*x]*
Cos[2*ArcTan[a*x]] - 180*ArcTan[a*x]^3*Cos[2*ArcTan[a*x]] - 6*ArcTan[a*x]*C
os[4*ArcTan[a*x]] + 9*ArcTan[a*x]^3*Cos[4*ArcTan[a*x]] + (648*ArcTan[a*x]^2
*Log[1 - I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - (648*ArcTan[a*x]^2*Log[1
 + I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + ((1296*I)*ArcTan[a*x]*PolyLog[
2, (-I)*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - ((1296*I)*ArcTan[a*x]*PolyL
og[2, I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - (1296*PolyLog[3, (-I)*E^(I*
ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + (1296*PolyLog[3, I*E^(I*ArcTan[a*x])])/S
qrt[1 + a^2*x^2] - 1132*Sin[2*ArcTan[a*x]] + 558*ArcTan[a*x]^2*Sin[2*ArcTan
[a*x]] + 2*Sin[4*ArcTan[a*x]] - 9*ArcTan[a*x]^2*Sin[4*ArcTan[a*x]]))/(216*a
^6*c*(c*(1 + a^2*x^2))^(3/2))
```

Maple [F] time = 1.638, size = 0, normalized size = 0.

$$\int x^5 (\arctan (ax))^3 \left(a^2cx^2 + c\right)^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*arctan(a*x)^3/(a^2*c*x^2+c)^(5/2),x)
```

```
[Out] int(x^5*arctan(a*x)^3/(a^2*c*x^2+c)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arctan(a*x)^3/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a^2cx^2 + c}x^5 \arctan (ax)^3}{a^6c^3x^6 + 3\,a^4c^3x^4 + 3\,a^2c^3x^2 + c^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arctan(a*x)^3/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^5*arctan(a*x)^3/(a^6*c^3*x^6 + 3*a^4*c^3*x^4
 + 3*a^2*c^3*x^2 + c^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5 \operatorname{atan}^3(ax)}{\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*atan(a*x)**3/(a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral(x**5*atan(a*x)**3/(c*(a**2*x**2 + 1))**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5 \arctan(ax)^3}{\left(a^2cx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arctan(a*x)^3/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x^5*arctan(a*x)^3/(a^2*c*x^2 + c)^(5/2), x)
```

3.451 $\int \frac{x^4 \tan^{-1}(ax)^3}{(c+a^2cx^2)^{5/2}} dx$

Optimal. Leaf size=622

$$\frac{3i\sqrt{a^2x^2+1}\tan^{-1}(ax)^2\text{PolyLog}\left(2,-ie^{i\tan^{-1}(ax)}\right)}{a^5c^2\sqrt{a^2cx^2+c}}-\frac{3i\sqrt{a^2x^2+1}\tan^{-1}(ax)^2\text{PolyLog}\left(2,ie^{i\tan^{-1}(ax)}\right)}{a^5c^2\sqrt{a^2cx^2+c}}-\frac{6\sqrt{a^2x^2+1}\tan^{-1}}{a}$$

```
[Out] -2/(27*a^5*c*(c + a^2*c*x^2)^(3/2)) + 68/(9*a^5*c^2*Sqrt[c + a^2*c*x^2]) +
(2*x^3*ArcTan[a*x])/(9*a^2*c*(c + a^2*c*x^2)^(3/2)) + (22*x*ArcTan[a*x])/(3
*a^4*c^2*Sqrt[c + a^2*c*x^2]) - (x^2*ArcTan[a*x]^2)/(3*a^3*c*(c + a^2*c*x^2
)^(3/2)) - (11*ArcTan[a*x]^2)/(3*a^5*c^2*Sqrt[c + a^2*c*x^2]) - (x^3*ArcTan
[a*x]^3)/(3*a^2*c*(c + a^2*c*x^2)^(3/2)) - (x*ArcTan[a*x]^3)/(a^4*c^2*Sqrt[
c + a^2*c*x^2]) - ((2*I)*Sqrt[1 + a^2*x^2]*ArcTan[E^(I*ArcTan[a*x])]*ArcTan
[a*x]^3)/(a^5*c^2*Sqrt[c + a^2*c*x^2]) + ((3*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*
x]^2*PolyLog[2, (-I)*E^(I*ArcTan[a*x])])/(a^5*c^2*Sqrt[c + a^2*c*x^2]) - ((
3*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog[2, I*E^(I*ArcTan[a*x])])/(a^5*
c^2*Sqrt[c + a^2*c*x^2]) - (6*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[3, (-I)
*E^(I*ArcTan[a*x])])/(a^5*c^2*Sqrt[c + a^2*c*x^2]) + (6*Sqrt[1 + a^2*x^2]*A
rcTan[a*x]*PolyLog[3, I*E^(I*ArcTan[a*x])])/(a^5*c^2*Sqrt[c + a^2*c*x^2]) -
 ((6*I)*Sqrt[1 + a^2*x^2]*PolyLog[4, (-I)*E^(I*ArcTan[a*x])])/(a^5*c^2*Sqrt
[c + a^2*c*x^2]) + ((6*I)*Sqrt[1 + a^2*x^2]*PolyLog[4, I*E^(I*ArcTan[a*x])]
)/(a^5*c^2*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.998001, antiderivative size = 622, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 15, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {4964, 4890, 4888, 4181, 2531, 6609, 2282, 6589, 4898, 4894, 4944, 4940, 4930, 266, 43}

$$\frac{3i\sqrt{a^2x^2+1}\tan^{-1}(ax)^2\text{PolyLog}\left(2,-ie^{i\tan^{-1}(ax)}\right)}{a^5c^2\sqrt{a^2cx^2+c}}-\frac{3i\sqrt{a^2x^2+1}\tan^{-1}(ax)^2\text{PolyLog}\left(2,ie^{i\tan^{-1}(ax)}\right)}{a^5c^2\sqrt{a^2cx^2+c}}-\frac{6\sqrt{a^2x^2+1}\tan^{-1}}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*ArcTan[a*x]^3)/(c + a^2*c*x^2)^(5/2),x]
```


```
[Out] -2/(27*a^5*c*(c + a^2*c*x^2)^(3/2)) + 68/(9*a^5*c^2*Sqrt[c + a^2*c*x^2]) +
(2*x^3*ArcTan[a*x])/(9*a^2*c*(c + a^2*c*x^2)^(3/2)) + (22*x*ArcTan[a*x])/(3
*a^4*c^2*Sqrt[c + a^2*c*x^2]) - (x^2*ArcTan[a*x]^2)/(3*a^3*c*(c + a^2*c*x^2
```

```
)^(3/2)) - (11*ArcTan[a*x]^2)/(3*a^5*c^2*Sqrt[c + a^2*c*x^2]) - (x^3*ArcTan
[a*x]^3)/(3*a^2*c*(c + a^2*c*x^2)^(3/2)) - (x*ArcTan[a*x]^3)/(a^4*c^2*Sqrt[
c + a^2*c*x^2]) - ((2*I)*Sqrt[1 + a^2*x^2]*ArcTan[E^(I*ArcTan[a*x])]*ArcTan
[a*x]^3)/(a^5*c^2*Sqrt[c + a^2*c*x^2]) + ((3*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*
x]^2*PolyLog[2, (-I)*E^(I*ArcTan[a*x])])/(a^5*c^2*Sqrt[c + a^2*c*x^2]) - ((
3*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog[2, I*E^(I*ArcTan[a*x])])/(a^5*
c^2*Sqrt[c + a^2*c*x^2]) - (6*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[3, (-I)
*E^(I*ArcTan[a*x])])/(a^5*c^2*Sqrt[c + a^2*c*x^2]) + (6*Sqrt[1 + a^2*x^2]*A
rcTan[a*x]*PolyLog[3, I*E^(I*ArcTan[a*x])])/(a^5*c^2*Sqrt[c + a^2*c*x^2]) -
 ((6*I)*Sqrt[1 + a^2*x^2]*PolyLog[4, (-I)*E^(I*ArcTan[a*x])])/(a^5*c^2*Sqrt
[c + a^2*c*x^2]) + ((6*I)*Sqrt[1 + a^2*x^2]*PolyLog[4, I*E^(I*ArcTan[a*x])]
)/(a^5*c^2*Sqrt[c + a^2*c*x^2])
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Arc
Tan[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan[c
*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p
, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rule 4890

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4888

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[c
*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && GtQ
[d, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4898

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[(b*p*(a + b*ArcTan[c*x])^(p - 1))/(c*d*Sqrt[d + e*x^2]), x]
 + (-Dist[b^2*p*(p - 1), Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2),
 x], x] + Simp[(x*(a + b*ArcTan[c*x])^p)/(d*Sqrt[d + e*x^2]), x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

Rule 4894

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[(x*(a + b*ArcTan[c*x]))/(d*Sq
rt[d + e*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

Rule 4944

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a +
 b*ArcTan[c*x])^p)/(d*f*(m + 1)), x] - Dist[(b*c*p)/(f*(m + 1)), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_)*((d_) + (e_.)
*(x_)^2)^(q_), x_Symbol] :> Simp[(b*p*(f*x)^m*(d + e*x^2)^(q + 1)*(a + b*Ar
cTan[c*x])^(p - 1))/(c*d*m^2), x] + (Dist[(f^2*(m - 1))/(c^2*d*m), Int[(f*x
)^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p, x], x] - Dist[(b^2*p*(
p - 1))/m^2, Int[(f*x)^m*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x]
- Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(c^2*d*m
), x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q +
 2, 0] && LtQ[q, -1] && GtQ[p, 1]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^4 \tan^{-1}(ax)^3}{\left(c+a^2cx^2\right)^{5/2}}\,dx = -\frac{\int \frac{x^2\tan^{-1}(ax)^3}{\left(c+a^2cx^2\right)^{5/2}}\,dx}{a^2} + \frac{\int \frac{x^2\tan^{-1}(ax)^3}{\left(c+a^2cx^2\right)^{3/2}}\,dx}{a^2c}$$

$$= -\frac{x^3\tan^{-1}(ax)^3}{3a^2c\left(c+a^2cx^2\right)^{3/2}} + \frac{\int \frac{x^3\tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^{5/2}}\,dx}{a} + \frac{\int \frac{\tan^{-1}(ax)^3}{\sqrt{c+a^2cx^2}}\,dx}{a^4c^2} - \frac{\int \frac{\tan^{-1}(ax)^3}{\left(c+a^2cx^2\right)^{3/2}}\,dx}{a^4c}$$

$$= \frac{2x^3\tan^{-1}(ax)}{9a^2c\left(c+a^2cx^2\right)^{3/2}} - \frac{x^2\tan^{-1}(ax)^2}{3a^3c\left(c+a^2cx^2\right)^{3/2}} - \frac{3\tan^{-1}(ax)^2}{a^5c^2\sqrt{c+a^2cx^2}} - \frac{x^3\tan^{-1}(ax)^3}{3a^2c\left(c+a^2cx^2\right)^{3/2}} - \frac{x\tan^{-1}(ax)^3}{a^4c^2\sqrt{c+a^2cx}}$$

$$= \frac{6}{a^5c^2\sqrt{c+a^2cx^2}} + \frac{2x^3\tan^{-1}(ax)}{9a^2c\left(c+a^2cx^2\right)^{3/2}} + \frac{6x\tan^{-1}(ax)}{a^4c^2\sqrt{c+a^2cx^2}} - \frac{x^2\tan^{-1}(ax)^2}{3a^3c\left(c+a^2cx^2\right)^{3/2}} - \frac{11\tan^{-1}(ax)^2}{3a^5c^2\sqrt{c+a^2cx^2}}$$

$$= \frac{22}{3a^5c^2\sqrt{c+a^2cx^2}} + \frac{2x^3\tan^{-1}(ax)}{9a^2c\left(c+a^2cx^2\right)^{3/2}} + \frac{22x\tan^{-1}(ax)}{3a^4c^2\sqrt{c+a^2cx^2}} - \frac{x^2\tan^{-1}(ax)^2}{3a^3c\left(c+a^2cx^2\right)^{3/2}} - \frac{11\tan^{-1}(ax)}{3a^5c^2\sqrt{c+a^2c}}$$

$$= -\frac{2}{27a^5c\left(c+a^2cx^2\right)^{3/2}} + \frac{68}{9a^5c^2\sqrt{c+a^2cx^2}} + \frac{2x^3\tan^{-1}(ax)}{9a^2c\left(c+a^2cx^2\right)^{3/2}} + \frac{22x\tan^{-1}(ax)}{3a^4c^2\sqrt{c+a^2cx^2}} - \frac{x^2\tan^{-1}}{3a^3c\left(c+a^2\right.}$$

$$= -\frac{2}{27a^5c\left(c+a^2cx^2\right)^{3/2}} + \frac{68}{9a^5c^2\sqrt{c+a^2cx^2}} + \frac{2x^3\tan^{-1}(ax)}{9a^2c\left(c+a^2cx^2\right)^{3/2}} + \frac{22x\tan^{-1}(ax)}{3a^4c^2\sqrt{c+a^2cx^2}} - \frac{x^2\tan^{-1}}{3a^3c\left(c+a^2\right.}$$

$$= -\frac{2}{27a^5c\left(c+a^2cx^2\right)^{3/2}} + \frac{68}{9a^5c^2\sqrt{c+a^2cx^2}} + \frac{2x^3\tan^{-1}(ax)}{9a^2c\left(c+a^2cx^2\right)^{3/2}} + \frac{22x\tan^{-1}(ax)}{3a^4c^2\sqrt{c+a^2cx^2}} - \frac{x^2\tan^{-1}}{3a^3c\left(c+a^2\right.}$$

$$= -\frac{2}{27a^5c\left(c+a^2cx^2\right)^{3/2}} + \frac{68}{9a^5c^2\sqrt{c+a^2cx^2}} + \frac{2x^3\tan^{-1}(ax)}{9a^2c\left(c+a^2cx^2\right)^{3/2}} + \frac{22x\tan^{-1}(ax)}{3a^4c^2\sqrt{c+a^2cx^2}} - \frac{x^2\tan^{-1}}{3a^3c\left(c+a^2\right.}$$

Mathematica [A] time = 2.62937, size = 691, normalized size = 1.11

$$-\frac{\sqrt{c\left(a^2x^2+1\right)}\left(-5184i\tan^{-1}(ax)^2\text{PolyLog}\left(2,-ie^{-i\tan^{-1}(ax)}\right)-5184i\tan^{-1}(ax)^2\text{PolyLog}\left(2,-ie^{i\tan^{-1}(ax)}\right)+5184i\pi\tan\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*ArcTan[a*x]^3)/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] -(Sqrt[c*(1 + a^2*x^2)]*((189*I)*Pi^4 - 12960/Sqrt[1 + a^2*x^2] + (216*I)*P
i^3*ArcTan[a*x] - (12960*a*x*ArcTan[a*x])/Sqrt[1 + a^2*x^2] - (648*I)*Pi^2*
ArcTan[a*x]^2 + (6480*ArcTan[a*x]^2)/Sqrt[1 + a^2*x^2] + (864*I)*Pi*ArcTan[
a*x]^3 + (2160*a*x*ArcTan[a*x]^3)/Sqrt[1 + a^2*x^2] - (432*I)*ArcTan[a*x]^4
 + 32*Cos[3*ArcTan[a*x]] - 144*ArcTan[a*x]^2*Cos[3*ArcTan[a*x]] - 1296*Pi^2
*ArcTan[a*x]*Log[1 - I/E^(I*ArcTan[a*x])] + 2592*Pi*ArcTan[a*x]^2*Log[1 - I
/E^(I*ArcTan[a*x])] + 216*Pi^3*Log[1 + I/E^(I*ArcTan[a*x])] - 1728*ArcTan[a
*x]^3*Log[1 + I/E^(I*ArcTan[a*x])] - 216*Pi^3*Log[1 + I*E^(I*ArcTan[a*x])]
+ 1296*Pi^2*ArcTan[a*x]*Log[1 + I*E^(I*ArcTan[a*x])] - 2592*Pi*ArcTan[a*x]^
2*Log[1 + I*E^(I*ArcTan[a*x])] + 1728*ArcTan[a*x]^3*Log[1 + I*E^(I*ArcTan[a
*x])] - 216*Pi^3*Log[Tan[(Pi + 2*ArcTan[a*x])/4]] - (5184*I)*ArcTan[a*x]^2*
PolyLog[2, (-I)/E^(I*ArcTan[a*x])] - (1296*I)*Pi*(Pi - 4*ArcTan[a*x])*PolyL
og[2, I/E^(I*ArcTan[a*x])] - (1296*I)*Pi^2*PolyLog[2, (-I)*E^(I*ArcTan[a*x]
)] + (5184*I)*Pi*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - (5184*I)*
ArcTan[a*x]^2*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - 10368*ArcTan[a*x]*PolyLo
g[3, (-I)/E^(I*ArcTan[a*x])] + 5184*Pi*PolyLog[3, I/E^(I*ArcTan[a*x])] - 51
84*Pi*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] + 10368*ArcTan[a*x]*PolyLog[3, (-I
)*E^(I*ArcTan[a*x])] + (10368*I)*PolyLog[4, (-I)/E^(I*ArcTan[a*x])] + (1036
8*I)*PolyLog[4, (-I)*E^(I*ArcTan[a*x])] + 96*ArcTan[a*x]*Sin[3*ArcTan[a*x]]
 - 144*ArcTan[a*x]^3*Sin[3*ArcTan[a*x]]))/(1728*a^5*c^3*Sqrt[1 + a^2*x^2])
```

Maple [F] time = 0.884, size = 0, normalized size = 0.

$$\int x^4 (\arctan(ax))^3 \left(a^2cx^2+c\right)^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arctan(a*x)^3/(a^2*c*x^2+c)^(5/2),x)
```

```
[Out] int(x^4*arctan(a*x)^3/(a^2*c*x^2+c)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctan(a*x)^3/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a^2cx^2+c}x^4\arctan\left(ax\right)^3}{a^6c^3x^6+3\,a^4c^3x^4+3\,a^2c^3x^2+c^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctan(a*x)^3/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^4*arctan(a*x)^3/(a^6*c^3*x^6 + 3*a^4*c^3*x^4
 + 3*a^2*c^3*x^2 + c^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^4\,\text{atan}^3\left(ax\right)}{\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*atan(a*x)**3/(a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral(x**4*atan(a*x)**3/(c*(a**2*x**2 + 1))**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^4\arctan\left(ax\right)^3}{\left(a^2cx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctan(a*x)^3/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x^4*arctan(a*x)^3/(a^2*c*x^2 + c)^(5/2), x)
```

3.452 $\int \frac{x^3 \tan^{-1}(ax)^3}{\left(c+a^2cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=237

$$-\frac{40x}{9a^3c^2\sqrt{a^2cx^2+c}}+\frac{2x\tan^{-1}(ax)^2}{a^3c^2\sqrt{a^2cx^2+c}}-\frac{2\tan^{-1}(ax)^3}{3a^4c^2\sqrt{a^2cx^2+c}}+\frac{40\tan^{-1}(ax)}{9a^4c^2\sqrt{a^2cx^2+c}}-\frac{2x^3}{27ac\left(a^2cx^2+c\right)^{3/2}}+\frac{x^3\tan^{-1}(ax)^2}{3ac\left(a^2cx^2+c\right)^{3/2}}$$

```
[Out] (-2*x^3)/(27*a*c*(c + a^2*c*x^2)^(3/2)) - (40*x)/(9*a^3*c^2*Sqrt[c + a^2*c*
x^2]) + (2*x^2*ArcTan[a*x])/(9*a^2*c*(c + a^2*c*x^2)^(3/2)) + (40*ArcTan[a*
x])/(9*a^4*c^2*Sqrt[c + a^2*c*x^2]) + (x^3*ArcTan[a*x]^2)/(3*a*c*(c + a^2*c
*x^2)^(3/2)) + (2*x*ArcTan[a*x]^2)/(a^3*c^2*Sqrt[c + a^2*c*x^2]) - (x^2*Arc
Tan[a*x]^3)/(3*a^2*c*(c + a^2*c*x^2)^(3/2)) - (2*ArcTan[a*x]^3)/(3*a^4*c^2*
Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.412489, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {4940, 4930, 4898, 191, 4938}

$$-\frac{40x}{9a^3c^2\sqrt{a^2cx^2+c}}+\frac{2x\tan^{-1}(ax)^2}{a^3c^2\sqrt{a^2cx^2+c}}-\frac{2\tan^{-1}(ax)^3}{3a^4c^2\sqrt{a^2cx^2+c}}+\frac{40\tan^{-1}(ax)}{9a^4c^2\sqrt{a^2cx^2+c}}-\frac{2x^3}{27ac\left(a^2cx^2+c\right)^{3/2}}+\frac{x^3\tan^{-1}(ax)^2}{3ac\left(a^2cx^2+c\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcTan[a*x]^3)/(c + a^2*c*x^2)^(5/2),x]
```



```
[Out] (-2*x^3)/(27*a*c*(c + a^2*c*x^2)^(3/2)) - (40*x)/(9*a^3*c^2*Sqrt[c + a^2*c*
x^2]) + (2*x^2*ArcTan[a*x])/(9*a^2*c*(c + a^2*c*x^2)^(3/2)) + (40*ArcTan[a*
x])/(9*a^4*c^2*Sqrt[c + a^2*c*x^2]) + (x^3*ArcTan[a*x]^2)/(3*a*c*(c + a^2*c
*x^2)^(3/2)) + (2*x*ArcTan[a*x]^2)/(a^3*c^2*Sqrt[c + a^2*c*x^2]) - (x^2*Arc
Tan[a*x]^3)/(3*a^2*c*(c + a^2*c*x^2)^(3/2)) - (2*ArcTan[a*x]^3)/(3*a^4*c^2*
Sqrt[c + a^2*c*x^2])
```


Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_)*((d_) + (e_.)
*(x_)^2)^(q_), x_Symbol] :> Simp[(b*p*(f*x)^m*(d + e*x^2)^(q + 1)*(a + b*Ar
cTan[c*x])^(p - 1))/(c*d*m^2), x] + (Dist[(f^2*(m - 1))/(c^2*d*m), Int[(f*x
)^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p, x], x] - Dist[(b^2*p*(
p - 1))/m^2, Int[(f*x)^m*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x]
```

```
- Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(c^2*d*m
), x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q +
 2, 0] && LtQ[q, -1] && GtQ[p, 1]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 4898

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[(b*p*(a + b*ArcTan[c*x])^(p - 1))/(c*d*Sqrt[d + e*x^2]), x]
 + (-Dist[b^2*p*(p - 1), Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2),
 x], x] + Simp[(x*(a + b*ArcTan[c*x])^p)/(d*Sqrt[d + e*x^2]), x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 4938

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[(b*(f*x)^m*(d + e*x^2)^(q + 1))/(c*d*m^2), x] +
 (Dist[(f^2*(m - 1))/(c^2*d*m), Int[(f*x)^(m - 2)*(d + e*x^2)^(q + 1)*(a +
b*ArcTan[c*x]), x], x] - Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(q + 1)*(a + b*A
rcTan[c*x]))/(c^2*d*m), x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d
] && EqQ[m + 2*q + 2, 0] && LtQ[q, -1]
```

Rubi steps

$$\int \frac{x^3 \tan^{-1}(ax)^3}{\left(c+a^2cx^2\right)^{5/2}}\,dx = \frac{x^3 \tan^{-1}(ax)^2}{3ac\left(c+a^2cx^2\right)^{3/2}} - \frac{x^2 \tan^{-1}(ax)^3}{3a^2c\left(c+a^2cx^2\right)^{3/2}} - \frac{2}{3}\int \frac{x^3 \tan^{-1}(ax)}{\left(c+a^2cx^2\right)^{5/2}}\,dx + \frac{2\int \frac{x\tan^{-1}(ax)^3}{\left(c+a^2cx^2\right)^{3/2}}\,dx}{3a^2c}$$

$$= -\frac{2x^3}{27ac\left(c+a^2cx^2\right)^{3/2}} + \frac{2x^2\tan^{-1}(ax)}{9a^2c\left(c+a^2cx^2\right)^{3/2}} + \frac{x^3\tan^{-1}(ax)^2}{3ac\left(c+a^2cx^2\right)^{3/2}} - \frac{x^2\tan^{-1}(ax)^3}{3a^2c\left(c+a^2cx^2\right)^{3/2}} - \frac{2\tan^{-1}}{3a^4c^2\sqrt{c-}}$$

$$= -\frac{2x^3}{27ac\left(c+a^2cx^2\right)^{3/2}} + \frac{2x^2\tan^{-1}(ax)}{9a^2c\left(c+a^2cx^2\right)^{3/2}} + \frac{40\tan^{-1}(ax)}{9a^4c^2\sqrt{c+a^2cx^2}} + \frac{x^3\tan^{-1}(ax)^2}{3ac\left(c+a^2cx^2\right)^{3/2}} + \frac{2x\tan^{-1}(a}{a^3c^2\sqrt{c+a}}$$

$$= -\frac{2x^3}{27ac\left(c+a^2cx^2\right)^{3/2}} - \frac{40x}{9a^3c^2\sqrt{c+a^2cx^2}} + \frac{2x^2\tan^{-1}(ax)}{9a^2c\left(c+a^2cx^2\right)^{3/2}} + \frac{40\tan^{-1}(ax)}{9a^4c^2\sqrt{c+a^2cx^2}} + \frac{x^3\tan^{-1}(a}{3ac\left(c+a^2c\right.}$$

Mathematica [A] time = 0.13304, size = 104, normalized size = 0.44

$$\frac{\sqrt{a^2cx^2+c}\left(-2ax\left(61a^2x^2+60\right)-9\left(3a^2x^2+2\right)\tan^{-1}(ax)^3+9ax\left(7a^2x^2+6\right)\tan^{-1}(ax)^2+6\left(21a^2x^2+20\right)\tan^{-1}(ax)\right)}{27a^4c^3\left(a^2x^2+1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*ArcTan[a*x]^3)/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] (Sqrt[c + a^2*c*x^2]*(-2*a*x*(60 + 61*a^2*x^2) + 6*(20 + 21*a^2*x^2)*ArcTan
[a*x] + 9*a*x*(6 + 7*a^2*x^2)*ArcTan[a*x]^2 - 9*(2 + 3*a^2*x^2)*ArcTan[a*x]
^3))/(27*a^4*c^3*(1 + a^2*x^2)^2)
```

Maple [C] time = 0.969, size = 312, normalized size = 1.3

$$-\frac{\left(9\,i\,(\arctan(ax))^2+9\,(\arctan(ax))^3-2\,i-6\,\arctan(ax)\right)\left(ix^3a^3+3\,a^2x^2-3\,iax-1\right)}{216\left(a^2x^2+1\right)^2c^3a^4}\sqrt{c\,(ax-i)\,(ax+i)} - \frac{\left(3\,(\arcta\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctan(a*x)^3/(a^2*c*x^2+c)^(5/2),x)
```

```
[Out] -1/216*(9*I*arctan(a*x)^2+9*arctan(a*x)^3-2*I-6*arctan(a*x))*(I*x^3*a^3+3*a
^2*x^2-3*I*a*x-1)*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)^2/c^3/a^4-3/8*(arct
an(a*x)^3-6*arctan(a*x)+3*I*arctan(a*x)^2-6*I)*(1+I*a*x)*(c*(a*x-I)*(a*x+I)
)^(1/2)/a^4/c^3/(a^2*x^2+1)+3/8*(c*(a*x-I)*(a*x+I))^(1/2)*(-1+I*a*x)*(arcta
n(a*x)^3-6*arctan(a*x)-3*I*arctan(a*x)^2+6*I)/a^4/c^3/(a^2*x^2+1)+1/216*(c*
(a*x-I)*(a*x+I))^(1/2)*(I*x^3*a^3-3*a^2*x^2-3*I*a*x+1)*(-9*I*arctan(a*x)^2+
9*arctan(a*x)^3+2*I-6*arctan(a*x))/a^4/c^3/(a^4*x^4+2*a^2*x^2+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \arctan (ax)^3}{\left(a^2cx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^3/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3*arctan(a*x)^3/(a^2*c*x^2 + c)^(5/2), x)
```

Fricas [A] time = 2.00609, size = 266, normalized size = 1.12

$$-\frac{\left(122\,a^3x^3+9\left(3\,a^2x^2+2\right)\arctan (ax)^3-9\left(7\,a^3x^3+6\,ax\right)\arctan (ax)^2+120\,ax-6\left(21\,a^2x^2+20\right)\arctan (ax)\right)\sqrt{a^2c\cdots}}{27\left(a^8c^3x^4+2\,a^6c^3x^2+a^4c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^3/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/27*(122*a^3*x^3 + 9*(3*a^2*x^2 + 2)*arctan(a*x)^3 - 9*(7*a^3*x^3 + 6*a*x
)*arctan(a*x)^2 + 120*a*x - 6*(21*a^2*x^2 + 20)*arctan(a*x))*sqrt(a^2*c*x^2
 + c)/(a^8*c^3*x^4 + 2*a^6*c^3*x^2 + a^4*c^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \operatorname{atan}^3 (ax)}{\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atan(a*x)**3/(a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral(x**3*atan(a*x)**3/(c*(a**2*x**2 + 1))**(5/2), x)
```

Giac [A] time = 1.3581, size = 207, normalized size = 0.87

$$\frac{x\left(\frac{7x^2}{ac}+\frac{6}{a^3c}\right)\arctan(ax)^2}{3\left(a^2cx^2+c\right)^{\frac{3}{2}}}-\frac{2x\left(\frac{61x^2}{ac}+\frac{60}{a^3c}\right)}{27\left(a^2cx^2+c\right)^{\frac{3}{2}}}-\frac{\left(3a^2cx^2+2c\right)\arctan(ax)^3}{3\left(a^2cx^2+c\right)^{\frac{3}{2}}a^4c^2}+\frac{2\left(21a^2cx^2+20c\right)\arctan(ax)}{9\left(a^2cx^2+c\right)^{\frac{3}{2}}a^4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^3/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*x*(7*x^2/(a*c) + 6/(a^3*c))*arctan(a*x)^2/(a^2*c*x^2 + c)^(3/2) - 2/27*
x*(61*x^2/(a*c) + 60/(a^3*c))/(a^2*c*x^2 + c)^(3/2) - 1/3*(3*a^2*c*x^2 + 2*
c)*arctan(a*x)^3/((a^2*c*x^2 + c)^(3/2)*a^4*c^2) + 2/9*(21*a^2*c*x^2 + 20*c
)*arctan(a*x)/((a^2*c*x^2 + c)^(3/2)*a^4*c^2)
```

3.453 $\int \frac{x^2 \tan^{-1}(ax)^3}{(c+a^2cx^2)^{5/2}}\, dx$

Optimal. Leaf size=199

$$-\frac{14}{9a^3c^2\sqrt{a^2cx^2+c}}-\frac{4x\tan^{-1}(ax)}{3a^2c^2\sqrt{a^2cx^2+c}}+\frac{2\tan^{-1}(ax)^2}{3a^3c^2\sqrt{a^2cx^2+c}}+\frac{2}{27a^3c\left(a^2cx^2+c\right)^{3/2}}+\frac{x^3\tan^{-1}(ax)^3}{3c\left(a^2cx^2+c\right)^{3/2}}-\frac{2x^3\tan^{-1}(ax)}{9c\left(a^2cx^2+c\right)^{3/2}}$$

```
[Out] 2/(27*a^3*c*(c + a^2*c*x^2)^(3/2)) - 14/(9*a^3*c^2*Sqrt[c + a^2*c*x^2]) - (
2*x^3*ArcTan[a*x])/(9*c*(c + a^2*c*x^2)^(3/2)) - (4*x*ArcTan[a*x])/(3*a^2*c
^2*Sqrt[c + a^2*c*x^2]) + (x^2*ArcTan[a*x]^2)/(3*a*c*(c + a^2*c*x^2)^(3/2))
 + (2*ArcTan[a*x]^2)/(3*a^3*c^2*Sqrt[c + a^2*c*x^2]) + (x^3*ArcTan[a*x]^3)/
(3*c*(c + a^2*c*x^2)^(3/2))
```

Rubi [A] time = 0.406532, antiderivative size = 199, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4944, 4940, 4930, 4894, 266, 43}

$$-\frac{14}{9a^3c^2\sqrt{a^2cx^2+c}}-\frac{4x\tan^{-1}(ax)}{3a^2c^2\sqrt{a^2cx^2+c}}+\frac{2\tan^{-1}(ax)^2}{3a^3c^2\sqrt{a^2cx^2+c}}+\frac{2}{27a^3c\left(a^2cx^2+c\right)^{3/2}}+\frac{x^3\tan^{-1}(ax)^3}{3c\left(a^2cx^2+c\right)^{3/2}}-\frac{2x^3\tan^{-1}(ax)}{9c\left(a^2cx^2+c\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcTan[a*x]^3)/(c + a^2*c*x^2)^(5/2),x]
```


```
[Out] 2/(27*a^3*c*(c + a^2*c*x^2)^(3/2)) - 14/(9*a^3*c^2*Sqrt[c + a^2*c*x^2]) - (
2*x^3*ArcTan[a*x])/(9*c*(c + a^2*c*x^2)^(3/2)) - (4*x*ArcTan[a*x])/(3*a^2*c
^2*Sqrt[c + a^2*c*x^2]) + (x^2*ArcTan[a*x]^2)/(3*a*c*(c + a^2*c*x^2)^(3/2))
 + (2*ArcTan[a*x]^2)/(3*a^3*c^2*Sqrt[c + a^2*c*x^2]) + (x^3*ArcTan[a*x]^3)/
(3*c*(c + a^2*c*x^2)^(3/2))
```


Rule 4944

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a +
 b*ArcTan[c*x])^p)/(d*f*(m + 1)), x] - Dist[(b*c*p)/(f*(m + 1)), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_)*((d_) + (e_.)
*(x_)^2)^(q_), x_Symbol] :> Simp[(b*p*(f*x)^m*(d + e*x^2)^(q + 1)*(a + b*Ar
cTan[c*x])^(p - 1))/(c*d*m^2), x] + (Dist[(f^2*(m - 1))/(c^2*d*m), Int[(f*x
)^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p, x], x] - Dist[(b^2*p*(
p - 1))/m^2, Int[(f*x)^m*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x]
- Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(c^2*d*m
), x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q +
 2, 0] && LtQ[q, -1] && GtQ[p, 1]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 4894

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[(x*(a + b*ArcTan[c*x]))/(d*Sq
rt[d + e*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^2\tan^{-1}(ax)^3}{\left(c+a^2cx^2\right)^{5/2}}\,dx = \frac{x^3\tan^{-1}(ax)^3}{3c\left(c+a^2cx^2\right)^{3/2}} - a\int \frac{x^3\tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^{5/2}}\,dx$$

$$= -\frac{2x^3\tan^{-1}(ax)}{9c\left(c+a^2cx^2\right)^{3/2}} + \frac{x^2\tan^{-1}(ax)^2}{3ac\left(c+a^2cx^2\right)^{3/2}} + \frac{x^3\tan^{-1}(ax)^3}{3c\left(c+a^2cx^2\right)^{3/2}} + \frac{1}{9}(2a)\int \frac{x^3}{\left(c+a^2cx^2\right)^{5/2}}\,dx - \frac{2\int \frac{x}{(c}}{}$$

$$= -\frac{2x^3\tan^{-1}(ax)}{9c\left(c+a^2cx^2\right)^{3/2}} + \frac{x^2\tan^{-1}(ax)^2}{3ac\left(c+a^2cx^2\right)^{3/2}} + \frac{2\tan^{-1}(ax)^2}{3a^3c^2\sqrt{c+a^2cx^2}} + \frac{x^3\tan^{-1}(ax)^3}{3c\left(c+a^2cx^2\right)^{3/2}} + \frac{1}{9}a\,\text{Subst}\left(\int\right.$$

$$= -\frac{4}{3a^3c^2\sqrt{c+a^2cx^2}} - \frac{2x^3\tan^{-1}(ax)}{9c\left(c+a^2cx^2\right)^{3/2}} - \frac{4x\tan^{-1}(ax)}{3a^2c^2\sqrt{c+a^2cx^2}} + \frac{x^2\tan^{-1}(ax)^2}{3ac\left(c+a^2cx^2\right)^{3/2}} + \frac{2\tan^{-1}(ax)^2}{3a^3c^2\sqrt{c+a^2c}}$$

$$= \frac{2}{27a^3c\left(c+a^2cx^2\right)^{3/2}} - \frac{14}{9a^3c^2\sqrt{c+a^2cx^2}} - \frac{2x^3\tan^{-1}(ax)}{9c\left(c+a^2cx^2\right)^{3/2}} - \frac{4x\tan^{-1}(ax)}{3a^2c^2\sqrt{c+a^2cx^2}} + \frac{x^2\tan^{-1}(ax}{3ac\left(c+a^2cx^2\right.}$$

Mathematica [A] time = 0.101736, size = 95, normalized size = 0.48

$$\frac{\sqrt{a^2cx^2+c}\left(-42a^2x^2+9a^3x^3\tan^{-1}(ax)^3+9\left(3a^2x^2+2\right)\tan^{-1}(ax)^2-6ax\left(7a^2x^2+6\right)\tan^{-1}(ax)-40\right)}{27a^3c^3\left(a^2x^2+1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*ArcTan[a*x]^3)/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] (Sqrt[c + a^2*c*x^2]*(-40 - 42*a^2*x^2 - 6*a*x*(6 + 7*a^2*x^2)*ArcTan[a*x]
+ 9*(2 + 3*a^2*x^2)*ArcTan[a*x]^2 + 9*a^3*x^3*ArcTan[a*x]^3))/(27*a^3*c^3*(
1 + a^2*x^2)^2)
```

Maple [C] time = 0.733, size = 308, normalized size = 1.6

$$\frac{\left(9\,i\left(\arctan\left(ax\right)\right)^2+9\,\left(\arctan\left(ax\right)\right)^3-2\,i-6\,\arctan\left(ax\right)\right)\left(a^3x^3-3\,ia^2x^2-3\,ax+i\right)}{216\,\left(a^2x^2+1\right)^2c^3a^3}\sqrt{c\left(ax-i\right)\left(ax+i\right)}+\frac{\left(\left(\arctan\left(\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(a*x)^3/(a^2*c*x^2+c)^(5/2),x)
```

```
[Out] 1/216*(9*I*arctan(a*x)^2+9*arctan(a*x)^3-2*I-6*arctan(a*x))*(a^3*x^3-3*I*a^
2*x^2-3*a*x+I)*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)^2/c^3/a^3+1/8*(arctan(
a*x)^3-6*arctan(a*x)+3*I*arctan(a*x)^2-6*I)*(a*x-I)*(c*(a*x-I)*(a*x+I))^(1/
2)/a^3/c^3/(a^2*x^2+1)+1/8*(c*(a*x-I)*(a*x+I))^(1/2)*(a*x+I)*(arctan(a*x)^3
-6*arctan(a*x)-3*I*arctan(a*x)^2+6*I)/a^3/c^3/(a^2*x^2+1)+1/216*(-9*I*arcta
n(a*x)^2+9*arctan(a*x)^3+2*I-6*arctan(a*x))*(c*(a*x-I)*(a*x+I))^(1/2)*(a^3*
x^3+3*I*a^2*x^2-3*a*x-I)/(a^4*x^4+2*a^2*x^2+1)/c^3/a^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \arctan(ax)^3}{\left(a^2cx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^3/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2*arctan(a*x)^3/(a^2*c*x^2 + c)^(5/2), x)
```

Fricas [A] time = 2.12679, size = 243, normalized size = 1.22

$$\frac{\left(9\,a^3x^3\arctan(ax)^3-42\,a^2x^2+9\left(3\,a^2x^2+2\right)\arctan(ax)^2-6\left(7\,a^3x^3+6\,ax\right)\arctan(ax)-40\right)\sqrt{a^2cx^2+c}}{27\left(a^7c^3x^4+2\,a^5c^3x^2+a^3c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^3/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/27*(9*a^3*x^3*arctan(a*x)^3 - 42*a^2*x^2 + 9*(3*a^2*x^2 + 2)*arctan(a*x)^
2 - 6*(7*a^3*x^3 + 6*a*x)*arctan(a*x) - 40)*sqrt(a^2*c*x^2 + c)/(a^7*c^3*x^
4 + 2*a^5*c^3*x^2 + a^3*c^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \operatorname{atan}^3(ax)}{\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(a*x)**3/(a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral(x**2*atan(a*x)**3/(c*(a**2*x**2 + 1))**(5/2), x)
```

Giac [A] time = 1.3084, size = 192, normalized size = 0.96

$$\frac{x^3 \arctan(ax)^3}{3\left(a^2cx^2+c\right)^{\frac{3}{2}}c} - \frac{1}{27}\left(\frac{6x\left(\frac{7x^2}{ac}+\frac{6}{a^3c}\right)\arctan(ax)}{\left(a^2cx^2+c\right)^{\frac{3}{2}}} - \frac{9\left(3a^2cx^2+2c\right)\arctan(ax)^2}{\left(a^2cx^2+c\right)^{\frac{3}{2}}a^4c^2} + \frac{2\left(21a^2cx^2+20c\right)}{\left(a^2cx^2+c\right)^{\frac{3}{2}}a^4c^2}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^3/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*x^3*arctan(a*x)^3/((a^2*c*x^2 + c)^(3/2)*c) - 1/27*(6*x*(7*x^2/(a*c) +
6/(a^3*c))*arctan(a*x)/(a^2*c*x^2 + c)^(3/2) - 9*(3*a^2*c*x^2 + 2*c)*arctan
(a*x)^2/((a^2*c*x^2 + c)^(3/2)*a^4*c^2) + 2*(21*a^2*c*x^2 + 20*c)/((a^2*c*x
^2 + c)^(3/2)*a^4*c^2))*a
```

3.454 $\int \frac{x\tan^{-1}(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=199

$$-\frac{40x}{27ac^2\sqrt{a^2cx^2+c}}+\frac{2x\tan^{-1}(ax)^2}{3ac^2\sqrt{a^2cx^2+c}}+\frac{4\tan^{-1}(ax)}{3a^2c^2\sqrt{a^2cx^2+c}}-\frac{2x}{27ac\left(a^2cx^2+c\right)^{3/2}}-\frac{\tan^{-1}(ax)^3}{3a^2c\left(a^2cx^2+c\right)^{3/2}}+\frac{x\tan^{-1}(ax)^2}{3ac\left(a^2cx^2+c\right)^{3/2}}$$

```
[Out] (-2*x)/(27*a*c*(c + a^2*c*x^2)^(3/2)) - (40*x)/(27*a*c^2*Sqrt[c + a^2*c*x^2
]) + (2*ArcTan[a*x])/(9*a^2*c*(c + a^2*c*x^2)^(3/2)) + (4*ArcTan[a*x])/(3*a
^2*c^2*Sqrt[c + a^2*c*x^2]) + (x*ArcTan[a*x]^2)/(3*a*c*(c + a^2*c*x^2)^(3/2
)) + (2*x*ArcTan[a*x]^2)/(3*a*c^2*Sqrt[c + a^2*c*x^2]) - ArcTan[a*x]^3/(3*a
^2*c*(c + a^2*c*x^2)^(3/2))
```

Rubi [A] time = 0.193512, antiderivative size = 199, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {4930, 4900, 4898, 191, 192}

$$-\frac{40x}{27ac^2\sqrt{a^2cx^2+c}}+\frac{2x\tan^{-1}(ax)^2}{3ac^2\sqrt{a^2cx^2+c}}+\frac{4\tan^{-1}(ax)}{3a^2c^2\sqrt{a^2cx^2+c}}-\frac{2x}{27ac\left(a^2cx^2+c\right)^{3/2}}-\frac{\tan^{-1}(ax)^3}{3a^2c\left(a^2cx^2+c\right)^{3/2}}+\frac{x\tan^{-1}(ax)^2}{3ac\left(a^2cx^2+c\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTan[a*x]^3)/(c + a^2*c*x^2)^(5/2),x]
```



```
[Out] (-2*x)/(27*a*c*(c + a^2*c*x^2)^(3/2)) - (40*x)/(27*a*c^2*Sqrt[c + a^2*c*x^2
]) + (2*ArcTan[a*x])/(9*a^2*c*(c + a^2*c*x^2)^(3/2)) + (4*ArcTan[a*x])/(3*a
^2*c^2*Sqrt[c + a^2*c*x^2]) + (x*ArcTan[a*x]^2)/(3*a*c*(c + a^2*c*x^2)^(3/2
)) + (2*x*ArcTan[a*x]^2)/(3*a*c^2*Sqrt[c + a^2*c*x^2]) - ArcTan[a*x]^3/(3*a
^2*c*(c + a^2*c*x^2)^(3/2))
```


Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 4900

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(b*p*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^(p - 1))/(4*c*d
*(q + 1)^2), x] + (Dist[(2*q + 3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a
 + b*ArcTan[c*x])^p, x], x] - Dist[(b^2*p*(p - 1))/(4*(q + 1)^2), Int[(d +
e*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x] - Simp[(x*(d + e*x^2)^(q + 1)*
(a + b*ArcTan[c*x])^p)/(2*d*(q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] && E
qQ[e, c^2*d] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

Rule 4898

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[(b*p*(a + b*ArcTan[c*x])^(p - 1))/(c*d*Sqrt[d + e*x^2]), x]
 + (-Dist[b^2*p*(p - 1), Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2),
 x], x] + Simp[(x*(a + b*ArcTan[c*x])^p)/(d*Sqrt[d + e*x^2]), x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rubi steps

$$\int \frac{x \tan^{-1}(ax)^3}{\left(c+a^2cx^2\right)^{5/2}}\,dx = -\frac{\tan^{-1}(ax)^3}{3a^2c\left(c+a^2cx^2\right)^{3/2}} + \frac{\int \frac{\tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^{5/2}}\,dx}{a}$$

$$= \frac{2\tan^{-1}(ax)}{9a^2c\left(c+a^2cx^2\right)^{3/2}} + \frac{x\tan^{-1}(ax)^2}{3ac\left(c+a^2cx^2\right)^{3/2}} - \frac{\tan^{-1}(ax)^3}{3a^2c\left(c+a^2cx^2\right)^{3/2}} - \frac{2\int \frac{1}{\left(c+a^2cx^2\right)^{5/2}}\,dx}{9a} + \frac{2\int \frac{\tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^{3/2}}}{3ac}$$

$$= -\frac{2x}{27ac\left(c+a^2cx^2\right)^{3/2}} + \frac{2\tan^{-1}(ax)}{9a^2c\left(c+a^2cx^2\right)^{3/2}} + \frac{4\tan^{-1}(ax)}{3a^2c^2\sqrt{c+a^2cx^2}} + \frac{x\tan^{-1}(ax)^2}{3ac\left(c+a^2cx^2\right)^{3/2}} + \frac{2x\tan^{-1}(a}{3ac^2\sqrt{c+a}}$$

$$= -\frac{2x}{27ac\left(c+a^2cx^2\right)^{3/2}} - \frac{40x}{27ac^2\sqrt{c+a^2cx^2}} + \frac{2\tan^{-1}(ax)}{9a^2c\left(c+a^2cx^2\right)^{3/2}} + \frac{4\tan^{-1}(ax)}{3a^2c^2\sqrt{c+a^2cx^2}} + \frac{x\tan^{-1}(a}{3ac\left(c+a^2c\right.}$$

Mathematica [A] time = 0.0929382, size = 91, normalized size = 0.46

$$\frac{\sqrt{a^2cx^2+c}\left(-2ax\left(20a^2x^2+21\right)+9ax\left(2a^2x^2+3\right)\tan^{-1}(ax)^2+6\left(6a^2x^2+7\right)\tan^{-1}(ax)-9\tan^{-1}(ax)^3\right)}{27c^3\left(a^3x^2+a\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcTan[a*x]^3)/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] (Sqrt[c + a^2*c*x^2]*(-2*a*x*(21 + 20*a^2*x^2) + 6*(7 + 6*a^2*x^2)*ArcTan[a
*x] + 9*a*x*(3 + 2*a^2*x^2)*ArcTan[a*x]^2 - 9*ArcTan[a*x]^3))/(27*c^3*(a +
a^3*x^2)^2)
```

Maple [C] time = 0.311, size = 312, normalized size = 1.6

$$\frac{\left(9i(\arctan(ax))^2+9(\arctan(ax))^3-2i-6\arctan(ax)\right)\left(ix^3a^3+3a^2x^2-3iax-1\right)}{216\left(a^2x^2+1\right)^2c^3a^2}\sqrt{c(ax-i)(ax+i)} - \frac{\left((\arctan(a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(a*x)^3/(a^2*c*x^2+c)^(5/2),x)
```

```
[Out] 1/216*(9*I*arctan(a*x)^2+9*arctan(a*x)^3-2*I-6*arctan(a*x))*(I*x^3*a^3+3*a^
2*x^2-3*I*a*x-1)*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)^2/c^3/a^2-1/8*(arcta
n(a*x)^3-6*arctan(a*x)+3*I*arctan(a*x)^2-6*I)*(1+I*a*x)*(c*(a*x-I)*(a*x+I))
^(1/2)/a^2/c^3/(a^2*x^2+1)+1/8*(c*(a*x-I)*(a*x+I))^(1/2)*(-1+I*a*x)*(arctan
(a*x)^3-6*arctan(a*x)-3*I*arctan(a*x)^2+6*I)/a^2/c^3/(a^2*x^2+1)-1/216*(c*(
a*x-I)*(a*x+I))^(1/2)*(I*x^3*a^3-3*a^2*x^2-3*I*a*x+1)*(-9*I*arctan(a*x)^2+9
*arctan(a*x)^3+2*I-6*arctan(a*x))/a^2/c^3/(a^4*x^4+2*a^2*x^2+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \arctan(ax)^3}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^3/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x*arctan(a*x)^3/(a^2*c*x^2 + c)^(5/2), x)
```

Fricas [A] time = 1.95946, size = 239, normalized size = 1.2

$$-\frac{\left(40\,a^3x^3-9\left(2\,a^3x^3+3\,ax\right)\arctan(ax)^2+9\,\arctan(ax)^3+42\,ax-6\left(6\,a^2x^2+7\right)\arctan(ax)\right)\sqrt{a^2cx^2+c}}{27\left(a^6c^3x^4+2\,a^4c^3x^2+a^2c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^3/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/27*(40*a^3*x^3 - 9*(2*a^3*x^3 + 3*a*x)*arctan(a*x)^2 + 9*arctan(a*x)^3 +
 42*a*x - 6*(6*a^2*x^2 + 7)*arctan(a*x))*sqrt(a^2*c*x^2 + c)/(a^6*c^3*x^4 +
 2*a^4*c^3*x^2 + a^2*c^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\,\mathrm{atan}^3(ax)}{\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(a*x)**3/(a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral(x*atan(a*x)**3/(c*(a**2*x**2 + 1))**(5/2), x)
```

Giac [A] time = 1.30652, size = 184, normalized size = 0.92

$$\frac{\left(\frac{2\,ax^2}{c}+\frac{3}{ac}\right)x\arctan\left(ax\right)^2}{3\left(a^2cx^2+c\right)^{\frac{3}{2}}}-\frac{2\left(\frac{20\,ax^2}{c}+\frac{21}{ac}\right)x}{27\left(a^2cx^2+c\right)^{\frac{3}{2}}}-\frac{\arctan\left(ax\right)^3}{3\left(a^2cx^2+c\right)^{\frac{3}{2}}a^2c}+\frac{2\left(6\,a^2cx^2+7\,c\right)\arctan\left(ax\right)}{9\left(a^2cx^2+c\right)^{\frac{3}{2}}a^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^3/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(2*a*x^2/c + 3/(a*c))*x*arctan(a*x)^2/(a^2*c*x^2 + c)^(3/2) - 2/27*(20*
a*x^2/c + 21/(a*c))*x/(a^2*c*x^2 + c)^(3/2) - 1/3*arctan(a*x)^3/((a^2*c*x^2
 + c)^(3/2)*a^2*c) + 2/9*(6*a^2*c*x^2 + 7*c)*arctan(a*x)/((a^2*c*x^2 + c)^(
3/2)*a^2*c^2)
```

3.455 $\int \frac{\tan^{-1}(ax)^3}{\left(c+a^2cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=215

$$-\frac{40}{9ac^2\sqrt{a^2cx^2+c}}+\frac{2x\tan^{-1}(ax)^3}{3c^2\sqrt{a^2cx^2+c}}+\frac{2\tan^{-1}(ax)^2}{ac^2\sqrt{a^2cx^2+c}}-\frac{40x\tan^{-1}(ax)}{9c^2\sqrt{a^2cx^2+c}}-\frac{2}{27ac\left(a^2cx^2+c\right)^{3/2}}+\frac{x\tan^{-1}(ax)^3}{3c\left(a^2cx^2+c\right)^{3/2}}+\frac{\text{t}}{3ac}$$

```
[Out] -2/(27*a*c*(c + a^2*c*x^2)^(3/2)) - 40/(9*a*c^2*Sqrt[c + a^2*c*x^2]) - (2*x
*ArcTan[a*x])/(9*c*(c + a^2*c*x^2)^(3/2)) - (40*x*ArcTan[a*x])/(9*c^2*Sqrt[
c + a^2*c*x^2]) + ArcTan[a*x]^2/(3*a*c*(c + a^2*c*x^2)^(3/2)) + (2*ArcTan[a
*x]^2)/(a*c^2*Sqrt[c + a^2*c*x^2]) + (x*ArcTan[a*x]^3)/(3*c*(c + a^2*c*x^2)
^(3/2)) + (2*x*ArcTan[a*x]^3)/(3*c^2*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.178554, antiderivative size = 215, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {4900, 4898, 4894, 4896}

$$-\frac{40}{9ac^2\sqrt{a^2cx^2+c}}+\frac{2x\tan^{-1}(ax)^3}{3c^2\sqrt{a^2cx^2+c}}+\frac{2\tan^{-1}(ax)^2}{ac^2\sqrt{a^2cx^2+c}}-\frac{40x\tan^{-1}(ax)}{9c^2\sqrt{a^2cx^2+c}}-\frac{2}{27ac\left(a^2cx^2+c\right)^{3/2}}+\frac{x\tan^{-1}(ax)^3}{3c\left(a^2cx^2+c\right)^{3/2}}+\frac{\text{t}}{3ac}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^3/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] -2/(27*a*c*(c + a^2*c*x^2)^(3/2)) - 40/(9*a*c^2*Sqrt[c + a^2*c*x^2]) - (2*x
*ArcTan[a*x])/(9*c*(c + a^2*c*x^2)^(3/2)) - (40*x*ArcTan[a*x])/(9*c^2*Sqrt[
c + a^2*c*x^2]) + ArcTan[a*x]^2/(3*a*c*(c + a^2*c*x^2)^(3/2)) + (2*ArcTan[a
*x]^2)/(a*c^2*Sqrt[c + a^2*c*x^2]) + (x*ArcTan[a*x]^3)/(3*c*(c + a^2*c*x^2)
^(3/2)) + (2*x*ArcTan[a*x]^3)/(3*c^2*Sqrt[c + a^2*c*x^2])
```

Rule 4900

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(b*p*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^(p - 1))/(4*c*d
*(q + 1)^2), x] + (Dist[(2*q + 3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a
 + b*ArcTan[c*x])^p, x], x] - Dist[(b^2*p*(p - 1))/(4*(q + 1)^2), Int[(d +
e*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x] - Simp[(x*(d + e*x^2)^(q + 1)*
(a + b*ArcTan[c*x])^p)/(2*d*(q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] && E
qQ[e, c^2*d] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

Rule 4898

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[(b*p*(a + b*ArcTan[c*x])^(p - 1))/(c*d*Sqrt[d + e*x^2]), x]
 + (-Dist[b^2*p*(p - 1), Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2),
 x], x] + Simp[(x*(a + b*ArcTan[c*x])^p)/(d*Sqrt[d + e*x^2]), x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

Rule 4894

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[(x*(a + b*ArcTan[c*x]))/(d*Sq
rt[d + e*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

Rule 4896

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbol
] :> Simp[(b*(d + e*x^2)^(q + 1))/(4*c*d*(q + 1)^2), x] + (Dist[(2*q + 3)/(
2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]), x], x] - Simp[(x
*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]))/(2*d*(q + 1)), x]) /; FreeQ[{a, b
, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -3/2]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^3}{\left(c+a^2cx^2\right)^{5/2}}\,dx = \frac{\tan^{-1}(ax)^2}{3ac\left(c+a^2cx^2\right)^{3/2}} + \frac{x\tan^{-1}(ax)^3}{3c\left(c+a^2cx^2\right)^{3/2}} - \frac{2}{3}\int \frac{\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^{5/2}}\,dx + \frac{2\int \frac{\tan^{-1}(ax)^3}{\left(c+a^2cx^2\right)^{3/2}}\,dx}{3c}$$

$$= -\frac{2}{27ac\left(c+a^2cx^2\right)^{3/2}} - \frac{2x\tan^{-1}(ax)}{9c\left(c+a^2cx^2\right)^{3/2}} + \frac{\tan^{-1}(ax)^2}{3ac\left(c+a^2cx^2\right)^{3/2}} + \frac{2\tan^{-1}(ax)^2}{ac^2\sqrt{c+a^2cx^2}} + \frac{x\tan^{-1}(ax)^3}{3c\left(c+a^2cx^2\right)^{3/2}}$$

$$= -\frac{2}{27ac\left(c+a^2cx^2\right)^{3/2}} - \frac{40}{9ac^2\sqrt{c+a^2cx^2}} - \frac{2x\tan^{-1}(ax)}{9c\left(c+a^2cx^2\right)^{3/2}} - \frac{40x\tan^{-1}(ax)}{9c^2\sqrt{c+a^2cx^2}} + \frac{\tan^{-1}(ax)^2}{3ac\left(c+a^2cx^2\right)^{3/2}}$$

Mathematica [A] time = 0.0847638, size = 104, normalized size = 0.48

$$\frac{\sqrt{a^2cx^2+c}\left(-2\left(60a^2x^2+61\right)+9ax\left(2a^2x^2+3\right)\tan^{-1}(ax)^3+9\left(6a^2x^2+7\right)\tan^{-1}(ax)^2-6ax\left(20a^2x^2+21\right)\tan^{-1}(ax)\right)}{27ac^3\left(a^2x^2+1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]^3/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] (Sqrt[c + a^2*c*x^2]*(-2*(61 + 60*a^2*x^2) - 6*a*x*(21 + 20*a^2*x^2)*ArcTan
[a*x] + 9*(7 + 6*a^2*x^2)*ArcTan[a*x]^2 + 9*a*x*(3 + 2*a^2*x^2)*ArcTan[a*x]
^3))/(27*a*c^3*(1 + a^2*x^2)^2)
```

Maple [C] time = 0.27, size = 308, normalized size = 1.4

$$-\frac{\left(9\,i\left(\arctan\left(ax\right)\right)^{2}+9\,\left(\arctan\left(ax\right)\right)^{3}-2\,i-6\,\arctan\left(ax\right)\right)\left(a^{3}x^{3}-3\,ia^{2}x^{2}-3\,ax+i\right)}{216\,\left(a^{2}x^{2}+1\right)^{2}ac^{3}}\sqrt{c\left(ax-i\right)\left(ax+i\right)}+\frac{\left(3\,(\mathrm{arcta}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^3/(a^2*c*x^2+c)^(5/2),x)
```

```
[Out] -1/216*(9*I*arctan(a*x)^2+9*arctan(a*x)^3-2*I-6*arctan(a*x))*(a^3*x^3-3*I*a
^2*x^2-3*a*x+I)*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)^2/a/c^3+3/8*(arctan(a
*x)^3-6*arctan(a*x)+3*I*arctan(a*x)^2-6*I)*(a*x-I)*(c*(a*x-I)*(a*x+I))^(1/2
)/c^3/a/(a^2*x^2+1)+3/8*(c*(a*x-I)*(a*x+I))^(1/2)*(a*x+I)*(arctan(a*x)^3-6*
arctan(a*x)-3*I*arctan(a*x)^2+6*I)/c^3/a/(a^2*x^2+1)-1/216*(-9*I*arctan(a*x
)^2+9*arctan(a*x)^3+2*I-6*arctan(a*x))*(c*(a*x-I)*(a*x+I))^(1/2)*(a^3*x^3+3
*I*a^2*x^2-3*a*x-I)/(a^4*x^4+2*a^2*x^2+1)/a/c^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan\left(ax\right)^{3}}{\left(a^{2}cx^{2}+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(arctan(a*x)^3/(a^2*c*x^2 + c)^(5/2), x)
```

Fricas [A] time = 1.72267, size = 263, normalized size = 1.22

$$-\frac{\sqrt{a^2cx^2+c}\left(120\,a^2x^2-9\left(2\,a^3x^3+3\,ax\right)\arctan\left(ax\right)^3-9\left(6\,a^2x^2+7\right)\arctan\left(ax\right)^2+6\left(20\,a^3x^3+21\,ax\right)\arctan\left(ax\right)+\right.}{27\left(a^5c^3x^4+2\,a^3c^3x^2+ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/27*sqrt(a^2*c*x^2 + c)*(120*a^2*x^2 - 9*(2*a^3*x^3 + 3*a*x)*arctan(a*x)^
3 - 9*(6*a^2*x^2 + 7)*arctan(a*x)^2 + 6*(20*a^3*x^3 + 21*a*x)*arctan(a*x) +
 122)/(a^5*c^3*x^4 + 2*a^3*c^3*x^2 + a*c^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atan}^3(ax)}{\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**3/(a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral(atan(a*x)**3/(c*(a**2*x**2 + 1))**(5/2), x)
```

Giac [A] time = 1.43972, size = 198, normalized size = 0.92

$$\frac{\left(\frac{2\,a^2x^2}{c}+\frac{3}{c}\right)x\arctan\left(ax\right)^3}{3\left(a^2cx^2+c\right)^{\frac{3}{2}}}-\frac{2\left(\frac{20\,a^2x^2}{c}+\frac{21}{c}\right)x\arctan\left(ax\right)}{9\left(a^2cx^2+c\right)^{\frac{3}{2}}}+\frac{\left(6\,a^2cx^2+7\,c\right)\arctan\left(ax\right)^2}{3\left(a^2cx^2+c\right)^{\frac{3}{2}}ac^2}-\frac{2\left(60\,a^2cx^2+61\,c\right)}{27\left(a^2cx^2+c\right)^{\frac{3}{2}}ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(2*a^2*x^2/c + 3/c)*x*arctan(a*x)^3/(a^2*c*x^2 + c)^(3/2) - 2/9*(20*a^2
*x^2/c + 21/c)*x*arctan(a*x)/(a^2*c*x^2 + c)^(3/2) + 1/3*(6*a^2*c*x^2 + 7*c
)*arctan(a*x)^2/((a^2*c*x^2 + c)^(3/2)*a*c^2) - 2/27*(60*a^2*c*x^2 + 61*c)/
((a^2*c*x^2 + c)^(3/2)*a*c^2)
```

3.456 $\int \frac{\tan^{-1}(ax)^3}{x\left(c+a^2cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=553

$$\frac{3i\sqrt{a^2x^2+1}\tan^{-1}(ax)^2\text{PolyLog}\left(2,-e^{i\tan^{-1}(ax)}\right)}{c^2\sqrt{a^2cx^2+c}}-\frac{3i\sqrt{a^2x^2+1}\tan^{-1}(ax)^2\text{PolyLog}\left(2,e^{i\tan^{-1}(ax)}\right)}{c^2\sqrt{a^2cx^2+c}}-6\sqrt{a^2x^2+1}\tan^{-}$$

```
[Out] (2*a*x)/(27*c*(c + a^2*c*x^2)^(3/2)) + (202*a*x)/(27*c^2*Sqrt[c + a^2*c*x^2
]) - (2*ArcTan[a*x])/(9*c*(c + a^2*c*x^2)^(3/2)) - (22*ArcTan[a*x])/(3*c^2*
Sqrt[c + a^2*c*x^2]) - (a*x*ArcTan[a*x]^2)/(3*c*(c + a^2*c*x^2)^(3/2)) - (1
1*a*x*ArcTan[a*x]^2)/(3*c^2*Sqrt[c + a^2*c*x^2]) + ArcTan[a*x]^3/(3*c*(c +
a^2*c*x^2)^(3/2)) + ArcTan[a*x]^3/(c^2*Sqrt[c + a^2*c*x^2]) - (2*Sqrt[1 + a
^2*x^2]*ArcTan[a*x]^3*ArcTanh[E^(I*ArcTan[a*x])])/(c^2*Sqrt[c + a^2*c*x^2])
 + ((3*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog[2, -E^(I*ArcTan[a*x])])/(
c^2*Sqrt[c + a^2*c*x^2]) - ((3*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog[2
, E^(I*ArcTan[a*x])])/(c^2*Sqrt[c + a^2*c*x^2]) - (6*Sqrt[1 + a^2*x^2]*ArcT
an[a*x]*PolyLog[3, -E^(I*ArcTan[a*x])])/(c^2*Sqrt[c + a^2*c*x^2]) + (6*Sqrt
[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[3, E^(I*ArcTan[a*x])])/(c^2*Sqrt[c + a^2*
c*x^2]) - ((6*I)*Sqrt[1 + a^2*x^2]*PolyLog[4, -E^(I*ArcTan[a*x])])/(c^2*Sqr
t[c + a^2*c*x^2]) + ((6*I)*Sqrt[1 + a^2*x^2]*PolyLog[4, E^(I*ArcTan[a*x])])
/(c^2*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.921598, antiderivative size = 553, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 13, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.542, Rules used = {4966, 4958, 4956, 4183, 2531, 6609, 2282, 6589, 4930, 4898, 191, 4900, 192}

$$\frac{3i\sqrt{a^2x^2+1}\tan^{-1}(ax)^2\text{PolyLog}\left(2,-e^{i\tan^{-1}(ax)}\right)}{c^2\sqrt{a^2cx^2+c}}-\frac{3i\sqrt{a^2x^2+1}\tan^{-1}(ax)^2\text{PolyLog}\left(2,e^{i\tan^{-1}(ax)}\right)}{c^2\sqrt{a^2cx^2+c}}-6\sqrt{a^2x^2+1}\tan^{-}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^3/(x*(c + a^2*c*x^2)^(5/2)),x]
```

```
[Out] (2*a*x)/(27*c*(c + a^2*c*x^2)^(3/2)) + (202*a*x)/(27*c^2*Sqrt[c + a^2*c*x^2
]) - (2*ArcTan[a*x])/(9*c*(c + a^2*c*x^2)^(3/2)) - (22*ArcTan[a*x])/(3*c^2*
Sqrt[c + a^2*c*x^2]) - (a*x*ArcTan[a*x]^2)/(3*c*(c + a^2*c*x^2)^(3/2)) - (1
1*a*x*ArcTan[a*x]^2)/(3*c^2*Sqrt[c + a^2*c*x^2]) + ArcTan[a*x]^3/(3*c*(c +
```

```
a^2*c*x^2)^(3/2)) + ArcTan[a*x]^3/(c^2*Sqrt[c + a^2*c*x^2]) - (2*Sqrt[1 + a
^2*x^2]*ArcTan[a*x]^3*ArcTanh[E^(I*ArcTan[a*x])])/(c^2*Sqrt[c + a^2*c*x^2])
 + ((3*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog[2, -E^(I*ArcTan[a*x])])/(
c^2*Sqrt[c + a^2*c*x^2]) - ((3*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog[2
, E^(I*ArcTan[a*x])])/(c^2*Sqrt[c + a^2*c*x^2]) - (6*Sqrt[1 + a^2*x^2]*ArcT
an[a*x]*PolyLog[3, -E^(I*ArcTan[a*x])])/(c^2*Sqrt[c + a^2*c*x^2]) + (6*Sqrt
[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[3, E^(I*ArcTan[a*x])])/(c^2*Sqrt[c + a^2*
c*x^2]) - ((6*I)*Sqrt[1 + a^2*x^2]*PolyLog[4, -E^(I*ArcTan[a*x])])/(c^2*Sqr
t[c + a^2*c*x^2]) + ((6*I)*Sqrt[1 + a^2*x^2]*PolyLog[4, E^(I*ArcTan[a*x])])
/(c^2*Sqrt[c + a^2*c*x^2])
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*
x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^p
, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*q]
 && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 4958

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4956

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Dist[1/Sqrt[d], Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTan
[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && G
tQ[d, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
```

```
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 4898

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[(b*p*(a + b*ArcTan[c*x])^(p - 1))/(c*d*Sqrt[d + e*x^2]), x]
 + (-Dist[b^2*p*(p - 1), Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2),
 x], x] + Simp[(x*(a + b*ArcTan[c*x])^p)/(d*Sqrt[d + e*x^2]), x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 4900

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(b*p*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^(p - 1))/(4*c*d
*(q + 1)^2), x] + (Dist[(2*q + 3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a
 + b*ArcTan[c*x])^p, x], x] - Dist[(b^2*p*(p - 1))/(4*(q + 1)^2), Int[(d +
e*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x] - Simp[(x*(d + e*x^2)^(q + 1)*
(a + b*ArcTan[c*x])^p)/(2*d*(q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] && E
qQ[e, c^2*d] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^3}{x\left(c+a^2cx^2\right)^{5/2}}\,dx = -\left(a^2\int \frac{x\tan^{-1}(ax)^3}{\left(c+a^2cx^2\right)^{5/2}}\,dx\right) + \frac{\int \frac{\tan^{-1}(ax)^3}{x\left(c+a^2cx^2\right)^{3/2}}\,dx}{c}$$

$$= \frac{\tan^{-1}(ax)^3}{3c\left(c+a^2cx^2\right)^{3/2}} - a\int \frac{\tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^{5/2}}\,dx + \frac{\int \frac{\tan^{-1}(ax)^3}{x\sqrt{c+a^2cx^2}}\,dx}{c^2} - \frac{a^2\int \frac{x\tan^{-1}(ax)^3}{\left(c+a^2cx^2\right)^{3/2}}\,dx}{c}$$

$$= -\frac{2\tan^{-1}(ax)}{9c\left(c+a^2cx^2\right)^{3/2}} - \frac{ax\tan^{-1}(ax)^2}{3c\left(c+a^2cx^2\right)^{3/2}} + \frac{\tan^{-1}(ax)^3}{3c\left(c+a^2cx^2\right)^{3/2}} + \frac{\tan^{-1}(ax)^3}{c^2\sqrt{c+a^2cx^2}} + \frac{1}{9}(2a)\int \frac{1}{\left(c+a^2c\right.}$$

$$= \frac{2ax}{27c\left(c+a^2cx^2\right)^{3/2}} - \frac{2\tan^{-1}(ax)}{9c\left(c+a^2cx^2\right)^{3/2}} - \frac{22\tan^{-1}(ax)}{3c^2\sqrt{c+a^2cx^2}} - \frac{ax\tan^{-1}(ax)^2}{3c\left(c+a^2cx^2\right)^{3/2}} - \frac{11ax\tan^{-1}(ax)^2}{3c^2\sqrt{c+a^2cx^2}}$$

$$= \frac{2ax}{27c\left(c+a^2cx^2\right)^{3/2}} + \frac{202ax}{27c^2\sqrt{c+a^2cx^2}} - \frac{2\tan^{-1}(ax)}{9c\left(c+a^2cx^2\right)^{3/2}} - \frac{22\tan^{-1}(ax)}{3c^2\sqrt{c+a^2cx^2}} - \frac{ax\tan^{-1}(ax)^2}{3c\left(c+a^2cx^2\right)^{3/2}}$$

$$= \frac{2ax}{27c\left(c+a^2cx^2\right)^{3/2}} + \frac{202ax}{27c^2\sqrt{c+a^2cx^2}} - \frac{2\tan^{-1}(ax)}{9c\left(c+a^2cx^2\right)^{3/2}} - \frac{22\tan^{-1}(ax)}{3c^2\sqrt{c+a^2cx^2}} - \frac{ax\tan^{-1}(ax)^2}{3c\left(c+a^2cx^2\right)^{3/2}}$$

$$= \frac{2ax}{27c\left(c+a^2cx^2\right)^{3/2}} + \frac{202ax}{27c^2\sqrt{c+a^2cx^2}} - \frac{2\tan^{-1}(ax)}{9c\left(c+a^2cx^2\right)^{3/2}} - \frac{22\tan^{-1}(ax)}{3c^2\sqrt{c+a^2cx^2}} - \frac{ax\tan^{-1}(ax)^2}{3c\left(c+a^2cx^2\right)^{3/2}}$$

$$= \frac{2ax}{27c\left(c+a^2cx^2\right)^{3/2}} + \frac{202ax}{27c^2\sqrt{c+a^2cx^2}} - \frac{2\tan^{-1}(ax)}{9c\left(c+a^2cx^2\right)^{3/2}} - \frac{22\tan^{-1}(ax)}{3c^2\sqrt{c+a^2cx^2}} - \frac{ax\tan^{-1}(ax)^2}{3c\left(c+a^2cx^2\right)^{3/2}}$$

$$= \frac{2ax}{27c\left(c+a^2cx^2\right)^{3/2}} + \frac{202ax}{27c^2\sqrt{c+a^2cx^2}} - \frac{2\tan^{-1}(ax)}{9c\left(c+a^2cx^2\right)^{3/2}} - \frac{22\tan^{-1}(ax)}{3c^2\sqrt{c+a^2cx^2}} - \frac{ax\tan^{-1}(ax)^2}{3c\left(c+a^2cx^2\right)^{3/2}}$$

Mathematica [A] time = 0.77725, size = 347, normalized size = 0.63

$$\frac{\left(a^2x^2+1\right)^{3/2}\left(648i\tan^{-1}(ax)^2\text{PolyLog}\left(2,e^{-i\tan^{-1}(ax)}\right)+648i\tan^{-1}(ax)^2\text{PolyLog}\left(2,-e^{i\tan^{-1}(ax)}\right)+1296\tan^{-1}(ax)\text{Po}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTan[a*x]^3/(x*(c + a^2*c*x^2)^(5/2)),x]
```

[Out] ((1 + a^2*x^2)^(3/2)*((-27*I)*Pi^4 + (1620*a*x)/Sqrt[1 + a^2*x^2] - (1620*ArcTan[a*x])/Sqrt[1 + a^2*x^2] - (810*a*x*ArcTan[a*x]^2)/Sqrt[1 + a^2*x^2] + (270*ArcTan[a*x]^3)/Sqrt[1 + a^2*x^2] + (54*I)*ArcTan[a*x]^4 - 12*ArcTan[a*x]*Cos[3*ArcTan[a*x]] + 18*ArcTan[a*x]^3*Cos[3*ArcTan[a*x]] + 216*ArcTan[a*x]^3*Log[1 - E^((-I)*ArcTan[a*x])] - 216*ArcTan[a*x]^3*Log[1 + E^(I*ArcTan[a*x])] + (648*I)*ArcTan[a*x]^2*PolyLog[2, E^((-I)*ArcTan[a*x])] + (648*I)*ArcTan[a*x]^2*PolyLog[2, -E^(I*ArcTan[a*x])] + 1296*ArcTan[a*x]*PolyLog[3, E^((-I)*ArcTan[a*x])] - 1296*ArcTan[a*x]*PolyLog[3, -E^(I*ArcTan[a*x])] - (1296*I)*PolyLog[4, E^((-I)*ArcTan[a*x])] - (1296*I)*PolyLog[4, -E^(I*ArcTan[a*x])] + 4*Sin[3*ArcTan[a*x]] - 18*ArcTan[a*x]^2*Sin[3*ArcTan[a*x]]))/(216*c*(c*(1 + a^2*x^2))^(3/2))

Maple [A] time = 0.353, size = 560, normalized size = 1.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arctan(a*x)^3/x/(a^2*c*x^2+c)^(5/2),x)

[Out] -1/216*(9*I*arctan(a*x)^2+9*arctan(a*x)^3-2*I-6*arctan(a*x))*(I*x^3*a^3+3*a^2*x^2-3*I*a*x-1)*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)^2/c^3+5/8*(arctan(a*x)^3-6*arctan(a*x)+3*I*arctan(a*x)^2-6*I)*(1+I*a*x)*(c*(a*x-I)*(a*x+I))^(1/2)/c^3/(a^2*x^2+1)-5/8*(c*(a*x-I)*(a*x+I))^(1/2)*(-1+I*a*x)*(arctan(a*x)^3-6*arctan(a*x)-3*I*arctan(a*x)^2+6*I)/c^3/(a^2*x^2+1)+1/216*(c*(a*x-I)*(a*x+I))^(1/2)*(I*x^3*a^3-3*a^2*x^2-3*I*a*x+1)*(-9*I*arctan(a*x)^2+9*arctan(a*x)^3+2*I-6*arctan(a*x))/c^3/(a^4*x^4+2*a^2*x^2+1)+I*(I*arctan(a*x)^3*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*arctan(a*x)^3*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))+3*arctan(a*x)^2*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-3*arctan(a*x)^2*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I*arctan(a*x)*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*I*arctan(a*x)*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*polylog(4,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*polylog(4,(1+I*a*x)/(a^2*x^2+1)^(1/2)))*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)^(1/2)/c^3

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{a^6c^3x^7+3\,a^4c^3x^5+3\,a^2c^3x^3+c^3x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^3/(a^6*c^3*x^7 + 3*a^4*c^3*x^5 + 3
*a^2*c^3*x^3 + c^3*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atan}^3(ax)}{x\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**3/x/(a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral(atan(a*x)**3/(x*(c*(a**2*x**2 + 1))**(5/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(ax)^3}{\left(a^2cx^2+c\right)^{\frac{5}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)^3/((a^2*c*x^2 + c)^(5/2)*x), x)
```

3.457 $\int \frac{\tan^{-1}(ax)^3}{x^2\left(c+a^2cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=493

$$\frac{6ia\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,-e^{i\tan^{-1}(ax)}\right)}{c^2\sqrt{a^2cx^2+c}}-\frac{6ia\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,e^{i\tan^{-1}(ax)}\right)}{c^2\sqrt{a^2cx^2+c}}-\frac{6a\sqrt{a^2x^2+1}\text{Pol}}{c^2\sqrt{}}$$

```
[Out] (2*a)/(27*c*(c + a^2*c*x^2)^(3/2)) + (94*a)/(9*c^2*Sqrt[c + a^2*c*x^2]) + (
2*a^2*x*ArcTan[a*x])/(9*c*(c + a^2*c*x^2)^(3/2)) + (94*a^2*x*ArcTan[a*x])/(
9*c^2*Sqrt[c + a^2*c*x^2]) - (a*ArcTan[a*x]^2)/(3*c*(c + a^2*c*x^2)^(3/2))
- (5*a*ArcTan[a*x]^2)/(c^2*Sqrt[c + a^2*c*x^2]) - (a^2*x*ArcTan[a*x]^3)/(3*
c*(c + a^2*c*x^2)^(3/2)) - (5*a^2*x*ArcTan[a*x]^3)/(3*c^2*Sqrt[c + a^2*c*x^
2]) - (Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/(c^3*x) - (6*a*Sqrt[1 + a^2*x^2]*
ArcTan[a*x]^2*ArcTanh[E^(I*ArcTan[a*x])])/(c^2*Sqrt[c + a^2*c*x^2]) + ((6*I
)*a*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])])/(c^2*Sqrt
[c + a^2*c*x^2]) - ((6*I)*a*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, E^(I*A
rcTan[a*x])])/(c^2*Sqrt[c + a^2*c*x^2]) - (6*a*Sqrt[1 + a^2*x^2]*PolyLog[3,
 -E^(I*ArcTan[a*x])])/(c^2*Sqrt[c + a^2*c*x^2]) + (6*a*Sqrt[1 + a^2*x^2]*Po
lyLog[3, E^(I*ArcTan[a*x])])/(c^2*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.951968, antiderivative size = 493, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 12, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4966, 4944, 4958, 4956, 4183, 2531, 2282, 6589, 4898, 4894, 4900, 4896}

$$\frac{6ia\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,-e^{i\tan^{-1}(ax)}\right)}{c^2\sqrt{a^2cx^2+c}}-\frac{6ia\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,e^{i\tan^{-1}(ax)}\right)}{c^2\sqrt{a^2cx^2+c}}-\frac{6a\sqrt{a^2x^2+1}\text{Pol}}{c^2\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^3/(x^2*(c + a^2*c*x^2)^(5/2)),x]
```

```
[Out] (2*a)/(27*c*(c + a^2*c*x^2)^(3/2)) + (94*a)/(9*c^2*Sqrt[c + a^2*c*x^2]) + (
2*a^2*x*ArcTan[a*x])/(9*c*(c + a^2*c*x^2)^(3/2)) + (94*a^2*x*ArcTan[a*x])/(
9*c^2*Sqrt[c + a^2*c*x^2]) - (a*ArcTan[a*x]^2)/(3*c*(c + a^2*c*x^2)^(3/2))
- (5*a*ArcTan[a*x]^2)/(c^2*Sqrt[c + a^2*c*x^2]) - (a^2*x*ArcTan[a*x]^3)/(3*
c*(c + a^2*c*x^2)^(3/2)) - (5*a^2*x*ArcTan[a*x]^3)/(3*c^2*Sqrt[c + a^2*c*x^
2]) - (Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/(c^3*x) - (6*a*Sqrt[1 + a^2*x^2]*
ArcTan[a*x]^2*ArcTanh[E^(I*ArcTan[a*x])])/(c^2*Sqrt[c + a^2*c*x^2]) + ((6*I
```

```
)*a*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])])/(c^2*Sqrt
[c + a^2*c*x^2]) - ((6*I)*a*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, E^(I*A
rcTan[a*x])])/(c^2*Sqrt[c + a^2*c*x^2]) - (6*a*Sqrt[1 + a^2*x^2]*PolyLog[3,
 -E^(I*ArcTan[a*x])])/(c^2*Sqrt[c + a^2*c*x^2]) + (6*a*Sqrt[1 + a^2*x^2]*Po
lyLog[3, E^(I*ArcTan[a*x])])/(c^2*Sqrt[c + a^2*c*x^2])
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*
x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^p
, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*q]
 && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 4944

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a +
 b*ArcTan[c*x])^p)/(d*f*(m + 1)), x] - Dist[(b*c*p)/(f*(m + 1)), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rule 4958

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4956

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Dist[1/Sqrt[d], Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTan
[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && G
tQ[d, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4898

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[(b*p*(a + b*ArcTan[c*x])^(p - 1))/(c*d*Sqrt[d + e*x^2]), x]
 + (-Dist[b^2*p*(p - 1), Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2),
 x], x] + Simp[(x*(a + b*ArcTan[c*x])^p)/(d*Sqrt[d + e*x^2]), x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

Rule 4894

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[(x*(a + b*ArcTan[c*x]))/(d*Sq
rt[d + e*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

Rule 4900

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(b*p*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^(p - 1))/(4*c*d
*(q + 1)^2), x] + (Dist[(2*q + 3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a
 + b*ArcTan[c*x])^p, x], x] - Dist[(b^2*p*(p - 1))/(4*(q + 1)^2), Int[(d +
e*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x] - Simp[(x*(d + e*x^2)^(q + 1)*
(a + b*ArcTan[c*x])^p)/(2*d*(q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] && E
```

```
qQ[e, c^2*d] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

Rule 4896

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbol
] :> Simp[(b*(d + e*x^2)^(q + 1))/(4*c*d*(q + 1)^2), x] + (Dist[(2*q + 3)/(
2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]), x], x] - Simp[(x
*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]))/(2*d*(q + 1)), x]) /; FreeQ[{a, b
, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -3/2]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^3}{x^2\left(c+a^2cx^2\right)^{5/2}}\,dx = -\left(a^2\int \frac{\tan^{-1}(ax)^3}{\left(c+a^2cx^2\right)^{5/2}}\,dx\right) + \frac{\int \frac{\tan^{-1}(ax)^3}{x^2\left(c+a^2cx^2\right)^{3/2}}\,dx}{c}$$

$$= -\frac{a\tan^{-1}(ax)^2}{3c\left(c+a^2cx^2\right)^{3/2}} - \frac{a^2x\tan^{-1}(ax)^3}{3c\left(c+a^2cx^2\right)^{3/2}} + \frac{1}{3}\left(2a^2\right)\int \frac{\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^{5/2}}\,dx + \frac{\int \frac{\tan^{-1}(ax)^3}{x^2\sqrt{c+a^2cx^2}}\,dx}{c^2} - \frac{(2a}{}$$

$$= \frac{2a}{27c\left(c+a^2cx^2\right)^{3/2}} + \frac{2a^2x\tan^{-1}(ax)}{9c\left(c+a^2cx^2\right)^{3/2}} - \frac{a\tan^{-1}(ax)^2}{3c\left(c+a^2cx^2\right)^{3/2}} - \frac{5a\tan^{-1}(ax)^2}{c^2\sqrt{c+a^2cx^2}} - \frac{a^2x\tan^{-1}(ax)^3}{3c\left(c+a^2cx^2\right)^{3/2}}$$

$$= \frac{2a}{27c\left(c+a^2cx^2\right)^{3/2}} + \frac{94a}{9c^2\sqrt{c+a^2cx^2}} + \frac{2a^2x\tan^{-1}(ax)}{9c\left(c+a^2cx^2\right)^{3/2}} + \frac{94a^2x\tan^{-1}(ax)}{9c^2\sqrt{c+a^2cx^2}} - \frac{a\tan^{-1}(ax)^2}{3c\left(c+a^2cx^2\right)^{3/}}$$

$$= \frac{2a}{27c\left(c+a^2cx^2\right)^{3/2}} + \frac{94a}{9c^2\sqrt{c+a^2cx^2}} + \frac{2a^2x\tan^{-1}(ax)}{9c\left(c+a^2cx^2\right)^{3/2}} + \frac{94a^2x\tan^{-1}(ax)}{9c^2\sqrt{c+a^2cx^2}} - \frac{a\tan^{-1}(ax)^2}{3c\left(c+a^2cx^2\right)^{3/}}$$

$$= \frac{2a}{27c\left(c+a^2cx^2\right)^{3/2}} + \frac{94a}{9c^2\sqrt{c+a^2cx^2}} + \frac{2a^2x\tan^{-1}(ax)}{9c\left(c+a^2cx^2\right)^{3/2}} + \frac{94a^2x\tan^{-1}(ax)}{9c^2\sqrt{c+a^2cx^2}} - \frac{a\tan^{-1}(ax)^2}{3c\left(c+a^2cx^2\right)^{3/}}$$

$$= \frac{2a}{27c\left(c+a^2cx^2\right)^{3/2}} + \frac{94a}{9c^2\sqrt{c+a^2cx^2}} + \frac{2a^2x\tan^{-1}(ax)}{9c\left(c+a^2cx^2\right)^{3/2}} + \frac{94a^2x\tan^{-1}(ax)}{9c^2\sqrt{c+a^2cx^2}} - \frac{a\tan^{-1}(ax)^2}{3c\left(c+a^2cx^2\right)^{3/}}$$

$$= \frac{2a}{27c\left(c+a^2cx^2\right)^{3/2}} + \frac{94a}{9c^2\sqrt{c+a^2cx^2}} + \frac{2a^2x\tan^{-1}(ax)}{9c\left(c+a^2cx^2\right)^{3/2}} + \frac{94a^2x\tan^{-1}(ax)}{9c^2\sqrt{c+a^2cx^2}} - \frac{a\tan^{-1}(ax)^2}{3c\left(c+a^2cx^2\right)^{3/}}$$

$$= \frac{2a}{27c\left(c+a^2cx^2\right)^{3/2}} + \frac{94a}{9c^2\sqrt{c+a^2cx^2}} + \frac{2a^2x\tan^{-1}(ax)}{9c\left(c+a^2cx^2\right)^{3/2}} + \frac{94a^2x\tan^{-1}(ax)}{9c^2\sqrt{c+a^2cx^2}} - \frac{a\tan^{-1}(ax)^2}{3c\left(c+a^2cx^2\right)^{3/}}$$

Mathematica [A] time = 2.32763, size = 399, normalized size = 0.81

$$-\frac{a\left(-648i\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,-e^{i\tan^{-1}(ax)}\right)+648i\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{PolyLog}\left(2,e^{i\tan^{-1}(ax)}\right)+648\sqrt{a^2x}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTan[a*x]^3/(x^2*(c + a^2*c*x^2)^(5/2)),x]
```

```
[Out] -(a*(-1134 - 1134*a*x*ArcTan[a*x] + 567*ArcTan[a*x]^2 + 189*a*x*ArcTan[a*x]
^3 - 2*Sqrt[1 + a^2*x^2]*Cos[3*ArcTan[a*x]] + 9*Sqrt[1 + a^2*x^2]*ArcTan[a*
x]^2*Cos[3*ArcTan[a*x]] + 27*a*x*ArcTan[a*x]^3*Csc[ArcTan[a*x]/2]^2 - 324*S
qrt[1 + a^2*x^2]*ArcTan[a*x]^2*Log[1 - E^(I*ArcTan[a*x])] + 324*Sqrt[1 + a^
2*x^2]*ArcTan[a*x]^2*Log[1 + E^(I*ArcTan[a*x])] - (648*I)*Sqrt[1 + a^2*x^2]
*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])] + (648*I)*Sqrt[1 + a^2*x^2]*Arc
Tan[a*x]*PolyLog[2, E^(I*ArcTan[a*x])] + 648*Sqrt[1 + a^2*x^2]*PolyLog[3, -
E^(I*ArcTan[a*x])] - 648*Sqrt[1 + a^2*x^2]*PolyLog[3, E^(I*ArcTan[a*x])] -
6*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*Sin[3*ArcTan[a*x]] + 9*Sqrt[1 + a^2*x^2]*Ar
cTan[a*x]^3*Sin[3*ArcTan[a*x]] + 54*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^3*Tan[Arc
Tan[a*x]/2]))/(108*c^2*Sqrt[c + a^2*c*x^2])
```

Maple [A] time = 0.362, size = 528, normalized size = 1.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^3/x^2/(a^2*c*x^2+c)^(5/2),x)
```

```
[Out] 1/216*a*(9*I*arctan(a*x)^2+9*arctan(a*x)^3-2*I-6*arctan(a*x))*(a^3*x^3-3*I*
a^2*x^2-3*a*x+I)*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)^2/c^3-7/8*a*(arctan(
a*x)^3-6*arctan(a*x)+3*I*arctan(a*x)^2-6*I)*(a*x-I)*(c*(a*x-I)*(a*x+I))^(1/
2)/c^3/(a^2*x^2+1)-7/8*(c*(a*x-I)*(a*x+I))^(1/2)*(a*x+I)*(arctan(a*x)^3-6*a
rctan(a*x)-3*I*arctan(a*x)^2+6*I)*a/c^3/(a^2*x^2+1)+1/216*(c*(a*x-I)*(a*x+I
))^(1/2)*(a^3*x^3+3*I*a^2*x^2-3*a*x-I)*(-9*I*arctan(a*x)^2+9*arctan(a*x)^3+
2*I-6*arctan(a*x))*a/c^3/(a^4*x^4+2*a^2*x^2+1)-arctan(a*x)^3*(c*(a*x-I)*(a*
x+I))^(1/2)/x/c^3-3*a*(arctan(a*x)^2*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-arct
an(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*I*arctan(a*x)*polylog(2,-(1+I
*a*x)/(a^2*x^2+1)^(1/2))+2*I*arctan(a*x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1
/2))+2*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*polylog(3,(1+I*a*x)/(a^2*x
^2+1)^(1/2)))*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)^(1/2)/c^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x^2/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{a^6c^3x^8+3\,a^4c^3x^6+3\,a^2c^3x^4+c^3x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x^2/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^3/(a^6*c^3*x^8 + 3*a^4*c^3*x^6 + 3
*a^2*c^3*x^4 + c^3*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{atan}^3(ax)}{x^2\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**3/x**2/(a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral(atan(a*x)**3/(x**2*(c*(a**2*x**2 + 1))**(5/2)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x^2/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.458 $\int x^m \left(c+a^2 c x^2\right)^2 \tan^{-1}(a x)^3 \, dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(x^m \left(a^2 c x^2+c\right)^2 \tan^{-1}(a x)^3,x\right)$$

```
[Out] Unintegrable[x^m*(c + a^2*c*x^2)^2*ArcTan[a*x]^3, x]
```

Rubi [A] time = 0.0544006, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int x^m \left(c+a^2 c x^2\right)^2 \tan^{-1}(a x)^3 \, dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*(c + a^2*c*x^2)^2*ArcTan[a*x]^3,x]
```

```
[Out] Defer[Int][x^m*(c + a^2*c*x^2)^2*ArcTan[a*x]^3, x]
```

Rubi steps

$$\int x^m \left(c+a^2 c x^2\right)^2 \tan^{-1}(a x)^3 \, dx=\int x^m \left(c+a^2 c x^2\right)^2 \tan^{-1}(a x)^3 \, dx$$

Mathematica [A] time = 1.84956, size = 0, normalized size = 0.

$$\int x^m \left(c+a^2 c x^2\right)^2 \tan^{-1}(a x)^3 \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*(c + a^2*c*x^2)^2*ArcTan[a*x]^3,x]
```

```
[Out] Integrate[x^m*(c + a^2*c*x^2)^2*ArcTan[a*x]^3, x]
```

Maple [A] time = 0.588, size = 0, normalized size = 0.

$$\int x^{m} \left(a^{2} c x^{2}+c\right)^{2} \left(\arctan \left(a x\right)\right)^{3}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^3,x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^3,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a^{4} c^{2} x^{4} + 2 \, a^{2} c^{2} x^{2} + c^{2}\right) x^{m} \arctan \left(a x\right)^{3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*x^m*arctan(a*x)^3, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$c^{2} \left(\int x^{m} \operatorname{atan}^{3}{\left(a x \right)}\, dx + \int 2 a^{2} x^{2} x^{m} \operatorname{atan}^{3}{\left(a x \right)}\, dx + \int a^{4} x^{4} x^{m} \operatorname{atan}^{3}{\left(a x \right)}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**2*atan(a*x)**3,x)
```

```
[Out] c**2*(Integral(x**m*atan(a*x)**3, x) + Integral(2*a**2*x**2*x**m*atan(a*x)*
*3, x) + Integral(a**4*x**4*x**m*atan(a*x)**3, x))
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2+c\right)^2 x^m \arctan(ax)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^2*x^m*arctan(a*x)^3, x)
```

3.459 $\int x^m \left(c+a^2 c x^2\right) \tan^{-1}(a x)^3 \, dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(x^m \left(a^2 c x^2+c\right) \tan^{-1}(a x)^3,x\right)$$

```
[Out] Unintegrable[x^m*(c + a^2*c*x^2)*ArcTan[a*x]^3, x]
```

Rubi [A] time = 0.034215, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int x^m \left(c+a^2 c x^2\right) \tan^{-1}(a x)^3 \, dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*(c + a^2*c*x^2)*ArcTan[a*x]^3,x]
```

```
[Out] Defer[Int][x^m*(c + a^2*c*x^2)*ArcTan[a*x]^3, x]
```

Rubi steps

$$\int x^m \left(c+a^2 c x^2\right) \tan^{-1}(a x)^3 \, dx = \int x^m \left(c+a^2 c x^2\right) \tan^{-1}(a x)^3 \, dx$$

Mathematica [A] time = 0.924008, size = 0, normalized size = 0.

$$\int x^m \left(c+a^2 c x^2\right) \tan^{-1}(a x)^3 \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*(c + a^2*c*x^2)*ArcTan[a*x]^3,x]
```

```
[Out] Integrate[x^m*(c + a^2*c*x^2)*ArcTan[a*x]^3, x]
```

Maple [A] time = 0.445, size = 0, normalized size = 0.

$$\int x^{m} \left(a^{2} c \,x^{2}+c \right) \left(\arctan \left(a x \right)\right)^{3}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)*arctan(a*x)^3,x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)*arctan(a*x)^3,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)*arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left({\left(a^{2} c x^{2} + c\right)} x^{m} \arctan\left(a x\right)^{3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)*arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)*x^m*arctan(a*x)^3, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$c \left(\int x^{m} \operatorname{atan}^{3}{\left(a x \right)}\, dx + \int a^{2} x^{2} x^{m} \operatorname{atan}^{3}{\left(a x \right)}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)*atan(a*x)**3,x)
```

```
[Out] c*(Integral(x**m*atan(a*x)**3, x) + Integral(a**2*x**2*x**m*atan(a*x)**3, x
))
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2+c\right)x^m \arctan(ax)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)*arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)*x^m*arctan(a*x)^3, x)
```

3.460 $\int \frac{x^m \tan^{-1}(ax)^3}{c+a^2cx^2} \, dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{x^m \tan^{-1}(ax)^3}{a^2cx^2+c},x\right)$$

```
[Out] Unintegrable[(x^m*ArcTan[a*x]^3)/(c + a^2*c*x^2), x]
```

Rubi [A] time = 0.0645185, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m \tan^{-1}(ax)^3}{c+a^2cx^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*ArcTan[a*x]^3)/(c + a^2*c*x^2),x]
```

```
[Out] Defer[Int][(x^m*ArcTan[a*x]^3)/(c + a^2*c*x^2), x]
```

Rubi steps

$$\int \frac{x^m \tan^{-1}(ax)^3}{c+a^2cx^2} \, dx = \int \frac{x^m \tan^{-1}(ax)^3}{c+a^2cx^2} \, dx$$

Mathematica [A] time = 0.864707, size = 0, normalized size = 0.

$$\int \frac{x^m \tan^{-1}(ax)^3}{c+a^2cx^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*ArcTan[a*x]^3)/(c + a^2*c*x^2),x]
```

```
[Out] Integrate[(x^m*ArcTan[a*x]^3)/(c + a^2*c*x^2), x]
```

Maple [A] time = 0.443, size = 0, normalized size = 0.

$$\int \frac{x^{m} \left(\arctan \left(a x\right)\right)^{3}}{a^{2} c \,x^{2}+c}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arctan(a*x)^3/(a^2*c*x^2+c),x)
```

```
[Out] int(x^m*arctan(a*x)^3/(a^2*c*x^2+c),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{m} \arctan \left(a x\right)^{3}}{a^{2} c x^{2} + c}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^3/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] integrate(x^m*arctan(a*x)^3/(a^2*c*x^2 + c), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$${\rm integral}\left(\frac{x^{m} \arctan \left(a x\right)^{3}}{a^{2} c x^{2} + c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^3/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(x^m*arctan(a*x)^3/(a^2*c*x^2 + c), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^{m} \operatorname{atan}^{3}{\left(a x \right)}}{a^{2} x^{2} + 1}\, dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*atan(a*x)**3/(a**2*c*x**2+c),x)
```

```
[Out] Integral(x**m*atan(a*x)**3/(a**2*x**2 + 1), x)/c
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m \arctan{(ax)}^3}{a^2cx^2 + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^3/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(x^m*arctan(a*x)^3/(a^2*c*x^2 + c), x)
```

3.461 $\int \frac{x^m \tan^{-1}(ax)^3}{\left(c+a^2cx^2\right)^2}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{x^m \tan^{-1}(ax)^3}{\left(a^2cx^2+c\right)^2},x\right)$$

```
[Out] Unintegrable[(x^m*ArcTan[a*x]^3)/(c + a^2*c*x^2)^2, x]
```

Rubi [A] time = 0.0618146, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m \tan^{-1}(ax)^3}{\left(c+a^2cx^2\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*ArcTan[a*x]^3)/(c + a^2*c*x^2)^2,x]
```

```
[Out] Defer[Int][(x^m*ArcTan[a*x]^3)/(c + a^2*c*x^2)^2, x]
```

Rubi steps

$$\int \frac{x^m \tan^{-1}(ax)^3}{\left(c+a^2cx^2\right)^2}\,dx=\int \frac{x^m \tan^{-1}(ax)^3}{\left(c+a^2cx^2\right)^2}\,dx$$

Mathematica [A] time = 0.680585, size = 0, normalized size = 0.

$$\int \frac{x^m \tan^{-1}(ax)^3}{\left(c+a^2cx^2\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*ArcTan[a*x]^3)/(c + a^2*c*x^2)^2,x]
```

```
[Out] Integrate[(x^m*ArcTan[a*x]^3)/(c + a^2*c*x^2)^2, x]
```

Maple [A] time = 1.095, size = 0, normalized size = 0.

$$\int \frac{x^m (\arctan (ax))^3}{\left(a^2cx^2 + c\right)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arctan(a*x)^3/(a^2*c*x^2+c)^2,x)
```

```
[Out] int(x^m*arctan(a*x)^3/(a^2*c*x^2+c)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m \arctan (ax)^3}{\left(a^2cx^2 + c\right)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^3/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^m*arctan(a*x)^3/(a^2*c*x^2 + c)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^m \arctan (ax)^3}{a^4c^2x^4 + 2\,a^2c^2x^2 + c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^3/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] integral(x^m*arctan(a*x)^3/(a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^m \operatorname{atan}^3(ax)}{a^4x^4+2a^2x^2+1}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*atan(a*x)**3/(a**2*c*x**2+c)**2,x)
```

```
[Out] Integral(x**m*atan(a*x)**3/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m \arctan(ax)^3}{\left(a^2cx^2+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^3/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(x^m*arctan(a*x)^3/(a^2*c*x^2 + c)^2, x)
```

3.462 $\int x^m \left(c+a^2 c x^2\right)^{3/2} \tan^{-1}(ax)^3 \, dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(x^m \left(a^2 c x^2+c\right)^{3/2} \tan^{-1}(ax)^3,x\right)$$

```
[Out] Unintegrable[x^m*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3, x]
```

Rubi [A] time = 0.109396, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^m \left(c+a^2 c x^2\right)^{3/2} \tan^{-1}(ax)^3 \, dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3,x]
```

```
[Out] Defer[Int][x^m*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3, x]
```

Rubi steps

$$\int x^m \left(c+a^2 c x^2\right)^{3/2} \tan^{-1}(ax)^3 \, dx=\int x^m \left(c+a^2 c x^2\right)^{3/2} \tan^{-1}(ax)^3 \, dx$$

Mathematica [A] time = 0.974488, size = 0, normalized size = 0.

$$\int x^m \left(c+a^2 c x^2\right)^{3/2} \tan^{-1}(ax)^3 \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3,x]
```

```
[Out] Integrate[x^m*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3, x]
```

Maple [A] time = 0.478, size = 0, normalized size = 0.

$$\int x^{m} \left(a^{2} c x^{2}+c\right)^{\frac{3}{2}} \left(\arctan \left(a x\right)\right)^{3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^3,x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(a^{2} c x^{2}+c\right)^{\frac{3}{2}} x^{m} \arctan \left(a x\right)^{3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)*x^m*arctan(a*x)^3, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a^{2} c x^{2}+c\right)^{\frac{3}{2}} x^{m} \arctan \left(a x\right)^{3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)^(3/2)*x^m*arctan(a*x)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**(3/2)*atan(a*x)**3,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.463 $\int x^m \sqrt{c+a^2cx^2} \tan^{-1}(ax)^3 \, dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(x^m \sqrt{a^2cx^2+c} \tan^{-1}(ax)^3, x\right)$$

```
[Out] Unintegrable[x^m*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3, x]
```

Rubi [A] time = 0.0945365, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int x^m \sqrt{c+a^2cx^2} \tan^{-1}(ax)^3 \, dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3,x]
```

```
[Out] Defer[Int][x^m*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3, x]
```

Rubi steps

$$\int x^m \sqrt{c+a^2cx^2} \tan^{-1}(ax)^3 \, dx = \int x^m \sqrt{c+a^2cx^2} \tan^{-1}(ax)^3 \, dx$$

Mathematica [A] time = 0.151649, size = 0, normalized size = 0.

$$\int x^m \sqrt{c+a^2cx^2} \tan^{-1}(ax)^3 \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3,x]
```

```
[Out] Integrate[x^m*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3, x]
```

Maple [A] time = 0.602, size = 0, normalized size = 0.

$$\int x^{m} (\arctan (ax))^{3} \sqrt{a^{2} c x^{2}+c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arctan(a*x)^3*(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(x^m*arctan(a*x)^3*(a^2*c*x^2+c)^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a^{2} c x^{2}+c}\, x^{m} \arctan (ax)^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^3*(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)*x^m*arctan(a*x)^3, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{a^{2} c x^{2}+c}\, x^{m} \arctan (ax)^{3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^3*(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^m*arctan(a*x)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*atan(a*x)**3*(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^3*(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.464 $\int \frac{x^m \tan^{-1}(ax)^3}{\sqrt{c+a^2cx^2}} \, dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{x^m \tan^{-1}(ax)^3}{\sqrt{a^2cx^2+c}}, x\right)$$

```
[Out] Unintegrable[(x^m*ArcTan[a*x]^3)/Sqrt[c + a^2*c*x^2], x]
```

Rubi [A] time = 0.103092, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m \tan^{-1}(ax)^3}{\sqrt{c+a^2cx^2}} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*ArcTan[a*x]^3)/Sqrt[c + a^2*c*x^2],x]

[Out] Defer[Int][(x^m*ArcTan[a*x]^3)/Sqrt[c + a^2*c*x^2], x]
```

Rubi steps

$$\int \frac{x^m \tan^{-1}(ax)^3}{\sqrt{c+a^2cx^2}} \, dx = \int \frac{x^m \tan^{-1}(ax)^3}{\sqrt{c+a^2cx^2}} \, dx$$

Mathematica [A] time = 0.503828, size = 0, normalized size = 0.

$$\int \frac{x^m \tan^{-1}(ax)^3}{\sqrt{c+a^2cx^2}} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*ArcTan[a*x]^3)/Sqrt[c + a^2*c*x^2],x]

[Out] Integrate[(x^m*ArcTan[a*x]^3)/Sqrt[c + a^2*c*x^2], x]
```

Maple [A] time = 0.925, size = 0, normalized size = 0.

$$\int x^m (\arctan(ax))^3 \frac{1}{\sqrt{a^2cx^2+c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(x^m*arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m \arctan(ax)^3}{\sqrt{a^2cx^2+c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m*arctan(a*x)^3/sqrt(a^2*c*x^2 + c), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^m \arctan(ax)^3}{\sqrt{a^2cx^2+c}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^m*arctan(a*x)^3/sqrt(a^2*c*x^2 + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*atan(a*x)**3/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m \arctan(ax)^3}{\sqrt{a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^m*arctan(a*x)^3/sqrt(a^2*c*x^2 + c), x)
```

3.465 $\int \frac{x^m \tan^{-1}(ax)^3}{\left(c+a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{x^m \tan^{-1}(ax)^3}{\left(a^2cx^2+c\right)^{3/2}},x\right)$$

```
[Out] Unintegrable[(x^m*ArcTan[a*x]^3)/(c + a^2*c*x^2)^(3/2), x]
```

Rubi [A] time = 0.116626, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m \tan^{-1}(ax)^3}{\left(c+a^2cx^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*ArcTan[a*x]^3)/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] Defer[Int][(x^m*ArcTan[a*x]^3)/(c + a^2*c*x^2)^(3/2), x]
```

Rubi steps

$$\int \frac{x^m \tan^{-1}(ax)^3}{\left(c+a^2cx^2\right)^{3/2}}\,dx=\int \frac{x^m \tan^{-1}(ax)^3}{\left(c+a^2cx^2\right)^{3/2}}\,dx$$

Mathematica [A] time = 0.644472, size = 0, normalized size = 0.

$$\int \frac{x^m \tan^{-1}(ax)^3}{\left(c+a^2cx^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*ArcTan[a*x]^3)/(c + a^2*c*x^2)^(3/2),x]
```

[Out] Integrate[(x^m*ArcTan[a*x]^3)/(c + a^2*c*x^2)^(3/2), x]

Maple [A] time = 1.05, size = 0, normalized size = 0.

$$\int x^{m} \left(\arctan \left(a x\right)\right)^{3} \left(a^{2} c \,x^{2}+c \right)^{-\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m*arctan(a*x)^3/(a^2*c*x^2+c)^(3/2),x)

[Out] int(x^m*arctan(a*x)^3/(a^2*c*x^2+c)^(3/2),x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{m} \arctan \left(a x\right)^{3}}{{\left(a^{2} c x^{2} + c\right)}^{\frac{3}{2}}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m*arctan(a*x)^3/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")

[Out] integrate(x^m*arctan(a*x)^3/(a^2*c*x^2 + c)^(3/2), x)

Fricas [A] time = 0., size = 0, normalized size = 0.

$${\rm integral}\left(\frac{\sqrt{a^{2} c x^{2} + c} x^{m} \arctan \left(a x\right)^{3}}{a^{4} c^{2} x^{4} + 2 \, a^{2} c^{2} x^{2} + c^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m*arctan(a*x)^3/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(a^2*c*x^2 + c)*x^m*arctan(a*x)^3/(a^4*c^2*x^4 + 2*a^2*c^2*x^2
 + c^2), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*atan(a*x)**3/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m \arctan(ax)^3}{\left(a^2cx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^3/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^m*arctan(a*x)^3/(a^2*c*x^2 + c)^(3/2), x)
```

3.466 $\int \frac{x(c+a^2cx^2)}{\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left(\frac{x\left(a^2cx^2+c\right)}{\tan^{-1}(ax)},x\right)$$

```
[Out] Unintegrable[(x*(c + a^2*c*x^2))/ArcTan[a*x], x]
```

Rubi [A] time = 0.0240839, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x\left(c+a^2cx^2\right)}{\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x*(c + a^2*c*x^2))/ArcTan[a*x],x]

[Out] Defer[Int][(x*(c + a^2*c*x^2))/ArcTan[a*x], x]
```

Rubi steps

$$\int \frac{x\left(c+a^2cx^2\right)}{\tan^{-1}(ax)}\,dx = \int \frac{x\left(c+a^2cx^2\right)}{\tan^{-1}(ax)}\,dx$$

Mathematica [A] time = 0.414979, size = 0, normalized size = 0.

$$\int \frac{x\left(c+a^2cx^2\right)}{\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x*(c + a^2*c*x^2))/ArcTan[a*x],x]
```

```
[Out] Integrate[(x*(c + a^2*c*x^2))/ArcTan[a*x], x]
```

Maple [A] time = 0.745, size = 0, normalized size = 0.

$$\int \frac{x\left(a^{2}c x^{2}+c\right)}{\arctan\left(a x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)/arctan(a*x),x)

[Out] int(x*(a^2*c*x^2+c)/arctan(a*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^{2}c x^{2}+c\right)x}{\arctan\left(a x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)/arctan(a*x),x, algorithm="maxima")

[Out] integrate((a^2*c*x^2 + c)*x/arctan(a*x), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{a^{2}c x^{3}+c x}{\arctan\left(a x\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)/arctan(a*x),x, algorithm="fricas")

[Out] integral((a^2*c*x^3 + c*x)/arctan(a*x), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$c\left(\int \frac{x}{\operatorname{atan}(ax)}\, dx + \int \frac{a^2x^3}{\operatorname{atan}(ax)}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)/atan(a*x),x)
```

```
[Out] c*(Integral(x/atan(a*x), x) + Integral(a**2*x**3/atan(a*x), x))
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2 + c\right)x}{\arctan(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)/arctan(a*x),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)*x/arctan(a*x), x)
```

3.467 $\int \frac{c+a^2cx^2}{\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=19

$$\text{Unintegrable}\left(\frac{a^2cx^2+c}{\tan^{-1}(ax)},x\right)$$

```
[Out] Unintegrable[(c + a^2*c*x^2)/ArcTan[a*x], x]
```

Rubi [A] time = 0.0127628, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{c+a^2cx^2}{\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)/ArcTan[a*x],x]
```

```
[Out] Defer[Int][(c + a^2*c*x^2)/ArcTan[a*x], x]
```

Rubi steps

$$\int \frac{c+a^2cx^2}{\tan^{-1}(ax)}\,dx=\int \frac{c+a^2cx^2}{\tan^{-1}(ax)}\,dx$$

Mathematica [A] time = 0.370999, size = 0, normalized size = 0.

$$\int \frac{c+a^2cx^2}{\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)/ArcTan[a*x],x]
```

```
[Out] Integrate[(c + a^2*c*x^2)/ArcTan[a*x], x]
```

Maple [A] time = 0.698, size = 0, normalized size = 0.

$$\int \frac{a^2 c x^2 + c}{\arctan(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)/arctan(a*x),x)
```

```
[Out] int((a^2*c*x^2+c)/arctan(a*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{a^2 c x^2 + c}{\arctan(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)/arctan(a*x), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{a^2 c x^2 + c}{\arctan(ax)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)/arctan(a*x), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$c\left(\int \frac{a^2 x^2}{\operatorname{atan}(ax)} dx + \int \frac{1}{\operatorname{atan}(ax)} dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)/atan(a*x),x)
```

```
[Out] c*(Integral(a**2*x**2/atan(a*x), x) + Integral(1/atan(a*x), x))
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{a^2cx^2 + c}{\arctan(ax)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)/arctan(a*x),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)/arctan(a*x), x)
```

3.468 $\int \frac{c+a^2cx^2}{x\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{a^2cx^2+c}{x\tan^{-1}(ax)},x\right)$$

```
[Out] Unintegrable[(c + a^2*c*x^2)/(x*ArcTan[a*x]), x]
```

Rubi [A] time = 0.0340749, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{c+a^2cx^2}{x\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)/(x*ArcTan[a*x]),x]
```

```
[Out] Defer[Int][(c + a^2*c*x^2)/(x*ArcTan[a*x]), x]
```

Rubi steps

$$\int \frac{c+a^2cx^2}{x\tan^{-1}(ax)}\,dx = \int \frac{c+a^2cx^2}{x\tan^{-1}(ax)}\,dx$$

Mathematica [A] time = 0.618755, size = 0, normalized size = 0.

$$\int \frac{c+a^2cx^2}{x\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)/(x*ArcTan[a*x]),x]
```

```
[Out] Integrate[(c + a^2*c*x^2)/(x*ArcTan[a*x]), x]
```

Maple [A] time = 0.816, size = 0, normalized size = 0.

$$\int \frac{a^2 c x^2 + c}{x \arctan(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)/x/arctan(a*x),x)
```

```
[Out] int((a^2*c*x^2+c)/x/arctan(a*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{a^2 c x^2 + c}{x \arctan(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)/x/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)/(x*arctan(a*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{a^2 c x^2 + c}{x \arctan(ax)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)/x/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)/(x*arctan(a*x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$c\left(\int \frac{1}{x \operatorname{atan}(ax)} dx + \int \frac{a^2 x}{\operatorname{atan}(ax)} dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)/x/atan(a*x),x)
```

```
[Out] c*(Integral(1/(x*atan(a*x)), x) + Integral(a**2*x/atan(a*x), x))
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{a^2cx^2 + c}{x \arctan(ax)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)/x/arctan(a*x),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)/(x*arctan(a*x)), x)
```

3.469 $\int \frac{x\left(c+a^2cx^2\right)^2}{\tan^{-1}(ax)} \, dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{x\left(a^2cx^2+c\right)^2}{\tan^{-1}(ax)},x\right)$$

```
[Out] Unintegrable[(x*(c + a^2*c*x^2)^2)/ArcTan[a*x], x]
```

Rubi [A] time = 0.0374872, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x\left(c+a^2cx^2\right)^2}{\tan^{-1}(ax)} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(x*(c + a^2*c*x^2)^2)/ArcTan[a*x],x]
```

```
[Out] Defer[Int][(x*(c + a^2*c*x^2)^2)/ArcTan[a*x], x]
```

Rubi steps

$$\int \frac{x\left(c+a^2cx^2\right)^2}{\tan^{-1}(ax)} \, dx = \int \frac{x\left(c+a^2cx^2\right)^2}{\tan^{-1}(ax)} \, dx$$

Mathematica [A] time = 0.557884, size = 0, normalized size = 0.

$$\int \frac{x\left(c+a^2cx^2\right)^2}{\tan^{-1}(ax)} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x*(c + a^2*c*x^2)^2)/ArcTan[a*x],x]
```

```
[Out] Integrate[(x*(c + a^2*c*x^2)^2)/ArcTan[a*x], x]
```

Maple [A] time = 0.905, size = 0, normalized size = 0.

$$\int \frac{x\left(a^{2}c x^{2}+c\right)^{2}}{\arctan\left(a x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^2/arctan(a*x),x)
```

```
[Out] int(x*(a^2*c*x^2+c)^2/arctan(a*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^{2}c x^{2}+c\right)^{2} x}{\arctan\left(a x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^2*x/arctan(a*x), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{a^{4}c^{2}x^{5}+2\,a^{2}c^{2}x^{3}+c^{2}x}{\arctan\left(a x\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^5 + 2*a^2*c^2*x^3 + c^2*x)/arctan(a*x), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$c^2\left(\int \frac{x}{\operatorname{atan}(ax)}\,dx+\int \frac{2a^2x^3}{\operatorname{atan}(ax)}\,dx+\int \frac{a^4x^5}{\operatorname{atan}(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**2/atan(a*x),x)
```

```
[Out] c**2*(Integral(x/atan(a*x), x) + Integral(2*a**2*x**3/atan(a*x), x) + Integ
ral(a**4*x**5/atan(a*x), x))
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^2x}{\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^2*x/arctan(a*x), x)
```

3.470 $\int \frac{(c+a^2cx^2)^2}{\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=21

$$\text{Unintegrable}\left(\frac{(a^2cx^2+c)^2}{\tan^{-1}(ax)},x\right)$$

```
[Out] Unintegrable[(c + a^2*c*x^2)^2/ArcTan[a*x], x]
```

Rubi [A] time = 0.0240531, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(c+a^2cx^2)^2}{\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^2/ArcTan[a*x],x]
```

```
[Out] Defer[Int][(c + a^2*c*x^2)^2/ArcTan[a*x], x]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^2}{\tan^{-1}(ax)}\,dx = \int \frac{(c+a^2cx^2)^2}{\tan^{-1}(ax)}\,dx$$

Mathematica [A] time = 0.433371, size = 0, normalized size = 0.

$$\int \frac{(c+a^2cx^2)^2}{\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^2/ArcTan[a*x],x]
```

```
[Out] Integrate[(c + a^2*c*x^2)^2/ArcTan[a*x], x]
```

Maple [A] time = 0.861, size = 0, normalized size = 0.

$$\int \frac{\left(a^2 c x^2 + c\right)^2}{\arctan\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^2/arctan(a*x),x)
```

```
[Out] int((a^2*c*x^2+c)^2/arctan(a*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2 c x^2 + c\right)^2}{\arctan\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^2/arctan(a*x), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{a^4 c^2 x^4 + 2\,a^2 c^2 x^2 + c^2}{\arctan\left(ax\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)/arctan(a*x), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$c^{2}\left(\int \frac{2a^{2}x^{2}}{\operatorname{atan}(ax)}\,dx+\int \frac{a^{4}x^{4}}{\operatorname{atan}(ax)}\,dx+\int \frac{1}{\operatorname{atan}(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**2/atan(a*x),x)
```

```
[Out] c**2*(Integral(2*a**2*x**2/atan(a*x), x) + Integral(a**4*x**4/atan(a*x), x)
 + Integral(1/atan(a*x), x))
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^{2}cx^{2}+c\right)^{2}}{\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^2/arctan(a*x), x)
```

3.471 $\int \frac{(c+a^2cx^2)^2}{x\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{(a^2cx^2+c)^2}{x\tan^{-1}(ax)},x\right)$$

```
[Out] Unintegrable[(c + a^2*c*x^2)^2/(x*ArcTan[a*x]), x]
```

Rubi [A] time = 0.0515064, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(c+a^2cx^2)^2}{x\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^2/(x*ArcTan[a*x]),x]

[Out] Defer[Int][(c + a^2*c*x^2)^2/(x*ArcTan[a*x]), x]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^2}{x\tan^{-1}(ax)}\,dx=\int \frac{(c+a^2cx^2)^2}{x\tan^{-1}(ax)}\,dx$$

Mathematica [A] time = 0.765755, size = 0, normalized size = 0.

$$\int \frac{(c+a^2cx^2)^2}{x\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^2/(x*ArcTan[a*x]),x]
```

```
[Out] Integrate[(c + a^2*c*x^2)^2/(x*ArcTan[a*x]), x]
```

Maple [A] time = 0.921, size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^2}{x\arctan\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^2/x/arctan(a*x),x)
```

```
[Out] int((a^2*c*x^2+c)^2/x/arctan(a*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^2}{x\arctan\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2/x/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^2/(x*arctan(a*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{a^4c^2x^4+2\,a^2c^2x^2+c^2}{x\arctan\left(ax\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2/x/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)/(x*arctan(a*x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$c^2\left(\int \frac{1}{x\operatorname{atan}(ax)}\,dx+\int \frac{2a^2x}{\operatorname{atan}(ax)}\,dx+\int \frac{a^4x^3}{\operatorname{atan}(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**2/x/atan(a*x),x)
```

```
[Out] c**2*(Integral(1/(x*atan(a*x)), x) + Integral(2*a**2*x/atan(a*x), x) + Inte
gral(a**4*x**3/atan(a*x), x))
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^2}{x\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2/x/arctan(a*x),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^2/(x*arctan(a*x)), x)
```

3.472 $\int \frac{x\left(c+a^2cx^2\right)^3}{\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{x\left(a^2cx^2+c\right)^3}{\tan^{-1}(ax)},x\right)$$

```
[Out] Unintegrable[(x*(c + a^2*c*x^2)^3)/ArcTan[a*x], x]
```

Rubi [A] time = 0.0381323, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x\left(c+a^2cx^2\right)^3}{\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x*(c + a^2*c*x^2)^3)/ArcTan[a*x],x]

[Out] Defer[Int][(x*(c + a^2*c*x^2)^3)/ArcTan[a*x], x]
```

Rubi steps

$$\int \frac{x\left(c+a^2cx^2\right)^3}{\tan^{-1}(ax)}\,dx=\int \frac{x\left(c+a^2cx^2\right)^3}{\tan^{-1}(ax)}\,dx$$

Mathematica [A] time = 0.532251, size = 0, normalized size = 0.

$$\int \frac{x\left(c+a^2cx^2\right)^3}{\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x*(c + a^2*c*x^2)^3)/ArcTan[a*x],x]
```

```
[Out] Integrate[(x*(c + a^2*c*x^2)^3)/ArcTan[a*x], x]
```

Maple [A] time = 1.16, size = 0, normalized size = 0.

$$\int \frac{x\left(a^{2} c x^{2}+c\right)^{3}}{\arctan\left(a x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^3/arctan(a*x),x)
```

```
[Out] int(x*(a^2*c*x^2+c)^3/arctan(a*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{{\left(a^{2} c x^{2}+c\right)}^{3} x}{\arctan\left(a x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^3*x/arctan(a*x), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{a^{6} c^{3} x^{7}+3 \, a^{4} c^{3} x^{5}+3 \, a^{2} c^{3} x^{3}+c^{3} x}{\arctan\left(a x\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral((a^6*c^3*x^7 + 3*a^4*c^3*x^5 + 3*a^2*c^3*x^3 + c^3*x)/arctan(a*x),
 x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$c^3\left(\int \frac{x}{\operatorname{atan}(ax)}\,dx + \int \frac{3a^2x^3}{\operatorname{atan}(ax)}\,dx + \int \frac{3a^4x^5}{\operatorname{atan}(ax)}\,dx + \int \frac{a^6x^7}{\operatorname{atan}(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**3/atan(a*x),x)
```

```
[Out] c**3*(Integral(x/atan(a*x), x) + Integral(3*a**2*x**3/atan(a*x), x) + Integ
ral(3*a**4*x**5/atan(a*x), x) + Integral(a**6*x**7/atan(a*x), x))
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^3 x}{\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^3*x/arctan(a*x), x)
```

3.473 $\int \frac{(c+a^2cx^2)^3}{\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=21

$$\text{Unintegrable}\left(\frac{(a^2cx^2+c)^3}{\tan^{-1}(ax)},x\right)$$

```
[Out] Unintegrable[(c + a^2*c*x^2)^3/ArcTan[a*x], x]
```

Rubi [A] time = 0.0231705, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{(c+a^2cx^2)^3}{\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^3/ArcTan[a*x],x]

[Out] Defer[Int][(c + a^2*c*x^2)^3/ArcTan[a*x], x]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^3}{\tan^{-1}(ax)}\,dx = \int \frac{(c+a^2cx^2)^3}{\tan^{-1}(ax)}\,dx$$

Mathematica [A] time = 0.453874, size = 0, normalized size = 0.

$$\int \frac{(c+a^2cx^2)^3}{\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^3/ArcTan[a*x],x]
```

```
[Out] Integrate[(c + a^2*c*x^2)^3/ArcTan[a*x], x]
```

Maple [A] time = 1.009, size = 0, normalized size = 0.

$$\int \frac{\left(a^{2} c \,x^{2}+c \right)^{3}}{\arctan \left(a x \right)}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^3/arctan(a*x),x)
```

```
[Out] int((a^2*c*x^2+c)^3/arctan(a*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^{2} c x^{2}+c\right)^{3}}{\arctan \left(a x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^3/arctan(a*x), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{a^{6} c^{3} x^{6}+3 \, a^{4} c^{3} x^{4}+3 \, a^{2} c^{3} x^{2}+c^{3}}{\arctan \left(a x\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)/arctan(a*x), x
)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$c^3\left(\int \frac{3a^2x^2}{\operatorname{atan}(ax)}\,dx + \int \frac{3a^4x^4}{\operatorname{atan}(ax)}\,dx + \int \frac{a^6x^6}{\operatorname{atan}(ax)}\,dx + \int \frac{1}{\operatorname{atan}(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**3/atan(a*x),x)
```

```
[Out] c**3*(Integral(3*a**2*x**2/atan(a*x), x) + Integral(3*a**4*x**4/atan(a*x),
x) + Integral(a**6*x**6/atan(a*x), x) + Integral(1/atan(a*x), x))
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2 + c\right)^3}{\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^3/arctan(a*x), x)
```

3.474 $\int \frac{(c+a^2cx^2)^3}{x\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{(a^2cx^2+c)^3}{x\tan^{-1}(ax)},x\right)$$

```
[Out] Unintegrable[(c + a^2*c*x^2)^3/(x*ArcTan[a*x]), x]
```

Rubi [A] time = 0.050382, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(c+a^2cx^2)^3}{x\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^3/(x*ArcTan[a*x]),x]
```

```
[Out] Defer[Int][(c + a^2*c*x^2)^3/(x*ArcTan[a*x]), x]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^3}{x\tan^{-1}(ax)}\,dx = \int \frac{(c+a^2cx^2)^3}{x\tan^{-1}(ax)}\,dx$$

Mathematica [A] time = 0.737893, size = 0, normalized size = 0.

$$\int \frac{(c+a^2cx^2)^3}{x\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^3/(x*ArcTan[a*x]),x]
```

```
[Out] Integrate[(c + a^2*c*x^2)^3/(x*ArcTan[a*x]), x]
```

Maple [A] time = 0.994, size = 0, normalized size = 0.

$$\int \frac{\left(a^{2} c\,x^{2}+c\right)^{3}}{x \arctan\left(a x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^3/x/arctan(a*x),x)
```

```
[Out] int((a^2*c*x^2+c)^3/x/arctan(a*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^{2} c\,x^{2}+c\right)^{3}}{x \arctan\left(a x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3/x/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^3/(x*arctan(a*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{a^{6} c^{3} x^{6}+3\, a^{4} c^{3} x^{4}+3\, a^{2} c^{3} x^{2}+c^{3}}{x \arctan\left(a x\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3/x/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)/(x*arctan(a*x)
), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$c^3\left(\int \frac{1}{x\,\mathrm{atan}\,(ax)}\,dx + \int \frac{3a^2x}{\mathrm{atan}\,(ax)}\,dx + \int \frac{3a^4x^3}{\mathrm{atan}\,(ax)}\,dx + \int \frac{a^6x^5}{\mathrm{atan}\,(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**3/x/atan(a*x),x)
```

```
[Out] c**3*(Integral(1/(x*atan(a*x)), x) + Integral(3*a**2*x/atan(a*x), x) + Inte
gral(3*a**4*x**3/atan(a*x), x) + Integral(a**6*x**5/atan(a*x), x))
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2 + c\right)^3}{x\arctan\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3/x/arctan(a*x),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^3/(x*arctan(a*x)), x)
```

3.475 $\int \frac{x^2}{(c+a^2cx^2)\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{x^2}{(a^2cx^2+c)\tan^{-1}(ax)},x\right)$$

```
[Out] Unintegrable[x^2/((c + a^2*c*x^2)*ArcTan[a*x]), x]
```

Rubi [A] time = 0.0660412, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^2}{(c+a^2cx^2)\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^2/((c + a^2*c*x^2)*ArcTan[a*x]),x]
```

```
[Out] Defer[Int][x^2/((c + a^2*c*x^2)*ArcTan[a*x]), x]
```

Rubi steps

$$\int \frac{x^2}{(c+a^2cx^2)\tan^{-1}(ax)}\,dx=\int \frac{x^2}{(c+a^2cx^2)\tan^{-1}(ax)}\,dx$$

Mathematica [A] time = 1.45228, size = 0, normalized size = 0.

$$\int \frac{x^2}{(c+a^2cx^2)\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^2/((c + a^2*c*x^2)*ArcTan[a*x]),x]
```

```
[Out] Integrate[x^2/((c + a^2*c*x^2)*ArcTan[a*x]), x]
```

Maple [A] time = 0.386, size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(a^2cx^2+c\right)\arctan\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a^2*c*x^2+c)/arctan(a*x),x)
```

```
[Out] int(x^2/(a^2*c*x^2+c)/arctan(a*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(a^2cx^2+c\right)\arctan\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((a^2*c*x^2 + c)*arctan(a*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x^2}{\left(a^2cx^2+c\right)\arctan\left(ax\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral(x^2/((a^2*c*x^2 + c)*arctan(a*x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^2}{a^2x^2 \operatorname{atan}(ax)+\operatorname{atan}(ax)}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a**2*c*x**2+c)/atan(a*x),x)
```

```
[Out] Integral(x**2/(a**2*x**2*atan(a*x) + atan(a*x)), x)/c
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(a^2cx^2+c)\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)/arctan(a*x),x, algorithm="giac")
```

```
[Out] integrate(x^2/((a^2*c*x^2 + c)*arctan(a*x)), x)
```

3.476 $\int \frac{x}{(c+a^2cx^2)\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{x}{(a^2cx^2+c)\tan^{-1}(ax)},x\right)$$

```
[Out] Unintegrable[x/((c + a^2*c*x^2)*ArcTan[a*x]), x]
```

Rubi [A] time = 0.0451765, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x}{(c+a^2cx^2)\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x/((c + a^2*c*x^2)*ArcTan[a*x]),x]
```

```
[Out] Defer[Int][x/((c + a^2*c*x^2)*ArcTan[a*x]), x]
```

Rubi steps

$$\int \frac{x}{(c+a^2cx^2)\tan^{-1}(ax)}\,dx=\int \frac{x}{(c+a^2cx^2)\tan^{-1}(ax)}\,dx$$

Mathematica [A] time = 0.520343, size = 0, normalized size = 0.

$$\int \frac{x}{(c+a^2cx^2)\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x/((c + a^2*c*x^2)*ArcTan[a*x]),x]
```

```
[Out] Integrate[x/((c + a^2*c*x^2)*ArcTan[a*x]), x]
```

Maple [A] time = 0.123, size = 0, normalized size = 0.

$$\int \frac{x}{\left(a^{2}c x^{2}+c\right) \arctan\left(a x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^2*c*x^2+c)/arctan(a*x),x)
```

```
[Out] int(x/(a^2*c*x^2+c)/arctan(a*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(a^{2}c x^{2}+c\right) \arctan\left(a x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x/((a^2*c*x^2 + c)*arctan(a*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x}{\left(a^{2}c x^{2}+c\right) \arctan\left(a x\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral(x/((a^2*c*x^2 + c)*arctan(a*x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x}{a^{2}x^{2}\operatorname{atan}\left(a x\right)+\operatorname{atan}\left(a x\right)}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a**2*c*x**2+c)/atan(a*x),x)
```

```
[Out] Integral(x/(a**2*x**2*atan(a*x) + atan(a*x)), x)/c
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(a^2cx^2+c\right)\arctan\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)/arctan(a*x),x, algorithm="giac")
```

```
[Out] integrate(x/((a^2*c*x^2 + c)*arctan(a*x)), x)
```

3.477 $\int \frac{1}{(c+a^2cx^2)\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=12

$$\frac{\log\left(\tan^{-1}(ax)\right)}{ac}$$

```
[Out] Log[ArcTan[a*x]]/(a*c)
```

Rubi [A] time = 0.0264302, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {4882}

$$\frac{\log\left(\tan^{-1}(ax)\right)}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a^2*c*x^2)*ArcTan[a*x]),x]
```

```
[Out] Log[ArcTan[a*x]]/(a*c)
```

Rule 4882

```
Int[1/(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)), x_Symbol]
 :> Simp[Log[RemoveContent[a + b*ArcTan[c*x], x]]/(b*c*d), x] /; FreeQ[{a,
b, c, d, e}, x] && EqQ[e, c^2*d]
```

Rubi steps

$$\int \frac{1}{\left(c+a^2cx^2\right)\tan^{-1}(ax)}\,dx=\frac{\log\left(\tan^{-1}(ax)\right)}{ac}$$

Mathematica [A] time = 0.0138428, size = 12, normalized size = 1.

$$\frac{\log\left(\tan^{-1}(ax)\right)}{ac}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a^2*c*x^2)*ArcTan[a*x]),x]
```

```
[Out] Log[ArcTan[a*x]]/(a*c)
```

Maple [A] time = 0.05, size = 13, normalized size = 1.1

$$\frac{\ln(\arctan(ax))}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2*c*x^2+c)/arctan(a*x),x)
```

```
[Out] ln(arctan(a*x))/a/c
```

Maxima [A] time = 1.05453, size = 20, normalized size = 1.67

$$\frac{\log(2|\arctan(ax)|)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)/arctan(a*x),x, algorithm="maxima")
```

```
[Out] log(2*abs(arctan(a*x)))/(a*c)
```

Fricas [A] time = 1.85804, size = 32, normalized size = 2.67

$$\frac{\log(\arctan(ax))}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)/arctan(a*x),x, algorithm="fricas")
```

```
[Out] log(arctan(a*x))/(a*c)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*c*x**2+c)/atan(a*x),x)
```

```
[Out] Exception raised: TypeError
```

Giac [A] time = 1.15777, size = 18, normalized size = 1.5

$$\frac{\log\left(|\arctan\left(ax\right)|\right)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)/arctan(a*x),x, algorithm="giac")
```

```
[Out] log(abs(arctan(a*x)))/(a*c)
```

3.478 $\int \frac{1}{x(c+a^2cx^2)\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{1}{x\left(a^2cx^2+c\right)\tan^{-1}(ax)},x\right)$$

```
[Out] Unintegrable[1/(x*(c + a^2*c*x^2)*ArcTan[a*x]), x]
```

Rubi [A] time = 0.0659304, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x\left(c+a^2cx^2\right)\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*(c + a^2*c*x^2)*ArcTan[a*x]),x]
```

```
[Out] Defer[Int][1/(x*(c + a^2*c*x^2)*ArcTan[a*x]), x]
```

Rubi steps

$$\int \frac{1}{x\left(c+a^2cx^2\right)\tan^{-1}(ax)}\,dx=\int \frac{1}{x\left(c+a^2cx^2\right)\tan^{-1}(ax)}\,dx$$

Mathematica [A] time = 0.183189, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(c+a^2cx^2\right)\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*(c + a^2*c*x^2)*ArcTan[a*x]),x]
```

```
[Out] Integrate[1/(x*(c + a^2*c*x^2)*ArcTan[a*x]), x]
```

Maple [A] time = 0.119, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(a^{2}c x^{2}+c\right)\arctan\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a^2*c*x^2+c)/arctan(a*x),x)
```

```
[Out] int(1/x/(a^2*c*x^2+c)/arctan(a*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^{2}c x^{2}+c\right)x\arctan\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)*x*arctan(a*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\left(a^{2}c x^{3}+cx\right)\arctan\left(ax\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral(1/((a^2*c*x^3 + c*x)*arctan(a*x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{a^2x^3\operatorname{atan}(ax)+x\operatorname{atan}(ax)}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a**2*c*x**2+c)/atan(a*x),x)
```

```
[Out] Integral(1/(a**2*x**3*atan(a*x) + x*atan(a*x)), x)/c
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a^2cx^2+c)x\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)/arctan(a*x),x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)*x*arctan(a*x)), x)
```

3.479 $\int \frac{1}{x^2(c+a^2cx^2)\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{1}{x^2\left(a^2cx^2+c\right)\tan^{-1}(ax)},x\right)$$

```
[Out] Unintegrable[1/(x^2*(c + a^2*c*x^2)*ArcTan[a*x]), x]
```

Rubi [A] time = 0.067958, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^2*(c + a^2*c*x^2)*ArcTan[a*x]),x]
```

```
[Out] Defer[Int][1/(x^2*(c + a^2*c*x^2)*ArcTan[a*x]), x]
```

Rubi steps

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)\tan^{-1}(ax)}\,dx=\int \frac{1}{x^2\left(c+a^2cx^2\right)\tan^{-1}(ax)}\,dx$$

Mathematica [A] time = 0.202235, size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^2*(c + a^2*c*x^2)*ArcTan[a*x]),x]
```

```
[Out] Integrate[1/(x^2*(c + a^2*c*x^2)*ArcTan[a*x]), x]
```

Maple [A] time = 0.187, size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \left(a^2 c x^2 + c\right) \arctan \left(a x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a^2*c*x^2+c)/arctan(a*x),x)
```

```
[Out] int(1/x^2/(a^2*c*x^2+c)/arctan(a*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2 c x^2 + c\right) x^2 \arctan \left(a x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)*x^2*arctan(a*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\left(a^2 c x^4 + c x^2\right) \arctan \left(a x\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral(1/((a^2*c*x^4 + c*x^2)*arctan(a*x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{a^2x^4\operatorname{atan}(ax)+x^2\operatorname{atan}(ax)}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a**2*c*x**2+c)/atan(a*x),x)
```

```
[Out] Integral(1/(a**2*x**4*atan(a*x) + x**2*atan(a*x)), x)/c
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a^2cx^2+c)x^2\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)/arctan(a*x),x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)*x^2*arctan(a*x)), x)
```

3.480 $\int \frac{x^4}{(c+a^2cx^2)^2 \tan^{-1}(ax)} dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{x^4}{(a^2cx^2+c)^2 \tan^{-1}(ax)}, x\right)$$

```
[Out] Unintegrable[x^4/((c + a^2*c*x^2)^2*ArcTan[a*x]), x]
```

Rubi [A] time = 0.068668, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^4}{(c+a^2cx^2)^2 \tan^{-1}(ax)} dx$$

Verification is Not applicable to the result.

```
[In] Int[x^4/((c + a^2*c*x^2)^2*ArcTan[a*x]),x]
```

```
[Out] Defer[Int][x^4/((c + a^2*c*x^2)^2*ArcTan[a*x]), x]
```

Rubi steps

$$\int \frac{x^4}{(c+a^2cx^2)^2 \tan^{-1}(ax)} dx = \int \frac{x^4}{(c+a^2cx^2)^2 \tan^{-1}(ax)} dx$$

Mathematica [A] time = 3.65353, size = 0, normalized size = 0.

$$\int \frac{x^4}{(c+a^2cx^2)^2 \tan^{-1}(ax)} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^4/((c + a^2*c*x^2)^2*ArcTan[a*x]),x]
```

```
[Out] Integrate[x^4/((c + a^2*c*x^2)^2*ArcTan[a*x]), x]
```

Maple [A] time = 0.46, size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(a^2cx^2+c\right)^2 \arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a^2*c*x^2+c)^2/arctan(a*x),x)
```

```
[Out] int(x^4/(a^2*c*x^2+c)^2/arctan(a*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(a^2cx^2+c\right)^2 \arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x^4/((a^2*c*x^2 + c)^2*arctan(a*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^4}{\left(a^4c^2x^4+2\,a^2c^2x^2+c^2\right)\arctan(ax)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral(x^4/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(a*x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^4}{a^4x^4 \operatorname{atan}(ax)+2a^2x^2 \operatorname{atan}(ax)+\operatorname{atan}(ax)}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(a**2*c*x**2+c)**2/atan(a*x),x)
```

```
[Out] Integral(x**4/(a**4*x**4*atan(a*x) + 2*a**2*x**2*atan(a*x) + atan(a*x)), x)
/c**2
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(a^2cx^2+c\right)^2 \arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="giac")
```

```
[Out] integrate(x^4/((a^2*c*x^2 + c)^2*arctan(a*x)), x)
```

3.481 $\int \frac{x^3}{\left(c+a^2cx^2\right)^2 \tan^{-1}(ax)}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{x^3}{\left(a^2cx^2+c\right)^2 \tan^{-1}(ax)},x\right)$$

```
[Out] Unintegrable[x^3/((c + a^2*c*x^2)^2*ArcTan[a*x]), x]
```

Rubi [A] time = 0.069281, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^3}{\left(c+a^2cx^2\right)^2 \tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^3/((c + a^2*c*x^2)^2*ArcTan[a*x]),x]

[Out] Defer[Int][x^3/((c + a^2*c*x^2)^2*ArcTan[a*x]), x]
```

Rubi steps

$$\int \frac{x^3}{\left(c+a^2cx^2\right)^2 \tan^{-1}(ax)}\,dx=\int \frac{x^3}{\left(c+a^2cx^2\right)^2 \tan^{-1}(ax)}\,dx$$

Mathematica [A] time = 2.06422, size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(c+a^2cx^2\right)^2 \tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^3/((c + a^2*c*x^2)^2*ArcTan[a*x]),x]
```

```
[Out] Integrate[x^3/((c + a^2*c*x^2)^2*ArcTan[a*x]), x]
```

Maple [A] time = 0.372, size = 0, normalized size = 0.

$$\int \frac{x^{3}}{\left(a^{2} c \,x^{2}+c \right)^{2} \arctan \left(a x \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a^2*c*x^2+c)^2/arctan(a*x),x)
```

```
[Out] int(x^3/(a^2*c*x^2+c)^2/arctan(a*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{3}}{\left(a^{2} c \,x^{2}+c \right)^{2} \arctan \left(a x \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x^3/((a^2*c*x^2 + c)^2*arctan(a*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x^{3}}{\left(a^{4} c^{2} x^{4} + 2 \, a^{2} c^{2} x^{2} + c^{2}\right) \arctan \left(a x\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral(x^3/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(a*x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^3}{a^4x^4\,\mathrm{atan}\,(ax)+2a^2x^2\,\mathrm{atan}\,(ax)+\mathrm{atan}\,(ax)}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a**2*c*x**2+c)**2/atan(a*x),x)
```

```
[Out] Integral(x**3/(a**4*x**4*atan(a*x) + 2*a**2*x**2*atan(a*x) + atan(a*x)), x)
/c**2
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(a^2cx^2+c\right)^2\arctan\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="giac")
```

```
[Out] integrate(x^3/((a^2*c*x^2 + c)^2*arctan(a*x)), x)
```

3.482 $\int \frac{x^2}{(c+a^2cx^2)^2 \tan^{-1}(ax)} dx$

Optimal. Leaf size=33

$$\frac{\log(\tan^{-1}(ax))}{2a^3c^2} - \frac{\text{CosIntegral}(2\tan^{-1}(ax))}{2a^3c^2}$$

```
[Out] -CosIntegral[2*ArcTan[a*x]]/(2*a^3*c^2) + Log[ArcTan[a*x]]/(2*a^3*c^2)
```

Rubi [A] time = 0.108464, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {4970, 3312, 3302}

$$\frac{\log(\tan^{-1}(ax))}{2a^3c^2} - \frac{\text{CosIntegral}(2\tan^{-1}(ax))}{2a^3c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((c + a^2*c*x^2)^2*ArcTan[a*x]),x]

[Out] -CosIntegral[2*ArcTan[a*x]]/(2*a^3*c^2) + Log[ArcTan[a*x]]/(2*a^3*c^2)
```

Rule 4970

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sin[x]^m)/
Cos[x]^(m + 2*(q + 1)), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
```

```
c*f, 0]
```

Rubi steps

$$\int \frac{x^2}{\left(c+a^2cx^2\right)^2\tan^{-1}(ax)}\,dx = \frac{\text{Subst}\left(\int \frac{\sin^2(x)}{x}\,dx, x, \tan^{-1}(ax)\right)}{a^3c^2}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{2x}-\frac{\cos(2x)}{2x}\right)dx, x, \tan^{-1}(ax)\right)}{a^3c^2}$$
$$= \frac{\log\left(\tan^{-1}(ax)\right)}{2a^3c^2} - \frac{\text{Subst}\left(\int \frac{\cos(2x)}{x}\,dx, x, \tan^{-1}(ax)\right)}{2a^3c^2}$$
$$= -\frac{\mathbf{Ci}\left(2\tan^{-1}(ax)\right)}{2a^3c^2} + \frac{\log\left(\tan^{-1}(ax)\right)}{2a^3c^2}$$

Mathematica [A] time = 0.108945, size = 25, normalized size = 0.76

$$\frac{\log\left(\tan^{-1}(ax)\right) - \text{CosIntegral}\left(2\tan^{-1}(ax)\right)}{2a^3c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((c + a^2*c*x^2)^2*ArcTan[a*x]),x]

[Out] (-CosIntegral[2*ArcTan[a*x]] + Log[ArcTan[a*x]])/(2*a^3*c^2)
```

Maple [A] time = 0.058, size = 30, normalized size = 0.9

$$-\frac{\text{Ci}\left(2\arctan(ax)\right)}{2a^3c^2} + \frac{\ln\left(\arctan(ax)\right)}{2a^3c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a^2*c*x^2+c)^2/arctan(a*x),x)

[Out] -1/2*Ci(2*arctan(a*x))/a^3/c^2+1/2*ln(arctan(a*x))/a^3/c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(a^2cx^2+c\right)^2 \arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((a^2*c*x^2 + c)^2*arctan(a*x)), x)
```

Fricas [C] time = 1.73265, size = 197, normalized size = 5.97

$$\frac{2\log(\arctan(ax)) - \text{log\_integral}\left(-\frac{a^2x^2+2i\,ax-1}{a^2x^2+1}\right) - \text{log\_integral}\left(-\frac{a^2x^2-2i\,ax-1}{a^2x^2+1}\right)}{4\,a^3c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="fricas")
```

```
[Out] 1/4*(2*log(arctan(a*x)) - log_integral(-(a^2*x^2 + 2*I*a*x - 1)/(a^2*x^2 +
1)) - log_integral(-(a^2*x^2 - 2*I*a*x - 1)/(a^2*x^2 + 1)))/(a^3*c^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^2}{a^4x^4\,\text{atan}(ax)+2a^2x^2\,\text{atan}(ax)+\text{atan}(ax)}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a**2*c*x**2+c)**2/atan(a*x),x)
```

```
[Out] Integral(x**2/(a**4*x**4*atan(a*x) + 2*a**2*x**2*atan(a*x) + atan(a*x)), x)
/c**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(a^2cx^2+c\right)^2 \arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="giac")
```

```
[Out] integrate(x^2/((a^2*c*x^2 + c)^2*arctan(a*x)), x)
```

3.483 $\int \frac{x}{\left(c+a^2cx^2\right)^2\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=17

$$\frac{\mathrm{Si}\left(2\tan^{-1}(ax)\right)}{2a^2c^2}$$

```
[Out] SinIntegral[2*ArcTan[a*x]]/(2*a^2*c^2)
```

Rubi [A] time = 0.0709479, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4970, 4406, 12, 3299}

$$\frac{\mathrm{Si}\left(2\tan^{-1}(ax)\right)}{2a^2c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/((c + a^2*c*x^2)^2*ArcTan[a*x]),x]
```

```
[Out] SinIntegral[2*ArcTan[a*x]]/(2*a^2*c^2)
```

Rule 4970

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sin[x]^m)/
Cos[x]^(m + 2*(q + 1)), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\left(c+a^2cx^2\right)^2\tan^{-1}(ax)}\,dx &= \frac{\text{Subst}\left(\int \frac{\cos(x)\sin(x)}{x}\,dx, x, \tan^{-1}(ax)\right)}{a^2c^2}\\ &= \frac{\text{Subst}\left(\int \frac{\sin(2x)}{2x}\,dx, x, \tan^{-1}(ax)\right)}{a^2c^2}\\ &= \frac{\text{Subst}\left(\int \frac{\sin(2x)}{x}\,dx, x, \tan^{-1}(ax)\right)}{2a^2c^2}\\ &= \frac{\text{Si}\left(2\tan^{-1}(ax)\right)}{2a^2c^2}\end{aligned}$$

Mathematica [A] time = 0.0380637, size = 17, normalized size = 1.

$$\frac{\text{Si}\left(2\tan^{-1}(ax)\right)}{2a^2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((c + a^2*c*x^2)^2*ArcTan[a*x]),x]
```

```
[Out] SinIntegral[2*ArcTan[a*x]]/(2*a^2*c^2)
```

Maple [A] time = 0.055, size = 16, normalized size = 0.9

$$\frac{\text{Si}\left(2\arctan(ax)\right)}{2a^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^2*c*x^2+c)^2/arctan(a*x),x)
```

```
[Out] 1/2*Si(2*arctan(a*x))/a^2/c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(a^2cx^2+c\right)^2\arctan\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x/((a^2*c*x^2 + c)^2*arctan(a*x)), x)
```

Fricas [C] time = 1.75612, size = 174, normalized size = 10.24

$$\frac{i\ \mathrm{log\_integral}\left(-\frac{a^2x^2+2i\,ax-1}{a^2x^2+1}\right)-i\ \mathrm{log\_integral}\left(-\frac{a^2x^2-2i\,ax-1}{a^2x^2+1}\right)}{4\,a^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="fricas")
```

```
[Out] 1/4*(I*log_integral(-(a^2*x^2 + 2*I*a*x - 1)/(a^2*x^2 + 1)) - I*log_integra
l(-(a^2*x^2 - 2*I*a*x - 1)/(a^2*x^2 + 1)))/(a^2*c^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x}{a^4x^4\operatorname{atan}\left(ax\right)+2a^2x^2\operatorname{atan}\left(ax\right)+\operatorname{atan}\left(ax\right)}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a**2*c*x**2+c)**2/atan(a*x),x)
```

```
[Out] Integral(x/(a**4*x**4*atan(a*x) + 2*a**2*x**2*atan(a*x) + atan(a*x)), x)/c*
*2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(a^2cx^2+c\right)^2\arctan\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="giac")
```

```
[Out] integrate(x/((a^2*c*x^2 + c)^2*arctan(a*x)), x)
```

3.484 $\int \frac{1}{\left(c+a^2cx^2\right)^2 \tan^{-1}(ax)}\,dx$

Optimal. Leaf size=33

$$\frac{\text{CosIntegral}\left(2\tan^{-1}(ax)\right)}{2ac^2}+\frac{\log\left(\tan^{-1}(ax)\right)}{2ac^2}$$

```
[Out] CosIntegral[2*ArcTan[a*x]]/(2*a*c^2) + Log[ArcTan[a*x]]/(2*a*c^2)
```

Rubi [A] time = 0.0664349, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {4904, 3312, 3302}

$$\frac{\text{CosIntegral}\left(2\tan^{-1}(ax)\right)}{2ac^2}+\frac{\log\left(\tan^{-1}(ax)\right)}{2ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a^2*c*x^2)^2*ArcTan[a*x]),x]
```

```
[Out] CosIntegral[2*ArcTan[a*x]]/(2*a*c^2) + Log[ArcTan[a*x]]/(2*a*c^2)
```

Rule 4904

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{1}{\left(c+a^2cx^2\right)^2\tan^{-1}(ax)}\,dx = \frac{\text{Subst}\left(\int \frac{\cos^2(x)}{x}\,dx, x, \tan^{-1}(ax)\right)}{ac^2}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{2x}+\frac{\cos(2x)}{2x}\right)dx, x, \tan^{-1}(ax)\right)}{ac^2}$$
$$= \frac{\log\left(\tan^{-1}(ax)\right)}{2ac^2} + \frac{\text{Subst}\left(\int \frac{\cos(2x)}{x}\,dx, x, \tan^{-1}(ax)\right)}{2ac^2}$$
$$= \frac{\text{Ci}\left(2\tan^{-1}(ax)\right)}{2ac^2} + \frac{\log\left(\tan^{-1}(ax)\right)}{2ac^2}$$

Mathematica [A] time = 0.0269469, size = 23, normalized size = 0.7

$$\frac{\text{CosIntegral}\left(2\tan^{-1}(ax)\right)+\log\left(\tan^{-1}(ax)\right)}{2ac^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a^2*c*x^2)^2*ArcTan[a*x]),x]
```

```
[Out] (CosIntegral[2*ArcTan[a*x]] + Log[ArcTan[a*x]])/(2*a*c^2)
```

Maple [A] time = 0.058, size = 30, normalized size = 0.9

$$\frac{\text{Ci}\left(2\arctan(ax)\right)}{2\,ac^2}+\frac{\ln\left(\arctan(ax)\right)}{2\,ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2*c*x^2+c)^2/arctan(a*x),x)
```

```
[Out] 1/2*Ci(2*arctan(a*x))/a/c^2+1/2*ln(arctan(a*x))/a/c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)^2\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^2*arctan(a*x)), x)
```

Fricas [C] time = 1.64666, size = 194, normalized size = 5.88

$$\frac{2\log(\arctan(ax))+\text{log\_integral}\left(-\frac{a^2x^2+2i\,ax-1}{a^2x^2+1}\right)+\text{log\_integral}\left(-\frac{a^2x^2-2i\,ax-1}{a^2x^2+1}\right)}{4\,ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="fricas")
```

```
[Out] 1/4*(2*log(arctan(a*x)) + log_integral(-(a^2*x^2 + 2*I*a*x - 1)/(a^2*x^2 +
1)) + log_integral(-(a^2*x^2 - 2*I*a*x - 1)/(a^2*x^2 + 1)))/(a*c^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{a^4x^4\operatorname{atan}(ax)+2a^2x^2\operatorname{atan}(ax)+\operatorname{atan}(ax)}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*c*x**2+c)**2/atan(a*x),x)
```

```
[Out] Integral(1/(a**4*x**4*atan(a*x) + 2*a**2*x**2*atan(a*x) + atan(a*x)), x)/c*
*2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)^2\arctan\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^2*arctan(a*x)), x)
```

3.485 $\int \frac{1}{x\left(c+a^2cx^2\right)^2\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{1}{x\left(a^2cx^2+c\right)^2\tan^{-1}(ax)},x\right)$$

```
[Out] Unintegrable[1/(x*(c + a^2*c*x^2)^2*ArcTan[a*x]), x]
```

Rubi [A] time = 0.0612764, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x\left(c+a^2cx^2\right)^2\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*(c + a^2*c*x^2)^2*ArcTan[a*x]),x]
```

```
[Out] Defer[Int][1/(x*(c + a^2*c*x^2)^2*ArcTan[a*x]), x]
```

Rubi steps

$$\int \frac{1}{x\left(c+a^2cx^2\right)^2\tan^{-1}(ax)}\,dx=\int \frac{1}{x\left(c+a^2cx^2\right)^2\tan^{-1}(ax)}\,dx$$

Mathematica [A] time = 0.715179, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(c+a^2cx^2\right)^2\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*(c + a^2*c*x^2)^2*ArcTan[a*x]),x]
```

```
[Out] Integrate[1/(x*(c + a^2*c*x^2)^2*ArcTan[a*x]), x]
```

Maple [A] time = 0.214, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(a^{2}c x^{2}+c\right)^{2}\arctan\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a^2*c*x^2+c)^2/arctan(a*x),x)
```

```
[Out] int(1/x/(a^2*c*x^2+c)^2/arctan(a*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^{2}c x^{2}+c\right)^{2}x\arctan\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^2*x*arctan(a*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{1}{\left(a^{4}c^{2}x^{5}+2\,a^{2}c^{2}x^{3}+c^{2}x\right)\arctan\left(ax\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral(1/((a^4*c^2*x^5 + 2*a^2*c^2*x^3 + c^2*x)*arctan(a*x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{a^4x^5 \operatorname{atan}(ax)+2a^2x^3 \operatorname{atan}(ax)+x \operatorname{atan}(ax)}\, dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a**2*c*x**2+c)**2/atan(a*x),x)
```

```
[Out] Integral(1/(a**4*x**5*atan(a*x) + 2*a**2*x**3*atan(a*x) + x*atan(a*x)), x)/
c**2
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2 + c\right)^2 x \arctan(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^2*x*arctan(a*x)), x)
```

3.486 $\int \frac{1}{x^2(c+a^2cx^2)^2\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{1}{x^2\left(a^2cx^2+c\right)^2\tan^{-1}(ax)},x\right)$$

```
[Out] Unintegrable[1/(x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]), x]
```

Rubi [A] time = 0.0684653, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^2\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]),x]

[Out] Defer[Int][1/(x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]), x]
```

Rubi steps

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^2\tan^{-1}(ax)}\,dx=\int \frac{1}{x^2\left(c+a^2cx^2\right)^2\tan^{-1}(ax)}\,dx$$

Mathematica [A] time = 1.08719, size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^2\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]),x]
```

```
[Out] Integrate[1/(x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]), x]
```

Maple [A] time = 0.25, size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \left(a^2 c x^2 + c\right)^2 \arctan\left(ax\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a^2*c*x^2+c)^2/arctan(a*x),x)
```

```
[Out] int(1/x^2/(a^2*c*x^2+c)^2/arctan(a*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2 c x^2 + c\right)^2 x^2 \arctan\left(ax\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^2*x^2*arctan(a*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\left(a^4 c^2 x^6 + 2\, a^2 c^2 x^4 + c^2 x^2\right) \arctan\left(ax\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral(1/((a^4*c^2*x^6 + 2*a^2*c^2*x^4 + c^2*x^2)*arctan(a*x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{a^4x^6\operatorname{atan}(ax)+2a^2x^4\operatorname{atan}(ax)+x^2\operatorname{atan}(ax)}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a**2*c*x**2+c)**2/atan(a*x),x)
```

```
[Out] Integral(1/(a**4*x**6*atan(a*x) + 2*a**2*x**4*atan(a*x) + x**2*atan(a*x)),
x)/c**2
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)^2x^2\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^2*x^2*arctan(a*x)), x)
```

3.487 $\int \frac{x^6}{\left(c+a^2cx^2\right)^3\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{x^6}{\left(a^2cx^2+c\right)^3\tan^{-1}(ax)},x\right)$$

```
[Out] Unintegrable[x^6/((c + a^2*c*x^2)^3*ArcTan[a*x]), x]
```

Rubi [A] time = 0.0665092, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{x^6}{\left(c+a^2cx^2\right)^3\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^6/((c + a^2*c*x^2)^3*ArcTan[a*x]),x]
```

```
[Out] Defer[Int][x^6/((c + a^2*c*x^2)^3*ArcTan[a*x]), x]
```

Rubi steps

$$\int \frac{x^6}{\left(c+a^2cx^2\right)^3\tan^{-1}(ax)}\,dx = \int \frac{x^6}{\left(c+a^2cx^2\right)^3\tan^{-1}(ax)}\,dx$$

Mathematica [A] time = 6.67685, size = 0, normalized size = 0.

$$\int \frac{x^6}{\left(c+a^2cx^2\right)^3\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^6/((c + a^2*c*x^2)^3*ArcTan[a*x]),x]
```

```
[Out] Integrate[x^6/((c + a^2*c*x^2)^3*ArcTan[a*x]), x]
```

Maple [A] time = 0.514, size = 0, normalized size = 0.

$$\int \frac{x^6}{\left(a^2cx^2+c\right)^3 \arctan\left(ax\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a^2*c*x^2+c)^3/arctan(a*x),x)
```

```
[Out] int(x^6/(a^2*c*x^2+c)^3/arctan(a*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{\left(a^2cx^2+c\right)^3 \arctan\left(ax\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x^6/((a^2*c*x^2 + c)^3*arctan(a*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x^6}{\left(a^6c^3x^6+3\,a^4c^3x^4+3\,a^2c^3x^2+c^3\right)\arctan\left(ax\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral(x^6/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*arctan(a*
x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^6}{a^6x^6\operatorname{atan}(ax)+3a^4x^4\operatorname{atan}(ax)+3a^2x^2\operatorname{atan}(ax)+\operatorname{atan}(ax)}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(a**2*c*x**2+c)**3/atan(a*x),x)
```

```
[Out] Integral(x**6/(a**6*x**6*atan(a*x) + 3*a**4*x**4*atan(a*x) + 3*a**2*x**2*at
an(a*x) + atan(a*x)), x)/c**3
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{\left(a^2cx^2+c\right)^3\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="giac")
```

```
[Out] integrate(x^6/((a^2*c*x^2 + c)^3*arctan(a*x)), x)
```

3.488 $\int \frac{x^5}{(c+a^2cx^2)^3 \tan^{-1}(ax)} dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{x^5}{(a^2cx^2+c)^3 \tan^{-1}(ax)}, x\right)$$

```
[Out] Unintegrable[x^5/((c + a^2*c*x^2)^3*ArcTan[a*x]), x]
```

Rubi [A] time = 0.0670864, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^5}{(c+a^2cx^2)^3 \tan^{-1}(ax)} dx$$

Verification is Not applicable to the result.

```
[In] Int[x^5/((c + a^2*c*x^2)^3*ArcTan[a*x]),x]
```

```
[Out] Defer[Int][x^5/((c + a^2*c*x^2)^3*ArcTan[a*x]), x]
```

Rubi steps

$$\int \frac{x^5}{(c+a^2cx^2)^3 \tan^{-1}(ax)} dx = \int \frac{x^5}{(c+a^2cx^2)^3 \tan^{-1}(ax)} dx$$

Mathematica [A] time = 7.69702, size = 0, normalized size = 0.

$$\int \frac{x^5}{(c+a^2cx^2)^3 \tan^{-1}(ax)} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^5/((c + a^2*c*x^2)^3*ArcTan[a*x]),x]
```

```
[Out] Integrate[x^5/((c + a^2*c*x^2)^3*ArcTan[a*x]), x]
```

Maple [A] time = 0.403, size = 0, normalized size = 0.

$$\int \frac{x^5}{\left(a^2cx^2+c\right)^3 \arctan\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a^2*c*x^2+c)^3/arctan(a*x),x)
```

```
[Out] int(x^5/(a^2*c*x^2+c)^3/arctan(a*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\left(a^2cx^2+c\right)^3 \arctan\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x^5/((a^2*c*x^2 + c)^3*arctan(a*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x^5}{\left(a^6c^3x^6+3\,a^4c^3x^4+3\,a^2c^3x^2+c^3\right)\arctan\left(ax\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral(x^5/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*arctan(a*
x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^5}{a^6x^6 \operatorname{atan}(ax)+3a^4x^4 \operatorname{atan}(ax)+3a^2x^2 \operatorname{atan}(ax)+\operatorname{atan}(ax)}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(a**2*c*x**2+c)**3/atan(a*x),x)
```

```
[Out] Integral(x**5/(a**6*x**6*atan(a*x) + 3*a**4*x**4*atan(a*x) + 3*a**2*x**2*at
an(a*x) + atan(a*x)), x)/c**3
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\left(a^2cx^2+c\right)^3 \arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="giac")
```

```
[Out] integrate(x^5/((a^2*c*x^2 + c)^3*arctan(a*x)), x)
```

3.489 $\int \frac{x^4}{\left(c+a^2cx^2\right)^3\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=50

$$-\frac{\text{CosIntegral}\left(2\tan^{-1}(ax)\right)}{2a^5c^3}+\frac{\text{CosIntegral}\left(4\tan^{-1}(ax)\right)}{8a^5c^3}+\frac{3\log\left(\tan^{-1}(ax)\right)}{8a^5c^3}$$

```
[Out] -CosIntegral[2*ArcTan[a*x]]/(2*a^5*c^3) + CosIntegral[4*ArcTan[a*x]]/(8*a^5
*c^3) + (3*Log[ArcTan[a*x]])/(8*a^5*c^3)
```

Rubi [A] time = 0.127434, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {4970, 3312, 3302}

$$-\frac{\text{CosIntegral}\left(2\tan^{-1}(ax)\right)}{2a^5c^3}+\frac{\text{CosIntegral}\left(4\tan^{-1}(ax)\right)}{8a^5c^3}+\frac{3\log\left(\tan^{-1}(ax)\right)}{8a^5c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((c + a^2*c*x^2)^3*ArcTan[a*x]),x]
```

```
[Out] -CosIntegral[2*ArcTan[a*x]]/(2*a^5*c^3) + CosIntegral[4*ArcTan[a*x]]/(8*a^5
*c^3) + (3*Log[ArcTan[a*x]])/(8*a^5*c^3)
```

Rule 4970

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sin[x]^m)/
Cos[x]^(m + 2*(q + 1)), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{x^4}{\left(c+a^2cx^2\right)^3\tan^{-1}(ax)}\,dx = \frac{\text{Subst}\left(\int \frac{\sin^4(x)}{x}\,dx, x, \tan^{-1}(ax)\right)}{a^5c^3}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{3}{8x}-\frac{\cos(2x)}{2x}+\frac{\cos(4x)}{8x}\right)dx, x, \tan^{-1}(ax)\right)}{a^5c^3}$$
$$= \frac{3\log\left(\tan^{-1}(ax)\right)}{8a^5c^3}+\frac{\text{Subst}\left(\int \frac{\cos(4x)}{x}\,dx, x, \tan^{-1}(ax)\right)}{8a^5c^3}-\frac{\text{Subst}\left(\int \frac{\cos(2x)}{x}\,dx, x, \tan^{-1}(\right.}{2a^5c^3}$$
$$= -\frac{\text{Ci}\left(2\tan^{-1}(ax)\right)}{2a^5c^3}+\frac{\text{Ci}\left(4\tan^{-1}(ax)\right)}{8a^5c^3}+\frac{3\log\left(\tan^{-1}(ax)\right)}{8a^5c^3}$$

Mathematica [A] time = 0.144777, size = 34, normalized size = 0.68

$$\frac{-4\text{CosIntegral}\left(2\tan^{-1}(ax)\right)+\text{CosIntegral}\left(4\tan^{-1}(ax)\right)+3\log\left(\tan^{-1}(ax)\right)}{8a^5c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((c + a^2*c*x^2)^3*ArcTan[a*x]),x]
```

```
[Out] (-4*CosIntegral[2*ArcTan[a*x]] + CosIntegral[4*ArcTan[a*x]] + 3*Log[ArcTan[
a*x]])/(8*a^5*c^3)
```

Maple [A] time = 0.059, size = 45, normalized size = 0.9

$$-\frac{\text{Ci}\left(2\arctan(ax)\right)}{2c^3a^5}+\frac{\text{Ci}\left(4\arctan(ax)\right)}{8c^3a^5}+\frac{3\ln\left(\arctan(ax)\right)}{8c^3a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a^2*c*x^2+c)^3/arctan(a*x),x)
```

[Out] -1/2*Ci(2*arctan(a*x))/a^5/c^3+1/8*Ci(4*arctan(a*x))/a^5/c^3+3/8*ln(arctan(a*x))/a^5/c^3

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(a^2cx^2+c\right)^3 \arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="maxima")

[Out] integrate(x^4/((a^2*c*x^2 + c)^3*arctan(a*x)), x)

Fricas [C] time = 1.67151, size = 452, normalized size = 9.04

$$\frac{6\log(\arctan(ax)) + \mathrm{log\_integral}\left(\frac{a^4x^4+4i\,a^3x^3-6\,a^2x^2-4i\,ax+1}{a^4x^4+2\,a^2x^2+1}\right) + \mathrm{log\_integral}\left(\frac{a^4x^4-4i\,a^3x^3-6\,a^2x^2+4i\,ax+1}{a^4x^4+2\,a^2x^2+1}\right) - 4\,\mathrm{log\_integral}}{16\,a^5c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="fricas")

[Out] 1/16*(6*log(arctan(a*x)) + log_integral((a^4*x^4 + 4*I*a^3*x^3 - 6*a^2*x^2 - 4*I*a*x + 1)/(a^4*x^4 + 2*a^2*x^2 + 1)) + log_integral((a^4*x^4 - 4*I*a^3*x^3 - 6*a^2*x^2 + 4*I*a*x + 1)/(a^4*x^4 + 2*a^2*x^2 + 1)) - 4*log_integral(-(a^2*x^2 + 2*I*a*x - 1)/(a^2*x^2 + 1)) - 4*log_integral(-(a^2*x^2 - 2*I*a*x - 1)/(a^2*x^2 + 1)))/(a^5*c^3)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^4}{a^6x^6\,\mathrm{atan}(ax)+3a^4x^4\,\mathrm{atan}(ax)+3a^2x^2\,\mathrm{atan}(ax)+\mathrm{atan}(ax)}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(a**2*c*x**2+c)**3/atan(a*x),x)
```

```
[Out] Integral(x**4/(a**6*x**6*atan(a*x) + 3*a**4*x**4*atan(a*x) + 3*a**2*x**2*at
an(a*x) + atan(a*x)), x)/c**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(a^2cx^2+c\right)^3 \arctan\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="giac")
```

```
[Out] integrate(x^4/((a^2*c*x^2 + c)^3*arctan(a*x)), x)
```

3.490 $\int \frac{x^3}{(c+a^2cx^2)^3 \tan^{-1}(ax)} \, dx$

Optimal. Leaf size=35

$$\frac{\text{Si}\left(2\tan^{-1}(ax)\right)}{4a^4c^3} - \frac{\text{Si}\left(4\tan^{-1}(ax)\right)}{8a^4c^3}$$

```
[Out] SinIntegral[2*ArcTan[a*x]]/(4*a^4*c^3) - SinIntegral[4*ArcTan[a*x]]/(8*a^4*
c^3)
```

Rubi [A] time = 0.113361, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {4970, 4406, 3299}

$$\frac{\text{Si}\left(2\tan^{-1}(ax)\right)}{4a^4c^3} - \frac{\text{Si}\left(4\tan^{-1}(ax)\right)}{8a^4c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((c + a^2*c*x^2)^3*ArcTan[a*x]),x]
```

```
[Out] SinIntegral[2*ArcTan[a*x]]/(4*a^4*c^3) - SinIntegral[4*ArcTan[a*x]]/(8*a^4*
c^3)
```

Rule 4970

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sin[x]^m)/
Cos[x]^(m + 2*(q + 1)), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\int \frac{x^3}{\left(c+a^2cx^2\right)^3\tan^{-1}(ax)}\,dx = \frac{\text{Subst}\left(\int \frac{\cos(x)\sin^3(x)}{x}\,dx, x, \tan^{-1}(ax)\right)}{a^4c^3}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{\sin(2x)}{4x}-\frac{\sin(4x)}{8x}\right)dx, x, \tan^{-1}(ax)\right)}{a^4c^3}$$
$$= -\frac{\text{Subst}\left(\int \frac{\sin(4x)}{x}\,dx, x, \tan^{-1}(ax)\right)}{8a^4c^3}+\frac{\text{Subst}\left(\int \frac{\sin(2x)}{x}\,dx, x, \tan^{-1}(ax)\right)}{4a^4c^3}$$
$$= \frac{\text{Si}\left(2\tan^{-1}(ax)\right)}{4a^4c^3}-\frac{\text{Si}\left(4\tan^{-1}(ax)\right)}{8a^4c^3}$$

Mathematica [A] time = 0.114585, size = 27, normalized size = 0.77

$$-\frac{\text{Si}\left(4\tan^{-1}(ax)\right)-2\text{Si}\left(2\tan^{-1}(ax)\right)}{8a^4c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((c + a^2*c*x^2)^3*ArcTan[a*x]),x]
```

```
[Out] -(-2*SinIntegral[2*ArcTan[a*x]] + SinIntegral[4*ArcTan[a*x]])/(8*a^4*c^3)
```

Maple [A] time = 0.06, size = 32, normalized size = 0.9

$$\frac{\text{Si}\left(2\arctan(ax)\right)}{4c^3a^4}-\frac{\text{Si}\left(4\arctan(ax)\right)}{8c^3a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a^2*c*x^2+c)^3/arctan(a*x),x)
```

```
[Out] 1/4*Si(2*arctan(a*x))/a^4/c^3-1/8*Si(4*arctan(a*x))/a^4/c^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(a^2cx^2+c\right)^3\arctan\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x^3/((a^2*c*x^2 + c)^3*arctan(a*x)), x)
```

Fricas [C] time = 1.69133, size = 436, normalized size = 12.46

$$\frac{-i\ \text{log\_integral}\left(\frac{a^4x^4+4i\,a^3x^3-6\,a^2x^2-4i\,ax+1}{a^4x^4+2\,a^2x^2+1}\right)+i\ \text{log\_integral}\left(\frac{a^4x^4-4i\,a^3x^3-6\,a^2x^2+4i\,ax+1}{a^4x^4+2\,a^2x^2+1}\right)+2i\ \text{log\_integral}\left(-\frac{a^2x^2+2i\,ax-1}{a^2x^2+1}\right)-}{16\,a^4c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="fricas")
```

```
[Out] 1/16*(-I*log_integral((a^4*x^4 + 4*I*a^3*x^3 - 6*a^2*x^2 - 4*I*a*x + 1)/(a^
4*x^4 + 2*a^2*x^2 + 1)) + I*log_integral((a^4*x^4 - 4*I*a^3*x^3 - 6*a^2*x^2
 + 4*I*a*x + 1)/(a^4*x^4 + 2*a^2*x^2 + 1)) + 2*I*log_integral(-(a^2*x^2 + 2
*I*a*x - 1)/(a^2*x^2 + 1)) - 2*I*log_integral(-(a^2*x^2 - 2*I*a*x - 1)/(a^2
*x^2 + 1)))/(a^4*c^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^3}{a^6x^6\operatorname{atan}(ax)+3a^4x^4\operatorname{atan}(ax)+3a^2x^2\operatorname{atan}(ax)+\operatorname{atan}(ax)}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a**2*c*x**2+c)**3/atan(a*x),x)
```

```
[Out] Integral(x**3/(a**6*x**6*atan(a*x) + 3*a**4*x**4*atan(a*x) + 3*a**2*x**2*at
an(a*x) + atan(a*x)), x)/c**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(a^2cx^2+c\right)^3 \arctan\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="giac")
```

```
[Out] integrate(x^3/((a^2*c*x^2 + c)^3*arctan(a*x)), x)
```

3.491 $\int \frac{x^2}{(c+a^2cx^2)^3 \tan^{-1}(ax)} dx$

Optimal. Leaf size=33

$$\frac{\log\left(\tan^{-1}(ax)\right)}{8a^3c^3} - \frac{\text{CosIntegral}\left(4\tan^{-1}(ax)\right)}{8a^3c^3}$$

```
[Out] -CosIntegral[4*ArcTan[a*x]]/(8*a^3*c^3) + Log[ArcTan[a*x]]/(8*a^3*c^3)
```

Rubi [A] time = 0.110074, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {4970, 4406, 3302}

$$\frac{\log\left(\tan^{-1}(ax)\right)}{8a^3c^3} - \frac{\text{CosIntegral}\left(4\tan^{-1}(ax)\right)}{8a^3c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((c + a^2*c*x^2)^3*ArcTan[a*x]),x]
```

```
[Out] -CosIntegral[4*ArcTan[a*x]]/(8*a^3*c^3) + Log[ArcTan[a*x]]/(8*a^3*c^3)
```

Rule 4970

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sin[x]^m)/
Cos[x]^(m + 2*(q + 1)), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
```

```
c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\left(c+a^2cx^2\right)^3\tan^{-1}(ax)}\,dx &= \frac{\text{Subst}\left(\int \frac{\cos^2(x)\sin^2(x)}{x}\,dx,x,\tan^{-1}(ax)\right)}{a^3c^3}\\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{8x}-\frac{\cos(4x)}{8x}\right)dx,x,\tan^{-1}(ax)\right)}{a^3c^3}\\ &= \frac{\log\left(\tan^{-1}(ax)\right)}{8a^3c^3}-\frac{\text{Subst}\left(\int \frac{\cos(4x)}{x}\,dx,x,\tan^{-1}(ax)\right)}{8a^3c^3}\\ &= -\frac{\text{Ci}\left(4\tan^{-1}(ax)\right)}{8a^3c^3}+\frac{\log\left(\tan^{-1}(ax)\right)}{8a^3c^3}\end{aligned}$$

Mathematica [A] time = 0.0645481, size = 25, normalized size = 0.76

$$\frac{\log\left(\tan^{-1}(ax)\right)-\text{CosIntegral}\left(4\tan^{-1}(ax)\right)}{8a^3c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((c + a^2*c*x^2)^3*ArcTan[a*x]),x]
```

```
[Out] (-CosIntegral[4*ArcTan[a*x]] + Log[ArcTan[a*x]])/(8*a^3*c^3)
```

Maple [A] time = 0.065, size = 30, normalized size = 0.9

$$-\frac{\text{Ci}\left(4\arctan(ax)\right)}{8\,c^3a^3}+\frac{\ln\left(\arctan(ax)\right)}{8\,c^3a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a^2*c*x^2+c)^3/arctan(a*x),x)
```

```
[Out] -1/8*Ci(4*arctan(a*x))/a^3/c^3+1/8*ln(arctan(a*x))/a^3/c^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(a^2cx^2+c\right)^3 \arctan\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((a^2*c*x^2 + c)^3*arctan(a*x)), x)
```

Fricas [C] time = 1.64494, size = 298, normalized size = 9.03

$$\frac{2\,\log\left(\arctan\left(ax\right)\right)-\mathrm{log\_integral}\left(\frac{a^4x^4+4i\,a^3x^3-6\,a^2x^2-4i\,ax+1}{a^4x^4+2\,a^2x^2+1}\right)-\mathrm{log\_integral}\left(\frac{a^4x^4-4i\,a^3x^3-6\,a^2x^2+4i\,ax+1}{a^4x^4+2\,a^2x^2+1}\right)}{16\,a^3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="fricas")
```

```
[Out] 1/16*(2*log(arctan(a*x)) - log_integral((a^4*x^4 + 4*I*a^3*x^3 - 6*a^2*x^2
- 4*I*a*x + 1)/(a^4*x^4 + 2*a^2*x^2 + 1)) - log_integral((a^4*x^4 - 4*I*a^3
*x^3 - 6*a^2*x^2 + 4*I*a*x + 1)/(a^4*x^4 + 2*a^2*x^2 + 1)))/(a^3*c^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^2}{a^6x^6\,\mathrm{atan}\left(ax\right)+3a^4x^4\,\mathrm{atan}\left(ax\right)+3a^2x^2\,\mathrm{atan}\left(ax\right)+\mathrm{atan}\left(ax\right)}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a**2*c*x**2+c)**3/atan(a*x),x)
```

```
[Out] Integral(x**2/(a**6*x**6*atan(a*x) + 3*a**4*x**4*atan(a*x) + 3*a**2*x**2*at
an(a*x) + atan(a*x)), x)/c**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(a^2cx^2+c\right)^3 \arctan\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="giac")
```

```
[Out] integrate(x^2/((a^2*c*x^2 + c)^3*arctan(a*x)), x)
```

3.492 $\int \frac{x}{\left(c+a^2cx^2\right)^3\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=35

$$\frac{\text{Si}\left(2\tan^{-1}(ax)\right)}{4a^2c^3}+\frac{\text{Si}\left(4\tan^{-1}(ax)\right)}{8a^2c^3}$$

```
[Out] SinIntegral[2*ArcTan[a*x]]/(4*a^2*c^3) + SinIntegral[4*ArcTan[a*x]]/(8*a^2*
c^3)
```

Rubi [A] time = 0.0899071, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {4970, 4406, 3299}

$$\frac{\text{Si}\left(2\tan^{-1}(ax)\right)}{4a^2c^3}+\frac{\text{Si}\left(4\tan^{-1}(ax)\right)}{8a^2c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x/((c + a^2*c*x^2)^3*ArcTan[a*x]),x]
```

```
[Out] SinIntegral[2*ArcTan[a*x]]/(4*a^2*c^3) + SinIntegral[4*ArcTan[a*x]]/(8*a^2*
c^3)
```

Rule 4970

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sin[x]^m)/
Cos[x]^(m + 2*(q + 1)), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\int \frac{x}{\left(c+a^2cx^2\right)^3\tan^{-1}(ax)}\,dx = \frac{\text{Subst}\left(\int \frac{\cos^3(x)\sin(x)}{x}\,dx,x,\tan^{-1}(ax)\right)}{a^2c^3}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{\sin(2x)}{4x}+\frac{\sin(4x)}{8x}\right)dx,x,\tan^{-1}(ax)\right)}{a^2c^3}$$
$$= \frac{\text{Subst}\left(\int \frac{\sin(4x)}{x}\,dx,x,\tan^{-1}(ax)\right)}{8a^2c^3}+\frac{\text{Subst}\left(\int \frac{\sin(2x)}{x}\,dx,x,\tan^{-1}(ax)\right)}{4a^2c^3}$$
$$= \frac{\text{Si}\left(2\tan^{-1}(ax)\right)}{4a^2c^3}+\frac{\text{Si}\left(4\tan^{-1}(ax)\right)}{8a^2c^3}$$

Mathematica [A] time = 0.0809775, size = 27, normalized size = 0.77

$$\frac{2\text{Si}\left(2\tan^{-1}(ax)\right)+\text{Si}\left(4\tan^{-1}(ax)\right)}{8a^2c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((c + a^2*c*x^2)^3*ArcTan[a*x]),x]
```

```
[Out] (2*SinIntegral[2*ArcTan[a*x]] + SinIntegral[4*ArcTan[a*x]])/(8*a^2*c^3)
```

Maple [A] time = 0.058, size = 32, normalized size = 0.9

$$\frac{\text{Si}\left(2\arctan(ax)\right)}{4c^3a^2}+\frac{\text{Si}\left(4\arctan(ax)\right)}{8c^3a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^2*c*x^2+c)^3/arctan(a*x),x)
```

```
[Out] 1/4*Si(2*arctan(a*x))/a^2/c^3+1/8*Si(4*arctan(a*x))/a^2/c^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(a^2cx^2+c\right)^3\arctan\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x/((a^2*c*x^2 + c)^3*arctan(a*x)), x)
```

Fricas [C] time = 1.61181, size = 435, normalized size = 12.43

$$\frac{i\,\mathrm{log\_integral}\left(\frac{a^4x^4+4i\,a^3x^3-6\,a^2x^2-4i\,ax+1}{a^4x^4+2\,a^2x^2+1}\right)-i\,\mathrm{log\_integral}\left(\frac{a^4x^4-4i\,a^3x^3-6\,a^2x^2+4i\,ax+1}{a^4x^4+2\,a^2x^2+1}\right)+2i\,\mathrm{log\_integral}\left(-\frac{a^2x^2+2i\,ax-1}{a^2x^2+1}\right)-2}{16\,a^2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="fricas")
```

```
[Out] 1/16*(I*log_integral((a^4*x^4 + 4*I*a^3*x^3 - 6*a^2*x^2 - 4*I*a*x + 1)/(a^4
*x^4 + 2*a^2*x^2 + 1)) - I*log_integral((a^4*x^4 - 4*I*a^3*x^3 - 6*a^2*x^2
+ 4*I*a*x + 1)/(a^4*x^4 + 2*a^2*x^2 + 1)) + 2*I*log_integral(-(a^2*x^2 + 2*
I*a*x - 1)/(a^2*x^2 + 1)) - 2*I*log_integral(-(a^2*x^2 - 2*I*a*x - 1)/(a^2*
x^2 + 1)))/(a^2*c^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x}{a^6x^6\operatorname{atan}\left(ax\right)+3a^4x^4\operatorname{atan}\left(ax\right)+3a^2x^2\operatorname{atan}\left(ax\right)+\operatorname{atan}\left(ax\right)}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a**2*c*x**2+c)**3/atan(a*x),x)
```

```
[Out] Integral(x/(a**6*x**6*atan(a*x) + 3*a**4*x**4*atan(a*x) + 3*a**2*x**2*atan(
a*x) + atan(a*x)), x)/c**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(a^2cx^2+c\right)^3 \arctan (ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="giac")
```

```
[Out] integrate(x/((a^2*c*x^2 + c)^3*arctan(a*x)), x)
```

3.493 $\int \frac{1}{\left(c+a^2cx^2\right)^3\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=50

$$\frac{\text{CosIntegral}\left(2\tan^{-1}(ax)\right)}{2ac^3}+\frac{\text{CosIntegral}\left(4\tan^{-1}(ax)\right)}{8ac^3}+\frac{3\log\left(\tan^{-1}(ax)\right)}{8ac^3}$$

```
[Out] CosIntegral[2*ArcTan[a*x]]/(2*a*c^3) + CosIntegral[4*ArcTan[a*x]]/(8*a*c^3)
 + (3*Log[ArcTan[a*x]])/(8*a*c^3)
```

Rubi [A] time = 0.0873405, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {4904, 3312, 3302}

$$\frac{\text{CosIntegral}\left(2\tan^{-1}(ax)\right)}{2ac^3}+\frac{\text{CosIntegral}\left(4\tan^{-1}(ax)\right)}{8ac^3}+\frac{3\log\left(\tan^{-1}(ax)\right)}{8ac^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a^2*c*x^2)^3*ArcTan[a*x]),x]
```

```
[Out] CosIntegral[2*ArcTan[a*x]]/(2*a*c^3) + CosIntegral[4*ArcTan[a*x]]/(8*a*c^3)
 + (3*Log[ArcTan[a*x]])/(8*a*c^3)
```

Rule 4904

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
```

```
c*f, 0]
```

Rubi steps

$$\int \frac{1}{\left(c+a^2cx^2\right)^3\tan^{-1}(ax)}\,dx = \frac{\text{Subst}\left(\int \frac{\cos^4(x)}{x}\,dx, x, \tan^{-1}(ax)\right)}{ac^3}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{3}{8x}+\frac{\cos(2x)}{2x}+\frac{\cos(4x)}{8x}\right)dx, x, \tan^{-1}(ax)\right)}{ac^3}$$
$$= \frac{3\log\left(\tan^{-1}(ax)\right)}{8ac^3}+\frac{\text{Subst}\left(\int \frac{\cos(4x)}{x}\,dx, x, \tan^{-1}(ax)\right)}{8ac^3}+\frac{\text{Subst}\left(\int \frac{\cos(2x)}{x}\,dx, x, \tan^{-1}(\right.}{2ac^3}$$
$$= \frac{\text{Ci}\left(2\tan^{-1}(ax)\right)}{2ac^3}+\frac{\text{Ci}\left(4\tan^{-1}(ax)\right)}{8ac^3}+\frac{3\log\left(\tan^{-1}(ax)\right)}{8ac^3}$$

Mathematica [A] time = 0.0298404, size = 34, normalized size = 0.68

$$\frac{4\text{CosIntegral}\left(2\tan^{-1}(ax)\right)+\text{CosIntegral}\left(4\tan^{-1}(ax)\right)+3\log\left(\tan^{-1}(ax)\right)}{8ac^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a^2*c*x^2)^3*ArcTan[a*x]),x]
```

```
[Out] (4*CosIntegral[2*ArcTan[a*x]] + CosIntegral[4*ArcTan[a*x]] + 3*Log[ArcTan[a
*x]])/(8*a*c^3)
```

Maple [A] time = 0.056, size = 45, normalized size = 0.9

$$\frac{\text{Ci}\left(2\arctan(ax)\right)}{2ac^3}+\frac{\text{Ci}\left(4\arctan(ax)\right)}{8ac^3}+\frac{3\ln\left(\arctan(ax)\right)}{8ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2*c*x^2+c)^3/arctan(a*x),x)
```

```
[Out] 1/2*Ci(2*arctan(a*x))/a/c^3+1/8*Ci(4*arctan(a*x))/a/c^3+3/8*ln(arctan(a*x))
/a/c^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)^3\arctan\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^3*arctan(a*x)), x)
```

Fricas [C] time = 1.73217, size = 450, normalized size = 9.

$$\frac{6\log\left(\arctan\left(ax\right)\right)+\log\_integral\left(\frac{a^4x^4+4i\,a^3x^3-6\,a^2x^2-4i\,ax+1}{a^4x^4+2\,a^2x^2+1}\right)+\log\_integral\left(\frac{a^4x^4-4i\,a^3x^3-6\,a^2x^2+4i\,ax+1}{a^4x^4+2\,a^2x^2+1}\right)+4\log\_integral}{16\,ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="fricas")
```

```
[Out] 1/16*(6*log(arctan(a*x)) + log_integral((a^4*x^4 + 4*I*a^3*x^3 - 6*a^2*x^2
- 4*I*a*x + 1)/(a^4*x^4 + 2*a^2*x^2 + 1)) + log_integral((a^4*x^4 - 4*I*a^3
*x^3 - 6*a^2*x^2 + 4*I*a*x + 1)/(a^4*x^4 + 2*a^2*x^2 + 1)) + 4*log_integral
(-(a^2*x^2 + 2*I*a*x - 1)/(a^2*x^2 + 1)) + 4*log_integral(-(a^2*x^2 - 2*I*a
*x - 1)/(a^2*x^2 + 1)))/(a*c^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{a^6x^6\operatorname{atan}\left(ax\right)+3a^4x^4\operatorname{atan}\left(ax\right)+3a^2x^2\operatorname{atan}\left(ax\right)+\operatorname{atan}\left(ax\right)}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*c*x**2+c)**3/atan(a*x),x)
```

```
[Out] Integral(1/(a**6*x**6*atan(a*x) + 3*a**4*x**4*atan(a*x) + 3*a**2*x**2*atan(
a*x) + atan(a*x)), x)/c**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)^3 \arctan\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^3*arctan(a*x)), x)
```

3.494 $\int \frac{1}{x\left(c+a^2cx^2\right)^3 \tan^{-1}(ax)} \, dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{1}{x\left(a^2cx^2+c\right)^3 \tan^{-1}(ax)},x\right)$$

```
[Out] Unintegrable[1/(x*(c + a^2*c*x^2)^3*ArcTan[a*x]), x]
```

Rubi [A] time = 0.064984, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x\left(c+a^2cx^2\right)^3 \tan^{-1}(ax)} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*(c + a^2*c*x^2)^3*ArcTan[a*x]),x]
```

```
[Out] Defer[Int][1/(x*(c + a^2*c*x^2)^3*ArcTan[a*x]), x]
```

Rubi steps

$$\int \frac{1}{x\left(c+a^2cx^2\right)^3 \tan^{-1}(ax)} \, dx = \int \frac{1}{x\left(c+a^2cx^2\right)^3 \tan^{-1}(ax)} \, dx$$

Mathematica [A] time = 0.877712, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(c+a^2cx^2\right)^3 \tan^{-1}(ax)} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*(c + a^2*c*x^2)^3*ArcTan[a*x]),x]
```

```
[Out] Integrate[1/(x*(c + a^2*c*x^2)^3*ArcTan[a*x]), x]
```

Maple [A] time = 0.323, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(a^2cx^2+c\right)^3\arctan\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a^2*c*x^2+c)^3/arctan(a*x),x)
```

```
[Out] int(1/x/(a^2*c*x^2+c)^3/arctan(a*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)^3 x\arctan\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^3*x*arctan(a*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\left(a^6c^3x^7+3\,a^4c^3x^5+3\,a^2c^3x^3+c^3x\right)\arctan\left(ax\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral(1/((a^6*c^3*x^7 + 3*a^4*c^3*x^5 + 3*a^2*c^3*x^3 + c^3*x)*arctan(a*
x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{a^6x^7\operatorname{atan}(ax)+3a^4x^5\operatorname{atan}(ax)+3a^2x^3\operatorname{atan}(ax)+x\operatorname{atan}(ax)}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a**2*c*x**2+c)**3/atan(a*x),x)
```

```
[Out] Integral(1/(a**6*x**7*atan(a*x) + 3*a**4*x**5*atan(a*x) + 3*a**2*x**3*atan(
a*x) + x*atan(a*x)), x)/c**3
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)^3x\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^3*x*arctan(a*x)), x)
```

3.495 $\int \frac{1}{x^2(c+a^2cx^2)^3\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{1}{x^2\left(a^2cx^2+c\right)^3\tan^{-1}(ax)},x\right)$$

```
[Out] Unintegrable[1/(x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]), x]
```

Rubi [A] time = 0.0779618, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^3\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]),x]

[Out] Defer[Int][1/(x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]), x]
```

Rubi steps

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^3\tan^{-1}(ax)}\,dx=\int \frac{1}{x^2\left(c+a^2cx^2\right)^3\tan^{-1}(ax)}\,dx$$

Mathematica [A] time = 1.27168, size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^3\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]),x]
```

```
[Out] Integrate[1/(x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]), x]
```

Maple [A] time = 0.282, size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \left(a^2 c x^2 + c\right)^3 \arctan\left(a x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a^2*c*x^2+c)^3/arctan(a*x),x)
```

```
[Out] int(1/x^2/(a^2*c*x^2+c)^3/arctan(a*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2 c x^2 + c\right)^3 x^2 \arctan\left(a x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^3*x^2*arctan(a*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\left(a^6 c^3 x^8 + 3\, a^4 c^3 x^6 + 3\, a^2 c^3 x^4 + c^3 x^2\right) \arctan\left(a x\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral(1/((a^6*c^3*x^8 + 3*a^4*c^3*x^6 + 3*a^2*c^3*x^4 + c^3*x^2)*arctan(
a*x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{a^6x^8 \operatorname{atan}(ax)+3a^4x^6 \operatorname{atan}(ax)+3a^2x^4 \operatorname{atan}(ax)+x^2 \operatorname{atan}(ax)}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a**2*c*x**2+c)**3/atan(a*x),x)
```

```
[Out] Integral(1/(a**6*x**8*atan(a*x) + 3*a**4*x**6*atan(a*x) + 3*a**2*x**4*atan(
a*x) + x**2*atan(a*x)), x)/c**3
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a^2cx^2+c)^3 x^2 \arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^3*x^2*arctan(a*x)), x)
```

3.496 $\int \frac{x\sqrt{c+a^2cx^2}}{\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{x\sqrt{a^2cx^2+c}}{\tan^{-1}(ax)},x\right)$$

```
[Out] Unintegrable[(x*Sqrt[c + a^2*c*x^2])/ArcTan[a*x], x]
```

Rubi [A] time = 0.0738188, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x\sqrt{c+a^2cx^2}}{\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x*Sqrt[c + a^2*c*x^2])/ArcTan[a*x],x]
```

```
[Out] Defer[Int][(x*Sqrt[c + a^2*c*x^2])/ArcTan[a*x], x]
```

Rubi steps

$$\int \frac{x\sqrt{c+a^2cx^2}}{\tan^{-1}(ax)}\,dx=\int \frac{x\sqrt{c+a^2cx^2}}{\tan^{-1}(ax)}\,dx$$

Mathematica [A] time = 2.12628, size = 0, normalized size = 0.

$$\int \frac{x\sqrt{c+a^2cx^2}}{\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x*Sqrt[c + a^2*c*x^2])/ArcTan[a*x],x]
```

```
[Out] Integrate[(x*Sqrt[c + a^2*c*x^2])/ArcTan[a*x], x]
```

Maple [A] time = 0.635, size = 0, normalized size = 0.

$$\int \frac{x}{\arctan(ax)}\sqrt{a^2cx^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x),x)
```

```
[Out] int(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a^2cx^2+c}x}{\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)*x/arctan(a*x), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{a^2cx^2+c}x}{\arctan(ax)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x/arctan(a*x), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sqrt{c\left(a^2x^2+1\right)}}{\operatorname{atan}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**(1/2)/atan(a*x),x)
```

```
[Out] Integral(x*sqrt(c*(a**2*x**2 + 1))/atan(a*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a^2cx^2+c}x}{\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)*x/arctan(a*x), x)
```

3.497 $\int \frac{\sqrt{c+a^2 c x^2}}{\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=23

$$\text{Unintegrable}\left(\frac{\sqrt{a^2 c x^2+c}}{\tan^{-1}(ax)},x\right)$$

```
[Out] Unintegrable[Sqrt[c + a^2*c*x^2]/ArcTan[a*x], x]
```

Rubi [A] time = 0.038524, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sqrt{c+a^2 c x^2}}{\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[c + a^2*c*x^2]/ArcTan[a*x],x]

[Out] Defer[Int][Sqrt[c + a^2*c*x^2]/ArcTan[a*x], x]
```

Rubi steps

$$\int \frac{\sqrt{c+a^2 c x^2}}{\tan^{-1}(ax)}\,dx=\int \frac{\sqrt{c+a^2 c x^2}}{\tan^{-1}(ax)}\,dx$$

Mathematica [A] time = 0.261209, size = 0, normalized size = 0.

$$\int \frac{\sqrt{c+a^2 c x^2}}{\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[c + a^2*c*x^2]/ArcTan[a*x],x]
```

```
[Out] Integrate[Sqrt[c + a^2*c*x^2]/ArcTan[a*x], x]
```

Maple [A] time = 0.638, size = 0, normalized size = 0.

$$\int \frac{1}{\arctan\left(ax\right)}\sqrt{a^{2}c\,x^{2}+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(1/2)/arctan(a*x),x)
```

```
[Out] int((a^2*c*x^2+c)^(1/2)/arctan(a*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a^{2}c\,x^{2}+c}}{\arctan\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)/arctan(a*x), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{a^{2}c\,x^{2}+c}}{\arctan\left(ax\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)/arctan(a*x), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c\left(a^2x^2+1\right)}}{\operatorname{atan}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(1/2)/atan(a*x),x)
```

```
[Out] Integral(sqrt(c*(a**2*x**2 + 1))/atan(a*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a^2cx^2+c}}{\arctan\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/arctan(a*x),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)/arctan(a*x), x)
```

3.498 $\int \frac{\sqrt{c+a^2cx^2}}{x\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{\sqrt{a^2cx^2+c}}{x\tan^{-1}(ax)},x\right)$$

```
[Out] Unintegrable[Sqrt[c + a^2*c*x^2]/(x*ArcTan[a*x]), x]
```

Rubi [A] time = 0.104632, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sqrt{c+a^2cx^2}}{x\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[c + a^2*c*x^2]/(x*ArcTan[a*x]),x]
```

```
[Out] Defer[Int][Sqrt[c + a^2*c*x^2]/(x*ArcTan[a*x]), x]
```

Rubi steps

$$\int \frac{\sqrt{c+a^2cx^2}}{x\tan^{-1}(ax)}\,dx = \int \frac{\sqrt{c+a^2cx^2}}{x\tan^{-1}(ax)}\,dx$$

Mathematica [A] time = 1.18985, size = 0, normalized size = 0.

$$\int \frac{\sqrt{c+a^2cx^2}}{x\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[c + a^2*c*x^2]/(x*ArcTan[a*x]),x]
```

```
[Out] Integrate[Sqrt[c + a^2*c*x^2]/(x*ArcTan[a*x]), x]
```

Maple [A] time = 0.695, size = 0, normalized size = 0.

$$\int \frac{1}{x \arctan (ax)} \sqrt{a^2cx^2 + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(1/2)/x/arctan(a*x),x)
```

```
[Out] int((a^2*c*x^2+c)^(1/2)/x/arctan(a*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a^2cx^2 + c}}{x \arctan (ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/x/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)/(x*arctan(a*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a^2cx^2 + c}}{x \arctan (ax)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/x/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)/(x*arctan(a*x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c\left(a^2x^2+1\right)}}{x\operatorname{atan}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(1/2)/x/atan(a*x),x)
```

```
[Out] Integral(sqrt(c*(a**2*x**2 + 1))/(x*atan(a*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a^2cx^2+c}}{x\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/x/arctan(a*x),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)/(x*arctan(a*x)), x)
```

3.499 $\int \frac{x\left(c+a^2cx^2\right)^{3/2}}{\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{x\left(a^2cx^2+c\right)^{3/2}}{\tan^{-1}(ax)},x\right)$$

```
[Out] Unintegrable[(x*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x], x]
```

Rubi [A] time = 0.0850088, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x\left(c+a^2cx^2\right)^{3/2}}{\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x],x]

[Out] Defer[Int][(x*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x], x]
```

Rubi steps

$$\int \frac{x\left(c+a^2cx^2\right)^{3/2}}{\tan^{-1}(ax)}\,dx=\int \frac{x\left(c+a^2cx^2\right)^{3/2}}{\tan^{-1}(ax)}\,dx$$

Mathematica [A] time = 2.21097, size = 0, normalized size = 0.

$$\int \frac{x\left(c+a^2cx^2\right)^{3/2}}{\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x],x]
```

```
[Out] Integrate[(x*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x], x]
```

Maple [A] time = 0.599, size = 0, normalized size = 0.

$$\int \frac{x}{\arctan\left(ax\right)}\left(a^{2}c\,x^{2}+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x),x)
```

```
[Out] int(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^{2}c x^{2}+c\right)^{\frac{3}{2}} x}{\arctan\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)*x/arctan(a*x), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(a^{2}c x^{3}+c x\right)\sqrt{a^{2}c x^{2}+c}}{\arctan\left(ax\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^3 + c*x)*sqrt(a^2*c*x^2 + c)/arctan(a*x), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}}{\operatorname{atan}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**(3/2)/atan(a*x),x)
```

```
[Out] Integral(x*(c*(a**2*x**2 + 1))**(3/2)/atan(a*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^{\frac{3}{2}}x}{\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)*x/arctan(a*x), x)
```

3.500 $\int \frac{(c+a^2cx^2)^{3/2}}{\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=23

$$\text{Unintegrable}\left(\frac{(a^2cx^2+c)^{3/2}}{\tan^{-1}(ax)},x\right)$$

```
[Out] Unintegrable[(c + a^2*c*x^2)^(3/2)/ArcTan[a*x], x]
```

Rubi [A] time = 0.0439783, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(c+a^2cx^2)^{3/2}}{\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^(3/2)/ArcTan[a*x],x]
```

```
[Out] Defer[Int][(c + a^2*c*x^2)^(3/2)/ArcTan[a*x], x]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^{3/2}}{\tan^{-1}(ax)}\,dx = \int \frac{(c+a^2cx^2)^{3/2}}{\tan^{-1}(ax)}\,dx$$

Mathematica [A] time = 0.344992, size = 0, normalized size = 0.

$$\int \frac{(c+a^2cx^2)^{3/2}}{\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^(3/2)/ArcTan[a*x],x]
```

```
[Out] Integrate[(c + a^2*c*x^2)^(3/2)/ArcTan[a*x], x]
```

Maple [A] time = 0.575, size = 0, normalized size = 0.

$$\int \frac{1}{\arctan(ax)}\left(a^2cx^2+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(3/2)/arctan(a*x),x)
```

```
[Out] int((a^2*c*x^2+c)^(3/2)/arctan(a*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^{\frac{3}{2}}}{\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)/arctan(a*x), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(a^2cx^2+c\right)^{\frac{3}{2}}}{\arctan(ax)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)^(3/2)/arctan(a*x), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}}{\operatorname{atan}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(3/2)/atan(a*x),x)

[Out] Integral((c*(a**2*x**2 + 1))**(3/2)/atan(a*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^{\frac{3}{2}}}{\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="giac")

[Out] integrate((a^2*c*x^2 + c)^(3/2)/arctan(a*x), x)
```

3.501 $\int \frac{(c+a^2cx^2)^{3/2}}{x\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{(a^2cx^2+c)^{3/2}}{x\tan^{-1}(ax)},x\right)$$

```
[Out] Unintegrable[(c + a^2*c*x^2)^(3/2)/(x*ArcTan[a*x]), x]
```

Rubi [A] time = 0.12855, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^(3/2)/(x*ArcTan[a*x]),x]

[Out] Defer[Int][(c + a^2*c*x^2)^(3/2)/(x*ArcTan[a*x]), x]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\tan^{-1}(ax)}\,dx = \int \frac{(c+a^2cx^2)^{3/2}}{x\tan^{-1}(ax)}\,dx$$

Mathematica [A] time = 1.37388, size = 0, normalized size = 0.

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^(3/2)/(x*ArcTan[a*x]),x]
```

```
[Out] Integrate[(c + a^2*c*x^2)^(3/2)/(x*ArcTan[a*x]), x]
```

Maple [A] time = 0.671, size = 0, normalized size = 0.

$$\int \frac{1}{x \arctan(ax)} \left(a^2 c x^2 + c\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(3/2)/x/arctan(a*x),x)
```

```
[Out] int((a^2*c*x^2+c)^(3/2)/x/arctan(a*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2 c x^2 + c\right)^{\frac{3}{2}}}{x \arctan(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/x/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)/(x*arctan(a*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^2 c x^2 + c\right)^{\frac{3}{2}}}{x \arctan(ax)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/x/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)^(3/2)/(x*arctan(a*x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}}{x\,\mathrm{atan}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(3/2)/x/atan(a*x),x)
```

```
[Out] Integral((c*(a**2*x**2 + 1))**(3/2)/(x*atan(a*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^{\frac{3}{2}}}{x\arctan\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/x/arctan(a*x),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)/(x*arctan(a*x)), x)
```

3.502 $\int \frac{x(c+a^2cx^2)^{5/2}}{\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{x\left(a^2cx^2+c\right)^{5/2}}{\tan^{-1}(ax)},x\right)$$

```
[Out] Unintegrable[(x*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x], x]
```

Rubi [A] time = 0.0945742, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x\left(c+a^2cx^2\right)^{5/2}}{\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x],x]
```

```
[Out] Defer[Int][(x*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x], x]
```

Rubi steps

$$\int \frac{x\left(c+a^2cx^2\right)^{5/2}}{\tan^{-1}(ax)}\,dx=\int \frac{x\left(c+a^2cx^2\right)^{5/2}}{\tan^{-1}(ax)}\,dx$$

Mathematica [A] time = 2.27153, size = 0, normalized size = 0.

$$\int \frac{x\left(c+a^2cx^2\right)^{5/2}}{\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x],x]
```

```
[Out] Integrate[(x*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x], x]
```

Maple [A] time = 0.72, size = 0, normalized size = 0.

$$\int \frac{x}{\arctan \left(a x\right)}\left(a^{2} c \,x^{2}+c \right)^{\frac{5}{2}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x),x)
```

```
[Out] int(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{{\left(a^{2} c x^{2} + c\right)}^{\frac{5}{2}} x}{\arctan\left(a x\right)}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(5/2)*x/arctan(a*x), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{{\left(a^{4} c^{2} x^{5} + 2 \, a^{2} c^{2} x^{3} + c^{2} x\right)} \sqrt{a^{2} c x^{2} + c}}{\arctan\left(a x\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^5 + 2*a^2*c^2*x^3 + c^2*x)*sqrt(a^2*c*x^2 + c)/arctan(a
*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**(5/2)/atan(a*x),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^{\frac{5}{2}}x}{\arctan\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(5/2)*x/arctan(a*x), x)
```

3.503 $\int \frac{(c+a^2cx^2)^{5/2}}{\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=23

$$\text{Unintegrable}\left(\frac{(a^2cx^2+c)^{5/2}}{\tan^{-1}(ax)},x\right)$$

```
[Out] Unintegrable[(c + a^2*c*x^2)^(5/2)/ArcTan[a*x], x]
```

Rubi [A] time = 0.0680472, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{(c+a^2cx^2)^{5/2}}{\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^(5/2)/ArcTan[a*x],x]
```

```
[Out] Defer[Int][(c + a^2*c*x^2)^(5/2)/ArcTan[a*x], x]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^{5/2}}{\tan^{-1}(ax)}\,dx = \int \frac{(c+a^2cx^2)^{5/2}}{\tan^{-1}(ax)}\,dx$$

Mathematica [A] time = 0.425774, size = 0, normalized size = 0.

$$\int \frac{(c+a^2cx^2)^{5/2}}{\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^(5/2)/ArcTan[a*x],x]
```

```
[Out] Integrate[(c + a^2*c*x^2)^(5/2)/ArcTan[a*x], x]
```

Maple [A] time = 0.766, size = 0, normalized size = 0.

$$\int \frac{1}{\arctan (ax)} \left(a^2cx^2+c\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(5/2)/arctan(a*x),x)
```

```
[Out] int((a^2*c*x^2+c)^(5/2)/arctan(a*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^{\frac{5}{2}}}{\arctan (ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(5/2)/arctan(a*x), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(a^4c^2x^4+2\,a^2c^2x^2+c^2\right)\sqrt{a^2cx^2+c}}{\arctan (ax)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)/arctan(a*x
), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}}{\operatorname{atan}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(5/2)/atan(a*x),x)
```

```
[Out] Integral((c*(a**2*x**2 + 1))**(5/2)/atan(a*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^{\frac{5}{2}}}{\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(5/2)/arctan(a*x), x)
```

3.504 $\int \frac{(c+a^2cx^2)^{5/2}}{x\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{(a^2cx^2+c)^{5/2}}{x\tan^{-1}(ax)},x\right)$$

```
[Out] Unintegrable[(c + a^2*c*x^2)^(5/2)/(x*ArcTan[a*x]), x]
```

Rubi [A] time = 0.13509, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(c+a^2cx^2)^{5/2}}{x\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^(5/2)/(x*ArcTan[a*x]),x]

[Out] Defer[Int][(c + a^2*c*x^2)^(5/2)/(x*ArcTan[a*x]), x]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^{5/2}}{x\tan^{-1}(ax)}\,dx = \int \frac{(c+a^2cx^2)^{5/2}}{x\tan^{-1}(ax)}\,dx$$

Mathematica [A] time = 1.43421, size = 0, normalized size = 0.

$$\int \frac{(c+a^2cx^2)^{5/2}}{x\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^(5/2)/(x*ArcTan[a*x]),x]
```

```
[Out] Integrate[(c + a^2*c*x^2)^(5/2)/(x*ArcTan[a*x]), x]
```

Maple [A] time = 0.729, size = 0, normalized size = 0.

$$\int \frac{1}{x \arctan(ax)} \left(a^2cx^2+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(5/2)/x/arctan(a*x),x)
```

```
[Out] int((a^2*c*x^2+c)^(5/2)/x/arctan(a*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^{\frac{5}{2}}}{x \arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)/x/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(5/2)/(x*arctan(a*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(a^4c^2x^4+2\,a^2c^2x^2+c^2\right)\sqrt{a^2cx^2+c}}{x \arctan(ax)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)/x/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)/(x*arctan(
a*x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}}{x\,\mathrm{atan}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(5/2)/x/atan(a*x),x)
```

```
[Out] Integral((c*(a**2*x**2 + 1))**(5/2)/(x*atan(a*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^{\frac{5}{2}}}{x\arctan\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)/x/arctan(a*x),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(5/2)/(x*arctan(a*x)), x)
```

3.505 $\int \frac{x}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{x}{\sqrt{a^2cx^2+c}\tan^{-1}(ax)},x\right)$$

```
[Out] Unintegrable[x/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]), x]
```

Rubi [A] time = 0.0815964, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]),x]
```

```
[Out] Defer[Int][x/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]), x]
```

Rubi steps

$$\int \frac{x}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}\,dx=\int \frac{x}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}\,dx$$

Mathematica [A] time = 0.795351, size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]),x]
```

```
[Out] Integrate[x/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]), x]
```

Maple [A] time = 0.596, size = 0, normalized size = 0.

$$\int \frac{x}{\arctan(ax)}\frac{1}{\sqrt{a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arctan(a*x)/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(x/arctan(a*x)/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{a^2cx^2+c}\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctan(a*x)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(a^2*c*x^2 + c)*arctan(a*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x}{\sqrt{a^2cx^2+c}\arctan(ax)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctan(a*x)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x/(sqrt(a^2*c*x^2 + c)*arctan(a*x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{c\left(a^2x^2+1\right)}\operatorname{atan}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/atan(a*x)/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(c*(a**2*x**2 + 1))*atan(a*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{a^2cx^2+c}\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctan(a*x)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/(sqrt(a^2*c*x^2 + c)*arctan(a*x)), x)
```

3.506 $\int \frac{1}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=23

$$\text{Unintegrable}\left(\frac{1}{\sqrt{a^2cx^2+c}\tan^{-1}(ax)},x\right)$$

```
[Out] Unintegrable[1/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]), x]
```

Rubi [A] time = 0.04321, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]),x]
```

```
[Out] Defer[Int][1/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]), x]
```

Rubi steps

$$\int \frac{1}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}\,dx=\int \frac{1}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}\,dx$$

Mathematica [A] time = 0.201257, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]),x]
```

```
[Out] Integrate[1/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]), x]
```

Maple [A] time = 0.483, size = 0, normalized size = 0.

$$\int \frac{1}{\arctan(ax)}\frac{1}{\sqrt{a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arctan(a*x)/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(1/arctan(a*x)/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a^2cx^2+c}\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(a*x)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a^2*c*x^2 + c)*arctan(a*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\sqrt{a^2cx^2+c}\arctan(ax)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(a*x)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/(sqrt(a^2*c*x^2 + c)*arctan(a*x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{c\left(a^2x^2+1\right)}\,\mathrm{atan}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/atan(a*x)/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(c*(a**2*x**2 + 1))*atan(a*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a^2cx^2+c}\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(a*x)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a^2*c*x^2 + c)*arctan(a*x)), x)
```

3.507 $\int \frac{1}{x\sqrt{c+a^2cx^2}\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{1}{x\sqrt{a^2cx^2+c}\tan^{-1}(ax)},x\right)$$

```
[Out] Unintegrable[1/(x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]), x]
```

Rubi [A] time = 0.113743, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]),x]
```

```
[Out] Defer[Int][1/(x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]), x]
```

Rubi steps

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\tan^{-1}(ax)}\,dx=\int \frac{1}{x\sqrt{c+a^2cx^2}\tan^{-1}(ax)}\,dx$$

Mathematica [A] time = 0.581009, size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]),x]
```

```
[Out] Integrate[1/(x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]), x]
```

Maple [A] time = 0.538, size = 0, normalized size = 0.

$$\int \frac{1}{x \arctan(ax)} \frac{1}{\sqrt{a^2 c x^2 + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/arctan(a*x)/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(1/x/arctan(a*x)/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a^2 c x^2 + c}\, x \arctan(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctan(a*x)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a^2*c*x^2 + c)*x*arctan(a*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{a^2 c x^2 + c}}{\left(a^2 c x^3 + c x\right) \arctan(ax)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctan(a*x)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)/((a^2*c*x^3 + c*x)*arctan(a*x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{c\left(a^2x^2+1\right)}\,\operatorname{atan}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/atan(a*x)/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(c*(a**2*x**2 + 1))*atan(a*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a^2cx^2+c}\,x\arctan\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctan(a*x)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a^2*c*x^2 + c)*x*arctan(a*x)), x)
```

3.508 $\int \frac{x^3}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{x^3}{\left(a^2cx^2+c\right)^{3/2}\tan^{-1}(ax)},x\right)$$

```
[Out] Unintegrable[x^3/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]), x]
```

Rubi [A] time = 0.132956, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^3}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^3/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]),x]
```

```
[Out] Defer[Int][x^3/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]), x]
```

Rubi steps

$$\int \frac{x^3}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}\,dx=\int \frac{x^3}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}\,dx$$

Mathematica [A] time = 4.75509, size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^3/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]),x]
```

```
[Out] Integrate[x^3/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]), x]
```

Maple [A] time = 1.117, size = 0, normalized size = 0.

$$\int \frac{x^3}{\arctan(ax)}\left(a^2cx^2+c\right)^{-\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x)
```

```
[Out] int(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(a^2cx^2+c\right)^{\frac{3}{2}}\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x^3/((a^2*c*x^2 + c)^(3/2)*arctan(a*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{a^2cx^2+c}\,x^3}{\left(a^4c^2x^4+2\,a^2c^2x^2+c^2\right)\arctan(ax)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^3/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arcta
n(a*x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}} \operatorname{atan}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a**2*c*x**2+c)**(3/2)/atan(a*x),x)
```

```
[Out] Integral(x**3/((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(a^2cx^2+c\right)^{\frac{3}{2}} \arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="giac")
```

```
[Out] integrate(x^3/((a^2*c*x^2 + c)^(3/2)*arctan(a*x)), x)
```

3.509 $\int \frac{x^2}{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{x^2}{(a^2cx^2+c)^{3/2}\tan^{-1}(ax)},x\right)$$

```
[Out] Unintegrable[x^2/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]), x]
```

Rubi [A] time = 0.131173, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^2}{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^2/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]),x]
```

```
[Out] Defer[Int][x^2/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]), x]
```

Rubi steps

$$\int \frac{x^2}{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)}\,dx = \int \frac{x^2}{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)}\,dx$$

Mathematica [F] time = 180.002, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[x^2/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]),x]
```

```
[Out] $Aborted
```

Maple [A] time = 0.847, size = 0, normalized size = 0.

$$\int \frac{x^2}{\arctan(ax)} \left(a^2 c x^2 + c\right)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x)
```

```
[Out] int(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(a^2 c x^2 + c\right)^{\frac{3}{2}} \arctan(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((a^2*c*x^2 + c)^(3/2)*arctan(a*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a^2 c x^2 + c}\, x^2}{\left(a^4 c^2 x^4 + 2\, a^2 c^2 x^2 + c^2\right) \arctan(ax)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^2/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arcta
n(a*x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\operatorname{atan}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a**2*c*x**2+c)**(3/2)/atan(a*x),x)
```

```
[Out] Integral(x**2/((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(a^2cx^2+c\right)^{\frac{3}{2}}\arctan\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="giac")
```

```
[Out] integrate(x^2/((a^2*c*x^2 + c)^(3/2)*arctan(a*x)), x)
```

3.510 $\int \frac{x}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=39

$$\frac{\sqrt{a^2x^2+1}\,\mathrm{Si}\left(\tan^{-1}(ax)\right)}{a^2c\sqrt{a^2cx^2+c}}$$

```
[Out] (Sqrt[1 + a^2*x^2]*SinIntegral[ArcTan[a*x]])/(a^2*c*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.168539, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {4971, 4970, 3299}

$$\frac{\sqrt{a^2x^2+1}\,\mathrm{Si}\left(\tan^{-1}(ax)\right)}{a^2c\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]),x]
```

```
[Out] (Sqrt[1 + a^2*x^2]*SinIntegral[ArcTan[a*x]])/(a^2*c*Sqrt[c + a^2*c*x^2])
```

Rule 4971

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[(d^(q + 1/2)*Sqrt[1 + c^2*x^2])/Sqrt[d + e*x^2],
 Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d
, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(In
tegerQ[q] || GtQ[d, 0])
```

Rule 4970

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sin[x]^m)/
Cos[x]^(m + 2*(q + 1)), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\int \frac{x}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}\,dx = \frac{\sqrt{1+a^2x^2}\int \frac{x}{\left(1+a^2x^2\right)^{3/2}\tan^{-1}(ax)}\,dx}{c\sqrt{c+a^2cx^2}}$$

$$= \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \frac{\sin(x)}{x}\,dx, x, \tan^{-1}(ax)\right)}{a^2c\sqrt{c+a^2cx^2}}$$

$$= \frac{\sqrt{1+a^2x^2}\,\text{Si}\left(\tan^{-1}(ax)\right)}{a^2c\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 0.11586, size = 37, normalized size = 0.95

$$\frac{\left(a^2x^2+1\right)^{3/2}\,\text{Si}\left(\tan^{-1}(ax)\right)}{a^2\left(c\left(a^2x^2+1\right)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]),x]
```

```
[Out] ((1 + a^2*x^2)^(3/2)*SinIntegral[ArcTan[a*x]])/(a^2*(c*(1 + a^2*x^2))^(3/2)
)
```

Maple [C] time = 0.296, size = 82, normalized size = 2.1

$$-\frac{\text{csgn}\left(\arctan(ax)\right)\pi}{2c^2a^2}\sqrt{c\left(ax-i\right)\left(ax+i\right)}\frac{1}{\sqrt{a^2x^2+1}}+\frac{\text{Si}\left(\arctan(ax)\right)}{c^2a^2}\sqrt{c\left(ax-i\right)\left(ax+i\right)}\frac{1}{\sqrt{a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x)
```

```
[Out] -1/2*csgn(arctan(a*x))*Pi/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(a*x+I))^(1/2)/c^2/a
^2+Si(arctan(a*x))/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(a*x+I))^(1/2)/c^2/a^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(a^2cx^2 + c\right)^{\frac{3}{2}} \arctan\left(ax\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x/((a^2*c*x^2 + c)^(3/2)*arctan(a*x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{a^2cx^2 + c}\,x}{\left(a^4c^2x^4 + 2\,a^2c^2x^2 + c^2\right)\arctan\left(ax\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(
a*x)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(c\left(a^2x^2 + 1\right)\right)^{\frac{3}{2}} \operatorname{atan}\left(ax\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a**2*c*x**2+c)**(3/2)/atan(a*x),x)
```

```
[Out] Integral(x/((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(a^2cx^2+c\right)^{\frac{3}{2}}\arctan\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="giac")
```

```
[Out] integrate(x/((a^2*c*x^2 + c)^(3/2)*arctan(a*x)), x)
```

3.511 $\int \frac{1}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=39

$$\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}\left(\tan^{-1}(ax)\right)}{ac\sqrt{a^2cx^2+c}}$$

```
[Out] (Sqrt[1 + a^2*x^2]*CosIntegral[ArcTan[a*x]])/(a*c*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.0904266, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4905, 4904, 3302}

$$\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}\left(\tan^{-1}(ax)\right)}{ac\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]),x]
```

```
[Out] (Sqrt[1 + a^2*x^2]*CosIntegral[ArcTan[a*x]])/(a*c*Sqrt[c + a^2*c*x^2])
```

Rule 4905

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[(d^(q + 1/2)*Sqrt[1 + c^2*x^2])/Sqrt[d + e*x^2], Int[(1 + c
^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

Rule 4904

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
```

```
c*f, 0]
```

Rubi steps

$$\int \frac{1}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}\,dx = \frac{\sqrt{1+a^2x^2}\int \frac{1}{\left(1+a^2x^2\right)^{3/2}\tan^{-1}(ax)}\,dx}{c\sqrt{c+a^2cx^2}}$$
$$= \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \frac{\cos(x)}{x}\,dx, x, \tan^{-1}(ax)\right)}{ac\sqrt{c+a^2cx^2}}$$
$$= \frac{\sqrt{1+a^2x^2}\,\text{Ci}\left(\tan^{-1}(ax)\right)}{ac\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 0.166972, size = 39, normalized size = 1.

$$\frac{\sqrt{a^2cx^2+c}\,\text{CosIntegral}\left(\tan^{-1}(ax)\right)}{ac^2\sqrt{a^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]),x]
```

```
[Out] (Sqrt[c + a^2*c*x^2]*CosIntegral[ArcTan[a*x]])/(a*c^2*Sqrt[1 + a^2*x^2])
```

Maple [C] time = 0.287, size = 136, normalized size = 3.5

$$\frac{-\frac{i}{2}\text{csgn}\left(\arctan(ax)\right)\text{csgn}\left(i\arctan(ax)\right)\pi}{c^2a}\sqrt{c\,(ax-i)\,(ax+i)}\frac{1}{\sqrt{a^2x^2+1}}+\frac{\frac{i}{2}\text{csgn}\left(i\arctan(ax)\right)\pi}{c^2a}\sqrt{c\,(ax-i)\,(ax+i)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x)
```

```
[Out] -1/2*I*csgn(arctan(a*x))*csgn(I*arctan(a*x))*Pi/(a^2*x^2+1)^(1/2)*(c*(a*x-I
)*(a*x+I))^(1/2)/c^2/a+1/2*I*csgn(I*arctan(a*x))*Pi/(a^2*x^2+1)^(1/2)*(c*(a
*x-I)*(a*x+I))^(1/2)/c^2/a+Ci(arctan(a*x))/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(a*
```

```
x+I))^(1/2)/c^2/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)^{\frac{3}{2}}\arctan\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^(3/2)*arctan(a*x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a^2cx^2+c}}{\left(a^4c^2x^4+2\,a^2c^2x^2+c^2\right)\arctan\left(ax\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(a*
x)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\operatorname{atan}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*c*x**2+c)**(3/2)/atan(a*x),x)
```

```
[Out] Integral(1/((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)^{\frac{3}{2}}\arctan\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^(3/2)*arctan(a*x)), x)
```

3.512 $\int \frac{1}{x(c+a^2cx^2)^{3/2}\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{1}{x\left(a^2cx^2+c\right)^{3/2}\tan^{-1}(ax)},x\right)$$

```
[Out] Unintegrable[1/(x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]), x]
```

Rubi [A] time = 0.125433, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]),x]
```

```
[Out] Defer[Int][1/(x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]), x]
```

Rubi steps

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}\,dx=\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}\,dx$$

Mathematica [A] time = 1.0901, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]),x]
```

```
[Out] Integrate[1/(x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]), x]
```

Maple [A] time = 0.456, size = 0, normalized size = 0.

$$\int \frac{1}{x \arctan (ax)} \left(a^2 c x^2 + c\right)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x)
```

```
[Out] int(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2 c x^2 + c\right)^{\frac{3}{2}} x \arctan (ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^(3/2)*x*arctan(a*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a^2 c x^2 + c}}{\left(a^4 c^2 x^5 + 2\, a^2 c^2 x^3 + c^2 x\right) \arctan (ax)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)/((a^4*c^2*x^5 + 2*a^2*c^2*x^3 + c^2*x)*arctan(
a*x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\operatorname{atan}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a**2*c*x**2+c)**(3/2)/atan(a*x),x)
```

```
[Out] Integral(1/(x*(c*(a**2*x**2 + 1))**(3/2)*atan(a*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)^{\frac{3}{2}}x\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^(3/2)*x*arctan(a*x)), x)
```

3.513 $\int \frac{1}{x^2\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{1}{x^2\left(a^2cx^2+c\right)^{3/2}\tan^{-1}(ax)},x\right)$$

```
[Out] Unintegrable[1/(x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]), x]
```

Rubi [A] time = 0.124562, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]),x]
```

```
[Out] Defer[Int][1/(x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]), x]
```

Rubi steps

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}\,dx=\int \frac{1}{x^2\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}\,dx$$

Mathematica [A] time = 1.01279, size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]),x]
```

```
[Out] Integrate[1/(x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]), x]
```

Maple [A] time = 0.493, size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \arctan(ax)} \left(a^2 c x^2 + c\right)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x)
```

```
[Out] int(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2 c x^2 + c\right)^{\frac{3}{2}} x^2 \arctan(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^(3/2)*x^2*arctan(a*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a^2 c x^2 + c}}{\left(a^4 c^2 x^6 + 2 a^2 c^2 x^4 + c^2 x^2\right) \arctan(ax)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)/((a^4*c^2*x^6 + 2*a^2*c^2*x^4 + c^2*x^2)*arcta
n(a*x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}} \operatorname{atan}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a**2*c*x**2+c)**(3/2)/atan(a*x),x)
```

```
[Out] Integral(1/(x**2*(c*(a**2*x**2 + 1))**(3/2)*atan(a*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)^{\frac{3}{2}} x^2 \arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^(3/2)*x^2*arctan(a*x)), x)
```

3.514 $\int \frac{x^5}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{x^5}{\left(a^2cx^2+c\right)^{5/2}\tan^{-1}(ax)},x\right)$$

```
[Out] Unintegrable[x^5/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]), x]
```

Rubi [A] time = 0.137514, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^5}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^5/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]),x]
```

```
[Out] Defer[Int][x^5/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]), x]
```

Rubi steps

$$\int \frac{x^5}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)}\,dx=\int \frac{x^5}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)}\,dx$$

Mathematica [A] time = 5.77527, size = 0, normalized size = 0.

$$\int \frac{x^5}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^5/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]),x]
```

```
[Out] Integrate[x^5/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]), x]
```

Maple [A] time = 1.595, size = 0, normalized size = 0.

$$\int \frac{x^5}{\arctan(ax)}\left(a^2cx^2+c\right)^{-\frac{5}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x)
```

```
[Out] int(x^5/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\left(a^2cx^2+c\right)^{\frac{5}{2}}\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x^5/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{a^2cx^2+c}\,x^5}{\left(a^6c^3x^6+3\,a^4c^3x^4+3\,a^2c^3x^2+c^3\right)\arctan(ax)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^5/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*
x^2 + c^3)*arctan(a*x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}\operatorname{atan}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(a**2*c*x**2+c)**(5/2)/atan(a*x),x)
```

```
[Out] Integral(x**5/((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\left(a^2cx^2+c\right)^{\frac{5}{2}}\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="giac")
```

```
[Out] integrate(x^5/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)), x)
```

3.515 $\int \frac{x^4}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{x^4}{\left(a^2cx^2+c\right)^{5/2}\tan^{-1}(ax)},x\right)$$

```
[Out] Unintegrable[x^4/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]), x]
```

Rubi [A] time = 0.138177, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^4}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^4/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]),x]
```

```
[Out] Defer[Int][x^4/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]), x]
```

Rubi steps

$$\int \frac{x^4}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)}\,dx=\int \frac{x^4}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)}\,dx$$

Mathematica [A] time = 20.0736, size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^4/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]),x]
```

```
[Out] Integrate[x^4/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]), x]
```

Maple [A] time = 0.915, size = 0, normalized size = 0.

$$\int \frac{x^4}{\arctan(ax)} \left(a^2cx^2+c\right)^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x)
```

```
[Out] int(x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(a^2cx^2+c\right)^{\frac{5}{2}} \arctan(ax)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x^4/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a^2cx^2+c}\,x^4}{\left(a^6c^3x^6+3\,a^4c^3x^4+3\,a^2c^3x^2+c^3\right)\arctan(ax)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^4/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*
x^2 + c^3)*arctan(a*x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}\operatorname{atan}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(a**2*c*x**2+c)**(5/2)/atan(a*x),x)
```

```
[Out] Integral(x**4/((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(a^2cx^2+c\right)^{\frac{5}{2}}\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="giac")
```

```
[Out] integrate(x^4/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)), x)
```

3.516 $\int \frac{x^3}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=87

$$\frac{3\sqrt{a^2x^2+1}\,\mathrm{Si}\left(\tan^{-1}(ax)\right)}{4a^4c^2\sqrt{a^2cx^2+c}}-\frac{\sqrt{a^2x^2+1}\,\mathrm{Si}\left(3\tan^{-1}(ax)\right)}{4a^4c^2\sqrt{a^2cx^2+c}}$$

```
[Out] (3*Sqrt[1 + a^2*x^2]*SinIntegral[ArcTan[a*x]])/(4*a^4*c^2*Sqrt[c + a^2*c*x^
2]) - (Sqrt[1 + a^2*x^2]*SinIntegral[3*ArcTan[a*x]])/(4*a^4*c^2*Sqrt[c + a^
2*c*x^2])
```

Rubi [A] time = 0.276325, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4971, 4970, 3312, 3299}

$$\frac{3\sqrt{a^2x^2+1}\,\mathrm{Si}\left(\tan^{-1}(ax)\right)}{4a^4c^2\sqrt{a^2cx^2+c}}-\frac{\sqrt{a^2x^2+1}\,\mathrm{Si}\left(3\tan^{-1}(ax)\right)}{4a^4c^2\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]),x]
```


```
[Out] (3*Sqrt[1 + a^2*x^2]*SinIntegral[ArcTan[a*x]])/(4*a^4*c^2*Sqrt[c + a^2*c*x^
2]) - (Sqrt[1 + a^2*x^2]*SinIntegral[3*ArcTan[a*x]])/(4*a^4*c^2*Sqrt[c + a^
2*c*x^2])
```


Rule 4971

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[(d^(q + 1/2)*Sqrt[1 + c^2*x^2])/Sqrt[d + e*x^2],
 Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d
, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(In
tegerQ[q] || GtQ[d, 0])
```

Rule 4970

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sin[x]^m)/
Cos[x]^(m + 2*(q + 1)), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
```

```
|| GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\int \frac{x^3}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)}\,dx = \frac{\sqrt{1+a^2x^2}\int \frac{x^3}{\left(1+a^2x^2\right)^{5/2}\tan^{-1}(ax)}\,dx}{c^2\sqrt{c+a^2cx^2}}$$

$$= \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \frac{\sin^3(x)}{x}\,dx, x, \tan^{-1}(ax)\right)}{a^4c^2\sqrt{c+a^2cx^2}}$$

$$= \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \left(\frac{3\sin(x)}{4x}-\frac{\sin(3x)}{4x}\right)dx, x, \tan^{-1}(ax)\right)}{a^4c^2\sqrt{c+a^2cx^2}}$$

$$= -\frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \frac{\sin(3x)}{x}\,dx, x, \tan^{-1}(ax)\right)}{4a^4c^2\sqrt{c+a^2cx^2}} + \frac{\left(3\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int \frac{\sin(x)}{x}\,dx, x, \text{t}\ldots}{4a^4c^2\sqrt{c+a^2cx^2}}$$

$$= \frac{3\sqrt{1+a^2x^2}\,\text{Si}\left(\tan^{-1}(ax)\right)}{4a^4c^2\sqrt{c+a^2cx^2}} - \frac{\sqrt{1+a^2x^2}\,\text{Si}\left(3\tan^{-1}(ax)\right)}{4a^4c^2\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 0.142239, size = 52, normalized size = 0.6

$$\frac{\left(a^2x^2+1\right)^{5/2}\left(3\text{Si}\left(\tan^{-1}(ax)\right)-\text{Si}\left(3\tan^{-1}(ax)\right)\right)}{4a^4\left(c\left(a^2x^2+1\right)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]),x]
```

```
[Out] ((1 + a^2*x^2)^(5/2)*(3*SinIntegral[ArcTan[a*x]] - SinIntegral[3*ArcTan[a*x
]]))/(4*a^4*(c*(1 + a^2*x^2))^(5/2))
```

Maple [C] time = 0.928, size = 125, normalized size = 1.4

$$-\frac{\operatorname{csgn}\left(\arctan\left(ax\right)\right)\pi}{4\,c^{3}a^{4}}\sqrt{c\left(ax-i\right)\left(ax+i\right)}\frac{1}{\sqrt{a^{2}x^{2}+1}}-\frac{\operatorname{Si}\left(3\,\arctan\left(ax\right)\right)}{4\,c^{3}a^{4}}\sqrt{c\left(ax-i\right)\left(ax+i\right)}\frac{1}{\sqrt{a^{2}x^{2}+1}}+\frac{3\operatorname{Si}\left(\arctan\left(a\right.\right.}{4\,c^{3}a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x)
```

```
[Out] -1/4*csgn(arctan(a*x))*Pi/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(a*x+I))^(1/2)/c^3/a
^4-1/4*Si(3*arctan(a*x))/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(a*x+I))^(1/2)/c^3/a^
4+3/4*Si(arctan(a*x))/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(a*x+I))^(1/2)/c^3/a^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{3}}{\left(a^{2}cx^{2}+c\right)^{\frac{5}{2}}\arctan\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x^3/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a^{2}cx^{2}+c}\,x^{3}}{\left(a^{6}c^{3}x^{6}+3\,a^{4}c^{3}x^{4}+3\,a^{2}c^{3}x^{2}+c^{3}\right)\arctan\left(ax\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^3/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*
x^2 + c^3)*arctan(a*x)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}} \operatorname{atan}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a**2*c*x**2+c)**(5/2)/atan(a*x),x)
```

```
[Out] Integral(x**3/((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(a^2cx^2+c\right)^{\frac{5}{2}} \arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="giac")
```

```
[Out] integrate(x^3/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)), x)
```

3.517 $\int \frac{x^2}{(c+a^2cx^2)^{5/2}\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=87

$$\frac{\sqrt{a^2x^2+1}\text{CosIntegral}\left(\tan^{-1}(ax)\right)}{4a^3c^2\sqrt{a^2cx^2+c}}-\frac{\sqrt{a^2x^2+1}\text{CosIntegral}\left(3\tan^{-1}(ax)\right)}{4a^3c^2\sqrt{a^2cx^2+c}}$$

```
[Out] (Sqrt[1 + a^2*x^2]*CosIntegral[ArcTan[a*x]])/(4*a^3*c^2*Sqrt[c + a^2*c*x^2]
) - (Sqrt[1 + a^2*x^2]*CosIntegral[3*ArcTan[a*x]])/(4*a^3*c^2*Sqrt[c + a^2*
c*x^2])
```

Rubi [A] time = 0.273019, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4971, 4970, 4406, 3302}

$$\frac{\sqrt{a^2x^2+1}\text{CosIntegral}\left(\tan^{-1}(ax)\right)}{4a^3c^2\sqrt{a^2cx^2+c}}-\frac{\sqrt{a^2x^2+1}\text{CosIntegral}\left(3\tan^{-1}(ax)\right)}{4a^3c^2\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]),x]
```

```
[Out] (Sqrt[1 + a^2*x^2]*CosIntegral[ArcTan[a*x]])/(4*a^3*c^2*Sqrt[c + a^2*c*x^2]
) - (Sqrt[1 + a^2*x^2]*CosIntegral[3*ArcTan[a*x]])/(4*a^3*c^2*Sqrt[c + a^2*
c*x^2])
```

Rule 4971

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[(d^(q + 1/2)*Sqrt[1 + c^2*x^2])/Sqrt[d + e*x^2],
 Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d
, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(In
tegerQ[q] || GtQ[d, 0])
```

Rule 4970

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sin[x]^m)/
Cos[x]^(m + 2*(q + 1)), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
```

```
|| GtQ[d, 0])
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)}\,dx &= \frac{\sqrt{1+a^2x^2}\int \frac{x^2}{\left(1+a^2x^2\right)^{5/2}\tan^{-1}(ax)}\,dx}{c^2\sqrt{c+a^2cx^2}}\\
&= \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \frac{\cos(x)\sin^2(x)}{x}\,dx,x,\tan^{-1}(ax)\right)}{a^3c^2\sqrt{c+a^2cx^2}}\\
&= \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \left(\frac{\cos(x)}{4x}-\frac{\cos(3x)}{4x}\right)dx,x,\tan^{-1}(ax)\right)}{a^3c^2\sqrt{c+a^2cx^2}}\\
&= \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \frac{\cos(x)}{x}\,dx,x,\tan^{-1}(ax)\right)}{4a^3c^2\sqrt{c+a^2cx^2}}-\frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \frac{\cos(3x)}{x}\,dx,x,\tan^{-1}\right.}{4a^3c^2\sqrt{c+a^2cx^2}}\\
&= \frac{\sqrt{1+a^2x^2}\,\text{Ci}\left(\tan^{-1}(ax)\right)}{4a^3c^2\sqrt{c+a^2cx^2}}-\frac{\sqrt{1+a^2x^2}\,\text{Ci}\left(3\tan^{-1}(ax)\right)}{4a^3c^2\sqrt{c+a^2cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.109361, size = 53, normalized size = 0.61

$$\frac{\sqrt{c\left(a^2x^2+1\right)}\left(\text{CosIntegral}\left(\tan^{-1}(ax)\right)-\text{CosIntegral}\left(3\tan^{-1}(ax)\right)\right)}{4a^3c^3\sqrt{a^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]),x]
```

```
[Out] (Sqrt[c*(1 + a^2*x^2)]*(CosIntegral[ArcTan[a*x]] - CosIntegral[3*ArcTan[a*x
]]))/(4*a^3*c^3*Sqrt[1 + a^2*x^2])
```

Maple [C] time = 0.655, size = 84, normalized size = 1.

$$-\frac{\mathrm{Ci}\left(3\arctan\left(ax\right)\right)}{4c^{3}a^{3}}\sqrt{c\left(ax-i\right)\left(ax+i\right)}\frac{1}{\sqrt{a^{2}x^{2}+1}}+\frac{\mathrm{Ci}\left(\arctan\left(ax\right)\right)}{4c^{3}a^{3}}\sqrt{c\left(ax-i\right)\left(ax+i\right)}\frac{1}{\sqrt{a^{2}x^{2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x)
```

```
[Out] -1/4*Ci(3*arctan(a*x))/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(a*x+I))^(1/2)/c^3/a^3+
1/4*Ci(arctan(a*x))/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(a*x+I))^(1/2)/c^3/a^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{2}}{\left(a^{2}cx^{2}+c\right)^{\frac{5}{2}}\arctan\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{a^{2}cx^{2}+c}\,x^{2}}{\left(a^{6}c^{3}x^{6}+3\,a^{4}c^{3}x^{4}+3\,a^{2}c^{3}x^{2}+c^{3}\right)\arctan\left(ax\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^2/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*
x^2 + c^3)*arctan(a*x)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}\operatorname{atan}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a**2*c*x**2+c)**(5/2)/atan(a*x),x)
```

```
[Out] Integral(x**2/((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(a^2cx^2+c\right)^{\frac{5}{2}}\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="giac")
```

```
[Out] integrate(x^2/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)), x)
```

3.518 $\int \frac{x}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=87

$$\frac{\sqrt{a^2x^2+1}\text{Si}\left(\tan^{-1}(ax)\right)}{4a^2c^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{a^2x^2+1}\text{Si}\left(3\tan^{-1}(ax)\right)}{4a^2c^2\sqrt{a^2cx^2+c}}$$

```
[Out] (Sqrt[1 + a^2*x^2]*SinIntegral[ArcTan[a*x]])/(4*a^2*c^2*Sqrt[c + a^2*c*x^2]
) + (Sqrt[1 + a^2*x^2]*SinIntegral[3*ArcTan[a*x]])/(4*a^2*c^2*Sqrt[c + a^2*
c*x^2])
```

Rubi [A] time = 0.198889, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4971, 4970, 4406, 3299}

$$\frac{\sqrt{a^2x^2+1}\text{Si}\left(\tan^{-1}(ax)\right)}{4a^2c^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{a^2x^2+1}\text{Si}\left(3\tan^{-1}(ax)\right)}{4a^2c^2\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]),x]
```

```
[Out] (Sqrt[1 + a^2*x^2]*SinIntegral[ArcTan[a*x]])/(4*a^2*c^2*Sqrt[c + a^2*c*x^2]
) + (Sqrt[1 + a^2*x^2]*SinIntegral[3*ArcTan[a*x]])/(4*a^2*c^2*Sqrt[c + a^2*
c*x^2])
```

Rule 4971

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[(d^(q + 1/2)*Sqrt[1 + c^2*x^2])/Sqrt[d + e*x^2],
 Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d
, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(In
tegerQ[q] || GtQ[d, 0])
```

Rule 4970

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sin[x]^m)/
Cos[x]^(m + 2*(q + 1)), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
```

```
 || GtQ[d, 0])
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\int \frac{x}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)}\,dx = \frac{\sqrt{1+a^2x^2}\int \frac{x}{\left(1+a^2x^2\right)^{5/2}\tan^{-1}(ax)}\,dx}{c^2\sqrt{c+a^2cx^2}}$$
$$= \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \frac{\cos^2(x)\sin(x)}{x}\,dx,x,\tan^{-1}(ax)\right)}{a^2c^2\sqrt{c+a^2cx^2}}$$
$$= \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \left(\frac{\sin(x)}{4x}+\frac{\sin(3x)}{4x}\right)dx,x,\tan^{-1}(ax)\right)}{a^2c^2\sqrt{c+a^2cx^2}}$$
$$= \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \frac{\sin(x)}{x}\,dx,x,\tan^{-1}(ax)\right)}{4a^2c^2\sqrt{c+a^2cx^2}} + \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \frac{\sin(3x)}{x}\,dx,x,\tan^{-1}(ax)\right)}{4a^2c^2\sqrt{c+a^2cx^2}}$$
$$= \frac{\sqrt{1+a^2x^2}\,\text{Si}\left(\tan^{-1}(ax)\right)}{4a^2c^2\sqrt{c+a^2cx^2}} + \frac{\sqrt{1+a^2x^2}\,\text{Si}\left(3\tan^{-1}(ax)\right)}{4a^2c^2\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 0.124767, size = 51, normalized size = 0.59

$$\frac{\sqrt{c\left(a^2x^2+1\right)}\left(\text{Si}\left(\tan^{-1}(ax)\right)+\text{Si}\left(3\tan^{-1}(ax)\right)\right)}{4a^2c^3\sqrt{a^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]),x]
```

```
[Out] (Sqrt[c*(1 + a^2*x^2)]*(SinIntegral[ArcTan[a*x]] + SinIntegral[3*ArcTan[a*x
]]))/(4*a^2*c^3*Sqrt[1 + a^2*x^2])
```

Maple [C] time = 0.283, size = 125, normalized size = 1.4

$$-\frac{\operatorname{csgn}(\arctan(ax))\,\pi}{4\,c^{3}a^{2}}\sqrt{c\,(ax-i)\,(ax+i)}\frac{1}{\sqrt{a^{2}x^{2}+1}}+\frac{\operatorname{Si}(3\arctan(ax))}{4\,c^{3}a^{2}}\sqrt{c\,(ax-i)\,(ax+i)}\frac{1}{\sqrt{a^{2}x^{2}+1}}+\frac{\operatorname{Si}(\arctan(ax}{4\,c^{3}a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x)
```

```
[Out] -1/4*csgn(arctan(a*x))*Pi/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(a*x+I))^(1/2)/c^3/a
^2+1/4*Si(3*arctan(a*x))/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(a*x+I))^(1/2)/c^3/a^
2+1/4*Si(arctan(a*x))/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(a*x+I))^(1/2)/c^3/a^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(a^{2}cx^{2}+c\right)^{\frac{5}{2}}\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{a^{2}cx^{2}+c}\,x}{\left(a^{6}c^{3}x^{6}+3\,a^{4}c^{3}x^{4}+3\,a^{2}c^{3}x^{2}+c^{3}\right)\arctan(ax)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^
2 + c^3)*arctan(a*x)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(c(a^2x^2+1))^{\frac{5}{2}}\operatorname{atan}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a**2*c*x**2+c)**(5/2)/atan(a*x),x)
```

```
[Out] Integral(x/((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(a^2cx^2+c)^{\frac{5}{2}}\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="giac")
```

```
[Out] integrate(x/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)), x)
```

3.519 $\int \frac{1}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=87

$$\frac{3\sqrt{a^2x^2+1}\text{CosIntegral}\left(\tan^{-1}(ax)\right)}{4ac^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{a^2x^2+1}\text{CosIntegral}\left(3\tan^{-1}(ax)\right)}{4ac^2\sqrt{a^2cx^2+c}}$$

```
[Out] (3*Sqrt[1 + a^2*x^2]*CosIntegral[ArcTan[a*x]])/(4*a*c^2*Sqrt[c + a^2*c*x^2]
) + (Sqrt[1 + a^2*x^2]*CosIntegral[3*ArcTan[a*x]])/(4*a*c^2*Sqrt[c + a^2*c*
x^2])
```

Rubi [A] time = 0.132726, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {4905, 4904, 3312, 3302}

$$\frac{3\sqrt{a^2x^2+1}\text{CosIntegral}\left(\tan^{-1}(ax)\right)}{4ac^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{a^2x^2+1}\text{CosIntegral}\left(3\tan^{-1}(ax)\right)}{4ac^2\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]),x]
```

```
[Out] (3*Sqrt[1 + a^2*x^2]*CosIntegral[ArcTan[a*x]])/(4*a*c^2*Sqrt[c + a^2*c*x^2]
) + (Sqrt[1 + a^2*x^2]*CosIntegral[3*ArcTan[a*x]])/(4*a*c^2*Sqrt[c + a^2*c*
x^2])
```

Rule 4905

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[(d^(q + 1/2)*Sqrt[1 + c^2*x^2])/Sqrt[d + e*x^2], Int[(1 + c
^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

Rule 4904

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)}\,dx &= \frac{\sqrt{1+a^2x^2}\int \frac{1}{\left(1+a^2x^2\right)^{5/2}\tan^{-1}(ax)}\,dx}{c^2\sqrt{c+a^2cx^2}}\\
&= \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \frac{\cos^3(x)}{x}\,dx, x, \tan^{-1}(ax)\right)}{ac^2\sqrt{c+a^2cx^2}}\\
&= \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \left(\frac{3\cos(x)}{4x}+\frac{\cos(3x)}{4x}\right)dx, x, \tan^{-1}(ax)\right)}{ac^2\sqrt{c+a^2cx^2}}\\
&= \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \frac{\cos(3x)}{x}\,dx, x, \tan^{-1}(ax)\right)}{4ac^2\sqrt{c+a^2cx^2}}+\frac{\left(3\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int \frac{\cos(x)}{x}\,dx, x, \text{ta}\right.}{4ac^2\sqrt{c+a^2cx^2}}\\
&= \frac{3\sqrt{1+a^2x^2}\,\text{Ci}\left(\tan^{-1}(ax)\right)}{4ac^2\sqrt{c+a^2cx^2}}+\frac{\sqrt{1+a^2x^2}\,\text{Ci}\left(3\tan^{-1}(ax)\right)}{4ac^2\sqrt{c+a^2cx^2}}\end{aligned}$$

Mathematica [A] time = 0.0431014, size = 50, normalized size = 0.57

$$\frac{\left(a^2x^2+1\right)^{5/2}\left(3\text{CosIntegral}\left(\tan^{-1}(ax)\right)+\text{CosIntegral}\left(3\tan^{-1}(ax)\right)\right)}{4a\left(c\left(a^2x^2+1\right)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]),x]
```

```
[Out] ((1 + a^2*x^2)^(5/2)*(3*CosIntegral[ArcTan[a*x]] + CosIntegral[3*ArcTan[a*x
]]))/(4*a*(c*(1 + a^2*x^2))^(5/2))
```

Maple [C] time = 0.283, size = 179, normalized size = 2.1

$$-\frac{\frac{i}{2}\operatorname{csgn}\left(\arctan\left(ax\right)\right)\operatorname{csgn}\left(i\arctan\left(ax\right)\right)\pi}{a c^{3}}\sqrt{c\left(ax-i\right)\left(ax+i\right)}\frac{1}{\sqrt{a^{2}x^{2}+1}}+\frac{\frac{i}{2}\operatorname{csgn}\left(i\arctan\left(ax\right)\right)\pi}{a c^{3}}\sqrt{c\left(ax-i\right)\left(ax+i\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x)
```

```
[Out] -1/2*I*csgn(arctan(a*x))*csgn(I*arctan(a*x))*Pi/(a^2*x^2+1)^(1/2)*(c*(a*x-I
)*(a*x+I))^(1/2)/a/c^3+1/2*I*csgn(I*arctan(a*x))*Pi/(a^2*x^2+1)^(1/2)*(c*(a
*x-I)*(a*x+I))^(1/2)/a/c^3+1/4*Ci(3*arctan(a*x))/(a^2*x^2+1)^(1/2)*(c*(a*x-
I)*(a*x+I))^(1/2)/a/c^3+3/4*Ci(arctan(a*x))/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(a
*x+I))^(1/2)/a/c^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^{2}cx^{2}+c\right)^{\frac{5}{2}}\arctan\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{a^{2}cx^{2}+c}}{\left(a^{6}c^{3}x^{6}+3\,a^{4}c^{3}x^{4}+3\,a^{2}c^{3}x^{2}+c^{3}\right)\arctan\left(ax\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2
+ c^3)*arctan(a*x)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}\operatorname{atan}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*c*x**2+c)**(5/2)/atan(a*x),x)
```

```
[Out] Integral(1/((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)^{\frac{5}{2}}\arctan\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)), x)
```

3.520 $\int \frac{1}{x(c+a^2cx^2)^{5/2}\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{1}{x\left(a^2cx^2+c\right)^{5/2}\tan^{-1}(ax)},x\right)$$

```
[Out] Unintegrable[1/(x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]), x]
```

Rubi [A] time = 0.124713, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]),x]
```

```
[Out] Defer[Int][1/(x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]), x]
```

Rubi steps

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)}\,dx=\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)}\,dx$$

Mathematica [A] time = 1.29615, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]),x]
```

```
[Out] Integrate[1/(x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]), x]
```

Maple [A] time = 0.47, size = 0, normalized size = 0.

$$\int \frac{1}{x \arctan (ax)} \left(a^2 c x^2 + c\right)^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x)
```

```
[Out] int(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2 c x^2 + c\right)^{\frac{5}{2}} x \arctan (ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^(5/2)*x*arctan(a*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a^2 c x^2 + c}}{\left(a^6 c^3 x^7 + 3\, a^4 c^3 x^5 + 3\, a^2 c^3 x^3 + c^3 x\right) \arctan (ax)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)/((a^6*c^3*x^7 + 3*a^4*c^3*x^5 + 3*a^2*c^3*x^3
+ c^3*x)*arctan(a*x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}\operatorname{atan}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a**2*c*x**2+c)**(5/2)/atan(a*x),x)
```

```
[Out] Integral(1/(x*(c*(a**2*x**2 + 1))**(5/2)*atan(a*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)^{\frac{5}{2}}x\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^(5/2)*x*arctan(a*x)), x)
```

3.521 $\int \frac{1}{x^2(c+a^2cx^2)^{5/2}\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{1}{x^2\left(a^2cx^2+c\right)^{5/2}\tan^{-1}(ax)},x\right)$$

```
[Out] Unintegrable[1/(x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]), x]
```

Rubi [A] time = 0.127961, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]),x]
```

```
[Out] Defer[Int][1/(x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]), x]
```

Rubi steps

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)}\,dx=\int \frac{1}{x^2\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)}\,dx$$

Mathematica [A] time = 1.34478, size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]),x]
```

```
[Out] Integrate[1/(x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]), x]
```

Maple [A] time = 0.535, size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \arctan(ax)} \left(a^2 c x^2 + c\right)^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x)
```

```
[Out] int(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2 c x^2 + c\right)^{\frac{5}{2}} x^2 \arctan(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^(5/2)*x^2*arctan(a*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{a^2 c x^2 + c}}{\left(a^6 c^3 x^8 + 3\, a^4 c^3 x^6 + 3\, a^2 c^3 x^4 + c^3 x^2\right) \arctan(ax)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)/((a^6*c^3*x^8 + 3*a^4*c^3*x^6 + 3*a^2*c^3*x^4
+ c^3*x^2)*arctan(a*x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^{2}\left(c\left(a^{2}x^{2}+1\right)\right)^{\frac{5}{2}}\operatorname{atan}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a**2*c*x**2+c)**(5/2)/atan(a*x),x)
```

```
[Out] Integral(1/(x**2*(c*(a**2*x**2 + 1))**(5/2)*atan(a*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^{2}cx^{2}+c\right)^{\frac{5}{2}}x^{2}\arctan\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^(5/2)*x^2*arctan(a*x)), x)
```

3.522 $\int \frac{x^m(c+a^2cx^2)^3}{\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{x^m\left(a^2cx^2+c\right)^3}{\tan^{-1}(ax)},x\right)$$

```
[Out] Unintegrable[(x^m*(c + a^2*c*x^2)^3)/ArcTan[a*x], x]
```

Rubi [A] time = 0.0573032, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m\left(c+a^2cx^2\right)^3}{\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*(c + a^2*c*x^2)^3)/ArcTan[a*x],x]
```

```
[Out] Defer[Int][(x^m*(c + a^2*c*x^2)^3)/ArcTan[a*x], x]
```

Rubi steps

$$\int \frac{x^m\left(c+a^2cx^2\right)^3}{\tan^{-1}(ax)}\,dx=\int \frac{x^m\left(c+a^2cx^2\right)^3}{\tan^{-1}(ax)}\,dx$$

Mathematica [A] time = 0.672111, size = 0, normalized size = 0.

$$\int \frac{x^m\left(c+a^2cx^2\right)^3}{\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*(c + a^2*c*x^2)^3)/ArcTan[a*x],x]
```

```
[Out] Integrate[(x^m*(c + a^2*c*x^2)^3)/ArcTan[a*x], x]
```

Maple [A] time = 0.572, size = 0, normalized size = 0.

$$\int \frac{x^m\left(a^2cx^2+c\right)^3}{\arctan\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)^3/arctan(a*x),x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)^3/arctan(a*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^3x^m}{\arctan\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^3*x^m/arctan(a*x), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(a^6c^3x^6+3\,a^4c^3x^4+3\,a^2c^3x^2+c^3\right)x^m}{\arctan\left(ax\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*x^m/arctan(a*x
), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$c^3\left(\int \frac{x^m}{\operatorname{atan}(ax)}\,dx + \int \frac{3a^2x^2x^m}{\operatorname{atan}(ax)}\,dx + \int \frac{3a^4x^4x^m}{\operatorname{atan}(ax)}\,dx + \int \frac{a^6x^6x^m}{\operatorname{atan}(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**3/atan(a*x),x)
```

```
[Out] c**3*(Integral(x**m/atan(a*x), x) + Integral(3*a**2*x**2*x**m/atan(a*x), x)
 + Integral(3*a**4*x**4*x**m/atan(a*x), x) + Integral(a**6*x**6*x**m/atan(a
*x), x))
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^3x^m}{\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^3*x^m/arctan(a*x), x)
```

3.523 $\int \frac{x^m(c+a^2cx^2)^2}{\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{x^m\left(a^2cx^2+c\right)^2}{\tan^{-1}(ax)},x\right)$$

```
[Out] Unintegrable[(x^m*(c + a^2*c*x^2)^2)/ArcTan[a*x], x]
```

Rubi [A] time = 0.0538758, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m\left(c+a^2cx^2\right)^2}{\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*(c + a^2*c*x^2)^2)/ArcTan[a*x],x]

[Out] Defer[Int][(x^m*(c + a^2*c*x^2)^2)/ArcTan[a*x], x]
```

Rubi steps

$$\int \frac{x^m\left(c+a^2cx^2\right)^2}{\tan^{-1}(ax)}\,dx=\int \frac{x^m\left(c+a^2cx^2\right)^2}{\tan^{-1}(ax)}\,dx$$

Mathematica [A] time = 0.765987, size = 0, normalized size = 0.

$$\int \frac{x^m\left(c+a^2cx^2\right)^2}{\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*(c + a^2*c*x^2)^2)/ArcTan[a*x],x]
```

[Out] Integrate[(x^m*(c + a^2*c*x^2)^2)/ArcTan[a*x], x]

Maple [A] time = 0.507, size = 0, normalized size = 0.

$$\int \frac{x^{m} \left(a^{2} c x^{2}+c\right)^{2}}{\arctan \left(a x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m*(a^2*c*x^2+c)^2/arctan(a*x),x)

[Out] int(x^m*(a^2*c*x^2+c)^2/arctan(a*x),x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{{\left(a^{2} c x^{2}+c\right)}^{2} x^{m}}{\arctan \left(a x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m*(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="maxima")

[Out] integrate((a^2*c*x^2 + c)^2*x^m/arctan(a*x), x)

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{{\left(a^{4} c^{2} x^{4}+2 \, a^{2} c^{2} x^{2}+c^{2}\right)} x^{m}}{\arctan \left(a x\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m*(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="fricas")

[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*x^m/arctan(a*x), x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$c^2\left(\int \frac{x^m}{\operatorname{atan}(ax)}\,dx + \int \frac{2a^2x^2x^m}{\operatorname{atan}(ax)}\,dx + \int \frac{a^4x^4x^m}{\operatorname{atan}(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**2/atan(a*x),x)
```

```
[Out] c**2*(Integral(x**m/atan(a*x), x) + Integral(2*a**2*x**2*x**m/atan(a*x), x)
 + Integral(a**4*x**4*x**m/atan(a*x), x))
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2 + c\right)^2 x^m}{\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^2*x^m/arctan(a*x), x)
```

3.524 $\int \frac{x^m(c+a^2cx^2)}{\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{x^m\left(a^2cx^2+c\right)}{\tan^{-1}(ax)},x\right)$$

```
[Out] Unintegrable[(x^m*(c + a^2*c*x^2))/ArcTan[a*x], x]
```

Rubi [A] time = 0.035248, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m\left(c+a^2cx^2\right)}{\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*(c + a^2*c*x^2))/ArcTan[a*x],x]

[Out] Defer[Int][(x^m*(c + a^2*c*x^2))/ArcTan[a*x], x]
```

Rubi steps

$$\int \frac{x^m\left(c+a^2cx^2\right)}{\tan^{-1}(ax)}\,dx=\int \frac{x^m\left(c+a^2cx^2\right)}{\tan^{-1}(ax)}\,dx$$

Mathematica [A] time = 0.497949, size = 0, normalized size = 0.

$$\int \frac{x^m\left(c+a^2cx^2\right)}{\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*(c + a^2*c*x^2))/ArcTan[a*x],x]
```

```
[Out] Integrate[(x^m*(c + a^2*c*x^2))/ArcTan[a*x], x]
```

Maple [A] time = 0.385, size = 0, normalized size = 0.

$$\int \frac{x^m \left(a^2 c x^2 + c\right)}{\arctan \left(a x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)/arctan(a*x),x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)/arctan(a*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2 c x^2 + c\right) x^m}{\arctan \left(a x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)*x^m/arctan(a*x), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^2 c x^2 + c\right) x^m}{\arctan \left(a x\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)*x^m/arctan(a*x), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$c\left(\int \frac{x^m}{\operatorname{atan}(ax)}\,dx + \int \frac{a^2x^2x^m}{\operatorname{atan}(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)/atan(a*x),x)
```

```
[Out] c*(Integral(x**m/atan(a*x), x) + Integral(a**2*x**2*x**m/atan(a*x), x))
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a^2cx^2 + c)x^m}{\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)/arctan(a*x),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)*x^m/arctan(a*x), x)
```

3.525 $\int \frac{x^m}{\left(c+a^2cx^2\right)\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{x^m}{\left(a^2cx^2+c\right)\tan^{-1}(ax)},x\right)$$

```
[Out] Unintegrable[x^m/((c + a^2*c*x^2)*ArcTan[a*x]), x]
```

Rubi [A] time = 0.0637915, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m}{\left(c+a^2cx^2\right)\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m/((c + a^2*c*x^2)*ArcTan[a*x]),x]
```

```
[Out] Defer[Int][x^m/((c + a^2*c*x^2)*ArcTan[a*x]), x]
```

Rubi steps

$$\int \frac{x^m}{\left(c+a^2cx^2\right)\tan^{-1}(ax)}\,dx=\int \frac{x^m}{\left(c+a^2cx^2\right)\tan^{-1}(ax)}\,dx$$

Mathematica [A] time = 0.294795, size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(c+a^2cx^2\right)\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m/((c + a^2*c*x^2)*ArcTan[a*x]),x]
```

```
[Out] Integrate[x^m/((c + a^2*c*x^2)*ArcTan[a*x]), x]
```

Maple [A] time = 0.4, size = 0, normalized size = 0.

$$\int \frac{x^m}{(a^2cx^2+c)\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a^2*c*x^2+c)/arctan(a*x),x)
```

```
[Out] int(x^m/(a^2*c*x^2+c)/arctan(a*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{(a^2cx^2+c)\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x^m/((a^2*c*x^2 + c)*arctan(a*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^m}{(a^2cx^2+c)\arctan(ax)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral(x^m/((a^2*c*x^2 + c)*arctan(a*x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^m}{a^2x^2\operatorname{atan}(ax)+\operatorname{atan}(ax)}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(a**2*c*x**2+c)/atan(a*x),x)
```

```
[Out] Integral(x**m/(a**2*x**2*atan(a*x) + atan(a*x)), x)/c
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{(a^2cx^2+c)\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)/arctan(a*x),x, algorithm="giac")
```

```
[Out] integrate(x^m/((a^2*c*x^2 + c)*arctan(a*x)), x)
```

3.526 $\int \frac{x^m}{\left(c+a^2cx^2\right)^2 \tan^{-1}(ax)} \, dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{x^m}{\left(a^2cx^2+c\right)^2 \tan^{-1}(ax)},x\right)$$

```
[Out] Unintegrable[x^m/((c + a^2*c*x^2)^2*ArcTan[a*x]), x]
```

Rubi [A] time = 0.0626612, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m}{\left(c+a^2cx^2\right)^2 \tan^{-1}(ax)} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m/((c + a^2*c*x^2)^2*ArcTan[a*x]),x]
```

```
[Out] Defer[Int][x^m/((c + a^2*c*x^2)^2*ArcTan[a*x]), x]
```

Rubi steps

$$\int \frac{x^m}{\left(c+a^2cx^2\right)^2 \tan^{-1}(ax)} \, dx=\int \frac{x^m}{\left(c+a^2cx^2\right)^2 \tan^{-1}(ax)} \, dx$$

Mathematica [A] time = 0.393671, size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(c+a^2cx^2\right)^2 \tan^{-1}(ax)} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m/((c + a^2*c*x^2)^2*ArcTan[a*x]),x]
```

```
[Out] Integrate[x^m/((c + a^2*c*x^2)^2*ArcTan[a*x]), x]
```

Maple [A] time = 1.061, size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(a^2 c x^2 + c\right)^2 \arctan\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a^2*c*x^2+c)^2/arctan(a*x),x)
```

```
[Out] int(x^m/(a^2*c*x^2+c)^2/arctan(a*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(a^2 c x^2 + c\right)^2 \arctan\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x^m/((a^2*c*x^2 + c)^2*arctan(a*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x^m}{\left(a^4 c^2 x^4 + 2\,a^2 c^2 x^2 + c^2\right) \arctan\left(ax\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral(x^m/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(a*x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^m}{a^4x^4 \operatorname{atan}(ax)+2a^2x^2 \operatorname{atan}(ax)+\operatorname{atan}(ax)}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(a**2*c*x**2+c)**2/atan(a*x),x)
```

```
[Out] Integral(x**m/(a**4*x**4*atan(a*x) + 2*a**2*x**2*atan(a*x) + atan(a*x)), x)
/c**2
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(a^2cx^2+c\right)^2 \arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="giac")
```

```
[Out] integrate(x^m/((a^2*c*x^2 + c)^2*arctan(a*x)), x)
```

3.527 $\int \frac{x^m}{\left(c+a^2cx^2\right)^3 \tan^{-1}(ax)} dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{x^m}{\left(a^2cx^2+c\right)^3 \tan^{-1}(ax)},x\right)$$

```
[Out] Unintegrable[x^m/((c + a^2*c*x^2)^3*ArcTan[a*x]), x]
```

Rubi [A] time = 0.0681296, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{x^m}{\left(c+a^2cx^2\right)^3 \tan^{-1}(ax)} dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m/((c + a^2*c*x^2)^3*ArcTan[a*x]),x]
```

```
[Out] Defer[Int][x^m/((c + a^2*c*x^2)^3*ArcTan[a*x]), x]
```

Rubi steps

$$\int \frac{x^m}{\left(c+a^2cx^2\right)^3 \tan^{-1}(ax)} dx = \int \frac{x^m}{\left(c+a^2cx^2\right)^3 \tan^{-1}(ax)} dx$$

Mathematica [A] time = 0.414543, size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(c+a^2cx^2\right)^3 \tan^{-1}(ax)} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m/((c + a^2*c*x^2)^3*ArcTan[a*x]),x]
```

```
[Out] Integrate[x^m/((c + a^2*c*x^2)^3*ArcTan[a*x]), x]
```

Maple [A] time = 1.09, size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(a^2cx^2+c\right)^3 \arctan\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a^2*c*x^2+c)^3/arctan(a*x),x)
```

```
[Out] int(x^m/(a^2*c*x^2+c)^3/arctan(a*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(a^2cx^2+c\right)^3 \arctan\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x^m/((a^2*c*x^2 + c)^3*arctan(a*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x^m}{\left(a^6c^3x^6+3\,a^4c^3x^4+3\,a^2c^3x^2+c^3\right)\arctan\left(ax\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral(x^m/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*arctan(a*
x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^m}{a^6x^6\operatorname{atan}(ax)+3a^4x^4\operatorname{atan}(ax)+3a^2x^2\operatorname{atan}(ax)+\operatorname{atan}(ax)}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(a**2*c*x**2+c)**3/atan(a*x),x)
```

```
[Out] Integral(x**m/(a**6*x**6*atan(a*x) + 3*a**4*x**4*atan(a*x) + 3*a**2*x**2*at
an(a*x) + atan(a*x)), x)/c**3
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{(a^2cx^2+c)^3\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="giac")
```

```
[Out] integrate(x^m/((a^2*c*x^2 + c)^3*arctan(a*x)), x)
```

3.528 $\int \frac{x^m\left(c+a^2cx^2\right)^{5/2}}{\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{x^m\left(a^2cx^2+c\right)^{5/2}}{\tan^{-1}(ax)},x\right)$$

```
[Out] Unintegrable[(x^m*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x], x]
```

Rubi [A] time = 0.114519, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m\left(c+a^2cx^2\right)^{5/2}}{\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x],x]

[Out] Defer[Int][(x^m*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x], x]
```

Rubi steps

$$\int \frac{x^m\left(c+a^2cx^2\right)^{5/2}}{\tan^{-1}(ax)}\,dx=\int \frac{x^m\left(c+a^2cx^2\right)^{5/2}}{\tan^{-1}(ax)}\,dx$$

Mathematica [A] time = 1.0732, size = 0, normalized size = 0.

$$\int \frac{x^m\left(c+a^2cx^2\right)^{5/2}}{\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x],x]
```

```
[Out] Integrate[(x^m*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x], x]
```

Maple [A] time = 0.531, size = 0, normalized size = 0.

$$\int \frac{x^m}{\arctan(ax)} \left(a^2 c x^2 + c\right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x),x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2 c x^2 + c\right)^{\frac{5}{2}} x^m}{\arctan(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(5/2)*x^m/arctan(a*x), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(a^4 c^2 x^4 + 2 a^2 c^2 x^2 + c^2\right)\sqrt{a^2 c x^2 + c}\, x^m}{\arctan(ax)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*x^m/arctan
(a*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**(5/2)/atan(a*x),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^{\frac{5}{2}}x^m}{\arctan\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(5/2)*x^m/arctan(a*x), x)
```

3.529 $\int \frac{x^m \left(c+a^2 c x^2\right)^{3/2}}{\tan^{-1}(ax)} \, dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{x^m \left(a^2 c x^2+c\right)^{3/2}}{\tan^{-1}(ax)},x\right)$$

```
[Out] Unintegrable[(x^m*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x], x]
```

Rubi [A] time = 0.113968, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m \left(c+a^2 c x^2\right)^{3/2}}{\tan^{-1}(ax)} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x],x]

[Out] Defer[Int][(x^m*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x], x]
```

Rubi steps

$$\int \frac{x^m \left(c+a^2 c x^2\right)^{3/2}}{\tan^{-1}(ax)} \, dx=\int \frac{x^m \left(c+a^2 c x^2\right)^{3/2}}{\tan^{-1}(ax)} \, dx$$

Mathematica [A] time = 0.577663, size = 0, normalized size = 0.

$$\int \frac{x^m \left(c+a^2 c x^2\right)^{3/2}}{\tan^{-1}(ax)} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x],x]
```

```
[Out] Integrate[(x^m*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x], x]
```

Maple [A] time = 0.503, size = 0, normalized size = 0.

$$\int \frac{x^m}{\arctan(ax)} \left(a^2 c x^2 + c\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x),x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2 c x^2 + c\right)^{\frac{3}{2}} x^m}{\arctan(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)*x^m/arctan(a*x), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^2 c x^2 + c\right)^{\frac{3}{2}} x^m}{\arctan(ax)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)^(3/2)*x^m/arctan(a*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**(3/2)/atan(a*x),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^{\frac{3}{2}}x^m}{\arctan\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)*x^m/arctan(a*x), x)
```

3.530 $\int \frac{x^m\sqrt{c+a^2cx^2}}{\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{x^m\sqrt{a^2cx^2+c}}{\tan^{-1}(ax)},x\right)$$

```
[Out] Unintegrable[(x^m*Sqrt[c + a^2*c*x^2])/ArcTan[a*x], x]
```

Rubi [A] time = 0.0988851, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*Sqrt[c + a^2*c*x^2])/ArcTan[a*x],x]
```

```
[Out] Defer[Int][(x^m*Sqrt[c + a^2*c*x^2])/ArcTan[a*x], x]
```

Rubi steps

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\tan^{-1}(ax)}\,dx=\int \frac{x^m\sqrt{c+a^2cx^2}}{\tan^{-1}(ax)}\,dx$$

Mathematica [A] time = 0.151949, size = 0, normalized size = 0.

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*Sqrt[c + a^2*c*x^2])/ArcTan[a*x],x]
```

```
[Out] Integrate[(x^m*Sqrt[c + a^2*c*x^2])/ArcTan[a*x], x]
```

Maple [A] time = 0.634, size = 0, normalized size = 0.

$$\int \frac{x^{m}}{\arctan \left(a x \right)}\sqrt{a^{2} c \,x^{2}+c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x),x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a^{2} c x^{2} + c} x^{m}}{\arctan \left(a x\right)}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)*x^m/arctan(a*x), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{a^{2} c x^{2} + c} x^{m}}{\arctan \left(a x\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^m/arctan(a*x), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m \sqrt{c\left(a^2x^2+1\right)}}{\operatorname{atan}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**(1/2)/atan(a*x),x)
```

```
[Out] Integral(x**m*sqrt(c*(a**2*x**2 + 1))/atan(a*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a^2cx^2+c}\,x^m}{\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)*x^m/arctan(a*x), x)
```

3.531 $\int \frac{x^m}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{x^m}{\sqrt{a^2cx^2+c}\tan^{-1}(ax)},x\right)$$

```
[Out] Unintegrable[x^m/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]), x]
```

Rubi [A] time = 0.107155, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]),x]
```

```
[Out] Defer[Int][x^m/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]), x]
```

Rubi steps

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}\,dx=\int \frac{x^m}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}\,dx$$

Mathematica [A] time = 0.430838, size = 0, normalized size = 0.

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]),x]
```

```
[Out] Integrate[x^m/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]), x]
```

Maple [A] time = 0.983, size = 0, normalized size = 0.

$$\int \frac{x^m}{\arctan(ax)} \frac{1}{\sqrt{a^2cx^2 + c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/arctan(a*x)/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(x^m/arctan(a*x)/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\sqrt{a^2cx^2 + c}\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arctan(a*x)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m/(sqrt(a^2*c*x^2 + c)*arctan(a*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^m}{\sqrt{a^2cx^2 + c}\arctan(ax)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arctan(a*x)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^m/(sqrt(a^2*c*x^2 + c)*arctan(a*x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\sqrt{c\left(a^2x^2 + 1\right)}\operatorname{atan}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/atan(a*x)/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**m/(sqrt(c*(a**2*x**2 + 1))*atan(a*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\sqrt{a^2cx^2+c}\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arctan(a*x)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^m/(sqrt(a^2*c*x^2 + c)*arctan(a*x)), x)
```

3.532 $\int \frac{x^m}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{x^m}{\left(a^2cx^2+c\right)^{3/2}\tan^{-1}(ax)},x\right)$$

```
[Out] Unintegrable[x^m/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]), x]
```

Rubi [A] time = 0.121125, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]),x]
```

```
[Out] Defer[Int][x^m/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]), x]
```

Rubi steps

$$\int \frac{x^m}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}\,dx=\int \frac{x^m}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}\,dx$$

Mathematica [A] time = 0.486529, size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]),x]
```

```
[Out] Integrate[x^m/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]), x]
```

Maple [A] time = 1.065, size = 0, normalized size = 0.

$$\int \frac{x^m}{\arctan(ax)} \left(a^2cx^2+c\right)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x)
```

```
[Out] int(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(a^2cx^2+c\right)^{\frac{3}{2}} \arctan(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x^m/((a^2*c*x^2 + c)^(3/2)*arctan(a*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a^2cx^2+c}\,x^m}{\left(a^4c^2x^4+2\,a^2c^2x^2+c^2\right)\arctan(ax)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^m/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arcta
n(a*x)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(a**2*c*x**2+c)**(3/2)/atan(a*x),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(a^2cx^2+c\right)^{\frac{3}{2}}\arctan\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="giac")
```

```
[Out] integrate(x^m/((a^2*c*x^2 + c)^(3/2)*arctan(a*x)), x)
```

3.533 $\int \frac{x^m}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{x^m}{\left(a^2cx^2+c\right)^{5/2}\tan^{-1}(ax)},x\right)$$

```
[Out] Unintegrable[x^m/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]), x]
```

Rubi [A] time = 0.123559, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]),x]
```

```
[Out] Defer[Int][x^m/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]), x]
```

Rubi steps

$$\int \frac{x^m}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)}\,dx=\int \frac{x^m}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)}\,dx$$

Mathematica [A] time = 0.526782, size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]),x]
```

```
[Out] Integrate[x^m/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]), x]
```

Maple [A] time = 1.069, size = 0, normalized size = 0.

$$\int \frac{x^m}{\arctan(ax)} \left(a^2cx^2+c\right)^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x)
```

```
[Out] int(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(a^2cx^2+c\right)^{\frac{5}{2}} \arctan(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x^m/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a^2cx^2+c}\,x^m}{\left(a^6c^3x^6+3\,a^4c^3x^4+3\,a^2c^3x^2+c^3\right)\arctan(ax)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^m/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*
x^2 + c^3)*arctan(a*x)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(a**2*c*x**2+c)**(5/2)/atan(a*x),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(a^2cx^2+c\right)^{\frac{5}{2}}\arctan\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="giac")
```

```
[Out] integrate(x^m/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)), x)
```

3.534 $\int \frac{x(c+a^2cx^2)}{\tan^{-1}(ax)^2}\,dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left(\frac{x\left(a^2cx^2+c\right)}{\tan^{-1}(ax)^2},x\right)$$

```
[Out] Unintegrable[(x*(c + a^2*c*x^2))/ArcTan[a*x]^2, x]
```

Rubi [A] time = 0.0234308, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x\left(c+a^2cx^2\right)}{\tan^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x*(c + a^2*c*x^2))/ArcTan[a*x]^2,x]

[Out] Defer[Int][(x*(c + a^2*c*x^2))/ArcTan[a*x]^2, x]
```

Rubi steps

$$\int \frac{x\left(c+a^2cx^2\right)}{\tan^{-1}(ax)^2}\,dx=\int \frac{x\left(c+a^2cx^2\right)}{\tan^{-1}(ax)^2}\,dx$$

Mathematica [A] time = 0.722272, size = 0, normalized size = 0.

$$\int \frac{x\left(c+a^2cx^2\right)}{\tan^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x*(c + a^2*c*x^2))/ArcTan[a*x]^2,x]
```

```
[Out] Integrate[(x*(c + a^2*c*x^2))/ArcTan[a*x]^2, x]
```

Maple [A] time = 0.787, size = 0, normalized size = 0.

$$\int \frac{x\left(a^{2}c x^{2}+c\right)}{\left(\arctan\left(ax\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)/arctan(a*x)^2,x)
```

```
[Out] int(x*(a^2*c*x^2+c)/arctan(a*x)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{a^{4}c x^{5}+2\,a^{2}c x^{3}-c\left(\int \frac{5\,a^{4}x^{4}}{\arctan(ax)}\,dx+\int \frac{6\,a^{2}x^{2}}{\arctan(ax)}\,dx+\int \frac{1}{\arctan(ax)}\,dx\right)\arctan\left(ax\right)+cx}{a\arctan\left(ax\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(a^4*c*x^5 + 2*a^2*c*x^3 + c*x - arctan(a*x)*integrate((5*a^4*c*x^4 + 6*a^
2*c*x^2 + c)/arctan(a*x), x))/(a*arctan(a*x))
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{a^{2}c x^{3}+cx}{\arctan\left(ax\right)^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^3 + c*x)/arctan(a*x)^2, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$c\left(\int \frac{x}{\operatorname{atan}^2(ax)}\,dx + \int \frac{a^2x^3}{\operatorname{atan}^2(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)/atan(a*x)**2,x)
```

```
[Out] c*(Integral(x/atan(a*x)**2, x) + Integral(a**2*x**3/atan(a*x)**2, x))
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a^2cx^2+c)x}{\arctan(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)*x/arctan(a*x)^2, x)
```

3.535 $\int \frac{c+a^2cx^2}{\tan^{-1}(ax)^2}\,dx$

Optimal. Leaf size=19

$$\text{Unintegrable}\left(\frac{a^2cx^2+c}{\tan^{-1}(ax)^2},x\right)$$

```
[Out] Unintegrable[(c + a^2*c*x^2)/ArcTan[a*x]^2, x]
```

Rubi [A] time = 0.0122972, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{c+a^2cx^2}{\tan^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)/ArcTan[a*x]^2,x]
```

```
[Out] Defer[Int][(c + a^2*c*x^2)/ArcTan[a*x]^2, x]
```

Rubi steps

$$\int \frac{c+a^2cx^2}{\tan^{-1}(ax)^2}\,dx=\int \frac{c+a^2cx^2}{\tan^{-1}(ax)^2}\,dx$$

Mathematica [A] time = 0.542987, size = 0, normalized size = 0.

$$\int \frac{c+a^2cx^2}{\tan^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)/ArcTan[a*x]^2,x]
```

```
[Out] Integrate[(c + a^2*c*x^2)/ArcTan[a*x]^2, x]
```

Maple [A] time = 0.757, size = 0, normalized size = 0.

$$\int \frac{a^2cx^2 + c}{\left(\arctan\left(ax\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)/arctan(a*x)^2,x)
```

```
[Out] int((a^2*c*x^2+c)/arctan(a*x)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{a^4cx^4 + 2\,a^2cx^2 - 4\,a\arctan\left(ax\right)\int \frac{a^3cx^3+acx}{\arctan(ax)}\,dx + c}{a\arctan\left(ax\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(a^4*c*x^4 + 2*a^2*c*x^2 - a*arctan(a*x)*integrate(4*(a^3*c*x^3 + a*c*x)/a
rctan(a*x), x) + c)/(a*arctan(a*x))
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{a^2cx^2 + c}{\arctan\left(ax\right)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)/arctan(a*x)^2, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$c\left(\int \frac{a^2x^2}{\operatorname{atan}^2(ax)}\,dx + \int \frac{1}{\operatorname{atan}^2(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)/atan(a*x)**2,x)
```

```
[Out] c*(Integral(a**2*x**2/atan(a*x)**2, x) + Integral(atan(a*x)**(-2), x))
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{a^2cx^2 + c}{\arctan(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)/arctan(a*x)^2, x)
```

3.536 $\int \frac{c+a^2cx^2}{x\tan^{-1}(ax)^2}\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{a^2cx^2+c}{x\tan^{-1}(ax)^2},x\right)$$

```
[Out] Unintegrable[(c + a^2*c*x^2)/(x*ArcTan[a*x]^2), x]
```

Rubi [A] time = 0.0320409, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{c+a^2cx^2}{x\tan^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)/(x*ArcTan[a*x]^2),x]
```

```
[Out] Defer[Int][(c + a^2*c*x^2)/(x*ArcTan[a*x]^2), x]
```

Rubi steps

$$\int \frac{c+a^2cx^2}{x\tan^{-1}(ax)^2}\,dx=\int \frac{c+a^2cx^2}{x\tan^{-1}(ax)^2}\,dx$$

Mathematica [A] time = 0.865468, size = 0, normalized size = 0.

$$\int \frac{c+a^2cx^2}{x\tan^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)/(x*ArcTan[a*x]^2),x]
```

```
[Out] Integrate[(c + a^2*c*x^2)/(x*ArcTan[a*x]^2), x]
```

Maple [A] time = 0.842, size = 0, normalized size = 0.

$$\int \frac{a^2cx^2+c}{x\left(\arctan \left(ax\right)\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)/x/arctan(a*x)^2,x)
```

```
[Out] int((a^2*c*x^2+c)/x/arctan(a*x)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{a^4cx^4+2\,a^2cx^2-cx\left(\int \frac{3\,a^4x^2}{\arctan(ax)}\, dx+\int \frac{2\,a^2}{\arctan(ax)}\, dx+\int -\frac{1}{x^2\arctan(ax)}\, dx\right)\arctan \left(ax\right)+c}{ax\arctan \left(ax\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)/x/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(a^4*c*x^4 + 2*a^2*c*x^2 - x*arctan(a*x)*integrate((3*a^4*c*x^4 + 2*a^2*c*
x^2 - c)/(x^2*arctan(a*x)), x) + c)/(a*x*arctan(a*x))
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{a^2cx^2+c}{x\arctan \left(ax\right)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)/x/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)/(x*arctan(a*x)^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$c\left(\int\frac{1}{x\,\mathrm{atan}^2\left(ax\right)}\,dx+\int\frac{a^2x}{\mathrm{atan}^2\left(ax\right)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)/x/atan(a*x)**2,x)
```

```
[Out] c*(Integral(1/(x*atan(a*x)**2), x) + Integral(a**2*x/atan(a*x)**2, x))
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{a^2cx^2+c}{x\arctan\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)/x/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)/(x*arctan(a*x)^2), x)
```

3.537 $\int \frac{x\left(c+a^2cx^2\right)^2}{\tan^{-1}(ax)^2}\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{x\left(a^2cx^2+c\right)^2}{\tan^{-1}(ax)^2},x\right)$$

```
[Out] Unintegrable[(x*(c + a^2*c*x^2)^2)/ArcTan[a*x]^2, x]
```

Rubi [A] time = 0.0362287, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x\left(c+a^2cx^2\right)^2}{\tan^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x*(c + a^2*c*x^2)^2)/ArcTan[a*x]^2,x]

[Out] Defer[Int][(x*(c + a^2*c*x^2)^2)/ArcTan[a*x]^2, x]
```

Rubi steps

$$\int \frac{x\left(c+a^2cx^2\right)^2}{\tan^{-1}(ax)^2}\,dx=\int \frac{x\left(c+a^2cx^2\right)^2}{\tan^{-1}(ax)^2}\,dx$$

Mathematica [A] time = 0.893431, size = 0, normalized size = 0.

$$\int \frac{x\left(c+a^2cx^2\right)^2}{\tan^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x*(c + a^2*c*x^2)^2)/ArcTan[a*x]^2,x]
```

```
[Out] Integrate[(x*(c + a^2*c*x^2)^2)/ArcTan[a*x]^2, x]
```

Maple [A] time = 0.934, size = 0, normalized size = 0.

$$\int \frac{x\left(a^{2}c x^{2}+c\right)^{2}}{\left(\arctan\left(a x\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^2/arctan(a*x)^2,x)
```

```
[Out] int(x*(a^2*c*x^2+c)^2/arctan(a*x)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{a^{6}c^{2}x^{7}+3\,a^{4}c^{2}x^{5}+3\,a^{2}c^{2}x^{3}-c^{2}\left(\int \frac{7\,a^{6}x^{6}}{\arctan(ax)}\,dx+\int \frac{15\,a^{4}x^{4}}{\arctan(ax)}\,dx+\int \frac{9\,a^{2}x^{2}}{\arctan(ax)}\,dx+\int \frac{1}{\arctan(ax)}\,dx\right)\arctan\left(ax\right)+c^{2}x}{a\arctan\left(ax\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(a^6*c^2*x^7 + 3*a^4*c^2*x^5 + 3*a^2*c^2*x^3 + c^2*x - arctan(a*x)*integra
te((7*a^6*c^2*x^6 + 15*a^4*c^2*x^4 + 9*a^2*c^2*x^2 + c^2)/arctan(a*x), x))/
(a*arctan(a*x))
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{a^{4}c^{2}x^{5}+2\,a^{2}c^{2}x^{3}+c^{2}x}{\arctan\left(ax\right)^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^5 + 2*a^2*c^2*x^3 + c^2*x)/arctan(a*x)^2, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$c^2\left(\int \frac{x}{\operatorname{atan}^2(ax)}\,dx + \int \frac{2a^2x^3}{\operatorname{atan}^2(ax)}\,dx + \int \frac{a^4x^5}{\operatorname{atan}^2(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**2/atan(a*x)**2,x)
```

```
[Out] c**2*(Integral(x/atan(a*x)**2, x) + Integral(2*a**2*x**3/atan(a*x)**2, x) +
 Integral(a**4*x**5/atan(a*x)**2, x))
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2 + c\right)^2 x}{\arctan(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^2*x/arctan(a*x)^2, x)
```

3.538 $\int \frac{\left(c+a^2 c x^2\right)^2}{\tan^{-1}(ax)^2} \, dx$

Optimal. Leaf size=21

$$\text{Unintegrable}\left(\frac{\left(a^2 c x^2+c\right)^2}{\tan^{-1}(ax)^2},x\right)$$

```
[Out] Unintegrable[(c + a^2*c*x^2)^2/ArcTan[a*x]^2, x]
```

Rubi [A] time = 0.0226466, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(c+a^2 c x^2\right)^2}{\tan^{-1}(ax)^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^2/ArcTan[a*x]^2,x]
```

```
[Out] Defer[Int][(c + a^2*c*x^2)^2/ArcTan[a*x]^2, x]
```

Rubi steps

$$\int \frac{\left(c+a^2 c x^2\right)^2}{\tan^{-1}(ax)^2} \, dx=\int \frac{\left(c+a^2 c x^2\right)^2}{\tan^{-1}(ax)^2} \, dx$$

Mathematica [A] time = 1.10692, size = 0, normalized size = 0.

$$\int \frac{\left(c+a^2 c x^2\right)^2}{\tan^{-1}(ax)^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^2/ArcTan[a*x]^2,x]
```

```
[Out] Integrate[(c + a^2*c*x^2)^2/ArcTan[a*x]^2, x]
```

Maple [A] time = 0.871, size = 0, normalized size = 0.

$$\int \frac{\left(a^{2} c x^{2}+c\right)^{2}}{\left(\arctan \left(a x\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^2/arctan(a*x)^2,x)
```

```
[Out] int((a^2*c*x^2+c)^2/arctan(a*x)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{a^{6} c^{2} x^{6}+3\, a^{4} c^{2} x^{4}+3\, a^{2} c^{2} x^{2}-6\, a \arctan \left(a x\right) \int \frac{a^{5} c^{2} x^{5}+2\, a^{3} c^{2} x^{3}+a c^{2} x}{\arctan(a x)}\,dx+c^{2}}{a \arctan \left(a x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(a^6*c^2*x^6 + 3*a^4*c^2*x^4 + 3*a^2*c^2*x^2 - a*arctan(a*x)*integrate(6*(
a^5*c^2*x^5 + 2*a^3*c^2*x^3 + a*c^2*x)/arctan(a*x), x) + c^2)/(a*arctan(a*x
))
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{a^{4} c^{2} x^{4}+2\, a^{2} c^{2} x^{2}+c^{2}}{\arctan \left(a x\right)^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)/arctan(a*x)^2, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$c^{2}\left(\int \frac{2a^{2}x^{2}}{\operatorname{atan}^{2}(ax)}\,dx+\int \frac{a^{4}x^{4}}{\operatorname{atan}^{2}(ax)}\,dx+\int \frac{1}{\operatorname{atan}^{2}(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**2/atan(a*x)**2,x)
```

```
[Out] c**2*(Integral(2*a**2*x**2/atan(a*x)**2, x) + Integral(a**4*x**4/atan(a*x)*
*2, x) + Integral(atan(a*x)**(-2), x))
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^{2}cx^{2}+c\right)^{2}}{\arctan(ax)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^2/arctan(a*x)^2, x)
```

3.539 $\int \frac{\left(c+a^2cx^2\right)^2}{x\tan^{-1}(ax)^2}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{\left(a^2cx^2+c\right)^2}{x\tan^{-1}(ax)^2},x\right)$$

```
[Out] Unintegrable[(c + a^2*c*x^2)^2/(x*ArcTan[a*x]^2), x]
```

Rubi [A] time = 0.0489749, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(c+a^2cx^2\right)^2}{x\tan^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^2/(x*ArcTan[a*x]^2),x]
```

```
[Out] Defer[Int][(c + a^2*c*x^2)^2/(x*ArcTan[a*x]^2), x]
```

Rubi steps

$$\int \frac{\left(c+a^2cx^2\right)^2}{x\tan^{-1}(ax)^2}\,dx=\int \frac{\left(c+a^2cx^2\right)^2}{x\tan^{-1}(ax)^2}\,dx$$

Mathematica [A] time = 1.10079, size = 0, normalized size = 0.

$$\int \frac{\left(c+a^2cx^2\right)^2}{x\tan^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^2/(x*ArcTan[a*x]^2),x]
```

```
[Out] Integrate[(c + a^2*c*x^2)^2/(x*ArcTan[a*x]^2), x]
```

Maple [A] time = 0.873, size = 0, normalized size = 0.

$$\int \frac{\left(a^{2} c \,x^{2}+c \right)^{2}}{x \arctan \left(a x \right)^{2}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^2/x/arctan(a*x)^2,x)
```

```
[Out] int((a^2*c*x^2+c)^2/x/arctan(a*x)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{a^{6} c^{2} x^{6}+3 \, a^{4} c^{2} x^{4}+3 \, a^{2} c^{2} x^{2}-c^{2} x \left(\int \frac{5 \, a^{6} x^{4}}{\arctan \left(a x\right)}\,{d x}+\int \frac{9 \, a^{4} x^{2}}{\arctan \left(a x\right)}\,{d x}+\int \frac{3 \, a^{2}}{\arctan \left(a x\right)}\,{d x}+\int -\frac{1}{x^{2} \arctan \left(a x\right)}\,{d x}\right) \arctan \left(a x\right)+}{a x \arctan \left(a x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2/x/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(a^6*c^2*x^6 + 3*a^4*c^2*x^4 + 3*a^2*c^2*x^2 - x*arctan(a*x)*integrate((5*
a^6*c^2*x^6 + 9*a^4*c^2*x^4 + 3*a^2*c^2*x^2 - c^2)/(x^2*arctan(a*x)), x) +
c^2)/(a*x*arctan(a*x))
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$${\rm integral}\left(\frac{a^{4} c^{2} x^{4}+2 \, a^{2} c^{2} x^{2}+c^{2}}{x \arctan \left(a x\right)^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2/x/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)/(x*arctan(a*x)^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$c^2\left(\int \frac{1}{x\operatorname{atan}^2(ax)}\,dx + \int \frac{2a^2x}{\operatorname{atan}^2(ax)}\,dx + \int \frac{a^4x^3}{\operatorname{atan}^2(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**2/x/atan(a*x)**2,x)
```

```
[Out] c**2*(Integral(1/(x*atan(a*x)**2), x) + Integral(2*a**2*x/atan(a*x)**2, x)
+ Integral(a**4*x**3/atan(a*x)**2, x))
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^2}{x\arctan(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2/x/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^2/(x*arctan(a*x)^2), x)
```

3.540 $\int \frac{x(c+a^2cx^2)^3}{\tan^{-1}(ax)^2} dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{x\left(a^2cx^2+c\right)^3}{\tan^{-1}(ax)^2},x\right)$$

[Out] Unintegrable[(x*(c + a^2*c*x^2)^3)/ArcTan[a*x]^2, x]

Rubi [A] time = 0.0358701, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x\left(c+a^2cx^2\right)^3}{\tan^{-1}(ax)^2} dx$$

Verification is Not applicable to the result.

[In] Int[(x*(c + a^2*c*x^2)^3)/ArcTan[a*x]^2,x]

[Out] Defer[Int][(x*(c + a^2*c*x^2)^3)/ArcTan[a*x]^2, x]

Rubi steps

$$\int \frac{x\left(c+a^2cx^2\right)^3}{\tan^{-1}(ax)^2} dx = \int \frac{x\left(c+a^2cx^2\right)^3}{\tan^{-1}(ax)^2} dx$$

Mathematica [A] time = 0.921422, size = 0, normalized size = 0.

$$\int \frac{x\left(c+a^2cx^2\right)^3}{\tan^{-1}(ax)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(x*(c + a^2*c*x^2)^3)/ArcTan[a*x]^2,x]

```
[Out] Integrate[(x*(c + a^2*c*x^2)^3)/ArcTan[a*x]^2, x]
```

Maple [A] time = 1.217, size = 0, normalized size = 0.

$$\int \frac{x\left(a^{2} c x^{2}+c\right)^{3}}{\left(\arctan \left(a x\right)\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^3/arctan(a*x)^2,x)
```

```
[Out] int(x*(a^2*c*x^2+c)^3/arctan(a*x)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{a^{8} c^{3} x^{9}+4\, a^{6} c^{3} x^{7}+6\, a^{4} c^{3} x^{5}+4\, a^{2} c^{3} x^{3}-c^{3}\left(\int \frac{9\, a^{8} x^{8}}{\arctan(ax)}\, dx+\int \frac{28\, a^{6} x^{6}}{\arctan(ax)}\, dx+\int \frac{30\, a^{4} x^{4}}{\arctan(ax)}\, dx+\int \frac{12\, a^{2} x^{2}}{\arctan(ax)}\, dx+\int \frac{}{\mathrm{arc}}\right.}{a \arctan \left(a x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(a^8*c^3*x^9 + 4*a^6*c^3*x^7 + 6*a^4*c^3*x^5 + 4*a^2*c^3*x^3 + c^3*x - arc
tan(a*x)*integrate((9*a^8*c^3*x^8 + 28*a^6*c^3*x^6 + 30*a^4*c^3*x^4 + 12*a^
2*c^3*x^2 + c^3)/arctan(a*x), x))/(a*arctan(a*x))
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{a^{6} c^{3} x^{7}+3\, a^{4} c^{3} x^{5}+3\, a^{2} c^{3} x^{3}+c^{3} x}{\arctan \left(a x\right)^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral((a^6*c^3*x^7 + 3*a^4*c^3*x^5 + 3*a^2*c^3*x^3 + c^3*x)/arctan(a*x)^
2, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$c^3\left(\int\frac{x}{\operatorname{atan}^2(ax)}\,dx+\int\frac{3a^2x^3}{\operatorname{atan}^2(ax)}\,dx+\int\frac{3a^4x^5}{\operatorname{atan}^2(ax)}\,dx+\int\frac{a^6x^7}{\operatorname{atan}^2(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**3/atan(a*x)**2,x)
```

```
[Out] c**3*(Integral(x/atan(a*x)**2, x) + Integral(3*a**2*x**3/atan(a*x)**2, x) +
 Integral(3*a**4*x**5/atan(a*x)**2, x) + Integral(a**6*x**7/atan(a*x)**2, x
))
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a^2cx^2+c\right)^3x}{\arctan(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^3*x/arctan(a*x)^2, x)
```

3.541 $\int \frac{\left(c+a^2 c x^2\right)^3}{\tan^{-1}(a x)^2} \, dx$

Optimal. Leaf size=21

$$\text{Unintegrable}\left(\frac{\left(a^2 c x^2+c\right)^3}{\tan^{-1}(a x)^2},x\right)$$

```
[Out] Unintegrable[(c + a^2*c*x^2)^3/ArcTan[a*x]^2, x]
```

Rubi [A] time = 0.0213235, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(c+a^2 c x^2\right)^3}{\tan^{-1}(a x)^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^3/ArcTan[a*x]^2,x]

[Out] Defer[Int][(c + a^2*c*x^2)^3/ArcTan[a*x]^2, x]
```

Rubi steps

$$\int \frac{\left(c+a^2 c x^2\right)^3}{\tan^{-1}(a x)^2} \, dx=\int \frac{\left(c+a^2 c x^2\right)^3}{\tan^{-1}(a x)^2} \, dx$$

Mathematica [A] time = 0.720318, size = 0, normalized size = 0.

$$\int \frac{\left(c+a^2 c x^2\right)^3}{\tan^{-1}(a x)^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^3/ArcTan[a*x]^2,x]
```

```
[Out] Integrate[(c + a^2*c*x^2)^3/ArcTan[a*x]^2, x]
```

Maple [A] time = 1.079, size = 0, normalized size = 0.

$$\int \frac{\left(a^{2} c\,x^{2}+c\right)^{3}}{\left(\arctan \left(a x\right)\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^3/arctan(a*x)^2,x)
```

```
[Out] int((a^2*c*x^2+c)^3/arctan(a*x)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{a^{8} c^{3} x^{8} + 4 \, a^{6} c^{3} x^{6} + 6 \, a^{4} c^{3} x^{4} + 4 \, a^{2} c^{3} x^{2} + c^{3} - 8 \, a \arctan \left(a x\right) \int \frac{a^{7} c^{3} x^{7} + 3 \, a^{5} c^{3} x^{5} + 3 \, a^{3} c^{3} x^{3} + a c^{3} x}{\arctan\left(a x\right)}\,{d x}}{a \arctan \left(a x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(a^8*c^3*x^8 + 4*a^6*c^3*x^6 + 6*a^4*c^3*x^4 + 4*a^2*c^3*x^2 + c^3 - a*arc
tan(a*x)*integrate(8*(a^7*c^3*x^7 + 3*a^5*c^3*x^5 + 3*a^3*c^3*x^3 + a*c^3*x
)/arctan(a*x), x))/(a*arctan(a*x))
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$${\rm integral}\left(\frac{a^{6} c^{3} x^{6} + 3 \, a^{4} c^{3} x^{4} + 3 \, a^{2} c^{3} x^{2} + c^{3}}{\arctan \left(a x\right)^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)/arctan(a*x)^2,
 x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$c^3\left(\int\frac{3a^2x^2}{\operatorname{atan}^2(ax)}\,dx+\int\frac{3a^4x^4}{\operatorname{atan}^2(ax)}\,dx+\int\frac{a^6x^6}{\operatorname{atan}^2(ax)}\,dx+\int\frac{1}{\operatorname{atan}^2(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**3/atan(a*x)**2,x)
```

```
[Out] c**3*(Integral(3*a**2*x**2/atan(a*x)**2, x) + Integral(3*a**4*x**4/atan(a*x
)**2, x) + Integral(a**6*x**6/atan(a*x)**2, x) + Integral(atan(a*x)**(-2),
x))
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a^2cx^2+c\right)^3}{\arctan(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^3/arctan(a*x)^2, x)
```

3.542 $\int \frac{\left(c+a^2cx^2\right)^3}{x\tan^{-1}(ax)^2}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{\left(a^2cx^2+c\right)^3}{x\tan^{-1}(ax)^2},x\right)$$

```
[Out] Unintegrable[(c + a^2*c*x^2)^3/(x*ArcTan[a*x]^2), x]
```

Rubi [A] time = 0.0505526, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(c+a^2cx^2\right)^3}{x\tan^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^3/(x*ArcTan[a*x]^2),x]
```

```
[Out] Defer[Int][(c + a^2*c*x^2)^3/(x*ArcTan[a*x]^2), x]
```

Rubi steps

$$\int \frac{\left(c+a^2cx^2\right)^3}{x\tan^{-1}(ax)^2}\,dx=\int \frac{\left(c+a^2cx^2\right)^3}{x\tan^{-1}(ax)^2}\,dx$$

Mathematica [A] time = 1.12445, size = 0, normalized size = 0.

$$\int \frac{\left(c+a^2cx^2\right)^3}{x\tan^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^3/(x*ArcTan[a*x]^2),x]
```

[Out] Integrate[(c + a^2*c*x^2)^3/(x*ArcTan[a*x]^2), x]

Maple [A] time = 1.024, size = 0, normalized size = 0.

$$\int \frac{\left(a^{2} c x^{2}+c\right)^{3}}{x \left(\arctan \left(a x\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^2*c*x^2+c)^3/x/arctan(a*x)^2,x)

[Out] int((a^2*c*x^2+c)^3/x/arctan(a*x)^2,x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{a^{8} c^{3} x^{8}+4\,a^{6} c^{3} x^{6}+6\,a^{4} c^{3} x^{4}+4\,a^{2} c^{3} x^{2}-c^{3} x\left(\int \frac{7\,a^{8} x^{6}}{\arctan(ax)}\,dx+\int \frac{20\,a^{6} x^{4}}{\arctan(ax)}\,dx+\int \frac{18\,a^{4} x^{2}}{\arctan(ax)}\,dx+\int \frac{4\,a^{2}}{\arctan(ax)}\,dx+\int -\right.}{a x \arctan\left(a x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2*c*x^2+c)^3/x/arctan(a*x)^2,x, algorithm="maxima")

[Out] -(a^8*c^3*x^8 + 4*a^6*c^3*x^6 + 6*a^4*c^3*x^4 + 4*a^2*c^3*x^2 + c^3 - x*arc
tan(a*x)*integrate((7*a^8*c^3*x^8 + 20*a^6*c^3*x^6 + 18*a^4*c^3*x^4 + 4*a^2
*c^3*x^2 - c^3)/(x^2*arctan(a*x)), x))/(a*x*arctan(a*x))

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{a^{6} c^{3} x^{6}+3\,a^{4} c^{3} x^{4}+3\,a^{2} c^{3} x^{2}+c^{3}}{x \arctan\left(a x\right)^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2*c*x^2+c)^3/x/arctan(a*x)^2,x, algorithm="fricas")

```
[Out] integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)/(x*arctan(a*x)
^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$c^3\left(\int \frac{1}{x\,\mathrm{atan}^2\,(ax)}\,dx+\int \frac{3a^2x}{\mathrm{atan}^2\,(ax)}\,dx+\int \frac{3a^4x^3}{\mathrm{atan}^2\,(ax)}\,dx+\int \frac{a^6x^5}{\mathrm{atan}^2\,(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**3/x/atan(a*x)**2,x)
```

```
[Out] c**3*(Integral(1/(x*atan(a*x)**2), x) + Integral(3*a**2*x/atan(a*x)**2, x)
+ Integral(3*a**4*x**3/atan(a*x)**2, x) + Integral(a**6*x**5/atan(a*x)**2,
x))
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^3}{x\arctan\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3/x/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^3/(x*arctan(a*x)^2), x)
```

3.543 $\int \frac{x^3}{\left(c+a^2cx^2\right)\tan^{-1}(ax)^2}\,dx$

Optimal. Leaf size=38

$$\frac{3\text{Unintegrable}\left(\frac{x^2}{\tan^{-1}(ax)},x\right)}{ac}-\frac{x^3}{ac\tan^{-1}(ax)}$$

```
[Out] -(x^3/(a*c*ArcTan[a*x])) + (3*Unintegrable[x^2/ArcTan[a*x], x])/(a*c)
```

Rubi [A] time = 0.0824813, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^3}{\left(c+a^2cx^2\right)\tan^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^3/((c + a^2*c*x^2)*ArcTan[a*x]^2),x]
```

```
[Out] -(x^3/(a*c*ArcTan[a*x])) + (3*Defer[Int][x^2/ArcTan[a*x], x])/(a*c)
```

Rubi steps

$$\int \frac{x^3}{\left(c+a^2cx^2\right)\tan^{-1}(ax)^2}\,dx=-\frac{x^3}{ac\tan^{-1}(ax)}+\frac{3\int \frac{x^2}{\tan^{-1}(ax)}\,dx}{ac}$$

Mathematica [A] time = 0.973472, size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(c+a^2cx^2\right)\tan^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^3/((c + a^2*c*x^2)*ArcTan[a*x]^2),x]
```

```
[Out] Integrate[x^3/((c + a^2*c*x^2)*ArcTan[a*x]^2), x]
```

Maple [A] time = 1.025, size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(a^2cx^2+c\right)\left(\arctan\left(ax\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a^2*c*x^2+c)/arctan(a*x)^2,x)
```

```
[Out] int(x^3/(a^2*c*x^2+c)/arctan(a*x)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{x^3 - 3\,\arctan\left(ax\right)\int \frac{x^2}{\arctan(ax)}\,dx}{ac\arctan\left(ax\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(x^3 - 3*arctan(a*x)*integrate(x^2/arctan(a*x), x))/(a*c*arctan(a*x))
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x^3}{\left(a^2cx^2+c\right)\arctan\left(ax\right)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(x^3/((a^2*c*x^2 + c)*arctan(a*x)^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^3}{a^2x^2\operatorname{atan}^2(ax)+\operatorname{atan}^2(ax)}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a**2*c*x**2+c)/atan(a*x)**2,x)
```

```
[Out] Integral(x**3/(a**2*x**2*atan(a*x)**2 + atan(a*x)**2), x)/c
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{(a^2cx^2+c)\arctan(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^3/((a^2*c*x^2 + c)*arctan(a*x)^2), x)
```

3.544 $\int \frac{x^2}{(c+a^2cx^2)\tan^{-1}(ax)^2}\,dx$

Optimal. Leaf size=36

$$\frac{2\text{Unintegrable}\left(\frac{x}{\tan^{-1}(ax)},x\right)}{ac}-\frac{x^2}{ac\tan^{-1}(ax)}$$

```
[Out] -(x^2/(a*c*ArcTan[a*x])) + (2*Unintegrable[x/ArcTan[a*x], x])/(a*c)
```

Rubi [A] time = 0.0730482, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^2}{(c+a^2cx^2)\tan^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^2/((c + a^2*c*x^2)*ArcTan[a*x]^2),x]

[Out] -(x^2/(a*c*ArcTan[a*x])) + (2*Defer[Int][x/ArcTan[a*x], x])/(a*c)
```

Rubi steps

$$\int \frac{x^2}{(c+a^2cx^2)\tan^{-1}(ax)^2}\,dx=-\frac{x^2}{ac\tan^{-1}(ax)}+\frac{2\int \frac{x}{\tan^{-1}(ax)}\,dx}{ac}$$

Mathematica [A] time = 0.549991, size = 0, normalized size = 0.

$$\int \frac{x^2}{(c+a^2cx^2)\tan^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^2/((c + a^2*c*x^2)*ArcTan[a*x]^2),x]
```

```
[Out] Integrate[x^2/((c + a^2*c*x^2)*ArcTan[a*x]^2), x]
```

Maple [A] time = 0.394, size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(a^2 c x^2 + c\right) \left(\arctan\left(a x\right)\right)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a^2*c*x^2+c)/arctan(a*x)^2,x)
```

```
[Out] int(x^2/(a^2*c*x^2+c)/arctan(a*x)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{x^2 - 2\,\arctan\left(a x\right) \int \frac{x}{\arctan(a x)} \, dx}{a c \arctan\left(a x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(x^2 - 2*arctan(a*x)*integrate(x/arctan(a*x), x))/(a*c*arctan(a*x))
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^2}{\left(a^2 c x^2 + c\right) \arctan\left(a x\right)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(x^2/((a^2*c*x^2 + c)*arctan(a*x)^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^2}{a^2x^2\,\mathrm{atan}^2\,(ax)+\mathrm{atan}^2\,(ax)}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a**2*c*x**2+c)/atan(a*x)**2,x)
```

```
[Out] Integral(x**2/(a**2*x**2*atan(a*x)**2 + atan(a*x)**2), x)/c
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(a^2cx^2+c\right)\arctan\,(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2/((a^2*c*x^2 + c)*arctan(a*x)^2), x)
```

3.545 $\int \frac{x}{\left(c+a^2cx^2\right)\tan^{-1}(ax)^2}\,dx$

Optimal. Leaf size=31

$$\frac{\text{Unintegrable}\left(\frac{1}{\tan^{-1}(ax)},x\right)}{ac}-\frac{x}{ac\tan^{-1}(ax)}$$

```
[Out] -(x/(a*c*ArcTan[a*x])) + Unintegrable[ArcTan[a*x]^(-1), x]/(a*c)
```

Rubi [A] time = 0.047012, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x}{\left(c+a^2cx^2\right)\tan^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x/((c + a^2*c*x^2)*ArcTan[a*x]^2),x]
```

```
[Out] -(x/(a*c*ArcTan[a*x])) + Defer[Int][ArcTan[a*x]^(-1), x]/(a*c)
```

Rubi steps

$$\int \frac{x}{\left(c+a^2cx^2\right)\tan^{-1}(ax)^2}\,dx=-\frac{x}{ac\tan^{-1}(ax)}+\frac{\int \frac{1}{\tan^{-1}(ax)}\,dx}{ac}$$

Mathematica [A] time = 0.281208, size = 0, normalized size = 0.

$$\int \frac{x}{\left(c+a^2cx^2\right)\tan^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x/((c + a^2*c*x^2)*ArcTan[a*x]^2),x]
```

```
[Out] Integrate[x/((c + a^2*c*x^2)*ArcTan[a*x]^2), x]
```

Maple [A] time = 0.134, size = 0, normalized size = 0.

$$\int \frac{x}{\left(a^{2} c\,x^{2}+c\right) \left(\arctan \left(a x\right)\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^2*c*x^2+c)/arctan(a*x)^2,x)
```

```
[Out] int(x/(a^2*c*x^2+c)/arctan(a*x)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{\arctan\left(ax\right)\int \frac{1}{\arctan\left(ax\right)}\, dx - x}{ac\arctan\left(ax\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] (arctan(a*x)*integrate(1/arctan(a*x), x) - x)/(a*c*arctan(a*x))
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x}{\left(a^{2} c x^{2} + c\right) \arctan\left(a x\right)^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(x/((a^2*c*x^2 + c)*arctan(a*x)^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x}{a^2x^2\,\mathrm{atan}^2(ax)+\mathrm{atan}^2(ax)}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a**2*c*x**2+c)/atan(a*x)**2,x)
```

```
[Out] Integral(x/(a**2*x**2*atan(a*x)**2 + atan(a*x)**2), x)/c
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(a^2cx^2+c)\arctan(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x/((a^2*c*x^2 + c)*arctan(a*x)^2), x)
```

3.546 $\int \frac{1}{(c+a^2cx^2)\tan^{-1}(ax)^2}\,dx$

Optimal. Leaf size=14

$$-\frac{1}{ac\tan^{-1}(ax)}$$

```
[Out] -(1/(a*c*ArcTan[a*x]))
```

Rubi [A] time = 0.0246541, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {4884}

$$-\frac{1}{ac\tan^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a^2*c*x^2)*ArcTan[a*x]^2),x]
```

```
[Out] -(1/(a*c*ArcTan[a*x]))
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(c+a^2cx^2\right)\tan^{-1}(ax)^2}\,dx = -\frac{1}{ac\tan^{-1}(ax)}$$

Mathematica [A] time = 0.0034184, size = 14, normalized size = 1.

$$-\frac{1}{ac\tan^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a^2*c*x^2)*ArcTan[a*x]^2),x]
```

```
[Out] -(1/(a*c*ArcTan[a*x]))
```

Maple [A] time = 0.055, size = 15, normalized size = 1.1

$$-\frac{1}{ac \arctan (ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2*c*x^2+c)/arctan(a*x)^2,x)
```

```
[Out] -1/a/c/arctan(a*x)
```

Maxima [A] time = 1.00949, size = 19, normalized size = 1.36

$$-\frac{1}{ac \arctan (ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] -1/(a*c*arctan(a*x))
```

Fricas [A] time = 1.60263, size = 30, normalized size = 2.14

$$-\frac{1}{ac \arctan (ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] -1/(a*c*arctan(a*x))
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*c*x**2+c)/atan(a*x)**2,x)
```

```
[Out] Exception raised: TypeError
```

Giac [A] time = 1.15975, size = 19, normalized size = 1.36

$$-\frac{1}{ac\arctan(ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] -1/(a*c*arctan(a*x))
```

3.547 $\int \frac{1}{x\left(c+a^2cx^2\right)\tan^{-1}(ax)^2}\,dx$

Optimal. Leaf size=38

$$-\frac{\text{Unintegrable}\left(\frac{1}{x^2\tan^{-1}(ax)},x\right)}{ac}-\frac{1}{acx\tan^{-1}(ax)}$$

```
[Out] -(1/(a*c*x*ArcTan[a*x])) - Unintegrable[1/(x^2*ArcTan[a*x]), x]/(a*c)
```

Rubi [A] time = 0.0778107, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x\left(c+a^2cx^2\right)\tan^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*(c + a^2*c*x^2)*ArcTan[a*x]^2),x]
```

```
[Out] -(1/(a*c*x*ArcTan[a*x])) - Defer[Int][1/(x^2*ArcTan[a*x]), x]/(a*c)
```

Rubi steps

$$\int \frac{1}{x\left(c+a^2cx^2\right)\tan^{-1}(ax)^2}\,dx=-\frac{1}{acx\tan^{-1}(ax)}-\frac{\int \frac{1}{x^2\tan^{-1}(ax)}\,dx}{ac}$$

Mathematica [A] time = 0.478529, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(c+a^2cx^2\right)\tan^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*(c + a^2*c*x^2)*ArcTan[a*x]^2),x]
```

[Out] Integrate[1/(x*(c + a^2*c*x^2)*ArcTan[a*x]^2), x]

Maple [A] time = 0.122, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(a^{2}c\,x^{2}+c\right)\left(\arctan\left(ax\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(a^2*c*x^2+c)/arctan(a*x)^2,x)

[Out] int(1/x/(a^2*c*x^2+c)/arctan(a*x)^2,x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{x\arctan\left(ax\right)\int\frac{1}{x^{2}\arctan\left(ax\right)}\,dx+1}{acx\arctan\left(ax\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="maxima")

[Out] -(x*arctan(a*x)*integrate(1/(x^2*arctan(a*x)), x) + 1)/(a*c*x*arctan(a*x))

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{1}{\left(a^{2}c\,x^{3}+cx\right)\arctan\left(ax\right)^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="fricas")

[Out] integral(1/((a^2*c*x^3 + c*x)*arctan(a*x)^2), x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{a^2x^3\operatorname{atan}^2(ax)+x\operatorname{atan}^2(ax)}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a**2*c*x**2+c)/atan(a*x)**2,x)
```

```
[Out] Integral(1/(a**2*x**3*atan(a*x)**2 + x*atan(a*x)**2), x)/c
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a^2cx^2+c)\,x\arctan(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)*x*arctan(a*x)^2), x)
```

3.548 $\int \frac{1}{x^2(c+a^2cx^2)\tan^{-1}(ax)^2}\,dx$

Optimal. Leaf size=38

$$-\frac{2\text{Unintegrable}\left(\frac{1}{x^3\tan^{-1}(ax)},x\right)}{ac}-\frac{1}{acx^2\tan^{-1}(ax)}$$

```
[Out] -(1/(a*c*x^2*ArcTan[a*x])) - (2*Unintegrable[1/(x^3*ArcTan[a*x]), x])/(a*c)
```

Rubi [A] time = 0.0818824, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)\tan^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^2*(c + a^2*c*x^2)*ArcTan[a*x]^2),x]
```

```
[Out] -(1/(a*c*x^2*ArcTan[a*x])) - (2*Defer[Int][1/(x^3*ArcTan[a*x]), x])/(a*c)
```

Rubi steps

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)\tan^{-1}(ax)^2}\,dx=-\frac{1}{acx^2\tan^{-1}(ax)}-\frac{2\int \frac{1}{x^3\tan^{-1}(ax)}\,dx}{ac}$$

Mathematica [A] time = 0.626985, size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)\tan^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^2*(c + a^2*c*x^2)*ArcTan[a*x]^2),x]
```

```
[Out] Integrate[1/(x^2*(c + a^2*c*x^2)*ArcTan[a*x]^2), x]
```

Maple [A] time = 0.391, size = 0, normalized size = 0.

$$\int \frac{1}{x^{2} \left(a^{2} c \,x^{2}+c \right) \left(\arctan \left(a x \right)\right)^{2}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a^2*c*x^2+c)/arctan(a*x)^2,x)
```

```
[Out] int(1/x^2/(a^2*c*x^2+c)/arctan(a*x)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{2 \, x^{2} \arctan \left(a x\right) \int \frac{1}{x^{3} \arctan\left(a x\right)}\,{d x} + 1}{a c x^{2} \arctan \left(a x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(2*x^2*arctan(a*x)*integrate(1/(x^3*arctan(a*x)), x) + 1)/(a*c*x^2*arctan(
a*x))
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$${\rm integral}\left(\frac{1}{{\left(a^{2} c x^{4} + c x^{2}\right)} \arctan \left(a x\right)^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(1/((a^2*c*x^4 + c*x^2)*arctan(a*x)^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{a^2x^4\operatorname{atan}^2(ax)+x^2\operatorname{atan}^2(ax)}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a**2*c*x**2+c)/atan(a*x)**2,x)
```

```
[Out] Integral(1/(a**2*x**4*atan(a*x)**2 + x**2*atan(a*x)**2), x)/c
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a^2cx^2+c)x^2\arctan(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)*x^2*arctan(a*x)^2), x)
```

3.549 $\int \frac{1}{x^3(c+a^2cx^2)\tan^{-1}(ax)^2}\,dx$

Optimal. Leaf size=38

$$-\frac{3\text{Unintegrable}\left(\frac{1}{x^4\tan^{-1}(ax)},x\right)}{ac}-\frac{1}{acx^3\tan^{-1}(ax)}$$

[Out] -(1/(a*c*x^3*ArcTan[a*x])) - (3*Unintegrable[1/(x^4*ArcTan[a*x]), x])/(a*c)

Rubi [A] time = 0.0823133, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^3\left(c+a^2cx^2\right)\tan^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

[In] Int[1/(x^3*(c + a^2*c*x^2)*ArcTan[a*x]^2),x]

[Out] -(1/(a*c*x^3*ArcTan[a*x])) - (3*Defer[Int][1/(x^4*ArcTan[a*x]), x])/(a*c)

Rubi steps

$$\int \frac{1}{x^3\left(c+a^2cx^2\right)\tan^{-1}(ax)^2}\,dx=-\frac{1}{acx^3\tan^{-1}(ax)}-\frac{3\int \frac{1}{x^4\tan^{-1}(ax)}\,dx}{ac}$$

Mathematica [A] time = 0.952813, size = 0, normalized size = 0.

$$\int \frac{1}{x^3\left(c+a^2cx^2\right)\tan^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

[In] Integrate[1/(x^3*(c + a^2*c*x^2)*ArcTan[a*x]^2),x]

```
[Out] Integrate[1/(x^3*(c + a^2*c*x^2)*ArcTan[a*x]^2), x]
```

Maple [A] time = 0.842, size = 0, normalized size = 0.

$$\int \frac{1}{x^{3} \left(a^{2} c\,x^{2}+c\right) \left(\arctan \left(a x\right)\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a^2*c*x^2+c)/arctan(a*x)^2,x)
```

```
[Out] int(1/x^3/(a^2*c*x^2+c)/arctan(a*x)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{3\,x^{3} \arctan \left(a x\right) \int \frac{1}{x^{4} \arctan\left(a x\right)}\, dx + 1}{a c x^{3} \arctan \left(a x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(3*x^3*arctan(a*x)*integrate(1/(x^4*arctan(a*x)), x) + 1)/(a*c*x^3*arctan(
a*x))
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\left(a^{2} c x^{5} + c x^{3}\right) \arctan\left(a x\right)^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(1/((a^2*c*x^5 + c*x^3)*arctan(a*x)^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{a^2x^5 \operatorname{atan}^2(ax)+x^3 \operatorname{atan}^2(ax)}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a**2*c*x**2+c)/atan(a*x)**2,x)
```

```
[Out] Integral(1/(a**2*x**5*atan(a*x)**2 + x**3*atan(a*x)**2), x)/c
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)x^3 \arctan(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)*x^3*arctan(a*x)^2), x)
```

3.550 $\int \frac{1}{x^4(c+a^2cx^2)\tan^{-1}(ax)^2} dx$

Optimal. Leaf size=38

$$-\frac{4\text{Unintegrable}\left(\frac{1}{x^5\tan^{-1}(ax)},x\right)}{ac}-\frac{1}{acx^4\tan^{-1}(ax)}$$

```
[Out] -(1/(a*c*x^4*ArcTan[a*x])) - (4*Unintegrable[1/(x^5*ArcTan[a*x]), x])/(a*c)
```

Rubi [A] time = 0.0818898, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^4(c+a^2cx^2)\tan^{-1}(ax)^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^4*(c + a^2*c*x^2)*ArcTan[a*x]^2),x]
```

```
[Out] -(1/(a*c*x^4*ArcTan[a*x])) - (4*Defer[Int][1/(x^5*ArcTan[a*x]), x])/(a*c)
```

Rubi steps

$$\int \frac{1}{x^4(c+a^2cx^2)\tan^{-1}(ax)^2} dx = -\frac{1}{acx^4\tan^{-1}(ax)}-\frac{4\int \frac{1}{x^5\tan^{-1}(ax)} dx}{ac}$$

Mathematica [A] time = 1.14125, size = 0, normalized size = 0.

$$\int \frac{1}{x^4(c+a^2cx^2)\tan^{-1}(ax)^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^4*(c + a^2*c*x^2)*ArcTan[a*x]^2),x]
```

```
[Out] Integrate[1/(x^4*(c + a^2*c*x^2)*ArcTan[a*x]^2), x]
```

Maple [A] time = 0.892, size = 0, normalized size = 0.

$$\int \frac{1}{x^{4} \left(a^{2} c x^{2}+c \right) \left(\arctan \left(a x \right)\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(a^2*c*x^2+c)/arctan(a*x)^2,x)
```

```
[Out] int(1/x^4/(a^2*c*x^2+c)/arctan(a*x)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{4 \, x^{4} \arctan \left(a x\right) \int \frac{1}{x^{5} \arctan\left(a x\right)}\,{d x} + 1}{a c x^{4} \arctan \left(a x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(4*x^4*arctan(a*x)*integrate(1/(x^5*arctan(a*x)), x) + 1)/(a*c*x^4*arctan(
a*x))
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$${\rm integral}\left(\frac{1}{{\left(a^{2} c x^{6} + c x^{4}\right)} \arctan \left(a x\right)^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(1/((a^2*c*x^6 + c*x^4)*arctan(a*x)^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{a^2x^6\,\mathrm{atan}^2\,(ax)+x^4\,\mathrm{atan}^2\,(ax)}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(a**2*c*x**2+c)/atan(a*x)**2,x)
```

```
[Out] Integral(1/(a**2*x**6*atan(a*x)**2 + x**4*atan(a*x)**2), x)/c
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)x^4\arctan\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)*x^4*arctan(a*x)^2), x)
```

3.551 $\int \frac{x^3}{\left(c+a^2cx^2\right)^2 \tan^{-1}(ax)^2} \, dx$

Optimal. Leaf size=71

$$\frac{\text{Unintegrable}\left(\frac{1}{\tan^{-1}(ax)},x\right)}{a^3c^2}-\frac{\text{CosIntegral}\left(2\tan^{-1}(ax)\right)}{a^4c^2}+\frac{x}{a^3c^2\left(a^2x^2+1\right)\tan^{-1}(ax)}-\frac{x}{a^3c^2\tan^{-1}(ax)}$$

```
[Out] -(x/(a^3*c^2*ArcTan[a*x])) + x/(a^3*c^2*(1 + a^2*x^2)*ArcTan[a*x]) - CosInt
egral[2*ArcTan[a*x]]/(a^4*c^2) + Unintegrable[ArcTan[a*x]^(-1), x]/(a^3*c^2
)
```

Rubi [A] time = 0.332047, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^3}{\left(c+a^2cx^2\right)^2 \tan^{-1}(ax)^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[x^3/((c + a^2*c*x^2)^2*ArcTan[a*x]^2),x]
```

```
[Out] -(x/(a^3*c^2*ArcTan[a*x])) + x/(a^3*c^2*(1 + a^2*x^2)*ArcTan[a*x]) - CosInt
egral[2*ArcTan[a*x]]/(a^4*c^2) + Defer[Int][ArcTan[a*x]^(-1), x]/(a^3*c^2)
```

Rubi steps

$$\int \frac{x^3}{\left(c+a^2cx^2\right)^2 \tan^{-1}(ax)^2}\,dx = -\frac{\int \frac{x}{\left(c+a^2cx^2\right)^2 \tan^{-1}(ax)^2}\,dx}{a^2} + \frac{\int \frac{x}{\left(c+a^2cx^2\right) \tan^{-1}(ax)^2}\,dx}{a^2c}$$

$$= -\frac{x}{a^3c^2\tan^{-1}(ax)} + \frac{x}{a^3c^2\left(1+a^2x^2\right)\tan^{-1}(ax)} - \frac{\int \frac{1}{\left(c+a^2cx^2\right)^2 \tan^{-1}(ax)}\,dx}{a^3} + \frac{\int \frac{x^2}{\left(c+a^2cx^2\right)^2 \tan^{-1}}}{a}$$

$$= -\frac{x}{a^3c^2\tan^{-1}(ax)} + \frac{x}{a^3c^2\left(1+a^2x^2\right)\tan^{-1}(ax)} - \frac{\text{Subst}\left(\int \frac{\cos^2(x)}{x}\,dx, x, \tan^{-1}(ax)\right)}{a^4c^2} + \frac{\text{Su}}{}$$

$$= -\frac{x}{a^3c^2\tan^{-1}(ax)} + \frac{x}{a^3c^2\left(1+a^2x^2\right)\tan^{-1}(ax)} + \frac{\text{Subst}\left(\int \left(\frac{1}{2x} - \frac{\cos(2x)}{2x}\right)dx, x, \tan^{-1}(ax\right.}{a^4c^2}$$

$$= -\frac{x}{a^3c^2\tan^{-1}(ax)} + \frac{x}{a^3c^2\left(1+a^2x^2\right)\tan^{-1}(ax)} - 2\frac{\text{Subst}\left(\int \frac{\cos(2x)}{x}\,dx, x, \tan^{-1}(ax)\right)}{2a^4c^2} + \frac{\int}{}$$

$$= -\frac{x}{a^3c^2\tan^{-1}(ax)} + \frac{x}{a^3c^2\left(1+a^2x^2\right)\tan^{-1}(ax)} - \frac{\text{Ci}\left(2\tan^{-1}(ax)\right)}{a^4c^2} + \frac{\int \frac{1}{\tan^{-1}(ax)}\,dx}{a^3c^2}$$

Mathematica [A] time = 4.063, size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(c+a^2cx^2\right)^2 \tan^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^3/((c + a^2*c*x^2)^2*ArcTan[a*x]^2),x]
```

```
[Out] Integrate[x^3/((c + a^2*c*x^2)^2*ArcTan[a*x]^2), x]
```

Maple [A] time = 0.482, size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(a^2cx^2+c\right)^2 \left(\arctan(ax)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a^2*c*x^2+c)^2/arctan(a*x)^2,x)
```

```
[Out] int(x^3/(a^2*c*x^2+c)^2/arctan(a*x)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{x^3 - \frac{\left(a^3c^2x^2+ac^2\right)\left(a^2\int \frac{x^4}{a^4x^4 \arctan(ax)+2\,a^2x^2 \arctan(ax)+\arctan(ax)}\,dx+3\int \frac{x^2}{a^4x^4 \arctan(ax)+2\,a^2x^2 \arctan(ax)+\arctan(ax)}\,dx\right)\arctan(ax)}{ac^2}}{\left(a^3c^2x^2 + ac^2\right)\arctan\left(ax\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(x^3 - (a^3*c^2*x^2 + a*c^2)*arctan(a*x)*integrate((a^2*x^4 + 3*x^2)/((a^5
*c^2*x^4 + 2*a^3*c^2*x^2 + a*c^2)*arctan(a*x)), x))/((a^3*c^2*x^2 + a*c^2)*
arctan(a*x))
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^3}{\left(a^4c^2x^4 + 2\,a^2c^2x^2 + c^2\right)\arctan\left(ax\right)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(x^3/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(a*x)^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^3}{a^4x^4 \operatorname{atan}^2(ax)+2a^2x^2 \operatorname{atan}^2(ax)+\operatorname{atan}^2(ax)}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a**2*c*x**2+c)**2/atan(a*x)**2,x)
```

```
[Out] Integral(x**3/(a**4*x**4*atan(a*x)**2 + 2*a**2*x**2*atan(a*x)**2 + atan(a*x
)**2), x)/c**2
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(a^2cx^2+c\right)^2 \arctan\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^3/((a^2*c*x^2 + c)^2*arctan(a*x)^2), x)
```

3.552 $\int \frac{x^2}{\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^2}\,dx$

Optimal. Leaf size=43

$$\frac{\operatorname{Si}\left(2\tan^{-1}(ax)\right)}{a^3c^2}-\frac{x^2}{ac^2\left(a^2x^2+1\right)\tan^{-1}(ax)}$$

```
[Out] -(x^2/(a*c^2*(1 + a^2*x^2)*ArcTan[a*x])) + SinIntegral[2*ArcTan[a*x]]/(a^3*
c^2)
```

Rubi [A] time = 0.138606, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {4942, 4970, 4406, 12, 3299}

$$\frac{\operatorname{Si}\left(2\tan^{-1}(ax)\right)}{a^3c^2}-\frac{x^2}{ac^2\left(a^2x^2+1\right)\tan^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((c + a^2*c*x^2)^2*ArcTan[a*x]^2),x]
```

```
[Out] -(x^2/(a*c^2*(1 + a^2*x^2)*ArcTan[a*x])) + SinIntegral[2*ArcTan[a*x]]/(a^3*
c^2)
```

Rule 4942

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^m*(d + e*x^2)^(q + 1)*(a + b*ArcT
an[c*x])^(p + 1))/(b*c*d*(p + 1)), x] - Dist[(f*m)/(b*c*(p + 1)), Int[(f*x)
^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 2, 0] && LtQ[p, -1]
```

Rule 4970

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sin[x]^m)/
Cos[x]^(m + 2*(q + 1)), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^2}\,dx &= -\frac{x^2}{ac^2\left(1+a^2x^2\right)\tan^{-1}(ax)}+\frac{2\int \frac{x}{\left(c+a^2cx^2\right)^2\tan^{-1}(ax)}\,dx}{a}\\
&= -\frac{x^2}{ac^2\left(1+a^2x^2\right)\tan^{-1}(ax)}+\frac{2\,\text{Subst}\left(\int \frac{\cos(x)\sin(x)}{x}\,dx,x,\tan^{-1}(ax)\right)}{a^3c^2}\\
&= -\frac{x^2}{ac^2\left(1+a^2x^2\right)\tan^{-1}(ax)}+\frac{2\,\text{Subst}\left(\int \frac{\sin(2x)}{2x}\,dx,x,\tan^{-1}(ax)\right)}{a^3c^2}\\
&= -\frac{x^2}{ac^2\left(1+a^2x^2\right)\tan^{-1}(ax)}+\frac{\text{Subst}\left(\int \frac{\sin(2x)}{x}\,dx,x,\tan^{-1}(ax)\right)}{a^3c^2}\\
&= -\frac{x^2}{ac^2\left(1+a^2x^2\right)\tan^{-1}(ax)}+\frac{\text{Si}\left(2\tan^{-1}(ax)\right)}{a^3c^2}
\end{aligned}$$

Mathematica [A] time = 0.109486, size = 40, normalized size = 0.93

$$\frac{\text{Si}\left(2\tan^{-1}(ax)\right)-\frac{a^2x^2}{\left(a^2x^2+1\right)\tan^{-1}(ax)}}{a^3c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((c + a^2*c*x^2)^2*ArcTan[a*x]^2),x]
```

```
[Out] (-((a^2*x^2)/((1 + a^2*x^2)*ArcTan[a*x])) + SinIntegral[2*ArcTan[a*x]])/(a^
3*c^2)
```

Maple [A] time = 0.065, size = 37, normalized size = 0.9

$$\frac{2\,\mathrm{Si}\left(2\arctan\left(ax\right)\right)\arctan\left(ax\right)+\cos\left(2\arctan\left(ax\right)\right)-1}{2\,a^{3}c^{2}\arctan\left(ax\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a^2*c*x^2+c)^2/arctan(a*x)^2,x)
```

```
[Out] 1/2/a^3/c^2*(2*Si(2*arctan(a*x))*arctan(a*x)+cos(2*arctan(a*x))-1)/arctan(a
*x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{2\left(a^{3}c^{2}x^{2}+ac^{2}\right)\arctan\left(ax\right)\int\frac{x}{\left(a^{5}c^{2}x^{4}+2\,a^{3}c^{2}x^{2}+ac^{2}\right)\arctan\left(ax\right)}\,dx-x^{2}}{\left(a^{3}c^{2}x^{2}+ac^{2}\right)\arctan\left(ax\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] (4*(a^3*c^2*x^2 + a*c^2)*arctan(a*x)*integrate(1/2*x/((a^5*c^2*x^4 + 2*a^3*
c^2*x^2 + a*c^2)*arctan(a*x)), x) - x^2)/((a^3*c^2*x^2 + a*c^2)*arctan(a*x)
)
```

Fricas [C] time = 1.97127, size = 301, normalized size = 7.

$$-\frac{2\,a^{2}x^{2}-\left(i\,a^{2}x^{2}+i\right)\arctan\left(ax\right)\mathrm{log\_integral}\left(-\frac{a^{2}x^{2}+2i\,ax-1}{a^{2}x^{2}+1}\right)-\left(-i\,a^{2}x^{2}-i\right)\arctan\left(ax\right)\mathrm{log\_integral}\left(-\frac{a^{2}x^{2}-2i\,ax-1}{a^{2}x^{2}+1}\right)}{2\left(a^{5}c^{2}x^{2}+a^{3}c^{2}\right)\arctan\left(ax\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^2*x^2 - (I*a^2*x^2 + I)*arctan(a*x)*log_integral(-(a^2*x^2 + 2*I*
a*x - 1)/(a^2*x^2 + 1)) - (-I*a^2*x^2 - I)*arctan(a*x)*log_integral(-(a^2*x
^2 - 2*I*a*x - 1)/(a^2*x^2 + 1)))/((a^5*c^2*x^2 + a^3*c^2)*arctan(a*x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^2}{a^4x^4\operatorname{atan}^2(ax)+2a^2x^2\operatorname{atan}^2(ax)+\operatorname{atan}^2(ax)}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a**2*c*x**2+c)**2/atan(a*x)**2,x)
```

```
[Out] Integral(x**2/(a**4*x**4*atan(a*x)**2 + 2*a**2*x**2*atan(a*x)**2 + atan(a*x
)**2), x)/c**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(a^2cx^2+c\right)^2\arctan(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2/((a^2*c*x^2 + c)^2*arctan(a*x)^2), x)
```

3.553 $\int \frac{x}{(c+a^2cx^2)^2 \tan^{-1}(ax)^2} dx$

Optimal. Leaf size=41

$$\frac{\text{CosIntegral}\left(2\tan^{-1}(ax)\right)}{a^2c^2} - \frac{x}{ac^2\left(a^2x^2+1\right)\tan^{-1}(ax)}$$

```
[Out] -(x/(a*c^2*(1 + a^2*x^2)*ArcTan[a*x])) + CosIntegral[2*ArcTan[a*x]]/(a^2*c^
2)
```

Rubi [A] time = 0.210444, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4968, 4970, 3312, 3302, 4904}

$$\frac{\text{CosIntegral}\left(2\tan^{-1}(ax)\right)}{a^2c^2} - \frac{x}{ac^2\left(a^2x^2+1\right)\tan^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[x/((c + a^2*c*x^2)^2*ArcTan[a*x]^2),x]
```

```
[Out] -(x/(a*c^2*(1 + a^2*x^2)*ArcTan[a*x])) + CosIntegral[2*ArcTan[a*x]]/(a^2*c^
2)
```

Rule 4968

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[(x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^(p
+ 1))/(b*c*d*(p + 1)), x] + (-Dist[(c*(m + 2*q + 2))/(b*(p + 1)), Int[x^(m
+ 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p + 1
)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; Fre
eQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &&
LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 4970

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sin[x]^m)/
Cos[x]^(m + 2*(q + 1)), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
```

```
|| GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 4904

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x}{\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^2}\,dx = -\frac{x}{ac^2\left(1+a^2x^2\right)\tan^{-1}(ax)}+\frac{\int \frac{1}{\left(c+a^2cx^2\right)^2\tan^{-1}(ax)}\,dx}{a}-a\int \frac{x^2}{\left(c+a^2cx^2\right)^2\tan^{-1}(ax)}\,dx$$

$$= -\frac{x}{ac^2\left(1+a^2x^2\right)\tan^{-1}(ax)}+\frac{\text{Subst}\left(\int \frac{\cos^2(x)}{x}\,dx,x,\tan^{-1}(ax)\right)}{a^2c^2}-\frac{\text{Subst}\left(\int \frac{\sin^2(x)}{x}\,dx,x,\right.}{a^2c^2}$$

$$= -\frac{x}{ac^2\left(1+a^2x^2\right)\tan^{-1}(ax)}-\frac{\text{Subst}\left(\int \left(\frac{1}{2x}-\frac{\cos(2x)}{2x}\right)dx,x,\tan^{-1}(ax)\right)}{a^2c^2}+\frac{\text{Subst}\left(\int \left(\frac{1}{2x}\right.\right.}{}$$

$$= -\frac{x}{ac^2\left(1+a^2x^2\right)\tan^{-1}(ax)}+2\frac{\text{Subst}\left(\int \frac{\cos(2x)}{x}\,dx,x,\tan^{-1}(ax)\right)}{2a^2c^2}$$

$$= -\frac{x}{ac^2\left(1+a^2x^2\right)\tan^{-1}(ax)}+\frac{\text{Ci}\left(2\tan^{-1}(ax)\right)}{a^2c^2}$$

Mathematica [A] time = 0.0696527, size = 36, normalized size = 0.88

$$\frac{\text{CosIntegral}\left(2\tan^{-1}(ax)\right)-\frac{ax}{\left(a^2x^2+1\right)\tan^{-1}(ax)}}{a^2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((c + a^2*c*x^2)^2*ArcTan[a*x]^2),x]
```

```
[Out] (-((a*x)/((1 + a^2*x^2)*ArcTan[a*x])) + CosIntegral[2*ArcTan[a*x]])/(a^2*c^
2)
```

Maple [A] time = 0.058, size = 38, normalized size = 0.9

$$\frac{2\,\mathrm{Ci}\left(2\,\arctan\left(ax\right)\right)\arctan\left(ax\right)-\sin\left(2\,\arctan\left(ax\right)\right)}{2\,a^2c^2\arctan\left(ax\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^2*c*x^2+c)^2/arctan(a*x)^2,x)
```

```
[Out] 1/2/a^2/c^2*(2*Ci(2*arctan(a*x))*arctan(a*x)-sin(2*arctan(a*x)))/arctan(a*x
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{x+\frac{\left(a^3c^2x^2+ac^2\right)\left(a^2\int\frac{x^2}{a^4x^4\arctan(ax)+2\,a^2x^2\arctan(ax)+\arctan(ax)}\,dx-\int\frac{1}{a^4x^4\arctan(ax)+2\,a^2x^2\arctan(ax)+\arctan(ax)}\,dx\right)\arctan(ax)}{ac^2}}{\left(a^3c^2x^2+ac^2\right)\arctan\left(ax\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] -((a^3*c^2*x^2 + a*c^2)*arctan(a*x)*integrate((a^2*x^2 - 1)/((a^5*c^2*x^4 +
 2*a^3*c^2*x^2 + a*c^2)*arctan(a*x)), x) + x)/((a^3*c^2*x^2 + a*c^2)*arctan
(a*x))
```

Fricas [C] time = 1.98861, size = 288, normalized size = 7.02

$$\frac{\left(a^2x^2+1\right)\arctan(ax)\,\text{log\_integral}\left(-\frac{a^2x^2+2i\,ax-1}{a^2x^2+1}\right)+\left(a^2x^2+1\right)\arctan(ax)\,\text{log\_integral}\left(-\frac{a^2x^2-2i\,ax-1}{a^2x^2+1}\right)-2\,ax}{2\left(a^4c^2x^2+a^2c^2\right)\arctan(ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/2*((a^2*x^2 + 1)*arctan(a*x)*log_integral(-(a^2*x^2 + 2*I*a*x - 1)/(a^2*x
^2 + 1)) + (a^2*x^2 + 1)*arctan(a*x)*log_integral(-(a^2*x^2 - 2*I*a*x - 1)/
(a^2*x^2 + 1)) - 2*a*x)/((a^4*c^2*x^2 + a^2*c^2)*arctan(a*x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x}{a^4x^4\,\text{atan}^2(ax)+2a^2x^2\,\text{atan}^2(ax)+\text{atan}^2(ax)}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a**2*c*x**2+c)**2/atan(a*x)**2,x)
```

```
[Out] Integral(x/(a**4*x**4*atan(a*x)**2 + 2*a**2*x**2*atan(a*x)**2 + atan(a*x)**
2), x)/c**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(a^2cx^2+c\right)^2\arctan(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x/((a^2*c*x^2 + c)^2*arctan(a*x)^2), x)
```

3.554 $\int \frac{1}{\left(c+a^2cx^2\right)^2 \tan^{-1}(ax)^2} dx$

Optimal. Leaf size=41

$$-\frac{1}{ac^2\left(a^2x^2+1\right)\tan^{-1}(ax)}-\frac{\text{Si}\left(2\tan^{-1}(ax)\right)}{ac^2}$$

```
[Out] -(1/(a*c^2*(1 + a^2*x^2)*ArcTan[a*x])) - SinIntegral[2*ArcTan[a*x]]/(a*c^2)
```

Rubi [A] time = 0.0930027, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {4902, 4970, 4406, 12, 3299}

$$-\frac{1}{ac^2\left(a^2x^2+1\right)\tan^{-1}(ax)}-\frac{\text{Si}\left(2\tan^{-1}(ax)\right)}{ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a^2*c*x^2)^2*ArcTan[a*x]^2),x]
```

```
[Out] -(1/(a*c^2*(1 + a^2*x^2)*ArcTan[a*x])) - SinIntegral[2*ArcTan[a*x]]/(a*c^2)
```

Rule 4902

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^(p + 1))/(b*c*d*(p
+ 1)), x] - Dist[(2*c*(q + 1))/(b*(p + 1)), Int[x*(d + e*x^2)^q*(a + b*ArcT
an[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && L
tQ[q, -1] && LtQ[p, -1]
```

Rule 4970

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sin[x]^m)/
Cos[x]^(m + 2*(q + 1)), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^2}\,dx &= -\frac{1}{ac^2\left(1+a^2x^2\right)\tan^{-1}(ax)}-(2a)\int \frac{x}{\left(c+a^2cx^2\right)^2\tan^{-1}(ax)}\,dx\\
&= -\frac{1}{ac^2\left(1+a^2x^2\right)\tan^{-1}(ax)}-\frac{2\,\text{Subst}\left(\int \frac{\cos(x)\sin(x)}{x}\,dx,x,\tan^{-1}(ax)\right)}{ac^2}\\
&= -\frac{1}{ac^2\left(1+a^2x^2\right)\tan^{-1}(ax)}-\frac{2\,\text{Subst}\left(\int \frac{\sin(2x)}{2x}\,dx,x,\tan^{-1}(ax)\right)}{ac^2}\\
&= -\frac{1}{ac^2\left(1+a^2x^2\right)\tan^{-1}(ax)}-\frac{\text{Subst}\left(\int \frac{\sin(2x)}{x}\,dx,x,\tan^{-1}(ax)\right)}{ac^2}\\
&= -\frac{1}{ac^2\left(1+a^2x^2\right)\tan^{-1}(ax)}-\frac{\text{Si}\left(2\tan^{-1}(ax)\right)}{ac^2}
\end{aligned}$$

Mathematica [A] time = 0.0641981, size = 34, normalized size = 0.83

$$-\frac{\frac{1}{a^2x^2\tan^{-1}(ax)+\tan^{-1}(ax)}+\text{Si}\left(2\tan^{-1}(ax)\right)}{ac^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a^2*c*x^2)^2*ArcTan[a*x]^2),x]
```

```
[Out] -(((ArcTan[a*x] + a^2*x^2*ArcTan[a*x])^(-1) + SinIntegral[2*ArcTan[a*x]])/(
a*c^2))
```

Maple [A] time = 0.061, size = 37, normalized size = 0.9

$$-\frac{2\,\mathrm{Si}\left(2\arctan\left(ax\right)\right)\arctan\left(ax\right)+\cos\left(2\arctan\left(ax\right)\right)+1}{2\,ac^{2}\arctan\left(ax\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2*c*x^2+c)^2/arctan(a*x)^2,x)
```

```
[Out] -1/2/a/c^2*(2*Si(2*arctan(a*x))*arctan(a*x)+cos(2*arctan(a*x))+1)/arctan(a*
x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2\left(a^{4}c^{2}x^{2}+a^{2}c^{2}\right)\arctan\left(ax\right)\int\frac{x}{\left(a^{4}c^{2}x^{4}+2\,a^{2}c^{2}x^{2}+c^{2}\right)\arctan(ax)}\,dx+1}{\left(a^{3}c^{2}x^{2}+ac^{2}\right)\arctan\left(ax\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(4*(a^4*c^2*x^2 + a^2*c^2)*arctan(a*x)*integrate(1/2*x/((a^4*c^2*x^4 + 2*a
^2*c^2*x^2 + c^2)*arctan(a*x)), x) + 1)/((a^3*c^2*x^2 + a*c^2)*arctan(a*x))
```

Fricas [C] time = 2.07139, size = 286, normalized size = 6.98

$$\frac{\left(-i\,a^{2}x^{2}-i\right)\arctan\left(ax\right)\mathrm{log\_integral}\left(-\frac{a^{2}x^{2}+2i\,ax-1}{a^{2}x^{2}+1}\right)+\left(i\,a^{2}x^{2}+i\right)\arctan\left(ax\right)\mathrm{log\_integral}\left(-\frac{a^{2}x^{2}-2i\,ax-1}{a^{2}x^{2}+1}\right)-2}{2\left(a^{3}c^{2}x^{2}+ac^{2}\right)\arctan\left(ax\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/2*((-I*a^2*x^2 - I)*arctan(a*x)*log_integral(-(a^2*x^2 + 2*I*a*x - 1)/(a^
2*x^2 + 1)) + (I*a^2*x^2 + I)*arctan(a*x)*log_integral(-(a^2*x^2 - 2*I*a*x
- 1)/(a^2*x^2 + 1)) - 2)/((a^3*c^2*x^2 + a*c^2)*arctan(a*x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{a^4x^4\operatorname{atan}^2(ax)+2a^2x^2\operatorname{atan}^2(ax)+\operatorname{atan}^2(ax)}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*c*x**2+c)**2/atan(a*x)**2,x)
```

```
[Out] Integral(1/(a**4*x**4*atan(a*x)**2 + 2*a**2*x**2*atan(a*x)**2 + atan(a*x)**
2), x)/c**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a^2cx^2+c)^2\arctan(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^2*arctan(a*x)^2), x)
```

3.555 $\int \frac{1}{x\left(c+a^2cx^2\right)^2 \tan^{-1}(ax)^2} dx$

Optimal. Leaf size=73

$$-\frac{\text{Unintegrable}\left(\frac{1}{x^2\tan^{-1}(ax)},x\right)}{ac^2}+\frac{ax}{c^2\left(a^2x^2+1\right)\tan^{-1}(ax)}-\frac{\text{CosIntegral}\left(2\tan^{-1}(ax)\right)}{c^2}-\frac{1}{ac^2x\tan^{-1}(ax)}$$

```
[Out] -(1/(a*c^2*x*ArcTan[a*x])) + (a*x)/(c^2*(1 + a^2*x^2)*ArcTan[a*x]) - CosInt
egral[2*ArcTan[a*x]]/c^2 - Unintegrable[1/(x^2*ArcTan[a*x]), x]/(a*c^2)
```

Rubi [A] time = 0.357018, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x\left(c+a^2cx^2\right)^2 \tan^{-1}(ax)^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*(c + a^2*c*x^2)^2*ArcTan[a*x]^2),x]
```

```
[Out] -(1/(a*c^2*x*ArcTan[a*x])) + (a*x)/(c^2*(1 + a^2*x^2)*ArcTan[a*x]) - CosInt
egral[2*ArcTan[a*x]]/c^2 - Defer[Int][1/(x^2*ArcTan[a*x]), x]/(a*c^2)
```

Rubi steps

$$\int \frac{1}{x\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^2}\,dx = -\left(a^2\int \frac{x}{\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^2}\,dx\right) + \frac{\int \frac{1}{x\left(c+a^2cx^2\right)\tan^{-1}(ax)^2}\,dx}{c}$$

$$= -\frac{1}{ac^2x\tan^{-1}(ax)} + \frac{ax}{c^2\left(1+a^2x^2\right)\tan^{-1}(ax)} - a\int \frac{1}{\left(c+a^2cx^2\right)^2\tan^{-1}(ax)}\,dx + a^3\int$$

$$= -\frac{1}{ac^2x\tan^{-1}(ax)} + \frac{ax}{c^2\left(1+a^2x^2\right)\tan^{-1}(ax)} - \frac{\text{Subst}\left(\int \frac{\cos^2(x)}{x}\,dx, x, \tan^{-1}(ax)\right)}{c^2} + \text{Su}$$

$$= -\frac{1}{ac^2x\tan^{-1}(ax)} + \frac{ax}{c^2\left(1+a^2x^2\right)\tan^{-1}(ax)} + \frac{\text{Subst}\left(\int \left(\frac{1}{2x} - \frac{\cos(2x)}{2x}\right)dx, x, \tan^{-1}(ax\right.}{c^2}$$

$$= -\frac{1}{ac^2x\tan^{-1}(ax)} + \frac{ax}{c^2\left(1+a^2x^2\right)\tan^{-1}(ax)} - 2\frac{\text{Subst}\left(\int \frac{\cos(2x)}{x}\,dx, x, \tan^{-1}(ax)\right)}{2c^2} - \int$$

$$= -\frac{1}{ac^2x\tan^{-1}(ax)} + \frac{ax}{c^2\left(1+a^2x^2\right)\tan^{-1}(ax)} - \frac{\text{Ci}\left(2\tan^{-1}(ax)\right)}{c^2} - \frac{\int \frac{1}{x^2\tan^{-1}(ax)}\,dx}{ac^2}$$

Mathematica [A] time = 1.48752, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*(c + a^2*c*x^2)^2*ArcTan[a*x]^2),x]

[Out] Integrate[1/(x*(c + a^2*c*x^2)^2*ArcTan[a*x]^2), x]
```

Maple [A] time = 0.246, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(a^2cx^2+c\right)^2(\arctan(ax))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^2,x)
```

```
[Out] int(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{\frac{\left(a^3c^2x^3+ac^2x\right)\left(3\,a^2\int\frac{x^2}{a^4x^6\arctan(ax)+2\,a^2x^4\arctan(ax)+x^2\arctan(ax)}\,dx+\int\frac{1}{\left(a^2x^2+1\right)^2x^2\arctan(ax)}\,dx\right)\arctan(ax)}{ac^2}+1}{\left(a^3c^2x^3+ac^2x\right)\arctan\left(ax\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] -((a^3*c^2*x^3 + a*c^2*x)*arctan(a*x)*integrate((3*a^2*x^2 + 1)/((a^5*c^2*x
^6 + 2*a^3*c^2*x^4 + a*c^2*x^2)*arctan(a*x)), x) + 1)/((a^3*c^2*x^3 + a*c^2
*x)*arctan(a*x))
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\left(a^4c^2x^5+2\,a^2c^2x^3+c^2x\right)\arctan\left(ax\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(1/((a^4*c^2*x^5 + 2*a^2*c^2*x^3 + c^2*x)*arctan(a*x)^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{1}{a^4x^5\operatorname{atan}^2(ax)+2a^2x^3\operatorname{atan}^2(ax)+x\operatorname{atan}^2(ax)}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a**2*c*x**2+c)**2/atan(a*x)**2,x)
```

```
[Out] Integral(1/(a**4*x**5*atan(a*x)**2 + 2*a**2*x**3*atan(a*x)**2 + x*atan(a*x)
**2), x)/c**2
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)^2 x \arctan\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^2*x*arctan(a*x)^2), x)
```

3.556 $\int \frac{1}{x^2(c+a^2cx^2)^2\tan^{-1}(ax)^2}\,dx$

Optimal. Leaf size=72

$$-\frac{2\text{Unintegrable}\left(\frac{1}{x^3\tan^{-1}(ax)},x\right)}{ac^2}+\frac{a}{c^2\left(a^2x^2+1\right)\tan^{-1}(ax)}+\frac{a\text{Si}\left(2\tan^{-1}(ax)\right)}{c^2}-\frac{1}{ac^2x^2\tan^{-1}(ax)}$$

```
[Out] -(1/(a*c^2*x^2*ArcTan[a*x])) + a/(c^2*(1 + a^2*x^2)*ArcTan[a*x]) + (a*SinIn
tegral[2*ArcTan[a*x]])/c^2 - (2*Unintegrable[1/(x^3*ArcTan[a*x]), x])/(a*c^
2)
```

Rubi [A] time = 0.244267, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]^2),x]
```

```
[Out] -(1/(a*c^2*x^2*ArcTan[a*x])) + a/(c^2*(1 + a^2*x^2)*ArcTan[a*x]) + (a*SinIn
tegral[2*ArcTan[a*x]])/c^2 - (2*Defer[Int][1/(x^3*ArcTan[a*x]), x])/(a*c^2)
```

Rubi steps

$$\int \frac{1}{x^2 \left(c+a^2 c x^2\right)^2 \tan^{-1}(ax)^2} \, dx = -\left(a^2 \int \frac{1}{\left(c+a^2 c x^2\right)^2 \tan^{-1}(ax)^2} \, dx\right) + \frac{\int \frac{1}{x^2\left(c+a^2 c x^2\right) \tan^{-1}(ax)^2} \, dx}{c}$$

$$= -\frac{1}{a c^2 x^2 \tan^{-1}(ax)} + \frac{a}{c^2 \left(1+a^2 x^2\right) \tan^{-1}(ax)} + \left(2a^3\right) \int \frac{x}{\left(c+a^2 c x^2\right)^2 \tan^{-1}(ax)} \, dx - \cdots$$

$$= -\frac{1}{a c^2 x^2 \tan^{-1}(ax)} + \frac{a}{c^2 \left(1+a^2 x^2\right) \tan^{-1}(ax)} - \frac{2 \int \frac{1}{x^3 \tan^{-1}(ax)} \, dx}{a c^2} + \frac{(2a) \operatorname{Subst}\left(\int \cos \cdots\right)}{\cdots}$$

$$= -\frac{1}{a c^2 x^2 \tan^{-1}(ax)} + \frac{a}{c^2 \left(1+a^2 x^2\right) \tan^{-1}(ax)} - \frac{2 \int \frac{1}{x^3 \tan^{-1}(ax)} \, dx}{a c^2} + \frac{(2a) \operatorname{Subst}\left(\int \frac{\sin \cdots}{2 \cdots}\right)}{\cdots}$$

$$= -\frac{1}{a c^2 x^2 \tan^{-1}(ax)} + \frac{a}{c^2 \left(1+a^2 x^2\right) \tan^{-1}(ax)} - \frac{2 \int \frac{1}{x^3 \tan^{-1}(ax)} \, dx}{a c^2} + \frac{a \operatorname{Subst}\left(\int \frac{\sin(2x)}{x} \cdots\right)}{c \cdots}$$

$$= -\frac{1}{a c^2 x^2 \tan^{-1}(ax)} + \frac{a}{c^2 \left(1+a^2 x^2\right) \tan^{-1}(ax)} + \frac{a \operatorname{Si}\left(2 \tan^{-1}(ax)\right)}{c^2} - \frac{2 \int \frac{1}{x^3 \tan^{-1}(ax)} \, dx}{a c^2} \cdots$$

Mathematica [A] time = 2.54306, size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \left(c+a^2 c x^2\right)^2 \tan^{-1}(ax)^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]^2),x]
```

```
[Out] Integrate[1/(x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]^2), x]
```

Maple [A] time = 0.29, size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \left(a^2 c x^2 + c\right)^2 (\arctan(ax))^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a^2*c*x^2+c)^2/arctan(a*x)^2,x)
```

```
[Out] int(1/x^2/(a^2*c*x^2+c)^2/arctan(a*x)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{2\left(a^3c^2x^4+ac^2x^2\right)\arctan\left(ax\right)\int\frac{2\,a^2x^2+1}{\left(a^5c^2x^7+2\,a^3c^2x^5+ac^2x^3\right)\arctan(ax)}\,dx+1}{\left(a^3c^2x^4+ac^2x^2\right)\arctan\left(ax\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] -((a^3*c^2*x^4 + a*c^2*x^2)*arctan(a*x)*integrate(2*(2*a^2*x^2 + 1)/((a^5*c
^2*x^7 + 2*a^3*c^2*x^5 + a*c^2*x^3)*arctan(a*x)), x) + 1)/((a^3*c^2*x^4 + a
*c^2*x^2)*arctan(a*x))
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\left(a^4c^2x^6+2\,a^2c^2x^4+c^2x^2\right)\arctan\left(ax\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(1/((a^4*c^2*x^6 + 2*a^2*c^2*x^4 + c^2*x^2)*arctan(a*x)^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{1}{a^4x^6\operatorname{atan}^2\left(ax\right)+2a^2x^4\operatorname{atan}^2\left(ax\right)+x^2\operatorname{atan}^2\left(ax\right)}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a**2*c*x**2+c)**2/atan(a*x)**2,x)
```

```
[Out] Integral(1/(a**4*x**6*atan(a*x)**2 + 2*a**2*x**4*atan(a*x)**2 + x**2*atan(a
*x)**2), x)/c**2
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)^2 x^2 \arctan\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^2*x^2*arctan(a*x)^2), x)
```

3.557 $\int \frac{1}{x^3(c+a^2cx^2)^2\tan^{-1}(ax)^2}\,dx$

Optimal. Leaf size=109

$$\frac{a\text{Unintegrable}\left(\frac{1}{x^2\tan^{-1}(ax)},x\right)}{c^2}-\frac{3\text{Unintegrable}\left(\frac{1}{x^4\tan^{-1}(ax)},x\right)}{ac^2}+\frac{a^2\text{CosIntegral}\left(2\tan^{-1}(ax)\right)}{c^2}-\frac{a^3x}{c^2\left(a^2x^2+1\right)\tan^{-}}$$

```
[Out] -(1/(a*c^2*x^3*ArcTan[a*x])) + a/(c^2*x*ArcTan[a*x]) - (a^3*x)/(c^2*(1 + a^
2*x^2)*ArcTan[a*x]) + (a^2*CosIntegral[2*ArcTan[a*x]])/c^2 - (3*Unintegrabl
e[1/(x^4*ArcTan[a*x]), x])/(a*c^2) + (a*Unintegrable[1/(x^2*ArcTan[a*x]), x
])/c^2
```

Rubi [A] time = 0.499298, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^3\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^3*(c + a^2*c*x^2)^2*ArcTan[a*x]^2),x]
```

```
[Out] -(1/(a*c^2*x^3*ArcTan[a*x])) + a/(c^2*x*ArcTan[a*x]) - (a^3*x)/(c^2*(1 + a^
2*x^2)*ArcTan[a*x]) + (a^2*CosIntegral[2*ArcTan[a*x]])/c^2 - (3*Defer[Int][
1/(x^4*ArcTan[a*x]), x])/(a*c^2) + (a*Defer[Int][1/(x^2*ArcTan[a*x]), x])/c
^2
```

Rubi steps

$$\int \frac{1}{x^3\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^2}\,dx = -\left(a^2\int \frac{1}{x\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^2}\,dx\right) + \frac{\int \frac{1}{x^3\left(c+a^2cx^2\right)\tan^{-1}(ax)^2}\,dx}{c}$$

$$= -\frac{1}{ac^2x^3\tan^{-1}(ax)} + a^4\int \frac{x}{\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^2}\,dx - \frac{3\int \frac{1}{x^4\tan^{-1}(ax)}\,dx}{ac^2} - \frac{a^2\int \frac{1}{x(c+}}{}$$

$$= -\frac{1}{ac^2x^3\tan^{-1}(ax)} + \frac{a}{c^2x\tan^{-1}(ax)} - \frac{a^3x}{c^2\left(1+a^2x^2\right)\tan^{-1}(ax)} + a^3\int \frac{1}{\left(c+a^2cx^2\right)^2\tan}$$

$$= -\frac{1}{ac^2x^3\tan^{-1}(ax)} + \frac{a}{c^2x\tan^{-1}(ax)} - \frac{a^3x}{c^2\left(1+a^2x^2\right)\tan^{-1}(ax)} - \frac{3\int \frac{1}{x^4\tan^{-1}(ax)}\,dx}{ac^2} + $$

$$= -\frac{1}{ac^2x^3\tan^{-1}(ax)} + \frac{a}{c^2x\tan^{-1}(ax)} - \frac{a^3x}{c^2\left(1+a^2x^2\right)\tan^{-1}(ax)} - \frac{3\int \frac{1}{x^4\tan^{-1}(ax)}\,dx}{ac^2} + $$

$$= -\frac{1}{ac^2x^3\tan^{-1}(ax)} + \frac{a}{c^2x\tan^{-1}(ax)} - \frac{a^3x}{c^2\left(1+a^2x^2\right)\tan^{-1}(ax)} - \frac{3\int \frac{1}{x^4\tan^{-1}(ax)}\,dx}{ac^2} + $$

$$= -\frac{1}{ac^2x^3\tan^{-1}(ax)} + \frac{a}{c^2x\tan^{-1}(ax)} - \frac{a^3x}{c^2\left(1+a^2x^2\right)\tan^{-1}(ax)} + \frac{a^2\mathrm{Ci}\left(2\tan^{-1}(ax)\right)}{c^2} - $$

Mathematica [A] time = 3.0934, size = 0, normalized size = 0.

$$\int \frac{1}{x^3\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^3*(c + a^2*c*x^2)^2*ArcTan[a*x]^2),x]
```

```
[Out] Integrate[1/(x^3*(c + a^2*c*x^2)^2*ArcTan[a*x]^2), x]
```

Maple [A] time = 0.986, size = 0, normalized size = 0.

$$\int \frac{1}{x^3\left(a^2cx^2+c\right)^2(\arctan(ax))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a^2*c*x^2+c)^2/arctan(a*x)^2,x)
```

```
[Out] int(1/x^3/(a^2*c*x^2+c)^2/arctan(a*x)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{\frac{\left(a^3c^2x^5+ac^2x^3\right)\left(5\,a^2\int\frac{x^2}{a^4x^8\arctan(ax)+2\,a^2x^6\arctan(ax)+x^4\arctan(ax)}\,dx+3\int\frac{1}{a^4x^8\arctan(ax)+2\,a^2x^6\arctan(ax)+x^4\arctan(ax)}\,dx\right)\arctan(ax)}{ac^2}+1}{\left(a^3c^2x^5+ac^2x^3\right)\arctan\left(ax\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] -((a^3*c^2*x^5 + a*c^2*x^3)*arctan(a*x)*integrate((5*a^2*x^2 + 3)/((a^5*c^2
*x^8 + 2*a^3*c^2*x^6 + a*c^2*x^4)*arctan(a*x)), x) + 1)/((a^3*c^2*x^5 + a*c
^2*x^3)*arctan(a*x))
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\left(a^4c^2x^7+2\,a^2c^2x^5+c^2x^3\right)\arctan\left(ax\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(1/((a^4*c^2*x^7 + 2*a^2*c^2*x^5 + c^2*x^3)*arctan(a*x)^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{1}{a^4x^7\operatorname{atan}^2(ax)+2a^2x^5\operatorname{atan}^2(ax)+x^3\operatorname{atan}^2(ax)}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a**2*c*x**2+c)**2/atan(a*x)**2,x)
```

```
[Out] Integral(1/(a**4*x**7*atan(a*x)**2 + 2*a**2*x**5*atan(a*x)**2 + x**3*atan(a
*x)**2), x)/c**2
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)^2x^3\arctan(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^2*x^3*arctan(a*x)^2), x)
```

3.558 $\int \frac{1}{x^4(c+a^2cx^2)^2 \tan^{-1}(ax)^2} dx$

Optimal. Leaf size=110

$$\frac{2a\text{Unintegrable}\left(\frac{1}{x^3\tan^{-1}(ax)},x\right)}{c^2}-\frac{4\text{Unintegrable}\left(\frac{1}{x^5\tan^{-1}(ax)},x\right)}{ac^2}-\frac{a^3\text{Si}\left(2\tan^{-1}(ax)\right)}{c^2}-\frac{a^3}{c^2\left(a^2x^2+1\right)\tan^{-1}(ax)}+\frac{}{c^2}$$

```
[Out] -(1/(a*c^2*x^4*ArcTan[a*x])) + a/(c^2*x^2*ArcTan[a*x]) - a^3/(c^2*(1 + a^2*
x^2)*ArcTan[a*x]) - (a^3*SinIntegral[2*ArcTan[a*x]])/c^2 - (4*Unintegrable[
1/(x^5*ArcTan[a*x]), x])/(a*c^2) + (2*a*Unintegrable[1/(x^3*ArcTan[a*x]), x
])/c^2
```

Rubi [A] time = 0.396449, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^4\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^4*(c + a^2*c*x^2)^2*ArcTan[a*x]^2),x]
```

```
[Out] -(1/(a*c^2*x^4*ArcTan[a*x])) + a/(c^2*x^2*ArcTan[a*x]) - a^3/(c^2*(1 + a^2*
x^2)*ArcTan[a*x]) - (a^3*SinIntegral[2*ArcTan[a*x]])/c^2 - (4*Defer[Int][1/
(x^5*ArcTan[a*x]), x])/(a*c^2) + (2*a*Defer[Int][1/(x^3*ArcTan[a*x]), x])/c
^2
```

Rubi steps

$$\int \frac{1}{x^4\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^2}\,dx = -\left(a^2\int \frac{1}{x^2\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^2}\,dx\right)+\frac{\int \frac{1}{x^4\left(c+a^2cx^2\right)\tan^{-1}(ax)^2}\,dx}{c}$$

$$= -\frac{1}{ac^2x^4\tan^{-1}(ax)}+a^4\int \frac{1}{\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^2}\,dx-\frac{4\int \frac{1}{x^5\tan^{-1}(ax)}\,dx}{ac^2}-\frac{a^2\int \frac{}{x^2(c\cdots}}{}$$

$$= -\frac{1}{ac^2x^4\tan^{-1}(ax)}+\frac{a}{c^2x^2\tan^{-1}(ax)}-\frac{a^3}{c^2\left(1+a^2x^2\right)\tan^{-1}(ax)}-\left(2a^5\right)\int \frac{}{\left(c+a^2cx^2\right)\cdots}$$

$$= -\frac{1}{ac^2x^4\tan^{-1}(ax)}+\frac{a}{c^2x^2\tan^{-1}(ax)}-\frac{a^3}{c^2\left(1+a^2x^2\right)\tan^{-1}(ax)}-\frac{4\int \frac{1}{x^5\tan^{-1}(ax)}\,dx}{ac^2}+$$

$$= -\frac{1}{ac^2x^4\tan^{-1}(ax)}+\frac{a}{c^2x^2\tan^{-1}(ax)}-\frac{a^3}{c^2\left(1+a^2x^2\right)\tan^{-1}(ax)}-\frac{4\int \frac{1}{x^5\tan^{-1}(ax)}\,dx}{ac^2}+$$

$$= -\frac{1}{ac^2x^4\tan^{-1}(ax)}+\frac{a}{c^2x^2\tan^{-1}(ax)}-\frac{a^3}{c^2\left(1+a^2x^2\right)\tan^{-1}(ax)}-\frac{4\int \frac{1}{x^5\tan^{-1}(ax)}\,dx}{ac^2}+$$

$$= -\frac{1}{ac^2x^4\tan^{-1}(ax)}+\frac{a}{c^2x^2\tan^{-1}(ax)}-\frac{a^3}{c^2\left(1+a^2x^2\right)\tan^{-1}(ax)}-\frac{a^3\mathrm{Si}\left(2\tan^{-1}(ax)\right)}{c^2}-$$

Mathematica [A] time = 2.9461, size = 0, normalized size = 0.

$$\int \frac{1}{x^4\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^4*(c + a^2*c*x^2)^2*ArcTan[a*x]^2),x]

[Out] Integrate[1/(x^4*(c + a^2*c*x^2)^2*ArcTan[a*x]^2), x]
```

Maple [A] time = 0.945, size = 0, normalized size = 0.

$$\int \frac{1}{x^4\left(a^2cx^2+c\right)^2(\arctan(ax))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(a^2*c*x^2+c)^2/arctan(a*x)^2,x)
```

```
[Out] int(1/x^4/(a^2*c*x^2+c)^2/arctan(a*x)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{2\left(a^{3}c^{2}x^{6}+ac^{2}x^{4}\right)\arctan\left(ax\right)\int\frac{3\,a^{2}x^{2}+2}{\left(a^{5}c^{2}x^{9}+2\,a^{3}c^{2}x^{7}+ac^{2}x^{5}\right)\arctan(ax)}\,dx+1}{\left(a^{3}c^{2}x^{6}+ac^{2}x^{4}\right)\arctan\left(ax\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] -((a^3*c^2*x^6 + a*c^2*x^4)*arctan(a*x)*integrate(2*(3*a^2*x^2 + 2)/((a^5*c
^2*x^9 + 2*a^3*c^2*x^7 + a*c^2*x^5)*arctan(a*x)), x) + 1)/((a^3*c^2*x^6 + a
*c^2*x^4)*arctan(a*x))
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{1}{\left(a^{4}c^{2}x^{8}+2\,a^{2}c^{2}x^{6}+c^{2}x^{4}\right)\arctan\left(ax\right)^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(1/((a^4*c^2*x^8 + 2*a^2*c^2*x^6 + c^2*x^4)*arctan(a*x)^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{1}{a^{4}x^{8}\operatorname{atan}^{2}\left(ax\right)+2a^{2}x^{6}\operatorname{atan}^{2}\left(ax\right)+x^{4}\operatorname{atan}^{2}\left(ax\right)}\,dx}{c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(a**2*c*x**2+c)**2/atan(a*x)**2,x)
```

```
[Out] Integral(1/(a**4*x**8*atan(a*x)**2 + 2*a**2*x**6*atan(a*x)**2 + x**4*atan(a
*x)**2), x)/c**2
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)^2 x^4 \arctan(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^2*x^4*arctan(a*x)^2), x)
```

3.559 $\int \frac{x^3}{\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^2}\,dx$

Optimal. Leaf size=86

$$\frac{\text{CosIntegral}\left(2\tan^{-1}(ax)\right)}{2a^4c^3}-\frac{\text{CosIntegral}\left(4\tan^{-1}(ax)\right)}{2a^4c^3}-\frac{x}{a^3c^3\left(a^2x^2+1\right)\tan^{-1}(ax)}+\frac{x}{a^3c^3\left(a^2x^2+1\right)^2\tan^{-1}(ax)}$$

```
[Out] x/(a^3*c^3*(1 + a^2*x^2)^2*ArcTan[a*x]) - x/(a^3*c^3*(1 + a^2*x^2)*ArcTan[a
*x]) + CosIntegral[2*ArcTan[a*x]]/(2*a^4*c^3) - CosIntegral[4*ArcTan[a*x]]/
(2*a^4*c^3)
```

Rubi [A] time = 0.517154, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {4964, 4968, 4970, 3312, 3302, 4904, 4406}

$$\frac{\text{CosIntegral}\left(2\tan^{-1}(ax)\right)}{2a^4c^3}-\frac{\text{CosIntegral}\left(4\tan^{-1}(ax)\right)}{2a^4c^3}-\frac{x}{a^3c^3\left(a^2x^2+1\right)\tan^{-1}(ax)}+\frac{x}{a^3c^3\left(a^2x^2+1\right)^2\tan^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((c + a^2*c*x^2)^3*ArcTan[a*x]^2),x]
```

```
[Out] x/(a^3*c^3*(1 + a^2*x^2)^2*ArcTan[a*x]) - x/(a^3*c^3*(1 + a^2*x^2)*ArcTan[a
*x]) + CosIntegral[2*ArcTan[a*x]]/(2*a^4*c^3) - CosIntegral[4*ArcTan[a*x]]/
(2*a^4*c^3)
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Arc
Tan[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan[c
*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p
, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rule 4968

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[(x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^(p
+ 1))/(b*c*d*(p + 1)), x] + (-Dist[(c*(m + 2*q + 2))/(b*(p + 1)), Int[x^(m
```

```
+ 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p + 1
)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; Fre
eQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &&
LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 4970

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sin[x]^m)/
Cos[x]^(m + 2*(q + 1)), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 4904

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \frac{x^3}{\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^2}\,dx = -\frac{\int \frac{x}{\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^2}\,dx}{a^2} + \frac{\int \frac{x}{\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^2}\,dx}{a^2c}$$

$$= \frac{x}{a^3c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)} - \frac{x}{a^3c^3\left(1+a^2x^2\right)\tan^{-1}(ax)} - \frac{\int \frac{1}{\left(c+a^2cx^2\right)^3\tan^{-1}(ax)}\,dx}{a^3} + \frac{3\int}{}$$

$$= \frac{x}{a^3c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)} - \frac{x}{a^3c^3\left(1+a^2x^2\right)\tan^{-1}(ax)} + \frac{\operatorname{Subst}\left(\int \frac{\cos^2(x)}{x}\,dx, x, \tan^{-1}\right.}{a^4c^3}$$

$$= \frac{x}{a^3c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)} - \frac{x}{a^3c^3\left(1+a^2x^2\right)\tan^{-1}(ax)} - \frac{\operatorname{Subst}\left(\int \left(\frac{1}{2x} - \frac{\cos(2x)}{2x}\right)dx,\right.}{a^4c^3}$$

$$= \frac{x}{a^3c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)} - \frac{x}{a^3c^3\left(1+a^2x^2\right)\tan^{-1}(ax)} - \frac{\operatorname{Subst}\left(\int \frac{\cos(4x)}{x}\,dx, x, \tan^{-1}\right.}{8a^4c^3}$$

$$= \frac{x}{a^3c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)} - \frac{x}{a^3c^3\left(1+a^2x^2\right)\tan^{-1}(ax)} + \frac{\operatorname{Ci}\left(2\tan^{-1}(ax)\right)}{2a^4c^3} - \frac{\operatorname{Ci}\left(4\,\mathrm{ta}\right.}{2a}$$

Mathematica [A] time = 0.133276, size = 83, normalized size = 0.97

$$\frac{\left(a^2x^2+1\right)^2\tan^{-1}(ax)\text{CosIntegral}\left(2\tan^{-1}(ax)\right) - \left(a^2x^2+1\right)^2\tan^{-1}(ax)\text{CosIntegral}\left(4\tan^{-1}(ax)\right) - 2a^3x^3}{2a^4c^3\left(a^2x^2+1\right)^2\tan^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((c + a^2*c*x^2)^3*ArcTan[a*x]^2),x]
```

```
[Out] (-2*a^3*x^3 + (1 + a^2*x^2)^2*ArcTan[a*x]*CosIntegral[2*ArcTan[a*x]] - (1 +
 a^2*x^2)^2*ArcTan[a*x]*CosIntegral[4*ArcTan[a*x]])/(2*a^4*c^3*(1 + a^2*x^2
)^2*ArcTan[a*x])
```

Maple [A] time = 0.061, size = 58, normalized size = 0.7

$$\frac{4\operatorname{Ci}(2\arctan(ax))\arctan(ax) - 4\operatorname{Ci}(4\arctan(ax))\arctan(ax) - 2\sin(2\arctan(ax)) + \sin(4\arctan(ax))}{8c^3a^4\arctan(ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a^2*c*x^2+c)^3/arctan(a*x)^2,x)
```

```
[Out] 1/8/a^4/c^3*(4*Ci(2*arctan(a*x))*arctan(a*x)-4*Ci(4*arctan(a*x))*arctan(a*x
)-2*sin(2*arctan(a*x))+sin(4*arctan(a*x)))/arctan(a*x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{x^3+\frac{(a^5c^3x^4+2a^3c^3x^2+ac^3)\left(a^2\int\frac{x^4}{a^6x^6\arctan(ax)+3a^4x^4\arctan(ax)+3a^2x^2\arctan(ax)+\arctan(ax)}\,dx-3\int\frac{x^2}{a^6x^6\arctan(ax)+3a^4x^4\arctan(ax)+3a^2x^2\arctan(ax)+\mathrm{arc}}\right.}{ac^3}}{(a^5c^3x^4+2a^3c^3x^2+ac^3)\arctan(ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(x^3 + (a^5*c^3*x^4 + 2*a^3*c^3*x^2 + a*c^3)*arctan(a*x)*integrate((a^2*x^
4 - 3*x^2)/((a^7*c^3*x^6 + 3*a^5*c^3*x^4 + 3*a^3*c^3*x^2 + a*c^3)*arctan(a*
x)), x))/((a^5*c^3*x^4 + 2*a^3*c^3*x^2 + a*c^3)*arctan(a*x))
```

Fricas [C] time = 2.01264, size = 699, normalized size = 8.13

$$-\frac{4a^3x^3+(a^4x^4+2a^2x^2+1)\arctan(ax)\operatorname{log\_integral}\left(\frac{a^4x^4+4i\,a^3x^3-6a^2x^2-4i\,ax+1}{a^4x^4+2a^2x^2+1}\right)+(a^4x^4+2a^2x^2+1)\arctan(ax)\operatorname{log\_i}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(4*a^3*x^3 + (a^4*x^4 + 2*a^2*x^2 + 1)*arctan(a*x)*log_integral((a^4*x
^4 + 4*I*a^3*x^3 - 6*a^2*x^2 - 4*I*a*x + 1)/(a^4*x^4 + 2*a^2*x^2 + 1)) + (a
^4*x^4 + 2*a^2*x^2 + 1)*arctan(a*x)*log_integral((a^4*x^4 - 4*I*a^3*x^3 - 6
*a^2*x^2 + 4*I*a*x + 1)/(a^4*x^4 + 2*a^2*x^2 + 1)) - (a^4*x^4 + 2*a^2*x^2 +
 1)*arctan(a*x)*log_integral(-(a^2*x^2 + 2*I*a*x - 1)/(a^2*x^2 + 1)) - (a^4
*x^4 + 2*a^2*x^2 + 1)*arctan(a*x)*log_integral(-(a^2*x^2 - 2*I*a*x - 1)/(a^
2*x^2 + 1)))/((a^8*c^3*x^4 + 2*a^6*c^3*x^2 + a^4*c^3)*arctan(a*x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^3}{a^6x^6 \operatorname{atan}^2(ax)+3a^4x^4 \operatorname{atan}^2(ax)+3a^2x^2 \operatorname{atan}^2(ax)+\operatorname{atan}^2(ax)}\, dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a**2*c*x**2+c)**3/atan(a*x)**2,x)
```

```
[Out] Integral(x**3/(a**6*x**6*atan(a*x)**2 + 3*a**4*x**4*atan(a*x)**2 + 3*a**2*x
**2*atan(a*x)**2 + atan(a*x)**2), x)/c**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{(a^2cx^2+c)^3 \arctan(ax)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^3/((a^2*c*x^2 + c)^3*arctan(a*x)^2), x)
```

3.560 $\int \frac{x^2}{\left(c+a^2cx^2\right)^3 \tan^{-1}(ax)^2} dx$

Optimal. Leaf size=67

$$\frac{\mathrm{Si}\left(4\tan^{-1}(ax)\right)}{2a^3c^3} - \frac{1}{a^3c^3\left(a^2x^2+1\right)\tan^{-1}(ax)} + \frac{1}{a^3c^3\left(a^2x^2+1\right)^2\tan^{-1}(ax)}$$

```
[Out] 1/(a^3*c^3*(1 + a^2*x^2)^2*ArcTan[a*x]) - 1/(a^3*c^3*(1 + a^2*x^2)*ArcTan[a
*x]) + SinIntegral[4*ArcTan[a*x]]/(2*a^3*c^3)
```

Rubi [A] time = 0.279959, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4964, 4902, 4970, 4406, 12, 3299}

$$\frac{\mathrm{Si}\left(4\tan^{-1}(ax)\right)}{2a^3c^3} - \frac{1}{a^3c^3\left(a^2x^2+1\right)\tan^{-1}(ax)} + \frac{1}{a^3c^3\left(a^2x^2+1\right)^2\tan^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((c + a^2*c*x^2)^3*ArcTan[a*x]^2),x]
```

```
[Out] 1/(a^3*c^3*(1 + a^2*x^2)^2*ArcTan[a*x]) - 1/(a^3*c^3*(1 + a^2*x^2)*ArcTan[a
*x]) + SinIntegral[4*ArcTan[a*x]]/(2*a^3*c^3)
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Arc
Tan[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan[c
*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p
, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rule 4902

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^(p + 1))/(b*c*d*(p
+ 1)), x] - Dist[(2*c*(q + 1))/(b*(p + 1)), Int[x*(d + e*x^2)^q*(a + b*ArcT
an[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && L
```

```
tQ[q, -1] && LtQ[p, -1]
```

Rule 4970

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sin[x]^m)/
Cos[x]^(m + 2*(q + 1)), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\int \frac{x^2}{\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^2}\,dx = -\frac{\int \frac{1}{\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^2}\,dx}{a^2} + \frac{\int \frac{1}{\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^2}\,dx}{a^2c}$$

$$= \frac{1}{a^3c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)} - \frac{1}{a^3c^3\left(1+a^2x^2\right)\tan^{-1}(ax)} + \frac{4\int \frac{x}{\left(c+a^2cx^2\right)^3\tan^{-1}(ax)}\,dx}{a} - \frac{2\int}{}$$

$$= \frac{1}{a^3c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)} - \frac{1}{a^3c^3\left(1+a^2x^2\right)\tan^{-1}(ax)} - \frac{2\,\text{Subst}\left(\int \frac{\cos(x)\sin(x)}{x}\,dx, x, \text{t}\right.}{a^3c^3}$$

$$= \frac{1}{a^3c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)} - \frac{1}{a^3c^3\left(1+a^2x^2\right)\tan^{-1}(ax)} - \frac{2\,\text{Subst}\left(\int \frac{\sin(2x)}{2x}\,dx, x, \tan^{-1}\right.}{a^3c^3}$$

$$= \frac{1}{a^3c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)} - \frac{1}{a^3c^3\left(1+a^2x^2\right)\tan^{-1}(ax)} + \frac{\text{Subst}\left(\int \frac{\sin(4x)}{x}\,dx, x, \tan^{-1}(\right.}{2a^3c^3}$$

$$= \frac{1}{a^3c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)} - \frac{1}{a^3c^3\left(1+a^2x^2\right)\tan^{-1}(ax)} + \frac{\text{Si}\left(4\tan^{-1}(ax)\right)}{2a^3c^3}$$

Mathematica [A] time = 0.170959, size = 59, normalized size = 0.88

$$\frac{\left(a^2x^2+1\right)^2\tan^{-1}(ax)\text{Si}\left(4\tan^{-1}(ax)\right)-2a^2x^2}{2a^3c^3\left(a^2x^2+1\right)^2\tan^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((c + a^2*c*x^2)^3*ArcTan[a*x]^2),x]
```

```
[Out] (-2*a^2*x^2 + (1 + a^2*x^2)^2*ArcTan[a*x]*SinIntegral[4*ArcTan[a*x]])/(2*a^
3*c^3*(1 + a^2*x^2)^2*ArcTan[a*x])
```

Maple [A] time = 0.071, size = 37, normalized size = 0.6

$$\frac{4\,\text{Si}\left(4\arctan(ax)\right)\arctan(ax)+\cos\left(4\arctan(ax)\right)-1}{8\,a^3c^3\arctan(ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a^2*c*x^2+c)^3/arctan(a*x)^2,x)
```

```
[Out] 1/8/a^3/c^3*(4*Si(4*arctan(a*x))*arctan(a*x)+cos(4*arctan(a*x))-1)/arctan(a
*x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2\left(a^5c^3x^4+2\,a^3c^3x^2+ac^3\right)\arctan\left(ax\right)\int\frac{a^2x^3-x}{\left(a^7c^3x^6+3\,a^5c^3x^4+3\,a^3c^3x^2+ac^3\right)\arctan(ax)}\,dx+x^2}{\left(a^5c^3x^4+2\,a^3c^3x^2+ac^3\right)\arctan\left(ax\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] -((a^5*c^3*x^4 + 2*a^3*c^3*x^2 + a*c^3)*arctan(a*x)*integrate(2*(a^2*x^3 -
x)/((a^7*c^3*x^6 + 3*a^5*c^3*x^4 + 3*a^3*c^3*x^2 + a*c^3)*arctan(a*x)), x)
+ x^2)/((a^5*c^3*x^4 + 2*a^3*c^3*x^2 + a*c^3)*arctan(a*x))
```

Fricas [C] time = 2.08475, size = 460, normalized size = 6.87

$$-\frac{4\,a^2x^2-\left(i\,a^4x^4+2i\,a^2x^2+i\right)\arctan\left(ax\right)\log\_integral\left(\frac{a^4x^4+4i\,a^3x^3-6\,a^2x^2-4i\,ax+1}{a^4x^4+2\,a^2x^2+1}\right)-\left(-i\,a^4x^4-2i\,a^2x^2-i\right)\arctan\left(ax\right)}{4\left(a^7c^3x^4+2\,a^5c^3x^2+a^3c^3\right)\arctan\left(ax\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(4*a^2*x^2 - (I*a^4*x^4 + 2*I*a^2*x^2 + I)*arctan(a*x)*log_integral((a
^4*x^4 + 4*I*a^3*x^3 - 6*a^2*x^2 - 4*I*a*x + 1)/(a^4*x^4 + 2*a^2*x^2 + 1))
- (-I*a^4*x^4 - 2*I*a^2*x^2 - I)*arctan(a*x)*log_integral((a^4*x^4 - 4*I*a^
3*x^3 - 6*a^2*x^2 + 4*I*a*x + 1)/(a^4*x^4 + 2*a^2*x^2 + 1)))/((a^7*c^3*x^4
+ 2*a^5*c^3*x^2 + a^3*c^3)*arctan(a*x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^2}{a^6x^6 \operatorname{atan}^2(ax)+3a^4x^4 \operatorname{atan}^2(ax)+3a^2x^2 \operatorname{atan}^2(ax)+\operatorname{atan}^2(ax)}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a**2*c*x**2+c)**3/atan(a*x)**2,x)
```

```
[Out] Integral(x**2/(a**6*x**6*atan(a*x)**2 + 3*a**4*x**4*atan(a*x)**2 + 3*a**2*x
**2*atan(a*x)**2 + atan(a*x)**2), x)/c**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(a^2cx^2+c\right)^3 \arctan(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2/((a^2*c*x^2 + c)^3*arctan(a*x)^2), x)
```

3.561 $\int \frac{x}{\left(c+a^2cx^2\right)^3 \tan^{-1}(ax)^2} dx$

Optimal. Leaf size=61

$$\frac{\text{CosIntegral}\left(2\tan^{-1}(ax)\right)}{2a^2c^3}+\frac{\text{CosIntegral}\left(4\tan^{-1}(ax)\right)}{2a^2c^3}-\frac{x}{ac^3\left(a^2x^2+1\right)^2\tan^{-1}(ax)}$$

```
[Out] -(x/(a*c^3*(1 + a^2*x^2)^2*ArcTan[a*x])) + CosIntegral[2*ArcTan[a*x]]/(2*a^
2*c^3) + CosIntegral[4*ArcTan[a*x]]/(2*a^2*c^3)
```

Rubi [A] time = 0.24642, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {4968, 4970, 4406, 3302, 4904, 3312}

$$\frac{\text{CosIntegral}\left(2\tan^{-1}(ax)\right)}{2a^2c^3}+\frac{\text{CosIntegral}\left(4\tan^{-1}(ax)\right)}{2a^2c^3}-\frac{x}{ac^3\left(a^2x^2+1\right)^2\tan^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[x/((c + a^2*c*x^2)^3*ArcTan[a*x]^2),x]
```

```
[Out] -(x/(a*c^3*(1 + a^2*x^2)^2*ArcTan[a*x])) + CosIntegral[2*ArcTan[a*x]]/(2*a^
2*c^3) + CosIntegral[4*ArcTan[a*x]]/(2*a^2*c^3)
```

Rule 4968

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[(x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^(p
+ 1))/(b*c*d*(p + 1)), x] + (-Dist[(c*(m + 2*q + 2))/(b*(p + 1)), Int[x^(m
+ 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p + 1
)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; Fre
eQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &&
LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 4970

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sin[x]^m)/
Cos[x]^(m + 2*(q + 1)), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
```

```
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 4904

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\int \frac{x}{\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^2}\,dx = -\frac{x}{ac^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)} + \frac{\int \frac{1}{\left(c+a^2cx^2\right)^3\tan^{-1}(ax)}\,dx}{a} - (3a)\int \frac{x^2}{\left(c+a^2cx^2\right)^3\tan^{-1}(ax)}$$

$$= -\frac{x}{ac^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)} + \frac{\text{Subst}\left(\int \frac{\cos^4(x)}{x}\,dx, x, \tan^{-1}(ax)\right)}{a^2c^3} - \frac{3\,\text{Subst}\left(\int \frac{\cos^2(x)\sin}{x}\right.}{a}$$

$$= -\frac{x}{ac^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)} + \frac{\text{Subst}\left(\int \left(\frac{3}{8x}+\frac{\cos(2x)}{2x}+\frac{\cos(4x)}{8x}\right)dx, x, \tan^{-1}(ax)\right)}{a^2c^3} - \frac{3\,S}{}$$

$$= -\frac{x}{ac^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)} + \frac{\text{Subst}\left(\int \frac{\cos(4x)}{x}\,dx, x, \tan^{-1}(ax)\right)}{8a^2c^3} + \frac{3\,\text{Subst}\left(\int \frac{\cos(4x)}{x}\right.}{8a^2c}$$

$$= -\frac{x}{ac^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)} + \frac{\text{Ci}\left(2\tan^{-1}(ax)\right)}{2a^2c^3} + \frac{\text{Ci}\left(4\tan^{-1}(ax)\right)}{2a^2c^3}$$

Mathematica [A] time = 0.066865, size = 75, normalized size = 1.23

$$\frac{\left(a^2x^2+1\right)^2\tan^{-1}(ax)\text{CosIntegral}\left(2\tan^{-1}(ax)\right)+\left(a^2x^2+1\right)^2\tan^{-1}(ax)\text{CosIntegral}\left(4\tan^{-1}(ax)\right)-2ax}{2c^3\left(a^3x^2+a\right)^2\tan^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((c + a^2*c*x^2)^3*ArcTan[a*x]^2),x]
```

```
[Out] (-2*a*x + (1 + a^2*x^2)^2*ArcTan[a*x]*CosIntegral[2*ArcTan[a*x]] + (1 + a^2
*x^2)^2*ArcTan[a*x]*CosIntegral[4*ArcTan[a*x]])/(2*c^3*(a + a^3*x^2)^2*ArcT
an[a*x])
```

Maple [A] time = 0.061, size = 60, normalized size = 1.

$$\frac{4\,\text{Ci}\left(2\arctan(ax)\right)\arctan(ax)+4\,\text{Ci}\left(4\arctan(ax)\right)\arctan(ax)-2\sin\left(2\arctan(ax)\right)-\sin\left(4\arctan(ax)\right)}{8\,a^2c^3\arctan(ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^2*c*x^2+c)^3/arctan(a*x)^2,x)
```

[Out] 1/8/a^2/c^3*(4*Ci(2*arctan(a*x))*arctan(a*x)+4*Ci(4*arctan(a*x))*arctan(a*x
)-2*sin(2*arctan(a*x))-sin(4*arctan(a*x)))/arctan(a*x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{x+\frac{(a^5c^3x^4+2\,a^3c^3x^2+ac^3)\left(3\,a^2\int\frac{x^2}{a^6x^6\arctan(ax)+3\,a^4x^4\arctan(ax)+3\,a^2x^2\arctan(ax)+\arctan(ax)}\,dx-\int\frac{1}{a^6x^6\arctan(ax)+3\,a^4x^4\arctan(ax)+3\,a^2x^2\arctan(ax)+\text{arcta}}\right.}{ac^3}}{\left(a^5c^3x^4+2\,a^3c^3x^2+ac^3\right)\arctan(ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="maxima")

[Out] -((a^5*c^3*x^4 + 2*a^3*c^3*x^2 + a*c^3)*arctan(a*x)*integrate((3*a^2*x^2 -
1)/((a^7*c^3*x^6 + 3*a^5*c^3*x^4 + 3*a^3*c^3*x^2 + a*c^3)*arctan(a*x)), x)
+ x)/((a^5*c^3*x^4 + 2*a^3*c^3*x^2 + a*c^3)*arctan(a*x))

Fricas [C] time = 2.0782, size = 693, normalized size = 11.36

$$\frac{\left(a^4x^4+2\,a^2x^2+1\right)\arctan(ax)\,\text{log\_integral}\left(\frac{a^4x^4+4i\,a^3x^3-6\,a^2x^2-4i\,ax+1}{a^4x^4+2\,a^2x^2+1}\right)+\left(a^4x^4+2\,a^2x^2+1\right)\arctan(ax)\,\text{log\_integral}\left(\right.}{4\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="fricas")

[Out] 1/4*((a^4*x^4 + 2*a^2*x^2 + 1)*arctan(a*x)*log_integral((a^4*x^4 + 4*I*a^3*
x^3 - 6*a^2*x^2 - 4*I*a*x + 1)/(a^4*x^4 + 2*a^2*x^2 + 1)) + (a^4*x^4 + 2*a^
2*x^2 + 1)*arctan(a*x)*log_integral((a^4*x^4 - 4*I*a^3*x^3 - 6*a^2*x^2 + 4*
I*a*x + 1)/(a^4*x^4 + 2*a^2*x^2 + 1)) + (a^4*x^4 + 2*a^2*x^2 + 1)*arctan(a*
x)*log_integral(-(a^2*x^2 + 2*I*a*x - 1)/(a^2*x^2 + 1)) + (a^4*x^4 + 2*a^2*
x^2 + 1)*arctan(a*x)*log_integral(-(a^2*x^2 - 2*I*a*x - 1)/(a^2*x^2 + 1)) -
 4*a*x)/((a^6*c^3*x^4 + 2*a^4*c^3*x^2 + a^2*c^3)*arctan(a*x))

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{x}{a^6x^6\operatorname{atan}^2(ax)+3a^4x^4\operatorname{atan}^2(ax)+3a^2x^2\operatorname{atan}^2(ax)+\operatorname{atan}^2(ax)}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a**2*c*x**2+c)**3/atan(a*x)**2,x)
```

```
[Out] Integral(x/(a**6*x**6*atan(a*x)**2 + 3*a**4*x**4*atan(a*x)**2 + 3*a**2*x**2
*atan(a*x)**2 + atan(a*x)**2), x)/c**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(a^2cx^2+c\right)^3\arctan\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x/((a^2*c*x^2 + c)^3*arctan(a*x)^2), x)
```

3.562 $\int \frac{1}{(c+a^2cx^2)^3\tan^{-1}(ax)^2}\,dx$

Optimal. Leaf size=58

$$-\frac{1}{ac^3\left(a^2x^2+1\right)^2\tan^{-1}(ax)}-\frac{\mathrm{Si}\left(2\tan^{-1}(ax)\right)}{ac^3}-\frac{\mathrm{Si}\left(4\tan^{-1}(ax)\right)}{2ac^3}$$

```
[Out] -(1/(a*c^3*(1 + a^2*x^2)^2*ArcTan[a*x])) - SinIntegral[2*ArcTan[a*x]]/(a*c^
3) - SinIntegral[4*ArcTan[a*x]]/(2*a*c^3)
```

Rubi [A] time = 0.114183, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {4902, 4970, 4406, 3299}

$$-\frac{1}{ac^3\left(a^2x^2+1\right)^2\tan^{-1}(ax)}-\frac{\mathrm{Si}\left(2\tan^{-1}(ax)\right)}{ac^3}-\frac{\mathrm{Si}\left(4\tan^{-1}(ax)\right)}{2ac^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a^2*c*x^2)^3*ArcTan[a*x]^2),x]
```

```
[Out] -(1/(a*c^3*(1 + a^2*x^2)^2*ArcTan[a*x])) - SinIntegral[2*ArcTan[a*x]]/(a*c^
3) - SinIntegral[4*ArcTan[a*x]]/(2*a*c^3)
```

Rule 4902

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^(p + 1))/(b*c*d*(p
+ 1)), x] - Dist[(2*c*(q + 1))/(b*(p + 1)), Int[x*(d + e*x^2)^q*(a + b*ArcT
an[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && L
tQ[q, -1] && LtQ[p, -1]
```

Rule 4970

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sin[x]^m)/
Cos[x]^(m + 2*(q + 1)), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\int \frac{1}{\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^2}\,dx = -\frac{1}{ac^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)}-(4a)\int \frac{x}{\left(c+a^2cx^2\right)^3\tan^{-1}(ax)}\,dx$$

$$= -\frac{1}{ac^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)}-\frac{4\,\mathrm{Subst}\left(\int \frac{\cos^3(x)\sin(x)}{x}\,dx,x,\tan^{-1}(ax)\right)}{ac^3}$$

$$= -\frac{1}{ac^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)}-\frac{4\,\mathrm{Subst}\left(\int \left(\frac{\sin(2x)}{4x}+\frac{\sin(4x)}{8x}\right)dx,x,\tan^{-1}(ax)\right)}{ac^3}$$

$$= -\frac{1}{ac^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)}-\frac{\mathrm{Subst}\left(\int \frac{\sin(4x)}{x}\,dx,x,\tan^{-1}(ax)\right)}{2ac^3}-\frac{\mathrm{Subst}\left(\int \frac{\sin(2x)}{x}\,dx,\right.}{ac^3}$$

$$= -\frac{1}{ac^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)}-\frac{\mathrm{Si}\left(2\tan^{-1}(ax)\right)}{ac^3}-\frac{\mathrm{Si}\left(4\tan^{-1}(ax)\right)}{2ac^3}$$

Mathematica [A] time = 0.0943938, size = 45, normalized size = 0.78

$$-\frac{\frac{1}{\left(a^2x^2+1\right)^2\tan^{-1}(ax)}+\mathrm{Si}\left(2\tan^{-1}(ax)\right)+\frac{1}{2}\mathrm{Si}\left(4\tan^{-1}(ax)\right)}{ac^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a^2*c*x^2)^3*ArcTan[a*x]^2),x]
```

```
[Out] -((1/((1 + a^2*x^2)^2*ArcTan[a*x]) + SinIntegral[2*ArcTan[a*x]] + SinIntegr
al[4*ArcTan[a*x]]/2)/(a*c^3))
```

Maple [A] time = 0.061, size = 59, normalized size = 1.

$$-\frac{8\,\mathrm{Si}\left(2\arctan\left(ax\right)\right)\arctan\left(ax\right)+4\,\mathrm{Si}\left(4\arctan\left(ax\right)\right)\arctan\left(ax\right)+4\cos\left(2\arctan\left(ax\right)\right)+\cos\left(4\arctan\left(ax\right)\right)+3}{8\,a c^{3}\arctan\left(ax\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2*c*x^2+c)^3/arctan(a*x)^2,x)
```

```
[Out] -1/8/a/c^3*(8*Si(2*arctan(a*x))*arctan(a*x)+4*Si(4*arctan(a*x))*arctan(a*x)
+4*cos(2*arctan(a*x))+cos(4*arctan(a*x))+3)/arctan(a*x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{4\left(a^{6}c^{3}x^{4}+2\,a^{4}c^{3}x^{2}+a^{2}c^{3}\right)\arctan\left(ax\right)\int\frac{x}{\left(a^{6}c^{3}x^{6}+3\,a^{4}c^{3}x^{4}+3\,a^{2}c^{3}x^{2}+c^{3}\right)\arctan\left(ax\right)}\,dx+1}{\left(a^{5}c^{3}x^{4}+2\,a^{3}c^{3}x^{2}+ac^{3}\right)\arctan\left(ax\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(8*(a^6*c^3*x^4 + 2*a^4*c^3*x^2 + a^2*c^3)*arctan(a*x)*integrate(1/2*x/((a
^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*arctan(a*x)), x) + 1)/((a
^5*c^3*x^4 + 2*a^3*c^3*x^2 + a*c^3)*arctan(a*x))
```

Fricas [C] time = 2.01078, size = 720, normalized size = 12.41

$$\left(-i\,a^{4}x^{4}-2i\,a^{2}x^{2}-i\right)\arctan\left(ax\right)\mathrm{log\_integral}\left(\frac{a^{4}x^{4}+4i\,a^{3}x^{3}-6\,a^{2}x^{2}-4i\,ax+1}{a^{4}x^{4}+2\,a^{2}x^{2}+1}\right)+\left(i\,a^{4}x^{4}+2i\,a^{2}x^{2}+i\right)\arctan\left(ax\right)\mathrm{log\_integr}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/4*((-I*a^4*x^4 - 2*I*a^2*x^2 - I)*arctan(a*x)*log_integral((a^4*x^4 + 4*I
*a^3*x^3 - 6*a^2*x^2 - 4*I*a*x + 1)/(a^4*x^4 + 2*a^2*x^2 + 1)) + (I*a^4*x^4
```

```
 + 2*I*a^2*x^2 + I)*arctan(a*x)*log_integral((a^4*x^4 - 4*I*a^3*x^3 - 6*a^2
*x^2 + 4*I*a*x + 1)/(a^4*x^4 + 2*a^2*x^2 + 1)) + (-2*I*a^4*x^4 - 4*I*a^2*x^
2 - 2*I)*arctan(a*x)*log_integral(-(a^2*x^2 + 2*I*a*x - 1)/(a^2*x^2 + 1)) +
 (2*I*a^4*x^4 + 4*I*a^2*x^2 + 2*I)*arctan(a*x)*log_integral(-(a^2*x^2 - 2*I
*a*x - 1)/(a^2*x^2 + 1)) - 4)/((a^5*c^3*x^4 + 2*a^3*c^3*x^2 + a*c^3)*arctan
(a*x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{a^{6}x^{6}\operatorname{atan}^{2}(ax)+3a^{4}x^{4}\operatorname{atan}^{2}(ax)+3a^{2}x^{2}\operatorname{atan}^{2}(ax)+\operatorname{atan}^{2}(ax)}\,dx}{c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*c*x**2+c)**3/atan(a*x)**2,x)
```

```
[Out] Integral(1/(a**6*x**6*atan(a*x)**2 + 3*a**4*x**4*atan(a*x)**2 + 3*a**2*x**2
*atan(a*x)**2 + atan(a*x)**2), x)/c**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^{2}cx^{2}+c\right)^{3}\arctan(ax)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^3*arctan(a*x)^2), x)
```

3.563 $\int \frac{1}{x(c+a^2cx^2)^3 \tan^{-1}(ax)^2} \, dx$

Optimal. Leaf size=112

$$-\frac{\text{Unintegrable}\left(\frac{1}{x^2 \tan^{-1}(ax)}, x\right)}{ac^3}+\frac{ax}{c^3 \left(a^2x^2+1\right) \tan^{-1}(ax)}+\frac{ax}{c^3 \left(a^2x^2+1\right)^2 \tan^{-1}(ax)}-\frac{3\text{CosIntegral}\left(2 \tan^{-1}(ax)\right)}{2c^3}-$$

```
[Out] -(1/(a*c^3*x*ArcTan[a*x])) + (a*x)/(c^3*(1 + a^2*x^2)^2*ArcTan[a*x]) + (a*x
)/(c^3*(1 + a^2*x^2)*ArcTan[a*x]) - (3*CosIntegral[2*ArcTan[a*x]])/(2*c^3)
- CosIntegral[4*ArcTan[a*x]]/(2*c^3) - Unintegrable[1/(x^2*ArcTan[a*x]), x]
/(a*c^3)
```

Rubi [A] time = 0.662014, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x\left(c+a^2cx^2\right)^3 \tan^{-1}(ax)^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*(c + a^2*c*x^2)^3*ArcTan[a*x]^2),x]
```

```
[Out] -(1/(a*c^3*x*ArcTan[a*x])) + (a*x)/(c^3*(1 + a^2*x^2)^2*ArcTan[a*x]) + (a*x
)/(c^3*(1 + a^2*x^2)*ArcTan[a*x]) - (3*CosIntegral[2*ArcTan[a*x]])/(2*c^3)
- CosIntegral[4*ArcTan[a*x]]/(2*c^3) - Defer[Int][1/(x^2*ArcTan[a*x]), x]/(
a*c^3)
```

Rubi steps

$$\int \frac{1}{x\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^2}\,dx = -\left(a^2\int \frac{x}{\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^2}\,dx\right) + \frac{\int \frac{1}{x\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^2}\,dx}{c}$$

$$= \frac{ax}{c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)} - a\int \frac{1}{\left(c+a^2cx^2\right)^3\tan^{-1}(ax)}\,dx + \left(3a^3\right)\int \frac{x^2}{\left(c+a^2cx^2\right)^3\ \text{ta}}$$

$$= -\frac{1}{ac^3x\tan^{-1}(ax)} + \frac{ax}{c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)} + \frac{ax}{c^3\left(1+a^2x^2\right)\tan^{-1}(ax)} - \frac{\text{Subst}\left(\int \right.}{}$$

$$= -\frac{1}{ac^3x\tan^{-1}(ax)} + \frac{ax}{c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)} + \frac{ax}{c^3\left(1+a^2x^2\right)\tan^{-1}(ax)} - \frac{\text{Subst}\left(\int \right.}{}$$

$$= -\frac{1}{ac^3x\tan^{-1}(ax)} + \frac{ax}{c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)} + \frac{ax}{c^3\left(1+a^2x^2\right)\tan^{-1}(ax)} - \frac{\text{Subst}\left(\int \right.}{}$$

$$= -\frac{1}{ac^3x\tan^{-1}(ax)} + \frac{ax}{c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)} + \frac{ax}{c^3\left(1+a^2x^2\right)\tan^{-1}(ax)} - \frac{\text{Ci}\left(2\tan^{-}\right.}{2c^3}$$

$$= -\frac{1}{ac^3x\tan^{-1}(ax)} + \frac{ax}{c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)} + \frac{ax}{c^3\left(1+a^2x^2\right)\tan^{-1}(ax)} - \frac{3\text{Ci}\left(2\tan\right.}{2c^3}$$

Mathematica [A] time = 1.601, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*(c + a^2*c*x^2)^3*ArcTan[a*x]^2),x]
```

```
[Out] Integrate[1/(x*(c + a^2*c*x^2)^3*ArcTan[a*x]^2), x]
```

Maple [A] time = 0.396, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(a^2cx^2+c\right)^3\left(\arctan(ax)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^2,x)
```

```
[Out] int(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{\frac{(a^5c^3x^5+2\,a^3c^3x^3+ac^3x)\left(5\,a^2\int\frac{x^2}{a^6x^8\arctan(ax)+3\,a^4x^6\arctan(ax)+3\,a^2x^4\arctan(ax)+x^2\arctan(ax)}\,dx+\int\frac{1}{(a^2x^2+1)^3x^2\arctan(ax)}\,dx\right)\arctan(ax)}{ac^3}+1}{(a^5c^3x^5+2\,a^3c^3x^3+ac^3x)\arctan(ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] -((a^5*c^3*x^5 + 2*a^3*c^3*x^3 + a*c^3*x)*arctan(a*x)*integrate((5*a^2*x^2
+ 1)/((a^7*c^3*x^8 + 3*a^5*c^3*x^6 + 3*a^3*c^3*x^4 + a*c^3*x^2)*arctan(a*x)
), x) + 1)/((a^5*c^3*x^5 + 2*a^3*c^3*x^3 + a*c^3*x)*arctan(a*x))
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{(a^6c^3x^7+3\,a^4c^3x^5+3\,a^2c^3x^3+c^3x)\arctan(ax)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(1/((a^6*c^3*x^7 + 3*a^4*c^3*x^5 + 3*a^2*c^3*x^3 + c^3*x)*arctan(a*
x)^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{1}{a^6x^7\operatorname{atan}^2(ax)+3a^4x^5\operatorname{atan}^2(ax)+3a^2x^3\operatorname{atan}^2(ax)+x\operatorname{atan}^2(ax)}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a**2*c*x**2+c)**3/atan(a*x)**2,x)
```

```
[Out] Integral(1/(a**6*x**7*atan(a*x)**2 + 3*a**4*x**5*atan(a*x)**2 + 3*a**2*x**3
*atan(a*x)**2 + x*atan(a*x)**2), x)/c**3
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)^3 x \arctan\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^3*x*arctan(a*x)^2), x)
```

3.564 $\int \frac{1}{x^2(c+a^2cx^2)^3\tan^{-1}(ax)^2}\,dx$

Optimal. Leaf size=110

$$-\frac{2\text{Unintegrable}\left(\frac{1}{x^3\tan^{-1}(ax)},x\right)}{ac^3}+\frac{a}{c^3\left(a^2x^2+1\right)\tan^{-1}(ax)}+\frac{a}{c^3\left(a^2x^2+1\right)^2\tan^{-1}(ax)}+\frac{2a\text{Si}\left(2\tan^{-1}(ax)\right)}{c^3}+\frac{a\text{Si}\left(4\tan^{-1}(ax)\right)}{2c^3}$$

```
[Out] -(1/(a*c^3*x^2*ArcTan[a*x])) + a/(c^3*(1 + a^2*x^2)^2*ArcTan[a*x]) + a/(c^3
*(1 + a^2*x^2)*ArcTan[a*x]) + (2*a*SinIntegral[2*ArcTan[a*x]])/c^3 + (a*Sin
Integral[4*ArcTan[a*x]])/(2*c^3) - (2*Unintegrable[1/(x^3*ArcTan[a*x]), x])
/(a*c^3)
```

Rubi [A] time = 0.425717, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]^2),x]
```

```
[Out] -(1/(a*c^3*x^2*ArcTan[a*x])) + a/(c^3*(1 + a^2*x^2)^2*ArcTan[a*x]) + a/(c^3
*(1 + a^2*x^2)*ArcTan[a*x]) + (2*a*SinIntegral[2*ArcTan[a*x]])/c^3 + (a*Sin
Integral[4*ArcTan[a*x]])/(2*c^3) - (2*Defer[Int][1/(x^3*ArcTan[a*x]), x])/(
a*c^3)
```

Rubi steps

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^2}\,dx = -\left(a^2\int \frac{1}{\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^2}\,dx\right) + \frac{\int \frac{1}{x^2\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^2}\,dx}{c}$$

$$= \frac{a}{c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)} + \left(4a^3\right)\int \frac{x}{\left(c+a^2cx^2\right)^3\tan^{-1}(ax)}\,dx + \frac{\int \frac{1}{x^2\left(c+a^2cx^2\right)\tan^{-1}(a}}{c^2}$$

$$= -\frac{1}{ac^3x^2\tan^{-1}(ax)} + \frac{a}{c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)} + \frac{a}{c^3\left(1+a^2x^2\right)\tan^{-1}(ax)} - \frac{2\int \frac{}{x^3\tan}}{ac}$$

$$= -\frac{1}{ac^3x^2\tan^{-1}(ax)} + \frac{a}{c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)} + \frac{a}{c^3\left(1+a^2x^2\right)\tan^{-1}(ax)} - \frac{2\int \frac{}{x^3\tan}}{ac}$$

$$= -\frac{1}{ac^3x^2\tan^{-1}(ax)} + \frac{a}{c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)} + \frac{a}{c^3\left(1+a^2x^2\right)\tan^{-1}(ax)} - \frac{2\int \frac{}{x^3\tan}}{ac}$$

$$= -\frac{1}{ac^3x^2\tan^{-1}(ax)} + \frac{a}{c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)} + \frac{a}{c^3\left(1+a^2x^2\right)\tan^{-1}(ax)} + \frac{a\mathrm{Si}\left(2\tan\right.}{c}$$

$$= -\frac{1}{ac^3x^2\tan^{-1}(ax)} + \frac{a}{c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)} + \frac{a}{c^3\left(1+a^2x^2\right)\tan^{-1}(ax)} + \frac{2a\mathrm{Si}\left(2\mathrm{t}\right.}{}$$

Mathematica [A] time = 2.12322, size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]^2),x]
```

```
[Out] Integrate[1/(x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]^2), x]
```

Maple [A] time = 0.343, size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(a^2cx^2+c\right)^3\left(\arctan(ax)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a^2*c*x^2+c)^3/arctan(a*x)^2,x)
```

```
[Out] int(1/x^2/(a^2*c*x^2+c)^3/arctan(a*x)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{2\left(a^5c^3x^6+2\,a^3c^3x^4+ac^3x^2\right)\arctan\left(ax\right)\int\frac{3\,a^2x^2+1}{\left(a^7c^3x^9+3\,a^5c^3x^7+3\,a^3c^3x^5+ac^3x^3\right)\arctan(ax)}\,dx+1}{\left(a^5c^3x^6+2\,a^3c^3x^4+ac^3x^2\right)\arctan\left(ax\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] -((a^5*c^3*x^6 + 2*a^3*c^3*x^4 + a*c^3*x^2)*arctan(a*x)*integrate(2*(3*a^2*
x^2 + 1)/((a^7*c^3*x^9 + 3*a^5*c^3*x^7 + 3*a^3*c^3*x^5 + a*c^3*x^3)*arctan(
a*x)), x) + 1)/((a^5*c^3*x^6 + 2*a^3*c^3*x^4 + a*c^3*x^2)*arctan(a*x))
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\left(a^6c^3x^8+3\,a^4c^3x^6+3\,a^2c^3x^4+c^3x^2\right)\arctan\left(ax\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(1/((a^6*c^3*x^8 + 3*a^4*c^3*x^6 + 3*a^2*c^3*x^4 + c^3*x^2)*arctan(
a*x)^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{1}{a^6x^8\operatorname{atan}^2\left(ax\right)+3a^4x^6\operatorname{atan}^2\left(ax\right)+3a^2x^4\operatorname{atan}^2\left(ax\right)+x^2\operatorname{atan}^2\left(ax\right)}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a**2*c*x**2+c)**3/atan(a*x)**2,x)
```

```
[Out] Integral(1/(a**6*x**8*atan(a*x)**2 + 3*a**4*x**6*atan(a*x)**2 + 3*a**2*x**4
*atan(a*x)**2 + x**2*atan(a*x)**2), x)/c**3
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)^3x^2\arctan\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^3*x^2*arctan(a*x)^2), x)
```

3.565 $\int \frac{1}{x^3\left(c+a^2cx^2\right)^3 \tan^{-1}(ax)^2} dx$

Optimal. Leaf size=157

$$\frac{2a\text{Unintegrable}\left(\frac{1}{x^2\tan^{-1}(ax)},x\right)}{c^3}-\frac{3\text{Unintegrable}\left(\frac{1}{x^4\tan^{-1}(ax)},x\right)}{ac^3}+\frac{5a^2\text{CosIntegral}\left(2\tan^{-1}(ax)\right)}{2c^3}+\frac{a^2\text{CosIntegral}\left(\dots\right.}{2c^3}$$

```
[Out] -(1/(a*c^3*x^3*ArcTan[a*x])) + (2*a)/(c^3*x*ArcTan[a*x]) - (a^3*x)/(c^3*(1
+ a^2*x^2)^2*ArcTan[a*x]) - (2*a^3*x)/(c^3*(1 + a^2*x^2)*ArcTan[a*x]) + (5*
a^2*CosIntegral[2*ArcTan[a*x]])/(2*c^3) + (a^2*CosIntegral[4*ArcTan[a*x]])/
(2*c^3) - (3*Unintegrable[1/(x^4*ArcTan[a*x]), x])/(a*c^3) + (2*a*Unintegra
ble[1/(x^2*ArcTan[a*x]), x])/c^3
```

Rubi [A] time = 1.2204, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^3\left(c+a^2cx^2\right)^3 \tan^{-1}(ax)^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^3*(c + a^2*c*x^2)^3*ArcTan[a*x]^2),x]
```

```
[Out] -(1/(a*c^3*x^3*ArcTan[a*x])) + (2*a)/(c^3*x*ArcTan[a*x]) - (a^3*x)/(c^3*(1
+ a^2*x^2)^2*ArcTan[a*x]) - (2*a^3*x)/(c^3*(1 + a^2*x^2)*ArcTan[a*x]) + (5*
a^2*CosIntegral[2*ArcTan[a*x]])/(2*c^3) + (a^2*CosIntegral[4*ArcTan[a*x]])/
(2*c^3) - (3*Defer[Int][1/(x^4*ArcTan[a*x]), x])/(a*c^3) + (2*a*Defer[Int][
1/(x^2*ArcTan[a*x]), x])/c^3
```

Rubi steps

$$\int \frac{1}{x^3\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^2}\,dx = -\left(a^2\int \frac{1}{x\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^2}\,dx\right)+\frac{\int \frac{1}{x^3\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^2}\,dx}{c}$$

$$= a^4\int \frac{x}{\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^2}\,dx+\frac{\int \frac{1}{x^3\left(c+a^2cx^2\right)\tan^{-1}(ax)^2}\,dx}{c^2}-2\frac{a^2\int \frac{1}{x\left(c+a^2cx^2\right)^2\tan^{-1}(}}{c}$$

$$= -\frac{1}{ac^3x^3\tan^{-1}(ax)}-\frac{a^3x}{c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)}+a^3\int \frac{1}{\left(c+a^2cx^2\right)^3\tan^{-1}(ax)}\,dx-($$

$$= -\frac{1}{ac^3x^3\tan^{-1}(ax)}-\frac{a^3x}{c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)}-\frac{3\int \frac{1}{x^4\tan^{-1}(ax)}\,dx}{ac^3}+\frac{a^2\,\text{Subst}\left(\int \frac{\cos}{}\right.}{}$$

$$= -\frac{1}{ac^3x^3\tan^{-1}(ax)}-\frac{a^3x}{c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)}-\frac{3\int \frac{1}{x^4\tan^{-1}(ax)}\,dx}{ac^3}+\frac{a^2\,\text{Subst}\left(\int\left(\frac{3}{8}\right.\right.}{}$$

$$= -\frac{1}{ac^3x^3\tan^{-1}(ax)}-\frac{a^3x}{c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)}-\frac{3\int \frac{1}{x^4\tan^{-1}(ax)}\,dx}{ac^3}+\frac{a^2\,\text{Subst}\left(\int \frac{\cos}{}\right.}{}$$

$$= -\frac{1}{ac^3x^3\tan^{-1}(ax)}-\frac{a^3x}{c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)}+\frac{a^2\text{Ci}\left(2\tan^{-1}(ax)\right)}{2c^3}+\frac{a^2\text{Ci}\left(4\tan^{-}\right.}{2c^3}$$

$$= -\frac{1}{ac^3x^3\tan^{-1}(ax)}-\frac{a^3x}{c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)}+\frac{a^2\text{Ci}\left(2\tan^{-1}(ax)\right)}{2c^3}+\frac{a^2\text{Ci}\left(4\tan^{-}\right.}{2c^3}$$

Mathematica [A] time = 2.91253, size = 0, normalized size = 0.

$$\int \frac{1}{x^3\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^3*(c + a^2*c*x^2)^3*ArcTan[a*x]^2),x]
```

```
[Out] Integrate[1/(x^3*(c + a^2*c*x^2)^3*ArcTan[a*x]^2), x]
```

Maple [A] time = 1.204, size = 0, normalized size = 0.

$$\int \frac{1}{x^3\left(a^2cx^2+c\right)^3\left(\arctan\left(ax\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a^2*c*x^2+c)^3/arctan(a*x)^2,x)
```

```
[Out] int(1/x^3/(a^2*c*x^2+c)^3/arctan(a*x)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{\frac{\left(a^5c^3x^7+2\,a^3c^3x^5+ac^3x^3\right)\left(7\,a^2\int\frac{x^2}{a^6x^{10}\arctan(ax)+3\,a^4x^8\arctan(ax)+3\,a^2x^6\arctan(ax)+x^4\arctan(ax)}\,dx+3\int\frac{1}{a^6x^{10}\arctan(ax)+3\,a^4x^8\arctan(ax)+3\,a^2x^6\arctan(ax)+x}\right.}{ac^3}}{\left(a^5c^3x^7+2\,a^3c^3x^5+ac^3x^3\right)\arctan\left(ax\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] -((a^5*c^3*x^7 + 2*a^3*c^3*x^5 + a*c^3*x^3)*arctan(a*x)*integrate((7*a^2*x^
2 + 3)/((a^7*c^3*x^10 + 3*a^5*c^3*x^8 + 3*a^3*c^3*x^6 + a*c^3*x^4)*arctan(a
*x)), x) + 1)/((a^5*c^3*x^7 + 2*a^3*c^3*x^5 + a*c^3*x^3)*arctan(a*x))
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\left(a^6c^3x^9+3\,a^4c^3x^7+3\,a^2c^3x^5+c^3x^3\right)\arctan\left(ax\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(1/((a^6*c^3*x^9 + 3*a^4*c^3*x^7 + 3*a^2*c^3*x^5 + c^3*x^3)*arctan(
a*x)^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{a^6x^9 \operatorname{atan}^2(ax)+3a^4x^7 \operatorname{atan}^2(ax)+3a^2x^5 \operatorname{atan}^2(ax)+x^3 \operatorname{atan}^2(ax)}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a**2*c*x**2+c)**3/atan(a*x)**2,x)
```

```
[Out] Integral(1/(a**6*x**9*atan(a*x)**2 + 3*a**4*x**7*atan(a*x)**2 + 3*a**2*x**5
*atan(a*x)**2 + x**3*atan(a*x)**2), x)/c**3
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a^2cx^2+c)^3 x^3 \arctan(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^3*x^3*arctan(a*x)^2), x)
```

3.566 $\int \frac{1}{x^4(c+a^2cx^2)^3 \tan^{-1}(ax)^2} dx$

Optimal. Leaf size=153

$$\frac{4a\text{Unintegrable}\left(\frac{1}{x^3\tan^{-1}(ax)},x\right)}{c^3}-\frac{4\text{Unintegrable}\left(\frac{1}{x^5\tan^{-1}(ax)},x\right)}{ac^3}-\frac{3a^3\text{Si}\left(2\tan^{-1}(ax)\right)}{c^3}-\frac{a^3\text{Si}\left(4\tan^{-1}(ax)\right)}{2c^3}-\frac{}{c^3\left(a^2x\right.}$$

```
[Out] -(1/(a*c^3*x^4*ArcTan[a*x])) + (2*a)/(c^3*x^2*ArcTan[a*x]) - a^3/(c^3*(1 +
a^2*x^2)^2*ArcTan[a*x]) - (2*a^3)/(c^3*(1 + a^2*x^2)*ArcTan[a*x]) - (3*a^3*
SinIntegral[2*ArcTan[a*x]])/c^3 - (a^3*SinIntegral[4*ArcTan[a*x]])/(2*c^3)
- (4*Unintegrable[1/(x^5*ArcTan[a*x]), x])/(a*c^3) + (4*a*Unintegrable[1/(x
^3*ArcTan[a*x]), x])/c^3
```

Rubi [A] time = 0.896924, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^4\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^4*(c + a^2*c*x^2)^3*ArcTan[a*x]^2),x]
```

```
[Out] -(1/(a*c^3*x^4*ArcTan[a*x])) + (2*a)/(c^3*x^2*ArcTan[a*x]) - a^3/(c^3*(1 +
a^2*x^2)^2*ArcTan[a*x]) - (2*a^3)/(c^3*(1 + a^2*x^2)*ArcTan[a*x]) - (3*a^3*
SinIntegral[2*ArcTan[a*x]])/c^3 - (a^3*SinIntegral[4*ArcTan[a*x]])/(2*c^3)
- (4*Defer[Int][1/(x^5*ArcTan[a*x]), x])/(a*c^3) + (4*a*Defer[Int][1/(x^3*A
rcTan[a*x]), x])/c^3
```

Rubi steps

$$\int \frac{1}{x^4\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^2}\,dx = -\left(a^2\int \frac{1}{x^2\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^2}\,dx\right)+\frac{\int \frac{1}{x^4\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^2}\,dx}{c}$$

$$= a^4\int \frac{1}{\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^2}\,dx+\frac{\int \frac{1}{x^4\left(c+a^2cx^2\right)\tan^{-1}(ax)^2}\,dx}{c^2}-2\frac{a^2\int \frac{1}{x^2\left(c+a^2cx^2\right)^2\tan^{-1}}}{c}$$

$$= -\frac{1}{ac^3x^4\tan^{-1}(ax)}-\frac{a^3}{c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)}-\left(4a^5\right)\int \frac{x}{\left(c+a^2cx^2\right)^3\tan^{-1}(ax)}\,dx$$

$$= -\frac{1}{ac^3x^4\tan^{-1}(ax)}-\frac{a^3}{c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)}-\frac{4\int \frac{1}{x^5\tan^{-1}(ax)}\,dx}{ac^3}-\frac{\left(4a^3\right)\mathrm{Subst}\left(\int\right.}{}$$

$$= -\frac{1}{ac^3x^4\tan^{-1}(ax)}-\frac{a^3}{c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)}-\frac{4\int \frac{1}{x^5\tan^{-1}(ax)}\,dx}{ac^3}-2\left(-\frac{a}{c^3x^2\tan^{-1}(}\right.$$

$$= -\frac{1}{ac^3x^4\tan^{-1}(ax)}-\frac{a^3}{c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)}-\frac{4\int \frac{1}{x^5\tan^{-1}(ax)}\,dx}{ac^3}-\frac{a^3\,\mathrm{Subst}\left(\int \frac{\sin}{}\right.}{}$$

$$= -\frac{1}{ac^3x^4\tan^{-1}(ax)}-\frac{a^3}{c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)}-\frac{a^3\mathrm{Si}\left(2\tan^{-1}(ax)\right)}{c^3}-\frac{a^3\mathrm{Si}\left(4\tan^{-1}(\right.}{2c^3}$$

$$= -\frac{1}{ac^3x^4\tan^{-1}(ax)}-\frac{a^3}{c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)}-\frac{a^3\mathrm{Si}\left(2\tan^{-1}(ax)\right)}{c^3}-\frac{a^3\mathrm{Si}\left(4\tan^{-1}(\right.}{2c^3}$$

Mathematica [A] time = 3.51735, size = 0, normalized size = 0.

$$\int \frac{1}{x^4\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^4*(c + a^2*c*x^2)^3*ArcTan[a*x]^2),x]
```

```
[Out] Integrate[1/(x^4*(c + a^2*c*x^2)^3*ArcTan[a*x]^2), x]
```

Maple [A] time = 1.036, size = 0, normalized size = 0.

$$\int \frac{1}{x^{4} \left(a^{2} c \,x^{2}+c \right)^{3} \arctan \left(a x \right)^{2}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(a^2*c*x^2+c)^3/arctan(a*x)^2,x)
```

```
[Out] int(1/x^4/(a^2*c*x^2+c)^3/arctan(a*x)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{4 \left(a^{5} c^{3} x^{8}+2 \, a^{3} c^{3} x^{6}+a c^{3} x^{4}\right) \arctan \left(a x\right) \int \frac{2 \, a^{2} x^{2}+1}{\left(a^{7} c^{3} x^{11}+3 \, a^{5} c^{3} x^{9}+3 \, a^{3} c^{3} x^{7}+a c^{3} x^{5}\right) \arctan \left(a x\right)}\,{d x} + 1}{\left(a^{5} c^{3} x^{8}+2 \, a^{3} c^{3} x^{6}+a c^{3} x^{4}\right) \arctan \left(a x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] -((a^5*c^3*x^8 + 2*a^3*c^3*x^6 + a*c^3*x^4)*arctan(a*x)*integrate(4*(2*a^2*
x^2 + 1)/((a^7*c^3*x^11 + 3*a^5*c^3*x^9 + 3*a^3*c^3*x^7 + a*c^3*x^5)*arctan
(a*x)), x) + 1)/((a^5*c^3*x^8 + 2*a^3*c^3*x^6 + a*c^3*x^4)*arctan(a*x))
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$${\rm integral}\left(\frac{1}{{\left(a^{6} c^{3} x^{10} + 3 \, a^{4} c^{3} x^{8} + 3 \, a^{2} c^{3} x^{6} + c^{3} x^{4}\right)} \arctan\left(a x\right)^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(1/((a^6*c^3*x^10 + 3*a^4*c^3*x^8 + 3*a^2*c^3*x^6 + c^3*x^4)*arctan
(a*x)^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{a^6x^{10}\operatorname{atan}^2(ax)+3a^4x^8\operatorname{atan}^2(ax)+3a^2x^6\operatorname{atan}^2(ax)+x^4\operatorname{atan}^2(ax)}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(a**2*c*x**2+c)**3/atan(a*x)**2,x)
```

```
[Out] Integral(1/(a**6*x**10*atan(a*x)**2 + 3*a**4*x**8*atan(a*x)**2 + 3*a**2*x**
6*atan(a*x)**2 + x**4*atan(a*x)**2), x)/c**3
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)^3x^4\arctan(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^3*x^4*arctan(a*x)^2), x)
```

3.567 $\int \frac{x\sqrt{c+a^2cx^2}}{\tan^{-1}(ax)^2}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{x\sqrt{a^2cx^2+c}}{\tan^{-1}(ax)^2},x\right)$$

```
[Out] Unintegrable[(x*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^2, x]
```

Rubi [A] time = 0.06794, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x\sqrt{c+a^2cx^2}}{\tan^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^2,x]

[Out] Defer[Int][(x*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^2, x]
```

Rubi steps

$$\int \frac{x\sqrt{c+a^2cx^2}}{\tan^{-1}(ax)^2}\,dx=\int \frac{x\sqrt{c+a^2cx^2}}{\tan^{-1}(ax)^2}\,dx$$

Mathematica [A] time = 1.5551, size = 0, normalized size = 0.

$$\int \frac{x\sqrt{c+a^2cx^2}}{\tan^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^2,x]
```

```
[Out] Integrate[(x*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^2, x]
```

Maple [A] time = 0.686, size = 0, normalized size = 0.

$$\int \frac{x}{\left(\arctan \left(a x\right)\right)^{2}}\sqrt{a^{2} c \,x^{2}+c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^2,x)
```

```
[Out] int(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a^{2} c x^{2} + c}\, x}{\arctan \left(a x\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)*x/arctan(a*x)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{a^{2} c x^{2} + c}\, x}{\arctan \left(a x\right)^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x/arctan(a*x)^2, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sqrt{c\left(a^2x^2+1\right)}}{\operatorname{atan}^2(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**(1/2)/atan(a*x)**2,x)
```

```
[Out] Integral(x*sqrt(c*(a**2*x**2 + 1))/atan(a*x)**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a^2cx^2+c}x}{\arctan(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)*x/arctan(a*x)^2, x)
```

3.568 $\int \frac{\sqrt{c+a^2cx^2}}{\tan^{-1}(ax)^2} \, dx$

Optimal. Leaf size=23

$$\text{Unintegrable}\left(\frac{\sqrt{a^2cx^2+c}}{\tan^{-1}(ax)^2},x\right)$$

```
[Out] Unintegrable[Sqrt[c + a^2*c*x^2]/ArcTan[a*x]^2, x]
```

Rubi [A] time = 0.0349173, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sqrt{c+a^2cx^2}}{\tan^{-1}(ax)^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[c + a^2*c*x^2]/ArcTan[a*x]^2,x]
```

```
[Out] Defer[Int][Sqrt[c + a^2*c*x^2]/ArcTan[a*x]^2, x]
```

Rubi steps

$$\int \frac{\sqrt{c+a^2cx^2}}{\tan^{-1}(ax)^2} \, dx = \int \frac{\sqrt{c+a^2cx^2}}{\tan^{-1}(ax)^2} \, dx$$

Mathematica [A] time = 0.423289, size = 0, normalized size = 0.

$$\int \frac{\sqrt{c+a^2cx^2}}{\tan^{-1}(ax)^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[c + a^2*c*x^2]/ArcTan[a*x]^2,x]
```

```
[Out] Integrate[Sqrt[c + a^2*c*x^2]/ArcTan[a*x]^2, x]
```

Maple [A] time = 0.645, size = 0, normalized size = 0.

$$\int \frac{1}{\left(\arctan\left(ax\right)\right)^{2}}\sqrt{a^{2}c\,x^{2}+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(1/2)/arctan(a*x)^2,x)
```

```
[Out] int((a^2*c*x^2+c)^(1/2)/arctan(a*x)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a^{2}c\,x^{2}+c}}{\arctan\left(ax\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)/arctan(a*x)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{a^{2}c\,x^{2}+c}}{\arctan\left(ax\right)^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)/arctan(a*x)^2, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c\left(a^2x^2+1\right)}}{\operatorname{atan}^2(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(1/2)/atan(a*x)**2,x)
```

```
[Out] Integral(sqrt(c*(a**2*x**2 + 1))/atan(a*x)**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a^2cx^2+c}}{\arctan(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)/arctan(a*x)^2, x)
```

3.569 $\int \frac{\sqrt{c+a^2cx^2}}{x\tan^{-1}(ax)^2}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{\sqrt{a^2cx^2+c}}{x\tan^{-1}(ax)^2},x\right)$$

```
[Out] Unintegrable[Sqrt[c + a^2*c*x^2]/(x*ArcTan[a*x]^2), x]
```

Rubi [A] time = 0.0962309, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sqrt{c+a^2cx^2}}{x\tan^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[c + a^2*c*x^2]/(x*ArcTan[a*x]^2),x]
```

```
[Out] Defer[Int][Sqrt[c + a^2*c*x^2]/(x*ArcTan[a*x]^2), x]
```

Rubi steps

$$\int \frac{\sqrt{c+a^2cx^2}}{x\tan^{-1}(ax)^2}\,dx=\int \frac{\sqrt{c+a^2cx^2}}{x\tan^{-1}(ax)^2}\,dx$$

Mathematica [A] time = 3.24059, size = 0, normalized size = 0.

$$\int \frac{\sqrt{c+a^2cx^2}}{x\tan^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[c + a^2*c*x^2]/(x*ArcTan[a*x]^2),x]
```

```
[Out] Integrate[Sqrt[c + a^2*c*x^2]/(x*ArcTan[a*x]^2), x]
```

Maple [A] time = 0.759, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(\arctan\left(ax\right)\right)^{2}}\sqrt{a^{2}cx^{2}+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^2,x)
```

```
[Out] int((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a^{2}cx^{2}+c}}{x\arctan\left(ax\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)/(x*arctan(a*x)^2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{a^{2}cx^{2}+c}}{x\arctan\left(ax\right)^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)/(x*arctan(a*x)^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c\left(a^2x^2+1\right)}}{x\,\mathrm{atan}^2\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(1/2)/x/atan(a*x)**2,x)
```

```
[Out] Integral(sqrt(c*(a**2*x**2 + 1))/(x*atan(a*x)**2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a^2cx^2+c}}{x\arctan\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)/(x*arctan(a*x)^2), x)
```

3.570 $\int \frac{x(c+a^2cx^2)^{3/2}}{\tan^{-1}(ax)^2} dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{x\left(a^2cx^2+c\right)^{3/2}}{\tan^{-1}(ax)^2},x\right)$$

```
[Out] Unintegrable[(x*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^2, x]
```

Rubi [A] time = 0.076589, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x\left(c+a^2cx^2\right)^{3/2}}{\tan^{-1}(ax)^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[(x*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^2,x]
```

```
[Out] Defer[Int][(x*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^2, x]
```

Rubi steps

$$\int \frac{x\left(c+a^2cx^2\right)^{3/2}}{\tan^{-1}(ax)^2} dx = \int \frac{x\left(c+a^2cx^2\right)^{3/2}}{\tan^{-1}(ax)^2} dx$$

Mathematica [A] time = 3.82949, size = 0, normalized size = 0.

$$\int \frac{x\left(c+a^2cx^2\right)^{3/2}}{\tan^{-1}(ax)^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^2,x]
```

```
[Out] Integrate[(x*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^2, x]
```

Maple [A] time = 0.653, size = 0, normalized size = 0.

$$\int \frac{x}{\left(\arctan \left(ax\right)\right)^{2}} \left(a^{2}cx^{2}+c\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x)
```

```
[Out] int(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^{2}cx^{2}+c\right)^{\frac{3}{2}}x}{\arctan \left(ax\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)*x/arctan(a*x)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(a^{2}cx^{3}+cx\right)\sqrt{a^{2}cx^{2}+c}}{\arctan \left(ax\right)^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^3 + c*x)*sqrt(a^2*c*x^2 + c)/arctan(a*x)^2, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}}{\operatorname{atan}^2(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**(3/2)/atan(a*x)**2,x)
```

```
[Out] Integral(x*(c*(a**2*x**2 + 1))**(3/2)/atan(a*x)**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^{\frac{3}{2}}x}{\arctan(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)*x/arctan(a*x)^2, x)
```

3.571 $\int \frac{\left(c+a^2cx^2\right)^{3/2}}{\tan^{-1}(ax)^2}\,dx$

Optimal. Leaf size=23

$$\text{Unintegrable}\left(\frac{\left(a^2cx^2+c\right)^{3/2}}{\tan^{-1}(ax)^2},x\right)$$

[Out] Unintegrable[(c + a^2*c*x^2)^(3/2)/ArcTan[a*x]^2, x]

Rubi [A] time = 0.0365105, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(c+a^2cx^2\right)^{3/2}}{\tan^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

[In] Int[(c + a^2*c*x^2)^(3/2)/ArcTan[a*x]^2,x]

[Out] Defer[Int][(c + a^2*c*x^2)^(3/2)/ArcTan[a*x]^2, x]

Rubi steps

$$\int \frac{\left(c+a^2cx^2\right)^{3/2}}{\tan^{-1}(ax)^2}\,dx=\int \frac{\left(c+a^2cx^2\right)^{3/2}}{\tan^{-1}(ax)^2}\,dx$$

Mathematica [A] time = 0.930142, size = 0, normalized size = 0.

$$\int \frac{\left(c+a^2cx^2\right)^{3/2}}{\tan^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

[In] Integrate[(c + a^2*c*x^2)^(3/2)/ArcTan[a*x]^2,x]

```
[Out] Integrate[(c + a^2*c*x^2)^(3/2)/ArcTan[a*x]^2, x]
```

Maple [A] time = 0.63, size = 0, normalized size = 0.

$$\int \frac{1}{\left(\arctan \left(a x\right)\right)^{2}} \left(a^{2} c \,x^{2}+c \right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x)
```

```
[Out] int((a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{{\left(a^{2} c x^{2} + c\right)}^{\frac{3}{2}}}{\arctan\left(a x\right)^{2}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)/arctan(a*x)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$${\rm integral}\left(\frac{{\left(a^{2} c x^{2} + c\right)}^{\frac{3}{2}}}{\arctan\left(a x\right)^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)^(3/2)/arctan(a*x)^2, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}}{\operatorname{atan}^2(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(3/2)/atan(a*x)**2,x)
```

```
[Out] Integral((c*(a**2*x**2 + 1))**(3/2)/atan(a*x)**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^{\frac{3}{2}}}{\arctan(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)/arctan(a*x)^2, x)
```

3.572 $\int \frac{(c+a^2cx^2)^{3/2}}{x\tan^{-1}(ax)^2}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{(a^2cx^2+c)^{3/2}}{x\tan^{-1}(ax)^2},x\right)$$

```
[Out] Unintegrable[(c + a^2*c*x^2)^(3/2)/(x*ArcTan[a*x]^2), x]
```

Rubi [A] time = 0.109323, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\tan^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^(3/2)/(x*ArcTan[a*x]^2),x]

[Out] Defer[Int][(c + a^2*c*x^2)^(3/2)/(x*ArcTan[a*x]^2), x]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\tan^{-1}(ax)^2}\,dx = \int \frac{(c+a^2cx^2)^{3/2}}{x\tan^{-1}(ax)^2}\,dx$$

Mathematica [A] time = 4.07741, size = 0, normalized size = 0.

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\tan^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^(3/2)/(x*ArcTan[a*x]^2),x]
```

```
[Out] Integrate[(c + a^2*c*x^2)^(3/2)/(x*ArcTan[a*x]^2), x]
```

Maple [A] time = 0.751, size = 0, normalized size = 0.

$$\int \frac{1}{x \left(\arctan\left(ax\right)\right)^{2}} \left(a^{2}cx^{2}+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^2,x)
```

```
[Out] int((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^{2}cx^{2}+c\right)^{\frac{3}{2}}}{x \arctan\left(ax\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)/(x*arctan(a*x)^2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^{2}cx^{2}+c\right)^{\frac{3}{2}}}{x \arctan\left(ax\right)^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)^(3/2)/(x*arctan(a*x)^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}}{x\,\mathrm{atan}^2\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(3/2)/x/atan(a*x)**2,x)
```

```
[Out] Integral((c*(a**2*x**2 + 1))**(3/2)/(x*atan(a*x)**2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^{\frac{3}{2}}}{x\arctan\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)/(x*arctan(a*x)^2), x)
```

3.573 $\int \frac{x(c+a^2cx^2)^{5/2}}{\tan^{-1}(ax)^2}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{x\left(a^2cx^2+c\right)^{5/2}}{\tan^{-1}(ax)^2},x\right)$$

```
[Out] Unintegrable[(x*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^2, x]
```

Rubi [A] time = 0.077516, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x\left(c+a^2cx^2\right)^{5/2}}{\tan^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^2,x]
```

```
[Out] Defer[Int][(x*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^2, x]
```

Rubi steps

$$\int \frac{x\left(c+a^2cx^2\right)^{5/2}}{\tan^{-1}(ax)^2}\,dx=\int \frac{x\left(c+a^2cx^2\right)^{5/2}}{\tan^{-1}(ax)^2}\,dx$$

Mathematica [A] time = 1.94567, size = 0, normalized size = 0.

$$\int \frac{x\left(c+a^2cx^2\right)^{5/2}}{\tan^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^2,x]
```

```
[Out] Integrate[(x*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^2, x]
```

Maple [A] time = 0.776, size = 0, normalized size = 0.

$$\int \frac{x}{\left(\arctan\left(ax\right)\right)^{2}}\left(a^{2}cx^{2}+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x)
```

```
[Out] int(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^{2}cx^{2}+c\right)^{\frac{5}{2}}x}{\arctan\left(ax\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(5/2)*x/arctan(a*x)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(a^{4}c^{2}x^{5}+2\,a^{2}c^{2}x^{3}+c^{2}x\right)\sqrt{a^{2}cx^{2}+c}}{\arctan\left(ax\right)^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^5 + 2*a^2*c^2*x^3 + c^2*x)*sqrt(a^2*c*x^2 + c)/arctan(a
*x)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**(5/2)/atan(a*x)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^{\frac{5}{2}}x}{\arctan\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(5/2)*x/arctan(a*x)^2, x)
```

3.574 $\int \frac{(c+a^2cx^2)^{5/2}}{\tan^{-1}(ax)^2} dx$

Optimal. Leaf size=23

$$\text{Unintegrable}\left(\frac{(a^2cx^2+c)^{5/2}}{\tan^{-1}(ax)^2}, x\right)$$

```
[Out] Unintegrable[(c + a^2*c*x^2)^(5/2)/ArcTan[a*x]^2, x]
```

Rubi [A] time = 0.036265, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(c+a^2cx^2)^{5/2}}{\tan^{-1}(ax)^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^(5/2)/ArcTan[a*x]^2,x]

[Out] Defer[Int][(c + a^2*c*x^2)^(5/2)/ArcTan[a*x]^2, x]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^{5/2}}{\tan^{-1}(ax)^2} dx = \int \frac{(c+a^2cx^2)^{5/2}}{\tan^{-1}(ax)^2} dx$$

Mathematica [A] time = 0.822757, size = 0, normalized size = 0.

$$\int \frac{(c+a^2cx^2)^{5/2}}{\tan^{-1}(ax)^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^(5/2)/ArcTan[a*x]^2,x]
```

```
[Out] Integrate[(c + a^2*c*x^2)^(5/2)/ArcTan[a*x]^2, x]
```

Maple [A] time = 0.728, size = 0, normalized size = 0.

$$\int \frac{1}{\left(\arctan\left(ax\right)\right)^{2}}\left(a^{2}cx^{2}+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x)
```

```
[Out] int((a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^{2}cx^{2}+c\right)^{\frac{5}{2}}}{\arctan\left(ax\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(5/2)/arctan(a*x)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^{4}c^{2}x^{4}+2\,a^{2}c^{2}x^{2}+c^{2}\right)\sqrt{a^{2}cx^{2}+c}}{\arctan\left(ax\right)^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)/arctan(a*x
)^2, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}}{\operatorname{atan}^2(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(5/2)/atan(a*x)**2,x)
```

```
[Out] Integral((c*(a**2*x**2 + 1))**(5/2)/atan(a*x)**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^{\frac{5}{2}}}{\arctan(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(5/2)/arctan(a*x)^2, x)
```

3.575 $\int \frac{\left(c+a^2 c x^2\right)^{5/2}}{x \tan^{-1}(a x)^2} \, dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{\left(a^2 c x^2+c\right)^{5/2}}{x \tan^{-1}(a x)^2},x\right)$$

```
[Out] Unintegrable[(c + a^2*c*x^2)^(5/2)/(x*ArcTan[a*x]^2), x]
```

Rubi [A] time = 0.110543, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(c+a^2 c x^2\right)^{5/2}}{x \tan^{-1}(a x)^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^(5/2)/(x*ArcTan[a*x]^2),x]

[Out] Defer[Int][(c + a^2*c*x^2)^(5/2)/(x*ArcTan[a*x]^2), x]
```

Rubi steps

$$\int \frac{\left(c+a^2 c x^2\right)^{5/2}}{x \tan^{-1}(a x)^2} \, dx=\int \frac{\left(c+a^2 c x^2\right)^{5/2}}{x \tan^{-1}(a x)^2} \, dx$$

Mathematica [A] time = 2.18771, size = 0, normalized size = 0.

$$\int \frac{\left(c+a^2 c x^2\right)^{5/2}}{x \tan^{-1}(a x)^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^(5/2)/(x*ArcTan[a*x]^2),x]
```

```
[Out] Integrate[(c + a^2*c*x^2)^(5/2)/(x*ArcTan[a*x]^2), x]
```

Maple [A] time = 0.756, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(\arctan \left(ax\right)\right)^{2}} \left(a^{2}c x^{2}+c\right)^{\frac{5}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^2,x)
```

```
[Out] int((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^{2}c x^{2}+c\right)^{\frac{5}{2}}}{x \arctan \left(ax\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(5/2)/(x*arctan(a*x)^2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(a^{4}c^{2}x^{4}+2\,a^{2}c^{2}x^{2}+c^{2}\right)\sqrt{a^{2}c x^{2}+c}}{x \arctan \left(ax\right)^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)/(x*arctan(
a*x)^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}}{x\,\mathrm{atan}^2\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(5/2)/x/atan(a*x)**2,x)
```

```
[Out] Integral((c*(a**2*x**2 + 1))**(5/2)/(x*atan(a*x)**2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^{\frac{5}{2}}}{x\arctan\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(5/2)/(x*arctan(a*x)^2), x)
```

3.576 $\int \frac{x}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{x}{\sqrt{a^2cx^2+c}\tan^{-1}(ax)^2},x\right)$$

```
[Out] Unintegrable[x/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2), x]
```

Rubi [A] time = 0.0758681, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2),x]
```

```
[Out] Defer[Int][x/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2), x]
```

Rubi steps

$$\int \frac{x}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}\,dx=\int \frac{x}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}\,dx$$

Mathematica [A] time = 1.14519, size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2),x]
```

```
[Out] Integrate[x/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2), x]
```

Maple [A] time = 0.628, size = 0, normalized size = 0.

$$\int \frac{x}{\left(\arctan\left(ax\right)\right)^{2}} \frac{1}{\sqrt{a^{2}c\,x^{2}+c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(x/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{a^{2}cx^{2}+c}\arctan\left(ax\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x}{\sqrt{a^{2}cx^{2}+c}\arctan\left(ax\right)^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x/(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{c\left(a^{2}x^{2}+1\right)}\operatorname{atan}^{2}\left(ax\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/atan(a*x)**2/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(c*(a**2*x**2 + 1))*atan(a*x)**2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{a^2cx^2+c}\arctan{(ax)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2), x)
```

3.577 $\int \frac{1}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}\,dx$

Optimal. Leaf size=23

$$\text{Unintegrable}\left(\frac{1}{\sqrt{a^2cx^2+c}\tan^{-1}(ax)^2},x\right)$$

```
[Out] Unintegrable[1/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2), x]
```

Rubi [A] time = 0.03548, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2),x]
```

```
[Out] Defer[Int][1/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2), x]
```

Rubi steps

$$\int \frac{1}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}\,dx=\int \frac{1}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}\,dx$$

Mathematica [A] time = 0.595513, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2),x]
```

```
[Out] Integrate[1/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2), x]
```

Maple [A] time = 0.714, size = 0, normalized size = 0.

$$\int \frac{1}{\left(\arctan\left(ax\right)\right)^{2}} \frac{1}{\sqrt{a^{2}c\,x^{2}+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(1/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a^{2}c\,x^{2}+c}\,\arctan\left(ax\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{1}{\sqrt{a^{2}c\,x^{2}+c}\,\arctan\left(ax\right)^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{c\left(a^{2}x^{2}+1\right)}\,\operatorname{atan}^{2}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/atan(a*x)**2/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(c*(a**2*x**2 + 1))*atan(a*x)**2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a^2cx^2 + c}\arctan{(ax)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2), x)
```

3.578 $\int \frac{1}{x\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}\,dx$

Optimal. Leaf size=63

$$-\frac{\text{Unintegrable}\left(\frac{1}{x^2\sqrt{a^2cx^2+c}\tan^{-1}(ax)},x\right)}{a}-\frac{\sqrt{a^2cx^2+c}}{acx\tan^{-1}(ax)}$$

```
[Out] -(Sqrt[c + a^2*c*x^2]/(a*c*x*ArcTan[a*x])) - Unintegrable[1/(x^2*Sqrt[c + a
^2*c*x^2]*ArcTan[a*x]), x]/a
```

Rubi [A] time = 0.216659, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2),x]
```

```
[Out] -(Sqrt[c + a^2*c*x^2]/(a*c*x*ArcTan[a*x])) - Defer[Int][1/(x^2*Sqrt[c + a^2
*c*x^2]*ArcTan[a*x]), x]/a
```

Rubi steps

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}\,dx=-\frac{\sqrt{c+a^2cx^2}}{acx\tan^{-1}(ax)}-\frac{\int \frac{1}{x^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}\,dx}{a}$$

Mathematica [A] time = 1.17693, size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2),x]
```

```
[Out] Integrate[1/(x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2), x]
```

Maple [A] time = 0.638, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(\arctan\left(ax\right)\right)^{2}}\frac{1}{\sqrt{a^{2}cx^{2}+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(1/x/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a^{2}cx^{2}+c}\,x\arctan\left(ax\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a^2*c*x^2 + c)*x*arctan(a*x)^2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{a^{2}cx^{2}+c}}{\left(a^{2}cx^{3}+cx\right)\arctan\left(ax\right)^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)/((a^2*c*x^3 + c*x)*arctan(a*x)^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{c\left(a^2x^2+1\right)}\,\mathrm{atan}^2\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/atan(a*x)**2/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(c*(a**2*x**2 + 1))*atan(a*x)**2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a^2cx^2+c}\,x\arctan\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a^2*c*x^2 + c)*x*arctan(a*x)^2), x)
```

3.579 $\int \frac{x^3}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2}\,dx$

Optimal. Leaf size=100

$$\frac{\text{Unintegrable}\left(\frac{x}{\sqrt{a^2cx^2+c}\tan^{-1}(ax)^2},x\right)}{a^2c}-\frac{\sqrt{a^2x^2+1}\text{CosIntegral}\left(\tan^{-1}(ax)\right)}{a^4c\sqrt{a^2cx^2+c}}+\frac{x}{a^3c\sqrt{a^2cx^2+c}\tan^{-1}(ax)}$$

```
[Out] x/(a^3*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]) - (Sqrt[1 + a^2*x^2]*CosIntegral[
ArcTan[a*x]])/(a^4*c*Sqrt[c + a^2*c*x^2]) + Unintegrable[x/(Sqrt[c + a^2*c*
x^2]*ArcTan[a*x]^2), x]/(a^2*c)
```

Rubi [A] time = 0.371759, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^3}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^3/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2),x]
```

```
[Out] x/(a^3*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]) - (Sqrt[1 + a^2*x^2]*CosIntegral[
ArcTan[a*x]])/(a^4*c*Sqrt[c + a^2*c*x^2]) + Defer[Int][x/(Sqrt[c + a^2*c*x^
2]*ArcTan[a*x]^2), x]/(a^2*c)
```

Rubi steps

$$\int \frac{x^3}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2}\,dx = -\frac{\int \frac{x}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2}\,dx}{a^2} + \frac{\int \frac{x}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}\,dx}{a^2c}$$

$$= \frac{x}{a^3c\sqrt{c+a^2cx^2}\tan^{-1}(ax)} - \frac{\int \frac{1}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}\,dx}{a^3} + \frac{\int \frac{x}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}\,dx}{a^2c}$$

$$= \frac{x}{a^3c\sqrt{c+a^2cx^2}\tan^{-1}(ax)} + \frac{\int \frac{x}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}\,dx}{a^2c} - \frac{\sqrt{1+a^2x^2}\int \frac{1}{\left(1+a^2x^2\right)^{3/2}\tan^{-1}(ax)}\,d}{a^3c\sqrt{c+a^2cx^2}}$$

$$= \frac{x}{a^3c\sqrt{c+a^2cx^2}\tan^{-1}(ax)} + \frac{\int \frac{x}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}\,dx}{a^2c} - \frac{\sqrt{1+a^2x^2}\,\mathrm{Subst}\left(\int \frac{\cos(x)}{x}\,dx, x\right.}{a^4c\sqrt{c+a^2cx^2}}$$

$$= \frac{x}{a^3c\sqrt{c+a^2cx^2}\tan^{-1}(ax)} - \frac{\sqrt{1+a^2x^2}\,\mathrm{Ci}\left(\tan^{-1}(ax)\right)}{a^4c\sqrt{c+a^2cx^2}} + \frac{\int \frac{x}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}\,dx}{a^2c}$$

Mathematica [A] time = 9.7761, size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^3/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2),x]
```

```
[Out] Integrate[x^3/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2), x]
```

Maple [A] time = 1.271, size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(\arctan(ax)\right)^2}\left(a^2cx^2+c\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x)
```

```
[Out] int(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(a^2cx^2+c\right)^{\frac{3}{2}}\arctan\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^3/((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a^2cx^2+c}\,x^3}{\left(a^4c^2x^4+2\,a^2c^2x^2+c^2\right)\arctan\left(ax\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^3/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arcta
n(a*x)^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\operatorname{atan}^2\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a**2*c*x**2+c)**(3/2)/atan(a*x)**2,x)
```

```
[Out] Integral(x**3/((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(a^2cx^2 + c\right)^{\frac{3}{2}} \arctan\left(ax\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^3/((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^2), x)
```

3.580 $\int \frac{x^2}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2}\,dx$

Optimal. Leaf size=97

$$\frac{\text{Unintegrable}\left(\frac{1}{\sqrt{a^2cx^2+c}\tan^{-1}(ax)^2},x\right)}{a^2c}+\frac{\sqrt{a^2x^2+1}\,\text{Si}\left(\tan^{-1}(ax)\right)}{a^3c\sqrt{a^2cx^2+c}}+\frac{1}{a^3c\sqrt{a^2cx^2+c}\tan^{-1}(ax)}$$

```
[Out] 1/(a^3*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]) + (Sqrt[1 + a^2*x^2]*SinIntegral[
ArcTan[a*x]])/(a^3*c*Sqrt[c + a^2*c*x^2]) + Unintegrable[1/(Sqrt[c + a^2*c*
x^2]*ArcTan[a*x]^2), x]/(a^2*c)
```

Rubi [A] time = 0.36594, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^2}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^2/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2),x]
```

```
[Out] 1/(a^3*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]) + (Sqrt[1 + a^2*x^2]*SinIntegral[
ArcTan[a*x]])/(a^3*c*Sqrt[c + a^2*c*x^2]) + Defer[Int][1/(Sqrt[c + a^2*c*x^
2]*ArcTan[a*x]^2), x]/(a^2*c)
```

Rubi steps

$$\int \frac{x^2}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2}\,dx = -\frac{\int \frac{1}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2}\,dx}{a^2} + \frac{\int \frac{1}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}\,dx}{a^2c}$$

$$= \frac{1}{a^3c\sqrt{c+a^2cx^2}\tan^{-1}(ax)} + \frac{\int \frac{x}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}\,dx}{a} + \frac{\int \frac{1}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}\,dx}{a^2c}$$

$$= \frac{1}{a^3c\sqrt{c+a^2cx^2}\tan^{-1}(ax)} + \frac{\int \frac{1}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}\,dx}{a^2c} + \frac{\sqrt{1+a^2x^2}\int \frac{x}{\left(1+a^2x^2\right)^{3/2}\tan^{-1}(ax)}\,d}{ac\sqrt{c+a^2cx^2}}$$

$$= \frac{1}{a^3c\sqrt{c+a^2cx^2}\tan^{-1}(ax)} + \frac{\int \frac{1}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}\,dx}{a^2c} + \frac{\sqrt{1+a^2x^2}\,\mathrm{Subst}\left(\int \frac{\sin(x)}{x}\,dx, x\right.}{a^3c\sqrt{c+a^2cx^2}}$$

$$= \frac{1}{a^3c\sqrt{c+a^2cx^2}\tan^{-1}(ax)} + \frac{\sqrt{1+a^2x^2}\,\mathrm{Si}\left(\tan^{-1}(ax)\right)}{a^3c\sqrt{c+a^2cx^2}} + \frac{\int \frac{1}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}\,dx}{a^2c}$$

Mathematica [A] time = 8.76207, size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^2/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2),x]

[Out] Integrate[x^2/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2), x]
```

Maple [A] time = 1.124, size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(\arctan(ax)\right)^2}\left(a^2cx^2+c\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x)

[Out] int(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(a^2cx^2+c\right)^{\frac{3}{2}}\arctan\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^2/((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a^2cx^2+c}\,x^2}{\left(a^4c^2x^4+2\,a^2c^2x^2+c^2\right)\arctan\left(ax\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^2/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arcta
n(a*x)^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\operatorname{atan}^2\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a**2*c*x**2+c)**(3/2)/atan(a*x)**2,x)
```

```
[Out] Integral(x**2/((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(a^2cx^2+c\right)^{\frac{3}{2}}\arctan\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2/((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^2), x)
```

3.581 $\int \frac{x}{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^2}\,dx$

Optimal. Leaf size=69

$$\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}\left(\tan^{-1}(ax)\right)}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\sqrt{a^2cx^2+c}\tan^{-1}(ax)}$$

```
[Out] -(x/(a*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])) + (Sqrt[1 + a^2*x^2]*CosIntegral
[ArcTan[a*x]])/(a^2*c*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.174492, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4942, 4905, 4904, 3302}

$$\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}\left(\tan^{-1}(ax)\right)}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\sqrt{a^2cx^2+c}\tan^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[x/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2),x]
```

```
[Out] -(x/(a*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])) + (Sqrt[1 + a^2*x^2]*CosIntegral
[ArcTan[a*x]])/(a^2*c*Sqrt[c + a^2*c*x^2])
```

Rule 4942

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^m*(d + e*x^2)^(q + 1)*(a + b*ArcT
an[c*x])^(p + 1))/(b*c*d*(p + 1)), x] - Dist[(f*m)/(b*c*(p + 1)), Int[(f*x)
^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 2, 0] && LtQ[p, -1]
```

Rule 4905

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[(d^(q + 1/2)*Sqrt[1 + c^2*x^2])/Sqrt[d + e*x^2], Int[(1 + c
^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

Rule 4904

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2}\,dx &= -\frac{x}{ac\sqrt{c+a^2cx^2}\tan^{-1}(ax)}+\frac{\int \frac{1}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}\,dx}{a}\\ &= -\frac{x}{ac\sqrt{c+a^2cx^2}\tan^{-1}(ax)}+\frac{\sqrt{1+a^2x^2}\int \frac{1}{\left(1+a^2x^2\right)^{3/2}\tan^{-1}(ax)}\,dx}{ac\sqrt{c+a^2cx^2}}\\ &= -\frac{x}{ac\sqrt{c+a^2cx^2}\tan^{-1}(ax)}+\frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \frac{\cos(x)}{x}\,dx,x,\tan^{-1}(ax)\right)}{a^2c\sqrt{c+a^2cx^2}}\\ &= -\frac{x}{ac\sqrt{c+a^2cx^2}\tan^{-1}(ax)}+\frac{\sqrt{1+a^2x^2}\,\text{Ci}\left(\tan^{-1}(ax)\right)}{a^2c\sqrt{c+a^2cx^2}}\end{aligned}$$

Mathematica [A] time = 0.100657, size = 55, normalized size = 0.8

$$\frac{\sqrt{a^2x^2+1}\tan^{-1}(ax)\text{CosIntegral}\left(\tan^{-1}(ax)\right)-ax}{a^2c\sqrt{a^2cx^2+c}\tan^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2),x]
```

```
[Out] (-(a*x) + Sqrt[1 + a^2*x^2]*ArcTan[a*x]*CosIntegral[ArcTan[a*x]])/(a^2*c*Sq
rt[c + a^2*c*x^2]*ArcTan[a*x])
```

Maple [C] time = 0.309, size = 210, normalized size = 3.

$$-\frac{1}{2\,c^{2}\arctan\left(ax\right)a^{2}}\left(\arctan\left(ax\right)\mathrm{Ei}\left(1,-i\arctan\left(ax\right)\right)x^{2}a^{2}+\mathrm{Ei}\left(1,-i\arctan\left(ax\right)\right)\arctan\left(ax\right)+\sqrt{a^{2}x^{2}+1}\,xa-i\sqrt{a^{2}x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x)
```

```
[Out] -1/2*(arctan(a*x)*Ei(1,-I*arctan(a*x))*x^2*a^2+Ei(1,-I*arctan(a*x))*arctan(
a*x)+(a^2*x^2+1)^(1/2)*x*a-I*(a^2*x^2+1)^(1/2))/(a^2*x^2+1)^(3/2)*(c*(a*x-I
)*(a*x+I))^(1/2)/arctan(a*x)/c^2/a^2-1/2*(arctan(a*x)*Ei(1,I*arctan(a*x))*x
^2*a^2+Ei(1,I*arctan(a*x))*arctan(a*x)+(a^2*x^2+1)^(1/2)*x*a+I*(a^2*x^2+1)^
(1/2))/(a^2*x^2+1)^(3/2)*(c*(a*x-I)*(a*x+I))^(1/2)/arctan(a*x)/c^2/a^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(a^{2}cx^{2}+c\right)^{\frac{3}{2}}\arctan\left(ax\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(x/((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{a^{2}cx^{2}+c}\,x}{\left(a^{4}c^{2}x^{4}+2\,a^{2}c^{2}x^{2}+c^{2}\right)\arctan\left(ax\right)^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(
a*x)^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\operatorname{atan}^2(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a**2*c*x**2+c)**(3/2)/atan(a*x)**2,x)
```

```
[Out] Integral(x/((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(a^2cx^2+c\right)^{\frac{3}{2}}\arctan(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x/((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^2), x)
```

3.582 $\int \frac{1}{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^2}\,dx$

Optimal. Leaf size=69

$$-\frac{\sqrt{a^2x^2+1}\,\mathrm{Si}\left(\tan^{-1}(ax)\right)}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\sqrt{a^2cx^2+c}\,\tan^{-1}(ax)}$$

```
[Out] -(1/(a*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])) - (Sqrt[1 + a^2*x^2]*SinIntegral
[ArcTan[a*x]])/(a*c*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.204715, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {4902, 4971, 4970, 3299}

$$-\frac{\sqrt{a^2x^2+1}\,\mathrm{Si}\left(\tan^{-1}(ax)\right)}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\sqrt{a^2cx^2+c}\,\tan^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2),x]
```

```
[Out] -(1/(a*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])) - (Sqrt[1 + a^2*x^2]*SinIntegral
[ArcTan[a*x]])/(a*c*Sqrt[c + a^2*c*x^2])
```

Rule 4902

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^(p + 1))/(b*c*d*(p
+ 1)), x] - Dist[(2*c*(q + 1))/(b*(p + 1)), Int[x*(d + e*x^2)^q*(a + b*ArcT
an[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && L
tQ[q, -1] && LtQ[p, -1]
```

Rule 4971

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[(d^(q + 1/2)*Sqrt[1 + c^2*x^2])/Sqrt[d + e*x^2],
 Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d
, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(In
tegerQ[q] || GtQ[d, 0])
```

Rule 4970

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sin[x]^m)/
Cos[x]^(m + 2*(q + 1)), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2}\,dx &= -\frac{1}{ac\sqrt{c+a^2cx^2}\tan^{-1}(ax)}-a\int \frac{x}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}\,dx\\
&= -\frac{1}{ac\sqrt{c+a^2cx^2}\tan^{-1}(ax)}-\frac{\left(a\sqrt{1+a^2x^2}\right)\int \frac{x}{\left(1+a^2x^2\right)^{3/2}\tan^{-1}(ax)}\,dx}{c\sqrt{c+a^2cx^2}}\\
&= -\frac{1}{ac\sqrt{c+a^2cx^2}\tan^{-1}(ax)}-\frac{\sqrt{1+a^2x^2}\,\mathrm{Subst}\left(\int \frac{\sin(x)}{x}\,dx,x,\tan^{-1}(ax)\right)}{ac\sqrt{c+a^2cx^2}}\\
&= -\frac{1}{ac\sqrt{c+a^2cx^2}\tan^{-1}(ax)}-\frac{\sqrt{1+a^2x^2}\,\mathrm{Si}\left(\tan^{-1}(ax)\right)}{ac\sqrt{c+a^2cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.100209, size = 53, normalized size = 0.77

$$-\frac{\sqrt{a^2x^2+1}\tan^{-1}(ax)\mathrm{Si}\left(\tan^{-1}(ax)\right)+1}{ac\sqrt{a^2cx^2+c}\tan^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2),x]
```

```
[Out] -((1 + Sqrt[1 + a^2*x^2]*ArcTan[a*x]*SinIntegral[ArcTan[a*x]])/(a*c*Sqrt[c
+ a^2*c*x^2]*ArcTan[a*x]))
```

Maple [C] time = 0.279, size = 212, normalized size = 3.1

$$\frac{\frac{i}{2}}{ac^2 \arctan(ax)} \left(\arctan(ax) \operatorname{Ei}(1, i\arctan(ax)) x^2 a^2 + \operatorname{Ei}(1, i\arctan(ax)) \arctan(ax) + \sqrt{a^2x^2+1}\,xa + i\sqrt{a^2x^2+1}\right) \sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x)
```

```
[Out] 1/2*I*(arctan(a*x)*Ei(1,I*arctan(a*x))*x^2*a^2+Ei(1,I*arctan(a*x))*arctan(a
*x)+(a^2*x^2+1)^(1/2)*x*a+I*(a^2*x^2+1)^(1/2))/(a^2*x^2+1)^(3/2)*(c*(a*x-I)
*(a*x+I))^(1/2)/arctan(a*x)/c^2/a-1/2*I*(arctan(a*x)*Ei(1,-I*arctan(a*x))*x
^2*a^2+Ei(1,-I*arctan(a*x))*arctan(a*x)+(a^2*x^2+1)^(1/2)*x*a-I*(a^2*x^2+1)
^(1/2))/(a^2*x^2+1)^(3/2)*(c*(a*x-I)*(a*x+I))^(1/2)/arctan(a*x)/c^2/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2 + c\right)^{\frac{3}{2}} \arctan(ax)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a^2cx^2 + c}}{\left(a^4c^2x^4 + 2\,a^2c^2x^2 + c^2\right)\arctan(ax)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(a*
x)^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\operatorname{atan}^2\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*c*x**2+c)**(3/2)/atan(a*x)**2,x)
```

```
[Out] Integral(1/((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)^{\frac{3}{2}}\arctan\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^2), x)
```

3.583 $\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2}\,dx$

Optimal. Leaf size=129

$$-\frac{\text{Unintegrable}\left(\frac{1}{x^2\sqrt{a^2cx^2+c}\tan^{-1}(ax)},x\right)}{ac}-\frac{\sqrt{a^2cx^2+c}}{ac^2x\tan^{-1}(ax)}-\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}\left(\tan^{-1}(ax)\right)}{c\sqrt{a^2cx^2+c}}+\frac{ax}{c\sqrt{a^2cx^2+c}\tan^{-1}(ax)}$$

```
[Out] (a*x)/(c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]) - Sqrt[c + a^2*c*x^2]/(a*c^2*x*Ar
cTan[a*x]) - (Sqrt[1 + a^2*x^2]*CosIntegral[ArcTan[a*x]])/(c*Sqrt[c + a^2*c
*x^2]) - Unintegrable[1/(x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]), x]/(a*c)
```

Rubi [A] time = 0.506161, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2),x]
```

```
[Out] (a*x)/(c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]) - Sqrt[c + a^2*c*x^2]/(a*c^2*x*Ar
cTan[a*x]) - (Sqrt[1 + a^2*x^2]*CosIntegral[ArcTan[a*x]])/(c*Sqrt[c + a^2*c
*x^2]) - Defer[Int][1/(x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]), x]/(a*c)
```

Rubi steps

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2}\,dx = -\left(a^2\int \frac{x}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2}\,dx\right)+\frac{\int \frac{1}{x\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}\,dx}{c}$$

$$= \frac{ax}{c\sqrt{c+a^2cx^2}\tan^{-1}(ax)}-\frac{\sqrt{c+a^2cx^2}}{ac^2x\tan^{-1}(ax)}-a\int \frac{1}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}\,dx-\frac{\int \frac{}{x^2}}{}$$

$$= \frac{ax}{c\sqrt{c+a^2cx^2}\tan^{-1}(ax)}-\frac{\sqrt{c+a^2cx^2}}{ac^2x\tan^{-1}(ax)}-\frac{\int \frac{1}{x^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}\,dx}{ac}-\frac{\left(a\sqrt{1+a^2x^2}\right)}{c}$$

$$= \frac{ax}{c\sqrt{c+a^2cx^2}\tan^{-1}(ax)}-\frac{\sqrt{c+a^2cx^2}}{ac^2x\tan^{-1}(ax)}-\frac{\int \frac{1}{x^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}\,dx}{ac}-\frac{\sqrt{1+a^2x^2}\,\mathrm{Su}}{}$$

$$= \frac{ax}{c\sqrt{c+a^2cx^2}\tan^{-1}(ax)}-\frac{\sqrt{c+a^2cx^2}}{ac^2x\tan^{-1}(ax)}-\frac{\sqrt{1+a^2x^2}\,\mathrm{Ci}\left(\tan^{-1}(ax)\right)}{c\sqrt{c+a^2cx^2}}-\frac{\int \frac{}{x^2\sqrt{c+a^2cx}}}{}$$

Mathematica [A] time = 1.85349, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2),x]
```

```
[Out] Integrate[1/(x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2), x]
```

Maple [A] time = 0.509, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(\arctan(ax)\right)^2}\left(a^2cx^2+c\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x)
```

```
[Out] int(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)^{\frac{3}{2}}x\arctan\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^(3/2)*x*arctan(a*x)^2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{a^2cx^2+c}}{\left(a^4c^2x^5+2\,a^2c^2x^3+c^2x\right)\arctan\left(ax\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)/((a^4*c^2*x^5 + 2*a^2*c^2*x^3 + c^2*x)*arctan(
a*x)^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\operatorname{atan}^2\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a**2*c*x**2+c)**(3/2)/atan(a*x)**2,x)
```

```
[Out] Integral(1/(x*(c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^{2}cx^{2}+c\right)^{\frac{3}{2}} x \arctan\left(ax\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^(3/2)*x*arctan(a*x)^2), x)
```

3.584 $\int \frac{1}{x^2\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2}\,dx$

Optimal. Leaf size=93

$$\frac{\text{Unintegrable}\left(\frac{1}{x^2\sqrt{a^2cx^2+c}\tan^{-1}(ax)^2},x\right)}{c}+\frac{a\sqrt{a^2x^2+1}\text{Si}\left(\tan^{-1}(ax)\right)}{c\sqrt{a^2cx^2+c}}+\frac{a}{c\sqrt{a^2cx^2+c}\tan^{-1}(ax)}$$

```
[Out] a/(c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]) + (a*Sqrt[1 + a^2*x^2]*SinIntegral[Ar
cTan[a*x]])/(c*Sqrt[c + a^2*c*x^2]) + Unintegrable[1/(x^2*Sqrt[c + a^2*c*x^
2]*ArcTan[a*x]^2), x]/c
```

Rubi [A] time = 0.425285, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2),x]
```

```
[Out] a/(c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]) + (a*Sqrt[1 + a^2*x^2]*SinIntegral[Ar
cTan[a*x]])/(c*Sqrt[c + a^2*c*x^2]) + Defer[Int][1/(x^2*Sqrt[c + a^2*c*x^2]
*ArcTan[a*x]^2), x]/c
```

Rubi steps

$$\int \frac{1}{x^2 \left(c+a^2cx^2\right)^{3/2} \tan^{-1}(ax)^2}\,dx = -\left(a^2 \int \frac{1}{\left(c+a^2cx^2\right)^{3/2} \tan^{-1}(ax)^2}\,dx\right) + \frac{\int \frac{1}{x^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}\,dx}{c}$$

$$= \frac{a}{c\sqrt{c+a^2cx^2}\tan^{-1}(ax)} + a^3 \int \frac{x}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}\,dx + \frac{\int \frac{1}{x^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}}{c}$$

$$= \frac{a}{c\sqrt{c+a^2cx^2}\tan^{-1}(ax)} + \frac{\int \frac{1}{x^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}\,dx}{c} + \frac{\left(a^3\sqrt{1+a^2x^2}\right)\int \frac{x}{\left(1+a^2x^2\right)^{3/2}\tan}}{c\sqrt{c+a^2cx^2}}$$

$$= \frac{a}{c\sqrt{c+a^2cx^2}\tan^{-1}(ax)} + \frac{\int \frac{1}{x^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}\,dx}{c} + \frac{\left(a\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int \frac{\sin(x)}{x}\right.}{c\sqrt{c+a^2cx^2}}$$

$$= \frac{a}{c\sqrt{c+a^2cx^2}\tan^{-1}(ax)} + \frac{a\sqrt{1+a^2x^2}\,\text{Si}\left(\tan^{-1}(ax)\right)}{c\sqrt{c+a^2cx^2}} + \frac{\int \frac{1}{x^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}\,dx}{c}$$

Mathematica [A] time = 3.45842, size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \left(c+a^2cx^2\right)^{3/2} \tan^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2),x]
```

```
[Out] Integrate[1/(x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2), x]
```

Maple [A] time = 0.534, size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \left(\arctan(ax)\right)^2} \left(a^2cx^2+c\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x)
```

```
[Out] int(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)^{\frac{3}{2}}x^2\arctan\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^(3/2)*x^2*arctan(a*x)^2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a^2cx^2+c}}{\left(a^4c^2x^6+2\,a^2c^2x^4+c^2x^2\right)\arctan\left(ax\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)/((a^4*c^2*x^6 + 2*a^2*c^2*x^4 + c^2*x^2)*arcta
n(a*x)^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\operatorname{atan}^2\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a**2*c*x**2+c)**(3/2)/atan(a*x)**2,x)
```

```
[Out] Integral(1/(x**2*(c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)^{\frac{3}{2}}x^2\arctan\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^(3/2)*x^2*arctan(a*x)^2), x)
```

3.585 $\int \frac{1}{x^3(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^2}\,dx$

Optimal. Leaf size=158

$$\frac{a\text{Unintegrable}\left(\frac{1}{x^2\sqrt{a^2cx^2+c}\tan^{-1}(ax)},x\right)}{c}+\frac{\text{Unintegrable}\left(\frac{1}{x^3\sqrt{a^2cx^2+c}\tan^{-1}(ax)^2},x\right)}{c}+\frac{a\sqrt{a^2cx^2+c}}{c^2x\tan^{-1}(ax)}+\frac{a^2\sqrt{a^2x^2+1}\text{CosInte}}{c\sqrt{a^2cx}}$$

```
[Out] -((a^3*x)/(c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])) + (a*Sqrt[c + a^2*c*x^2])/(c
^2*x*ArcTan[a*x]) + (a^2*Sqrt[1 + a^2*x^2]*CosIntegral[ArcTan[a*x]])/(c*Sqr
t[c + a^2*c*x^2]) + Unintegrable[1/(x^3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2),
 x]/c + (a*Unintegrable[1/(x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]), x])/c
```

Rubi [A] time = 0.7405, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{1}{x^3\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^3*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2),x]
```

```
[Out] -((a^3*x)/(c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])) + (a*Sqrt[c + a^2*c*x^2])/(c
^2*x*ArcTan[a*x]) + (a^2*Sqrt[1 + a^2*x^2]*CosIntegral[ArcTan[a*x]])/(c*Sqr
t[c + a^2*c*x^2]) + Defer[Int][1/(x^3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2), x
]/c + (a*Defer[Int][1/(x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]), x])/c
```

Rubi steps

$$\int \frac{1}{x^3 \left(c+a^2 c x^2\right)^{3/2} \tan^{-1}(ax)^2}\,dx = -\left(a^2 \int \frac{1}{x\left(c+a^2 c x^2\right)^{3/2} \tan^{-1}(ax)^2}\,dx\right) + \frac{\int \frac{1}{x^3\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}\,dx}{c}$$

$$= a^4 \int \frac{x}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2}\,dx + \frac{\int \frac{1}{x^3\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}\,dx}{c} - \frac{a^2\int \frac{1}{x\sqrt{c+a^2cx^2}\tan^{-1}}}{c}$$

$$= -\frac{a^3 x}{c\sqrt{c+a^2cx^2}\tan^{-1}(ax)} + \frac{a\sqrt{c+a^2cx^2}}{c^2 x\tan^{-1}(ax)} + a^3\int \frac{1}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}\,dx + \frac{\int}{}$$

$$= -\frac{a^3 x}{c\sqrt{c+a^2cx^2}\tan^{-1}(ax)} + \frac{a\sqrt{c+a^2cx^2}}{c^2 x\tan^{-1}(ax)} + \frac{\int \frac{1}{x^3\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}\,dx}{c} + \frac{a\int \frac{}{x^2\sqrt{c+a^2}}}{}$$

$$= -\frac{a^3 x}{c\sqrt{c+a^2cx^2}\tan^{-1}(ax)} + \frac{a\sqrt{c+a^2cx^2}}{c^2 x\tan^{-1}(ax)} + \frac{\int \frac{1}{x^3\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}\,dx}{c} + \frac{a\int \frac{}{x^2\sqrt{c+a^2}}}{}$$

$$= -\frac{a^3 x}{c\sqrt{c+a^2cx^2}\tan^{-1}(ax)} + \frac{a\sqrt{c+a^2cx^2}}{c^2 x\tan^{-1}(ax)} + \frac{a^2\sqrt{1+a^2x^2}\,\mathrm{Ci}\left(\tan^{-1}(ax)\right)}{c\sqrt{c+a^2cx^2}} + \frac{\int \frac{}{x^3\sqrt{c}}}{}$$

Mathematica [A] time = 5.80862, size = 0, normalized size = 0.

$$\int \frac{1}{x^3\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^3*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2),x]
```

```
[Out] Integrate[1/(x^3*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2), x]
```

Maple [A] time = 0.698, size = 0, normalized size = 0.

$$\int \frac{1}{x^3\left(\arctan(ax)\right)^2}\left(a^2cx^2+c\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x)
```

```
[Out] int(1/x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)^{\frac{3}{2}}x^3\arctan\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^(3/2)*x^3*arctan(a*x)^2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{a^2cx^2+c}}{\left(a^4c^2x^7+2\,a^2c^2x^5+c^2x^3\right)\arctan\left(ax\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)/((a^4*c^2*x^7 + 2*a^2*c^2*x^5 + c^2*x^3)*arcta
n(a*x)^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\operatorname{atan}^2\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a**2*c*x**2+c)**(3/2)/atan(a*x)**2,x)
```

```
[Out] Integral(1/(x**3*(c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)^{\frac{3}{2}}x^3\arctan\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^(3/2)*x^3*arctan(a*x)^2), x)
```

3.586 $\int \frac{1}{x^4\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2}\,dx$

Optimal. Leaf size=133

$$-\frac{a^2\text{Unintegrable}\left(\frac{1}{x^2\sqrt{a^2cx^2+c}\tan^{-1}(ax)^2},x\right)}{c}+\frac{\text{Unintegrable}\left(\frac{1}{x^4\sqrt{a^2cx^2+c}\tan^{-1}(ax)^2},x\right)}{c}-\frac{a^3\sqrt{a^2x^2+1}\text{Si}\left(\tan^{-1}(ax)\right)}{c\sqrt{a^2cx^2+c}}-\frac{}{c\sqrt{a}}$$

```
[Out] -(a^3/(c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])) - (a^3*Sqrt[1 + a^2*x^2]*SinInte
gral[ArcTan[a*x]])/(c*Sqrt[c + a^2*c*x^2]) + Unintegrable[1/(x^4*Sqrt[c + a
^2*c*x^2]*ArcTan[a*x]^2), x]/c - (a^2*Unintegrable[1/(x^2*Sqrt[c + a^2*c*x^
2]*ArcTan[a*x]^2), x])/c
```

Rubi [A] time = 0.665191, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^4\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^4*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2),x]
```

```
[Out] -(a^3/(c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])) - (a^3*Sqrt[1 + a^2*x^2]*SinInte
gral[ArcTan[a*x]])/(c*Sqrt[c + a^2*c*x^2]) + Defer[Int][1/(x^4*Sqrt[c + a^2
*c*x^2]*ArcTan[a*x]^2), x]/c - (a^2*Defer[Int][1/(x^2*Sqrt[c + a^2*c*x^2]*A
rcTan[a*x]^2), x])/c
```

Rubi steps

$$\int \frac{1}{x^4 \left(c+a^2 c x^2\right)^{3/2} \tan^{-1}(ax)^2} \, dx = -\left(a^2 \int \frac{1}{x^2 \left(c+a^2 c x^2\right)^{3/2} \tan^{-1}(ax)^2} \, dx\right) + \frac{\int \frac{1}{x^4 \sqrt{c+a^2 c x^2} \tan^{-1}(ax)^2} \, dx}{c}$$

$$= a^4 \int \frac{1}{\left(c+a^2 c x^2\right)^{3/2} \tan^{-1}(ax)^2} \, dx + \frac{\int \frac{1}{x^4 \sqrt{c+a^2 c x^2} \tan^{-1}(ax)^2} \, dx}{c} - \frac{a^2 \int \frac{1}{x^2 \sqrt{c+a^2 c x^2} \tan^{-1}}}{c}$$

$$= -\frac{a^3}{c \sqrt{c+a^2 c x^2} \tan^{-1}(ax)} - a^5 \int \frac{x}{\left(c+a^2 c x^2\right)^{3/2} \tan^{-1}(ax)} \, dx + \frac{\int \frac{1}{x^4 \sqrt{c+a^2 c x^2} \tan^{-1}(ax}}{c}$$

$$= -\frac{a^3}{c \sqrt{c+a^2 c x^2} \tan^{-1}(ax)} + \frac{\int \frac{1}{x^4 \sqrt{c+a^2 c x^2} \tan^{-1}(ax)^2} \, dx}{c} - \frac{a^2 \int \frac{1}{x^2 \sqrt{c+a^2 c x^2} \tan^{-1}(ax)^2} \, dx}{c} -$$

$$= -\frac{a^3}{c \sqrt{c+a^2 c x^2} \tan^{-1}(ax)} + \frac{\int \frac{1}{x^4 \sqrt{c+a^2 c x^2} \tan^{-1}(ax)^2} \, dx}{c} - \frac{a^2 \int \frac{1}{x^2 \sqrt{c+a^2 c x^2} \tan^{-1}(ax)^2} \, dx}{c} -$$

$$= -\frac{a^3}{c \sqrt{c+a^2 c x^2} \tan^{-1}(ax)} - \frac{a^3 \sqrt{1+a^2 x^2} \, \mathrm{Si}\left(\tan^{-1}(ax)\right)}{c \sqrt{c+a^2 c x^2}} + \frac{\int \frac{1}{x^4 \sqrt{c+a^2 c x^2} \tan^{-1}(ax)^2} \, dx}{c}$$

Mathematica [A] time = 6.57957, size = 0, normalized size = 0.

$$\int \frac{1}{x^4 \left(c+a^2 c x^2\right)^{3/2} \tan^{-1}(ax)^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^4*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2),x]
```

```
[Out] Integrate[1/(x^4*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2), x]
```

Maple [A] time = 1.17, size = 0, normalized size = 0.

$$\int \frac{1}{x^4 \left(\arctan(ax)\right)^2} \left(a^2 c x^2 + c\right)^{-\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x)
```

```
[Out] int(1/x^4/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)^{\frac{3}{2}}x^4\arctan\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^(3/2)*x^4*arctan(a*x)^2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{a^2cx^2+c}}{\left(a^4c^2x^8+2\,a^2c^2x^6+c^2x^4\right)\arctan\left(ax\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)/((a^4*c^2*x^8 + 2*a^2*c^2*x^6 + c^2*x^4)*arcta
n(a*x)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(a**2*c*x**2+c)**(3/2)/atan(a*x)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)^{\frac{3}{2}}x^4\arctan\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^(3/2)*x^4*arctan(a*x)^2), x)
```

3.587 $\int \frac{x^5}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^2}\,dx$

Optimal. Leaf size=176

$$\frac{\text{Unintegrable}\left(\frac{x}{\sqrt{a^2cx^2+c}\tan^{-1}(ax)^2},x\right)}{a^4c^2}-\frac{7\sqrt{a^2x^2+1}\text{CosIntegral}\left(\tan^{-1}(ax)\right)}{4a^6c^2\sqrt{a^2cx^2+c}}+\frac{3\sqrt{a^2x^2+1}\text{CosIntegral}\left(3\tan^{-1}(ax)\right)}{4a^6c^2\sqrt{a^2cx^2+c}}+$$

```
[Out] x^3/(a^3*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]) + x/(a^5*c^2*Sqrt[c + a^2*c*x
^2]*ArcTan[a*x]) - (7*Sqrt[1 + a^2*x^2]*CosIntegral[ArcTan[a*x]])/(4*a^6*c^
2*Sqrt[c + a^2*c*x^2]) + (3*Sqrt[1 + a^2*x^2]*CosIntegral[3*ArcTan[a*x]])/(
4*a^6*c^2*Sqrt[c + a^2*c*x^2]) + Unintegrable[x/(Sqrt[c + a^2*c*x^2]*ArcTan
[a*x]^2), x]/(a^4*c^2)
```

Rubi [A] time = 0.911595, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^5}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^5/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2),x]
```

```
[Out] x^3/(a^3*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]) + x/(a^5*c^2*Sqrt[c + a^2*c*x
^2]*ArcTan[a*x]) - (7*Sqrt[1 + a^2*x^2]*CosIntegral[ArcTan[a*x]])/(4*a^6*c^
2*Sqrt[c + a^2*c*x^2]) + (3*Sqrt[1 + a^2*x^2]*CosIntegral[3*ArcTan[a*x]])/(
4*a^6*c^2*Sqrt[c + a^2*c*x^2]) + Defer[Int][x/(Sqrt[c + a^2*c*x^2]*ArcTan[a
*x]^2), x]/(a^4*c^2)
```

Rubi steps

$$\int \frac{x^5}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^2}\,dx = -\frac{\int \frac{x^3}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^2}\,dx}{a^2} + \frac{\int \frac{x^3}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2}\,dx}{a^2c}$$

$$= \frac{x^3}{a^3c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)} - \frac{3\int \frac{x^2}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)}\,dx}{a^3} + \frac{\int \frac{x}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}\,dx}{a^4c^2} - \cdots$$

$$= \frac{x^3}{a^3c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)} + \frac{x}{a^5c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)} + \frac{\int \frac{x}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}\,dx}{a^4c^2} - \cdots$$

$$= \frac{x^3}{a^3c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)} + \frac{x}{a^5c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)} + \frac{\int \frac{x}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}\,dx}{a^4c^2} - \cdots$$

$$= \frac{x^3}{a^3c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)} + \frac{x}{a^5c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)} + \frac{\int \frac{x}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}\,dx}{a^4c^2} - \cdots$$

$$= \frac{x^3}{a^3c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)} + \frac{x}{a^5c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)} - \frac{\sqrt{1+a^2x^2}\,\mathrm{Ci}\left(\tan^{-1}(ax)\right)}{a^6c^2\sqrt{c+a^2cx^2}} \cdots$$

$$= \frac{x^3}{a^3c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)} + \frac{x}{a^5c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)} - \frac{7\sqrt{1+a^2x^2}\,\mathrm{Ci}\left(\tan^{-1}(ax\right.}{4a^6c^2\sqrt{c+a^2cx^2}} \cdots$$

Mathematica [A] time = 12.3128, size = 0, normalized size = 0.

$$\int \frac{x^5}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^5/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2),x]
```

```
[Out] Integrate[x^5/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2), x]
```

Maple [A] time = 1.791, size = 0, normalized size = 0.

$$\int \frac{x^5}{(\arctan(ax))^2}\left(a^2cx^2+c\right)^{-\frac{5}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x)
```

```
[Out] int(x^5/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\left(a^2cx^2+c\right)^{\frac{5}{2}}\arctan(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^5/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{a^2cx^2+c}\,x^5}{\left(a^6c^3x^6+3\,a^4c^3x^4+3\,a^2c^3x^2+c^3\right)\arctan(ax)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^5/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*
x^2 + c^3)*arctan(a*x)^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}\operatorname{atan}^2(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(a**2*c*x**2+c)**(5/2)/atan(a*x)**2,x)
```

```
[Out] Integral(x**5/((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\left(a^2cx^2+c\right)^{\frac{5}{2}}\arctan(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^5/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^2), x)
```

3.588 $\int \frac{x^4}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^2}\,dx$

Optimal. Leaf size=173

$$\frac{\text{Unintegrable}\left(\frac{1}{\sqrt{a^2cx^2+c}\tan^{-1}(ax)^2},x\right)}{a^4c^2}+\frac{5\sqrt{a^2x^2+1}\,\text{Si}\left(\tan^{-1}(ax)\right)}{4a^5c^2\sqrt{a^2cx^2+c}}-\frac{3\sqrt{a^2x^2+1}\,\text{Si}\left(3\tan^{-1}(ax)\right)}{4a^5c^2\sqrt{a^2cx^2+c}}+\frac{2}{a^5c^2\sqrt{a^2cx^2+c}\tan^{-}}$$

```
[Out] -(1/(a^5*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x])) + 2/(a^5*c^2*Sqrt[c + a^2*c*
x^2]*ArcTan[a*x]) + (5*Sqrt[1 + a^2*x^2]*SinIntegral[ArcTan[a*x]])/(4*a^5*c
^2*Sqrt[c + a^2*c*x^2]) - (3*Sqrt[1 + a^2*x^2]*SinIntegral[3*ArcTan[a*x]])/
(4*a^5*c^2*Sqrt[c + a^2*c*x^2]) + Unintegrable[1/(Sqrt[c + a^2*c*x^2]*ArcTa
n[a*x]^2), x]/(a^4*c^2)
```

Rubi [A] time = 1.08074, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^4}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^4/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2),x]
```

```
[Out] -(1/(a^5*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x])) + 2/(a^5*c^2*Sqrt[c + a^2*c*
x^2]*ArcTan[a*x]) + (5*Sqrt[1 + a^2*x^2]*SinIntegral[ArcTan[a*x]])/(4*a^5*c
^2*Sqrt[c + a^2*c*x^2]) - (3*Sqrt[1 + a^2*x^2]*SinIntegral[3*ArcTan[a*x]])/
(4*a^5*c^2*Sqrt[c + a^2*c*x^2]) + Defer[Int][1/(Sqrt[c + a^2*c*x^2]*ArcTan[
a*x]^2), x]/(a^4*c^2)
```

Rubi steps

$$\int \frac{x^4}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^2}\,dx = -\frac{\int \frac{x^2}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^2}\,dx}{a^2} + \frac{\int \frac{x^2}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2}\,dx}{a^2c}$$

$$= \frac{\int \frac{1}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^2}\,dx}{a^4} + \frac{\int \frac{1}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}\,dx}{a^4c^2} - 2\frac{\int \frac{1}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2}\,dx}{a^4c}$$

$$= -\frac{1}{a^5c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)} - \frac{3\int \frac{x}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)}\,dx}{a^3} + \frac{\int \frac{1}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}\,dx}{a^4c^2} -$$

$$= -\frac{1}{a^5c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)} + \frac{\int \frac{1}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}\,dx}{a^4c^2} - 2\left(-\frac{1}{a^5c^2\sqrt{c+a^2cx^2}\tan^{-1}(\right.$$

$$= -\frac{1}{a^5c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)} + \frac{\int \frac{1}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}\,dx}{a^4c^2} - 2\left(-\frac{1}{a^5c^2\sqrt{c+a^2cx^2}\tan^{-1}(\right.$$

$$= -\frac{1}{a^5c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)} - 2\left(-\frac{1}{a^5c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)} - \frac{\sqrt{1+a^2x^2}\,\text{Si}\left(\tan^{-}\right.}{a^5c^2\sqrt{c+a^2cx}}\right.$$

$$= -\frac{1}{a^5c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)} - 2\left(-\frac{1}{a^5c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)} - \frac{\sqrt{1+a^2x^2}\,\text{Si}\left(\tan^{-}\right.}{a^5c^2\sqrt{c+a^2cx}}\right.$$

$$= -\frac{1}{a^5c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)} - \frac{3\sqrt{1+a^2x^2}\,\text{Si}\left(\tan^{-1}(ax)\right)}{4a^5c^2\sqrt{c+a^2cx^2}} - 2\left(-\frac{1}{a^5c^2\sqrt{c+a^2cx^2}\tan}\right.$$

Mathematica [A] time = 11.2259, size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^4/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2),x]
```

```
[Out] Integrate[x^4/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2), x]
```

Maple [A] time = 1.131, size = 0, normalized size = 0.

$$\int \frac{x^{4}}{\left(\arctan \left(a x\right)\right)^{2}} \left(a^{2} c \,x^{2}+c\right)^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x)
```

```
[Out] int(x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{4}}{{\left(a^{2} c x^{2} + c\right)}^{\frac{5}{2}} \arctan \left(a x\right)^{2}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^4/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$${\rm integral}\left(\frac{\sqrt{a^{2} c x^{2} + c} x^{4}}{{\left(a^{6} c^{3} x^{6} + 3 \, a^{4} c^{3} x^{4} + 3 \, a^{2} c^{3} x^{2} + c^{3}\right)} \arctan \left(a x\right)^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^4/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*
x^2 + c^3)*arctan(a*x)^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}\operatorname{atan}^2(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(a**2*c*x**2+c)**(5/2)/atan(a*x)**2,x)
```

```
[Out] Integral(x**4/((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(a^2cx^2+c\right)^{\frac{5}{2}}\arctan(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^4/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^2), x)
```

3.589 $\int \frac{x^3}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^2}\,dx$

Optimal. Leaf size=118

$$\frac{3\sqrt{a^2x^2+1}\,\text{CosIntegral}\left(\tan^{-1}(ax)\right)}{4a^4c^2\sqrt{a^2cx^2+c}}-\frac{3\sqrt{a^2x^2+1}\,\text{CosIntegral}\left(3\tan^{-1}(ax)\right)}{4a^4c^2\sqrt{a^2cx^2+c}}-\frac{x^3}{ac\left(a^2cx^2+c\right)^{3/2}\tan^{-1}(ax)}$$

```
[Out] -(x^3/(a*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x])) + (3*Sqrt[1 + a^2*x^2]*CosIn
tegral[ArcTan[a*x]])/(4*a^4*c^2*Sqrt[c + a^2*c*x^2]) - (3*Sqrt[1 + a^2*x^2]
*CosIntegral[3*ArcTan[a*x]])/(4*a^4*c^2*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.402728, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {4942, 4971, 4970, 4406, 3302}

$$\frac{3\sqrt{a^2x^2+1}\,\text{CosIntegral}\left(\tan^{-1}(ax)\right)}{4a^4c^2\sqrt{a^2cx^2+c}}-\frac{3\sqrt{a^2x^2+1}\,\text{CosIntegral}\left(3\tan^{-1}(ax)\right)}{4a^4c^2\sqrt{a^2cx^2+c}}-\frac{x^3}{ac\left(a^2cx^2+c\right)^{3/2}\tan^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2),x]
```

```
[Out] -(x^3/(a*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x])) + (3*Sqrt[1 + a^2*x^2]*CosIn
tegral[ArcTan[a*x]])/(4*a^4*c^2*Sqrt[c + a^2*c*x^2]) - (3*Sqrt[1 + a^2*x^2]
*CosIntegral[3*ArcTan[a*x]])/(4*a^4*c^2*Sqrt[c + a^2*c*x^2])
```

Rule 4942

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^m*(d + e*x^2)^(q + 1)*(a + b*ArcT
an[c*x])^(p + 1))/(b*c*d*(p + 1)), x] - Dist[(f*m)/(b*c*(p + 1)), Int[(f*x)
^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 2, 0] && LtQ[p, -1]
```

Rule 4971

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[(d^(q + 1/2)*Sqrt[1 + c^2*x^2])/Sqrt[d + e*x^2],
 Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d
```

```
, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(In
tegerQ[q] || GtQ[d, 0])
```

Rule 4970

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sin[x]^m)/
Cos[x]^(m + 2*(q + 1)), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{x^3}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^2}\,dx = -\frac{x^3}{ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)} + \frac{3\int \frac{x^2}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)}\,dx}{a}$$

$$= -\frac{x^3}{ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)} + \frac{\left(3\sqrt{1+a^2x^2}\right)\int \frac{x^2}{\left(1+a^2x^2\right)^{5/2}\tan^{-1}(ax)}\,dx}{ac^2\sqrt{c+a^2cx^2}}$$

$$= -\frac{x^3}{ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)} + \frac{\left(3\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int \frac{\cos(x)\sin^2(x)}{x}\,dx, x, \tan^{-1}(ax)\right)}{a^4c^2\sqrt{c+a^2cx^2}}$$

$$= -\frac{x^3}{ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)} + \frac{\left(3\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int \left(\frac{\cos(x)}{4x}-\frac{\cos(3x)}{4x}\right)dx, x, \tan^{-1}(a\right.}{a^4c^2\sqrt{c+a^2cx^2}}$$

$$= -\frac{x^3}{ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)} + \frac{\left(3\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int \frac{\cos(x)}{x}\,dx, x, \tan^{-1}(ax)\right)}{4a^4c^2\sqrt{c+a^2cx^2}} - (3\sqrt{1}$$

$$= -\frac{x^3}{ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)} + \frac{3\sqrt{1+a^2x^2}\,\text{Ci}\left(\tan^{-1}(ax)\right)}{4a^4c^2\sqrt{c+a^2cx^2}} - \frac{3\sqrt{1+a^2x^2}\,\text{Ci}\left(3\tan^{-1}(a\right.}{4a^4c^2\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 0.204778, size = 82, normalized size = 0.69

$$\frac{3c\sqrt{a^2x^2+1}\left(\text{CosIntegral}\left(\tan^{-1}(ax)\right)-\text{CosIntegral}\left(3\tan^{-1}(ax)\right)\right)-\frac{4a^3cx^3}{\left(a^2x^2+1\right)\tan^{-1}(ax)}}{4a^4c^3\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2),x]
```

```
[Out] ((-4*a^3*c*x^3)/((1 + a^2*x^2)*ArcTan[a*x]) + 3*c*Sqrt[1 + a^2*x^2]*(CosInt
egral[ArcTan[a*x]] - CosIntegral[3*ArcTan[a*x]]))/(4*a^4*c^3*Sqrt[c + a^2*c
*x^2])
```

Maple [C] time = 1.072, size = 582, normalized size = 4.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x)
```

```
[Out] 1/8*(3*arctan(a*x)*Ei(1,-3*I*arctan(a*x))*x^4*a^4+6*arctan(a*x)*Ei(1,-3*I*a
rctan(a*x))*x^2*a^2-(a^2*x^2+1)^(1/2)*x^3*a^3+3*I*(a^2*x^2+1)^(1/2)*x^2*a^2
+3*Ei(1,-3*I*arctan(a*x))*arctan(a*x)+3*(a^2*x^2+1)^(1/2)*x*a-I*(a^2*x^2+1)
^(1/2))/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(a*x+I))^(1/2)/(a^4*x^4+2*a^2*x^2+1)/a
rctan(a*x)/c^3/a^4+1/8*(3*arctan(a*x)*Ei(1,3*I*arctan(a*x))*x^4*a^4-(a^2*x^
2+1)^(1/2)*x^3*a^3+6*arctan(a*x)*Ei(1,3*I*arctan(a*x))*x^2*a^2-3*I*(a^2*x^2
+1)^(1/2)*x^2*a^2+3*(a^2*x^2+1)^(1/2)*x*a+3*Ei(1,3*I*arctan(a*x))*arctan(a*
x)+I*(a^2*x^2+1)^(1/2))/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(a*x+I))^(1/2)/(a^4*x^
4+2*a^2*x^2+1)/arctan(a*x)/c^3/a^4-3/8*(arctan(a*x)*Ei(1,I*arctan(a*x))*x^2
*a^2+Ei(1,I*arctan(a*x))*arctan(a*x)+(a^2*x^2+1)^(1/2)*x*a+I*(a^2*x^2+1)^(1
/2))/(a^2*x^2+1)^(3/2)*(c*(a*x-I)*(a*x+I))^(1/2)/arctan(a*x)/c^3/a^4-3/8*(a
rctan(a*x)*Ei(1,-I*arctan(a*x))*x^2*a^2+Ei(1,-I*arctan(a*x))*arctan(a*x)+(a
^2*x^2+1)^(1/2)*x*a-I*(a^2*x^2+1)^(1/2))/(a^2*x^2+1)^(3/2)*(c*(a*x-I)*(a*x+
I))^(1/2)/arctan(a*x)/c^3/a^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(a^2cx^2+c\right)^{\frac{5}{2}}\arctan\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^3/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{a^2cx^2+c}\,x^3}{\left(a^6c^3x^6+3\,a^4c^3x^4+3\,a^2c^3x^2+c^3\right)\arctan\left(ax\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^3/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*
x^2 + c^3)*arctan(a*x)^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}\operatorname{atan}^2(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a**2*c*x**2+c)**(5/2)/atan(a*x)**2,x)
```

```
[Out] Integral(x**3/((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(a^2cx^2+c\right)^{\frac{5}{2}}\arctan(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^3/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^2), x)
```

3.590 $\int \frac{x^2}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^2}\,dx$

Optimal. Leaf size=142

$$-\frac{\sqrt{a^2x^2+1}\,\mathrm{Si}\left(\tan^{-1}(ax)\right)}{4a^3c^2\sqrt{a^2cx^2+c}}+\frac{3\sqrt{a^2x^2+1}\,\mathrm{Si}\left(3\tan^{-1}(ax)\right)}{4a^3c^2\sqrt{a^2cx^2+c}}-\frac{1}{a^3c^2\sqrt{a^2cx^2+c}\tan^{-1}(ax)}+\frac{1}{a^3c\left(a^2cx^2+c\right)^{3/2}\tan^{-1}(ax)}$$

```
[Out] 1/(a^3*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]) - 1/(a^3*c^2*Sqrt[c + a^2*c*x^2
]*ArcTan[a*x]) - (Sqrt[1 + a^2*x^2]*SinIntegral[ArcTan[a*x]])/(4*a^3*c^2*Sq
rt[c + a^2*c*x^2]) + (3*Sqrt[1 + a^2*x^2]*SinIntegral[3*ArcTan[a*x]])/(4*a^
3*c^2*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.580778, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4964, 4902, 4971, 4970, 3299, 4406}

$$-\frac{\sqrt{a^2x^2+1}\,\mathrm{Si}\left(\tan^{-1}(ax)\right)}{4a^3c^2\sqrt{a^2cx^2+c}}+\frac{3\sqrt{a^2x^2+1}\,\mathrm{Si}\left(3\tan^{-1}(ax)\right)}{4a^3c^2\sqrt{a^2cx^2+c}}-\frac{1}{a^3c^2\sqrt{a^2cx^2+c}\tan^{-1}(ax)}+\frac{1}{a^3c\left(a^2cx^2+c\right)^{3/2}\tan^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2),x]
```

```
[Out] 1/(a^3*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]) - 1/(a^3*c^2*Sqrt[c + a^2*c*x^2
]*ArcTan[a*x]) - (Sqrt[1 + a^2*x^2]*SinIntegral[ArcTan[a*x]])/(4*a^3*c^2*Sq
rt[c + a^2*c*x^2]) + (3*Sqrt[1 + a^2*x^2]*SinIntegral[3*ArcTan[a*x]])/(4*a^
3*c^2*Sqrt[c + a^2*c*x^2])
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Arc
Tan[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan[c
*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p
, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rule 4902

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^(p + 1))/(b*c*d*(p
 + 1)), x] - Dist[(2*c*(q + 1))/(b*(p + 1)), Int[x*(d + e*x^2)^q*(a + b*ArcT
an[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && L
tQ[q, -1] && LtQ[p, -1]
```

Rule 4971

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[(d^(q + 1/2)*Sqrt[1 + c^2*x^2])/Sqrt[d + e*x^2],
 Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d
, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(In
tegerQ[q] || GtQ[d, 0])
```

Rule 4970

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sin[x]^m)/
Cos[x]^(m + 2*(q + 1)), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \frac{x^2}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^2}\,dx = -\frac{\int \frac{1}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^2}\,dx}{a^2} + \frac{\int \frac{1}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2}\,dx}{a^2c}$$

$$= \frac{1}{a^3c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)} - \frac{1}{a^3c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)} + \frac{3\int \frac{x}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)}\,dx}{a}$$

$$= \frac{1}{a^3c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)} - \frac{1}{a^3c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)} - \frac{\sqrt{1+a^2x^2}\int \frac{x}{\left(1+a^2x^2\right)^{3/2}\tan}}{ac^2\sqrt{c+a^2cx^2}}$$

$$= \frac{1}{a^3c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)} - \frac{1}{a^3c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)} - \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \frac{\sin(}{x}\right.}{a^3c^2\sqrt{c+a}}$$

$$= \frac{1}{a^3c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)} - \frac{1}{a^3c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)} - \frac{\sqrt{1+a^2x^2}\,\text{Si}\left(\tan^{-1}(ax)\right)}{a^3c^2\sqrt{c+a^2cx^2}}$$

$$= \frac{1}{a^3c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)} - \frac{1}{a^3c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)} - \frac{\sqrt{1+a^2x^2}\,\text{Si}\left(\tan^{-1}(ax)\right)}{a^3c^2\sqrt{c+a^2cx^2}}$$

$$= \frac{1}{a^3c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)} - \frac{1}{a^3c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)} - \frac{\sqrt{1+a^2x^2}\,\text{Si}\left(\tan^{-1}(ax)\right)}{4a^3c^2\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 0.266633, size = 99, normalized size = 0.7

$$-\frac{\left(a^2x^2+1\right)^{3/2}\tan^{-1}(ax)\text{Si}\left(\tan^{-1}(ax)\right) - 3\left(a^2x^2+1\right)^{3/2}\tan^{-1}(ax)\text{Si}\left(3\tan^{-1}(ax)\right) + 4a^2x^2}{4a^3c^2\left(a^2x^2+1\right)\sqrt{a^2cx^2+c}\tan^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2),x]
```

```
[Out] -(4*a^2*x^2 + (1 + a^2*x^2)^(3/2)*ArcTan[a*x]*SinIntegral[ArcTan[a*x]] - 3*
(1 + a^2*x^2)^(3/2)*ArcTan[a*x]*SinIntegral[3*ArcTan[a*x]])/(4*a^3*c^2*(1 +
 a^2*x^2)*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])
```

Maple [C] time = 0.977, size = 586, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x)
```

```
[Out] -1/8*I*(3*arctan(a*x)*Ei(1,3*I*arctan(a*x))*x^4*a^4-(a^2*x^2+1)^(1/2)*x^3*a
^3+6*arctan(a*x)*Ei(1,3*I*arctan(a*x))*x^2*a^2-3*I*(a^2*x^2+1)^(1/2)*x^2*a^
2+3*(a^2*x^2+1)^(1/2)*x*a+3*Ei(1,3*I*arctan(a*x))*arctan(a*x)+I*(a^2*x^2+1)
^(1/2))/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(a*x+I))^(1/2)/(a^4*x^4+2*a^2*x^2+1)/a
rctan(a*x)/c^3/a^3+1/8*I*(3*arctan(a*x)*Ei(1,-3*I*arctan(a*x))*x^4*a^4+6*ar
ctan(a*x)*Ei(1,-3*I*arctan(a*x))*x^2*a^2-(a^2*x^2+1)^(1/2)*x^3*a^3+3*I*(a^2
*x^2+1)^(1/2)*x^2*a^2+3*Ei(1,-3*I*arctan(a*x))*arctan(a*x)+3*(a^2*x^2+1)^(1
/2)*x*a-I*(a^2*x^2+1)^(1/2))/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(a*x+I))^(1/2)/(a
^4*x^4+2*a^2*x^2+1)/arctan(a*x)/c^3/a^3+1/8*I*(arctan(a*x)*Ei(1,I*arctan(a*
x))*x^2*a^2+Ei(1,I*arctan(a*x))*arctan(a*x)+(a^2*x^2+1)^(1/2)*x*a+I*(a^2*x^
2+1)^(1/2))/(a^2*x^2+1)^(3/2)*(c*(a*x-I)*(a*x+I))^(1/2)/arctan(a*x)/c^3/a^3
-1/8*I*(arctan(a*x)*Ei(1,-I*arctan(a*x))*x^2*a^2+Ei(1,-I*arctan(a*x))*arcta
n(a*x)+(a^2*x^2+1)^(1/2)*x*a-I*(a^2*x^2+1)^(1/2))/(a^2*x^2+1)^(3/2)*(c*(a*x
-I)*(a*x+I))^(1/2)/arctan(a*x)/c^3/a^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(a^2cx^2+c\right)^{\frac{5}{2}}\arctan\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^2/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{a^2cx^2+c}\,x^2}{\left(a^6c^3x^6+3\,a^4c^3x^4+3\,a^2c^3x^2+c^3\right)\arctan\left(ax\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^2/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*
x^2 + c^3)*arctan(a*x)^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}\operatorname{atan}^2(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a**2*c*x**2+c)**(5/2)/atan(a*x)**2,x)
```

```
[Out] Integral(x**2/((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(a^2cx^2+c\right)^{\frac{5}{2}}\arctan(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^2), x)
```

3.591 $\int \frac{x}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^2}\,dx$

Optimal. Leaf size=116

$$\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}\left(\tan^{-1}(ax)\right)}{4a^2c^2\sqrt{a^2cx^2+c}}+\frac{3\sqrt{a^2x^2+1}\,\text{CosIntegral}\left(3\tan^{-1}(ax)\right)}{4a^2c^2\sqrt{a^2cx^2+c}}-\frac{x}{ac\left(a^2cx^2+c\right)^{3/2}\tan^{-1}(ax)}$$

```
[Out] -(x/(a*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x])) + (Sqrt[1 + a^2*x^2]*CosIntegr
al[ArcTan[a*x]])/(4*a^2*c^2*Sqrt[c + a^2*c*x^2]) + (3*Sqrt[1 + a^2*x^2]*Cos
Integral[3*ArcTan[a*x]])/(4*a^2*c^2*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.501346, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {4968, 4971, 4970, 4406, 3302, 4905, 4904, 3312}

$$\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}\left(\tan^{-1}(ax)\right)}{4a^2c^2\sqrt{a^2cx^2+c}}+\frac{3\sqrt{a^2x^2+1}\,\text{CosIntegral}\left(3\tan^{-1}(ax)\right)}{4a^2c^2\sqrt{a^2cx^2+c}}-\frac{x}{ac\left(a^2cx^2+c\right)^{3/2}\tan^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[x/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2),x]
```


```
[Out] -(x/(a*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x])) + (Sqrt[1 + a^2*x^2]*CosIntegr
al[ArcTan[a*x]])/(4*a^2*c^2*Sqrt[c + a^2*c*x^2]) + (3*Sqrt[1 + a^2*x^2]*Cos
Integral[3*ArcTan[a*x]])/(4*a^2*c^2*Sqrt[c + a^2*c*x^2])
```


Rule 4968

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[(x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^(p
+ 1))/(b*c*d*(p + 1)), x] + (-Dist[(c*(m + 2*q + 2))/(b*(p + 1)), Int[x^(m
+ 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p + 1
)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; Fre
eQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &&
LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 4971

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[(d^(q + 1/2)*Sqrt[1 + c^2*x^2])/Sqrt[d + e*x^2],
```

```
Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d
, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(In
tegerQ[q] || GtQ[d, 0])
```

Rule 4970

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sin[x]^m)/
Cos[x]^(m + 2*(q + 1)), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 4905

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[(d^(q + 1/2)*Sqrt[1 + c^2*x^2])/Sqrt[d + e*x^2], Int[(1 + c
^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

Rule 4904

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\int \frac{x}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^2}\,dx = -\frac{x}{ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)} + \frac{\int \frac{1}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)}\,dx}{a} - (2a)\int \frac{x^2}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(}$$

$$= -\frac{x}{ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)} + \frac{\sqrt{1+a^2x^2}\int \frac{1}{\left(1+a^2x^2\right)^{5/2}\tan^{-1}(ax)}\,dx}{ac^2\sqrt{c+a^2cx^2}} - \frac{\left(2a\sqrt{1+a^2x^2}\right)\int}{c^2\sqrt{c}}$$

$$= -\frac{x}{ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)} + \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \frac{\cos^3(x)}{x}\,dx, x, \tan^{-1}(ax)\right)}{a^2c^2\sqrt{c+a^2cx^2}} - \frac{\left(2\sqrt{1+}\right.}{}$$

$$= -\frac{x}{ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)} + \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \left(\frac{3\cos(x)}{4x}+\frac{\cos(3x)}{4x}\right)dx, x, \tan^{-1}(ax\right.}{a^2c^2\sqrt{c+a^2cx^2}}$$

$$= -\frac{x}{ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)} + \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \frac{\cos(3x)}{x}\,dx, x, \tan^{-1}(ax)\right)}{4a^2c^2\sqrt{c+a^2cx^2}} - \frac{\sqrt{1+a}}{}$$

$$= -\frac{x}{ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)} + \frac{\sqrt{1+a^2x^2}\,\text{Ci}\left(\tan^{-1}(ax)\right)}{4a^2c^2\sqrt{c+a^2cx^2}} + \frac{3\sqrt{1+a^2x^2}\,\text{Ci}\left(3\tan^{-1}(ax\right.}{4a^2c^2\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 0.179799, size = 95, normalized size = 0.82

$$\frac{\left(a^2x^2+1\right)^{3/2}\tan^{-1}(ax)\text{CosIntegral}\left(\tan^{-1}(ax)\right)+3\left(a^2x^2+1\right)^{3/2}\tan^{-1}(ax)\text{CosIntegral}\left(3\tan^{-1}(ax)\right)-4ax}{4a^2c^2\left(a^2x^2+1\right)\sqrt{a^2cx^2+c}\tan^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2),x]
```

```
[Out] (-4*a*x + (1 + a^2*x^2)^(3/2)*ArcTan[a*x]*CosIntegral[ArcTan[a*x]] + 3*(1 +
 a^2*x^2)^(3/2)*ArcTan[a*x]*CosIntegral[3*ArcTan[a*x]])/(4*a^2*c^2*(1 + a^2
*x^2)*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])
```

Maple [C] time = 0.379, size = 582, normalized size = 5.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x)
```

```
[Out] -1/8*(arctan(a*x)*Ei(1,-I*arctan(a*x))*x^2*a^2+Ei(1,-I*arctan(a*x))*arctan(
a*x)+(a^2*x^2+1)^(1/2)*x*a-I*(a^2*x^2+1)^(1/2))/(a^2*x^2+1)^(3/2)*(c*(a*x-I
)*(a*x+I))^(1/2)/arctan(a*x)/c^3/a^2-1/8*(3*arctan(a*x)*Ei(1,-3*I*arctan(a*
x))*x^4*a^4+6*arctan(a*x)*Ei(1,-3*I*arctan(a*x))*x^2*a^2-(a^2*x^2+1)^(1/2)*
x^3*a^3+3*I*(a^2*x^2+1)^(1/2)*x^2*a^2+3*Ei(1,-3*I*arctan(a*x))*arctan(a*x)+
3*(a^2*x^2+1)^(1/2)*x*a-I*(a^2*x^2+1)^(1/2))/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(
a*x+I))^(1/2)/arctan(a*x)/(a^4*x^4+2*a^2*x^2+1)/c^3/a^2-1/8*(3*arctan(a*x)*
Ei(1,3*I*arctan(a*x))*x^4*a^4-(a^2*x^2+1)^(1/2)*x^3*a^3+6*arctan(a*x)*Ei(1,
3*I*arctan(a*x))*x^2*a^2-3*I*(a^2*x^2+1)^(1/2)*x^2*a^2+3*(a^2*x^2+1)^(1/2)*
x*a+3*Ei(1,3*I*arctan(a*x))*arctan(a*x)+I*(a^2*x^2+1)^(1/2))/(a^2*x^2+1)^(1
/2)*(c*(a*x-I)*(a*x+I))^(1/2)/arctan(a*x)/(a^4*x^4+2*a^2*x^2+1)/c^3/a^2-1/8
*(arctan(a*x)*Ei(1,I*arctan(a*x))*x^2*a^2+Ei(1,I*arctan(a*x))*arctan(a*x)+(
a^2*x^2+1)^(1/2)*x*a+I*(a^2*x^2+1)^(1/2))/(a^2*x^2+1)^(3/2)*(c*(a*x-I)*(a*x
+I))^(1/2)/arctan(a*x)/c^3/a^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(a^2cx^2+c\right)^{\frac{5}{2}}\arctan\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(x/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a^2cx^2+cx}}{\left(a^6c^3x^6+3\,a^4c^3x^4+3\,a^2c^3x^2+c^3\right)\arctan\left(ax\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^
2 + c^3)*arctan(a*x)^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}\operatorname{atan}^2(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a**2*c*x**2+c)**(5/2)/atan(a*x)**2,x)
```

```
[Out] Integral(x/((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(a^2cx^2+c\right)^{\frac{5}{2}}\arctan(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^2), x)
```

3.592 $\int \frac{1}{(c+a^2cx^2)^{5/2}\tan^{-1}(ax)^2}\,dx$

Optimal. Leaf size=115

$$-\frac{3\sqrt{a^2x^2+1}\,\mathrm{Si}\left(\tan^{-1}(ax)\right)}{4ac^2\sqrt{a^2cx^2+c}}-\frac{3\sqrt{a^2x^2+1}\,\mathrm{Si}\left(3\tan^{-1}(ax)\right)}{4ac^2\sqrt{a^2cx^2+c}}-\frac{1}{ac\left(a^2cx^2+c\right)^{3/2}\tan^{-1}(ax)}$$

```
[Out] -(1/(a*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x])) - (3*Sqrt[1 + a^2*x^2]*SinInte
gral[ArcTan[a*x]])/(4*a*c^2*Sqrt[c + a^2*c*x^2]) - (3*Sqrt[1 + a^2*x^2]*Sin
Integral[3*ArcTan[a*x]])/(4*a*c^2*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.244665, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {4902, 4971, 4970, 4406, 3299}

$$-\frac{3\sqrt{a^2x^2+1}\,\mathrm{Si}\left(\tan^{-1}(ax)\right)}{4ac^2\sqrt{a^2cx^2+c}}-\frac{3\sqrt{a^2x^2+1}\,\mathrm{Si}\left(3\tan^{-1}(ax)\right)}{4ac^2\sqrt{a^2cx^2+c}}-\frac{1}{ac\left(a^2cx^2+c\right)^{3/2}\tan^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2),x]
```

```
[Out] -(1/(a*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x])) - (3*Sqrt[1 + a^2*x^2]*SinInte
gral[ArcTan[a*x]])/(4*a*c^2*Sqrt[c + a^2*c*x^2]) - (3*Sqrt[1 + a^2*x^2]*Sin
Integral[3*ArcTan[a*x]])/(4*a*c^2*Sqrt[c + a^2*c*x^2])
```

Rule 4902

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^(p + 1))/(b*c*d*(p
+ 1)), x] - Dist[(2*c*(q + 1))/(b*(p + 1)), Int[x*(d + e*x^2)^q*(a + b*ArcT
an[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && L
tQ[q, -1] && LtQ[p, -1]
```

Rule 4971

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[(d^(q + 1/2)*Sqrt[1 + c^2*x^2])/Sqrt[d + e*x^2],
 Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d
```

```
, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(In
tegerQ[q] || GtQ[d, 0])
```

Rule 4970

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sin[x]^m)/
Cos[x]^(m + 2*(q + 1)), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\int \frac{1}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^2}\,dx = -\frac{1}{ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)} - (3a)\int \frac{x}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)}\,dx$$

$$= -\frac{1}{ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)} - \frac{\left(3a\sqrt{1+a^2x^2}\right)\int \frac{x}{\left(1+a^2x^2\right)^{5/2}\tan^{-1}(ax)}\,dx}{c^2\sqrt{c+a^2cx^2}}$$

$$= -\frac{1}{ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)} - \frac{\left(3\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int \frac{\cos^2(x)\sin(x)}{x}\,dx, x, \tan^{-1}(ax)\right)}{ac^2\sqrt{c+a^2cx^2}}$$

$$= -\frac{1}{ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)} - \frac{\left(3\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int \left(\frac{\sin(x)}{4x}+\frac{\sin(3x)}{4x}\right)dx, x, \tan^{-1}(\right.}{ac^2\sqrt{c+a^2cx^2}}$$

$$= -\frac{1}{ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)} - \frac{\left(3\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int \frac{\sin(x)}{x}\,dx, x, \tan^{-1}(ax)\right)}{4ac^2\sqrt{c+a^2cx^2}} - \frac{(3\sqrt{}}{}$$

$$= -\frac{1}{ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)} - \frac{3\sqrt{1+a^2x^2}\,\text{Si}\left(\tan^{-1}(ax)\right)}{4ac^2\sqrt{c+a^2cx^2}} - \frac{3\sqrt{1+a^2x^2}\,\text{Si}\left(3\tan^{-1}(\right.}{4ac^2\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 0.186117, size = 61, normalized size = 0.53

$$\frac{-3\left(a^2x^2+1\right)^{3/2}\left(\text{Si}\left(\tan^{-1}(ax)\right)+\text{Si}\left(3\tan^{-1}(ax)\right)\right)-\frac{4}{\tan^{-1}(ax)}}{4ac\left(a^2cx^2+c\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2),x]
```

```
[Out] (-4/ArcTan[a*x] - 3*(1 + a^2*x^2)^(3/2)*(SinIntegral[ArcTan[a*x]] + SinInte
gral[3*ArcTan[a*x]]))/(4*a*c*(c + a^2*c*x^2)^(3/2))
```

Maple [C] time = 0.341, size = 586, normalized size = 5.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x)
```

```
[Out] 1/8*I*(3*arctan(a*x)*Ei(1,3*I*arctan(a*x))*x^4*a^4-(a^2*x^2+1)^(1/2)*x^3*a^
3+6*arctan(a*x)*Ei(1,3*I*arctan(a*x))*x^2*a^2-3*I*(a^2*x^2+1)^(1/2)*x^2*a^2
+3*(a^2*x^2+1)^(1/2)*x*a+3*Ei(1,3*I*arctan(a*x))*arctan(a*x)+I*(a^2*x^2+1)^
(1/2))/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(a*x+I))^(1/2)/(a^4*x^4+2*a^2*x^2+1)/ar
ctan(a*x)/a/c^3-1/8*I*(3*arctan(a*x)*Ei(1,-3*I*arctan(a*x))*x^4*a^4+6*arcta
n(a*x)*Ei(1,-3*I*arctan(a*x))*x^2*a^2-(a^2*x^2+1)^(1/2)*x^3*a^3+3*I*(a^2*x^
2+1)^(1/2)*x^2*a^2+3*Ei(1,-3*I*arctan(a*x))*arctan(a*x)+3*(a^2*x^2+1)^(1/2)
*x*a-I*(a^2*x^2+1)^(1/2))/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(a*x+I))^(1/2)/(a^4*
x^4+2*a^2*x^2+1)/arctan(a*x)/a/c^3+3/8*I*(arctan(a*x)*Ei(1,I*arctan(a*x))*x
^2*a^2+Ei(1,I*arctan(a*x))*arctan(a*x)+(a^2*x^2+1)^(1/2)*x*a+I*(a^2*x^2+1)^
(1/2))/(a^2*x^2+1)^(3/2)*(c*(a*x-I)*(a*x+I))^(1/2)/arctan(a*x)/a/c^3-3/8*I*
(arctan(a*x)*Ei(1,-I*arctan(a*x))*x^2*a^2+Ei(1,-I*arctan(a*x))*arctan(a*x)+
(a^2*x^2+1)^(1/2)*x*a-I*(a^2*x^2+1)^(1/2))/(a^2*x^2+1)^(3/2)*(c*(a*x-I)*(a*
x+I))^(1/2)/arctan(a*x)/a/c^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)^{\frac{5}{2}}\arctan\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a^2cx^2+c}}{\left(a^6c^3x^6+3\,a^4c^3x^4+3\,a^2c^3x^2+c^3\right)\arctan\left(ax\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2
+ c^3)*arctan(a*x)^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}\operatorname{atan}^2(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*c*x**2+c)**(5/2)/atan(a*x)**2,x)
```

```
[Out] Integral(1/((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)^{\frac{5}{2}}\arctan(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^2), x)
```

3.593 $\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^2}\,dx$

Optimal. Leaf size=198

$$-\frac{\text{Unintegrable}\left(\frac{1}{x^2\sqrt{a^2cx^2+c}\tan^{-1}(ax)},x\right)}{ac^2}-\frac{5\sqrt{a^2x^2+1}\,\text{CosIntegral}\left(\tan^{-1}(ax)\right)}{4c^2\sqrt{a^2cx^2+c}}-\frac{3\sqrt{a^2x^2+1}\,\text{CosIntegral}\left(3\tan^{-1}(ax)\right)}{4c^2\sqrt{a^2cx^2+c}}$$

```
[Out] (a*x)/(c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]) + (a*x)/(c^2*Sqrt[c + a^2*c*x^2
]*ArcTan[a*x]) - Sqrt[c + a^2*c*x^2]/(a*c^3*x*ArcTan[a*x]) - (5*Sqrt[1 + a^
2*x^2]*CosIntegral[ArcTan[a*x]])/(4*c^2*Sqrt[c + a^2*c*x^2]) - (3*Sqrt[1 +
a^2*x^2]*CosIntegral[3*ArcTan[a*x]])/(4*c^2*Sqrt[c + a^2*c*x^2]) - Unintegr
able[1/(x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]), x]/(a*c^2)
```

Rubi [A] time = 1.13725, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2),x]
```

```
[Out] (a*x)/(c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]) + (a*x)/(c^2*Sqrt[c + a^2*c*x^2
]*ArcTan[a*x]) - Sqrt[c + a^2*c*x^2]/(a*c^3*x*ArcTan[a*x]) - (5*Sqrt[1 + a^
2*x^2]*CosIntegral[ArcTan[a*x]])/(4*c^2*Sqrt[c + a^2*c*x^2]) - (3*Sqrt[1 +
a^2*x^2]*CosIntegral[3*ArcTan[a*x]])/(4*c^2*Sqrt[c + a^2*c*x^2]) - Defer[In
t][1/(x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]), x]/(a*c^2)
```

Rubi steps

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^2}\,dx = -\left(a^2\int \frac{x}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^2}\,dx\right) + \frac{\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2}\,dx}{c}$$

$$= \frac{ax}{c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)} - a\int \frac{1}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)}\,dx + \left(2a^3\right)\int \frac{}{\left(c+a^2cx^2\right.}$$

$$= \frac{ax}{c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)} + \frac{ax}{c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)} - \frac{\sqrt{c+a^2cx^2}}{ac^3x\tan^{-1}(ax)} - \frac{\int \frac{}{x^2\sqrt{c+a^2}}}{}$$

$$= \frac{ax}{c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)} + \frac{ax}{c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)} - \frac{\sqrt{c+a^2cx^2}}{ac^3x\tan^{-1}(ax)} - \frac{\int \frac{}{x^2\sqrt{c+a^2}}}{}$$

$$= \frac{ax}{c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)} + \frac{ax}{c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)} - \frac{\sqrt{c+a^2cx^2}}{ac^3x\tan^{-1}(ax)} - \frac{\int \frac{}{x^2\sqrt{c+a^2}}}{}$$

$$= \frac{ax}{c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)} + \frac{ax}{c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)} - \frac{\sqrt{c+a^2cx^2}}{ac^3x\tan^{-1}(ax)} - \frac{\sqrt{1+a^2x}}{c^2\sqrt{}}$$

$$= \frac{ax}{c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)} + \frac{ax}{c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)} - \frac{\sqrt{c+a^2cx^2}}{ac^3x\tan^{-1}(ax)} - \frac{5\sqrt{1+a^2}}{4c^2}$$

Mathematica [A] time = 2.20655, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2),x]
```

```
[Out] Integrate[1/(x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2), x]
```

Maple [A] time = 0.577, size = 0, normalized size = 0.

$$\int \frac{1}{x \left(\arctan \left(a x\right)\right)^{2}} \left(a^{2} c \,x^{2}+c \right)^{-\frac{5}{2}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x)
```

```
[Out] int(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{{\left(a^{2} c x^{2} + c\right)}^{\frac{5}{2}} x \arctan\left(a x\right)^{2}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^(5/2)*x*arctan(a*x)^2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{a^{2} c x^{2} + c}}{{\left(a^{6} c^{3} x^{7} + 3 \, a^{4} c^{3} x^{5} + 3 \, a^{2} c^{3} x^{3} + c^{3} x\right)} \arctan\left(a x\right)^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)/((a^6*c^3*x^7 + 3*a^4*c^3*x^5 + 3*a^2*c^3*x^3
+ c^3*x)*arctan(a*x)^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}\operatorname{atan}^2(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a**2*c*x**2+c)**(5/2)/atan(a*x)**2,x)
```

```
[Out] Integral(1/(x*(c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)^{\frac{5}{2}}x\arctan(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^(5/2)*x*arctan(a*x)^2), x)
```

3.594 $\int \frac{1}{x^2(c+a^2cx^2)^{5/2}\tan^{-1}(ax)^2}\,dx$

Optimal. Leaf size=163

$$\frac{\text{Unintegrable}\left(\frac{1}{x^2\sqrt{a^2cx^2+c}\tan^{-1}(ax)^2},x\right)}{c^2}+\frac{7a\sqrt{a^2x^2+1}\text{Si}\left(\tan^{-1}(ax)\right)}{4c^2\sqrt{a^2cx^2+c}}+\frac{3a\sqrt{a^2x^2+1}\text{Si}\left(3\tan^{-1}(ax)\right)}{4c^2\sqrt{a^2cx^2+c}}+\frac{a}{c^2\sqrt{a^2cx^2+c}\tan}$$

```
[Out] a/(c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]) + a/(c^2*Sqrt[c + a^2*c*x^2]*ArcTan
[a*x]) + (7*a*Sqrt[1 + a^2*x^2]*SinIntegral[ArcTan[a*x]])/(4*c^2*Sqrt[c + a
^2*c*x^2]) + (3*a*Sqrt[1 + a^2*x^2]*SinIntegral[3*ArcTan[a*x]])/(4*c^2*Sqrt
[c + a^2*c*x^2]) + Unintegrable[1/(x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2),
x]/c^2
```

Rubi [A] time = 0.804084, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2),x]
```

```
[Out] a/(c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]) + a/(c^2*Sqrt[c + a^2*c*x^2]*ArcTan
[a*x]) + (7*a*Sqrt[1 + a^2*x^2]*SinIntegral[ArcTan[a*x]])/(4*c^2*Sqrt[c + a
^2*c*x^2]) + (3*a*Sqrt[1 + a^2*x^2]*SinIntegral[3*ArcTan[a*x]])/(4*c^2*Sqrt
[c + a^2*c*x^2]) + Defer[Int][1/(x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2), x]
/c^2
```

Rubi steps

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^2}\,dx = -\left(a^2\int \frac{1}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^2}\,dx\right) + \frac{\int \frac{1}{x^2\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2}\,dx}{c}$$

$$= \frac{a}{c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)} + \left(3a^3\right)\int \frac{x}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)}\,dx + \frac{\int \frac{1}{x^2\sqrt{c+a^2cx^2}\,\tan}}{c^2}$$

$$= \frac{a}{c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)} + \frac{a}{c^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)} + \frac{\int \frac{1}{x^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}\,dx}{c^2} +$$

$$= \frac{a}{c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)} + \frac{a}{c^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)} + \frac{\int \frac{1}{x^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}\,dx}{c^2} +$$

$$= \frac{a}{c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)} + \frac{a}{c^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)} + \frac{\int \frac{1}{x^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}\,dx}{c^2} +$$

$$= \frac{a}{c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)} + \frac{a}{c^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)} + \frac{a\sqrt{1+a^2x^2}\,\mathrm{Si}\left(\tan^{-1}(ax)\right)}{c^2\sqrt{c+a^2cx^2}}$$

$$= \frac{a}{c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)} + \frac{a}{c^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)} + \frac{7a\sqrt{1+a^2x^2}\,\mathrm{Si}\left(\tan^{-1}(ax\right.}{4c^2\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 3.95863, size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2),x]
```

```
[Out] Integrate[1/(x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2), x]
```

Maple [A] time = 0.596, size = 0, normalized size = 0.

$$\int \frac{1}{x^{2} \left(\arctan \left(a x\right)\right)^{2}} \left(a^{2} c \,x^{2}+c \right)^{-\frac{5}{2}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x)
```

```
[Out] int(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{{\left(a^{2} c x^{2} + c\right)}^{\frac{5}{2}} x^{2} \arctan\left(a x\right)^{2}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^(5/2)*x^2*arctan(a*x)^2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$${\rm integral}\left(\frac{\sqrt{a^{2} c x^{2} + c}}{{\left(a^{6} c^{3} x^{8} + 3 \, a^{4} c^{3} x^{6} + 3 \, a^{2} c^{3} x^{4} + c^{3} x^{2}\right)} \arctan\left(a x\right)^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)/((a^6*c^3*x^8 + 3*a^4*c^3*x^6 + 3*a^2*c^3*x^4
+ c^3*x^2)*arctan(a*x)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a**2*c*x**2+c)**(5/2)/atan(a*x)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)^{\frac{5}{2}}x^2\arctan\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^(5/2)*x^2*arctan(a*x)^2), x)
```

3.595 $\int \frac{1}{x^3\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^2}\,dx$

Optimal. Leaf size=236

$$\frac{2a\text{Unintegrable}\left(\frac{1}{x^2\sqrt{a^2cx^2+c}\tan^{-1}(ax)},x\right)}{c^2}+\frac{\text{Unintegrable}\left(\frac{1}{x^3\sqrt{a^2cx^2+c}\tan^{-1}(ax)^2},x\right)}{c^2}+\frac{9a^2\sqrt{a^2x^2+1}\text{CosIntegral}\left(\tan^{-1}(a\right.}{4c^2\sqrt{a^2cx^2+c}}$$

```
[Out] -((a^3*x)/(c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x])) - (2*a^3*x)/(c^2*Sqrt[c +
a^2*c*x^2]*ArcTan[a*x]) + (2*a*Sqrt[c + a^2*c*x^2])/(c^3*x*ArcTan[a*x]) + (
9*a^2*Sqrt[1 + a^2*x^2]*CosIntegral[ArcTan[a*x]])/(4*c^2*Sqrt[c + a^2*c*x^2
]) + (3*a^2*Sqrt[1 + a^2*x^2]*CosIntegral[3*ArcTan[a*x]])/(4*c^2*Sqrt[c + a
^2*c*x^2]) + Unintegrable[1/(x^3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2), x]/c^2
 + (2*a*Unintegrable[1/(x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]), x])/c^2
```

Rubi [A] time = 2.04828, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^3\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^3*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2),x]
```

```
[Out] -((a^3*x)/(c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x])) - (2*a^3*x)/(c^2*Sqrt[c +
a^2*c*x^2]*ArcTan[a*x]) + (2*a*Sqrt[c + a^2*c*x^2])/(c^3*x*ArcTan[a*x]) + (
9*a^2*Sqrt[1 + a^2*x^2]*CosIntegral[ArcTan[a*x]])/(4*c^2*Sqrt[c + a^2*c*x^2
]) + (3*a^2*Sqrt[1 + a^2*x^2]*CosIntegral[3*ArcTan[a*x]])/(4*c^2*Sqrt[c + a
^2*c*x^2]) + Defer[Int][1/(x^3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2), x]/c^2 +
 (2*a*Defer[Int][1/(x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]), x])/c^2
```

Rubi steps

$$\int \frac{1}{x^3 \left(c+a^2 c x^2\right)^{5/2} \tan^{-1}(ax)^2} \, dx = -\left(a^2 \int \frac{1}{x \left(c+a^2 c x^2\right)^{5/2} \tan^{-1}(ax)^2} \, dx\right) + \frac{\int \frac{1}{x^3 \left(c+a^2 c x^2\right)^{3/2} \tan^{-1}(ax)^2} \, dx}{c}$$

$$= a^4 \int \frac{x}{\left(c+a^2 c x^2\right)^{5/2} \tan^{-1}(ax)^2} \, dx + \frac{\int \frac{1}{x^3 \sqrt{c+a^2 c x^2} \tan^{-1}(ax)^2} \, dx}{c^2} - 2\frac{a^2 \int \frac{1}{x \left(c+a^2 c x^2\right)^{3/2} \mathrm{ta}}}{c}$$

$$= -\frac{a^3 x}{c \left(c+a^2 c x^2\right)^{3/2} \tan^{-1}(ax)} + a^3 \int \frac{1}{\left(c+a^2 c x^2\right)^{5/2} \tan^{-1}(ax)} \, dx - \left(2a^5\right) \int \frac{}{\left(c+a^2\right.}$$

$$= -\frac{a^3 x}{c \left(c+a^2 c x^2\right)^{3/2} \tan^{-1}(ax)} + \frac{\int \frac{1}{x^3 \sqrt{c+a^2 c x^2} \tan^{-1}(ax)^2} \, dx}{c^2} - 2\left(\frac{a^3 x}{c^2 \sqrt{c+a^2 c x^2} \tan^{-1}(a}\right.$$

$$= -\frac{a^3 x}{c \left(c+a^2 c x^2\right)^{3/2} \tan^{-1}(ax)} + \frac{\int \frac{1}{x^3 \sqrt{c+a^2 c x^2} \tan^{-1}(ax)^2} \, dx}{c^2} + \frac{\left(a^2 \sqrt{1+a^2 x^2}\right) \mathrm{Subst}\left(\int\right.}{c^2 \sqrt{c+}}$$

$$= -\frac{a^3 x}{c \left(c+a^2 c x^2\right)^{3/2} \tan^{-1}(ax)} + \frac{\int \frac{1}{x^3 \sqrt{c+a^2 c x^2} \tan^{-1}(ax)^2} \, dx}{c^2} - 2\left(\frac{a^3 x}{c^2 \sqrt{c+a^2 c x^2} \tan^{-1}(a}\right.$$

$$= -\frac{a^3 x}{c \left(c+a^2 c x^2\right)^{3/2} \tan^{-1}(ax)} + \frac{\int \frac{1}{x^3 \sqrt{c+a^2 c x^2} \tan^{-1}(ax)^2} \, dx}{c^2} - 2\left(\frac{a^3 x}{c^2 \sqrt{c+a^2 c x^2} \tan^{-1}(a}\right.$$

$$= -\frac{a^3 x}{c \left(c+a^2 c x^2\right)^{3/2} \tan^{-1}(ax)} + \frac{a^2 \sqrt{1+a^2 x^2} \mathrm{Ci}\left(\tan^{-1}(ax)\right)}{4c^2 \sqrt{c+a^2 c x^2}} + \frac{3a^2 \sqrt{1+a^2 x^2} \mathrm{Ci}\left(3 \mathrm{t}\right.}{4c^2 \sqrt{c+a^2 c}}$$

Mathematica [A] time = 6.79928, size = 0, normalized size = 0.

$$\int \frac{1}{x^3 \left(c+a^2 c x^2\right)^{5/2} \tan^{-1}(ax)^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^3*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2),x]
```

```
[Out] Integrate[1/(x^3*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2), x]
```

Maple [A] time = 0.762, size = 0, normalized size = 0.

$$\int \frac{1}{x^3 (\arctan (ax))^2} \left(a^2 c x^2 + c\right)^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x)
```

```
[Out] int(1/x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2 c x^2 + c\right)^{\frac{5}{2}} x^3 \arctan (ax)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^(5/2)*x^3*arctan(a*x)^2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a^2 c x^2 + c}}{\left(a^6 c^3 x^9 + 3\, a^4 c^3 x^7 + 3\, a^2 c^3 x^5 + c^3 x^3\right) \arctan (ax)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)/((a^6*c^3*x^9 + 3*a^4*c^3*x^7 + 3*a^2*c^3*x^5
+ c^3*x^3)*arctan(a*x)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a**2*c*x**2+c)**(5/2)/atan(a*x)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)^{\frac{5}{2}}x^3\arctan\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^(5/2)*x^3*arctan(a*x)^2), x)
```

3.596 $\int \frac{1}{x^4\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^2}\,dx$

Optimal. Leaf size=207

$$-\frac{2a^2\text{Unintegrable}\left(\frac{1}{x^2\sqrt{a^2cx^2+c}\tan^{-1}(ax)^2},x\right)}{c^2}+\frac{\text{Unintegrable}\left(\frac{1}{x^4\sqrt{a^2cx^2+c}\tan^{-1}(ax)^2},x\right)}{c^2}-\frac{11a^3\sqrt{a^2x^2+1}\text{Si}\left(\tan^{-1}(ax)\right)}{4c^2\sqrt{a^2cx^2+c}}-$$

```
[Out] -(a^3/(c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x])) - (2*a^3)/(c^2*Sqrt[c + a^2*c*
x^2]*ArcTan[a*x]) - (11*a^3*Sqrt[1 + a^2*x^2]*SinIntegral[ArcTan[a*x]])/(4*
c^2*Sqrt[c + a^2*c*x^2]) - (3*a^3*Sqrt[1 + a^2*x^2]*SinIntegral[3*ArcTan[a*
x]])/(4*c^2*Sqrt[c + a^2*c*x^2]) + Unintegrable[1/(x^4*Sqrt[c + a^2*c*x^2]*
ArcTan[a*x]^2), x]/c^2 - (2*a^2*Unintegrable[1/(x^2*Sqrt[c + a^2*c*x^2]*Arc
Tan[a*x]^2), x])/c^2
```

Rubi [A] time = 1.60702, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^4\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^4*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2),x]
```

```
[Out] -(a^3/(c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x])) - (2*a^3)/(c^2*Sqrt[c + a^2*c*
x^2]*ArcTan[a*x]) - (11*a^3*Sqrt[1 + a^2*x^2]*SinIntegral[ArcTan[a*x]])/(4*
c^2*Sqrt[c + a^2*c*x^2]) - (3*a^3*Sqrt[1 + a^2*x^2]*SinIntegral[3*ArcTan[a*
x]])/(4*c^2*Sqrt[c + a^2*c*x^2]) + Defer[Int][1/(x^4*Sqrt[c + a^2*c*x^2]*Ar
cTan[a*x]^2), x]/c^2 - (2*a^2*Defer[Int][1/(x^2*Sqrt[c + a^2*c*x^2]*ArcTan[
a*x]^2), x])/c^2
```

Rubi steps

$$\int \frac{1}{x^4\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^2}\,dx = -\left(a^2\int \frac{1}{x^2\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^2}\,dx\right)+\frac{\int \frac{1}{x^4\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2}\,dx}{c}$$

$$= a^4\int \frac{1}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^2}\,dx+\frac{\int \frac{1}{x^4\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}\,dx}{c^2}-2\frac{a^2\int \frac{1}{x^2\left(c+a^2cx^2\right)^{3/2}}}{c}$$

$$= -\frac{a^3}{c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}-\left(3a^5\right)\int \frac{x}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)}\,dx+\frac{\int \frac{1}{x^4\sqrt{c+a^2cx^2}}}{c^2}$$

$$= -\frac{a^3}{c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}+\frac{\int \frac{1}{x^4\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}\,dx}{c^2}-2\left(\frac{a^3}{c^2\sqrt{c+a^2cx^2}\tan^{-1}(a}\right.$$

$$= -\frac{a^3}{c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}+\frac{\int \frac{1}{x^4\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}\,dx}{c^2}-\frac{\left(3a^3\sqrt{1+a^2x^2}\right)\text{Subst}\left(\right.}{c^2\sqrt{}}$$

$$= -\frac{a^3}{c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}+\frac{\int \frac{1}{x^4\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}\,dx}{c^2}-2\left(\frac{a^3}{c^2\sqrt{c+a^2cx^2}\tan^{-1}(a}\right.$$

$$= -\frac{a^3}{c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}+\frac{\int \frac{1}{x^4\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}\,dx}{c^2}-2\left(\frac{a^3}{c^2\sqrt{c+a^2cx^2}\tan^{-1}(a}\right.$$

$$= -\frac{a^3}{c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}-\frac{3a^3\sqrt{1+a^2x^2}\,\text{Si}\left(\tan^{-1}(ax)\right)}{4c^2\sqrt{c+a^2cx^2}}-\frac{3a^3\sqrt{1+a^2x^2}\,\text{Si}\left(3\,\text{t}\right.}{4c^2\sqrt{c+a^2c}}$$

<u>Mathematica</u> [A] time = 6.34402, size = 0, normalized size = 0.

$$\int \frac{1}{x^4\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^4*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2),x]
```

```
[Out] Integrate[1/(x^4*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2), x]
```

Maple [A] time = 1.217, size = 0, normalized size = 0.

$$\int \frac{1}{x^{4} \left(\arctan \left(a x\right)\right)^{2}} \left(a^{2} c \,x^{2}+c \right)^{-\frac{5}{2}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x)
```

```
[Out] int(1/x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{{\left(a^{2} c x^{2} + c\right)}^{\frac{5}{2}} x^{4} \arctan \left(a x\right)^{2}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^(5/2)*x^4*arctan(a*x)^2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$${\rm integral}\left(\frac{\sqrt{a^{2} c x^{2} + c}}{{\left(a^{6} c^{3} x^{10} + 3 \, a^{4} c^{3} x^{8} + 3 \, a^{2} c^{3} x^{6} + c^{3} x^{4}\right)} \arctan \left(a x\right)^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)/((a^6*c^3*x^10 + 3*a^4*c^3*x^8 + 3*a^2*c^3*x^6
 + c^3*x^4)*arctan(a*x)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(a**2*c*x**2+c)**(5/2)/atan(a*x)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)^{\frac{5}{2}}x^4\arctan\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^(5/2)*x^4*arctan(a*x)^2), x)
```

3.597 $\int \frac{\sqrt{fx}}{(d+c^2dx^2)^2(a+b\tan^{-1}(cx))^2}\,dx$

Optimal. Leaf size=32

$$\text{Unintegrable}\left(\frac{\sqrt{fx}}{(c^2dx^2+d)^2\,(a+b\tan^{-1}(cx))^2},x\right)$$

```
[Out] Unintegrable[Sqrt[f*x]/((d + c^2*d*x^2)^2*(a + b*ArcTan[c*x])^2), x]
```

Rubi [A] time = 0.100492, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sqrt{fx}}{(d+c^2dx^2)^2\,(a+b\tan^{-1}(cx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[f*x]/((d + c^2*d*x^2)^2*(a + b*ArcTan[c*x])^2),x]
```

```
[Out] Defer[Int][Sqrt[f*x]/((d + c^2*d*x^2)^2*(a + b*ArcTan[c*x])^2), x]
```

Rubi steps

$$\int \frac{\sqrt{fx}}{(d+c^2dx^2)^2\,(a+b\tan^{-1}(cx))^2}\,dx = \int \frac{\sqrt{fx}}{(d+c^2dx^2)^2\,(a+b\tan^{-1}(cx))^2}\,dx$$

Mathematica [A] time = 28.9607, size = 0, normalized size = 0.

$$\int \frac{\sqrt{fx}}{(d+c^2dx^2)^2\,(a+b\tan^{-1}(cx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[f*x]/((d + c^2*d*x^2)^2*(a + b*ArcTan[c*x])^2),x]
```

```
[Out] Integrate[Sqrt[f*x]/((d + c^2*d*x^2)^2*(a + b*ArcTan[c*x])^2), x]
```

Maple [A] time = 1.223, size = 0, normalized size = 0.

$$\int \frac{1}{\left(c^2dx^2+d\right)^2 \left(a+b\arctan\left(cx\right)\right)^2}\sqrt{fx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^(1/2)/(c^2*d*x^2+d)^2/(a+b*arctan(c*x))^2,x)
```

```
[Out] int((f*x)^(1/2)/(c^2*d*x^2+d)^2/(a+b*arctan(c*x))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{\frac{1}{2}\left(a^2c^2d^2x^2+a^2d^2+\left(b^2c^2d^2x^2+b^2d^2\right)\arctan\left(cx\right)^2+2\left(abc^2d^2x^2+abd^2\right)\arctan\left(cx\right)\right)\sqrt{f}\int\frac{}{a^3c^4d^2x^4+2\,a^3c^2d^2x^2+a^3d^2+\left(b^3c\right.}}{2\left(a^2c^2d^2x^2+a^2d^2+\left(b^2c^2d^2x^2+b^2d^2\right)\arctan\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(1/2)/(c^2*d*x^2+d)^2/(a+b*arctan(c*x))^2,x, algorithm="max
ima")
```

```
[Out] 1/2*(2*(a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^2*d^2*x^2 + b^2*d^2)*arctan(c*x)
^2 + 2*(a*b*c^2*d^2*x^2 + a*b*d^2)*arctan(c*x))*sqrt(f)*integrate(1/4*(a*c^
2*x^2 + 4*b*c*x + (b*c^2*x^2 + b)*arctan(c*x) + a)*sqrt(x)/(a^3*c^4*d^2*x^4
 + 2*a^3*c^2*d^2*x^2 + a^3*d^2 + (b^3*c^4*d^2*x^4 + 2*b^3*c^2*d^2*x^2 + b^3
*d^2)*arctan(c*x)^3 + 3*(a*b^2*c^4*d^2*x^4 + 2*a*b^2*c^2*d^2*x^2 + a*b^2*d^
2)*arctan(c*x)^2 + 3*(a^2*b*c^4*d^2*x^4 + 2*a^2*b*c^2*d^2*x^2 + a^2*b*d^2)*
arctan(c*x)), x) + sqrt(f)*x^(3/2))/(a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^2*d
^2*x^2 + b^2*d^2)*arctan(c*x)^2 + 2*(a*b*c^2*d^2*x^2 + a*b*d^2)*arctan(c*x)
)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{fx}}{a^2c^4d^2x^4 + 2\,a^2c^2d^2x^2 + a^2d^2 + \left(b^2c^4d^2x^4 + 2\,b^2c^2d^2x^2 + b^2d^2\right)\arctan\left(cx\right)^2 + 2\,\left(abc^4d^2x^4 + 2\,abc^2d^2x^2 + abd^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(1/2)/(c^2*d*x^2+d)^2/(a+b*arctan(c*x))^2,x, algorithm="fri
cas")
```

```
[Out] integral(sqrt(f*x)/(a^2*c^4*d^2*x^4 + 2*a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^
4*d^2*x^4 + 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arctan(c*x)^2 + 2*(a*b*c^4*d^2*x^4
 + 2*a*b*c^2*d^2*x^2 + a*b*d^2)*arctan(c*x)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**(1/2)/(c**2*d*x**2+d)**2/(a+b*atan(c*x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{fx}}{\left(c^2dx^2 + d\right)^2\left(b\arctan\left(cx\right) + a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(1/2)/(c^2*d*x^2+d)^2/(a+b*arctan(c*x))^2,x, algorithm="gia
c")
```

```
[Out] integrate(sqrt(f*x)/((c^2*d*x^2 + d)^2*(b*arctan(c*x) + a)^2), x)
```

3.598 $\int \frac{x^m(c+a^2cx^2)^3}{\tan^{-1}(ax)^2}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{x^m\left(a^2cx^2+c\right)^3}{\tan^{-1}(ax)^2},x\right)$$

```
[Out] Unintegrable[(x^m*(c + a^2*c*x^2)^3)/ArcTan[a*x]^2, x]
```

Rubi [A] time = 0.0547625, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m\left(c+a^2cx^2\right)^3}{\tan^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*(c + a^2*c*x^2)^3)/ArcTan[a*x]^2,x]
```

```
[Out] Defer[Int][(x^m*(c + a^2*c*x^2)^3)/ArcTan[a*x]^2, x]
```

Rubi steps

$$\int \frac{x^m\left(c+a^2cx^2\right)^3}{\tan^{-1}(ax)^2}\,dx=\int \frac{x^m\left(c+a^2cx^2\right)^3}{\tan^{-1}(ax)^2}\,dx$$

Mathematica [A] time = 0.738601, size = 0, normalized size = 0.

$$\int \frac{x^m\left(c+a^2cx^2\right)^3}{\tan^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*(c + a^2*c*x^2)^3)/ArcTan[a*x]^2,x]
```

```
[Out] Integrate[(x^m*(c + a^2*c*x^2)^3)/ArcTan[a*x]^2, x]
```

Maple [A] time = 0.565, size = 0, normalized size = 0.

$$\int \frac{x^{m} \left(a^{2} c x^{2}+c\right)^{3}}{\left(\arctan \left(a x\right)\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^2,x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^2,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^{6} c^{3} x^{6}+3 \, a^{4} c^{3} x^{4}+3 \, a^{2} c^{3} x^{2}+c^{3}\right) x^{m}}{\arctan \left(a x\right)^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*x^m/arctan(a*x
)^2, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$c^3\left(\int\frac{x^m}{\operatorname{atan}^2(ax)}\,dx+\int\frac{3a^2x^2x^m}{\operatorname{atan}^2(ax)}\,dx+\int\frac{3a^4x^4x^m}{\operatorname{atan}^2(ax)}\,dx+\int\frac{a^6x^6x^m}{\operatorname{atan}^2(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**3/atan(a*x)**2,x)
```

```
[Out] c**3*(Integral(x**m/atan(a*x)**2, x) + Integral(3*a**2*x**2*x**m/atan(a*x)*
*2, x) + Integral(3*a**4*x**4*x**m/atan(a*x)**2, x) + Integral(a**6*x**6*x*
*m/atan(a*x)**2, x))
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a^2cx^2+c\right)^3x^m}{\arctan(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^3*x^m/arctan(a*x)^2, x)
```

3.599 $\int \frac{x^m(c+a^2cx^2)^2}{\tan^{-1}(ax)^2}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{x^m\left(a^2cx^2+c\right)^2}{\tan^{-1}(ax)^2},x\right)$$

```
[Out] Unintegrable[(x^m*(c + a^2*c*x^2)^2)/ArcTan[a*x]^2, x]
```

Rubi [A] time = 0.0547921, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m\left(c+a^2cx^2\right)^2}{\tan^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*(c + a^2*c*x^2)^2)/ArcTan[a*x]^2,x]
```

```
[Out] Defer[Int][(x^m*(c + a^2*c*x^2)^2)/ArcTan[a*x]^2, x]
```

Rubi steps

$$\int \frac{x^m\left(c+a^2cx^2\right)^2}{\tan^{-1}(ax)^2}\,dx=\int \frac{x^m\left(c+a^2cx^2\right)^2}{\tan^{-1}(ax)^2}\,dx$$

Mathematica [A] time = 0.80589, size = 0, normalized size = 0.

$$\int \frac{x^m\left(c+a^2cx^2\right)^2}{\tan^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*(c + a^2*c*x^2)^2)/ArcTan[a*x]^2,x]
```

```
[Out] Integrate[(x^m*(c + a^2*c*x^2)^2)/ArcTan[a*x]^2, x]
```

Maple [A] time = 0.524, size = 0, normalized size = 0.

$$\int \frac{x^m \left(a^2 c x^2 + c\right)^2}{\left(\arctan\left(a x\right)\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^2,x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^2,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(a^4 c^2 x^4 + 2\, a^2 c^2 x^2 + c^2\right) x^m}{\arctan\left(a x\right)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*x^m/arctan(a*x)^2, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$c^2\left(\int \frac{x^m}{\operatorname{atan}^2(ax)}\,dx+\int \frac{2a^2x^2x^m}{\operatorname{atan}^2(ax)}\,dx+\int \frac{a^4x^4x^m}{\operatorname{atan}^2(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**2/atan(a*x)**2,x)
```

```
[Out] c**2*(Integral(x**m/atan(a*x)**2, x) + Integral(2*a**2*x**2*x**m/atan(a*x)*
*2, x) + Integral(a**4*x**4*x**m/atan(a*x)**2, x))
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^2x^m}{\arctan(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^2*x^m/arctan(a*x)^2, x)
```

3.600 $\int \frac{x^m\left(c+a^2cx^2\right)}{\tan^{-1}(ax)^2}\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{x^m\left(a^2cx^2+c\right)}{\tan^{-1}(ax)^2},x\right)$$

```
[Out] Unintegrable[(x^m*(c + a^2*c*x^2))/ArcTan[a*x]^2, x]
```

Rubi [A] time = 0.0339223, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m\left(c+a^2cx^2\right)}{\tan^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*(c + a^2*c*x^2))/ArcTan[a*x]^2,x]
```

```
[Out] Defer[Int][(x^m*(c + a^2*c*x^2))/ArcTan[a*x]^2, x]
```

Rubi steps

$$\int \frac{x^m\left(c+a^2cx^2\right)}{\tan^{-1}(ax)^2}\,dx=\int \frac{x^m\left(c+a^2cx^2\right)}{\tan^{-1}(ax)^2}\,dx$$

Mathematica [A] time = 0.530269, size = 0, normalized size = 0.

$$\int \frac{x^m\left(c+a^2cx^2\right)}{\tan^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*(c + a^2*c*x^2))/ArcTan[a*x]^2,x]
```

```
[Out] Integrate[(x^m*(c + a^2*c*x^2))/ArcTan[a*x]^2, x]
```

Maple [A] time = 0.392, size = 0, normalized size = 0.

$$\int \frac{x^m \left(a^2 c x^2 + c\right)}{\left(\arctan\left(ax\right)\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)/arctan(a*x)^2,x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)/arctan(a*x)^2,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(a^2 c x^2 + c\right) x^m}{\arctan\left(ax\right)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)*x^m/arctan(a*x)^2, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$c\left(\int \frac{x^m}{\operatorname{atan}^2(ax)}\,dx + \int \frac{a^2x^2x^m}{\operatorname{atan}^2(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)/atan(a*x)**2,x)
```

```
[Out] c*(Integral(x**m/atan(a*x)**2, x) + Integral(a**2*x**2*x**m/atan(a*x)**2, x
))
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2 + c\right)x^m}{\arctan(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)*x^m/arctan(a*x)^2, x)
```

3.601 $\int \frac{x^m}{\left(c+a^2cx^2\right)\tan^{-1}(ax)^2}\,dx$

Optimal. Leaf size=40

$$\frac{m\text{Unintegrable}\left(\frac{x^{m-1}}{\tan^{-1}(ax)},x\right)}{ac}-\frac{x^m}{ac\tan^{-1}(ax)}$$

```
[Out] -(x^m/(a*c*ArcTan[a*x])) + (m*Unintegrable[x^(-1 + m)/ArcTan[a*x], x])/(a*c
)
```

Rubi [A] time = 0.0846464, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m}{\left(c+a^2cx^2\right)\tan^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m/((c + a^2*c*x^2)*ArcTan[a*x]^2),x]
```

```
[Out] -(x^m/(a*c*ArcTan[a*x])) + (m*Defer[Int][x^(-1 + m)/ArcTan[a*x], x])/(a*c)
```

Rubi steps

$$\int \frac{x^m}{\left(c+a^2cx^2\right)\tan^{-1}(ax)^2}\,dx=-\frac{x^m}{ac\tan^{-1}(ax)}+\frac{m\int \frac{x^{-1+m}}{\tan^{-1}(ax)}\,dx}{ac}$$

Mathematica [A] time = 0.332036, size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(c+a^2cx^2\right)\tan^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m/((c + a^2*c*x^2)*ArcTan[a*x]^2),x]
```

```
[Out] Integrate[x^m/((c + a^2*c*x^2)*ArcTan[a*x]^2), x]
```

Maple [A] time = 0.437, size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(a^2cx^2+c\right)\left(\arctan\left(ax\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a^2*c*x^2+c)/arctan(a*x)^2,x)
```

```
[Out] int(x^m/(a^2*c*x^2+c)/arctan(a*x)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{m\arctan\left(ax\right)\int\frac{x^m}{x\arctan(ax)}\,dx-x^m}{ac\arctan\left(ax\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] (m*arctan(a*x)*integrate(x^m/(x*arctan(a*x)), x) - x^m)/(a*c*arctan(a*x))
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{x^m}{\left(a^2cx^2+c\right)\arctan\left(ax\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(x^m/((a^2*c*x^2 + c)*arctan(a*x)^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^m}{a^2x^2 \operatorname{atan}^2(ax)+\operatorname{atan}^2(ax)}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(a**2*c*x**2+c)/atan(a*x)**2,x)
```

```
[Out] Integral(x**m/(a**2*x**2*atan(a*x)**2 + atan(a*x)**2), x)/c
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{(a^2cx^2+c)\arctan(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^m/((a^2*c*x^2 + c)*arctan(a*x)^2), x)
```

3.602 $\int \frac{x^m}{\left(c+a^2cx^2\right)^2 \tan^{-1}(ax)^2} \, dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{x^m}{\left(a^2cx^2+c\right)^2 \tan^{-1}(ax)^2},x\right)$$

```
[Out] Unintegrable[x^m/((c + a^2*c*x^2)^2*ArcTan[a*x]^2), x]
```

Rubi [A] time = 0.063152, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m}{\left(c+a^2cx^2\right)^2 \tan^{-1}(ax)^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m/((c + a^2*c*x^2)^2*ArcTan[a*x]^2),x]
```

```
[Out] Defer[Int][x^m/((c + a^2*c*x^2)^2*ArcTan[a*x]^2), x]
```

Rubi steps

$$\int \frac{x^m}{\left(c+a^2cx^2\right)^2 \tan^{-1}(ax)^2} \, dx = \int \frac{x^m}{\left(c+a^2cx^2\right)^2 \tan^{-1}(ax)^2} \, dx$$

Mathematica [A] time = 0.450396, size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(c+a^2cx^2\right)^2 \tan^{-1}(ax)^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m/((c + a^2*c*x^2)^2*ArcTan[a*x]^2),x]
```

```
[Out] Integrate[x^m/((c + a^2*c*x^2)^2*ArcTan[a*x]^2), x]
```

Maple [A] time = 1.14, size = 0, normalized size = 0.

$$\int \frac{x^{m}}{\left(a^{2} c x^{2}+c\right)^{2} \left(\arctan \left(a x\right)\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^2,x)
```

```
[Out] int(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^2,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x^{m}}{\left(a^{4} c^{2} x^{4}+2\, a^{2} c^{2} x^{2}+c^{2}\right) \arctan \left(a x\right)^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(x^m/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(a*x)^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^m}{a^4x^4\,\mathrm{atan}^2\,(ax)+2a^2x^2\,\mathrm{atan}^2\,(ax)+\mathrm{atan}^2\,(ax)}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(a**2*c*x**2+c)**2/atan(a*x)**2,x)
```

```
[Out] Integral(x**m/(a**4*x**4*atan(a*x)**2 + 2*a**2*x**2*atan(a*x)**2 + atan(a*x
)**2), x)/c**2
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(a^2cx^2+c\right)^2\arctan\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^m/((a^2*c*x^2 + c)^2*arctan(a*x)^2), x)
```

3.603 $\int \frac{x^m}{\left(c+a^2cx^2\right)^3 \tan^{-1}(ax)^2} \, dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{x^m}{\left(a^2cx^2+c\right)^3 \tan^{-1}(ax)^2},x\right)$$

```
[Out] Unintegrable[x^m/((c + a^2*c*x^2)^3*ArcTan[a*x]^2), x]
```

Rubi [A] time = 0.0632904, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m}{\left(c+a^2cx^2\right)^3 \tan^{-1}(ax)^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m/((c + a^2*c*x^2)^3*ArcTan[a*x]^2),x]
```

```
[Out] Defer[Int][x^m/((c + a^2*c*x^2)^3*ArcTan[a*x]^2), x]
```

Rubi steps

$$\int \frac{x^m}{\left(c+a^2cx^2\right)^3 \tan^{-1}(ax)^2} \, dx = \int \frac{x^m}{\left(c+a^2cx^2\right)^3 \tan^{-1}(ax)^2} \, dx$$

Mathematica [A] time = 0.466337, size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(c+a^2cx^2\right)^3 \tan^{-1}(ax)^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m/((c + a^2*c*x^2)^3*ArcTan[a*x]^2),x]
```

```
[Out] Integrate[x^m/((c + a^2*c*x^2)^3*ArcTan[a*x]^2), x]
```

Maple [A] time = 1.261, size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(a^2cx^2+c\right)^3 \left(\arctan\left(ax\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^2,x)
```

```
[Out] int(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^2,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^m}{\left(a^6c^3x^6+3\,a^4c^3x^4+3\,a^2c^3x^2+c^3\right)\arctan\left(ax\right)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(x^m/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*arctan(a*
x)^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^m}{a^6x^6\operatorname{atan}^2(ax)+3a^4x^4\operatorname{atan}^2(ax)+3a^2x^2\operatorname{atan}^2(ax)+\operatorname{atan}^2(ax)}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(a**2*c*x**2+c)**3/atan(a*x)**2,x)
```

```
[Out] Integral(x**m/(a**6*x**6*atan(a*x)**2 + 3*a**4*x**4*atan(a*x)**2 + 3*a**2*x
**2*atan(a*x)**2 + atan(a*x)**2), x)/c**3
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{(a^2cx^2+c)^3\arctan(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^m/((a^2*c*x^2 + c)^3*arctan(a*x)^2), x)
```

3.604 $\int \frac{x^m(c+a^2cx^2)^{5/2}}{\tan^{-1}(ax)^2}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{x^m\left(a^2cx^2+c\right)^{5/2}}{\tan^{-1}(ax)^2},x\right)$$

```
[Out] Unintegrable[(x^m*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^2, x]
```

Rubi [A] time = 0.111763, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m\left(c+a^2cx^2\right)^{5/2}}{\tan^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^2,x]
```

```
[Out] Defer[Int][(x^m*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^2, x]
```

Rubi steps

$$\int \frac{x^m\left(c+a^2cx^2\right)^{5/2}}{\tan^{-1}(ax)^2}\,dx=\int \frac{x^m\left(c+a^2cx^2\right)^{5/2}}{\tan^{-1}(ax)^2}\,dx$$

Mathematica [A] time = 1.22053, size = 0, normalized size = 0.

$$\int \frac{x^m\left(c+a^2cx^2\right)^{5/2}}{\tan^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^2,x]
```

```
[Out] Integrate[(x^m*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^2, x]
```

Maple [A] time = 0.615, size = 0, normalized size = 0.

$$\int \frac{x^m}{(\arctan(ax))^2}\left(a^2cx^2+c\right)^{\frac{5}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^{\frac{5}{2}}x^m}{\arctan(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(5/2)*x^m/arctan(a*x)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^4c^2x^4+2\,a^2c^2x^2+c^2\right)\sqrt{a^2cx^2+c}\,x^m}{\arctan(ax)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*x^m/arctan
(a*x)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**(5/2)/atan(a*x)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^{\frac{5}{2}}x^m}{\arctan\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(5/2)*x^m/arctan(a*x)^2, x)
```

3.605 $\int \frac{x^m\left(c+a^2cx^2\right)^{3/2}}{\tan^{-1}(ax)^2}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{x^m\left(a^2cx^2+c\right)^{3/2}}{\tan^{-1}(ax)^2},x\right)$$

```
[Out] Unintegrable[(x^m*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^2, x]
```

Rubi [A] time = 0.114093, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m\left(c+a^2cx^2\right)^{3/2}}{\tan^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^2,x]
```

```
[Out] Defer[Int][(x^m*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^2, x]
```

Rubi steps

$$\int \frac{x^m\left(c+a^2cx^2\right)^{3/2}}{\tan^{-1}(ax)^2}\,dx=\int \frac{x^m\left(c+a^2cx^2\right)^{3/2}}{\tan^{-1}(ax)^2}\,dx$$

Mathematica [A] time = 0.657256, size = 0, normalized size = 0.

$$\int \frac{x^m\left(c+a^2cx^2\right)^{3/2}}{\tan^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^2,x]
```

```
[Out] Integrate[(x^m*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^2, x]
```

Maple [A] time = 0.586, size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(\arctan\left(ax\right)\right)^2}\left(a^2cx^2+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^{\frac{3}{2}}x^m}{\arctan\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)*x^m/arctan(a*x)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(a^2cx^2+c\right)^{\frac{3}{2}}x^m}{\arctan\left(ax\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)^(3/2)*x^m/arctan(a*x)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**(3/2)/atan(a*x)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^{\frac{3}{2}}x^m}{\arctan\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)*x^m/arctan(a*x)^2, x)
```

3.606 $\int \frac{x^m\sqrt{c+a^2cx^2}}{\tan^{-1}(ax)^2}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{x^m\sqrt{a^2cx^2+c}}{\tan^{-1}(ax)^2},x\right)$$

```
[Out] Unintegrable[(x^m*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^2, x]
```

Rubi [A] time = 0.0964304, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\tan^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^2,x]
```

```
[Out] Defer[Int][(x^m*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^2, x]
```

Rubi steps

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\tan^{-1}(ax)^2}\,dx=\int \frac{x^m\sqrt{c+a^2cx^2}}{\tan^{-1}(ax)^2}\,dx$$

Mathematica [A] time = 0.176551, size = 0, normalized size = 0.

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\tan^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^2,x]
```

```
[Out] Integrate[(x^m*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^2, x]
```

Maple [A] time = 0.703, size = 0, normalized size = 0.

$$\int \frac{x^{m}}{\left(\arctan \left(a x\right)\right)^{2}}\sqrt{a^{2} c \,x^{2}+c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^2,x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a^{2} c x^{2} + c} x^{m}}{\arctan \left(a x\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)*x^m/arctan(a*x)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{a^{2} c x^{2} + c} x^{m}}{\arctan \left(a x\right)^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^m/arctan(a*x)^2, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m \sqrt{c\left(a^2x^2+1\right)}}{\operatorname{atan}^2(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**(1/2)/atan(a*x)**2,x)
```

```
[Out] Integral(x**m*sqrt(c*(a**2*x**2 + 1))/atan(a*x)**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a^2cx^2+c}\,x^m}{\arctan(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)*x^m/arctan(a*x)^2, x)
```

3.607 $\int \frac{x^m}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{x^m}{\sqrt{a^2cx^2+c}\tan^{-1}(ax)^2},x\right)$$

```
[Out] Unintegrable[x^m/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2), x]
```

Rubi [A] time = 0.104568, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2),x]
```

```
[Out] Defer[Int][x^m/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2), x]
```

Rubi steps

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}\,dx=\int \frac{x^m}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}\,dx$$

Mathematica [A] time = 0.469357, size = 0, normalized size = 0.

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2),x]
```

```
[Out] Integrate[x^m/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2), x]
```

Maple [A] time = 1.109, size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(\arctan \left(ax\right)\right)^2} \frac{1}{\sqrt{a^2 c x^2 + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(x^m/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\sqrt{a^2 c x^2 + c} \arctan \left(ax\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m/(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x^m}{\sqrt{a^2 c x^2 + c} \arctan \left(ax\right)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^m/(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/atan(a*x)**2/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\sqrt{a^2cx^2+c}\arctan{(ax)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^m/(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2), x)
```

3.608 $\int \frac{x^m}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{x^m}{\left(a^2cx^2+c\right)^{3/2}\tan^{-1}(ax)^2},x\right)$$

```
[Out] Unintegrable[x^m/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2), x]
```

Rubi [A] time = 0.119258, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2),x]
```

```
[Out] Defer[Int][x^m/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2), x]
```

Rubi steps

$$\int \frac{x^m}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2}\,dx=\int \frac{x^m}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2}\,dx$$

Mathematica [A] time = 0.53358, size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2),x]
```

```
[Out] Integrate[x^m/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2), x]
```

Maple [A] time = 1.208, size = 0, normalized size = 0.

$$\int \frac{x^m}{(\arctan(ax))^2}\left(a^2cx^2+c\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x)
```

```
[Out] int(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(a^2cx^2+c\right)^{\frac{3}{2}}\arctan(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^m/((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{a^2cx^2+c}\,x^m}{\left(a^4c^2x^4+2\,a^2c^2x^2+c^2\right)\arctan(ax)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^m/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arcta
n(a*x)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(a**2*c*x**2+c)**(3/2)/atan(a*x)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(a^2cx^2+c\right)^{\frac{3}{2}}\arctan\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^m/((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^2), x)
```

3.609 $\int \frac{x^m}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^2}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{x^m}{\left(a^2cx^2+c\right)^{5/2}\tan^{-1}(ax)^2},x\right)$$

```
[Out] Unintegrable[x^m/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2), x]
```

Rubi [A] time = 0.117504, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2),x]
```

```
[Out] Defer[Int][x^m/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2), x]
```

Rubi steps

$$\int \frac{x^m}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^2}\,dx=\int \frac{x^m}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^2}\,dx$$

Mathematica [A] time = 0.581358, size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2),x]
```

```
[Out] Integrate[x^m/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2), x]
```

Maple [A] time = 1.236, size = 0, normalized size = 0.

$$\int \frac{x^{m}}{\left(\arctan \left(a x\right)\right)^{2}} \left(a^{2} c \,x^{2}+c \right)^{-\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x)
```

```
[Out] int(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{m}}{{\left(a^{2} c x^{2} + c\right)}^{\frac{5}{2}} \arctan \left(a x\right)^{2}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^m/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$${\rm integral}\left(\frac{\sqrt{a^{2} c x^{2} + c} x^{m}}{{\left(a^{6} c^{3} x^{6} + 3 \, a^{4} c^{3} x^{4} + 3 \, a^{2} c^{3} x^{2} + c^{3}\right)} \arctan \left(a x\right)^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^m/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*
x^2 + c^3)*arctan(a*x)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(a**2*c*x**2+c)**(5/2)/atan(a*x)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(a^2cx^2+c\right)^{\frac{5}{2}}\arctan\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^m/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^2), x)
```

3.610 $\int \frac{x(c+a^2cx^2)}{\tan^{-1}(ax)^3} \, dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left(\frac{x\left(a^2cx^2+c\right)}{\tan^{-1}(ax)^3},x\right)$$

```
[Out] Unintegrable[(x*(c + a^2*c*x^2))/ArcTan[a*x]^3, x]
```

Rubi [A] time = 0.0235311, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x\left(c+a^2cx^2\right)}{\tan^{-1}(ax)^3} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(x*(c + a^2*c*x^2))/ArcTan[a*x]^3,x]

[Out] Defer[Int][(x*(c + a^2*c*x^2))/ArcTan[a*x]^3, x]
```

Rubi steps

$$\int \frac{x\left(c+a^2cx^2\right)}{\tan^{-1}(ax)^3} \, dx=\int \frac{x\left(c+a^2cx^2\right)}{\tan^{-1}(ax)^3} \, dx$$

Mathematica [A] time = 1.09627, size = 0, normalized size = 0.

$$\int \frac{x\left(c+a^2cx^2\right)}{\tan^{-1}(ax)^3} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x*(c + a^2*c*x^2))/ArcTan[a*x]^3,x]
```

[Out] Integrate[(x*(c + a^2*c*x^2))/ArcTan[a*x]^3, x]

Maple [A] time = 0.871, size = 0, normalized size = 0.

$$\int \frac{x\left(a^{2}c x^{2}+c\right)}{\left(\arctan\left(ax\right)\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)/arctan(a*x)^3,x)
```

```
[Out] int(x*(a^2*c*x^2+c)/arctan(a*x)^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{a^{5}cx^{5}+2\,a^{3}cx^{3}-2\,a^{2}c\left(\int \frac{15\,a^{4}x^{5}}{\arctan(ax)}\,dx+\int \frac{22\,a^{2}x^{3}}{\arctan(ax)}\,dx+\int \frac{7\,x}{\arctan(ax)}\,dx\right)\arctan\left(ax\right)^{2}+acx+\left(5\,a^{6}cx^{6}+11\,a^{4}cx^{4}+7\,a\right.}{2\,a^{2}\arctan\left(ax\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(a^5*c*x^5 + 2*a^3*c*x^3 - 2*a^2*arctan(a*x)^2*integrate((15*a^4*c*x^5
 + 22*a^2*c*x^3 + 7*c*x)/arctan(a*x), x) + a*c*x + (5*a^6*c*x^6 + 11*a^4*c*
x^4 + 7*a^2*c*x^2 + c)*arctan(a*x))/(a^2*arctan(a*x)^2)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{a^{2}cx^{3}+cx}{\arctan\left(ax\right)^{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^3 + c*x)/arctan(a*x)^3, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$c\left(\int \frac{x}{\operatorname{atan}^{3}(ax)}\,dx + \int \frac{a^{2}x^{3}}{\operatorname{atan}^{3}(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)/atan(a*x)**3,x)
```

```
[Out] c*(Integral(x/atan(a*x)**3, x) + Integral(a**2*x**3/atan(a*x)**3, x))
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a^{2}cx^{2} + c)x}{\arctan(ax)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)*x/arctan(a*x)^3, x)
```

3.611 $\int \frac{c+a^2cx^2}{\tan^{-1}(ax)^3} \, dx$

Optimal. Leaf size=19

$$\text{Unintegrable}\left(\frac{a^2cx^2+c}{\tan^{-1}(ax)^3},x\right)$$

```
[Out] Unintegrable[(c + a^2*c*x^2)/ArcTan[a*x]^3, x]
```

Rubi [A] time = 0.0126611, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{c+a^2cx^2}{\tan^{-1}(ax)^3} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)/ArcTan[a*x]^3,x]

[Out] Defer[Int][(c + a^2*c*x^2)/ArcTan[a*x]^3, x]
```

Rubi steps

$$\int \frac{c+a^2cx^2}{\tan^{-1}(ax)^3} \, dx = \int \frac{c+a^2cx^2}{\tan^{-1}(ax)^3} \, dx$$

Mathematica [A] time = 1.15084, size = 0, normalized size = 0.

$$\int \frac{c+a^2cx^2}{\tan^{-1}(ax)^3} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)/ArcTan[a*x]^3,x]

[Out] Integrate[(c + a^2*c*x^2)/ArcTan[a*x]^3, x]
```

Maple [A] time = 0.793, size = 0, normalized size = 0.

$$\int \frac{a^{2} c\,x^{2}+c}{\left(\arctan \left(a x \right)\right)^{3}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)/arctan(a*x)^3,x)
```

```
[Out] int((a^2*c*x^2+c)/arctan(a*x)^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{a^{4} c x^{4}+2 \, a^{2} c x^{2}-4 \, a \arctan \left(a x\right)^{2} \int \frac{5 \, a^{4} c x^{4}+6 \, a^{2} c x^{2}+c}{\arctan \left(a x\right)}\,{d x}+4 \, {\left(a^{5} c x^{5}+2 \, a^{3} c x^{3}+a c x\right)} \arctan \left(a x\right)+c}{2 \, a \arctan \left(a x\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(a^4*c*x^4 + 2*a^2*c*x^2 - 2*a*arctan(a*x)^2*integrate(2*(5*a^4*c*x^4
+ 6*a^2*c*x^2 + c)/arctan(a*x), x) + 4*(a^5*c*x^5 + 2*a^3*c*x^3 + a*c*x)*ar
ctan(a*x) + c)/(a*arctan(a*x)^2)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$${\rm integral}\left(\frac{a^{2} c x^{2}+c}{\arctan \left(a x\right)^{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)/arctan(a*x)^3, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$c\left(\int \frac{a^2x^2}{\operatorname{atan}^3(ax)}\,dx + \int \frac{1}{\operatorname{atan}^3(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)/atan(a*x)**3,x)
```

```
[Out] c*(Integral(a**2*x**2/atan(a*x)**3, x) + Integral(atan(a*x)**(-3), x))
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{a^2cx^2 + c}{\arctan(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)/arctan(a*x)^3, x)
```

3.612 $\int \frac{c+a^2cx^2}{x\tan^{-1}(ax)^3}\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{a^2cx^2+c}{x\tan^{-1}(ax)^3},x\right)$$

```
[Out] Unintegrable[(c + a^2*c*x^2)/(x*ArcTan[a*x]^3), x]
```

Rubi [A] time = 0.0320454, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{c+a^2cx^2}{x\tan^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)/(x*ArcTan[a*x]^3),x]
```

```
[Out] Defer[Int][(c + a^2*c*x^2)/(x*ArcTan[a*x]^3), x]
```

Rubi steps

$$\int \frac{c+a^2cx^2}{x\tan^{-1}(ax)^3}\,dx=\int \frac{c+a^2cx^2}{x\tan^{-1}(ax)^3}\,dx$$

Mathematica [A] time = 1.50604, size = 0, normalized size = 0.

$$\int \frac{c+a^2cx^2}{x\tan^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)/(x*ArcTan[a*x]^3),x]
```

```
[Out] Integrate[(c + a^2*c*x^2)/(x*ArcTan[a*x]^3), x]
```

Maple [A] time = 0.96, size = 0, normalized size = 0.

$$\int \frac{a^{2} c \,x^{2}+c}{x \arctan \left(a x \right)^{3}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)/x/arctan(a*x)^3,x)
```

```
[Out] int((a^2*c*x^2+c)/x/arctan(a*x)^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{a^{5} c x^{5}+2 a^{3} c x^{3}-2 c x^{2}\left(\int \frac{6 a^{6} x^{3}}{\arctan(ax)}\,dx+\int \frac{5 a^{4} x}{\arctan(ax)}\,dx+\int \frac{1}{x^{3} \arctan(ax)}\,dx\right)\arctan\left(ax\right)^{2}+acx+\left(3 a^{6} c x^{6}+5 a^{4} c x^{4}+a}{2 a^{2} x^{2} \arctan\left(ax\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)/x/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(a^5*c*x^5 + 2*a^3*c*x^3 - 2*x^2*arctan(a*x)^2*integrate((6*a^6*c*x^6
+ 5*a^4*c*x^4 + c)/(x^3*arctan(a*x)), x) + a*c*x + (3*a^6*c*x^6 + 5*a^4*c*x
^4 + a^2*c*x^2 - c)*arctan(a*x))/(a^2*x^2*arctan(a*x)^2)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{a^{2} c x^{2}+c}{x \arctan\left(ax\right)^{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)/x/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)/(x*arctan(a*x)^3), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$c\left(\int \frac{1}{x\,\mathrm{atan}^{3}(ax)}\,dx+\int \frac{a^{2}x}{\mathrm{atan}^{3}(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)/x/atan(a*x)**3,x)
```

```
[Out] c*(Integral(1/(x*atan(a*x)**3), x) + Integral(a**2*x/atan(a*x)**3, x))
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{a^{2}cx^{2}+c}{x\arctan(ax)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)/x/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)/(x*arctan(a*x)^3), x)
```

3.613 $\int \frac{x\left(c+a^2cx^2\right)^2}{\tan^{-1}(ax)^3}\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{x\left(a^2cx^2+c\right)^2}{\tan^{-1}(ax)^3},x\right)$$

```
[Out] Unintegrable[(x*(c + a^2*c*x^2)^2)/ArcTan[a*x]^3, x]
```

Rubi [A] time = 0.0354829, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x\left(c+a^2cx^2\right)^2}{\tan^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x*(c + a^2*c*x^2)^2)/ArcTan[a*x]^3,x]
```

```
[Out] Defer[Int][(x*(c + a^2*c*x^2)^2)/ArcTan[a*x]^3, x]
```

Rubi steps

$$\int \frac{x\left(c+a^2cx^2\right)^2}{\tan^{-1}(ax)^3}\,dx=\int \frac{x\left(c+a^2cx^2\right)^2}{\tan^{-1}(ax)^3}\,dx$$

Mathematica [A] time = 0.980394, size = 0, normalized size = 0.

$$\int \frac{x\left(c+a^2cx^2\right)^2}{\tan^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x*(c + a^2*c*x^2)^2)/ArcTan[a*x]^3,x]
```

```
[Out] Integrate[(x*(c + a^2*c*x^2)^2)/ArcTan[a*x]^3, x]
```

Maple [A] time = 1.066, size = 0, normalized size = 0.

$$\int \frac{x\left(a^{2} c x^{2}+c\right)^{2}}{\left(\arctan \left(a x\right)\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^2/arctan(a*x)^3,x)
```

```
[Out] int(x*(a^2*c*x^2+c)^2/arctan(a*x)^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{a^{7} c^{2} x^{7}+3\, a^{5} c^{2} x^{5}+3\, a^{3} c^{2} x^{3}-4\, a^{2} \arctan \left(a x\right)^{2} \int \frac{14\, a^{6} c^{2} x^{7}+33\, a^{4} c^{2} x^{5}+24\, a^{2} c^{2} x^{3}+5\, c^{2} x}{\arctan(ax)}\, dx+a c^{2} x+\left(7\, a^{8} c^{2} x^{8}+22\, a^{6} c^{2} x^{6}+2\right.}{2\, a^{2} \arctan \left(a x\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(a^7*c^2*x^7 + 3*a^5*c^2*x^5 + 3*a^3*c^2*x^3 - 2*a^2*arctan(a*x)^2*int
egrate(2*(14*a^6*c^2*x^7 + 33*a^4*c^2*x^5 + 24*a^2*c^2*x^3 + 5*c^2*x)/arcta
n(a*x), x) + a*c^2*x + (7*a^8*c^2*x^8 + 22*a^6*c^2*x^6 + 24*a^4*c^2*x^4 + 1
0*a^2*c^2*x^2 + c^2)*arctan(a*x))/(a^2*arctan(a*x)^2)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$${\rm integral}\left(\frac{a^{4} c^{2} x^{5}+2\, a^{2} c^{2} x^{3}+c^{2} x}{\arctan \left(a x\right)^{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^5 + 2*a^2*c^2*x^3 + c^2*x)/arctan(a*x)^3, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$c^2\left(\int \frac{x}{\operatorname{atan}^3(ax)}\,dx + \int \frac{2a^2x^3}{\operatorname{atan}^3(ax)}\,dx + \int \frac{a^4x^5}{\operatorname{atan}^3(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**2/atan(a*x)**3,x)
```

```
[Out] c**2*(Integral(x/atan(a*x)**3, x) + Integral(2*a**2*x**3/atan(a*x)**3, x) +
 Integral(a**4*x**5/atan(a*x)**3, x))
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2 + c\right)^2 x}{\arctan(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^2*x/arctan(a*x)^3, x)
```

3.614 $\int \frac{\left(c+a^2cx^2\right)^2}{\tan^{-1}(ax)^3}\,dx$

Optimal. Leaf size=21

$$\text{Unintegrable}\left(\frac{\left(a^2cx^2+c\right)^2}{\tan^{-1}(ax)^3},x\right)$$

```
[Out] Unintegrable[(c + a^2*c*x^2)^2/ArcTan[a*x]^3, x]
```

Rubi [A] time = 0.0216738, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(c+a^2cx^2\right)^2}{\tan^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^2/ArcTan[a*x]^3,x]
```

```
[Out] Defer[Int][(c + a^2*c*x^2)^2/ArcTan[a*x]^3, x]
```

Rubi steps

$$\int \frac{\left(c+a^2cx^2\right)^2}{\tan^{-1}(ax)^3}\,dx=\int \frac{\left(c+a^2cx^2\right)^2}{\tan^{-1}(ax)^3}\,dx$$

Mathematica [A] time = 0.698005, size = 0, normalized size = 0.

$$\int \frac{\left(c+a^2cx^2\right)^2}{\tan^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^2/ArcTan[a*x]^3,x]
```

```
[Out] Integrate[(c + a^2*c*x^2)^2/ArcTan[a*x]^3, x]
```

Maple [A] time = 0.929, size = 0, normalized size = 0.

$$\int \frac{\left(a^{2} c x^{2}+c\right)^{2}}{\left(\arctan \left(a x\right)\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^2/arctan(a*x)^3,x)
```

```
[Out] int((a^2*c*x^2+c)^2/arctan(a*x)^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{a^{6} c^{2} x^{6}+3 \, a^{4} c^{2} x^{4}+3 \, a^{2} c^{2} x^{2}-6 \, a \arctan \left(a x\right)^{2} \int \frac{7 \, a^{6} c^{2} x^{6}+15 \, a^{4} c^{2} x^{4}+9 \, a^{2} c^{2} x^{2}+c^{2}}{\arctan\left(a x\right)}\,{d x}+c^{2}+6 \, {\left(a^{7} c^{2} x^{7}+3 \, a^{5} c^{2} x^{5}+3 \, a^{3} c^{2} x^{3}+\right.}}{2 \, a \arctan \left(a x\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(a^6*c^2*x^6 + 3*a^4*c^2*x^4 + 3*a^2*c^2*x^2 - 2*a*arctan(a*x)^2*integ
rate(3*(7*a^6*c^2*x^6 + 15*a^4*c^2*x^4 + 9*a^2*c^2*x^2 + c^2)/arctan(a*x),
x) + c^2 + 6*(a^7*c^2*x^7 + 3*a^5*c^2*x^5 + 3*a^3*c^2*x^3 + a*c^2*x)*arctan
(a*x))/(a*arctan(a*x)^2)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$${\rm integral}\left(\frac{a^{4} c^{2} x^{4}+2 \, a^{2} c^{2} x^{2}+c^{2}}{\arctan\left(a x\right)^{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)/arctan(a*x)^3, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$c^2\left(\int \frac{2a^2x^2}{\operatorname{atan}^3(ax)}\,dx + \int \frac{a^4x^4}{\operatorname{atan}^3(ax)}\,dx + \int \frac{1}{\operatorname{atan}^3(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**2/atan(a*x)**3,x)
```

```
[Out] c**2*(Integral(2*a**2*x**2/atan(a*x)**3, x) + Integral(a**4*x**4/atan(a*x)*
*3, x) + Integral(atan(a*x)**(-3), x))
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2 + c\right)^2}{\arctan(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^2/arctan(a*x)^3, x)
```

3.615 $\int \frac{\left(c+a^2cx^2\right)^2}{x\tan^{-1}(ax)^3}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{\left(a^2cx^2+c\right)^2}{x\tan^{-1}(ax)^3},x\right)$$

```
[Out] Unintegrable[(c + a^2*c*x^2)^2/(x*ArcTan[a*x]^3), x]
```

Rubi [A] time = 0.0483794, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(c+a^2cx^2\right)^2}{x\tan^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^2/(x*ArcTan[a*x]^3),x]

[Out] Defer[Int][(c + a^2*c*x^2)^2/(x*ArcTan[a*x]^3), x]
```

Rubi steps

$$\int \frac{\left(c+a^2cx^2\right)^2}{x\tan^{-1}(ax)^3}\,dx=\int \frac{\left(c+a^2cx^2\right)^2}{x\tan^{-1}(ax)^3}\,dx$$

Mathematica [A] time = 1.23289, size = 0, normalized size = 0.

$$\int \frac{\left(c+a^2cx^2\right)^2}{x\tan^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^2/(x*ArcTan[a*x]^3),x]
```

```
[Out] Integrate[(c + a^2*c*x^2)^2/(x*ArcTan[a*x]^3), x]
```

Maple [A] time = 0.949, size = 0, normalized size = 0.

$$\int \frac{\left(a^{2} c x^{2}+c\right)^{2}}{x \left(\arctan \left(a x\right)\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^2/x/arctan(a*x)^3,x)
```

```
[Out] int((a^2*c*x^2+c)^2/x/arctan(a*x)^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{a^{7} c^{2} x^{7}+3 \, a^{5} c^{2} x^{5}+3 \, a^{3} c^{2} x^{3}-2 \, c^{2} x^{2} \left(\int \frac{15 \, a^{8} x^{5}}{\arctan\left(a x\right)}\, dx+\int \frac{28 \, a^{6} x^{3}}{\arctan\left(a x\right)}\, dx+\int \frac{12 \, a^{4} x}{\arctan\left(a x\right)}\, dx+\int \frac{1}{x^{3} \arctan\left(a x\right)}\, dx\right) \arctan \left(a x\right.}{2 \, a^{2} x^{2} \arctan \left(a x\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2/x/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(a^7*c^2*x^7 + 3*a^5*c^2*x^5 + 3*a^3*c^2*x^3 - 2*x^2*arctan(a*x)^2*int
egrate((15*a^8*c^2*x^8 + 28*a^6*c^2*x^6 + 12*a^4*c^2*x^4 + c^2)/(x^3*arctan
(a*x)), x) + a*c^2*x + (5*a^8*c^2*x^8 + 14*a^6*c^2*x^6 + 12*a^4*c^2*x^4 + 2
*a^2*c^2*x^2 - c^2)*arctan(a*x))/(a^2*x^2*arctan(a*x)^2)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$${\rm integral}\left(\frac{a^{4} c^{2} x^{4}+2 \, a^{2} c^{2} x^{2}+c^{2}}{x \arctan \left(a x\right)^{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2/x/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)/(x*arctan(a*x)^3), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$c^2\left(\int \frac{1}{x\,\mathrm{atan}^3\,(ax)}\,dx + \int \frac{2a^2x}{\mathrm{atan}^3\,(ax)}\,dx + \int \frac{a^4x^3}{\mathrm{atan}^3\,(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**2/x/atan(a*x)**3,x)
```

```
[Out] c**2*(Integral(1/(x*atan(a*x)**3), x) + Integral(2*a**2*x/atan(a*x)**3, x)
+ Integral(a**4*x**3/atan(a*x)**3, x))
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2 + c\right)^2}{x\arctan\left(ax\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2/x/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^2/(x*arctan(a*x)^3), x)
```

3.616 $\int \frac{x(c+a^2cx^2)^3}{\tan^{-1}(ax)^3} dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{x\left(a^2cx^2+c\right)^3}{\tan^{-1}(ax)^3},x\right)$$

```
[Out] Unintegrable[(x*(c + a^2*c*x^2)^3)/ArcTan[a*x]^3, x]
```

Rubi [A] time = 0.0355532, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x\left(c+a^2cx^2\right)^3}{\tan^{-1}(ax)^3} dx$$

Verification is Not applicable to the result.

```
[In] Int[(x*(c + a^2*c*x^2)^3)/ArcTan[a*x]^3,x]
```

```
[Out] Defer[Int][(x*(c + a^2*c*x^2)^3)/ArcTan[a*x]^3, x]
```

Rubi steps

$$\int \frac{x\left(c+a^2cx^2\right)^3}{\tan^{-1}(ax)^3} dx = \int \frac{x\left(c+a^2cx^2\right)^3}{\tan^{-1}(ax)^3} dx$$

Mathematica [A] time = 1.01826, size = 0, normalized size = 0.

$$\int \frac{x\left(c+a^2cx^2\right)^3}{\tan^{-1}(ax)^3} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x*(c + a^2*c*x^2)^3)/ArcTan[a*x]^3,x]
```

```
[Out] Integrate[(x*(c + a^2*c*x^2)^3)/ArcTan[a*x]^3, x]
```

Maple [A] time = 1.348, size = 0, normalized size = 0.

$$\int \frac{x\left(a^{2}c x^{2}+c\right)^{3}}{\left(\arctan\left(ax\right)\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^3/arctan(a*x)^3,x)
```

```
[Out] int(x*(a^2*c*x^2+c)^3/arctan(a*x)^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{a^{9}c^{3}x^{9}+4\,a^{7}c^{3}x^{7}+6\,a^{5}c^{3}x^{5}+4\,a^{3}c^{3}x^{3}-2\,a^{2}c^{3}\left(\int\frac{45\,a^{8}x^{9}}{\arctan(ax)}\,dx+\int\frac{148\,a^{6}x^{7}}{\arctan(ax)}\,dx+\int\frac{174\,a^{4}x^{5}}{\arctan(ax)}\,dx+\int\frac{84\,a^{2}x^{3}}{\arctan(ax)}\,dx+\int\right.}{2\,a^{2}\arctan}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(a^9*c^3*x^9 + 4*a^7*c^3*x^7 + 6*a^5*c^3*x^5 + 4*a^3*c^3*x^3 + a*c^3*x
 - 2*a^2*arctan(a*x)^2*integrate((45*a^8*c^3*x^9 + 148*a^6*c^3*x^7 + 174*a^
4*c^3*x^5 + 84*a^2*c^3*x^3 + 13*c^3*x)/arctan(a*x), x) + (9*a^10*c^3*x^10 +
 37*a^8*c^3*x^8 + 58*a^6*c^3*x^6 + 42*a^4*c^3*x^4 + 13*a^2*c^3*x^2 + c^3)*a
rctan(a*x))/(a^2*arctan(a*x)^2)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{a^{6}c^{3}x^{7}+3\,a^{4}c^{3}x^{5}+3\,a^{2}c^{3}x^{3}+c^{3}x}{\arctan\left(ax\right)^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral((a^6*c^3*x^7 + 3*a^4*c^3*x^5 + 3*a^2*c^3*x^3 + c^3*x)/arctan(a*x)^
3, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$c^3\left(\int \frac{x}{\operatorname{atan}^3(ax)}\,dx + \int \frac{3a^2x^3}{\operatorname{atan}^3(ax)}\,dx + \int \frac{3a^4x^5}{\operatorname{atan}^3(ax)}\,dx + \int \frac{a^6x^7}{\operatorname{atan}^3(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**3/atan(a*x)**3,x)
```

```
[Out] c**3*(Integral(x/atan(a*x)**3, x) + Integral(3*a**2*x**3/atan(a*x)**3, x) +
 Integral(3*a**4*x**5/atan(a*x)**3, x) + Integral(a**6*x**7/atan(a*x)**3, x
))
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^3 x}{\arctan(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^3*x/arctan(a*x)^3, x)
```

3.617 $\int \frac{(c+a^2cx^2)^3}{\tan^{-1}(ax)^3} dx$

Optimal. Leaf size=21

$$\text{Unintegrable}\left(\frac{(a^2cx^2+c)^3}{\tan^{-1}(ax)^3}, x\right)$$

```
[Out] Unintegrable[(c + a^2*c*x^2)^3/ArcTan[a*x]^3, x]
```

Rubi [A] time = 0.0218144, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(c+a^2cx^2)^3}{\tan^{-1}(ax)^3} dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^3/ArcTan[a*x]^3,x]
```

```
[Out] Defer[Int][(c + a^2*c*x^2)^3/ArcTan[a*x]^3, x]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^3}{\tan^{-1}(ax)^3} dx = \int \frac{(c+a^2cx^2)^3}{\tan^{-1}(ax)^3} dx$$

Mathematica [A] time = 1.1501, size = 0, normalized size = 0.

$$\int \frac{(c+a^2cx^2)^3}{\tan^{-1}(ax)^3} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^3/ArcTan[a*x]^3,x]
```

```
[Out] Integrate[(c + a^2*c*x^2)^3/ArcTan[a*x]^3, x]
```

Maple [A] time = 1.2, size = 0, normalized size = 0.

$$\int \frac{\left(a^{2} c x^{2}+c\right)^{3}}{\left(\arctan \left(a x\right)\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^3/arctan(a*x)^3,x)
```

```
[Out] int((a^2*c*x^2+c)^3/arctan(a*x)^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{a^{8} c^{3} x^{8}+4\, a^{6} c^{3} x^{6}+6\, a^{4} c^{3} x^{4}+4\, a^{2} c^{3} x^{2}-8\, a \arctan \left(a x\right)^{2} \int \frac{9\, a^{8} c^{3} x^{8}+28\, a^{6} c^{3} x^{6}+30\, a^{4} c^{3} x^{4}+12\, a^{2} c^{3} x^{2}+c^{3}}{\arctan(ax)}\, dx+c^{3}+8\left(a^{9} c^{3} x^{9}+\right.}{2\, a \arctan \left(a x\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(a^8*c^3*x^8 + 4*a^6*c^3*x^6 + 6*a^4*c^3*x^4 + 4*a^2*c^3*x^2 - 2*a*arc
tan(a*x)^2*integrate(4*(9*a^8*c^3*x^8 + 28*a^6*c^3*x^6 + 30*a^4*c^3*x^4 + 1
2*a^2*c^3*x^2 + c^3)/arctan(a*x), x) + c^3 + 8*(a^9*c^3*x^9 + 4*a^7*c^3*x^7
 + 6*a^5*c^3*x^5 + 4*a^3*c^3*x^3 + a*c^3*x)*arctan(a*x))/(a*arctan(a*x)^2)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{a^{6} c^{3} x^{6}+3\, a^{4} c^{3} x^{4}+3\, a^{2} c^{3} x^{2}+c^{3}}{\arctan \left(a x\right)^{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)/arctan(a*x)^3,
 x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$c^3\left(\int \frac{3a^2x^2}{\operatorname{atan}^3(ax)}\,dx + \int \frac{3a^4x^4}{\operatorname{atan}^3(ax)}\,dx + \int \frac{a^6x^6}{\operatorname{atan}^3(ax)}\,dx + \int \frac{1}{\operatorname{atan}^3(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**3/atan(a*x)**3,x)
```

```
[Out] c**3*(Integral(3*a**2*x**2/atan(a*x)**3, x) + Integral(3*a**4*x**4/atan(a*x
)**3, x) + Integral(a**6*x**6/atan(a*x)**3, x) + Integral(atan(a*x)**(-3),
x))
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^3}{\arctan(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^3/arctan(a*x)^3, x)
```

3.618 $\int \frac{(c+a^2cx^2)^3}{x\tan^{-1}(ax)^3}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{(a^2cx^2+c)^3}{x\tan^{-1}(ax)^3},x\right)$$

```
[Out] Unintegrable[(c + a^2*c*x^2)^3/(x*ArcTan[a*x]^3), x]
```

Rubi [A] time = 0.0489339, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(c+a^2cx^2)^3}{x\tan^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^3/(x*ArcTan[a*x]^3),x]
```

```
[Out] Defer[Int][(c + a^2*c*x^2)^3/(x*ArcTan[a*x]^3), x]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^3}{x\tan^{-1}(ax)^3}\,dx=\int \frac{(c+a^2cx^2)^3}{x\tan^{-1}(ax)^3}\,dx$$

Mathematica [A] time = 1.21194, size = 0, normalized size = 0.

$$\int \frac{(c+a^2cx^2)^3}{x\tan^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^3/(x*ArcTan[a*x]^3),x]
```

```
[Out] Integrate[(c + a^2*c*x^2)^3/(x*ArcTan[a*x]^3), x]
```

Maple [A] time = 1.163, size = 0, normalized size = 0.

$$\int \frac{\left(a^{2} c\,x^{2}+c\right)^{3}}{x \left(\arctan \left(a x\right)\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^3/x/arctan(a*x)^3,x)
```

```
[Out] int((a^2*c*x^2+c)^3/x/arctan(a*x)^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{a^{9} c^{3} x^{9} + 4 \, a^{7} c^{3} x^{7} + 6 \, a^{5} c^{3} x^{5} + 4 \, a^{3} c^{3} x^{3} - 2 \, c^{3} x^{2} \left(\int \frac{28 \, a^{10} x^{7}}{\arctan(ax)} \, dx + \int \frac{81 \, a^{8} x^{5}}{\arctan(ax)} \, dx + \int \frac{76 \, a^{6} x^{3}}{\arctan(ax)} \, dx + \int \frac{22 \, a^{4} x}{\arctan(ax)} \, dx + \int \right.}{2 \, a^{2} x^{2} \operatorname{arcta}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3/x/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(a^9*c^3*x^9 + 4*a^7*c^3*x^7 + 6*a^5*c^3*x^5 + 4*a^3*c^3*x^3 + a*c^3*x
 - 2*x^2*arctan(a*x)^2*integrate((28*a^10*c^3*x^10 + 81*a^8*c^3*x^8 + 76*a^
6*c^3*x^6 + 22*a^4*c^3*x^4 + c^3)/(x^3*arctan(a*x)), x) + (7*a^10*c^3*x^10
+ 27*a^8*c^3*x^8 + 38*a^6*c^3*x^6 + 22*a^4*c^3*x^4 + 3*a^2*c^3*x^2 - c^3)*a
rctan(a*x))/(a^2*x^2*arctan(a*x)^2)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{a^{6} c^{3} x^{6} + 3 \, a^{4} c^{3} x^{4} + 3 \, a^{2} c^{3} x^{2} + c^{3}}{x \arctan\left(a x\right)^{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3/x/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)/(x*arctan(a*x)
^3), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$c^3\left(\int \frac{1}{x\,\mathrm{atan}^3\,(ax)}\,dx+\int \frac{3a^2x}{\mathrm{atan}^3\,(ax)}\,dx+\int \frac{3a^4x^3}{\mathrm{atan}^3\,(ax)}\,dx+\int \frac{a^6x^5}{\mathrm{atan}^3\,(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**3/x/atan(a*x)**3,x)
```

```
[Out] c**3*(Integral(1/(x*atan(a*x)**3), x) + Integral(3*a**2*x/atan(a*x)**3, x)
+ Integral(3*a**4*x**3/atan(a*x)**3, x) + Integral(a**6*x**5/atan(a*x)**3,
x))
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^3}{x\arctan\,(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3/x/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^3/(x*arctan(a*x)^3), x)
```

3.619 $\int \frac{x^3}{(c+a^2cx^2)\tan^{-1}(ax)^3}\,dx$

Optimal. Leaf size=42

$$\frac{3\text{Unintegrable}\left(\frac{x^2}{\tan^{-1}(ax)^2},x\right)}{2ac}-\frac{x^3}{2ac\tan^{-1}(ax)^2}$$

```
[Out] -x^3/(2*a*c*ArcTan[a*x]^2) + (3*Unintegrable[x^2/ArcTan[a*x]^2, x])/(2*a*c)
```

Rubi [A] time = 0.0962606, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^3}{(c+a^2cx^2)\tan^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^3/((c + a^2*c*x^2)*ArcTan[a*x]^3),x]
```

```
[Out] -x^3/(2*a*c*ArcTan[a*x]^2) + (3*Defer[Int][x^2/ArcTan[a*x]^2, x])/(2*a*c)
```

Rubi steps

$$\int \frac{x^3}{(c+a^2cx^2)\tan^{-1}(ax)^3}\,dx=-\frac{x^3}{2ac\tan^{-1}(ax)^2}+\frac{3\int \frac{x^2}{\tan^{-1}(ax)^2}\,dx}{2ac}$$

Mathematica [A] time = 0.861201, size = 0, normalized size = 0.

$$\int \frac{x^3}{(c+a^2cx^2)\tan^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^3/((c + a^2*c*x^2)*ArcTan[a*x]^3),x]
```

```
[Out] Integrate[x^3/((c + a^2*c*x^2)*ArcTan[a*x]^3), x]
```

Maple [A] time = 0.963, size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(a^2cx^2+c\right)\left(\arctan\left(ax\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a^2*c*x^2+c)/arctan(a*x)^3,x)
```

```
[Out] int(x^3/(a^2*c*x^2+c)/arctan(a*x)^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{ax^3 - 6\arctan\left(ax\right)^2 \int \frac{2\,a^2x^3+x}{\arctan(ax)}\,dx + 3\left(a^2x^4+x^2\right)\arctan\left(ax\right)}{2\,a^2c\arctan\left(ax\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(a*x^3 - 2*arctan(a*x)^2*integrate(3*(2*a^2*x^3 + x)/arctan(a*x), x) +
 3*(a^2*x^4 + x^2)*arctan(a*x))/(a^2*c*arctan(a*x)^2)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^3}{\left(a^2cx^2+c\right)\arctan\left(ax\right)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(x^3/((a^2*c*x^2 + c)*arctan(a*x)^3), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^3}{a^2x^2 \operatorname{atan}^3(ax)+\operatorname{atan}^3(ax)}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a**2*c*x**2+c)/atan(a*x)**3,x)
```

```
[Out] Integral(x**3/(a**2*x**2*atan(a*x)**3 + atan(a*x)**3), x)/c
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{(a^2cx^2 + c)\arctan(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(x^3/((a^2*c*x^2 + c)*arctan(a*x)^3), x)
```

3.620 $\int \frac{x^2}{\left(c+a^2cx^2\right)\tan^{-1}(ax)^3}\,dx$

Optimal. Leaf size=37

$$\frac{\text{Unintegrable}\left(\frac{x}{\tan^{-1}(ax)^2},x\right)}{ac}-\frac{x^2}{2ac\tan^{-1}(ax)^2}$$

```
[Out] -x^2/(2*a*c*ArcTan[a*x]^2) + Unintegrable[x/ArcTan[a*x]^2, x]/(a*c)
```

Rubi [A] time = 0.0750696, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^2}{\left(c+a^2cx^2\right)\tan^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^2/((c + a^2*c*x^2)*ArcTan[a*x]^3),x]
```

```
[Out] -x^2/(2*a*c*ArcTan[a*x]^2) + Defer[Int][x/ArcTan[a*x]^2, x]/(a*c)
```

Rubi steps

$$\int \frac{x^2}{\left(c+a^2cx^2\right)\tan^{-1}(ax)^3}\,dx=-\frac{x^2}{2ac\tan^{-1}(ax)^2}+\frac{\int \frac{x}{\tan^{-1}(ax)^2}\,dx}{ac}$$

Mathematica [A] time = 0.574142, size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(c+a^2cx^2\right)\tan^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^2/((c + a^2*c*x^2)*ArcTan[a*x]^3),x]
```

```
[Out] Integrate[x^2/((c + a^2*c*x^2)*ArcTan[a*x]^3), x]
```

Maple [A] time = 0.463, size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(a^2cx^2+c\right)\left(\arctan\left(ax\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a^2*c*x^2+c)/arctan(a*x)^3,x)
```

```
[Out] int(x^2/(a^2*c*x^2+c)/arctan(a*x)^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{ax^2-2\,\arctan\left(ax\right)^2\int\frac{3\,a^2x^2+1}{\arctan(ax)}\,dx+2\left(a^2x^3+x\right)\arctan\left(ax\right)}{2\,a^2c\arctan\left(ax\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(a*x^2 - 2*arctan(a*x)^2*integrate((3*a^2*x^2 + 1)/arctan(a*x), x) + 2
*(a^2*x^3 + x)*arctan(a*x))/(a^2*c*arctan(a*x)^2)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^2}{\left(a^2cx^2+c\right)\arctan\left(ax\right)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(x^2/((a^2*c*x^2 + c)*arctan(a*x)^3), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^2}{a^2x^2 \operatorname{atan}^3(ax)+\operatorname{atan}^3(ax)}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a**2*c*x**2+c)/atan(a*x)**3,x)
```

```
[Out] Integral(x**2/(a**2*x**2*atan(a*x)**3 + atan(a*x)**3), x)/c
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(a^2cx^2 + c) \arctan(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(x^2/((a^2*c*x^2 + c)*arctan(a*x)^3), x)
```

3.621 $\int \frac{x}{(c+a^2cx^2)\tan^{-1}(ax)^3}\,dx$

Optimal. Leaf size=36

$$\frac{\text{Unintegrable}\left(\frac{1}{\tan^{-1}(ax)^2},x\right)}{2ac}-\frac{x}{2ac\tan^{-1}(ax)^2}$$

```
[Out] -x/(2*a*c*ArcTan[a*x]^2) + Unintegrable[ArcTan[a*x]^(-2), x]/(2*a*c)
```

Rubi [A] time = 0.0488842, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x}{\left(c+a^2cx^2\right)\tan^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x/((c + a^2*c*x^2)*ArcTan[a*x]^3),x]

[Out] -x/(2*a*c*ArcTan[a*x]^2) + Defer[Int][ArcTan[a*x]^(-2), x]/(2*a*c)
```

Rubi steps

$$\int \frac{x}{\left(c+a^2cx^2\right)\tan^{-1}(ax)^3}\,dx=-\frac{x}{2ac\tan^{-1}(ax)^2}+\frac{\int \frac{1}{\tan^{-1}(ax)^2}\,dx}{2ac}$$

Mathematica [A] time = 0.449724, size = 0, normalized size = 0.

$$\int \frac{x}{\left(c+a^2cx^2\right)\tan^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x/((c + a^2*c*x^2)*ArcTan[a*x]^3),x]
```

```
[Out] Integrate[x/((c + a^2*c*x^2)*ArcTan[a*x]^3), x]
```

Maple [A] time = 0.158, size = 0, normalized size = 0.

$$\int \frac{x}{\left(a^{2} c x^{2}+c\right) \left(\arctan \left(a x\right)\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^2*c*x^2+c)/arctan(a*x)^3,x)
```

```
[Out] int(x/(a^2*c*x^2+c)/arctan(a*x)^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{2\, a^{2} \arctan \left(a x\right)^{2} \int \frac{x}{\arctan\left(a x\right)}\, dx - a x - \left(a^{2} x^{2} + 1\right) \arctan \left(a x\right)}{2\, a^{2} c \arctan \left(a x\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*a^2*arctan(a*x)^2*integrate(x/arctan(a*x), x) - a*x - (a^2*x^2 + 1)*
arctan(a*x))/(a^2*c*arctan(a*x)^2)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$${\rm integral}\left(\frac{x}{\left(a^{2} c x^{2} + c\right)} \arctan\left(a x\right)^{3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(x/((a^2*c*x^2 + c)*arctan(a*x)^3), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x}{a^2x^2\operatorname{atan}^3(ax)+\operatorname{atan}^3(ax)}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a**2*c*x**2+c)/atan(a*x)**3,x)
```

```
[Out] Integral(x/(a**2*x**2*atan(a*x)**3 + atan(a*x)**3), x)/c
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(a^2cx^2+c)\arctan(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(x/((a^2*c*x^2 + c)*arctan(a*x)^3), x)
```

3.622 $\int \frac{1}{(c+a^2cx^2)\tan^{-1}(ax)^3}\,dx$

Optimal. Leaf size=16

$$-\frac{1}{2ac\tan^{-1}(ax)^2}$$

```
[Out] -1/(2*a*c*ArcTan[a*x]^2)
```

Rubi [A] time = 0.0245363, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {4884}

$$-\frac{1}{2ac\tan^{-1}(ax)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a^2*c*x^2)*ArcTan[a*x]^3),x]
```

```
[Out] -1/(2*a*c*ArcTan[a*x]^2)
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{(c+a^2cx^2)\tan^{-1}(ax)^3}\,dx = -\frac{1}{2ac\tan^{-1}(ax)^2}$$

Mathematica [A] time = 0.0037157, size = 16, normalized size = 1.

$$-\frac{1}{2ac\tan^{-1}(ax)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a^2*c*x^2)*ArcTan[a*x]^3),x]
```

```
[Out] -1/(2*a*c*ArcTan[a*x]^2)
```

Maple [A] time = 0.059, size = 15, normalized size = 0.9

$$-\frac{1}{2\,ac\,(\arctan(ax))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2*c*x^2+c)/arctan(a*x)^3,x)
```

```
[Out] -1/2/a/c/arctan(a*x)^2
```

Maxima [A] time = 0.991771, size = 19, normalized size = 1.19

$$-\frac{1}{2\,ac\arctan(ax)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] -1/2/(a*c*arctan(a*x)^2)
```

Fricas [A] time = 1.56614, size = 35, normalized size = 2.19

$$-\frac{1}{2\,ac\arctan(ax)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] -1/2/(a*c*arctan(a*x)^2)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*c*x**2+c)/atan(a*x)**3,x)
```

```
[Out] Exception raised: TypeError
```

Giac [A] time = 1.1233, size = 19, normalized size = 1.19

$$-\frac{1}{2\,ac\arctan\left(ax\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] -1/2/(a*c*arctan(a*x)^2)
```

3.623 $\int \frac{1}{x\left(c+a^2cx^2\right)\tan^{-1}(ax)^3}\,dx$

Optimal. Leaf size=42

$$-\frac{\text{Unintegrable}\left(\frac{1}{x^2\tan^{-1}(ax)^2},x\right)}{2ac}-\frac{1}{2acx\tan^{-1}(ax)^2}$$

```
[Out] -1/(2*a*c*x*ArcTan[a*x]^2) - Unintegrable[1/(x^2*ArcTan[a*x]^2), x]/(2*a*c)
```

Rubi [A] time = 0.0769726, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x\left(c+a^2cx^2\right)\tan^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*(c + a^2*c*x^2)*ArcTan[a*x]^3),x]
```

```
[Out] -1/(2*a*c*x*ArcTan[a*x]^2) - Defer[Int][1/(x^2*ArcTan[a*x]^2), x]/(2*a*c)
```

Rubi steps

$$\int \frac{1}{x\left(c+a^2cx^2\right)\tan^{-1}(ax)^3}\,dx=-\frac{1}{2acx\tan^{-1}(ax)^2}-\frac{\int \frac{1}{x^2\tan^{-1}(ax)^2}\,dx}{2ac}$$

Mathematica [A] time = 0.439704, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(c+a^2cx^2\right)\tan^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*(c + a^2*c*x^2)*ArcTan[a*x]^3),x]
```

```
[Out] Integrate[1/(x*(c + a^2*c*x^2)*ArcTan[a*x]^3), x]
```

Maple [A] time = 0.132, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(a^{2}c x^{2}+c\right)\left(\arctan\left(ax\right)\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a^2*c*x^2+c)/arctan(a*x)^3,x)
```

```
[Out] int(1/x/(a^2*c*x^2+c)/arctan(a*x)^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{2\,x^{2}\arctan\left(ax\right)^{2}\int\frac{1}{x^{3}\arctan(ax)}\,dx-ax+\left(a^{2}x^{2}+1\right)\arctan\left(ax\right)}{2\,a^{2}c x^{2}\arctan\left(ax\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*x^2*arctan(a*x)^2*integrate(1/(x^3*arctan(a*x)), x) - a*x + (a^2*x^2
 + 1)*arctan(a*x))/(a^2*c*x^2*arctan(a*x)^2)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\left(a^{2}c x^{3}+c x\right)\arctan\left(ax\right)^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(1/((a^2*c*x^3 + c*x)*arctan(a*x)^3), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{a^2x^3\operatorname{atan}^3(ax)+x\operatorname{atan}^3(ax)}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a**2*c*x**2+c)/atan(a*x)**3,x)
```

```
[Out] Integral(1/(a**2*x**3*atan(a*x)**3 + x*atan(a*x)**3), x)/c
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)x\arctan(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)*x*arctan(a*x)^3), x)
```

3.624 $\int \frac{1}{x^2(c+a^2cx^2)\tan^{-1}(ax)^3}\,dx$

Optimal. Leaf size=40

$$-\frac{\text{Unintegrable}\left(\frac{1}{x^3\tan^{-1}(ax)^2},x\right)}{ac}-\frac{1}{2acx^2\tan^{-1}(ax)^2}$$

```
[Out] -1/(2*a*c*x^2*ArcTan[a*x]^2) - Unintegrable[1/(x^3*ArcTan[a*x]^2), x]/(a*c)
```

Rubi [A] time = 0.081877, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)\tan^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^2*(c + a^2*c*x^2)*ArcTan[a*x]^3),x]
```

```
[Out] -1/(2*a*c*x^2*ArcTan[a*x]^2) - Defer[Int][1/(x^3*ArcTan[a*x]^2), x]/(a*c)
```

Rubi steps

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)\tan^{-1}(ax)^3}\,dx=-\frac{1}{2acx^2\tan^{-1}(ax)^2}-\frac{\int \frac{1}{x^3\tan^{-1}(ax)^2}\,dx}{ac}$$

Mathematica [A] time = 1.04358, size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)\tan^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^2*(c + a^2*c*x^2)*ArcTan[a*x]^3),x]
```

```
[Out] Integrate[1/(x^2*(c + a^2*c*x^2)*ArcTan[a*x]^3), x]
```

Maple [A] time = 0.441, size = 0, normalized size = 0.

$$\int \frac{1}{x^{2} \left(a^{2} c \,x^{2}+c \right) \left(\arctan \left(a x \right)\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a^2*c*x^2+c)/arctan(a*x)^3,x)
```

```
[Out] int(1/x^2/(a^2*c*x^2+c)/arctan(a*x)^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{2 \, x^{3} \arctan \left(a x\right)^{2} \int \frac{a^{2} x^{2} + 3}{x^{4} \arctan \left(a x\right)}\,{d x} - a x + 2 \, {\left(a^{2} x^{2} + 1\right)} \arctan \left(a x\right)}{2 \, a^{2} c x^{3} \arctan \left(a x\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*x^3*arctan(a*x)^2*integrate((a^2*x^2 + 3)/(x^4*arctan(a*x)), x) - a*
x + 2*(a^2*x^2 + 1)*arctan(a*x))/(a^2*c*x^3*arctan(a*x)^2)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$${\rm integral}\left(\frac{1}{{\left(a^{2} c x^{4} + c x^{2}\right)} \arctan \left(a x\right)^{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(1/((a^2*c*x^4 + c*x^2)*arctan(a*x)^3), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{a^2x^4\operatorname{atan}^3(ax)+x^2\operatorname{atan}^3(ax)}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a**2*c*x**2+c)/atan(a*x)**3,x)
```

```
[Out] Integral(1/(a**2*x**4*atan(a*x)**3 + x**2*atan(a*x)**3), x)/c
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)x^2\arctan\left(ax\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)*x^2*arctan(a*x)^3), x)
```

3.625 $\int \frac{1}{x^3(c+a^2cx^2)\tan^{-1}(ax)^3}\,dx$

Optimal. Leaf size=42

$$-\frac{3\text{Unintegrable}\left(\frac{1}{x^4\tan^{-1}(ax)^2},x\right)}{2ac}-\frac{1}{2acx^3\tan^{-1}(ax)^2}$$

```
[Out] -1/(2*a*c*x^3*ArcTan[a*x]^2) - (3*Unintegrable[1/(x^4*ArcTan[a*x]^2), x])/(
2*a*c)
```

Rubi [A] time = 0.08074, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^3\left(c+a^2cx^2\right)\tan^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^3*(c + a^2*c*x^2)*ArcTan[a*x]^3),x]
```

```
[Out] -1/(2*a*c*x^3*ArcTan[a*x]^2) - (3*Defer[Int][1/(x^4*ArcTan[a*x]^2), x])/(2*
a*c)
```

Rubi steps

$$\int \frac{1}{x^3\left(c+a^2cx^2\right)\tan^{-1}(ax)^3}\,dx=-\frac{1}{2acx^3\tan^{-1}(ax)^2}-\frac{3\int \frac{1}{x^4\tan^{-1}(ax)^2}\,dx}{2ac}$$

Mathematica [A] time = 1.23822, size = 0, normalized size = 0.

$$\int \frac{1}{x^3\left(c+a^2cx^2\right)\tan^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^3*(c + a^2*c*x^2)*ArcTan[a*x]^3),x]
```

```
[Out] Integrate[1/(x^3*(c + a^2*c*x^2)*ArcTan[a*x]^3), x]
```

Maple [A] time = 0.947, size = 0, normalized size = 0.

$$\int \frac{1}{x^{3}\left(a^{2}c x^{2}+c\right)\left(\arctan\left(a x\right)\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a^2*c*x^2+c)/arctan(a*x)^3,x)
```

```
[Out] int(1/x^3/(a^2*c*x^2+c)/arctan(a*x)^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{6\, x^{4} \arctan\left(a x\right)^{2} \int \frac{a^{2} x^{2}+2}{x^{5} \arctan\left(a x\right)}\, dx - a x + 3\, \left(a^{2} x^{2}+1\right) \arctan\left(a x\right)}{2\, a^{2} c x^{4} \arctan\left(a x\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*x^4*arctan(a*x)^2*integrate(3*(a^2*x^2 + 2)/(x^5*arctan(a*x)), x) -
a*x + 3*(a^2*x^2 + 1)*arctan(a*x))/(a^2*c*x^4*arctan(a*x)^2)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\left(a^{2} c x^{5}+c x^{3}\right) \arctan\left(a x\right)^{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(1/((a^2*c*x^5 + c*x^3)*arctan(a*x)^3), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{a^2x^5\operatorname{atan}^3(ax)+x^3\operatorname{atan}^3(ax)}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a**2*c*x**2+c)/atan(a*x)**3,x)
```

```
[Out] Integral(1/(a**2*x**5*atan(a*x)**3 + x**3*atan(a*x)**3), x)/c
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a^2cx^2+c)x^3\arctan(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)*x^3*arctan(a*x)^3), x)
```

3.626 $\int \frac{1}{x^4(c+a^2cx^2)\tan^{-1}(ax)^3}\,dx$

Optimal. Leaf size=40

$$-\frac{2\text{Unintegrable}\left(\frac{1}{x^5\tan^{-1}(ax)^2},x\right)}{ac}-\frac{1}{2acx^4\tan^{-1}(ax)^2}$$

```
[Out] -1/(2*a*c*x^4*ArcTan[a*x]^2) - (2*Unintegrable[1/(x^5*ArcTan[a*x]^2), x])/(
a*c)
```

Rubi [A] time = 0.0819838, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^4\left(c+a^2cx^2\right)\tan^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^4*(c + a^2*c*x^2)*ArcTan[a*x]^3),x]
```

```
[Out] -1/(2*a*c*x^4*ArcTan[a*x]^2) - (2*Defer[Int][1/(x^5*ArcTan[a*x]^2), x])/(a*
c)
```

Rubi steps

$$\int \frac{1}{x^4\left(c+a^2cx^2\right)\tan^{-1}(ax)^3}\,dx=-\frac{1}{2acx^4\tan^{-1}(ax)^2}-\frac{2\int \frac{1}{x^5\tan^{-1}(ax)^2}\,dx}{ac}$$

Mathematica [A] time = 2.32177, size = 0, normalized size = 0.

$$\int \frac{1}{x^4\left(c+a^2cx^2\right)\tan^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^4*(c + a^2*c*x^2)*ArcTan[a*x]^3),x]
```

```
[Out] Integrate[1/(x^4*(c + a^2*c*x^2)*ArcTan[a*x]^3), x]
```

Maple [A] time = 0.974, size = 0, normalized size = 0.

$$\int \frac{1}{x^4 \left(a^2 c x^2 + c\right) \left(\arctan \left(a x\right)\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(a^2*c*x^2+c)/arctan(a*x)^3,x)
```

```
[Out] int(1/x^4/(a^2*c*x^2+c)/arctan(a*x)^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{4\, x^5 \arctan \left(a x\right)^2 \int \frac{3\, a^2 x^2 + 5}{x^6 \arctan(a x)}\, dx - a x + 4 \left(a^2 x^2 + 1\right) \arctan \left(a x\right)}{2\, a^2 c x^5 \arctan \left(a x\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*x^5*arctan(a*x)^2*integrate(2*(3*a^2*x^2 + 5)/(x^6*arctan(a*x)), x)
- a*x + 4*(a^2*x^2 + 1)*arctan(a*x))/(a^2*c*x^5*arctan(a*x)^2)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\left(a^2 c x^6 + c x^4\right) \arctan \left(a x\right)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(1/((a^2*c*x^6 + c*x^4)*arctan(a*x)^3), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{a^2x^6\operatorname{atan}^3(ax)+x^4\operatorname{atan}^3(ax)}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(a**2*c*x**2+c)/atan(a*x)**3,x)
```

```
[Out] Integral(1/(a**2*x**6*atan(a*x)**3 + x**4*atan(a*x)**3), x)/c
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a^2cx^2+c)x^4\arctan(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)*x^4*arctan(a*x)^3), x)
```

3.627 $\int \frac{x^3}{\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^3}\,dx$

Optimal. Leaf size=115

$$\frac{\text{Unintegrable}\left(\frac{1}{\tan^{-1}(ax)^2},x\right)}{2a^3c^2}+\frac{\text{Si}\left(2\tan^{-1}(ax)\right)}{a^4c^2}+\frac{x}{2a^3c^2\left(a^2x^2+1\right)\tan^{-1}(ax)^2}+\frac{1-a^2x^2}{2a^4c^2\left(a^2x^2+1\right)\tan^{-1}(ax)}-\frac{x}{2a^3c^2\tan^{-1}(ax)^2}$$

```
[Out] -x/(2*a^3*c^2*ArcTan[a*x]^2) + x/(2*a^3*c^2*(1 + a^2*x^2)*ArcTan[a*x]^2) +
(1 - a^2*x^2)/(2*a^4*c^2*(1 + a^2*x^2)*ArcTan[a*x]) + SinIntegral[2*ArcTan[
a*x]]/(a^4*c^2) + Unintegrable[ArcTan[a*x]^(-2), x]/(2*a^3*c^2)
```

Rubi [A] time = 0.243868, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{x^3}{\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^3/((c + a^2*c*x^2)^2*ArcTan[a*x]^3),x]
```

```
[Out] -x/(2*a^3*c^2*ArcTan[a*x]^2) + x/(2*a^3*c^2*(1 + a^2*x^2)*ArcTan[a*x]^2) +
(1 - a^2*x^2)/(2*a^4*c^2*(1 + a^2*x^2)*ArcTan[a*x]) + SinIntegral[2*ArcTan[
a*x]]/(a^4*c^2) + Defer[Int][ArcTan[a*x]^(-2), x]/(2*a^3*c^2)
```

Rubi steps

$$\int \frac{x^3}{\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^3}\,dx = -\frac{\int \frac{x}{\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^3}\,dx}{a^2} + \frac{\int \frac{x}{\left(c+a^2cx^2\right)\tan^{-1}(ax)^3}\,dx}{a^2c}$$

$$= -\frac{x}{2a^3c^2\tan^{-1}(ax)^2} + \frac{x}{2a^3c^2\left(1+a^2x^2\right)\tan^{-1}(ax)^2} + \frac{1-a^2x^2}{2a^4c^2\left(1+a^2x^2\right)\tan^{-1}(ax)} + \frac{2\int}{}$$

$$= -\frac{x}{2a^3c^2\tan^{-1}(ax)^2} + \frac{x}{2a^3c^2\left(1+a^2x^2\right)\tan^{-1}(ax)^2} + \frac{1-a^2x^2}{2a^4c^2\left(1+a^2x^2\right)\tan^{-1}(ax)} + \frac{2\,\mathrm{S}}{}$$

$$= -\frac{x}{2a^3c^2\tan^{-1}(ax)^2} + \frac{x}{2a^3c^2\left(1+a^2x^2\right)\tan^{-1}(ax)^2} + \frac{1-a^2x^2}{2a^4c^2\left(1+a^2x^2\right)\tan^{-1}(ax)} + \frac{2\,\mathrm{S}}{}$$

$$= -\frac{x}{2a^3c^2\tan^{-1}(ax)^2} + \frac{x}{2a^3c^2\left(1+a^2x^2\right)\tan^{-1}(ax)^2} + \frac{1-a^2x^2}{2a^4c^2\left(1+a^2x^2\right)\tan^{-1}(ax)} + \frac{\mathrm{Sul}}{}$$

$$= -\frac{x}{2a^3c^2\tan^{-1}(ax)^2} + \frac{x}{2a^3c^2\left(1+a^2x^2\right)\tan^{-1}(ax)^2} + \frac{1-a^2x^2}{2a^4c^2\left(1+a^2x^2\right)\tan^{-1}(ax)} + \frac{\mathrm{Si}(}{}$$

Mathematica [A] time = 9.38248, size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^3/((c + a^2*c*x^2)^2*ArcTan[a*x]^3),x]
```

```
[Out] Integrate[x^3/((c + a^2*c*x^2)^2*ArcTan[a*x]^3), x]
```

Maple [A] time = 0.525, size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(a^2cx^2+c\right)^2(\arctan(ax))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a^2*c*x^2+c)^2/arctan(a*x)^3,x)
```

```
[Out] int(x^3/(a^2*c*x^2+c)^2/arctan(a*x)^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{ax^3+\left(a^2x^4+3\,x^2\right)\arctan\left(ax\right)-\frac{2\left(a^4c^2x^2+a^2c^2\right)\left(a^4\int\frac{x^5}{a^4x^4\arctan(ax)+2\,a^2x^2\arctan(ax)+\arctan(ax)}\,dx+2\,a^2\int\frac{x^3}{a^4x^4\arctan(ax)+2\,a^2x^2\arctan(ax)+\mathrm{arc}}\right.}{a^2c^2}}{2\left(a^4c^2x^2+a^2c^2\right)\arctan\left(ax\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(a*x^3 - 2*(a^4*c^2*x^2 + a^2*c^2)*arctan(a*x)^2*integrate((a^4*x^5 +
2*a^2*x^3 + 3*x)/((a^6*c^2*x^4 + 2*a^4*c^2*x^2 + a^2*c^2)*arctan(a*x)), x)
+ (a^2*x^4 + 3*x^2)*arctan(a*x))/((a^4*c^2*x^2 + a^2*c^2)*arctan(a*x)^2)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x^3}{\left(a^4c^2x^4+2\,a^2c^2x^2+c^2\right)\arctan\left(ax\right)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(x^3/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(a*x)^3), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{x^3}{a^4x^4\operatorname{atan}^3(ax)+2a^2x^2\operatorname{atan}^3(ax)+\operatorname{atan}^3(ax)}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a**2*c*x**2+c)**2/atan(a*x)**3,x)
```

```
[Out] Integral(x**3/(a**4*x**4*atan(a*x)**3 + 2*a**2*x**2*atan(a*x)**3 + atan(a*x
)**3), x)/c**2
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(a^2cx^2+c\right)^2 \arctan\left(ax\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(x^3/((a^2*c*x^2 + c)^2*arctan(a*x)^3), x)
```

3.628 $\int \frac{x^2}{\left(c+a^2cx^2\right)^2 \tan^{-1}(ax)^3} \, dx$

Optimal. Leaf size=71

$$\frac{\text{CosIntegral}\left(2\tan^{-1}(ax)\right)}{a^3c^2} - \frac{x^2}{2ac^2\left(a^2x^2+1\right)\tan^{-1}(ax)^2} - \frac{x}{a^2c^2\left(a^2x^2+1\right)\tan^{-1}(ax)}$$

```
[Out] -x^2/(2*a*c^2*(1 + a^2*x^2)*ArcTan[a*x]^2) - x/(a^2*c^2*(1 + a^2*x^2)*ArcTa
n[a*x]) + CosIntegral[2*ArcTan[a*x]]/(a^3*c^2)
```

Rubi [A] time = 0.287846, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4942, 4968, 4970, 3312, 3302, 4904}

$$\frac{\text{CosIntegral}\left(2\tan^{-1}(ax)\right)}{a^3c^2} - \frac{x^2}{2ac^2\left(a^2x^2+1\right)\tan^{-1}(ax)^2} - \frac{x}{a^2c^2\left(a^2x^2+1\right)\tan^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((c + a^2*c*x^2)^2*ArcTan[a*x]^3),x]
```

```
[Out] -x^2/(2*a*c^2*(1 + a^2*x^2)*ArcTan[a*x]^2) - x/(a^2*c^2*(1 + a^2*x^2)*ArcTa
n[a*x]) + CosIntegral[2*ArcTan[a*x]]/(a^3*c^2)
```

Rule 4942

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^m*(d + e*x^2)^(q + 1)*(a + b*ArcT
an[c*x])^(p + 1))/(b*c*d*(p + 1)), x] - Dist[(f*m)/(b*c*(p + 1)), Int[(f*x)
^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 2, 0] && LtQ[p, -1]
```

Rule 4968

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[(x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^(p
+ 1))/(b*c*d*(p + 1)), x] + (-Dist[(c*(m + 2*q + 2))/(b*(p + 1)), Int[x^(m
+ 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p + 1
)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; Fre
```

```
eQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &&
LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 4970

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sin[x]^m)/
Cos[x]^(m + 2*(q + 1)), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 4904

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x^2}{\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^3}\,dx = -\frac{x^2}{2ac^2\left(1+a^2x^2\right)\tan^{-1}(ax)^2}+\frac{\int \frac{x}{\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^2}\,dx}{a}$$

$$= -\frac{x^2}{2ac^2\left(1+a^2x^2\right)\tan^{-1}(ax)^2}-\frac{x}{a^2c^2\left(1+a^2x^2\right)\tan^{-1}(ax)}+\frac{\int \frac{1}{\left(c+a^2cx^2\right)^2\tan^{-1}(ax)}\,dx}{a^2}-\int$$

$$= -\frac{x^2}{2ac^2\left(1+a^2x^2\right)\tan^{-1}(ax)^2}-\frac{x}{a^2c^2\left(1+a^2x^2\right)\tan^{-1}(ax)}+\frac{\text{Subst}\left(\int \frac{\cos^2(x)}{x}\,dx, x, \tan^{-}\right.}{a^3c^2}$$

$$= -\frac{x^2}{2ac^2\left(1+a^2x^2\right)\tan^{-1}(ax)^2}-\frac{x}{a^2c^2\left(1+a^2x^2\right)\tan^{-1}(ax)}-\frac{\text{Subst}\left(\int \left(\frac{1}{2x}-\frac{\cos(2x)}{2x}\right)dx,\right.}{a^3c^2}$$

$$= -\frac{x^2}{2ac^2\left(1+a^2x^2\right)\tan^{-1}(ax)^2}-\frac{x}{a^2c^2\left(1+a^2x^2\right)\tan^{-1}(ax)}+2\frac{\text{Subst}\left(\int \frac{\cos(2x)}{x}\,dx, x, \tan\right.}{2a^3c^2}$$

$$= -\frac{x^2}{2ac^2\left(1+a^2x^2\right)\tan^{-1}(ax)^2}-\frac{x}{a^2c^2\left(1+a^2x^2\right)\tan^{-1}(ax)}+\frac{\text{Ci}\left(2\tan^{-1}(ax)\right)}{a^3c^2}$$

Mathematica [A] time = 0.106058, size = 51, normalized size = 0.72

$$\frac{2\text{CosIntegral}\left(2\tan^{-1}(ax)\right)-\frac{ax\left(ax+2\tan^{-1}(ax)\right)}{\left(a^2x^2+1\right)\tan^{-1}(ax)^2}}{2a^3c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((c + a^2*c*x^2)^2*ArcTan[a*x]^3),x]
```

```
[Out] (-((a*x*(a*x + 2*ArcTan[a*x]))/((1 + a^2*x^2)*ArcTan[a*x]^2)) + 2*CosIntegr
al[2*ArcTan[a*x]])/(2*a^3*c^2)
```

Maple [A] time = 0.075, size = 52, normalized size = 0.7

$$\frac{4\,\text{Ci}\left(2\arctan(ax)\right)\left(\arctan(ax)\right)^2-2\sin\left(2\arctan(ax)\right)\arctan(ax)+\cos\left(2\arctan(ax)\right)-1}{4\,a^3c^2\left(\arctan(ax)\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a^2*c*x^2+c)^2/arctan(a*x)^3,x)
```

```
[Out] 1/4/a^3/c^2*(4*Ci(2*arctan(a*x))*arctan(a*x)^2-2*sin(2*arctan(a*x))*arctan(
a*x)+cos(2*arctan(a*x))-1)/arctan(a*x)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{ax^2 + 2\,x\arctan(ax) + \frac{2\left(a^4c^2x^2+a^2c^2\right)\left(a^2\int \frac{x^2}{a^4x^4\arctan(ax)+2\,a^2x^2\arctan(ax)+\arctan(ax)}\,dx-\int \frac{1}{a^4x^4\arctan(ax)+2\,a^2x^2\arctan(ax)+\arctan(ax)}\,dx\right)\operatorname{arc}}{a^2c^2}}{2\left(a^4c^2x^2 + a^2c^2\right)\arctan(ax)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(2*(a^4*c^2*x^2 + a^2*c^2)*arctan(a*x)^2*integrate((a^2*x^2 - 1)/((a^6
*c^2*x^4 + 2*a^4*c^2*x^2 + a^2*c^2)*arctan(a*x)), x) + a*x^2 + 2*x*arctan(a
*x))/((a^4*c^2*x^2 + a^2*c^2)*arctan(a*x)^2)
```

Fricas [C] time = 1.74136, size = 327, normalized size = 4.61

$$-\frac{a^2x^2 - \left(a^2x^2+1\right)\arctan(ax)^2\operatorname{log\_integral}\left(-\frac{a^2x^2+2i\,ax-1}{a^2x^2+1}\right) - \left(a^2x^2+1\right)\arctan(ax)^2\operatorname{log\_integral}\left(-\frac{a^2x^2-2i\,ax-1}{a^2x^2+1}\right) + 2}{2\left(a^5c^2x^2 + a^3c^2\right)\arctan(ax)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(a^2*x^2 - (a^2*x^2 + 1)*arctan(a*x)^2*log_integral(-(a^2*x^2 + 2*I*a*
x - 1)/(a^2*x^2 + 1)) - (a^2*x^2 + 1)*arctan(a*x)^2*log_integral(-(a^2*x^2
- 2*I*a*x - 1)/(a^2*x^2 + 1)) + 2*a*x*arctan(a*x))/((a^5*c^2*x^2 + a^3*c^2)
*arctan(a*x)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^2}{a^4x^4\operatorname{atan}^3(ax)+2a^2x^2\operatorname{atan}^3(ax)+\operatorname{atan}^3(ax)}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a**2*c*x**2+c)**2/atan(a*x)**3,x)
```

```
[Out] Integral(x**2/(a**4*x**4*atan(a*x)**3 + 2*a**2*x**2*atan(a*x)**3 + atan(a*x
)**3), x)/c**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(a^2cx^2+c\right)^2 \arctan (ax)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(x^2/((a^2*c*x^2 + c)^2*arctan(a*x)^3), x)
```

3.629 $\int \frac{x}{\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^3}\,dx$

Optimal. Leaf size=81

$$-\frac{\mathrm{Si}\left(2\tan^{-1}(ax)\right)}{a^2c^2}-\frac{x}{2ac^2\left(a^2x^2+1\right)\tan^{-1}(ax)^2}-\frac{1-a^2x^2}{2a^2c^2\left(a^2x^2+1\right)\tan^{-1}(ax)}$$

```
[Out] -x/(2*a*c^2*(1 + a^2*x^2)*ArcTan[a*x]^2) - (1 - a^2*x^2)/(2*a^2*c^2*(1 + a^
2*x^2)*ArcTan[a*x]) - SinIntegral[2*ArcTan[a*x]]/(a^2*c^2)
```

Rubi [A] time = 0.119267, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4932, 4970, 4406, 12, 3299}

$$-\frac{\mathrm{Si}\left(2\tan^{-1}(ax)\right)}{a^2c^2}-\frac{x}{2ac^2\left(a^2x^2+1\right)\tan^{-1}(ax)^2}-\frac{1-a^2x^2}{2a^2c^2\left(a^2x^2+1\right)\tan^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[x/((c + a^2*c*x^2)^2*ArcTan[a*x]^3),x]
```

```
[Out] -x/(2*a*c^2*(1 + a^2*x^2)*ArcTan[a*x]^2) - (1 - a^2*x^2)/(2*a^2*c^2*(1 + a^
2*x^2)*ArcTan[a*x]) - SinIntegral[2*ArcTan[a*x]]/(a^2*c^2)
```

Rule 4932

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*(x_))/((d_) + (e_.)*(x_)^2)^2,
 x_Symbol] :> Simp[(x*(a + b*ArcTan[c*x])^(p + 1))/(b*c*d*(p + 1)*(d + e*x^
2)), x] + (-Dist[4/(b^2*(p + 1)*(p + 2)), Int[(x*(a + b*ArcTan[c*x])^(p + 2
))/(d + e*x^2)^2, x], x] - Simp[((1 - c^2*x^2)*(a + b*ArcTan[c*x])^(p + 2))
/(b^2*e*(p + 1)*(p + 2)*(d + e*x^2)), x]) /; FreeQ[{a, b, c, d, e}, x] && E
qQ[e, c^2*d] && LtQ[p, -1] && NeQ[p, -2]
```

Rule 4970

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sin[x]^m)/
Cos[x]^(m + 2*(q + 1)), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
```

```
|| GtQ[d, 0])
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\int \frac{x}{\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^3}\,dx = -\frac{x}{2ac^2\left(1+a^2x^2\right)\tan^{-1}(ax)^2}-\frac{1-a^2x^2}{2a^2c^2\left(1+a^2x^2\right)\tan^{-1}(ax)}-2\int \frac{x}{\left(c+a^2cx^2\right)^2\tan^{-1}(ax}$$

$$= -\frac{x}{2ac^2\left(1+a^2x^2\right)\tan^{-1}(ax)^2}-\frac{1-a^2x^2}{2a^2c^2\left(1+a^2x^2\right)\tan^{-1}(ax)}-\frac{2\,\text{Subst}\left(\int \frac{\cos(x)\sin(x)}{x}\,dx,x}{a^2c^2}$$

$$= -\frac{x}{2ac^2\left(1+a^2x^2\right)\tan^{-1}(ax)^2}-\frac{1-a^2x^2}{2a^2c^2\left(1+a^2x^2\right)\tan^{-1}(ax)}-\frac{2\,\text{Subst}\left(\int \frac{\sin(2x)}{2x}\,dx,x,\text{ta}}{a^2c^2}$$

$$= -\frac{x}{2ac^2\left(1+a^2x^2\right)\tan^{-1}(ax)^2}-\frac{1-a^2x^2}{2a^2c^2\left(1+a^2x^2\right)\tan^{-1}(ax)}-\frac{\text{Subst}\left(\int \frac{\sin(2x)}{x}\,dx,x,\tan}{a^2c^2}$$

$$= -\frac{x}{2ac^2\left(1+a^2x^2\right)\tan^{-1}(ax)^2}-\frac{1-a^2x^2}{2a^2c^2\left(1+a^2x^2\right)\tan^{-1}(ax)}-\frac{\text{Si}\left(2\tan^{-1}(ax)\right)}{a^2c^2}$$

Mathematica [A] time = 0.0524829, size = 70, normalized size = 0.86

$$\frac{-2\left(a^2x^2+1\right)\tan^{-1}(ax)^2\text{Si}\left(2\tan^{-1}(ax)\right)+\left(a^2x^2-1\right)\tan^{-1}(ax)-ax}{2a^2c^2\left(a^2x^2+1\right)\tan^{-1}(ax)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((c + a^2*c*x^2)^2*ArcTan[a*x]^3),x]
```

```
[Out] (-(a*x) + (-1 + a^2*x^2)*ArcTan[a*x] - 2*(1 + a^2*x^2)*ArcTan[a*x]^2*SinInt
egral[2*ArcTan[a*x]])/(2*a^2*c^2*(1 + a^2*x^2)*ArcTan[a*x]^2)
```

Maple [A] time = 0.065, size = 51, normalized size = 0.6

$$-\frac{4\,\mathrm{Si}\left(2\arctan\left(ax\right)\right)\left(\arctan\left(ax\right)\right)^{2}+2\cos\left(2\arctan\left(ax\right)\right)\arctan\left(ax\right)+\sin\left(2\arctan\left(ax\right)\right)}{4\,a^{2}c^{2}\left(\arctan\left(ax\right)\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^2*c*x^2+c)^2/arctan(a*x)^3,x)
```

```
[Out] -1/4/a^2/c^2*(4*Si(2*arctan(a*x))*arctan(a*x)^2+2*cos(2*arctan(a*x))*arctan
(a*x)+sin(2*arctan(a*x)))/arctan(a*x)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{4\left(a^{4}c^{2}x^{2}+a^{2}c^{2}\right)\arctan\left(ax\right)^{2}\int\frac{x}{\left(a^{4}c^{2}x^{4}+2\,a^{2}c^{2}x^{2}+c^{2}\right)\arctan\left(ax\right)}\,dx+ax-\left(a^{2}x^{2}-1\right)\arctan\left(ax\right)}{2\left(a^{4}c^{2}x^{2}+a^{2}c^{2}\right)\arctan\left(ax\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(8*(a^4*c^2*x^2 + a^2*c^2)*arctan(a*x)^2*integrate(1/2*x/((a^4*c^2*x^4
 + 2*a^2*c^2*x^2 + c^2)*arctan(a*x)), x) + a*x - (a^2*x^2 - 1)*arctan(a*x))
/((a^4*c^2*x^2 + a^2*c^2)*arctan(a*x)^2)
```

Fricas [C] time = 1.77001, size = 338, normalized size = 4.17

$$\frac{\left(-i\,a^{2}x^{2}-i\right)\arctan\left(ax\right)^{2}\log\_integral\left(-\frac{a^{2}x^{2}+2i\,ax-1}{a^{2}x^{2}+1}\right)+\left(i\,a^{2}x^{2}+i\right)\arctan\left(ax\right)^{2}\log\_integral\left(-\frac{a^{2}x^{2}-2i\,ax-1}{a^{2}x^{2}+1}\right)-ax+\left(a\right.}{2\left(a^{4}c^{2}x^{2}+a^{2}c^{2}\right)\arctan\left(ax\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] 1/2*((-I*a^2*x^2 - I)*arctan(a*x)^2*log_integral(-(a^2*x^2 + 2*I*a*x - 1)/(
a^2*x^2 + 1)) + (I*a^2*x^2 + I)*arctan(a*x)^2*log_integral(-(a^2*x^2 - 2*I*
a*x - 1)/(a^2*x^2 + 1)) - a*x + (a^2*x^2 - 1)*arctan(a*x))/((a^4*c^2*x^2 +
a^2*c^2)*arctan(a*x)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x}{a^4x^4\operatorname{atan}^3(ax)+2a^2x^2\operatorname{atan}^3(ax)+\operatorname{atan}^3(ax)}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a**2*c*x**2+c)**2/atan(a*x)**3,x)
```

```
[Out] Integral(x/(a**4*x**4*atan(a*x)**3 + 2*a**2*x**2*atan(a*x)**3 + atan(a*x)**
3), x)/c**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(a^2cx^2+c)^2\arctan(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(x/((a^2*c*x^2 + c)^2*arctan(a*x)^3), x)
```

3.630 $\int \frac{1}{(c+a^2cx^2)^2 \tan^{-1}(ax)^3} dx$

Optimal. Leaf size=65

$$\frac{x}{c^2 (a^2x^2+1) \tan^{-1}(ax)} - \frac{1}{2ac^2 (a^2x^2+1) \tan^{-1}(ax)^2} - \frac{\text{CosIntegral}(2\tan^{-1}(ax))}{ac^2}$$

```
[Out] -1/(2*a*c^2*(1 + a^2*x^2)*ArcTan[a*x]^2) + x/(c^2*(1 + a^2*x^2)*ArcTan[a*x]
) - CosIntegral[2*ArcTan[a*x]]/(a*c^2)
```

Rubi [A] time = 0.245925, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {4902, 4968, 4970, 3312, 3302, 4904}

$$\frac{x}{c^2 (a^2x^2+1) \tan^{-1}(ax)} - \frac{1}{2ac^2 (a^2x^2+1) \tan^{-1}(ax)^2} - \frac{\text{CosIntegral}(2\tan^{-1}(ax))}{ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a^2*c*x^2)^2*ArcTan[a*x]^3),x]
```

```
[Out] -1/(2*a*c^2*(1 + a^2*x^2)*ArcTan[a*x]^2) + x/(c^2*(1 + a^2*x^2)*ArcTan[a*x]
) - CosIntegral[2*ArcTan[a*x]]/(a*c^2)
```

Rule 4902

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^(p + 1))/(b*c*d*(p
+ 1)), x] - Dist[(2*c*(q + 1))/(b*(p + 1)), Int[x*(d + e*x^2)^q*(a + b*ArcT
an[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && L
tQ[q, -1] && LtQ[p, -1]
```

Rule 4968

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[(x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^(p
+ 1))/(b*c*d*(p + 1)), x] + (-Dist[(c*(m + 2*q + 2))/(b*(p + 1)), Int[x^(m
+ 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p + 1
)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; Fre
eQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &&
```

```
LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 4970

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sin[x]^m)/
Cos[x]^(m + 2*(q + 1)), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 4904

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{1}{\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^3}\,dx = -\frac{1}{2ac^2\left(1+a^2x^2\right)\tan^{-1}(ax)^2} - a\int \frac{x}{\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^2}\,dx$$

$$= -\frac{1}{2ac^2\left(1+a^2x^2\right)\tan^{-1}(ax)^2} + \frac{x}{c^2\left(1+a^2x^2\right)\tan^{-1}(ax)} + a^2\int \frac{x^2}{\left(c+a^2cx^2\right)^2\tan^{-1}(ax)}$$

$$= -\frac{1}{2ac^2\left(1+a^2x^2\right)\tan^{-1}(ax)^2} + \frac{x}{c^2\left(1+a^2x^2\right)\tan^{-1}(ax)} - \frac{\text{Subst}\left(\int \frac{\cos^2(x)}{x}\,dx, x, \tan^{-1}\right.}{ac^2}$$

$$= -\frac{1}{2ac^2\left(1+a^2x^2\right)\tan^{-1}(ax)^2} + \frac{x}{c^2\left(1+a^2x^2\right)\tan^{-1}(ax)} + \frac{\text{Subst}\left(\int \left(\frac{1}{2x} - \frac{\cos(2x)}{2x}\right)dx,\right.}{ac^2}$$

$$= -\frac{1}{2ac^2\left(1+a^2x^2\right)\tan^{-1}(ax)^2} + \frac{x}{c^2\left(1+a^2x^2\right)\tan^{-1}(ax)} - 2\frac{\text{Subst}\left(\int \frac{\cos(2x)}{x}\,dx, x, \tan^{-1}\right.}{2ac^2}$$

$$= -\frac{1}{2ac^2\left(1+a^2x^2\right)\tan^{-1}(ax)^2} + \frac{x}{c^2\left(1+a^2x^2\right)\tan^{-1}(ax)} - \frac{\text{Ci}\left(2\tan^{-1}(ax)\right)}{ac^2}$$

Mathematica [A] time = 0.0653508, size = 58, normalized size = 0.89

$$\frac{-2\left(a^2x^2+1\right)\tan^{-1}(ax)^2\text{CosIntegral}\left(2\tan^{-1}(ax)\right)+2ax\tan^{-1}(ax)-1}{2c^2\left(a^3x^2+a\right)\tan^{-1}(ax)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a^2*c*x^2)^2*ArcTan[a*x]^3),x]
```

```
[Out] (-1 + 2*a*x*ArcTan[a*x] - 2*(1 + a^2*x^2)*ArcTan[a*x]^2*CosIntegral[2*ArcTa
n[a*x]])/(2*c^2*(a + a^3*x^2)*ArcTan[a*x]^2)
```

Maple [A] time = 0.071, size = 52, normalized size = 0.8

$$-\frac{4\,\text{Ci}\,(2\,\arctan(ax))\,(\arctan(ax))^2 - 2\,\sin(2\,\arctan(ax))\arctan(ax) + \cos(2\,\arctan(ax)) + 1}{4\,ac^2\,(\arctan(ax))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2*c*x^2+c)^2/arctan(a*x)^3,x)
```

```
[Out] -1/4/a/c^2*(4*Ci(2*arctan(a*x))*arctan(a*x)^2-2*sin(2*arctan(a*x))*arctan(a
*x)+cos(2*arctan(a*x))+1)/arctan(a*x)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{2\,ax\arctan(ax) + \frac{2\left(a^3c^2x^2+ac^2\right)\left(a^2\int\frac{x^2}{a^4x^4\arctan(ax)+2\,a^2x^2\arctan(ax)+\arctan(ax)}\,dx-\int\frac{1}{a^4x^4\arctan(ax)+2\,a^2x^2\arctan(ax)+\arctan(ax)}\,dx\right)\arctan(ax)^2}{c^2} - }{2\left(a^3c^2x^2+ac^2\right)\arctan(ax)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(a^3*c^2*x^2 + a*c^2)*arctan(a*x)^2*integrate((a^2*x^2 - 1)/((a^4*c^
2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(a*x)), x) + 2*a*x*arctan(a*x) - 1)/((a^
3*c^2*x^2 + a*c^2)*arctan(a*x)^2)
```

Fricas [C] time = 1.74219, size = 316, normalized size = 4.86

$$-\frac{\left(a^2x^2+1\right)\arctan(ax)^2\,\mathrm{log\_integral}\left(-\frac{a^2x^2+2i\,ax-1}{a^2x^2+1}\right) + \left(a^2x^2+1\right)\arctan(ax)^2\,\mathrm{log\_integral}\left(-\frac{a^2x^2-2i\,ax-1}{a^2x^2+1}\right) - 2\,ax\,\mathrm{arcta}}{2\left(a^3c^2x^2+ac^2\right)\arctan(ax)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] -1/2*((a^2*x^2 + 1)*arctan(a*x)^2*log_integral(-(a^2*x^2 + 2*I*a*x - 1)/(a^
2*x^2 + 1)) + (a^2*x^2 + 1)*arctan(a*x)^2*log_integral(-(a^2*x^2 - 2*I*a*x
- 1)/(a^2*x^2 + 1)) - 2*a*x*arctan(a*x) + 1)/((a^3*c^2*x^2 + a*c^2)*arctan(
a*x)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{1}{a^4x^4\,\mathrm{atan}^3(ax)+2a^2x^2\,\mathrm{atan}^3(ax)+\mathrm{atan}^3(ax)}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*c*x**2+c)**2/atan(a*x)**3,x)
```

```
[Out] Integral(1/(a**4*x**4*atan(a*x)**3 + 2*a**2*x**2*atan(a*x)**3 + atan(a*x)**
3), x)/c**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)^2 \arctan\left(ax\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^2*arctan(a*x)^3), x)
```

3.631 $\int \frac{1}{x(c+a^2cx^2)^2 \tan^{-1}(ax)^3} dx$

Optimal. Leaf size=113

$$-\frac{\text{Unintegrable}\left(\frac{1}{x^2\tan^{-1}(ax)^2},x\right)}{2ac^2}+\frac{ax}{2c^2\left(a^2x^2+1\right)\tan^{-1}(ax)^2}+\frac{1-a^2x^2}{2c^2\left(a^2x^2+1\right)\tan^{-1}(ax)}+\frac{\text{Si}\left(2\tan^{-1}(ax)\right)}{c^2}-\frac{1}{2ac^2x\tan$$

```
[Out] -1/(2*a*c^2*x*ArcTan[a*x]^2) + (a*x)/(2*c^2*(1 + a^2*x^2)*ArcTan[a*x]^2) +
(1 - a^2*x^2)/(2*c^2*(1 + a^2*x^2)*ArcTan[a*x]) + SinIntegral[2*ArcTan[a*x]
]/c^2 - Unintegrable[1/(x^2*ArcTan[a*x]^2), x]/(2*a*c^2)
```

Rubi [A] time = 0.259754, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*(c + a^2*c*x^2)^2*ArcTan[a*x]^3),x]
```

```
[Out] -1/(2*a*c^2*x*ArcTan[a*x]^2) + (a*x)/(2*c^2*(1 + a^2*x^2)*ArcTan[a*x]^2) +
(1 - a^2*x^2)/(2*c^2*(1 + a^2*x^2)*ArcTan[a*x]) + SinIntegral[2*ArcTan[a*x]
]/c^2 - Defer[Int][1/(x^2*ArcTan[a*x]^2), x]/(2*a*c^2)
```

Rubi steps

$$\int \frac{1}{x\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^3}\,dx = -\left(a^2\int \frac{x}{\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^3}\,dx\right)+\frac{\int \frac{1}{x\left(c+a^2cx^2\right)\tan^{-1}(ax)^3}\,dx}{c}$$

$$= -\frac{1}{2ac^2x\tan^{-1}(ax)^2}+\frac{ax}{2c^2\left(1+a^2x^2\right)\tan^{-1}(ax)^2}+\frac{1-a^2x^2}{2c^2\left(1+a^2x^2\right)\tan^{-1}(ax)}+\left(2a^2\right)$$

$$= -\frac{1}{2ac^2x\tan^{-1}(ax)^2}+\frac{ax}{2c^2\left(1+a^2x^2\right)\tan^{-1}(ax)^2}+\frac{1-a^2x^2}{2c^2\left(1+a^2x^2\right)\tan^{-1}(ax)}+\frac{2\,\mathrm{Sub}}{}$$

$$= -\frac{1}{2ac^2x\tan^{-1}(ax)^2}+\frac{ax}{2c^2\left(1+a^2x^2\right)\tan^{-1}(ax)^2}+\frac{1-a^2x^2}{2c^2\left(1+a^2x^2\right)\tan^{-1}(ax)}+\frac{2\,\mathrm{Sub}}{}$$

$$= -\frac{1}{2ac^2x\tan^{-1}(ax)^2}+\frac{ax}{2c^2\left(1+a^2x^2\right)\tan^{-1}(ax)^2}+\frac{1-a^2x^2}{2c^2\left(1+a^2x^2\right)\tan^{-1}(ax)}+\frac{\mathrm{Subst}}{}$$

$$= -\frac{1}{2ac^2x\tan^{-1}(ax)^2}+\frac{ax}{2c^2\left(1+a^2x^2\right)\tan^{-1}(ax)^2}+\frac{1-a^2x^2}{2c^2\left(1+a^2x^2\right)\tan^{-1}(ax)}+\frac{\mathrm{Si}\left(2\,\mathrm{t}\right.}{}$$

Mathematica [A] time = 1.77278, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*(c + a^2*c*x^2)^2*ArcTan[a*x]^3),x]
```

```
[Out] Integrate[1/(x*(c + a^2*c*x^2)^2*ArcTan[a*x]^3), x]
```

Maple [A] time = 0.275, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(a^2cx^2+c\right)^2(\arctan(ax))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^3,x)
```

```
[Out] int(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{-ax+\left(3\,a^2x^2+1\right)\arctan\left(ax\right)+\frac{2\left(a^4c^2x^4+a^2c^2x^2\right)\bigg(3\,a^4\int\frac{x^4}{a^4x^7\arctan(ax)+2\,a^2x^5\arctan(ax)+x^3\arctan(ax)}\,dx+2\,a^2\int\frac{x^2}{a^4x^7\arctan(ax)+2\,a^2x^5\arctan(ax)+\cdots}}{a^2c^2}}{2\left(a^4c^2x^4+a^2c^2x^2\right)\arctan\left(ax\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(a^4*c^2*x^4 + a^2*c^2*x^2)*arctan(a*x)^2*integrate((3*a^4*x^4 + 2*a
^2*x^2 + 1)/((a^6*c^2*x^7 + 2*a^4*c^2*x^5 + a^2*c^2*x^3)*arctan(a*x)), x) -
 a*x + (3*a^2*x^2 + 1)*arctan(a*x))/((a^4*c^2*x^4 + a^2*c^2*x^2)*arctan(a*x
)^2)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\left(a^4c^2x^5+2\,a^2c^2x^3+c^2x\right)\arctan\left(ax\right)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(1/((a^4*c^2*x^5 + 2*a^2*c^2*x^3 + c^2*x)*arctan(a*x)^3), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{1}{a^4x^5\operatorname{atan}^3(ax)+2a^2x^3\operatorname{atan}^3(ax)+x\operatorname{atan}^3(ax)}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a**2*c*x**2+c)**2/atan(a*x)**3,x)
```

```
[Out] Integral(1/(a**4*x**5*atan(a*x)**3 + 2*a**2*x**3*atan(a*x)**3 + x*atan(a*x)
**3), x)/c**2
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)^2 x \arctan(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^2*x*arctan(a*x)^3), x)
```

3.632 $\int \frac{1}{x^2(c+a^2cx^2)^2 \tan^{-1}(ax)^3} dx$

Optimal. Leaf size=103

$$-\frac{\text{Unintegrable}\left(\frac{1}{x^3\tan^{-1}(ax)^2},x\right)}{ac^2}-\frac{a^2x}{c^2\left(a^2x^2+1\right)\tan^{-1}(ax)}+\frac{a}{2c^2\left(a^2x^2+1\right)\tan^{-1}(ax)^2}+\frac{a\text{CosIntegral}\left(2\tan^{-1}(ax)\right)}{c^2}$$

```
[Out] -1/(2*a*c^2*x^2*ArcTan[a*x]^2) + a/(2*c^2*(1 + a^2*x^2)*ArcTan[a*x]^2) - (a
^2*x)/(c^2*(1 + a^2*x^2)*ArcTan[a*x]) + (a*CosIntegral[2*ArcTan[a*x]])/c^2
- Unintegrable[1/(x^3*ArcTan[a*x]^2), x]/(a*c^2)
```

Rubi [A] time = 0.390234, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]^3),x]
```

```
[Out] -1/(2*a*c^2*x^2*ArcTan[a*x]^2) + a/(2*c^2*(1 + a^2*x^2)*ArcTan[a*x]^2) - (a
^2*x)/(c^2*(1 + a^2*x^2)*ArcTan[a*x]) + (a*CosIntegral[2*ArcTan[a*x]])/c^2
- Defer[Int][1/(x^3*ArcTan[a*x]^2), x]/(a*c^2)
```

Rubi steps

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^3}\,dx = -\left(a^2\int \frac{1}{\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^3}\,dx\right)+\frac{\int \frac{1}{x^2\left(c+a^2cx^2\right)\tan^{-1}(ax)^3}\,dx}{c}$$

$$= -\frac{1}{2ac^2x^2\tan^{-1}(ax)^2}+\frac{a}{2c^2\left(1+a^2x^2\right)\tan^{-1}(ax)^2}+a^3\int \frac{x}{\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^2}\,dx$$

$$= -\frac{1}{2ac^2x^2\tan^{-1}(ax)^2}+\frac{a}{2c^2\left(1+a^2x^2\right)\tan^{-1}(ax)^2}-\frac{a^2x}{c^2\left(1+a^2x^2\right)\tan^{-1}(ax)}+a^2\int$$

$$= -\frac{1}{2ac^2x^2\tan^{-1}(ax)^2}+\frac{a}{2c^2\left(1+a^2x^2\right)\tan^{-1}(ax)^2}-\frac{a^2x}{c^2\left(1+a^2x^2\right)\tan^{-1}(ax)}-\frac{\int \frac{}{x^3 t}}{}$$

$$= -\frac{1}{2ac^2x^2\tan^{-1}(ax)^2}+\frac{a}{2c^2\left(1+a^2x^2\right)\tan^{-1}(ax)^2}-\frac{a^2x}{c^2\left(1+a^2x^2\right)\tan^{-1}(ax)}-\frac{\int \frac{}{x^3 t}}{}$$

$$= -\frac{1}{2ac^2x^2\tan^{-1}(ax)^2}+\frac{a}{2c^2\left(1+a^2x^2\right)\tan^{-1}(ax)^2}-\frac{a^2x}{c^2\left(1+a^2x^2\right)\tan^{-1}(ax)}-\frac{\int \frac{}{x^3 t}}{}$$

$$= -\frac{1}{2ac^2x^2\tan^{-1}(ax)^2}+\frac{a}{2c^2\left(1+a^2x^2\right)\tan^{-1}(ax)^2}-\frac{a^2x}{c^2\left(1+a^2x^2\right)\tan^{-1}(ax)}+\frac{a\mathrm{Ci}(}{}$$

Mathematica [A] time = 2.52671, size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]^3),x]
```

```
[Out] Integrate[1/(x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]^3), x]
```

Maple [A] time = 0.319, size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(a^2cx^2+c\right)^2(\arctan(ax))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a^2*c*x^2+c)^2/arctan(a*x)^3,x)
```

```
[Out] int(1/x^2/(a^2*c*x^2+c)^2/arctan(a*x)^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{-ax+2\left(2\,a^2x^2+1\right)\arctan\left(ax\right)+\frac{2\left(a^4c^2x^5+a^2c^2x^3\right)\left(6\,a^4\int\frac{x^4}{a^4x^8\arctan(ax)+2\,a^2x^6\arctan(ax)+x^4\arctan(ax)}\,dx+7\,a^2\int\frac{x^2}{a^4x^8\arctan(ax)+2\,a^2x^6\arctan(a}\right.}{a^2c^2}}{2\left(a^4c^2x^5+a^2c^2x^3\right)\arctan\left(ax\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(a^4*c^2*x^5 + a^2*c^2*x^3)*arctan(a*x)^2*integrate((6*a^4*x^4 + 7*a
^2*x^2 + 3)/((a^6*c^2*x^8 + 2*a^4*c^2*x^6 + a^2*c^2*x^4)*arctan(a*x)), x) -
 a*x + 2*(2*a^2*x^2 + 1)*arctan(a*x))/((a^4*c^2*x^5 + a^2*c^2*x^3)*arctan(a
*x)^2)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\left(a^4c^2x^6+2\,a^2c^2x^4+c^2x^2\right)\arctan\left(ax\right)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(1/((a^4*c^2*x^6 + 2*a^2*c^2*x^4 + c^2*x^2)*arctan(a*x)^3), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{1}{a^4x^6\operatorname{atan}^3(ax)+2a^2x^4\operatorname{atan}^3(ax)+x^2\operatorname{atan}^3(ax)}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a**2*c*x**2+c)**2/atan(a*x)**3,x)
```

```
[Out] Integral(1/(a**4*x**6*atan(a*x)**3 + 2*a**2*x**4*atan(a*x)**3 + x**2*atan(a
*x)**3), x)/c**2
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)^2x^2\arctan\left(ax\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^2*x^2*arctan(a*x)^3), x)
```

3.633 $\int \frac{1}{x^3(c+a^2cx^2)^2\tan^{-1}(ax)^3}\,dx$

Optimal. Leaf size=159

$$\frac{a\text{Unintegrable}\left(\frac{1}{x^2\tan^{-1}(ax)^2},x\right)}{2c^2}-\frac{3\text{Unintegrable}\left(\frac{1}{x^4\tan^{-1}(ax)^2},x\right)}{2ac^2}-\frac{a^2\text{Si}\left(2\tan^{-1}(ax)\right)}{c^2}-\frac{a^3x}{2c^2\left(a^2x^2+1\right)\tan^{-1}(ax)^2}-$$

```
[Out] -1/(2*a*c^2*x^3*ArcTan[a*x]^2) + a/(2*c^2*x*ArcTan[a*x]^2) - (a^3*x)/(2*c^2
*(1 + a^2*x^2)*ArcTan[a*x]^2) - (a^2*(1 - a^2*x^2))/(2*c^2*(1 + a^2*x^2)*Ar
cTan[a*x]) - (a^2*SinIntegral[2*ArcTan[a*x]])/c^2 - (3*Unintegrable[1/(x^4*
ArcTan[a*x]^2), x])/(2*a*c^2) + (a*Unintegrable[1/(x^2*ArcTan[a*x]^2), x])/
(2*c^2)
```

Rubi [A] time = 0.407787, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^3\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^3*(c + a^2*c*x^2)^2*ArcTan[a*x]^3),x]
```

```
[Out] -1/(2*a*c^2*x^3*ArcTan[a*x]^2) + a/(2*c^2*x*ArcTan[a*x]^2) - (a^3*x)/(2*c^2
*(1 + a^2*x^2)*ArcTan[a*x]^2) - (a^2*(1 - a^2*x^2))/(2*c^2*(1 + a^2*x^2)*Ar
cTan[a*x]) - (a^2*SinIntegral[2*ArcTan[a*x]])/c^2 - (3*Defer[Int][1/(x^4*Ar
cTan[a*x]^2), x])/(2*a*c^2) + (a*Defer[Int][1/(x^2*ArcTan[a*x]^2), x])/(2*c
^2)
```

Rubi steps

$$\int \frac{1}{x^3\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^3}\,dx = -\left(a^2\int \frac{1}{x\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^3}\,dx\right)+\frac{\int \frac{1}{x^3\left(c+a^2cx^2\right)\tan^{-1}(ax)^3}\,dx}{c}$$

$$= -\frac{1}{2ac^2x^3\tan^{-1}(ax)^2}+a^4\int \frac{x}{\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^3}\,dx-\frac{3\int \frac{1}{x^4\tan^{-1}(ax)^2}\,dx}{2ac^2}-\frac{a^2\int}{}$$

$$= -\frac{1}{2ac^2x^3\tan^{-1}(ax)^2}+\frac{a}{2c^2x\tan^{-1}(ax)^2}-\frac{a^3x}{2c^2\left(1+a^2x^2\right)\tan^{-1}(ax)^2}-\frac{a^2\left(1-\right.}{2c^2\left(1+a^2x^2\right.}$$

$$= -\frac{1}{2ac^2x^3\tan^{-1}(ax)^2}+\frac{a}{2c^2x\tan^{-1}(ax)^2}-\frac{a^3x}{2c^2\left(1+a^2x^2\right)\tan^{-1}(ax)^2}-\frac{a^2\left(1-\right.}{2c^2\left(1+a^2x^2\right.}$$

$$= -\frac{1}{2ac^2x^3\tan^{-1}(ax)^2}+\frac{a}{2c^2x\tan^{-1}(ax)^2}-\frac{a^3x}{2c^2\left(1+a^2x^2\right)\tan^{-1}(ax)^2}-\frac{a^2\left(1-\right.}{2c^2\left(1+a^2x^2\right.}$$

$$= -\frac{1}{2ac^2x^3\tan^{-1}(ax)^2}+\frac{a}{2c^2x\tan^{-1}(ax)^2}-\frac{a^3x}{2c^2\left(1+a^2x^2\right)\tan^{-1}(ax)^2}-\frac{a^2\left(1-\right.}{2c^2\left(1+a^2x^2\right.}$$

$$= -\frac{1}{2ac^2x^3\tan^{-1}(ax)^2}+\frac{a}{2c^2x\tan^{-1}(ax)^2}-\frac{a^3x}{2c^2\left(1+a^2x^2\right)\tan^{-1}(ax)^2}-\frac{a^2\left(1-\right.}{2c^2\left(1+a^2x^2\right.}$$

Mathematica [A] time = 2.06653, size = 0, normalized size = 0.

$$\int \frac{1}{x^3\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^3*(c + a^2*c*x^2)^2*ArcTan[a*x]^3),x]
```

```
[Out] Integrate[1/(x^3*(c + a^2*c*x^2)^2*ArcTan[a*x]^3), x]
```

Maple [A] time = 1.105, size = 0, normalized size = 0.

$$\int \frac{1}{x^3\left(a^2cx^2+c\right)^2(\arctan(ax))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a^2*c*x^2+c)^2/arctan(a*x)^3,x)
```

```
[Out] int(1/x^3/(a^2*c*x^2+c)^2/arctan(a*x)^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{4\left(a^{4}c^{2}x^{6}+a^{2}c^{2}x^{4}\right)\arctan\left(ax\right)^{2}\int\frac{5\,a^{4}x^{4}+7\,a^{2}x^{2}+3}{\left(a^{6}c^{2}x^{9}+2\,a^{4}c^{2}x^{7}+a^{2}c^{2}x^{5}\right)\arctan(ax)}\,dx-ax+\left(5\,a^{2}x^{2}+3\right)\arctan\left(ax\right)}{2\left(a^{4}c^{2}x^{6}+a^{2}c^{2}x^{4}\right)\arctan\left(ax\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(a^4*c^2*x^6 + a^2*c^2*x^4)*arctan(a*x)^2*integrate(2*(5*a^4*x^4 + 7
*a^2*x^2 + 3)/((a^6*c^2*x^9 + 2*a^4*c^2*x^7 + a^2*c^2*x^5)*arctan(a*x)), x)
 - a*x + (5*a^2*x^2 + 3)*arctan(a*x))/((a^4*c^2*x^6 + a^2*c^2*x^4)*arctan(a
*x)^2)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{1}{\left(a^{4}c^{2}x^{7}+2\,a^{2}c^{2}x^{5}+c^{2}x^{3}\right)\arctan\left(ax\right)^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(1/((a^4*c^2*x^7 + 2*a^2*c^2*x^5 + c^2*x^3)*arctan(a*x)^3), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{1}{a^{4}x^{7}\operatorname{atan}^{3}\left(ax\right)+2a^{2}x^{5}\operatorname{atan}^{3}\left(ax\right)+x^{3}\operatorname{atan}^{3}\left(ax\right)}\,dx}{c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a**2*c*x**2+c)**2/atan(a*x)**3,x)
```

```
[Out] Integral(1/(a**4*x**7*atan(a*x)**3 + 2*a**2*x**5*atan(a*x)**3 + x**3*atan(a
*x)**3), x)/c**2
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)^2x^3\arctan\left(ax\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^2*x^3*arctan(a*x)^3), x)
```

3.634 $\int \frac{1}{x^4(c+a^2cx^2)^2\tan^{-1}(ax)^3}\,dx$

Optimal. Leaf size=141

$$\frac{a\text{Unintegrable}\left(\frac{1}{x^3\tan^{-1}(ax)^2},x\right)}{c^2}-\frac{2\text{Unintegrable}\left(\frac{1}{x^5\tan^{-1}(ax)^2},x\right)}{ac^2}-\frac{a^3\text{CosIntegral}\left(2\tan^{-1}(ax)\right)}{c^2}+\frac{a^4x}{c^2\left(a^2x^2+1\right)\tan^{-}}$$

```
[Out] -1/(2*a*c^2*x^4*ArcTan[a*x]^2) + a/(2*c^2*x^2*ArcTan[a*x]^2) - a^3/(2*c^2*(
1 + a^2*x^2)*ArcTan[a*x]^2) + (a^4*x)/(c^2*(1 + a^2*x^2)*ArcTan[a*x]) - (a^
3*CosIntegral[2*ArcTan[a*x]])/c^2 - (2*Unintegrable[1/(x^5*ArcTan[a*x]^2),
x])/(a*c^2) + (a*Unintegrable[1/(x^3*ArcTan[a*x]^2), x])/c^2
```

Rubi [A] time = 0.546747, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^4\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^4*(c + a^2*c*x^2)^2*ArcTan[a*x]^3),x]
```

```
[Out] -1/(2*a*c^2*x^4*ArcTan[a*x]^2) + a/(2*c^2*x^2*ArcTan[a*x]^2) - a^3/(2*c^2*(
1 + a^2*x^2)*ArcTan[a*x]^2) + (a^4*x)/(c^2*(1 + a^2*x^2)*ArcTan[a*x]) - (a^
3*CosIntegral[2*ArcTan[a*x]])/c^2 - (2*Defer[Int][1/(x^5*ArcTan[a*x]^2), x]
)/(a*c^2) + (a*Defer[Int][1/(x^3*ArcTan[a*x]^2), x])/c^2
```

Rubi steps

$$\int \frac{1}{x^4\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^3}\,dx = -\left(a^2\int \frac{1}{x^2\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^3}\,dx\right)+\frac{\int \frac{1}{x^4\left(c+a^2cx^2\right)\tan^{-1}(ax)^3}\,dx}{c}$$

$$= -\frac{1}{2ac^2x^4\tan^{-1}(ax)^2}+a^4\int \frac{1}{\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^3}\,dx-\frac{2\int \frac{1}{x^5\tan^{-1}(ax)^2}\,dx}{ac^2}-\frac{a^2\int}{}$$

$$= -\frac{1}{2ac^2x^4\tan^{-1}(ax)^2}+\frac{a}{2c^2x^2\tan^{-1}(ax)^2}-\frac{a^3}{2c^2\left(1+a^2x^2\right)\tan^{-1}(ax)^2}-a^5\int \frac{}{\left(c+a\right.}$$

$$= -\frac{1}{2ac^2x^4\tan^{-1}(ax)^2}+\frac{a}{2c^2x^2\tan^{-1}(ax)^2}-\frac{a^3}{2c^2\left(1+a^2x^2\right)\tan^{-1}(ax)^2}+\frac{a^4}{c^2\left(1+a^2x^2\right.}$$

$$= -\frac{1}{2ac^2x^4\tan^{-1}(ax)^2}+\frac{a}{2c^2x^2\tan^{-1}(ax)^2}-\frac{a^3}{2c^2\left(1+a^2x^2\right)\tan^{-1}(ax)^2}+\frac{a^4}{c^2\left(1+a^2x^2\right.}$$

$$= -\frac{1}{2ac^2x^4\tan^{-1}(ax)^2}+\frac{a}{2c^2x^2\tan^{-1}(ax)^2}-\frac{a^3}{2c^2\left(1+a^2x^2\right)\tan^{-1}(ax)^2}+\frac{a^4}{c^2\left(1+a^2x^2\right.}$$

$$= -\frac{1}{2ac^2x^4\tan^{-1}(ax)^2}+\frac{a}{2c^2x^2\tan^{-1}(ax)^2}-\frac{a^3}{2c^2\left(1+a^2x^2\right)\tan^{-1}(ax)^2}+\frac{a^4}{c^2\left(1+a^2x^2\right.}$$

$$= -\frac{1}{2ac^2x^4\tan^{-1}(ax)^2}+\frac{a}{2c^2x^2\tan^{-1}(ax)^2}-\frac{a^3}{2c^2\left(1+a^2x^2\right)\tan^{-1}(ax)^2}+\frac{a^4}{c^2\left(1+a^2x^2\right.}$$

Mathematica [A] time = 6.36486, size = 0, normalized size = 0.

$$\int \frac{1}{x^4\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^4*(c + a^2*c*x^2)^2*ArcTan[a*x]^3),x]
```

```
[Out] Integrate[1/(x^4*(c + a^2*c*x^2)^2*ArcTan[a*x]^3), x]
```

Maple [A] time = 1.066, size = 0, normalized size = 0.

$$\int \frac{1}{x^4\left(a^2cx^2+c\right)^2 \left(\arctan\left(ax\right)\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(a^2*c*x^2+c)^2/arctan(a*x)^3,x)
```

```
[Out] int(1/x^4/(a^2*c*x^2+c)^2/arctan(a*x)^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{-ax+2\left(3\,a^2x^2+2\right)\arctan\left(ax\right)+\frac{2\left(a^4c^2x^7+a^2c^2x^5\right)\left(15\,a^4\int \frac{x^4}{a^4x^{10}\arctan(ax)+2\,a^2x^8\arctan(ax)+x^6\arctan(ax)}\,dx+23\,a^2\int \frac{x^2}{a^4x^{10}\arctan(ax)+2\,a^2x^8\,\mathrm{arct}\cdots}\right.}{a^2c^2}}{2\left(a^4c^2x^7+a^2c^2x^5\right)\arctan\left(ax\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(a^4*c^2*x^7 + a^2*c^2*x^5)*arctan(a*x)^2*integrate((15*a^4*x^4 + 23
*a^2*x^2 + 10)/((a^6*c^2*x^10 + 2*a^4*c^2*x^8 + a^2*c^2*x^6)*arctan(a*x)),
x) - a*x + 2*(3*a^2*x^2 + 2)*arctan(a*x))/((a^4*c^2*x^7 + a^2*c^2*x^5)*arct
an(a*x)^2)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\left(a^4c^2x^8+2\,a^2c^2x^6+c^2x^4\right)\arctan\left(ax\right)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(1/((a^4*c^2*x^8 + 2*a^2*c^2*x^6 + c^2*x^4)*arctan(a*x)^3), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{a^4x^8 \operatorname{atan}^3(ax)+2a^2x^6 \operatorname{atan}^3(ax)+x^4 \operatorname{atan}^3(ax)}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(a**2*c*x**2+c)**2/atan(a*x)**3,x)
```

```
[Out] Integral(1/(a**4*x**8*atan(a*x)**3 + 2*a**2*x**6*atan(a*x)**3 + x**4*atan(a
*x)**3), x)/c**2
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)^2 x^4 \arctan(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^2*x^4*arctan(a*x)^3), x)
```

3.635 $\int \frac{x^3}{(c+a^2cx^2)^3 \tan^{-1}(ax)^3} dx$

Optimal. Leaf size=177

$$-\frac{\text{Si}\left(2\tan^{-1}(ax)\right)}{2a^4c^3}+\frac{\text{Si}\left(4\tan^{-1}(ax)\right)}{a^4c^3}-\frac{x}{2a^3c^3\left(a^2x^2+1\right)\tan^{-1}(ax)^2}+\frac{x}{2a^3c^3\left(a^2x^2+1\right)^2\tan^{-1}(ax)^2}-\frac{1-a^2x^2}{2a^4c^3\left(a^2x^2+1\right)\tan^{-1}(ax)}$$

```
[Out] x/(2*a^3*c^3*(1 + a^2*x^2)^2*ArcTan[a*x]^2) - x/(2*a^3*c^3*(1 + a^2*x^2)*Ar
cTan[a*x]^2) + 2/(a^4*c^3*(1 + a^2*x^2)^2*ArcTan[a*x]) - 3/(2*a^4*c^3*(1 +
a^2*x^2)*ArcTan[a*x]) - (1 - a^2*x^2)/(2*a^4*c^3*(1 + a^2*x^2)*ArcTan[a*x])
 - SinIntegral[2*ArcTan[a*x]]/(2*a^4*c^3) + SinIntegral[4*ArcTan[a*x]]/(a^4
*c^3)
```

Rubi [A] time = 0.642054, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 25, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {4964, 4932, 4970, 4406, 12, 3299, 4968, 4902}

$$-\frac{\text{Si}\left(2\tan^{-1}(ax)\right)}{2a^4c^3}+\frac{\text{Si}\left(4\tan^{-1}(ax)\right)}{a^4c^3}-\frac{x}{2a^3c^3\left(a^2x^2+1\right)\tan^{-1}(ax)^2}+\frac{x}{2a^3c^3\left(a^2x^2+1\right)^2\tan^{-1}(ax)^2}-\frac{1-a^2x^2}{2a^4c^3\left(a^2x^2+1\right)\tan^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((c + a^2*c*x^2)^3*ArcTan[a*x]^3),x]
```


```
[Out] x/(2*a^3*c^3*(1 + a^2*x^2)^2*ArcTan[a*x]^2) - x/(2*a^3*c^3*(1 + a^2*x^2)*Ar
cTan[a*x]^2) + 2/(a^4*c^3*(1 + a^2*x^2)^2*ArcTan[a*x]) - 3/(2*a^4*c^3*(1 +
a^2*x^2)*ArcTan[a*x]) - (1 - a^2*x^2)/(2*a^4*c^3*(1 + a^2*x^2)*ArcTan[a*x])
 - SinIntegral[2*ArcTan[a*x]]/(2*a^4*c^3) + SinIntegral[4*ArcTan[a*x]]/(a^4
*c^3)
```


Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Arc
Tan[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan[c
*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p
, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rule 4932

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*(x_))/((d_) + (e_.)*(x_)^2)^2,
 x_Symbol] :> Simp[(x*(a + b*ArcTan[c*x])^(p + 1))/(b*c*d*(p + 1)*(d + e*x^
2)), x] + (-Dist[4/(b^2*(p + 1)*(p + 2)), Int[(x*(a + b*ArcTan[c*x])^(p + 2
))/(d + e*x^2)^2, x], x] - Simp[((1 - c^2*x^2)*(a + b*ArcTan[c*x])^(p + 2))
/(b^2*e*(p + 1)*(p + 2)*(d + e*x^2)), x]) /; FreeQ[{a, b, c, d, e}, x] && E
qQ[e, c^2*d] && LtQ[p, -1] && NeQ[p, -2]
```

Rule 4970

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sin[x]^m)/
Cos[x]^(m + 2*(q + 1)), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 4968

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[(x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^(p
+ 1))/(b*c*d*(p + 1)), x] + (-Dist[(c*(m + 2*q + 2))/(b*(p + 1)), Int[x^(m
+ 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p + 1
)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; Fre
eQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &&
LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 4902

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^(p + 1))/(b*c*d*(p
+ 1)), x] - Dist[(2*c*(q + 1))/(b*(p + 1)), Int[x*(d + e*x^2)^q*(a + b*ArcT
an[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && L
tQ[q, -1] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{x^3}{\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^3}\,dx = -\frac{\int \frac{x}{\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^3}\,dx}{a^2}+\frac{\int \frac{x}{\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^3}\,dx}{a^2c}$$

$$= \frac{x}{2a^3c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)^2}-\frac{x}{2a^3c^3\left(1+a^2x^2\right)\tan^{-1}(ax)^2}-\frac{1-a^2x^2}{2a^4c^3\left(1+a^2x^2\right)\tan^{-1}(a$$

$$= \frac{x}{2a^3c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)^2}-\frac{x}{2a^3c^3\left(1+a^2x^2\right)\tan^{-1}(ax)^2}+\frac{1}{2a^4c^3\left(1+a^2x^2\right)^2\tan^{-1}($$

$$= \frac{x}{2a^3c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)^2}-\frac{x}{2a^3c^3\left(1+a^2x^2\right)\tan^{-1}(ax)^2}+\frac{2}{a^4c^3\left(1+a^2x^2\right)^2\tan^{-1}(a$$

$$= \frac{x}{2a^3c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)^2}-\frac{x}{2a^3c^3\left(1+a^2x^2\right)\tan^{-1}(ax)^2}+\frac{2}{a^4c^3\left(1+a^2x^2\right)^2\tan^{-1}(a$$

$$= \frac{x}{2a^3c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)^2}-\frac{x}{2a^3c^3\left(1+a^2x^2\right)\tan^{-1}(ax)^2}+\frac{2}{a^4c^3\left(1+a^2x^2\right)^2\tan^{-1}(a$$

$$= \frac{x}{2a^3c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)^2}-\frac{x}{2a^3c^3\left(1+a^2x^2\right)\tan^{-1}(ax)^2}+\frac{2}{a^4c^3\left(1+a^2x^2\right)^2\tan^{-1}(a$$

$$= \frac{x}{2a^3c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)^2}-\frac{x}{2a^3c^3\left(1+a^2x^2\right)\tan^{-1}(ax)^2}+\frac{2}{a^4c^3\left(1+a^2x^2\right)^2\tan^{-1}(a$$

Mathematica [A] time = 0.233105, size = 72, normalized size = 0.41

$$\frac{\frac{a^2x^2\left(\left(a^2x^2-3\right)\tan^{-1}(ax)-ax\right)}{\left(a^2x^2+1\right)^2\tan^{-1}(ax)^2}-\text{Si}\left(2\tan^{-1}(ax)\right)+2\text{Si}\left(4\tan^{-1}(ax)\right)}{2a^4c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((c + a^2*c*x^2)^3*ArcTan[a*x]^3),x]
```

```
[Out] ((a^2*x^2*(-(a*x) + (-3 + a^2*x^2)*ArcTan[a*x]))/((1 + a^2*x^2)^2*ArcTan[a*
x]^2) - SinIntegral[2*ArcTan[a*x]] + 2*SinIntegral[4*ArcTan[a*x]])/(2*a^4*c
^3)
```

Maple [A] time = 0.069, size = 90, normalized size = 0.5

$$-\frac{8\,\mathrm{Si}\left(2\arctan\left(ax\right)\right)\left(\arctan\left(ax\right)\right)^{2}-16\,\mathrm{Si}\left(4\arctan\left(ax\right)\right)\left(\arctan\left(ax\right)\right)^{2}+4\cos\left(2\arctan\left(ax\right)\right)\arctan\left(ax\right)-4\cos}{16\,c^{3}a^{4}\left(\arctan\left(ax\right)\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a^2*c*x^2+c)^3/arctan(a*x)^3,x)
```

```
[Out] -1/16/a^4/c^3*(8*Si(2*arctan(a*x))*arctan(a*x)^2-16*Si(4*arctan(a*x))*arcta
n(a*x)^2+4*cos(2*arctan(a*x))*arctan(a*x)-4*cos(4*arctan(a*x))*arctan(a*x)+
2*sin(2*arctan(a*x))-sin(4*arctan(a*x)))/arctan(a*x)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{ax^{3}-\left(a^{2}x^{4}-3\,x^{2}\right)\arctan\left(ax\right)+\frac{2\left(a^{6}c^{3}x^{4}+2\,a^{4}c^{3}x^{2}+a^{2}c^{3}\right)\left(5\,a^{2}\int\frac{x^{3}}{a^{6}x^{6}\arctan(ax)+3\,a^{4}x^{4}\arctan(ax)+3\,a^{2}x^{2}\arctan(ax)+\arctan(ax)}\,dx-3\int\frac{}{a^{6}x^{6}\arc}\right.}{a^{2}c^{3}}}{2\left(a^{6}c^{3}x^{4}+2\,a^{4}c^{3}x^{2}+a^{2}c^{3}\right)\arctan\left(ax\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(a*x^3 + 2*(a^6*c^3*x^4 + 2*a^4*c^3*x^2 + a^2*c^3)*arctan(a*x)^2*integ
rate((5*a^2*x^3 - 3*x)/((a^8*c^3*x^6 + 3*a^6*c^3*x^4 + 3*a^4*c^3*x^2 + a^2*
c^3)*arctan(a*x)), x) - (a^2*x^4 - 3*x^2)*arctan(a*x))/((a^6*c^3*x^4 + 2*a^
4*c^3*x^2 + a^2*c^3)*arctan(a*x)^2)
```

Fricas [C] time = 1.74747, size = 799, normalized size = 4.51

$$-\frac{2\,a^3x^3-\left(2i\,a^4x^4+4i\,a^2x^2+2i\right)\arctan\left(ax\right)^2\,\mathrm{log\_integral}\left(\frac{a^4x^4+4i\,a^3x^3-6\,a^2x^2-4i\,ax+1}{a^4x^4+2\,a^2x^2+1}\right)-\left(-2i\,a^4x^4-4i\,a^2x^2-2i\right)\arctan}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*a^3*x^3 - (2*I*a^4*x^4 + 4*I*a^2*x^2 + 2*I)*arctan(a*x)^2*log_integ
ral((a^4*x^4 + 4*I*a^3*x^3 - 6*a^2*x^2 - 4*I*a*x + 1)/(a^4*x^4 + 2*a^2*x^2
+ 1)) - (-2*I*a^4*x^4 - 4*I*a^2*x^2 - 2*I)*arctan(a*x)^2*log_integral((a^4*
x^4 - 4*I*a^3*x^3 - 6*a^2*x^2 + 4*I*a*x + 1)/(a^4*x^4 + 2*a^2*x^2 + 1)) - (
-I*a^4*x^4 - 2*I*a^2*x^2 - I)*arctan(a*x)^2*log_integral(-(a^2*x^2 + 2*I*a*
x - 1)/(a^2*x^2 + 1)) - (I*a^4*x^4 + 2*I*a^2*x^2 + I)*arctan(a*x)^2*log_int
egral(-(a^2*x^2 - 2*I*a*x - 1)/(a^2*x^2 + 1)) - 2*(a^4*x^4 - 3*a^2*x^2)*arc
tan(a*x))/((a^8*c^3*x^4 + 2*a^6*c^3*x^2 + a^4*c^3)*arctan(a*x)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^3}{a^6x^6\operatorname{atan}^3(ax)+3a^4x^4\operatorname{atan}^3(ax)+3a^2x^2\operatorname{atan}^3(ax)+\operatorname{atan}^3(ax)}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a**2*c*x**2+c)**3/atan(a*x)**3,x)
```

```
[Out] Integral(x**3/(a**6*x**6*atan(a*x)**3 + 3*a**4*x**4*atan(a*x)**3 + 3*a**2*x
**2*atan(a*x)**3 + atan(a*x)**3), x)/c**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(a^2cx^2+c\right)^3\arctan\left(ax\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(x^3/((a^2*c*x^2 + c)^3*arctan(a*x)^3), x)
```

3.636 $\int \frac{x^2}{(c+a^2cx^2)^3 \tan^{-1}(ax)^3} dx$

Optimal. Leaf size=120

$$\frac{\text{CosIntegral}\left(4\tan^{-1}(ax)\right)}{a^3c^3}+\frac{x}{a^2c^3\left(a^2x^2+1\right)\tan^{-1}(ax)}-\frac{2x}{a^2c^3\left(a^2x^2+1\right)^2\tan^{-1}(ax)}-\frac{1}{2a^3c^3\left(a^2x^2+1\right)\tan^{-1}(ax)^2}+\frac{}{2}$$

```
[Out] 1/(2*a^3*c^3*(1 + a^2*x^2)^2*ArcTan[a*x]^2) - 1/(2*a^3*c^3*(1 + a^2*x^2)*Ar
cTan[a*x]^2) - (2*x)/(a^2*c^3*(1 + a^2*x^2)^2*ArcTan[a*x]) + x/(a^2*c^3*(1
+ a^2*x^2)*ArcTan[a*x]) + CosIntegral[4*ArcTan[a*x]]/(a^3*c^3)
```

Rubi [A] time = 0.596477, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {4964, 4902, 4968, 4970, 3312, 3302, 4904, 4406}

$$\frac{\text{CosIntegral}\left(4\tan^{-1}(ax)\right)}{a^3c^3}+\frac{x}{a^2c^3\left(a^2x^2+1\right)\tan^{-1}(ax)}-\frac{2x}{a^2c^3\left(a^2x^2+1\right)^2\tan^{-1}(ax)}-\frac{1}{2a^3c^3\left(a^2x^2+1\right)\tan^{-1}(ax)^2}+\frac{}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((c + a^2*c*x^2)^3*ArcTan[a*x]^3),x]
```

```
[Out] 1/(2*a^3*c^3*(1 + a^2*x^2)^2*ArcTan[a*x]^2) - 1/(2*a^3*c^3*(1 + a^2*x^2)*Ar
cTan[a*x]^2) - (2*x)/(a^2*c^3*(1 + a^2*x^2)^2*ArcTan[a*x]) + x/(a^2*c^3*(1
+ a^2*x^2)*ArcTan[a*x]) + CosIntegral[4*ArcTan[a*x]]/(a^3*c^3)
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Arc
Tan[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan[c
*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p
, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rule 4902

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^(p + 1))/(b*c*d*(p
+ 1)), x] - Dist[(2*c*(q + 1))/(b*(p + 1)), Int[x*(d + e*x^2)^q*(a + b*ArcT
```

```
an[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && L
tQ[q, -1] && LtQ[p, -1]
```

Rule 4968

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[(x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^(p
+ 1))/(b*c*d*(p + 1)), x] + (-Dist[(c*(m + 2*q + 2))/(b*(p + 1)), Int[x^(m
+ 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p + 1
)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; Fre
eQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &&
LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 4970

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sin[x]^m)/
Cos[x]^(m + 2*(q + 1)), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 4904

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
```

tQ[p, 0]

Rubi steps

$$\int \frac{x^2}{\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^3}\,dx = -\frac{\int \frac{1}{\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^3}\,dx}{a^2} + \frac{\int \frac{1}{\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^3}\,dx}{a^2c}$$

$$= \frac{1}{2a^3c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)^2} - \frac{1}{2a^3c^3\left(1+a^2x^2\right)\tan^{-1}(ax)^2} + \frac{2\int \frac{x}{\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^2}\,dx}{a}$$

$$= \frac{1}{2a^3c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)^2} - \frac{1}{2a^3c^3\left(1+a^2x^2\right)\tan^{-1}(ax)^2} - \frac{2x}{a^2c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax}$$

$$= \frac{1}{2a^3c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)^2} - \frac{1}{2a^3c^3\left(1+a^2x^2\right)\tan^{-1}(ax)^2} - \frac{2x}{a^2c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax}$$

$$= \frac{1}{2a^3c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)^2} - \frac{1}{2a^3c^3\left(1+a^2x^2\right)\tan^{-1}(ax)^2} - \frac{2x}{a^2c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax}$$

$$= \frac{1}{2a^3c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)^2} - \frac{1}{2a^3c^3\left(1+a^2x^2\right)\tan^{-1}(ax)^2} - \frac{2x}{a^2c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax}$$

$$= \frac{1}{2a^3c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)^2} - \frac{1}{2a^3c^3\left(1+a^2x^2\right)\tan^{-1}(ax)^2} - \frac{2x}{a^2c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax}$$

Mathematica [A] time = 0.132692, size = 60, normalized size = 0.5

$$\frac{\frac{ax\left(2\left(a^2x^2-1\right)\tan^{-1}(ax)-ax\right)}{\left(a^2x^2+1\right)^2\tan^{-1}(ax)^2} + 2\text{CosIntegral}\left(4\tan^{-1}(ax)\right)}{2a^3c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((c + a^2*c*x^2)^3*ArcTan[a*x]^3),x]
```

```
[Out] ((a*x*(-(a*x) + 2*(-1 + a^2*x^2)*ArcTan[a*x]))/((1 + a^2*x^2)^2*ArcTan[a*x]
^2) + 2*CosIntegral[4*ArcTan[a*x]])/(2*a^3*c^3)
```

Maple [A] time = 0.075, size = 52, normalized size = 0.4

$$\frac{16\,\mathrm{Ci}\left(4\arctan\left(ax\right)\right)\left(\arctan\left(ax\right)\right)^{2}-4\sin\left(4\arctan\left(ax\right)\right)\arctan\left(ax\right)+\cos\left(4\arctan\left(ax\right)\right)-1}{16\,a^{3}c^{3}\left(\arctan\left(ax\right)\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a^2*c*x^2+c)^3/arctan(a*x)^3,x)
```

```
[Out] 1/16/a^3/c^3*(16*Ci(4*arctan(a*x))*arctan(a*x)^2-4*sin(4*arctan(a*x))*arcta
n(a*x)+cos(4*arctan(a*x))-1)/arctan(a*x)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{-ax^{2}+2\left(a^{2}x^{3}-x\right)\arctan\left(ax\right)+\frac{2\left(a^{6}c^{3}x^{4}+2\,a^{4}c^{3}x^{2}+a^{2}c^{3}\right)\left(a^{4}\int\frac{x^{4}}{a^{6}x^{6}\arctan(ax)+3\,a^{4}x^{4}\arctan(ax)+3\,a^{2}x^{2}\arctan(ax)+\arctan(ax)}\,dx-6\,a^{2}\int\frac{}{a^{6}x^{6}\,\mathrm{arct}}\right.}{a^{2}c^{3}}}{2\left(a^{6}c^{3}x^{4}+2\,a^{4}c^{3}x^{2}+a^{2}c^{3}\right)\arctan\left(ax\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(a^6*c^3*x^4 + 2*a^4*c^3*x^2 + a^2*c^3)*arctan(a*x)^2*integrate((a^4
*x^4 - 6*a^2*x^2 + 1)/((a^8*c^3*x^6 + 3*a^6*c^3*x^4 + 3*a^4*c^3*x^2 + a^2*c
^3)*arctan(a*x)), x) - a*x^2 + 2*(a^2*x^3 - x)*arctan(a*x))/((a^6*c^3*x^4 +
 2*a^4*c^3*x^2 + a^2*c^3)*arctan(a*x)^2)
```

Fricas [C] time = 1.7319, size = 497, normalized size = 4.14

$$-\frac{a^{2}x^{2}-\left(a^{4}x^{4}+2\,a^{2}x^{2}+1\right)\arctan\left(ax\right)^{2}\mathrm{log\_integral}\left(\frac{a^{4}x^{4}+4i\,a^{3}x^{3}-6\,a^{2}x^{2}-4i\,ax+1}{a^{4}x^{4}+2\,a^{2}x^{2}+1}\right)-\left(a^{4}x^{4}+2\,a^{2}x^{2}+1\right)\arctan\left(ax\right)^{2}\mathrm{log}}{2\left(a^{7}c^{3}x^{4}+2\,a^{5}c^{3}x^{2}+a^{3}c^{3}\right)\arctan\left(ax\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(a^2*x^2 - (a^4*x^4 + 2*a^2*x^2 + 1)*arctan(a*x)^2*log_integral((a^4*x
^4 + 4*I*a^3*x^3 - 6*a^2*x^2 - 4*I*a*x + 1)/(a^4*x^4 + 2*a^2*x^2 + 1)) - (a
^4*x^4 + 2*a^2*x^2 + 1)*arctan(a*x)^2*log_integral((a^4*x^4 - 4*I*a^3*x^3 -
 6*a^2*x^2 + 4*I*a*x + 1)/(a^4*x^4 + 2*a^2*x^2 + 1)) - 2*(a^3*x^3 - a*x)*ar
ctan(a*x))/((a^7*c^3*x^4 + 2*a^5*c^3*x^2 + a^3*c^3)*arctan(a*x)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^2}{a^6x^6\operatorname{atan}^3(ax)+3a^4x^4\operatorname{atan}^3(ax)+3a^2x^2\operatorname{atan}^3(ax)+\operatorname{atan}^3(ax)}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a**2*c*x**2+c)**3/atan(a*x)**3,x)
```

```
[Out] Integral(x**2/(a**6*x**6*atan(a*x)**3 + 3*a**4*x**4*atan(a*x)**3 + 3*a**2*x
**2*atan(a*x)**3 + atan(a*x)**3), x)/c**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(a^2cx^2+c\right)^3\arctan(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(x^2/((a^2*c*x^2 + c)^3*arctan(a*x)^3), x)
```

3.637 $\int \frac{x}{\left(c+a^2cx^2\right)^3 \tan^{-1}(ax)^3}\,dx$

Optimal. Leaf size=113

$$-\frac{\mathrm{Si}\left(2\tan^{-1}(ax)\right)}{2a^2c^3}-\frac{\mathrm{Si}\left(4\tan^{-1}(ax)\right)}{a^2c^3}-\frac{x}{2ac^3\left(a^2x^2+1\right)^2\tan^{-1}(ax)^2}+\frac{3}{2a^2c^3\left(a^2x^2+1\right)\tan^{-1}(ax)}-\frac{2}{a^2c^3\left(a^2x^2+1\right)^2\mathrm{ta}$$

```
[Out] -x/(2*a*c^3*(1 + a^2*x^2)^2*ArcTan[a*x]^2) - 2/(a^2*c^3*(1 + a^2*x^2)^2*Arc
Tan[a*x]) + 3/(2*a^2*c^3*(1 + a^2*x^2)*ArcTan[a*x]) - SinIntegral[2*ArcTan[
a*x]]/(2*a^2*c^3) - SinIntegral[4*ArcTan[a*x]]/(a^2*c^3)
```

Rubi [A] time = 0.449375, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {4968, 4964, 4902, 4970, 4406, 12, 3299}

$$-\frac{\mathrm{Si}\left(2\tan^{-1}(ax)\right)}{2a^2c^3}-\frac{\mathrm{Si}\left(4\tan^{-1}(ax)\right)}{a^2c^3}-\frac{x}{2ac^3\left(a^2x^2+1\right)^2\tan^{-1}(ax)^2}+\frac{3}{2a^2c^3\left(a^2x^2+1\right)\tan^{-1}(ax)}-\frac{2}{a^2c^3\left(a^2x^2+1\right)^2\mathrm{ta}$$

Antiderivative was successfully verified.

```
[In] Int[x/((c + a^2*c*x^2)^3*ArcTan[a*x]^3),x]
```

```
[Out] -x/(2*a*c^3*(1 + a^2*x^2)^2*ArcTan[a*x]^2) - 2/(a^2*c^3*(1 + a^2*x^2)^2*Arc
Tan[a*x]) + 3/(2*a^2*c^3*(1 + a^2*x^2)*ArcTan[a*x]) - SinIntegral[2*ArcTan[
a*x]]/(2*a^2*c^3) - SinIntegral[4*ArcTan[a*x]]/(a^2*c^3)
```

Rule 4968

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[(x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^(p
+ 1))/(b*c*d*(p + 1)), x] + (-Dist[(c*(m + 2*q + 2))/(b*(p + 1)), Int[x^(m
+ 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p + 1
)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; Fre
eQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &&
LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Arc
```

```
Tan[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan[c
*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p
, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rule 4902

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^(p + 1))/(b*c*d*(p
+ 1)), x] - Dist[(2*c*(q + 1))/(b*(p + 1)), Int[x*(d + e*x^2)^q*(a + b*ArcT
an[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && L
tQ[q, -1] && LtQ[p, -1]
```

Rule 4970

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sin[x]^m)/
Cos[x]^(m + 2*(q + 1)), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\int \frac{x}{\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^3}\,dx = -\frac{x}{2ac^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)^2} + \frac{\int \frac{1}{\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^2}\,dx}{2a} - \frac{1}{2}(3a)\int \frac{x^2}{\left(c+a^2cx^2\right)^3\tan^{-1}}$$

$$= -\frac{x}{2ac^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)^2} - \frac{1}{2a^2c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)} - 2\int \frac{x}{\left(c+a^2cx^2\right)^3\tan^{-1}}$$

$$= -\frac{x}{2ac^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)^2} - \frac{2}{a^2c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)} + \frac{3}{2a^2c^3\left(1+a^2x^2\right)\tan^{-1}(a}$$

$$= -\frac{x}{2ac^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)^2} - \frac{2}{a^2c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)} + \frac{3}{2a^2c^3\left(1+a^2x^2\right)\tan^{-1}(a}$$

$$= -\frac{x}{2ac^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)^2} - \frac{2}{a^2c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)} + \frac{3}{2a^2c^3\left(1+a^2x^2\right)\tan^{-1}(a}$$

$$= -\frac{x}{2ac^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)^2} - \frac{2}{a^2c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)} + \frac{3}{2a^2c^3\left(1+a^2x^2\right)\tan^{-1}(a}$$

$$= -\frac{x}{2ac^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)^2} - \frac{2}{a^2c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)} + \frac{3}{2a^2c^3\left(1+a^2x^2\right)\tan^{-1}(a}$$

Mathematica [A] time = 0.162507, size = 98, normalized size = 0.87

$$-\frac{\left(a^2x^2+1\right)^2\tan^{-1}(ax)^2\operatorname{Si}\left(2\tan^{-1}(ax)\right) + 2\left(a^2x^2+1\right)^2\tan^{-1}(ax)^2\operatorname{Si}\left(4\tan^{-1}(ax)\right) - 3a^2x^2\tan^{-1}(ax) + ax + \tan^{-1}(a}{2a^2c^3\left(a^2x^2+1\right)^2\tan^{-1}(ax)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((c + a^2*c*x^2)^3*ArcTan[a*x]^3),x]
```

```
[Out] -(a*x + ArcTan[a*x] - 3*a^2*x^2*ArcTan[a*x] + (1 + a^2*x^2)^2*ArcTan[a*x]^2
*SinIntegral[2*ArcTan[a*x]] + 2*(1 + a^2*x^2)^2*ArcTan[a*x]^2*SinIntegral[4
*ArcTan[a*x]])/(2*a^2*c^3*(1 + a^2*x^2)^2*ArcTan[a*x]^2)
```

Maple [A] time = 0.066, size = 88, normalized size = 0.8

$$-\frac{8\,\mathrm{Si}\left(2\arctan\left(ax\right)\right)\left(\arctan\left(ax\right)\right)^{2}+16\,\mathrm{Si}\left(4\arctan\left(ax\right)\right)\left(\arctan\left(ax\right)\right)^{2}+4\cos\left(2\arctan\left(ax\right)\right)\arctan\left(ax\right)+4\cos\left(\right.}{16\,a^{2}c^{3}\left(\arctan\left(ax\right)\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^2*c*x^2+c)^3/arctan(a*x)^3,x)
```

```
[Out] -1/16/a^2/c^3*(8*Si(2*arctan(a*x))*arctan(a*x)^2+16*Si(4*arctan(a*x))*arcta
n(a*x)^2+4*cos(2*arctan(a*x))*arctan(a*x)+4*cos(4*arctan(a*x))*arctan(a*x)+
2*sin(2*arctan(a*x))+sin(4*arctan(a*x)))/arctan(a*x)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{-ax+\left(3\,a^{2}x^{2}-1\right)\arctan\left(ax\right)+\frac{2\left(a^{6}c^{3}x^{4}+2\,a^{4}c^{3}x^{2}+a^{2}c^{3}\right)\left(3\,a^{2}\int\frac{x^{3}}{a^{6}x^{6}\arctan(ax)+3\,a^{4}x^{4}\arctan(ax)+3\,a^{2}x^{2}\arctan(ax)+\arctan(ax)}\,dx-5\int\frac{}{a^{6}x^{6}\arctan(}\right.}{c^{3}}}{2\left(a^{6}c^{3}x^{4}+2\,a^{4}c^{3}x^{2}+a^{2}c^{3}\right)\arctan\left(ax\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(a^6*c^3*x^4 + 2*a^4*c^3*x^2 + a^2*c^3)*arctan(a*x)^2*integrate((3*a
^2*x^3 - 5*x)/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*arctan(a
*x)), x) - a*x + (3*a^2*x^2 - 1)*arctan(a*x))/((a^6*c^3*x^4 + 2*a^4*c^3*x^2
 + a^2*c^3)*arctan(a*x)^2)
```

Fricas [C] time = 1.87409, size = 784, normalized size = 6.94

$$\frac{\left(-2i\,a^{4}x^{4}-4i\,a^{2}x^{2}-2i\right)\arctan\left(ax\right)^{2}\,\mathrm{log\_integral}\left(\frac{a^{4}x^{4}+4i\,a^{3}x^{3}-6\,a^{2}x^{2}-4i\,ax+1}{a^{4}x^{4}+2\,a^{2}x^{2}+1}\right)+\left(2i\,a^{4}x^{4}+4i\,a^{2}x^{2}+2i\right)\arctan\left(ax\right)^{2}\,\mathrm{log\_}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] 1/4*((-2*I*a^4*x^4 - 4*I*a^2*x^2 - 2*I)*arctan(a*x)^2*log_integral((a^4*x^4
 + 4*I*a^3*x^3 - 6*a^2*x^2 - 4*I*a*x + 1)/(a^4*x^4 + 2*a^2*x^2 + 1)) + (2*I
*a^4*x^4 + 4*I*a^2*x^2 + 2*I)*arctan(a*x)^2*log_integral((a^4*x^4 - 4*I*a^3
*x^3 - 6*a^2*x^2 + 4*I*a*x + 1)/(a^4*x^4 + 2*a^2*x^2 + 1)) + (-I*a^4*x^4 -
2*I*a^2*x^2 - I)*arctan(a*x)^2*log_integral(-(a^2*x^2 + 2*I*a*x - 1)/(a^2*x
^2 + 1)) + (I*a^4*x^4 + 2*I*a^2*x^2 + I)*arctan(a*x)^2*log_integral(-(a^2*x
^2 - 2*I*a*x - 1)/(a^2*x^2 + 1)) - 2*a*x + 2*(3*a^2*x^2 - 1)*arctan(a*x))/(
(a^6*c^3*x^4 + 2*a^4*c^3*x^2 + a^2*c^3)*arctan(a*x)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x}{a^6x^6\operatorname{atan}^3(ax)+3a^4x^4\operatorname{atan}^3(ax)+3a^2x^2\operatorname{atan}^3(ax)+\operatorname{atan}^3(ax)}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a**2*c*x**2+c)**3/atan(a*x)**3,x)
```

```
[Out] Integral(x/(a**6*x**6*atan(a*x)**3 + 3*a**4*x**4*atan(a*x)**3 + 3*a**2*x**2
*atan(a*x)**3 + atan(a*x)**3), x)/c**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(a^2cx^2+c\right)^3\arctan\left(ax\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(x/((a^2*c*x^2 + c)^3*arctan(a*x)^3), x)
```

3.638 $\int \frac{1}{(c+a^2cx^2)^3 \tan^{-1}(ax)^3} dx$

Optimal. Leaf size=81

$$\frac{2x}{c^3 (a^2x^2+1)^2 \tan^{-1}(ax)} - \frac{1}{2ac^3 (a^2x^2+1)^2 \tan^{-1}(ax)^2} - \frac{\text{CosIntegral}(2\tan^{-1}(ax))}{ac^3} - \frac{\text{CosIntegral}(4\tan^{-1}(ax))}{ac^3}$$

```
[Out] -1/(2*a*c^3*(1 + a^2*x^2)^2*ArcTan[a*x]^2) + (2*x)/(c^3*(1 + a^2*x^2)^2*Arc
Tan[a*x]) - CosIntegral[2*ArcTan[a*x]]/(a*c^3) - CosIntegral[4*ArcTan[a*x]]
/(a*c^3)
```

Rubi [A] time = 0.265952, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {4902, 4968, 4970, 4406, 3302, 4904, 3312}

$$\frac{2x}{c^3 (a^2x^2+1)^2 \tan^{-1}(ax)} - \frac{1}{2ac^3 (a^2x^2+1)^2 \tan^{-1}(ax)^2} - \frac{\text{CosIntegral}(2\tan^{-1}(ax))}{ac^3} - \frac{\text{CosIntegral}(4\tan^{-1}(ax))}{ac^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a^2*c*x^2)^3*ArcTan[a*x]^3),x]
```

```
[Out] -1/(2*a*c^3*(1 + a^2*x^2)^2*ArcTan[a*x]^2) + (2*x)/(c^3*(1 + a^2*x^2)^2*Arc
Tan[a*x]) - CosIntegral[2*ArcTan[a*x]]/(a*c^3) - CosIntegral[4*ArcTan[a*x]]
/(a*c^3)
```

Rule 4902

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^(p + 1))/(b*c*d*(p
+ 1)), x] - Dist[(2*c*(q + 1))/(b*(p + 1)), Int[x*(d + e*x^2)^q*(a + b*ArcT
an[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && L
tQ[q, -1] && LtQ[p, -1]
```

Rule 4968

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[(x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^(p
+ 1))/(b*c*d*(p + 1)), x] + (-Dist[(c*(m + 2*q + 2))/(b*(p + 1)), Int[x^(m
```

```
+ 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p + 1
)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; Fre
eQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &&
LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 4970

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sin[x]^m)/
Cos[x]^(m + 2*(q + 1)), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 4904

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\int \frac{1}{\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^3}\,dx = -\frac{1}{2ac^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)^2} - (2a)\int \frac{x}{\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^2}\,dx$$

$$= -\frac{1}{2ac^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)^2} + \frac{2x}{c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)} - 2\int \frac{1}{\left(c+a^2cx^2\right)^3\tan^{-1}(ax)}$$

$$= -\frac{1}{2ac^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)^2} + \frac{2x}{c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)} - \frac{2\,\text{Subst}\left(\int \frac{\cos^4(x)}{x}\,dx, x, \tan\right.}{ac^3}$$

$$= -\frac{1}{2ac^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)^2} + \frac{2x}{c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)} - \frac{2\,\text{Subst}\left(\int\left(\frac{3}{8x}+\frac{\cos(2x)}{2x}+\right.\right.}{ac^3}$$

$$= -\frac{1}{2ac^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)^2} + \frac{2x}{c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)} - \frac{\text{Subst}\left(\int \frac{\cos(4x)}{x}\,dx, x, \tan^{-}\right.}{4ac^3}$$

$$= -\frac{1}{2ac^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)^2} + \frac{2x}{c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)} - \frac{\text{Ci}\left(2\tan^{-1}(ax)\right)}{ac^3} - \frac{\text{Ci}\left(4\,\text{ta}\right.}{a}$$

Mathematica [A] time = 0.0934862, size = 89, normalized size = 1.1

$$-\frac{2\left(a^2x^2+1\right)^2\tan^{-1}(ax)^2\text{CosIntegral}\left(2\tan^{-1}(ax)\right)+2\left(a^2x^2+1\right)^2\tan^{-1}(ax)^2\text{CosIntegral}\left(4\tan^{-1}(ax)\right)-4ax\tan^{-1}}{2ac^3\left(a^2x^2+1\right)^2\tan^{-1}(ax)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a^2*c*x^2)^3*ArcTan[a*x]^3),x]
```

```
[Out] -(1 - 4*a*x*ArcTan[a*x] + 2*(1 + a^2*x^2)^2*ArcTan[a*x]^2*CosIntegral[2*Arc
Tan[a*x]] + 2*(1 + a^2*x^2)^2*ArcTan[a*x]^2*CosIntegral[4*ArcTan[a*x]])/(2*
a*c^3*(1 + a^2*x^2)^2*ArcTan[a*x]^2)
```

Maple [A] time = 0.067, size = 89, normalized size = 1.1

$$-\frac{16\,\text{Ci}\,(2\,\arctan(ax))\,(\arctan(ax))^2+16\,\text{Ci}\,(4\,\arctan(ax))\,(\arctan(ax))^2-8\,\sin(2\,\arctan(ax))\arctan(ax)-4\,\sin}{16\,ac^3\,(\arctan(ax))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2*c*x^2+c)^3/arctan(a*x)^3,x)
```

```
[Out] -1/16/a/c^3*(16*Ci(2*arctan(a*x))*arctan(a*x)^2+16*Ci(4*arctan(a*x))*arctan
(a*x)^2-8*sin(2*arctan(a*x))*arctan(a*x)-4*sin(4*arctan(a*x))*arctan(a*x)+4
*cos(2*arctan(a*x))+cos(4*arctan(a*x))+3)/arctan(a*x)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{4\left(a^{5}c^{3}x^{4}+2\,a^{3}c^{3}x^{2}+ac^{3}\right)\arctan\left(ax\right)^{2}\int\frac{3\,a^{2}x^{2}-1}{\left(a^{6}c^{3}x^{6}+3\,a^{4}c^{3}x^{4}+3\,a^{2}c^{3}x^{2}+c^{3}\right)\arctan(ax)}\,dx+4\,ax\arctan\left(ax\right)-1}{2\left(a^{5}c^{3}x^{4}+2\,a^{3}c^{3}x^{2}+ac^{3}\right)\arctan\left(ax\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(a^5*c^3*x^4 + 2*a^3*c^3*x^2 + a*c^3)*arctan(a*x)^2*integrate(2*(3*a
^2*x^2 - 1)/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*arctan(a*x
)), x) + 4*a*x*arctan(a*x) - 1)/((a^5*c^3*x^4 + 2*a^3*c^3*x^2 + a*c^3)*arct
an(a*x)^2)
```

Fricas [C] time = 1.75905, size = 726, normalized size = 8.96

$$-\frac{\left(a^{4}x^{4}+2\,a^{2}x^{2}+1\right)\arctan\left(ax\right)^{2}\operatorname{log\_integral}\left(\frac{a^{4}x^{4}+4i\,a^{3}x^{3}-6\,a^{2}x^{2}-4i\,ax+1}{a^{4}x^{4}+2\,a^{2}x^{2}+1}\right)+\left(a^{4}x^{4}+2\,a^{2}x^{2}+1\right)\arctan\left(ax\right)^{2}\operatorname{log\_integ}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] -1/2*((a^4*x^4 + 2*a^2*x^2 + 1)*arctan(a*x)^2*log_integral((a^4*x^4 + 4*I*a
^3*x^3 - 6*a^2*x^2 - 4*I*a*x + 1)/(a^4*x^4 + 2*a^2*x^2 + 1)) + (a^4*x^4 + 2
*a^2*x^2 + 1)*arctan(a*x)^2*log_integral((a^4*x^4 - 4*I*a^3*x^3 - 6*a^2*x^2
 + 4*I*a*x + 1)/(a^4*x^4 + 2*a^2*x^2 + 1)) + (a^4*x^4 + 2*a^2*x^2 + 1)*arct
an(a*x)^2*log_integral(-(a^2*x^2 + 2*I*a*x - 1)/(a^2*x^2 + 1)) + (a^4*x^4 +
 2*a^2*x^2 + 1)*arctan(a*x)^2*log_integral(-(a^2*x^2 - 2*I*a*x - 1)/(a^2*x^
2 + 1)) - 4*a*x*arctan(a*x) + 1)/((a^5*c^3*x^4 + 2*a^3*c^3*x^2 + a*c^3)*arc
tan(a*x)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{a^6x^6\operatorname{atan}^3(ax)+3a^4x^4\operatorname{atan}^3(ax)+3a^2x^2\operatorname{atan}^3(ax)+\operatorname{atan}^3(ax)}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*c*x**2+c)**3/atan(a*x)**3,x)
```

```
[Out] Integral(1/(a**6*x**6*atan(a*x)**3 + 3*a**4*x**4*atan(a*x)**3 + 3*a**2*x**2
*atan(a*x)**3 + atan(a*x)**3), x)/c**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a^2cx^2+c)^3\arctan(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^3*arctan(a*x)^3), x)
```

3.639 $\int \frac{1}{x\left(c+a^2cx^2\right)^3 \tan^{-1}(ax)^3} dx$

Optimal. Leaf size=199

$$-\frac{\text{Unintegrable}\left(\frac{1}{x^2\tan^{-1}(ax)^2},x\right)}{2ac^3}+\frac{ax}{2c^3\left(a^2x^2+1\right)\tan^{-1}(ax)^2}+\frac{ax}{2c^3\left(a^2x^2+1\right)^2\tan^{-1}(ax)^2}+\frac{1-a^2x^2}{2c^3\left(a^2x^2+1\right)\tan^{-1}(ax)}$$

```
[Out] -1/(2*a*c^3*x*ArcTan[a*x]^2) + (a*x)/(2*c^3*(1 + a^2*x^2)^2*ArcTan[a*x]^2)
+ (a*x)/(2*c^3*(1 + a^2*x^2)*ArcTan[a*x]^2) + 2/(c^3*(1 + a^2*x^2)^2*ArcTan
[a*x]) - 3/(2*c^3*(1 + a^2*x^2)*ArcTan[a*x]) + (1 - a^2*x^2)/(2*c^3*(1 + a^
2*x^2)*ArcTan[a*x]) + (3*SinIntegral[2*ArcTan[a*x]])/(2*c^3) + SinIntegral[
4*ArcTan[a*x]]/c^3 - Unintegrable[1/(x^2*ArcTan[a*x]^2), x]/(2*a*c^3)
```

Rubi [A] time = 0.765388, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x\left(c+a^2cx^2\right)^3 \tan^{-1}(ax)^3} dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*(c + a^2*c*x^2)^3*ArcTan[a*x]^3),x]
```

```
[Out] -1/(2*a*c^3*x*ArcTan[a*x]^2) + (a*x)/(2*c^3*(1 + a^2*x^2)^2*ArcTan[a*x]^2)
+ (a*x)/(2*c^3*(1 + a^2*x^2)*ArcTan[a*x]^2) + 2/(c^3*(1 + a^2*x^2)^2*ArcTan
[a*x]) - 3/(2*c^3*(1 + a^2*x^2)*ArcTan[a*x]) + (1 - a^2*x^2)/(2*c^3*(1 + a^
2*x^2)*ArcTan[a*x]) + (3*SinIntegral[2*ArcTan[a*x]])/(2*c^3) + SinIntegral[
4*ArcTan[a*x]]/c^3 - Defer[Int][1/(x^2*ArcTan[a*x]^2), x]/(2*a*c^3)
```

Rubi steps

$$\int \frac{1}{x\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^3}\,dx = -\left(a^2\int \frac{x}{\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^3}\,dx\right) + \frac{\int \frac{1}{x\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^3}\,dx}{c}$$

$$= \frac{ax}{2c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)^2} - \frac{1}{2}a\int \frac{1}{\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^2}\,dx + \frac{1}{2}\left(3a^3\right)\int \frac{}{\left(c+a^2cx\right.}$$

$$= -\frac{1}{2ac^3x\tan^{-1}(ax)^2} + \frac{ax}{2c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)^2} + \frac{ax}{2c^3\left(1+a^2x^2\right)\tan^{-1}(ax)^2} + \frac{}{2c^3\left(1\right.}$$

$$= -\frac{1}{2ac^3x\tan^{-1}(ax)^2} + \frac{ax}{2c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)^2} + \frac{ax}{2c^3\left(1+a^2x^2\right)\tan^{-1}(ax)^2} + \frac{}{c^3\left(1\right.}$$

$$= -\frac{1}{2ac^3x\tan^{-1}(ax)^2} + \frac{ax}{2c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)^2} + \frac{ax}{2c^3\left(1+a^2x^2\right)\tan^{-1}(ax)^2} + \frac{}{c^3\left(1\right.}$$

$$= -\frac{1}{2ac^3x\tan^{-1}(ax)^2} + \frac{ax}{2c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)^2} + \frac{ax}{2c^3\left(1+a^2x^2\right)\tan^{-1}(ax)^2} + \frac{}{c^3\left(1\right.}$$

$$= -\frac{1}{2ac^3x\tan^{-1}(ax)^2} + \frac{ax}{2c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)^2} + \frac{ax}{2c^3\left(1+a^2x^2\right)\tan^{-1}(ax)^2} + \frac{}{c^3\left(1\right.}$$

$$= -\frac{1}{2ac^3x\tan^{-1}(ax)^2} + \frac{ax}{2c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)^2} + \frac{ax}{2c^3\left(1+a^2x^2\right)\tan^{-1}(ax)^2} + \frac{}{c^3\left(1\right.}$$

Mathematica [A] time = 2.57424, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*(c + a^2*c*x^2)^3*ArcTan[a*x]^3),x]
```

```
[Out] Integrate[1/(x*(c + a^2*c*x^2)^3*ArcTan[a*x]^3), x]
```

Maple [A] time = 0.436, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(a^{2}c x^{2}+c\right)^{3} \left(\arctan\left(a x\right)\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^3,x)
```

```
[Out] int(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{-ax+\left(5\,a^{2}x^{2}+1\right)\arctan\left(ax\right)+\frac{2\left(a^{6}c^{3}x^{6}+2\,a^{4}c^{3}x^{4}+a^{2}c^{3}x^{2}\right)\left(10\,a^{4}\int \frac{x^{4}}{a^{6}x^{9}\arctan(ax)+3\,a^{4}x^{7}\arctan(ax)+3\,a^{2}x^{5}\arctan(ax)+x^{3}\arctan(ax)}\,dx+3\,a^{2}\int \frac{}{a^{6}\cdots}\right.}{a^{2}c^{3}}}{2\left(a^{6}c^{3}x^{6}+2\,a^{4}c^{3}x^{4}+a^{2}c^{3}x^{2}\right)\arctan\left(ax\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(a^6*c^3*x^6 + 2*a^4*c^3*x^4 + a^2*c^3*x^2)*arctan(a*x)^2*integrate(
(10*a^4*x^4 + 3*a^2*x^2 + 1)/((a^8*c^3*x^9 + 3*a^6*c^3*x^7 + 3*a^4*c^3*x^5
+ a^2*c^3*x^3)*arctan(a*x)), x) - a*x + (5*a^2*x^2 + 1)*arctan(a*x))/((a^6*
c^3*x^6 + 2*a^4*c^3*x^4 + a^2*c^3*x^2)*arctan(a*x)^2)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\left(a^{6}c^{3}x^{7}+3\,a^{4}c^{3}x^{5}+3\,a^{2}c^{3}x^{3}+c^{3}x\right)\arctan\left(ax\right)^{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(1/((a^6*c^3*x^7 + 3*a^4*c^3*x^5 + 3*a^2*c^3*x^3 + c^3*x)*arctan(a*
x)^3), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{a^6x^7\operatorname{atan}^3(ax)+3a^4x^5\operatorname{atan}^3(ax)+3a^2x^3\operatorname{atan}^3(ax)+x\operatorname{atan}^3(ax)}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a**2*c*x**2+c)**3/atan(a*x)**3,x)
```

```
[Out] Integral(1/(a**6*x**7*atan(a*x)**3 + 3*a**4*x**5*atan(a*x)**3 + 3*a**2*x**3
*atan(a*x)**3 + x*atan(a*x)**3), x)/c**3
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)^3 x\arctan(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^3*x*arctan(a*x)^3), x)
```

3.640 $\int \frac{1}{x^2\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^3}\,dx$

Optimal. Leaf size=167

$$-\frac{\text{Unintegrable}\left(\frac{1}{x^3\tan^{-1}(ax)^2},x\right)}{ac^3}-\frac{a^2x}{c^3\left(a^2x^2+1\right)\tan^{-1}(ax)}-\frac{2a^2x}{c^3\left(a^2x^2+1\right)^2\tan^{-1}(ax)}+\frac{a}{2c^3\left(a^2x^2+1\right)\tan^{-1}(ax)^2}+\frac{}{2}$$

```
[Out] -1/(2*a*c^3*x^2*ArcTan[a*x]^2) + a/(2*c^3*(1 + a^2*x^2)^2*ArcTan[a*x]^2) +
a/(2*c^3*(1 + a^2*x^2)*ArcTan[a*x]^2) - (2*a^2*x)/(c^3*(1 + a^2*x^2)^2*ArcT
an[a*x]) - (a^2*x)/(c^3*(1 + a^2*x^2)*ArcTan[a*x]) + (2*a*CosIntegral[2*Arc
Tan[a*x]])/c^3 + (a*CosIntegral[4*ArcTan[a*x]])/c^3 - Unintegrable[1/(x^3*A
rcTan[a*x]^2), x]/(a*c^3)
```

Rubi [A] time = 0.732928, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]^3),x]
```

```
[Out] -1/(2*a*c^3*x^2*ArcTan[a*x]^2) + a/(2*c^3*(1 + a^2*x^2)^2*ArcTan[a*x]^2) +
a/(2*c^3*(1 + a^2*x^2)*ArcTan[a*x]^2) - (2*a^2*x)/(c^3*(1 + a^2*x^2)^2*ArcT
an[a*x]) - (a^2*x)/(c^3*(1 + a^2*x^2)*ArcTan[a*x]) + (2*a*CosIntegral[2*Arc
Tan[a*x]])/c^3 + (a*CosIntegral[4*ArcTan[a*x]])/c^3 - Defer[Int][1/(x^3*Arc
Tan[a*x]^2), x]/(a*c^3)
```

Rubi steps

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^3}\,dx = -\left(a^2\int \frac{1}{\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^3}\,dx\right) + \frac{\int \frac{1}{x^2\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^3}\,dx}{c}$$

$$= \frac{a}{2c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)^2} + \left(2a^3\right)\int \frac{x}{\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^2}\,dx + \frac{\int \frac{1}{x^2\left(c+a^2cx^2\right)\tan^{-1}}}{c^2}$$

$$= -\frac{1}{2ac^3x^2\tan^{-1}(ax)^2} + \frac{a}{2c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)^2} + \frac{a}{2c^3\left(1+a^2x^2\right)\tan^{-1}(ax)^2} - \frac{}{c^3(}$$

$$= -\frac{1}{2ac^3x^2\tan^{-1}(ax)^2} + \frac{a}{2c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)^2} + \frac{a}{2c^3\left(1+a^2x^2\right)\tan^{-1}(ax)^2} - \frac{}{c^3(}$$

$$= -\frac{1}{2ac^3x^2\tan^{-1}(ax)^2} + \frac{a}{2c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)^2} + \frac{a}{2c^3\left(1+a^2x^2\right)\tan^{-1}(ax)^2} - \frac{}{c^3(}$$

$$= -\frac{1}{2ac^3x^2\tan^{-1}(ax)^2} + \frac{a}{2c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)^2} + \frac{a}{2c^3\left(1+a^2x^2\right)\tan^{-1}(ax)^2} - \frac{}{c^3(}$$

$$= -\frac{1}{2ac^3x^2\tan^{-1}(ax)^2} + \frac{a}{2c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)^2} + \frac{a}{2c^3\left(1+a^2x^2\right)\tan^{-1}(ax)^2} - \frac{}{c^3(}$$

$$= -\frac{1}{2ac^3x^2\tan^{-1}(ax)^2} + \frac{a}{2c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)^2} + \frac{a}{2c^3\left(1+a^2x^2\right)\tan^{-1}(ax)^2} - \frac{}{c^3(}$$

Mathematica [A] time = 3.64928, size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]^3),x]
```

```
[Out] Integrate[1/(x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]^3), x]
```

Maple [A] time = 0.391, size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \left(a^2 c x^2 + c\right)^3 \left(\arctan\left(ax\right)\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a^2*c*x^2+c)^3/arctan(a*x)^3,x)
```

```
[Out] int(1/x^2/(a^2*c*x^2+c)^3/arctan(a*x)^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{-ax + 2\left(3\,a^2x^2 + 1\right)\arctan\left(ax\right) + \frac{2\left(a^6c^3x^7 + 2\,a^4c^3x^5 + a^2c^3x^3\right)\left(15\,a^4 \int \frac{x^4}{a^6x^{10}\arctan(ax) + 3\,a^4x^8\arctan(ax) + 3\,a^2x^6\arctan(ax) + x^4\arctan(ax)}\,dx + 10\,a^2}{}}{2\left(a^6c^3x^7 + 2\,a^4c^3x^5 + \right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(a^6*c^3*x^7 + 2*a^4*c^3*x^5 + a^2*c^3*x^3)*arctan(a*x)^2*integrate(
(15*a^4*x^4 + 10*a^2*x^2 + 3)/((a^8*c^3*x^10 + 3*a^6*c^3*x^8 + 3*a^4*c^3*x^
6 + a^2*c^3*x^4)*arctan(a*x)), x) - a*x + 2*(3*a^2*x^2 + 1)*arctan(a*x))/((
a^6*c^3*x^7 + 2*a^4*c^3*x^5 + a^2*c^3*x^3)*arctan(a*x)^2)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\left(a^6c^3x^8 + 3\,a^4c^3x^6 + 3\,a^2c^3x^4 + c^3x^2\right)\arctan\left(ax\right)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(1/((a^6*c^3*x^8 + 3*a^4*c^3*x^6 + 3*a^2*c^3*x^4 + c^3*x^2)*arctan(
a*x)^3), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{a^6x^8\operatorname{atan}^3(ax)+3a^4x^6\operatorname{atan}^3(ax)+3a^2x^4\operatorname{atan}^3(ax)+x^2\operatorname{atan}^3(ax)}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a**2*c*x**2+c)**3/atan(a*x)**3,x)
```

```
[Out] Integral(1/(a**6*x**8*atan(a*x)**3 + 3*a**4*x**6*atan(a*x)**3 + 3*a**2*x**4
*atan(a*x)**3 + x**2*atan(a*x)**3), x)/c**3
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)^3x^2\arctan(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^3*x^2*arctan(a*x)^3), x)
```

3.641 $\int \frac{1}{x^3(c+a^2cx^2)^3 \tan^{-1}(ax)^3} dx$

Optimal. Leaf size=246

$$\frac{a\text{Unintegrable}\left(\frac{1}{x^2\tan^{-1}(ax)^2},x\right)}{c^3}-\frac{3\text{Unintegrable}\left(\frac{1}{x^4\tan^{-1}(ax)^2},x\right)}{2ac^3}-\frac{5a^2\text{Si}\left(2\tan^{-1}(ax)\right)}{2c^3}-\frac{a^2\text{Si}\left(4\tan^{-1}(ax)\right)}{c^3}-\frac{}{c^3\left(a\right.}$$

```
[Out] -1/(2*a*c^3*x^3*ArcTan[a*x]^2) + a/(c^3*x*ArcTan[a*x]^2) - (a^3*x)/(2*c^3*(
1 + a^2*x^2)^2*ArcTan[a*x]^2) - (a^3*x)/(c^3*(1 + a^2*x^2)*ArcTan[a*x]^2) -
 (2*a^2)/(c^3*(1 + a^2*x^2)^2*ArcTan[a*x]) + (3*a^2)/(2*c^3*(1 + a^2*x^2)*A
rcTan[a*x]) - (a^2*(1 - a^2*x^2))/(c^3*(1 + a^2*x^2)*ArcTan[a*x]) - (5*a^2*
SinIntegral[2*ArcTan[a*x]])/(2*c^3) - (a^2*SinIntegral[4*ArcTan[a*x]])/c^3
- (3*Unintegrable[1/(x^4*ArcTan[a*x]^2), x])/(2*a*c^3) + (a*Unintegrable[1/
(x^2*ArcTan[a*x]^2), x])/c^3
```

Rubi [A] time = 1.28108, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^3\left(c+a^2cx^2\right)^3 \tan^{-1}(ax)^3} dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^3*(c + a^2*c*x^2)^3*ArcTan[a*x]^3),x]
```

```
[Out] -1/(2*a*c^3*x^3*ArcTan[a*x]^2) + a/(c^3*x*ArcTan[a*x]^2) - (a^3*x)/(2*c^3*(
1 + a^2*x^2)^2*ArcTan[a*x]^2) - (a^3*x)/(c^3*(1 + a^2*x^2)*ArcTan[a*x]^2) -
 (2*a^2)/(c^3*(1 + a^2*x^2)^2*ArcTan[a*x]) + (3*a^2)/(2*c^3*(1 + a^2*x^2)*A
rcTan[a*x]) - (a^2*(1 - a^2*x^2))/(c^3*(1 + a^2*x^2)*ArcTan[a*x]) - (5*a^2*
SinIntegral[2*ArcTan[a*x]])/(2*c^3) - (a^2*SinIntegral[4*ArcTan[a*x]])/c^3
- (3*Defer[Int][1/(x^4*ArcTan[a*x]^2), x])/(2*a*c^3) + (a*Defer[Int][1/(x^2
*ArcTan[a*x]^2), x])/c^3
```

Rubi steps

$$\int \frac{1}{x^3\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^3}\,dx = -\left(a^2\int \frac{1}{x\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^3}\,dx\right) + \frac{\int \frac{1}{x^3\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^3}\,dx}{c}$$

$$= a^4\int \frac{x}{\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^3}\,dx + \frac{\int \frac{1}{x^3\left(c+a^2cx^2\right)\tan^{-1}(ax)^3}\,dx}{c^2} - 2\frac{a^2\int \frac{1}{x\left(c+a^2cx^2\right)^2\tan^{-1}(ax}}{c}$$

$$= -\frac{1}{2ac^3x^3\tan^{-1}(ax)^2} - \frac{a^3x}{2c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)^2} + \frac{1}{2}a^3\int \frac{1}{\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^2}$$

$$= -\frac{1}{2ac^3x^3\tan^{-1}(ax)^2} - \frac{a^3x}{2c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)^2} - \frac{a^2}{2c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)} + \frac{1}{2}\left(3\right.$$

$$= -\frac{1}{2ac^3x^3\tan^{-1}(ax)^2} - \frac{a^3x}{2c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)^2} - \frac{2a^2}{c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)} + \frac{}{2c^3\left(\right.}$$

$$= -\frac{1}{2ac^3x^3\tan^{-1}(ax)^2} - \frac{a^3x}{2c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)^2} - \frac{2a^2}{c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)} + \frac{}{2c^3\left(\right.}$$

$$= -\frac{1}{2ac^3x^3\tan^{-1}(ax)^2} - \frac{a^3x}{2c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)^2} - \frac{2a^2}{c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)} + \frac{}{2c^3\left(\right.}$$

$$= -\frac{1}{2ac^3x^3\tan^{-1}(ax)^2} - \frac{a^3x}{2c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)^2} - \frac{2a^2}{c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)} + \frac{}{2c^3\left(\right.}$$

$$= -\frac{1}{2ac^3x^3\tan^{-1}(ax)^2} - \frac{a^3x}{2c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)^2} - \frac{2a^2}{c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)} + \frac{}{2c^3\left(\right.}$$

Mathematica [A] time = 4.81972, size = 0, normalized size = 0.

$$\int \frac{1}{x^3\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^3*(c + a^2*c*x^2)^3*ArcTan[a*x]^3),x]
```

```
[Out] Integrate[1/(x^3*(c + a^2*c*x^2)^3*ArcTan[a*x]^3), x]
```

Maple [A] time = 1.508, size = 0, normalized size = 0.

$$\int \frac{1}{x^{3} \left(a^{2} c \,x^{2}+c \right)^{3} \arctan \left(a x \right)^{3}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a^2*c*x^2+c)^3/arctan(a*x)^3,x)
```

```
[Out] int(1/x^3/(a^2*c*x^2+c)^3/arctan(a*x)^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{-ax + \left(7\,a^{2}x^{2}+3\right)\arctan\left(ax\right) + \frac{2\left(a^{6}c^{3}x^{8}+2\,a^{4}c^{3}x^{6}+a^{2}c^{3}x^{4}\right)\left(21\,a^{4}\int \frac{x^{4}}{a^{6}x^{11}\arctan\left(ax\right)+3\,a^{4}x^{9}\arctan\left(ax\right)+3\,a^{2}x^{7}\arctan\left(ax\right)+x^{5}\arctan\left(ax\right)}\,dx+19\,a^{2}\int}{}}{2\left(a^{6}c^{3}x^{8}+2\,a^{4}c^{3}x^{6}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(a^6*c^3*x^8 + 2*a^4*c^3*x^6 + a^2*c^3*x^4)*arctan(a*x)^2*integrate(
(21*a^4*x^4 + 19*a^2*x^2 + 6)/((a^8*c^3*x^11 + 3*a^6*c^3*x^9 + 3*a^4*c^3*x^
7 + a^2*c^3*x^5)*arctan(a*x)), x) - a*x + (7*a^2*x^2 + 3)*arctan(a*x))/((a^
6*c^3*x^8 + 2*a^4*c^3*x^6 + a^2*c^3*x^4)*arctan(a*x)^2)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$${\rm integral}\left(\frac{1}{{\left(a^{6} c^{3} x^{9} + 3 \, a^{4} c^{3} x^{7} + 3 \, a^{2} c^{3} x^{5} + c^{3} x^{3}\right)} \arctan\left(a x\right)^{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(1/((a^6*c^3*x^9 + 3*a^4*c^3*x^7 + 3*a^2*c^3*x^5 + c^3*x^3)*arctan(
a*x)^3), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{a^6x^9 \operatorname{atan}^3(ax)+3a^4x^7 \operatorname{atan}^3(ax)+3a^2x^5 \operatorname{atan}^3(ax)+x^3 \operatorname{atan}^3(ax)}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a**2*c*x**2+c)**3/atan(a*x)**3,x)
```

```
[Out] Integral(1/(a**6*x**9*atan(a*x)**3 + 3*a**4*x**7*atan(a*x)**3 + 3*a**2*x**5
*atan(a*x)**3 + x**3*atan(a*x)**3), x)/c**3
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)^3 x^3 \arctan(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^3*x^3*arctan(a*x)^3), x)
```

3.642 $\int \frac{1}{x^4(c+a^2cx^2)^3 \tan^{-1}(ax)^3} dx$

Optimal. Leaf size=206

$$\frac{2a\text{Unintegrable}\left(\frac{1}{x^3\tan^{-1}(ax)^2},x\right)}{c^3}-\frac{2\text{Unintegrable}\left(\frac{1}{x^5\tan^{-1}(ax)^2},x\right)}{ac^3}-\frac{3a^3\text{CosIntegral}\left(2\tan^{-1}(ax)\right)}{c^3}-\frac{a^3\text{CosIntegra}}{}$$

```
[Out] -1/(2*a*c^3*x^4*ArcTan[a*x]^2) + a/(c^3*x^2*ArcTan[a*x]^2) - a^3/(2*c^3*(1
+ a^2*x^2)^2*ArcTan[a*x]^2) - a^3/(c^3*(1 + a^2*x^2)*ArcTan[a*x]^2) + (2*a^
4*x)/(c^3*(1 + a^2*x^2)^2*ArcTan[a*x]) + (2*a^4*x)/(c^3*(1 + a^2*x^2)*ArcTa
n[a*x]) - (3*a^3*CosIntegral[2*ArcTan[a*x]])/c^3 - (a^3*CosIntegral[4*ArcTa
n[a*x]])/c^3 - (2*Unintegrable[1/(x^5*ArcTan[a*x]^2), x])/(a*c^3) + (2*a*Un
integrable[1/(x^3*ArcTan[a*x]^2), x])/c^3
```

Rubi [A] time = 1.40316, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^4\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^4*(c + a^2*c*x^2)^3*ArcTan[a*x]^3),x]
```

```
[Out] -1/(2*a*c^3*x^4*ArcTan[a*x]^2) + a/(c^3*x^2*ArcTan[a*x]^2) - a^3/(2*c^3*(1
+ a^2*x^2)^2*ArcTan[a*x]^2) - a^3/(c^3*(1 + a^2*x^2)*ArcTan[a*x]^2) + (2*a^
4*x)/(c^3*(1 + a^2*x^2)^2*ArcTan[a*x]) + (2*a^4*x)/(c^3*(1 + a^2*x^2)*ArcTa
n[a*x]) - (3*a^3*CosIntegral[2*ArcTan[a*x]])/c^3 - (a^3*CosIntegral[4*ArcTa
n[a*x]])/c^3 - (2*Defer[Int][1/(x^5*ArcTan[a*x]^2), x])/(a*c^3) + (2*a*Defe
r[Int][1/(x^3*ArcTan[a*x]^2), x])/c^3
```

Rubi steps

$$\int \frac{1}{x^4 \left(c+a^2cx^2\right)^3 \tan^{-1}(ax)^3}\,dx = -\left(a^2 \int \frac{1}{x^2 \left(c+a^2cx^2\right)^3 \tan^{-1}(ax)^3}\,dx\right) + \frac{\int \frac{1}{x^4\left(c+a^2cx^2\right)^2 \tan^{-1}(ax)^3}\,dx}{c}$$

$$= a^4 \int \frac{1}{\left(c+a^2cx^2\right)^3 \tan^{-1}(ax)^3}\,dx + \frac{\int \frac{1}{x^4\left(c+a^2cx^2\right) \tan^{-1}(ax)^3}\,dx}{c^2} - 2\frac{a^2 \int \frac{1}{x^2\left(c+a^2cx^2\right)^2 \tan^{-1}(a}}{c}$$

$$= -\frac{1}{2ac^3x^4 \tan^{-1}(ax)^2} - \frac{a^3}{2c^3\left(1+a^2x^2\right)^2 \tan^{-1}(ax)^2} - \left(2a^5\right)\int \frac{x}{\left(c+a^2cx^2\right)^3 \tan^{-1}(ax)^2}$$

$$= -\frac{1}{2ac^3x^4 \tan^{-1}(ax)^2} - \frac{a^3}{2c^3\left(1+a^2x^2\right)^2 \tan^{-1}(ax)^2} + \frac{2a^4x}{c^3\left(1+a^2x^2\right)^2 \tan^{-1}(ax)} - \left(2a^4\right)$$

$$= -\frac{1}{2ac^3x^4 \tan^{-1}(ax)^2} - \frac{a^3}{2c^3\left(1+a^2x^2\right)^2 \tan^{-1}(ax)^2} + \frac{2a^4x}{c^3\left(1+a^2x^2\right)^2 \tan^{-1}(ax)} - \frac{2\int}{}$$

$$= -\frac{1}{2ac^3x^4 \tan^{-1}(ax)^2} - \frac{a^3}{2c^3\left(1+a^2x^2\right)^2 \tan^{-1}(ax)^2} + \frac{2a^4x}{c^3\left(1+a^2x^2\right)^2 \tan^{-1}(ax)} - \frac{2\int}{}$$

$$= -\frac{1}{2ac^3x^4 \tan^{-1}(ax)^2} - \frac{a^3}{2c^3\left(1+a^2x^2\right)^2 \tan^{-1}(ax)^2} + \frac{2a^4x}{c^3\left(1+a^2x^2\right)^2 \tan^{-1}(ax)} - \frac{2\int}{}$$

$$= -\frac{1}{2ac^3x^4 \tan^{-1}(ax)^2} - \frac{a^3}{2c^3\left(1+a^2x^2\right)^2 \tan^{-1}(ax)^2} + \frac{2a^4x}{c^3\left(1+a^2x^2\right)^2 \tan^{-1}(ax)} - \frac{a^3\mathrm{Ci}}{}$$

$$= -\frac{1}{2ac^3x^4 \tan^{-1}(ax)^2} - \frac{a^3}{2c^3\left(1+a^2x^2\right)^2 \tan^{-1}(ax)^2} + \frac{2a^4x}{c^3\left(1+a^2x^2\right)^2 \tan^{-1}(ax)} - \frac{a^3\mathrm{Ci}}{}$$

Mathematica [A] time = 9.02906, size = 0, normalized size = 0.

$$\int \frac{1}{x^4 \left(c+a^2cx^2\right)^3 \tan^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^4*(c + a^2*c*x^2)^3*ArcTan[a*x]^3),x]
```

[Out] Integrate[1/(x^4*(c + a^2*c*x^2)^3*ArcTan[a*x]^3), x]

Maple [A] time = 1.156, size = 0, normalized size = 0.

$$\int \frac{1}{x^4 \left(a^2 c x^2+c\right)^3 \left(\arctan \left(a x\right)\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^4/(a^2*c*x^2+c)^3/arctan(a*x)^3,x)

[Out] int(1/x^4/(a^2*c*x^2+c)^3/arctan(a*x)^3,x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{4 \left(a^6 c^3 x^9+2\, a^4 c^3 x^7+a^2 c^3 x^5\right) \arctan \left(a x\right)^2 \int \frac{14\, a^4 x^4+15\, a^2 x^2+5}{\left(a^8 c^3 x^{12}+3\, a^6 c^3 x^{10}+3\, a^4 c^3 x^8+a^2 c^3 x^6\right) \arctan(ax)}\, dx - a x + 4 \left(2\, a^2 x^2+1\right) \arctan \left(a\right.}{2 \left(a^6 c^3 x^9+2\, a^4 c^3 x^7+a^2 c^3 x^5\right) \arctan \left(a x\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="maxima")

[Out] 1/2*(2*(a^6*c^3*x^9 + 2*a^4*c^3*x^7 + a^2*c^3*x^5)*arctan(a*x)^2*integrate(
2*(14*a^4*x^4 + 15*a^2*x^2 + 5)/((a^8*c^3*x^12 + 3*a^6*c^3*x^10 + 3*a^4*c^3
*x^8 + a^2*c^3*x^6)*arctan(a*x)), x) - a*x + 4*(2*a^2*x^2 + 1)*arctan(a*x))
/((a^6*c^3*x^9 + 2*a^4*c^3*x^7 + a^2*c^3*x^5)*arctan(a*x)^2)

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\left(a^6 c^3 x^{10}+3\, a^4 c^3 x^8+3\, a^2 c^3 x^6+c^3 x^4\right) \arctan \left(a x\right)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="fricas")

```
[Out] integral(1/((a^6*c^3*x^10 + 3*a^4*c^3*x^8 + 3*a^2*c^3*x^6 + c^3*x^4)*arctan
(a*x)^3), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{a^{6}x^{10}\operatorname{atan}^{3}(ax)+3a^{4}x^{8}\operatorname{atan}^{3}(ax)+3a^{2}x^{6}\operatorname{atan}^{3}(ax)+x^{4}\operatorname{atan}^{3}(ax)}\,dx}{c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(a**2*c*x**2+c)**3/atan(a*x)**3,x)
```

```
[Out] Integral(1/(a**6*x**10*atan(a*x)**3 + 3*a**4*x**8*atan(a*x)**3 + 3*a**2*x**
6*atan(a*x)**3 + x**4*atan(a*x)**3), x)/c**3
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^{2}cx^{2}+c\right)^{3}x^{4}\arctan\left(ax\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^3*x^4*arctan(a*x)^3), x)
```

3.643 $\int \left(\frac{x^3}{\left(1+a^2x^2\right)\tan^{-1}(ax)^3}-\frac{3x^2}{2a\tan^{-1}(ax)^2}\right)dx$

Optimal. Leaf size=16

$$-\frac{x^3}{2a\tan^{-1}(ax)^2}$$

```
[Out] -x^3/(2*a*ArcTan[a*x]^2)
```

Rubi [A] time = 0.090723, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {4926}

$$-\frac{x^3}{2a\tan^{-1}(ax)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((1 + a^2*x^2)*ArcTan[a*x]^3) - (3*x^2)/(2*a*ArcTan[a*x]^2),x]
```

```
[Out] -x^3/(2*a*ArcTan[a*x]^2)
```

Rule 4926

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_))/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[((f*x)^m*(a + b*ArcTan[c*x])^(p + 1))/(b*c*d*
(p + 1)), x] - Dist[(f*m)/(b*c*d*(p + 1)), Int[(f*x)^(m - 1)*(a + b*ArcTan[
c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] &
& LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \left(\frac{x^3}{\left(1+a^2x^2\right)\tan^{-1}(ax)^3}-\frac{3x^2}{2a\tan^{-1}(ax)^2}\right)dx &= -\frac{3\int \frac{x^2}{\tan^{-1}(ax)^2}\,dx}{2a}+\int \frac{x^3}{\left(1+a^2x^2\right)\tan^{-1}(ax)^3}\,dx\\ &= -\frac{x^3}{2a\tan^{-1}(ax)^2}\end{aligned}$$

Mathematica [A] time = 0.151827, size = 16, normalized size = 1.

$$-\frac{x^3}{2a\tan^{-1}(ax)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((1 + a^2*x^2)*ArcTan[a*x]^3) - (3*x^2)/(2*a*ArcTan[a*x]^2),x
]
```

```
[Out] -x^3/(2*a*ArcTan[a*x]^2)
```

Maple [F] time = 1.108, size = 0, normalized size = 0.

$$\int \frac{x^3}{(a^2x^2+1)(\arctan(ax))^3} - \frac{3x^2}{2a(\arctan(ax))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a^2*x^2+1)/arctan(a*x)^3-3/2*x^2/a/arctan(a*x)^2,x)
```

```
[Out] int(x^3/(a^2*x^2+1)/arctan(a*x)^3-3/2*x^2/a/arctan(a*x)^2,x)
```

Maxima [A] time = 1.36384, size = 19, normalized size = 1.19

$$-\frac{x^3}{2a\arctan(ax)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*x^2+1)/arctan(a*x)^3-3/2*x^2/a/arctan(a*x)^2,x, algorith
m="maxima")
```

```
[Out] -1/2*x^3/(a*arctan(a*x)^2)
```

Fricas [A] time = 1.58476, size = 38, normalized size = 2.38

$$-\frac{x^3}{2\,a\arctan{(ax)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*x^2+1)/arctan(a*x)^3-3/2*x^2/a/arctan(a*x)^2,x, algorith
m="fricas")
```

```
[Out] -1/2*x^3/(a*arctan(a*x)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int -\frac{2ax^3}{a^2x^2\operatorname{atan}^3{(ax)}+\operatorname{atan}^3{(ax)}}\,dx+\int \frac{3x^2\operatorname{atan}{(ax)}}{a^2x^2\operatorname{atan}^3{(ax)}+\operatorname{atan}^3{(ax)}}\,dx+\int \frac{3a^2x^4\operatorname{atan}{(ax)}}{a^2x^2\operatorname{atan}^3{(ax)}+\operatorname{atan}^3{(ax)}}\,dx}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a**2*x**2+1)/atan(a*x)**3-3/2*x**2/a/atan(a*x)**2,x)
```

```
[Out] -(Integral(-2*a*x**3/(a**2*x**2*atan(a*x)**3 + atan(a*x)**3), x) + Integral
(3*x**2*atan(a*x)/(a**2*x**2*atan(a*x)**3 + atan(a*x)**3), x) + Integral(3*
a**2*x**4*atan(a*x)/(a**2*x**2*atan(a*x)**3 + atan(a*x)**3), x))/(2*a)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{(a^2x^2+1)\arctan{(ax)}^3}-\frac{3\,x^2}{2\,a\arctan{(ax)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*x^2+1)/arctan(a*x)^3-3/2*x^2/a/arctan(a*x)^2,x, algorith
m="giac")
```

```
[Out] integrate(x^3/((a^2*x^2 + 1)*arctan(a*x)^3) - 3/2*x^2/(a*arctan(a*x)^2), x)
```

3.644 $\int \frac{x\sqrt{c+a^2cx^2}}{\tan^{-1}(ax)^3}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{x\sqrt{a^2cx^2+c}}{\tan^{-1}(ax)^3},x\right)$$

```
[Out] Unintegrable[(x*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^3, x]
```

Rubi [A] time = 0.0673714, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x\sqrt{c+a^2cx^2}}{\tan^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^3,x]
```

```
[Out] Defer[Int][(x*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^3, x]
```

Rubi steps

$$\int \frac{x\sqrt{c+a^2cx^2}}{\tan^{-1}(ax)^3}\,dx = \int \frac{x\sqrt{c+a^2cx^2}}{\tan^{-1}(ax)^3}\,dx$$

Mathematica [A] time = 1.79358, size = 0, normalized size = 0.

$$\int \frac{x\sqrt{c+a^2cx^2}}{\tan^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^3,x]
```

```
[Out] Integrate[(x*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^3, x]
```

Maple [A] time = 0.77, size = 0, normalized size = 0.

$$\int \frac{x}{\left(\arctan\left(ax\right)\right)^{3}}\sqrt{a^{2}c\,x^{2}+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x)
```

```
[Out] int(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a^{2}c x^{2}+c}\,x}{\arctan\left(ax\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)*x/arctan(a*x)^3, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{a^{2}c x^{2}+c}\,x}{\arctan\left(ax\right)^{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x/arctan(a*x)^3, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sqrt{c\left(a^2x^2+1\right)}}{\operatorname{atan}^3(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**(1/2)/atan(a*x)**3,x)
```

```
[Out] Integral(x*sqrt(c*(a**2*x**2 + 1))/atan(a*x)**3, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a^2cx^2+c}\,x}{\arctan(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)*x/arctan(a*x)^3, x)
```

3.645 $\int \frac{\sqrt{c+a^2cx^2}}{\tan^{-1}(ax)^3}\,dx$

Optimal. Leaf size=23

$$\text{Unintegrable}\left(\frac{\sqrt{a^2cx^2+c}}{\tan^{-1}(ax)^3},x\right)$$

```
[Out] Unintegrable[Sqrt[c + a^2*c*x^2]/ArcTan[a*x]^3, x]
```

Rubi [A] time = 0.0350956, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sqrt{c+a^2cx^2}}{\tan^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[c + a^2*c*x^2]/ArcTan[a*x]^3,x]
```

```
[Out] Defer[Int][Sqrt[c + a^2*c*x^2]/ArcTan[a*x]^3, x]
```

Rubi steps

$$\int \frac{\sqrt{c+a^2cx^2}}{\tan^{-1}(ax)^3}\,dx=\int \frac{\sqrt{c+a^2cx^2}}{\tan^{-1}(ax)^3}\,dx$$

Mathematica [A] time = 1.02541, size = 0, normalized size = 0.

$$\int \frac{\sqrt{c+a^2cx^2}}{\tan^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[c + a^2*c*x^2]/ArcTan[a*x]^3,x]
```

```
[Out] Integrate[Sqrt[c + a^2*c*x^2]/ArcTan[a*x]^3, x]
```

Maple [A] time = 0.762, size = 0, normalized size = 0.

$$\int \frac{1}{\left(\arctan \left(ax\right)\right)^{3}}\sqrt{a^{2}cx^{2}+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x)
```

```
[Out] int((a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a^{2}cx^{2}+c}}{\arctan \left(ax\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)/arctan(a*x)^3, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a^{2}cx^{2}+c}}{\arctan \left(ax\right)^{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)/arctan(a*x)^3, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c\left(a^2x^2+1\right)}}{\operatorname{atan}^3\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(1/2)/atan(a*x)**3,x)
```

```
[Out] Integral(sqrt(c*(a**2*x**2 + 1))/atan(a*x)**3, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a^2cx^2+c}}{\arctan\left(ax\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)/arctan(a*x)^3, x)
```

3.646 $\int \frac{\sqrt{c+a^2cx^2}}{x\tan^{-1}(ax)^3}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{\sqrt{a^2cx^2+c}}{x\tan^{-1}(ax)^3},x\right)$$

```
[Out] Unintegrable[Sqrt[c + a^2*c*x^2]/(x*ArcTan[a*x]^3), x]
```

Rubi [A] time = 0.115324, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sqrt{c+a^2cx^2}}{x\tan^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[c + a^2*c*x^2]/(x*ArcTan[a*x]^3),x]

[Out] Defer[Int][Sqrt[c + a^2*c*x^2]/(x*ArcTan[a*x]^3), x]
```

Rubi steps

$$\int \frac{\sqrt{c+a^2cx^2}}{x\tan^{-1}(ax)^3}\,dx=\int \frac{\sqrt{c+a^2cx^2}}{x\tan^{-1}(ax)^3}\,dx$$

Mathematica [A] time = 3.37709, size = 0, normalized size = 0.

$$\int \frac{\sqrt{c+a^2cx^2}}{x\tan^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[c + a^2*c*x^2]/(x*ArcTan[a*x]^3),x]
```

```
[Out] Integrate[Sqrt[c + a^2*c*x^2]/(x*ArcTan[a*x]^3), x]
```

Maple [A] time = 0.877, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(\arctan\left(ax\right)\right)^{3}}\sqrt{a^{2}c x^{2}+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^3,x)
```

```
[Out] int((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a^{2}c x^{2}+c}}{x\arctan\left(ax\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)/(x*arctan(a*x)^3), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{a^{2}c x^{2}+c}}{x\arctan\left(ax\right)^{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)/(x*arctan(a*x)^3), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c\left(a^2x^2+1\right)}}{x\,\mathrm{atan}^3\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(1/2)/x/atan(a*x)**3,x)
```

```
[Out] Integral(sqrt(c*(a**2*x**2 + 1))/(x*atan(a*x)**3), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a^2cx^2+c}}{x\arctan\left(ax\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)/(x*arctan(a*x)^3), x)
```

3.647 $\int \frac{x(c+a^2cx^2)^{3/2}}{\tan^{-1}(ax)^3}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{x\left(a^2cx^2+c\right)^{3/2}}{\tan^{-1}(ax)^3},x\right)$$

```
[Out] Unintegrable[(x*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^3, x]
```

Rubi [A] time = 0.0800006, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x\left(c+a^2cx^2\right)^{3/2}}{\tan^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^3,x]
```

```
[Out] Defer[Int][(x*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^3, x]
```

Rubi steps

$$\int \frac{x\left(c+a^2cx^2\right)^{3/2}}{\tan^{-1}(ax)^3}\,dx=\int \frac{x\left(c+a^2cx^2\right)^{3/2}}{\tan^{-1}(ax)^3}\,dx$$

Mathematica [A] time = 6.04173, size = 0, normalized size = 0.

$$\int \frac{x\left(c+a^2cx^2\right)^{3/2}}{\tan^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^3,x]
```

```
[Out] Integrate[(x*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^3, x]
```

Maple [A] time = 0.696, size = 0, normalized size = 0.

$$\int \frac{x}{\left(\arctan \left(ax\right)\right)^{3}} \left(a^{2}c x^{2}+c\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x)
```

```
[Out] int(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^{2}c x^{2}+c\right)^{\frac{3}{2}} x}{\arctan \left(ax\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)*x/arctan(a*x)^3, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(a^{2}c x^{3}+c x\right)\sqrt{a^{2}c x^{2}+c}}{\arctan \left(ax\right)^{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^3 + c*x)*sqrt(a^2*c*x^2 + c)/arctan(a*x)^3, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}}{\operatorname{atan}^3(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**(3/2)/atan(a*x)**3,x)
```

```
[Out] Integral(x*(c*(a**2*x**2 + 1))**(3/2)/atan(a*x)**3, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^{\frac{3}{2}}x}{\arctan(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)*x/arctan(a*x)^3, x)
```

3.648 $\int \frac{\left(c+a^2 c x^2\right)^{3/2}}{\tan^{-1}(a x)^3} \, dx$

Optimal. Leaf size=23

$$\text{Unintegrable}\left(\frac{\left(a^2 c x^2+c\right)^{3/2}}{\tan^{-1}(a x)^3},x\right)$$

```
[Out] Unintegrable[(c + a^2*c*x^2)^(3/2)/ArcTan[a*x]^3, x]
```

Rubi [A] time = 0.0369999, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(c+a^2 c x^2\right)^{3/2}}{\tan^{-1}(a x)^3} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^(3/2)/ArcTan[a*x]^3,x]
```

```
[Out] Defer[Int][(c + a^2*c*x^2)^(3/2)/ArcTan[a*x]^3, x]
```

Rubi steps

$$\int \frac{\left(c+a^2 c x^2\right)^{3/2}}{\tan^{-1}(a x)^3} \, dx=\int \frac{\left(c+a^2 c x^2\right)^{3/2}}{\tan^{-1}(a x)^3} \, dx$$

Mathematica [A] time = 1.07163, size = 0, normalized size = 0.

$$\int \frac{\left(c+a^2 c x^2\right)^{3/2}}{\tan^{-1}(a x)^3} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^(3/2)/ArcTan[a*x]^3,x]
```

[Out] Integrate[(c + a^2*c*x^2)^(3/2)/ArcTan[a*x]^3, x]

Maple [A] time = 0.68, size = 0, normalized size = 0.

$$\int \frac{1}{\left(\arctan \left(ax\right)\right)^{3}} \left(a^{2}c x^{2}+c\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x)

[Out] int((a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^{2}c x^{2}+c\right)^{\frac{3}{2}}}{\arctan \left(ax\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="maxima")

[Out] integrate((a^2*c*x^2 + c)^(3/2)/arctan(a*x)^3, x)

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(a^{2}c x^{2}+c\right)^{\frac{3}{2}}}{\arctan \left(ax\right)^{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="fricas")

[Out] integral((a^2*c*x^2 + c)^(3/2)/arctan(a*x)^3, x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}}{\operatorname{atan}^3(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(3/2)/atan(a*x)**3,x)
```

```
[Out] Integral((c*(a**2*x**2 + 1))**(3/2)/atan(a*x)**3, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^{\frac{3}{2}}}{\arctan(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)/arctan(a*x)^3, x)
```

3.649 $\int \frac{\left(c+a^2 c x^2\right)^{3/2}}{x \tan^{-1}(a x)^3} \, dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{\left(a^2 c x^2+c\right)^{3/2}}{x \tan^{-1}(a x)^3},x\right)$$

```
[Out] Unintegrable[(c + a^2*c*x^2)^(3/2)/(x*ArcTan[a*x]^3), x]
```

Rubi [A] time = 0.112814, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(c+a^2 c x^2\right)^{3/2}}{x \tan^{-1}(a x)^3} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^(3/2)/(x*ArcTan[a*x]^3),x]

[Out] Defer[Int][(c + a^2*c*x^2)^(3/2)/(x*ArcTan[a*x]^3), x]
```

Rubi steps

$$\int \frac{\left(c+a^2 c x^2\right)^{3/2}}{x \tan^{-1}(a x)^3} \, dx=\int \frac{\left(c+a^2 c x^2\right)^{3/2}}{x \tan^{-1}(a x)^3} \, dx$$

Mathematica [A] time = 4.07662, size = 0, normalized size = 0.

$$\int \frac{\left(c+a^2 c x^2\right)^{3/2}}{x \tan^{-1}(a x)^3} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^(3/2)/(x*ArcTan[a*x]^3),x]
```

```
[Out] Integrate[(c + a^2*c*x^2)^(3/2)/(x*ArcTan[a*x]^3), x]
```

Maple [A] time = 0.696, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(\arctan\left(ax\right)\right)^{3}}\left(a^{2}c x^{2}+c\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^3,x)
```

```
[Out] int((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^{2}c x^{2}+c\right)^{\frac{3}{2}}}{x\arctan\left(ax\right)^{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)/(x*arctan(a*x)^3), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(a^{2}c x^{2}+c\right)^{\frac{3}{2}}}{x\arctan\left(ax\right)^{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)^(3/2)/(x*arctan(a*x)^3), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}}{x\,\mathrm{atan}^3\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(3/2)/x/atan(a*x)**3,x)
```

```
[Out] Integral((c*(a**2*x**2 + 1))**(3/2)/(x*atan(a*x)**3), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^{\frac{3}{2}}}{x\arctan\left(ax\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)/(x*arctan(a*x)^3), x)
```

3.650 $\int \frac{x\left(c+a^2cx^2\right)^{5/2}}{\tan^{-1}(ax)^3}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{x\left(a^2cx^2+c\right)^{5/2}}{\tan^{-1}(ax)^3},x\right)$$

```
[Out] Unintegrable[(x*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^3, x]
```

Rubi [A] time = 0.0993194, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x\left(c+a^2cx^2\right)^{5/2}}{\tan^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^3,x]
```

```
[Out] Defer[Int][(x*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^3, x]
```

Rubi steps

$$\int \frac{x\left(c+a^2cx^2\right)^{5/2}}{\tan^{-1}(ax)^3}\,dx=\int \frac{x\left(c+a^2cx^2\right)^{5/2}}{\tan^{-1}(ax)^3}\,dx$$

Mathematica [A] time = 2.19122, size = 0, normalized size = 0.

$$\int \frac{x\left(c+a^2cx^2\right)^{5/2}}{\tan^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^3,x]
```

```
[Out] Integrate[(x*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^3, x]
```

Maple [A] time = 0.825, size = 0, normalized size = 0.

$$\int \frac{x}{(\arctan (ax))^3} \left(a^2cx^2+c\right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x)
```

```
[Out] int(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^{\frac{5}{2}}x}{\arctan (ax)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(5/2)*x/arctan(a*x)^3, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^4c^2x^5+2\,a^2c^2x^3+c^2x\right)\sqrt{a^2cx^2+c}}{\arctan (ax)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^5 + 2*a^2*c^2*x^3 + c^2*x)*sqrt(a^2*c*x^2 + c)/arctan(a
*x)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**(5/2)/atan(a*x)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2 c x^2+c\right)^{\frac{5}{2}} x}{\arctan \left(a x\right)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(5/2)*x/arctan(a*x)^3, x)
```

3.651 $\int \frac{\left(c+a^2cx^2\right)^{5/2}}{\tan^{-1}(ax)^3}\,dx$

Optimal. Leaf size=23

$$\text{Unintegrable}\left(\frac{\left(a^2cx^2+c\right)^{5/2}}{\tan^{-1}(ax)^3},x\right)$$

```
[Out] Unintegrable[(c + a^2*c*x^2)^(5/2)/ArcTan[a*x]^3, x]
```

Rubi [A] time = 0.0472571, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(c+a^2cx^2\right)^{5/2}}{\tan^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^(5/2)/ArcTan[a*x]^3,x]
```

```
[Out] Defer[Int][(c + a^2*c*x^2)^(5/2)/ArcTan[a*x]^3, x]
```

Rubi steps

$$\int \frac{\left(c+a^2cx^2\right)^{5/2}}{\tan^{-1}(ax)^3}\,dx=\int \frac{\left(c+a^2cx^2\right)^{5/2}}{\tan^{-1}(ax)^3}\,dx$$

Mathematica [A] time = 1.17616, size = 0, normalized size = 0.

$$\int \frac{\left(c+a^2cx^2\right)^{5/2}}{\tan^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^(5/2)/ArcTan[a*x]^3,x]
```

```
[Out] Integrate[(c + a^2*c*x^2)^(5/2)/ArcTan[a*x]^3, x]
```

Maple [A] time = 0.77, size = 0, normalized size = 0.

$$\int \frac{1}{\left(\arctan \left(ax\right)\right)^{3}}\left(a^{2}c x^{2}+c\right)^{\frac{5}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x)
```

```
[Out] int((a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^{2} c x^{2}+c\right)^{\frac{5}{2}}}{\arctan \left(ax\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(5/2)/arctan(a*x)^3, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(a^{4} c^{2} x^{4}+2\,a^{2} c^{2} x^{2}+c^{2}\right)\sqrt{a^{2} c x^{2}+c}}{\arctan \left(ax\right)^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)/arctan(a*x
)^3, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}}{\operatorname{atan}^3(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(5/2)/atan(a*x)**3,x)
```

```
[Out] Integral((c*(a**2*x**2 + 1))**(5/2)/atan(a*x)**3, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^{\frac{5}{2}}}{\arctan(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(5/2)/arctan(a*x)^3, x)
```

3.652 $\int \frac{(c+a^2cx^2)^{5/2}}{x\tan^{-1}(ax)^3}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{(a^2cx^2+c)^{5/2}}{x\tan^{-1}(ax)^3},x\right)$$

```
[Out] Unintegrable[(c + a^2*c*x^2)^(5/2)/(x*ArcTan[a*x]^3), x]
```

Rubi [A] time = 0.126013, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(c+a^2cx^2)^{5/2}}{x\tan^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^(5/2)/(x*ArcTan[a*x]^3),x]
```

```
[Out] Defer[Int][(c + a^2*c*x^2)^(5/2)/(x*ArcTan[a*x]^3), x]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^{5/2}}{x\tan^{-1}(ax)^3}\,dx = \int \frac{(c+a^2cx^2)^{5/2}}{x\tan^{-1}(ax)^3}\,dx$$

Mathematica [A] time = 2.53549, size = 0, normalized size = 0.

$$\int \frac{(c+a^2cx^2)^{5/2}}{x\tan^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^(5/2)/(x*ArcTan[a*x]^3),x]
```

```
[Out] Integrate[(c + a^2*c*x^2)^(5/2)/(x*ArcTan[a*x]^3), x]
```

Maple [A] time = 0.77, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(\arctan\left(ax\right)\right)^{3}}\left(a^{2}c x^{2}+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^3,x)
```

```
[Out] int((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^{2}c x^{2}+c\right)^{\frac{5}{2}}}{x \arctan\left(ax\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(5/2)/(x*arctan(a*x)^3), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(a^{4}c^{2}x^{4}+2\,a^{2}c^{2}x^{2}+c^{2}\right)\sqrt{a^{2}c x^{2}+c}}{x \arctan\left(ax\right)^{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)/(x*arctan(
a*x)^3), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c\left(a^{2}x^{2}+1\right)\right)^{\frac{5}{2}}}{x\,\mathrm{atan}^{3}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(5/2)/x/atan(a*x)**3,x)
```

```
[Out] Integral((c*(a**2*x**2 + 1))**(5/2)/(x*atan(a*x)**3), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^{2}cx^{2}+c\right)^{\frac{5}{2}}}{x\arctan\left(ax\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(5/2)/(x*arctan(a*x)^3), x)
```

3.653 $\int \frac{x}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{x}{\sqrt{a^2cx^2+c}\tan^{-1}(ax)^3},x\right)$$

```
[Out] Unintegrable[x/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3), x]
```

Rubi [A] time = 0.0824185, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3),x]
```

```
[Out] Defer[Int][x/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3), x]
```

Rubi steps

$$\int \frac{x}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}\,dx=\int \frac{x}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}\,dx$$

Mathematica [A] time = 1.26591, size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3),x]
```

```
[Out] Integrate[x/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3), x]
```

Maple [A] time = 0.693, size = 0, normalized size = 0.

$$\int \frac{x}{\left(\arctan \left(ax\right)\right)^{3}}\frac{1}{\sqrt{a^{2}c\,x^{2}+c}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(x/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{a^{2}cx^{2}+c}\arctan \left(ax\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(a^2*c*x^2 + c)*arctan(a*x)^3), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x}{\sqrt{a^{2}cx^{2}+c}\arctan \left(ax\right)^{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x/(sqrt(a^2*c*x^2 + c)*arctan(a*x)^3), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{c\left(a^{2}x^{2}+1\right)}\operatorname{atan}^{3}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/atan(a*x)**3/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(c*(a**2*x**2 + 1))*atan(a*x)**3), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{a^2cx^2+c}\arctan{(ax)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/(sqrt(a^2*c*x^2 + c)*arctan(a*x)^3), x)
```

3.654 $\int \frac{1}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}\,dx$

Optimal. Leaf size=23

$$\text{Unintegrable}\left(\frac{1}{\sqrt{a^2cx^2+c}\tan^{-1}(ax)^3},x\right)$$

```
[Out] Unintegrable[1/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3), x]
```

Rubi [A] time = 0.036585, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3),x]
```

```
[Out] Defer[Int][1/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3), x]
```

Rubi steps

$$\int \frac{1}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}\,dx=\int \frac{1}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}\,dx$$

Mathematica [A] time = 0.677937, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3),x]
```

```
[Out] Integrate[1/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3), x]
```

Maple [A] time = 0.803, size = 0, normalized size = 0.

$$\int \frac{1}{\left(\arctan\left(ax\right)\right)^{3}}\frac{1}{\sqrt{a^{2}c\,x^{2}+c}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(1/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a^{2}cx^{2}+c}\arctan\left(ax\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a^2*c*x^2 + c)*arctan(a*x)^3), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\sqrt{a^{2}cx^{2}+c}\arctan\left(ax\right)^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/(sqrt(a^2*c*x^2 + c)*arctan(a*x)^3), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{c\left(a^{2}x^{2}+1\right)}\operatorname{atan}^{3}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/atan(a*x)**3/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(c*(a**2*x**2 + 1))*atan(a*x)**3), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a^2cx^2+c}\arctan{(ax)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a^2*c*x^2 + c)*arctan(a*x)^3), x)
```

3.655 $\int \frac{1}{x\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}\,dx$

Optimal. Leaf size=67

$$-\frac{\text{Unintegrable}\left(\frac{1}{x^2\sqrt{a^2cx^2+c}\tan^{-1}(ax)^2},x\right)}{2a}-\frac{\sqrt{a^2cx^2+c}}{2acx\tan^{-1}(ax)^2}$$

```
[Out] -Sqrt[c + a^2*c*x^2]/(2*a*c*x*ArcTan[a*x]^2) - Unintegrable[1/(x^2*Sqrt[c +
 a^2*c*x^2]*ArcTan[a*x]^2), x]/(2*a)
```

Rubi [A] time = 0.216458, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3),x]
```

```
[Out] -Sqrt[c + a^2*c*x^2]/(2*a*c*x*ArcTan[a*x]^2) - Defer[Int][1/(x^2*Sqrt[c + a
^2*c*x^2]*ArcTan[a*x]^2), x]/(2*a)
```

Rubi steps

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}\,dx=-\frac{\sqrt{c+a^2cx^2}}{2acx\tan^{-1}(ax)^2}-\frac{\int \frac{1}{x^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}\,dx}{2a}$$

Mathematica [A] time = 3.57479, size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3),x]
```

```
[Out] Integrate[1/(x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3), x]
```

Maple [A] time = 0.838, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(\arctan\left(ax\right)\right)^{3}}\frac{1}{\sqrt{a^{2}cx^{2}+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(1/x/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a^{2}cx^{2}+c}\,x\arctan\left(ax\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a^2*c*x^2 + c)*x*arctan(a*x)^3), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{a^{2}cx^{2}+c}}{\left(a^{2}cx^{3}+cx\right)\arctan\left(ax\right)^{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)/((a^2*c*x^3 + c*x)*arctan(a*x)^3), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{c\left(a^2x^2+1\right)}\,\mathrm{atan}^3\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/atan(a*x)**3/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(c*(a**2*x**2 + 1))*atan(a*x)**3), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a^2cx^2+c}\,x\arctan\left(ax\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a^2*c*x^2 + c)*x*arctan(a*x)^3), x)
```

3.656 $\int \frac{1}{x^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{1}{x^2\sqrt{a^2cx^2+c}\tan^{-1}(ax)^3},x\right)$$

```
[Out] Unintegrable[1/(x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3), x]
```

Rubi [A] time = 0.10828, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3),x]

[Out] Defer[Int][1/(x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3), x]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}\,dx=\int \frac{1}{x^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}\,dx$$

Mathematica [A] time = 5.11106, size = 0, normalized size = 0.

$$\int \frac{1}{x^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3),x]

[Out] Integrate[1/(x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3), x]
```

Maple [A] time = 0.629, size = 0, normalized size = 0.

$$\int \frac{1}{x^2 (\arctan (ax))^3} \frac{1}{\sqrt{a^2 c x^2 + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(1/x^2/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a^2 c x^2 + c} x^2 \arctan (ax)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a^2*c*x^2 + c)*x^2*arctan(a*x)^3), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a^2 c x^2 + c}}{\left(a^2 c x^4 + c x^2\right) \arctan (ax)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)/((a^2*c*x^4 + c*x^2)*arctan(a*x)^3), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\sqrt{c\left(a^2x^2+1\right)}\operatorname{atan}^3\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/atan(a*x)**3/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(c*(a**2*x**2 + 1))*atan(a*x)**3), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a^2cx^2+c}x^2\arctan\left(ax\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a^2*c*x^2 + c)*x^2*arctan(a*x)^3), x)
```

3.657 $\int \frac{1}{x^3\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{1}{x^3\sqrt{a^2cx^2+c}\tan^{-1}(ax)^3},x\right)$$

```
[Out] Unintegrable[1/(x^3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3), x]
```

Rubi [A] time = 0.11056, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^3\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3),x]
```

```
[Out] Defer[Int][1/(x^3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3), x]
```

Rubi steps

$$\int \frac{1}{x^3\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}\,dx=\int \frac{1}{x^3\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}\,dx$$

Mathematica [A] time = 6.43649, size = 0, normalized size = 0.

$$\int \frac{1}{x^3\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3),x]
```

```
[Out] Integrate[1/(x^3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3), x]
```

Maple [A] time = 0.859, size = 0, normalized size = 0.

$$\int \frac{1}{x^{3} \left(\arctan \left(a x\right)\right)^{3}}\frac{1}{\sqrt{a^{2} c \,x^{2}+c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(1/x^3/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a^{2} c x^{2} + c} x^{3} \arctan \left(a x\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a^2*c*x^2 + c)*x^3*arctan(a*x)^3), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{a^{2} c x^{2} + c}}{{\left(a^{2} c x^{5} + c x^{3}\right)} \arctan \left(a x\right)^{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)/((a^2*c*x^5 + c*x^3)*arctan(a*x)^3), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3\sqrt{c\left(a^2x^2+1\right)}\operatorname{atan}^3\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/atan(a*x)**3/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt(c*(a**2*x**2 + 1))*atan(a*x)**3), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a^2cx^2+c}x^3\arctan\left(ax\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a^2*c*x^2 + c)*x^3*arctan(a*x)^3), x)
```

3.658 $\int \frac{x^3}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^3}\,dx$

Optimal. Leaf size=135

$$\frac{\text{Unintegrable}\left(\frac{x}{\sqrt{a^2cx^2+c}\tan^{-1}(ax)^3},x\right)}{a^2c}+\frac{\sqrt{a^2x^2+1}\,\text{Si}\left(\tan^{-1}(ax)\right)}{2a^4c\sqrt{a^2cx^2+c}}+\frac{x}{2a^3c\sqrt{a^2cx^2+c}\tan^{-1}(ax)^2}+\frac{1}{2a^4c\sqrt{a^2cx^2+c}\tan^{-1}(ax)}$$

```
[Out] x/(2*a^3*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2) + 1/(2*a^4*c*Sqrt[c + a^2*c*x
^2]*ArcTan[a*x]) + (Sqrt[1 + a^2*x^2]*SinIntegral[ArcTan[a*x]])/(2*a^4*c*Sq
rt[c + a^2*c*x^2]) + Unintegrable[x/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3), x]
/(a^2*c)
```

Rubi [A] time = 0.50715, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^3}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^3/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3),x]
```

```
[Out] x/(2*a^3*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2) + 1/(2*a^4*c*Sqrt[c + a^2*c*x
^2]*ArcTan[a*x]) + (Sqrt[1 + a^2*x^2]*SinIntegral[ArcTan[a*x]])/(2*a^4*c*Sq
rt[c + a^2*c*x^2]) + Defer[Int][x/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3), x]/(
a^2*c)
```

Rubi steps

$$\int \frac{x^3}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^3}\,dx = -\frac{\int \frac{x}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^3}\,dx}{a^2} + \frac{\int \frac{x}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}\,dx}{a^2c}$$

$$= \frac{x}{2a^3c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2} - \frac{\int \frac{1}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2}\,dx}{2a^3} + \frac{\int \frac{x}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}\,dx}{a^2c}$$

$$= \frac{x}{2a^3c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2} + \frac{1}{2a^4c\sqrt{c+a^2cx^2}\tan^{-1}(ax)} + \frac{\int \frac{x}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}\,dx}{2a^2} +$$

$$= \frac{x}{2a^3c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2} + \frac{1}{2a^4c\sqrt{c+a^2cx^2}\tan^{-1}(ax)} + \frac{\int \frac{x}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}\,dx}{a^2c} +$$

$$= \frac{x}{2a^3c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2} + \frac{1}{2a^4c\sqrt{c+a^2cx^2}\tan^{-1}(ax)} + \frac{\int \frac{x}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}\,dx}{a^2c} +$$

$$= \frac{x}{2a^3c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2} + \frac{1}{2a^4c\sqrt{c+a^2cx^2}\tan^{-1}(ax)} + \frac{\sqrt{1+a^2x^2}\,\mathrm{Si}\left(\tan^{-1}(ax)\right)}{2a^4c\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 3.37592, size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^3/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3),x]
```

```
[Out] Integrate[x^3/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3), x]
```

Maple [A] time = 1.299, size = 0, normalized size = 0.

$$\int \frac{x^3}{(\arctan(ax))^3}\left(a^2cx^2+c\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x)
```

```
[Out] int(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(a^2cx^2+c\right)^{\frac{3}{2}}\arctan\left(ax\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] integrate(x^3/((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^3), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a^2cx^2+c}\,x^3}{\left(a^4c^2x^4+2\,a^2c^2x^2+c^2\right)\arctan\left(ax\right)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^3/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arcta
n(a*x)^3), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\operatorname{atan}^3\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a**2*c*x**2+c)**(3/2)/atan(a*x)**3,x)
```

```
[Out] Integral(x**3/((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**3), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(a^2cx^2+c\right)^{\frac{3}{2}}\arctan\left(ax\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(x^3/((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^3), x)
```

3.659 $\int \frac{x^2}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^3}\,dx$

Optimal. Leaf size=134

$$\frac{\text{Unintegrable}\left(\frac{1}{\sqrt{a^2cx^2+c}\tan^{-1}(ax)^3},x\right)}{a^2c}+\frac{\sqrt{a^2x^2+1}\text{CosIntegral}\left(\tan^{-1}(ax)\right)}{2a^3c\sqrt{a^2cx^2+c}}-\frac{x}{2a^2c\sqrt{a^2cx^2+c}\tan^{-1}(ax)}+\frac{}{2a^3c\sqrt{a^2cx^2}}$$

```
[Out] 1/(2*a^3*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2) - x/(2*a^2*c*Sqrt[c + a^2*c*x
^2]*ArcTan[a*x]) + (Sqrt[1 + a^2*x^2]*CosIntegral[ArcTan[a*x]])/(2*a^3*c*Sq
rt[c + a^2*c*x^2]) + Unintegrable[1/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3), x]
/(a^2*c)
```

Rubi [A] time = 0.411795, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^2}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^2/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3),x]
```

```
[Out] 1/(2*a^3*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2) - x/(2*a^2*c*Sqrt[c + a^2*c*x
^2]*ArcTan[a*x]) + (Sqrt[1 + a^2*x^2]*CosIntegral[ArcTan[a*x]])/(2*a^3*c*Sq
rt[c + a^2*c*x^2]) + Defer[Int][1/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3), x]/(
a^2*c)
```

Rubi steps

$$\int \frac{x^2}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^3}\,dx = -\frac{\int \frac{1}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^3}\,dx}{a^2} + \frac{\int \frac{1}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}\,dx}{a^2c}$$

$$= \frac{1}{2a^3c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2} + \frac{\int \frac{x}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2}\,dx}{2a} + \frac{\int \frac{1}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}\,dx}{a^2c}$$

$$= \frac{1}{2a^3c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2} - \frac{x}{2a^2c\sqrt{c+a^2cx^2}\tan^{-1}(ax)} + \frac{\int \frac{1}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}\,dx}{2a^2} +$$

$$= \frac{1}{2a^3c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2} - \frac{x}{2a^2c\sqrt{c+a^2cx^2}\tan^{-1}(ax)} + \frac{\int \frac{1}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}\,dx}{a^2c} +$$

$$= \frac{1}{2a^3c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2} - \frac{x}{2a^2c\sqrt{c+a^2cx^2}\tan^{-1}(ax)} + \frac{\int \frac{1}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}\,dx}{a^2c} +$$

$$= \frac{1}{2a^3c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2} - \frac{x}{2a^2c\sqrt{c+a^2cx^2}\tan^{-1}(ax)} + \frac{\sqrt{1+a^2x^2}\,\mathrm{Ci}\left(\tan^{-1}(ax)\right)}{2a^3c\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 3.16582, size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^2/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3),x]
```

```
[Out] Integrate[x^2/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3), x]
```

Maple [A] time = 1.103, size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(\arctan(ax)\right)^3}\left(a^2cx^2+c\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x)
```

```
[Out] int(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(a^2cx^2+c\right)^{\frac{3}{2}}\arctan\left(ax\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] integrate(x^2/((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^3), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a^2cx^2+c}\,x^2}{\left(a^4c^2x^4+2\,a^2c^2x^2+c^2\right)\arctan\left(ax\right)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^2/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arcta
n(a*x)^3), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\operatorname{atan}^3\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a**2*c*x**2+c)**(3/2)/atan(a*x)**3,x)
```

```
[Out] Integral(x**2/((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**3), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(a^2cx^2+c\right)^{\frac{3}{2}}\arctan\left(ax\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(x^2/((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^3), x)
```

3.660 $\int \frac{x}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^3}\,dx$

Optimal. Leaf size=104

$$-\frac{\sqrt{a^2x^2+1}\,\mathrm{Si}\left(\tan^{-1}(ax)\right)}{2a^2c\sqrt{a^2cx^2+c}}-\frac{x}{2ac\sqrt{a^2cx^2+c}\tan^{-1}(ax)^2}-\frac{1}{2a^2c\sqrt{a^2cx^2+c}\tan^{-1}(ax)}$$

```
[Out] -x/(2*a*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2) - 1/(2*a^2*c*Sqrt[c + a^2*c*x^
2]*ArcTan[a*x]) - (Sqrt[1 + a^2*x^2]*SinIntegral[ArcTan[a*x]])/(2*a^2*c*Sqr
t[c + a^2*c*x^2])
```

Rubi [A] time = 0.322079, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {4942, 4902, 4971, 4970, 3299}

$$-\frac{\sqrt{a^2x^2+1}\,\mathrm{Si}\left(\tan^{-1}(ax)\right)}{2a^2c\sqrt{a^2cx^2+c}}-\frac{x}{2ac\sqrt{a^2cx^2+c}\tan^{-1}(ax)^2}-\frac{1}{2a^2c\sqrt{a^2cx^2+c}\tan^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[x/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3),x]
```


```
[Out] -x/(2*a*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2) - 1/(2*a^2*c*Sqrt[c + a^2*c*x^
2]*ArcTan[a*x]) - (Sqrt[1 + a^2*x^2]*SinIntegral[ArcTan[a*x]])/(2*a^2*c*Sqr
t[c + a^2*c*x^2])
```


Rule 4942

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^m*(d + e*x^2)^(q + 1)*(a + b*ArcT
an[c*x])^(p + 1))/(b*c*d*(p + 1)), x] - Dist[(f*m)/(b*c*(p + 1)), Int[(f*x)
^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 2, 0] && LtQ[p, -1]
```

Rule 4902

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^(p + 1))/(b*c*d*(p
+ 1)), x] - Dist[(2*c*(q + 1))/(b*(p + 1)), Int[x*(d + e*x^2)^q*(a + b*ArcT
an[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && L
```

```
tQ[q, -1] && LtQ[p, -1]
```

Rule 4971

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[(d^(q + 1/2)*Sqrt[1 + c^2*x^2])/Sqrt[d + e*x^2],
 Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d
, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(In
tegerQ[q] || GtQ[d, 0])
```

Rule 4970

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sin[x]^m)/
Cos[x]^(m + 2*(q + 1)), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\int \frac{x}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^3}\,dx = -\frac{x}{2ac\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}+\frac{\int \frac{1}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2}\,dx}{2a}$$

$$= -\frac{x}{2ac\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}-\frac{1}{2a^2c\sqrt{c+a^2cx^2}\tan^{-1}(ax)}-\frac{1}{2}\int \frac{x}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}$$

$$= -\frac{x}{2ac\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}-\frac{1}{2a^2c\sqrt{c+a^2cx^2}\tan^{-1}(ax)}-\frac{\sqrt{1+a^2x^2}\int \frac{x}{\left(1+a^2x^2\right)^{3/2}\tan}{2c\sqrt{c+a^2cx^2}}$$

$$= -\frac{x}{2ac\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}-\frac{1}{2a^2c\sqrt{c+a^2cx^2}\tan^{-1}(ax)}-\frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \frac{\sin(x}{x}}{2a^2c\sqrt{c+a}$$

$$= -\frac{x}{2ac\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}-\frac{1}{2a^2c\sqrt{c+a^2cx^2}\tan^{-1}(ax)}-\frac{\sqrt{1+a^2x^2}\,\text{Si}\left(\tan^{-1}(ax)\right)}{2a^2c\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 0.129822, size = 63, normalized size = 0.61

$$-\frac{\sqrt{a^2x^2+1}\tan^{-1}(ax)^2\mathrm{Si}\left(\tan^{-1}(ax)\right)+ax+\tan^{-1}(ax)}{2a^2c\sqrt{a^2cx^2+c}\tan^{-1}(ax)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3),x]
```

```
[Out] -(a*x + ArcTan[a*x] + Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*SinIntegral[ArcTan[a*
x]])/(2*a^2*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)
```

Maple [C] time = 0.331, size = 294, normalized size = 2.8

$$\frac{-\frac{i}{4}}{c^2(\arctan(ax))^2a^2}\left((\arctan(ax))^2\operatorname{Ei}(1,-i\arctan(ax))x^2a^2+\arctan(ax)\sqrt{a^2x^2+1}xa+\operatorname{Ei}(1,-i\arctan(ax))(\mathrm{arcta}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x)
```

```
[Out] -1/4*I*(arctan(a*x)^2*Ei(1,-I*arctan(a*x))*x^2*a^2+arctan(a*x)*(a^2*x^2+1)^
(1/2)*x*a+Ei(1,-I*arctan(a*x))*arctan(a*x)^2-I*(a^2*x^2+1)^(1/2)*x*a-I*arct
an(a*x)*(a^2*x^2+1)^(1/2)-(a^2*x^2+1)^(1/2))/(a^2*x^2+1)^(3/2)*(c*(a*x-I)*(
a*x+I))^(1/2)/arctan(a*x)^2/c^2/a^2+1/4*I*(arctan(a*x)^2*Ei(1,I*arctan(a*x)
)*x^2*a^2+arctan(a*x)*(a^2*x^2+1)^(1/2)*x*a+I*(a^2*x^2+1)^(1/2)*x*a+Ei(1,I*
arctan(a*x))*arctan(a*x)^2+I*arctan(a*x)*(a^2*x^2+1)^(1/2)-(a^2*x^2+1)^(1/2
))/(a^2*x^2+1)^(3/2)*(c*(a*x-I)*(a*x+I))^(1/2)/arctan(a*x)^2/c^2/a^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x}{\left(a^2cx^2+c\right)^{\frac{3}{2}}\arctan(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] integrate(x/((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a^2cx^2+c}\,x}{\left(a^4c^2x^4+2\,a^2c^2x^2+c^2\right)\arctan\left(ax\right)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(
a*x)^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\operatorname{atan}^3\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a**2*c*x**2+c)**(3/2)/atan(a*x)**3,x)
```

```
[Out] Integral(x/((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(a^2cx^2+c\right)^{\frac{3}{2}}\arctan\left(ax\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(x/((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^3), x)
```

3.661 $\int \frac{1}{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^3}\,dx$

Optimal. Leaf size=101

$$-\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}\left(\tan^{-1}(ax)\right)}{2ac\sqrt{a^2cx^2+c}}+\frac{x}{2c\sqrt{a^2cx^2+c}\tan^{-1}(ax)}-\frac{1}{2ac\sqrt{a^2cx^2+c}\tan^{-1}(ax)^2}$$

```
[Out] -1/(2*a*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2) + x/(2*c*Sqrt[c + a^2*c*x^2]*A
rcTan[a*x]) - (Sqrt[1 + a^2*x^2]*CosIntegral[ArcTan[a*x]])/(2*a*c*Sqrt[c +
a^2*c*x^2])
```

Rubi [A] time = 0.223709, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {4902, 4942, 4905, 4904, 3302}

$$-\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}\left(\tan^{-1}(ax)\right)}{2ac\sqrt{a^2cx^2+c}}+\frac{x}{2c\sqrt{a^2cx^2+c}\tan^{-1}(ax)}-\frac{1}{2ac\sqrt{a^2cx^2+c}\tan^{-1}(ax)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3),x]
```


```
[Out] -1/(2*a*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2) + x/(2*c*Sqrt[c + a^2*c*x^2]*A
rcTan[a*x]) - (Sqrt[1 + a^2*x^2]*CosIntegral[ArcTan[a*x]])/(2*a*c*Sqrt[c +
a^2*c*x^2])
```


Rule 4902

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^(p + 1))/(b*c*d*(p
+ 1)), x] - Dist[(2*c*(q + 1))/(b*(p + 1)), Int[x*(d + e*x^2)^q*(a + b*ArcT
an[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && L
tQ[q, -1] && LtQ[p, -1]
```

Rule 4942

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^m*(d + e*x^2)^(q + 1)*(a + b*ArcT
an[c*x])^(p + 1))/(b*c*d*(p + 1)), x] - Dist[(f*m)/(b*c*(p + 1)), Int[(f*x)
^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b,
```

```
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 2, 0] && LtQ[p, -1]
```

Rule 4905

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[(d^(q + 1/2)*Sqrt[1 + c^2*x^2])/Sqrt[d + e*x^2], Int[(1 + c
^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

Rule 4904

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{1}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^3}\,dx = -\frac{1}{2ac\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2} - \frac{1}{2}a\int \frac{x}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2}\,dx$$

$$= -\frac{1}{2ac\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2} + \frac{x}{2c\sqrt{c+a^2cx^2}\tan^{-1}(ax)} - \frac{1}{2}\int \frac{1}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(a}$$

$$= -\frac{1}{2ac\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2} + \frac{x}{2c\sqrt{c+a^2cx^2}\tan^{-1}(ax)} - \frac{\sqrt{1+a^2x^2}\int \frac{1}{\left(1+a^2x^2\right)^{3/2}\tan^{-1}}}{2c\sqrt{c+a^2cx^2}}$$

$$= -\frac{1}{2ac\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2} + \frac{x}{2c\sqrt{c+a^2cx^2}\tan^{-1}(ax)} - \frac{\sqrt{1+a^2x^2}\,\mathrm{Subst}\left(\int \frac{\cos(x)}{x}\right.}{2ac\sqrt{c+a^2c}}$$

$$= -\frac{1}{2ac\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2} + \frac{x}{2c\sqrt{c+a^2cx^2}\tan^{-1}(ax)} - \frac{\sqrt{1+a^2x^2}\,\mathbf{Ci}\left(\tan^{-1}(ax)\right)}{2ac\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 0.0750852, size = 65, normalized size = 0.64

$$\frac{-\sqrt{a^2x^2+1}\tan^{-1}(ax)^2\text{CosIntegral}\left(\tan^{-1}(ax)\right)+ax\tan^{-1}(ax)-1}{2ac\sqrt{a^2cx^2+c}\tan^{-1}(ax)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3),x]
```

```
[Out] (-1 + a*x*ArcTan[a*x] - Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*CosIntegral[ArcTan[
a*x]])/(2*a*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)
```

Maple [C] time = 0.303, size = 292, normalized size = 2.9

$$\frac{1}{4\,ac^2\left(\arctan(ax)\right)^2}\left(\left(\arctan(ax)\right)^2\text{Ei}\left(1,i\arctan(ax)\right)x^2a^2+\arctan(ax)\sqrt{a^2x^2+1}xa+i\sqrt{a^2x^2+1}xa+\text{Ei}\left(1,i\arctan\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x)
```

```
[Out] 1/4*(arctan(a*x)^2*Ei(1,I*arctan(a*x))*x^2*a^2+arctan(a*x)*(a^2*x^2+1)^(1/2
)*x*a+I*(a^2*x^2+1)^(1/2)*x*a+Ei(1,I*arctan(a*x))*arctan(a*x)^2+I*arctan(a*
x)*(a^2*x^2+1)^(1/2)-(a^2*x^2+1)^(1/2))/(a^2*x^2+1)^(3/2)*(c*(a*x-I)*(a*x+I
))^(1/2)/arctan(a*x)^2/c^2/a+1/4*(arctan(a*x)^2*Ei(1,-I*arctan(a*x))*x^2*a^
2+arctan(a*x)*(a^2*x^2+1)^(1/2)*x*a+Ei(1,-I*arctan(a*x))*arctan(a*x)^2-I*(a
^2*x^2+1)^(1/2)*x*a-I*arctan(a*x)*(a^2*x^2+1)^(1/2)-(a^2*x^2+1)^(1/2))/(a^2
*x^2+1)^(3/2)*(c*(a*x-I)*(a*x+I))^(1/2)/arctan(a*x)^2/c^2/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(a^2cx^2+c\right)^{\frac{3}{2}}\arctan(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a^2cx^2+c}}{\left(a^4c^2x^4+2\,a^2c^2x^2+c^2\right)\arctan\left(ax\right)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(a*
x)^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\operatorname{atan}^3\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*c*x**2+c)**(3/2)/atan(a*x)**3,x)
```

```
[Out] Integral(1/((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(a^2cx^2+c\right)^{\frac{3}{2}}\arctan\left(ax\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^3), x)
```

3.662 $\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^3}\,dx$

Optimal. Leaf size=165

$$-\frac{\text{Unintegrable}\left(\frac{1}{x^2\sqrt{a^2cx^2+c}\tan^{-1}(ax)^2},x\right)}{2ac}-\frac{\sqrt{a^2cx^2+c}}{2ac^2x\tan^{-1}(ax)^2}+\frac{\sqrt{a^2x^2+1}\text{Si}\left(\tan^{-1}(ax)\right)}{2c\sqrt{a^2cx^2+c}}+\frac{ax}{2c\sqrt{a^2cx^2+c}\tan^{-1}(ax)^2}+\frac{}{2}$$

```
[Out] (a*x)/(2*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2) - Sqrt[c + a^2*c*x^2]/(2*a*c^
2*x*ArcTan[a*x]^2) + 1/(2*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]) + (Sqrt[1 + a^
2*x^2]*SinIntegral[ArcTan[a*x]])/(2*c*Sqrt[c + a^2*c*x^2]) - Unintegrable[1
/(x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2), x]/(2*a*c)
```

Rubi [A] time = 0.629789, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3),x]
```

```
[Out] (a*x)/(2*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2) - Sqrt[c + a^2*c*x^2]/(2*a*c^
2*x*ArcTan[a*x]^2) + 1/(2*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]) + (Sqrt[1 + a^
2*x^2]*SinIntegral[ArcTan[a*x]])/(2*c*Sqrt[c + a^2*c*x^2]) - Defer[Int][1/(
x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2), x]/(2*a*c)
```

Rubi steps

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^3}\,dx = -\left(a^2\int \frac{x}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^3}\,dx\right) + \frac{\int \frac{1}{x\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}\,dx}{c}$$

$$= \frac{ax}{2c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2} - \frac{\sqrt{c+a^2cx^2}}{2ac^2x\tan^{-1}(ax)^2} - \frac{1}{2}a\int \frac{1}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2}\,dx$$

$$= \frac{ax}{2c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2} - \frac{\sqrt{c+a^2cx^2}}{2ac^2x\tan^{-1}(ax)^2} + \frac{1}{2c\sqrt{c+a^2cx^2}\tan^{-1}(ax)} + \frac{1}{2}a^2\int$$

$$= \frac{ax}{2c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2} - \frac{\sqrt{c+a^2cx^2}}{2ac^2x\tan^{-1}(ax)^2} + \frac{1}{2c\sqrt{c+a^2cx^2}\tan^{-1}(ax)} - \frac{\int \frac{}{x^2\sqrt{c+}}}{}$$

$$= \frac{ax}{2c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2} - \frac{\sqrt{c+a^2cx^2}}{2ac^2x\tan^{-1}(ax)^2} + \frac{1}{2c\sqrt{c+a^2cx^2}\tan^{-1}(ax)} - \frac{\int \frac{}{x^2\sqrt{c+}}}{}$$

$$= \frac{ax}{2c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2} - \frac{\sqrt{c+a^2cx^2}}{2ac^2x\tan^{-1}(ax)^2} + \frac{1}{2c\sqrt{c+a^2cx^2}\tan^{-1}(ax)} + \frac{\sqrt{1+a}}{2}$$

Mathematica [A] time = 2.405, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3),x]
```

```
[Out] Integrate[1/(x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3), x]
```

Maple [A] time = 0.559, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(\arctan(ax)\right)^3}\left(a^2cx^2+c\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x)
```

```
[Out] int(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)^{\frac{3}{2}}x\arctan\left(ax\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^(3/2)*x*arctan(a*x)^3), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{a^2cx^2+c}}{\left(a^4c^2x^5+2\,a^2c^2x^3+c^2x\right)\arctan\left(ax\right)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)/((a^4*c^2*x^5 + 2*a^2*c^2*x^3 + c^2*x)*arctan(
a*x)^3), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\operatorname{atan}^3\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a**2*c*x**2+c)**(3/2)/atan(a*x)**3,x)
```

```
[Out] Integral(1/(x*(c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**3), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)^{\frac{3}{2}}x\arctan\left(ax\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^(3/2)*x*arctan(a*x)^3), x)
```

3.663 $\int \frac{1}{x^2\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^3}\,dx$

Optimal. Leaf size=130

$$\frac{\text{Unintegrable}\left(\frac{1}{x^2\sqrt{a^2cx^2+c}\tan^{-1}(ax)^3},x\right)}{c}+\frac{a\sqrt{a^2x^2+1}\text{CosIntegral}\left(\tan^{-1}(ax)\right)}{2c\sqrt{a^2cx^2+c}}-\frac{a^2x}{2c\sqrt{a^2cx^2+c}\tan^{-1}(ax)}+\frac{}{2c\sqrt{a^2cx^2+}}$$

```
[Out] a/(2*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2) - (a^2*x)/(2*c*Sqrt[c + a^2*c*x^2
]*ArcTan[a*x]) + (a*Sqrt[1 + a^2*x^2]*CosIntegral[ArcTan[a*x]])/(2*c*Sqrt[c
 + a^2*c*x^2]) + Unintegrable[1/(x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3), x]
/c
```

Rubi [A] time = 0.462229, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3),x]
```

```
[Out] a/(2*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2) - (a^2*x)/(2*c*Sqrt[c + a^2*c*x^2
]*ArcTan[a*x]) + (a*Sqrt[1 + a^2*x^2]*CosIntegral[ArcTan[a*x]])/(2*c*Sqrt[c
 + a^2*c*x^2]) + Defer[Int][1/(x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3), x]/c
```

Rubi steps

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^3}\,dx = -\left(a^2\int \frac{1}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^3}\,dx\right)+\frac{\int \frac{1}{x^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}\,dx}{c}$$

$$= \frac{a}{2c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}+\frac{1}{2}a^3\int \frac{x}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2}\,dx+\frac{\int \frac{1}{x^2\sqrt{c+a^2cx^2}\tan}}{c}$$

$$= \frac{a}{2c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}-\frac{a^2x}{2c\sqrt{c+a^2cx^2}\tan^{-1}(ax)}+\frac{1}{2}a^2\int \frac{1}{\left(c+a^2cx^2\right)^{3/2}\tan^{-}}$$

$$= \frac{a}{2c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}-\frac{a^2x}{2c\sqrt{c+a^2cx^2}\tan^{-1}(ax)}+\frac{\int \frac{1}{x^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}\,dx}{c}+$$

$$= \frac{a}{2c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}-\frac{a^2x}{2c\sqrt{c+a^2cx^2}\tan^{-1}(ax)}+\frac{\int \frac{1}{x^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}\,dx}{c}+$$

$$= \frac{a}{2c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}-\frac{a^2x}{2c\sqrt{c+a^2cx^2}\tan^{-1}(ax)}+\frac{a\sqrt{1+a^2x^2}\,\mathrm{Ci}\left(\tan^{-1}(ax)\right)}{2c\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 2.47986, size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3),x]
```

```
[Out] Integrate[1/(x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3), x]
```

Maple [A] time = 0.595, size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(\arctan(ax)\right)^3}\left(a^2cx^2+c\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x)
```

```
[Out] int(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)^{\frac{3}{2}}x^2\arctan\left(ax\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^(3/2)*x^2*arctan(a*x)^3), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{a^2cx^2+c}}{\left(a^4c^2x^6+2\,a^2c^2x^4+c^2x^2\right)\arctan\left(ax\right)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)/((a^4*c^2*x^6 + 2*a^2*c^2*x^4 + c^2*x^2)*arcta
n(a*x)^3), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\operatorname{atan}^3\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a**2*c*x**2+c)**(3/2)/atan(a*x)**3,x)
```

```
[Out] Integral(1/(x**2*(c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**3), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2 + c\right)^{\frac{3}{2}} x^2 \arctan\left(ax\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^(3/2)*x^2*arctan(a*x)^3), x)
```

3.664 $\int \frac{1}{x^3(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^3}\,dx$

Optimal. Leaf size=199

$$\frac{a\text{Unintegrable}\left(\frac{1}{x^2\sqrt{a^2cx^2+c}\tan^{-1}(ax)^2},x\right)}{2c}+\frac{\text{Unintegrable}\left(\frac{1}{x^3\sqrt{a^2cx^2+c}\tan^{-1}(ax)^3},x\right)}{c}+\frac{a\sqrt{a^2cx^2+c}}{2c^2x\tan^{-1}(ax)^2}-\frac{a^2\sqrt{a^2x^2+1}\text{Si}\left(\text{ta}\right.}{2c\sqrt{a^2cx^2}}$$

```
[Out] -(a^3*x)/(2*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2) + (a*Sqrt[c + a^2*c*x^2])/
(2*c^2*x*ArcTan[a*x]^2) - a^2/(2*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]) - (a^2*
Sqrt[1 + a^2*x^2]*SinIntegral[ArcTan[a*x]])/(2*c*Sqrt[c + a^2*c*x^2]) + Uni
ntegrable[1/(x^3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3), x]/c + (a*Unintegrable
[1/(x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2), x])/(2*c)
```

Rubi [A] time = 0.877665, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^3\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^3*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3),x]
```

```
[Out] -(a^3*x)/(2*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2) + (a*Sqrt[c + a^2*c*x^2])/
(2*c^2*x*ArcTan[a*x]^2) - a^2/(2*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]) - (a^2*
Sqrt[1 + a^2*x^2]*SinIntegral[ArcTan[a*x]])/(2*c*Sqrt[c + a^2*c*x^2]) + Def
er[Int][1/(x^3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3), x]/c + (a*Defer[Int][1/(
x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2), x])/(2*c)
```

Rubi steps

$$\int \frac{1}{x^3\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^3}\,dx = -\left(a^2\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^3}\,dx\right) + \frac{\int \frac{1}{x^3\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}\,dx}{c}$$

$$= a^4\int \frac{x}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^3}\,dx + \frac{\int \frac{1}{x^3\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}\,dx}{c} - \frac{a^2\int \frac{1}{x\sqrt{c+a^2cx^2}\tan^{-1}}}{c}$$

$$= -\frac{a^3x}{2c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2} + \frac{a\sqrt{c+a^2cx^2}}{2c^2x\tan^{-1}(ax)^2} + \frac{1}{2}a^3\int \frac{1}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2}$$

$$= -\frac{a^3x}{2c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2} + \frac{a\sqrt{c+a^2cx^2}}{2c^2x\tan^{-1}(ax)^2} - \frac{a^2}{2c\sqrt{c+a^2cx^2}\tan^{-1}(ax)} - \frac{1}{2}a^4\int$$

$$= -\frac{a^3x}{2c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2} + \frac{a\sqrt{c+a^2cx^2}}{2c^2x\tan^{-1}(ax)^2} - \frac{a^2}{2c\sqrt{c+a^2cx^2}\tan^{-1}(ax)} + \frac{\int \frac{}{x^3\sqrt{c}}}{}$$

$$= -\frac{a^3x}{2c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2} + \frac{a\sqrt{c+a^2cx^2}}{2c^2x\tan^{-1}(ax)^2} - \frac{a^2}{2c\sqrt{c+a^2cx^2}\tan^{-1}(ax)} + \frac{\int \frac{}{x^3\sqrt{c}}}{}$$

$$= -\frac{a^3x}{2c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2} + \frac{a\sqrt{c+a^2cx^2}}{2c^2x\tan^{-1}(ax)^2} - \frac{a^2}{2c\sqrt{c+a^2cx^2}\tan^{-1}(ax)} - \frac{a^2\sqrt{1}}{}$$

Mathematica [A] time = 3.13534, size = 0, normalized size = 0.

$$\int \frac{1}{x^3\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^3*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3),x]
```

```
[Out] Integrate[1/(x^3*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3), x]
```

Maple [A] time = 0.764, size = 0, normalized size = 0.

$$\int \frac{1}{x^3\left(\arctan(ax)\right)^3}\left(a^2cx^2+c\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x)
```

```
[Out] int(1/x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)^{\frac{3}{2}}x^3\arctan\left(ax\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^(3/2)*x^3*arctan(a*x)^3), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a^2cx^2+c}}{\left(a^4c^2x^7+2\,a^2c^2x^5+c^2x^3\right)\arctan\left(ax\right)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)/((a^4*c^2*x^7 + 2*a^2*c^2*x^5 + c^2*x^3)*arcta
n(a*x)^3), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\operatorname{atan}^3\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a**2*c*x**2+c)**(3/2)/atan(a*x)**3,x)
```

```
[Out] Integral(1/(x**3*(c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**3), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)^{\frac{3}{2}}x^3\arctan\left(ax\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^(3/2)*x^3*arctan(a*x)^3), x)
```

3.665 $\int \frac{1}{x^4(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^3}\,dx$

Optimal. Leaf size=168

$$-\frac{a^2\text{Unintegrable}\left(\frac{1}{x^2\sqrt{a^2cx^2+c}\tan^{-1}(ax)^3},x\right)}{c}+\frac{\text{Unintegrable}\left(\frac{1}{x^4\sqrt{a^2cx^2+c}\tan^{-1}(ax)^3},x\right)}{c}-\frac{a^3\sqrt{a^2x^2+1}\text{CosIntegral}\left(\tan^{-1}(a\right.}{2c\sqrt{a^2cx^2+c}}$$

```
[Out] -a^3/(2*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2) + (a^4*x)/(2*c*Sqrt[c + a^2*c*
x^2]*ArcTan[a*x]) - (a^3*Sqrt[1 + a^2*x^2]*CosIntegral[ArcTan[a*x]])/(2*c*S
qrt[c + a^2*c*x^2]) + Unintegrable[1/(x^4*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3
), x]/c - (a^2*Unintegrable[1/(x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3), x])/
c
```

Rubi [A] time = 0.683791, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^4\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^4*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3),x]
```

```
[Out] -a^3/(2*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2) + (a^4*x)/(2*c*Sqrt[c + a^2*c*
x^2]*ArcTan[a*x]) - (a^3*Sqrt[1 + a^2*x^2]*CosIntegral[ArcTan[a*x]])/(2*c*S
qrt[c + a^2*c*x^2]) + Defer[Int][1/(x^4*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3),
 x]/c - (a^2*Defer[Int][1/(x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3), x])/c
```

Rubi steps

$$\int \frac{1}{x^4 \left(c+a^2cx^2\right)^{3/2} \tan^{-1}(ax)^3}\,dx = -\left(a^2 \int \frac{1}{x^2 \left(c+a^2cx^2\right)^{3/2} \tan^{-1}(ax)^3}\,dx\right) + \frac{\int \frac{1}{x^4\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}\,dx}{c}$$

$$= a^4 \int \frac{1}{\left(c+a^2cx^2\right)^{3/2} \tan^{-1}(ax)^3}\,dx + \frac{\int \frac{1}{x^4\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}\,dx}{c} - \frac{a^2 \int \frac{1}{x^2\sqrt{c+a^2cx^2}\tan^{-}}}{c}$$

$$= -\frac{a^3}{2c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2} - \frac{1}{2}a^5 \int \frac{x}{\left(c+a^2cx^2\right)^{3/2} \tan^{-1}(ax)^2}\,dx + \frac{\int \frac{1}{x^4\sqrt{c+a^2cx^2}\,\mathrm{ta}}}{c}$$

$$= -\frac{a^3}{2c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2} + \frac{a^4x}{2c\sqrt{c+a^2cx^2}\tan^{-1}(ax)} - \frac{1}{2}a^4 \int \frac{1}{\left(c+a^2cx^2\right)^{3/2}\tan}$$

$$= -\frac{a^3}{2c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2} + \frac{a^4x}{2c\sqrt{c+a^2cx^2}\tan^{-1}(ax)} + \frac{\int \frac{1}{x^4\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}\,dx}{c} -$$

$$= -\frac{a^3}{2c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2} + \frac{a^4x}{2c\sqrt{c+a^2cx^2}\tan^{-1}(ax)} + \frac{\int \frac{1}{x^4\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}\,dx}{c} -$$

$$= -\frac{a^3}{2c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2} + \frac{a^4x}{2c\sqrt{c+a^2cx^2}\tan^{-1}(ax)} - \frac{a^3\sqrt{1+a^2x^2}\,\mathrm{Ci}\left(\tan^{-1}(ax\right.}{2c\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 6.98047, size = 0, normalized size = 0.

$$\int \frac{1}{x^4 \left(c+a^2cx^2\right)^{3/2} \tan^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^4*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3),x]
```

```
[Out] Integrate[1/(x^4*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3), x]
```

Maple [A] time = 1.14, size = 0, normalized size = 0.

$$\int \frac{1}{x^4 \left(\arctan(ax)\right)^3} \left(a^2cx^2+c\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x)
```

```
[Out] int(1/x^4/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)^{\frac{3}{2}}x^4\arctan\left(ax\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^(3/2)*x^4*arctan(a*x)^3), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a^2cx^2+c}}{\left(a^4c^2x^8+2\,a^2c^2x^6+c^2x^4\right)\arctan\left(ax\right)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)/((a^4*c^2*x^8 + 2*a^2*c^2*x^6 + c^2*x^4)*arcta
n(a*x)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(a**2*c*x**2+c)**(3/2)/atan(a*x)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)^{\frac{3}{2}}x^4\arctan\left(ax\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^(3/2)*x^4*arctan(a*x)^3), x)
```

3.666 $\int \frac{x^5}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^3}\,dx$

Optimal. Leaf size=240

$$\frac{\text{Unintegrable}\left(\frac{x}{\sqrt{a^2cx^2+c}\tan^{-1}(ax)^3},x\right)}{a^4c^2}+\frac{7\sqrt{a^2x^2+1}\text{Si}\left(\tan^{-1}(ax)\right)}{8a^6c^2\sqrt{a^2cx^2+c}}-\frac{9\sqrt{a^2x^2+1}\text{Si}\left(3\tan^{-1}(ax)\right)}{8a^6c^2\sqrt{a^2cx^2+c}}+\frac{x}{2a^5c^2\sqrt{a^2cx^2+c}\tan$$

```
[Out] x^3/(2*a^3*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2) + x/(2*a^5*c^2*Sqrt[c + a
^2*c*x^2]*ArcTan[a*x]^2) - 3/(2*a^6*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]) +
2/(a^6*c^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]) + (7*Sqrt[1 + a^2*x^2]*SinInteg
ral[ArcTan[a*x]])/(8*a^6*c^2*Sqrt[c + a^2*c*x^2]) - (9*Sqrt[1 + a^2*x^2]*Si
nIntegral[3*ArcTan[a*x]])/(8*a^6*c^2*Sqrt[c + a^2*c*x^2]) + Unintegrable[x/
(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3), x]/(a^4*c^2)
```

Rubi [A] time = 1.35826, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^5}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^5/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3),x]
```

```
[Out] x^3/(2*a^3*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2) + x/(2*a^5*c^2*Sqrt[c + a
^2*c*x^2]*ArcTan[a*x]^2) - 3/(2*a^6*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]) +
2/(a^6*c^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]) + (7*Sqrt[1 + a^2*x^2]*SinInteg
ral[ArcTan[a*x]])/(8*a^6*c^2*Sqrt[c + a^2*c*x^2]) - (9*Sqrt[1 + a^2*x^2]*Si
nIntegral[3*ArcTan[a*x]])/(8*a^6*c^2*Sqrt[c + a^2*c*x^2]) + Defer[Int][x/(S
qrt[c + a^2*c*x^2]*ArcTan[a*x]^3), x]/(a^4*c^2)
```

Rubi steps

$$\int \frac{x^5}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^3}\,dx = -\frac{\int \frac{x^3}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^3}\,dx}{a^2} + \frac{\int \frac{x^3}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^3}\,dx}{a^2c}$$

$$= \frac{x^3}{2a^3c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2} - \frac{3\int \frac{x^2}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^2}\,dx}{2a^3} + \frac{\int \frac{x}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}\,dx}{a^4c^2}$$

$$= \frac{x^3}{2a^3c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2} + \frac{x}{2a^5c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2} + \frac{3\int \frac{1}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(a}}{2a^5}$$

$$= \frac{x^3}{2a^3c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2} + \frac{x}{2a^5c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2} - \frac{3}{2a^6c\left(c+a^2cx^2\right)^{3/2}\mathrm{t}}$$

$$= \frac{x^3}{2a^3c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2} + \frac{x}{2a^5c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2} - \frac{3}{2a^6c\left(c+a^2cx^2\right)^{3/2}\mathrm{t}}$$

$$= \frac{x^3}{2a^3c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2} + \frac{x}{2a^5c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2} - \frac{3}{2a^6c\left(c+a^2cx^2\right)^{3/2}\mathrm{t}}$$

$$= \frac{x^3}{2a^3c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2} + \frac{x}{2a^5c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2} - \frac{3}{2a^6c\left(c+a^2cx^2\right)^{3/2}\mathrm{t}}$$

$$= \frac{x^3}{2a^3c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2} + \frac{x}{2a^5c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2} - \frac{3}{2a^6c\left(c+a^2cx^2\right)^{3/2}\mathrm{t}}$$

$$= \frac{x^3}{2a^3c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2} + \frac{x}{2a^5c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2} - \frac{3}{2a^6c\left(c+a^2cx^2\right)^{3/2}\mathrm{t}}$$

Mathematica [A] time = 7.71025, size = 0, normalized size = 0.

$$\int \frac{x^5}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^5/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3),x]
```

```
[Out] Integrate[x^5/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3), x]
```

Maple [A] time = 1.859, size = 0, normalized size = 0.

$$\int \frac{x^5}{\left(\arctan\left(ax\right)\right)^3}\left(a^2cx^2+c\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x)
```

```
[Out] int(x^5/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\left(a^2cx^2+c\right)^{\frac{5}{2}}\arctan\left(ax\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] integrate(x^5/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^3), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a^2cx^2+c}\,x^5}{\left(a^6c^3x^6+3\,a^4c^3x^4+3\,a^2c^3x^2+c^3\right)\arctan\left(ax\right)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^5/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*
x^2 + c^3)*arctan(a*x)^3), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}\operatorname{atan}^3\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(a**2*c*x**2+c)**(5/2)/atan(a*x)**3,x)
```

```
[Out] Integral(x**5/((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**3), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\left(a^2cx^2+c\right)^{\frac{5}{2}}\arctan\left(ax\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(x^5/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^3), x)
```

3.667 $\int \frac{x^4}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^3}\,dx$

Optimal. Leaf size=234

$$\frac{\text{Unintegrable}\left(\frac{1}{\sqrt{a^2cx^2+c}\tan^{-1}(ax)^3},x\right)}{a^4c^2}+\frac{5\sqrt{a^2x^2+1}\text{CosIntegral}\left(\tan^{-1}(ax)\right)}{8a^5c^2\sqrt{a^2cx^2+c}}-\frac{9\sqrt{a^2x^2+1}\text{CosIntegral}\left(3\tan^{-1}(ax)\right)}{8a^5c^2\sqrt{a^2cx^2+c}}-$$

```
[Out] -1/(2*a^5*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2) + 1/(a^5*c^2*Sqrt[c + a^2*
c*x^2]*ArcTan[a*x]^2) + (3*x)/(2*a^4*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]) -
 x/(a^4*c^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]) + (5*Sqrt[1 + a^2*x^2]*CosInte
gral[ArcTan[a*x]])/(8*a^5*c^2*Sqrt[c + a^2*c*x^2]) - (9*Sqrt[1 + a^2*x^2]*C
osIntegral[3*ArcTan[a*x]])/(8*a^5*c^2*Sqrt[c + a^2*c*x^2]) + Unintegrable[1
/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3), x]/(a^4*c^2)
```

Rubi [A] time = 1.44342, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^4}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^4/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3),x]
```

```
[Out] -1/(2*a^5*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2) + 1/(a^5*c^2*Sqrt[c + a^2*
c*x^2]*ArcTan[a*x]^2) + (3*x)/(2*a^4*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]) -
 x/(a^4*c^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]) + (5*Sqrt[1 + a^2*x^2]*CosInte
gral[ArcTan[a*x]])/(8*a^5*c^2*Sqrt[c + a^2*c*x^2]) - (9*Sqrt[1 + a^2*x^2]*C
osIntegral[3*ArcTan[a*x]])/(8*a^5*c^2*Sqrt[c + a^2*c*x^2]) + Defer[Int][1/(
Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3), x]/(a^4*c^2)
```

Rubi steps

$$\int \frac{x^4}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^3}\,dx = -\frac{\int \frac{x^2}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^3}\,dx}{a^2} + \frac{\int \frac{x^2}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^3}\,dx}{a^2c}$$

$$= \frac{\int \frac{1}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^3}\,dx}{a^4} + \frac{\int \frac{1}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}\,dx}{a^4c^2} - 2\frac{\int \frac{1}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^3}\,dx}{a^4c}$$

$$= -\frac{1}{2a^5c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2} - \frac{3\int \frac{x}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^2}\,dx}{2a^3} + \frac{\int \frac{1}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}\,d}{a^4c^2}$$

$$= -\frac{1}{2a^5c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2} + \frac{3x}{2a^4c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)} - \frac{3\int \frac{1}{\left(c+a^2cx^2\right)^{5/2}\tan^{-}}}{2a^4}$$

$$= -\frac{1}{2a^5c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2} + \frac{3x}{2a^4c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)} + \frac{\int \frac{1}{\sqrt{c+a^2cx^2}\tan^{-1}(ax}}{a^4c^2}$$

$$= -\frac{1}{2a^5c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2} + \frac{3x}{2a^4c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)} + \frac{\int \frac{1}{\sqrt{c+a^2cx^2}\tan^{-1}(ax}}{a^4c^2}$$

$$= -\frac{1}{2a^5c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2} + \frac{3x}{2a^4c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)} - 2\left(-\frac{}{2a^5c^2\sqrt{c+a^2}}\right.$$

$$= -\frac{1}{2a^5c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2} + \frac{3x}{2a^4c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)} - 2\left(-\frac{}{2a^5c^2\sqrt{c+a^2}}\right.$$

$$= -\frac{1}{2a^5c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2} + \frac{3x}{2a^4c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)} - \frac{3\sqrt{1+a^2x^2}\,\mathrm{Ci}\left(\mathrm{ta}\right.}{8a^5c^2\sqrt{c+a}}$$

Mathematica [A] time = 7.41571, size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^4/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3),x]
```

```
[Out] Integrate[x^4/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3), x]
```

Maple [A] time = 1.25, size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(\arctan\left(ax\right)\right)^3}\left(a^2cx^2+c\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x)
```

```
[Out] int(x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(a^2cx^2+c\right)^{\frac{5}{2}}\arctan\left(ax\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] integrate(x^4/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^3), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a^2cx^2+c}\,x^4}{\left(a^6c^3x^6+3\,a^4c^3x^4+3\,a^2c^3x^2+c^3\right)\arctan\left(ax\right)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^4/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*
x^2 + c^3)*arctan(a*x)^3), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}} \operatorname{atan}^3(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(a**2*c*x**2+c)**(5/2)/atan(a*x)**3,x)
```

```
[Out] Integral(x**4/((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**3), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(a^2cx^2+c\right)^{\frac{5}{2}} \arctan(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(x^4/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^3), x)
```

3.668 $\int \frac{x^3}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^3}\,dx$

Optimal. Leaf size=180

$$-\frac{3\sqrt{a^2x^2+1}\operatorname{Si}\left(\tan^{-1}(ax)\right)}{8a^4c^2\sqrt{a^2cx^2+c}}+\frac{9\sqrt{a^2x^2+1}\operatorname{Si}\left(3\tan^{-1}(ax)\right)}{8a^4c^2\sqrt{a^2cx^2+c}}-\frac{3}{2a^4c^2\sqrt{a^2cx^2+c}\tan^{-1}(ax)}-\frac{x^3}{2ac\left(a^2cx^2+c\right)^{3/2}\tan^{-1}(ax)^2}$$

```
[Out] -x^3/(2*a*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2) + 3/(2*a^4*c*(c + a^2*c*x^
2)^(3/2)*ArcTan[a*x]) - 3/(2*a^4*c^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]) - (3*
Sqrt[1 + a^2*x^2]*SinIntegral[ArcTan[a*x]])/(8*a^4*c^2*Sqrt[c + a^2*c*x^2])
 + (9*Sqrt[1 + a^2*x^2]*SinIntegral[3*ArcTan[a*x]])/(8*a^4*c^2*Sqrt[c + a^2
*c*x^2])
```

Rubi [A] time = 0.714658, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {4942, 4964, 4902, 4971, 4970, 3299, 4406}

$$-\frac{3\sqrt{a^2x^2+1}\operatorname{Si}\left(\tan^{-1}(ax)\right)}{8a^4c^2\sqrt{a^2cx^2+c}}+\frac{9\sqrt{a^2x^2+1}\operatorname{Si}\left(3\tan^{-1}(ax)\right)}{8a^4c^2\sqrt{a^2cx^2+c}}-\frac{3}{2a^4c^2\sqrt{a^2cx^2+c}\tan^{-1}(ax)}-\frac{x^3}{2ac\left(a^2cx^2+c\right)^{3/2}\tan^{-1}(ax)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3),x]
```

```
[Out] -x^3/(2*a*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2) + 3/(2*a^4*c*(c + a^2*c*x^
2)^(3/2)*ArcTan[a*x]) - 3/(2*a^4*c^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]) - (3*
Sqrt[1 + a^2*x^2]*SinIntegral[ArcTan[a*x]])/(8*a^4*c^2*Sqrt[c + a^2*c*x^2])
 + (9*Sqrt[1 + a^2*x^2]*SinIntegral[3*ArcTan[a*x]])/(8*a^4*c^2*Sqrt[c + a^2
*c*x^2])
```

Rule 4942

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^m*(d + e*x^2)^(q + 1)*(a + b*ArcT
an[c*x])^(p + 1))/(b*c*d*(p + 1)), x] - Dist[(f*m)/(b*c*(p + 1)), Int[(f*x)
^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 2, 0] && LtQ[p, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Arc
Tan[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan[c
*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p
, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rule 4902

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^(p + 1))/(b*c*d*(p
+ 1)), x] - Dist[(2*c*(q + 1))/(b*(p + 1)), Int[x*(d + e*x^2)^q*(a + b*ArcT
an[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && L
tQ[q, -1] && LtQ[p, -1]
```

Rule 4971

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[(d^(q + 1/2)*Sqrt[1 + c^2*x^2])/Sqrt[d + e*x^2],
 Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d
, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(In
tegerQ[q] || GtQ[d, 0])
```

Rule 4970

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sin[x]^m)/
Cos[x]^(m + 2*(q + 1)), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \frac{x^3}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^3}\,dx = -\frac{x^3}{2ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2} + \frac{3\int \frac{x^2}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^2}\,dx}{2a}$$

$$= -\frac{x^3}{2ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2} - \frac{3\int \frac{1}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^2}\,dx}{2a^3} + \frac{3\int \frac{1}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2}\,d}{2a^3c}$$

$$= -\frac{x^3}{2ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2} + \frac{3}{2a^4c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)} - \frac{3}{2a^4c^2\sqrt{c+a^2cx^2}\tan}$$

$$= -\frac{x^3}{2ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2} + \frac{3}{2a^4c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)} - \frac{3}{2a^4c^2\sqrt{c+a^2cx^2}\tan}$$

$$= -\frac{x^3}{2ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2} + \frac{3}{2a^4c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)} - \frac{3}{2a^4c^2\sqrt{c+a^2cx^2}\tan}$$

$$= -\frac{x^3}{2ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2} + \frac{3}{2a^4c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)} - \frac{3}{2a^4c^2\sqrt{c+a^2cx^2}\tan}$$

$$= -\frac{x^3}{2ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2} + \frac{3}{2a^4c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)} - \frac{3}{2a^4c^2\sqrt{c+a^2cx^2}\tan}$$

$$= -\frac{x^3}{2ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2} + \frac{3}{2a^4c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)} - \frac{3}{2a^4c^2\sqrt{c+a^2cx^2}\tan}$$

Mathematica [A] time = 0.312128, size = 114, normalized size = 0.63

$$\frac{-3\left(a^2x^2+1\right)^{3/2}\tan^{-1}(ax)^2\mathrm{Si}\left(\tan^{-1}(ax)\right)+9\left(a^2x^2+1\right)^{3/2}\tan^{-1}(ax)^2\mathrm{Si}\left(3\tan^{-1}(ax)\right)-4a^2x^2\left(ax+3\tan^{-1}(ax)\right)}{8a^4c^2\left(a^2x^2+1\right)\sqrt{a^2cx^2+c}\tan^{-1}(ax)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3),x]
```

```
[Out] (-4*a^2*x^2*(a*x + 3*ArcTan[a*x]) - 3*(1 + a^2*x^2)^(3/2)*ArcTan[a*x]^2*Sin
Integral[ArcTan[a*x]] + 9*(1 + a^2*x^2)^(3/2)*ArcTan[a*x]^2*SinIntegral[3*A
```

```
rcTan[a*x]])/(8*a^4*c^2*(1 + a^2*x^2)*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)
```

Maple [C] time = 1.096, size = 848, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x)
```

```
[Out] 1/16*I*(9*arctan(a*x)^2*Ei(1,-3*I*arctan(a*x))*x^4*a^4-3*arctan(a*x)*(a^2*x
^2+1)^(1/2)*x^3*a^3+18*arctan(a*x)^2*Ei(1,-3*I*arctan(a*x))*x^2*a^2+I*(a^2*
x^2+1)^(1/2)*x^3*a^3+9*I*arctan(a*x)*(a^2*x^2+1)^(1/2)*x^2*a^2+3*(a^2*x^2+1
)^(1/2)*x^2*a^2+9*arctan(a*x)*(a^2*x^2+1)^(1/2)*x*a-3*I*(a^2*x^2+1)^(1/2)*x
*a+9*Ei(1,-3*I*arctan(a*x))*arctan(a*x)^2-3*I*arctan(a*x)*(a^2*x^2+1)^(1/2)
-(a^2*x^2+1)^(1/2))*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)^(1/2)/(a^4*x^4+2*
a^2*x^2+1)/arctan(a*x)^2/c^3/a^4-1/16*I*(9*arctan(a*x)^2*Ei(1,3*I*arctan(a*
x))*x^4*a^4-3*arctan(a*x)*(a^2*x^2+1)^(1/2)*x^3*a^3+18*arctan(a*x)^2*Ei(1,3
*I*arctan(a*x))*x^2*a^2-I*(a^2*x^2+1)^(1/2)*x^3*a^3-9*I*arctan(a*x)*(a^2*x^
2+1)^(1/2)*x^2*a^2+3*(a^2*x^2+1)^(1/2)*x^2*a^2+9*arctan(a*x)*(a^2*x^2+1)^(1
/2)*x*a+9*Ei(1,3*I*arctan(a*x))*arctan(a*x)^2+3*I*(a^2*x^2+1)^(1/2)*x*a+3*I
*arctan(a*x)*(a^2*x^2+1)^(1/2)-(a^2*x^2+1)^(1/2))*(c*(a*x-I)*(a*x+I))^(1/2)
/(a^2*x^2+1)^(1/2)/(a^4*x^4+2*a^2*x^2+1)/arctan(a*x)^2/c^3/a^4+3/16*I*(arct
an(a*x)^2*Ei(1,I*arctan(a*x))*x^2*a^2+arctan(a*x)*(a^2*x^2+1)^(1/2)*x*a+I*(
a^2*x^2+1)^(1/2)*x*a+Ei(1,I*arctan(a*x))*arctan(a*x)^2+I*arctan(a*x)*(a^2*x
^2+1)^(1/2)-(a^2*x^2+1)^(1/2))*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)^(3/2)/
arctan(a*x)^2/c^3/a^4-3/16*I*(arctan(a*x)^2*Ei(1,-I*arctan(a*x))*x^2*a^2+ar
ctan(a*x)*(a^2*x^2+1)^(1/2)*x*a+Ei(1,-I*arctan(a*x))*arctan(a*x)^2-I*(a^2*x
^2+1)^(1/2)*x*a-I*arctan(a*x)*(a^2*x^2+1)^(1/2)-(a^2*x^2+1)^(1/2))*(c*(a*x-
I)*(a*x+I))^(1/2)/(a^2*x^2+1)^(3/2)/arctan(a*x)^2/c^3/a^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(a^2cx^2+c\right)^{\frac{5}{2}}\arctan\left(ax\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] integrate(x^3/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a^2cx^2+c}\,x^3}{\left(a^6c^3x^6+3\,a^4c^3x^4+3\,a^2c^3x^2+c^3\right)\arctan\left(ax\right)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^3/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*
x^2 + c^3)*arctan(a*x)^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^3}{\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}\operatorname{atan}^3\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a**2*c*x**2+c)**(5/2)/atan(a*x)**3,x)
```

```
[Out] Integral(x**3/((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^3}{\left(a^2cx^2+c\right)^{\frac{5}{2}}\arctan\left(ax\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(x^3/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^3), x)
```

3.669 $\int \frac{x^2}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^3}\,dx$

Optimal. Leaf size=209

$$-\frac{\sqrt{a^2x^2+1}\text{CosIntegral}\left(\tan^{-1}(ax)\right)}{8a^3c^2\sqrt{a^2cx^2+c}}+\frac{9\sqrt{a^2x^2+1}\text{CosIntegral}\left(3\tan^{-1}(ax)\right)}{8a^3c^2\sqrt{a^2cx^2+c}}+\frac{x}{2a^2c^2\sqrt{a^2cx^2+c}\tan^{-1}(ax)}-\frac{}{2a^3c^2}$$

```
[Out] 1/(2*a^3*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2) - 1/(2*a^3*c^2*Sqrt[c + a^2
*c*x^2]*ArcTan[a*x]^2) - (3*x)/(2*a^2*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x])
+ x/(2*a^2*c^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]) - (Sqrt[1 + a^2*x^2]*CosInt
egral[ArcTan[a*x]])/(8*a^3*c^2*Sqrt[c + a^2*c*x^2]) + (9*Sqrt[1 + a^2*x^2]*
CosIntegral[3*ArcTan[a*x]])/(8*a^3*c^2*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.907102, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {4964, 4902, 4942, 4905, 4904, 3302, 4968, 4971, 4970, 4406, 3312}

$$-\frac{\sqrt{a^2x^2+1}\text{CosIntegral}\left(\tan^{-1}(ax)\right)}{8a^3c^2\sqrt{a^2cx^2+c}}+\frac{9\sqrt{a^2x^2+1}\text{CosIntegral}\left(3\tan^{-1}(ax)\right)}{8a^3c^2\sqrt{a^2cx^2+c}}+\frac{x}{2a^2c^2\sqrt{a^2cx^2+c}\tan^{-1}(ax)}-\frac{}{2a^3c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3),x]
```

```
[Out] 1/(2*a^3*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2) - 1/(2*a^3*c^2*Sqrt[c + a^2
*c*x^2]*ArcTan[a*x]^2) - (3*x)/(2*a^2*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x])
+ x/(2*a^2*c^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]) - (Sqrt[1 + a^2*x^2]*CosInt
egral[ArcTan[a*x]])/(8*a^3*c^2*Sqrt[c + a^2*c*x^2]) + (9*Sqrt[1 + a^2*x^2]*
CosIntegral[3*ArcTan[a*x]])/(8*a^3*c^2*Sqrt[c + a^2*c*x^2])
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Arc
Tan[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan[c
*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p
, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rule 4902

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^(p + 1))/(b*c*d*(p
 + 1)), x] - Dist[(2*c*(q + 1))/(b*(p + 1)), Int[x*(d + e*x^2)^q*(a + b*ArcT
an[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && L
tQ[q, -1] && LtQ[p, -1]
```

Rule 4942

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^m*(d + e*x^2)^(q + 1)*(a + b*ArcT
an[c*x])^(p + 1))/(b*c*d*(p + 1)), x] - Dist[(f*m)/(b*c*(p + 1)), Int[(f*x)
^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 2, 0] && LtQ[p, -1]
```

Rule 4905

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[(d^(q + 1/2)*Sqrt[1 + c^2*x^2])/Sqrt[d + e*x^2], Int[(1 + c
^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

Rule 4904

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 4968

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[(x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^(p
 + 1))/(b*c*d*(p + 1)), x] + (-Dist[(c*(m + 2*q + 2))/(b*(p + 1)), Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p + 1
)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; Fre
eQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &&
LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 4971

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[(d^(q + 1/2)*Sqrt[1 + c^2*x^2])/Sqrt[d + e*x^2],
 Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d
, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(In
tegerQ[q] || GtQ[d, 0])
```

Rule 4970

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sin[x]^m)/
Cos[x]^(m + 2*(q + 1)), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\int \frac{x^2}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^3}\,dx = -\frac{\int \frac{1}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^3}\,dx}{a^2} + \frac{\int \frac{1}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^3}\,dx}{a^2c}$$

$$= \frac{1}{2a^3c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2} - \frac{1}{2a^3c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2} + \frac{3\int \frac{x}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)}}{2a}$$

$$= \frac{1}{2a^3c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2} - \frac{1}{2a^3c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2} - \frac{3x}{2a^2c\left(c+a^2cx^2\right)^{3/2}\tan}$$

$$= \frac{1}{2a^3c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2} - \frac{1}{2a^3c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2} - \frac{3x}{2a^2c\left(c+a^2cx^2\right)^{3/2}\tan}$$

$$= \frac{1}{2a^3c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2} - \frac{1}{2a^3c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2} - \frac{3x}{2a^2c\left(c+a^2cx^2\right)^{3/2}\tan}$$

$$= \frac{1}{2a^3c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2} - \frac{1}{2a^3c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2} - \frac{3x}{2a^2c\left(c+a^2cx^2\right)^{3/2}\tan}$$

$$= \frac{1}{2a^3c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2} - \frac{1}{2a^3c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2} - \frac{3x}{2a^2c\left(c+a^2cx^2\right)^{3/2}\tan}$$

$$= \frac{1}{2a^3c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2} - \frac{1}{2a^3c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2} - \frac{3x}{2a^2c\left(c+a^2cx^2\right)^{3/2}\tan}$$

Mathematica [A] time = 0.22747, size = 119, normalized size = 0.57

$$\frac{-\left(a^2x^2+1\right)^{3/2}\tan^{-1}(ax)^2\text{CosIntegral}\left(\tan^{-1}(ax)\right)+9\left(a^2x^2+1\right)^{3/2}\tan^{-1}(ax)^2\text{CosIntegral}\left(3\tan^{-1}(ax)\right)+4ax\left(\left(a^2x^2\right.\right.}{8a^3c^2\left(a^2x^2+1\right)\sqrt{a^2cx^2+c}\tan^{-1}(ax)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3),x]
```

```
[Out] (4*a*x*(-(a*x) + (-2 + a^2*x^2)*ArcTan[a*x]) - (1 + a^2*x^2)^(3/2)*ArcTan[a
*x]^2*CosIntegral[ArcTan[a*x]] + 9*(1 + a^2*x^2)^(3/2)*ArcTan[a*x]^2*CosInt
egral[3*ArcTan[a*x]])/(8*a^3*c^2*(1 + a^2*x^2)*Sqrt[c + a^2*c*x^2]*ArcTan[a
```

```
*x]^2)
```

Maple [C] time = 0.961, size = 844, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x)
```

```
[Out] -1/16*(9*arctan(a*x)^2*Ei(1,3*I*arctan(a*x))*x^4*a^4-3*arctan(a*x)*(a^2*x^2
+1)^(1/2)*x^3*a^3+18*arctan(a*x)^2*Ei(1,3*I*arctan(a*x))*x^2*a^2-I*(a^2*x^2
+1)^(1/2)*x^3*a^3-9*I*arctan(a*x)*(a^2*x^2+1)^(1/2)*x^2*a^2+3*(a^2*x^2+1)^(
1/2)*x^2*a^2+9*arctan(a*x)*(a^2*x^2+1)^(1/2)*x*a+9*Ei(1,3*I*arctan(a*x))*ar
ctan(a*x)^2+3*I*(a^2*x^2+1)^(1/2)*x*a+3*I*arctan(a*x)*(a^2*x^2+1)^(1/2)-(a^
2*x^2+1)^(1/2))/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(a*x+I))^(1/2)/(a^4*x^4+2*a^2*
x^2+1)/arctan(a*x)^2/c^3/a^3-1/16*(9*arctan(a*x)^2*Ei(1,-3*I*arctan(a*x))*x
^4*a^4-3*arctan(a*x)*(a^2*x^2+1)^(1/2)*x^3*a^3+18*arctan(a*x)^2*Ei(1,-3*I*a
rctan(a*x))*x^2*a^2+I*(a^2*x^2+1)^(1/2)*x^3*a^3+9*I*arctan(a*x)*(a^2*x^2+1)
^(1/2)*x^2*a^2+3*(a^2*x^2+1)^(1/2)*x^2*a^2+9*arctan(a*x)*(a^2*x^2+1)^(1/2)*
x*a-3*I*(a^2*x^2+1)^(1/2)*x*a+9*Ei(1,-3*I*arctan(a*x))*arctan(a*x)^2-3*I*ar
ctan(a*x)*(a^2*x^2+1)^(1/2)-(a^2*x^2+1)^(1/2))/(a^2*x^2+1)^(1/2)*(c*(a*x-I)
*(a*x+I))^(1/2)/(a^4*x^4+2*a^2*x^2+1)/arctan(a*x)^2/c^3/a^3+1/16*(arctan(a*
x)^2*Ei(1,I*arctan(a*x))*x^2*a^2+arctan(a*x)*(a^2*x^2+1)^(1/2)*x*a+I*(a^2*x
^2+1)^(1/2)*x*a+Ei(1,I*arctan(a*x))*arctan(a*x)^2+I*arctan(a*x)*(a^2*x^2+1)
^(1/2)-(a^2*x^2+1)^(1/2))/(a^2*x^2+1)^(3/2)*(c*(a*x-I)*(a*x+I))^(1/2)/arcta
n(a*x)^2/c^3/a^3+1/16*(arctan(a*x)^2*Ei(1,-I*arctan(a*x))*x^2*a^2+arctan(a*
x)*(a^2*x^2+1)^(1/2)*x*a+Ei(1,-I*arctan(a*x))*arctan(a*x)^2-I*(a^2*x^2+1)^(
1/2)*x*a-I*arctan(a*x)*(a^2*x^2+1)^(1/2)-(a^2*x^2+1)^(1/2))/(a^2*x^2+1)^(3/
2)*(c*(a*x-I)*(a*x+I))^(1/2)/arctan(a*x)^2/c^3/a^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(a^2cx^2+c\right)^{\frac{5}{2}}\arctan\left(ax\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] integrate(x^2/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a^2cx^2+c}x^2}{\left(a^6c^3x^6+3\,a^4c^3x^4+3\,a^2c^3x^2+c^3\right)\arctan\left(ax\right)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^2/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*
x^2 + c^3)*arctan(a*x)^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}\operatorname{atan}^3\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a**2*c*x**2+c)**(5/2)/atan(a*x)**3,x)
```

```
[Out] Integral(x**2/((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(a^2cx^2+c\right)^{\frac{5}{2}}\arctan\left(ax\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(x^2/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^3), x)
```

3.670 $\int \frac{x}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^3}\,dx$

Optimal. Leaf size=175

$$-\frac{\sqrt{a^2x^2+1}\,\mathrm{Si}\left(\tan^{-1}(ax)\right)}{8a^2c^2\sqrt{a^2cx^2+c}}-\frac{9\sqrt{a^2x^2+1}\,\mathrm{Si}\left(3\tan^{-1}(ax)\right)}{8a^2c^2\sqrt{a^2cx^2+c}}+\frac{1}{a^2c^2\sqrt{a^2cx^2+c}\tan^{-1}(ax)}-\frac{x}{2ac\left(a^2cx^2+c\right)^{3/2}\tan^{-1}(ax)^2}$$

```
[Out] -x/(2*a*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2) - 3/(2*a^2*c*(c + a^2*c*x^2)
^(3/2)*ArcTan[a*x]) + 1/(a^2*c^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]) - (Sqrt[1
 + a^2*x^2]*SinIntegral[ArcTan[a*x]])/(8*a^2*c^2*Sqrt[c + a^2*c*x^2]) - (9*
Sqrt[1 + a^2*x^2]*SinIntegral[3*ArcTan[a*x]])/(8*a^2*c^2*Sqrt[c + a^2*c*x^2
])
```

Rubi [A] time = 0.931144, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {4968, 4964, 4902, 4971, 4970, 3299, 4406}

$$-\frac{\sqrt{a^2x^2+1}\,\mathrm{Si}\left(\tan^{-1}(ax)\right)}{8a^2c^2\sqrt{a^2cx^2+c}}-\frac{9\sqrt{a^2x^2+1}\,\mathrm{Si}\left(3\tan^{-1}(ax)\right)}{8a^2c^2\sqrt{a^2cx^2+c}}+\frac{1}{a^2c^2\sqrt{a^2cx^2+c}\tan^{-1}(ax)}-\frac{x}{2ac\left(a^2cx^2+c\right)^{3/2}\tan^{-1}(ax)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3),x]
```

```
[Out] -x/(2*a*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2) - 3/(2*a^2*c*(c + a^2*c*x^2)
^(3/2)*ArcTan[a*x]) + 1/(a^2*c^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]) - (Sqrt[1
 + a^2*x^2]*SinIntegral[ArcTan[a*x]])/(8*a^2*c^2*Sqrt[c + a^2*c*x^2]) - (9*
Sqrt[1 + a^2*x^2]*SinIntegral[3*ArcTan[a*x]])/(8*a^2*c^2*Sqrt[c + a^2*c*x^2
])
```

Rule 4968

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[(x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^(p
+ 1))/(b*c*d*(p + 1)), x] + (-Dist[(c*(m + 2*q + 2))/(b*(p + 1)), Int[x^(m
+ 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p + 1
)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; Fre
eQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &&
LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Arc
Tan[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan[c
*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p
, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rule 4902

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^(p + 1))/(b*c*d*(p
+ 1)), x] - Dist[(2*c*(q + 1))/(b*(p + 1)), Int[x*(d + e*x^2)^q*(a + b*ArcT
an[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && L
tQ[q, -1] && LtQ[p, -1]
```

Rule 4971

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[(d^(q + 1/2)*Sqrt[1 + c^2*x^2])/Sqrt[d + e*x^2],
 Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d
, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(In
tegerQ[q] || GtQ[d, 0])
```

Rule 4970

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sin[x]^m)/
Cos[x]^(m + 2*(q + 1)), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \frac{x}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^3}\,dx = -\frac{x}{2ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2} + \frac{\int \frac{1}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^2}\,dx}{2a} - a\int \frac{x^2}{\left(c+a^2cx^2\right)^{5/2}\tan^{-}}$$

$$= -\frac{x}{2ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2} - \frac{1}{2a^2c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)} - \frac{3}{2}\int \frac{x}{\left(c+a^2cx^2\right)^{5/2}}$$

$$= -\frac{x}{2ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2} - \frac{3}{2a^2c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)} + \frac{1}{a^2c^2\sqrt{c+a^2cx^2}\tan}$$

$$= -\frac{x}{2ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2} - \frac{3}{2a^2c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)} + \frac{1}{a^2c^2\sqrt{c+a^2cx^2}\tan}$$

$$= -\frac{x}{2ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2} - \frac{3}{2a^2c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)} + \frac{1}{a^2c^2\sqrt{c+a^2cx^2}\tan}$$

$$= -\frac{x}{2ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2} - \frac{3}{2a^2c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)} + \frac{1}{a^2c^2\sqrt{c+a^2cx^2}\tan}$$

$$= -\frac{x}{2ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2} - \frac{3}{2a^2c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)} + \frac{1}{a^2c^2\sqrt{c+a^2cx^2}\tan}$$

$$= -\frac{x}{2ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2} - \frac{3}{2a^2c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)} + \frac{1}{a^2c^2\sqrt{c+a^2cx^2}\tan}$$

Mathematica [A] time = 0.270236, size = 118, normalized size = 0.67

$$\frac{-\left(a^2x^2+1\right)^{3/2}\tan^{-1}(ax)^2\text{Si}\left(\tan^{-1}(ax)\right) - 9\left(a^2x^2+1\right)^{3/2}\tan^{-1}(ax)^2\text{Si}\left(3\tan^{-1}(ax)\right) + 8a^2x^2\tan^{-1}(ax) - 4ax - 4\tan}{8a^2c^2\left(a^2x^2+1\right)\sqrt{a^2cx^2+c}\tan^{-1}(ax)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3),x]
```

```
[Out] (-4*a*x - 4*ArcTan[a*x] + 8*a^2*x^2*ArcTan[a*x] - (1 + a^2*x^2)^(3/2)*ArcTa
n[a*x]^2*SinIntegral[ArcTan[a*x]] - 9*(1 + a^2*x^2)^(3/2)*ArcTan[a*x]^2*Sin
```

```
Integral[3*ArcTan[a*x]])/(8*a^2*c^2*(1 + a^2*x^2)*Sqrt[c + a^2*c*x^2]*ArcTa
n[a*x]^2)
```

Maple [C] time = 0.401, size = 848, normalized size = 4.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x)
```

```
[Out] -1/16*I*(arctan(a*x)^2*Ei(1,-I*arctan(a*x))*x^2*a^2+arctan(a*x)*(a^2*x^2+1)
^(1/2)*x*a+Ei(1,-I*arctan(a*x))*arctan(a*x)^2-I*(a^2*x^2+1)^(1/2)*x*a-I*arc
tan(a*x)*(a^2*x^2+1)^(1/2)-(a^2*x^2+1)^(1/2))/(a^2*x^2+1)^(3/2)*(c*(a*x-I)*
(a*x+I))^(1/2)/arctan(a*x)^2/c^3/a^2-1/16*I*(9*arctan(a*x)^2*Ei(1,-3*I*arct
an(a*x))*x^4*a^4-3*arctan(a*x)*(a^2*x^2+1)^(1/2)*x^3*a^3+18*arctan(a*x)^2*E
i(1,-3*I*arctan(a*x))*x^2*a^2+I*(a^2*x^2+1)^(1/2)*x^3*a^3+9*I*arctan(a*x)*(
a^2*x^2+1)^(1/2)*x^2*a^2+3*(a^2*x^2+1)^(1/2)*x^2*a^2+9*arctan(a*x)*(a^2*x^2
+1)^(1/2)*x*a-3*I*(a^2*x^2+1)^(1/2)*x*a+9*Ei(1,-3*I*arctan(a*x))*arctan(a*x
)^2-3*I*arctan(a*x)*(a^2*x^2+1)^(1/2)-(a^2*x^2+1)^(1/2))/(a^2*x^2+1)^(1/2)*
(c*(a*x-I)*(a*x+I))^(1/2)/arctan(a*x)^2/(a^4*x^4+2*a^2*x^2+1)/c^3/a^2+1/16*
I*(9*arctan(a*x)^2*Ei(1,3*I*arctan(a*x))*x^4*a^4-3*arctan(a*x)*(a^2*x^2+1)^
(1/2)*x^3*a^3+18*arctan(a*x)^2*Ei(1,3*I*arctan(a*x))*x^2*a^2-I*(a^2*x^2+1)^
(1/2)*x^3*a^3-9*I*arctan(a*x)*(a^2*x^2+1)^(1/2)*x^2*a^2+3*(a^2*x^2+1)^(1/2)
*x^2*a^2+9*arctan(a*x)*(a^2*x^2+1)^(1/2)*x*a+9*Ei(1,3*I*arctan(a*x))*arctan
(a*x)^2+3*I*(a^2*x^2+1)^(1/2)*x*a+3*I*arctan(a*x)*(a^2*x^2+1)^(1/2)-(a^2*x^
2+1)^(1/2))/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(a*x+I))^(1/2)/arctan(a*x)^2/(a^4*
x^4+2*a^2*x^2+1)/c^3/a^2+1/16*I*(arctan(a*x)^2*Ei(1,I*arctan(a*x))*x^2*a^2+
arctan(a*x)*(a^2*x^2+1)^(1/2)*x*a+I*(a^2*x^2+1)^(1/2)*x*a+Ei(1,I*arctan(a*x
))*arctan(a*x)^2+I*arctan(a*x)*(a^2*x^2+1)^(1/2)-(a^2*x^2+1)^(1/2))/(a^2*x^
2+1)^(3/2)*(c*(a*x-I)*(a*x+I))^(1/2)/arctan(a*x)^2/c^3/a^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(a^2cx^2+c\right)^{\frac{5}{2}}\arctan\left(ax\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="maxima")
```

[Out] integrate(x/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^3), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a^2cx^2+c}\,x}{\left(a^6c^3x^6+3\,a^4c^3x^4+3\,a^2c^3x^2+c^3\right)\arctan\left(ax\right)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="fricas")

[Out] integral(sqrt(a^2*c*x^2 + c)*x/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*arctan(a*x)^3), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x}{\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}\operatorname{atan}^3\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a**2*c*x**2+c)**(5/2)/atan(a*x)**3,x)

[Out] Integral(x/((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**3), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x}{\left(a^2cx^2+c\right)^{\frac{5}{2}}\arctan\left(ax\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="giac")

[Out] integrate(x/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^3), x)

3.671 $\int \frac{1}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^3}\,dx$

Optimal. Leaf size=145

$$-\frac{3\sqrt{a^2x^2+1}\,\mathrm{CosIntegral}\left(\tan^{-1}(ax)\right)}{8ac^2\sqrt{a^2cx^2+c}}-\frac{9\sqrt{a^2x^2+1}\,\mathrm{CosIntegral}\left(3\tan^{-1}(ax)\right)}{8ac^2\sqrt{a^2cx^2+c}}+\frac{3x}{2c\left(a^2cx^2+c\right)^{3/2}\tan^{-1}(ax)}-\frac{\quad}{2ac\left(a^2\right.}$$

```
[Out] -1/(2*a*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2) + (3*x)/(2*c*(c + a^2*c*x^2)
^(3/2)*ArcTan[a*x]) - (3*Sqrt[1 + a^2*x^2]*CosIntegral[ArcTan[a*x]])/(8*a*c
^2*Sqrt[c + a^2*c*x^2]) - (9*Sqrt[1 + a^2*x^2]*CosIntegral[3*ArcTan[a*x]])/
(8*a*c^2*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.550504, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4902, 4968, 4971, 4970, 4406, 3302, 4905, 4904, 3312}

$$-\frac{3\sqrt{a^2x^2+1}\,\mathrm{CosIntegral}\left(\tan^{-1}(ax)\right)}{8ac^2\sqrt{a^2cx^2+c}}-\frac{9\sqrt{a^2x^2+1}\,\mathrm{CosIntegral}\left(3\tan^{-1}(ax)\right)}{8ac^2\sqrt{a^2cx^2+c}}+\frac{3x}{2c\left(a^2cx^2+c\right)^{3/2}\tan^{-1}(ax)}-\frac{\quad}{2ac\left(a^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3),x]
```

```
[Out] -1/(2*a*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2) + (3*x)/(2*c*(c + a^2*c*x^2)
^(3/2)*ArcTan[a*x]) - (3*Sqrt[1 + a^2*x^2]*CosIntegral[ArcTan[a*x]])/(8*a*c
^2*Sqrt[c + a^2*c*x^2]) - (9*Sqrt[1 + a^2*x^2]*CosIntegral[3*ArcTan[a*x]])/
(8*a*c^2*Sqrt[c + a^2*c*x^2])
```

Rule 4902

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^(p + 1))/(b*c*d*(p
+ 1)), x] - Dist[(2*c*(q + 1))/(b*(p + 1)), Int[x*(d + e*x^2)^q*(a + b*ArcT
an[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && L
tQ[q, -1] && LtQ[p, -1]
```

Rule 4968

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[(x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^(p
```

```
+ 1))/(b*c*d*(p + 1)), x] + (-Dist[(c*(m + 2*q + 2))/(b*(p + 1)), Int[x^(m
+ 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p + 1
)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; Fre
eQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &&
LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 4971

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[(d^(q + 1/2)*Sqrt[1 + c^2*x^2])/Sqrt[d + e*x^2],
 Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d
, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(In
tegerQ[q] || GtQ[d, 0])
```

Rule 4970

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sin[x]^m)/
Cos[x]^(m + 2*(q + 1)), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 4905

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[(d^(q + 1/2)*Sqrt[1 + c^2*x^2])/Sqrt[d + e*x^2], Int[(1 + c
^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

Rule 4904

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\int \frac{1}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^3}\,dx = -\frac{1}{2ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2}-\frac{1}{2}(3a)\int \frac{x}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^2}\,dx$$

$$= -\frac{1}{2ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2}+\frac{3x}{2c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}-\frac{3}{2}\int \frac{1}{\left(c+a^2cx^2\right)^{5/2}\tan}$$

$$= -\frac{1}{2ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2}+\frac{3x}{2c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}-\frac{\left(3\sqrt{1+a^2x^2}\right)\int \frac{}{\left(1+a^2x^2\right.}}{2c^2\sqrt{c+a^2}}$$

$$= -\frac{1}{2ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2}+\frac{3x}{2c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}-\frac{\left(3\sqrt{1+a^2x^2}\right)\text{Subst}\left(\right.}{2ac^2\sqrt{}}$$

$$= -\frac{1}{2ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2}+\frac{3x}{2c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}-\frac{\left(3\sqrt{1+a^2x^2}\right)\text{Subst}\left(\right.}{2}$$

$$= -\frac{1}{2ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2}+\frac{3x}{2c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}-\frac{\left(3\sqrt{1+a^2x^2}\right)\text{Subst}\left(\right.}{8ac^2\sqrt{}}$$

$$= -\frac{1}{2ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2}+\frac{3x}{2c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}-\frac{3\sqrt{1+a^2x^2}\,\text{Ci}\left(\tan^{-1}(\right.}{8ac^2\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 0.19457, size = 102, normalized size = 0.7

$$\frac{-3\left(a^2x^2+1\right)^{3/2}\tan^{-1}(ax)^2\text{CosIntegral}\left(\tan^{-1}(ax)\right)-9\left(a^2x^2+1\right)^{3/2}\tan^{-1}(ax)^2\text{CosIntegral}\left(3\tan^{-1}(ax)\right)+12ax\tan}{8c^2\left(a^3x^2+a\right)\sqrt{a^2cx^2+c}\tan^{-1}(ax)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3),x]
```

```
[Out] (-4 + 12*a*x*ArcTan[a*x] - 3*(1 + a^2*x^2)^(3/2)*ArcTan[a*x]^2*CosIntegral[
ArcTan[a*x]] - 9*(1 + a^2*x^2)^(3/2)*ArcTan[a*x]^2*CosIntegral[3*ArcTan[a*x
]])/(8*c^2*(a + a^3*x^2)*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)
```

Maple [C] time = 0.389, size = 844, normalized size = 5.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x)
```

```
[Out] 1/16*(9*arctan(a*x)^2*Ei(1,3*I*arctan(a*x))*x^4*a^4-3*arctan(a*x)*(a^2*x^2+
1)^(1/2)*x^3*a^3+18*arctan(a*x)^2*Ei(1,3*I*arctan(a*x))*x^2*a^2-I*(a^2*x^2+
1)^(1/2)*x^3*a^3-9*I*arctan(a*x)*(a^2*x^2+1)^(1/2)*x^2*a^2+3*(a^2*x^2+1)^(1
/2)*x^2*a^2+9*arctan(a*x)*(a^2*x^2+1)^(1/2)*x*a+9*Ei(1,3*I*arctan(a*x))*arc
tan(a*x)^2+3*I*(a^2*x^2+1)^(1/2)*x*a+3*I*arctan(a*x)*(a^2*x^2+1)^(1/2)-(a^2
*x^2+1)^(1/2))/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(a*x+I))^(1/2)/(a^4*x^4+2*a^2*x
^2+1)/arctan(a*x)^2/a/c^3+1/16*(9*arctan(a*x)^2*Ei(1,-3*I*arctan(a*x))*x^4*
a^4-3*arctan(a*x)*(a^2*x^2+1)^(1/2)*x^3*a^3+18*arctan(a*x)^2*Ei(1,-3*I*arct
an(a*x))*x^2*a^2+I*(a^2*x^2+1)^(1/2)*x^3*a^3+9*I*arctan(a*x)*(a^2*x^2+1)^(1
/2)*x^2*a^2+3*(a^2*x^2+1)^(1/2)*x^2*a^2+9*arctan(a*x)*(a^2*x^2+1)^(1/2)*x*a
-3*I*(a^2*x^2+1)^(1/2)*x*a+9*Ei(1,-3*I*arctan(a*x))*arctan(a*x)^2-3*I*arcta
n(a*x)*(a^2*x^2+1)^(1/2)-(a^2*x^2+1)^(1/2))/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(a
*x+I))^(1/2)/(a^4*x^4+2*a^2*x^2+1)/arctan(a*x)^2/a/c^3+3/16*(arctan(a*x)^2*
Ei(1,I*arctan(a*x))*x^2*a^2+arctan(a*x)*(a^2*x^2+1)^(1/2)*x*a+I*(a^2*x^2+1)
^(1/2)*x*a+Ei(1,I*arctan(a*x))*arctan(a*x)^2+I*arctan(a*x)*(a^2*x^2+1)^(1/2
)-(a^2*x^2+1)^(1/2))/(a^2*x^2+1)^(3/2)*(c*(a*x-I)*(a*x+I))^(1/2)/arctan(a*x
)^2/a/c^3+3/16*(arctan(a*x)^2*Ei(1,-I*arctan(a*x))*x^2*a^2+arctan(a*x)*(a^2
*x^2+1)^(1/2)*x*a+Ei(1,-I*arctan(a*x))*arctan(a*x)^2-I*(a^2*x^2+1)^(1/2)*x*
a-I*arctan(a*x)*(a^2*x^2+1)^(1/2)-(a^2*x^2+1)^(1/2))/(a^2*x^2+1)^(3/2)*(c*(
a*x-I)*(a*x+I))^(1/2)/arctan(a*x)^2/a/c^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)^{\frac{5}{2}}\arctan\left(ax\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a^2cx^2+c}}{\left(a^6c^3x^6+3\,a^4c^3x^4+3\,a^2c^3x^2+c^3\right)\arctan\left(ax\right)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2
+ c^3)*arctan(a*x)^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}\operatorname{atan}^3\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*c*x**2+c)**(5/2)/atan(a*x)**3,x)
```

```
[Out] Integral(1/((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)^{\frac{5}{2}}\arctan\left(ax\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^3), x)
```

3.672 $\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^3}\,dx$

Optimal. Leaf size=262

$$-\frac{\text{Unintegrable}\left(\frac{1}{x^2\sqrt{a^2cx^2+c}\tan^{-1}(ax)^2},x\right)}{2ac^2}+\frac{5\sqrt{a^2x^2+1}\text{Si}\left(\tan^{-1}(ax)\right)}{8c^2\sqrt{a^2cx^2+c}}+\frac{9\sqrt{a^2x^2+1}\text{Si}\left(3\tan^{-1}(ax)\right)}{8c^2\sqrt{a^2cx^2+c}}+\frac{ax}{2c^2\sqrt{a^2cx^2+c}\,\text{ta}}$$

```
[Out] (a*x)/(2*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2) + (a*x)/(2*c^2*Sqrt[c + a^2
*c*x^2]*ArcTan[a*x]^2) - Sqrt[c + a^2*c*x^2]/(2*a*c^3*x*ArcTan[a*x]^2) + 3/
(2*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]) - 1/(2*c^2*Sqrt[c + a^2*c*x^2]*ArcT
an[a*x]) + (5*Sqrt[1 + a^2*x^2]*SinIntegral[ArcTan[a*x]])/(8*c^2*Sqrt[c + a
^2*c*x^2]) + (9*Sqrt[1 + a^2*x^2]*SinIntegral[3*ArcTan[a*x]])/(8*c^2*Sqrt[c
 + a^2*c*x^2]) - Unintegrable[1/(x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2), x]
/(2*a*c^2)
```

Rubi [A] time = 1.72045, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3),x]
```

```
[Out] (a*x)/(2*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2) + (a*x)/(2*c^2*Sqrt[c + a^2
*c*x^2]*ArcTan[a*x]^2) - Sqrt[c + a^2*c*x^2]/(2*a*c^3*x*ArcTan[a*x]^2) + 3/
(2*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]) - 1/(2*c^2*Sqrt[c + a^2*c*x^2]*ArcT
an[a*x]) + (5*Sqrt[1 + a^2*x^2]*SinIntegral[ArcTan[a*x]])/(8*c^2*Sqrt[c + a
^2*c*x^2]) + (9*Sqrt[1 + a^2*x^2]*SinIntegral[3*ArcTan[a*x]])/(8*c^2*Sqrt[c
 + a^2*c*x^2]) - Defer[Int][1/(x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2), x]/(
2*a*c^2)
```

Rubi steps

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^3}\,dx = -\left(a^2\int \frac{x}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^3}\,dx\right) + \frac{\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^3}\,dx}{c}$$

$$= \frac{ax}{2c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2} - \frac{1}{2}a\int \frac{1}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^2}\,dx + a^3\int \frac{\ }{\left(c+a^2cx\right.}$$

$$= \frac{ax}{2c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2} + \frac{ax}{2c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2} - \frac{\sqrt{c+a^2cx^2}}{2ac^3x\tan^{-1}(ax)^2} + \frac{\ }{2c}$$

$$= \frac{ax}{2c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2} + \frac{ax}{2c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2} - \frac{\sqrt{c+a^2cx^2}}{2ac^3x\tan^{-1}(ax)^2} + \frac{\ }{2c}$$

$$= \frac{ax}{2c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2} + \frac{ax}{2c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2} - \frac{\sqrt{c+a^2cx^2}}{2ac^3x\tan^{-1}(ax)^2} + \frac{\ }{2c}$$

$$= \frac{ax}{2c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2} + \frac{ax}{2c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2} - \frac{\sqrt{c+a^2cx^2}}{2ac^3x\tan^{-1}(ax)^2} + \frac{\ }{2c}$$

$$= \frac{ax}{2c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2} + \frac{ax}{2c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2} - \frac{\sqrt{c+a^2cx^2}}{2ac^3x\tan^{-1}(ax)^2} + \frac{\ }{2c}$$

$$= \frac{ax}{2c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2} + \frac{ax}{2c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2} - \frac{\sqrt{c+a^2cx^2}}{2ac^3x\tan^{-1}(ax)^2} + \frac{\ }{2c}$$

$$= \frac{ax}{2c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2} + \frac{ax}{2c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2} - \frac{\sqrt{c+a^2cx^2}}{2ac^3x\tan^{-1}(ax)^2} + \frac{\ }{2c}$$

Mathematica [A] time = 3.46554, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3),x]
```

```
[Out] Integrate[1/(x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3), x]
```

Maple [A] time = 0.653, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(\arctan\left(ax\right)\right)^{3}}\left(a^{2}cx^{2}+c\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x)
```

```
[Out] int(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^{2}cx^{2}+c\right)^{\frac{5}{2}}x\arctan\left(ax\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^(5/2)*x*arctan(a*x)^3), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{a^{2}cx^{2}+c}}{\left(a^{6}c^{3}x^{7}+3\,a^{4}c^{3}x^{5}+3\,a^{2}c^{3}x^{3}+c^{3}x\right)\arctan\left(ax\right)^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)/((a^6*c^3*x^7 + 3*a^4*c^3*x^5 + 3*a^2*c^3*x^3
+ c^3*x)*arctan(a*x)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a**2*c*x**2+c)**(5/2)/atan(a*x)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)^{\frac{5}{2}}x\arctan\left(ax\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^(5/2)*x*arctan(a*x)^3), x)
```

3.673 $\int \frac{1}{x^2\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^3}\,dx$

Optimal. Leaf size=231

$$\frac{\text{Unintegrable}\left(\frac{1}{x^2\sqrt{a^2cx^2+c}\tan^{-1}(ax)^3},x\right)}{c^2}+\frac{7a\sqrt{a^2x^2+1}\text{CosIntegral}\left(\tan^{-1}(ax)\right)}{8c^2\sqrt{a^2cx^2+c}}+\frac{9a\sqrt{a^2x^2+1}\text{CosIntegral}\left(3\tan^{-1}(ax)\right)}{8c^2\sqrt{a^2cx^2+c}}$$

```
[Out] a/(2*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2) + a/(2*c^2*Sqrt[c + a^2*c*x^2]*
ArcTan[a*x]^2) - (3*a^2*x)/(2*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]) - (a^2*x
)/(2*c^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]) + (7*a*Sqrt[1 + a^2*x^2]*CosInteg
ral[ArcTan[a*x]])/(8*c^2*Sqrt[c + a^2*c*x^2]) + (9*a*Sqrt[1 + a^2*x^2]*CosI
ntegral[3*ArcTan[a*x]])/(8*c^2*Sqrt[c + a^2*c*x^2]) + Unintegrable[1/(x^2*S
qrt[c + a^2*c*x^2]*ArcTan[a*x]^3), x]/c^2
```

Rubi [A] time = 1.15175, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3),x]
```

```
[Out] a/(2*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2) + a/(2*c^2*Sqrt[c + a^2*c*x^2]*
ArcTan[a*x]^2) - (3*a^2*x)/(2*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]) - (a^2*x
)/(2*c^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]) + (7*a*Sqrt[1 + a^2*x^2]*CosInteg
ral[ArcTan[a*x]])/(8*c^2*Sqrt[c + a^2*c*x^2]) + (9*a*Sqrt[1 + a^2*x^2]*CosI
ntegral[3*ArcTan[a*x]])/(8*c^2*Sqrt[c + a^2*c*x^2]) + Defer[Int][1/(x^2*Sqr
t[c + a^2*c*x^2]*ArcTan[a*x]^3), x]/c^2
```

Rubi steps

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^3}\,dx = -\left(a^2\int \frac{1}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^3}\,dx\right) + \frac{\int \frac{1}{x^2\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^3}\,dx}{c}$$

$$= \frac{a}{2c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2} + \frac{1}{2}\left(3a^3\right)\int \frac{x}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^2}\,dx + \frac{\int \frac{}{x^2\sqrt{c+a}}}{}$$

$$= \frac{a}{2c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2} + \frac{a}{2c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2} - \frac{3a^2x}{2c\left(c+a^2cx^2\right)^{3/2}\tan^{-}}$$

$$= \frac{a}{2c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2} + \frac{a}{2c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2} - \frac{3a^2x}{2c\left(c+a^2cx^2\right)^{3/2}\tan^{-}}$$

$$= \frac{a}{2c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2} + \frac{a}{2c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2} - \frac{3a^2x}{2c\left(c+a^2cx^2\right)^{3/2}\tan^{-}}$$

$$= \frac{a}{2c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2} + \frac{a}{2c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2} - \frac{3a^2x}{2c\left(c+a^2cx^2\right)^{3/2}\tan^{-}}$$

$$= \frac{a}{2c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2} + \frac{a}{2c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2} - \frac{3a^2x}{2c\left(c+a^2cx^2\right)^{3/2}\tan^{-}}$$

$$= \frac{a}{2c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2} + \frac{a}{2c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2} - \frac{3a^2x}{2c\left(c+a^2cx^2\right)^{3/2}\tan^{-}}$$

Mathematica [A] time = 5.09301, size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3),x]
```

```
[Out] Integrate[1/(x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3), x]
```

Maple [A] time = 0.689, size = 0, normalized size = 0.

$$\int \frac{1}{x^{2} \left(\arctan \left(a x\right)\right)^{3}} \left(a^{2} c \,x^{2}+c \right)^{-\frac{5}{2}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x)
```

```
[Out] int(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{{\left(a^{2} c x^{2} + c\right)}^{\frac{5}{2}} x^{2} \arctan \left(a x\right)^{3}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^(5/2)*x^2*arctan(a*x)^3), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$${\rm integral}\left(\frac{\sqrt{a^{2} c x^{2} + c}}{{\left(a^{6} c^{3} x^{8} + 3 \, a^{4} c^{3} x^{6} + 3 \, a^{2} c^{3} x^{4} + c^{3} x^{2}\right)} \arctan \left(a x\right)^{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)/((a^6*c^3*x^8 + 3*a^4*c^3*x^6 + 3*a^2*c^3*x^4
+ c^3*x^2)*arctan(a*x)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a**2*c*x**2+c)**(5/2)/atan(a*x)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2 + c\right)^{\frac{5}{2}} x^2 \arctan\left(ax\right)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^(5/2)*x^2*arctan(a*x)^3), x)
```

3.674 $\int \frac{x^m(c+a^2cx^2)^3}{\tan^{-1}(ax)^3} dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{x^m\left(a^2cx^2+c\right)^3}{\tan^{-1}(ax)^3},x\right)$$

```
[Out] Unintegrable[(x^m*(c + a^2*c*x^2)^3)/ArcTan[a*x]^3, x]
```

Rubi [A] time = 0.0564415, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m\left(c+a^2cx^2\right)^3}{\tan^{-1}(ax)^3} dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*(c + a^2*c*x^2)^3)/ArcTan[a*x]^3,x]
```

```
[Out] Defer[Int][(x^m*(c + a^2*c*x^2)^3)/ArcTan[a*x]^3, x]
```

Rubi steps

$$\int \frac{x^m\left(c+a^2cx^2\right)^3}{\tan^{-1}(ax)^3} dx=\int \frac{x^m\left(c+a^2cx^2\right)^3}{\tan^{-1}(ax)^3} dx$$

Mathematica [A] time = 0.754781, size = 0, normalized size = 0.

$$\int \frac{x^m\left(c+a^2cx^2\right)^3}{\tan^{-1}(ax)^3} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*(c + a^2*c*x^2)^3)/ArcTan[a*x]^3,x]
```

```
[Out] Integrate[(x^m*(c + a^2*c*x^2)^3)/ArcTan[a*x]^3, x]
```

Maple [A] time = 0.573, size = 0, normalized size = 0.

$$\int \frac{x^m \left(a^2 c x^2 + c\right)^3}{\left(\arctan\left(ax\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^3,x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^3,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^6 c^3 x^6 + 3\,a^4 c^3 x^4 + 3\,a^2 c^3 x^2 + c^3\right) x^m}{\arctan\left(ax\right)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*x^m/arctan(a*x
)^3, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$c^3\left(\int \frac{x^m}{\operatorname{atan}^3(ax)}\,dx + \int \frac{3a^2x^2x^m}{\operatorname{atan}^3(ax)}\,dx + \int \frac{3a^4x^4x^m}{\operatorname{atan}^3(ax)}\,dx + \int \frac{a^6x^6x^m}{\operatorname{atan}^3(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**3/atan(a*x)**3,x)
```

```
[Out] c**3*(Integral(x**m/atan(a*x)**3, x) + Integral(3*a**2*x**2*x**m/atan(a*x)*
*3, x) + Integral(3*a**4*x**4*x**m/atan(a*x)**3, x) + Integral(a**6*x**6*x*
*m/atan(a*x)**3, x))
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2 + c\right)^3 x^m}{\arctan(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^3*x^m/arctan(a*x)^3, x)
```

3.675 $\int \frac{x^m\left(c+a^2cx^2\right)^2}{\tan^{-1}(ax)^3}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{x^m\left(a^2cx^2+c\right)^2}{\tan^{-1}(ax)^3},x\right)$$

```
[Out] Unintegrable[(x^m*(c + a^2*c*x^2)^2)/ArcTan[a*x]^3, x]
```

Rubi [A] time = 0.0542615, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m\left(c+a^2cx^2\right)^2}{\tan^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*(c + a^2*c*x^2)^2)/ArcTan[a*x]^3,x]
```

```
[Out] Defer[Int][(x^m*(c + a^2*c*x^2)^2)/ArcTan[a*x]^3, x]
```

Rubi steps

$$\int \frac{x^m\left(c+a^2cx^2\right)^2}{\tan^{-1}(ax)^3}\,dx=\int \frac{x^m\left(c+a^2cx^2\right)^2}{\tan^{-1}(ax)^3}\,dx$$

Mathematica [A] time = 0.825364, size = 0, normalized size = 0.

$$\int \frac{x^m\left(c+a^2cx^2\right)^2}{\tan^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*(c + a^2*c*x^2)^2)/ArcTan[a*x]^3,x]
```

[Out] Integrate[(x^m*(c + a^2*c*x^2)^2)/ArcTan[a*x]^3, x]

Maple [A] time = 0.535, size = 0, normalized size = 0.

$$\int \frac{x^m \left(a^2 c x^2 + c\right)^2}{\left(\arctan\left(ax\right)\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^3,x)

[Out] int(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^3,x)

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="maxima")

[Out] Timed out

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^4 c^2 x^4 + 2\, a^2 c^2 x^2 + c^2\right) x^m}{\arctan\left(ax\right)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="fricas")

[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*x^m/arctan(a*x)^3, x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$c^2\left(\int \frac{x^m}{\operatorname{atan}^3(ax)}\,dx+\int \frac{2a^2x^2x^m}{\operatorname{atan}^3(ax)}\,dx+\int \frac{a^4x^4x^m}{\operatorname{atan}^3(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**2/atan(a*x)**3,x)
```

```
[Out] c**2*(Integral(x**m/atan(a*x)**3, x) + Integral(2*a**2*x**2*x**m/atan(a*x)*
*3, x) + Integral(a**4*x**4*x**m/atan(a*x)**3, x))
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^2x^m}{\arctan(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^2*x^m/arctan(a*x)^3, x)
```

3.676 $\int \frac{x^m(c+a^2cx^2)}{\tan^{-1}(ax)^3}\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{x^m\left(a^2cx^2+c\right)}{\tan^{-1}(ax)^3},x\right)$$

```
[Out] Unintegrable[(x^m*(c + a^2*c*x^2))/ArcTan[a*x]^3, x]
```

Rubi [A] time = 0.0338105, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m\left(c+a^2cx^2\right)}{\tan^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*(c + a^2*c*x^2))/ArcTan[a*x]^3,x]

[Out] Defer[Int][(x^m*(c + a^2*c*x^2))/ArcTan[a*x]^3, x]
```

Rubi steps

$$\int \frac{x^m\left(c+a^2cx^2\right)}{\tan^{-1}(ax)^3}\,dx=\int \frac{x^m\left(c+a^2cx^2\right)}{\tan^{-1}(ax)^3}\,dx$$

Mathematica [A] time = 0.553764, size = 0, normalized size = 0.

$$\int \frac{x^m\left(c+a^2cx^2\right)}{\tan^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*(c + a^2*c*x^2))/ArcTan[a*x]^3,x]
```

```
[Out] Integrate[(x^m*(c + a^2*c*x^2))/ArcTan[a*x]^3, x]
```

Maple [A] time = 0.372, size = 0, normalized size = 0.

$$\int \frac{x^m \left(a^2 c x^2 + c\right)}{\left(\arctan\left(a x\right)\right)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)/arctan(a*x)^3,x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)/arctan(a*x)^3,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^2 c x^2 + c\right) x^m}{\arctan\left(a x\right)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)*x^m/arctan(a*x)^3, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$c\left(\int \frac{x^m}{\operatorname{atan}^3(ax)}\,dx + \int \frac{a^2x^2x^m}{\operatorname{atan}^3(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)/atan(a*x)**3,x)
```

```
[Out] c*(Integral(x**m/atan(a*x)**3, x) + Integral(a**2*x**2*x**m/atan(a*x)**3, x
))
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a^2cx^2 + c)x^m}{\arctan(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)*x^m/arctan(a*x)^3, x)
```

3.677 $\int \frac{x^m}{(c+a^2cx^2)\tan^{-1}(ax)^3}\,dx$

Optimal. Leaf size=45

$$\frac{m\text{Unintegrable}\left(\frac{x^{m-1}}{\tan^{-1}(ax)^2},x\right)}{2ac}-\frac{x^m}{2ac\tan^{-1}(ax)^2}$$

```
[Out] -x^m/(2*a*c*ArcTan[a*x]^2) + (m*Unintegrable[x^(-1 + m)/ArcTan[a*x]^2, x])/
(2*a*c)
```

Rubi [A] time = 0.0816005, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m}{(c+a^2cx^2)\tan^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m/((c + a^2*c*x^2)*ArcTan[a*x]^3),x]
```

```
[Out] -x^m/(2*a*c*ArcTan[a*x]^2) + (m*Defer[Int][x^(-1 + m)/ArcTan[a*x]^2, x])/(2
*a*c)
```

Rubi steps

$$\int \frac{x^m}{(c+a^2cx^2)\tan^{-1}(ax)^3}\,dx=-\frac{x^m}{2ac\tan^{-1}(ax)^2}+\frac{m\int\frac{x^{-1+m}}{\tan^{-1}(ax)^2}\,dx}{2ac}$$

Mathematica [A] time = 0.374935, size = 0, normalized size = 0.

$$\int \frac{x^m}{(c+a^2cx^2)\tan^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m/((c + a^2*c*x^2)*ArcTan[a*x]^3),x]
```

```
[Out] Integrate[x^m/((c + a^2*c*x^2)*ArcTan[a*x]^3), x]
```

Maple [A] time = 0.422, size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(a^2cx^2+c\right)\left(\arctan\left(ax\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a^2*c*x^2+c)/arctan(a*x)^3,x)
```

```
[Out] int(x^m/(a^2*c*x^2+c)/arctan(a*x)^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{mx\left(\int \frac{a^2mx^m}{\arctan(ax)}\,dx+\int \frac{a^2x^m}{\arctan(ax)}\,dx+\int \frac{mx^m}{x^2\arctan(ax)}\,dx+\int -\frac{x^m}{x^2\arctan(ax)}\,dx\right)\arctan\left(ax\right)^2-axx^m-\left(a^2mx^2+m\right)x^m\,\mathrm{arct}}{2\,a^2cx\arctan\left(ax\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(x*arctan(a*x)^2*integrate(((a^2*m^2 + a^2*m)*x^2 + m^2 - m)*x^m/(x^2*a
rctan(a*x)), x) - a*x*x^m - (a^2*m*x^2 + m)*x^m*arctan(a*x))/(a^2*c*x*arcta
n(a*x)^2)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x^m}{\left(a^2cx^2+c\right)\arctan\left(ax\right)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(x^m/((a^2*c*x^2 + c)*arctan(a*x)^3), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^m}{a^2x^2\operatorname{atan}^3(ax)+\operatorname{atan}^3(ax)}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(a**2*c*x**2+c)/atan(a*x)**3,x)
```

```
[Out] Integral(x**m/(a**2*x**2*atan(a*x)**3 + atan(a*x)**3), x)/c
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{(a^2cx^2+c)\arctan(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(x^m/((a^2*c*x^2 + c)*arctan(a*x)^3), x)
```

3.678 $\int \frac{x^m}{\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^3}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{x^m}{\left(a^2cx^2+c\right)^2\tan^{-1}(ax)^3},x\right)$$

```
[Out] Unintegrable[x^m/((c + a^2*c*x^2)^2*ArcTan[a*x]^3), x]
```

Rubi [A] time = 0.0667678, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m}{\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m/((c + a^2*c*x^2)^2*ArcTan[a*x]^3),x]
```

```
[Out] Defer[Int][x^m/((c + a^2*c*x^2)^2*ArcTan[a*x]^3), x]
```

Rubi steps

$$\int \frac{x^m}{\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^3}\,dx=\int \frac{x^m}{\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^3}\,dx$$

Mathematica [A] time = 0.492035, size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m/((c + a^2*c*x^2)^2*ArcTan[a*x]^3),x]
```

```
[Out] Integrate[x^m/((c + a^2*c*x^2)^2*ArcTan[a*x]^3), x]
```

Maple [A] time = 1.17, size = 0, normalized size = 0.

$$\int \frac{x^{m}}{\left(a^{2} c \,x^{2}+c \right)^{2} \left(\arctan \left(a x \right)\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^3,x)
```

```
[Out] int(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{\left(a^{4} c^{2} x^{3} + a^{2} c^{2} x\right) \arctan \left(a x\right)^{2} \int \frac{{\left({\left(a^{4} m^{2} - 3 \, a^{4} m + 2 \, a^{4}\right)} x^{4} + 2 \, {\left(a^{2} m^{2} - 2 \, a^{2} m - a^{2}\right)} x^{2} + m^{2} - m\right)} x^{m}}{{\left(a^{6} c^{2} x^{6} + 2 \, a^{4} c^{2} x^{4} + a^{2} c^{2} x^{2}\right)} \arctan\left(a x\right)}\,{d x} - a x x^{m} - {\left({\left(a^{2} m - 2 \, a^{2}\right)} x^{2} + m\right)} x^{m} \mathrm{arct}}{2 \, {\left(a^{4} c^{2} x^{3} + a^{2} c^{2} x\right)} \arctan \left(a x\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(a^4*c^2*x^3 + a^2*c^2*x)*arctan(a*x)^2*integrate(1/2*((a^4*m^2 - 3*
a^4*m + 2*a^4)*x^4 + 2*(a^2*m^2 - 2*a^2*m - a^2)*x^2 + m^2 - m)*x^m/((a^6*c
^2*x^6 + 2*a^4*c^2*x^4 + a^2*c^2*x^2)*arctan(a*x)), x) - a*x*x^m - ((a^2*m
- 2*a^2)*x^2 + m)*x^m*arctan(a*x))/((a^4*c^2*x^3 + a^2*c^2*x)*arctan(a*x)^2
)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x^{m}}{{\left(a^{4} c^{2} x^{4} + 2 \, a^{2} c^{2} x^{2} + c^{2}\right)} \arctan\left(a x\right)^{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(x^m/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(a*x)^3), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^m}{a^4x^4\operatorname{atan}^3(ax)+2a^2x^2\operatorname{atan}^3(ax)+\operatorname{atan}^3(ax)}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(a**2*c*x**2+c)**2/atan(a*x)**3,x)
```

```
[Out] Integral(x**m/(a**4*x**4*atan(a*x)**3 + 2*a**2*x**2*atan(a*x)**3 + atan(a*x
)**3), x)/c**2
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(a^2cx^2+c\right)^2\arctan(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(x^m/((a^2*c*x^2 + c)^2*arctan(a*x)^3), x)
```

3.679 $\int \frac{x^m}{\left(c+a^2cx^2\right)^3 \tan^{-1}(ax)^3} \, dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{x^m}{\left(a^2cx^2+c\right)^3 \tan^{-1}(ax)^3},x\right)$$

```
[Out] Unintegrable[x^m/((c + a^2*c*x^2)^3*ArcTan[a*x]^3), x]
```

Rubi [A] time = 0.0630917, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{x^m}{\left(c+a^2cx^2\right)^3 \tan^{-1}(ax)^3} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m/((c + a^2*c*x^2)^3*ArcTan[a*x]^3),x]
```

```
[Out] Defer[Int][x^m/((c + a^2*c*x^2)^3*ArcTan[a*x]^3), x]
```

Rubi steps

$$\int \frac{x^m}{\left(c+a^2cx^2\right)^3 \tan^{-1}(ax)^3} \, dx = \int \frac{x^m}{\left(c+a^2cx^2\right)^3 \tan^{-1}(ax)^3} \, dx$$

Mathematica [A] time = 0.510576, size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(c+a^2cx^2\right)^3 \tan^{-1}(ax)^3} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m/((c + a^2*c*x^2)^3*ArcTan[a*x]^3),x]
```

```
[Out] Integrate[x^m/((c + a^2*c*x^2)^3*ArcTan[a*x]^3), x]
```

Maple [A] time = 1.22, size = 0, normalized size = 0.

$$\int \frac{x^{m}}{\left(a^{2} c x^{2}+c\right)^{3} \left(\arctan \left(a x\right)\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^3,x)
```

```
[Out] int(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{\left(a^{6} c^{3} x^{5}+2\, a^{4} c^{3} x^{3}+a^{2} c^{3} x\right) \arctan \left(a x\right)^{2} \int \frac{\left(\left(a^{4} m^{2}-7\, a^{4} m+12\, a^{4}\right) x^{4}+2\, \left(a^{2} m^{2}-4\, a^{2} m-2\, a^{2}\right) x^{2}+m^{2}-m\right) x^{m}}{\left(a^{8} c^{3} x^{8}+3\, a^{6} c^{3} x^{6}+3\, a^{4} c^{3} x^{4}+a^{2} c^{3} x^{2}\right) \arctan(a x)}\, dx - a x x^{m} - \left(\left(a^{2} m - 4\, a^{2}\right) x^{2} -\right.}{2 \left(a^{6} c^{3} x^{5}+2\, a^{4} c^{3} x^{3}+a^{2} c^{3} x\right) \arctan \left(a x\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(a^6*c^3*x^5 + 2*a^4*c^3*x^3 + a^2*c^3*x)*arctan(a*x)^2*integrate(1/
2*((a^4*m^2 - 7*a^4*m + 12*a^4)*x^4 + 2*(a^2*m^2 - 4*a^2*m - 2*a^2)*x^2 + m
^2 - m)*x^m/((a^8*c^3*x^8 + 3*a^6*c^3*x^6 + 3*a^4*c^3*x^4 + a^2*c^3*x^2)*ar
ctan(a*x)), x) - a*x*x^m - ((a^2*m - 4*a^2)*x^2 + m)*x^m*arctan(a*x))/((a^6
*c^3*x^5 + 2*a^4*c^3*x^3 + a^2*c^3*x)*arctan(a*x)^2)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x^{m}}{\left(a^{6} c^{3} x^{6}+3\, a^{4} c^{3} x^{4}+3\, a^{2} c^{3} x^{2}+c^{3}\right) \arctan \left(a x\right)^{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(x^m/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*arctan(a*
x)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(a**2*c*x**2+c)**3/atan(a*x)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(a^2cx^2+c\right)^3\arctan\left(ax\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(x^m/((a^2*c*x^2 + c)^3*arctan(a*x)^3), x)
```

3.680 $\int \frac{x^m (c+a^2cx^2)^{5/2}}{\tan^{-1}(ax)^3} \, dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{x^m (a^2cx^2+c)^{5/2}}{\tan^{-1}(ax)^3},x\right)$$

```
[Out] Unintegrable[(x^m*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^3, x]
```

Rubi [A] time = 0.113625, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m (c+a^2cx^2)^{5/2}}{\tan^{-1}(ax)^3} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^3,x]
```

```
[Out] Defer[Int][(x^m*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^3, x]
```

Rubi steps

$$\int \frac{x^m (c+a^2cx^2)^{5/2}}{\tan^{-1}(ax)^3} \, dx = \int \frac{x^m (c+a^2cx^2)^{5/2}}{\tan^{-1}(ax)^3} \, dx$$

Mathematica [A] time = 1.47127, size = 0, normalized size = 0.

$$\int \frac{x^m (c+a^2cx^2)^{5/2}}{\tan^{-1}(ax)^3} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^3,x]
```

```
[Out] Integrate[(x^m*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^3, x]
```

Maple [A] time = 0.618, size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(\arctan \left(ax\right)\right)^3} \left(a^2cx^2 + c\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2 + c\right)^{\frac{5}{2}} x^m}{\arctan \left(ax\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(5/2)*x^m/arctan(a*x)^3, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^4c^2x^4 + 2\,a^2c^2x^2 + c^2\right)\sqrt{a^2cx^2 + c}\,x^m}{\arctan \left(ax\right)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*x^m/arctan
(a*x)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**(5/2)/atan(a*x)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^{\frac{5}{2}}x^m}{\arctan\left(ax\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(5/2)*x^m/arctan(a*x)^3, x)
```

3.681 $\int \frac{x^m \left(c+a^2 c x^2\right)^{3/2}}{\tan^{-1}(ax)^3} \, dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{x^m \left(a^2 c x^2+c\right)^{3/2}}{\tan^{-1}(ax)^3},x\right)$$

```
[Out] Unintegrable[(x^m*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^3, x]
```

Rubi [A] time = 0.113165, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m \left(c+a^2 c x^2\right)^{3/2}}{\tan^{-1}(ax)^3} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^3,x]

[Out] Defer[Int][(x^m*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^3, x]
```

Rubi steps

$$\int \frac{x^m \left(c+a^2 c x^2\right)^{3/2}}{\tan^{-1}(ax)^3} \, dx=\int \frac{x^m \left(c+a^2 c x^2\right)^{3/2}}{\tan^{-1}(ax)^3} \, dx$$

Mathematica [A] time = 0.770978, size = 0, normalized size = 0.

$$\int \frac{x^m \left(c+a^2 c x^2\right)^{3/2}}{\tan^{-1}(ax)^3} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^3,x]
```

```
[Out] Integrate[(x^m*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^3, x]
```

Maple [A] time = 0.585, size = 0, normalized size = 0.

$$\int \frac{x^m}{(\arctan(ax))^3}\left(a^2cx^2+c\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^{\frac{3}{2}}x^m}{\arctan(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)*x^m/arctan(a*x)^3, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(a^2cx^2+c\right)^{\frac{3}{2}}x^m}{\arctan(ax)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)^(3/2)*x^m/arctan(a*x)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**(3/2)/atan(a*x)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^{\frac{3}{2}}x^m}{\arctan\left(ax\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)*x^m/arctan(a*x)^3, x)
```

3.682 $\int \frac{x^m\sqrt{c+a^2cx^2}}{\tan^{-1}(ax)^3}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{x^m\sqrt{a^2cx^2+c}}{\tan^{-1}(ax)^3},x\right)$$

```
[Out] Unintegrable[(x^m*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^3, x]
```

Rubi [A] time = 0.116844, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\tan^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^3,x]

[Out] Defer[Int][(x^m*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^3, x]
```

Rubi steps

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\tan^{-1}(ax)^3}\,dx = \int \frac{x^m\sqrt{c+a^2cx^2}}{\tan^{-1}(ax)^3}\,dx$$

Mathematica [A] time = 0.213247, size = 0, normalized size = 0.

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\tan^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^3,x]
```

```
[Out] Integrate[(x^m*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^3, x]
```

Maple [A] time = 0.753, size = 0, normalized size = 0.

$$\int \frac{x^{m}}{\left(\arctan \left(a x\right)\right)^{3}}\sqrt{a^{2} c \,x^{2}+c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a^{2} c x^{2} + c} x^{m}}{\arctan \left(a x\right)^{3}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)*x^m/arctan(a*x)^3, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$${\rm integral}\left(\frac{\sqrt{a^{2} c x^{2} + c} x^{m}}{\arctan \left(a x\right)^{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^m/arctan(a*x)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**(1/2)/atan(a*x)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a^2cx^2 + cx^m}}{\arctan{(ax)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)*x^m/arctan(a*x)^3, x)
```

3.683 $\int \frac{x^m}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{x^m}{\sqrt{a^2cx^2+c}\tan^{-1}(ax)^3},x\right)$$

```
[Out] Unintegrable[x^m/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3), x]
```

Rubi [A] time = 0.10424, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3),x]
```

```
[Out] Defer[Int][x^m/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3), x]
```

Rubi steps

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}\,dx = \int \frac{x^m}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}\,dx$$

Mathematica [A] time = 0.515232, size = 0, normalized size = 0.

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3),x]
```

```
[Out] Integrate[x^m/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3), x]
```

Maple [A] time = 1.19, size = 0, normalized size = 0.

$$\int \frac{x^{m}}{\left(\arctan \left(a x\right)\right)^{3}}\frac{1}{\sqrt{a^{2} c \,x^{2}+c}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(x^m/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{m}}{\sqrt{a^{2} c x^{2} + c} \arctan \left(a x\right)^{3}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m/(sqrt(a^2*c*x^2 + c)*arctan(a*x)^3), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$${\rm integral}\left(\frac{x^{m}}{\sqrt{a^{2} c x^{2} + c} \arctan \left(a x\right)^{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^m/(sqrt(a^2*c*x^2 + c)*arctan(a*x)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/atan(a*x)**3/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\sqrt{a^2cx^2+c}\arctan{(ax)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^m/(sqrt(a^2*c*x^2 + c)*arctan(a*x)^3), x)
```

3.684 $\int \frac{x^m}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^3}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{x^m}{\left(a^2cx^2+c\right)^{3/2}\tan^{-1}(ax)^3},x\right)$$

```
[Out] Unintegrable[x^m/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3), x]
```

Rubi [A] time = 0.116083, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3),x]
```

```
[Out] Defer[Int][x^m/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3), x]
```

Rubi steps

$$\int \frac{x^m}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^3}\,dx=\int \frac{x^m}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^3}\,dx$$

Mathematica [A] time = 0.570858, size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3),x]
```

```
[Out] Integrate[x^m/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3), x]
```

Maple [A] time = 1.271, size = 0, normalized size = 0.

$$\int \frac{x^m}{(\arctan(ax))^3}\left(a^2cx^2+c\right)^{-\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x)
```

```
[Out] int(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(a^2cx^2+c\right)^{\frac{3}{2}}\arctan(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] integrate(x^m/((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^3), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{a^2cx^2+c}\,x^m}{\left(a^4c^2x^4+2\,a^2c^2x^2+c^2\right)\arctan(ax)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^m/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arcta
n(a*x)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(a**2*c*x**2+c)**(3/2)/atan(a*x)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(a^2cx^2+c\right)^{\frac{3}{2}}\arctan\left(ax\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(x^m/((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^3), x)
```

3.685 $\int \frac{x^m}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^3}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{x^m}{\left(a^2cx^2+c\right)^{5/2}\tan^{-1}(ax)^3},x\right)$$

```
[Out] Unintegrable[x^m/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3), x]
```

Rubi [A] time = 0.117249, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{x^m}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3),x]
```

```
[Out] Defer[Int][x^m/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3), x]
```

Rubi steps

$$\int \frac{x^m}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^3}\,dx=\int \frac{x^m}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^3}\,dx$$

Mathematica [A] time = 0.617376, size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3),x]
```

[Out] Integrate[x^m/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3), x]

Maple [A] time = 1.25, size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(\arctan\left(ax\right)\right)^3}\left(a^2cx^2+c\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x)

[Out] int(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(a^2cx^2+c\right)^{\frac{5}{2}}\arctan\left(ax\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="maxima")

[Out] integrate(x^m/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^3), x)

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a^2cx^2+c}\,x^m}{\left(a^6c^3x^6+3\,a^4c^3x^4+3\,a^2c^3x^2+c^3\right)\arctan\left(ax\right)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="fricas")

[Out] integral(sqrt(a^2*c*x^2 + c)*x^m/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*
x^2 + c^3)*arctan(a*x)^3), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(a**2*c*x**2+c)**(5/2)/atan(a*x)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(a^2cx^2+c\right)^{\frac{5}{2}}\arctan\left(ax\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(x^m/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^3), x)
```

3.686 $\int x^m \left(c+a^2 c x^2\right) \sqrt{\tan^{-1}(ax)} \, dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(x^m \left(a^2 c x^2+c\right) \sqrt{\tan^{-1}(ax)},x\right)$$

```
[Out] Unintegrable[x^m*(c + a^2*c*x^2)*Sqrt[ArcTan[a*x]], x]
```

Rubi [A] time = 0.0341483, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^m \left(c+a^2 c x^2\right) \sqrt{\tan^{-1}(ax)} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*(c + a^2*c*x^2)*Sqrt[ArcTan[a*x]],x]
```

```
[Out] Defer[Int][x^m*(c + a^2*c*x^2)*Sqrt[ArcTan[a*x]], x]
```

Rubi steps

$$\int x^m \left(c+a^2 c x^2\right) \sqrt{\tan^{-1}(ax)} \, dx=\int x^m \left(c+a^2 c x^2\right) \sqrt{\tan^{-1}(ax)} \, dx$$

Mathematica [A] time = 1.95812, size = 0, normalized size = 0.

$$\int x^m \left(c+a^2 c x^2\right) \sqrt{\tan^{-1}(ax)} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*(c + a^2*c*x^2)*Sqrt[ArcTan[a*x]],x]
```

```
[Out] Integrate[x^m*(c + a^2*c*x^2)*Sqrt[ArcTan[a*x]], x]
```

Maple [A] time = 0.765, size = 0, normalized size = 0.

$$\int x^{m} \left(a^{2} c\,x^{2}+c\right) \sqrt{\arctan \left(a x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)*arctan(a*x)^(1/2),x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)*arctan(a*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)*arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^{2} c x^{2}+c\right) x^{m} \sqrt{\arctan\left(a x\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)*arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)*x^m*sqrt(arctan(a*x)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)*atan(a*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2+c\right)x^m\sqrt{\arctan\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)*arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)*x^m*sqrt(arctan(a*x)), x)
```

3.687 $\int x\left(c+a^2cx^2\right)\sqrt{\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=56

$$\frac{c\left(a^2x^2+1\right)^2\sqrt{\tan^{-1}(ax)}}{4a^2}-\frac{\text{Unintegrable}\left(\frac{a^2cx^2+c}{\sqrt{\tan^{-1}(ax)}},x\right)}{8a}$$

```
[Out] (c*(1 + a^2*x^2)^2*Sqrt[ArcTan[a*x]])/(4*a^2) - Unintegrable[(c + a^2*c*x^2
)/Sqrt[ArcTan[a*x]], x]/(8*a)
```

Rubi [A] time = 0.0461141, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x\left(c+a^2cx^2\right)\sqrt{\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x*(c + a^2*c*x^2)*Sqrt[ArcTan[a*x]],x]
```

```
[Out] (c*(1 + a^2*x^2)^2*Sqrt[ArcTan[a*x]])/(4*a^2) - Defer[Int][(c + a^2*c*x^2)/
Sqrt[ArcTan[a*x]], x]/(8*a)
```

Rubi steps

$$\int x\left(c+a^2cx^2\right)\sqrt{\tan^{-1}(ax)}\,dx=\frac{c\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}}{4a^2}-\frac{\int\frac{c+a^2cx^2}{\sqrt{\tan^{-1}(ax)}}\,dx}{8a}$$

Mathematica [A] time = 2.28078, size = 0, normalized size = 0.

$$\int x\left(c+a^2cx^2\right)\sqrt{\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x*(c + a^2*c*x^2)*Sqrt[ArcTan[a*x]],x]
```

```
[Out] Integrate[x*(c + a^2*c*x^2)*Sqrt[ArcTan[a*x]], x]
```

Maple [A] time = 0.377, size = 0, normalized size = 0.

$$\int x\left(a^{2}c x^{2}+c\right)\sqrt{\arctan\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)*arctan(a*x)^(1/2),x)
```

```
[Out] int(x*(a^2*c*x^2+c)*arctan(a*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)*arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)*arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$c\left(\int x\sqrt{\operatorname{atan}\left(ax\right)}\,dx+\int a^{2}x^{3}\sqrt{\operatorname{atan}\left(ax\right)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)*atan(a*x)**(1/2),x)
```

```
[Out] c*(Integral(x*sqrt(atan(a*x)), x) + Integral(a**2*x**3*sqrt(atan(a*x)), x))
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (a^2cx^2 + c)x\sqrt{\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)*arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)*x*sqrt(arctan(a*x)), x)
```

3.688 $\int (c + a^2cx^2) \sqrt{\tan^{-1}(ax)}\, dx$

Optimal. Leaf size=21

$$\text{Unintegrable}\left(\left(a^2cx^2 + c\right) \sqrt{\tan^{-1}(ax)}, x\right)$$

```
[Out] Unintegrable[(c + a^2*c*x^2)*Sqrt[ArcTan[a*x]], x]
```

Rubi [A] time = 0.0131612, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \left(c + a^2cx^2\right) \sqrt{\tan^{-1}(ax)}\, dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)*Sqrt[ArcTan[a*x]],x]
```

```
[Out] Defer[Int][(c + a^2*c*x^2)*Sqrt[ArcTan[a*x]], x]
```

Rubi steps

$$\int \left(c + a^2cx^2\right) \sqrt{\tan^{-1}(ax)}\, dx = \int \left(c + a^2cx^2\right) \sqrt{\tan^{-1}(ax)}\, dx$$

Mathematica [A] time = 3.10737, size = 0, normalized size = 0.

$$\int \left(c + a^2cx^2\right) \sqrt{\tan^{-1}(ax)}\, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)*Sqrt[ArcTan[a*x]],x]
```

```
[Out] Integrate[(c + a^2*c*x^2)*Sqrt[ArcTan[a*x]], x]
```

Maple [A] time = 0.256, size = 0, normalized size = 0.

$$\int \left(a^{2}c x^{2}+c\right)\sqrt{\arctan\left(a x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)*arctan(a*x)^(1/2),x)
```

```
[Out] int((a^2*c*x^2+c)*arctan(a*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$c\left(\int a^{2}x^{2}\sqrt{\operatorname{atan}\left(a x\right)}\, dx+\int \sqrt{\operatorname{atan}\left(a x\right)}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)*atan(a*x)**(1/2),x)
```

```
[Out] c*(Integral(a**2*x**2*sqrt(atan(a*x)), x) + Integral(sqrt(atan(a*x)), x))
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2 + c\right)\sqrt{\arctan\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)*sqrt(arctan(a*x)), x)
```

3.689 $\int \frac{(c+a^2cx^2)\sqrt{\tan^{-1}(ax)}}{x}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{(a^2cx^2+c)\sqrt{\tan^{-1}(ax)}}{x},x\right)$$

```
[Out] Unintegrable[((c + a^2*c*x^2)*Sqrt[ArcTan[a*x]])/x, x]
```

Rubi [A] time = 0.039548, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(c+a^2cx^2)\sqrt{\tan^{-1}(ax)}}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((c + a^2*c*x^2)*Sqrt[ArcTan[a*x]])/x,x]
```

```
[Out] Defer[Int][((c + a^2*c*x^2)*Sqrt[ArcTan[a*x]])/x, x]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)\sqrt{\tan^{-1}(ax)}}{x}\,dx = \int \frac{(c+a^2cx^2)\sqrt{\tan^{-1}(ax)}}{x}\,dx$$

Mathematica [A] time = 1.61191, size = 0, normalized size = 0.

$$\int \frac{(c+a^2cx^2)\sqrt{\tan^{-1}(ax)}}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((c + a^2*c*x^2)*Sqrt[ArcTan[a*x]])/x,x]
```

```
[Out] Integrate[((c + a^2*c*x^2)*Sqrt[ArcTan[a*x]])/x, x]
```

Maple [A] time = 0.401, size = 0, normalized size = 0.

$$\int \frac{a^{2} c \,x^{2}+c}{x}\sqrt{\arctan \left(a x \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)*arctan(a*x)^(1/2)/x,x)
```

```
[Out] int((a^2*c*x^2+c)*arctan(a*x)^(1/2)/x,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$c \left(\int \frac{\sqrt{\operatorname{atan}{\left(a x \right)}}}{x}\, dx + \int a^{2} x \sqrt{\operatorname{atan}{\left(a x \right)}}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)*atan(a*x)**(1/2)/x,x)
```

```
[Out] c*(Integral(sqrt(atan(a*x))/x, x) + Integral(a**2*x*sqrt(atan(a*x)), x))
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)\sqrt{\arctan\left(ax\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)*sqrt(arctan(a*x))/x, x)
```

3.690 $\int x^m \left(c+a^2 c x^2\right)^2 \sqrt{\tan^{-1}(a x)}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(x^m \left(a^2 c x^2+c\right)^2 \sqrt{\tan^{-1}(a x)},x\right)$$

```
[Out] Unintegrable[x^m*(c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]], x]
```

Rubi [A] time = 0.0854464, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^m \left(c+a^2 c x^2\right)^2 \sqrt{\tan^{-1}(a x)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*(c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]],x]
```

```
[Out] Defer[Int][x^m*(c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]], x]
```

Rubi steps

$$\int x^m \left(c+a^2 c x^2\right)^2 \sqrt{\tan^{-1}(a x)}\,dx=\int x^m \left(c+a^2 c x^2\right)^2 \sqrt{\tan^{-1}(a x)}\,dx$$

Mathematica [A] time = 1.25971, size = 0, normalized size = 0.

$$\int x^m \left(c+a^2 c x^2\right)^2 \sqrt{\tan^{-1}(a x)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*(c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]],x]
```

```
[Out] Integrate[x^m*(c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]], x]
```

Maple [A] time = 0.904, size = 0, normalized size = 0.

$$\int x^m \left(a^2 c x^2 + c\right)^2 \sqrt{\arctan (ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^(1/2),x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^4 c^2 x^4 + 2\, a^2 c^2 x^2 + c^2\right) x^m \sqrt{\arctan (ax)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*x^m*sqrt(arctan(a*x)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**2*atan(a*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2+c\right)^2 x^m \sqrt{\arctan\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^2*x^m*sqrt(arctan(a*x)), x)
```

3.691 $\int x\left(c+a^2cx^2\right)^2\sqrt{\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=60

$$\frac{c^2\left(a^2x^2+1\right)^3\sqrt{\tan^{-1}(ax)}}{6a^2}-\frac{\text{Unintegrable}\left(\frac{\left(a^2cx^2+c\right)^2}{\sqrt{\tan^{-1}(ax)}},x\right)}{12a}$$

```
[Out] (c^2*(1 + a^2*x^2)^3*Sqrt[ArcTan[a*x]])/(6*a^2) - Unintegrable[(c + a^2*c*x
^2)^2/Sqrt[ArcTan[a*x]], x]/(12*a)
```

Rubi [A] time = 0.0720387, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x\left(c+a^2cx^2\right)^2\sqrt{\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x*(c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]],x]
```

```
[Out] (c^2*(1 + a^2*x^2)^3*Sqrt[ArcTan[a*x]])/(6*a^2) - Defer[Int][(c + a^2*c*x^2
)^2/Sqrt[ArcTan[a*x]], x]/(12*a)
```

Rubi steps

$$\int x\left(c+a^2cx^2\right)^2\sqrt{\tan^{-1}(ax)}\,dx=\frac{c^2\left(1+a^2x^2\right)^3\sqrt{\tan^{-1}(ax)}}{6a^2}-\frac{\int\frac{\left(c+a^2cx^2\right)^2}{\sqrt{\tan^{-1}(ax)}}\,dx}{12a}$$

Mathematica [A] time = 1.77285, size = 0, normalized size = 0.

$$\int x\left(c+a^2cx^2\right)^2\sqrt{\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x*(c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]],x]
```

```
[Out] Integrate[x*(c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]], x]
```

Maple [A] time = 0.527, size = 0, normalized size = 0.

$$\int x\left(a^{2}c\,x^{2}+c\right)^{2}\sqrt{\arctan\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^2*arctan(a*x)^(1/2),x)
```

```
[Out] int(x*(a^2*c*x^2+c)^2*arctan(a*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^2*arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^2*arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$c^{2}\left(\int x\sqrt{\operatorname{atan}\left(ax\right)}\,dx+\int 2a^{2}x^{3}\sqrt{\operatorname{atan}\left(ax\right)}\,dx+\int a^{4}x^{5}\sqrt{\operatorname{atan}\left(ax\right)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**2*atan(a*x)**(1/2),x)
```

```
[Out] c**2*(Integral(x*sqrt(atan(a*x)), x) + Integral(2*a**2*x**3*sqrt(atan(a*x))
, x) + Integral(a**4*x**5*sqrt(atan(a*x)), x))
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\left(a^{2}cx^{2}+c\right)^{2}x\sqrt{\arctan\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^2*arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^2*x*sqrt(arctan(a*x)), x)
```

3.692 $\int \left(c+a^2 c x^2\right)^2 \sqrt{\tan^{-1}(ax)}\, dx$

Optimal. Leaf size=23

$$\text{Unintegrable}\left(\left(a^2 c x^2+c\right)^2 \sqrt{\tan^{-1}(ax)},x\right)$$

```
[Out] Unintegrable[(c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]], x]
```

Rubi [A] time = 0.0226769, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \left(c+a^2 c x^2\right)^2 \sqrt{\tan^{-1}(ax)}\, dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]],x]
```

```
[Out] Defer[Int][(c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]], x]
```

Rubi steps

$$\int \left(c+a^2 c x^2\right)^2 \sqrt{\tan^{-1}(ax)}\, dx=\int \left(c+a^2 c x^2\right)^2 \sqrt{\tan^{-1}(ax)}\, dx$$

Mathematica [A] time = 1.64386, size = 0, normalized size = 0.

$$\int \left(c+a^2 c x^2\right)^2 \sqrt{\tan^{-1}(ax)}\, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]],x]
```

```
[Out] Integrate[(c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]], x]
```

Maple [A] time = 0.413, size = 0, normalized size = 0.

$$\int \left(a^{2} c\,x^{2}+c\right)^{2} \sqrt{\arctan \left(a x \right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^2*arctan(a*x)^(1/2),x)
```

```
[Out] int((a^2*c*x^2+c)^2*arctan(a*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$c^{2} \left(\int 2 a^{2} x^{2} \sqrt{\operatorname{atan}{\left(a x \right)}}\, dx + \int a^{4} x^{4} \sqrt{\operatorname{atan}{\left(a x \right)}}\, dx + \int \sqrt{\operatorname{atan}{\left(a x \right)}}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**2*atan(a*x)**(1/2),x)
```

```
[Out] c**2*(Integral(2*a**2*x**2*sqrt(atan(a*x)), x) + Integral(a**4*x**4*sqrt(at
an(a*x)), x) + Integral(sqrt(atan(a*x)), x))
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2+c\right)^2\sqrt{\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^2*sqrt(arctan(a*x)), x)
```

3.693 $\int \frac{\left(c+a^2cx^2\right)^2\sqrt{\tan^{-1}(ax)}}{x}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{\left(a^2cx^2+c\right)^2\sqrt{\tan^{-1}(ax)}}{x},x\right)$$

```
[Out] Unintegrable[((c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]])/x, x]
```

Rubi [A] time = 0.0607245, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(c+a^2cx^2\right)^2\sqrt{\tan^{-1}(ax)}}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]])/x,x]
```

```
[Out] Defer[Int][((c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]])/x, x]
```

Rubi steps

$$\int \frac{\left(c+a^2cx^2\right)^2\sqrt{\tan^{-1}(ax)}}{x}\,dx=\int \frac{\left(c+a^2cx^2\right)^2\sqrt{\tan^{-1}(ax)}}{x}\,dx$$

Mathematica [A] time = 1.26086, size = 0, normalized size = 0.

$$\int \frac{\left(c+a^2cx^2\right)^2\sqrt{\tan^{-1}(ax)}}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]])/x,x]
```

```
[Out] Integrate[((c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]])/x, x]
```

Maple [A] time = 0.493, size = 0, normalized size = 0.

$$\int \frac{\left(a^2 c x^2 + c\right)^2}{x} \sqrt{\arctan\left(a x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^2*arctan(a*x)^(1/2)/x,x)
```

```
[Out] int((a^2*c*x^2+c)^2*arctan(a*x)^(1/2)/x,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$c^2\left(\int \frac{\sqrt{\operatorname{atan}(ax)}}{x}\,dx + \int 2a^2x\sqrt{\operatorname{atan}(ax)}\,dx + \int a^4x^3\sqrt{\operatorname{atan}(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**2*atan(a*x)**(1/2)/x,x)
```

```
[Out] c**2*(Integral(sqrt(atan(a*x))/x, x) + Integral(2*a**2*x*sqrt(atan(a*x)), x
) + Integral(a**4*x**3*sqrt(atan(a*x)), x))
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^2\sqrt{\arctan(ax)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^2*sqrt(arctan(a*x))/x, x)
```

3.694 $\int x^m \left(c+a^2 c x^2\right)^3 \sqrt{\tan^{-1}(a x)}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(x^m \left(a^2 c x^2+c\right)^3 \sqrt{\tan^{-1}(a x)},x\right)$$

```
[Out] Unintegrable[x^m*(c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]], x]
```

Rubi [A] time = 0.0583095, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^m \left(c+a^2 c x^2\right)^3 \sqrt{\tan^{-1}(a x)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*(c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]],x]
```

```
[Out] Defer[Int][x^m*(c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]], x]
```

Rubi steps

$$\int x^m \left(c+a^2 c x^2\right)^3 \sqrt{\tan^{-1}(a x)}\,dx=\int x^m \left(c+a^2 c x^2\right)^3 \sqrt{\tan^{-1}(a x)}\,dx$$

Mathematica [A] time = 0.841356, size = 0, normalized size = 0.

$$\int x^m \left(c+a^2 c x^2\right)^3 \sqrt{\tan^{-1}(a x)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*(c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]],x]
```

```
[Out] Integrate[x^m*(c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]], x]
```

Maple [A] time = 0.951, size = 0, normalized size = 0.

$$\int x^{m} \left(a^{2} c x^{2}+c\right)^{3} \sqrt{\arctan \left(a x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)^3*arctan(a*x)^(1/2),x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)^3*arctan(a*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^3*arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a^{6} c^{3} x^{6}+3\, a^{4} c^{3} x^{4}+3\, a^{2} c^{3} x^{2}+c^{3}\right) x^{m} \sqrt{\arctan \left(a x\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^3*arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*x^m*sqrt(arcta
n(a*x)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**3*atan(a*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2 + c\right)^3 x^m \sqrt{\arctan(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^3*arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^3*x^m*sqrt(arctan(a*x)), x)
```

3.695 $\int x\left(c+a^2cx^2\right)^3\sqrt{\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=60

$$\frac{c^3\left(a^2x^2+1\right)^4\sqrt{\tan^{-1}(ax)}}{8a^2}-\frac{\text{Unintegrable}\left(\frac{\left(a^2cx^2+c\right)^3}{\sqrt{\tan^{-1}(ax)}},x\right)}{16a}$$

```
[Out] (c^3*(1 + a^2*x^2)^4*Sqrt[ArcTan[a*x]])/(8*a^2) - Unintegrable[(c + a^2*c*x
^2)^3/Sqrt[ArcTan[a*x]], x]/(16*a)
```

Rubi [A] time = 0.0643505, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x\left(c+a^2cx^2\right)^3\sqrt{\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x*(c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]],x]
```

```
[Out] (c^3*(1 + a^2*x^2)^4*Sqrt[ArcTan[a*x]])/(8*a^2) - Defer[Int][(c + a^2*c*x^2
)^3/Sqrt[ArcTan[a*x]], x]/(16*a)
```

Rubi steps

$$\int x\left(c+a^2cx^2\right)^3\sqrt{\tan^{-1}(ax)}\,dx=\frac{c^3\left(1+a^2x^2\right)^4\sqrt{\tan^{-1}(ax)}}{8a^2}-\frac{\int\frac{\left(c+a^2cx^2\right)^3}{\sqrt{\tan^{-1}(ax)}}\,dx}{16a}$$

Mathematica [A] time = 1.81601, size = 0, normalized size = 0.

$$\int x\left(c+a^2cx^2\right)^3\sqrt{\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x*(c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]],x]
```

```
[Out] Integrate[x*(c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]], x]
```

Maple [A] time = 0.734, size = 0, normalized size = 0.

$$\int x\left(a^{2}c x^{2}+c\right)^{3}\sqrt{\arctan\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^3*arctan(a*x)^(1/2),x)
```

```
[Out] int(x*(a^2*c*x^2+c)^3*arctan(a*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^3*arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^3*arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$c^{3}\left(\int x\sqrt{\operatorname{atan}{\left(ax\right)}}\,dx+\int 3a^{2}x^{3}\sqrt{\operatorname{atan}{\left(ax\right)}}\,dx+\int 3a^{4}x^{5}\sqrt{\operatorname{atan}{\left(ax\right)}}\,dx+\int a^{6}x^{7}\sqrt{\operatorname{atan}{\left(ax\right)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**3*atan(a*x)**(1/2),x)
```

```
[Out] c**3*(Integral(x*sqrt(atan(a*x)), x) + Integral(3*a**2*x**3*sqrt(atan(a*x))
, x) + Integral(3*a**4*x**5*sqrt(atan(a*x)), x) + Integral(a**6*x**7*sqrt(a
tan(a*x)), x))
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(a^{2}cx^{2}+c\right)^{3}x\sqrt{\arctan\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^3*arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^3*x*sqrt(arctan(a*x)), x)
```

3.696 $\int \left(c+a^2 c x^2\right)^3 \sqrt{\tan^{-1}(a x)}\,dx$

Optimal. Leaf size=23

$$\text{Unintegrable}\left(\left(a^2 c x^2+c\right)^3 \sqrt{\tan^{-1}(a x)},x\right)$$

```
[Out] Unintegrable[(c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]], x]
```

Rubi [A] time = 0.0240414, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \left(c+a^2 c x^2\right)^3 \sqrt{\tan^{-1}(a x)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]],x]
```

```
[Out] Defer[Int][(c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]], x]
```

Rubi steps

$$\int \left(c+a^2 c x^2\right)^3 \sqrt{\tan^{-1}(a x)}\,dx=\int \left(c+a^2 c x^2\right)^3 \sqrt{\tan^{-1}(a x)}\,dx$$

Mathematica [A] time = 1.7122, size = 0, normalized size = 0.

$$\int \left(c+a^2 c x^2\right)^3 \sqrt{\tan^{-1}(a x)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]],x]
```

```
[Out] Integrate[(c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]], x]
```

Maple [A] time = 0.599, size = 0, normalized size = 0.

$$\int \left(a^{2} c \,x^{2}+c \right)^{3} \sqrt{\arctan \left(a x \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^3*arctan(a*x)^(1/2),x)
```

```
[Out] int((a^2*c*x^2+c)^3*arctan(a*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$c^{3} \left(\int 3 a^{2} x^{2} \sqrt{\operatorname{atan}{\left(a x \right)}}\, dx + \int 3 a^{4} x^{4} \sqrt{\operatorname{atan}{\left(a x \right)}}\, dx + \int a^{6} x^{6} \sqrt{\operatorname{atan}{\left(a x \right)}}\, dx + \int \sqrt{\operatorname{atan}{\left(a x \right)}}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**3*atan(a*x)**(1/2),x)
```

```
[Out] c**3*(Integral(3*a**2*x**2*sqrt(atan(a*x)), x) + Integral(3*a**4*x**4*sqrt(
atan(a*x)), x) + Integral(a**6*x**6*sqrt(atan(a*x)), x) + Integral(sqrt(ata
n(a*x)), x))
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2+c\right)^3\sqrt{\arctan\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^3*sqrt(arctan(a*x)), x)
```

3.697 $\int \frac{(c+a^2cx^2)^3\sqrt{\tan^{-1}(ax)}}{x}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{(a^2cx^2+c)^3\sqrt{\tan^{-1}(ax)}}{x},x\right)$$

```
[Out] Unintegrable[((c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]])/x, x]
```

Rubi [A] time = 0.052464, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(c+a^2cx^2)^3\sqrt{\tan^{-1}(ax)}}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]])/x,x]
```

```
[Out] Defer[Int][((c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]])/x, x]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^3\sqrt{\tan^{-1}(ax)}}{x}\,dx=\int \frac{(c+a^2cx^2)^3\sqrt{\tan^{-1}(ax)}}{x}\,dx$$

Mathematica [A] time = 1.31801, size = 0, normalized size = 0.

$$\int \frac{(c+a^2cx^2)^3\sqrt{\tan^{-1}(ax)}}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]])/x,x]
```

```
[Out] Integrate[((c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]])/x, x]
```

Maple [A] time = 0.672, size = 0, normalized size = 0.

$$\int \frac{\left(a^{2} c\,x^{2}+c\right)^{3}}{x}\sqrt{\arctan \left(a x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^3*arctan(a*x)^(1/2)/x,x)
```

```
[Out] int((a^2*c*x^2+c)^3*arctan(a*x)^(1/2)/x,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$c^{3}\left(\int \frac{\sqrt{\operatorname{atan}(ax)}}{x}\,dx + \int 3a^{2}x\sqrt{\operatorname{atan}(ax)}\,dx + \int 3a^{4}x^{3}\sqrt{\operatorname{atan}(ax)}\,dx + \int a^{6}x^{5}\sqrt{\operatorname{atan}(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**3*atan(a*x)**(1/2)/x,x)
```

```
[Out] c**3*(Integral(sqrt(atan(a*x))/x, x) + Integral(3*a**2*x*sqrt(atan(a*x)), x
) + Integral(3*a**4*x**3*sqrt(atan(a*x)), x) + Integral(a**6*x**5*sqrt(atan
(a*x)), x))
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^{2}cx^{2}+c\right)^{3}\sqrt{\arctan(ax)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^3*sqrt(arctan(a*x))/x, x)
```

3.698 $\int \frac{x^m\sqrt{\tan^{-1}(ax)}}{c+a^2cx^2}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{x^m\sqrt{\tan^{-1}(ax)}}{a^2cx^2+c},x\right)$$

```
[Out] Unintegrable[(x^m*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2), x]
```

Rubi [A] time = 0.0639933, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m\sqrt{\tan^{-1}(ax)}}{c+a^2cx^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2),x]

[Out] Defer[Int][(x^m*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2), x]
```

Rubi steps

$$\int \frac{x^m\sqrt{\tan^{-1}(ax)}}{c+a^2cx^2}\,dx=\int \frac{x^m\sqrt{\tan^{-1}(ax)}}{c+a^2cx^2}\,dx$$

Mathematica [A] time = 0.774744, size = 0, normalized size = 0.

$$\int \frac{x^m\sqrt{\tan^{-1}(ax)}}{c+a^2cx^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2),x]
```

```
[Out] Integrate[(x^m*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2), x]
```

Maple [A] time = 0.572, size = 0, normalized size = 0.

$$\int \frac{x^m}{a^2cx^2+c}\sqrt{\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c),x)
```

```
[Out] int(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^m\sqrt{\arctan(ax)}}{a^2cx^2+c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(x^m*sqrt(arctan(a*x))/(a^2*c*x^2 + c), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^m\sqrt{\operatorname{atan}(ax)}}{a^2x^2+1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*atan(a*x)**(1/2)/(a**2*c*x**2+c),x)
```

```
[Out] Integral(x**m*sqrt(atan(a*x))/(a**2*x**2 + 1), x)/c
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m\sqrt{\arctan(ax)}}{a^2cx^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(x^m*sqrt(arctan(a*x))/(a^2*c*x^2 + c), x)
```

3.699 $\int \frac{x^3\sqrt{\tan^{-1}(ax)}}{c+a^2cx^2}\,dx$

Optimal. Leaf size=59

$$\frac{\text{Unintegrable}\left(x\sqrt{\tan^{-1}(ax)},x\right)}{a^2c}+\frac{2\text{Unintegrable}\left(\tan^{-1}(ax)^{3/2},x\right)}{3a^3c}-\frac{2x\tan^{-1}(ax)^{3/2}}{3a^3c}$$

```
[Out] (-2*x*ArcTan[a*x]^(3/2))/(3*a^3*c) + Unintegrable[x*Sqrt[ArcTan[a*x]], x]/(
a^2*c) + (2*Unintegrable[ArcTan[a*x]^(3/2), x])/(3*a^3*c)
```

Rubi [A] time = 0.122434, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^3\sqrt{\tan^{-1}(ax)}}{c+a^2cx^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^3*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2),x]
```

```
[Out] (-2*x*ArcTan[a*x]^(3/2))/(3*a^3*c) + Defer[Int][x*Sqrt[ArcTan[a*x]], x]/(a^
2*c) + (2*Defer[Int][ArcTan[a*x]^(3/2), x])/(3*a^3*c)
```

Rubi steps

$$\begin{aligned}\int \frac{x^3\sqrt{\tan^{-1}(ax)}}{c+a^2cx^2}\,dx &= -\frac{\int \frac{x\sqrt{\tan^{-1}(ax)}}{c+a^2cx^2}\,dx}{a^2}+\frac{\int x\sqrt{\tan^{-1}(ax)}\,dx}{a^2c}\\ &= -\frac{2x\tan^{-1}(ax)^{3/2}}{3a^3c}+\frac{2\int \tan^{-1}(ax)^{3/2}\,dx}{3a^3c}+\frac{\int x\sqrt{\tan^{-1}(ax)}\,dx}{a^2c}\end{aligned}$$

Mathematica [A] time = 2.64351, size = 0, normalized size = 0.

$$\int \frac{x^3\sqrt{\tan^{-1}(ax)}}{c+a^2cx^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^3*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2),x]
```

```
[Out] Integrate[(x^3*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2), x]
```

Maple [A] time = 0.555, size = 0, normalized size = 0.

$$\int \frac{x^{3}}{a^{2} c \,x^{2}+c}\sqrt{\arctan \left(a x \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c),x)
```

```
[Out] int(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^3\sqrt{\operatorname{atan}(ax)}}{a^2x^2+1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atan(a*x)**(1/2)/(a**2*c*x**2+c),x)
```

```
[Out] Integral(x**3*sqrt(atan(a*x))/(a**2*x**2 + 1), x)/c
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3\sqrt{\arctan(ax)}}{a^2cx^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(x^3*sqrt(arctan(a*x))/(a^2*c*x^2 + c), x)
```

3.700 $\int \frac{x^2\sqrt{\tan^{-1}(ax)}}{c+a^2cx^2}\,dx$

Optimal. Leaf size=36

$$\frac{\text{Unintegrable}\left(\sqrt{\tan^{-1}(ax)},x\right)}{a^2c}-\frac{2\tan^{-1}(ax)^{3/2}}{3a^3c}$$

```
[Out] (-2*ArcTan[a*x]^(3/2))/(3*a^3*c) + Unintegrable[Sqrt[ArcTan[a*x]], x]/(a^2*
c)
```

Rubi [A] time = 0.0977936, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^2\sqrt{\tan^{-1}(ax)}}{c+a^2cx^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^2*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2),x]
```

```
[Out] (-2*ArcTan[a*x]^(3/2))/(3*a^3*c) + Defer[Int][Sqrt[ArcTan[a*x]], x]/(a^2*c)
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\sqrt{\tan^{-1}(ax)}}{c+a^2cx^2}\,dx &= -\frac{\int \frac{\sqrt{\tan^{-1}(ax)}}{c+a^2cx^2}\,dx}{a^2}+\frac{\int \sqrt{\tan^{-1}(ax)}\,dx}{a^2c}\\ &= -\frac{2\tan^{-1}(ax)^{3/2}}{3a^3c}+\frac{\int \sqrt{\tan^{-1}(ax)}\,dx}{a^2c}\end{aligned}$$

Mathematica [A] time = 1.06475, size = 0, normalized size = 0.

$$\int \frac{x^2\sqrt{\tan^{-1}(ax)}}{c+a^2cx^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^2*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2),x]
```

```
[Out] Integrate[(x^2*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2), x]
```

Maple [A] time = 0.257, size = 0, normalized size = 0.

$$\int \frac{x^2}{a^2cx^2+c}\sqrt{\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c),x)
```

```
[Out] int(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^2\sqrt{\operatorname{atan}(ax)}}{a^2x^2+1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(a*x)**(1/2)/(a**2*c*x**2+c),x)
```

```
[Out] Integral(x**2*sqrt(atan(a*x))/(a**2*x**2 + 1), x)/c
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\sqrt{\arctan(ax)}}{a^2cx^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(x^2*sqrt(arctan(a*x))/(a^2*c*x^2 + c), x)
```

3.701 $\int \frac{x\sqrt{\tan^{-1}(ax)}}{c+a^2cx^2}\,dx$

Optimal. Leaf size=40

$$\frac{2x\tan^{-1}(ax)^{3/2}}{3ac}-\frac{2\text{Unintegrable}\left(\tan^{-1}(ax)^{3/2},x\right)}{3ac}$$

```
[Out] (2*x*ArcTan[a*x]^(3/2))/(3*a*c) - (2*Unintegrable[ArcTan[a*x]^(3/2), x])/(3
*a*c)
```

Rubi [A] time = 0.0488572, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x\sqrt{\tan^{-1}(ax)}}{c+a^2cx^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2),x]
```

```
[Out] (2*x*ArcTan[a*x]^(3/2))/(3*a*c) - (2*Defer[Int][ArcTan[a*x]^(3/2), x])/(3*a
*c)
```

Rubi steps

$$\int \frac{x\sqrt{\tan^{-1}(ax)}}{c+a^2cx^2}\,dx=\frac{2x\tan^{-1}(ax)^{3/2}}{3ac}-\frac{2\int \tan^{-1}(ax)^{3/2}\,dx}{3ac}$$

Mathematica [A] time = 0.973775, size = 0, normalized size = 0.

$$\int \frac{x\sqrt{\tan^{-1}(ax)}}{c+a^2cx^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2),x]
```

```
[Out] Integrate[(x*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2), x]
```

Maple [A] time = 0.142, size = 0, normalized size = 0.

$$\int \frac{x}{a^2cx^2+c}\sqrt{\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c),x)
```

```
[Out] int(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x\sqrt{\operatorname{atan}(ax)}}{a^2x^2+1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(a*x)**(1/2)/(a**2*c*x**2+c),x)
```

```
[Out] Integral(x*sqrt(atan(a*x))/(a**2*x**2 + 1), x)/c
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sqrt{\arctan(ax)}}{a^2cx^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(x*sqrt(arctan(a*x))/(a^2*c*x^2 + c), x)
```

3.702 $\int \frac{\sqrt{\tan^{-1}(ax)}}{c+a^2cx^2}\,dx$

Optimal. Leaf size=18

$$\frac{2\tan^{-1}(ax)^{3/2}}{3ac}$$

```
[Out] (2*ArcTan[a*x]^(3/2))/(3*a*c)
```

Rubi [A] time = 0.0242699, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {4884}

$$\frac{2\tan^{-1}(ax)^{3/2}}{3ac}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[ArcTan[a*x]]/(c + a^2*c*x^2),x]

[Out] (2*ArcTan[a*x]^(3/2))/(3*a*c)
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\sqrt{\tan^{-1}(ax)}}{c+a^2cx^2}\,dx=\frac{2\tan^{-1}(ax)^{3/2}}{3ac}$$

Mathematica [A] time = 0.0034562, size = 18, normalized size = 1.

$$\frac{2\tan^{-1}(ax)^{3/2}}{3ac}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[ArcTan[a*x]]/(c + a^2*c*x^2),x]
```

```
[Out] (2*ArcTan[a*x]^(3/2))/(3*a*c)
```

Maple [A] time = 0.092, size = 15, normalized size = 0.8

$$\frac{2}{3\,ac}\left(\arctan\left(ax\right)\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(1/2)/(a^2*c*x^2+c),x)
```

```
[Out] 2/3*arctan(a*x)^(3/2)/a/c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 1.71705, size = 39, normalized size = 2.17

$$\frac{2\,\arctan\left(ax\right)^{\frac{3}{2}}}{3\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] 2/3*arctan(a*x)^(3/2)/(a*c)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sqrt{\operatorname{atan}(ax)}}{a^2x^2+1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(1/2)/(a**2*c*x**2+c),x)
```

```
[Out] Integral(sqrt(atan(a*x))/(a**2*x**2 + 1), x)/c
```

Giac [A] time = 1.08891, size = 19, normalized size = 1.06

$$\frac{2\,\arctan(ax)^{\frac{3}{2}}}{3\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] 2/3*arctan(a*x)^(3/2)/(a*c)
```

3.703 $\int \frac{\sqrt{\tan^{-1}(ax)}}{x(c+a^2cx^2)}\,dx$

Optimal. Leaf size=48

$$\frac{i\,\text{Unintegrable}\left(\frac{\sqrt{\tan^{-1}(ax)}}{x(ax+i)},x\right)}{c}-\frac{2i\tan^{-1}(ax)^{3/2}}{3c}$$

```
[Out] (((-2*I)/3)*ArcTan[a*x]^(3/2))/c + (I*Unintegrable[Sqrt[ArcTan[a*x]]/(x*(I
+ a*x)), x])/c
```

Rubi [A] time = 0.107228, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\sqrt{\tan^{-1}(ax)}}{x\left(c+a^2cx^2\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[ArcTan[a*x]]/(x*(c + a^2*c*x^2)),x]
```

```
[Out] (((-2*I)/3)*ArcTan[a*x]^(3/2))/c + (I*Defer[Int][Sqrt[ArcTan[a*x]]/(x*(I +
a*x)), x])/c
```

Rubi steps

$$\int \frac{\sqrt{\tan^{-1}(ax)}}{x\left(c+a^2cx^2\right)}\,dx=-\frac{2i\tan^{-1}(ax)^{3/2}}{3c}+\frac{i\int \frac{\sqrt{\tan^{-1}(ax)}}{x(i+ax)}\,dx}{c}$$

Mathematica [A] time = 0.584482, size = 0, normalized size = 0.

$$\int \frac{\sqrt{\tan^{-1}(ax)}}{x\left(c+a^2cx^2\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[ArcTan[a*x]]/(x*(c + a^2*c*x^2)),x]
```

```
[Out] Integrate[Sqrt[ArcTan[a*x]]/(x*(c + a^2*c*x^2)), x]
```

Maple [A] time = 0.139, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(a^{2}c x^{2}+c\right)}\sqrt{\arctan\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c),x)
```

```
[Out] int(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sqrt{\operatorname{atan}(ax)}}{a^2x^3+x}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(1/2)/x/(a**2*c*x**2+c),x)
```

```
[Out] Integral(sqrt(atan(a*x))/(a**2*x**3 + x), x)/c
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\arctan(ax)}}{(a^2cx^2+c)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(sqrt(arctan(a*x))/((a^2*c*x^2 + c)*x), x)
```

3.704 $\int \frac{\sqrt{\tan^{-1}(ax)}}{x^2(c+a^2cx^2)}\,dx$

Optimal. Leaf size=35

$$\frac{\text{Unintegrable}\left(\frac{\sqrt{\tan^{-1}(ax)}}{x^2},x\right)}{c}-\frac{2a\tan^{-1}(ax)^{3/2}}{3c}$$

```
[Out] (-2*a*ArcTan[a*x]^(3/2))/(3*c) + Unintegrable[Sqrt[ArcTan[a*x]]/x^2, x]/c
```

Rubi [A] time = 0.103873, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sqrt{\tan^{-1}(ax)}}{x^2\left(c+a^2cx^2\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[ArcTan[a*x]]/(x^2*(c + a^2*c*x^2)),x]
```

```
[Out] (-2*a*ArcTan[a*x]^(3/2))/(3*c) + Defer[Int][Sqrt[ArcTan[a*x]]/x^2, x]/c
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\tan^{-1}(ax)}}{x^2\left(c+a^2cx^2\right)}\,dx&=-\left(a^2\int \frac{\sqrt{\tan^{-1}(ax)}}{c+a^2cx^2}\,dx\right)+\frac{\int \frac{\sqrt{\tan^{-1}(ax)}}{x^2}\,dx}{c}\\&=-\frac{2a\tan^{-1}(ax)^{3/2}}{3c}+\frac{\int \frac{\sqrt{\tan^{-1}(ax)}}{x^2}\,dx}{c}\end{aligned}$$

Mathematica [A] time = 1.45247, size = 0, normalized size = 0.

$$\int \frac{\sqrt{\tan^{-1}(ax)}}{x^2\left(c+a^2cx^2\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[ArcTan[a*x]]/(x^2*(c + a^2*c*x^2)),x]
```

```
[Out] Integrate[Sqrt[ArcTan[a*x]]/(x^2*(c + a^2*c*x^2)), x]
```

Maple [A] time = 0.248, size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \left(a^2 c x^2 + c\right)} \sqrt{\arctan(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(1/2)/x^2/(a^2*c*x^2+c),x)
```

```
[Out] int(arctan(a*x)^(1/2)/x^2/(a^2*c*x^2+c),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/x^2/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/x^2/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sqrt{\operatorname{atan}(ax)}}{a^2x^4+x^2}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(1/2)/x**2/(a**2*c*x**2+c),x)
```

```
[Out] Integral(sqrt(atan(a*x))/(a**2*x**4 + x**2), x)/c
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\arctan(ax)}}{(a^2cx^2+c)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/x^2/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(sqrt(arctan(a*x))/((a^2*c*x^2 + c)*x^2), x)
```

3.705 $\int \frac{\sqrt{\tan^{-1}(ax)}}{x^3(c+a^2cx^2)}\,dx$

Optimal. Leaf size=72

$$-\frac{ia^2\text{Unintegrable}\left(\frac{\sqrt{\tan^{-1}(ax)}}{x(ax+i)},x\right)}{c}+\frac{\text{Unintegrable}\left(\frac{\sqrt{\tan^{-1}(ax)}}{x^3},x\right)}{c}+\frac{2ia^2\tan^{-1}(ax)^{3/2}}{3c}$$

```
[Out] (((2*I)/3)*a^2*ArcTan[a*x]^(3/2))/c + Unintegrable[Sqrt[ArcTan[a*x]]/x^3, x
]/c - (I*a^2*Unintegrable[Sqrt[ArcTan[a*x]]/(x*(I + a*x)), x])/c
```

Rubi [A] time = 0.193768, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sqrt{\tan^{-1}(ax)}}{x^3\left(c+a^2cx^2\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[ArcTan[a*x]]/(x^3*(c + a^2*c*x^2)),x]
```

```
[Out] (((2*I)/3)*a^2*ArcTan[a*x]^(3/2))/c + Defer[Int][Sqrt[ArcTan[a*x]]/x^3, x]/
c - (I*a^2*Defer[Int][Sqrt[ArcTan[a*x]]/(x*(I + a*x)), x])/c
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{\tan^{-1}(ax)}}{x^3\left(c+a^2cx^2\right)}\,dx &= -\left(a^2\int \frac{\sqrt{\tan^{-1}(ax)}}{x\left(c+a^2cx^2\right)}\,dx\right)+\frac{\int \frac{\sqrt{\tan^{-1}(ax)}}{x^3}\,dx}{c}\\
&= \frac{2ia^2\tan^{-1}(ax)^{3/2}}{3c}+\frac{\int \frac{\sqrt{\tan^{-1}(ax)}}{x^3}\,dx}{c}-\frac{\left(ia^2\right)\int \frac{\sqrt{\tan^{-1}(ax)}}{x(i+ax)}\,dx}{c}
\end{aligned}$$

Mathematica [A] time = 2.15093, size = 0, normalized size = 0.

$$\int \frac{\sqrt{\tan^{-1}(ax)}}{x^3 \left(c + a^2 c x^2\right)} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[ArcTan[a*x]]/(x^3*(c + a^2*c*x^2)),x]
```

```
[Out] Integrate[Sqrt[ArcTan[a*x]]/(x^3*(c + a^2*c*x^2)), x]
```

Maple [A] time = 0.605, size = 0, normalized size = 0.

$$\int \frac{1}{x^3 \left(a^2 c x^2 + c\right)} \sqrt{\arctan (ax)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(1/2)/x^3/(a^2*c*x^2+c),x)
```

```
[Out] int(arctan(a*x)^(1/2)/x^3/(a^2*c*x^2+c),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/x^3/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/x^3/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sqrt{\operatorname{atan}(ax)}}{a^2x^5+x^3}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(1/2)/x**3/(a**2*c*x**2+c),x)
```

```
[Out] Integral(sqrt(atan(a*x))/(a**2*x**5 + x**3), x)/c
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\arctan(ax)}}{\left(a^2cx^2+c\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/x^3/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(sqrt(arctan(a*x))/((a^2*c*x^2 + c)*x^3), x)
```

3.706 $\int \frac{\sqrt{\tan^{-1}(ax)}}{x^4(c+a^2cx^2)} dx$

Optimal. Leaf size=59

$$-\frac{a^2\text{Unintegrable}\left(\frac{\sqrt{\tan^{-1}(ax)}}{x^2},x\right)}{c}+\frac{\text{Unintegrable}\left(\frac{\sqrt{\tan^{-1}(ax)}}{x^4},x\right)}{c}+\frac{2a^3\tan^{-1}(ax)^{3/2}}{3c}$$

[Out] (2*a^3*ArcTan[a*x]^(3/2))/(3*c) + Unintegrable[Sqrt[ArcTan[a*x]]/x^4, x]/c - (a^2*Unintegrable[Sqrt[ArcTan[a*x]]/x^2, x])/c

---

Rubi [A] time = 0.187223, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sqrt{\tan^{-1}(ax)}}{x^4\left(c+a^2cx^2\right)}\,dx$$

Verification is Not applicable to the result.

[In] Int[Sqrt[ArcTan[a*x]]/(x^4*(c + a^2*c*x^2)),x]

[Out] (2*a^3*ArcTan[a*x]^(3/2))/(3*c) + Defer[Int][Sqrt[ArcTan[a*x]]/x^4, x]/c - (a^2*Defer[Int][Sqrt[ArcTan[a*x]]/x^2, x])/c

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{\tan^{-1}(ax)}}{x^4\left(c+a^2cx^2\right)}\,dx &= -\left(a^2\int \frac{\sqrt{\tan^{-1}(ax)}}{x^2\left(c+a^2cx^2\right)}\,dx\right)+\frac{\int \frac{\sqrt{\tan^{-1}(ax)}}{x^4}\,dx}{c}\\
&= a^4\int \frac{\sqrt{\tan^{-1}(ax)}}{c+a^2cx^2}\,dx+\frac{\int \frac{\sqrt{\tan^{-1}(ax)}}{x^4}\,dx}{c}-\frac{a^2\int \frac{\sqrt{\tan^{-1}(ax)}}{x^2}\,dx}{c}\\
&= \frac{2a^3\tan^{-1}(ax)^{3/2}}{3c}+\frac{\int \frac{\sqrt{\tan^{-1}(ax)}}{x^4}\,dx}{c}-\frac{a^2\int \frac{\sqrt{\tan^{-1}(ax)}}{x^2}\,dx}{c}
\end{aligned}$$

Mathematica [A] time = 6.02801, size = 0, normalized size = 0.

$$\int \frac{\sqrt{\tan^{-1}(ax)}}{x^4\left(c+a^2cx^2\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[ArcTan[a*x]]/(x^4*(c + a^2*c*x^2)),x]
```

```
[Out] Integrate[Sqrt[ArcTan[a*x]]/(x^4*(c + a^2*c*x^2)), x]
```

Maple [A] time = 0.5, size = 0, normalized size = 0.

$$\int \frac{1}{x^4\left(a^2cx^2+c\right)}\sqrt{\arctan\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(1/2)/x^4/(a^2*c*x^2+c),x)
```

```
[Out] int(arctan(a*x)^(1/2)/x^4/(a^2*c*x^2+c),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/x^4/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/x^4/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sqrt{\operatorname{atan}(ax)}}{a^2x^6+x^4}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(1/2)/x**4/(a**2*c*x**2+c),x)
```

```
[Out] Integral(sqrt(atan(a*x))/(a**2*x**6 + x**4), x)/c
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\arctan(ax)}}{\left(a^2cx^2+c\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/x^4/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(sqrt(arctan(a*x))/((a^2*c*x^2 + c)*x^4), x)
```

3.707 $\int \frac{x^m\sqrt{\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^2}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{x^m\sqrt{\tan^{-1}(ax)}}{\left(a^2cx^2+c\right)^2},x\right)$$

```
[Out] Unintegrable[(x^m*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^2, x]
```

Rubi [A] time = 0.0636876, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m\sqrt{\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^2,x]

[Out] Defer[Int][(x^m*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^2, x]
```

Rubi steps

$$\int \frac{x^m\sqrt{\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^2}\,dx=\int \frac{x^m\sqrt{\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^2}\,dx$$

Mathematica [A] time = 1.63709, size = 0, normalized size = 0.

$$\int \frac{x^m\sqrt{\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^2,x]
```

```
[Out] Integrate[(x^m*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^2, x]
```

Maple [A] time = 0.786, size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(a^2cx^2+c\right)^2}\sqrt{\arctan\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^2,x)
```

```
[Out] int(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x^m\sqrt{\arctan\left(ax\right)}}{a^4c^2x^4+2\,a^2c^2x^2+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] integral(x^m*sqrt(arctan(a*x))/(a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*atan(a*x)**(1/2)/(a**2*c*x**2+c)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m \sqrt{\arctan(ax)}}{(a^2cx^2 + c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(x^m*sqrt(arctan(a*x))/(a^2*c*x^2 + c)^2, x)
```

3.708 $\int \frac{x^3\sqrt{\tan^{-1}(ax)}}{(c+a^2cx^2)^2}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{x^3\sqrt{\tan^{-1}(ax)}}{\left(a^2cx^2+c\right)^2},x\right)$$

```
[Out] Unintegrable[(x^3*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^2, x]
```

Rubi [A] time = 0.0637315, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{x^3\sqrt{\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^3*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^2,x]

[Out] Defer[Int][(x^3*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^2, x]
```

Rubi steps

$$\int \frac{x^3\sqrt{\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^2}\,dx = \int \frac{x^3\sqrt{\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^2}\,dx$$

Mathematica [A] time = 3.68064, size = 0, normalized size = 0.

$$\int \frac{x^3\sqrt{\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^3*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^2,x]
```

```
[Out] Integrate[(x^3*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^2, x]
```

Maple [A] time = 0.562, size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(a^2 c x^2 + c\right)^2} \sqrt{\arctan\left(ax\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^2,x)
```

```
[Out] int(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^3\sqrt{\operatorname{atan}(ax)}}{a^4x^4+2a^2x^2+1}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atan(a*x)**(1/2)/(a**2*c*x**2+c)**2,x)

[Out] Integral(x**3*sqrt(atan(a*x))/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3\sqrt{\arctan(ax)}}{(a^2cx^2+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^2,x, algorithm="giac")

[Out] integrate(x^3*sqrt(arctan(a*x))/(a^2*c*x^2 + c)^2, x)
```

3.709 $\int \frac{x^2\sqrt{\tan^{-1}(ax)}}{(c+a^2cx^2)^2}\,dx$

Optimal. Leaf size=80

$$\frac{\sqrt{\pi}S\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{8a^3c^2}-\frac{x\sqrt{\tan^{-1}(ax)}}{2a^2c^2\left(a^2x^2+1\right)}+\frac{\tan^{-1}(ax)^{3/2}}{3a^3c^2}$$

```
[Out] -(x*Sqrt[ArcTan[a*x]])/(2*a^2*c^2*(1 + a^2*x^2)) + ArcTan[a*x]^(3/2)/(3*a^3
*c^2) + (Sqrt[Pi]*FresnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(8*a^3*c^2)
```

Rubi [A] time = 0.146863, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4936, 4970, 4406, 12, 3305, 3351}

$$\frac{\sqrt{\pi}S\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{8a^3c^2}-\frac{x\sqrt{\tan^{-1}(ax)}}{2a^2c^2\left(a^2x^2+1\right)}+\frac{\tan^{-1}(ax)^{3/2}}{3a^3c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^2,x]
```

```
[Out] -(x*Sqrt[ArcTan[a*x]])/(2*a^2*c^2*(1 + a^2*x^2)) + ArcTan[a*x]^(3/2)/(3*a^3
*c^2) + (Sqrt[Pi]*FresnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(8*a^3*c^2)
```

Rule 4936

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^2)/((d_) + (e_.)*(x_)^2)
^2, x_Symbol] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(2*b*c^3*d^2*(p + 1)), x]
 + (Dist[(b*p)/(2*c), Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(d + e*x^2)^2, x]
, x] - Simp[(x*(a + b*ArcTan[c*x])^p)/(2*c^2*d*(d + e*x^2)), x]) /; FreeQ[{
a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4970

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sin[x]^m)/
```

```
Cos[x]^(m + 2*(q + 1)), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{x^2\sqrt{\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^2}\,dx = -\frac{x\sqrt{\tan^{-1}(ax)}}{2a^2c^2\left(1+a^2x^2\right)} + \frac{\tan^{-1}(ax)^{3/2}}{3a^3c^2} + \frac{\int \frac{x}{\left(c+a^2cx^2\right)^2\sqrt{\tan^{-1}(ax)}}\,dx}{4a}$$

$$= -\frac{x\sqrt{\tan^{-1}(ax)}}{2a^2c^2\left(1+a^2x^2\right)} + \frac{\tan^{-1}(ax)^{3/2}}{3a^3c^2} + \frac{\text{Subst}\left(\int \frac{\cos(x)\sin(x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{4a^3c^2}$$

$$= -\frac{x\sqrt{\tan^{-1}(ax)}}{2a^2c^2\left(1+a^2x^2\right)} + \frac{\tan^{-1}(ax)^{3/2}}{3a^3c^2} + \frac{\text{Subst}\left(\int \frac{\sin(2x)}{2\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{4a^3c^2}$$

$$= -\frac{x\sqrt{\tan^{-1}(ax)}}{2a^2c^2\left(1+a^2x^2\right)} + \frac{\tan^{-1}(ax)^{3/2}}{3a^3c^2} + \frac{\text{Subst}\left(\int \frac{\sin(2x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{8a^3c^2}$$

$$= -\frac{x\sqrt{\tan^{-1}(ax)}}{2a^2c^2\left(1+a^2x^2\right)} + \frac{\tan^{-1}(ax)^{3/2}}{3a^3c^2} + \frac{\text{Subst}\left(\int \sin\left(2x^2\right)\,dx, x, \sqrt{\tan^{-1}(ax)}\right)}{4a^3c^2}$$

$$= -\frac{x\sqrt{\tan^{-1}(ax)}}{2a^2c^2\left(1+a^2x^2\right)} + \frac{\tan^{-1}(ax)^{3/2}}{3a^3c^2} + \frac{\sqrt{\pi}S\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{8a^3c^2}$$

Mathematica [A] time = 0.203515, size = 66, normalized size = 0.82

$$\frac{4\sqrt{\tan^{-1}(ax)}\left(2\tan^{-1}(ax)-\frac{3ax}{a^2x^2+1}\right)+3\sqrt{\pi}S\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{24a^3c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^2,x]
```

```
[Out] (4*Sqrt[ArcTan[a*x]]*((-3*a*x)/(1 + a^2*x^2) + 2*ArcTan[a*x]) + 3*Sqrt[Pi]*
FresnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(24*a^3*c^2)
```

Maple [A] time = 0.105, size = 60, normalized size = 0.8

$$\frac{1}{24\,a^3c^2}\left(3\,\sqrt{\arctan(ax)}\sqrt{\pi}\text{FresnelS}\left(2\,\frac{\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+8\,(\arctan(ax))^2-6\,\sin(2\,\arctan(ax))\arctan(ax)\right)\frac{}{\sqrt{\text{arct}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^2,x)
```

```
[Out] 1/24/a^3/c^2*(3*arctan(a*x)^(1/2)*Pi^(1/2)*FresnelS(2*arctan(a*x)^(1/2)/Pi^
(1/2))+8*arctan(a*x)^2-6*sin(2*arctan(a*x))*arctan(a*x))/arctan(a*x)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^{2}\sqrt{\operatorname{atan}{\left(ax \right)}}}{a^{4}x^{4}+2a^{2}x^{2}+1}\, dx}{c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(a*x)**(1/2)/(a**2*c*x**2+c)**2,x)
```

```
[Out] Integral(x**2*sqrt(atan(a*x))/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \sqrt{\arctan(ax)}}{(a^2cx^2 + c)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*sqrt(arctan(a*x))/(a^2*c*x^2 + c)^2, x)
```

3.710 $\int \frac{x\sqrt{\tan^{-1}(ax)}}{(c+a^2cx^2)^2}\,dx$

Optimal. Leaf size=79

$$\frac{\sqrt{\pi}\text{FresnelC}\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{8a^2c^2}-\frac{\sqrt{\tan^{-1}(ax)}}{2a^2c^2\left(a^2x^2+1\right)}+\frac{\sqrt{\tan^{-1}(ax)}}{4a^2c^2}$$

```
[Out] Sqrt[ArcTan[a*x]]/(4*a^2*c^2) - Sqrt[ArcTan[a*x]]/(2*a^2*c^2*(1 + a^2*x^2))
 + (Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(8*a^2*c^2)
```

Rubi [A] time = 0.120755, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {4930, 4904, 3312, 3304, 3352}

$$\frac{\sqrt{\pi}\text{FresnelC}\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{8a^2c^2}-\frac{\sqrt{\tan^{-1}(ax)}}{2a^2c^2\left(a^2x^2+1\right)}+\frac{\sqrt{\tan^{-1}(ax)}}{4a^2c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^2,x]
```

```
[Out] Sqrt[ArcTan[a*x]]/(4*a^2*c^2) - Sqrt[ArcTan[a*x]]/(2*a^2*c^2*(1 + a^2*x^2))
 + (Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(8*a^2*c^2)
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 4904

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
```

```
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{x\sqrt{\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^2}\,dx = -\frac{\sqrt{\tan^{-1}(ax)}}{2a^2c^2\left(1+a^2x^2\right)} + \frac{\int \frac{1}{\left(c+a^2cx^2\right)^2\sqrt{\tan^{-1}(ax)}}\,dx}{4a}$$

$$= -\frac{\sqrt{\tan^{-1}(ax)}}{2a^2c^2\left(1+a^2x^2\right)} + \frac{\text{Subst}\left(\int \frac{\cos^2(x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{4a^2c^2}$$

$$= -\frac{\sqrt{\tan^{-1}(ax)}}{2a^2c^2\left(1+a^2x^2\right)} + \frac{\text{Subst}\left(\int \left(\frac{1}{2\sqrt{x}}+\frac{\cos(2x)}{2\sqrt{x}}\right)dx, x, \tan^{-1}(ax)\right)}{4a^2c^2}$$

$$= \frac{\sqrt{\tan^{-1}(ax)}}{4a^2c^2} - \frac{\sqrt{\tan^{-1}(ax)}}{2a^2c^2\left(1+a^2x^2\right)} + \frac{\text{Subst}\left(\int \frac{\cos(2x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{8a^2c^2}$$

$$= \frac{\sqrt{\tan^{-1}(ax)}}{4a^2c^2} - \frac{\sqrt{\tan^{-1}(ax)}}{2a^2c^2\left(1+a^2x^2\right)} + \frac{\text{Subst}\left(\int \cos\left(2x^2\right)dx, x, \sqrt{\tan^{-1}(ax)}\right)}{4a^2c^2}$$

$$= \frac{\sqrt{\tan^{-1}(ax)}}{4a^2c^2} - \frac{\sqrt{\tan^{-1}(ax)}}{2a^2c^2\left(1+a^2x^2\right)} + \frac{\sqrt{\pi}C\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{8a^2c^2}$$

Mathematica [C] time = 0.280483, size = 136, normalized size = 1.72

$$\frac{4\sqrt{\pi}\text{FresnelC}\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right) + \frac{-i\sqrt{2}\sqrt{-i\tan^{-1}(ax)}\text{Gamma}\left(\frac{1}{2},-2i\tan^{-1}(ax)\right)+i\sqrt{2}\sqrt{i\tan^{-1}(ax)}\text{Gamma}\left(\frac{1}{2},2i\tan^{-1}(ax)\right)+\frac{16\left(a^2x^2-1\right)\tan^{-1}(ax)}{a^2x^2+1}}{\sqrt{\tan^{-1}(ax)}}}{64a^2c^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^2,x]
```

```
[Out] (4*Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]] + ((16*(-1 + a^2*x^2)*
ArcTan[a*x])/(1 + a^2*x^2) - I*Sqrt[2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-
2*I)*ArcTan[a*x]] + I*Sqrt[2]*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (2*I)*ArcTan[a
*x]])/Sqrt[ArcTan[a*x]])/(64*a^2*c^2)
```

Maple [A] time = 0.112, size = 46, normalized size = 0.6

$$-\frac{\cos\left(2\arctan\left(ax\right)\right)}{4a^{2}c^{2}}\sqrt{\arctan\left(ax\right)}+\frac{\sqrt{\pi}}{8a^{2}c^{2}}\mathrm{FresnelC}\left(2\frac{\sqrt{\arctan\left(ax\right)}}{\sqrt{\pi}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^2,x)
```

```
[Out] -1/4/a^2/c^2*arctan(a*x)^(1/2)*cos(2*arctan(a*x))+1/8*FresnelC(2*arctan(a*x
)^(1/2)/Pi^(1/2))*Pi^(1/2)/a^2/c^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{x\sqrt{\operatorname{atan}\left(ax\right)}}{a^{4}x^{4}+2a^{2}x^{2}+1}\,dx}{c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(a*x)**(1/2)/(a**2*c*x**2+c)**2,x)
```

```
[Out] Integral(x*sqrt(atan(a*x))/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sqrt{\arctan(ax)}}{(a^2cx^2+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(x*sqrt(arctan(a*x))/(a^2*c*x^2 + c)^2, x)
```

3.711 $\int \frac{\sqrt{\tan^{-1}(ax)}}{(c+a^2cx^2)^2}\,dx$

Optimal. Leaf size=77

$$\frac{x\sqrt{\tan^{-1}(ax)}}{2c^2(a^2x^2+1)}-\frac{\sqrt{\pi}S\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{8ac^2}+\frac{\tan^{-1}(ax)^{3/2}}{3ac^2}$$

```
[Out] (x*Sqrt[ArcTan[a*x]])/(2*c^2*(1 + a^2*x^2)) + ArcTan[a*x]^(3/2)/(3*a*c^2) -
 (Sqrt[Pi]*FresnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(8*a*c^2)
```

Rubi [A] time = 0.104373, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4892, 4970, 4406, 12, 3305, 3351}

$$\frac{x\sqrt{\tan^{-1}(ax)}}{2c^2(a^2x^2+1)}-\frac{\sqrt{\pi}S\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{8ac^2}+\frac{\tan^{-1}(ax)^{3/2}}{3ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[ArcTan[a*x]]/(c + a^2*c*x^2)^2,x]
```

```
[Out] (x*Sqrt[ArcTan[a*x]])/(2*c^2*(1 + a^2*x^2)) + ArcTan[a*x]^(3/2)/(3*a*c^2) -
 (Sqrt[Pi]*FresnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(8*a*c^2)
```

Rule 4892

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[(x*(a + b*ArcTan[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*c*
p)/2, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(a +
 b*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4970

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sin[x]^m)/
```

```
Cos[x]^(m + 2*(q + 1)), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{\sqrt{\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^2}\,dx = \frac{x\sqrt{\tan^{-1}(ax)}}{2c^2\left(1+a^2x^2\right)} + \frac{\tan^{-1}(ax)^{3/2}}{3ac^2} - \frac{1}{4}a\int \frac{x}{\left(c+a^2cx^2\right)^2\sqrt{\tan^{-1}(ax)}}\,dx$$

$$= \frac{x\sqrt{\tan^{-1}(ax)}}{2c^2\left(1+a^2x^2\right)} + \frac{\tan^{-1}(ax)^{3/2}}{3ac^2} - \frac{\text{Subst}\left(\int \frac{\cos(x)\sin(x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{4ac^2}$$

$$= \frac{x\sqrt{\tan^{-1}(ax)}}{2c^2\left(1+a^2x^2\right)} + \frac{\tan^{-1}(ax)^{3/2}}{3ac^2} - \frac{\text{Subst}\left(\int \frac{\sin(2x)}{2\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{4ac^2}$$

$$= \frac{x\sqrt{\tan^{-1}(ax)}}{2c^2\left(1+a^2x^2\right)} + \frac{\tan^{-1}(ax)^{3/2}}{3ac^2} - \frac{\text{Subst}\left(\int \frac{\sin(2x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{8ac^2}$$

$$= \frac{x\sqrt{\tan^{-1}(ax)}}{2c^2\left(1+a^2x^2\right)} + \frac{\tan^{-1}(ax)^{3/2}}{3ac^2} - \frac{\text{Subst}\left(\int \sin\left(2x^2\right)\,dx, x, \sqrt{\tan^{-1}(ax)}\right)}{4ac^2}$$

$$= \frac{x\sqrt{\tan^{-1}(ax)}}{2c^2\left(1+a^2x^2\right)} + \frac{\tan^{-1}(ax)^{3/2}}{3ac^2} - \frac{\sqrt{\pi}S\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{8ac^2}$$

Mathematica [C] time = 0.1591, size = 89, normalized size = 1.16

$$\frac{3\sqrt{2}\sqrt{-i\tan^{-1}(ax)}\text{Gamma}\left(\frac{3}{2},-2i\tan^{-1}(ax)\right)+3\sqrt{2}\sqrt{i\tan^{-1}(ax)}\text{Gamma}\left(\frac{3}{2},2i\tan^{-1}(ax)\right)+16\tan^{-1}(ax)^2}{48ac^2\sqrt{\tan^{-1}(ax)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[ArcTan[a*x]]/(c + a^2*c*x^2)^2,x]
```

```
[Out] (16*ArcTan[a*x]^2 + 3*Sqrt[2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma[3/2, (-2*I)*ArcT
an[a*x]] + 3*Sqrt[2]*Sqrt[I*ArcTan[a*x]]*Gamma[3/2, (2*I)*ArcTan[a*x]])/(48
*a*c^2*Sqrt[ArcTan[a*x]])
```

Maple [A] time = 0.108, size = 60, normalized size = 0.8

$$\frac{1}{24\,ac^2}\left(8\,(\arctan(ax))^2+6\,\sin(2\,\arctan(ax))\arctan(ax)-3\,\sqrt{\arctan(ax)}\sqrt{\pi}\text{FresnelS}\left(2\,\frac{\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)\right)\frac{1}{\sqrt{\text{arcta}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^2,x)
```

```
[Out] 1/24/a/c^2/arctan(a*x)^(1/2)*(8*arctan(a*x)^2+6*sin(2*arctan(a*x))*arctan(a
*x)-3*arctan(a*x)^(1/2)*Pi^(1/2)*FresnelS(2*arctan(a*x)^(1/2)/Pi^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sqrt{\operatorname{atan}{\left(a x \right)}}}{a^{4} x^{4} + 2 a^{2} x^{2} + 1}\, dx}{c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(1/2)/(a**2*c*x**2+c)**2,x)
```

```
[Out] Integral(sqrt(atan(a*x))/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\arctan(ax)}}{(a^2cx^2+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(arctan(a*x))/(a^2*c*x^2 + c)^2, x)
```

3.712 $\int \frac{\sqrt{\tan^{-1}(ax)}}{x(c+a^2cx^2)^2} dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{\sqrt{\tan^{-1}(ax)}}{x\left(a^2cx^2+c\right)^2},x\right)$$

```
[Out] Unintegrable[Sqrt[ArcTan[a*x]]/(x*(c + a^2*c*x^2)^2), x]
```

Rubi [A] time = 0.0595246, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sqrt{\tan^{-1}(ax)}}{x\left(c+a^2cx^2\right)^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[ArcTan[a*x]]/(x*(c + a^2*c*x^2)^2),x]

[Out] Defer[Int][Sqrt[ArcTan[a*x]]/(x*(c + a^2*c*x^2)^2), x]
```

Rubi steps

$$\int \frac{\sqrt{\tan^{-1}(ax)}}{x\left(c+a^2cx^2\right)^2} dx = \int \frac{\sqrt{\tan^{-1}(ax)}}{x\left(c+a^2cx^2\right)^2} dx$$

Mathematica [A] time = 1.68618, size = 0, normalized size = 0.

$$\int \frac{\sqrt{\tan^{-1}(ax)}}{x\left(c+a^2cx^2\right)^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[ArcTan[a*x]]/(x*(c + a^2*c*x^2)^2),x]
```

```
[Out] Integrate[Sqrt[ArcTan[a*x]]/(x*(c + a^2*c*x^2)^2), x]
```

Maple [A] time = 0.54, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(a^2cx^2+c\right)^2}\sqrt{\arctan\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^2,x)
```

```
[Out] int(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sqrt{\operatorname{atan}(ax)}}{a^4x^5+2a^2x^3+x}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(1/2)/x/(a**2*c*x**2+c)**2,x)
```

```
[Out] Integral(sqrt(atan(a*x))/(a**4*x**5 + 2*a**2*x**3 + x), x)/c**2
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\arctan(ax)}}{(a^2cx^2+c)^2 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(arctan(a*x))/((a^2*c*x^2 + c)^2*x), x)
```

3.713 $\int \frac{x^m\sqrt{\tan^{-1}(ax)}}{(c+a^2cx^2)^3}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{x^m\sqrt{\tan^{-1}(ax)}}{\left(a^2cx^2+c\right)^3},x\right)$$

```
[Out] Unintegrable[(x^m*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^3, x]
```

Rubi [A] time = 0.0654586, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m\sqrt{\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^3,x]

[Out] Defer[Int][(x^m*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^3, x]
```

Rubi steps

$$\int \frac{x^m\sqrt{\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^3}\,dx=\int \frac{x^m\sqrt{\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^3}\,dx$$

Mathematica [A] time = 2.04745, size = 0, normalized size = 0.

$$\int \frac{x^m\sqrt{\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^3,x]
```

```
[Out] Integrate[(x^m*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^3, x]
```

Maple [A] time = 0.849, size = 0, normalized size = 0.

$$\int \frac{x^{m}}{\left(a^{2} c \,x^{2}+c \right)^{3}}\sqrt{\arctan \left(a x \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x)
```

```
[Out] int(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x^{m} \sqrt{\arctan \left(a x\right)}}{a^{6} c^{3} x^{6} + 3 \, a^{4} c^{3} x^{4} + 3 \, a^{2} c^{3} x^{2} + c^{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] integral(x^m*sqrt(arctan(a*x))/(a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2
 + c^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*atan(a*x)**(1/2)/(a**2*c*x**2+c)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m\sqrt{\arctan(ax)}}{\left(a^2cx^2+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] integrate(x^m*sqrt(arctan(a*x))/(a^2*c*x^2 + c)^3, x)
```

3.714 $\int \frac{x^5\sqrt{\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^3}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{x^5\sqrt{\tan^{-1}(ax)}}{\left(a^2cx^2+c\right)^3},x\right)$$

```
[Out] Unintegrable[(x^5*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^3, x]
```

Rubi [A] time = 0.0658467, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^5\sqrt{\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^5*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^3,x]

[Out] Defer[Int][(x^5*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^3, x]
```

Rubi steps

$$\int \frac{x^5\sqrt{\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^3}\,dx=\int \frac{x^5\sqrt{\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^3}\,dx$$

Mathematica [A] time = 5.13811, size = 0, normalized size = 0.

$$\int \frac{x^5\sqrt{\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^5*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^3,x]
```

```
[Out] Integrate[(x^5*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^3, x]
```

Maple [A] time = 0.584, size = 0, normalized size = 0.

$$\int \frac{x^5}{\left(a^2cx^2+c\right)^3}\sqrt{\arctan\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x)
```

```
[Out] int(x^5*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^5\sqrt{\operatorname{atan}(ax)}}{a^6x^6+3a^4x^4+3a^2x^2+1}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*atan(a*x)**(1/2)/(a**2*c*x**2+c)**3,x)
```

```
[Out] Integral(x**5*sqrt(atan(a*x))/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1),
x)/c**3
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5\sqrt{\arctan(ax)}}{\left(a^2cx^2+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] integrate(x^5*sqrt(arctan(a*x))/(a^2*c*x^2 + c)^3, x)
```

3.715 $\int \frac{x^4\sqrt{\tan^{-1}(ax)}}{(c+a^2cx^2)^3}\,dx$

Optimal. Leaf size=139

$$-\frac{\sqrt{\frac{\pi}{2}}S\left(2\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{64a^5c^3}+\frac{\sqrt{\pi}S\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{8a^5c^3}+\frac{\tan^{-1}(ax)^{3/2}}{4a^5c^3}-\frac{\sqrt{\tan^{-1}(ax)}\sin\left(2\tan^{-1}(ax)\right)}{4a^5c^3}+\frac{\sqrt{\tan^{-1}(ax)}\sin\left(4\right.}{32a^5c^3}$$

```
[Out] ArcTan[a*x]^(3/2)/(4*a^5*c^3) - (Sqrt[Pi/2]*FresnelS[2*Sqrt[2/Pi]*Sqrt[ArcT
an[a*x]]])/(64*a^5*c^3) + (Sqrt[Pi]*FresnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]
])/(8*a^5*c^3) - (Sqrt[ArcTan[a*x]]*Sin[2*ArcTan[a*x]])/(4*a^5*c^3) + (Sqrt
[ArcTan[a*x]]*Sin[4*ArcTan[a*x]])/(32*a^5*c^3)
```

Rubi [A] time = 0.178834, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {4970, 3312, 3296, 3305, 3351}

$$-\frac{\sqrt{\frac{\pi}{2}}S\left(2\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{64a^5c^3}+\frac{\sqrt{\pi}S\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{8a^5c^3}+\frac{\tan^{-1}(ax)^{3/2}}{4a^5c^3}-\frac{\sqrt{\tan^{-1}(ax)}\sin\left(2\tan^{-1}(ax)\right)}{4a^5c^3}+\frac{\sqrt{\tan^{-1}(ax)}\sin\left(4\right.}{32a^5c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^3,x]
```


```
[Out] ArcTan[a*x]^(3/2)/(4*a^5*c^3) - (Sqrt[Pi/2]*FresnelS[2*Sqrt[2/Pi]*Sqrt[ArcT
an[a*x]]])/(64*a^5*c^3) + (Sqrt[Pi]*FresnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]
])/(8*a^5*c^3) - (Sqrt[ArcTan[a*x]]*Sin[2*ArcTan[a*x]])/(4*a^5*c^3) + (Sqrt
[ArcTan[a*x]]*Sin[4*ArcTan[a*x]])/(32*a^5*c^3)
```


Rule 4970

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sin[x]^m)/
Cos[x]^(m + 2*(q + 1)), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^4\sqrt{\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^3}\,dx &= \frac{\text{Subst}\left(\int \sqrt{x}\sin^4(x)\,dx, x, \tan^{-1}(ax)\right)}{a^5c^3}\\
&= \frac{\text{Subst}\left(\int \left(\frac{3\sqrt{x}}{8}-\frac{1}{2}\sqrt{x}\cos(2x)+\frac{1}{8}\sqrt{x}\cos(4x)\right)dx, x, \tan^{-1}(ax)\right)}{a^5c^3}\\
&= \frac{\tan^{-1}(ax)^{3/2}}{4a^5c^3}+\frac{\text{Subst}\left(\int \sqrt{x}\cos(4x)\,dx, x, \tan^{-1}(ax)\right)}{8a^5c^3}-\frac{\text{Subst}\left(\int \sqrt{x}\cos(2x)\,dx, x, \tan^{-1}(ax)\right)}{2a^5c^3}\\
&= \frac{\tan^{-1}(ax)^{3/2}}{4a^5c^3}-\frac{\sqrt{\tan^{-1}(ax)}\sin\left(2\tan^{-1}(ax)\right)}{4a^5c^3}+\frac{\sqrt{\tan^{-1}(ax)}\sin\left(4\tan^{-1}(ax)\right)}{32a^5c^3}-\frac{\text{Subst}\left(\int \frac{\sin(4x}{\sqrt{x}}\right.}{64}\\
&= \frac{\tan^{-1}(ax)^{3/2}}{4a^5c^3}-\frac{\sqrt{\tan^{-1}(ax)}\sin\left(2\tan^{-1}(ax)\right)}{4a^5c^3}+\frac{\sqrt{\tan^{-1}(ax)}\sin\left(4\tan^{-1}(ax)\right)}{32a^5c^3}-\frac{\text{Subst}\left(\int \sin\left(4\right.\right.}{}\\
&= \frac{\tan^{-1}(ax)^{3/2}}{4a^5c^3}-\frac{\sqrt{\frac{\pi}{2}}S\left(2\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{64a^5c^3}+\frac{\sqrt{\pi}S\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{8a^5c^3}-\frac{\sqrt{\tan^{-1}(ax)}\sin\left(2\tan^{-1}(ax)\right)}{4a^5c^3}+
\end{aligned}
$$

Mathematica [C] time = 0.48253, size = 181, normalized size = 1.3

$$\frac{-8\sqrt{2}\sqrt{-i\tan^{-1}(ax)}\text{Gamma}\left(\frac{1}{2},-2i\tan^{-1}(ax)\right)-8\sqrt{2}\sqrt{i\tan^{-1}(ax)}\text{Gamma}\left(\frac{1}{2},2i\tan^{-1}(ax)\right)+\sqrt{-i\tan^{-1}(ax)}\text{Gamm}}{256a^5c^3\sqrt{\tan}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^3,x]
```

```
[Out] ((-96*a*x*ArcTan[a*x])/(1 + a^2*x^2)^2 - (160*a^3*x^3*ArcTan[a*x])/(1 + a^2
*x^2)^2 + 64*ArcTan[a*x]^2 - 8*Sqrt[2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-
2*I)*ArcTan[a*x]] - 8*Sqrt[2]*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (2*I)*ArcTan[a
*x]] + Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-4*I)*ArcTan[a*x]] + Sqrt[I*ArcTa
n[a*x]]*Gamma[1/2, (4*I)*ArcTan[a*x]])/(256*a^5*c^3*Sqrt[ArcTan[a*x]])
```

Maple [A] time = 0.125, size = 102, normalized size = 0.7

$$\frac{1}{128\,c^3a^5}\left(-\sqrt{2}\sqrt{\arctan(ax)}\sqrt{\pi}\text{FresnelS}\left(2\,\frac{\sqrt{2}\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+16\,\sqrt{\arctan(ax)}\sqrt{\pi}\text{FresnelS}\left(2\,\frac{\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+32$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x)
```

```
[Out] 1/128/a^5/c^3*(-2^(1/2)*arctan(a*x)^(1/2)*Pi^(1/2)*FresnelS(2*2^(1/2)/Pi^(1
/2)*arctan(a*x)^(1/2))+16*arctan(a*x)^(1/2)*Pi^(1/2)*FresnelS(2*arctan(a*x)
^(1/2)/Pi^(1/2))+32*arctan(a*x)^2-32*sin(2*arctan(a*x))*arctan(a*x)+4*sin(4
*arctan(a*x))*arctan(a*x))/arctan(a*x)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^4\sqrt{\operatorname{atan}(ax)}}{a^6x^6+3a^4x^4+3a^2x^2+1}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*atan(a*x)**(1/2)/(a**2*c*x**2+c)**3,x)
```

```
[Out] Integral(x**4*sqrt(atan(a*x))/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1),
x)/c**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4\sqrt{\arctan(ax)}}{(a^2cx^2+c)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] integrate(x^4*sqrt(arctan(a*x))/(a^2*c*x^2 + c)^3, x)
```

3.716 $\int \frac{x^3\sqrt{\tan^{-1}(ax)}}{(c+a^2cx^2)^3}\,dx$

Optimal. Leaf size=118

$$-\frac{\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{64a^4c^3}+\frac{\sqrt{\pi}\text{FresnelC}\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{16a^4c^3}+\frac{x^4\sqrt{\tan^{-1}(ax)}}{4c^3\left(a^2x^2+1\right)^2}-\frac{3\sqrt{\tan^{-1}(ax)}}{32a^4c^3}$$

```
[Out] (-3*Sqrt[ArcTan[a*x]])/(32*a^4*c^3) + (x^4*Sqrt[ArcTan[a*x]])/(4*c^3*(1 + a
^2*x^2)^2) - (Sqrt[Pi/2]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(64*a^4*
c^3) + (Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(16*a^4*c^3)
```

Rubi [A] time = 0.214544, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {4944, 4970, 3312, 3304, 3352}

$$-\frac{\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{64a^4c^3}+\frac{\sqrt{\pi}\text{FresnelC}\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{16a^4c^3}+\frac{x^4\sqrt{\tan^{-1}(ax)}}{4c^3\left(a^2x^2+1\right)^2}-\frac{3\sqrt{\tan^{-1}(ax)}}{32a^4c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^3,x]
```


```
[Out] (-3*Sqrt[ArcTan[a*x]])/(32*a^4*c^3) + (x^4*Sqrt[ArcTan[a*x]])/(4*c^3*(1 + a
^2*x^2)^2) - (Sqrt[Pi/2]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(64*a^4*
c^3) + (Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(16*a^4*c^3)
```


Rule 4944

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a +
 b*ArcTan[c*x])^p)/(d*f*(m + 1)), x] - Dist[(b*c*p)/(f*(m + 1)), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rule 4970

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sin[x]^m)/
Cos[x]^(m + 2*(q + 1)), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{x^3\sqrt{\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^3}\,dx = \frac{x^4\sqrt{\tan^{-1}(ax)}}{4c^3\left(1+a^2x^2\right)^2} - \frac{1}{8}a\int \frac{x^4}{\left(c+a^2cx^2\right)^3\sqrt{\tan^{-1}(ax)}}\,dx$$

$$= \frac{x^4\sqrt{\tan^{-1}(ax)}}{4c^3\left(1+a^2x^2\right)^2} - \frac{\text{Subst}\left(\int \frac{\sin^4(x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{8a^4c^3}$$

$$= \frac{x^4\sqrt{\tan^{-1}(ax)}}{4c^3\left(1+a^2x^2\right)^2} - \frac{\text{Subst}\left(\int \left(\frac{3}{8\sqrt{x}} - \frac{\cos(2x)}{2\sqrt{x}} + \frac{\cos(4x)}{8\sqrt{x}}\right)dx, x, \tan^{-1}(ax)\right)}{8a^4c^3}$$

$$= -\frac{3\sqrt{\tan^{-1}(ax)}}{32a^4c^3} + \frac{x^4\sqrt{\tan^{-1}(ax)}}{4c^3\left(1+a^2x^2\right)^2} - \frac{\text{Subst}\left(\int \frac{\cos(4x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{64a^4c^3} + \frac{\text{Subst}\left(\int \frac{\cos(2x)}{\sqrt{x}}\,dx, x,\right.}{16a^4c^3}$$

$$= -\frac{3\sqrt{\tan^{-1}(ax)}}{32a^4c^3} + \frac{x^4\sqrt{\tan^{-1}(ax)}}{4c^3\left(1+a^2x^2\right)^2} - \frac{\text{Subst}\left(\int \cos\left(4x^2\right)dx, x, \sqrt{\tan^{-1}(ax)}\right)}{32a^4c^3} + \frac{\text{Subst}\left(\int \cos\left(2\right.\right.}{}$$

$$= -\frac{3\sqrt{\tan^{-1}(ax)}}{32a^4c^3} + \frac{x^4\sqrt{\tan^{-1}(ax)}}{4c^3\left(1+a^2x^2\right)^2} - \frac{\sqrt{\frac{\pi}{2}}C\left(2\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{64a^4c^3} + \frac{\sqrt{\pi}C\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{16a^4c^3}$$

Mathematica [C] time = 0.649943, size = 230, normalized size = 1.95

$$\frac{\frac{-12i\sqrt{2}\sqrt{-i\tan^{-1}(ax)}\text{Gamma}\left(\frac{1}{2},-2i\tan^{-1}(ax)\right)+12i\sqrt{2}\sqrt{i\tan^{-1}(ax)}\text{Gamma}\left(\frac{1}{2},2i\tan^{-1}(ax)\right)+3i\sqrt{-i\tan^{-1}(ax)}\text{Gamma}\left(\frac{1}{2},-4i\tan^{-1}(ax)\right)-3i\sqrt{i\tan^{-1}(ax)}}{\sqrt{\tan^{-1}(ax)}}}{2048a^4c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^3,x]
```

```
[Out] (-10*Sqrt[2*Pi]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]] + 80*Sqrt[Pi]*Fres
nelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]] + ((64*(-3 - 6*a^2*x^2 + 5*a^4*x^4)*Ar
cTan[a*x])/(1 + a^2*x^2)^2 - (12*I)*Sqrt[2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/
2, (-2*I)*ArcTan[a*x]] + (12*I)*Sqrt[2]*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (2*I
)*ArcTan[a*x]] + (3*I)*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-4*I)*ArcTan[a*x]
] - (3*I)*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (4*I)*ArcTan[a*x]])/Sqrt[ArcTan[a*
x]])/(2048*a^4*c^3)
```

Maple [A] time = 0.114, size = 93, normalized size = 0.8

$$-\frac{1}{128\,c^{3}a^{4}}\left(\sqrt{2}\sqrt{\arctan\left(ax\right)}\sqrt{\pi}\mathrm{FresnelC}\left(2\,\frac{\sqrt{2}\sqrt{\arctan\left(ax\right)}}{\sqrt{\pi}}\right)+16\,\cos\left(2\,\arctan\left(ax\right)\right)\arctan\left(ax\right)-4\,\cos\left(4\,\arctan\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x)
```

```
[Out] -1/128/a^4/c^3/arctan(a*x)^(1/2)*(2^(1/2)*arctan(a*x)^(1/2)*Pi^(1/2)*Fresne
lC(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))+16*cos(2*arctan(a*x))*arctan(a*x)-
4*cos(4*arctan(a*x))*arctan(a*x)-8*arctan(a*x)^(1/2)*Pi^(1/2)*FresnelC(2*ar
ctan(a*x)^(1/2)/Pi^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^3\sqrt{\operatorname{atan}(ax)}}{a^6x^6+3a^4x^4+3a^2x^2+1}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atan(a*x)**(1/2)/(a**2*c*x**2+c)**3,x)
```

```
[Out] Integral(x**3*sqrt(atan(a*x))/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1),
x)/c**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3\sqrt{\arctan(ax)}}{\left(a^2cx^2+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] integrate(x^3*sqrt(arctan(a*x))/(a^2*c*x^2 + c)^3, x)
```

3.717 $\int \frac{x^2\sqrt{\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^3}\,dx$

Optimal. Leaf size=83

$$\frac{\sqrt{\frac{\pi}{2}}S\left(2\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{64a^3c^3}+\frac{\tan^{-1}(ax)^{3/2}}{12a^3c^3}-\frac{\sqrt{\tan^{-1}(ax)}\sin\left(4\tan^{-1}(ax)\right)}{32a^3c^3}$$

```
[Out] ArcTan[a*x]^(3/2)/(12*a^3*c^3) + (Sqrt[Pi/2]*FresnelS[2*Sqrt[2/Pi]*Sqrt[Arc
Tan[a*x]]])/(64*a^3*c^3) - (Sqrt[ArcTan[a*x]]*Sin[4*ArcTan[a*x]])/(32*a^3*c
^3)
```

Rubi [A] time = 0.13635, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {4970, 4406, 3296, 3305, 3351}

$$\frac{\sqrt{\frac{\pi}{2}}S\left(2\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{64a^3c^3}+\frac{\tan^{-1}(ax)^{3/2}}{12a^3c^3}-\frac{\sqrt{\tan^{-1}(ax)}\sin\left(4\tan^{-1}(ax)\right)}{32a^3c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^3,x]
```

```
[Out] ArcTan[a*x]^(3/2)/(12*a^3*c^3) + (Sqrt[Pi/2]*FresnelS[2*Sqrt[2/Pi]*Sqrt[Arc
Tan[a*x]]])/(64*a^3*c^3) - (Sqrt[ArcTan[a*x]]*Sin[4*ArcTan[a*x]])/(32*a^3*c
^3)
```

Rule 4970

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sin[x]^m)/
Cos[x]^(m + 2*(q + 1)), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
```

```
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2\sqrt{\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^3}\,dx &= \frac{\text{Subst}\left(\int \sqrt{x}\cos^2(x)\sin^2(x)\,dx, x, \tan^{-1}(ax)\right)}{a^3c^3}\\
&= \frac{\text{Subst}\left(\int \left(\frac{\sqrt{x}}{8}-\frac{1}{8}\sqrt{x}\cos(4x)\right)\,dx, x, \tan^{-1}(ax)\right)}{a^3c^3}\\
&= \frac{\tan^{-1}(ax)^{3/2}}{12a^3c^3}-\frac{\text{Subst}\left(\int \sqrt{x}\cos(4x)\,dx, x, \tan^{-1}(ax)\right)}{8a^3c^3}\\
&= \frac{\tan^{-1}(ax)^{3/2}}{12a^3c^3}-\frac{\sqrt{\tan^{-1}(ax)}\sin\left(4\tan^{-1}(ax)\right)}{32a^3c^3}+\frac{\text{Subst}\left(\int \frac{\sin(4x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{64a^3c^3}\\
&= \frac{\tan^{-1}(ax)^{3/2}}{12a^3c^3}-\frac{\sqrt{\tan^{-1}(ax)}\sin\left(4\tan^{-1}(ax)\right)}{32a^3c^3}+\frac{\text{Subst}\left(\int \sin\left(4x^2\right)\,dx, x, \sqrt{\tan^{-1}(ax)}\right)}{32a^3c^3}\\
&= \frac{\tan^{-1}(ax)^{3/2}}{12a^3c^3}+\frac{\sqrt{\frac{\pi}{2}}S\left(2\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{64a^3c^3}-\frac{\sqrt{\tan^{-1}(ax)}\sin\left(4\tan^{-1}(ax)\right)}{32a^3c^3}
\end{aligned}$$

Mathematica [C] time = 0.404652, size = 141, normalized size = 1.7

$$\frac{-3\left(a^2x^2+1\right)^2\sqrt{-i\tan^{-1}(ax)}\text{Gamma}\left(\frac{1}{2},-4i\tan^{-1}(ax)\right)-3\left(a^2x^2+1\right)^2\sqrt{i\tan^{-1}(ax)}\text{Gamma}\left(\frac{1}{2},4i\tan^{-1}(ax)\right)+32\ \mathrm{ta}}{768a^3c^3\left(a^2x^2+1\right)^2\sqrt{\tan^{-1}(ax)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^3,x]
```

```
[Out] (32*ArcTan[a*x]*(3*a*x*(-1 + a^2*x^2) + 2*(1 + a^2*x^2)^2*ArcTan[a*x]) - 3*
(1 + a^2*x^2)^2*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-4*I)*ArcTan[a*x]] - 3*(
1 + a^2*x^2)^2*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (4*I)*ArcTan[a*x]])/(768*a^3*
c^3*(1 + a^2*x^2)^2*Sqrt[ArcTan[a*x]])
```

Maple [A] time = 0.115, size = 66, normalized size = 0.8

$$\frac{1}{384\,c^3a^3}\left(3\,\sqrt{2}\sqrt{\arctan(ax)}\sqrt{\pi}\text{FresnelS}\left(2\,\frac{\sqrt{2}\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+32\,(\arctan(ax))^2-12\,\sin(4\,\arctan(ax))\arctan(ax\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x)
```

```
[Out] 1/384/a^3/c^3*(3*2^(1/2)*arctan(a*x)^(1/2)*Pi^(1/2)*FresnelS(2*2^(1/2)/Pi^(
1/2)*arctan(a*x)^(1/2))+32*arctan(a*x)^2-12*sin(4*arctan(a*x))*arctan(a*x))
/arctan(a*x)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^2\sqrt{\operatorname{atan}(ax)}}{a^6x^6+3a^4x^4+3a^2x^2+1}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(a*x)**(1/2)/(a**2*c*x**2+c)**3,x)
```

```
[Out] Integral(x**2*sqrt(atan(a*x))/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1),
x)/c**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\sqrt{\arctan(ax)}}{(a^2cx^2+c)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] integrate(x^2*sqrt(arctan(a*x))/(a^2*c*x^2 + c)^3, x)
```

3.718 $\int \frac{x\sqrt{\tan^{-1}(ax)}}{(c+a^2cx^2)^3}\,dx$

Optimal. Leaf size=118

$$\frac{\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{64a^2c^3}+\frac{\sqrt{\pi}\text{FresnelC}\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{16a^2c^3}-\frac{\sqrt{\tan^{-1}(ax)}}{4a^2c^3\left(a^2x^2+1\right)^2}+\frac{3\sqrt{\tan^{-1}(ax)}}{32a^2c^3}$$

```
[Out] (3*Sqrt[ArcTan[a*x]])/(32*a^2*c^3) - Sqrt[ArcTan[a*x]]/(4*a^2*c^3*(1 + a^2*
x^2)^2) + (Sqrt[Pi/2]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(64*a^2*c^3
) + (Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(16*a^2*c^3)
```

Rubi [A] time = 0.14963, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {4930, 4904, 3312, 3304, 3352}

$$\frac{\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{64a^2c^3}+\frac{\sqrt{\pi}\text{FresnelC}\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{16a^2c^3}-\frac{\sqrt{\tan^{-1}(ax)}}{4a^2c^3\left(a^2x^2+1\right)^2}+\frac{3\sqrt{\tan^{-1}(ax)}}{32a^2c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^3,x]
```


```
[Out] (3*Sqrt[ArcTan[a*x]])/(32*a^2*c^3) - Sqrt[ArcTan[a*x]]/(4*a^2*c^3*(1 + a^2*
x^2)^2) + (Sqrt[Pi/2]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(64*a^2*c^3
) + (Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(16*a^2*c^3)
```


Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 4904

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{x\sqrt{\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^3}\,dx = -\frac{\sqrt{\tan^{-1}(ax)}}{4a^2c^3\left(1+a^2x^2\right)^2}+\frac{\int \frac{1}{\left(c+a^2cx^2\right)^3\sqrt{\tan^{-1}(ax)}}\,dx}{8a}$$

$$= -\frac{\sqrt{\tan^{-1}(ax)}}{4a^2c^3\left(1+a^2x^2\right)^2}+\frac{\text{Subst}\left(\int \frac{\cos^4(x)}{\sqrt{x}}\,dx,x,\tan^{-1}(ax)\right)}{8a^2c^3}$$

$$= -\frac{\sqrt{\tan^{-1}(ax)}}{4a^2c^3\left(1+a^2x^2\right)^2}+\frac{\text{Subst}\left(\int \left(\frac{3}{8\sqrt{x}}+\frac{\cos(2x)}{2\sqrt{x}}+\frac{\cos(4x)}{8\sqrt{x}}\right)dx,x,\tan^{-1}(ax)\right)}{8a^2c^3}$$

$$= \frac{3\sqrt{\tan^{-1}(ax)}}{32a^2c^3}-\frac{\sqrt{\tan^{-1}(ax)}}{4a^2c^3\left(1+a^2x^2\right)^2}+\frac{\text{Subst}\left(\int \frac{\cos(4x)}{\sqrt{x}}\,dx,x,\tan^{-1}(ax)\right)}{64a^2c^3}+\frac{\text{Subst}\left(\int \frac{\cos(2x)}{\sqrt{x}}\,dx,x,\text{t}\right.}{16a^2c^3}$$

$$= \frac{3\sqrt{\tan^{-1}(ax)}}{32a^2c^3}-\frac{\sqrt{\tan^{-1}(ax)}}{4a^2c^3\left(1+a^2x^2\right)^2}+\frac{\text{Subst}\left(\int \cos\left(4x^2\right)dx,x,\sqrt{\tan^{-1}(ax)}\right)}{32a^2c^3}+\frac{\text{Subst}\left(\int \cos\left(2x^2\right.\right.}{8}$$

$$= \frac{3\sqrt{\tan^{-1}(ax)}}{32a^2c^3}-\frac{\sqrt{\tan^{-1}(ax)}}{4a^2c^3\left(1+a^2x^2\right)^2}+\frac{\sqrt{\frac{\pi}{2}}C\left(2\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{64a^2c^3}+\frac{\sqrt{\pi}C\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{16a^2c^3}$$

Mathematica [C] time = 0.64178, size = 230, normalized size = 1.95

$$\frac{\frac{-20i\sqrt{2}\sqrt{-i\tan^{-1}(ax)}\text{Gamma}\left(\frac{1}{2},-2i\tan^{-1}(ax)\right)+20i\sqrt{2}\sqrt{i\tan^{-1}(ax)}\text{Gamma}\left(\frac{1}{2},2i\tan^{-1}(ax)\right)-11i\sqrt{-i\tan^{-1}(ax)}\text{Gamma}\left(\frac{1}{2},-4i\tan^{-1}(ax)\right)+11i\sqrt{i\tan^{-1}(ax)}}{\sqrt{\tan^{-1}(ax)}}}{2048a^2c^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^3,x]
```

```
[Out] (-6*Sqrt[2*Pi]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]] + 48*Sqrt[Pi]*Fresn
elC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]] + ((64*(-5 + 6*a^2*x^2 + 3*a^4*x^4)*Arc
Tan[a*x])/(1 + a^2*x^2)^2 - (20*I)*Sqrt[2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2
, (-2*I)*ArcTan[a*x]] + (20*I)*Sqrt[2]*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (2*I)
*ArcTan[a*x]] - (11*I)*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-4*I)*ArcTan[a*x]
] + (11*I)*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (4*I)*ArcTan[a*x]])/Sqrt[ArcTan[a
*x]])/(2048*a^2*c^3)
```

Maple [A] time = 0.115, size = 94, normalized size = 0.8

$$-\frac{1}{128\,c^3a^2}\left(-\sqrt{2}\sqrt{\arctan(ax)}\sqrt{\pi}\,\mathrm{FresnelC}\left(2\,\frac{\sqrt{2}\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+16\,\cos(2\,\arctan(ax))\arctan(ax)-8\,\sqrt{\arctan(a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x)
```

```
[Out] -1/128/a^2/c^3/arctan(a*x)^(1/2)*(-2^(1/2)*arctan(a*x)^(1/2)*Pi^(1/2)*Fresn
elC(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))+16*cos(2*arctan(a*x))*arctan(a*x)
-8*arctan(a*x)^(1/2)*Pi^(1/2)*FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2))+4*cos(
4*arctan(a*x))*arctan(a*x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x\sqrt{\operatorname{atan}(ax)}}{a^6x^6+3a^4x^4+3a^2x^2+1}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(a*x)**(1/2)/(a**2*c*x**2+c)**3,x)
```

```
[Out] Integral(x*sqrt(atan(a*x))/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1), x)/
c**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sqrt{\arctan(ax)}}{\left(a^2cx^2+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] integrate(x*sqrt(arctan(a*x))/(a^2*c*x^2 + c)^3, x)
```

3.719 $\int \frac{\sqrt{\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^3}\,dx$

Optimal. Leaf size=139

$$-\frac{\sqrt{\frac{\pi}{2}}S\left(2\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{64ac^3}-\frac{\sqrt{\pi}S\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{8ac^3}+\frac{\tan^{-1}(ax)^{3/2}}{4ac^3}+\frac{\sqrt{\tan^{-1}(ax)}\sin\left(2\tan^{-1}(ax)\right)}{4ac^3}+\frac{\sqrt{\tan^{-1}(ax)}\sin\left(4\right.}{32ac^3}$$

```
[Out] ArcTan[a*x]^(3/2)/(4*a*c^3) - (Sqrt[Pi/2]*FresnelS[2*Sqrt[2/Pi]*Sqrt[ArcTan
[a*x]]])/(64*a*c^3) - (Sqrt[Pi]*FresnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(
8*a*c^3) + (Sqrt[ArcTan[a*x]]*Sin[2*ArcTan[a*x]])/(4*a*c^3) + (Sqrt[ArcTan[
a*x]]*Sin[4*ArcTan[a*x]])/(32*a*c^3)
```

Rubi [A] time = 0.144651, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {4904, 3312, 3296, 3305, 3351}

$$-\frac{\sqrt{\frac{\pi}{2}}S\left(2\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{64ac^3}-\frac{\sqrt{\pi}S\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{8ac^3}+\frac{\tan^{-1}(ax)^{3/2}}{4ac^3}+\frac{\sqrt{\tan^{-1}(ax)}\sin\left(2\tan^{-1}(ax)\right)}{4ac^3}+\frac{\sqrt{\tan^{-1}(ax)}\sin\left(4\right.}{32ac^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[ArcTan[a*x]]/(c + a^2*c*x^2)^3,x]
```

```
[Out] ArcTan[a*x]^(3/2)/(4*a*c^3) - (Sqrt[Pi/2]*FresnelS[2*Sqrt[2/Pi]*Sqrt[ArcTan
[a*x]]])/(64*a*c^3) - (Sqrt[Pi]*FresnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(
8*a*c^3) + (Sqrt[ArcTan[a*x]]*Sin[2*ArcTan[a*x]])/(4*a*c^3) + (Sqrt[ArcTan[
a*x]]*Sin[4*ArcTan[a*x]])/(32*a*c^3)
```

Rule 4904

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{\sqrt{\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^3}\,dx = \frac{\text{Subst}\left(\int \sqrt{x}\cos^4(x)\,dx, x, \tan^{-1}(ax)\right)}{ac^3}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{3\sqrt{x}}{8}+\frac{1}{2}\sqrt{x}\cos(2x)+\frac{1}{8}\sqrt{x}\cos(4x)\right)dx, x, \tan^{-1}(ax)\right)}{ac^3}$$

$$= \frac{\tan^{-1}(ax)^{3/2}}{4ac^3}+\frac{\text{Subst}\left(\int \sqrt{x}\cos(4x)\,dx, x, \tan^{-1}(ax)\right)}{8ac^3}+\frac{\text{Subst}\left(\int \sqrt{x}\cos(2x)\,dx, x, \tan^{-1}(ax)\right)}{2ac^3}$$

$$= \frac{\tan^{-1}(ax)^{3/2}}{4ac^3}+\frac{\sqrt{\tan^{-1}(ax)}\sin\left(2\tan^{-1}(ax)\right)}{4ac^3}+\frac{\sqrt{\tan^{-1}(ax)}\sin\left(4\tan^{-1}(ax)\right)}{32ac^3}-\frac{\text{Subst}\left(\int \frac{\sin(4x)}{\sqrt{x}}\right.}{64a}$$

$$= \frac{\tan^{-1}(ax)^{3/2}}{4ac^3}+\frac{\sqrt{\tan^{-1}(ax)}\sin\left(2\tan^{-1}(ax)\right)}{4ac^3}+\frac{\sqrt{\tan^{-1}(ax)}\sin\left(4\tan^{-1}(ax)\right)}{32ac^3}-\frac{\text{Subst}\left(\int \sin\left(4x\right.\right.}{}$$

$$= \frac{\tan^{-1}(ax)^{3/2}}{4ac^3}-\frac{\sqrt{\frac{\pi}{2}}S\left(2\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{64ac^3}-\frac{\sqrt{\pi}S\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{8ac^3}+\frac{\sqrt{\tan^{-1}(ax)}\sin\left(2\tan^{-1}(ax)\right)}{4ac^3}+$$

Mathematica [C] time = 0.419533, size = 192, normalized size = 1.38

$$\frac{\frac{8\sqrt{2}\sqrt{-i\tan^{-1}(ax)}\text{Gamma}\left(\frac{1}{2},-2i\tan^{-1}(ax)\right)}{a}+\frac{8\sqrt{2}\sqrt{i\tan^{-1}(ax)}\text{Gamma}\left(\frac{1}{2},2i\tan^{-1}(ax)\right)}{a}+\frac{\sqrt{-i\tan^{-1}(ax)}\text{Gamma}\left(\frac{1}{2},-4i\tan^{-1}(ax)\right)}{a}+\frac{\sqrt{i\tan^{-1}(ax)}\text{(}}{}}{256c^3\sqrt{\tan^{-1}(ax)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[ArcTan[a*x]]/(c + a^2*c*x^2)^3,x]
```

```
[Out] ((160*x*ArcTan[a*x])/(1 + a^2*x^2)^2 + (96*a^2*x^3*ArcTan[a*x])/(1 + a^2*x^
2)^2 + (64*ArcTan[a*x]^2)/a + (8*Sqrt[2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2,
(-2*I)*ArcTan[a*x]])/a + (8*Sqrt[2]*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (2*I)*Ar
cTan[a*x]])/a + (Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-4*I)*ArcTan[a*x]])/a +
 (Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (4*I)*ArcTan[a*x]])/a)/(256*c^3*Sqrt[ArcTa
n[a*x]])
```

Maple [A] time = 0.118, size = 102, normalized size = 0.7

$$\frac{1}{128\,ac^3}\left(-\sqrt{2}\sqrt{\arctan(ax)}\sqrt{\pi}\text{FresnelS}\left(2\,\frac{\sqrt{2}\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+32\,(\arctan(ax))^2+32\,\sin(2\,\arctan(ax))\arctan(ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x)
```

```
[Out] 1/128/a/c^3/arctan(a*x)^(1/2)*(-2^(1/2)*arctan(a*x)^(1/2)*Pi^(1/2)*FresnelS
(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))+32*arctan(a*x)^2+32*sin(2*arctan(a*x
))*arctan(a*x)+4*sin(4*arctan(a*x))*arctan(a*x)-16*arctan(a*x)^(1/2)*Pi^(1/
2)*FresnelS(2*arctan(a*x)^(1/2)/Pi^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sqrt{\operatorname{atan}(ax)}}{a^6x^6+3a^4x^4+3a^2x^2+1}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(1/2)/(a**2*c*x**2+c)**3,x)
```

```
[Out] Integral(sqrt(atan(a*x))/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1), x)/c*
*3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\arctan(ax)}}{(a^2cx^2+c)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(arctan(a*x))/(a^2*c*x^2 + c)^3, x)
```

3.720 $\int \frac{\sqrt{\tan^{-1}(ax)}}{x(c+a^2cx^2)^3} \, dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{\sqrt{\tan^{-1}(ax)}}{x\left(a^2cx^2+c\right)^3},x\right)$$

```
[Out] Unintegrable[Sqrt[ArcTan[a*x]]/(x*(c + a^2*c*x^2)^3), x]
```

Rubi [A] time = 0.0623277, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sqrt{\tan^{-1}(ax)}}{x\left(c+a^2cx^2\right)^3} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[ArcTan[a*x]]/(x*(c + a^2*c*x^2)^3),x]

[Out] Defer[Int][Sqrt[ArcTan[a*x]]/(x*(c + a^2*c*x^2)^3), x]
```

Rubi steps

$$\int \frac{\sqrt{\tan^{-1}(ax)}}{x\left(c+a^2cx^2\right)^3} \, dx = \int \frac{\sqrt{\tan^{-1}(ax)}}{x\left(c+a^2cx^2\right)^3} \, dx$$

Mathematica [A] time = 2.20391, size = 0, normalized size = 0.

$$\int \frac{\sqrt{\tan^{-1}(ax)}}{x\left(c+a^2cx^2\right)^3} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[ArcTan[a*x]]/(x*(c + a^2*c*x^2)^3),x]
```

```
[Out] Integrate[Sqrt[ArcTan[a*x]]/(x*(c + a^2*c*x^2)^3), x]
```

Maple [A] time = 0.813, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(a^{2}c\,x^{2}+c\right)^{3}}\sqrt{\arctan\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^3,x)
```

```
[Out] int(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^3,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sqrt{\operatorname{atan}(ax)}}{a^6x^7+3a^4x^5+3a^2x^3+x}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(1/2)/x/(a**2*c*x**2+c)**3,x)
```

```
[Out] Integral(sqrt(atan(a*x))/(a**6*x**7 + 3*a**4*x**5 + 3*a**2*x**3 + x), x)/c*
*3
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\arctan(ax)}}{(a^2cx^2+c)^3x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(arctan(a*x))/((a^2*c*x^2 + c)^3*x), x)
```

3.721 $\int x^m \sqrt{c+a^2cx^2} \sqrt{\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(x^m \sqrt{a^2cx^2+c} \sqrt{\tan^{-1}(ax)},x\right)$$

```
[Out] Unintegrable[x^m*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]], x]
```

Rubi [A] time = 0.1007, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^m \sqrt{c+a^2cx^2} \sqrt{\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]],x]

[Out] Defer[Int][x^m*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]], x]
```

Rubi steps

$$\int x^m \sqrt{c+a^2cx^2} \sqrt{\tan^{-1}(ax)}\,dx = \int x^m \sqrt{c+a^2cx^2} \sqrt{\tan^{-1}(ax)}\,dx$$

Mathematica [A] time = 0.921544, size = 0, normalized size = 0.

$$\int x^m \sqrt{c+a^2cx^2} \sqrt{\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]],x]

[Out] Integrate[x^m*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]], x]
```

Maple [A] time = 1.26, size = 0, normalized size = 0.

$$\int x^{m}\sqrt{a^{2}c\,x^{2}+c}\,\sqrt{\arctan \left(a x \right)}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2),x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{a^{2}cx^{2}+c}x^{m}\sqrt{\arctan\left(ax\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^m*sqrt(arctan(a*x)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**(1/2)*atan(a*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.722 $\int x^2\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(x^2\sqrt{a^2cx^2+c}\sqrt{\tan^{-1}(ax)},x\right)$$

```
[Out] Unintegrable[x^2*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]], x]
```

Rubi [A] time = 0.103086, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^2\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^2*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]],x]
```

```
[Out] Defer[Int][x^2*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]], x]
```

Rubi steps

$$\int x^2\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}\,dx=\int x^2\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}\,dx$$

Mathematica [A] time = 3.60945, size = 0, normalized size = 0.

$$\int x^2\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^2*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]],x]
```

```
[Out] Integrate[x^2*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]], x]
```

Maple [A] time = 1.5, size = 0, normalized size = 0.

$$\int x^2\sqrt{a^2cx^2+c}\,\sqrt{\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2),x)
```

```
[Out] int(x^2*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int x^2\sqrt{c\left(a^2x^2+1\right)}\,\sqrt{\operatorname{atan}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a**2*c*x**2+c)**(1/2)*atan(a*x)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(c*(a**2*x**2 + 1))*sqrt(atan(a*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a^2cx^2 + c}x^2\sqrt{\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)*x^2*sqrt(arctan(a*x)), x)
```

3.723 $\int x\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=65

$$\frac{\left(a^2cx^2+c\right)^{3/2}\sqrt{\tan^{-1}(ax)}}{3a^2c}-\frac{\text{Unintegrable}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{\tan^{-1}(ax)}},x\right)}{6a}$$

```
[Out] ((c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]])/(3*a^2*c) - Unintegrable[Sqrt[c +
 a^2*c*x^2]/Sqrt[ArcTan[a*x]], x]/(6*a)
```

Rubi [A] time = 0.103673, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]],x]
```

```
[Out] ((c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]])/(3*a^2*c) - Defer[Int][Sqrt[c + a
^2*c*x^2]/Sqrt[ArcTan[a*x]], x]/(6*a)
```

Rubi steps

$$\int x\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}\,dx=\frac{\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}}{3a^2c}-\frac{\int\frac{\sqrt{c+a^2cx^2}}{\sqrt{\tan^{-1}(ax)}}\,dx}{6a}$$

Mathematica [A] time = 6.86393, size = 0, normalized size = 0.

$$\int x\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]],x]
```

[Out] Integrate[x*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]], x]

Maple [A] time = 0.974, size = 0, normalized size = 0.

$$\int x\sqrt{a^2cx^2+c}\,\sqrt{\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2),x)

[Out] int(x*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2),x)

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2),x, algorithm="maxima")

[Out] Exception raised: RuntimeError

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{c\left(a^2x^2+1\right)}\,\sqrt{\operatorname{atan}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**(1/2)*atan(a*x)**(1/2),x)
```

```
[Out] Integral(x*sqrt(c*(a**2*x**2 + 1))*sqrt(atan(a*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a^2cx^2 + c}x\sqrt{\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)*x*sqrt(arctan(a*x)), x)
```

3.724 $\int \sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=25

$$\text{Unintegrable}\left(\sqrt{a^2cx^2+c}\sqrt{\tan^{-1}(ax)},x\right)$$

```
[Out] Unintegrable[Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]], x]
```

Rubi [A] time = 0.0320874, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]],x]
```

```
[Out] Defer[Int][Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]], x]
```

Rubi steps

$$\int \sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}\,dx=\int \sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}\,dx$$

Mathematica [A] time = 0.417004, size = 0, normalized size = 0.

$$\int \sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]],x]
```

```
[Out] Integrate[Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]], x]
```

Maple [A] time = 0.862, size = 0, normalized size = 0.

$$\int \sqrt{a^2 c x^2 + c}\,\sqrt{\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2),x)
```

```
[Out] int((a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c\left(a^2 x^2 + 1\right)}\,\sqrt{\operatorname{atan}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(1/2)*atan(a*x)**(1/2),x)
```

```
[Out] Integral(sqrt(c*(a**2*x**2 + 1))*sqrt(atan(a*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a^2cx^2 + c}\sqrt{\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)*sqrt(arctan(a*x)), x)
```

3.725 $\int x^m \left(c+a^2 c x^2\right)^{3/2} \sqrt{\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(x^m \left(a^2 c x^2+c\right)^{3/2} \sqrt{\tan^{-1}(ax)},x\right)$$

```
[Out] Unintegrable[x^m*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]], x]
```

Rubi [A] time = 0.108997, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^m \left(c+a^2 c x^2\right)^{3/2} \sqrt{\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]],x]

[Out] Defer[Int][x^m*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]], x]
```

Rubi steps

$$\int x^m \left(c+a^2 c x^2\right)^{3/2} \sqrt{\tan^{-1}(ax)}\,dx=\int x^m \left(c+a^2 c x^2\right)^{3/2} \sqrt{\tan^{-1}(ax)}\,dx$$

Mathematica [A] time = 1.00142, size = 0, normalized size = 0.

$$\int x^m \left(c+a^2 c x^2\right)^{3/2} \sqrt{\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]],x]

[Out] Integrate[x^m*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]], x]
```

Maple [A] time = 0.987, size = 0, normalized size = 0.

$$\int x^{m} \left(a^{2} c \,x^{2}+c \right)^{\frac{3}{2}} \sqrt{\arctan \left(a x \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(1/2),x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a^{2} c x^{2} + c\right)^{\frac{3}{2}} x^{m} \sqrt{\arctan\left(a x\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)^(3/2)*x^m*sqrt(arctan(a*x)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**(3/2)*atan(a*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.726 $\int x^2 \left(c+a^2 c x^2\right)^{3/2} \sqrt{\tan^{-1}(a x)} \, dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(x^2 \left(a^2 c x^2+c\right)^{3/2} \sqrt{\tan^{-1}(a x)},x\right)$$

```
[Out] Unintegrable[x^2*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]], x]
```

Rubi [A] time = 0.123861, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^2 \left(c+a^2 c x^2\right)^{3/2} \sqrt{\tan^{-1}(a x)} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[x^2*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]],x]
```

```
[Out] Defer[Int][x^2*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]], x]
```

Rubi steps

$$\int x^2 \left(c+a^2 c x^2\right)^{3/2} \sqrt{\tan^{-1}(a x)} \, dx=\int x^2 \left(c+a^2 c x^2\right)^{3/2} \sqrt{\tan^{-1}(a x)} \, dx$$

Mathematica [A] time = 3.96448, size = 0, normalized size = 0.

$$\int x^2 \left(c+a^2 c x^2\right)^{3/2} \sqrt{\tan^{-1}(a x)} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^2*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]],x]
```

```
[Out] Integrate[x^2*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]], x]
```

Maple [A] time = 1.306, size = 0, normalized size = 0.

$$\int x^2 \left(a^2 c x^2 + c\right)^{\frac{3}{2}} \sqrt{\arctan (ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(1/2),x)
```

```
[Out] int(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a**2*c*x**2+c)**(3/2)*atan(a*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2+c\right)^{\frac{3}{2}}x^2\sqrt{\arctan\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)*x^2*sqrt(arctan(a*x)), x)
```

3.727 $\int x\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=65

$$\frac{\left(a^2cx^2+c\right)^{5/2}\sqrt{\tan^{-1}(ax)}}{5a^2c}-\frac{\text{Unintegrable}\left(\frac{\left(a^2cx^2+c\right)^{3/2}}{\sqrt{\tan^{-1}(ax)}},x\right)}{10a}$$

```
[Out] ((c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]])/(5*a^2*c) - Unintegrable[(c + a^2
*c*x^2)^(3/2)/Sqrt[ArcTan[a*x]], x]/(10*a)
```

Rubi [A] time = 0.12153, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]],x]
```

```
[Out] ((c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]])/(5*a^2*c) - Defer[Int][(c + a^2*c
*x^2)^(3/2)/Sqrt[ArcTan[a*x]], x]/(10*a)
```

Rubi steps

$$\int x\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}\,dx=\frac{\left(c+a^2cx^2\right)^{5/2}\sqrt{\tan^{-1}(ax)}}{5a^2c}-\frac{\int\frac{\left(c+a^2cx^2\right)^{3/2}}{\sqrt{\tan^{-1}(ax)}}\,dx}{10a}$$

Mathematica [A] time = 7.82895, size = 0, normalized size = 0.

$$\int x\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]],x]
```

```
[Out] Integrate[x*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]], x]
```

Maple [A] time = 0.783, size = 0, normalized size = 0.

$$\int x \left(a^{2} c \,x^{2}+c \right)^{\frac{3}{2}} \sqrt{\arctan \left(a x \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(1/2),x)
```

```
[Out] int(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**(3/2)*atan(a*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2+c\right)^{\frac{3}{2}} x\sqrt{\arctan\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)*x*sqrt(arctan(a*x)), x)
```

3.728 $\int \left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=25

$$\text{Unintegrable}\left(\left(a^2cx^2+c\right)^{3/2}\sqrt{\tan^{-1}(ax)},x\right)$$

```
[Out] Unintegrable[(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]], x]
```

Rubi [A] time = 0.0357409, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]],x]
```

```
[Out] Defer[Int][(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]], x]
```

Rubi steps

$$\int \left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}\,dx=\int \left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}\,dx$$

Mathematica [A] time = 1.68885, size = 0, normalized size = 0.

$$\int \left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]],x]
```

```
[Out] Integrate[(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]], x]
```

Maple [A] time = 0.691, size = 0, normalized size = 0.

$$\int \left(a^{2} c \,x^{2}+c \right)^{\frac{3}{2}} \sqrt{\arctan \left(a x \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(1/2),x)
```

```
[Out] int((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(3/2)*atan(a*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2+c\right)^{\frac{3}{2}}\sqrt{\arctan\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)*sqrt(arctan(a*x)), x)
```

3.729 $\int x^m \left(c+a^2cx^2\right)^{5/2} \sqrt{\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(x^m \left(a^2cx^2+c\right)^{5/2} \sqrt{\tan^{-1}(ax)},x\right)$$

```
[Out] Unintegrable[x^m*(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]], x]
```

Rubi [A] time = 0.10932, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^m \left(c+a^2cx^2\right)^{5/2} \sqrt{\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]],x]
```

```
[Out] Defer[Int][x^m*(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]], x]
```

Rubi steps

$$\int x^m \left(c+a^2cx^2\right)^{5/2} \sqrt{\tan^{-1}(ax)}\,dx=\int x^m \left(c+a^2cx^2\right)^{5/2} \sqrt{\tan^{-1}(ax)}\,dx$$

Mathematica [A] time = 1.34454, size = 0, normalized size = 0.

$$\int x^m \left(c+a^2cx^2\right)^{5/2} \sqrt{\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]],x]
```

```
[Out] Integrate[x^m*(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]], x]
```

Maple [A] time = 1.094, size = 0, normalized size = 0.

$$\int x^m \left(a^2 c x^2 + c\right)^{\frac{5}{2}} \sqrt{\arctan\left(a x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(1/2),x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^4 c^2 x^4 + 2\, a^2 c^2 x^2 + c^2\right) \sqrt{a^2 c x^2 + c} x^m \sqrt{\arctan\left(a x\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*x^m*sqrt(a
rctan(a*x)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**(5/2)*atan(a*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.730 $\int x^2 \left(c+a^2 c x^2\right)^{5/2} \sqrt{\tan^{-1}(a x)} \, dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(x^2 \left(a^2 c x^2+c\right)^{5/2} \sqrt{\tan^{-1}(a x)},x\right)$$

```
[Out] Unintegrable[x^2*(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]], x]
```

Rubi [A] time = 0.121058, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^2 \left(c+a^2 c x^2\right)^{5/2} \sqrt{\tan^{-1}(a x)} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[x^2*(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]],x]
```

```
[Out] Defer[Int][x^2*(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]], x]
```

Rubi steps

$$\int x^2 \left(c+a^2 c x^2\right)^{5/2} \sqrt{\tan^{-1}(a x)} \, dx=\int x^2 \left(c+a^2 c x^2\right)^{5/2} \sqrt{\tan^{-1}(a x)} \, dx$$

Mathematica [A] time = 3.42196, size = 0, normalized size = 0.

$$\int x^2 \left(c+a^2 c x^2\right)^{5/2} \sqrt{\tan^{-1}(a x)} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^2*(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]],x]
```

```
[Out] Integrate[x^2*(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]], x]
```

Maple [A] time = 1.474, size = 0, normalized size = 0.

$$\int x^2 \left(a^2 c x^2 + c\right)^{\frac{5}{2}} \sqrt{\arctan \left(a x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(1/2),x)
```

```
[Out] int(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a**2*c*x**2+c)**(5/2)*atan(a*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2 + c\right)^{\frac{5}{2}} x^2 \sqrt{\arctan\left(ax\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(5/2)*x^2*sqrt(arctan(a*x)), x)
```

3.731 $\int x\left(c+a^2cx^2\right)^{5/2}\sqrt{\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=65

$$\frac{\left(a^2cx^2+c\right)^{7/2}\sqrt{\tan^{-1}(ax)}}{7a^2c}-\frac{\text{Unintegrable}\left(\frac{\left(a^2cx^2+c\right)^{5/2}}{\sqrt{\tan^{-1}(ax)}},x\right)}{14a}$$

```
[Out] ((c + a^2*c*x^2)^(7/2)*Sqrt[ArcTan[a*x]])/(7*a^2*c) - Unintegrable[(c + a^2
*c*x^2)^(5/2)/Sqrt[ArcTan[a*x]], x]/(14*a)
```

Rubi [A] time = 0.11663, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x\left(c+a^2cx^2\right)^{5/2}\sqrt{\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x*(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]],x]
```

```
[Out] ((c + a^2*c*x^2)^(7/2)*Sqrt[ArcTan[a*x]])/(7*a^2*c) - Defer[Int][(c + a^2*c
*x^2)^(5/2)/Sqrt[ArcTan[a*x]], x]/(14*a)
```

Rubi steps

$$\int x\left(c+a^2cx^2\right)^{5/2}\sqrt{\tan^{-1}(ax)}\,dx=\frac{\left(c+a^2cx^2\right)^{7/2}\sqrt{\tan^{-1}(ax)}}{7a^2c}-\frac{\int\frac{\left(c+a^2cx^2\right)^{5/2}}{\sqrt{\tan^{-1}(ax)}}\,dx}{14a}$$

Mathematica [A] time = 7.40609, size = 0, normalized size = 0.

$$\int x\left(c+a^2cx^2\right)^{5/2}\sqrt{\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x*(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]],x]
```

```
[Out] Integrate[x*(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]], x]
```

Maple [A] time = 0.915, size = 0, normalized size = 0.

$$\int x\left(a^2cx^2+c\right)^{\frac{5}{2}}\sqrt{\arctan\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(1/2),x)
```

```
[Out] int(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**(5/2)*atan(a*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2+c\right)^{\frac{5}{2}} x\sqrt{\arctan\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(5/2)*x*sqrt(arctan(a*x)), x)
```

3.732 $\int \left(c+a^2cx^2\right)^{5/2} \sqrt{\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=25

$$\text{Unintegrable}\left(\left(a^2cx^2+c\right)^{5/2} \sqrt{\tan^{-1}(ax)},x\right)$$

```
[Out] Unintegrable[(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]], x]
```

Rubi [A] time = 0.0372892, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \left(c+a^2cx^2\right)^{5/2} \sqrt{\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]],x]
```

```
[Out] Defer[Int][(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]], x]
```

Rubi steps

$$\int \left(c+a^2cx^2\right)^{5/2} \sqrt{\tan^{-1}(ax)}\,dx=\int \left(c+a^2cx^2\right)^{5/2} \sqrt{\tan^{-1}(ax)}\,dx$$

Mathematica [A] time = 0.486408, size = 0, normalized size = 0.

$$\int \left(c+a^2cx^2\right)^{5/2} \sqrt{\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]],x]
```

```
[Out] Integrate[(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]], x]
```

Maple [A] time = 0.791, size = 0, normalized size = 0.

$$\int \left(a^2cx^2+c\right)^{\frac{5}{2}} \sqrt{\arctan\left(ax\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(1/2),x)

[Out] int((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(1/2),x, algorithm="maxima")

[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(1/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(5/2)*atan(a*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2+c\right)^{\frac{5}{2}}\sqrt{\arctan\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(5/2)*sqrt(arctan(a*x)), x)
```

3.733 $\int \frac{x^m \sqrt{\tan^{-1}(ax)}}{\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(\frac{x^m \sqrt{\tan^{-1}(ax)}}{\sqrt{a^2cx^2+c}},x\right)$$

```
[Out] Unintegrable[(x^m*Sqrt[ArcTan[a*x]])/Sqrt[c + a^2*c*x^2], x]
```

Rubi [A] time = 0.104949, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m \sqrt{\tan^{-1}(ax)}}{\sqrt{c+a^2cx^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*Sqrt[ArcTan[a*x]])/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] Defer[Int][(x^m*Sqrt[ArcTan[a*x]])/Sqrt[c + a^2*c*x^2], x]
```

Rubi steps

$$\int \frac{x^m \sqrt{\tan^{-1}(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^m \sqrt{\tan^{-1}(ax)}}{\sqrt{c+a^2cx^2}}\,dx$$

Mathematica [A] time = 0.877464, size = 0, normalized size = 0.

$$\int \frac{x^m \sqrt{\tan^{-1}(ax)}}{\sqrt{c+a^2cx^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*Sqrt[ArcTan[a*x]])/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] Integrate[(x^m*Sqrt[ArcTan[a*x]])/Sqrt[c + a^2*c*x^2], x]
```

Maple [A] time = 1.339, size = 0, normalized size = 0.

$$\int x^m \sqrt{\arctan(ax)} \frac{1}{\sqrt{a^2cx^2 + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^m\sqrt{\arctan(ax)}}{\sqrt{a^2cx^2 + c}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^m*sqrt(arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*atan(a*x)**(1/2)/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m \sqrt{\arctan(ax)}}{\sqrt{a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^m*sqrt(arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

3.734 $\int \frac{x^3\sqrt{\tan^{-1}(ax)}}{\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=135

$$\frac{\text{Unintegrable}\left(\frac{1}{\sqrt{a^2cx^2+c}\sqrt{\tan^{-1}(ax)}},x\right)}{3a^3}-\frac{\text{Unintegrable}\left(\frac{x^2}{\sqrt{a^2cx^2+c}\sqrt{\tan^{-1}(ax)}},x\right)}{6a}+\frac{x^2\sqrt{a^2cx^2+c}\sqrt{\tan^{-1}(ax)}}{3a^2c}-\frac{2\sqrt{a^2cx^2+}}{3}$$

```
[Out] (-2*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]])/(3*a^4*c) + (x^2*Sqrt[c + a^2*c*
x^2]*Sqrt[ArcTan[a*x]])/(3*a^2*c) + Unintegrable[1/(Sqrt[c + a^2*c*x^2]*Sqr
t[ArcTan[a*x]]), x]/(3*a^3) - Unintegrable[x^2/(Sqrt[c + a^2*c*x^2]*Sqrt[Ar
cTan[a*x]]), x]/(6*a)
```

Rubi [A] time = 0.332401, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^3\sqrt{\tan^{-1}(ax)}}{\sqrt{c+a^2cx^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^3*Sqrt[ArcTan[a*x]])/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] (-2*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]])/(3*a^4*c) + (x^2*Sqrt[c + a^2*c*
x^2]*Sqrt[ArcTan[a*x]])/(3*a^2*c) + Defer[Int][1/(Sqrt[c + a^2*c*x^2]*Sqrt[
ArcTan[a*x]]), x]/(3*a^3) - Defer[Int][x^2/(Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan
[a*x]]), x]/(6*a)
```

Rubi steps

$$\int \frac{x^3\sqrt{\tan^{-1}(ax)}}{\sqrt{c+a^2cx^2}}\,dx=\frac{x^2\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}}{3a^2c}-\frac{2\int \frac{x\sqrt{\tan^{-1}(ax)}}{\sqrt{c+a^2cx^2}}\,dx}{3a^2}-\frac{\int \frac{x^2}{\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}}\,dx}{6a}$$

$$=-\frac{2\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}}{3a^4c}+\frac{x^2\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}}{3a^2c}+\frac{\int \frac{1}{\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}}\,dx}{3a^3}-\frac{\int \frac{x^2}{\sqrt{c+a^2cx^2}\sqrt{}}}{6}$$

Mathematica [A] time = 4.24728, size = 0, normalized size = 0.

$$\int \frac{x^3\sqrt{\tan^{-1}(ax)}}{\sqrt{c+a^2cx^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^3*Sqrt[ArcTan[a*x]])/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] Integrate[(x^3*Sqrt[ArcTan[a*x]])/Sqrt[c + a^2*c*x^2], x]
```

Maple [A] time = 4.891, size = 0, normalized size = 0.

$$\int x^3\sqrt{\arctan(ax)}\frac{1}{\sqrt{a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3\sqrt{\operatorname{atan}(ax)}}{\sqrt{c(a^2x^2+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atan(a*x)**(1/2)/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(atan(a*x))/sqrt(c*(a**2*x**2 + 1)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3\sqrt{\arctan(ax)}}{\sqrt{a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^3*sqrt(arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

3.735 $\int \frac{x^2\sqrt{\tan^{-1}(ax)}}{\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=99

$$-\frac{\text{Unintegrable}\left(\frac{x}{\sqrt{a^2cx^2+c}\sqrt{\tan^{-1}(ax)}},x\right)}{4a}-\frac{\text{Unintegrable}\left(\frac{\sqrt{\tan^{-1}(ax)}}{\sqrt{a^2cx^2+c}},x\right)}{2a^2}+\frac{x\sqrt{a^2cx^2+c}\sqrt{\tan^{-1}(ax)}}{2a^2c}$$

```
[Out] (x*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]])/(2*a^2*c) - Unintegrable[x/(Sqrt[
c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]), x]/(4*a) - Unintegrable[Sqrt[ArcTan[a*x]
]/Sqrt[c + a^2*c*x^2], x]/(2*a^2)
```

Rubi [A] time = 0.216172, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^2\sqrt{\tan^{-1}(ax)}}{\sqrt{c+a^2cx^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^2*Sqrt[ArcTan[a*x]])/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] (x*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]])/(2*a^2*c) - Defer[Int][x/(Sqrt[c
+ a^2*c*x^2]*Sqrt[ArcTan[a*x]]), x]/(4*a) - Defer[Int][Sqrt[ArcTan[a*x]]/Sq
rt[c + a^2*c*x^2], x]/(2*a^2)
```

Rubi steps

$$\int \frac{x^2\sqrt{\tan^{-1}(ax)}}{\sqrt{c+a^2cx^2}}\,dx=\frac{x\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}}{2a^2c}-\frac{\int \frac{\sqrt{\tan^{-1}(ax)}}{\sqrt{c+a^2cx^2}}\,dx}{2a^2}-\frac{\int \frac{x}{\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}}\,dx}{4a}$$

Mathematica [A] time = 2.5137, size = 0, normalized size = 0.

$$\int \frac{x^2\sqrt{\tan^{-1}(ax)}}{\sqrt{c+a^2cx^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^2*Sqrt[ArcTan[a*x]])/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] Integrate[(x^2*Sqrt[ArcTan[a*x]])/Sqrt[c + a^2*c*x^2], x]
```

Maple [A] time = 3.52, size = 0, normalized size = 0.

$$\int x^{2} \sqrt{\arctan (a x)}\, \frac{1}{\sqrt{a^{2} c \,x^{2}+c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\sqrt{\operatorname{atan}(ax)}}{\sqrt{c\left(a^2x^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(a*x)**(1/2)/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(atan(a*x))/sqrt(c*(a**2*x**2 + 1)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\sqrt{\arctan(ax)}}{\sqrt{a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2*sqrt(arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

3.736 $\int \frac{x\sqrt{\tan^{-1}(ax)}}{\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=62

$$\frac{\sqrt{a^2cx^2+c}\sqrt{\tan^{-1}(ax)}}{a^2c}-\frac{\text{Unintegrable}\left(\frac{1}{\sqrt{a^2cx^2+c}\sqrt{\tan^{-1}(ax)}},x\right)}{2a}$$

```
[Out] (Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]])/(a^2*c) - Unintegrable[1/(Sqrt[c +
a^2*c*x^2]*Sqrt[ArcTan[a*x]]), x]/(2*a)
```

Rubi [A] time = 0.108974, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x\sqrt{\tan^{-1}(ax)}}{\sqrt{c+a^2cx^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x*Sqrt[ArcTan[a*x]])/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] (Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]])/(a^2*c) - Defer[Int][1/(Sqrt[c + a^
2*c*x^2]*Sqrt[ArcTan[a*x]]), x]/(2*a)
```

Rubi steps

$$\int \frac{x\sqrt{\tan^{-1}(ax)}}{\sqrt{c+a^2cx^2}}\,dx=\frac{\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}}{a^2c}-\frac{\int \frac{1}{\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}}\,dx}{2a}$$

Mathematica [A] time = 0.79745, size = 0, normalized size = 0.

$$\int \frac{x\sqrt{\tan^{-1}(ax)}}{\sqrt{c+a^2cx^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x*Sqrt[ArcTan[a*x]])/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] Integrate[(x*Sqrt[ArcTan[a*x]])/Sqrt[c + a^2*c*x^2], x]
```

Maple [A] time = 1.201, size = 0, normalized size = 0.

$$\int x\sqrt{\arctan \left(a x \right)}\frac{1}{\sqrt{a^{2} c \,x^{2}+c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sqrt{\operatorname{atan}(ax)}}{\sqrt{c(a^2x^2+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(a*x)**(1/2)/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x*sqrt(atan(a*x))/sqrt(c*(a**2*x**2 + 1)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sqrt{\arctan(ax)}}{\sqrt{a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x*sqrt(arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

3.737 $\int \frac{\sqrt{\tan^{-1}(ax)}}{\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=25

$$\text{Unintegrable}\left(\frac{\sqrt{\tan^{-1}(ax)}}{\sqrt{a^2cx^2+c}},x\right)$$

```
[Out] Unintegrable[Sqrt[ArcTan[a*x]]/Sqrt[c + a^2*c*x^2], x]
```

Rubi [A] time = 0.0343619, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sqrt{\tan^{-1}(ax)}}{\sqrt{c+a^2cx^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[ArcTan[a*x]]/Sqrt[c + a^2*c*x^2],x]

[Out] Defer[Int][Sqrt[ArcTan[a*x]]/Sqrt[c + a^2*c*x^2], x]
```

Rubi steps

$$\int \frac{\sqrt{\tan^{-1}(ax)}}{\sqrt{c+a^2cx^2}}\,dx=\int \frac{\sqrt{\tan^{-1}(ax)}}{\sqrt{c+a^2cx^2}}\,dx$$

Mathematica [A] time = 0.203536, size = 0, normalized size = 0.

$$\int \frac{\sqrt{\tan^{-1}(ax)}}{\sqrt{c+a^2cx^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[ArcTan[a*x]]/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] Integrate[Sqrt[ArcTan[a*x]]/Sqrt[c + a^2*c*x^2], x]
```

Maple [A] time = 0.94, size = 0, normalized size = 0.

$$\int \sqrt{\arctan (ax)}\frac{1}{\sqrt{a^{2}c x^{2}+c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\operatorname{atan}(ax)}}{\sqrt{c(a^2x^2+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(1/2)/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(atan(a*x))/sqrt(c*(a**2*x**2 + 1)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\arctan(ax)}}{\sqrt{a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

3.738 $\int \frac{\sqrt{\tan^{-1}(ax)}}{x\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(\frac{\sqrt{\tan^{-1}(ax)}}{x\sqrt{a^2cx^2+c}},x\right)$$

```
[Out] Unintegrable[Sqrt[ArcTan[a*x]]/(x*Sqrt[c + a^2*c*x^2]), x]
```

Rubi [A] time = 0.106914, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sqrt{\tan^{-1}(ax)}}{x\sqrt{c+a^2cx^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[ArcTan[a*x]]/(x*Sqrt[c + a^2*c*x^2]),x]

[Out] Defer[Int][Sqrt[ArcTan[a*x]]/(x*Sqrt[c + a^2*c*x^2]), x]
```

Rubi steps

$$\int \frac{\sqrt{\tan^{-1}(ax)}}{x\sqrt{c+a^2cx^2}}\,dx=\int \frac{\sqrt{\tan^{-1}(ax)}}{x\sqrt{c+a^2cx^2}}\,dx$$

Mathematica [A] time = 1.04832, size = 0, normalized size = 0.

$$\int \frac{\sqrt{\tan^{-1}(ax)}}{x\sqrt{c+a^2cx^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[ArcTan[a*x]]/(x*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] Integrate[Sqrt[ArcTan[a*x]]/(x*Sqrt[c + a^2*c*x^2]), x]
```

Maple [A] time = 1.052, size = 0, normalized size = 0.

$$\int \frac{1}{x}\sqrt{\arctan(ax)}\frac{1}{\sqrt{a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\operatorname{atan}(ax)}}{x\sqrt{c(a^2x^2+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(1/2)/x/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(atan(a*x))/(x*sqrt(c*(a**2*x**2 + 1))), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\arctan(ax)}}{\sqrt{a^2cx^2+cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(arctan(a*x))/(sqrt(a^2*c*x^2 + c)*x), x)
```

3.739 $\int \frac{\sqrt{\tan^{-1}(ax)}}{x^2\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=64

$$\frac{1}{2}a\text{Unintegrable}\left(\frac{1}{x\sqrt{a^2cx^2+c}\sqrt{\tan^{-1}(ax)}},x\right)-\frac{\sqrt{a^2cx^2+c}\sqrt{\tan^{-1}(ax)}}{cx}$$

```
[Out] -((Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]])/(c*x)) + (a*Unintegrable[1/(x*Sqr
t[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]), x])/2
```

Rubi [A] time = 0.21421, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sqrt{\tan^{-1}(ax)}}{x^2\sqrt{c+a^2cx^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[ArcTan[a*x]]/(x^2*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] -((Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]])/(c*x)) + (a*Defer[Int][1/(x*Sqrt[
c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]), x])/2
```

Rubi steps

$$\int \frac{\sqrt{\tan^{-1}(ax)}}{x^2\sqrt{c+a^2cx^2}}\,dx=-\frac{\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}}{cx}+\frac{1}{2}a\int \frac{1}{x\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}}\,dx$$

Mathematica [A] time = 1.73093, size = 0, normalized size = 0.

$$\int \frac{\sqrt{\tan^{-1}(ax)}}{x^2\sqrt{c+a^2cx^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[ArcTan[a*x]]/(x^2*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] Integrate[Sqrt[ArcTan[a*x]]/(x^2*Sqrt[c + a^2*c*x^2]), x]
```

Maple [A] time = 0.801, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\sqrt{\arctan (ax)}\frac{1}{\sqrt{a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(1/2)/x^2/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(arctan(a*x)^(1/2)/x^2/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/x^2/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/x^2/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\operatorname{atan}(ax)}}{x^2\sqrt{c\left(a^2x^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(1/2)/x**2/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(atan(a*x))/(x**2*sqrt(c*(a**2*x**2 + 1))), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\arctan(ax)}}{\sqrt{a^2cx^2+c}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/x^2/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(arctan(a*x))/(sqrt(a^2*c*x^2 + c)*x^2), x)
```

3.740 $\int \frac{\sqrt{\tan^{-1}(ax)}}{x^3\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=101

$$-\frac{1}{2}a^2\text{Unintegrable}\left(\frac{\sqrt{\tan^{-1}(ax)}}{x\sqrt{a^2cx^2+c}},x\right)+\frac{1}{4}a\text{Unintegrable}\left(\frac{1}{x^2\sqrt{a^2cx^2+c}\sqrt{\tan^{-1}(ax)}},x\right)-\frac{\sqrt{a^2cx^2+c}\sqrt{\tan^{-1}(ax)}}{2cx^2}$$

```
[Out] -(Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]])/(2*c*x^2) + (a*Unintegrable[1/(x^2
*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]), x])/4 - (a^2*Unintegrable[Sqrt[Arc
Tan[a*x]]/(x*Sqrt[c + a^2*c*x^2]), x])/2
```

Rubi [A] time = 0.329239, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sqrt{\tan^{-1}(ax)}}{x^3\sqrt{c+a^2cx^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[ArcTan[a*x]]/(x^3*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] -(Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]])/(2*c*x^2) + (a*Defer[Int][1/(x^2*S
qrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]), x])/4 - (a^2*Defer[Int][Sqrt[ArcTan[
a*x]]/(x*Sqrt[c + a^2*c*x^2]), x])/2
```

Rubi steps

$$\int \frac{\sqrt{\tan^{-1}(ax)}}{x^3\sqrt{c+a^2cx^2}}\,dx=-\frac{\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}}{2cx^2}+\frac{1}{4}a\int \frac{1}{x^2\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}}\,dx-\frac{1}{2}a^2\int \frac{\sqrt{\tan^{-1}(ax)}}{x\sqrt{c+a^2cx^2}}\,dx$$

Mathematica [A] time = 3.51652, size = 0, normalized size = 0.

$$\int \frac{\sqrt{\tan^{-1}(ax)}}{x^3\sqrt{c+a^2cx^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[ArcTan[a*x]]/(x^3*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] Integrate[Sqrt[ArcTan[a*x]]/(x^3*Sqrt[c + a^2*c*x^2]), x]
```

Maple [A] time = 1.257, size = 0, normalized size = 0.

$$\int \frac{1}{x^3}\sqrt{\arctan(ax)}\frac{1}{\sqrt{a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(1/2)/x^3/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(arctan(a*x)^(1/2)/x^3/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/x^3/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/x^3/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\operatorname{atan}(ax)}}{x^3\sqrt{c\left(a^2x^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(1/2)/x**3/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(atan(a*x))/(x**3*sqrt(c*(a**2*x**2 + 1))), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\arctan(ax)}}{\sqrt{a^2cx^2+c}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/x^3/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(arctan(a*x))/(sqrt(a^2*c*x^2 + c)*x^3), x)
```

3.741 $\int \frac{\sqrt{\tan^{-1}(ax)}}{x^4\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=136

$$-\frac{1}{3}a^3\text{Unintegrable}\left(\frac{1}{x\sqrt{a^2cx^2+c}\sqrt{\tan^{-1}(ax)}},x\right)+\frac{1}{6}a\text{Unintegrable}\left(\frac{1}{x^3\sqrt{a^2cx^2+c}\sqrt{\tan^{-1}(ax)}},x\right)+\frac{2a^2\sqrt{a^2cx^2+c}\sqrt{}}{3cx}$$

```
[Out] -(Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]])/(3*c*x^3) + (2*a^2*Sqrt[c + a^2*c*
x^2]*Sqrt[ArcTan[a*x]])/(3*c*x) + (a*Unintegrable[1/(x^3*Sqrt[c + a^2*c*x^2
]*Sqrt[ArcTan[a*x]]), x])/6 - (a^3*Unintegrable[1/(x*Sqrt[c + a^2*c*x^2]*Sq
rt[ArcTan[a*x]]), x])/3
```

Rubi [A] time = 0.424138, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sqrt{\tan^{-1}(ax)}}{x^4\sqrt{c+a^2cx^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[ArcTan[a*x]]/(x^4*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] -(Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]])/(3*c*x^3) + (2*a^2*Sqrt[c + a^2*c*
x^2]*Sqrt[ArcTan[a*x]])/(3*c*x) + (a*Defer[Int][1/(x^3*Sqrt[c + a^2*c*x^2]*
Sqrt[ArcTan[a*x]]), x])/6 - (a^3*Defer[Int][1/(x*Sqrt[c + a^2*c*x^2]*Sqrt[A
rcTan[a*x]]), x])/3
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\tan^{-1}(ax)}}{x^4\sqrt{c+a^2cx^2}}\,dx &= -\frac{\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}}{3cx^3}+\frac{1}{6}a\int \frac{1}{x^3\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}}\,dx-\frac{1}{3}\left(2a^2\right)\int \frac{\sqrt{\tan^{-1}(ax)}}{x^2\sqrt{c+a^2cx^2}}\,dx\\ &= -\frac{\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}}{3cx^3}+\frac{2a^2\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}}{3cx}+\frac{1}{6}a\int \frac{1}{x^3\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}}\,dx-\frac{1}{3}\end{aligned}$$

Mathematica [A] time = 19.2338, size = 0, normalized size = 0.

$$\int \frac{\sqrt{\tan^{-1}(ax)}}{x^4\sqrt{c+a^2cx^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[ArcTan[a*x]]/(x^4*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] Integrate[Sqrt[ArcTan[a*x]]/(x^4*Sqrt[c + a^2*c*x^2]), x]
```

Maple [A] time = 3.262, size = 0, normalized size = 0.

$$\int \frac{1}{x^4}\sqrt{\arctan(ax)}\frac{1}{\sqrt{a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(1/2)/x^4/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(arctan(a*x)^(1/2)/x^4/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/x^4/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/x^4/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(1/2)/x**4/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\arctan(ax)}}{\sqrt{a^2cx^2+c}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/x^4/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(arctan(a*x))/(sqrt(a^2*c*x^2 + c)*x^4), x)
```

3.742 $\int \frac{x^m\sqrt{\tan^{-1}(ax)}}{(c+a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(\frac{x^m\sqrt{\tan^{-1}(ax)}}{(a^2cx^2+c)^{3/2}},x\right)$$

```
[Out] Unintegrable[(x^m*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(3/2), x]
```

Rubi [A] time = 0.115156, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m\sqrt{\tan^{-1}(ax)}}{(c+a^2cx^2)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(3/2),x]

[Out] Defer[Int][(x^m*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(3/2), x]
```

Rubi steps

$$\int \frac{x^m\sqrt{\tan^{-1}(ax)}}{(c+a^2cx^2)^{3/2}}\,dx=\int \frac{x^m\sqrt{\tan^{-1}(ax)}}{(c+a^2cx^2)^{3/2}}\,dx$$

Mathematica [A] time = 1.01526, size = 0, normalized size = 0.

$$\int \frac{x^m\sqrt{\tan^{-1}(ax)}}{(c+a^2cx^2)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] Integrate[(x^m*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(3/2), x]
```

Maple [A] time = 0.981, size = 0, normalized size = 0.

$$\int x^m \sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{-\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] int(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a^2cx^2+c}\,x^m\sqrt{\arctan(ax)}}{a^4c^2x^4+2\,a^2c^2x^2+c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^m*sqrt(arctan(a*x))/(a^4*c^2*x^4 + 2*a^2*c^2
*x^2 + c^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*atan(a*x)**(1/2)/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m \sqrt{\arctan(ax)}}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^m*sqrt(arctan(a*x))/(a^2*c*x^2 + c)^(3/2), x)
```

3.743 $\int \frac{x^3 \sqrt{\tan^{-1}(ax)}}{(c+a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(\frac{x^3 \sqrt{\tan^{-1}(ax)}}{(a^2cx^2+c)^{3/2}},x\right)$$

```
[Out] Unintegrable[(x^3*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(3/2), x]
```

Rubi [A] time = 0.123784, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^3 \sqrt{\tan^{-1}(ax)}}{(c+a^2cx^2)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^3*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(3/2),x]

[Out] Defer[Int][(x^3*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(3/2), x]
```

Rubi steps

$$\int \frac{x^3 \sqrt{\tan^{-1}(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^3 \sqrt{\tan^{-1}(ax)}}{(c+a^2cx^2)^{3/2}}\,dx$$

Mathematica [A] time = 24.9396, size = 0, normalized size = 0.

$$\int \frac{x^3 \sqrt{\tan^{-1}(ax)}}{(c+a^2cx^2)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^3*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] Integrate[(x^3*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(3/2), x]
```

Maple [A] time = 3.121, size = 0, normalized size = 0.

$$\int x^3\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{-\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] int(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3\sqrt{\operatorname{atan}\left(ax\right)}}{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atan(a*x)**(1/2)/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(x**3*sqrt(atan(a*x))/(c*(a**2*x**2 + 1))**(3/2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3\sqrt{\arctan\left(ax\right)}}{\left(a^2cx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^3*sqrt(arctan(a*x))/(a^2*c*x^2 + c)^(3/2), x)
```

3.744 $\int \frac{x^2 \sqrt{\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{3/2}} \, dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(\frac{x^2 \sqrt{\tan^{-1}(ax)}}{\left(a^2cx^2+c\right)^{3/2}},x\right)$$

```
[Out] Unintegrable[(x^2*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(3/2), x]
```

Rubi [A] time = 0.116569, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^2 \sqrt{\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{3/2}} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^2*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] Defer[Int][(x^2*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(3/2), x]
```

Rubi steps

$$\int \frac{x^2 \sqrt{\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{3/2}} \, dx = \int \frac{x^2 \sqrt{\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{3/2}} \, dx$$

Mathematica [A] time = 3.62174, size = 0, normalized size = 0.

$$\int \frac{x^2 \sqrt{\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{3/2}} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^2*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] Integrate[(x^2*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(3/2), x]
```

Maple [A] time = 2.974, size = 0, normalized size = 0.

$$\int x^{2} \sqrt{\arctan \left(a x\right)}\, \left(a^{2} c \,x^{2}+c \right)^{-\frac{3}{2}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] int(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\sqrt{\operatorname{atan}(ax)}}{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(a*x)**(1/2)/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(x**2*sqrt(atan(a*x))/(c*(a**2*x**2 + 1))**(3/2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\sqrt{\arctan(ax)}}{\left(a^2cx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^2*sqrt(arctan(a*x))/(a^2*c*x^2 + c)^(3/2), x)
```

3.745 $\int \frac{x\sqrt{\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=93

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{a^2x^2+1}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{a^2c\sqrt{a^2cx^2+c}}-\frac{\sqrt{\tan^{-1}(ax)}}{a^2c\sqrt{a^2cx^2+c}}$$

```
[Out] -(Sqrt[ArcTan[a*x]]/(a^2*c*Sqrt[c + a^2*c*x^2])) + (Sqrt[Pi/2]*Sqrt[1 + a^2
*x^2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(a^2*c*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.182426, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {4930, 4905, 4904, 3304, 3352}

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{a^2x^2+1}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{a^2c\sqrt{a^2cx^2+c}}-\frac{\sqrt{\tan^{-1}(ax)}}{a^2c\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] -(Sqrt[ArcTan[a*x]]/(a^2*c*Sqrt[c + a^2*c*x^2])) + (Sqrt[Pi/2]*Sqrt[1 + a^2
*x^2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(a^2*c*Sqrt[c + a^2*c*x^2])
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 4905

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[(d^(q + 1/2)*Sqrt[1 + c^2*x^2])/Sqrt[d + e*x^2], Int[(1 + c
^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
```

```
EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

Rule 4904

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x\sqrt{\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{3/2}}\,dx &= -\frac{\sqrt{\tan^{-1}(ax)}}{a^2c\sqrt{c+a^2cx^2}}+\frac{\int \frac{1}{\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}}\,dx}{2a}\\
&= -\frac{\sqrt{\tan^{-1}(ax)}}{a^2c\sqrt{c+a^2cx^2}}+\frac{\sqrt{1+a^2x^2}\int \frac{1}{\left(1+a^2x^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}}\,dx}{2ac\sqrt{c+a^2cx^2}}\\
&= -\frac{\sqrt{\tan^{-1}(ax)}}{a^2c\sqrt{c+a^2cx^2}}+\frac{\sqrt{1+a^2x^2}\,\mathrm{Subst}\left(\int \frac{\cos(x)}{\sqrt{x}}\,dx,x,\tan^{-1}(ax)\right)}{2a^2c\sqrt{c+a^2cx^2}}\\
&= -\frac{\sqrt{\tan^{-1}(ax)}}{a^2c\sqrt{c+a^2cx^2}}+\frac{\sqrt{1+a^2x^2}\,\mathrm{Subst}\left(\int \cos\left(x^2\right)\,dx,x,\sqrt{\tan^{-1}(ax)}\right)}{a^2c\sqrt{c+a^2cx^2}}\\
&= -\frac{\sqrt{\tan^{-1}(ax)}}{a^2c\sqrt{c+a^2cx^2}}+\frac{\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}\,C\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{a^2c\sqrt{c+a^2cx^2}}
\end{aligned}$$

Mathematica [C] time = 0.1514, size = 121, normalized size = 1.3

$$\frac{-i\sqrt{a^2x^2+1}\sqrt{-i\tan^{-1}(ax)}\text{Gamma}\left(\frac{1}{2},-i\tan^{-1}(ax)\right)+i\sqrt{a^2x^2+1}\sqrt{i\tan^{-1}(ax)}\text{Gamma}\left(\frac{1}{2},i\tan^{-1}(ax)\right)-4\tan^{-1}(ax)}{4a^2c\sqrt{a^2cx^2+c}\sqrt{\tan^{-1}(ax)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] (-4*ArcTan[a*x] - I*Sqrt[1 + a^2*x^2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-I
)*ArcTan[a*x]] + I*Sqrt[1 + a^2*x^2]*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, I*ArcTa
n[a*x]])/(4*a^2*c*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]])
```

Maple [F] time = 0.882, size = 0, normalized size = 0.

$$\int x\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{-\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] int(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sqrt{\operatorname{atan}(ax)}}{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(a*x)**(1/2)/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(x*sqrt(atan(a*x))/(c*(a**2*x**2 + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sqrt{\arctan(ax)}}{\left(a^2cx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x*sqrt(arctan(a*x))/(a^2*c*x^2 + c)^(3/2), x)
```

3.746 $\int \frac{\sqrt{\tan^{-1}(ax)}}{(c+a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=91

$$\frac{x\sqrt{\tan^{-1}(ax)}}{c\sqrt{a^2cx^2+c}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{a^2x^2+1}S\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{ac\sqrt{a^2cx^2+c}}$$

```
[Out] (x*Sqrt[ArcTan[a*x]])/(c*Sqrt[c + a^2*c*x^2]) - (Sqrt[Pi/2]*Sqrt[1 + a^2*x^
2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(a*c*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.111199, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4905, 4904, 3296, 3305, 3351}

$$\frac{x\sqrt{\tan^{-1}(ax)}}{c\sqrt{a^2cx^2+c}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{a^2x^2+1}S\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{ac\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[ArcTan[a*x]]/(c + a^2*c*x^2)^(3/2),x]
```


```
[Out] (x*Sqrt[ArcTan[a*x]])/(c*Sqrt[c + a^2*c*x^2]) - (Sqrt[Pi/2]*Sqrt[1 + a^2*x^
2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(a*c*Sqrt[c + a^2*c*x^2])
```


Rule 4905

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[(d^(q + 1/2)*Sqrt[1 + c^2*x^2])/Sqrt[d + e*x^2], Int[(1 + c
^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

Rule 4904

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{3/2}}\,dx &= \frac{\sqrt{1+a^2x^2}\int \frac{\sqrt{\tan^{-1}(ax)}}{\left(1+a^2x^2\right)^{3/2}}\,dx}{c\sqrt{c+a^2cx^2}}\\
&= \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \sqrt{x}\cos(x)\,dx, x, \tan^{-1}(ax)\right)}{ac\sqrt{c+a^2cx^2}}\\
&= \frac{x\sqrt{\tan^{-1}(ax)}}{c\sqrt{c+a^2cx^2}} - \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \frac{\sin(x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{2ac\sqrt{c+a^2cx^2}}\\
&= \frac{x\sqrt{\tan^{-1}(ax)}}{c\sqrt{c+a^2cx^2}} - \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \sin\left(x^2\right)\,dx, x, \sqrt{\tan^{-1}(ax)}\right)}{ac\sqrt{c+a^2cx^2}}\\
&= \frac{x\sqrt{\tan^{-1}(ax)}}{c\sqrt{c+a^2cx^2}} - \frac{\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}\,S\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{ac\sqrt{c+a^2cx^2}}
\end{aligned}
$$

Mathematica [A] time = 0.0717767, size = 78, normalized size = 0.86

$$
\frac{2ax\sqrt{\tan^{-1}(ax)} - \sqrt{2\pi}\sqrt{a^2x^2+1}\,S\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{2ac\sqrt{a^2cx^2+c}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[ArcTan[a*x]]/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] (2*a*x*Sqrt[ArcTan[a*x]] - Sqrt[2*Pi]*Sqrt[1 + a^2*x^2]*FresnelS[Sqrt[2/Pi]
*Sqrt[ArcTan[a*x]]])/(2*a*c*Sqrt[c + a^2*c*x^2])
```

Maple [F] time = 0.705, size = 0, normalized size = 0.

$$\int \sqrt{\arctan \left(a x\right)}\, \left(a^{2} c \,x^{2}+c \right)^{-\frac{3}{2}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] int(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\operatorname{atan}\left(ax\right)}}{\left(c\left(a^{2}x^{2}+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(1/2)/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(sqrt(atan(a*x))/(c*(a**2*x**2 + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\arctan\left(ax\right)}}{\left(a^{2}cx^{2}+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(arctan(a*x))/(a^2*c*x^2 + c)^(3/2), x)
```

3.747 $\int \frac{\sqrt{\tan^{-1}(ax)}}{x(c+a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(\frac{\sqrt{\tan^{-1}(ax)}}{x\left(a^2cx^2+c\right)^{3/2}},x\right)$$

```
[Out] Unintegrable[Sqrt[ArcTan[a*x]]/(x*(c + a^2*c*x^2)^(3/2)), x]
```

Rubi [A] time = 0.116241, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sqrt{\tan^{-1}(ax)}}{x\left(c+a^2cx^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[ArcTan[a*x]]/(x*(c + a^2*c*x^2)^(3/2)),x]

[Out] Defer[Int][Sqrt[ArcTan[a*x]]/(x*(c + a^2*c*x^2)^(3/2)), x]
```

Rubi steps

$$\int \frac{\sqrt{\tan^{-1}(ax)}}{x\left(c+a^2cx^2\right)^{3/2}}\,dx=\int \frac{\sqrt{\tan^{-1}(ax)}}{x\left(c+a^2cx^2\right)^{3/2}}\,dx$$

Mathematica [A] time = 1.94337, size = 0, normalized size = 0.

$$\int \frac{\sqrt{\tan^{-1}(ax)}}{x\left(c+a^2cx^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[ArcTan[a*x]]/(x*(c + a^2*c*x^2)^(3/2)),x]
```

```
[Out] Integrate[Sqrt[ArcTan[a*x]]/(x*(c + a^2*c*x^2)^(3/2)), x]
```

Maple [A] time = 0.733, size = 0, normalized size = 0.

$$\int \frac{1}{x}\sqrt{\arctan\left(ax\right)}\left(a^{2}c x^{2}+c\right)^{-\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] int(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\operatorname{atan}(ax)}}{x\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(1/2)/x/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(sqrt(atan(a*x))/(x*(c*(a**2*x**2 + 1))**(3/2)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\arctan(ax)}}{\left(a^2cx^2+c\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(arctan(a*x))/((a^2*c*x^2 + c)^(3/2)*x), x)
```

3.748 $\int \frac{\sqrt{\tan^{-1}(ax)}}{x^2(c+a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(\frac{\sqrt{\tan^{-1}(ax)}}{x^2\left(a^2cx^2+c\right)^{3/2}},x\right)$$

```
[Out] Unintegrable[Sqrt[ArcTan[a*x]]/(x^2*(c + a^2*c*x^2)^(3/2)), x]
```

Rubi [A] time = 0.118102, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sqrt{\tan^{-1}(ax)}}{x^2\left(c+a^2cx^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[ArcTan[a*x]]/(x^2*(c + a^2*c*x^2)^(3/2)),x]
```

```
[Out] Defer[Int][Sqrt[ArcTan[a*x]]/(x^2*(c + a^2*c*x^2)^(3/2)), x]
```

Rubi steps

$$\int \frac{\sqrt{\tan^{-1}(ax)}}{x^2\left(c+a^2cx^2\right)^{3/2}}\,dx=\int \frac{\sqrt{\tan^{-1}(ax)}}{x^2\left(c+a^2cx^2\right)^{3/2}}\,dx$$

Mathematica [A] time = 5.75692, size = 0, normalized size = 0.

$$\int \frac{\sqrt{\tan^{-1}(ax)}}{x^2\left(c+a^2cx^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[ArcTan[a*x]]/(x^2*(c + a^2*c*x^2)^(3/2)),x]
```

```
[Out] Integrate[Sqrt[ArcTan[a*x]]/(x^2*(c + a^2*c*x^2)^(3/2)), x]
```

Maple [A] time = 0.776, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\sqrt{\arctan (ax)}\left(a^2cx^2+c\right)^{-\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(1/2)/x^2/(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] int(arctan(a*x)^(1/2)/x^2/(a^2*c*x^2+c)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/x^2/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/x^2/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(1/2)/x**2/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\arctan(ax)}}{(a^2cx^2+c)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/x^2/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(arctan(a*x))/((a^2*c*x^2 + c)^(3/2)*x^2), x)
```

3.749 $\int \frac{x^m \sqrt{\tan^{-1}(ax)}}{(c+a^2cx^2)^{5/2}} dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(\frac{x^m \sqrt{\tan^{-1}(ax)}}{(a^2cx^2+c)^{5/2}}, x\right)$$

```
[Out] Unintegrable[(x^m*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(5/2), x]
```

Rubi [A] time = 0.114243, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m \sqrt{\tan^{-1}(ax)}}{(c+a^2cx^2)^{5/2}} dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(5/2),x]

[Out] Defer[Int][(x^m*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(5/2), x]
```

Rubi steps

$$\int \frac{x^m \sqrt{\tan^{-1}(ax)}}{(c+a^2cx^2)^{5/2}} dx = \int \frac{x^m \sqrt{\tan^{-1}(ax)}}{(c+a^2cx^2)^{5/2}} dx$$

Mathematica [A] time = 1.68085, size = 0, normalized size = 0.

$$\int \frac{x^m \sqrt{\tan^{-1}(ax)}}{(c+a^2cx^2)^{5/2}} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] Integrate[(x^m*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(5/2), x]
```

Maple [A] time = 0.947, size = 0, normalized size = 0.

$$\int x^m \sqrt{\arctan(ax)} \left(a^2 c x^2 + c\right)^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x)
```

```
[Out] int(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a^2 c x^2 + c}\, x^m \sqrt{\arctan(ax)}}{a^6 c^3 x^6 + 3\,a^4 c^3 x^4 + 3\,a^2 c^3 x^2 + c^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^m*sqrt(arctan(a*x))/(a^6*c^3*x^6 + 3*a^4*c^3
*x^4 + 3*a^2*c^3*x^2 + c^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*atan(a*x)**(1/2)/(a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m \sqrt{\arctan(ax)}}{\left(a^2cx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x^m*sqrt(arctan(a*x))/(a^2*c*x^2 + c)^(5/2), x)
```

3.750 $\int \frac{x^4 \sqrt{\tan^{-1}(ax)}}{\left(c+a^2 c x^2\right)^{5/2}} \, dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(\frac{x^4 \sqrt{\tan^{-1}(ax)}}{\left(a^2 c x^2+c\right)^{5/2}},x\right)$$

```
[Out] Unintegrable[(x^4*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(5/2), x]
```

Rubi [A] time = 0.122244, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^4 \sqrt{\tan^{-1}(ax)}}{\left(c+a^2 c x^2\right)^{5/2}} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^4*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(5/2),x]

[Out] Defer[Int][(x^4*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(5/2), x]
```

Rubi steps

$$\int \frac{x^4 \sqrt{\tan^{-1}(ax)}}{\left(c+a^2 c x^2\right)^{5/2}} \, dx = \int \frac{x^4 \sqrt{\tan^{-1}(ax)}}{\left(c+a^2 c x^2\right)^{5/2}} \, dx$$

Mathematica [A] time = 3.85382, size = 0, normalized size = 0.

$$\int \frac{x^4 \sqrt{\tan^{-1}(ax)}}{\left(c+a^2 c x^2\right)^{5/2}} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^4*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] Integrate[(x^4*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(5/2), x]
```

Maple [A] time = 2.079, size = 0, normalized size = 0.

$$\int x^{4} \sqrt{\arctan\left(a x\right)} \left(a^{2} c \,x^{2}+c \right)^{-\frac{5}{2}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x)
```

```
[Out] int(x^4*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*atan(a*x)**(1/2)/(a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4\sqrt{\arctan(ax)}}{\left(a^2cx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x^4*sqrt(arctan(a*x))/(a^2*c*x^2 + c)^(5/2), x)
```

3.751 $\int \frac{x^3\sqrt{\tan^{-1}(ax)}}{(c+a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=215

$$\frac{3\sqrt{\frac{\pi}{2}}\sqrt{a^2x^2+1}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{4a^4c^2\sqrt{a^2cx^2+c}}-\frac{\sqrt{\frac{\pi}{6}}\sqrt{a^2x^2+1}\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{12a^4c^2\sqrt{a^2cx^2+c}}-\frac{3\sqrt{\tan^{-1}(ax)}}{4a^4c^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{a^2x^2-}}{}$$

```
[Out] (-3*Sqrt[ArcTan[a*x]])/(4*a^4*c^2*Sqrt[c + a^2*c*x^2]) + (Sqrt[1 + a^2*x^2]
*Sqrt[ArcTan[a*x]]*Cos[3*ArcTan[a*x]])/(12*a^4*c^2*Sqrt[c + a^2*c*x^2]) + (
3*Sqrt[Pi/2]*Sqrt[1 + a^2*x^2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(4*a
^4*c^2*Sqrt[c + a^2*c*x^2]) - (Sqrt[Pi/6]*Sqrt[1 + a^2*x^2]*FresnelC[Sqrt[6
/Pi]*Sqrt[ArcTan[a*x]]])/(12*a^4*c^2*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.347323, antiderivative size = 215, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {4971, 4970, 3312, 3296, 3304, 3352}

$$\frac{3\sqrt{\frac{\pi}{2}}\sqrt{a^2x^2+1}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{4a^4c^2\sqrt{a^2cx^2+c}}-\frac{\sqrt{\frac{\pi}{6}}\sqrt{a^2x^2+1}\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{12a^4c^2\sqrt{a^2cx^2+c}}-\frac{3\sqrt{\tan^{-1}(ax)}}{4a^4c^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{a^2x^2-}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] (-3*Sqrt[ArcTan[a*x]])/(4*a^4*c^2*Sqrt[c + a^2*c*x^2]) + (Sqrt[1 + a^2*x^2]
*Sqrt[ArcTan[a*x]]*Cos[3*ArcTan[a*x]])/(12*a^4*c^2*Sqrt[c + a^2*c*x^2]) + (
3*Sqrt[Pi/2]*Sqrt[1 + a^2*x^2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(4*a
^4*c^2*Sqrt[c + a^2*c*x^2]) - (Sqrt[Pi/6]*Sqrt[1 + a^2*x^2]*FresnelC[Sqrt[6
/Pi]*Sqrt[ArcTan[a*x]]])/(12*a^4*c^2*Sqrt[c + a^2*c*x^2])
```

Rule 4971

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[(d^(q + 1/2)*Sqrt[1 + c^2*x^2])/Sqrt[d + e*x^2],
 Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d
, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(In
tegerQ[q] || GtQ[d, 0])
```

Rule 4970

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sin[x]^m)/
Cos[x]^(m + 2*(q + 1)), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{x^3\sqrt{\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{5/2}}\,dx = \frac{\sqrt{1+a^2x^2}\int \frac{x^3\sqrt{\tan^{-1}(ax)}}{\left(1+a^2x^2\right)^{5/2}}\,dx}{c^2\sqrt{c+a^2cx^2}}$$

$$= \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \sqrt{x}\sin^3(x)\,dx, x, \tan^{-1}(ax)\right)}{a^4c^2\sqrt{c+a^2cx^2}}$$

$$= \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \left(\frac{3}{4}\sqrt{x}\sin(x)-\frac{1}{4}\sqrt{x}\sin(3x)\right)dx, x, \tan^{-1}(ax)\right)}{a^4c^2\sqrt{c+a^2cx^2}}$$

$$= -\frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \sqrt{x}\sin(3x)\,dx, x, \tan^{-1}(ax)\right)}{4a^4c^2\sqrt{c+a^2cx^2}} + \frac{\left(3\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int \sqrt{x}\sin(x)\,dx, x, \text{tan}\right.}{4a^4c^2\sqrt{c+a^2cx^2}}$$

$$= -\frac{3\sqrt{\tan^{-1}(ax)}}{4a^4c^2\sqrt{c+a^2cx^2}} + \frac{\sqrt{1+a^2x^2}\sqrt{\tan^{-1}(ax)}\cos\left(3\tan^{-1}(ax)\right)}{12a^4c^2\sqrt{c+a^2cx^2}} - \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \frac{\cos(3x)}{\sqrt{x}}\,dx, x\right.}{24a^4c^2\sqrt{c+a^2cx^2}}$$

$$= -\frac{3\sqrt{\tan^{-1}(ax)}}{4a^4c^2\sqrt{c+a^2cx^2}} + \frac{\sqrt{1+a^2x^2}\sqrt{\tan^{-1}(ax)}\cos\left(3\tan^{-1}(ax)\right)}{12a^4c^2\sqrt{c+a^2cx^2}} - \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \cos\left(3x^2\right)\right.}{12a^4c^2\sqrt{c+a^2c}}$$

$$= -\frac{3\sqrt{\tan^{-1}(ax)}}{4a^4c^2\sqrt{c+a^2cx^2}} + \frac{\sqrt{1+a^2x^2}\sqrt{\tan^{-1}(ax)}\cos\left(3\tan^{-1}(ax)\right)}{12a^4c^2\sqrt{c+a^2cx^2}} + \frac{3\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}\,C\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(\right.}{4a^4c^2\sqrt{c+a^2cx^2}}$$

<u>**Mathematica**</u> **[C]** time = 0.50442, size = 324, normalized size = 1.51

$$\frac{ia^2x^2\sqrt{3a^2x^2+3}\sqrt{-i\tan^{-1}(ax)}\text{Gamma}\left(\frac{1}{2},-3i\tan^{-1}(ax)\right) - ia^2x^2\sqrt{3a^2x^2+3}\sqrt{i\tan^{-1}(ax)}\text{Gamma}\left(\frac{1}{2},3i\tan^{-1}(ax)\right) - }{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] (-96*ArcTan[a*x] - 144*a^2*x^2*ArcTan[a*x] - (27*I)*(1 + a^2*x^2)^(3/2)*Sqr
t[(-I)*ArcTan[a*x]]*Gamma[1/2, (-I)*ArcTan[a*x]] + (27*I)*(1 + a^2*x^2)^(3/
2)*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, I*ArcTan[a*x]] + I*Sqrt[3 + 3*a^2*x^2]*Sq
rt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-3*I)*ArcTan[a*x]] + I*a^2*x^2*Sqrt[3 + 3*
a^2*x^2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-3*I)*ArcTan[a*x]] - I*Sqrt[3 +
 3*a^2*x^2]*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (3*I)*ArcTan[a*x]] - I*a^2*x^2*S
qrt[3 + 3*a^2*x^2]*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (3*I)*ArcTan[a*x]])/(144*
```

```
a^4*c^2*(1 + a^2*x^2)*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]])
```

Maple [F] time = 3.116, size = 0, normalized size = 0.

$$\int x^3 \sqrt{\arctan(ax)} \left(a^2 c x^2 + c\right)^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x)
```

```
[Out] int(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atan(a*x)**(1/2)/(a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3\sqrt{\arctan(ax)}}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x^3*sqrt(arctan(a*x))/(a^2*c*x^2 + c)^(5/2), x)
```

3.752 $\int \frac{x^2\sqrt{\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=163

$$-\frac{\sqrt{\frac{\pi}{2}}\sqrt{a^2x^2+1}S\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{4a^3c^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{\frac{\pi}{6}}\sqrt{a^2x^2+1}S\left(\sqrt{\frac{6}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{12a^3c^2\sqrt{a^2cx^2+c}}+\frac{x^3\sqrt{\tan^{-1}(ax)}}{3c\left(a^2cx^2+c\right)^{3/2}}$$

```
[Out] (x^3*Sqrt[ArcTan[a*x]])/(3*c*(c + a^2*c*x^2)^(3/2)) - (Sqrt[Pi/2]*Sqrt[1 +
a^2*x^2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(4*a^3*c^2*Sqrt[c + a^2*c*
x^2]) + (Sqrt[Pi/6]*Sqrt[1 + a^2*x^2]*FresnelS[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]
])/(12*a^3*c^2*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.4249, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {4944, 4971, 4970, 3312, 3305, 3351}

$$-\frac{\sqrt{\frac{\pi}{2}}\sqrt{a^2x^2+1}S\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{4a^3c^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{\frac{\pi}{6}}\sqrt{a^2x^2+1}S\left(\sqrt{\frac{6}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{12a^3c^2\sqrt{a^2cx^2+c}}+\frac{x^3\sqrt{\tan^{-1}(ax)}}{3c\left(a^2cx^2+c\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] (x^3*Sqrt[ArcTan[a*x]])/(3*c*(c + a^2*c*x^2)^(3/2)) - (Sqrt[Pi/2]*Sqrt[1 +
a^2*x^2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(4*a^3*c^2*Sqrt[c + a^2*c*
x^2]) + (Sqrt[Pi/6]*Sqrt[1 + a^2*x^2]*FresnelS[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]
])/(12*a^3*c^2*Sqrt[c + a^2*c*x^2])
```

Rule 4944

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a +
 b*ArcTan[c*x])^p)/(d*f*(m + 1)), x] - Dist[(b*c*p)/(f*(m + 1)), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rule 4971

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[(d^(q + 1/2)*Sqrt[1 + c^2*x^2])/Sqrt[d + e*x^2],
 Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d
, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(In
tegerQ[q] || GtQ[d, 0])
```

Rule 4970

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sin[x]^m)/
Cos[x]^(m + 2*(q + 1)), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{x^2\sqrt{\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{5/2}}\,dx = \frac{x^3\sqrt{\tan^{-1}(ax)}}{3c\left(c+a^2cx^2\right)^{3/2}} - \frac{1}{6}a\int \frac{x^3}{\left(c+a^2cx^2\right)^{5/2}\sqrt{\tan^{-1}(ax)}}\,dx$$

$$= \frac{x^3\sqrt{\tan^{-1}(ax)}}{3c\left(c+a^2cx^2\right)^{3/2}} - \frac{\left(a\sqrt{1+a^2x^2}\right)\int \frac{x^3}{\left(1+a^2x^2\right)^{5/2}\sqrt{\tan^{-1}(ax)}}\,dx}{6c^2\sqrt{c+a^2cx^2}}$$

$$= \frac{x^3\sqrt{\tan^{-1}(ax)}}{3c\left(c+a^2cx^2\right)^{3/2}} - \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \frac{\sin^3(x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{6a^3c^2\sqrt{c+a^2cx^2}}$$

$$= \frac{x^3\sqrt{\tan^{-1}(ax)}}{3c\left(c+a^2cx^2\right)^{3/2}} - \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \left(\frac{3\sin(x)}{4\sqrt{x}} - \frac{\sin(3x)}{4\sqrt{x}}\right)dx, x, \tan^{-1}(ax)\right)}{6a^3c^2\sqrt{c+a^2cx^2}}$$

$$= \frac{x^3\sqrt{\tan^{-1}(ax)}}{3c\left(c+a^2cx^2\right)^{3/2}} + \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \frac{\sin(3x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{24a^3c^2\sqrt{c+a^2cx^2}} - \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \frac{\sin(x)}{\sqrt{x}}\,dx\right.}{8a^3c^2\sqrt{c+a^2cx}}$$

$$= \frac{x^3\sqrt{\tan^{-1}(ax)}}{3c\left(c+a^2cx^2\right)^{3/2}} + \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \sin\left(3x^2\right)dx, x, \sqrt{\tan^{-1}(ax)}\right)}{12a^3c^2\sqrt{c+a^2cx^2}} - \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \text{si}\right.}{4a^3c^2\sqrt{}}$$

$$= \frac{x^3\sqrt{\tan^{-1}(ax)}}{3c\left(c+a^2cx^2\right)^{3/2}} - \frac{\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}\,S\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{4a^3c^2\sqrt{c+a^2cx^2}} + \frac{\sqrt{\frac{\pi}{6}}\sqrt{1+a^2x^2}\,S\left(\sqrt{\frac{6}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{12a^3c^2\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 0.263236, size = 133, normalized size = 0.82

$$\frac{-9\sqrt{2\pi}\left(a^2x^2+1\right)^{3/2}S\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right) + \sqrt{6\pi}\left(a^2x^2+1\right)^{3/2}S\left(\sqrt{\frac{6}{\pi}}\sqrt{\tan^{-1}(ax)}\right) + 24a^3x^3\sqrt{\tan^{-1}(ax)}}{72a^3c^2\left(a^2x^2+1\right)\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] (24*a^3*x^3*Sqrt[ArcTan[a*x]] - 9*Sqrt[2*Pi]*(1 + a^2*x^2)^(3/2)*FresnelS[S
qrt[2/Pi]*Sqrt[ArcTan[a*x]]] + Sqrt[6*Pi]*(1 + a^2*x^2)^(3/2)*FresnelS[Sqrt
[6/Pi]*Sqrt[ArcTan[a*x]]])/(72*a^3*c^2*(1 + a^2*x^2)*Sqrt[c + a^2*c*x^2])
```

Maple [F] time = 2.87, size = 0, normalized size = 0.

$$\int x^{2} \sqrt{\arctan (a x)}\, \left(a^{2} c \,x^{2}+c \right)^{-\frac{5}{2}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x)
```

```
[Out] int(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(a*x)**(1/2)/(a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \sqrt{\arctan(ax)}}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x^2*sqrt(arctan(a*x))/(a^2*c*x^2 + c)^(5/2), x)
```

3.753 $\int \frac{x\sqrt{\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=163

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{a^2x^2+1}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{4a^2c^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{\frac{\pi}{6}}\sqrt{a^2x^2+1}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{12a^2c^2\sqrt{a^2cx^2+c}}-\frac{\sqrt{\tan^{-1}(ax)}}{3a^2c\left(a^2cx^2+c\right)^{3/2}}$$

```
[Out] -Sqrt[ArcTan[a*x]]/(3*a^2*c*(c + a^2*c*x^2)^(3/2)) + (Sqrt[Pi/2]*Sqrt[1 + a
^2*x^2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(4*a^2*c^2*Sqrt[c + a^2*c*x
^2]) + (Sqrt[Pi/6]*Sqrt[1 + a^2*x^2]*FresnelC[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]]
)/(12*a^2*c^2*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.250406, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4930, 4905, 4904, 3312, 3304, 3352}

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{a^2x^2+1}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{4a^2c^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{\frac{\pi}{6}}\sqrt{a^2x^2+1}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{12a^2c^2\sqrt{a^2cx^2+c}}-\frac{\sqrt{\tan^{-1}(ax)}}{3a^2c\left(a^2cx^2+c\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(5/2),x]
```


```
[Out] -Sqrt[ArcTan[a*x]]/(3*a^2*c*(c + a^2*c*x^2)^(3/2)) + (Sqrt[Pi/2]*Sqrt[1 + a
^2*x^2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(4*a^2*c^2*Sqrt[c + a^2*c*x
^2]) + (Sqrt[Pi/6]*Sqrt[1 + a^2*x^2]*FresnelC[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]]
)/(12*a^2*c^2*Sqrt[c + a^2*c*x^2])
```


Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 4905

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[(d^(q + 1/2)*Sqrt[1 + c^2*x^2])/Sqrt[d + e*x^2], Int[(1 + c
^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

Rule 4904

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{x\sqrt{\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{5/2}}\,dx = -\frac{\sqrt{\tan^{-1}(ax)}}{3a^2c\left(c+a^2cx^2\right)^{3/2}} + \frac{\int \frac{1}{\left(c+a^2cx^2\right)^{5/2}\sqrt{\tan^{-1}(ax)}}\,dx}{6a}$$

$$= -\frac{\sqrt{\tan^{-1}(ax)}}{3a^2c\left(c+a^2cx^2\right)^{3/2}} + \frac{\sqrt{1+a^2x^2}\int \frac{1}{\left(1+a^2x^2\right)^{5/2}\sqrt{\tan^{-1}(ax)}}\,dx}{6ac^2\sqrt{c+a^2cx^2}}$$

$$= -\frac{\sqrt{\tan^{-1}(ax)}}{3a^2c\left(c+a^2cx^2\right)^{3/2}} + \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \frac{\cos^3(x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{6a^2c^2\sqrt{c+a^2cx^2}}$$

$$= -\frac{\sqrt{\tan^{-1}(ax)}}{3a^2c\left(c+a^2cx^2\right)^{3/2}} + \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \left(\frac{3\cos(x)}{4\sqrt{x}}+\frac{\cos(3x)}{4\sqrt{x}}\right)dx, x, \tan^{-1}(ax)\right)}{6a^2c^2\sqrt{c+a^2cx^2}}$$

$$= -\frac{\sqrt{\tan^{-1}(ax)}}{3a^2c\left(c+a^2cx^2\right)^{3/2}} + \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \frac{\cos(3x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{24a^2c^2\sqrt{c+a^2cx^2}} + \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \frac{\cos(x)}{\sqrt{x}}\right.}{8a^2c^2\sqrt{c+a^2}}$$

$$= -\frac{\sqrt{\tan^{-1}(ax)}}{3a^2c\left(c+a^2cx^2\right)^{3/2}} + \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \cos\left(3x^2\right)dx, x, \sqrt{\tan^{-1}(ax)}\right)}{12a^2c^2\sqrt{c+a^2cx^2}} + \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int\right.}{4a^2c}$$

$$= -\frac{\sqrt{\tan^{-1}(ax)}}{3a^2c\left(c+a^2cx^2\right)^{3/2}} + \frac{\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}\,C\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{4a^2c^2\sqrt{c+a^2cx^2}} + \frac{\sqrt{\frac{\pi}{6}}\sqrt{1+a^2x^2}\,C\left(\sqrt{\frac{6}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{12a^2c^2\sqrt{c+a^2cx^2}}$$

Mathematica [C] time = 0.416646, size = 167, normalized size = 1.02

$$\frac{-48\tan^{-1}(ax) - i\left(a^2x^2+1\right)^{3/2}\left(9\sqrt{-i\tan^{-1}(ax)}\,\text{Gamma}\left(\frac{1}{2}, -i\tan^{-1}(ax)\right) - 9\sqrt{i\tan^{-1}(ax)}\,\text{Gamma}\left(\frac{1}{2}, i\tan^{-1}(ax)\right) + \right.}{144a^2c\left(a^2cx^2+c\right)^{3/2}\sqrt{\tan^{-1}(a}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] (-48*ArcTan[a*x] - I*(1 + a^2*x^2)^(3/2)*(9*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/
2, (-I)*ArcTan[a*x]] - 9*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, I*ArcTan[a*x]] + Sq
rt[3]*(Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-3*I)*ArcTan[a*x]] - Sqrt[I*ArcTa
n[a*x]]*Gamma[1/2, (3*I)*ArcTan[a*x]])))/(144*a^2*c*(c + a^2*c*x^2)^(3/2)*S
```

```
qrt[ArcTan[a*x]])
```

Maple [F] time = 0.823, size = 0, normalized size = 0.

$$\int x\sqrt{\arctan(ax)}\,\left(a^{2}c\,x^{2}+c\right)^{-\frac{5}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x)

[Out] int(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")

[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(a*x)**(1/2)/(a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sqrt{\arctan(ax)}}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x*sqrt(arctan(a*x))/(a^2*c*x^2 + c)^(5/2), x)
```

3.754 $\int \frac{\sqrt{\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=213

$$-\frac{3\sqrt{\frac{\pi}{2}}\sqrt{a^2x^2+1}S\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{4ac^2\sqrt{a^2cx^2+c}}-\frac{\sqrt{\frac{\pi}{6}}\sqrt{a^2x^2+1}S\left(\sqrt{\frac{6}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{12ac^2\sqrt{a^2cx^2+c}}+\frac{3x\sqrt{\tan^{-1}(ax)}}{4c^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{a^2x^2+1}\sqrt{\tan^{-1}(ax)}\,\mathrm{s}}{12ac^2\sqrt{a^2cx}}$$

```
[Out] (3*x*Sqrt[ArcTan[a*x]])/(4*c^2*Sqrt[c + a^2*c*x^2]) - (3*Sqrt[Pi/2]*Sqrt[1
+ a^2*x^2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(4*a*c^2*Sqrt[c + a^2*c*
x^2]) - (Sqrt[Pi/6]*Sqrt[1 + a^2*x^2]*FresnelS[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]
])/(12*a*c^2*Sqrt[c + a^2*c*x^2]) + (Sqrt[1 + a^2*x^2]*Sqrt[ArcTan[a*x]]*Si
n[3*ArcTan[a*x]])/(12*a*c^2*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.187895, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4905, 4904, 3312, 3296, 3305, 3351}

$$-\frac{3\sqrt{\frac{\pi}{2}}\sqrt{a^2x^2+1}S\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{4ac^2\sqrt{a^2cx^2+c}}-\frac{\sqrt{\frac{\pi}{6}}\sqrt{a^2x^2+1}S\left(\sqrt{\frac{6}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{12ac^2\sqrt{a^2cx^2+c}}+\frac{3x\sqrt{\tan^{-1}(ax)}}{4c^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{a^2x^2+1}\sqrt{\tan^{-1}(ax)}\,\mathrm{s}}{12ac^2\sqrt{a^2cx}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[ArcTan[a*x]]/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] (3*x*Sqrt[ArcTan[a*x]])/(4*c^2*Sqrt[c + a^2*c*x^2]) - (3*Sqrt[Pi/2]*Sqrt[1
+ a^2*x^2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(4*a*c^2*Sqrt[c + a^2*c*
x^2]) - (Sqrt[Pi/6]*Sqrt[1 + a^2*x^2]*FresnelS[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]
])/(12*a*c^2*Sqrt[c + a^2*c*x^2]) + (Sqrt[1 + a^2*x^2]*Sqrt[ArcTan[a*x]]*Si
n[3*ArcTan[a*x]])/(12*a*c^2*Sqrt[c + a^2*c*x^2])
```

Rule 4905

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[(d^(q + 1/2)*Sqrt[1 + c^2*x^2])/Sqrt[d + e*x^2], Int[(1 + c
^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] && !(IntegerQ[q] || GtQ[d, 0])
```

Rule 4904

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{\sqrt{\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{5/2}}\,dx = \frac{\sqrt{1+a^2x^2}\int \frac{\sqrt{\tan^{-1}(ax)}}{\left(1+a^2x^2\right)^{5/2}}\,dx}{c^2\sqrt{c+a^2cx^2}}$$
$$= \frac{\sqrt{1+a^2x^2}\,\mathrm{Subst}\left(\int \sqrt{x}\cos^3(x)\,dx, x, \tan^{-1}(ax)\right)}{ac^2\sqrt{c+a^2cx^2}}$$
$$= \frac{\sqrt{1+a^2x^2}\,\mathrm{Subst}\left(\int \left(\frac{3}{4}\sqrt{x}\cos(x)+\frac{1}{4}\sqrt{x}\cos(3x)\right)dx, x, \tan^{-1}(ax)\right)}{ac^2\sqrt{c+a^2cx^2}}$$
$$= \frac{\sqrt{1+a^2x^2}\,\mathrm{Subst}\left(\int \sqrt{x}\cos(3x)\,dx, x, \tan^{-1}(ax)\right)}{4ac^2\sqrt{c+a^2cx^2}} + \frac{\left(3\sqrt{1+a^2x^2}\right)\mathrm{Subst}\left(\int \sqrt{x}\cos(x)\,dx, x, \tan\right.}{4ac^2\sqrt{c+a^2cx^2}}$$
$$= \frac{3x\sqrt{\tan^{-1}(ax)}}{4c^2\sqrt{c+a^2cx^2}} + \frac{\sqrt{1+a^2x^2}\sqrt{\tan^{-1}(ax)}\sin\left(3\tan^{-1}(ax)\right)}{12ac^2\sqrt{c+a^2cx^2}} - \frac{\sqrt{1+a^2x^2}\,\mathrm{Subst}\left(\int \frac{\sin(3x)}{\sqrt{x}}\,dx, x, \mathrm{ta}\right.}{24ac^2\sqrt{c+a^2cx^2}}$$
$$= \frac{3x\sqrt{\tan^{-1}(ax)}}{4c^2\sqrt{c+a^2cx^2}} + \frac{\sqrt{1+a^2x^2}\sqrt{\tan^{-1}(ax)}\sin\left(3\tan^{-1}(ax)\right)}{12ac^2\sqrt{c+a^2cx^2}} - \frac{\sqrt{1+a^2x^2}\,\mathrm{Subst}\left(\int \sin\left(3x^2\right)dx,\right.}{12ac^2\sqrt{c+a^2cx^2}}$$
$$= \frac{3x\sqrt{\tan^{-1}(ax)}}{4c^2\sqrt{c+a^2cx^2}} - \frac{3\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}S\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{4ac^2\sqrt{c+a^2cx^2}} - \frac{\sqrt{\frac{\pi}{6}}\sqrt{1+a^2x^2}S\left(\sqrt{\frac{6}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{12ac^2\sqrt{c+a^2cx^2}} + \frac{\sqrt{}}{}$$

Mathematica [A] time = 0.166428, size = 137, normalized size = 0.64

$$\frac{-27\sqrt{2\pi}\left(a^2x^2+1\right)^{3/2}S\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)-\sqrt{6\pi}\left(a^2x^2+1\right)^{3/2}S\left(\sqrt{\frac{6}{\pi}}\sqrt{\tan^{-1}(ax)}\right)+24ax\left(2a^2x^2+3\right)\sqrt{\tan^{-1}(ax)}}{72c^2\left(a^3x^2+a\right)\sqrt{a^2cx^2+c}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[ArcTan[a*x]]/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] (24*a*x*(3 + 2*a^2*x^2)*Sqrt[ArcTan[a*x]] - 27*Sqrt[2*Pi]*(1 + a^2*x^2)^(3/
2)*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]] - Sqrt[6*Pi]*(1 + a^2*x^2)^(3/2)*
FresnelS[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]])/(72*c^2*(a + a^3*x^2)*Sqrt[c + a^2*
c*x^2])
```

Maple [F] time = 0.709, size = 0, normalized size = 0.

$$\int \sqrt{\arctan (ax)}\left(a^2cx^2+c\right)^{-\frac{5}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x)

[Out] int(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")

[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(1/2)/(a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\arctan(ax)}}{\left(a^2cx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(arctan(a*x))/(a^2*c*x^2 + c)^(5/2), x)
```

3.755 $\int \frac{\sqrt{\tan^{-1}(ax)}}{x(c+a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(\frac{\sqrt{\tan^{-1}(ax)}}{x\left(a^2cx^2+c\right)^{5/2}},x\right)$$

```
[Out] Unintegrable[Sqrt[ArcTan[a*x]]/(x*(c + a^2*c*x^2)^(5/2)), x]
```

Rubi [A] time = 0.1215, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sqrt{\tan^{-1}(ax)}}{x\left(c+a^2cx^2\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[ArcTan[a*x]]/(x*(c + a^2*c*x^2)^(5/2)),x]

[Out] Defer[Int][Sqrt[ArcTan[a*x]]/(x*(c + a^2*c*x^2)^(5/2)), x]
```

Rubi steps

$$\int \frac{\sqrt{\tan^{-1}(ax)}}{x\left(c+a^2cx^2\right)^{5/2}}\,dx=\int \frac{\sqrt{\tan^{-1}(ax)}}{x\left(c+a^2cx^2\right)^{5/2}}\,dx$$

Mathematica [A] time = 2.23298, size = 0, normalized size = 0.

$$\int \frac{\sqrt{\tan^{-1}(ax)}}{x\left(c+a^2cx^2\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[ArcTan[a*x]]/(x*(c + a^2*c*x^2)^(5/2)),x]
```

```
[Out] Integrate[Sqrt[ArcTan[a*x]]/(x*(c + a^2*c*x^2)^(5/2)), x]
```

Maple [A] time = 0.721, size = 0, normalized size = 0.

$$\int \frac{1}{x}\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{-\frac{5}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^(5/2),x)
```

```
[Out] int(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(1/2)/x/(a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\arctan(ax)}}{\left(a^2cx^2+c\right)^{\frac{5}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(arctan(a*x))/((a^2*c*x^2 + c)^(5/2)*x), x)
```

3.756 $\int x^m \left(c+a^2 c x^2\right) \tan^{-1}(a x)^{3/2} \, dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(x^m \left(a^2 c x^2+c\right) \tan^{-1}(a x)^{3/2},x\right)$$

```
[Out] Unintegrable[x^m*(c + a^2*c*x^2)*ArcTan[a*x]^(3/2), x]
```

Rubi [A] time = 0.0351817, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^m \left(c+a^2 c x^2\right) \tan^{-1}(a x)^{3/2} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*(c + a^2*c*x^2)*ArcTan[a*x]^(3/2),x]
```

```
[Out] Defer[Int][x^m*(c + a^2*c*x^2)*ArcTan[a*x]^(3/2), x]
```

Rubi steps

$$\int x^m \left(c+a^2 c x^2\right) \tan^{-1}(a x)^{3/2} \, dx=\int x^m \left(c+a^2 c x^2\right) \tan^{-1}(a x)^{3/2} \, dx$$

Mathematica [A] time = 1.82312, size = 0, normalized size = 0.

$$\int x^m \left(c+a^2 c x^2\right) \tan^{-1}(a x)^{3/2} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*(c + a^2*c*x^2)*ArcTan[a*x]^(3/2),x]
```

```
[Out] Integrate[x^m*(c + a^2*c*x^2)*ArcTan[a*x]^(3/2), x]
```

Maple [A] time = 0.819, size = 0, normalized size = 0.

$$\int x^{m} \left(a^{2} c \,x^{2}+c \right) \arctan \left(a x \right)^{\frac{3}{2}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)*arctan(a*x)^(3/2),x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)*arctan(a*x)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)*arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^{2} c x^{2} + c\right) x^{m} \arctan\left(a x\right)^{\frac{3}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)*arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)*x^m*arctan(a*x)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)*atan(a*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2 + c\right)x^m \arctan\left(ax\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)*arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)*x^m*arctan(a*x)^(3/2), x)
```

3.757 $\int x^2 \left(c+a^2 c x^2\right) \tan^{-1}(a x)^{3/2} \, dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(x^2 \left(a^2 c x^2+c\right) \tan^{-1}(a x)^{3/2},x\right)$$

```
[Out] Unintegrable[x^2*(c + a^2*c*x^2)*ArcTan[a*x]^(3/2), x]
```

Rubi [A] time = 0.0361001, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^2 \left(c+a^2 c x^2\right) \tan^{-1}(a x)^{3/2} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[x^2*(c + a^2*c*x^2)*ArcTan[a*x]^(3/2),x]
```

```
[Out] Defer[Int][x^2*(c + a^2*c*x^2)*ArcTan[a*x]^(3/2), x]
```

Rubi steps

$$\int x^2 \left(c+a^2 c x^2\right) \tan^{-1}(a x)^{3/2} \, dx=\int x^2 \left(c+a^2 c x^2\right) \tan^{-1}(a x)^{3/2} \, dx$$

Mathematica [A] time = 4.04176, size = 0, normalized size = 0.

$$\int x^2 \left(c+a^2 c x^2\right) \tan^{-1}(a x)^{3/2} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^2*(c + a^2*c*x^2)*ArcTan[a*x]^(3/2),x]
```

```
[Out] Integrate[x^2*(c + a^2*c*x^2)*ArcTan[a*x]^(3/2), x]
```

Maple [A] time = 0.64, size = 0, normalized size = 0.

$$\int x^{2} \left(a^{2} c \,x^{2}+c \right) \arctan \left(a x \right)^{\frac{3}{2}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a^2*c*x^2+c)*arctan(a*x)^(3/2),x)
```

```
[Out] int(x^2*(a^2*c*x^2+c)*arctan(a*x)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)*arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)*arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a**2*c*x**2+c)*atan(a*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2 + c\right)x^2 \arctan\left(ax\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)*arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)*x^2*arctan(a*x)^(3/2), x)
```

3.758 $\int x\left(c+a^2cx^2\right)\tan^{-1}(ax)^{3/2}\,dx$

Optimal. Leaf size=56

$$\frac{c\left(a^2x^2+1\right)^2\tan^{-1}(ax)^{3/2}}{4a^2}-\frac{3\text{Unintegrable}\left(\left(a^2cx^2+c\right)\sqrt{\tan^{-1}(ax)},x\right)}{8a}$$

```
[Out] (c*(1 + a^2*x^2)^2*ArcTan[a*x]^(3/2))/(4*a^2) - (3*Unintegrable[(c + a^2*c*
x^2)*Sqrt[ArcTan[a*x]], x])/(8*a)
```

Rubi [A] time = 0.0382859, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x\left(c+a^2cx^2\right)\tan^{-1}(ax)^{3/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x*(c + a^2*c*x^2)*ArcTan[a*x]^(3/2),x]
```

```
[Out] (c*(1 + a^2*x^2)^2*ArcTan[a*x]^(3/2))/(4*a^2) - (3*Defer[Int][(c + a^2*c*x^
2)*Sqrt[ArcTan[a*x]], x])/(8*a)
```

Rubi steps

$$\int x\left(c+a^2cx^2\right)\tan^{-1}(ax)^{3/2}\,dx=\frac{c\left(1+a^2x^2\right)^2\tan^{-1}(ax)^{3/2}}{4a^2}-\frac{3\int\left(c+a^2cx^2\right)\sqrt{\tan^{-1}(ax)}\,dx}{8a}$$

Mathematica [A] time = 1.26044, size = 0, normalized size = 0.

$$\int x\left(c+a^2cx^2\right)\tan^{-1}(ax)^{3/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x*(c + a^2*c*x^2)*ArcTan[a*x]^(3/2),x]
```

```
[Out] Integrate[x*(c + a^2*c*x^2)*ArcTan[a*x]^(3/2), x]
```

Maple [A] time = 0.39, size = 0, normalized size = 0.

$$\int x\left(a^{2}c x^{2}+c\right)\left(\arctan\left(a x\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)*arctan(a*x)^(3/2),x)
```

```
[Out] int(x*(a^2*c*x^2+c)*arctan(a*x)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)*arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)*arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$c\left(\int x\operatorname{atan}^{\frac{3}{2}}\left(a x\right)dx+\int a^{2}x^{3}\operatorname{atan}^{\frac{3}{2}}\left(a x\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)*atan(a*x)**(3/2),x)
```

```
[Out] c*(Integral(x*atan(a*x)**(3/2), x) + Integral(a**2*x**3*atan(a*x)**(3/2), x
))
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2+c\right)x\arctan\left(ax\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)*arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)*x*arctan(a*x)^(3/2), x)
```

3.759 $\int \left(c+a^2cx^2\right)\tan^{-1}(ax)^{3/2}\,dx$

Optimal. Leaf size=79

$$\frac{1}{8}c\text{Unintegrable}\left(\frac{1}{\sqrt{\tan^{-1}(ax)}},x\right)+\frac{2}{3}c\text{Unintegrable}\left(\tan^{-1}(ax)^{3/2},x\right)+\frac{1}{3}cx\left(a^2x^2+1\right)\tan^{-1}(ax)^{3/2}-\frac{c\left(a^2x^2+1\right)\sqrt{\tan^{-1}(ax)}}{4a}$$

```
[Out] -(c*(1 + a^2*x^2)*Sqrt[ArcTan[a*x]])/(4*a) + (c*x*(1 + a^2*x^2)*ArcTan[a*x]
^(3/2))/3 + (c*Unintegrable[1/Sqrt[ArcTan[a*x]], x])/8 + (2*c*Unintegrable[
ArcTan[a*x]^(3/2), x])/3
```

Rubi [A] time = 0.0235658, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \left(c+a^2cx^2\right)\tan^{-1}(ax)^{3/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)*ArcTan[a*x]^(3/2),x]
```

```
[Out] -(c*(1 + a^2*x^2)*Sqrt[ArcTan[a*x]])/(4*a) + (c*x*(1 + a^2*x^2)*ArcTan[a*x]
^(3/2))/3 + (c*Defer[Int][1/Sqrt[ArcTan[a*x]], x])/8 + (2*c*Defer[Int][ArcT
an[a*x]^(3/2), x])/3
```

Rubi steps

$$\int \left(c+a^2cx^2\right)\tan^{-1}(ax)^{3/2}\,dx=-\frac{c\left(1+a^2x^2\right)\sqrt{\tan^{-1}(ax)}}{4a}+\frac{1}{3}cx\left(1+a^2x^2\right)\tan^{-1}(ax)^{3/2}+\frac{1}{8}c\int \frac{1}{\sqrt{\tan^{-1}(ax)}}\,dx+\frac{1}{3}(2$$

Mathematica [A] time = 3.92188, size = 0, normalized size = 0.

$$\int \left(c+a^2cx^2\right)\tan^{-1}(ax)^{3/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)*ArcTan[a*x]^(3/2),x]
```

```
[Out] Integrate[(c + a^2*c*x^2)*ArcTan[a*x]^(3/2), x]
```

Maple [A] time = 0.317, size = 0, normalized size = 0.

$$\int \left(a^2cx^2 + c\right)(\arctan(ax))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)*arctan(a*x)^(3/2),x)
```

```
[Out] int((a^2*c*x^2+c)*arctan(a*x)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$c\left(\int a^2x^2\operatorname{atan}^{\frac{3}{2}}(ax)\,dx+\int\operatorname{atan}^{\frac{3}{2}}(ax)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)*atan(a*x)**(3/2),x)
```

```
[Out] c*(Integral(a**2*x**2*atan(a*x)**(3/2), x) + Integral(atan(a*x)**(3/2), x))
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\left(a^2cx^2+c\right)\arctan(ax)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)*arctan(a*x)^(3/2), x)
```

3.760 $\int \frac{(c+a^2cx^2)\tan^{-1}(ax)^{3/2}}{x}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{(a^2cx^2+c)\tan^{-1}(ax)^{3/2}}{x},x\right)$$

```
[Out] Unintegrable[((c + a^2*c*x^2)*ArcTan[a*x]^(3/2))/x, x]
```

Rubi [A] time = 0.0320078, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{(c+a^2cx^2)\tan^{-1}(ax)^{3/2}}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((c + a^2*c*x^2)*ArcTan[a*x]^(3/2))/x,x]

[Out] Defer[Int][((c + a^2*c*x^2)*ArcTan[a*x]^(3/2))/x, x]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)\tan^{-1}(ax)^{3/2}}{x}\,dx = \int \frac{(c+a^2cx^2)\tan^{-1}(ax)^{3/2}}{x}\,dx$$

Mathematica [A] time = 1.71671, size = 0, normalized size = 0.

$$\int \frac{(c+a^2cx^2)\tan^{-1}(ax)^{3/2}}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((c + a^2*c*x^2)*ArcTan[a*x]^(3/2))/x,x]

[Out] Integrate[((c + a^2*c*x^2)*ArcTan[a*x]^(3/2))/x, x]
```

Maple [A] time = 0.413, size = 0, normalized size = 0.

$$\int \frac{a^2cx^2 + c}{x} (\arctan (ax))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)*arctan(a*x)^(3/2)/x,x)
```

```
[Out] int((a^2*c*x^2+c)*arctan(a*x)^(3/2)/x,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$c\left(\int \frac{\operatorname{atan}^{\frac{3}{2}}(ax)}{x}\, dx + \int a^2 x \operatorname{atan}^{\frac{3}{2}}(ax)\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)*atan(a*x)**(3/2)/x,x)
```

```
[Out] c*(Integral(atan(a*x)**(3/2)/x, x) + Integral(a**2*x*atan(a*x)**(3/2), x))
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)\arctan\left(ax\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^(3/2)/x,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)*arctan(a*x)^(3/2)/x, x)
```

3.761 $\int \frac{(c+a^2cx^2)\tan^{-1}(ax)^{3/2}}{x^2}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{(a^2cx^2+c)\tan^{-1}(ax)^{3/2}}{x^2},x\right)$$

```
[Out] Unintegrable[((c + a^2*c*x^2)*ArcTan[a*x]^(3/2))/x^2, x]
```

Rubi [A] time = 0.0350468, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{(c+a^2cx^2)\tan^{-1}(ax)^{3/2}}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((c + a^2*c*x^2)*ArcTan[a*x]^(3/2))/x^2,x]
```

```
[Out] Defer[Int][((c + a^2*c*x^2)*ArcTan[a*x]^(3/2))/x^2, x]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)\tan^{-1}(ax)^{3/2}}{x^2}\,dx = \int \frac{(c+a^2cx^2)\tan^{-1}(ax)^{3/2}}{x^2}\,dx$$

Mathematica [A] time = 1.5803, size = 0, normalized size = 0.

$$\int \frac{(c+a^2cx^2)\tan^{-1}(ax)^{3/2}}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((c + a^2*c*x^2)*ArcTan[a*x]^(3/2))/x^2,x]
```

```
[Out] Integrate[((c + a^2*c*x^2)*ArcTan[a*x]^(3/2))/x^2, x]
```

Maple [A] time = 0.234, size = 0, normalized size = 0.

$$\int \frac{a^2cx^2 + c}{x^2} (\arctan (ax))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)*arctan(a*x)^(3/2)/x^2,x)
```

```
[Out] int((a^2*c*x^2+c)*arctan(a*x)^(3/2)/x^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$c \left(\int a^2 \operatorname{atan}^{\frac{3}{2}} (ax)\, dx + \int \frac{\operatorname{atan}^{\frac{3}{2}} (ax)}{x^2}\, dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)*atan(a*x)**(3/2)/x**2,x)
```

```
[Out] c*(Integral(a**2*atan(a*x)**(3/2), x) + Integral(atan(a*x)**(3/2)/x**2, x))
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2 + c\right)\arctan\left(ax\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)*arctan(a*x)^(3/2)/x^2, x)
```

3.762 $\int x^m \left(c+a^2 c x^2\right)^2 \tan^{-1}(a x)^{3/2} \, dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(x^m \left(a^2 c x^2+c\right)^2 \tan^{-1}(a x)^{3/2},x\right)$$

```
[Out] Unintegrable[x^m*(c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2), x]
```

Rubi [A] time = 0.0541324, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^m \left(c+a^2 c x^2\right)^2 \tan^{-1}(a x)^{3/2} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*(c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2),x]
```

```
[Out] Defer[Int][x^m*(c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2), x]
```

Rubi steps

$$\int x^m \left(c+a^2 c x^2\right)^2 \tan^{-1}(a x)^{3/2} \, dx=\int x^m \left(c+a^2 c x^2\right)^2 \tan^{-1}(a x)^{3/2} \, dx$$

Mathematica [A] time = 1.25354, size = 0, normalized size = 0.

$$\int x^m \left(c+a^2 c x^2\right)^2 \tan^{-1}(a x)^{3/2} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*(c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2),x]
```

```
[Out] Integrate[x^m*(c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2), x]
```

Maple [A] time = 0.944, size = 0, normalized size = 0.

$$\int x^{m} \left(a^{2} c \,x^{2}+c \right)^{2} \left(\arctan \left(a x \right)\right)^{\frac{3}{2}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^(3/2),x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^{4} c^{2} x^{4}+2 \, a^{2} c^{2} x^{2}+c^{2}\right) x^{m} \arctan \left(a x\right)^{\frac{3}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*x^m*arctan(a*x)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**2*atan(a*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2 + c\right)^2 x^m \arctan\left(ax\right)^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^2*x^m*arctan(a*x)^(3/2), x)
```

3.763 $\int x^2 \left(c+a^2 c x^2\right)^2 \tan^{-1}(ax)^{3/2} \, dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(x^2 \left(a^2 c x^2+c\right)^2 \tan^{-1}(ax)^{3/2},x\right)$$

```
[Out] Unintegrable[x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2), x]
```

Rubi [A] time = 0.0560478, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^2 \left(c+a^2 c x^2\right)^2 \tan^{-1}(ax)^{3/2} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2),x]

[Out] Defer[Int][x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2), x]
```

Rubi steps

$$\int x^2 \left(c+a^2 c x^2\right)^2 \tan^{-1}(ax)^{3/2} \, dx=\int x^2 \left(c+a^2 c x^2\right)^2 \tan^{-1}(ax)^{3/2} \, dx$$

Mathematica [A] time = 3.05284, size = 0, normalized size = 0.

$$\int x^2 \left(c+a^2 c x^2\right)^2 \tan^{-1}(ax)^{3/2} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2),x]

[Out] Integrate[x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2), x]
```

Maple [A] time = 0.934, size = 0, normalized size = 0.

$$\int x^2 \left(a^2 c x^2 + c\right)^2 (\arctan (ax))^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a^2*c*x^2+c)^2*arctan(a*x)^(3/2),x)
```

```
[Out] int(x^2*(a^2*c*x^2+c)^2*arctan(a*x)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^2*arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^2*arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a**2*c*x**2+c)**2*atan(a*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2+c\right)^2 x^2 \arctan\left(ax\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^2*arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^2*x^2*arctan(a*x)^(3/2), x)
```

3.764 $\int x\left(c+a^2cx^2\right)^2 \tan^{-1}(ax)^{3/2}\,dx$

Optimal. Leaf size=60

$$\frac{c^2\left(a^2x^2+1\right)^3\tan^{-1}(ax)^{3/2}}{6a^2}-\frac{\text{Unintegrable}\left(\left(a^2cx^2+c\right)^2\sqrt{\tan^{-1}(ax)},x\right)}{4a}$$

```
[Out] (c^2*(1 + a^2*x^2)^3*ArcTan[a*x]^(3/2))/(6*a^2) - Unintegrable[(c + a^2*c*x
^2)^2*Sqrt[ArcTan[a*x]], x]/(4*a)
```

Rubi [A] time = 0.0617982, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^{3/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x*(c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2),x]
```

```
[Out] (c^2*(1 + a^2*x^2)^3*ArcTan[a*x]^(3/2))/(6*a^2) - Defer[Int][(c + a^2*c*x^2
)^2*Sqrt[ArcTan[a*x]], x]/(4*a)
```

Rubi steps

$$\int x\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^{3/2}\,dx=\frac{c^2\left(1+a^2x^2\right)^3\tan^{-1}(ax)^{3/2}}{6a^2}-\frac{\int\left(c+a^2cx^2\right)^2\sqrt{\tan^{-1}(ax)}\,dx}{4a}$$

Mathematica [A] time = 1.27597, size = 0, normalized size = 0.

$$\int x\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^{3/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x*(c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2),x]
```

```
[Out] Integrate[x*(c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2), x]
```

Maple [A] time = 0.543, size = 0, normalized size = 0.

$$\int x\left(a^{2}c\,x^{2}+c\right)^{2}\left(\arctan\left(ax\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^2*arctan(a*x)^(3/2),x)
```

```
[Out] int(x*(a^2*c*x^2+c)^2*arctan(a*x)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^2*arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^2*arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**2*atan(a*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2+c\right)^2 x \arctan\left(ax\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^2*arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^2*x*arctan(a*x)^(3/2), x)
```

3.765 $\int \left(c+a^2cx^2\right)^2 \tan^{-1}(ax)^{3/2}\,dx$

Optimal. Leaf size=169

$$\frac{3}{80}c\text{Unintegrable}\left(\frac{a^2cx^2+c}{\sqrt{\tan^{-1}(ax)}},x\right)+\frac{1}{10}c^2\text{Unintegrable}\left(\frac{1}{\sqrt{\tan^{-1}(ax)}},x\right)+\frac{8}{15}c^2\text{Unintegrable}\left(\tan^{-1}(ax)^{3/2},x\right)+\frac{1}{5}c$$

```
[Out] -(c^2*(1 + a^2*x^2)*Sqrt[ArcTan[a*x]])/(5*a) - (3*c^2*(1 + a^2*x^2)^2*Sqrt[
ArcTan[a*x]])/(40*a) + (4*c^2*x*(1 + a^2*x^2)*ArcTan[a*x]^(3/2))/15 + (c^2*
x*(1 + a^2*x^2)^2*ArcTan[a*x]^(3/2))/5 + (c^2*Unintegrable[1/Sqrt[ArcTan[a*
x]], x])/10 + (3*c*Unintegrable[(c + a^2*c*x^2)/Sqrt[ArcTan[a*x]], x])/80 +
 (8*c^2*Unintegrable[ArcTan[a*x]^(3/2), x])/15
```

Rubi [A] time = 0.0663589, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \left(c+a^2cx^2\right)^2 \tan^{-1}(ax)^{3/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2),x]
```

```
[Out] -(c^2*(1 + a^2*x^2)*Sqrt[ArcTan[a*x]])/(5*a) - (3*c^2*(1 + a^2*x^2)^2*Sqrt[
ArcTan[a*x]])/(40*a) + (4*c^2*x*(1 + a^2*x^2)*ArcTan[a*x]^(3/2))/15 + (c^2*
x*(1 + a^2*x^2)^2*ArcTan[a*x]^(3/2))/5 + (c^2*Defer[Int][1/Sqrt[ArcTan[a*x]
], x])/10 + (3*c*Defer[Int][(c + a^2*c*x^2)/Sqrt[ArcTan[a*x]], x])/80 + (8*
c^2*Defer[Int][ArcTan[a*x]^(3/2), x])/15
```

Rubi steps

$$\int \left(c+a^2cx^2\right)^2 \tan^{-1}(ax)^{3/2}\,dx=-\frac{3c^2\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}}{40a}+\frac{1}{5}c^2x\left(1+a^2x^2\right)^2\tan^{-1}(ax)^{3/2}+\frac{1}{80}(3c)\int\frac{c+a^2cx^2}{\sqrt{\tan^{-1}(ax)}}$$

$$=-\frac{c^2\left(1+a^2x^2\right)\sqrt{\tan^{-1}(ax)}}{5a}-\frac{3c^2\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}}{40a}+\frac{4}{15}c^2x\left(1+a^2x^2\right)\tan^{-1}(ax$$

Mathematica [A] time = 2.12461, size = 0, normalized size = 0.

$$\int \left(c + a^2cx^2\right)^2 \tan^{-1}(ax)^{3/2}\, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2),x]
```

```
[Out] Integrate[(c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2), x]
```

Maple [A] time = 0.431, size = 0, normalized size = 0.

$$\int \left(a^2cx^2 + c\right)^2 (\arctan(ax))^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^2*arctan(a*x)^(3/2),x)
```

```
[Out] int((a^2*c*x^2+c)^2*arctan(a*x)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**2*atan(a*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2+c\right)^2 \arctan\left(ax\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^2*arctan(a*x)^(3/2), x)
```

3.766 $\int \frac{\left(c+a^2cx^2\right)^2 \tan^{-1}(ax)^{3/2}}{x} \, dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{\left(a^2cx^2+c\right)^2 \tan^{-1}(ax)^{3/2}}{x},x\right)$$

```
[Out] Unintegrable[((c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2))/x, x]
```

Rubi [A] time = 0.0503347, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(c+a^2cx^2\right)^2 \tan^{-1}(ax)^{3/2}}{x} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[((c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2))/x,x]
```

```
[Out] Defer[Int][((c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2))/x, x]
```

Rubi steps

$$\int \frac{\left(c+a^2cx^2\right)^2 \tan^{-1}(ax)^{3/2}}{x} \, dx=\int \frac{\left(c+a^2cx^2\right)^2 \tan^{-1}(ax)^{3/2}}{x} \, dx$$

Mathematica [A] time = 1.69833, size = 0, normalized size = 0.

$$\int \frac{\left(c+a^2cx^2\right)^2 \tan^{-1}(ax)^{3/2}}{x} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2))/x,x]
```

```
[Out] Integrate[((c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2))/x, x]
```

Maple [A] time = 0.517, size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2 + c\right)^2}{x} \left(\arctan\left(ax\right)\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^2*arctan(a*x)^(3/2)/x,x)
```

```
[Out] int((a^2*c*x^2+c)^2*arctan(a*x)^(3/2)/x,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$c^{2}\left(\int \frac{\operatorname{atan}^{\frac{3}{2}}(ax)}{x}\,dx+\int 2a^{2}x\operatorname{atan}^{\frac{3}{2}}(ax)\,dx+\int a^{4}x^{3}\operatorname{atan}^{\frac{3}{2}}(ax)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**2*atan(a*x)**(3/2)/x,x)
```

```
[Out] c**2*(Integral(atan(a*x)**(3/2)/x, x) + Integral(2*a**2*x*atan(a*x)**(3/2),
 x) + Integral(a**4*x**3*atan(a*x)**(3/2), x))
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^{2}cx^{2}+c\right)^{2}\arctan\left(ax\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^(3/2)/x,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^2*arctan(a*x)^(3/2)/x, x)
```

3.767 $\int \frac{\left(c+a^2cx^2\right)^2 \tan^{-1}(ax)^{3/2}}{x^2} \, dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{\left(a^2cx^2+c\right)^2 \tan^{-1}(ax)^{3/2}}{x^2},x\right)$$

```
[Out] Unintegrable[((c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2))/x^2, x]
```

Rubi [A] time = 0.0560523, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(c+a^2cx^2\right)^2 \tan^{-1}(ax)^{3/2}}{x^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[((c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2))/x^2,x]
```

```
[Out] Defer[Int][((c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2))/x^2, x]
```

Rubi steps

$$\int \frac{\left(c+a^2cx^2\right)^2 \tan^{-1}(ax)^{3/2}}{x^2} \, dx=\int \frac{\left(c+a^2cx^2\right)^2 \tan^{-1}(ax)^{3/2}}{x^2} \, dx$$

Mathematica [A] time = 2.07765, size = 0, normalized size = 0.

$$\int \frac{\left(c+a^2cx^2\right)^2 \tan^{-1}(ax)^{3/2}}{x^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2))/x^2,x]
```

```
[Out] Integrate[((c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2))/x^2, x]
```

Maple [A] time = 0.444, size = 0, normalized size = 0.

$$\int \frac{(a^2cx^2+c)^2}{x^2} (\arctan(ax))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^2*arctan(a*x)^(3/2)/x^2,x)
```

```
[Out] int((a^2*c*x^2+c)^2*arctan(a*x)^(3/2)/x^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$c^{2}\left(\int 2a^{2}\operatorname{atan}^{\frac{3}{2}}(ax)\,dx+\int \frac{\operatorname{atan}^{\frac{3}{2}}(ax)}{x^{2}}\,dx+\int a^{4}x^{2}\operatorname{atan}^{\frac{3}{2}}(ax)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**2*atan(a*x)**(3/2)/x**2,x)
```

```
[Out] c**2*(Integral(2*a**2*atan(a*x)**(3/2), x) + Integral(atan(a*x)**(3/2)/x**2
, x) + Integral(a**4*x**2*atan(a*x)**(3/2), x))
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^{2}cx^{2}+c\right)^{2}\arctan(ax)^{\frac{3}{2}}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^2*arctan(a*x)^(3/2)/x^2, x)
```

3.768 $\int x^m \left(c+a^2cx^2\right)^3 \tan^{-1}(ax)^{3/2}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(x^m \left(a^2cx^2+c\right)^3 \tan^{-1}(ax)^{3/2},x\right)$$

```
[Out] Unintegrable[x^m*(c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2), x]
```

Rubi [A] time = 0.0573169, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^m \left(c+a^2cx^2\right)^3 \tan^{-1}(ax)^{3/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*(c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2),x]
```

```
[Out] Defer[Int][x^m*(c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2), x]
```

Rubi steps

$$\int x^m \left(c+a^2cx^2\right)^3 \tan^{-1}(ax)^{3/2}\,dx = \int x^m \left(c+a^2cx^2\right)^3 \tan^{-1}(ax)^{3/2}\,dx$$

Mathematica [A] time = 0.845527, size = 0, normalized size = 0.

$$\int x^m \left(c+a^2cx^2\right)^3 \tan^{-1}(ax)^{3/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*(c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2),x]
```

```
[Out] Integrate[x^m*(c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2), x]
```

Maple [A] time = 1.001, size = 0, normalized size = 0.

$$\int x^{m} \left(a^{2} c \,x^{2}+c \right)^{3} \left(\arctan \left(a x \right)\right)^{\frac{3}{2}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)^3*arctan(a*x)^(3/2),x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)^3*arctan(a*x)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^3*arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a^{6} c^{3} x^{6} + 3\, a^{4} c^{3} x^{4} + 3\, a^{2} c^{3} x^{2} + c^{3}\right) x^{m} \arctan\left(a x\right)^{\frac{3}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^3*arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*x^m*arctan(a*x
)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**3*atan(a*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2 + c\right)^3 x^m \arctan\left(ax\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^3*arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^3*x^m*arctan(a*x)^(3/2), x)
```

3.769 $\int x^2 \left(c+a^2 c x^2\right)^3 \tan^{-1}(a x)^{3/2} \, dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(x^2 \left(a^2 c x^2+c\right)^3 \tan^{-1}(a x)^{3/2},x\right)$$

```
[Out] Unintegrable[x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2), x]
```

Rubi [A] time = 0.0566498, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^2 \left(c+a^2 c x^2\right)^3 \tan^{-1}(a x)^{3/2} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2),x]
```

```
[Out] Defer[Int][x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2), x]
```

Rubi steps

$$\int x^2 \left(c+a^2 c x^2\right)^3 \tan^{-1}(a x)^{3/2} \, dx=\int x^2 \left(c+a^2 c x^2\right)^3 \tan^{-1}(a x)^{3/2} \, dx$$

Mathematica [A] time = 2.9096, size = 0, normalized size = 0.

$$\int x^2 \left(c+a^2 c x^2\right)^3 \tan^{-1}(a x)^{3/2} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2),x]
```

```
[Out] Integrate[x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2), x]
```

Maple [A] time = 1.284, size = 0, normalized size = 0.

$$\int x^2 \left(a^2 c x^2 + c\right)^3 (\arctan(ax))^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a^2*c*x^2+c)^3*arctan(a*x)^(3/2),x)
```

```
[Out] int(x^2*(a^2*c*x^2+c)^3*arctan(a*x)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^3*arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^3*arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a**2*c*x**2+c)**3*atan(a*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2+c\right)^3 x^2 \arctan{(ax)}^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^3*arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^3*x^2*arctan(a*x)^(3/2), x)
```

3.770 $\int x\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^{3/2}\,dx$

Optimal. Leaf size=60

$$\frac{c^3\left(a^2x^2+1\right)^4\tan^{-1}(ax)^{3/2}}{8a^2}-\frac{3\text{Unintegrable}\left(\left(a^2cx^2+c\right)^3\sqrt{\tan^{-1}(ax)},x\right)}{16a}$$

```
[Out] (c^3*(1 + a^2*x^2)^4*ArcTan[a*x]^(3/2))/(8*a^2) - (3*Unintegrable[(c + a^2*
c*x^2)^3*Sqrt[ArcTan[a*x]], x])/(16*a)
```

Rubi [A] time = 0.0620576, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^{3/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x*(c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2),x]
```

```
[Out] (c^3*(1 + a^2*x^2)^4*ArcTan[a*x]^(3/2))/(8*a^2) - (3*Defer[Int][(c + a^2*c*
x^2)^3*Sqrt[ArcTan[a*x]], x])/(16*a)
```

Rubi steps

$$\int x\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^{3/2}\,dx=\frac{c^3\left(1+a^2x^2\right)^4\tan^{-1}(ax)^{3/2}}{8a^2}-\frac{3\int\left(c+a^2cx^2\right)^3\sqrt{\tan^{-1}(ax)}\,dx}{16a}$$

Mathematica [A] time = 1.3656, size = 0, normalized size = 0.

$$\int x\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^{3/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x*(c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2),x]
```

```
[Out] Integrate[x*(c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2), x]
```

Maple [A] time = 0.778, size = 0, normalized size = 0.

$$\int x\left(a^{2} c x^{2}+c\right)^{3} (\arctan (a x))^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^3*arctan(a*x)^(3/2),x)
```

```
[Out] int(x*(a^2*c*x^2+c)^3*arctan(a*x)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^3*arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^3*arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**3*atan(a*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2+c\right)^3 x \arctan(ax)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^3*arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^3*x*arctan(a*x)^(3/2), x)
```

3.771 $\int \left(c+a^2cx^2\right)^3 \tan^{-1}(ax)^{3/2}\,dx$

Optimal. Leaf size=255

$$\frac{9}{280}c^2\text{Unintegrable}\left(\frac{a^2cx^2+c}{\sqrt{\tan^{-1}(ax)}},x\right)+\frac{1}{56}c\text{Unintegrable}\left(\frac{\left(a^2cx^2+c\right)^2}{\sqrt{\tan^{-1}(ax)}},x\right)+\frac{3}{35}c^3\text{Unintegrable}\left(\frac{1}{\sqrt{\tan^{-1}(ax)}},x\right)+$$

```
[Out] (-6*c^3*(1 + a^2*x^2)*Sqrt[ArcTan[a*x]])/(35*a) - (9*c^3*(1 + a^2*x^2)^2*Sq
rt[ArcTan[a*x]])/(140*a) - (c^3*(1 + a^2*x^2)^3*Sqrt[ArcTan[a*x]])/(28*a) +
 (8*c^3*x*(1 + a^2*x^2)*ArcTan[a*x]^(3/2))/35 + (6*c^3*x*(1 + a^2*x^2)^2*Ar
cTan[a*x]^(3/2))/35 + (c^3*x*(1 + a^2*x^2)^3*ArcTan[a*x]^(3/2))/7 + (3*c^3*
Unintegrable[1/Sqrt[ArcTan[a*x]], x])/35 + (9*c^2*Unintegrable[(c + a^2*c*x
^2)/Sqrt[ArcTan[a*x]], x])/280 + (c*Unintegrable[(c + a^2*c*x^2)^2/Sqrt[Arc
Tan[a*x]], x])/56 + (16*c^3*Unintegrable[ArcTan[a*x]^(3/2), x])/35
```

Rubi [A] time = 0.124399, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \left(c+a^2cx^2\right)^3 \tan^{-1}(ax)^{3/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2),x]
```

```
[Out] (-6*c^3*(1 + a^2*x^2)*Sqrt[ArcTan[a*x]])/(35*a) - (9*c^3*(1 + a^2*x^2)^2*Sq
rt[ArcTan[a*x]])/(140*a) - (c^3*(1 + a^2*x^2)^3*Sqrt[ArcTan[a*x]])/(28*a) +
 (8*c^3*x*(1 + a^2*x^2)*ArcTan[a*x]^(3/2))/35 + (6*c^3*x*(1 + a^2*x^2)^2*Ar
cTan[a*x]^(3/2))/35 + (c^3*x*(1 + a^2*x^2)^3*ArcTan[a*x]^(3/2))/7 + (3*c^3*
Defer[Int][1/Sqrt[ArcTan[a*x]], x])/35 + (9*c^2*Defer[Int][(c + a^2*c*x^2)/
Sqrt[ArcTan[a*x]], x])/280 + (c*Defer[Int][(c + a^2*c*x^2)^2/Sqrt[ArcTan[a*
x]], x])/56 + (16*c^3*Defer[Int][ArcTan[a*x]^(3/2), x])/35
```

Rubi steps

$$\int \left(c+a^2cx^2\right)^3 \tan^{-1}(ax)^{3/2}\,dx = -\frac{c^3\left(1+a^2x^2\right)^3\sqrt{\tan^{-1}(ax)}}{28a} + \frac{1}{7}c^3x\left(1+a^2x^2\right)^3\tan^{-1}(ax)^{3/2} + \frac{1}{56}c\int \frac{\left(c+a^2cx^2\right)^2}{\sqrt{\tan^{-1}(ax)}}\,d$$

$$= -\frac{9c^3\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}}{140a} - \frac{c^3\left(1+a^2x^2\right)^3\sqrt{\tan^{-1}(ax)}}{28a} + \frac{6}{35}c^3x\left(1+a^2x^2\right)^2\tan^{-}$$

$$= -\frac{6c^3\left(1+a^2x^2\right)\sqrt{\tan^{-1}(ax)}}{35a} - \frac{9c^3\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}}{140a} - \frac{c^3\left(1+a^2x^2\right)^3\sqrt{\tan^{-1}(}}{28a}$$

Mathematica [A] time = 2.19559, size = 0, normalized size = 0.

$$\int \left(c+a^2cx^2\right)^3 \tan^{-1}(ax)^{3/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2),x]
```

```
[Out] Integrate[(c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2), x]
```

Maple [A] time = 0.671, size = 0, normalized size = 0.

$$\int \left(a^2cx^2+c\right)^3 (\arctan(ax))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^3*arctan(a*x)^(3/2),x)
```

```
[Out] int((a^2*c*x^2+c)^3*arctan(a*x)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**3*atan(a*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2+c\right)^3 \arctan\left(ax\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^3*arctan(a*x)^(3/2), x)
```

3.772 $\int \frac{(c+a^2cx^2)^3 \tan^{-1}(ax)^{3/2}}{x} dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{(a^2cx^2+c)^3 \tan^{-1}(ax)^{3/2}}{x}, x\right)$$

```
[Out] Unintegrable[((c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2))/x, x]
```

Rubi [A] time = 0.052133, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{(c+a^2cx^2)^3 \tan^{-1}(ax)^{3/2}}{x} dx$$

Verification is Not applicable to the result.

```
[In] Int[((c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2))/x,x]

[Out] Defer[Int][((c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2))/x, x]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^3 \tan^{-1}(ax)^{3/2}}{x} dx = \int \frac{(c+a^2cx^2)^3 \tan^{-1}(ax)^{3/2}}{x} dx$$

Mathematica [A] time = 1.63856, size = 0, normalized size = 0.

$$\int \frac{(c+a^2cx^2)^3 \tan^{-1}(ax)^{3/2}}{x} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2))/x,x]
```

```
[Out] Integrate[((c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2))/x, x]
```

Maple [A] time = 0.822, size = 0, normalized size = 0.

$$\int \frac{(a^2cx^2+c)^3}{x}(\arctan(ax))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^3*arctan(a*x)^(3/2)/x,x)
```

```
[Out] int((a^2*c*x^2+c)^3*arctan(a*x)^(3/2)/x,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**3*atan(a*x)**(3/2)/x,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^3\arctan\left(ax\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^(3/2)/x,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^3*arctan(a*x)^(3/2)/x, x)
```

3.773 $\int \frac{\left(c+a^2cx^2\right)^3 \tan^{-1}(ax)^{3/2}}{x^2} \, dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{\left(a^2cx^2+c\right)^3 \tan^{-1}(ax)^{3/2}}{x^2},x\right)$$

```
[Out] Unintegrable[((c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2))/x^2, x]
```

Rubi [A] time = 0.0571167, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(c+a^2cx^2\right)^3 \tan^{-1}(ax)^{3/2}}{x^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[((c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2))/x^2,x]
```

```
[Out] Defer[Int][((c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2))/x^2, x]
```

Rubi steps

$$\int \frac{\left(c+a^2cx^2\right)^3 \tan^{-1}(ax)^{3/2}}{x^2} \, dx = \int \frac{\left(c+a^2cx^2\right)^3 \tan^{-1}(ax)^{3/2}}{x^2} \, dx$$

Mathematica [A] time = 2.4771, size = 0, normalized size = 0.

$$\int \frac{\left(c+a^2cx^2\right)^3 \tan^{-1}(ax)^{3/2}}{x^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2))/x^2,x]
```

```
[Out] Integrate[((c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2))/x^2, x]
```

Maple [A] time = 0.557, size = 0, normalized size = 0.

$$\int \frac{(a^2cx^2+c)^3}{x^2} (\arctan(ax))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^3*arctan(a*x)^(3/2)/x^2,x)
```

```
[Out] int((a^2*c*x^2+c)^3*arctan(a*x)^(3/2)/x^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**3*atan(a*x)**(3/2)/x**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^3 \arctan\left(ax\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^3*arctan(a*x)^(3/2)/x^2, x)
```

3.774 $\int \frac{x^m \tan^{-1}(ax)^{3/2}}{c+a^2cx^2} \, dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{x^m \tan^{-1}(ax)^{3/2}}{a^2cx^2+c},x\right)$$

```
[Out] Unintegrable[(x^m*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2), x]
```

Rubi [A] time = 0.0627471, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m \tan^{-1}(ax)^{3/2}}{c+a^2cx^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2),x]

[Out] Defer[Int][(x^m*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2), x]
```

Rubi steps

$$\int \frac{x^m \tan^{-1}(ax)^{3/2}}{c+a^2cx^2} \, dx=\int \frac{x^m \tan^{-1}(ax)^{3/2}}{c+a^2cx^2} \, dx$$

Mathematica [A] time = 0.670854, size = 0, normalized size = 0.

$$\int \frac{x^m \tan^{-1}(ax)^{3/2}}{c+a^2cx^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2),x]

[Out] Integrate[(x^m*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2), x]
```

Maple [A] time = 0.566, size = 0, normalized size = 0.

$$\int \frac{x^{m}}{a^{2} c \,x^{2}+c} \left(\arctan \left(a x \right)\right)^{\frac{3}{2}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c),x)
```

```
[Out] int(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x^{m} \arctan \left(ax\right)^{\frac{3}{2}}}{a^{2} c x^{2} + c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2 + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*atan(a*x)**(3/2)/(a**2*c*x**2+c),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m \arctan(ax)^{\frac{3}{2}}}{a^2cx^2 + c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2 + c), x)
```

3.775 $\int \frac{x^3 \tan^{-1}(ax)^{3/2}}{c+a^2cx^2} dx$

Optimal. Leaf size=59

$$\frac{\text{Unintegrable}\left(x\tan^{-1}(ax)^{3/2},x\right)}{a^2c}+\frac{2\text{Unintegrable}\left(\tan^{-1}(ax)^{5/2},x\right)}{5a^3c}-\frac{2x\tan^{-1}(ax)^{5/2}}{5a^3c}$$

```
[Out] (-2*x*ArcTan[a*x]^(5/2))/(5*a^3*c) + Unintegrable[x*ArcTan[a*x]^(3/2), x]/(
a^2*c) + (2*Unintegrable[ArcTan[a*x]^(5/2), x])/(5*a^3*c)
```

Rubi [A] time = 0.122881, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^3\tan^{-1}(ax)^{3/2}}{c+a^2cx^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^3*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2),x]
```

```
[Out] (-2*x*ArcTan[a*x]^(5/2))/(5*a^3*c) + Defer[Int][x*ArcTan[a*x]^(3/2), x]/(a^
2*c) + (2*Defer[Int][ArcTan[a*x]^(5/2), x])/(5*a^3*c)
```

Rubi steps

$$\begin{aligned}\int \frac{x^3\tan^{-1}(ax)^{3/2}}{c+a^2cx^2}\,dx &= -\frac{\int \frac{x\tan^{-1}(ax)^{3/2}}{c+a^2cx^2}\,dx}{a^2}+\frac{\int x\tan^{-1}(ax)^{3/2}\,dx}{a^2c}\\ &= -\frac{2x\tan^{-1}(ax)^{5/2}}{5a^3c}+\frac{2\int \tan^{-1}(ax)^{5/2}\,dx}{5a^3c}+\frac{\int x\tan^{-1}(ax)^{3/2}\,dx}{a^2c}\end{aligned}$$

Mathematica [A] time = 3.95321, size = 0, normalized size = 0.

$$\int \frac{x^3\tan^{-1}(ax)^{3/2}}{c+a^2cx^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^3*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2),x]
```

```
[Out] Integrate[(x^3*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2), x]
```

Maple [A] time = 0.562, size = 0, normalized size = 0.

$$\int \frac{x^3}{a^2cx^2+c} (\arctan (ax))^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c),x)
```

```
[Out] int(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^3 \operatorname{atan}^{\frac{3}{2}}(ax)}{a^2x^2+1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atan(a*x)**(3/2)/(a**2*c*x**2+c),x)
```

```
[Out] Integral(x**3*atan(a*x)**(3/2)/(a**2*x**2 + 1), x)/c
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \arctan(ax)^{\frac{3}{2}}}{a^2cx^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2 + c), x)
```

3.776 $\int \frac{x^2 \tan^{-1}(ax)^{3/2}}{c+a^2cx^2} dx$

Optimal. Leaf size=36

$$\frac{\text{Unintegrable}\left(\tan^{-1}(ax)^{3/2},x\right)}{a^2c}-\frac{2\tan^{-1}(ax)^{5/2}}{5a^3c}$$

```
[Out] (-2*ArcTan[a*x]^(5/2))/(5*a^3*c) + Unintegrable[ArcTan[a*x]^(3/2), x]/(a^2*
c)
```

Rubi [A] time = 0.0964175, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^2 \tan^{-1}(ax)^{3/2}}{c+a^2cx^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^2*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2),x]
```

```
[Out] (-2*ArcTan[a*x]^(5/2))/(5*a^3*c) + Defer[Int][ArcTan[a*x]^(3/2), x]/(a^2*c)
```

Rubi steps

$$\begin{aligned}\int \frac{x^2 \tan^{-1}(ax)^{3/2}}{c+a^2cx^2} dx &= -\frac{\int \frac{\tan^{-1}(ax)^{3/2}}{c+a^2cx^2} dx}{a^2}+\frac{\int \tan^{-1}(ax)^{3/2} dx}{a^2c}\\ &= -\frac{2\tan^{-1}(ax)^{5/2}}{5a^3c}+\frac{\int \tan^{-1}(ax)^{3/2} dx}{a^2c}\end{aligned}$$

Mathematica [A] time = 1.16721, size = 0, normalized size = 0.

$$\int \frac{x^2 \tan^{-1}(ax)^{3/2}}{c+a^2cx^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^2*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2),x]
```

```
[Out] Integrate[(x^2*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2), x]
```

Maple [A] time = 0.255, size = 0, normalized size = 0.

$$\int \frac{x^2}{a^2cx^2 + c} (\arctan (ax))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c),x)
```

```
[Out] int(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^2 \operatorname{atan}^{\frac{3}{2}}(ax)}{a^2x^2+1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(a*x)**(3/2)/(a**2*c*x**2+c),x)
```

```
[Out] Integral(x**2*atan(a*x)**(3/2)/(a**2*x**2 + 1), x)/c
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \arctan(ax)^{\frac{3}{2}}}{a^2cx^2 + c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2 + c), x)
```

3.777 $\int \frac{x \tan^{-1}(ax)^{3/2}}{c+a^2cx^2} \, dx$

Optimal. Leaf size=40

$$\frac{2x \tan^{-1}(ax)^{5/2}}{5ac} - \frac{2\text{Unintegrable}\left(\tan^{-1}(ax)^{5/2}, x\right)}{5ac}$$

```
[Out] (2*x*ArcTan[a*x]^(5/2))/(5*a*c) - (2*Unintegrable[ArcTan[a*x]^(5/2), x])/(5
*a*c)
```

Rubi [A] time = 0.047974, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{x \tan^{-1}(ax)^{3/2}}{c + a^2cx^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(x*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2),x]
```

```
[Out] (2*x*ArcTan[a*x]^(5/2))/(5*a*c) - (2*Defer[Int][ArcTan[a*x]^(5/2), x])/(5*a
*c)
```

Rubi steps

$$\int \frac{x \tan^{-1}(ax)^{3/2}}{c + a^2cx^2} \, dx = \frac{2x \tan^{-1}(ax)^{5/2}}{5ac} - \frac{2 \int \tan^{-1}(ax)^{5/2} \, dx}{5ac}$$

Mathematica [A] time = 0.972139, size = 0, normalized size = 0.

$$\int \frac{x \tan^{-1}(ax)^{3/2}}{c + a^2cx^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2),x]
```

```
[Out] Integrate[(x*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2), x]
```

Maple [A] time = 0.141, size = 0, normalized size = 0.

$$\int \frac{x}{a^2cx^2+c}\left(\arctan\left(ax\right)\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c),x)
```

```
[Out] int(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x\operatorname{atan}^{\frac{3}{2}}\left(ax\right)}{a^2x^2+1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(a*x)**(3/2)/(a**2*c*x**2+c),x)
```

```
[Out] Integral(x*atan(a*x)**(3/2)/(a**2*x**2 + 1), x)/c
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\arctan{(ax)}^{\frac{3}{2}}}{a^2cx^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(x*arctan(a*x)^(3/2)/(a^2*c*x^2 + c), x)
```

3.778 $\int \frac{\tan^{-1}(ax)^{3/2}}{c+a^2cx^2} dx$

Optimal. Leaf size=18

$$\frac{2\tan^{-1}(ax)^{5/2}}{5ac}$$

```
[Out] (2*ArcTan[a*x]^(5/2))/(5*a*c)
```

Rubi [A] time = 0.0251957, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {4884}

$$\frac{2\tan^{-1}(ax)^{5/2}}{5ac}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^(3/2)/(c + a^2*c*x^2),x]
```

```
[Out] (2*ArcTan[a*x]^(5/2))/(5*a*c)
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^{3/2}}{c+a^2cx^2} dx = \frac{2\tan^{-1}(ax)^{5/2}}{5ac}$$

Mathematica [A] time = 0.0034325, size = 18, normalized size = 1.

$$\frac{2\tan^{-1}(ax)^{5/2}}{5ac}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]^(3/2)/(c + a^2*c*x^2),x]
```

```
[Out] (2*ArcTan[a*x]^(5/2))/(5*a*c)
```

Maple [A] time = 0.083, size = 15, normalized size = 0.8

$$\frac{2}{5\,ac}\left(\arctan\left(ax\right)\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(3/2)/(a^2*c*x^2+c),x)
```

```
[Out] 2/5*arctan(a*x)^(5/2)/a/c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 1.69903, size = 39, normalized size = 2.17

$$\frac{2\,\arctan\left(ax\right)^{\frac{5}{2}}}{5\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] 2/5*arctan(a*x)^(5/2)/(a*c)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\operatorname{atan}^{\frac{3}{2}}(ax)}{a^2x^2+1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(3/2)/(a**2*c*x**2+c),x)
```

```
[Out] Integral(atan(a*x)**(3/2)/(a**2*x**2 + 1), x)/c
```

Giac [A] time = 1.09879, size = 19, normalized size = 1.06

$$\frac{2\arctan(ax)^{\frac{5}{2}}}{5\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] 2/5*arctan(a*x)^(5/2)/(a*c)
```

3.779 $\int \frac{\tan^{-1}(ax)^{3/2}}{x(c+a^2cx^2)}\,dx$

Optimal. Leaf size=48

$$\frac{i\text{Unintegrable}\left(\frac{\tan^{-1}(ax)^{3/2}}{x(ax+i)},x\right)}{c}-\frac{2i\tan^{-1}(ax)^{5/2}}{5c}$$

```
[Out] (((-2*I)/5)*ArcTan[a*x]^(5/2))/c + (I*Unintegrable[ArcTan[a*x]^(3/2)/(x*(I
+ a*x)), x])/c
```

Rubi [A] time = 0.10831, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\tan^{-1}(ax)^{3/2}}{x\left(c+a^2cx^2\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcTan[a*x]^(3/2)/(x*(c + a^2*c*x^2)),x]
```

```
[Out] (((-2*I)/5)*ArcTan[a*x]^(5/2))/c + (I*Defer[Int][ArcTan[a*x]^(3/2)/(x*(I +
a*x)), x])/c
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^{3/2}}{x\left(c+a^2cx^2\right)}\,dx=-\frac{2i\tan^{-1}(ax)^{5/2}}{5c}+\frac{i\int \frac{\tan^{-1}(ax)^{3/2}}{x(i+ax)}\,dx}{c}$$

Mathematica [A] time = 0.57485, size = 0, normalized size = 0.

$$\int \frac{\tan^{-1}(ax)^{3/2}}{x\left(c+a^2cx^2\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcTan[a*x]^(3/2)/(x*(c + a^2*c*x^2)),x]
```

```
[Out] Integrate[ArcTan[a*x]^(3/2)/(x*(c + a^2*c*x^2)), x]
```

Maple [A] time = 0.14, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(a^{2}c x^{2}+c\right)} \left(\arctan \left(a x\right)\right)^{\frac{3}{2}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c),x)
```

```
[Out] int(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\operatorname{atan}^{\frac{3}{2}}(ax)}{a^2x^3+x}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(3/2)/x/(a**2*c*x**2+c),x)
```

```
[Out] Integral(atan(a*x)**(3/2)/(a**2*x**3 + x), x)/c
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(ax)^{\frac{3}{2}}}{(a^2cx^2+c)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)^(3/2)/((a^2*c*x^2 + c)*x), x)
```

3.780 $\int \frac{\tan^{-1}(ax)^{3/2}}{x^2(c+a^2cx^2)}\,dx$

Optimal. Leaf size=35

$$\frac{\text{Unintegrable}\left(\frac{\tan^{-1}(ax)^{3/2}}{x^2},x\right)}{c}-\frac{2a\tan^{-1}(ax)^{5/2}}{5c}$$

```
[Out] (-2*a*ArcTan[a*x]^(5/2))/(5*c) + Unintegrable[ArcTan[a*x]^(3/2)/x^2, x]/c
```

Rubi [A] time = 0.102409, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\tan^{-1}(ax)^{3/2}}{x^2\left(c+a^2cx^2\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcTan[a*x]^(3/2)/(x^2*(c + a^2*c*x^2)),x]
```

```
[Out] (-2*a*ArcTan[a*x]^(5/2))/(5*c) + Defer[Int][ArcTan[a*x]^(3/2)/x^2, x]/c
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^{-1}(ax)^{3/2}}{x^2\left(c+a^2cx^2\right)}\,dx &= -\left(a^2\int \frac{\tan^{-1}(ax)^{3/2}}{c+a^2cx^2}\,dx\right)+\frac{\int \frac{\tan^{-1}(ax)^{3/2}}{x^2}\,dx}{c}\\ &= -\frac{2a\tan^{-1}(ax)^{5/2}}{5c}+\frac{\int \frac{\tan^{-1}(ax)^{3/2}}{x^2}\,dx}{c}\end{aligned}$$

Mathematica [A] time = 1.23577, size = 0, normalized size = 0.

$$\int \frac{\tan^{-1}(ax)^{3/2}}{x^2\left(c+a^2cx^2\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcTan[a*x]^(3/2)/(x^2*(c + a^2*c*x^2)),x]
```

```
[Out] Integrate[ArcTan[a*x]^(3/2)/(x^2*(c + a^2*c*x^2)), x]
```

Maple [A] time = 0.244, size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \left(a^2 c x^2 + c\right)} (\arctan (ax))^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(3/2)/x^2/(a^2*c*x^2+c),x)
```

```
[Out] int(arctan(a*x)^(3/2)/x^2/(a^2*c*x^2+c),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/x^2/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/x^2/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\operatorname{atan}^{\frac{3}{2}}(ax)}{a^2x^4+x^2}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(3/2)/x**2/(a**2*c*x**2+c),x)
```

```
[Out] Integral(atan(a*x)**(3/2)/(a**2*x**4 + x**2), x)/c
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(ax)^{\frac{3}{2}}}{(a^2cx^2+c)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/x^2/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)^(3/2)/((a^2*c*x^2 + c)*x^2), x)
```

3.781 $\int \frac{\tan^{-1}(ax)^{3/2}}{x^3(c+a^2cx^2)}\,dx$

Optimal. Leaf size=72

$$-\frac{ia^2\text{Unintegrable}\left(\frac{\tan^{-1}(ax)^{3/2}}{x(ax+i)},x\right)}{c}+\frac{\text{Unintegrable}\left(\frac{\tan^{-1}(ax)^{3/2}}{x^3},x\right)}{c}+\frac{2ia^2\tan^{-1}(ax)^{5/2}}{5c}$$

```
[Out] (((2*I)/5)*a^2*ArcTan[a*x]^(5/2))/c + Unintegrable[ArcTan[a*x]^(3/2)/x^3, x
]/c - (I*a^2*Unintegrable[ArcTan[a*x]^(3/2)/(x*(I + a*x)), x])/c
```

Rubi [A] time = 0.192333, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\tan^{-1}(ax)^{3/2}}{x^3\left(c+a^2cx^2\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcTan[a*x]^(3/2)/(x^3*(c + a^2*c*x^2)),x]
```

```
[Out] (((2*I)/5)*a^2*ArcTan[a*x]^(5/2))/c + Defer[Int][ArcTan[a*x]^(3/2)/x^3, x]/
c - (I*a^2*Defer[Int][ArcTan[a*x]^(3/2)/(x*(I + a*x)), x])/c
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^{-1}(ax)^{3/2}}{x^3\left(c+a^2cx^2\right)}\,dx&=-\left(a^2\int \frac{\tan^{-1}(ax)^{3/2}}{x\left(c+a^2cx^2\right)}\,dx\right)+\frac{\int \frac{\tan^{-1}(ax)^{3/2}}{x^3}\,dx}{c}\\&=\frac{2ia^2\tan^{-1}(ax)^{5/2}}{5c}+\frac{\int \frac{\tan^{-1}(ax)^{3/2}}{x^3}\,dx}{c}-\frac{\left(ia^2\right)\int \frac{\tan^{-1}(ax)^{3/2}}{x(i+ax)}\,dx}{c}\end{aligned}$$

Mathematica [A] time = 1.71906, size = 0, normalized size = 0.

$$\int \frac{\tan^{-1}(ax)^{3/2}}{x^3\left(c+a^2cx^2\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcTan[a*x]^(3/2)/(x^3*(c + a^2*c*x^2)),x]
```

```
[Out] Integrate[ArcTan[a*x]^(3/2)/(x^3*(c + a^2*c*x^2)), x]
```

Maple [A] time = 0.622, size = 0, normalized size = 0.

$$\int \frac{1}{x^{3}\left(a^{2}c x^{2}+c\right)}\left(\arctan\left(ax\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(3/2)/x^3/(a^2*c*x^2+c),x)
```

```
[Out] int(arctan(a*x)^(3/2)/x^3/(a^2*c*x^2+c),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/x^3/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/x^3/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\operatorname{atan}^{\frac{3}{2}}(ax)}{a^2x^5+x^3}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(3/2)/x**3/(a**2*c*x**2+c),x)
```

```
[Out] Integral(atan(a*x)**(3/2)/(a**2*x**5 + x**3), x)/c
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(ax)^{\frac{3}{2}}}{(a^2cx^2+c)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/x^3/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)^(3/2)/((a^2*c*x^2 + c)*x^3), x)
```

3.782 $\int \frac{\tan^{-1}(ax)^{3/2}}{x^4(c+a^2cx^2)} dx$

Optimal. Leaf size=59

$$-\frac{a^2\text{Unintegrable}\left(\frac{\tan^{-1}(ax)^{3/2}}{x^2},x\right)}{c}+\frac{\text{Unintegrable}\left(\frac{\tan^{-1}(ax)^{3/2}}{x^4},x\right)}{c}+\frac{2a^3\tan^{-1}(ax)^{5/2}}{5c}$$

```
[Out] (2*a^3*ArcTan[a*x]^(5/2))/(5*c) + Unintegrable[ArcTan[a*x]^(3/2)/x^4, x]/c
- (a^2*Unintegrable[ArcTan[a*x]^(3/2)/x^2, x])/c
```

Rubi [A] time = 0.186247, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\tan^{-1}(ax)^{3/2}}{x^4\left(c+a^2cx^2\right)} dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcTan[a*x]^(3/2)/(x^4*(c + a^2*c*x^2)),x]
```

```
[Out] (2*a^3*ArcTan[a*x]^(5/2))/(5*c) + Defer[Int][ArcTan[a*x]^(3/2)/x^4, x]/c -
(a^2*Defer[Int][ArcTan[a*x]^(3/2)/x^2, x])/c
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^{-1}(ax)^{3/2}}{x^4\left(c+a^2cx^2\right)} dx &= -\left(a^2\int \frac{\tan^{-1}(ax)^{3/2}}{x^2\left(c+a^2cx^2\right)} dx\right)+\frac{\int \frac{\tan^{-1}(ax)^{3/2}}{x^4} dx}{c}\\
&= a^4\int \frac{\tan^{-1}(ax)^{3/2}}{c+a^2cx^2} dx+\frac{\int \frac{\tan^{-1}(ax)^{3/2}}{x^4} dx}{c}-\frac{a^2\int \frac{\tan^{-1}(ax)^{3/2}}{x^2} dx}{c}\\
&= \frac{2a^3\tan^{-1}(ax)^{5/2}}{5c}+\frac{\int \frac{\tan^{-1}(ax)^{3/2}}{x^4} dx}{c}-\frac{a^2\int \frac{\tan^{-1}(ax)^{3/2}}{x^2} dx}{c}
\end{aligned}$$

Mathematica [A] time = 3.73934, size = 0, normalized size = 0.

$$\int \frac{\tan^{-1}(ax)^{3/2}}{x^4\left(c+a^2cx^2\right)} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcTan[a*x]^(3/2)/(x^4*(c + a^2*c*x^2)),x]
```

```
[Out] Integrate[ArcTan[a*x]^(3/2)/(x^4*(c + a^2*c*x^2)), x]
```

Maple [A] time = 0.498, size = 0, normalized size = 0.

$$\int \frac{1}{x^4 \left(a^2 c x^2 + c\right)} (\arctan(ax))^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(3/2)/x^4/(a^2*c*x^2+c),x)
```

```
[Out] int(arctan(a*x)^(3/2)/x^4/(a^2*c*x^2+c),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/x^4/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/x^4/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\operatorname{atan}^{\frac{3}{2}}(ax)}{a^2x^6+x^4}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(3/2)/x**4/(a**2*c*x**2+c),x)
```

```
[Out] Integral(atan(a*x)**(3/2)/(a**2*x**6 + x**4), x)/c
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(ax)^{\frac{3}{2}}}{(a^2cx^2+c)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/x^4/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)^(3/2)/((a^2*c*x^2 + c)*x^4), x)
```

3.783 $\int \frac{x^m \tan^{-1}(ax)^{3/2}}{\left(c+a^2cx^2\right)^2} dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{x^m \tan^{-1}(ax)^{3/2}}{\left(a^2cx^2+c\right)^2},x\right)$$

```
[Out] Unintegrable[(x^m*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^2, x]
```

Rubi [A] time = 0.0640575, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m \tan^{-1}(ax)^{3/2}}{\left(c+a^2cx^2\right)^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^2,x]

[Out] Defer[Int][(x^m*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^2, x]
```

Rubi steps

$$\int \frac{x^m \tan^{-1}(ax)^{3/2}}{\left(c+a^2cx^2\right)^2} dx = \int \frac{x^m \tan^{-1}(ax)^{3/2}}{\left(c+a^2cx^2\right)^2} dx$$

Mathematica [A] time = 1.35661, size = 0, normalized size = 0.

$$\int \frac{x^m \tan^{-1}(ax)^{3/2}}{\left(c+a^2cx^2\right)^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^2,x]
```

```
[Out] Integrate[(x^m*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^2, x]
```

Maple [A] time = 0.766, size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(a^2cx^2+c\right)^2}\left(\arctan\left(ax\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^2,x)
```

```
[Out] int(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^m\arctan\left(ax\right)^{\frac{3}{2}}}{a^4c^2x^4+2\,a^2c^2x^2+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] integral(x^m*arctan(a*x)^(3/2)/(a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*atan(a*x)**(3/2)/(a**2*c*x**2+c)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m \arctan(ax)^{\frac{3}{2}}}{\left(a^2cx^2+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2 + c)^2, x)
```

3.784 $\int \frac{x^3 \tan^{-1}(ax)^{3/2}}{\left(c+a^2cx^2\right)^2}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{x^3 \tan^{-1}(ax)^{3/2}}{\left(a^2cx^2+c\right)^2},x\right)$$

```
[Out] Unintegrable[(x^3*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^2, x]
```

Rubi [A] time = 0.0643157, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^3 \tan^{-1}(ax)^{3/2}}{\left(c+a^2cx^2\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^3*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^2,x]

[Out] Defer[Int][(x^3*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^2, x]
```

Rubi steps

$$\int \frac{x^3 \tan^{-1}(ax)^{3/2}}{\left(c+a^2cx^2\right)^2}\,dx=\int \frac{x^3 \tan^{-1}(ax)^{3/2}}{\left(c+a^2cx^2\right)^2}\,dx$$

Mathematica [A] time = 3.87674, size = 0, normalized size = 0.

$$\int \frac{x^3 \tan^{-1}(ax)^{3/2}}{\left(c+a^2cx^2\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^3*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^2,x]
```

```
[Out] Integrate[(x^3*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^2, x]
```

Maple [A] time = 0.523, size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(a^2cx^2+c\right)^2} \left(\arctan\left(ax\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^2,x)
```

```
[Out] int(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^3 \operatorname{atan}^{\frac{3}{2}}(ax)}{a^4x^4+2a^2x^2+1}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atan(a*x)**(3/2)/(a**2*c*x**2+c)**2,x)
```

```
[Out] Integral(x**3*atan(a*x)**(3/2)/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \arctan(ax)^{\frac{3}{2}}}{(a^2cx^2+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2 + c)^2, x)
```

3.785 $\int \frac{x^2 \tan^{-1}(ax)^{3/2}}{(c+a^2cx^2)^2} dx$

Optimal. Leaf size=127

$$\frac{3\sqrt{\pi}\text{FresnelC}\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{32a^3c^2} - \frac{x\tan^{-1}(ax)^{3/2}}{2a^2c^2(a^2x^2+1)} - \frac{3\sqrt{\tan^{-1}(ax)}}{8a^3c^2(a^2x^2+1)} + \frac{\tan^{-1}(ax)^{5/2}}{5a^3c^2} + \frac{3\sqrt{\tan^{-1}(ax)}}{16a^3c^2}$$

```
[Out] (3*Sqrt[ArcTan[a*x]])/(16*a^3*c^2) - (3*Sqrt[ArcTan[a*x]])/(8*a^3*c^2*(1 +
a^2*x^2)) - (x*ArcTan[a*x]^(3/2))/(2*a^2*c^2*(1 + a^2*x^2)) + ArcTan[a*x]^(
5/2)/(5*a^3*c^2) + (3*Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(3
2*a^3*c^2)
```

Rubi [A] time = 0.188436, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4936, 4930, 4904, 3312, 3304, 3352}

$$\frac{3\sqrt{\pi}\text{FresnelC}\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{32a^3c^2} - \frac{x\tan^{-1}(ax)^{3/2}}{2a^2c^2(a^2x^2+1)} - \frac{3\sqrt{\tan^{-1}(ax)}}{8a^3c^2(a^2x^2+1)} + \frac{\tan^{-1}(ax)^{5/2}}{5a^3c^2} + \frac{3\sqrt{\tan^{-1}(ax)}}{16a^3c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^2,x]
```

```
[Out] (3*Sqrt[ArcTan[a*x]])/(16*a^3*c^2) - (3*Sqrt[ArcTan[a*x]])/(8*a^3*c^2*(1 +
a^2*x^2)) - (x*ArcTan[a*x]^(3/2))/(2*a^2*c^2*(1 + a^2*x^2)) + ArcTan[a*x]^(
5/2)/(5*a^3*c^2) + (3*Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(3
2*a^3*c^2)
```

Rule 4936

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^2)/((d_) + (e_.)*(x_)^2)
^2, x_Symbol] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(2*b*c^3*d^2*(p + 1)), x]
 + (Dist[(b*p)/(2*c), Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(d + e*x^2)^2, x]
, x] - Simp[(x*(a + b*ArcTan[c*x])^p)/(2*c^2*d*(d + e*x^2)), x]) /; FreeQ[{
a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 4904

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{x^2 \tan^{-1}(ax)^{3/2}}{\left(c+a^2cx^2\right)^2} dx = -\frac{x\tan^{-1}(ax)^{3/2}}{2a^2c^2\left(1+a^2x^2\right)} + \frac{\tan^{-1}(ax)^{5/2}}{5a^3c^2} + \frac{3\int \frac{x\sqrt{\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^2} dx}{4a}$$

$$= -\frac{3\sqrt{\tan^{-1}(ax)}}{8a^3c^2\left(1+a^2x^2\right)} - \frac{x\tan^{-1}(ax)^{3/2}}{2a^2c^2\left(1+a^2x^2\right)} + \frac{\tan^{-1}(ax)^{5/2}}{5a^3c^2} + \frac{3\int \frac{1}{\left(c+a^2cx^2\right)^2\sqrt{\tan^{-1}(ax)}} dx}{16a^2}$$

$$= -\frac{3\sqrt{\tan^{-1}(ax)}}{8a^3c^2\left(1+a^2x^2\right)} - \frac{x\tan^{-1}(ax)^{3/2}}{2a^2c^2\left(1+a^2x^2\right)} + \frac{\tan^{-1}(ax)^{5/2}}{5a^3c^2} + \frac{3\,\text{Subst}\left(\int \frac{\cos^2(x)}{\sqrt{x}} dx, x, \tan^{-1}(ax)\right)}{16a^3c^2}$$

$$= -\frac{3\sqrt{\tan^{-1}(ax)}}{8a^3c^2\left(1+a^2x^2\right)} - \frac{x\tan^{-1}(ax)^{3/2}}{2a^2c^2\left(1+a^2x^2\right)} + \frac{\tan^{-1}(ax)^{5/2}}{5a^3c^2} + \frac{3\,\text{Subst}\left(\int \left(\frac{1}{2\sqrt{x}} + \frac{\cos(2x)}{2\sqrt{x}}\right) dx, x, \tan^{-1}(\right.}{16a^3c^2}$$

$$= \frac{3\sqrt{\tan^{-1}(ax)}}{16a^3c^2} - \frac{3\sqrt{\tan^{-1}(ax)}}{8a^3c^2\left(1+a^2x^2\right)} - \frac{x\tan^{-1}(ax)^{3/2}}{2a^2c^2\left(1+a^2x^2\right)} + \frac{\tan^{-1}(ax)^{5/2}}{5a^3c^2} + \frac{3\,\text{Subst}\left(\int \frac{\cos(2x)}{\sqrt{x}} dx, x, \text{t}\right.}{32a^3c^2}$$

$$= \frac{3\sqrt{\tan^{-1}(ax)}}{16a^3c^2} - \frac{3\sqrt{\tan^{-1}(ax)}}{8a^3c^2\left(1+a^2x^2\right)} - \frac{x\tan^{-1}(ax)^{3/2}}{2a^2c^2\left(1+a^2x^2\right)} + \frac{\tan^{-1}(ax)^{5/2}}{5a^3c^2} + \frac{3\,\text{Subst}\left(\int \cos\left(2x^2\right) dx,\right.}{16a^3c^2}$$

$$= \frac{3\sqrt{\tan^{-1}(ax)}}{16a^3c^2} - \frac{3\sqrt{\tan^{-1}(ax)}}{8a^3c^2\left(1+a^2x^2\right)} - \frac{x\tan^{-1}(ax)^{3/2}}{2a^2c^2\left(1+a^2x^2\right)} + \frac{\tan^{-1}(ax)^{5/2}}{5a^3c^2} + \frac{3\sqrt{\pi}C\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{32a^3c^2}$$

Mathematica [C] time = 0.353764, size = 187, normalized size = 1.47

$$\frac{\frac{15\left(-i\sqrt{2}\sqrt{-i\tan^{-1}(ax)}\text{Gamma}\left(\frac{1}{2},-2i\tan^{-1}(ax)\right)+i\sqrt{2}\sqrt{i\tan^{-1}(ax)}\text{Gamma}\left(\frac{1}{2},2i\tan^{-1}(ax)\right)+8\tan^{-1}(ax)\right)}{\sqrt{\tan^{-1}(ax)}} + \frac{16\sqrt{\tan^{-1}(ax)}\left(15\left(a^2x^2-1\right)+16\left(a^2x^2+1\right)\tan^{-1}\right.}{a^2x^2+1}}{1280a^3c^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^2,x]
```

```
[Out] ((16*Sqrt[ArcTan[a*x]]*(15*(-1 + a^2*x^2) - 40*a*x*ArcTan[a*x] + 16*(1 + a^
2*x^2)*ArcTan[a*x]^2))/(1 + a^2*x^2) + 60*(-2*Sqrt[ArcTan[a*x]] + Sqrt[Pi]*
FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]]) + (15*(8*ArcTan[a*x] - I*Sqrt[2]*
Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-2*I)*ArcTan[a*x]] + I*Sqrt[2]*Sqrt[I*Ar
cTan[a*x]]*Gamma[1/2, (2*I)*ArcTan[a*x]]))/Sqrt[ArcTan[a*x]])/(1280*a^3*c^2
```

)

Maple [A] time = 0.102, size = 81, normalized size = 0.6

$$\frac{1}{5\,a^3c^2}\left(\arctan\left(ax\right)\right)^{\frac{5}{2}}-\frac{\sin\left(2\,\arctan\left(ax\right)\right)}{4\,a^3c^2}\left(\arctan\left(ax\right)\right)^{\frac{3}{2}}-\frac{3\,\cos\left(2\,\arctan\left(ax\right)\right)}{16\,a^3c^2}\sqrt{\arctan\left(ax\right)}+\frac{3\,\sqrt{\pi}}{32\,a^3c^2}\mathrm{FresnelC}\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^2,x)
```

```
[Out] 1/5*arctan(a*x)^(5/2)/a^3/c^2-1/4/a^3/c^2*arctan(a*x)^(3/2)*sin(2*arctan(a*
x))-3/16/a^3/c^2*arctan(a*x)^(1/2)*cos(2*arctan(a*x))+3/32*FresnelC(2*arcta
n(a*x)^(1/2)/Pi^(1/2))*Pi^(1/2)/a^3/c^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^2 \operatorname{atan}^{\frac{3}{2}}(ax)}{a^4x^4+2a^2x^2+1}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(a*x)**(3/2)/(a**2*c*x**2+c)**2,x)
```

```
[Out] Integral(x**2*atan(a*x)**(3/2)/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \arctan(ax)^{\frac{3}{2}}}{(a^2cx^2+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2 + c)^2, x)
```

3.786 $\int \frac{x\tan^{-1}(ax)^{3/2}}{(c+a^2cx^2)^2}\,dx$

Optimal. Leaf size=109

$$-\frac{3\sqrt{\pi}S\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{32a^2c^2}-\frac{\tan^{-1}(ax)^{3/2}}{2a^2c^2\left(a^2x^2+1\right)}+\frac{3x\sqrt{\tan^{-1}(ax)}}{8ac^2\left(a^2x^2+1\right)}+\frac{\tan^{-1}(ax)^{3/2}}{4a^2c^2}$$

```
[Out] (3*x*Sqrt[ArcTan[a*x]])/(8*a*c^2*(1 + a^2*x^2)) + ArcTan[a*x]^(3/2)/(4*a^2*
c^2) - ArcTan[a*x]^(3/2)/(2*a^2*c^2*(1 + a^2*x^2)) - (3*Sqrt[Pi]*FresnelS[(
2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(32*a^2*c^2)
```

Rubi [A] time = 0.152058, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {4930, 4892, 4970, 4406, 12, 3305, 3351}

$$-\frac{3\sqrt{\pi}S\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{32a^2c^2}-\frac{\tan^{-1}(ax)^{3/2}}{2a^2c^2\left(a^2x^2+1\right)}+\frac{3x\sqrt{\tan^{-1}(ax)}}{8ac^2\left(a^2x^2+1\right)}+\frac{\tan^{-1}(ax)^{3/2}}{4a^2c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^2,x]
```

```
[Out] (3*x*Sqrt[ArcTan[a*x]])/(8*a*c^2*(1 + a^2*x^2)) + ArcTan[a*x]^(3/2)/(4*a^2*
c^2) - ArcTan[a*x]^(3/2)/(2*a^2*c^2*(1 + a^2*x^2)) - (3*Sqrt[Pi]*FresnelS[(
2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(32*a^2*c^2)
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 4892

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[(x*(a + b*ArcTan[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*c*
```

```
p)/2, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(a +
 b*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4970

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sin[x]^m)/
Cos[x]^(m + 2*(q + 1)), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{x\tan^{-1}(ax)^{3/2}}{\left(c+a^2cx^2\right)^2}\,dx = -\frac{\tan^{-1}(ax)^{3/2}}{2a^2c^2\left(1+a^2x^2\right)} + \frac{3\int \frac{\sqrt{\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^2}\,dx}{4a}$$

$$= \frac{3x\sqrt{\tan^{-1}(ax)}}{8ac^2\left(1+a^2x^2\right)} + \frac{\tan^{-1}(ax)^{3/2}}{4a^2c^2} - \frac{\tan^{-1}(ax)^{3/2}}{2a^2c^2\left(1+a^2x^2\right)} - \frac{3}{16}\int \frac{x}{\left(c+a^2cx^2\right)^2\sqrt{\tan^{-1}(ax)}}\,dx$$

$$= \frac{3x\sqrt{\tan^{-1}(ax)}}{8ac^2\left(1+a^2x^2\right)} + \frac{\tan^{-1}(ax)^{3/2}}{4a^2c^2} - \frac{\tan^{-1}(ax)^{3/2}}{2a^2c^2\left(1+a^2x^2\right)} - \frac{3\,\text{Subst}\left(\int \frac{\cos(x)\sin(x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{16a^2c^2}$$

$$= \frac{3x\sqrt{\tan^{-1}(ax)}}{8ac^2\left(1+a^2x^2\right)} + \frac{\tan^{-1}(ax)^{3/2}}{4a^2c^2} - \frac{\tan^{-1}(ax)^{3/2}}{2a^2c^2\left(1+a^2x^2\right)} - \frac{3\,\text{Subst}\left(\int \frac{\sin(2x)}{2\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{16a^2c^2}$$

$$= \frac{3x\sqrt{\tan^{-1}(ax)}}{8ac^2\left(1+a^2x^2\right)} + \frac{\tan^{-1}(ax)^{3/2}}{4a^2c^2} - \frac{\tan^{-1}(ax)^{3/2}}{2a^2c^2\left(1+a^2x^2\right)} - \frac{3\,\text{Subst}\left(\int \frac{\sin(2x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{32a^2c^2}$$

$$= \frac{3x\sqrt{\tan^{-1}(ax)}}{8ac^2\left(1+a^2x^2\right)} + \frac{\tan^{-1}(ax)^{3/2}}{4a^2c^2} - \frac{\tan^{-1}(ax)^{3/2}}{2a^2c^2\left(1+a^2x^2\right)} - \frac{3\,\text{Subst}\left(\int \sin\left(2x^2\right)dx, x, \sqrt{\tan^{-1}(ax)}\right)}{16a^2c^2}$$

$$= \frac{3x\sqrt{\tan^{-1}(ax)}}{8ac^2\left(1+a^2x^2\right)} + \frac{\tan^{-1}(ax)^{3/2}}{4a^2c^2} - \frac{\tan^{-1}(ax)^{3/2}}{2a^2c^2\left(1+a^2x^2\right)} - \frac{3\sqrt{\pi}S\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{32a^2c^2}$$

<u>Mathematica</u> [A] time = 0.0923423, size = 75, normalized size = 0.69

$$\frac{\frac{4\sqrt{\tan^{-1}(ax)}\left(2\left(a^2x^2-1\right)\tan^{-1}(ax)+3ax\right)}{a^2x^2+1} - 3\sqrt{\pi}S\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{32a^2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^2,x]
```

```
[Out] ((4*Sqrt[ArcTan[a*x]]*(3*a*x + 2*(-1 + a^2*x^2)*ArcTan[a*x]))/(1 + a^2*x^2)
 - 3*Sqrt[Pi]*FresnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(32*a^2*c^2)
```

Maple [A] time = 0.099, size = 67, normalized size = 0.6

$$-\frac{1}{32\,a^{2}c^{2}}\left(8\,\left(\arctan\left(ax\right)\right)^{2}\cos\left(2\,\arctan\left(ax\right)\right)+3\,\sqrt{\arctan\left(ax\right)}\sqrt{\pi}\mathrm{FresnelS}\left(2\,\frac{\sqrt{\arctan\left(ax\right)}}{\sqrt{\pi}}\right)-6\,\sin\left(2\,\arctan\left(ax\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^2,x)
```

```
[Out] -1/32/a^2/c^2*(8*arctan(a*x)^2*cos(2*arctan(a*x))+3*arctan(a*x)^(1/2)*Pi^(1
/2)*FresnelS(2*arctan(a*x)^(1/2)/Pi^(1/2))-6*sin(2*arctan(a*x))*arctan(a*x)
)/arctan(a*x)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x\operatorname{atan}^{\frac{3}{2}}\left(ax\right)}{a^{4}x^{4}+2a^{2}x^{2}+1}\,dx}{c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(a*x)**(3/2)/(a**2*c*x**2+c)**2,x)
```

```
[Out] Integral(x*atan(a*x)**(3/2)/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \arctan(ax)^{\frac{3}{2}}}{(a^2cx^2 + c)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(x*arctan(a*x)^(3/2)/(a^2*c*x^2 + c)^2, x)
```

3.787 $\int \frac{\tan^{-1}(ax)^{3/2}}{\left(c+a^2cx^2\right)^2}\,dx$

Optimal. Leaf size=124

$$\frac{x\tan^{-1}(ax)^{3/2}}{2c^2\left(a^2x^2+1\right)}+\frac{3\sqrt{\tan^{-1}(ax)}}{8ac^2\left(a^2x^2+1\right)}-\frac{3\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{32ac^2}+\frac{\tan^{-1}(ax)^{5/2}}{5ac^2}-\frac{3\sqrt{\tan^{-1}(ax)}}{16ac^2}$$

```
[Out] (-3*Sqrt[ArcTan[a*x]])/(16*a*c^2) + (3*Sqrt[ArcTan[a*x]])/(8*a*c^2*(1 + a^2
*x^2)) + (x*ArcTan[a*x]^(3/2))/(2*c^2*(1 + a^2*x^2)) + ArcTan[a*x]^(5/2)/(5
*a*c^2) - (3*Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(32*a*c^2)
```

Rubi [A] time = 0.147686, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4892, 4930, 4904, 3312, 3304, 3352}

$$\frac{x\tan^{-1}(ax)^{3/2}}{2c^2\left(a^2x^2+1\right)}+\frac{3\sqrt{\tan^{-1}(ax)}}{8ac^2\left(a^2x^2+1\right)}-\frac{3\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{32ac^2}+\frac{\tan^{-1}(ax)^{5/2}}{5ac^2}-\frac{3\sqrt{\tan^{-1}(ax)}}{16ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^(3/2)/(c + a^2*c*x^2)^2,x]
```


```
[Out] (-3*Sqrt[ArcTan[a*x]])/(16*a*c^2) + (3*Sqrt[ArcTan[a*x]])/(8*a*c^2*(1 + a^2
*x^2)) + (x*ArcTan[a*x]^(3/2))/(2*c^2*(1 + a^2*x^2)) + ArcTan[a*x]^(5/2)/(5
*a*c^2) - (3*Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(32*a*c^2)
```


Rule 4892

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[(x*(a + b*ArcTan[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*c*
p)/2, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(a +
 b*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
```

```
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 4904

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^{3/2}}{\left(c+a^2cx^2\right)^2}\,dx = \frac{x\tan^{-1}(ax)^{3/2}}{2c^2\left(1+a^2x^2\right)} + \frac{\tan^{-1}(ax)^{5/2}}{5ac^2} - \frac{1}{4}(3a)\int \frac{x\sqrt{\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^2}\,dx$$

$$= \frac{3\sqrt{\tan^{-1}(ax)}}{8ac^2\left(1+a^2x^2\right)} + \frac{x\tan^{-1}(ax)^{3/2}}{2c^2\left(1+a^2x^2\right)} + \frac{\tan^{-1}(ax)^{5/2}}{5ac^2} - \frac{3}{16}\int \frac{1}{\left(c+a^2cx^2\right)^2\sqrt{\tan^{-1}(ax)}}\,dx$$

$$= \frac{3\sqrt{\tan^{-1}(ax)}}{8ac^2\left(1+a^2x^2\right)} + \frac{x\tan^{-1}(ax)^{3/2}}{2c^2\left(1+a^2x^2\right)} + \frac{\tan^{-1}(ax)^{5/2}}{5ac^2} - \frac{3\,\mathrm{Subst}\left(\int \frac{\cos^2(x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{16ac^2}$$

$$= \frac{3\sqrt{\tan^{-1}(ax)}}{8ac^2\left(1+a^2x^2\right)} + \frac{x\tan^{-1}(ax)^{3/2}}{2c^2\left(1+a^2x^2\right)} + \frac{\tan^{-1}(ax)^{5/2}}{5ac^2} - \frac{3\,\mathrm{Subst}\left(\int \left(\frac{1}{2\sqrt{x}} + \frac{\cos(2x)}{2\sqrt{x}}\right)\,dx, x, \tan^{-1}(ax)\right)}{16ac^2}$$

$$= -\frac{3\sqrt{\tan^{-1}(ax)}}{16ac^2} + \frac{3\sqrt{\tan^{-1}(ax)}}{8ac^2\left(1+a^2x^2\right)} + \frac{x\tan^{-1}(ax)^{3/2}}{2c^2\left(1+a^2x^2\right)} + \frac{\tan^{-1}(ax)^{5/2}}{5ac^2} - \frac{3\,\mathrm{Subst}\left(\int \frac{\cos(2x)}{\sqrt{x}}\,dx, x, \tan^{-}\right.}{32ac^2}$$

$$= -\frac{3\sqrt{\tan^{-1}(ax)}}{16ac^2} + \frac{3\sqrt{\tan^{-1}(ax)}}{8ac^2\left(1+a^2x^2\right)} + \frac{x\tan^{-1}(ax)^{3/2}}{2c^2\left(1+a^2x^2\right)} + \frac{\tan^{-1}(ax)^{5/2}}{5ac^2} - \frac{3\,\mathrm{Subst}\left(\int \cos\left(2x^2\right)\,dx, x, \sqrt{}\right.}{16ac^2}$$

$$= -\frac{3\sqrt{\tan^{-1}(ax)}}{16ac^2} + \frac{3\sqrt{\tan^{-1}(ax)}}{8ac^2\left(1+a^2x^2\right)} + \frac{x\tan^{-1}(ax)^{3/2}}{2c^2\left(1+a^2x^2\right)} + \frac{\tan^{-1}(ax)^{5/2}}{5ac^2} - \frac{3\sqrt{\pi}C\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{32ac^2}$$

Mathematica [A] time = 0.167113, size = 90, normalized size = 0.73

$$\frac{\frac{2\sqrt{\tan^{-1}(ax)}\left(-15a^2x^2+16\left(a^2x^2+1\right)\tan^{-1}(ax)^2+40ax\tan^{-1}(ax)+15\right)}{a^2x^2+1} - 15\sqrt{\pi}\,\mathrm{FresnelC}\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{160ac^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]^(3/2)/(c + a^2*c*x^2)^2,x]
```

```
[Out] ((2*Sqrt[ArcTan[a*x]]*(15 - 15*a^2*x^2 + 40*a*x*ArcTan[a*x] + 16*(1 + a^2*x
^2)*ArcTan[a*x]^2))/(1 + a^2*x^2) - 15*Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x
]])/Sqrt[Pi]])/(160*a*c^2)
```

Maple [A] time = 0.109, size = 75, normalized size = 0.6

$$\frac{1}{160\,ac^2}\left(32\,(\arctan(ax))^3+40\,(\arctan(ax))^2\sin(2\,\arctan(ax))+30\,\cos(2\,\arctan(ax))\arctan(ax)-15\,\sqrt{\arctan}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^2,x)
```

```
[Out] 1/160/a/c^2/arctan(a*x)^(1/2)*(32*arctan(a*x)^3+40*arctan(a*x)^2*sin(2*arct
an(a*x))+30*cos(2*arctan(a*x))*arctan(a*x)-15*arctan(a*x)^(1/2)*Pi^(1/2)*Fr
esnelC(2*arctan(a*x)^(1/2)/Pi^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\operatorname{atan}^{\frac{3}{2}}(ax)}{a^4x^4+2a^2x^2+1}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(3/2)/(a**2*c*x**2+c)**2,x)
```

```
[Out] Integral(atan(a*x)**(3/2)/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(ax)^{\frac{3}{2}}}{(a^2cx^2+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)^(3/2)/(a^2*c*x^2 + c)^2, x)
```

3.788 $\int \frac{\tan^{-1}(ax)^{3/2}}{x(c+a^2cx^2)^2} \, dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{\tan^{-1}(ax)^{3/2}}{x\left(a^2cx^2+c\right)^2},x\right)$$

```
[Out] Unintegrable[ArcTan[a*x]^(3/2)/(x*(c + a^2*c*x^2)^2), x]
```

Rubi [A] time = 0.0594933, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\tan^{-1}(ax)^{3/2}}{x\left(c+a^2cx^2\right)^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcTan[a*x]^(3/2)/(x*(c + a^2*c*x^2)^2),x]

[Out] Defer[Int][ArcTan[a*x]^(3/2)/(x*(c + a^2*c*x^2)^2), x]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^{3/2}}{x\left(c+a^2cx^2\right)^2} \, dx = \int \frac{\tan^{-1}(ax)^{3/2}}{x\left(c+a^2cx^2\right)^2} \, dx$$

Mathematica [A] time = 1.73513, size = 0, normalized size = 0.

$$\int \frac{\tan^{-1}(ax)^{3/2}}{x\left(c+a^2cx^2\right)^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcTan[a*x]^(3/2)/(x*(c + a^2*c*x^2)^2),x]
```

```
[Out] Integrate[ArcTan[a*x]^(3/2)/(x*(c + a^2*c*x^2)^2), x]
```

Maple [A] time = 0.575, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(a^{2}c\,x^{2}+c\right)^{2}}\left(\arctan\left(ax\right)\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^2,x)
```

```
[Out] int(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\operatorname{atan}^{\frac{3}{2}}(ax)}{a^4x^5+2a^2x^3+x}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(3/2)/x/(a**2*c*x**2+c)**2,x)
```

```
[Out] Integral(atan(a*x)**(3/2)/(a**4*x**5 + 2*a**2*x**3 + x), x)/c**2
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(ax)^{\frac{3}{2}}}{\left(a^2cx^2+c\right)^2x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)^(3/2)/((a^2*c*x^2 + c)^2*x), x)
```

3.789 $\int \frac{x^m \tan^{-1}(ax)^{3/2}}{\left(c+a^2cx^2\right)^3} \, dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{x^m \tan^{-1}(ax)^{3/2}}{\left(a^2cx^2+c\right)^3},x\right)$$

```
[Out] Unintegrable[(x^m*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^3, x]
```

Rubi [A] time = 0.0629787, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m \tan^{-1}(ax)^{3/2}}{\left(c+a^2cx^2\right)^3} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^3,x]
```

```
[Out] Defer[Int][(x^m*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^3, x]
```

Rubi steps

$$\int \frac{x^m \tan^{-1}(ax)^{3/2}}{\left(c+a^2cx^2\right)^3} \, dx = \int \frac{x^m \tan^{-1}(ax)^{3/2}}{\left(c+a^2cx^2\right)^3} \, dx$$

Mathematica [A] time = 1.75964, size = 0, normalized size = 0.

$$\int \frac{x^m \tan^{-1}(ax)^{3/2}}{\left(c+a^2cx^2\right)^3} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^3,x]
```

```
[Out] Integrate[(x^m*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^3, x]
```

Maple [A] time = 0.835, size = 0, normalized size = 0.

$$\int \frac{x^{m}}{\left(a^{2} c \,x^{2}+c \right)^{3}} \left(\arctan \left(a x \right)\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x)
```

```
[Out] int(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^{m} \arctan \left(a x\right)^{\frac{3}{2}}}{a^{6} c^{3} x^{6} + 3 \, a^{4} c^{3} x^{4} + 3 \, a^{2} c^{3} x^{2} + c^{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] integral(x^m*arctan(a*x)^(3/2)/(a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2
 + c^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*atan(a*x)**(3/2)/(a**2*c*x**2+c)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m \arctan(ax)^{\frac{3}{2}}}{\left(a^2cx^2+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] integrate(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2 + c)^3, x)
```

3.790 $\int \frac{x^5 \tan^{-1}(ax)^{3/2}}{(c+a^2cx^2)^3}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{x^5 \tan^{-1}(ax)^{3/2}}{(a^2cx^2+c)^3},x\right)$$

```
[Out] Unintegrable[(x^5*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^3, x]
```

Rubi [A] time = 0.064012, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^5 \tan^{-1}(ax)^{3/2}}{(c+a^2cx^2)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^5*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^3,x]
```

```
[Out] Defer[Int][(x^5*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^3, x]
```

Rubi steps

$$\int \frac{x^5 \tan^{-1}(ax)^{3/2}}{(c+a^2cx^2)^3}\,dx = \int \frac{x^5 \tan^{-1}(ax)^{3/2}}{(c+a^2cx^2)^3}\,dx$$

Mathematica [A] time = 6.33719, size = 0, normalized size = 0.

$$\int \frac{x^5 \tan^{-1}(ax)^{3/2}}{(c+a^2cx^2)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^5*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^3,x]
```

```
[Out] Integrate[(x^5*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^3, x]
```

Maple [A] time = 0.762, size = 0, normalized size = 0.

$$\int \frac{x^5}{\left(a^2cx^2+c\right)^3} \left(\arctan\left(ax\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x)
```

```
[Out] int(x^5*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^5 \operatorname{atan}^{\frac{3}{2}}(ax)}{a^6x^6+3a^4x^4+3a^2x^2+1}\, dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*atan(a*x)**(3/2)/(a**2*c*x**2+c)**3,x)
```

```
[Out] Integral(x**5*atan(a*x)**(3/2)/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1),
 x)/c**3
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5 \arctan(ax)^{\frac{3}{2}}}{(a^2cx^2+c)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] integrate(x^5*arctan(a*x)^(3/2)/(a^2*c*x^2 + c)^3, x)
```

3.791 $\int \frac{x^4 \tan^{-1}(ax)^{3/2}}{(c+a^2cx^2)^3} dx$

Optimal. Leaf size=230

$$-\frac{3\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{512a^5c^3}+\frac{3\sqrt{\pi}\text{FresnelC}\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{32a^5c^3}+\frac{3x^4\sqrt{\tan^{-1}(ax)}}{32ac^3\left(a^2x^2+1\right)^2}-\frac{x^3\tan^{-1}(ax)^{3/2}}{4a^2c^3\left(a^2x^2+1\right)^2}-\frac{3x\tan^{-1}}{8a^4c^3\left(a^2\right.}$$

```
[Out] (27*Sqrt[ArcTan[a*x]])/(256*a^5*c^3) + (3*x^4*Sqrt[ArcTan[a*x]])/(32*a*c^3*
(1 + a^2*x^2)^2) - (9*Sqrt[ArcTan[a*x]])/(32*a^5*c^3*(1 + a^2*x^2)) - (x^3*
ArcTan[a*x]^(3/2))/(4*a^2*c^3*(1 + a^2*x^2)^2) - (3*x*ArcTan[a*x]^(3/2))/(8
*a^4*c^3*(1 + a^2*x^2)) + (3*ArcTan[a*x]^(5/2))/(20*a^5*c^3) - (3*Sqrt[Pi/2
]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(512*a^5*c^3) + (3*Sqrt[Pi]*Fre
snelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(32*a^5*c^3)
```

Rubi [A] time = 0.41113, antiderivative size = 230, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4940, 4936, 4930, 4904, 3312, 3304, 3352, 4970}

$$-\frac{3\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{512a^5c^3}+\frac{3\sqrt{\pi}\text{FresnelC}\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{32a^5c^3}+\frac{3x^4\sqrt{\tan^{-1}(ax)}}{32ac^3\left(a^2x^2+1\right)^2}-\frac{x^3\tan^{-1}(ax)^{3/2}}{4a^2c^3\left(a^2x^2+1\right)^2}-\frac{3x\tan^{-1}}{8a^4c^3\left(a^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^3,x]
```

```
[Out] (27*Sqrt[ArcTan[a*x]])/(256*a^5*c^3) + (3*x^4*Sqrt[ArcTan[a*x]])/(32*a*c^3*
(1 + a^2*x^2)^2) - (9*Sqrt[ArcTan[a*x]])/(32*a^5*c^3*(1 + a^2*x^2)) - (x^3*
ArcTan[a*x]^(3/2))/(4*a^2*c^3*(1 + a^2*x^2)^2) - (3*x*ArcTan[a*x]^(3/2))/(8
*a^4*c^3*(1 + a^2*x^2)) + (3*ArcTan[a*x]^(5/2))/(20*a^5*c^3) - (3*Sqrt[Pi/2
]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(512*a^5*c^3) + (3*Sqrt[Pi]*Fre
snelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(32*a^5*c^3)
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_)*((d_) + (e_.)
*(x_)^2)^(q_), x_Symbol] :> Simp[(b*p*(f*x)^m*(d + e*x^2)^(q + 1)*(a + b*Ar
```

```
cTan[c*x])^(p - 1))/(c*d*m^2), x] + (Dist[(f^2*(m - 1))/(c^2*d*m), Int[(f*x
)^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p, x], x] - Dist[(b^2*p*(
p - 1))/m^2, Int[(f*x)^m*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x]
- Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(c^2*d*m
), x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q +
 2, 0] && LtQ[q, -1] && GtQ[p, 1]
```

Rule 4936

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^2)/((d_) + (e_.)*(x_)^2)
^2, x_Symbol] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(2*b*c^3*d^2*(p + 1)), x]
 + (Dist[(b*p)/(2*c), Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(d + e*x^2)^2, x]
, x] - Simp[(x*(a + b*ArcTan[c*x])^p)/(2*c^2*d*(d + e*x^2)), x]) /; FreeQ[{
a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 4904

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4970

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sin[x]^m)/
Cos[x]^(m + 2*(q + 1)), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x^4 \tan^{-1}(ax)^{3/2}}{\left(c+a^2cx^2\right)^3}\,dx = \frac{3x^4\sqrt{\tan^{-1}(ax)}}{32ac^3\left(1+a^2x^2\right)^2} - \frac{x^3\tan^{-1}(ax)^{3/2}}{4a^2c^3\left(1+a^2x^2\right)^2} - \frac{3}{64}\int \frac{x^4}{\left(c+a^2cx^2\right)^3\sqrt{\tan^{-1}(ax)}}\,dx + \frac{3\int \frac{x^2\tan^{-1}(ax)^{3/2}}{\left(c+a^2cx^2\right)^2}}{4a^2c}$$

$$= \frac{3x^4\sqrt{\tan^{-1}(ax)}}{32ac^3\left(1+a^2x^2\right)^2} - \frac{x^3\tan^{-1}(ax)^{3/2}}{4a^2c^3\left(1+a^2x^2\right)^2} - \frac{3x\tan^{-1}(ax)^{3/2}}{8a^4c^3\left(1+a^2x^2\right)} + \frac{3\tan^{-1}(ax)^{5/2}}{20a^5c^3} - \frac{3\,\mathrm{Subst}\left(\int \frac{\sin^4(x}{\sqrt{x}}\right.}{64}$$

$$= \frac{3x^4\sqrt{\tan^{-1}(ax)}}{32ac^3\left(1+a^2x^2\right)^2} - \frac{9\sqrt{\tan^{-1}(ax)}}{32a^5c^3\left(1+a^2x^2\right)} - \frac{x^3\tan^{-1}(ax)^{3/2}}{4a^2c^3\left(1+a^2x^2\right)^2} - \frac{3x\tan^{-1}(ax)^{3/2}}{8a^4c^3\left(1+a^2x^2\right)} + \frac{3\tan^{-1}(ax)^{5/}}{20a^5c^3}$$

$$= -\frac{9\sqrt{\tan^{-1}(ax)}}{256a^5c^3} + \frac{3x^4\sqrt{\tan^{-1}(ax)}}{32ac^3\left(1+a^2x^2\right)^2} - \frac{9\sqrt{\tan^{-1}(ax)}}{32a^5c^3\left(1+a^2x^2\right)} - \frac{x^3\tan^{-1}(ax)^{3/2}}{4a^2c^3\left(1+a^2x^2\right)^2} - \frac{3x\tan^{-1}(ax)^{3/}}{8a^4c^3\left(1+a^2x^2\right.}$$

$$= -\frac{9\sqrt{\tan^{-1}(ax)}}{256a^5c^3} + \frac{3x^4\sqrt{\tan^{-1}(ax)}}{32ac^3\left(1+a^2x^2\right)^2} - \frac{9\sqrt{\tan^{-1}(ax)}}{32a^5c^3\left(1+a^2x^2\right)} - \frac{x^3\tan^{-1}(ax)^{3/2}}{4a^2c^3\left(1+a^2x^2\right)^2} - \frac{3x\tan^{-1}(ax)^{3/}}{8a^4c^3\left(1+a^2x^2\right.}$$

$$= \frac{27\sqrt{\tan^{-1}(ax)}}{256a^5c^3} + \frac{3x^4\sqrt{\tan^{-1}(ax)}}{32ac^3\left(1+a^2x^2\right)^2} - \frac{9\sqrt{\tan^{-1}(ax)}}{32a^5c^3\left(1+a^2x^2\right)} - \frac{x^3\tan^{-1}(ax)^{3/2}}{4a^2c^3\left(1+a^2x^2\right)^2} - \frac{3x\tan^{-1}(ax)^{3/}}{8a^4c^3\left(1+a^2x^2\right.}$$

$$= \frac{27\sqrt{\tan^{-1}(ax)}}{256a^5c^3} + \frac{3x^4\sqrt{\tan^{-1}(ax)}}{32ac^3\left(1+a^2x^2\right)^2} - \frac{9\sqrt{\tan^{-1}(ax)}}{32a^5c^3\left(1+a^2x^2\right)} - \frac{x^3\tan^{-1}(ax)^{3/2}}{4a^2c^3\left(1+a^2x^2\right)^2} - \frac{3x\tan^{-1}(ax)^{3/}}{8a^4c^3\left(1+a^2x^2\right.}$$

$$= \frac{27\sqrt{\tan^{-1}(ax)}}{256a^5c^3} + \frac{3x^4\sqrt{\tan^{-1}(ax)}}{32ac^3\left(1+a^2x^2\right)^2} - \frac{9\sqrt{\tan^{-1}(ax)}}{32a^5c^3\left(1+a^2x^2\right)} - \frac{x^3\tan^{-1}(ax)^{3/2}}{4a^2c^3\left(1+a^2x^2\right)^2} - \frac{3x\tan^{-1}(ax)^{3/}}{8a^4c^3\left(1+a^2x^2\right.}$$

Mathematica [C] time = 0.775524, size = 355, normalized size = 1.54

$$\frac{90\sqrt{\tan^{-1}(ax)}\left(\frac{\mathrm{Gamma}\left(\frac{1}{2},-4i\tan^{-1}(ax)\right)}{\sqrt{-i\tan^{-1}(ax)}} + \frac{\mathrm{Gamma}\left(\frac{1}{2},4i\tan^{-1}(ax)\right)}{\sqrt{i\tan^{-1}(ax)}} + 8\right) + \frac{225\left(-4i\sqrt{2}\sqrt{-i\tan^{-1}(ax)}\mathrm{Gamma}\left(\frac{1}{2},-2i\tan^{-1}(ax)\right) + 4i\sqrt{2}\sqrt{i\tan^{-1}\right.}{}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^3,x]
```

```
[Out] ((64*Sqrt[ArcTan[a*x]]*(15*(-15 - 6*a^2*x^2 + 17*a^4*x^4) - 160*a*x*(3 + 5*
a^2*x^2)*ArcTan[a*x] + 192*(1 + a^2*x^2)^2*ArcTan[a*x]^2))/(1 + a^2*x^2)^2
- 510*(12*Sqrt[ArcTan[a*x]] + Sqrt[2*Pi]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcTan[
a*x]]] - 8*Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]]) + 90*Sqrt[Arc
Tan[a*x]]*(8 + Gamma[1/2, (-4*I)*ArcTan[a*x]]/Sqrt[(-I)*ArcTan[a*x]] + Gamm
a[1/2, (4*I)*ArcTan[a*x]]/Sqrt[I*ArcTan[a*x]]) + (225*(24*ArcTan[a*x] - (4*
I)*Sqrt[2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-2*I)*ArcTan[a*x]] + (4*I)*Sq
rt[2]*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (2*I)*ArcTan[a*x]] - I*Sqrt[(-I)*ArcTa
n[a*x]]*Gamma[1/2, (-4*I)*ArcTan[a*x]] + I*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (
4*I)*ArcTan[a*x]]))/Sqrt[ArcTan[a*x]])/(81920*a^5*c^3)
```

Maple [A] time = 0.168, size = 132, normalized size = 0.6

$$\frac{1}{5120\,c^3a^5}\left(768\,(\arctan(ax))^3-1280\,(\arctan(ax))^2\sin(2\arctan(ax))+160\,(\arctan(ax))^2\sin(4\arctan(ax))-15\,\mathrm{v}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x)
```

```
[Out] 1/5120/a^5/c^3/arctan(a*x)^(1/2)*(768*arctan(a*x)^3-1280*arctan(a*x)^2*sin(
2*arctan(a*x))+160*arctan(a*x)^2*sin(4*arctan(a*x))-15*2^(1/2)*arctan(a*x)^
(1/2)*Pi^(1/2)*FresnelC(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))+60*cos(4*arct
an(a*x))*arctan(a*x)-960*cos(2*arctan(a*x))*arctan(a*x)+480*arctan(a*x)^(1/
2)*Pi^(1/2)*FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^4 \operatorname{atan}^{\frac{3}{2}}(ax)}{a^6x^6+3a^4x^4+3a^2x^2+1}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*atan(a*x)**(3/2)/(a**2*c*x**2+c)**3,x)
```

```
[Out] Integral(x**4*atan(a*x)**(3/2)/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1),
 x)/c**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4 \arctan(ax)^{\frac{3}{2}}}{\left(a^2cx^2+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] integrate(x^4*arctan(a*x)^(3/2)/(a^2*c*x^2 + c)^3, x)
```

3.792 $\int \frac{x^3 \tan^{-1}(ax)^{3/2}}{\left(c+a^2cx^2\right)^3} dx$

Optimal. Leaf size=168

$$\frac{3\sqrt{\frac{\pi}{2}}S\left(2\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{512a^4c^3} - \frac{3\sqrt{\pi}S\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{64a^4c^3} + \frac{x^4\tan^{-1}(ax)^{3/2}}{4c^3\left(a^2x^2+1\right)^2} - \frac{3\tan^{-1}(ax)^{3/2}}{32a^4c^3} + \frac{3\sqrt{\tan^{-1}(ax)}\sin\left(2\tan^{-1}(ax)\right)}{32a^4c^3} -$$

```
[Out] (-3*ArcTan[a*x]^(3/2))/(32*a^4*c^3) + (x^4*ArcTan[a*x]^(3/2))/(4*c^3*(1 + a
^2*x^2)^2) + (3*Sqrt[Pi/2]*FresnelS[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(512*a
^4*c^3) - (3*Sqrt[Pi]*FresnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(64*a^4*c^3
) + (3*Sqrt[ArcTan[a*x]]*Sin[2*ArcTan[a*x]])/(32*a^4*c^3) - (3*Sqrt[ArcTan[
a*x]]*Sin[4*ArcTan[a*x]])/(256*a^4*c^3)
```

Rubi [A] time = 0.247115, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4944, 4970, 3312, 3296, 3305, 3351}

$$\frac{3\sqrt{\frac{\pi}{2}}S\left(2\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{512a^4c^3} - \frac{3\sqrt{\pi}S\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{64a^4c^3} + \frac{x^4\tan^{-1}(ax)^{3/2}}{4c^3\left(a^2x^2+1\right)^2} - \frac{3\tan^{-1}(ax)^{3/2}}{32a^4c^3} + \frac{3\sqrt{\tan^{-1}(ax)}\sin\left(2\tan^{-1}(ax)\right)}{32a^4c^3} -$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^3,x]
```


```
[Out] (-3*ArcTan[a*x]^(3/2))/(32*a^4*c^3) + (x^4*ArcTan[a*x]^(3/2))/(4*c^3*(1 + a
^2*x^2)^2) + (3*Sqrt[Pi/2]*FresnelS[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(512*a
^4*c^3) - (3*Sqrt[Pi]*FresnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(64*a^4*c^3
) + (3*Sqrt[ArcTan[a*x]]*Sin[2*ArcTan[a*x]])/(32*a^4*c^3) - (3*Sqrt[ArcTan[
a*x]]*Sin[4*ArcTan[a*x]])/(256*a^4*c^3)
```


Rule 4944

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a +
 b*ArcTan[c*x])^p)/(d*f*(m + 1)), x] - Dist[(b*c*p)/(f*(m + 1)), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
```

```
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rule 4970

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sin[x]^m)/
Cos[x]^(m + 2*(q + 1)), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{x^3 \tan^{-1}(ax)^{3/2}}{\left(c+a^2cx^2\right)^3}\,dx = \frac{x^4 \tan^{-1}(ax)^{3/2}}{4c^3\left(1+a^2x^2\right)^2} - \frac{1}{8}(3a)\int \frac{x^4\sqrt{\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^3}\,dx$$
$$= \frac{x^4 \tan^{-1}(ax)^{3/2}}{4c^3\left(1+a^2x^2\right)^2} - \frac{3\,\mathrm{Subst}\left(\int \sqrt{x}\sin^4(x)\,dx, x, \tan^{-1}(ax)\right)}{8a^4c^3}$$
$$= \frac{x^4 \tan^{-1}(ax)^{3/2}}{4c^3\left(1+a^2x^2\right)^2} - \frac{3\,\mathrm{Subst}\left(\int \left(\frac{3\sqrt{x}}{8} - \frac{1}{2}\sqrt{x}\cos(2x) + \frac{1}{8}\sqrt{x}\cos(4x)\right)dx, x, \tan^{-1}(ax)\right)}{8a^4c^3}$$
$$= -\frac{3\tan^{-1}(ax)^{3/2}}{32a^4c^3} + \frac{x^4 \tan^{-1}(ax)^{3/2}}{4c^3\left(1+a^2x^2\right)^2} - \frac{3\,\mathrm{Subst}\left(\int \sqrt{x}\cos(4x)\,dx, x, \tan^{-1}(ax)\right)}{64a^4c^3} + \frac{3\,\mathrm{Subst}\left(\int \sqrt{x}\right.}{}$$
$$= -\frac{3\tan^{-1}(ax)^{3/2}}{32a^4c^3} + \frac{x^4 \tan^{-1}(ax)^{3/2}}{4c^3\left(1+a^2x^2\right)^2} + \frac{3\sqrt{\tan^{-1}(ax)}\sin\left(2\tan^{-1}(ax)\right)}{32a^4c^3} - \frac{3\sqrt{\tan^{-1}(ax)}\sin\left(4\tan^{-1}\right.}{256a^4c^3}$$
$$= -\frac{3\tan^{-1}(ax)^{3/2}}{32a^4c^3} + \frac{x^4 \tan^{-1}(ax)^{3/2}}{4c^3\left(1+a^2x^2\right)^2} + \frac{3\sqrt{\tan^{-1}(ax)}\sin\left(2\tan^{-1}(ax)\right)}{32a^4c^3} - \frac{3\sqrt{\tan^{-1}(ax)}\sin\left(4\tan^{-1}\right.}{256a^4c^3}$$
$$= -\frac{3\tan^{-1}(ax)^{3/2}}{32a^4c^3} + \frac{x^4 \tan^{-1}(ax)^{3/2}}{4c^3\left(1+a^2x^2\right)^2} + \frac{3\sqrt{\frac{\pi}{2}}S\left(2\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{512a^4c^3} - \frac{3\sqrt{\pi}S\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{64a^4c^3} + \frac{3\sqrt{\tan}}{}$$

Mathematica [C] time = 0.280525, size = 350, normalized size = 2.08

$$-\frac{9\left(-2\sqrt{2}\sqrt{-i\tan^{-1}(ax)}\mathrm{Gamma}\left(\frac{1}{2},-2i\tan^{-1}(ax)\right) - 2\sqrt{2}\sqrt{i\tan^{-1}(ax)}\mathrm{Gamma}\left(\frac{1}{2},2i\tan^{-1}(ax)\right) - \sqrt{-i\tan^{-1}(ax)}\mathrm{Ga}\right.}{4096a^4c^3\sqrt{\tan^{-1}(ax)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^3,x]
```

```
[Out] Sqrt[ArcTan[a*x]]*((3*x*(3 + 5*a^2*x^2))/(64*a^3*c^3*(1 + a^2*x^2)^2) + ((-
3 - 6*a^2*x^2 + 5*a^4*x^4)*ArcTan[a*x])/(32*a^4*c^3*(1 + a^2*x^2)^2)) - (9*
(-2*Sqrt[2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-2*I)*ArcTan[a*x]] - 2*Sqrt[
2]*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (2*I)*ArcTan[a*x]] - Sqrt[(-I)*ArcTan[a*x
]]*Gamma[1/2, (-4*I)*ArcTan[a*x]] - Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (4*I)*Ar
cTan[a*x]]))/(4096*a^4*c^3*Sqrt[ArcTan[a*x]]) - (15*(-2*Sqrt[2]*Sqrt[(-I)*A
```

```
rcTan[a*x]]*Gamma[1/2, (-2*I)*ArcTan[a*x]] - 2*Sqrt[2]*Sqrt[I*ArcTan[a*x]]*
Gamma[1/2, (2*I)*ArcTan[a*x]] + Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-4*I)*Ar
cTan[a*x]] + Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (4*I)*ArcTan[a*x]]))/(4096*a^4*
c^3*Sqrt[ArcTan[a*x]])
```

Maple [A] time = 0.12, size = 124, normalized size = 0.7

$$-\frac{1}{1024\,c^3a^4}\left(-3\,\sqrt{2}\sqrt{\arctan(ax)}\sqrt{\pi}\mathrm{FresnelS}\left(2\,\frac{\sqrt{2}\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+128\,(\arctan(ax))^2\cos(2\,\arctan(ax))-32\,(\mathrm{arc}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x)
```

```
[Out] -1/1024/a^4/c^3*(-3*2^(1/2)*arctan(a*x)^(1/2)*Pi^(1/2)*FresnelS(2*2^(1/2)/P
i^(1/2)*arctan(a*x)^(1/2))+128*arctan(a*x)^2*cos(2*arctan(a*x))-32*arctan(a
*x)^2*cos(4*arctan(a*x))+48*arctan(a*x)^(1/2)*Pi^(1/2)*FresnelS(2*arctan(a*
x)^(1/2)/Pi^(1/2))-96*sin(2*arctan(a*x))*arctan(a*x)+12*sin(4*arctan(a*x))*
arctan(a*x))/arctan(a*x)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^3 \operatorname{atan}^{\frac{3}{2}}(ax)}{a^6x^6+3a^4x^4+3a^2x^2+1}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atan(a*x)**(3/2)/(a**2*c*x**2+c)**3,x)
```

```
[Out] Integral(x**3*atan(a*x)**(3/2)/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1),
 x)/c**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \arctan(ax)^{\frac{3}{2}}}{\left(a^2cx^2+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] integrate(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2 + c)^3, x)
```

3.793 $\int \frac{x^2 \tan^{-1}(ax)^{3/2}}{(c+a^2cx^2)^3} dx$

Optimal. Leaf size=108

$$\frac{3\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{512a^3c^3}+\frac{\tan^{-1}(ax)^{5/2}}{20a^3c^3}-\frac{\tan^{-1}(ax)^{3/2}\sin\left(4\tan^{-1}(ax)\right)}{32a^3c^3}-\frac{3\sqrt{\tan^{-1}(ax)}\cos\left(4\tan^{-1}(ax)\right)}{256a^3c^3}$$

```
[Out] ArcTan[a*x]^(5/2)/(20*a^3*c^3) - (3*Sqrt[ArcTan[a*x]]*Cos[4*ArcTan[a*x]])/(
256*a^3*c^3) + (3*Sqrt[Pi/2]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(512
*a^3*c^3) - (ArcTan[a*x]^(3/2)*Sin[4*ArcTan[a*x]])/(32*a^3*c^3)
```

Rubi [A] time = 0.162614, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {4970, 4406, 3296, 3304, 3352}

$$\frac{3\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{512a^3c^3}+\frac{\tan^{-1}(ax)^{5/2}}{20a^3c^3}-\frac{\tan^{-1}(ax)^{3/2}\sin\left(4\tan^{-1}(ax)\right)}{32a^3c^3}-\frac{3\sqrt{\tan^{-1}(ax)}\cos\left(4\tan^{-1}(ax)\right)}{256a^3c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^3,x]
```

```
[Out] ArcTan[a*x]^(5/2)/(20*a^3*c^3) - (3*Sqrt[ArcTan[a*x]]*Cos[4*ArcTan[a*x]])/(
256*a^3*c^3) + (3*Sqrt[Pi/2]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(512
*a^3*c^3) - (ArcTan[a*x]^(3/2)*Sin[4*ArcTan[a*x]])/(32*a^3*c^3)
```

Rule 4970

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sin[x]^m)/
Cos[x]^(m + 2*(q + 1)), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
```

```
tQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{x^2 \tan^{-1}(ax)^{3/2}}{\left(c+a^2cx^2\right)^3}\,dx = \frac{\text{Subst}\left(\int x^{3/2}\cos^2(x)\sin^2(x)\,dx, x, \tan^{-1}(ax)\right)}{a^3c^3}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{x^{3/2}}{8}-\frac{1}{8}x^{3/2}\cos(4x)\right)\,dx, x, \tan^{-1}(ax)\right)}{a^3c^3}$$

$$= \frac{\tan^{-1}(ax)^{5/2}}{20a^3c^3} - \frac{\text{Subst}\left(\int x^{3/2}\cos(4x)\,dx, x, \tan^{-1}(ax)\right)}{8a^3c^3}$$

$$= \frac{\tan^{-1}(ax)^{5/2}}{20a^3c^3} - \frac{\tan^{-1}(ax)^{3/2}\sin\left(4\tan^{-1}(ax)\right)}{32a^3c^3} + \frac{3\,\text{Subst}\left(\int \sqrt{x}\sin(4x)\,dx, x, \tan^{-1}(ax)\right)}{64a^3c^3}$$

$$= \frac{\tan^{-1}(ax)^{5/2}}{20a^3c^3} - \frac{3\sqrt{\tan^{-1}(ax)}\cos\left(4\tan^{-1}(ax)\right)}{256a^3c^3} - \frac{\tan^{-1}(ax)^{3/2}\sin\left(4\tan^{-1}(ax)\right)}{32a^3c^3} + \frac{3\,\text{Subst}\left(\int \right.}{}$$

$$= \frac{\tan^{-1}(ax)^{5/2}}{20a^3c^3} - \frac{3\sqrt{\tan^{-1}(ax)}\cos\left(4\tan^{-1}(ax)\right)}{256a^3c^3} - \frac{\tan^{-1}(ax)^{3/2}\sin\left(4\tan^{-1}(ax)\right)}{32a^3c^3} + \frac{3\,\text{Subst}\left(\int \right.}{}$$

$$= \frac{\tan^{-1}(ax)^{5/2}}{20a^3c^3} - \frac{3\sqrt{\tan^{-1}(ax)}\cos\left(4\tan^{-1}(ax)\right)}{256a^3c^3} + \frac{3\sqrt{\frac{\pi}{2}}C\left(2\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{512a^3c^3} - \frac{\tan^{-1}(ax)^{3/2}\sin}{32a^3}$$

Mathematica [C] time = 0.745112, size = 353, normalized size = 3.27

$$\frac{-90\sqrt{\tan^{-1}(ax)}\left(\frac{\text{Gamma}\left(\frac{1}{2},-4i\tan^{-1}(ax)\right)}{\sqrt{-i\tan^{-1}(ax)}}+\frac{\text{Gamma}\left(\frac{1}{2},4i\tan^{-1}(ax)\right)}{\sqrt{i\tan^{-1}(ax)}}+8\right)+\frac{15\left(-4i\sqrt{2}\sqrt{-i\tan^{-1}(ax)}\text{Gamma}\left(\frac{1}{2},-2i\tan^{-1}(ax)\right)+4i\sqrt{2}\sqrt{i\tan^{-1}\cdots}\right.}{\cdots}}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^3,x]
```

```
[Out] ((64*Sqrt[ArcTan[a*x]]*(-15*(1 - 6*a^2*x^2 + a^4*x^4) + 160*a*x*(-1 + a^2*x
^2)*ArcTan[a*x] + 64*(1 + a^2*x^2)^2*ArcTan[a*x]^2))/(1 + a^2*x^2)^2 + 30*(
12*Sqrt[ArcTan[a*x]] + Sqrt[2*Pi]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]]
- 8*Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]]) - 90*Sqrt[ArcTan[a*x
]]*(8 + Gamma[1/2, (-4*I)*ArcTan[a*x]]/Sqrt[(-I)*ArcTan[a*x]] + Gamma[1/2,
(4*I)*ArcTan[a*x]]/Sqrt[I*ArcTan[a*x]]) + (15*(24*ArcTan[a*x] - (4*I)*Sqrt[
2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-2*I)*ArcTan[a*x]] + (4*I)*Sqrt[2]*Sq
rt[I*ArcTan[a*x]]*Gamma[1/2, (2*I)*ArcTan[a*x]] - I*Sqrt[(-I)*ArcTan[a*x]]*
Gamma[1/2, (-4*I)*ArcTan[a*x]] + I*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (4*I)*Arc
Tan[a*x]]))/Sqrt[ArcTan[a*x]])/(81920*a^3*c^3)
```

Maple [A] time = 0.111, size = 81, normalized size = 0.8

$$\frac{1}{5120\,c^3a^3}\left(15\,\sqrt{2}\sqrt{\arctan(ax)}\sqrt{\pi}\text{FresnelC}\left(2\,\frac{\sqrt{2}\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+256\,(\arctan(ax))^3-160\,(\arctan(ax))^2\sin(4\,\text{ar}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x)
```

```
[Out] 1/5120/a^3/c^3*(15*2^(1/2)*arctan(a*x)^(1/2)*Pi^(1/2)*FresnelC(2*2^(1/2)/Pi
^(1/2)*arctan(a*x)^(1/2))+256*arctan(a*x)^3-160*arctan(a*x)^2*sin(4*arctan(
a*x))-60*cos(4*arctan(a*x))*arctan(a*x))/arctan(a*x)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^2 \operatorname{atan}^{\frac{3}{2}}(ax)}{a^6x^6+3a^4x^4+3a^2x^2+1}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(a*x)**(3/2)/(a**2*c*x**2+c)**3,x)
```

```
[Out] Integral(x**2*atan(a*x)**(3/2)/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1),
 x)/c**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \arctan(ax)^{\frac{3}{2}}}{\left(a^2cx^2+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] integrate(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2 + c)^3, x)
```

3.794 $\int \frac{x\tan^{-1}(ax)^{3/2}}{(c+a^2cx^2)^3}\,dx$

Optimal. Leaf size=168

$$-\frac{3\sqrt{\frac{\pi}{2}}S\left(2\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{512a^2c^3}-\frac{3\sqrt{\pi}S\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{64a^2c^3}-\frac{\tan^{-1}(ax)^{3/2}}{4a^2c^3\left(a^2x^2+1\right)^2}+\frac{3\tan^{-1}(ax)^{3/2}}{32a^2c^3}+\frac{3\sqrt{\tan^{-1}(ax)}\sin\left(2\tan^{-1}(a\right.}{32a^2c^3}$$

```
[Out] (3*ArcTan[a*x]^(3/2))/(32*a^2*c^3) - ArcTan[a*x]^(3/2)/(4*a^2*c^3*(1 + a^2*
x^2)^2) - (3*Sqrt[Pi/2]*FresnelS[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(512*a^2*
c^3) - (3*Sqrt[Pi]*FresnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(64*a^2*c^3) +
 (3*Sqrt[ArcTan[a*x]]*Sin[2*ArcTan[a*x]])/(32*a^2*c^3) + (3*Sqrt[ArcTan[a*x
]]*Sin[4*ArcTan[a*x]])/(256*a^2*c^3)
```

Rubi [A] time = 0.185752, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4930, 4904, 3312, 3296, 3305, 3351}

$$-\frac{3\sqrt{\frac{\pi}{2}}S\left(2\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{512a^2c^3}-\frac{3\sqrt{\pi}S\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{64a^2c^3}-\frac{\tan^{-1}(ax)^{3/2}}{4a^2c^3\left(a^2x^2+1\right)^2}+\frac{3\tan^{-1}(ax)^{3/2}}{32a^2c^3}+\frac{3\sqrt{\tan^{-1}(ax)}\sin\left(2\tan^{-1}(a\right.}{32a^2c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^3,x]
```


```
[Out] (3*ArcTan[a*x]^(3/2))/(32*a^2*c^3) - ArcTan[a*x]^(3/2)/(4*a^2*c^3*(1 + a^2*
x^2)^2) - (3*Sqrt[Pi/2]*FresnelS[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(512*a^2*
c^3) - (3*Sqrt[Pi]*FresnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(64*a^2*c^3) +
 (3*Sqrt[ArcTan[a*x]]*Sin[2*ArcTan[a*x]])/(32*a^2*c^3) + (3*Sqrt[ArcTan[a*x
]]*Sin[4*ArcTan[a*x]])/(256*a^2*c^3)
```


Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
```

```
0] && NeQ[q, -1]
```

Rule 4904

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{x\tan^{-1}(ax)^{3/2}}{\left(c+a^2cx^2\right)^3}\,dx = -\frac{\tan^{-1}(ax)^{3/2}}{4a^2c^3\left(1+a^2x^2\right)^2} + \frac{3\int \frac{\sqrt{\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^3}\,dx}{8a}$$

$$= -\frac{\tan^{-1}(ax)^{3/2}}{4a^2c^3\left(1+a^2x^2\right)^2} + \frac{3\,\text{Subst}\left(\int \sqrt{x}\cos^4(x)\,dx, x, \tan^{-1}(ax)\right)}{8a^2c^3}$$

$$= -\frac{\tan^{-1}(ax)^{3/2}}{4a^2c^3\left(1+a^2x^2\right)^2} + \frac{3\,\text{Subst}\left(\int \left(\frac{3\sqrt{x}}{8} + \frac{1}{2}\sqrt{x}\cos(2x) + \frac{1}{8}\sqrt{x}\cos(4x)\right)dx, x, \tan^{-1}(ax)\right)}{8a^2c^3}$$

$$= \frac{3\tan^{-1}(ax)^{3/2}}{32a^2c^3} - \frac{\tan^{-1}(ax)^{3/2}}{4a^2c^3\left(1+a^2x^2\right)^2} + \frac{3\,\text{Subst}\left(\int \sqrt{x}\cos(4x)\,dx, x, \tan^{-1}(ax)\right)}{64a^2c^3} + \frac{3\,\text{Subst}\left(\int \sqrt{x}\right.}{}$$

$$= \frac{3\tan^{-1}(ax)^{3/2}}{32a^2c^3} - \frac{\tan^{-1}(ax)^{3/2}}{4a^2c^3\left(1+a^2x^2\right)^2} + \frac{3\sqrt{\tan^{-1}(ax)}\sin\left(2\tan^{-1}(ax)\right)}{32a^2c^3} + \frac{3\sqrt{\tan^{-1}(ax)}\sin\left(4\tan\right.}{256a^2c^3}$$

$$= \frac{3\tan^{-1}(ax)^{3/2}}{32a^2c^3} - \frac{\tan^{-1}(ax)^{3/2}}{4a^2c^3\left(1+a^2x^2\right)^2} + \frac{3\sqrt{\tan^{-1}(ax)}\sin\left(2\tan^{-1}(ax)\right)}{32a^2c^3} + \frac{3\sqrt{\tan^{-1}(ax)}\sin\left(4\tan\right.}{256a^2c^3}$$

$$= \frac{3\tan^{-1}(ax)^{3/2}}{32a^2c^3} - \frac{\tan^{-1}(ax)^{3/2}}{4a^2c^3\left(1+a^2x^2\right)^2} - \frac{3\sqrt{\frac{\pi}{2}}S\left(2\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{512a^2c^3} - \frac{3\sqrt{\pi}S\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{64a^2c^3} + \frac{3\sqrt{\text{ta}}}{}$$

Mathematica [C] time = 0.234078, size = 347, normalized size = 2.07

$$\frac{3a^4x^4\sqrt{-i\tan^{-1}(ax)}\text{Gamma}\left(\frac{1}{2}, -4i\tan^{-1}(ax)\right) + 3a^4x^4\sqrt{i\tan^{-1}(ax)}\text{Gamma}\left(\frac{1}{2}, 4i\tan^{-1}(ax)\right) + 6a^2x^2\sqrt{-i\tan^{-1}(ax}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^3,x]
```

```
[Out] (480*a*x*ArcTan[a*x] + 288*a^3*x^3*ArcTan[a*x] - 320*ArcTan[a*x]^2 + 384*a^
2*x^2*ArcTan[a*x]^2 + 192*a^4*x^4*ArcTan[a*x]^2 + 24*Sqrt[2]*(1 + a^2*x^2)^
2*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-2*I)*ArcTan[a*x]] + 24*Sqrt[2]*(1 + a
^2*x^2)^2*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (2*I)*ArcTan[a*x]] + 3*Sqrt[(-I)*A
rcTan[a*x]]*Gamma[1/2, (-4*I)*ArcTan[a*x]] + 6*a^2*x^2*Sqrt[(-I)*ArcTan[a*x
```

```
]]*Gamma[1/2, (-4*I)*ArcTan[a*x]] + 3*a^4*x^4*Sqrt[(-I)*ArcTan[a*x]]*Gamma[
1/2, (-4*I)*ArcTan[a*x]] + 3*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (4*I)*ArcTan[a*
x]] + 6*a^2*x^2*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (4*I)*ArcTan[a*x]] + 3*a^4*x
^4*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (4*I)*ArcTan[a*x]])/(2048*c^3*(a + a^3*x^
2)^2*Sqrt[ArcTan[a*x]])
```

Maple [A] time = 0.14, size = 124, normalized size = 0.7

$$-\frac{1}{1024\,c^{3}a^{2}}\left(3\,\sqrt{2}\sqrt{\arctan\left(ax\right)}\sqrt{\pi}\mathrm{FresnelS}\left(2\,\frac{\sqrt{2}\sqrt{\arctan\left(ax\right)}}{\sqrt{\pi}}\right)+128\,\left(\arctan\left(ax\right)\right)^{2}\cos\left(2\,\arctan\left(ax\right)\right)+32\,\left(\mathrm{arcta}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x)
```

```
[Out] -1/1024/a^2/c^3*(3*2^(1/2)*arctan(a*x)^(1/2)*Pi^(1/2)*FresnelS(2*2^(1/2)/Pi
^(1/2)*arctan(a*x)^(1/2))+128*arctan(a*x)^2*cos(2*arctan(a*x))+32*arctan(a*
x)^2*cos(4*arctan(a*x))+48*arctan(a*x)^(1/2)*Pi^(1/2)*FresnelS(2*arctan(a*x
)^(1/2)/Pi^(1/2))-96*sin(2*arctan(a*x))*arctan(a*x)-12*sin(4*arctan(a*x))*a
rctan(a*x))/arctan(a*x)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x \operatorname{atan}^{\frac{3}{2}}(ax)}{a^6x^6+3a^4x^4+3a^2x^2+1}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(a*x)**(3/2)/(a**2*c*x**2+c)**3,x)
```

```
[Out] Integral(x*atan(a*x)**(3/2)/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1), x)
/c**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \arctan(ax)^{\frac{3}{2}}}{\left(a^2cx^2+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] integrate(x*arctan(a*x)^(3/2)/(a^2*c*x^2 + c)^3, x)
```

3.795 $\int \frac{\tan^{-1}(ax)^{3/2}}{(c+a^2cx^2)^3}\,dx$

Optimal. Leaf size=219

$$\frac{3x\tan^{-1}(ax)^{3/2}}{8c^3\left(a^2x^2+1\right)}+\frac{x\tan^{-1}(ax)^{3/2}}{4c^3\left(a^2x^2+1\right)^2}+\frac{9\sqrt{\tan^{-1}(ax)}}{32ac^3\left(a^2x^2+1\right)}+\frac{3\sqrt{\tan^{-1}(ax)}}{32ac^3\left(a^2x^2+1\right)^2}-\frac{3\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{512ac^3}-\frac{3\sqrt{\pi}}{}$$

```
[Out] (-45*Sqrt[ArcTan[a*x]])/(256*a*c^3) + (3*Sqrt[ArcTan[a*x]])/(32*a*c^3*(1 +
a^2*x^2)^2) + (9*Sqrt[ArcTan[a*x]])/(32*a*c^3*(1 + a^2*x^2)) + (x*ArcTan[a*
x]^(3/2))/(4*c^3*(1 + a^2*x^2)^2) + (3*x*ArcTan[a*x]^(3/2))/(8*c^3*(1 + a^2
*x^2)) + (3*ArcTan[a*x]^(5/2))/(20*a*c^3) - (3*Sqrt[Pi/2]*FresnelC[2*Sqrt[2
/Pi]*Sqrt[ArcTan[a*x]]])/(512*a*c^3) - (3*Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[
a*x]])/Sqrt[Pi]])/(32*a*c^3)
```

Rubi [A] time = 0.290959, antiderivative size = 219, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4900, 4892, 4930, 4904, 3312, 3304, 3352}

$$\frac{3x\tan^{-1}(ax)^{3/2}}{8c^3\left(a^2x^2+1\right)}+\frac{x\tan^{-1}(ax)^{3/2}}{4c^3\left(a^2x^2+1\right)^2}+\frac{9\sqrt{\tan^{-1}(ax)}}{32ac^3\left(a^2x^2+1\right)}+\frac{3\sqrt{\tan^{-1}(ax)}}{32ac^3\left(a^2x^2+1\right)^2}-\frac{3\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{512ac^3}-\frac{3\sqrt{\pi}}{}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^(3/2)/(c + a^2*c*x^2)^3,x]
```


```
[Out] (-45*Sqrt[ArcTan[a*x]])/(256*a*c^3) + (3*Sqrt[ArcTan[a*x]])/(32*a*c^3*(1 +
a^2*x^2)^2) + (9*Sqrt[ArcTan[a*x]])/(32*a*c^3*(1 + a^2*x^2)) + (x*ArcTan[a*
x]^(3/2))/(4*c^3*(1 + a^2*x^2)^2) + (3*x*ArcTan[a*x]^(3/2))/(8*c^3*(1 + a^2
*x^2)) + (3*ArcTan[a*x]^(5/2))/(20*a*c^3) - (3*Sqrt[Pi/2]*FresnelC[2*Sqrt[2
/Pi]*Sqrt[ArcTan[a*x]]])/(512*a*c^3) - (3*Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[
a*x]])/Sqrt[Pi]])/(32*a*c^3)
```


Rule 4900

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(b*p*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^(p - 1))/(4*c*d
```

```
*(q + 1)^2), x] + (Dist[(2*q + 3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a
 + b*ArcTan[c*x])^p, x], x] - Dist[(b^2*p*(p - 1))/(4*(q + 1)^2), Int[(d +
e*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x] - Simp[(x*(d + e*x^2)^(q + 1)*
(a + b*ArcTan[c*x])^p)/(2*d*(q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] && E
qQ[e, c^2*d] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

Rule 4892

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[(x*(a + b*ArcTan[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*c*
p)/2, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(a +
 b*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 4904

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^{3/2}}{\left(c+a^2cx^2\right)^3}\,dx = \frac{3\sqrt{\tan^{-1}(ax)}}{32ac^3\left(1+a^2x^2\right)^2}+\frac{x\tan^{-1}(ax)^{3/2}}{4c^3\left(1+a^2x^2\right)^2}-\frac{3}{64}\int \frac{1}{\left(c+a^2cx^2\right)^3\sqrt{\tan^{-1}(ax)}}\,dx+\frac{3\int \frac{\tan^{-1}(ax)^{3/2}}{\left(c+a^2cx^2\right)^2}\,dx}{4c}$$

$$= \frac{3\sqrt{\tan^{-1}(ax)}}{32ac^3\left(1+a^2x^2\right)^2}+\frac{x\tan^{-1}(ax)^{3/2}}{4c^3\left(1+a^2x^2\right)^2}+\frac{3x\tan^{-1}(ax)^{3/2}}{8c^3\left(1+a^2x^2\right)}+\frac{3\tan^{-1}(ax)^{5/2}}{20ac^3}-\frac{3\,\mathrm{Subst}\left(\int \frac{\cos^4(x)}{\sqrt{x}}\,dx,x,\right.}{64ac^3}$$

$$= \frac{3\sqrt{\tan^{-1}(ax)}}{32ac^3\left(1+a^2x^2\right)^2}+\frac{9\sqrt{\tan^{-1}(ax)}}{32ac^3\left(1+a^2x^2\right)}+\frac{x\tan^{-1}(ax)^{3/2}}{4c^3\left(1+a^2x^2\right)^2}+\frac{3x\tan^{-1}(ax)^{3/2}}{8c^3\left(1+a^2x^2\right)}+\frac{3\tan^{-1}(ax)^{5/2}}{20ac^3}-\frac{3\,\mathrm{Su}}{}$$

$$= -\frac{9\sqrt{\tan^{-1}(ax)}}{256ac^3}+\frac{3\sqrt{\tan^{-1}(ax)}}{32ac^3\left(1+a^2x^2\right)^2}+\frac{9\sqrt{\tan^{-1}(ax)}}{32ac^3\left(1+a^2x^2\right)}+\frac{x\tan^{-1}(ax)^{3/2}}{4c^3\left(1+a^2x^2\right)^2}+\frac{3x\tan^{-1}(ax)^{3/2}}{8c^3\left(1+a^2x^2\right)}+\frac{3\,\mathrm{ta}}{}$$

$$= -\frac{9\sqrt{\tan^{-1}(ax)}}{256ac^3}+\frac{3\sqrt{\tan^{-1}(ax)}}{32ac^3\left(1+a^2x^2\right)^2}+\frac{9\sqrt{\tan^{-1}(ax)}}{32ac^3\left(1+a^2x^2\right)}+\frac{x\tan^{-1}(ax)^{3/2}}{4c^3\left(1+a^2x^2\right)^2}+\frac{3x\tan^{-1}(ax)^{3/2}}{8c^3\left(1+a^2x^2\right)}+\frac{3\,\mathrm{ta}}{}$$

$$= -\frac{45\sqrt{\tan^{-1}(ax)}}{256ac^3}+\frac{3\sqrt{\tan^{-1}(ax)}}{32ac^3\left(1+a^2x^2\right)^2}+\frac{9\sqrt{\tan^{-1}(ax)}}{32ac^3\left(1+a^2x^2\right)}+\frac{x\tan^{-1}(ax)^{3/2}}{4c^3\left(1+a^2x^2\right)^2}+\frac{3x\tan^{-1}(ax)^{3/2}}{8c^3\left(1+a^2x^2\right)}+\frac{3}{}$$

$$= -\frac{45\sqrt{\tan^{-1}(ax)}}{256ac^3}+\frac{3\sqrt{\tan^{-1}(ax)}}{32ac^3\left(1+a^2x^2\right)^2}+\frac{9\sqrt{\tan^{-1}(ax)}}{32ac^3\left(1+a^2x^2\right)}+\frac{x\tan^{-1}(ax)^{3/2}}{4c^3\left(1+a^2x^2\right)^2}+\frac{3x\tan^{-1}(ax)^{3/2}}{8c^3\left(1+a^2x^2\right)}+\frac{3}{}$$

$$= -\frac{45\sqrt{\tan^{-1}(ax)}}{256ac^3}+\frac{3\sqrt{\tan^{-1}(ax)}}{32ac^3\left(1+a^2x^2\right)^2}+\frac{9\sqrt{\tan^{-1}(ax)}}{32ac^3\left(1+a^2x^2\right)}+\frac{x\tan^{-1}(ax)^{3/2}}{4c^3\left(1+a^2x^2\right)^2}+\frac{3x\tan^{-1}(ax)^{3/2}}{8c^3\left(1+a^2x^2\right)}+\frac{3}{}$$

Mathematica [A] time = 0.571426, size = 123, normalized size = 0.56

$$\frac{-15\sqrt{2\pi}\,\mathrm{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)-480\sqrt{\pi}\,\mathrm{FresnelC}\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)+4\sqrt{\tan^{-1}(ax)}\left(8\tan^{-1}(ax)\left(24\tan^{-1}(ax)+40\right.\right.}{5120ac^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]^(3/2)/(c + a^2*c*x^2)^3,x]
```

```
[Out] (-15*Sqrt[2*Pi]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]] - 480*Sqrt[Pi]*Fre
snelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]] + 4*Sqrt[ArcTan[a*x]]*(240*Cos[2*ArcT
an[a*x]] + 15*Cos[4*ArcTan[a*x]] + 8*ArcTan[a*x]*(24*ArcTan[a*x] + 40*Sin[2
*ArcTan[a*x]] + 5*Sin[4*ArcTan[a*x]])))/(5120*a*c^3)
```

Maple [A] time = 0.118, size = 132, normalized size = 0.6

$$\frac{1}{5120\,ac^3}\Bigl(768\,(\arctan(ax))^3+1280\,(\arctan(ax))^2\sin(2\arctan(ax))+160\,(\arctan(ax))^2\sin(4\arctan(ax))-15$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x)
```

```
[Out] 1/5120/a/c^3/arctan(a*x)^(1/2)*(768*arctan(a*x)^3+1280*arctan(a*x)^2*sin(2*
arctan(a*x))+160*arctan(a*x)^2*sin(4*arctan(a*x))-15*2^(1/2)*arctan(a*x)^(1
/2)*Pi^(1/2)*FresnelC(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))+60*cos(4*arctan
(a*x))*arctan(a*x)+960*cos(2*arctan(a*x))*arctan(a*x)-480*arctan(a*x)^(1/2)
*Pi^(1/2)*FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\operatorname{atan}^{\frac{3}{2}}(ax)}{a^6x^6+3a^4x^4+3a^2x^2+1}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(3/2)/(a**2*c*x**2+c)**3,x)
```

```
[Out] Integral(atan(a*x)**(3/2)/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1), x)/c
**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(ax)^{\frac{3}{2}}}{\left(a^2cx^2+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)^(3/2)/(a^2*c*x^2 + c)^3, x)
```

3.796 $\int \frac{\tan^{-1}(ax)^{3/2}}{x\left(c+a^2cx^2\right)^3} dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{\tan^{-1}(ax)^{3/2}}{x\left(a^2cx^2+c\right)^3},x\right)$$

```
[Out] Unintegrable[ArcTan[a*x]^(3/2)/(x*(c + a^2*c*x^2)^3), x]
```

Rubi [A] time = 0.0606894, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\tan^{-1}(ax)^{3/2}}{x\left(c+a^2cx^2\right)^3} dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcTan[a*x]^(3/2)/(x*(c + a^2*c*x^2)^3),x]
```

```
[Out] Defer[Int][ArcTan[a*x]^(3/2)/(x*(c + a^2*c*x^2)^3), x]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^{3/2}}{x\left(c+a^2cx^2\right)^3} dx = \int \frac{\tan^{-1}(ax)^{3/2}}{x\left(c+a^2cx^2\right)^3} dx$$

Mathematica [A] time = 2.30922, size = 0, normalized size = 0.

$$\int \frac{\tan^{-1}(ax)^{3/2}}{x\left(c+a^2cx^2\right)^3} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcTan[a*x]^(3/2)/(x*(c + a^2*c*x^2)^3),x]
```

```
[Out] Integrate[ArcTan[a*x]^(3/2)/(x*(c + a^2*c*x^2)^3), x]
```

Maple [A] time = 0.801, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(a^2cx^2+c\right)^3}(\arctan(ax))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^3,x)
```

```
[Out] int(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^3,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\operatorname{atan}^{\frac{3}{2}}(ax)}{a^6x^7+3a^4x^5+3a^2x^3+x}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(3/2)/x/(a**2*c*x**2+c)**3,x)
```

```
[Out] Integral(atan(a*x)**(3/2)/(a**6*x**7 + 3*a**4*x**5 + 3*a**2*x**3 + x), x)/c
**3
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(ax)^{\frac{3}{2}}}{\left(a^2cx^2+c\right)^3 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)^(3/2)/((a^2*c*x^2 + c)^3*x), x)
```

3.797 $\int x^m \sqrt{c+a^2 c x^2} \tan^{-1}(ax)^{3/2} \, dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(x^m \sqrt{a^2 c x^2+c} \tan^{-1}(ax)^{3/2},x\right)$$

```
[Out] Unintegrable[x^m*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2), x]
```

Rubi [A] time = 0.0980957, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int x^m \sqrt{c+a^2 c x^2} \tan^{-1}(ax)^{3/2} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2),x]
```

```
[Out] Defer[Int][x^m*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2), x]
```

Rubi steps

$$\int x^m \sqrt{c+a^2 c x^2} \tan^{-1}(ax)^{3/2} \, dx = \int x^m \sqrt{c+a^2 c x^2} \tan^{-1}(ax)^{3/2} \, dx$$

Mathematica [A] time = 0.65001, size = 0, normalized size = 0.

$$\int x^m \sqrt{c+a^2 c x^2} \tan^{-1}(ax)^{3/2} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2),x]
```

```
[Out] Integrate[x^m*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2), x]
```

Maple [A] time = 1.234, size = 0, normalized size = 0.

$$\int x^{m} \left(\arctan \left(a x\right)\right)^{\frac{3}{2}} \sqrt{a^{2} c \,x^{2}+c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(x^m*arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{a^{2} c x^{2} + c} x^{m} \arctan\left(a x\right)^{\frac{3}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^m*arctan(a*x)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*atan(a*x)**(3/2)*(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.798 $\int x^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}\,dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(x^2\sqrt{a^2cx^2+c}\tan^{-1}(ax)^{3/2},x\right)$$

```
[Out] Unintegrable[x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2), x]
```

Rubi [A] time = 0.101085, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2),x]
```

```
[Out] Defer[Int][x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2), x]
```

Rubi steps

$$\int x^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}\,dx=\int x^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}\,dx$$

Mathematica [A] time = 2.96972, size = 0, normalized size = 0.

$$\int x^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2),x]
```

```
[Out] Integrate[x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2), x]
```

Maple [A] time = 1.598, size = 0, normalized size = 0.

$$\int x^2 (\arctan (ax))^{\frac{3}{2}} \sqrt{a^2 c x^2 + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(x^2*arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(a*x)**(3/2)*(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a^2cx^2 + c}x^2 \arctan(ax)^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)*x^2*arctan(a*x)^(3/2), x)
```

3.799 $\int x\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}\,dx$

Optimal. Leaf size=65

$$\frac{(a^2cx^2+c)^{3/2}\tan^{-1}(ax)^{3/2}}{3a^2c}-\frac{\text{Unintegrable}\left(\sqrt{a^2cx^2+c}\sqrt{\tan^{-1}(ax)},x\right)}{2a}$$

```
[Out] ((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2))/(3*a^2*c) - Unintegrable[Sqrt[c +
 a^2*c*x^2]*Sqrt[ArcTan[a*x]], x]/(2*a)
```

Rubi [A] time = 0.105623, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2),x]
```

```
[Out] ((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2))/(3*a^2*c) - Defer[Int][Sqrt[c + a
^2*c*x^2]*Sqrt[ArcTan[a*x]], x]/(2*a)
```

Rubi steps

$$\int x\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}\,dx=\frac{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^{3/2}}{3a^2c}-\frac{\int\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}\,dx}{2a}$$

Mathematica [A] time = 6.06444, size = 0, normalized size = 0.

$$\int x\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2),x]
```

```
[Out] Integrate[x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2), x]
```

Maple [A] time = 1.02, size = 0, normalized size = 0.

$$\int x\left(\arctan\left(ax\right)\right)^{\frac{3}{2}}\sqrt{a^{2}c x^{2}+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(x*arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(a*x)**(3/2)*(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a^2cx^2 + c}x \arctan(ax)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)*x*arctan(a*x)^(3/2), x)
```

3.800 $\int \sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}\,dx$

Optimal. Leaf size=117

$$\frac{3}{8}c\text{Unintegrable}\left(\frac{1}{\sqrt{a^2cx^2+c}\sqrt{\tan^{-1}(ax)}},x\right)+\frac{1}{2}c\text{Unintegrable}\left(\frac{\tan^{-1}(ax)^{3/2}}{\sqrt{a^2cx^2+c}},x\right)+\frac{1}{2}x\sqrt{a^2cx^2+c}\tan^{-1}(ax)^{3/2}-\frac{3\sqrt{}}{}$$

```
[Out] (-3*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]])/(4*a) + (x*Sqrt[c + a^2*c*x^2]*A
rcTan[a*x]^(3/2))/2 + (3*c*Unintegrable[1/(Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[
a*x]]), x])/8 + (c*Unintegrable[ArcTan[a*x]^(3/2)/Sqrt[c + a^2*c*x^2], x])/
2
```

Rubi [A] time = 0.105027, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2),x]
```

```
[Out] (-3*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]])/(4*a) + (x*Sqrt[c + a^2*c*x^2]*A
rcTan[a*x]^(3/2))/2 + (3*c*Defer[Int][1/(Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*
x]]), x])/8 + (c*Defer[Int][ArcTan[a*x]^(3/2)/Sqrt[c + a^2*c*x^2], x])/2
```

Rubi steps

$$\int \sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}\,dx=-\frac{3\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}}{4a}+\frac{1}{2}x\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}+\frac{1}{8}(3c)\int\frac{1}{\sqrt{c+a^2cx^2}\sqrt{\tan^{-}}}$$

Mathematica [A] time = 0.277892, size = 0, normalized size = 0.

$$\int \sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2),x]
```

```
[Out] Integrate[Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2), x]
```

Maple [A] time = 0.773, size = 0, normalized size = 0.

$$\int (\arctan (ax))^{\frac{3}{2}} \sqrt{a^2cx^2 + c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(3/2)*(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a^2cx^2 + c}\arctan{(ax)}^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)*arctan(a*x)^(3/2), x)
```

3.801 $\int \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}}{x}\,dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(\frac{\sqrt{a^2cx^2+c}\tan^{-1}(ax)^{3/2}}{x},x\right)$$

```
[Out] Unintegrable[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2))/x, x]
```

Rubi [A] time = 0.0966777, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2))/x,x]
```

```
[Out] Defer[Int][(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2))/x, x]
```

Rubi steps

$$\int \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}}{x}\,dx=\int \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}}{x}\,dx$$

Mathematica [A] time = 2.88512, size = 0, normalized size = 0.

$$\int \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2))/x,x]
```

```
[Out] Integrate[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2))/x, x]
```

Maple [A] time = 0.886, size = 0, normalized size = 0.

$$\int \frac{1}{x} (\arctan (ax))^{\frac{3}{2}} \sqrt{a^2cx^2 + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2)/x,x)
```

```
[Out] int(arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2)/x,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(3/2)*(a**2*c*x**2+c)**(1/2)/x,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)*arctan(a*x)^(3/2)/x, x)
```

3.802 $\int x^m \left(c+a^2 c x^2\right)^{3/2} \tan^{-1}(a x)^{3/2} \, dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(x^m \left(a^2 c x^2+c\right)^{3/2} \tan^{-1}(a x)^{3/2},x\right)$$

```
[Out] Unintegrable[x^m*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2), x]
```

Rubi [A] time = 0.109922, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^m \left(c+a^2 c x^2\right)^{3/2} \tan^{-1}(a x)^{3/2} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2),x]
```

```
[Out] Defer[Int][x^m*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2), x]
```

Rubi steps

$$\int x^m \left(c+a^2 c x^2\right)^{3/2} \tan^{-1}(a x)^{3/2} \, dx=\int x^m \left(c+a^2 c x^2\right)^{3/2} \tan^{-1}(a x)^{3/2} \, dx$$

Mathematica [A] time = 0.985064, size = 0, normalized size = 0.

$$\int x^m \left(c+a^2 c x^2\right)^{3/2} \tan^{-1}(a x)^{3/2} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2),x]
```

```
[Out] Integrate[x^m*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2), x]
```

Maple [A] time = 0.963, size = 0, normalized size = 0.

$$\int x^{m} \left(a^{2} c \,x^{2}+c \right)^{\frac{3}{2}} \left(\arctan \left(a x \right)\right)^{\frac{3}{2}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2),x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a^{2} c x^{2} + c\right)^{\frac{3}{2}} x^{m} \arctan\left(a x\right)^{\frac{3}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)^(3/2)*x^m*arctan(a*x)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**(3/2)*atan(a*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.803 $\int x^2 \left(c+a^2 c x^2\right)^{3/2} \tan^{-1}(ax)^{3/2} \, dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(x^2 \left(a^2 c x^2+c\right)^{3/2} \tan^{-1}(ax)^{3/2},x\right)$$

```
[Out] Unintegrable[x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2), x]
```

Rubi [A] time = 0.115181, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^2 \left(c+a^2 c x^2\right)^{3/2} \tan^{-1}(ax)^{3/2} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2),x]

[Out] Defer[Int][x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2), x]
```

Rubi steps

$$\int x^2 \left(c+a^2 c x^2\right)^{3/2} \tan^{-1}(ax)^{3/2} \, dx=\int x^2 \left(c+a^2 c x^2\right)^{3/2} \tan^{-1}(ax)^{3/2} \, dx$$

Mathematica [A] time = 3.76918, size = 0, normalized size = 0.

$$\int x^2 \left(c+a^2 c x^2\right)^{3/2} \tan^{-1}(ax)^{3/2} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2),x]

[Out] Integrate[x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2), x]
```

Maple [A] time = 1.235, size = 0, normalized size = 0.

$$\int x^{2} \left(a^{2} c x^{2}+c\right)^{\frac{3}{2}} \left(\arctan \left(a x\right)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2),x)
```

```
[Out] int(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a**2*c*x**2+c)**(3/2)*atan(a*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2+c\right)^{\frac{3}{2}} x^2 \arctan\left(ax\right)^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)*x^2*arctan(a*x)^(3/2), x)
```

3.804 $\int x \left(c+a^2 c x^2\right)^{3/2} \tan^{-1}(ax)^{3/2} \, dx$

Optimal. Leaf size=65

$$\frac{\left(a^2cx^2+c\right)^{5/2}\tan^{-1}(ax)^{3/2}}{5a^2c}-\frac{3\text{Unintegrable}\left(\left(a^2cx^2+c\right)^{3/2}\sqrt{\tan^{-1}(ax)},x\right)}{10a}$$

```
[Out] ((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2))/(5*a^2*c) - (3*Unintegrable[(c +
a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]], x])/(10*a)
```

Rubi [A] time = 0.117776, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int x \left(c+a^2 c x^2\right)^{3/2} \tan^{-1}(ax)^{3/2} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2),x]
```

```
[Out] ((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2))/(5*a^2*c) - (3*Defer[Int][(c + a^
2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]], x])/(10*a)
```

Rubi steps

$$\int x \left(c+a^2cx^2\right)^{3/2} \tan^{-1}(ax)^{3/2} \, dx = \frac{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{3/2}}{5a^2c}-\frac{3\int \left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}\, dx}{10a}$$

Mathematica [A] time = 2.38796, size = 0, normalized size = 0.

$$\int x \left(c+a^2 c x^2\right)^{3/2} \tan^{-1}(ax)^{3/2} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2),x]
```

```
[Out] Integrate[x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2), x]
```

Maple [A] time = 0.757, size = 0, normalized size = 0.

$$\int x\left(a^{2}c x^{2}+c\right)^{\frac{3}{2}}\left(\arctan\left(ax\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2),x)
```

```
[Out] int(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**(3/2)*atan(a*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2+c\right)^{\frac{3}{2}} x \arctan\left(ax\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)*x*arctan(a*x)^(3/2), x)
```

3.805 $\int \left(c+a^2 c x^2\right)^{3/2} \tan^{-1}(ax)^{3/2} \, dx$

Optimal. Leaf size=209

$$\frac{9}{32}c^2\text{Unintegrable}\left(\frac{1}{\sqrt{a^2cx^2+c}\sqrt{\tan^{-1}(ax)}},x\right)+\frac{3}{8}c^2\text{Unintegrable}\left(\frac{\tan^{-1}(ax)^{3/2}}{\sqrt{a^2cx^2+c}},x\right)+\frac{1}{16}c\text{Unintegrable}\left(\frac{\sqrt{a^2cx^2+}}{\sqrt{\tan^{-1}(a}}\right.$$

```
[Out] (-9*c*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]])/(16*a) - ((c + a^2*c*x^2)^(3/2
)*Sqrt[ArcTan[a*x]])/(8*a) + (3*c*x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2))/
8 + (x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2))/4 + (9*c^2*Unintegrable[1/(
Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]), x])/32 + (c*Unintegrable[Sqrt[c + a
^2*c*x^2]/Sqrt[ArcTan[a*x]], x])/16 + (3*c^2*Unintegrable[ArcTan[a*x]^(3/2)
/Sqrt[c + a^2*c*x^2], x])/8
```

Rubi [A] time = 0.178488, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \left(c+a^2 c x^2\right)^{3/2} \tan^{-1}(ax)^{3/2} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2),x]
```

```
[Out] (-9*c*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]])/(16*a) - ((c + a^2*c*x^2)^(3/2
)*Sqrt[ArcTan[a*x]])/(8*a) + (3*c*x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2))/
8 + (x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2))/4 + (9*c^2*Defer[Int][1/(Sq
rt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]), x])/32 + (c*Defer[Int][Sqrt[c + a^2*c
*x^2]/Sqrt[ArcTan[a*x]], x])/16 + (3*c^2*Defer[Int][ArcTan[a*x]^(3/2)/Sqrt[
c + a^2*c*x^2], x])/8
```

Rubi steps

$$\int \left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}\,dx = -\frac{\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}}{8a}+\frac{1}{4}x\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}+\frac{1}{16}c\int\frac{\sqrt{c+a^2cx^2}}{\sqrt{\tan^{-1}(ax)}}$$

$$= -\frac{9c\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}}{16a}-\frac{\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}}{8a}+\frac{3}{8}cx\sqrt{c+a^2cx^2}\tan^{-1}(ax$$

Mathematica [A] time = 1.4241, size = 0, normalized size = 0.

$$\int \left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2),x]
```

```
[Out] Integrate[(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2), x]
```

Maple [A] time = 0.642, size = 0, normalized size = 0.

$$\int \left(a^2cx^2+c\right)^{\frac{3}{2}}(\arctan(ax))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2),x)
```

```
[Out] int((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(3/2)*atan(a*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2+c\right)^{\frac{3}{2}} \arctan\left(ax\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^(3/2), x)
```

3.806 $\int \frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}}{x}\,dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(\frac{\left(a^2cx^2+c\right)^{3/2}\tan^{-1}(ax)^{3/2}}{x},x\right)$$

```
[Out] Unintegrable[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2))/x, x]
```

Rubi [A] time = 0.113296, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2))/x,x]
```

```
[Out] Defer[Int][((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2))/x, x]
```

Rubi steps

$$\int \frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}}{x}\,dx=\int \frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}}{x}\,dx$$

Mathematica [A] time = 1.9971, size = 0, normalized size = 0.

$$\int \frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2))/x,x]
```

```
[Out] Integrate[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2))/x, x]
```

Maple [A] time = 0.671, size = 0, normalized size = 0.

$$\int \frac{1}{x}\left(a^2cx^2+c\right)^{\frac{3}{2}}(\arctan(ax))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2)/x,x)
```

```
[Out] int((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2)/x,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(3/2)*atan(a*x)**(3/2)/x,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^{\frac{3}{2}}\arctan\left(ax\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2)/x,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^(3/2)/x, x)
```

3.807 $\int x^m \left(c+a^2 c x^2\right)^{5/2} \tan^{-1}(a x)^{3/2} \, dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(x^m \left(a^2 c x^2+c\right)^{5/2} \tan^{-1}(a x)^{3/2},x\right)$$

```
[Out] Unintegrable[x^m*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2), x]
```

Rubi [A] time = 0.113466, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^m \left(c+a^2 c x^2\right)^{5/2} \tan^{-1}(a x)^{3/2} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2),x]
```

```
[Out] Defer[Int][x^m*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2), x]
```

Rubi steps

$$\int x^m \left(c+a^2 c x^2\right)^{5/2} \tan^{-1}(a x)^{3/2} \, dx=\int x^m \left(c+a^2 c x^2\right)^{5/2} \tan^{-1}(a x)^{3/2} \, dx$$

Mathematica [A] time = 1.28566, size = 0, normalized size = 0.

$$\int x^m \left(c+a^2 c x^2\right)^{5/2} \tan^{-1}(a x)^{3/2} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2),x]
```

```
[Out] Integrate[x^m*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2), x]
```

Maple [A] time = 0.938, size = 0, normalized size = 0.

$$\int x^m \left(a^2 c x^2 + c\right)^{\frac{5}{2}} (\arctan(ax))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2),x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a^4 c^2 x^4 + 2\,a^2 c^2 x^2 + c^2\right)\sqrt{a^2 c x^2 + c}\,x^m \arctan(ax)^{\frac{3}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*x^m*arctan
(a*x)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**(5/2)*atan(a*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.808 $\int x^2 \left(c+a^2 c x^2\right)^{5/2} \tan^{-1}(ax)^{3/2} \, dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(x^2 \left(a^2 c x^2+c\right)^{5/2} \tan^{-1}(ax)^{3/2},x\right)$$

```
[Out] Unintegrable[x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2), x]
```

Rubi [A] time = 0.114154, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^2 \left(c+a^2 c x^2\right)^{5/2} \tan^{-1}(ax)^{3/2} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2),x]
```

```
[Out] Defer[Int][x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2), x]
```

Rubi steps

$$\int x^2 \left(c+a^2 c x^2\right)^{5/2} \tan^{-1}(ax)^{3/2} \, dx=\int x^2 \left(c+a^2 c x^2\right)^{5/2} \tan^{-1}(ax)^{3/2} \, dx$$

Mathematica [A] time = 3.10501, size = 0, normalized size = 0.

$$\int x^2 \left(c+a^2 c x^2\right)^{5/2} \tan^{-1}(ax)^{3/2} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2),x]
```

```
[Out] Integrate[x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2), x]
```

Maple [A] time = 1.398, size = 0, normalized size = 0.

$$\int x^{2} \left(a^{2} c\,x^{2}+c \right)^{\frac{5}{2}} \left(\arctan \left(a x \right)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2),x)
```

```
[Out] int(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a**2*c*x**2+c)**(5/2)*atan(a*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2+c\right)^{\frac{5}{2}} x^2 \arctan\left(ax\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(5/2)*x^2*arctan(a*x)^(3/2), x)
```

3.809 $\int x\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{3/2}\,dx$

Optimal. Leaf size=65

$$\frac{\left(a^2cx^2+c\right)^{7/2}\tan^{-1}(ax)^{3/2}}{7a^2c}-\frac{3\text{Unintegrable}\left(\left(a^2cx^2+c\right)^{5/2}\sqrt{\tan^{-1}(ax)},x\right)}{14a}$$

```
[Out] ((c + a^2*c*x^2)^(7/2)*ArcTan[a*x]^(3/2))/(7*a^2*c) - (3*Unintegrable[(c +
a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]], x])/(14*a)
```

Rubi [A] time = 0.116419, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{3/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2),x]
```

```
[Out] ((c + a^2*c*x^2)^(7/2)*ArcTan[a*x]^(3/2))/(7*a^2*c) - (3*Defer[Int][(c + a^
2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]], x])/(14*a)
```

Rubi steps

$$\int x\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{3/2}\,dx=\frac{\left(c+a^2cx^2\right)^{7/2}\tan^{-1}(ax)^{3/2}}{7a^2c}-\frac{3\int\left(c+a^2cx^2\right)^{5/2}\sqrt{\tan^{-1}(ax)}\,dx}{14a}$$

Mathematica [A] time = 6.80532, size = 0, normalized size = 0.

$$\int x\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{3/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2),x]
```

```
[Out] Integrate[x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2), x]
```

Maple [A] time = 0.884, size = 0, normalized size = 0.

$$\int x\left(a^{2}c x^{2}+c\right)^{\frac{5}{2}} \left(\arctan \left(a x\right)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2),x)
```

```
[Out] int(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**(5/2)*atan(a*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2+c\right)^{\frac{5}{2}} x \arctan(ax)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(5/2)*x*arctan(a*x)^(3/2), x)
```

3.810 $\int \left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{3/2}\,dx$

Optimal. Leaf size=303

$$\frac{15}{64}c^3\text{Unintegrable}\left(\frac{1}{\sqrt{a^2cx^2+c}\sqrt{\tan^{-1}(ax)}},x\right)+\frac{5}{16}c^3\text{Unintegrable}\left(\frac{\tan^{-1}(ax)^{3/2}}{\sqrt{a^2cx^2+c}},x\right)+\frac{5}{96}c^2\text{Unintegrable}\left(\frac{\sqrt{a^2cx}}{\sqrt{\tan}}\right.$$

```
[Out] (-15*c^2*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]])/(32*a) - (5*c*(c + a^2*c*x^
2)^(3/2)*Sqrt[ArcTan[a*x]])/(48*a) - ((c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x
]])/(20*a) + (5*c^2*x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2))/16 + (5*c*x*(c
 + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2))/24 + (x*(c + a^2*c*x^2)^(5/2)*ArcTan
[a*x]^(3/2))/6 + (15*c^3*Unintegrable[1/(Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*
x]]), x])/64 + (5*c^2*Unintegrable[Sqrt[c + a^2*c*x^2]/Sqrt[ArcTan[a*x]], x
])/96 + (c*Unintegrable[(c + a^2*c*x^2)^(3/2)/Sqrt[ArcTan[a*x]], x])/40 + (
5*c^3*Unintegrable[ArcTan[a*x]^(3/2)/Sqrt[c + a^2*c*x^2], x])/16
```

Rubi [A] time = 0.273812, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{3/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2),x]
```

```
[Out] (-15*c^2*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]])/(32*a) - (5*c*(c + a^2*c*x^
2)^(3/2)*Sqrt[ArcTan[a*x]])/(48*a) - ((c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x
]])/(20*a) + (5*c^2*x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2))/16 + (5*c*x*(c
 + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2))/24 + (x*(c + a^2*c*x^2)^(5/2)*ArcTan
[a*x]^(3/2))/6 + (15*c^3*Defer[Int][1/(Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]
]), x])/64 + (5*c^2*Defer[Int][Sqrt[c + a^2*c*x^2]/Sqrt[ArcTan[a*x]], x])/9
6 + (c*Defer[Int][(c + a^2*c*x^2)^(3/2)/Sqrt[ArcTan[a*x]], x])/40 + (5*c^3*
Defer[Int][ArcTan[a*x]^(3/2)/Sqrt[c + a^2*c*x^2], x])/16
```

Rubi steps

$$\int \left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{3/2}\,dx = -\frac{\left(c+a^2cx^2\right)^{5/2}\sqrt{\tan^{-1}(ax)}}{20a}+\frac{1}{6}x\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{3/2}+\frac{1}{40}c\int \frac{\left(c+a^2cx^2\right)^{3/2}}{\sqrt{\tan^{-1}(ax)}}$$

$$= -\frac{5c\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}}{48a}-\frac{\left(c+a^2cx^2\right)^{5/2}\sqrt{\tan^{-1}(ax)}}{20a}+\frac{5}{24}cx\left(c+a^2cx^2\right)^{3/2}\text{ta}$$

$$= -\frac{15c^2\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}}{32a}-\frac{5c\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}}{48a}-\frac{\left(c+a^2cx^2\right)^{5/2}\sqrt{\tan^{-}}}{20a}$$

Mathematica [A] time = 0.45528, size = 0, normalized size = 0.

$$\int \left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{3/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2),x]
```

```
[Out] Integrate[(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2), x]
```

Maple [A] time = 0.737, size = 0, normalized size = 0.

$$\int \left(a^2cx^2+c\right)^{\frac{5}{2}}(\arctan(ax))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2),x)
```

```
[Out] int((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(5/2)*atan(a*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2 + c\right)^{\frac{5}{2}} \arctan\left(ax\right)^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^(3/2), x)
```

3.811 $\int \frac{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{3/2}}{x}\,dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(\frac{\left(a^2cx^2+c\right)^{5/2}\tan^{-1}(ax)^{3/2}}{x},x\right)$$

```
[Out] Unintegrable[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2))/x, x]
```

Rubi [A] time = 0.111838, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{3/2}}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2))/x,x]
```

```
[Out] Defer[Int][((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2))/x, x]
```

Rubi steps

$$\int \frac{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{3/2}}{x}\,dx=\int \frac{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{3/2}}{x}\,dx$$

Mathematica [A] time = 2.15885, size = 0, normalized size = 0.

$$\int \frac{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{3/2}}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2))/x,x]
```

```
[Out] Integrate[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2))/x, x]
```

Maple [A] time = 0.842, size = 0, normalized size = 0.

$$\int \frac{1}{x}\left(a^{2}c\,x^{2}+c\right)^{\frac{5}{2}}\left(\arctan\left(ax\right)\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2)/x,x)
```

```
[Out] int((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2)/x,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(5/2)*atan(a*x)**(3/2)/x,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^{\frac{5}{2}}\arctan\left(ax\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2)/x,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^(3/2)/x, x)
```

3.812 $\int \frac{x^m \tan^{-1}(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(\frac{x^m \tan^{-1}(ax)^{3/2}}{\sqrt{a^2cx^2+c}},x\right)$$

```
[Out] Unintegrable[(x^m*ArcTan[a*x]^(3/2))/Sqrt[c + a^2*c*x^2], x]
```

Rubi [A] time = 0.101575, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m \tan^{-1}(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*ArcTan[a*x]^(3/2))/Sqrt[c + a^2*c*x^2],x]

[Out] Defer[Int][(x^m*ArcTan[a*x]^(3/2))/Sqrt[c + a^2*c*x^2], x]
```

Rubi steps

$$\int \frac{x^m \tan^{-1}(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^m \tan^{-1}(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx$$

Mathematica [A] time = 0.806173, size = 0, normalized size = 0.

$$\int \frac{x^m \tan^{-1}(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*ArcTan[a*x]^(3/2))/Sqrt[c + a^2*c*x^2],x]

[Out] Integrate[(x^m*ArcTan[a*x]^(3/2))/Sqrt[c + a^2*c*x^2], x]
```

Maple [A] time = 1.44, size = 0, normalized size = 0.

$$\int x^{m} \left(\arctan \left(a x\right)\right)^{\frac{3}{2}} \frac{1}{\sqrt{a^{2} c \,x^{2}+c}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x^{m} \arctan \left(a x\right)^{\frac{3}{2}}}{\sqrt{a^{2} c x^{2} + c}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^m*arctan(a*x)^(3/2)/sqrt(a^2*c*x^2 + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*atan(a*x)**(3/2)/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m \arctan{(ax)}^{\frac{3}{2}}}{\sqrt{a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^m*arctan(a*x)^(3/2)/sqrt(a^2*c*x^2 + c), x)
```

3.813 $\int \frac{x^3 \tan^{-1}(ax)^{3/2}}{\sqrt{c+a^2cx^2}} \, dx$

Optimal. Leaf size=166

$$\frac{\text{Unintegrable}\left(\frac{x}{\sqrt{a^2cx^2+c}\sqrt{\tan^{-1}(ax)}},x\right)}{8a^2}+\frac{5\text{Unintegrable}\left(\frac{\sqrt{\tan^{-1}(ax)}}{\sqrt{a^2cx^2+c}},x\right)}{4a^3}+\frac{x^2\sqrt{a^2cx^2+c}\tan^{-1}(ax)^{3/2}}{3a^2c}-\frac{x\sqrt{a^2cx^2+c}\sqrt{\tan}}{4a^3c}$$

```
[Out] -(x*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]])/(4*a^3*c) - (2*Sqrt[c + a^2*c*x^
2]*ArcTan[a*x]^(3/2))/(3*a^4*c) + (x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2
))/(3*a^2*c) + Unintegrable[x/(Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]), x]/(
8*a^2) + (5*Unintegrable[Sqrt[ArcTan[a*x]]/Sqrt[c + a^2*c*x^2], x])/(4*a^3)
```

Rubi [A] time = 0.450052, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^3 \tan^{-1}(ax)^{3/2}}{\sqrt{c+a^2cx^2}} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^3*ArcTan[a*x]^(3/2))/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] -(x*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]])/(4*a^3*c) - (2*Sqrt[c + a^2*c*x^
2]*ArcTan[a*x]^(3/2))/(3*a^4*c) + (x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2
))/(3*a^2*c) + Defer[Int][x/(Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]), x]/(8*
a^2) + (5*Defer[Int][Sqrt[ArcTan[a*x]]/Sqrt[c + a^2*c*x^2], x])/(4*a^3)
```

Rubi steps

$$\begin{aligned}\int \frac{x^3 \tan^{-1}(ax)^{3/2}}{\sqrt{c+a^2cx^2}} \, dx &= \frac{x^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}}{3a^2c}-\frac{2\int \frac{x\tan^{-1}(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx}{3a^2}-\frac{\int \frac{x^2\sqrt{\tan^{-1}(ax)}}{\sqrt{c+a^2cx^2}}\,dx}{2a}\\ &= -\frac{x\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}}{4a^3c}-\frac{2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}}{3a^4c}+\frac{x^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}}{3a^2c}+\frac{\int \frac{\sqrt{\tan^{-1}}}{\sqrt{c+a^2c}}}{4a^3}\end{aligned}$$

Mathematica [A] time = 3.48949, size = 0, normalized size = 0.

$$\int \frac{x^3 \tan^{-1}(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^3*ArcTan[a*x]^(3/2))/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] Integrate[(x^3*ArcTan[a*x]^(3/2))/Sqrt[c + a^2*c*x^2], x]
```

Maple [A] time = 4.641, size = 0, normalized size = 0.

$$\int x^3 (\arctan(ax))^{\frac{3}{2}} \frac{1}{\sqrt{a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atan(a*x)**(3/2)/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \arctan(ax)^{\frac{3}{2}}}{\sqrt{a^2cx^2 + c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^3*arctan(a*x)^(3/2)/sqrt(a^2*c*x^2 + c), x)
```

3.814 $\int \frac{x^2 \tan^{-1}(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=130

$$\frac{3\text{Unintegrable}\left(\frac{1}{\sqrt{a^2cx^2+c}\sqrt{\tan^{-1}(ax)}},x\right)}{8a^2}-\frac{\text{Unintegrable}\left(\frac{\tan^{-1}(ax)^{3/2}}{\sqrt{a^2cx^2+c}},x\right)}{2a^2}+\frac{x\sqrt{a^2cx^2+c}\tan^{-1}(ax)^{3/2}}{2a^2c}-\frac{3\sqrt{a^2cx^2+c}\sqrt{\tan^{-1}(ax)}}{4a^3c}$$

```
[Out] (-3*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]])/(4*a^3*c) + (x*Sqrt[c + a^2*c*x^
2]*ArcTan[a*x]^(3/2))/(2*a^2*c) + (3*Unintegrable[1/(Sqrt[c + a^2*c*x^2]*Sq
rt[ArcTan[a*x]]), x])/(8*a^2) - Unintegrable[ArcTan[a*x]^(3/2)/Sqrt[c + a^2
*c*x^2], x]/(2*a^2)
```

Rubi [A] time = 0.25866, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^2 \tan^{-1}(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^2*ArcTan[a*x]^(3/2))/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] (-3*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]])/(4*a^3*c) + (x*Sqrt[c + a^2*c*x^
2]*ArcTan[a*x]^(3/2))/(2*a^2*c) + (3*Defer[Int][1/(Sqrt[c + a^2*c*x^2]*Sqrt
[ArcTan[a*x]]), x])/(8*a^2) - Defer[Int][ArcTan[a*x]^(3/2)/Sqrt[c + a^2*c*x
^2], x]/(2*a^2)
```

Rubi steps

$$\begin{aligned}\int \frac{x^2 \tan^{-1}(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx &= \frac{x\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}}{2a^2c}-\frac{\int \frac{\tan^{-1}(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx}{2a^2}-\frac{3\int \frac{x\sqrt{\tan^{-1}(ax)}}{\sqrt{c+a^2cx^2}}\,dx}{4a}\\ &= -\frac{3\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}}{4a^3c}+\frac{x\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}}{2a^2c}+\frac{3\int \frac{1}{\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}}\,dx}{8a^2}-\frac{\int \frac{\tan^{-1}(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx}{2a^2}\end{aligned}$$

Mathematica [A] time = 2.33421, size = 0, normalized size = 0.

$$\int \frac{x^2 \tan^{-1}(ax)^{3/2}}{\sqrt{c + a^2cx^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^2*ArcTan[a*x]^(3/2))/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] Integrate[(x^2*ArcTan[a*x]^(3/2))/Sqrt[c + a^2*c*x^2], x]
```

Maple [A] time = 3.53, size = 0, normalized size = 0.

$$\int x^2 (\arctan(ax))^{\frac{3}{2}} \frac{1}{\sqrt{a^2cx^2 + c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(a*x)**(3/2)/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \arctan(ax)^{\frac{3}{2}}}{\sqrt{a^2cx^2 + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctan(a*x)^(3/2)/sqrt(a^2*c*x^2 + c), x)
```

3.815 $\int \frac{x\tan^{-1}(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=62

$$\frac{\sqrt{a^2cx^2+c}\tan^{-1}(ax)^{3/2}}{a^2c}-\frac{3\text{Unintegrable}\left(\frac{\sqrt{\tan^{-1}(ax)}}{\sqrt{a^2cx^2+c}},x\right)}{2a}$$

```
[Out] (Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2))/(a^2*c) - (3*Unintegrable[Sqrt[ArcT
an[a*x]]/Sqrt[c + a^2*c*x^2], x])/(2*a)
```

Rubi [A] time = 0.107876, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x\tan^{-1}(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x*ArcTan[a*x]^(3/2))/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] (Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2))/(a^2*c) - (3*Defer[Int][Sqrt[ArcTan
[a*x]]/Sqrt[c + a^2*c*x^2], x])/(2*a)
```

Rubi steps

$$\int \frac{x\tan^{-1}(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx=\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}}{a^2c}-\frac{3\int \frac{\sqrt{\tan^{-1}(ax)}}{\sqrt{c+a^2cx^2}}\,dx}{2a}$$

Mathematica [A] time = 0.654257, size = 0, normalized size = 0.

$$\int \frac{x\tan^{-1}(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x*ArcTan[a*x]^(3/2))/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] Integrate[(x*ArcTan[a*x]^(3/2))/Sqrt[c + a^2*c*x^2], x]
```

Maple [A] time = 1.283, size = 0, normalized size = 0.

$$\int x (\arctan (ax))^{\frac{3}{2}} \frac{1}{\sqrt{a^2 c x^2 + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(a*x)**(3/2)/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \arctan (ax)^{\frac{3}{2}}}{\sqrt{a^2cx^2 + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x*arctan(a*x)^(3/2)/sqrt(a^2*c*x^2 + c), x)
```

3.816 $\int \frac{\tan^{-1}(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=25

$$\text{Unintegrable}\left(\frac{\tan^{-1}(ax)^{3/2}}{\sqrt{a^2cx^2+c}},x\right)$$

```
[Out] Unintegrable[ArcTan[a*x]^(3/2)/Sqrt[c + a^2*c*x^2], x]
```

Rubi [A] time = 0.0353936, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\tan^{-1}(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcTan[a*x]^(3/2)/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] Defer[Int][ArcTan[a*x]^(3/2)/Sqrt[c + a^2*c*x^2], x]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{\tan^{-1}(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx$$

Mathematica [A] time = 0.207248, size = 0, normalized size = 0.

$$\int \frac{\tan^{-1}(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcTan[a*x]^(3/2)/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] Integrate[ArcTan[a*x]^(3/2)/Sqrt[c + a^2*c*x^2], x]
```

Maple [A] time = 0.915, size = 0, normalized size = 0.

$$\int (\arctan (ax))^{\frac{3}{2}} \frac{1}{\sqrt{a^2 c x^2 + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(3/2)/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(ax)^{\frac{3}{2}}}{\sqrt{a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)^(3/2)/sqrt(a^2*c*x^2 + c), x)
```

3.817 $\int \frac{\tan^{-1}(ax)^{3/2}}{x\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(\frac{\tan^{-1}(ax)^{3/2}}{x\sqrt{a^2cx^2+c}},x\right)$$

```
[Out] Unintegrable[ArcTan[a*x]^(3/2)/(x*Sqrt[c + a^2*c*x^2]), x]
```

Rubi [A] time = 0.105312, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\tan^{-1}(ax)^{3/2}}{x\sqrt{c+a^2cx^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcTan[a*x]^(3/2)/(x*Sqrt[c + a^2*c*x^2]),x]

[Out] Defer[Int][ArcTan[a*x]^(3/2)/(x*Sqrt[c + a^2*c*x^2]), x]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^{3/2}}{x\sqrt{c+a^2cx^2}}\,dx = \int \frac{\tan^{-1}(ax)^{3/2}}{x\sqrt{c+a^2cx^2}}\,dx$$

Mathematica [A] time = 1.21858, size = 0, normalized size = 0.

$$\int \frac{\tan^{-1}(ax)^{3/2}}{x\sqrt{c+a^2cx^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcTan[a*x]^(3/2)/(x*Sqrt[c + a^2*c*x^2]),x]

[Out] Integrate[ArcTan[a*x]^(3/2)/(x*Sqrt[c + a^2*c*x^2]), x]
```

Maple [A] time = 0.986, size = 0, normalized size = 0.

$$\int \frac{1}{x} (\arctan (ax))^{\frac{3}{2}} \frac{1}{\sqrt{a^2cx^2 + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(3/2)/x/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan{(ax)}^{\frac{3}{2}}}{\sqrt{a^2cx^2+c}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)^(3/2)/(sqrt(a^2*c*x^2 + c)*x), x)
```

3.818 $\int \frac{\tan^{-1}(ax)^{3/2}}{x^2\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=64

$$\frac{3}{2}a\text{Unintegrable}\left(\frac{\sqrt{\tan^{-1}(ax)}}{x\sqrt{a^2cx^2+c}},x\right)-\frac{\sqrt{a^2cx^2+c}\tan^{-1}(ax)^{3/2}}{cx}$$

```
[Out] -((Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2))/(c*x)) + (3*a*Unintegrable[Sqrt[A
rcTan[a*x]]/(x*Sqrt[c + a^2*c*x^2]), x])/2
```

Rubi [A] time = 0.213437, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\tan^{-1}(ax)^{3/2}}{x^2\sqrt{c+a^2cx^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcTan[a*x]^(3/2)/(x^2*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] -((Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2))/(c*x)) + (3*a*Defer[Int][Sqrt[Arc
Tan[a*x]]/(x*Sqrt[c + a^2*c*x^2]), x])/2
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^{3/2}}{x^2\sqrt{c+a^2cx^2}}\,dx=-\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}}{cx}+\frac{1}{2}(3a)\int \frac{\sqrt{\tan^{-1}(ax)}}{x\sqrt{c+a^2cx^2}}\,dx$$

Mathematica [A] time = 1.24526, size = 0, normalized size = 0.

$$\int \frac{\tan^{-1}(ax)^{3/2}}{x^2\sqrt{c+a^2cx^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcTan[a*x]^(3/2)/(x^2*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] Integrate[ArcTan[a*x]^(3/2)/(x^2*Sqrt[c + a^2*c*x^2]), x]
```

Maple [A] time = 0.8, size = 0, normalized size = 0.

$$\int \frac{1}{x^2} (\arctan (ax))^{\frac{3}{2}} \frac{1}{\sqrt{a^2 c x^2 + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(3/2)/x^2/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(arctan(a*x)^(3/2)/x^2/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/x^2/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/x^2/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(3/2)/x**2/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan (ax)^{\frac{3}{2}}}{\sqrt{a^2cx^2 + c}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/x^2/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)^(3/2)/(sqrt(a^2*c*x^2 + c)*x^2), x)
```

3.819 $\int \frac{\tan^{-1}(ax)^{3/2}}{x^3\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=136

$$\frac{3}{8}a^2\text{Unintegrable}\left(\frac{1}{x\sqrt{a^2cx^2+c}\sqrt{\tan^{-1}(ax)}},x\right)-\frac{1}{2}a^2\text{Unintegrable}\left(\frac{\tan^{-1}(ax)^{3/2}}{x\sqrt{a^2cx^2+c}},x\right)-\frac{3a\sqrt{a^2cx^2+c}\sqrt{\tan^{-1}(ax)}}{4cx}-$$

```
[Out] (-3*a*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]])/(4*c*x) - (Sqrt[c + a^2*c*x^2]
*ArcTan[a*x]^(3/2))/(2*c*x^2) + (3*a^2*Unintegrable[1/(x*Sqrt[c + a^2*c*x^2
]*Sqrt[ArcTan[a*x]]), x])/8 - (a^2*Unintegrable[ArcTan[a*x]^(3/2)/(x*Sqrt[c
 + a^2*c*x^2]), x])/2
```

Rubi [A] time = 0.422847, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\tan^{-1}(ax)^{3/2}}{x^3\sqrt{c+a^2cx^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcTan[a*x]^(3/2)/(x^3*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] (-3*a*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]])/(4*c*x) - (Sqrt[c + a^2*c*x^2]
*ArcTan[a*x]^(3/2))/(2*c*x^2) + (3*a^2*Defer[Int][1/(x*Sqrt[c + a^2*c*x^2]*
Sqrt[ArcTan[a*x]]), x])/8 - (a^2*Defer[Int][ArcTan[a*x]^(3/2)/(x*Sqrt[c + a
^2*c*x^2]), x])/2
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^{-1}(ax)^{3/2}}{x^3\sqrt{c+a^2cx^2}}\,dx &= -\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}}{2cx^2}+\frac{1}{4}(3a)\int \frac{\sqrt{\tan^{-1}(ax)}}{x^2\sqrt{c+a^2cx^2}}\,dx-\frac{1}{2}a^2\int \frac{\tan^{-1}(ax)^{3/2}}{x\sqrt{c+a^2cx^2}}\,dx\\ &= -\frac{3a\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}}{4cx}-\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}}{2cx^2}+\frac{1}{8}\left(3a^2\right)\int \frac{1}{x\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}}\,dx-\end{aligned}$$

Mathematica [A] time = 3.95466, size = 0, normalized size = 0.

$$\int \frac{\tan^{-1}(ax)^{3/2}}{x^3\sqrt{c+a^2cx^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcTan[a*x]^(3/2)/(x^3*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] Integrate[ArcTan[a*x]^(3/2)/(x^3*Sqrt[c + a^2*c*x^2]), x]
```

Maple [A] time = 1.401, size = 0, normalized size = 0.

$$\int \frac{1}{x^3}\left(\arctan(ax)\right)^{\frac{3}{2}}\frac{1}{\sqrt{a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(3/2)/x^3/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(arctan(a*x)^(3/2)/x^3/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/x^3/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/x^3/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(3/2)/x**3/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan{(ax)}^{\frac{3}{2}}}{\sqrt{a^2cx^2 + c}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/x^3/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)^(3/2)/(sqrt(a^2*c*x^2 + c)*x^3), x)
```

3.820 $\int \frac{\tan^{-1}(ax)^{3/2}}{x^4\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=171

$$-\frac{5}{4}a^3\text{Unintegrable}\left(\frac{\sqrt{\tan^{-1}(ax)}}{x\sqrt{a^2cx^2+c}},x\right)+\frac{1}{8}a^2\text{Unintegrable}\left(\frac{1}{x^2\sqrt{a^2cx^2+c}\sqrt{\tan^{-1}(ax)}},x\right)+\frac{2a^2\sqrt{a^2cx^2+c}\tan^{-1}(ax)^3}{3cx}$$

```
[Out] -(a*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]])/(4*c*x^2) - (Sqrt[c + a^2*c*x^2]
*ArcTan[a*x]^(3/2))/(3*c*x^3) + (2*a^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2
))/(3*c*x) + (a^2*Unintegrable[1/(x^2*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]
), x])/8 - (5*a^3*Unintegrable[Sqrt[ArcTan[a*x]]/(x*Sqrt[c + a^2*c*x^2]), x
])/4
```

Rubi [A] time = 0.655827, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\tan^{-1}(ax)^{3/2}}{x^4\sqrt{c+a^2cx^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcTan[a*x]^(3/2)/(x^4*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] -(a*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]])/(4*c*x^2) - (Sqrt[c + a^2*c*x^2]
*ArcTan[a*x]^(3/2))/(3*c*x^3) + (2*a^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2
))/(3*c*x) + (a^2*Defer[Int][1/(x^2*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]),
 x])/8 - (5*a^3*Defer[Int][Sqrt[ArcTan[a*x]]/(x*Sqrt[c + a^2*c*x^2]), x])/4
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^{3/2}}{x^4\sqrt{c+a^2cx^2}}\,dx=-\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}}{3cx^3}+\frac{1}{2}a\int \frac{\sqrt{\tan^{-1}(ax)}}{x^3\sqrt{c+a^2cx^2}}\,dx-\frac{1}{3}\left(2a^2\right)\int \frac{\tan^{-1}(ax)^{3/2}}{x^2\sqrt{c+a^2cx^2}}\,dx$$

$$=-\frac{a\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}}{4cx^2}-\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}}{3cx^3}+\frac{2a^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}}{3cx}+\frac{1}{8}a^2\int$$

Mathematica [A] time = 18.204, size = 0, normalized size = 0.

$$\int \frac{\tan^{-1}(ax)^{3/2}}{x^4\sqrt{c+a^2cx^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcTan[a*x]^(3/2)/(x^4*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] Integrate[ArcTan[a*x]^(3/2)/(x^4*Sqrt[c + a^2*c*x^2]), x]
```

Maple [A] time = 3.905, size = 0, normalized size = 0.

$$\int \frac{1}{x^4}(\arctan(ax))^{\frac{3}{2}}\frac{1}{\sqrt{a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(3/2)/x^4/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(arctan(a*x)^(3/2)/x^4/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/x^4/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/x^4/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(3/2)/x**4/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(ax)^{\frac{3}{2}}}{\sqrt{a^2cx^2 + cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/x^4/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)^(3/2)/(sqrt(a^2*c*x^2 + c)*x^4), x)
```

3.821 $\int \frac{x^m \tan^{-1}(ax)^{3/2}}{\left(c+a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(\frac{x^m \tan^{-1}(ax)^{3/2}}{\left(a^2cx^2+c\right)^{3/2}},x\right)$$

```
[Out] Unintegrable[(x^m*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(3/2), x]
```

Rubi [A] time = 0.115381, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m \tan^{-1}(ax)^{3/2}}{\left(c+a^2cx^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] Defer[Int][(x^m*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(3/2), x]
```

Rubi steps

$$\int \frac{x^m \tan^{-1}(ax)^{3/2}}{\left(c+a^2cx^2\right)^{3/2}}\,dx=\int \frac{x^m \tan^{-1}(ax)^{3/2}}{\left(c+a^2cx^2\right)^{3/2}}\,dx$$

Mathematica [A] time = 0.931034, size = 0, normalized size = 0.

$$\int \frac{x^m \tan^{-1}(ax)^{3/2}}{\left(c+a^2cx^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(3/2),x]
```

[Out] Integrate[(x^m*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(3/2), x]

Maple [A] time = 1.016, size = 0, normalized size = 0.

$$\int x^m (\arctan (ax))^{\frac{3}{2}} \left(a^2cx^2 + c\right)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x)

[Out] int(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x)

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")

[Out] Exception raised: RuntimeError

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a^2cx^2 + c}x^m \arctan (ax)^{\frac{3}{2}}}{a^4c^2x^4 + 2\,a^2c^2x^2 + c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(a^2*c*x^2 + c)*x^m*arctan(a*x)^(3/2)/(a^4*c^2*x^4 + 2*a^2*c^2
*x^2 + c^2), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*atan(a*x)**(3/2)/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m \arctan(ax)^{\frac{3}{2}}}{(a^2cx^2 + c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2 + c)^(3/2), x)
```

3.822 $\int \frac{x^3 \tan^{-1}(ax)^{3/2}}{\left(c+a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(\frac{x^3 \tan^{-1}(ax)^{3/2}}{\left(a^2cx^2+c\right)^{3/2}},x\right)$$

```
[Out] Unintegrable[(x^3*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(3/2), x]
```

Rubi [A] time = 0.135235, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^3 \tan^{-1}(ax)^{3/2}}{\left(c+a^2cx^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^3*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] Defer[Int][(x^3*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(3/2), x]
```

Rubi steps

$$\int \frac{x^3 \tan^{-1}(ax)^{3/2}}{\left(c+a^2cx^2\right)^{3/2}}\,dx=\int \frac{x^3 \tan^{-1}(ax)^{3/2}}{\left(c+a^2cx^2\right)^{3/2}}\,dx$$

Mathematica [A] time = 6.27946, size = 0, normalized size = 0.

$$\int \frac{x^3 \tan^{-1}(ax)^{3/2}}{\left(c+a^2cx^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^3*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] Integrate[(x^3*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(3/2), x]
```

Maple [A] time = 3.514, size = 0, normalized size = 0.

$$\int x^3 (\arctan (ax))^{\frac{3}{2}} \left(a^2 c x^2 + c\right)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] int(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atan(a*x)**(3/2)/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \arctan(ax)^{\frac{3}{2}}}{(a^2cx^2 + c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2 + c)^(3/2), x)
```

3.823 $\int \frac{x^2 \tan^{-1}(ax)^{3/2}}{\left(c+a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(\frac{x^2 \tan^{-1}(ax)^{3/2}}{\left(a^2cx^2+c\right)^{3/2}},x\right)$$

```
[Out] Unintegrable[(x^2*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(3/2), x]
```

Rubi [A] time = 0.151754, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^2 \tan^{-1}(ax)^{3/2}}{\left(c+a^2cx^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^2*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] Defer[Int][(x^2*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(3/2), x]
```

Rubi steps

$$\int \frac{x^2 \tan^{-1}(ax)^{3/2}}{\left(c+a^2cx^2\right)^{3/2}}\,dx=\int \frac{x^2 \tan^{-1}(ax)^{3/2}}{\left(c+a^2cx^2\right)^{3/2}}\,dx$$

Mathematica [A] time = 3.67725, size = 0, normalized size = 0.

$$\int \frac{x^2 \tan^{-1}(ax)^{3/2}}{\left(c+a^2cx^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^2*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] Integrate[(x^2*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(3/2), x]
```

Maple [A] time = 3.1, size = 0, normalized size = 0.

$$\int x^{2} \left(\arctan \left(a x\right)\right)^{\frac{3}{2}} \left(a^{2} c \,x^{2}+c \right)^{-\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] int(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(a*x)**(3/2)/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \arctan(ax)^{\frac{3}{2}}}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2 + c)^(3/2), x)
```

3.824 $\int \frac{x\tan^{-1}(ax)^{3/2}}{\left(c+a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=129

$$-\frac{3\sqrt{\frac{\pi}{2}}\sqrt{a^2x^2+1}S\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{2a^2c\sqrt{a^2cx^2+c}}-\frac{\tan^{-1}(ax)^{3/2}}{a^2c\sqrt{a^2cx^2+c}}+\frac{3x\sqrt{\tan^{-1}(ax)}}{2ac\sqrt{a^2cx^2+c}}$$

```
[Out] (3*x*Sqrt[ArcTan[a*x]])/(2*a*c*Sqrt[c + a^2*c*x^2]) - ArcTan[a*x]^(3/2)/(a^
2*c*Sqrt[c + a^2*c*x^2]) - (3*Sqrt[Pi/2]*Sqrt[1 + a^2*x^2]*FresnelS[Sqrt[2/
Pi]*Sqrt[ArcTan[a*x]]])/(2*a^2*c*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.211357, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4930, 4905, 4904, 3296, 3305, 3351}

$$-\frac{3\sqrt{\frac{\pi}{2}}\sqrt{a^2x^2+1}S\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{2a^2c\sqrt{a^2cx^2+c}}-\frac{\tan^{-1}(ax)^{3/2}}{a^2c\sqrt{a^2cx^2+c}}+\frac{3x\sqrt{\tan^{-1}(ax)}}{2ac\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(3/2),x]
```


```
[Out] (3*x*Sqrt[ArcTan[a*x]])/(2*a*c*Sqrt[c + a^2*c*x^2]) - ArcTan[a*x]^(3/2)/(a^
2*c*Sqrt[c + a^2*c*x^2]) - (3*Sqrt[Pi/2]*Sqrt[1 + a^2*x^2]*FresnelS[Sqrt[2/
Pi]*Sqrt[ArcTan[a*x]]])/(2*a^2*c*Sqrt[c + a^2*c*x^2])
```


Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 4905

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[(d^(q + 1/2)*Sqrt[1 + c^2*x^2])/Sqrt[d + e*x^2], Int[(1 + c
```

```
^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

Rule 4904

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{x\tan^{-1}(ax)^{3/2}}{\left(c+a^2cx^2\right)^{3/2}}\,dx = -\frac{\tan^{-1}(ax)^{3/2}}{a^2c\sqrt{c+a^2cx^2}}+\frac{3\int \frac{\sqrt{\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{3/2}}\,dx}{2a}$$

$$= -\frac{\tan^{-1}(ax)^{3/2}}{a^2c\sqrt{c+a^2cx^2}}+\frac{\left(3\sqrt{1+a^2x^2}\right)\int \frac{\sqrt{\tan^{-1}(ax)}}{\left(1+a^2x^2\right)^{3/2}}\,dx}{2ac\sqrt{c+a^2cx^2}}$$

$$= -\frac{\tan^{-1}(ax)^{3/2}}{a^2c\sqrt{c+a^2cx^2}}+\frac{\left(3\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int \sqrt{x}\cos(x)\,dx,x,\tan^{-1}(ax)\right)}{2a^2c\sqrt{c+a^2cx^2}}$$

$$= \frac{3x\sqrt{\tan^{-1}(ax)}}{2ac\sqrt{c+a^2cx^2}}-\frac{\tan^{-1}(ax)^{3/2}}{a^2c\sqrt{c+a^2cx^2}}-\frac{\left(3\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int \frac{\sin(x)}{\sqrt{x}}\,dx,x,\tan^{-1}(ax)\right)}{4a^2c\sqrt{c+a^2cx^2}}$$

$$= \frac{3x\sqrt{\tan^{-1}(ax)}}{2ac\sqrt{c+a^2cx^2}}-\frac{\tan^{-1}(ax)^{3/2}}{a^2c\sqrt{c+a^2cx^2}}-\frac{\left(3\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int \sin\left(x^2\right)\,dx,x,\sqrt{\tan^{-1}(ax)}\right)}{2a^2c\sqrt{c+a^2cx^2}}$$

$$= \frac{3x\sqrt{\tan^{-1}(ax)}}{2ac\sqrt{c+a^2cx^2}}-\frac{\tan^{-1}(ax)^{3/2}}{a^2c\sqrt{c+a^2cx^2}}-\frac{3\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}\,S\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{2a^2c\sqrt{c+a^2cx^2}}$$

Mathematica [C] time = 0.172261, size = 128, normalized size = 0.99

$$\frac{3\sqrt{a^2x^2+1}\sqrt{-i\tan^{-1}(ax)}\,\text{Gamma}\left(\frac{1}{2},-i\tan^{-1}(ax)\right)+3\sqrt{a^2x^2+1}\sqrt{i\tan^{-1}(ax)}\,\text{Gamma}\left(\frac{1}{2},i\tan^{-1}(ax)\right)+4\left(3ax-2\right.}{8a^2c\sqrt{a^2cx^2+c}\sqrt{\tan^{-1}(ax)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] (4*(3*a*x - 2*ArcTan[a*x])*ArcTan[a*x] + 3*Sqrt[1 + a^2*x^2]*Sqrt[(-I)*ArcT
an[a*x]]*Gamma[1/2, (-I)*ArcTan[a*x]] + 3*Sqrt[1 + a^2*x^2]*Sqrt[I*ArcTan[a
*x]]*Gamma[1/2, I*ArcTan[a*x]])/(8*a^2*c*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*
x]])
```

Maple [F] time = 0.906, size = 0, normalized size = 0.

$$\int x\left(\arctan(ax)\right)^{\frac{3}{2}}\left(a^2cx^2+c\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] int(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(a*x)**(3/2)/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \arctan(ax)^{\frac{3}{2}}}{\left(a^2cx^2 + c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x*arctan(a*x)^(3/2)/(a^2*c*x^2 + c)^(3/2), x)
```

3.825 $\int \frac{\tan^{-1}(ax)^{3/2}}{\left(c+a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=125

$$-\frac{3\sqrt{\frac{\pi}{2}}\sqrt{a^2x^2+1}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{2ac\sqrt{a^2cx^2+c}}+\frac{x\tan^{-1}(ax)^{3/2}}{c\sqrt{a^2cx^2+c}}+\frac{3\sqrt{\tan^{-1}(ax)}}{2ac\sqrt{a^2cx^2+c}}$$

```
[Out] (3*Sqrt[ArcTan[a*x]])/(2*a*c*Sqrt[c + a^2*c*x^2]) + (x*ArcTan[a*x]^(3/2))/(
c*Sqrt[c + a^2*c*x^2]) - (3*Sqrt[Pi/2]*Sqrt[1 + a^2*x^2]*FresnelC[Sqrt[2/Pi
]*Sqrt[ArcTan[a*x]]])/(2*a*c*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.14225, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4898, 4905, 4904, 3304, 3352}

$$-\frac{3\sqrt{\frac{\pi}{2}}\sqrt{a^2x^2+1}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{2ac\sqrt{a^2cx^2+c}}+\frac{x\tan^{-1}(ax)^{3/2}}{c\sqrt{a^2cx^2+c}}+\frac{3\sqrt{\tan^{-1}(ax)}}{2ac\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^(3/2)/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] (3*Sqrt[ArcTan[a*x]])/(2*a*c*Sqrt[c + a^2*c*x^2]) + (x*ArcTan[a*x]^(3/2))/(
c*Sqrt[c + a^2*c*x^2]) - (3*Sqrt[Pi/2]*Sqrt[1 + a^2*x^2]*FresnelC[Sqrt[2/Pi
]*Sqrt[ArcTan[a*x]]])/(2*a*c*Sqrt[c + a^2*c*x^2])
```

Rule 4898

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[(b*p*(a + b*ArcTan[c*x])^(p - 1))/(c*d*Sqrt[d + e*x^2]), x]
 + (-Dist[b^2*p*(p - 1), Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2),
 x], x] + Simp[(x*(a + b*ArcTan[c*x])^p)/(d*Sqrt[d + e*x^2]), x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

Rule 4905

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[(d^(q + 1/2)*Sqrt[1 + c^2*x^2])/Sqrt[d + e*x^2], Int[(1 + c
```

```
^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

Rule 4904

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^{-1}(ax)^{3/2}}{\left(c+a^2cx^2\right)^{3/2}}\,dx &= \frac{3\sqrt{\tan^{-1}(ax)}}{2ac\sqrt{c+a^2cx^2}}+\frac{x\tan^{-1}(ax)^{3/2}}{c\sqrt{c+a^2cx^2}}-\frac{3}{4}\int \frac{1}{\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}}\,dx\\
&= \frac{3\sqrt{\tan^{-1}(ax)}}{2ac\sqrt{c+a^2cx^2}}+\frac{x\tan^{-1}(ax)^{3/2}}{c\sqrt{c+a^2cx^2}}-\frac{\left(3\sqrt{1+a^2x^2}\right)\int \frac{1}{\left(1+a^2x^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}}\,dx}{4c\sqrt{c+a^2cx^2}}\\
&= \frac{3\sqrt{\tan^{-1}(ax)}}{2ac\sqrt{c+a^2cx^2}}+\frac{x\tan^{-1}(ax)^{3/2}}{c\sqrt{c+a^2cx^2}}-\frac{\left(3\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int \frac{\cos(x)}{\sqrt{x}}\,dx,x,\tan^{-1}(ax)\right)}{4ac\sqrt{c+a^2cx^2}}\\
&= \frac{3\sqrt{\tan^{-1}(ax)}}{2ac\sqrt{c+a^2cx^2}}+\frac{x\tan^{-1}(ax)^{3/2}}{c\sqrt{c+a^2cx^2}}-\frac{\left(3\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int \cos\left(x^2\right)\,dx,x,\sqrt{\tan^{-1}(ax)}\right)}{2ac\sqrt{c+a^2cx^2}}\\
&= \frac{3\sqrt{\tan^{-1}(ax)}}{2ac\sqrt{c+a^2cx^2}}+\frac{x\tan^{-1}(ax)^{3/2}}{c\sqrt{c+a^2cx^2}}-\frac{3\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}\,C\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{2ac\sqrt{c+a^2cx^2}}
\end{aligned}$$

Mathematica [C] time = 0.167468, size = 104, normalized size = 0.83

$$\frac{\left(a^2x^2+1\right)^{3/2}\sqrt{\tan^{-1}(ax)}\left(\sqrt{i\tan^{-1}(ax)}\text{Gamma}\left(\frac{5}{2},-i\tan^{-1}(ax)\right)+\sqrt{-i\tan^{-1}(ax)}\text{Gamma}\left(\frac{5}{2},i\tan^{-1}(ax)\right)\right)}{2a\left(c\left(a^2x^2+1\right)\right)^{3/2}\sqrt{\tan^{-1}(ax)^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTan[a*x]^(3/2)/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] ((1 + a^2*x^2)^(3/2)*Sqrt[ArcTan[a*x]]*(Sqrt[I*ArcTan[a*x]]*Gamma[5/2, (-I)
*ArcTan[a*x]] + Sqrt[(-I)*ArcTan[a*x]]*Gamma[5/2, I*ArcTan[a*x]]))/(2*a*(c*
(1 + a^2*x^2))^(3/2)*Sqrt[ArcTan[a*x]^2])
```

Maple [F] time = 0.885, size = 0, normalized size = 0.

$$\int \left(\arctan\left(ax\right)\right)^{\frac{3}{2}}\left(a^2cx^2+c\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] int(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(3/2)/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(ax)^{\frac{3}{2}}}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)^(3/2)/(a^2*c*x^2 + c)^(3/2), x)
```

3.826 $\int \frac{\tan^{-1}(ax)^{3/2}}{x\left(c+a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(\frac{\tan^{-1}(ax)^{3/2}}{x\left(a^2cx^2+c\right)^{3/2}},x\right)$$

```
[Out] Unintegrable[ArcTan[a*x]^(3/2)/(x*(c + a^2*c*x^2)^(3/2)), x]
```

Rubi [A] time = 0.117206, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\tan^{-1}(ax)^{3/2}}{x\left(c+a^2cx^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcTan[a*x]^(3/2)/(x*(c + a^2*c*x^2)^(3/2)),x]
```

```
[Out] Defer[Int][ArcTan[a*x]^(3/2)/(x*(c + a^2*c*x^2)^(3/2)), x]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^{3/2}}{x\left(c+a^2cx^2\right)^{3/2}}\,dx=\int \frac{\tan^{-1}(ax)^{3/2}}{x\left(c+a^2cx^2\right)^{3/2}}\,dx$$

Mathematica [A] time = 2.20839, size = 0, normalized size = 0.

$$\int \frac{\tan^{-1}(ax)^{3/2}}{x\left(c+a^2cx^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcTan[a*x]^(3/2)/(x*(c + a^2*c*x^2)^(3/2)),x]
```

```
[Out] Integrate[ArcTan[a*x]^(3/2)/(x*(c + a^2*c*x^2)^(3/2)), x]
```

Maple [A] time = 0.743, size = 0, normalized size = 0.

$$\int \frac{1}{x} (\arctan (ax))^{\frac{3}{2}} \left(a^2cx^2+c\right)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] int(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(3/2)/x/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan{(ax)}^{\frac{3}{2}}}{\left(a^2cx^2+c\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)^(3/2)/((a^2*c*x^2 + c)^(3/2)*x), x)
```

3.827 $\int \frac{\tan^{-1}(ax)^{3/2}}{x^2(c+a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(\frac{\tan^{-1}(ax)^{3/2}}{x^2\left(a^2cx^2+c\right)^{3/2}},x\right)$$

```
[Out] Unintegrable[ArcTan[a*x]^(3/2)/(x^2*(c + a^2*c*x^2)^(3/2)), x]
```

Rubi [A] time = 0.117861, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\tan^{-1}(ax)^{3/2}}{x^2\left(c+a^2cx^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcTan[a*x]^(3/2)/(x^2*(c + a^2*c*x^2)^(3/2)),x]
```

```
[Out] Defer[Int][ArcTan[a*x]^(3/2)/(x^2*(c + a^2*c*x^2)^(3/2)), x]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^{3/2}}{x^2\left(c+a^2cx^2\right)^{3/2}}\,dx=\int \frac{\tan^{-1}(ax)^{3/2}}{x^2\left(c+a^2cx^2\right)^{3/2}}\,dx$$

Mathematica [A] time = 7.11128, size = 0, normalized size = 0.

$$\int \frac{\tan^{-1}(ax)^{3/2}}{x^2\left(c+a^2cx^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcTan[a*x]^(3/2)/(x^2*(c + a^2*c*x^2)^(3/2)),x]
```

```
[Out] Integrate[ArcTan[a*x]^(3/2)/(x^2*(c + a^2*c*x^2)^(3/2)), x]
```

Maple [A] time = 0.762, size = 0, normalized size = 0.

$$\int \frac{1}{x^2} (\arctan (ax))^{\frac{3}{2}} \left(a^2cx^2 + c\right)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(3/2)/x^2/(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] int(arctan(a*x)^(3/2)/x^2/(a^2*c*x^2+c)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/x^2/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/x^2/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(3/2)/x**2/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(ax)^{\frac{3}{2}}}{\left(a^2cx^2+c\right)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/x^2/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)^(3/2)/((a^2*c*x^2 + c)^(3/2)*x^2), x)
```

3.828 $\int \frac{x^m \tan^{-1}(ax)^{3/2}}{\left(c+a^2cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(\frac{x^m \tan^{-1}(ax)^{3/2}}{\left(a^2cx^2+c\right)^{5/2}},x\right)$$

```
[Out] Unintegrable[(x^m*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(5/2), x]
```

Rubi [A] time = 0.117957, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m \tan^{-1}(ax)^{3/2}}{\left(c+a^2cx^2\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] Defer[Int][(x^m*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(5/2), x]
```

Rubi steps

$$\int \frac{x^m \tan^{-1}(ax)^{3/2}}{\left(c+a^2cx^2\right)^{5/2}}\,dx=\int \frac{x^m \tan^{-1}(ax)^{3/2}}{\left(c+a^2cx^2\right)^{5/2}}\,dx$$

Mathematica [A] time = 1.41864, size = 0, normalized size = 0.

$$\int \frac{x^m \tan^{-1}(ax)^{3/2}}{\left(c+a^2cx^2\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] Integrate[(x^m*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(5/2), x]
```

Maple [A] time = 0.97, size = 0, normalized size = 0.

$$\int x^m (\arctan(ax))^{\frac{3}{2}} \left(a^2cx^2+c\right)^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x)
```

```
[Out] int(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a^2cx^2+c}\,x^m \arctan(ax)^{\frac{3}{2}}}{a^6c^3x^6+3\,a^4c^3x^4+3\,a^2c^3x^2+c^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^m*arctan(a*x)^(3/2)/(a^6*c^3*x^6 + 3*a^4*c^3
*x^4 + 3*a^2*c^3*x^2 + c^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*atan(a*x)**(3/2)/(a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m \arctan(ax)^{\frac{3}{2}}}{(a^2cx^2 + c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2 + c)^(5/2), x)
```

3.829 $\int \frac{x^5 \tan^{-1}(ax)^{3/2}}{\left(c+a^2cx^2\right)^{5/2}} dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(\frac{x^5 \tan^{-1}(ax)^{3/2}}{\left(a^2cx^2+c\right)^{5/2}},x\right)$$

```
[Out] Unintegrable[(x^5*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(5/2), x]
```

Rubi [A] time = 0.123934, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^5 \tan^{-1}(ax)^{3/2}}{\left(c+a^2cx^2\right)^{5/2}} dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^5*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] Defer[Int][(x^5*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(5/2), x]
```

Rubi steps

$$\int \frac{x^5 \tan^{-1}(ax)^{3/2}}{\left(c+a^2cx^2\right)^{5/2}} dx = \int \frac{x^5 \tan^{-1}(ax)^{3/2}}{\left(c+a^2cx^2\right)^{5/2}} dx$$

Mathematica [A] time = 8.90971, size = 0, normalized size = 0.

$$\int \frac{x^5 \tan^{-1}(ax)^{3/2}}{\left(c+a^2cx^2\right)^{5/2}} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^5*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] Integrate[(x^5*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(5/2), x]
```

Maple [A] time = 4.625, size = 0, normalized size = 0.

$$\int x^5 (\arctan(ax))^{\frac{3}{2}} \left(a^2cx^2+c\right)^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x)
```

```
[Out] int(x^5*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*atan(a*x)**(3/2)/(a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5 \arctan(ax)^{\frac{3}{2}}}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x^5*arctan(a*x)^(3/2)/(a^2*c*x^2 + c)^(5/2), x)
```

3.830 $\int \frac{x^4 \tan^{-1}(ax)^{3/2}}{\left(c+a^2cx^2\right)^{5/2}} dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(\frac{x^4 \tan^{-1}(ax)^{3/2}}{\left(a^2cx^2+c\right)^{5/2}},x\right)$$

```
[Out] Unintegrable[(x^4*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(5/2), x]
```

Rubi [A] time = 0.122993, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{x^4 \tan^{-1}(ax)^{3/2}}{\left(c+a^2cx^2\right)^{5/2}} dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^4*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] Defer[Int][(x^4*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(5/2), x]
```

Rubi steps

$$\int \frac{x^4 \tan^{-1}(ax)^{3/2}}{\left(c+a^2cx^2\right)^{5/2}} dx = \int \frac{x^4 \tan^{-1}(ax)^{3/2}}{\left(c+a^2cx^2\right)^{5/2}} dx$$

Mathematica [A] time = 3.82887, size = 0, normalized size = 0.

$$\int \frac{x^4 \tan^{-1}(ax)^{3/2}}{\left(c+a^2cx^2\right)^{5/2}} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^4*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(5/2),x]
```

[Out] Integrate[(x^4*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(5/2), x]

Maple [A] time = 2.106, size = 0, normalized size = 0.

$$\int x^{4} \left(\arctan \left(a x\right)\right)^{\frac{3}{2}} \left(a^{2} c \,x^{2}+c \right)^{-\frac{5}{2}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x)

[Out] int(x^4*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x)

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")

[Out] Exception raised: RuntimeError

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*atan(a*x)**(3/2)/(a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4 \arctan(ax)^{\frac{3}{2}}}{(a^2cx^2 + c)^{\frac{5}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x^4*arctan(a*x)^(3/2)/(a^2*c*x^2 + c)^(5/2), x)
```

3.831 $\int \frac{x^3 \tan^{-1}(ax)^{3/2}}{\left(c+a^2cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=263

$$-\frac{9\sqrt{\frac{\pi}{2}}\sqrt{a^2x^2+1}S\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{8a^4c^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{\frac{\pi}{6}}\sqrt{a^2x^2+1}S\left(\sqrt{\frac{6}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{24a^4c^2\sqrt{a^2cx^2+c}}+\frac{x\sqrt{\tan^{-1}(ax)}}{a^3c^2\sqrt{a^2cx^2+c}}-\frac{2\tan^{-1}(ax)^{3/2}}{3a^4c^2\sqrt{a^2cx^2+c}}+\frac{x}{6ac}$$

```
[Out] (x^3*Sqrt[ArcTan[a*x]])/(6*a*c*(c + a^2*c*x^2)^(3/2)) + (x*Sqrt[ArcTan[a*x]
])/(a^3*c^2*Sqrt[c + a^2*c*x^2]) - (x^2*ArcTan[a*x]^(3/2))/(3*a^2*c*(c + a^
2*c*x^2)^(3/2)) - (2*ArcTan[a*x]^(3/2))/(3*a^4*c^2*Sqrt[c + a^2*c*x^2]) - (
9*Sqrt[Pi/2]*Sqrt[1 + a^2*x^2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(8*a
^4*c^2*Sqrt[c + a^2*c*x^2]) + (Sqrt[Pi/6]*Sqrt[1 + a^2*x^2]*FresnelS[Sqrt[6
/Pi]*Sqrt[ArcTan[a*x]]])/(24*a^4*c^2*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.64147, antiderivative size = 263, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {4940, 4930, 4905, 4904, 3296, 3305, 3351, 4971, 4970, 3312}

$$-\frac{9\sqrt{\frac{\pi}{2}}\sqrt{a^2x^2+1}S\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{8a^4c^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{\frac{\pi}{6}}\sqrt{a^2x^2+1}S\left(\sqrt{\frac{6}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{24a^4c^2\sqrt{a^2cx^2+c}}+\frac{x\sqrt{\tan^{-1}(ax)}}{a^3c^2\sqrt{a^2cx^2+c}}-\frac{2\tan^{-1}(ax)^{3/2}}{3a^4c^2\sqrt{a^2cx^2+c}}+\frac{x}{6ac}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(5/2),x]
```


```
[Out] (x^3*Sqrt[ArcTan[a*x]])/(6*a*c*(c + a^2*c*x^2)^(3/2)) + (x*Sqrt[ArcTan[a*x]
])/(a^3*c^2*Sqrt[c + a^2*c*x^2]) - (x^2*ArcTan[a*x]^(3/2))/(3*a^2*c*(c + a^
2*c*x^2)^(3/2)) - (2*ArcTan[a*x]^(3/2))/(3*a^4*c^2*Sqrt[c + a^2*c*x^2]) - (
9*Sqrt[Pi/2]*Sqrt[1 + a^2*x^2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(8*a
^4*c^2*Sqrt[c + a^2*c*x^2]) + (Sqrt[Pi/6]*Sqrt[1 + a^2*x^2]*FresnelS[Sqrt[6
/Pi]*Sqrt[ArcTan[a*x]]])/(24*a^4*c^2*Sqrt[c + a^2*c*x^2])
```


Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_)*((d_) + (e_.)
*(x_)^2)^(q_), x_Symbol] :> Simp[(b*p*(f*x)^m*(d + e*x^2)^(q + 1)*(a + b*Ar
cTan[c*x])^(p - 1))/(c*d*m^2), x] + (Dist[(f^2*(m - 1))/(c^2*d*m), Int[(f*x
```

```
)^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p, x], x] - Dist[(b^2*p*(
p - 1))/m^2, Int[(f*x)^m*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x]
- Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(c^2*d*m
), x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q +
 2, 0] && LtQ[q, -1] && GtQ[p, 1]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 4905

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[(d^(q + 1/2)*Sqrt[1 + c^2*x^2])/Sqrt[d + e*x^2], Int[(1 + c
^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

Rule 4904

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4971

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[(d^(q + 1/2)*Sqrt[1 + c^2*x^2])/Sqrt[d + e*x^2],
 Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d
, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(In
tegerQ[q] || GtQ[d, 0])
```

Rule 4970

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sin[x]^m)/
Cos[x]^(m + 2*(q + 1)), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\int \frac{x^3 \tan^{-1}(ax)^{3/2}}{\left(c+a^2cx^2\right)^{5/2}}\,dx = \frac{x^3\sqrt{\tan^{-1}(ax)}}{6ac\left(c+a^2cx^2\right)^{3/2}} - \frac{x^2\tan^{-1}(ax)^{3/2}}{3a^2c\left(c+a^2cx^2\right)^{3/2}} - \frac{1}{12}\int \frac{x^3}{\left(c+a^2cx^2\right)^{5/2}\sqrt{\tan^{-1}(ax)}}\,dx + \frac{2\int \frac{x\tan^{-1}(ax)^{3/2}}{\left(c+a^2cx^2\right)^{3/2}}}{3a^2c}$$

$$= \frac{x^3\sqrt{\tan^{-1}(ax)}}{6ac\left(c+a^2cx^2\right)^{3/2}} - \frac{x^2\tan^{-1}(ax)^{3/2}}{3a^2c\left(c+a^2cx^2\right)^{3/2}} - \frac{2\tan^{-1}(ax)^{3/2}}{3a^4c^2\sqrt{c+a^2cx^2}} + \frac{\int \frac{\sqrt{\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{3/2}}\,dx}{a^3c} - \frac{\sqrt{1+a^2x^2}\int}{12c^2\sqrt{}}$$

$$= \frac{x^3\sqrt{\tan^{-1}(ax)}}{6ac\left(c+a^2cx^2\right)^{3/2}} - \frac{x^2\tan^{-1}(ax)^{3/2}}{3a^2c\left(c+a^2cx^2\right)^{3/2}} - \frac{2\tan^{-1}(ax)^{3/2}}{3a^4c^2\sqrt{c+a^2cx^2}} - \frac{\sqrt{1+a^2x^2}\,\mathrm{Subst}\left(\int \frac{\sin^3(x)}{\sqrt{x}}\,dx, x, \mathrm{t}\right.}{12a^4c^2\sqrt{c+a^2cx^2}}$$

$$= \frac{x^3\sqrt{\tan^{-1}(ax)}}{6ac\left(c+a^2cx^2\right)^{3/2}} - \frac{x^2\tan^{-1}(ax)^{3/2}}{3a^2c\left(c+a^2cx^2\right)^{3/2}} - \frac{2\tan^{-1}(ax)^{3/2}}{3a^4c^2\sqrt{c+a^2cx^2}} - \frac{\sqrt{1+a^2x^2}\,\mathrm{Subst}\left(\int\left(\frac{3\sin(x)}{4\sqrt{x}} - \frac{\sin(}{4\sqrt{}}\right.\right.}{12a^4c^2\sqrt{c+a^2c}}$$

$$= \frac{x^3\sqrt{\tan^{-1}(ax)}}{6ac\left(c+a^2cx^2\right)^{3/2}} + \frac{x\sqrt{\tan^{-1}(ax)}}{a^3c^2\sqrt{c+a^2cx^2}} - \frac{x^2\tan^{-1}(ax)^{3/2}}{3a^2c\left(c+a^2cx^2\right)^{3/2}} - \frac{2\tan^{-1}(ax)^{3/2}}{3a^4c^2\sqrt{c+a^2cx^2}} + \frac{\sqrt{1+a^2x^2}\,\mathrm{Sub}}{48}$$

$$= \frac{x^3\sqrt{\tan^{-1}(ax)}}{6ac\left(c+a^2cx^2\right)^{3/2}} + \frac{x\sqrt{\tan^{-1}(ax)}}{a^3c^2\sqrt{c+a^2cx^2}} - \frac{x^2\tan^{-1}(ax)^{3/2}}{3a^2c\left(c+a^2cx^2\right)^{3/2}} - \frac{2\tan^{-1}(ax)^{3/2}}{3a^4c^2\sqrt{c+a^2cx^2}} + \frac{\sqrt{1+a^2x^2}\,\mathrm{Sub}}{}$$

$$= \frac{x^3\sqrt{\tan^{-1}(ax)}}{6ac\left(c+a^2cx^2\right)^{3/2}} + \frac{x\sqrt{\tan^{-1}(ax)}}{a^3c^2\sqrt{c+a^2cx^2}} - \frac{x^2\tan^{-1}(ax)^{3/2}}{3a^2c\left(c+a^2cx^2\right)^{3/2}} - \frac{2\tan^{-1}(ax)^{3/2}}{3a^4c^2\sqrt{c+a^2cx^2}} - \frac{9\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}}{8a^4c^2}$$

Mathematica [C] time = 1.05531, size = 272, normalized size = 1.03

$$\frac{3\left(a^2x^2+1\right)^{3/2}\left(3\sqrt{-i\tan^{-1}(ax)}\mathrm{Gamma}\left(\frac{1}{2},-i\tan^{-1}(ax)\right)+3\sqrt{i\tan^{-1}(ax)}\mathrm{Gamma}\left(\frac{1}{2},i\tan^{-1}(ax)\right)+\sqrt{3}\left(\sqrt{-i\tan^{-1}(\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] (24*ArcTan[a*x]*(a*x*(6 + 7*a^2*x^2) - 2*(2 + 3*a^2*x^2)*ArcTan[a*x]) - 7*S
qrt[6*Pi]*(1 + a^2*x^2)^(3/2)*Sqrt[ArcTan[a*x]]*(3*Sqrt[3]*FresnelS[Sqrt[2/
Pi]*Sqrt[ArcTan[a*x]]] - FresnelS[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]]) + 3*(1 + a
```

```
^2*x^2)^(3/2)*(3*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-I)*ArcTan[a*x]] + 3*Sq
rt[I*ArcTan[a*x]]*Gamma[1/2, I*ArcTan[a*x]] + Sqrt[3]*(Sqrt[(-I)*ArcTan[a*x
]]*Gamma[1/2, (-3*I)*ArcTan[a*x]] + Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (3*I)*Ar
cTan[a*x]])))/(144*a^4*c*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]])
```

Maple [F] time = 3.464, size = 0, normalized size = 0.

$$\int x^{3} \left(\arctan \left(a x\right)\right)^{\frac{3}{2}} \left(a^{2} c \,x^{2}+c \right)^{-\frac{5}{2}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x)
```

```
[Out] int(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atan(a*x)**(3/2)/(a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \arctan(ax)^{\frac{3}{2}}}{\left(a^2cx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2 + c)^(5/2), x)
```

3.832 $\int \frac{x^2 \tan^{-1}(ax)^{3/2}}{(c+a^2cx^2)^{5/2}} dx$

Optimal. Leaf size=247

$$-\frac{3\sqrt{\frac{\pi}{2}}\sqrt{a^2x^2+1}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{8a^3c^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{\frac{\pi}{6}}\sqrt{a^2x^2+1}\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{24a^3c^2\sqrt{a^2cx^2+c}}+\frac{3\sqrt{\tan^{-1}(ax)}}{8a^3c^2\sqrt{a^2cx^2+c}}-\frac{\sqrt{a^2}}{}$$

```
[Out] (3*Sqrt[ArcTan[a*x]])/(8*a^3*c^2*Sqrt[c + a^2*c*x^2]) + (x^3*ArcTan[a*x]^(3
/2))/(3*c*(c + a^2*c*x^2)^(3/2)) - (Sqrt[1 + a^2*x^2]*Sqrt[ArcTan[a*x]]*Cos
[3*ArcTan[a*x]])/(24*a^3*c^2*Sqrt[c + a^2*c*x^2]) - (3*Sqrt[Pi/2]*Sqrt[1 +
a^2*x^2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(8*a^3*c^2*Sqrt[c + a^2*c*
x^2]) + (Sqrt[Pi/6]*Sqrt[1 + a^2*x^2]*FresnelC[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]
])/(24*a^3*c^2*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.476209, antiderivative size = 247, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {4944, 4971, 4970, 3312, 3296, 3304, 3352}

$$-\frac{3\sqrt{\frac{\pi}{2}}\sqrt{a^2x^2+1}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{8a^3c^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{\frac{\pi}{6}}\sqrt{a^2x^2+1}\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{24a^3c^2\sqrt{a^2cx^2+c}}+\frac{3\sqrt{\tan^{-1}(ax)}}{8a^3c^2\sqrt{a^2cx^2+c}}-\frac{\sqrt{a^2}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(5/2),x]
```


```
[Out] (3*Sqrt[ArcTan[a*x]])/(8*a^3*c^2*Sqrt[c + a^2*c*x^2]) + (x^3*ArcTan[a*x]^(3
/2))/(3*c*(c + a^2*c*x^2)^(3/2)) - (Sqrt[1 + a^2*x^2]*Sqrt[ArcTan[a*x]]*Cos
[3*ArcTan[a*x]])/(24*a^3*c^2*Sqrt[c + a^2*c*x^2]) - (3*Sqrt[Pi/2]*Sqrt[1 +
a^2*x^2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(8*a^3*c^2*Sqrt[c + a^2*c*
x^2]) + (Sqrt[Pi/6]*Sqrt[1 + a^2*x^2]*FresnelC[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]
])/(24*a^3*c^2*Sqrt[c + a^2*c*x^2])
```


Rule 4944

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a +
 b*ArcTan[c*x])^p)/(d*f*(m + 1)), x] - Dist[(b*c*p)/(f*(m + 1)), Int[(f*x)^
```

```
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rule 4971

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[(d^(q + 1/2)*Sqrt[1 + c^2*x^2])/Sqrt[d + e*x^2],
 Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d
, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(In
tegerQ[q] || GtQ[d, 0])
```

Rule 4970

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sin[x]^m)/
Cos[x]^(m + 2*(q + 1)), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{x^2 \tan^{-1}(ax)^{3/2}}{\left(c+a^2cx^2\right)^{5/2}}\,dx = \frac{x^3 \tan^{-1}(ax)^{3/2}}{3c\left(c+a^2cx^2\right)^{3/2}} - \frac{1}{2}a\int \frac{x^3\sqrt{\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{5/2}}\,dx$$

$$= \frac{x^3 \tan^{-1}(ax)^{3/2}}{3c\left(c+a^2cx^2\right)^{3/2}} - \frac{\left(a\sqrt{1+a^2x^2}\right)\int \frac{x^3\sqrt{\tan^{-1}(ax)}}{\left(1+a^2x^2\right)^{5/2}}\,dx}{2c^2\sqrt{c+a^2cx^2}}$$

$$= \frac{x^3 \tan^{-1}(ax)^{3/2}}{3c\left(c+a^2cx^2\right)^{3/2}} - \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \sqrt{x}\sin^3(x)\,dx, x, \tan^{-1}(ax)\right)}{2a^3c^2\sqrt{c+a^2cx^2}}$$

$$= \frac{x^3 \tan^{-1}(ax)^{3/2}}{3c\left(c+a^2cx^2\right)^{3/2}} - \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \left(\frac{3}{4}\sqrt{x}\sin(x) - \frac{1}{4}\sqrt{x}\sin(3x)\right)dx, x, \tan^{-1}(ax)\right)}{2a^3c^2\sqrt{c+a^2cx^2}}$$

$$= \frac{x^3 \tan^{-1}(ax)^{3/2}}{3c\left(c+a^2cx^2\right)^{3/2}} + \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \sqrt{x}\sin(3x)\,dx, x, \tan^{-1}(ax)\right)}{8a^3c^2\sqrt{c+a^2cx^2}} - \frac{\left(3\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int\right.}{8a^3c^2}$$

$$= \frac{3\sqrt{\tan^{-1}(ax)}}{8a^3c^2\sqrt{c+a^2cx^2}} + \frac{x^3 \tan^{-1}(ax)^{3/2}}{3c\left(c+a^2cx^2\right)^{3/2}} - \frac{\sqrt{1+a^2x^2}\sqrt{\tan^{-1}(ax)}\cos\left(3\tan^{-1}(ax)\right)}{24a^3c^2\sqrt{c+a^2cx^2}} + \frac{\sqrt{1+a^2x^2}\,\text{S}}{}$$

$$= \frac{3\sqrt{\tan^{-1}(ax)}}{8a^3c^2\sqrt{c+a^2cx^2}} + \frac{x^3 \tan^{-1}(ax)^{3/2}}{3c\left(c+a^2cx^2\right)^{3/2}} - \frac{\sqrt{1+a^2x^2}\sqrt{\tan^{-1}(ax)}\cos\left(3\tan^{-1}(ax)\right)}{24a^3c^2\sqrt{c+a^2cx^2}} + \frac{\sqrt{1+a^2x^2}\,\text{S}}{}$$

$$= \frac{3\sqrt{\tan^{-1}(ax)}}{8a^3c^2\sqrt{c+a^2cx^2}} + \frac{x^3 \tan^{-1}(ax)^{3/2}}{3c\left(c+a^2cx^2\right)^{3/2}} - \frac{\sqrt{1+a^2x^2}\sqrt{\tan^{-1}(ax)}\cos\left(3\tan^{-1}(ax)\right)}{24a^3c^2\sqrt{c+a^2cx^2}} - \frac{3\sqrt{\frac{\pi}{2}}\sqrt{1+a}}{8a}$$

Mathematica [C] time = 0.545978, size = 338, normalized size = 1.37

$$\frac{-ia^2x^2\sqrt{3a^2x^2+3}\sqrt{-i\tan^{-1}(ax)}\text{Gamma}\left(\frac{1}{2}, -3i\tan^{-1}(ax)\right) + ia^2x^2\sqrt{3a^2x^2+3}\sqrt{i\tan^{-1}(ax)}\text{Gamma}\left(\frac{1}{2}, 3i\tan^{-1}(ax)\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] (96*ArcTan[a*x] + 144*a^2*x^2*ArcTan[a*x] + 96*a^3*x^3*ArcTan[a*x]^2 + (27*
I)*(1 + a^2*x^2)^(3/2)*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-I)*ArcTan[a*x]]
```

```
- (27*I)*(1 + a^2*x^2)^(3/2)*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, I*ArcTan[a*x]]
- I*Sqrt[3 + 3*a^2*x^2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-3*I)*ArcTan[a*x
]] - I*a^2*x^2*Sqrt[3 + 3*a^2*x^2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-3*I)
*ArcTan[a*x]] + I*Sqrt[3 + 3*a^2*x^2]*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (3*I)*
ArcTan[a*x]] + I*a^2*x^2*Sqrt[3 + 3*a^2*x^2]*Sqrt[I*ArcTan[a*x]]*Gamma[1/2,
 (3*I)*ArcTan[a*x]])/(288*a^3*c^2*(1 + a^2*x^2)*Sqrt[c + a^2*c*x^2]*Sqrt[Ar
cTan[a*x]])
```

Maple [F] time = 2.95, size = 0, normalized size = 0.

$$\int x^2 (\arctan(ax))^{\frac{3}{2}} \left(a^2 c x^2 + c\right)^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x)
```

```
[Out] int(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(a*x)**(3/2)/(a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \arctan (ax)^{\frac{3}{2}}}{\left(a^2cx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2 + c)^(5/2), x)
```

3.833 $\int \frac{x\tan^{-1}(ax)^{3/2}}{\left(c+a^2cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=248

$$-\frac{3\sqrt{\frac{\pi}{2}}\sqrt{a^2x^2+1}S\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{8a^2c^2\sqrt{a^2cx^2+c}}-\frac{\sqrt{\frac{\pi}{6}}\sqrt{a^2x^2+1}S\left(\sqrt{\frac{6}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{24a^2c^2\sqrt{a^2cx^2+c}}+\frac{3x\sqrt{\tan^{-1}(ax)}}{8ac^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{a^2x^2+1}\sqrt{\tan^{-1}(ax)}\,\text{si}}{24a^2c^2\sqrt{a^2cx}}$$

```
[Out] (3*x*Sqrt[ArcTan[a*x]])/(8*a*c^2*Sqrt[c + a^2*c*x^2]) - ArcTan[a*x]^(3/2)/(
3*a^2*c*(c + a^2*c*x^2)^(3/2)) - (3*Sqrt[Pi/2]*Sqrt[1 + a^2*x^2]*FresnelS[S
qrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(8*a^2*c^2*Sqrt[c + a^2*c*x^2]) - (Sqrt[Pi/6]
*Sqrt[1 + a^2*x^2]*FresnelS[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]])/(24*a^2*c^2*Sqrt
[c + a^2*c*x^2]) + (Sqrt[1 + a^2*x^2]*Sqrt[ArcTan[a*x]]*Sin[3*ArcTan[a*x]])
/(24*a^2*c^2*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.281474, antiderivative size = 248, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {4930, 4905, 4904, 3312, 3296, 3305, 3351}

$$-\frac{3\sqrt{\frac{\pi}{2}}\sqrt{a^2x^2+1}S\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{8a^2c^2\sqrt{a^2cx^2+c}}-\frac{\sqrt{\frac{\pi}{6}}\sqrt{a^2x^2+1}S\left(\sqrt{\frac{6}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{24a^2c^2\sqrt{a^2cx^2+c}}+\frac{3x\sqrt{\tan^{-1}(ax)}}{8ac^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{a^2x^2+1}\sqrt{\tan^{-1}(ax)}\,\text{si}}{24a^2c^2\sqrt{a^2cx}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(5/2),x]
```


```
[Out] (3*x*Sqrt[ArcTan[a*x]])/(8*a*c^2*Sqrt[c + a^2*c*x^2]) - ArcTan[a*x]^(3/2)/(
3*a^2*c*(c + a^2*c*x^2)^(3/2)) - (3*Sqrt[Pi/2]*Sqrt[1 + a^2*x^2]*FresnelS[S
qrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(8*a^2*c^2*Sqrt[c + a^2*c*x^2]) - (Sqrt[Pi/6]
*Sqrt[1 + a^2*x^2]*FresnelS[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]])/(24*a^2*c^2*Sqrt
[c + a^2*c*x^2]) + (Sqrt[1 + a^2*x^2]*Sqrt[ArcTan[a*x]]*Sin[3*ArcTan[a*x]])
/(24*a^2*c^2*Sqrt[c + a^2*c*x^2])
```


Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
```

```
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 4905

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[(d^(q + 1/2)*Sqrt[1 + c^2*x^2])/Sqrt[d + e*x^2], Int[(1 + c
^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

Rule 4904

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{x\tan^{-1}(ax)^{3/2}}{\left(c+a^2cx^2\right)^{5/2}}\,dx = -\frac{\tan^{-1}(ax)^{3/2}}{3a^2c\left(c+a^2cx^2\right)^{3/2}} + \frac{\int \frac{\sqrt{\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{5/2}}\,dx}{2a}$$

$$= -\frac{\tan^{-1}(ax)^{3/2}}{3a^2c\left(c+a^2cx^2\right)^{3/2}} + \frac{\sqrt{1+a^2x^2}\int \frac{\sqrt{\tan^{-1}(ax)}}{\left(1+a^2x^2\right)^{5/2}}\,dx}{2ac^2\sqrt{c+a^2cx^2}}$$

$$= -\frac{\tan^{-1}(ax)^{3/2}}{3a^2c\left(c+a^2cx^2\right)^{3/2}} + \frac{\sqrt{1+a^2x^2}\,\mathrm{Subst}\left(\int \sqrt{x}\cos^3(x)\,dx, x, \tan^{-1}(ax)\right)}{2a^2c^2\sqrt{c+a^2cx^2}}$$

$$= -\frac{\tan^{-1}(ax)^{3/2}}{3a^2c\left(c+a^2cx^2\right)^{3/2}} + \frac{\sqrt{1+a^2x^2}\,\mathrm{Subst}\left(\int \left(\frac{3}{4}\sqrt{x}\cos(x)+\frac{1}{4}\sqrt{x}\cos(3x)\right)dx, x, \tan^{-1}(ax)\right)}{2a^2c^2\sqrt{c+a^2cx^2}}$$

$$= -\frac{\tan^{-1}(ax)^{3/2}}{3a^2c\left(c+a^2cx^2\right)^{3/2}} + \frac{\sqrt{1+a^2x^2}\,\mathrm{Subst}\left(\int \sqrt{x}\cos(3x)\,dx, x, \tan^{-1}(ax)\right)}{8a^2c^2\sqrt{c+a^2cx^2}} + \frac{\left(3\sqrt{1+a^2x^2}\right)\mathrm{Subst}\left(\right.}{8a^2c}$$

$$= \frac{3x\sqrt{\tan^{-1}(ax)}}{8ac^2\sqrt{c+a^2cx^2}} - \frac{\tan^{-1}(ax)^{3/2}}{3a^2c\left(c+a^2cx^2\right)^{3/2}} + \frac{\sqrt{1+a^2x^2}\sqrt{\tan^{-1}(ax)}\sin\left(3\tan^{-1}(ax)\right)}{24a^2c^2\sqrt{c+a^2cx^2}} - \frac{\sqrt{1+a^2x^2}\,\mathrm{Su}}{}$$

$$= \frac{3x\sqrt{\tan^{-1}(ax)}}{8ac^2\sqrt{c+a^2cx^2}} - \frac{\tan^{-1}(ax)^{3/2}}{3a^2c\left(c+a^2cx^2\right)^{3/2}} + \frac{\sqrt{1+a^2x^2}\sqrt{\tan^{-1}(ax)}\sin\left(3\tan^{-1}(ax)\right)}{24a^2c^2\sqrt{c+a^2cx^2}} - \frac{\sqrt{1+a^2x^2}\,\mathrm{Su}}{}$$

$$= \frac{3x\sqrt{\tan^{-1}(ax)}}{8ac^2\sqrt{c+a^2cx^2}} - \frac{\tan^{-1}(ax)^{3/2}}{3a^2c\left(c+a^2cx^2\right)^{3/2}} - \frac{3\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}\,S\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{8a^2c^2\sqrt{c+a^2cx^2}} - \frac{\sqrt{\frac{\pi}{6}}\sqrt{1+a^2x^2}\,S\left(\right.}{24a^2c^2\sqrt{c}}$$

Mathematica [C] time = 1.00984, size = 261, normalized size = 1.05

$$\frac{3\left(a^2x^2+1\right)^{3/2}\left(3\sqrt{-i\tan^{-1}(ax)}\mathrm{Gamma}\left(\frac{1}{2},-i\tan^{-1}(ax)\right)+3\sqrt{i\tan^{-1}(ax)}\mathrm{Gamma}\left(\frac{1}{2},i\tan^{-1}(ax)\right)+\sqrt{3}\left(\sqrt{-i\tan^{-1}(a\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] (48*(3*a*x + 2*a^3*x^3 - 2*ArcTan[a*x])*ArcTan[a*x] - 4*Sqrt[6*Pi]*(1 + a^2
*x^2)^(3/2)*Sqrt[ArcTan[a*x]]*(3*Sqrt[3]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*
x]]] - FresnelS[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]]) + 3*(1 + a^2*x^2)^(3/2)*(3*S
qrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-I)*ArcTan[a*x]] + 3*Sqrt[I*ArcTan[a*x]]*
Gamma[1/2, I*ArcTan[a*x]] + Sqrt[3]*(Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-3*
I)*ArcTan[a*x]] + Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (3*I)*ArcTan[a*x]])))/(288
*a^2*c*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]])
```

Maple [F] time = 0.772, size = 0, normalized size = 0.

$$\int x\left(\arctan\left(ax\right)\right)^{\frac{3}{2}}\left(a^{2}c x^{2}+c\right)^{-\frac{5}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x)
```

```
[Out] int(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(a*x)**(3/2)/(a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \arctan{(ax)}^{\frac{3}{2}}}{\left(a^2cx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x*arctan(a*x)^(3/2)/(a^2*c*x^2 + c)^(5/2), x)
```

3.834 $\int \frac{\tan^{-1}(ax)^{3/2}}{\left(c+a^2cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=252

$$-\frac{9\sqrt{\frac{\pi}{2}}\sqrt{a^2x^2+1}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{8ac^2\sqrt{a^2cx^2+c}}-\frac{\sqrt{\frac{\pi}{6}}\sqrt{a^2x^2+1}\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{24ac^2\sqrt{a^2cx^2+c}}+\frac{2x\tan^{-1}(ax)^{3/2}}{3c^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{\text{ta}}}{ac^2\sqrt{a}}$$

```
[Out] Sqrt[ArcTan[a*x]]/(6*a*c*(c + a^2*c*x^2)^(3/2)) + Sqrt[ArcTan[a*x]]/(a*c^2*
Sqrt[c + a^2*c*x^2]) + (x*ArcTan[a*x]^(3/2))/(3*c*(c + a^2*c*x^2)^(3/2)) +
(2*x*ArcTan[a*x]^(3/2))/(3*c^2*Sqrt[c + a^2*c*x^2]) - (9*Sqrt[Pi/2]*Sqrt[1
+ a^2*x^2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(8*a*c^2*Sqrt[c + a^2*c*
x^2]) - (Sqrt[Pi/6]*Sqrt[1 + a^2*x^2]*FresnelC[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]
])/(24*a*c^2*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.349053, antiderivative size = 252, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4900, 4898, 4905, 4904, 3304, 3352, 3312}

$$-\frac{9\sqrt{\frac{\pi}{2}}\sqrt{a^2x^2+1}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{8ac^2\sqrt{a^2cx^2+c}}-\frac{\sqrt{\frac{\pi}{6}}\sqrt{a^2x^2+1}\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{24ac^2\sqrt{a^2cx^2+c}}+\frac{2x\tan^{-1}(ax)^{3/2}}{3c^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{\text{ta}}}{ac^2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^(3/2)/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] Sqrt[ArcTan[a*x]]/(6*a*c*(c + a^2*c*x^2)^(3/2)) + Sqrt[ArcTan[a*x]]/(a*c^2*
Sqrt[c + a^2*c*x^2]) + (x*ArcTan[a*x]^(3/2))/(3*c*(c + a^2*c*x^2)^(3/2)) +
(2*x*ArcTan[a*x]^(3/2))/(3*c^2*Sqrt[c + a^2*c*x^2]) - (9*Sqrt[Pi/2]*Sqrt[1
+ a^2*x^2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(8*a*c^2*Sqrt[c + a^2*c*
x^2]) - (Sqrt[Pi/6]*Sqrt[1 + a^2*x^2]*FresnelC[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]
])/(24*a*c^2*Sqrt[c + a^2*c*x^2])
```

Rule 4900

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(b*p*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^(p - 1))/(4*c*d
*(q + 1)^2), x] + (Dist[(2*q + 3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a
```

```
 + b*ArcTan[c*x])^p, x], x] - Dist[(b^2*p*(p - 1))/(4*(q + 1)^2), Int[(d +
e*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x] - Simp[(x*(d + e*x^2)^(q + 1)*
(a + b*ArcTan[c*x])^p)/(2*d*(q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] && E
qQ[e, c^2*d] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

Rule 4898

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[(b*p*(a + b*ArcTan[c*x])^(p - 1))/(c*d*Sqrt[d + e*x^2]), x]
 + (-Dist[b^2*p*(p - 1), Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2),
 x], x] + Simp[(x*(a + b*ArcTan[c*x])^p)/(d*Sqrt[d + e*x^2]), x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

Rule 4905

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[(d^(q + 1/2)*Sqrt[1 + c^2*x^2])/Sqrt[d + e*x^2], Int[(1 + c
^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

Rule 4904

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^{3/2}}{\left(c+a^2cx^2\right)^{5/2}}\,dx = \frac{\sqrt{\tan^{-1}(ax)}}{6ac\left(c+a^2cx^2\right)^{3/2}} + \frac{x\tan^{-1}(ax)^{3/2}}{3c\left(c+a^2cx^2\right)^{3/2}} - \frac{1}{12}\int \frac{1}{\left(c+a^2cx^2\right)^{5/2}\sqrt{\tan^{-1}(ax)}}\,dx + \frac{2\int \frac{\tan^{-1}(ax)^{3/2}}{\left(c+a^2cx^2\right)^{3/2}}\,dx}{3c}$$

$$= \frac{\sqrt{\tan^{-1}(ax)}}{6ac\left(c+a^2cx^2\right)^{3/2}} + \frac{\sqrt{\tan^{-1}(ax)}}{ac^2\sqrt{c+a^2cx^2}} + \frac{x\tan^{-1}(ax)^{3/2}}{3c\left(c+a^2cx^2\right)^{3/2}} + \frac{2x\tan^{-1}(ax)^{3/2}}{3c^2\sqrt{c+a^2cx^2}} - \frac{\int \frac{1}{\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(}}}{2c}$$

$$= \frac{\sqrt{\tan^{-1}(ax)}}{6ac\left(c+a^2cx^2\right)^{3/2}} + \frac{\sqrt{\tan^{-1}(ax)}}{ac^2\sqrt{c+a^2cx^2}} + \frac{x\tan^{-1}(ax)^{3/2}}{3c\left(c+a^2cx^2\right)^{3/2}} + \frac{2x\tan^{-1}(ax)^{3/2}}{3c^2\sqrt{c+a^2cx^2}} - \frac{\sqrt{1+a^2x^2}\int \frac{}{\left(1+a^2x^2\right.}}{2c^2\sqrt{c+}}$$

$$= \frac{\sqrt{\tan^{-1}(ax)}}{6ac\left(c+a^2cx^2\right)^{3/2}} + \frac{\sqrt{\tan^{-1}(ax)}}{ac^2\sqrt{c+a^2cx^2}} + \frac{x\tan^{-1}(ax)^{3/2}}{3c\left(c+a^2cx^2\right)^{3/2}} + \frac{2x\tan^{-1}(ax)^{3/2}}{3c^2\sqrt{c+a^2cx^2}} - \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\right.}{}$$

$$= \frac{\sqrt{\tan^{-1}(ax)}}{6ac\left(c+a^2cx^2\right)^{3/2}} + \frac{\sqrt{\tan^{-1}(ax)}}{ac^2\sqrt{c+a^2cx^2}} + \frac{x\tan^{-1}(ax)^{3/2}}{3c\left(c+a^2cx^2\right)^{3/2}} + \frac{2x\tan^{-1}(ax)^{3/2}}{3c^2\sqrt{c+a^2cx^2}} - \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\right.}{48ac^2}$$

$$= \frac{\sqrt{\tan^{-1}(ax)}}{6ac\left(c+a^2cx^2\right)^{3/2}} + \frac{\sqrt{\tan^{-1}(ax)}}{ac^2\sqrt{c+a^2cx^2}} + \frac{x\tan^{-1}(ax)^{3/2}}{3c\left(c+a^2cx^2\right)^{3/2}} + \frac{2x\tan^{-1}(ax)^{3/2}}{3c^2\sqrt{c+a^2cx^2}} - \frac{\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}C\left(\sqrt{\right.}}{ac^2\sqrt{c+}}$$

$$= \frac{\sqrt{\tan^{-1}(ax)}}{6ac\left(c+a^2cx^2\right)^{3/2}} + \frac{\sqrt{\tan^{-1}(ax)}}{ac^2\sqrt{c+a^2cx^2}} + \frac{x\tan^{-1}(ax)^{3/2}}{3c\left(c+a^2cx^2\right)^{3/2}} + \frac{2x\tan^{-1}(ax)^{3/2}}{3c^2\sqrt{c+a^2cx^2}} - \frac{9\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}C\left(\right.}{8ac^2\sqrt{c+}}$$

Mathematica [A] time = 0.202902, size = 153, normalized size = 0.61

$$\frac{-81\sqrt{2\pi}\left(a^2x^2+1\right)^{3/2}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)-\sqrt{6\pi}\left(a^2x^2+1\right)^{3/2}\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\tan^{-1}(ax)}\right)+24\sqrt{\tan^{-1}(ax)}\left(\right.}{144c^2\left(a^3x^2+a\right)\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]^(3/2)/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] (24*Sqrt[ArcTan[a*x]]*(7 + 6*a^2*x^2 + (6*a*x + 4*a^3*x^3)*ArcTan[a*x]) - 8
1*Sqrt[2*Pi]*(1 + a^2*x^2)^(3/2)*FresnelC[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]] - S
```

```
qrt[6*Pi]*(1 + a^2*x^2)^(3/2)*FresnelC[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]])/(144*
c^2*(a + a^3*x^2)*Sqrt[c + a^2*c*x^2])
```

Maple [F] time = 0.67, size = 0, normalized size = 0.

$$\int (\arctan(ax))^{\frac{3}{2}} \left(a^2cx^2 + c\right)^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x)
```

```
[Out] int(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(3/2)/(a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(ax)^{\frac{3}{2}}}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)^(3/2)/(a^2*c*x^2 + c)^(5/2), x)
```

3.835 $\int \frac{\tan^{-1}(ax)^{3/2}}{x(c+a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(\frac{\tan^{-1}(ax)^{3/2}}{x\left(a^2cx^2+c\right)^{5/2}},x\right)$$

```
[Out] Unintegrable[ArcTan[a*x]^(3/2)/(x*(c + a^2*c*x^2)^(5/2)), x]
```

Rubi [A] time = 0.115561, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\tan^{-1}(ax)^{3/2}}{x\left(c+a^2cx^2\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcTan[a*x]^(3/2)/(x*(c + a^2*c*x^2)^(5/2)),x]

[Out] Defer[Int][ArcTan[a*x]^(3/2)/(x*(c + a^2*c*x^2)^(5/2)), x]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^{3/2}}{x\left(c+a^2cx^2\right)^{5/2}}\,dx=\int \frac{\tan^{-1}(ax)^{3/2}}{x\left(c+a^2cx^2\right)^{5/2}}\,dx$$

Mathematica [A] time = 2.39662, size = 0, normalized size = 0.

$$\int \frac{\tan^{-1}(ax)^{3/2}}{x\left(c+a^2cx^2\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcTan[a*x]^(3/2)/(x*(c + a^2*c*x^2)^(5/2)),x]
```

```
[Out] Integrate[ArcTan[a*x]^(3/2)/(x*(c + a^2*c*x^2)^(5/2)), x]
```

Maple [A] time = 0.743, size = 0, normalized size = 0.

$$\int \frac{1}{x} (\arctan (ax))^{\frac{3}{2}} \left(a^2cx^2 + c\right)^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^(5/2),x)
```

```
[Out] int(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(3/2)/x/(a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan{(ax)}^{\frac{3}{2}}}{\left(a^2cx^2+c\right)^{\frac{5}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)^(3/2)/((a^2*c*x^2 + c)^(5/2)*x), x)
```

3.836 $\int \frac{\tan^{-1}(ax)^{3/2}}{x^2(c+a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(\frac{\tan^{-1}(ax)^{3/2}}{x^2\left(a^2cx^2+c\right)^{5/2}},x\right)$$

```
[Out] Unintegrable[ArcTan[a*x]^(3/2)/(x^2*(c + a^2*c*x^2)^(5/2)), x]
```

Rubi [A] time = 0.11669, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\tan^{-1}(ax)^{3/2}}{x^2\left(c+a^2cx^2\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcTan[a*x]^(3/2)/(x^2*(c + a^2*c*x^2)^(5/2)),x]

[Out] Defer[Int][ArcTan[a*x]^(3/2)/(x^2*(c + a^2*c*x^2)^(5/2)), x]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^{3/2}}{x^2\left(c+a^2cx^2\right)^{5/2}}\,dx=\int \frac{\tan^{-1}(ax)^{3/2}}{x^2\left(c+a^2cx^2\right)^{5/2}}\,dx$$

Mathematica [A] time = 8.2133, size = 0, normalized size = 0.

$$\int \frac{\tan^{-1}(ax)^{3/2}}{x^2\left(c+a^2cx^2\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcTan[a*x]^(3/2)/(x^2*(c + a^2*c*x^2)^(5/2)),x]
```

```
[Out] Integrate[ArcTan[a*x]^(3/2)/(x^2*(c + a^2*c*x^2)^(5/2)), x]
```

Maple [A] time = 0.721, size = 0, normalized size = 0.

$$\int \frac{1}{x^2} (\arctan (ax))^{\frac{3}{2}} \left(a^2cx^2 + c\right)^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(3/2)/x^2/(a^2*c*x^2+c)^(5/2),x)
```

```
[Out] int(arctan(a*x)^(3/2)/x^2/(a^2*c*x^2+c)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/x^2/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/x^2/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(3/2)/x**2/(a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan{(ax)}^{\frac{3}{2}}}{\left(a^2cx^2+c\right)^{\frac{5}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/x^2/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)^(3/2)/((a^2*c*x^2 + c)^(5/2)*x^2), x)
```

3.837 $\int x^m \left(c + a^2 c x^2\right) \tan^{-1}(ax)^{5/2} \, dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(x^m \left(a^2 c x^2 + c\right) \tan^{-1}(ax)^{5/2}, x\right)$$

```
[Out] Unintegrable[x^m*(c + a^2*c*x^2)*ArcTan[a*x]^(5/2), x]
```

Rubi [A] time = 0.0344447, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^m \left(c + a^2 c x^2\right) \tan^{-1}(ax)^{5/2} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*(c + a^2*c*x^2)*ArcTan[a*x]^(5/2),x]
```

```
[Out] Defer[Int][x^m*(c + a^2*c*x^2)*ArcTan[a*x]^(5/2), x]
```

Rubi steps

$$\int x^m \left(c + a^2 c x^2\right) \tan^{-1}(ax)^{5/2} \, dx = \int x^m \left(c + a^2 c x^2\right) \tan^{-1}(ax)^{5/2} \, dx$$

Mathematica [A] time = 1.81788, size = 0, normalized size = 0.

$$\int x^m \left(c + a^2 c x^2\right) \tan^{-1}(ax)^{5/2} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*(c + a^2*c*x^2)*ArcTan[a*x]^(5/2),x]
```

```
[Out] Integrate[x^m*(c + a^2*c*x^2)*ArcTan[a*x]^(5/2), x]
```

Maple [A] time = 0.743, size = 0, normalized size = 0.

$$\int x^{m} \left(a^{2} c \,x^{2}+c \right) \arctan \left(a x \right)^{\frac{5}{2}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)*arctan(a*x)^(5/2),x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)*arctan(a*x)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)*arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a^{2} c x^{2} + c\right) x^{m} \arctan\left(a x\right)^{\frac{5}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)*arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)*x^m*arctan(a*x)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)*atan(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2+c\right)x^m \arctan\left(ax\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)*arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)*x^m*arctan(a*x)^(5/2), x)
```

3.838 $\int x^2 \left(c+a^2 c x^2\right) \tan^{-1}(a x)^{5/2} \, dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(x^2 \left(a^2 c x^2+c\right) \tan^{-1}(a x)^{5/2},x\right)$$

```
[Out] Unintegrable[x^2*(c + a^2*c*x^2)*ArcTan[a*x]^(5/2), x]
```

Rubi [A] time = 0.0349865, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int x^2 \left(c+a^2 c x^2\right) \tan^{-1}(a x)^{5/2} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[x^2*(c + a^2*c*x^2)*ArcTan[a*x]^(5/2),x]
```

```
[Out] Defer[Int][x^2*(c + a^2*c*x^2)*ArcTan[a*x]^(5/2), x]
```

Rubi steps

$$\int x^2 \left(c+a^2 c x^2\right) \tan^{-1}(a x)^{5/2} \, dx=\int x^2 \left(c+a^2 c x^2\right) \tan^{-1}(a x)^{5/2} \, dx$$

Mathematica [A] time = 3.42324, size = 0, normalized size = 0.

$$\int x^2 \left(c+a^2 c x^2\right) \tan^{-1}(a x)^{5/2} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^2*(c + a^2*c*x^2)*ArcTan[a*x]^(5/2),x]
```

```
[Out] Integrate[x^2*(c + a^2*c*x^2)*ArcTan[a*x]^(5/2), x]
```

Maple [A] time = 0.657, size = 0, normalized size = 0.

$$\int x^2 \left(a^2 c x^2 + c\right) (\arctan (a x))^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a^2*c*x^2+c)*arctan(a*x)^(5/2),x)
```

```
[Out] int(x^2*(a^2*c*x^2+c)*arctan(a*x)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)*arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)*arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a**2*c*x**2+c)*atan(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2+c\right)x^2\arctan\left(ax\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)*arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)*x^2*arctan(a*x)^(5/2), x)
```

3.839 $\int x\left(c+a^2cx^2\right)\tan^{-1}(ax)^{5/2}\,dx$

Optimal. Leaf size=115

$$-\frac{5c\text{Unintegrable}\left(\frac{1}{\sqrt{\tan^{-1}(ax)}},x\right)}{64a}-\frac{5c\text{Unintegrable}\left(\tan^{-1}(ax)^{3/2},x\right)}{12a}+\frac{c\left(a^2x^2+1\right)^2\tan^{-1}(ax)^{5/2}}{4a^2}-\frac{5cx\left(a^2x^2+1\right)\tan}{24a}$$

```
[Out] (5*c*(1 + a^2*x^2)*Sqrt[ArcTan[a*x]])/(32*a^2) - (5*c*x*(1 + a^2*x^2)*ArcTa
n[a*x]^(3/2))/(24*a) + (c*(1 + a^2*x^2)^2*ArcTan[a*x]^(5/2))/(4*a^2) - (5*c
*Unintegrable[1/Sqrt[ArcTan[a*x]], x])/(64*a) - (5*c*Unintegrable[ArcTan[a*
x]^(3/2), x])/(12*a)
```

Rubi [A] time = 0.0551784, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x\left(c+a^2cx^2\right)\tan^{-1}(ax)^{5/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x*(c + a^2*c*x^2)*ArcTan[a*x]^(5/2),x]
```

```
[Out] (5*c*(1 + a^2*x^2)*Sqrt[ArcTan[a*x]])/(32*a^2) - (5*c*x*(1 + a^2*x^2)*ArcTa
n[a*x]^(3/2))/(24*a) + (c*(1 + a^2*x^2)^2*ArcTan[a*x]^(5/2))/(4*a^2) - (5*c
*Defer[Int][1/Sqrt[ArcTan[a*x]], x])/(64*a) - (5*c*Defer[Int][ArcTan[a*x]^(
3/2), x])/(12*a)
```

Rubi steps

$$\begin{aligned}\int x\left(c+a^2cx^2\right)\tan^{-1}(ax)^{5/2}\,dx &= \frac{c\left(1+a^2x^2\right)^2\tan^{-1}(ax)^{5/2}}{4a^2}-\frac{5\int\left(c+a^2cx^2\right)\tan^{-1}(ax)^{3/2}\,dx}{8a}\\ &= \frac{5c\left(1+a^2x^2\right)\sqrt{\tan^{-1}(ax)}}{32a^2}-\frac{5cx\left(1+a^2x^2\right)\tan^{-1}(ax)^{3/2}}{24a}+\frac{c\left(1+a^2x^2\right)^2\tan^{-1}(ax)^{5/2}}{4a^2}\end{aligned}$$

Mathematica [A] time = 1.92466, size = 0, normalized size = 0.

$$\int x\left(c+a^2cx^2\right)\tan^{-1}(ax)^{5/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x*(c + a^2*c*x^2)*ArcTan[a*x]^(5/2),x]
```

```
[Out] Integrate[x*(c + a^2*c*x^2)*ArcTan[a*x]^(5/2), x]
```

Maple [A] time = 0.377, size = 0, normalized size = 0.

$$\int x\left(a^{2}c x^{2}+c\right)\left(\arctan\left(a x\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)*arctan(a*x)^(5/2),x)
```

```
[Out] int(x*(a^2*c*x^2+c)*arctan(a*x)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)*arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)*arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)*atan(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2+c\right)x\arctan\left(ax\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)*arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)*x*arctan(a*x)^(5/2), x)
```

3.840 $\int \left(c+a^2 c x^2\right) \tan^{-1}(a x)^{5/2} \, dx$

Optimal. Leaf size=79

$$\frac{5}{8} c \text{Unintegrable}\left(\sqrt{\tan^{-1}(a x)},x\right)+\frac{2}{3} c \text{Unintegrable}\left(\tan^{-1}(a x)^{5/2},x\right)+\frac{1}{3} c x \left(a^2 x^2+1\right) \tan^{-1}(a x)^{5/2}-\frac{5 c \left(a^2 x^2+1\right) \text{t}}{12 a}$$

```
[Out] (-5*c*(1 + a^2*x^2)*ArcTan[a*x]^(3/2))/(12*a) + (c*x*(1 + a^2*x^2)*ArcTan[a
*x]^(5/2))/3 + (5*c*Unintegrable[Sqrt[ArcTan[a*x]], x])/8 + (2*c*Unintegrab
le[ArcTan[a*x]^(5/2), x])/3
```

Rubi [A] time = 0.0249849, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \left(c+a^2 c x^2\right) \tan^{-1}(a x)^{5/2} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)*ArcTan[a*x]^(5/2),x]
```

```
[Out] (-5*c*(1 + a^2*x^2)*ArcTan[a*x]^(3/2))/(12*a) + (c*x*(1 + a^2*x^2)*ArcTan[a
*x]^(5/2))/3 + (5*c*Defer[Int][Sqrt[ArcTan[a*x]], x])/8 + (2*c*Defer[Int][A
rcTan[a*x]^(5/2), x])/3
```

Rubi steps

$$\int \left(c+a^2 c x^2\right) \tan^{-1}(a x)^{5/2} \, dx=-\frac{5 c \left(1+a^2 x^2\right) \tan^{-1}(a x)^{3/2}}{12 a}+\frac{1}{3} c x \left(1+a^2 x^2\right) \tan^{-1}(a x)^{5/2}+\frac{1}{8} (5 c) \int \sqrt{\tan^{-1}(a x)} \, dx$$

Mathematica [A] time = 3.56711, size = 0, normalized size = 0.

$$\int \left(c+a^2 c x^2\right) \tan^{-1}(a x)^{5/2} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)*ArcTan[a*x]^(5/2),x]
```

```
[Out] Integrate[(c + a^2*c*x^2)*ArcTan[a*x]^(5/2), x]
```

Maple [A] time = 0.323, size = 0, normalized size = 0.

$$\int \left(a^2 c x^2 + c\right) (\arctan (ax))^{\frac{5}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)*arctan(a*x)^(5/2),x)
```

```
[Out] int((a^2*c*x^2+c)*arctan(a*x)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)*atan(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2 + c\right)\arctan\left(ax\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)*arctan(a*x)^(5/2), x)
```

3.841 $\int \frac{(c+a^2cx^2)\tan^{-1}(ax)^{5/2}}{x}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{(a^2cx^2+c)\tan^{-1}(ax)^{5/2}}{x},x\right)$$

```
[Out] Unintegrable[((c + a^2*c*x^2)*ArcTan[a*x]^(5/2))/x, x]
```

Rubi [A] time = 0.0323027, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{(c+a^2cx^2)\tan^{-1}(ax)^{5/2}}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((c + a^2*c*x^2)*ArcTan[a*x]^(5/2))/x,x]
```

```
[Out] Defer[Int][((c + a^2*c*x^2)*ArcTan[a*x]^(5/2))/x, x]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)\tan^{-1}(ax)^{5/2}}{x}\,dx = \int \frac{(c+a^2cx^2)\tan^{-1}(ax)^{5/2}}{x}\,dx$$

Mathematica [A] time = 2.84917, size = 0, normalized size = 0.

$$\int \frac{(c+a^2cx^2)\tan^{-1}(ax)^{5/2}}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((c + a^2*c*x^2)*ArcTan[a*x]^(5/2))/x,x]
```

```
[Out] Integrate[((c + a^2*c*x^2)*ArcTan[a*x]^(5/2))/x, x]
```

Maple [A] time = 0.386, size = 0, normalized size = 0.

$$\int \frac{a^{2} c \,x^{2}+c}{x} \left(\arctan \left(a x \right)\right)^{\frac{5}{2}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)*arctan(a*x)^(5/2)/x,x)
```

```
[Out] int((a^2*c*x^2+c)*arctan(a*x)^(5/2)/x,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^(5/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^(5/2)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)*atan(a*x)**(5/2)/x,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a^2cx^2 + c)\arctan(ax)^{\frac{5}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^(5/2)/x,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)*arctan(a*x)^(5/2)/x, x)
```

3.842 $\int \frac{(c+a^2cx^2)\tan^{-1}(ax)^{5/2}}{x^2}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{\left(a^2cx^2+c\right)\tan^{-1}(ax)^{5/2}}{x^2},x\right)$$

```
[Out] Unintegrable[((c + a^2*c*x^2)*ArcTan[a*x]^(5/2))/x^2, x]
```

Rubi [A] time = 0.0332529, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(c+a^2cx^2\right)\tan^{-1}(ax)^{5/2}}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((c + a^2*c*x^2)*ArcTan[a*x]^(5/2))/x^2,x]
```

```
[Out] Defer[Int][((c + a^2*c*x^2)*ArcTan[a*x]^(5/2))/x^2, x]
```

Rubi steps

$$\int \frac{\left(c+a^2cx^2\right)\tan^{-1}(ax)^{5/2}}{x^2}\,dx=\int \frac{\left(c+a^2cx^2\right)\tan^{-1}(ax)^{5/2}}{x^2}\,dx$$

Mathematica [A] time = 1.61715, size = 0, normalized size = 0.

$$\int \frac{\left(c+a^2cx^2\right)\tan^{-1}(ax)^{5/2}}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((c + a^2*c*x^2)*ArcTan[a*x]^(5/2))/x^2,x]
```

```
[Out] Integrate[((c + a^2*c*x^2)*ArcTan[a*x]^(5/2))/x^2, x]
```

Maple [A] time = 0.23, size = 0, normalized size = 0.

$$\int \frac{a^2cx^2 + c}{x^2} (\arctan(ax))^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)*arctan(a*x)^(5/2)/x^2,x)
```

```
[Out] int((a^2*c*x^2+c)*arctan(a*x)^(5/2)/x^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^(5/2)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^(5/2)/x^2,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)*atan(a*x)**(5/2)/x**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a^2cx^2 + c)\arctan(ax)^{\frac{5}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^(5/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)*arctan(a*x)^(5/2)/x^2, x)
```

3.843 $\int x^m \left(c+a^2 c x^2\right)^2 \tan^{-1}(a x)^{5/2} \, dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(x^m \left(a^2 c x^2+c\right)^2 \tan^{-1}(a x)^{5/2},x\right)$$

```
[Out] Unintegrable[x^m*(c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2), x]
```

Rubi [A] time = 0.0540261, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^m \left(c+a^2 c x^2\right)^2 \tan^{-1}(a x)^{5/2} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*(c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2),x]
```

```
[Out] Defer[Int][x^m*(c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2), x]
```

Rubi steps

$$\int x^m \left(c+a^2 c x^2\right)^2 \tan^{-1}(a x)^{5/2} \, dx=\int x^m \left(c+a^2 c x^2\right)^2 \tan^{-1}(a x)^{5/2} \, dx$$

Mathematica [A] time = 1.31349, size = 0, normalized size = 0.

$$\int x^m \left(c+a^2 c x^2\right)^2 \tan^{-1}(a x)^{5/2} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*(c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2),x]
```

```
[Out] Integrate[x^m*(c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2), x]
```

Maple [A] time = 0.914, size = 0, normalized size = 0.

$$\int x^m \left(a^2 c x^2 + c\right)^2 (\arctan (ax))^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^(5/2),x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^4 c^2 x^4 + 2\, a^2 c^2 x^2 + c^2\right) x^m \arctan (ax)^{\frac{5}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*x^m*arctan(a*x)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**2*atan(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2+c\right)^2 x^m \arctan(ax)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^2*x^m*arctan(a*x)^(5/2), x)
```

3.844 $\int x^2 \left(c+a^2 c x^2\right)^2 \tan^{-1}(a x)^{5/2} \, dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(x^2 \left(a^2 c x^2+c\right)^2 \tan^{-1}(a x)^{5/2},x\right)$$

```
[Out] Unintegrable[x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2), x]
```

Rubi [A] time = 0.0556613, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^2 \left(c+a^2 c x^2\right)^2 \tan^{-1}(a x)^{5/2} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2),x]
```

```
[Out] Defer[Int][x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2), x]
```

Rubi steps

$$\int x^2 \left(c+a^2 c x^2\right)^2 \tan^{-1}(a x)^{5/2} \, dx=\int x^2 \left(c+a^2 c x^2\right)^2 \tan^{-1}(a x)^{5/2} \, dx$$

Mathematica [A] time = 2.5248, size = 0, normalized size = 0.

$$\int x^2 \left(c+a^2 c x^2\right)^2 \tan^{-1}(a x)^{5/2} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2),x]
```

```
[Out] Integrate[x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2), x]
```

Maple [A] time = 0.905, size = 0, normalized size = 0.

$$\int x^{2} \left(a^{2} c x^{2}+c \right)^{2} \left(\arctan \left(a x \right)\right)^{\frac{5}{2}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a^2*c*x^2+c)^2*arctan(a*x)^(5/2),x)
```

```
[Out] int(x^2*(a^2*c*x^2+c)^2*arctan(a*x)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^2*arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^2*arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a**2*c*x**2+c)**2*atan(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2+c\right)^2x^2\arctan\left(ax\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^2*arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^2*x^2*arctan(a*x)^(5/2), x)
```

3.845 $\int x\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^{5/2}\,dx$

Optimal. Leaf size=213

$$-\frac{c\text{Unintegrable}\left(\frac{a^2cx^2+c}{\sqrt{\tan^{-1}(ax)}},x\right)}{64a}-\frac{c^2\text{Unintegrable}\left(\frac{1}{\sqrt{\tan^{-1}(ax)}},x\right)}{24a}-\frac{2c^2\text{Unintegrable}\left(\tan^{-1}(ax)^{3/2},x\right)}{9a}+\frac{c^2\left(a^2x^2+1\right)}{6}$$

```
[Out] (c^2*(1 + a^2*x^2)*Sqrt[ArcTan[a*x]])/(12*a^2) + (c^2*(1 + a^2*x^2)^2*Sqrt[
ArcTan[a*x]])/(32*a^2) - (c^2*x*(1 + a^2*x^2)*ArcTan[a*x]^(3/2))/(9*a) - (c
^2*x*(1 + a^2*x^2)^2*ArcTan[a*x]^(3/2))/(12*a) + (c^2*(1 + a^2*x^2)^3*ArcTa
n[a*x]^(5/2))/(6*a^2) - (c^2*Unintegrable[1/Sqrt[ArcTan[a*x]], x])/(24*a) -
 (c*Unintegrable[(c + a^2*c*x^2)/Sqrt[ArcTan[a*x]], x])/(64*a) - (2*c^2*Uni
ntegrable[ArcTan[a*x]^(3/2), x])/(9*a)
```

Rubi [A] time = 0.113981, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^{5/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x*(c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2),x]
```

```
[Out] (c^2*(1 + a^2*x^2)*Sqrt[ArcTan[a*x]])/(12*a^2) + (c^2*(1 + a^2*x^2)^2*Sqrt[
ArcTan[a*x]])/(32*a^2) - (c^2*x*(1 + a^2*x^2)*ArcTan[a*x]^(3/2))/(9*a) - (c
^2*x*(1 + a^2*x^2)^2*ArcTan[a*x]^(3/2))/(12*a) + (c^2*(1 + a^2*x^2)^3*ArcTa
n[a*x]^(5/2))/(6*a^2) - (c^2*Defer[Int][1/Sqrt[ArcTan[a*x]], x])/(24*a) - (
c*Defer[Int][(c + a^2*c*x^2)/Sqrt[ArcTan[a*x]], x])/(64*a) - (2*c^2*Defer[I
nt][ArcTan[a*x]^(3/2), x])/(9*a)
```

Rubi steps

$$\int x\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^{5/2}\,dx = \frac{c^2\left(1+a^2x^2\right)^3\tan^{-1}(ax)^{5/2}}{6a^2} - \frac{5\int\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^{3/2}\,dx}{12a}$$
$$= \frac{c^2\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}}{32a^2} - \frac{c^2x\left(1+a^2x^2\right)^2\tan^{-1}(ax)^{3/2}}{12a} + \frac{c^2\left(1+a^2x^2\right)^3\tan^{-1}(ax)}{6a^2}$$
$$= \frac{c^2\left(1+a^2x^2\right)\sqrt{\tan^{-1}(ax)}}{12a^2} + \frac{c^2\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}}{32a^2} - \frac{c^2x\left(1+a^2x^2\right)\tan^{-1}(ax)^3}{9a}$$

Mathematica [A] time = 1.42827, size = 0, normalized size = 0.

$$\int x\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^{5/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x*(c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2),x]
```

```
[Out] Integrate[x*(c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2), x]
```

Maple [A] time = 0.532, size = 0, normalized size = 0.

$$\int x\left(a^2cx^2+c\right)^2(\arctan(ax))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^2*arctan(a*x)^(5/2),x)
```

```
[Out] int(x*(a^2*c*x^2+c)^2*arctan(a*x)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^2*arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^2*arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**2*atan(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2+c\right)^2 x \arctan\left(ax\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^2*arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^2*x*arctan(a*x)^(5/2), x)
```

3.846 $\int \left(c+a^2cx^2\right)^2 \tan^{-1}(ax)^{5/2}\,dx$

Optimal. Leaf size=169

$$\frac{3}{16}c\text{Unintegrable}\left(\left(a^2cx^2+c\right)\sqrt{\tan^{-1}(ax)},x\right)+\frac{1}{2}c^2\text{Unintegrable}\left(\sqrt{\tan^{-1}(ax)},x\right)+\frac{8}{15}c^2\text{Unintegrable}\left(\tan^{-1}(ax)\right.$$

```
[Out] -(c^2*(1 + a^2*x^2)*ArcTan[a*x]^(3/2))/(3*a) - (c^2*(1 + a^2*x^2)^2*ArcTan[
a*x]^(3/2))/(8*a) + (4*c^2*x*(1 + a^2*x^2)*ArcTan[a*x]^(5/2))/15 + (c^2*x*(
1 + a^2*x^2)^2*ArcTan[a*x]^(5/2))/5 + (c^2*Unintegrable[Sqrt[ArcTan[a*x]],
x])/2 + (3*c*Unintegrable[(c + a^2*c*x^2)*Sqrt[ArcTan[a*x]], x])/16 + (8*c^
2*Unintegrable[ArcTan[a*x]^(5/2), x])/15
```

Rubi [A] time = 0.0698693, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \left(c+a^2cx^2\right)^2 \tan^{-1}(ax)^{5/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2),x]
```

```
[Out] -(c^2*(1 + a^2*x^2)*ArcTan[a*x]^(3/2))/(3*a) - (c^2*(1 + a^2*x^2)^2*ArcTan[
a*x]^(3/2))/(8*a) + (4*c^2*x*(1 + a^2*x^2)*ArcTan[a*x]^(5/2))/15 + (c^2*x*(
1 + a^2*x^2)^2*ArcTan[a*x]^(5/2))/5 + (c^2*Defer[Int][Sqrt[ArcTan[a*x]], x]
)/2 + (3*c*Defer[Int][(c + a^2*c*x^2)*Sqrt[ArcTan[a*x]], x])/16 + (8*c^2*De
fer[Int][ArcTan[a*x]^(5/2), x])/15
```

Rubi steps

$$\int \left(c+a^2cx^2\right)^2 \tan^{-1}(ax)^{5/2}\,dx = -\frac{c^2\left(1+a^2x^2\right)^2\tan^{-1}(ax)^{3/2}}{8a}+\frac{1}{5}c^2x\left(1+a^2x^2\right)^2\tan^{-1}(ax)^{5/2}+\frac{1}{16}(3c)\int \left(c+a^2cx^2\right)$$

$$= -\frac{c^2\left(1+a^2x^2\right)\tan^{-1}(ax)^{3/2}}{3a}-\frac{c^2\left(1+a^2x^2\right)^2\tan^{-1}(ax)^{3/2}}{8a}+\frac{4}{15}c^2x\left(1+a^2x^2\right)\tan^{-1}(a$$

Mathematica [A] time = 2.19038, size = 0, normalized size = 0.

$$\int \left(c + a^2cx^2\right)^2 \tan^{-1}(ax)^{5/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2),x]
```

```
[Out] Integrate[(c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2), x]
```

Maple [A] time = 0.419, size = 0, normalized size = 0.

$$\int \left(a^2cx^2 + c\right)^2 (\arctan(ax))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^2*arctan(a*x)^(5/2),x)
```

```
[Out] int((a^2*c*x^2+c)^2*arctan(a*x)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**2*atan(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2 + c\right)^2 \arctan\left(ax\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^2*arctan(a*x)^(5/2), x)
```

3.847 $\int \frac{\left(c+a^2cx^2\right)^2 \tan^{-1}(ax)^{5/2}}{x}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{\left(a^2cx^2+c\right)^2 \tan^{-1}(ax)^{5/2}}{x},x\right)$$

```
[Out] Unintegrable[((c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2))/x, x]
```

Rubi [A] time = 0.0482354, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\left(c+a^2cx^2\right)^2 \tan^{-1}(ax)^{5/2}}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2))/x,x]
```

```
[Out] Defer[Int][((c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2))/x, x]
```

Rubi steps

$$\int \frac{\left(c+a^2cx^2\right)^2 \tan^{-1}(ax)^{5/2}}{x}\,dx = \int \frac{\left(c+a^2cx^2\right)^2 \tan^{-1}(ax)^{5/2}}{x}\,dx$$

Mathematica [A] time = 1.81931, size = 0, normalized size = 0.

$$\int \frac{\left(c+a^2cx^2\right)^2 \tan^{-1}(ax)^{5/2}}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2))/x,x]
```

```
[Out] Integrate[((c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2))/x, x]
```

Maple [A] time = 0.504, size = 0, normalized size = 0.

$$\int \frac{(a^2cx^2+c)^2}{x} (\arctan(ax))^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^2*arctan(a*x)^(5/2)/x,x)
```

```
[Out] int((a^2*c*x^2+c)^2*arctan(a*x)^(5/2)/x,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^(5/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^(5/2)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**2*atan(a*x)**(5/2)/x,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2 c x^2 + c\right)^2 \arctan\left(ax\right)^{\frac{5}{2}}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^(5/2)/x,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^2*arctan(a*x)^(5/2)/x, x)
```

3.848 $\int \frac{\left(c+a^2cx^2\right)^2 \tan^{-1}(ax)^{5/2}}{x^2}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{\left(a^2cx^2+c\right)^2\tan^{-1}(ax)^{5/2}}{x^2},x\right)$$

```
[Out] Unintegrable[((c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2))/x^2, x]
```

Rubi [A] time = 0.0542138, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^{5/2}}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2))/x^2,x]
```

```
[Out] Defer[Int][((c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2))/x^2, x]
```

Rubi steps

$$\int \frac{\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^{5/2}}{x^2}\,dx=\int \frac{\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^{5/2}}{x^2}\,dx$$

Mathematica [A] time = 2.51974, size = 0, normalized size = 0.

$$\int \frac{\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^{5/2}}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2))/x^2,x]
```

```
[Out] Integrate[((c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2))/x^2, x]
```

Maple [A] time = 0.422, size = 0, normalized size = 0.

$$\int \frac{(a^2cx^2+c)^2}{x^2} (\arctan(ax))^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^2*arctan(a*x)^(5/2)/x^2,x)
```

```
[Out] int((a^2*c*x^2+c)^2*arctan(a*x)^(5/2)/x^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^(5/2)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^(5/2)/x^2,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**2*atan(a*x)**(5/2)/x**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^2\arctan\left(ax\right)^{\frac{5}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^(5/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^2*arctan(a*x)^(5/2)/x^2, x)
```

3.849 $\int x^m \left(c+a^2 c x^2\right)^3 \tan^{-1}(ax)^{5/2} \, dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(x^m \left(a^2 c x^2+c\right)^3 \tan^{-1}(ax)^{5/2},x\right)$$

[Out] Unintegrable[x^m*(c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2), x]

Rubi [A] time = 0.0546779, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^m \left(c+a^2 c x^2\right)^3 \tan^{-1}(ax)^{5/2} \, dx$$

Verification is Not applicable to the result.

[In] Int[x^m*(c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2),x]

[Out] Defer[Int][x^m*(c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2), x]

Rubi steps

$$\int x^m \left(c+a^2 c x^2\right)^3 \tan^{-1}(ax)^{5/2} \, dx=\int x^m \left(c+a^2 c x^2\right)^3 \tan^{-1}(ax)^{5/2} \, dx$$

Mathematica [A] time = 0.884281, size = 0, normalized size = 0.

$$\int x^m \left(c+a^2 c x^2\right)^3 \tan^{-1}(ax)^{5/2} \, dx$$

Verification is Not applicable to the result.

[In] Integrate[x^m*(c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2),x]

[Out] Integrate[x^m*(c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2), x]

Maple [A] time = 0.954, size = 0, normalized size = 0.

$$\int x^{m} \left(a^{2} c x^{2}+c \right)^{3} \left(\arctan \left(a x \right)\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)^3*arctan(a*x)^(5/2),x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)^3*arctan(a*x)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^3*arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a^{6} c^{3} x^{6} + 3 \, a^{4} c^{3} x^{4} + 3 \, a^{2} c^{3} x^{2} + c^{3}\right) x^{m} \arctan \left(a x\right)^{\frac{5}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^3*arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*x^m*arctan(a*x
)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**3*atan(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2+c\right)^3 x^m \arctan(ax)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^3*arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^3*x^m*arctan(a*x)^(5/2), x)
```

3.850 $\int x^2 \left(c+a^2 c x^2\right)^3 \tan^{-1}(a x)^{5/2} \, dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(x^2 \left(a^2 c x^2+c\right)^3 \tan^{-1}(a x)^{5/2},x\right)$$

```
[Out] Unintegrable[x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2), x]
```

Rubi [A] time = 0.054122, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^2 \left(c+a^2 c x^2\right)^3 \tan^{-1}(a x)^{5/2} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2),x]
```

```
[Out] Defer[Int][x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2), x]
```

Rubi steps

$$\int x^2 \left(c+a^2 c x^2\right)^3 \tan^{-1}(a x)^{5/2} \, dx=\int x^2 \left(c+a^2 c x^2\right)^3 \tan^{-1}(a x)^{5/2} \, dx$$

Mathematica [A] time = 2.41466, size = 0, normalized size = 0.

$$\int x^2 \left(c+a^2 c x^2\right)^3 \tan^{-1}(a x)^{5/2} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2),x]
```

```
[Out] Integrate[x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2), x]
```

Maple [A] time = 1.273, size = 0, normalized size = 0.

$$\int x^2 \left(a^2 c x^2 + c\right)^3 (\arctan(ax))^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a^2*c*x^2+c)^3*arctan(a*x)^(5/2),x)
```

```
[Out] int(x^2*(a^2*c*x^2+c)^3*arctan(a*x)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^3*arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^3*arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a**2*c*x**2+c)**3*atan(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2+c\right)^3x^2\arctan\left(ax\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^3*arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^3*x^2*arctan(a*x)^(5/2), x)
```

3.851 $\int x\left(c+a^2cx^2\right)^3 \tan^{-1}(ax)^{5/2}\,dx$

Optimal. Leaf size=305

$$-\frac{9c^2\text{Unintegrable}\left(\frac{a^2cx^2+c}{\sqrt{\tan^{-1}(ax)}},x\right)}{896a}-\frac{5c\text{Unintegrable}\left(\frac{\left(a^2cx^2+c\right)^2}{\sqrt{\tan^{-1}(ax)}},x\right)}{896a}-\frac{3c^3\text{Unintegrable}\left(\frac{1}{\sqrt{\tan^{-1}(ax)}},x\right)}{112a}-\frac{c^3\text{Unintegra}}{}$$

```
[Out] (3*c^3*(1 + a^2*x^2)*Sqrt[ArcTan[a*x]])/(56*a^2) + (9*c^3*(1 + a^2*x^2)^2*S
qrt[ArcTan[a*x]])/(448*a^2) + (5*c^3*(1 + a^2*x^2)^3*Sqrt[ArcTan[a*x]])/(44
8*a^2) - (c^3*x*(1 + a^2*x^2)*ArcTan[a*x]^(3/2))/(14*a) - (3*c^3*x*(1 + a^2
*x^2)^2*ArcTan[a*x]^(3/2))/(56*a) - (5*c^3*x*(1 + a^2*x^2)^3*ArcTan[a*x]^(3
/2))/(112*a) + (c^3*(1 + a^2*x^2)^4*ArcTan[a*x]^(5/2))/(8*a^2) - (3*c^3*Uni
ntegrable[1/Sqrt[ArcTan[a*x]], x])/(112*a) - (9*c^2*Unintegrable[(c + a^2*c
*x^2)/Sqrt[ArcTan[a*x]], x])/(896*a) - (5*c*Unintegrable[(c + a^2*c*x^2)^2/
Sqrt[ArcTan[a*x]], x])/(896*a) - (c^3*Unintegrable[ArcTan[a*x]^(3/2), x])/(
7*a)
```

Rubi [A] time = 0.187492, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x\left(c+a^2cx^2\right)^3 \tan^{-1}(ax)^{5/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x*(c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2),x]
```

```
[Out] (3*c^3*(1 + a^2*x^2)*Sqrt[ArcTan[a*x]])/(56*a^2) + (9*c^3*(1 + a^2*x^2)^2*S
qrt[ArcTan[a*x]])/(448*a^2) + (5*c^3*(1 + a^2*x^2)^3*Sqrt[ArcTan[a*x]])/(44
8*a^2) - (c^3*x*(1 + a^2*x^2)*ArcTan[a*x]^(3/2))/(14*a) - (3*c^3*x*(1 + a^2
*x^2)^2*ArcTan[a*x]^(3/2))/(56*a) - (5*c^3*x*(1 + a^2*x^2)^3*ArcTan[a*x]^(3
/2))/(112*a) + (c^3*(1 + a^2*x^2)^4*ArcTan[a*x]^(5/2))/(8*a^2) - (3*c^3*Def
er[Int][1/Sqrt[ArcTan[a*x]], x])/(112*a) - (9*c^2*Defer[Int][(c + a^2*c*x^2
)/Sqrt[ArcTan[a*x]], x])/(896*a) - (5*c*Defer[Int][(c + a^2*c*x^2)^2/Sqrt[A
rcTan[a*x]], x])/(896*a) - (c^3*Defer[Int][ArcTan[a*x]^(3/2), x])/(7*a)
```

Rubi steps

$$\int x\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^{5/2}\,dx = \frac{c^3\left(1+a^2x^2\right)^4\tan^{-1}(ax)^{5/2}}{8a^2} - \frac{5\int\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^{3/2}\,dx}{16a}$$

$$= \frac{5c^3\left(1+a^2x^2\right)^3\sqrt{\tan^{-1}(ax)}}{448a^2} - \frac{5c^3x\left(1+a^2x^2\right)^3\tan^{-1}(ax)^{3/2}}{112a} + \frac{c^3\left(1+a^2x^2\right)^4\tan^{-1}$$

$$= \frac{9c^3\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}}{448a^2} + \frac{5c^3\left(1+a^2x^2\right)^3\sqrt{\tan^{-1}(ax)}}{448a^2} - \frac{3c^3x\left(1+a^2x^2\right)^2\tan^{-}}{56a}$$

$$= \frac{3c^3\left(1+a^2x^2\right)\sqrt{\tan^{-1}(ax)}}{56a^2} + \frac{9c^3\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}}{448a^2} + \frac{5c^3\left(1+a^2x^2\right)^3\sqrt{\tan^{-}}}{448a^2}$$

Mathematica [A] time = 1.50749, size = 0, normalized size = 0.

$$\int x\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^{5/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x*(c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2),x]
```

```
[Out] Integrate[x*(c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2), x]
```

Maple [A] time = 0.76, size = 0, normalized size = 0.

$$\int x\left(a^2cx^2+c\right)^3(\arctan(ax))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^3*arctan(a*x)^(5/2),x)
```

```
[Out] int(x*(a^2*c*x^2+c)^3*arctan(a*x)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^3*arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^3*arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**3*atan(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2+c\right)^3 x \arctan\left(ax\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^3*arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^3*x*arctan(a*x)^(5/2), x)
```

3.852 $\int \left(c+a^2 c x^2\right)^3 \tan^{-1}(ax)^{5/2}\,dx$

Optimal. Leaf size=255

$$\frac{9}{56}c^2\text{Unintegrable}\left(\left(a^2cx^2+c\right)\sqrt{\tan^{-1}(ax)},x\right)+\frac{5}{56}c\text{Unintegrable}\left(\left(a^2cx^2+c\right)^2\sqrt{\tan^{-1}(ax)},x\right)+\frac{3}{7}c^3\text{Unintegrab}$$

```
[Out] (-2*c^3*(1 + a^2*x^2)*ArcTan[a*x]^(3/2))/(7*a) - (3*c^3*(1 + a^2*x^2)^2*Arc
Tan[a*x]^(3/2))/(28*a) - (5*c^3*(1 + a^2*x^2)^3*ArcTan[a*x]^(3/2))/(84*a) +
 (8*c^3*x*(1 + a^2*x^2)*ArcTan[a*x]^(5/2))/35 + (6*c^3*x*(1 + a^2*x^2)^2*Ar
cTan[a*x]^(5/2))/35 + (c^3*x*(1 + a^2*x^2)^3*ArcTan[a*x]^(5/2))/7 + (3*c^3*
Unintegrable[Sqrt[ArcTan[a*x]], x])/7 + (9*c^2*Unintegrable[(c + a^2*c*x^2)
*Sqrt[ArcTan[a*x]], x])/56 + (5*c*Unintegrable[(c + a^2*c*x^2)^2*Sqrt[ArcTa
n[a*x]], x])/56 + (16*c^3*Unintegrable[ArcTan[a*x]^(5/2), x])/35
```

Rubi [A] time = 0.126401, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \left(c+a^2cx^2\right)^3 \tan^{-1}(ax)^{5/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2),x]
```

```
[Out] (-2*c^3*(1 + a^2*x^2)*ArcTan[a*x]^(3/2))/(7*a) - (3*c^3*(1 + a^2*x^2)^2*Arc
Tan[a*x]^(3/2))/(28*a) - (5*c^3*(1 + a^2*x^2)^3*ArcTan[a*x]^(3/2))/(84*a) +
 (8*c^3*x*(1 + a^2*x^2)*ArcTan[a*x]^(5/2))/35 + (6*c^3*x*(1 + a^2*x^2)^2*Ar
cTan[a*x]^(5/2))/35 + (c^3*x*(1 + a^2*x^2)^3*ArcTan[a*x]^(5/2))/7 + (3*c^3*
Defer[Int][Sqrt[ArcTan[a*x]], x])/7 + (9*c^2*Defer[Int][(c + a^2*c*x^2)*Sqr
t[ArcTan[a*x]], x])/56 + (5*c*Defer[Int][(c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]
], x])/56 + (16*c^3*Defer[Int][ArcTan[a*x]^(5/2), x])/35
```

Rubi steps

$$\int \left(c+a^2cx^2\right)^3\tan^{-1}(ax)^{5/2}\,dx = -\frac{5c^3\left(1+a^2x^2\right)^3\tan^{-1}(ax)^{3/2}}{84a}+\frac{1}{7}c^3x\left(1+a^2x^2\right)^3\tan^{-1}(ax)^{5/2}+\frac{1}{56}(5c)\int \left(c+a^2cx^2\right)$$

$$= -\frac{3c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)^{3/2}}{28a}-\frac{5c^3\left(1+a^2x^2\right)^3\tan^{-1}(ax)^{3/2}}{84a}+\frac{6}{35}c^3x\left(1+a^2x^2\right)^2\tan^{-}$$

$$= -\frac{2c^3\left(1+a^2x^2\right)\tan^{-1}(ax)^{3/2}}{7a}-\frac{3c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)^{3/2}}{28a}-\frac{5c^3\left(1+a^2x^2\right)^3\tan^{-1}(ax)}{84a}$$

Mathematica [A] time = 2.1729, size = 0, normalized size = 0.

$$\int \left(c+a^2cx^2\right)^3\tan^{-1}(ax)^{5/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2),x]
```

```
[Out] Integrate[(c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2), x]
```

Maple [A] time = 0.592, size = 0, normalized size = 0.

$$\int \left(a^2cx^2+c\right)^3(\arctan(ax))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^3*arctan(a*x)^(5/2),x)
```

```
[Out] int((a^2*c*x^2+c)^3*arctan(a*x)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**3*atan(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2+c\right)^3 \arctan\left(ax\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^3*arctan(a*x)^(5/2), x)
```

3.853 $\int \frac{\left(c+a^2cx^2\right)^3 \tan^{-1}(ax)^{5/2}}{x} \, dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{\left(a^2cx^2+c\right)^3 \tan^{-1}(ax)^{5/2}}{x},x\right)$$

```
[Out] Unintegrable[((c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2))/x, x]
```

Rubi [A] time = 0.0499419, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(c+a^2cx^2\right)^3 \tan^{-1}(ax)^{5/2}}{x} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[((c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2))/x,x]

[Out] Defer[Int][((c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2))/x, x]
```

Rubi steps

$$\int \frac{\left(c+a^2cx^2\right)^3 \tan^{-1}(ax)^{5/2}}{x} \, dx = \int \frac{\left(c+a^2cx^2\right)^3 \tan^{-1}(ax)^{5/2}}{x} \, dx$$

Mathematica [A] time = 1.51496, size = 0, normalized size = 0.

$$\int \frac{\left(c+a^2cx^2\right)^3 \tan^{-1}(ax)^{5/2}}{x} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2))/x,x]
```

```
[Out] Integrate[((c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2))/x, x]
```

Maple [A] time = 0.669, size = 0, normalized size = 0.

$$\int \frac{\left(a^2 c x^2 + c\right)^3}{x} (\arctan(ax))^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^3*arctan(a*x)^(5/2)/x,x)
```

```
[Out] int((a^2*c*x^2+c)^3*arctan(a*x)^(5/2)/x,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^(5/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^(5/2)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**3*atan(a*x)**(5/2)/x,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^3 \arctan\left(ax\right)^{\frac{5}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^(5/2)/x,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^3*arctan(a*x)^(5/2)/x, x)
```

3.854 $\int \frac{\left(c+a^2cx^2\right)^3 \tan^{-1}(ax)^{5/2}}{x^2} \, dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{\left(a^2cx^2+c\right)^3 \tan^{-1}(ax)^{5/2}}{x^2},x\right)$$

```
[Out] Unintegrable[((c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2))/x^2, x]
```

Rubi [A] time = 0.0538417, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(c+a^2cx^2\right)^3 \tan^{-1}(ax)^{5/2}}{x^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[((c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2))/x^2,x]

[Out] Defer[Int][((c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2))/x^2, x]
```

Rubi steps

$$\int \frac{\left(c+a^2cx^2\right)^3 \tan^{-1}(ax)^{5/2}}{x^2} \, dx = \int \frac{\left(c+a^2cx^2\right)^3 \tan^{-1}(ax)^{5/2}}{x^2} \, dx$$

Mathematica [A] time = 2.54449, size = 0, normalized size = 0.

$$\int \frac{\left(c+a^2cx^2\right)^3 \tan^{-1}(ax)^{5/2}}{x^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2))/x^2,x]
```

```
[Out] Integrate[((c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2))/x^2, x]
```

Maple [A] time = 0.56, size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^3}{x^2}\left(\arctan\left(ax\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^3*arctan(a*x)^(5/2)/x^2,x)
```

```
[Out] int((a^2*c*x^2+c)^3*arctan(a*x)^(5/2)/x^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^(5/2)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^(5/2)/x^2,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**3*atan(a*x)**(5/2)/x**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^3 \arctan(ax)^{\frac{5}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^(5/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^3*arctan(a*x)^(5/2)/x^2, x)
```

3.855 $\int \frac{x^m \tan^{-1}(ax)^{5/2}}{c+a^2cx^2} dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{x^m \tan^{-1}(ax)^{5/2}}{a^2cx^2 + c}, x\right)$$

```
[Out] Unintegrable[(x^m*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2), x]
```

Rubi [A] time = 0.0629983, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m \tan^{-1}(ax)^{5/2}}{c + a^2cx^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2),x]

[Out] Defer[Int][(x^m*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2), x]
```

Rubi steps

$$\int \frac{x^m \tan^{-1}(ax)^{5/2}}{c + a^2cx^2} dx = \int \frac{x^m \tan^{-1}(ax)^{5/2}}{c + a^2cx^2} dx$$

Mathematica [A] time = 0.676898, size = 0, normalized size = 0.

$$\int \frac{x^m \tan^{-1}(ax)^{5/2}}{c + a^2cx^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2),x]

[Out] Integrate[(x^m*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2), x]
```

Maple [A] time = 0.556, size = 0, normalized size = 0.

$$\int \frac{x^{m}}{a^{2} c \,x^{2}+c} \left(\arctan \left(a x \right)\right)^{\frac{5}{2}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c),x)
```

```
[Out] int(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x^{m} \arctan\left(a x\right)^{\frac{5}{2}}}{a^{2} c x^{2} + c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2 + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*atan(a*x)**(5/2)/(a**2*c*x**2+c),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m \arctan(ax)^{\frac{5}{2}}}{a^2cx^2 + c} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2 + c), x)
```

3.856 $\int \frac{x^3 \tan^{-1}(ax)^{5/2}}{c+a^2cx^2} \, dx$

Optimal. Leaf size=59

$$\frac{\text{Unintegrable}\left(x\tan^{-1}(ax)^{5/2},x\right)}{a^2c}+\frac{2\text{Unintegrable}\left(\tan^{-1}(ax)^{7/2},x\right)}{7a^3c}-\frac{2x\tan^{-1}(ax)^{7/2}}{7a^3c}$$

```
[Out] (-2*x*ArcTan[a*x]^(7/2))/(7*a^3*c) + Unintegrable[x*ArcTan[a*x]^(5/2), x]/(
a^2*c) + (2*Unintegrable[ArcTan[a*x]^(7/2), x])/(7*a^3*c)
```

Rubi [A] time = 0.119754, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^3 \tan^{-1}(ax)^{5/2}}{c+a^2cx^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^3*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2),x]
```

```
[Out] (-2*x*ArcTan[a*x]^(7/2))/(7*a^3*c) + Defer[Int][x*ArcTan[a*x]^(5/2), x]/(a^
2*c) + (2*Defer[Int][ArcTan[a*x]^(7/2), x])/(7*a^3*c)
```

Rubi steps

$$\begin{aligned}\int \frac{x^3 \tan^{-1}(ax)^{5/2}}{c+a^2cx^2} \, dx &= -\frac{\int \frac{x\tan^{-1}(ax)^{5/2}}{c+a^2cx^2} \, dx}{a^2}+\frac{\int x\tan^{-1}(ax)^{5/2} \, dx}{a^2c}\\ &= -\frac{2x\tan^{-1}(ax)^{7/2}}{7a^3c}+\frac{2\int \tan^{-1}(ax)^{7/2} \, dx}{7a^3c}+\frac{\int x\tan^{-1}(ax)^{5/2} \, dx}{a^2c}\end{aligned}$$

Mathematica [A] time = 4.25563, size = 0, normalized size = 0.

$$\int \frac{x^3 \tan^{-1}(ax)^{5/2}}{c+a^2cx^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^3*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2),x]
```

```
[Out] Integrate[(x^3*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2), x]
```

Maple [A] time = 0.609, size = 0, normalized size = 0.

$$\int \frac{x^3}{a^2cx^2+c}(\arctan(ax))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2+c),x)
```

```
[Out] int(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2+c),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atan(a*x)**(5/2)/(a**2*c*x**2+c),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \arctan{(ax)}^{\frac{5}{2}}}{a^2cx^2 + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2 + c), x)
```

3.857 $\int \frac{x^2 \tan^{-1}(ax)^{5/2}}{c+a^2cx^2} dx$

Optimal. Leaf size=36

$$\frac{\text{Unintegrable}\left(\tan^{-1}(ax)^{5/2},x\right)}{a^2c}-\frac{2\tan^{-1}(ax)^{7/2}}{7a^3c}$$

```
[Out] (-2*ArcTan[a*x]^(7/2))/(7*a^3*c) + Unintegrable[ArcTan[a*x]^(5/2), x]/(a^2*
c)
```

Rubi [A] time = 0.0911277, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^2\tan^{-1}(ax)^{5/2}}{c+a^2cx^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^2*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2),x]
```

```
[Out] (-2*ArcTan[a*x]^(7/2))/(7*a^3*c) + Defer[Int][ArcTan[a*x]^(5/2), x]/(a^2*c)
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\tan^{-1}(ax)^{5/2}}{c+a^2cx^2}\,dx &= -\frac{\int \frac{\tan^{-1}(ax)^{5/2}}{c+a^2cx^2}\,dx}{a^2}+\frac{\int \tan^{-1}(ax)^{5/2}\,dx}{a^2c}\\ &= -\frac{2\tan^{-1}(ax)^{7/2}}{7a^3c}+\frac{\int \tan^{-1}(ax)^{5/2}\,dx}{a^2c}\end{aligned}$$

Mathematica [A] time = 1.19794, size = 0, normalized size = 0.

$$\int \frac{x^2\tan^{-1}(ax)^{5/2}}{c+a^2cx^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^2*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2),x]
```

```
[Out] Integrate[(x^2*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2), x]
```

Maple [A] time = 0.244, size = 0, normalized size = 0.

$$\int \frac{x^2}{a^2cx^2+c}\left(\arctan\left(ax\right)\right)^{\frac{5}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c),x)
```

```
[Out] int(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(a*x)**(5/2)/(a**2*c*x**2+c),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \arctan(ax)^{\frac{5}{2}}}{a^2cx^2 + c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2 + c), x)
```

3.858 $\int \frac{x\tan^{-1}(ax)^{5/2}}{c+a^2cx^2}\,dx$

Optimal. Leaf size=40

$$\frac{2x\tan^{-1}(ax)^{7/2}}{7ac}-\frac{2\text{Unintegrable}\left(\tan^{-1}(ax)^{7/2},x\right)}{7ac}$$

```
[Out] (2*x*ArcTan[a*x]^(7/2))/(7*a*c) - (2*Unintegrable[ArcTan[a*x]^(7/2), x])/(7
*a*c)
```

Rubi [A] time = 0.0538538, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x\tan^{-1}(ax)^{5/2}}{c+a^2cx^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2),x]
```

```
[Out] (2*x*ArcTan[a*x]^(7/2))/(7*a*c) - (2*Defer[Int][ArcTan[a*x]^(7/2), x])/(7*a
*c)
```

Rubi steps

$$\int \frac{x\tan^{-1}(ax)^{5/2}}{c+a^2cx^2}\,dx=\frac{2x\tan^{-1}(ax)^{7/2}}{7ac}-\frac{2\int \tan^{-1}(ax)^{7/2}\,dx}{7ac}$$

Mathematica [A] time = 0.991519, size = 0, normalized size = 0.

$$\int \frac{x\tan^{-1}(ax)^{5/2}}{c+a^2cx^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2),x]
```

```
[Out] Integrate[(x*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2), x]
```

Maple [A] time = 0.136, size = 0, normalized size = 0.

$$\int \frac{x}{a^{2}c\,x^{2}+c} \left(\arctan \left(a x \right)\right)^{\frac{5}{2}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c),x)
```

```
[Out] int(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x \operatorname{atan}^{\frac{5}{2}}{\left(a x \right)}}{a^{2} x^{2} + 1}\, dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(a*x)**(5/2)/(a**2*c*x**2+c),x)
```

```
[Out] Integral(x*atan(a*x)**(5/2)/(a**2*x**2 + 1), x)/c
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \arctan (ax)^{\frac{5}{2}}}{a^2cx^2 + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(x*arctan(a*x)^(5/2)/(a^2*c*x^2 + c), x)
```

3.859 $\int \frac{\tan^{-1}(ax)^{5/2}}{c+a^2cx^2}\,dx$

Optimal. Leaf size=18

$$\frac{2\tan^{-1}(ax)^{7/2}}{7ac}$$

```
[Out] (2*ArcTan[a*x]^(7/2))/(7*a*c)
```

Rubi [A] time = 0.0252277, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {4884}

$$\frac{2\tan^{-1}(ax)^{7/2}}{7ac}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^(5/2)/(c + a^2*c*x^2),x]
```

```
[Out] (2*ArcTan[a*x]^(7/2))/(7*a*c)
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^{5/2}}{c+a^2cx^2}\,dx = \frac{2\tan^{-1}(ax)^{7/2}}{7ac}$$

Mathematica [A] time = 0.0029866, size = 18, normalized size = 1.

$$\frac{2\tan^{-1}(ax)^{7/2}}{7ac}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]^(5/2)/(c + a^2*c*x^2),x]
```

```
[Out] (2*ArcTan[a*x]^(7/2))/(7*a*c)
```

Maple [A] time = 0.079, size = 15, normalized size = 0.8

$$\frac{2}{7\,ac}\left(\arctan\left(ax\right)\right)^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(5/2)/(a^2*c*x^2+c),x)
```

```
[Out] 2/7*arctan(a*x)^(7/2)/a/c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 1.61919, size = 39, normalized size = 2.17

$$\frac{2\,\arctan\left(ax\right)^{\frac{7}{2}}}{7\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] 2/7*arctan(a*x)^(7/2)/(a*c)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(5/2)/(a**2*c*x**2+c),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.08444, size = 19, normalized size = 1.06

$$\frac{2\arctan\left(ax\right)^{\frac{7}{2}}}{7\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] 2/7*arctan(a*x)^(7/2)/(a*c)
```

3.860 $\int \frac{\tan^{-1}(ax)^{5/2}}{x\left(c+a^2cx^2\right)}\,dx$

Optimal. Leaf size=48

$$\frac{i\text{Unintegrable}\left(\frac{\tan^{-1}(ax)^{5/2}}{x(ax+i)},x\right)}{c}-\frac{2i\tan^{-1}(ax)^{7/2}}{7c}$$

```
[Out] (((-2*I)/7)*ArcTan[a*x]^(7/2))/c + (I*Unintegrable[ArcTan[a*x]^(5/2)/(x*(I
+ a*x)), x])/c
```

Rubi [A] time = 0.114966, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\tan^{-1}(ax)^{5/2}}{x\left(c+a^2cx^2\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcTan[a*x]^(5/2)/(x*(c + a^2*c*x^2)),x]
```

```
[Out] (((-2*I)/7)*ArcTan[a*x]^(7/2))/c + (I*Defer[Int][ArcTan[a*x]^(5/2)/(x*(I +
a*x)), x])/c
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^{5/2}}{x\left(c+a^2cx^2\right)}\,dx=-\frac{2i\tan^{-1}(ax)^{7/2}}{7c}+\frac{i\int \frac{\tan^{-1}(ax)^{5/2}}{x(i+ax)}\,dx}{c}$$

Mathematica [A] time = 0.593584, size = 0, normalized size = 0.

$$\int \frac{\tan^{-1}(ax)^{5/2}}{x\left(c+a^2cx^2\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcTan[a*x]^(5/2)/(x*(c + a^2*c*x^2)),x]
```

```
[Out] Integrate[ArcTan[a*x]^(5/2)/(x*(c + a^2*c*x^2)), x]
```

Maple [A] time = 0.14, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(a^{2} c x^{2}+c\right)} \left(\arctan\left(a x\right)\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c),x)
```

```
[Out] int(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\operatorname{atan}^{\frac{5}{2}}(ax)}{a^2x^3+x}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(5/2)/x/(a**2*c*x**2+c),x)
```

```
[Out] Integral(atan(a*x)**(5/2)/(a**2*x**3 + x), x)/c
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(ax)^{\frac{5}{2}}}{(a^2cx^2+c)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)^(5/2)/((a^2*c*x^2 + c)*x), x)
```

3.861 $\int \frac{\tan^{-1}(ax)^{5/2}}{x^2(c+a^2cx^2)}\,dx$

Optimal. Leaf size=35

$$\frac{\text{Unintegrable}\left(\frac{\tan^{-1}(ax)^{5/2}}{x^2},x\right)}{c}-\frac{2a\tan^{-1}(ax)^{7/2}}{7c}$$

```
[Out] (-2*a*ArcTan[a*x]^(7/2))/(7*c) + Unintegrable[ArcTan[a*x]^(5/2)/x^2, x]/c
```

Rubi [A] time = 0.103763, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\tan^{-1}(ax)^{5/2}}{x^2\left(c+a^2cx^2\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcTan[a*x]^(5/2)/(x^2*(c + a^2*c*x^2)),x]
```

```
[Out] (-2*a*ArcTan[a*x]^(7/2))/(7*c) + Defer[Int][ArcTan[a*x]^(5/2)/x^2, x]/c
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^{-1}(ax)^{5/2}}{x^2\left(c+a^2cx^2\right)}\,dx &= -\left(a^2\int \frac{\tan^{-1}(ax)^{5/2}}{c+a^2cx^2}\,dx\right)+\frac{\int \frac{\tan^{-1}(ax)^{5/2}}{x^2}\,dx}{c}\\ &= -\frac{2a\tan^{-1}(ax)^{7/2}}{7c}+\frac{\int \frac{\tan^{-1}(ax)^{5/2}}{x^2}\,dx}{c}\end{aligned}$$

Mathematica [A] time = 1.26704, size = 0, normalized size = 0.

$$\int \frac{\tan^{-1}(ax)^{5/2}}{x^2\left(c+a^2cx^2\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcTan[a*x]^(5/2)/(x^2*(c + a^2*c*x^2)),x]
```

```
[Out] Integrate[ArcTan[a*x]^(5/2)/(x^2*(c + a^2*c*x^2)), x]
```

Maple [A] time = 0.242, size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(a^2cx^2+c\right)}\left(\arctan\left(ax\right)\right)^{\frac{5}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(5/2)/x^2/(a^2*c*x^2+c),x)
```

```
[Out] int(arctan(a*x)^(5/2)/x^2/(a^2*c*x^2+c),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/x^2/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/x^2/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\operatorname{atan}^{\frac{5}{2}}(ax)}{a^2x^4+x^2}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(5/2)/x**2/(a**2*c*x**2+c),x)
```

```
[Out] Integral(atan(a*x)**(5/2)/(a**2*x**4 + x**2), x)/c
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(ax)^{\frac{5}{2}}}{(a^2cx^2+c)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/x^2/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)^(5/2)/((a^2*c*x^2 + c)*x^2), x)
```

3.862 $\int \frac{\tan^{-1}(ax)^{5/2}}{x^3(c+a^2cx^2)}\,dx$

Optimal. Leaf size=72

$$-\frac{ia^2\text{Unintegrable}\left(\frac{\tan^{-1}(ax)^{5/2}}{x(ax+i)},x\right)}{c}+\frac{\text{Unintegrable}\left(\frac{\tan^{-1}(ax)^{5/2}}{x^3},x\right)}{c}+\frac{2ia^2\tan^{-1}(ax)^{7/2}}{7c}$$

```
[Out] (((2*I)/7)*a^2*ArcTan[a*x]^(7/2))/c + Unintegrable[ArcTan[a*x]^(5/2)/x^3, x
]/c - (I*a^2*Unintegrable[ArcTan[a*x]^(5/2)/(x*(I + a*x)), x])/c
```

Rubi [A] time = 0.186641, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\tan^{-1}(ax)^{5/2}}{x^3\left(c+a^2cx^2\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcTan[a*x]^(5/2)/(x^3*(c + a^2*c*x^2)),x]
```

```
[Out] (((2*I)/7)*a^2*ArcTan[a*x]^(7/2))/c + Defer[Int][ArcTan[a*x]^(5/2)/x^3, x]/
c - (I*a^2*Defer[Int][ArcTan[a*x]^(5/2)/(x*(I + a*x)), x])/c
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^{-1}(ax)^{5/2}}{x^3\left(c+a^2cx^2\right)}\,dx &= -\left(a^2\int \frac{\tan^{-1}(ax)^{5/2}}{x\left(c+a^2cx^2\right)}\,dx\right)+\frac{\int \frac{\tan^{-1}(ax)^{5/2}}{x^3}\,dx}{c}\\ &= \frac{2ia^2\tan^{-1}(ax)^{7/2}}{7c}+\frac{\int \frac{\tan^{-1}(ax)^{5/2}}{x^3}\,dx}{c}-\frac{\left(ia^2\right)\int \frac{\tan^{-1}(ax)^{5/2}}{x(i+ax)}\,dx}{c}\end{aligned}$$

Mathematica [A] time = 1.98426, size = 0, normalized size = 0.

$$\int \frac{\tan^{-1}(ax)^{5/2}}{x^3\left(c+a^2cx^2\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcTan[a*x]^(5/2)/(x^3*(c + a^2*c*x^2)),x]
```

```
[Out] Integrate[ArcTan[a*x]^(5/2)/(x^3*(c + a^2*c*x^2)), x]
```

Maple [A] time = 0.604, size = 0, normalized size = 0.

$$\int \frac{1}{x^{3}\left(a^{2} c\,x^{2}+c\right)}\left(\arctan \left(a x\right)\right)^{\frac{5}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(5/2)/x^3/(a^2*c*x^2+c),x)
```

```
[Out] int(arctan(a*x)^(5/2)/x^3/(a^2*c*x^2+c),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/x^3/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/x^3/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\operatorname{atan}^{\frac{5}{2}}(ax)}{a^2x^5+x^3}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(5/2)/x**3/(a**2*c*x**2+c),x)
```

```
[Out] Integral(atan(a*x)**(5/2)/(a**2*x**5 + x**3), x)/c
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(ax)^{\frac{5}{2}}}{(a^2cx^2+c)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/x^3/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)^(5/2)/((a^2*c*x^2 + c)*x^3), x)
```

3.863 $\int \frac{\tan^{-1}(ax)^{5/2}}{x^4\left(c+a^2cx^2\right)}\,dx$

Optimal. Leaf size=59

$$-\frac{a^2\text{Unintegrable}\left(\frac{\tan^{-1}(ax)^{5/2}}{x^2},x\right)}{c}+\frac{\text{Unintegrable}\left(\frac{\tan^{-1}(ax)^{5/2}}{x^4},x\right)}{c}+\frac{2a^3\tan^{-1}(ax)^{7/2}}{7c}$$

```
[Out] (2*a^3*ArcTan[a*x]^(7/2))/(7*c) + Unintegrable[ArcTan[a*x]^(5/2)/x^4, x]/c
- (a^2*Unintegrable[ArcTan[a*x]^(5/2)/x^2, x])/c
```

Rubi [A] time = 0.17904, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\tan^{-1}(ax)^{5/2}}{x^4\left(c+a^2cx^2\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcTan[a*x]^(5/2)/(x^4*(c + a^2*c*x^2)),x]
```

```
[Out] (2*a^3*ArcTan[a*x]^(7/2))/(7*c) + Defer[Int][ArcTan[a*x]^(5/2)/x^4, x]/c -
(a^2*Defer[Int][ArcTan[a*x]^(5/2)/x^2, x])/c
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^{-1}(ax)^{5/2}}{x^4\left(c+a^2cx^2\right)}\,dx &= -\left(a^2\int \frac{\tan^{-1}(ax)^{5/2}}{x^2\left(c+a^2cx^2\right)}\,dx\right)+\frac{\int \frac{\tan^{-1}(ax)^{5/2}}{x^4}\,dx}{c}\\ &= a^4\int \frac{\tan^{-1}(ax)^{5/2}}{c+a^2cx^2}\,dx+\frac{\int \frac{\tan^{-1}(ax)^{5/2}}{x^4}\,dx}{c}-\frac{a^2\int \frac{\tan^{-1}(ax)^{5/2}}{x^2}\,dx}{c}\\ &= \frac{2a^3\tan^{-1}(ax)^{7/2}}{7c}+\frac{\int \frac{\tan^{-1}(ax)^{5/2}}{x^4}\,dx}{c}-\frac{a^2\int \frac{\tan^{-1}(ax)^{5/2}}{x^2}\,dx}{c}\end{aligned}$$

Mathematica [A] time = 3.45598, size = 0, normalized size = 0.

$$\int \frac{\tan^{-1}(ax)^{5/2}}{x^4\left(c+a^2cx^2\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcTan[a*x]^(5/2)/(x^4*(c + a^2*c*x^2)),x]
```

```
[Out] Integrate[ArcTan[a*x]^(5/2)/(x^4*(c + a^2*c*x^2)), x]
```

Maple [A] time = 0.487, size = 0, normalized size = 0.

$$\int \frac{1}{x^{4}\left(a^{2}c\,x^{2}+c\right)}\left(\arctan\left(ax\right)\right)^{\frac{5}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(5/2)/x^4/(a^2*c*x^2+c),x)
```

```
[Out] int(arctan(a*x)^(5/2)/x^4/(a^2*c*x^2+c),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/x^4/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/x^4/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\operatorname{atan}^{\frac{5}{2}}(ax)}{a^2x^6+x^4}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(5/2)/x**4/(a**2*c*x**2+c),x)
```

```
[Out] Integral(atan(a*x)**(5/2)/(a**2*x**6 + x**4), x)/c
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(ax)^{\frac{5}{2}}}{(a^2cx^2+c)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/x^4/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)^(5/2)/((a^2*c*x^2 + c)*x^4), x)
```

3.864 $\int \frac{x^m \tan^{-1}(ax)^{5/2}}{\left(c+a^2cx^2\right)^2} \, dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{x^m \tan^{-1}(ax)^{5/2}}{\left(a^2cx^2+c\right)^2},x\right)$$

```
[Out] Unintegrable[(x^m*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^2, x]
```

Rubi [A] time = 0.0638905, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m \tan^{-1}(ax)^{5/2}}{\left(c+a^2cx^2\right)^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^2,x]

[Out] Defer[Int][(x^m*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^2, x]
```

Rubi steps

$$\int \frac{x^m \tan^{-1}(ax)^{5/2}}{\left(c+a^2cx^2\right)^2} \, dx=\int \frac{x^m \tan^{-1}(ax)^{5/2}}{\left(c+a^2cx^2\right)^2} \, dx$$

Mathematica [A] time = 1.36425, size = 0, normalized size = 0.

$$\int \frac{x^m \tan^{-1}(ax)^{5/2}}{\left(c+a^2cx^2\right)^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^2,x]
```

```
[Out] Integrate[(x^m*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^2, x]
```

Maple [A] time = 0.74, size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(a^2 c x^2 + c\right)^2} \left(\arctan\left(a x\right)\right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^2,x)
```

```
[Out] int(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^m \arctan\left(a x\right)^{\frac{5}{2}}}{a^4 c^2 x^4 + 2\, a^2 c^2 x^2 + c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] integral(x^m*arctan(a*x)^(5/2)/(a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*atan(a*x)**(5/2)/(a**2*c*x**2+c)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m \arctan(ax)^{\frac{5}{2}}}{(a^2cx^2+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2 + c)^2, x)
```

3.865 $\int \frac{x^3 \tan^{-1}(ax)^{5/2}}{(c+a^2cx^2)^2} \, dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{x^3 \tan^{-1}(ax)^{5/2}}{(a^2cx^2+c)^2}, x\right)$$

```
[Out] Unintegrable[(x^3*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^2, x]
```

Rubi [A] time = 0.06566, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^3 \tan^{-1}(ax)^{5/2}}{(c+a^2cx^2)^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^3*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^2,x]

[Out] Defer[Int][(x^3*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^2, x]
```

Rubi steps

$$\int \frac{x^3 \tan^{-1}(ax)^{5/2}}{(c+a^2cx^2)^2} \, dx = \int \frac{x^3 \tan^{-1}(ax)^{5/2}}{(c+a^2cx^2)^2} \, dx$$

Mathematica [A] time = 4.30624, size = 0, normalized size = 0.

$$\int \frac{x^3 \tan^{-1}(ax)^{5/2}}{(c+a^2cx^2)^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^3*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^2,x]
```

```
[Out] Integrate[(x^3*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^2, x]
```

Maple [A] time = 0.507, size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(a^2cx^2+c\right)^2} (\arctan (ax))^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^2,x)
```

```
[Out] int(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atan(a*x)**(5/2)/(a**2*c*x**2+c)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \arctan(ax)^{\frac{5}{2}}}{\left(a^2cx^2 + c\right)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2 + c)^2, x)
```

3.866 $\int \frac{x^2 \tan^{-1}(ax)^{5/2}}{\left(c+a^2cx^2\right)^2}\,dx$

Optimal. Leaf size=157

$$-\frac{15\sqrt{\pi}S\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{128a^3c^2}-\frac{x\tan^{-1}(ax)^{5/2}}{2a^2c^2\left(a^2x^2+1\right)}-\frac{5\tan^{-1}(ax)^{3/2}}{8a^3c^2\left(a^2x^2+1\right)}+\frac{15x\sqrt{\tan^{-1}(ax)}}{32a^2c^2\left(a^2x^2+1\right)}+\frac{\tan^{-1}(ax)^{7/2}}{7a^3c^2}+\frac{5\tan^{-1}(ax)^{3/2}}{16a^3c^2}$$

```
[Out] (15*x*Sqrt[ArcTan[a*x]])/(32*a^2*c^2*(1 + a^2*x^2)) + (5*ArcTan[a*x]^(3/2))
/(16*a^3*c^2) - (5*ArcTan[a*x]^(3/2))/(8*a^3*c^2*(1 + a^2*x^2)) - (x*ArcTan
[a*x]^(5/2))/(2*a^2*c^2*(1 + a^2*x^2)) + ArcTan[a*x]^(7/2)/(7*a^3*c^2) - (1
5*Sqrt[Pi]*FresnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(128*a^3*c^2)
```

Rubi [A] time = 0.224478, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4936, 4930, 4892, 4970, 4406, 12, 3305, 3351}

$$-\frac{15\sqrt{\pi}S\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{128a^3c^2}-\frac{x\tan^{-1}(ax)^{5/2}}{2a^2c^2\left(a^2x^2+1\right)}-\frac{5\tan^{-1}(ax)^{3/2}}{8a^3c^2\left(a^2x^2+1\right)}+\frac{15x\sqrt{\tan^{-1}(ax)}}{32a^2c^2\left(a^2x^2+1\right)}+\frac{\tan^{-1}(ax)^{7/2}}{7a^3c^2}+\frac{5\tan^{-1}(ax)^{3/2}}{16a^3c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^2,x]
```

```
[Out] (15*x*Sqrt[ArcTan[a*x]])/(32*a^2*c^2*(1 + a^2*x^2)) + (5*ArcTan[a*x]^(3/2))
/(16*a^3*c^2) - (5*ArcTan[a*x]^(3/2))/(8*a^3*c^2*(1 + a^2*x^2)) - (x*ArcTan
[a*x]^(5/2))/(2*a^2*c^2*(1 + a^2*x^2)) + ArcTan[a*x]^(7/2)/(7*a^3*c^2) - (1
5*Sqrt[Pi]*FresnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(128*a^3*c^2)
```

Rule 4936

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^2)/((d_) + (e_.)*(x_)^2)
^2, x_Symbol] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(2*b*c^3*d^2*(p + 1)), x]
 + (Dist[(b*p)/(2*c), Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(d + e*x^2)^2, x]
, x] - Simp[(x*(a + b*ArcTan[c*x])^p)/(2*c^2*d*(d + e*x^2)), x]) /; FreeQ[{
a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 4892

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[(x*(a + b*ArcTan[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*c*
p)/2, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(a +
 b*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4970

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sin[x]^m)/
Cos[x]^(m + 2*(q + 1)), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{x^2 \tan^{-1}(ax)^{5/2}}{\left(c+a^2cx^2\right)^2}\,dx = -\frac{x\tan^{-1}(ax)^{5/2}}{2a^2c^2\left(1+a^2x^2\right)} + \frac{\tan^{-1}(ax)^{7/2}}{7a^3c^2} + \frac{5\int \frac{x\tan^{-1}(ax)^{3/2}}{\left(c+a^2cx^2\right)^2}\,dx}{4a}$$

$$= -\frac{5\tan^{-1}(ax)^{3/2}}{8a^3c^2\left(1+a^2x^2\right)} - \frac{x\tan^{-1}(ax)^{5/2}}{2a^2c^2\left(1+a^2x^2\right)} + \frac{\tan^{-1}(ax)^{7/2}}{7a^3c^2} + \frac{15\int \frac{\sqrt{\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^2}\,dx}{16a^2}$$

$$= \frac{15x\sqrt{\tan^{-1}(ax)}}{32a^2c^2\left(1+a^2x^2\right)} + \frac{5\tan^{-1}(ax)^{3/2}}{16a^3c^2} - \frac{5\tan^{-1}(ax)^{3/2}}{8a^3c^2\left(1+a^2x^2\right)} - \frac{x\tan^{-1}(ax)^{5/2}}{2a^2c^2\left(1+a^2x^2\right)} + \frac{\tan^{-1}(ax)^{7/2}}{7a^3c^2} - \frac{15}{}$$

$$= \frac{15x\sqrt{\tan^{-1}(ax)}}{32a^2c^2\left(1+a^2x^2\right)} + \frac{5\tan^{-1}(ax)^{3/2}}{16a^3c^2} - \frac{5\tan^{-1}(ax)^{3/2}}{8a^3c^2\left(1+a^2x^2\right)} - \frac{x\tan^{-1}(ax)^{5/2}}{2a^2c^2\left(1+a^2x^2\right)} + \frac{\tan^{-1}(ax)^{7/2}}{7a^3c^2} - \frac{15}{}$$

$$= \frac{15x\sqrt{\tan^{-1}(ax)}}{32a^2c^2\left(1+a^2x^2\right)} + \frac{5\tan^{-1}(ax)^{3/2}}{16a^3c^2} - \frac{5\tan^{-1}(ax)^{3/2}}{8a^3c^2\left(1+a^2x^2\right)} - \frac{x\tan^{-1}(ax)^{5/2}}{2a^2c^2\left(1+a^2x^2\right)} + \frac{\tan^{-1}(ax)^{7/2}}{7a^3c^2} - \frac{15}{}$$

$$= \frac{15x\sqrt{\tan^{-1}(ax)}}{32a^2c^2\left(1+a^2x^2\right)} + \frac{5\tan^{-1}(ax)^{3/2}}{16a^3c^2} - \frac{5\tan^{-1}(ax)^{3/2}}{8a^3c^2\left(1+a^2x^2\right)} - \frac{x\tan^{-1}(ax)^{5/2}}{2a^2c^2\left(1+a^2x^2\right)} + \frac{\tan^{-1}(ax)^{7/2}}{7a^3c^2} - \frac{15}{}$$

$$= \frac{15x\sqrt{\tan^{-1}(ax)}}{32a^2c^2\left(1+a^2x^2\right)} + \frac{5\tan^{-1}(ax)^{3/2}}{16a^3c^2} - \frac{5\tan^{-1}(ax)^{3/2}}{8a^3c^2\left(1+a^2x^2\right)} - \frac{x\tan^{-1}(ax)^{5/2}}{2a^2c^2\left(1+a^2x^2\right)} + \frac{\tan^{-1}(ax)^{7/2}}{7a^3c^2} - \frac{15}{}$$

$$= \frac{15x\sqrt{\tan^{-1}(ax)}}{32a^2c^2\left(1+a^2x^2\right)} + \frac{5\tan^{-1}(ax)^{3/2}}{16a^3c^2} - \frac{5\tan^{-1}(ax)^{3/2}}{8a^3c^2\left(1+a^2x^2\right)} - \frac{x\tan^{-1}(ax)^{5/2}}{2a^2c^2\left(1+a^2x^2\right)} + \frac{\tan^{-1}(ax)^{7/2}}{7a^3c^2} - \frac{15}{}$$

Mathematica [A] time = 0.200296, size = 111, normalized size = 0.71

$$\frac{4\sqrt{\tan^{-1}(ax)}\left(32\left(a^2x^2+1\right)\tan^{-1}(ax)^3+70\left(a^2x^2-1\right)\tan^{-1}(ax)+105ax-112ax\tan^{-1}(ax)^2\right)-105\sqrt{\pi}\left(a^2x^2+1\right)S\left(\right.}{896a^3c^2\left(a^2x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^2,x]
```

```
[Out] (4*Sqrt[ArcTan[a*x]]*(105*a*x + 70*(-1 + a^2*x^2)*ArcTan[a*x] - 112*a*x*Arc
Tan[a*x]^2 + 32*(1 + a^2*x^2)*ArcTan[a*x]^3) - 105*Sqrt[Pi]*(1 + a^2*x^2)*F
resnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(896*a^3*c^2*(1 + a^2*x^2))
```

Maple [A] time = 0.106, size = 102, normalized size = 0.7

$$\frac{1}{7\,a^3c^2}\,(\arctan(ax))^{\frac{7}{2}} - \frac{\sin(2\,\arctan(ax))}{4\,a^3c^2}\,(\arctan(ax))^{\frac{5}{2}} - \frac{5\,\cos(2\,\arctan(ax))}{16\,a^3c^2}\,(\arctan(ax))^{\frac{3}{2}} + \frac{15\,\sin(2\,\arctan(a}{64\,a^3c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^2,x)
```

```
[Out] 1/7*arctan(a*x)^(7/2)/a^3/c^2-1/4/a^3/c^2*arctan(a*x)^(5/2)*sin(2*arctan(a*
x))-5/16/a^3/c^2*arctan(a*x)^(3/2)*cos(2*arctan(a*x))+15/64/a^3/c^2*arctan(
a*x)^(1/2)*sin(2*arctan(a*x))-15/128*FresnelS(2*arctan(a*x)^(1/2)/Pi^(1/2))
*Pi^(1/2)/a^3/c^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(a*x)**(5/2)/(a**2*c*x**2+c)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \arctan(ax)^{\frac{5}{2}}}{(a^2cx^2+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2 + c)^2, x)
```

3.867 $\int \frac{x\tan^{-1}(ax)^{5/2}}{\left(c+a^2cx^2\right)^2}\,dx$

Optimal. Leaf size=156

$$-\frac{15\sqrt{\pi}\text{FresnelC}\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{128a^2c^2}-\frac{\tan^{-1}(ax)^{5/2}}{2a^2c^2\left(a^2x^2+1\right)}+\frac{5x\tan^{-1}(ax)^{3/2}}{8ac^2\left(a^2x^2+1\right)}+\frac{15\sqrt{\tan^{-1}(ax)}}{32a^2c^2\left(a^2x^2+1\right)}+\frac{\tan^{-1}(ax)^{5/2}}{4a^2c^2}-\frac{15\sqrt{\tan^{-1}(a}}{64a^2c^2}$$

```
[Out] (-15*Sqrt[ArcTan[a*x]])/(64*a^2*c^2) + (15*Sqrt[ArcTan[a*x]])/(32*a^2*c^2*(
1 + a^2*x^2)) + (5*x*ArcTan[a*x]^(3/2))/(8*a*c^2*(1 + a^2*x^2)) + ArcTan[a*
x]^(5/2)/(4*a^2*c^2) - ArcTan[a*x]^(5/2)/(2*a^2*c^2*(1 + a^2*x^2)) - (15*Sq
rt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(128*a^2*c^2)
```

Rubi [A] time = 0.199346, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4930, 4892, 4904, 3312, 3304, 3352}

$$-\frac{15\sqrt{\pi}\text{FresnelC}\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{128a^2c^2}-\frac{\tan^{-1}(ax)^{5/2}}{2a^2c^2\left(a^2x^2+1\right)}+\frac{5x\tan^{-1}(ax)^{3/2}}{8ac^2\left(a^2x^2+1\right)}+\frac{15\sqrt{\tan^{-1}(ax)}}{32a^2c^2\left(a^2x^2+1\right)}+\frac{\tan^{-1}(ax)^{5/2}}{4a^2c^2}-\frac{15\sqrt{\tan^{-1}(a}}{64a^2c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^2,x]
```

```
[Out] (-15*Sqrt[ArcTan[a*x]])/(64*a^2*c^2) + (15*Sqrt[ArcTan[a*x]])/(32*a^2*c^2*(
1 + a^2*x^2)) + (5*x*ArcTan[a*x]^(3/2))/(8*a*c^2*(1 + a^2*x^2)) + ArcTan[a*
x]^(5/2)/(4*a^2*c^2) - ArcTan[a*x]^(5/2)/(2*a^2*c^2*(1 + a^2*x^2)) - (15*Sq
rt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(128*a^2*c^2)
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 4892

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[(x*(a + b*ArcTan[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*c*
p)/2, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(a +
 b*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4904

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{x\tan^{-1}(ax)^{5/2}}{\left(c+a^2cx^2\right)^2}\,dx = -\frac{\tan^{-1}(ax)^{5/2}}{2a^2c^2\left(1+a^2x^2\right)} + \frac{5\int \frac{\tan^{-1}(ax)^{3/2}}{\left(c+a^2cx^2\right)^2}\,dx}{4a}$$

$$= \frac{5x\tan^{-1}(ax)^{3/2}}{8ac^2\left(1+a^2x^2\right)} + \frac{\tan^{-1}(ax)^{5/2}}{4a^2c^2} - \frac{\tan^{-1}(ax)^{5/2}}{2a^2c^2\left(1+a^2x^2\right)} - \frac{15}{16}\int \frac{x\sqrt{\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^2}\,dx$$

$$= \frac{15\sqrt{\tan^{-1}(ax)}}{32a^2c^2\left(1+a^2x^2\right)} + \frac{5x\tan^{-1}(ax)^{3/2}}{8ac^2\left(1+a^2x^2\right)} + \frac{\tan^{-1}(ax)^{5/2}}{4a^2c^2} - \frac{\tan^{-1}(ax)^{5/2}}{2a^2c^2\left(1+a^2x^2\right)} - \frac{15\int \frac{1}{\left(c+a^2cx^2\right)^2\sqrt{\tan^{-1}(ax)}}}{64a}$$

$$= \frac{15\sqrt{\tan^{-1}(ax)}}{32a^2c^2\left(1+a^2x^2\right)} + \frac{5x\tan^{-1}(ax)^{3/2}}{8ac^2\left(1+a^2x^2\right)} + \frac{\tan^{-1}(ax)^{5/2}}{4a^2c^2} - \frac{\tan^{-1}(ax)^{5/2}}{2a^2c^2\left(1+a^2x^2\right)} - \frac{15\,\text{Subst}\left(\int \frac{\cos^2(x)}{\sqrt{x}}\,dx,\right.}{64a^2c^2}$$

$$= \frac{15\sqrt{\tan^{-1}(ax)}}{32a^2c^2\left(1+a^2x^2\right)} + \frac{5x\tan^{-1}(ax)^{3/2}}{8ac^2\left(1+a^2x^2\right)} + \frac{\tan^{-1}(ax)^{5/2}}{4a^2c^2} - \frac{\tan^{-1}(ax)^{5/2}}{2a^2c^2\left(1+a^2x^2\right)} - \frac{15\,\text{Subst}\left(\int \left(\frac{1}{2\sqrt{x}} + \frac{\text{co}}{2}\right.\right.}{64a}$$

$$= -\frac{15\sqrt{\tan^{-1}(ax)}}{64a^2c^2} + \frac{15\sqrt{\tan^{-1}(ax)}}{32a^2c^2\left(1+a^2x^2\right)} + \frac{5x\tan^{-1}(ax)^{3/2}}{8ac^2\left(1+a^2x^2\right)} + \frac{\tan^{-1}(ax)^{5/2}}{4a^2c^2} - \frac{\tan^{-1}(ax)^{5/2}}{2a^2c^2\left(1+a^2x^2\right)} - \frac{15\,\text{S}}{}$$

$$= -\frac{15\sqrt{\tan^{-1}(ax)}}{64a^2c^2} + \frac{15\sqrt{\tan^{-1}(ax)}}{32a^2c^2\left(1+a^2x^2\right)} + \frac{5x\tan^{-1}(ax)^{3/2}}{8ac^2\left(1+a^2x^2\right)} + \frac{\tan^{-1}(ax)^{5/2}}{4a^2c^2} - \frac{\tan^{-1}(ax)^{5/2}}{2a^2c^2\left(1+a^2x^2\right)} - \frac{15\,\text{S}}{}$$

$$= -\frac{15\sqrt{\tan^{-1}(ax)}}{64a^2c^2} + \frac{15\sqrt{\tan^{-1}(ax)}}{32a^2c^2\left(1+a^2x^2\right)} + \frac{5x\tan^{-1}(ax)^{3/2}}{8ac^2\left(1+a^2x^2\right)} + \frac{\tan^{-1}(ax)^{5/2}}{4a^2c^2} - \frac{\tan^{-1}(ax)^{5/2}}{2a^2c^2\left(1+a^2x^2\right)} - \frac{15\sqrt{}}{}$$

Mathematica [C] time = 0.169337, size = 234, normalized size = 1.5

$$\frac{15i\sqrt{2}\left(a^2x^2+1\right)\sqrt{-i\tan^{-1}(ax)}\text{Gamma}\left(\frac{1}{2},-2i\tan^{-1}(ax)\right) - 15i\sqrt{2}a^2x^2\sqrt{i\tan^{-1}(ax)}\text{Gamma}\left(\frac{1}{2},2i\tan^{-1}(ax)\right) - 15i\sqrt{}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^2,x]
```

```
[Out] (240*ArcTan[a*x] - 240*a^2*x^2*ArcTan[a*x] + 640*a*x*ArcTan[a*x]^2 - 256*Ar
cTan[a*x]^3 + 256*a^2*x^2*ArcTan[a*x]^3 - 60*Sqrt[Pi]*(1 + a^2*x^2)*Sqrt[Ar
```

```
cTan[a*x]]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]] + (15*I)*Sqrt[2]*(1 + a
^2*x^2)*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-2*I)*ArcTan[a*x]] - (15*I)*Sqrt
[2]*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (2*I)*ArcTan[a*x]] - (15*I)*Sqrt[2]*a^2*
x^2*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (2*I)*ArcTan[a*x]])/(1024*a^2*c^2*(1 + a
^2*x^2)*Sqrt[ArcTan[a*x]])
```

Maple [A] time = 0.103, size = 88, normalized size = 0.6

$$-\frac{\cos(2\arctan(ax))}{4a^2c^2}(\arctan(ax))^{\frac{5}{2}}+\frac{5\sin(2\arctan(ax))}{16a^2c^2}(\arctan(ax))^{\frac{3}{2}}+\frac{15\cos(2\arctan(ax))}{64a^2c^2}\sqrt{\arctan(ax)}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^2,x)
```

```
[Out] -1/4/a^2/c^2*arctan(a*x)^(5/2)*cos(2*arctan(a*x))+5/16/a^2/c^2*arctan(a*x)^
(3/2)*sin(2*arctan(a*x))+15/64/a^2/c^2*arctan(a*x)^(1/2)*cos(2*arctan(a*x))
-15/128*FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2))*Pi^(1/2)/a^2/c^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F]   time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x \operatorname{atan}^{\frac{5}{2}}(ax)}{a^4x^4+2a^2x^2+1}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(a*x)**(5/2)/(a**2*c*x**2+c)**2,x)
```

```
[Out] Integral(x*atan(a*x)**(5/2)/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

Giac [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{x \arctan(ax)^{\frac{5}{2}}}{\left(a^2cx^2+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(x*arctan(a*x)^(5/2)/(a^2*c*x^2 + c)^2, x)
```

3.868 $\int \frac{\tan^{-1}(ax)^{5/2}}{\left(c+a^2cx^2\right)^2} dx$

Optimal. Leaf size=151

$$\frac{x\tan^{-1}(ax)^{5/2}}{2c^2\left(a^2x^2+1\right)}+\frac{5\tan^{-1}(ax)^{3/2}}{8ac^2\left(a^2x^2+1\right)}-\frac{15x\sqrt{\tan^{-1}(ax)}}{32c^2\left(a^2x^2+1\right)}+\frac{15\sqrt{\pi}S\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{128ac^2}+\frac{\tan^{-1}(ax)^{7/2}}{7ac^2}-\frac{5\tan^{-1}(ax)^{3/2}}{16ac^2}$$

```
[Out] (-15*x*Sqrt[ArcTan[a*x]])/(32*c^2*(1 + a^2*x^2)) - (5*ArcTan[a*x]^(3/2))/(1
6*a*c^2) + (5*ArcTan[a*x]^(3/2))/(8*a*c^2*(1 + a^2*x^2)) + (x*ArcTan[a*x]^(
5/2))/(2*c^2*(1 + a^2*x^2)) + ArcTan[a*x]^(7/2)/(7*a*c^2) + (15*Sqrt[Pi]*Fr
esnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(128*a*c^2)
```

Rubi [A] time = 0.183928, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4892, 4930, 4970, 4406, 12, 3305, 3351}

$$\frac{x\tan^{-1}(ax)^{5/2}}{2c^2\left(a^2x^2+1\right)}+\frac{5\tan^{-1}(ax)^{3/2}}{8ac^2\left(a^2x^2+1\right)}-\frac{15x\sqrt{\tan^{-1}(ax)}}{32c^2\left(a^2x^2+1\right)}+\frac{15\sqrt{\pi}S\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{128ac^2}+\frac{\tan^{-1}(ax)^{7/2}}{7ac^2}-\frac{5\tan^{-1}(ax)^{3/2}}{16ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^(5/2)/(c + a^2*c*x^2)^2,x]
```

```
[Out] (-15*x*Sqrt[ArcTan[a*x]])/(32*c^2*(1 + a^2*x^2)) - (5*ArcTan[a*x]^(3/2))/(1
6*a*c^2) + (5*ArcTan[a*x]^(3/2))/(8*a*c^2*(1 + a^2*x^2)) + (x*ArcTan[a*x]^(
5/2))/(2*c^2*(1 + a^2*x^2)) + ArcTan[a*x]^(7/2)/(7*a*c^2) + (15*Sqrt[Pi]*Fr
esnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(128*a*c^2)
```

Rule 4892

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[(x*(a + b*ArcTan[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*c*
p)/2, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(a +
 b*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 4970

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sin[x]^m)/
Cos[x]^(m + 2*(q + 1)), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^{5/2}}{\left(c+a^2cx^2\right)^2}\,dx = \frac{x\tan^{-1}(ax)^{5/2}}{2c^2\left(1+a^2x^2\right)} + \frac{\tan^{-1}(ax)^{7/2}}{7ac^2} - \frac{1}{4}(5a)\int \frac{x\tan^{-1}(ax)^{3/2}}{\left(c+a^2cx^2\right)^2}\,dx$$

$$= \frac{5\tan^{-1}(ax)^{3/2}}{8ac^2\left(1+a^2x^2\right)} + \frac{x\tan^{-1}(ax)^{5/2}}{2c^2\left(1+a^2x^2\right)} + \frac{\tan^{-1}(ax)^{7/2}}{7ac^2} - \frac{15}{16}\int \frac{\sqrt{\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^2}\,dx$$

$$= -\frac{15x\sqrt{\tan^{-1}(ax)}}{32c^2\left(1+a^2x^2\right)} - \frac{5\tan^{-1}(ax)^{3/2}}{16ac^2} + \frac{5\tan^{-1}(ax)^{3/2}}{8ac^2\left(1+a^2x^2\right)} + \frac{x\tan^{-1}(ax)^{5/2}}{2c^2\left(1+a^2x^2\right)} + \frac{\tan^{-1}(ax)^{7/2}}{7ac^2} + \frac{1}{64}(15a)$$

$$= -\frac{15x\sqrt{\tan^{-1}(ax)}}{32c^2\left(1+a^2x^2\right)} - \frac{5\tan^{-1}(ax)^{3/2}}{16ac^2} + \frac{5\tan^{-1}(ax)^{3/2}}{8ac^2\left(1+a^2x^2\right)} + \frac{x\tan^{-1}(ax)^{5/2}}{2c^2\left(1+a^2x^2\right)} + \frac{\tan^{-1}(ax)^{7/2}}{7ac^2} + \frac{15\,\mathrm{Subs}}{}$$

$$= -\frac{15x\sqrt{\tan^{-1}(ax)}}{32c^2\left(1+a^2x^2\right)} - \frac{5\tan^{-1}(ax)^{3/2}}{16ac^2} + \frac{5\tan^{-1}(ax)^{3/2}}{8ac^2\left(1+a^2x^2\right)} + \frac{x\tan^{-1}(ax)^{5/2}}{2c^2\left(1+a^2x^2\right)} + \frac{\tan^{-1}(ax)^{7/2}}{7ac^2} + \frac{15\,\mathrm{Subs}}{}$$

$$= -\frac{15x\sqrt{\tan^{-1}(ax)}}{32c^2\left(1+a^2x^2\right)} - \frac{5\tan^{-1}(ax)^{3/2}}{16ac^2} + \frac{5\tan^{-1}(ax)^{3/2}}{8ac^2\left(1+a^2x^2\right)} + \frac{x\tan^{-1}(ax)^{5/2}}{2c^2\left(1+a^2x^2\right)} + \frac{\tan^{-1}(ax)^{7/2}}{7ac^2} + \frac{15\,\mathrm{Subs}}{}$$

$$= -\frac{15x\sqrt{\tan^{-1}(ax)}}{32c^2\left(1+a^2x^2\right)} - \frac{5\tan^{-1}(ax)^{3/2}}{16ac^2} + \frac{5\tan^{-1}(ax)^{3/2}}{8ac^2\left(1+a^2x^2\right)} + \frac{x\tan^{-1}(ax)^{5/2}}{2c^2\left(1+a^2x^2\right)} + \frac{\tan^{-1}(ax)^{7/2}}{7ac^2} + \frac{15\,\mathrm{Subs}}{}$$

$$= -\frac{15x\sqrt{\tan^{-1}(ax)}}{32c^2\left(1+a^2x^2\right)} - \frac{5\tan^{-1}(ax)^{3/2}}{16ac^2} + \frac{5\tan^{-1}(ax)^{3/2}}{8ac^2\left(1+a^2x^2\right)} + \frac{x\tan^{-1}(ax)^{5/2}}{2c^2\left(1+a^2x^2\right)} + \frac{\tan^{-1}(ax)^{7/2}}{7ac^2} + \frac{15\sqrt{\pi}S}{1}$$

Mathematica [A] time = 0.266976, size = 85, normalized size = 0.56

$$\frac{105\sqrt{\pi}S\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right) + 2\sqrt{\tan^{-1}(ax)}\left(64\tan^{-1}(ax)^3 + 7\left(16\tan^{-1}(ax)^2 - 15\right)\sin\left(2\tan^{-1}(ax)\right) + 140\tan^{-1}(ax)\cos\left(2\right.\right.}{896ac^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]^(5/2)/(c + a^2*c*x^2)^2,x]
```

```
[Out] (105*Sqrt[Pi]*FresnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]] + 2*Sqrt[ArcTan[a*x]
]*(64*ArcTan[a*x]^3 + 140*ArcTan[a*x]*Cos[2*ArcTan[a*x]] + 7*(-15 + 16*ArcT
an[a*x]^2)*Sin[2*ArcTan[a*x]]))/(896*a*c^2)
```

Maple [A] time = 0.109, size = 102, normalized size = 0.7

$$\frac{1}{7\,ac^{2}}\left(\arctan\left(ax\right)\right)^{\frac{7}{2}}+\frac{\sin\left(2\,\arctan\left(ax\right)\right)}{4\,ac^{2}}\left(\arctan\left(ax\right)\right)^{\frac{5}{2}}+\frac{5\,\cos\left(2\,\arctan\left(ax\right)\right)}{16\,ac^{2}}\left(\arctan\left(ax\right)\right)^{\frac{3}{2}}-\frac{15\,\sin\left(2\,\arctan\left(a\right.\right.}{64\,ac^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(5/2)/(a^2*c*x^2+c)^2,x)
```

```
[Out] 1/7*arctan(a*x)^(7/2)/a/c^2+1/4/a/c^2*arctan(a*x)^(5/2)*sin(2*arctan(a*x))+
5/16/a/c^2*arctan(a*x)^(3/2)*cos(2*arctan(a*x))-15/64/a/c^2*arctan(a*x)^(1/
2)*sin(2*arctan(a*x))+15/128*FresnelS(2*arctan(a*x)^(1/2)/Pi^(1/2))*Pi^(1/2
)/a/c^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\operatorname{atan}^{\frac{5}{2}}(ax)}{a^4x^4+2a^2x^2+1}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(5/2)/(a**2*c*x**2+c)**2,x)
```

```
[Out] Integral(atan(a*x)**(5/2)/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(ax)^{\frac{5}{2}}}{\left(a^2cx^2+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)^(5/2)/(a^2*c*x^2 + c)^2, x)
```

3.869 $\int \frac{\tan^{-1}(ax)^{5/2}}{x(c+a^2cx^2)^2} dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{\tan^{-1}(ax)^{5/2}}{x\left(a^2cx^2+c\right)^2},x\right)$$

```
[Out] Unintegrable[ArcTan[a*x]^(5/2)/(x*(c + a^2*c*x^2)^2), x]
```

Rubi [A] time = 0.0590948, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\tan^{-1}(ax)^{5/2}}{x\left(c+a^2cx^2\right)^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcTan[a*x]^(5/2)/(x*(c + a^2*c*x^2)^2),x]
```

```
[Out] Defer[Int][ArcTan[a*x]^(5/2)/(x*(c + a^2*c*x^2)^2), x]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^{5/2}}{x\left(c+a^2cx^2\right)^2} dx = \int \frac{\tan^{-1}(ax)^{5/2}}{x\left(c+a^2cx^2\right)^2} dx$$

Mathematica [A] time = 1.9198, size = 0, normalized size = 0.

$$\int \frac{\tan^{-1}(ax)^{5/2}}{x\left(c+a^2cx^2\right)^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcTan[a*x]^(5/2)/(x*(c + a^2*c*x^2)^2),x]
```

```
[Out] Integrate[ArcTan[a*x]^(5/2)/(x*(c + a^2*c*x^2)^2), x]
```

Maple [A] time = 0.526, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(a^{2} c\,x^{2}+c\right)^{2}} \left(\arctan\left(a x\right)\right)^{\frac{5}{2}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^2,x)
```

```
[Out] int(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\operatorname{atan}^{\frac{5}{2}}(ax)}{a^4x^5+2a^2x^3+x}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(5/2)/x/(a**2*c*x**2+c)**2,x)
```

```
[Out] Integral(atan(a*x)**(5/2)/(a**4*x**5 + 2*a**2*x**3 + x), x)/c**2
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(ax)^{\frac{5}{2}}}{\left(a^2cx^2+c\right)^2 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)^(5/2)/((a^2*c*x^2 + c)^2*x), x)
```

3.870 $\int \frac{x^m \tan^{-1}(ax)^{5/2}}{\left(c+a^2cx^2\right)^3} dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{x^m \tan^{-1}(ax)^{5/2}}{\left(a^2cx^2+c\right)^3},x\right)$$

```
[Out] Unintegrable[(x^m*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^3, x]
```

Rubi [A] time = 0.0631842, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m \tan^{-1}(ax)^{5/2}}{\left(c+a^2cx^2\right)^3} dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^3,x]
```

```
[Out] Defer[Int][(x^m*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^3, x]
```

Rubi steps

$$\int \frac{x^m \tan^{-1}(ax)^{5/2}}{\left(c+a^2cx^2\right)^3} dx = \int \frac{x^m \tan^{-1}(ax)^{5/2}}{\left(c+a^2cx^2\right)^3} dx$$

Mathematica [A] time = 1.7485, size = 0, normalized size = 0.

$$\int \frac{x^m \tan^{-1}(ax)^{5/2}}{\left(c+a^2cx^2\right)^3} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^3,x]
```

```
[Out] Integrate[(x^m*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^3, x]
```

Maple [A] time = 0.805, size = 0, normalized size = 0.

$$\int \frac{x^{m}}{\left(a^{2} c x^{2}+c\right)^{3}} \left(\arctan \left(a x\right)\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x)
```

```
[Out] int(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x^{m} \arctan \left(a x\right)^{\frac{5}{2}}}{a^{6} c^{3} x^{6} + 3 \, a^{4} c^{3} x^{4} + 3 \, a^{2} c^{3} x^{2} + c^{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] integral(x^m*arctan(a*x)^(5/2)/(a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2
 + c^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*atan(a*x)**(5/2)/(a**2*c*x**2+c)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m \arctan(ax)^{\frac{5}{2}}}{(a^2cx^2 + c)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] integrate(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2 + c)^3, x)
```

3.871 $\int \frac{x^5 \tan^{-1}(ax)^{5/2}}{\left(c+a^2cx^2\right)^3} dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{x^5 \tan^{-1}(ax)^{5/2}}{\left(a^2cx^2+c\right)^3},x\right)$$

```
[Out] Unintegrable[(x^5*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^3, x]
```

Rubi [A] time = 0.0615847, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^5 \tan^{-1}(ax)^{5/2}}{\left(c+a^2cx^2\right)^3} dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^5*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^3,x]
```

```
[Out] Defer[Int][(x^5*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^3, x]
```

Rubi steps

$$\int \frac{x^5 \tan^{-1}(ax)^{5/2}}{\left(c+a^2cx^2\right)^3} dx = \int \frac{x^5 \tan^{-1}(ax)^{5/2}}{\left(c+a^2cx^2\right)^3} dx$$

Mathematica [A] time = 8.17172, size = 0, normalized size = 0.

$$\int \frac{x^5 \tan^{-1}(ax)^{5/2}}{\left(c+a^2cx^2\right)^3} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^5*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^3,x]
```

```
[Out] Integrate[(x^5*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^3, x]
```

Maple [A] time = 0.566, size = 0, normalized size = 0.

$$\int \frac{x^5}{\left(a^2cx^2+c\right)^3} \left(\arctan \left(ax\right)\right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x)
```

```
[Out] int(x^5*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*atan(a*x)**(5/2)/(a**2*c*x**2+c)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5 \arctan(ax)^{\frac{5}{2}}}{(a^2cx^2+c)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] integrate(x^5*arctan(a*x)^(5/2)/(a^2*c*x^2 + c)^3, x)
```

3.872 $\int \frac{x^4 \tan^{-1}(ax)^{5/2}}{(c+a^2cx^2)^3} dx$

Optimal. Leaf size=310

$$\frac{15\sqrt{\frac{\pi}{2}}S\left(2\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{4096a^5c^3}-\frac{15\sqrt{\pi}S\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{128a^5c^3}+\frac{5x^4\tan^{-1}(ax)^{3/2}}{32ac^3\left(a^2x^2+1\right)^2}-\frac{x^3\tan^{-1}(ax)^{5/2}}{4a^2c^3\left(a^2x^2+1\right)^2}-\frac{3x\tan^{-1}(ax)^{5/2}}{8a^4c^3\left(a^2x^2+1\right)}+\frac{45}{128}$$

```
[Out] (45*x*Sqrt[ArcTan[a*x]])/(128*a^4*c^3*(1 + a^2*x^2)) + (45*ArcTan[a*x]^(3/2
))/(256*a^5*c^3) + (5*x^4*ArcTan[a*x]^(3/2))/(32*a*c^3*(1 + a^2*x^2)^2) - (
15*ArcTan[a*x]^(3/2))/(32*a^5*c^3*(1 + a^2*x^2)) - (x^3*ArcTan[a*x]^(5/2))/
(4*a^2*c^3*(1 + a^2*x^2)^2) - (3*x*ArcTan[a*x]^(5/2))/(8*a^4*c^3*(1 + a^2*x
^2)) + (3*ArcTan[a*x]^(7/2))/(28*a^5*c^3) + (15*Sqrt[Pi/2]*FresnelS[2*Sqrt[
2/Pi]*Sqrt[ArcTan[a*x]]])/(4096*a^5*c^3) - (15*Sqrt[Pi]*FresnelS[(2*Sqrt[Ar
cTan[a*x]])/Sqrt[Pi]])/(128*a^5*c^3) + (15*Sqrt[ArcTan[a*x]]*Sin[2*ArcTan[a
*x]])/(256*a^5*c^3) - (15*Sqrt[ArcTan[a*x]]*Sin[4*ArcTan[a*x]])/(2048*a^5*c
^3)
```

Rubi [A] time = 0.486393, antiderivative size = 310, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {4940, 4936, 4930, 4892, 4970, 4406, 12, 3305, 3351, 3312, 3296}

$$\frac{15\sqrt{\frac{\pi}{2}}S\left(2\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{4096a^5c^3}-\frac{15\sqrt{\pi}S\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{128a^5c^3}+\frac{5x^4\tan^{-1}(ax)^{3/2}}{32ac^3\left(a^2x^2+1\right)^2}-\frac{x^3\tan^{-1}(ax)^{5/2}}{4a^2c^3\left(a^2x^2+1\right)^2}-\frac{3x\tan^{-1}(ax)^{5/2}}{8a^4c^3\left(a^2x^2+1\right)}+\frac{45}{128}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^3,x]
```


```
[Out] (45*x*Sqrt[ArcTan[a*x]])/(128*a^4*c^3*(1 + a^2*x^2)) + (45*ArcTan[a*x]^(3/2
))/(256*a^5*c^3) + (5*x^4*ArcTan[a*x]^(3/2))/(32*a*c^3*(1 + a^2*x^2)^2) - (
15*ArcTan[a*x]^(3/2))/(32*a^5*c^3*(1 + a^2*x^2)) - (x^3*ArcTan[a*x]^(5/2))/
(4*a^2*c^3*(1 + a^2*x^2)^2) - (3*x*ArcTan[a*x]^(5/2))/(8*a^4*c^3*(1 + a^2*x
^2)) + (3*ArcTan[a*x]^(7/2))/(28*a^5*c^3) + (15*Sqrt[Pi/2]*FresnelS[2*Sqrt[
2/Pi]*Sqrt[ArcTan[a*x]]])/(4096*a^5*c^3) - (15*Sqrt[Pi]*FresnelS[(2*Sqrt[Ar
cTan[a*x]])/Sqrt[Pi]])/(128*a^5*c^3) + (15*Sqrt[ArcTan[a*x]]*Sin[2*ArcTan[a
*x]])/(256*a^5*c^3) - (15*Sqrt[ArcTan[a*x]]*Sin[4*ArcTan[a*x]])/(2048*a^5*c
```

```
^3)
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_)*((d_) + (e_.)
*(x_)^2)^(q_), x_Symbol] :> Simp[(b*p*(f*x)^m*(d + e*x^2)^(q + 1)*(a + b*Ar
cTan[c*x])^(p - 1))/(c*d*m^2), x] + (Dist[(f^2*(m - 1))/(c^2*d*m), Int[(f*x
)^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p, x], x] - Dist[(b^2*p*(
p - 1))/m^2, Int[(f*x)^m*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x]
- Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(c^2*d*m
), x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q +
 2, 0] && LtQ[q, -1] && GtQ[p, 1]
```

Rule 4936

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^2)/((d_) + (e_.)*(x_)^2)
^2, x_Symbol] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(2*b*c^3*d^2*(p + 1)), x]
 + (Dist[(b*p)/(2*c), Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(d + e*x^2)^2, x]
, x] - Simp[(x*(a + b*ArcTan[c*x])^p)/(2*c^2*d*(d + e*x^2)), x]) /; FreeQ[{
a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 4892

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[(x*(a + b*ArcTan[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*c*
p)/2, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(a +
 b*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4970

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sin[x]^m)/
Cos[x]^(m + 2*(q + 1)), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^4 \tan^{-1}(ax)^{5/2}}{\left(c+a^2cx^2\right)^3}\,dx = \frac{5x^4\tan^{-1}(ax)^{3/2}}{32ac^3\left(1+a^2x^2\right)^2} - \frac{x^3\tan^{-1}(ax)^{5/2}}{4a^2c^3\left(1+a^2x^2\right)^2} - \frac{15}{64}\int \frac{x^4\sqrt{\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^3}\,dx + \frac{3\int \frac{x^2\tan^{-1}(ax)^{5/2}}{\left(c+a^2cx^2\right)^2}\,dx}{4a^2c}$$

$$= \frac{5x^4\tan^{-1}(ax)^{3/2}}{32ac^3\left(1+a^2x^2\right)^2} - \frac{x^3\tan^{-1}(ax)^{5/2}}{4a^2c^3\left(1+a^2x^2\right)^2} - \frac{3x\tan^{-1}(ax)^{5/2}}{8a^4c^3\left(1+a^2x^2\right)} + \frac{3\tan^{-1}(ax)^{7/2}}{28a^5c^3} - \frac{15\,\text{Subst}\left(\int \sqrt{x}\,\text{si}\right.}{6}$$

$$= \frac{5x^4\tan^{-1}(ax)^{3/2}}{32ac^3\left(1+a^2x^2\right)^2} - \frac{15\tan^{-1}(ax)^{3/2}}{32a^5c^3\left(1+a^2x^2\right)} - \frac{x^3\tan^{-1}(ax)^{5/2}}{4a^2c^3\left(1+a^2x^2\right)^2} - \frac{3x\tan^{-1}(ax)^{5/2}}{8a^4c^3\left(1+a^2x^2\right)} + \frac{3\tan^{-1}(ax)^{7/2}}{28a^5c^3}$$

$$= \frac{45x\sqrt{\tan^{-1}(ax)}}{128a^4c^3\left(1+a^2x^2\right)} + \frac{45\tan^{-1}(ax)^{3/2}}{256a^5c^3} + \frac{5x^4\tan^{-1}(ax)^{3/2}}{32ac^3\left(1+a^2x^2\right)^2} - \frac{15\tan^{-1}(ax)^{3/2}}{32a^5c^3\left(1+a^2x^2\right)} - \frac{x^3\tan^{-1}(ax)^{5}}{4a^2c^3\left(1+a^2x\right.}$$

$$= \frac{45x\sqrt{\tan^{-1}(ax)}}{128a^4c^3\left(1+a^2x^2\right)} + \frac{45\tan^{-1}(ax)^{3/2}}{256a^5c^3} + \frac{5x^4\tan^{-1}(ax)^{3/2}}{32ac^3\left(1+a^2x^2\right)^2} - \frac{15\tan^{-1}(ax)^{3/2}}{32a^5c^3\left(1+a^2x^2\right)} - \frac{x^3\tan^{-1}(ax)^{5}}{4a^2c^3\left(1+a^2x\right.}$$

$$= \frac{45x\sqrt{\tan^{-1}(ax)}}{128a^4c^3\left(1+a^2x^2\right)} + \frac{45\tan^{-1}(ax)^{3/2}}{256a^5c^3} + \frac{5x^4\tan^{-1}(ax)^{3/2}}{32ac^3\left(1+a^2x^2\right)^2} - \frac{15\tan^{-1}(ax)^{3/2}}{32a^5c^3\left(1+a^2x^2\right)} - \frac{x^3\tan^{-1}(ax)^{5}}{4a^2c^3\left(1+a^2x\right.}$$

$$= \frac{45x\sqrt{\tan^{-1}(ax)}}{128a^4c^3\left(1+a^2x^2\right)} + \frac{45\tan^{-1}(ax)^{3/2}}{256a^5c^3} + \frac{5x^4\tan^{-1}(ax)^{3/2}}{32ac^3\left(1+a^2x^2\right)^2} - \frac{15\tan^{-1}(ax)^{3/2}}{32a^5c^3\left(1+a^2x^2\right)} - \frac{x^3\tan^{-1}(ax)^{5}}{4a^2c^3\left(1+a^2x\right.}$$

$$= \frac{45x\sqrt{\tan^{-1}(ax)}}{128a^4c^3\left(1+a^2x^2\right)} + \frac{45\tan^{-1}(ax)^{3/2}}{256a^5c^3} + \frac{5x^4\tan^{-1}(ax)^{3/2}}{32ac^3\left(1+a^2x^2\right)^2} - \frac{15\tan^{-1}(ax)^{3/2}}{32a^5c^3\left(1+a^2x^2\right)} - \frac{x^3\tan^{-1}(ax)^{5}}{4a^2c^3\left(1+a^2x\right.}$$

$$= \frac{45x\sqrt{\tan^{-1}(ax)}}{128a^4c^3\left(1+a^2x^2\right)} + \frac{45\tan^{-1}(ax)^{3/2}}{256a^5c^3} + \frac{5x^4\tan^{-1}(ax)^{3/2}}{32ac^3\left(1+a^2x^2\right)^2} - \frac{15\tan^{-1}(ax)^{3/2}}{32a^5c^3\left(1+a^2x^2\right)} - \frac{x^3\tan^{-1}(ax)^{5}}{4a^2c^3\left(1+a^2x\right.}$$

Mathematica [C] time = 0.598959, size = 287, normalized size = 0.93

$$3360\sqrt{2}\left(a^2x^2+1\right)^2\sqrt{-i\tan^{-1}(ax)}\,\text{Gamma}\left(\frac{1}{2},-2i\tan^{-1}(ax)\right) + 3360\sqrt{2}\left(a^2x^2+1\right)^2\sqrt{i\tan^{-1}(ax)}\,\text{Gamma}\left(\frac{1}{2},2i\tan^{-1}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^3,x]
```

```
[Out] (50400*a*x*ArcTan[a*x] + 57120*a^3*x^3*ArcTan[a*x] - 33600*ArcTan[a*x]^2 -
13440*a^2*x^2*ArcTan[a*x]^2 + 38080*a^4*x^4*ArcTan[a*x]^2 - 43008*a*x*ArcTa
n[a*x]^3 - 71680*a^3*x^3*ArcTan[a*x]^3 + 12288*(1 + a^2*x^2)^2*ArcTan[a*x]^
4 + 3360*Sqrt[2]*(1 + a^2*x^2)^2*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-2*I)*A
rcTan[a*x]] + 3360*Sqrt[2]*(1 + a^2*x^2)^2*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (
2*I)*ArcTan[a*x]] - 105*(1 + a^2*x^2)^2*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (
-4*I)*ArcTan[a*x]] - 105*(1 + a^2*x^2)^2*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (4*
I)*ArcTan[a*x]])/(114688*a^5*c^3*(1 + a^2*x^2)^2*Sqrt[ArcTan[a*x]])
```

Maple [A] time = 0.122, size = 194, normalized size = 0.6

$$\frac{3}{28\,c^3a^5}\left(\arctan\left(ax\right)\right)^{\frac{7}{2}}-\frac{\sin\left(2\,\arctan\left(ax\right)\right)}{4\,c^3a^5}\left(\arctan\left(ax\right)\right)^{\frac{5}{2}}+\frac{\sin\left(4\,\arctan\left(ax\right)\right)}{32\,c^3a^5}\left(\arctan\left(ax\right)\right)^{\frac{5}{2}}-\frac{5\,\cos\left(2\,\arctan\left(a\right.\right.}{16\,c^3a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x)
```

```
[Out] 3/28*arctan(a*x)^(7/2)/a^5/c^3-1/4/a^5/c^3*arctan(a*x)^(5/2)*sin(2*arctan(a
*x))+1/32/a^5/c^3*arctan(a*x)^(5/2)*sin(4*arctan(a*x))-5/16/a^5/c^3*arctan(
a*x)^(3/2)*cos(2*arctan(a*x))+5/256/a^5/c^3*arctan(a*x)^(3/2)*cos(4*arctan(
a*x))+15/64*sin(2*arctan(a*x))*arctan(a*x)^(1/2)/a^5/c^3-15/2048*sin(4*arct
an(a*x))*arctan(a*x)^(1/2)/a^5/c^3+15/8192*FresnelS(2*2^(1/2)/Pi^(1/2)*arct
an(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^5/c^3-15/128*FresnelS(2*arctan(a*x)^(1/2)
/Pi^(1/2))*Pi^(1/2)/a^5/c^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*atan(a*x)**(5/2)/(a**2*c*x**2+c)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4 \arctan(ax)^{\frac{5}{2}}}{\left(a^2cx^2+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] integrate(x^4*arctan(a*x)^(5/2)/(a^2*c*x^2 + c)^3, x)
```

3.873 $\int \frac{x^3 \tan^{-1}(ax)^{5/2}}{(c+a^2cx^2)^3} \, dx$

Optimal. Leaf size=256

$$\frac{15\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{4096a^4c^3}-\frac{15\sqrt{\pi}\text{FresnelC}\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{256a^4c^3}+\frac{x^4\tan^{-1}(ax)^{5/2}}{4c^3(a^2x^2+1)^2}-\frac{15x^4\sqrt{\tan^{-1}(ax)}}{256c^3(a^2x^2+1)^2}+\frac{5x^3\tan}{32ac^3(a}$$

```
[Out] (-135*Sqrt[ArcTan[a*x]])/(2048*a^4*c^3) - (15*x^4*Sqrt[ArcTan[a*x]])/(256*c
^3*(1 + a^2*x^2)^2) + (45*Sqrt[ArcTan[a*x]])/(256*a^4*c^3*(1 + a^2*x^2)) +
(5*x^3*ArcTan[a*x]^(3/2))/(32*a*c^3*(1 + a^2*x^2)^2) + (15*x*ArcTan[a*x]^(3
/2))/(64*a^3*c^3*(1 + a^2*x^2)) - (3*ArcTan[a*x]^(5/2))/(32*a^4*c^3) + (x^4
*ArcTan[a*x]^(5/2))/(4*c^3*(1 + a^2*x^2)^2) + (15*Sqrt[Pi/2]*FresnelC[2*Sqr
t[2/Pi]*Sqrt[ArcTan[a*x]]])/(4096*a^4*c^3) - (15*Sqrt[Pi]*FresnelC[(2*Sqrt[
ArcTan[a*x]])/Sqrt[Pi]])/(256*a^4*c^3)
```

Rubi [A] time = 0.491625, antiderivative size = 256, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4944, 4940, 4936, 4930, 4904, 3312, 3304, 3352, 4970}

$$\frac{15\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{4096a^4c^3}-\frac{15\sqrt{\pi}\text{FresnelC}\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{256a^4c^3}+\frac{x^4\tan^{-1}(ax)^{5/2}}{4c^3(a^2x^2+1)^2}-\frac{15x^4\sqrt{\tan^{-1}(ax)}}{256c^3(a^2x^2+1)^2}+\frac{5x^3\tan}{32ac^3(a}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^3,x]
```


```
[Out] (-135*Sqrt[ArcTan[a*x]])/(2048*a^4*c^3) - (15*x^4*Sqrt[ArcTan[a*x]])/(256*c
^3*(1 + a^2*x^2)^2) + (45*Sqrt[ArcTan[a*x]])/(256*a^4*c^3*(1 + a^2*x^2)) +
(5*x^3*ArcTan[a*x]^(3/2))/(32*a*c^3*(1 + a^2*x^2)^2) + (15*x*ArcTan[a*x]^(3
/2))/(64*a^3*c^3*(1 + a^2*x^2)) - (3*ArcTan[a*x]^(5/2))/(32*a^4*c^3) + (x^4
*ArcTan[a*x]^(5/2))/(4*c^3*(1 + a^2*x^2)^2) + (15*Sqrt[Pi/2]*FresnelC[2*Sqr
t[2/Pi]*Sqrt[ArcTan[a*x]]])/(4096*a^4*c^3) - (15*Sqrt[Pi]*FresnelC[(2*Sqrt[
ArcTan[a*x]])/Sqrt[Pi]])/(256*a^4*c^3)
```


Rule 4944

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a +
 b*ArcTan[c*x])^p)/(d*f*(m + 1)), x] - Dist[(b*c*p)/(f*(m + 1)), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_)*((d_) + (e_.)
*(x_)^2)^(q_), x_Symbol] :> Simp[(b*p*(f*x)^m*(d + e*x^2)^(q + 1)*(a + b*Ar
cTan[c*x])^(p - 1))/(c*d*m^2), x] + (Dist[(f^2*(m - 1))/(c^2*d*m), Int[(f*x
)^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p, x], x] - Dist[(b^2*p*(
p - 1))/m^2, Int[(f*x)^m*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x]
- Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(c^2*d*m
), x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q +
 2, 0] && LtQ[q, -1] && GtQ[p, 1]
```

Rule 4936

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^2)/((d_) + (e_.)*(x_)^2)
^2, x_Symbol] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(2*b*c^3*d^2*(p + 1)), x]
 + (Dist[(b*p)/(2*c), Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(d + e*x^2)^2, x]
, x] - Simp[(x*(a + b*ArcTan[c*x])^p)/(2*c^2*d*(d + e*x^2)), x]) /; FreeQ[{
a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 4904

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
```

```
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4970

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sin[x]^m)/
Cos[x]^(m + 2*(q + 1)), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x^3 \tan^{-1}(ax)^{5/2}}{\left(c+a^2cx^2\right)^3}\,dx = \frac{x^4\tan^{-1}(ax)^{5/2}}{4c^3\left(1+a^2x^2\right)^2} - \frac{1}{8}(5a)\int \frac{x^4\tan^{-1}(ax)^{3/2}}{\left(c+a^2cx^2\right)^3}\,dx$$

$$= -\frac{15x^4\sqrt{\tan^{-1}(ax)}}{256c^3\left(1+a^2x^2\right)^2} + \frac{5x^3\tan^{-1}(ax)^{3/2}}{32ac^3\left(1+a^2x^2\right)^2} + \frac{x^4\tan^{-1}(ax)^{5/2}}{4c^3\left(1+a^2x^2\right)^2} + \frac{1}{512}(15a)\int \frac{x^4}{\left(c+a^2cx^2\right)^3\sqrt{\tan^{-1}(}}$$

$$= -\frac{15x^4\sqrt{\tan^{-1}(ax)}}{256c^3\left(1+a^2x^2\right)^2} + \frac{5x^3\tan^{-1}(ax)^{3/2}}{32ac^3\left(1+a^2x^2\right)^2} + \frac{15x\tan^{-1}(ax)^{3/2}}{64a^3c^3\left(1+a^2x^2\right)} - \frac{3\tan^{-1}(ax)^{5/2}}{32a^4c^3} + \frac{x^4\tan^{-1}(ax)^{5/2}}{4c^3\left(1+a^2x^2\right)^2}$$

$$= -\frac{15x^4\sqrt{\tan^{-1}(ax)}}{256c^3\left(1+a^2x^2\right)^2} + \frac{45\sqrt{\tan^{-1}(ax)}}{256a^4c^3\left(1+a^2x^2\right)} + \frac{5x^3\tan^{-1}(ax)^{3/2}}{32ac^3\left(1+a^2x^2\right)^2} + \frac{15x\tan^{-1}(ax)^{3/2}}{64a^3c^3\left(1+a^2x^2\right)} - \frac{3\tan^{-1}(ax}{32a^4c^3}$$

$$= \frac{45\sqrt{\tan^{-1}(ax)}}{2048a^4c^3} - \frac{15x^4\sqrt{\tan^{-1}(ax)}}{256c^3\left(1+a^2x^2\right)^2} + \frac{45\sqrt{\tan^{-1}(ax)}}{256a^4c^3\left(1+a^2x^2\right)} + \frac{5x^3\tan^{-1}(ax)^{3/2}}{32ac^3\left(1+a^2x^2\right)^2} + \frac{15x\tan^{-1}(ax)}{64a^3c^3\left(1+a^2\right.}$$

$$= \frac{45\sqrt{\tan^{-1}(ax)}}{2048a^4c^3} - \frac{15x^4\sqrt{\tan^{-1}(ax)}}{256c^3\left(1+a^2x^2\right)^2} + \frac{45\sqrt{\tan^{-1}(ax)}}{256a^4c^3\left(1+a^2x^2\right)} + \frac{5x^3\tan^{-1}(ax)^{3/2}}{32ac^3\left(1+a^2x^2\right)^2} + \frac{15x\tan^{-1}(ax)}{64a^3c^3\left(1+a^2\right.}$$

$$= -\frac{135\sqrt{\tan^{-1}(ax)}}{2048a^4c^3} - \frac{15x^4\sqrt{\tan^{-1}(ax)}}{256c^3\left(1+a^2x^2\right)^2} + \frac{45\sqrt{\tan^{-1}(ax)}}{256a^4c^3\left(1+a^2x^2\right)} + \frac{5x^3\tan^{-1}(ax)^{3/2}}{32ac^3\left(1+a^2x^2\right)^2} + \frac{15x\tan^{-1}(}{64a^3c^3\left(1+\right.}$$

$$= -\frac{135\sqrt{\tan^{-1}(ax)}}{2048a^4c^3} - \frac{15x^4\sqrt{\tan^{-1}(ax)}}{256c^3\left(1+a^2x^2\right)^2} + \frac{45\sqrt{\tan^{-1}(ax)}}{256a^4c^3\left(1+a^2x^2\right)} + \frac{5x^3\tan^{-1}(ax)^{3/2}}{32ac^3\left(1+a^2x^2\right)^2} + \frac{15x\tan^{-1}(}{64a^3c^3\left(1+\right.}$$

$$= -\frac{135\sqrt{\tan^{-1}(ax)}}{2048a^4c^3} - \frac{15x^4\sqrt{\tan^{-1}(ax)}}{256c^3\left(1+a^2x^2\right)^2} + \frac{45\sqrt{\tan^{-1}(ax)}}{256a^4c^3\left(1+a^2x^2\right)} + \frac{5x^3\tan^{-1}(ax)^{3/2}}{32ac^3\left(1+a^2x^2\right)^2} + \frac{15x\tan^{-1}(}{64a^3c^3\left(1+\right.}$$

Mathematica [C] time = 0.701617, size = 359, normalized size = 1.4

$$\frac{510\sqrt{2\pi}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right) + \frac{900i\sqrt{2}\left(a^2x^2+1\right)^2\sqrt{-i\tan^{-1}(ax)}\,\text{Gamma}\left(\frac{1}{2},-2i\tan^{-1}(ax)\right) - 900i\sqrt{2}\left(a^2x^2+1\right)^2\sqrt{i\tan^{-1}(ax)}\,\text{Gamma}\left(\frac{1}{2},2\right.}{}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^3,x]
```

```
[Out] (510*Sqrt[2*Pi]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]] + (14400*ArcTan[a*
x] + 5760*a^2*x^2*ArcTan[a*x] - 16320*a^4*x^4*ArcTan[a*x] + 30720*a*x*ArcTa
n[a*x]^2 + 51200*a^3*x^3*ArcTan[a*x]^2 - 12288*ArcTan[a*x]^3 - 24576*a^2*x^
2*ArcTan[a*x]^3 + 20480*a^4*x^4*ArcTan[a*x]^3 - 4080*Sqrt[Pi]*(1 + a^2*x^2)
^2*Sqrt[ArcTan[a*x]]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]] + (900*I)*Sqr
t[2]*(1 + a^2*x^2)^2*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-2*I)*ArcTan[a*x]]
- (900*I)*Sqrt[2]*(1 + a^2*x^2)^2*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (2*I)*ArcT
an[a*x]] + (135*I)*(1 + a^2*x^2)^2*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-4*I)
*ArcTan[a*x]] - (135*I)*(1 + a^2*x^2)^2*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (4*I
)*ArcTan[a*x]])/((1 + a^2*x^2)^2*Sqrt[ArcTan[a*x]]))/(131072*a^4*c^3)
```

Maple [A] time = 0.12, size = 180, normalized size = 0.7

$$-\frac{\cos\left(2\arctan\left(ax\right)\right)}{8c^{3}a^{4}}\left(\arctan\left(ax\right)\right)^{\frac{5}{2}}+\frac{\cos\left(4\arctan\left(ax\right)\right)}{32c^{3}a^{4}}\left(\arctan\left(ax\right)\right)^{\frac{5}{2}}+\frac{5\sin\left(2\arctan\left(ax\right)\right)}{32c^{3}a^{4}}\left(\arctan\left(ax\right)\right)^{\frac{3}{2}}-\frac{5}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x)
```

```
[Out] -1/8/a^4/c^3*arctan(a*x)^(5/2)*cos(2*arctan(a*x))+1/32/a^4/c^3*arctan(a*x)^
(5/2)*cos(4*arctan(a*x))+5/32/a^4/c^3*arctan(a*x)^(3/2)*sin(2*arctan(a*x))-
5/256/a^4/c^3*arctan(a*x)^(3/2)*sin(4*arctan(a*x))+15/128/a^4/c^3*arctan(a*
x)^(1/2)*cos(2*arctan(a*x))-15/2048/a^4/c^3*arctan(a*x)^(1/2)*cos(4*arctan(
a*x))+15/8192*FresnelC(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*2^(1/2)*Pi^(1/
2)/a^4/c^3-15/256*FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2))*Pi^(1/2)/a^4/c^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atan(a*x)**(5/2)/(a**2*c*x**2+c)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \arctan(ax)^{\frac{5}{2}}}{\left(a^2cx^2+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] integrate(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2 + c)^3, x)
```

3.874 $\int \frac{x^2 \tan^{-1}(ax)^{5/2}}{(c+a^2cx^2)^3} dx$

Optimal. Leaf size=133

$$-\frac{15\sqrt{\frac{\pi}{2}}S\left(2\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{4096a^3c^3}+\frac{\tan^{-1}(ax)^{7/2}}{28a^3c^3}-\frac{\tan^{-1}(ax)^{5/2}\sin\left(4\tan^{-1}(ax)\right)}{32a^3c^3}+\frac{15\sqrt{\tan^{-1}(ax)}\sin\left(4\tan^{-1}(ax)\right)}{2048a^3c^3}-\frac{5\,\mathrm{t}}{}$$

```
[Out] ArcTan[a*x]^(7/2)/(28*a^3*c^3) - (5*ArcTan[a*x]^(3/2)*Cos[4*ArcTan[a*x]])/(
256*a^3*c^3) - (15*Sqrt[Pi/2]*FresnelS[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(40
96*a^3*c^3) + (15*Sqrt[ArcTan[a*x]]*Sin[4*ArcTan[a*x]])/(2048*a^3*c^3) - (A
rcTan[a*x]^(5/2)*Sin[4*ArcTan[a*x]])/(32*a^3*c^3)
```

Rubi [A] time = 0.175099, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {4970, 4406, 3296, 3305, 3351}

$$-\frac{15\sqrt{\frac{\pi}{2}}S\left(2\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{4096a^3c^3}+\frac{\tan^{-1}(ax)^{7/2}}{28a^3c^3}-\frac{\tan^{-1}(ax)^{5/2}\sin\left(4\tan^{-1}(ax)\right)}{32a^3c^3}+\frac{15\sqrt{\tan^{-1}(ax)}\sin\left(4\tan^{-1}(ax)\right)}{2048a^3c^3}-\frac{5\,\mathrm{t}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^3,x]
```

```
[Out] ArcTan[a*x]^(7/2)/(28*a^3*c^3) - (5*ArcTan[a*x]^(3/2)*Cos[4*ArcTan[a*x]])/(
256*a^3*c^3) - (15*Sqrt[Pi/2]*FresnelS[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(40
96*a^3*c^3) + (15*Sqrt[ArcTan[a*x]]*Sin[4*ArcTan[a*x]])/(2048*a^3*c^3) - (A
rcTan[a*x]^(5/2)*Sin[4*ArcTan[a*x]])/(32*a^3*c^3)
```

Rule 4970

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sin[x]^m)/
Cos[x]^(m + 2*(q + 1)), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{x^2 \tan^{-1}(ax)^{5/2}}{\left(c+a^2cx^2\right)^3}\,dx = \frac{\text{Subst}\left(\int x^{5/2}\cos^2(x)\sin^2(x)\,dx, x, \tan^{-1}(ax)\right)}{a^3c^3}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{x^{5/2}}{8} - \frac{1}{8}x^{5/2}\cos(4x)\right)dx, x, \tan^{-1}(ax)\right)}{a^3c^3}$$

$$= \frac{\tan^{-1}(ax)^{7/2}}{28a^3c^3} - \frac{\text{Subst}\left(\int x^{5/2}\cos(4x)\,dx, x, \tan^{-1}(ax)\right)}{8a^3c^3}$$

$$= \frac{\tan^{-1}(ax)^{7/2}}{28a^3c^3} - \frac{\tan^{-1}(ax)^{5/2}\sin\left(4\tan^{-1}(ax)\right)}{32a^3c^3} + \frac{5\,\text{Subst}\left(\int x^{3/2}\sin(4x)\,dx, x, \tan^{-1}(ax)\right)}{64a^3c^3}$$

$$= \frac{\tan^{-1}(ax)^{7/2}}{28a^3c^3} - \frac{5\tan^{-1}(ax)^{3/2}\cos\left(4\tan^{-1}(ax)\right)}{256a^3c^3} - \frac{\tan^{-1}(ax)^{5/2}\sin\left(4\tan^{-1}(ax)\right)}{32a^3c^3} + \frac{15\,\text{Subst}\left(\right.}{}$$

$$= \frac{\tan^{-1}(ax)^{7/2}}{28a^3c^3} - \frac{5\tan^{-1}(ax)^{3/2}\cos\left(4\tan^{-1}(ax)\right)}{256a^3c^3} + \frac{15\sqrt{\tan^{-1}(ax)}\sin\left(4\tan^{-1}(ax)\right)}{2048a^3c^3} - \frac{\tan^{-1}(a}{}$$

$$= \frac{\tan^{-1}(ax)^{7/2}}{28a^3c^3} - \frac{5\tan^{-1}(ax)^{3/2}\cos\left(4\tan^{-1}(ax)\right)}{256a^3c^3} + \frac{15\sqrt{\tan^{-1}(ax)}\sin\left(4\tan^{-1}(ax)\right)}{2048a^3c^3} - \frac{\tan^{-1}(a}{}$$

$$= \frac{\tan^{-1}(ax)^{7/2}}{28a^3c^3} - \frac{5\tan^{-1}(ax)^{3/2}\cos\left(4\tan^{-1}(ax)\right)}{256a^3c^3} - \frac{15\sqrt{\frac{\pi}{2}}S\left(2\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{4096a^3c^3} + \frac{15\sqrt{\tan^{-1}(a}}{2}$$

Mathematica [C] time = 0.439953, size = 185, normalized size = 1.39

$$\frac{105\left(a^2x^2+1\right)^2\sqrt{-i\tan^{-1}(ax)}\text{Gamma}\left(\frac{1}{2},-4i\tan^{-1}(ax)\right)+105\left(a^2x^2+1\right)^2\sqrt{i\tan^{-1}(ax)}\text{Gamma}\left(\frac{1}{2},4i\tan^{-1}(ax)\right)+}{114688a^3c^3\left(\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^3,x]
```

```
[Out] (32*ArcTan[a*x]*(-105*a*x*(-1 + a^2*x^2) - 70*(1 - 6*a^2*x^2 + a^4*x^4)*Arc
Tan[a*x] + 448*a*x*(-1 + a^2*x^2)*ArcTan[a*x]^2 + 128*(1 + a^2*x^2)^2*ArcTa
n[a*x]^3) + 105*(1 + a^2*x^2)^2*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-4*I)*Ar
cTan[a*x]] + 105*(1 + a^2*x^2)^2*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (4*I)*ArcTa
n[a*x]])/(114688*a^3*c^3*(1 + a^2*x^2)^2*Sqrt[ArcTan[a*x]])
```

Maple [A] time = 0.114, size = 96, normalized size = 0.7

$$\frac{1}{57344\,c^3a^3}\left(2048\,(\arctan(ax))^4-1792\,(\arctan(ax))^3\sin(4\arctan(ax))-105\sqrt{2}\sqrt{\arctan(ax)}\sqrt{\pi}\,\mathrm{FresnelS}\left(2\,\frac{\sqrt{2}\sqrt{\cdot}}{\cdot}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x)
```

```
[Out] 1/57344/a^3/c^3*(2048*arctan(a*x)^4-1792*arctan(a*x)^3*sin(4*arctan(a*x))-1
05*2^(1/2)*arctan(a*x)^(1/2)*Pi^(1/2)*FresnelS(2*2^(1/2)/Pi^(1/2)*arctan(a*
x)^(1/2))-1120*arctan(a*x)^2*cos(4*arctan(a*x))+420*sin(4*arctan(a*x))*arct
an(a*x))/arctan(a*x)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(a*x)**(5/2)/(a**2*c*x**2+c)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \arctan(ax)^{\frac{5}{2}}}{(a^2cx^2 + c)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] integrate(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2 + c)^3, x)
```

3.875 $\int \frac{x\tan^{-1}(ax)^{5/2}}{(c+a^2cx^2)^3}\,dx$

Optimal. Leaf size=254

$$-\frac{15\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{4096a^2c^3}-\frac{15\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{256a^2c^3}-\frac{\tan^{-1}(ax)^{5/2}}{4a^2c^3\left(a^2x^2+1\right)^2}+\frac{15x\tan^{-1}(ax)^{3/2}}{64ac^3\left(a^2x^2+1\right)}+\frac{5x\tan}{32ac^3\,(}$$

```
[Out] (-225*Sqrt[ArcTan[a*x]])/(2048*a^2*c^3) + (15*Sqrt[ArcTan[a*x]])/(256*a^2*c
^3*(1 + a^2*x^2)^2) + (45*Sqrt[ArcTan[a*x]])/(256*a^2*c^3*(1 + a^2*x^2)) +
(5*x*ArcTan[a*x]^(3/2))/(32*a*c^3*(1 + a^2*x^2)^2) + (15*x*ArcTan[a*x]^(3/2
))/(64*a*c^3*(1 + a^2*x^2)) + (3*ArcTan[a*x]^(5/2))/(32*a^2*c^3) - ArcTan[a
*x]^(5/2)/(4*a^2*c^3*(1 + a^2*x^2)^2) - (15*Sqrt[Pi/2]*FresnelC[2*Sqrt[2/Pi
]*Sqrt[ArcTan[a*x]]])/(4096*a^2*c^3) - (15*Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan
[a*x]])/Sqrt[Pi]])/(256*a^2*c^3)
```

Rubi [A] time = 0.337698, antiderivative size = 254, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {4930, 4900, 4892, 4904, 3312, 3304, 3352}

$$-\frac{15\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{4096a^2c^3}-\frac{15\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{256a^2c^3}-\frac{\tan^{-1}(ax)^{5/2}}{4a^2c^3\left(a^2x^2+1\right)^2}+\frac{15x\tan^{-1}(ax)^{3/2}}{64ac^3\left(a^2x^2+1\right)}+\frac{5x\tan}{32ac^3\,(}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^3,x]
```


```
[Out] (-225*Sqrt[ArcTan[a*x]])/(2048*a^2*c^3) + (15*Sqrt[ArcTan[a*x]])/(256*a^2*c
^3*(1 + a^2*x^2)^2) + (45*Sqrt[ArcTan[a*x]])/(256*a^2*c^3*(1 + a^2*x^2)) +
(5*x*ArcTan[a*x]^(3/2))/(32*a*c^3*(1 + a^2*x^2)^2) + (15*x*ArcTan[a*x]^(3/2
))/(64*a*c^3*(1 + a^2*x^2)) + (3*ArcTan[a*x]^(5/2))/(32*a^2*c^3) - ArcTan[a
*x]^(5/2)/(4*a^2*c^3*(1 + a^2*x^2)^2) - (15*Sqrt[Pi/2]*FresnelC[2*Sqrt[2/Pi
]*Sqrt[ArcTan[a*x]]])/(4096*a^2*c^3) - (15*Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan
[a*x]])/Sqrt[Pi]])/(256*a^2*c^3)
```


Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 4900

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(b*p*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^(p - 1))/(4*c*d
*(q + 1)^2), x] + (Dist[(2*q + 3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a
 + b*ArcTan[c*x])^p, x], x] - Dist[(b^2*p*(p - 1))/(4*(q + 1)^2), Int[(d +
e*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x] - Simp[(x*(d + e*x^2)^(q + 1)*
(a + b*ArcTan[c*x])^p)/(2*d*(q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] && E
qQ[e, c^2*d] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

Rule 4892

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[(x*(a + b*ArcTan[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*c*
p)/2, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(a +
 b*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4904

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{x\tan^{-1}(ax)^{5/2}}{\left(c+a^2cx^2\right)^3}\,dx = -\frac{\tan^{-1}(ax)^{5/2}}{4a^2c^3\left(1+a^2x^2\right)^2}+\frac{5\int\frac{\tan^{-1}(ax)^{3/2}}{\left(c+a^2cx^2\right)^3}\,dx}{8a}$$

$$= \frac{15\sqrt{\tan^{-1}(ax)}}{256a^2c^3\left(1+a^2x^2\right)^2}+\frac{5x\tan^{-1}(ax)^{3/2}}{32ac^3\left(1+a^2x^2\right)^2}-\frac{\tan^{-1}(ax)^{5/2}}{4a^2c^3\left(1+a^2x^2\right)^2}-\frac{15\int\frac{1}{\left(c+a^2cx^2\right)^3\sqrt{\tan^{-1}(ax)}}\,dx}{512a}+ 15\int$$

$$= \frac{15\sqrt{\tan^{-1}(ax)}}{256a^2c^3\left(1+a^2x^2\right)^2}+\frac{5x\tan^{-1}(ax)^{3/2}}{32ac^3\left(1+a^2x^2\right)^2}+\frac{15x\tan^{-1}(ax)^{3/2}}{64ac^3\left(1+a^2x^2\right)}+\frac{3\tan^{-1}(ax)^{5/2}}{32a^2c^3}-\frac{\tan^{-1}(ax)^{5/2}}{4a^2c^3\left(1+a^2x^2\right)}$$

$$= \frac{15\sqrt{\tan^{-1}(ax)}}{256a^2c^3\left(1+a^2x^2\right)^2}+\frac{45\sqrt{\tan^{-1}(ax)}}{256a^2c^3\left(1+a^2x^2\right)}+\frac{5x\tan^{-1}(ax)^{3/2}}{32ac^3\left(1+a^2x^2\right)^2}+\frac{15x\tan^{-1}(ax)^{3/2}}{64ac^3\left(1+a^2x^2\right)}+\frac{3\tan^{-1}(ax)}{32a^2c^3}$$

$$= -\frac{45\sqrt{\tan^{-1}(ax)}}{2048a^2c^3}+\frac{15\sqrt{\tan^{-1}(ax)}}{256a^2c^3\left(1+a^2x^2\right)^2}+\frac{45\sqrt{\tan^{-1}(ax)}}{256a^2c^3\left(1+a^2x^2\right)}+\frac{5x\tan^{-1}(ax)^{3/2}}{32ac^3\left(1+a^2x^2\right)^2}+\frac{15x\tan^{-1}(a}{64ac^3(1+a}$$

$$= -\frac{45\sqrt{\tan^{-1}(ax)}}{2048a^2c^3}+\frac{15\sqrt{\tan^{-1}(ax)}}{256a^2c^3\left(1+a^2x^2\right)^2}+\frac{45\sqrt{\tan^{-1}(ax)}}{256a^2c^3\left(1+a^2x^2\right)}+\frac{5x\tan^{-1}(ax)^{3/2}}{32ac^3\left(1+a^2x^2\right)^2}+\frac{15x\tan^{-1}(a}{64ac^3(1+a}$$

$$= -\frac{225\sqrt{\tan^{-1}(ax)}}{2048a^2c^3}+\frac{15\sqrt{\tan^{-1}(ax)}}{256a^2c^3\left(1+a^2x^2\right)^2}+\frac{45\sqrt{\tan^{-1}(ax)}}{256a^2c^3\left(1+a^2x^2\right)}+\frac{5x\tan^{-1}(ax)^{3/2}}{32ac^3\left(1+a^2x^2\right)^2}+\frac{15x\tan^{-1}(}{64ac^3(1+}$$

$$= -\frac{225\sqrt{\tan^{-1}(ax)}}{2048a^2c^3}+\frac{15\sqrt{\tan^{-1}(ax)}}{256a^2c^3\left(1+a^2x^2\right)^2}+\frac{45\sqrt{\tan^{-1}(ax)}}{256a^2c^3\left(1+a^2x^2\right)}+\frac{5x\tan^{-1}(ax)^{3/2}}{32ac^3\left(1+a^2x^2\right)^2}+\frac{15x\tan^{-1}(}{64ac^3(1+}$$

$$= -\frac{225\sqrt{\tan^{-1}(ax)}}{2048a^2c^3}+\frac{15\sqrt{\tan^{-1}(ax)}}{256a^2c^3\left(1+a^2x^2\right)^2}+\frac{45\sqrt{\tan^{-1}(ax)}}{256a^2c^3\left(1+a^2x^2\right)}+\frac{5x\tan^{-1}(ax)^{3/2}}{32ac^3\left(1+a^2x^2\right)^2}+\frac{15x\tan^{-1}(}{64ac^3(1+}$$

Mathematica [C] time = 0.650959, size = 359, normalized size = 1.41

$$450\sqrt{2\pi}\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)+\frac{1020i\sqrt{2}\left(a^2x^2+1\right)^2\sqrt{-i\tan^{-1}(ax)}\text{Gamma}\left(\frac{1}{2},-2i\tan^{-1}(ax)\right)-1020i\sqrt{2}\left(a^2x^2+1\right)^2\sqrt{i\tan^{-1}(ax)}\text{Gamma}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^3,x]
```

```
[Out] (450*Sqrt[2*Pi]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]] + (16320*ArcTan[a*
x] - 5760*a^2*x^2*ArcTan[a*x] - 14400*a^4*x^4*ArcTan[a*x] + 51200*a*x*ArcTa
n[a*x]^2 + 30720*a^3*x^3*ArcTan[a*x]^2 - 20480*ArcTan[a*x]^3 + 24576*a^2*x^
2*ArcTan[a*x]^3 + 12288*a^4*x^4*ArcTan[a*x]^3 - 3600*Sqrt[Pi]*(1 + a^2*x^2)
^2*Sqrt[ArcTan[a*x]]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]] + (1020*I)*Sq
rt[2]*(1 + a^2*x^2)^2*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-2*I)*ArcTan[a*x]]
 - (1020*I)*Sqrt[2]*(1 + a^2*x^2)^2*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (2*I)*Ar
cTan[a*x]] + (345*I)*(1 + a^2*x^2)^2*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-4*
I)*ArcTan[a*x]] - (345*I)*(1 + a^2*x^2)^2*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (4
*I)*ArcTan[a*x]])/((1 + a^2*x^2)^2*Sqrt[ArcTan[a*x]]))/(131072*a^2*c^3)
```

Maple [A] time = 0.133, size = 180, normalized size = 0.7

$$-\frac{\cos(2\arctan(ax))}{8c^3a^2}(\arctan(ax))^{\frac{5}{2}}-\frac{\cos(4\arctan(ax))}{32c^3a^2}(\arctan(ax))^{\frac{5}{2}}+\frac{5\sin(2\arctan(ax))}{32c^3a^2}(\arctan(ax))^{\frac{3}{2}}+\frac{5}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x)
```

```
[Out] -1/8/a^2/c^3*arctan(a*x)^(5/2)*cos(2*arctan(a*x))-1/32/a^2/c^3*arctan(a*x)^
(5/2)*cos(4*arctan(a*x))+5/32/a^2/c^3*arctan(a*x)^(3/2)*sin(2*arctan(a*x))+
5/256/a^2/c^3*arctan(a*x)^(3/2)*sin(4*arctan(a*x))+15/128/a^2/c^3*arctan(a*
x)^(1/2)*cos(2*arctan(a*x))+15/2048/a^2/c^3*arctan(a*x)^(1/2)*cos(4*arctan(
a*x))-15/8192*FresnelC(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*2^(1/2)*Pi^(1/
2)/a^2/c^3-15/256*FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2))*Pi^(1/2)/a^2/c^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(a*x)**(5/2)/(a**2*c*x**2+c)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \arctan(ax)^{\frac{5}{2}}}{\left(a^2cx^2 + c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] integrate(x*arctan(a*x)^(5/2)/(a^2*c*x^2 + c)^3, x)
```

3.876 $\int \frac{\tan^{-1}(ax)^{5/2}}{\left(c+a^2cx^2\right)^3}\,dx$

Optimal. Leaf size=296

$$\frac{3x\tan^{-1}(ax)^{5/2}}{8c^3\left(a^2x^2+1\right)}+\frac{x\tan^{-1}(ax)^{5/2}}{4c^3\left(a^2x^2+1\right)^2}+\frac{15\tan^{-1}(ax)^{3/2}}{32ac^3\left(a^2x^2+1\right)}+\frac{5\tan^{-1}(ax)^{3/2}}{32ac^3\left(a^2x^2+1\right)^2}-\frac{45x\sqrt{\tan^{-1}(ax)}}{128c^3\left(a^2x^2+1\right)}+\frac{15\sqrt{\frac{\pi}{2}}S\left(2\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(a}\right.}{4096ac^3}$$

```
[Out] (-45*x*Sqrt[ArcTan[a*x]])/(128*c^3*(1 + a^2*x^2)) - (75*ArcTan[a*x]^(3/2))/
(256*a*c^3) + (5*ArcTan[a*x]^(3/2))/(32*a*c^3*(1 + a^2*x^2)^2) + (15*ArcTan
[a*x]^(3/2))/(32*a*c^3*(1 + a^2*x^2)) + (x*ArcTan[a*x]^(5/2))/(4*c^3*(1 + a
^2*x^2)^2) + (3*x*ArcTan[a*x]^(5/2))/(8*c^3*(1 + a^2*x^2)) + (3*ArcTan[a*x]
^(7/2))/(28*a*c^3) + (15*Sqrt[Pi/2]*FresnelS[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]
])/(4096*a*c^3) + (15*Sqrt[Pi]*FresnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(1
28*a*c^3) - (15*Sqrt[ArcTan[a*x]]*Sin[2*ArcTan[a*x]])/(256*a*c^3) - (15*Sqr
t[ArcTan[a*x]]*Sin[4*ArcTan[a*x]])/(2048*a*c^3)
```

Rubi [A] time = 0.368857, antiderivative size = 296, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 11, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.524, Rules used = {4900, 4892, 4930, 4970, 4406, 12, 3305, 3351, 4904, 3312, 3296}

$$\frac{3x\tan^{-1}(ax)^{5/2}}{8c^3\left(a^2x^2+1\right)}+\frac{x\tan^{-1}(ax)^{5/2}}{4c^3\left(a^2x^2+1\right)^2}+\frac{15\tan^{-1}(ax)^{3/2}}{32ac^3\left(a^2x^2+1\right)}+\frac{5\tan^{-1}(ax)^{3/2}}{32ac^3\left(a^2x^2+1\right)^2}-\frac{45x\sqrt{\tan^{-1}(ax)}}{128c^3\left(a^2x^2+1\right)}+\frac{15\sqrt{\frac{\pi}{2}}S\left(2\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(a}\right.}{4096ac^3}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^(5/2)/(c + a^2*c*x^2)^3,x]
```


```
[Out] (-45*x*Sqrt[ArcTan[a*x]])/(128*c^3*(1 + a^2*x^2)) - (75*ArcTan[a*x]^(3/2))/
(256*a*c^3) + (5*ArcTan[a*x]^(3/2))/(32*a*c^3*(1 + a^2*x^2)^2) + (15*ArcTan
[a*x]^(3/2))/(32*a*c^3*(1 + a^2*x^2)) + (x*ArcTan[a*x]^(5/2))/(4*c^3*(1 + a
^2*x^2)^2) + (3*x*ArcTan[a*x]^(5/2))/(8*c^3*(1 + a^2*x^2)) + (3*ArcTan[a*x]
^(7/2))/(28*a*c^3) + (15*Sqrt[Pi/2]*FresnelS[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]
])/(4096*a*c^3) + (15*Sqrt[Pi]*FresnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(1
28*a*c^3) - (15*Sqrt[ArcTan[a*x]]*Sin[2*ArcTan[a*x]])/(256*a*c^3) - (15*Sqr
t[ArcTan[a*x]]*Sin[4*ArcTan[a*x]])/(2048*a*c^3)
```

Rule 4900

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(b*p*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^(p - 1))/(4*c*d
*(q + 1)^2), x] + (Dist[(2*q + 3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a
 + b*ArcTan[c*x])^p, x], x] - Dist[(b^2*p*(p - 1))/(4*(q + 1)^2), Int[(d +
e*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x] - Simp[(x*(d + e*x^2)^(q + 1)*
(a + b*ArcTan[c*x])^p)/(2*d*(q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] && E
qQ[e, c^2*d] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

Rule 4892

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[(x*(a + b*ArcTan[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*c*
p)/2, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(a +
 b*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 4970

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sin[x]^m)/
Cos[x]^(m + 2*(q + 1)), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4904

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^{5/2}}{\left(c+a^2cx^2\right)^3}\,dx = \frac{5\tan^{-1}(ax)^{3/2}}{32ac^3\left(1+a^2x^2\right)^2}+\frac{x\tan^{-1}(ax)^{5/2}}{4c^3\left(1+a^2x^2\right)^2}-\frac{15}{64}\int\frac{\sqrt{\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^3}\,dx+\frac{3\int\frac{\tan^{-1}(ax)^{5/2}}{\left(c+a^2cx^2\right)^2}\,dx}{4c}$$

$$= \frac{5\tan^{-1}(ax)^{3/2}}{32ac^3\left(1+a^2x^2\right)^2}+\frac{x\tan^{-1}(ax)^{5/2}}{4c^3\left(1+a^2x^2\right)^2}+\frac{3x\tan^{-1}(ax)^{5/2}}{8c^3\left(1+a^2x^2\right)}+\frac{3\tan^{-1}(ax)^{7/2}}{28ac^3}-\frac{15\,\mathrm{Subst}\left(\int\sqrt{x}\cos^4(x\right.}{64ac}$$

$$= \frac{5\tan^{-1}(ax)^{3/2}}{32ac^3\left(1+a^2x^2\right)^2}+\frac{15\tan^{-1}(ax)^{3/2}}{32ac^3\left(1+a^2x^2\right)}+\frac{x\tan^{-1}(ax)^{5/2}}{4c^3\left(1+a^2x^2\right)^2}+\frac{3x\tan^{-1}(ax)^{5/2}}{8c^3\left(1+a^2x^2\right)}+\frac{3\tan^{-1}(ax)^{7/2}}{28ac^3}-\frac{15}{}$$

$$= -\frac{45x\sqrt{\tan^{-1}(ax)}}{128c^3\left(1+a^2x^2\right)}-\frac{75\tan^{-1}(ax)^{3/2}}{256ac^3}+\frac{5\tan^{-1}(ax)^{3/2}}{32ac^3\left(1+a^2x^2\right)^2}+\frac{15\tan^{-1}(ax)^{3/2}}{32ac^3\left(1+a^2x^2\right)}+\frac{x\tan^{-1}(ax)^{5/2}}{4c^3\left(1+a^2x^2\right)^2}+$$

$$= -\frac{45x\sqrt{\tan^{-1}(ax)}}{128c^3\left(1+a^2x^2\right)}-\frac{75\tan^{-1}(ax)^{3/2}}{256ac^3}+\frac{5\tan^{-1}(ax)^{3/2}}{32ac^3\left(1+a^2x^2\right)^2}+\frac{15\tan^{-1}(ax)^{3/2}}{32ac^3\left(1+a^2x^2\right)}+\frac{x\tan^{-1}(ax)^{5/2}}{4c^3\left(1+a^2x^2\right)^2}+$$

$$= -\frac{45x\sqrt{\tan^{-1}(ax)}}{128c^3\left(1+a^2x^2\right)}-\frac{75\tan^{-1}(ax)^{3/2}}{256ac^3}+\frac{5\tan^{-1}(ax)^{3/2}}{32ac^3\left(1+a^2x^2\right)^2}+\frac{15\tan^{-1}(ax)^{3/2}}{32ac^3\left(1+a^2x^2\right)}+\frac{x\tan^{-1}(ax)^{5/2}}{4c^3\left(1+a^2x^2\right)^2}+$$

$$= -\frac{45x\sqrt{\tan^{-1}(ax)}}{128c^3\left(1+a^2x^2\right)}-\frac{75\tan^{-1}(ax)^{3/2}}{256ac^3}+\frac{5\tan^{-1}(ax)^{3/2}}{32ac^3\left(1+a^2x^2\right)^2}+\frac{15\tan^{-1}(ax)^{3/2}}{32ac^3\left(1+a^2x^2\right)}+\frac{x\tan^{-1}(ax)^{5/2}}{4c^3\left(1+a^2x^2\right)^2}+$$

$$= -\frac{45x\sqrt{\tan^{-1}(ax)}}{128c^3\left(1+a^2x^2\right)}-\frac{75\tan^{-1}(ax)^{3/2}}{256ac^3}+\frac{5\tan^{-1}(ax)^{3/2}}{32ac^3\left(1+a^2x^2\right)^2}+\frac{15\tan^{-1}(ax)^{3/2}}{32ac^3\left(1+a^2x^2\right)}+\frac{x\tan^{-1}(ax)^{5/2}}{4c^3\left(1+a^2x^2\right)^2}+$$

$$= -\frac{45x\sqrt{\tan^{-1}(ax)}}{128c^3\left(1+a^2x^2\right)}-\frac{75\tan^{-1}(ax)^{3/2}}{256ac^3}+\frac{5\tan^{-1}(ax)^{3/2}}{32ac^3\left(1+a^2x^2\right)^2}+\frac{15\tan^{-1}(ax)^{3/2}}{32ac^3\left(1+a^2x^2\right)}+\frac{x\tan^{-1}(ax)^{5/2}}{4c^3\left(1+a^2x^2\right)^2}+$$

Mathematica [A] time = 0.361495, size = 162, normalized size = 0.55

$$\frac{\frac{16\sqrt{\tan^{-1}(ax)}\left(-105ax\left(15a^2x^2+17\right)+384\left(a^2x^2+1\right)^2\tan^{-1}(ax)^3+448ax\left(3a^2x^2+5\right)\tan^{-1}(ax)^2-70\left(15a^4x^4+6a^2x^2-17\right)\tan^{-1}(ax)\right)}{\left(a^2x^2+1\right)^2}+105\sqrt{2\pi}S\left(2\sqrt{\frac{2}{\pi}}\sqrt{\right.}{57344ac^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTan[a*x]^(5/2)/(c + a^2*c*x^2)^3,x]
```

```
[Out] ((16*Sqrt[ArcTan[a*x]]*(-105*a*x*(17 + 15*a^2*x^2) - 70*(-17 + 6*a^2*x^2 +
15*a^4*x^4)*ArcTan[a*x] + 448*a*x*(5 + 3*a^2*x^2)*ArcTan[a*x]^2 + 384*(1 +
a^2*x^2)^2*ArcTan[a*x]^3))/(1 + a^2*x^2)^2 + 105*Sqrt[2*Pi]*FresnelS[2*Sqrt
[2/Pi]*Sqrt[ArcTan[a*x]]] + 6720*Sqrt[Pi]*FresnelS[(2*Sqrt[ArcTan[a*x]])/Sq
rt[Pi]])/(57344*a*c^3)
```

Maple [A] time = 0.117, size = 194, normalized size = 0.7

$$\frac{3}{28\,ac^3}\left(\arctan\left(ax\right)\right)^{\frac{7}{2}}+\frac{\sin\left(2\,\arctan\left(ax\right)\right)}{4\,ac^3}\left(\arctan\left(ax\right)\right)^{\frac{5}{2}}+\frac{\sin\left(4\,\arctan\left(ax\right)\right)}{32\,ac^3}\left(\arctan\left(ax\right)\right)^{\frac{5}{2}}+\frac{5\,\cos\left(2\,\arctan\left(ax\right)\right)}{16\,ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x)
```

```
[Out] 3/28*arctan(a*x)^(7/2)/a/c^3+1/4/a/c^3*arctan(a*x)^(5/2)*sin(2*arctan(a*x))
+1/32/a/c^3*arctan(a*x)^(5/2)*sin(4*arctan(a*x))+5/16/a/c^3*arctan(a*x)^(3/
2)*cos(2*arctan(a*x))+5/256/a/c^3*arctan(a*x)^(3/2)*cos(4*arctan(a*x))-15/6
4*sin(2*arctan(a*x))*arctan(a*x)^(1/2)/a/c^3-15/2048*sin(4*arctan(a*x))*arc
tan(a*x)^(1/2)/a/c^3+15/8192*FresnelS(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))
*2^(1/2)*Pi^(1/2)/a/c^3+15/128*FresnelS(2*arctan(a*x)^(1/2)/Pi^(1/2))*Pi^(1
/2)/a/c^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\operatorname{atan}^{\frac{5}{2}}(ax)}{a^6x^6+3a^4x^4+3a^2x^2+1}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(5/2)/(a**2*c*x**2+c)**3,x)
```

```
[Out] Integral(atan(a*x)**(5/2)/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1), x)/c
**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(ax)^{\frac{5}{2}}}{\left(a^2cx^2+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)^(5/2)/(a^2*c*x^2 + c)^3, x)
```

3.877 $\int \frac{\tan^{-1}(ax)^{5/2}}{x\left(c+a^2cx^2\right)^3} \, dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{\tan^{-1}(ax)^{5/2}}{x\left(a^2cx^2+c\right)^3},x\right)$$

```
[Out] Unintegrable[ArcTan[a*x]^(5/2)/(x*(c + a^2*c*x^2)^3), x]
```

Rubi [A] time = 0.0571301, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\tan^{-1}(ax)^{5/2}}{x\left(c+a^2cx^2\right)^3} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcTan[a*x]^(5/2)/(x*(c + a^2*c*x^2)^3),x]
```

```
[Out] Defer[Int][ArcTan[a*x]^(5/2)/(x*(c + a^2*c*x^2)^3), x]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^{5/2}}{x\left(c+a^2cx^2\right)^3} \, dx=\int \frac{\tan^{-1}(ax)^{5/2}}{x\left(c+a^2cx^2\right)^3} \, dx$$

Mathematica [A] time = 3.19862, size = 0, normalized size = 0.

$$\int \frac{\tan^{-1}(ax)^{5/2}}{x\left(c+a^2cx^2\right)^3} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcTan[a*x]^(5/2)/(x*(c + a^2*c*x^2)^3),x]
```

```
[Out] Integrate[ArcTan[a*x]^(5/2)/(x*(c + a^2*c*x^2)^3), x]
```

Maple [A] time = 0.78, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(a^{2}c x^{2}+c\right)^{3}} \left(\arctan\left(a x\right)\right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^3,x)
```

```
[Out] int(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^3,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(5/2)/x/(a**2*c*x**2+c)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(ax)^{\frac{5}{2}}}{(a^2cx^2+c)^3 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)^(5/2)/((a^2*c*x^2 + c)^3*x), x)
```

3.878 $\int x^m \sqrt{c+a^2 c x^2} \tan^{-1}(a x)^{5/2} \, dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(x^m \sqrt{a^2 c x^2+c} \tan^{-1}(a x)^{5/2},x\right)$$

```
[Out] Unintegrable[x^m*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2), x]
```

Rubi [A] time = 0.0951104, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^m \sqrt{c+a^2 c x^2} \tan^{-1}(a x)^{5/2} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2),x]

[Out] Defer[Int][x^m*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2), x]
```

Rubi steps

$$\int x^m \sqrt{c+a^2 c x^2} \tan^{-1}(a x)^{5/2} \, dx=\int x^m \sqrt{c+a^2 c x^2} \tan^{-1}(a x)^{5/2} \, dx$$

Mathematica [A] time = 0.66196, size = 0, normalized size = 0.

$$\int x^m \sqrt{c+a^2 c x^2} \tan^{-1}(a x)^{5/2} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2),x]

[Out] Integrate[x^m*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2), x]
```

Maple [A] time = 1.262, size = 0, normalized size = 0.

$$\int x^{m} (\arctan (ax))^{\frac{5}{2}} \sqrt{a^{2}c x^{2}+c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arctan(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(x^m*arctan(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{a^{2}c x^{2}+c}\, x^{m} \arctan (ax)^{\frac{5}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^m*arctan(a*x)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*atan(a*x)**(5/2)*(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.879 $\int x^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{5/2}\,dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(x^2\sqrt{a^2cx^2+c}\tan^{-1}(ax)^{5/2},x\right)$$

```
[Out] Unintegrable[x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2), x]
```

Rubi [A] time = 0.108693, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{5/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2),x]
```

```
[Out] Defer[Int][x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2), x]
```

Rubi steps

$$\int x^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{5/2}\,dx=\int x^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{5/2}\,dx$$

Mathematica [A] time = 3.27563, size = 0, normalized size = 0.

$$\int x^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{5/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2),x]
```

```
[Out] Integrate[x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2), x]
```

Maple [A] time = 1.536, size = 0, normalized size = 0.

$$\int x^2 (\arctan (ax))^{\frac{5}{2}} \sqrt{a^2 c x^2 + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(x^2*arctan(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(a*x)**(5/2)*(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a^2cx^2 + c}x^2 \arctan{(ax)}^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)*x^2*arctan(a*x)^(5/2), x)
```

3.880 $\int x\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{5/2}\,dx$

Optimal. Leaf size=158

$$-\frac{5c\text{Unintegrable}\left(\frac{1}{\sqrt{a^2cx^2+c}\sqrt{\tan^{-1}(ax)}},x\right)}{16a}-\frac{5c\text{Unintegrable}\left(\frac{\tan^{-1}(ax)^{3/2}}{\sqrt{a^2cx^2+c}},x\right)}{12a}+\frac{\left(a^2cx^2+c\right)^{3/2}\tan^{-1}(ax)^{5/2}}{3a^2c}-\frac{5x\sqrt{a^2cx^2}}{}$$

```
[Out] (5*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]])/(8*a^2) - (5*x*Sqrt[c + a^2*c*x^2
]*ArcTan[a*x]^(3/2))/(12*a) + ((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2))/(3*
a^2*c) - (5*c*Unintegrable[1/(Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]), x])/(
16*a) - (5*c*Unintegrable[ArcTan[a*x]^(3/2)/Sqrt[c + a^2*c*x^2], x])/(12*a)
```

Rubi [A] time = 0.178721, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{5/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2),x]
```

```
[Out] (5*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]])/(8*a^2) - (5*x*Sqrt[c + a^2*c*x^2
]*ArcTan[a*x]^(3/2))/(12*a) + ((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2))/(3*
a^2*c) - (5*c*Defer[Int][1/(Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]), x])/(16
*a) - (5*c*Defer[Int][ArcTan[a*x]^(3/2)/Sqrt[c + a^2*c*x^2], x])/(12*a)
```

Rubi steps

$$\int x\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{5/2}\,dx=\frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{5/2}}{3a^2c}-\frac{5\int\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}\,dx}{6a}$$

$$=\frac{5\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}}{8a^2}-\frac{5x\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}}{12a}+\frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{5/2}}{3a^2c}$$

Mathematica [A] time = 4.82122, size = 0, normalized size = 0.

$$\int x\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{5/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2),x]
```

```
[Out] Integrate[x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2), x]
```

Maple [A] time = 0.995, size = 0, normalized size = 0.

$$\int x\left(\arctan\left(ax\right)\right)^{\frac{5}{2}}\sqrt{a^{2}c\,x^{2}+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(x*arctan(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(a*x)**(5/2)*(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a^2cx^2+c}\,x\arctan(ax)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)*x*arctan(a*x)^(5/2), x)
```

3.881 $\int \sqrt{c+a^2cx^2}\tan^{-1}(ax)^{5/2}\,dx$

Optimal. Leaf size=117

$$\frac{15}{8}c\text{Unintegrable}\left(\frac{\sqrt{\tan^{-1}(ax)}}{\sqrt{a^2cx^2+c}},x\right)+\frac{1}{2}c\text{Unintegrable}\left(\frac{\tan^{-1}(ax)^{5/2}}{\sqrt{a^2cx^2+c}},x\right)+\frac{1}{2}x\sqrt{a^2cx^2+c}\tan^{-1}(ax)^{5/2}-\frac{5\sqrt{a^2cx^2+c}\,\mathrm{t}}{4a}$$

```
[Out] (-5*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2))/(4*a) + (x*Sqrt[c + a^2*c*x^2]*A
rcTan[a*x]^(5/2))/2 + (15*c*Unintegrable[Sqrt[ArcTan[a*x]]/Sqrt[c + a^2*c*x
^2], x])/8 + (c*Unintegrable[ArcTan[a*x]^(5/2)/Sqrt[c + a^2*c*x^2], x])/2
```

Rubi [A] time = 0.103224, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \sqrt{c+a^2cx^2}\tan^{-1}(ax)^{5/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2),x]
```

```
[Out] (-5*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2))/(4*a) + (x*Sqrt[c + a^2*c*x^2]*A
rcTan[a*x]^(5/2))/2 + (15*c*Defer[Int][Sqrt[ArcTan[a*x]]/Sqrt[c + a^2*c*x^2
], x])/8 + (c*Defer[Int][ArcTan[a*x]^(5/2)/Sqrt[c + a^2*c*x^2], x])/2
```

Rubi steps

$$\int \sqrt{c+a^2cx^2}\tan^{-1}(ax)^{5/2}\,dx=-\frac{5\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}}{4a}+\frac{1}{2}x\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{5/2}+\frac{1}{2}c\int\frac{\tan^{-1}(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx+\frac{1}{8}(1$$

Mathematica [A] time = 0.293931, size = 0, normalized size = 0.

$$\int \sqrt{c+a^2cx^2}\tan^{-1}(ax)^{5/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2),x]
```

```
[Out] Integrate[Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2), x]
```

Maple [A] time = 0.869, size = 0, normalized size = 0.

$$\int (\arctan (ax))^{\frac{5}{2}} \sqrt{a^{2}c x^{2}+c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(arctan(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(5/2)*(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a^2cx^2 + c}\arctan(ax)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)*arctan(a*x)^(5/2), x)
```

3.882 $\int \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{5/2}}{x}\,dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(\frac{\sqrt{a^2cx^2+c}\tan^{-1}(ax)^{5/2}}{x},x\right)$$

```
[Out] Unintegrable[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2))/x, x]
```

Rubi [A] time = 0.0940205, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{5/2}}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2))/x,x]
```

```
[Out] Defer[Int][(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2))/x, x]
```

Rubi steps

$$\int \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{5/2}}{x}\,dx=\int \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{5/2}}{x}\,dx$$

Mathematica [A] time = 2.28417, size = 0, normalized size = 0.

$$\int \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{5/2}}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2))/x,x]
```

```
[Out] Integrate[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2))/x, x]
```

Maple [A] time = 0.938, size = 0, normalized size = 0.

$$\int \frac{1}{x} (\arctan (ax))^{\frac{5}{2}} \sqrt{a^2cx^2 + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2)/x,x)
```

```
[Out] int(arctan(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2)/x,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(5/2)*(a**2*c*x**2+c)**(1/2)/x,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a^2cx^2 + c}\arctan{(ax)}^{\frac{5}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)*arctan(a*x)^(5/2)/x, x)
```

3.883 $\int x^m \left(c+a^2 c x^2\right)^{3/2} \tan^{-1}(a x)^{5/2} \, dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(x^m \left(a^2 c x^2+c\right)^{3/2} \tan^{-1}(a x)^{5/2},x\right)$$

```
[Out] Unintegrable[x^m*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2), x]
```

Rubi [A] time = 0.106928, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^m \left(c+a^2 c x^2\right)^{3/2} \tan^{-1}(a x)^{5/2} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2),x]
```

```
[Out] Defer[Int][x^m*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2), x]
```

Rubi steps

$$\int x^m \left(c+a^2 c x^2\right)^{3/2} \tan^{-1}(a x)^{5/2} \, dx=\int x^m \left(c+a^2 c x^2\right)^{3/2} \tan^{-1}(a x)^{5/2} \, dx$$

Mathematica [A] time = 1.0369, size = 0, normalized size = 0.

$$\int x^m \left(c+a^2 c x^2\right)^{3/2} \tan^{-1}(a x)^{5/2} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2),x]
```

```
[Out] Integrate[x^m*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2), x]
```

Maple [A] time = 1.021, size = 0, normalized size = 0.

$$\int x^m \left(a^2 c x^2 + c\right)^{\frac{3}{2}} (\arctan(ax))^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2),x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^2 c x^2 + c\right)^{\frac{3}{2}} x^m \arctan(ax)^{\frac{5}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)^(3/2)*x^m*arctan(a*x)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**(3/2)*atan(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.884 $\int x^2 \left(c+a^2 c x^2\right)^{3/2} \tan^{-1}(a x)^{5/2} \, dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(x^2 \left(a^2 c x^2+c\right)^{3/2} \tan^{-1}(a x)^{5/2},x\right)$$

```
[Out] Unintegrable[x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2), x]
```

Rubi [A] time = 0.116219, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^2 \left(c+a^2 c x^2\right)^{3/2} \tan^{-1}(a x)^{5/2} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2),x]
```

```
[Out] Defer[Int][x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2), x]
```

Rubi steps

$$\int x^2 \left(c+a^2 c x^2\right)^{3/2} \tan^{-1}(a x)^{5/2} \, dx=\int x^2 \left(c+a^2 c x^2\right)^{3/2} \tan^{-1}(a x)^{5/2} \, dx$$

Mathematica [A] time = 3.96046, size = 0, normalized size = 0.

$$\int x^2 \left(c+a^2 c x^2\right)^{3/2} \tan^{-1}(a x)^{5/2} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2),x]
```

```
[Out] Integrate[x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2), x]
```

Maple [A] time = 1.33, size = 0, normalized size = 0.

$$\int x^2 \left(a^2 c x^2 + c\right)^{\frac{3}{2}} (\arctan (a x))^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2),x)
```

```
[Out] int(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a**2*c*x**2+c)**(3/2)*atan(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2 + c\right)^{\frac{3}{2}} x^2 \arctan(ax)^{\frac{5}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)*x^2*arctan(a*x)^(5/2), x)
```

3.885 $\int x\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{5/2}\,dx$

Optimal. Leaf size=256

$$-\frac{9c^2\text{Unintegrable}\left(\frac{1}{\sqrt{a^2cx^2+c}\sqrt{\tan^{-1}(ax)}},x\right)}{64a}-\frac{3c^2\text{Unintegrable}\left(\frac{\tan^{-1}(ax)^{3/2}}{\sqrt{a^2cx^2+c}},x\right)}{16a}-\frac{c\text{Unintegrable}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{\tan^{-1}(ax)}},x\right)}{32a}+\frac{\left(a^2cx^2\right.}{}$$

[Out] (9*c*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]])/(32*a^2) + ((c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]])/(16*a^2) - (3*c*x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2))/(16*a) - (x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2))/(8*a) + ((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2))/(5*a^2*c) - (9*c^2*Unintegrable[1/(Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]), x])/(64*a) - (c*Unintegrable[Sqrt[c + a^2*c*x^2]/Sqrt[ArcTan[a*x]], x])/(32*a) - (3*c^2*Unintegrable[ArcTan[a*x]^(3/2)/Sqrt[c + a^2*c*x^2], x])/(16*a)

Rubi [A] time = 0.272472, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{5/2}\,dx$$

Verification is Not applicable to the result.

[In] Int[x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2),x]

[Out] (9*c*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]])/(32*a^2) + ((c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]])/(16*a^2) - (3*c*x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2))/(16*a) - (x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2))/(8*a) + ((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2))/(5*a^2*c) - (9*c^2*Defer[Int][1/(Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]), x])/(64*a) - (c*Defer[Int][Sqrt[c + a^2*c*x^2]/Sqrt[ArcTan[a*x]], x])/(32*a) - (3*c^2*Defer[Int][ArcTan[a*x]^(3/2)/Sqrt[c + a^2*c*x^2], x])/(16*a)

Rubi steps

$$\int x\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{5/2}\,dx=\frac{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{5/2}}{5a^2c}-\frac{\int\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}\,dx}{2a}$$

$$=\frac{\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}}{16a^2}-\frac{x\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}}{8a}+\frac{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(}{5a^2c}$$

$$=\frac{9c\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}}{32a^2}+\frac{\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}}{16a^2}-\frac{3cx\sqrt{c+a^2cx^2}\tan^{-1}(ax}{16a}$$

Mathematica [A] time = 2.55925, size = 0, normalized size = 0.

$$\int x\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{5/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2),x]
```

```
[Out] Integrate[x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2), x]
```

Maple [A] time = 0.796, size = 0, normalized size = 0.

$$\int x\left(a^2cx^2+c\right)^{\frac{3}{2}}\left(\arctan(ax)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2),x)
```

```
[Out] int(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**(3/2)*atan(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2 + c\right)^{\frac{3}{2}} x \arctan\left(ax\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)*x*arctan(a*x)^(5/2), x)
```

3.886 $\int \left(c+a^2cx^2\right)^{3/2} \tan^{-1}(ax)^{5/2} \, dx$

Optimal. Leaf size=209

$$\frac{45}{32}c^2\text{Unintegrable}\left(\frac{\sqrt{\tan^{-1}(ax)}}{\sqrt{a^2cx^2+c}},x\right)+\frac{3}{8}c^2\text{Unintegrable}\left(\frac{\tan^{-1}(ax)^{5/2}}{\sqrt{a^2cx^2+c}},x\right)+\frac{5}{16}c\text{Unintegrable}\left(\sqrt{a^2cx^2+c}\sqrt{\tan^{-1}(\right.$$

```
[Out] (-15*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2))/(16*a) - (5*(c + a^2*c*x^2)^(
3/2)*ArcTan[a*x]^(3/2))/(24*a) + (3*c*x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/
2))/8 + (x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2))/4 + (45*c^2*Unintegrabl
e[Sqrt[ArcTan[a*x]]/Sqrt[c + a^2*c*x^2], x])/32 + (5*c*Unintegrable[Sqrt[c
+ a^2*c*x^2]*Sqrt[ArcTan[a*x]], x])/16 + (3*c^2*Unintegrable[ArcTan[a*x]^(5
/2)/Sqrt[c + a^2*c*x^2], x])/8
```

Rubi [A] time = 0.179346, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \left(c+a^2cx^2\right)^{3/2} \tan^{-1}(ax)^{5/2} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2),x]
```

```
[Out] (-15*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2))/(16*a) - (5*(c + a^2*c*x^2)^(
3/2)*ArcTan[a*x]^(3/2))/(24*a) + (3*c*x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/
2))/8 + (x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2))/4 + (45*c^2*Defer[Int][
Sqrt[ArcTan[a*x]]/Sqrt[c + a^2*c*x^2], x])/32 + (5*c*Defer[Int][Sqrt[c + a^
2*c*x^2]*Sqrt[ArcTan[a*x]], x])/16 + (3*c^2*Defer[Int][ArcTan[a*x]^(5/2)/Sq
rt[c + a^2*c*x^2], x])/8
```

Rubi steps

$$\int \left(c+a^2cx^2\right)^{3/2} \tan^{-1}(ax)^{5/2} \, dx = -\frac{5\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}}{24a}+\frac{1}{4}x\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{5/2}+\frac{1}{16}(5c)\int \sqrt{c+a^2}$$

$$= -\frac{15c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}}{16a}-\frac{5\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}}{24a}+\frac{3}{8}cx\sqrt{c+a^2cx^2}\tan^{-}$$

Mathematica [A] time = 1.51022, size = 0, normalized size = 0.

$$\int \left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{5/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2),x]
```

```
[Out] Integrate[(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2), x]
```

Maple [A] time = 0.741, size = 0, normalized size = 0.

$$\int \left(a^2cx^2+c\right)^{\frac{3}{2}}(\arctan(ax))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2),x)
```

```
[Out] int((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(3/2)*atan(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2+c\right)^{\frac{3}{2}}\arctan\left(ax\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^(5/2), x)
```

3.887 $\int \frac{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^{5/2}}{x}\,dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(\frac{(a^2cx^2+c)^{3/2}\tan^{-1}(ax)^{5/2}}{x},x\right)$$

```
[Out] Unintegrable[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2))/x, x]
```

Rubi [A] time = 0.107989, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^{5/2}}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2))/x,x]
```

```
[Out] Defer[Int][((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2))/x, x]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^{5/2}}{x}\,dx=\int \frac{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^{5/2}}{x}\,dx$$

Mathematica [A] time = 2.06908, size = 0, normalized size = 0.

$$\int \frac{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^{5/2}}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2))/x,x]
```

```
[Out] Integrate[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2))/x, x]
```

Maple [A] time = 0.74, size = 0, normalized size = 0.

$$\int \frac{1}{x} \left(a^{2} c x^{2}+c\right)^{\frac{3}{2}} \left(\arctan \left(a x\right)\right)^{\frac{5}{2}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2)/x,x)
```

```
[Out] int((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2)/x,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(3/2)*atan(a*x)**(5/2)/x,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^{\frac{3}{2}}\arctan\left(ax\right)^{\frac{5}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2)/x,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^(5/2)/x, x)
```

3.888 $\int x^m \left(c+a^2cx^2\right)^{5/2} \tan^{-1}(ax)^{5/2}\,dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(x^m \left(a^2cx^2+c\right)^{5/2} \tan^{-1}(ax)^{5/2},x\right)$$

```
[Out] Unintegrable[x^m*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2), x]
```

Rubi [A] time = 0.108217, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int x^m \left(c+a^2cx^2\right)^{5/2} \tan^{-1}(ax)^{5/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2),x]
```

```
[Out] Defer[Int][x^m*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2), x]
```

Rubi steps

$$\int x^m \left(c+a^2cx^2\right)^{5/2} \tan^{-1}(ax)^{5/2}\,dx=\int x^m \left(c+a^2cx^2\right)^{5/2} \tan^{-1}(ax)^{5/2}\,dx$$

Mathematica [A] time = 1.39494, size = 0, normalized size = 0.

$$\int x^m \left(c+a^2cx^2\right)^{5/2} \tan^{-1}(ax)^{5/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2),x]
```

```
[Out] Integrate[x^m*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2), x]
```

Maple [A] time = 1.03, size = 0, normalized size = 0.

$$\int x^{m} \left(a^{2} c \,x^{2}+c \right)^{\frac{5}{2}} \left(\arctan \left(a x \right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2),x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a^{4} c^{2} x^{4}+2 \, a^{2} c^{2} x^{2}+c^{2}\right) \sqrt{a^{2} c x^{2}+c} x^{m} \arctan \left(a x\right)^{\frac{5}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*x^m*arctan
(a*x)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**(5/2)*atan(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.889 $\int x^2 \left(c+a^2 c x^2\right)^{5/2} \tan^{-1}(a x)^{5/2} \, dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(x^2 \left(a^2 c x^2+c\right)^{5/2} \tan^{-1}(a x)^{5/2},x\right)$$

```
[Out] Unintegrable[x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2), x]
```

Rubi [A] time = 0.113945, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^2 \left(c+a^2 c x^2\right)^{5/2} \tan^{-1}(a x)^{5/2} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2),x]
```

```
[Out] Defer[Int][x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2), x]
```

Rubi steps

$$\int x^2 \left(c+a^2 c x^2\right)^{5/2} \tan^{-1}(a x)^{5/2} \, dx=\int x^2 \left(c+a^2 c x^2\right)^{5/2} \tan^{-1}(a x)^{5/2} \, dx$$

Mathematica [A] time = 3.37949, size = 0, normalized size = 0.

$$\int x^2 \left(c+a^2 c x^2\right)^{5/2} \tan^{-1}(a x)^{5/2} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2),x]
```

```
[Out] Integrate[x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2), x]
```

Maple [A] time = 1.628, size = 0, normalized size = 0.

$$\int x^{2} \left(a^{2} c \,x^{2}+c \right)^{\frac{5}{2}} \left(\arctan \left(a x \right)\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2),x)
```

```
[Out] int(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a**2*c*x**2+c)**(5/2)*atan(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2+c\right)^{\frac{5}{2}}x^2\arctan\left(ax\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(5/2)*x^2*arctan(a*x)^(5/2), x)
```

3.890 $\int x \left(c + a^2cx^2\right)^{5/2} \tan^{-1}(ax)^{5/2}\,dx$

Optimal. Leaf size=356

$$-\frac{75c^3\text{Unintegrable}\left(\frac{1}{\sqrt{a^2cx^2+c}\sqrt{\tan^{-1}(ax)}},x\right)}{896a}-\frac{25c^2\text{Unintegrable}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{\tan^{-1}(ax)}},x\right)}{1344a}-\frac{25c^3\text{Unintegrable}\left(\frac{\tan^{-1}(ax)^{3/2}}{\sqrt{a^2cx^2+c}},x\right)}{224a}-$$

```
[Out] (75*c^2*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]])/(448*a^2) + (25*c*(c + a^2*c
*x^2)^(3/2)*Sqrt[ArcTan[a*x]])/(672*a^2) + ((c + a^2*c*x^2)^(5/2)*Sqrt[ArcT
an[a*x]])/(56*a^2) - (25*c^2*x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2))/(224*
a) - (25*c*x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2))/(336*a) - (5*x*(c + a
^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2))/(84*a) + ((c + a^2*c*x^2)^(7/2)*ArcTan[a
*x]^(5/2))/(7*a^2*c) - (75*c^3*Unintegrable[1/(Sqrt[c + a^2*c*x^2]*Sqrt[Arc
Tan[a*x]]), x])/(896*a) - (25*c^2*Unintegrable[Sqrt[c + a^2*c*x^2]/Sqrt[Arc
Tan[a*x]], x])/(1344*a) - (c*Unintegrable[(c + a^2*c*x^2)^(3/2)/Sqrt[ArcTan
[a*x]], x])/(112*a) - (25*c^3*Unintegrable[ArcTan[a*x]^(3/2)/Sqrt[c + a^2*c
*x^2], x])/(224*a)
```

Rubi [A] time = 0.363545, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int x \left(c + a^2cx^2\right)^{5/2} \tan^{-1}(ax)^{5/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2),x]
```

```
[Out] (75*c^2*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]])/(448*a^2) + (25*c*(c + a^2*c
*x^2)^(3/2)*Sqrt[ArcTan[a*x]])/(672*a^2) + ((c + a^2*c*x^2)^(5/2)*Sqrt[ArcT
an[a*x]])/(56*a^2) - (25*c^2*x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2))/(224*
a) - (25*c*x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2))/(336*a) - (5*x*(c + a
^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2))/(84*a) + ((c + a^2*c*x^2)^(7/2)*ArcTan[a
*x]^(5/2))/(7*a^2*c) - (75*c^3*Defer[Int][1/(Sqrt[c + a^2*c*x^2]*Sqrt[ArcTa
n[a*x]]), x])/(896*a) - (25*c^2*Defer[Int][Sqrt[c + a^2*c*x^2]/Sqrt[ArcTan[
a*x]], x])/(1344*a) - (c*Defer[Int][(c + a^2*c*x^2)^(3/2)/Sqrt[ArcTan[a*x]]
, x])/(112*a) - (25*c^3*Defer[Int][ArcTan[a*x]^(3/2)/Sqrt[c + a^2*c*x^2], x
])/(224*a)
```

Rubi steps

$$\int x\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{5/2}\,dx=\frac{\left(c+a^2cx^2\right)^{7/2}\tan^{-1}(ax)^{5/2}}{7a^2c}-\frac{5\int\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{3/2}\,dx}{14a}$$

$$=\frac{\left(c+a^2cx^2\right)^{5/2}\sqrt{\tan^{-1}(ax)}}{56a^2}-\frac{5x\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{3/2}}{84a}+\frac{\left(c+a^2cx^2\right)^{7/2}\tan^{-1}(a}{7a^2c}$$

$$=\frac{25c\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}}{672a^2}+\frac{\left(c+a^2cx^2\right)^{5/2}\sqrt{\tan^{-1}(ax)}}{56a^2}-\frac{25cx\left(c+a^2cx^2\right)^{3/2}\text{t}}{336a}$$

$$=\frac{75c^2\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}}{448a^2}+\frac{25c\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}}{672a^2}+\frac{\left(c+a^2cx^2\right)^{5/2}\sqrt{\tan}}{56a^2}$$

Mathematica [A] time = 5.80239, size = 0, normalized size = 0.

$$\int x\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{5/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2),x]
```

```
[Out] Integrate[x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2), x]
```

Maple [A] time = 0.964, size = 0, normalized size = 0.

$$\int x\left(a^2cx^2+c\right)^{\frac{5}{2}}(\arctan(ax))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2),x)
```

```
[Out] int(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**(5/2)*atan(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2+c\right)^{\frac{5}{2}} x \arctan\left(ax\right)^{\frac{5}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(5/2)*x*arctan(a*x)^(5/2), x)
```

3.891 $\int \left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{5/2}\,dx$

Optimal. Leaf size=303

$$\frac{75}{64}c^3\text{Unintegrable}\left(\frac{\sqrt{\tan^{-1}(ax)}}{\sqrt{a^2cx^2+c}},x\right)+\frac{5}{16}c^3\text{Unintegrable}\left(\frac{\tan^{-1}(ax)^{5/2}}{\sqrt{a^2cx^2+c}},x\right)+\frac{25}{96}c^2\text{Unintegrable}\left(\sqrt{a^2cx^2+c}\sqrt{\tan^{-1}}\right.$$

```
[Out] (-25*c^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2))/(32*a) - (25*c*(c + a^2*c*x
^2)^(3/2)*ArcTan[a*x]^(3/2))/(144*a) - ((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(
3/2))/(12*a) + (5*c^2*x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2))/16 + (5*c*x*
(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2))/24 + (x*(c + a^2*c*x^2)^(5/2)*ArcT
an[a*x]^(5/2))/6 + (75*c^3*Unintegrable[Sqrt[ArcTan[a*x]]/Sqrt[c + a^2*c*x^
2], x])/64 + (25*c^2*Unintegrable[Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]], x]
)/96 + (c*Unintegrable[(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]], x])/8 + (5*
c^3*Unintegrable[ArcTan[a*x]^(5/2)/Sqrt[c + a^2*c*x^2], x])/16
```

Rubi [A] time = 0.266523, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{5/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2),x]
```

```
[Out] (-25*c^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2))/(32*a) - (25*c*(c + a^2*c*x
^2)^(3/2)*ArcTan[a*x]^(3/2))/(144*a) - ((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(
3/2))/(12*a) + (5*c^2*x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2))/16 + (5*c*x*
(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2))/24 + (x*(c + a^2*c*x^2)^(5/2)*ArcT
an[a*x]^(5/2))/6 + (75*c^3*Defer[Int][Sqrt[ArcTan[a*x]]/Sqrt[c + a^2*c*x^2]
, x])/64 + (25*c^2*Defer[Int][Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]], x])/96
 + (c*Defer[Int][(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]], x])/8 + (5*c^3*De
fer[Int][ArcTan[a*x]^(5/2)/Sqrt[c + a^2*c*x^2], x])/16
```

Rubi steps

$$\int \left(c+a^2cx^2\right)^{5/2} \tan^{-1}(ax)^{5/2}\,dx = -\frac{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{3/2}}{12a} + \frac{1}{6}x\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{5/2} + \frac{1}{8}c\int \left(c+a^2cx^2\right)^{3/2}$$

$$= -\frac{25c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}}{144a} - \frac{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{3/2}}{12a} + \frac{5}{24}cx\left(c+a^2cx^2\right)^{3/2}$$

$$= -\frac{25c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}}{32a} - \frac{25c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}}{144a} - \frac{\left(c+a^2cx^2\right)^{5/2}\tan}{12a}$$

Mathematica [A] time = 0.484942, size = 0, normalized size = 0.

$$\int \left(c+a^2cx^2\right)^{5/2} \tan^{-1}(ax)^{5/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2),x]
```

```
[Out] Integrate[(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2), x]
```

Maple [A] time = 0.844, size = 0, normalized size = 0.

$$\int \left(a^2cx^2+c\right)^{\frac{5}{2}} (\arctan(ax))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2),x)
```

```
[Out] int((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(5/2)*atan(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2 + c\right)^{\frac{5}{2}} \arctan\left(ax\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^(5/2), x)
```

3.892 $\int \frac{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{5/2}}{x}\,dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(\frac{\left(a^2cx^2+c\right)^{5/2}\tan^{-1}(ax)^{5/2}}{x},x\right)$$

```
[Out] Unintegrable[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2))/x, x]
```

Rubi [A] time = 0.108953, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{5/2}}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2))/x,x]
```

```
[Out] Defer[Int][((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2))/x, x]
```

Rubi steps

$$\int \frac{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{5/2}}{x}\,dx=\int \frac{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{5/2}}{x}\,dx$$

Mathematica [A] time = 2.36034, size = 0, normalized size = 0.

$$\int \frac{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{5/2}}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2))/x,x]
```

```
[Out] Integrate[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2))/x, x]
```

Maple [A] time = 0.842, size = 0, normalized size = 0.

$$\int \frac{1}{x}\left(a^{2}c x^{2}+c\right)^{\frac{5}{2}} \left(\arctan \left(a x\right)\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2)/x,x)
```

```
[Out] int((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2)/x,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(5/2)*atan(a*x)**(5/2)/x,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^{\frac{5}{2}}\arctan\left(ax\right)^{\frac{5}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2)/x,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^(5/2)/x, x)
```

3.893 $\int \frac{x^m \tan^{-1}(ax)^{5/2}}{\sqrt{c+a^2cx^2}} dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(\frac{x^m \tan^{-1}(ax)^{5/2}}{\sqrt{a^2cx^2+c}}, x\right)$$

```
[Out] Unintegrable[(x^m*ArcTan[a*x]^(5/2))/Sqrt[c + a^2*c*x^2], x]
```

Rubi [A] time = 0.102222, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m \tan^{-1}(ax)^{5/2}}{\sqrt{c+a^2cx^2}} dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*ArcTan[a*x]^(5/2))/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] Defer[Int][(x^m*ArcTan[a*x]^(5/2))/Sqrt[c + a^2*c*x^2], x]
```

Rubi steps

$$\int \frac{x^m \tan^{-1}(ax)^{5/2}}{\sqrt{c+a^2cx^2}} dx = \int \frac{x^m \tan^{-1}(ax)^{5/2}}{\sqrt{c+a^2cx^2}} dx$$

Mathematica [A] time = 0.805661, size = 0, normalized size = 0.

$$\int \frac{x^m \tan^{-1}(ax)^{5/2}}{\sqrt{c+a^2cx^2}} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*ArcTan[a*x]^(5/2))/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] Integrate[(x^m*ArcTan[a*x]^(5/2))/Sqrt[c + a^2*c*x^2], x]
```

Maple [A] time = 1.338, size = 0, normalized size = 0.

$$\int x^{m} \left(\arctan \left(a x\right)\right)^{\frac{5}{2}} \frac{1}{\sqrt{a^{2} c \,x^{2}+c}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x^{m} \arctan\left(a x\right)^{\frac{5}{2}}}{\sqrt{a^{2} c x^{2} + c}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^m*arctan(a*x)^(5/2)/sqrt(a^2*c*x^2 + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*atan(a*x)**(5/2)/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m \arctan{(ax)}^{\frac{5}{2}}}{\sqrt{a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^m*arctan(a*x)^(5/2)/sqrt(a^2*c*x^2 + c), x)
```

3.894 $\int \frac{x^3 \tan^{-1}(ax)^{5/2}}{\sqrt{c+a^2cx^2}} dx$

Optimal. Leaf size=197

$$-\frac{5\text{Unintegrable}\left(\frac{1}{\sqrt{a^2cx^2+c}\sqrt{\tan^{-1}(ax)}},x\right)}{16a^3}+\frac{25\text{Unintegrable}\left(\frac{\tan^{-1}(ax)^{3/2}}{\sqrt{a^2cx^2+c}},x\right)}{12a^3}+\frac{x^2\sqrt{a^2cx^2+c}\tan^{-1}(ax)^{5/2}}{3a^2c}-\frac{2\sqrt{a^2cx^2+c}}{3a}$$

```
[Out] (5*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]])/(8*a^4*c) - (5*x*Sqrt[c + a^2*c*x
^2]*ArcTan[a*x]^(3/2))/(12*a^3*c) - (2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2
))/(3*a^4*c) + (x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2))/(3*a^2*c) - (5*U
nintegrable[1/(Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]), x])/(16*a^3) + (25*U
nintegrable[ArcTan[a*x]^(3/2)/Sqrt[c + a^2*c*x^2], x])/(12*a^3)
```

Rubi [A] time = 0.472049, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{x^3 \tan^{-1}(ax)^{5/2}}{\sqrt{c+a^2cx^2}} dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^3*ArcTan[a*x]^(5/2))/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] (5*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]])/(8*a^4*c) - (5*x*Sqrt[c + a^2*c*x
^2]*ArcTan[a*x]^(3/2))/(12*a^3*c) - (2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2
))/(3*a^4*c) + (x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2))/(3*a^2*c) - (5*D
efer[Int][1/(Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]), x])/(16*a^3) + (25*Def
er[Int][ArcTan[a*x]^(3/2)/Sqrt[c + a^2*c*x^2], x])/(12*a^3)
```

Rubi steps

$$\int \frac{x^3 \tan^{-1}(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx = \frac{x^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{5/2}}{3a^2c} - \frac{2\int \frac{x\tan^{-1}(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx}{3a^2} - \frac{5\int \frac{x^2\tan^{-1}(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx}{6a}$$

$$= -\frac{5x\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}}{12a^3c} - \frac{2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{5/2}}{3a^4c} + \frac{x^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{5/2}}{3a^2c} + \frac{5\int \frac{\tan}{}}{}$$

$$= \frac{5\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}}{8a^4c} - \frac{5x\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}}{12a^3c} - \frac{2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{5/2}}{3a^4c} + \frac{x^2\sqrt{c+}}{}$$

Mathematica [A] time = 3.85488, size = 0, normalized size = 0.

$$\int \frac{x^3 \tan^{-1}(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^3*ArcTan[a*x]^(5/2))/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] Integrate[(x^3*ArcTan[a*x]^(5/2))/Sqrt[c + a^2*c*x^2], x]
```

Maple [A] time = 4.998, size = 0, normalized size = 0.

$$\int x^3 (\arctan(ax))^{\frac{5}{2}} \frac{1}{\sqrt{a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atan(a*x)**(5/2)/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \arctan (ax)^{\frac{5}{2}}}{\sqrt{a^2cx^2 + c}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^3*arctan(a*x)^(5/2)/sqrt(a^2*c*x^2 + c), x)
```

3.895 $\int \frac{x^2 \tan^{-1}(ax)^{5/2}}{\sqrt{c+a^2cx^2}} \, dx$

Optimal. Leaf size=130

$$\frac{15\text{Unintegrable}\left(\frac{\sqrt{\tan^{-1}(ax)}}{\sqrt{a^2cx^2+c}},x\right)}{8a^2}-\frac{\text{Unintegrable}\left(\frac{\tan^{-1}(ax)^{5/2}}{\sqrt{a^2cx^2+c}},x\right)}{2a^2}+\frac{x\sqrt{a^2cx^2+c}\tan^{-1}(ax)^{5/2}}{2a^2c}-\frac{5\sqrt{a^2cx^2+c}\tan^{-1}(ax)}{4a^3c}$$

```
[Out] (-5*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2))/(4*a^3*c) + (x*Sqrt[c + a^2*c*x^
2]*ArcTan[a*x]^(5/2))/(2*a^2*c) + (15*Unintegrable[Sqrt[ArcTan[a*x]]/Sqrt[c
 + a^2*c*x^2], x])/(8*a^2) - Unintegrable[ArcTan[a*x]^(5/2)/Sqrt[c + a^2*c*
x^2], x]/(2*a^2)
```

Rubi [A] time = 0.253795, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^2 \tan^{-1}(ax)^{5/2}}{\sqrt{c+a^2cx^2}} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^2*ArcTan[a*x]^(5/2))/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] (-5*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2))/(4*a^3*c) + (x*Sqrt[c + a^2*c*x^
2]*ArcTan[a*x]^(5/2))/(2*a^2*c) + (15*Defer[Int][Sqrt[ArcTan[a*x]]/Sqrt[c +
 a^2*c*x^2], x])/(8*a^2) - Defer[Int][ArcTan[a*x]^(5/2)/Sqrt[c + a^2*c*x^2]
, x]/(2*a^2)
```

Rubi steps

$$\int \frac{x^2 \tan^{-1}(ax)^{5/2}}{\sqrt{c+a^2cx^2}} \, dx = \frac{x\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{5/2}}{2a^2c}-\frac{\int \frac{\tan^{-1}(ax)^{5/2}}{\sqrt{c+a^2cx^2}} \, dx}{2a^2}-\frac{5\int \frac{x\tan^{-1}(ax)^{3/2}}{\sqrt{c+a^2cx^2}} \, dx}{4a}$$

$$=-\frac{5\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}}{4a^3c}+\frac{x\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{5/2}}{2a^2c}-\frac{\int \frac{\tan^{-1}(ax)^{5/2}}{\sqrt{c+a^2cx^2}} \, dx}{2a^2}+\frac{15\int \frac{\sqrt{\tan^{-1}(ax)}}{\sqrt{c+a^2cx^2}} \, dx}{8a^2}$$

Mathematica [A] time = 2.46559, size = 0, normalized size = 0.

$$\int \frac{x^2 \tan^{-1}(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^2*ArcTan[a*x]^(5/2))/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] Integrate[(x^2*ArcTan[a*x]^(5/2))/Sqrt[c + a^2*c*x^2], x]
```

Maple [A] time = 3.563, size = 0, normalized size = 0.

$$\int x^2 (\arctan(ax))^{\frac{5}{2}} \frac{1}{\sqrt{a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(a*x)**(5/2)/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \arctan (ax)^{\frac{5}{2}}}{\sqrt{a^2cx^2 + c}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctan(a*x)^(5/2)/sqrt(a^2*c*x^2 + c), x)
```

3.896 $\int \frac{x\tan^{-1}(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=62

$$\frac{\sqrt{a^2cx^2+c}\tan^{-1}(ax)^{5/2}}{a^2c}-\frac{5\text{Unintegrable}\left(\frac{\tan^{-1}(ax)^{3/2}}{\sqrt{a^2cx^2+c}},x\right)}{2a}$$

```
[Out] (Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2))/(a^2*c) - (5*Unintegrable[ArcTan[a*
x]^(3/2)/Sqrt[c + a^2*c*x^2], x])/(2*a)
```

Rubi [A] time = 0.109268, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x\tan^{-1}(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x*ArcTan[a*x]^(5/2))/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] (Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2))/(a^2*c) - (5*Defer[Int][ArcTan[a*x]
^(3/2)/Sqrt[c + a^2*c*x^2], x])/(2*a)
```

Rubi steps

$$\int \frac{x\tan^{-1}(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx=\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{5/2}}{a^2c}-\frac{5\int \frac{\tan^{-1}(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx}{2a}$$

Mathematica [A] time = 0.662033, size = 0, normalized size = 0.

$$\int \frac{x\tan^{-1}(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x*ArcTan[a*x]^(5/2))/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] Integrate[(x*ArcTan[a*x]^(5/2))/Sqrt[c + a^2*c*x^2], x]
```

Maple [A] time = 1.2, size = 0, normalized size = 0.

$$\int x (\arctan (ax))^{\frac{5}{2}} \frac{1}{\sqrt{a^2cx^2+c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(a*x)**(5/2)/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \arctan (ax)^{\frac{5}{2}}}{\sqrt{a^2cx^2 + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x*arctan(a*x)^(5/2)/sqrt(a^2*c*x^2 + c), x)
```

3.897 $\int \frac{\tan^{-1}(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=25

$$\text{Unintegrable}\left(\frac{\tan^{-1}(ax)^{5/2}}{\sqrt{a^2cx^2+c}},x\right)$$

```
[Out] Unintegrable[ArcTan[a*x]^(5/2)/Sqrt[c + a^2*c*x^2], x]
```

Rubi [A] time = 0.0349281, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\tan^{-1}(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcTan[a*x]^(5/2)/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] Defer[Int][ArcTan[a*x]^(5/2)/Sqrt[c + a^2*c*x^2], x]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx=\int \frac{\tan^{-1}(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx$$

Mathematica [A] time = 0.209275, size = 0, normalized size = 0.

$$\int \frac{\tan^{-1}(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcTan[a*x]^(5/2)/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] Integrate[ArcTan[a*x]^(5/2)/Sqrt[c + a^2*c*x^2], x]
```

Maple [A] time = 0.927, size = 0, normalized size = 0.

$$\int (\arctan (ax))^{\frac{5}{2}} \frac{1}{\sqrt{a^2 c x^2 + c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x)

[Out] int(arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")

[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(5/2)/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan{(ax)}^{\frac{5}{2}}}{\sqrt{a^2cx^2 + c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)^(5/2)/sqrt(a^2*c*x^2 + c), x)
```

3.898 $\int \frac{\tan^{-1}(ax)^{5/2}}{x\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(\frac{\tan^{-1}(ax)^{5/2}}{x\sqrt{a^2cx^2+c}},x\right)$$

```
[Out] Unintegrable[ArcTan[a*x]^(5/2)/(x*Sqrt[c + a^2*c*x^2]), x]
```

Rubi [A] time = 0.103126, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\tan^{-1}(ax)^{5/2}}{x\sqrt{c+a^2cx^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcTan[a*x]^(5/2)/(x*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] Defer[Int][ArcTan[a*x]^(5/2)/(x*Sqrt[c + a^2*c*x^2]), x]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^{5/2}}{x\sqrt{c+a^2cx^2}}\,dx = \int \frac{\tan^{-1}(ax)^{5/2}}{x\sqrt{c+a^2cx^2}}\,dx$$

Mathematica [A] time = 1.22339, size = 0, normalized size = 0.

$$\int \frac{\tan^{-1}(ax)^{5/2}}{x\sqrt{c+a^2cx^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcTan[a*x]^(5/2)/(x*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] Integrate[ArcTan[a*x]^(5/2)/(x*Sqrt[c + a^2*c*x^2]), x]
```

Maple [A] time = 1.101, size = 0, normalized size = 0.

$$\int \frac{1}{x} (\arctan (ax))^{\frac{5}{2}} \frac{1}{\sqrt{a^2cx^2 + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(5/2)/x/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(ax)^{\frac{5}{2}}}{\sqrt{a^2cx^2 + c}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)^(5/2)/(sqrt(a^2*c*x^2 + c)*x), x)
```

3.899 $\int \frac{\tan^{-1}(ax)^{5/2}}{x^2\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=64

$$\frac{5}{2}a\text{Unintegrable}\left(\frac{\tan^{-1}(ax)^{3/2}}{x\sqrt{a^2cx^2+c}},x\right)-\frac{\sqrt{a^2cx^2+c}\tan^{-1}(ax)^{5/2}}{cx}$$

```
[Out] -((Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2))/(c*x)) + (5*a*Unintegrable[ArcTan
[a*x]^(3/2)/(x*Sqrt[c + a^2*c*x^2]), x])/2
```

Rubi [A] time = 0.210198, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\tan^{-1}(ax)^{5/2}}{x^2\sqrt{c+a^2cx^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcTan[a*x]^(5/2)/(x^2*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] -((Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2))/(c*x)) + (5*a*Defer[Int][ArcTan[a
*x]^(3/2)/(x*Sqrt[c + a^2*c*x^2]), x])/2
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^{5/2}}{x^2\sqrt{c+a^2cx^2}}\,dx=-\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{5/2}}{cx}+\frac{1}{2}(5a)\int \frac{\tan^{-1}(ax)^{3/2}}{x\sqrt{c+a^2cx^2}}\,dx$$

Mathematica [A] time = 1.37024, size = 0, normalized size = 0.

$$\int \frac{\tan^{-1}(ax)^{5/2}}{x^2\sqrt{c+a^2cx^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcTan[a*x]^(5/2)/(x^2*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] Integrate[ArcTan[a*x]^(5/2)/(x^2*Sqrt[c + a^2*c*x^2]), x]
```

Maple [A] time = 0.918, size = 0, normalized size = 0.

$$\int \frac{1}{x^2} (\arctan (ax))^{\frac{5}{2}} \frac{1}{\sqrt{a^2cx^2 + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(5/2)/x^2/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(arctan(a*x)^(5/2)/x^2/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/x^2/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/x^2/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(5/2)/x**2/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(ax)^{\frac{5}{2}}}{\sqrt{a^2cx^2+c}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/x^2/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)^(5/2)/(sqrt(a^2*c*x^2 + c)*x^2), x)
```

3.900 $\int \frac{\tan^{-1}(ax)^{5/2}}{x^3\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=136

$$\frac{15}{8}a^2\text{Unintegrable}\left(\frac{\sqrt{\tan^{-1}(ax)}}{x\sqrt{a^2cx^2+c}},x\right)-\frac{1}{2}a^2\text{Unintegrable}\left(\frac{\tan^{-1}(ax)^{5/2}}{x\sqrt{a^2cx^2+c}},x\right)-\frac{\sqrt{a^2cx^2+c}\tan^{-1}(ax)^{5/2}}{2cx^2}-\frac{5a\sqrt{a^2cx^2+c}}{4c}$$

```
[Out] (-5*a*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2))/(4*c*x) - (Sqrt[c + a^2*c*x^2]
*ArcTan[a*x]^(5/2))/(2*c*x^2) + (15*a^2*Unintegrable[Sqrt[ArcTan[a*x]]/(x*S
qrt[c + a^2*c*x^2]), x])/8 - (a^2*Unintegrable[ArcTan[a*x]^(5/2)/(x*Sqrt[c
+ a^2*c*x^2]), x])/2
```

Rubi [A] time = 0.423259, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\tan^{-1}(ax)^{5/2}}{x^3\sqrt{c+a^2cx^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcTan[a*x]^(5/2)/(x^3*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] (-5*a*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2))/(4*c*x) - (Sqrt[c + a^2*c*x^2]
*ArcTan[a*x]^(5/2))/(2*c*x^2) + (15*a^2*Defer[Int][Sqrt[ArcTan[a*x]]/(x*Sqr
t[c + a^2*c*x^2]), x])/8 - (a^2*Defer[Int][ArcTan[a*x]^(5/2)/(x*Sqrt[c + a^
2*c*x^2]), x])/2
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^{5/2}}{x^3\sqrt{c+a^2cx^2}}\,dx=-\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{5/2}}{2cx^2}+\frac{1}{4}(5a)\int \frac{\tan^{-1}(ax)^{3/2}}{x^2\sqrt{c+a^2cx^2}}\,dx-\frac{1}{2}a^2\int \frac{\tan^{-1}(ax)^{5/2}}{x\sqrt{c+a^2cx^2}}\,dx$$

$$=-\frac{5a\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}}{4cx}-\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{5/2}}{2cx^2}-\frac{1}{2}a^2\int \frac{\tan^{-1}(ax)^{5/2}}{x\sqrt{c+a^2cx^2}}\,dx+\frac{1}{8}\left(15a^2\right)\int \frac{\sqrt{}}{x}$$

Mathematica [A] time = 4.51711, size = 0, normalized size = 0.

$$\int \frac{\tan^{-1}(ax)^{5/2}}{x^3\sqrt{c+a^2cx^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcTan[a*x]^(5/2)/(x^3*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] Integrate[ArcTan[a*x]^(5/2)/(x^3*Sqrt[c + a^2*c*x^2]), x]
```

Maple [A] time = 1.404, size = 0, normalized size = 0.

$$\int \frac{1}{x^3}\left(\arctan\left(ax\right)\right)^{\frac{5}{2}}\frac{1}{\sqrt{a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(5/2)/x^3/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(arctan(a*x)^(5/2)/x^3/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/x^3/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/x^3/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(5/2)/x**3/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(ax)^{\frac{5}{2}}}{\sqrt{a^2cx^2+c}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/x^3/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)^(5/2)/(sqrt(a^2*c*x^2 + c)*x^3), x)
```

3.901 $\int \frac{\tan^{-1}(ax)^{5/2}}{x^4\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=206

$$\frac{5}{16}a^3\text{Unintegrable}\left(\frac{1}{x\sqrt{a^2cx^2+c}\sqrt{\tan^{-1}(ax)}},x\right)-\frac{25}{12}a^3\text{Unintegrable}\left(\frac{\tan^{-1}(ax)^{3/2}}{x\sqrt{a^2cx^2+c}},x\right)+\frac{2a^2\sqrt{a^2cx^2+c}\tan^{-1}(ax)^{5/2}}{3cx}$$

```
[Out] (-5*a^2*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]])/(8*c*x) - (5*a*Sqrt[c + a^2*
c*x^2]*ArcTan[a*x]^(3/2))/(12*c*x^2) - (Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/
2))/(3*c*x^3) + (2*a^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2))/(3*c*x) + (5*
a^3*Unintegrable[1/(x*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]), x])/16 - (25*
a^3*Unintegrable[ArcTan[a*x]^(3/2)/(x*Sqrt[c + a^2*c*x^2]), x])/12
```

Rubi [A] time = 0.743608, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\tan^{-1}(ax)^{5/2}}{x^4\sqrt{c+a^2cx^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcTan[a*x]^(5/2)/(x^4*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] (-5*a^2*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]])/(8*c*x) - (5*a*Sqrt[c + a^2*
c*x^2]*ArcTan[a*x]^(3/2))/(12*c*x^2) - (Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/
2))/(3*c*x^3) + (2*a^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2))/(3*c*x) + (5*
a^3*Defer[Int][1/(x*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]), x])/16 - (25*a^
3*Defer[Int][ArcTan[a*x]^(3/2)/(x*Sqrt[c + a^2*c*x^2]), x])/12
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^{5/2}}{x^4\sqrt{c+a^2cx^2}}\,dx = -\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{5/2}}{3cx^3} + \frac{1}{6}(5a)\int \frac{\tan^{-1}(ax)^{3/2}}{x^3\sqrt{c+a^2cx^2}}\,dx - \frac{1}{3}\left(2a^2\right)\int \frac{\tan^{-1}(ax)^{5/2}}{x^2\sqrt{c+a^2cx^2}}\,dx$$

$$= -\frac{5a\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}}{12cx^2} - \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{5/2}}{3cx^3} + \frac{2a^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{5/2}}{3cx} + \frac{1}{8}\left(5a^2\right)\int$$

$$= -\frac{5a^2\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}}{8cx} - \frac{5a\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}}{12cx^2} - \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{5/2}}{3cx^3} + \frac{2a^2\sqrt{c+}}{}$$

Mathematica [A] time = 16.0537, size = 0, normalized size = 0.

$$\int \frac{\tan^{-1}(ax)^{5/2}}{x^4\sqrt{c+a^2cx^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcTan[a*x]^(5/2)/(x^4*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] Integrate[ArcTan[a*x]^(5/2)/(x^4*Sqrt[c + a^2*c*x^2]), x]
```

Maple [A] time = 3.984, size = 0, normalized size = 0.

$$\int \frac{1}{x^4}(\arctan(ax))^{\frac{5}{2}}\frac{1}{\sqrt{a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(5/2)/x^4/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(arctan(a*x)^(5/2)/x^4/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/x^4/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/x^4/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(5/2)/x**4/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(ax)^{\frac{5}{2}}}{\sqrt{a^2cx^2 + cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/x^4/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)^(5/2)/(sqrt(a^2*c*x^2 + c)*x^4), x)
```

3.902 $\int \frac{x^m \tan^{-1}(ax)^{5/2}}{\left(c+a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(\frac{x^m \tan^{-1}(ax)^{5/2}}{\left(a^2cx^2+c\right)^{3/2}},x\right)$$

```
[Out] Unintegrable[(x^m*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^(3/2), x]
```

Rubi [A] time = 0.113673, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m \tan^{-1}(ax)^{5/2}}{\left(c+a^2cx^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^(3/2),x]

[Out] Defer[Int][(x^m*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^(3/2), x]
```

Rubi steps

$$\int \frac{x^m \tan^{-1}(ax)^{5/2}}{\left(c+a^2cx^2\right)^{3/2}}\,dx=\int \frac{x^m \tan^{-1}(ax)^{5/2}}{\left(c+a^2cx^2\right)^{3/2}}\,dx$$

Mathematica [A] time = 0.93631, size = 0, normalized size = 0.

$$\int \frac{x^m \tan^{-1}(ax)^{5/2}}{\left(c+a^2cx^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] Integrate[(x^m*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^(3/2), x]
```

Maple [A] time = 1.059, size = 0, normalized size = 0.

$$\int x^m (\arctan(ax))^{\frac{5}{2}} \left(a^2cx^2 + c\right)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] int(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a^2cx^2 + c}x^m \arctan(ax)^{\frac{5}{2}}}{a^4c^2x^4 + 2a^2c^2x^2 + c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^m*arctan(a*x)^(5/2)/(a^4*c^2*x^4 + 2*a^2*c^2
*x^2 + c^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*atan(a*x)**(5/2)/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m \arctan(ax)^{\frac{5}{2}}}{(a^2cx^2 + c)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2 + c)^(3/2), x)
```

3.903 $\int \frac{x^2 \tan^{-1}(ax)^{5/2}}{\left(c+a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(\frac{x^2 \tan^{-1}(ax)^{5/2}}{\left(a^2cx^2+c\right)^{3/2}},x\right)$$

```
[Out] Unintegrable[(x^2*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^(3/2), x]
```

Rubi [A] time = 0.121548, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^2 \tan^{-1}(ax)^{5/2}}{\left(c+a^2cx^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^2*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^(3/2),x]

[Out] Defer[Int][(x^2*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^(3/2), x]
```

Rubi steps

$$\int \frac{x^2 \tan^{-1}(ax)^{5/2}}{\left(c+a^2cx^2\right)^{3/2}}\,dx=\int \frac{x^2 \tan^{-1}(ax)^{5/2}}{\left(c+a^2cx^2\right)^{3/2}}\,dx$$

Mathematica [A] time = 3.65366, size = 0, normalized size = 0.

$$\int \frac{x^2 \tan^{-1}(ax)^{5/2}}{\left(c+a^2cx^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^2*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] Integrate[(x^2*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^(3/2), x]
```

Maple [A] time = 2.978, size = 0, normalized size = 0.

$$\int x^2 (\arctan(ax))^{\frac{5}{2}} \left(a^2cx^2 + c\right)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] int(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(a*x)**(5/2)/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \arctan(ax)^{\frac{5}{2}}}{\left(a^2cx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2 + c)^(3/2), x)
```

3.904 $\int \frac{x\tan^{-1}(ax)^{5/2}}{\left(c+a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=161

$$-\frac{15\sqrt{\frac{\pi}{2}}\sqrt{a^2x^2+1}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{4a^2c\sqrt{a^2cx^2+c}}-\frac{\tan^{-1}(ax)^{5/2}}{a^2c\sqrt{a^2cx^2+c}}+\frac{5x\tan^{-1}(ax)^{3/2}}{2ac\sqrt{a^2cx^2+c}}+\frac{15\sqrt{\tan^{-1}(ax)}}{4a^2c\sqrt{a^2cx^2+c}}$$

```
[Out] (15*Sqrt[ArcTan[a*x]])/(4*a^2*c*Sqrt[c + a^2*c*x^2]) + (5*x*ArcTan[a*x]^(3/
2))/(2*a*c*Sqrt[c + a^2*c*x^2]) - ArcTan[a*x]^(5/2)/(a^2*c*Sqrt[c + a^2*c*x
^2]) - (15*Sqrt[Pi/2]*Sqrt[1 + a^2*x^2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcTan[a*x
]]])/(4*a^2*c*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.22684, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4930, 4898, 4905, 4904, 3304, 3352}

$$-\frac{15\sqrt{\frac{\pi}{2}}\sqrt{a^2x^2+1}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{4a^2c\sqrt{a^2cx^2+c}}-\frac{\tan^{-1}(ax)^{5/2}}{a^2c\sqrt{a^2cx^2+c}}+\frac{5x\tan^{-1}(ax)^{3/2}}{2ac\sqrt{a^2cx^2+c}}+\frac{15\sqrt{\tan^{-1}(ax)}}{4a^2c\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] (15*Sqrt[ArcTan[a*x]])/(4*a^2*c*Sqrt[c + a^2*c*x^2]) + (5*x*ArcTan[a*x]^(3/
2))/(2*a*c*Sqrt[c + a^2*c*x^2]) - ArcTan[a*x]^(5/2)/(a^2*c*Sqrt[c + a^2*c*x
^2]) - (15*Sqrt[Pi/2]*Sqrt[1 + a^2*x^2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcTan[a*x
]]])/(4*a^2*c*Sqrt[c + a^2*c*x^2])
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 4898

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[(b*p*(a + b*ArcTan[c*x])^(p - 1))/(c*d*Sqrt[d + e*x^2]), x]
 + (-Dist[b^2*p*(p - 1), Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2),
 x], x] + Simp[(x*(a + b*ArcTan[c*x])^p)/(d*Sqrt[d + e*x^2]), x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

Rule 4905

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[(d^(q + 1/2)*Sqrt[1 + c^2*x^2])/Sqrt[d + e*x^2], Int[(1 + c
^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

Rule 4904

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{x \tan^{-1}(ax)^{5/2}}{\left(c+a^2cx^2\right)^{3/2}}\,dx = -\frac{\tan^{-1}(ax)^{5/2}}{a^2c\sqrt{c+a^2cx^2}} + \frac{5\int \frac{\tan^{-1}(ax)^{3/2}}{\left(c+a^2cx^2\right)^{3/2}}\,dx}{2a}$$

$$= \frac{15\sqrt{\tan^{-1}(ax)}}{4a^2c\sqrt{c+a^2cx^2}} + \frac{5x\tan^{-1}(ax)^{3/2}}{2ac\sqrt{c+a^2cx^2}} - \frac{\tan^{-1}(ax)^{5/2}}{a^2c\sqrt{c+a^2cx^2}} - \frac{15\int \frac{1}{\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}}\,dx}{8a}$$

$$= \frac{15\sqrt{\tan^{-1}(ax)}}{4a^2c\sqrt{c+a^2cx^2}} + \frac{5x\tan^{-1}(ax)^{3/2}}{2ac\sqrt{c+a^2cx^2}} - \frac{\tan^{-1}(ax)^{5/2}}{a^2c\sqrt{c+a^2cx^2}} - \frac{\left(15\sqrt{1+a^2x^2}\right)\int \frac{1}{\left(1+a^2x^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}}\,dx}{8ac\sqrt{c+a^2cx^2}}$$

$$= \frac{15\sqrt{\tan^{-1}(ax)}}{4a^2c\sqrt{c+a^2cx^2}} + \frac{5x\tan^{-1}(ax)^{3/2}}{2ac\sqrt{c+a^2cx^2}} - \frac{\tan^{-1}(ax)^{5/2}}{a^2c\sqrt{c+a^2cx^2}} - \frac{\left(15\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int \frac{\cos(x)}{\sqrt{x}}\,dx, x, \tan^{-1}\right.}{8a^2c\sqrt{c+a^2cx^2}}$$

$$= \frac{15\sqrt{\tan^{-1}(ax)}}{4a^2c\sqrt{c+a^2cx^2}} + \frac{5x\tan^{-1}(ax)^{3/2}}{2ac\sqrt{c+a^2cx^2}} - \frac{\tan^{-1}(ax)^{5/2}}{a^2c\sqrt{c+a^2cx^2}} - \frac{\left(15\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int \cos\left(x^2\right)\,dx, x, \sqrt{\right.}{4a^2c\sqrt{c+a^2cx^2}}$$

$$= \frac{15\sqrt{\tan^{-1}(ax)}}{4a^2c\sqrt{c+a^2cx^2}} + \frac{5x\tan^{-1}(ax)^{3/2}}{2ac\sqrt{c+a^2cx^2}} - \frac{\tan^{-1}(ax)^{5/2}}{a^2c\sqrt{c+a^2cx^2}} - \frac{15\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}\,C\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{4a^2c\sqrt{c+a^2cx^2}}$$

Mathematica [C] time = 0.191129, size = 139, normalized size = 0.86

$$\frac{15i\sqrt{a^2x^2+1}\sqrt{-i\tan^{-1}(ax)}\text{Gamma}\left(\frac{1}{2},-i\tan^{-1}(ax)\right) - 15i\sqrt{a^2x^2+1}\sqrt{i\tan^{-1}(ax)}\text{Gamma}\left(\frac{1}{2},i\tan^{-1}(ax)\right) + 4\tan^{-1}(\ldots}{16a^2c\sqrt{a^2cx^2+c}\sqrt{\tan^{-1}(ax)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] (4*ArcTan[a*x]*(15 + 10*a*x*ArcTan[a*x] - 4*ArcTan[a*x]^2) + (15*I)*Sqrt[1
+ a^2*x^2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-I)*ArcTan[a*x]] - (15*I)*Sqr
t[1 + a^2*x^2]*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, I*ArcTan[a*x]])/(16*a^2*c*Sqr
t[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]])
```

Maple [F] time = 0.946, size = 0, normalized size = 0.

$$\int x\left(\arctan(ax)\right)^{\frac{5}{2}}\left(a^2cx^2+c\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] int(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(a*x)**(5/2)/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \arctan(ax)^{\frac{5}{2}}}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x*arctan(a*x)^(5/2)/(a^2*c*x^2 + c)^(3/2), x)
```

3.905 $\int \frac{\tan^{-1}(ax)^{5/2}}{\left(c+a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=155

$$\frac{15\sqrt{\frac{\pi}{2}}\sqrt{a^2x^2+1}S\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{4ac\sqrt{a^2cx^2+c}}+\frac{x\tan^{-1}(ax)^{5/2}}{c\sqrt{a^2cx^2+c}}+\frac{5\tan^{-1}(ax)^{3/2}}{2ac\sqrt{a^2cx^2+c}}-\frac{15x\sqrt{\tan^{-1}(ax)}}{4c\sqrt{a^2cx^2+c}}$$

```
[Out] (-15*x*Sqrt[ArcTan[a*x]])/(4*c*Sqrt[c + a^2*c*x^2]) + (5*ArcTan[a*x]^(3/2))
/(2*a*c*Sqrt[c + a^2*c*x^2]) + (x*ArcTan[a*x]^(5/2))/(c*Sqrt[c + a^2*c*x^2]
) + (15*Sqrt[Pi/2]*Sqrt[1 + a^2*x^2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]]
)/(4*a*c*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.158385, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4898, 4905, 4904, 3296, 3305, 3351}

$$\frac{15\sqrt{\frac{\pi}{2}}\sqrt{a^2x^2+1}S\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{4ac\sqrt{a^2cx^2+c}}+\frac{x\tan^{-1}(ax)^{5/2}}{c\sqrt{a^2cx^2+c}}+\frac{5\tan^{-1}(ax)^{3/2}}{2ac\sqrt{a^2cx^2+c}}-\frac{15x\sqrt{\tan^{-1}(ax)}}{4c\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^(5/2)/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] (-15*x*Sqrt[ArcTan[a*x]])/(4*c*Sqrt[c + a^2*c*x^2]) + (5*ArcTan[a*x]^(3/2))
/(2*a*c*Sqrt[c + a^2*c*x^2]) + (x*ArcTan[a*x]^(5/2))/(c*Sqrt[c + a^2*c*x^2]
) + (15*Sqrt[Pi/2]*Sqrt[1 + a^2*x^2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]]
)/(4*a*c*Sqrt[c + a^2*c*x^2])
```

Rule 4898

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[(b*p*(a + b*ArcTan[c*x])^(p - 1))/(c*d*Sqrt[d + e*x^2]), x]
 + (-Dist[b^2*p*(p - 1), Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2),
 x], x] + Simp[(x*(a + b*ArcTan[c*x])^p)/(d*Sqrt[d + e*x^2]), x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

Rule 4905

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[(d^(q + 1/2)*Sqrt[1 + c^2*x^2])/Sqrt[d + e*x^2], Int[(1 + c
^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

Rule 4904

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^{5/2}}{\left(c+a^2cx^2\right)^{3/2}}\,dx = \frac{5\tan^{-1}(ax)^{3/2}}{2ac\sqrt{c+a^2cx^2}} + \frac{x\tan^{-1}(ax)^{5/2}}{c\sqrt{c+a^2cx^2}} - \frac{15}{4}\int \frac{\sqrt{\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{3/2}}\,dx$$

$$= \frac{5\tan^{-1}(ax)^{3/2}}{2ac\sqrt{c+a^2cx^2}} + \frac{x\tan^{-1}(ax)^{5/2}}{c\sqrt{c+a^2cx^2}} - \frac{\left(15\sqrt{1+a^2x^2}\right)\int \frac{\sqrt{\tan^{-1}(ax)}}{\left(1+a^2x^2\right)^{3/2}}\,dx}{4c\sqrt{c+a^2cx^2}}$$

$$= \frac{5\tan^{-1}(ax)^{3/2}}{2ac\sqrt{c+a^2cx^2}} + \frac{x\tan^{-1}(ax)^{5/2}}{c\sqrt{c+a^2cx^2}} - \frac{\left(15\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int \sqrt{x}\cos(x)\,dx, x, \tan^{-1}(ax)\right)}{4ac\sqrt{c+a^2cx^2}}$$

$$= -\frac{15x\sqrt{\tan^{-1}(ax)}}{4c\sqrt{c+a^2cx^2}} + \frac{5\tan^{-1}(ax)^{3/2}}{2ac\sqrt{c+a^2cx^2}} + \frac{x\tan^{-1}(ax)^{5/2}}{c\sqrt{c+a^2cx^2}} + \frac{\left(15\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int \frac{\sin(x)}{\sqrt{x}}\,dx, x, \tan\right.}{8ac\sqrt{c+a^2cx^2}}$$

$$= -\frac{15x\sqrt{\tan^{-1}(ax)}}{4c\sqrt{c+a^2cx^2}} + \frac{5\tan^{-1}(ax)^{3/2}}{2ac\sqrt{c+a^2cx^2}} + \frac{x\tan^{-1}(ax)^{5/2}}{c\sqrt{c+a^2cx^2}} + \frac{\left(15\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int \sin\left(x^2\right)\,dx, x, \right.}{4ac\sqrt{c+a^2cx^2}}$$

$$= -\frac{15x\sqrt{\tan^{-1}(ax)}}{4c\sqrt{c+a^2cx^2}} + \frac{5\tan^{-1}(ax)^{3/2}}{2ac\sqrt{c+a^2cx^2}} + \frac{x\tan^{-1}(ax)^{5/2}}{c\sqrt{c+a^2cx^2}} + \frac{15\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}S\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{4ac\sqrt{c+a^2cx^2}}$$

Mathematica [C] time = 0.150204, size = 94, normalized size = 0.61

$$-\frac{\left(a^2x^2+1\right)^{3/2}\left(\sqrt{-i\tan^{-1}(ax)}\text{Gamma}\left(\frac{7}{2},-i\tan^{-1}(ax)\right)+\sqrt{i\tan^{-1}(ax)}\text{Gamma}\left(\frac{7}{2},i\tan^{-1}(ax)\right)\right)}{2a\left(c\left(a^2x^2+1\right)\right)^{3/2}\sqrt{\tan^{-1}(ax)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTan[a*x]^(5/2)/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] -((1 + a^2*x^2)^(3/2)*(Sqrt[(-I)*ArcTan[a*x]]*Gamma[7/2, (-I)*ArcTan[a*x]]
+ Sqrt[I*ArcTan[a*x]]*Gamma[7/2, I*ArcTan[a*x]]))/(2*a*(c*(1 + a^2*x^2))^(3
/2)*Sqrt[ArcTan[a*x]])
```

Maple [F] time = 0.693, size = 0, normalized size = 0.

$$\int (\arctan(ax))^{\frac{5}{2}}\left(a^2cx^2+c\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] int(arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(5/2)/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(ax)^{\frac{5}{2}}}{\left(a^2cx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)^(5/2)/(a^2*c*x^2 + c)^(3/2), x)
```

3.906 $\int \frac{\tan^{-1}(ax)^{5/2}}{x\left(c+a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(\frac{\tan^{-1}(ax)^{5/2}}{x\left(a^2cx^2+c\right)^{3/2}},x\right)$$

```
[Out] Unintegrable[ArcTan[a*x]^(5/2)/(x*(c + a^2*c*x^2)^(3/2)), x]
```

Rubi [A] time = 0.119166, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\tan^{-1}(ax)^{5/2}}{x\left(c+a^2cx^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcTan[a*x]^(5/2)/(x*(c + a^2*c*x^2)^(3/2)),x]

[Out] Defer[Int][ArcTan[a*x]^(5/2)/(x*(c + a^2*c*x^2)^(3/2)), x]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^{5/2}}{x\left(c+a^2cx^2\right)^{3/2}}\,dx=\int \frac{\tan^{-1}(ax)^{5/2}}{x\left(c+a^2cx^2\right)^{3/2}}\,dx$$

Mathematica [A] time = 2.22746, size = 0, normalized size = 0.

$$\int \frac{\tan^{-1}(ax)^{5/2}}{x\left(c+a^2cx^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcTan[a*x]^(5/2)/(x*(c + a^2*c*x^2)^(3/2)),x]
```

```
[Out] Integrate[ArcTan[a*x]^(5/2)/(x*(c + a^2*c*x^2)^(3/2)), x]
```

Maple [A] time = 0.757, size = 0, normalized size = 0.

$$\int \frac{1}{x} (\arctan (ax))^{\frac{5}{2}} \left(a^{2}c x^{2}+c\right)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] int(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(5/2)/x/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan (ax)^{\frac{5}{2}}}{\left(a^2cx^2+c\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)^(5/2)/((a^2*c*x^2 + c)^(3/2)*x), x)
```

3.907 $\int \frac{x^m \tan^{-1}(ax)^{5/2}}{\left(c+a^2cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(\frac{x^m \tan^{-1}(ax)^{5/2}}{\left(a^2cx^2+c\right)^{5/2}},x\right)$$

```
[Out] Unintegrable[(x^m*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^(5/2), x]
```

Rubi [A] time = 0.116672, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m \tan^{-1}(ax)^{5/2}}{\left(c+a^2cx^2\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] Defer[Int][(x^m*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^(5/2), x]
```

Rubi steps

$$\int \frac{x^m \tan^{-1}(ax)^{5/2}}{\left(c+a^2cx^2\right)^{5/2}}\,dx = \int \frac{x^m \tan^{-1}(ax)^{5/2}}{\left(c+a^2cx^2\right)^{5/2}}\,dx$$

Mathematica [A] time = 1.40564, size = 0, normalized size = 0.

$$\int \frac{x^m \tan^{-1}(ax)^{5/2}}{\left(c+a^2cx^2\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] Integrate[(x^m*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^(5/2), x]
```

Maple [A] time = 0.953, size = 0, normalized size = 0.

$$\int x^m (\arctan(ax))^{\frac{5}{2}} \left(a^2cx^2 + c\right)^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x)
```

```
[Out] int(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a^2cx^2 + c}x^m \arctan(ax)^{\frac{5}{2}}}{a^6c^3x^6 + 3\,a^4c^3x^4 + 3\,a^2c^3x^2 + c^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^m*arctan(a*x)^(5/2)/(a^6*c^3*x^6 + 3*a^4*c^3
*x^4 + 3*a^2*c^3*x^2 + c^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*atan(a*x)**(5/2)/(a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m \arctan{(ax)}^{\frac{5}{2}}}{\left(a^2cx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2 + c)^(5/2), x)
```

3.908 $\int \frac{x^4 \tan^{-1}(ax)^{5/2}}{\left(c+a^2cx^2\right)^{5/2}} dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(\frac{x^4 \tan^{-1}(ax)^{5/2}}{\left(a^2cx^2+c\right)^{5/2}},x\right)$$

```
[Out] Unintegrable[(x^4*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^(5/2), x]
```

Rubi [A] time = 0.118527, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^4 \tan^{-1}(ax)^{5/2}}{\left(c+a^2cx^2\right)^{5/2}} dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^4*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] Defer[Int][(x^4*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^(5/2), x]
```

Rubi steps

$$\int \frac{x^4 \tan^{-1}(ax)^{5/2}}{\left(c+a^2cx^2\right)^{5/2}} dx = \int \frac{x^4 \tan^{-1}(ax)^{5/2}}{\left(c+a^2cx^2\right)^{5/2}} dx$$

Mathematica [A] time = 4.12198, size = 0, normalized size = 0.

$$\int \frac{x^4 \tan^{-1}(ax)^{5/2}}{\left(c+a^2cx^2\right)^{5/2}} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^4*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] Integrate[(x^4*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^(5/2), x]
```

Maple [A] time = 2.063, size = 0, normalized size = 0.

$$\int x^4 (\arctan (ax))^{\frac{5}{2}} \left(a^2cx^2 + c\right)^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x)
```

```
[Out] int(x^4*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*atan(a*x)**(5/2)/(a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4 \arctan(ax)^{\frac{5}{2}}}{\left(a^2cx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x^4*arctan(a*x)^(5/2)/(a^2*c*x^2 + c)^(5/2), x)
```

3.909 $\int \frac{x^3 \tan^{-1}(ax)^{5/2}}{\left(c+a^2cx^2\right)^{5/2}} \, dx$

Optimal. Leaf size=350

$$-\frac{45\sqrt{\frac{\pi}{2}}\sqrt{a^2x^2+1}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{16a^4c^2\sqrt{a^2cx^2+c}}+\frac{5\sqrt{\frac{\pi}{6}}\sqrt{a^2x^2+1}\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{144a^4c^2\sqrt{a^2cx^2+c}}+\frac{5x\tan^{-1}(ax)^{3/2}}{3a^3c^2\sqrt{a^2cx^2+c}}-$$

```
[Out] (45*Sqrt[ArcTan[a*x]])/(16*a^4*c^2*Sqrt[c + a^2*c*x^2]) + (5*x^3*ArcTan[a*x
]^(3/2))/(18*a*c*(c + a^2*c*x^2)^(3/2)) + (5*x*ArcTan[a*x]^(3/2))/(3*a^3*c^
2*Sqrt[c + a^2*c*x^2]) - (x^2*ArcTan[a*x]^(5/2))/(3*a^2*c*(c + a^2*c*x^2)^(
3/2)) - (2*ArcTan[a*x]^(5/2))/(3*a^4*c^2*Sqrt[c + a^2*c*x^2]) - (5*Sqrt[1 +
 a^2*x^2]*Sqrt[ArcTan[a*x]]*Cos[3*ArcTan[a*x]])/(144*a^4*c^2*Sqrt[c + a^2*c
*x^2]) - (45*Sqrt[Pi/2]*Sqrt[1 + a^2*x^2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcTan[a
*x]]])/(16*a^4*c^2*Sqrt[c + a^2*c*x^2]) + (5*Sqrt[Pi/6]*Sqrt[1 + a^2*x^2]*F
resnelC[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]])/(144*a^4*c^2*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.708065, antiderivative size = 350, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 11, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {4940, 4930, 4898, 4905, 4904, 3304, 3352, 4971, 4970, 3312, 3296}

$$-\frac{45\sqrt{\frac{\pi}{2}}\sqrt{a^2x^2+1}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{16a^4c^2\sqrt{a^2cx^2+c}}+\frac{5\sqrt{\frac{\pi}{6}}\sqrt{a^2x^2+1}\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{144a^4c^2\sqrt{a^2cx^2+c}}+\frac{5x\tan^{-1}(ax)^{3/2}}{3a^3c^2\sqrt{a^2cx^2+c}}-$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^(5/2),x]
```


```
[Out] (45*Sqrt[ArcTan[a*x]])/(16*a^4*c^2*Sqrt[c + a^2*c*x^2]) + (5*x^3*ArcTan[a*x
]^(3/2))/(18*a*c*(c + a^2*c*x^2)^(3/2)) + (5*x*ArcTan[a*x]^(3/2))/(3*a^3*c^
2*Sqrt[c + a^2*c*x^2]) - (x^2*ArcTan[a*x]^(5/2))/(3*a^2*c*(c + a^2*c*x^2)^(
3/2)) - (2*ArcTan[a*x]^(5/2))/(3*a^4*c^2*Sqrt[c + a^2*c*x^2]) - (5*Sqrt[1 +
 a^2*x^2]*Sqrt[ArcTan[a*x]]*Cos[3*ArcTan[a*x]])/(144*a^4*c^2*Sqrt[c + a^2*c
*x^2]) - (45*Sqrt[Pi/2]*Sqrt[1 + a^2*x^2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcTan[a
*x]]])/(16*a^4*c^2*Sqrt[c + a^2*c*x^2]) + (5*Sqrt[Pi/6]*Sqrt[1 + a^2*x^2]*F
resnelC[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]])/(144*a^4*c^2*Sqrt[c + a^2*c*x^2])
```


Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_)*((d_) + (e_.)
*(x_)^2)^(q_), x_Symbol] :> Simp[(b*p*(f*x)^m*(d + e*x^2)^(q + 1)*(a + b*Ar
cTan[c*x])^(p - 1))/(c*d*m^2), x] + (Dist[(f^2*(m - 1))/(c^2*d*m), Int[(f*x
)^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p, x], x] - Dist[(b^2*p*(
p - 1))/m^2, Int[(f*x)^m*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x]
- Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(c^2*d*m
), x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q +
 2, 0] && LtQ[q, -1] && GtQ[p, 1]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 4898

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[(b*p*(a + b*ArcTan[c*x])^(p - 1))/(c*d*Sqrt[d + e*x^2]), x]
 + (-Dist[b^2*p*(p - 1), Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2),
 x], x] + Simp[(x*(a + b*ArcTan[c*x])^p)/(d*Sqrt[d + e*x^2]), x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

Rule 4905

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[(d^(q + 1/2)*Sqrt[1 + c^2*x^2])/Sqrt[d + e*x^2], Int[(1 + c
^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

Rule 4904

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4971

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[(d^(q + 1/2)*Sqrt[1 + c^2*x^2])/Sqrt[d + e*x^2],
 Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d
, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(In
tegerQ[q] || GtQ[d, 0])
```

Rule 4970

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sin[x]^m)/
Cos[x]^(m + 2*(q + 1)), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^3\tan^{-1}(ax)^{5/2}}{\left(c+a^2cx^2\right)^{5/2}}\,dx = \frac{5x^3\tan^{-1}(ax)^{3/2}}{18ac\left(c+a^2cx^2\right)^{3/2}} - \frac{x^2\tan^{-1}(ax)^{5/2}}{3a^2c\left(c+a^2cx^2\right)^{3/2}} - \frac{5}{12}\int \frac{x^3\sqrt{\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{5/2}}\,dx + \frac{2\int \frac{x\tan^{-1}(ax)^{5/2}}{\left(c+a^2cx^2\right)^{3/2}}\,dx}{3a^2c}$$

$$= \frac{5x^3\tan^{-1}(ax)^{3/2}}{18ac\left(c+a^2cx^2\right)^{3/2}} - \frac{x^2\tan^{-1}(ax)^{5/2}}{3a^2c\left(c+a^2cx^2\right)^{3/2}} - \frac{2\tan^{-1}(ax)^{5/2}}{3a^4c^2\sqrt{c+a^2cx^2}} + \frac{5\int \frac{\tan^{-1}(ax)^{3/2}}{\left(c+a^2cx^2\right)^{3/2}}\,dx}{3a^3c} - \frac{\left(5\sqrt{1+a^2x^2}\cdots\right.}{12c^2\cdots}$$

$$= \frac{5\sqrt{\tan^{-1}(ax)}}{2a^4c^2\sqrt{c+a^2cx^2}} + \frac{5x^3\tan^{-1}(ax)^{3/2}}{18ac\left(c+a^2cx^2\right)^{3/2}} + \frac{5x\tan^{-1}(ax)^{3/2}}{3a^3c^2\sqrt{c+a^2cx^2}} - \frac{x^2\tan^{-1}(ax)^{5/2}}{3a^2c\left(c+a^2cx^2\right)^{3/2}} - \frac{2\tan^{-1}(ax)\cdots}{3a^4c^2\sqrt{c+a^2\cdots}}$$

$$= \frac{5\sqrt{\tan^{-1}(ax)}}{2a^4c^2\sqrt{c+a^2cx^2}} + \frac{5x^3\tan^{-1}(ax)^{3/2}}{18ac\left(c+a^2cx^2\right)^{3/2}} + \frac{5x\tan^{-1}(ax)^{3/2}}{3a^3c^2\sqrt{c+a^2cx^2}} - \frac{x^2\tan^{-1}(ax)^{5/2}}{3a^2c\left(c+a^2cx^2\right)^{3/2}} - \frac{2\tan^{-1}(ax)\cdots}{3a^4c^2\sqrt{c+a^2\cdots}}$$

$$= \frac{5\sqrt{\tan^{-1}(ax)}}{2a^4c^2\sqrt{c+a^2cx^2}} + \frac{5x^3\tan^{-1}(ax)^{3/2}}{18ac\left(c+a^2cx^2\right)^{3/2}} + \frac{5x\tan^{-1}(ax)^{3/2}}{3a^3c^2\sqrt{c+a^2cx^2}} - \frac{x^2\tan^{-1}(ax)^{5/2}}{3a^2c\left(c+a^2cx^2\right)^{3/2}} - \frac{2\tan^{-1}(ax)\cdots}{3a^4c^2\sqrt{c+a^2\cdots}}$$

$$= \frac{45\sqrt{\tan^{-1}(ax)}}{16a^4c^2\sqrt{c+a^2cx^2}} + \frac{5x^3\tan^{-1}(ax)^{3/2}}{18ac\left(c+a^2cx^2\right)^{3/2}} + \frac{5x\tan^{-1}(ax)^{3/2}}{3a^3c^2\sqrt{c+a^2cx^2}} - \frac{x^2\tan^{-1}(ax)^{5/2}}{3a^2c\left(c+a^2cx^2\right)^{3/2}} - \frac{2\tan^{-1}(a\cdots}{3a^4c^2\sqrt{c+\cdots}}$$

$$= \frac{45\sqrt{\tan^{-1}(ax)}}{16a^4c^2\sqrt{c+a^2cx^2}} + \frac{5x^3\tan^{-1}(ax)^{3/2}}{18ac\left(c+a^2cx^2\right)^{3/2}} + \frac{5x\tan^{-1}(ax)^{3/2}}{3a^3c^2\sqrt{c+a^2cx^2}} - \frac{x^2\tan^{-1}(ax)^{5/2}}{3a^2c\left(c+a^2cx^2\right)^{3/2}} - \frac{2\tan^{-1}(a\cdots}{3a^4c^2\sqrt{c+\cdots}}$$

$$= \frac{45\sqrt{\tan^{-1}(ax)}}{16a^4c^2\sqrt{c+a^2cx^2}} + \frac{5x^3\tan^{-1}(ax)^{3/2}}{18ac\left(c+a^2cx^2\right)^{3/2}} + \frac{5x\tan^{-1}(ax)^{3/2}}{3a^3c^2\sqrt{c+a^2cx^2}} - \frac{x^2\tan^{-1}(ax)^{5/2}}{3a^2c\left(c+a^2cx^2\right)^{3/2}} - \frac{2\tan^{-1}(a\cdots}{3a^4c^2\sqrt{c+\cdots}}$$

Mathematica [C] time = 0.553307, size = 370, normalized size = 1.06

$$\frac{-5ia^2x^2\sqrt{3a^2x^2+3}\sqrt{-i\tan^{-1}(ax)}\,\mathrm{Gamma}\left(\frac{1}{2},-3i\tan^{-1}(ax)\right)+5ia^2x^2\sqrt{3a^2x^2+3}\sqrt{i\tan^{-1}(ax)}\,\mathrm{Gamma}\left(\frac{1}{2},3i\tan^{-1}(ax\cdots\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] (4800*ArcTan[a*x] + 5040*a^2*x^2*ArcTan[a*x] + 2880*a*x*ArcTan[a*x]^2 + 336
0*a^3*x^3*ArcTan[a*x]^2 - 1152*ArcTan[a*x]^3 - 1728*a^2*x^2*ArcTan[a*x]^3 +
 (1215*I)*(1 + a^2*x^2)^(3/2)*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-I)*ArcTan
[a*x]] - (1215*I)*(1 + a^2*x^2)^(3/2)*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, I*ArcT
an[a*x]] - (5*I)*Sqrt[3 + 3*a^2*x^2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-3*
I)*ArcTan[a*x]] - (5*I)*a^2*x^2*Sqrt[3 + 3*a^2*x^2]*Sqrt[(-I)*ArcTan[a*x]]*
Gamma[1/2, (-3*I)*ArcTan[a*x]] + (5*I)*Sqrt[3 + 3*a^2*x^2]*Sqrt[I*ArcTan[a*
x]]*Gamma[1/2, (3*I)*ArcTan[a*x]] + (5*I)*a^2*x^2*Sqrt[3 + 3*a^2*x^2]*Sqrt[
I*ArcTan[a*x]]*Gamma[1/2, (3*I)*ArcTan[a*x]])/(1728*a^4*c^2*(1 + a^2*x^2)*S
qrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]])
```

Maple [F] time = 3.497, size = 0, normalized size = 0.

$$\int x^3 (\arctan (ax))^{\frac{5}{2}} \left(a^2cx^2+c\right)^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x)
```

```
[Out] int(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atan(a*x)**(5/2)/(a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \arctan (ax)^{\frac{5}{2}}}{\left(a^2cx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2 + c)^(5/2), x)
```

3.910 $\int \frac{x^2 \tan^{-1}(ax)^{5/2}}{\left(c+a^2cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=295

$$\frac{15\sqrt{\frac{\pi}{2}}\sqrt{a^2x^2+1}S\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{16a^3c^2\sqrt{a^2cx^2+c}}-\frac{5\sqrt{\frac{\pi}{6}}\sqrt{a^2x^2+1}S\left(\sqrt{\frac{6}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{144a^3c^2\sqrt{a^2cx^2+c}}-\frac{5x\sqrt{\tan^{-1}(ax)}}{6a^2c^2\sqrt{a^2cx^2+c}}+\frac{5\tan^{-1}(ax)^{3/2}}{9a^3c^2\sqrt{a^2cx^2+c}}+$$

```
[Out] (-5*x^3*Sqrt[ArcTan[a*x]])/(36*c*(c + a^2*c*x^2)^(3/2)) - (5*x*Sqrt[ArcTan[
a*x]])/(6*a^2*c^2*Sqrt[c + a^2*c*x^2]) + (5*x^2*ArcTan[a*x]^(3/2))/(18*a*c*
(c + a^2*c*x^2)^(3/2)) + (5*ArcTan[a*x]^(3/2))/(9*a^3*c^2*Sqrt[c + a^2*c*x^
2]) + (x^3*ArcTan[a*x]^(5/2))/(3*c*(c + a^2*c*x^2)^(3/2)) + (15*Sqrt[Pi/2]*
Sqrt[1 + a^2*x^2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(16*a^3*c^2*Sqrt[
c + a^2*c*x^2]) - (5*Sqrt[Pi/6]*Sqrt[1 + a^2*x^2]*FresnelS[Sqrt[6/Pi]*Sqrt[
ArcTan[a*x]]])/(144*a^3*c^2*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.775387, antiderivative size = 295, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 11, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {4944, 4940, 4930, 4905, 4904, 3296, 3305, 3351, 4971, 4970, 3312}

$$\frac{15\sqrt{\frac{\pi}{2}}\sqrt{a^2x^2+1}S\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{16a^3c^2\sqrt{a^2cx^2+c}}-\frac{5\sqrt{\frac{\pi}{6}}\sqrt{a^2x^2+1}S\left(\sqrt{\frac{6}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{144a^3c^2\sqrt{a^2cx^2+c}}-\frac{5x\sqrt{\tan^{-1}(ax)}}{6a^2c^2\sqrt{a^2cx^2+c}}+\frac{5\tan^{-1}(ax)^{3/2}}{9a^3c^2\sqrt{a^2cx^2+c}}+$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] (-5*x^3*Sqrt[ArcTan[a*x]])/(36*c*(c + a^2*c*x^2)^(3/2)) - (5*x*Sqrt[ArcTan[
a*x]])/(6*a^2*c^2*Sqrt[c + a^2*c*x^2]) + (5*x^2*ArcTan[a*x]^(3/2))/(18*a*c*
(c + a^2*c*x^2)^(3/2)) + (5*ArcTan[a*x]^(3/2))/(9*a^3*c^2*Sqrt[c + a^2*c*x^
2]) + (x^3*ArcTan[a*x]^(5/2))/(3*c*(c + a^2*c*x^2)^(3/2)) + (15*Sqrt[Pi/2]*
Sqrt[1 + a^2*x^2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(16*a^3*c^2*Sqrt[
c + a^2*c*x^2]) - (5*Sqrt[Pi/6]*Sqrt[1 + a^2*x^2]*FresnelS[Sqrt[6/Pi]*Sqrt[
ArcTan[a*x]]])/(144*a^3*c^2*Sqrt[c + a^2*c*x^2])
```

Rule 4944

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a +
```

```
b*ArcTan[c*x])^p)/(d*f*(m + 1)), x] - Dist[(b*c*p)/(f*(m + 1)), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_)*((d_) + (e_.)
*(x_)^2)^(q_), x_Symbol] :> Simp[(b*p*(f*x)^m*(d + e*x^2)^(q + 1)*(a + b*Ar
cTan[c*x])^(p - 1))/(c*d*m^2), x] + (Dist[(f^2*(m - 1))/(c^2*d*m), Int[(f*x
)^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p, x], x] - Dist[(b^2*p*(
p - 1))/m^2, Int[(f*x)^m*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x]
- Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(c^2*d*m
), x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q +
 2, 0] && LtQ[q, -1] && GtQ[p, 1]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 4905

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[(d^(q + 1/2)*Sqrt[1 + c^2*x^2])/Sqrt[d + e*x^2], Int[(1 + c
^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

Rule 4904

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4971

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[(d^(q + 1/2)*Sqrt[1 + c^2*x^2])/Sqrt[d + e*x^2],
 Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d
, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(In
tegerQ[q] || GtQ[d, 0])
```

Rule 4970

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sin[x]^m)/
Cos[x]^(m + 2*(q + 1)), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\int \frac{x^2 \tan^{-1}(ax)^{5/2}}{\left(c+a^2cx^2\right)^{5/2}}\,dx = \frac{x^3 \tan^{-1}(ax)^{5/2}}{3c\left(c+a^2cx^2\right)^{3/2}} - \frac{1}{6}(5a)\int \frac{x^3 \tan^{-1}(ax)^{3/2}}{\left(c+a^2cx^2\right)^{5/2}}\,dx$$

$$= -\frac{5x^3\sqrt{\tan^{-1}(ax)}}{36c\left(c+a^2cx^2\right)^{3/2}} + \frac{5x^2\tan^{-1}(ax)^{3/2}}{18ac\left(c+a^2cx^2\right)^{3/2}} + \frac{x^3\tan^{-1}(ax)^{5/2}}{3c\left(c+a^2cx^2\right)^{3/2}} + \frac{1}{72}(5a)\int \frac{x^3}{\left(c+a^2cx^2\right)^{5/2}\sqrt{\tan^{-1}\cdots}}$$

$$= -\frac{5x^3\sqrt{\tan^{-1}(ax)}}{36c\left(c+a^2cx^2\right)^{3/2}} + \frac{5x^2\tan^{-1}(ax)^{3/2}}{18ac\left(c+a^2cx^2\right)^{3/2}} + \frac{5\tan^{-1}(ax)^{3/2}}{9a^3c^2\sqrt{c+a^2cx^2}} + \frac{x^3\tan^{-1}(ax)^{5/2}}{3c\left(c+a^2cx^2\right)^{3/2}} - \frac{5\int \frac{\sqrt{\tan^{-1}(a\cdots}}{(c+a^2cx^2)^{3\cdots}}}{6a^2c}$$

$$= -\frac{5x^3\sqrt{\tan^{-1}(ax)}}{36c\left(c+a^2cx^2\right)^{3/2}} + \frac{5x^2\tan^{-1}(ax)^{3/2}}{18ac\left(c+a^2cx^2\right)^{3/2}} + \frac{5\tan^{-1}(ax)^{3/2}}{9a^3c^2\sqrt{c+a^2cx^2}} + \frac{x^3\tan^{-1}(ax)^{5/2}}{3c\left(c+a^2cx^2\right)^{3/2}} + \frac{\left(5\sqrt{1+a^2x^2}\cdots\right.}{\cdots}$$

$$= -\frac{5x^3\sqrt{\tan^{-1}(ax)}}{36c\left(c+a^2cx^2\right)^{3/2}} + \frac{5x^2\tan^{-1}(ax)^{3/2}}{18ac\left(c+a^2cx^2\right)^{3/2}} + \frac{5\tan^{-1}(ax)^{3/2}}{9a^3c^2\sqrt{c+a^2cx^2}} + \frac{x^3\tan^{-1}(ax)^{5/2}}{3c\left(c+a^2cx^2\right)^{3/2}} + \frac{\left(5\sqrt{1+a^2x^2}\cdots\right.}{\cdots}$$

$$= -\frac{5x^3\sqrt{\tan^{-1}(ax)}}{36c\left(c+a^2cx^2\right)^{3/2}} - \frac{5x\sqrt{\tan^{-1}(ax)}}{6a^2c^2\sqrt{c+a^2cx^2}} + \frac{5x^2\tan^{-1}(ax)^{3/2}}{18ac\left(c+a^2cx^2\right)^{3/2}} + \frac{5\tan^{-1}(ax)^{3/2}}{9a^3c^2\sqrt{c+a^2cx^2}} + \frac{x^3\tan^{-1}(a\cdots}{3c\left(c+a^2cx\cdots\right.}$$

$$= -\frac{5x^3\sqrt{\tan^{-1}(ax)}}{36c\left(c+a^2cx^2\right)^{3/2}} - \frac{5x\sqrt{\tan^{-1}(ax)}}{6a^2c^2\sqrt{c+a^2cx^2}} + \frac{5x^2\tan^{-1}(ax)^{3/2}}{18ac\left(c+a^2cx^2\right)^{3/2}} + \frac{5\tan^{-1}(ax)^{3/2}}{9a^3c^2\sqrt{c+a^2cx^2}} + \frac{x^3\tan^{-1}(a\cdots}{3c\left(c+a^2cx\cdots\right.}$$

$$= -\frac{5x^3\sqrt{\tan^{-1}(ax)}}{36c\left(c+a^2cx^2\right)^{3/2}} - \frac{5x\sqrt{\tan^{-1}(ax)}}{6a^2c^2\sqrt{c+a^2cx^2}} + \frac{5x^2\tan^{-1}(ax)^{3/2}}{18ac\left(c+a^2cx^2\right)^{3/2}} + \frac{5\tan^{-1}(ax)^{3/2}}{9a^3c^2\sqrt{c+a^2cx^2}} + \frac{x^3\tan^{-1}(a\cdots}{3c\left(c+a^2cx\cdots\right.}$$

Mathematica [C] time = 1.0817, size = 287, normalized size = 0.97

$$\frac{-15\left(a^2x^2+1\right)^{3/2}\left(3\sqrt{-i\tan^{-1}(ax)}\,\text{Gamma}\left(\frac{1}{2},-i\tan^{-1}(ax)\right)+3\sqrt{i\tan^{-1}(ax)}\,\text{Gamma}\left(\frac{1}{2},i\tan^{-1}(ax)\right)+\sqrt{3}\left(\sqrt{-i\tan^{-}\cdots}\right.\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] (-24*ArcTan[a*x]*(5*a*x*(6 + 7*a^2*x^2) - 10*(2 + 3*a^2*x^2)*ArcTan[a*x] -
12*a^3*x^3*ArcTan[a*x]^2) + 35*Sqrt[6*Pi]*(1 + a^2*x^2)^(3/2)*Sqrt[ArcTan[a
*x]]*(3*Sqrt[3]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]] - FresnelS[Sqrt[6/Pi
]*Sqrt[ArcTan[a*x]]]) - 15*(1 + a^2*x^2)^(3/2)*(3*Sqrt[(-I)*ArcTan[a*x]]*Ga
mma[1/2, (-I)*ArcTan[a*x]] + 3*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, I*ArcTan[a*x]
] + Sqrt[3]*(Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-3*I)*ArcTan[a*x]] + Sqrt[I
*ArcTan[a*x]]*Gamma[1/2, (3*I)*ArcTan[a*x]])))/(864*a^3*c*(c + a^2*c*x^2)^(
3/2)*Sqrt[ArcTan[a*x]])
```

Maple [F] time = 2.843, size = 0, normalized size = 0.

$$\int x^2 (\arctan(ax))^{\frac{5}{2}} \left(a^2cx^2 + c\right)^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x)
```

```
[Out] int(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(a*x)**(5/2)/(a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \arctan(ax)^{\frac{5}{2}}}{\left(a^2cx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2 + c)^(5/2), x)
```

3.911 $\int \frac{x \tan^{-1}(ax)^{5/2}}{\left(c+a^2cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=293

$$-\frac{15\sqrt{\frac{\pi}{2}}\sqrt{a^2x^2+1}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{16a^2c^2\sqrt{a^2cx^2+c}}-\frac{5\sqrt{\frac{\pi}{6}}\sqrt{a^2x^2+1}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{144a^2c^2\sqrt{a^2cx^2+c}}+\frac{5x\tan^{-1}(ax)^{3/2}}{9ac^2\sqrt{a^2cx^2+c}}+\frac{5}{6a^2}$$

```
[Out] (5*Sqrt[ArcTan[a*x]])/(36*a^2*c*(c + a^2*c*x^2)^(3/2)) + (5*Sqrt[ArcTan[a*x
]])/(6*a^2*c^2*Sqrt[c + a^2*c*x^2]) + (5*x*ArcTan[a*x]^(3/2))/(18*a*c*(c +
a^2*c*x^2)^(3/2)) + (5*x*ArcTan[a*x]^(3/2))/(9*a*c^2*Sqrt[c + a^2*c*x^2]) -
 ArcTan[a*x]^(5/2)/(3*a^2*c*(c + a^2*c*x^2)^(3/2)) - (15*Sqrt[Pi/2]*Sqrt[1
+ a^2*x^2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(16*a^2*c^2*Sqrt[c + a^2
*c*x^2]) - (5*Sqrt[Pi/6]*Sqrt[1 + a^2*x^2]*FresnelC[Sqrt[6/Pi]*Sqrt[ArcTan[
a*x]]])/(144*a^2*c^2*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.450979, antiderivative size = 293, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4930, 4900, 4898, 4905, 4904, 3304, 3352, 3312}

$$-\frac{15\sqrt{\frac{\pi}{2}}\sqrt{a^2x^2+1}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{16a^2c^2\sqrt{a^2cx^2+c}}-\frac{5\sqrt{\frac{\pi}{6}}\sqrt{a^2x^2+1}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{144a^2c^2\sqrt{a^2cx^2+c}}+\frac{5x\tan^{-1}(ax)^{3/2}}{9ac^2\sqrt{a^2cx^2+c}}+\frac{5}{6a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^(5/2),x]
```


```
[Out] (5*Sqrt[ArcTan[a*x]])/(36*a^2*c*(c + a^2*c*x^2)^(3/2)) + (5*Sqrt[ArcTan[a*x
]])/(6*a^2*c^2*Sqrt[c + a^2*c*x^2]) + (5*x*ArcTan[a*x]^(3/2))/(18*a*c*(c +
a^2*c*x^2)^(3/2)) + (5*x*ArcTan[a*x]^(3/2))/(9*a*c^2*Sqrt[c + a^2*c*x^2]) -
 ArcTan[a*x]^(5/2)/(3*a^2*c*(c + a^2*c*x^2)^(3/2)) - (15*Sqrt[Pi/2]*Sqrt[1
+ a^2*x^2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(16*a^2*c^2*Sqrt[c + a^2
*c*x^2]) - (5*Sqrt[Pi/6]*Sqrt[1 + a^2*x^2]*FresnelC[Sqrt[6/Pi]*Sqrt[ArcTan[
a*x]]])/(144*a^2*c^2*Sqrt[c + a^2*c*x^2])
```


Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
```

```
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 4900

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(b*p*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^(p - 1))/(4*c*d
*(q + 1)^2), x] + (Dist[(2*q + 3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a
 + b*ArcTan[c*x])^p, x], x] - Dist[(b^2*p*(p - 1))/(4*(q + 1)^2), Int[(d +
e*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x] - Simp[(x*(d + e*x^2)^(q + 1)*
(a + b*ArcTan[c*x])^p)/(2*d*(q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] && E
qQ[e, c^2*d] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

Rule 4898

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[(b*p*(a + b*ArcTan[c*x])^(p - 1))/(c*d*Sqrt[d + e*x^2]), x]
 + (-Dist[b^2*p*(p - 1), Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2),
 x], x] + Simp[(x*(a + b*ArcTan[c*x])^p)/(d*Sqrt[d + e*x^2]), x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

Rule 4905

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[(d^(q + 1/2)*Sqrt[1 + c^2*x^2])/Sqrt[d + e*x^2], Int[(1 + c
^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

Rule 4904

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\int \frac{x\tan^{-1}(ax)^{5/2}}{\left(c+a^2cx^2\right)^{5/2}}\,dx = -\frac{\tan^{-1}(ax)^{5/2}}{3a^2c\left(c+a^2cx^2\right)^{3/2}}+\frac{5\int \frac{\tan^{-1}(ax)^{3/2}}{\left(c+a^2cx^2\right)^{5/2}}\,dx}{6a}$$

$$=\frac{5\sqrt{\tan^{-1}(ax)}}{36a^2c\left(c+a^2cx^2\right)^{3/2}}+\frac{5x\tan^{-1}(ax)^{3/2}}{18ac\left(c+a^2cx^2\right)^{3/2}}-\frac{\tan^{-1}(ax)^{5/2}}{3a^2c\left(c+a^2cx^2\right)^{3/2}}-\frac{5\int \frac{1}{\left(c+a^2cx^2\right)^{5/2}\sqrt{\tan^{-1}(ax)}}\,dx}{72a}+$$

$$=\frac{5\sqrt{\tan^{-1}(ax)}}{36a^2c\left(c+a^2cx^2\right)^{3/2}}+\frac{5\sqrt{\tan^{-1}(ax)}}{6a^2c^2\sqrt{c+a^2cx^2}}+\frac{5x\tan^{-1}(ax)^{3/2}}{18ac\left(c+a^2cx^2\right)^{3/2}}+\frac{5x\tan^{-1}(ax)^{3/2}}{9ac^2\sqrt{c+a^2cx^2}}-\frac{\tan^{-1}(ax}{3a^2c\left(c+a^2\right.}$$

$$=\frac{5\sqrt{\tan^{-1}(ax)}}{36a^2c\left(c+a^2cx^2\right)^{3/2}}+\frac{5\sqrt{\tan^{-1}(ax)}}{6a^2c^2\sqrt{c+a^2cx^2}}+\frac{5x\tan^{-1}(ax)^{3/2}}{18ac\left(c+a^2cx^2\right)^{3/2}}+\frac{5x\tan^{-1}(ax)^{3/2}}{9ac^2\sqrt{c+a^2cx^2}}-\frac{\tan^{-1}(ax}{3a^2c\left(c+a^2\right.}$$

$$=\frac{5\sqrt{\tan^{-1}(ax)}}{36a^2c\left(c+a^2cx^2\right)^{3/2}}+\frac{5\sqrt{\tan^{-1}(ax)}}{6a^2c^2\sqrt{c+a^2cx^2}}+\frac{5x\tan^{-1}(ax)^{3/2}}{18ac\left(c+a^2cx^2\right)^{3/2}}+\frac{5x\tan^{-1}(ax)^{3/2}}{9ac^2\sqrt{c+a^2cx^2}}-\frac{\tan^{-1}(ax}{3a^2c\left(c+a^2\right.}$$

$$=\frac{5\sqrt{\tan^{-1}(ax)}}{36a^2c\left(c+a^2cx^2\right)^{3/2}}+\frac{5\sqrt{\tan^{-1}(ax)}}{6a^2c^2\sqrt{c+a^2cx^2}}+\frac{5x\tan^{-1}(ax)^{3/2}}{18ac\left(c+a^2cx^2\right)^{3/2}}+\frac{5x\tan^{-1}(ax)^{3/2}}{9ac^2\sqrt{c+a^2cx^2}}-\frac{\tan^{-1}(ax}{3a^2c\left(c+a^2\right.}$$

$$=\frac{5\sqrt{\tan^{-1}(ax)}}{36a^2c\left(c+a^2cx^2\right)^{3/2}}+\frac{5\sqrt{\tan^{-1}(ax)}}{6a^2c^2\sqrt{c+a^2cx^2}}+\frac{5x\tan^{-1}(ax)^{3/2}}{18ac\left(c+a^2cx^2\right)^{3/2}}+\frac{5x\tan^{-1}(ax)^{3/2}}{9ac^2\sqrt{c+a^2cx^2}}-\frac{\tan^{-1}(ax}{3a^2c\left(c+a^2\right.}$$

$$=\frac{5\sqrt{\tan^{-1}(ax)}}{36a^2c\left(c+a^2cx^2\right)^{3/2}}+\frac{5\sqrt{\tan^{-1}(ax)}}{6a^2c^2\sqrt{c+a^2cx^2}}+\frac{5x\tan^{-1}(ax)^{3/2}}{18ac\left(c+a^2cx^2\right)^{3/2}}+\frac{5x\tan^{-1}(ax)^{3/2}}{9ac^2\sqrt{c+a^2cx^2}}-\frac{\tan^{-1}(ax}{3a^2c\left(c+a^2\right.}$$

Mathematica [C] time = 0.5335, size = 356, normalized size = 1.22

$$5ia^2x^2\sqrt{3a^2x^2+3}\sqrt{-i\tan^{-1}(ax)}\text{Gamma}\left(\frac{1}{2},-3i\tan^{-1}(ax)\right)-5ia^2x^2\sqrt{3a^2x^2+3}\sqrt{i\tan^{-1}(ax)}\text{Gamma}\left(\frac{1}{2},3i\tan^{-1}(ax)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] (1680*ArcTan[a*x] + 1440*a^2*x^2*ArcTan[a*x] + 1440*a*x*ArcTan[a*x]^2 + 960
*a^3*x^3*ArcTan[a*x]^2 - 576*ArcTan[a*x]^3 + (405*I)*(1 + a^2*x^2)^(3/2)*Sq
rt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-I)*ArcTan[a*x]] - (405*I)*(1 + a^2*x^2)^(
3/2)*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, I*ArcTan[a*x]] + (5*I)*Sqrt[3 + 3*a^2*x
^2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-3*I)*ArcTan[a*x]] + (5*I)*a^2*x^2*S
qrt[3 + 3*a^2*x^2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-3*I)*ArcTan[a*x]] -
(5*I)*Sqrt[3 + 3*a^2*x^2]*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (3*I)*ArcTan[a*x]]
 - (5*I)*a^2*x^2*Sqrt[3 + 3*a^2*x^2]*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (3*I)*A
rcTan[a*x]])/(1728*a^2*c^2*(1 + a^2*x^2)*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*
x]])
```

Maple [F] time = 0.799, size = 0, normalized size = 0.

$$\int x\left(\arctan\left(ax\right)\right)^{\frac{5}{2}}\left(a^{2}cx^{2}+c\right)^{-\frac{5}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x)
```

```
[Out] int(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(a*x)**(5/2)/(a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \arctan (ax)^{\frac{5}{2}}}{\left(a^2cx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x*arctan(a*x)^(5/2)/(a^2*c*x^2 + c)^(5/2), x)
```

3.912 $\int \frac{\tan^{-1}(ax)^{5/2}}{\left(c+a^2cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=337

$$\frac{45\sqrt{\frac{\pi}{2}}\sqrt{a^2x^2+1}S\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{16ac^2\sqrt{a^2cx^2+c}}+\frac{5\sqrt{\frac{\pi}{6}}\sqrt{a^2x^2+1}S\left(\sqrt{\frac{6}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{144ac^2\sqrt{a^2cx^2+c}}+\frac{2x\tan^{-1}(ax)^{5/2}}{3c^2\sqrt{a^2cx^2+c}}+\frac{5\tan^{-1}(ax)^{3/2}}{3ac^2\sqrt{a^2cx^2+c}}-\frac{45x\cdots}{16c^2\cdots}$$

```
[Out] (-45*x*Sqrt[ArcTan[a*x]])/(16*c^2*Sqrt[c + a^2*c*x^2]) + (5*ArcTan[a*x]^(3/
2))/(18*a*c*(c + a^2*c*x^2)^(3/2)) + (5*ArcTan[a*x]^(3/2))/(3*a*c^2*Sqrt[c
+ a^2*c*x^2]) + (x*ArcTan[a*x]^(5/2))/(3*c*(c + a^2*c*x^2)^(3/2)) + (2*x*Ar
cTan[a*x]^(5/2))/(3*c^2*Sqrt[c + a^2*c*x^2]) + (45*Sqrt[Pi/2]*Sqrt[1 + a^2*
x^2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(16*a*c^2*Sqrt[c + a^2*c*x^2])
 + (5*Sqrt[Pi/6]*Sqrt[1 + a^2*x^2]*FresnelS[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]])/
(144*a*c^2*Sqrt[c + a^2*c*x^2]) - (5*Sqrt[1 + a^2*x^2]*Sqrt[ArcTan[a*x]]*Si
n[3*ArcTan[a*x]])/(144*a*c^2*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.414009, antiderivative size = 337, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {4900, 4898, 4905, 4904, 3296, 3305, 3351, 3312}

$$\frac{45\sqrt{\frac{\pi}{2}}\sqrt{a^2x^2+1}S\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{16ac^2\sqrt{a^2cx^2+c}}+\frac{5\sqrt{\frac{\pi}{6}}\sqrt{a^2x^2+1}S\left(\sqrt{\frac{6}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{144ac^2\sqrt{a^2cx^2+c}}+\frac{2x\tan^{-1}(ax)^{5/2}}{3c^2\sqrt{a^2cx^2+c}}+\frac{5\tan^{-1}(ax)^{3/2}}{3ac^2\sqrt{a^2cx^2+c}}-\frac{45x\cdots}{16c^2\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^(5/2)/(c + a^2*c*x^2)^(5/2),x]
```


```
[Out] (-45*x*Sqrt[ArcTan[a*x]])/(16*c^2*Sqrt[c + a^2*c*x^2]) + (5*ArcTan[a*x]^(3/
2))/(18*a*c*(c + a^2*c*x^2)^(3/2)) + (5*ArcTan[a*x]^(3/2))/(3*a*c^2*Sqrt[c
+ a^2*c*x^2]) + (x*ArcTan[a*x]^(5/2))/(3*c*(c + a^2*c*x^2)^(3/2)) + (2*x*Ar
cTan[a*x]^(5/2))/(3*c^2*Sqrt[c + a^2*c*x^2]) + (45*Sqrt[Pi/2]*Sqrt[1 + a^2*
x^2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(16*a*c^2*Sqrt[c + a^2*c*x^2])
 + (5*Sqrt[Pi/6]*Sqrt[1 + a^2*x^2]*FresnelS[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]])/
(144*a*c^2*Sqrt[c + a^2*c*x^2]) - (5*Sqrt[1 + a^2*x^2]*Sqrt[ArcTan[a*x]]*Si
n[3*ArcTan[a*x]])/(144*a*c^2*Sqrt[c + a^2*c*x^2])
```


Rule 4900

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(b*p*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^(p - 1))/(4*c*d
*(q + 1)^2), x] + (Dist[(2*q + 3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a
 + b*ArcTan[c*x])^p, x], x] - Dist[(b^2*p*(p - 1))/(4*(q + 1)^2), Int[(d +
e*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x] - Simp[(x*(d + e*x^2)^(q + 1)*
(a + b*ArcTan[c*x])^p)/(2*d*(q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] && E
qQ[e, c^2*d] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

Rule 4898

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[(b*p*(a + b*ArcTan[c*x])^(p - 1))/(c*d*Sqrt[d + e*x^2]), x]
 + (-Dist[b^2*p*(p - 1), Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2),
 x], x] + Simp[(x*(a + b*ArcTan[c*x])^p)/(d*Sqrt[d + e*x^2]), x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

Rule 4905

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[(d^(q + 1/2)*Sqrt[1 + c^2*x^2])/Sqrt[d + e*x^2], Int[(1 + c
^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

Rule 4904

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^{5/2}}{\left(c+a^2cx^2\right)^{5/2}}\,dx = \frac{5\tan^{-1}(ax)^{3/2}}{18ac\left(c+a^2cx^2\right)^{3/2}} + \frac{x\tan^{-1}(ax)^{5/2}}{3c\left(c+a^2cx^2\right)^{3/2}} - \frac{5}{12}\int \frac{\sqrt{\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{5/2}}\,dx + \frac{2\int \frac{\tan^{-1}(ax)^{5/2}}{\left(c+a^2cx^2\right)^{3/2}}\,dx}{3c}$$

$$= \frac{5\tan^{-1}(ax)^{3/2}}{18ac\left(c+a^2cx^2\right)^{3/2}} + \frac{5\tan^{-1}(ax)^{3/2}}{3ac^2\sqrt{c+a^2cx^2}} + \frac{x\tan^{-1}(ax)^{5/2}}{3c\left(c+a^2cx^2\right)^{3/2}} + \frac{2x\tan^{-1}(ax)^{5/2}}{3c^2\sqrt{c+a^2cx^2}} - \frac{5\int \frac{\sqrt{\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{3/2}}\,dx}{2c} -$$

$$= \frac{5\tan^{-1}(ax)^{3/2}}{18ac\left(c+a^2cx^2\right)^{3/2}} + \frac{5\tan^{-1}(ax)^{3/2}}{3ac^2\sqrt{c+a^2cx^2}} + \frac{x\tan^{-1}(ax)^{5/2}}{3c\left(c+a^2cx^2\right)^{3/2}} + \frac{2x\tan^{-1}(ax)^{5/2}}{3c^2\sqrt{c+a^2cx^2}} - \frac{\left(5\sqrt{1+a^2x^2}\right)\int \frac{\sqrt{t}}{(1}}{2c^2\sqrt{c+a^2}}$$

$$= \frac{5\tan^{-1}(ax)^{3/2}}{18ac\left(c+a^2cx^2\right)^{3/2}} + \frac{5\tan^{-1}(ax)^{3/2}}{3ac^2\sqrt{c+a^2cx^2}} + \frac{x\tan^{-1}(ax)^{5/2}}{3c\left(c+a^2cx^2\right)^{3/2}} + \frac{2x\tan^{-1}(ax)^{5/2}}{3c^2\sqrt{c+a^2cx^2}} - \frac{\left(5\sqrt{1+a^2x^2}\right)\text{Sub}}{}$$

$$= -\frac{5x\sqrt{\tan^{-1}(ax)}}{2c^2\sqrt{c+a^2cx^2}} + \frac{5\tan^{-1}(ax)^{3/2}}{18ac\left(c+a^2cx^2\right)^{3/2}} + \frac{5\tan^{-1}(ax)^{3/2}}{3ac^2\sqrt{c+a^2cx^2}} + \frac{x\tan^{-1}(ax)^{5/2}}{3c\left(c+a^2cx^2\right)^{3/2}} + \frac{2x\tan^{-1}(ax)^{5/2}}{3c^2\sqrt{c+a^2cx^2}} -$$

$$= -\frac{45x\sqrt{\tan^{-1}(ax)}}{16c^2\sqrt{c+a^2cx^2}} + \frac{5\tan^{-1}(ax)^{3/2}}{18ac\left(c+a^2cx^2\right)^{3/2}} + \frac{5\tan^{-1}(ax)^{3/2}}{3ac^2\sqrt{c+a^2cx^2}} + \frac{x\tan^{-1}(ax)^{5/2}}{3c\left(c+a^2cx^2\right)^{3/2}} + \frac{2x\tan^{-1}(ax)^{5/2}}{3c^2\sqrt{c+a^2cx^2}}$$

$$= -\frac{45x\sqrt{\tan^{-1}(ax)}}{16c^2\sqrt{c+a^2cx^2}} + \frac{5\tan^{-1}(ax)^{3/2}}{18ac\left(c+a^2cx^2\right)^{3/2}} + \frac{5\tan^{-1}(ax)^{3/2}}{3ac^2\sqrt{c+a^2cx^2}} + \frac{x\tan^{-1}(ax)^{5/2}}{3c\left(c+a^2cx^2\right)^{3/2}} + \frac{2x\tan^{-1}(ax)^{5/2}}{3c^2\sqrt{c+a^2cx^2}}$$

$$= -\frac{45x\sqrt{\tan^{-1}(ax)}}{16c^2\sqrt{c+a^2cx^2}} + \frac{5\tan^{-1}(ax)^{3/2}}{18ac\left(c+a^2cx^2\right)^{3/2}} + \frac{5\tan^{-1}(ax)^{3/2}}{3ac^2\sqrt{c+a^2cx^2}} + \frac{x\tan^{-1}(ax)^{5/2}}{3c\left(c+a^2cx^2\right)^{3/2}} + \frac{2x\tan^{-1}(ax)^{5/2}}{3c^2\sqrt{c+a^2cx^2}}$$

Mathematica [C] time = 1.0915, size = 293, normalized size = 0.87

$$\frac{-105\left(a^2x^2+1\right)^{3/2}\left(3\sqrt{-i\tan^{-1}(ax)}\text{Gamma}\left(\frac{1}{2},-i\tan^{-1}(ax)\right)+3\sqrt{i\tan^{-1}(ax)}\text{Gamma}\left(\frac{1}{2},i\tan^{-1}(ax)\right)+\sqrt{3}\left(\sqrt{-i\tan\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTan[a*x]^(5/2)/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] (-48*ArcTan[a*x]*(5*a*x*(21 + 20*a^2*x^2) - 10*(7 + 6*a^2*x^2)*ArcTan[a*x]
- 12*a*x*(3 + 2*a^2*x^2)*ArcTan[a*x]^2) + 200*Sqrt[6*Pi]*(1 + a^2*x^2)^(3/2
)*Sqrt[ArcTan[a*x]]*(3*Sqrt[3]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]] - Fre
snelS[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]]) - 105*(1 + a^2*x^2)^(3/2)*(3*Sqrt[(-I)
*ArcTan[a*x]]*Gamma[1/2, (-I)*ArcTan[a*x]] + 3*Sqrt[I*ArcTan[a*x]]*Gamma[1/
2, I*ArcTan[a*x]] + Sqrt[3]*(Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-3*I)*ArcTa
n[a*x]] + Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (3*I)*ArcTan[a*x]])))/(1728*a*c*(c
 + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]])
```

Maple [F] time = 0.741, size = 0, normalized size = 0.

$$\int (\arctan (ax))^{\frac{5}{2}} \left(a^2cx^2+c\right)^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x)
```

```
[Out] int(arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(5/2)/(a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan\left(ax\right)^{\frac{5}{2}}}{\left(a^2cx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)^(5/2)/(a^2*c*x^2 + c)^(5/2), x)
```

3.913 $\int \frac{\tan^{-1}(ax)^{5/2}}{x\left(c+a^2cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(\frac{\tan^{-1}(ax)^{5/2}}{x\left(a^2cx^2+c\right)^{5/2}},x\right)$$

```
[Out] Unintegrable[ArcTan[a*x]^(5/2)/(x*(c + a^2*c*x^2)^(5/2)), x]
```

Rubi [A] time = 0.11856, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\tan^{-1}(ax)^{5/2}}{x\left(c+a^2cx^2\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcTan[a*x]^(5/2)/(x*(c + a^2*c*x^2)^(5/2)),x]
```

```
[Out] Defer[Int][ArcTan[a*x]^(5/2)/(x*(c + a^2*c*x^2)^(5/2)), x]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^{5/2}}{x\left(c+a^2cx^2\right)^{5/2}}\,dx=\int \frac{\tan^{-1}(ax)^{5/2}}{x\left(c+a^2cx^2\right)^{5/2}}\,dx$$

Mathematica [A] time = 2.53711, size = 0, normalized size = 0.

$$\int \frac{\tan^{-1}(ax)^{5/2}}{x\left(c+a^2cx^2\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcTan[a*x]^(5/2)/(x*(c + a^2*c*x^2)^(5/2)),x]
```

```
[Out] Integrate[ArcTan[a*x]^(5/2)/(x*(c + a^2*c*x^2)^(5/2)), x]
```

Maple [A] time = 0.807, size = 0, normalized size = 0.

$$\int \frac{1}{x} (\arctan (ax))^{\frac{5}{2}} \left(a^2cx^2+c\right)^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^(5/2),x)
```

```
[Out] int(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(5/2)/x/(a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(ax)^{\frac{5}{2}}}{\left(a^2cx^2+c\right)^{\frac{5}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)^(5/2)/((a^2*c*x^2 + c)^(5/2)*x), x)
```

3.914 $\int \frac{x^m(c+a^2cx^2)}{\sqrt{\tan^{-1}(ax)}}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{x^m\left(a^2cx^2+c\right)}{\sqrt{\tan^{-1}(ax)}},x\right)$$

```
[Out] Unintegrable[(x^m*(c + a^2*c*x^2))/Sqrt[ArcTan[a*x]], x]
```

Rubi [A] time = 0.034926, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m\left(c+a^2cx^2\right)}{\sqrt{\tan^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*(c + a^2*c*x^2))/Sqrt[ArcTan[a*x]],x]

[Out] Defer[Int][(x^m*(c + a^2*c*x^2))/Sqrt[ArcTan[a*x]], x]
```

Rubi steps

$$\int \frac{x^m\left(c+a^2cx^2\right)}{\sqrt{\tan^{-1}(ax)}}\,dx=\int \frac{x^m\left(c+a^2cx^2\right)}{\sqrt{\tan^{-1}(ax)}}\,dx$$

Mathematica [A] time = 1.94782, size = 0, normalized size = 0.

$$\int \frac{x^m\left(c+a^2cx^2\right)}{\sqrt{\tan^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*(c + a^2*c*x^2))/Sqrt[ArcTan[a*x]],x]
```

```
[Out] Integrate[(x^m*(c + a^2*c*x^2))/Sqrt[ArcTan[a*x]], x]
```

Maple [A] time = 0.745, size = 0, normalized size = 0.

$$\int x^m \left(a^2 c x^2 + c\right) \frac{1}{\sqrt{\arctan\left(ax\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)/arctan(a*x)^(1/2),x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)/arctan(a*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^2 c x^2 + c\right) x^m}{\sqrt{\arctan\left(ax\right)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)*x^m/sqrt(arctan(a*x)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)/atan(a*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a^2cx^2+c)x^m}{\sqrt{\arctan(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)*x^m/sqrt(arctan(a*x)), x)
```

3.915 $\int \frac{x(c+a^2cx^2)}{\sqrt{\tan^{-1}(ax)}}\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{x\left(a^2cx^2+c\right)}{\sqrt{\tan^{-1}(ax)}},x\right)$$

```
[Out] Unintegrable[(x*(c + a^2*c*x^2))/Sqrt[ArcTan[a*x]], x]
```

Rubi [A] time = 0.0230053, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x\left(c+a^2cx^2\right)}{\sqrt{\tan^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x*(c + a^2*c*x^2))/Sqrt[ArcTan[a*x]],x]

[Out] Defer[Int][(x*(c + a^2*c*x^2))/Sqrt[ArcTan[a*x]], x]
```

Rubi steps

$$\int \frac{x\left(c+a^2cx^2\right)}{\sqrt{\tan^{-1}(ax)}}\,dx=\int \frac{x\left(c+a^2cx^2\right)}{\sqrt{\tan^{-1}(ax)}}\,dx$$

Mathematica [A] time = 0.989938, size = 0, normalized size = 0.

$$\int \frac{x\left(c+a^2cx^2\right)}{\sqrt{\tan^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x*(c + a^2*c*x^2))/Sqrt[ArcTan[a*x]],x]
```

```
[Out] Integrate[(x*(c + a^2*c*x^2))/Sqrt[ArcTan[a*x]], x]
```

Maple [A] time = 0.382, size = 0, normalized size = 0.

$$\int x\left(a^2cx^2+c\right)\frac{1}{\sqrt{\arctan\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)/arctan(a*x)^(1/2),x)
```

```
[Out] int(x*(a^2*c*x^2+c)/arctan(a*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$c\left(\int \frac{x}{\sqrt{\operatorname{atan}(ax)}}\,dx+\int \frac{a^2x^3}{\sqrt{\operatorname{atan}(ax)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)/atan(a*x)**(1/2),x)
```

```
[Out] c*(Integral(x/sqrt(atan(a*x)), x) + Integral(a**2*x**3/sqrt(atan(a*x)), x))
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a^2cx^2+c)x}{\sqrt{\arctan(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)*x/sqrt(arctan(a*x)), x)
```

3.916 $\int \frac{c+a^2cx^2}{\sqrt{\tan^{-1}(ax)}}\,dx$

Optimal. Leaf size=21

$$\text{Unintegrable}\left(\frac{a^2cx^2+c}{\sqrt{\tan^{-1}(ax)}},x\right)$$

```
[Out] Unintegrable[(c + a^2*c*x^2)/Sqrt[ArcTan[a*x]], x]
```

Rubi [A] time = 0.0122409, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{c+a^2cx^2}{\sqrt{\tan^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)/Sqrt[ArcTan[a*x]],x]

[Out] Defer[Int][(c + a^2*c*x^2)/Sqrt[ArcTan[a*x]], x]
```

Rubi steps

$$\int \frac{c+a^2cx^2}{\sqrt{\tan^{-1}(ax)}}\,dx=\int \frac{c+a^2cx^2}{\sqrt{\tan^{-1}(ax)}}\,dx$$

Mathematica [A] time = 0.107862, size = 0, normalized size = 0.

$$\int \frac{c+a^2cx^2}{\sqrt{\tan^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)/Sqrt[ArcTan[a*x]],x]
```

```
[Out] Integrate[(c + a^2*c*x^2)/Sqrt[ArcTan[a*x]], x]
```

Maple [A] time = 0.274, size = 0, normalized size = 0.

$$\int \left(a^{2} c \,x^{2}+c \right) \frac{1}{\sqrt{\arctan \left(a x \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)/arctan(a*x)^(1/2),x)
```

```
[Out] int((a^2*c*x^2+c)/arctan(a*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$c\left(\int \frac{a^2x^2}{\sqrt{\operatorname{atan}(ax)}}\,dx + \int \frac{1}{\sqrt{\operatorname{atan}(ax)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)/atan(a*x)**(1/2),x)
```

```
[Out] c*(Integral(a**2*x**2/sqrt(atan(a*x)), x) + Integral(1/sqrt(atan(a*x)), x))
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{a^2cx^2 + c}{\sqrt{\arctan(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)/sqrt(arctan(a*x)), x)
```

3.917 $\int \frac{c+a^2cx^2}{x\sqrt{\tan^{-1}(ax)}}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{a^2cx^2+c}{x\sqrt{\tan^{-1}(ax)}},x\right)$$

```
[Out] Unintegrable[(c + a^2*c*x^2)/(x*Sqrt[ArcTan[a*x]]), x]
```

Rubi [A] time = 0.0322063, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{c+a^2cx^2}{x\sqrt{\tan^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)/(x*Sqrt[ArcTan[a*x]]),x]
```

```
[Out] Defer[Int][(c + a^2*c*x^2)/(x*Sqrt[ArcTan[a*x]]), x]
```

Rubi steps

$$\int \frac{c+a^2cx^2}{x\sqrt{\tan^{-1}(ax)}}\,dx=\int \frac{c+a^2cx^2}{x\sqrt{\tan^{-1}(ax)}}\,dx$$

Mathematica [A] time = 1.38055, size = 0, normalized size = 0.

$$\int \frac{c+a^2cx^2}{x\sqrt{\tan^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)/(x*Sqrt[ArcTan[a*x]]),x]
```

```
[Out] Integrate[(c + a^2*c*x^2)/(x*Sqrt[ArcTan[a*x]]), x]
```

Maple [A] time = 0.427, size = 0, normalized size = 0.

$$\int \frac{a^2cx^2 + c}{x} \frac{1}{\sqrt{\arctan(ax)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)/x/arctan(a*x)^(1/2),x)
```

```
[Out] int((a^2*c*x^2+c)/x/arctan(a*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)/x/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)/x/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$c\left(\int \frac{1}{x\sqrt{\operatorname{atan}(ax)}}\,dx + \int \frac{a^2x}{\sqrt{\operatorname{atan}(ax)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)/x/atan(a*x)**(1/2),x)
```

```
[Out] c*(Integral(1/(x*sqrt(atan(a*x))), x) + Integral(a**2*x/sqrt(atan(a*x)), x)
)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{a^2cx^2 + c}{x\sqrt{\arctan(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)/x/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)/(x*sqrt(arctan(a*x))), x)
```

3.918 $\int \frac{x^m\left(c+a^2cx^2\right)^2}{\sqrt{\tan^{-1}(ax)}}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{x^m\left(a^2cx^2+c\right)^2}{\sqrt{\tan^{-1}(ax)}},x\right)$$

```
[Out] Unintegrable[(x^m*(c + a^2*c*x^2)^2)/Sqrt[ArcTan[a*x]], x]
```

Rubi [A] time = 0.0553061, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m\left(c+a^2cx^2\right)^2}{\sqrt{\tan^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*(c + a^2*c*x^2)^2)/Sqrt[ArcTan[a*x]],x]

[Out] Defer[Int][(x^m*(c + a^2*c*x^2)^2)/Sqrt[ArcTan[a*x]], x]
```

Rubi steps

$$\int \frac{x^m\left(c+a^2cx^2\right)^2}{\sqrt{\tan^{-1}(ax)}}\,dx=\int \frac{x^m\left(c+a^2cx^2\right)^2}{\sqrt{\tan^{-1}(ax)}}\,dx$$

Mathematica [A] time = 1.31504, size = 0, normalized size = 0.

$$\int \frac{x^m\left(c+a^2cx^2\right)^2}{\sqrt{\tan^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*(c + a^2*c*x^2)^2)/Sqrt[ArcTan[a*x]],x]
```

```
[Out] Integrate[(x^m*(c + a^2*c*x^2)^2)/Sqrt[ArcTan[a*x]], x]
```

Maple [A] time = 0.883, size = 0, normalized size = 0.

$$\int x^m \left(a^2 c x^2 + c\right)^2 \frac{1}{\sqrt{\arctan(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^4c^2x^4 + 2\,a^2c^2x^2 + c^2\right)x^m}{\sqrt{\arctan(ax)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*x^m/sqrt(arctan(a*x)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**2/atan(a*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^2x^m}{\sqrt{\arctan\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^2*x^m/sqrt(arctan(a*x)), x)
```

3.919 $\int \frac{x\left(c+a^2cx^2\right)^2}{\sqrt{\tan^{-1}(ax)}} \, dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{x\left(a^2cx^2+c\right)^2}{\sqrt{\tan^{-1}(ax)}},x\right)$$

```
[Out] Unintegrable[(x*(c + a^2*c*x^2)^2)/Sqrt[ArcTan[a*x]], x]
```

Rubi [A] time = 0.0387091, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x\left(c+a^2cx^2\right)^2}{\sqrt{\tan^{-1}(ax)}} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(x*(c + a^2*c*x^2)^2)/Sqrt[ArcTan[a*x]],x]

[Out] Defer[Int][(x*(c + a^2*c*x^2)^2)/Sqrt[ArcTan[a*x]], x]
```

Rubi steps

$$\int \frac{x\left(c+a^2cx^2\right)^2}{\sqrt{\tan^{-1}(ax)}} \, dx=\int \frac{x\left(c+a^2cx^2\right)^2}{\sqrt{\tan^{-1}(ax)}} \, dx$$

Mathematica [A] time = 1.17321, size = 0, normalized size = 0.

$$\int \frac{x\left(c+a^2cx^2\right)^2}{\sqrt{\tan^{-1}(ax)}} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x*(c + a^2*c*x^2)^2)/Sqrt[ArcTan[a*x]],x]
```

```
[Out] Integrate[(x*(c + a^2*c*x^2)^2)/Sqrt[ArcTan[a*x]], x]
```

Maple [A] time = 0.514, size = 0, normalized size = 0.

$$\int x\left(a^{2}c x^{2}+c\right)^{2} \frac{1}{\sqrt{\arctan\left(a x\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x)
```

```
[Out] int(x*(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$c^2\left(\int \frac{x}{\sqrt{\operatorname{atan}(ax)}}\,dx + \int \frac{2a^2x^3}{\sqrt{\operatorname{atan}(ax)}}\,dx + \int \frac{a^4x^5}{\sqrt{\operatorname{atan}(ax)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**2/atan(a*x)**(1/2),x)
```

```
[Out] c**2*(Integral(x/sqrt(atan(a*x)), x) + Integral(2*a**2*x**3/sqrt(atan(a*x))
, x) + Integral(a**4*x**5/sqrt(atan(a*x)), x))
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2 + c\right)^2 x}{\sqrt{\arctan(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^2*x/sqrt(arctan(a*x)), x)
```

3.920 $\int \frac{(c+a^2cx^2)^2}{\sqrt{\tan^{-1}(ax)}} dx$

Optimal. Leaf size=23

$$\text{Unintegrable}\left(\frac{(a^2cx^2+c)^2}{\sqrt{\tan^{-1}(ax)}},x\right)$$

[Out] Unintegrable[(c + a^2*c*x^2)^2/Sqrt[ArcTan[a*x]], x]

Rubi [A] time = 0.0219361, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(c+a^2cx^2)^2}{\sqrt{\tan^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

[In] Int[(c + a^2*c*x^2)^2/Sqrt[ArcTan[a*x]],x]

[Out] Defer[Int][(c + a^2*c*x^2)^2/Sqrt[ArcTan[a*x]], x]

Rubi steps

$$\int \frac{(c+a^2cx^2)^2}{\sqrt{\tan^{-1}(ax)}}\,dx = \int \frac{(c+a^2cx^2)^2}{\sqrt{\tan^{-1}(ax)}}\,dx$$

Mathematica [A] time = 0.72273, size = 0, normalized size = 0.

$$\int \frac{(c+a^2cx^2)^2}{\sqrt{\tan^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^2/Sqrt[ArcTan[a*x]],x]
```

```
[Out] Integrate[(c + a^2*c*x^2)^2/Sqrt[ArcTan[a*x]], x]
```

Maple [A] time = 0.406, size = 0, normalized size = 0.

$$\int \left(a^{2} c \,x^{2}+c \right)^{2} \frac{1}{\sqrt{\arctan \left(a x \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x)
```

```
[Out] int((a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$c^{2}\left(\int \frac{2a^{2}x^{2}}{\sqrt{\operatorname{atan}{(ax)}}}\,dx+\int \frac{a^{4}x^{4}}{\sqrt{\operatorname{atan}{(ax)}}}\,dx+\int \frac{1}{\sqrt{\operatorname{atan}{(ax)}}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**2/atan(a*x)**(1/2),x)
```

```
[Out] c**2*(Integral(2*a**2*x**2/sqrt(atan(a*x)), x) + Integral(a**4*x**4/sqrt(at
an(a*x)), x) + Integral(1/sqrt(atan(a*x)), x))
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^{2}cx^{2}+c\right)^{2}}{\sqrt{\arctan\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^2/sqrt(arctan(a*x)), x)
```

3.921 $\int \frac{(c+a^2cx^2)^2}{x\sqrt{\tan^{-1}(ax)}} \, dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{(a^2cx^2+c)^2}{x\sqrt{\tan^{-1}(ax)}},x\right)$$

```
[Out] Unintegrable[(c + a^2*c*x^2)^2/(x*Sqrt[ArcTan[a*x]]), x]
```

Rubi [A] time = 0.0501793, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{(c+a^2cx^2)^2}{x\sqrt{\tan^{-1}(ax)}} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^2/(x*Sqrt[ArcTan[a*x]]),x]
```

```
[Out] Defer[Int][(c + a^2*c*x^2)^2/(x*Sqrt[ArcTan[a*x]]), x]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^2}{x\sqrt{\tan^{-1}(ax)}} \, dx = \int \frac{(c+a^2cx^2)^2}{x\sqrt{\tan^{-1}(ax)}} \, dx$$

Mathematica [A] time = 1.30113, size = 0, normalized size = 0.

$$\int \frac{(c+a^2cx^2)^2}{x\sqrt{\tan^{-1}(ax)}} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^2/(x*Sqrt[ArcTan[a*x]]),x]
```

```
[Out] Integrate[(c + a^2*c*x^2)^2/(x*Sqrt[ArcTan[a*x]]), x]
```

Maple [A] time = 0.461, size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^2}{x} \frac{1}{\sqrt{\arctan\left(ax\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^2/x/arctan(a*x)^(1/2),x)
```

```
[Out] int((a^2*c*x^2+c)^2/x/arctan(a*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2/x/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2/x/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$c^2\left(\int \frac{1}{x\sqrt{\operatorname{atan}(ax)}}\,dx + \int \frac{2a^2x}{\sqrt{\operatorname{atan}(ax)}}\,dx + \int \frac{a^4x^3}{\sqrt{\operatorname{atan}(ax)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**2/x/atan(a*x)**(1/2),x)
```

```
[Out] c**2*(Integral(1/(x*sqrt(atan(a*x))), x) + Integral(2*a**2*x/sqrt(atan(a*x)
), x) + Integral(a**4*x**3/sqrt(atan(a*x)), x))
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2 + c\right)^2}{x\sqrt{\arctan(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2/x/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^2/(x*sqrt(arctan(a*x))), x)
```

3.922 $\int \frac{x^m\left(c+a^2cx^2\right)^3}{\sqrt{\tan^{-1}(ax)}}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{x^m\left(a^2cx^2+c\right)^3}{\sqrt{\tan^{-1}(ax)}},x\right)$$

```
[Out] Unintegrable[(x^m*(c + a^2*c*x^2)^3)/Sqrt[ArcTan[a*x]], x]
```

Rubi [A] time = 0.0557871, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m\left(c+a^2cx^2\right)^3}{\sqrt{\tan^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*(c + a^2*c*x^2)^3)/Sqrt[ArcTan[a*x]],x]

[Out] Defer[Int][(x^m*(c + a^2*c*x^2)^3)/Sqrt[ArcTan[a*x]], x]
```

Rubi steps

$$\int \frac{x^m\left(c+a^2cx^2\right)^3}{\sqrt{\tan^{-1}(ax)}}\,dx=\int \frac{x^m\left(c+a^2cx^2\right)^3}{\sqrt{\tan^{-1}(ax)}}\,dx$$

Mathematica [A] time = 0.792792, size = 0, normalized size = 0.

$$\int \frac{x^m\left(c+a^2cx^2\right)^3}{\sqrt{\tan^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*(c + a^2*c*x^2)^3)/Sqrt[ArcTan[a*x]],x]
```

```
[Out] Integrate[(x^m*(c + a^2*c*x^2)^3)/Sqrt[ArcTan[a*x]], x]
```

Maple [A] time = 0.959, size = 0, normalized size = 0.

$$\int x^{m} \left(a^{2} c x^{2}+c\right)^{3} \frac{1}{\sqrt{\arctan \left(a x\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(a^{6} c^{3} x^{6}+3 \, a^{4} c^{3} x^{4}+3 \, a^{2} c^{3} x^{2}+c^{3}\right) x^{m}}{\sqrt{\arctan \left(a x\right)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*x^m/sqrt(arcta
n(a*x)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**3/atan(a*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^3x^m}{\sqrt{\arctan\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^3*x^m/sqrt(arctan(a*x)), x)
```

3.923 $\int \frac{x(c+a^2cx^2)^3}{\sqrt{\tan^{-1}(ax)}}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{x\left(a^2cx^2+c\right)^3}{\sqrt{\tan^{-1}(ax)}},x\right)$$

```
[Out] Unintegrable[(x*(c + a^2*c*x^2)^3)/Sqrt[ArcTan[a*x]], x]
```

Rubi [A] time = 0.0365296, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x\left(c+a^2cx^2\right)^3}{\sqrt{\tan^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x*(c + a^2*c*x^2)^3)/Sqrt[ArcTan[a*x]],x]

[Out] Defer[Int][(x*(c + a^2*c*x^2)^3)/Sqrt[ArcTan[a*x]], x]
```

Rubi steps

$$\int \frac{x\left(c+a^2cx^2\right)^3}{\sqrt{\tan^{-1}(ax)}}\,dx=\int \frac{x\left(c+a^2cx^2\right)^3}{\sqrt{\tan^{-1}(ax)}}\,dx$$

Mathematica [A] time = 1.20067, size = 0, normalized size = 0.

$$\int \frac{x\left(c+a^2cx^2\right)^3}{\sqrt{\tan^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x*(c + a^2*c*x^2)^3)/Sqrt[ArcTan[a*x]],x]
```

```
[Out] Integrate[(x*(c + a^2*c*x^2)^3)/Sqrt[ArcTan[a*x]], x]
```

Maple [A] time = 0.696, size = 0, normalized size = 0.

$$\int x\left(a^{2}c x^{2}+c\right)^{3}\frac{1}{\sqrt{\arctan\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x)
```

```
[Out] int(x*(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$c^3\left(\int \frac{x}{\sqrt{\operatorname{atan}(ax)}}\,dx + \int \frac{3a^2x^3}{\sqrt{\operatorname{atan}(ax)}}\,dx + \int \frac{3a^4x^5}{\sqrt{\operatorname{atan}(ax)}}\,dx + \int \frac{a^6x^7}{\sqrt{\operatorname{atan}(ax)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**3/atan(a*x)**(1/2),x)
```

```
[Out] c**3*(Integral(x/sqrt(atan(a*x)), x) + Integral(3*a**2*x**3/sqrt(atan(a*x))
, x) + Integral(3*a**4*x**5/sqrt(atan(a*x)), x) + Integral(a**6*x**7/sqrt(a
tan(a*x)), x))
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^3 x}{\sqrt{\arctan(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^3*x/sqrt(arctan(a*x)), x)
```

3.924 $\int \frac{\left(c+a^2cx^2\right)^3}{\sqrt{\tan^{-1}(ax)}}\,dx$

Optimal. Leaf size=23

$$\text{Unintegrable}\left(\frac{\left(a^2cx^2+c\right)^3}{\sqrt{\tan^{-1}(ax)}},x\right)$$

```
[Out] Unintegrable[(c + a^2*c*x^2)^3/Sqrt[ArcTan[a*x]], x]
```

Rubi [A] time = 0.0216785, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(c+a^2cx^2\right)^3}{\sqrt{\tan^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^3/Sqrt[ArcTan[a*x]],x]

[Out] Defer[Int][(c + a^2*c*x^2)^3/Sqrt[ArcTan[a*x]], x]
```

Rubi steps

$$\int \frac{\left(c+a^2cx^2\right)^3}{\sqrt{\tan^{-1}(ax)}}\,dx = \int \frac{\left(c+a^2cx^2\right)^3}{\sqrt{\tan^{-1}(ax)}}\,dx$$

Mathematica [A] time = 0.741524, size = 0, normalized size = 0.

$$\int \frac{\left(c+a^2cx^2\right)^3}{\sqrt{\tan^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^3/Sqrt[ArcTan[a*x]],x]
```

```
[Out] Integrate[(c + a^2*c*x^2)^3/Sqrt[ArcTan[a*x]], x]
```

Maple [A] time = 0.531, size = 0, normalized size = 0.

$$\int \left(a^2cx^2+c\right)^3\frac{1}{\sqrt{\arctan\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x)
```

```
[Out] int((a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$c^3\left(\int \frac{3a^2x^2}{\sqrt{\operatorname{atan}(ax)}}\,dx + \int \frac{3a^4x^4}{\sqrt{\operatorname{atan}(ax)}}\,dx + \int \frac{a^6x^6}{\sqrt{\operatorname{atan}(ax)}}\,dx + \int \frac{1}{\sqrt{\operatorname{atan}(ax)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**3/atan(a*x)**(1/2),x)
```

```
[Out] c**3*(Integral(3*a**2*x**2/sqrt(atan(a*x)), x) + Integral(3*a**4*x**4/sqrt(
atan(a*x)), x) + Integral(a**6*x**6/sqrt(atan(a*x)), x) + Integral(1/sqrt(a
tan(a*x)), x))
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2 + c\right)^3}{\sqrt{\arctan(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^3/sqrt(arctan(a*x)), x)
```

3.925 $\int \frac{(c+a^2cx^2)^3}{x\sqrt{\tan^{-1}(ax)}}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{(a^2cx^2+c)^3}{x\sqrt{\tan^{-1}(ax)}},x\right)$$

```
[Out] Unintegrable[(c + a^2*c*x^2)^3/(x*Sqrt[ArcTan[a*x]]), x]
```

Rubi [A] time = 0.0515009, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(c+a^2cx^2)^3}{x\sqrt{\tan^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^3/(x*Sqrt[ArcTan[a*x]]),x]
```

```
[Out] Defer[Int][(c + a^2*c*x^2)^3/(x*Sqrt[ArcTan[a*x]]), x]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^3}{x\sqrt{\tan^{-1}(ax)}}\,dx=\int \frac{(c+a^2cx^2)^3}{x\sqrt{\tan^{-1}(ax)}}\,dx$$

Mathematica [A] time = 1.33833, size = 0, normalized size = 0.

$$\int \frac{(c+a^2cx^2)^3}{x\sqrt{\tan^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^3/(x*Sqrt[ArcTan[a*x]]),x]
```

```
[Out] Integrate[(c + a^2*c*x^2)^3/(x*Sqrt[ArcTan[a*x]]), x]
```

Maple [A] time = 0.631, size = 0, normalized size = 0.

$$\int \frac{\left(a^{2} c \,x^{2}+c \right)^{3}}{x}\frac{1}{\sqrt{\arctan \left(a x \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^3/x/arctan(a*x)^(1/2),x)
```

```
[Out] int((a^2*c*x^2+c)^3/x/arctan(a*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3/x/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3/x/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$c^3\left(\int \frac{1}{x\sqrt{\operatorname{atan}(ax)}}\,dx + \int \frac{3a^2x}{\sqrt{\operatorname{atan}(ax)}}\,dx + \int \frac{3a^4x^3}{\sqrt{\operatorname{atan}(ax)}}\,dx + \int \frac{a^6x^5}{\sqrt{\operatorname{atan}(ax)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**3/x/atan(a*x)**(1/2),x)
```

```
[Out] c**3*(Integral(1/(x*sqrt(atan(a*x))), x) + Integral(3*a**2*x/sqrt(atan(a*x)
), x) + Integral(3*a**4*x**3/sqrt(atan(a*x)), x) + Integral(a**6*x**5/sqrt(
atan(a*x)), x))
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2 + c\right)^3}{x\sqrt{\arctan(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3/x/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^3/(x*sqrt(arctan(a*x))), x)
```

3.926 $\int \frac{x^m}{(c+a^2cx^2)\sqrt{\tan^{-1}(ax)}}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{x^m}{(a^2cx^2+c)\sqrt{\tan^{-1}(ax)}},x\right)$$

```
[Out] Unintegrable[x^m/((c + a^2*c*x^2)*Sqrt[ArcTan[a*x]]), x]
```

Rubi [A] time = 0.0677465, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m}{(c+a^2cx^2)\sqrt{\tan^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m/((c + a^2*c*x^2)*Sqrt[ArcTan[a*x]]),x]
```

```
[Out] Defer[Int][x^m/((c + a^2*c*x^2)*Sqrt[ArcTan[a*x]]), x]
```

Rubi steps

$$\int \frac{x^m}{(c+a^2cx^2)\sqrt{\tan^{-1}(ax)}}\,dx=\int \frac{x^m}{(c+a^2cx^2)\sqrt{\tan^{-1}(ax)}}\,dx$$

Mathematica [A] time = 0.602541, size = 0, normalized size = 0.

$$\int \frac{x^m}{(c+a^2cx^2)\sqrt{\tan^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m/((c + a^2*c*x^2)*Sqrt[ArcTan[a*x]]),x]
```

```
[Out] Integrate[x^m/((c + a^2*c*x^2)*Sqrt[ArcTan[a*x]]), x]
```

Maple [A] time = 0.566, size = 0, normalized size = 0.

$$\int \frac{x^m}{a^2 c x^2 + c} \frac{1}{\sqrt{\arctan (ax)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a^2*c*x^2+c)/arctan(a*x)^(1/2),x)
```

```
[Out] int(x^m/(a^2*c*x^2+c)/arctan(a*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^m}{\left(a^2 c x^2 + c\right)\sqrt{\arctan (ax)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^m/((a^2*c*x^2 + c)*sqrt(arctan(a*x))), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^m}{a^2x^2\sqrt{\operatorname{atan}(ax)}+\sqrt{\operatorname{atan}(ax)}}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(a**2*c*x**2+c)/atan(a*x)**(1/2),x)
```

```
[Out] Integral(x**m/(a**2*x**2*sqrt(atan(a*x)) + sqrt(atan(a*x))), x)/c
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{(a^2cx^2+c)\sqrt{\arctan(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^m/((a^2*c*x^2 + c)*sqrt(arctan(a*x))), x)
```

3.927 $\int \frac{x}{\left(c+a^2cx^2\right)\sqrt{\tan^{-1}(ax)}}\,dx$

Optimal. Leaf size=36

$$\frac{2x\sqrt{\tan^{-1}(ax)}}{ac}-\frac{2\text{Unintegrable}\left(\sqrt{\tan^{-1}(ax)},x\right)}{ac}$$

```
[Out] (2*x*Sqrt[ArcTan[a*x]])/(a*c) - (2*Unintegrable[Sqrt[ArcTan[a*x]], x])/(a*c
)
```

Rubi [A] time = 0.0490175, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x}{\left(c+a^2cx^2\right)\sqrt{\tan^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x/((c + a^2*c*x^2)*Sqrt[ArcTan[a*x]]),x]
```

```
[Out] (2*x*Sqrt[ArcTan[a*x]])/(a*c) - (2*Defer[Int][Sqrt[ArcTan[a*x]], x])/(a*c)
```

Rubi steps

$$\int \frac{x}{\left(c+a^2cx^2\right)\sqrt{\tan^{-1}(ax)}}\,dx=\frac{2x\sqrt{\tan^{-1}(ax)}}{ac}-\frac{2\int \sqrt{\tan^{-1}(ax)}\,dx}{ac}$$

Mathematica [A] time = 0.81425, size = 0, normalized size = 0.

$$\int \frac{x}{\left(c+a^2cx^2\right)\sqrt{\tan^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x/((c + a^2*c*x^2)*Sqrt[ArcTan[a*x]]),x]
```

```
[Out] Integrate[x/((c + a^2*c*x^2)*Sqrt[ArcTan[a*x]]), x]
```

Maple [A] time = 0.148, size = 0, normalized size = 0.

$$\int \frac{x}{a^2cx^2 + c} \frac{1}{\sqrt{\arctan(ax)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^2*c*x^2+c)/arctan(a*x)^(1/2),x)
```

```
[Out] int(x/(a^2*c*x^2+c)/arctan(a*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x}{a^2x^2\sqrt{\operatorname{atan}(ax)}+\sqrt{\operatorname{atan}(ax)}}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a**2*c*x**2+c)/atan(a*x)**(1/2),x)
```

```
[Out] Integral(x/(a**2*x**2*sqrt(atan(a*x)) + sqrt(atan(a*x))), x)/c
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(a^2cx^2+c)\sqrt{\arctan(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/((a^2*c*x^2 + c)*sqrt(arctan(a*x))), x)
```

3.928 $\int \frac{1}{(c+a^2cx^2)\sqrt{\tan^{-1}(ax)}}\,dx$

Optimal. Leaf size=16

$$\frac{2\sqrt{\tan^{-1}(ax)}}{ac}$$

```
[Out] (2*Sqrt[ArcTan[a*x]])/(a*c)
```

Rubi [A] time = 0.0243045, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {4884}

$$\frac{2\sqrt{\tan^{-1}(ax)}}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a^2*c*x^2)*Sqrt[ArcTan[a*x]]),x]
```

```
[Out] (2*Sqrt[ArcTan[a*x]])/(a*c)
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(c+a^2cx^2\right)\sqrt{\tan^{-1}(ax)}}\,dx = \frac{2\sqrt{\tan^{-1}(ax)}}{ac}$$

Mathematica [A] time = 0.0031818, size = 16, normalized size = 1.

$$\frac{2\sqrt{\tan^{-1}(ax)}}{ac}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a^2*c*x^2)*Sqrt[ArcTan[a*x]]),x]
```

```
[Out] (2*Sqrt[ArcTan[a*x]])/(a*c)
```

Maple [A] time = 0.087, size = 15, normalized size = 0.9

$$2\,{\frac {\sqrt {\arctan \left(ax \right) }}{ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2*c*x^2+c)/arctan(a*x)^(1/2),x)
```

```
[Out] 2*arctan(a*x)^(1/2)/a/c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 1.88247, size = 36, normalized size = 2.25

$$\frac{2 \, \sqrt{\arctan\left(a x\right)}}{a c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(arctan(a*x))/(a*c)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*c*x**2+c)/atan(a*x)**(1/2),x)
```

```
[Out] Exception raised: TypeError
```

Giac [A] time = 1.1066, size = 19, normalized size = 1.19

$$\frac{2\sqrt{\arctan(ax)}}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(arctan(a*x))/(a*c)
```

3.929 $\int \frac{1}{x(c+a^2cx^2)\sqrt{\tan^{-1}(ax)}}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{1}{x\left(a^2cx^2+c\right)\sqrt{\tan^{-1}(ax)}},x\right)$$

```
[Out] Unintegrable[1/(x*(c + a^2*c*x^2)*Sqrt[ArcTan[a*x]]), x]
```

Rubi [A] time = 0.0645273, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x\left(c+a^2cx^2\right)\sqrt{\tan^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*(c + a^2*c*x^2)*Sqrt[ArcTan[a*x]]),x]
```

```
[Out] Defer[Int][1/(x*(c + a^2*c*x^2)*Sqrt[ArcTan[a*x]]), x]
```

Rubi steps

$$\int \frac{1}{x\left(c+a^2cx^2\right)\sqrt{\tan^{-1}(ax)}}\,dx=\int \frac{1}{x\left(c+a^2cx^2\right)\sqrt{\tan^{-1}(ax)}}\,dx$$

Mathematica [A] time = 0.251681, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(c+a^2cx^2\right)\sqrt{\tan^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*(c + a^2*c*x^2)*Sqrt[ArcTan[a*x]]),x]
```

```
[Out] Integrate[1/(x*(c + a^2*c*x^2)*Sqrt[ArcTan[a*x]]), x]
```

Maple [A] time = 0.137, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(a^{2}c x^{2}+c\right)}\frac{1}{\sqrt{\arctan\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a^2*c*x^2+c)/arctan(a*x)^(1/2),x)
```

```
[Out] int(1/x/(a^2*c*x^2+c)/arctan(a*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{a^2x^3\sqrt{\operatorname{atan}(ax)}+x\sqrt{\operatorname{atan}(ax)}}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a**2*c*x**2+c)/atan(a*x)**(1/2),x)
```

```
[Out] Integral(1/(a**2*x**3*sqrt(atan(a*x)) + x*sqrt(atan(a*x))), x)/c
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)x\sqrt{\arctan(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)*x*sqrt(arctan(a*x))), x)
```

3.930 $\int \frac{x^m}{\left(c+a^2cx^2\right)^2\sqrt{\tan^{-1}(ax)}}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{x^m}{\left(a^2cx^2+c\right)^2\sqrt{\tan^{-1}(ax)}},x\right)$$

```
[Out] Unintegrable[x^m/((c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]), x]
```

Rubi [A] time = 0.0671878, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m}{\left(c+a^2cx^2\right)^2\sqrt{\tan^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m/((c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]),x]
```

```
[Out] Defer[Int][x^m/((c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]), x]
```

Rubi steps

$$\int \frac{x^m}{\left(c+a^2cx^2\right)^2\sqrt{\tan^{-1}(ax)}}\,dx=\int \frac{x^m}{\left(c+a^2cx^2\right)^2\sqrt{\tan^{-1}(ax)}}\,dx$$

Mathematica [A] time = 1.45679, size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(c+a^2cx^2\right)^2\sqrt{\tan^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m/((c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]),x]
```

```
[Out] Integrate[x^m/((c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]), x]
```

Maple [A] time = 0.779, size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(a^2cx^2+c\right)^2}\frac{1}{\sqrt{\arctan\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x)
```

```
[Out] int(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x^m}{\left(a^4c^2x^4+2\,a^2c^2x^2+c^2\right)\sqrt{\arctan\left(ax\right)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^m/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(arctan(a*x))), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(a**2*c*x**2+c)**2/atan(a*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(a^2cx^2+c\right)^2\sqrt{\arctan\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^m/((a^2*c*x^2 + c)^2*sqrt(arctan(a*x))), x)
```

3.931 $\int \frac{x^3}{\left(c+a^2cx^2\right)^2\sqrt{\tan^{-1}(ax)}}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{x^3}{\left(a^2cx^2+c\right)^2\sqrt{\tan^{-1}(ax)}},x\right)$$

```
[Out] Unintegrable[x^3/((c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]), x]
```

Rubi [A] time = 0.0656968, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^3}{\left(c+a^2cx^2\right)^2\sqrt{\tan^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^3/((c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]),x]

[Out] Defer[Int][x^3/((c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]), x]
```

Rubi steps

$$\int \frac{x^3}{\left(c+a^2cx^2\right)^2\sqrt{\tan^{-1}(ax)}}\,dx=\int \frac{x^3}{\left(c+a^2cx^2\right)^2\sqrt{\tan^{-1}(ax)}}\,dx$$

Mathematica [A] time = 3.55544, size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(c+a^2cx^2\right)^2\sqrt{\tan^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^3/((c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]),x]
```

```
[Out] Integrate[x^3/((c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]), x]
```

Maple [A] time = 0.645, size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(a^2cx^2+c\right)^2}\frac{1}{\sqrt{\arctan\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x)
```

```
[Out] int(x^3/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^3}{a^4x^4\sqrt{\operatorname{atan}(ax)}+2a^2x^2\sqrt{\operatorname{atan}(ax)}+\sqrt{\operatorname{atan}(ax)}}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a**2*c*x**2+c)**2/atan(a*x)**(1/2),x)
```

```
[Out] Integral(x**3/(a**4*x**4*sqrt(atan(a*x)) + 2*a**2*x**2*sqrt(atan(a*x)) + sq
rt(atan(a*x))), x)/c**2
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(a^2cx^2+c\right)^2\sqrt{\arctan(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/((a^2*c*x^2 + c)^2*sqrt(arctan(a*x))), x)
```

3.932 $\int \frac{x^2}{\left(c+a^2cx^2\right)^2\sqrt{\tan^{-1}(ax)}}\,dx$

Optimal. Leaf size=47

$$\frac{\sqrt{\tan^{-1}(ax)}}{a^3c^2}-\frac{\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{2a^3c^2}$$

```
[Out] Sqrt[ArcTan[a*x]]/(a^3*c^2) - (Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt
[Pi]])/(2*a^3*c^2)
```

Rubi [A] time = 0.106711, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4970, 3312, 3304, 3352}

$$\frac{\sqrt{\tan^{-1}(ax)}}{a^3c^2}-\frac{\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{2a^3c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]),x]
```

```
[Out] Sqrt[ArcTan[a*x]]/(a^3*c^2) - (Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt
[Pi]])/(2*a^3*c^2)
```

Rule 4970

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sin[x]^m)/
Cos[x]^(m + 2*(q + 1)), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{x^2}{\left(c+a^2cx^2\right)^2\sqrt{\tan^{-1}(ax)}}\,dx = \frac{\text{Subst}\left(\int \frac{\sin^2(x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{a^3c^2}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{2\sqrt{x}}-\frac{\cos(2x)}{2\sqrt{x}}\right)dx, x, \tan^{-1}(ax)\right)}{a^3c^2}$$

$$= \frac{\sqrt{\tan^{-1}(ax)}}{a^3c^2}-\frac{\text{Subst}\left(\int \frac{\cos(2x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{2a^3c^2}$$

$$= \frac{\sqrt{\tan^{-1}(ax)}}{a^3c^2}-\frac{\text{Subst}\left(\int \cos\left(2x^2\right)dx, x, \sqrt{\tan^{-1}(ax)}\right)}{a^3c^2}$$

$$= \frac{\sqrt{\tan^{-1}(ax)}}{a^3c^2}-\frac{\sqrt{\pi}C\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{2a^3c^2}$$

Mathematica [C] time = 0.204202, size = 122, normalized size = 2.6

$$\frac{i\sqrt{2}\sqrt{-i\tan^{-1}(ax)}\text{Gamma}\left(\frac{1}{2},-2i\tan^{-1}(ax)\right)-i\sqrt{2}\sqrt{i\tan^{-1}(ax)}\text{Gamma}\left(\frac{1}{2},2i\tan^{-1}(ax)\right)-4\sqrt{\pi}\sqrt{\tan^{-1}(ax)}\text{Fresne}}{16a^3c^2\sqrt{\tan^{-1}(ax)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/((c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]),x]
```

```
[Out] (16*ArcTan[a*x] - 4*Sqrt[Pi]*Sqrt[ArcTan[a*x]]*FresnelC[(2*Sqrt[ArcTan[a*x]
])/Sqrt[Pi]] + I*Sqrt[2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-2*I)*ArcTan[a*
```

```
x]] - I*Sqrt[2]*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (2*I)*ArcTan[a*x]])/(16*a^3*
c^2*Sqrt[ArcTan[a*x]])
```

Maple [A] time = 0.099, size = 38, normalized size = 0.8

$$-\frac{\sqrt{\pi}}{2\,a^3c^2}\mathrm{FresnelC}\left(2\,\frac{\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{1}{a^3c^2}\sqrt{\arctan(ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x)
```

```
[Out] -1/2*FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2))*Pi^(1/2)/a^3/c^2+arctan(a*x)^(1
/2)/a^3/c^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^2}{a^4x^4\sqrt{\operatorname{atan}(ax)}+2a^2x^2\sqrt{\operatorname{atan}(ax)}+\sqrt{\operatorname{atan}(ax)}}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a**2*c*x**2+c)**2/atan(a*x)**(1/2),x)
```

```
[Out] Integral(x**2/(a**4*x**4*sqrt(atan(a*x)) + 2*a**2*x**2*sqrt(atan(a*x)) + sq
rt(atan(a*x))), x)/c**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(a^2cx^2+c\right)^2\sqrt{\arctan(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/((a^2*c*x^2 + c)^2*sqrt(arctan(a*x))), x)
```

3.933 $\int \frac{x}{\left(c+a^2cx^2\right)^2\sqrt{\tan^{-1}(ax)}}\,dx$

Optimal. Leaf size=31

$$\frac{\sqrt{\pi}S\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{2a^2c^2}$$

```
[Out] (Sqrt[Pi]*FresnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(2*a^2*c^2)
```

Rubi [A] time = 0.0744631, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {4970, 4406, 12, 3305, 3351}

$$\frac{\sqrt{\pi}S\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{2a^2c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/((c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]),x]
```

```
[Out] (Sqrt[Pi]*FresnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(2*a^2*c^2)
```

Rule 4970

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sin[x]^m)/
Cos[x]^(m + 2*(q + 1)), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{x}{\left(c+a^2cx^2\right)^2\sqrt{\tan^{-1}(ax)}}\,dx = \frac{\text{Subst}\left(\int \frac{\cos(x)\sin(x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{a^2c^2}$$
$$= \frac{\text{Subst}\left(\int \frac{\sin(2x)}{2\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{a^2c^2}$$
$$= \frac{\text{Subst}\left(\int \frac{\sin(2x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{2a^2c^2}$$
$$= \frac{\text{Subst}\left(\int \sin\left(2x^2\right)dx, x, \sqrt{\tan^{-1}(ax)}\right)}{a^2c^2}$$
$$= \frac{\sqrt{\pi}S\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{2a^2c^2}$$

Mathematica [A] time = 0.0525929, size = 31, normalized size = 1.

$$\frac{\sqrt{\pi}S\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{2a^2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]),x]
```

```
[Out] (Sqrt[Pi]*FresnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(2*a^2*c^2)
```

Maple [A] time = 0.079, size = 24, normalized size = 0.8

$$\frac{\sqrt{\pi}}{2\,a^2c^2}\mathrm{FresnelS}\left(2\,\frac{\sqrt{\arctan\left(ax\right)}}{\sqrt{\pi}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x)
```

```
[Out] 1/2*FresnelS(2*arctan(a*x)^(1/2)/Pi^(1/2))*Pi^(1/2)/a^2/c^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x}{a^4x^4\sqrt{\operatorname{atan}(ax)}+2a^2x^2\sqrt{\operatorname{atan}(ax)}+\sqrt{\operatorname{atan}(ax)}}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a**2*c*x**2+c)**2/atan(a*x)**(1/2),x)
```

```
[Out] Integral(x/(a**4*x**4*sqrt(atan(a*x)) + 2*a**2*x**2*sqrt(atan(a*x)) + sqrt(
atan(a*x))), x)/c**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(a^2cx^2+c\right)^2\sqrt{\arctan(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/((a^2*c*x^2 + c)^2*sqrt(arctan(a*x))), x)
```

3.934 $\int \frac{1}{(c+a^2cx^2)^2 \sqrt{\tan^{-1}(ax)}} dx$

Optimal. Leaf size=47

$$\frac{\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{2ac^2}+\frac{\sqrt{\tan^{-1}(ax)}}{ac^2}$$

```
[Out] Sqrt[ArcTan[a*x]]/(a*c^2) + (Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[P
i]])/(2*a*c^2)
```

Rubi [A] time = 0.0695104, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {4904, 3312, 3304, 3352}

$$\frac{\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{2ac^2}+\frac{\sqrt{\tan^{-1}(ax)}}{ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]),x]
```

```
[Out] Sqrt[ArcTan[a*x]]/(a*c^2) + (Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[P
i]])/(2*a*c^2)
```

Rule 4904

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\left(c+a^2cx^2\right)^2\sqrt{\tan^{-1}(ax)}}\,dx &= \frac{\text{Subst}\left(\int \frac{\cos^2(x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{ac^2}\\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{2\sqrt{x}}+\frac{\cos(2x)}{2\sqrt{x}}\right)dx, x, \tan^{-1}(ax)\right)}{ac^2}\\
&= \frac{\sqrt{\tan^{-1}(ax)}}{ac^2}+\frac{\text{Subst}\left(\int \frac{\cos(2x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{2ac^2}\\
&= \frac{\sqrt{\tan^{-1}(ax)}}{ac^2}+\frac{\text{Subst}\left(\int \cos\left(2x^2\right)dx, x, \sqrt{\tan^{-1}(ax)}\right)}{ac^2}\\
&= \frac{\sqrt{\tan^{-1}(ax)}}{ac^2}+\frac{\sqrt{\pi}C\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{2ac^2}
\end{aligned}
$$

Mathematica [A] time = 0.232991, size = 43, normalized size = 0.91

$$\frac{\sqrt{\pi}\text{FresnelC}\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)+2\sqrt{\tan^{-1}(ax)}}{2ac^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]),x]
```

```
[Out] (2*Sqrt[ArcTan[a*x]] + Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(
2*a*c^2)
```

Maple [A] time = 0.138, size = 38, normalized size = 0.8

$$\frac{\sqrt{\pi}}{2\,ac^2}\mathrm{FresnelC}\left(2\,\frac{\sqrt{\arctan\left(ax\right)}}{\sqrt{\pi}}\right)+\frac{1}{ac^2}\sqrt{\arctan\left(ax\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x)
```

```
[Out] 1/2*FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2))*Pi^(1/2)/a/c^2+arctan(a*x)^(1/2)
/a/c^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{a^4x^4\sqrt{\operatorname{atan}\left(ax\right)}+2a^2x^2\sqrt{\operatorname{atan}\left(ax\right)}+\sqrt{\operatorname{atan}\left(ax\right)}}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*c*x**2+c)**2/atan(a*x)**(1/2),x)
```

```
[Out] Integral(1/(a**4*x**4*sqrt(atan(a*x)) + 2*a**2*x**2*sqrt(atan(a*x)) + sqrt(
atan(a*x))), x)/c**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)^2\sqrt{\arctan\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^2*sqrt(arctan(a*x))), x)
```

3.935 $\int \frac{1}{x\left(c+a^2cx^2\right)^2\sqrt{\tan^{-1}(ax)}}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{1}{x\left(a^2cx^2+c\right)^2\sqrt{\tan^{-1}(ax)}},x\right)$$

```
[Out] Unintegrable[1/(x*(c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]), x]
```

Rubi [A] time = 0.0595416, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x\left(c+a^2cx^2\right)^2\sqrt{\tan^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*(c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]),x]

[Out] Defer[Int][1/(x*(c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]), x]
```

Rubi steps

$$\int \frac{1}{x\left(c+a^2cx^2\right)^2\sqrt{\tan^{-1}(ax)}}\,dx=\int \frac{1}{x\left(c+a^2cx^2\right)^2\sqrt{\tan^{-1}(ax)}}\,dx$$

Mathematica [A] time = 1.35283, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(c+a^2cx^2\right)^2\sqrt{\tan^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*(c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]),x]
```

```
[Out] Integrate[1/(x*(c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]), x]
```

Maple [A] time = 0.688, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(a^2cx^2+c\right)^2}\frac{1}{\sqrt{\arctan\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x)
```

```
[Out] int(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{a^4x^5\sqrt{\operatorname{atan}(ax)}+2a^2x^3\sqrt{\operatorname{atan}(ax)}+x\sqrt{\operatorname{atan}(ax)}}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a**2*c*x**2+c)**2/atan(a*x)**(1/2),x)
```

```
[Out] Integral(1/(a**4*x**5*sqrt(atan(a*x)) + 2*a**2*x**3*sqrt(atan(a*x)) + x*sqr
t(atan(a*x))), x)/c**2
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)^2 x\sqrt{\arctan(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^2*x*sqrt(arctan(a*x))), x)
```

3.936 $\int \frac{x^m}{(c+a^2cx^2)^3\sqrt{\tan^{-1}(ax)}}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{x^m}{(a^2cx^2+c)^3\sqrt{\tan^{-1}(ax)}},x\right)$$

```
[Out] Unintegrable[x^m/((c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]), x]
```

Rubi [A] time = 0.0635359, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{x^m}{(c+a^2cx^2)^3\sqrt{\tan^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m/((c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]),x]
```

```
[Out] Defer[Int][x^m/((c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]), x]
```

Rubi steps

$$\int \frac{x^m}{(c+a^2cx^2)^3\sqrt{\tan^{-1}(ax)}}\,dx = \int \frac{x^m}{(c+a^2cx^2)^3\sqrt{\tan^{-1}(ax)}}\,dx$$

Mathematica [A] time = 1.87729, size = 0, normalized size = 0.

$$\int \frac{x^m}{(c+a^2cx^2)^3\sqrt{\tan^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m/((c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]),x]
```

[Out] Integrate[x^m/((c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]), x]

Maple [A] time = 0.941, size = 0, normalized size = 0.

$$\int \frac{x^{m}}{\left(a^{2} c \,x^{2}+c \right)^{3}} \frac{1}{\sqrt{\arctan \left(a x \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x)
```

```
[Out] int(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x^{m}}{\left(a^{6} c^{3} x^{6} + 3 \, a^{4} c^{3} x^{4} + 3 \, a^{2} c^{3} x^{2} + c^{3}\right)\sqrt{\arctan \left(a x\right)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^m/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*sqrt(arct
an(a*x))), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(a**2*c*x**2+c)**3/atan(a*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(a^2cx^2+c\right)^3\sqrt{\arctan\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^m/((a^2*c*x^2 + c)^3*sqrt(arctan(a*x))), x)
```

3.937 $\int \frac{x^5}{\left(c+a^2cx^2\right)^3\sqrt{\tan^{-1}(ax)}}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{x^5}{\left(a^2cx^2+c\right)^3\sqrt{\tan^{-1}(ax)}},x\right)$$

```
[Out] Unintegrable[x^5/((c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]), x]
```

Rubi [A] time = 0.0633327, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^5}{\left(c+a^2cx^2\right)^3\sqrt{\tan^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^5/((c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]),x]
```

```
[Out] Defer[Int][x^5/((c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]), x]
```

Rubi steps

$$\int \frac{x^5}{\left(c+a^2cx^2\right)^3\sqrt{\tan^{-1}(ax)}}\,dx=\int \frac{x^5}{\left(c+a^2cx^2\right)^3\sqrt{\tan^{-1}(ax)}}\,dx$$

Mathematica [A] time = 4.3683, size = 0, normalized size = 0.

$$\int \frac{x^5}{\left(c+a^2cx^2\right)^3\sqrt{\tan^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^5/((c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]),x]
```

```
[Out] Integrate[x^5/((c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]), x]
```

Maple [A] time = 0.641, size = 0, normalized size = 0.

$$\int \frac{x^5}{\left(a^2cx^2+c\right)^3}\frac{1}{\sqrt{\arctan\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x)
```

```
[Out] int(x^5/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^5}{a^6x^6\sqrt{\operatorname{atan}(ax)}+3a^4x^4\sqrt{\operatorname{atan}(ax)}+3a^2x^2\sqrt{\operatorname{atan}(ax)}+\sqrt{\operatorname{atan}(ax)}}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(a**2*c*x**2+c)**3/atan(a*x)**(1/2),x)
```

```
[Out] Integral(x**5/(a**6*x**6*sqrt(atan(a*x)) + 3*a**4*x**4*sqrt(atan(a*x)) + 3*
a**2*x**2*sqrt(atan(a*x)) + sqrt(atan(a*x))), x)/c**3
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\left(a^2cx^2+c\right)^3\sqrt{\arctan(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^5/((a^2*c*x^2 + c)^3*sqrt(arctan(a*x))), x)
```

3.938 $\int \frac{x^4}{\left(c+a^2cx^2\right)^3\sqrt{\tan^{-1}(ax)}}\,dx$

Optimal. Leaf size=89

$$\frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{8a^5c^3}-\frac{\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{2a^5c^3}+\frac{3\sqrt{\tan^{-1}(ax)}}{4a^5c^3}$$

```
[Out] (3*Sqrt[ArcTan[a*x]])/(4*a^5*c^3) + (Sqrt[Pi/2]*FresnelC[2*Sqrt[2/Pi]*Sqrt[
ArcTan[a*x]]])/(8*a^5*c^3) - (Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[
Pi]])/(2*a^5*c^3)
```

Rubi [A] time = 0.137362, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4970, 3312, 3304, 3352}

$$\frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{8a^5c^3}-\frac{\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{2a^5c^3}+\frac{3\sqrt{\tan^{-1}(ax)}}{4a^5c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]),x]
```

```
[Out] (3*Sqrt[ArcTan[a*x]])/(4*a^5*c^3) + (Sqrt[Pi/2]*FresnelC[2*Sqrt[2/Pi]*Sqrt[
ArcTan[a*x]]])/(8*a^5*c^3) - (Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[
Pi]])/(2*a^5*c^3)
```

Rule 4970

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sin[x]^m)/
Cos[x]^(m + 2*(q + 1)), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
```

```
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{x^4}{\left(c+a^2cx^2\right)^3\sqrt{\tan^{-1}(ax)}}\,dx = \frac{\text{Subst}\left(\int \frac{\sin^4(x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{a^5c^3}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{3}{8\sqrt{x}} - \frac{\cos(2x)}{2\sqrt{x}} + \frac{\cos(4x)}{8\sqrt{x}}\right)dx, x, \tan^{-1}(ax)\right)}{a^5c^3}$$

$$= \frac{3\sqrt{\tan^{-1}(ax)}}{4a^5c^3} + \frac{\text{Subst}\left(\int \frac{\cos(4x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{8a^5c^3} - \frac{\text{Subst}\left(\int \frac{\cos(2x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{2a^5c^3}$$

$$= \frac{3\sqrt{\tan^{-1}(ax)}}{4a^5c^3} + \frac{\text{Subst}\left(\int \cos\left(4x^2\right)dx, x, \sqrt{\tan^{-1}(ax)}\right)}{4a^5c^3} - \frac{\text{Subst}\left(\int \cos\left(2x^2\right)dx, x, \sqrt{\tan^{-1}(ax)}\right)}{a^5c^3}$$

$$= \frac{3\sqrt{\tan^{-1}(ax)}}{4a^5c^3} + \frac{\sqrt{\frac{\pi}{2}}C\left(2\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{8a^5c^3} - \frac{\sqrt{\pi}C\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{2a^5c^3}$$

Mathematica [C] time = 0.494803, size = 230, normalized size = 2.58

$$3\sqrt{\tan^{-1}(ax)}\left(4\sqrt{2}\sqrt{i\tan^{-1}(ax)}\text{Gamma}\left(\frac{1}{2},-2i\tan^{-1}(ax)\right) + 4\sqrt{2}\sqrt{-i\tan^{-1}(ax)}\text{Gamma}\left(\frac{1}{2},2i\tan^{-1}(ax)\right) - \sqrt{i\tan^{-1}}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4/((c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]),x]
```

```
[Out] (10*Sqrt[2*Pi]*Sqrt[ArcTan[a*x]^2]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]]
 - 80*Sqrt[Pi]*Sqrt[ArcTan[a*x]^2]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]]
 + 3*Sqrt[ArcTan[a*x]]*(64*Sqrt[ArcTan[a*x]^2] + 4*Sqrt[2]*Sqrt[I*ArcTan[a*
x]]*Gamma[1/2, (-2*I)*ArcTan[a*x]] + 4*Sqrt[2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma
[1/2, (2*I)*ArcTan[a*x]] - Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (-4*I)*ArcTan[a*x
]] - Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (4*I)*ArcTan[a*x]]))/(256*a^5*c^3*Sq
rt[ArcTan[a*x]^2])
```

Maple [A] time = 0.117, size = 68, normalized size = 0.8

$$\frac{\sqrt{2}\sqrt{\pi}}{16\,c^3a^5}\mathrm{FresnelC}\left(2\,\frac{\sqrt{2}\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)-\frac{\sqrt{\pi}}{2\,c^3a^5}\mathrm{FresnelC}\left(2\,\frac{\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{3}{4\,c^3a^5}\sqrt{\arctan(ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x)
```

```
[Out] 1/16*FresnelC(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^5/c^
3-1/2*FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2))*Pi^(1/2)/a^5/c^3+3/4*arctan(a*
x)^(1/2)/a^5/c^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^4}{a^6x^6\sqrt{\operatorname{atan}(ax)}+3a^4x^4\sqrt{\operatorname{atan}(ax)}+3a^2x^2\sqrt{\operatorname{atan}(ax)}+\sqrt{\operatorname{atan}(ax)}}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(a**2*c*x**2+c)**3/atan(a*x)**(1/2),x)
```

```
[Out] Integral(x**4/(a**6*x**6*sqrt(atan(a*x)) + 3*a**4*x**4*sqrt(atan(a*x)) + 3*
a**2*x**2*sqrt(atan(a*x)) + sqrt(atan(a*x))), x)/c**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(a^2cx^2+c\right)^3\sqrt{\arctan(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/((a^2*c*x^2 + c)^3*sqrt(arctan(a*x))), x)
```

3.939 $\int \frac{x^3}{\left(c+a^2cx^2\right)^3\sqrt{\tan^{-1}(ax)}}\,dx$

Optimal. Leaf size=71

$$\frac{\sqrt{\pi}S\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{4a^4c^3}-\frac{\sqrt{\frac{\pi}{2}}S\left(2\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{8a^4c^3}$$

```
[Out] -(Sqrt[Pi/2]*FresnelS[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(8*a^4*c^3) + (Sqrt[
Pi]*FresnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(4*a^4*c^3)
```

Rubi [A] time = 0.12819, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4970, 4406, 3305, 3351}

$$\frac{\sqrt{\pi}S\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{4a^4c^3}-\frac{\sqrt{\frac{\pi}{2}}S\left(2\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{8a^4c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]),x]
```

```
[Out] -(Sqrt[Pi/2]*FresnelS[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(8*a^4*c^3) + (Sqrt[
Pi]*FresnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(4*a^4*c^3)
```

Rule 4970

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sin[x]^m)/
Cos[x]^(m + 2*(q + 1)), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
```

```
tQ[p, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{x^3}{\left(c+a^2cx^2\right)^3\sqrt{\tan^{-1}(ax)}}\,dx=\frac{\text{Subst}\left(\int\frac{\cos(x)\sin^3(x)}{\sqrt{x}}\,dx,x,\tan^{-1}(ax)\right)}{a^4c^3}$$
$$=\frac{\text{Subst}\left(\int\left(\frac{\sin(2x)}{4\sqrt{x}}-\frac{\sin(4x)}{8\sqrt{x}}\right)dx,x,\tan^{-1}(ax)\right)}{a^4c^3}$$
$$=-\frac{\text{Subst}\left(\int\frac{\sin(4x)}{\sqrt{x}}\,dx,x,\tan^{-1}(ax)\right)}{8a^4c^3}+\frac{\text{Subst}\left(\int\frac{\sin(2x)}{\sqrt{x}}\,dx,x,\tan^{-1}(ax)\right)}{4a^4c^3}$$
$$=-\frac{\text{Subst}\left(\int\sin\left(4x^2\right)dx,x,\sqrt{\tan^{-1}(ax)}\right)}{4a^4c^3}+\frac{\text{Subst}\left(\int\sin\left(2x^2\right)dx,x,\sqrt{\tan^{-1}(ax)}\right)}{2a^4c^3}$$
$$=-\frac{\sqrt{\frac{\pi}{2}}S\left(2\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{8a^4c^3}+\frac{\sqrt{\pi}S\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{4a^4c^3}$$

Mathematica [C] time = 0.143132, size = 131, normalized size = 1.85

$$\frac{-2\sqrt{2}\sqrt{-i\tan^{-1}(ax)}\text{Gamma}\left(\frac{1}{2},-2i\tan^{-1}(ax)\right)-2\sqrt{2}\sqrt{i\tan^{-1}(ax)}\text{Gamma}\left(\frac{1}{2},2i\tan^{-1}(ax)\right)+\sqrt{-i\tan^{-1}(ax)}\text{Gamm}}{32a^4c^3\sqrt{\tan^{-1}(ax)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/((c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]),x]
```

```
[Out] (-2*Sqrt[2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-2*I)*ArcTan[a*x]] - 2*Sqrt[
2]*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (2*I)*ArcTan[a*x]] + Sqrt[(-I)*ArcTan[a*x
]]*Gamma[1/2, (-4*I)*ArcTan[a*x]] + Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (4*I)*Ar
cTan[a*x]])/(32*a^4*c^3*Sqrt[ArcTan[a*x]])
```

Maple [A] time = 0.11, size = 54, normalized size = 0.8

$$-\frac{\sqrt{2}\sqrt{\pi}}{16\,c^3a^4}\mathrm{FresnelS}\left(2\,\frac{\sqrt{2}\sqrt{\arctan{(ax)}}}{\sqrt{\pi}}\right)+\frac{\sqrt{\pi}}{4\,c^3a^4}\mathrm{FresnelS}\left(2\,\frac{\sqrt{\arctan{(ax)}}}{\sqrt{\pi}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x)
```

```
[Out] -1/16*FresnelS(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^4/c
^3+1/4*FresnelS(2*arctan(a*x)^(1/2)/Pi^(1/2))*Pi^(1/2)/a^4/c^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^3}{a^6x^6\sqrt{\operatorname{atan}(ax)}+3a^4x^4\sqrt{\operatorname{atan}(ax)}+3a^2x^2\sqrt{\operatorname{atan}(ax)}+\sqrt{\operatorname{atan}(ax)}}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a**2*c*x**2+c)**3/atan(a*x)**(1/2),x)
```

```
[Out] Integral(x**3/(a**6*x**6*sqrt(atan(a*x)) + 3*a**4*x**4*sqrt(atan(a*x)) + 3*
a**2*x**2*sqrt(atan(a*x)) + sqrt(atan(a*x))), x)/c**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{(a^2cx^2+c)^3\sqrt{\arctan(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/((a^2*c*x^2 + c)^3*sqrt(arctan(a*x))), x)
```

3.940 $\int \frac{x^2}{(c+a^2cx^2)^3\sqrt{\tan^{-1}(ax)}}\,dx$

Optimal. Leaf size=58

$$\frac{\sqrt{\tan^{-1}(ax)}}{4a^3c^3}-\frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{8a^3c^3}$$

```
[Out] Sqrt[ArcTan[a*x]]/(4*a^3*c^3) - (Sqrt[Pi/2]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcT
an[a*x]]])/(8*a^3*c^3)
```

Rubi [A] time = 0.118924, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4970, 4406, 3304, 3352}

$$\frac{\sqrt{\tan^{-1}(ax)}}{4a^3c^3}-\frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{8a^3c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]),x]
```

```
[Out] Sqrt[ArcTan[a*x]]/(4*a^3*c^3) - (Sqrt[Pi/2]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcT
an[a*x]]])/(8*a^3*c^3)
```

Rule 4970

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sin[x]^m)/
Cos[x]^(m + 2*(q + 1)), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\left(c+a^2cx^2\right)^3\sqrt{\tan^{-1}(ax)}}\,dx &= \frac{\text{Subst}\left(\int \frac{\cos^2(x)\sin^2(x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{a^3c^3}\\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{8\sqrt{x}}-\frac{\cos(4x)}{8\sqrt{x}}\right)dx, x, \tan^{-1}(ax)\right)}{a^3c^3}\\
&= \frac{\sqrt{\tan^{-1}(ax)}}{4a^3c^3}-\frac{\text{Subst}\left(\int \frac{\cos(4x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{8a^3c^3}\\
&= \frac{\sqrt{\tan^{-1}(ax)}}{4a^3c^3}-\frac{\text{Subst}\left(\int \cos\left(4x^2\right)dx, x, \sqrt{\tan^{-1}(ax)}\right)}{4a^3c^3}\\
&= \frac{\sqrt{\tan^{-1}(ax)}}{4a^3c^3}-\frac{\sqrt{\frac{\pi}{2}}C\left(2\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{8a^3c^3}\end{aligned}$$

Mathematica [C] time = 0.453344, size = 229, normalized size = 3.95

$$\frac{\sqrt{\tan^{-1}(ax)}\left(4\sqrt{2}\sqrt{i\tan^{-1}(ax)}\text{Gamma}\left(\frac{1}{2},-2i\tan^{-1}(ax)\right)+4\sqrt{2}\sqrt{-i\tan^{-1}(ax)}\text{Gamma}\left(\frac{1}{2},2i\tan^{-1}(ax)\right)+7\sqrt{i\tan^{-1}}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/((c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]),x]
```

```
[Out] (-2*Sqrt[2*Pi]*Sqrt[ArcTan[a*x]^2]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]]
 + 16*Sqrt[Pi]*Sqrt[ArcTan[a*x]^2]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]]
```

```
 + Sqrt[ArcTan[a*x]]*(64*Sqrt[ArcTan[a*x]^2] + 4*Sqrt[2]*Sqrt[I*ArcTan[a*x]
]*Gamma[1/2, (-2*I)*ArcTan[a*x]] + 4*Sqrt[2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1
/2, (2*I)*ArcTan[a*x]] + 7*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (-4*I)*ArcTan[a*x
]] + 7*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (4*I)*ArcTan[a*x]]))/(256*a^3*c^3*
Sqrt[ArcTan[a*x]^2])
```

Maple [A] time = 0.117, size = 45, normalized size = 0.8

$$-\frac{\sqrt{2}\sqrt{\pi}}{16\,c^3a^3}\mathrm{FresnelC}\left(2\,\frac{\sqrt{2}\sqrt{\arctan\left(ax\right)}}{\sqrt{\pi}}\right)+\frac{1}{4\,c^3a^3}\sqrt{\arctan\left(ax\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x)
```

```
[Out] -1/16*FresnelC(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^3/c
^3+1/4*arctan(a*x)^(1/2)/a^3/c^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^2}{a^6x^6\sqrt{\operatorname{atan}(ax)}+3a^4x^4\sqrt{\operatorname{atan}(ax)}+3a^2x^2\sqrt{\operatorname{atan}(ax)}+\sqrt{\operatorname{atan}(ax)}}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a**2*c*x**2+c)**3/atan(a*x)**(1/2),x)
```

```
[Out] Integral(x**2/(a**6*x**6*sqrt(atan(a*x)) + 3*a**4*x**4*sqrt(atan(a*x)) + 3*
a**2*x**2*sqrt(atan(a*x)) + sqrt(atan(a*x))), x)/c**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(a^2cx^2+c\right)^3\sqrt{\arctan(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/((a^2*c*x^2 + c)^3*sqrt(arctan(a*x))), x)
```

3.941 $\int \frac{x}{\left(c+a^2cx^2\right)^3\sqrt{\tan^{-1}(ax)}}\,dx$

Optimal. Leaf size=71

$$\frac{\sqrt{\frac{\pi}{2}}S\left(2\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{8a^2c^3}+\frac{\sqrt{\pi}S\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{4a^2c^3}$$

```
[Out] (Sqrt[Pi/2]*FresnelS[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(8*a^2*c^3) + (Sqrt[P
i]*FresnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(4*a^2*c^3)
```

Rubi [A] time = 0.106192, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4970, 4406, 3305, 3351}

$$\frac{\sqrt{\frac{\pi}{2}}S\left(2\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{8a^2c^3}+\frac{\sqrt{\pi}S\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{4a^2c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x/((c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]),x]
```

```
[Out] (Sqrt[Pi/2]*FresnelS[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(8*a^2*c^3) + (Sqrt[P
i]*FresnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(4*a^2*c^3)
```

Rule 4970

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sin[x]^m)/
Cos[x]^(m + 2*(q + 1)), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{x}{\left(c+a^2cx^2\right)^3\sqrt{\tan^{-1}(ax)}}\,dx = \frac{\text{Subst}\left(\int \frac{\cos^3(x)\sin(x)}{\sqrt{x}}\,dx,x,\tan^{-1}(ax)\right)}{a^2c^3}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{\sin(2x)}{4\sqrt{x}}+\frac{\sin(4x)}{8\sqrt{x}}\right)dx,x,\tan^{-1}(ax)\right)}{a^2c^3}$$

$$= \frac{\text{Subst}\left(\int \frac{\sin(4x)}{\sqrt{x}}\,dx,x,\tan^{-1}(ax)\right)}{8a^2c^3}+\frac{\text{Subst}\left(\int \frac{\sin(2x)}{\sqrt{x}}\,dx,x,\tan^{-1}(ax)\right)}{4a^2c^3}$$

$$= \frac{\text{Subst}\left(\int \sin\left(4x^2\right)dx,x,\sqrt{\tan^{-1}(ax)}\right)}{4a^2c^3}+\frac{\text{Subst}\left(\int \sin\left(2x^2\right)dx,x,\sqrt{\tan^{-1}(ax)}\right)}{2a^2c^3}$$

$$= \frac{\sqrt{\frac{\pi}{2}}S\left(2\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{8a^2c^3}+\frac{\sqrt{\pi}S\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{4a^2c^3}$$

Mathematica [C] time = 0.106355, size = 133, normalized size = 1.87

$$\frac{-2\sqrt{2}\sqrt{-i\tan^{-1}(ax)}\text{Gamma}\left(\frac{1}{2},-2i\tan^{-1}(ax)\right)-2\sqrt{2}\sqrt{i\tan^{-1}(ax)}\text{Gamma}\left(\frac{1}{2},2i\tan^{-1}(ax)\right)-\sqrt{-i\tan^{-1}(ax)}\text{Gamm}}{32a^2c^3\sqrt{\tan^{-1}(ax)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]),x]
```

```
[Out] (-2*Sqrt[2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-2*I)*ArcTan[a*x]] - 2*Sqrt[
2]*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (2*I)*ArcTan[a*x]] - Sqrt[(-I)*ArcTan[a*x
```

```
]]*Gamma[1/2, (-4*I)*ArcTan[a*x]] - Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (4*I)*Ar
cTan[a*x]])/(32*a^2*c^3*Sqrt[ArcTan[a*x]])
```

Maple [A] time = 0.09, size = 54, normalized size = 0.8

$$\frac{\sqrt{2}\sqrt{\pi}}{16\,c^3a^2}\mathrm{FresnelS}\left(2\,\frac{\sqrt{2}\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{\sqrt{\pi}}{4\,c^3a^2}\mathrm{FresnelS}\left(2\,\frac{\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x)
```

```
[Out] 1/16*FresnelS(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^2/c^
3+1/4*FresnelS(2*arctan(a*x)^(1/2)/Pi^(1/2))*Pi^(1/2)/a^2/c^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x}{a^6x^6\sqrt{\operatorname{atan}(ax)}+3a^4x^4\sqrt{\operatorname{atan}(ax)}+3a^2x^2\sqrt{\operatorname{atan}(ax)}+\sqrt{\operatorname{atan}(ax)}}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a**2*c*x**2+c)**3/atan(a*x)**(1/2),x)
```

```
[Out] Integral(x/(a**6*x**6*sqrt(atan(a*x)) + 3*a**4*x**4*sqrt(atan(a*x)) + 3*a**
2*x**2*sqrt(atan(a*x)) + sqrt(atan(a*x))), x)/c**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(a^2cx^2+c\right)^3\sqrt{\arctan(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/((a^2*c*x^2 + c)^3*sqrt(arctan(a*x))), x)
```

3.942 $\int \frac{1}{(c+a^2cx^2)^3\sqrt{\tan^{-1}(ax)}}\,dx$

Optimal. Leaf size=89

$$\frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{8ac^3}+\frac{\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{2ac^3}+\frac{3\sqrt{\tan^{-1}(ax)}}{4ac^3}$$

```
[Out] (3*Sqrt[ArcTan[a*x]])/(4*a*c^3) + (Sqrt[Pi/2]*FresnelC[2*Sqrt[2/Pi]*Sqrt[Ar
cTan[a*x]]])/(8*a*c^3) + (Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]]
)/(2*a*c^3)
```

Rubi [A] time = 0.0955638, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {4904, 3312, 3304, 3352}

$$\frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{8ac^3}+\frac{\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{2ac^3}+\frac{3\sqrt{\tan^{-1}(ax)}}{4ac^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]),x]
```


```
[Out] (3*Sqrt[ArcTan[a*x]])/(4*a*c^3) + (Sqrt[Pi/2]*FresnelC[2*Sqrt[2/Pi]*Sqrt[Ar
cTan[a*x]]])/(8*a*c^3) + (Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]]
)/(2*a*c^3)
```


Rule 4904

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
```

```
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{1}{\left(c+a^2cx^2\right)^3\sqrt{\tan^{-1}(ax)}}\,dx = \frac{\text{Subst}\left(\int \frac{\cos^4(x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{ac^3}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{3}{8\sqrt{x}}+\frac{\cos(2x)}{2\sqrt{x}}+\frac{\cos(4x)}{8\sqrt{x}}\right)dx, x, \tan^{-1}(ax)\right)}{ac^3}$$

$$= \frac{3\sqrt{\tan^{-1}(ax)}}{4ac^3}+\frac{\text{Subst}\left(\int \frac{\cos(4x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{8ac^3}+\frac{\text{Subst}\left(\int \frac{\cos(2x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{2ac^3}$$

$$= \frac{3\sqrt{\tan^{-1}(ax)}}{4ac^3}+\frac{\text{Subst}\left(\int \cos\left(4x^2\right)dx, x, \sqrt{\tan^{-1}(ax)}\right)}{4ac^3}+\frac{\text{Subst}\left(\int \cos\left(2x^2\right)dx, x, \right.}{ac^3}$$

$$= \frac{3\sqrt{\tan^{-1}(ax)}}{4ac^3}+\frac{\sqrt{\frac{\pi}{2}}C\left(2\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{8ac^3}+\frac{\sqrt{\pi}C\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{2ac^3}$$

Mathematica [C] time = 0.266006, size = 147, normalized size = 1.65

$$\frac{-4i\sqrt{2}\sqrt{-i\tan^{-1}(ax)}\text{Gamma}\left(\frac{1}{2},-2i\tan^{-1}(ax)\right)+4i\sqrt{2}\sqrt{i\tan^{-1}(ax)}\text{Gamma}\left(\frac{1}{2},2i\tan^{-1}(ax)\right)-i\sqrt{-i\tan^{-1}(ax)}\text{Gam}}{32ac^3\sqrt{\tan^{-1}(ax)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]),x]
```

```
[Out] (24*ArcTan[a*x] - (4*I)*Sqrt[2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-2*I)*Ar
cTan[a*x]] + (4*I)*Sqrt[2]*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (2*I)*ArcTan[a*x]
] - I*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-4*I)*ArcTan[a*x]] + I*Sqrt[I*ArcT
an[a*x]]*Gamma[1/2, (4*I)*ArcTan[a*x]])/(32*a*c^3*Sqrt[ArcTan[a*x]])
```

Maple [A] time = 0.115, size = 68, normalized size = 0.8

$$\frac{\sqrt{2}\sqrt{\pi}}{16\,ac^{3}}\text{FresnelC}\left(2\,\frac{\sqrt{2}\sqrt{\arctan\left(ax\right)}}{\sqrt{\pi}}\right)+\frac{\sqrt{\pi}}{2\,ac^{3}}\text{FresnelC}\left(2\,\frac{\sqrt{\arctan\left(ax\right)}}{\sqrt{\pi}}\right)+\frac{3}{4\,ac^{3}}\sqrt{\arctan\left(ax\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x)
```

```
[Out] 1/16*FresnelC(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a/c^3+
1/2*FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2))*Pi^(1/2)/a/c^3+3/4*arctan(a*x)^(
1/2)/a/c^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{a^6x^6\sqrt{\operatorname{atan}(ax)}+3a^4x^4\sqrt{\operatorname{atan}(ax)}+3a^2x^2\sqrt{\operatorname{atan}(ax)}+\sqrt{\operatorname{atan}(ax)}}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*c*x**2+c)**3/atan(a*x)**(1/2),x)
```

```
[Out] Integral(1/(a**6*x**6*sqrt(atan(a*x)) + 3*a**4*x**4*sqrt(atan(a*x)) + 3*a**
2*x**2*sqrt(atan(a*x)) + sqrt(atan(a*x))), x)/c**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)^3\sqrt{\arctan(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^3*sqrt(arctan(a*x))), x)
```

3.943 $\int \frac{1}{x(c+a^2cx^2)^3\sqrt{\tan^{-1}(ax)}}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{1}{x\left(a^2cx^2+c\right)^3\sqrt{\tan^{-1}(ax)}},x\right)$$

```
[Out] Unintegrable[1/(x*(c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]), x]
```

Rubi [A] time = 0.0594507, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x\left(c+a^2cx^2\right)^3\sqrt{\tan^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*(c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]),x]
```

```
[Out] Defer[Int][1/(x*(c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]), x]
```

Rubi steps

$$\int \frac{1}{x\left(c+a^2cx^2\right)^3\sqrt{\tan^{-1}(ax)}}\,dx=\int \frac{1}{x\left(c+a^2cx^2\right)^3\sqrt{\tan^{-1}(ax)}}\,dx$$

Mathematica [A] time = 1.6342, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(c+a^2cx^2\right)^3\sqrt{\tan^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*(c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]),x]
```

```
[Out] Integrate[1/(x*(c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]), x]
```

Maple [A] time = 0.808, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(a^{2} c\,x^{2}+c\right)^{3}}\frac{1}{\sqrt{\arctan\left(a x\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x)
```

```
[Out] int(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{a^{6}x^{7}\sqrt{\operatorname{atan}(ax)}+3a^{4}x^{5}\sqrt{\operatorname{atan}(ax)}+3a^{2}x^{3}\sqrt{\operatorname{atan}(ax)}+x\sqrt{\operatorname{atan}(ax)}}\,dx}{c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a**2*c*x**2+c)**3/atan(a*x)**(1/2),x)
```

```
[Out] Integral(1/(a**6*x**7*sqrt(atan(a*x)) + 3*a**4*x**5*sqrt(atan(a*x)) + 3*a**
2*x**3*sqrt(atan(a*x)) + x*sqrt(atan(a*x))), x)/c**3
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^{2}cx^{2}+c\right)^{3}x\sqrt{\arctan(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^3*x*sqrt(arctan(a*x))), x)
```

3.944 $\int \frac{x^m\sqrt{c+a^2cx^2}}{\sqrt{\tan^{-1}(ax)}}\,dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(\frac{x^m\sqrt{a^2cx^2+c}}{\sqrt{\tan^{-1}(ax)}},x\right)$$

```
[Out] Unintegrable[(x^m*Sqrt[c + a^2*c*x^2])/Sqrt[ArcTan[a*x]], x]
```

Rubi [A] time = 0.094579, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\sqrt{\tan^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*Sqrt[c + a^2*c*x^2])/Sqrt[ArcTan[a*x]],x]

[Out] Defer[Int][(x^m*Sqrt[c + a^2*c*x^2])/Sqrt[ArcTan[a*x]], x]
```

Rubi steps

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\sqrt{\tan^{-1}(ax)}}\,dx=\int \frac{x^m\sqrt{c+a^2cx^2}}{\sqrt{\tan^{-1}(ax)}}\,dx$$

Mathematica [A] time = 0.797557, size = 0, normalized size = 0.

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\sqrt{\tan^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*Sqrt[c + a^2*c*x^2])/Sqrt[ArcTan[a*x]],x]
```

```
[Out] Integrate[(x^m*Sqrt[c + a^2*c*x^2])/Sqrt[ArcTan[a*x]], x]
```

Maple [A] time = 1.248, size = 0, normalized size = 0.

$$\int x^m \sqrt{a^2 c x^2 + c}\frac{1}{\sqrt{\arctan(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a^2 c x^2 + c}\, x^m}{\sqrt{\arctan(ax)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^m/sqrt(arctan(a*x)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**(1/2)/atan(a*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a^2cx^2 + c}x^m}{\sqrt{\arctan (ax)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)*x^m/sqrt(arctan(a*x)), x)
```

3.945 $\int \frac{x\sqrt{c+a^2cx^2}}{\sqrt{\tan^{-1}(ax)}}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{x\sqrt{a^2cx^2+c}}{\sqrt{\tan^{-1}(ax)}},x\right)$$

```
[Out] Unintegrable[(x*Sqrt[c + a^2*c*x^2])/Sqrt[ArcTan[a*x]], x]
```

Rubi [A] time = 0.0667061, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x\sqrt{c+a^2cx^2}}{\sqrt{\tan^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x*Sqrt[c + a^2*c*x^2])/Sqrt[ArcTan[a*x]],x]

[Out] Defer[Int][(x*Sqrt[c + a^2*c*x^2])/Sqrt[ArcTan[a*x]], x]
```

Rubi steps

$$\int \frac{x\sqrt{c+a^2cx^2}}{\sqrt{\tan^{-1}(ax)}}\,dx=\int \frac{x\sqrt{c+a^2cx^2}}{\sqrt{\tan^{-1}(ax)}}\,dx$$

Mathematica [A] time = 1.85682, size = 0, normalized size = 0.

$$\int \frac{x\sqrt{c+a^2cx^2}}{\sqrt{\tan^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x*Sqrt[c + a^2*c*x^2])/Sqrt[ArcTan[a*x]],x]
```

```
[Out] Integrate[(x*Sqrt[c + a^2*c*x^2])/Sqrt[ArcTan[a*x]], x]
```

Maple [A] time = 1.023, size = 0, normalized size = 0.

$$\int x\sqrt{a^2cx^2+c}\frac{1}{\sqrt{\arctan(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)
```

```
[Out] int(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sqrt{c\left(a^2x^2+1\right)}}{\sqrt{\operatorname{atan}\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**(1/2)/atan(a*x)**(1/2),x)
```

```
[Out] Integral(x*sqrt(c*(a**2*x**2 + 1))/sqrt(atan(a*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a^2cx^2+cx}}{\sqrt{\arctan\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)*x/sqrt(arctan(a*x)), x)
```

3.946 $\int \frac{\sqrt{c+a^2cx^2}}{\sqrt{\tan^{-1}(ax)}}\,dx$

Optimal. Leaf size=25

$$\text{Unintegrable}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{\tan^{-1}(ax)}},x\right)$$

```
[Out] Unintegrable[Sqrt[c + a^2*c*x^2]/Sqrt[ArcTan[a*x]], x]
```

Rubi [A] time = 0.0338757, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sqrt{c+a^2cx^2}}{\sqrt{\tan^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[c + a^2*c*x^2]/Sqrt[ArcTan[a*x]],x]
```

```
[Out] Defer[Int][Sqrt[c + a^2*c*x^2]/Sqrt[ArcTan[a*x]], x]
```

Rubi steps

$$\int \frac{\sqrt{c+a^2cx^2}}{\sqrt{\tan^{-1}(ax)}}\,dx=\int \frac{\sqrt{c+a^2cx^2}}{\sqrt{\tan^{-1}(ax)}}\,dx$$

Mathematica [A] time = 0.264905, size = 0, normalized size = 0.

$$\int \frac{\sqrt{c+a^2cx^2}}{\sqrt{\tan^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[c + a^2*c*x^2]/Sqrt[ArcTan[a*x]],x]
```

```
[Out] Integrate[Sqrt[c + a^2*c*x^2]/Sqrt[ArcTan[a*x]], x]
```

Maple [A] time = 0.912, size = 0, normalized size = 0.

$$\int \sqrt{a^2cx^2 + c}\,\frac{1}{\sqrt{\arctan\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)
```

```
[Out] int((a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c\left(a^{2}x^{2}+1\right)}}{\sqrt{\operatorname{atan}\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(1/2)/atan(a*x)**(1/2),x)
```

```
[Out] Integral(sqrt(c*(a**2*x**2 + 1))/sqrt(atan(a*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a^{2}cx^{2}+c}}{\sqrt{\arctan\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)/sqrt(arctan(a*x)), x)
```

3.947 $\int \frac{\sqrt{c+a^2cx^2}}{x\sqrt{\tan^{-1}(ax)}}\,dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(\frac{\sqrt{a^2cx^2+c}}{x\sqrt{\tan^{-1}(ax)}},x\right)$$

```
[Out] Unintegrable[Sqrt[c + a^2*c*x^2]/(x*Sqrt[ArcTan[a*x]]), x]
```

Rubi [A] time = 0.0972284, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sqrt{c+a^2cx^2}}{x\sqrt{\tan^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[c + a^2*c*x^2]/(x*Sqrt[ArcTan[a*x]]),x]
```

```
[Out] Defer[Int][Sqrt[c + a^2*c*x^2]/(x*Sqrt[ArcTan[a*x]]), x]
```

Rubi steps

$$\int \frac{\sqrt{c+a^2cx^2}}{x\sqrt{\tan^{-1}(ax)}}\,dx=\int \frac{\sqrt{c+a^2cx^2}}{x\sqrt{\tan^{-1}(ax)}}\,dx$$

Mathematica [A] time = 2.78155, size = 0, normalized size = 0.

$$\int \frac{\sqrt{c+a^2cx^2}}{x\sqrt{\tan^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[c + a^2*c*x^2]/(x*Sqrt[ArcTan[a*x]]),x]
```

```
[Out] Integrate[Sqrt[c + a^2*c*x^2]/(x*Sqrt[ArcTan[a*x]]), x]
```

Maple [A] time = 1.029, size = 0, normalized size = 0.

$$\int \frac{1}{x}\sqrt{a^2cx^2+c}\frac{1}{\sqrt{\arctan\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^(1/2),x)
```

```
[Out] int((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c\left(a^2x^2+1\right)}}{x\sqrt{\operatorname{atan}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(1/2)/x/atan(a*x)**(1/2),x)
```

```
[Out] Integral(sqrt(c*(a**2*x**2 + 1))/(x*sqrt(atan(a*x))), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a^2cx^2+c}}{x\sqrt{\arctan(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)/(x*sqrt(arctan(a*x))), x)
```

3.948 $\int \frac{x^m\left(c+a^2cx^2\right)^{3/2}}{\sqrt{\tan^{-1}(ax)}}\,dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(\frac{x^m\left(a^2cx^2+c\right)^{3/2}}{\sqrt{\tan^{-1}(ax)}},x\right)$$

```
[Out] Unintegrable[(x^m*(c + a^2*c*x^2)^(3/2))/Sqrt[ArcTan[a*x]], x]
```

Rubi [A] time = 0.108527, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m\left(c+a^2cx^2\right)^{3/2}}{\sqrt{\tan^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*(c + a^2*c*x^2)^(3/2))/Sqrt[ArcTan[a*x]],x]

[Out] Defer[Int][(x^m*(c + a^2*c*x^2)^(3/2))/Sqrt[ArcTan[a*x]], x]
```

Rubi steps

$$\int \frac{x^m\left(c+a^2cx^2\right)^{3/2}}{\sqrt{\tan^{-1}(ax)}}\,dx=\int \frac{x^m\left(c+a^2cx^2\right)^{3/2}}{\sqrt{\tan^{-1}(ax)}}\,dx$$

Mathematica [A] time = 0.97965, size = 0, normalized size = 0.

$$\int \frac{x^m\left(c+a^2cx^2\right)^{3/2}}{\sqrt{\tan^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*(c + a^2*c*x^2)^(3/2))/Sqrt[ArcTan[a*x]],x]
```

```
[Out] Integrate[(x^m*(c + a^2*c*x^2)^(3/2))/Sqrt[ArcTan[a*x]], x]
```

Maple [A] time = 1.023, size = 0, normalized size = 0.

$$\int x^m \left(a^2 c x^2 + c\right)^{\frac{3}{2}} \frac{1}{\sqrt{\arctan\left(ax\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^2 c x^2 + c\right)^{\frac{3}{2}} x^m}{\sqrt{\arctan\left(ax\right)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)^(3/2)*x^m/sqrt(arctan(a*x)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**(3/2)/atan(a*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^{\frac{3}{2}}x^m}{\sqrt{\arctan\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)*x^m/sqrt(arctan(a*x)), x)
```

3.949 $\int \frac{x(c+a^2cx^2)^{3/2}}{\sqrt{\tan^{-1}(ax)}}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{x\left(a^2cx^2+c\right)^{3/2}}{\sqrt{\tan^{-1}(ax)}},x\right)$$

```
[Out] Unintegrable[(x*(c + a^2*c*x^2)^(3/2))/Sqrt[ArcTan[a*x]], x]
```

Rubi [A] time = 0.08012, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x\left(c+a^2cx^2\right)^{3/2}}{\sqrt{\tan^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x*(c + a^2*c*x^2)^(3/2))/Sqrt[ArcTan[a*x]],x]
```

```
[Out] Defer[Int][(x*(c + a^2*c*x^2)^(3/2))/Sqrt[ArcTan[a*x]], x]
```

Rubi steps

$$\int \frac{x\left(c+a^2cx^2\right)^{3/2}}{\sqrt{\tan^{-1}(ax)}}\,dx=\int \frac{x\left(c+a^2cx^2\right)^{3/2}}{\sqrt{\tan^{-1}(ax)}}\,dx$$

Mathematica [A] time = 2.90591, size = 0, normalized size = 0.

$$\int \frac{x\left(c+a^2cx^2\right)^{3/2}}{\sqrt{\tan^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x*(c + a^2*c*x^2)^(3/2))/Sqrt[ArcTan[a*x]],x]
```

```
[Out] Integrate[(x*(c + a^2*c*x^2)^(3/2))/Sqrt[ArcTan[a*x]], x]
```

Maple [A] time = 0.858, size = 0, normalized size = 0.

$$\int x\left(a^{2}c x^{2}+c\right)^{\frac{3}{2}}\frac{1}{\sqrt{\arctan\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)
```

```
[Out] int(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**(3/2)/atan(a*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^{\frac{3}{2}}x}{\sqrt{\arctan\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)*x/sqrt(arctan(a*x)), x)
```

3.950 $\int \frac{\left(c+a^2 c x^2\right)^{3/2}}{\sqrt{\tan^{-1}(a x)}} \, dx$

Optimal. Leaf size=25

$$\text{Unintegrable}\left(\frac{\left(a^2 c x^2+c\right)^{3/2}}{\sqrt{\tan^{-1}(a x)}},x\right)$$

```
[Out] Unintegrable[(c + a^2*c*x^2)^(3/2)/Sqrt[ArcTan[a*x]], x]
```

Rubi [A] time = 0.0363264, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(c+a^2 c x^2\right)^{3/2}}{\sqrt{\tan^{-1}(a x)}} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^(3/2)/Sqrt[ArcTan[a*x]],x]

[Out] Defer[Int][(c + a^2*c*x^2)^(3/2)/Sqrt[ArcTan[a*x]], x]
```

Rubi steps

$$\int \frac{\left(c+a^2 c x^2\right)^{3/2}}{\sqrt{\tan^{-1}(a x)}} \, dx=\int \frac{\left(c+a^2 c x^2\right)^{3/2}}{\sqrt{\tan^{-1}(a x)}} \, dx$$

Mathematica [A] time = 0.36435, size = 0, normalized size = 0.

$$\int \frac{\left(c+a^2 c x^2\right)^{3/2}}{\sqrt{\tan^{-1}(a x)}} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^(3/2)/Sqrt[ArcTan[a*x]],x]
```

```
[Out] Integrate[(c + a^2*c*x^2)^(3/2)/Sqrt[ArcTan[a*x]], x]
```

Maple [A] time = 0.668, size = 0, normalized size = 0.

$$\int \left(a^2cx^2+c\right)^{\frac{3}{2}} \frac{1}{\sqrt{\arctan\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)
```

```
[Out] int((a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(3/2)/atan(a*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^{\frac{3}{2}}}{\sqrt{\arctan\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)/sqrt(arctan(a*x)), x)
```

3.951 $\int \frac{\left(c+a^2 c x^2\right)^{3/2}}{x \sqrt{\tan^{-1}(a x)}} \, dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(\frac{\left(a^2 c x^2+c\right)^{3/2}}{x \sqrt{\tan^{-1}(a x)}},x\right)$$

```
[Out] Unintegrable[(c + a^2*c*x^2)^(3/2)/(x*Sqrt[ArcTan[a*x]]), x]
```

Rubi [A] time = 0.109824, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(c+a^2 c x^2\right)^{3/2}}{x \sqrt{\tan^{-1}(a x)}} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^(3/2)/(x*Sqrt[ArcTan[a*x]]),x]

[Out] Defer[Int][(c + a^2*c*x^2)^(3/2)/(x*Sqrt[ArcTan[a*x]]), x]
```

Rubi steps

$$\int \frac{\left(c+a^2 c x^2\right)^{3/2}}{x \sqrt{\tan^{-1}(a x)}} \, dx = \int \frac{\left(c+a^2 c x^2\right)^{3/2}}{x \sqrt{\tan^{-1}(a x)}} \, dx$$

Mathematica [A] time = 2.4056, size = 0, normalized size = 0.

$$\int \frac{\left(c+a^2 c x^2\right)^{3/2}}{x \sqrt{\tan^{-1}(a x)}} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^(3/2)/(x*Sqrt[ArcTan[a*x]]),x]
```

```
[Out] Integrate[(c + a^2*c*x^2)^(3/2)/(x*Sqrt[ArcTan[a*x]]), x]
```

Maple [A] time = 0.724, size = 0, normalized size = 0.

$$\int \frac{1}{x}\left(a^{2}c x^{2}+c\right)^{\frac{3}{2}}\frac{1}{\sqrt{\arctan\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^(1/2),x)
```

```
[Out] int((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(3/2)/x/atan(a*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^{\frac{3}{2}}}{x\sqrt{\arctan\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)/(x*sqrt(arctan(a*x))), x)
```

3.952 $\int \frac{x^m(c+a^2cx^2)^{5/2}}{\sqrt{\tan^{-1}(ax)}}\,dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(\frac{x^m\left(a^2cx^2+c\right)^{5/2}}{\sqrt{\tan^{-1}(ax)}},x\right)$$

```
[Out] Unintegrable[(x^m*(c + a^2*c*x^2)^(5/2))/Sqrt[ArcTan[a*x]], x]
```

Rubi [A] time = 0.111195, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m\left(c+a^2cx^2\right)^{5/2}}{\sqrt{\tan^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*(c + a^2*c*x^2)^(5/2))/Sqrt[ArcTan[a*x]],x]

[Out] Defer[Int][(x^m*(c + a^2*c*x^2)^(5/2))/Sqrt[ArcTan[a*x]], x]
```

Rubi steps

$$\int \frac{x^m\left(c+a^2cx^2\right)^{5/2}}{\sqrt{\tan^{-1}(ax)}}\,dx=\int \frac{x^m\left(c+a^2cx^2\right)^{5/2}}{\sqrt{\tan^{-1}(ax)}}\,dx$$

Mathematica [A] time = 1.31876, size = 0, normalized size = 0.

$$\int \frac{x^m\left(c+a^2cx^2\right)^{5/2}}{\sqrt{\tan^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*(c + a^2*c*x^2)^(5/2))/Sqrt[ArcTan[a*x]],x]
```

```
[Out] Integrate[(x^m*(c + a^2*c*x^2)^(5/2))/Sqrt[ArcTan[a*x]], x]
```

Maple [A] time = 1.014, size = 0, normalized size = 0.

$$\int x^{m} \left(a^{2} c \,x^{2}+c \right)^{\frac{5}{2}} \frac{1}{\sqrt{\arctan \left(a x \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^{4} c^{2} x^{4} + 2 \, a^{2} c^{2} x^{2} + c^{2}\right) \sqrt{a^{2} c x^{2} + c} x^{m}}{\sqrt{\arctan\left(a x\right)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*x^m/sqrt(a
rctan(a*x)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**(5/2)/atan(a*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^{\frac{5}{2}}x^m}{\sqrt{\arctan\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(5/2)*x^m/sqrt(arctan(a*x)), x)
```

3.953 $\int \frac{x\left(c+a^2cx^2\right)^{5/2}}{\sqrt{\tan^{-1}(ax)}}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{x\left(a^2cx^2+c\right)^{5/2}}{\sqrt{\tan^{-1}(ax)}},x\right)$$

```
[Out] Unintegrable[(x*(c + a^2*c*x^2)^(5/2))/Sqrt[ArcTan[a*x]], x]
```

Rubi [A] time = 0.0778781, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x\left(c+a^2cx^2\right)^{5/2}}{\sqrt{\tan^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x*(c + a^2*c*x^2)^(5/2))/Sqrt[ArcTan[a*x]],x]

[Out] Defer[Int][(x*(c + a^2*c*x^2)^(5/2))/Sqrt[ArcTan[a*x]], x]
```

Rubi steps

$$\int \frac{x\left(c+a^2cx^2\right)^{5/2}}{\sqrt{\tan^{-1}(ax)}}\,dx=\int \frac{x\left(c+a^2cx^2\right)^{5/2}}{\sqrt{\tan^{-1}(ax)}}\,dx$$

Mathematica [A] time = 2.49274, size = 0, normalized size = 0.

$$\int \frac{x\left(c+a^2cx^2\right)^{5/2}}{\sqrt{\tan^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x*(c + a^2*c*x^2)^(5/2))/Sqrt[ArcTan[a*x]],x]
```

```
[Out] Integrate[(x*(c + a^2*c*x^2)^(5/2))/Sqrt[ArcTan[a*x]], x]
```

Maple [A] time = 0.899, size = 0, normalized size = 0.

$$\int x\left(a^{2}c x^{2}+c\right)^{\frac{5}{2}}\frac{1}{\sqrt{\arctan\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

```
[Out] int(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**(5/2)/atan(a*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^{2} c x^{2}+c\right)^{\frac{5}{2}} x}{\sqrt{\arctan\left(a x\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(5/2)*x/sqrt(arctan(a*x)), x)
```

3.954 $\int \frac{\left(c+a^2 c x^2\right)^{5/2}}{\sqrt{\tan^{-1}(ax)}}\,dx$

Optimal. Leaf size=25

$$\text{Unintegrable}\left(\frac{\left(a^2 c x^2+c\right)^{5/2}}{\sqrt{\tan^{-1}(ax)}},x\right)$$

```
[Out] Unintegrable[(c + a^2*c*x^2)^(5/2)/Sqrt[ArcTan[a*x]], x]
```

Rubi [A] time = 0.0372537, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(c+a^2 c x^2\right)^{5/2}}{\sqrt{\tan^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^(5/2)/Sqrt[ArcTan[a*x]],x]

[Out] Defer[Int][(c + a^2*c*x^2)^(5/2)/Sqrt[ArcTan[a*x]], x]
```

Rubi steps

$$\int \frac{\left(c+a^2 c x^2\right)^{5/2}}{\sqrt{\tan^{-1}(ax)}}\,dx=\int \frac{\left(c+a^2 c x^2\right)^{5/2}}{\sqrt{\tan^{-1}(ax)}}\,dx$$

Mathematica [A] time = 0.44021, size = 0, normalized size = 0.

$$\int \frac{\left(c+a^2 c x^2\right)^{5/2}}{\sqrt{\tan^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^(5/2)/Sqrt[ArcTan[a*x]],x]
```

```
[Out] Integrate[(c + a^2*c*x^2)^(5/2)/Sqrt[ArcTan[a*x]], x]
```

Maple [A] time = 0.739, size = 0, normalized size = 0.

$$\int \left(a^2 c x^2 + c\right)^{\frac{5}{2}} \frac{1}{\sqrt{\arctan\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

```
[Out] int((a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(5/2)/atan(a*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^{\frac{5}{2}}}{\sqrt{\arctan\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(5/2)/sqrt(arctan(a*x)), x)
```

3.955 $\int \frac{\left(c+a^2cx^2\right)^{5/2}}{x\sqrt{\tan^{-1}(ax)}}\,dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(\frac{\left(a^2cx^2+c\right)^{5/2}}{x\sqrt{\tan^{-1}(ax)}},x\right)$$

```
[Out] Unintegrable[(c + a^2*c*x^2)^(5/2)/(x*Sqrt[ArcTan[a*x]]), x]
```

Rubi [A] time = 0.120096, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(c+a^2cx^2\right)^{5/2}}{x\sqrt{\tan^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^(5/2)/(x*Sqrt[ArcTan[a*x]]),x]

[Out] Defer[Int][(c + a^2*c*x^2)^(5/2)/(x*Sqrt[ArcTan[a*x]]), x]
```

Rubi steps

$$\int \frac{\left(c+a^2cx^2\right)^{5/2}}{x\sqrt{\tan^{-1}(ax)}}\,dx=\int \frac{\left(c+a^2cx^2\right)^{5/2}}{x\sqrt{\tan^{-1}(ax)}}\,dx$$

Mathematica [A] time = 2.42865, size = 0, normalized size = 0.

$$\int \frac{\left(c+a^2cx^2\right)^{5/2}}{x\sqrt{\tan^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^(5/2)/(x*Sqrt[ArcTan[a*x]]),x]
```

```
[Out] Integrate[(c + a^2*c*x^2)^(5/2)/(x*Sqrt[ArcTan[a*x]]), x]
```

Maple [A] time = 0.753, size = 0, normalized size = 0.

$$\int \frac{1}{x} \left(a^2 c x^2 + c\right)^{\frac{5}{2}} \frac{1}{\sqrt{\arctan\left(ax\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^(1/2),x)
```

```
[Out] int((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(5/2)/x/atan(a*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^{\frac{5}{2}}}{x\sqrt{\arctan\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(5/2)/(x*sqrt(arctan(a*x))), x)
```

3.956 $\int \frac{x^m}{\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}}\,dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(\frac{x^m}{\sqrt{a^2cx^2+c}\sqrt{\tan^{-1}(ax)}},x\right)$$

```
[Out] Unintegrable[x^m/(Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]), x]
```

Rubi [A] time = 0.105653, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m/(Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]),x]
```

```
[Out] Defer[Int][x^m/(Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]), x]
```

Rubi steps

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}}\,dx=\int \frac{x^m}{\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}}\,dx$$

Mathematica [A] time = 0.839541, size = 0, normalized size = 0.

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m/(Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]),x]
```

```
[Out] Integrate[x^m/(Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]), x]
```

Maple [A] time = 1.307, size = 0, normalized size = 0.

$$\int x^{m} \frac{1}{\sqrt{a^{2}c\,x^{2}+c}}\frac{1}{\sqrt{\arctan \left(a x \right)}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)
```

```
[Out] int(x^m/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^{m}}{\sqrt{a^{2}cx^{2}+c}\sqrt{\arctan\left(ax\right)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^m/(sqrt(a^2*c*x^2 + c)*sqrt(arctan(a*x))), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(a**2*c*x**2+c)**(1/2)/atan(a*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^m/(sqrt(a^2*c*x^2 + c)*sqrt(arctan(a*x))), x)
```

3.957 $\int \frac{x}{\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}} \, dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{x}{\sqrt{a^2cx^2+c}\sqrt{\tan^{-1}(ax)}},x\right)$$

```
[Out] Unintegrable[x/(Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]), x]
```

Rubi [A] time = 0.070693, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x}{\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[x/(Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]),x]
```

```
[Out] Defer[Int][x/(Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]), x]
```

Rubi steps

$$\int \frac{x}{\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}} \, dx = \int \frac{x}{\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}} \, dx$$

Mathematica [A] time = 1.0799, size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x/(Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]),x]
```

```
[Out] Integrate[x/(Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]), x]
```

Maple [A] time = 1.086, size = 0, normalized size = 0.

$$\int x\frac{1}{\sqrt{a^{2}c x^{2}+c}}\frac{1}{\sqrt{\arctan\left(a x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)
```

```
[Out] int(x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{c\left(a^2x^2+1\right)}\sqrt{\operatorname{atan}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a**2*c*x**2+c)**(1/2)/atan(a*x)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(c*(a**2*x**2 + 1))*sqrt(atan(a*x))), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/(sqrt(a^2*c*x^2 + c)*sqrt(arctan(a*x))), x)
```

3.958 $\int \frac{1}{\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}}\,dx$

Optimal. Leaf size=25

$$\text{Unintegrable}\left(\frac{1}{\sqrt{a^2cx^2+c}\sqrt{\tan^{-1}(ax)}},x\right)$$

```
[Out] Unintegrable[1/(Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]), x]
```

Rubi [A] time = 0.0349825, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]),x]

[Out] Defer[Int][1/(Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]), x]
```

Rubi steps

$$\int \frac{1}{\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}}\,dx=\int \frac{1}{\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}}\,dx$$

Mathematica [A] time = 0.222611, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]),x]
```

```
[Out] Integrate[1/(Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]), x]
```

Maple [A] time = 0.868, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a^{2} c \,x^{2}+c}}\frac{1}{\sqrt{\arctan \left(a x \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)
```

```
[Out] int(1/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{c\left(a^2x^2+1\right)}\sqrt{\operatorname{atan}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*c*x**2+c)**(1/2)/atan(a*x)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(c*(a**2*x**2 + 1))*sqrt(atan(a*x))), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a^2*c*x^2 + c)*sqrt(arctan(a*x))), x)
```

3.959 $\int \frac{1}{x\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}}\,dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(\frac{1}{x\sqrt{a^2cx^2+c}\sqrt{\tan^{-1}(ax)}},x\right)$$

```
[Out] Unintegrable[1/(x*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]), x]
```

Rubi [A] time = 0.106716, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]),x]
```

```
[Out] Defer[Int][1/(x*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]), x]
```

Rubi steps

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}}\,dx=\int \frac{1}{x\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}}\,dx$$

Mathematica [A] time = 0.946745, size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]),x]
```

```
[Out] Integrate[1/(x*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]), x]
```

Maple [A] time = 1.012, size = 0, normalized size = 0.

$$\int \frac{1}{x}\frac{1}{\sqrt{a^2cx^2+c}}\frac{1}{\sqrt{\arctan(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)
```

```
[Out] int(1/x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{c\left(a^{2}x^{2}+1\right)}\sqrt{\operatorname{atan}\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a**2*c*x**2+c)**(1/2)/atan(a*x)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(c*(a**2*x**2 + 1))*sqrt(atan(a*x))), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a^{2}cx^{2}+c}\,x\sqrt{\arctan\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a^2*c*x^2 + c)*x*sqrt(arctan(a*x))), x)
```

3.960 $\int \frac{x^m}{(c+a^2cx^2)^{3/2}\sqrt{\tan^{-1}(ax)}}\,dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(\frac{x^m}{(a^2cx^2+c)^{3/2}\sqrt{\tan^{-1}(ax)}},x\right)$$

```
[Out] Unintegrable[x^m/((c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]), x]
```

Rubi [A] time = 0.115696, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m}{(c+a^2cx^2)^{3/2}\sqrt{\tan^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m/((c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]),x]

[Out] Defer[Int][x^m/((c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]), x]
```

Rubi steps

$$\int \frac{x^m}{(c+a^2cx^2)^{3/2}\sqrt{\tan^{-1}(ax)}}\,dx = \int \frac{x^m}{(c+a^2cx^2)^{3/2}\sqrt{\tan^{-1}(ax)}}\,dx$$

Mathematica [A] time = 0.96978, size = 0, normalized size = 0.

$$\int \frac{x^m}{(c+a^2cx^2)^{3/2}\sqrt{\tan^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m/((c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]),x]
```

```
[Out] Integrate[x^m/((c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]), x]
```

Maple [A] time = 0.923, size = 0, normalized size = 0.

$$\int x^{m} \left(a^{2} c \,x^{2}+c \right)^{-\frac{3}{2}} \frac{1}{\sqrt{\arctan \left(a x \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)
```

```
[Out] int(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a^{2} c x^{2} + c} x^{m}}{{\left(a^{4} c^{2} x^{4} + 2 \, a^{2} c^{2} x^{2} + c^{2}\right)} \sqrt{\arctan\left(a x\right)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^m/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(
arctan(a*x))), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(a**2*c*x**2+c)**(3/2)/atan(a*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{(a^2cx^2+c)^{\frac{3}{2}}\sqrt{\arctan(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^m/((a^2*c*x^2 + c)^(3/2)*sqrt(arctan(a*x))), x)
```

3.961 $\int \frac{x^2}{\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}}\,dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(\frac{x^2}{\left(a^2cx^2+c\right)^{3/2}\sqrt{\tan^{-1}(ax)}},x\right)$$

```
[Out] Unintegrable[x^2/((c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]), x]
```

Rubi [A] time = 0.118322, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^2}{\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^2/((c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]),x]

[Out] Defer[Int][x^2/((c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]), x]
```

Rubi steps

$$\int \frac{x^2}{\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}}\,dx=\int \frac{x^2}{\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}}\,dx$$

Mathematica [A] time = 3.47943, size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^2/((c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]),x]
```

```
[Out] Integrate[x^2/((c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]), x]
```

Maple [A] time = 3.299, size = 0, normalized size = 0.

$$\int x^2 \left(a^2 c x^2 + c\right)^{-\frac{3}{2}} \frac{1}{\sqrt{\arctan (ax)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)
```

```
[Out] int(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\sqrt{\operatorname{atan}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a**2*c*x**2+c)**(3/2)/atan(a*x)**(1/2),x)
```

```
[Out] Integral(x**2/((c*(a**2*x**2 + 1))**(3/2)*sqrt(atan(a*x))), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(a^2cx^2+c\right)^{\frac{3}{2}}\sqrt{\arctan(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/((a^2*c*x^2 + c)^(3/2)*sqrt(arctan(a*x))), x)
```

3.962 $\int \frac{x}{\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}}\,dx$

Optimal. Leaf size=60

$$\frac{\sqrt{2\pi}\sqrt{a^2x^2+1}S\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{a^2c\sqrt{a^2cx^2+c}}$$

```
[Out] (Sqrt[2*Pi]*Sqrt[1 + a^2*x^2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(a^2*
c*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.16481, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4971, 4970, 3305, 3351}

$$\frac{\sqrt{2\pi}\sqrt{a^2x^2+1}S\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{a^2c\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]),x]
```

```
[Out] (Sqrt[2*Pi]*Sqrt[1 + a^2*x^2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(a^2*
c*Sqrt[c + a^2*c*x^2])
```

Rule 4971

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[(d^(q + 1/2)*Sqrt[1 + c^2*x^2])/Sqrt[d + e*x^2],
 Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d
, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(In
tegerQ[q] || GtQ[d, 0])
```

Rule 4970

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sin[x]^m)/
Cos[x]^(m + 2*(q + 1)), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
```

```
|| GtQ[d, 0])
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}}\,dx &= \frac{\sqrt{1+a^2x^2}\int \frac{x}{\left(1+a^2x^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}}\,dx}{c\sqrt{c+a^2cx^2}}\\
&= \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \frac{\sin(x)}{\sqrt{x}}\,dx,x,\tan^{-1}(ax)\right)}{a^2c\sqrt{c+a^2cx^2}}\\
&= \frac{\left(2\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int \sin\left(x^2\right)\,dx,x,\sqrt{\tan^{-1}(ax)}\right)}{a^2c\sqrt{c+a^2cx^2}}\\
&= \frac{\sqrt{2\pi}\sqrt{1+a^2x^2}S\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{a^2c\sqrt{c+a^2cx^2}}
\end{aligned}$$

Mathematica [C] time = 0.134121, size = 97, normalized size = 1.62

$$-\frac{\sqrt{a^2x^2+1}\left(\sqrt{-i\tan^{-1}(ax)}\text{Gamma}\left(\frac{1}{2},-i\tan^{-1}(ax)\right)+\sqrt{i\tan^{-1}(ax)}\text{Gamma}\left(\frac{1}{2},i\tan^{-1}(ax)\right)\right)}{2a^2c\sqrt{c\left(a^2x^2+1\right)}\sqrt{\tan^{-1}(ax)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]),x]
```

```
[Out] -(Sqrt[1 + a^2*x^2]*(Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-I)*ArcTan[a*x]] +
Sqrt[I*ArcTan[a*x]]*Gamma[1/2, I*ArcTan[a*x]]))/(2*a^2*c*Sqrt[c*(1 + a^2*x^
```

```
2)]*Sqrt[ArcTan[a*x]])
```

Maple [F] time = 0.859, size = 0, normalized size = 0.

$$\int x \left(a^2 c x^2 + c\right)^{-\frac{3}{2}} \frac{1}{\sqrt{\arctan\left(a x\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)
```

```
[Out] int(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(c\left(a^{2}x^{2}+1\right)\right)^{\frac{3}{2}}\sqrt{\operatorname{atan}\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a**2*c*x**2+c)**(3/2)/atan(a*x)**(1/2),x)
```

```
[Out] Integral(x/((c*(a**2*x**2 + 1))**(3/2)*sqrt(atan(a*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(a^{2}cx^{2}+c\right)^{\frac{3}{2}}\sqrt{\arctan\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/((a^2*c*x^2 + c)^(3/2)*sqrt(arctan(a*x))), x)
```

3.963 $\int \frac{1}{(c+a^2cx^2)^{3/2}\sqrt{\tan^{-1}(ax)}}\,dx$

Optimal. Leaf size=60

$$\frac{\sqrt{2\pi}\sqrt{a^2x^2+1}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{ac\sqrt{a^2cx^2+c}}$$

```
[Out] (Sqrt[2*Pi]*Sqrt[1 + a^2*x^2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(a*c*
Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.0931624, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4905, 4904, 3304, 3352}

$$\frac{\sqrt{2\pi}\sqrt{a^2x^2+1}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{ac\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]),x]
```

```
[Out] (Sqrt[2*Pi]*Sqrt[1 + a^2*x^2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(a*c*
Sqrt[c + a^2*c*x^2])
```

Rule 4905

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[(d^(q + 1/2)*Sqrt[1 + c^2*x^2])/Sqrt[d + e*x^2], Int[(1 + c
^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

Rule 4904

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}}\,dx &= \frac{\sqrt{1+a^2x^2}\int \frac{1}{\left(1+a^2x^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}}\,dx}{c\sqrt{c+a^2cx^2}}\\ &= \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \frac{\cos(x)}{\sqrt{x}}\,dx,x,\tan^{-1}(ax)\right)}{ac\sqrt{c+a^2cx^2}}\\ &= \frac{\left(2\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int \cos\left(x^2\right)\,dx,x,\sqrt{\tan^{-1}(ax)}\right)}{ac\sqrt{c+a^2cx^2}}\\ &= \frac{\sqrt{2\pi}\sqrt{1+a^2x^2}C\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{ac\sqrt{c+a^2cx^2}}\end{aligned}$$

Mathematica [A] time = 0.253614, size = 60, normalized size = 1.

$$\frac{\sqrt{2\pi}\sqrt{a^2cx^2+c}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{ac^2\sqrt{a^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]),x]
```

```
[Out] (Sqrt[2*Pi]*Sqrt[c + a^2*c*x^2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(a*
c^2*Sqrt[1 + a^2*x^2])
```

Maple [F] time = 0.719, size = 0, normalized size = 0.

$$\int \left(a^2 c x^2 + c\right)^{-\frac{3}{2}} \frac{1}{\sqrt{\arctan\left(ax\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)
```

```
[Out] int(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}} \sqrt{\operatorname{atan}\left(ax\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*c*x**2+c)**(3/2)/atan(a*x)**(1/2),x)
```

```
[Out] Integral(1/((c*(a**2*x**2 + 1))**(3/2)*sqrt(atan(a*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)^{\frac{3}{2}}\sqrt{\arctan\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^(3/2)*sqrt(arctan(a*x))), x)
```

3.964 $\int \frac{1}{x(c+a^2cx^2)^{3/2}\sqrt{\tan^{-1}(ax)}}\,dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(\frac{1}{x\left(a^2cx^2+c\right)^{3/2}\sqrt{\tan^{-1}(ax)}},x\right)$$

```
[Out] Unintegrable[1/(x*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]), x]
```

Rubi [A] time = 0.116805, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]),x]
```

```
[Out] Defer[Int][1/(x*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]), x]
```

Rubi steps

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}}\,dx=\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}}\,dx$$

Mathematica [A] time = 1.8003, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]),x]
```

```
[Out] Integrate[1/(x*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]), x]
```

Maple [A] time = 0.704, size = 0, normalized size = 0.

$$\int \frac{1}{x}\left(a^{2} c x^{2}+c\right)^{-\frac{3}{2}} \frac{1}{\sqrt{\arctan\left(a x\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)
```

```
[Out] int(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a**2*c*x**2+c)**(3/2)/atan(a*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)^{\frac{3}{2}}x\sqrt{\arctan\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^(3/2)*x*sqrt(arctan(a*x))), x)
```

3.965 $\int \frac{x^m}{\left(c+a^2cx^2\right)^{5/2}\sqrt{\tan^{-1}(ax)}}\,dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(\frac{x^m}{\left(a^2cx^2+c\right)^{5/2}\sqrt{\tan^{-1}(ax)}},x\right)$$

```
[Out] Unintegrable[x^m/((c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]), x]
```

Rubi [A] time = 0.117916, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{x^m}{\left(c+a^2cx^2\right)^{5/2}\sqrt{\tan^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m/((c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]),x]
```

```
[Out] Defer[Int][x^m/((c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]), x]
```

Rubi steps

$$\int \frac{x^m}{\left(c+a^2cx^2\right)^{5/2}\sqrt{\tan^{-1}(ax)}}\,dx = \int \frac{x^m}{\left(c+a^2cx^2\right)^{5/2}\sqrt{\tan^{-1}(ax)}}\,dx$$

Mathematica [A] time = 1.55653, size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(c+a^2cx^2\right)^{5/2}\sqrt{\tan^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m/((c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]),x]
```

[Out] Integrate[x^m/((c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]), x]

Maple [A] time = 0.961, size = 0, normalized size = 0.

$$\int x^{m} \left(a^{2} c x^{2}+c \right)^{-\frac{5}{2}} \frac{1}{\sqrt{\arctan \left(a x \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)

[Out] int(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="maxima")

[Out] Exception raised: RuntimeError

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a^{2} c x^{2} + c} x^{m}}{{\left(a^{6} c^{3} x^{6} + 3 \, a^{4} c^{3} x^{4} + 3 \, a^{2} c^{3} x^{2} + c^{3}\right)} \sqrt{\arctan \left(a x\right)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(a^2*c*x^2 + c)*x^m/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*
x^2 + c^3)*sqrt(arctan(a*x))), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(a**2*c*x**2+c)**(5/2)/atan(a*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(a^2cx^2+c\right)^{\frac{5}{2}}\sqrt{\arctan\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^m/((a^2*c*x^2 + c)^(5/2)*sqrt(arctan(a*x))), x)
```

3.966 $\int \frac{x^4}{\left(c+a^2cx^2\right)^{5/2}\sqrt{\tan^{-1}(ax)}}\,dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(\frac{x^4}{\left(a^2cx^2+c\right)^{5/2}\sqrt{\tan^{-1}(ax)}},x\right)$$

```
[Out] Unintegrable[x^4/((c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]), x]
```

Rubi [A] time = 0.120415, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^4}{\left(c+a^2cx^2\right)^{5/2}\sqrt{\tan^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^4/((c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]),x]

[Out] Defer[Int][x^4/((c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]), x]
```

Rubi steps

$$\int \frac{x^4}{\left(c+a^2cx^2\right)^{5/2}\sqrt{\tan^{-1}(ax)}}\,dx=\int \frac{x^4}{\left(c+a^2cx^2\right)^{5/2}\sqrt{\tan^{-1}(ax)}}\,dx$$

Mathematica [A] time = 3.64429, size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(c+a^2cx^2\right)^{5/2}\sqrt{\tan^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^4/((c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]),x]
```

```
[Out] Integrate[x^4/((c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]), x]
```

Maple [A] time = 2.194, size = 0, normalized size = 0.

$$\int x^4 \left(a^2 c x^2 + c\right)^{-\frac{5}{2}} \frac{1}{\sqrt{\arctan\left(ax\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

```
[Out] int(x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(a**2*c*x**2+c)**(5/2)/atan(a*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(a^2cx^2+c\right)^{\frac{5}{2}}\sqrt{\arctan\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/((a^2*c*x^2 + c)^(5/2)*sqrt(arctan(a*x))), x)
```

3.967 $\int \frac{x^3}{(c+a^2cx^2)^{5/2}\sqrt{\tan^{-1}(ax)}}\,dx$

Optimal. Leaf size=131

$$\frac{3\sqrt{\frac{\pi}{2}}\sqrt{a^2x^2+1}S\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{2a^4c^2\sqrt{a^2cx^2+c}}-\frac{\sqrt{\frac{\pi}{6}}\sqrt{a^2x^2+1}S\left(\sqrt{\frac{6}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{2a^4c^2\sqrt{a^2cx^2+c}}$$

```
[Out] (3*Sqrt[Pi/2]*Sqrt[1 + a^2*x^2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(2*
a^4*c^2*Sqrt[c + a^2*c*x^2]) - (Sqrt[Pi/6]*Sqrt[1 + a^2*x^2]*FresnelS[Sqrt[
6/Pi]*Sqrt[ArcTan[a*x]]])/(2*a^4*c^2*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.291452, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {4971, 4970, 3312, 3305, 3351}

$$\frac{3\sqrt{\frac{\pi}{2}}\sqrt{a^2x^2+1}S\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{2a^4c^2\sqrt{a^2cx^2+c}}-\frac{\sqrt{\frac{\pi}{6}}\sqrt{a^2x^2+1}S\left(\sqrt{\frac{6}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{2a^4c^2\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]),x]
```


```
[Out] (3*Sqrt[Pi/2]*Sqrt[1 + a^2*x^2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(2*
a^4*c^2*Sqrt[c + a^2*c*x^2]) - (Sqrt[Pi/6]*Sqrt[1 + a^2*x^2]*FresnelS[Sqrt[
6/Pi]*Sqrt[ArcTan[a*x]]])/(2*a^4*c^2*Sqrt[c + a^2*c*x^2])
```


Rule 4971

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[(d^(q + 1/2)*Sqrt[1 + c^2*x^2])/Sqrt[d + e*x^2],
 Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d
, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(In
tegerQ[q] || GtQ[d, 0])
```

Rule 4970

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sin[x]^m)/
```

```
Cos[x]^(m + 2*(q + 1)), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{x^3}{\left(c+a^2cx^2\right)^{5/2}\sqrt{\tan^{-1}(ax)}}\,dx = \frac{\sqrt{1+a^2x^2}\int \frac{x^3}{\left(1+a^2x^2\right)^{5/2}\sqrt{\tan^{-1}(ax)}}\,dx}{c^2\sqrt{c+a^2cx^2}}$$

$$= \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \frac{\sin^3(x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{a^4c^2\sqrt{c+a^2cx^2}}$$

$$= \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \left(\frac{3\sin(x)}{4\sqrt{x}} - \frac{\sin(3x)}{4\sqrt{x}}\right)dx, x, \tan^{-1}(ax)\right)}{a^4c^2\sqrt{c+a^2cx^2}}$$

$$= -\frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \frac{\sin(3x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{4a^4c^2\sqrt{c+a^2cx^2}} + \frac{\left(3\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int \frac{\sin(x)}{\sqrt{x}}\,dx, x\right.}{4a^4c^2\sqrt{c+a^2cx^2}}$$

$$= -\frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \sin\left(3x^2\right)dx, x, \sqrt{\tan^{-1}(ax)}\right)}{2a^4c^2\sqrt{c+a^2cx^2}} + \frac{\left(3\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int \sin\left(\right.\right.}{2a^4c^2\sqrt{c+}}$$

$$= \frac{3\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}\,S\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{2a^4c^2\sqrt{c+a^2cx^2}} - \frac{\sqrt{\frac{\pi}{6}}\sqrt{1+a^2x^2}\,S\left(\sqrt{\frac{6}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{2a^4c^2\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 0.207917, size = 95, normalized size = 0.73

$$\frac{\sqrt{\frac{\pi}{6}}\left(a^2x^2+1\right)^{3/2}\left(3\sqrt{3}S\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right) - S\left(\sqrt{\frac{6}{\pi}}\sqrt{\tan^{-1}(ax)}\right)\right)}{2a^4c\left(c\left(a^2x^2+1\right)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]),x]
```

```
[Out] (Sqrt[Pi/6]*(1 + a^2*x^2)^(3/2)*(3*Sqrt[3]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[
a*x]]] - FresnelS[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]]))/(2*a^4*c*(c*(1 + a^2*x^2)
)^(3/2))
```

Maple [F] time = 3.53, size = 0, normalized size = 0.

$$\int x^3\left(a^2cx^2+c\right)^{-\frac{5}{2}}\frac{1}{\sqrt{\arctan(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

```
[Out] int(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a**2*c*x**2+c)**(5/2)/atan(a*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(a^2cx^2+c\right)^{\frac{5}{2}}\sqrt{\arctan\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/((a^2*c*x^2 + c)^(5/2)*sqrt(arctan(a*x))), x)
```

3.968 $\int \frac{x^2}{\left(c+a^2cx^2\right)^{5/2}\sqrt{\tan^{-1}(ax)}}\,dx$

Optimal. Leaf size=131

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{a^2x^2+1}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{2a^3c^2\sqrt{a^2cx^2+c}}-\frac{\sqrt{\frac{\pi}{6}}\sqrt{a^2x^2+1}\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{2a^3c^2\sqrt{a^2cx^2+c}}$$

```
[Out] (Sqrt[Pi/2]*Sqrt[1 + a^2*x^2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(2*a^
3*c^2*Sqrt[c + a^2*c*x^2]) - (Sqrt[Pi/6]*Sqrt[1 + a^2*x^2]*FresnelC[Sqrt[6/
Pi]*Sqrt[ArcTan[a*x]]])/(2*a^3*c^2*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.296279, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {4971, 4970, 4406, 3304, 3352}

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{a^2x^2+1}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{2a^3c^2\sqrt{a^2cx^2+c}}-\frac{\sqrt{\frac{\pi}{6}}\sqrt{a^2x^2+1}\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{2a^3c^2\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]),x]
```

```
[Out] (Sqrt[Pi/2]*Sqrt[1 + a^2*x^2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(2*a^
3*c^2*Sqrt[c + a^2*c*x^2]) - (Sqrt[Pi/6]*Sqrt[1 + a^2*x^2]*FresnelC[Sqrt[6/
Pi]*Sqrt[ArcTan[a*x]]])/(2*a^3*c^2*Sqrt[c + a^2*c*x^2])
```

Rule 4971

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[(d^(q + 1/2)*Sqrt[1 + c^2*x^2])/Sqrt[d + e*x^2],
 Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d
, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(In
tegerQ[q] || GtQ[d, 0])
```

Rule 4970

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sin[x]^m)/
```

```
Cos[x]^(m + 2*(q + 1)), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{x^2}{\left(c+a^2cx^2\right)^{5/2}\sqrt{\tan^{-1}(ax)}}\,dx = \frac{\sqrt{1+a^2x^2}\int \frac{x^2}{\left(1+a^2x^2\right)^{5/2}\sqrt{\tan^{-1}(ax)}}\,dx}{c^2\sqrt{c+a^2cx^2}}$$

$$= \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \frac{\cos(x)\sin^2(x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{a^3c^2\sqrt{c+a^2cx^2}}$$

$$= \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \left(\frac{\cos(x)}{4\sqrt{x}} - \frac{\cos(3x)}{4\sqrt{x}}\right) dx, x, \tan^{-1}(ax)\right)}{a^3c^2\sqrt{c+a^2cx^2}}$$

$$= \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \frac{\cos(x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{4a^3c^2\sqrt{c+a^2cx^2}} - \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \frac{\cos(3x)}{\sqrt{x}}\,dx, x, \tan^{-1}\right.}{4a^3c^2\sqrt{c+a^2cx^2}}$$

$$= \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \cos\left(x^2\right) dx, x, \sqrt{\tan^{-1}(ax)}\right)}{2a^3c^2\sqrt{c+a^2cx^2}} - \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \cos\left(3x^2\right) dx\right.}{2a^3c^2\sqrt{c+a^2cx^2}}$$

$$= \frac{\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}\,C\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{2a^3c^2\sqrt{c+a^2cx^2}} - \frac{\sqrt{\frac{\pi}{6}}\sqrt{1+a^2x^2}\,C\left(\sqrt{\frac{6}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{2a^3c^2\sqrt{c+a^2cx^2}}$$

Mathematica [C] time = 0.19218, size = 159, normalized size = 1.21

$$-\frac{i\sqrt{a^2x^2+1}\left(3\sqrt{-i\tan^{-1}(ax)}\text{Gamma}\left(\frac{1}{2}, -i\tan^{-1}(ax)\right) - 3\sqrt{i\tan^{-1}(ax)}\text{Gamma}\left(\frac{1}{2}, i\tan^{-1}(ax)\right) + \sqrt{3}\left(\sqrt{i\tan^{-1}(ax)}\right.\right.}{24a^3c^2\sqrt{c\left(a^2x^2+1\right)}\sqrt{\tan^{-1}(ax)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/((c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]),x]
```

```
[Out] ((-I/24)*Sqrt[1 + a^2*x^2]*(3*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-I)*ArcTan
[a*x]] - 3*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, I*ArcTan[a*x]] + Sqrt[3]*(-(Sqrt[
(-I)*ArcTan[a*x]]*Gamma[1/2, (-3*I)*ArcTan[a*x]]) + Sqrt[I*ArcTan[a*x]]*Gam
ma[1/2, (3*I)*ArcTan[a*x]])))/(a^3*c^2*Sqrt[c*(1 + a^2*x^2)]*Sqrt[ArcTan[a*
x]])
```

Maple [F] time = 2.98, size = 0, normalized size = 0.

$$\int x^2 \left(a^2 c x^2 + c\right)^{-\frac{5}{2}} \frac{1}{\sqrt{\arctan\left(ax\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

```
[Out] int(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a**2*c*x**2+c)**(5/2)/atan(a*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(a^2cx^2+c\right)^{\frac{5}{2}}\sqrt{\arctan\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/((a^2*c*x^2 + c)^(5/2)*sqrt(arctan(a*x))), x)
```

3.969 $\int \frac{x}{(c+a^2cx^2)^{5/2}\sqrt{\tan^{-1}(ax)}}\,dx$

Optimal. Leaf size=131

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{a^2x^2+1}S\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{2a^2c^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{\frac{\pi}{6}}\sqrt{a^2x^2+1}S\left(\sqrt{\frac{6}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{2a^2c^2\sqrt{a^2cx^2+c}}$$

```
[Out] (Sqrt[Pi/2]*Sqrt[1 + a^2*x^2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(2*a^
2*c^2*Sqrt[c + a^2*c*x^2]) + (Sqrt[Pi/6]*Sqrt[1 + a^2*x^2]*FresnelS[Sqrt[6/
Pi]*Sqrt[ArcTan[a*x]]])/(2*a^2*c^2*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.214189, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {4971, 4970, 4406, 3305, 3351}

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{a^2x^2+1}S\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{2a^2c^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{\frac{\pi}{6}}\sqrt{a^2x^2+1}S\left(\sqrt{\frac{6}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{2a^2c^2\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]),x]
```


```
[Out] (Sqrt[Pi/2]*Sqrt[1 + a^2*x^2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(2*a^
2*c^2*Sqrt[c + a^2*c*x^2]) + (Sqrt[Pi/6]*Sqrt[1 + a^2*x^2]*FresnelS[Sqrt[6/
Pi]*Sqrt[ArcTan[a*x]]])/(2*a^2*c^2*Sqrt[c + a^2*c*x^2])
```


Rule 4971

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[(d^(q + 1/2)*Sqrt[1 + c^2*x^2])/Sqrt[d + e*x^2],
 Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d
, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(In
tegerQ[q] || GtQ[d, 0])
```

Rule 4970

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sin[x]^m)/
```

```
Cos[x]^(m + 2*(q + 1)), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{x}{\left(c+a^2cx^2\right)^{5/2}\sqrt{\tan^{-1}(ax)}}\,dx = \frac{\sqrt{1+a^2x^2}\int \frac{x}{\left(1+a^2x^2\right)^{5/2}\sqrt{\tan^{-1}(ax)}}\,dx}{c^2\sqrt{c+a^2cx^2}}$$

$$= \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \frac{\cos^2(x)\sin(x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{a^2c^2\sqrt{c+a^2cx^2}}$$

$$= \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \left(\frac{\sin(x)}{4\sqrt{x}}+\frac{\sin(3x)}{4\sqrt{x}}\right)dx, x, \tan^{-1}(ax)\right)}{a^2c^2\sqrt{c+a^2cx^2}}$$

$$= \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \frac{\sin(x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{4a^2c^2\sqrt{c+a^2cx^2}} + \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \frac{\sin(3x)}{\sqrt{x}}\,dx, x, \tan\right.}{4a^2c^2\sqrt{c+a^2cx^2}}$$

$$= \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \sin\left(x^2\right)dx, x, \sqrt{\tan^{-1}(ax)}\right)}{2a^2c^2\sqrt{c+a^2cx^2}} + \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \sin\left(3x^2\right)\right.}{2a^2c^2\sqrt{c+a^2c}}$$

$$= \frac{\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}\,S\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{2a^2c^2\sqrt{c+a^2cx^2}} + \frac{\sqrt{\frac{\pi}{6}}\sqrt{1+a^2x^2}\,S\left(\sqrt{\frac{6}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{2a^2c^2\sqrt{c+a^2cx^2}}$$

Mathematica [C] time = 0.207549, size = 156, normalized size = 1.19

$$-\frac{\left(a^2x^2+1\right)^{3/2}\left(3\sqrt{-i\tan^{-1}(ax)}\text{Gamma}\left(\frac{1}{2},-i\tan^{-1}(ax)\right)+3\sqrt{i\tan^{-1}(ax)}\text{Gamma}\left(\frac{1}{2},i\tan^{-1}(ax)\right)+\sqrt{3}\left(\sqrt{-i\tan^{-1}}\right.\right.}{24a^2c\left(c\left(a^2x^2+1\right)\right)^{3/2}\sqrt{\tan^{-1}(ax)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]),x]
```

```
[Out] -((1 + a^2*x^2)^(3/2)*(3*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-I)*ArcTan[a*x]
] + 3*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, I*ArcTan[a*x]] + Sqrt[3]*(Sqrt[(-I)*Ar
cTan[a*x]]*Gamma[1/2, (-3*I)*ArcTan[a*x]] + Sqrt[I*ArcTan[a*x]]*Gamma[1/2,
(3*I)*ArcTan[a*x]])))/(24*a^2*c*(c*(1 + a^2*x^2))^(3/2)*Sqrt[ArcTan[a*x]])
```

Maple [F] time = 0.815, size = 0, normalized size = 0.

$$\int x\left(a^2cx^2+c\right)^{-\frac{5}{2}}\frac{1}{\sqrt{\arctan(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

```
[Out] int(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a**2*c*x**2+c)**(5/2)/atan(a*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(a^2cx^2+c\right)^{\frac{5}{2}}\sqrt{\arctan\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/((a^2*c*x^2 + c)^(5/2)*sqrt(arctan(a*x))), x)
```

3.970 $\int \frac{1}{\left(c+a^2cx^2\right)^{5/2}\sqrt{\tan^{-1}(ax)}}\,dx$

Optimal. Leaf size=131

$$\frac{3\sqrt{\frac{\pi}{2}}\sqrt{a^2x^2+1}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{2ac^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{\frac{\pi}{6}}\sqrt{a^2x^2+1}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{2ac^2\sqrt{a^2cx^2+c}}$$

```
[Out] (3*Sqrt[Pi/2]*Sqrt[1 + a^2*x^2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(2*
a*c^2*Sqrt[c + a^2*c*x^2]) + (Sqrt[Pi/6]*Sqrt[1 + a^2*x^2]*FresnelC[Sqrt[6/
Pi]*Sqrt[ArcTan[a*x]]])/(2*a*c^2*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.146135, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4905, 4904, 3312, 3304, 3352}

$$\frac{3\sqrt{\frac{\pi}{2}}\sqrt{a^2x^2+1}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{2ac^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{\frac{\pi}{6}}\sqrt{a^2x^2+1}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{2ac^2\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]),x]
```

```
[Out] (3*Sqrt[Pi/2]*Sqrt[1 + a^2*x^2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(2*
a*c^2*Sqrt[c + a^2*c*x^2]) + (Sqrt[Pi/6]*Sqrt[1 + a^2*x^2]*FresnelC[Sqrt[6/
Pi]*Sqrt[ArcTan[a*x]]])/(2*a*c^2*Sqrt[c + a^2*c*x^2])
```

Rule 4905

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[(d^(q + 1/2)*Sqrt[1 + c^2*x^2])/Sqrt[d + e*x^2], Int[(1 + c
^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

Rule 4904

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
```

```
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{1}{\left(c+a^2cx^2\right)^{5/2}\sqrt{\tan^{-1}(ax)}}\,dx = \frac{\sqrt{1+a^2x^2}\int \frac{1}{\left(1+a^2x^2\right)^{5/2}\sqrt{\tan^{-1}(ax)}}\,dx}{c^2\sqrt{c+a^2cx^2}}$$

$$= \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \frac{\cos^3(x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{ac^2\sqrt{c+a^2cx^2}}$$

$$= \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \left(\frac{3\cos(x)}{4\sqrt{x}}+\frac{\cos(3x)}{4\sqrt{x}}\right)dx, x, \tan^{-1}(ax)\right)}{ac^2\sqrt{c+a^2cx^2}}$$

$$= \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \frac{\cos(3x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{4ac^2\sqrt{c+a^2cx^2}} + \frac{\left(3\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int \frac{\cos(x)}{\sqrt{x}}\,dx, x,\right.}{4ac^2\sqrt{c+a^2cx^2}}$$

$$= \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \cos\left(3x^2\right)dx, x, \sqrt{\tan^{-1}(ax)}\right)}{2ac^2\sqrt{c+a^2cx^2}} + \frac{\left(3\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int \cos\left(x\right.\right.}{2ac^2\sqrt{c+a}}$$

$$= \frac{3\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}\,C\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{2ac^2\sqrt{c+a^2cx^2}} + \frac{\sqrt{\frac{\pi}{6}}\sqrt{1+a^2x^2}\,C\left(\sqrt{\frac{6}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{2ac^2\sqrt{c+a^2cx^2}}$$

Mathematica [C] time = 0.214688, size = 159, normalized size = 1.21

$$-\frac{i\sqrt{c\left(a^2x^2+1\right)}\left(9\sqrt{-i\tan^{-1}(ax)}\text{Gamma}\left(\frac{1}{2},-i\tan^{-1}(ax)\right)-9\sqrt{i\tan^{-1}(ax)}\text{Gamma}\left(\frac{1}{2},i\tan^{-1}(ax)\right)+\sqrt{3}\left(\sqrt{-i\tan^{-1}\right.\right.}{24ac^3\sqrt{a^2x^2+1}\sqrt{\tan^{-1}(ax)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]),x]
```

```
[Out] ((-I/24)*Sqrt[c*(1 + a^2*x^2)]*(9*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-I)*Ar
cTan[a*x]] - 9*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, I*ArcTan[a*x]] + Sqrt[3]*(Sqr
t[(-I)*ArcTan[a*x]]*Gamma[1/2, (-3*I)*ArcTan[a*x]] - Sqrt[I*ArcTan[a*x]]*Ga
mma[1/2, (3*I)*ArcTan[a*x]])))/(a*c^3*Sqrt[1 + a^2*x^2]*Sqrt[ArcTan[a*x]])
```

Maple [F] time = 0.7, size = 0, normalized size = 0.

$$\int\left(a^2cx^2+c\right)^{-\frac{5}{2}}\frac{1}{\sqrt{\arctan\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

```
[Out] int(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*c*x**2+c)**(5/2)/atan(a*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)^{\frac{5}{2}}\sqrt{\arctan\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^(5/2)*sqrt(arctan(a*x))), x)
```

3.971 $\int \frac{1}{x(c+a^2cx^2)^{5/2}\sqrt{\tan^{-1}(ax)}}\,dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(\frac{1}{x\left(a^2cx^2+c\right)^{5/2}\sqrt{\tan^{-1}(ax)}},x\right)$$

```
[Out] Unintegrable[1/(x*(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]), x]
```

Rubi [A] time = 0.117957, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\sqrt{\tan^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]),x]
```

```
[Out] Defer[Int][1/(x*(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]), x]
```

Rubi steps

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\sqrt{\tan^{-1}(ax)}}\,dx=\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\sqrt{\tan^{-1}(ax)}}\,dx$$

Mathematica [A] time = 2.07828, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\sqrt{\tan^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]),x]
```

```
[Out] Integrate[1/(x*(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]), x]
```

Maple [A] time = 0.727, size = 0, normalized size = 0.

$$\int \frac{1}{x} \left(a^{2} c \,x^{2}+c \right)^{-\frac{5}{2}} \frac{1}{\sqrt{\arctan \left(a x \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

```
[Out] int(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a**2*c*x**2+c)**(5/2)/atan(a*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)^{\frac{5}{2}}x\sqrt{\arctan\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^(5/2)*x*sqrt(arctan(a*x))), x)
```

3.972 $\int \frac{x^m(c+a^2cx^2)}{\tan^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{x^m\left(a^2cx^2+c\right)}{\tan^{-1}(ax)^{3/2}},x\right)$$

```
[Out] Unintegrable[(x^m*(c + a^2*c*x^2))/ArcTan[a*x]^(3/2), x]
```

Rubi [A] time = 0.0351689, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m\left(c+a^2cx^2\right)}{\tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*(c + a^2*c*x^2))/ArcTan[a*x]^(3/2),x]
```

```
[Out] Defer[Int][(x^m*(c + a^2*c*x^2))/ArcTan[a*x]^(3/2), x]
```

Rubi steps

$$\int \frac{x^m\left(c+a^2cx^2\right)}{\tan^{-1}(ax)^{3/2}}\,dx=\int \frac{x^m\left(c+a^2cx^2\right)}{\tan^{-1}(ax)^{3/2}}\,dx$$

Mathematica [A] time = 1.9637, size = 0, normalized size = 0.

$$\int \frac{x^m\left(c+a^2cx^2\right)}{\tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*(c + a^2*c*x^2))/ArcTan[a*x]^(3/2),x]
```

```
[Out] Integrate[(x^m*(c + a^2*c*x^2))/ArcTan[a*x]^(3/2), x]
```

Maple [A] time = 0.722, size = 0, normalized size = 0.

$$\int x^{m} \left(a^{2} c \,x^{2}+c \right) \left(\arctan \left(a x \right)\right)^{-\frac{3}{2}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)/arctan(a*x)^(3/2),x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)/arctan(a*x)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(a^{2} c x^{2} + c\right)x^{m}}{\arctan \left(a x\right)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)*x^m/arctan(a*x)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)/atan(a*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a^2cx^2 + c)x^m}{\arctan(ax)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)*x^m/arctan(a*x)^(3/2), x)
```

3.973 $\int \frac{x(c+a^2cx^2)}{\tan^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{x\left(a^2cx^2+c\right)}{\tan^{-1}(ax)^{3/2}},x\right)$$

```
[Out] Unintegrable[(x*(c + a^2*c*x^2))/ArcTan[a*x]^(3/2), x]
```

Rubi [A] time = 0.0229703, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x\left(c+a^2cx^2\right)}{\tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x*(c + a^2*c*x^2))/ArcTan[a*x]^(3/2),x]

[Out] Defer[Int][(x*(c + a^2*c*x^2))/ArcTan[a*x]^(3/2), x]
```

Rubi steps

$$\int \frac{x\left(c+a^2cx^2\right)}{\tan^{-1}(ax)^{3/2}}\,dx=\int \frac{x\left(c+a^2cx^2\right)}{\tan^{-1}(ax)^{3/2}}\,dx$$

Mathematica [A] time = 1.51548, size = 0, normalized size = 0.

$$\int \frac{x\left(c+a^2cx^2\right)}{\tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x*(c + a^2*c*x^2))/ArcTan[a*x]^(3/2),x]
```

```
[Out] Integrate[(x*(c + a^2*c*x^2))/ArcTan[a*x]^(3/2), x]
```

Maple [A] time = 0.386, size = 0, normalized size = 0.

$$\int x\left(a^{2}c\,x^{2}+c\right)\left(\arctan\left(ax\right)\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)/arctan(a*x)^(3/2),x)
```

```
[Out] int(x*(a^2*c*x^2+c)/arctan(a*x)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$c\left(\int\frac{x}{\operatorname{atan}^{\frac{3}{2}}\left(ax\right)}\,dx+\int\frac{a^{2}x^{3}}{\operatorname{atan}^{\frac{3}{2}}\left(ax\right)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)/atan(a*x)**(3/2),x)
```

```
[Out] c*(Integral(x/atan(a*x)**(3/2), x) + Integral(a**2*x**3/atan(a*x)**(3/2), x
))
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)x}{\arctan\left(ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)*x/arctan(a*x)^(3/2), x)
```

3.974 $\int \frac{c+a^2cx^2}{\tan^{-1}(ax)^{3/2}} \, dx$

Optimal. Leaf size=21

$$\text{Unintegrable}\left(\frac{a^2cx^2+c}{\tan^{-1}(ax)^{3/2}},x\right)$$

```
[Out] Unintegrable[(c + a^2*c*x^2)/ArcTan[a*x]^(3/2), x]
```

Rubi [A] time = 0.0120097, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{c+a^2cx^2}{\tan^{-1}(ax)^{3/2}} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)/ArcTan[a*x]^(3/2),x]
```

```
[Out] Defer[Int][(c + a^2*c*x^2)/ArcTan[a*x]^(3/2), x]
```

Rubi steps

$$\int \frac{c+a^2cx^2}{\tan^{-1}(ax)^{3/2}} \, dx = \int \frac{c+a^2cx^2}{\tan^{-1}(ax)^{3/2}} \, dx$$

Mathematica [A] time = 1.41159, size = 0, normalized size = 0.

$$\int \frac{c+a^2cx^2}{\tan^{-1}(ax)^{3/2}} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)/ArcTan[a*x]^(3/2),x]
```

```
[Out] Integrate[(c + a^2*c*x^2)/ArcTan[a*x]^(3/2), x]
```

Maple [A] time = 0.268, size = 0, normalized size = 0.

$$\int \left(a^{2} c\,x^{2}+c\right) \left(\arctan \left(a x \right)\right)^{-\frac{3}{2}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)/arctan(a*x)^(3/2),x)
```

```
[Out] int((a^2*c*x^2+c)/arctan(a*x)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$c \left(\int \frac{a^{2} x^{2}}{\operatorname{atan}^{\frac{3}{2}}{\left(a x \right)}}\, dx + \int \frac{1}{\operatorname{atan}^{\frac{3}{2}}{\left(a x \right)}}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)/atan(a*x)**(3/2),x)
```

```
[Out] c*(Integral(a**2*x**2/atan(a*x)**(3/2), x) + Integral(atan(a*x)**(-3/2), x)
)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{a^2cx^2 + c}{\arctan (ax)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)/arctan(a*x)^(3/2), x)
```

3.975 $\int \frac{c+a^2cx^2}{x\tan^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{a^2cx^2+c}{x\tan^{-1}(ax)^{3/2}},x\right)$$

```
[Out] Unintegrable[(c + a^2*c*x^2)/(x*ArcTan[a*x]^(3/2)), x]
```

Rubi [A] time = 0.0318866, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{c+a^2cx^2}{x\tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)/(x*ArcTan[a*x]^(3/2)),x]
```

```
[Out] Defer[Int][(c + a^2*c*x^2)/(x*ArcTan[a*x]^(3/2)), x]
```

Rubi steps

$$\int \frac{c+a^2cx^2}{x\tan^{-1}(ax)^{3/2}}\,dx=\int \frac{c+a^2cx^2}{x\tan^{-1}(ax)^{3/2}}\,dx$$

Mathematica [A] time = 1.9196, size = 0, normalized size = 0.

$$\int \frac{c+a^2cx^2}{x\tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)/(x*ArcTan[a*x]^(3/2)),x]
```

```
[Out] Integrate[(c + a^2*c*x^2)/(x*ArcTan[a*x]^(3/2)), x]
```

Maple [A] time = 0.399, size = 0, normalized size = 0.

$$\int \frac{a^2 c x^2 + c}{x} (\arctan (ax))^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)/x/arctan(a*x)^(3/2),x)
```

```
[Out] int((a^2*c*x^2+c)/x/arctan(a*x)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)/x/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)/x/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$c \left(\int \frac{1}{x \operatorname{atan}^{\frac{3}{2}} (ax)} dx + \int \frac{a^2 x}{\operatorname{atan}^{\frac{3}{2}} (ax)} dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)/x/atan(a*x)**(3/2),x)
```

```
[Out] c*(Integral(1/(x*atan(a*x)**(3/2)), x) + Integral(a**2*x/atan(a*x)**(3/2),
x))
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{a^2cx^2 + c}{x \arctan(ax)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)/x/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)/(x*arctan(a*x)^(3/2)), x)
```

3.976 $\int \frac{x^m(c+a^2cx^2)^2}{\tan^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{x^m\left(a^2cx^2+c\right)^2}{\tan^{-1}(ax)^{3/2}},x\right)$$

```
[Out] Unintegrable[(x^m*(c + a^2*c*x^2)^2)/ArcTan[a*x]^(3/2), x]
```

Rubi [A] time = 0.0540973, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m\left(c+a^2cx^2\right)^2}{\tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*(c + a^2*c*x^2)^2)/ArcTan[a*x]^(3/2),x]
```

```
[Out] Defer[Int][(x^m*(c + a^2*c*x^2)^2)/ArcTan[a*x]^(3/2), x]
```

Rubi steps

$$\int \frac{x^m\left(c+a^2cx^2\right)^2}{\tan^{-1}(ax)^{3/2}}\,dx=\int \frac{x^m\left(c+a^2cx^2\right)^2}{\tan^{-1}(ax)^{3/2}}\,dx$$

Mathematica [A] time = 1.346, size = 0, normalized size = 0.

$$\int \frac{x^m\left(c+a^2cx^2\right)^2}{\tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*(c + a^2*c*x^2)^2)/ArcTan[a*x]^(3/2),x]
```

```
[Out] Integrate[(x^m*(c + a^2*c*x^2)^2)/ArcTan[a*x]^(3/2), x]
```

Maple [A] time = 0.932, size = 0, normalized size = 0.

$$\int x^m \left(a^2 c x^2 + c\right)^2 (\arctan(ax))^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^4 c^2 x^4 + 2\, a^2 c^2 x^2 + c^2\right) x^m}{\arctan(ax)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*x^m/arctan(a*x)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**2/atan(a*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^2 x^m}{\arctan\left(ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^2*x^m/arctan(a*x)^(3/2), x)
```

3.977 $\int \frac{x(c+a^2cx^2)^2}{\tan^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{x\left(a^2cx^2+c\right)^2}{\tan^{-1}(ax)^{3/2}},x\right)$$

```
[Out] Unintegrable[(x*(c + a^2*c*x^2)^2)/ArcTan[a*x]^(3/2), x]
```

Rubi [A] time = 0.0360699, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x\left(c+a^2cx^2\right)^2}{\tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x*(c + a^2*c*x^2)^2)/ArcTan[a*x]^(3/2),x]
```

```
[Out] Defer[Int][(x*(c + a^2*c*x^2)^2)/ArcTan[a*x]^(3/2), x]
```

Rubi steps

$$\int \frac{x\left(c+a^2cx^2\right)^2}{\tan^{-1}(ax)^{3/2}}\,dx=\int \frac{x\left(c+a^2cx^2\right)^2}{\tan^{-1}(ax)^{3/2}}\,dx$$

Mathematica [A] time = 1.63208, size = 0, normalized size = 0.

$$\int \frac{x\left(c+a^2cx^2\right)^2}{\tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x*(c + a^2*c*x^2)^2)/ArcTan[a*x]^(3/2),x]
```

```
[Out] Integrate[(x*(c + a^2*c*x^2)^2)/ArcTan[a*x]^(3/2), x]
```

Maple [A] time = 0.558, size = 0, normalized size = 0.

$$\int x \left(a^{2} c \,x^{2}+c \right)^{2} \left(\arctan \left(a x \right)\right)^{-\frac{3}{2}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x)
```

```
[Out] int(x*(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$c^{2} \left(\int \frac{x}{\operatorname{atan}^{\frac{3}{2}}{\left(a x \right)}}\, dx + \int \frac{2 a^{2} x^{3}}{\operatorname{atan}^{\frac{3}{2}}{\left(a x \right)}}\, dx + \int \frac{a^{4} x^{5}}{\operatorname{atan}^{\frac{3}{2}}{\left(a x \right)}}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**2/atan(a*x)**(3/2),x)
```

```
[Out] c**2*(Integral(x/atan(a*x)**(3/2), x) + Integral(2*a**2*x**3/atan(a*x)**(3/
2), x) + Integral(a**4*x**5/atan(a*x)**(3/2), x))
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^2x}{\arctan\left(ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^2*x/arctan(a*x)^(3/2), x)
```

3.978 $\int \frac{(c+a^2cx^2)^2}{\tan^{-1}(ax)^{3/2}} \, dx$

Optimal. Leaf size=23

$$\text{Unintegrable}\left(\frac{(a^2cx^2+c)^2}{\tan^{-1}(ax)^{3/2}},x\right)$$

```
[Out] Unintegrable[(c + a^2*c*x^2)^2/ArcTan[a*x]^(3/2), x]
```

Rubi [A] time = 0.0218943, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(c+a^2cx^2)^2}{\tan^{-1}(ax)^{3/2}} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^2/ArcTan[a*x]^(3/2),x]
```

```
[Out] Defer[Int][(c + a^2*c*x^2)^2/ArcTan[a*x]^(3/2), x]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^2}{\tan^{-1}(ax)^{3/2}} \, dx = \int \frac{(c+a^2cx^2)^2}{\tan^{-1}(ax)^{3/2}} \, dx$$

Mathematica [A] time = 1.4939, size = 0, normalized size = 0.

$$\int \frac{(c+a^2cx^2)^2}{\tan^{-1}(ax)^{3/2}} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^2/ArcTan[a*x]^(3/2),x]
```

```
[Out] Integrate[(c + a^2*c*x^2)^2/ArcTan[a*x]^(3/2), x]
```

Maple [A] time = 0.44, size = 0, normalized size = 0.

$$\int \left(a^2cx^2+c\right)^2 (\arctan(ax))^{-\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x)
```

```
[Out] int((a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$c^2\left(\int \frac{2a^2x^2}{\operatorname{atan}^{\frac{3}{2}}(ax)}\, dx + \int \frac{a^4x^4}{\operatorname{atan}^{\frac{3}{2}}(ax)}\, dx + \int \frac{1}{\operatorname{atan}^{\frac{3}{2}}(ax)}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**2/atan(a*x)**(3/2),x)
```

```
[Out] c**2*(Integral(2*a**2*x**2/atan(a*x)**(3/2), x) + Integral(a**4*x**4/atan(a
*x)**(3/2), x) + Integral(atan(a*x)**(-3/2), x))
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^2}{\arctan\left(ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^2/arctan(a*x)^(3/2), x)
```

3.979 $\int \frac{\left(c+a^2 c x^2\right)^2}{x \tan^{-1}(a x)^{3/2}} \, dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{\left(a^2 c x^2+c\right)^2}{x \tan^{-1}(a x)^{3/2}},x\right)$$

```
[Out] Unintegrable[(c + a^2*c*x^2)^2/(x*ArcTan[a*x]^(3/2)), x]
```

Rubi [A] time = 0.048979, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(c+a^2 c x^2\right)^2}{x \tan^{-1}(a x)^{3/2}} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^2/(x*ArcTan[a*x]^(3/2)),x]
```

```
[Out] Defer[Int][(c + a^2*c*x^2)^2/(x*ArcTan[a*x]^(3/2)), x]
```

Rubi steps

$$\int \frac{\left(c+a^2 c x^2\right)^2}{x \tan^{-1}(a x)^{3/2}} \, dx=\int \frac{\left(c+a^2 c x^2\right)^2}{x \tan^{-1}(a x)^{3/2}} \, dx$$

Mathematica [A] time = 2.03162, size = 0, normalized size = 0.

$$\int \frac{\left(c+a^2 c x^2\right)^2}{x \tan^{-1}(a x)^{3/2}} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^2/(x*ArcTan[a*x]^(3/2)),x]
```

```
[Out] Integrate[(c + a^2*c*x^2)^2/(x*ArcTan[a*x]^(3/2)), x]
```

Maple [A] time = 0.49, size = 0, normalized size = 0.

$$\int \frac{\left(a^2 c x^2 + c\right)^2}{x} (\arctan (ax))^{-\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^2/x/arctan(a*x)^(3/2),x)
```

```
[Out] int((a^2*c*x^2+c)^2/x/arctan(a*x)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2/x/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2/x/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$c^2\left(\int \frac{1}{x\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx+\int \frac{2a^2x}{\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx+\int \frac{a^4x^3}{\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**2/x/atan(a*x)**(3/2),x)
```

```
[Out] c**2*(Integral(1/(x*atan(a*x)**(3/2)), x) + Integral(2*a**2*x/atan(a*x)**(3
/2), x) + Integral(a**4*x**3/atan(a*x)**(3/2), x))
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^2}{x\arctan(ax)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2/x/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^2/(x*arctan(a*x)^(3/2)), x)
```

3.980 $\int \frac{x^m(c+a^2cx^2)^3}{\tan^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{x^m\left(a^2cx^2+c\right)^3}{\tan^{-1}(ax)^{3/2}},x\right)$$

```
[Out] Unintegrable[(x^m*(c + a^2*c*x^2)^3)/ArcTan[a*x]^(3/2), x]
```

Rubi [A] time = 0.055548, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m\left(c+a^2cx^2\right)^3}{\tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*(c + a^2*c*x^2)^3)/ArcTan[a*x]^(3/2),x]

[Out] Defer[Int][(x^m*(c + a^2*c*x^2)^3)/ArcTan[a*x]^(3/2), x]
```

Rubi steps

$$\int \frac{x^m\left(c+a^2cx^2\right)^3}{\tan^{-1}(ax)^{3/2}}\,dx=\int \frac{x^m\left(c+a^2cx^2\right)^3}{\tan^{-1}(ax)^{3/2}}\,dx$$

Mathematica [A] time = 0.856113, size = 0, normalized size = 0.

$$\int \frac{x^m\left(c+a^2cx^2\right)^3}{\tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*(c + a^2*c*x^2)^3)/ArcTan[a*x]^(3/2),x]
```

```
[Out] Integrate[(x^m*(c + a^2*c*x^2)^3)/ArcTan[a*x]^(3/2), x]
```

Maple [A] time = 0.989, size = 0, normalized size = 0.

$$\int x^m \left(a^2 c x^2 + c\right)^3 (\arctan (a x))^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^6 c^3 x^6 + 3\, a^4 c^3 x^4 + 3\, a^2 c^3 x^2 + c^3\right) x^m}{\arctan (a x)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*x^m/arctan(a*x
)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**3/atan(a*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^3x^m}{\arctan\left(ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^3*x^m/arctan(a*x)^(3/2), x)
```

3.981 $\int \frac{x\left(c+a^2cx^2\right)^3}{\tan^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{x\left(a^2cx^2+c\right)^3}{\tan^{-1}(ax)^{3/2}},x\right)$$

```
[Out] Unintegrable[(x*(c + a^2*c*x^2)^3)/ArcTan[a*x]^(3/2), x]
```

Rubi [A] time = 0.0366894, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x\left(c+a^2cx^2\right)^3}{\tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x*(c + a^2*c*x^2)^3)/ArcTan[a*x]^(3/2),x]
```

```
[Out] Defer[Int][(x*(c + a^2*c*x^2)^3)/ArcTan[a*x]^(3/2), x]
```

Rubi steps

$$\int \frac{x\left(c+a^2cx^2\right)^3}{\tan^{-1}(ax)^{3/2}}\,dx = \int \frac{x\left(c+a^2cx^2\right)^3}{\tan^{-1}(ax)^{3/2}}\,dx$$

Mathematica [A] time = 1.67053, size = 0, normalized size = 0.

$$\int \frac{x\left(c+a^2cx^2\right)^3}{\tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x*(c + a^2*c*x^2)^3)/ArcTan[a*x]^(3/2),x]
```

```
[Out] Integrate[(x*(c + a^2*c*x^2)^3)/ArcTan[a*x]^(3/2), x]
```

Maple [A] time = 0.837, size = 0, normalized size = 0.

$$\int x\left(a^{2}c x^{2}+c\right)^{3}(\arctan(ax))^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x)
```

```
[Out] int(x*(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$c^{3}\left(\int\frac{x}{\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx+\int\frac{3a^{2}x^{3}}{\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx+\int\frac{3a^{4}x^{5}}{\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx+\int\frac{a^{6}x^{7}}{\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**3/atan(a*x)**(3/2),x)
```

```
[Out] c**3*(Integral(x/atan(a*x)**(3/2), x) + Integral(3*a**2*x**3/atan(a*x)**(3/
2), x) + Integral(3*a**4*x**5/atan(a*x)**(3/2), x) + Integral(a**6*x**7/ata
n(a*x)**(3/2), x))
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^3 x}{\arctan\left(ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^3*x/arctan(a*x)^(3/2), x)
```

3.982 $\int \frac{(c+a^2cx^2)^3}{\tan^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=23

$$\text{Unintegrable}\left(\frac{(a^2cx^2+c)^3}{\tan^{-1}(ax)^{3/2}},x\right)$$

```
[Out] Unintegrable[(c + a^2*c*x^2)^3/ArcTan[a*x]^(3/2), x]
```

Rubi [A] time = 0.02225, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(c+a^2cx^2)^3}{\tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^3/ArcTan[a*x]^(3/2),x]
```

```
[Out] Defer[Int][(c + a^2*c*x^2)^3/ArcTan[a*x]^(3/2), x]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^3}{\tan^{-1}(ax)^{3/2}}\,dx = \int \frac{(c+a^2cx^2)^3}{\tan^{-1}(ax)^{3/2}}\,dx$$

Mathematica [A] time = 1.53046, size = 0, normalized size = 0.

$$\int \frac{(c+a^2cx^2)^3}{\tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^3/ArcTan[a*x]^(3/2),x]
```

```
[Out] Integrate[(c + a^2*c*x^2)^3/ArcTan[a*x]^(3/2), x]
```

Maple [A] time = 0.657, size = 0, normalized size = 0.

$$\int \left(a^{2} c \,x^{2}+c \right)^{3} \left(\arctan \left(a x \right)\right)^{-\frac{3}{2}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x)
```

```
[Out] int((a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$c^{3} \left(\int \frac{3 a^{2} x^{2}}{\operatorname{atan}^{\frac{3}{2}}{\left(a x \right)}}\, dx + \int \frac{3 a^{4} x^{4}}{\operatorname{atan}^{\frac{3}{2}}{\left(a x \right)}}\, dx + \int \frac{a^{6} x^{6}}{\operatorname{atan}^{\frac{3}{2}}{\left(a x \right)}}\, dx + \int \frac{1}{\operatorname{atan}^{\frac{3}{2}}{\left(a x \right)}}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**3/atan(a*x)**(3/2),x)
```

```
[Out] c**3*(Integral(3*a**2*x**2/atan(a*x)**(3/2), x) + Integral(3*a**4*x**4/atan
(a*x)**(3/2), x) + Integral(a**6*x**6/atan(a*x)**(3/2), x) + Integral(atan(
a*x)**(-3/2), x))
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^3}{\arctan\left(ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^3/arctan(a*x)^(3/2), x)
```

3.983 $\int \frac{\left(c+a^2cx^2\right)^3}{x\tan^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{\left(a^2cx^2+c\right)^3}{x\tan^{-1}(ax)^{3/2}},x\right)$$

```
[Out] Unintegrable[(c + a^2*c*x^2)^3/(x*ArcTan[a*x]^(3/2)), x]
```

Rubi [A] time = 0.0497706, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(c+a^2cx^2\right)^3}{x\tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^3/(x*ArcTan[a*x]^(3/2)),x]
```

```
[Out] Defer[Int][(c + a^2*c*x^2)^3/(x*ArcTan[a*x]^(3/2)), x]
```

Rubi steps

$$\int \frac{\left(c+a^2cx^2\right)^3}{x\tan^{-1}(ax)^{3/2}}\,dx=\int \frac{\left(c+a^2cx^2\right)^3}{x\tan^{-1}(ax)^{3/2}}\,dx$$

Mathematica [A] time = 2.088, size = 0, normalized size = 0.

$$\int \frac{\left(c+a^2cx^2\right)^3}{x\tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^3/(x*ArcTan[a*x]^(3/2)),x]
```

```
[Out] Integrate[(c + a^2*c*x^2)^3/(x*ArcTan[a*x]^(3/2)), x]
```

Maple [A] time = 0.697, size = 0, normalized size = 0.

$$\int \frac{\left(a^{2} c \,x^{2}+c \right)^{3}}{x}\left(\arctan \left(a x \right)\right)^{-\frac{3}{2}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^3/x/arctan(a*x)^(3/2),x)
```

```
[Out] int((a^2*c*x^2+c)^3/x/arctan(a*x)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3/x/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3/x/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$c^3\left(\int \frac{1}{x\,\mathrm{atan}^{\frac{3}{2}}(ax)}\,dx+\int \frac{3a^2x}{\mathrm{atan}^{\frac{3}{2}}(ax)}\,dx+\int \frac{3a^4x^3}{\mathrm{atan}^{\frac{3}{2}}(ax)}\,dx+\int \frac{a^6x^5}{\mathrm{atan}^{\frac{3}{2}}(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**3/x/atan(a*x)**(3/2),x)
```

```
[Out] c**3*(Integral(1/(x*atan(a*x)**(3/2)), x) + Integral(3*a**2*x/atan(a*x)**(3
/2), x) + Integral(3*a**4*x**3/atan(a*x)**(3/2), x) + Integral(a**6*x**5/at
an(a*x)**(3/2), x))
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^3}{x\arctan(ax)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3/x/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^3/(x*arctan(a*x)^(3/2)), x)
```

3.984 $\int \frac{x^m}{(c+a^2cx^2)\tan^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=45

$$\frac{2m\text{Unintegrable}\left(\frac{x^{m-1}}{\sqrt{\tan^{-1}(ax)}},x\right)}{ac}-\frac{2x^m}{ac\sqrt{\tan^{-1}(ax)}}$$

```
[Out] (-2*x^m)/(a*c*Sqrt[ArcTan[a*x]]) + (2*m*Unintegrable[x^(-1 + m)/Sqrt[ArcTan
[a*x]], x])/(a*c)
```

Rubi [A] time = 0.0805591, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m}{(c+a^2cx^2)\tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m/((c + a^2*c*x^2)*ArcTan[a*x]^(3/2)),x]
```

```
[Out] (-2*x^m)/(a*c*Sqrt[ArcTan[a*x]]) + (2*m*Defer[Int][x^(-1 + m)/Sqrt[ArcTan[a
*x]], x])/(a*c)
```

Rubi steps

$$\int \frac{x^m}{(c+a^2cx^2)\tan^{-1}(ax)^{3/2}}\,dx=-\frac{2x^m}{ac\sqrt{\tan^{-1}(ax)}}+\frac{(2m)\int \frac{x^{-1+m}}{\sqrt{\tan^{-1}(ax)}}\,dx}{ac}$$

Mathematica [A] time = 0.700901, size = 0, normalized size = 0.

$$\int \frac{x^m}{(c+a^2cx^2)\tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m/((c + a^2*c*x^2)*ArcTan[a*x]^(3/2)),x]
```

```
[Out] Integrate[x^m/((c + a^2*c*x^2)*ArcTan[a*x]^(3/2)), x]
```

Maple [A] time = 0.572, size = 0, normalized size = 0.

$$\int \frac{x^m}{a^2cx^2 + c} (\arctan(ax))^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a^2*c*x^2+c)/arctan(a*x)^(3/2),x)
```

```
[Out] int(x^m/(a^2*c*x^2+c)/arctan(a*x)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x^m}{\left(a^2cx^2 + c\right)\arctan(ax)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(x^m/((a^2*c*x^2 + c)*arctan(a*x)^(3/2)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(a**2*c*x**2+c)/atan(a*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{(a^2cx^2 + c)\arctan(ax)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^m/((a^2*c*x^2 + c)*arctan(a*x)^(3/2)), x)
```

3.985 $\int \frac{x}{(c+a^2cx^2)\tan^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=36

$$\frac{2\text{Unintegrable}\left(\frac{1}{\sqrt{\tan^{-1}(ax)}},x\right)}{ac}-\frac{2x}{ac\sqrt{\tan^{-1}(ax)}}$$

```
[Out] (-2*x)/(a*c*Sqrt[ArcTan[a*x]]) + (2*Unintegrable[1/Sqrt[ArcTan[a*x]], x])/(
a*c)
```

Rubi [A] time = 0.0480899, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x}{\left(c+a^2cx^2\right)\tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x/((c + a^2*c*x^2)*ArcTan[a*x]^(3/2)),x]
```

```
[Out] (-2*x)/(a*c*Sqrt[ArcTan[a*x]]) + (2*Defer[Int][1/Sqrt[ArcTan[a*x]], x])/(a*
c)
```

Rubi steps

$$\int \frac{x}{\left(c+a^2cx^2\right)\tan^{-1}(ax)^{3/2}}\,dx=-\frac{2x}{ac\sqrt{\tan^{-1}(ax)}}+\frac{2\int \frac{1}{\sqrt{\tan^{-1}(ax)}}\,dx}{ac}$$

Mathematica [A] time = 1.01886, size = 0, normalized size = 0.

$$\int \frac{x}{\left(c+a^2cx^2\right)\tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x/((c + a^2*c*x^2)*ArcTan[a*x]^(3/2)),x]
```

```
[Out] Integrate[x/((c + a^2*c*x^2)*ArcTan[a*x]^(3/2)), x]
```

Maple [A] time = 0.141, size = 0, normalized size = 0.

$$\int \frac{x}{a^2cx^2+c} (\arctan(ax))^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^2*c*x^2+c)/arctan(a*x)^(3/2),x)
```

```
[Out] int(x/(a^2*c*x^2+c)/arctan(a*x)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x}{a^2x^2\operatorname{atan}^{\frac{3}{2}}(ax)+\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a**2*c*x**2+c)/atan(a*x)**(3/2),x)
```

```
[Out] Integral(x/(a**2*x**2*atan(a*x)**(3/2) + atan(a*x)**(3/2)), x)/c
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(a^2cx^2+c)\arctan(ax)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x/((a^2*c*x^2 + c)*arctan(a*x)^(3/2)), x)
```

3.986 $\int \frac{1}{\left(c+a^2cx^2\right)\tan^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=16

$$-\frac{2}{ac\sqrt{\tan^{-1}(ax)}}$$

```
[Out] -2/(a*c*Sqrt[ArcTan[a*x]])
```

Rubi [A] time = 0.0249069, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {4884}

$$-\frac{2}{ac\sqrt{\tan^{-1}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a^2*c*x^2)*ArcTan[a*x]^(3/2)),x]
```

```
[Out] -2/(a*c*Sqrt[ArcTan[a*x]])
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(c+a^2cx^2\right)\tan^{-1}(ax)^{3/2}}\,dx = -\frac{2}{ac\sqrt{\tan^{-1}(ax)}}$$

Mathematica [A] time = 0.0049823, size = 16, normalized size = 1.

$$-\frac{2}{ac\sqrt{\tan^{-1}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a^2*c*x^2)*ArcTan[a*x]^(3/2)),x]
```

```
[Out] -2/(a*c*Sqrt[ArcTan[a*x]])
```

Maple [A] time = 0.087, size = 15, normalized size = 0.9

$$-2\,\frac{1}{ac\sqrt{\arctan\left(ax\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2*c*x^2+c)/arctan(a*x)^(3/2),x)
```

```
[Out] -2/a/c/arctan(a*x)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 1.61517, size = 38, normalized size = 2.38

$$-\frac{2}{ac\sqrt{\arctan\left(ax\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] -2/(a*c*sqrt(arctan(a*x)))
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*c*x**2+c)/atan(a*x)**(3/2),x)
```

```
[Out] Exception raised: TypeError
```

Giac [A] time = 1.11232, size = 19, normalized size = 1.19

$$-\frac{2}{ac\sqrt{\arctan\left(ax\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] -2/(a*c*sqrt(arctan(a*x)))
```

3.987 $\int \frac{1}{x(c+a^2cx^2)\tan^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=42

$$-\frac{2\text{Unintegrable}\left(\frac{1}{x^2\sqrt{\tan^{-1}(ax)}},x\right)}{ac}-\frac{2}{acx\sqrt{\tan^{-1}(ax)}}$$

```
[Out] -2/(a*c*x*Sqrt[ArcTan[a*x]]) - (2*Unintegrable[1/(x^2*Sqrt[ArcTan[a*x]]), x
])/(a*c)
```

Rubi [A] time = 0.0764422, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x\left(c+a^2cx^2\right)\tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*(c + a^2*c*x^2)*ArcTan[a*x]^(3/2)),x]
```

```
[Out] -2/(a*c*x*Sqrt[ArcTan[a*x]]) - (2*Defer[Int][1/(x^2*Sqrt[ArcTan[a*x]]), x])
/(a*c)
```

Rubi steps

$$\int \frac{1}{x\left(c+a^2cx^2\right)\tan^{-1}(ax)^{3/2}}\,dx=-\frac{2}{acx\sqrt{\tan^{-1}(ax)}}-\frac{2\int \frac{1}{x^2\sqrt{\tan^{-1}(ax)}}\,dx}{ac}$$

Mathematica [A] time = 0.200762, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(c+a^2cx^2\right)\tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*(c + a^2*c*x^2)*ArcTan[a*x]^(3/2)),x]
```

```
[Out] Integrate[1/(x*(c + a^2*c*x^2)*ArcTan[a*x]^(3/2)), x]
```

Maple [A] time = 0.145, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(a^{2}c x^{2}+c\right)}\left(\arctan\left(ax\right)\right)^{-\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a^2*c*x^2+c)/arctan(a*x)^(3/2),x)
```

```
[Out] int(1/x/(a^2*c*x^2+c)/arctan(a*x)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{a^2x^3 \operatorname{atan}^{\frac{3}{2}}(ax)+x \operatorname{atan}^{\frac{3}{2}}(ax)}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a**2*c*x**2+c)/atan(a*x)**(3/2),x)
```

```
[Out] Integral(1/(a**2*x**3*atan(a*x)**(3/2) + x*atan(a*x)**(3/2)), x)/c
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2 + c\right)x \arctan\left(ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)*x*arctan(a*x)^(3/2)), x)
```

3.988 $\int \frac{x^m}{\left(c+a^2cx^2\right)^2 \tan^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{x^m}{\left(a^2cx^2+c\right)^2 \tan^{-1}(ax)^{3/2}},x\right)$$

```
[Out] Unintegrable[x^m/((c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)), x]
```

Rubi [A] time = 0.0623191, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m}{\left(c+a^2cx^2\right)^2 \tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m/((c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)),x]
```

```
[Out] Defer[Int][x^m/((c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)), x]
```

Rubi steps

$$\int \frac{x^m}{\left(c+a^2cx^2\right)^2 \tan^{-1}(ax)^{3/2}}\,dx = \int \frac{x^m}{\left(c+a^2cx^2\right)^2 \tan^{-1}(ax)^{3/2}}\,dx$$

Mathematica [A] time = 1.11633, size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(c+a^2cx^2\right)^2 \tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m/((c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)),x]
```

```
[Out] Integrate[x^m/((c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)), x]
```

Maple [A] time = 0.765, size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(a^2 c x^2 + c\right)^2} \left(\arctan\left(a x\right)\right)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x)
```

```
[Out] int(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^m}{\left(a^4 c^2 x^4 + 2\, a^2 c^2 x^2 + c^2\right) \arctan\left(a x\right)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(x^m/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(a*x)^(3/2)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(a**2*c*x**2+c)**2/atan(a*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{(a^2cx^2+c)^2 \arctan(ax)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^m/((a^2*c*x^2 + c)^2*arctan(a*x)^(3/2)), x)
```

3.989 $\int \frac{x^4}{\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=91

$$\frac{8\text{Unintegrable}\left(\frac{x^3}{\left(a^2cx^2+c\right)^2\sqrt{\tan^{-1}(ax)}},x\right)}{a}+4a\text{Unintegrable}\left(\frac{x^5}{\left(a^2cx^2+c\right)^2\sqrt{\tan^{-1}(ax)}},x\right)-\frac{2x^4}{ac^2\left(a^2x^2+1\right)\sqrt{\tan^{-1}(ax)}}$$

```
[Out] (-2*x^4)/(a*c^2*(1 + a^2*x^2)*Sqrt[ArcTan[a*x]]) + (8*Unintegrable[x^3/((c
+ a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]), x])/a + 4*a*Unintegrable[x^5/((c + a^2*c
*x^2)^2*Sqrt[ArcTan[a*x]]), x]
```

Rubi [A] time = 0.198846, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^4}{\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^4/((c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)),x]
```

```
[Out] (-2*x^4)/(a*c^2*(1 + a^2*x^2)*Sqrt[ArcTan[a*x]]) + (8*Defer[Int][x^3/((c +
a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]), x])/a + 4*a*Defer[Int][x^5/((c + a^2*c*x^2
)^2*Sqrt[ArcTan[a*x]]), x]
```

Rubi steps

$$\int \frac{x^4}{\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^{3/2}}\,dx=-\frac{2x^4}{ac^2\left(1+a^2x^2\right)\sqrt{\tan^{-1}(ax)}}+\frac{8\int \frac{x^3}{\left(c+a^2cx^2\right)^2\sqrt{\tan^{-1}(ax)}}\,dx}{a}+(4a)\int \frac{x^5}{\left(c+a^2cx^2\right)^2\sqrt{\tan^{-1}(ax)}}$$

Mathematica [A] time = 4.16415, size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^4/((c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)),x]
```

```
[Out] Integrate[x^4/((c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)), x]
```

Maple [A] time = 0.501, size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(a^2cx^2+c\right)^2}\left(\arctan\left(ax\right)\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x)
```

```
[Out] int(x^4/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^4}{a^4x^4\operatorname{atan}^{\frac{3}{2}}(ax)+2a^2x^2\operatorname{atan}^{\frac{3}{2}}(ax)+\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(a**2*c*x**2+c)**2/atan(a*x)**(3/2),x)
```

```
[Out] Integral(x**4/(a**4*x**4*atan(a*x)**(3/2) + 2*a**2*x**2*atan(a*x)**(3/2) +
atan(a*x)**(3/2)), x)/c**2
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(a^2cx^2+c\right)^2\arctan(ax)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/((a^2*c*x^2 + c)^2*arctan(a*x)^(3/2)), x)
```

3.990 $\int \frac{x^3}{\left(c+a^2cx^2\right)^2 \tan^{-1}(ax)^{3/2}} \, dx$

Optimal. Leaf size=105

$$2a\text{Unintegrable}\left(\frac{x^4}{\left(a^2cx^2+c\right)^2\sqrt{\tan^{-1}(ax)}},x\right)-\frac{3\sqrt{\pi}\text{FresnelC}\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{a^4c^2}-\frac{2x^3}{ac^2\left(a^2x^2+1\right)\sqrt{\tan^{-1}(ax)}}+\frac{6\sqrt{\tan^{-1}(ax)}}{a^4c^2}$$

```
[Out] (-2*x^3)/(a*c^2*(1 + a^2*x^2)*Sqrt[ArcTan[a*x]]) + (6*Sqrt[ArcTan[a*x]])/(a
^4*c^2) - (3*Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(a^4*c^2) +
 2*a*Unintegrable[x^4/((c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]), x]
```

Rubi [A] time = 0.246323, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^3}{\left(c+a^2cx^2\right)^2 \tan^{-1}(ax)^{3/2}} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[x^3/((c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)),x]
```

```
[Out] (-2*x^3)/(a*c^2*(1 + a^2*x^2)*Sqrt[ArcTan[a*x]]) + (6*Sqrt[ArcTan[a*x]])/(a
^4*c^2) - (3*Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(a^4*c^2) +
 2*a*Defer[Int][x^4/((c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]), x]
```

Rubi steps

$$\int \frac{x^3}{\left(c+a^2cx^2\right)^2 \tan^{-1}(ax)^{3/2}}\,dx = -\frac{2x^3}{ac^2\left(1+a^2x^2\right)\sqrt{\tan^{-1}(ax)}} + \frac{6\int \frac{x^2}{\left(c+a^2cx^2\right)^2\sqrt{\tan^{-1}(ax)}}\,dx}{a} + (2a)\int \frac{x^4}{\left(c+a^2cx^2\right)^2\sqrt{\tan^{-1}}}$$

$$= -\frac{2x^3}{ac^2\left(1+a^2x^2\right)\sqrt{\tan^{-1}(ax)}} + (2a)\int \frac{x^4}{\left(c+a^2cx^2\right)^2\sqrt{\tan^{-1}(ax)}}\,dx + \frac{6\,\mathrm{Subst}\left(\int \frac{\sin^2(}{\sqrt{x}}\right.}{a}$$

$$= -\frac{2x^3}{ac^2\left(1+a^2x^2\right)\sqrt{\tan^{-1}(ax)}} + (2a)\int \frac{x^4}{\left(c+a^2cx^2\right)^2\sqrt{\tan^{-1}(ax)}}\,dx + \frac{6\,\mathrm{Subst}\left(\int\left(\frac{1}{2\sqrt{x}}\right.\right.}{}$$

$$= -\frac{2x^3}{ac^2\left(1+a^2x^2\right)\sqrt{\tan^{-1}(ax)}} + \frac{6\sqrt{\tan^{-1}(ax)}}{a^4c^2} + (2a)\int \frac{x^4}{\left(c+a^2cx^2\right)^2\sqrt{\tan^{-1}(ax)}}\,dx -$$

$$= -\frac{2x^3}{ac^2\left(1+a^2x^2\right)\sqrt{\tan^{-1}(ax)}} + \frac{6\sqrt{\tan^{-1}(ax)}}{a^4c^2} + (2a)\int \frac{x^4}{\left(c+a^2cx^2\right)^2\sqrt{\tan^{-1}(ax)}}\,dx -$$

$$= -\frac{2x^3}{ac^2\left(1+a^2x^2\right)\sqrt{\tan^{-1}(ax)}} + \frac{6\sqrt{\tan^{-1}(ax)}}{a^4c^2} - \frac{3\sqrt{\pi}C\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{a^4c^2} + (2a)\int \frac{}{\left(c+a^2c\right.}$$

Mathematica [A] time = 4.95874, size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(c+a^2cx^2\right)^2 \tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^3/((c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)),x]
```

```
[Out] Integrate[x^3/((c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)), x]
```

Maple [A] time = 0.505, size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(a^2cx^2+c\right)^2}\left(\arctan(ax)\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x)
```

```
[Out] int(x^3/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^3}{a^4x^4\operatorname{atan}^{\frac{3}{2}}(ax)+2a^2x^2\operatorname{atan}^{\frac{3}{2}}(ax)+\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a**2*c*x**2+c)**2/atan(a*x)**(3/2),x)
```

```
[Out] Integral(x**3/(a**4*x**4*atan(a*x)**(3/2) + 2*a**2*x**2*atan(a*x)**(3/2) +
atan(a*x)**(3/2)), x)/c**2
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(a^2cx^2+c\right)^2\arctan\left(ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/((a^2*c*x^2 + c)^2*arctan(a*x)^(3/2)), x)
```

3.991 $\int \frac{x^2}{\left(c+a^2cx^2\right)^2 \tan^{-1}(ax)^{3/2}} \, dx$

Optimal. Leaf size=60

$$\frac{2\sqrt{\pi}S\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{a^3c^2}-\frac{2x^2}{ac^2\left(a^2x^2+1\right)\sqrt{\tan^{-1}(ax)}}$$

```
[Out] (-2*x^2)/(a*c^2*(1 + a^2*x^2)*Sqrt[ArcTan[a*x]]) + (2*Sqrt[Pi]*FresnelS[(2*
Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(a^3*c^2)
```

Rubi [A] time = 0.144906, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4942, 4970, 4406, 12, 3305, 3351}

$$\frac{2\sqrt{\pi}S\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{a^3c^2}-\frac{2x^2}{ac^2\left(a^2x^2+1\right)\sqrt{\tan^{-1}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)),x]
```

```
[Out] (-2*x^2)/(a*c^2*(1 + a^2*x^2)*Sqrt[ArcTan[a*x]]) + (2*Sqrt[Pi]*FresnelS[(2*
Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(a^3*c^2)
```

Rule 4942

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^m*(d + e*x^2)^(q + 1)*(a + b*ArcT
an[c*x])^(p + 1))/(b*c*d*(p + 1)), x] - Dist[(f*m)/(b*c*(p + 1)), Int[(f*x)
^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 2, 0] && LtQ[p, -1]
```

Rule 4970

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sin[x]^m)/
```

```
Cos[x]^(m + 2*(q + 1)), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{x^2}{(c+a^2cx^2)^2 \tan^{-1}(ax)^{3/2}}\,dx = -\frac{2x^2}{ac^2(1+a^2x^2)\sqrt{\tan^{-1}(ax)}} + \frac{4\int \frac{x}{(c+a^2cx^2)^2\sqrt{\tan^{-1}(ax)}}\,dx}{a}$$

$$= -\frac{2x^2}{ac^2(1+a^2x^2)\sqrt{\tan^{-1}(ax)}} + \frac{4\,\text{Subst}\left(\int \frac{\cos(x)\sin(x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{a^3c^2}$$

$$= -\frac{2x^2}{ac^2(1+a^2x^2)\sqrt{\tan^{-1}(ax)}} + \frac{4\,\text{Subst}\left(\int \frac{\sin(2x)}{2\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{a^3c^2}$$

$$= -\frac{2x^2}{ac^2(1+a^2x^2)\sqrt{\tan^{-1}(ax)}} + \frac{2\,\text{Subst}\left(\int \frac{\sin(2x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{a^3c^2}$$

$$= -\frac{2x^2}{ac^2(1+a^2x^2)\sqrt{\tan^{-1}(ax)}} + \frac{4\,\text{Subst}\left(\int \sin(2x^2)\,dx, x, \sqrt{\tan^{-1}(ax)}\right)}{a^3c^2}$$

$$= -\frac{2x^2}{ac^2(1+a^2x^2)\sqrt{\tan^{-1}(ax)}} + \frac{2\sqrt{\pi}S\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{a^3c^2}$$

Mathematica [A] time = 0.182232, size = 60, normalized size = 1.

$$\frac{2\sqrt{\pi}S\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{a^3c^2} - \frac{2x^2}{ac^2(a^2x^2+1)\sqrt{\tan^{-1}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)),x]

[Out] (-2*x^2)/(a*c^2*(1 + a^2*x^2)*Sqrt[ArcTan[a*x]]) + (2*Sqrt[Pi]*FresnelS[(2*
Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(a^3*c^2)
```

Maple [A] time = 0.107, size = 46, normalized size = 0.8

$$\frac{1}{a^3c^2}\left(2\sqrt{\arctan(ax)}\sqrt{\pi}\,\mathrm{FresnelS}\left(2\frac{\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\cos(2\arctan(ax))-1\right)\frac{1}{\sqrt{\arctan(ax)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x)
```

```
[Out] 1/a^3/c^2*(2*arctan(a*x)^(1/2)*Pi^(1/2)*FresnelS(2*arctan(a*x)^(1/2)/Pi^(1/
2))+cos(2*arctan(a*x))-1)/arctan(a*x)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^2}{a^4x^4\operatorname{atan}^{\frac{3}{2}}(ax)+2a^2x^2\operatorname{atan}^{\frac{3}{2}}(ax)+\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a**2*c*x**2+c)**2/atan(a*x)**(3/2),x)
```

```
[Out] Integral(x**2/(a**4*x**4*atan(a*x)**(3/2) + 2*a**2*x**2*atan(a*x)**(3/2) +
atan(a*x)**(3/2)), x)/c**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(a^2cx^2+c\right)^2\arctan\left(ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/((a^2*c*x^2 + c)^2*arctan(a*x)^(3/2)), x)
```

3.992 $\int \frac{x}{\left(c+a^2cx^2\right)^2 \tan^{-1}(ax)^{3/2}} dx$

Optimal. Leaf size=138

$$\frac{2\sqrt{\pi}\text{FresnelC}\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{a^2c^2}-\frac{2x}{ac^2\left(a^2x^2+1\right)\sqrt{\tan^{-1}(ax)}}+\frac{4\left(1-a^2x^2\right)\sqrt{\tan^{-1}(ax)}}{a^2c^2\left(a^2x^2+1\right)}-\frac{8\sqrt{\tan^{-1}(ax)}}{a^2c^2\left(a^2x^2+1\right)}+\frac{4\sqrt{\tan^{-1}(ax)}}{a^2c^2}$$

```
[Out] (-2*x)/(a*c^2*(1 + a^2*x^2)*Sqrt[ArcTan[a*x]]) + (4*Sqrt[ArcTan[a*x]])/(a^2
*c^2) - (8*Sqrt[ArcTan[a*x]])/(a^2*c^2*(1 + a^2*x^2)) + (4*(1 - a^2*x^2)*Sq
rt[ArcTan[a*x]])/(a^2*c^2*(1 + a^2*x^2)) + (2*Sqrt[Pi]*FresnelC[(2*Sqrt[Arc
Tan[a*x]])/Sqrt[Pi]])/(a^2*c^2)
```

Rubi [A] time = 0.169586, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4932, 4930, 4904, 3312, 3304, 3352}

$$\frac{2\sqrt{\pi}\text{FresnelC}\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{a^2c^2}-\frac{2x}{ac^2\left(a^2x^2+1\right)\sqrt{\tan^{-1}(ax)}}+\frac{4\left(1-a^2x^2\right)\sqrt{\tan^{-1}(ax)}}{a^2c^2\left(a^2x^2+1\right)}-\frac{8\sqrt{\tan^{-1}(ax)}}{a^2c^2\left(a^2x^2+1\right)}+\frac{4\sqrt{\tan^{-1}(ax)}}{a^2c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/((c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)),x]
```


```
[Out] (-2*x)/(a*c^2*(1 + a^2*x^2)*Sqrt[ArcTan[a*x]]) + (4*Sqrt[ArcTan[a*x]])/(a^2
*c^2) - (8*Sqrt[ArcTan[a*x]])/(a^2*c^2*(1 + a^2*x^2)) + (4*(1 - a^2*x^2)*Sq
rt[ArcTan[a*x]])/(a^2*c^2*(1 + a^2*x^2)) + (2*Sqrt[Pi]*FresnelC[(2*Sqrt[Arc
Tan[a*x]])/Sqrt[Pi]])/(a^2*c^2)
```


Rule 4932

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*(x_))/((d_) + (e_.)*(x_)^2)^2,
 x_Symbol] :> Simp[(x*(a + b*ArcTan[c*x])^(p + 1))/(b*c*d*(p + 1)*(d + e*x^
2)), x] + (-Dist[4/(b^2*(p + 1)*(p + 2)), Int[(x*(a + b*ArcTan[c*x])^(p + 2
))/(d + e*x^2)^2, x], x] - Simp[((1 - c^2*x^2)*(a + b*ArcTan[c*x])^(p + 2))
/(b^2*e*(p + 1)*(p + 2)*(d + e*x^2)), x]) /; FreeQ[{a, b, c, d, e}, x] && E
qQ[e, c^2*d] && LtQ[p, -1] && NeQ[p, -2]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 4904

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{x}{\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^{3/2}}\,dx = -\frac{2x}{ac^2\left(1+a^2x^2\right)\sqrt{\tan^{-1}(ax)}}+\frac{4\left(1-a^2x^2\right)\sqrt{\tan^{-1}(ax)}}{a^2c^2\left(1+a^2x^2\right)}+16\int \frac{x\sqrt{\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^2}\,dx$$

$$= -\frac{2x}{ac^2\left(1+a^2x^2\right)\sqrt{\tan^{-1}(ax)}}-\frac{8\sqrt{\tan^{-1}(ax)}}{a^2c^2\left(1+a^2x^2\right)}+\frac{4\left(1-a^2x^2\right)\sqrt{\tan^{-1}(ax)}}{a^2c^2\left(1+a^2x^2\right)}+\frac{4\int \frac{}{\left(c+a^2c\right.}}{}$$

$$= -\frac{2x}{ac^2\left(1+a^2x^2\right)\sqrt{\tan^{-1}(ax)}}-\frac{8\sqrt{\tan^{-1}(ax)}}{a^2c^2\left(1+a^2x^2\right)}+\frac{4\left(1-a^2x^2\right)\sqrt{\tan^{-1}(ax)}}{a^2c^2\left(1+a^2x^2\right)}+\frac{4\,\mathrm{Subst}\,(}{}$$

$$= -\frac{2x}{ac^2\left(1+a^2x^2\right)\sqrt{\tan^{-1}(ax)}}-\frac{8\sqrt{\tan^{-1}(ax)}}{a^2c^2\left(1+a^2x^2\right)}+\frac{4\left(1-a^2x^2\right)\sqrt{\tan^{-1}(ax)}}{a^2c^2\left(1+a^2x^2\right)}+\frac{4\,\mathrm{Subst}\,(}{}$$

$$= -\frac{2x}{ac^2\left(1+a^2x^2\right)\sqrt{\tan^{-1}(ax)}}+\frac{4\sqrt{\tan^{-1}(ax)}}{a^2c^2}-\frac{8\sqrt{\tan^{-1}(ax)}}{a^2c^2\left(1+a^2x^2\right)}+\frac{4\left(1-a^2x^2\right)\sqrt{\tan^{-1}}}{a^2c^2\left(1+a^2x^2\right)}$$

$$= -\frac{2x}{ac^2\left(1+a^2x^2\right)\sqrt{\tan^{-1}(ax)}}+\frac{4\sqrt{\tan^{-1}(ax)}}{a^2c^2}-\frac{8\sqrt{\tan^{-1}(ax)}}{a^2c^2\left(1+a^2x^2\right)}+\frac{4\left(1-a^2x^2\right)\sqrt{\tan^{-1}}}{a^2c^2\left(1+a^2x^2\right)}$$

$$= -\frac{2x}{ac^2\left(1+a^2x^2\right)\sqrt{\tan^{-1}(ax)}}+\frac{4\sqrt{\tan^{-1}(ax)}}{a^2c^2}-\frac{8\sqrt{\tan^{-1}(ax)}}{a^2c^2\left(1+a^2x^2\right)}+\frac{4\left(1-a^2x^2\right)\sqrt{\tan^{-1}}}{a^2c^2\left(1+a^2x^2\right)}$$

Mathematica [A] time = 0.0917111, size = 48, normalized size = 0.35

$$\frac{2\sqrt{\pi}\text{FresnelC}\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)-\frac{\sin\left(2\tan^{-1}(ax)\right)}{\sqrt{\tan^{-1}(ax)}}}{a^2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)),x]
```

```
[Out] (2*Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]] - Sin[2*ArcTan[a*x]]/S
qrt[ArcTan[a*x]])/(a^2*c^2)
```

Maple [A] time = 0.098, size = 47, normalized size = 0.3

$$\frac{1}{a^{2}c^{2}}\left(2\sqrt{\arctan\left(ax\right)}\sqrt{\pi}\,\mathrm{FresnelC}\left(2\frac{\sqrt{\arctan\left(ax\right)}}{\sqrt{\pi}}\right)-\sin\left(2\arctan\left(ax\right)\right)\right)\frac{1}{\sqrt{\arctan\left(ax\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x)
```

```
[Out] 1/a^2/c^2*(2*arctan(a*x)^(1/2)*Pi^(1/2)*FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/
2))-sin(2*arctan(a*x)))/arctan(a*x)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x}{a^{4}x^{4}\operatorname{atan}^{\frac{3}{2}}{\left(ax\right)}+2a^{2}x^{2}\operatorname{atan}^{\frac{3}{2}}{\left(ax\right)}+\operatorname{atan}^{\frac{3}{2}}{\left(ax\right)}}\,dx}{c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a**2*c*x**2+c)**2/atan(a*x)**(3/2),x)
```

```
[Out] Integral(x/(a**4*x**4*atan(a*x)**(3/2) + 2*a**2*x**2*atan(a*x)**(3/2) + ata
n(a*x)**(3/2)), x)/c**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(a^2cx^2+c\right)^2\arctan\left(ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x/((a^2*c*x^2 + c)^2*arctan(a*x)^(3/2)), x)
```

3.993 $\int \frac{1}{(c+a^2cx^2)^2 \tan^{-1}(ax)^{3/2}} dx$

Optimal. Leaf size=57

$$-\frac{2}{ac^2(a^2x^2+1)\sqrt{\tan^{-1}(ax)}}-\frac{2\sqrt{\pi}S\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{ac^2}$$

```
[Out] -2/(a*c^2*(1 + a^2*x^2)*Sqrt[ArcTan[a*x]]) - (2*Sqrt[Pi]*FresnelS[(2*Sqrt[A
rcTan[a*x]])/Sqrt[Pi]])/(a*c^2)
```

Rubi [A] time = 0.103588, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4902, 4970, 4406, 12, 3305, 3351}

$$-\frac{2}{ac^2(a^2x^2+1)\sqrt{\tan^{-1}(ax)}}-\frac{2\sqrt{\pi}S\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)),x]
```

```
[Out] -2/(a*c^2*(1 + a^2*x^2)*Sqrt[ArcTan[a*x]]) - (2*Sqrt[Pi]*FresnelS[(2*Sqrt[A
rcTan[a*x]])/Sqrt[Pi]])/(a*c^2)
```

Rule 4902

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^(p + 1))/(b*c*d*(p
+ 1)), x] - Dist[(2*c*(q + 1))/(b*(p + 1)), Int[x*(d + e*x^2)^q*(a + b*ArcT
an[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && L
tQ[q, -1] && LtQ[p, -1]
```

Rule 4970

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sin[x]^m)/
```

```
Cos[x]^(m + 2*(q + 1)), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{1}{\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^{3/2}}\,dx = -\frac{2}{ac^2\left(1+a^2x^2\right)\sqrt{\tan^{-1}(ax)}} - (4a)\int \frac{x}{\left(c+a^2cx^2\right)^2\sqrt{\tan^{-1}(ax)}}\,dx$$

$$= -\frac{2}{ac^2\left(1+a^2x^2\right)\sqrt{\tan^{-1}(ax)}} - \frac{4\,\mathrm{Subst}\left(\int \frac{\cos(x)\sin(x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{ac^2}$$

$$= -\frac{2}{ac^2\left(1+a^2x^2\right)\sqrt{\tan^{-1}(ax)}} - \frac{4\,\mathrm{Subst}\left(\int \frac{\sin(2x)}{2\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{ac^2}$$

$$= -\frac{2}{ac^2\left(1+a^2x^2\right)\sqrt{\tan^{-1}(ax)}} - \frac{2\,\mathrm{Subst}\left(\int \frac{\sin(2x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{ac^2}$$

$$= -\frac{2}{ac^2\left(1+a^2x^2\right)\sqrt{\tan^{-1}(ax)}} - \frac{4\,\mathrm{Subst}\left(\int \sin\left(2x^2\right)\,dx, x, \sqrt{\tan^{-1}(ax)}\right)}{ac^2}$$

$$= -\frac{2}{ac^2\left(1+a^2x^2\right)\sqrt{\tan^{-1}(ax)}} - \frac{2\sqrt{\pi}S\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{ac^2}$$

Mathematica [A] time = 0.149662, size = 52, normalized size = 0.91

$$\frac{-\frac{2}{\left(a^2x^2+1\right)\sqrt{\tan^{-1}(ax)}} - 2\sqrt{\pi}S\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{ac^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)),x]
```

```
[Out] (-2/((1 + a^2*x^2)*Sqrt[ArcTan[a*x]]) - 2*Sqrt[Pi]*FresnelS[(2*Sqrt[ArcTan[
a*x]])/Sqrt[Pi]])/(a*c^2)
```

Maple [A] time = 0.102, size = 47, normalized size = 0.8

$$-\frac{1}{ac^2}\left(2\sqrt{\arctan(ax)}\sqrt{\pi}\mathrm{FresnelS}\left(2\frac{\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\cos(2\arctan(ax))+1\right)\frac{1}{\sqrt{\arctan(ax)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x)
```

```
[Out] -1/a/c^2*(2*arctan(a*x)^(1/2)*Pi^(1/2)*FresnelS(2*arctan(a*x)^(1/2)/Pi^(1/2
))+cos(2*arctan(a*x))+1)/arctan(a*x)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{a^{4}x^{4}\operatorname{atan}^{\frac{3}{2}}(ax)+2a^{2}x^{2}\operatorname{atan}^{\frac{3}{2}}(ax)+\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx}{c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*c*x**2+c)**2/atan(a*x)**(3/2),x)
```

```
[Out] Integral(1/(a**4*x**4*atan(a*x)**(3/2) + 2*a**2*x**2*atan(a*x)**(3/2) + ata
n(a*x)**(3/2)), x)/c**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)^2 \arctan(ax)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^2*arctan(a*x)^(3/2)), x)
```

3.994 $\int \frac{1}{x\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=101

$$-\frac{2\text{Unintegrable}\left(\frac{1}{x^2\left(a^2cx^2+c\right)^2\sqrt{\tan^{-1}(ax)}},x\right)}{a}-\frac{2}{ac^2x\left(a^2x^2+1\right)\sqrt{\tan^{-1}(ax)}}-\frac{3\sqrt{\pi}\text{FresnelC}\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{c^2}-\frac{6\sqrt{\tan^{-1}(ax)}}{c^2}$$

```
[Out] -2/(a*c^2*x*(1 + a^2*x^2)*Sqrt[ArcTan[a*x]]) - (6*Sqrt[ArcTan[a*x]])/c^2 -
(3*Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/c^2 - (2*Unintegrable
[1/(x^2*(c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]), x])/a
```

Rubi [A] time = 0.199116, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*(c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)),x]
```

```
[Out] -2/(a*c^2*x*(1 + a^2*x^2)*Sqrt[ArcTan[a*x]]) - (6*Sqrt[ArcTan[a*x]])/c^2 -
(3*Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/c^2 - (2*Defer[Int][1
/(x^2*(c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]), x])/a
```

Rubi steps

$$\int \frac{1}{x\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^{3/2}}\,dx = -\frac{2}{ac^2x\left(1+a^2x^2\right)\sqrt{\tan^{-1}(ax)}} - \frac{2\int \frac{1}{x^2\left(c+a^2cx^2\right)^2\sqrt{\tan^{-1}(ax)}}\,dx}{a} - (6a)\int \frac{1}{\left(c+a^2cx^2\right)^2 \cdots}$$

$$= -\frac{2}{ac^2x\left(1+a^2x^2\right)\sqrt{\tan^{-1}(ax)}} - \frac{2\int \frac{1}{x^2\left(c+a^2cx^2\right)^2\sqrt{\tan^{-1}(ax)}}\,dx}{a} - \frac{6\,\text{Subst}\left(\int \frac{\cos^2(x)}{\sqrt{x}}\,dx \cdots\right.}{c^2}$$

$$= -\frac{2}{ac^2x\left(1+a^2x^2\right)\sqrt{\tan^{-1}(ax)}} - \frac{2\int \frac{1}{x^2\left(c+a^2cx^2\right)^2\sqrt{\tan^{-1}(ax)}}\,dx}{a} - \frac{6\,\text{Subst}\left(\int\left(\frac{1}{2\sqrt{x}} + \cdots\right.\right.}{\cdots}$$

$$= -\frac{2}{ac^2x\left(1+a^2x^2\right)\sqrt{\tan^{-1}(ax)}} - \frac{6\sqrt{\tan^{-1}(ax)}}{c^2} - \frac{2\int \frac{1}{x^2\left(c+a^2cx^2\right)^2\sqrt{\tan^{-1}(ax)}}\,dx}{a} - \frac{3\,\text{Su}\cdots}{\cdots}$$

$$= -\frac{2}{ac^2x\left(1+a^2x^2\right)\sqrt{\tan^{-1}(ax)}} - \frac{6\sqrt{\tan^{-1}(ax)}}{c^2} - \frac{2\int \frac{1}{x^2\left(c+a^2cx^2\right)^2\sqrt{\tan^{-1}(ax)}}\,dx}{a} - \frac{6\,\text{Su}\cdots}{\cdots}$$

$$= -\frac{2}{ac^2x\left(1+a^2x^2\right)\sqrt{\tan^{-1}(ax)}} - \frac{6\sqrt{\tan^{-1}(ax)}}{c^2} - \frac{3\sqrt{\pi}C\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{c^2} - \frac{2\int \frac{\cdots}{x^2\left(c+a^2c\cdots\right.}}{\cdots}$$

Mathematica [A] time = 4.5515, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*(c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)),x]
```

```
[Out] Integrate[1/(x*(c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)), x]
```

Maple [A] time = 0.525, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(a^2cx^2+c\right)^2}\left(\arctan(ax)\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x)
```

```
[Out] int(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{a^4x^5\operatorname{atan}^{\frac{3}{2}}(ax)+2a^2x^3\operatorname{atan}^{\frac{3}{2}}(ax)+x\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a**2*c*x**2+c)**2/atan(a*x)**(3/2),x)
```

```
[Out] Integral(1/(a**4*x**5*atan(a*x)**(3/2) + 2*a**2*x**3*atan(a*x)**(3/2) + x*a
tan(a*x)**(3/2)), x)/c**2
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)^2 x \arctan\left(ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^2*x*arctan(a*x)^(3/2)), x)
```

3.995 $\int \frac{1}{x^2(c+a^2cx^2)^2 \tan^{-1}(ax)^{3/2}} dx$

Optimal. Leaf size=91

$$-\frac{4\text{Unintegrable}\left(\frac{1}{x^3(a^2cx^2+c)^2\sqrt{\tan^{-1}(ax)}},x\right)}{a}-8a\text{Unintegrable}\left(\frac{1}{x\left(a^2cx^2+c\right)^2\sqrt{\tan^{-1}(ax)}},x\right)-\frac{2}{ac^2x^2\left(a^2x^2+1\right)\sqrt{\tan^{-1}(ax)}}$$

```
[Out] -2/(a*c^2*x^2*(1 + a^2*x^2)*Sqrt[ArcTan[a*x]]) - (4*Unintegrable[1/(x^3*(c
+ a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]), x])/a - 8*a*Unintegrable[1/(x*(c + a^2*c
*x^2)^2*Sqrt[ArcTan[a*x]]), x]
```

Rubi [A] time = 0.192871, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^2 \left(c+a^2cx^2\right)^2 \tan^{-1}(ax)^{3/2}} dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)),x]
```

```
[Out] -2/(a*c^2*x^2*(1 + a^2*x^2)*Sqrt[ArcTan[a*x]]) - (4*Defer[Int][1/(x^3*(c +
a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]), x])/a - 8*a*Defer[Int][1/(x*(c + a^2*c*x^2
)^2*Sqrt[ArcTan[a*x]]), x]
```

Rubi steps

$$\int \frac{1}{x^2 \left(c+a^2cx^2\right)^2 \tan^{-1}(ax)^{3/2}} dx = -\frac{2}{ac^2x^2\left(1+a^2x^2\right)\sqrt{\tan^{-1}(ax)}}-\frac{4\int \frac{1}{x^3\left(c+a^2cx^2\right)^2\sqrt{\tan^{-1}(ax)}}dx}{a}-(8a)\int \frac{1}{x\left(c+a^2cx^2\right)^2\sqrt{\tan^{-1}(ax)}}dx$$

Mathematica [A] time = 4.48279, size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \left(c+a^2cx^2\right)^2 \tan^{-1}(ax)^{3/2}} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)),x]
```

```
[Out] Integrate[1/(x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)), x]
```

Maple [A] time = 0.51, size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \left(a^2 c x^2 + c\right)^2} (\arctan(ax))^{-\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x)
```

```
[Out] int(1/x^2/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\frac{a^4x^6 \operatorname{atan}^{\frac{3}{2}}(ax)+2a^2x^4 \operatorname{atan}^{\frac{3}{2}}(ax)+x^2 \operatorname{atan}^{\frac{3}{2}}(ax)}{c^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a**2*c*x**2+c)**2/atan(a*x)**(3/2),x)
```

```
[Out] Integral(1/(a**4*x**6*atan(a*x)**(3/2) + 2*a**2*x**4*atan(a*x)**(3/2) + x**
2*atan(a*x)**(3/2)), x)/c**2
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)^2 x^2 \arctan\left(ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^2*x^2*arctan(a*x)^(3/2)), x)
```

3.996 $\int \frac{1}{x^3\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=91

$$-\frac{6\text{Unintegrable}\left(\frac{1}{x^4\left(a^2cx^2+c\right)^2\sqrt{\tan^{-1}(ax)}},x\right)}{a}-10a\text{Unintegrable}\left(\frac{1}{x^2\left(a^2cx^2+c\right)^2\sqrt{\tan^{-1}(ax)}},x\right)-\frac{2}{ac^2x^3\left(a^2x^2+1\right)\sqrt{\text{t}}}$$

```
[Out] -2/(a*c^2*x^3*(1 + a^2*x^2)*Sqrt[ArcTan[a*x]]) - (6*Unintegrable[1/(x^4*(c
+ a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]), x])/a - 10*a*Unintegrable[1/(x^2*(c + a^
2*c*x^2)^2*Sqrt[ArcTan[a*x]]), x]
```

Rubi [A] time = 0.198758, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^3\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^3*(c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)),x]
```

```
[Out] -2/(a*c^2*x^3*(1 + a^2*x^2)*Sqrt[ArcTan[a*x]]) - (6*Defer[Int][1/(x^4*(c +
a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]), x])/a - 10*a*Defer[Int][1/(x^2*(c + a^2*c*
x^2)^2*Sqrt[ArcTan[a*x]]), x]
```

Rubi steps

$$\int \frac{1}{x^3\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^{3/2}}\,dx=-\frac{2}{ac^2x^3\left(1+a^2x^2\right)\sqrt{\tan^{-1}(ax)}}-\frac{6\int \frac{1}{x^4\left(c+a^2cx^2\right)^2\sqrt{\tan^{-1}(ax)}}\,dx}{a}-(10a)\int \frac{}{x^2\left(c+a^2c\right.}$$

Mathematica [A] time = 6.72678, size = 0, normalized size = 0.

$$\int \frac{1}{x^3\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^3*(c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)),x]
```

```
[Out] Integrate[1/(x^3*(c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)), x]
```

Maple [A] time = 0.869, size = 0, normalized size = 0.

$$\int \frac{1}{x^3\left(a^2cx^2+c\right)^2}(\arctan(ax))^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x)
```

```
[Out] int(1/x^3/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{a^4x^7\operatorname{atan}^{\frac{3}{2}}(ax)+2a^2x^5\operatorname{atan}^{\frac{3}{2}}(ax)+x^3\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a**2*c*x**2+c)**2/atan(a*x)**(3/2),x)
```

```
[Out] Integral(1/(a**4*x**7*atan(a*x)**(3/2) + 2*a**2*x**5*atan(a*x)**(3/2) + x**
3*atan(a*x)**(3/2)), x)/c**2
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a^2cx^2+c)^2 x^3 \arctan(ax)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^2*x^3*arctan(a*x)^(3/2)), x)
```

3.997 $\int \frac{1}{x^4\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=91

$$-\frac{8\text{Unintegrable}\left(\frac{1}{x^5\left(a^2cx^2+c\right)^2\sqrt{\tan^{-1}(ax)}},x\right)}{a}-12a\text{Unintegrable}\left(\frac{1}{x^3\left(a^2cx^2+c\right)^2\sqrt{\tan^{-1}(ax)}},x\right)-\frac{2}{ac^2x^4\left(a^2x^2+1\right)\sqrt{\tan^{-1}(ax)}}$$

```
[Out] -2/(a*c^2*x^4*(1 + a^2*x^2)*Sqrt[ArcTan[a*x]]) - (8*Unintegrable[1/(x^5*(c
+ a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]), x])/a - 12*a*Unintegrable[1/(x^3*(c + a^
2*c*x^2)^2*Sqrt[ArcTan[a*x]]), x]
```

Rubi [A] time = 0.201475, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^4\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^4*(c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)),x]
```

```
[Out] -2/(a*c^2*x^4*(1 + a^2*x^2)*Sqrt[ArcTan[a*x]]) - (8*Defer[Int][1/(x^5*(c +
a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]), x])/a - 12*a*Defer[Int][1/(x^3*(c + a^2*c*
x^2)^2*Sqrt[ArcTan[a*x]]), x]
```

Rubi steps

$$\int \frac{1}{x^4\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^{3/2}}\,dx=-\frac{2}{ac^2x^4\left(1+a^2x^2\right)\sqrt{\tan^{-1}(ax)}}-\frac{8\int \frac{1}{x^5\left(c+a^2cx^2\right)^2\sqrt{\tan^{-1}(ax)}}\,dx}{a}-(12a)\int \frac{1}{x^3\left(c+a^2cx^2\right)^2\sqrt{\tan^{-1}(ax)}}\,dx$$

Mathematica [A] time = 6.77262, size = 0, normalized size = 0.

$$\int \frac{1}{x^4\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^4*(c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)),x]
```

```
[Out] Integrate[1/(x^4*(c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)), x]
```

Maple [A] time = 0.66, size = 0, normalized size = 0.

$$\int \frac{1}{x^4 \left(a^2 c x^2 + c\right)^2} (\arctan (ax))^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x)
```

```
[Out] int(1/x^4/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(a**2*c*x**2+c)**2/atan(a*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)^2x^4\arctan\left(ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^2*x^4*arctan(a*x)^(3/2)), x)
```

3.998 $\int \frac{x^m}{\left(c+a^2cx^2\right)^3 \tan^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{x^m}{\left(a^2cx^2+c\right)^3 \tan^{-1}(ax)^{3/2}},x\right)$$

```
[Out] Unintegrable[x^m/((c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2)), x]
```

Rubi [A] time = 0.0633806, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{x^m}{\left(c+a^2cx^2\right)^3 \tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m/((c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2)),x]
```

```
[Out] Defer[Int][x^m/((c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2)), x]
```

Rubi steps

$$\int \frac{x^m}{\left(c+a^2cx^2\right)^3 \tan^{-1}(ax)^{3/2}}\,dx = \int \frac{x^m}{\left(c+a^2cx^2\right)^3 \tan^{-1}(ax)^{3/2}}\,dx$$

Mathematica [A] time = 1.35523, size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(c+a^2cx^2\right)^3 \tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m/((c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2)),x]
```

```
[Out] Integrate[x^m/((c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2)), x]
```

Maple [A] time = 0.839, size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(a^2 c x^2 + c\right)^3} \left(\arctan\left(ax\right)\right)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x)
```

```
[Out] int(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^m}{\left(a^6 c^3 x^6 + 3\, a^4 c^3 x^4 + 3\, a^2 c^3 x^2 + c^3\right) \arctan\left(ax\right)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(x^m/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*arctan(a*
x)^(3/2)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(a**2*c*x**2+c)**3/atan(a*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{(a^2cx^2+c)^3 \arctan(ax)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^m/((a^2*c*x^2 + c)^3*arctan(a*x)^(3/2)), x)
```

3.999 $\int \frac{x^3}{\left(c+a^2cx^2\right)^3 \tan^{-1}(ax)^{3/2}} \, dx$

Optimal. Leaf size=96

$$-\frac{\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{a^4c^3}+\frac{\sqrt{\pi}\text{FresnelC}\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{a^4c^3}-\frac{2x^3}{ac^3\left(a^2x^2+1\right)^2\sqrt{\tan^{-1}(ax)}}$$

```
[Out] (-2*x^3)/(a*c^3*(1 + a^2*x^2)^2*Sqrt[ArcTan[a*x]]) - (Sqrt[Pi/2]*FresnelC[2
*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(a^4*c^3) + (Sqrt[Pi]*FresnelC[(2*Sqrt[ArcT
an[a*x]])/Sqrt[Pi]])/(a^4*c^3)
```

Rubi [A] time = 0.334085, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4968, 4970, 3312, 3304, 3352, 4406}

$$-\frac{\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{a^4c^3}+\frac{\sqrt{\pi}\text{FresnelC}\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{a^4c^3}-\frac{2x^3}{ac^3\left(a^2x^2+1\right)^2\sqrt{\tan^{-1}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2)),x]
```

```
[Out] (-2*x^3)/(a*c^3*(1 + a^2*x^2)^2*Sqrt[ArcTan[a*x]]) - (Sqrt[Pi/2]*FresnelC[2
*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(a^4*c^3) + (Sqrt[Pi]*FresnelC[(2*Sqrt[ArcT
an[a*x]])/Sqrt[Pi]])/(a^4*c^3)
```

Rule 4968

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[(x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^(p
+ 1))/(b*c*d*(p + 1)), x] + (-Dist[(c*(m + 2*q + 2))/(b*(p + 1)), Int[x^(m
+ 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p + 1
)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; Fre
eQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &&
LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 4970

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sin[x]^m)/
Cos[x]^(m + 2*(q + 1)), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \frac{x^3}{\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^{3/2}}\,dx = -\frac{2x^3}{ac^3\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}} + \frac{6\int \frac{x^2}{\left(c+a^2cx^2\right)^3\sqrt{\tan^{-1}(ax)}}\,dx}{a} - (2a)\int \frac{x^4}{\left(c+a^2cx^2\right)^3\sqrt{\tan}}$$

$$= -\frac{2x^3}{ac^3\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}} - \frac{2\,\text{Subst}\left(\int \frac{\sin^4(x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{a^4c^3} + \frac{6\,\text{Subst}\left(\int \frac{\cos^2}{}\right.}{}$$

$$= -\frac{2x^3}{ac^3\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}} - \frac{2\,\text{Subst}\left(\int\left(\frac{3}{8\sqrt{x}} - \frac{\cos(2x)}{2\sqrt{x}} + \frac{\cos(4x)}{8\sqrt{x}}\right)dx, x, \tan^{-1}(ax)\right)}{a^4c^3}$$

$$= -\frac{2x^3}{ac^3\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}} - \frac{\text{Subst}\left(\int \frac{\cos(4x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{4a^4c^3} - \frac{3\,\text{Subst}\left(\int \frac{\cos(4x}{\sqrt{x}}\right.}{4a}$$

$$= -\frac{2x^3}{ac^3\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}} - \frac{\text{Subst}\left(\int \cos\left(4x^2\right)dx, x, \sqrt{\tan^{-1}(ax)}\right)}{2a^4c^3} - \frac{3\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{2x^3}{ac^3\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}} - \frac{\sqrt{\frac{\pi}{2}}C\left(2\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{a^4c^3} + \frac{\sqrt{\pi}C\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{a^4c^3}$$

<u>Mathematica</u> [C] time = 0.347452, size = 148, normalized size = 1.54

$$\frac{\frac{3i\sqrt{-i\tan^{-1}(ax)}\text{Gamma}\left(\frac{1}{2},-4i\tan^{-1}(ax)\right)-3i\sqrt{i\tan^{-1}(ax)}\text{Gamma}\left(\frac{1}{2},4i\tan^{-1}(ax)\right)-\frac{32a^3x^3}{\left(a^2x^2+1\right)^2}}{\sqrt{\tan^{-1}(ax)}} - 2\sqrt{2\pi}\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right) + 16\sqrt{\pi}}{16a^4c^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/((c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2)),x]
```

```
[Out] (-2*Sqrt[2*Pi]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]] + 16*Sqrt[Pi]*Fresn
elC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]] + ((-32*a^3*x^3)/(1 + a^2*x^2)^2 + (3*I
)*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-4*I)*ArcTan[a*x]] - (3*I)*Sqrt[I*ArcT
an[a*x]]*Gamma[1/2, (4*I)*ArcTan[a*x]])/Sqrt[ArcTan[a*x]])/(16*a^4*c^3)
```

Maple [A] time = 0.112, size = 86, normalized size = 0.9

$$-\frac{1}{4\,c^{3}a^{4}}\left(2\,\sqrt{2}\sqrt{\arctan\left(ax\right)}\sqrt{\pi}\mathrm{FresnelC}\left(2\,\frac{\sqrt{2}\sqrt{\arctan\left(ax\right)}}{\sqrt{\pi}}\right)-4\,\sqrt{\arctan\left(ax\right)}\sqrt{\pi}\mathrm{FresnelC}\left(2\,\frac{\sqrt{\arctan\left(ax\right)}}{\sqrt{\pi}}\right)+2\,\mathrm{s}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x)
```

```
[Out] -1/4/a^4/c^3/arctan(a*x)^(1/2)*(2*2^(1/2)*arctan(a*x)^(1/2)*Pi^(1/2)*Fresne
lC(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))-4*arctan(a*x)^(1/2)*Pi^(1/2)*Fresn
elC(2*arctan(a*x)^(1/2)/Pi^(1/2))+2*sin(2*arctan(a*x))-sin(4*arctan(a*x)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^{3}}{a^{6}x^{6}\operatorname{atan}^{\frac{3}{2}}{\left(ax\right)}+3a^{4}x^{4}\operatorname{atan}^{\frac{3}{2}}{\left(ax\right)}+3a^{2}x^{2}\operatorname{atan}^{\frac{3}{2}}{\left(ax\right)}+\operatorname{atan}^{\frac{3}{2}}{\left(ax\right)}}\,dx}{c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a**2*c*x**2+c)**3/atan(a*x)**(3/2),x)
```

```
[Out] Integral(x**3/(a**6*x**6*atan(a*x)**(3/2) + 3*a**4*x**4*atan(a*x)**(3/2) +
3*a**2*x**2*atan(a*x)**(3/2) + atan(a*x)**(3/2)), x)/c**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(a^2cx^2+c\right)^3 \arctan\left(ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/((a^2*c*x^2 + c)^3*arctan(a*x)^(3/2)), x)
```

3.1000 $\int \frac{x^2}{\left(c+a^2cx^2\right)^3 \tan^{-1}(ax)^{3/2}} \, dx$

Optimal. Leaf size=67

$$\frac{\sqrt{\frac{\pi}{2}}S\left(2\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{a^3c^3}-\frac{2x^2}{ac^3\left(a^2x^2+1\right)^2\sqrt{\tan^{-1}(ax)}}$$

```
[Out] (-2*x^2)/(a*c^3*(1 + a^2*x^2)^2*Sqrt[ArcTan[a*x]]) + (Sqrt[Pi/2]*FresnelS[2
*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(a^3*c^3)
```

Rubi [A] time = 0.313015, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {4968, 4970, 4406, 3305, 3351}

$$\frac{\sqrt{\frac{\pi}{2}}S\left(2\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{a^3c^3}-\frac{2x^2}{ac^3\left(a^2x^2+1\right)^2\sqrt{\tan^{-1}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2)),x]
```

```
[Out] (-2*x^2)/(a*c^3*(1 + a^2*x^2)^2*Sqrt[ArcTan[a*x]]) + (Sqrt[Pi/2]*FresnelS[2
*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(a^3*c^3)
```

Rule 4968

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[(x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^(p
+ 1))/(b*c*d*(p + 1)), x] + (-Dist[(c*(m + 2*q + 2))/(b*(p + 1)), Int[x^(m
+ 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p + 1
)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; Fre
eQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &&
LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 4970

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sin[x]^m)/
```

```
Cos[x]^(m + 2*(q + 1)), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{x^2}{\left(c+a^2cx^2\right)^3 \tan^{-1}(ax)^{3/2}}\,dx = -\frac{2x^2}{ac^3\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}} + \frac{4\int \frac{x}{\left(c+a^2cx^2\right)^3\sqrt{\tan^{-1}(ax)}}\,dx}{a} - (4a)\int \frac{x^3}{\left(c+a^2cx^2\right)^3\sqrt{\tan}}$$

$$= -\frac{2x^2}{ac^3\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}} + \frac{4\,\mathrm{Subst}\left(\int \frac{\cos^3(x)\sin(x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{a^3c^3} - \frac{4\,\mathrm{Subst}\,(}{}$$

$$= -\frac{2x^2}{ac^3\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}} - \frac{4\,\mathrm{Subst}\left(\int \left(\frac{\sin(2x)}{4\sqrt{x}} - \frac{\sin(4x)}{8\sqrt{x}}\right)dx, x, \tan^{-1}(ax)\right)}{a^3c^3} + \frac{4\,\mathrm{S}}{}$$

$$= -\frac{2x^2}{ac^3\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}} + 2\frac{\mathrm{Subst}\left(\int \frac{\sin(4x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{2a^3c^3}$$

$$= -\frac{2x^2}{ac^3\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}} + 2\frac{\mathrm{Subst}\left(\int \sin\left(4x^2\right)dx, x, \sqrt{\tan^{-1}(ax)}\right)}{a^3c^3}$$

$$= -\frac{2x^2}{ac^3\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}} + \frac{\sqrt{\frac{\pi}{2}}S\left(2\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{a^3c^3}$$

Mathematica [C] time = 0.391103, size = 112, normalized size = 1.67

$$\frac{-\left(a^2x^2+1\right)^2\sqrt{-i\tan^{-1}(ax)}\,\mathrm{Gamma}\left(\frac{1}{2},-4i\tan^{-1}(ax)\right) - \left(a^2x^2+1\right)^2\sqrt{i\tan^{-1}(ax)}\,\mathrm{Gamma}\left(\frac{1}{2},4i\tan^{-1}(ax)\right) - 8a^2x^2}{4a^3c^3\left(a^2x^2+1\right)^2\sqrt{\tan^{-1}(ax)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/((c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2)),x]
```

```
[Out] (-8*a^2*x^2 - (1 + a^2*x^2)^2*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-4*I)*ArcT
an[a*x]] - (1 + a^2*x^2)^2*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (4*I)*ArcTan[a*x]
])/(4*a^3*c^3*(1 + a^2*x^2)^2*Sqrt[ArcTan[a*x]])
```

Maple [A] time = 0.115, size = 53, normalized size = 0.8

$$\frac{1}{4\,c^3a^3}\left(2\,\sqrt{2}\sqrt{\arctan\left(ax\right)}\sqrt{\pi}\mathrm{FresnelS}\left(2\,\frac{\sqrt{2}\sqrt{\arctan\left(ax\right)}}{\sqrt{\pi}}\right)+\cos\left(4\,\arctan\left(ax\right)\right)-1\right)\frac{1}{\sqrt{\arctan\left(ax\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x)
```

```
[Out] 1/4/a^3/c^3*(2*2^(1/2)*arctan(a*x)^(1/2)*Pi^(1/2)*FresnelS(2*2^(1/2)/Pi^(1/
2)*arctan(a*x)^(1/2))+cos(4*arctan(a*x))-1)/arctan(a*x)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^2}{\frac{a^6x^6\operatorname{atan}^{\frac{3}{2}}(ax)+3a^4x^4\operatorname{atan}^{\frac{3}{2}}(ax)+3a^2x^2\operatorname{atan}^{\frac{3}{2}}(ax)+\operatorname{atan}^{\frac{3}{2}}(ax)}{c^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a**2*c*x**2+c)**3/atan(a*x)**(3/2),x)
```

```
[Out] Integral(x**2/(a**6*x**6*atan(a*x)**(3/2) + 3*a**4*x**4*atan(a*x)**(3/2) +
3*a**2*x**2*atan(a*x)**(3/2) + atan(a*x)**(3/2)), x)/c**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(a^2cx^2+c\right)^3\arctan\left(ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/((a^2*c*x^2 + c)^3*arctan(a*x)^(3/2)), x)
```

3.1001 $\int \frac{x}{(c+a^2cx^2)^3 \tan^{-1}(ax)^{3/2}} \, dx$

Optimal. Leaf size=93

$$\frac{\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{a^2c^3}+\frac{\sqrt{\pi}\text{FresnelC}\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{a^2c^3}-\frac{2x}{ac^3\left(a^2x^2+1\right)^2\sqrt{\tan^{-1}(ax)}}$$

```
[Out] (-2*x)/(a*c^3*(1 + a^2*x^2)^2*Sqrt[ArcTan[a*x]]) + (Sqrt[Pi/2]*FresnelC[2*S
qrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(a^2*c^3) + (Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan
[a*x]])/Sqrt[Pi]])/(a^2*c^3)
```

Rubi [A] time = 0.271273, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {4968, 4970, 4406, 3304, 3352, 4904, 3312}

$$\frac{\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{a^2c^3}+\frac{\sqrt{\pi}\text{FresnelC}\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{a^2c^3}-\frac{2x}{ac^3\left(a^2x^2+1\right)^2\sqrt{\tan^{-1}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2)),x]
```



```
[Out] (-2*x)/(a*c^3*(1 + a^2*x^2)^2*Sqrt[ArcTan[a*x]]) + (Sqrt[Pi/2]*FresnelC[2*S
qrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(a^2*c^3) + (Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan
[a*x]])/Sqrt[Pi]])/(a^2*c^3)
```


Rule 4968

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[(x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^(p
+ 1))/(b*c*d*(p + 1)), x] + (-Dist[(c*(m + 2*q + 2))/(b*(p + 1)), Int[x^(m
+ 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p + 1
)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; Fre
eQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &&
LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 4970

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sin[x]^m)/
Cos[x]^(m + 2*(q + 1)), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4904

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\int \frac{x}{\left(c+a^2cx^2\right)^3 \tan^{-1}(ax)^{3/2}}\,dx = -\frac{2x}{ac^3\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}} + \frac{2\int \frac{1}{\left(c+a^2cx^2\right)^3\sqrt{\tan^{-1}(ax)}}\,dx}{a} - (6a)\int \frac{x^2}{\left(c+a^2cx^2\right)^3\sqrt{\tan}}$$

$$= -\frac{2x}{ac^3\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}} + \frac{2\,\text{Subst}\left(\int \frac{\cos^4(x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{a^2c^3} - \frac{6\,\text{Subst}\left(\int \frac{\cos^2}{}\right.}{}$$

$$= -\frac{2x}{ac^3\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}} + \frac{2\,\text{Subst}\left(\int \left(\frac{3}{8\sqrt{x}} + \frac{\cos(2x)}{2\sqrt{x}} + \frac{\cos(4x)}{8\sqrt{x}}\right)dx, x, \tan^{-1}(ax)\right)}{a^2c^3}$$

$$= -\frac{2x}{ac^3\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}} + \frac{\text{Subst}\left(\int \frac{\cos(4x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{4a^2c^3} + \frac{3\,\text{Subst}\left(\int \frac{\cos(4x}{\sqrt{x}}\right.}{4a}$$

$$= -\frac{2x}{ac^3\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}} + \frac{\text{Subst}\left(\int \cos\left(4x^2\right)dx, x, \sqrt{\tan^{-1}(ax)}\right)}{2a^2c^3} + \frac{3\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{2x}{ac^3\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}} + \frac{\sqrt{\frac{\pi}{2}}C\left(2\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{a^2c^3} + \frac{\sqrt{\pi}C\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{a^2c^3}$$

Mathematica [C] time = 0.235884, size = 156, normalized size = 1.68

$$\frac{-i\sqrt{2}\sqrt{-i\tan^{-1}(ax)}\text{Gamma}\left(\frac{1}{2}, -2i\tan^{-1}(ax)\right) + i\sqrt{2}\sqrt{i\tan^{-1}(ax)}\text{Gamma}\left(\frac{1}{2}, 2i\tan^{-1}(ax)\right) - i\sqrt{-i\tan^{-1}(ax)}\text{Gamma}}{4a^2c^3\sqrt{\tan^{-1}(ax)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2)),x]
```

```
[Out] ((-8*a*x)/(1 + a^2*x^2)^2 - I*Sqrt[2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-2
*I)*ArcTan[a*x]] + I*Sqrt[2]*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (2*I)*ArcTan[a*
x]] - I*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-4*I)*ArcTan[a*x]] + I*Sqrt[I*Ar
cTan[a*x]]*Gamma[1/2, (4*I)*ArcTan[a*x]])/(4*a^2*c^3*Sqrt[ArcTan[a*x]])
```

Maple [A] time = 0.113, size = 84, normalized size = 0.9

$$-\frac{1}{4c^3a^2}\left(-2\sqrt{2}\sqrt{\arctan(ax)}\sqrt{\pi}\,\mathrm{FresnelC}\left(2\frac{\sqrt{2}\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)-4\sqrt{\arctan(ax)}\sqrt{\pi}\,\mathrm{FresnelC}\left(2\frac{\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x)
```

```
[Out] -1/4/a^2/c^3/arctan(a*x)^(1/2)*(-2*2^(1/2)*arctan(a*x)^(1/2)*Pi^(1/2)*Fresn
elC(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))-4*arctan(a*x)^(1/2)*Pi^(1/2)*Fres
nelC(2*arctan(a*x)^(1/2)/Pi^(1/2))+2*sin(2*arctan(a*x))+sin(4*arctan(a*x)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x}{a^6x^6\operatorname{atan}^{\frac{3}{2}}(ax)+3a^4x^4\operatorname{atan}^{\frac{3}{2}}(ax)+3a^2x^2\operatorname{atan}^{\frac{3}{2}}(ax)+\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a**2*c*x**2+c)**3/atan(a*x)**(3/2),x)
```

```
[Out] Integral(x/(a**6*x**6*atan(a*x)**(3/2) + 3*a**4*x**4*atan(a*x)**(3/2) + 3*a
**2*x**2*atan(a*x)**(3/2) + atan(a*x)**(3/2)), x)/c**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(a^2cx^2+c\right)^3 \arctan(ax)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x/((a^2*c*x^2 + c)^3*arctan(a*x)^(3/2)), x)
```

3.1002 $\int \frac{1}{\left(c+a^2cx^2\right)^3 \tan^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=94

$$-\frac{2}{ac^3\left(a^2x^2+1\right)^2\sqrt{\tan^{-1}(ax)}}-\frac{\sqrt{\frac{\pi}{2}}S\left(2\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{ac^3}-\frac{2\sqrt{\pi}S\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{ac^3}$$

```
[Out] -2/(a*c^3*(1 + a^2*x^2)^2*Sqrt[ArcTan[a*x]]) - (Sqrt[Pi/2]*FresnelS[2*Sqrt[
2/Pi]*Sqrt[ArcTan[a*x]]])/(a*c^3) - (2*Sqrt[Pi]*FresnelS[(2*Sqrt[ArcTan[a*x
]])/Sqrt[Pi]])/(a*c^3)
```

Rubi [A] time = 0.137698, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {4902, 4970, 4406, 3305, 3351}

$$-\frac{2}{ac^3\left(a^2x^2+1\right)^2\sqrt{\tan^{-1}(ax)}}-\frac{\sqrt{\frac{\pi}{2}}S\left(2\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{ac^3}-\frac{2\sqrt{\pi}S\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{ac^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2)),x]
```


```
[Out] -2/(a*c^3*(1 + a^2*x^2)^2*Sqrt[ArcTan[a*x]]) - (Sqrt[Pi/2]*FresnelS[2*Sqrt[
2/Pi]*Sqrt[ArcTan[a*x]]])/(a*c^3) - (2*Sqrt[Pi]*FresnelS[(2*Sqrt[ArcTan[a*x
]])/Sqrt[Pi]])/(a*c^3)
```


Rule 4902

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^(p + 1))/(b*c*d*(p
+ 1)), x] - Dist[(2*c*(q + 1))/(b*(p + 1)), Int[x*(d + e*x^2)^q*(a + b*ArcT
an[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && L
tQ[q, -1] && LtQ[p, -1]
```

Rule 4970

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sin[x]^m)/
Cos[x]^(m + 2*(q + 1)), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{1}{\left(c+a^2cx^2\right)^3 \tan^{-1}(ax)^{3/2}}\,dx = -\frac{2}{ac^3\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}} - (8a)\int \frac{x}{\left(c+a^2cx^2\right)^3\sqrt{\tan^{-1}(ax)}}\,dx$$

$$= -\frac{2}{ac^3\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}} - \frac{8\,\text{Subst}\left(\int \frac{\cos^3(x)\sin(x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{ac^3}$$

$$= -\frac{2}{ac^3\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}} - \frac{8\,\text{Subst}\left(\int \left(\frac{\sin(2x)}{4\sqrt{x}}+\frac{\sin(4x)}{8\sqrt{x}}\right)dx, x, \tan^{-1}(ax)\right)}{ac^3}$$

$$= -\frac{2}{ac^3\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}} - \frac{\text{Subst}\left(\int \frac{\sin(4x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{ac^3} - \frac{2\,\text{Subst}\left(\int \frac{\sin(2}{\sqrt{x}}\right.}{}$$

$$= -\frac{2}{ac^3\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}} - \frac{2\,\text{Subst}\left(\int \sin\left(4x^2\right)dx, x, \sqrt{\tan^{-1}(ax)}\right)}{ac^3} - \frac{4\,\text{Subst}}{}$$

$$= -\frac{2}{ac^3\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}} - \frac{\sqrt{\frac{\pi}{2}}S\left(2\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{ac^3} - \frac{2\sqrt{\pi}S\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{ac^3}$$

Mathematica [C] time = 0.298549, size = 144, normalized size = 1.53

$$\frac{2\sqrt{2}\sqrt{-i\tan^{-1}(ax)}\text{Gamma}\left(\frac{1}{2},-2i\tan^{-1}(ax)\right)+2\sqrt{2}\sqrt{i\tan^{-1}(ax)}\text{Gamma}\left(\frac{1}{2},2i\tan^{-1}(ax)\right)+\sqrt{-i\tan^{-1}(ax)}\text{Gamm}}{4ac^3\sqrt{\tan^{-1}(ax)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2)),x]
```

```
[Out] (-8/(1 + a^2*x^2)^2 + 2*Sqrt[2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-2*I)*Ar
cTan[a*x]] + 2*Sqrt[2]*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (2*I)*ArcTan[a*x]] +
Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-4*I)*ArcTan[a*x]] + Sqrt[I*ArcTan[a*x]]
*Gamma[1/2, (4*I)*ArcTan[a*x]])/(4*a*c^3*Sqrt[ArcTan[a*x]])
```

Maple [A] time = 0.112, size = 85, normalized size = 0.9

$$-\frac{1}{4\,ac^{3}}\left(2\,\sqrt{2}\sqrt{\arctan\left(ax\right)}\sqrt{\pi}\mathrm{FresnelS}\left(2\,\frac{\sqrt{2}\sqrt{\arctan\left(ax\right)}}{\sqrt{\pi}}\right)+8\,\sqrt{\arctan\left(ax\right)}\sqrt{\pi}\mathrm{FresnelS}\left(2\,\frac{\sqrt{\arctan\left(ax\right)}}{\sqrt{\pi}}\right)+4\,\cos\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x)
```

```
[Out] -1/4/a/c^3*(2*2^(1/2)*arctan(a*x)^(1/2)*Pi^(1/2)*FresnelS(2*2^(1/2)/Pi^(1/2
)*arctan(a*x)^(1/2))+8*arctan(a*x)^(1/2)*Pi^(1/2)*FresnelS(2*arctan(a*x)^(1
/2)/Pi^(1/2))+4*cos(2*arctan(a*x))+cos(4*arctan(a*x))+3)/arctan(a*x)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{a^{6}x^{6}\operatorname{atan}^{\frac{3}{2}}{\left(ax \right)}+3a^{4}x^{4}\operatorname{atan}^{\frac{3}{2}}{\left(ax \right)}+3a^{2}x^{2}\operatorname{atan}^{\frac{3}{2}}{\left(ax \right)}+\operatorname{atan}^{\frac{3}{2}}{\left(ax \right)}}\,dx}{c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*c*x**2+c)**3/atan(a*x)**(3/2),x)
```

```
[Out] Integral(1/(a**6*x**6*atan(a*x)**(3/2) + 3*a**4*x**4*atan(a*x)**(3/2) + 3*a
**2*x**2*atan(a*x)**(3/2) + atan(a*x)**(3/2)), x)/c**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)^3\arctan\left(ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^3*arctan(a*x)^(3/2)), x)
```

3.1003 $\int \frac{1}{x(c+a^2cx^2)^3\tan^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=139

$$-\frac{2\text{Unintegrable}\left(\frac{1}{x^2(a^2cx^2+c)^3\sqrt{\tan^{-1}(ax)}},x\right)}{a}-\frac{2}{ac^3x\left(a^2x^2+1\right)^2\sqrt{\tan^{-1}(ax)}}-\frac{5\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{4c^3}-\frac{5\sqrt{\pi}\text{F}}{}$$

```
[Out] -2/(a*c^3*x*(1 + a^2*x^2)^2*Sqrt[ArcTan[a*x]]) - (15*Sqrt[ArcTan[a*x]])/(2*
c^3) - (5*Sqrt[Pi/2]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(4*c^3) - (5
*Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/c^3 - (2*Unintegrable[1
/(x^2*(c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]), x])/a
```

Rubi [A] time = 0.232738, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*(c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2)),x]
```

```
[Out] -2/(a*c^3*x*(1 + a^2*x^2)^2*Sqrt[ArcTan[a*x]]) - (15*Sqrt[ArcTan[a*x]])/(2*
c^3) - (5*Sqrt[Pi/2]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(4*c^3) - (5
*Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/c^3 - (2*Defer[Int][1/(
x^2*(c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]), x])/a
```

Rubi steps

$$\int \frac{1}{x\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^{3/2}}\,dx = -\frac{2}{ac^3x\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}} - \frac{2\int \frac{1}{x^2\left(c+a^2cx^2\right)^3\sqrt{\tan^{-1}(ax)}}\,dx}{a} - (10a)\int \frac{}{\left(c+a^2cx^2\right)}$$

$$= -\frac{2}{ac^3x\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}} - \frac{2\int \frac{1}{x^2\left(c+a^2cx^2\right)^3\sqrt{\tan^{-1}(ax)}}\,dx}{a} - \frac{10\,\mathrm{Subst}\left(\int \frac{\cos^4(x)}{\sqrt{x}}\right.}{c^3}$$

$$= -\frac{2}{ac^3x\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}} - \frac{2\int \frac{1}{x^2\left(c+a^2cx^2\right)^3\sqrt{\tan^{-1}(ax)}}\,dx}{a} - \frac{10\,\mathrm{Subst}\left(\int \left(\frac{3}{8\sqrt{x}}+\right.\right.}{}$$

$$= -\frac{2}{ac^3x\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}} - \frac{15\sqrt{\tan^{-1}(ax)}}{2c^3} - \frac{2\int \frac{1}{x^2\left(c+a^2cx^2\right)^3\sqrt{\tan^{-1}(ax)}}\,dx}{a} - \frac{5}{}$$

$$= -\frac{2}{ac^3x\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}} - \frac{15\sqrt{\tan^{-1}(ax)}}{2c^3} - \frac{2\int \frac{1}{x^2\left(c+a^2cx^2\right)^3\sqrt{\tan^{-1}(ax)}}\,dx}{a} - \frac{5}{}$$

$$= -\frac{2}{ac^3x\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}} - \frac{15\sqrt{\tan^{-1}(ax)}}{2c^3} - \frac{5\sqrt{\frac{\pi}{2}}C\left(2\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{4c^3} - \frac{5}{}$$

Mathematica [A] time = 5.08192, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*(c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2)),x]
```

```
[Out] Integrate[1/(x*(c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2)), x]
```

Maple [A] time = 0.777, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(a^2cx^2+c\right)^3}\left(\arctan(ax)\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x)
```

```
[Out] int(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{a^6x^7\operatorname{atan}^{\frac{3}{2}}(ax)+3a^4x^5\operatorname{atan}^{\frac{3}{2}}(ax)+3a^2x^3\operatorname{atan}^{\frac{3}{2}}(ax)+x\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a**2*c*x**2+c)**3/atan(a*x)**(3/2),x)
```

```
[Out] Integral(1/(a**6*x**7*atan(a*x)**(3/2) + 3*a**4*x**5*atan(a*x)**(3/2) + 3*a
**2*x**3*atan(a*x)**(3/2) + x*atan(a*x)**(3/2)), x)/c**3
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)^3 x \arctan(ax)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^3*x*arctan(a*x)^(3/2)), x)
```

3.1004 $\int \frac{1}{x^2(c+a^2cx^2)^3\tan^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=91

$$-\frac{4\text{Unintegrable}\left(\frac{1}{x^3(a^2cx^2+c)^3\sqrt{\tan^{-1}(ax)}},x\right)}{a}-12a\text{Unintegrable}\left(\frac{1}{x\left(a^2cx^2+c\right)^3\sqrt{\tan^{-1}(ax)}},x\right)-\frac{2}{ac^3x^2\left(a^2x^2+1\right)^2\sqrt{\tan\ldots}}$$

```
[Out] -2/(a*c^3*x^2*(1 + a^2*x^2)^2*Sqrt[ArcTan[a*x]]) - (4*Unintegrable[1/(x^3*(
c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]), x])/a - 12*a*Unintegrable[1/(x*(c + a^
2*c*x^2)^3*Sqrt[ArcTan[a*x]]), x]
```

Rubi [A] time = 0.195114, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2)),x]
```

```
[Out] -2/(a*c^3*x^2*(1 + a^2*x^2)^2*Sqrt[ArcTan[a*x]]) - (4*Defer[Int][1/(x^3*(c
+ a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]), x])/a - 12*a*Defer[Int][1/(x*(c + a^2*c*
x^2)^3*Sqrt[ArcTan[a*x]]), x]
```

Rubi steps

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^{3/2}}\,dx=-\frac{2}{ac^3x^2\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}}-\frac{4\int \frac{1}{x^3(c+a^2cx^2)^3\sqrt{\tan^{-1}(ax)}}\,dx}{a}-(12a)\int \frac{\ldots}{x\left(c+a^2cx\ldots\right.}$$

Mathematica [A] time = 6.08334, size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2)),x]
```

```
[Out] Integrate[1/(x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2)), x]
```

Maple [A] time = 0.562, size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \left(a^2 c x^2 + c\right)^3} \left(\arctan \left(ax\right)\right)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x)
```

```
[Out] int(1/x^2/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a**2*c*x**2+c)**3/atan(a*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)^3x^2\arctan\left(ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^3*x^2*arctan(a*x)^(3/2)), x)
```

3.1005 $\int \frac{1}{x^3(c+a^2cx^2)^3 \tan^{-1}(ax)^{3/2}} \, dx$

Optimal. Leaf size=91

$$-\frac{6\text{Unintegrable}\left(\frac{1}{x^4(a^2cx^2+c)^3\sqrt{\tan^{-1}(ax)}},x\right)}{a}-14a\text{Unintegrable}\left(\frac{1}{x^2\left(a^2cx^2+c\right)^3\sqrt{\tan^{-1}(ax)}},x\right)-\frac{2}{ac^3x^3\left(a^2x^2+1\right)^2\sqrt{\tan^{-1}(ax)}}$$

```
[Out] -2/(a*c^3*x^3*(1 + a^2*x^2)^2*Sqrt[ArcTan[a*x]]) - (6*Unintegrable[1/(x^4*(
c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]), x])/a - 14*a*Unintegrable[1/(x^2*(c +
a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]), x]
```

Rubi [A] time = 0.21723, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{1}{x^3\left(c+a^2cx^2\right)^3 \tan^{-1}(ax)^{3/2}} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^3*(c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2)),x]
```

```
[Out] -2/(a*c^3*x^3*(1 + a^2*x^2)^2*Sqrt[ArcTan[a*x]]) - (6*Defer[Int][1/(x^4*(c
+ a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]), x])/a - 14*a*Defer[Int][1/(x^2*(c + a^2*
c*x^2)^3*Sqrt[ArcTan[a*x]]), x]
```

Rubi steps

$$\int \frac{1}{x^3\left(c+a^2cx^2\right)^3 \tan^{-1}(ax)^{3/2}} \, dx = -\frac{2}{ac^3x^3\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}}-\frac{6\int \frac{1}{x^4(c+a^2cx^2)^3\sqrt{\tan^{-1}(ax)}} \, dx}{a}-(14a)\int \frac{1}{x^2\left(c+a^2\right.}$$

Mathematica [A] time = 7.28469, size = 0, normalized size = 0.

$$\int \frac{1}{x^3\left(c+a^2cx^2\right)^3 \tan^{-1}(ax)^{3/2}} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^3*(c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2)),x]
```

```
[Out] Integrate[1/(x^3*(c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2)), x]
```

Maple [A] time = 1.398, size = 0, normalized size = 0.

$$\int \frac{1}{x^{3} \left(a^{2} c \,x^{2}+c \right)^{3}} \left(\arctan \left(a x \right)\right)^{-\frac{3}{2}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x)
```

```
[Out] int(1/x^3/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a**2*c*x**2+c)**3/atan(a*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)^3 x^3 \arctan\left(ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^3*x^3*arctan(a*x)^(3/2)), x)
```

3.1006 $\int \frac{1}{x^4(c+a^2cx^2)^3\tan^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=91

$$-\frac{8\text{Unintegrable}\left(\frac{1}{x^5(a^2cx^2+c)^3\sqrt{\tan^{-1}(ax)}},x\right)}{a}-16a\text{Unintegrable}\left(\frac{1}{x^3\left(a^2cx^2+c\right)^3\sqrt{\tan^{-1}(ax)}},x\right)-\frac{2}{ac^3x^4\left(a^2x^2+1\right)^2\sqrt{\tan^{-1}(ax)}}$$

```
[Out] -2/(a*c^3*x^4*(1 + a^2*x^2)^2*Sqrt[ArcTan[a*x]]) - (8*Unintegrable[1/(x^5*(
c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]), x])/a - 16*a*Unintegrable[1/(x^3*(c +
a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]), x]
```

Rubi [A] time = 0.203617, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^4\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^4*(c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2)),x]
```

```
[Out] -2/(a*c^3*x^4*(1 + a^2*x^2)^2*Sqrt[ArcTan[a*x]]) - (8*Defer[Int][1/(x^5*(c
+ a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]), x])/a - 16*a*Defer[Int][1/(x^3*(c + a^2*
c*x^2)^3*Sqrt[ArcTan[a*x]]), x]
```

Rubi steps

$$\int \frac{1}{x^4\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^{3/2}}\,dx=-\frac{2}{ac^3x^4\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}}-\frac{8\int \frac{1}{x^5(c+a^2cx^2)^3\sqrt{\tan^{-1}(ax)}}\,dx}{a}-(16a)\int \frac{1}{x^3\left(c+a^2c\right.}$$

Mathematica [A] time = 7.03226, size = 0, normalized size = 0.

$$\int \frac{1}{x^4\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^4*(c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2)),x]
```

```
[Out] Integrate[1/(x^4*(c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2)), x]
```

Maple [A] time = 0.754, size = 0, normalized size = 0.

$$\int \frac{1}{x^{4}\left(a^{2}c\,x^{2}+c\right)^{3}}\left(\arctan\left(ax\right)\right)^{-\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x)
```

```
[Out] int(1/x^4/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(a**2*c*x**2+c)**3/atan(a*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)^3 x^4 \arctan\left(ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^3*x^4*arctan(a*x)^(3/2)), x)
```

3.1007 $\int \frac{x^m\sqrt{c+a^2cx^2}}{\tan^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(\frac{x^m\sqrt{a^2cx^2+c}}{\tan^{-1}(ax)^{3/2}},x\right)$$

```
[Out] Unintegrable[(x^m*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^(3/2), x]
```

Rubi [A] time = 0.0983858, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^(3/2),x]
```

```
[Out] Defer[Int][(x^m*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^(3/2), x]
```

Rubi steps

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\tan^{-1}(ax)^{3/2}}\,dx=\int \frac{x^m\sqrt{c+a^2cx^2}}{\tan^{-1}(ax)^{3/2}}\,dx$$

Mathematica [A] time = 0.688899, size = 0, normalized size = 0.

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^(3/2),x]
```

```
[Out] Integrate[(x^m*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^(3/2), x]
```

Maple [A] time = 1.216, size = 0, normalized size = 0.

$$\int x^m \sqrt{a^2cx^2 + c}\, (\arctan (ax))^{-\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2),x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{a^2cx^2 + c}x^m}{\arctan (ax)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^m/arctan(a*x)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**(1/2)/atan(a*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a^2cx^2+c}\,x^m}{\arctan(ax)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)*x^m/arctan(a*x)^(3/2), x)
```

3.1008 $\int \frac{x\sqrt{c+a^2cx^2}}{\tan^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{x\sqrt{a^2cx^2+c}}{\tan^{-1}(ax)^{3/2}},x\right)$$

```
[Out] Unintegrable[(x*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^(3/2), x]
```

Rubi [A] time = 0.0696731, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x\sqrt{c+a^2cx^2}}{\tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^(3/2),x]
```

```
[Out] Defer[Int][(x*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^(3/2), x]
```

Rubi steps

$$\int \frac{x\sqrt{c+a^2cx^2}}{\tan^{-1}(ax)^{3/2}}\,dx=\int \frac{x\sqrt{c+a^2cx^2}}{\tan^{-1}(ax)^{3/2}}\,dx$$

Mathematica [A] time = 1.81469, size = 0, normalized size = 0.

$$\int \frac{x\sqrt{c+a^2cx^2}}{\tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^(3/2),x]
```

```
[Out] Integrate[(x*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^(3/2), x]
```

Maple [A] time = 1.009, size = 0, normalized size = 0.

$$\int x\sqrt{a^2cx^2+c}\,(\arctan(ax))^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2),x)
```

```
[Out] int(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sqrt{c\left(a^{2}x^{2}+1\right)}}{\operatorname{atan}^{\frac{3}{2}}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**(1/2)/atan(a*x)**(3/2),x)
```

```
[Out] Integral(x*sqrt(c*(a**2*x**2 + 1))/atan(a*x)**(3/2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a^{2}cx^{2}+c}x}{\arctan\left(ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)*x/arctan(a*x)^(3/2), x)
```

3.1009 $\int \frac{\sqrt{c+a^2cx^2}}{\tan^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=25

$$\text{Unintegrable}\left(\frac{\sqrt{a^2cx^2+c}}{\tan^{-1}(ax)^{3/2}},x\right)$$

```
[Out] Unintegrable[Sqrt[c + a^2*c*x^2]/ArcTan[a*x]^(3/2), x]
```

Rubi [A] time = 0.0362968, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sqrt{c+a^2cx^2}}{\tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[c + a^2*c*x^2]/ArcTan[a*x]^(3/2),x]
```

```
[Out] Defer[Int][Sqrt[c + a^2*c*x^2]/ArcTan[a*x]^(3/2), x]
```

Rubi steps

$$\int \frac{\sqrt{c+a^2cx^2}}{\tan^{-1}(ax)^{3/2}}\,dx=\int \frac{\sqrt{c+a^2cx^2}}{\tan^{-1}(ax)^{3/2}}\,dx$$

Mathematica [A] time = 0.424586, size = 0, normalized size = 0.

$$\int \frac{\sqrt{c+a^2cx^2}}{\tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[c + a^2*c*x^2]/ArcTan[a*x]^(3/2),x]
```

```
[Out] Integrate[Sqrt[c + a^2*c*x^2]/ArcTan[a*x]^(3/2), x]
```

Maple [A] time = 0.902, size = 0, normalized size = 0.

$$\int \sqrt{a^2cx^2 + c}\,(\arctan(ax))^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2),x)
```

```
[Out] int((a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c\left(a^2x^2+1\right)}}{\operatorname{atan}^{\frac{3}{2}}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(1/2)/atan(a*x)**(3/2),x)
```

```
[Out] Integral(sqrt(c*(a**2*x**2 + 1))/atan(a*x)**(3/2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a^2cx^2+c}}{\arctan\left(ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)/arctan(a*x)^(3/2), x)
```

3.1010 $\int \frac{\sqrt{c+a^2cx^2}}{x\tan^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(\frac{\sqrt{a^2cx^2+c}}{x\tan^{-1}(ax)^{3/2}},x\right)$$

```
[Out] Unintegrable[Sqrt[c + a^2*c*x^2]/(x*ArcTan[a*x]^(3/2)), x]
```

Rubi [A] time = 0.10035, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sqrt{c+a^2cx^2}}{x\tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[c + a^2*c*x^2]/(x*ArcTan[a*x]^(3/2)),x]
```

```
[Out] Defer[Int][Sqrt[c + a^2*c*x^2]/(x*ArcTan[a*x]^(3/2)), x]
```

Rubi steps

$$\int \frac{\sqrt{c+a^2cx^2}}{x\tan^{-1}(ax)^{3/2}}\,dx=\int \frac{\sqrt{c+a^2cx^2}}{x\tan^{-1}(ax)^{3/2}}\,dx$$

Mathematica [A] time = 4.70213, size = 0, normalized size = 0.

$$\int \frac{\sqrt{c+a^2cx^2}}{x\tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[c + a^2*c*x^2]/(x*ArcTan[a*x]^(3/2)),x]
```

```
[Out] Integrate[Sqrt[c + a^2*c*x^2]/(x*ArcTan[a*x]^(3/2)), x]
```

Maple [A] time = 0.931, size = 0, normalized size = 0.

$$\int \frac{1}{x}\sqrt{a^2cx^2+c}\,(\arctan(ax))^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^(3/2),x)
```

```
[Out] int((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c\left(a^2x^2+1\right)}}{x\,\mathrm{atan}^{\frac{3}{2}}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(1/2)/x/atan(a*x)**(3/2),x)
```

```
[Out] Integral(sqrt(c*(a**2*x**2 + 1))/(x*atan(a*x)**(3/2)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a^2cx^2+c}}{x\arctan\left(ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)/(x*arctan(a*x)^(3/2)), x)
```

3.1011 $\int \frac{x^m\left(c+a^2cx^2\right)^{3/2}}{\tan^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(\frac{x^m\left(a^2cx^2+c\right)^{3/2}}{\tan^{-1}(ax)^{3/2}},x\right)$$

```
[Out] Unintegrable[(x^m*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^(3/2), x]
```

Rubi [A] time = 0.112226, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m\left(c+a^2cx^2\right)^{3/2}}{\tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^(3/2),x]
```

```
[Out] Defer[Int][(x^m*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^(3/2), x]
```

Rubi steps

$$\int \frac{x^m\left(c+a^2cx^2\right)^{3/2}}{\tan^{-1}(ax)^{3/2}}\,dx=\int \frac{x^m\left(c+a^2cx^2\right)^{3/2}}{\tan^{-1}(ax)^{3/2}}\,dx$$

Mathematica [A] time = 1.03778, size = 0, normalized size = 0.

$$\int \frac{x^m\left(c+a^2cx^2\right)^{3/2}}{\tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^(3/2),x]
```

```
[Out] Integrate[(x^m*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^(3/2), x]
```

Maple [A] time = 1.01, size = 0, normalized size = 0.

$$\int x^m \left(a^2 c x^2+c\right)^{\frac{3}{2}} (\arctan (a x))^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^2 c x^2+c\right)^{\frac{3}{2}} x^m}{\arctan (a x)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)^(3/2)*x^m/arctan(a*x)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**(3/2)/atan(a*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^{\frac{3}{2}}x^m}{\arctan\left(ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)*x^m/arctan(a*x)^(3/2), x)
```

3.1012 $\int \frac{x(c+a^2cx^2)^{3/2}}{\tan^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{x\left(a^2cx^2+c\right)^{3/2}}{\tan^{-1}(ax)^{3/2}},x\right)$$

```
[Out] Unintegrable[(x*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^(3/2), x]
```

Rubi [A] time = 0.0773068, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x\left(c+a^2cx^2\right)^{3/2}}{\tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^(3/2),x]
```

```
[Out] Defer[Int][(x*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^(3/2), x]
```

Rubi steps

$$\int \frac{x\left(c+a^2cx^2\right)^{3/2}}{\tan^{-1}(ax)^{3/2}}\,dx=\int \frac{x\left(c+a^2cx^2\right)^{3/2}}{\tan^{-1}(ax)^{3/2}}\,dx$$

Mathematica [A] time = 6.78127, size = 0, normalized size = 0.

$$\int \frac{x\left(c+a^2cx^2\right)^{3/2}}{\tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^(3/2),x]
```

```
[Out] Integrate[(x*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^(3/2), x]
```

Maple [A] time = 0.806, size = 0, normalized size = 0.

$$\int x\left(a^{2}c x^{2}+c\right)^{\frac{3}{2}} \left(\arctan\left(a x\right)\right)^{-\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

```
[Out] int(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**(3/2)/atan(a*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^{\frac{3}{2}}x}{\arctan\left(ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)*x/arctan(a*x)^(3/2), x)
```

3.1013 $\int \frac{\left(c+a^2cx^2\right)^{3/2}}{\tan^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=25

$$\text{Unintegrable}\left(\frac{\left(a^2cx^2+c\right)^{3/2}}{\tan^{-1}(ax)^{3/2}},x\right)$$

```
[Out] Unintegrable[(c + a^2*c*x^2)^(3/2)/ArcTan[a*x]^(3/2), x]
```

Rubi [A] time = 0.035829, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(c+a^2cx^2\right)^{3/2}}{\tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^(3/2)/ArcTan[a*x]^(3/2),x]
```

```
[Out] Defer[Int][(c + a^2*c*x^2)^(3/2)/ArcTan[a*x]^(3/2), x]
```

Rubi steps

$$\int \frac{\left(c+a^2cx^2\right)^{3/2}}{\tan^{-1}(ax)^{3/2}}\,dx=\int \frac{\left(c+a^2cx^2\right)^{3/2}}{\tan^{-1}(ax)^{3/2}}\,dx$$

Mathematica [A] time = 1.34847, size = 0, normalized size = 0.

$$\int \frac{\left(c+a^2cx^2\right)^{3/2}}{\tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^(3/2)/ArcTan[a*x]^(3/2),x]
```

```
[Out] Integrate[(c + a^2*c*x^2)^(3/2)/ArcTan[a*x]^(3/2), x]
```

Maple [A] time = 0.679, size = 0, normalized size = 0.

$$\int \left(a^2cx^2+c\right)^{\frac{3}{2}} \left(\arctan \left(ax\right)\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

```
[Out] int((a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(3/2)/atan(a*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^{\frac{3}{2}}}{\arctan\left(ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)/arctan(a*x)^(3/2), x)
```

3.1014 $\int \frac{\left(c+a^2cx^2\right)^{3/2}}{x\tan^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(\frac{\left(a^2cx^2+c\right)^{3/2}}{x\tan^{-1}(ax)^{3/2}},x\right)$$

```
[Out] Unintegrable[(c + a^2*c*x^2)^(3/2)/(x*ArcTan[a*x]^(3/2)), x]
```

Rubi [A] time = 0.111514, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(c+a^2cx^2\right)^{3/2}}{x\tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^(3/2)/(x*ArcTan[a*x]^(3/2)),x]
```

```
[Out] Defer[Int][(c + a^2*c*x^2)^(3/2)/(x*ArcTan[a*x]^(3/2)), x]
```

Rubi steps

$$\int \frac{\left(c+a^2cx^2\right)^{3/2}}{x\tan^{-1}(ax)^{3/2}}\,dx=\int \frac{\left(c+a^2cx^2\right)^{3/2}}{x\tan^{-1}(ax)^{3/2}}\,dx$$

Mathematica [A] time = 9.47789, size = 0, normalized size = 0.

$$\int \frac{\left(c+a^2cx^2\right)^{3/2}}{x\tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^(3/2)/(x*ArcTan[a*x]^(3/2)),x]
```

```
[Out] Integrate[(c + a^2*c*x^2)^(3/2)/(x*ArcTan[a*x]^(3/2)), x]
```

Maple [A] time = 0.714, size = 0, normalized size = 0.

$$\int \frac{1}{x} \left(a^{2} c x^{2}+c\right)^{\frac{3}{2}} \left(\arctan \left(a x\right)\right)^{-\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^(3/2),x)
```

```
[Out] int((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(3/2)/x/atan(a*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^{\frac{3}{2}}}{x\arctan\left(ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)/(x*arctan(a*x)^(3/2)), x)
```

3.1015 $\int \frac{x^m\left(c+a^2cx^2\right)^{5/2}}{\tan^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(\frac{x^m\left(a^2cx^2+c\right)^{5/2}}{\tan^{-1}(ax)^{3/2}},x\right)$$

```
[Out] Unintegrable[(x^m*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^(3/2), x]
```

Rubi [A] time = 0.111533, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m\left(c+a^2cx^2\right)^{5/2}}{\tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^(3/2),x]
```

```
[Out] Defer[Int][(x^m*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^(3/2), x]
```

Rubi steps

$$\int \frac{x^m\left(c+a^2cx^2\right)^{5/2}}{\tan^{-1}(ax)^{3/2}}\,dx=\int \frac{x^m\left(c+a^2cx^2\right)^{5/2}}{\tan^{-1}(ax)^{3/2}}\,dx$$

Mathematica [A] time = 1.40023, size = 0, normalized size = 0.

$$\int \frac{x^m\left(c+a^2cx^2\right)^{5/2}}{\tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^(3/2),x]
```

[Out] Integrate[(x^m*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^(3/2), x]

Maple [A] time = 1.028, size = 0, normalized size = 0.

$$\int x^m \left(a^2 c x^2 + c\right)^{\frac{5}{2}} \left(\arctan\left(ax\right)\right)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)

[Out] int(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="maxima")

[Out] Exception raised: RuntimeError

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^4 c^2 x^4 + 2\, a^2 c^2 x^2 + c^2\right) \sqrt{a^2 c x^2 + c} x^m}{\arctan\left(ax\right)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="fricas")

[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*x^m/arctan
(a*x)^(3/2), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**(5/2)/atan(a*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^{\frac{5}{2}}x^m}{\arctan\left(ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(5/2)*x^m/arctan(a*x)^(3/2), x)
```

3.1016 $\int \frac{x\left(c+a^2cx^2\right)^{5/2}}{\tan^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{x\left(a^2cx^2+c\right)^{5/2}}{\tan^{-1}(ax)^{3/2}},x\right)$$

```
[Out] Unintegrable[(x*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^(3/2), x]
```

Rubi [A] time = 0.0785105, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x\left(c+a^2cx^2\right)^{5/2}}{\tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^(3/2),x]
```

```
[Out] Defer[Int][(x*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^(3/2), x]
```

Rubi steps

$$\int \frac{x\left(c+a^2cx^2\right)^{5/2}}{\tan^{-1}(ax)^{3/2}}\,dx=\int \frac{x\left(c+a^2cx^2\right)^{5/2}}{\tan^{-1}(ax)^{3/2}}\,dx$$

Mathematica [A] time = 2.65301, size = 0, normalized size = 0.

$$\int \frac{x\left(c+a^2cx^2\right)^{5/2}}{\tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^(3/2),x]
```

```
[Out] Integrate[(x*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^(3/2), x]
```

Maple [A] time = 1.01, size = 0, normalized size = 0.

$$\int x \left(a^{2} c \,x^{2}+c \right)^{\frac{5}{2}} \left(\arctan \left(a x \right)\right)^{-\frac{3}{2}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
```

```
[Out] int(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**(5/2)/atan(a*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^{\frac{5}{2}}x}{\arctan\left(ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(5/2)*x/arctan(a*x)^(3/2), x)
```

3.1017 $\int \frac{\left(c+a^2 c x^2\right)^{5/2}}{\tan^{-1}(a x)^{3/2}} \, dx$

Optimal. Leaf size=25

$$\text{Unintegrable}\left(\frac{\left(a^2 c x^2+c\right)^{5/2}}{\tan^{-1}(a x)^{3/2}},x\right)$$

```
[Out] Unintegrable[(c + a^2*c*x^2)^(5/2)/ArcTan[a*x]^(3/2), x]
```

Rubi [A] time = 0.0377011, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(c+a^2 c x^2\right)^{5/2}}{\tan^{-1}(a x)^{3/2}} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^(5/2)/ArcTan[a*x]^(3/2),x]
```

```
[Out] Defer[Int][(c + a^2*c*x^2)^(5/2)/ArcTan[a*x]^(3/2), x]
```

Rubi steps

$$\int \frac{\left(c+a^2 c x^2\right)^{5/2}}{\tan^{-1}(a x)^{3/2}} \, dx=\int \frac{\left(c+a^2 c x^2\right)^{5/2}}{\tan^{-1}(a x)^{3/2}} \, dx$$

Mathematica [A] time = 1.12167, size = 0, normalized size = 0.

$$\int \frac{\left(c+a^2 c x^2\right)^{5/2}}{\tan^{-1}(a x)^{3/2}} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^(5/2)/ArcTan[a*x]^(3/2),x]
```

```
[Out] Integrate[(c + a^2*c*x^2)^(5/2)/ArcTan[a*x]^(3/2), x]
```

Maple [A] time = 0.837, size = 0, normalized size = 0.

$$\int \left(a^{2} c \,x^{2}+c \right)^{\frac{5}{2}} \left(\arctan \left(a x \right)\right)^{-\frac{3}{2}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
```

```
[Out] int((a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(5/2)/atan(a*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^{\frac{5}{2}}}{\arctan\left(ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(5/2)/arctan(a*x)^(3/2), x)
```

3.1018 $\int \frac{(c+a^2cx^2)^{5/2}}{x\tan^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(\frac{(a^2cx^2+c)^{5/2}}{x\tan^{-1}(ax)^{3/2}},x\right)$$

```
[Out] Unintegrable[(c + a^2*c*x^2)^(5/2)/(x*ArcTan[a*x]^(3/2)), x]
```

Rubi [A] time = 0.113243, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(c+a^2cx^2)^{5/2}}{x\tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^(5/2)/(x*ArcTan[a*x]^(3/2)),x]
```

```
[Out] Defer[Int][(c + a^2*c*x^2)^(5/2)/(x*ArcTan[a*x]^(3/2)), x]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^{5/2}}{x\tan^{-1}(ax)^{3/2}}\,dx=\int \frac{(c+a^2cx^2)^{5/2}}{x\tan^{-1}(ax)^{3/2}}\,dx$$

Mathematica [A] time = 4.70925, size = 0, normalized size = 0.

$$\int \frac{(c+a^2cx^2)^{5/2}}{x\tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^(5/2)/(x*ArcTan[a*x]^(3/2)),x]
```

```
[Out] Integrate[(c + a^2*c*x^2)^(5/2)/(x*ArcTan[a*x]^(3/2)), x]
```

Maple [A] time = 0.924, size = 0, normalized size = 0.

$$\int \frac{1}{x}\left(a^2cx^2+c\right)^{\frac{5}{2}}\left(\arctan\left(ax\right)\right)^{-\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^(3/2),x)
```

```
[Out] int((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(5/2)/x/atan(a*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a^2cx^2+c)^{\frac{5}{2}}}{x\arctan{(ax)}^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(5/2)/(x*arctan(a*x)^(3/2)), x)
```

3.1019 $\int \frac{x^m}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(\frac{x^m}{\sqrt{a^2cx^2+c}\tan^{-1}(ax)^{3/2}},x\right)$$

```
[Out] Unintegrable[x^m/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2)), x]
```

Rubi [A] time = 0.10761, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2)),x]
```

```
[Out] Defer[Int][x^m/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2)), x]
```

Rubi steps

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}}\,dx=\int \frac{x^m}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}}\,dx$$

Mathematica [A] time = 0.839773, size = 0, normalized size = 0.

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2)),x]
```

```
[Out] Integrate[x^m/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2)), x]
```

Maple [A] time = 1.389, size = 0, normalized size = 0.

$$\int x^{m} (\arctan (ax))^{-\frac{3}{2}} \frac{1}{\sqrt{a^{2} c x^{2}+c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(x^m/arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^{m}}{\sqrt{a^{2} c x^{2}+c} \arctan (ax)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^m/(sqrt(a^2*c*x^2 + c)*arctan(a*x)^(3/2)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/atan(a*x)**(3/2)/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\sqrt{a^2cx^2+c}\arctan{(ax)}^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^m/(sqrt(a^2*c*x^2 + c)*arctan(a*x)^(3/2)), x)
```

3.1020 $\int \frac{x}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{x}{\sqrt{a^2cx^2+c}\tan^{-1}(ax)^{3/2}},x\right)$$

```
[Out] Unintegrable[x/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2)), x]
```

Rubi [A] time = 0.0722918, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2)),x]

[Out] Defer[Int][x/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2)), x]
```

Rubi steps

$$\int \frac{x}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}}\,dx=\int \frac{x}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}}\,dx$$

Mathematica [A] time = 1.03024, size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2)),x]

[Out] Integrate[x/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2)), x]
```

Maple [A] time = 1.2, size = 0, normalized size = 0.

$$\int x\left(\arctan\left(ax\right)\right)^{-\frac{3}{2}}\frac{1}{\sqrt{a^{2}c\,x^{2}+c}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(x/arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{x}{\sqrt{c\left(a^{2}x^{2}+1\right)}\operatorname{atan}^{\frac{3}{2}}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/atan(a*x)**(3/2)/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(c*(a**2*x**2 + 1))*atan(a*x)**(3/2)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{a^2cx^2+c}\arctan(ax)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/(sqrt(a^2*c*x^2 + c)*arctan(a*x)^(3/2)), x)
```

3.1021 $\int \frac{1}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=25

$$\text{Unintegrable}\left(\frac{1}{\sqrt{a^2cx^2+c}\tan^{-1}(ax)^{3/2}},x\right)$$

```
[Out] Unintegrable[1/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2)), x]
```

Rubi [A] time = 0.0350177, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2)),x]
```

```
[Out] Defer[Int][1/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2)), x]
```

Rubi steps

$$\int \frac{1}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}}\,dx=\int \frac{1}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}}\,dx$$

Mathematica [A] time = 0.632402, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2)),x]
```

```
[Out] Integrate[1/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2)), x]
```

Maple [A] time = 0.914, size = 0, normalized size = 0.

$$\int (\arctan(ax))^{-\frac{3}{2}} \frac{1}{\sqrt{a^2cx^2 + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(1/arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{c\left(a^2x^2 + 1\right)}\operatorname{atan}^{\frac{3}{2}}(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/atan(a*x)**(3/2)/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(c*(a**2*x**2 + 1))*atan(a*x)**(3/2)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a^2cx^2 + c}\arctan\left(ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a^2*c*x^2 + c)*arctan(a*x)^(3/2)), x)
```

3.1022 $\int \frac{1}{x\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=67

$$-\frac{2\text{Unintegrable}\left(\frac{1}{x^2\sqrt{a^2cx^2+c}\sqrt{\tan^{-1}(ax)}},x\right)}{a}-\frac{2\sqrt{a^2cx^2+c}}{acx\sqrt{\tan^{-1}(ax)}}$$

```
[Out] (-2*Sqrt[c + a^2*c*x^2])/(a*c*x*Sqrt[ArcTan[a*x]]) - (2*Unintegrable[1/(x^2
*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]), x])/a
```

Rubi [A] time = 0.210974, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2)),x]
```

```
[Out] (-2*Sqrt[c + a^2*c*x^2])/(a*c*x*Sqrt[ArcTan[a*x]]) - (2*Defer[Int][1/(x^2*S
qrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]), x])/a
```

Rubi steps

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}}\,dx=-\frac{2\sqrt{c+a^2cx^2}}{acx\sqrt{\tan^{-1}(ax)}}-\frac{2\int \frac{1}{x^2\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}}\,dx}{a}$$

Mathematica [A] time = 5.14239, size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2)),x]
```

```
[Out] Integrate[1/(x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2)), x]
```

Maple [A] time = 1.042, size = 0, normalized size = 0.

$$\int \frac{1}{x} \left(\arctan \left(ax\right)\right)^{-\frac{3}{2}} \frac{1}{\sqrt{a^2 c x^2 + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(1/x/arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/atan(a*x)**(3/2)/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a^2cx^2 + cx} \arctan{(ax)}^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a^2*c*x^2 + c)*x*arctan(a*x)^(3/2)), x)
```

3.1023 $\int \frac{1}{x^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(\frac{1}{x^2\sqrt{a^2cx^2+c}\tan^{-1}(ax)^{3/2}},x\right)$$

```
[Out] Unintegrable[1/(x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2)), x]
```

Rubi [A] time = 0.10533, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2)),x]
```

```
[Out] Defer[Int][1/(x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2)), x]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}}\,dx=\int \frac{1}{x^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}}\,dx$$

Mathematica [A] time = 10.3594, size = 0, normalized size = 0.

$$\int \frac{1}{x^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2)),x]
```

```
[Out] Integrate[1/(x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2)), x]
```

Maple [A] time = 0.832, size = 0, normalized size = 0.

$$\int \frac{1}{x^2} (\arctan(ax))^{-\frac{3}{2}} \frac{1}{\sqrt{a^2 c x^2 + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(1/x^2/arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas"
)
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/atan(a*x)**(3/2)/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a^2cx^2 + c}x^2 \arctan{(ax)}^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a^2*c*x^2 + c)*x^2*arctan(a*x)^(3/2)), x)
```

3.1024 $\int \frac{x^m}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(\frac{x^m}{\left(a^2cx^2+c\right)^{3/2}\tan^{-1}(ax)^{3/2}},x\right)$$

[Out] Unintegrable[x^m/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)), x]

Rubi [A] time = 0.116532, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

[In] Int[x^m/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)),x]

[Out] Defer[Int][x^m/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)), x]

Rubi steps

$$\int \frac{x^m}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}}\,dx=\int \frac{x^m}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}}\,dx$$

Mathematica [A] time = 0.988014, size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

[In] Integrate[x^m/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)),x]

```
[Out] Integrate[x^m/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)), x]
```

Maple [A] time = 0.945, size = 0, normalized size = 0.

$$\int x^{m} \left(a^{2} c x^{2}+c\right)^{-\frac{3}{2}} (\arctan (a x))^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

```
[Out] int(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a^{2} c x^{2}+c} x^{m}}{\left(a^{4} c^{2} x^{4}+2 a^{2} c^{2} x^{2}+c^{2}\right) \arctan (a x)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^m/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arcta
n(a*x)^(3/2)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(a**2*c*x**2+c)**(3/2)/atan(a*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{(a^2cx^2 + c)^{\frac{3}{2}} \arctan(ax)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^m/((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^(3/2)), x)
```

3.1025 $\int \frac{x^3}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=98

$$\frac{6\text{Unintegrable}\left(\frac{x^2}{\left(a^2cx^2+c\right)^{3/2}\sqrt{\tan^{-1}(ax)}},x\right)}{a}+4a\text{Unintegrable}\left(\frac{x^4}{\left(a^2cx^2+c\right)^{3/2}\sqrt{\tan^{-1}(ax)}},x\right)-\frac{2x^3}{ac\sqrt{a^2cx^2+c}\sqrt{\tan^{-1}(ax)}}$$

```
[Out] (-2*x^3)/(a*c*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]) + (6*Unintegrable[x^2/
((c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]), x])/a + 4*a*Unintegrable[x^4/((c
 + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]), x]
```

Rubi [A] time = 0.373211, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^3}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^3/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)),x]
```

```
[Out] (-2*x^3)/(a*c*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]) + (6*Defer[Int][x^2/((
c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]), x])/a + 4*a*Defer[Int][x^4/((c + a
^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]), x]
```

Rubi steps

$$\int \frac{x^3}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}}\,dx=-\frac{2x^3}{ac\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}}+\frac{6\int \frac{x^2}{\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}}\,dx}{a}+(4a)\int \frac{x^4}{\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}}$$

Mathematica [A] time = 5.97907, size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^3/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)),x]
```

```
[Out] Integrate[x^3/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)), x]
```

Maple [A] time = 2.855, size = 0, normalized size = 0.

$$\int x^3 \left(a^2cx^2+c\right)^{-\frac{3}{2}} (\arctan(ax))^{-\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

```
[Out] int(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a**2*c*x**2+c)**(3/2)/atan(a*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(a^2cx^2+c\right)^{\frac{3}{2}}\arctan\left(ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^(3/2)), x)
```

3.1026 $\int \frac{x^2}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=126

$$2a\text{Unintegrable}\left(\frac{x^3}{\left(a^2cx^2+c\right)^{3/2}\sqrt{\tan^{-1}(ax)}},x\right)+\frac{4\sqrt{2\pi}\sqrt{a^2x^2+1}S\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{a^3c\sqrt{a^2cx^2+c}}-\frac{2x^2}{ac\sqrt{a^2cx^2+c}\sqrt{\tan^{-1}(ax)}}$$

```
[Out] (-2*x^2)/(a*c*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]) + (4*Sqrt[2*Pi]*Sqrt[1
 + a^2*x^2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(a^3*c*Sqrt[c + a^2*c*x
^2]) + 2*a*Unintegrable[x^3/((c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]), x]
```

Rubi [A] time = 0.431362, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^2}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^2/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)),x]
```

```
[Out] (-2*x^2)/(a*c*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]) + (4*Sqrt[2*Pi]*Sqrt[1
 + a^2*x^2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(a^3*c*Sqrt[c + a^2*c*x
^2]) + 2*a*Defer[Int][x^3/((c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]), x]
```

Rubi steps

$$\int \frac{x^2}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}}\,dx = -\frac{2x^2}{ac\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}} + \frac{4\int \frac{x}{\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}}\,dx}{a} + (2a)\int \frac{x^3}{\left(c+a^2cx^2\right)^{3/2}\sqrt{\ldots}}$$

$$= -\frac{2x^2}{ac\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}} + (2a)\int \frac{x^3}{\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}}\,dx + \frac{\left(4\sqrt{1+a^2x^2}\ldots\right.}{\ldots}$$

$$= -\frac{2x^2}{ac\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}} + (2a)\int \frac{x^3}{\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}}\,dx + \frac{\left(4\sqrt{1+a^2x^2}\ldots\right.}{\ldots}$$

$$= -\frac{2x^2}{ac\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}} + (2a)\int \frac{x^3}{\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}}\,dx + \frac{\left(8\sqrt{1+a^2x^2}\ldots\right.}{\ldots}$$

$$= -\frac{2x^2}{ac\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}} + \frac{4\sqrt{2\pi}\sqrt{1+a^2x^2}S\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{a^3c\sqrt{c+a^2cx^2}} + (2a)\int \frac{\ldots}{\left(c+a\ldots\right.}$$

Mathematica [A] time = 2.2166, size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^2/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)),x]
```

```
[Out] Integrate[x^2/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)), x]
```

Maple [A] time = 2.893, size = 0, normalized size = 0.

$$\int x^2\left(a^2cx^2+c\right)^{-\frac{3}{2}}\left(\arctan(ax)\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

```
[Out] int(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a**2*c*x**2+c)**(3/2)/atan(a*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(a^2cx^2+c\right)^{\frac{3}{2}} \arctan\left(ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^(3/2)), x)
```

3.1027 $\int \frac{x}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=93

$$\frac{2\sqrt{2\pi}\sqrt{a^2x^2+1}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{a^2c\sqrt{a^2cx^2+c}}-\frac{2x}{ac\sqrt{a^2cx^2+c}\sqrt{\tan^{-1}(ax)}}$$

```
[Out] (-2*x)/(a*c*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]) + (2*Sqrt[2*Pi]*Sqrt[1 +
 a^2*x^2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(a^2*c*Sqrt[c + a^2*c*x^2
])
```

Rubi [A] time = 0.178184, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {4942, 4905, 4904, 3304, 3352}

$$\frac{2\sqrt{2\pi}\sqrt{a^2x^2+1}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{a^2c\sqrt{a^2cx^2+c}}-\frac{2x}{ac\sqrt{a^2cx^2+c}\sqrt{\tan^{-1}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)),x]
```


```
[Out] (-2*x)/(a*c*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]) + (2*Sqrt[2*Pi]*Sqrt[1 +
 a^2*x^2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(a^2*c*Sqrt[c + a^2*c*x^2
])
```


Rule 4942

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^m*(d + e*x^2)^(q + 1)*(a + b*ArcT
an[c*x])^(p + 1))/(b*c*d*(p + 1)), x] - Dist[(f*m)/(b*c*(p + 1)), Int[(f*x)
^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 2, 0] && LtQ[p, -1]
```

Rule 4905

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[(d^(q + 1/2)*Sqrt[1 + c^2*x^2])/Sqrt[d + e*x^2], Int[(1 + c
```

```
^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

Rule 4904

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}}\,dx &= -\frac{2x}{ac\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}}+\frac{2\int \frac{1}{\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}}\,dx}{a}\\
&= -\frac{2x}{ac\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}}+\frac{\left(2\sqrt{1+a^2x^2}\right)\int \frac{1}{\left(1+a^2x^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}}\,dx}{ac\sqrt{c+a^2cx^2}}\\
&= -\frac{2x}{ac\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}}+\frac{\left(2\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int \frac{\cos(x)}{\sqrt{x}}\,dx,x,\tan^{-1}(ax)\right)}{a^2c\sqrt{c+a^2cx^2}}\\
&= -\frac{2x}{ac\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}}+\frac{\left(4\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int \cos\left(x^2\right)\,dx,x,\sqrt{\tan^{-1}(ax)}\right)}{a^2c\sqrt{c+a^2cx^2}}\\
&= -\frac{2x}{ac\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}}+\frac{2\sqrt{2\pi}\sqrt{1+a^2x^2}\,C\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{a^2c\sqrt{c+a^2cx^2}}
\end{aligned}$$

Mathematica [F] time = 0.341656, size = 0, normalized size = 0.

$$\int \frac{x}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)),x]
```

```
[Out] Integrate[x/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)), x]
```

Maple [F] time = 0.83, size = 0, normalized size = 0.

$$\int x\left(a^2cx^2+c\right)^{-\frac{3}{2}}(\arctan(ax))^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

```
[Out] int(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a**2*c*x**2+c)**(3/2)/atan(a*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(a^2cx^2+c\right)^{\frac{3}{2}}\arctan\left(ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x/((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^(3/2)), x)
```

3.1028 $\int \frac{1}{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=92

$$-\frac{2\sqrt{2\pi}\sqrt{a^2x^2+1}S\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{ac\sqrt{a^2cx^2+c}}-\frac{2}{ac\sqrt{a^2cx^2+c}\sqrt{\tan^{-1}(ax)}}$$

```
[Out] -2/(a*c*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]) - (2*Sqrt[2*Pi]*Sqrt[1 + a^2
*x^2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(a*c*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.212764, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4902, 4971, 4970, 3305, 3351}

$$-\frac{2\sqrt{2\pi}\sqrt{a^2x^2+1}S\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{ac\sqrt{a^2cx^2+c}}-\frac{2}{ac\sqrt{a^2cx^2+c}\sqrt{\tan^{-1}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)),x]
```

```
[Out] -2/(a*c*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]) - (2*Sqrt[2*Pi]*Sqrt[1 + a^2
*x^2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(a*c*Sqrt[c + a^2*c*x^2])
```

Rule 4902

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^(p + 1))/(b*c*d*(p
+ 1)), x] - Dist[(2*c*(q + 1))/(b*(p + 1)), Int[x*(d + e*x^2)^q*(a + b*ArcT
an[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && L
tQ[q, -1] && LtQ[p, -1]
```

Rule 4971

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[(d^(q + 1/2)*Sqrt[1 + c^2*x^2])/Sqrt[d + e*x^2],
 Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d
```

```
, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(In
tegerQ[q] || GtQ[d, 0])
```

Rule 4970

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sin[x]^m)/
Cos[x]^(m + 2*(q + 1)), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}}\,dx &= -\frac{2}{ac\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}}-(2a)\int \frac{x}{\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}}\,dx\\
&= -\frac{2}{ac\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}}-\frac{\left(2a\sqrt{1+a^2x^2}\right)\int \frac{x}{\left(1+a^2x^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}}\,dx}{c\sqrt{c+a^2cx^2}}\\
&= -\frac{2}{ac\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}}-\frac{\left(2\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int \frac{\sin(x)}{\sqrt{x}}\,dx,x,\tan^{-1}(ax)\right)}{ac\sqrt{c+a^2cx^2}}\\
&= -\frac{2}{ac\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}}-\frac{\left(4\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int \sin\left(x^2\right)\,dx,x,\sqrt{\tan^{-1}(ax)}\right)}{ac\sqrt{c+a^2cx^2}}\\
&= -\frac{2}{ac\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}}-\frac{2\sqrt{2\pi}\sqrt{1+a^2x^2}S\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{ac\sqrt{c+a^2cx^2}}
\end{aligned}$$

Mathematica [C] time = 0.127484, size = 107, normalized size = 1.16

$$\frac{\sqrt{a^2x^2+1}\sqrt{-i\tan^{-1}(ax)}\text{Gamma}\left(\frac{1}{2},-i\tan^{-1}(ax)\right)+\sqrt{a^2x^2+1}\sqrt{i\tan^{-1}(ax)}\text{Gamma}\left(\frac{1}{2},i\tan^{-1}(ax)\right)-2}{ac\sqrt{a^2cx^2+c}\sqrt{\tan^{-1}(ax)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)),x]
```

```
[Out] (-2 + Sqrt[1 + a^2*x^2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-I)*ArcTan[a*x]]
 + Sqrt[1 + a^2*x^2]*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, I*ArcTan[a*x]])/(a*c*Sq
rt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]])
```

Maple [F] time = 0.723, size = 0, normalized size = 0.

$$\int \left(a^2cx^2+c\right)^{-\frac{3}{2}} (\arctan(ax))^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

```
[Out] int(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*c*x**2+c)**(3/2)/atan(a*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)^{\frac{3}{2}}\arctan\left(ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^(3/2)), x)
```

3.1029 $\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=125

$$-\frac{2\text{Unintegrable}\left(\frac{1}{x^2\left(a^2cx^2+c\right)^{3/2}\sqrt{\tan^{-1}(ax)}},x\right)}{a}-\frac{4\sqrt{2\pi}\sqrt{a^2x^2+1}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{c\sqrt{a^2cx^2+c}}-\frac{2}{acx\sqrt{a^2cx^2+c}\sqrt{\tan^{-1}(ax)}}$$

```
[Out] -2/(a*c*x*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]) - (4*Sqrt[2*Pi]*Sqrt[1 + a
^2*x^2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(c*Sqrt[c + a^2*c*x^2]) - (
2*Unintegrable[1/(x^2*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]), x])/a
```

Rubi [A] time = 0.332313, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)),x]
```

```
[Out] -2/(a*c*x*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]) - (4*Sqrt[2*Pi]*Sqrt[1 + a
^2*x^2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(c*Sqrt[c + a^2*c*x^2]) - (
2*Defer[Int][1/(x^2*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]), x])/a
```

Rubi steps

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}}\,dx = -\frac{2}{acx\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}} - \frac{2\int \frac{1}{x^2\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}}\,dx}{a} - (4a)\int \frac{}{\left(c+a^2cx^2\right)^{3}}$$

$$= -\frac{2}{acx\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}} - \frac{2\int \frac{1}{x^2\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}}\,dx}{a} - \frac{\left(4a\sqrt{1+a^2x^2}\right)\int \frac{}{(1}}{c\sqrt{c+}}$$

$$= -\frac{2}{acx\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}} - \frac{2\int \frac{1}{x^2\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}}\,dx}{a} - \frac{\left(4\sqrt{1+a^2x^2}\right)\text{Subs}}{c}$$

$$= -\frac{2}{acx\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}} - \frac{2\int \frac{1}{x^2\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}}\,dx}{a} - \frac{\left(8\sqrt{1+a^2x^2}\right)\text{Subs}}{}$$

$$= -\frac{2}{acx\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}} - \frac{4\sqrt{2\pi}\sqrt{1+a^2x^2}\,C\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{c\sqrt{c+a^2cx^2}} - \frac{2\int}{x^2\left(c+a^2\right.}$$

Mathematica [A] time = 5.29389, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)),x]

[Out] Integrate[1/(x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)), x]
```

Maple [A] time = 0.809, size = 0, normalized size = 0.

$$\int \frac{1}{x}\left(a^2cx^2+c\right)^{-\frac{3}{2}}(\arctan(ax))^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

```
[Out] int(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a**2*c*x**2+c)**(3/2)/atan(a*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)^{\frac{3}{2}}x\arctan\left(ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^(3/2)*x*arctan(a*x)^(3/2)), x)
```

3.1030 $\int \frac{1}{x^2(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=98

$$-\frac{4\text{Unintegrable}\left(\frac{1}{x^3(a^2cx^2+c)^{3/2}\sqrt{\tan^{-1}(ax)}},x\right)}{a}-6a\text{Unintegrable}\left(\frac{1}{x\left(a^2cx^2+c\right)^{3/2}\sqrt{\tan^{-1}(ax)}},x\right)-\frac{2}{acx^2\sqrt{a^2cx^2+c}\sqrt{\tan^{-1}(ax)}}$$

```
[Out] -2/(a*c*x^2*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]) - (4*Unintegrable[1/(x^3
*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]), x])/a - 6*a*Unintegrable[1/(x*(c
 + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]), x]
```

Rubi [A] time = 0.35991, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)),x]
```

```
[Out] -2/(a*c*x^2*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]) - (4*Defer[Int][1/(x^3*(
c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]), x])/a - 6*a*Defer[Int][1/(x*(c + a
^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]), x]
```

Rubi steps

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}}\,dx=-\frac{2}{acx^2\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}}-\frac{4\int \frac{1}{x^3(c+a^2cx^2)^{3/2}\sqrt{\tan^{-1}(ax)}}\,dx}{a}-(6a)\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}}\,dx$$

Mathematica [A] time = 19.4375, size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)),x]
```

```
[Out] Integrate[1/(x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)), x]
```

Maple [A] time = 0.81, size = 0, normalized size = 0.

$$\int \frac{1}{x^{2}} \left(a^{2} c \,x^{2}+c \right)^{-\frac{3}{2}} \left(\arctan \left(a x \right)\right)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

```
[Out] int(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="fricas"
)
```

[Out] Exception raised: UnboundLocalError

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a**2*c*x**2+c)**(3/2)/atan(a*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)^{\frac{3}{2}}x^2\arctan\left(ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^(3/2)*x^2*arctan(a*x)^(3/2)), x)
```

3.1031 $\int \frac{1}{x^3(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=98

$$-\frac{6\text{Unintegrable}\left(\frac{1}{x^4(a^2cx^2+c)^{3/2}\sqrt{\tan^{-1}(ax)}},x\right)}{a}-8a\text{Unintegrable}\left(\frac{1}{x^2\left(a^2cx^2+c\right)^{3/2}\sqrt{\tan^{-1}(ax)}},x\right)-\frac{2}{acx^3\sqrt{a^2cx^2+c}\sqrt{\tan^{-1}(ax)}}$$

```
[Out] -2/(a*c*x^3*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]) - (6*Unintegrable[1/(x^4
*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]), x])/a - 8*a*Unintegrable[1/(x^2*
(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]), x]
```

Rubi [A] time = 0.365251, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^3\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^3*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)),x]
```

```
[Out] -2/(a*c*x^3*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]) - (6*Defer[Int][1/(x^4*(
c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]), x])/a - 8*a*Defer[Int][1/(x^2*(c +
 a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]), x]
```

Rubi steps

$$\int \frac{1}{x^3\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}}\,dx=-\frac{2}{acx^3\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}}-\frac{6\int \frac{1}{x^4(c+a^2cx^2)^{3/2}\sqrt{\tan^{-1}(ax)}}\,dx}{a}-(8a)\int \frac{1}{x^2\left(c+a^2\right.}$$

Mathematica [A] time = 16.2427, size = 0, normalized size = 0.

$$\int \frac{1}{x^3\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^3*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)),x]
```

```
[Out] Integrate[1/(x^3*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)), x]
```

Maple [A] time = 0.94, size = 0, normalized size = 0.

$$\int \frac{1}{x^{3}} \left(a^{2} c x^{2}+c\right)^{-\frac{3}{2}} \left(\arctan \left(a x\right)\right)^{-\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

```
[Out] int(1/x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="fricas"
)
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a**2*c*x**2+c)**(3/2)/atan(a*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)^{\frac{3}{2}}x^3\arctan\left(ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^(3/2)*x^3*arctan(a*x)^(3/2)), x)
```

3.1032 $\int \frac{1}{x^4\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=98

$$-\frac{8\text{Unintegrable}\left(\frac{1}{x^5\left(a^2cx^2+c\right)^{3/2}\sqrt{\tan^{-1}(ax)}},x\right)}{a}-10a\text{Unintegrable}\left(\frac{1}{x^3\left(a^2cx^2+c\right)^{3/2}\sqrt{\tan^{-1}(ax)}},x\right)-\frac{2}{acx^4\sqrt{a^2cx^2+c}\sqrt{t\ldots}}$$

```
[Out] -2/(a*c*x^4*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]) - (8*Unintegrable[1/(x^5
*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]), x])/a - 10*a*Unintegrable[1/(x^3
*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]), x]
```

Rubi [A] time = 0.36305, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{1}{x^4\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^4*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)),x]
```

```
[Out] -2/(a*c*x^4*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]) - (8*Defer[Int][1/(x^5*(
c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]), x])/a - 10*a*Defer[Int][1/(x^3*(c
+ a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]), x]
```

Rubi steps

$$\int \frac{1}{x^4\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}}\,dx=-\frac{2}{acx^4\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}}-\frac{8\int \frac{1}{x^5\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}}\,dx}{a}-(10a)\int \frac{\ldots}{x^3\left(c+a^2c\ldots\right.}$$

Mathematica [A] time = 18.2584, size = 0, normalized size = 0.

$$\int \frac{1}{x^4\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^4*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)),x]
```

```
[Out] Integrate[1/(x^4*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)), x]
```

Maple [A] time = 1.993, size = 0, normalized size = 0.

$$\int \frac{1}{x^4} \left(a^2 c x^2 + c\right)^{-\frac{3}{2}} (\arctan (ax))^{-\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

```
[Out] int(1/x^4/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="fricas"
)
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(a**2*c*x**2+c)**(3/2)/atan(a*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)^{\frac{3}{2}}x^4\arctan\left(ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^(3/2)*x^4*arctan(a*x)^(3/2)), x)
```

3.1033 $\int \frac{x^m}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(\frac{x^m}{\left(a^2cx^2+c\right)^{5/2}\tan^{-1}(ax)^{3/2}},x\right)$$

```
[Out] Unintegrable[x^m/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2)), x]
```

Rubi [A] time = 0.114877, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2)),x]
```

```
[Out] Defer[Int][x^m/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2)), x]
```

Rubi steps

$$\int \frac{x^m}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{3/2}}\,dx=\int \frac{x^m}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{3/2}}\,dx$$

Mathematica [A] time = 1.07632, size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2)),x]
```

```
[Out] Integrate[x^m/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2)), x]
```

Maple [A] time = 1.112, size = 0, normalized size = 0.

$$\int x^m \left(a^2 c x^2 + c\right)^{-\frac{5}{2}} (\arctan(ax))^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
```

```
[Out] int(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a^2cx^2 + c}x^m}{\left(a^6c^3x^6 + 3a^4c^3x^4 + 3a^2c^3x^2 + c^3\right)\arctan(ax)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^m/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*
x^2 + c^3)*arctan(a*x)^(3/2)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(a**2*c*x**2+c)**(5/2)/atan(a*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(a^2cx^2+c\right)^{\frac{5}{2}}\arctan\left(ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^m/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^(3/2)), x)
```

3.1034 $\int \frac{x^3}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=160

$$\frac{3\sqrt{\frac{\pi}{2}}\sqrt{a^2x^2+1}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{a^4c^2\sqrt{a^2cx^2+c}}-\frac{\sqrt{\frac{3\pi}{2}}\sqrt{a^2x^2+1}\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{a^4c^2\sqrt{a^2cx^2+c}}-\frac{2x^3}{ac\left(a^2cx^2+c\right)^{3/2}\sqrt{\tan^{-1}(a}}$$

```
[Out] (-2*x^3)/(a*c*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]) + (3*Sqrt[Pi/2]*Sqrt
[1 + a^2*x^2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(a^4*c^2*Sqrt[c + a^2
*c*x^2]) - (Sqrt[(3*Pi)/2]*Sqrt[1 + a^2*x^2]*FresnelC[Sqrt[6/Pi]*Sqrt[ArcTa
n[a*x]]])/(a^4*c^2*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.433501, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {4942, 4971, 4970, 4406, 3304, 3352}

$$\frac{3\sqrt{\frac{\pi}{2}}\sqrt{a^2x^2+1}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{a^4c^2\sqrt{a^2cx^2+c}}-\frac{\sqrt{\frac{3\pi}{2}}\sqrt{a^2x^2+1}\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{a^4c^2\sqrt{a^2cx^2+c}}-\frac{2x^3}{ac\left(a^2cx^2+c\right)^{3/2}\sqrt{\tan^{-1}(a}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2)),x]
```


```
[Out] (-2*x^3)/(a*c*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]) + (3*Sqrt[Pi/2]*Sqrt
[1 + a^2*x^2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(a^4*c^2*Sqrt[c + a^2
*c*x^2]) - (Sqrt[(3*Pi)/2]*Sqrt[1 + a^2*x^2]*FresnelC[Sqrt[6/Pi]*Sqrt[ArcTa
n[a*x]]])/(a^4*c^2*Sqrt[c + a^2*c*x^2])
```


Rule 4942

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^m*(d + e*x^2)^(q + 1)*(a + b*ArcT
an[c*x])^(p + 1))/(b*c*d*(p + 1)), x] - Dist[(f*m)/(b*c*(p + 1)), Int[(f*x)
^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 2, 0] && LtQ[p, -1]
```

Rule 4971

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[(d^(q + 1/2)*Sqrt[1 + c^2*x^2])/Sqrt[d + e*x^2],
 Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d
, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(In
tegerQ[q] || GtQ[d, 0])
```

Rule 4970

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sin[x]^m)/
Cos[x]^(m + 2*(q + 1)), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{x^3}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{3/2}}\,dx = -\frac{2x^3}{ac\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}} + \frac{6\int \frac{x^2}{\left(c+a^2cx^2\right)^{5/2}\sqrt{\tan^{-1}(ax)}}\,dx}{a}$$

$$= -\frac{2x^3}{ac\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}} + \frac{\left(6\sqrt{1+a^2x^2}\right)\int \frac{x^2}{\left(1+a^2x^2\right)^{5/2}\sqrt{\tan^{-1}(ax)}}\,dx}{ac^2\sqrt{c+a^2cx^2}}$$

$$= -\frac{2x^3}{ac\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}} + \frac{\left(6\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int \frac{\cos(x)\sin^2(x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax\right.}{a^4c^2\sqrt{c+a^2cx^2}}$$

$$= -\frac{2x^3}{ac\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}} + \frac{\left(6\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int \left(\frac{\cos(x)}{4\sqrt{x}} - \frac{\cos(3x)}{4\sqrt{x}}\right)dx, x, \tan\right.}{a^4c^2\sqrt{c+a^2cx^2}}$$

$$= -\frac{2x^3}{ac\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}} + \frac{\left(3\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int \frac{\cos(x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{2a^4c^2\sqrt{c+a^2cx^2}} - $$

$$= -\frac{2x^3}{ac\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}} + \frac{\left(3\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int \cos\left(x^2\right)dx, x, \sqrt{\tan^{-1}(ax\right.}{a^4c^2\sqrt{c+a^2cx^2}}$$

$$= -\frac{2x^3}{ac\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}} + \frac{3\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}C\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{a^4c^2\sqrt{c+a^2cx^2}} - \frac{\sqrt{\frac{3\pi}{2}}\sqrt{1+a^2}}{a^4}$$

Mathematica [C] time = 0.517108, size = 182, normalized size = 1.14

$$\frac{-\frac{8a^3cx^3}{a^2x^2+1} - ic\sqrt{a^2x^2+1}\left(3\sqrt{-i\tan^{-1}(ax)}\text{Gamma}\left(\frac{1}{2}, -i\tan^{-1}(ax)\right) - 3\sqrt{i\tan^{-1}(ax)}\text{Gamma}\left(\frac{1}{2}, i\tan^{-1}(ax)\right) + \sqrt{3}\left(\sqrt{i\,t\right.\right.}{4a^4c^3\sqrt{a^2cx^2+c}\sqrt{\tan^{-1}(ax)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2)),x]
```

```
[Out] ((-8*a^3*c*x^3)/(1 + a^2*x^2) - I*c*Sqrt[1 + a^2*x^2]*(3*Sqrt[(-I)*ArcTan[a
*x]]*Gamma[1/2, (-I)*ArcTan[a*x]] - 3*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, I*ArcT
an[a*x]] + Sqrt[3]*(-(Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-3*I)*ArcTan[a*x]]
```

```
) + Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (3*I)*ArcTan[a*x]])))/(4*a^4*c^3*Sqrt[c
+ a^2*c*x^2]*Sqrt[ArcTan[a*x]])
```

Maple [F] time = 4.371, size = 0, normalized size = 0.

$$\int x^3 \left(a^2 c x^2 + c\right)^{-\frac{5}{2}} (\arctan (ax))^{-\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)

[Out] int(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="maxima")

[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a**2*c*x**2+c)**(5/2)/atan(a*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(a^2cx^2+c\right)^{\frac{5}{2}}\arctan\left(ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^(3/2)), x)
```

3.1035 $\int \frac{x^2}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=281

$$\frac{\sqrt{2\pi}\sqrt{a^2x^2+1}S\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{a^3c^2\sqrt{a^2cx^2+c}}-\frac{3\sqrt{\frac{\pi}{2}}\sqrt{a^2x^2+1}S\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{a^3c^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{\frac{2\pi}{3}}\sqrt{a^2x^2+1}S\left(\sqrt{\frac{6}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{a^3c^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{\frac{\pi}{6}}\ldots}{}$$

```
[Out] (-2*x^2)/(a*c*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]) - (3*Sqrt[Pi/2]*Sqrt
[1 + a^2*x^2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(a^3*c^2*Sqrt[c + a^2
*c*x^2]) + (Sqrt[2*Pi]*Sqrt[1 + a^2*x^2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*
x]]])/(a^3*c^2*Sqrt[c + a^2*c*x^2]) + (Sqrt[Pi/6]*Sqrt[1 + a^2*x^2]*Fresnel
S[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]])/(a^3*c^2*Sqrt[c + a^2*c*x^2]) + (Sqrt[(2*P
i)/3]*Sqrt[1 + a^2*x^2]*FresnelS[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]])/(a^3*c^2*Sq
rt[c + a^2*c*x^2])
```

Rubi [A] time = 0.666494, antiderivative size = 281, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {4968, 4971, 4970, 3312, 3305, 3351, 4406}

$$\frac{\sqrt{2\pi}\sqrt{a^2x^2+1}S\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{a^3c^2\sqrt{a^2cx^2+c}}-\frac{3\sqrt{\frac{\pi}{2}}\sqrt{a^2x^2+1}S\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{a^3c^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{\frac{2\pi}{3}}\sqrt{a^2x^2+1}S\left(\sqrt{\frac{6}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{a^3c^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{\frac{\pi}{6}}\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2)),x]
```


```
[Out] (-2*x^2)/(a*c*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]) - (3*Sqrt[Pi/2]*Sqrt
[1 + a^2*x^2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(a^3*c^2*Sqrt[c + a^2
*c*x^2]) + (Sqrt[2*Pi]*Sqrt[1 + a^2*x^2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*
x]]])/(a^3*c^2*Sqrt[c + a^2*c*x^2]) + (Sqrt[Pi/6]*Sqrt[1 + a^2*x^2]*Fresnel
S[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]])/(a^3*c^2*Sqrt[c + a^2*c*x^2]) + (Sqrt[(2*P
i)/3]*Sqrt[1 + a^2*x^2]*FresnelS[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]])/(a^3*c^2*Sq
rt[c + a^2*c*x^2])
```


Rule 4968

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[(x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^(p
```

```
+ 1))/(b*c*d*(p + 1)), x] + (-Dist[(c*(m + 2*q + 2))/(b*(p + 1)), Int[x^(m
+ 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p + 1
)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; Fre
eQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &&
LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 4971

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[(d^(q + 1/2)*Sqrt[1 + c^2*x^2])/Sqrt[d + e*x^2],
 Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d
, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(In
tegerQ[q] || GtQ[d, 0])
```

Rule 4970

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sin[x]^m)/
Cos[x]^(m + 2*(q + 1)), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
```

```
tQ[p, 0]
```

Rubi steps

$$\int \frac{x^2}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{3/2}}\,dx = -\frac{2x^2}{ac\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}} + \frac{4\int \frac{x}{\left(c+a^2cx^2\right)^{5/2}\sqrt{\tan^{-1}(ax)}}\,dx}{a} - (2a)\int \frac{x}{\left(c+a^2cx^2\right)^{5/}}$$

$$= -\frac{2x^2}{ac\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}} + \frac{\left(4\sqrt{1+a^2x^2}\right)\int \frac{x}{\left(1+a^2x^2\right)^{5/2}\sqrt{\tan^{-1}(ax)}}\,dx}{ac^2\sqrt{c+a^2cx^2}} - \frac{\left(2a\sqrt{1+\right.}{}$$

$$= -\frac{2x^2}{ac\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}} - \frac{\left(2\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int \frac{\sin^3(x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{a^3c^2\sqrt{c+a^2cx^2}} +$$

$$= -\frac{2x^2}{ac\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}} - \frac{\left(2\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int \left(\frac{3\sin(x)}{4\sqrt{x}} - \frac{\sin(3x)}{4\sqrt{x}}\right)dx, x, \text{ta}\right.}{a^3c^2\sqrt{c+a^2cx^2}}$$

$$= -\frac{2x^2}{ac\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}} + \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \frac{\sin(3x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{2a^3c^2\sqrt{c+a^2cx^2}} + \frac{\sqrt{}}{}$$

$$= -\frac{2x^2}{ac\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}} + \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \sin\left(3x^2\right)dx, x, \sqrt{\tan^{-1}(ax)}\right)}{a^3c^2\sqrt{c+a^2cx^2}}$$

$$= -\frac{2x^2}{ac\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}} - \frac{3\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}\,S\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{a^3c^2\sqrt{c+a^2cx^2}} + \frac{\sqrt{2\pi}\sqrt{1+}}{a}$$

Mathematica [C] time = 0.504035, size = 241, normalized size = 0.86

$$-\frac{\frac{\left(a^2x^2+1\right)^{3/2}\left(3\sqrt{-i\tan^{-1}(ax)}\,\text{Gamma}\left(\frac{1}{2},-i\tan^{-1}(ax)\right)+3\sqrt{i\tan^{-1}(ax)}\,\text{Gamma}\left(\frac{1}{2},i\tan^{-1}(ax)\right)+\sqrt{3}\left(\sqrt{-i\tan^{-1}(ax)}\,\text{Gamma}\left(\frac{1}{2},-3i\tan^{-1}(ax)\right)+\sqrt{i\tan^{-1}(a}\right.\right.}{\sqrt{\tan^{-1}(ax)}}}{6a^3c\left(a^2cx^2+c\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2)),x]
```

```
[Out] ((-12*a^2*x^2)/Sqrt[ArcTan[a*x]] + Sqrt[6*Pi]*(1 + a^2*x^2)^(3/2)*(-3*Sqrt[
3]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]] + FresnelS[Sqrt[6/Pi]*Sqrt[ArcTan
[a*x]]]) - ((1 + a^2*x^2)^(3/2)*(3*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-I)*A
rcTan[a*x]] + 3*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, I*ArcTan[a*x]] + Sqrt[3]*(Sq
rt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-3*I)*ArcTan[a*x]] + Sqrt[I*ArcTan[a*x]]*G
amma[1/2, (3*I)*ArcTan[a*x]])))/Sqrt[ArcTan[a*x]])/(6*a^3*c*(c + a^2*c*x^2)
^(3/2))
```

Maple [F] time = 3.001, size = 0, normalized size = 0.

$$\int x^2 \left(a^2 c x^2 + c\right)^{-\frac{5}{2}} (\arctan(ax))^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
```

```
[Out] int(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a**2*c*x**2+c)**(5/2)/atan(a*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(a^2cx^2+c\right)^{\frac{5}{2}}\arctan\left(ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^(3/2)), x)
```

3.1036 $\int \frac{x}{\left(c+a^2cx^2\right)^{5/2} \tan^{-1}(ax)^{3/2}} dx$

Optimal. Leaf size=280

$$-\frac{\sqrt{2\pi}\sqrt{a^2x^2+1}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{a^2c^2\sqrt{a^2cx^2+c}}+\frac{3\sqrt{\frac{\pi}{2}}\sqrt{a^2x^2+1}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{a^2c^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{\frac{2\pi}{3}}\sqrt{a^2x^2+1}\text{FresnelC}}{a^2c^2\sqrt{a^2cx}}$$

```
[Out] (-2*x)/(a*c*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]) + (3*Sqrt[Pi/2]*Sqrt[1
 + a^2*x^2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(a^2*c^2*Sqrt[c + a^2*c
*x^2]) - (Sqrt[2*Pi]*Sqrt[1 + a^2*x^2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]
]])/(a^2*c^2*Sqrt[c + a^2*c*x^2]) + (Sqrt[Pi/6]*Sqrt[1 + a^2*x^2]*FresnelC[
Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]])/(a^2*c^2*Sqrt[c + a^2*c*x^2]) + (Sqrt[(2*Pi)
/3]*Sqrt[1 + a^2*x^2]*FresnelC[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]])/(a^2*c^2*Sqrt
[c + a^2*c*x^2])
```

Rubi [A] time = 0.546427, antiderivative size = 280, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4968, 4971, 4970, 4406, 3304, 3352, 4905, 4904, 3312}

$$-\frac{\sqrt{2\pi}\sqrt{a^2x^2+1}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{a^2c^2\sqrt{a^2cx^2+c}}+\frac{3\sqrt{\frac{\pi}{2}}\sqrt{a^2x^2+1}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{a^2c^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{\frac{2\pi}{3}}\sqrt{a^2x^2+1}\text{FresnelC}}{a^2c^2\sqrt{a^2cx}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2)),x]
```

```
[Out] (-2*x)/(a*c*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]) + (3*Sqrt[Pi/2]*Sqrt[1
 + a^2*x^2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(a^2*c^2*Sqrt[c + a^2*c
*x^2]) - (Sqrt[2*Pi]*Sqrt[1 + a^2*x^2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]
]])/(a^2*c^2*Sqrt[c + a^2*c*x^2]) + (Sqrt[Pi/6]*Sqrt[1 + a^2*x^2]*FresnelC[
Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]])/(a^2*c^2*Sqrt[c + a^2*c*x^2]) + (Sqrt[(2*Pi)
/3]*Sqrt[1 + a^2*x^2]*FresnelC[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]])/(a^2*c^2*Sqrt
[c + a^2*c*x^2])
```

Rule 4968

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[(x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^(p
```

```
+ 1))/(b*c*d*(p + 1)), x] + (-Dist[(c*(m + 2*q + 2))/(b*(p + 1)), Int[x^(m
+ 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p + 1
)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; Fre
eQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &&
LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 4971

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[(d^(q + 1/2)*Sqrt[1 + c^2*x^2])/Sqrt[d + e*x^2],
 Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d
, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(In
tegerQ[q] || GtQ[d, 0])
```

Rule 4970

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sin[x]^m)/
Cos[x]^(m + 2*(q + 1)), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4905

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[(d^(q + 1/2)*Sqrt[1 + c^2*x^2])/Sqrt[d + e*x^2], Int[(1 + c
^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
```

```
EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

Rule 4904

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\int \frac{x}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{3/2}}\,dx = -\frac{2x}{ac\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}} + \frac{2\int \frac{1}{\left(c+a^2cx^2\right)^{5/2}\sqrt{\tan^{-1}(ax)}}\,dx}{a} - (4a)\int \frac{x}{\left(c+a^2cx^2\right)^{5/}}$$

$$= -\frac{2x}{ac\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}} + \frac{\left(2\sqrt{1+a^2x^2}\right)\int \frac{1}{\left(1+a^2x^2\right)^{5/2}\sqrt{\tan^{-1}(ax)}}\,dx}{ac^2\sqrt{c+a^2cx^2}} - \frac{\left(4a\sqrt{1+}\right.}{}$$

$$= -\frac{2x}{ac\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}} + \frac{\left(2\sqrt{1+a^2x^2}\right)\mathrm{Subst}\left(\int \frac{\cos^3(x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{a^2c^2\sqrt{c+a^2cx^2}} -$$

$$= -\frac{2x}{ac\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}} + \frac{\left(2\sqrt{1+a^2x^2}\right)\mathrm{Subst}\left(\int \left(\frac{3\cos(x)}{4\sqrt{x}} + \frac{\cos(3x)}{4\sqrt{x}}\right)dx, x, \mathrm{t}\right.}{a^2c^2\sqrt{c+a^2cx^2}}$$

$$= -\frac{2x}{ac\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}} + \frac{\sqrt{1+a^2x^2}\,\mathrm{Subst}\left(\int \frac{\cos(3x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{2a^2c^2\sqrt{c+a^2cx^2}} - \frac{\sqrt{}}{}$$

$$= -\frac{2x}{ac\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}} + \frac{\sqrt{1+a^2x^2}\,\mathrm{Subst}\left(\int \cos\left(3x^2\right)dx, x, \sqrt{\tan^{-1}(ax)}\right)}{a^2c^2\sqrt{c+a^2cx^2}}$$

$$= -\frac{2x}{ac\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}} + \frac{3\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}\,\mathrm{C}\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{a^2c^2\sqrt{c+a^2cx^2}} - \frac{\sqrt{2\pi}\sqrt{1+}}{}$$

Mathematica [C] time = 0.482562, size = 299, normalized size = 1.07

$$-\frac{i\left(a^2x^2\sqrt{3a^2x^2+3}\sqrt{-i\tan^{-1}(ax)}\mathrm{Gamma}\left(\frac{1}{2},-3i\tan^{-1}(ax)\right) - a^2x^2\sqrt{3a^2x^2+3}\sqrt{i\tan^{-1}(ax)}\mathrm{Gamma}\left(\frac{1}{2},3i\tan^{-1}(ax\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2)),x]
```

```
[Out] ((-I/4)*((-8*I)*a*x + (1 + a^2*x^2)^(3/2)*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2,
 (-I)*ArcTan[a*x]] - (1 + a^2*x^2)^(3/2)*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, I*A
rcTan[a*x]] + Sqrt[3 + 3*a^2*x^2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-3*I)*
```

```
ArcTan[a*x]] + a^2*x^2*Sqrt[3 + 3*a^2*x^2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2
, (-3*I)*ArcTan[a*x]] - Sqrt[3 + 3*a^2*x^2]*Sqrt[I*ArcTan[a*x]]*Gamma[1/2,
(3*I)*ArcTan[a*x]] - a^2*x^2*Sqrt[3 + 3*a^2*x^2]*Sqrt[I*ArcTan[a*x]]*Gamma[
1/2, (3*I)*ArcTan[a*x]]))/(a^2*c^2*(1 + a^2*x^2)*Sqrt[c + a^2*c*x^2]*Sqrt[A
rcTan[a*x]])
```

Maple [F] time = 0.855, size = 0, normalized size = 0.

$$\int x \left(a^2 c x^2 + c\right)^{-\frac{5}{2}} \left(\arctan\left(ax\right)\right)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
```

```
[Out] int(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a**2*c*x**2+c)**(5/2)/atan(a*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(a^2cx^2+c\right)^{\frac{5}{2}}\arctan\left(ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^(3/2)), x)
```

3.1037 $\int \frac{1}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=157

$$-\frac{3\sqrt{\frac{\pi}{2}}\sqrt{a^2x^2+1}S\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{ac^2\sqrt{a^2cx^2+c}}-\frac{\sqrt{\frac{3\pi}{2}}\sqrt{a^2x^2+1}S\left(\sqrt{\frac{6}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{ac^2\sqrt{a^2cx^2+c}}-\frac{2}{ac\left(a^2cx^2+c\right)^{3/2}\sqrt{\tan^{-1}(ax)}}$$

```
[Out] -2/(a*c*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]) - (3*Sqrt[Pi/2]*Sqrt[1 + a
^2*x^2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(a*c^2*Sqrt[c + a^2*c*x^2])
 - (Sqrt[(3*Pi)/2]*Sqrt[1 + a^2*x^2]*FresnelS[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]]
)/(a*c^2*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.265272, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4902, 4971, 4970, 4406, 3305, 3351}

$$-\frac{3\sqrt{\frac{\pi}{2}}\sqrt{a^2x^2+1}S\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{ac^2\sqrt{a^2cx^2+c}}-\frac{\sqrt{\frac{3\pi}{2}}\sqrt{a^2x^2+1}S\left(\sqrt{\frac{6}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{ac^2\sqrt{a^2cx^2+c}}-\frac{2}{ac\left(a^2cx^2+c\right)^{3/2}\sqrt{\tan^{-1}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2)),x]
```

```
[Out] -2/(a*c*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]) - (3*Sqrt[Pi/2]*Sqrt[1 + a
^2*x^2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(a*c^2*Sqrt[c + a^2*c*x^2])
 - (Sqrt[(3*Pi)/2]*Sqrt[1 + a^2*x^2]*FresnelS[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]]
)/(a*c^2*Sqrt[c + a^2*c*x^2])
```

Rule 4902

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^(p + 1))/(b*c*d*(p
+ 1)), x] - Dist[(2*c*(q + 1))/(b*(p + 1)), Int[x*(d + e*x^2)^q*(a + b*ArcT
an[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && L
tQ[q, -1] && LtQ[p, -1]
```

Rule 4971

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[(d^(q + 1/2)*Sqrt[1 + c^2*x^2])/Sqrt[d + e*x^2],
 Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d
, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(In
tegerQ[q] || GtQ[d, 0])
```

Rule 4970

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sin[x]^m)/
Cos[x]^(m + 2*(q + 1)), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{1}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{3/2}}\,dx = -\frac{2}{ac\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}} - (6a)\int \frac{x}{\left(c+a^2cx^2\right)^{5/2}\sqrt{\tan^{-1}(ax)}}\,dx$$

$$= -\frac{2}{ac\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}} - \frac{\left(6a\sqrt{1+a^2x^2}\right)\int \frac{x}{\left(1+a^2x^2\right)^{5/2}\sqrt{\tan^{-1}(ax)}}\,dx}{c^2\sqrt{c+a^2cx^2}}$$

$$= -\frac{2}{ac\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}} - \frac{\left(6\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int \frac{\cos^2(x)\sin(x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{ac^2\sqrt{c+a^2cx^2}}$$

$$= -\frac{2}{ac\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}} - \frac{\left(6\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int \left(\frac{\sin(x)}{4\sqrt{x}} + \frac{\sin(3x)}{4\sqrt{x}}\right)\,dx, x, \tan^{-1}(ax)\right)}{ac^2\sqrt{c+a^2cx^2}}$$

$$= -\frac{2}{ac\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}} - \frac{\left(3\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int \frac{\sin(x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{2ac^2\sqrt{c+a^2cx^2}} - \ldots$$

$$= -\frac{2}{ac\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}} - \frac{\left(3\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int \sin\left(x^2\right)\,dx, x, \sqrt{\tan^{-1}(ax)}\right)}{ac^2\sqrt{c+a^2cx^2}} \ldots$$

$$= -\frac{2}{ac\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}} - \frac{3\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}S\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{ac^2\sqrt{c+a^2cx^2}} - \frac{\sqrt{\frac{3\pi}{2}}\sqrt{1+a^2 \ldots}}{ac \ldots}$$

<u>Mathematica</u> [C] time = 0.382963, size = 158, normalized size = 1.01

$$\frac{-8+\left(a^2x^2+1\right)^{3/2}\left(3\sqrt{-i\tan^{-1}(ax)}\text{Gamma}\left(\frac{1}{2},-i\tan^{-1}(ax)\right)+3\sqrt{i\tan^{-1}(ax)}\text{Gamma}\left(\frac{1}{2},i\tan^{-1}(ax)\right)+\sqrt{3}\left(\sqrt{-i\tan \ldots}\right.\right.}{4ac\left(a^2cx^2+c\right)^{3/2}\sqrt{\tan^{-1}(ax)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2)),x]
```

```
[Out] (-8 + (1 + a^2*x^2)^(3/2)*(3*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-I)*ArcTan[
a*x]] + 3*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, I*ArcTan[a*x]] + Sqrt[3]*(Sqrt[(-I
)*ArcTan[a*x]]*Gamma[1/2, (-3*I)*ArcTan[a*x]] + Sqrt[I*ArcTan[a*x]]*Gamma[1
/2, (3*I)*ArcTan[a*x]])))/(4*a*c*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]])
```

Maple [F] time = 0.763, size = 0, normalized size = 0.

$$\int \left(a^2cx^2+c\right)^{-\frac{5}{2}} \left(\arctan\left(ax\right)\right)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
```

```
[Out] int(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*c*x**2+c)**(5/2)/atan(a*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)^{\frac{5}{2}}\arctan\left(ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^(3/2)), x)
```

3.1038 $\int \frac{1}{x(c+a^2cx^2)^{5/2}\tan^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=185

$$-\frac{2\text{Unintegrable}\left(\frac{1}{x^2(a^2cx^2+c)^{5/2}\sqrt{\tan^{-1}(ax)}},x\right)}{a}-\frac{6\sqrt{2\pi}\sqrt{a^2x^2+1}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{c^2\sqrt{a^2cx^2+c}}-\frac{2\sqrt{\frac{2\pi}{3}}\sqrt{a^2x^2+1}\text{Fresnel}}{c^2\sqrt{a^2cx}}$$

```
[Out] -2/(a*c*x*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]) - (6*Sqrt[2*Pi]*Sqrt[1 +
 a^2*x^2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(c^2*Sqrt[c + a^2*c*x^2])
 - (2*Sqrt[(2*Pi)/3]*Sqrt[1 + a^2*x^2]*FresnelC[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]
]])/(c^2*Sqrt[c + a^2*c*x^2]) - (2*Unintegrable[1/(x^2*(c + a^2*c*x^2)^(5/2
)*Sqrt[ArcTan[a*x]]), x])/a
```

Rubi [A] time = 0.396891, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2)),x]
```

```
[Out] -2/(a*c*x*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]) - (6*Sqrt[2*Pi]*Sqrt[1 +
 a^2*x^2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(c^2*Sqrt[c + a^2*c*x^2])
 - (2*Sqrt[(2*Pi)/3]*Sqrt[1 + a^2*x^2]*FresnelC[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]
]])/(c^2*Sqrt[c + a^2*c*x^2]) - (2*Defer[Int][1/(x^2*(c + a^2*c*x^2)^(5/2)*
Sqrt[ArcTan[a*x]]), x])/a
```

Rubi steps

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{3/2}}\,dx = -\frac{2}{acx\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}} - \frac{2\int \frac{1}{x^2\left(c+a^2cx^2\right)^{5/2}\sqrt{\tan^{-1}(ax)}}\,dx}{a} - (8a)\int \frac{}{\left(c+a^2cx^2\right.}$$

$$= -\frac{2}{acx\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}} - \frac{2\int \frac{1}{x^2\left(c+a^2cx^2\right)^{5/2}\sqrt{\tan^{-1}(ax)}}\,dx}{a} - \frac{\left(8a\sqrt{1+a^2x^2}\right)\int}{c^2\sqrt{c}}$$

$$= -\frac{2}{acx\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}} - \frac{2\int \frac{1}{x^2\left(c+a^2cx^2\right)^{5/2}\sqrt{\tan^{-1}(ax)}}\,dx}{a} - \frac{\left(8\sqrt{1+a^2x^2}\right)\mathrm{Sul}}{}$$

$$= -\frac{2}{acx\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}} - \frac{2\int \frac{1}{x^2\left(c+a^2cx^2\right)^{5/2}\sqrt{\tan^{-1}(ax)}}\,dx}{a} - \frac{\left(8\sqrt{1+a^2x^2}\right)\mathrm{Sul}}{}$$

$$= -\frac{2}{acx\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}} - \frac{2\int \frac{1}{x^2\left(c+a^2cx^2\right)^{5/2}\sqrt{\tan^{-1}(ax)}}\,dx}{a} - \frac{\left(2\sqrt{1+a^2x^2}\right)\mathrm{Sul}}{}$$

$$= -\frac{2}{acx\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}} - \frac{2\int \frac{1}{x^2\left(c+a^2cx^2\right)^{5/2}\sqrt{\tan^{-1}(ax)}}\,dx}{a} - \frac{\left(4\sqrt{1+a^2x^2}\right)\mathrm{Sul}}{}$$

$$= -\frac{2}{acx\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}} - \frac{6\sqrt{2\pi}\sqrt{1+a^2x^2}\,C\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{c^2\sqrt{c+a^2cx^2}} - \frac{2\sqrt{\frac{2\pi}{3}}\sqrt{1}}{}$$

Mathematica [A] time = 5.54463, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2)),x]
```

```
[Out] Integrate[1/(x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2)), x]
```

Maple [A] time = 0.746, size = 0, normalized size = 0.

$$\int \frac{1}{x} \left(a^2 c x^2 + c\right)^{-\frac{5}{2}} (\arctan (ax))^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
```

```
[Out] int(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a**2*c*x**2+c)**(5/2)/atan(a*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)^{\frac{5}{2}} x \arctan\left(ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^(5/2)*x*arctan(a*x)^(3/2)), x)
```

3.1039 $\int \frac{1}{x^2\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=98

$$-\frac{4\text{Unintegrable}\left(\frac{1}{x^3\left(a^2cx^2+c\right)^{5/2}\sqrt{\tan^{-1}(ax)}},x\right)}{a}-10a\text{Unintegrable}\left(\frac{1}{x\left(a^2cx^2+c\right)^{5/2}\sqrt{\tan^{-1}(ax)}},x\right)-\frac{2}{acx^2\left(a^2cx^2+c\right)^{3/2}\ldots}$$

```
[Out] -2/(a*c*x^2*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]) - (4*Unintegrable[1/(x
^3*(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]), x])/a - 10*a*Unintegrable[1/(x
*(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]), x]
```

Rubi [A] time = 0.369258, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2)),x]
```

```
[Out] -2/(a*c*x^2*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]) - (4*Defer[Int][1/(x^3
*(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]), x])/a - 10*a*Defer[Int][1/(x*(c
+ a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]), x]
```

Rubi steps

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{3/2}}\,dx=-\frac{2}{acx^2\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}}-\frac{4\int \frac{1}{x^3\left(c+a^2cx^2\right)^{5/2}\sqrt{\tan^{-1}(ax)}}\,dx}{a}-(10a)\int \frac{\ldots}{x\left(c+\ldots\right.}$$

Mathematica [A] time = 14.5737, size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2)),x]
```

```
[Out] Integrate[1/(x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2)), x]
```

Maple [A] time = 0.797, size = 0, normalized size = 0.

$$\int \frac{1}{x^2} \left(a^2 c x^2 + c\right)^{-\frac{5}{2}} (\arctan (ax))^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
```

```
[Out] int(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="fricas"
)
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a**2*c*x**2+c)**(5/2)/atan(a*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)^{\frac{5}{2}}x^2\arctan\left(ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^(5/2)*x^2*arctan(a*x)^(3/2)), x)
```

3.1040 $\int \frac{1}{x^3\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=98

$$-\frac{6\text{Unintegrable}\left(\frac{1}{x^4\left(a^2cx^2+c\right)^{5/2}\sqrt{\tan^{-1}(ax)}},x\right)}{a}-12a\text{Unintegrable}\left(\frac{1}{x^2\left(a^2cx^2+c\right)^{5/2}\sqrt{\tan^{-1}(ax)}},x\right)-\frac{2}{acx^3\left(a^2cx^2+c\right)^{3/2}}$$

```
[Out] -2/(a*c*x^3*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]) - (6*Unintegrable[1/(x
^4*(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]), x])/a - 12*a*Unintegrable[1/(x
^2*(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]), x]
```

Rubi [A] time = 0.364653, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^3\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^3*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2)),x]
```

```
[Out] -2/(a*c*x^3*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]) - (6*Defer[Int][1/(x^4
*(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]), x])/a - 12*a*Defer[Int][1/(x^2*(
c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]), x]
```

Rubi steps

$$\int \frac{1}{x^3\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{3/2}}\,dx=-\frac{2}{acx^3\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}}-\frac{6\int \frac{1}{x^4\left(c+a^2cx^2\right)^{5/2}\sqrt{\tan^{-1}(ax)}}\,dx}{a}-(12a)\int \frac{}{x^2\left(c+\right.}$$

Mathematica [A] time = 24.6788, size = 0, normalized size = 0.

$$\int \frac{1}{x^3\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^3*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2)),x]
```

```
[Out] Integrate[1/(x^3*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2)), x]
```

Maple [A] time = 0.909, size = 0, normalized size = 0.

$$\int \frac{1}{x^3} \left(a^2 c x^2 + c\right)^{-\frac{5}{2}} (\arctan (ax))^{-\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
```

```
[Out] int(1/x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="fricas"
)
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a**2*c*x**2+c)**(5/2)/atan(a*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)^{\frac{5}{2}}x^3\arctan\left(ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^(5/2)*x^3*arctan(a*x)^(3/2)), x)
```

3.1041 $\int \frac{1}{x^4\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=98

$$-\frac{8\text{Unintegrable}\left(\frac{1}{x^5\left(a^2cx^2+c\right)^{5/2}\sqrt{\tan^{-1}(ax)}},x\right)}{a}-14a\text{Unintegrable}\left(\frac{1}{x^3\left(a^2cx^2+c\right)^{5/2}\sqrt{\tan^{-1}(ax)}},x\right)-\frac{2}{acx^4\left(a^2cx^2+c\right)^{3/2}\ldots}$$

```
[Out] -2/(a*c*x^4*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]) - (8*Unintegrable[1/(x
^5*(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]), x])/a - 14*a*Unintegrable[1/(x
^3*(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]), x]
```

Rubi [A] time = 0.368755, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^4\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^4*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2)),x]
```

```
[Out] -2/(a*c*x^4*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]) - (8*Defer[Int][1/(x^5
*(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]), x])/a - 14*a*Defer[Int][1/(x^3*(
c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]), x]
```

Rubi steps

$$\int \frac{1}{x^4\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{3/2}}\,dx=-\frac{2}{acx^4\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}}-\frac{8\int \frac{1}{x^5\left(c+a^2cx^2\right)^{5/2}\sqrt{\tan^{-1}(ax)}}\,dx}{a}-(14a)\int \frac{}{x^3\left(c\ldots\right.}$$

Mathematica [A] time = 20.7907, size = 0, normalized size = 0.

$$\int \frac{1}{x^4\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^4*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2)),x]
```

```
[Out] Integrate[1/(x^4*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2)), x]
```

Maple [A] time = 2.022, size = 0, normalized size = 0.

$$\int \frac{1}{x^4}\left(a^2cx^2+c\right)^{-\frac{5}{2}}\left(\arctan\left(ax\right)\right)^{-\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
```

```
[Out] int(1/x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="fricas"
)
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(a**2*c*x**2+c)**(5/2)/atan(a*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)^{\frac{5}{2}}x^4\arctan\left(ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^(5/2)*x^4*arctan(a*x)^(3/2)), x)
```

3.1042 $\int \frac{x^m(c+a^2cx^2)}{\tan^{-1}(ax)^{5/2}} dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{x^m\left(a^2cx^2+c\right)}{\tan^{-1}(ax)^{5/2}},x\right)$$

```
[Out] Unintegrable[(x^m*(c + a^2*c*x^2))/ArcTan[a*x]^(5/2), x]
```

Rubi [A] time = 0.0360636, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m\left(c+a^2cx^2\right)}{\tan^{-1}(ax)^{5/2}} dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*(c + a^2*c*x^2))/ArcTan[a*x]^(5/2),x]

[Out] Defer[Int][(x^m*(c + a^2*c*x^2))/ArcTan[a*x]^(5/2), x]
```

Rubi steps

$$\int \frac{x^m\left(c+a^2cx^2\right)}{\tan^{-1}(ax)^{5/2}} dx = \int \frac{x^m\left(c+a^2cx^2\right)}{\tan^{-1}(ax)^{5/2}} dx$$

Mathematica [A] time = 1.95451, size = 0, normalized size = 0.

$$\int \frac{x^m\left(c+a^2cx^2\right)}{\tan^{-1}(ax)^{5/2}} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*(c + a^2*c*x^2))/ArcTan[a*x]^(5/2),x]
```

[Out] Integrate[(x^m*(c + a^2*c*x^2))/ArcTan[a*x]^(5/2), x]

Maple [A] time = 0.764, size = 0, normalized size = 0.

$$\int x^{m} \left(a^{2} c\,x^{2}+c \right) \left(\arctan \left(a x \right)\right)^{-\frac{5}{2}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m*(a^2*c*x^2+c)/arctan(a*x)^(5/2),x)

[Out] int(x^m*(a^2*c*x^2+c)/arctan(a*x)^(5/2),x)

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m*(a^2*c*x^2+c)/arctan(a*x)^(5/2),x, algorithm="maxima")

[Out] Exception raised: RuntimeError

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^{2} c x^{2} + c\right)x^{m}}{\arctan\left(a x\right)^{\frac{5}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m*(a^2*c*x^2+c)/arctan(a*x)^(5/2),x, algorithm="fricas")

[Out] integral((a^2*c*x^2 + c)*x^m/arctan(a*x)^(5/2), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)/atan(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a^2cx^2 + c)x^m}{\arctan(ax)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)*x^m/arctan(a*x)^(5/2), x)
```

3.1043 $\int \frac{x(c+a^2cx^2)}{\tan^{-1}(ax)^{5/2}}\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{x\left(a^2cx^2+c\right)}{\tan^{-1}(ax)^{5/2}},x\right)$$

```
[Out] Unintegrable[(x*(c + a^2*c*x^2))/ArcTan[a*x]^(5/2), x]
```

Rubi [A] time = 0.0227916, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x\left(c+a^2cx^2\right)}{\tan^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x*(c + a^2*c*x^2))/ArcTan[a*x]^(5/2),x]

[Out] Defer[Int][(x*(c + a^2*c*x^2))/ArcTan[a*x]^(5/2), x]
```

Rubi steps

$$\int \frac{x\left(c+a^2cx^2\right)}{\tan^{-1}(ax)^{5/2}}\,dx=\int \frac{x\left(c+a^2cx^2\right)}{\tan^{-1}(ax)^{5/2}}\,dx$$

Mathematica [A] time = 2.54147, size = 0, normalized size = 0.

$$\int \frac{x\left(c+a^2cx^2\right)}{\tan^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x*(c + a^2*c*x^2))/ArcTan[a*x]^(5/2),x]
```

```
[Out] Integrate[(x*(c + a^2*c*x^2))/ArcTan[a*x]^(5/2), x]
```

Maple [A] time = 0.389, size = 0, normalized size = 0.

$$\int x \left(a^{2} c x^{2}+c \right) \left(\arctan \left(a x \right)\right)^{-\frac{5}{2}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)/arctan(a*x)^(5/2),x)
```

```
[Out] int(x*(a^2*c*x^2+c)/arctan(a*x)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$c \left(\int \frac{x}{\operatorname{atan}^{\frac{5}{2}}{\left(a x \right)}}\, dx + \int \frac{a^{2} x^{3}}{\operatorname{atan}^{\frac{5}{2}}{\left(a x \right)}}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)/atan(a*x)**(5/2),x)
```

```
[Out] c*(Integral(x/atan(a*x)**(5/2), x) + Integral(a**2*x**3/atan(a*x)**(5/2), x
))
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a^2cx^2+c)x}{\arctan(ax)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)*x/arctan(a*x)^(5/2), x)
```

3.1044 $\int \frac{c+a^2cx^2}{\tan^{-1}(ax)^{5/2}}\,dx$

Optimal. Leaf size=21

$$\text{Unintegrable}\left(\frac{a^2cx^2+c}{\tan^{-1}(ax)^{5/2}},x\right)$$

```
[Out] Unintegrable[(c + a^2*c*x^2)/ArcTan[a*x]^(5/2), x]
```

Rubi [A] time = 0.0127875, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{c+a^2cx^2}{\tan^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)/ArcTan[a*x]^(5/2),x]
```

```
[Out] Defer[Int][(c + a^2*c*x^2)/ArcTan[a*x]^(5/2), x]
```

Rubi steps

$$\int \frac{c+a^2cx^2}{\tan^{-1}(ax)^{5/2}}\,dx=\int \frac{c+a^2cx^2}{\tan^{-1}(ax)^{5/2}}\,dx$$

Mathematica [A] time = 1.95498, size = 0, normalized size = 0.

$$\int \frac{c+a^2cx^2}{\tan^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)/ArcTan[a*x]^(5/2),x]
```

```
[Out] Integrate[(c + a^2*c*x^2)/ArcTan[a*x]^(5/2), x]
```

Maple [A] time = 0.266, size = 0, normalized size = 0.

$$\int \left(a^{2} c \,x^{2}+c \right) \arctan \left(a x \right)^{-\frac{5}{2}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)/arctan(a*x)^(5/2),x)
```

```
[Out] int((a^2*c*x^2+c)/arctan(a*x)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$c \left(\int \frac{a^{2} x^{2}}{\operatorname{atan}^{\frac{5}{2}}{\left(a x \right)}}\, dx + \int \frac{1}{\operatorname{atan}^{\frac{5}{2}}{\left(a x \right)}}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)/atan(a*x)**(5/2),x)
```

```
[Out] c*(Integral(a**2*x**2/atan(a*x)**(5/2), x) + Integral(atan(a*x)**(-5/2), x)
)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{a^2cx^2 + c}{\arctan (ax)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)/arctan(a*x)^(5/2), x)
```

3.1045 $\int \frac{c+a^2cx^2}{x\tan^{-1}(ax)^{5/2}}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{a^2cx^2+c}{x\tan^{-1}(ax)^{5/2}},x\right)$$

```
[Out] Unintegrable[(c + a^2*c*x^2)/(x*ArcTan[a*x]^(5/2)), x]
```

Rubi [A] time = 0.0319057, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{c+a^2cx^2}{x\tan^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)/(x*ArcTan[a*x]^(5/2)),x]
```

```
[Out] Defer[Int][(c + a^2*c*x^2)/(x*ArcTan[a*x]^(5/2)), x]
```

Rubi steps

$$\int \frac{c+a^2cx^2}{x\tan^{-1}(ax)^{5/2}}\,dx=\int \frac{c+a^2cx^2}{x\tan^{-1}(ax)^{5/2}}\,dx$$

Mathematica [A] time = 5.29517, size = 0, normalized size = 0.

$$\int \frac{c+a^2cx^2}{x\tan^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)/(x*ArcTan[a*x]^(5/2)),x]
```

```
[Out] Integrate[(c + a^2*c*x^2)/(x*ArcTan[a*x]^(5/2)), x]
```

Maple [A] time = 0.391, size = 0, normalized size = 0.

$$\int \frac{a^{2} c x^{2}+c}{x}\left(\arctan \left(a x\right)\right)^{-\frac{5}{2}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)/x/arctan(a*x)^(5/2),x)
```

```
[Out] int((a^2*c*x^2+c)/x/arctan(a*x)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)/x/arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)/x/arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$c\left(\int \frac{1}{x \operatorname{atan}^{\frac{5}{2}}{\left(a x \right)}}\, dx + \int \frac{a^{2} x}{\operatorname{atan}^{\frac{5}{2}}{\left(a x \right)}}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)/x/atan(a*x)**(5/2),x)
```

```
[Out] c*(Integral(1/(x*atan(a*x)**(5/2)), x) + Integral(a**2*x/atan(a*x)**(5/2),
x))
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{a^2cx^2 + c}{x \arctan{(ax)}^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)/x/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)/(x*arctan(a*x)^(5/2)), x)
```

3.1046 $\int \frac{x^m(c+a^2cx^2)^2}{\tan^{-1}(ax)^{5/2}}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{x^m\left(a^2cx^2+c\right)^2}{\tan^{-1}(ax)^{5/2}},x\right)$$

```
[Out] Unintegrable[(x^m*(c + a^2*c*x^2)^2)/ArcTan[a*x]^(5/2), x]
```

Rubi [A] time = 0.0556037, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m\left(c+a^2cx^2\right)^2}{\tan^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*(c + a^2*c*x^2)^2)/ArcTan[a*x]^(5/2),x]
```

```
[Out] Defer[Int][(x^m*(c + a^2*c*x^2)^2)/ArcTan[a*x]^(5/2), x]
```

Rubi steps

$$\int \frac{x^m\left(c+a^2cx^2\right)^2}{\tan^{-1}(ax)^{5/2}}\,dx=\int \frac{x^m\left(c+a^2cx^2\right)^2}{\tan^{-1}(ax)^{5/2}}\,dx$$

Mathematica [A] time = 1.3597, size = 0, normalized size = 0.

$$\int \frac{x^m\left(c+a^2cx^2\right)^2}{\tan^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*(c + a^2*c*x^2)^2)/ArcTan[a*x]^(5/2),x]
```

```
[Out] Integrate[(x^m*(c + a^2*c*x^2)^2)/ArcTan[a*x]^(5/2), x]
```

Maple [A] time = 0.922, size = 0, normalized size = 0.

$$\int x^m \left(a^2 c x^2 + c\right)^2 (\arctan (ax))^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^4 c^2 x^4 + 2 a^2 c^2 x^2 + c^2\right) x^m}{\arctan (ax)^{\frac{5}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*x^m/arctan(a*x)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**2/atan(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2 + c\right)^2 x^m}{\arctan\left(ax\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^2*x^m/arctan(a*x)^(5/2), x)
```

3.1047 $\int \frac{x(c+a^2cx^2)^2}{\tan^{-1}(ax)^{5/2}}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{x\left(a^2cx^2+c\right)^2}{\tan^{-1}(ax)^{5/2}},x\right)$$

```
[Out] Unintegrable[(x*(c + a^2*c*x^2)^2)/ArcTan[a*x]^(5/2), x]
```

Rubi [A] time = 0.0372311, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x\left(c+a^2cx^2\right)^2}{\tan^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x*(c + a^2*c*x^2)^2)/ArcTan[a*x]^(5/2),x]
```

```
[Out] Defer[Int][(x*(c + a^2*c*x^2)^2)/ArcTan[a*x]^(5/2), x]
```

Rubi steps

$$\int \frac{x\left(c+a^2cx^2\right)^2}{\tan^{-1}(ax)^{5/2}}\,dx=\int \frac{x\left(c+a^2cx^2\right)^2}{\tan^{-1}(ax)^{5/2}}\,dx$$

Mathematica [A] time = 2.0289, size = 0, normalized size = 0.

$$\int \frac{x\left(c+a^2cx^2\right)^2}{\tan^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x*(c + a^2*c*x^2)^2)/ArcTan[a*x]^(5/2),x]
```

```
[Out] Integrate[(x*(c + a^2*c*x^2)^2)/ArcTan[a*x]^(5/2), x]
```

Maple [A] time = 0.559, size = 0, normalized size = 0.

$$\int x\left(a^{2}c x^{2}+c\right)^{2} (\arctan(ax))^{-\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x)
```

```
[Out] int(x*(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$c^{2}\left(\int \frac{x}{\operatorname{atan}^{\frac{5}{2}}(ax)}\, dx + \int \frac{2a^{2}x^{3}}{\operatorname{atan}^{\frac{5}{2}}(ax)}\, dx + \int \frac{a^{4}x^{5}}{\operatorname{atan}^{\frac{5}{2}}(ax)}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**2/atan(a*x)**(5/2),x)
```

```
[Out] c**2*(Integral(x/atan(a*x)**(5/2), x) + Integral(2*a**2*x**3/atan(a*x)**(5/
2), x) + Integral(a**4*x**5/atan(a*x)**(5/2), x))
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^2x}{\arctan\left(ax\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^2*x/arctan(a*x)^(5/2), x)
```

3.1048 $\int \frac{(c+a^2cx^2)^2}{\tan^{-1}(ax)^{5/2}}\,dx$

Optimal. Leaf size=23

$$\text{Unintegrable}\left(\frac{(a^2cx^2+c)^2}{\tan^{-1}(ax)^{5/2}},x\right)$$

[Out] Unintegrable[(c + a^2*c*x^2)^2/ArcTan[a*x]^(5/2), x]

Rubi [A] time = 0.0220499, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(c+a^2cx^2)^2}{\tan^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

[In] Int[(c + a^2*c*x^2)^2/ArcTan[a*x]^(5/2),x]

[Out] Defer[Int][(c + a^2*c*x^2)^2/ArcTan[a*x]^(5/2), x]

Rubi steps

$$\int \frac{(c+a^2cx^2)^2}{\tan^{-1}(ax)^{5/2}}\,dx = \int \frac{(c+a^2cx^2)^2}{\tan^{-1}(ax)^{5/2}}\,dx$$

Mathematica [A] time = 1.4929, size = 0, normalized size = 0.

$$\int \frac{(c+a^2cx^2)^2}{\tan^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

[In] Integrate[(c + a^2*c*x^2)^2/ArcTan[a*x]^(5/2),x]

```
[Out] Integrate[(c + a^2*c*x^2)^2/ArcTan[a*x]^(5/2), x]
```

Maple [A] time = 0.437, size = 0, normalized size = 0.

$$\int \left(a^{2} c \,x^{2}+c \right)^{2} \left(\arctan \left(a x \right)\right)^{-\frac{5}{2}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x)
```

```
[Out] int((a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$c^{2} \left(\int \frac{2 a^{2} x^{2}}{\operatorname{atan}^{\frac{5}{2}}{\left(a x \right)}}\, dx + \int \frac{a^{4} x^{4}}{\operatorname{atan}^{\frac{5}{2}}{\left(a x \right)}}\, dx + \int \frac{1}{\operatorname{atan}^{\frac{5}{2}}{\left(a x \right)}}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**2/atan(a*x)**(5/2),x)
```

```
[Out] c**2*(Integral(2*a**2*x**2/atan(a*x)**(5/2), x) + Integral(a**4*x**4/atan(a
*x)**(5/2), x) + Integral(atan(a*x)**(-5/2), x))
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2 c x^2 + c\right)^2}{\arctan\left(a x\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^2/arctan(a*x)^(5/2), x)
```

3.1049 $\int \frac{\left(c+a^2cx^2\right)^2}{x\tan^{-1}(ax)^{5/2}}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{\left(a^2cx^2+c\right)^2}{x\tan^{-1}(ax)^{5/2}},x\right)$$

```
[Out] Unintegrable[(c + a^2*c*x^2)^2/(x*ArcTan[a*x]^(5/2)), x]
```

Rubi [A] time = 0.0504415, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(c+a^2cx^2\right)^2}{x\tan^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^2/(x*ArcTan[a*x]^(5/2)),x]

[Out] Defer[Int][(c + a^2*c*x^2)^2/(x*ArcTan[a*x]^(5/2)), x]
```

Rubi steps

$$\int \frac{\left(c+a^2cx^2\right)^2}{x\tan^{-1}(ax)^{5/2}}\,dx=\int \frac{\left(c+a^2cx^2\right)^2}{x\tan^{-1}(ax)^{5/2}}\,dx$$

Mathematica [A] time = 2.99239, size = 0, normalized size = 0.

$$\int \frac{\left(c+a^2cx^2\right)^2}{x\tan^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^2/(x*ArcTan[a*x]^(5/2)),x]
```

[Out] Integrate[(c + a^2*c*x^2)^2/(x*ArcTan[a*x]^(5/2)), x]

Maple [A] time = 0.506, size = 0, normalized size = 0.

$$\int \frac{\left(a^{2} c \,x^{2}+c \right)^{2}}{x}\left(\arctan \left(a x \right)\right)^{-\frac{5}{2}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^2*c*x^2+c)^2/x/arctan(a*x)^(5/2),x)

[Out] int((a^2*c*x^2+c)^2/x/arctan(a*x)^(5/2),x)

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2*c*x^2+c)^2/x/arctan(a*x)^(5/2),x, algorithm="maxima")

[Out] Exception raised: RuntimeError

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2*c*x^2+c)^2/x/arctan(a*x)^(5/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

Sympy [A] time = 0., size = 0, normalized size = 0.

$$c^2\left(\int \frac{1}{x\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx + \int \frac{2a^2x}{\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx + \int \frac{a^4x^3}{\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**2/x/atan(a*x)**(5/2),x)
```

```
[Out] c**2*(Integral(1/(x*atan(a*x)**(5/2)), x) + Integral(2*a**2*x/atan(a*x)**(5
/2), x) + Integral(a**4*x**3/atan(a*x)**(5/2), x))
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2 + c\right)^2}{x\arctan(ax)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2/x/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^2/(x*arctan(a*x)^(5/2)), x)
```

3.1050 $\int \frac{x^m(c+a^2cx^2)^3}{\tan^{-1}(ax)^{5/2}}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{x^m\left(a^2cx^2+c\right)^3}{\tan^{-1}(ax)^{5/2}},x\right)$$

```
[Out] Unintegrable[(x^m*(c + a^2*c*x^2)^3)/ArcTan[a*x]^(5/2), x]
```

Rubi [A] time = 0.0564165, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m\left(c+a^2cx^2\right)^3}{\tan^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*(c + a^2*c*x^2)^3)/ArcTan[a*x]^(5/2),x]
```

```
[Out] Defer[Int][(x^m*(c + a^2*c*x^2)^3)/ArcTan[a*x]^(5/2), x]
```

Rubi steps

$$\int \frac{x^m\left(c+a^2cx^2\right)^3}{\tan^{-1}(ax)^{5/2}}\,dx=\int \frac{x^m\left(c+a^2cx^2\right)^3}{\tan^{-1}(ax)^{5/2}}\,dx$$

Mathematica [A] time = 0.866928, size = 0, normalized size = 0.

$$\int \frac{x^m\left(c+a^2cx^2\right)^3}{\tan^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*(c + a^2*c*x^2)^3)/ArcTan[a*x]^(5/2),x]
```

```
[Out] Integrate[(x^m*(c + a^2*c*x^2)^3)/ArcTan[a*x]^(5/2), x]
```

Maple [A] time = 0.964, size = 0, normalized size = 0.

$$\int x^m \left(a^2 c x^2 + c\right)^3 (\arctan (ax))^{-\frac{5}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^6 c^3 x^6 + 3\, a^4 c^3 x^4 + 3\, a^2 c^3 x^2 + c^3\right) x^m}{\arctan (ax)^{\frac{5}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*x^m/arctan(a*x
)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**3/atan(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^3x^m}{\arctan\left(ax\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^3*x^m/arctan(a*x)^(5/2), x)
```

3.1051 $\int \frac{x\left(c+a^2cx^2\right)^3}{\tan^{-1}(ax)^{5/2}}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{x\left(a^2cx^2+c\right)^3}{\tan^{-1}(ax)^{5/2}},x\right)$$

```
[Out] Unintegrable[(x*(c + a^2*c*x^2)^3)/ArcTan[a*x]^(5/2), x]
```

Rubi [A] time = 0.0366252, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x\left(c+a^2cx^2\right)^3}{\tan^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x*(c + a^2*c*x^2)^3)/ArcTan[a*x]^(5/2),x]
```

```
[Out] Defer[Int][(x*(c + a^2*c*x^2)^3)/ArcTan[a*x]^(5/2), x]
```

Rubi steps

$$\int \frac{x\left(c+a^2cx^2\right)^3}{\tan^{-1}(ax)^{5/2}}\,dx=\int \frac{x\left(c+a^2cx^2\right)^3}{\tan^{-1}(ax)^{5/2}}\,dx$$

Mathematica [A] time = 2.05577, size = 0, normalized size = 0.

$$\int \frac{x\left(c+a^2cx^2\right)^3}{\tan^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x*(c + a^2*c*x^2)^3)/ArcTan[a*x]^(5/2),x]
```

```
[Out] Integrate[(x*(c + a^2*c*x^2)^3)/ArcTan[a*x]^(5/2), x]
```

Maple [A] time = 0.817, size = 0, normalized size = 0.

$$\int x \left(a^2 c x^2 + c\right)^3 (\arctan (ax))^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x)
```

```
[Out] int(x*(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**3/atan(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^3 x}{\arctan\left(ax\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^3*x/arctan(a*x)^(5/2), x)
```

3.1052 $\int \frac{(c+a^2cx^2)^3}{\tan^{-1}(ax)^{5/2}}\,dx$

Optimal. Leaf size=23

$$\text{Unintegrable}\left(\frac{(a^2cx^2+c)^3}{\tan^{-1}(ax)^{5/2}},x\right)$$

```
[Out] Unintegrable[(c + a^2*c*x^2)^3/ArcTan[a*x]^(5/2), x]
```

Rubi [A] time = 0.0217256, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{(c+a^2cx^2)^3}{\tan^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^3/ArcTan[a*x]^(5/2),x]
```

```
[Out] Defer[Int][(c + a^2*c*x^2)^3/ArcTan[a*x]^(5/2), x]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^3}{\tan^{-1}(ax)^{5/2}}\,dx = \int \frac{(c+a^2cx^2)^3}{\tan^{-1}(ax)^{5/2}}\,dx$$

Mathematica [A] time = 1.53599, size = 0, normalized size = 0.

$$\int \frac{(c+a^2cx^2)^3}{\tan^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^3/ArcTan[a*x]^(5/2),x]
```

```
[Out] Integrate[(c + a^2*c*x^2)^3/ArcTan[a*x]^(5/2), x]
```

Maple [A] time = 0.638, size = 0, normalized size = 0.

$$\int \left(a^{2} c x^{2}+c\right)^{3} \left(\arctan \left(a x\right)\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x)
```

```
[Out] int((a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$c^{3} \left(\int \frac{3 a^{2} x^{2}}{\operatorname{atan}^{\frac{5}{2}}{\left(a x \right)}}\, dx + \int \frac{3 a^{4} x^{4}}{\operatorname{atan}^{\frac{5}{2}}{\left(a x \right)}}\, dx + \int \frac{a^{6} x^{6}}{\operatorname{atan}^{\frac{5}{2}}{\left(a x \right)}}\, dx + \int \frac{1}{\operatorname{atan}^{\frac{5}{2}}{\left(a x \right)}}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**3/atan(a*x)**(5/2),x)
```

```
[Out] c**3*(Integral(3*a**2*x**2/atan(a*x)**(5/2), x) + Integral(3*a**4*x**4/atan
(a*x)**(5/2), x) + Integral(a**6*x**6/atan(a*x)**(5/2), x) + Integral(atan(
a*x)**(-5/2), x))
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^3}{\arctan\left(ax\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^3/arctan(a*x)^(5/2), x)
```

3.1053 $\int \frac{(c+a^2cx^2)^3}{x\tan^{-1}(ax)^{5/2}}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{(a^2cx^2+c)^3}{x\tan^{-1}(ax)^{5/2}},x\right)$$

```
[Out] Unintegrable[(c + a^2*c*x^2)^3/(x*ArcTan[a*x]^(5/2)), x]
```

Rubi [A] time = 0.0511725, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(c+a^2cx^2)^3}{x\tan^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^3/(x*ArcTan[a*x]^(5/2)),x]
```

```
[Out] Defer[Int][(c + a^2*c*x^2)^3/(x*ArcTan[a*x]^(5/2)), x]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^3}{x\tan^{-1}(ax)^{5/2}}\,dx=\int \frac{(c+a^2cx^2)^3}{x\tan^{-1}(ax)^{5/2}}\,dx$$

Mathematica [A] time = 3.78255, size = 0, normalized size = 0.

$$\int \frac{(c+a^2cx^2)^3}{x\tan^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^3/(x*ArcTan[a*x]^(5/2)),x]
```

```
[Out] Integrate[(c + a^2*c*x^2)^3/(x*ArcTan[a*x]^(5/2)), x]
```

Maple [A] time = 0.705, size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^3}{x}(\arctan(ax))^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^3/x/arctan(a*x)^(5/2),x)
```

```
[Out] int((a^2*c*x^2+c)^3/x/arctan(a*x)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3/x/arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3/x/arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$c^{3}\left(\int \frac{1}{x\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx+\int \frac{3a^{2}x}{\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx+\int \frac{3a^{4}x^{3}}{\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx+\int \frac{a^{6}x^{5}}{\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**3/x/atan(a*x)**(5/2),x)
```

```
[Out] c**3*(Integral(1/(x*atan(a*x)**(5/2)), x) + Integral(3*a**2*x/atan(a*x)**(5
/2), x) + Integral(3*a**4*x**3/atan(a*x)**(5/2), x) + Integral(a**6*x**5/at
an(a*x)**(5/2), x))
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^{2}cx^{2}+c\right)^{3}}{x\arctan(ax)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3/x/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^3/(x*arctan(a*x)^(5/2)), x)
```

3.1054 $\int \frac{x^m}{(c+a^2cx^2)\tan^{-1}(ax)^{5/2}}\,dx$

Optimal. Leaf size=49

$$\frac{2m\text{Unintegrable}\left(\frac{x^{m-1}}{\tan^{-1}(ax)^{3/2}},x\right)}{3ac}-\frac{2x^m}{3ac\tan^{-1}(ax)^{3/2}}$$

```
[Out] (-2*x^m)/(3*a*c*ArcTan[a*x]^(3/2)) + (2*m*Unintegrable[x^(-1 + m)/ArcTan[a*
x]^(3/2), x])/(3*a*c)
```

Rubi [A] time = 0.0817706, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m}{(c+a^2cx^2)\tan^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m/((c + a^2*c*x^2)*ArcTan[a*x]^(5/2)),x]
```

```
[Out] (-2*x^m)/(3*a*c*ArcTan[a*x]^(3/2)) + (2*m*Defer[Int][x^(-1 + m)/ArcTan[a*x]
^(3/2), x])/(3*a*c)
```

Rubi steps

$$\int \frac{x^m}{(c+a^2cx^2)\tan^{-1}(ax)^{5/2}}\,dx=-\frac{2x^m}{3ac\tan^{-1}(ax)^{3/2}}+\frac{(2m)\int \frac{x^{-1+m}}{\tan^{-1}(ax)^{3/2}}\,dx}{3ac}$$

Mathematica [A] time = 0.725964, size = 0, normalized size = 0.

$$\int \frac{x^m}{(c+a^2cx^2)\tan^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m/((c + a^2*c*x^2)*ArcTan[a*x]^(5/2)),x]
```

```
[Out] Integrate[x^m/((c + a^2*c*x^2)*ArcTan[a*x]^(5/2)), x]
```

Maple [A] time = 0.575, size = 0, normalized size = 0.

$$\int \frac{x^m}{a^2cx^2+c}(\arctan(ax))^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a^2*c*x^2+c)/arctan(a*x)^(5/2),x)
```

```
[Out] int(x^m/(a^2*c*x^2+c)/arctan(a*x)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^m}{\left(a^2cx^2+c\right)\arctan(ax)^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(x^m/((a^2*c*x^2 + c)*arctan(a*x)^(5/2)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(a**2*c*x**2+c)/atan(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(a^2cx^2+c\right)\arctan\left(ax\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x^m/((a^2*c*x^2 + c)*arctan(a*x)^(5/2)), x)
```

3.1055 $\int \frac{x}{(c+a^2cx^2)\tan^{-1}(ax)^{5/2}}\,dx$

Optimal. Leaf size=40

$$\frac{2\text{Unintegrable}\left(\frac{1}{\tan^{-1}(ax)^{3/2}},x\right)}{3ac}-\frac{2x}{3ac\tan^{-1}(ax)^{3/2}}$$

```
[Out] (-2*x)/(3*a*c*ArcTan[a*x]^(3/2)) + (2*Unintegrable[ArcTan[a*x]^(-3/2), x])/
(3*a*c)
```

Rubi [A] time = 0.0488155, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x}{(c+a^2cx^2)\tan^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x/((c + a^2*c*x^2)*ArcTan[a*x]^(5/2)),x]
```

```
[Out] (-2*x)/(3*a*c*ArcTan[a*x]^(3/2)) + (2*Defer[Int][ArcTan[a*x]^(-3/2), x])/(3
*a*c)
```

Rubi steps

$$\int \frac{x}{(c+a^2cx^2)\tan^{-1}(ax)^{5/2}}\,dx=-\frac{2x}{3ac\tan^{-1}(ax)^{3/2}}+\frac{2\int \frac{1}{\tan^{-1}(ax)^{3/2}}\,dx}{3ac}$$

Mathematica [A] time = 1.37679, size = 0, normalized size = 0.

$$\int \frac{x}{(c+a^2cx^2)\tan^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x/((c + a^2*c*x^2)*ArcTan[a*x]^(5/2)),x]
```

```
[Out] Integrate[x/((c + a^2*c*x^2)*ArcTan[a*x]^(5/2)), x]
```

Maple [A] time = 0.142, size = 0, normalized size = 0.

$$\int \frac{x}{a^2cx^2 + c} (\arctan(ax))^{-\frac{5}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^2*c*x^2+c)/arctan(a*x)^(5/2),x)
```

```
[Out] int(x/(a^2*c*x^2+c)/arctan(a*x)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x}{a^2x^2\operatorname{atan}^{\frac{5}{2}}(ax)+\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a**2*c*x**2+c)/atan(a*x)**(5/2),x)
```

```
[Out] Integral(x/(a**2*x**2*atan(a*x)**(5/2) + atan(a*x)**(5/2)), x)/c
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(a^2cx^2+c)\arctan(ax)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x/((a^2*c*x^2 + c)*arctan(a*x)^(5/2)), x)
```

3.1056 $\int \frac{1}{(c+a^2cx^2)\tan^{-1}(ax)^{5/2}}\,dx$

Optimal. Leaf size=18

$$-\frac{2}{3ac\tan^{-1}(ax)^{3/2}}$$

```
[Out] -2/(3*a*c*ArcTan[a*x]^(3/2))
```

Rubi [A] time = 0.0243907, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {4884}

$$-\frac{2}{3ac\tan^{-1}(ax)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a^2*c*x^2)*ArcTan[a*x]^(5/2)),x]
```

```
[Out] -2/(3*a*c*ArcTan[a*x]^(3/2))
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(c+a^2cx^2\right)\tan^{-1}(ax)^{5/2}}\,dx=-\frac{2}{3ac\tan^{-1}(ax)^{3/2}}$$

Mathematica [A] time = 0.0059077, size = 18, normalized size = 1.

$$-\frac{2}{3ac\tan^{-1}(ax)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a^2*c*x^2)*ArcTan[a*x]^(5/2)),x]
```

```
[Out] -2/(3*a*c*ArcTan[a*x]^(3/2))
```

Maple [A] time = 0.08, size = 15, normalized size = 0.8

$$-\frac{2}{3\,ac}\left(\arctan\left(ax\right)\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2*c*x^2+c)/arctan(a*x)^(5/2),x)
```

```
[Out] -2/3/a/c/arctan(a*x)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 1.60513, size = 41, normalized size = 2.28

$$-\frac{2}{3\,ac\arctan\left(ax\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3/(a*c*arctan(a*x)^(3/2))
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*c*x**2+c)/atan(a*x)**(5/2),x)
```

```
[Out] Exception raised: TypeError
```

Giac [A] time = 1.10679, size = 19, normalized size = 1.06

$$-\frac{2}{3\,ac\arctan\left(ax\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] -2/3/(a*c*arctan(a*x)^(3/2))
```

3.1057 $\int \frac{1}{x(c+a^2cx^2)\tan^{-1}(ax)^{5/2}}\,dx$

Optimal. Leaf size=46

$$-\frac{2\text{Unintegrable}\left(\frac{1}{x^2\tan^{-1}(ax)^{3/2}},x\right)}{3ac}-\frac{2}{3acx\tan^{-1}(ax)^{3/2}}$$

```
[Out] -2/(3*a*c*x*ArcTan[a*x]^(3/2)) - (2*Unintegrable[1/(x^2*ArcTan[a*x]^(3/2)),
 x])/(3*a*c)
```

Rubi [A] time = 0.0772083, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x\left(c+a^2cx^2\right)\tan^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*(c + a^2*c*x^2)*ArcTan[a*x]^(5/2)),x]
```

```
[Out] -2/(3*a*c*x*ArcTan[a*x]^(3/2)) - (2*Defer[Int][1/(x^2*ArcTan[a*x]^(3/2)), x
])/(3*a*c)
```

Rubi steps

$$\int \frac{1}{x\left(c+a^2cx^2\right)\tan^{-1}(ax)^{5/2}}\,dx=-\frac{2}{3acx\tan^{-1}(ax)^{3/2}}-\frac{2\int \frac{1}{x^2\tan^{-1}(ax)^{3/2}}\,dx}{3ac}$$

Mathematica [A] time = 1.29838, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(c+a^2cx^2\right)\tan^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

[In] Integrate[1/(x*(c + a^2*c*x^2)*ArcTan[a*x]^(5/2)),x]

[Out] Integrate[1/(x*(c + a^2*c*x^2)*ArcTan[a*x]^(5/2)), x]

Maple [A] time = 0.142, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(a^{2}c\,x^{2}+c\right)}\left(\arctan \left(ax\right)\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(a^2*c*x^2+c)/arctan(a*x)^(5/2),x)

[Out] int(1/x/(a^2*c*x^2+c)/arctan(a*x)^(5/2),x)

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a^2*c*x^2+c)/arctan(a*x)^(5/2),x, algorithm="maxima")

[Out] Exception raised: RuntimeError

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a^2*c*x^2+c)/arctan(a*x)^(5/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{a^2x^3\operatorname{atan}^{\frac{5}{2}}(ax)+x\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a**2*c*x**2+c)/atan(a*x)**(5/2),x)
```

```
[Out] Integral(1/(a**2*x**3*atan(a*x)**(5/2) + x*atan(a*x)**(5/2)), x)/c
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a^2cx^2+c)x\arctan(ax)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)*x*arctan(a*x)^(5/2)), x)
```

3.1058 $\int \frac{x^m}{\left(c+a^2cx^2\right)^2 \tan^{-1}(ax)^{5/2}} \, dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{x^m}{\left(a^2cx^2+c\right)^2 \tan^{-1}(ax)^{5/2}},x\right)$$

```
[Out] Unintegrable[x^m/((c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2)), x]
```

Rubi [A] time = 0.063027, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m}{\left(c+a^2cx^2\right)^2 \tan^{-1}(ax)^{5/2}} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m/((c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2)),x]
```

```
[Out] Defer[Int][x^m/((c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2)), x]
```

Rubi steps

$$\int \frac{x^m}{\left(c+a^2cx^2\right)^2 \tan^{-1}(ax)^{5/2}} \, dx = \int \frac{x^m}{\left(c+a^2cx^2\right)^2 \tan^{-1}(ax)^{5/2}} \, dx$$

Mathematica [A] time = 1.15639, size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(c+a^2cx^2\right)^2 \tan^{-1}(ax)^{5/2}} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m/((c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2)),x]
```

```
[Out] Integrate[x^m/((c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2)), x]
```

Maple [A] time = 0.77, size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(a^2 c x^2 + c\right)^2} (\arctan(ax))^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x)
```

```
[Out] int(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x^m}{\left(a^4 c^2 x^4 + 2\,a^2 c^2 x^2 + c^2\right) \arctan(ax)^{\frac{5}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(x^m/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(a*x)^(5/2)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(a**2*c*x**2+c)**2/atan(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(a^2cx^2+c\right)^2 \arctan\left(ax\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x^m/((a^2*c*x^2 + c)^2*arctan(a*x)^(5/2)), x)
```

3.1059 $\int \frac{x^3}{\left(c+a^2cx^2\right)^2 \tan^{-1}(ax)^{5/2}}\,dx$

Optimal. Leaf size=184

$$\frac{16}{3}\text{Unintegrable}\left(\frac{x^3}{\left(a^2cx^2+c\right)^2\sqrt{\tan^{-1}(ax)}},x\right)+\frac{8}{3}a^2\text{Unintegrable}\left(\frac{x^5}{\left(a^2cx^2+c\right)^2\sqrt{\tan^{-1}(ax)}},x\right)+\frac{4\sqrt{\pi}S\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{a^4c^2}$$

```
[Out] (-2*x^3)/(3*a*c^2*(1 + a^2*x^2)*ArcTan[a*x]^(3/2)) - (4*x^2)/(a^2*c^2*(1 +
a^2*x^2)*Sqrt[ArcTan[a*x]]) - (4*x^4)/(3*c^2*(1 + a^2*x^2)*Sqrt[ArcTan[a*x]
]) + (4*Sqrt[Pi]*FresnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(a^4*c^2) + (16*
Unintegrable[x^3/((c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]), x])/3 + (8*a^2*Unin
tegrable[x^5/((c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]), x])/3
```

Rubi [A] time = 0.415337, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^3}{\left(c+a^2cx^2\right)^2 \tan^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^3/((c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2)),x]
```

```
[Out] (-2*x^3)/(3*a*c^2*(1 + a^2*x^2)*ArcTan[a*x]^(3/2)) - (4*x^2)/(a^2*c^2*(1 +
a^2*x^2)*Sqrt[ArcTan[a*x]]) - (4*x^4)/(3*c^2*(1 + a^2*x^2)*Sqrt[ArcTan[a*x]
]) + (4*Sqrt[Pi]*FresnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(a^4*c^2) + (16*
Defer[Int][x^3/((c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]), x])/3 + (8*a^2*Defer[
Int][x^5/((c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]), x])/3
```

Rubi steps

$$\int \frac{x^3}{\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^{5/2}}\,dx = -\frac{2x^3}{3ac^2\left(1+a^2x^2\right)\tan^{-1}(ax)^{3/2}} + \frac{2\int \frac{x^2}{\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^{3/2}}\,dx}{a} + \frac{1}{3}(2a)\int \frac{x^4}{\left(c+a^2cx^2\right)^2 \text{ta}}$$

$$= -\frac{2x^3}{3ac^2\left(1+a^2x^2\right)\tan^{-1}(ax)^{3/2}} - \frac{4x^2}{a^2c^2\left(1+a^2x^2\right)\sqrt{\tan^{-1}(ax)}} - \frac{4x^4}{3c^2\left(1+a^2x^2\right)\sqrt{\tan^{-1}}}$$

$$= -\frac{2x^3}{3ac^2\left(1+a^2x^2\right)\tan^{-1}(ax)^{3/2}} - \frac{4x^2}{a^2c^2\left(1+a^2x^2\right)\sqrt{\tan^{-1}(ax)}} - \frac{4x^4}{3c^2\left(1+a^2x^2\right)\sqrt{\tan^{-1}}}$$

$$= -\frac{2x^3}{3ac^2\left(1+a^2x^2\right)\tan^{-1}(ax)^{3/2}} - \frac{4x^2}{a^2c^2\left(1+a^2x^2\right)\sqrt{\tan^{-1}(ax)}} - \frac{4x^4}{3c^2\left(1+a^2x^2\right)\sqrt{\tan^{-1}}}$$

$$= -\frac{2x^3}{3ac^2\left(1+a^2x^2\right)\tan^{-1}(ax)^{3/2}} - \frac{4x^2}{a^2c^2\left(1+a^2x^2\right)\sqrt{\tan^{-1}(ax)}} - \frac{4x^4}{3c^2\left(1+a^2x^2\right)\sqrt{\tan^{-1}}}$$

$$= -\frac{2x^3}{3ac^2\left(1+a^2x^2\right)\tan^{-1}(ax)^{3/2}} - \frac{4x^2}{a^2c^2\left(1+a^2x^2\right)\sqrt{\tan^{-1}(ax)}} - \frac{4x^4}{3c^2\left(1+a^2x^2\right)\sqrt{\tan^{-1}}}$$

$$= -\frac{2x^3}{3ac^2\left(1+a^2x^2\right)\tan^{-1}(ax)^{3/2}} - \frac{4x^2}{a^2c^2\left(1+a^2x^2\right)\sqrt{\tan^{-1}(ax)}} - \frac{4x^4}{3c^2\left(1+a^2x^2\right)\sqrt{\tan^{-1}}}$$

Mathematica [A] time = 4.55844, size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^3/((c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2)),x]
```

```
[Out] Integrate[x^3/((c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2)), x]
```

Maple [A] time = 0.496, size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(a^2cx^2+c\right)^2}\left(\arctan\left(ax\right)\right)^{-\frac{5}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x)
```

```
[Out] int(x^3/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a**2*c*x**2+c)**2/atan(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(a^2cx^2+c\right)^2 \arctan\left(ax\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/((a^2*c*x^2 + c)^2*arctan(a*x)^(5/2)), x)
```

3.1060 $\int \frac{x^2}{\left(c+a^2cx^2\right)^2 \tan^{-1}(ax)^{5/2}} \, dx$

Optimal. Leaf size=180

$$\frac{8\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{3a^3c^2} - \frac{2x^2}{3ac^2\left(a^2x^2+1\right)\tan^{-1}(ax)^{3/2}} - \frac{8x}{3a^2c^2\left(a^2x^2+1\right)\sqrt{\tan^{-1}(ax)}} + \frac{16\left(1-a^2x^2\right)\sqrt{\tan^{-1}(ax}}{3a^3c^2\left(a^2x^2+1\right)}$$

```
[Out] (-2*x^2)/(3*a*c^2*(1 + a^2*x^2)*ArcTan[a*x]^(3/2)) - (8*x)/(3*a^2*c^2*(1 +
a^2*x^2)*Sqrt[ArcTan[a*x]]) + (16*Sqrt[ArcTan[a*x]])/(3*a^3*c^2) - (32*Sqrt
[ArcTan[a*x]])/(3*a^3*c^2*(1 + a^2*x^2)) + (16*(1 - a^2*x^2)*Sqrt[ArcTan[a*
x]])/(3*a^3*c^2*(1 + a^2*x^2)) + (8*Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])
/Sqrt[Pi]])/(3*a^3*c^2)
```

Rubi [A] time = 0.2631, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {4942, 4932, 4930, 4904, 3312, 3304, 3352}

$$\frac{8\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{3a^3c^2} - \frac{2x^2}{3ac^2\left(a^2x^2+1\right)\tan^{-1}(ax)^{3/2}} - \frac{8x}{3a^2c^2\left(a^2x^2+1\right)\sqrt{\tan^{-1}(ax)}} + \frac{16\left(1-a^2x^2\right)\sqrt{\tan^{-1}(ax}}{3a^3c^2\left(a^2x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2)),x]
```


```
[Out] (-2*x^2)/(3*a*c^2*(1 + a^2*x^2)*ArcTan[a*x]^(3/2)) - (8*x)/(3*a^2*c^2*(1 +
a^2*x^2)*Sqrt[ArcTan[a*x]]) + (16*Sqrt[ArcTan[a*x]])/(3*a^3*c^2) - (32*Sqrt
[ArcTan[a*x]])/(3*a^3*c^2*(1 + a^2*x^2)) + (16*(1 - a^2*x^2)*Sqrt[ArcTan[a*
x]])/(3*a^3*c^2*(1 + a^2*x^2)) + (8*Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])
/Sqrt[Pi]])/(3*a^3*c^2)
```


Rule 4942

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^m*(d + e*x^2)^(q + 1)*(a + b*ArcT
an[c*x])^(p + 1))/(b*c*d*(p + 1)), x] - Dist[(f*m)/(b*c*(p + 1)), Int[(f*x)
^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b,
```

```
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 2, 0] && LtQ[p, -1]
```

Rule 4932

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*(x_))/((d_) + (e_.)*(x_)^2)^2,
 x_Symbol] :> Simp[(x*(a + b*ArcTan[c*x])^(p + 1))/(b*c*d*(p + 1)*(d + e*x^
2)), x] + (-Dist[4/(b^2*(p + 1)*(p + 2)), Int[(x*(a + b*ArcTan[c*x])^(p + 2
))/(d + e*x^2)^2, x], x] - Simp[((1 - c^2*x^2)*(a + b*ArcTan[c*x])^(p + 2))
/(b^2*e*(p + 1)*(p + 2)*(d + e*x^2)), x]) /; FreeQ[{a, b, c, d, e}, x] && E
qQ[e, c^2*d] && LtQ[p, -1] && NeQ[p, -2]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 4904

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{x^2}{\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^{5/2}}\,dx = -\frac{2x^2}{3ac^2\left(1+a^2x^2\right)\tan^{-1}(ax)^{3/2}} + \frac{4\int \frac{x}{\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^{3/2}}\,dx}{3a}$$

$$= -\frac{2x^2}{3ac^2\left(1+a^2x^2\right)\tan^{-1}(ax)^{3/2}} - \frac{8x}{3a^2c^2\left(1+a^2x^2\right)\sqrt{\tan^{-1}(ax)}} + \frac{16\left(1-a^2x^2\right)\sqrt{\tan}}{3a^3c^2\left(1+a^2x^2\right.}$$

$$= -\frac{2x^2}{3ac^2\left(1+a^2x^2\right)\tan^{-1}(ax)^{3/2}} - \frac{8x}{3a^2c^2\left(1+a^2x^2\right)\sqrt{\tan^{-1}(ax)}} - \frac{32\sqrt{\tan^{-1}(ax)}}{3a^3c^2\left(1+a^2x^2\right)} +$$

$$= -\frac{2x^2}{3ac^2\left(1+a^2x^2\right)\tan^{-1}(ax)^{3/2}} - \frac{8x}{3a^2c^2\left(1+a^2x^2\right)\sqrt{\tan^{-1}(ax)}} - \frac{32\sqrt{\tan^{-1}(ax)}}{3a^3c^2\left(1+a^2x^2\right)} +$$

$$= -\frac{2x^2}{3ac^2\left(1+a^2x^2\right)\tan^{-1}(ax)^{3/2}} - \frac{8x}{3a^2c^2\left(1+a^2x^2\right)\sqrt{\tan^{-1}(ax)}} - \frac{32\sqrt{\tan^{-1}(ax)}}{3a^3c^2\left(1+a^2x^2\right)} +$$

$$= -\frac{2x^2}{3ac^2\left(1+a^2x^2\right)\tan^{-1}(ax)^{3/2}} - \frac{8x}{3a^2c^2\left(1+a^2x^2\right)\sqrt{\tan^{-1}(ax)}} + \frac{16\sqrt{\tan^{-1}(ax)}}{3a^3c^2} - \frac{3}{3a}$$

$$= -\frac{2x^2}{3ac^2\left(1+a^2x^2\right)\tan^{-1}(ax)^{3/2}} - \frac{8x}{3a^2c^2\left(1+a^2x^2\right)\sqrt{\tan^{-1}(ax)}} + \frac{16\sqrt{\tan^{-1}(ax)}}{3a^3c^2} - \frac{3}{3a}$$

$$= -\frac{2x^2}{3ac^2\left(1+a^2x^2\right)\tan^{-1}(ax)^{3/2}} - \frac{8x}{3a^2c^2\left(1+a^2x^2\right)\sqrt{\tan^{-1}(ax)}} + \frac{16\sqrt{\tan^{-1}(ax)}}{3a^3c^2} - \frac{3}{3a}$$

Mathematica [C] time = 0.341508, size = 162, normalized size = 0.9

$$\frac{\sqrt{2}\left(a^2x^2+1\right)\left(-i\tan^{-1}(ax)\right)^{3/2}\text{Gamma}\left(\frac{1}{2},-2i\tan^{-1}(ax)\right)+\sqrt{2}\left(a^2x^2+1\right)\left(i\tan^{-1}(ax)\right)^{3/2}\text{Gamma}\left(\frac{1}{2},2i\tan^{-1}(ax)\right)}{3a^3c^2\left(a^2x^2+1\right)\tan^{-1}(ax)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/((c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2)),x]
```

```
[Out] (-2*a*x*(a*x + 4*ArcTan[a*x]) + 4*Sqrt[Pi]*(1 + a^2*x^2)*ArcTan[a*x]^(3/2)*
FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]] + Sqrt[2]*(1 + a^2*x^2)*((-I)*ArcT
an[a*x])^(3/2)*Gamma[1/2, (-2*I)*ArcTan[a*x]] + Sqrt[2]*(1 + a^2*x^2)*(I*Ar
cTan[a*x])^(3/2)*Gamma[1/2, (2*I)*ArcTan[a*x]])/(3*a^3*c^2*(1 + a^2*x^2)*Ar
cTan[a*x]^(3/2))
```

Maple [A] time = 0.11, size = 62, normalized size = 0.3

$$-\frac{1}{3\,a^3c^2}\left(-8\,\sqrt{\pi}\mathrm{FresnelC}\left(2\,\frac{\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)(\arctan(ax))^{3/2}+4\,\sin(2\,\arctan(ax))\arctan(ax)-\cos(2\,\arctan(ax))+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x)
```

```
[Out] -1/3/a^3/c^2*(-8*Pi^(1/2)*FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2))*arctan(a*x
)^(3/2)+4*sin(2*arctan(a*x))*arctan(a*x)-cos(2*arctan(a*x))+1)/arctan(a*x)^
(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a**2*c*x**2+c)**2/atan(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(a^2cx^2+c\right)^2 \arctan\left(ax\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/((a^2*c*x^2 + c)^2*arctan(a*x)^(5/2)), x)
```

3.1061 $\int \frac{x}{(c+a^2cx^2)^2 \tan^{-1}(ax)^{5/2}} dx$

Optimal. Leaf size=101

$$-\frac{8\sqrt{\pi}S\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{3a^2c^2}-\frac{2x}{3ac^2\left(a^2x^2+1\right)\tan^{-1}(ax)^{3/2}}-\frac{4\left(1-a^2x^2\right)}{3a^2c^2\left(a^2x^2+1\right)\sqrt{\tan^{-1}(ax)}}$$

```
[Out] (-2*x)/(3*a*c^2*(1 + a^2*x^2)*ArcTan[a*x]^(3/2)) - (4*(1 - a^2*x^2))/(3*a^2
*c^2*(1 + a^2*x^2)*Sqrt[ArcTan[a*x]]) - (8*Sqrt[Pi]*FresnelS[(2*Sqrt[ArcTan
[a*x]])/Sqrt[Pi]])/(3*a^2*c^2)
```

Rubi [A] time = 0.121547, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4932, 4970, 4406, 12, 3305, 3351}

$$-\frac{8\sqrt{\pi}S\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{3a^2c^2}-\frac{2x}{3ac^2\left(a^2x^2+1\right)\tan^{-1}(ax)^{3/2}}-\frac{4\left(1-a^2x^2\right)}{3a^2c^2\left(a^2x^2+1\right)\sqrt{\tan^{-1}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2)),x]
```

```
[Out] (-2*x)/(3*a*c^2*(1 + a^2*x^2)*ArcTan[a*x]^(3/2)) - (4*(1 - a^2*x^2))/(3*a^2
*c^2*(1 + a^2*x^2)*Sqrt[ArcTan[a*x]]) - (8*Sqrt[Pi]*FresnelS[(2*Sqrt[ArcTan
[a*x]])/Sqrt[Pi]])/(3*a^2*c^2)
```

Rule 4932

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*(x_))/((d_) + (e_.)*(x_)^2)^2,
 x_Symbol] :> Simp[(x*(a + b*ArcTan[c*x])^(p + 1))/(b*c*d*(p + 1)*(d + e*x^
2)), x] + (-Dist[4/(b^2*(p + 1)*(p + 2)), Int[(x*(a + b*ArcTan[c*x])^(p + 2
))/(d + e*x^2)^2, x], x] - Simp[((1 - c^2*x^2)*(a + b*ArcTan[c*x])^(p + 2))
/(b^2*e*(p + 1)*(p + 2)*(d + e*x^2)), x]) /; FreeQ[{a, b, c, d, e}, x] && E
qQ[e, c^2*d] && LtQ[p, -1] && NeQ[p, -2]
```

Rule 4970

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sin[x]^m)/
Cos[x]^(m + 2*(q + 1)), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{x}{\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^{5/2}}\,dx = -\frac{2x}{3ac^2\left(1+a^2x^2\right)\tan^{-1}(ax)^{3/2}} - \frac{4\left(1-a^2x^2\right)}{3a^2c^2\left(1+a^2x^2\right)\sqrt{\tan^{-1}(ax)}} - \frac{16}{3}\int \frac{x}{\left(c+a^2cx^2\right)^2\sqrt{t}}$$

$$= -\frac{2x}{3ac^2\left(1+a^2x^2\right)\tan^{-1}(ax)^{3/2}} - \frac{4\left(1-a^2x^2\right)}{3a^2c^2\left(1+a^2x^2\right)\sqrt{\tan^{-1}(ax)}} - \frac{16\,\mathrm{Subst}\left(\int \frac{\cos(x)\sin(}{\sqrt{x}}\right.}{3a^2c}$$

$$= -\frac{2x}{3ac^2\left(1+a^2x^2\right)\tan^{-1}(ax)^{3/2}} - \frac{4\left(1-a^2x^2\right)}{3a^2c^2\left(1+a^2x^2\right)\sqrt{\tan^{-1}(ax)}} - \frac{16\,\mathrm{Subst}\left(\int \frac{\sin(2x)}{2\sqrt{x}}\,dx\right.}{3a^2c^2}$$

$$= -\frac{2x}{3ac^2\left(1+a^2x^2\right)\tan^{-1}(ax)^{3/2}} - \frac{4\left(1-a^2x^2\right)}{3a^2c^2\left(1+a^2x^2\right)\sqrt{\tan^{-1}(ax)}} - \frac{8\,\mathrm{Subst}\left(\int \frac{\sin(2x)}{\sqrt{x}}\,dx,\right.}{3a^2c^2}$$

$$= -\frac{2x}{3ac^2\left(1+a^2x^2\right)\tan^{-1}(ax)^{3/2}} - \frac{4\left(1-a^2x^2\right)}{3a^2c^2\left(1+a^2x^2\right)\sqrt{\tan^{-1}(ax)}} - \frac{16\,\mathrm{Subst}\left(\int \sin\left(2x^2\right.\right.}{3a^2}$$

$$= -\frac{2x}{3ac^2\left(1+a^2x^2\right)\tan^{-1}(ax)^{3/2}} - \frac{4\left(1-a^2x^2\right)}{3a^2c^2\left(1+a^2x^2\right)\sqrt{\tan^{-1}(ax)}} - \frac{8\sqrt{\pi}S\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{3a^2c^2}$$

Mathematica [A] time = 0.0779631, size = 88, normalized size = 0.87

$$-\frac{2\left(4\sqrt{\pi}\left(a^2x^2+1\right)\tan^{-1}(ax)^{3/2}S\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)+\left(2-2a^2x^2\right)\tan^{-1}(ax)+ax\right)}{3a^2c^2\left(a^2x^2+1\right)\tan^{-1}(ax)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2)),x]
```

```
[Out] (-2*(a*x + (2 - 2*a^2*x^2)*ArcTan[a*x] + 4*Sqrt[Pi]*(1 + a^2*x^2)*ArcTan[a*
x]^(3/2)*FresnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]]))/(3*a^2*c^2*(1 + a^2*x^2
)*ArcTan[a*x]^(3/2))
```

Maple [A] time = 0.102, size = 59, normalized size = 0.6

$$-\frac{1}{3\,a^{2}c^{2}}\left(8\,\sqrt{\pi}\mathrm{FresnelS}\left(2\,\frac{\sqrt{\arctan\left(ax\right)}}{\sqrt{\pi}}\right)\left(\arctan\left(ax\right)\right)^{3/2}+4\,\cos\left(2\,\arctan\left(ax\right)\right)\arctan\left(ax\right)+\sin\left(2\,\arctan\left(ax\right)\right)\right)($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x)
```

```
[Out] -1/3/a^2/c^2*(8*Pi^(1/2)*FresnelS(2*arctan(a*x)^(1/2)/Pi^(1/2))*arctan(a*x)
^(3/2)+4*cos(2*arctan(a*x))*arctan(a*x)+sin(2*arctan(a*x)))/arctan(a*x)^(3/
2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a**2*c*x**2+c)**2/atan(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(a^2cx^2+c\right)^2 \arctan\left(ax\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x/((a^2*c*x^2 + c)^2*arctan(a*x)^(5/2)), x)
```

3.1062 $\int \frac{1}{\left(c+a^2cx^2\right)^2 \tan^{-1}(ax)^{5/2}}\,dx$

Optimal. Leaf size=174

$$\frac{8x}{3c^2\left(a^2x^2+1\right)\sqrt{\tan^{-1}(ax)}}-\frac{16\left(1-a^2x^2\right)\sqrt{\tan^{-1}(ax)}}{3ac^2\left(a^2x^2+1\right)}+\frac{32\sqrt{\tan^{-1}(ax)}}{3ac^2\left(a^2x^2+1\right)}-\frac{2}{3ac^2\left(a^2x^2+1\right)\tan^{-1}(ax)^{3/2}}-\frac{8\sqrt{\pi}\text{Fresn}}{}$$

```
[Out] -2/(3*a*c^2*(1 + a^2*x^2)*ArcTan[a*x]^(3/2)) + (8*x)/(3*c^2*(1 + a^2*x^2)*S
qrt[ArcTan[a*x]]) - (16*Sqrt[ArcTan[a*x]])/(3*a*c^2) + (32*Sqrt[ArcTan[a*x]
])/(3*a*c^2*(1 + a^2*x^2)) - (16*(1 - a^2*x^2)*Sqrt[ArcTan[a*x]])/(3*a*c^2*
(1 + a^2*x^2)) - (8*Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(3*a
*c^2)
```

Rubi [A] time = 0.206001, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4902, 4932, 4930, 4904, 3312, 3304, 3352}

$$\frac{8x}{3c^2\left(a^2x^2+1\right)\sqrt{\tan^{-1}(ax)}}-\frac{16\left(1-a^2x^2\right)\sqrt{\tan^{-1}(ax)}}{3ac^2\left(a^2x^2+1\right)}+\frac{32\sqrt{\tan^{-1}(ax)}}{3ac^2\left(a^2x^2+1\right)}-\frac{2}{3ac^2\left(a^2x^2+1\right)\tan^{-1}(ax)^{3/2}}-\frac{8\sqrt{\pi}\text{Fresn}}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2)),x]
```

```
[Out] -2/(3*a*c^2*(1 + a^2*x^2)*ArcTan[a*x]^(3/2)) + (8*x)/(3*c^2*(1 + a^2*x^2)*S
qrt[ArcTan[a*x]]) - (16*Sqrt[ArcTan[a*x]])/(3*a*c^2) + (32*Sqrt[ArcTan[a*x]
])/(3*a*c^2*(1 + a^2*x^2)) - (16*(1 - a^2*x^2)*Sqrt[ArcTan[a*x]])/(3*a*c^2*
(1 + a^2*x^2)) - (8*Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(3*a
*c^2)
```

Rule 4902

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^(p + 1))/(b*c*d*(p
+ 1)), x] - Dist[(2*c*(q + 1))/(b*(p + 1)), Int[x*(d + e*x^2)^q*(a + b*ArcT
an[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && L
```

```
tQ[q, -1] && LtQ[p, -1]
```

Rule 4932

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*(x_))/((d_) + (e_.)*(x_)^2)^2,
 x_Symbol] :> Simp[(x*(a + b*ArcTan[c*x])^(p + 1))/(b*c*d*(p + 1)*(d + e*x^
2)), x] + (-Dist[4/(b^2*(p + 1)*(p + 2)), Int[(x*(a + b*ArcTan[c*x])^(p + 2
))/(d + e*x^2)^2, x], x] - Simp[((1 - c^2*x^2)*(a + b*ArcTan[c*x])^(p + 2))
/(b^2*e*(p + 1)*(p + 2)*(d + e*x^2)), x]) /; FreeQ[{a, b, c, d, e}, x] && E
qQ[e, c^2*d] && LtQ[p, -1] && NeQ[p, -2]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 4904

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{1}{\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^{5/2}}\,dx = -\frac{2}{3ac^2\left(1+a^2x^2\right)\tan^{-1}(ax)^{3/2}} - \frac{1}{3}(4a)\int \frac{x}{\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^{3/2}}\,dx$$

$$= -\frac{2}{3ac^2\left(1+a^2x^2\right)\tan^{-1}(ax)^{3/2}} + \frac{8x}{3c^2\left(1+a^2x^2\right)\sqrt{\tan^{-1}(ax)}} - \frac{16\left(1-a^2x^2\right)\sqrt{\tan^{-1}(}}{3ac^2\left(1+a^2x^2\right)}$$

$$= -\frac{2}{3ac^2\left(1+a^2x^2\right)\tan^{-1}(ax)^{3/2}} + \frac{8x}{3c^2\left(1+a^2x^2\right)\sqrt{\tan^{-1}(ax)}} + \frac{32\sqrt{\tan^{-1}(ax)}}{3ac^2\left(1+a^2x^2\right)} - 16($$

$$= -\frac{2}{3ac^2\left(1+a^2x^2\right)\tan^{-1}(ax)^{3/2}} + \frac{8x}{3c^2\left(1+a^2x^2\right)\sqrt{\tan^{-1}(ax)}} + \frac{32\sqrt{\tan^{-1}(ax)}}{3ac^2\left(1+a^2x^2\right)} - 16($$

$$= -\frac{2}{3ac^2\left(1+a^2x^2\right)\tan^{-1}(ax)^{3/2}} + \frac{8x}{3c^2\left(1+a^2x^2\right)\sqrt{\tan^{-1}(ax)}} + \frac{32\sqrt{\tan^{-1}(ax)}}{3ac^2\left(1+a^2x^2\right)} - 16($$

$$= -\frac{2}{3ac^2\left(1+a^2x^2\right)\tan^{-1}(ax)^{3/2}} + \frac{8x}{3c^2\left(1+a^2x^2\right)\sqrt{\tan^{-1}(ax)}} - \frac{16\sqrt{\tan^{-1}(ax)}}{3ac^2} + \frac{32\sqrt{}}{3ac^2}$$

$$= -\frac{2}{3ac^2\left(1+a^2x^2\right)\tan^{-1}(ax)^{3/2}} + \frac{8x}{3c^2\left(1+a^2x^2\right)\sqrt{\tan^{-1}(ax)}} - \frac{16\sqrt{\tan^{-1}(ax)}}{3ac^2} + \frac{32\sqrt{}}{3ac^2}$$

$$= -\frac{2}{3ac^2\left(1+a^2x^2\right)\tan^{-1}(ax)^{3/2}} + \frac{8x}{3c^2\left(1+a^2x^2\right)\sqrt{\tan^{-1}(ax)}} - \frac{16\sqrt{\tan^{-1}(ax)}}{3ac^2} + \frac{32\sqrt{}}{3ac^2}$$

Mathematica [C] time = 0.416717, size = 170, normalized size = 0.98

$$\frac{\frac{\sqrt{2}\left(a^2x^2+1\right)\tan^{-1}(ax)^2\text{Gamma}\left(\frac{1}{2},2i\tan^{-1}(ax)\right)}{\sqrt{i\tan^{-1}(ax)}} + \sqrt{2}\left(a^2x^2+1\right)\sqrt{i\tan^{-1}(ax)}\sqrt{\tan^{-1}(ax)^2}\text{Gamma}\left(\frac{1}{2},-2i\tan^{-1}(ax)\right) - 4\sqrt{\pi}\left(a\right.}{3c^2\left(a^3x^2+a\right)\tan^{-1}(ax)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2)),x]
```

```
[Out] (-2 + 8*a*x*ArcTan[a*x] - 4*Sqrt[Pi]*(1 + a^2*x^2)*ArcTan[a*x]^(3/2)*Fresne
lC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]] + Sqrt[2]*(1 + a^2*x^2)*Sqrt[I*ArcTan[a*
```

```
x]]*Sqrt[ArcTan[a*x]^2]*Gamma[1/2, (-2*I)*ArcTan[a*x]] + (Sqrt[2]*(1 + a^2*
x^2)*ArcTan[a*x]^2*Gamma[1/2, (2*I)*ArcTan[a*x]])/Sqrt[I*ArcTan[a*x]])/(3*c
^2*(a + a^3*x^2)*ArcTan[a*x]^(3/2))
```

Maple [A] time = 0.106, size = 62, normalized size = 0.4

$$\frac{1}{3\,ac^2}\left(-8\,\sqrt{\pi}\mathrm{FresnelC}\left(2\,\frac{\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)(\arctan(ax))^{3/2}+4\,\sin(2\,\arctan(ax))\arctan(ax)-\cos(2\,\arctan(ax))-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x)
```

```
[Out] 1/3/a/c^2*(-8*Pi^(1/2)*FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2))*arctan(a*x)^(
3/2)+4*sin(2*arctan(a*x))*arctan(a*x)-cos(2*arctan(a*x))-1)/arctan(a*x)^(3/
2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\frac{1}{a^4x^4\operatorname{atan}^{\frac{5}{2}}(ax)+2a^2x^2\operatorname{atan}^{\frac{5}{2}}(ax)+\operatorname{atan}^{\frac{5}{2}}(ax)}}{c^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*c*x**2+c)**2/atan(a*x)**(5/2),x)
```

```
[Out] Integral(1/(a**4*x**4*atan(a*x)**(5/2) + 2*a**2*x**2*atan(a*x)**(5/2) + ata
n(a*x)**(5/2)), x)/c**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)^2\arctan\left(ax\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^2*arctan(a*x)^(5/2)), x)
```

3.1063 $\int \frac{1}{x(c+a^2cx^2)^2 \tan^{-1}(ax)^{5/2}} dx$

Optimal. Leaf size=178

$$\frac{8\text{Unintegrable}\left(\frac{1}{x^3(a^2cx^2+c)^2\sqrt{\tan^{-1}(ax)}},x\right)}{3a^2}+\frac{16}{3}\text{Unintegrable}\left(\frac{1}{x\left(a^2cx^2+c\right)^2\sqrt{\tan^{-1}(ax)}},x\right)+\frac{4}{c^2\left(a^2x^2+1\right)\sqrt{\tan^{-1}(ax}}$$

```
[Out] -2/(3*a*c^2*x*(1 + a^2*x^2)*ArcTan[a*x]^(3/2)) + 4/(c^2*(1 + a^2*x^2)*Sqrt[
ArcTan[a*x]]) + 4/(3*a^2*c^2*x^2*(1 + a^2*x^2)*Sqrt[ArcTan[a*x]]) + (4*Sqrt
[Pi]*FresnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/c^2 + (8*Unintegrable[1/(x^3
*(c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]), x])/(3*a^2) + (16*Unintegrable[1/(x*
(c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]), x])/3
```

Rubi [A] time = 0.36048, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x\left(c+a^2cx^2\right)^2 \tan^{-1}(ax)^{5/2}} dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*(c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2)),x]
```

```
[Out] -2/(3*a*c^2*x*(1 + a^2*x^2)*ArcTan[a*x]^(3/2)) + 4/(c^2*(1 + a^2*x^2)*Sqrt[
ArcTan[a*x]]) + 4/(3*a^2*c^2*x^2*(1 + a^2*x^2)*Sqrt[ArcTan[a*x]]) + (4*Sqrt
[Pi]*FresnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/c^2 + (8*Defer[Int][1/(x^3*(
c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]), x])/(3*a^2) + (16*Defer[Int][1/(x*(c +
 a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]), x])/3
```

Rubi steps

$$\int \frac{1}{x\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^{5/2}}\,dx = -\frac{2}{3ac^2x\left(1+a^2x^2\right)\tan^{-1}(ax)^{3/2}} - \frac{2\int \frac{1}{x^2\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^{3/2}}\,dx}{3a} - (2a)\int \frac{}{\left(c+a^2cx^2\right)}$$

$$= -\frac{2}{3ac^2x\left(1+a^2x^2\right)\tan^{-1}(ax)^{3/2}} + \frac{4}{c^2\left(1+a^2x^2\right)\sqrt{\tan^{-1}(ax)}} + \frac{4}{3a^2c^2x^2\left(1+a^2x^2\right)}$$

$$= -\frac{2}{3ac^2x\left(1+a^2x^2\right)\tan^{-1}(ax)^{3/2}} + \frac{4}{c^2\left(1+a^2x^2\right)\sqrt{\tan^{-1}(ax)}} + \frac{4}{3a^2c^2x^2\left(1+a^2x^2\right)}$$

$$= -\frac{2}{3ac^2x\left(1+a^2x^2\right)\tan^{-1}(ax)^{3/2}} + \frac{4}{c^2\left(1+a^2x^2\right)\sqrt{\tan^{-1}(ax)}} + \frac{4}{3a^2c^2x^2\left(1+a^2x^2\right)}$$

$$= -\frac{2}{3ac^2x\left(1+a^2x^2\right)\tan^{-1}(ax)^{3/2}} + \frac{4}{c^2\left(1+a^2x^2\right)\sqrt{\tan^{-1}(ax)}} + \frac{4}{3a^2c^2x^2\left(1+a^2x^2\right)}$$

$$= -\frac{2}{3ac^2x\left(1+a^2x^2\right)\tan^{-1}(ax)^{3/2}} + \frac{4}{c^2\left(1+a^2x^2\right)\sqrt{\tan^{-1}(ax)}} + \frac{4}{3a^2c^2x^2\left(1+a^2x^2\right)}$$

$$= -\frac{2}{3ac^2x\left(1+a^2x^2\right)\tan^{-1}(ax)^{3/2}} + \frac{4}{c^2\left(1+a^2x^2\right)\sqrt{\tan^{-1}(ax)}} + \frac{4}{3a^2c^2x^2\left(1+a^2x^2\right)}$$

Mathematica [A] time = 4.25049, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*(c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2)),x]
```

```
[Out] Integrate[1/(x*(c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2)), x]
```

Maple [A] time = 0.535, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(a^{2} c\,x^{2}+c\right)^{2}} \left(\arctan \left(a x\right)\right)^{-\frac{5}{2}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x)

[Out] int(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="maxima")

[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a**2*c*x**2+c)**2/atan(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^{2}cx^{2}+c\right)^{2}x\arctan\left(ax\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^2*x*arctan(a*x)^(5/2)), x)
```

3.1064 $\int \frac{1}{x^2\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^{5/2}}\,dx$

Optimal. Leaf size=196

$$\frac{56}{3}\text{Unintegrable}\left(\frac{1}{x^2\left(a^2cx^2+c\right)^2\sqrt{\tan^{-1}(ax)}},x\right)+\frac{8\text{Unintegrable}\left(\frac{1}{x^4\left(a^2cx^2+c\right)^2\sqrt{\tan^{-1}(ax)}},x\right)}{a^2}+\frac{16}{3c^2x\left(a^2x^2+1\right)\sqrt{\tan^{-1}}}$$

```
[Out] -2/(3*a*c^2*x^2*(1 + a^2*x^2)*ArcTan[a*x]^(3/2)) + 8/(3*a^2*c^2*x^3*(1 + a^
2*x^2)*Sqrt[ArcTan[a*x]]) + 16/(3*c^2*x*(1 + a^2*x^2)*Sqrt[ArcTan[a*x]]) +
(16*a*Sqrt[ArcTan[a*x]])/c^2 + (8*a*Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])
/Sqrt[Pi]])/c^2 + (8*Unintegrable[1/(x^4*(c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]
]), x])/a^2 + (56*Unintegrable[1/(x^2*(c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]),
 x])/3
```

Rubi [A] time = 0.483022, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2)),x]
```

```
[Out] -2/(3*a*c^2*x^2*(1 + a^2*x^2)*ArcTan[a*x]^(3/2)) + 8/(3*a^2*c^2*x^3*(1 + a^
2*x^2)*Sqrt[ArcTan[a*x]]) + 16/(3*c^2*x*(1 + a^2*x^2)*Sqrt[ArcTan[a*x]]) +
(16*a*Sqrt[ArcTan[a*x]])/c^2 + (8*a*Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])
/Sqrt[Pi]])/c^2 + (8*Defer[Int][1/(x^4*(c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]])
, x])/a^2 + (56*Defer[Int][1/(x^2*(c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]), x])
/3
```

Rubi steps

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^{5/2}}\,dx = -\frac{2}{3ac^2x^2\left(1+a^2x^2\right)\tan^{-1}(ax)^{3/2}} - \frac{4\int \frac{1}{x^3\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^{3/2}}\,dx}{3a} - \frac{1}{3}(8a)\int \frac{}{x\left(c+a\right.}$$

$$= -\frac{2}{3ac^2x^2\left(1+a^2x^2\right)\tan^{-1}(ax)^{3/2}} + \frac{8}{3a^2c^2x^3\left(1+a^2x^2\right)\sqrt{\tan^{-1}(ax)}} + \frac{}{3c^2x\left(1+a^2\right.}$$

$$= -\frac{2}{3ac^2x^2\left(1+a^2x^2\right)\tan^{-1}(ax)^{3/2}} + \frac{8}{3a^2c^2x^3\left(1+a^2x^2\right)\sqrt{\tan^{-1}(ax)}} + \frac{}{3c^2x\left(1+a^2\right.}$$

$$= -\frac{2}{3ac^2x^2\left(1+a^2x^2\right)\tan^{-1}(ax)^{3/2}} + \frac{8}{3a^2c^2x^3\left(1+a^2x^2\right)\sqrt{\tan^{-1}(ax)}} + \frac{}{3c^2x\left(1+a^2\right.}$$

$$= -\frac{2}{3ac^2x^2\left(1+a^2x^2\right)\tan^{-1}(ax)^{3/2}} + \frac{8}{3a^2c^2x^3\left(1+a^2x^2\right)\sqrt{\tan^{-1}(ax)}} + \frac{}{3c^2x\left(1+a^2\right.}$$

$$= -\frac{2}{3ac^2x^2\left(1+a^2x^2\right)\tan^{-1}(ax)^{3/2}} + \frac{8}{3a^2c^2x^3\left(1+a^2x^2\right)\sqrt{\tan^{-1}(ax)}} + \frac{}{3c^2x\left(1+a^2\right.}$$

$$= -\frac{2}{3ac^2x^2\left(1+a^2x^2\right)\tan^{-1}(ax)^{3/2}} + \frac{8}{3a^2c^2x^3\left(1+a^2x^2\right)\sqrt{\tan^{-1}(ax)}} + \frac{}{3c^2x\left(1+a^2\right.}$$

Mathematica [A] time = 6.39176, size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2)),x]
```

```
[Out] Integrate[1/(x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2)), x]
```

Maple [A] time = 0.51, size = 0, normalized size = 0.

$$\int \frac{1}{x^{2} \left(a^{2} c\,x^{2}+c\right)^{2}} \left(\arctan \left(a x\right)\right)^{-\frac{5}{2}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x)
```

```
[Out] int(1/x^2/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a**2*c*x**2+c)**2/atan(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)^2 x^2 \arctan\left(ax\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^2*x^2*arctan(a*x)^(5/2)), x)
```

3.1065 $\int \frac{1}{x^3(c+a^2cx^2)^2 \tan^{-1}(ax)^{5/2}}\,dx$

Optimal. Leaf size=186

$$\frac{80}{3}a^2\text{Unintegrable}\left(\frac{1}{x\left(a^2cx^2+c\right)^2\sqrt{\tan^{-1}(ax)}},x\right)+\frac{112}{3}\text{Unintegrable}\left(\frac{1}{x^3\left(a^2cx^2+c\right)^2\sqrt{\tan^{-1}(ax)}},x\right)+\frac{16\text{Unintegr}}{}$$

```
[Out] -2/(3*a*c^2*x^3*(1 + a^2*x^2)*ArcTan[a*x]^(3/2)) + 4/(a^2*c^2*x^4*(1 + a^2*
x^2)*Sqrt[ArcTan[a*x]]) + 20/(3*c^2*x^2*(1 + a^2*x^2)*Sqrt[ArcTan[a*x]]) +
(16*Unintegrable[1/(x^5*(c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]), x])/a^2 + (11
2*Unintegrable[1/(x^3*(c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]), x])/3 + (80*a^2
*Unintegrable[1/(x*(c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]), x])/3
```

Rubi [A] time = 0.463937, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{1}{x^3\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^3*(c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2)),x]
```

```
[Out] -2/(3*a*c^2*x^3*(1 + a^2*x^2)*ArcTan[a*x]^(3/2)) + 4/(a^2*c^2*x^4*(1 + a^2*
x^2)*Sqrt[ArcTan[a*x]]) + 20/(3*c^2*x^2*(1 + a^2*x^2)*Sqrt[ArcTan[a*x]]) +
(16*Defer[Int][1/(x^5*(c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]), x])/a^2 + (112*
Defer[Int][1/(x^3*(c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]), x])/3 + (80*a^2*Def
er[Int][1/(x*(c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]), x])/3
```

Rubi steps

$$\int \frac{1}{x^3 \left(c+a^2cx^2\right)^2 \tan^{-1}(ax)^{5/2}}\,dx = -\frac{2}{3ac^2x^3\left(1+a^2x^2\right)\tan^{-1}(ax)^{3/2}} - \frac{2\int \frac{1}{x^4\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^{3/2}}\,dx}{a} - \frac{1}{3}(10a)\int \frac{}{x^2\left(c+\right.}$$

$$= -\frac{2}{3ac^2x^3\left(1+a^2x^2\right)\tan^{-1}(ax)^{3/2}} + \frac{4}{a^2c^2x^4\left(1+a^2x^2\right)\sqrt{\tan^{-1}(ax)}} + \frac{}{3c^2x^2\left(1+a^2\right.}$$

Mathematica [A] time = 5.28382, size = 0, normalized size = 0.

$$\int \frac{1}{x^3\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^3*(c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2)),x]
```

```
[Out] Integrate[1/(x^3*(c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2)), x]
```

Maple [A] time = 0.867, size = 0, normalized size = 0.

$$\int \frac{1}{x^3\left(a^2cx^2+c\right)^2}\left(\arctan(ax)\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x)
```

```
[Out] int(1/x^3/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a**2*c*x**2+c)**2/atan(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)^2 x^3 \arctan\left(ax\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^2*x^3*arctan(a*x)^(5/2)), x)
```

3.1066 $\int \frac{1}{x^4(c+a^2cx^2)^2\tan^{-1}(ax)^{5/2}}\,dx$

Optimal. Leaf size=186

$$40a^2\text{Unintegrable}\left(\frac{1}{x^2\left(a^2cx^2+c\right)^2\sqrt{\tan^{-1}(ax)}},x\right)+\frac{184}{3}\text{Unintegrable}\left(\frac{1}{x^4\left(a^2cx^2+c\right)^2\sqrt{\tan^{-1}(ax)}},x\right)+\frac{80\text{Uninte}}{}$$

```
[Out] -2/(3*a*c^2*x^4*(1 + a^2*x^2)*ArcTan[a*x]^(3/2)) + 16/(3*a^2*c^2*x^5*(1 + a
^2*x^2)*Sqrt[ArcTan[a*x]]) + 8/(c^2*x^3*(1 + a^2*x^2)*Sqrt[ArcTan[a*x]]) +
(80*Unintegrable[1/(x^6*(c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]), x])/(3*a^2) +
 (184*Unintegrable[1/(x^4*(c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]), x])/3 + 40*
a^2*Unintegrable[1/(x^2*(c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]), x]
```

Rubi [A] time = 0.471433, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^4\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^4*(c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2)),x]
```

```
[Out] -2/(3*a*c^2*x^4*(1 + a^2*x^2)*ArcTan[a*x]^(3/2)) + 16/(3*a^2*c^2*x^5*(1 + a
^2*x^2)*Sqrt[ArcTan[a*x]]) + 8/(c^2*x^3*(1 + a^2*x^2)*Sqrt[ArcTan[a*x]]) +
(80*Defer[Int][1/(x^6*(c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]), x])/(3*a^2) + (
184*Defer[Int][1/(x^4*(c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]), x])/3 + 40*a^2*
Defer[Int][1/(x^2*(c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]), x]
```

Rubi steps

$$\int \frac{1}{x^4\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^{5/2}}\,dx = -\frac{2}{3ac^2x^4\left(1+a^2x^2\right)\tan^{-1}(ax)^{3/2}} - \frac{8\int \frac{1}{x^5\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^{3/2}}\,dx}{3a} - (4a)\int \frac{}{x^3\left(c+a^2c\right.}$$

$$= -\frac{2}{3ac^2x^4\left(1+a^2x^2\right)\tan^{-1}(ax)^{3/2}} + \frac{16}{3a^2c^2x^5\left(1+a^2x^2\right)\sqrt{\tan^{-1}(ax)}} + \frac{8}{c^2x^3\left(1+a^2x^2\right.}$$

Mathematica [A] time = 10.9998, size = 0, normalized size = 0.

$$\int \frac{1}{x^4\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^4*(c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2)),x]
```

```
[Out] Integrate[1/(x^4*(c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2)), x]
```

Maple [A] time = 0.662, size = 0, normalized size = 0.

$$\int \frac{1}{x^4\left(a^2cx^2+c\right)^2}\left(\arctan(ax)\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x)
```

```
[Out] int(1/x^4/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(a**2*c*x**2+c)**2/atan(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)^2x^4\arctan\left(ax\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^2*x^4*arctan(a*x)^(5/2)), x)
```

3.1067 $\int \frac{x^m}{\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^{5/2}}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{x^m}{\left(a^2cx^2+c\right)^3\tan^{-1}(ax)^{5/2}},x\right)$$

```
[Out] Unintegrable[x^m/((c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2)), x]
```

Rubi [A] time = 0.0647661, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m}{\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m/((c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2)),x]
```

```
[Out] Defer[Int][x^m/((c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2)), x]
```

Rubi steps

$$\int \frac{x^m}{\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^{5/2}}\,dx=\int \frac{x^m}{\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^{5/2}}\,dx$$

Mathematica [A] time = 1.36501, size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m/((c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2)),x]
```

```
[Out] Integrate[x^m/((c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2)), x]
```

Maple [A] time = 0.826, size = 0, normalized size = 0.

$$\int \frac{x^m}{(a^2cx^2+c)^3} (\arctan(ax))^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x)
```

```
[Out] int(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^m}{(a^6c^3x^6+3\,a^4c^3x^4+3\,a^2c^3x^2+c^3)\arctan(ax)^{\frac{5}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(x^m/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*arctan(a*
x)^(5/2)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(a**2*c*x**2+c)**3/atan(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{(a^2cx^2 + c)^3 \arctan(ax)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x^m/((a^2*c*x^2 + c)^3*arctan(a*x)^(5/2)), x)
```

3.1068 $\int \frac{x^3}{\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^{5/2}}\,dx$

Optimal. Leaf size=160

$$\frac{4\sqrt{2\pi}S\left(2\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{3a^4c^3}-\frac{4\sqrt{\pi}S\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{3a^4c^3}+\frac{4x^4}{3c^3\left(a^2x^2+1\right)^2\sqrt{\tan^{-1}(ax)}}-\frac{2x^3}{3ac^3\left(a^2x^2+1\right)^2\tan^{-1}(ax)^{3/2}}-\frac{}{a^2c}$$

```
[Out] (-2*x^3)/(3*a*c^3*(1 + a^2*x^2)^2*ArcTan[a*x]^(3/2)) - (4*x^2)/(a^2*c^3*(1
+ a^2*x^2)^2*Sqrt[ArcTan[a*x]]) + (4*x^4)/(3*c^3*(1 + a^2*x^2)^2*Sqrt[ArcTa
n[a*x]]) + (4*Sqrt[2*Pi]*FresnelS[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(3*a^4*c
^3) - (4*Sqrt[Pi]*FresnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(3*a^4*c^3)
```

Rubi [A] time = 0.590159, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4968, 4942, 4970, 4406, 3305, 3351}

$$\frac{4\sqrt{2\pi}S\left(2\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{3a^4c^3}-\frac{4\sqrt{\pi}S\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{3a^4c^3}+\frac{4x^4}{3c^3\left(a^2x^2+1\right)^2\sqrt{\tan^{-1}(ax)}}-\frac{2x^3}{3ac^3\left(a^2x^2+1\right)^2\tan^{-1}(ax)^{3/2}}-\frac{}{a^2c}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2)),x]
```


```
[Out] (-2*x^3)/(3*a*c^3*(1 + a^2*x^2)^2*ArcTan[a*x]^(3/2)) - (4*x^2)/(a^2*c^3*(1
+ a^2*x^2)^2*Sqrt[ArcTan[a*x]]) + (4*x^4)/(3*c^3*(1 + a^2*x^2)^2*Sqrt[ArcTa
n[a*x]]) + (4*Sqrt[2*Pi]*FresnelS[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(3*a^4*c
^3) - (4*Sqrt[Pi]*FresnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(3*a^4*c^3)
```


Rule 4968

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[(x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^(p
+ 1))/(b*c*d*(p + 1)), x] + (-Dist[(c*(m + 2*q + 2))/(b*(p + 1)), Int[x^(m
+ 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p + 1
)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; Fre
eQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &&
```

```
LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 4942

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^m*(d + e*x^2)^(q + 1)*(a + b*ArcT
an[c*x])^(p + 1))/(b*c*d*(p + 1)), x] - Dist[(f*m)/(b*c*(p + 1)), Int[(f*x)
^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 2, 0] && LtQ[p, -1]
```

Rule 4970

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sin[x]^m)/
Cos[x]^(m + 2*(q + 1)), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{x^3}{\left(c+a^2cx^2\right)^3 \tan^{-1}(ax)^{5/2}}\,dx = -\frac{2x^3}{3ac^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)^{3/2}} + \frac{2\int \frac{x^2}{\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^{3/2}}\,dx}{a} - \frac{1}{3}(2a)\int \frac{x}{\left(c+a^2cx^2\right)^3}$$

$$= -\frac{2x^3}{3ac^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)^{3/2}} - \frac{4x^2}{a^2c^3\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}} + \frac{4x^4}{3c^3\left(1+a^2x^2\right)^2\sqrt{\mathrm{t}}}$$

$$= -\frac{2x^3}{3ac^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)^{3/2}} - \frac{4x^2}{a^2c^3\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}} + \frac{4x^4}{3c^3\left(1+a^2x^2\right)^2\sqrt{\mathrm{t}}}$$

$$= -\frac{2x^3}{3ac^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)^{3/2}} - \frac{4x^2}{a^2c^3\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}} + \frac{4x^4}{3c^3\left(1+a^2x^2\right)^2\sqrt{\mathrm{t}}}$$

$$= -\frac{2x^3}{3ac^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)^{3/2}} - \frac{4x^2}{a^2c^3\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}} + \frac{4x^4}{3c^3\left(1+a^2x^2\right)^2\sqrt{\mathrm{t}}}$$

$$= -\frac{2x^3}{3ac^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)^{3/2}} - \frac{4x^2}{a^2c^3\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}} + \frac{4x^4}{3c^3\left(1+a^2x^2\right)^2\sqrt{\mathrm{t}}}$$

$$= -\frac{2x^3}{3ac^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)^{3/2}} - \frac{4x^2}{a^2c^3\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}} + \frac{4x^4}{3c^3\left(1+a^2x^2\right)^2\sqrt{\mathrm{t}}}$$

Mathematica [C] time = 0.442479, size = 227, normalized size = 1.42

$$\frac{i\sqrt{2}\left(a^2x^2+1\right)^2\left(-i\tan^{-1}(ax)\right)^{3/2}\mathrm{Gamma}\left(\frac{1}{2},-2i\tan^{-1}(ax)\right)+\sqrt{2}\left(a^2x^2+1\right)^2\sqrt{i\tan^{-1}(ax)}\tan^{-1}(ax)\mathrm{Gamma}\left(\frac{1}{2},2i\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/((c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2)),x]
```

```
[Out] (I*Sqrt[2]*(1 + a^2*x^2)^2*((-I)*ArcTan[a*x])^(3/2)*Gamma[1/2, (-2*I)*ArcTa
n[a*x]] + Sqrt[2]*(1 + a^2*x^2)^2*Sqrt[I*ArcTan[a*x]]*ArcTan[a*x]*Gamma[1/2
, (2*I)*ArcTan[a*x]] - 2*(a^2*x^2*(a*x + (6 - 2*a^2*x^2)*ArcTan[a*x]) + I*(
```

```
1 + a^2*x^2)^2*((-I)*ArcTan[a*x])^(3/2)*Gamma[1/2, (-4*I)*ArcTan[a*x]] + (1
 + a^2*x^2)^2*Sqrt[I*ArcTan[a*x]]*ArcTan[a*x]*Gamma[1/2, (4*I)*ArcTan[a*x]]
))/(3*a^4*c^3*(1 + a^2*x^2)^2*ArcTan[a*x]^(3/2))
```

Maple [A] time = 0.119, size = 112, normalized size = 0.7

$$-\frac{1}{12\,c^3a^4}\left(-16\,\sqrt{2}\sqrt{\pi}\mathrm{FresnelS}\left(2\,\frac{\sqrt{2}\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)(\arctan(ax))^{3/2}+16\,\sqrt{\pi}\mathrm{FresnelS}\left(2\,\frac{\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)(\arctan(ax))^{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x)
```

```
[Out] -1/12/a^4/c^3*(-16*2^(1/2)*Pi^(1/2)*FresnelS(2*2^(1/2)/Pi^(1/2)*arctan(a*x)
^(1/2))*arctan(a*x)^(3/2)+16*Pi^(1/2)*FresnelS(2*arctan(a*x)^(1/2)/Pi^(1/2)
)*arctan(a*x)^(3/2)+8*cos(2*arctan(a*x))*arctan(a*x)-8*cos(4*arctan(a*x))*a
rctan(a*x)+2*sin(2*arctan(a*x))-sin(4*arctan(a*x)))/arctan(a*x)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a**2*c*x**2+c)**3/atan(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(a^2cx^2+c\right)^3 \arctan\left(ax\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/((a^2*c*x^2 + c)^3*arctan(a*x)^(5/2)), x)
```

3.1069 $\int \frac{x^2}{\left(c+a^2cx^2\right)^3 \tan^{-1}(ax)^{5/2}}\,dx$

Optimal. Leaf size=129

$$\frac{4\sqrt{2\pi}\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{3a^3c^3}+\frac{8x^3}{3c^3\left(a^2x^2+1\right)^2\sqrt{\tan^{-1}(ax)}}-\frac{2x^2}{3ac^3\left(a^2x^2+1\right)^2\tan^{-1}(ax)^{3/2}}-\frac{8x}{3a^2c^3\left(a^2x^2+1\right)^2}$$

```
[Out] (-2*x^2)/(3*a*c^3*(1 + a^2*x^2)^2*ArcTan[a*x]^(3/2)) - (8*x)/(3*a^2*c^3*(1
+ a^2*x^2)^2*Sqrt[ArcTan[a*x]]) + (8*x^3)/(3*c^3*(1 + a^2*x^2)^2*Sqrt[ArcTa
n[a*x]]) + (4*Sqrt[2*Pi]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(3*a^3*c
^3)
```

Rubi [A] time = 0.688409, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 27, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {4968, 4970, 3312, 3304, 3352, 4406, 4904}

$$\frac{4\sqrt{2\pi}\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{3a^3c^3}+\frac{8x^3}{3c^3\left(a^2x^2+1\right)^2\sqrt{\tan^{-1}(ax)}}-\frac{2x^2}{3ac^3\left(a^2x^2+1\right)^2\tan^{-1}(ax)^{3/2}}-\frac{8x}{3a^2c^3\left(a^2x^2+1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2)),x]
```

```
[Out] (-2*x^2)/(3*a*c^3*(1 + a^2*x^2)^2*ArcTan[a*x]^(3/2)) - (8*x)/(3*a^2*c^3*(1
+ a^2*x^2)^2*Sqrt[ArcTan[a*x]]) + (8*x^3)/(3*c^3*(1 + a^2*x^2)^2*Sqrt[ArcTa
n[a*x]]) + (4*Sqrt[2*Pi]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(3*a^3*c
^3)
```

Rule 4968

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[(x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^(p
+ 1))/(b*c*d*(p + 1)), x] + (-Dist[(c*(m + 2*q + 2))/(b*(p + 1)), Int[x^(m
+ 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p + 1
)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; Fre
eQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &&
```

```
LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 4970

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sin[x]^m)/
Cos[x]^(m + 2*(q + 1)), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 4904

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x^2}{\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^{5/2}}\,dx = -\frac{2x^2}{3ac^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)^{3/2}} + \frac{4\int \frac{x}{\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^{3/2}}\,dx}{3a} - \frac{1}{3}(4a)\int \frac{x^3}{\left(c+a^2cx^2\right)^3\text{ta}}$$

$$= -\frac{2x^2}{3ac^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)^{3/2}} - \frac{8x}{3a^2c^3\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}} + \frac{8x^3}{3c^3\left(1+a^2x^2\right)^2\sqrt{\text{ta}}}$$

$$= -\frac{2x^2}{3ac^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)^{3/2}} - \frac{8x}{3a^2c^3\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}} + \frac{8x^3}{3c^3\left(1+a^2x^2\right)^2\sqrt{\text{ta}}}$$

$$= -\frac{2x^2}{3ac^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)^{3/2}} - \frac{8x}{3a^2c^3\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}} + \frac{8x^3}{3c^3\left(1+a^2x^2\right)^2\sqrt{\text{ta}}}$$

$$= -\frac{2x^2}{3ac^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)^{3/2}} - \frac{8x}{3a^2c^3\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}} + \frac{8x^3}{3c^3\left(1+a^2x^2\right)^2\sqrt{\text{ta}}}$$

$$= -\frac{2x^2}{3ac^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)^{3/2}} - \frac{8x}{3a^2c^3\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}} + \frac{8x^3}{3c^3\left(1+a^2x^2\right)^2\sqrt{\text{ta}}}$$

$$= -\frac{2x^2}{3ac^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)^{3/2}} - \frac{8x}{3a^2c^3\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}} + \frac{8x^3}{3c^3\left(1+a^2x^2\right)^2\sqrt{\text{ta}}}$$

Mathematica [C] time = 0.899716, size = 259, normalized size = 2.01

$$\frac{\frac{\frac{4\sqrt{2}\left(-i\tan^{-1}(ax)\right)^{3/2}\text{Gamma}\left(\frac{1}{2},-2i\tan^{-1}(ax)\right)}{a^3}+\frac{4\sqrt{2}\left(i\tan^{-1}(ax)\right)^{3/2}\text{Gamma}\left(\frac{1}{2},2i\tan^{-1}(ax)\right)}{a^3}+\frac{7\left(-i\tan^{-1}(ax)\right)^{3/2}\text{Gamma}\left(\frac{1}{2},-4i\tan^{-1}(ax)\right)}{a^3}+\frac{7\left(i\tan^{-1}(ax)\right)^{3/2}\text{Gamma}\left(\frac{1}{2},4i\,\text{tan}\right.}{a^3}}{\tan^{-1}(ax)^{3/2}}}{12c^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/((c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2)),x]
```

```
[Out] ((2*Sqrt[2*Pi]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/a^3 - (16*Sqrt[Pi]
*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/a^3 + ((-8*x^2)/(a*(1 + a^2*x^2)
^2) + (32*x^3*ArcTan[a*x])/(1 + a^2*x^2)^2 - (32*x*ArcTan[a*x])/(a + a^3*x^
```

```
2)^2 + (4*Sqrt[2]*((-I)*ArcTan[a*x])^(3/2)*Gamma[1/2, (-2*I)*ArcTan[a*x]])/
a^3 + (4*Sqrt[2]*(I*ArcTan[a*x])^(3/2)*Gamma[1/2, (2*I)*ArcTan[a*x]])/a^3 +
 (7*((-I)*ArcTan[a*x])^(3/2)*Gamma[1/2, (-4*I)*ArcTan[a*x]])/a^3 + (7*(I*Ar
cTan[a*x])^(3/2)*Gamma[1/2, (4*I)*ArcTan[a*x]])/a^3)/ArcTan[a*x]^(3/2))/(12
*c^3)
```

Maple [A] time = 0.115, size = 68, normalized size = 0.5

$$-\frac{1}{12\,c^{3}a^{3}}\left(-16\,\sqrt{2}\sqrt{\pi}\mathrm{FresnelC}\left(2\,\frac{\sqrt{2}\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)(\arctan(ax))^{3/2}+8\,\sin(4\,\arctan(ax))\arctan(ax)-\cos(4\,\mathrm{arct}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x)
```

```
[Out] -1/12/a^3/c^3*(-16*2^(1/2)*Pi^(1/2)*FresnelC(2*2^(1/2)/Pi^(1/2)*arctan(a*x)
^(1/2))*arctan(a*x)^(3/2)+8*sin(4*arctan(a*x))*arctan(a*x)-cos(4*arctan(a*x
))+1)/arctan(a*x)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a**2*c*x**2+c)**3/atan(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(a^2cx^2+c\right)^3 \arctan\left(ax\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/((a^2*c*x^2 + c)^3*arctan(a*x)^(5/2)), x)
```

3.1070 $\int \frac{x}{\left(c+a^2cx^2\right)^3 \tan^{-1}(ax)^{5/2}}\,dx$

Optimal. Leaf size=155

$$-\frac{4\sqrt{2\pi}S\left(2\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{3a^2c^3}-\frac{4\sqrt{\pi}S\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{3a^2c^3}+\frac{4x^2}{c^3\left(a^2x^2+1\right)^2\sqrt{\tan^{-1}(ax)}}-\frac{2x}{3ac^3\left(a^2x^2+1\right)^2\tan^{-1}(ax)^{3/2}}-\frac{}{3a}$$

```
[Out] (-2*x)/(3*a*c^3*(1 + a^2*x^2)^2*ArcTan[a*x]^(3/2)) - 4/(3*a^2*c^3*(1 + a^2*
x^2)^2*Sqrt[ArcTan[a*x]]) + (4*x^2)/(c^3*(1 + a^2*x^2)^2*Sqrt[ArcTan[a*x]])
 - (4*Sqrt[2*Pi]*FresnelS[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(3*a^2*c^3) - (4
*Sqrt[Pi]*FresnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(3*a^2*c^3)
```

Rubi [A] time = 0.499861, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4968, 4970, 4406, 3305, 3351, 4902}

$$-\frac{4\sqrt{2\pi}S\left(2\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{3a^2c^3}-\frac{4\sqrt{\pi}S\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{3a^2c^3}+\frac{4x^2}{c^3\left(a^2x^2+1\right)^2\sqrt{\tan^{-1}(ax)}}-\frac{2x}{3ac^3\left(a^2x^2+1\right)^2\tan^{-1}(ax)^{3/2}}-\frac{}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[x/((c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2)),x]
```

```
[Out] (-2*x)/(3*a*c^3*(1 + a^2*x^2)^2*ArcTan[a*x]^(3/2)) - 4/(3*a^2*c^3*(1 + a^2*
x^2)^2*Sqrt[ArcTan[a*x]]) + (4*x^2)/(c^3*(1 + a^2*x^2)^2*Sqrt[ArcTan[a*x]])
 - (4*Sqrt[2*Pi]*FresnelS[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(3*a^2*c^3) - (4
*Sqrt[Pi]*FresnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(3*a^2*c^3)
```

Rule 4968

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[(x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^(p
+ 1))/(b*c*d*(p + 1)), x] + (-Dist[(c*(m + 2*q + 2))/(b*(p + 1)), Int[x^(m
+ 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p + 1
)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; Fre
eQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &&
```

```
LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 4970

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sin[x]^m)/
Cos[x]^(m + 2*(q + 1)), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4902

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^(p + 1))/(b*c*d*(p
+ 1)), x] - Dist[(2*c*(q + 1))/(b*(p + 1)), Int[x*(d + e*x^2)^q*(a + b*ArcT
an[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && L
tQ[q, -1] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{x}{\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^{5/2}}\,dx = -\frac{2x}{3ac^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)^{3/2}} + \frac{2\int \frac{1}{\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^{3/2}}\,dx}{3a} - (2a)\int \frac{x^2}{\left(c+a^2cx^2\right)^3\,\mathrm{ta}}$$

$$= -\frac{2x}{3ac^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)^{3/2}} - \frac{4}{3a^2c^3\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}} + \frac{4x^2}{c^3\left(1+a^2x^2\right)^2\sqrt{\mathrm{t}}}$$

$$= -\frac{2x}{3ac^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)^{3/2}} - \frac{4}{3a^2c^3\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}} + \frac{4x^2}{c^3\left(1+a^2x^2\right)^2\sqrt{\mathrm{t}}}$$

$$= -\frac{2x}{3ac^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)^{3/2}} - \frac{4}{3a^2c^3\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}} + \frac{4x^2}{c^3\left(1+a^2x^2\right)^2\sqrt{\mathrm{t}}}$$

$$= -\frac{2x}{3ac^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)^{3/2}} - \frac{4}{3a^2c^3\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}} + \frac{4x^2}{c^3\left(1+a^2x^2\right)^2\sqrt{\mathrm{t}}}$$

$$= -\frac{2x}{3ac^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)^{3/2}} - \frac{4}{3a^2c^3\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}} + \frac{4x^2}{c^3\left(1+a^2x^2\right)^2\sqrt{\mathrm{t}}}$$

$$= -\frac{2x}{3ac^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)^{3/2}} - \frac{4}{3a^2c^3\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}} + \frac{4x^2}{c^3\left(1+a^2x^2\right)^2\sqrt{\mathrm{t}}}$$

Mathematica [C] time = 0.379553, size = 220, normalized size = 1.42

$$\frac{i\sqrt{2}\left(a^2x^2+1\right)^2\left(-i\tan^{-1}(ax)\right)^{3/2}\mathrm{Gamma}\left(\frac{1}{2},-2i\tan^{-1}(ax)\right)+\sqrt{2}\left(a^2x^2+1\right)^2\sqrt{i\tan^{-1}(ax)}\tan^{-1}(ax)\mathrm{Gamma}\left(\frac{1}{2},2i\,\mathrm{t}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2)),x]
```

```
[Out] (I*Sqrt[2]*(1 + a^2*x^2)^2*((-I)*ArcTan[a*x])^(3/2)*Gamma[1/2, (-2*I)*ArcTa
n[a*x]] + Sqrt[2]*(1 + a^2*x^2)^2*Sqrt[I*ArcTan[a*x]]*ArcTan[a*x]*Gamma[1/2
, (2*I)*ArcTan[a*x]] + 2*(-(a*x) - 2*ArcTan[a*x] + 6*a^2*x^2*ArcTan[a*x] +
I*(1 + a^2*x^2)^2*((-I)*ArcTan[a*x])^(3/2)*Gamma[1/2, (-4*I)*ArcTan[a*x]] +
```

```
(1 + a^2*x^2)^2*Sqrt[I*ArcTan[a*x]]*ArcTan[a*x]*Gamma[1/2, (4*I)*ArcTan[a*
x]]))/(3*c^3*(a + a^3*x^2)^2*ArcTan[a*x]^(3/2))
```

Maple [A] time = 0.116, size = 110, normalized size = 0.7

$$-\frac{1}{12\,c^3a^2}\left(16\,\sqrt{2}\sqrt{\pi}\text{FresnelS}\left(2\,\frac{\sqrt{2}\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)(\arctan(ax))^{3/2}+16\,\sqrt{\pi}\text{FresnelS}\left(2\,\frac{\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)(\arctan(ax))^{3/2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x)
```

```
[Out] -1/12/a^2/c^3*(16*2^(1/2)*Pi^(1/2)*FresnelS(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^
(1/2))*arctan(a*x)^(3/2)+16*Pi^(1/2)*FresnelS(2*arctan(a*x)^(1/2)/Pi^(1/2))
*arctan(a*x)^(3/2)+8*cos(2*arctan(a*x))*arctan(a*x)+8*cos(4*arctan(a*x))*ar
ctan(a*x)+2*sin(2*arctan(a*x))+sin(4*arctan(a*x)))/arctan(a*x)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a**2*c*x**2+c)**3/atan(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(a^2cx^2+c\right)^3 \arctan\left(ax\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x/((a^2*c*x^2 + c)^3*arctan(a*x)^(5/2)), x)
```

3.1071 $\int \frac{1}{(c+a^2cx^2)^3 \tan^{-1}(ax)^{5/2}} dx$

Optimal. Leaf size=125

$$\frac{16x}{3c^3(a^2x^2+1)^2\sqrt{\tan^{-1}(ax)}} - \frac{2}{3ac^3(a^2x^2+1)^2\tan^{-1}(ax)^{3/2}} - \frac{4\sqrt{2\pi}\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{3ac^3} - \frac{8\sqrt{\pi}\text{FresnelC}\left(\frac{2\sqrt{}}{}\right.}{3ac^3}$$

```
[Out] -2/(3*a*c^3*(1 + a^2*x^2)^2*ArcTan[a*x]^(3/2)) + (16*x)/(3*c^3*(1 + a^2*x^2
)^2*Sqrt[ArcTan[a*x]]) - (4*Sqrt[2*Pi]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*
x]]])/(3*a*c^3) - (8*Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(3*
a*c^3)
```

Rubi [A] time = 0.296887, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {4902, 4968, 4970, 4406, 3304, 3352, 4904, 3312}

$$\frac{16x}{3c^3(a^2x^2+1)^2\sqrt{\tan^{-1}(ax)}} - \frac{2}{3ac^3(a^2x^2+1)^2\tan^{-1}(ax)^{3/2}} - \frac{4\sqrt{2\pi}\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{3ac^3} - \frac{8\sqrt{\pi}\text{FresnelC}\left(\frac{2\sqrt{}}{}\right.}{3ac^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2)),x]
```

```
[Out] -2/(3*a*c^3*(1 + a^2*x^2)^2*ArcTan[a*x]^(3/2)) + (16*x)/(3*c^3*(1 + a^2*x^2
)^2*Sqrt[ArcTan[a*x]]) - (4*Sqrt[2*Pi]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*
x]]])/(3*a*c^3) - (8*Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(3*
a*c^3)
```

Rule 4902

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^(p + 1))/(b*c*d*(p
+ 1)), x] - Dist[(2*c*(q + 1))/(b*(p + 1)), Int[x*(d + e*x^2)^q*(a + b*ArcT
an[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && L
tQ[q, -1] && LtQ[p, -1]
```

Rule 4968

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[(x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^(p
+ 1))/(b*c*d*(p + 1)), x] + (-Dist[(c*(m + 2*q + 2))/(b*(p + 1)), Int[x^(m
+ 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p + 1
)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; Fre
eQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &&
LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 4970

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sin[x]^m)/
Cos[x]^(m + 2*(q + 1)), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4904

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\int \frac{1}{\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^{5/2}}\,dx = -\frac{2}{3ac^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)^{3/2}} - \frac{1}{3}(8a)\int \frac{x}{\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^{3/2}}\,dx$$

$$= -\frac{2}{3ac^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)^{3/2}} + \frac{16x}{3c^3\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}} - \frac{16}{3}\int \frac{1}{\left(c+a^2cx^2\right)^3\sqrt{t}}$$

$$= -\frac{2}{3ac^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)^{3/2}} + \frac{16x}{3c^3\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}} - \frac{16\,\text{Subst}\left(\int \frac{\cos^4(x)}{\sqrt{x}}\,dx\right.}{3ac^3}$$

$$= -\frac{2}{3ac^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)^{3/2}} + \frac{16x}{3c^3\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}} - \frac{16\,\text{Subst}\left(\int \left(\frac{3}{8\sqrt{x}}+\right.\right.}{}$$

$$= -\frac{2}{3ac^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)^{3/2}} + \frac{16x}{3c^3\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}} - \frac{2\,\text{Subst}\left(\int \frac{\cos(4x)}{\sqrt{x}}\,dx,\right.}{3ac^3}$$

$$= -\frac{2}{3ac^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)^{3/2}} + \frac{16x}{3c^3\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}} - \frac{4\,\text{Subst}\left(\int \cos\left(4x^2\right)\right.}{3ac}$$

$$= -\frac{2}{3ac^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)^{3/2}} + \frac{16x}{3c^3\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}} - \frac{4\sqrt{2\pi}C\left(2\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-}}\right.}{3ac^3}$$

Mathematica [C] time = 0.697245, size = 186, normalized size = 1.49

$$\frac{2\left(\frac{\sqrt{2}\tan^{-1}(ax)^2\text{Gamma}\left(\frac{1}{2},2i\tan^{-1}(ax)\right)}{a\sqrt{i\tan^{-1}(ax)}} + \frac{\tan^{-1}(ax)^2\text{Gamma}\left(\frac{1}{2},4i\tan^{-1}(ax)\right)}{a\sqrt{i\tan^{-1}(ax)}} - \frac{\sqrt{2}\left(-i\tan^{-1}(ax)\right)^{3/2}\text{Gamma}\left(\frac{1}{2},-2i\tan^{-1}(ax)\right)}{a} - \frac{\left(-i\tan^{-1}(ax)\right)^{3/2}\text{G}}{}\right.}{3c^3\tan^{-1}(ax)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2)),x]
```

```
[Out] (2*(-(1/(a*(1 + a^2*x^2)^2)) + (8*x*ArcTan[a*x])/(1 + a^2*x^2)^2 - (Sqrt[2]
*((-I)*ArcTan[a*x])^(3/2)*Gamma[1/2, (-2*I)*ArcTan[a*x]])/a + (Sqrt[2]*ArcT
an[a*x]^2*Gamma[1/2, (2*I)*ArcTan[a*x]])/(a*Sqrt[I*ArcTan[a*x]]) - (((-I)*A
rcTan[a*x])^(3/2)*Gamma[1/2, (-4*I)*ArcTan[a*x]])/a + (ArcTan[a*x]^2*Gamma[
1/2, (4*I)*ArcTan[a*x]])/(a*Sqrt[I*ArcTan[a*x]])))/(3*c^3*ArcTan[a*x]^(3/2)
)
```

Maple [A] time = 0.121, size = 113, normalized size = 0.9

$$\frac{1}{12\,ac^3}\left(-16\,\sqrt{2}\sqrt{\pi}\mathrm{FresnelC}\left(2\,\frac{\sqrt{2}\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)(\arctan(ax))^{3/2}-32\,\sqrt{\pi}\mathrm{FresnelC}\left(2\,\frac{\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)(\arctan(ax))^{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x)
```

```
[Out] 1/12/a/c^3*(-16*2^(1/2)*Pi^(1/2)*FresnelC(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1
/2))*arctan(a*x)^(3/2)-32*Pi^(1/2)*FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2))*a
rctan(a*x)^(3/2)+16*sin(2*arctan(a*x))*arctan(a*x)+8*sin(4*arctan(a*x))*arc
tan(a*x)-4*cos(2*arctan(a*x))-cos(4*arctan(a*x))-3)/arctan(a*x)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*c*x**2+c)**3/atan(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2 c x^2 + c\right)^3 \arctan\left(ax\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^3*arctan(a*x)^(5/2)), x)
```

3.1072 $\int \frac{1}{x(c+a^2cx^2)^3 \tan^{-1}(ax)^{5/2}} dx$

Optimal. Leaf size=214

$$\frac{8\text{Unintegrable}\left(\frac{1}{x^3(a^2cx^2+c)^3\sqrt{\tan^{-1}(ax)}},x\right)}{3a^2}+8\text{Unintegrable}\left(\frac{1}{x\left(a^2cx^2+c\right)^3\sqrt{\tan^{-1}(ax)}},x\right)+\frac{4}{3a^2c^3x^2\left(a^2x^2+1\right)^2\sqrt{\text{ta}}}$$

```
[Out] -2/(3*a*c^3*x*(1 + a^2*x^2)^2*ArcTan[a*x]^(3/2)) + 20/(3*c^3*(1 + a^2*x^2)^
2*Sqrt[ArcTan[a*x]]) + 4/(3*a^2*c^3*x^2*(1 + a^2*x^2)^2*Sqrt[ArcTan[a*x]])
+ (5*Sqrt[2*Pi]*FresnelS[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(3*c^3) + (20*Sqr
t[Pi]*FresnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(3*c^3) + (8*Unintegrable[1
/(x^3*(c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]), x])/(3*a^2) + 8*Unintegrable[1/
(x*(c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]), x]
```

Rubi [A] time = 0.411624, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x\left(c+a^2cx^2\right)^3 \tan^{-1}(ax)^{5/2}} dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*(c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2)),x]
```

```
[Out] -2/(3*a*c^3*x*(1 + a^2*x^2)^2*ArcTan[a*x]^(3/2)) + 20/(3*c^3*(1 + a^2*x^2)^
2*Sqrt[ArcTan[a*x]]) + 4/(3*a^2*c^3*x^2*(1 + a^2*x^2)^2*Sqrt[ArcTan[a*x]])
+ (5*Sqrt[2*Pi]*FresnelS[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(3*c^3) + (20*Sqr
t[Pi]*FresnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(3*c^3) + (8*Defer[Int][1/(
x^3*(c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]), x])/(3*a^2) + 8*Defer[Int][1/(x*(
c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]), x]
```

Rubi steps

$$\int \frac{1}{x\left(c+a^2cx^2\right)^3 \tan^{-1}(ax)^{5/2}}\,dx = -\frac{2}{3ac^3x\left(1+a^2x^2\right)^2 \tan^{-1}(ax)^{3/2}} - \frac{2\int \frac{1}{x^2\left(c+a^2cx^2\right)^3 \tan^{-1}(ax)^{3/2}}\,dx}{3a} - \frac{1}{3}(10a)\int \frac{}{\left(c+a^2cx\right.}$$

$$= -\frac{2}{3ac^3x\left(1+a^2x^2\right)^2 \tan^{-1}(ax)^{3/2}} + \frac{20}{3c^3\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}} + \frac{4}{3a^2c^3x^2\left(1+a^2x^2\right)}$$

$$= -\frac{2}{3ac^3x\left(1+a^2x^2\right)^2 \tan^{-1}(ax)^{3/2}} + \frac{20}{3c^3\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}} + \frac{4}{3a^2c^3x^2\left(1+a^2x^2\right)}$$

$$= -\frac{2}{3ac^3x\left(1+a^2x^2\right)^2 \tan^{-1}(ax)^{3/2}} + \frac{20}{3c^3\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}} + \frac{4}{3a^2c^3x^2\left(1+a^2x^2\right)}$$

$$= -\frac{2}{3ac^3x\left(1+a^2x^2\right)^2 \tan^{-1}(ax)^{3/2}} + \frac{20}{3c^3\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}} + \frac{4}{3a^2c^3x^2\left(1+a^2x^2\right)}$$

$$= -\frac{2}{3ac^3x\left(1+a^2x^2\right)^2 \tan^{-1}(ax)^{3/2}} + \frac{20}{3c^3\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}} + \frac{4}{3a^2c^3x^2\left(1+a^2x^2\right)}$$

$$= -\frac{2}{3ac^3x\left(1+a^2x^2\right)^2 \tan^{-1}(ax)^{3/2}} + \frac{20}{3c^3\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}} + \frac{4}{3a^2c^3x^2\left(1+a^2x^2\right)}$$

Mathematica [A] time = 5.7692, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(c+a^2cx^2\right)^3 \tan^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*(c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2)),x]
```

```
[Out] Integrate[1/(x*(c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2)), x]
```

Maple [A] time = 0.816, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(a^{2}c x^{2}+c\right)^{3}} \left(\arctan\left(ax\right)\right)^{-\frac{5}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x)
```

```
[Out] int(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a**2*c*x**2+c)**3/atan(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)^3 x \arctan\left(ax\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^3*x*arctan(a*x)^(5/2)), x)
```

3.1073 $\int \frac{1}{x^2(c+a^2cx^2)^3 \tan^{-1}(ax)^{5/2}} \, dx$

Optimal. Leaf size=229

$$\frac{80}{3}\text{Unintegrable}\left(\frac{1}{x^2\left(a^2cx^2+c\right)^3\sqrt{\tan^{-1}(ax)}},x\right)+\frac{8\text{Unintegrable}\left(\frac{1}{x^4\left(a^2cx^2+c\right)^3\sqrt{\tan^{-1}(ax)}},x\right)}{a^2}+\frac{8}{c^3x\left(a^2x^2+1\right)^2\sqrt{\tan^{-1}(ax)}}$$

[Out] -2/(3*a*c^3*x^2*(1 + a^2*x^2)^2*ArcTan[a*x]^(3/2)) + 8/(3*a^2*c^3*x^3*(1 + a^2*x^2)^2*Sqrt[ArcTan[a*x]]) + 8/(c^3*x*(1 + a^2*x^2)^2*Sqrt[ArcTan[a*x]]) + (30*a*Sqrt[ArcTan[a*x]])/c^3 + (5*a*Sqrt[Pi/2]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/c^3 + (20*a*Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/c^3 + (8*Unintegrable[1/(x^4*(c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]), x])/a^2 + (80*Unintegrable[1/(x^2*(c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]), x])/3

Rubi [A] time = 0.508072, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^3 \tan^{-1}(ax)^{5/2}} \, dx$$

Verification is Not applicable to the result.

[In] Int[1/(x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2)),x]

[Out] -2/(3*a*c^3*x^2*(1 + a^2*x^2)^2*ArcTan[a*x]^(3/2)) + 8/(3*a^2*c^3*x^3*(1 + a^2*x^2)^2*Sqrt[ArcTan[a*x]]) + 8/(c^3*x*(1 + a^2*x^2)^2*Sqrt[ArcTan[a*x]]) + (30*a*Sqrt[ArcTan[a*x]])/c^3 + (5*a*Sqrt[Pi/2]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/c^3 + (20*a*Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/c^3 + (8*Defer[Int][1/(x^4*(c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]), x])/a^2 + (80*Defer[Int][1/(x^2*(c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]), x])/3

Rubi steps

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^3 \tan^{-1}(ax)^{5/2}}\,dx = -\frac{2}{3ac^3x^2\left(1+a^2x^2\right)^2\tan^{-1}(ax)^{3/2}} - \frac{4\int \frac{1}{x^3\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^{3/2}}\,dx}{3a} - (4a)\int \frac{}{x\left(c+a^2c\right.}$$

$$= -\frac{2}{3ac^3x^2\left(1+a^2x^2\right)^2\tan^{-1}(ax)^{3/2}} + \frac{8}{3a^2c^3x^3\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}} + \frac{}{c^3x\left(1+a^2x\right.}$$

$$= -\frac{2}{3ac^3x^2\left(1+a^2x^2\right)^2\tan^{-1}(ax)^{3/2}} + \frac{8}{3a^2c^3x^3\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}} + \frac{}{c^3x\left(1+a^2x\right.}$$

$$= -\frac{2}{3ac^3x^2\left(1+a^2x^2\right)^2\tan^{-1}(ax)^{3/2}} + \frac{8}{3a^2c^3x^3\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}} + \frac{}{c^3x\left(1+a^2x\right.}$$

$$= -\frac{2}{3ac^3x^2\left(1+a^2x^2\right)^2\tan^{-1}(ax)^{3/2}} + \frac{8}{3a^2c^3x^3\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}} + \frac{}{c^3x\left(1+a^2x\right.}$$

$$= -\frac{2}{3ac^3x^2\left(1+a^2x^2\right)^2\tan^{-1}(ax)^{3/2}} + \frac{8}{3a^2c^3x^3\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}} + \frac{}{c^3x\left(1+a^2x\right.}$$

$$= -\frac{2}{3ac^3x^2\left(1+a^2x^2\right)^2\tan^{-1}(ax)^{3/2}} + \frac{8}{3a^2c^3x^3\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}} + \frac{}{c^3x\left(1+a^2x\right.}$$

Mathematica [A] time = 7.28295, size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^3 \tan^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2)),x]
```

```
[Out] Integrate[1/(x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2)), x]
```

Maple [A] time = 0.584, size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \left(a^2 c x^2 + c\right)^3} \left(\arctan \left(a x\right)\right)^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x)
```

```
[Out] int(1/x^2/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a**2*c*x**2+c)**3/atan(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)^3 x^2 \arctan\left(ax\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^3*x^2*arctan(a*x)^(5/2)), x)
```

3.1074 $\int \frac{1}{x^3(c+a^2cx^2)^3\tan^{-1}(ax)^{5/2}}\,dx$

Optimal. Leaf size=184

$$56a^2\text{Unintegrable}\left(\frac{1}{x\left(a^2cx^2+c\right)^3\sqrt{\tan^{-1}(ax)}},x\right)+\frac{152}{3}\text{Unintegrable}\left(\frac{1}{x^3\left(a^2cx^2+c\right)^3\sqrt{\tan^{-1}(ax)}},x\right)+\frac{16\text{Uninte}}{}$$

```
[Out] -2/(3*a*c^3*x^3*(1 + a^2*x^2)^2*ArcTan[a*x]^(3/2)) + 4/(a^2*c^3*x^4*(1 + a^
2*x^2)^2*Sqrt[ArcTan[a*x]]) + 28/(3*c^3*x^2*(1 + a^2*x^2)^2*Sqrt[ArcTan[a*x
]]) + (16*Unintegrable[1/(x^5*(c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]), x])/a^2
 + (152*Unintegrable[1/(x^3*(c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]), x])/3 + 5
6*a^2*Unintegrable[1/(x*(c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]), x]
```

Rubi [A] time = 0.461449, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{1}{x^3\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^3*(c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2)),x]
```

```
[Out] -2/(3*a*c^3*x^3*(1 + a^2*x^2)^2*ArcTan[a*x]^(3/2)) + 4/(a^2*c^3*x^4*(1 + a^
2*x^2)^2*Sqrt[ArcTan[a*x]]) + 28/(3*c^3*x^2*(1 + a^2*x^2)^2*Sqrt[ArcTan[a*x
]]) + (16*Defer[Int][1/(x^5*(c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]), x])/a^2 +
 (152*Defer[Int][1/(x^3*(c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]), x])/3 + 56*a^
2*Defer[Int][1/(x*(c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]), x]
```

Rubi steps

$$\int \frac{1}{x^3\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^{5/2}}\,dx = -\frac{2}{3ac^3x^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)^{3/2}} - \frac{2\int \frac{1}{x^4\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^{3/2}}\,dx}{a} - \frac{1}{3}(14a)\int \frac{}{x^2\left(c+\right.}$$

$$= -\frac{2}{3ac^3x^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)^{3/2}} + \frac{4}{a^2c^3x^4\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}} + \frac{}{3c^3x^2\left(1+a^2\right.}$$

Mathematica [A] time = 5.78539, size = 0, normalized size = 0.

$$\int \frac{1}{x^3\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^3*(c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2)),x]
```

```
[Out] Integrate[1/(x^3*(c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2)), x]
```

Maple [A] time = 1.507, size = 0, normalized size = 0.

$$\int \frac{1}{x^3\left(a^2cx^2+c\right)^3}\left(\arctan(ax)\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x)
```

```
[Out] int(1/x^3/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a**2*c*x**2+c)**3/atan(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)^3 x^3 \arctan\left(ax\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^3*x^3*arctan(a*x)^(5/2)), x)
```

3.1075 $\int \frac{1}{x^4\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^{5/2}}\,dx$

Optimal. Leaf size=188

$$\frac{224}{3}a^2\text{Unintegrable}\left(\frac{1}{x^2\left(a^2cx^2+c\right)^3\sqrt{\tan^{-1}(ax)}},x\right)+80\text{Unintegrable}\left(\frac{1}{x^4\left(a^2cx^2+c\right)^3\sqrt{\tan^{-1}(ax)}},x\right)+\frac{80\text{Uninteg}}{}$$

```
[Out] -2/(3*a*c^3*x^4*(1 + a^2*x^2)^2*ArcTan[a*x]^(3/2)) + 16/(3*a^2*c^3*x^5*(1 +
 a^2*x^2)^2*Sqrt[ArcTan[a*x]]) + 32/(3*c^3*x^3*(1 + a^2*x^2)^2*Sqrt[ArcTan[
a*x]]) + (80*Unintegrable[1/(x^6*(c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]), x])/
(3*a^2) + 80*Unintegrable[1/(x^4*(c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]), x] +
 (224*a^2*Unintegrable[1/(x^2*(c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]), x])/3
```

Rubi [A] time = 0.469182, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^4\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^4*(c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2)),x]
```

```
[Out] -2/(3*a*c^3*x^4*(1 + a^2*x^2)^2*ArcTan[a*x]^(3/2)) + 16/(3*a^2*c^3*x^5*(1 +
 a^2*x^2)^2*Sqrt[ArcTan[a*x]]) + 32/(3*c^3*x^3*(1 + a^2*x^2)^2*Sqrt[ArcTan[
a*x]]) + (80*Defer[Int][1/(x^6*(c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]), x])/(3
*a^2) + 80*Defer[Int][1/(x^4*(c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]), x] + (22
4*a^2*Defer[Int][1/(x^2*(c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]), x])/3
```

Rubi steps

$$\int \frac{1}{x^4\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^{5/2}}\,dx = -\frac{2}{3ac^3x^4\left(1+a^2x^2\right)^2\tan^{-1}(ax)^{3/2}} - \frac{8\int \frac{1}{x^5\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^{3/2}}\,dx}{3a} - \frac{1}{3}(16a)\int \frac{}{x^3\left(c\right.}$$

$$= -\frac{2}{3ac^3x^4\left(1+a^2x^2\right)^2\tan^{-1}(ax)^{3/2}} + \frac{16}{3a^2c^3x^5\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}} + \frac{}{3c^3x^3\left(1+\right.}$$

Mathematica [A] time = 12.7624, size = 0, normalized size = 0.

$$\int \frac{1}{x^4\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^4*(c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2)),x]
```

```
[Out] Integrate[1/(x^4*(c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2)), x]
```

Maple [A] time = 0.755, size = 0, normalized size = 0.

$$\int \frac{1}{x^4\left(a^2cx^2+c\right)^3}\left(\arctan(ax)\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x)
```

```
[Out] int(1/x^4/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(a**2*c*x**2+c)**3/atan(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)^3 x^4 \arctan\left(ax\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^3*x^4*arctan(a*x)^(5/2)), x)
```

3.1076 $\int \frac{x^m\sqrt{c+a^2cx^2}}{\tan^{-1}(ax)^{5/2}}\,dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(\frac{x^m\sqrt{a^2cx^2+c}}{\tan^{-1}(ax)^{5/2}},x\right)$$

```
[Out] Unintegrable[(x^m*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^(5/2), x]
```

Rubi [A] time = 0.0981857, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\tan^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^(5/2),x]
```

```
[Out] Defer[Int][(x^m*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^(5/2), x]
```

Rubi steps

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\tan^{-1}(ax)^{5/2}}\,dx = \int \frac{x^m\sqrt{c+a^2cx^2}}{\tan^{-1}(ax)^{5/2}}\,dx$$

Mathematica [A] time = 0.68651, size = 0, normalized size = 0.

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\tan^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^(5/2),x]
```

```
[Out] Integrate[(x^m*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^(5/2), x]
```

Maple [A] time = 1.264, size = 0, normalized size = 0.

$$\int x^m \sqrt{a^2 c x^2 + c}\, (\arctan(ax))^{-\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(5/2),x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a^2 c x^2 + c}\, x^m}{\arctan(ax)^{\frac{5}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^m/arctan(a*x)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**(1/2)/atan(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a^2cx^2 + c}x^m}{\arctan(ax)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)*x^m/arctan(a*x)^(5/2), x)
```

3.1077 $\int \frac{x\sqrt{c+a^2cx^2}}{\tan^{-1}(ax)^{5/2}}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{x\sqrt{a^2cx^2+c}}{\tan^{-1}(ax)^{5/2}},x\right)$$

```
[Out] Unintegrable[(x*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^(5/2), x]
```

Rubi [A] time = 0.0665015, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x\sqrt{c+a^2cx^2}}{\tan^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^(5/2),x]
```

```
[Out] Defer[Int][(x*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^(5/2), x]
```

Rubi steps

$$\int \frac{x\sqrt{c+a^2cx^2}}{\tan^{-1}(ax)^{5/2}}\,dx = \int \frac{x\sqrt{c+a^2cx^2}}{\tan^{-1}(ax)^{5/2}}\,dx$$

Mathematica [A] time = 2.56806, size = 0, normalized size = 0.

$$\int \frac{x\sqrt{c+a^2cx^2}}{\tan^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^(5/2),x]
```

```
[Out] Integrate[(x*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^(5/2), x]
```

Maple [A] time = 1.01, size = 0, normalized size = 0.

$$\int x\sqrt{a^2cx^2+c}\,(\arctan(ax))^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(5/2),x)
```

```
[Out] int(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**(1/2)/atan(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a^2cx^2+c}\,x}{\arctan{(ax)}^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)*x/arctan(a*x)^(5/2), x)
```

3.1078 $\int \frac{\sqrt{c+a^2cx^2}}{\tan^{-1}(ax)^{5/2}}\,dx$

Optimal. Leaf size=25

$$\text{Unintegrable}\left(\frac{\sqrt{a^2cx^2+c}}{\tan^{-1}(ax)^{5/2}},x\right)$$

```
[Out] Unintegrable[Sqrt[c + a^2*c*x^2]/ArcTan[a*x]^(5/2), x]
```

Rubi [A] time = 0.0336495, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sqrt{c+a^2cx^2}}{\tan^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[c + a^2*c*x^2]/ArcTan[a*x]^(5/2),x]
```

```
[Out] Defer[Int][Sqrt[c + a^2*c*x^2]/ArcTan[a*x]^(5/2), x]
```

Rubi steps

$$\int \frac{\sqrt{c+a^2cx^2}}{\tan^{-1}(ax)^{5/2}}\,dx=\int \frac{\sqrt{c+a^2cx^2}}{\tan^{-1}(ax)^{5/2}}\,dx$$

Mathematica [A] time = 1.50746, size = 0, normalized size = 0.

$$\int \frac{\sqrt{c+a^2cx^2}}{\tan^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[c + a^2*c*x^2]/ArcTan[a*x]^(5/2),x]
```

```
[Out] Integrate[Sqrt[c + a^2*c*x^2]/ArcTan[a*x]^(5/2), x]
```

Maple [A] time = 0.878, size = 0, normalized size = 0.

$$\int \sqrt{a^2 c x^2 + c}\,(\arctan(ax))^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(1/2)/arctan(a*x)^(5/2),x)
```

```
[Out] int((a^2*c*x^2+c)^(1/2)/arctan(a*x)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(1/2)/atan(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a^2cx^2+c}}{\arctan(ax)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)/arctan(a*x)^(5/2), x)
```

3.1079 $\int \frac{\sqrt{c+a^2cx^2}}{x\tan^{-1}(ax)^{5/2}}\,dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(\frac{\sqrt{a^2cx^2+c}}{x\tan^{-1}(ax)^{5/2}},x\right)$$

```
[Out] Unintegrable[Sqrt[c + a^2*c*x^2]/(x*ArcTan[a*x]^(5/2)), x]
```

Rubi [A] time = 0.0987067, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sqrt{c+a^2cx^2}}{x\tan^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[c + a^2*c*x^2]/(x*ArcTan[a*x]^(5/2)),x]

[Out] Defer[Int][Sqrt[c + a^2*c*x^2]/(x*ArcTan[a*x]^(5/2)), x]
```

Rubi steps

$$\int \frac{\sqrt{c+a^2cx^2}}{x\tan^{-1}(ax)^{5/2}}\,dx=\int \frac{\sqrt{c+a^2cx^2}}{x\tan^{-1}(ax)^{5/2}}\,dx$$

Mathematica [A] time = 17.757, size = 0, normalized size = 0.

$$\int \frac{\sqrt{c+a^2cx^2}}{x\tan^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[c + a^2*c*x^2]/(x*ArcTan[a*x]^(5/2)),x]
```

```
[Out] Integrate[Sqrt[c + a^2*c*x^2]/(x*ArcTan[a*x]^(5/2)), x]
```

Maple [A] time = 0.918, size = 0, normalized size = 0.

$$\int \frac{1}{x}\sqrt{a^2cx^2+c}\,(\arctan(ax))^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^(5/2),x)
```

```
[Out] int((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(1/2)/x/atan(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a^2cx^2+c}}{x\arctan{(ax)}^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)/(x*arctan(a*x)^(5/2)), x)
```

3.1080 $\int \frac{x^m(c+a^2cx^2)^{3/2}}{\tan^{-1}(ax)^{5/2}}\,dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(\frac{x^m\left(a^2cx^2+c\right)^{3/2}}{\tan^{-1}(ax)^{5/2}},x\right)$$

```
[Out] Unintegrable[(x^m*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^(5/2), x]
```

Rubi [A] time = 0.108828, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m\left(c+a^2cx^2\right)^{3/2}}{\tan^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^(5/2),x]
```

```
[Out] Defer[Int][(x^m*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^(5/2), x]
```

Rubi steps

$$\int \frac{x^m\left(c+a^2cx^2\right)^{3/2}}{\tan^{-1}(ax)^{5/2}}\,dx=\int \frac{x^m\left(c+a^2cx^2\right)^{3/2}}{\tan^{-1}(ax)^{5/2}}\,dx$$

Mathematica [A] time = 1.05634, size = 0, normalized size = 0.

$$\int \frac{x^m\left(c+a^2cx^2\right)^{3/2}}{\tan^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^(5/2),x]
```

```
[Out] Integrate[(x^m*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^(5/2), x]
```

Maple [A] time = 0.98, size = 0, normalized size = 0.

$$\int x^{m} \left(a^{2} c x^{2}+c\right)^{\frac{3}{2}} \left(\arctan \left(a x\right)\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(a^{2} c x^{2}+c\right)^{\frac{3}{2}} x^{m}}{\arctan\left(a x\right)^{\frac{5}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)^(3/2)*x^m/arctan(a*x)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**(3/2)/atan(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^{\frac{3}{2}}x^m}{\arctan\left(ax\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)*x^m/arctan(a*x)^(5/2), x)
```

3.1081 $\int \frac{x(c+a^2cx^2)^{3/2}}{\tan^{-1}(ax)^{5/2}} dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{x\left(a^2cx^2+c\right)^{3/2}}{\tan^{-1}(ax)^{5/2}},x\right)$$

```
[Out] Unintegrable[(x*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^(5/2), x]
```

Rubi [A] time = 0.0764499, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x\left(c+a^2cx^2\right)^{3/2}}{\tan^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^(5/2),x]
```

```
[Out] Defer[Int][(x*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^(5/2), x]
```

Rubi steps

$$\int \frac{x\left(c+a^2cx^2\right)^{3/2}}{\tan^{-1}(ax)^{5/2}}\,dx = \int \frac{x\left(c+a^2cx^2\right)^{3/2}}{\tan^{-1}(ax)^{5/2}}\,dx$$

Mathematica [A] time = 2.59537, size = 0, normalized size = 0.

$$\int \frac{x\left(c+a^2cx^2\right)^{3/2}}{\tan^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^(5/2),x]
```

```
[Out] Integrate[(x*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^(5/2), x]
```

Maple [A] time = 0.799, size = 0, normalized size = 0.

$$\int x \left(a^{2} c \,x^{2}+c \right)^{\frac{3}{2}} \left(\arctan \left(a x \right)\right)^{-\frac{5}{2}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
```

```
[Out] int(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**(3/2)/atan(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^{\frac{3}{2}}x}{\arctan\left(ax\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)*x/arctan(a*x)^(5/2), x)
```

3.1082 $\int \frac{\left(c+a^2cx^2\right)^{3/2}}{\tan^{-1}(ax)^{5/2}}\,dx$

Optimal. Leaf size=25

$$\text{Unintegrable}\left(\frac{\left(a^2cx^2+c\right)^{3/2}}{\tan^{-1}(ax)^{5/2}},x\right)$$

```
[Out] Unintegrable[(c + a^2*c*x^2)^(3/2)/ArcTan[a*x]^(5/2), x]
```

Rubi [A] time = 0.0364513, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(c+a^2cx^2\right)^{3/2}}{\tan^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^(3/2)/ArcTan[a*x]^(5/2),x]
```

```
[Out] Defer[Int][(c + a^2*c*x^2)^(3/2)/ArcTan[a*x]^(5/2), x]
```

Rubi steps

$$\int \frac{\left(c+a^2cx^2\right)^{3/2}}{\tan^{-1}(ax)^{5/2}}\,dx=\int \frac{\left(c+a^2cx^2\right)^{3/2}}{\tan^{-1}(ax)^{5/2}}\,dx$$

Mathematica [A] time = 1.63012, size = 0, normalized size = 0.

$$\int \frac{\left(c+a^2cx^2\right)^{3/2}}{\tan^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^(3/2)/ArcTan[a*x]^(5/2),x]
```

```
[Out] Integrate[(c + a^2*c*x^2)^(3/2)/ArcTan[a*x]^(5/2), x]
```

Maple [A] time = 0.678, size = 0, normalized size = 0.

$$\int \left(a^{2} c\,x^{2}+c\right)^{\frac{3}{2}} \left(\arctan \left(a x\right)\right)^{-\frac{5}{2}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
```

```
[Out] int((a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(3/2)/atan(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^{\frac{3}{2}}}{\arctan\left(ax\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)/arctan(a*x)^(5/2), x)
```

3.1083 $\int \frac{\left(c+a^2cx^2\right)^{3/2}}{x\tan^{-1}(ax)^{5/2}}\,dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(\frac{\left(a^2cx^2+c\right)^{3/2}}{x\tan^{-1}(ax)^{5/2}},x\right)$$

```
[Out] Unintegrable[(c + a^2*c*x^2)^(3/2)/(x*ArcTan[a*x]^(5/2)), x]
```

Rubi [A] time = 0.111282, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(c+a^2cx^2\right)^{3/2}}{x\tan^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^(3/2)/(x*ArcTan[a*x]^(5/2)),x]
```

```
[Out] Defer[Int][(c + a^2*c*x^2)^(3/2)/(x*ArcTan[a*x]^(5/2)), x]
```

Rubi steps

$$\int \frac{\left(c+a^2cx^2\right)^{3/2}}{x\tan^{-1}(ax)^{5/2}}\,dx=\int \frac{\left(c+a^2cx^2\right)^{3/2}}{x\tan^{-1}(ax)^{5/2}}\,dx$$

Mathematica [A] time = 10.9058, size = 0, normalized size = 0.

$$\int \frac{\left(c+a^2cx^2\right)^{3/2}}{x\tan^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^(3/2)/(x*ArcTan[a*x]^(5/2)),x]
```

```
[Out] Integrate[(c + a^2*c*x^2)^(3/2)/(x*ArcTan[a*x]^(5/2)), x]
```

Maple [A] time = 0.741, size = 0, normalized size = 0.

$$\int \frac{1}{x}\left(a^{2}c x^{2}+c\right)^{\frac{3}{2}}\left(\arctan\left(a x\right)\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^(5/2),x)
```

```
[Out] int((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(3/2)/x/atan(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2 + c\right)^{\frac{3}{2}}}{x \arctan\left(ax\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)/(x*arctan(a*x)^(5/2)), x)
```

3.1084 $\int \frac{x^m \left(c+a^2 c x^2\right)^{5/2}}{\tan^{-1}(ax)^{5/2}} \, dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(\frac{x^m \left(a^2 c x^2+c\right)^{5/2}}{\tan^{-1}(ax)^{5/2}},x\right)$$

```
[Out] Unintegrable[(x^m*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^(5/2), x]
```

Rubi [A] time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m \left(c+a^2 c x^2\right)^{5/2}}{\tan^{-1}(ax)^{5/2}} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^(5/2),x]
```

```
[Out] Defer[Int][(x^m*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^(5/2), x]
```

Rubi steps

$$\int \frac{x^m \left(c+a^2 c x^2\right)^{5/2}}{\tan^{-1}(ax)^{5/2}} \, dx = \int \frac{x^m \left(c+a^2 c x^2\right)^{5/2}}{\tan^{-1}(ax)^{5/2}} \, dx$$

Mathematica [A] time = 1.39443, size = 0, normalized size = 0.

$$\int \frac{x^m \left(c+a^2 c x^2\right)^{5/2}}{\tan^{-1}(ax)^{5/2}} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^(5/2),x]
```

```
[Out] Integrate[(x^m*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^(5/2), x]
```

Maple [A] time = 1.077, size = 0, normalized size = 0.

$$\int x^{m} \left(a^{2} c x^{2}+c\right)^{\frac{5}{2}} \left(\arctan \left(a x\right)\right)^{-\frac{5}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(a^{4} c^{2} x^{4}+2 \, a^{2} c^{2} x^{2}+c^{2}\right) \sqrt{a^{2} c x^{2}+c} x^{m}}{\arctan \left(a x\right)^{\frac{5}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*x^m/arctan
(a*x)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**(5/2)/atan(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^{\frac{5}{2}}x^m}{\arctan\left(ax\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(5/2)*x^m/arctan(a*x)^(5/2), x)
```

3.1085 $\int \frac{x(c+a^2cx^2)^{5/2}}{\tan^{-1}(ax)^{5/2}}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{x\left(a^2cx^2+c\right)^{5/2}}{\tan^{-1}(ax)^{5/2}},x\right)$$

```
[Out] Unintegrable[(x*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^(5/2), x]
```

Rubi [A] time = 0.0791245, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x\left(c+a^2cx^2\right)^{5/2}}{\tan^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^(5/2),x]
```

```
[Out] Defer[Int][(x*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^(5/2), x]
```

Rubi steps

$$\int \frac{x\left(c+a^2cx^2\right)^{5/2}}{\tan^{-1}(ax)^{5/2}}\,dx=\int \frac{x\left(c+a^2cx^2\right)^{5/2}}{\tan^{-1}(ax)^{5/2}}\,dx$$

Mathematica [A] time = 3.02425, size = 0, normalized size = 0.

$$\int \frac{x\left(c+a^2cx^2\right)^{5/2}}{\tan^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^(5/2),x]
```

```
[Out] Integrate[(x*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^(5/2), x]
```

Maple [A] time = 0.974, size = 0, normalized size = 0.

$$\int x\left(a^2cx^2+c\right)^{\frac{5}{2}}(\arctan(ax))^{-\frac{5}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

```
[Out] int(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**(5/2)/atan(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^{\frac{5}{2}}x}{\arctan\left(ax\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(5/2)*x/arctan(a*x)^(5/2), x)
```

3.1086 $\int \frac{(c+a^2cx^2)^{5/2}}{\tan^{-1}(ax)^{5/2}}\,dx$

Optimal. Leaf size=25

$$\text{Unintegrable}\left(\frac{(a^2cx^2+c)^{5/2}}{\tan^{-1}(ax)^{5/2}},x\right)$$

```
[Out] Unintegrable[(c + a^2*c*x^2)^(5/2)/ArcTan[a*x]^(5/2), x]
```

Rubi [A] time = 0.0425756, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(c+a^2cx^2)^{5/2}}{\tan^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^(5/2)/ArcTan[a*x]^(5/2),x]
```

```
[Out] Defer[Int][(c + a^2*c*x^2)^(5/2)/ArcTan[a*x]^(5/2), x]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^{5/2}}{\tan^{-1}(ax)^{5/2}}\,dx = \int \frac{(c+a^2cx^2)^{5/2}}{\tan^{-1}(ax)^{5/2}}\,dx$$

Mathematica [A] time = 1.73513, size = 0, normalized size = 0.

$$\int \frac{(c+a^2cx^2)^{5/2}}{\tan^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^(5/2)/ArcTan[a*x]^(5/2),x]
```

```
[Out] Integrate[(c + a^2*c*x^2)^(5/2)/ArcTan[a*x]^(5/2), x]
```

Maple [A] time = 0.803, size = 0, normalized size = 0.

$$\int \left(a^2 c x^2 + c\right)^{\frac{5}{2}} \left(\arctan\left(a x\right)\right)^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

```
[Out] int((a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(5/2)/atan(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^{\frac{5}{2}}}{\arctan\left(ax\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(5/2)/arctan(a*x)^(5/2), x)
```

3.1087 $\int \frac{\left(c+a^2cx^2\right)^{5/2}}{x\tan^{-1}(ax)^{5/2}}\,dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(\frac{\left(a^2cx^2+c\right)^{5/2}}{x\tan^{-1}(ax)^{5/2}},x\right)$$

```
[Out] Unintegrable[(c + a^2*c*x^2)^(5/2)/(x*ArcTan[a*x]^(5/2)), x]
```

Rubi [A] time = 0.113891, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(c+a^2cx^2\right)^{5/2}}{x\tan^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^(5/2)/(x*ArcTan[a*x]^(5/2)),x]
```

```
[Out] Defer[Int][(c + a^2*c*x^2)^(5/2)/(x*ArcTan[a*x]^(5/2)), x]
```

Rubi steps

$$\int \frac{\left(c+a^2cx^2\right)^{5/2}}{x\tan^{-1}(ax)^{5/2}}\,dx=\int \frac{\left(c+a^2cx^2\right)^{5/2}}{x\tan^{-1}(ax)^{5/2}}\,dx$$

Mathematica [A] time = 11.0498, size = 0, normalized size = 0.

$$\int \frac{\left(c+a^2cx^2\right)^{5/2}}{x\tan^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^(5/2)/(x*ArcTan[a*x]^(5/2)),x]
```

```
[Out] Integrate[(c + a^2*c*x^2)^(5/2)/(x*ArcTan[a*x]^(5/2)), x]
```

Maple [A] time = 0.921, size = 0, normalized size = 0.

$$\int \frac{1}{x}\left(a^{2}c x^{2}+c\right)^{\frac{5}{2}} \left(\arctan \left(a x\right)\right)^{-\frac{5}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^(5/2),x)
```

```
[Out] int((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(5/2)/x/atan(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a^2cx^2 + c)^{\frac{5}{2}}}{x \arctan(ax)^{\frac{5}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(5/2)/(x*arctan(a*x)^(5/2)), x)
```

3.1088 $\int \frac{x^m}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{5/2}}\,dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(\frac{x^m}{\sqrt{a^2cx^2+c}\tan^{-1}(ax)^{5/2}},x\right)$$

```
[Out] Unintegrable[x^m/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2)), x]
```

Rubi [A] time = 0.114348, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2)),x]
```

```
[Out] Defer[Int][x^m/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2)), x]
```

Rubi steps

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{5/2}}\,dx = \int \frac{x^m}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{5/2}}\,dx$$

Mathematica [A] time = 0.849283, size = 0, normalized size = 0.

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2)),x]
```

```
[Out] Integrate[x^m/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2)), x]
```

Maple [A] time = 1.395, size = 0, normalized size = 0.

$$\int x^{m} (\arctan (ax))^{-\frac{5}{2}} \frac{1}{\sqrt{a^{2}c x^{2}+c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(x^m/arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^{m}}{\sqrt{a^{2}c x^{2}+c}\arctan (ax)^{\frac{5}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^m/(sqrt(a^2*c*x^2 + c)*arctan(a*x)^(5/2)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/atan(a*x)**(5/2)/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\sqrt{a^2cx^2 + c}\arctan(ax)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^m/(sqrt(a^2*c*x^2 + c)*arctan(a*x)^(5/2)), x)
```

3.1089 $\int \frac{x}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{5/2}}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{x}{\sqrt{a^2cx^2+c}\tan^{-1}(ax)^{5/2}},x\right)$$

```
[Out] Unintegrable[x/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2)), x]
```

Rubi [A] time = 0.0752147, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2)),x]
```

```
[Out] Defer[Int][x/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2)), x]
```

Rubi steps

$$\int \frac{x}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{5/2}}\,dx=\int \frac{x}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{5/2}}\,dx$$

Mathematica [A] time = 1.73838, size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2)),x]
```

```
[Out] Integrate[x/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2)), x]
```

Maple [A] time = 1.203, size = 0, normalized size = 0.

$$\int x (\arctan (ax))^{-\frac{5}{2}} \frac{1}{\sqrt{a^2cx^2 + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(x/arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/atan(a*x)**(5/2)/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{a^2cx^2+c}\arctan{(ax)}^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/(sqrt(a^2*c*x^2 + c)*arctan(a*x)^(5/2)), x)
```

3.1090 $\int \frac{1}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{5/2}}\,dx$

Optimal. Leaf size=25

$$\text{Unintegrable}\left(\frac{1}{\sqrt{a^2cx^2+c}\tan^{-1}(ax)^{5/2}},x\right)$$

```
[Out] Unintegrable[1/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2)), x]
```

Rubi [A] time = 0.0361965, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2)),x]
```

```
[Out] Defer[Int][1/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2)), x]
```

Rubi steps

$$\int \frac{1}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{5/2}}\,dx=\int \frac{1}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{5/2}}\,dx$$

Mathematica [A] time = 0.798492, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2)),x]
```

```
[Out] Integrate[1/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2)), x]
```

Maple [A] time = 0.962, size = 0, normalized size = 0.

$$\int (\arctan (ax))^{-\frac{5}{2}} \frac{1}{\sqrt{a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(1/arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/atan(a*x)**(5/2)/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a^2cx^2+c}\arctan(ax)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a^2*c*x^2 + c)*arctan(a*x)^(5/2)), x)
```

3.1091 $\int \frac{1}{x\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{5/2}}\,dx$

Optimal. Leaf size=71

$$-\frac{2\text{Unintegrable}\left(\frac{1}{x^2\sqrt{a^2cx^2+c}\tan^{-1}(ax)^{3/2}},x\right)}{3a}-\frac{2\sqrt{a^2cx^2+c}}{3acx\tan^{-1}(ax)^{3/2}}$$

```
[Out] (-2*Sqrt[c + a^2*c*x^2])/(3*a*c*x*ArcTan[a*x]^(3/2)) - (2*Unintegrable[1/(x
^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2)), x])/(3*a)
```

Rubi [A] time = 0.216067, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2)),x]
```

```
[Out] (-2*Sqrt[c + a^2*c*x^2])/(3*a*c*x*ArcTan[a*x]^(3/2)) - (2*Defer[Int][1/(x^2
*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2)), x])/(3*a)
```

Rubi steps

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{5/2}}\,dx=-\frac{2\sqrt{c+a^2cx^2}}{3acx\tan^{-1}(ax)^{3/2}}-\frac{2\int \frac{1}{x^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}}\,dx}{3a}$$

Mathematica [A] time = 3.35126, size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2)),x]
```

```
[Out] Integrate[1/(x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2)), x]
```

Maple [A] time = 1.277, size = 0, normalized size = 0.

$$\int \frac{1}{x} (\arctan(ax))^{-\frac{5}{2}} \frac{1}{\sqrt{a^2cx^2 + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(1/x/arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/atan(a*x)**(5/2)/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a^2cx^2+cx}\arctan{(ax)}^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a^2*c*x^2 + c)*x*arctan(a*x)^(5/2)), x)
```

3.1092 $\int \frac{1}{x^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{5/2}}\,dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(\frac{1}{x^2\sqrt{a^2cx^2+c}\tan^{-1}(ax)^{5/2}},x\right)$$

```
[Out] Unintegrable[1/(x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2)), x]
```

Rubi [A] time = 0.108675, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2)),x]
```

```
[Out] Defer[Int][1/(x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2)), x]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{5/2}}\,dx=\int \frac{1}{x^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{5/2}}\,dx$$

Mathematica [A] time = 13.0982, size = 0, normalized size = 0.

$$\int \frac{1}{x^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2)),x]
```

```
[Out] Integrate[1/(x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2)), x]
```

Maple [A] time = 0.898, size = 0, normalized size = 0.

$$\int \frac{1}{x^2} (\arctan (ax))^{-\frac{5}{2}} \frac{1}{\sqrt{a^2 c x^2 + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x)

[Out] int(1/x^2/arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima"
)

[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas"
)

[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/atan(a*x)**(5/2)/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a^2cx^2+c}\,x^2\arctan{(ax)}^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a^2*c*x^2 + c)*x^2*arctan(a*x)^(5/2)), x)
```

3.1093 $\int \frac{x^m}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{5/2}}\,dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(\frac{x^m}{\left(a^2cx^2+c\right)^{3/2}\tan^{-1}(ax)^{5/2}},x\right)$$

```
[Out] Unintegrable[x^m/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2)), x]
```

Rubi [A] time = 0.118238, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2)),x]
```

```
[Out] Defer[Int][x^m/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2)), x]
```

Rubi steps

$$\int \frac{x^m}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{5/2}}\,dx=\int \frac{x^m}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{5/2}}\,dx$$

Mathematica [A] time = 0.989379, size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2)),x]
```

[Out] Integrate[x^m/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2)), x]

Maple [A] time = 1.246, size = 0, normalized size = 0.

$$\int x^{m} \left(a^{2} c x^{2}+c \right)^{-\frac{3}{2}} \left(\arctan \left(a x \right)\right)^{-\frac{5}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)

[Out] int(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="maxima")

[Out] Exception raised: RuntimeError

Fricas [A] time = 0., size = 0, normalized size = 0.

$${\rm integral}\left(\frac{\sqrt{a^{2} c x^{2} + c} x^{m}}{{\left(a^{4} c^{2} x^{4} + 2 \, a^{2} c^{2} x^{2} + c^{2}\right)} \arctan\left(a x\right)^{\frac{5}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="fricas")

[Out] integral(sqrt(a^2*c*x^2 + c)*x^m/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arcta
n(a*x)^(5/2)), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(a**2*c*x**2+c)**(3/2)/atan(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(a^2cx^2+c\right)^{\frac{3}{2}}\arctan\left(ax\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x^m/((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^(5/2)), x)
```

3.1094 $\int \frac{x^3}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{5/2}}\,dx$

Optimal. Leaf size=227

$$\frac{44}{3}\text{Unintegrable}\left(\frac{x^3}{\left(a^2cx^2+c\right)^{3/2}\sqrt{\tan^{-1}(ax)}},x\right)+8a^2\text{Unintegrable}\left(\frac{x^5}{\left(a^2cx^2+c\right)^{3/2}\sqrt{\tan^{-1}(ax)}},x\right)+\frac{8\sqrt{2\pi}\sqrt{a^2x^2+}}{a^4c}$$

```
[Out] (-2*x^3)/(3*a*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2)) - (4*x^2)/(a^2*c*Sqr
t[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]) - (8*x^4)/(3*c*Sqrt[c + a^2*c*x^2]*Sqrt
[ArcTan[a*x]]) + (8*Sqrt[2*Pi]*Sqrt[1 + a^2*x^2]*FresnelS[Sqrt[2/Pi]*Sqrt[A
rcTan[a*x]]])/(a^4*c*Sqrt[c + a^2*c*x^2]) + (44*Unintegrable[x^3/((c + a^2*
c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]), x])/3 + 8*a^2*Unintegrable[x^5/((c + a^2*c
*x^2)^(3/2)*Sqrt[ArcTan[a*x]]), x]
```

Rubi [A] time = 0.927643, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^3}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^3/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2)),x]
```

```
[Out] (-2*x^3)/(3*a*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2)) - (4*x^2)/(a^2*c*Sqr
t[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]) - (8*x^4)/(3*c*Sqrt[c + a^2*c*x^2]*Sqrt
[ArcTan[a*x]]) + (8*Sqrt[2*Pi]*Sqrt[1 + a^2*x^2]*FresnelS[Sqrt[2/Pi]*Sqrt[A
rcTan[a*x]]])/(a^4*c*Sqrt[c + a^2*c*x^2]) + (44*Defer[Int][x^3/((c + a^2*c*
x^2)^(3/2)*Sqrt[ArcTan[a*x]]), x])/3 + 8*a^2*Defer[Int][x^5/((c + a^2*c*x^2
)^(3/2)*Sqrt[ArcTan[a*x]]), x]
```

Rubi steps

$$\int \frac{x^3}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{5/2}}\,dx = -\frac{2x^3}{3ac\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}} + \frac{2\int \frac{x^2}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}}\,dx}{a} + \frac{1}{3}(4a)\int \frac{x}{\left(c+a^2cx^2\right)^{3/2}}$$

$$= -\frac{2x^3}{3ac\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}} - \frac{4x^2}{a^2c\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}} - \frac{8x^4}{3c\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(a}}$$

$$= -\frac{2x^3}{3ac\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}} - \frac{4x^2}{a^2c\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}} - \frac{8x^4}{3c\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(a}}$$

$$= -\frac{2x^3}{3ac\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}} - \frac{4x^2}{a^2c\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}} - \frac{8x^4}{3c\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(a}}$$

$$= -\frac{2x^3}{3ac\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}} - \frac{4x^2}{a^2c\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}} - \frac{8x^4}{3c\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(a}}$$

$$= -\frac{2x^3}{3ac\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}} - \frac{4x^2}{a^2c\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}} - \frac{8x^4}{3c\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(a}}$$

Mathematica [A] time = 2.57426, size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^3/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2)),x]
```

```
[Out] Integrate[x^3/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2)), x]
```

Maple [A] time = 3.161, size = 0, normalized size = 0.

$$\int x^3\left(a^2cx^2+c\right)^{-\frac{3}{2}}(\arctan(ax))^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
```

```
[Out] int(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a**2*c*x**2+c)**(3/2)/atan(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(a^2cx^2+c\right)^{\frac{3}{2}}\arctan\left(ax\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^(5/2)), x)
```

3.1095 $\int \frac{x^2}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{5/2}}\,dx$

Optimal. Leaf size=229

$$4\text{Unintegrable}\left(\frac{x^2}{\left(a^2cx^2+c\right)^{3/2}\sqrt{\tan^{-1}(ax)}},x\right)+\frac{8}{3}a^2\text{Unintegrable}\left(\frac{x^4}{\left(a^2cx^2+c\right)^{3/2}\sqrt{\tan^{-1}(ax)}},x\right)+\frac{8\sqrt{2\pi}\sqrt{a^2x^2+\cdots}}{3\cdots}$$

```
[Out] (-2*x^2)/(3*a*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2)) - (8*x)/(3*a^2*c*Sqr
t[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]) - (4*x^3)/(3*c*Sqrt[c + a^2*c*x^2]*Sqrt
[ArcTan[a*x]]) + (8*Sqrt[2*Pi]*Sqrt[1 + a^2*x^2]*FresnelC[Sqrt[2/Pi]*Sqrt[A
rcTan[a*x]]])/(3*a^3*c*Sqrt[c + a^2*c*x^2]) + 4*Unintegrable[x^2/((c + a^2*
c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]), x] + (8*a^2*Unintegrable[x^4/((c + a^2*c*x
^2)^(3/2)*Sqrt[ArcTan[a*x]]), x])/3
```

Rubi [A] time = 0.680397, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^2}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^2/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2)),x]
```

```
[Out] (-2*x^2)/(3*a*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2)) - (8*x)/(3*a^2*c*Sqr
t[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]) - (4*x^3)/(3*c*Sqrt[c + a^2*c*x^2]*Sqrt
[ArcTan[a*x]]) + (8*Sqrt[2*Pi]*Sqrt[1 + a^2*x^2]*FresnelC[Sqrt[2/Pi]*Sqrt[A
rcTan[a*x]]])/(3*a^3*c*Sqrt[c + a^2*c*x^2]) + 4*Defer[Int][x^2/((c + a^2*c*
x^2)^(3/2)*Sqrt[ArcTan[a*x]]), x] + (8*a^2*Defer[Int][x^4/((c + a^2*c*x^2)^
(3/2)*Sqrt[ArcTan[a*x]]), x])/3
```

Rubi steps

$$\int \frac{x^2}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{5/2}}\,dx = -\frac{2x^2}{3ac\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}} + \frac{4\int \frac{x}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}}\,dx}{3a} + \frac{1}{3}(2a)\int \frac{x}{\left(c+a^2cx^2\right)^{3/2}}$$

$$= -\frac{2x^2}{3ac\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}} - \frac{8x}{3a^2c\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}} - \frac{4x^3}{3c\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}}}$$

$$= -\frac{2x^2}{3ac\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}} - \frac{8x}{3a^2c\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}} - \frac{4x^3}{3c\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}}}$$

$$= -\frac{2x^2}{3ac\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}} - \frac{8x}{3a^2c\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}} - \frac{4x^3}{3c\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}}}$$

$$= -\frac{2x^2}{3ac\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}} - \frac{8x}{3a^2c\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}} - \frac{4x^3}{3c\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}}}$$

$$= -\frac{2x^2}{3ac\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}} - \frac{8x}{3a^2c\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}} - \frac{4x^3}{3c\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}}}$$

Mathematica [A] time = 2.10563, size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^2/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2)),x]
```

```
[Out] Integrate[x^2/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2)), x]
```

Maple [A] time = 3.109, size = 0, normalized size = 0.

$$\int x^2\left(a^2cx^2+c\right)^{-\frac{3}{2}}(\arctan(ax))^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
```

```
[Out] int(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a**2*c*x**2+c)**(3/2)/atan(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(a^2cx^2+c\right)^{\frac{3}{2}}\arctan\left(ax\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^(5/2)), x)
```

3.1096 $\int \frac{x}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{5/2}}\,dx$

Optimal. Leaf size=129

$$-\frac{4\sqrt{2\pi}\sqrt{a^2x^2+1}S\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{3a^2c\sqrt{a^2cx^2+c}}-\frac{2x}{3ac\sqrt{a^2cx^2+c}\tan^{-1}(ax)^{3/2}}-\frac{4}{3a^2c\sqrt{a^2cx^2+c}\sqrt{\tan^{-1}(ax)}}$$

```
[Out] (-2*x)/(3*a*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2)) - 4/(3*a^2*c*Sqrt[c +
a^2*c*x^2]*Sqrt[ArcTan[a*x]]) - (4*Sqrt[2*Pi]*Sqrt[1 + a^2*x^2]*FresnelS[Sq
rt[2/Pi]*Sqrt[ArcTan[a*x]]])/(3*a^2*c*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.303309, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4942, 4902, 4971, 4970, 3305, 3351}

$$-\frac{4\sqrt{2\pi}\sqrt{a^2x^2+1}S\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{3a^2c\sqrt{a^2cx^2+c}}-\frac{2x}{3ac\sqrt{a^2cx^2+c}\tan^{-1}(ax)^{3/2}}-\frac{4}{3a^2c\sqrt{a^2cx^2+c}\sqrt{\tan^{-1}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2)),x]
```


```
[Out] (-2*x)/(3*a*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2)) - 4/(3*a^2*c*Sqrt[c +
a^2*c*x^2]*Sqrt[ArcTan[a*x]]) - (4*Sqrt[2*Pi]*Sqrt[1 + a^2*x^2]*FresnelS[Sq
rt[2/Pi]*Sqrt[ArcTan[a*x]]])/(3*a^2*c*Sqrt[c + a^2*c*x^2])
```


Rule 4942

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^m*(d + e*x^2)^(q + 1)*(a + b*ArcT
an[c*x])^(p + 1))/(b*c*d*(p + 1)), x] - Dist[(f*m)/(b*c*(p + 1)), Int[(f*x)
^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 2, 0] && LtQ[p, -1]
```

Rule 4902

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^(p + 1))/(b*c*d*(p
```

```
+ 1)), x] - Dist[(2*c*(q + 1))/(b*(p + 1)), Int[x*(d + e*x^2)^q*(a + b*ArcT
an[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && L
tQ[q, -1] && LtQ[p, -1]
```

Rule 4971

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[(d^(q + 1/2)*Sqrt[1 + c^2*x^2])/Sqrt[d + e*x^2],
 Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d
, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(In
tegerQ[q] || GtQ[d, 0])
```

Rule 4970

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sin[x]^m)/
Cos[x]^(m + 2*(q + 1)), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{x}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{5/2}}\,dx = -\frac{2x}{3ac\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}} + \frac{2\int \frac{1}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}}\,dx}{3a}$$

$$= -\frac{2x}{3ac\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}} - \frac{4}{3a^2c\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}} - \frac{4}{3}\int \frac{x}{\left(c+a^2cx^2\right)^{3/2}}$$

$$= -\frac{2x}{3ac\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}} - \frac{4}{3a^2c\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}} - \frac{\left(4\sqrt{1+a^2x^2}\right)\int \frac{}{(1+a}}{3c\sqrt{c+}}$$

$$= -\frac{2x}{3ac\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}} - \frac{4}{3a^2c\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}} - \frac{\left(4\sqrt{1+a^2x^2}\right)\text{Subst}}{3a^2c}$$

$$= -\frac{2x}{3ac\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}} - \frac{4}{3a^2c\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}} - \frac{\left(8\sqrt{1+a^2x^2}\right)\text{Subst}}{3a}$$

$$= -\frac{2x}{3ac\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}} - \frac{4}{3a^2c\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}} - \frac{4\sqrt{2\pi}\sqrt{1+a^2x^2}S\left(\right.}{3a^2c\sqrt{c+}}$$

Mathematica [C] time = 0.185774, size = 124, normalized size = 0.96

$$-\frac{2\left(-i\sqrt{a^2x^2+1}\left(-i\tan^{-1}(ax)\right)^{3/2}\text{Gamma}\left(\frac{1}{2},-i\tan^{-1}(ax)\right)+i\sqrt{a^2x^2+1}\left(i\tan^{-1}(ax)\right)^{3/2}\text{Gamma}\left(\frac{1}{2},i\tan^{-1}(ax)\right)+\right.}{3a^2c\sqrt{a^2cx^2+c}\tan^{-1}(ax)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2)),x]
```

```
[Out] (-2*(a*x + 2*ArcTan[a*x] - I*Sqrt[1 + a^2*x^2]*((-I)*ArcTan[a*x])^(3/2)*Gam
ma[1/2, (-I)*ArcTan[a*x]] + I*Sqrt[1 + a^2*x^2]*(I*ArcTan[a*x])^(3/2)*Gamma
[1/2, I*ArcTan[a*x]]))/(3*a^2*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2))
```

Maple [F] time = 0.904, size = 0, normalized size = 0.

$$\int x\left(a^2cx^2+c\right)^{-\frac{3}{2}}\left(\arctan(ax)\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
```

```
[Out] int(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a**2*c*x**2+c)**(3/2)/atan(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(a^2cx^2+c\right)^{\frac{3}{2}}\arctan\left(ax\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x/((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^(5/2)), x)
```

3.1097 $\int \frac{1}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{5/2}}\,dx$

Optimal. Leaf size=126

$$-\frac{4\sqrt{2\pi}\sqrt{a^2x^2+1}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{3ac\sqrt{a^2cx^2+c}}+\frac{4x}{3c\sqrt{a^2cx^2+c}\sqrt{\tan^{-1}(ax)}}-\frac{2}{3ac\sqrt{a^2cx^2+c}\tan^{-1}(ax)^{3/2}}$$

```
[Out] -2/(3*a*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2)) + (4*x)/(3*c*Sqrt[c + a^2*
c*x^2]*Sqrt[ArcTan[a*x]]) - (4*Sqrt[2*Pi]*Sqrt[1 + a^2*x^2]*FresnelC[Sqrt[2
/Pi]*Sqrt[ArcTan[a*x]]])/(3*a*c*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.228462, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4902, 4942, 4905, 4904, 3304, 3352}

$$-\frac{4\sqrt{2\pi}\sqrt{a^2x^2+1}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{3ac\sqrt{a^2cx^2+c}}+\frac{4x}{3c\sqrt{a^2cx^2+c}\sqrt{\tan^{-1}(ax)}}-\frac{2}{3ac\sqrt{a^2cx^2+c}\tan^{-1}(ax)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2)),x]
```


```
[Out] -2/(3*a*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2)) + (4*x)/(3*c*Sqrt[c + a^2*
c*x^2]*Sqrt[ArcTan[a*x]]) - (4*Sqrt[2*Pi]*Sqrt[1 + a^2*x^2]*FresnelC[Sqrt[2
/Pi]*Sqrt[ArcTan[a*x]]])/(3*a*c*Sqrt[c + a^2*c*x^2])
```


Rule 4902

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^(p + 1))/(b*c*d*(p
+ 1)), x] - Dist[(2*c*(q + 1))/(b*(p + 1)), Int[x*(d + e*x^2)^q*(a + b*ArcT
an[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && L
tQ[q, -1] && LtQ[p, -1]
```

Rule 4942

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^m*(d + e*x^2)^(q + 1)*(a + b*ArcT
```

```
an[c*x])^(p + 1))/(b*c*d*(p + 1)), x] - Dist[(f*m)/(b*c*(p + 1)), Int[(f*x)
^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 2, 0] && LtQ[p, -1]
```

Rule 4905

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[(d^(q + 1/2)*Sqrt[1 + c^2*x^2])/Sqrt[d + e*x^2], Int[(1 + c
^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

Rule 4904

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{1}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{5/2}}\,dx = -\frac{2}{3ac\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}} - \frac{1}{3}(2a)\int \frac{x}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}}\,dx$$

$$= -\frac{2}{3ac\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}} + \frac{4x}{3c\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}} - \frac{4}{3}\int \frac{1}{\left(c+a^2cx^2\right)^{3/2}\sqrt{\mathrm{ta}}}$$

$$= -\frac{2}{3ac\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}} + \frac{4x}{3c\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}} - \frac{\left(4\sqrt{1+a^2x^2}\right)\int \frac{}{(1+a^2x^2)}}{3c\sqrt{c+a^2c}}$$

$$= -\frac{2}{3ac\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}} + \frac{4x}{3c\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}} - \frac{\left(4\sqrt{1+a^2x^2}\right)\mathrm{Subst}\left(\int\right.}{3ac\sqrt{c}}$$

$$= -\frac{2}{3ac\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}} + \frac{4x}{3c\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}} - \frac{\left(8\sqrt{1+a^2x^2}\right)\mathrm{Subst}\left(\int\right.}{3ac}$$

$$= -\frac{2}{3ac\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}} + \frac{4x}{3c\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}} - \frac{4\sqrt{2\pi}\sqrt{1+a^2x^2}C\left(\sqrt{\frac{2}{\pi}}\right.}{3ac\sqrt{c+a^2c}}$$

Mathematica [C] time = 0.144226, size = 120, normalized size = 0.95

$$\frac{-2\sqrt{a^2x^2+1}\left(-i\tan^{-1}(ax)\right)^{3/2}\mathrm{Gamma}\left(\frac{1}{2},-i\tan^{-1}(ax)\right)-2\sqrt{a^2x^2+1}\left(i\tan^{-1}(ax)\right)^{3/2}\mathrm{Gamma}\left(\frac{1}{2},i\tan^{-1}(ax)\right)+4ax\,\mathrm{t}}{3ac\sqrt{a^2cx^2+c}\tan^{-1}(ax)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2)),x]
```

```
[Out] (-2 + 4*a*x*ArcTan[a*x] - 2*Sqrt[1 + a^2*x^2]*((-I)*ArcTan[a*x])^(3/2)*Gamm
a[1/2, (-I)*ArcTan[a*x]] - 2*Sqrt[1 + a^2*x^2]*(I*ArcTan[a*x])^(3/2)*Gamma[
1/2, I*ArcTan[a*x]])/(3*a*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2))
```

Maple [F] time = 0.82, size = 0, normalized size = 0.

$$\int \left(a^2cx^2+c\right)^{-\frac{3}{2}}(\arctan(ax))^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
```

```
[Out] int(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*c*x**2+c)**(3/2)/atan(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^{2}c x^{2}+c\right)^{\frac{3}{2}} \arctan\left(a x\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^(5/2)), x)
```

3.1098 $\int \frac{1}{x(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^{5/2}}\,dx$

Optimal. Leaf size=225

$$\frac{8\text{Unintegrable}\left(\frac{1}{x^3(a^2cx^2+c)^{3/2}\sqrt{\tan^{-1}(ax)}},x\right)}{3a^2}+4\text{Unintegrable}\left(\frac{1}{x\left(a^2cx^2+c\right)^{3/2}\sqrt{\tan^{-1}(ax)}},x\right)+\frac{8\sqrt{2\pi}\sqrt{a^2x^2+1}S\left(\sqrt{\right.}}{3c\sqrt{a^2cx^2}}$$

```
[Out] -2/(3*a*c*x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2)) + 8/(3*c*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]) + 4/(3*a^2*c*x^2*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]) + (8*Sqrt[2*Pi]*Sqrt[1 + a^2*x^2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(3*c*Sqrt[c + a^2*c*x^2]) + (8*Unintegrable[1/(x^3*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]), x])/(3*a^2) + 4*Unintegrable[1/(x*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]), x]
```

Rubi [A] time = 0.697471, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2)),x]
```

```
[Out] -2/(3*a*c*x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2)) + 8/(3*c*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]) + 4/(3*a^2*c*x^2*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]) + (8*Sqrt[2*Pi]*Sqrt[1 + a^2*x^2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(3*c*Sqrt[c + a^2*c*x^2]) + (8*Defer[Int][1/(x^3*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]), x])/(3*a^2) + 4*Defer[Int][1/(x*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]), x]
```

Rubi steps

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{5/2}}\,dx = -\frac{2}{3acx\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}} - \frac{2\int \frac{1}{x^2\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}}\,dx}{3a} - \frac{1}{3}(4a)\int \frac{}{\left(c+a^2cx\right.}$$

$$= -\frac{2}{3acx\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}} + \frac{8}{3c\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}} + \frac{4}{3a^2cx^2\sqrt{c+a^2cx^2}\sqrt{t}}$$

$$= -\frac{2}{3acx\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}} + \frac{8}{3c\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}} + \frac{4}{3a^2cx^2\sqrt{c+a^2cx^2}\sqrt{t}}$$

$$= -\frac{2}{3acx\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}} + \frac{8}{3c\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}} + \frac{4}{3a^2cx^2\sqrt{c+a^2cx^2}\sqrt{t}}$$

$$= -\frac{2}{3acx\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}} + \frac{8}{3c\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}} + \frac{4}{3a^2cx^2\sqrt{c+a^2cx^2}\sqrt{t}}$$

$$= -\frac{2}{3acx\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}} + \frac{8}{3c\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}} + \frac{4}{3a^2cx^2\sqrt{c+a^2cx^2}\sqrt{t}}$$

Mathematica [A] time = 11.4864, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2)),x]
```

```
[Out] Integrate[1/(x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2)), x]
```

Maple [A] time = 0.805, size = 0, normalized size = 0.

$$\int \frac{1}{x}\left(a^2cx^2+c\right)^{-\frac{3}{2}}(\arctan(ax))^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
```

```
[Out] int(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a**2*c*x**2+c)**(3/2)/atan(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)^{\frac{3}{2}}x\arctan\left(ax\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="giac")

[Out] integrate(1/((a^2*c*x^2 + c)^(3/2)*x*arctan(a*x)^(5/2)), x)
```

3.1099 $\int \frac{1}{x^2\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{5/2}}\,dx$

Optimal. Leaf size=225

$$\frac{8\text{Unintegrable}\left(\frac{1}{x^4\left(a^2cx^2+c\right)^{3/2}\sqrt{\tan^{-1}(ax)}},x\right)}{a^2}+\frac{44}{3}\text{Unintegrable}\left(\frac{1}{x^2\left(a^2cx^2+c\right)^{3/2}\sqrt{\tan^{-1}(ax)}},x\right)+\frac{8\sqrt{2\pi}a\sqrt{a^2x^2+1}\text{F}}{c\sqrt{}}$$

[Out] -2/(3*a*c*x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2)) + 8/(3*a^2*c*x^3*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]) + 4/(c*x*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]) + (8*a*Sqrt[2*Pi]*Sqrt[1 + a^2*x^2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(c*Sqrt[c + a^2*c*x^2]) + (8*Unintegrable[1/(x^4*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]), x])/a^2 + (44*Unintegrable[1/(x^2*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]), x])/3

Rubi [A] time = 0.833288, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

[In] Int[1/(x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2)),x]

[Out] -2/(3*a*c*x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2)) + 8/(3*a^2*c*x^3*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]) + 4/(c*x*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]) + (8*a*Sqrt[2*Pi]*Sqrt[1 + a^2*x^2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(c*Sqrt[c + a^2*c*x^2]) + (8*Defer[Int][1/(x^4*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]), x])/a^2 + (44*Defer[Int][1/(x^2*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]), x])/3

Rubi steps

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{5/2}}\,dx = -\frac{2}{3acx^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}} - \frac{4\int \frac{1}{x^3\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}}\,dx}{3a} - (2a)\int \frac{\cdots}{x\left(c+a^2\cdots\right.}$$

$$= -\frac{2}{3acx^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}} + \frac{8}{3a^2cx^3\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}} + \frac{4}{cx\sqrt{c+a^2cx^2}\cdots}$$

$$= -\frac{2}{3acx^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}} + \frac{8}{3a^2cx^3\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}} + \frac{4}{cx\sqrt{c+a^2cx^2}\cdots}$$

$$= -\frac{2}{3acx^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}} + \frac{8}{3a^2cx^3\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}} + \frac{4}{cx\sqrt{c+a^2cx^2}\cdots}$$

$$= -\frac{2}{3acx^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}} + \frac{8}{3a^2cx^3\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}} + \frac{4}{cx\sqrt{c+a^2cx^2}\cdots}$$

$$= -\frac{2}{3acx^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}} + \frac{8}{3a^2cx^3\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}} + \frac{4}{cx\sqrt{c+a^2cx^2}\cdots}$$

Mathematica [A] time = 13.0346, size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2)),x]
```

```
[Out] Integrate[1/(x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2)), x]
```

Maple [A] time = 0.803, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\left(a^2cx^2+c\right)^{-\frac{3}{2}}\left(\arctan(ax)\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
```

```
[Out] int(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="fricas"
)
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a**2*c*x**2+c)**(3/2)/atan(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)^{\frac{3}{2}}x^2\arctan\left(ax\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^(3/2)*x^2*arctan(a*x)^(5/2)), x)
```

3.1100 $\int \frac{1}{x^3(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^{5/2}}\,dx$

Optimal. Leaf size=199

$$16a^2\text{Unintegrable}\left(\frac{1}{x\left(a^2cx^2+c\right)^{3/2}\sqrt{\tan^{-1}(ax)}},x\right)+\frac{92}{3}\text{Unintegrable}\left(\frac{1}{x^3\left(a^2cx^2+c\right)^{3/2}\sqrt{\tan^{-1}(ax)}},x\right)+\frac{16\text{Uninte}}{}$$

```
[Out] -2/(3*a*c*x^3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2)) + 4/(a^2*c*x^4*Sqrt[c
+ a^2*c*x^2]*Sqrt[ArcTan[a*x]]) + 16/(3*c*x^2*Sqrt[c + a^2*c*x^2]*Sqrt[ArcT
an[a*x]]) + (16*Unintegrable[1/(x^5*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]
), x])/a^2 + (92*Unintegrable[1/(x^3*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]
]), x])/3 + 16*a^2*Unintegrable[1/(x*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]
]), x]
```

Rubi [A] time = 0.84894, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^3\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^3*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2)),x]
```

```
[Out] -2/(3*a*c*x^3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2)) + 4/(a^2*c*x^4*Sqrt[c
+ a^2*c*x^2]*Sqrt[ArcTan[a*x]]) + 16/(3*c*x^2*Sqrt[c + a^2*c*x^2]*Sqrt[ArcT
an[a*x]]) + (16*Defer[Int][1/(x^5*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]),
 x])/a^2 + (92*Defer[Int][1/(x^3*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]),
x])/3 + 16*a^2*Defer[Int][1/(x*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]), x]
```

Rubi steps

$$\int \frac{1}{x^3\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{5/2}}\,dx = -\frac{2}{3acx^3\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}} - \frac{2\int \frac{1}{x^4\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}}\,dx}{a} - \frac{1}{3}(8a)\int \frac{}{x^2\left(c+\right.}$$

$$= -\frac{2}{3acx^3\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}} + \frac{4}{a^2cx^4\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}} + \frac{16}{3cx^2\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 15.0857, size = 0, normalized size = 0.

$$\int \frac{1}{x^3\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^3*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2)),x]
```

```
[Out] Integrate[1/(x^3*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2)), x]
```

Maple [A] time = 0.983, size = 0, normalized size = 0.

$$\int \frac{1}{x^3}\left(a^2cx^2+c\right)^{-\frac{3}{2}}(\arctan(ax))^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
```

```
[Out] int(1/x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="fricas"
)
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a**2*c*x**2+c)**(3/2)/atan(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)^{\frac{3}{2}}x^3\arctan\left(ax\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^(3/2)*x^3*arctan(a*x)^(5/2)), x)
```

3.1101 $\int \frac{1}{x^4(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^{5/2}}\,dx$

Optimal. Leaf size=203

$$\frac{80}{3}a^2\text{Unintegrable}\left(\frac{1}{x^2\left(a^2cx^2+c\right)^{3/2}\sqrt{\tan^{-1}(ax)}},x\right)+52\text{Unintegrable}\left(\frac{1}{x^4\left(a^2cx^2+c\right)^{3/2}\sqrt{\tan^{-1}(ax)}},x\right)+\frac{80\text{Uninte}}{}$$

```
[Out] -2/(3*a*c*x^4*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2)) + 16/(3*a^2*c*x^5*Sqrt
[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]) + 20/(3*c*x^3*Sqrt[c + a^2*c*x^2]*Sqrt[A
rcTan[a*x]]) + (80*Unintegrable[1/(x^6*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*
x]]), x])/(3*a^2) + 52*Unintegrable[1/(x^4*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTa
n[a*x]]), x] + (80*a^2*Unintegrable[1/(x^2*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTa
n[a*x]]), x])/3
```

Rubi [A] time = 0.852086, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^4\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^4*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2)),x]
```

```
[Out] -2/(3*a*c*x^4*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2)) + 16/(3*a^2*c*x^5*Sqrt
[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]) + 20/(3*c*x^3*Sqrt[c + a^2*c*x^2]*Sqrt[A
rcTan[a*x]]) + (80*Defer[Int][1/(x^6*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]
]), x])/(3*a^2) + 52*Defer[Int][1/(x^4*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*
x]]), x] + (80*a^2*Defer[Int][1/(x^2*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]
]), x])/3
```

Rubi steps

$$\int \frac{1}{x^4\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{5/2}}\,dx = -\frac{2}{3acx^4\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}} - \frac{8\int \frac{1}{x^5\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}}\,dx}{3a} - \frac{1}{3}(10a)\int \frac{}{x^3\left(\right.}$$

$$= -\frac{2}{3acx^4\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}} + \frac{16}{3a^2cx^5\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}} + \frac{}{3cx^3\sqrt{c+a^2}}$$

Mathematica [A] time = 26.8702, size = 0, normalized size = 0.

$$\int \frac{1}{x^4\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^4*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2)),x]
```

```
[Out] Integrate[1/(x^4*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2)), x]
```

Maple [A] time = 2.049, size = 0, normalized size = 0.

$$\int \frac{1}{x^4}\left(a^2cx^2+c\right)^{-\frac{3}{2}}(\arctan(ax))^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
```

```
[Out] int(1/x^4/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="fricas"
)
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(a**2*c*x**2+c)**(3/2)/atan(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)^{\frac{3}{2}}x^4\arctan\left(ax\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^(3/2)*x^4*arctan(a*x)^(5/2)), x)
```

3.1102 $\int \frac{x^m}{(c+a^2cx^2)^{5/2}\tan^{-1}(ax)^{5/2}}\,dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(\frac{x^m}{(a^2cx^2+c)^{5/2}\tan^{-1}(ax)^{5/2}},x\right)$$

```
[Out] Unintegrable[x^m/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2)), x]
```

Rubi [A] time = 0.118695, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m}{(c+a^2cx^2)^{5/2}\tan^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2)),x]
```

```
[Out] Defer[Int][x^m/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2)), x]
```

Rubi steps

$$\int \frac{x^m}{(c+a^2cx^2)^{5/2}\tan^{-1}(ax)^{5/2}}\,dx=\int \frac{x^m}{(c+a^2cx^2)^{5/2}\tan^{-1}(ax)^{5/2}}\,dx$$

Mathematica [A] time = 1.08492, size = 0, normalized size = 0.

$$\int \frac{x^m}{(c+a^2cx^2)^{5/2}\tan^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2)),x]
```

```
[Out] Integrate[x^m/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2)), x]
```

Maple [A] time = 1.023, size = 0, normalized size = 0.

$$\int x^{m} \left(a^{2} c \,x^{2}+c \right)^{-\frac{5}{2}} \left(\arctan \left(a x \right)\right)^{-\frac{5}{2}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

```
[Out] int(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a^{2} c x^{2} + c} x^{m}}{{\left(a^{6} c^{3} x^{6} + 3 \, a^{4} c^{3} x^{4} + 3 \, a^{2} c^{3} x^{2} + c^{3}\right)} \arctan\left(a x\right)^{\frac{5}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^m/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*
x^2 + c^3)*arctan(a*x)^(5/2)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(a**2*c*x**2+c)**(5/2)/atan(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(a^2cx^2+c\right)^{\frac{5}{2}}\arctan\left(ax\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x^m/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^(5/2)), x)
```

3.1103 $\int \frac{x^3}{\left(c+a^2cx^2\right)^{5/2} \tan^{-1}(ax)^{5/2}} \, dx$

Optimal. Leaf size=190

$$-\frac{\sqrt{2\pi}\sqrt{a^2x^2+1}S\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{a^4c^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{6\pi}\sqrt{a^2x^2+1}S\left(\sqrt{\frac{6}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{a^4c^2\sqrt{a^2cx^2+c}}-\frac{2x^3}{3ac\left(a^2cx^2+c\right)^{3/2}\tan^{-1}(ax)^{3/2}}-\frac{}{a^2c\left(a^2cx\right.}$$

```
[Out] (-2*x^3)/(3*a*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)) - (4*x^2)/(a^2*c*(
c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]) - (Sqrt[2*Pi]*Sqrt[1 + a^2*x^2]*Fre
snelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(a^4*c^2*Sqrt[c + a^2*c*x^2]) + (Sqrt[
6*Pi]*Sqrt[1 + a^2*x^2]*FresnelS[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]])/(a^4*c^2*Sq
rt[c + a^2*c*x^2])
```

Rubi [A] time = 0.808129, antiderivative size = 190, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {4942, 4968, 4971, 4970, 3312, 3305, 3351, 4406}

$$-\frac{\sqrt{2\pi}\sqrt{a^2x^2+1}S\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{a^4c^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{6\pi}\sqrt{a^2x^2+1}S\left(\sqrt{\frac{6}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{a^4c^2\sqrt{a^2cx^2+c}}-\frac{2x^3}{3ac\left(a^2cx^2+c\right)^{3/2}\tan^{-1}(ax)^{3/2}}-\frac{}{a^2c\left(a^2cx\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2)),x]
```

```
[Out] (-2*x^3)/(3*a*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)) - (4*x^2)/(a^2*c*(
c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]) - (Sqrt[2*Pi]*Sqrt[1 + a^2*x^2]*Fre
snelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(a^4*c^2*Sqrt[c + a^2*c*x^2]) + (Sqrt[
6*Pi]*Sqrt[1 + a^2*x^2]*FresnelS[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]])/(a^4*c^2*Sq
rt[c + a^2*c*x^2])
```

Rule 4942

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^m*(d + e*x^2)^(q + 1)*(a + b*ArcT
an[c*x])^(p + 1))/(b*c*d*(p + 1)), x] - Dist[(f*m)/(b*c*(p + 1)), Int[(f*x)
^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b,
```

```
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 2, 0] && LtQ[p, -1]
```

Rule 4968

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[(x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^(p
+ 1))/(b*c*d*(p + 1)), x] + (-Dist[(c*(m + 2*q + 2))/(b*(p + 1)), Int[x^(m
+ 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p + 1
)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; Fre
eQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &&
LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 4971

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[(d^(q + 1/2)*Sqrt[1 + c^2*x^2])/Sqrt[d + e*x^2],
 Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d
, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(In
tegerQ[q] || GtQ[d, 0])
```

Rule 4970

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sin[x]^m)/
Cos[x]^(m + 2*(q + 1)), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \frac{x^3}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{5/2}}\,dx = -\frac{2x^3}{3ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}} + \frac{2\int \frac{x^2}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{3/2}}\,dx}{a}$$

$$= -\frac{2x^3}{3ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}} - \frac{4x^2}{a^2c\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}} - 4\int \frac{}{\left(c+a^2cx^2\right)^{5}}$$

$$= -\frac{2x^3}{3ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}} - \frac{4x^2}{a^2c\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}} - \frac{\left(4\sqrt{1+a^2x^2}\right)\int}{c^2\sqrt{}}$$

$$= -\frac{2x^3}{3ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}} - \frac{4x^2}{a^2c\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}} - \frac{\left(4\sqrt{1+a^2x^2}\right)\mathrm{Su}}{a}$$

$$= -\frac{2x^3}{3ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}} - \frac{4x^2}{a^2c\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}} - \frac{\left(4\sqrt{1+a^2x^2}\right)\mathrm{Su}}{}$$

$$= -\frac{2x^3}{3ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}} - \frac{4x^2}{a^2c\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}} + \frac{\sqrt{1+a^2x^2}\,\mathrm{Subst}}{a^4}$$

$$= -\frac{2x^3}{3ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}} - \frac{4x^2}{a^2c\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}} + \frac{\left(2\sqrt{1+a^2x^2}\right)\mathrm{Su}}{}$$

$$= -\frac{2x^3}{3ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}} - \frac{4x^2}{a^2c\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}} - \frac{\sqrt{2\pi}\sqrt{1+a^2x^2}S}{a^4c^2\sqrt{}}$$

Mathematica [C] time = 0.852121, size = 255, normalized size = 1.34

$$-\frac{\left(a^2x^2+1\right)^{3/2}\tan^{-1}(ax)\left(3\sqrt{-i\tan^{-1}(ax)}\text{Gamma}\left(\frac{1}{2},-i\tan^{-1}(ax)\right)+3\sqrt{i\tan^{-1}(ax)}\text{Gamma}\left(\frac{1}{2},i\tan^{-1}(ax)\right)+\sqrt{3}\left(\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2)),x]
```

```
[Out] (-2*a^2*x^2*(a*x + 6*ArcTan[a*x]) + Sqrt[6*Pi]*(1 + a^2*x^2)^(3/2)*ArcTan[a
*x]^(3/2)*(-3*Sqrt[3]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]] + FresnelS[Sqr
t[6/Pi]*Sqrt[ArcTan[a*x]]]) - (1 + a^2*x^2)^(3/2)*ArcTan[a*x]*(3*Sqrt[(-I)*
ArcTan[a*x]]*Gamma[1/2, (-I)*ArcTan[a*x]] + 3*Sqrt[I*ArcTan[a*x]]*Gamma[1/2
, I*ArcTan[a*x]] + Sqrt[3]*(Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-3*I)*ArcTan
[a*x]] + Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (3*I)*ArcTan[a*x]])))/(3*a^4*c*(c +
 a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2))
```

Maple [F] time = 3.796, size = 0, normalized size = 0.

$$\int x^3\left(a^2cx^2+c\right)^{-\frac{5}{2}}(\arctan(ax))^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

```
[Out] int(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a**2*c*x**2+c)**(5/2)/atan(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(a^2cx^2 + c\right)^{\frac{5}{2}} \arctan\left(ax\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^(5/2)), x)
```

3.1104 $\int \frac{x^2}{\left(c+a^2cx^2\right)^{5/2} \tan^{-1}(ax)^{5/2}} \, dx$

Optimal. Leaf size=224

$$-\frac{\sqrt{2\pi}\sqrt{a^2x^2+1}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{3a^3c^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{6\pi}\sqrt{a^2x^2+1}\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{a^3c^2\sqrt{a^2cx^2+c}}+\frac{4x^3}{3c\left(a^2cx^2+c\right)^{3/2}\sqrt{\tan^{-}}}$$

```
[Out] (-2*x^2)/(3*a*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)) - (8*x)/(3*a^2*c*(
c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]) + (4*x^3)/(3*c*(c + a^2*c*x^2)^(3/2
)*Sqrt[ArcTan[a*x]]) - (Sqrt[2*Pi]*Sqrt[1 + a^2*x^2]*FresnelC[Sqrt[2/Pi]*Sq
rt[ArcTan[a*x]]])/(3*a^3*c^2*Sqrt[c + a^2*c*x^2]) + (Sqrt[6*Pi]*Sqrt[1 + a^
2*x^2]*FresnelC[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]])/(a^3*c^2*Sqrt[c + a^2*c*x^2]
)
```

Rubi [A] time = 1.13956, antiderivative size = 224, normalized size of antiderivative = 1., number of steps used = 27, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {4968, 4942, 4971, 4970, 4406, 3304, 3352, 4905, 4904, 3312}

$$-\frac{\sqrt{2\pi}\sqrt{a^2x^2+1}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{3a^3c^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{6\pi}\sqrt{a^2x^2+1}\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{a^3c^2\sqrt{a^2cx^2+c}}+\frac{4x^3}{3c\left(a^2cx^2+c\right)^{3/2}\sqrt{\tan^{-}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2)),x]
```


```
[Out] (-2*x^2)/(3*a*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)) - (8*x)/(3*a^2*c*(
c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]) + (4*x^3)/(3*c*(c + a^2*c*x^2)^(3/2
)*Sqrt[ArcTan[a*x]]) - (Sqrt[2*Pi]*Sqrt[1 + a^2*x^2]*FresnelC[Sqrt[2/Pi]*Sq
rt[ArcTan[a*x]]])/(3*a^3*c^2*Sqrt[c + a^2*c*x^2]) + (Sqrt[6*Pi]*Sqrt[1 + a^
2*x^2]*FresnelC[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]])/(a^3*c^2*Sqrt[c + a^2*c*x^2]
)
```


Rule 4968

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[(x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^(p
+ 1))/(b*c*d*(p + 1)), x] + (-Dist[(c*(m + 2*q + 2))/(b*(p + 1)), Int[x^(m
```

```
+ 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p + 1
)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; Fre
eQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &&
LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 4942

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^m*(d + e*x^2)^(q + 1)*(a + b*ArcT
an[c*x])^(p + 1))/(b*c*d*(p + 1)), x] - Dist[(f*m)/(b*c*(p + 1)), Int[(f*x)
^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 2, 0] && LtQ[p, -1]
```

Rule 4971

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[(d^(q + 1/2)*Sqrt[1 + c^2*x^2])/Sqrt[d + e*x^2],
 Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d
, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(In
tegerQ[q] || GtQ[d, 0])
```

Rule 4970

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sin[x]^m)/
Cos[x]^(m + 2*(q + 1)), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4905

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[(d^(q + 1/2)*Sqrt[1 + c^2*x^2])/Sqrt[d + e*x^2], Int[(1 + c
^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

Rule 4904

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\int \frac{x^2}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{5/2}}\,dx = -\frac{2x^2}{3ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}} + \frac{4\int \frac{x}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{3/2}}\,dx}{3a} - \frac{1}{3}(2a)\int \frac{}{\left(c+a^2cx^2\right.}$$

$$= -\frac{2x^2}{3ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}} - \frac{8x}{3a^2c\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}} + \frac{4x}{3c\left(c+a^2cx^2\right)^{3/}}$$

$$= -\frac{2x^2}{3ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}} - \frac{8x}{3a^2c\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}} + \frac{4x}{3c\left(c+a^2cx^2\right)^{3/}}$$

$$= -\frac{2x^2}{3ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}} - \frac{8x}{3a^2c\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}} + \frac{4x}{3c\left(c+a^2cx^2\right)^{3/}}$$

$$= -\frac{2x^2}{3ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}} - \frac{8x}{3a^2c\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}} + \frac{4x}{3c\left(c+a^2cx^2\right)^{3/}}$$

$$= -\frac{2x^2}{3ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}} - \frac{8x}{3a^2c\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}} + \frac{4x}{3c\left(c+a^2cx^2\right)^{3/}}$$

$$= -\frac{2x^2}{3ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}} - \frac{8x}{3a^2c\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}} + \frac{4x}{3c\left(c+a^2cx^2\right)^{3/}}$$

$$= -\frac{2x^2}{3ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}} - \frac{8x}{3a^2c\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}} + \frac{4x}{3c\left(c+a^2cx^2\right)^{3/}}$$

Mathematica [C] time = 0.814797, size = 311, normalized size = 1.39

$$\frac{-3a^2x^2\sqrt{3a^2x^2+3}\tan^{-1}(ax)^2\text{Gamma}\left(\frac{1}{2},3i\tan^{-1}(ax)\right)+\left(a^2x^2+1\right)^{3/2}\tan^{-1}(ax)^2\text{Gamma}\left(\frac{1}{2},i\tan^{-1}(ax)\right)-3i\sqrt{3}\left(a^2x^2+1\right)^{3/2}\tan^{-1}(ax)\sqrt{\tan^{-1}(ax)^2}\text{Gamma}\left(\frac{1}{2},-\right.}{\sqrt{i\tan^{-1}(ax)}}$$

6

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2)),x]
```

```
[Out] (-((1 + a^2*x^2)^(3/2)*((-I)*ArcTan[a*x])^(3/2)*Gamma[1/2, (-I)*ArcTan[a*x]
]) + (-4*a^2*x^2*Sqrt[I*ArcTan[a*x]] + (16*I)*a*x*(I*ArcTan[a*x])^(3/2) - (
8*I)*a^3*x^3*(I*ArcTan[a*x])^(3/2) + (1 + a^2*x^2)^(3/2)*ArcTan[a*x]^2*Gamm
a[1/2, I*ArcTan[a*x]] - (3*I)*Sqrt[3]*(1 + a^2*x^2)^(3/2)*ArcTan[a*x]*Sqrt[
ArcTan[a*x]^2]*Gamma[1/2, (-3*I)*ArcTan[a*x]] - 3*Sqrt[3 + 3*a^2*x^2]*ArcTa
n[a*x]^2*Gamma[1/2, (3*I)*ArcTan[a*x]] - 3*a^2*x^2*Sqrt[3 + 3*a^2*x^2]*ArcT
an[a*x]^2*Gamma[1/2, (3*I)*ArcTan[a*x]])/Sqrt[I*ArcTan[a*x]])/(6*a^3*c^2*(1
 + a^2*x^2)*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2))
```

Maple [F] time = 3.125, size = 0, normalized size = 0.

$$\int x^2 \left(a^2cx^2+c\right)^{-\frac{5}{2}} (\arctan(ax))^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

```
[Out] int(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a**2*c*x**2+c)**(5/2)/atan(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(a^2cx^2+c\right)^{\frac{5}{2}}\arctan\left(ax\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^(5/2)), x)
```

3.1105 $\int \frac{x}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{5/2}}\,dx$

Optimal. Leaf size=222

$$-\frac{\sqrt{2\pi}\sqrt{a^2x^2+1}S\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{3a^2c^2\sqrt{a^2cx^2+c}}-\frac{\sqrt{6\pi}\sqrt{a^2x^2+1}S\left(\sqrt{\frac{6}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{a^2c^2\sqrt{a^2cx^2+c}}+\frac{8x^2}{3c\left(a^2cx^2+c\right)^{3/2}\sqrt{\tan^{-1}(ax)}}-\frac{}{3ac\left(a^2cx\right.}$$

```
[Out] (-2*x)/(3*a*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)) - 4/(3*a^2*c*(c + a^
2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]) + (8*x^2)/(3*c*(c + a^2*c*x^2)^(3/2)*Sqrt
[ArcTan[a*x]]) - (Sqrt[2*Pi]*Sqrt[1 + a^2*x^2]*FresnelS[Sqrt[2/Pi]*Sqrt[Arc
Tan[a*x]]])/(3*a^2*c^2*Sqrt[c + a^2*c*x^2]) - (Sqrt[6*Pi]*Sqrt[1 + a^2*x^2]
*FresnelS[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]])/(a^2*c^2*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 1.05731, antiderivative size = 222, normalized size of antiderivative = 1., number of steps used = 27, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4968, 4971, 4970, 3312, 3305, 3351, 4406, 4902}

$$-\frac{\sqrt{2\pi}\sqrt{a^2x^2+1}S\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{3a^2c^2\sqrt{a^2cx^2+c}}-\frac{\sqrt{6\pi}\sqrt{a^2x^2+1}S\left(\sqrt{\frac{6}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{a^2c^2\sqrt{a^2cx^2+c}}+\frac{8x^2}{3c\left(a^2cx^2+c\right)^{3/2}\sqrt{\tan^{-1}(ax)}}-\frac{}{3ac\left(a^2cx\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2)),x]
```

```
[Out] (-2*x)/(3*a*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)) - 4/(3*a^2*c*(c + a^
2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]) + (8*x^2)/(3*c*(c + a^2*c*x^2)^(3/2)*Sqrt
[ArcTan[a*x]]) - (Sqrt[2*Pi]*Sqrt[1 + a^2*x^2]*FresnelS[Sqrt[2/Pi]*Sqrt[Arc
Tan[a*x]]])/(3*a^2*c^2*Sqrt[c + a^2*c*x^2]) - (Sqrt[6*Pi]*Sqrt[1 + a^2*x^2]
*FresnelS[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]])/(a^2*c^2*Sqrt[c + a^2*c*x^2])
```

Rule 4968

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[(x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^(p
+ 1))/(b*c*d*(p + 1)), x] + (-Dist[(c*(m + 2*q + 2))/(b*(p + 1)), Int[x^(m
+ 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p + 1
)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; Fre
```

```
eQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &&
LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 4971

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[(d^(q + 1/2)*Sqrt[1 + c^2*x^2])/Sqrt[d + e*x^2],
 Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d
, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(In
tegerQ[q] || GtQ[d, 0])
```

Rule 4970

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sin[x]^m)/
Cos[x]^(m + 2*(q + 1)), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 4902

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^(p + 1))/(b*c*d*(p + 1)), x] - Dist[(2*c*(q + 1))/(b*(p + 1)), Int[x*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{x}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{5/2}}\,dx = -\frac{2x}{3ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}} + \frac{2\int \frac{1}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{3/2}}\,dx}{3a} - \frac{1}{3}(4a)\int \frac{}{\left(c+a^2cx\right.}$$

$$= -\frac{2x}{3ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}} - \frac{4}{3a^2c\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}} + \frac{8}{3c\left(c+a^2cx^2\right)}$$

$$= -\frac{2x}{3ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}} - \frac{4}{3a^2c\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}} + \frac{8}{3c\left(c+a^2cx^2\right)}$$

$$= -\frac{2x}{3ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}} - \frac{4}{3a^2c\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}} + \frac{8}{3c\left(c+a^2cx^2\right)}$$

$$= -\frac{2x}{3ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}} - \frac{4}{3a^2c\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}} + \frac{8}{3c\left(c+a^2cx^2\right)}$$

$$= -\frac{2x}{3ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}} - \frac{4}{3a^2c\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}} + \frac{8}{3c\left(c+a^2cx^2\right)}$$

$$= -\frac{2x}{3ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}} - \frac{4}{3a^2c\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}} + \frac{8}{3c\left(c+a^2cx^2\right)}$$

$$= -\frac{2x}{3ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}} - \frac{4}{3a^2c\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}} + \frac{8}{3c\left(c+a^2cx^2\right)}$$

Mathematica [C] time = 0.829771, size = 261, normalized size = 1.18

$$\frac{7\left(a^2x^2+1\right)^{3/2}\tan^{-1}(ax)\left(3\sqrt{-i\tan^{-1}(ax)}\text{Gamma}\left(\frac{1}{2},-i\tan^{-1}(ax)\right)+3\sqrt{i\tan^{-1}(ax)}\text{Gamma}\left(\frac{1}{2},i\tan^{-1}(ax)\right)+\sqrt{3}\left(\sqrt{\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2)),x]
```

```
[Out] (-12*(a*x + (2 - 4*a^2*x^2)*ArcTan[a*x]) + 4*Sqrt[6*Pi]*(1 + a^2*x^2)^(3/2)
*ArcTan[a*x]^(3/2)*(3*Sqrt[3]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]] - Fres
nelS[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]]) + 7*(1 + a^2*x^2)^(3/2)*ArcTan[a*x]*(3*
Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-I)*ArcTan[a*x]] + 3*Sqrt[I*ArcTan[a*x]]
*Gamma[1/2, I*ArcTan[a*x]] + Sqrt[3]*(Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-3
*I)*ArcTan[a*x]] + Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (3*I)*ArcTan[a*x]])))/(18
*a^2*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2))
```

Maple [F] time = 0.898, size = 0, normalized size = 0.

$$\int x\left(a^2cx^2+c\right)^{-\frac{5}{2}}\left(\arctan\left(ax\right)\right)^{-\frac{5}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

```
[Out] int(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a**2*c*x**2+c)**(5/2)/atan(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(a^2cx^2+c\right)^{\frac{5}{2}}\arctan\left(ax\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^(5/2)), x)
```

3.1106 $\int \frac{1}{(c+a^2cx^2)^{5/2}\tan^{-1}(ax)^{5/2}}\,dx$

Optimal. Leaf size=183

$$-\frac{\sqrt{2\pi}\sqrt{a^2x^2+1}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{ac^2\sqrt{a^2cx^2+c}}-\frac{\sqrt{6\pi}\sqrt{a^2x^2+1}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{ac^2\sqrt{a^2cx^2+c}}+\frac{4x}{c\left(a^2cx^2+c\right)^{3/2}\sqrt{\tan^{-1}(a}}$$

```
[Out] -2/(3*a*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)) + (4*x)/(c*(c + a^2*c*x^
2)^(3/2)*Sqrt[ArcTan[a*x]]) - (Sqrt[2*Pi]*Sqrt[1 + a^2*x^2]*FresnelC[Sqrt[2
/Pi]*Sqrt[ArcTan[a*x]]])/(a*c^2*Sqrt[c + a^2*c*x^2]) - (Sqrt[6*Pi]*Sqrt[1 +
 a^2*x^2]*FresnelC[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]])/(a*c^2*Sqrt[c + a^2*c*x^2
])
```

Rubi [A] time = 0.616814, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {4902, 4968, 4971, 4970, 4406, 3304, 3352, 4905, 4904, 3312}

$$-\frac{\sqrt{2\pi}\sqrt{a^2x^2+1}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{ac^2\sqrt{a^2cx^2+c}}-\frac{\sqrt{6\pi}\sqrt{a^2x^2+1}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{ac^2\sqrt{a^2cx^2+c}}+\frac{4x}{c\left(a^2cx^2+c\right)^{3/2}\sqrt{\tan^{-1}(a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2)),x]
```

```
[Out] -2/(3*a*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)) + (4*x)/(c*(c + a^2*c*x^
2)^(3/2)*Sqrt[ArcTan[a*x]]) - (Sqrt[2*Pi]*Sqrt[1 + a^2*x^2]*FresnelC[Sqrt[2
/Pi]*Sqrt[ArcTan[a*x]]])/(a*c^2*Sqrt[c + a^2*c*x^2]) - (Sqrt[6*Pi]*Sqrt[1 +
 a^2*x^2]*FresnelC[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]])/(a*c^2*Sqrt[c + a^2*c*x^2
])
```

Rule 4902

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^(p + 1))/(b*c*d*(p
+ 1)), x] - Dist[(2*c*(q + 1))/(b*(p + 1)), Int[x*(d + e*x^2)^q*(a + b*ArcT
an[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && L
tQ[q, -1] && LtQ[p, -1]
```

Rule 4968

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[(x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^(p
+ 1))/(b*c*d*(p + 1)), x] + (-Dist[(c*(m + 2*q + 2))/(b*(p + 1)), Int[x^(m
+ 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p + 1
)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; Fre
eQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &&
LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 4971

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[(d^(q + 1/2)*Sqrt[1 + c^2*x^2])/Sqrt[d + e*x^2],
 Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d
, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(In
tegerQ[q] || GtQ[d, 0])
```

Rule 4970

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sin[x]^m)/
Cos[x]^(m + 2*(q + 1)), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4905

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[(d^(q + 1/2)*Sqrt[1 + c^2*x^2])/Sqrt[d + e*x^2], Int[(1 + c
^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

Rule 4904

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\int \frac{1}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{5/2}}\,dx = -\frac{2}{3ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}} - (2a)\int \frac{x}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{3/2}}\,dx$$

$$= -\frac{2}{3ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}} + \frac{4x}{c\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}} - 4\int \frac{}{\left(c+a^2cx^2\right)^{5/}}$$

$$= -\frac{2}{3ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}} + \frac{4x}{c\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}} - \frac{\left(4\sqrt{1+a^2x^2}\right)\int}{c^2\sqrt{c}}$$

$$= -\frac{2}{3ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}} + \frac{4x}{c\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}} - \frac{\left(4\sqrt{1+a^2x^2}\right)\mathrm{Su}}{}$$

$$= -\frac{2}{3ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}} + \frac{4x}{c\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}} - \frac{\left(4\sqrt{1+a^2x^2}\right)\mathrm{Su}}{}$$

$$= -\frac{2}{3ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}} + \frac{4x}{c\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}} - \frac{\sqrt{1+a^2x^2}\,\mathrm{Subst}}{ac}$$

$$= -\frac{2}{3ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}} + \frac{4x}{c\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}} - \frac{\left(2\sqrt{1+a^2x^2}\right)\mathrm{Su}}{}$$

$$= -\frac{2}{3ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}} + \frac{4x}{c\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}} - \frac{\sqrt{2\pi}\sqrt{1+a^2x^2}\,C}{ac^2\sqrt{c}}$$

Mathematica [C] time = 0.513151, size = 300, normalized size = 1.64

$$\frac{-3a^2x^2\sqrt{3a^2x^2+3}\left(-i\tan^{-1}(ax)\right)^{3/2}\mathrm{Gamma}\left(\frac{1}{2},-3i\tan^{-1}(ax)\right) - 3a^2x^2\sqrt{3a^2x^2+3}\left(i\tan^{-1}(ax)\right)^{3/2}\mathrm{Gamma}\left(\frac{1}{2},3i\tan\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2)),x]
```

```
[Out] (-4 + 24*a*x*ArcTan[a*x] - 3*(1 + a^2*x^2)^(3/2)*((-I)*ArcTan[a*x])^(3/2)*G
amma[1/2, (-I)*ArcTan[a*x]] - 3*(1 + a^2*x^2)^(3/2)*(I*ArcTan[a*x])^(3/2)*G
```

```
amma[1/2, I*ArcTan[a*x]] - 3*Sqrt[3 + 3*a^2*x^2]*((-I)*ArcTan[a*x])^(3/2)*G
amma[1/2, (-3*I)*ArcTan[a*x]] - 3*a^2*x^2*Sqrt[3 + 3*a^2*x^2]*((-I)*ArcTan[
a*x])^(3/2)*Gamma[1/2, (-3*I)*ArcTan[a*x]] - 3*Sqrt[3 + 3*a^2*x^2]*(I*ArcTa
n[a*x])^(3/2)*Gamma[1/2, (3*I)*ArcTan[a*x]] - 3*a^2*x^2*Sqrt[3 + 3*a^2*x^2]
*(I*ArcTan[a*x])^(3/2)*Gamma[1/2, (3*I)*ArcTan[a*x]])/(6*c^2*(a + a^3*x^2)*
Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2))
```

Maple [F] time = 0.761, size = 0, normalized size = 0.

$$\int \left(a^2cx^2+c\right)^{-\frac{5}{2}} (\arctan(ax))^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

```
[Out] int(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*c*x**2+c)**(5/2)/atan(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)^{\frac{5}{2}}\arctan\left(ax\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^(5/2)), x)
```

3.1107 $\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{5/2}}\,dx$

Optimal. Leaf size=285

$$\frac{8\text{Unintegrable}\left(\frac{1}{x^3\left(a^2cx^2+c\right)^{5/2}\sqrt{\tan^{-1}(ax)}},x\right)}{3a^2}+\frac{20}{3}\text{Unintegrable}\left(\frac{1}{x\left(a^2cx^2+c\right)^{5/2}\sqrt{\tan^{-1}(ax)}},x\right)+\frac{4\sqrt{2\pi}\sqrt{a^2x^2+1}S\left(\sqrt{\cdots}\right.}{c^2\sqrt{a^2cx^2\cdots}}$$

```
[Out] -2/(3*a*c*x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)) + 16/(3*c*(c + a^2*c*x
^2)^(3/2)*Sqrt[ArcTan[a*x]]) + 4/(3*a^2*c*x^2*(c + a^2*c*x^2)^(3/2)*Sqrt[Ar
cTan[a*x]]) + (4*Sqrt[2*Pi]*Sqrt[1 + a^2*x^2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcT
an[a*x]]])/(c^2*Sqrt[c + a^2*c*x^2]) + (4*Sqrt[(2*Pi)/3]*Sqrt[1 + a^2*x^2]*
FresnelS[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]])/(c^2*Sqrt[c + a^2*c*x^2]) + (8*Unin
tegrable[1/(x^3*(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]), x])/(3*a^2) + (20
*Unintegrable[1/(x*(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]), x])/3
```

Rubi [A] time = 0.767769, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2)),x]
```

```
[Out] -2/(3*a*c*x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)) + 16/(3*c*(c + a^2*c*x
^2)^(3/2)*Sqrt[ArcTan[a*x]]) + 4/(3*a^2*c*x^2*(c + a^2*c*x^2)^(3/2)*Sqrt[Ar
cTan[a*x]]) + (4*Sqrt[2*Pi]*Sqrt[1 + a^2*x^2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcT
an[a*x]]])/(c^2*Sqrt[c + a^2*c*x^2]) + (4*Sqrt[(2*Pi)/3]*Sqrt[1 + a^2*x^2]*
FresnelS[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]])/(c^2*Sqrt[c + a^2*c*x^2]) + (8*Defe
r[Int][1/(x^3*(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]), x])/(3*a^2) + (20*D
efer[Int][1/(x*(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]), x])/3
```

Rubi steps

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{5/2}}\,dx = -\frac{2}{3acx\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}} - \frac{2\int \frac{1}{x^2\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{3/2}}\,dx}{3a} - \frac{1}{3}(8a)\int \frac{}{\left(c+\right.}$$

$$= -\frac{2}{3acx\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}} + \frac{16}{3c\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}} + \frac{}{3a^2cx^2\left(c+a\right.}$$

$$= -\frac{2}{3acx\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}} + \frac{16}{3c\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}} + \frac{}{3a^2cx^2\left(c+a\right.}$$

$$= -\frac{2}{3acx\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}} + \frac{16}{3c\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}} + \frac{}{3a^2cx^2\left(c+a\right.}$$

$$= -\frac{2}{3acx\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}} + \frac{16}{3c\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}} + \frac{}{3a^2cx^2\left(c+a\right.}$$

$$= -\frac{2}{3acx\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}} + \frac{16}{3c\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}} + \frac{}{3a^2cx^2\left(c+a\right.}$$

$$= -\frac{2}{3acx\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}} + \frac{16}{3c\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}} + \frac{}{3a^2cx^2\left(c+a\right.}$$

$$= -\frac{2}{3acx\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}} + \frac{16}{3c\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}} + \frac{}{3a^2cx^2\left(c+a\right.}$$

Mathematica [A] time = 12.1814, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2)),x]
```

```
[Out] Integrate[1/(x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2)), x]
```

Maple [A] time = 0.783, size = 0, normalized size = 0.

$$\int \frac{1}{x} \left(a^2 c x^2 + c\right)^{-\frac{5}{2}} (\arctan(ax))^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

```
[Out] int(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a**2*c*x**2+c)**(5/2)/atan(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)^{\frac{5}{2}}x\arctan\left(ax\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^(5/2)*x*arctan(a*x)^(5/2)), x)
```

3.1108 $\int \frac{1}{x^2(c+a^2cx^2)^{5/2}\tan^{-1}(ax)^{5/2}}\,dx$

Optimal. Leaf size=290

$$\frac{8\text{Unintegrable}\left(\frac{1}{x^4(a^2cx^2+c)^{5/2}\sqrt{\tan^{-1}(ax)}},x\right)}{a^2}+\frac{68}{3}\text{Unintegrable}\left(\frac{1}{x^2\left(a^2cx^2+c\right)^{5/2}\sqrt{\tan^{-1}(ax)}},x\right)+\frac{20\sqrt{2\pi}a\sqrt{a^2x^2+1}\text{Fr}}{c^2\sqrt{}}$$

```
[Out] -2/(3*a*c*x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)) + 8/(3*a^2*c*x^3*(c
+ a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]) + 20/(3*c*x*(c + a^2*c*x^2)^(3/2)*Sqr
t[ArcTan[a*x]]) + (20*a*Sqrt[2*Pi]*Sqrt[1 + a^2*x^2]*FresnelC[Sqrt[2/Pi]*Sq
rt[ArcTan[a*x]]])/(c^2*Sqrt[c + a^2*c*x^2]) + (20*a*Sqrt[(2*Pi)/3]*Sqrt[1 +
 a^2*x^2]*FresnelC[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]])/(3*c^2*Sqrt[c + a^2*c*x^2
]) + (8*Unintegrable[1/(x^4*(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]), x])/a
^2 + (68*Unintegrable[1/(x^2*(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]), x])/
3
```

Rubi [A] time = 0.931795, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2)),x]
```

```
[Out] -2/(3*a*c*x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)) + 8/(3*a^2*c*x^3*(c
+ a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]) + 20/(3*c*x*(c + a^2*c*x^2)^(3/2)*Sqr
t[ArcTan[a*x]]) + (20*a*Sqrt[2*Pi]*Sqrt[1 + a^2*x^2]*FresnelC[Sqrt[2/Pi]*Sq
rt[ArcTan[a*x]]])/(c^2*Sqrt[c + a^2*c*x^2]) + (20*a*Sqrt[(2*Pi)/3]*Sqrt[1 +
 a^2*x^2]*FresnelC[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]])/(3*c^2*Sqrt[c + a^2*c*x^2
]) + (8*Defer[Int][1/(x^4*(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]), x])/a^2
 + (68*Defer[Int][1/(x^2*(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]), x])/3
```

Rubi steps

$$\int \frac{1}{x^2 \left(c+a^2cx^2\right)^{5/2} \tan^{-1}(ax)^{5/2}}\,dx = -\frac{2}{3acx^2 \left(c+a^2cx^2\right)^{3/2} \tan^{-1}(ax)^{3/2}} - \frac{4\int \frac{1}{x^3\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{3/2}}\,dx}{3a} - \frac{1}{3}(10a)\int \frac{}{x}$$

$$= -\frac{2}{3acx^2 \left(c+a^2cx^2\right)^{3/2} \tan^{-1}(ax)^{3/2}} + \frac{8}{3a^2cx^3 \left(c+a^2cx^2\right)^{3/2} \sqrt{\tan^{-1}(ax)}} + \frac{}{3cx\left(c\right.}$$

$$= -\frac{2}{3acx^2 \left(c+a^2cx^2\right)^{3/2} \tan^{-1}(ax)^{3/2}} + \frac{8}{3a^2cx^3 \left(c+a^2cx^2\right)^{3/2} \sqrt{\tan^{-1}(ax)}} + \frac{}{3cx\left(c\right.}$$

$$= -\frac{2}{3acx^2 \left(c+a^2cx^2\right)^{3/2} \tan^{-1}(ax)^{3/2}} + \frac{8}{3a^2cx^3 \left(c+a^2cx^2\right)^{3/2} \sqrt{\tan^{-1}(ax)}} + \frac{}{3cx\left(c\right.}$$

$$= -\frac{2}{3acx^2 \left(c+a^2cx^2\right)^{3/2} \tan^{-1}(ax)^{3/2}} + \frac{8}{3a^2cx^3 \left(c+a^2cx^2\right)^{3/2} \sqrt{\tan^{-1}(ax)}} + \frac{}{3cx\left(c\right.}$$

$$= -\frac{2}{3acx^2 \left(c+a^2cx^2\right)^{3/2} \tan^{-1}(ax)^{3/2}} + \frac{8}{3a^2cx^3 \left(c+a^2cx^2\right)^{3/2} \sqrt{\tan^{-1}(ax)}} + \frac{}{3cx\left(c\right.}$$

$$= -\frac{2}{3acx^2 \left(c+a^2cx^2\right)^{3/2} \tan^{-1}(ax)^{3/2}} + \frac{8}{3a^2cx^3 \left(c+a^2cx^2\right)^{3/2} \sqrt{\tan^{-1}(ax)}} + \frac{}{3cx\left(c\right.}$$

$$= -\frac{2}{3acx^2 \left(c+a^2cx^2\right)^{3/2} \tan^{-1}(ax)^{3/2}} + \frac{8}{3a^2cx^3 \left(c+a^2cx^2\right)^{3/2} \sqrt{\tan^{-1}(ax)}} + \frac{}{3cx\left(c\right.}$$

Mathematica [A] time = 13.9876, size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \left(c+a^2cx^2\right)^{5/2} \tan^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2)),x]
```

```
[Out] Integrate[1/(x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2)), x]
```

Maple [A] time = 0.834, size = 0, normalized size = 0.

$$\int \frac{1}{x^{2}} \left(a^{2} c x^{2}+c\right)^{-\frac{5}{2}} \left(\arctan \left(a x\right)\right)^{-\frac{5}{2}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

```
[Out] int(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="fricas"
)
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a**2*c*x**2+c)**(5/2)/atan(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)^{\frac{5}{2}}x^2\arctan\left(ax\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^(5/2)*x^2*arctan(a*x)^(5/2)), x)
```

3.1109 $\int \frac{1}{x^3(c+a^2cx^2)^{5/2}\tan^{-1}(ax)^{5/2}}\,dx$

Optimal. Leaf size=195

$$40a^2\text{Unintegrable}\left(\frac{1}{x\left(a^2cx^2+c\right)^{5/2}\sqrt{\tan^{-1}(ax)}},x\right)+44\text{Unintegrable}\left(\frac{1}{x^3\left(a^2cx^2+c\right)^{5/2}\sqrt{\tan^{-1}(ax)}},x\right)+\frac{16\text{Uninteg}}{}$$

```
[Out] -2/(3*a*c*x^3*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)) + 4/(a^2*c*x^4*(c +
a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]) + 8/(c*x^2*(c + a^2*c*x^2)^(3/2)*Sqrt[A
rcTan[a*x]]) + (16*Unintegrable[1/(x^5*(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*
x]]), x])/a^2 + 44*Unintegrable[1/(x^3*(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*
x]]), x] + 40*a^2*Unintegrable[1/(x*(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]
), x]
```

Rubi [A] time = 0.899205, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^3\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^3*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2)),x]
```

```
[Out] -2/(3*a*c*x^3*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)) + 4/(a^2*c*x^4*(c +
a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]) + 8/(c*x^2*(c + a^2*c*x^2)^(3/2)*Sqrt[A
rcTan[a*x]]) + (16*Defer[Int][1/(x^5*(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]
]), x])/a^2 + 44*Defer[Int][1/(x^3*(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]])
, x] + 40*a^2*Defer[Int][1/(x*(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]), x]
```

Rubi steps

$$\int \frac{1}{x^3\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{5/2}}\,dx = -\frac{2}{3acx^3\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}} - \frac{2\int \frac{1}{x^4\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{3/2}}\,dx}{a} - (4a)\int \frac{}{x^2\left(\right.}$$

$$= -\frac{2}{3acx^3\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}} + \frac{4}{a^2cx^4\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}} + \frac{}{cx^2\left(c+\right.}$$

Mathematica [A] time = 16.5181, size = 0, normalized size = 0.

$$\int \frac{1}{x^3\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^3*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2)),x]
```

```
[Out] Integrate[1/(x^3*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2)), x]
```

Maple [A] time = 0.951, size = 0, normalized size = 0.

$$\int \frac{1}{x^3}\left(a^2cx^2+c\right)^{-\frac{5}{2}}(\arctan(ax))^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

```
[Out] int(1/x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="fricas"
)
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a**2*c*x**2+c)**(5/2)/atan(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)^{\frac{5}{2}}x^3\arctan\left(ax\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^(5/2)*x^3*arctan(a*x)^(5/2)), x)
```

3.1110 $\int \frac{1}{x^4(c+a^2cx^2)^{5/2}\tan^{-1}(ax)^{5/2}}\,dx$

Optimal. Leaf size=203

$$56a^2\text{Unintegrable}\left(\frac{1}{x^2\left(a^2cx^2+c\right)^{5/2}\sqrt{\tan^{-1}(ax)}},x\right)+\frac{212}{3}\text{Unintegrable}\left(\frac{1}{x^4\left(a^2cx^2+c\right)^{5/2}\sqrt{\tan^{-1}(ax)}},x\right)+\frac{80\text{Uni}}{}$$

```
[Out] -2/(3*a*c*x^4*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)) + 16/(3*a^2*c*x^5*(c
 + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]) + 28/(3*c*x^3*(c + a^2*c*x^2)^(3/2)*
Sqrt[ArcTan[a*x]]) + (80*Unintegrable[1/(x^6*(c + a^2*c*x^2)^(5/2)*Sqrt[Arc
Tan[a*x]]), x])/(3*a^2) + (212*Unintegrable[1/(x^4*(c + a^2*c*x^2)^(5/2)*Sq
rt[ArcTan[a*x]]), x])/3 + 56*a^2*Unintegrable[1/(x^2*(c + a^2*c*x^2)^(5/2)*
Sqrt[ArcTan[a*x]]), x]
```

Rubi [A] time = 0.917727, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^4\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^4*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2)),x]
```

```
[Out] -2/(3*a*c*x^4*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)) + 16/(3*a^2*c*x^5*(c
 + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]) + 28/(3*c*x^3*(c + a^2*c*x^2)^(3/2)*
Sqrt[ArcTan[a*x]]) + (80*Defer[Int][1/(x^6*(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTa
n[a*x]]), x])/(3*a^2) + (212*Defer[Int][1/(x^4*(c + a^2*c*x^2)^(5/2)*Sqrt[A
rcTan[a*x]]), x])/3 + 56*a^2*Defer[Int][1/(x^2*(c + a^2*c*x^2)^(5/2)*Sqrt[A
rcTan[a*x]]), x]
```

Rubi steps

$$\int \frac{1}{x^4\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{5/2}}\,dx = -\frac{2}{3acx^4\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}} - \frac{8\int \frac{1}{x^5\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{3/2}}\,dx}{3a} - \frac{1}{3}(14a)\int \frac{}{x^3}$$

$$= -\frac{2}{3acx^4\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}} + \frac{16}{3a^2cx^5\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}} + \frac{}{3cx^3\left(c\right.}$$

Mathematica [A] time = 31.1297, size = 0, normalized size = 0.

$$\int \frac{1}{x^4\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^4*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2)),x]
```

```
[Out] Integrate[1/(x^4*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2)), x]
```

Maple [A] time = 2.015, size = 0, normalized size = 0.

$$\int \frac{1}{x^4}\left(a^2cx^2+c\right)^{-\frac{5}{2}}(\arctan(ax))^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

```
[Out] int(1/x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="fricas"
)
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(a**2*c*x**2+c)**(5/2)/atan(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)^{\frac{5}{2}} x^4 \arctan\left(ax\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^(5/2)*x^4*arctan(a*x)^(5/2)), x)
```

3.1111 $\int \frac{x\tan^{-1}(ax)^n}{c+a^2cx^2}\,dx$

Optimal. Leaf size=45

$$\frac{x\tan^{-1}(ax)^{n+1}}{ac(n+1)}-\frac{\text{Unintegrable}\left(\tan^{-1}(ax)^{n+1},x\right)}{ac(n+1)}$$

```
[Out] (x*ArcTan[a*x]^(1 + n))/(a*c*(1 + n)) - Unintegrable[ArcTan[a*x]^(1 + n), x
]/(a*c*(1 + n))
```

Rubi [A] time = 0.084999, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x\tan^{-1}(ax)^n}{c+a^2cx^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x*ArcTan[a*x]^n)/(c + a^2*c*x^2),x]
```

```
[Out] (x*ArcTan[a*x]^(1 + n))/(a*c*(1 + n)) - Defer[Int][ArcTan[a*x]^(1 + n), x]/
(a*c*(1 + n))
```

Rubi steps

$$\int \frac{x\tan^{-1}(ax)^n}{c+a^2cx^2}\,dx=\frac{x\tan^{-1}(ax)^{1+n}}{ac(1+n)}-\frac{\int \tan^{-1}(ax)^{1+n}\,dx}{ac(1+n)}$$

Mathematica [A] time = 0.89217, size = 0, normalized size = 0.

$$\int \frac{x\tan^{-1}(ax)^n}{c+a^2cx^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x*ArcTan[a*x]^n)/(c + a^2*c*x^2),x]
```

```
[Out] Integrate[(x*ArcTan[a*x]^n)/(c + a^2*c*x^2), x]
```

Maple [A] time = 0.602, size = 0, normalized size = 0.

$$\int \frac{x\left(\arctan\left(ax\right)\right)^{n}}{a^{2}c\,x^{2}+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(a*x)^n/(a^2*c*x^2+c),x)
```

```
[Out] int(x*arctan(a*x)^n/(a^2*c*x^2+c),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^n/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x\arctan\left(ax\right)^{n}}{a^{2}c\,x^{2}+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^n/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(x*arctan(a*x)^n/(a^2*c*x^2 + c), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x \operatorname{atan}^{n}(ax)}{a^{2}x^{2}+1}\, dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(a*x)**n/(a**2*c*x**2+c),x)
```

```
[Out] Integral(x*atan(a*x)**n/(a**2*x**2 + 1), x)/c
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \arctan(ax)^{n}}{a^{2}cx^{2}+c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^n/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(x*arctan(a*x)^n/(a^2*c*x^2 + c), x)
```

3.1112 $\int \frac{\tan^{-1}(ax)^n}{c+a^2cx^2}\,dx$

Optimal. Leaf size=20

$$\frac{\tan^{-1}(ax)^{n+1}}{ac(n+1)}$$

```
[Out] ArcTan[a*x]^(1 + n)/(a*c*(1 + n))
```

Rubi [A] time = 0.0402534, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {4884}

$$\frac{\tan^{-1}(ax)^{n+1}}{ac(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^n/(c + a^2*c*x^2),x]
```

```
[Out] ArcTan[a*x]^(1 + n)/(a*c*(1 + n))
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^n}{c+a^2cx^2}\,dx=\frac{\tan^{-1}(ax)^{1+n}}{ac(1+n)}$$

Mathematica [A] time = 0.0057055, size = 20, normalized size = 1.

$$\frac{\tan^{-1}(ax)^{n+1}}{ac(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]^n/(c + a^2*c*x^2),x]
```

```
[Out] ArcTan[a*x]^(1 + n)/(a*c*(1 + n))
```

Maple [A] time = 0.066, size = 21, normalized size = 1.1

$$\frac{(\arctan(ax))^{1+n}}{(1+n)\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^n/(a^2*c*x^2+c),x)
```

```
[Out] arctan(a*x)^(1+n)/a/c/(1+n)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^n/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 1.73808, size = 55, normalized size = 2.75

$$\frac{\arctan(ax)^{n}\arctan(ax)}{acn+ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^n/(a^2*c*x^2+c),x, algorithm="fricas")
```

[Out] arctan(a*x)^n*arctan(a*x)/(a*c*n + a*c)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\operatorname{atan}^n(ax)}{a^2x^2+1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(atan(a*x)**n/(a**2*c*x**2+c),x)

[Out] Integral(atan(a*x)**n/(a**2*x**2 + 1), x)/c

Giac [A] time = 1.07713, size = 27, normalized size = 1.35

$$\frac{\arctan(ax)^{n+1}}{ac(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctan(a*x)^n/(a^2*c*x^2+c),x, algorithm="giac")

[Out] arctan(a*x)^(n + 1)/(a*c*(n + 1))

3.1113 $\int (fx)^m \left(d+c^2dx^2\right)^q \left(a+b\tan^{-1}(cx)\right)^p\,dx$

Optimal. Leaf size=30

$$\text{Unintegrable}\left((fx)^m \left(c^2dx^2+d\right)^q \left(a+b\tan^{-1}(cx)\right)^p,x\right)$$

```
[Out] Unintegrable[(f*x)^m*(d + c^2*d*x^2)^q*(a + b*ArcTan[c*x])^p, x]
```

Rubi [A] time = 0.0986459, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (fx)^m \left(d+c^2dx^2\right)^q \left(a+b\tan^{-1}(cx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(f*x)^m*(d + c^2*d*x^2)^q*(a + b*ArcTan[c*x])^p,x]
```

```
[Out] Defer[Int][(f*x)^m*(d + c^2*d*x^2)^q*(a + b*ArcTan[c*x])^p, x]
```

Rubi steps

$$\int (fx)^m \left(d+c^2dx^2\right)^q \left(a+b\tan^{-1}(cx)\right)^p\,dx=\int (fx)^m \left(d+c^2dx^2\right)^q \left(a+b\tan^{-1}(cx)\right)^p\,dx$$

Mathematica [A] time = 0.630113, size = 0, normalized size = 0.

$$\int (fx)^m \left(d+c^2dx^2\right)^q \left(a+b\tan^{-1}(cx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(f*x)^m*(d + c^2*d*x^2)^q*(a + b*ArcTan[c*x])^p,x]
```

```
[Out] Integrate[(f*x)^m*(d + c^2*d*x^2)^q*(a + b*ArcTan[c*x])^p, x]
```

Maple [A] time = 5.934, size = 0, normalized size = 0.

$$\int (fx)^m \left(c^2dx^2 + d\right)^q (a + b\arctan(cx))^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(c^2*d*x^2+d)^q*(a+b*arctan(c*x))^p,x)
```

```
[Out] int((f*x)^m*(c^2*d*x^2+d)^q*(a+b*arctan(c*x))^p,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(c^2dx^2 + d\right)^q (fx)^m (b\arctan(cx) + a)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(c^2*d*x^2+d)^q*(a+b*arctan(c*x))^p,x, algorithm="maxima"
)
```

```
[Out] integrate((c^2*d*x^2 + d)^q*(f*x)^m*(b*arctan(c*x) + a)^p, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(c^2*d*x^2+d)^q*(a+b*arctan(c*x))^p,x, algorithm="fricas"
)
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(c**2*d*x**2+d)**q*(a+b*atan(c*x))**p,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(c^2dx^2+d\right)^q \left(fx\right)^m \left(b\arctan\left(cx\right)+a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(c^2*d*x^2+d)^q*(a+b*arctan(c*x))^p,x, algorithm="giac")
```

```
[Out] integrate((c^2*d*x^2 + d)^q*(f*x)^m*(b*arctan(c*x) + a)^p, x)
```

3.1114 $\int x^3 \left(d+ex^2\right) \left(a+b\tan^{-1}(cx)\right) dx$

Optimal. Leaf size=107

$$\frac{1}{4}dx^4\left(a+b\tan^{-1}(cx)\right)+\frac{1}{6}ex^6\left(a+b\tan^{-1}(cx)\right)-\frac{bx^3\left(3c^2d-2e\right)}{36c^3}+\frac{bx\left(3c^2d-2e\right)}{12c^5}-\frac{b\left(3c^2d-2e\right)\tan^{-1}(cx)}{12c^6}-\frac{bex^5}{30c}$$

```
[Out] (b*(3*c^2*d - 2*e)*x)/(12*c^5) - (b*(3*c^2*d - 2*e)*x^3)/(36*c^3) - (b*e*x^
5)/(30*c) - (b*(3*c^2*d - 2*e)*ArcTan[c*x])/(12*c^6) + (d*x^4*(a + b*ArcTan
[c*x]))/4 + (e*x^6*(a + b*ArcTan[c*x]))/6
```

Rubi [A] time = 0.110298, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {14, 4976, 459, 302, 203}

$$\frac{1}{4}dx^4\left(a+b\tan^{-1}(cx)\right)+\frac{1}{6}ex^6\left(a+b\tan^{-1}(cx)\right)-\frac{bx^3\left(3c^2d-2e\right)}{36c^3}+\frac{bx\left(3c^2d-2e\right)}{12c^5}-\frac{b\left(3c^2d-2e\right)\tan^{-1}(cx)}{12c^6}-\frac{bex^5}{30c}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + e*x^2)*(a + b*ArcTan[c*x]),x]
```

```
[Out] (b*(3*c^2*d - 2*e)*x)/(12*c^5) - (b*(3*c^2*d - 2*e)*x^3)/(36*c^3) - (b*e*x^
5)/(30*c) - (b*(3*c^2*d - 2*e)*ArcTan[c*x])/(12*c^6) + (d*x^4*(a + b*ArcTan
[c*x]))/4 + (e*x^6*(a + b*ArcTan[c*x]))/6
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 4976

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Dis
t[a + b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&  !(
ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(IL
tQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILtQ[(m
 - 1)/2, 0]))
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int x^3\left(d+ex^2\right)\left(a+b\tan^{-1}(cx)\right)dx &= \frac{1}{4}dx^4\left(a+b\tan^{-1}(cx)\right)+\frac{1}{6}ex^6\left(a+b\tan^{-1}(cx)\right)-(bc)\int\frac{x^4\left(3d+2ex^2\right)}{12+12c^2x^2}\,dx\\
&= -\frac{bex^5}{30c}+\frac{1}{4}dx^4\left(a+b\tan^{-1}(cx)\right)+\frac{1}{6}ex^6\left(a+b\tan^{-1}(cx)\right)+\left(bc\left(-3d+\frac{2e}{c^2}\right)\right)\int\frac{}{1}\\
&= -\frac{bex^5}{30c}+\frac{1}{4}dx^4\left(a+b\tan^{-1}(cx)\right)+\frac{1}{6}ex^6\left(a+b\tan^{-1}(cx)\right)+\left(bc\left(-3d+\frac{2e}{c^2}\right)\right)\int\Big(\\
&= \frac{b\left(3d-\frac{2e}{c^2}\right)x}{12c^3}-\frac{b\left(3d-\frac{2e}{c^2}\right)x^3}{36c}-\frac{bex^5}{30c}+\frac{1}{4}dx^4\left(a+b\tan^{-1}(cx)\right)+\frac{1}{6}ex^6\left(a+b\tan\right.\\
&= \frac{b\left(3d-\frac{2e}{c^2}\right)x}{12c^3}-\frac{b\left(3d-\frac{2e}{c^2}\right)x^3}{36c}-\frac{bex^5}{30c}-\frac{b\left(3d-\frac{2e}{c^2}\right)\tan^{-1}(cx)}{12c^4}+\frac{1}{4}dx^4\left(a+b\tan^{-1}\right.\end{aligned}$$

Mathematica [A] time = 0.0054455, size = 127, normalized size = 1.19

$$\frac{1}{4}adx^4+\frac{1}{6}aex^6+\frac{bdx}{4c^3}-\frac{bd\tan^{-1}(cx)}{4c^4}+\frac{bex^3}{18c^3}-\frac{bex}{6c^5}+\frac{be\tan^{-1}(cx)}{6c^6}-\frac{bdx^3}{12c}+\frac{1}{4}bdx^4\tan^{-1}(cx)-\frac{bex^5}{30c}+\frac{1}{6}bex^6\tan^{-1}(cx$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + e*x^2)*(a + b*ArcTan[c*x]),x]
```

```
[Out] (b*d*x)/(4*c^3) - (b*e*x)/(6*c^5) - (b*d*x^3)/(12*c) + (b*e*x^3)/(18*c^3) +
 (a*d*x^4)/4 - (b*e*x^5)/(30*c) + (a*e*x^6)/6 - (b*d*ArcTan[c*x])/(4*c^4) +
 (b*e*ArcTan[c*x])/(6*c^6) + (b*d*x^4*ArcTan[c*x])/4 + (b*e*x^6*ArcTan[c*x]
)/6
```

Maple [A] time = 0.038, size = 106, normalized size = 1.

$$\frac{aex^6}{6}+\frac{ax^4d}{4}+\frac{b\arctan(cx)\,ex^6}{6}+\frac{b\arctan(cx)\,x^4d}{4}-\frac{bex^5}{30c}-\frac{bdx^3}{12c}+\frac{bx^3e}{18c^3}+\frac{bdx}{4c^3}-\frac{bex}{6c^5}-\frac{\arctan(cx)\,bd}{4c^4}+\frac{b\arctan}{6c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x^2+d)*(a+b*arctan(c*x)),x)
```

```
[Out] 1/6*a*e*x^6+1/4*a*x^4*d+1/6*b*arctan(c*x)*e*x^6+1/4*b*arctan(c*x)*x^4*d-1/3
0*b*e*x^5/c-1/12*b*d*x^3/c+1/18/c^3*b*x^3*e+1/4*b*d*x/c^3-1/6/c^5*b*e*x-1/4
*b*d*arctan(c*x)/c^4+1/6/c^6*b*arctan(c*x)*e
```

Maxima [A] time = 1.47799, size = 146, normalized size = 1.36

$$\frac{1}{6}\,aex^6+\frac{1}{4}\,adx^4+\frac{1}{12}\left(3\,x^4\arctan(cx)-c\left(\frac{c^2x^3-3\,x}{c^4}+\frac{3\,\arctan(cx)}{c^5}\right)\right)bd+\frac{1}{90}\left(15\,x^6\arctan(cx)-c\left(\frac{3\,c^4x^5-5\,c^2x^3}{c^6}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] 1/6*a*e*x^6 + 1/4*a*d*x^4 + 1/12*(3*x^4*arctan(c*x) - c*((c^2*x^3 - 3*x)/c^
4 + 3*arctan(c*x)/c^5))*b*d + 1/90*(15*x^6*arctan(c*x) - c*((3*c^4*x^5 - 5*
c^2*x^3 + 15*x)/c^6 - 15*arctan(c*x)/c^7))*b*e
```

Fricas [A] time = 1.64274, size = 258, normalized size = 2.41

$$\frac{30\,ac^6ex^6+45\,ac^6dx^4-6\,bc^5ex^5-5\left(3\,bc^5d-2\,bc^3e\right)x^3+15\left(3\,bc^3d-2\,bce\right)x+15\left(2\,bc^6ex^6+3\,bc^6dx^4-3\,bc^2d+2\,be\right)}{180\,c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] 1/180*(30*a*c^6*e*x^6 + 45*a*c^6*d*x^4 - 6*b*c^5*e*x^5 - 5*(3*b*c^5*d - 2*b
*c^3*e)*x^3 + 15*(3*b*c^3*d - 2*b*c*e)*x + 15*(2*b*c^6*e*x^6 + 3*b*c^6*d*x^
4 - 3*b*c^2*d + 2*b*e)*arctan(c*x))/c^6
```

Sympy [A] time = 2.86752, size = 138, normalized size = 1.29

$$\begin{cases} \frac{adx^4}{4} + \frac{aex^6}{6} + \frac{bdx^4\operatorname{atan}(cx)}{4} + \frac{bex^6\operatorname{atan}(cx)}{6} - \frac{bdx^3}{12c} - \frac{bex^5}{30c} + \frac{bdx}{4c^3} + \frac{bex^3}{18c^3} - \frac{bd\operatorname{atan}(cx)}{4c^4} - \frac{bex}{6c^5} + \frac{be\operatorname{atan}(cx)}{6c^6} & \text{for } c \neq 0 \\ a\left(\frac{dx^4}{4} + \frac{ex^6}{6}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x**2+d)*(a+b*atan(c*x)),x)
```

```
[Out] Piecewise((a*d*x**4/4 + a*e*x**6/6 + b*d*x**4*atan(c*x)/4 + b*e*x**6*atan(c
*x)/6 - b*d*x**3/(12*c) - b*e*x**5/(30*c) + b*d*x/(4*c**3) + b*e*x**3/(18*c
**3) - b*d*atan(c*x)/(4*c**4) - b*e*x/(6*c**5) + b*e*atan(c*x)/(6*c**6), Ne
(c, 0)), (a*(d*x**4/4 + e*x**6/6), True))
```

Giac [A] time = 1.18206, size = 180, normalized size = 1.68

$$\frac{30\,bc^6x^6\arctan(cx)\,e + 30\,ac^6x^6e + 45\,bc^6dx^4\arctan(cx) + 45\,ac^6dx^4 - 6\,bc^5x^5e - 15\,bc^5dx^3 + 10\,bc^3x^3e + 45\,bc^3dx - 4}{180\,c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] 1/180*(30*b*c^6*x^6*arctan(c*x)*e + 30*a*c^6*x^6*e + 45*b*c^6*d*x^4*arctan(
c*x) + 45*a*c^6*d*x^4 - 6*b*c^5*x^5*e - 15*b*c^5*d*x^3 + 10*b*c^3*x^3*e + 4
5*b*c^3*d*x - 45*b*c^2*d*arctan(c*x) - 30*pi*b*e*sgn(c)*sgn(x) - 30*b*c*x*e
 + 30*b*arctan(c*x)*e)/c^6
```

3.1115 $\int x^2 \left(d+ex^2\right) \left(a+b\tan^{-1}(cx)\right) \, dx$

Optimal. Leaf size=94

$$\frac{1}{3}dx^3\left(a+b\tan^{-1}(cx)\right)+\frac{1}{5}ex^5\left(a+b\tan^{-1}(cx)\right)-\frac{bx^2\left(5c^2d-3e\right)}{30c^3}+\frac{b\left(5c^2d-3e\right)\log\left(c^2x^2+1\right)}{30c^5}-\frac{bex^4}{20c}$$

```
[Out] -(b*(5*c^2*d - 3*e)*x^2)/(30*c^3) - (b*e*x^4)/(20*c) + (d*x^3*(a + b*ArcTan
[c*x]))/3 + (e*x^5*(a + b*ArcTan[c*x]))/5 + (b*(5*c^2*d - 3*e)*Log[1 + c^2*
x^2])/(30*c^5)
```

Rubi [A] time = 0.139407, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {14, 4976, 446, 77}

$$\frac{1}{3}dx^3\left(a+b\tan^{-1}(cx)\right)+\frac{1}{5}ex^5\left(a+b\tan^{-1}(cx)\right)-\frac{bx^2\left(5c^2d-3e\right)}{30c^3}+\frac{b\left(5c^2d-3e\right)\log\left(c^2x^2+1\right)}{30c^5}-\frac{bex^4}{20c}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + e*x^2)*(a + b*ArcTan[c*x]),x]
```

```
[Out] -(b*(5*c^2*d - 3*e)*x^2)/(30*c^3) - (b*e*x^4)/(20*c) + (d*x^3*(a + b*ArcTan
[c*x]))/3 + (e*x^5*(a + b*ArcTan[c*x]))/5 + (b*(5*c^2*d - 3*e)*Log[1 + c^2*
x^2])/(30*c^5)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 4976

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Dis
t[a + b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] && !(
ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(IL
tQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] && !ILtQ[(m
 - 1)/2, 0]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int x^2\left(d+ex^2\right)\left(a+b\tan^{-1}(cx)\right)dx=\frac{1}{3}dx^3\left(a+b\tan^{-1}(cx)\right)+\frac{1}{5}ex^5\left(a+b\tan^{-1}(cx)\right)-(bc)\int\frac{x^3\left(5d+3ex^2\right)}{15+15c^2x^2}dx$$

$$=\frac{1}{3}dx^3\left(a+b\tan^{-1}(cx)\right)+\frac{1}{5}ex^5\left(a+b\tan^{-1}(cx)\right)-\frac{1}{2}(bc)\,\mathrm{Subst}\left(\int\frac{x(5d+3ex)}{15+15c^2x}\,d\right.$$

$$=\frac{1}{3}dx^3\left(a+b\tan^{-1}(cx)\right)+\frac{1}{5}ex^5\left(a+b\tan^{-1}(cx)\right)-\frac{1}{2}(bc)\,\mathrm{Subst}\left(\int\left(\frac{5c^2d-3e}{15c^4}+\right.\right.$$

$$=-\frac{b\left(5c^2d-3e\right)x^2}{30c^3}-\frac{bex^4}{20c}+\frac{1}{3}dx^3\left(a+b\tan^{-1}(cx)\right)+\frac{1}{5}ex^5\left(a+b\tan^{-1}(cx)\right)+\frac{b(}{}$$

Mathematica [A] time = 0.0212439, size = 119, normalized size = 1.27

$$\frac{1}{3}adx^3+\frac{1}{5}aex^5+\frac{bd\log\left(c^2x^2+1\right)}{6c^3}+\frac{bex^2}{10c^3}-\frac{be\log\left(c^2x^2+1\right)}{10c^5}-\frac{bdx^2}{6c}+\frac{1}{3}bdx^3\tan^{-1}(cx)-\frac{bex^4}{20c}+\frac{1}{5}bex^5\tan^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + e*x^2)*(a + b*ArcTan[c*x]),x]
```

```
[Out] -(b*d*x^2)/(6*c) + (b*e*x^2)/(10*c^3) + (a*d*x^3)/3 - (b*e*x^4)/(20*c) + (a
*e*x^5)/5 + (b*d*x^3*ArcTan[c*x])/3 + (b*e*x^5*ArcTan[c*x])/5 + (b*d*Log[1
+ c^2*x^2])/(6*c^3) - (b*e*Log[1 + c^2*x^2])/(10*c^5)
```

Maple [A] time = 0.037, size = 102, normalized size = 1.1

$$\frac{ae x^5}{5}+\frac{ad x^3}{3}+\frac{be x^5 \arctan \left(cx\right)}{5}+\frac{b \arctan \left(cx\right) d x^3}{3}-\frac{bd x^2}{6c}-\frac{be x^4}{20c}+\frac{be x^2}{10c^3}+\frac{bd \ln \left(c^2x^2+1\right)}{6c^3}-\frac{be \ln \left(c^2x^2+1\right)}{10c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^2+d)*(a+b*arctan(c*x)),x)
```

```
[Out] 1/5*a*e*x^5+1/3*a*d*x^3+1/5*b*e*x^5*arctan(c*x)+1/3*b*arctan(c*x)*d*x^3-1/6
*b*d*x^2/c-1/20*b*e*x^4/c+1/10*b*e*x^2/c^3+1/6*b*d*ln(c^2*x^2+1)/c^3-1/10*b
*e*ln(c^2*x^2+1)/c^5
```

Maxima [A] time = 1.083, size = 142, normalized size = 1.51

$$\frac{1}{5}\,aex^5+\frac{1}{3}\,adx^3+\frac{1}{6}\left(2\,x^3\arctan\left(cx\right)-c\left(\frac{x^2}{c^2}-\frac{\log\left(c^2x^2+1\right)}{c^4}\right)\right)bd+\frac{1}{20}\left(4\,x^5\arctan\left(cx\right)-c\left(\frac{c^2x^4-2\,x^2}{c^4}+\frac{2\,\log\left(c^2x\right.}{c^6}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] 1/5*a*e*x^5 + 1/3*a*d*x^3 + 1/6*(2*x^3*arctan(c*x) - c*(x^2/c^2 - log(c^2*x
^2 + 1)/c^4))*b*d + 1/20*(4*x^5*arctan(c*x) - c*((c^2*x^4 - 2*x^2)/c^4 + 2*
log(c^2*x^2 + 1)/c^6))*b*e
```

Fricas [A] time = 1.7496, size = 244, normalized size = 2.6

$$\frac{12\,ac^5ex^5+20\,ac^5dx^3-3\,bc^4ex^4-2\left(5\,bc^4d-3\,bc^2e\right)x^2+4\left(3\,bc^5ex^5+5\,bc^5dx^3\right)\arctan\left(cx\right)+2\left(5\,bc^2d-3\,be\right)\log\left(c^2\right.}{60\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] 1/60*(12*a*c^5*e*x^5 + 20*a*c^5*d*x^3 - 3*b*c^4*e*x^4 - 2*(5*b*c^4*d - 3*b*
c^2*e)*x^2 + 4*(3*b*c^5*e*x^5 + 5*b*c^5*d*x^3)*arctan(c*x) + 2*(5*b*c^2*d -
```

```
3*b*e)*log(c^2*x^2 + 1))/c^5
```

Sympy [A] time = 2.05625, size = 128, normalized size = 1.36

$$\begin{cases} \frac{adx^3}{3} + \frac{aex^5}{5} + \frac{bdx^3 \operatorname{atan}(cx)}{3} + \frac{bex^5 \operatorname{atan}(cx)}{5} - \frac{bdx^2}{6c} - \frac{bex^4}{20c} + \frac{bd \log\left(x^2+\frac{1}{c^2}\right)}{6c^3} + \frac{bex^2}{10c^3} - \frac{be \log\left(x^2+\frac{1}{c^2}\right)}{10c^5} & \text{for } c \neq 0 \\ a\left(\frac{dx^3}{3} + \frac{ex^5}{5}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d)*(a+b*atan(c*x)),x)
```

```
[Out] Piecewise((a*d*x**3/3 + a*e*x**5/5 + b*d*x**3*atan(c*x)/3 + b*e*x**5*atan(c
*x)/5 - b*d*x**2/(6*c) - b*e*x**4/(20*c) + b*d*log(x**2 + c**(-2))/(6*c**3)
 + b*e*x**2/(10*c**3) - b*e*log(x**2 + c**(-2))/(10*c**5), Ne(c, 0)), (a*(d
*x**3/3 + e*x**5/5), True))
```

Giac [A] time = 1.09012, size = 162, normalized size = 1.72

$$\frac{12\, bc^5x^5 \arctan(cx)\, e + 12\, ac^5x^5e + 20\, bc^5dx^3 \arctan(cx) + 20\, ac^5dx^3 - 3\, bc^4x^4e - 10\, bc^4dx^2 + 6\, bc^2x^2e + 10\, bc^2d \log\left(c^2\right.}{60\, c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] 1/60*(12*b*c^5*x^5*arctan(c*x)*e + 12*a*c^5*x^5*e + 20*b*c^5*d*x^3*arctan(c
*x) + 20*a*c^5*d*x^3 - 3*b*c^4*x^4*e - 10*b*c^4*d*x^2 + 6*b*c^2*x^2*e + 10*
b*c^2*d*log(c^2*x^2 + 1) - 6*b*e*log(c^2*x^2 + 1))/c^5
```

3.1116 $\int x\left(d+ex^2\right)\left(a+b\tan^{-1}(cx)\right)\,dx$

Optimal. Leaf size=82

$$\frac{\left(d+ex^2\right)^2\left(a+b\tan^{-1}(cx)\right)}{4e}-\frac{bx\left(2c^2d-e\right)}{4c^3}-\frac{b\left(c^2d-e\right)^2\tan^{-1}(cx)}{4c^4e}-\frac{bex^3}{12c}$$

```
[Out] -(b*(2*c^2*d - e)*x)/(4*c^3) - (b*e*x^3)/(12*c) - (b*(c^2*d - e)^2*ArcTan[c
*x])/(4*c^4*e) + ((d + e*x^2)^2*(a + b*ArcTan[c*x]))/(4*e)
```

Rubi [A] time = 0.0698475, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {4974, 390, 203}

$$\frac{\left(d+ex^2\right)^2\left(a+b\tan^{-1}(cx)\right)}{4e}-\frac{bx\left(2c^2d-e\right)}{4c^3}-\frac{b\left(c^2d-e\right)^2\tan^{-1}(cx)}{4c^4e}-\frac{bex^3}{12c}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + e*x^2)*(a + b*ArcTan[c*x]),x]
```


```
[Out] -(b*(2*c^2*d - e)*x)/(4*c^3) - (b*e*x^3)/(12*c) - (b*(c^2*d - e)^2*ArcTan[c
*x])/(4*c^4*e) + ((d + e*x^2)^2*(a + b*ArcTan[c*x]))/(4*e)
```


Rule 4974

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]))/(2*e*(q + 1)), x
] - Dist[(b*c)/(2*e*(q + 1)), Int[(d + e*x^2)^(q + 1)/(1 + c^2*x^2), x], x]
 /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int x\left(d+ex^2\right)\left(a+b\tan^{-1}(cx)\right)dx &= \frac{\left(d+ex^2\right)^2\left(a+b\tan^{-1}(cx)\right)}{4e}-\frac{(bc)\int\frac{\left(d+ex^2\right)^2}{1+c^2x^2}\,dx}{4e}\\
&= \frac{\left(d+ex^2\right)^2\left(a+b\tan^{-1}(cx)\right)}{4e}-\frac{(bc)\int\left(\frac{\left(2c^2d-e\right)e}{c^4}+\frac{e^2x^2}{c^2}+\frac{c^4d^2-2c^2de+e^2}{c^4\left(1+c^2x^2\right)}\right)dx}{4e}\\
&= -\frac{b\left(2c^2d-e\right)x}{4c^3}-\frac{bex^3}{12c}+\frac{\left(d+ex^2\right)^2\left(a+b\tan^{-1}(cx)\right)}{4e}-\frac{\left(b\left(c^2d-e\right)^2\right)\int\frac{1}{1+c^2x^2}\,dx}{4c^3e}\\
&= -\frac{b\left(2c^2d-e\right)x}{4c^3}-\frac{bex^3}{12c}-\frac{b\left(c^2d-e\right)^2\tan^{-1}(cx)}{4c^4e}+\frac{\left(d+ex^2\right)^2\left(a+b\tan^{-1}(cx)\right)}{4e}
\end{aligned}
$$

Mathematica [A] time = 0.0043604, size = 103, normalized size = 1.26

$$\frac{1}{2}adx^2+\frac{1}{4}aex^4+\frac{bd\tan^{-1}(cx)}{2c^2}+\frac{bex}{4c^3}-\frac{be\tan^{-1}(cx)}{4c^4}+\frac{1}{2}bdx^2\tan^{-1}(cx)-\frac{bdx}{2c}-\frac{bex^3}{12c}+\frac{1}{4}bex^4\tan^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + e*x^2)*(a + b*ArcTan[c*x]),x]
```

```
[Out] -(b*d*x)/(2*c) + (b*e*x)/(4*c^3) + (a*d*x^2)/2 - (b*e*x^3)/(12*c) + (a*e*x^
4)/4 + (b*d*ArcTan[c*x])/(2*c^2) - (b*e*ArcTan[c*x])/(4*c^4) + (b*d*x^2*Arc
Tan[c*x])/2 + (b*e*x^4*ArcTan[c*x])/4
```

Maple [A] time = 0.037, size = 86, normalized size = 1.1

$$\frac{aex^4}{4}+\frac{ax^2d}{2}+\frac{b\arctan(cx)ex^4}{4}+\frac{b\arctan(cx)dx^2}{2}-\frac{bex^3}{12c}-\frac{bdx}{2c}+\frac{bex}{4c^3}+\frac{\arctan(cx)bd}{2c^2}-\frac{b\arctan(cx)e}{4c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d)*(a+b*arctan(c*x)),x)
```

```
[Out] 1/4*a*e*x^4+1/2*a*x^2*d+1/4*b*arctan(c*x)*e*x^4+1/2*b*arctan(c*x)*d*x^2-1/1
2*b*e*x^3/c-1/2*b*d*x/c+1/4*b*e*x/c^3+1/2*b*d*arctan(c*x)/c^2-1/4*b*e*arcta
```

```
n(c*x)/c^4
```

Maxima [A] time = 1.41794, size = 119, normalized size = 1.45

$$\frac{1}{4}\,aex^4 + \frac{1}{2}\,adx^2 + \frac{1}{2}\left(x^2\arctan\left(cx\right) - c\left(\frac{x}{c^2} - \frac{\arctan\left(cx\right)}{c^3}\right)\right)bd + \frac{1}{12}\left(3\,x^4\arctan\left(cx\right) - c\left(\frac{c^2x^3 - 3\,x}{c^4} + \frac{3\,\arctan\left(cx\right)}{c^5}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] 1/4*a*e*x^4 + 1/2*a*d*x^2 + 1/2*(x^2*arctan(c*x) - c*(x/c^2 - arctan(c*x)/c
^3))*b*d + 1/12*(3*x^4*arctan(c*x) - c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)
/c^5))*b*e
```

Fricas [A] time = 1.75088, size = 197, normalized size = 2.4

$$\frac{3\,ac^4ex^4 + 6\,ac^4dx^2 - bc^3ex^3 - 3\left(2\,bc^3d - bce\right)x + 3\left(bc^4ex^4 + 2\,bc^4dx^2 + 2\,bc^2d - be\right)\arctan\left(cx\right)}{12\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] 1/12*(3*a*c^4*e*x^4 + 6*a*c^4*d*x^2 - b*c^3*e*x^3 - 3*(2*b*c^3*d - b*c*e)*x
 + 3*(b*c^4*e*x^4 + 2*b*c^4*d*x^2 + 2*b*c^2*d - b*e)*arctan(c*x))/c^4
```

Sympy [A] time = 1.56302, size = 114, normalized size = 1.39

$$\begin{cases} \frac{adx^2}{2} + \frac{aex^4}{4} + \frac{bdx^2\operatorname{atan}\left(cx\right)}{2} + \frac{bex^4\operatorname{atan}\left(cx\right)}{4} - \frac{bdx}{2c} - \frac{bex^3}{12c} + \frac{bd\operatorname{atan}\left(cx\right)}{2c^2} + \frac{bex}{4c^3} - \frac{be\operatorname{atan}\left(cx\right)}{4c^4} & \text{for}\: c \neq 0 \\ a\left(\frac{dx^2}{2} + \frac{ex^4}{4}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d)*(a+b*atan(c*x)),x)
```

```
[Out] Piecewise((a*d*x**2/2 + a*e*x**4/4 + b*d*x**2*atan(c*x)/2 + b*e*x**4*atan(c
*x)/4 - b*d*x/(2*c) - b*e*x**3/(12*c) + b*d*atan(c*x)/(2*c**2) + b*e*x/(4*c
**3) - b*e*atan(c*x)/(4*c**4), Ne(c, 0)), (a*(d*x**2/2 + e*x**4/4), True))
```

Giac [A] time = 1.1876, size = 154, normalized size = 1.88

$$\frac{3\,bc^4x^4\arctan\left(cx\right)e+3\,ac^4x^4e+6\,bc^4dx^2\arctan\left(cx\right)+6\,ac^4dx^2-bc^3x^3e-6\,\pi bc^2d\mathrm{sgn}\left(c\right)\mathrm{sgn}\left(x\right)-6\,bc^3dx+6\,bc^2d\,\mathrm{ar}}{12\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] 1/12*(3*b*c^4*x^4*arctan(c*x)*e + 3*a*c^4*x^4*e + 6*b*c^4*d*x^2*arctan(c*x)
 + 6*a*c^4*d*x^2 - b*c^3*x^3*e - 6*pi*b*c^2*d*sgn(c)*sgn(x) - 6*b*c^3*d*x +
 6*b*c^2*d*arctan(c*x) + 3*b*c*x*e - 3*b*arctan(c*x)*e)/c^4
```

3.1117 $\int \left(d+ex^2\right)\left(a+b\tan^{-1}(cx)\right)\,dx$

Optimal. Leaf size=68

$$dx\left(a+b\tan^{-1}(cx)\right)+\frac{1}{3}ex^3\left(a+b\tan^{-1}(cx)\right)-\frac{b\left(3c^2d-e\right)\log\left(c^2x^2+1\right)}{6c^3}-\frac{bex^2}{6c}$$

```
[Out] -(b*e*x^2)/(6*c) + d*x*(a + b*ArcTan[c*x]) + (e*x^3*(a + b*ArcTan[c*x]))/3
- (b*(3*c^2*d - e)*Log[1 + c^2*x^2])/(6*c^3)
```

Rubi [A] time = 0.0718078, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4912, 1593, 444, 43}

$$dx\left(a+b\tan^{-1}(cx)\right)+\frac{1}{3}ex^3\left(a+b\tan^{-1}(cx)\right)-\frac{b\left(3c^2d-e\right)\log\left(c^2x^2+1\right)}{6c^3}-\frac{bex^2}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)*(a + b*ArcTan[c*x]),x]
```

```
[Out] -(b*e*x^2)/(6*c) + d*x*(a + b*ArcTan[c*x]) + (e*x^3*(a + b*ArcTan[c*x]))/3
- (b*(3*c^2*d - e)*Log[1 + c^2*x^2])/(6*c^3)
```

Rule 4912

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcTan[c*x], u, x]
- Dist[b*c, Int[u/(1 + c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && (I
ntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
```

```
1, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \left(d+ex^2\right)\left(a+b\tan^{-1}(cx)\right)dx = dx\left(a+b\tan^{-1}(cx)\right)+\frac{1}{3}ex^3\left(a+b\tan^{-1}(cx)\right)-(bc)\int\frac{dx+\frac{ex^3}{3}}{1+c^2x^2}\,dx$$

$$= dx\left(a+b\tan^{-1}(cx)\right)+\frac{1}{3}ex^3\left(a+b\tan^{-1}(cx)\right)-(bc)\int\frac{x\left(d+\frac{ex^2}{3}\right)}{1+c^2x^2}\,dx$$

$$= dx\left(a+b\tan^{-1}(cx)\right)+\frac{1}{3}ex^3\left(a+b\tan^{-1}(cx)\right)-\frac{1}{2}(bc)\,\mathrm{Subst}\left(\int\frac{d+\frac{ex}{3}}{1+c^2x}\,dx,x,x^2\right)$$

$$= dx\left(a+b\tan^{-1}(cx)\right)+\frac{1}{3}ex^3\left(a+b\tan^{-1}(cx)\right)-\frac{1}{2}(bc)\,\mathrm{Subst}\left(\int\left(\frac{e}{3c^2}+\frac{3c^2d-e}{3c^2\left(1+c^2x\right.}\right.\right.$$

$$= -\frac{bex^2}{6c}+dx\left(a+b\tan^{-1}(cx)\right)+\frac{1}{3}ex^3\left(a+b\tan^{-1}(cx)\right)-\frac{b\left(3c^2d-e\right)\log\left(1+c^2x^2\right)}{6c^3}$$

Mathematica [A] time = 0.0100485, size = 85, normalized size = 1.25

$$adx+\frac{1}{3}aex^3-\frac{bd\log\left(c^2x^2+1\right)}{2c}+\frac{be\log\left(c^2x^2+1\right)}{6c^3}+bdx\tan^{-1}(cx)-\frac{bex^2}{6c}+\frac{1}{3}bex^3\tan^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)*(a + b*ArcTan[c*x]),x]
```

```
[Out] a*d*x - (b*e*x^2)/(6*c) + (a*e*x^3)/3 + b*d*x*ArcTan[c*x] + (b*e*x^3*ArcTan
[c*x])/3 - (b*d*Log[1 + c^2*x^2])/(2*c) + (b*e*Log[1 + c^2*x^2])/(6*c^3)
```

Maple [A] time = 0.035, size = 76, normalized size = 1.1

$$\frac{aex^3}{3}+adx+\frac{bex^3\arctan(cx)}{3}+b\arctan(cx)\,dx-\frac{bex^2}{6c}-\frac{b\ln\left(c^2x^2+1\right)d}{2c}+\frac{be\ln\left(c^2x^2+1\right)}{6c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(a+b*arctan(c*x)),x)
```

```
[Out] 1/3*a*e*x^3+a*d*x+1/3*b*e*x^3*arctan(c*x)+b*arctan(c*x)*d*x-1/6*b*e*x^2/c-1
/2/c*b*ln(c^2*x^2+1)*d+1/6*b*e*ln(c^2*x^2+1)/c^3
```

Maxima [A] time = 1.05791, size = 108, normalized size = 1.59

$$\frac{1}{3}\,aex^3+\frac{1}{6}\left(2\,x^3\arctan\left(cx\right)-c\left(\frac{x^2}{c^2}-\frac{\log\left(c^2x^2+1\right)}{c^4}\right)\right)be+adx+\frac{\left(2\,cx\arctan\left(cx\right)-\log\left(c^2x^2+1\right)\right)bd}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] 1/3*a*e*x^3 + 1/6*(2*x^3*arctan(c*x) - c*(x^2/c^2 - log(c^2*x^2 + 1)/c^4))*
b*e + a*d*x + 1/2*(2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*b*d/c
```

Fricas [A] time = 1.69946, size = 181, normalized size = 2.66

$$\frac{2\,ac^3ex^3+6\,ac^3dx-bc^2ex^2+2\left(bc^3ex^3+3\,bc^3dx\right)\arctan\left(cx\right)-\left(3\,bc^2d-be\right)\log\left(c^2x^2+1\right)}{6\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*a*c^3*e*x^3 + 6*a*c^3*d*x - b*c^2*e*x^2 + 2*(b*c^3*e*x^3 + 3*b*c^3*d
*x)*arctan(c*x) - (3*b*c^2*d - b*e)*log(c^2*x^2 + 1))/c^3
```

Sympy [A] time = 1.03202, size = 94, normalized size = 1.38

$$\begin{cases} adx+\frac{aex^3}{3}+bdx\,\mathrm{atan}\,(cx)+\frac{bex^3\,\mathrm{atan}\,(cx)}{3}-\frac{bd\log\left(x^2+\frac{1}{c^2}\right)}{2c}-\frac{bex^2}{6c}+\frac{be\log\left(x^2+\frac{1}{c^2}\right)}{6c^3} & \text{for}\ c\neq 0\\ a\left(dx+\frac{ex^3}{3}\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(a+b*atan(c*x)),x)
```

```
[Out] Piecewise((a*d*x + a*e*x**3/3 + b*d*x*atan(c*x) + b*e*x**3*atan(c*x)/3 - b*
d*log(x**2 + c**(-2))/(2*c) - b*e*x**2/(6*c) + b*e*log(x**2 + c**(-2))/(6*c
**3), Ne(c, 0)), (a*(d*x + e*x**3/3), True))
```

Giac [A] time = 1.10571, size = 127, normalized size = 1.87

$$\frac{2\,bc^3x^3\arctan\left(cx\right)e + 2\,ac^3x^3e + 6\,bc^3dx\arctan\left(cx\right) + 6\,ac^3dx - bc^2x^2e - 3\,bc^2d\log\left(c^2x^2+1\right) + be\log\left(c^2x^2+1\right)}{6\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] 1/6*(2*b*c^3*x^3*arctan(c*x)*e + 2*a*c^3*x^3*e + 6*b*c^3*d*x*arctan(c*x) +
6*a*c^3*d*x - b*c^2*x^2*e - 3*b*c^2*d*log(c^2*x^2 + 1) + b*e*log(c^2*x^2 +
1))/c^3
```

3.1118 $\int \frac{(d+ex^2)(a+b\tan^{-1}(cx))}{x}\,dx$

Optimal. Leaf size=77

$$\frac{1}{2}ibd\text{PolyLog}(2,-icx)-\frac{1}{2}ibd\text{PolyLog}(2,icx)+\frac{1}{2}ex^2\left(a+b\tan^{-1}(cx)\right)+ad\log(x)+\frac{be\tan^{-1}(cx)}{2c^2}-\frac{bex}{2c}$$

```
[Out] -(b*e*x)/(2*c) + (b*e*ArcTan[c*x])/(2*c^2) + (e*x^2*(a + b*ArcTan[c*x]))/2
+ a*d*Log[x] + (I/2)*b*d*PolyLog[2, (-I)*c*x] - (I/2)*b*d*PolyLog[2, I*c*x]
```

Rubi [A] time = 0.0928198, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {4980, 4848, 2391, 4852, 321, 203}

$$\frac{1}{2}ibd\text{PolyLog}(2,-icx)-\frac{1}{2}ibd\text{PolyLog}(2,icx)+\frac{1}{2}ex^2\left(a+b\tan^{-1}(cx)\right)+ad\log(x)+\frac{be\tan^{-1}(cx)}{2c^2}-\frac{bex}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)*(a + b*ArcTan[c*x]))/x,x]
```

```
[Out] -(b*e*x)/(2*c) + (b*e*ArcTan[c*x])/(2*c^2) + (e*x^2*(a + b*ArcTan[c*x]))/2
+ a*d*Log[x] + (I/2)*b*d*PolyLog[2, (-I)*c*x] - (I/2)*b*d*PolyLog[2, I*c*x]
```

Rule 4980

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(d+ex^2\right)\left(a+b\tan^{-1}(cx)\right)}{x}\,dx &= \int \left(\frac{d\left(a+b\tan^{-1}(cx)\right)}{x}+ex\left(a+b\tan^{-1}(cx)\right)\right)dx\\ &= d\int \frac{a+b\tan^{-1}(cx)}{x}\,dx+e\int x\left(a+b\tan^{-1}(cx)\right)dx\\ &= \frac{1}{2}ex^2\left(a+b\tan^{-1}(cx)\right)+ad\log(x)+\frac{1}{2}(ibd)\int \frac{\log(1-icx)}{x}\,dx-\frac{1}{2}(ibd)\int \frac{\log(1+ic}{x}\\ &= -\frac{bex}{2c}+\frac{1}{2}ex^2\left(a+b\tan^{-1}(cx)\right)+ad\log(x)+\frac{1}{2}ibd\mathrm{Li}_2(-icx)-\frac{1}{2}ibd\mathrm{Li}_2(icx)+\frac{(be)\int}{}\\ &= -\frac{bex}{2c}+\frac{be\tan^{-1}(cx)}{2c^2}+\frac{1}{2}ex^2\left(a+b\tan^{-1}(cx)\right)+ad\log(x)+\frac{1}{2}ibd\mathrm{Li}_2(-icx)-\frac{1}{2}ibd\mathrm{L}\end{aligned}$$

Mathematica [A] time = 0.0040815, size = 83, normalized size = 1.08

$$\frac{1}{2}ibd\,\text{PolyLog}(2,-icx)-\frac{1}{2}ibd\,\text{PolyLog}(2,icx)+ad\log(x)+\frac{1}{2}aex^2+\frac{be\tan^{-1}(cx)}{2c^2}+\frac{1}{2}bex^2\tan^{-1}(cx)-\frac{bex}{2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)*(a + b*ArcTan[c*x]))/x,x]
```

```
[Out] -(b*e*x)/(2*c) + (a*e*x^2)/2 + (b*e*ArcTan[c*x])/(2*c^2) + (b*e*x^2*ArcTan[
c*x])/2 + a*d*Log[x] + (I/2)*b*d*PolyLog[2, (-I)*c*x] - (I/2)*b*d*PolyLog[2
, I*c*x]
```

Maple [A] time = 0.049, size = 117, normalized size = 1.5

$$\frac{ax^2e}{2}+ad\ln(cx)+\frac{\arctan(cx)\,bex^2}{2}+b\arctan(cx)\,d\ln(cx)+\frac{b\arctan(cx)\,e}{2c^2}-\frac{bex}{2c}+\frac{i}{2}bd\ln(cx)\ln(1+icx)-\frac{i}{2}bd\,\mathrm{l}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(a+b*arctan(c*x))/x,x)
```

```
[Out] 1/2*a*x^2*e+a*d*ln(c*x)+1/2*arctan(c*x)*b*e*x^2+b*arctan(c*x)*d*ln(c*x)+1/2
/c^2*b*e*arctan(c*x)-1/2*b*e*x/c+1/2*I*b*d*ln(c*x)*ln(1+I*c*x)-1/2*I*b*d*ln
(c*x)*ln(1-I*c*x)+1/2*I*b*d*dilog(1+I*c*x)-1/2*I*b*d*dilog(1-I*c*x)
```

Maxima [B] time = 2.15175, size = 158, normalized size = 2.05

$$\frac{1}{2}aex^2+ad\log(x)-\frac{\pi bc^2d\log\left(c^2x^2+1\right)-4bc^2d\arctan(cx)\log(x|c|)+2ibc^2d\mathrm{Li}_2(icx+1)-2ibc^2d\mathrm{Li}_2(-icx+1)-}{4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arctan(c*x))/x,x, algorithm="maxima")
```

```
[Out] 1/2*a*e*x^2 + a*d*log(x) - 1/4*(pi*b*c^2*d*log(c^2*x^2 + 1) - 4*b*c^2*d*arc
tan(c*x)*log(x*abs(c)) + 2*I*b*c^2*d*dilog(I*c*x + 1) - 2*I*b*c^2*d*dilog(-
I*c*x + 1) + 2*b*c*e*x - (2*b*c^2*e*x^2 + 4*I*b*c^2*d*arctan2(0, c) + 2*b*e
)*arctan(c*x))/c^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{aex^2+ad+\left(bex^2+bd\right)\arctan(cx)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arctan(c*x))/x,x, algorithm="fricas")
```

```
[Out] integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arctan(c*x))/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b\operatorname{atan}(cx))\left(d + ex^2\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(a+b*atan(c*x))/x,x)
```

```
[Out] Integral((a + b*atan(c*x))*(d + e*x**2)/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2 + d\right)(b\arctan(cx) + a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arctan(c*x))/x,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*(b*arctan(c*x) + a)/x, x)
```

3.1119 $\int \frac{(d+ex^2)(a+b\tan^{-1}(cx))}{x^2}\,dx$

Optimal. Leaf size=57

$$-\frac{d\left(a+b\tan^{-1}(cx)\right)}{x}+ex\left(a+b\tan^{-1}(cx)\right)-\frac{b\left(c^2d+e\right)\log\left(c^2x^2+1\right)}{2c}+bcd\log(x)$$

```
[Out] -((d*(a + b*ArcTan[c*x]))/x) + e*x*(a + b*ArcTan[c*x]) + b*c*d*Log[x] - (b*
(c^2*d + e)*Log[1 + c^2*x^2])/(2*c)
```

Rubi [A] time = 0.0765387, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {14, 4976, 446, 72}

$$-\frac{d\left(a+b\tan^{-1}(cx)\right)}{x}+ex\left(a+b\tan^{-1}(cx)\right)-\frac{b\left(c^2d+e\right)\log\left(c^2x^2+1\right)}{2c}+bcd\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)*(a + b*ArcTan[c*x]))/x^2,x]
```

```
[Out] -((d*(a + b*ArcTan[c*x]))/x) + e*x*(a + b*ArcTan[c*x]) + b*c*d*Log[x] - (b*
(c^2*d + e)*Log[1 + c^2*x^2])/(2*c)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 4976

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Dis
t[a + b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&  !(
ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(IL
tQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILtQ[(m
 - 1)/2, 0]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex^2)\left(a+b\tan^{-1}(cx)\right)}{x^2}\,dx &= -\frac{d\left(a+b\tan^{-1}(cx)\right)}{x}+ex\left(a+b\tan^{-1}(cx)\right)-(bc)\int \frac{-d+ex^2}{x\left(1+c^2x^2\right)}\,dx\\ &= -\frac{d\left(a+b\tan^{-1}(cx)\right)}{x}+ex\left(a+b\tan^{-1}(cx)\right)-\frac{1}{2}(bc)\,\text{Subst}\left(\int \frac{-d+ex}{x\left(1+c^2x\right)}\,dx,x,x^2\right)\\ &= -\frac{d\left(a+b\tan^{-1}(cx)\right)}{x}+ex\left(a+b\tan^{-1}(cx)\right)-\frac{1}{2}(bc)\,\text{Subst}\left(\int \left(-\frac{d}{x}+\frac{c^2d+e}{1+c^2x}\right)dx,x\right.\\ &= -\frac{d\left(a+b\tan^{-1}(cx)\right)}{x}+ex\left(a+b\tan^{-1}(cx)\right)+bcd\log(x)-\frac{b\left(c^2d+e\right)\log\left(1+c^2x^2\right)}{2c}\end{aligned}$$

Mathematica [A] time = 0.0049527, size = 73, normalized size = 1.28

$$-\frac{ad}{x}+aex-\frac{1}{2}bcd\log\left(c^2x^2+1\right)-\frac{be\log\left(c^2x^2+1\right)}{2c}+bcd\log(x)-\frac{bd\tan^{-1}(cx)}{x}+bex\tan^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)*(a + b*ArcTan[c*x]))/x^2,x]
```

```
[Out] -((a*d)/x) + a*e*x - (b*d*ArcTan[c*x])/x + b*e*x*ArcTan[c*x] + b*c*d*Log[x]
 - (b*c*d*Log[1 + c^2*x^2])/2 - (b*e*Log[1 + c^2*x^2])/(2*c)
```

Maple [A] time = 0.04, size = 72, normalized size = 1.3

$$aex-\frac{ad}{x}+bex\arctan(cx)-\frac{\arctan(cx)\,bd}{x}-\frac{bcd\ln\left(c^2x^2+1\right)}{2}-\frac{be\ln\left(c^2x^2+1\right)}{2c}+cbd\ln(cx)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(a+b*arctan(c*x))/x^2,x)
```

```
[Out] a*e*x-a*d/x+b*e*x*arctan(c*x)-b*arctan(c*x)*d/x-1/2*b*c*d*ln(c^2*x^2+1)-1/2
*b*e*ln(c^2*x^2+1)/c+c*b*d*ln(c*x)
```

Maxima [A] time = 0.944748, size = 99, normalized size = 1.74

$$-\frac{1}{2}\left(c\left(\log\left(c^2x^2+1\right)-\log\left(x^2\right)\right)+\frac{2\arctan\left(cx\right)}{x}\right)bd+aex+\frac{\left(2\,cx\arctan\left(cx\right)-\log\left(c^2x^2+1\right)\right)be}{2\,c}-\frac{ad}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arctan(c*x))/x^2,x, algorithm="maxima")
```

```
[Out] -1/2*(c*(log(c^2*x^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*b*d + a*e*x + 1/2*
(2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*b*e/c - a*d/x
```

Fricas [A] time = 1.66704, size = 174, normalized size = 3.05

$$\frac{2\,bc^2dx\log\left(x\right)+2\,acex^2-2\,acd-\left(bc^2d+be\right)x\log\left(c^2x^2+1\right)+2\left(bcex^2-bcd\right)\arctan\left(cx\right)}{2\,cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arctan(c*x))/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*c^2*d*x*log(x) + 2*a*c*e*x^2 - 2*a*c*d - (b*c^2*d + b*e)*x*log(c^2
*x^2 + 1) + 2*(b*c*e*x^2 - b*c*d)*arctan(c*x))/(c*x)
```

Sympy [A] time = 1.43454, size = 80, normalized size = 1.4

$$\begin{cases} -\frac{ad}{x}+aex+bcd\log\left(x\right)-\frac{bcd\log\left(x^2+\frac{1}{c^2}\right)}{2}-\frac{bd\operatorname{atan}\left(cx\right)}{x}+bex\operatorname{atan}\left(cx\right)-\frac{be\log\left(x^2+\frac{1}{c^2}\right)}{2c} & \text{for}\ c\neq 0 \\ a\left(-\frac{d}{x}+ex\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(a+b*atan(c*x))/x**2,x)
```

```
[Out] Piecewise((-a*d/x + a*e*x + b*c*d*log(x) - b*c*d*log(x**2 + c**(-2))/2 - b*
d*atan(c*x)/x + b*e*x*atan(c*x) - b*e*log(x**2 + c**(-2))/(2*c), Ne(c, 0)),
 (a*(-d/x + e*x), True))
```

Giac [A] time = 1.09957, size = 120, normalized size = 2.11

$$\frac{2\,bcx^2\arctan\left(cx\right)e-bc^2dx\log\left(c^2x^2+1\right)+2\,bc^2dx\log\left(x\right)+2\,acx^2e-2\,bcd\arctan\left(cx\right)-bxe\log\left(c^2x^2+1\right)-2\,acd}{2\,cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arctan(c*x))/x^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*b*c*x^2*arctan(c*x)*e - b*c^2*d*x*log(c^2*x^2 + 1) + 2*b*c^2*d*x*log
(x) + 2*a*c*x^2*e - 2*b*c*d*arctan(c*x) - b*x*e*log(c^2*x^2 + 1) - 2*a*c*d)
/(c*x)
```

3.1120 $\int \frac{(d+ex^2)(a+b\tan^{-1}(cx))}{x^3}\,dx$

Optimal. Leaf size=77

$$\frac{1}{2}ibe\text{PolyLog}(2,-icx)-\frac{1}{2}ibe\text{PolyLog}(2,icx)-\frac{d\left(a+b\tan^{-1}(cx)\right)}{2x^2}+ae\log(x)-\frac{1}{2}bc^2d\tan^{-1}(cx)-\frac{bcd}{2x}$$

```
[Out] -(b*c*d)/(2*x) - (b*c^2*d*ArcTan[c*x])/2 - (d*(a + b*ArcTan[c*x]))/(2*x^2)
+ a*e*Log[x] + (I/2)*b*e*PolyLog[2, (-I)*c*x] - (I/2)*b*e*PolyLog[2, I*c*x]
```

Rubi [A] time = 0.0991435, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {4980, 4852, 325, 203, 4848, 2391}

$$\frac{1}{2}ibe\text{PolyLog}(2,-icx)-\frac{1}{2}ibe\text{PolyLog}(2,icx)-\frac{d\left(a+b\tan^{-1}(cx)\right)}{2x^2}+ae\log(x)-\frac{1}{2}bc^2d\tan^{-1}(cx)-\frac{bcd}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)*(a + b*ArcTan[c*x]))/x^3,x]
```


```
[Out] -(b*c*d)/(2*x) - (b*c^2*d*ArcTan[c*x])/2 - (d*(a + b*ArcTan[c*x]))/(2*x^2)
+ a*e*Log[x] + (I/2)*b*e*PolyLog[2, (-I)*c*x] - (I/2)*b*e*PolyLog[2, I*c*x]
```


Rule 4980

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(d+ex^2\right)\left(a+b\tan^{-1}(cx)\right)}{x^3}\,dx &= \int \left(\frac{d\left(a+b\tan^{-1}(cx)\right)}{x^3}+\frac{e\left(a+b\tan^{-1}(cx)\right)}{x}\right)dx\\
&= d\int \frac{a+b\tan^{-1}(cx)}{x^3}\,dx+e\int \frac{a+b\tan^{-1}(cx)}{x}\,dx\\
&= -\frac{d\left(a+b\tan^{-1}(cx)\right)}{2x^2}+ae\log(x)+\frac{1}{2}(bcd)\int \frac{1}{x^2\left(1+c^2x^2\right)}\,dx+\frac{1}{2}(ibe)\int \frac{\log(1-ic
\\
&= -\frac{bcd}{2x}-\frac{d\left(a+b\tan^{-1}(cx)\right)}{2x^2}+ae\log(x)+\frac{1}{2}ibe\text{Li}_2(-icx)-\frac{1}{2}ibe\text{Li}_2(icx)-\frac{1}{2}\left(bc^3d\right)\int
\\
&= -\frac{bcd}{2x}-\frac{1}{2}bc^2d\tan^{-1}(cx)-\frac{d\left(a+b\tan^{-1}(cx)\right)}{2x^2}+ae\log(x)+\frac{1}{2}ibe\text{Li}_2(-icx)-\frac{1}{2}ibe\text{Li}
\end{aligned}$$

Mathematica [C] time = 0.0047646, size = 86, normalized size = 1.12

$$-\frac{bcd\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-c^2x^2\right)}{2x}+\frac{1}{2}ibe\text{PolyLog}(2,-icx)-\frac{1}{2}ibe\text{PolyLog}(2,icx)-\frac{ad}{2x^2}+ae\log(x)-\frac{bd\tan}{2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + e*x^2)*(a + b*ArcTan[c*x]))/x^3,x]
```

```
[Out] -(a*d)/(2*x^2) - (b*d*ArcTan[c*x])/(2*x^2) - (b*c*d*Hypergeometric2F1[-1/2,
 1, 1/2, -(c^2*x^2)])/(2*x) + a*e*Log[x] + (I/2)*b*e*PolyLog[2, (-I)*c*x] -
 (I/2)*b*e*PolyLog[2, I*c*x]
```

Maple [A] time = 0.053, size = 117, normalized size = 1.5

$$-\frac{ad}{2\,x^2}+ae\ln(cx)-\frac{\arctan(cx)\,bd}{2\,x^2}+b\arctan(cx)\,e\ln(cx)+\frac{i}{2}be\ln(cx)\ln(1+icx)-\frac{i}{2}be\ln(cx)\ln(1-icx)+\frac{i}{2}bedil$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(a+b*arctan(c*x))/x^3,x)
```

```
[Out] -1/2*a*d/x^2+a*e*ln(c*x)-1/2*b*arctan(c*x)*d/x^2+b*arctan(c*x)*e*ln(c*x)+1/
2*I*b*e*ln(c*x)*ln(1+I*c*x)-1/2*I*b*e*ln(c*x)*ln(1-I*c*x)+1/2*I*b*e*dilog(1
+I*c*x)-1/2*I*b*e*dilog(1-I*c*x)-1/2*b*c^2*d*arctan(c*x)-1/2*b*c*d/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2}\left(\left(c\arctan(cx)+\frac{1}{x}\right)c+\frac{\arctan(cx)}{x^2}\right)bd+be\int\frac{\arctan(cx)}{x}\,dx+ae\log(x)-\frac{ad}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arctan(c*x))/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*((c*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*b*d + b*e*integrate(arctan
(c*x)/x, x) + a*e*log(x) - 1/2*a*d/x^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{aex^2+ad+\left(bex^2+bd\right)\arctan(cx)}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arctan(c*x))/x^3,x, algorithm="fricas")
```

```
[Out] integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arctan(c*x))/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+b\,\mathrm{atan}\,(cx))\left(d+ex^2\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(a+b*atan(c*x))/x**3,x)
```

```
[Out] Integral((a + b*atan(c*x))*(d + e*x**2)/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2+d\right)(b\arctan(cx)+a)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arctan(c*x))/x^3,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*(b*arctan(c*x) + a)/x^3, x)
```

3.1121 $\int \frac{(d+ex^2)(a+b\tan^{-1}(cx))}{x^4}\,dx$

Optimal. Leaf size=83

$$-\frac{d\left(a+b\tan^{-1}(cx)\right)}{3x^3}-\frac{e\left(a+b\tan^{-1}(cx)\right)}{x}+\frac{1}{6}bc\left(c^2d-3e\right)\log\left(c^2x^2+1\right)-\frac{1}{3}bc\log(x)\left(c^2d-3e\right)-\frac{bcd}{6x^2}$$

```
[Out] -(b*c*d)/(6*x^2) - (d*(a + b*ArcTan[c*x]))/(3*x^3) - (e*(a + b*ArcTan[c*x])
)/x - (b*c*(c^2*d - 3*e)*Log[x])/3 + (b*c*(c^2*d - 3*e)*Log[1 + c^2*x^2])/6
```

Rubi [A] time = 0.119147, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {14, 4976, 12, 446, 77}

$$-\frac{d\left(a+b\tan^{-1}(cx)\right)}{3x^3}-\frac{e\left(a+b\tan^{-1}(cx)\right)}{x}+\frac{1}{6}bc\left(c^2d-3e\right)\log\left(c^2x^2+1\right)-\frac{1}{3}bc\log(x)\left(c^2d-3e\right)-\frac{bcd}{6x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)*(a + b*ArcTan[c*x]))/x^4,x]
```

```
[Out] -(b*c*d)/(6*x^2) - (d*(a + b*ArcTan[c*x]))/(3*x^3) - (e*(a + b*ArcTan[c*x])
)/x - (b*c*(c^2*d - 3*e)*Log[x])/3 + (b*c*(c^2*d - 3*e)*Log[1 + c^2*x^2])/6
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 4976

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Dis
t[a + b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&  !(
ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(IL
tQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILtQ[(m
 - 1)/2, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex^2)\left(a+b\tan^{-1}(cx)\right)}{x^4}\,dx &= -\frac{d\left(a+b\tan^{-1}(cx)\right)}{3x^3}-\frac{e\left(a+b\tan^{-1}(cx)\right)}{x}-(bc)\int \frac{-d-3ex^2}{3x^3\left(1+c^2x^2\right)}\,dx \\
&= -\frac{d\left(a+b\tan^{-1}(cx)\right)}{3x^3}-\frac{e\left(a+b\tan^{-1}(cx)\right)}{x}-\frac{1}{3}(bc)\int \frac{-d-3ex^2}{x^3\left(1+c^2x^2\right)}\,dx \\
&= -\frac{d\left(a+b\tan^{-1}(cx)\right)}{3x^3}-\frac{e\left(a+b\tan^{-1}(cx)\right)}{x}-\frac{1}{6}(bc)\,\mathrm{Subst}\left(\int \frac{-d-3ex}{x^2\left(1+c^2x\right)}\,dx,x,x^2\right) \\
&= -\frac{d\left(a+b\tan^{-1}(cx)\right)}{3x^3}-\frac{e\left(a+b\tan^{-1}(cx)\right)}{x}-\frac{1}{6}(bc)\,\mathrm{Subst}\left(\int \left(-\frac{d}{x^2}+\frac{c^2d-3e}{x}+\frac{-c
\end{aligned}$$

$$= -\frac{bcd}{6x^2}-\frac{d\left(a+b\tan^{-1}(cx)\right)}{3x^3}-\frac{e\left(a+b\tan^{-1}(cx)\right)}{x}-\frac{1}{3}bc\left(c^2d-3e\right)\log(x)+\frac{1}{6}bc\left(c^2d\right.$$

Mathematica [A] time = 0.0335666, size = 98, normalized size = 1.18

$$-\frac{ad}{3x^3}-\frac{ae}{x}+\frac{1}{6}bcd\left(c^2\log\left(c^2x^2+1\right)-2c^2\log(x)-\frac{1}{x^2}\right)-\frac{1}{2}bce\log\left(c^2x^2+1\right)-\frac{bd\tan^{-1}(cx)}{3x^3}+bce\log(x)-\frac{be\tan^{-1}(cx)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)*(a + b*ArcTan[c*x]))/x^4,x]
```

```
[Out] -(a*d)/(3*x^3) - (a*e)/x - (b*d*ArcTan[c*x])/(3*x^3) - (b*e*ArcTan[c*x])/x
+ b*c*e*Log[x] - (b*c*e*Log[1 + c^2*x^2])/2 + (b*c*d*(-x^(-2) - 2*c^2*Log[x
] + c^2*Log[1 + c^2*x^2]))/6
```

Maple [A] time = 0.046, size = 97, normalized size = 1.2

$$-\frac{ae}{x}-\frac{ad}{3\,x^3}-\frac{b\arctan(cx)\,e}{x}-\frac{\arctan(cx)\,bd}{3\,x^3}+\frac{c^3b\ln\left(c^2x^2+1\right)d}{6}-\frac{cb\ln\left(c^2x^2+1\right)e}{2}-\frac{c^3bd\ln(cx)}{3}+cb\ln(cx)\,e-\frac{b}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(a+b*arctan(c*x))/x^4,x)
```

```
[Out] -a*e/x-1/3*a*d/x^3-b*arctan(c*x)*e/x-1/3*b*arctan(c*x)*d/x^3+1/6*c^3*b*ln(c
^2*x^2+1)*d-1/2*c*b*ln(c^2*x^2+1)*e-1/3*c^3*b*d*ln(c*x)+c*b*ln(c*x)*e-1/6*b
*c*d/x^2
```

Maxima [A] time = 0.942301, size = 126, normalized size = 1.52

$$\frac{1}{6}\left(\left(c^2\log\left(c^2x^2+1\right)-c^2\log\left(x^2\right)-\frac{1}{x^2}\right)c-\frac{2\arctan(cx)}{x^3}\right)bd-\frac{1}{2}\left(c\left(\log\left(c^2x^2+1\right)-\log\left(x^2\right)\right)+\frac{2\arctan(cx)}{x}\right)be-\frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arctan(c*x))/x^4,x, algorithm="maxima")
```

```
[Out] 1/6*((c^2*log(c^2*x^2 + 1) - c^2*log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^3)*b
*d - 1/2*(c*(log(c^2*x^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*b*e - a*e/x -
1/3*a*d/x^3
```

Fricas [A] time = 1.72895, size = 204, normalized size = 2.46

$$\frac{\left(bc^3d-3\,bce\right)x^3\log\left(c^2x^2+1\right)-2\left(bc^3d-3\,bce\right)x^3\log(x)-bcdx-6\,aex^2-2\,ad-2\left(3\,bex^2+bd\right)\arctan(cx)}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arctan(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*((b*c^3*d - 3*b*c*e)*x^3*log(c^2*x^2 + 1) - 2*(b*c^3*d - 3*b*c*e)*x^3*l
og(x) - b*c*d*x - 6*a*e*x^2 - 2*a*d - 2*(3*b*e*x^2 + b*d)*arctan(c*x))/x^3
```

Sympy [A] time = 1.84981, size = 116, normalized size = 1.4

$$\begin{cases} -\frac{ad}{3x^3} - \frac{ae}{x} - \frac{bc^3d\log(x)}{3} + \frac{bc^3d\log\left(x^2+\frac{1}{c^2}\right)}{6} - \frac{bcd}{6x^2} + bce\log(x) - \frac{bce\log\left(x^2+\frac{1}{c^2}\right)}{2} - \frac{bd\,\mathrm{atan}(cx)}{3x^3} - \frac{be\,\mathrm{atan}(cx)}{x} & \text{for } c \neq 0 \\ a\left(-\frac{d}{3x^3} - \frac{e}{x}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(a+b*atan(c*x))/x**4,x)
```

```
[Out] Piecewise((-a*d/(3*x**3) - a*e/x - b*c**3*d*log(x)/3 + b*c**3*d*log(x**2 +
c**(-2))/6 - b*c*d/(6*x**2) + b*c*e*log(x) - b*c*e*log(x**2 + c**(-2))/2 -
b*d*atan(c*x)/(3*x**3) - b*e*atan(c*x)/x, Ne(c, 0)), (a*(-d/(3*x**3) - e/x)
, True))
```

Giac [A] time = 1.09708, size = 142, normalized size = 1.71

$$\frac{bc^3dx^3\log\left(c^2x^2+1\right) - 2\,bc^3dx^3\log(x) - 3\,bcx^3e\log\left(c^2x^2+1\right) + 6\,bcx^3e\log(x) - 6\,bx^2\arctan(cx)\,e - bcdx - 6\,ax^2e - 2}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arctan(c*x))/x^4,x, algorithm="giac")
```

```
[Out] 1/6*(b*c^3*d*x^3*log(c^2*x^2 + 1) - 2*b*c^3*d*x^3*log(x) - 3*b*c*x^3*e*log(
c^2*x^2 + 1) + 6*b*c*x^3*e*log(x) - 6*b*x^2*arctan(c*x)*e - b*c*d*x - 6*a*x
^2*e - 2*b*d*arctan(c*x) - 2*a*d)/x^3
```

3.1122 $\int \frac{(d+ex^2)(a+b\tan^{-1}(cx))}{x^5}\,dx$

Optimal. Leaf size=82

$$-\frac{d\left(a+b\tan^{-1}(cx)\right)}{4x^4}-\frac{e\left(a+b\tan^{-1}(cx)\right)}{2x^2}+\frac{bc\left(c^2d-2e\right)}{4x}+\frac{1}{4}bc^2\left(c^2d-2e\right)\tan^{-1}(cx)-\frac{bcd}{12x^3}$$

```
[Out] -(b*c*d)/(12*x^3) + (b*c*(c^2*d - 2*e))/(4*x) + (b*c^2*(c^2*d - 2*e)*ArcTan
[c*x])/4 - (d*(a + b*ArcTan[c*x]))/(4*x^4) - (e*(a + b*ArcTan[c*x]))/(2*x^2
)
```

Rubi [A] time = 0.0904979, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {14, 4976, 12, 453, 325, 203}

$$-\frac{d\left(a+b\tan^{-1}(cx)\right)}{4x^4}-\frac{e\left(a+b\tan^{-1}(cx)\right)}{2x^2}+\frac{bc\left(c^2d-2e\right)}{4x}+\frac{1}{4}bc^2\left(c^2d-2e\right)\tan^{-1}(cx)-\frac{bcd}{12x^3}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)*(a + b*ArcTan[c*x]))/x^5,x]
```

```
[Out] -(b*c*d)/(12*x^3) + (b*c*(c^2*d - 2*e))/(4*x) + (b*c^2*(c^2*d - 2*e)*ArcTan
[c*x])/4 - (d*(a + b*ArcTan[c*x]))/(4*x^4) - (e*(a + b*ArcTan[c*x]))/(2*x^2
)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 4976

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Dis
t[a + b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&  !(
ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(IL
tQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILtQ[(m
```

```
- 1)/2, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{(d+ex^2)(a+b\tan^{-1}(cx))}{x^5}\,dx = -\frac{d(a+b\tan^{-1}(cx))}{4x^4} - \frac{e(a+b\tan^{-1}(cx))}{2x^2} - (bc)\int \frac{-d-2ex^2}{4x^4(1+c^2x^2)}\,dx$$

$$= -\frac{d(a+b\tan^{-1}(cx))}{4x^4} - \frac{e(a+b\tan^{-1}(cx))}{2x^2} - \frac{1}{4}(bc)\int \frac{-d-2ex^2}{x^4(1+c^2x^2)}\,dx$$

$$= -\frac{bcd}{12x^3} - \frac{d(a+b\tan^{-1}(cx))}{4x^4} - \frac{e(a+b\tan^{-1}(cx))}{2x^2} - \frac{1}{4}\left(bc\left(c^2d-2e\right)\right)\int \frac{1}{x^2(1+c^2$$

$$= -\frac{bcd}{12x^3} + \frac{bc(c^2d-2e)}{4x} - \frac{d(a+b\tan^{-1}(cx))}{4x^4} - \frac{e(a+b\tan^{-1}(cx))}{2x^2} + \frac{1}{4}\left(bc^3\left(c^2d-\right.\right.$$

$$= -\frac{bcd}{12x^3} + \frac{bc(c^2d-2e)}{4x} + \frac{1}{4}bc^2\left(c^2d-2e\right)\tan^{-1}(cx) - \frac{d(a+b\tan^{-1}(cx))}{4x^4} - \frac{e(a+$$

Mathematica [C] time = 0.0052514, size = 97, normalized size = 1.18

$$-\frac{bcd\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-c^2x^2\right)}{12x^3} - \frac{bce\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-c^2x^2\right)}{2x} - \frac{ad}{4x^4} - \frac{ae}{2x^2} - \frac{bd\tan^{-1}(c}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)*(a + b*ArcTan[c*x]))/x^5,x]
```

```
[Out] -(a*d)/(4*x^4) - (a*e)/(2*x^2) - (b*d*ArcTan[c*x])/(4*x^4) - (b*e*ArcTan[c*
x])/(2*x^2) - (b*c*d*Hypergeometric2F1[-3/2, 1, -1/2, -(c^2*x^2)])/(12*x^3)
 - (b*c*e*Hypergeometric2F1[-1/2, 1, 1/2, -(c^2*x^2)])/(2*x)
```

Maple [A] time = 0.044, size = 86, normalized size = 1.1

$$-\frac{ae}{2x^2} - \frac{ad}{4x^4} - \frac{b\arctan(cx)e}{2x^2} - \frac{\arctan(cx)bd}{4x^4} + \frac{c^4b\arctan(cx)d}{4} - \frac{bc^2e\arctan(cx)}{2} + \frac{bc^3d}{4x} - \frac{bce}{2x} - \frac{bcd}{12x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(a+b*arctan(c*x))/x^5,x)
```

```
[Out] -1/2*a*e/x^2-1/4*a*d/x^4-1/2*b*arctan(c*x)*e/x^2-1/4*b*arctan(c*x)*d/x^4+1/
4*c^4*b*arctan(c*x)*d-1/2*b*c^2*e*arctan(c*x)+1/4*b*c^3*d/x-1/2*c*b*e/x-1/1
```

```
2*b*c*d/x^3
```

Maxima [A] time = 1.43652, size = 108, normalized size = 1.32

$$\frac{1}{12}\left(\left(3c^3\arctan(cx)+\frac{3c^2x^2-1}{x^3}\right)c-\frac{3\arctan(cx)}{x^4}\right)bd-\frac{1}{2}\left(\left(c\arctan(cx)+\frac{1}{x}\right)c+\frac{\arctan(cx)}{x^2}\right)be-\frac{ae}{2x^2}-\frac{ad}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arctan(c*x))/x^5,x, algorithm="maxima")
```

```
[Out] 1/12*((3*c^3*arctan(c*x) + (3*c^2*x^2 - 1)/x^3)*c - 3*arctan(c*x)/x^4)*b*d
- 1/2*((c*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*b*e - 1/2*a*e/x^2 - 1/4*a
*d/x^4
```

Fricas [A] time = 1.68077, size = 177, normalized size = 2.16

$$-\frac{bcdx+6aex^2-3(bc^3d-2bce)x^3+3ad-3((bc^4d-2bc^2e)x^4-2bex^2-bd)\arctan(cx)}{12x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arctan(c*x))/x^5,x, algorithm="fricas")
```

```
[Out] -1/12*(b*c*d*x + 6*a*e*x^2 - 3*(b*c^3*d - 2*b*c*e)*x^3 + 3*a*d - 3*((b*c^4*
d - 2*b*c^2*e)*x^4 - 2*b*e*x^2 - b*d)*arctan(c*x))/x^4
```

Sympy [A] time = 1.49037, size = 99, normalized size = 1.21

$$-\frac{ad}{4x^4}-\frac{ae}{2x^2}+\frac{bc^4d\operatorname{atan}(cx)}{4}+\frac{bc^3d}{4x}-\frac{bc^2e\operatorname{atan}(cx)}{2}-\frac{bcd}{12x^3}-\frac{bce}{2x}-\frac{bd\operatorname{atan}(cx)}{4x^4}-\frac{be\operatorname{atan}(cx)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(a+b*atan(c*x))/x**5,x)
```

```
[Out] -a*d/(4*x**4) - a*e/(2*x**2) + b*c**4*d*atan(c*x)/4 + b*c**3*d/(4*x) - b*c*
*2*e*atan(c*x)/2 - b*c*d/(12*x**3) - b*c*e/(2*x) - b*d*atan(c*x)/(4*x**4) -
```

```
b*e*atan(c*x)/(2*x**2)
```

Giac [A] time = 1.16072, size = 143, normalized size = 1.74

$$-\frac{3\,\pi bc^4dx^4\operatorname{sgn}(c)\operatorname{sgn}(x)-3\,bc^4dx^4\arctan(cx)+6\,bc^2x^4\arctan(cx)\,e-3\,bc^3dx^3+6\,bcx^3e+6\,bx^2\arctan(cx)\,e+bc}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arctan(c*x))/x^5,x, algorithm="giac")
```

```
[Out] -1/12*(3*pi*b*c^4*d*x^4*sgn(c)*sgn(x) - 3*b*c^4*d*x^4*arctan(c*x) + 6*b*c^2
*x^4*arctan(c*x)*e - 3*b*c^3*d*x^3 + 6*b*c*x^3*e + 6*b*x^2*arctan(c*x)*e +
b*c*d*x + 6*a*x^2*e + 3*b*d*arctan(c*x) + 3*a*d)/x^4
```

3.1123 $\int \frac{(d+ex^2)(a+b\tan^{-1}(cx))}{x^6}\,dx$

Optimal. Leaf size=110

$$-\frac{d\left(a+b\tan^{-1}(cx)\right)}{5x^5}-\frac{e\left(a+b\tan^{-1}(cx)\right)}{3x^3}+\frac{bc\left(3c^2d-5e\right)}{30x^2}-\frac{1}{30}bc^3\left(3c^2d-5e\right)\log\left(c^2x^2+1\right)+\frac{1}{15}bc^3\log(x)\left(3c^2d-5e\right)$$

```
[Out] -(b*c*d)/(20*x^4) + (b*c*(3*c^2*d - 5*e))/(30*x^2) - (d*(a + b*ArcTan[c*x])
)/(5*x^5) - (e*(a + b*ArcTan[c*x]))/(3*x^3) + (b*c^3*(3*c^2*d - 5*e)*Log[x]
)/15 - (b*c^3*(3*c^2*d - 5*e)*Log[1 + c^2*x^2])/30
```

Rubi [A] time = 0.128825, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {14, 4976, 12, 446, 77}

$$-\frac{d\left(a+b\tan^{-1}(cx)\right)}{5x^5}-\frac{e\left(a+b\tan^{-1}(cx)\right)}{3x^3}+\frac{bc\left(3c^2d-5e\right)}{30x^2}-\frac{1}{30}bc^3\left(3c^2d-5e\right)\log\left(c^2x^2+1\right)+\frac{1}{15}bc^3\log(x)\left(3c^2d-5e\right)$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)*(a + b*ArcTan[c*x]))/x^6,x]
```

```
[Out] -(b*c*d)/(20*x^4) + (b*c*(3*c^2*d - 5*e))/(30*x^2) - (d*(a + b*ArcTan[c*x])
)/(5*x^5) - (e*(a + b*ArcTan[c*x]))/(3*x^3) + (b*c^3*(3*c^2*d - 5*e)*Log[x]
)/15 - (b*c^3*(3*c^2*d - 5*e)*Log[1 + c^2*x^2])/30
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 4976

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Dis
t[a + b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&  !(
ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(IL
tQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILtQ[(m
```

```
 - 1)/2, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)\left(a+b\tan^{-1}(cx)\right)}{x^6}\,dx = -\frac{d\left(a+b\tan^{-1}(cx)\right)}{5x^5}-\frac{e\left(a+b\tan^{-1}(cx)\right)}{3x^3}-(bc)\int \frac{-3d-5ex^2}{15x^5\left(1+c^2x^2\right)}\,dx$$

$$= -\frac{d\left(a+b\tan^{-1}(cx)\right)}{5x^5}-\frac{e\left(a+b\tan^{-1}(cx)\right)}{3x^3}-\frac{1}{15}(bc)\int \frac{-3d-5ex^2}{x^5\left(1+c^2x^2\right)}\,dx$$

$$= -\frac{d\left(a+b\tan^{-1}(cx)\right)}{5x^5}-\frac{e\left(a+b\tan^{-1}(cx)\right)}{3x^3}-\frac{1}{30}(bc)\,\mathrm{Subst}\left(\int \frac{-3d-5ex}{x^3\left(1+c^2x\right)}\,dx, x, x$$

$$= -\frac{d\left(a+b\tan^{-1}(cx)\right)}{5x^5}-\frac{e\left(a+b\tan^{-1}(cx)\right)}{3x^3}-\frac{1}{30}(bc)\,\mathrm{Subst}\left(\int\left(-\frac{3d}{x^3}+\frac{3c^2d-5e}{x^2}\right.\right.$$

$$= -\frac{bcd}{20x^4}+\frac{bc\left(3c^2d-5e\right)}{30x^2}-\frac{d\left(a+b\tan^{-1}(cx)\right)}{5x^5}-\frac{e\left(a+b\tan^{-1}(cx)\right)}{3x^3}+\frac{1}{15}bc^3\left(3c^2d\right.$$

Mathematica [A] time = 0.0422192, size = 123, normalized size = 1.12

$$-\frac{ad}{5x^5}-\frac{ae}{3x^3}+\frac{1}{10}bcd\left(\frac{c^2}{x^2}-c^4\log\left(c^2x^2+1\right)+2c^4\log(x)-\frac{1}{2x^4}\right)+\frac{1}{6}bce\left(c^2\log\left(c^2x^2+1\right)-2c^2\log(x)-\frac{1}{x^2}\right)-\frac{bd\tan}{5x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)*(a + b*ArcTan[c*x]))/x^6,x]
```

```
[Out] -(a*d)/(5*x^5) - (a*e)/(3*x^3) - (b*d*ArcTan[c*x])/(5*x^5) - (b*e*ArcTan[c*
x])/(3*x^3) + (b*c*e*(-x^(-2) - 2*c^2*Log[x] + c^2*Log[1 + c^2*x^2]))/6 + (
b*c*d*(-1/(2*x^4) + c^2/x^2 + 2*c^4*Log[x] - c^4*Log[1 + c^2*x^2]))/10
```

Maple [A] time = 0.044, size = 120, normalized size = 1.1

$$-\frac{ad}{5\,x^5}-\frac{ae}{3\,x^3}-\frac{\arctan(cx)\,bd}{5\,x^5}-\frac{b\arctan(cx)\,e}{3\,x^3}-\frac{c^5b\ln\left(c^2x^2+1\right)d}{10}+\frac{bc^3e\ln\left(c^2x^2+1\right)}{6}+\frac{c^5bd\ln(cx)}{5}-\frac{c^3b\ln(cx)\,e}{3}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(a+b*arctan(c*x))/x^6,x)
```

```
[Out] -1/5*a*d/x^5-1/3*a*e/x^3-1/5*b*arctan(c*x)*d/x^5-1/3*b*arctan(c*x)*e/x^3-1/
10*c^5*b*ln(c^2*x^2+1)*d+1/6*b*c^3*e*ln(c^2*x^2+1)+1/5*c^5*b*d*ln(c*x)-1/3*
c^3*b*ln(c*x)*e+1/10*c^3*b*d/x^2-1/6*c*b*e/x^2-1/20*b*c*d/x^4
```

Maxima [A] time = 0.955741, size = 157, normalized size = 1.43

$$-\frac{1}{20}\left(\left(2\,c^4\log\left(c^2x^2+1\right)-2\,c^4\log\left(x^2\right)-\frac{2\,c^2x^2-1}{x^4}\right)c+\frac{4\,\arctan(cx)}{x^5}\right)bd+\frac{1}{6}\left(\left(c^2\log\left(c^2x^2+1\right)-c^2\log\left(x^2\right)-\frac{1}{x^2}\right)c-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arctan(c*x))/x^6,x, algorithm="maxima")
```

```
[Out] -1/20*((2*c^4*log(c^2*x^2 + 1) - 2*c^4*log(x^2) - (2*c^2*x^2 - 1)/x^4)*c +
4*arctan(c*x)/x^5)*b*d + 1/6*((c^2*log(c^2*x^2 + 1) - c^2*log(x^2) - 1/x^2)
*c - 2*arctan(c*x)/x^3)*b*e - 1/3*a*e/x^3 - 1/5*a*d/x^5
```

Fricas [A] time = 1.85409, size = 269, normalized size = 2.45

$$-\frac{2\left(3\,bc^5d-5\,bc^3e\right)x^5\log\left(c^2x^2+1\right)-4\left(3\,bc^5d-5\,bc^3e\right)x^5\log(x)+3\,bcdx+20\,aex^2-2\left(3\,bc^3d-5\,bce\right)x^3+12\,ad+4\left(\right.}{60\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arctan(c*x))/x^6,x, algorithm="fricas")
```

```
[Out] -1/60*(2*(3*b*c^5*d - 5*b*c^3*e)*x^5*log(c^2*x^2 + 1) - 4*(3*b*c^5*d - 5*b*
c^3*e)*x^5*log(x) + 3*b*c*d*x + 20*a*e*x^2 - 2*(3*b*c^3*d - 5*b*c*e)*x^3 +
12*a*d + 4*(5*b*e*x^2 + 3*b*d)*arctan(c*x))/x^5
```

Sympy [A] time = 3.17923, size = 153, normalized size = 1.39

$$\begin{cases} -\frac{ad}{5x^5} - \frac{ae}{3x^3} + \frac{bc^5 d \log(x)}{5} - \frac{bc^5 d \log\left(x^2 + \frac{1}{c^2}\right)}{10} + \frac{bc^3 d}{10x^2} - \frac{bc^3 e \log(x)}{3} + \frac{bc^3 e \log\left(x^2 + \frac{1}{c^2}\right)}{6} - \frac{bcd}{20x^4} - \frac{bce}{6x^2} - \frac{bd \operatorname{atan}(cx)}{5x^5} - \frac{be \operatorname{atan}(cx)}{3x^3} & \text{for } c \\ a\left(-\frac{d}{5x^5} - \frac{e}{3x^3}\right) & \text{other} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(a+b*atan(c*x))/x**6,x)
```

```
[Out] Piecewise((-a*d/(5*x**5) - a*e/(3*x**3) + b*c**5*d*log(x)/5 - b*c**5*d*log(
x**2 + c**(-2))/10 + b*c**3*d/(10*x**2) - b*c**3*e*log(x)/3 + b*c**3*e*log(
x**2 + c**(-2))/6 - b*c*d/(20*x**4) - b*c*e/(6*x**2) - b*d*atan(c*x)/(5*x**
5) - b*e*atan(c*x)/(3*x**3), Ne(c, 0)), (a*(-d/(5*x**5) - e/(3*x**3)), True
))
```

Giac [A] time = 1.10962, size = 174, normalized size = 1.58

$$-\frac{6\,bc^5dx^5\log\left(c^2x^2+1\right) - 12\,bc^5dx^5\log(x) - 10\,bc^3x^5e\log\left(c^2x^2+1\right) + 20\,bc^3x^5e\log(x) - 6\,bc^3dx^3 + 10\,bcx^3e + 20\,bx^2}{60\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arctan(c*x))/x^6,x, algorithm="giac")
```

```
[Out] -1/60*(6*b*c^5*d*x^5*log(c^2*x^2 + 1) - 12*b*c^5*d*x^5*log(x) - 10*b*c^3*x^
5*e*log(c^2*x^2 + 1) + 20*b*c^3*x^5*e*log(x) - 6*b*c^3*d*x^3 + 10*b*c*x^3*e
 + 20*b*x^2*arctan(c*x)*e + 3*b*c*d*x + 20*a*x^2*e + 12*b*d*arctan(c*x) + 1
2*a*d)/x^5
```

3.1124 $\int \frac{(d+ex^2)(a+b\tan^{-1}(cx))}{x^7}\,dx$

Optimal. Leaf size=105

$$-\frac{d\left(a+b\tan^{-1}(cx)\right)}{6x^6}-\frac{e\left(a+b\tan^{-1}(cx)\right)}{4x^4}+\frac{bc\left(2c^2d-3e\right)}{36x^3}-\frac{bc^3\left(2c^2d-3e\right)}{12x}-\frac{1}{12}bc^4\left(2c^2d-3e\right)\tan^{-1}(cx)-\frac{bcd}{30x^5}$$

```
[Out] -(b*c*d)/(30*x^5) + (b*c*(2*c^2*d - 3*e))/(36*x^3) - (b*c^3*(2*c^2*d - 3*e)
)/(12*x) - (b*c^4*(2*c^2*d - 3*e)*ArcTan[c*x])/12 - (d*(a + b*ArcTan[c*x]))
/(6*x^6) - (e*(a + b*ArcTan[c*x]))/(4*x^4)
```

Rubi [A] time = 0.109091, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {14, 4976, 12, 453, 325, 203}

$$-\frac{d\left(a+b\tan^{-1}(cx)\right)}{6x^6}-\frac{e\left(a+b\tan^{-1}(cx)\right)}{4x^4}+\frac{bc\left(2c^2d-3e\right)}{36x^3}-\frac{bc^3\left(2c^2d-3e\right)}{12x}-\frac{1}{12}bc^4\left(2c^2d-3e\right)\tan^{-1}(cx)-\frac{bcd}{30x^5}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)*(a + b*ArcTan[c*x]))/x^7,x]
```

```
[Out] -(b*c*d)/(30*x^5) + (b*c*(2*c^2*d - 3*e))/(36*x^3) - (b*c^3*(2*c^2*d - 3*e)
)/(12*x) - (b*c^4*(2*c^2*d - 3*e)*ArcTan[c*x])/12 - (d*(a + b*ArcTan[c*x]))
/(6*x^6) - (e*(a + b*ArcTan[c*x]))/(4*x^4)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 4976

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Dis
t[a + b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&  !(
ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(IL
tQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILtQ[(m
```

```
- 1)/2, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{(d+ex^2)(a+b\tan^{-1}(cx))}{x^7}\,dx = -\frac{d(a+b\tan^{-1}(cx))}{6x^6} - \frac{e(a+b\tan^{-1}(cx))}{4x^4} - (bc)\int \frac{-2d-3ex^2}{12x^6(1+c^2x^2)}\,dx$$

$$= -\frac{d(a+b\tan^{-1}(cx))}{6x^6} - \frac{e(a+b\tan^{-1}(cx))}{4x^4} - \frac{1}{12}(bc)\int \frac{-2d-3ex^2}{x^6(1+c^2x^2)}\,dx$$

$$= -\frac{bcd}{30x^5} - \frac{d(a+b\tan^{-1}(cx))}{6x^6} - \frac{e(a+b\tan^{-1}(cx))}{4x^4} - \frac{1}{12}\left(bc\left(2c^2d-3e\right)\right)\int \frac{1}{x^4(1+c$$

$$= -\frac{bcd}{30x^5} + \frac{bc(2c^2d-3e)}{36x^3} - \frac{d(a+b\tan^{-1}(cx))}{6x^6} - \frac{e(a+b\tan^{-1}(cx))}{4x^4} + \frac{1}{12}\left(bc^3\left(2c^2d$$

$$= -\frac{bcd}{30x^5} + \frac{bc(2c^2d-3e)}{36x^3} - \frac{bc^3(2c^2d-3e)}{12x} - \frac{d(a+b\tan^{-1}(cx))}{6x^6} - \frac{e(a+b\tan^{-1}(cx)}{4x^4}$$

$$= -\frac{bcd}{30x^5} + \frac{bc(2c^2d-3e)}{36x^3} - \frac{bc^3(2c^2d-3e)}{12x} - \frac{1}{12}bc^4\left(2c^2d-3e\right)\tan^{-1}(cx) - \frac{d(a+b\,t}{6}$$

Mathematica [C] time = 0.0051924, size = 97, normalized size = 0.92

$$-\frac{bcd\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},-c^2x^2\right)}{30x^5} - \frac{bce\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-c^2x^2\right)}{12x^3} - \frac{ad}{6x^6} - \frac{ae}{4x^4} - \frac{bd\tan^{-1}(c}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)*(a + b*ArcTan[c*x]))/x^7,x]
```

```
[Out] -(a*d)/(6*x^6) - (a*e)/(4*x^4) - (b*d*ArcTan[c*x])/(6*x^6) - (b*e*ArcTan[c*
x])/(4*x^4) - (b*c*d*Hypergeometric2F1[-5/2, 1, -3/2, -(c^2*x^2)])/(30*x^5)
 - (b*c*e*Hypergeometric2F1[-3/2, 1, -1/2, -(c^2*x^2)])/(12*x^3)
```

Maple [A] time = 0.044, size = 106, normalized size = 1.

$$-\frac{ae}{4\,x^4} - \frac{ad}{6\,x^6} - \frac{b\arctan(cx)\,e}{4\,x^4} - \frac{\arctan(cx)\,bd}{6\,x^6} - \frac{c^6b\arctan(cx)\,d}{6} + \frac{bc^4e\arctan(cx)}{4} - \frac{c^5bd}{6\,x} + \frac{bc^3e}{4\,x} + \frac{bc^3d}{18\,x^3} - \frac{bce}{12\,x^3} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(a+b*arctan(c*x))/x^7,x)
```

```
[Out] -1/4*a*e/x^4-1/6*a*d/x^6-1/4*b*arctan(c*x)*e/x^4-1/6*b*arctan(c*x)*d/x^6-1/
6*c^6*b*arctan(c*x)*d+1/4*b*c^4*e*arctan(c*x)-1/6*c^5*b*d/x+1/4*b*c^3*e/x+1
/18*c^3*b*d/x^3-1/12*c*b*e/x^3-1/30*b*c*d/x^5
```

Maxima [A] time = 1.45785, size = 139, normalized size = 1.32

$$-\frac{1}{90}\left(\left(15\,c^5\arctan\left(cx\right)+\frac{15\,c^4x^4-5\,c^2x^2+3}{x^5}\right)c+\frac{15\,\arctan\left(cx\right)}{x^6}\right)bd+\frac{1}{12}\left(\left(3\,c^3\arctan\left(cx\right)+\frac{3\,c^2x^2-1}{x^3}\right)c-\frac{3\,\mathrm{arct}}{x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arctan(c*x))/x^7,x, algorithm="maxima")
```

```
[Out] -1/90*((15*c^5*arctan(c*x) + (15*c^4*x^4 - 5*c^2*x^2 + 3)/x^5)*c + 15*arcta
n(c*x)/x^6)*b*d + 1/12*((3*c^3*arctan(c*x) + (3*c^2*x^2 - 1)/x^3)*c - 3*arc
tan(c*x)/x^4)*b*e - 1/4*a*e/x^4 - 1/6*a*d/x^6
```

Fricas [A] time = 1.76981, size = 238, normalized size = 2.27

$$-\frac{15\left(2\,bc^5d-3\,bc^3e\right)x^5+6\,bcdx+45\,aex^2-5\left(2\,bc^3d-3\,bce\right)x^3+30\,ad+15\left(\left(2\,bc^6d-3\,bc^4e\right)x^6+3\,bex^2+2\,bd\right)\mathrm{arct}}{180\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arctan(c*x))/x^7,x, algorithm="fricas")
```

```
[Out] -1/180*(15*(2*b*c^5*d - 3*b*c^3*e)*x^5 + 6*b*c*d*x + 45*a*e*x^2 - 5*(2*b*c^
3*d - 3*b*c*e)*x^3 + 30*a*d + 15*((2*b*c^6*d - 3*b*c^4*e)*x^6 + 3*b*e*x^2 +
 2*b*d)*arctan(c*x))/x^6
```

Sympy [A] time = 2.25256, size = 122, normalized size = 1.16

$$-\frac{ad}{6x^6}-\frac{ae}{4x^4}-\frac{bc^6d\,\mathrm{atan}\left(cx\right)}{6}-\frac{bc^5d}{6x}+\frac{bc^4e\,\mathrm{atan}\left(cx\right)}{4}+\frac{bc^3d}{18x^3}+\frac{bc^3e}{4x}-\frac{bcd}{30x^5}-\frac{bce}{12x^3}-\frac{bd\,\mathrm{atan}\left(cx\right)}{6x^6}-\frac{be\,\mathrm{atan}\left(cx\right)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(a+b*atan(c*x))/x**7,x)
```

```
[Out] -a*d/(6*x**6) - a*e/(4*x**4) - b*c**6*d*atan(c*x)/6 - b*c**5*d/(6*x) + b*c*
*4*e*atan(c*x)/4 + b*c**3*d/(18*x**3) + b*c**3*e/(4*x) - b*c*d/(30*x**5) -
b*c*e/(12*x**3) - b*d*atan(c*x)/(6*x**6) - b*e*atan(c*x)/(4*x**4)
```

Giac [A] time = 1.19593, size = 174, normalized size = 1.66

$$-\frac{30\,bc^6dx^6\arctan(cx)+45\,\pi bc^4x^6e\operatorname{sgn}(c)\operatorname{sgn}(x)-45\,bc^4x^6\arctan(cx)\,e+30\,bc^5dx^5-45\,bc^3x^5e-10\,bc^3dx^3+15\,bcx}{180\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arctan(c*x))/x^7,x, algorithm="giac")
```

```
[Out] -1/180*(30*b*c^6*d*x^6*arctan(c*x) + 45*pi*b*c^4*x^6*e*sgn(c)*sgn(x) - 45*b
*c^4*x^6*arctan(c*x)*e + 30*b*c^5*d*x^5 - 45*b*c^3*x^5*e - 10*b*c^3*d*x^3 +
 15*b*c*x^3*e + 45*b*x^2*arctan(c*x)*e + 6*b*c*d*x + 45*a*x^2*e + 30*b*d*ar
ctan(c*x) + 30*a*d)/x^6
```

3.1125 $\int x^3 \left(d+ex^2\right)^2 \left(a+b\tan^{-1}(cx)\right)\,dx$

Optimal. Leaf size=185

$$\frac{1}{4}d^2x^4\left(a+b\tan^{-1}(cx)\right)+\frac{1}{3}dex^6\left(a+b\tan^{-1}(cx)\right)+\frac{1}{8}e^2x^8\left(a+b\tan^{-1}(cx)\right)-\frac{bx^3\left(6c^4d^2-8c^2de+3e^2\right)}{72c^5}+\frac{bx\left(6c^4d^2\right.}{2}$$

```
[Out] (b*(6*c^4*d^2 - 8*c^2*d*e + 3*e^2)*x)/(24*c^7) - (b*(6*c^4*d^2 - 8*c^2*d*e
+ 3*e^2)*x^3)/(72*c^5) - (b*(8*c^2*d - 3*e)*e*x^5)/(120*c^3) - (b*e^2*x^7)/
(56*c) - (b*(6*c^4*d^2 - 8*c^2*d*e + 3*e^2)*ArcTan[c*x])/(24*c^8) + (d^2*x^
4*(a + b*ArcTan[c*x]))/4 + (d*e*x^6*(a + b*ArcTan[c*x]))/3 + (e^2*x^8*(a +
b*ArcTan[c*x]))/8
```

Rubi [A] time = 0.193197, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {266, 43, 4976, 1261, 203}

$$\frac{1}{4}d^2x^4\left(a+b\tan^{-1}(cx)\right)+\frac{1}{3}dex^6\left(a+b\tan^{-1}(cx)\right)+\frac{1}{8}e^2x^8\left(a+b\tan^{-1}(cx)\right)-\frac{bx^3\left(6c^4d^2-8c^2de+3e^2\right)}{72c^5}+\frac{bx\left(6c^4d^2\right.}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + e*x^2)^2*(a + b*ArcTan[c*x]),x]
```

```
[Out] (b*(6*c^4*d^2 - 8*c^2*d*e + 3*e^2)*x)/(24*c^7) - (b*(6*c^4*d^2 - 8*c^2*d*e
+ 3*e^2)*x^3)/(72*c^5) - (b*(8*c^2*d - 3*e)*e*x^5)/(120*c^3) - (b*e^2*x^7)/
(56*c) - (b*(6*c^4*d^2 - 8*c^2*d*e + 3*e^2)*ArcTan[c*x])/(24*c^8) + (d^2*x^
4*(a + b*ArcTan[c*x]))/4 + (d*e*x^6*(a + b*ArcTan[c*x]))/3 + (e^2*x^8*(a +
b*ArcTan[c*x]))/8
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 4976

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Dis
t[a + b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&  !(
ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(IL
tQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILtQ[(m
 - 1)/2, 0]))
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int x^3\left(d+ex^2\right)^2\left(a+b\tan^{-1}(cx)\right)dx=\frac{1}{4}d^2x^4\left(a+b\tan^{-1}(cx)\right)+\frac{1}{3}dex^6\left(a+b\tan^{-1}(cx)\right)+\frac{1}{8}e^2x^8\left(a+b\tan^{-1}(cx)\right)-($$

$$=\frac{1}{4}d^2x^4\left(a+b\tan^{-1}(cx)\right)+\frac{1}{3}dex^6\left(a+b\tan^{-1}(cx)\right)+\frac{1}{8}e^2x^8\left(a+b\tan^{-1}(cx)\right)-($$

$$=\frac{b\left(6c^4d^2-8c^2de+3e^2\right)x}{24c^7}-\frac{b\left(6c^4d^2-8c^2de+3e^2\right)x^3}{72c^5}-\frac{b\left(8c^2d-3e\right)ex^5}{120c^3}-\frac{be^2x}{56c}$$

$$=\frac{b\left(6c^4d^2-8c^2de+3e^2\right)x}{24c^7}-\frac{b\left(6c^4d^2-8c^2de+3e^2\right)x^3}{72c^5}-\frac{b\left(8c^2d-3e\right)ex^5}{120c^3}-\frac{be^2x}{56c}$$

Mathematica [A] time = 0.139875, size = 174, normalized size = 0.94

$$\frac{105ac^8x^4\left(6d^2+8dex^2+3e^2x^4\right)+bcx\left(-3c^6\left(70d^2x^2+56dex^4+15e^2x^6\right)+7c^4\left(90d^2+40dex^2+9e^2x^4\right)-105c^2e\left(8d+e\right.\right.}{2520c^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + e*x^2)^2*(a + b*ArcTan[c*x]),x]
```

```
[Out] (105*a*c^8*x^4*(6*d^2 + 8*d*e*x^2 + 3*e^2*x^4) + b*c*x*(315*e^2 - 105*c^2*e
*(8*d + e*x^2) + 7*c^4*(90*d^2 + 40*d*e*x^2 + 9*e^2*x^4) - 3*c^6*(70*d^2*x^
2 + 56*d*e*x^4 + 15*e^2*x^6)) + 105*b*(-6*c^4*d^2 + 8*c^2*d*e - 3*e^2 + c^8
*(6*d^2*x^4 + 8*d*e*x^6 + 3*e^2*x^8))*ArcTan[c*x])/(2520*c^8)
```

Maple [A] time = 0.038, size = 203, normalized size = 1.1

$$\frac{ae^2x^8}{8}+\frac{aedx^6}{3}+\frac{ax^4d^2}{4}+\frac{b\arctan(cx)e^2x^8}{8}+\frac{b\arctan(cx)edx^6}{3}+\frac{b\arctan(cx)d^2x^4}{4}-\frac{be^2x^7}{56c}-\frac{bedx^5}{15c}-\frac{bd^2x^3}{12c}+\frac{bx}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x^2+d)^2*(a+b*arctan(c*x)),x)
```

```
[Out] 1/8*a*e^2*x^8+1/3*a*e*d*x^6+1/4*a*x^4*d^2+1/8*b*arctan(c*x)*e^2*x^8+1/3*b*a
rctan(c*x)*e*d*x^6+1/4*b*arctan(c*x)*d^2*x^4-1/56*b*e^2*x^7/c-1/15/c*b*e*d*
x^5-1/12*b*d^2*x^3/c+1/40/c^3*b*x^5*e^2+1/9/c^3*b*x^3*d*e+1/4*b*d^2*x/c^3-1
/24/c^5*b*x^3*e^2-1/3/c^5*b*e*d*x+1/8/c^7*b*x*e^2-1/4*b*d^2*arctan(c*x)/c^4
+1/3/c^6*b*arctan(c*x)*e*d-1/8/c^8*b*arctan(c*x)*e^2
```

Maxima [A] time = 1.47338, size = 248, normalized size = 1.34

$$\frac{1}{8}ae^2x^8+\frac{1}{3}adex^6+\frac{1}{4}ad^2x^4+\frac{1}{12}\left(3x^4\arctan(cx)-c\left(\frac{c^2x^3-3x}{c^4}+\frac{3\arctan(cx)}{c^5}\right)\right)bd^2+\frac{1}{45}\left(15x^6\arctan(cx)-c\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^2*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] 1/8*a*e^2*x^8 + 1/3*a*d*e*x^6 + 1/4*a*d^2*x^4 + 1/12*(3*x^4*arctan(c*x) - c
*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5))*b*d^2 + 1/45*(15*x^6*arctan(c*x
) - c*((3*c^4*x^5 - 5*c^2*x^3 + 15*x)/c^6 - 15*arctan(c*x)/c^7))*b*d*e + 1/
840*(105*x^8*arctan(c*x) - c*((15*c^6*x^7 - 21*c^4*x^5 + 35*c^2*x^3 - 105*x
)/c^8 + 105*arctan(c*x)/c^9))*b*e^2
```

Fricas [A] time = 1.77162, size = 455, normalized size = 2.46

$$\frac{315\,ac^8e^2x^8 + 840\,ac^8dex^6 - 45\,bc^7e^2x^7 + 630\,ac^8d^2x^4 - 21\left(8\,bc^7de - 3\,bc^5e^2\right)x^5 - 35\left(6\,bc^7d^2 - 8\,bc^5de + 3\,bc^3e^2\right)x^3 + 10}{2520\,c^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^2*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] 1/2520*(315*a*c^8*e^2*x^8 + 840*a*c^8*d*e*x^6 - 45*b*c^7*e^2*x^7 + 630*a*c^
8*d^2*x^4 - 21*(8*b*c^7*d*e - 3*b*c^5*e^2)*x^5 - 35*(6*b*c^7*d^2 - 8*b*c^5*
d*e + 3*b*c^3*e^2)*x^3 + 105*(6*b*c^5*d^2 - 8*b*c^3*d*e + 3*b*c*e^2)*x + 10
5*(3*b*c^8*e^2*x^8 + 8*b*c^8*d*e*x^6 + 6*b*c^8*d^2*x^4 - 6*b*c^4*d^2 + 8*b*
c^2*d*e - 3*b*e^2)*arctan(c*x))/c^8
```

Sympy [A] time = 5.68769, size = 260, normalized size = 1.41

$$\begin{cases} \frac{ad^2x^4}{4} + \frac{adex^6}{3} + \frac{ae^2x^8}{8} + \frac{bd^2x^4\operatorname{atan}(cx)}{4} + \frac{bdex^6\operatorname{atan}(cx)}{3} + \frac{be^2x^8\operatorname{atan}(cx)}{8} - \frac{bd^2x^3}{12c} - \frac{bdex^5}{15c} - \frac{be^2x^7}{56c} + \frac{bd^2x}{4c^3} + \frac{bdex^3}{9c^3} + \frac{be^2x^5}{40c^3} - \frac{bd^2\operatorname{atan}(c}{4c^4} \\ a\left(\frac{d^2x^4}{4} + \frac{dex^6}{3} + \frac{e^2x^8}{8}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x**2+d)**2*(a+b*atan(c*x)),x)
```

```
[Out] Piecewise((a*d**2*x**4/4 + a*d*e*x**6/3 + a*e**2*x**8/8 + b*d**2*x**4*atan(
c*x)/4 + b*d*e*x**6*atan(c*x)/3 + b*e**2*x**8*atan(c*x)/8 - b*d**2*x**3/(12
*c) - b*d*e*x**5/(15*c) - b*e**2*x**7/(56*c) + b*d**2*x/(4*c**3) + b*d*e*x*
*3/(9*c**3) + b*e**2*x**5/(40*c**3) - b*d**2*atan(c*x)/(4*c**4) - b*d*e*x/(
3*c**5) - b*e**2*x**3/(24*c**5) + b*d*e*atan(c*x)/(3*c**6) + b*e**2*x/(8*c*
*7) - b*e**2*atan(c*x)/(8*c**8), Ne(c, 0)), (a*(d**2*x**4/4 + d*e*x**6/3 +
e**2*x**8/8), True))
```

Giac [A] time = 1.36381, size = 315, normalized size = 1.7

$$315\,bc^8x^8\arctan(cx)\,e^2 + 315\,ac^8x^8e^2 + 840\,bc^8dx^6\arctan(cx)\,e + 840\,ac^8dx^6e + 630\,bc^8d^2x^4\arctan(cx) - 45\,bc^7x^7e^2 -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^2*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] 1/2520*(315*b*c^8*x^8*arctan(c*x)*e^2 + 315*a*c^8*x^8*e^2 + 840*b*c^8*d*x^6
*arctan(c*x)*e + 840*a*c^8*d*x^6*e + 630*b*c^8*d^2*x^4*arctan(c*x) - 45*b*c
^7*x^7*e^2 + 630*a*c^8*d^2*x^4 - 168*b*c^7*d*x^5*e - 210*b*c^7*d^2*x^3 + 63
*b*c^5*x^5*e^2 + 280*b*c^5*d*x^3*e + 630*b*c^5*d^2*x - 630*b*c^4*d^2*arctan
(c*x) - 105*b*c^3*x^3*e^2 - 840*pi*b*c^2*d*e*sgn(c)*sgn(x) - 840*b*c^3*d*x*
e + 840*b*c^2*d*arctan(c*x)*e + 315*b*c*x*e^2 - 315*b*arctan(c*x)*e^2)/c^8
```

3.1126 $\int x^2 \left(d+ex^2\right)^2 \left(a+b\tan^{-1}(cx)\right) dx$

Optimal. Leaf size=161

$$\frac{1}{3}d^2x^3\left(a+b\tan^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a+b\tan^{-1}(cx)\right)+\frac{1}{7}e^2x^7\left(a+b\tan^{-1}(cx)\right)-\frac{bx^2\left(35c^4d^2-42c^2de+15e^2\right)}{210c^5}+\frac{b\left(35c^4d\right.}{}$$

```
[Out] -(b*(35*c^4*d^2 - 42*c^2*d*e + 15*e^2)*x^2)/(210*c^5) - (b*(14*c^2*d - 5*e)
*e*x^4)/(140*c^3) - (b*e^2*x^6)/(42*c) + (d^2*x^3*(a + b*ArcTan[c*x]))/3 +
(2*d*e*x^5*(a + b*ArcTan[c*x]))/5 + (e^2*x^7*(a + b*ArcTan[c*x]))/7 + (b*(3
5*c^4*d^2 - 42*c^2*d*e + 15*e^2)*Log[1 + c^2*x^2])/(210*c^7)
```

Rubi [A] time = 0.245806, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {270, 4976, 12, 1251, 771}

$$\frac{1}{3}d^2x^3\left(a+b\tan^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a+b\tan^{-1}(cx)\right)+\frac{1}{7}e^2x^7\left(a+b\tan^{-1}(cx)\right)-\frac{bx^2\left(35c^4d^2-42c^2de+15e^2\right)}{210c^5}+\frac{b\left(35c^4d\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + e*x^2)^2*(a + b*ArcTan[c*x]),x]
```

```
[Out] -(b*(35*c^4*d^2 - 42*c^2*d*e + 15*e^2)*x^2)/(210*c^5) - (b*(14*c^2*d - 5*e)
*e*x^4)/(140*c^3) - (b*e^2*x^6)/(42*c) + (d^2*x^3*(a + b*ArcTan[c*x]))/3 +
(2*d*e*x^5*(a + b*ArcTan[c*x]))/5 + (e^2*x^7*(a + b*ArcTan[c*x]))/7 + (b*(3
5*c^4*d^2 - 42*c^2*d*e + 15*e^2)*Log[1 + c^2*x^2])/(210*c^7)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 4976

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Dis
t[a + b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&  !(
ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(IL
```

```
tQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILtQ[(m
 - 1)/2, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 771

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^
2 - 4*a*c, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0] && IntegerQ[m]))
```

Rubi steps

$$\int x^2 \left(d+ex^2\right)^2 \left(a+b\tan^{-1}(cx)\right) dx = \frac{1}{3}d^2x^3\left(a+b\tan^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a+b\tan^{-1}(cx)\right)+\frac{1}{7}e^2x^7\left(a+b\tan^{-1}(cx)\right)-$$

$$= \frac{1}{3}d^2x^3\left(a+b\tan^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a+b\tan^{-1}(cx)\right)+\frac{1}{7}e^2x^7\left(a+b\tan^{-1}(cx)\right)-$$

$$= \frac{1}{3}d^2x^3\left(a+b\tan^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a+b\tan^{-1}(cx)\right)+\frac{1}{7}e^2x^7\left(a+b\tan^{-1}(cx)\right)-$$

$$= \frac{1}{3}d^2x^3\left(a+b\tan^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a+b\tan^{-1}(cx)\right)+\frac{1}{7}e^2x^7\left(a+b\tan^{-1}(cx)\right)-$$

$$= -\frac{b\left(35c^4d^2-42c^2de+15e^2\right)x^2}{210c^5}-\frac{b\left(14c^2d-5e\right)ex^4}{140c^3}-\frac{be^2x^6}{42c}+\frac{1}{3}d^2x^3\left(a+b\,\mathrm{ta}\right.$$

Mathematica [A] time = 0.117087, size = 162, normalized size = 1.01

$$\frac{c^2x^2\left(4ac^5x\left(35d^2+42dex^2+15e^2x^4\right)-2bc^4\left(35d^2+21dex^2+5e^2x^4\right)+3bc^2e\left(28d+5ex^2\right)-30be^2\right)+2b\left(35c^4d^2-42\right.}{420c^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + e*x^2)^2*(a + b*ArcTan[c*x]),x]
```

```
[Out] (c^2*x^2*(-30*b*e^2 + 3*b*c^2*e*(28*d + 5*e*x^2) - 2*b*c^4*(35*d^2 + 21*d*e
*x^2 + 5*e^2*x^4) + 4*a*c^5*x*(35*d^2 + 42*d*e*x^2 + 15*e^2*x^4)) + 4*b*c^7
*x^3*(35*d^2 + 42*d*e*x^2 + 15*e^2*x^4)*ArcTan[c*x] + 2*b*(35*c^4*d^2 - 42*
c^2*d*e + 15*e^2)*Log[1 + c^2*x^2])/(420*c^7)
```

Maple [A] time = 0.036, size = 192, normalized size = 1.2

$$\frac{ae^2x^7}{7}+\frac{2\,aedx^5}{5}+\frac{ad^2x^3}{3}+\frac{b\arctan(cx)\,e^2x^7}{7}+\frac{2\,b\arctan(cx)\,edx^5}{5}+\frac{b\arctan(cx)\,d^2x^3}{3}-\frac{bd^2x^2}{6\,c}-\frac{bdex^4}{10\,c}-\frac{be^2x^6}{42\,c}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^2+d)^2*(a+b*arctan(c*x)),x)
```

```
[Out] 1/7*a*e^2*x^7+2/5*a*e*d*x^5+1/3*a*d^2*x^3+1/7*b*arctan(c*x)*e^2*x^7+2/5*b*a
rctan(c*x)*e*d*x^5+1/3*b*arctan(c*x)*d^2*x^3-1/6*b*d^2*x^2/c-1/10/c*b*e*d*x
^4-1/42*b*e^2*x^6/c+1/5/c^3*b*x^2*d*e+1/28/c^3*b*x^4*e^2-1/14/c^5*b*x^2*e^2
+1/6*b*d^2*ln(c^2*x^2+1)/c^3-1/5/c^5*b*ln(c^2*x^2+1)*e*d+1/14/c^7*b*ln(c^2*
x^2+1)*e^2
```

Maxima [A] time = 0.972984, size = 244, normalized size = 1.52

$$\frac{1}{7}\,ae^2x^7+\frac{2}{5}\,adex^5+\frac{1}{3}\,ad^2x^3+\frac{1}{6}\left(2\,x^3\arctan(cx)-c\left(\frac{x^2}{c^2}-\frac{\log\left(c^2x^2+1\right)}{c^4}\right)\right)bd^2+\frac{1}{10}\left(4\,x^5\arctan(cx)-c\left(\frac{c^2x^4-2\,x^2}{c^4}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^2*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] 1/7*a*e^2*x^7 + 2/5*a*d*e*x^5 + 1/3*a*d^2*x^3 + 1/6*(2*x^3*arctan(c*x) - c*
(x^2/c^2 - log(c^2*x^2 + 1)/c^4))*b*d^2 + 1/10*(4*x^5*arctan(c*x) - c*((c^2
*x^4 - 2*x^2)/c^4 + 2*log(c^2*x^2 + 1)/c^6))*b*d*e + 1/84*(12*x^7*arctan(c*
x) - c*((2*c^4*x^6 - 3*c^2*x^4 + 6*x^2)/c^6 - 6*log(c^2*x^2 + 1)/c^8))*b*e^
2
```

Fricas [A] time = 1.68684, size = 424, normalized size = 2.63

$$\frac{60\,ac^7e^2x^7+168\,ac^7dex^5-10\,bc^6e^2x^6+140\,ac^7d^2x^3-3\left(14\,bc^6de-5\,bc^4e^2\right)x^4-2\left(35\,bc^6d^2-42\,bc^4de+15\,bc^2e^2\right)x^2+}{420\,c^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^2*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] 1/420*(60*a*c^7*e^2*x^7 + 168*a*c^7*d*e*x^5 - 10*b*c^6*e^2*x^6 + 140*a*c^7*
d^2*x^3 - 3*(14*b*c^6*d*e - 5*b*c^4*e^2)*x^4 - 2*(35*b*c^6*d^2 - 42*b*c^4*d
*e + 15*b*c^2*e^2)*x^2 + 4*(15*b*c^7*e^2*x^7 + 42*b*c^7*d*e*x^5 + 35*b*c^7*
d^2*x^3)*arctan(c*x) + 2*(35*b*c^4*d^2 - 42*b*c^2*d*e + 15*b*e^2)*log(c^2*x
^2 + 1))/c^7
```

Sympy [A] time = 4.09677, size = 245, normalized size = 1.52

$$\begin{cases} \frac{ad^2x^3}{3}+\frac{2adex^5}{5}+\frac{ae^2x^7}{7}+\frac{bd^2x^3\operatorname{atan}(cx)}{3}+\frac{2bdex^5\operatorname{atan}(cx)}{5}+\frac{be^2x^7\operatorname{atan}(cx)}{7}-\frac{bd^2x^2}{6c}-\frac{bdex^4}{10c}-\frac{be^2x^6}{42c}+\frac{bd^2\log\left(x^2+\frac{1}{c^2}\right)}{6c^3}+\frac{bdex^2}{5c^3}+\frac{be^2x}{28c^3} \\ a\left(\frac{d^2x^3}{3}+\frac{2dex^5}{5}+\frac{e^2x^7}{7}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d)**2*(a+b*atan(c*x)),x)
```

```
[Out] Piecewise((a*d**2*x**3/3 + 2*a*d*e*x**5/5 + a*e**2*x**7/7 + b*d**2*x**3*ata
n(c*x)/3 + 2*b*d*e*x**5*atan(c*x)/5 + b*e**2*x**7*atan(c*x)/7 - b*d**2*x**2
/(6*c) - b*d*e*x**4/(10*c) - b*e**2*x**6/(42*c) + b*d**2*log(x**2 + c**(-2)
)/(6*c**3) + b*d*e*x**2/(5*c**3) + b*e**2*x**4/(28*c**3) - b*d*e*log(x**2 +
 c**(-2))/(5*c**5) - b*e**2*x**2/(14*c**5) + b*e**2*log(x**2 + c**(-2))/(14
*c**7), Ne(c, 0)), (a*(d**2*x**3/3 + 2*d*e*x**5/5 + e**2*x**7/7), True))
```

Giac [A] time = 1.1001, size = 284, normalized size = 1.76

$$\frac{60\,bc^7x^7\arctan(cx)\,e^2+60\,ac^7x^7e^2+168\,bc^7dx^5\arctan(cx)\,e+168\,ac^7dx^5e+140\,bc^7d^2x^3\arctan(cx)-10\,bc^6x^6e^2+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^2*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] 1/420*(60*b*c^7*x^7*arctan(c*x)*e^2 + 60*a*c^7*x^7*e^2 + 168*b*c^7*d*x^5*ar
ctan(c*x)*e + 168*a*c^7*d*x^5*e + 140*b*c^7*d^2*x^3*arctan(c*x) - 10*b*c^6*
x^6*e^2 + 140*a*c^7*d^2*x^3 - 42*b*c^6*d*x^4*e - 70*b*c^6*d^2*x^2 + 15*b*c^
4*x^4*e^2 + 84*b*c^4*d*x^2*e + 70*b*c^4*d^2*log(c^2*x^2 + 1) - 30*b*c^2*x^2
*e^2 - 84*b*c^2*d*e*log(c^2*x^2 + 1) + 30*b*e^2*log(c^2*x^2 + 1))/c^7
```

3.1127 $\int x\left(d+ex^2\right)^2\left(a+b\tan^{-1}(cx)\right)\,dx$

Optimal. Leaf size=115

$$\frac{\left(d+ex^2\right)^3\left(a+b\tan^{-1}(cx)\right)}{6e}-\frac{bx\left(3c^4d^2-3c^2de+e^2\right)}{6c^5}-\frac{bex^3\left(3c^2d-e\right)}{18c^3}-\frac{b\left(c^2d-e\right)^3\tan^{-1}(cx)}{6c^6e}-\frac{be^2x^5}{30c}$$

```
[Out] -(b*(3*c^4*d^2 - 3*c^2*d*e + e^2)*x)/(6*c^5) - (b*(3*c^2*d - e)*e*x^3)/(18*
c^3) - (b*e^2*x^5)/(30*c) - (b*(c^2*d - e)^3*ArcTan[c*x])/(6*c^6*e) + ((d +
 e*x^2)^3*(a + b*ArcTan[c*x]))/(6*e)
```

Rubi [A] time = 0.113821, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {4974, 390, 203}

$$\frac{\left(d+ex^2\right)^3\left(a+b\tan^{-1}(cx)\right)}{6e}-\frac{bx\left(3c^4d^2-3c^2de+e^2\right)}{6c^5}-\frac{bex^3\left(3c^2d-e\right)}{18c^3}-\frac{b\left(c^2d-e\right)^3\tan^{-1}(cx)}{6c^6e}-\frac{be^2x^5}{30c}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + e*x^2)^2*(a + b*ArcTan[c*x]),x]
```

```
[Out] -(b*(3*c^4*d^2 - 3*c^2*d*e + e^2)*x)/(6*c^5) - (b*(3*c^2*d - e)*e*x^3)/(18*
c^3) - (b*e^2*x^5)/(30*c) - (b*(c^2*d - e)^3*ArcTan[c*x])/(6*c^6*e) + ((d +
 e*x^2)^3*(a + b*ArcTan[c*x]))/(6*e)
```

Rule 4974

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]))/(2*e*(q + 1)), x
] - Dist[(b*c)/(2*e*(q + 1)), Int[(d + e*x^2)^(q + 1)/(1 + c^2*x^2), x], x]
 /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int x\left(d+ex^2\right)^2\left(a+b\tan^{-1}(cx)\right)dx=\frac{\left(d+ex^2\right)^3\left(a+b\tan^{-1}(cx)\right)}{6e}-\frac{(bc)\int\frac{\left(d+ex^2\right)^3}{1+c^2x^2}dx}{6e}$$

$$=\frac{\left(d+ex^2\right)^3\left(a+b\tan^{-1}(cx)\right)}{6e}-\frac{(bc)\int\left(\frac{e\left(3c^4d^2-3c^2de+e^2\right)}{c^6}+\frac{\left(3c^2d-e\right)e^2x^2}{c^4}+\frac{e^3x^4}{c^2}+\frac{c^6d^3-3}{c}\right.}{6e}$$

$$=-\frac{b\left(3c^4d^2-3c^2de+e^2\right)x}{6c^5}-\frac{b\left(3c^2d-e\right)ex^3}{18c^3}-\frac{be^2x^5}{30c}+\frac{\left(d+ex^2\right)^3\left(a+b\tan^{-1}(cx\right.}{6e}$$

$$=-\frac{b\left(3c^4d^2-3c^2de+e^2\right)x}{6c^5}-\frac{b\left(3c^2d-e\right)ex^3}{18c^3}-\frac{be^2x^5}{30c}-\frac{b\left(c^2d-e\right)^3\tan^{-1}(cx)}{6c^6e}+\frac{(}{}$$

Mathematica [A] time = 0.0892871, size = 140, normalized size = 1.22

$$\frac{cx\left(15ac^5x\left(3d^2+3dex^2+e^2x^4\right)-3bc^4\left(15d^2+5dex^2+e^2x^4\right)+5bc^2e\left(9d+ex^2\right)-15be^2\right)+15b\tan^{-1}(cx)\left(c^6\left(3d^2x^2+3\right.\right.}{90c^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + e*x^2)^2*(a + b*ArcTan[c*x]),x]
```

```
[Out] (c*x*(-15*b*e^2 + 5*b*c^2*e*(9*d + e*x^2) + 15*a*c^5*x*(3*d^2 + 3*d*e*x^2 +
 e^2*x^4) - 3*b*c^4*(15*d^2 + 5*d*e*x^2 + e^2*x^4)) + 15*b*(3*c^4*d^2 - 3*c
^2*d*e + e^2 + c^6*(3*d^2*x^2 + 3*d*e*x^4 + e^2*x^6))*ArcTan[c*x])/(90*c^6)
```

Maple [A] time = 0.038, size = 168, normalized size = 1.5

$$\frac{ae^2x^6}{6}+\frac{aedx^4}{2}+\frac{ax^2d^2}{2}+\frac{b\arctan(cx)e^2x^6}{6}+\frac{b\arctan(cx)edx^4}{2}+\frac{b\arctan(cx)d^2x^2}{2}-\frac{be^2x^5}{30c}-\frac{bx^3de}{6c}-\frac{bd^2x}{2c}+\frac{bx^3e^2}{18c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d)^2*(a+b*arctan(c*x)),x)
```

```
[Out] 1/6*a*e^2*x^6+1/2*a*e*d*x^4+1/2*a*x^2*d^2+1/6*b*arctan(c*x)*e^2*x^6+1/2*b*a
rctan(c*x)*e*d*x^4+1/2*b*arctan(c*x)*d^2*x^2-1/30*b*e^2*x^5/c-1/6/c*b*x^3*d
*e-1/2*b*d^2*x/c+1/18/c^3*b*x^3*e^2+1/2/c^3*b*e*d*x-1/6/c^5*b*x*e^2+1/2*b*d
^2*arctan(c*x)/c^2-1/2/c^4*b*arctan(c*x)*e*d+1/6/c^6*b*arctan(c*x)*e^2
```

Maxima [A] time = 1.44669, size = 211, normalized size = 1.83

$$\frac{1}{6}\,ae^2x^6+\frac{1}{2}\,adex^4+\frac{1}{2}\,ad^2x^2+\frac{1}{2}\left(x^2\arctan\left(cx\right)-c\left(\frac{x}{c^2}-\frac{\arctan\left(cx\right)}{c^3}\right)\right)bd^2+\frac{1}{6}\left(3\,x^4\arctan\left(cx\right)-c\left(\frac{c^2x^3-3\,x}{c^4}+\frac{3}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^2*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] 1/6*a*e^2*x^6 + 1/2*a*d*e*x^4 + 1/2*a*d^2*x^2 + 1/2*(x^2*arctan(c*x) - c*(x
/c^2 - arctan(c*x)/c^3))*b*d^2 + 1/6*(3*x^4*arctan(c*x) - c*((c^2*x^3 - 3*x
)/c^4 + 3*arctan(c*x)/c^5))*b*d*e + 1/90*(15*x^6*arctan(c*x) - c*((3*c^4*x^
5 - 5*c^2*x^3 + 15*x)/c^6 - 15*arctan(c*x)/c^7))*b*e^2
```

Fricas [A] time = 1.63242, size = 363, normalized size = 3.16

$$\frac{15\,ac^6e^2x^6+45\,ac^6dex^4-3\,bc^5e^2x^5+45\,ac^6d^2x^2-5\left(3\,bc^5de-bc^3e^2\right)x^3-15\left(3\,bc^5d^2-3\,bc^3de+bce^2\right)x+15\left(bc^6e^2x^6\right.}{90\,c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^2*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] 1/90*(15*a*c^6*e^2*x^6 + 45*a*c^6*d*e*x^4 - 3*b*c^5*e^2*x^5 + 45*a*c^6*d^2*
x^2 - 5*(3*b*c^5*d*e - b*c^3*e^2)*x^3 - 15*(3*b*c^5*d^2 - 3*b*c^3*d*e + b*c
*e^2)*x + 15*(b*c^6*e^2*x^6 + 3*b*c^6*d*e*x^4 + 3*b*c^6*d^2*x^2 + 3*b*c^4*d
^2 - 3*b*c^2*d*e + b*e^2)*arctan(c*x))/c^6
```

Sympy [A] time = 3.53508, size = 219, normalized size = 1.9

$$\begin{cases} \frac{ad^2x^2}{2} + \frac{adex^4}{2} + \frac{ae^2x^6}{6} + \frac{bd^2x^2\operatorname{atan}(cx)}{2} + \frac{bdex^4\operatorname{atan}(cx)}{2} + \frac{be^2x^6\operatorname{atan}(cx)}{6} - \frac{bd^2x}{2c} - \frac{bdex^3}{6c} - \frac{be^2x^5}{30c} + \frac{bd^2\operatorname{atan}(cx)}{2c^2} + \frac{bdex}{2c^3} + \frac{be^2x^3}{18c^3} - \frac{bde\,a \\ a\left(\frac{d^2x^2}{2} + \frac{dex^4}{2} + \frac{e^2x^6}{6}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d)**2*(a+b*atan(c*x)),x)
```

```
[Out] Piecewise((a*d**2*x**2/2 + a*d*e*x**4/2 + a*e**2*x**6/6 + b*d**2*x**2*atan(
c*x)/2 + b*d*e*x**4*atan(c*x)/2 + b*e**2*x**6*atan(c*x)/6 - b*d**2*x/(2*c)
- b*d*e*x**3/(6*c) - b*e**2*x**5/(30*c) + b*d**2*atan(c*x)/(2*c**2) + b*d*e
*x/(2*c**3) + b*e**2*x**3/(18*c**3) - b*d*e*atan(c*x)/(2*c**4) - b*e**2*x/(
6*c**5) + b*e**2*atan(c*x)/(6*c**6), Ne(c, 0)), (a*(d**2*x**2/2 + d*e*x**4/
2 + e**2*x**6/6), True))
```

Giac [A] time = 1.26916, size = 281, normalized size = 2.44

$$\frac{15\,bc^6x^6\arctan(cx)\,e^2 + 15\,ac^6x^6e^2 + 45\,bc^6dx^4\arctan(cx)\,e + 45\,ac^6dx^4e + 45\,bc^6d^2x^2\arctan(cx) - 3\,bc^5x^5e^2 + 45\,ac^6}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^2*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] 1/90*(15*b*c^6*x^6*arctan(c*x)*e^2 + 15*a*c^6*x^6*e^2 + 45*b*c^6*d*x^4*arct
an(c*x)*e + 45*a*c^6*d*x^4*e + 45*b*c^6*d^2*x^2*arctan(c*x) - 3*b*c^5*x^5*e
^2 + 45*a*c^6*d^2*x^2 - 15*b*c^5*d*x^3*e - 45*pi*b*c^4*d^2*sgn(c)*sgn(x) -
45*b*c^5*d^2*x + 45*b*c^4*d^2*arctan(c*x) + 5*b*c^3*x^3*e^2 + 45*b*c^3*d*x*
e - 45*b*c^2*d*arctan(c*x)*e - 15*pi*b*e^2*sgn(c)*sgn(x) - 15*b*c*x*e^2 + 1
5*b*arctan(c*x)*e^2)/c^6
```

3.1128 $\int \left(d+ex^2\right)^2 \left(a+b\tan^{-1}(cx)\right)\,dx$

Optimal. Leaf size=124

$$d^2x\left(a+b\tan^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\tan^{-1}(cx)\right)+\frac{1}{5}e^2x^5\left(a+b\tan^{-1}(cx)\right)-\frac{b\left(15c^4d^2-10c^2de+3e^2\right)\log\left(c^2x^2+1\right)}{30c^5}-$$

```
[Out] -(b*(10*c^2*d - 3*e)*e*x^2)/(30*c^3) - (b*e^2*x^4)/(20*c) + d^2*x*(a + b*Ar
cTan[c*x]) + (2*d*e*x^3*(a + b*ArcTan[c*x]))/3 + (e^2*x^5*(a + b*ArcTan[c*x
]))/5 - (b*(15*c^4*d^2 - 10*c^2*d*e + 3*e^2)*Log[1 + c^2*x^2])/(30*c^5)
```

Rubi [A] time = 0.159716, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {194, 4912, 1594, 1247, 698}

$$d^2x\left(a+b\tan^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\tan^{-1}(cx)\right)+\frac{1}{5}e^2x^5\left(a+b\tan^{-1}(cx)\right)-\frac{b\left(15c^4d^2-10c^2de+3e^2\right)\log\left(c^2x^2+1\right)}{30c^5}-$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^2*(a + b*ArcTan[c*x]),x]
```

```
[Out] -(b*(10*c^2*d - 3*e)*e*x^2)/(30*c^3) - (b*e^2*x^4)/(20*c) + d^2*x*(a + b*Ar
cTan[c*x]) + (2*d*e*x^3*(a + b*ArcTan[c*x]))/3 + (e^2*x^5*(a + b*ArcTan[c*x
]))/5 - (b*(15*c^4*d^2 - 10*c^2*d*e + 3*e^2)*Log[1 + c^2*x^2])/(30*c^5)
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4912

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcTan[c*x], u, x]
- Dist[b*c, Int[u/(1 + c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && (I
ntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
```

```
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int \left(d+ex^2\right)^2 \left(a+b\tan^{-1}(cx)\right) dx = d^2x\left(a+b\tan^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\tan^{-1}(cx)\right)+\frac{1}{5}e^2x^5\left(a+b\tan^{-1}(cx)\right)-(bc)\int$$

$$= d^2x\left(a+b\tan^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\tan^{-1}(cx)\right)+\frac{1}{5}e^2x^5\left(a+b\tan^{-1}(cx)\right)-(bc)\int$$

$$= d^2x\left(a+b\tan^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\tan^{-1}(cx)\right)+\frac{1}{5}e^2x^5\left(a+b\tan^{-1}(cx)\right)-\frac{1}{2}(bc)$$

$$= d^2x\left(a+b\tan^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\tan^{-1}(cx)\right)+\frac{1}{5}e^2x^5\left(a+b\tan^{-1}(cx)\right)-\frac{1}{2}(bc)$$

$$= -\frac{b\left(10c^2d-3e\right)ex^2}{30c^3}-\frac{be^2x^4}{20c}+d^2x\left(a+b\tan^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\tan^{-1}(cx)\right)+\frac{1}{5}$$

Mathematica [A] time = 0.0923648, size = 130, normalized size = 1.05

$$\frac{c^2x\left(4ac^3\left(15d^2+10dex^2+3e^2x^4\right)+bex\left(6e-c^2\left(20d+3ex^2\right)\right)\right)-2b\left(15c^4d^2-10c^2de+3e^2\right)\log\left(c^2x^2+1\right)+4bc^5x\tan}{60c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^2*(a + b*ArcTan[c*x]),x]
```

```
[Out] (c^2*x*(4*a*c^3*(15*d^2 + 10*d*e*x^2 + 3*e^2*x^4) + b*e*x*(6*e - c^2*(20*d
+ 3*e*x^2))) + 4*b*c^5*x*(15*d^2 + 10*d*e*x^2 + 3*e^2*x^4)*ArcTan[c*x] - 2*
b*(15*c^4*d^2 - 10*c^2*d*e + 3*e^2)*Log[1 + c^2*x^2])/(60*c^5)
```

Maple [A] time = 0.038, size = 151, normalized size = 1.2

$$\frac{ax^5e^2}{5}+\frac{2\,ax^3de}{3}+ad^2x+\frac{b\arctan\left(cx\right)x^5e^2}{5}+\frac{2\,b\arctan\left(cx\right)x^3de}{3}+b\arctan\left(cx\right)d^2x-\frac{bx^2de}{3\,c}-\frac{be^2x^4}{20\,c}+\frac{bx^2e^2}{10\,c^3}-\frac{b\,\mathrm{l}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(a+b*arctan(c*x)),x)
```

```
[Out] 1/5*a*x^5*e^2+2/3*a*x^3*d*e+a*d^2*x+1/5*b*arctan(c*x)*x^5*e^2+2/3*b*arctan(
c*x)*x^3*d*e+b*arctan(c*x)*d^2*x-1/3/c*b*x^2*d*e-1/20*b*e^2*x^4/c+1/10/c^3*
b*x^2*e^2-1/2/c*b*ln(c^2*x^2+1)*d^2+1/3/c^3*b*ln(c^2*x^2+1)*e*d-1/10/c^5*b*
ln(c^2*x^2+1)*e^2
```

Maxima [A] time = 0.971701, size = 198, normalized size = 1.6

$$\frac{1}{5}\,ae^2x^5+\frac{2}{3}\,adex^3+\frac{1}{3}\left(2\,x^3\arctan\left(cx\right)-c\left(\frac{x^2}{c^2}-\frac{\log\left(c^2x^2+1\right)}{c^4}\right)\right)bde+\frac{1}{20}\left(4\,x^5\arctan\left(cx\right)-c\left(\frac{c^2x^4-2\,x^2}{c^4}+\frac{2\,\log\left(\right.}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] 1/5*a*e^2*x^5 + 2/3*a*d*e*x^3 + 1/3*(2*x^3*arctan(c*x) - c*(x^2/c^2 - log(c
^2*x^2 + 1)/c^4))*b*d*e + 1/20*(4*x^5*arctan(c*x) - c*((c^2*x^4 - 2*x^2)/c^
4 + 2*log(c^2*x^2 + 1)/c^6))*b*e^2 + a*d^2*x + 1/2*(2*c*x*arctan(c*x) - log
(c^2*x^2 + 1))*b*d^2/c
```

Fricas [A] time = 1.5331, size = 339, normalized size = 2.73

$$\frac{12\,ac^5e^2x^5+40\,ac^5dex^3-3\,bc^4e^2x^4+60\,ac^5d^2x-2\left(10\,bc^4de-3\,bc^2e^2\right)x^2+4\left(3\,bc^5e^2x^5+10\,bc^5dex^3+15\,bc^5d^2x\right)\mathrm{arct}}{60\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] 1/60*(12*a*c^5*e^2*x^5 + 40*a*c^5*d*e*x^3 - 3*b*c^4*e^2*x^4 + 60*a*c^5*d^2*
x - 2*(10*b*c^4*d*e - 3*b*c^2*e^2)*x^2 + 4*(3*b*c^5*e^2*x^5 + 10*b*c^5*d*e*
x^3 + 15*b*c^5*d^2*x)*arctan(c*x) - 2*(15*b*c^4*d^2 - 10*b*c^2*d*e + 3*b*e^
2)*log(c^2*x^2 + 1))/c^5
```

Sympy [A] time = 2.32238, size = 194, normalized size = 1.56

$$\begin{cases} ad^2x + \frac{2adex^3}{3} + \frac{ae^2x^5}{5} + bd^2x\,\mathrm{atan}\,(cx) + \frac{2bdex^3\,\mathrm{atan}\,(cx)}{3} + \frac{be^2x^5\,\mathrm{atan}\,(cx)}{5} - \frac{bd^2\log\left(x^2+\frac{1}{c^2}\right)}{2c} - \frac{bdex^2}{3c} - \frac{be^2x^4}{20c} + \frac{bde\log\left(x^2+\frac{1}{c^2}\right)}{3c^3} + \frac{be^2 \cdots}{10c \cdots} \\ a\left(d^2x + \frac{2dex^3}{3} + \frac{e^2x^5}{5}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(a+b*atan(c*x)),x)
```

```
[Out] Piecewise((a*d**2*x + 2*a*d*e*x**3/3 + a*e**2*x**5/5 + b*d**2*x*atan(c*x) +
 2*b*d*e*x**3*atan(c*x)/3 + b*e**2*x**5*atan(c*x)/5 - b*d**2*log(x**2 + c**
(-2))/(2*c) - b*d*e*x**2/(3*c) - b*e**2*x**4/(20*c) + b*d*e*log(x**2 + c**(
-2))/(3*c**3) + b*e**2*x**2/(10*c**3) - b*e**2*log(x**2 + c**(-2))/(10*c**5
), Ne(c, 0)), (a*(d**2*x + 2*d*e*x**3/3 + e**2*x**5/5), True))
```

Giac [A] time = 1.13019, size = 231, normalized size = 1.86

$$\frac{12\,bc^5x^5\arctan(cx)\,e^2 + 12\,ac^5x^5e^2 + 40\,bc^5dx^3\arctan(cx)\,e + 40\,ac^5dx^3e + 60\,bc^5d^2x\arctan(cx) - 3\,bc^4x^4e^2 + 60\,ac^5d \cdots}{60\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] 1/60*(12*b*c^5*x^5*arctan(c*x)*e^2 + 12*a*c^5*x^5*e^2 + 40*b*c^5*d*x^3*arct
an(c*x)*e + 40*a*c^5*d*x^3*e + 60*b*c^5*d^2*x*arctan(c*x) - 3*b*c^4*x^4*e^2
 + 60*a*c^5*d^2*x - 20*b*c^4*d*x^2*e - 30*b*c^4*d^2*log(c^2*x^2 + 1) + 6*b*
c^2*x^2*e^2 + 20*b*c^2*d*e*log(c^2*x^2 + 1) - 6*b*e^2*log(c^2*x^2 + 1))/c^5
```

3.1129 $\int \frac{(d+ex^2)^2(a+b\tan^{-1}(cx))}{x}\,dx$

Optimal. Leaf size=137

$$\frac{1}{2}ibd^2\text{PolyLog}(2,-icx)-\frac{1}{2}ibd^2\text{PolyLog}(2,icx)+dex^2\left(a+b\tan^{-1}(cx)\right)+\frac{1}{4}e^2x^4\left(a+b\tan^{-1}(cx)\right)+ad^2\log(x)+\frac{bde}{}$$

```
[Out] -((b*d*e*x)/c) + (b*e^2*x)/(4*c^3) - (b*e^2*x^3)/(12*c) + (b*d*e*ArcTan[c*x
])/c^2 - (b*e^2*ArcTan[c*x])/(4*c^4) + d*e*x^2*(a + b*ArcTan[c*x]) + (e^2*x
^4*(a + b*ArcTan[c*x]))/4 + a*d^2*Log[x] + (I/2)*b*d^2*PolyLog[2, (-I)*c*x]
 - (I/2)*b*d^2*PolyLog[2, I*c*x]
```

Rubi [A] time = 0.179517, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4980, 4848, 2391, 4852, 321, 203, 302}

$$\frac{1}{2}ibd^2\text{PolyLog}(2,-icx)-\frac{1}{2}ibd^2\text{PolyLog}(2,icx)+dex^2\left(a+b\tan^{-1}(cx)\right)+\frac{1}{4}e^2x^4\left(a+b\tan^{-1}(cx)\right)+ad^2\log(x)+\frac{bde}{}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^2*(a + b*ArcTan[c*x]))/x,x]
```


```
[Out] -((b*d*e*x)/c) + (b*e^2*x)/(4*c^3) - (b*e^2*x^3)/(12*c) + (b*d*e*ArcTan[c*x
])/c^2 - (b*e^2*ArcTan[c*x])/(4*c^4) + d*e*x^2*(a + b*ArcTan[c*x]) + (e^2*x
^4*(a + b*ArcTan[c*x]))/4 + a*d^2*Log[x] + (I/2)*b*d^2*PolyLog[2, (-I)*c*x]
 - (I/2)*b*d^2*PolyLog[2, I*c*x]
```


Rule 4980

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
```

```
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\int \frac{(d+ex^2)^2 \left(a+b\tan^{-1}(cx)\right)}{x}\,dx = \int \left(\frac{d^2\left(a+b\tan^{-1}(cx)\right)}{x} + 2dex\left(a+b\tan^{-1}(cx)\right) + e^2x^3\left(a+b\tan^{-1}(cx)\right)\right)dx$$

$$= d^2\int \frac{a+b\tan^{-1}(cx)}{x}\,dx + (2de)\int x\left(a+b\tan^{-1}(cx)\right)dx + e^2\int x^3\left(a+b\tan^{-1}(\right.$$

$$= dex^2\left(a+b\tan^{-1}(cx)\right) + \frac{1}{4}e^2x^4\left(a+b\tan^{-1}(cx)\right) + ad^2\log(x) + \frac{1}{2}\left(ibd^2\right)\int \frac{\log(1}{}$$

$$= -\frac{bdex}{c} + dex^2\left(a+b\tan^{-1}(cx)\right) + \frac{1}{4}e^2x^4\left(a+b\tan^{-1}(cx)\right) + ad^2\log(x) + \frac{1}{2}ibd^2\text{Li}$$

$$= -\frac{bdex}{c} + \frac{be^2x}{4c^3} - \frac{be^2x^3}{12c} + \frac{bde\tan^{-1}(cx)}{c^2} + dex^2\left(a+b\tan^{-1}(cx)\right) + \frac{1}{4}e^2x^4\left(a+b\,\text{t}\right.$$

$$= -\frac{bdex}{c} + \frac{be^2x}{4c^3} - \frac{be^2x^3}{12c} + \frac{bde\tan^{-1}(cx)}{c^2} - \frac{be^2\tan^{-1}(cx)}{4c^4} + dex^2\left(a+b\tan^{-1}(cx)\right)$$

Mathematica [A] time = 0.101077, size = 123, normalized size = 0.9

$$\frac{1}{2}ibd^2\text{PolyLog}(2,-icx) - \frac{1}{2}ibd^2\text{PolyLog}(2,icx) + dex^2\left(a+b\tan^{-1}(cx)\right) + \frac{1}{4}e^2x^4\left(a+b\tan^{-1}(cx)\right) + ad^2\log(x) - \frac{bde}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)^2*(a + b*ArcTan[c*x]))/x,x]
```

```
[Out] -((b*d*e*(c*x - ArcTan[c*x]))/c^2) - (b*e^2*(-3*c*x + c^3*x^3 + 3*ArcTan[c*
x]))/(12*c^4) + d*e*x^2*(a + b*ArcTan[c*x]) + (e^2*x^4*(a + b*ArcTan[c*x]))
/4 + a*d^2*Log[x] + (I/2)*b*d^2*PolyLog[2, (-I)*c*x] - (I/2)*b*d^2*PolyLog[
2, I*c*x]
```

Maple [A] time = 0.051, size = 187, normalized size = 1.4

$$\frac{ax^4e^2}{4} + ax^2de + ad^2\ln(cx) + \frac{b\arctan(cx)\,x^4e^2}{4} + b\arctan(cx)\,x^2de + b\arctan(cx)\,d^2\ln(cx) - \frac{be^2x^3}{12\,c} - \frac{bdex}{c} + \frac{be^2x}{4\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(a+b*arctan(c*x))/x,x)
```

[Out] 1/4*a*x^4*e^2+a*x^2*d*e+a*d^2*ln(c*x)+1/4*b*arctan(c*x)*x^4*e^2+b*arctan(c*x)*x^2*d*e+b*arctan(c*x)*d^2*ln(c*x)-1/12*b*e^2*x^3/c-b*d*e*x/c+1/4*b*e^2*x/c^3+b*d*e*arctan(c*x)/c^2-1/4*b*e^2*arctan(c*x)/c^4+1/2*I*b*d^2*ln(c*x)*ln(1+I*c*x)-1/2*I*b*d^2*ln(c*x)*ln(1-I*c*x)+1/2*I*b*d^2*dilog(1+I*c*x)-1/2*I*b*d^2*dilog(1-I*c*x)

Maxima [A] time = 2.14796, size = 251, normalized size = 1.83

$$\frac{1}{4}\,ae^2x^4 + adex^2 + ad^2\log(x) - \frac{bc^3e^2x^3 + 3\,\pi bc^4d^2\log\left(c^2x^2+1\right) - 12\,bc^4d^2\arctan(cx)\log(x|c|) + 6i\,bc^4d^2\mathrm{Li}_2\,(i\,cx+1)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^2*(a+b*arctan(c*x))/x,x, algorithm="maxima")

[Out] 1/4*a*e^2*x^4 + a*d*e*x^2 + a*d^2*log(x) - 1/12*(b*c^3*e^2*x^3 + 3*pi*b*c^4*d^2*log(c^2*x^2 + 1) - 12*b*c^4*d^2*arctan(c*x)*log(x*abs(c)) + 6*I*b*c^4*d^2*dilog(I*c*x + 1) - 6*I*b*c^4*d^2*dilog(-I*c*x + 1) + 3*(4*b*c^3*d*e - b*c*e^2)*x - (3*b*c^4*e^2*x^4 + 12*b*c^4*d*e*x^2 + 12*I*b*c^4*d^2*arctan2(0, c) + 12*b*c^2*d*e - 3*b*e^2)*arctan(c*x))/c^4

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{ae^2x^4 + 2\,adex^2 + ad^2 + \left(be^2x^4 + 2\,bdex^2 + bd^2\right)\arctan(cx)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^2*(a+b*arctan(c*x))/x,x, algorithm="fricas")

[Out] integral((a*e^2*x^4 + 2*a*d*e*x^2 + a*d^2 + (b*e^2*x^4 + 2*b*d*e*x^2 + b*d^2)*arctan(c*x))/x, x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b\,\mathrm{atan}\,(cx))\left(d + ex^2\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(a+b*atan(c*x))/x,x)
```

```
[Out] Integral((a + b*atan(c*x))*(d + e*x**2)**2/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2+d\right)^2\left(b\arctan\left(cx\right)+a\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arctan(c*x))/x,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^2*(b*arctan(c*x) + a)/x, x)
```

3.1130 $\int \frac{(d+ex^2)^2(a+b\tan^{-1}(cx))}{x^2}\,dx$

Optimal. Leaf size=109

$$-\frac{d^2\left(a+b\tan^{-1}(cx)\right)}{x}+2dex\left(a+b\tan^{-1}(cx)\right)+\frac{1}{3}e^2x^3\left(a+b\tan^{-1}(cx)\right)-\frac{b\left(3c^4d^2+6c^2de-e^2\right)\log\left(c^2x^2+1\right)}{6c^3}+bcd^2$$

```
[Out] -(b*e^2*x^2)/(6*c) - (d^2*(a + b*ArcTan[c*x]))/x + 2*d*e*x*(a + b*ArcTan[c*
x]) + (e^2*x^3*(a + b*ArcTan[c*x]))/3 + b*c*d^2*Log[x] - (b*(3*c^4*d^2 + 6*
c^2*d*e - e^2)*Log[1 + c^2*x^2])/(6*c^3)
```

Rubi [A] time = 0.155636, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {270, 4976, 1251, 893}

$$-\frac{d^2\left(a+b\tan^{-1}(cx)\right)}{x}+2dex\left(a+b\tan^{-1}(cx)\right)+\frac{1}{3}e^2x^3\left(a+b\tan^{-1}(cx)\right)-\frac{b\left(3c^4d^2+6c^2de-e^2\right)\log\left(c^2x^2+1\right)}{6c^3}+bcd^2$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^2*(a + b*ArcTan[c*x]))/x^2,x]
```


```
[Out] -(b*e^2*x^2)/(6*c) - (d^2*(a + b*ArcTan[c*x]))/x + 2*d*e*x*(a + b*ArcTan[c*
x]) + (e^2*x^3*(a + b*ArcTan[c*x]))/3 + b*c*d^2*Log[x] - (b*(3*c^4*d^2 + 6*
c^2*d*e - e^2)*Log[1 + c^2*x^2])/(6*c^3)
```


Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 4976

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Dis
t[a + b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&  !(
ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(IL
tQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILtQ[(m
```

```
 - 1)/2, 0]))
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^2\left(a+b\tan^{-1}(cx)\right)}{x^2}\,dx = -\frac{d^2\left(a+b\tan^{-1}(cx)\right)}{x} + 2dex\left(a+b\tan^{-1}(cx)\right) + \frac{1}{3}e^2x^3\left(a+b\tan^{-1}(cx)\right) - (bc)\int$$

$$= -\frac{d^2\left(a+b\tan^{-1}(cx)\right)}{x} + 2dex\left(a+b\tan^{-1}(cx)\right) + \frac{1}{3}e^2x^3\left(a+b\tan^{-1}(cx)\right) - \frac{1}{2}(bc)$$

$$= -\frac{d^2\left(a+b\tan^{-1}(cx)\right)}{x} + 2dex\left(a+b\tan^{-1}(cx)\right) + \frac{1}{3}e^2x^3\left(a+b\tan^{-1}(cx)\right) - \frac{1}{2}(bc)$$

$$= -\frac{be^2x^2}{6c} - \frac{d^2\left(a+b\tan^{-1}(cx)\right)}{x} + 2dex\left(a+b\tan^{-1}(cx)\right) + \frac{1}{3}e^2x^3\left(a+b\tan^{-1}(cx)\right)$$

Mathematica [A] time = 0.106088, size = 114, normalized size = 1.05

$$\frac{1}{6}\left(-\frac{6ad^2}{x} + 12adex + 2ae^2x^3 + \frac{b\left(-3c^4d^2-6c^2de+e^2\right)\log\left(c^2x^2+1\right)}{c^3} + \frac{2b\tan^{-1}(cx)\left(-3d^2+6dex^2+e^2x^4\right)}{x} + 6bcd^2\log\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)^2*(a + b*ArcTan[c*x]))/x^2,x]
```

```
[Out] ((-6*a*d^2)/x + 12*a*d*e*x - (b*e^2*x^2)/c + 2*a*e^2*x^3 + (2*b*(-3*d^2 + 6
*d*e*x^2 + e^2*x^4)*ArcTan[c*x])/x + 6*b*c*d^2*Log[x] + (b*(-3*c^4*d^2 - 6*
c^2*d*e + e^2)*Log[1 + c^2*x^2])/c^3)/6
```

Maple [A] time = 0.046, size = 138, normalized size = 1.3

$$\frac{ax^3e^2}{3}+2\,aedx-\frac{ad^2}{x}+\frac{b\arctan(cx)\,x^3e^2}{3}+2\,b\arctan(cx)\,edx-\frac{bd^2\arctan(cx)}{x}-\frac{be^2x^2}{6\,c}-\frac{cb\ln\left(c^2x^2+1\right)d^2}{2}-\frac{b\ln\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(a+b*arctan(c*x))/x^2,x)
```

```
[Out] 1/3*a*x^3*e^2+2*a*e*d*x-a*d^2/x+1/3*b*arctan(c*x)*x^3*e^2+2*b*arctan(c*x)*e
*d*x-b*arctan(c*x)*d^2/x-1/6*b*e^2*x^2/c-1/2*c*b*ln(c^2*x^2+1)*d^2-b/c*ln(c
^2*x^2+1)*e*d+1/6*b/c^3*ln(c^2*x^2+1)*e^2+c*b*d^2*ln(c*x)
```

Maxima [A] time = 0.970993, size = 176, normalized size = 1.61

$$\frac{1}{3}\,ae^2x^3-\frac{1}{2}\left(c\left(\log\left(c^2x^2+1\right)-\log\left(x^2\right)\right)+\frac{2\,\arctan(cx)}{x}\right)bd^2+\frac{1}{6}\left(2\,x^3\arctan(cx)-c\left(\frac{x^2}{c^2}-\frac{\log\left(c^2x^2+1\right)}{c^4}\right)\right)be^2+2\,a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arctan(c*x))/x^2,x, algorithm="maxima")
```

```
[Out] 1/3*a*e^2*x^3 - 1/2*(c*(log(c^2*x^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*b*d
^2 + 1/6*(2*x^3*arctan(c*x) - c*(x^2/c^2 - log(c^2*x^2 + 1)/c^4))*b*e^2 + 2
*a*d*e*x + (2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*b*d*e/c - a*d^2/x
```

Fricas [A] time = 1.59273, size = 302, normalized size = 2.77

$$\frac{2\,ac^3e^2x^4+6\,bc^4d^2x\log(x)+12\,ac^3dex^2-bc^2e^2x^3-6\,ac^3d^2-\left(3\,bc^4d^2+6\,bc^2de-be^2\right)x\log\left(c^2x^2+1\right)+2\left(bc^3e^2x^4+6\right.}{6\,c^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arctan(c*x))/x^2,x, algorithm="fricas")
```

```
[Out] 1/6*(2*a*c^3*e^2*x^4 + 6*b*c^4*d^2*x*log(x) + 12*a*c^3*d*e*x^2 - b*c^2*e^2*
x^3 - 6*a*c^3*d^2 - (3*b*c^4*d^2 + 6*b*c^2*d*e - b*e^2)*x*log(c^2*x^2 + 1)
+ 2*(b*c^3*e^2*x^4 + 6*b*c^3*d*e*x^2 - 3*b*c^3*d^2)*arctan(c*x))/(c^3*x)
```

Sympy [A] time = 2.5094, size = 165, normalized size = 1.51

$$\begin{cases} -\frac{ad^2}{x} + 2adex + \frac{ae^2x^3}{3} + bcd^2\log(x) - \frac{bcd^2\log\left(x^2+\frac{1}{c^2}\right)}{2} - \frac{bd^2\operatorname{atan}(cx)}{x} + 2bdex\operatorname{atan}(cx) + \frac{be^2x^3\operatorname{atan}(cx)}{3} - \frac{bde\log\left(x^2+\frac{1}{c^2}\right)}{c} - \frac{be^2}{6} \\ a\left(-\frac{d^2}{x} + 2dex + \frac{e^2x^3}{3}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(a+b*atan(c*x))/x**2,x)
```

```
[Out] Piecewise((-a*d**2/x + 2*a*d*e*x + a*e**2*x**3/3 + b*c*d**2*log(x) - b*c*d*
*2*log(x**2 + c**(-2))/2 - b*d**2*atan(c*x)/x + 2*b*d*e*x*atan(c*x) + b*e**
2*x**3*atan(c*x)/3 - b*d*e*log(x**2 + c**(-2))/c - b*e**2*x**2/(6*c) + b*e*
*2*log(x**2 + c**(-2))/(6*c**3), Ne(c, 0)), (a*(-d**2/x + 2*d*e*x + e**2*x*
*3/3), True))
```

Giac [A] time = 1.08155, size = 220, normalized size = 2.02

$$\frac{2\,bc^3x^4\arctan(cx)\,e^2 + 2\,ac^3x^4e^2 + 12\,bc^3dx^2\arctan(cx)\,e - 3\,bc^4d^2x\log\left(c^2x^2+1\right) + 6\,bc^4d^2x\log(x) + 12\,ac^3dx^2e - 6}{6\,c^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arctan(c*x))/x^2,x, algorithm="giac")
```

```
[Out] 1/6*(2*b*c^3*x^4*arctan(c*x)*e^2 + 2*a*c^3*x^4*e^2 + 12*b*c^3*d*x^2*arctan(
c*x)*e - 3*b*c^4*d^2*x*log(c^2*x^2 + 1) + 6*b*c^4*d^2*x*log(x) + 12*a*c^3*d
*x^2*e - 6*b*c^3*d^2*arctan(c*x) - b*c^2*x^3*e^2 - 6*b*c^2*d*x*e*log(c^2*x^
2 + 1) - 6*a*c^3*d^2 + b*x*e^2*log(c^2*x^2 + 1))/(c^3*x)
```

3.1131 $\int \frac{(d+ex^2)^2(a+b\tan^{-1}(cx))}{x^3}\,dx$

Optimal. Leaf size=128

$$ibde\text{PolyLog}(2,-icx) - ibde\text{PolyLog}(2,icx) - \frac{d^2\left(a+b\tan^{-1}(cx)\right)}{2x^2} + \frac{1}{2}e^2x^2\left(a+b\tan^{-1}(cx)\right) + 2ade\log(x) - \frac{1}{2}bc^2d^2\text{ ta}$$

```
[Out] -(b*c*d^2)/(2*x) - (b*e^2*x)/(2*c) - (b*c^2*d^2*ArcTan[c*x])/2 + (b*e^2*Arc
Tan[c*x])/(2*c^2) - (d^2*(a + b*ArcTan[c*x]))/(2*x^2) + (e^2*x^2*(a + b*Arc
Tan[c*x]))/2 + 2*a*d*e*Log[x] + I*b*d*e*PolyLog[2, (-I)*c*x] - I*b*d*e*Poly
Log[2, I*c*x]
```

Rubi [A] time = 0.163266, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4980, 4852, 325, 203, 4848, 2391, 321}

$$ibde\text{PolyLog}(2,-icx) - ibde\text{PolyLog}(2,icx) - \frac{d^2\left(a+b\tan^{-1}(cx)\right)}{2x^2} + \frac{1}{2}e^2x^2\left(a+b\tan^{-1}(cx)\right) + 2ade\log(x) - \frac{1}{2}bc^2d^2\text{ ta}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^2*(a + b*ArcTan[c*x]))/x^3,x]
```


```
[Out] -(b*c*d^2)/(2*x) - (b*e^2*x)/(2*c) - (b*c^2*d^2*ArcTan[c*x])/2 + (b*e^2*Arc
Tan[c*x])/(2*c^2) - (d^2*(a + b*ArcTan[c*x]))/(2*x^2) + (e^2*x^2*(a + b*Arc
Tan[c*x]))/2 + 2*a*d*e*Log[x] + I*b*d*e*PolyLog[2, (-I)*c*x] - I*b*d*e*Poly
Log[2, I*c*x]
```


Rule 4980

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
```

```
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{(d+ex^2)^2 (a+b\tan^{-1}(cx))}{x^3}\,dx = \int \left(\frac{d^2 (a+b\tan^{-1}(cx))}{x^3} + \frac{2de(a+b\tan^{-1}(cx))}{x} + e^2x(a+b\tan^{-1}(cx))\right)dx$$
$$= d^2\int \frac{a+b\tan^{-1}(cx)}{x^3}\,dx + (2de)\int \frac{a+b\tan^{-1}(cx)}{x}\,dx + e^2\int x(a+b\tan^{-1}(cx))\,d$$
$$= -\frac{d^2(a+b\tan^{-1}(cx))}{2x^2} + \frac{1}{2}e^2x^2(a+b\tan^{-1}(cx)) + 2ade\log(x) + \frac{1}{2}(bcd^2)\int \frac{}{x^2(1}$$
$$= -\frac{bcd^2}{2x} - \frac{be^2x}{2c} - \frac{d^2(a+b\tan^{-1}(cx))}{2x^2} + \frac{1}{2}e^2x^2(a+b\tan^{-1}(cx)) + 2ade\log(x) + ibd$$
$$= -\frac{bcd^2}{2x} - \frac{be^2x}{2c} - \frac{1}{2}bc^2d^2\tan^{-1}(cx) + \frac{be^2\tan^{-1}(cx)}{2c^2} - \frac{d^2(a+b\tan^{-1}(cx))}{2x^2} + \frac{1}{2}e^2x^2($$

Mathematica [C] time = 0.102198, size = 118, normalized size = 0.92

$$\frac{1}{2}\left(-\frac{bcd^2\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-c^2x^2\right)}{x} + 2ibde\text{PolyLog}(2,-icx) - 2ibde\text{PolyLog}(2,icx) - \frac{d^2(a+b\tan^{-1}(cx)}{x^2}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + e*x^2)^2*(a + b*ArcTan[c*x]))/x^3,x]
```

```
[Out] (-((b*e^2*(c*x - ArcTan[c*x]))/c^2) - (d^2*(a + b*ArcTan[c*x]))/x^2 + e^2*x
^2*(a + b*ArcTan[c*x]) - (b*c*d^2*Hypergeometric2F1[-1/2, 1, 1/2, -(c^2*x^2
)])/x + 4*a*d*e*Log[x] + (2*I)*b*d*e*PolyLog[2, (-I)*c*x] - (2*I)*b*d*e*Pol
yLog[2, I*c*x])/2
```

Maple [A] time = 0.058, size = 178, normalized size = 1.4

$$\frac{ax^2e^2}{2} - \frac{ad^2}{2x^2} + 2aed\ln(cx) + \frac{b\arctan(cx)x^2e^2}{2} - \frac{bd^2\arctan(cx)}{2x^2} + 2b\arctan(cx)ed\ln(cx) - \frac{be^2x}{2c} - \frac{bc^2d^2\arctan(c}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(a+b*arctan(c*x))/x^3,x)
```

```
[Out] 1/2*a*x^2*e^2-1/2*a*d^2/x^2+2*a*e*d*ln(c*x)+1/2*b*arctan(c*x)*x^2*e^2-1/2*b
*arctan(c*x)*d^2/x^2+2*b*arctan(c*x)*e*d*ln(c*x)-1/2*b*e^2*x/c-1/2*b*c^2*d^
```

```
2*arctan(c*x)+1/2*b*e^2*arctan(c*x)/c^2-1/2*b*c*d^2/x+I*b*e*d*ln(c*x)*ln(1+
I*c*x)-I*b*e*d*ln(c*x)*ln(1-I*c*x)+I*b*e*d*dilog(1+I*c*x)-I*b*e*d*dilog(1-I
*c*x)
```

Maxima [A] time = 2.14647, size = 223, normalized size = 1.74

$$\frac{1}{2}ae^2x^2 - \frac{1}{2}\left(\left(c\arctan(cx) + \frac{1}{x}\right)c + \frac{\arctan(cx)}{x^2}\right)bd^2 + 2\,ade\log(x) - \frac{ad^2}{2\,x^2} - \frac{\pi bc^2de\log\left(c^2x^2+1\right) - 4\,bc^2de\arctan(c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arctan(c*x))/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*a*e^2*x^2 - 1/2*((c*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*b*d^2 + 2*a
*d*e*log(x) - 1/2*a*d^2/x^2 - 1/2*(pi*b*c^2*d*e*log(c^2*x^2 + 1) - 4*b*c^2*
d*e*arctan(c*x)*log(x*abs(c)) + 2*I*b*c^2*d*e*dilog(I*c*x + 1) - 2*I*b*c^2*
d*e*dilog(-I*c*x + 1) + b*c*e^2*x - (b*c^2*e^2*x^2 + 4*I*b*c^2*d*e*arctan2(
0, c) + b*e^2)*arctan(c*x))/c^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{ae^2x^4 + 2\,adex^2 + ad^2 + \left(be^2x^4 + 2\,bdex^2 + bd^2\right)\arctan(cx)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arctan(c*x))/x^3,x, algorithm="fricas")
```

```
[Out] integral((a*e^2*x^4 + 2*a*d*e*x^2 + a*d^2 + (b*e^2*x^4 + 2*b*d*e*x^2 + b*d^
2)*arctan(c*x))/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b\,\mathrm{atan}(cx))\left(d + ex^2\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(a+b*atan(c*x))/x**3,x)
```

```
[Out] Integral((a + b*atan(c*x))*(d + e*x**2)**2/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2+d\right)^2 (b\arctan(cx)+a)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arctan(c*x))/x^3,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^2*(b*arctan(c*x) + a)/x^3, x)
```

3.1132 $\int \frac{(d+ex^2)^2(a+b\tan^{-1}(cx))}{x^4}\,dx$

Optimal. Leaf size=115

$$-\frac{d^2\left(a+b\tan^{-1}(cx)\right)}{3x^3}-\frac{2de\left(a+b\tan^{-1}(cx)\right)}{x}+e^2x\left(a+b\tan^{-1}(cx)\right)+\frac{b\left(c^4d^2-6c^2de-3e^2\right)\log\left(c^2x^2+1\right)}{6c}-\frac{1}{3}bcd\,\mathrm{lo}$$

```
[Out] -(b*c*d^2)/(6*x^2) - (d^2*(a + b*ArcTan[c*x]))/(3*x^3) - (2*d*e*(a + b*ArcT
an[c*x]))/x + e^2*x*(a + b*ArcTan[c*x]) - (b*c*d*(c^2*d - 6*e)*Log[x])/3 +
(b*(c^4*d^2 - 6*c^2*d*e - 3*e^2)*Log[1 + c^2*x^2])/(6*c)
```

Rubi [A] time = 0.168281, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {270, 4976, 12, 1251, 893}

$$-\frac{d^2\left(a+b\tan^{-1}(cx)\right)}{3x^3}-\frac{2de\left(a+b\tan^{-1}(cx)\right)}{x}+e^2x\left(a+b\tan^{-1}(cx)\right)+\frac{b\left(c^4d^2-6c^2de-3e^2\right)\log\left(c^2x^2+1\right)}{6c}-\frac{1}{3}bcd\,\mathrm{lo}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^2*(a + b*ArcTan[c*x]))/x^4,x]
```

```
[Out] -(b*c*d^2)/(6*x^2) - (d^2*(a + b*ArcTan[c*x]))/(3*x^3) - (2*d*e*(a + b*ArcT
an[c*x]))/x + e^2*x*(a + b*ArcTan[c*x]) - (b*c*d*(c^2*d - 6*e)*Log[x])/3 +
(b*(c^4*d^2 - 6*c^2*d*e - 3*e^2)*Log[1 + c^2*x^2])/(6*c)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 4976

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Dis
t[a + b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&  !(
ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(IL
tQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILtQ[(m
```

```
 - 1)/2, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(d+ex^2\right)^2\left(a+b\tan^{-1}(cx)\right)}{x^4}\,dx &= -\frac{d^2\left(a+b\tan^{-1}(cx)\right)}{3x^3}-\frac{2de\left(a+b\tan^{-1}(cx)\right)}{x}+e^2x\left(a+b\tan^{-1}(cx)\right)-(bc)\int \frac{-d^2
\\
&= -\frac{d^2\left(a+b\tan^{-1}(cx)\right)}{3x^3}-\frac{2de\left(a+b\tan^{-1}(cx)\right)}{x}+e^2x\left(a+b\tan^{-1}(cx)\right)-\frac{1}{3}(bc)\int \frac{-
\\
&= -\frac{d^2\left(a+b\tan^{-1}(cx)\right)}{3x^3}-\frac{2de\left(a+b\tan^{-1}(cx)\right)}{x}+e^2x\left(a+b\tan^{-1}(cx)\right)-\frac{1}{6}(bc)\,\mathrm{Subs}
\\
&= -\frac{d^2\left(a+b\tan^{-1}(cx)\right)}{3x^3}-\frac{2de\left(a+b\tan^{-1}(cx)\right)}{x}+e^2x\left(a+b\tan^{-1}(cx)\right)-\frac{1}{6}(bc)\,\mathrm{Subs}
\\
&= -\frac{bcd^2}{6x^2}-\frac{d^2\left(a+b\tan^{-1}(cx)\right)}{3x^3}-\frac{2de\left(a+b\tan^{-1}(cx)\right)}{x}+e^2x\left(a+b\tan^{-1}(cx)\right)-\frac{1}{3}b
\end{aligned}
$$

Mathematica [A] time = 0.115644, size = 119, normalized size = 1.03

$$\frac{1}{6}\left(-\frac{2ad^2}{x^3}-\frac{12ade}{x}+6ae^2x+\frac{b\left(c^4d^2-6c^2de-3e^2\right)\log\left(c^2x^2+1\right)}{c}-2bcd\log(x)\left(c^2d-6e\right)-\frac{2b\tan^{-1}(cx)\left(d^2+6dex^2\right.}{x^3}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)^2*(a + b*ArcTan[c*x]))/x^4,x]
```

```
[Out] ((-2*a*d^2)/x^3 - (b*c*d^2)/x^2 - (12*a*d*e)/x + 6*a*e^2*x - (2*b*(d^2 + 6*
d*e*x^2 - 3*e^2*x^4)*ArcTan[c*x])/x^3 - 2*b*c*d*(c^2*d - 6*e)*Log[x] + (b*(
c^4*d^2 - 6*c^2*d*e - 3*e^2)*Log[1 + c^2*x^2])/c)/6
```

Maple [A] time = 0.046, size = 147, normalized size = 1.3

$$axe^2-2\,\frac{aed}{x}-\frac{ad^2}{3\,x^3}+b\arctan(cx)\,xe^2-2\,\frac{b\arctan(cx)\,ed}{x}-\frac{bd^2\arctan(cx)}{3\,x^3}+\frac{c^3b\ln\left(c^2x^2+1\right)d^2}{6}-cb\ln\left(c^2x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(a+b*arctan(c*x))/x^4,x)
```

```
[Out] a*x*e^2-2*a*e*d/x-1/3*a*d^2/x^3+b*arctan(c*x)*x*e^2-2*b*arctan(c*x)*e*d/x-1
/3*b*arctan(c*x)*d^2/x^3+1/6*c^3*b*ln(c^2*x^2+1)*d^2-c*b*ln(c^2*x^2+1)*e*d-
1/2/c*b*ln(c^2*x^2+1)*e^2-1/3*c^3*b*d^2*ln(c*x)+2*c*b*ln(c*x)*d*e-1/6*b*c*d
^2/x^2
```

Maxima [A] time = 0.963767, size = 182, normalized size = 1.58

$$\frac{1}{6}\left(\left(\left(c^2\log\left(c^2x^2+1\right)-c^2\log\left(x^2\right)-\frac{1}{x^2}\right)c-\frac{2\arctan(cx)}{x^3}\right)bd^2-\left(c\left(\log\left(c^2x^2+1\right)-\log\left(x^2\right)\right)+\frac{2\arctan(cx)}{x}\right)bde+a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arctan(c*x))/x^4,x, algorithm="maxima")
```

```
[Out] 1/6*((c^2*log(c^2*x^2 + 1) - c^2*log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^3)*b
*d^2 - (c*(log(c^2*x^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*b*d*e + a*e^2*x
```

```
+ 1/2*(2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*b*e^2/c - 2*a*d*e/x - 1/3*a*d^
2/x^3
```

Fricas [A] time = 1.38202, size = 311, normalized size = 2.7

$$\frac{6\,ace^2x^4 - bc^2d^2x - 12\,acdex^2 + \left(bc^4d^2 - 6\,bc^2de - 3\,be^2\right)x^3\log\left(c^2x^2+1\right) - 2\left(bc^4d^2 - 6\,bc^2de\right)x^3\log\left(x\right) - 2\,acd^2 + 2\left(3\,b\right.}{6\,cx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arctan(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*(6*a*c*e^2*x^4 - b*c^2*d^2*x - 12*a*c*d*e*x^2 + (b*c^4*d^2 - 6*b*c^2*d*
e - 3*b*e^2)*x^3*log(c^2*x^2 + 1) - 2*(b*c^4*d^2 - 6*b*c^2*d*e)*x^3*log(x)
- 2*a*c*d^2 + 2*(3*b*c*e^2*x^4 - 6*b*c*d*e*x^2 - b*c*d^2)*arctan(c*x))/(c*x
^3)
```

Sympy [A] time = 2.52562, size = 180, normalized size = 1.57

$$\begin{cases} -\frac{ad^2}{3x^3} - \frac{2ade}{x} + ae^2x - \frac{bc^3d^2\log\left(x\right)}{3} + \frac{bc^3d^2\log\left(x^2+\frac{1}{c^2}\right)}{6} - \frac{bcd^2}{6x^2} + 2bcde\log\left(x\right) - bcde\log\left(x^2+\frac{1}{c^2}\right) - \frac{bd^2\operatorname{atan}\left(cx\right)}{3x^3} - \frac{2bde\operatorname{atan}\left(cx\right)}{x} + \\ a\left(-\frac{d^2}{3x^3} - \frac{2de}{x} + e^2x\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(a+b*atan(c*x))/x**4,x)
```

```
[Out] Piecewise((-a*d**2/(3*x**3) - 2*a*d*e/x + a*e**2*x - b*c**3*d**2*log(x)/3 +
 b*c**3*d**2*log(x**2 + c**(-2))/6 - b*c*d**2/(6*x**2) + 2*b*c*d*e*log(x) -
 b*c*d*e*log(x**2 + c**(-2)) - b*d**2*atan(c*x)/(3*x**3) - 2*b*d*e*atan(c*x
)/x + b*e**2*x*atan(c*x) - b*e**2*log(x**2 + c**(-2))/(2*c), Ne(c, 0)), (a*
(-d**2/(3*x**3) - 2*d*e/x + e**2*x), True))
```

Giac [A] time = 1.09679, size = 232, normalized size = 2.02

$$\frac{bc^4d^2x^3\log\left(c^2x^2+1\right) - 2\,bc^4d^2x^3\log\left(x\right) - 6\,bc^2dx^3e\log\left(c^2x^2+1\right) + 12\,bc^2dx^3e\log\left(x\right) + 6\,bcx^4\arctan\left(cx\right)e^2 + 6\,acx^4e}{6\,cx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arctan(c*x))/x^4,x, algorithm="giac")
```

```
[Out] 1/6*(b*c^4*d^2*x^3*log(c^2*x^2 + 1) - 2*b*c^4*d^2*x^3*log(x) - 6*b*c^2*d*x^
3*e*log(c^2*x^2 + 1) + 12*b*c^2*d*x^3*e*log(x) + 6*b*c*x^4*arctan(c*x)*e^2
+ 6*a*c*x^4*e^2 - 12*b*c*d*x^2*arctan(c*x)*e - b*c^2*d^2*x - 12*a*c*d*x^2*e
 - 3*b*x^3*e^2*log(c^2*x^2 + 1) - 2*b*c*d^2*arctan(c*x) - 2*a*c*d^2)/(c*x^3
)
```

3.1133 $\int \frac{(d+ex^2)^2(a+b\tan^{-1}(cx))}{x^5}\,dx$

Optimal. Leaf size=139

$$\frac{1}{2}ibe^2\text{PolyLog}(2,-icx)-\frac{1}{2}ibe^2\text{PolyLog}(2,icx)-\frac{d^2\left(a+b\tan^{-1}(cx)\right)}{4x^4}-\frac{de\left(a+b\tan^{-1}(cx)\right)}{x^2}+ae^2\log(x)+\frac{bc^3d^2}{4x}+\frac{1}{4}b$$

```
[Out] -(b*c*d^2)/(12*x^3) + (b*c^3*d^2)/(4*x) - (b*c*d*e)/x + (b*c^4*d^2*ArcTan[c
*x])/4 - b*c^2*d*e*ArcTan[c*x] - (d^2*(a + b*ArcTan[c*x]))/(4*x^4) - (d*e*(
a + b*ArcTan[c*x]))/x^2 + a*e^2*Log[x] + (I/2)*b*e^2*PolyLog[2, (-I)*c*x] -
 (I/2)*b*e^2*PolyLog[2, I*c*x]
```

Rubi [A] time = 0.168279, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4980, 4852, 325, 203, 4848, 2391}

$$\frac{1}{2}ibe^2\text{PolyLog}(2,-icx)-\frac{1}{2}ibe^2\text{PolyLog}(2,icx)-\frac{d^2\left(a+b\tan^{-1}(cx)\right)}{4x^4}-\frac{de\left(a+b\tan^{-1}(cx)\right)}{x^2}+ae^2\log(x)+\frac{bc^3d^2}{4x}+\frac{1}{4}b$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^2*(a + b*ArcTan[c*x]))/x^5,x]
```


```
[Out] -(b*c*d^2)/(12*x^3) + (b*c^3*d^2)/(4*x) - (b*c*d*e)/x + (b*c^4*d^2*ArcTan[c
*x])/4 - b*c^2*d*e*ArcTan[c*x] - (d^2*(a + b*ArcTan[c*x]))/(4*x^4) - (d*e*(
a + b*ArcTan[c*x]))/x^2 + a*e^2*Log[x] + (I/2)*b*e^2*PolyLog[2, (-I)*c*x] -
 (I/2)*b*e^2*PolyLog[2, I*c*x]
```


Rule 4980

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
```

```
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{(d+ex^2)^2 (a+b\tan^{-1}(cx))}{x^5}\,dx = \int \left(\frac{d^2 (a+b\tan^{-1}(cx))}{x^5} + \frac{2de (a+b\tan^{-1}(cx))}{x^3} + \frac{e^2 (a+b\tan^{-1}(cx))}{x}\right) dx$$
$$= d^2 \int \frac{a+b\tan^{-1}(cx)}{x^5}\,dx + (2de) \int \frac{a+b\tan^{-1}(cx)}{x^3}\,dx + e^2 \int \frac{a+b\tan^{-1}(cx)}{x}\,dx$$
$$= -\frac{d^2 (a+b\tan^{-1}(cx))}{4x^4} - \frac{de (a+b\tan^{-1}(cx))}{x^2} + ae^2 \log(x) + \frac{1}{4}\left(bcd^2\right) \int \frac{1}{x^4 (1+c^2x$$
$$= -\frac{bcd^2}{12x^3} - \frac{bcde}{x} - \frac{d^2 (a+b\tan^{-1}(cx))}{4x^4} - \frac{de (a+b\tan^{-1}(cx))}{x^2} + ae^2 \log(x) + \frac{1}{2} ibe^2\text{Li}$$
$$= -\frac{bcd^2}{12x^3} + \frac{bc^3d^2}{4x} - \frac{bcde}{x} - bc^2de\tan^{-1}(cx) - \frac{d^2 (a+b\tan^{-1}(cx))}{4x^4} - \frac{de (a+b\tan^{-1}(c}{x^2}$$
$$= -\frac{bcd^2}{12x^3} + \frac{bc^3d^2}{4x} - \frac{bcde}{x} + \frac{1}{4} bc^4d^2\tan^{-1}(cx) - bc^2de\tan^{-1}(cx) - \frac{d^2 (a+b\tan^{-1}(cx)}{4x^4}$$

Mathematica [C] time = 0.0945496, size = 130, normalized size = 0.94

$$-\frac{bcd^2 \text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-c^2x^2\right)}{12x^3} - \frac{bcde\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-c^2x^2\right)}{x} + \frac{1}{2} ibe^2\text{PolyLog}(2,-icx)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + e*x^2)^2*(a + b*ArcTan[c*x]))/x^5,x]
```

```
[Out] -(d^2*(a + b*ArcTan[c*x]))/(4*x^4) - (d*e*(a + b*ArcTan[c*x]))/x^2 - (b*c*d
^2*Hypergeometric2F1[-3/2, 1, -1/2, -(c^2*x^2)])/(12*x^3) - (b*c*d*e*Hyperg
eometric2F1[-1/2, 1, 1/2, -(c^2*x^2)])/x + a*e^2*Log[x] + (I/2)*b*e^2*PolyL
og[2, (-I)*c*x] - (I/2)*b*e^2*PolyLog[2, I*c*x]
```

Maple [A] time = 0.059, size = 190, normalized size = 1.4

$$-\frac{aed}{x^2} - \frac{ad^2}{4\,x^4} + ae^2\ln(cx) - \frac{b\arctan(cx)\,ed}{x^2} - \frac{bd^2\arctan(cx)}{4\,x^4} + b\arctan(cx)\,e^2\ln(cx) + \frac{i}{2}be^2\ln(cx)\ln(1+icx) - \frac{i}{2}be$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(a+b*arctan(c*x))/x^5,x)
```

```
[Out] -a*e*d/x^2-1/4*a*d^2/x^4+a*e^2*ln(c*x)-b*arctan(c*x)*e*d/x^2-1/4*b*arctan(c
*x)*d^2/x^4+b*arctan(c*x)*e^2*ln(c*x)+1/2*I*b*e^2*ln(c*x)*ln(1+I*c*x)-1/2*I
*b*e^2*ln(c*x)*ln(1-I*c*x)+1/2*I*b*e^2*dilog(1+I*c*x)-1/2*I*b*e^2*dilog(1-I
*c*x)+1/4*b*c^4*d^2*arctan(c*x)-b*c^2*d*e*arctan(c*x)+1/4*b*c^3*d^2/x-b*c*d
*e/x-1/12*b*c*d^2/x^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{12}\left(\left(3\,c^3\arctan\left(cx\right)+\frac{3\,c^2x^2-1}{x^3}\right)c-\frac{3\,\arctan\left(cx\right)}{x^4}\right)bd^2-\left(\left(c\arctan\left(cx\right)+\frac{1}{x}\right)c+\frac{\arctan\left(cx\right)}{x^2}\right)bde+be^2\int\frac{\arctan\left(\right.}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arctan(c*x))/x^5,x, algorithm="maxima")
```

```
[Out] 1/12*((3*c^3*arctan(c*x) + (3*c^2*x^2 - 1)/x^3)*c - 3*arctan(c*x)/x^4)*b*d^
2 - ((c*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*b*d*e + b*e^2*integrate(arc
tan(c*x)/x, x) + a*e^2*log(x) - a*d*e/x^2 - 1/4*a*d^2/x^4
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{ae^2x^4+2\,adex^2+ad^2+\left(be^2x^4+2\,bdex^2+bd^2\right)\arctan\left(cx\right)}{x^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arctan(c*x))/x^5,x, algorithm="fricas")
```

```
[Out] integral((a*e^2*x^4 + 2*a*d*e*x^2 + a*d^2 + (b*e^2*x^4 + 2*b*d*e*x^2 + b*d^
2)*arctan(c*x))/x^5, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a+b\,\text{atan}\left(cx\right)\right)\left(d+ex^2\right)^2}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(a+b*atan(c*x))/x**5,x)
```

```
[Out] Integral((a + b*atan(c*x))*(d + e*x**2)**2/x**5, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2 + d)^2 (b \arctan(cx) + a)}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arctan(c*x))/x^5,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^2*(b*arctan(c*x) + a)/x^5, x)
```

3.1134 $\int \frac{(d+ex^2)^2 (a+b\tan^{-1}(cx))}{x^6} dx$

Optimal. Leaf size=150

$$-\frac{d^2 \left(a+b\tan^{-1}(cx)\right)}{5x^5}-\frac{2de\left(a+b\tan^{-1}(cx)\right)}{3x^3}-\frac{e^2\left(a+b\tan^{-1}(cx)\right)}{x}-\frac{1}{30}bc\left(3c^4d^2-10c^2de+15e^2\right)\log\left(c^2x^2+1\right)+$$

```
[Out] -(b*c*d^2)/(20*x^4) + (b*c*d*(3*c^2*d - 10*e))/(30*x^2) - (d^2*(a + b*ArcTa
n[c*x]))/(5*x^5) - (2*d*e*(a + b*ArcTan[c*x]))/(3*x^3) - (e^2*(a + b*ArcTan
[c*x]))/x + (b*c*(3*c^4*d^2 - 10*c^2*d*e + 15*e^2)*Log[x])/15 - (b*c*(3*c^4
*d^2 - 10*c^2*d*e + 15*e^2)*Log[1 + c^2*x^2])/30
```

Rubi [A] time = 0.185044, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {270, 4976, 12, 1251, 893}

$$-\frac{d^2 \left(a+b\tan^{-1}(cx)\right)}{5x^5}-\frac{2de\left(a+b\tan^{-1}(cx)\right)}{3x^3}-\frac{e^2\left(a+b\tan^{-1}(cx)\right)}{x}-\frac{1}{30}bc\left(3c^4d^2-10c^2de+15e^2\right)\log\left(c^2x^2+1\right)+$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^2*(a + b*ArcTan[c*x]))/x^6,x]
```


```
[Out] -(b*c*d^2)/(20*x^4) + (b*c*d*(3*c^2*d - 10*e))/(30*x^2) - (d^2*(a + b*ArcTa
n[c*x]))/(5*x^5) - (2*d*e*(a + b*ArcTan[c*x]))/(3*x^3) - (e^2*(a + b*ArcTan
[c*x]))/x + (b*c*(3*c^4*d^2 - 10*c^2*d*e + 15*e^2)*Log[x])/15 - (b*c*(3*c^4
*d^2 - 10*c^2*d*e + 15*e^2)*Log[1 + c^2*x^2])/30
```


Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 4976

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Dis
t[a + b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&  !(
ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(IL
```

```
tQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILtQ[(m
 - 1)/2, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^2\left(a+b\tan^{-1}(cx)\right)}{x^6}\,dx = -\frac{d^2\left(a+b\tan^{-1}(cx)\right)}{5x^5}-\frac{2de\left(a+b\tan^{-1}(cx)\right)}{3x^3}-\frac{e^2\left(a+b\tan^{-1}(cx)\right)}{x}-(bc)\int\frac{-3d$$

$$= -\frac{d^2\left(a+b\tan^{-1}(cx)\right)}{5x^5}-\frac{2de\left(a+b\tan^{-1}(cx)\right)}{3x^3}-\frac{e^2\left(a+b\tan^{-1}(cx)\right)}{x}-\frac{1}{15}(bc)\int$$

$$= -\frac{d^2\left(a+b\tan^{-1}(cx)\right)}{5x^5}-\frac{2de\left(a+b\tan^{-1}(cx)\right)}{3x^3}-\frac{e^2\left(a+b\tan^{-1}(cx)\right)}{x}-\frac{1}{30}(bc)\,\mathrm{Sub}$$

$$= -\frac{d^2\left(a+b\tan^{-1}(cx)\right)}{5x^5}-\frac{2de\left(a+b\tan^{-1}(cx)\right)}{3x^3}-\frac{e^2\left(a+b\tan^{-1}(cx)\right)}{x}-\frac{1}{30}(bc)\,\mathrm{Sub}$$

$$= -\frac{bcd^2}{20x^4}+\frac{bcd\left(3c^2d-10e\right)}{30x^2}-\frac{d^2\left(a+b\tan^{-1}(cx)\right)}{5x^5}-\frac{2de\left(a+b\tan^{-1}(cx)\right)}{3x^3}-\frac{e^2\left(a+\right.}{}$$

Mathematica [A] time = 0.128078, size = 149, normalized size = 0.99

$$\frac{1}{60}\left(-\frac{12d^2\left(a+b\tan^{-1}(cx)\right)}{x^5}-\frac{40de\left(a+b\tan^{-1}(cx)\right)}{x^3}-\frac{60e^2\left(a+b\tan^{-1}(cx)\right)}{x}-3bcd^2\left(-\frac{2c^2}{x^2}+2c^4\log\left(c^2x^2+1\right)-4\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)^2*(a + b*ArcTan[c*x]))/x^6,x]
```

```
[Out] ((-12*d^2*(a + b*ArcTan[c*x]))/x^5 - (40*d*e*(a + b*ArcTan[c*x]))/x^3 - (60
*e^2*(a + b*ArcTan[c*x]))/x + 30*b*c*e^2*(2*Log[x] - Log[1 + c^2*x^2]) - 20
*b*c*d*e*(x^(-2) + 2*c^2*Log[x] - c^2*Log[1 + c^2*x^2]) - 3*b*c*d^2*(x^(-4)
 - (2*c^2)/x^2 - 4*c^4*Log[x] + 2*c^4*Log[1 + c^2*x^2]))/60
```

Maple [A] time = 0.049, size = 186, normalized size = 1.2

$$-\frac{ae^2}{x}-\frac{ad^2}{5x^5}-\frac{2aed}{3x^3}-\frac{b\arctan(cx)e^2}{x}-\frac{bd^2\arctan(cx)}{5x^5}-\frac{2b\arctan(cx)ed}{3x^3}-\frac{c^5b\ln\left(c^2x^2+1\right)d^2}{10}+\frac{c^3b\ln\left(c^2x^2+1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(a+b*arctan(c*x))/x^6,x)
```

```
[Out] -a*e^2/x-1/5*a*d^2/x^5-2/3*a*e*d/x^3-b*arctan(c*x)*e^2/x-1/5*b*arctan(c*x)*
d^2/x^5-2/3*b*arctan(c*x)*e*d/x^3-1/10*c^5*b*ln(c^2*x^2+1)*d^2+1/3*c^3*b*ln
(c^2*x^2+1)*e*d-1/2*c*b*ln(c^2*x^2+1)*e^2+1/5*c^5*b*d^2*ln(c*x)-2/3*c^3*b*l
n(c*x)*d*e+c*b*ln(c*x)*e^2+1/10*b*c^3*d^2/x^2-1/3*c*b*e*d/x^2-1/20*b*c*d^2/
x^4
```

Maxima [A] time = 0.982012, size = 224, normalized size = 1.49

$$-\frac{1}{20}\left(\left(2c^4\log\left(c^2x^2+1\right)-2c^4\log\left(x^2\right)-\frac{2c^2x^2-1}{x^4}\right)c+\frac{4\arctan(cx)}{x^5}\right)bd^2+\frac{1}{3}\left(\left(c^2\log\left(c^2x^2+1\right)-c^2\log\left(x^2\right)-\frac{1}{x^2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arctan(c*x))/x^6,x, algorithm="maxima")
```

```
[Out] -1/20*((2*c^4*log(c^2*x^2 + 1) - 2*c^4*log(x^2) - (2*c^2*x^2 - 1)/x^4)*c +
4*arctan(c*x)/x^5)*b*d^2 + 1/3*((c^2*log(c^2*x^2 + 1) - c^2*log(x^2) - 1/x^
2)*c - 2*arctan(c*x)/x^3)*b*d*e - 1/2*(c*(log(c^2*x^2 + 1) - log(x^2)) + 2*
arctan(c*x)/x)*b*e^2 - a*e^2/x - 2/3*a*d*e/x^3 - 1/5*a*d^2/x^5
```

Fricas [A] time = 1.49382, size = 379, normalized size = 2.53

$$-\frac{60\,ae^2x^4+2\left(3\,bc^5d^2-10\,bc^3de+15\,bce^2\right)x^5\log\left(c^2x^2+1\right)-4\left(3\,bc^5d^2-10\,bc^3de+15\,bce^2\right)x^5\log\left(x\right)+3\,bcd^2x+40\,a}{60\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arctan(c*x))/x^6,x, algorithm="fricas")
```

```
[Out] -1/60*(60*a*e^2*x^4 + 2*(3*b*c^5*d^2 - 10*b*c^3*d*e + 15*b*c*e^2)*x^5*log(c
^2*x^2 + 1) - 4*(3*b*c^5*d^2 - 10*b*c^3*d*e + 15*b*c*e^2)*x^5*log(x) + 3*b*
c*d^2*x + 40*a*d*e*x^2 - 2*(3*b*c^3*d^2 - 10*b*c*d*e)*x^3 + 12*a*d^2 + 4*(1
5*b*e^2*x^4 + 10*b*d*e*x^2 + 3*b*d^2)*arctan(c*x))/x^5
```

Sympy [A] time = 3.43594, size = 235, normalized size = 1.57

$$\begin{cases} -\frac{ad^2}{5x^5}-\frac{2ade}{3x^3}-\frac{ae^2}{x}+\frac{bc^5d^2\log\left(x\right)}{5}-\frac{bc^5d^2\log\left(x^2+\frac{1}{c^2}\right)}{10}+\frac{bc^3d^2}{10x^2}-\frac{2bc^3de\log\left(x\right)}{3}+\frac{bc^3de\log\left(x^2+\frac{1}{c^2}\right)}{3}-\frac{bcd^2}{20x^4}-\frac{bcde}{3x^2}+bce^2\log\left(x\right)-\frac{bce^2\log}{} \\ a\left(-\frac{d^2}{5x^5}-\frac{2de}{3x^3}-\frac{e^2}{x}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(a+b*atan(c*x))/x**6,x)
```

```
[Out] Piecewise((-a*d**2/(5*x**5) - 2*a*d*e/(3*x**3) - a*e**2/x + b*c**5*d**2*log
(x)/5 - b*c**5*d**2*log(x**2 + c**(-2))/10 + b*c**3*d**2/(10*x**2) - 2*b*c*
*3*d*e*log(x)/3 + b*c**3*d*e*log(x**2 + c**(-2))/3 - b*c*d**2/(20*x**4) - b
*c*d*e/(3*x**2) + b*c*e**2*log(x) - b*c*e**2*log(x**2 + c**(-2))/2 - b*d**2
*atan(c*x)/(5*x**5) - 2*b*d*e*atan(c*x)/(3*x**3) - b*e**2*atan(c*x)/x, Ne(c
, 0)), (a*(-d**2/(5*x**5) - 2*d*e/(3*x**3) - e**2/x), True))
```

Giac [A] time = 1.09799, size = 265, normalized size = 1.77

$$-\frac{6\,bc^5d^2x^5\log\left(c^2x^2+1\right)-12\,bc^5d^2x^5\log\left(x\right)-20\,bc^3dx^5e\log\left(c^2x^2+1\right)+40\,bc^3dx^5e\log\left(x\right)-6\,bc^3d^2x^3+30\,bcx^5e^2\log}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arctan(c*x))/x^6,x, algorithm="giac")
```

```
[Out] -1/60*(6*b*c^5*d^2*x^5*log(c^2*x^2 + 1) - 12*b*c^5*d^2*x^5*log(x) - 20*b*c^
3*d*x^5*e*log(c^2*x^2 + 1) + 40*b*c^3*d*x^5*e*log(x) - 6*b*c^3*d^2*x^3 + 30
*b*c*x^5*e^2*log(c^2*x^2 + 1) - 60*b*c*x^5*e^2*log(x) + 60*b*x^4*arctan(c*x
)*e^2 + 20*b*c*d*x^3*e + 60*a*x^4*e^2 + 40*b*d*x^2*arctan(c*x)*e + 3*b*c*d^
2*x + 40*a*d*x^2*e + 12*b*d^2*arctan(c*x) + 12*a*d^2)/x^5
```

3.1135 $\int \frac{(d+ex^2)^2(a+b\tan^{-1}(cx))}{x^7}\,dx$

Optimal. Leaf size=111

$$-\frac{(d+ex^2)^3\left(a+b\tan^{-1}(cx)\right)}{6dx^6}-\frac{bc\left(c^4d^2-3c^2de+3e^2\right)}{6x}+\frac{bcd\left(c^2d-3e\right)}{18x^3}-\frac{b\left(c^2d-e\right)^3\tan^{-1}(cx)}{6d}-\frac{bcd^2}{30x^5}$$

```
[Out] -(b*c*d^2)/(30*x^5) + (b*c*d*(c^2*d - 3*e))/(18*x^3) - (b*c*(c^4*d^2 - 3*c^
2*d*e + 3*e^2))/(6*x) - (b*(c^2*d - e)^3*ArcTan[c*x])/(6*d) - ((d + e*x^2)^
3*(a + b*ArcTan[c*x]))/(6*d*x^6)
```

Rubi [A] time = 0.147815, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {264, 4976, 12, 461, 203}

$$-\frac{(d+ex^2)^3\left(a+b\tan^{-1}(cx)\right)}{6dx^6}-\frac{bc\left(c^4d^2-3c^2de+3e^2\right)}{6x}+\frac{bcd\left(c^2d-3e\right)}{18x^3}-\frac{b\left(c^2d-e\right)^3\tan^{-1}(cx)}{6d}-\frac{bcd^2}{30x^5}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^2*(a + b*ArcTan[c*x]))/x^7,x]
```

```
[Out] -(b*c*d^2)/(30*x^5) + (b*c*d*(c^2*d - 3*e))/(18*x^3) - (b*c*(c^4*d^2 - 3*c^
2*d*e + 3*e^2))/(6*x) - (b*(c^2*d - e)^3*ArcTan[c*x])/(6*d) - ((d + e*x^2)^
3*(a + b*ArcTan[c*x]))/(6*d*x^6)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 4976

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Dis
t[a + b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&  !(
ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(IL
tQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILtQ[(m
```

```
 - 1)/2, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(d+ex^2\right)^2\left(a+b\tan^{-1}(cx)\right)}{x^7}\,dx &= -\frac{\left(d+ex^2\right)^3\left(a+b\tan^{-1}(cx)\right)}{6dx^6}-(bc)\int \frac{\left(d+ex^2\right)^3}{6x^6\left(-d-c^2dx^2\right)}\,dx\\
&= -\frac{\left(d+ex^2\right)^3\left(a+b\tan^{-1}(cx)\right)}{6dx^6}-\frac{1}{6}(bc)\int \frac{\left(d+ex^2\right)^3}{x^6\left(-d-c^2dx^2\right)}\,dx\\
&= -\frac{\left(d+ex^2\right)^3\left(a+b\tan^{-1}(cx)\right)}{6dx^6}-\frac{1}{6}(bc)\int \left(-\frac{d^2}{x^6}+\frac{d\left(c^2d-3e\right)}{x^4}+\frac{-c^4d^2+3c^2de-}{x^2}\right.\\
&= -\frac{bcd^2}{30x^5}+\frac{bcd\left(c^2d-3e\right)}{18x^3}-\frac{bc\left(c^4d^2-3c^2de+3e^2\right)}{6x}-\frac{\left(d+ex^2\right)^3\left(a+b\tan^{-1}(cx)\right)}{6dx^6}\\
&= -\frac{bcd^2}{30x^5}+\frac{bcd\left(c^2d-3e\right)}{18x^3}-\frac{bc\left(c^4d^2-3c^2de+3e^2\right)}{6x}-\frac{b\left(c^2d-e\right)^3\tan^{-1}(cx)}{6d}-\frac{(d+}{}
\end{aligned}$$

Mathematica [C] time = 0.096282, size = 112, normalized size = 1.01

$$-\frac{5\left(bcdex^3\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-c^2x^2\right)+3bce^2x^5\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-c^2x^2\right)+\left(d^2+3dex^2+\right.\right.}{30x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)^2*(a + b*ArcTan[c*x]))/x^7,x]
```

```
[Out] -(b*c*d^2*x*Hypergeometric2F1[-5/2, 1, -3/2, -(c^2*x^2)] + 5*((d^2 + 3*d*e*
x^2 + 3*e^2*x^4)*(a + b*ArcTan[c*x]) + b*c*d*e*x^3*Hypergeometric2F1[-3/2,
1, -1/2, -(c^2*x^2)] + 3*b*c*e^2*x^5*Hypergeometric2F1[-1/2, 1, 1/2, -(c^2*
x^2)]))/(30*x^6)
```

Maple [A] time = 0.048, size = 168, normalized size = 1.5

$$-\frac{ae^2}{2x^2}-\frac{aed}{2x^4}-\frac{ad^2}{6x^6}-\frac{b\arctan(cx)e^2}{2x^2}-\frac{b\arctan(cx)ed}{2x^4}-\frac{bd^2\arctan(cx)}{6x^6}-\frac{c^6b\arctan(cx)d^2}{6}+\frac{c^4b\arctan(cx)ed}{2}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(a+b*arctan(c*x))/x^7,x)
```

```
[Out] -1/2*a*e^2/x^2-1/2*a*e*d/x^4-1/6*a*d^2/x^6-1/2*b*arctan(c*x)*e^2/x^2-1/2*b*
arctan(c*x)*e*d/x^4-1/6*b*arctan(c*x)*d^2/x^6-1/6*c^6*b*arctan(c*x)*d^2+1/2
*c^4*b*arctan(c*x)*e*d-1/2*c^2*b*arctan(c*x)*e^2-1/6*c^5*b*d^2/x+1/2*c^3*b*
e*d/x-1/2*c*b*e^2/x+1/18*c^3*b*d^2/x^3-1/6*c*b*e*d/x^3-1/30*b*c*d^2/x^5
```

Maxima [A] time = 1.5355, size = 196, normalized size = 1.77

$$-\frac{1}{90}\left(\left(15\,c^5\arctan(cx)+\frac{15\,c^4x^4-5\,c^2x^2+3}{x^5}\right)c+\frac{15\arctan(cx)}{x^6}\right)bd^2+\frac{1}{6}\left(\left(3\,c^3\arctan(cx)+\frac{3\,c^2x^2-1}{x^3}\right)c-\frac{3\arctan}{x^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arctan(c*x))/x^7,x, algorithm="maxima")
```

```
[Out] -1/90*((15*c^5*arctan(c*x) + (15*c^4*x^4 - 5*c^2*x^2 + 3)/x^5)*c + 15*arcta
n(c*x)/x^6)*b*d^2 + 1/6*((3*c^3*arctan(c*x) + (3*c^2*x^2 - 1)/x^3)*c - 3*ar
ctan(c*x)/x^4)*b*d*e - 1/2*((c*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*b*e^
2 - 1/2*a*e^2/x^2 - 1/2*a*d*e/x^4 - 1/6*a*d^2/x^6
```

Fricas [A] time = 1.4593, size = 329, normalized size = 2.96

$$-\frac{45\,ae^2x^4+15\left(bc^5d^2-3\,bc^3de+3\,bce^2\right)x^5+3\,bcd^2x+45\,adex^2-5\left(bc^3d^2-3\,bcde\right)x^3+15\,ad^2+15\left(3\,be^2x^4+\left(bc^6d^2\right.\right.}{90\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arctan(c*x))/x^7,x, algorithm="fricas")
```

```
[Out] -1/90*(45*a*e^2*x^4 + 15*(b*c^5*d^2 - 3*b*c^3*d*e + 3*b*c*e^2)*x^5 + 3*b*c*
d^2*x + 45*a*d*e*x^2 - 5*(b*c^3*d^2 - 3*b*c*d*e)*x^3 + 15*a*d^2 + 15*(3*b*e
^2*x^4 + (b*c^6*d^2 - 3*b*c^4*d*e + 3*b*c^2*e^2)*x^6 + 3*b*d*e*x^2 + b*d^2)
*arctan(c*x))/x^6
```

Sympy [A] time = 2.51614, size = 192, normalized size = 1.73

$$-\frac{ad^2}{6x^6}-\frac{ade}{2x^4}-\frac{ae^2}{2x^2}-\frac{bc^6d^2\operatorname{atan}(cx)}{6}-\frac{bc^5d^2}{6x}+\frac{bc^4de\operatorname{atan}(cx)}{2}+\frac{bc^3d^2}{18x^3}+\frac{bc^3de}{2x}-\frac{bc^2e^2\operatorname{atan}(cx)}{2}-\frac{bcd^2}{30x^5}-\frac{bcde}{6x^3}-\frac{b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(a+b*atan(c*x))/x**7,x)
```

```
[Out] -a*d**2/(6*x**6) - a*d*e/(2*x**4) - a*e**2/(2*x**2) - b*c**6*d**2*atan(c*x)
/6 - b*c**5*d**2/(6*x) + b*c**4*d*e*atan(c*x)/2 + b*c**3*d**2/(18*x**3) + b
*c**3*d*e/(2*x) - b*c**2*e**2*atan(c*x)/2 - b*c*d**2/(30*x**5) - b*c*d*e/(6
*x**3) - b*c*e**2/(2*x) - b*d**2*atan(c*x)/(6*x**6) - b*d*e*atan(c*x)/(2*x*
*4) - b*e**2*atan(c*x)/(2*x**2)
```

Giac [A] time = 1.33979, size = 258, normalized size = 2.32

$$-\frac{15\,bc^6d^2x^6\arctan(cx)+45\,\pi bc^4dx^6e\operatorname{sgn}(c)\operatorname{sgn}(x)-45\,bc^4dx^6\arctan(cx)\,e+15\,bc^5d^2x^5+45\,bc^2x^6\arctan(cx)\,e^2-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arctan(c*x))/x^7,x, algorithm="giac")
```

```
[Out] -1/90*(15*b*c^6*d^2*x^6*arctan(c*x) + 45*pi*b*c^4*d*x^6*e*sgn(c)*sgn(x) - 4
5*b*c^4*d*x^6*arctan(c*x)*e + 15*b*c^5*d^2*x^5 + 45*b*c^2*x^6*arctan(c*x)*e
```

```
^2 - 45*b*c^3*d*x^5*e - 5*b*c^3*d^2*x^3 + 45*b*c*x^5*e^2 + 45*b*x^4*arctan(
c*x)*e^2 + 15*b*c*d*x^3*e + 45*a*x^4*e^2 + 45*b*d*x^2*arctan(c*x)*e + 3*b*c
*d^2*x + 45*a*d*x^2*e + 15*b*d^2*arctan(c*x) + 15*a*d^2)/x^6
```

3.1136 $\int \frac{(d+ex^2)^2(a+b\tan^{-1}(cx))}{x^8}\,dx$

Optimal. Leaf size=186

$$-\frac{d^2\left(a+b\tan^{-1}(cx)\right)}{7x^7}-\frac{2de\left(a+b\tan^{-1}(cx)\right)}{5x^5}-\frac{e^2\left(a+b\tan^{-1}(cx)\right)}{3x^3}-\frac{bc\left(15c^4d^2-42c^2de+35e^2\right)}{210x^2}+\frac{1}{210}bc^3\left(15c^4d^2\right.$$

```
[Out] -(b*c*d^2)/(42*x^6) + (b*c*d*(5*c^2*d - 14*e))/(140*x^4) - (b*c*(15*c^4*d^2
 - 42*c^2*d*e + 35*e^2))/(210*x^2) - (d^2*(a + b*ArcTan[c*x]))/(7*x^7) - (2
*d*e*(a + b*ArcTan[c*x]))/(5*x^5) - (e^2*(a + b*ArcTan[c*x]))/(3*x^3) - (b*
c^3*(15*c^4*d^2 - 42*c^2*d*e + 35*e^2)*Log[x])/105 + (b*c^3*(15*c^4*d^2 - 4
2*c^2*d*e + 35*e^2)*Log[1 + c^2*x^2])/210
```

Rubi [A] time = 0.232365, antiderivative size = 186, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {270, 4976, 12, 1251, 893}

$$-\frac{d^2\left(a+b\tan^{-1}(cx)\right)}{7x^7}-\frac{2de\left(a+b\tan^{-1}(cx)\right)}{5x^5}-\frac{e^2\left(a+b\tan^{-1}(cx)\right)}{3x^3}-\frac{bc\left(15c^4d^2-42c^2de+35e^2\right)}{210x^2}+\frac{1}{210}bc^3\left(15c^4d^2\right.$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^2*(a + b*ArcTan[c*x]))/x^8,x]
```

```
[Out] -(b*c*d^2)/(42*x^6) + (b*c*d*(5*c^2*d - 14*e))/(140*x^4) - (b*c*(15*c^4*d^2
 - 42*c^2*d*e + 35*e^2))/(210*x^2) - (d^2*(a + b*ArcTan[c*x]))/(7*x^7) - (2
*d*e*(a + b*ArcTan[c*x]))/(5*x^5) - (e^2*(a + b*ArcTan[c*x]))/(3*x^3) - (b*
c^3*(15*c^4*d^2 - 42*c^2*d*e + 35*e^2)*Log[x])/105 + (b*c^3*(15*c^4*d^2 - 4
2*c^2*d*e + 35*e^2)*Log[1 + c^2*x^2])/210
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 4976

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Dis
t[a + b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2
```

```
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] && !(
ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(IL
tQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] && !ILtQ[(m
 - 1)/2, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^2\left(a+b\tan^{-1}(cx)\right)}{x^8}\,dx = -\frac{d^2\left(a+b\tan^{-1}(cx)\right)}{7x^7}-\frac{2de\left(a+b\tan^{-1}(cx)\right)}{5x^5}-\frac{e^2\left(a+b\tan^{-1}(cx)\right)}{3x^3}-(bc)\int \frac{-15}{}$$

$$= -\frac{d^2\left(a+b\tan^{-1}(cx)\right)}{7x^7}-\frac{2de\left(a+b\tan^{-1}(cx)\right)}{5x^5}-\frac{e^2\left(a+b\tan^{-1}(cx)\right)}{3x^3}-\frac{1}{105}(bc)\int$$

$$= -\frac{d^2\left(a+b\tan^{-1}(cx)\right)}{7x^7}-\frac{2de\left(a+b\tan^{-1}(cx)\right)}{5x^5}-\frac{e^2\left(a+b\tan^{-1}(cx)\right)}{3x^3}-\frac{1}{210}(bc)\,\mathrm{Su}$$

$$= -\frac{d^2\left(a+b\tan^{-1}(cx)\right)}{7x^7}-\frac{2de\left(a+b\tan^{-1}(cx)\right)}{5x^5}-\frac{e^2\left(a+b\tan^{-1}(cx)\right)}{3x^3}-\frac{1}{210}(bc)\,\mathrm{Su}$$

$$= -\frac{bcd^2}{42x^6}+\frac{bcd\left(5c^2d-14e\right)}{140x^4}-\frac{bc\left(15c^4d^2-42c^2de+35e^2\right)}{210x^2}-\frac{d^2\left(a+b\tan^{-1}(cx)\right)}{7x^7}-$$

Mathematica [A] time = 0.172982, size = 177, normalized size = 0.95

$$\frac{1}{420}\left(-\frac{60d^2\left(a+b\tan^{-1}(cx)\right)}{x^7}-\frac{168de\left(a+b\tan^{-1}(cx)\right)}{x^5}-\frac{140e^2\left(a+b\tan^{-1}(cx)\right)}{x^3}-5bcd^2\left(\frac{6c^4x^4-3c^2x^2+2}{x^6}-6c^6\log\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)^2*(a + b*ArcTan[c*x]))/x^8,x]
```

```
[Out] ((-60*d^2*(a + b*ArcTan[c*x]))/x^7 - (168*d*e*(a + b*ArcTan[c*x]))/x^5 - (1
40*e^2*(a + b*ArcTan[c*x]))/x^3 - 70*b*c*e^2*(x^(-2) + 2*c^2*Log[x] - c^2*L
og[1 + c^2*x^2]) - 42*b*c*d*e*(x^(-4) - (2*c^2)/x^2 - 4*c^4*Log[x] + 2*c^4*
Log[1 + c^2*x^2]) - 5*b*c*d^2*((2 - 3*c^2*x^2 + 6*c^4*x^4)/x^6 + 12*c^6*Log
[x] - 6*c^6*Log[1 + c^2*x^2]))/420
```

Maple [A] time = 0.047, size = 224, normalized size = 1.2

$$-\frac{ad^2}{7x^7}-\frac{2aed}{5x^5}-\frac{ae^2}{3x^3}-\frac{bd^2\arctan(cx)}{7x^7}-\frac{2b\arctan(cx)ed}{5x^5}-\frac{b\arctan(cx)e^2}{3x^3}+\frac{c^7b\ln\left(c^2x^2+1\right)d^2}{14}-\frac{c^5b\ln\left(c^2x^2+1\right.}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(a+b*arctan(c*x))/x^8,x)
```

```
[Out] -1/7*a*d^2/x^7-2/5*a*e*d/x^5-1/3*a*e^2/x^3-1/7*b*arctan(c*x)*d^2/x^7-2/5*b*
arctan(c*x)*e*d/x^5-1/3*b*arctan(c*x)*e^2/x^3+1/14*c^7*b*ln(c^2*x^2+1)*d^2-
1/5*c^5*b*ln(c^2*x^2+1)*e*d+1/6*c^3*b*ln(c^2*x^2+1)*e^2-1/14*c^5*b*d^2/x^2+
1/5*c^3*b*e*d/x^2-1/6*c*b*e^2/x^2-1/7*c^7*b*d^2*ln(c*x)+2/5*c^5*b*ln(c*x)*d
*e-1/3*c^3*b*ln(c*x)*e^2-1/42*b*c*d^2/x^6+1/28*c^3*b*d^2/x^4-1/10*c*b*e*d/x
^4
```

Maxima [A] time = 0.974972, size = 266, normalized size = 1.43

$$\frac{1}{84}\left(\left(\left(6c^6\log\left(c^2x^2+1\right)-6c^6\log\left(x^2\right)-\frac{6c^4x^4-3c^2x^2+2}{x^6}\right)c-\frac{12\arctan(cx)}{x^7}\right)bd^2-\frac{1}{10}\left(\left(\left(2c^4\log\left(c^2x^2+1\right)-2c^4\log\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arctan(c*x))/x^8,x, algorithm="maxima")
```

```
[Out] 1/84*((6*c^6*log(c^2*x^2 + 1) - 6*c^6*log(x^2) - (6*c^4*x^4 - 3*c^2*x^2 + 2
)/x^6)*c - 12*arctan(c*x)/x^7)*b*d^2 - 1/10*((2*c^4*log(c^2*x^2 + 1) - 2*c^
4*log(x^2) - (2*c^2*x^2 - 1)/x^4)*c + 4*arctan(c*x)/x^5)*b*d*e + 1/6*((c^2*
log(c^2*x^2 + 1) - c^2*log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^3)*b*e^2 - 1/3
*a*e^2/x^3 - 2/5*a*d*e/x^5 - 1/7*a*d^2/x^7
```

Fricas [A] time = 1.56819, size = 462, normalized size = 2.48

$$\frac{2\left(15\,bc^7d^2 - 42\,bc^5de + 35\,bc^3e^2\right)x^7\log\left(c^2x^2+1\right) - 4\left(15\,bc^7d^2 - 42\,bc^5de + 35\,bc^3e^2\right)x^7\log\left(x\right) - 140\,ae^2x^4 - 2\left(15\,bc^5d^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arctan(c*x))/x^8,x, algorithm="fricas")
```

```
[Out] 1/420*(2*(15*b*c^7*d^2 - 42*b*c^5*d*e + 35*b*c^3*e^2)*x^7*log(c^2*x^2 + 1)
- 4*(15*b*c^7*d^2 - 42*b*c^5*d*e + 35*b*c^3*e^2)*x^7*log(x) - 140*a*e^2*x^4
 - 2*(15*b*c^5*d^2 - 42*b*c^3*d*e + 35*b*c*e^2)*x^5 - 10*b*c*d^2*x - 168*a*
d*e*x^2 + 3*(5*b*c^3*d^2 - 14*b*c*d*e)*x^3 - 60*a*d^2 - 4*(35*b*e^2*x^4 + 4
2*b*d*e*x^2 + 15*b*d^2)*arctan(c*x))/x^7
```

Sympy [A] time = 5.4085, size = 289, normalized size = 1.55

$$\begin{cases} -\frac{ad^2}{7x^7} - \frac{2ade}{5x^5} - \frac{ae^2}{3x^3} - \frac{bc^7d^2\log(x)}{7} + \frac{bc^7d^2\log\left(x^2+\frac{1}{c^2}\right)}{14} - \frac{bc^5d^2}{14x^2} + \frac{2bc^5de\log(x)}{5} - \frac{bc^5de\log\left(x^2+\frac{1}{c^2}\right)}{5} + \frac{bc^3d^2}{28x^4} + \frac{bc^3de}{5x^2} - \frac{bc^3e^2\log(x)}{3} + \cdots \\ a\left(-\frac{d^2}{7x^7} - \frac{2de}{5x^5} - \frac{e^2}{3x^3}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(a+b*atan(c*x))/x**8,x)
```

```
[Out] Piecewise((-a*d**2/(7*x**7) - 2*a*d*e/(5*x**5) - a*e**2/(3*x**3) - b*c**7*d
**2*log(x)/7 + b*c**7*d**2*log(x**2 + c**(-2))/14 - b*c**5*d**2/(14*x**2) +
 2*b*c**5*d*e*log(x)/5 - b*c**5*d*e*log(x**2 + c**(-2))/5 + b*c**3*d**2/(28
*x**4) + b*c**3*d*e/(5*x**2) - b*c**3*e**2*log(x)/3 + b*c**3*e**2*log(x**2
+ c**(-2))/6 - b*c*d**2/(42*x**6) - b*c*d*e/(10*x**4) - b*c*e**2/(6*x**2) -
 b*d**2*atan(c*x)/(7*x**7) - 2*b*d*e*atan(c*x)/(5*x**5) - b*e**2*atan(c*x)/
(3*x**3), Ne(c, 0)), (a*(-d**2/(7*x**7) - 2*d*e/(5*x**5) - e**2/(3*x**3)),
True))
```

Giac [A] time = 1.09629, size = 315, normalized size = 1.69

$$\frac{30\,bc^7d^2x^7\log\left(c^2x^2+1\right)-60\,bc^7d^2x^7\log\left(x\right)-84\,bc^5dx^7e\log\left(c^2x^2+1\right)+168\,bc^5dx^7e\log\left(x\right)-30\,bc^5d^2x^5+70\,bc^3x^7e \cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arctan(c*x))/x^8,x, algorithm="giac")
```

```
[Out] 1/420*(30*b*c^7*d^2*x^7*log(c^2*x^2 + 1) - 60*b*c^7*d^2*x^7*log(x) - 84*b*c
^5*d*x^7*e*log(c^2*x^2 + 1) + 168*b*c^5*d*x^7*e*log(x) - 30*b*c^5*d^2*x^5 +
 70*b*c^3*x^7*e^2*log(c^2*x^2 + 1) - 140*b*c^3*x^7*e^2*log(x) + 84*b*c^3*d*
x^5*e + 15*b*c^3*d^2*x^3 - 70*b*c*x^5*e^2 - 140*b*x^4*arctan(c*x)*e^2 - 42*
b*c*d*x^3*e - 140*a*x^4*e^2 - 168*b*d*x^2*arctan(c*x)*e - 10*b*c*d^2*x - 16
8*a*d*x^2*e - 60*b*d^2*arctan(c*x) - 60*a*d^2)/x^7
```

3.1137 $\int x^3 \left(d+ex^2\right)^3 \left(a+b\tan^{-1}(cx)\right) dx$

Optimal. Leaf size=240

$$\frac{\left(d+ex^2\right)^5 \left(a+b\tan^{-1}(cx)\right)}{10e^2}-\frac{d\left(d+ex^2\right)^4 \left(a+b\tan^{-1}(cx)\right)}{8e^2}-\frac{bex^5\left(20c^4d^2-15c^2de+4e^2\right)}{200c^5}-\frac{bx^3\left(-20c^4d^2e+10c^6d^3\right.}{120c^7}$$

```
[Out] (b*(10*c^6*d^3 - 20*c^4*d^2*e + 15*c^2*d*e^2 - 4*e^3)*x)/(40*c^9) - (b*(10*
c^6*d^3 - 20*c^4*d^2*e + 15*c^2*d*e^2 - 4*e^3)*x^3)/(120*c^7) - (b*e*(20*c^
4*d^2 - 15*c^2*d*e + 4*e^2)*x^5)/(200*c^5) - (b*(15*c^2*d - 4*e)*e^2*x^7)/(
280*c^3) - (b*e^3*x^9)/(90*c) + (b*(c^2*d - e)^4*(c^2*d + 4*e)*ArcTan[c*x])
/(40*c^10*e^2) - (d*(d + e*x^2)^4*(a + b*ArcTan[c*x]))/(8*e^2) + ((d + e*x^
2)^5*(a + b*ArcTan[c*x]))/(10*e^2)
```

Rubi [A] time = 0.458801, antiderivative size = 285, normalized size of antiderivative = 1.19, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {266, 43, 4976, 12, 528, 388, 203}

$$\frac{\left(d+ex^2\right)^5 \left(a+b\tan^{-1}(cx)\right)}{10e^2}-\frac{d\left(d+ex^2\right)^4 \left(a+b\tan^{-1}(cx)\right)}{8e^2}-\frac{bx\left(25c^4d^2-135c^2de+84e^2\right)\left(d+ex^2\right)^2}{4200c^5e}+\frac{bx\left(750c^4d^2e\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + e*x^2)^3*(a + b*ArcTan[c*x]),x]
```

```
[Out] (b*(325*c^8*d^4 + 1815*c^6*d^3*e - 4977*c^4*d^2*e^2 + 4305*c^2*d*e^3 - 1260
*e^4)*x)/(12600*c^9*e) + (b*(5*c^6*d^3 + 750*c^4*d^2*e - 1071*c^2*d*e^2 + 4
20*e^3)*x*(d + e*x^2))/(12600*c^7*e) - (b*(25*c^4*d^2 - 135*c^2*d*e + 84*e^
2)*x*(d + e*x^2)^2)/(4200*c^5*e) - (b*(23*c^2*d - 36*e)*x*(d + e*x^2)^3)/(2
520*c^3*e) - (b*x*(d + e*x^2)^4)/(90*c*e) + (b*(c^2*d - e)^4*(c^2*d + 4*e)*
ArcTan[c*x])/(40*c^10*e^2) - (d*(d + e*x^2)^4*(a + b*ArcTan[c*x]))/(8*e^2)
+ ((d + e*x^2)^5*(a + b*ArcTan[c*x]))/(10*e^2)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 4976

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Dis
t[a + b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&  !(
ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(IL
tQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILtQ[(m
 - 1)/2, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int x^3\left(d+ex^2\right)^3\left(a+b\tan^{-1}(cx)\right)\,dx=-\frac{d\left(d+ex^2\right)^4\left(a+b\tan^{-1}(cx)\right)}{8e^2}+\frac{\left(d+ex^2\right)^5\left(a+b\tan^{-1}(cx)\right)}{10e^2}-(bc)\int\frac{\left(d+ex^2\right.}{40e^2}$$

$$=-\frac{d\left(d+ex^2\right)^4\left(a+b\tan^{-1}(cx)\right)}{8e^2}+\frac{\left(d+ex^2\right)^5\left(a+b\tan^{-1}(cx)\right)}{10e^2}-\frac{(bc)\int\frac{\left(d+ex^2\right)^4(-}{1+c^2}}{40e^2}$$

$$=-\frac{bx\left(d+ex^2\right)^4}{90ce}-\frac{d\left(d+ex^2\right)^4\left(a+b\tan^{-1}(cx)\right)}{8e^2}+\frac{\left(d+ex^2\right)^5\left(a+b\tan^{-1}(cx)\right)}{10e^2}-$$

$$=-\frac{b\left(23c^2d-36e\right)x\left(d+ex^2\right)^3}{2520c^3e}-\frac{bx\left(d+ex^2\right)^4}{90ce}-\frac{d\left(d+ex^2\right)^4\left(a+b\tan^{-1}(cx)\right)}{8e^2}+$$

$$=-\frac{b\left(25c^4d^2-135c^2de+84e^2\right)x\left(d+ex^2\right)^2}{4200c^5e}-\frac{b\left(23c^2d-36e\right)x\left(d+ex^2\right)^3}{2520c^3e}-\frac{bx\left(d\right.}{9}$$

$$=\frac{b\left(5c^6d^3+750c^4d^2e-1071c^2de^2+420e^3\right)x\left(d+ex^2\right)}{12600c^7e}-\frac{b\left(25c^4d^2-135c^2de+84\right.}{4200c^5e}$$

$$=\frac{b\left(325c^8d^4+1815c^6d^3e-4977c^4d^2e^2+4305c^2de^3-1260e^4\right)x}{12600c^9e}+\frac{b\left(5c^6d^3+750c\right.}{}$$

$$=\frac{b\left(325c^8d^4+1815c^6d^3e-4977c^4d^2e^2+4305c^2de^3-1260e^4\right)x}{12600c^9e}+\frac{b\left(5c^6d^3+750c\right.}{}$$

Mathematica [A] time = 0.226935, size = 262, normalized size = 1.09

$$\frac{cx\left(315ac^9x^3\left(20d^2ex^2+10d^3+15de^2x^4+4e^3x^6\right)-b\left(5c^8\left(252d^2ex^4+210d^3x^2+135de^2x^6+28e^3x^8\right)-15c^6\left(140d^2ex^2+\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + e*x^2)^3*(a + b*ArcTan[c*x]),x]
```

```
[Out] (c*x*(315*a*c^9*x^3*(10*d^3 + 20*d^2*e*x^2 + 15*d*e^2*x^4 + 4*e^3*x^6) - b*
(1260*e^3 - 105*c^2*e^2*(45*d + 4*e*x^2) + 63*c^4*e*(100*d^2 + 25*d*e*x^2 +
 4*e^2*x^4) - 15*c^6*(210*d^3 + 140*d^2*e*x^2 + 63*d*e^2*x^4 + 12*e^3*x^6)
+ 5*c^8*(210*d^3*x^2 + 252*d^2*e*x^4 + 135*d*e^2*x^6 + 28*e^3*x^8))) + 315*
b*(-10*c^6*d^3 + 20*c^4*d^2*e - 15*c^2*d*e^2 + 4*e^3 + c^10*x^4*(10*d^3 + 2
0*d^2*e*x^2 + 15*d*e^2*x^4 + 4*e^3*x^6))*ArcTan[c*x])/(12600*c^10)
```

Maple [A] time = 0.038, size = 315, normalized size = 1.3

$$-\frac{bd^2ex^5}{10c}+\frac{bx^3d^2e}{6c^3}-\frac{bx^3de^2}{8c^5}-\frac{bd^2ex}{2c^5}-\frac{3\arctan(cx)\,bde^2}{8c^8}+\frac{bd^2\arctan(cx)\,e}{2c^6}+\frac{3b\arctan(cx)\,de^2x^8}{8}+\frac{b\arctan(cx)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x^2+d)^3*(a+b*arctan(c*x)),x)
```

```
[Out] -1/10/c*b*d^2*e*x^5+1/6/c^3*b*x^3*d^2*e-1/8/c^5*b*x^3*d*e^2-1/2/c^5*b*d^2*e
*x-3/8/c^8*b*arctan(c*x)*d*e^2+1/2/c^6*b*arctan(c*x)*d^2*e+3/8*b*arctan(c*x
)*d*e^2*x^8+1/2*b*arctan(c*x)*d^2*e*x^6-3/56/c*b*d*e^2*x^7+3/8/c^7*b*d*e^2*
x+3/40/c^3*b*x^5*d*e^2+1/10*a*e^3*x^10+1/4*a*x^4*d^3-1/4*b*d^3*arctan(c*x)/
c^4+1/70/c^3*b*x^7*e^3+1/4*b*arctan(c*x)*x^4*d^3+1/10*b*arctan(c*x)*e^3*x^1
0-1/10/c^9*b*e^3*x+1/30/c^7*b*e^3*x^3-1/50/c^5*b*x^5*e^3+1/2*a*d^2*e*x^6+3/
8*a*d*e^2*x^8+1/10/c^10*b*arctan(c*x)*e^3+1/4*b*d^3*x/c^3-1/12*b*d^3*x^3/c-
1/90*b*e^3*x^9/c
```

Maxima [A] time = 1.48413, size = 362, normalized size = 1.51

$$\frac{1}{10}ae^3x^{10}+\frac{3}{8}ade^2x^8+\frac{1}{2}ad^2ex^6+\frac{1}{4}ad^3x^4+\frac{1}{12}\left(3x^4\arctan(cx)-c\left(\frac{c^2x^3-3x}{c^4}+\frac{3\arctan(cx)}{c^5}\right)\right)bd^3+\frac{1}{30}\left(15x^6\text{ar}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^3*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] 1/10*a*e^3*x^10 + 3/8*a*d*e^2*x^8 + 1/2*a*d^2*e*x^6 + 1/4*a*d^3*x^4 + 1/12*
(3*x^4*arctan(c*x) - c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5))*b*d^3 + 1
/30*(15*x^6*arctan(c*x) - c*((3*c^4*x^5 - 5*c^2*x^3 + 15*x)/c^6 - 15*arctan
(c*x)/c^7))*b*d^2*e + 1/280*(105*x^8*arctan(c*x) - c*((15*c^6*x^7 - 21*c^4*
x^5 + 35*c^2*x^3 - 105*x)/c^8 + 105*arctan(c*x)/c^9))*b*d*e^2 + 1/3150*(315
*x^10*arctan(c*x) - c*((35*c^8*x^9 - 45*c^6*x^7 + 63*c^4*x^5 - 105*c^2*x^3
+ 315*x)/c^10 - 315*arctan(c*x)/c^11))*b*e^3
```

Fricas [A] time = 1.42384, size = 711, normalized size = 2.96

$$1260\,ac^{10}e^3x^{10}+4725\,ac^{10}de^2x^8-140\,bc^9e^3x^9+6300\,ac^{10}d^2ex^6+3150\,ac^{10}d^3x^4-45\left(15\,bc^9de^2-4\,bc^7e^3\right)x^7-63\left(20\,b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^3*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] 1/12600*(1260*a*c^10*e^3*x^10 + 4725*a*c^10*d*e^2*x^8 - 140*b*c^9*e^3*x^9 +
 6300*a*c^10*d^2*e*x^6 + 3150*a*c^10*d^3*x^4 - 45*(15*b*c^9*d*e^2 - 4*b*c^7
*e^3)*x^7 - 63*(20*b*c^9*d^2*e - 15*b*c^7*d*e^2 + 4*b*c^5*e^3)*x^5 - 105*(1
0*b*c^9*d^3 - 20*b*c^7*d^2*e + 15*b*c^5*d*e^2 - 4*b*c^3*e^3)*x^3 + 315*(10*
b*c^7*d^3 - 20*b*c^5*d^2*e + 15*b*c^3*d*e^2 - 4*b*c*e^3)*x + 315*(4*b*c^10*
e^3*x^10 + 15*b*c^10*d*e^2*x^8 + 20*b*c^10*d^2*e*x^6 + 10*b*c^10*d^3*x^4 -
10*b*c^6*d^3 + 20*b*c^4*d^2*e - 15*b*c^2*d*e^2 + 4*b*e^3)*arctan(c*x))/c^10
```

Sympy [A] time = 9.67284, size = 411, normalized size = 1.71

$$\begin{cases} \frac{ad^3x^4}{4} + \frac{ad^2ex^6}{2} + \frac{3ade^2x^8}{8} + \frac{ae^3x^{10}}{10} + \frac{bd^3x^4\operatorname{atan}(cx)}{4} + \frac{bd^2ex^6\operatorname{atan}(cx)}{2} + \frac{3bde^2x^8\operatorname{atan}(cx)}{8} + \frac{be^3x^{10}\operatorname{atan}(cx)}{10} - \frac{bd^3x^3}{12c} - \frac{bd^2ex^5}{10c} - \frac{3bde^2x^7}{56c} - \\ a\left(\frac{d^3x^4}{4} + \frac{d^2ex^6}{2} + \frac{3de^2x^8}{8} + \frac{e^3x^{10}}{10}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x**2+d)**3*(a+b*atan(c*x)),x)
```

```
[Out] Piecewise((a*d**3*x**4/4 + a*d**2*e*x**6/2 + 3*a*d*e**2*x**8/8 + a*e**3*x**
10/10 + b*d**3*x**4*atan(c*x)/4 + b*d**2*e*x**6*atan(c*x)/2 + 3*b*d*e**2*x*
*8*atan(c*x)/8 + b*e**3*x**10*atan(c*x)/10 - b*d**3*x**3/(12*c) - b*d**2*e*
x**5/(10*c) - 3*b*d*e**2*x**7/(56*c) - b*e**3*x**9/(90*c) + b*d**3*x/(4*c**
3) + b*d**2*e*x**3/(6*c**3) + 3*b*d*e**2*x**5/(40*c**3) + b*e**3*x**7/(70*c
**3) - b*d**3*atan(c*x)/(4*c**4) - b*d**2*e*x/(2*c**5) - b*d*e**2*x**3/(8*c
**5) - b*e**3*x**5/(50*c**5) + b*d**2*e*atan(c*x)/(2*c**6) + 3*b*d*e**2*x/(
8*c**7) + b*e**3*x**3/(30*c**7) - 3*b*d*e**2*atan(c*x)/(8*c**8) - b*e**3*x/
(10*c**9) + b*e**3*atan(c*x)/(10*c**10), Ne(c, 0)), (a*(d**3*x**4/4 + d**2*
e*x**6/2 + 3*d*e**2*x**8/8 + e**3*x**10/10), True))
```

Giac [A] time = 1.64996, size = 479, normalized size = 2.

$$1260\,bc^{10}x^{10}\arctan(cx)\,e^3 + 1260\,ac^{10}x^{10}e^3 + 4725\,bc^{10}dx^8\arctan(cx)\,e^2 + 4725\,ac^{10}dx^8e^2 + 6300\,bc^{10}d^2x^6\arctan(cx)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^3*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] 1/12600*(1260*b*c^10*x^10*arctan(c*x)*e^3 + 1260*a*c^10*x^10*e^3 + 4725*b*c
^10*d*x^8*arctan(c*x)*e^2 + 4725*a*c^10*d*x^8*e^2 + 6300*b*c^10*d^2*x^6*arc
tan(c*x)*e - 140*b*c^9*x^9*e^3 + 6300*a*c^10*d^2*x^6*e + 3150*b*c^10*d^3*x^
4*arctan(c*x) - 675*b*c^9*d*x^7*e^2 + 3150*a*c^10*d^3*x^4 - 1260*b*c^9*d^2*
x^5*e - 1050*b*c^9*d^3*x^3 + 180*b*c^7*x^7*e^3 + 945*b*c^7*d*x^5*e^2 + 2100
*b*c^7*d^2*x^3*e + 3150*b*c^7*d^3*x - 252*b*c^5*x^5*e^3 - 3150*b*c^6*d^3*ar
ctan(c*x) - 1575*b*c^5*d*x^3*e^2 - 6300*pi*b*c^4*d^2*e*sgn(c)*sgn(x) - 6300
*b*c^5*d^2*x*e + 6300*b*c^4*d^2*arctan(c*x)*e + 420*b*c^3*x^3*e^3 + 4725*b*
c^3*d*x*e^2 - 4725*b*c^2*d*arctan(c*x)*e^2 - 1260*pi*b*e^3*sgn(c)*sgn(x) -
1260*b*c*x*e^3 + 1260*b*arctan(c*x)*e^3)/c^10
```

3.1138 $\int x^2 \left(d + ex^2\right)^3 \left(a + b\tan^{-1}(cx)\right) dx$

Optimal. Leaf size=239

$$\frac{3}{5}d^2ex^5\left(a + b\tan^{-1}(cx)\right) + \frac{1}{3}d^3x^3\left(a + b\tan^{-1}(cx)\right) + \frac{3}{7}de^2x^7\left(a + b\tan^{-1}(cx)\right) + \frac{1}{9}e^3x^9\left(a + b\tan^{-1}(cx)\right) - \frac{bex^4\left(189c^4d\right.}{}$$

```
[Out] -(b*(105*c^6*d^3 - 189*c^4*d^2*e + 135*c^2*d*e^2 - 35*e^3)*x^2)/(630*c^7) -
 (b*e*(189*c^4*d^2 - 135*c^2*d*e + 35*e^2)*x^4)/(1260*c^5) - (b*(27*c^2*d -
 7*e)*e^2*x^6)/(378*c^3) - (b*e^3*x^8)/(72*c) + (d^3*x^3*(a + b*ArcTan[c*x]
))/3 + (3*d^2*e*x^5*(a + b*ArcTan[c*x]))/5 + (3*d*e^2*x^7*(a + b*ArcTan[c*x
]))/7 + (e^3*x^9*(a + b*ArcTan[c*x]))/9 + (b*(105*c^6*d^3 - 189*c^4*d^2*e +
 135*c^2*d*e^2 - 35*e^3)*Log[1 + c^2*x^2])/(630*c^9)
```

Rubi [A] time = 0.384412, antiderivative size = 239, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {270, 4976, 12, 1799, 1620}

$$\frac{3}{5}d^2ex^5\left(a + b\tan^{-1}(cx)\right) + \frac{1}{3}d^3x^3\left(a + b\tan^{-1}(cx)\right) + \frac{3}{7}de^2x^7\left(a + b\tan^{-1}(cx)\right) + \frac{1}{9}e^3x^9\left(a + b\tan^{-1}(cx)\right) - \frac{bex^4\left(189c^4d\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + e*x^2)^3*(a + b*ArcTan[c*x]),x]
```


```
[Out] -(b*(105*c^6*d^3 - 189*c^4*d^2*e + 135*c^2*d*e^2 - 35*e^3)*x^2)/(630*c^7) -
 (b*e*(189*c^4*d^2 - 135*c^2*d*e + 35*e^2)*x^4)/(1260*c^5) - (b*(27*c^2*d -
 7*e)*e^2*x^6)/(378*c^3) - (b*e^3*x^8)/(72*c) + (d^3*x^3*(a + b*ArcTan[c*x]
))/3 + (3*d^2*e*x^5*(a + b*ArcTan[c*x]))/5 + (3*d*e^2*x^7*(a + b*ArcTan[c*x
]))/7 + (e^3*x^9*(a + b*ArcTan[c*x]))/9 + (b*(105*c^6*d^3 - 189*c^4*d^2*e +
 135*c^2*d*e^2 - 35*e^3)*Log[1 + c^2*x^2])/(630*c^9)
```


Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 4976

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Dis
```

```
t[a + b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] && !(
ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(IL
tQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] && !ILtQ[(m
 - 1)/2, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1799

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\int x^2\left(d+ex^2\right)^3\left(a+b\tan^{-1}(cx)\right)dx=\frac{1}{3}d^3x^3\left(a+b\tan^{-1}(cx)\right)+\frac{3}{5}d^2ex^5\left(a+b\tan^{-1}(cx)\right)+\frac{3}{7}de^2x^7\left(a+b\tan^{-1}(cx)\right)$$

$$=\frac{1}{3}d^3x^3\left(a+b\tan^{-1}(cx)\right)+\frac{3}{5}d^2ex^5\left(a+b\tan^{-1}(cx)\right)+\frac{3}{7}de^2x^7\left(a+b\tan^{-1}(cx)\right)$$

$$=\frac{1}{3}d^3x^3\left(a+b\tan^{-1}(cx)\right)+\frac{3}{5}d^2ex^5\left(a+b\tan^{-1}(cx)\right)+\frac{3}{7}de^2x^7\left(a+b\tan^{-1}(cx)\right)$$

$$=\frac{1}{3}d^3x^3\left(a+b\tan^{-1}(cx)\right)+\frac{3}{5}d^2ex^5\left(a+b\tan^{-1}(cx)\right)+\frac{3}{7}de^2x^7\left(a+b\tan^{-1}(cx)\right)$$

$$=-\frac{b\left(105c^6d^3-189c^4d^2e+135c^2de^2-35e^3\right)x^2}{630c^7}-\frac{be\left(189c^4d^2-135c^2de+35e^2\right)}{1260c^5}$$

Mathematica [A] time = 0.208617, size = 252, normalized size = 1.05

$$\frac{3}{5}d^2ex^5\left(a+b\tan^{-1}(cx)\right)+\frac{1}{3}d^3x^3\left(a+b\tan^{-1}(cx)\right)+\frac{3}{7}de^2x^7\left(a+b\tan^{-1}(cx)\right)+\frac{1}{9}e^3x^9\left(a+b\tan^{-1}(cx)\right)+\frac{3}{20}bd^2e\left(\frac{2x^2}{c^3}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + e*x^2)^3*(a + b*ArcTan[c*x]),x]
```

```
[Out] (d^3*x^3*(a + b*ArcTan[c*x]))/3 + (3*d^2*e*x^5*(a + b*ArcTan[c*x]))/5 + (3*
d*e^2*x^7*(a + b*ArcTan[c*x]))/7 + (e^3*x^9*(a + b*ArcTan[c*x]))/9 + (b*e^3
*((12*x^2)/c^7 - (6*x^4)/c^5 + (4*x^6)/c^3 - (3*x^8)/c - (12*Log[1 + c^2*x^
2])/c^9))/216 - (b*d*e^2*((6*x^2)/c^5 - (3*x^4)/c^3 + (2*x^6)/c - (6*Log[1
+ c^2*x^2])/c^7))/28 + (3*b*d^2*e*((2*x^2)/c^3 - x^4/c - (2*Log[1 + c^2*x^2
])/c^5))/20 - (b*d^3*(x^2/c - Log[1 + c^2*x^2]/c^3))/6
```

Maple [A] time = 0.039, size = 297, normalized size = 1.2

$$\frac{ae^3x^9}{9}+\frac{3\,ade^2x^7}{7}+\frac{3\,ad^2ex^5}{5}+\frac{ad^3x^3}{3}+\frac{b\arctan(cx)\,e^3x^9}{9}+\frac{3\,b\arctan(cx)\,de^2x^7}{7}+\frac{3\,b\arctan(cx)\,d^2ex^5}{5}+\frac{b\arctan(}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^2+d)^3*(a+b*arctan(c*x)),x)
```

```
[Out] 1/9*a*e^3*x^9+3/7*a*d*e^2*x^7+3/5*a*d^2*e*x^5+1/3*a*d^3*x^3+1/9*b*arctan(c*
x)*e^3*x^9+3/7*b*arctan(c*x)*d*e^2*x^7+3/5*b*arctan(c*x)*d^2*e*x^5+1/3*b*ar
ctan(c*x)*d^3*x^3-1/6*b*d^3*x^2/c-3/20/c*b*d^2*e*x^4-1/14/c*b*d*e^2*x^6+3/1
0/c^3*b*x^2*d^2*e-1/72*b*e^3*x^8/c+3/28/c^3*b*x^4*d*e^2+1/54/c^3*b*x^6*e^3-
3/14/c^5*b*x^2*d*e^2-1/36/c^5*b*e^3*x^4+1/18/c^7*b*e^3*x^2+1/6*b*d^3*ln(c^2
*x^2+1)/c^3-3/10/c^5*b*ln(c^2*x^2+1)*d^2*e+3/14/c^7*b*ln(c^2*x^2+1)*d*e^2-1
/18/c^9*b*ln(c^2*x^2+1)*e^3
```

Maxima [A] time = 1.04122, size = 358, normalized size = 1.5

$$\frac{1}{9}ae^3x^9+\frac{3}{7}ade^2x^7+\frac{3}{5}ad^2ex^5+\frac{1}{3}ad^3x^3+\frac{1}{6}\left(2\,x^3\arctan(cx)-c\left(\frac{x^2}{c^2}-\frac{\log\left(c^2x^2+1\right)}{c^4}\right)\right)bd^3+\frac{3}{20}\left(4\,x^5\arctan(cx)-c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^3*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] 1/9*a*e^3*x^9 + 3/7*a*d*e^2*x^7 + 3/5*a*d^2*e*x^5 + 1/3*a*d^3*x^3 + 1/6*(2*
x^3*arctan(c*x) - c*(x^2/c^2 - log(c^2*x^2 + 1)/c^4))*b*d^3 + 3/20*(4*x^5*a
rctan(c*x) - c*((c^2*x^4 - 2*x^2)/c^4 + 2*log(c^2*x^2 + 1)/c^6))*b*d^2*e +
1/28*(12*x^7*arctan(c*x) - c*((2*c^4*x^6 - 3*c^2*x^4 + 6*x^2)/c^6 - 6*log(c
^2*x^2 + 1)/c^8))*b*d*e^2 + 1/216*(24*x^9*arctan(c*x) - c*((3*c^6*x^8 - 4*c
^4*x^6 + 6*c^2*x^4 - 12*x^2)/c^8 + 12*log(c^2*x^2 + 1)/c^10))*b*e^3
```

Fricas [A] time = 1.35348, size = 647, normalized size = 2.71

$$\frac{840\, ac^9e^3x^9 + 3240\, ac^9de^2x^7 - 105\, bc^8e^3x^8 + 4536\, ac^9d^2ex^5 + 2520\, ac^9d^3x^3 - 20\left(27\, bc^8de^2 - 7\, bc^6e^3\right)x^6 - 6\left(189\, bc^8d^2e\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^3*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] 1/7560*(840*a*c^9*e^3*x^9 + 3240*a*c^9*d*e^2*x^7 - 105*b*c^8*e^3*x^8 + 4536
*a*c^9*d^2*e*x^5 + 2520*a*c^9*d^3*x^3 - 20*(27*b*c^8*d*e^2 - 7*b*c^6*e^3)*x
^6 - 6*(189*b*c^8*d^2*e - 135*b*c^6*d*e^2 + 35*b*c^4*e^3)*x^4 - 12*(105*b*c
^8*d^3 - 189*b*c^6*d^2*e + 135*b*c^4*d*e^2 - 35*b*c^2*e^3)*x^2 + 24*(35*b*c
^9*e^3*x^9 + 135*b*c^9*d*e^2*x^7 + 189*b*c^9*d^2*e*x^5 + 105*b*c^9*d^3*x^3)
*arctan(c*x) + 12*(105*b*c^6*d^3 - 189*b*c^4*d^2*e + 135*b*c^2*d*e^2 - 35*b
*e^3)*log(c^2*x^2 + 1))/c^9
```

Sympy [A] time = 6.92135, size = 389, normalized size = 1.63

$$\begin{cases} \frac{ad^3x^3}{3} + \frac{3ad^2ex^5}{5} + \frac{3ade^2x^7}{7} + \frac{ae^3x^9}{9} + \frac{bd^3x^3 \operatorname{atan}(cx)}{3} + \frac{3bd^2ex^5 \operatorname{atan}(cx)}{5} + \frac{3bde^2x^7 \operatorname{atan}(cx)}{7} + \frac{be^3x^9 \operatorname{atan}(cx)}{9} - \frac{bd^3x^2}{6c} - \frac{3bd^2ex^4}{20c} - \frac{bde^2x^6}{14c} \\ a\left(\frac{d^3x^3}{3} + \frac{3d^2ex^5}{5} + \frac{3de^2x^7}{7} + \frac{e^3x^9}{9}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d)**3*(a+b*atan(c*x)),x)
```

```
[Out] Piecewise((a*d**3*x**3/3 + 3*a*d**2*e*x**5/5 + 3*a*d*e**2*x**7/7 + a*e**3*x
**9/9 + b*d**3*x**3*atan(c*x)/3 + 3*b*d**2*e*x**5*atan(c*x)/5 + 3*b*d*e**2*
x**7*atan(c*x)/7 + b*e**3*x**9*atan(c*x)/9 - b*d**3*x**2/(6*c) - 3*b*d**2*e
```

```
*x**4/(20*c) - b*d*e**2*x**6/(14*c) - b*e**3*x**8/(72*c) + b*d**3*log(x**2
+ c**(-2))/(6*c**3) + 3*b*d**2*e*x**2/(10*c**3) + 3*b*d*e**2*x**4/(28*c**3)
 + b*e**3*x**6/(54*c**3) - 3*b*d**2*e*log(x**2 + c**(-2))/(10*c**5) - 3*b*d
*e**2*x**2/(14*c**5) - b*e**3*x**4/(36*c**5) + 3*b*d*e**2*log(x**2 + c**(-2
))/(14*c**7) + b*e**3*x**2/(18*c**7) - b*e**3*log(x**2 + c**(-2))/(18*c**9)
, Ne(c, 0)), (a*(d**3*x**3/3 + 3*d**2*e*x**5/5 + 3*d*e**2*x**7/7 + e**3*x**
9/9), True))
```

Giac [A] time = 1.10018, size = 424, normalized size = 1.77

$$\frac{840\,bc^9x^9\arctan(cx)\,e^3 + 840\,ac^9x^9e^3 + 3240\,bc^9dx^7\arctan(cx)\,e^2 + 3240\,ac^9dx^7e^2 + 4536\,bc^9d^2x^5\arctan(cx)\,e - 105\,\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^3*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] 1/7560*(840*b*c^9*x^9*arctan(c*x)*e^3 + 840*a*c^9*x^9*e^3 + 3240*b*c^9*d*x^
7*arctan(c*x)*e^2 + 3240*a*c^9*d*x^7*e^2 + 4536*b*c^9*d^2*x^5*arctan(c*x)*e
 - 105*b*c^8*x^8*e^3 + 4536*a*c^9*d^2*x^5*e + 2520*b*c^9*d^3*x^3*arctan(c*x
) - 540*b*c^8*d*x^6*e^2 + 2520*a*c^9*d^3*x^3 - 1134*b*c^8*d^2*x^4*e - 1260*
b*c^8*d^3*x^2 + 140*b*c^6*x^6*e^3 + 810*b*c^6*d*x^4*e^2 + 2268*b*c^6*d^2*x^
2*e + 1260*b*c^6*d^3*log(c^2*x^2 + 1) - 210*b*c^4*x^4*e^3 - 1620*b*c^4*d*x^
2*e^2 - 2268*b*c^4*d^2*e*log(c^2*x^2 + 1) + 420*b*c^2*x^2*e^3 + 1620*b*c^2*
d*e^2*log(c^2*x^2 + 1) - 420*b*e^3*log(c^2*x^2 + 1))/c^9
```

3.1139 $\int x \left(d+ex^2\right)^3 \left(a+b\tan^{-1}(cx)\right)\,dx$

Optimal. Leaf size=158

$$\frac{\left(d+ex^2\right)^4\left(a+b\tan^{-1}(cx)\right)}{8e}-\frac{bex^3\left(6c^4d^2-4c^2de+e^2\right)}{24c^5}-\frac{bx\left(2c^2d-e\right)\left(2c^4d^2-2c^2de+e^2\right)}{8c^7}-\frac{be^2x^5\left(4c^2d-e\right)}{40c^3}-\frac{b\left(c\right.}{}$$

```
[Out] -(b*(2*c^2*d - e)*(2*c^4*d^2 - 2*c^2*d*e + e^2)*x)/(8*c^7) - (b*e*(6*c^4*d^
2 - 4*c^2*d*e + e^2)*x^3)/(24*c^5) - (b*(4*c^2*d - e)*e^2*x^5)/(40*c^3) - (
b*e^3*x^7)/(56*c) - (b*(c^2*d - e)^4*ArcTan[c*x])/(8*c^8*e) + ((d + e*x^2)^
4*(a + b*ArcTan[c*x]))/(8*e)
```

Rubi [A] time = 0.146663, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {4974, 390, 203}

$$\frac{\left(d+ex^2\right)^4\left(a+b\tan^{-1}(cx)\right)}{8e}-\frac{bex^3\left(6c^4d^2-4c^2de+e^2\right)}{24c^5}-\frac{bx\left(2c^2d-e\right)\left(2c^4d^2-2c^2de+e^2\right)}{8c^7}-\frac{be^2x^5\left(4c^2d-e\right)}{40c^3}-\frac{b\left(c\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + e*x^2)^3*(a + b*ArcTan[c*x]),x]
```



```
[Out] -(b*(2*c^2*d - e)*(2*c^4*d^2 - 2*c^2*d*e + e^2)*x)/(8*c^7) - (b*e*(6*c^4*d^
2 - 4*c^2*d*e + e^2)*x^3)/(24*c^5) - (b*(4*c^2*d - e)*e^2*x^5)/(40*c^3) - (
b*e^3*x^7)/(56*c) - (b*(c^2*d - e)^4*ArcTan[c*x])/(8*c^8*e) + ((d + e*x^2)^
4*(a + b*ArcTan[c*x]))/(8*e)
```


Rule 4974

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]))/(2*e*(q + 1)), x
] - Dist[(b*c)/(2*e*(q + 1)), Int[(d + e*x^2)^(q + 1)/(1 + c^2*x^2), x], x]
 /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
```

```
0] && GeQ[p, -q]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int x\left(d+ex^2\right)^3\left(a+b\tan^{-1}(cx)\right)dx=\frac{\left(d+ex^2\right)^4\left(a+b\tan^{-1}(cx)\right)}{8e}-\frac{(bc)\int\frac{\left(d+ex^2\right)^4}{1+c^2x^2}\,dx}{8e}$$

$$=\frac{\left(d+ex^2\right)^4\left(a+b\tan^{-1}(cx)\right)}{8e}-\frac{(bc)\int\left(\frac{\left(2c^2d-e\right)e\left(2c^4d^2-2c^2de+e^2\right)}{c^8}+\frac{e^2\left(6c^4d^2-4c^2de+e^2\right)x^2}{c^6}\right.}{8}$$

$$=-\frac{b\left(2c^2d-e\right)\left(2c^4d^2-2c^2de+e^2\right)x}{8c^7}-\frac{be\left(6c^4d^2-4c^2de+e^2\right)x^3}{24c^5}-\frac{b\left(4c^2d-e\right)e^2}{40c^3}$$

$$=-\frac{b\left(2c^2d-e\right)\left(2c^4d^2-2c^2de+e^2\right)x}{8c^7}-\frac{be\left(6c^4d^2-4c^2de+e^2\right)x^3}{24c^5}-\frac{b\left(4c^2d-e\right)e^2}{40c^3}$$

Mathematica [A] time = 0.159754, size = 217, normalized size = 1.37

$$\frac{cx\left(105ac^7x\left(6d^2ex^2+4d^3+4de^2x^4+e^3x^6\right)-3bc^6\left(70d^2ex^2+140d^3+28de^2x^4+5e^3x^6\right)+7bc^4e\left(90d^2+20dex^2+3e^2x^4\right.}{84}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + e*x^2)^3*(a + b*ArcTan[c*x]),x]
```

```
[Out] (c*x*(105*b*e^3 - 35*b*c^2*e^2*(12*d + e*x^2) + 7*b*c^4*e*(90*d^2 + 20*d*e*
x^2 + 3*e^2*x^4) + 105*a*c^7*x*(4*d^3 + 6*d^2*e*x^2 + 4*d*e^2*x^4 + e^3*x^6
) - 3*b*c^6*(140*d^3 + 70*d^2*e*x^2 + 28*d*e^2*x^4 + 5*e^3*x^6)) + 105*b*(4
*c^6*d^3 - 6*c^4*d^2*e + 4*c^2*d*e^2 - e^3 + c^8*(4*d^3*x^2 + 6*d^2*e*x^4 +
 4*d*e^2*x^6 + e^3*x^8))*ArcTan[c*x])/(840*c^8)
```

Maple [A] time = 0.037, size = 265, normalized size = 1.7

$$\frac{ae^3x^8}{8}+\frac{ade^2x^6}{2}+\frac{3\,ad^2ex^4}{4}+\frac{ax^2d^3}{2}+\frac{b\arctan(cx)\,e^3x^8}{8}+\frac{b\arctan(cx)\,de^2x^6}{2}+\frac{3\,b\arctan(cx)\,d^2ex^4}{4}+\frac{b\arctan(cx)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d)^3*(a+b*arctan(c*x)),x)
```

```
[Out] 1/8*a*e^3*x^8+1/2*a*d*e^2*x^6+3/4*a*d^2*e*x^4+1/2*a*x^2*d^3+1/8*b*arctan(c*
x)*e^3*x^8+1/2*b*arctan(c*x)*d*e^2*x^6+3/4*b*arctan(c*x)*d^2*e*x^4+1/2*b*ar
ctan(c*x)*d^3*x^2-1/56*b*e^3*x^7/c-1/10/c*b*x^5*d*e^2-1/4/c*b*x^3*d^2*e-1/2
*b*d^3*x/c+1/40/c^3*b*x^5*e^3+1/6/c^3*b*x^3*d*e^2+3/4/c^3*b*d^2*e*x-1/24/c^
5*b*e^3*x^3-1/2/c^5*b*d*e^2*x+1/8/c^7*b*e^3*x+1/2/c^2*b*arctan(c*x)*d^3-3/4
/c^4*b*arctan(c*x)*d^2*e+1/2/c^6*b*arctan(c*x)*d*e^2-1/8/c^8*b*arctan(c*x)*
e^3
```

Maxima [A] time = 1.46397, size = 313, normalized size = 1.98

$$\frac{1}{8}ae^3x^8+\frac{1}{2}ade^2x^6+\frac{3}{4}ad^2ex^4+\frac{1}{2}ad^3x^2+\frac{1}{2}\left(x^2\arctan\left(cx\right)-c\left(\frac{x}{c^2}-\frac{\arctan\left(cx\right)}{c^3}\right)\right)bd^3+\frac{1}{4}\left(3x^4\arctan\left(cx\right)-c\left(\frac{c^2x}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^3*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] 1/8*a*e^3*x^8 + 1/2*a*d*e^2*x^6 + 3/4*a*d^2*e*x^4 + 1/2*a*d^3*x^2 + 1/2*(x^
2*arctan(c*x) - c*(x/c^2 - arctan(c*x)/c^3))*b*d^3 + 1/4*(3*x^4*arctan(c*x)
 - c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5))*b*d^2*e + 1/30*(15*x^6*arct
an(c*x) - c*((3*c^4*x^5 - 5*c^2*x^3 + 15*x)/c^6 - 15*arctan(c*x)/c^7))*b*d*
e^2 + 1/840*(105*x^8*arctan(c*x) - c*((15*c^6*x^7 - 21*c^4*x^5 + 35*c^2*x^3
 - 105*x)/c^8 + 105*arctan(c*x)/c^9))*b*e^3
```

Fricas [A] time = 1.54233, size = 559, normalized size = 3.54

$$\frac{105\,ac^8e^3x^8+420\,ac^8de^2x^6-15\,bc^7e^3x^7+630\,ac^8d^2ex^4+420\,ac^8d^3x^2-21\left(4\,bc^7de^2-bc^5e^3\right)x^5-35\left(6\,bc^7d^2e-4\,bc^5d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^3*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] 1/840*(105*a*c^8*e^3*x^8 + 420*a*c^8*d*e^2*x^6 - 15*b*c^7*e^3*x^7 + 630*a*c
^8*d^2*e*x^4 + 420*a*c^8*d^3*x^2 - 21*(4*b*c^7*d*e^2 - b*c^5*e^3)*x^5 - 35*
```

```
(6*b*c^7*d^2*e - 4*b*c^5*d*e^2 + b*c^3*e^3)*x^3 - 105*(4*b*c^7*d^3 - 6*b*c^
5*d^2*e + 4*b*c^3*d*e^2 - b*c*e^3)*x + 105*(b*c^8*e^3*x^8 + 4*b*c^8*d*e^2*x
^6 + 6*b*c^8*d^2*e*x^4 + 4*b*c^8*d^3*x^2 + 4*b*c^6*d^3 - 6*b*c^4*d^2*e + 4*
b*c^2*d*e^2 - b*e^3)*arctan(c*x))/c^8
```

Sympy [A] time = 6.38401, size = 350, normalized size = 2.22

$$\begin{cases} \frac{ad^3x^2}{2} + \frac{3ad^2ex^4}{4} + \frac{ade^2x^6}{2} + \frac{ae^3x^8}{8} + \frac{bd^3x^2\operatorname{atan}(cx)}{2} + \frac{3bd^2ex^4\operatorname{atan}(cx)}{4} + \frac{bde^2x^6\operatorname{atan}(cx)}{2} + \frac{be^3x^8\operatorname{atan}(cx)}{8} - \frac{bd^3x}{2c} - \frac{bd^2ex^3}{4c} - \frac{bde^2x^5}{10c} - \frac{be}{5} \\ a\left(\frac{d^3x^2}{2} + \frac{3d^2ex^4}{4} + \frac{de^2x^6}{2} + \frac{e^3x^8}{8}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d)**3*(a+b*atan(c*x)),x)
```

```
[Out] Piecewise((a*d**3*x**2/2 + 3*a*d**2*e*x**4/4 + a*d*e**2*x**6/2 + a*e**3*x**
8/8 + b*d**3*x**2*atan(c*x)/2 + 3*b*d**2*e*x**4*atan(c*x)/4 + b*d*e**2*x**6
*atan(c*x)/2 + b*e**3*x**8*atan(c*x)/8 - b*d**3*x/(2*c) - b*d**2*e*x**3/(4*
c) - b*d*e**2*x**5/(10*c) - b*e**3*x**7/(56*c) + b*d**3*atan(c*x)/(2*c**2)
+ 3*b*d**2*e*x/(4*c**3) + b*d*e**2*x**3/(6*c**3) + b*e**3*x**5/(40*c**3) -
3*b*d**2*e*atan(c*x)/(4*c**4) - b*d*e**2*x/(2*c**5) - b*e**3*x**3/(24*c**5)
 + b*d*e**2*atan(c*x)/(2*c**6) + b*e**3*x/(8*c**7) - b*e**3*atan(c*x)/(8*c*
*8), Ne(c, 0)), (a*(d**3*x**2/2 + 3*d**2*e*x**4/4 + d*e**2*x**6/2 + e**3*x*
*8/8), True))
```

Giac [B] time = 1.46809, size = 416, normalized size = 2.63

$$\frac{105\,bc^8x^8\arctan(cx)\,e^3 + 105\,ac^8x^8e^3 + 420\,bc^8dx^6\arctan(cx)\,e^2 + 420\,ac^8dx^6e^2 + 630\,bc^8d^2x^4\arctan(cx)\,e - 15\,bc^7x^7}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^3*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] 1/840*(105*b*c^8*x^8*arctan(c*x)*e^3 + 105*a*c^8*x^8*e^3 + 420*b*c^8*d*x^6*
arctan(c*x)*e^2 + 420*a*c^8*d*x^6*e^2 + 630*b*c^8*d^2*x^4*arctan(c*x)*e - 1
5*b*c^7*x^7*e^3 + 630*a*c^8*d^2*x^4*e + 420*b*c^8*d^3*x^2*arctan(c*x) - 84*
b*c^7*d*x^5*e^2 + 420*a*c^8*d^3*x^2 - 210*b*c^7*d^2*x^3*e - 420*pi*b*c^6*d^
3*sgn(c)*sgn(x) - 420*b*c^7*d^3*x + 21*b*c^5*x^5*e^3 + 420*b*c^6*d^3*arctan
(c*x) + 140*b*c^5*d*x^3*e^2 + 630*b*c^5*d^2*x*e - 630*b*c^4*d^2*arctan(c*x)
```

```
*e - 35*b*c^3*x^3*e^3 - 420*pi*b*c^2*d*e^2*sgn(c)*sgn(x) - 420*b*c^3*d*x*e^
2 + 420*b*c^2*d*arctan(c*x)*e^2 + 105*b*c*x*e^3 - 105*b*arctan(c*x)*e^3)/c^
8
```

3.1140 $\int \left(d+ex^2\right)^3 \left(a+b\tan^{-1}(cx)\right) dx$

Optimal. Leaf size=188

$$d^2ex^3\left(a+b\tan^{-1}(cx)\right)+d^3x\left(a+b\tan^{-1}(cx)\right)+\frac{3}{5}de^2x^5\left(a+b\tan^{-1}(cx)\right)+\frac{1}{7}e^3x^7\left(a+b\tan^{-1}(cx)\right)-\frac{bex^2\left(35c^4d^2-2\right.}{70c^5}$$

```
[Out] -(b*e*(35*c^4*d^2 - 21*c^2*d*e + 5*e^2)*x^2)/(70*c^5) - (b*(21*c^2*d - 5*e)
*e^2*x^4)/(140*c^3) - (b*e^3*x^6)/(42*c) + d^3*x*(a + b*ArcTan[c*x]) + d^2*
e*x^3*(a + b*ArcTan[c*x]) + (3*d*e^2*x^5*(a + b*ArcTan[c*x]))/5 + (e^3*x^7*
(a + b*ArcTan[c*x]))/7 - (b*(35*c^6*d^3 - 35*c^4*d^2*e + 21*c^2*d*e^2 - 5*e
^3)*Log[1 + c^2*x^2])/(70*c^7)
```

Rubi [A] time = 0.150537, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {194, 4912, 1810, 260}

$$d^2ex^3\left(a+b\tan^{-1}(cx)\right)+d^3x\left(a+b\tan^{-1}(cx)\right)+\frac{3}{5}de^2x^5\left(a+b\tan^{-1}(cx)\right)+\frac{1}{7}e^3x^7\left(a+b\tan^{-1}(cx)\right)-\frac{bex^2\left(35c^4d^2-2\right.}{70c^5}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^3*(a + b*ArcTan[c*x]),x]
```


```
[Out] -(b*e*(35*c^4*d^2 - 21*c^2*d*e + 5*e^2)*x^2)/(70*c^5) - (b*(21*c^2*d - 5*e)
*e^2*x^4)/(140*c^3) - (b*e^3*x^6)/(42*c) + d^3*x*(a + b*ArcTan[c*x]) + d^2*
e*x^3*(a + b*ArcTan[c*x]) + (3*d*e^2*x^5*(a + b*ArcTan[c*x]))/5 + (e^3*x^7*
(a + b*ArcTan[c*x]))/7 - (b*(35*c^6*d^3 - 35*c^4*d^2*e + 21*c^2*d*e^2 - 5*e
^3)*Log[1 + c^2*x^2])/(70*c^7)
```


Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4912

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcTan[c*x], u, x]
- Dist[b*c, Int[u/(1 + c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && (I
ntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \left(d+ex^2\right)^3 \left(a+b\tan^{-1}(cx)\right) dx = d^3x\left(a+b\tan^{-1}(cx)\right)+d^2ex^3\left(a+b\tan^{-1}(cx)\right)+\frac{3}{5}de^2x^5\left(a+b\tan^{-1}(cx)\right)+\frac{1}{7}e^3$$

$$= d^3x\left(a+b\tan^{-1}(cx)\right)+d^2ex^3\left(a+b\tan^{-1}(cx)\right)+\frac{3}{5}de^2x^5\left(a+b\tan^{-1}(cx)\right)+\frac{1}{7}e^3$$

$$= -\frac{be\left(35c^4d^2-21c^2de+5e^2\right)x^2}{70c^5}-\frac{b\left(21c^2d-5e\right)e^2x^4}{140c^3}-\frac{be^3x^6}{42c}+d^3x\left(a+b\tan^{-1}(\right.$$

$$= -\frac{be\left(35c^4d^2-21c^2de+5e^2\right)x^2}{70c^5}-\frac{b\left(21c^2d-5e\right)e^2x^4}{140c^3}-\frac{be^3x^6}{42c}+d^3x\left(a+b\tan^{-1}(\right.$$

Mathematica [A] time = 0.140756, size = 192, normalized size = 1.02

$$\frac{c^2x\left(12ac^5\left(35d^2ex^2+35d^3+21de^2x^4+5e^3x^6\right)-bex\left(c^4\left(210d^2+63dex^2+10e^2x^4\right)-3c^2e\left(42d+5ex^2\right)+30e^2\right)\right)-6b}{420c^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^3*(a + b*ArcTan[c*x]),x]
```

```
[Out] (c^2*x*(12*a*c^5*(35*d^3 + 35*d^2*e*x^2 + 21*d*e^2*x^4 + 5*e^3*x^6) - b*e*x
*(30*e^2 - 3*c^2*e*(42*d + 5*e*x^2) + c^4*(210*d^2 + 63*d*e*x^2 + 10*e^2*x^
4))) + 12*b*c^7*x*(35*d^3 + 35*d^2*e*x^2 + 21*d*e^2*x^4 + 5*e^3*x^6)*ArcTan
[c*x] - 6*b*(35*c^6*d^3 - 35*c^4*d^2*e + 21*c^2*d*e^2 - 5*e^3)*Log[1 + c^2*
x^2])/(420*c^7)
```

Maple [A] time = 0.038, size = 239, normalized size = 1.3

$$\frac{ax^7e^3}{7}+\frac{3\,ax^5de^2}{5}+ax^3d^2e+ad^3x+\frac{b\arctan(cx)\,x^7e^3}{7}+\frac{3\,b\arctan(cx)\,x^5de^2}{5}+b\arctan(cx)\,x^3d^2e+b\arctan(cx)\,d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^3*(a+b*arctan(c*x)),x)
```

```
[Out] 1/7*a*x^7*e^3+3/5*a*x^5*d*e^2+a*x^3*d^2*e+a*d^3*x+1/7*b*arctan(c*x)*x^7*e^3
+3/5*b*arctan(c*x)*x^5*d*e^2+b*arctan(c*x)*x^3*d^2*e+b*arctan(c*x)*d^3*x-1/
2/c*b*x^2*d^2*e-3/20/c*b*x^4*d*e^2-1/42*b*e^3*x^6/c+3/10/c^3*b*x^2*d*e^2+1/
28/c^3*b*e^3*x^4-1/14/c^5*b*e^3*x^2-1/2/c*b*ln(c^2*x^2+1)*d^3+1/2/c^3*b*ln(
c^2*x^2+1)*d^2*e-3/10/c^5*b*ln(c^2*x^2+1)*d*e^2+1/14/c^7*b*ln(c^2*x^2+1)*e^
3
```

Maxima [A] time = 0.962492, size = 300, normalized size = 1.6

$$\frac{1}{7}ae^3x^7+\frac{3}{5}ade^2x^5+ad^2ex^3+\frac{1}{2}\left(2x^3\arctan(cx)-c\left(\frac{x^2}{c^2}-\frac{\log\left(c^2x^2+1\right)}{c^4}\right)\right)bd^2e+\frac{3}{20}\left(4x^5\arctan(cx)-c\left(\frac{c^2x^4-2x^2}{c^4}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] 1/7*a*e^3*x^7 + 3/5*a*d*e^2*x^5 + a*d^2*e*x^3 + 1/2*(2*x^3*arctan(c*x) - c*
(x^2/c^2 - log(c^2*x^2 + 1)/c^4))*b*d^2*e + 3/20*(4*x^5*arctan(c*x) - c*((c
^2*x^4 - 2*x^2)/c^4 + 2*log(c^2*x^2 + 1)/c^6))*b*d*e^2 + 1/84*(12*x^7*arcta
n(c*x) - c*((2*c^4*x^6 - 3*c^2*x^4 + 6*x^2)/c^6 - 6*log(c^2*x^2 + 1)/c^8))*
b*e^3 + a*d^3*x + 1/2*(2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*b*d^3/c
```

Fricas [A] time = 1.78603, size = 513, normalized size = 2.73

$$\frac{60\,ac^7e^3x^7+252\,ac^7de^2x^5-10\,bc^6e^3x^6+420\,ac^7d^2ex^3+420\,ac^7d^3x-3\left(21\,bc^6de^2-5\,bc^4e^3\right)x^4-6\left(35\,bc^6d^2e-21\,bc^4de\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] 1/420*(60*a*c^7*e^3*x^7 + 252*a*c^7*d*e^2*x^5 - 10*b*c^6*e^3*x^6 + 420*a*c^
7*d^2*e*x^3 + 420*a*c^7*d^3*x - 3*(21*b*c^6*d*e^2 - 5*b*c^4*e^3)*x^4 - 6*(3
5*b*c^6*d^2*e - 21*b*c^4*d*e^2 + 5*b*c^2*e^3)*x^2 + 12*(5*b*c^7*e^3*x^7 + 2
1*b*c^7*d*e^2*x^5 + 35*b*c^7*d^2*e*x^3 + 35*b*c^7*d^3*x)*arctan(c*x) - 6*(3
```

5*b*c^6*d^3 - 35*b*c^4*d^2*e + 21*b*c^2*d*e^2 - 5*b*e^3)*log(c^2*x^2 + 1))/
c^7

Sympy [A] time = 4.23916, size = 306, normalized size = 1.63

$$\begin{cases} ad^3x + ad^2ex^3 + \frac{3ade^2x^5}{5} + \frac{ae^3x^7}{7} + bd^3x\operatorname{atan}(cx) + bd^2ex^3\operatorname{atan}(cx) + \frac{3bde^2x^5\operatorname{atan}(cx)}{5} + \frac{be^3x^7\operatorname{atan}(cx)}{7} - \frac{bd^3\log\left(x^2+\frac{1}{c^2}\right)}{2c} - \cdots \\ a\left(d^3x + d^2ex^3 + \frac{3de^2x^5}{5} + \frac{e^3x^7}{7}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**3*(a+b*atan(c*x)),x)
```

```
[Out] Piecewise((a*d**3*x + a*d**2*e*x**3 + 3*a*d*e**2*x**5/5 + a*e**3*x**7/7 + b
*d**3*x*atan(c*x) + b*d**2*e*x**3*atan(c*x) + 3*b*d*e**2*x**5*atan(c*x)/5 +
 b*e**3*x**7*atan(c*x)/7 - b*d**3*log(x**2 + c**(-2))/(2*c) - b*d**2*e*x**2
/(2*c) - 3*b*d*e**2*x**4/(20*c) - b*e**3*x**6/(42*c) + b*d**2*e*log(x**2 +
c**(-2))/(2*c**3) + 3*b*d*e**2*x**2/(10*c**3) + b*e**3*x**4/(28*c**3) - 3*b
*d*e**2*log(x**2 + c**(-2))/(10*c**5) - b*e**3*x**2/(14*c**5) + b*e**3*log(
x**2 + c**(-2))/(14*c**7), Ne(c, 0)), (a*(d**3*x + d**2*e*x**3 + 3*d*e**2*x
**5/5 + e**3*x**7/7), True))
```

Giac [A] time = 1.11154, size = 352, normalized size = 1.87

$$\frac{60\,bc^7x^7\arctan(cx)e^3 + 60\,ac^7x^7e^3 + 252\,bc^7dx^5\arctan(cx)e^2 + 252\,ac^7dx^5e^2 + 420\,bc^7d^2x^3\arctan(cx)e - 10\,bc^6x^6e \cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] 1/420*(60*b*c^7*x^7*arctan(c*x)*e^3 + 60*a*c^7*x^7*e^3 + 252*b*c^7*d*x^5*ar
ctan(c*x)*e^2 + 252*a*c^7*d*x^5*e^2 + 420*b*c^7*d^2*x^3*arctan(c*x)*e - 10*
b*c^6*x^6*e^3 + 420*a*c^7*d^2*x^3*e + 420*b*c^7*d^3*x*arctan(c*x) - 63*b*c^
6*d*x^4*e^2 + 420*a*c^7*d^3*x - 210*b*c^6*d^2*x^2*e - 210*b*c^6*d^3*log(c^2
*x^2 + 1) + 15*b*c^4*x^4*e^3 + 126*b*c^4*d*x^2*e^2 + 210*b*c^4*d^2*e*log(c^
2*x^2 + 1) - 30*b*c^2*x^2*e^3 - 126*b*c^2*d*e^2*log(c^2*x^2 + 1) + 30*b*e^3
*log(c^2*x^2 + 1))/c^7
```

3.1141 $\int \frac{(d+ex^2)^3 (a+b\tan^{-1}(cx))}{x} dx$

Optimal. Leaf size=228

$$\frac{1}{2}ibd^3\text{PolyLog}(2,-icx)-\frac{1}{2}ibd^3\text{PolyLog}(2,icx)+\frac{3}{2}d^2ex^2\left(a+b\tan^{-1}(cx)\right)+\frac{3}{4}de^2x^4\left(a+b\tan^{-1}(cx)\right)+\frac{1}{6}e^3x^6\left(a+b\tan^{-1}(cx)\right)$$

```
[Out] (-3*b*d^2*e*x)/(2*c) + (3*b*d*e^2*x)/(4*c^3) - (b*e^3*x)/(6*c^5) - (b*d*e^2
*x^3)/(4*c) + (b*e^3*x^3)/(18*c^3) - (b*e^3*x^5)/(30*c) + (3*b*d^2*e*ArcTan
[c*x])/(2*c^2) - (3*b*d*e^2*ArcTan[c*x])/(4*c^4) + (b*e^3*ArcTan[c*x])/(6*c
^6) + (3*d^2*e*x^2*(a + b*ArcTan[c*x]))/2 + (3*d*e^2*x^4*(a + b*ArcTan[c*x]
))/4 + (e^3*x^6*(a + b*ArcTan[c*x]))/6 + a*d^3*Log[x] + (I/2)*b*d^3*PolyLog
[2, (-I)*c*x] - (I/2)*b*d^3*PolyLog[2, I*c*x]
```

Rubi [A] time = 0.221279, antiderivative size = 228, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4980, 4848, 2391, 4852, 321, 203, 302}

$$\frac{1}{2}ibd^3\text{PolyLog}(2,-icx)-\frac{1}{2}ibd^3\text{PolyLog}(2,icx)+\frac{3}{2}d^2ex^2\left(a+b\tan^{-1}(cx)\right)+\frac{3}{4}de^2x^4\left(a+b\tan^{-1}(cx)\right)+\frac{1}{6}e^3x^6\left(a+b\tan^{-1}(cx)\right)$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^3*(a + b*ArcTan[c*x]))/x,x]
```

```
[Out] (-3*b*d^2*e*x)/(2*c) + (3*b*d*e^2*x)/(4*c^3) - (b*e^3*x)/(6*c^5) - (b*d*e^2
*x^3)/(4*c) + (b*e^3*x^3)/(18*c^3) - (b*e^3*x^5)/(30*c) + (3*b*d^2*e*ArcTan
[c*x])/(2*c^2) - (3*b*d*e^2*ArcTan[c*x])/(4*c^4) + (b*e^3*ArcTan[c*x])/(6*c
^6) + (3*d^2*e*x^2*(a + b*ArcTan[c*x]))/2 + (3*d*e^2*x^4*(a + b*ArcTan[c*x]
))/4 + (e^3*x^6*(a + b*ArcTan[c*x]))/6 + a*d^3*Log[x] + (I/2)*b*d^3*PolyLog
[2, (-I)*c*x] - (I/2)*b*d^3*PolyLog[2, I*c*x]
```

Rule 4980

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\int \frac{(d+ex^2)^3 (a+b\tan^{-1}(cx))}{x}\,dx = \int \left(\frac{d^3 (a+b\tan^{-1}(cx))}{x} + 3d^2ex (a+b\tan^{-1}(cx)) + 3de^2x^3 (a+b\tan^{-1}(cx)) + e^3x\right.$$
$$= d^3 \int \frac{a+b\tan^{-1}(cx)}{x}\,dx + (3d^2e)\int x (a+b\tan^{-1}(cx))\,dx + (3de^2)\int x^3 (a+b\tan$$
$$= \frac{3}{2}d^2ex^2 (a+b\tan^{-1}(cx)) + \frac{3}{4}de^2x^4 (a+b\tan^{-1}(cx)) + \frac{1}{6}e^3x^6 (a+b\tan^{-1}(cx)) + a$$
$$= -\frac{3bd^2ex}{2c} + \frac{3}{2}d^2ex^2 (a+b\tan^{-1}(cx)) + \frac{3}{4}de^2x^4 (a+b\tan^{-1}(cx)) + \frac{1}{6}e^3x^6 (a+b\tan$$
$$= -\frac{3bd^2ex}{2c} + \frac{3bde^2x}{4c^3} - \frac{be^3x}{6c^5} - \frac{bde^2x^3}{4c} + \frac{be^3x^3}{18c^3} - \frac{be^3x^5}{30c} + \frac{3bd^2e\tan^{-1}(cx)}{2c^2} + \frac{3}{2}d^2ex^2$$
$$= -\frac{3bd^2ex}{2c} + \frac{3bde^2x}{4c^3} - \frac{be^3x}{6c^5} - \frac{bde^2x^3}{4c} + \frac{be^3x^3}{18c^3} - \frac{be^3x^5}{30c} + \frac{3bd^2e\tan^{-1}(cx)}{2c^2} - \frac{3bde^2\,\mathrm{ta}}{4}$$

Mathematica [A] time = 0.169533, size = 190, normalized size = 0.83

$$\frac{1}{2}ibd^3\text{PolyLog}(2,-icx) - \frac{1}{2}ibd^3\text{PolyLog}(2,icx) + \frac{3}{2}d^2ex^2 (a+b\tan^{-1}(cx)) + \frac{3}{4}de^2x^4 (a+b\tan^{-1}(cx)) + \frac{1}{6}e^3x^6 (a+b\,\mathrm{t}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)^3*(a + b*ArcTan[c*x]))/x,x]
```

```
[Out] -(b*e^3*(15*c*x - 5*c^3*x^3 + 3*c^5*x^5 - 15*ArcTan[c*x]))/(90*c^6) - (3*b*
d^2*e*(c*x - ArcTan[c*x]))/(2*c^2) - (b*d*e^2*(-3*c*x + c^3*x^3 + 3*ArcTan[
c*x]))/(4*c^4) + (3*d^2*e*x^2*(a + b*ArcTan[c*x]))/2 + (3*d*e^2*x^4*(a + b*
ArcTan[c*x]))/4 + (e^3*x^6*(a + b*ArcTan[c*x]))/6 + a*d^3*Log[x] + (I/2)*b*
d^3*PolyLog[2, (-I)*c*x] - (I/2)*b*d^3*PolyLog[2, I*c*x]
```

Maple [A] time = 0.052, size = 272, normalized size = 1.2

$$\frac{ax^6e^3}{6} + \frac{3\,ax^4de^2}{4} + \frac{3\,ax^2d^2e}{2} + ad^3\ln(cx) + \frac{b\arctan(cx)\,x^6e^3}{6} + \frac{3\,b\arctan(cx)\,x^4de^2}{4} + \frac{3\,b\arctan(cx)\,x^2d^2e}{2} + b\,\mathrm{arcta}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^3*(a+b*arctan(c*x))/x,x)
```

```
[Out] 1/6*a*x^6*e^3+3/4*a*x^4*d*e^2+3/2*a*x^2*d^2*e+a*d^3*ln(c*x)+1/6*b*arctan(c*
x)*x^6*e^3+3/4*b*arctan(c*x)*x^4*d*e^2+3/2*b*arctan(c*x)*x^2*d^2*e+b*arctan
(c*x)*d^3*ln(c*x)-1/30*b*e^3*x^5/c-1/4*b*d*e^2*x^3/c-3/2*b*d^2*e*x/c+1/18*b
*e^3*x^3/c^3+3/4*b*d*e^2*x/c^3-1/6*b*e^3*x/c^5+3/2*b*d^2*e*arctan(c*x)/c^2-
3/4*b*d*e^2*arctan(c*x)/c^4+1/6*b*e^3*arctan(c*x)/c^6-1/2*I*b*d^3*ln(c*x)*l
n(1-I*c*x)+1/2*I*b*d^3*ln(c*x)*ln(1+I*c*x)-1/2*I*b*d^3*dilog(1-I*c*x)+1/2*I
*b*d^3*dilog(1+I*c*x)
```

Maxima [A] time = 2.22607, size = 356, normalized size = 1.56

$$\frac{1}{6}ae^3x^6 + \frac{3}{4}ade^2x^4 + \frac{3}{2}ad^2ex^2 + ad^3\log(x) - \frac{6\,bc^5e^3x^5 + 45\,\pi bc^6d^3\log\left(c^2x^2+1\right) - 180\,bc^6d^3\arctan(cx)\log(x|c|) +}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x,x, algorithm="maxima")
```

```
[Out] 1/6*a*e^3*x^6 + 3/4*a*d*e^2*x^4 + 3/2*a*d^2*e*x^2 + a*d^3*log(x) - 1/180*(6
*b*c^5*e^3*x^5 + 45*pi*b*c^6*d^3*log(c^2*x^2 + 1) - 180*b*c^6*d^3*arctan(c*
x)*log(x*abs(c)) + 90*I*b*c^6*d^3*dilog(I*c*x + 1) - 90*I*b*c^6*d^3*dilog(-
I*c*x + 1) + 5*(9*b*c^5*d*e^2 - 2*b*c^3*e^3)*x^3 + 15*(18*b*c^5*d^2*e - 9*b
*c^3*d*e^2 + 2*b*c*e^3)*x - (30*b*c^6*e^3*x^6 + 135*b*c^6*d*e^2*x^4 + 270*b
*c^6*d^2*e*x^2 + 180*I*b*c^6*d^3*arctan2(0, c) + 270*b*c^4*d^2*e - 135*b*c^
2*d*e^2 + 30*b*e^3)*arctan(c*x))/c^6
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{ae^3x^6 + 3\,ade^2x^4 + 3\,ad^2ex^2 + ad^3 + \left(be^3x^6 + 3\,bde^2x^4 + 3\,bd^2ex^2 + bd^3\right)\arctan(cx)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x,x, algorithm="fricas")
```

```
[Out] integral((a*e^3*x^6 + 3*a*d*e^2*x^4 + 3*a*d^2*e*x^2 + a*d^3 + (b*e^3*x^6 +
3*b*d*e^2*x^4 + 3*b*d^2*e*x^2 + b*d^3)*arctan(c*x))/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b\operatorname{atan}(cx))\left(d + ex^2\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**3*(a+b*atan(c*x))/x,x)
```

```
[Out] Integral((a + b*atan(c*x))*(d + e*x**2)**3/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2 + d\right)^3 (b\arctan(cx) + a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^3*(b*arctan(c*x) + a)/x, x)
```

3.1142 $\int \frac{(d+ex^2)^3(a+b\tan^{-1}(cx))}{x^2}\,dx$

Optimal. Leaf size=160

$$3d^2ex\left(a+b\tan^{-1}(cx)\right)-\frac{d^3\left(a+b\tan^{-1}(cx)\right)}{x}+de^2x^3\left(a+b\tan^{-1}(cx)\right)+\frac{1}{5}e^3x^5\left(a+b\tan^{-1}(cx)\right)-\frac{b\left(15c^4d^2e+5c^6d\right.}{}$$

```
[Out] -(b*(5*c^2*d - e)*e^2*x^2)/(10*c^3) - (b*e^3*x^4)/(20*c) - (d^3*(a + b*ArcT
an[c*x]))/x + 3*d^2*e*x*(a + b*ArcTan[c*x]) + d*e^2*x^3*(a + b*ArcTan[c*x])
 + (e^3*x^5*(a + b*ArcTan[c*x]))/5 + b*c*d^3*Log[x] - (b*(5*c^6*d^3 + 15*c^
4*d^2*e - 5*c^2*d*e^2 + e^3)*Log[1 + c^2*x^2])/(10*c^5)
```

Rubi [A] time = 0.258139, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {270, 4976, 1799, 1620}

$$3d^2ex\left(a+b\tan^{-1}(cx)\right)-\frac{d^3\left(a+b\tan^{-1}(cx)\right)}{x}+de^2x^3\left(a+b\tan^{-1}(cx)\right)+\frac{1}{5}e^3x^5\left(a+b\tan^{-1}(cx)\right)-\frac{b\left(15c^4d^2e+5c^6d\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^3*(a + b*ArcTan[c*x]))/x^2,x]
```


```
[Out] -(b*(5*c^2*d - e)*e^2*x^2)/(10*c^3) - (b*e^3*x^4)/(20*c) - (d^3*(a + b*ArcT
an[c*x]))/x + 3*d^2*e*x*(a + b*ArcTan[c*x]) + d*e^2*x^3*(a + b*ArcTan[c*x])
 + (e^3*x^5*(a + b*ArcTan[c*x]))/5 + b*c*d^3*Log[x] - (b*(5*c^6*d^3 + 15*c^
4*d^2*e - 5*c^2*d*e^2 + e^3)*Log[1 + c^2*x^2])/(10*c^5)
```


Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 4976

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Dis
t[a + b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] && !(
ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(IL
```

```
tQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILtQ[(m
 - 1)/2, 0]))
```

Rule 1799

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^3 \left(a+b\tan^{-1}(cx)\right)}{x^2}\,dx = -\frac{d^3\left(a+b\tan^{-1}(cx)\right)}{x}+3d^2ex\left(a+b\tan^{-1}(cx)\right)+de^2x^3\left(a+b\tan^{-1}(cx)\right)+\frac{1}{5}e^3x^5$$

$$= -\frac{d^3\left(a+b\tan^{-1}(cx)\right)}{x}+3d^2ex\left(a+b\tan^{-1}(cx)\right)+de^2x^3\left(a+b\tan^{-1}(cx)\right)+\frac{1}{5}e^3x^5$$

$$= -\frac{d^3\left(a+b\tan^{-1}(cx)\right)}{x}+3d^2ex\left(a+b\tan^{-1}(cx)\right)+de^2x^3\left(a+b\tan^{-1}(cx)\right)+\frac{1}{5}e^3x^5$$

$$= -\frac{b\left(5c^2d-e\right)e^2x^2}{10c^3}-\frac{be^3x^4}{20c}-\frac{d^3\left(a+b\tan^{-1}(cx)\right)}{x}+3d^2ex\left(a+b\tan^{-1}(cx)\right)+de^2x$$

Mathematica [A] time = 0.149035, size = 169, normalized size = 1.06

$$\frac{1}{20}\left(60ad^2ex-\frac{20ad^3}{x}+20ade^2x^3+4ae^3x^5-\frac{2b\left(15c^4d^2e+5c^6d^3-5c^2de^2+e^3\right)\log\left(c^2x^2+1\right)}{c^5}+\frac{2be^2x^2\left(e-5c^2d\right)}{c^3}+\frac{4b}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)^3*(a + b*ArcTan[c*x]))/x^2,x]
```

```
[Out] ((-20*a*d^3)/x + 60*a*d^2*e*x + (2*b*e^2*(-5*c^2*d + e)*x^2)/c^3 + 20*a*d*e
^2*x^3 - (b*e^3*x^4)/c + 4*a*e^3*x^5 + (4*b*(-5*d^3 + 15*d^2*e*x^2 + 5*d*e^
```

```
2*x^4 + e^3*x^6)*ArcTan[c*x])/x + 20*b*c*d^3*Log[x] - (2*b*(5*c^6*d^3 + 15*
c^4*d^2*e - 5*c^2*d*e^2 + e^3)*Log[1 + c^2*x^2])/c^5)/20
```

Maple [A] time = 0.044, size = 211, normalized size = 1.3

$$\frac{ax^5e^3}{5} + ax^3de^2 + 3\,ad^2ex - \frac{ad^3}{x} + \frac{b\arctan(cx)\,x^5e^3}{5} + b\arctan(cx)\,x^3de^2 + 3\,b\arctan(cx)\,d^2ex - \frac{bd^3\arctan(cx)}{x} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^3*(a+b*arctan(c*x))/x^2,x)
```

```
[Out] 1/5*a*x^5*e^3+a*x^3*d*e^2+3*a*d^2*e*x-a*d^3/x+1/5*b*arctan(c*x)*x^5*e^3+b*a
rctan(c*x)*x^3*d*e^2+3*b*arctan(c*x)*d^2*e*x-b*arctan(c*x)*d^3/x-1/20*b*e^3
*x^4/c-1/2*b/c*x^2*d*e^2+1/10*b/c^3*e^3*x^2-1/2*b*c*d^3*ln(c^2*x^2+1)-3/2*b
/c*ln(c^2*x^2+1)*d^2*e+1/2*b/c^3*ln(c^2*x^2+1)*d*e^2-1/10*b/c^5*ln(c^2*x^2+
1)*e^3+c*b*d^3*ln(c*x)
```

Maxima [A] time = 0.965819, size = 266, normalized size = 1.66

$$\frac{1}{5}\,ae^3x^5 + ade^2x^3 - \frac{1}{2}\left(c\left(\log\left(c^2x^2+1\right) - \log\left(x^2\right)\right) + \frac{2\,\arctan(cx)}{x}\right)bd^3 + \frac{1}{2}\left(2\,x^3\arctan(cx) - c\left(\frac{x^2}{c^2} - \frac{\log\left(c^2x^2+1\right)}{c^4}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x^2,x, algorithm="maxima")
```

```
[Out] 1/5*a*e^3*x^5 + a*d*e^2*x^3 - 1/2*(c*(log(c^2*x^2 + 1) - log(x^2)) + 2*arct
an(c*x)/x)*b*d^3 + 1/2*(2*x^3*arctan(c*x) - c*(x^2/c^2 - log(c^2*x^2 + 1)/c
^4))*b*d*e^2 + 1/20*(4*x^5*arctan(c*x) - c*((c^2*x^4 - 2*x^2)/c^4 + 2*log(c
^2*x^2 + 1)/c^6))*b*e^3 + 3*a*d^2*e*x + 3/2*(2*c*x*arctan(c*x) - log(c^2*x^
2 + 1))*b*d^2*e/c - a*d^3/x
```

Fricas [A] time = 1.95102, size = 446, normalized size = 2.79

$$\frac{4\,ac^5e^3x^6 + 20\,ac^5de^2x^4 - bc^4e^3x^5 + 20\,bc^6d^3x\log(x) + 60\,ac^5d^2ex^2 - 20\,ac^5d^3 - 2\left(5\,bc^4de^2 - bc^2e^3\right)x^3 - 2\left(5\,bc^6d^3 + 1\right.}{20\,c^5x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x^2,x, algorithm="fricas")
```

```
[Out] 1/20*(4*a*c^5*e^3*x^6 + 20*a*c^5*d*e^2*x^4 - b*c^4*e^3*x^5 + 20*b*c^6*d^3*x
*log(x) + 60*a*c^5*d^2*e*x^2 - 20*a*c^5*d^3 - 2*(5*b*c^4*d*e^2 - b*c^2*e^3)
*x^3 - 2*(5*b*c^6*d^3 + 15*b*c^4*d^2*e - 5*b*c^2*d*e^2 + b*e^3)*x*log(c^2*x
^2 + 1) + 4*(b*c^5*e^3*x^6 + 5*b*c^5*d*e^2*x^4 + 15*b*c^5*d^2*e*x^2 - 5*b*c
^5*d^3)*arctan(c*x))/(c^5*x)
```

Sympy [A] time = 4.54261, size = 258, normalized size = 1.61

$$\begin{cases} -\frac{ad^3}{x} + 3ad^2ex + ade^2x^3 + \frac{ae^3x^5}{5} + bcd^3\log(x) - \frac{bcd^3\log\left(x^2+\frac{1}{c^2}\right)}{2} - \frac{bd^3\operatorname{atan}(cx)}{x} + 3bd^2ex\operatorname{atan}(cx) + bde^2x^3\operatorname{atan}(cx) + \frac{be^3x \\ a\left(-\frac{d^3}{x} + 3d^2ex + de^2x^3 + \frac{e^3x^5}{5}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**3*(a+b*atan(c*x))/x**2,x)
```

```
[Out] Piecewise((-a*d**3/x + 3*a*d**2*e*x + a*d*e**2*x**3 + a*e**3*x**5/5 + b*c*d
**3*log(x) - b*c*d**3*log(x**2 + c**(-2))/2 - b*d**3*atan(c*x)/x + 3*b*d**2
*e*x*atan(c*x) + b*d*e**2*x**3*atan(c*x) + b*e**3*x**5*atan(c*x)/5 - 3*b*d*
*2*e*log(x**2 + c**(-2))/(2*c) - b*d*e**2*x**2/(2*c) - b*e**3*x**4/(20*c) +
 b*d*e**2*log(x**2 + c**(-2))/(2*c**3) + b*e**3*x**2/(10*c**3) - b*e**3*log
(x**2 + c**(-2))/(10*c**5), Ne(c, 0)), (a*(-d**3/x + 3*d**2*e*x + d*e**2*x*
*3 + e**3*x**5/5), True))
```

Giac [A] time = 1.11454, size = 325, normalized size = 2.03

$$\frac{4\,bc^5x^6\arctan(cx)\,e^3 + 4\,ac^5x^6e^3 + 20\,bc^5dx^4\arctan(cx)\,e^2 + 20\,ac^5dx^4e^2 + 60\,bc^5d^2x^2\arctan(cx)\,e - 10\,bc^6d^3x\log\left(c^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x^2,x, algorithm="giac")
```

```
[Out] 1/20*(4*b*c^5*x^6*arctan(c*x)*e^3 + 4*a*c^5*x^6*e^3 + 20*b*c^5*d*x^4*arctan
(c*x)*e^2 + 20*a*c^5*d*x^4*e^2 + 60*b*c^5*d^2*x^2*arctan(c*x)*e - 10*b*c^6*
```

```
d^3*x*log(c^2*x^2 + 1) + 20*b*c^6*d^3*x*log(x) - b*c^4*x^5*e^3 + 60*a*c^5*d
^2*x^2*e - 20*b*c^5*d^3*arctan(c*x) - 10*b*c^4*d*x^3*e^2 - 30*b*c^4*d^2*x*e
*log(c^2*x^2 + 1) - 20*a*c^5*d^3 + 2*b*c^2*x^3*e^3 + 10*b*c^2*d*x*e^2*log(c
^2*x^2 + 1) - 2*b*x*e^3*log(c^2*x^2 + 1))/(c^5*x)
```

3.1143 $\int \frac{(d+ex^2)^3 (a+b\tan^{-1}(cx))}{x^3} \, dx$

Optimal. Leaf size=200

$$\frac{3}{2}ibd^2e\text{PolyLog}(2,-icx)-\frac{3}{2}ibd^2e\text{PolyLog}(2,icx)-\frac{d^3\left(a+b\tan^{-1}(cx)\right)}{2x^2}+\frac{3}{2}de^2x^2\left(a+b\tan^{-1}(cx)\right)+\frac{1}{4}e^3x^4\left(a+b\tan\right.$$

```
[Out] -(b*c*d^3)/(2*x) - (3*b*d*e^2*x)/(2*c) + (b*e^3*x)/(4*c^3) - (b*e^3*x^3)/(1
2*c) - (b*c^2*d^3*ArcTan[c*x])/2 + (3*b*d*e^2*ArcTan[c*x])/(2*c^2) - (b*e^3
*ArcTan[c*x])/(4*c^4) - (d^3*(a + b*ArcTan[c*x]))/(2*x^2) + (3*d*e^2*x^2*(a
 + b*ArcTan[c*x]))/2 + (e^3*x^4*(a + b*ArcTan[c*x]))/4 + 3*a*d^2*e*Log[x] +
 ((3*I)/2)*b*d^2*e*PolyLog[2, (-I)*c*x] - ((3*I)/2)*b*d^2*e*PolyLog[2, I*c*
x]
```

Rubi [A] time = 0.210537, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {4980, 4852, 325, 203, 4848, 2391, 321, 302}

$$\frac{3}{2}ibd^2e\text{PolyLog}(2,-icx)-\frac{3}{2}ibd^2e\text{PolyLog}(2,icx)-\frac{d^3\left(a+b\tan^{-1}(cx)\right)}{2x^2}+\frac{3}{2}de^2x^2\left(a+b\tan^{-1}(cx)\right)+\frac{1}{4}e^3x^4\left(a+b\tan\right.$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^3*(a + b*ArcTan[c*x]))/x^3,x]
```

```
[Out] -(b*c*d^3)/(2*x) - (3*b*d*e^2*x)/(2*c) + (b*e^3*x)/(4*c^3) - (b*e^3*x^3)/(1
2*c) - (b*c^2*d^3*ArcTan[c*x])/2 + (3*b*d*e^2*ArcTan[c*x])/(2*c^2) - (b*e^3
*ArcTan[c*x])/(4*c^4) - (d^3*(a + b*ArcTan[c*x]))/(2*x^2) + (3*d*e^2*x^2*(a
 + b*ArcTan[c*x]))/2 + (e^3*x^4*(a + b*ArcTan[c*x]))/4 + 3*a*d^2*e*Log[x] +
 ((3*I)/2)*b*d^2*e*PolyLog[2, (-I)*c*x] - ((3*I)/2)*b*d^2*e*PolyLog[2, I*c*
x]
```

Rule 4980

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
```

```
Q[m, 2*n - 1]
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^3\left(a+b\tan^{-1}(cx)\right)}{x^3}\,dx = \int \left(\frac{d^3\left(a+b\tan^{-1}(cx)\right)}{x^3}+\frac{3d^2e\left(a+b\tan^{-1}(cx)\right)}{x}+3de^2x\left(a+b\tan^{-1}(cx)\right)+e^3x^3\right.$$

$$= d^3\int \frac{a+b\tan^{-1}(cx)}{x^3}\,dx+\left(3d^2e\right)\int \frac{a+b\tan^{-1}(cx)}{x}\,dx+\left(3de^2\right)\int x\left(a+b\tan^{-1}(\right.$$

$$= -\frac{d^3\left(a+b\tan^{-1}(cx)\right)}{2x^2}+\frac{3}{2}de^2x^2\left(a+b\tan^{-1}(cx)\right)+\frac{1}{4}e^3x^4\left(a+b\tan^{-1}(cx)\right)+3ad^2$$

$$= -\frac{bcd^3}{2x}-\frac{3bde^2x}{2c}-\frac{d^3\left(a+b\tan^{-1}(cx)\right)}{2x^2}+\frac{3}{2}de^2x^2\left(a+b\tan^{-1}(cx)\right)+\frac{1}{4}e^3x^4\left(a+b\right.$$

$$= -\frac{bcd^3}{2x}-\frac{3bde^2x}{2c}+\frac{be^3x}{4c^3}-\frac{be^3x^3}{12c}-\frac{1}{2}bc^2d^3\tan^{-1}(cx)+\frac{3bde^2\tan^{-1}(cx)}{2c^2}-\frac{d^3\left(a+b\right.}{}$$

$$= -\frac{bcd^3}{2x}-\frac{3bde^2x}{2c}+\frac{be^3x}{4c^3}-\frac{be^3x^3}{12c}-\frac{1}{2}bc^2d^3\tan^{-1}(cx)+\frac{3bde^2\tan^{-1}(cx)}{2c^2}-\frac{be^3\tan^{-}}{4c^4}$$

Mathematica [C] time = 0.160364, size = 170, normalized size = 0.85

$$\frac{1}{12}\left(-\frac{6bcd^3\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-c^2x^2\right)}{x}+18ibd^2e\text{PolyLog}(2,-icx)-18ibd^2e\text{PolyLog}(2,icx)-\frac{6d^3\left(a+b\tan\right.}{x^2}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + e*x^2)^3*(a + b*ArcTan[c*x]))/x^3,x]
```

```
[Out] ((-18*b*d*e^2*(c*x - ArcTan[c*x]))/c^2 - (b*e^3*(-3*c*x + c^3*x^3 + 3*ArcTa
n[c*x]))/c^4 - (6*d^3*(a + b*ArcTan[c*x]))/x^2 + 18*d*e^2*x^2*(a + b*ArcTan
[c*x]) + 3*e^3*x^4*(a + b*ArcTan[c*x]) - (6*b*c*d^3*Hypergeometric2F1[-1/2,
 1, 1/2, -(c^2*x^2)])/x + 36*a*d^2*e*Log[x] + (18*I)*b*d^2*e*PolyLog[2, (-I
)*c*x] - (18*I)*b*d^2*e*PolyLog[2, I*c*x])/12
```

Maple [A] time = 0.056, size = 251, normalized size = 1.3

$$\frac{ae^3x^4}{4}+\frac{3\,ax^2de^2}{2}-\frac{d^3a}{2\,x^2}+3\,ad^2e\ln(cx)+\frac{b\arctan(cx)\,e^3x^4}{4}+\frac{3\,b\arctan(cx)\,x^2de^2}{2}-\frac{bd^3\arctan(cx)}{2\,x^2}+3\,b\arctan(cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^3*(a+b*arctan(c*x))/x^3,x)
```

```
[Out] 1/4*a*e^3*x^4+3/2*a*x^2*d*e^2-1/2*d^3*a/x^2+3*a*d^2*e*ln(c*x)+1/4*b*arctan(
c*x)*e^3*x^4+3/2*b*arctan(c*x)*x^2*d*e^2-1/2*b*arctan(c*x)*d^3/x^2+3*b*arct
an(c*x)*d^2*e*ln(c*x)-1/12*b*e^3*x^3/c-3/2*b*d*e^2*x/c+1/4*b*e^3*x/c^3-1/2*
b*c^2*d^3*arctan(c*x)+3/2*b*d*e^2*arctan(c*x)/c^2-1/4*b*e^3*arctan(c*x)/c^4
-1/2*b*c*d^3/x-3/2*I*b*d^2*e*dilog(1-I*c*x)+3/2*I*b*d^2*e*dilog(1+I*c*x)-3/
2*I*b*d^2*e*ln(c*x)*ln(1-I*c*x)+3/2*I*b*d^2*e*ln(c*x)*ln(1+I*c*x)
```

Maxima [A] time = 2.20896, size = 321, normalized size = 1.6

$$\frac{1}{4}ae^3x^4+\frac{3}{2}ade^2x^2-\frac{1}{2}\left(\left(c\arctan\left(cx\right)+\frac{1}{x}\right)c+\frac{\arctan\left(cx\right)}{x^2}\right)bd^3+3\,ad^2e\log\left(x\right)-\frac{ad^3}{2\,x^2}-\frac{bc^3e^3x^3+9\,\pi bc^4d^2e\log\left(c^2x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*a*e^3*x^4 + 3/2*a*d*e^2*x^2 - 1/2*((c*arctan(c*x) + 1/x)*c + arctan(c*x
)/x^2)*b*d^3 + 3*a*d^2*e*log(x) - 1/2*a*d^3/x^2 - 1/12*(b*c^3*e^3*x^3 + 9*p
i*b*c^4*d^2*e*log(c^2*x^2 + 1) - 36*b*c^4*d^2*e*arctan(c*x)*log(x*abs(c)) +
 18*I*b*c^4*d^2*e*dilog(I*c*x + 1) - 18*I*b*c^4*d^2*e*dilog(-I*c*x + 1) + 3
*(6*b*c^3*d*e^2 - b*c*e^3)*x - (3*b*c^4*e^3*x^4 + 18*b*c^4*d*e^2*x^2 + 36*I
*b*c^4*d^2*e*arctan2(0, c) + 18*b*c^2*d*e^2 - 3*b*e^3)*arctan(c*x))/c^4
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{ae^3x^6+3\,ade^2x^4+3\,ad^2ex^2+ad^3+\left(be^3x^6+3\,bde^2x^4+3\,bd^2ex^2+bd^3\right)\arctan\left(cx\right)}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x^3,x, algorithm="fricas")
```

```
[Out] integral((a*e^3*x^6 + 3*a*d*e^2*x^4 + 3*a*d^2*e*x^2 + a*d^3 + (b*e^3*x^6 +
3*b*d*e^2*x^4 + 3*b*d^2*e*x^2 + b*d^3)*arctan(c*x))/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b\,\mathrm{atan}\,(cx))\left(d + ex^2\right)^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**3*(a+b*atan(c*x))/x**3,x)
```

```
[Out] Integral((a + b*atan(c*x))*(d + e*x**2)**3/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2 + d\right)^3 (b\arctan\,(cx) + a)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x^3,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^3*(b*arctan(c*x) + a)/x^3, x)
```

3.1144 $\int \frac{\left(d+ex^2\right)^3\left(a+b\tan^{-1}(cx)\right)}{x^4}\,dx$

Optimal. Leaf size=158

$$-\frac{3d^2e\left(a+b\tan^{-1}(cx)\right)}{x}-\frac{d^3\left(a+b\tan^{-1}(cx)\right)}{3x^3}+3de^2x\left(a+b\tan^{-1}(cx)\right)+\frac{1}{3}e^3x^3\left(a+b\tan^{-1}(cx)\right)+\frac{b\left(c^2d+e\right)\left(c^4d^2\right.}{}$$

```
[Out] -(b*c*d^3)/(6*x^2) - (b*e^3*x^2)/(6*c) - (d^3*(a + b*ArcTan[c*x]))/(3*x^3)
- (3*d^2*e*(a + b*ArcTan[c*x]))/x + 3*d*e^2*x*(a + b*ArcTan[c*x]) + (e^3*x^
3*(a + b*ArcTan[c*x]))/3 - (b*c*d^2*(c^2*d - 9*e)*Log[x])/3 + (b*(c^2*d + e
)*(c^4*d^2 - 10*c^2*d*e + e^2)*Log[1 + c^2*x^2])/(6*c^3)
```

Rubi [A] time = 0.264665, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {270, 4976, 12, 1799, 1620}

$$-\frac{3d^2e\left(a+b\tan^{-1}(cx)\right)}{x}-\frac{d^3\left(a+b\tan^{-1}(cx)\right)}{3x^3}+3de^2x\left(a+b\tan^{-1}(cx)\right)+\frac{1}{3}e^3x^3\left(a+b\tan^{-1}(cx)\right)+\frac{b\left(c^2d+e\right)\left(c^4d^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^3*(a + b*ArcTan[c*x]))/x^4,x]
```


```
[Out] -(b*c*d^3)/(6*x^2) - (b*e^3*x^2)/(6*c) - (d^3*(a + b*ArcTan[c*x]))/(3*x^3)
- (3*d^2*e*(a + b*ArcTan[c*x]))/x + 3*d*e^2*x*(a + b*ArcTan[c*x]) + (e^3*x^
3*(a + b*ArcTan[c*x]))/3 - (b*c*d^2*(c^2*d - 9*e)*Log[x])/3 + (b*(c^2*d + e
)*(c^4*d^2 - 10*c^2*d*e + e^2)*Log[1 + c^2*x^2])/(6*c^3)
```


Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 4976

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Dis
t[a + b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&  !(
ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(IL
```

```
tQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILtQ[(m
 - 1)/2, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1799

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^3\left(a+b\tan^{-1}(cx)\right)}{x^4}\,dx = -\frac{d^3\left(a+b\tan^{-1}(cx)\right)}{3x^3}-\frac{3d^2e\left(a+b\tan^{-1}(cx)\right)}{x}+3de^2x\left(a+b\tan^{-1}(cx)\right)+\frac{1}{3}e^3x^3\left(\right.$$

$$= -\frac{d^3\left(a+b\tan^{-1}(cx)\right)}{3x^3}-\frac{3d^2e\left(a+b\tan^{-1}(cx)\right)}{x}+3de^2x\left(a+b\tan^{-1}(cx)\right)+\frac{1}{3}e^3x^3\left(\right.$$

$$= -\frac{d^3\left(a+b\tan^{-1}(cx)\right)}{3x^3}-\frac{3d^2e\left(a+b\tan^{-1}(cx)\right)}{x}+3de^2x\left(a+b\tan^{-1}(cx)\right)+\frac{1}{3}e^3x^3\left(\right.$$

$$= -\frac{d^3\left(a+b\tan^{-1}(cx)\right)}{3x^3}-\frac{3d^2e\left(a+b\tan^{-1}(cx)\right)}{x}+3de^2x\left(a+b\tan^{-1}(cx)\right)+\frac{1}{3}e^3x^3\left(\right.$$

$$= -\frac{bcd^3}{6x^2}-\frac{be^3x^2}{6c}-\frac{d^3\left(a+b\tan^{-1}(cx)\right)}{3x^3}-\frac{3d^2e\left(a+b\tan^{-1}(cx)\right)}{x}+3de^2x\left(a+b\tan\right.$$

Mathematica [A] time = 0.164613, size = 166, normalized size = 1.05

$$\frac{1}{6}\left(-\frac{18ad^2e}{x}-\frac{2ad^3}{x^3}+18ade^2x+2ae^3x^3+\frac{b\left(-9c^4d^2e+c^6d^3-9c^2de^2+e^3\right)\log\left(c^2x^2+1\right)}{c^3}-2bcd^2\log(x)\left(c^2d-9e\right)+\frac{2b}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)^3*(a + b*ArcTan[c*x]))/x^4,x]
```

```
[Out] ((-2*a*d^3)/x^3 - (b*c*d^3)/x^2 - (18*a*d^2*e)/x + 18*a*d*e^2*x - (b*e^3*x^
2)/c + 2*a*e^3*x^3 + (2*b*(-d^3 - 9*d^2*e*x^2 + 9*d*e^2*x^4 + e^3*x^6)*ArcT
an[c*x])/x^3 - 2*b*c*d^2*(c^2*d - 9*e)*Log[x] + (b*(c^6*d^3 - 9*c^4*d^2*e -
 9*c^2*d*e^2 + e^3)*Log[1 + c^2*x^2])/c^3)/6
```

Maple [A] time = 0.048, size = 213, normalized size = 1.4

$$\frac{ae^3x^3}{3} + 3\,ade^2x - 3\,\frac{ad^2e}{x} - \frac{ad^3}{3\,x^3} + \frac{b\arctan(cx)\,e^3x^3}{3} + 3\,b\arctan(cx)\,de^2x - 3\,\frac{bd^2\arctan(cx)\,e}{x} - \frac{bd^3\arctan(cx)}{3\,x^3} - $$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^3*(a+b*arctan(c*x))/x^4,x)
```

```
[Out] 1/3*a*e^3*x^3+3*a*d*e^2*x-3*a*d^2*e/x-1/3*a*d^3/x^3+1/3*b*arctan(c*x)*e^3*x
^3+3*b*arctan(c*x)*d*e^2*x-3*b*arctan(c*x)*d^2*e/x-1/3*b*arctan(c*x)*d^3/x^
3-1/6*b*e^3*x^2/c+1/6*b*c^3*d^3*ln(c^2*x^2+1)-3/2*c*b*ln(c^2*x^2+1)*d^2*e-3
/2/c*b*ln(c^2*x^2+1)*d*e^2+1/6/c^3*b*ln(c^2*x^2+1)*e^3-1/3*c^3*b*d^3*ln(c*x
)+3*c*b*ln(c*x)*d^2*e-1/6*b*c*d^3/x^2
```

Maxima [A] time = 0.981411, size = 261, normalized size = 1.65

$$\frac{1}{3}\,ae^3x^3 + \frac{1}{6}\left(\left(c^2\log\left(c^2x^2+1\right) - c^2\log\left(x^2\right) - \frac{1}{x^2}\right)c - \frac{2\,\arctan(cx)}{x^3}\right)bd^3 - \frac{3}{2}\left(c\left(\log\left(c^2x^2+1\right) - \log\left(x^2\right)\right) + \frac{2\,\arctan}{x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x^4,x, algorithm="maxima")
```

```
[Out] 1/3*a*e^3*x^3 + 1/6*((c^2*log(c^2*x^2 + 1) - c^2*log(x^2) - 1/x^2)*c - 2*ar
ctan(c*x)/x^3)*b*d^3 - 3/2*(c*(log(c^2*x^2 + 1) - log(x^2)) + 2*arctan(c*x)
/x)*b*d^2*e + 1/6*(2*x^3*arctan(c*x) - c*(x^2/c^2 - log(c^2*x^2 + 1)/c^4))*
b*e^3 + 3*a*d*e^2*x + 3/2*(2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*b*d*e^2/c
- 3*a*d^2*e/x - 1/3*a*d^3/x^3
```

Fricas [A] time = 1.72967, size = 433, normalized size = 2.74

$$\frac{2\,ac^3e^3x^6 + 18\,ac^3de^2x^4 - bc^2e^3x^5 - bc^4d^3x - 18\,ac^3d^2ex^2 - 2\,ac^3d^3 + \left(bc^6d^3 - 9\,bc^4d^2e - 9\,bc^2de^2 + be^3\right)x^3\log\left(c^2x^2+1\right)}{6\,c^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*(2*a*c^3*e^3*x^6 + 18*a*c^3*d*e^2*x^4 - b*c^2*e^3*x^5 - b*c^4*d^3*x - 1
8*a*c^3*d^2*e*x^2 - 2*a*c^3*d^3 + (b*c^6*d^3 - 9*b*c^4*d^2*e - 9*b*c^2*d*e^
2 + b*e^3)*x^3*log(c^2*x^2 + 1) - 2*(b*c^6*d^3 - 9*b*c^4*d^2*e)*x^3*log(x)
+ 2*(b*c^3*e^3*x^6 + 9*b*c^3*d*e^2*x^4 - 9*b*c^3*d^2*e*x^2 - b*c^3*d^3)*arc
tan(c*x))/(c^3*x^3)
```

Sympy [A] time = 4.60556, size = 272, normalized size = 1.72

$$\begin{cases} -\frac{ad^3}{3x^3} - \frac{3ad^2e}{x} + 3ade^2x + \frac{ae^3x^3}{3} - \frac{bc^3d^3\log(x)}{3} + \frac{bc^3d^3\log\left(x^2+\frac{1}{c^2}\right)}{6} - \frac{bcd^3}{6x^2} + 3bcd^2e\log(x) - \frac{3bcd^2e\log\left(x^2+\frac{1}{c^2}\right)}{2} - \frac{bd^3\operatorname{atan}(cx)}{3x^3} - \frac{3bd^2e \\ a\left(-\frac{d^3}{3x^3} - \frac{3d^2e}{x} + 3de^2x + \frac{e^3x^3}{3}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**3*(a+b*atan(c*x))/x**4,x)
```

```
[Out] Piecewise((-a*d**3/(3*x**3) - 3*a*d**2*e/x + 3*a*d*e**2*x + a*e**3*x**3/3 -
 b*c**3*d**3*log(x)/3 + b*c**3*d**3*log(x**2 + c**(-2))/6 - b*c*d**3/(6*x**
2) + 3*b*c*d**2*e*log(x) - 3*b*c*d**2*e*log(x**2 + c**(-2))/2 - b*d**3*atan
(c*x)/(3*x**3) - 3*b*d**2*e*atan(c*x)/x + 3*b*d*e**2*x*atan(c*x) + b*e**3*x
**3*atan(c*x)/3 - 3*b*d*e**2*log(x**2 + c**(-2))/(2*c) - b*e**3*x**2/(6*c)
+ b*e**3*log(x**2 + c**(-2))/(6*c**3), Ne(c, 0)), (a*(-d**3/(3*x**3) - 3*d*
*2*e/x + 3*d*e**2*x + e**3*x**3/3), True))
```

Giac [A] time = 1.10084, size = 340, normalized size = 2.15

$$\frac{bc^6d^3x^3\log\left(c^2x^2+1\right) - 2\,bc^6d^3x^3\log(x) + 2\,bc^3x^6\arctan(cx)\,e^3 - 9\,bc^4d^2x^3e\log\left(c^2x^2+1\right) + 18\,bc^4d^2x^3e\log(x) + 2\,ac}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x^4,x, algorithm="giac")
```

```
[Out] 1/6*(b*c^6*d^3*x^3*log(c^2*x^2 + 1) - 2*b*c^6*d^3*x^3*log(x) + 2*b*c^3*x^6*
arctan(c*x)*e^3 - 9*b*c^4*d^2*x^3*e*log(c^2*x^2 + 1) + 18*b*c^4*d^2*x^3*e*l
og(x) + 2*a*c^3*x^6*e^3 + 18*b*c^3*d*x^4*arctan(c*x)*e^2 + 18*a*c^3*d*x^4*e
^2 - 18*b*c^3*d^2*x^2*arctan(c*x)*e - b*c^4*d^3*x - b*c^2*x^5*e^3 - 18*a*c^
3*d^2*x^2*e - 9*b*c^2*d*x^3*e^2*log(c^2*x^2 + 1) - 2*b*c^3*d^3*arctan(c*x)
- 2*a*c^3*d^3 + b*x^3*e^3*log(c^2*x^2 + 1))/(c^3*x^3)
```

3.1145 $\int \frac{(d+ex^2)^3(a+b\tan^{-1}(cx))}{x^5}\,dx$

Optimal. Leaf size=200

$$\frac{3}{2}ibde^2\text{PolyLog}(2,-icx)-\frac{3}{2}ibde^2\text{PolyLog}(2,icx)-\frac{3d^2e\left(a+b\tan^{-1}(cx)\right)}{2x^2}-\frac{d^3\left(a+b\tan^{-1}(cx)\right)}{4x^4}+\frac{1}{2}e^3x^2\left(a+b\tan^{-1}\right.$$

```
[Out] -(b*c*d^3)/(12*x^3) + (b*c^3*d^3)/(4*x) - (3*b*c*d^2*e)/(2*x) - (b*e^3*x)/(
2*c) + (b*c^4*d^3*ArcTan[c*x])/4 - (3*b*c^2*d^2*e*ArcTan[c*x])/2 + (b*e^3*A
rcTan[c*x])/(2*c^2) - (d^3*(a + b*ArcTan[c*x]))/(4*x^4) - (3*d^2*e*(a + b*A
rcTan[c*x]))/(2*x^2) + (e^3*x^2*(a + b*ArcTan[c*x]))/2 + 3*a*d*e^2*Log[x] +
 ((3*I)/2)*b*d*e^2*PolyLog[2, (-I)*c*x] - ((3*I)/2)*b*d*e^2*PolyLog[2, I*c*
x]
```

Rubi [A] time = 0.20703, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4980, 4852, 325, 203, 4848, 2391, 321}

$$\frac{3}{2}ibde^2\text{PolyLog}(2,-icx)-\frac{3}{2}ibde^2\text{PolyLog}(2,icx)-\frac{3d^2e\left(a+b\tan^{-1}(cx)\right)}{2x^2}-\frac{d^3\left(a+b\tan^{-1}(cx)\right)}{4x^4}+\frac{1}{2}e^3x^2\left(a+b\tan^{-1}\right.$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^3*(a + b*ArcTan[c*x]))/x^5,x]
```


```
[Out] -(b*c*d^3)/(12*x^3) + (b*c^3*d^3)/(4*x) - (3*b*c*d^2*e)/(2*x) - (b*e^3*x)/(
2*c) + (b*c^4*d^3*ArcTan[c*x])/4 - (3*b*c^2*d^2*e*ArcTan[c*x])/2 + (b*e^3*A
rcTan[c*x])/(2*c^2) - (d^3*(a + b*ArcTan[c*x]))/(4*x^4) - (3*d^2*e*(a + b*A
rcTan[c*x]))/(2*x^2) + (e^3*x^2*(a + b*ArcTan[c*x]))/2 + 3*a*d*e^2*Log[x] +
 ((3*I)/2)*b*d*e^2*PolyLog[2, (-I)*c*x] - ((3*I)/2)*b*d*e^2*PolyLog[2, I*c*
x]
```


Rule 4980

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^3\left(a+b\tan^{-1}(cx)\right)}{x^5}dx = \int \left(\frac{d^3\left(a+b\tan^{-1}(cx)\right)}{x^5}+\frac{3d^2e\left(a+b\tan^{-1}(cx)\right)}{x^3}+\frac{3de^2\left(a+b\tan^{-1}(cx)\right)}{x}+e^3x\left(a\right.\right.$$

$$= d^3\int \frac{a+b\tan^{-1}(cx)}{x^5}dx+\left(3d^2e\right)\int \frac{a+b\tan^{-1}(cx)}{x^3}dx+\left(3de^2\right)\int \frac{a+b\tan^{-1}(cx)}{x}$$

$$= -\frac{d^3\left(a+b\tan^{-1}(cx)\right)}{4x^4}-\frac{3d^2e\left(a+b\tan^{-1}(cx)\right)}{2x^2}+\frac{1}{2}e^3x^2\left(a+b\tan^{-1}(cx)\right)+3ade^2\,\mathrm{l}$$

$$= -\frac{bcd^3}{12x^3}-\frac{3bcd^2e}{2x}-\frac{be^3x}{2c}-\frac{d^3\left(a+b\tan^{-1}(cx)\right)}{4x^4}-\frac{3d^2e\left(a+b\tan^{-1}(cx)\right)}{2x^2}+\frac{1}{2}e^3x^2\left(a\right.$$

$$= -\frac{bcd^3}{12x^3}+\frac{bc^3d^3}{4x}-\frac{3bcd^2e}{2x}-\frac{be^3x}{2c}-\frac{3}{2}bc^2d^2e\tan^{-1}(cx)+\frac{be^3\tan^{-1}(cx)}{2c^2}-\frac{d^3\left(a+b\,\mathrm{t}\right.}{4x}$$

$$= -\frac{bcd^3}{12x^3}+\frac{bc^3d^3}{4x}-\frac{3bcd^2e}{2x}-\frac{be^3x}{2c}+\frac{1}{4}bc^4d^3\tan^{-1}(cx)-\frac{3}{2}bc^2d^2e\tan^{-1}(cx)+\frac{be^3\tan}{2}$$

Mathematica [C] time = 0.206362, size = 169, normalized size = 0.84

$$\frac{1}{12}\left(-\frac{18bcd^2e\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-c^2x^2\right)}{x}-\frac{bcd^3\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-c^2x^2\right)}{x^3}+18ibde^2\text{PolyLog}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + e*x^2)^3*(a + b*ArcTan[c*x]))/x^5,x]
```

```
[Out] ((-6*b*e^3*(c*x - ArcTan[c*x]))/c^2 - (3*d^3*(a + b*ArcTan[c*x]))/x^4 - (18
*d^2*e*(a + b*ArcTan[c*x]))/x^2 + 6*e^3*x^2*(a + b*ArcTan[c*x]) - (b*c*d^3*
Hypergeometric2F1[-3/2, 1, -1/2, -(c^2*x^2)])/x^3 - (18*b*c*d^2*e*Hypergeom
etric2F1[-1/2, 1, 1/2, -(c^2*x^2)])/x + 36*a*d*e^2*Log[x] + (18*I)*b*d*e^2*
PolyLog[2, (-I)*c*x] - (18*I)*b*d*e^2*PolyLog[2, I*c*x])/12
```

Maple [A] time = 0.056, size = 251, normalized size = 1.3

$$\frac{ae^3x^2}{2}-\frac{3\,ad^2e}{2\,x^2}-\frac{ad^3}{4\,x^4}+3\,ade^2\ln(cx)+\frac{b\arctan(cx)\,e^3x^2}{2}-\frac{3\,bd^2\arctan(cx)\,e}{2\,x^2}-\frac{bd^3\arctan(cx)}{4\,x^4}+3\,b\arctan(cx)\,de^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^3*(a+b*arctan(c*x))/x^5,x)
```

```
[Out] 1/2*a*e^3*x^2-3/2*a*d^2*e/x^2-1/4*a*d^3/x^4+3*a*d*e^2*ln(c*x)+1/2*b*arctan(
c*x)*e^3*x^2-3/2*b*arctan(c*x)*d^2*e/x^2-1/4*b*arctan(c*x)*d^3/x^4+3*b*arct
an(c*x)*d*e^2*ln(c*x)-1/2*b*e^3*x/c+1/4*b*c^4*d^3*arctan(c*x)-3/2*b*c^2*d^2
*e*arctan(c*x)+1/2*b*e^3*arctan(c*x)/c^2+1/4*b*c^3*d^3/x-3/2*b*c*d^2*e/x-1/
12*b*c*d^3/x^3+3/2*I*b*d*e^2*dilog(1+I*c*x)-3/2*I*b*d*e^2*dilog(1-I*c*x)+3/
2*I*b*d*e^2*ln(c*x)*ln(1+I*c*x)-3/2*I*b*d*e^2*ln(c*x)*ln(1-I*c*x)
```

Maxima [A] time = 2.16394, size = 316, normalized size = 1.58

$$\frac{1}{2}ae^3x^2 + \frac{1}{12}\left(\left(\left(3c^3\arctan(cx) + \frac{3c^2x^2-1}{x^3}\right)c - \frac{3\arctan(cx)}{x^4}\right)bd^3 - \frac{3}{2}\left(\left(c\arctan(cx) + \frac{1}{x}\right)c + \frac{\arctan(cx)}{x^2}\right)bd^2e + 3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x^5,x, algorithm="maxima")
```

```
[Out] 1/2*a*e^3*x^2 + 1/12*((3*c^3*arctan(c*x) + (3*c^2*x^2 - 1)/x^3)*c - 3*arcta
n(c*x)/x^4)*b*d^3 - 3/2*((c*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*b*d^2*e
 + 3*a*d*e^2*log(x) - 3/2*a*d^2*e/x^2 - 1/4*a*d^3/x^4 - 1/4*(3*pi*b*c^2*d*e
^2*log(c^2*x^2 + 1) - 12*b*c^2*d*e^2*arctan(c*x)*log(x*abs(c)) + 6*I*b*c^2*
d*e^2*dilog(I*c*x + 1) - 6*I*b*c^2*d*e^2*dilog(-I*c*x + 1) + 2*b*c*e^3*x -
(2*b*c^2*e^3*x^2 + 12*I*b*c^2*d*e^2*arctan2(0, c) + 2*b*e^3)*arctan(c*x))/c
^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{ae^3x^6 + 3\,ade^2x^4 + 3\,ad^2ex^2 + ad^3 + \left(be^3x^6 + 3\,bde^2x^4 + 3\,bd^2ex^2 + bd^3\right)\arctan(cx)}{x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x^5,x, algorithm="fricas")
```

```
[Out] integral((a*e^3*x^6 + 3*a*d*e^2*x^4 + 3*a*d^2*e*x^2 + a*d^3 + (b*e^3*x^6 +
3*b*d*e^2*x^4 + 3*b*d^2*e*x^2 + b*d^3)*arctan(c*x))/x^5, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b\,\mathrm{atan}\,(cx))\left(d + ex^2\right)^3}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**3*(a+b*atan(c*x))/x**5,x)
```

```
[Out] Integral((a + b*atan(c*x))*(d + e*x**2)**3/x**5, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2 + d\right)^3 (b\arctan\,(cx) + a)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x^5,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^3*(b*arctan(c*x) + a)/x^5, x)
```

3.1146 $\int \frac{(d+ex^2)^3(a+b\tan^{-1}(cx))}{x^6}\,dx$

Optimal. Leaf size=177

$$-\frac{d^2e\left(a+b\tan^{-1}(cx)\right)}{x^3}-\frac{d^3\left(a+b\tan^{-1}(cx)\right)}{5x^5}-\frac{3de^2\left(a+b\tan^{-1}(cx)\right)}{x}+e^3x\left(a+b\tan^{-1}(cx)\right)-\frac{b\left(-5c^4d^2e+c^6d^3+1\right.}{}$$

```
[Out] -(b*c*d^3)/(20*x^4) + (b*c*d^2*(c^2*d - 5*e))/(10*x^2) - (d^3*(a + b*ArcTan
[c*x]))/(5*x^5) - (d^2*e*(a + b*ArcTan[c*x]))/x^3 - (3*d*e^2*(a + b*ArcTan[
c*x]))/x + e^3*x*(a + b*ArcTan[c*x]) + (b*c*d*(c^4*d^2 - 5*c^2*d*e + 15*e^2
)*Log[x])/5 - (b*(c^6*d^3 - 5*c^4*d^2*e + 15*c^2*d*e^2 + 5*e^3)*Log[1 + c^2
*x^2])/(10*c)
```

Rubi [A] time = 0.285452, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {270, 4976, 12, 1799, 1620}

$$-\frac{d^2e\left(a+b\tan^{-1}(cx)\right)}{x^3}-\frac{d^3\left(a+b\tan^{-1}(cx)\right)}{5x^5}-\frac{3de^2\left(a+b\tan^{-1}(cx)\right)}{x}+e^3x\left(a+b\tan^{-1}(cx)\right)-\frac{b\left(-5c^4d^2e+c^6d^3+1\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^3*(a + b*ArcTan[c*x]))/x^6,x]
```


```
[Out] -(b*c*d^3)/(20*x^4) + (b*c*d^2*(c^2*d - 5*e))/(10*x^2) - (d^3*(a + b*ArcTan
[c*x]))/(5*x^5) - (d^2*e*(a + b*ArcTan[c*x]))/x^3 - (3*d*e^2*(a + b*ArcTan[
c*x]))/x + e^3*x*(a + b*ArcTan[c*x]) + (b*c*d*(c^4*d^2 - 5*c^2*d*e + 15*e^2
)*Log[x])/5 - (b*(c^6*d^3 - 5*c^4*d^2*e + 15*c^2*d*e^2 + 5*e^3)*Log[1 + c^2
*x^2])/(10*c)
```


Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 4976

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Dis
t[a + b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2
```

```
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&  !(
ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(IL
tQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILtQ[(m
 - 1)/2, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1799

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^3 \left(a+b\tan^{-1}(cx)\right)}{x^6}\,dx = -\frac{d^3\left(a+b\tan^{-1}(cx)\right)}{5x^5}-\frac{d^2e\left(a+b\tan^{-1}(cx)\right)}{x^3}-\frac{3de^2\left(a+b\tan^{-1}(cx)\right)}{x}+e^3x\left(a+b\right.$$

$$=-\frac{d^3\left(a+b\tan^{-1}(cx)\right)}{5x^5}-\frac{d^2e\left(a+b\tan^{-1}(cx)\right)}{x^3}-\frac{3de^2\left(a+b\tan^{-1}(cx)\right)}{x}+e^3x\left(a+b\right.$$

$$=-\frac{d^3\left(a+b\tan^{-1}(cx)\right)}{5x^5}-\frac{d^2e\left(a+b\tan^{-1}(cx)\right)}{x^3}-\frac{3de^2\left(a+b\tan^{-1}(cx)\right)}{x}+e^3x\left(a+b\right.$$

$$=-\frac{d^3\left(a+b\tan^{-1}(cx)\right)}{5x^5}-\frac{d^2e\left(a+b\tan^{-1}(cx)\right)}{x^3}-\frac{3de^2\left(a+b\tan^{-1}(cx)\right)}{x}+e^3x\left(a+b\right.$$

$$=-\frac{bcd^3}{20x^4}+\frac{bcd^2\left(c^2d-5e\right)}{10x^2}-\frac{d^3\left(a+b\tan^{-1}(cx)\right)}{5x^5}-\frac{d^2e\left(a+b\tan^{-1}(cx)\right)}{x^3}-\frac{3de^2\left(a\right.}{}$$

Mathematica [A] time = 0.172457, size = 184, normalized size = 1.04

$$\frac{1}{20}\left(-\frac{20ad^2e}{x^3}-\frac{4ad^3}{x^5}-\frac{60ade^2}{x}+20ae^3x-\frac{2b\left(-5c^4d^2e+c^6d^3+15c^2de^2+5e^3\right)\log\left(c^2x^2+1\right)}{c}+4bcd\log(x)\left(c^4d^2-5c\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)^3*(a + b*ArcTan[c*x]))/x^6,x]
```

```
[Out] ((-4*a*d^3)/x^5 - (b*c*d^3)/x^4 - (20*a*d^2*e)/x^3 + (2*b*c*d^2*(c^2*d - 5*
e))/x^2 - (60*a*d*e^2)/x + 20*a*e^3*x - (4*b*(d^3 + 5*d^2*e*x^2 + 15*d*e^2*
x^4 - 5*e^3*x^6)*ArcTan[c*x])/x^5 + 4*b*c*d*(c^4*d^2 - 5*c^2*d*e + 15*e^2)*
Log[x] - (2*b*(c^6*d^3 - 5*c^4*d^2*e + 15*c^2*d*e^2 + 5*e^3)*Log[1 + c^2*x^
2])/c)/20
```

Maple [A] time = 0.047, size = 236, normalized size = 1.3

$$ae^3x-3\,\frac{ade^2}{x}-\frac{ad^3}{5\,x^5}-\frac{ad^2e}{x^3}+b\arctan(cx)\,e^3x-3\,\frac{\arctan(cx)\,bde^2}{x}-\frac{bd^3\arctan(cx)}{5\,x^5}-\frac{bd^2\arctan(cx)\,e}{x^3}-\frac{c^5b\ln\left(c^2\right.}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^3*(a+b*arctan(c*x))/x^6,x)
```

```
[Out] a*e^3*x-3*a*d*e^2/x-1/5*a*d^3/x^5-a*d^2*e/x^3+b*arctan(c*x)*e^3*x-3*b*arcta
n(c*x)*d*e^2/x-1/5*b*arctan(c*x)*d^3/x^5-b*arctan(c*x)*d^2*e/x^3-1/10*c^5*b
*ln(c^2*x^2+1)*d^3+1/2*c^3*b*ln(c^2*x^2+1)*d^2*e-3/2*c*b*ln(c^2*x^2+1)*d*e^
2-1/2/c*b*ln(c^2*x^2+1)*e^3+1/10*b*c^3*d^3/x^2-1/2*c*b*d^2*e/x^2-1/20*b*c*d
^3/x^4+1/5*c^5*b*d^3*ln(c*x)-c^3*b*ln(c*x)*d^2*e+3*c*b*ln(c*x)*d*e^2
```

Maxima [A] time = 0.998314, size = 281, normalized size = 1.59

$$-\frac{1}{20}\left(\left(\left(2\,c^4\log\left(c^2x^2+1\right)-2\,c^4\log\left(x^2\right)-\frac{2\,c^2x^2-1}{x^4}\right)c+\frac{4\,\arctan(cx)}{x^5}\right)bd^3+\frac{1}{2}\left(\left(c^2\log\left(c^2x^2+1\right)-c^2\log\left(x^2\right)-\frac{1}{x^2}\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x^6,x, algorithm="maxima")
```

```
[Out] -1/20*((2*c^4*log(c^2*x^2 + 1) - 2*c^4*log(x^2) - (2*c^2*x^2 - 1)/x^4)*c +
4*arctan(c*x)/x^5)*b*d^3 + 1/2*((c^2*log(c^2*x^2 + 1) - c^2*log(x^2) - 1/x^
2)*c - 2*arctan(c*x)/x^3)*b*d^2*e - 3/2*(c*(log(c^2*x^2 + 1) - log(x^2)) +
2*arctan(c*x)/x)*b*d*e^2 + a*e^3*x + 1/2*(2*c*x*arctan(c*x) - log(c^2*x^2 +
 1))*b*e^3/c - 3*a*d*e^2/x - a*d^2*e/x^3 - 1/5*a*d^3/x^5
```

Fricas [A] time = 1.78263, size = 473, normalized size = 2.67

$$\frac{20\,ace^3x^6 - 60\,acde^2x^4 - bc^2d^3x - 20\,acd^2ex^2 - 2\left(bc^6d^3 - 5\,bc^4d^2e + 15\,bc^2de^2 + 5\,be^3\right)x^5\log\left(c^2x^2+1\right) + 4\left(bc^6d^3 - 5\,bc\right.}{20\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x^6,x, algorithm="fricas")
```

```
[Out] 1/20*(20*a*c*e^3*x^6 - 60*a*c*d*e^2*x^4 - b*c^2*d^3*x - 20*a*c*d^2*e*x^2 -
2*(b*c^6*d^3 - 5*b*c^4*d^2*e + 15*b*c^2*d*e^2 + 5*b*e^3)*x^5*log(c^2*x^2 +
1) + 4*(b*c^6*d^3 - 5*b*c^4*d^2*e + 15*b*c^2*d*e^2)*x^5*log(x) - 4*a*c*d^3
+ 2*(b*c^4*d^3 - 5*b*c^2*d^2*e)*x^3 + 4*(5*b*c*e^3*x^6 - 15*b*c*d*e^2*x^4 -
 5*b*c*d^2*e*x^2 - b*c*d^3)*arctan(c*x))/(c*x^5)
```

Sympy [A] time = 4.91528, size = 289, normalized size = 1.63

$$\begin{cases} -\frac{ad^3}{5x^5} - \frac{ad^2e}{x^3} - \frac{3ade^2}{x} + ae^3x + \frac{bc^5d^3\log(x)}{5} - \frac{bc^5d^3\log\left(x^2+\frac{1}{c^2}\right)}{10} + \frac{bc^3d^3}{10x^2} - bc^3d^2e\log(x) + \frac{bc^3d^2e\log\left(x^2+\frac{1}{c^2}\right)}{2} - \frac{bcd^3}{20x^4} - \frac{bcd^2e}{2x^2} + 3bcde^2 \\ a\left(-\frac{d^3}{5x^5} - \frac{d^2e}{x^3} - \frac{3de^2}{x} + e^3x\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**3*(a+b*atan(c*x))/x**6,x)
```

```
[Out] Piecewise((-a*d**3/(5*x**5) - a*d**2*e/x**3 - 3*a*d*e**2/x + a*e**3*x + b*c
**5*d**3*log(x)/5 - b*c**5*d**3*log(x**2 + c**(-2))/10 + b*c**3*d**3/(10*x*
*2) - b*c**3*d**2*e*log(x) + b*c**3*d**2*e*log(x**2 + c**(-2))/2 - b*c*d**3
/(20*x**4) - b*c*d**2*e/(2*x**2) + 3*b*c*d*e**2*log(x) - 3*b*c*d*e**2*log(x
**2 + c**(-2))/2 - b*d**3*atan(c*x)/(5*x**5) - b*d**2*e*atan(c*x)/x**3 - 3*
b*d*e**2*atan(c*x)/x + b*e**3*x*atan(c*x) - b*e**3*log(x**2 + c**(-2))/(2*c
), Ne(c, 0)), (a*(-d**3/(5*x**5) - d**2*e/x**3 - 3*d*e**2/x + e**3*x), True
))
```

Giac [A] time = 1.10351, size = 359, normalized size = 2.03

$$-\frac{2 bc^6 d^3 x^5 \log\left(c^2 x^2+1\right) - 4 bc^6 d^3 x^5 \log\left(x\right) - 10 bc^4 d^2 x^5 e \log\left(c^2 x^2+1\right) + 20 bc^4 d^2 x^5 e \log\left(x\right) - 2 bc^4 d^3 x^3 + 30 bc^2 d x^5 e}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x^6,x, algorithm="giac")
```

```
[Out] -1/20*(2*b*c^6*d^3*x^5*log(c^2*x^2 + 1) - 4*b*c^6*d^3*x^5*log(x) - 10*b*c^4
*d^2*x^5*e*log(c^2*x^2 + 1) + 20*b*c^4*d^2*x^5*e*log(x) - 2*b*c^4*d^3*x^3 +
 30*b*c^2*d*x^5*e^2*log(c^2*x^2 + 1) - 60*b*c^2*d*x^5*e^2*log(x) - 20*b*c*x
^6*arctan(c*x)*e^3 - 20*a*c*x^6*e^3 + 60*b*c*d*x^4*arctan(c*x)*e^2 + 10*b*c
^2*d^2*x^3*e + 60*a*c*d*x^4*e^2 + 20*b*c*d^2*x^2*arctan(c*x)*e + 10*b*x^5*e
^3*log(c^2*x^2 + 1) + b*c^2*d^3*x + 20*a*c*d^2*x^2*e + 4*b*c*d^3*arctan(c*x
) + 4*a*c*d^3)/(c*x^5)
```

3.1147 $\int \frac{(d+ex^2)^3(a+b\tan^{-1}(cx))}{x^7}\,dx$

Optimal. Leaf size=228

$$\frac{1}{2}ibe^3\text{PolyLog}(2,-icx)-\frac{1}{2}ibe^3\text{PolyLog}(2,icx)-\frac{3d^2e\left(a+b\tan^{-1}(cx)\right)}{4x^4}-\frac{d^3\left(a+b\tan^{-1}(cx)\right)}{6x^6}-\frac{3de^2\left(a+b\tan^{-1}(cx)\right)}{2x^2}$$

```
[Out] -(b*c*d^3)/(30*x^5) + (b*c^3*d^3)/(18*x^3) - (b*c*d^2*e)/(4*x^3) - (b*c^5*d
^3)/(6*x) + (3*b*c^3*d^2*e)/(4*x) - (3*b*c*d*e^2)/(2*x) - (b*c^6*d^3*ArcTan
[c*x])/6 + (3*b*c^4*d^2*e*ArcTan[c*x])/4 - (3*b*c^2*d*e^2*ArcTan[c*x])/2 -
(d^3*(a + b*ArcTan[c*x]))/(6*x^6) - (3*d^2*e*(a + b*ArcTan[c*x]))/(4*x^4) -
 (3*d*e^2*(a + b*ArcTan[c*x]))/(2*x^2) + a*e^3*Log[x] + (I/2)*b*e^3*PolyLog
[2, (-I)*c*x] - (I/2)*b*e^3*PolyLog[2, I*c*x]
```

Rubi [A] time = 0.230883, antiderivative size = 228, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4980, 4852, 325, 203, 4848, 2391}

$$\frac{1}{2}ibe^3\text{PolyLog}(2,-icx)-\frac{1}{2}ibe^3\text{PolyLog}(2,icx)-\frac{3d^2e\left(a+b\tan^{-1}(cx)\right)}{4x^4}-\frac{d^3\left(a+b\tan^{-1}(cx)\right)}{6x^6}-\frac{3de^2\left(a+b\tan^{-1}(cx)\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^3*(a + b*ArcTan[c*x]))/x^7,x]
```



```
[Out] -(b*c*d^3)/(30*x^5) + (b*c^3*d^3)/(18*x^3) - (b*c*d^2*e)/(4*x^3) - (b*c^5*d
^3)/(6*x) + (3*b*c^3*d^2*e)/(4*x) - (3*b*c*d*e^2)/(2*x) - (b*c^6*d^3*ArcTan
[c*x])/6 + (3*b*c^4*d^2*e*ArcTan[c*x])/4 - (3*b*c^2*d*e^2*ArcTan[c*x])/2 -
(d^3*(a + b*ArcTan[c*x]))/(6*x^6) - (3*d^2*e*(a + b*ArcTan[c*x]))/(4*x^4) -
 (3*d*e^2*(a + b*ArcTan[c*x]))/(2*x^2) + a*e^3*Log[x] + (I/2)*b*e^3*PolyLog
[2, (-I)*c*x] - (I/2)*b*e^3*PolyLog[2, I*c*x]
```


Rule 4980

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{(d+ex^2)^3 (a+b\tan^{-1}(cx))}{x^7}\,dx = \int \left(\frac{d^3 (a+b\tan^{-1}(cx))}{x^7} + \frac{3d^2e (a+b\tan^{-1}(cx))}{x^5} + \frac{3de^2 (a+b\tan^{-1}(cx))}{x^3} + \frac{e^3 (a\right.$$

$$= d^3 \int \frac{a+b\tan^{-1}(cx)}{x^7}\,dx + (3d^2e) \int \frac{a+b\tan^{-1}(cx)}{x^5}\,dx + (3de^2) \int \frac{a+b\tan^{-1}(cx}{x^3}$$

$$= -\frac{d^3 (a+b\tan^{-1}(cx))}{6x^6} - \frac{3d^2e (a+b\tan^{-1}(cx))}{4x^4} - \frac{3de^2 (a+b\tan^{-1}(cx))}{2x^2} + ae^3 \log($$

$$= -\frac{bcd^3}{30x^5} - \frac{bcd^2e}{4x^3} - \frac{3bcde^2}{2x} - \frac{d^3 (a+b\tan^{-1}(cx))}{6x^6} - \frac{3d^2e (a+b\tan^{-1}(cx))}{4x^4} - \frac{3de^2 (a}{}$$

$$= -\frac{bcd^3}{30x^5} + \frac{bc^3d^3}{18x^3} - \frac{bcd^2e}{4x^3} + \frac{3bc^3d^2e}{4x} - \frac{3bcde^2}{2x} - \frac{3}{2}bc^2de^2\tan^{-1}(cx) - \frac{d^3 (a+b\tan^{-1}}{6x^6}$$

$$= -\frac{bcd^3}{30x^5} + \frac{bc^3d^3}{18x^3} - \frac{bcd^2e}{4x^3} - \frac{bc^5d^3}{6x} + \frac{3bc^3d^2e}{4x} - \frac{3bcde^2}{2x} + \frac{3}{4}bc^4d^2e\tan^{-1}(cx) - \frac{3}{2}bc^2d$$

$$= -\frac{bcd^3}{30x^5} + \frac{bc^3d^3}{18x^3} - \frac{bcd^2e}{4x^3} - \frac{bc^5d^3}{6x} + \frac{3bc^3d^2e}{4x} - \frac{3bcde^2}{2x} - \frac{1}{6}bc^6d^3\tan^{-1}(cx) + \frac{3}{4}bc^4d^2$$

Mathematica [C] time = 0.138102, size = 175, normalized size = 0.77

$$\frac{1}{60}\left(-\frac{15bcd^2e\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-c^2x^2\right)}{x^3} - \frac{2bcd^3\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},-c^2x^2\right)}{x^5} - \frac{90bcde^2\text{Hyp}}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + e*x^2)^3*(a + b*ArcTan[c*x]))/x^7,x]
```

```
[Out] ((-10*d^3*(a + b*ArcTan[c*x]))/x^6 - (45*d^2*e*(a + b*ArcTan[c*x]))/x^4 - (
90*d*e^2*(a + b*ArcTan[c*x]))/x^2 - (2*b*c*d^3*Hypergeometric2F1[-5/2, 1, -
3/2, -(c^2*x^2)])/x^5 - (15*b*c*d^2*e*Hypergeometric2F1[-3/2, 1, -1/2, -(c^
2*x^2)])/x^3 - (90*b*c*d*e^2*Hypergeometric2F1[-1/2, 1, 1/2, -(c^2*x^2)])/x
 + 60*a*e^3*Log[x] + (30*I)*b*e^3*PolyLog[2, (-I)*c*x] - (30*I)*b*e^3*PolyL
og[2, I*c*x])/60
```

Maple [A] time = 0.06, size = 272, normalized size = 1.2

$$-\frac{3\,ade^2}{2\,x^2} - \frac{3\,ad^2e}{4\,x^4} - \frac{ad^3}{6\,x^6} + ae^3\ln(cx) - \frac{3\,\arctan(cx)\,bde^2}{2\,x^2} - \frac{3\,bd^2\arctan(cx)\,e}{4\,x^4} - \frac{bd^3\arctan(cx)}{6\,x^6} + b\arctan(cx)\,e^3\ln($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^3*(a+b*arctan(c*x))/x^7,x)
```

```
[Out] -3/2*a*d*e^2/x^2-3/4*a*d^2*e/x^4-1/6*a*d^3/x^6+a*e^3*ln(c*x)-3/2*b*arctan(c
*x)*d*e^2/x^2-3/4*b*arctan(c*x)*d^2*e/x^4-1/6*b*arctan(c*x)*d^3/x^6+b*arcta
n(c*x)*e^3*ln(c*x)-1/2*I*b*e^3*ln(c*x)*ln(1-I*c*x)+1/2*I*b*e^3*ln(c*x)*ln(1
+I*c*x)-1/2*I*b*e^3*dilog(1-I*c*x)+1/2*I*b*e^3*dilog(1+I*c*x)-1/6*b*c^6*d^3
*arctan(c*x)+3/4*b*c^4*d^2*e*arctan(c*x)-3/2*b*c^2*d*e^2*arctan(c*x)-1/6*b*
c^5*d^3/x+3/4*b*c^3*d^2*e/x-3/2*b*c*d*e^2/x-1/30*b*c*d^3/x^5+1/18*b*c^3*d^3
/x^3-1/4*b*c*d^2*e/x^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{90}\left(\left(15\,c^5\arctan(cx)+\frac{15\,c^4x^4-5\,c^2x^2+3}{x^5}\right)c+\frac{15\,\arctan(cx)}{x^6}\right)bd^3+\frac{1}{4}\left(\left(3\,c^3\arctan(cx)+\frac{3\,c^2x^2-1}{x^3}\right)c-\frac{3\,\text{arct}}{x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x^7,x, algorithm="maxima")
```

```
[Out] -1/90*((15*c^5*arctan(c*x) + (15*c^4*x^4 - 5*c^2*x^2 + 3)/x^5)*c + 15*arcta
n(c*x)/x^6)*b*d^3 + 1/4*((3*c^3*arctan(c*x) + (3*c^2*x^2 - 1)/x^3)*c - 3*ar
ctan(c*x)/x^4)*b*d^2*e - 3/2*((c*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*b*
d*e^2 + b*e^3*integrate(arctan(c*x)/x, x) + a*e^3*log(x) - 3/2*a*d*e^2/x^2
- 3/4*a*d^2*e/x^4 - 1/6*a*d^3/x^6
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{ae^3x^6+3\,ade^2x^4+3\,ad^2ex^2+ad^3+\left(be^3x^6+3\,bde^2x^4+3\,bd^2ex^2+bd^3\right)\arctan(cx)}{x^7},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x^7,x, algorithm="fricas")
```

```
[Out] integral((a*e^3*x^6 + 3*a*d*e^2*x^4 + 3*a*d^2*e*x^2 + a*d^3 + (b*e^3*x^6 +
3*b*d*e^2*x^4 + 3*b*d^2*e*x^2 + b*d^3)*arctan(c*x))/x^7, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b\operatorname{atan}(cx))\left(d + ex^2\right)^3}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**3*(a+b*atan(c*x))/x**7,x)
```

```
[Out] Integral((a + b*atan(c*x))*(d + e*x**2)**3/x**7, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2 + d\right)^3 (b\arctan(cx) + a)}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x^7,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^3*(b*arctan(c*x) + a)/x^7, x)
```

3.1148 $\int \frac{(d+ex^2)^3(a+b\tan^{-1}(cx))}{x^8}\,dx$

Optimal. Leaf size=224

$$-\frac{3d^2e\left(a+b\tan^{-1}(cx)\right)}{5x^5}-\frac{d^3\left(a+b\tan^{-1}(cx)\right)}{7x^7}-\frac{de^2\left(a+b\tan^{-1}(cx)\right)}{x^3}-\frac{e^3\left(a+b\tan^{-1}(cx)\right)}{x}-\frac{bcd\left(5c^4d^2-21c^2de+\right.}{70x^2}$$

```
[Out] -(b*c*d^3)/(42*x^6) + (b*c*d^2*(5*c^2*d - 21*e))/(140*x^4) - (b*c*d*(5*c^4*
d^2 - 21*c^2*d*e + 35*e^2))/(70*x^2) - (d^3*(a + b*ArcTan[c*x]))/(7*x^7) -
(3*d^2*e*(a + b*ArcTan[c*x]))/(5*x^5) - (d*e^2*(a + b*ArcTan[c*x]))/x^3 - (
e^3*(a + b*ArcTan[c*x]))/x - (b*c*(5*c^6*d^3 - 21*c^4*d^2*e + 35*c^2*d*e^2
- 35*e^3)*Log[x])/35 + (b*c*(5*c^6*d^3 - 21*c^4*d^2*e + 35*c^2*d*e^2 - 35*e
^3)*Log[1 + c^2*x^2])/70
```

Rubi [A] time = 0.327039, antiderivative size = 224, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {270, 4976, 12, 1799, 1620}

$$-\frac{3d^2e\left(a+b\tan^{-1}(cx)\right)}{5x^5}-\frac{d^3\left(a+b\tan^{-1}(cx)\right)}{7x^7}-\frac{de^2\left(a+b\tan^{-1}(cx)\right)}{x^3}-\frac{e^3\left(a+b\tan^{-1}(cx)\right)}{x}-\frac{bcd\left(5c^4d^2-21c^2de+\right.}{70x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^3*(a + b*ArcTan[c*x]))/x^8,x]
```


```
[Out] -(b*c*d^3)/(42*x^6) + (b*c*d^2*(5*c^2*d - 21*e))/(140*x^4) - (b*c*d*(5*c^4*
d^2 - 21*c^2*d*e + 35*e^2))/(70*x^2) - (d^3*(a + b*ArcTan[c*x]))/(7*x^7) -
(3*d^2*e*(a + b*ArcTan[c*x]))/(5*x^5) - (d*e^2*(a + b*ArcTan[c*x]))/x^3 - (
e^3*(a + b*ArcTan[c*x]))/x - (b*c*(5*c^6*d^3 - 21*c^4*d^2*e + 35*c^2*d*e^2
- 35*e^3)*Log[x])/35 + (b*c*(5*c^6*d^3 - 21*c^4*d^2*e + 35*c^2*d*e^2 - 35*e
^3)*Log[1 + c^2*x^2])/70
```


Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 4976

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Dis
t[a + b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&  !(
ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(IL
tQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILtQ[(m
 - 1)/2, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1799

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^3\left(a+b\tan^{-1}(cx)\right)}{x^8}\,dx = -\frac{d^3\left(a+b\tan^{-1}(cx)\right)}{7x^7}-\frac{3d^2e\left(a+b\tan^{-1}(cx)\right)}{5x^5}-\frac{de^2\left(a+b\tan^{-1}(cx)\right)}{x^3}-\frac{e^3\left(a+b\right.}{}$$

$$= -\frac{d^3\left(a+b\tan^{-1}(cx)\right)}{7x^7}-\frac{3d^2e\left(a+b\tan^{-1}(cx)\right)}{5x^5}-\frac{de^2\left(a+b\tan^{-1}(cx)\right)}{x^3}-\frac{e^3\left(a+b\right.}{}$$

$$= -\frac{d^3\left(a+b\tan^{-1}(cx)\right)}{7x^7}-\frac{3d^2e\left(a+b\tan^{-1}(cx)\right)}{5x^5}-\frac{de^2\left(a+b\tan^{-1}(cx)\right)}{x^3}-\frac{e^3\left(a+b\right.}{}$$

$$= -\frac{d^3\left(a+b\tan^{-1}(cx)\right)}{7x^7}-\frac{3d^2e\left(a+b\tan^{-1}(cx)\right)}{5x^5}-\frac{de^2\left(a+b\tan^{-1}(cx)\right)}{x^3}-\frac{e^3\left(a+b\right.}{}$$

$$= -\frac{bcd^3}{42x^6}+\frac{bcd^2\left(5c^2d-21e\right)}{140x^4}-\frac{bcd\left(5c^4d^2-21c^2de+35e^2\right)}{70x^2}-\frac{d^3\left(a+b\tan^{-1}(cx)\right)}{7x^7}-$$

Mathematica [A] time = 0.171879, size = 230, normalized size = 1.03

$$-\frac{3d^2e\left(a+b\tan^{-1}(cx)\right)}{5x^5}-\frac{d^3\left(a+b\tan^{-1}(cx)\right)}{7x^7}-\frac{de^2\left(a+b\tan^{-1}(cx)\right)}{x^3}-\frac{e^3\left(a+b\tan^{-1}(cx)\right)}{x}-\frac{3}{20}bcd^2e\left(-\frac{2c^2}{x^2}+2c^4\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)^3*(a + b*ArcTan[c*x]))/x^8,x]
```

```
[Out] -(d^3*(a + b*ArcTan[c*x]))/(7*x^7) - (3*d^2*e*(a + b*ArcTan[c*x]))/(5*x^5)
- (d*e^2*(a + b*ArcTan[c*x]))/x^3 - (e^3*(a + b*ArcTan[c*x]))/x + (b*c*e^3*
(2*Log[x] - Log[1 + c^2*x^2]))/2 - (b*c*d*e^2*(x^(-2) + 2*c^2*Log[x] - c^2*
Log[1 + c^2*x^2]))/2 - (3*b*c*d^2*e*(x^(-4) - (2*c^2)/x^2 - 4*c^4*Log[x] +
2*c^4*Log[1 + c^2*x^2]))/20 - (b*c*d^3*(2/x^6 - (3*c^2)/x^4 + (6*c^4)/x^2 +
 12*c^6*Log[x] - 6*c^6*Log[1 + c^2*x^2]))/84
```

Maple [A] time = 0.047, size = 290, normalized size = 1.3

$$-\frac{ad^3}{7x^7}-\frac{ae^3}{x}-\frac{3ad^2e}{5x^5}-\frac{ade^2}{x^3}-\frac{bd^3\arctan(cx)}{7x^7}-\frac{b\arctan(cx)e^3}{x}-\frac{3bd^2\arctan(cx)e}{5x^5}-\frac{\arctan(cx)bde^2}{x^3}+\frac{c^7b\ln\left(c^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^3*(a+b*arctan(c*x))/x^8,x)
```

```
[Out] -1/7*a*d^3/x^7-a*e^3/x-3/5*a*d^2*e/x^5-a*d*e^2/x^3-1/7*b*arctan(c*x)*d^3/x^
7-b*arctan(c*x)*e^3/x-3/5*b*arctan(c*x)*d^2*e/x^5-b*arctan(c*x)*d*e^2/x^3+1
/14*c^7*b*ln(c^2*x^2+1)*d^3-3/10*c^5*b*ln(c^2*x^2+1)*d^2*e+1/2*c^3*b*ln(c^2
*x^2+1)*d*e^2-1/2*c*b*ln(c^2*x^2+1)*e^3-1/7*c^7*b*d^3*ln(c*x)+3/5*c^5*b*ln(
c*x)*d^2*e-c^3*b*ln(c*x)*d*e^2+c*b*ln(c*x)*e^3+1/28*c^3*b*d^3/x^4-3/20*c*b*
d^2*e/x^4-1/42*b*c*d^3/x^6-1/14*c^5*b*d^3/x^2+3/10*c^3*b*d^2*e/x^2-1/2*c*b*
d*e^2/x^2
```

Maxima [A] time = 1.03388, size = 333, normalized size = 1.49

$$\frac{1}{84}\left(\left(6c^6\log\left(c^2x^2+1\right)-6c^6\log\left(x^2\right)-\frac{6c^4x^4-3c^2x^2+2}{x^6}\right)c-\frac{12\arctan(cx)}{x^7}\right)bd^3-\frac{3}{20}\left(\left(2c^4\log\left(c^2x^2+1\right)-2c^4\log\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x^8,x, algorithm="maxima")
```

```
[Out] 1/84*((6*c^6*log(c^2*x^2 + 1) - 6*c^6*log(x^2) - (6*c^4*x^4 - 3*c^2*x^2 + 2
)/x^6)*c - 12*arctan(c*x)/x^7)*b*d^3 - 3/20*((2*c^4*log(c^2*x^2 + 1) - 2*c^
4*log(x^2) - (2*c^2*x^2 - 1)/x^4)*c + 4*arctan(c*x)/x^5)*b*d^2*e + 1/2*((c^
2*log(c^2*x^2 + 1) - c^2*log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^3)*b*d*e^2 -
 1/2*(c*(log(c^2*x^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*b*e^3 - a*e^3/x -
a*d*e^2/x^3 - 3/5*a*d^2*e/x^5 - 1/7*a*d^3/x^7
```

Fricas [A] time = 2.00347, size = 567, normalized size = 2.53

$$-\frac{420\,ae^3x^6 - 6\left(5\,bc^7d^3 - 21\,bc^5d^2e + 35\,bc^3de^2 - 35\,bce^3\right)x^7\log\left(c^2x^2+1\right) + 12\left(5\,bc^7d^3 - 21\,bc^5d^2e + 35\,bc^3de^2 - 35\,bce\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x^8,x, algorithm="fricas")
```

```
[Out] -1/420*(420*a*e^3*x^6 - 6*(5*b*c^7*d^3 - 21*b*c^5*d^2*e + 35*b*c^3*d*e^2 -
35*b*c*e^3)*x^7*log(c^2*x^2 + 1) + 12*(5*b*c^7*d^3 - 21*b*c^5*d^2*e + 35*b*
c^3*d*e^2 - 35*b*c*e^3)*x^7*log(x) + 420*a*d*e^2*x^4 + 10*b*c*d^3*x + 252*a
*d^2*e*x^2 + 6*(5*b*c^5*d^3 - 21*b*c^3*d^2*e + 35*b*c*d*e^2)*x^5 + 60*a*d^3
 - 3*(5*b*c^3*d^3 - 21*b*c*d^2*e)*x^3 + 12*(35*b*e^3*x^6 + 35*b*d*e^2*x^4 +
 21*b*d^2*e*x^2 + 5*b*d^3)*arctan(c*x))/x^7
```

Sympy [A] time = 6.39111, size = 362, normalized size = 1.62

$$\begin{cases} -\frac{ad^3}{7x^7} - \frac{3ad^2e}{5x^5} - \frac{ade^2}{x^3} - \frac{ae^3}{x} - \frac{bc^7d^3\log(x)}{7} + \frac{bc^7d^3\log\left(x^2+\frac{1}{c^2}\right)}{14} - \frac{bc^5d^3}{14x^2} + \frac{3bc^5d^2e\log(x)}{5} - \frac{3bc^5d^2e\log\left(x^2+\frac{1}{c^2}\right)}{10} + \frac{bc^3d^3}{28x^4} + \frac{3bc^3d^2e}{10x^2} - bc^3de^2 \\ a\left(-\frac{d^3}{7x^7} - \frac{3d^2e}{5x^5} - \frac{de^2}{x^3} - \frac{e^3}{x}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**3*(a+b*atan(c*x))/x**8,x)
```

```
[Out] Piecewise((-a*d**3/(7*x**7) - 3*a*d**2*e/(5*x**5) - a*d*e**2/x**3 - a*e**3/
x - b*c**7*d**3*log(x)/7 + b*c**7*d**3*log(x**2 + c**(-2))/14 - b*c**5*d**3
/(14*x**2) + 3*b*c**5*d**2*e*log(x)/5 - 3*b*c**5*d**2*e*log(x**2 + c**(-2))
/10 + b*c**3*d**3/(28*x**4) + 3*b*c**3*d**2*e/(10*x**2) - b*c**3*d*e**2*log
```

```
(x) + b*c**3*d*e**2*log(x**2 + c**(-2))/2 - b*c*d**3/(42*x**6) - 3*b*c*d**2
*e/(20*x**4) - b*c*d*e**2/(2*x**2) + b*c*e**3*log(x) - b*c*e**3*log(x**2 +
c**(-2))/2 - b*d**3*atan(c*x)/(7*x**7) - 3*b*d**2*e*atan(c*x)/(5*x**5) - b*
d*e**2*atan(c*x)/x**3 - b*e**3*atan(c*x)/x, Ne(c, 0)), (a*(-d**3/(7*x**7) -
 3*d**2*e/(5*x**5) - d*e**2/x**3 - e**3/x), True))
```

Giac [A] time = 1.10426, size = 405, normalized size = 1.81

$$\frac{30\,bc^7d^3x^7\log\left(c^2x^2+1\right)-60\,bc^7d^3x^7\log\left(x\right)-126\,bc^5d^2x^7e\log\left(c^2x^2+1\right)+252\,bc^5d^2x^7e\log\left(x\right)-30\,bc^5d^3x^5+210\,b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x^8,x, algorithm="giac")
```

```
[Out] 1/420*(30*b*c^7*d^3*x^7*log(c^2*x^2 + 1) - 60*b*c^7*d^3*x^7*log(x) - 126*b*
c^5*d^2*x^7*e*log(c^2*x^2 + 1) + 252*b*c^5*d^2*x^7*e*log(x) - 30*b*c^5*d^3*
x^5 + 210*b*c^3*d*x^7*e^2*log(c^2*x^2 + 1) - 420*b*c^3*d*x^7*e^2*log(x) + 1
26*b*c^3*d^2*x^5*e - 210*b*c*x^7*e^3*log(c^2*x^2 + 1) + 420*b*c*x^7*e^3*log
(x) + 15*b*c^3*d^3*x^3 - 420*b*x^6*arctan(c*x)*e^3 - 210*b*c*d*x^5*e^2 - 42
0*a*x^6*e^3 - 420*b*d*x^4*arctan(c*x)*e^2 - 63*b*c*d^2*x^3*e - 420*a*d*x^4*
e^2 - 252*b*d^2*x^2*arctan(c*x)*e - 10*b*c*d^3*x - 252*a*d^2*x^2*e - 60*b*d
^3*arctan(c*x) - 60*a*d^3)/x^7
```

3.1149 $\int \frac{(d+ex^2)^3(a+b\tan^{-1}(cx))}{x^9}\,dx$

Optimal. Leaf size=152

$$-\frac{(d+ex^2)^4(a+b\tan^{-1}(cx))}{8dx^8}-\frac{bcd(c^4d^2-4c^2de+6e^2)}{24x^3}+\frac{bc(c^2d-2e)(c^4d^2-2c^2de+2e^2)}{8x}+\frac{bcd^2(c^2d-4e)}{40x^5}+\frac{b(c^2d$$

```
[Out] -(b*c*d^3)/(56*x^7) + (b*c*d^2*(c^2*d - 4*e))/(40*x^5) - (b*c*d*(c^4*d^2 -
4*c^2*d*e + 6*e^2))/(24*x^3) + (b*c*(c^2*d - 2*e)*(c^4*d^2 - 2*c^2*d*e + 2*
e^2))/(8*x) + (b*(c^2*d - e)^4*ArcTan[c*x])/(8*d) - ((d + e*x^2)^4*(a + b*A
rcTan[c*x]))/(8*d*x^8)
```

Rubi [A] time = 0.195143, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {264, 4976, 12, 461, 203}

$$-\frac{(d+ex^2)^4(a+b\tan^{-1}(cx))}{8dx^8}-\frac{bcd(c^4d^2-4c^2de+6e^2)}{24x^3}+\frac{bc(c^2d-2e)(c^4d^2-2c^2de+2e^2)}{8x}+\frac{bcd^2(c^2d-4e)}{40x^5}+\frac{b(c^2d$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^3*(a + b*ArcTan[c*x]))/x^9,x]
```

```
[Out] -(b*c*d^3)/(56*x^7) + (b*c*d^2*(c^2*d - 4*e))/(40*x^5) - (b*c*d*(c^4*d^2 -
4*c^2*d*e + 6*e^2))/(24*x^3) + (b*c*(c^2*d - 2*e)*(c^4*d^2 - 2*c^2*d*e + 2*
e^2))/(8*x) + (b*(c^2*d - e)^4*ArcTan[c*x])/(8*d) - ((d + e*x^2)^4*(a + b*A
rcTan[c*x]))/(8*d*x^8)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 4976

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Dis
t[a + b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&  !(
```

```
ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(IL
tQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILtQ[(m
 - 1)/2, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^3\left(a+b\tan^{-1}(cx)\right)}{x^9}\,dx = -\frac{\left(d+ex^2\right)^4\left(a+b\tan^{-1}(cx)\right)}{8dx^8}-(bc)\int \frac{\left(d+ex^2\right)^4}{8x^8\left(-d-c^2dx^2\right)}\,dx$$

$$= -\frac{\left(d+ex^2\right)^4\left(a+b\tan^{-1}(cx)\right)}{8dx^8}-\frac{1}{8}(bc)\int \frac{\left(d+ex^2\right)^4}{x^8\left(-d-c^2dx^2\right)}\,dx$$

$$= -\frac{\left(d+ex^2\right)^4\left(a+b\tan^{-1}(cx)\right)}{8dx^8}-\frac{1}{8}(bc)\int\left(-\frac{d^3}{x^8}+\frac{d^2\left(c^2d-4e\right)}{x^6}-\frac{d\left(c^4d^2-4c^2de\right.}{x^4}\right.$$

$$= -\frac{bcd^3}{56x^7}+\frac{bcd^2\left(c^2d-4e\right)}{40x^5}-\frac{bcd\left(c^4d^2-4c^2de+6e^2\right)}{24x^3}+\frac{bc\left(c^2d-2e\right)\left(c^4d^2-2c^2de\right.}{8x}$$

$$= -\frac{bcd^3}{56x^7}+\frac{bcd^2\left(c^2d-4e\right)}{40x^5}-\frac{bcd\left(c^4d^2-4c^2de+6e^2\right)}{24x^3}+\frac{bc\left(c^2d-2e\right)\left(c^4d^2-2c^2de\right.}{8x}$$

Mathematica [C] time = 0.177316, size = 154, normalized size = 1.01

$$-\frac{35\left(2bcde^2x^5\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-c^2x^2\right)+4bce^3x^7\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-c^2x^2\right)+\left(4d^2ex^2+d^3\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)^3*(a + b*ArcTan[c*x]))/x^9,x]
```

```
[Out] -(5*b*c*d^3*x*Hypergeometric2F1[-7/2, 1, -5/2, -(c^2*x^2)] + 28*b*c*d^2*e*x
^3*Hypergeometric2F1[-5/2, 1, -3/2, -(c^2*x^2)] + 35*((d^3 + 4*d^2*e*x^2 +
6*d*e^2*x^4 + 4*e^3*x^6)*(a + b*ArcTan[c*x]) + 2*b*c*d*e^2*x^5*Hypergeometr
ic2F1[-3/2, 1, -1/2, -(c^2*x^2)] + 4*b*c*e^3*x^7*Hypergeometric2F1[-1/2, 1,
 1/2, -(c^2*x^2)]))/(280*x^8)
```

Maple [A] time = 0.049, size = 265, normalized size = 1.7

$$-\frac{ae^3}{2x^2}-\frac{3ade^2}{4x^4}-\frac{ad^2e}{2x^6}-\frac{ad^3}{8x^8}-\frac{b\arctan(cx)e^3}{2x^2}-\frac{3\arctan(cx)bde^2}{4x^4}-\frac{bd^2\arctan(cx)e}{2x^6}-\frac{bd^3\arctan(cx)}{8x^8}+\frac{c^8b\arctan}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^3*(a+b*arctan(c*x))/x^9,x)
```

```
[Out] -1/2*a*e^3/x^2-3/4*a*d*e^2/x^4-1/2*a*d^2*e/x^6-1/8*a*d^3/x^8-1/2*b*arctan(c
*x)*e^3/x^2-3/4*b*arctan(c*x)*d*e^2/x^4-1/2*b*arctan(c*x)*d^2*e/x^6-1/8*b*a
rctan(c*x)*d^3/x^8+1/8*c^8*b*arctan(c*x)*d^3-1/2*c^6*b*arctan(c*x)*d^2*e+3/
4*c^4*b*arctan(c*x)*d*e^2-1/2*c^2*b*arctan(c*x)*e^3+1/8*c^7*b*d^3/x-1/2*c^5
*b*d^2*e/x+3/4*c^3*b*d*e^2/x-1/2*c*b*e^3/x+1/40*c^3*b*d^3/x^5-1/10*c*b*d^2*
e/x^5-1/56*b*c*d^3/x^7-1/24*c^5*b*d^3/x^3+1/6*c^3*b*d^2*e/x^3-1/4*c*b*d*e^2
/x^3
```

Maxima [A] time = 1.47127, size = 294, normalized size = 1.93

$$\frac{1}{840}\left(\left(\left(105c^7\arctan(cx)+\frac{105c^6x^6-35c^4x^4+21c^2x^2-15}{x^7}\right)c-\frac{105\arctan(cx)}{x^8}\right)bd^3-\frac{1}{30}\left(\left(\left(15c^5\arctan(cx)+\frac{15c^4x}{}\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x^9,x, algorithm="maxima")
```

```
[Out] 1/840*((105*c^7*arctan(c*x) + (105*c^6*x^6 - 35*c^4*x^4 + 21*c^2*x^2 - 15)/
x^7)*c - 105*arctan(c*x)/x^8)*b*d^3 - 1/30*((15*c^5*arctan(c*x) + (15*c^4*x
^4 - 5*c^2*x^2 + 3)/x^5)*c + 15*arctan(c*x)/x^6)*b*d^2*e + 1/4*((3*c^3*arct
an(c*x) + (3*c^2*x^2 - 1)/x^3)*c - 3*arctan(c*x)/x^4)*b*d*e^2 - 1/2*((c*arc
tan(c*x) + 1/x)*c + arctan(c*x)/x^2)*b*e^3 - 1/2*a*e^3/x^2 - 3/4*a*d*e^2/x^
4 - 1/2*a*d^2*e/x^6 - 1/8*a*d^3/x^8
```

Fricas [A] time = 1.822, size = 512, normalized size = 3.37

$$-\frac{420\,ae^3x^6+630\,ade^2x^4-105\left(bc^7d^3-4\,bc^5d^2e+6\,bc^3de^2-4\,bce^3\right)x^7+15\,bcd^3x+420\,ad^2ex^2+35\left(bc^5d^3-4\,bc^3d^2e\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x^9,x, algorithm="fricas")
```

```
[Out] -1/840*(420*a*e^3*x^6 + 630*a*d*e^2*x^4 - 105*(b*c^7*d^3 - 4*b*c^5*d^2*e +
6*b*c^3*d*e^2 - 4*b*c*e^3)*x^7 + 15*b*c*d^3*x + 420*a*d^2*e*x^2 + 35*(b*c^5
*d^3 - 4*b*c^3*d^2*e + 6*b*c*d*e^2)*x^5 + 105*a*d^3 - 21*(b*c^3*d^3 - 4*b*c
*d^2*e)*x^3 + 105*(4*b*e^3*x^6 - (b*c^8*d^3 - 4*b*c^6*d^2*e + 6*b*c^4*d*e^2
 - 4*b*c^2*e^3)*x^8 + 6*b*d*e^2*x^4 + 4*b*d^2*e*x^2 + b*d^3)*arctan(c*x))/x
^8
```

Sympy [B] time = 4.46187, size = 309, normalized size = 2.03

$$-\frac{ad^3}{8x^8}-\frac{ad^2e}{2x^6}-\frac{3ade^2}{4x^4}-\frac{ae^3}{2x^2}+\frac{bc^8d^3\operatorname{atan}(cx)}{8}+\frac{bc^7d^3}{8x}-\frac{bc^6d^2e\operatorname{atan}(cx)}{2}-\frac{bc^5d^3}{24x^3}-\frac{bc^5d^2e}{2x}+\frac{3bc^4de^2\operatorname{atan}(cx)}{4}+\frac{bc}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**3*(a+b*atan(c*x))/x**9,x)
```

```
[Out] -a*d**3/(8*x**8) - a*d**2*e/(2*x**6) - 3*a*d*e**2/(4*x**4) - a*e**3/(2*x**2
) + b*c**8*d**3*atan(c*x)/8 + b*c**7*d**3/(8*x) - b*c**6*d**2*e*atan(c*x)/2
 - b*c**5*d**3/(24*x**3) - b*c**5*d**2*e/(2*x) + 3*b*c**4*d*e**2*atan(c*x)/
4 + b*c**3*d**3/(40*x**5) + b*c**3*d**2*e/(6*x**3) + 3*b*c**3*d*e**2/(4*x)
- b*c**2*e**3*atan(c*x)/2 - b*c*d**3/(56*x**7) - b*c*d**2*e/(10*x**5) - b*c
```

```
*d*e**2/(4*x**3) - b*c*e**3/(2*x) - b*d**3*atan(c*x)/(8*x**8) - b*d**2*e*at
an(c*x)/(2*x**6) - 3*b*d*e**2*atan(c*x)/(4*x**4) - b*e**3*atan(c*x)/(2*x**2
)
```

Giac [B] time = 2.1154, size = 410, normalized size = 2.7

$$-\frac{105\,\pi bc^8d^3x^8\mathrm{sgn}\,(c)\,\mathrm{sgn}\,(x) - 105\,bc^8d^3x^8\arctan\,(cx) + 420\,bc^6d^2x^8\arctan\,(cx)\,e - 105\,bc^7d^3x^7 + 630\,\pi bc^4dx^8e^2\mathrm{sgn}\,(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x^9,x, algorithm="giac")
```

```
[Out] -1/840*(105*pi*b*c^8*d^3*x^8*sgn(c)*sgn(x) - 105*b*c^8*d^3*x^8*arctan(c*x)
+ 420*b*c^6*d^2*x^8*arctan(c*x)*e - 105*b*c^7*d^3*x^7 + 630*pi*b*c^4*d*x^8*
e^2*sgn(c)*sgn(x) - 630*b*c^4*d*x^8*arctan(c*x)*e^2 + 420*b*c^5*d^2*x^7*e +
 35*b*c^5*d^3*x^5 + 420*b*c^2*x^8*arctan(c*x)*e^3 - 630*b*c^3*d*x^7*e^2 - 1
40*b*c^3*d^2*x^5*e - 21*b*c^3*d^3*x^3 + 420*b*c*x^7*e^3 + 420*b*x^6*arctan(
c*x)*e^3 + 210*b*c*d*x^5*e^2 + 420*a*x^6*e^3 + 630*b*d*x^4*arctan(c*x)*e^2
+ 84*b*c*d^2*x^3*e + 630*a*d*x^4*e^2 + 420*b*d^2*x^2*arctan(c*x)*e + 15*b*c
*d^3*x + 420*a*d^2*x^2*e + 105*b*d^3*arctan(c*x) + 105*a*d^3)/x^8
```

3.1150 $\int \left(c+dx^2\right)^4 \tan^{-1}(ax)\,dx$

Optimal. Leaf size=244

$$-\frac{d^2x^4\left(378a^4c^2-180a^2cd+35d^2\right)}{1260a^5}-\frac{dx^2\left(-378a^4c^2d+420a^6c^3+180a^2cd^2-35d^3\right)}{630a^7}-\frac{\left(378a^4c^2d^2-420a^6c^3d+315a^8\right.}{}$$

```
[Out] -(d*(420*a^6*c^3 - 378*a^4*c^2*d + 180*a^2*c*d^2 - 35*d^3)*x^2)/(630*a^7) -
 (d^2*(378*a^4*c^2 - 180*a^2*c*d + 35*d^2)*x^4)/(1260*a^5) - ((36*a^2*c - 7
*d)*d^3*x^6)/(378*a^3) - (d^4*x^8)/(72*a) + c^4*x*ArcTan[a*x] + (4*c^3*d*x^
3*ArcTan[a*x])/3 + (6*c^2*d^2*x^5*ArcTan[a*x])/5 + (4*c*d^3*x^7*ArcTan[a*x]
)/7 + (d^4*x^9*ArcTan[a*x])/9 - ((315*a^8*c^4 - 420*a^6*c^3*d + 378*a^4*c^2
*d^2 - 180*a^2*c*d^3 + 35*d^4)*Log[1 + a^2*x^2])/(630*a^9)
```

Rubi [A] time = 0.176337, antiderivative size = 244, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {194, 4912, 1810, 260}

$$-\frac{d^2x^4\left(378a^4c^2-180a^2cd+35d^2\right)}{1260a^5}-\frac{dx^2\left(-378a^4c^2d+420a^6c^3+180a^2cd^2-35d^3\right)}{630a^7}-\frac{\left(378a^4c^2d^2-420a^6c^3d+315a^8\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^4*ArcTan[a*x],x]
```

```
[Out] -(d*(420*a^6*c^3 - 378*a^4*c^2*d + 180*a^2*c*d^2 - 35*d^3)*x^2)/(630*a^7) -
 (d^2*(378*a^4*c^2 - 180*a^2*c*d + 35*d^2)*x^4)/(1260*a^5) - ((36*a^2*c - 7
*d)*d^3*x^6)/(378*a^3) - (d^4*x^8)/(72*a) + c^4*x*ArcTan[a*x] + (4*c^3*d*x^
3*ArcTan[a*x])/3 + (6*c^2*d^2*x^5*ArcTan[a*x])/5 + (4*c*d^3*x^7*ArcTan[a*x]
)/7 + (d^4*x^9*ArcTan[a*x])/9 - ((315*a^8*c^4 - 420*a^6*c^3*d + 378*a^4*c^2
*d^2 - 180*a^2*c*d^3 + 35*d^4)*Log[1 + a^2*x^2])/(630*a^9)
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4912

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcTan[c*x], u, x]
```

```
- Dist[b*c, Int[u/(1 + c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && (I
ntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \left(c+dx^2\right)^4 \tan^{-1}(ax)\,dx = c^4x\tan^{-1}(ax)+\frac{4}{3}c^3dx^3\tan^{-1}(ax)+\frac{6}{5}c^2d^2x^5\tan^{-1}(ax)+\frac{4}{7}cd^3x^7\tan^{-1}(ax)+\frac{1}{9}d^4x^9\tan^{-1}$$

$$= c^4x\tan^{-1}(ax)+\frac{4}{3}c^3dx^3\tan^{-1}(ax)+\frac{6}{5}c^2d^2x^5\tan^{-1}(ax)+\frac{4}{7}cd^3x^7\tan^{-1}(ax)+\frac{1}{9}d^4x^9\tan^{-1}$$

$$= -\frac{d\left(420a^6c^3-378a^4c^2d+180a^2cd^2-35d^3\right)x^2}{630a^7}-\frac{d^2\left(378a^4c^2-180a^2cd+35d^2\right)x^4}{1260a^5}-\frac{(36a}{}$$

$$= -\frac{d\left(420a^6c^3-378a^4c^2d+180a^2cd^2-35d^3\right)x^2}{630a^7}-\frac{d^2\left(378a^4c^2-180a^2cd+35d^2\right)x^4}{1260a^5}-\frac{(36a}{}$$

Mathematica [A] time = 0.174467, size = 212, normalized size = 0.87

$$-\frac{a^2dx^2\left(3a^6\left(756c^2dx^2+1680c^3+240cd^2x^4+35d^3x^6\right)-4a^4d\left(1134c^2+270cdx^2+35d^2x^4\right)+30a^2d^2\left(72c+7dx^2\right)-42}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^4*ArcTan[a*x],x]
```

```
[Out] -(a^2*d*x^2*(-420*d^3 + 30*a^2*d^2*(72*c + 7*d*x^2) - 4*a^4*d*(1134*c^2 + 2
70*c*d*x^2 + 35*d^2*x^4) + 3*a^6*(1680*c^3 + 756*c^2*d*x^2 + 240*c*d^2*x^4
+ 35*d^3*x^6)) - 24*a^9*x*(315*c^4 + 420*c^3*d*x^2 + 378*c^2*d^2*x^4 + 180*
c*d^3*x^6 + 35*d^4*x^8)*ArcTan[a*x] + 12*(315*a^8*c^4 - 420*a^6*c^3*d + 378
*a^4*c^2*d^2 - 180*a^2*c*d^3 + 35*d^4)*Log[1 + a^2*x^2])/(7560*a^9)
```

Maple [A] time = 0.039, size = 279, normalized size = 1.1

$$\frac{d^4x^9 \arctan(ax)}{9} + \frac{4\,cd^3x^7 \arctan(ax)}{7} + \frac{6\,c^2d^2x^5 \arctan(ax)}{5} + \frac{4\,c^3dx^3 \arctan(ax)}{3} + c^4x \arctan(ax) - \frac{2\,dc^3x^2}{3\,a} - \frac{3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^4*arctan(a*x),x)
```

```
[Out] 1/9*d^4*x^9*arctan(a*x)+4/7*c*d^3*x^7*arctan(a*x)+6/5*c^2*d^2*x^5*arctan(a*
x)+4/3*c^3*d*x^3*arctan(a*x)+c^4*x*arctan(a*x)-2/3/a*c^3*d*x^2-3/10/a*c^2*d
^2*x^4-2/21/a*c*d^3*x^6+3/5/a^3*c^2*d^2*x^2-1/72*d^4*x^8/a+1/7/a^3*x^4*c*d^
3+1/54/a^3*d^4*x^6-2/7/a^5*x^2*c*d^3-1/36/a^5*d^4*x^4+1/18/a^7*x^2*d^4-1/2/
a*ln(a^2*x^2+1)*c^4+2/3/a^3*ln(a^2*x^2+1)*c^3*d-3/5/a^5*ln(a^2*x^2+1)*c^2*d
^2+2/7/a^7*ln(a^2*x^2+1)*c*d^3-1/18/a^9*ln(a^2*x^2+1)*d^4
```

Maxima [A] time = 0.979488, size = 305, normalized size = 1.25

$$-\frac{1}{7560}\,a\left(\frac{105\,a^6d^4x^8 + 20\left(36\,a^6cd^3 - 7\,a^4d^4\right)x^6 + 6\left(378\,a^6c^2d^2 - 180\,a^4cd^3 + 35\,a^2d^4\right)x^4 + 12\left(420\,a^6c^3d - 378\,a^4c^2d^2\right.}{a^8}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^4*arctan(a*x),x, algorithm="maxima")
```

```
[Out] -1/7560*a*((105*a^6*d^4*x^8 + 20*(36*a^6*c*d^3 - 7*a^4*d^4)*x^6 + 6*(378*a^
6*c^2*d^2 - 180*a^4*c*d^3 + 35*a^2*d^4)*x^4 + 12*(420*a^6*c^3*d - 378*a^4*c
^2*d^2 + 180*a^2*c*d^3 - 35*d^4)*x^2)/a^8 + 12*(315*a^8*c^4 - 420*a^6*c^3*d
 + 378*a^4*c^2*d^2 - 180*a^2*c*d^3 + 35*d^4)*log(a^2*x^2 + 1)/a^10) + 1/315
*(35*d^4*x^9 + 180*c*d^3*x^7 + 378*c^2*d^2*x^5 + 420*c^3*d*x^3 + 315*c^4*x)
*arctan(a*x)
```

Fricas [A] time = 1.77139, size = 541, normalized size = 2.22

$$-\frac{105\,a^8d^4x^8 + 20\left(36\,a^8cd^3 - 7\,a^6d^4\right)x^6 + 6\left(378\,a^8c^2d^2 - 180\,a^6cd^3 + 35\,a^4d^4\right)x^4 + 12\left(420\,a^8c^3d - 378\,a^6c^2d^2 + 180\,a^4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^4*arctan(a*x),x, algorithm="fricas")
```

```
[Out] -1/7560*(105*a^8*d^4*x^8 + 20*(36*a^8*c*d^3 - 7*a^6*d^4)*x^6 + 6*(378*a^8*c
^2*d^2 - 180*a^6*c*d^3 + 35*a^4*d^4)*x^4 + 12*(420*a^8*c^3*d - 378*a^6*c^2*
d^2 + 180*a^4*c*d^3 - 35*a^2*d^4)*x^2 - 24*(35*a^9*d^4*x^9 + 180*a^9*c*d^3*
x^7 + 378*a^9*c^2*d^2*x^5 + 420*a^9*c^3*d*x^3 + 315*a^9*c^4*x)*arctan(a*x)
+ 12*(315*a^8*c^4 - 420*a^6*c^3*d + 378*a^4*c^2*d^2 - 180*a^2*c*d^3 + 35*d^
4)*log(a^2*x^2 + 1))/a^9
```

Sympy [A] time = 6.62667, size = 314, normalized size = 1.29

$$\begin{cases} c^4x\operatorname{atan}(ax) + \frac{4c^3dx^3\operatorname{atan}(ax)}{3} + \frac{6c^2d^2x^5\operatorname{atan}(ax)}{5} + \frac{4cd^3x^7\operatorname{atan}(ax)}{7} + \frac{d^4x^9\operatorname{atan}(ax)}{9} - \frac{c^4\log\left(x^2+\frac{1}{a^2}\right)}{2a} - \frac{2c^3dx^2}{3a} - \frac{3c^2d^2x^4}{10a} - \frac{2cd^3x^6}{21a} - \\ 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**4*atan(a*x),x)
```

```
[Out] Piecewise((c**4*x*atan(a*x) + 4*c**3*d*x**3*atan(a*x)/3 + 6*c**2*d**2*x**5*
atan(a*x)/5 + 4*c*d**3*x**7*atan(a*x)/7 + d**4*x**9*atan(a*x)/9 - c**4*log(
x**2 + a**(-2))/(2*a) - 2*c**3*d*x**2/(3*a) - 3*c**2*d**2*x**4/(10*a) - 2*c
*d**3*x**6/(21*a) - d**4*x**8/(72*a) + 2*c**3*d*log(x**2 + a**(-2))/(3*a**3
) + 3*c**2*d**2*x**2/(5*a**3) + c*d**3*x**4/(7*a**3) + d**4*x**6/(54*a**3)
- 3*c**2*d**2*log(x**2 + a**(-2))/(5*a**5) - 2*c*d**3*x**2/(7*a**5) - d**4*
x**4/(36*a**5) + 2*c*d**3*log(x**2 + a**(-2))/(7*a**7) + d**4*x**2/(18*a**7
) - d**4*log(x**2 + a**(-2))/(18*a**9), Ne(a, 0)), (0, True))
```

Giac [A] time = 1.09783, size = 315, normalized size = 1.29

$$\frac{1}{315}\left(35\,d^4x^9 + 180\,cd^3x^7 + 378\,c^2d^2x^5 + 420\,c^3dx^3 + 315\,c^4x\right)\arctan(ax) - \frac{105\,a^7d^4x^8 + 720\,a^7cd^3x^6 + 2268\,a^7c^2d^2x^4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^4*arctan(a*x),x, algorithm="giac")
```

```
[Out] 1/315*(35*d^4*x^9 + 180*c*d^3*x^7 + 378*c^2*d^2*x^5 + 420*c^3*d*x^3 + 315*c
^4*x)*arctan(a*x) - 1/7560*(105*a^7*d^4*x^8 + 720*a^7*c*d^3*x^6 + 2268*a^7*
```

```
c^2*d^2*x^4 - 140*a^5*d^4*x^6 + 5040*a^7*c^3*d*x^2 - 1080*a^5*c*d^3*x^4 - 4
536*a^5*c^2*d^2*x^2 + 210*a^3*d^4*x^4 + 2160*a^3*c*d^3*x^2 - 420*a*d^4*x^2)
/a^8 - 1/630*(315*a^8*c^4 - 420*a^6*c^3*d + 378*a^4*c^2*d^2 - 180*a^2*c*d^3
 + 35*d^4)*log(a^2*x^2 + 1)/a^9
```

3.1151 $\int \frac{x^3(a+b\tan^{-1}(cx))}{d+ex^2}\,dx$

Optimal. Leaf size=361

$$-\frac{ibd\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2e^2}+\frac{ibd\text{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{(1-icx)\left(c\sqrt{-d}-i\sqrt{e}\right)}\right)}{4e^2}+\frac{ibd\text{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-d}+\sqrt{e}x\right)}{(1-icx)\left(c\sqrt{-d}+i\sqrt{e}\right)}\right)}{4e^2}+\frac{d\log\left(\frac{2}{1-icx}\right.}{}$$

```
[Out] -(b*x)/(2*c*e) + (b*ArcTan[c*x])/(2*c^2*e) + (x^2*(a + b*ArcTan[c*x]))/(2*e
) + (d*(a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e^2 - (d*(a + b*ArcTan[c*x])
*Log[(2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/
(2*e^2) - (d*(a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[
-d] + I*Sqrt[e])*(1 - I*c*x))])/(2*e^2) - ((I/2)*b*d*PolyLog[2, 1 - 2/(1 -
I*c*x)])/e^2 + ((I/4)*b*d*PolyLog[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*S
qrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/e^2 + ((I/4)*b*d*PolyLog[2, 1 - (2*c*(S
qrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/e^2
```

Rubi [A] time = 0.369632, antiderivative size = 361, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4916, 4852, 321, 203, 4980, 4856, 2402, 2315, 2447}

$$-\frac{ibd\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2e^2}+\frac{ibd\text{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{(1-icx)\left(c\sqrt{-d}-i\sqrt{e}\right)}\right)}{4e^2}+\frac{ibd\text{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-d}+\sqrt{e}x\right)}{(1-icx)\left(c\sqrt{-d}+i\sqrt{e}\right)}\right)}{4e^2}+\frac{d\log\left(\frac{2}{1-icx}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcTan[c*x]))/(d + e*x^2),x]
```

```
[Out] -(b*x)/(2*c*e) + (b*ArcTan[c*x])/(2*c^2*e) + (x^2*(a + b*ArcTan[c*x]))/(2*e
) + (d*(a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e^2 - (d*(a + b*ArcTan[c*x])
*Log[(2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/
(2*e^2) - (d*(a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[
-d] + I*Sqrt[e])*(1 - I*c*x))])/(2*e^2) - ((I/2)*b*d*PolyLog[2, 1 - 2/(1 -
I*c*x)])/e^2 + ((I/4)*b*d*PolyLog[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*S
qrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/e^2 + ((I/4)*b*d*PolyLog[2, 1 - (2*c*(S
qrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/e^2
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
```

```
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4980

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{x^3\left(a+b\tan^{-1}(cx)\right)}{d+ex^2}\,dx = \frac{\int x\left(a+b\tan^{-1}(cx)\right)dx}{e} - \frac{d\int \frac{x\left(a+b\tan^{-1}(cx)\right)}{d+ex^2}\,dx}{e}$$

$$= \frac{x^2\left(a+b\tan^{-1}(cx)\right)}{2e} - \frac{(bc)\int \frac{x^2}{1+c^2x^2}\,dx}{2e} - \frac{d\int\left(-\frac{a+b\tan^{-1}(cx)}{2\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{a+b\tan^{-1}(cx)}{2\sqrt{e}\left(\sqrt{-d}+\sqrt{e}x\right)}\right)dx}{e}$$

$$= -\frac{bx}{2ce} + \frac{x^2\left(a+b\tan^{-1}(cx)\right)}{2e} + \frac{d\int \frac{a+b\tan^{-1}(cx)}{\sqrt{-d}-\sqrt{e}x}\,dx}{2e^{3/2}} - \frac{d\int \frac{a+b\tan^{-1}(cx)}{\sqrt{-d}+\sqrt{e}x}\,dx}{2e^{3/2}} + \frac{b\int \frac{1}{1+c^2x^2}\,dx}{2ce}$$

$$= -\frac{bx}{2ce} + \frac{b\tan^{-1}(cx)}{2c^2e} + \frac{x^2\left(a+b\tan^{-1}(cx)\right)}{2e} + \frac{d\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2}{1-icx}\right)}{e^2} - \frac{d\left(a+b\tan\right.}{}$$

$$= -\frac{bx}{2ce} + \frac{b\tan^{-1}(cx)}{2c^2e} + \frac{x^2\left(a+b\tan^{-1}(cx)\right)}{2e} + \frac{d\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2}{1-icx}\right)}{e^2} - \frac{d\left(a+b\tan\right.}{}$$

$$= -\frac{bx}{2ce} + \frac{b\tan^{-1}(cx)}{2c^2e} + \frac{x^2\left(a+b\tan^{-1}(cx)\right)}{2e} + \frac{d\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2}{1-icx}\right)}{e^2} - \frac{d\left(a+b\tan\right.}{}$$

Mathematica [A] time = 0.259825, size = 503, normalized size = 1.39

$$-\frac{ibd\text{PolyLog}\left(2,-\frac{\sqrt{e}(1-icx)}{-\sqrt{e}+ic\sqrt{-d}}\right)}{4e^2} - \frac{ibd\text{PolyLog}\left(2,\frac{\sqrt{e}(1-icx)}{\sqrt{e}+ic\sqrt{-d}}\right)}{4e^2} + \frac{ibd\text{PolyLog}\left(2,-\frac{\sqrt{e}(1+icx)}{-\sqrt{e}+ic\sqrt{-d}}\right)}{4e^2} + \frac{ibd\text{PolyLog}\left(2,\frac{\sqrt{e}(1+icx)}{\sqrt{e}+ic\sqrt{-d}}\right)}{4e^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(a + b*ArcTan[c*x]))/(d + e*x^2),x]
```

```
[Out] -(b*x)/(2*c*e) + (a*x^2)/(2*e) + (b*ArcTan[c*x])/(2*c^2*e) + (b*x^2*ArcTan[
c*x])/(2*e) + ((I/4)*b*d*Log[1 + I*c*x]*Log[(c*(Sqrt[-d] - Sqrt[e]*x))/(c*S
qrt[-d] - I*Sqrt[e])])/e^2 - ((I/4)*b*d*Log[1 - I*c*x]*Log[(c*(Sqrt[-d] - S
qrt[e]*x))/(c*Sqrt[-d] + I*Sqrt[e])])/e^2 - ((I/4)*b*d*Log[1 - I*c*x]*Log[(
c*(Sqrt[-d] + Sqrt[e]*x))/(c*Sqrt[-d] - I*Sqrt[e])])/e^2 + ((I/4)*b*d*Log[1
 + I*c*x]*Log[(c*(Sqrt[-d] + Sqrt[e]*x))/(c*Sqrt[-d] + I*Sqrt[e])])/e^2 - (
a*d*Log[d + e*x^2])/(2*e^2) - ((I/4)*b*d*PolyLog[2, -((Sqrt[e]*(1 - I*c*x))
/(I*c*Sqrt[-d] - Sqrt[e]))])/e^2 - ((I/4)*b*d*PolyLog[2, (Sqrt[e]*(1 - I*c*
x))/(I*c*Sqrt[-d] + Sqrt[e])])/e^2 + ((I/4)*b*d*PolyLog[2, -((Sqrt[e]*(1 +
I*c*x))/(I*c*Sqrt[-d] - Sqrt[e]))])/e^2 + ((I/4)*b*d*PolyLog[2, (Sqrt[e]*(1
 + I*c*x))/(I*c*Sqrt[-d] + Sqrt[e])])/e^2
```

Maple [C] time = 0.211, size = 703, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctan(c*x))/(e*x^2+d),x)
```

```
[Out] 1/2*a*x^2/e-1/2*a/e^2*d*ln(c^2*e*x^2+c^2*d)+1/2*b*arctan(c*x)*x^2/e-1/2*b*a
rctan(c*x)/e^2*d*ln(c^2*e*x^2+c^2*d)-1/2*b*x/c/e+1/2*b*arctan(c*x)/c^2/e+1/
4*I*b/e^2*d*dilog((RootOf(e*_Z^2+2*I*_Z*e+c^2*d-e,index=2)-c*x+I)/RootOf(e*
_Z^2+2*I*_Z*e+c^2*d-e,index=2))+1/4*I*b/e^2*d*ln(c*x+I)*ln(c^2*e*x^2+c^2*d)
-1/4*I*b/e^2*d*dilog((RootOf(e*_Z^2-2*I*_Z*e+c^2*d-e,index=1)-c*x-I)/RootOf
(e*_Z^2-2*I*_Z*e+c^2*d-e,index=1))-1/4*I*b/e^2*d*ln(c*x-I)*ln(c^2*e*x^2+c^2
*d)+1/4*I*b/e^2*d*ln(c*x-I)*ln((RootOf(e*_Z^2+2*I*_Z*e+c^2*d-e,index=2)-c*x
+I)/RootOf(e*_Z^2+2*I*_Z*e+c^2*d-e,index=2))-1/4*I*b/e^2*d*dilog((RootOf(e*
_Z^2-2*I*_Z*e+c^2*d-e,index=2)-c*x-I)/RootOf(e*_Z^2-2*I*_Z*e+c^2*d-e,index=
2))+1/4*I*b/e^2*d*ln(c*x-I)*ln((RootOf(e*_Z^2+2*I*_Z*e+c^2*d-e,index=1)-c*x
+I)/RootOf(e*_Z^2+2*I*_Z*e+c^2*d-e,index=1))+1/4*I*b/e^2*d*dilog((RootOf(e*
_Z^2+2*I*_Z*e+c^2*d-e,index=1)-c*x+I)/RootOf(e*_Z^2+2*I*_Z*e+c^2*d-e,index=
1))-1/4*I*b/e^2*d*ln(c*x+I)*ln((RootOf(e*_Z^2-2*I*_Z*e+c^2*d-e,index=2)-c*x
-I)/RootOf(e*_Z^2-2*I*_Z*e+c^2*d-e,index=2))-1/4*I*b/e^2*d*ln(c*x+I)*ln((Ro
otOf(e*_Z^2-2*I*_Z*e+c^2*d-e,index=1)-c*x-I)/RootOf(e*_Z^2-2*I*_Z*e+c^2*d-e
,index=1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}\,a\left(\frac{x^2}{e}-\frac{d\log\left(ex^2+d\right)}{e^2}\right)+2\,b\int\frac{x^3\arctan\left(cx\right)}{2\left(ex^2+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] 1/2*a*(x^2/e - d*log(e*x^2 + d)/e^2) + 2*b*integrate(1/2*x^3*arctan(c*x)/(e
*x^2 + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{bx^3\arctan\left(cx\right)+ax^3}{ex^2+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*x^3*arctan(c*x) + a*x^3)/(e*x^2 + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^3\left(a+b\,\mathrm{atan}\left(cx\right)\right)}{d+ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atan(c*x))/(e*x**2+d),x)
```

```
[Out] Integral(x**3*(a + b*atan(c*x))/(d + e*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \arctan (cx) + a)x^3}{ex^2 + d} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))/(e*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)*x^3/(e*x^2 + d), x)
```

3.1152 $\int \frac{x\left(a+b\tan^{-1}(cx)\right)}{d+ex^2}\,dx$

Optimal. Leaf size=311

$$-\frac{ib\text{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{(1-icx)\left(c\sqrt{-d}-i\sqrt{e}\right)}\right)}{4e}-\frac{ib\text{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-d}+\sqrt{e}x\right)}{(1-icx)\left(c\sqrt{-d}+i\sqrt{e}\right)}\right)}{4e}+\frac{ib\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2e}+\frac{\left(a+b\tan^{-1}(cx\right.}{}$$

```
[Out] -(((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e) + ((a + b*ArcTan[c*x])*Log[(2
*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/(2*e) +
 ((a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqr
t[e])*(1 - I*c*x))])/(2*e) + ((I/2)*b*PolyLog[2, 1 - 2/(1 - I*c*x)])/e - ((
I/4)*b*PolyLog[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e]
)*(1 - I*c*x))])/e - ((I/4)*b*PolyLog[2, 1 - (2*c*(Sqrt[-d] + Sqrt[e]*x))/(
(c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/e
```

Rubi [A] time = 0.243335, antiderivative size = 311, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {4980, 4856, 2402, 2315, 2447}

$$-\frac{ib\text{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{(1-icx)\left(c\sqrt{-d}-i\sqrt{e}\right)}\right)}{4e}-\frac{ib\text{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-d}+\sqrt{e}x\right)}{(1-icx)\left(c\sqrt{-d}+i\sqrt{e}\right)}\right)}{4e}+\frac{ib\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2e}+\frac{\left(a+b\tan^{-1}(cx\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcTan[c*x]))/(d + e*x^2),x]
```


```
[Out] -(((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e) + ((a + b*ArcTan[c*x])*Log[(2
*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/(2*e) +
 ((a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqr
t[e])*(1 - I*c*x))])/(2*e) + ((I/2)*b*PolyLog[2, 1 - 2/(1 - I*c*x)])/e - ((
I/4)*b*PolyLog[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e]
)*(1 - I*c*x))])/e - ((I/4)*b*PolyLog[2, 1 - (2*c*(Sqrt[-d] + Sqrt[e]*x))/(
(c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/e
```


Rule 4980

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
```

```
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{x\left(a+b\tan^{-1}(cx)\right)}{d+ex^2}\,dx = \int\left(-\frac{a+b\tan^{-1}(cx)}{2\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{a+b\tan^{-1}(cx)}{2\sqrt{e}\left(\sqrt{-d}+\sqrt{e}x\right)}\right)dx$$

$$= -\frac{\int\frac{a+b\tan^{-1}(cx)}{\sqrt{-d}-\sqrt{e}x}\,dx}{2\sqrt{e}}+\frac{\int\frac{a+b\tan^{-1}(cx)}{\sqrt{-d}+\sqrt{e}x}\,dx}{2\sqrt{e}}$$

$$= -\frac{\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2}{1-icx}\right)}{e}+\frac{\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(c\sqrt{-d}-i\sqrt{e}\right)(1-icx)}\right)}{2e}+\frac{\left(a+b\tan^{-1}(cx)\right)}{}$$

$$= -\frac{\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2}{1-icx}\right)}{e}+\frac{\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(c\sqrt{-d}-i\sqrt{e}\right)(1-icx)}\right)}{2e}+\frac{\left(a+b\tan^{-1}(cx)\right)}{}$$

$$= -\frac{\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2}{1-icx}\right)}{e}+\frac{\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(c\sqrt{-d}-i\sqrt{e}\right)(1-icx)}\right)}{2e}+\frac{\left(a+b\tan^{-1}(cx)\right)}{}$$

Mathematica [A] time = 0.113027, size = 441, normalized size = 1.42

$$\frac{ib\text{PolyLog}\left(2,-\frac{\sqrt{e}(1-icx)}{-\sqrt{e}+ic\sqrt{-d}}\right)}{4e}+\frac{ib\text{PolyLog}\left(2,\frac{\sqrt{e}(1-icx)}{\sqrt{e}+ic\sqrt{-d}}\right)}{4e}-\frac{ib\text{PolyLog}\left(2,-\frac{\sqrt{e}(1+icx)}{-\sqrt{e}+ic\sqrt{-d}}\right)}{4e}-\frac{ib\text{PolyLog}\left(2,\frac{\sqrt{e}(1+icx)}{\sqrt{e}+ic\sqrt{-d}}\right)}{4e}+\frac{a\log}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(a + b*ArcTan[c*x]))/(d + e*x^2),x]
```

```
[Out] ((-I/4)*b*Log[1 + I*c*x]*Log[(c*(Sqrt[-d] - Sqrt[e]*x))/(c*Sqrt[-d] - I*Sqr
t[e])])/e + ((I/4)*b*Log[1 - I*c*x]*Log[(c*(Sqrt[-d] - Sqrt[e]*x))/(c*Sqrt[
-d] + I*Sqrt[e])])/e + ((I/4)*b*Log[1 - I*c*x]*Log[(c*(Sqrt[-d] + Sqrt[e]*x
))/(c*Sqrt[-d] - I*Sqrt[e])])/e - ((I/4)*b*Log[1 + I*c*x]*Log[(c*(Sqrt[-d]
+ Sqrt[e]*x))/(c*Sqrt[-d] + I*Sqrt[e])])/e + (a*Log[d + e*x^2])/(2*e) + ((I
/4)*b*PolyLog[2, -((Sqrt[e]*(1 - I*c*x))/(I*c*Sqrt[-d] - Sqrt[e]))])/e + ((
I/4)*b*PolyLog[2, (Sqrt[e]*(1 - I*c*x))/(I*c*Sqrt[-d] + Sqrt[e])])/e - ((I/
4)*b*PolyLog[2, -((Sqrt[e]*(1 + I*c*x))/(I*c*Sqrt[-d] - Sqrt[e]))])/e - ((I
/4)*b*PolyLog[2, (Sqrt[e]*(1 + I*c*x))/(I*c*Sqrt[-d] + Sqrt[e])])/e
```

Maple [C] time = 0.193, size = 646, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctan(c*x))/(e*x^2+d),x)
```

```
[Out] 1/2*a/e*ln(c^2*e*x^2+c^2*d)+1/2*b/e*ln(c^2*e*x^2+c^2*d)*arctan(c*x)-1/4*I*b
/e*ln(c*x-I)*ln((RootOf(e*_Z^2+2*I*_Z*e+c^2*d-e,index=1)-c*x+I)/RootOf(e*_Z
^2+2*I*_Z*e+c^2*d-e,index=1))-1/4*I*b/e*ln(c*x-I)*ln((RootOf(e*_Z^2+2*I*_Z*
e+c^2*d-e,index=2)-c*x+I)/RootOf(e*_Z^2+2*I*_Z*e+c^2*d-e,index=2))+1/4*I*b/
e*ln(c*x-I)*ln(c^2*e*x^2+c^2*d)-1/4*I*b/e*dilog((RootOf(e*_Z^2+2*I*_Z*e+c^2
*d-e,index=1)-c*x+I)/RootOf(e*_Z^2+2*I*_Z*e+c^2*d-e,index=1))-1/4*I*b/e*dil
og((RootOf(e*_Z^2+2*I*_Z*e+c^2*d-e,index=2)-c*x+I)/RootOf(e*_Z^2+2*I*_Z*e+c
^2*d-e,index=2))+1/4*I*b/e*ln(c*x+I)*ln((RootOf(e*_Z^2-2*I*_Z*e+c^2*d-e,ind
ex=1)-c*x-I)/RootOf(e*_Z^2-2*I*_Z*e+c^2*d-e,index=1))+1/4*I*b/e*ln(c*x+I)*l
n((RootOf(e*_Z^2-2*I*_Z*e+c^2*d-e,index=2)-c*x-I)/RootOf(e*_Z^2-2*I*_Z*e+c^
2*d-e,index=2))-1/4*I*b/e*ln(c*x+I)*ln(c^2*e*x^2+c^2*d)+1/4*I*b/e*dilog((Ro
otOf(e*_Z^2-2*I*_Z*e+c^2*d-e,index=1)-c*x-I)/RootOf(e*_Z^2-2*I*_Z*e+c^2*d-e
,index=1))+1/4*I*b/e*dilog((RootOf(e*_Z^2-2*I*_Z*e+c^2*d-e,index=2)-c*x-I)/
RootOf(e*_Z^2-2*I*_Z*e+c^2*d-e,index=2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$2\,b\int\frac{x\arctan\left(cx\right)}{2\left(ex^2+d\right)}\,dx+\frac{a\log\left(ex^2+d\right)}{2\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] 2*b*integrate(1/2*x*arctan(c*x)/(e*x^2 + d), x) + 1/2*a*log(e*x^2 + d)/e
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{bx\arctan\left(cx\right)+ax}{ex^2+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*x*arctan(c*x) + a*x)/(e*x^2 + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(a+b\operatorname{atan}\left(cx\right)\right)}{d+ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atan(c*x))/(e*x**2+d),x)
```

```
[Out] Integral(x*(a + b*atan(c*x))/(d + e*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\arctan\left(cx\right)+a\right)x}{ex^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))/(e*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)*x/(e*x^2 + d), x)
```

3.1153 $\int \frac{a+b\tan^{-1}(cx)}{x(d+ex^2)}\,dx$

Optimal. Leaf size=353

$$\frac{ib\text{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{(1-icx)\left(c\sqrt{-d}-i\sqrt{e}\right)}\right)}{4d}+\frac{ib\text{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-d}+\sqrt{e}x\right)}{(1-icx)\left(c\sqrt{-d}+i\sqrt{e}\right)}\right)}{4d}+\frac{ib\text{PolyLog}(2,-icx)}{2d}-\frac{ib\text{PolyLog}(2,icx)}{2d}$$

```
[Out] (a*Log[x])/d + ((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/d - ((a + b*ArcTan[
c*x])*Log[(2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x
))])/(2*d) - ((a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt
[-d] + I*Sqrt[e])*(1 - I*c*x))])/(2*d) + ((I/2)*b*PolyLog[2, (-I)*c*x])/d -
 ((I/2)*b*PolyLog[2, I*c*x])/d - ((I/2)*b*PolyLog[2, 1 - 2/(1 - I*c*x)])/d
+ ((I/4)*b*PolyLog[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqr
t[e])*(1 - I*c*x))])/d + ((I/4)*b*PolyLog[2, 1 - (2*c*(Sqrt[-d] + Sqrt[e]*x
))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/d
```

Rubi [A] time = 0.385525, antiderivative size = 353, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {4928, 4848, 2391, 4980, 4856, 2402, 2315, 2447}

$$\frac{ib\text{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{(1-icx)\left(c\sqrt{-d}-i\sqrt{e}\right)}\right)}{4d}+\frac{ib\text{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-d}+\sqrt{e}x\right)}{(1-icx)\left(c\sqrt{-d}+i\sqrt{e}\right)}\right)}{4d}+\frac{ib\text{PolyLog}(2,-icx)}{2d}-\frac{ib\text{PolyLog}(2,icx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])/(x*(d + e*x^2)),x]
```


```
[Out] (a*Log[x])/d + ((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/d - ((a + b*ArcTan[
c*x])*Log[(2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x
))])/(2*d) - ((a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt
[-d] + I*Sqrt[e])*(1 - I*c*x))])/(2*d) + ((I/2)*b*PolyLog[2, (-I)*c*x])/d -
 ((I/2)*b*PolyLog[2, I*c*x])/d - ((I/2)*b*PolyLog[2, 1 - 2/(1 - I*c*x)])/d
+ ((I/4)*b*PolyLog[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqr
t[e])*(1 - I*c*x))])/d + ((I/4)*b*PolyLog[2, 1 - (2*c*(Sqrt[-d] + Sqrt[e]*x
))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/d
```


Rule 4928

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
```

```
/; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4980

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
```

```
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{a+b\tan^{-1}(cx)}{x\left(d+ex^2\right)}\,dx = \int \left(\frac{a+b\tan^{-1}(cx)}{dx} - \frac{ex\left(a+b\tan^{-1}(cx)\right)}{d\left(d+ex^2\right)}\right)dx$$

$$= \frac{\int \frac{a+b\tan^{-1}(cx)}{x}\,dx}{d} - \frac{e\int \frac{x\left(a+b\tan^{-1}(cx)\right)}{d+ex^2}\,dx}{d}$$

$$= \frac{a\log(x)}{d} + \frac{(ib)\int \frac{\log(1-icx)}{x}\,dx}{2d} - \frac{(ib)\int \frac{\log(1+icx)}{x}\,dx}{2d} - \frac{e\int \left(-\frac{a+b\tan^{-1}(cx)}{2\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)} + \frac{a+b\tan^{-1}(cx)}{2\sqrt{e}\left(\sqrt{-d}+\sqrt{e}x\right)}\right)dx}{d}$$

$$= \frac{a\log(x)}{d} + \frac{ib\mathrm{Li}_2(-icx)}{2d} - \frac{ib\mathrm{Li}_2(icx)}{2d} + \frac{\sqrt{e}\int \frac{a+b\tan^{-1}(cx)}{\sqrt{-d}-\sqrt{e}x}\,dx}{2d} - \frac{\sqrt{e}\int \frac{a+b\tan^{-1}(cx)}{\sqrt{-d}+\sqrt{e}x}\,dx}{2d}$$

$$= \frac{a\log(x)}{d} + \frac{\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2}{1-icx}\right)}{d} - \frac{\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(c\sqrt{-d}-i\sqrt{e}\right)(1-icx)}\right)}{2d} - \frac{\left(a+b\tan\right.}{}$$

$$= \frac{a\log(x)}{d} + \frac{\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2}{1-icx}\right)}{d} - \frac{\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(c\sqrt{-d}-i\sqrt{e}\right)(1-icx)}\right)}{2d} - \frac{\left(a+b\tan\right.}{}$$

$$= \frac{a\log(x)}{d} + \frac{\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2}{1-icx}\right)}{d} - \frac{\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(c\sqrt{-d}-i\sqrt{e}\right)(1-icx)}\right)}{2d} - \frac{\left(a+b\tan\right.}{}$$

Mathematica [A] time = 0.241868, size = 429, normalized size = 1.22

$$\frac{ib\mathrm{PolyLog}\left(2,\frac{\sqrt{e}(-cx+i)}{c\sqrt{-d}+i\sqrt{e}}\right) - ib\mathrm{PolyLog}\left(2,\frac{\sqrt{e}(1-icx)}{\sqrt{e}+ic\sqrt{-d}}\right) + ib\mathrm{PolyLog}\left(2,\frac{\sqrt{e}(1+icx)}{\sqrt{e}+ic\sqrt{-d}}\right) - ib\mathrm{PolyLog}\left(2,\frac{\sqrt{e}(cx+i)}{c\sqrt{-d}+i\sqrt{e}}\right) + 2ib\mathrm{PolyLo}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c*x])/(x*(d + e*x^2)),x]
```

```
[Out] (4*a*Log[x] + I*b*Log[1 + I*c*x]*Log[(c*(Sqrt[-d] - Sqrt[e]*x))/(c*Sqrt[-d]
 - I*Sqrt[e])] - I*b*Log[1 - I*c*x]*Log[(c*(Sqrt[-d] - Sqrt[e]*x))/(c*Sqrt[
-d] + I*Sqrt[e])] - I*b*Log[1 - I*c*x]*Log[(c*(Sqrt[-d] + Sqrt[e]*x))/(c*Sq
rt[-d] - I*Sqrt[e])] + I*b*Log[1 + I*c*x]*Log[(c*(Sqrt[-d] + Sqrt[e]*x))/(c
```

```
*Sqrt[-d] + I*Sqrt[e])] - 2*a*Log[d + e*x^2] + (2*I)*b*PolyLog[2, (-I)*c*x]
 - (2*I)*b*PolyLog[2, I*c*x] + I*b*PolyLog[2, (Sqrt[e]*(I - c*x))/(c*Sqrt[-
d] + I*Sqrt[e])] - I*b*PolyLog[2, (Sqrt[e]*(1 - I*c*x))/(I*c*Sqrt[-d] + Sqr
t[e])] + I*b*PolyLog[2, (Sqrt[e]*(1 + I*c*x))/(I*c*Sqrt[-d] + Sqrt[e])] - I
*b*PolyLog[2, (Sqrt[e]*(I + c*x))/(c*Sqrt[-d] + I*Sqrt[e])])/(4*d)
```

Maple [C] time = 0.208, size = 736, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/x/(e*x^2+d),x)
```

```
[Out] -1/2*a/d*ln(c^2*e*x^2+c^2*d)+a/d*ln(c*x)-1/2*b*arctan(c*x)/d*ln(c^2*e*x^2+c
^2*d)+b*arctan(c*x)/d*ln(c*x)+1/4*I*b/d*ln(c*x-I)*ln((RootOf(e*_Z^2+2*I*_Z*
e+c^2*d-e,index=2)-c*x+I)/RootOf(e*_Z^2+2*I*_Z*e+c^2*d-e,index=2))+1/4*I*b/
d*dilog((RootOf(e*_Z^2+2*I*_Z*e+c^2*d-e,index=1)-c*x+I)/RootOf(e*_Z^2+2*I*_
Z*e+c^2*d-e,index=1))+1/2*I*b/d*ln(c*x)*ln(1+I*c*x)-1/4*I*b/d*ln(c*x+I)*ln(
(RootOf(e*_Z^2-2*I*_Z*e+c^2*d-e,index=2)-c*x-I)/RootOf(e*_Z^2-2*I*_Z*e+c^2*
d-e,index=2))-1/4*I*b/d*dilog((RootOf(e*_Z^2-2*I*_Z*e+c^2*d-e,index=1)-c*x-
I)/RootOf(e*_Z^2-2*I*_Z*e+c^2*d-e,index=1))-1/4*I*b/d*ln(c*x+I)*ln((RootOf(
e*_Z^2-2*I*_Z*e+c^2*d-e,index=1)-c*x-I)/RootOf(e*_Z^2-2*I*_Z*e+c^2*d-e,inde
x=1))-1/4*I*b/d*ln(c*x-I)*ln(c^2*e*x^2+c^2*d)-1/2*I*b/d*dilog(1-I*c*x)+1/2*
I*b/d*dilog(1+I*c*x)+1/4*I*b/d*ln(c*x+I)*ln(c^2*e*x^2+c^2*d)+1/4*I*b/d*ln(c
*x-I)*ln((RootOf(e*_Z^2+2*I*_Z*e+c^2*d-e,index=1)-c*x+I)/RootOf(e*_Z^2+2*I*
_Z*e+c^2*d-e,index=1))-1/2*I*b/d*ln(c*x)*ln(1-I*c*x)-1/4*I*b/d*dilog((RootO
f(e*_Z^2-2*I*_Z*e+c^2*d-e,index=2)-c*x-I)/RootOf(e*_Z^2-2*I*_Z*e+c^2*d-e,in
dex=2))+1/4*I*b/d*dilog((RootOf(e*_Z^2+2*I*_Z*e+c^2*d-e,index=2)-c*x+I)/Roo
tOf(e*_Z^2+2*I*_Z*e+c^2*d-e,index=2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2}\,a\left(\frac{\log\left(ex^{2}+d\right)}{d}-\frac{2\,\log\left(x\right)}{d}\right)+2\,b\int\frac{\arctan\left(cx\right)}{2\left(ex^{3}+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] -1/2*a*(log(e*x^2 + d)/d - 2*log(x)/d) + 2*b*integrate(1/2*arctan(c*x)/(e*x
^3 + d*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b\arctan(cx)+a}{ex^3+dx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*arctan(c*x) + a)/(e*x^3 + d*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/x/(e*x**2+d),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\arctan(cx)+a}{(ex^2+d)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x/(e*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)/((e*x^2 + d)*x), x)
```

3.1154 $\int \frac{a+b\tan^{-1}(cx)}{x^3(d+ex^2)}\,dx$

Optimal. Leaf size=409

$$-\frac{ibe\text{PolyLog}(2,-icx)}{2d^2}+\frac{ibe\text{PolyLog}(2,icx)}{2d^2}+\frac{ibe\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2d^2}-\frac{ibe\text{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{(1-icx)\left(c\sqrt{-d}-i\sqrt{e}\right)}\right)}{4d^2}-\frac{ibe\text{Po}}{}$$

```
[Out] -(b*c)/(2*d*x) - (b*c^2*ArcTan[c*x])/(2*d) - (a + b*ArcTan[c*x])/(2*d*x^2)
- (a*e*Log[x])/d^2 - (e*(a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/d^2 + (e*(a
 + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e]
)*(1 - I*c*x))])/(2*d^2) + (e*(a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] + Sqrt
[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(2*d^2) - ((I/2)*b*e*PolyL
og[2, (-I)*c*x])/d^2 + ((I/2)*b*e*PolyLog[2, I*c*x])/d^2 + ((I/2)*b*e*PolyL
og[2, 1 - 2/(1 - I*c*x)])/d^2 - ((I/4)*b*e*PolyLog[2, 1 - (2*c*(Sqrt[-d] -
Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/d^2 - ((I/4)*b*e*PolyL
og[2, 1 - (2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x
))])/d^2
```

Rubi [A] time = 0.482376, antiderivative size = 409, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 12, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {4918, 4852, 325, 203, 4928, 4848, 2391, 4980, 4856, 2402, 2315, 2447}

$$-\frac{ibe\text{PolyLog}(2,-icx)}{2d^2}+\frac{ibe\text{PolyLog}(2,icx)}{2d^2}+\frac{ibe\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2d^2}-\frac{ibe\text{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{(1-icx)\left(c\sqrt{-d}-i\sqrt{e}\right)}\right)}{4d^2}-\frac{ibe\text{Po}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])/(x^3*(d + e*x^2)),x]
```


```
[Out] -(b*c)/(2*d*x) - (b*c^2*ArcTan[c*x])/(2*d) - (a + b*ArcTan[c*x])/(2*d*x^2)
- (a*e*Log[x])/d^2 - (e*(a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/d^2 + (e*(a
 + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e]
)*(1 - I*c*x))])/(2*d^2) + (e*(a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] + Sqrt
[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(2*d^2) - ((I/2)*b*e*PolyL
og[2, (-I)*c*x])/d^2 + ((I/2)*b*e*PolyLog[2, I*c*x])/d^2 + ((I/2)*b*e*PolyL
og[2, 1 - 2/(1 - I*c*x)])/d^2 - ((I/4)*b*e*PolyLog[2, 1 - (2*c*(Sqrt[-d] -
Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/d^2 - ((I/4)*b*e*PolyL
og[2, 1 - (2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x
```

```
))])/d^2
```

Rule 4918

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/(d + e*x^2),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4928

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4980

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{a+b\tan^{-1}(cx)}{x^3(d+ex^2)}\,dx = \frac{\int \frac{a+b\tan^{-1}(cx)}{x^3}\,dx}{d} - \frac{e\int \frac{a+b\tan^{-1}(cx)}{x(d+ex^2)}\,dx}{d}$$

$$= -\frac{a+b\tan^{-1}(cx)}{2dx^2} + \frac{(bc)\int \frac{1}{x^2(1+c^2x^2)}\,dx}{2d} - \frac{e\int \left(\frac{a+b\tan^{-1}(cx)}{dx} - \frac{ex(a+b\tan^{-1}(cx))}{d(d+ex^2)}\right)dx}{d}$$

$$= -\frac{bc}{2dx} - \frac{a+b\tan^{-1}(cx)}{2dx^2} - \frac{(bc^3)\int \frac{1}{1+c^2x^2}\,dx}{2d} - \frac{e\int \frac{a+b\tan^{-1}(cx)}{x}\,dx}{d^2} + \frac{e^2\int \frac{x(a+b\tan^{-1}(cx))}{d+ex^2}\,dx}{d^2}$$

$$= -\frac{bc}{2dx} - \frac{bc^2\tan^{-1}(cx)}{2d} - \frac{a+b\tan^{-1}(cx)}{2dx^2} - \frac{ae\log(x)}{d^2} - \frac{(ibe)\int \frac{\log(1-icx)}{x}\,dx}{2d^2} + \frac{(ibe)\int \frac{\log(1+icx)}{x}\,d}{2d^2}$$

$$= -\frac{bc}{2dx} - \frac{bc^2\tan^{-1}(cx)}{2d} - \frac{a+b\tan^{-1}(cx)}{2dx^2} - \frac{ae\log(x)}{d^2} - \frac{ibe\mathrm{Li}_2(-icx)}{2d^2} + \frac{ibe\mathrm{Li}_2(icx)}{2d^2} - \frac{e^{3/2}\int \frac{a+b}{\sqrt{}}}{2}$$

$$= -\frac{bc}{2dx} - \frac{bc^2\tan^{-1}(cx)}{2d} - \frac{a+b\tan^{-1}(cx)}{2dx^2} - \frac{ae\log(x)}{d^2} - \frac{e(a+b\tan^{-1}(cx))\log\left(\frac{2}{1-icx}\right)}{d^2} + \frac{e(a+}{}$$

$$= -\frac{bc}{2dx} - \frac{bc^2\tan^{-1}(cx)}{2d} - \frac{a+b\tan^{-1}(cx)}{2dx^2} - \frac{ae\log(x)}{d^2} - \frac{e(a+b\tan^{-1}(cx))\log\left(\frac{2}{1-icx}\right)}{d^2} + \frac{e(a+}{}$$

$$= -\frac{bc}{2dx} - \frac{bc^2\tan^{-1}(cx)}{2d} - \frac{a+b\tan^{-1}(cx)}{2dx^2} - \frac{ae\log(x)}{d^2} - \frac{e(a+b\tan^{-1}(cx))\log\left(\frac{2}{1-icx}\right)}{d^2} + \frac{e(a+}{}$$

Mathematica [C] time = 0.272978, size = 504, normalized size = 1.23

$$\frac{-\frac{bc\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-c^2x^2\right)}{2x} - \frac{a+b\tan^{-1}(cx)}{2x^2}}{d} - \frac{e\left(-\frac{ib\left(\text{PolyLog}\left(2,-\frac{\sqrt{e}(1-icx)}{-\sqrt{e}+ic\sqrt{-d}}\right)+\log(1-icx)\log\left(\frac{c(\sqrt{-d}-\sqrt{e}x)}{c\sqrt{-d}+i\sqrt{e}}\right)\right)}{4d} - \frac{ib\left(\text{PolyLog}\left(2,\frac{\sqrt{e}(1-icx)}{\sqrt{e}+ic\sqrt{-d}}\right)\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c*x])/(x^3*(d + e*x^2)),x]
```

```
[Out] (-(a + b*ArcTan[c*x])/(2*x^2) - (b*c*Hypergeometric2F1[-1/2, 1, 1/2, -(c^2*
x^2)])/(2*x))/d - (e*((a*Log[x])/d - (a*Log[d + e*x^2])/(2*d) + ((I/2)*b*Po
lyLog[2, (-I)*c*x])/d - ((I/2)*b*PolyLog[2, I*c*x])/d - ((I/4)*b*(Log[1 - I
*c*x]*Log[(c*(Sqrt[-d] - Sqrt[e]*x))/(c*Sqrt[-d] + I*Sqrt[e])] + PolyLog[2,
 -((Sqrt[e]*(1 - I*c*x))/(I*c*Sqrt[-d] - Sqrt[e]))]))/d - ((I/4)*b*(Log[1 -
```

```
I*c*x]*Log[(c*(Sqrt[-d] + Sqrt[e]*x))/(c*Sqrt[-d] - I*Sqrt[e])] + PolyLog[
2, (Sqrt[e]*(1 - I*c*x))/(I*c*Sqrt[-d] + Sqrt[e])]))/d + ((I/4)*b*(Log[1 +
I*c*x]*Log[(c*(Sqrt[-d] + Sqrt[e]*x))/(c*Sqrt[-d] + I*Sqrt[e])] + PolyLog[2
, -((Sqrt[e]*(1 + I*c*x))/(I*c*Sqrt[-d] - Sqrt[e]))]))/d + ((I/4)*b*(Log[1
+ I*c*x]*Log[(c*(Sqrt[-d] - Sqrt[e]*x))/(c*Sqrt[-d] - I*Sqrt[e])] + PolyLog
[2, (Sqrt[e]*(1 + I*c*x))/(I*c*Sqrt[-d] + Sqrt[e])]))/d))/d
```

Maple [C] time = 0.2, size = 801, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/x^3/(e*x^2+d),x)
```

```
[Out] 1/2*a*e/d^2*ln(c^2*e*x^2+c^2*d)-1/2*a/d/x^2-a/d^2*e*ln(c*x)+1/2*b*arctan(c*
x)*e/d^2*ln(c^2*e*x^2+c^2*d)-1/2*b*arctan(c*x)/d/x^2-b*arctan(c*x)/d^2*e*ln
(c*x)-1/4*I*b/d^2*e*dilog((RootOf(e*_Z^2+2*I*_Z*e+c^2*d-e,index=1)-c*x+I)/R
ootOf(e*_Z^2+2*I*_Z*e+c^2*d-e,index=1))+1/4*I*b/d^2*e*ln(c*x-I)*ln(c^2*e*x^
2+c^2*d)+1/2*I*b/d^2*e*dilog(1-I*c*x)-1/2*I*b/d^2*e*ln(c*x)*ln(1+I*c*x)-1/4
*I*b/d^2*e*ln(c*x-I)*ln((RootOf(e*_Z^2+2*I*_Z*e+c^2*d-e,index=2)-c*x+I)/Roo
tOf(e*_Z^2+2*I*_Z*e+c^2*d-e,index=2))+1/4*I*b/d^2*e*dilog((RootOf(e*_Z^2-2*
I*_Z*e+c^2*d-e,index=2)-c*x-I)/RootOf(e*_Z^2-2*I*_Z*e+c^2*d-e,index=2))-1/4
*I*b/d^2*e*ln(c*x-I)*ln((RootOf(e*_Z^2+2*I*_Z*e+c^2*d-e,index=1)-c*x+I)/Roo
tOf(e*_Z^2+2*I*_Z*e+c^2*d-e,index=1))-1/4*I*b/d^2*e*dilog((RootOf(e*_Z^2+2*
I*_Z*e+c^2*d-e,index=2)-c*x+I)/RootOf(e*_Z^2+2*I*_Z*e+c^2*d-e,index=2))+1/4
*I*b/d^2*e*dilog((RootOf(e*_Z^2-2*I*_Z*e+c^2*d-e,index=1)-c*x-I)/RootOf(e*_
Z^2-2*I*_Z*e+c^2*d-e,index=1))+1/2*I*b/d^2*e*ln(c*x)*ln(1-I*c*x)-1/2*b*c^2*
arctan(c*x)/d-1/2*b*c/d/x-1/4*I*b/d^2*e*ln(c*x+I)*ln(c^2*e*x^2+c^2*d)+1/4*I
*b/d^2*e*ln(c*x+I)*ln((RootOf(e*_Z^2-2*I*_Z*e+c^2*d-e,index=2)-c*x-I)/RootO
f(e*_Z^2-2*I*_Z*e+c^2*d-e,index=2))+1/4*I*b/d^2*e*ln(c*x+I)*ln((RootOf(e*_Z
^2-2*I*_Z*e+c^2*d-e,index=1)-c*x-I)/RootOf(e*_Z^2-2*I*_Z*e+c^2*d-e,index=1)
)-1/2*I*b/d^2*e*dilog(1+I*c*x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}\,a\left(\frac{e\log\left(ex^2+d\right)}{d^2}-\frac{2\,e\log\left(x\right)}{d^2}-\frac{1}{dx^2}\right)+2\,b\int\frac{\arctan\left(cx\right)}{2\left(ex^5+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^3/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] 1/2*a*(e*log(e*x^2 + d)/d^2 - 2*e*log(x)/d^2 - 1/(d*x^2)) + 2*b*integrate(1
/2*arctan(c*x)/(e*x^5 + d*x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b\arctan(cx) + a}{ex^5 + dx^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^3/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*arctan(c*x) + a)/(e*x^5 + d*x^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/x**3/(e*x**2+d),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\arctan(cx) + a}{(ex^2 + d)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^3/(e*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)/((e*x^2 + d)*x^3), x)
```

3.1155 $\int \frac{x^2\left(a+b\tan^{-1}(cx)\right)}{d+ex^2}\,dx$

Optimal. Leaf size=555

$$\frac{ib\sqrt{-d}\text{PolyLog}\left(2,\frac{\sqrt{e}(-cx+i)}{c\sqrt{-d}+i\sqrt{e}}\right)}{4e^{3/2}}-\frac{ib\sqrt{-d}\text{PolyLog}\left(2,\frac{\sqrt{e}(1-icx)}{\sqrt{e}+ic\sqrt{-d}}\right)}{4e^{3/2}}-\frac{ib\sqrt{-d}\text{PolyLog}\left(2,\frac{\sqrt{e}(1+icx)}{\sqrt{e}+ic\sqrt{-d}}\right)}{4e^{3/2}}+\frac{ib\sqrt{-d}\text{PolyLog}\left(2,\frac{\sqrt{e}(}{c\sqrt{-}}\right.}{4e^{3/2}}$$

```
[Out] (a*x)/e + (b*x*ArcTan[c*x])/e - (a*Sqrt[d]*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/e^(
3/2) - ((I/4)*b*Sqrt[-d]*Log[1 + I*c*x]*Log[(c*(Sqrt[-d] - Sqrt[e]*x))/(c*S
qrt[-d] - I*Sqrt[e])])/e^(3/2) + ((I/4)*b*Sqrt[-d]*Log[1 - I*c*x]*Log[(c*(S
qrt[-d] - Sqrt[e]*x))/(c*Sqrt[-d] + I*Sqrt[e])])/e^(3/2) - ((I/4)*b*Sqrt[-d
]*Log[1 - I*c*x]*Log[(c*(Sqrt[-d] + Sqrt[e]*x))/(c*Sqrt[-d] - I*Sqrt[e])])/
e^(3/2) + ((I/4)*b*Sqrt[-d]*Log[1 + I*c*x]*Log[(c*(Sqrt[-d] + Sqrt[e]*x))/(
c*Sqrt[-d] + I*Sqrt[e])])/e^(3/2) - (b*Log[1 + c^2*x^2])/(2*c*e) + ((I/4)*b
*Sqrt[-d]*PolyLog[2, (Sqrt[e]*(I - c*x))/(c*Sqrt[-d] + I*Sqrt[e])])/e^(3/2)
 - ((I/4)*b*Sqrt[-d]*PolyLog[2, (Sqrt[e]*(1 - I*c*x))/(I*c*Sqrt[-d] + Sqrt[
e])])/e^(3/2) - ((I/4)*b*Sqrt[-d]*PolyLog[2, (Sqrt[e]*(1 + I*c*x))/(I*c*Sqr
t[-d] + Sqrt[e])])/e^(3/2) + ((I/4)*b*Sqrt[-d]*PolyLog[2, (Sqrt[e]*(I + c*x
))/(c*Sqrt[-d] + I*Sqrt[e])])/e^(3/2)
```

Rubi [A] time = 0.630897, antiderivative size = 555, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {4916, 4846, 260, 4910, 205, 4908, 2409, 2394, 2393, 2391}

$$\frac{ib\sqrt{-d}\text{PolyLog}\left(2,\frac{\sqrt{e}(-cx+i)}{c\sqrt{-d}+i\sqrt{e}}\right)}{4e^{3/2}}-\frac{ib\sqrt{-d}\text{PolyLog}\left(2,\frac{\sqrt{e}(1-icx)}{\sqrt{e}+ic\sqrt{-d}}\right)}{4e^{3/2}}-\frac{ib\sqrt{-d}\text{PolyLog}\left(2,\frac{\sqrt{e}(1+icx)}{\sqrt{e}+ic\sqrt{-d}}\right)}{4e^{3/2}}+\frac{ib\sqrt{-d}\text{PolyLog}\left(2,\frac{\sqrt{e}(}{c\sqrt{-}}\right.}{4e^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcTan[c*x]))/(d + e*x^2),x]
```



```
[Out] (a*x)/e + (b*x*ArcTan[c*x])/e - (a*Sqrt[d]*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/e^(
3/2) - ((I/4)*b*Sqrt[-d]*Log[1 + I*c*x]*Log[(c*(Sqrt[-d] - Sqrt[e]*x))/(c*S
qrt[-d] - I*Sqrt[e])])/e^(3/2) + ((I/4)*b*Sqrt[-d]*Log[1 - I*c*x]*Log[(c*(S
qrt[-d] - Sqrt[e]*x))/(c*Sqrt[-d] + I*Sqrt[e])])/e^(3/2) - ((I/4)*b*Sqrt[-d
]*Log[1 - I*c*x]*Log[(c*(Sqrt[-d] + Sqrt[e]*x))/(c*Sqrt[-d] - I*Sqrt[e])])/
e^(3/2) + ((I/4)*b*Sqrt[-d]*Log[1 + I*c*x]*Log[(c*(Sqrt[-d] + Sqrt[e]*x))/(
c*Sqrt[-d] + I*Sqrt[e])])/e^(3/2) - (b*Log[1 + c^2*x^2])/(2*c*e) + ((I/4)*b
*Sqrt[-d]*PolyLog[2, (Sqrt[e]*(I - c*x))/(c*Sqrt[-d] + I*Sqrt[e])])/e^(3/2)
```

```
 - ((I/4)*b*Sqrt[-d]*PolyLog[2, (Sqrt[e]*(1 - I*c*x))/(I*c*Sqrt[-d] + Sqrt[
e])])/e^(3/2) - ((I/4)*b*Sqrt[-d]*PolyLog[2, (Sqrt[e]*(1 + I*c*x))/(I*c*Sqr
t[-d] + Sqrt[e])])/e^(3/2) + ((I/4)*b*Sqrt[-d]*PolyLog[2, (Sqrt[e]*(I + c*x
))/(c*Sqrt[-d] + I*Sqrt[e])])/e^(3/2)
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4910

```
Int[(ArcTan[(c_.)*(x_)]*(b_.) + (a_))/((d_.) + (e_.)*(x_)^2), x_Symbol] :>
Dist[a, Int[1/(d + e*x^2), x], x] + Dist[b, Int[ArcTan[c*x]/(d + e*x^2), x]
, x] /; FreeQ[{a, b, c, d, e}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 4908

```
Int[ArcTan[(c_.)*(x_)]/((d_.) + (e_.)*(x_)^2), x_Symbol] :> Dist[I/2, Int[L
og[1 - I*c*x]/(d + e*x^2), x], x] - Dist[I/2, Int[Log[1 + I*c*x]/(d + e*x^2
), x], x] /; FreeQ[{c, d, e}, x]
```

Rule 2409

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
```

```
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{x^2\left(a+b\tan^{-1}(cx)\right)}{d+ex^2}\,dx = \frac{\int\left(a+b\tan^{-1}(cx)\right)dx}{e} - \frac{d\int\frac{a+b\tan^{-1}(cx)}{d+ex^2}\,dx}{e}$$

$$= \frac{ax}{e} + \frac{b\int\tan^{-1}(cx)\,dx}{e} - \frac{(ad)\int\frac{1}{d+ex^2}\,dx}{e} - \frac{(bd)\int\frac{\tan^{-1}(cx)}{d+ex^2}\,dx}{e}$$

$$= \frac{ax}{e} + \frac{bx\tan^{-1}(cx)}{e} - \frac{a\sqrt{d}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{3/2}} - \frac{(bc)\int\frac{x}{1+c^2x^2}\,dx}{e} - \frac{(ibd)\int\frac{\log(1-icx)}{d+ex^2}\,dx}{2e} + \frac{(ibd)\int}{}$$

$$= \frac{ax}{e} + \frac{bx\tan^{-1}(cx)}{e} - \frac{a\sqrt{d}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{3/2}} - \frac{b\log\left(1+c^2x^2\right)}{2ce} - \frac{(ibd)\int\left(\frac{\sqrt{-d}\log(1-icx)}{2d\left(\sqrt{-d}-\sqrt{e}x\right)} + \frac{\sqrt{-d}\log}{2d(\sqrt{-}}\right.}{2e}$$

$$= \frac{ax}{e} + \frac{bx\tan^{-1}(cx)}{e} - \frac{a\sqrt{d}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{3/2}} - \frac{b\log\left(1+c^2x^2\right)}{2ce} - \frac{\left(ib\sqrt{-d}\right)\int\frac{\log(1-icx)}{\sqrt{-d}-\sqrt{e}x}\,dx}{4e} - \frac{(ib}{}$$

$$= \frac{ax}{e} + \frac{bx\tan^{-1}(cx)}{e} - \frac{a\sqrt{d}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{3/2}} - \frac{ib\sqrt{-d}\log(1+icx)\log\left(\frac{c\left(\sqrt{-d}-\sqrt{e}x\right)}{c\sqrt{-d}-i\sqrt{e}}\right)}{4e^{3/2}} + \frac{ib\sqrt{-d}\log}{}$$

$$= \frac{ax}{e} + \frac{bx\tan^{-1}(cx)}{e} - \frac{a\sqrt{d}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{3/2}} - \frac{ib\sqrt{-d}\log(1+icx)\log\left(\frac{c\left(\sqrt{-d}-\sqrt{e}x\right)}{c\sqrt{-d}-i\sqrt{e}}\right)}{4e^{3/2}} + \frac{ib\sqrt{-d}\log}{}$$

$$= \frac{ax}{e} + \frac{bx\tan^{-1}(cx)}{e} - \frac{a\sqrt{d}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{3/2}} - \frac{ib\sqrt{-d}\log(1+icx)\log\left(\frac{c\left(\sqrt{-d}-\sqrt{e}x\right)}{c\sqrt{-d}-i\sqrt{e}}\right)}{4e^{3/2}} + \frac{ib\sqrt{-d}\log}{}$$

Mathematica [A] time = 3.38658, size = 776, normalized size = 1.4

$$b\left(\frac{c^2d\left(i\left(\text{PolyLog}\left(2,\frac{\left(2i\sqrt{-c^2de}+c^2d+e\right)\left(x\sqrt{-c^2de}+cd\right)}{\left(c^2d-e\right)\left(cd-x\sqrt{-c^2de}\right)}\right)-\text{PolyLog}\left(2,\frac{\left(-2i\sqrt{-c^2de}+c^2d+e\right)\left(x\sqrt{-c^2de}+cd\right)}{\left(c^2d-e\right)\left(cd-x\sqrt{-c^2de}\right)}\right)\right)-2\cos^{-1}\left(-\frac{c^2d+e}{c^2d-e}\right)\tanh^{-1}\left(\frac{cex}{\sqrt{-c^2de}}\right)-4\tan^{-1}(cx)\tanh^{-1}\left(\frac{c}{x\sqrt{-}}\right.\right.}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(a + b*ArcTan[c*x]))/(d + e*x^2),x]
```

```
[Out] (a*x)/e - (a*Sqrt[d]*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/e^(3/2) + (b*(4*c*x*ArcTa
n[c*x] - 2*Log[1 + c^2*x^2] + (c^2*d*(-4*ArcTan[c*x]*ArcTanh[(c*d)/(Sqrt[-(
c^2*d*e)]*x)] - 2*ArcCos[-((c^2*d + e)/(c^2*d - e))]*ArcTanh[(c*e*x)/Sqrt[-
```

```
(c^2*d*e)]] - (ArcCos[-((c^2*d + e)/(c^2*d - e))] - (2*I)*ArcTanh[(c*e*x)/S
qrt[-(c^2*d*e)]])*Log[(2*c*d*(I*e + Sqrt[-(c^2*d*e)])*(-I + c*x))/((c^2*d -
 e)*(-(c*d) + Sqrt[-(c^2*d*e)]*x))] - (ArcCos[-((c^2*d + e)/(c^2*d - e))] +
 (2*I)*ArcTanh[(c*e*x)/Sqrt[-(c^2*d*e)]])*Log[(2*c*d*((-I)*e + Sqrt[-(c^2*d
*e)])*(I + c*x))/((c^2*d - e)*(-(c*d) + Sqrt[-(c^2*d*e)]*x))] + (ArcCos[-((
c^2*d + e)/(c^2*d - e))] + (2*I)*ArcTanh[(c*d)/(Sqrt[-(c^2*d*e)]*x)] + (2*I
)*ArcTanh[(c*e*x)/Sqrt[-(c^2*d*e)]])*Log[(Sqrt[2]*Sqrt[-(c^2*d*e)])/(Sqrt[-
(c^2*d) + e]*E^(I*ArcTan[c*x])*Sqrt[-(c^2*d) - e + (-(c^2*d) + e)*Cos[2*Arc
Tan[c*x]]])] + (ArcCos[-((c^2*d + e)/(c^2*d - e))] - (2*I)*ArcTanh[(c*d)/(S
qrt[-(c^2*d*e)]*x)] - (2*I)*ArcTanh[(c*e*x)/Sqrt[-(c^2*d*e)]])*Log[(Sqrt[2]
*Sqrt[-(c^2*d*e)]*E^(I*ArcTan[c*x]))/(Sqrt[-(c^2*d) + e]*Sqrt[-(c^2*d) - e
+ (-(c^2*d) + e)*Cos[2*ArcTan[c*x]]])] + I*(-PolyLog[2, ((c^2*d + e - (2*I)
*Sqrt[-(c^2*d*e)])*(c*d + Sqrt[-(c^2*d*e)]*x))/((c^2*d - e)*(c*d - Sqrt[-(c
^2*d*e)]*x))] + PolyLog[2, ((c^2*d + e + (2*I)*Sqrt[-(c^2*d*e)])*(c*d + Sqr
t[-(c^2*d*e)]*x))/((c^2*d - e)*(c*d - Sqrt[-(c^2*d*e)]*x))])))/Sqrt[-(c^2*d
*e)]))/(4*c*e)
```

Maple [C] time = 0.501, size = 2409, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctan(c*x))/(e*x^2+d),x)
```

```
[Out] 1/2*b*(d*e)^(1/2)/e*arctanh(1/4*(2*(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)+2*c^2*
d+2*e)/c/(d*e)^(1/2))/(c^2*d-e)+1/8/c*b/(c^2*d-e)*ln((1+I*c*x)^4/(c^2*x^2+1
)^2*c^2*d+2*c^2*d*(1+I*c*x)^2/(c^2*x^2+1)-(1+I*c*x)^4/(c^2*x^2+1)^2*e+c^2*d
+2*(1+I*c*x)^2/(c^2*x^2+1)*e-e)+1/c*b/e*ln((1+I*c*x)^2/(c^2*x^2+1)+1)+2/c*b
/(c^2*d-e)*ln((1+I*c*x)/(c^2*x^2+1)^(1/2))-1/4/c*b/e*ln((1+I*c*x)^4/(c^2*x^
2+1)^2*c^2*d+2*c^2*d*(1+I*c*x)^2/(c^2*x^2+1)-(1+I*c*x)^4/(c^2*x^2+1)^2*e+c^
2*d+2*(1+I*c*x)^2/(c^2*x^2+1)*e-e)+3/4*b*(d*e)^(1/2)/e^2*arctanh(1/4*(2*(c^
2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)+2*c^2*d+2*e)/c/(d*e)^(1/2))-1/4*b*(d*e)^(1/2
)*arctanh(1/4*(2*(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)+2*c^2*d+2*e)/c/(d*e)^(1/
2))/(c^2*d-e)^2-a*d/e/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))+a*x/e+b*x*arctan(
c*x)/e-2*c*b/e*d/(c^2*d-e)*ln((1+I*c*x)/(c^2*x^2+1)^(1/2))-1/8*c^2*b*(d*e)^
(1/2)/e^3*d*arctanh(1/4*(2*(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)+2*c^2*d+2*e)/c
/(d*e)^(1/2))+1/8*c^5*b/e^2*d^3/(c^2*d-e)^2*ln((1+I*c*x)^4/(c^2*x^2+1)^2*c^
2*d+2*c^2*d*(1+I*c*x)^2/(c^2*x^2+1)-(1+I*c*x)^4/(c^2*x^2+1)^2*e+c^2*d+2*(1+
I*c*x)^2/(c^2*x^2+1)*e-e)+3/8/c^2*b*(d*e)^(1/2)/d/e*arctanh(1/4*(2*(c^2*d-e
)*(1+I*c*x)^2/(c^2*x^2+1)+2*c^2*d+2*e)/c/(d*e)^(1/2))-1/4*c*b/e*d/(c^2*d-e)
*ln((1+I*c*x)^4/(c^2*x^2+1)^2*c^2*d+2*c^2*d*(1+I*c*x)^2/(c^2*x^2+1)-(1+I*c*
```

```
x)^4/(c^2*x^2+1)^2*e+c^2*d+2*(1+I*c*x)^2/(c^2*x^2+1)*e-e)+1/8*c^3*b/e^2*d^2
/(c^2*d-e)*ln((1+I*c*x)^4/(c^2*x^2+1)^2*c^2*d+2*c^2*d*(1+I*c*x)^2/(c^2*x^2+
1)-(1+I*c*x)^4/(c^2*x^2+1)^2*e+c^2*d+2*(1+I*c*x)^2/(c^2*x^2+1)*e-e)+1/8*c^3
*b/e*d^2/(c^2*d-e)^2*ln((1+I*c*x)^4/(c^2*x^2+1)^2*c^2*d+2*c^2*d*(1+I*c*x)^2
/(c^2*x^2+1)-(1+I*c*x)^4/(c^2*x^2+1)^2*e+c^2*d+2*(1+I*c*x)^2/(c^2*x^2+1)*e-
e)+3/4/c^2*b*(d*e)^(1/2)/d*arctanh(1/4*(2*(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)
+2*c^2*d+2*e)/c/(d*e)^(1/2))/(c^2*d-e)+3/8/c*b*e/(c^2*d-e)^2*ln((1+I*c*x)^4
/(c^2*x^2+1)^2*c^2*d+2*c^2*d*(1+I*c*x)^2/(c^2*x^2+1)-(1+I*c*x)^4/(c^2*x^2+1
)^2*e+c^2*d+2*(1+I*c*x)^2/(c^2*x^2+1)*e-e)+I/c*b*arctan(c*x)/e+1/4*c*b/e^2*
d*sum((_R1^2*c^2*d-_R1^2*e+c^2*d+3*e)/(_R1^2*c^2*d-_R1^2*e+c^2*d+e)*(I*arct
an(c*x)*ln((_R1-(1+I*c*x)/(c^2*x^2+1)^(1/2))/_R1)+dilog((_R1-(1+I*c*x)/(c^2
*x^2+1)^(1/2))/_R1)),_R1=RootOf((c^2*d-e)*_Z^4+(2*c^2*d+2*e)*_Z^2+c^2*d-e))
-5/8*c*b*d/(c^2*d-e)^2*ln((1+I*c*x)^4/(c^2*x^2+1)^2*c^2*d+2*c^2*d*(1+I*c*x)
^2/(c^2*x^2+1)-(1+I*c*x)^4/(c^2*x^2+1)^2*e+c^2*d+2*(1+I*c*x)^2/(c^2*x^2+1)*
e-e)-1/4*c*b/e^2*d*ln((1+I*c*x)^4/(c^2*x^2+1)^2*c^2*d+2*c^2*d*(1+I*c*x)^2/(
c^2*x^2+1)-(1+I*c*x)^4/(c^2*x^2+1)^2*e+c^2*d+2*(1+I*c*x)^2/(c^2*x^2+1)*e-e)
-1/4*c*b/e^2*d*sum((_R1^2*c^2*d-_R1^2*e+c^2*d-e)/(_R1^2*c^2*d-_R1^2*e+c^2*d
+e)*(I*arctan(c*x)*ln((_R1-(1+I*c*x)/(c^2*x^2+1)^(1/2))/_R1)+dilog((_R1-(1+
I*c*x)/(c^2*x^2+1)^(1/2))/_R1)),_R1=RootOf((c^2*d-e)*_Z^4+(2*c^2*d+2*e)*_Z^
2+c^2*d-e))+1/8*c^6*b*(d*e)^(1/2)/e^3*d^3*arctanh(1/4*(2*(c^2*d-e)*(1+I*c*x
)^2/(c^2*x^2+1)+2*c^2*d+2*e)/c/(d*e)^(1/2))/(c^2*d-e)^2+3/8/c^2*b*(d*e)^(1/
2)/d*e*arctanh(1/4*(2*(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)+2*c^2*d+2*e)/c/(d*e
)^(1/2))/(c^2*d-e)^2-5/4*c^2*b*(d*e)^(1/2)/e^2*d*arctanh(1/4*(2*(c^2*d-e)*(
1+I*c*x)^2/(c^2*x^2+1)+2*c^2*d+2*e)/c/(d*e)^(1/2))/(c^2*d-e)+1/4*c^4*b*(d*e
)^(1/2)/e^2*d^2*arctanh(1/4*(2*(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)+2*c^2*d+2*
e)/c/(d*e)^(1/2))/(c^2*d-e)^2-1/2*c^2*b*(d*e)^(1/2)*d/e*arctanh(1/4*(2*(c^2
*d-e)*(1+I*c*x)^2/(c^2*x^2+1)+2*c^2*d+2*e)/c/(d*e)^(1/2))/(c^2*d-e)^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{bx^2\arctan(cx)+ax^2}{ex^2+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*x^2*arctan(c*x) + a*x^2)/(e*x^2 + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^2\,(a+b\,\text{atan}\,(cx))}{d+ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atan(c*x))/(e*x**2+d),x)
```

```
[Out] Integral(x**2*(a + b*atan(c*x))/(d + e*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(b\arctan(cx)+a)x^2}{ex^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))/(e*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)*x^2/(e*x^2 + d), x)
```

3.1156 $\int \frac{a+b\tan^{-1}(cx)}{d+ex^2}\,dx$

Optimal. Leaf size=517

$$\frac{ib\text{PolyLog}\left(2,\frac{\sqrt{e}(-cx+i)}{c\sqrt{-d}+i\sqrt{e}}\right)}{4\sqrt{-d}\sqrt{e}}-\frac{ib\text{PolyLog}\left(2,\frac{\sqrt{e}(1-icx)}{\sqrt{e}+ic\sqrt{-d}}\right)}{4\sqrt{-d}\sqrt{e}}-\frac{ib\text{PolyLog}\left(2,\frac{\sqrt{e}(1+icx)}{\sqrt{e}+ic\sqrt{-d}}\right)}{4\sqrt{-d}\sqrt{e}}+\frac{ib\text{PolyLog}\left(2,\frac{\sqrt{e}(cx+i)}{c\sqrt{-d}+i\sqrt{e}}\right)}{4\sqrt{-d}\sqrt{e}}+\frac{a\tan^{-1}}{\sqrt{d}\sqrt{}}$$

```
[Out] (a*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*Sqrt[e]) - ((I/4)*b*Log[1 + I*c*x]
*Log[(c*(Sqrt[-d] - Sqrt[e]*x))/(c*Sqrt[-d] - I*Sqrt[e])])/(Sqrt[-d]*Sqrt[e
]) + ((I/4)*b*Log[1 - I*c*x]*Log[(c*(Sqrt[-d] - Sqrt[e]*x))/(c*Sqrt[-d] + I
*Sqrt[e])])/(Sqrt[-d]*Sqrt[e]) - ((I/4)*b*Log[1 - I*c*x]*Log[(c*(Sqrt[-d] +
 Sqrt[e]*x))/(c*Sqrt[-d] - I*Sqrt[e])])/(Sqrt[-d]*Sqrt[e]) + ((I/4)*b*Log[1
 + I*c*x]*Log[(c*(Sqrt[-d] + Sqrt[e]*x))/(c*Sqrt[-d] + I*Sqrt[e])])/(Sqrt[-
d]*Sqrt[e]) + ((I/4)*b*PolyLog[2, (Sqrt[e]*(I - c*x))/(c*Sqrt[-d] + I*Sqrt[
e])])/(Sqrt[-d]*Sqrt[e]) - ((I/4)*b*PolyLog[2, (Sqrt[e]*(1 - I*c*x))/(I*c*S
qrt[-d] + Sqrt[e])])/(Sqrt[-d]*Sqrt[e]) - ((I/4)*b*PolyLog[2, (Sqrt[e]*(1 +
 I*c*x))/(I*c*Sqrt[-d] + Sqrt[e])])/(Sqrt[-d]*Sqrt[e]) + ((I/4)*b*PolyLog[2
, (Sqrt[e]*(I + c*x))/(c*Sqrt[-d] + I*Sqrt[e])])/(Sqrt[-d]*Sqrt[e])
```

Rubi [A] time = 0.405913, antiderivative size = 517, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {4910, 205, 4908, 2409, 2394, 2393, 2391}

$$\frac{ib\text{PolyLog}\left(2,\frac{\sqrt{e}(-cx+i)}{c\sqrt{-d}+i\sqrt{e}}\right)}{4\sqrt{-d}\sqrt{e}}-\frac{ib\text{PolyLog}\left(2,\frac{\sqrt{e}(1-icx)}{\sqrt{e}+ic\sqrt{-d}}\right)}{4\sqrt{-d}\sqrt{e}}-\frac{ib\text{PolyLog}\left(2,\frac{\sqrt{e}(1+icx)}{\sqrt{e}+ic\sqrt{-d}}\right)}{4\sqrt{-d}\sqrt{e}}+\frac{ib\text{PolyLog}\left(2,\frac{\sqrt{e}(cx+i)}{c\sqrt{-d}+i\sqrt{e}}\right)}{4\sqrt{-d}\sqrt{e}}+\frac{a\tan^{-1}}{\sqrt{d}\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])/(d + e*x^2),x]
```

```
[Out] (a*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*Sqrt[e]) - ((I/4)*b*Log[1 + I*c*x]
*Log[(c*(Sqrt[-d] - Sqrt[e]*x))/(c*Sqrt[-d] - I*Sqrt[e])])/(Sqrt[-d]*Sqrt[e
]) + ((I/4)*b*Log[1 - I*c*x]*Log[(c*(Sqrt[-d] - Sqrt[e]*x))/(c*Sqrt[-d] + I
*Sqrt[e])])/(Sqrt[-d]*Sqrt[e]) - ((I/4)*b*Log[1 - I*c*x]*Log[(c*(Sqrt[-d] +
 Sqrt[e]*x))/(c*Sqrt[-d] - I*Sqrt[e])])/(Sqrt[-d]*Sqrt[e]) + ((I/4)*b*Log[1
 + I*c*x]*Log[(c*(Sqrt[-d] + Sqrt[e]*x))/(c*Sqrt[-d] + I*Sqrt[e])])/(Sqrt[-
d]*Sqrt[e]) + ((I/4)*b*PolyLog[2, (Sqrt[e]*(I - c*x))/(c*Sqrt[-d] + I*Sqrt[
e])])/(Sqrt[-d]*Sqrt[e]) - ((I/4)*b*PolyLog[2, (Sqrt[e]*(1 - I*c*x))/(I*c*S
qrt[-d] + Sqrt[e])])/(Sqrt[-d]*Sqrt[e]) - ((I/4)*b*PolyLog[2, (Sqrt[e]*(1 +
```

```
 I*c*x))/(I*c*Sqrt[-d] + Sqrt[e])])/(Sqrt[-d]*Sqrt[e]) + ((I/4)*b*PolyLog[2
, (Sqrt[e]*(I + c*x))/(c*Sqrt[-d] + I*Sqrt[e])])/(Sqrt[-d]*Sqrt[e])
```

Rule 4910

```
Int[(ArcTan[(c_.)*(x_)]*(b_.) + (a_))/((d_.) + (e_.)*(x_)^2), x_Symbol] :>
Dist[a, Int[1/(d + e*x^2), x], x] + Dist[b, Int[ArcTan[c*x]/(d + e*x^2), x]
, x] /; FreeQ[{a, b, c, d, e}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 4908

```
Int[ArcTan[(c_.)*(x_)]/((d_.) + (e_.)*(x_)^2), x_Symbol] :> Dist[I/2, Int[L
og[1 - I*c*x]/(d + e*x^2), x], x] - Dist[I/2, Int[Log[1 + I*c*x]/(d + e*x^2
), x], x] /; FreeQ[{c, d, e}, x]
```

Rule 2409

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{a+b\tan^{-1}(cx)}{d+ex^2}\,dx = a\int \frac{1}{d+ex^2}\,dx + b\int \frac{\tan^{-1}(cx)}{d+ex^2}\,dx$$

$$= \frac{a\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}} + \frac{1}{2}(ib)\int \frac{\log(1-icx)}{d+ex^2}\,dx - \frac{1}{2}(ib)\int \frac{\log(1+icx)}{d+ex^2}\,dx$$

$$= \frac{a\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}} + \frac{1}{2}(ib)\int \left(\frac{\sqrt{-d}\log(1-icx)}{2d\left(\sqrt{-d}-\sqrt{e}x\right)} + \frac{\sqrt{-d}\log(1-icx)}{2d\left(\sqrt{-d}+\sqrt{e}x\right)}\right)dx - \frac{1}{2}(ib)\int \left(\frac{\sqrt{-d}\log(1+i}{2d\left(\sqrt{-d}-\sqrt{e}\right.}\right.$$

$$= \frac{a\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}} - \frac{(ib)\int \frac{\log(1-icx)}{\sqrt{-d}-\sqrt{e}x}\,dx}{4\sqrt{-d}} - \frac{(ib)\int \frac{\log(1-icx)}{\sqrt{-d}+\sqrt{e}x}\,dx}{4\sqrt{-d}} + \frac{(ib)\int \frac{\log(1+icx)}{\sqrt{-d}-\sqrt{e}x}\,dx}{4\sqrt{-d}} + \frac{(ib)\int \frac{\log(1+icx)}{\sqrt{-d}+\sqrt{e}x}}{4\sqrt{-d}}$$

$$= \frac{a\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}} - \frac{ib\log(1+icx)\log\left(\frac{c\left(\sqrt{-d}-\sqrt{e}x\right)}{c\sqrt{-d}-i\sqrt{e}}\right)}{4\sqrt{-d}\sqrt{e}} + \frac{ib\log(1-icx)\log\left(\frac{c\left(\sqrt{-d}-\sqrt{e}x\right)}{c\sqrt{-d}+i\sqrt{e}}\right)}{4\sqrt{-d}\sqrt{e}} - \frac{ib\log(1-}{}$$

$$= \frac{a\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}} - \frac{ib\log(1+icx)\log\left(\frac{c\left(\sqrt{-d}-\sqrt{e}x\right)}{c\sqrt{-d}-i\sqrt{e}}\right)}{4\sqrt{-d}\sqrt{e}} + \frac{ib\log(1-icx)\log\left(\frac{c\left(\sqrt{-d}-\sqrt{e}x\right)}{c\sqrt{-d}+i\sqrt{e}}\right)}{4\sqrt{-d}\sqrt{e}} - \frac{ib\log(1-}{}$$

$$= \frac{a\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}} - \frac{ib\log(1+icx)\log\left(\frac{c\left(\sqrt{-d}-\sqrt{e}x\right)}{c\sqrt{-d}-i\sqrt{e}}\right)}{4\sqrt{-d}\sqrt{e}} + \frac{ib\log(1-icx)\log\left(\frac{c\left(\sqrt{-d}-\sqrt{e}x\right)}{c\sqrt{-d}+i\sqrt{e}}\right)}{4\sqrt{-d}\sqrt{e}} - \frac{ib\log(1-}{}$$

Mathematica [A] time = 0.232943, size = 461, normalized size = 0.89

$$ib\sqrt{d}\text{PolyLog}\left(2,\frac{\sqrt{e}(-cx+i)}{c\sqrt{-d}+i\sqrt{e}}\right) - ib\sqrt{d}\text{PolyLog}\left(2,\frac{\sqrt{e}(1-icx)}{\sqrt{e}+ic\sqrt{-d}}\right) - ib\sqrt{d}\text{PolyLog}\left(2,\frac{\sqrt{e}(1+icx)}{\sqrt{e}+ic\sqrt{-d}}\right) + ib\sqrt{d}\text{PolyLog}\left(2,\frac{\sqrt{e}(cx+i)}{c\sqrt{-d}+i\sqrt{e}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])/(d + e*x^2),x]
```

```
[Out] (4*a*Sqrt[-d]*ArcTan[(Sqrt[e]*x)/Sqrt[d]] - I*b*Sqrt[d]*Log[1 + I*c*x]*Log[
(c*(Sqrt[-d] - Sqrt[e]*x))/(c*Sqrt[-d] - I*Sqrt[e])] + I*b*Sqrt[d]*Log[1 -
I*c*x]*Log[(c*(Sqrt[-d] - Sqrt[e]*x))/(c*Sqrt[-d] + I*Sqrt[e])] - I*b*Sqrt[
d]*Log[1 - I*c*x]*Log[(c*(Sqrt[-d] + Sqrt[e]*x))/(c*Sqrt[-d] - I*Sqrt[e])]
```

```
+ I*b*Sqrt[d]*Log[1 + I*c*x]*Log[(c*(Sqrt[-d] + Sqrt[e]*x))/(c*Sqrt[-d] + I
*Sqrt[e])] + I*b*Sqrt[d]*PolyLog[2, (Sqrt[e]*(I - c*x))/(c*Sqrt[-d] + I*Sqr
t[e])] - I*b*Sqrt[d]*PolyLog[2, (Sqrt[e]*(1 - I*c*x))/(I*c*Sqrt[-d] + Sqrt[
e])] - I*b*Sqrt[d]*PolyLog[2, (Sqrt[e]*(1 + I*c*x))/(I*c*Sqrt[-d] + Sqrt[e]
)] + I*b*Sqrt[d]*PolyLog[2, (Sqrt[e]*(I + c*x))/(c*Sqrt[-d] + I*Sqrt[e])])/
(4*Sqrt[-d^2]*Sqrt[e])
```

Maple [B] time = 0.219, size = 886, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/(e*x^2+d),x)
```

```
[Out] a/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))-I*c*b*ln(1-(c^2*d-e)*(1+I*c*x)^2/(c^2
*x^2+1)/(-c^2*d-2*(c^2*e*d)^(1/2)-e))*arctan(c*x)/(c^4*d^2-2*c^2*d*e+e^2)*(
c^2*e*d)^(1/2)+1/2*I/c*b*ln(1-(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d-2*(
c^2*e*d)^(1/2)-e))*arctan(c*x)/d/(c^4*d^2-2*c^2*d*e+e^2)*(c^2*e*d)^(1/2)*e-
1/2/c*b*(c^2*e*d)^(1/2)/e/d*arctan(c*x)^2-1/4/c*b*(c^2*e*d)^(1/2)/e/d*polyl
og(2,(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d+2*(c^2*e*d)^(1/2)-e))+1/2*c^
3*b/e/(c^4*d^2-2*c^2*d*e+e^2)*arctan(c*x)^2*(c^2*e*d)^(1/2)*d-c*b/(c^4*d^2-
2*c^2*d*e+e^2)*arctan(c*x)^2*(c^2*e*d)^(1/2)+1/2*I*c^3*b*ln(1-(c^2*d-e)*(1+
I*c*x)^2/(c^2*x^2+1)/(-c^2*d-2*(c^2*e*d)^(1/2)-e))*arctan(c*x)/e/(c^4*d^2-2
*c^2*d*e+e^2)*(c^2*e*d)^(1/2)*d-1/2*I/c*b*(c^2*e*d)^(1/2)/e/d*arctan(c*x)*l
n(1-(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d+2*(c^2*e*d)^(1/2)-e))+1/4*c^3
*b/e/(c^4*d^2-2*c^2*d*e+e^2)*polylog(2,(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-
c^2*d-2*(c^2*e*d)^(1/2)-e))*(c^2*e*d)^(1/2)*d-1/2*c*b/(c^4*d^2-2*c^2*d*e+e^
2)*polylog(2,(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d-2*(c^2*e*d)^(1/2)-e)
)*(c^2*e*d)^(1/2)+1/2/c*b/d/(c^4*d^2-2*c^2*d*e+e^2)*arctan(c*x)^2*(c^2*e*d)
^(1/2)*e+1/4/c*b/d/(c^4*d^2-2*c^2*d*e+e^2)*polylog(2,(c^2*d-e)*(1+I*c*x)^2/
(c^2*x^2+1)/(-c^2*d-2*(c^2*e*d)^(1/2)-e))*(c^2*e*d)^(1/2)*e
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b\arctan(cx)+a}{ex^2+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*arctan(c*x) + a)/(e*x^2 + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{a+b\,\text{atan}(cx)}{d+ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/(e*x**2+d),x)
```

```
[Out] Integral((a + b*atan(c*x))/(d + e*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{b\arctan(cx)+a}{ex^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/(e*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)/(e*x^2 + d), x)
```

3.1157 $\int \frac{a+b\tan^{-1}(cx)}{x^2(d+ex^2)}\,dx$

Optimal. Leaf size=561

$$\frac{ib\sqrt{e}\text{PolyLog}\left(2,\frac{\sqrt{e}(-cx+i)}{c\sqrt{-d}+i\sqrt{e}}\right)}{4(-d)^{3/2}}-\frac{ib\sqrt{e}\text{PolyLog}\left(2,\frac{\sqrt{e}(1-icx)}{\sqrt{e}+ic\sqrt{-d}}\right)}{4(-d)^{3/2}}-\frac{ib\sqrt{e}\text{PolyLog}\left(2,\frac{\sqrt{e}(1+icx)}{\sqrt{e}+ic\sqrt{-d}}\right)}{4(-d)^{3/2}}+\frac{ib\sqrt{e}\text{PolyLog}\left(2,\frac{\sqrt{e}(cx+i)}{c\sqrt{-d}+i\sqrt{e}}\right)}{4(-d)^{3/2}}$$

```
[Out] -((a + b*ArcTan[c*x])/(d*x)) - (a*Sqrt[e]*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/d^(3
/2) + (b*c*Log[x])/d - ((I/4)*b*Sqrt[e]*Log[1 + I*c*x]*Log[(c*(Sqrt[-d] - S
qrt[e]*x))/(c*Sqrt[-d] - I*Sqrt[e])])/(-d)^(3/2) + ((I/4)*b*Sqrt[e]*Log[1 -
 I*c*x]*Log[(c*(Sqrt[-d] - Sqrt[e]*x))/(c*Sqrt[-d] + I*Sqrt[e])])/(-d)^(3/2
) - ((I/4)*b*Sqrt[e]*Log[1 - I*c*x]*Log[(c*(Sqrt[-d] + Sqrt[e]*x))/(c*Sqrt[
-d] - I*Sqrt[e])])/(-d)^(3/2) + ((I/4)*b*Sqrt[e]*Log[1 + I*c*x]*Log[(c*(Sqr
t[-d] + Sqrt[e]*x))/(c*Sqrt[-d] + I*Sqrt[e])])/(-d)^(3/2) - (b*c*Log[1 + c^
2*x^2])/(2*d) + ((I/4)*b*Sqrt[e]*PolyLog[2, (Sqrt[e]*(I - c*x))/(c*Sqrt[-d]
 + I*Sqrt[e])])/(-d)^(3/2) - ((I/4)*b*Sqrt[e]*PolyLog[2, (Sqrt[e]*(1 - I*c*
x))/(I*c*Sqrt[-d] + Sqrt[e])])/(-d)^(3/2) - ((I/4)*b*Sqrt[e]*PolyLog[2, (Sq
rt[e]*(1 + I*c*x))/(I*c*Sqrt[-d] + Sqrt[e])])/(-d)^(3/2) + ((I/4)*b*Sqrt[e]
*PolyLog[2, (Sqrt[e]*(I + c*x))/(c*Sqrt[-d] + I*Sqrt[e])])/(-d)^(3/2)
```

Rubi [A] time = 0.528472, antiderivative size = 561, normalized size of antiderivative = 1., number of steps used = 25, number of rules used = 13, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.619, Rules used = {4918, 4852, 266, 36, 29, 31, 4910, 205, 4908, 2409, 2394, 2393, 2391}

$$\frac{ib\sqrt{e}\text{PolyLog}\left(2,\frac{\sqrt{e}(-cx+i)}{c\sqrt{-d}+i\sqrt{e}}\right)}{4(-d)^{3/2}}-\frac{ib\sqrt{e}\text{PolyLog}\left(2,\frac{\sqrt{e}(1-icx)}{\sqrt{e}+ic\sqrt{-d}}\right)}{4(-d)^{3/2}}-\frac{ib\sqrt{e}\text{PolyLog}\left(2,\frac{\sqrt{e}(1+icx)}{\sqrt{e}+ic\sqrt{-d}}\right)}{4(-d)^{3/2}}+\frac{ib\sqrt{e}\text{PolyLog}\left(2,\frac{\sqrt{e}(cx+i)}{c\sqrt{-d}+i\sqrt{e}}\right)}{4(-d)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])/(x^2*(d + e*x^2)),x]
```

```
[Out] -((a + b*ArcTan[c*x])/(d*x)) - (a*Sqrt[e]*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/d^(3
/2) + (b*c*Log[x])/d - ((I/4)*b*Sqrt[e]*Log[1 + I*c*x]*Log[(c*(Sqrt[-d] - S
qrt[e]*x))/(c*Sqrt[-d] - I*Sqrt[e])])/(-d)^(3/2) + ((I/4)*b*Sqrt[e]*Log[1 -
 I*c*x]*Log[(c*(Sqrt[-d] - Sqrt[e]*x))/(c*Sqrt[-d] + I*Sqrt[e])])/(-d)^(3/2
) - ((I/4)*b*Sqrt[e]*Log[1 - I*c*x]*Log[(c*(Sqrt[-d] + Sqrt[e]*x))/(c*Sqrt[
-d] - I*Sqrt[e])])/(-d)^(3/2) + ((I/4)*b*Sqrt[e]*Log[1 + I*c*x]*Log[(c*(Sqr
t[-d] + Sqrt[e]*x))/(c*Sqrt[-d] + I*Sqrt[e])])/(-d)^(3/2) - (b*c*Log[1 + c^
2*x^2])/(2*d) + ((I/4)*b*Sqrt[e]*PolyLog[2, (Sqrt[e]*(I - c*x))/(c*Sqrt[-d]
```

```
 + I*Sqrt[e])])/(-d)^(3/2) - ((I/4)*b*Sqrt[e]*PolyLog[2, (Sqrt[e]*(1 - I*c*
x))/(I*c*Sqrt[-d] + Sqrt[e])])/(-d)^(3/2) - ((I/4)*b*Sqrt[e]*PolyLog[2, (Sq
rt[e]*(1 + I*c*x))/(I*c*Sqrt[-d] + Sqrt[e])])/(-d)^(3/2) + ((I/4)*b*Sqrt[e]
*PolyLog[2, (Sqrt[e]*(I + c*x))/(c*Sqrt[-d] + I*Sqrt[e])])/(-d)^(3/2)
```

Rule 4918

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/(d + e*x^2),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 4910

```
Int[(ArcTan[(c_.)*(x_)]*(b_.) + (a_))/((d_.) + (e_.)*(x_)^2), x_Symbol] :>
Dist[a, Int[1/(d + e*x^2), x], x] + Dist[b, Int[ArcTan[c*x]/(d + e*x^2), x]
```

```
, x] /; FreeQ[{a, b, c, d, e}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 4908

```
Int[ArcTan[(c_.)*(x_)]/((d_.) + (e_.)*(x_)^2), x_Symbol] :> Dist[I/2, Int[L
og[1 - I*c*x]/(d + e*x^2), x], x] - Dist[I/2, Int[Log[1 + I*c*x]/(d + e*x^2
), x], x] /; FreeQ[{c, d, e}, x]
```

Rule 2409

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{a+b\tan^{-1}(cx)}{x^2\left(d+ex^2\right)}\,dx = \frac{\int \frac{a+b\tan^{-1}(cx)}{x^2}\,dx}{d} - \frac{e\int \frac{a+b\tan^{-1}(cx)}{d+ex^2}\,dx}{d}$$

$$= -\frac{a+b\tan^{-1}(cx)}{dx} + \frac{(bc)\int \frac{1}{x\left(1+c^2x^2\right)}\,dx}{d} - \frac{(ae)\int \frac{1}{d+ex^2}\,dx}{d} - \frac{(be)\int \frac{\tan^{-1}(cx)}{d+ex^2}\,dx}{d}$$

$$= -\frac{a+b\tan^{-1}(cx)}{dx} - \frac{a\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{3/2}} + \frac{(bc)\,\text{Subst}\left(\int \frac{1}{x\left(1+c^2x\right)}\,dx, x, x^2\right)}{2d} - \frac{(ibe)\int \frac{\log(1-icx)}{d+ex^2}\,dx}{2d}$$

$$= -\frac{a+b\tan^{-1}(cx)}{dx} - \frac{a\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{3/2}} + \frac{(bc)\,\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^2\right)}{2d} - \frac{\left(bc^3\right)\text{Subst}\left(\int \frac{1}{1+c^2x}\,dx, x\right.}{2d}$$

$$= -\frac{a+b\tan^{-1}(cx)}{dx} - \frac{a\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{3/2}} + \frac{bc\log(x)}{d} - \frac{bc\log\left(1+c^2x^2\right)}{2d} - \frac{(ibe)\int \frac{\log(1-icx)}{\sqrt{-d}-\sqrt{e}x}\,dx}{4(-d)^{3/2}} - $$

$$= -\frac{a+b\tan^{-1}(cx)}{dx} - \frac{a\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{3/2}} + \frac{bc\log(x)}{d} - \frac{ib\sqrt{e}\log(1+icx)\log\left(\frac{c\left(\sqrt{-d}-\sqrt{e}x\right)}{c\sqrt{-d}-i\sqrt{e}}\right)}{4(-d)^{3/2}} + $$

$$= -\frac{a+b\tan^{-1}(cx)}{dx} - \frac{a\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{3/2}} + \frac{bc\log(x)}{d} - \frac{ib\sqrt{e}\log(1+icx)\log\left(\frac{c\left(\sqrt{-d}-\sqrt{e}x\right)}{c\sqrt{-d}-i\sqrt{e}}\right)}{4(-d)^{3/2}} + $$

$$= -\frac{a+b\tan^{-1}(cx)}{dx} - \frac{a\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{3/2}} + \frac{bc\log(x)}{d} - \frac{ib\sqrt{e}\log(1+icx)\log\left(\frac{c\left(\sqrt{-d}-\sqrt{e}x\right)}{c\sqrt{-d}-i\sqrt{e}}\right)}{4(-d)^{3/2}} + $$

Mathematica [A] time = 0.776603, size = 468, normalized size = 0.83

$$-\frac{\sqrt{e}\left(ib\sqrt{d}\left(\text{PolyLog}\left(2,\frac{\sqrt{e}(-cx+i)}{c\sqrt{-d}+i\sqrt{e}}\right)+\log(1+icx)\log\left(\frac{c\left(\sqrt{-d}+\sqrt{e}x\right)}{c\sqrt{-d}+i\sqrt{e}}\right)\right)-ib\sqrt{d}\left(\text{PolyLog}\left(2,\frac{\sqrt{e}(1-icx)}{\sqrt{e}+ic\sqrt{-d}}\right)+\log(1-icx)\log\left(\frac{c\left(\sqrt{-d}+\sqrt{e}x\right)}{c\sqrt{-d}-i\sqrt{e}}\right)\right)-ib\sqrt{d}\left(\text{PolyLog}\left(2,\frac{\sqrt{e}(1+icx)}{\sqrt{e}+ic\sqrt{-d}}\right)+\cdots\right.}{4\sqrt{-d^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c*x])/(x^2*(d + e*x^2)),x]
```

```
[Out] (-((a + b*ArcTan[c*x])/x) + b*c*Log[x] - (b*c*Log[1 + c^2*x^2])/2 - (Sqrt[e
]*(4*a*Sqrt[-d]*ArcTan[(Sqrt[e]*x)/Sqrt[d]] + I*b*Sqrt[d]*(Log[1 + I*c*x]*L
og[(c*(Sqrt[-d] + Sqrt[e]*x))/(c*Sqrt[-d] + I*Sqrt[e])] + PolyLog[2, (Sqrt[
e]*(I - c*x))/(c*Sqrt[-d] + I*Sqrt[e])]) - I*b*Sqrt[d]*(Log[1 - I*c*x]*Log[
```

```
(c*(Sqrt[-d] + Sqrt[e]*x))/(c*Sqrt[-d] - I*Sqrt[e])] + PolyLog[2, (Sqrt[e]*
(1 - I*c*x))/(I*c*Sqrt[-d] + Sqrt[e])]) - I*b*Sqrt[d]*(Log[1 + I*c*x]*Log[(
c*(Sqrt[-d] - Sqrt[e]*x))/(c*Sqrt[-d] - I*Sqrt[e])] + PolyLog[2, (Sqrt[e]*(
1 + I*c*x))/(I*c*Sqrt[-d] + Sqrt[e])]) + I*b*Sqrt[d]*(Log[1 - I*c*x]*Log[(c
*(Sqrt[-d] - Sqrt[e]*x))/(c*Sqrt[-d] + I*Sqrt[e])] + PolyLog[2, (Sqrt[e]*(I
 + c*x))/(c*Sqrt[-d] + I*Sqrt[e])])))/(4*Sqrt[-d^2]))/d
```

Maple [C] time = 0.453, size = 2439, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/x^2/(e*x^2+d),x)
```

```
[Out] c*b/d*ln((1+I*c*x)/(c^2*x^2+1)^(1/2)-1)-1/8*b*c^3/(c^2*d-e)*ln((1+I*c*x)^4/
(c^2*x^2+1)^2*c^2*d+2*c^2*d*(1+I*c*x)^2/(c^2*x^2+1)-(1+I*c*x)^4/(c^2*x^2+1)
^2*e+c^2*d+2*(1+I*c*x)^2/(c^2*x^2+1)*e-e)-2*b*c^3/(c^2*d-e)*ln((1+I*c*x)/(c
^2*x^2+1)^(1/2))+c*b/d*ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))-1/4*c*b/d*ln((1+I*
c*x)^4/(c^2*x^2+1)^2*c^2*d+2*c^2*d*(1+I*c*x)^2/(c^2*x^2+1)-(1+I*c*x)^4/(c^2
*x^2+1)^2*e+c^2*d+2*(1+I*c*x)^2/(c^2*x^2+1)*e-e)+3/4*b*(d*e)^(1/2)/d^2*arct
anh(1/4*(2*(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)+2*c^2*d+2*e)/c/(d*e)^(1/2))-b*
arctan(c*x)/x/d+1/4*b/c/d^2*e*sum((_R1^2*c^2*d-_R1^2*e+3*c^2*d+e)/(_R1^2*c^
2*d-_R1^2*e+c^2*d+e)*(I*arctan(c*x)*ln((_R1-(1+I*c*x)/(c^2*x^2+1)^(1/2))/_R
1)+dilog((_R1-(1+I*c*x)/(c^2*x^2+1)^(1/2))/_R1)),_R1=RootOf((c^2*d-e)*_Z^4+
(2*c^2*d+2*e)*_Z^2+c^2*d-e))-1/4*b/c/d^2*e*ln((1+I*c*x)^4/(c^2*x^2+1)^2*c^2
*d+2*c^2*d*(1+I*c*x)^2/(c^2*x^2+1)-(1+I*c*x)^4/(c^2*x^2+1)^2*e+c^2*d+2*(1+I
*c*x)^2/(c^2*x^2+1)*e-e)-1/4*b*c^4*(d*e)^(1/2)*arctanh(1/4*(2*(c^2*d-e)*(1+
I*c*x)^2/(c^2*x^2+1)+2*c^2*d+2*e)/c/(d*e)^(1/2))/(c^2*d-e)^2-5/8*b*c^3*e/(c
^2*d-e)^2*ln((1+I*c*x)^4/(c^2*x^2+1)^2*c^2*d+2*c^2*d*(1+I*c*x)^2/(c^2*x^2+1
)-(1+I*c*x)^4/(c^2*x^2+1)^2*e+c^2*d+2*(1+I*c*x)^2/(c^2*x^2+1)*e-e)+3/8*b*c^
5*d/(c^2*d-e)^2*ln((1+I*c*x)^4/(c^2*x^2+1)^2*c^2*d+2*c^2*d*(1+I*c*x)^2/(c^2
*x^2+1)-(1+I*c*x)^4/(c^2*x^2+1)^2*e+c^2*d+2*(1+I*c*x)^2/(c^2*x^2+1)*e-e)+I*
c*b*arctan(c*x)/d+2*c*b/d*e/(c^2*d-e)*ln((1+I*c*x)/(c^2*x^2+1)^(1/2))+1/8*c
*b/d*e^2/(c^2*d-e)^2*ln((1+I*c*x)^4/(c^2*x^2+1)^2*c^2*d+2*c^2*d*(1+I*c*x)^2
/(c^2*x^2+1)-(1+I*c*x)^4/(c^2*x^2+1)^2*e+c^2*d+2*(1+I*c*x)^2/(c^2*x^2+1)*e-
e)+1/4*c*b/d*e/(c^2*d-e)*ln((1+I*c*x)^4/(c^2*x^2+1)^2*c^2*d+2*c^2*d*(1+I*c*
x)^2/(c^2*x^2+1)-(1+I*c*x)^4/(c^2*x^2+1)^2*e+c^2*d+2*(1+I*c*x)^2/(c^2*x^2+1
)*e-e)+1/8*b/c^2*(d*e)^(1/2)/d^3*e^3*arctanh(1/4*(2*(c^2*d-e)*(1+I*c*x)^2/(
c^2*x^2+1)+2*c^2*d+2*e)/c/(d*e)^(1/2))/(c^2*d-e)^2+3/8*b*c^6*(d*e)^(1/2)*d/
e*arctanh(1/4*(2*(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)+2*c^2*d+2*e)/c/(d*e)^(1/
2))/(c^2*d-e)^2-1/2*b*c^2*(d*e)^(1/2)/d*e*arctanh(1/4*(2*(c^2*d-e)*(1+I*c*x
```

```
)^2/(c^2*x^2+1)+2*c^2*d+2*e)/c/(d*e)^(1/2))/(c^2*d-e)^2-a*e/d/(d*e)^(1/2)*a
rctan(e*x/(d*e)^(1/2))-1/4*b/c/d^2*e*sum((_R1^2*c^2*d-_R1^2*e-c^2*d+e)/(_R1
^2*c^2*d-_R1^2*e+c^2*d+e)*(I*arctan(c*x)*ln((_R1-(1+I*c*x)/(c^2*x^2+1)^(1/2
))/_R1)+dilog((_R1-(1+I*c*x)/(c^2*x^2+1)^(1/2))/_R1)),_R1=RootOf((c^2*d-e)*
_Z^4+(2*c^2*d+2*e)*_Z^2+c^2*d-e))-a/d/x-1/8*b/c/d^2*e^2/(c^2*d-e)*ln((1+I*c
*x)^4/(c^2*x^2+1)^2*c^2*d+2*c^2*d*(1+I*c*x)^2/(c^2*x^2+1)-(1+I*c*x)^4/(c^2*
x^2+1)^2*e+c^2*d+2*(1+I*c*x)^2/(c^2*x^2+1)*e-e)+1/8*b/c/d^2*e^3/(c^2*d-e)^2
*ln((1+I*c*x)^4/(c^2*x^2+1)^2*c^2*d+2*c^2*d*(1+I*c*x)^2/(c^2*x^2+1)-(1+I*c*
x)^4/(c^2*x^2+1)^2*e+c^2*d+2*(1+I*c*x)^2/(c^2*x^2+1)*e-e)-3/4*b*c^4*(d*e)^(
1/2)/e*arctanh(1/4*(2*(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)+2*c^2*d+2*e)/c/(d*e
)^(1/2))/(c^2*d-e)-1/2*b*c^2*(d*e)^(1/2)/d*arctanh(1/4*(2*(c^2*d-e)*(1+I*c*
x)^2/(c^2*x^2+1)+2*c^2*d+2*e)/c/(d*e)^(1/2))/(c^2*d-e)+3/8*b*c^2*(d*e)^(1/2
)/d/e*arctanh(1/4*(2*(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)+2*c^2*d+2*e)/c/(d*e)
^(1/2))-1/8*b/c^2*(d*e)^(1/2)/d^3*e*arctanh(1/4*(2*(c^2*d-e)*(1+I*c*x)^2/(c
^2*x^2+1)+2*c^2*d+2*e)/c/(d*e)^(1/2))+1/4*b*(d*e)^(1/2)/d^2*e^2*arctanh(1/4
*(2*(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)+2*c^2*d+2*e)/c/(d*e)^(1/2))/(c^2*d-e)
^2+5/4*b*(d*e)^(1/2)/d^2*e*arctanh(1/4*(2*(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)
+2*c^2*d+2*e)/c/(d*e)^(1/2))/(c^2*d-e)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^2/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b\arctan(cx)+a}{ex^4+dx^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^2/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*arctan(c*x) + a)/(e*x^4 + d*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/x**2/(e*x**2+d),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \arctan (cx) + a}{(ex^2 + d)x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^2/(e*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)/((e*x^2 + d)*x^2), x)
```

3.1158 $\int \frac{x^3\left(a+b\tan^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$

Optimal. Leaf size=403

$$-\frac{ib\text{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{(1-icx)\left(c\sqrt{-d}-i\sqrt{e}\right)}\right)}{4e^2}-\frac{ib\text{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-d}+\sqrt{e}x\right)}{(1-icx)\left(c\sqrt{-d}+i\sqrt{e}\right)}\right)}{4e^2}+\frac{ib\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2e^2}+\frac{d\left(a+b\tan^{-}\right.}{2e^2\left(d+e\right.}$$

```
[Out] -(b*c^2*d*ArcTan[c*x])/(2*(c^2*d - e)*e^2) + (d*(a + b*ArcTan[c*x]))/(2*e^2
*(d + e*x^2)) + (b*c*Sqrt[d]*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*(c^2*d - e)*e^
(3/2)) - ((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e^2 + ((a + b*ArcTan[c*x]
)*Log[(2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])
/(2*e^2) + ((a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-
d] + I*Sqrt[e])*(1 - I*c*x))])/(2*e^2) + ((I/2)*b*PolyLog[2, 1 - 2/(1 - I*c
*x)])/e^2 - ((I/4)*b*PolyLog[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-
d] - I*Sqrt[e])*(1 - I*c*x))])/e^2 - ((I/4)*b*PolyLog[2, 1 - (2*c*(Sqrt[-d]
 + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/e^2
```

Rubi [A] time = 0.448857, antiderivative size = 403, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4980, 4974, 391, 203, 205, 4856, 2402, 2315, 2447}

$$-\frac{ib\text{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{(1-icx)\left(c\sqrt{-d}-i\sqrt{e}\right)}\right)}{4e^2}-\frac{ib\text{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-d}+\sqrt{e}x\right)}{(1-icx)\left(c\sqrt{-d}+i\sqrt{e}\right)}\right)}{4e^2}+\frac{ib\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2e^2}+\frac{d\left(a+b\tan^{-}\right.}{2e^2\left(d+e\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcTan[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] -(b*c^2*d*ArcTan[c*x])/(2*(c^2*d - e)*e^2) + (d*(a + b*ArcTan[c*x]))/(2*e^2
*(d + e*x^2)) + (b*c*Sqrt[d]*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*(c^2*d - e)*e^
(3/2)) - ((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e^2 + ((a + b*ArcTan[c*x]
)*Log[(2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])
/(2*e^2) + ((a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-
d] + I*Sqrt[e])*(1 - I*c*x))])/(2*e^2) + ((I/2)*b*PolyLog[2, 1 - 2/(1 - I*c
*x)])/e^2 - ((I/4)*b*PolyLog[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-
d] - I*Sqrt[e])*(1 - I*c*x))])/e^2 - ((I/4)*b*PolyLog[2, 1 - (2*c*(Sqrt[-d]
 + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/e^2
```

Rule 4980

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

Rule 4974

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]))/(2*e*(q + 1)), x
] - Dist[(b*c)/(2*e*(q + 1)), Int[(d + e*x^2)^(q + 1)/(1 + c^2*x^2), x], x]
 /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 391

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
```

```
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{x^3\left(a+b\tan^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx = \int\left(-\frac{dx\left(a+b\tan^{-1}(cx)\right)}{e\left(d+ex^2\right)^2}+\frac{x\left(a+b\tan^{-1}(cx)\right)}{e\left(d+ex^2\right)}\right)dx$$

$$= \frac{\int \frac{x\left(a+b\tan^{-1}(cx)\right)}{d+ex^2}\,dx}{e} - \frac{d\int \frac{x\left(a+b\tan^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx}{e}$$

$$= \frac{d\left(a+b\tan^{-1}(cx)\right)}{2e^2\left(d+ex^2\right)} - \frac{(bcd)\int \frac{1}{\left(1+c^2x^2\right)\left(d+ex^2\right)}\,dx}{2e^2} + \frac{\int\left(-\frac{a+b\tan^{-1}(cx)}{2\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{a+b\tan^{-1}(cx)}{2\sqrt{e}\left(\sqrt{-d}+\sqrt{e}x\right)}\right)dx}{e}$$

$$= \frac{d\left(a+b\tan^{-1}(cx)\right)}{2e^2\left(d+ex^2\right)} - \frac{\left(bc^3d\right)\int \frac{1}{1+c^2x^2}\,dx}{2\left(c^2d-e\right)e^2} - \frac{\int \frac{a+b\tan^{-1}(cx)}{\sqrt{-d}-\sqrt{e}x}\,dx}{2e^{3/2}} + \frac{\int \frac{a+b\tan^{-1}(cx)}{\sqrt{-d}+\sqrt{e}x}\,dx}{2e^{3/2}} + \frac{(bcd)\int}{2\left(c^2d\right.}$$

$$= -\frac{bc^2d\tan^{-1}(cx)}{2\left(c^2d-e\right)e^2} + \frac{d\left(a+b\tan^{-1}(cx)\right)}{2e^2\left(d+ex^2\right)} + \frac{bc\sqrt{d}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2\left(c^2d-e\right)e^{3/2}} - \frac{\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2}{1-icx}\right.}{e^2}$$

$$= -\frac{bc^2d\tan^{-1}(cx)}{2\left(c^2d-e\right)e^2} + \frac{d\left(a+b\tan^{-1}(cx)\right)}{2e^2\left(d+ex^2\right)} + \frac{bc\sqrt{d}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2\left(c^2d-e\right)e^{3/2}} - \frac{\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2}{1-icx}\right.}{e^2}$$

$$= -\frac{bc^2d\tan^{-1}(cx)}{2\left(c^2d-e\right)e^2} + \frac{d\left(a+b\tan^{-1}(cx)\right)}{2e^2\left(d+ex^2\right)} + \frac{bc\sqrt{d}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2\left(c^2d-e\right)e^{3/2}} - \frac{\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2}{1-icx}\right.}{e^2}$$

Mathematica [A] time = 7.91023, size = 522, normalized size = 1.3

$$2a\left(\frac{d}{d+ex^2}+\log\left(d+ex^2\right)\right)+b\left(-i\text{PolyLog}\left(2,\frac{c\left(\sqrt{d}-i\sqrt{e}x\right)}{c\sqrt{d}-\sqrt{e}}\right)+i\text{PolyLog}\left(2,\frac{c\left(\sqrt{d}-i\sqrt{e}x\right)}{c\sqrt{d}+\sqrt{e}}\right)+i\text{PolyLog}\left(2,\frac{c\left(\sqrt{d}+i\sqrt{e}x\right)}{c\sqrt{d}-\sqrt{e}}\right)-i\text{Poly}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcTan[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] (2*a*(d/(d + e*x^2) + Log[d + e*x^2]) + b*((-2*c^2*d*ArcTan[c*x])/(c^2*d -
e) + (2*d*ArcTan[c*x])/(d + e*x^2) + (2*c*Sqrt[d]*Sqrt[e]*ArcTan[(Sqrt[e]*x
)/Sqrt[d]])/(c^2*d - e) + 2*ArcTan[c*x]*Log[((-I)*Sqrt[d])/Sqrt[e] + x] + 2
*ArcTan[c*x]*Log[(I*Sqrt[d])/Sqrt[e] + x] + I*Log[((-I)*Sqrt[d])/Sqrt[e] +
x]*Log[(Sqrt[e]*(-1 - I*c*x))/(c*Sqrt[d] - Sqrt[e])] - I*Log[((-I)*Sqrt[d])
/Sqrt[e] + x]*Log[(Sqrt[e]*(1 - I*c*x))/(c*Sqrt[d] + Sqrt[e])] - I*Log[(I*S
qrt[d])/Sqrt[e] + x]*Log[(Sqrt[e]*(-1 + I*c*x))/(c*Sqrt[d] - Sqrt[e])] + I*
Log[(I*Sqrt[d])/Sqrt[e] + x]*Log[(Sqrt[e]*(1 + I*c*x))/(c*Sqrt[d] + Sqrt[e]
)] - I*PolyLog[2, (c*(Sqrt[d] - I*Sqrt[e]*x))/(c*Sqrt[d] - Sqrt[e])] + I*Po
lyLog[2, (c*(Sqrt[d] - I*Sqrt[e]*x))/(c*Sqrt[d] + Sqrt[e])] + I*PolyLog[2,
(c*(Sqrt[d] + I*Sqrt[e]*x))/(c*Sqrt[d] - Sqrt[e])] - I*PolyLog[2, (c*(Sqrt[
d] + I*Sqrt[e]*x))/(c*Sqrt[d] + Sqrt[e])]))/(4*e^2)
```

Maple [C] time = 0.203, size = 760, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctan(c*x))/(e*x^2+d)^2,x)
```

```
[Out] 1/2*a/e^2*ln(c^2*e*x^2+c^2*d)+1/2*c^2*a/e^2*d/(c^2*e*x^2+c^2*d)+1/2*b*arcta
n(c*x)/e^2*ln(c^2*e*x^2+c^2*d)+1/2*c^2*b*arctan(c*x)/e^2*d/(c^2*e*x^2+c^2*d
)+1/4*I*b/e^2*ln(c*x+I)*ln((RootOf(e*_Z^2-2*I*_Z*e+c^2*d-e,index=2)-c*x-I)/
RootOf(e*_Z^2-2*I*_Z*e+c^2*d-e,index=2))-1/4*I*b/e^2*dilog((RootOf(e*_Z^2+2
*I*_Z*e+c^2*d-e,index=1)-c*x+I)/RootOf(e*_Z^2+2*I*_Z*e+c^2*d-e,index=1))-1/
4*I*b/e^2*ln(c*x-I)*ln((RootOf(e*_Z^2+2*I*_Z*e+c^2*d-e,index=2)-c*x+I)/Root
Of(e*_Z^2+2*I*_Z*e+c^2*d-e,index=2))-1/4*I*b/e^2*ln(c*x-I)*ln((RootOf(e*_Z^
2+2*I*_Z*e+c^2*d-e,index=1)-c*x+I)/RootOf(e*_Z^2+2*I*_Z*e+c^2*d-e,index=1))
+1/4*I*b/e^2*dilog((RootOf(e*_Z^2-2*I*_Z*e+c^2*d-e,index=1)-c*x-I)/RootOf(e
*_Z^2-2*I*_Z*e+c^2*d-e,index=1))-1/4*I*b/e^2*ln(c*x+I)*ln(c^2*e*x^2+c^2*d)+
```

```
1/4*I*b/e^2*ln(c^2*e*x^2+c^2*d)*ln(c*x-I)-1/4*I*b/e^2*dilog((RootOf(e*_Z^2+
2*I*_Z*e+c^2*d-e,index=2)-c*x+I)/RootOf(e*_Z^2+2*I*_Z*e+c^2*d-e,index=2))+1
/4*I*b/e^2*dilog((RootOf(e*_Z^2-2*I*_Z*e+c^2*d-e,index=2)-c*x-I)/RootOf(e*_
Z^2-2*I*_Z*e+c^2*d-e,index=2))+1/4*I*b/e^2*ln(c*x+I)*ln((RootOf(e*_Z^2-2*I*
_Z*e+c^2*d-e,index=1)-c*x-I)/RootOf(e*_Z^2-2*I*_Z*e+c^2*d-e,index=1))+1/2*c
*b/e*d/(c^2*d-e)/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))-1/2*b*c^2*d*arctan(c*x
)/(c^2*d-e)/e^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}\,a\left(\frac{d}{e^3x^2+de^2}+\frac{\log\left(ex^2+d\right)}{e^2}\right)+2\,b\int\frac{x^3\arctan\left(cx\right)}{2\left(e^2x^4+2\,dex^2+d^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/2*a*(d/(e^3*x^2 + d*e^2) + log(e*x^2 + d)/e^2) + 2*b*integrate(1/2*x^3*ar
ctan(c*x)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{bx^3\arctan\left(cx\right)+ax^3}{e^2x^4+2\,dex^2+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^3*arctan(c*x) + a*x^3)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atan(c*x))/(e*x**2+d)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \arctan(cx) + a)x^3}{(ex^2 + d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)*x^3/(e*x^2 + d)^2, x)
```

3.1159 $\int \frac{x(a+b\tan^{-1}(cx))}{(d+ex^2)^2}\,dx$

Optimal. Leaf size=91

$$-\frac{a+b\tan^{-1}(cx)}{2e\left(d+ex^2\right)}+\frac{bc^2\tan^{-1}(cx)}{2e\left(c^2d-e\right)}-\frac{bc\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2\sqrt{d}\sqrt{e}\left(c^2d-e\right)}$$

```
[Out] (b*c^2*ArcTan[c*x])/(2*(c^2*d - e)*e) - (a + b*ArcTan[c*x])/(2*e*(d + e*x^2
)) - (b*c*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*Sqrt[d]*(c^2*d - e)*Sqrt[e])
```

Rubi [A] time = 0.0662689, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {4974, 391, 203, 205}

$$-\frac{a+b\tan^{-1}(cx)}{2e\left(d+ex^2\right)}+\frac{bc^2\tan^{-1}(cx)}{2e\left(c^2d-e\right)}-\frac{bc\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2\sqrt{d}\sqrt{e}\left(c^2d-e\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcTan[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] (b*c^2*ArcTan[c*x])/(2*(c^2*d - e)*e) - (a + b*ArcTan[c*x])/(2*e*(d + e*x^2
)) - (b*c*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*Sqrt[d]*(c^2*d - e)*Sqrt[e])
```

Rule 4974

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]))/(2*e*(q + 1)), x
] - Dist[(b*c)/(2*e*(q + 1)), Int[(d + e*x^2)^(q + 1)/(1 + c^2*x^2), x], x]
 /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 391

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{x\left(a+b\tan^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx &= -\frac{a+b\tan^{-1}(cx)}{2e\left(d+ex^2\right)}+\frac{(bc)\int \frac{1}{\left(1+c^2x^2\right)\left(d+ex^2\right)}\,dx}{2e}\\ &= -\frac{a+b\tan^{-1}(cx)}{2e\left(d+ex^2\right)}-\frac{(bc)\int \frac{1}{d+ex^2}\,dx}{2\left(c^2d-e\right)}+\frac{\left(bc^3\right)\int \frac{1}{1+c^2x^2}\,dx}{2\left(c^2d-e\right)e}\\ &= \frac{bc^2\tan^{-1}(cx)}{2\left(c^2d-e\right)e}-\frac{a+b\tan^{-1}(cx)}{2e\left(d+ex^2\right)}-\frac{bc\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2\sqrt{d}\left(c^2d-e\right)\sqrt{e}}\end{aligned}$$

Mathematica [A] time = 0.148354, size = 98, normalized size = 1.08

$$\frac{a\sqrt{d}\left(c^2d-e\right)-b\sqrt{d}e\left(c^2x^2+1\right)\tan^{-1}(cx)+bc\sqrt{e}\left(d+ex^2\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2\sqrt{d}e\left(e-c^2d\right)\left(d+ex^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcTan[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] (a*Sqrt[d]*(c^2*d - e) - b*Sqrt[d]*e*(1 + c^2*x^2)*ArcTan[c*x] + b*c*Sqrt[e
]*(d + e*x^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*Sqrt[d]*e*(-(c^2*d) + e)*(d +
 e*x^2))
```

Maple [A] time = 0.043, size = 109, normalized size = 1.2

$$-\frac{c^2a}{2\,e\left(c^2ex^2+c^2d\right)}-\frac{c^2b\arctan(cx)}{2\,e\left(c^2ex^2+c^2d\right)}-\frac{cb}{2\,c^2d-2\,e}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}+\frac{c^2b\arctan(cx)}{\left(2\,c^2d-2\,e\right)e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctan(c*x))/(e*x^2+d)^2,x)
```

```
[Out] -1/2*c^2*a/e/(c^2*e*x^2+c^2*d)-1/2*c^2*b/e/(c^2*e*x^2+c^2*d)*arctan(c*x)-1/
2*c*b/(c^2*d-e)/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))+1/2*b*c^2*arctan(c*x)/(
c^2*d-e)/e
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.88815, size = 501, normalized size = 5.51

$$\left[-\frac{2\,ac^2d^2-2\,ade-\left(bcex^2+bcd\right)\sqrt{-de}\log\left(\frac{ex^2-2\,\sqrt{-de}x-d}{ex^2+d}\right)-2\left(bc^2dex^2+bde\right)\arctan\left(cx\right)}{4\left(c^2d^3e-d^2e^2+\left(c^2d^2e^2-de^3\right)x^2\right)},-\frac{ac^2d^2-ade+\left(bcex^2+bc\right.}{2\left(c^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*a*c^2*d^2 - 2*a*d*e - (b*c*e*x^2 + b*c*d)*sqrt(-d*e)*log((e*x^2 -
2*sqrt(-d*e)*x - d)/(e*x^2 + d)) - 2*(b*c^2*d*e*x^2 + b*d*e)*arctan(c*x))/(
c^2*d^3*e - d^2*e^2 + (c^2*d^2*e^2 - d*e^3)*x^2), -1/2*(a*c^2*d^2 - a*d*e +
 (b*c*e*x^2 + b*c*d)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) - (b*c^2*d*e*x^2 + b*d
*e)*arctan(c*x))/(c^2*d^3*e - d^2*e^2 + (c^2*d^2*e^2 - d*e^3)*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atan(c*x))/(e*x**2+d)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.28288, size = 157, normalized size = 1.73

$$-\frac{bc\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}}{2\left(c^2d-e\right)\sqrt{d}}-\frac{ae^{(-1)}}{2\left(x^2e+d\right)}+\frac{bc^2x^2\arctan\left(cx\right)e-2\,ac^2d+b\arctan\left(cx\right)e+2\,ae}{2\left(c^2dx^2e^2+c^2d^2e-x^2e^3-de^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] -1/2*b*c*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/((c^2*d - e)*sqrt(d)) - 1/2*a*e
^(-1)/(x^2*e + d) + 1/2*(b*c^2*x^2*arctan(c*x)*e - 2*a*c^2*d + b*arctan(c*x
)*e + 2*a*e)/(c^2*d*x^2*e^2 + c^2*d^2*e - x^2*e^3 - d*e^2)
```

3.1160 $\int \frac{a+b\tan^{-1}(cx)}{x(d+ex^2)^2}\,dx$

Optimal. Leaf size=443

$$\frac{ib\text{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{(1-icx)\left(c\sqrt{-d}-i\sqrt{e}\right)}\right)}{4d^2}+\frac{ib\text{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-d}+\sqrt{e}x\right)}{(1-icx)\left(c\sqrt{-d}+i\sqrt{e}\right)}\right)}{4d^2}+\frac{ib\text{PolyLog}(2,-icx)}{2d^2}-\frac{ib\text{PolyLog}(2,icx)}{2d^2}$$

```
[Out] -(b*c^2*ArcTan[c*x])/(2*d*(c^2*d - e)) + (a + b*ArcTan[c*x])/(2*d*(d + e*x^
2)) + (b*c*Sqrt[e]*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*(c^2*d - e)) + (
a*Log[x])/d^2 + ((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/d^2 - ((a + b*ArcT
an[c*x])*Log[(2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*
c*x))])/(2*d^2) - ((a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c
*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(2*d^2) + ((I/2)*b*PolyLog[2, (-I)*c*
x])/d^2 - ((I/2)*b*PolyLog[2, I*c*x])/d^2 - ((I/2)*b*PolyLog[2, 1 - 2/(1 -
I*c*x)])/d^2 + ((I/4)*b*PolyLog[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqr
t[-d] - I*Sqrt[e])*(1 - I*c*x))])/d^2 + ((I/4)*b*PolyLog[2, 1 - (2*c*(Sqrt[
-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/d^2
```

Rubi [A] time = 0.48941, antiderivative size = 443, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 11, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.524, Rules used = {4980, 4848, 2391, 4974, 391, 203, 205, 4856, 2402, 2315, 2447}

$$\frac{ib\text{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{(1-icx)\left(c\sqrt{-d}-i\sqrt{e}\right)}\right)}{4d^2}+\frac{ib\text{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-d}+\sqrt{e}x\right)}{(1-icx)\left(c\sqrt{-d}+i\sqrt{e}\right)}\right)}{4d^2}+\frac{ib\text{PolyLog}(2,-icx)}{2d^2}-\frac{ib\text{PolyLog}(2,icx)}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])/(x*(d + e*x^2)^2),x]
```


```
[Out] -(b*c^2*ArcTan[c*x])/(2*d*(c^2*d - e)) + (a + b*ArcTan[c*x])/(2*d*(d + e*x^
2)) + (b*c*Sqrt[e]*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*(c^2*d - e)) + (
a*Log[x])/d^2 + ((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/d^2 - ((a + b*ArcT
an[c*x])*Log[(2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*
c*x))])/(2*d^2) - ((a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c
*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(2*d^2) + ((I/2)*b*PolyLog[2, (-I)*c*
x])/d^2 - ((I/2)*b*PolyLog[2, I*c*x])/d^2 - ((I/2)*b*PolyLog[2, 1 - 2/(1 -
I*c*x)])/d^2 + ((I/4)*b*PolyLog[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqr
t[-d] - I*Sqrt[e])*(1 - I*c*x))])/d^2 + ((I/4)*b*PolyLog[2, 1 - (2*c*(Sqrt[
```

```
-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/d^2
```

Rule 4980

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4974

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]))/(2*e*(q + 1)), x
] - Dist[(b*c)/(2*e*(q + 1)), Int[(d + e*x^2)^(q + 1)/(1 + c^2*x^2), x], x]
 /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 391

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{a+b\tan^{-1}(cx)}{x\left(d+ex^2\right)^2}\,dx = \int \left(\frac{a+b\tan^{-1}(cx)}{d^2x} - \frac{ex\left(a+b\tan^{-1}(cx)\right)}{d\left(d+ex^2\right)^2} - \frac{ex\left(a+b\tan^{-1}(cx)\right)}{d^2\left(d+ex^2\right)}\right)dx$$

$$= \frac{\int \frac{a+b\tan^{-1}(cx)}{x}\,dx}{d^2} - \frac{e\int \frac{x\left(a+b\tan^{-1}(cx)\right)}{d+ex^2}\,dx}{d^2} - \frac{e\int \frac{x\left(a+b\tan^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx}{d}$$

$$= \frac{a+b\tan^{-1}(cx)}{2d\left(d+ex^2\right)} + \frac{a\log(x)}{d^2} + \frac{(ib)\int \frac{\log(1-icx)}{x}\,dx}{2d^2} - \frac{(ib)\int \frac{\log(1+icx)}{x}\,dx}{2d^2} - \frac{(bc)\int \frac{1}{\left(1+c^2x^2\right)\left(d+ex^2\right)}\,dx}{2d} -$$

$$= \frac{a+b\tan^{-1}(cx)}{2d\left(d+ex^2\right)} + \frac{a\log(x)}{d^2} + \frac{ib\text{Li}_2(-icx)}{2d^2} - \frac{ib\text{Li}_2(icx)}{2d^2} - \frac{\left(bc^3\right)\int \frac{1}{1+c^2x^2}\,dx}{2d\left(c^2d-e\right)} + \frac{\sqrt{e}\int \frac{a+b\tan^{-1}(cx)}{\sqrt{-d}-\sqrt{e}x}\,d}{2d^2}$$

$$= -\frac{bc^2\tan^{-1}(cx)}{2d\left(c^2d-e\right)} + \frac{a+b\tan^{-1}(cx)}{2d\left(d+ex^2\right)} + \frac{bc\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}\left(c^2d-e\right)} + \frac{a\log(x)}{d^2} + \frac{\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2}{1-icx}\right.}{d^2}$$

$$= -\frac{bc^2\tan^{-1}(cx)}{2d\left(c^2d-e\right)} + \frac{a+b\tan^{-1}(cx)}{2d\left(d+ex^2\right)} + \frac{bc\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}\left(c^2d-e\right)} + \frac{a\log(x)}{d^2} + \frac{\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2}{1-icx}\right.}{d^2}$$

$$= -\frac{bc^2\tan^{-1}(cx)}{2d\left(c^2d-e\right)} + \frac{a+b\tan^{-1}(cx)}{2d\left(d+ex^2\right)} + \frac{bc\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}\left(c^2d-e\right)} + \frac{a\log(x)}{d^2} + \frac{\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2}{1-icx}\right.}{d^2}$$

Mathematica [A] time = 5.87583, size = 590, normalized size = 1.33

$$2a\left(\frac{d}{d+ex^2} - \log\left(d+ex^2\right) + 2\log(x)\right) + b\left(i\text{PolyLog}\left(2, \frac{c\left(\sqrt{d}-i\sqrt{e}x\right)}{c\sqrt{d}-\sqrt{e}}\right) - i\text{PolyLog}\left(2, \frac{c\left(\sqrt{d}-i\sqrt{e}x\right)}{c\sqrt{d}+\sqrt{e}}\right) - i\text{PolyLog}\left(2, \frac{c\left(\sqrt{d}+i\sqrt{e}x\right)}{c\sqrt{d}-\sqrt{e}}\right)\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])/(x*(d + e*x^2)^2),x]
```

```
[Out] (2*a*(d/(d + e*x^2) + 2*Log[x] - Log[d + e*x^2]) + b*((-2*c^2*d*ArcTan[c*x]
)/(c^2*d - e) + (2*d*ArcTan[c*x])/(d + e*x^2) + (2*c*Sqrt[d]*Sqrt[e]*ArcTan
[(Sqrt[e]*x)/Sqrt[d]])/(c^2*d - e) + 4*ArcTan[c*x]*Log[x] - 2*ArcTan[c*x]*L
og[((-I)*Sqrt[d])/Sqrt[e] + x] - 2*ArcTan[c*x]*Log[(I*Sqrt[d])/Sqrt[e] + x]
 - I*Log[((-I)*Sqrt[d])/Sqrt[e] + x]*Log[(Sqrt[e]*(-1 - I*c*x))/(c*Sqrt[d]
- Sqrt[e])] - (2*I)*Log[x]*Log[1 - I*c*x] + I*Log[((-I)*Sqrt[d])/Sqrt[e] +
```

```
x]*Log[(Sqrt[e]*(1 - I*c*x))/(c*Sqrt[d] + Sqrt[e])] + I*Log[(I*Sqrt[d])/Sqr
t[e] + x]*Log[(Sqrt[e]*(-1 + I*c*x))/(c*Sqrt[d] - Sqrt[e])] + (2*I)*Log[x]*
Log[1 + I*c*x] - I*Log[(I*Sqrt[d])/Sqrt[e] + x]*Log[(Sqrt[e]*(1 + I*c*x))/(
c*Sqrt[d] + Sqrt[e])] + (2*I)*PolyLog[2, (-I)*c*x] - (2*I)*PolyLog[2, I*c*x
] + I*PolyLog[2, (c*(Sqrt[d] - I*Sqrt[e]*x))/(c*Sqrt[d] - Sqrt[e])] - I*Pol
yLog[2, (c*(Sqrt[d] - I*Sqrt[e]*x))/(c*Sqrt[d] + Sqrt[e])] - I*PolyLog[2, (
c*(Sqrt[d] + I*Sqrt[e]*x))/(c*Sqrt[d] - Sqrt[e])] + I*PolyLog[2, (c*(Sqrt[d
] + I*Sqrt[e]*x))/(c*Sqrt[d] + Sqrt[e])]))/(4*d^2)
```

Maple [C] time = 0.212, size = 847, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/x/(e*x^2+d)^2,x)
```

```
[Out] -1/2*a/d^2*ln(c^2*e*x^2+c^2*d)+1/2*a*c^2/d/(c^2*e*x^2+c^2*d)+a/d^2*ln(c*x)-
1/2*b*arctan(c*x)/d^2*ln(c^2*e*x^2+c^2*d)+1/2*b*c^2*arctan(c*x)/d/(c^2*e*x^
2+c^2*d)+b*arctan(c*x)/d^2*ln(c*x)+1/2*b*c/d*e/(c^2*d-e)/(d*e)^(1/2)*arctan
(e*x/(d*e)^(1/2))-1/2*b*c^2*arctan(c*x)/d/(c^2*d-e)+1/4*I*b/d^2*ln(c*x-I)*l
n((RootOf(e*_Z^2+2*I*_Z*e+c^2*d-e,index=1)-c*x+I)/RootOf(e*_Z^2+2*I*_Z*e+c^
2*d-e,index=1))-1/4*I*b/d^2*dilog((RootOf(e*_Z^2-2*I*_Z*e+c^2*d-e,index=2)-
c*x-I)/RootOf(e*_Z^2-2*I*_Z*e+c^2*d-e,index=2))-1/4*I*b/d^2*ln(c*x+I)*ln((R
ootOf(e*_Z^2-2*I*_Z*e+c^2*d-e,index=1)-c*x-I)/RootOf(e*_Z^2-2*I*_Z*e+c^2*d-
e,index=1))+1/4*I*b/d^2*dilog((RootOf(e*_Z^2+2*I*_Z*e+c^2*d-e,index=2)-c*x+
I)/RootOf(e*_Z^2+2*I*_Z*e+c^2*d-e,index=2))+1/2*I*b/d^2*ln(c*x)*ln(1+I*c*x)
-1/2*I*b/d^2*dilog(1-I*c*x)+1/4*I*b/d^2*ln(c*x-I)*ln((RootOf(e*_Z^2+2*I*_Z*
e+c^2*d-e,index=2)-c*x+I)/RootOf(e*_Z^2+2*I*_Z*e+c^2*d-e,index=2))-1/4*I*b/
d^2*ln(c*x+I)*ln((RootOf(e*_Z^2-2*I*_Z*e+c^2*d-e,index=2)-c*x-I)/RootOf(e*_
Z^2-2*I*_Z*e+c^2*d-e,index=2))-1/4*I*b/d^2*ln(c*x-I)*ln(c^2*e*x^2+c^2*d)-1/
2*I*b/d^2*ln(c*x)*ln(1-I*c*x)-1/4*I*b/d^2*dilog((RootOf(e*_Z^2-2*I*_Z*e+c^2
*d-e,index=1)-c*x-I)/RootOf(e*_Z^2-2*I*_Z*e+c^2*d-e,index=1))+1/4*I*b/d^2*l
n(c*x+I)*ln(c^2*e*x^2+c^2*d)+1/2*I*b/d^2*dilog(1+I*c*x)+1/4*I*b/d^2*dilog((
RootOf(e*_Z^2+2*I*_Z*e+c^2*d-e,index=1)-c*x+I)/RootOf(e*_Z^2+2*I*_Z*e+c^2*d
-e,index=1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}\,a\left(\frac{1}{dex^2+d^2}-\frac{\log\left(ex^2+d\right)}{d^2}+\frac{2\,\log\left(x\right)}{d^2}\right)+2\,b\int\frac{\arctan\left(cx\right)}{2\left(e^2x^5+2\,dex^3+d^2x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/2*a*(1/(d*e*x^2 + d^2) - log(e*x^2 + d)/d^2 + 2*log(x)/d^2) + 2*b*integra
te(1/2*arctan(c*x)/(e^2*x^5 + 2*d*e*x^3 + d^2*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b\arctan\left(cx\right)+a}{e^2x^5+2\,dex^3+d^2x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*arctan(c*x) + a)/(e^2*x^5 + 2*d*e*x^3 + d^2*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/x/(e*x**2+d)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{b\arctan\left(cx\right)+a}{\left(ex^2+d\right)^2x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)/((e*x^2 + d)^2*x), x)
```

3.1161 $\int \frac{a+b\tan^{-1}(cx)}{x^3\left(d+ex^2\right)^2}\,dx$

Optimal. Leaf size=489

$$-\frac{ibe\text{PolyLog}(2,-icx)}{d^3}+\frac{ibe\text{PolyLog}(2,icx)}{d^3}+\frac{ibe\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{d^3}-\frac{ibe\text{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{(1-icx)\left(c\sqrt{-d}-i\sqrt{e}\right)}\right)}{2d^3}-\frac{ibe\text{Po}}{}$$

```
[Out] -(b*c)/(2*d^2*x) - (b*c^2*ArcTan[c*x])/(2*d^2) + (b*c^2*e*ArcTan[c*x])/(2*d
^2*(c^2*d - e)) - (a + b*ArcTan[c*x])/(2*d^2*x^2) - (e*(a + b*ArcTan[c*x]))
/(2*d^2*(d + e*x^2)) - (b*c*e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(5/2)
*(c^2*d - e)) - (2*a*e*Log[x])/d^3 - (2*e*(a + b*ArcTan[c*x])*Log[2/(1 - I*
c*x)])/d^3 + (e*(a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sq
rt[-d] - I*Sqrt[e])*(1 - I*c*x))])/d^3 + (e*(a + b*ArcTan[c*x])*Log[(2*c*(S
qrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/d^3 - (I*b*e
*PolyLog[2, (-I)*c*x])/d^3 + (I*b*e*PolyLog[2, I*c*x])/d^3 + (I*b*e*PolyLog
[2, 1 - 2/(1 - I*c*x)])/d^3 - ((I/2)*b*e*PolyLog[2, 1 - (2*c*(Sqrt[-d] - Sq
rt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/d^3 - ((I/2)*b*e*PolyLog
[2, 1 - (2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))
])/d^3
```

Rubi [A] time = 0.512779, antiderivative size = 489, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 13, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.619, Rules used = {4980, 4852, 325, 203, 4848, 2391, 4974, 391, 205, 4856, 2402, 2315, 2447}

$$-\frac{ibe\text{PolyLog}(2,-icx)}{d^3}+\frac{ibe\text{PolyLog}(2,icx)}{d^3}+\frac{ibe\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{d^3}-\frac{ibe\text{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{(1-icx)\left(c\sqrt{-d}-i\sqrt{e}\right)}\right)}{2d^3}-\frac{ibe\text{Po}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])/(x^3*(d + e*x^2)^2),x]
```


```
[Out] -(b*c)/(2*d^2*x) - (b*c^2*ArcTan[c*x])/(2*d^2) + (b*c^2*e*ArcTan[c*x])/(2*d
^2*(c^2*d - e)) - (a + b*ArcTan[c*x])/(2*d^2*x^2) - (e*(a + b*ArcTan[c*x]))
/(2*d^2*(d + e*x^2)) - (b*c*e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(5/2)
*(c^2*d - e)) - (2*a*e*Log[x])/d^3 - (2*e*(a + b*ArcTan[c*x])*Log[2/(1 - I*
c*x)])/d^3 + (e*(a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sq
rt[-d] - I*Sqrt[e])*(1 - I*c*x))])/d^3 + (e*(a + b*ArcTan[c*x])*Log[(2*c*(S
```

```
qrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/d^3 - (I*b*e
*PolyLog[2, (-I)*c*x])/d^3 + (I*b*e*PolyLog[2, I*c*x])/d^3 + (I*b*e*PolyLog
[2, 1 - 2/(1 - I*c*x)])/d^3 - ((I/2)*b*e*PolyLog[2, 1 - (2*c*(Sqrt[-d] - Sq
rt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/d^3 - ((I/2)*b*e*PolyLog
[2, 1 - (2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))
])/d^3
```

Rule 4980

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4974

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]))/(2*e*(q + 1)), x
] - Dist[(b*c)/(2*e*(q + 1)), Int[(d + e*x^2)^(q + 1)/(1 + c^2*x^2), x], x]
 /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 391

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
```

```
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{a+b\tan^{-1}(cx)}{x^3\left(d+ex^2\right)^2}\,dx = \int\left(\frac{a+b\tan^{-1}(cx)}{d^2x^3}-\frac{2e\left(a+b\tan^{-1}(cx)\right)}{d^3x}+\frac{e^2x\left(a+b\tan^{-1}(cx)\right)}{d^2\left(d+ex^2\right)^2}+\frac{2e^2x\left(a+b\tan^{-1}(cx)\right)}{d^3\left(d+ex^2\right)}\right)$$

$$=\frac{\int\frac{a+b\tan^{-1}(cx)}{x^3}\,dx}{d^2}-\frac{(2e)\int\frac{a+b\tan^{-1}(cx)}{x}\,dx}{d^3}+\frac{\left(2e^2\right)\int\frac{x\left(a+b\tan^{-1}(cx)\right)}{d+ex^2}\,dx}{d^3}+\frac{e^2\int\frac{x\left(a+b\tan^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx}{d^2}$$

$$=-\frac{a+b\tan^{-1}(cx)}{2d^2x^2}-\frac{e\left(a+b\tan^{-1}(cx)\right)}{2d^2\left(d+ex^2\right)}-\frac{2ae\log(x)}{d^3}+\frac{(bc)\int\frac{1}{x^2\left(1+c^2x^2\right)}\,dx}{2d^2}-\frac{(ibe)\int\frac{\log(1-icx)}{x}\,d}{d^3}$$

$$=-\frac{bc}{2d^2x}-\frac{a+b\tan^{-1}(cx)}{2d^2x^2}-\frac{e\left(a+b\tan^{-1}(cx)\right)}{2d^2\left(d+ex^2\right)}-\frac{2ae\log(x)}{d^3}-\frac{ibe\text{Li}_2(-icx)}{d^3}+\frac{ibe\text{Li}_2(icx)}{d^3}-$$

$$=-\frac{bc}{2d^2x}-\frac{bc^2\tan^{-1}(cx)}{2d^2}+\frac{bc^2e\tan^{-1}(cx)}{2d^2\left(c^2d-e\right)}-\frac{a+b\tan^{-1}(cx)}{2d^2x^2}-\frac{e\left(a+b\tan^{-1}(cx)\right)}{2d^2\left(d+ex^2\right)}-\frac{bce^{3/2}\tan}{2d^{5/2}\left(c^2\right.}$$

$$=-\frac{bc}{2d^2x}-\frac{bc^2\tan^{-1}(cx)}{2d^2}+\frac{bc^2e\tan^{-1}(cx)}{2d^2\left(c^2d-e\right)}-\frac{a+b\tan^{-1}(cx)}{2d^2x^2}-\frac{e\left(a+b\tan^{-1}(cx)\right)}{2d^2\left(d+ex^2\right)}-\frac{bce^{3/2}\tan}{2d^{5/2}\left(c^2\right.}$$

$$=-\frac{bc}{2d^2x}-\frac{bc^2\tan^{-1}(cx)}{2d^2}+\frac{bc^2e\tan^{-1}(cx)}{2d^2\left(c^2d-e\right)}-\frac{a+b\tan^{-1}(cx)}{2d^2x^2}-\frac{e\left(a+b\tan^{-1}(cx)\right)}{2d^2\left(d+ex^2\right)}-\frac{bce^{3/2}\tan}{2d^{5/2}\left(c^2\right.}$$

Mathematica [A] time = 12.4007, size = 643, normalized size = 1.31

$$-\frac{a\left(d\left(\frac{e}{d+ex^2}+\frac{1}{x^2}\right)-2e\log\left(d+ex^2\right)+4e\log(x)\right)+b\left(-e\left(-i\text{PolyLog}\left(2,\frac{c\left(\sqrt{d}-i\sqrt{e}x\right)}{c\sqrt{d}-\sqrt{e}}\right)+i\text{PolyLog}\left(2,\frac{c\left(\sqrt{d}-i\sqrt{e}x\right)}{c\sqrt{d}+\sqrt{e}}\right)+i\text{Pol}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])/(x^3*(d + e*x^2)^2),x]
```

```
[Out] -(a*(d*(x^(-2) + e/(d + e*x^2)) + 4*e*Log[x] - 2*e*Log[d + e*x^2]) + b*((c*
d)/x + (c^2*d*(c^2*d - 2*e)*ArcTan[c*x])/(c^2*d - e) + d*(x^(-2) + e/(d + e
*x^2))*ArcTan[c*x] + (c*Sqrt[d]*e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(c^2*d
 - e) + 4*e*ArcTan[c*x]*Log[x] - 2*e*ArcTan[c*x]*Log[d + e*x^2] - (2*I)*e*(
Log[x]*(Log[1 - I*c*x] - Log[1 + I*c*x]) - PolyLog[2, (-I)*c*x] + PolyLog[2
, I*c*x]) - e*(2*ArcTan[c*x]*Log[((-I)*Sqrt[d])/Sqrt[e] + x] + 2*ArcTan[c*x
]*Log[(I*Sqrt[d])/Sqrt[e] + x] + I*Log[((-I)*Sqrt[d])/Sqrt[e] + x]*Log[(Sqr
t[e]*(-1 - I*c*x))/(c*Sqrt[d] - Sqrt[e])] - I*Log[((-I)*Sqrt[d])/Sqrt[e] +
x]*Log[(Sqrt[e]*(1 - I*c*x))/(c*Sqrt[d] + Sqrt[e])] - I*Log[(I*Sqrt[d])/Sqr
t[e] + x]*Log[(Sqrt[e]*(-1 + I*c*x))/(c*Sqrt[d] - Sqrt[e])] + I*Log[(I*Sqrt
[d])/Sqrt[e] + x]*Log[(Sqrt[e]*(1 + I*c*x))/(c*Sqrt[d] + Sqrt[e])] - 2*ArcT
an[c*x]*Log[d + e*x^2] - I*PolyLog[2, (c*(Sqrt[d] - I*Sqrt[e]*x))/(c*Sqrt[d
] - Sqrt[e])] + I*PolyLog[2, (c*(Sqrt[d] - I*Sqrt[e]*x))/(c*Sqrt[d] + Sqrt[
e])] + I*PolyLog[2, (c*(Sqrt[d] + I*Sqrt[e]*x))/(c*Sqrt[d] - Sqrt[e])] - I*
PolyLog[2, (c*(Sqrt[d] + I*Sqrt[e]*x))/(c*Sqrt[d] + Sqrt[e])])))/(2*d^3)
```

Maple [C] time = 0.217, size = 925, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/x^3/(e*x^2+d)^2,x)
```

```
[Out] I*b/d^3*e*ln(c*x)*ln(1-I*c*x)+1/2*I*b/d^3*e*ln(c*x+I)*ln((RootOf(e*_Z^2-2*I
*_Z*e+c^2*d-e,index=1)-c*x-I)/RootOf(e*_Z^2-2*I*_Z*e+c^2*d-e,index=1))-1/2*
I*b/d^3*e*ln(c*x-I)*ln((RootOf(e*_Z^2+2*I*_Z*e+c^2*d-e,index=1)-c*x+I)/Root
Of(e*_Z^2+2*I*_Z*e+c^2*d-e,index=1))-I*b/d^3*e*ln(c*x)*ln(1+I*c*x)-1/2*I*b/
d^3*e*ln(c*x-I)*ln((RootOf(e*_Z^2+2*I*_Z*e+c^2*d-e,index=2)-c*x+I)/RootOf(e
*_Z^2+2*I*_Z*e+c^2*d-e,index=2))+1/2*I*b/d^3*e*ln(c*x-I)*ln(c^2*e*x^2+c^2*d
)-1/2*I*b/d^3*e*ln(c*x+I)*ln(c^2*e*x^2+c^2*d)+1/2*I*b/d^3*e*ln(c*x+I)*ln((R
ootOf(e*_Z^2-2*I*_Z*e+c^2*d-e,index=2)-c*x-I)/RootOf(e*_Z^2-2*I*_Z*e+c^2*d-
e,index=2))-1/2*a/d^2/x^2-1/2*c^2*b*arctan(c*x)*e/d^2/(c^2*e*x^2+c^2*d)-1/2
*b*c/d^2/x-1/2*c*b/d^2*e^2/(c^2*d-e)/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))-1/
2*b*arctan(c*x)/d^2/x^2+b*arctan(c*x)*e/d^3*ln(c^2*e*x^2+c^2*d)-2*b*arctan(
c*x)/d^3*e*ln(c*x)+I*b/d^3*e*dilog(1-I*c*x)-1/2*c^4*b/d/(c^2*d-e)*arctan(c*
x)+1/2*I*b/d^3*e*dilog((RootOf(e*_Z^2-2*I*_Z*e+c^2*d-e,index=2)-c*x-I)/Root
Of(e*_Z^2-2*I*_Z*e+c^2*d-e,index=2))-I*b/d^3*e*dilog(1+I*c*x)-1/2*I*b/d^3*e
*dilog((RootOf(e*_Z^2+2*I*_Z*e+c^2*d-e,index=1)-c*x+I)/RootOf(e*_Z^2+2*I*_Z
*e+c^2*d-e,index=1))+1/2*I*b/d^3*e*dilog((RootOf(e*_Z^2-2*I*_Z*e+c^2*d-e,in
dex=1)-c*x-I)/RootOf(e*_Z^2-2*I*_Z*e+c^2*d-e,index=1))-1/2*I*b/d^3*e*dilog(
(RootOf(e*_Z^2+2*I*_Z*e+c^2*d-e,index=2)-c*x+I)/RootOf(e*_Z^2+2*I*_Z*e+c^2*
```

```
d-e,index=2))+b*c^2*e*arctan(c*x)/d^2/(c^2*d-e)+a*e/d^3*ln(c^2*e*x^2+c^2*d)
-2*a/d^3*e*ln(c*x)-1/2*c^2*a*e/d^2/(c^2*e*x^2+c^2*d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2}a\left(\frac{2ex^2+d}{d^2ex^4+d^3x^2}-\frac{2e\log\left(ex^2+d\right)}{d^3}+\frac{4e\log(x)}{d^3}\right)+2b\int\frac{\arctan(cx)}{2\left(e^2x^7+2dex^5+d^2x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^3/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] -1/2*a*((2*e*x^2 + d)/(d^2*e*x^4 + d^3*x^2) - 2*e*log(e*x^2 + d)/d^3 + 4*e*
log(x)/d^3) + 2*b*integrate(1/2*arctan(c*x)/(e^2*x^7 + 2*d*e*x^5 + d^2*x^3)
, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b\arctan(cx)+a}{e^2x^7+2dex^5+d^2x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^3/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*arctan(c*x) + a)/(e^2*x^7 + 2*d*e*x^5 + d^2*x^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/x**3/(e*x**2+d)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \arctan(cx) + a}{\left(ex^2 + d\right)^2 x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^3/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)/((e*x^2 + d)^2*x^3), x)
```

3.1162 $\int \frac{x^2(a+b\tan^{-1}(cx))}{(d+ex^2)^2}\,dx$

Optimal. Leaf size=1335

result too large to display

```
[Out] -(x*(a + b*ArcTan[c*x]))/(2*e*(d + e*x^2)) + (a*ArcTan[(Sqrt[e]*x)/Sqrt[d]]
)/(Sqrt[d]*e^(3/2)) - ((a + b*ArcTan[c*x])*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*
Sqrt[d]*e^(3/2)) - ((I/4)*b*Log[1 + I*c*x]*Log[(c*(Sqrt[-d] - Sqrt[e]*x))/(
c*Sqrt[-d] - I*Sqrt[e])])/(Sqrt[-d]*e^(3/2)) + ((I/4)*b*Log[1 - I*c*x]*Log[
(c*(Sqrt[-d] - Sqrt[e]*x))/(c*Sqrt[-d] + I*Sqrt[e])])/(Sqrt[-d]*e^(3/2)) -
((I/4)*b*Log[1 - I*c*x]*Log[(c*(Sqrt[-d] + Sqrt[e]*x))/(c*Sqrt[-d] - I*Sqrt
[e])])/(Sqrt[-d]*e^(3/2)) + ((I/4)*b*Log[1 + I*c*x]*Log[(c*(Sqrt[-d] + Sqrt
[e]*x))/(c*Sqrt[-d] + I*Sqrt[e])])/(Sqrt[-d]*e^(3/2)) - ((I/8)*b*c*Log[(Sqr
t[e]*(1 - Sqrt[-c^2]*x))/(I*Sqrt[-c^2]*Sqrt[d] + Sqrt[e])]*Log[1 - (I*Sqrt[
e]*x)/Sqrt[d]])/(Sqrt[-c^2]*Sqrt[d]*e^(3/2)) + ((I/8)*b*c*Log[-((Sqrt[e]*(1
 + Sqrt[-c^2]*x))/(I*Sqrt[-c^2]*Sqrt[d] - Sqrt[e]))]*Log[1 - (I*Sqrt[e]*x)/
Sqrt[d]])/(Sqrt[-c^2]*Sqrt[d]*e^(3/2)) + ((I/8)*b*c*Log[-((Sqrt[e]*(1 - Sqr
t[-c^2]*x))/(I*Sqrt[-c^2]*Sqrt[d] - Sqrt[e]))]*Log[1 + (I*Sqrt[e]*x)/Sqrt[d
]])/(Sqrt[-c^2]*Sqrt[d]*e^(3/2)) - ((I/8)*b*c*Log[(Sqrt[e]*(1 + Sqrt[-c^2]*
x))/(I*Sqrt[-c^2]*Sqrt[d] + Sqrt[e])]*Log[1 + (I*Sqrt[e]*x)/Sqrt[d]])/(Sqrt
[-c^2]*Sqrt[d]*e^(3/2)) + (b*c*Log[1 + c^2*x^2])/(4*(c^2*d - e)*e) - (b*c*L
og[d + e*x^2])/(4*(c^2*d - e)*e) + ((I/4)*b*PolyLog[2, (Sqrt[e]*(I - c*x))/
(c*Sqrt[-d] + I*Sqrt[e])])/(Sqrt[-d]*e^(3/2)) - ((I/4)*b*PolyLog[2, (Sqrt[e
]*(1 - I*c*x))/(I*c*Sqrt[-d] + Sqrt[e])])/(Sqrt[-d]*e^(3/2)) - ((I/4)*b*Pol
yLog[2, (Sqrt[e]*(1 + I*c*x))/(I*c*Sqrt[-d] + Sqrt[e])])/(Sqrt[-d]*e^(3/2))
 + ((I/4)*b*PolyLog[2, (Sqrt[e]*(I + c*x))/(c*Sqrt[-d] + I*Sqrt[e])])/(Sqrt
[-d]*e^(3/2)) - ((I/8)*b*c*PolyLog[2, (Sqrt[-c^2]*(Sqrt[d] - I*Sqrt[e]*x))/
(Sqrt[-c^2]*Sqrt[d] - I*Sqrt[e])])/(Sqrt[-c^2]*Sqrt[d]*e^(3/2)) + ((I/8)*b*
c*PolyLog[2, (Sqrt[-c^2]*(Sqrt[d] - I*Sqrt[e]*x))/(Sqrt[-c^2]*Sqrt[d] + I*S
qrt[e])])/(Sqrt[-c^2]*Sqrt[d]*e^(3/2)) - ((I/8)*b*c*PolyLog[2, (Sqrt[-c^2]*
(Sqrt[d] + I*Sqrt[e]*x))/(Sqrt[-c^2]*Sqrt[d] - I*Sqrt[e])])/(Sqrt[-c^2]*Sqr
t[d]*e^(3/2)) + ((I/8)*b*c*PolyLog[2, (Sqrt[-c^2]*(Sqrt[d] + I*Sqrt[e]*x))/
(Sqrt[-c^2]*Sqrt[d] + I*Sqrt[e])])/(Sqrt[-c^2]*Sqrt[d]*e^(3/2))
```

Rubi [A] time = 1.96414, antiderivative size = 1335, normalized size of antiderivative = 1., number of steps used = 45, number of rules used = 14, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {4980, 199, 205, 4912, 6725, 444, 36, 31, 4908, 2409, 2394, 2393, 2391, 4910}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(a+b\tan^{-1}(cx)\right)}{2\sqrt{d}e^{3/2}}-\frac{x\left(a+b\tan^{-1}(cx)\right)}{2e\left(ex^2+d\right)}+\frac{a\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}e^{3/2}}-\frac{ib\log(icx+1)\log\left(\frac{c\left(\sqrt{-d}-\sqrt{e}x\right)}{c\sqrt{-d}-i\sqrt{e}}\right)}{4\sqrt{-d}e^{3/2}}+\frac{ib\log(1-icx}{4\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcTan[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] -(x*(a + b*ArcTan[c*x]))/(2*e*(d + e*x^2)) + (a*ArcTan[(Sqrt[e]*x)/Sqrt[d]]
)/(Sqrt[d]*e^(3/2)) - ((a + b*ArcTan[c*x])*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*
Sqrt[d]*e^(3/2)) - ((I/4)*b*Log[1 + I*c*x]*Log[(c*(Sqrt[-d] - Sqrt[e]*x))/(
c*Sqrt[-d] - I*Sqrt[e])])/(Sqrt[-d]*e^(3/2)) + ((I/4)*b*Log[1 - I*c*x]*Log[
(c*(Sqrt[-d] - Sqrt[e]*x))/(c*Sqrt[-d] + I*Sqrt[e])])/(Sqrt[-d]*e^(3/2)) -
((I/4)*b*Log[1 - I*c*x]*Log[(c*(Sqrt[-d] + Sqrt[e]*x))/(c*Sqrt[-d] - I*Sqrt
[e])])/(Sqrt[-d]*e^(3/2)) + ((I/4)*b*Log[1 + I*c*x]*Log[(c*(Sqrt[-d] + Sqrt
[e]*x))/(c*Sqrt[-d] + I*Sqrt[e])])/(Sqrt[-d]*e^(3/2)) - ((I/8)*b*c*Log[(Sqr
t[e]*(1 - Sqrt[-c^2]*x))/(I*Sqrt[-c^2]*Sqrt[d] + Sqrt[e])]*Log[1 - (I*Sqrt[
e]*x)/Sqrt[d]])/(Sqrt[-c^2]*Sqrt[d]*e^(3/2)) + ((I/8)*b*c*Log[-((Sqrt[e]*(1
 + Sqrt[-c^2]*x))/(I*Sqrt[-c^2]*Sqrt[d] - Sqrt[e]))]*Log[1 - (I*Sqrt[e]*x)/
Sqrt[d]])/(Sqrt[-c^2]*Sqrt[d]*e^(3/2)) + ((I/8)*b*c*Log[-((Sqrt[e]*(1 - Sqr
t[-c^2]*x))/(I*Sqrt[-c^2]*Sqrt[d] - Sqrt[e]))]*Log[1 + (I*Sqrt[e]*x)/Sqrt[d
]])/(Sqrt[-c^2]*Sqrt[d]*e^(3/2)) - ((I/8)*b*c*Log[(Sqrt[e]*(1 + Sqrt[-c^2]*
x))/(I*Sqrt[-c^2]*Sqrt[d] + Sqrt[e])]*Log[1 + (I*Sqrt[e]*x)/Sqrt[d]])/(Sqrt
[-c^2]*Sqrt[d]*e^(3/2)) + (b*c*Log[1 + c^2*x^2])/(4*(c^2*d - e)*e) - (b*c*L
og[d + e*x^2])/(4*(c^2*d - e)*e) + ((I/4)*b*PolyLog[2, (Sqrt[e]*(I - c*x))/
(c*Sqrt[-d] + I*Sqrt[e])])/(Sqrt[-d]*e^(3/2)) - ((I/4)*b*PolyLog[2, (Sqrt[e
]*(1 - I*c*x))/(I*c*Sqrt[-d] + Sqrt[e])])/(Sqrt[-d]*e^(3/2)) - ((I/4)*b*Pol
yLog[2, (Sqrt[e]*(1 + I*c*x))/(I*c*Sqrt[-d] + Sqrt[e])])/(Sqrt[-d]*e^(3/2))
 + ((I/4)*b*PolyLog[2, (Sqrt[e]*(I + c*x))/(c*Sqrt[-d] + I*Sqrt[e])])/(Sqrt
[-d]*e^(3/2)) - ((I/8)*b*c*PolyLog[2, (Sqrt[-c^2]*(Sqrt[d] - I*Sqrt[e]*x))/
(Sqrt[-c^2]*Sqrt[d] - I*Sqrt[e])])/(Sqrt[-c^2]*Sqrt[d]*e^(3/2)) + ((I/8)*b*
c*PolyLog[2, (Sqrt[-c^2]*(Sqrt[d] - I*Sqrt[e]*x))/(Sqrt[-c^2]*Sqrt[d] + I*S
qrt[e])])/(Sqrt[-c^2]*Sqrt[d]*e^(3/2)) - ((I/8)*b*c*PolyLog[2, (Sqrt[-c^2]*
(Sqrt[d] + I*Sqrt[e]*x))/(Sqrt[-c^2]*Sqrt[d] - I*Sqrt[e])])/(Sqrt[-c^2]*Sqr
t[d]*e^(3/2)) + ((I/8)*b*c*PolyLog[2, (Sqrt[-c^2]*(Sqrt[d] + I*Sqrt[e]*x))/
(Sqrt[-c^2]*Sqrt[d] + I*Sqrt[e])])/(Sqrt[-c^2]*Sqrt[d]*e^(3/2))
```

Rule 4980

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
```

```
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 4912

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcTan[c*x], u, x]
- Dist[b*c, Int[u/(1 + c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && (I
ntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 4908

```
Int[ArcTan[(c_.)*(x_)]/((d_.) + (e_.)*(x_)^2), x_Symbol] :> Dist[I/2, Int[L
og[1 - I*c*x]/(d + e*x^2), x], x] - Dist[I/2, Int[Log[1 + I*c*x]/(d + e*x^2
), x], x] /; FreeQ[{c, d, e}, x]
```

Rule 2409

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4910

```
Int[(ArcTan[(c_.)*(x_)]*(b_.) + (a_))/((d_.) + (e_.)*(x_)^2), x_Symbol] :>
Dist[a, Int[1/(d + e*x^2), x], x] + Dist[b, Int[ArcTan[c*x]/(d + e*x^2), x]
, x] /; FreeQ[{a, b, c, d, e}, x]
```

Rubi steps

$$\int \frac{x^2\left(a+b\tan^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx=\int\left(-\frac{d\left(a+b\tan^{-1}(cx)\right)}{e\left(d+ex^2\right)^2}+\frac{a+b\tan^{-1}(cx)}{e\left(d+ex^2\right)}\right)dx$$

$$=\frac{\int\frac{a+b\tan^{-1}(cx)}{d+ex^2}\,dx}{e}-\frac{d\int\frac{a+b\tan^{-1}(cx)}{\left(d+ex^2\right)^2}\,dx}{e}$$

$$=-\frac{x\left(a+b\tan^{-1}(cx)\right)}{2e\left(d+ex^2\right)}-\frac{\left(a+b\tan^{-1}(cx)\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2\sqrt{d}e^{3/2}}+\frac{a\int\frac{1}{d+ex^2}\,dx}{e}+\frac{b\int\frac{\tan^{-1}(cx)}{d+ex^2}\,dx}{e}+\cdots$$

$$=-\frac{x\left(a+b\tan^{-1}(cx)\right)}{2e\left(d+ex^2\right)}+\frac{a\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}e^{3/2}}-\frac{\left(a+b\tan^{-1}(cx)\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2\sqrt{d}e^{3/2}}+\frac{(ib)\int\frac{\log(1-icx)}{d+ex^2}\,d\cdots}{2e}$$

$$=-\frac{x\left(a+b\tan^{-1}(cx)\right)}{2e\left(d+ex^2\right)}+\frac{a\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}e^{3/2}}-\frac{\left(a+b\tan^{-1}(cx)\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2\sqrt{d}e^{3/2}}+\frac{(bc)\int\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\cdots}{1+c^2x^2}}{2\sqrt{d}e^{3/2}}$$

$$=-\frac{x\left(a+b\tan^{-1}(cx)\right)}{2e\left(d+ex^2\right)}+\frac{a\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}e^{3/2}}-\frac{\left(a+b\tan^{-1}(cx)\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2\sqrt{d}e^{3/2}}+\frac{(ibc)\int\frac{\log\left(1-\frac{i\sqrt{e}x}{\sqrt{d}}\cdots\right.}{1+c^2x^2}}{4\sqrt{d}e^{3/2}}$$

$$=-\frac{x\left(a+b\tan^{-1}(cx)\right)}{2e\left(d+ex^2\right)}+\frac{a\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}e^{3/2}}-\frac{\left(a+b\tan^{-1}(cx)\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2\sqrt{d}e^{3/2}}-\frac{ib\log(1+icx)\,\mathrm{lo}\cdots}{4\sqrt{-d}\cdots}$$

$$=-\frac{x\left(a+b\tan^{-1}(cx)\right)}{2e\left(d+ex^2\right)}+\frac{a\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}e^{3/2}}-\frac{\left(a+b\tan^{-1}(cx)\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2\sqrt{d}e^{3/2}}-\frac{ib\log(1+icx)\,\mathrm{lo}\cdots}{4\sqrt{-d}\cdots}$$

$$=-\frac{x\left(a+b\tan^{-1}(cx)\right)}{2e\left(d+ex^2\right)}+\frac{a\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}e^{3/2}}-\frac{\left(a+b\tan^{-1}(cx)\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2\sqrt{d}e^{3/2}}-\frac{ib\log(1+icx)\,\mathrm{lo}\cdots}{4\sqrt{-d}\cdots}$$

$$=-\frac{x\left(a+b\tan^{-1}(cx)\right)}{2e\left(d+ex^2\right)}+\frac{a\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}e^{3/2}}-\frac{\left(a+b\tan^{-1}(cx)\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2\sqrt{d}e^{3/2}}-\frac{ib\log(1+icx)\,\mathrm{lo}\cdots}{4\sqrt{-d}\cdots}$$

$$=-\frac{x\left(a+b\tan^{-1}(cx)\right)}{2e\left(d+ex^2\right)}+\frac{a\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}e^{3/2}}-\frac{\left(a+b\tan^{-1}(cx)\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2\sqrt{d}e^{3/2}}-\frac{ib\log(1+icx)\,\mathrm{lo}\cdots}{4\sqrt{-d}\cdots}$$

Mathematica [A] time = 10.2903, size = 881, normalized size = 0.66

$$-\frac{ax}{2e\left(ex^2+d\right)}+\frac{a\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2\sqrt{d}e^{3/2}}+\frac{bc\left(-\frac{2\log\left(\frac{\left(c^2d-e\right)\cos\left(2\tan^{-1}(cx)\right)}{dc^2+e}+1\right)}{c^2d-e}+\frac{-4\tan^{-1}(cx)\tanh^{-1}\left(\frac{\sqrt{-c^2de}}{cex}\right)+2\cos^{-1}\left(-\frac{dc^2+e}{c^2d-e}\right)\tanh^{-1}\left(\frac{cex}{\sqrt{-c^2de}}\right)+\left(\cos\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(a + b*ArcTan[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] -(a*x)/(2*e*(d + e*x^2)) + (a*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*Sqrt[d]*e^(3/
2)) + (b*c*((-2*Log[1 + ((c^2*d - e)*Cos[2*ArcTan[c*x]])/(c^2*d + e)])/(c^2
*d - e) + (-4*ArcTan[c*x]*ArcTanh[Sqrt[-(c^2*d*e)]/(c*e*x)] + 2*ArcCos[-((c
^2*d + e)/(c^2*d - e))]*ArcTanh[(c*e*x)/Sqrt[-(c^2*d*e)]] + (ArcCos[-((c^2*
d + e)/(c^2*d - e))] - (2*I)*ArcTanh[(c*e*x)/Sqrt[-(c^2*d*e)]])*Log[(-2*c^2
*d*(I*e + Sqrt[-(c^2*d*e)])*(-I + c*x))/((c^2*d - e)*(c^2*d - c*Sqrt[-(c^2*
d*e)]*x))] + (ArcCos[-((c^2*d + e)/(c^2*d - e))] + (2*I)*ArcTanh[(c*e*x)/Sq
rt[-(c^2*d*e)]])*Log[((2*I)*c^2*d*(e + I*Sqrt[-(c^2*d*e)])*(I + c*x))/((c^2
*d - e)*(c^2*d - c*Sqrt[-(c^2*d*e)]*x))] - (ArcCos[-((c^2*d + e)/(c^2*d - e
))] - (2*I)*ArcTanh[Sqrt[-(c^2*d*e)]/(c*e*x)] + (2*I)*ArcTanh[(c*e*x)/Sqrt[
-(c^2*d*e)]])*Log[(Sqrt[2]*Sqrt[-(c^2*d*e)])/(Sqrt[-(c^2*d) + e]*E^(I*ArcTa
n[c*x])*Sqrt[-(c^2*d) - e + (-(c^2*d) + e)*Cos[2*ArcTan[c*x]]])] - (ArcCos[
-((c^2*d + e)/(c^2*d - e))] + (2*I)*ArcTanh[Sqrt[-(c^2*d*e)]/(c*e*x)] - (2*
I)*ArcTanh[(c*e*x)/Sqrt[-(c^2*d*e)]])*Log[(Sqrt[2]*Sqrt[-(c^2*d*e)]*E^(I*Ar
cTan[c*x]))/(Sqrt[-(c^2*d) + e]*Sqrt[-(c^2*d) - e + (-(c^2*d) + e)*Cos[2*Ar
cTan[c*x]]])] + I*(PolyLog[2, ((c^2*d + e - (2*I)*Sqrt[-(c^2*d*e)])*(c^2*d
+ c*Sqrt[-(c^2*d*e)]*x))/((c^2*d - e)*(c^2*d - c*Sqrt[-(c^2*d*e)]*x))] - Po
lyLog[2, ((c^2*d + e + (2*I)*Sqrt[-(c^2*d*e)])*(c^2*d + c*Sqrt[-(c^2*d*e)]*
x))/((c^2*d - e)*(c^2*d - c*Sqrt[-(c^2*d*e)]*x))]))/Sqrt[-(c^2*d*e)] - (4*A
rcTan[c*x]*Sin[2*ArcTan[c*x]])/(c^2*d + e + (c^2*d - e)*Cos[2*ArcTan[c*x]])
))/(8*e)
```

Maple [B] time = 0.815, size = 2315, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctan(c*x))/(e*x^2+d)^2,x)
```

[Out] -1/2*c^4*b*arctan(c*x)/(c^2*d-e)/e/(c^2*e*x^2+c^2*d)*x*d+3/4*c^3*b*d*arctan
(c*x)^2/e/(c^2*d-e)/(c^4*d^2-2*c^2*d*e+e^2)*(c^2*e*d)^(1/2)+1/8/c*b*polylog
(2,(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d+2*(c^2*e*d)^(1/2)-e))/d/(c^2*d
-e)/(c^4*d^2-2*c^2*d*e+e^2)*(c^2*e*d)^(1/2)*e+c^3*b/(c^2*d-e)^2/e*d*ln((1+I
*c*x)/(c^2*x^2+1)^(1/2))-3/8*c*b*polylog(2,(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1
)/(-c^2*d+2*(c^2*e*d)^(1/2)-e))/(c^4*d^2-2*c^2*d*e+e^2)/(c^2*d-e)*(c^2*e*d)
^(1/2)-3/4*c*b*arctan(c*x)^2/(c^4*d^2-2*c^2*d*e+e^2)/(c^2*d-e)*(c^2*e*d)^(1
/2)+1/4*c*b*(c^2*e*d)^(1/2)/e^2/(c^2*d-e)*arctan(c*x)^2+1/8*c*b*(c^2*e*d)^(
1/2)/e^2/(c^2*d-e)*polylog(2,(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d-2*(c
^2*e*d)^(1/2)-e))+1/2*c^2*b*arctan(c*x)/(c^2*d-e)/(c^2*e*x^2+c^2*d)*x-1/4*b
*(d*e)^(1/2)/d/e*arctanh(1/4*(2*(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)+2*c^2*d+2
*e)/c/(d*e)^(1/2))/(c^2*d-e)-1/4*c^2*b*(d*e)^(1/2)/e^2*arctanh(1/4*(2*(c^2*
d-e)*(1+I*c*x)^2/(c^2*x^2+1)+2*c^2*d+2*e)/c/(d*e)^(1/2))/(c^2*d-e)-1/4*c^3*
b/(c^2*d-e)^2/e*d*ln((1+I*c*x)^4/(c^2*x^2+1)^2*c^2*d+2*c^2*d*(1+I*c*x)^2/(c
^2*x^2+1)-(1+I*c*x)^4/(c^2*x^2+1)^2*e+c^2*d+2*(1+I*c*x)^2/(c^2*x^2+1)*e-e)-
1/4*I*c^5*b*d^2*ln(1-(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d+2*(c^2*e*d)^
(1/2)-e))*arctan(c*x)/e^2/(c^2*d-e)/(c^4*d^2-2*c^2*d*e+e^2)*(c^2*e*d)^(1/2)
+3/4*I*c^3*b*ln(1-(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d+2*(c^2*e*d)^(1/
2)-e))*arctan(c*x)*d/(c^2*d-e)/e/(c^4*d^2-2*c^2*d*e+e^2)*(c^2*e*d)^(1/2)+1/
4*I/c*b*ln(1-(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d+2*(c^2*e*d)^(1/2)-e)
)*arctan(c*x)/d/(c^2*d-e)/(c^4*d^2-2*c^2*d*e+e^2)*(c^2*e*d)^(1/2)*e-1/4*I/c
b(c^2*e*d)^(1/2)/(c^2*d-e)/d/e*arctan(c*x)*ln(1-(c^2*d-e)*(1+I*c*x)^2/(c^
2*x^2+1)/(-c^2*d-2*(c^2*e*d)^(1/2)-e))+1/4*c*b/(c^2*d-e)^2*ln((1+I*c*x)^4/(
c^2*x^2+1)^2*c^2*d+2*c^2*d*(1+I*c*x)^2/(c^2*x^2+1)-(1+I*c*x)^4/(c^2*x^2+1)^
2*e+c^2*d+2*(1+I*c*x)^2/(c^2*x^2+1)*e-e)-c*b/(c^2*d-e)^2*ln((1+I*c*x)/(c^2*
x^2+1)^(1/2))+1/2*a/e/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))+1/4/c*b*arctan(c*
x)^2/d/(c^2*d-e)/(c^4*d^2-2*c^2*d*e+e^2)*(c^2*e*d)^(1/2)*e-1/8*c^5*b*d^2*po
lylog(2,(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d+2*(c^2*e*d)^(1/2)-e))/e^2
/(c^2*d-e)/(c^4*d^2-2*c^2*d*e+e^2)*(c^2*e*d)^(1/2)+3/8*c^3*b*d*polylog(2,(c
^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d+2*(c^2*e*d)^(1/2)-e))/e/(c^2*d-e)/(
c^4*d^2-2*c^2*d*e+e^2)*(c^2*e*d)^(1/2)-1/4*c^5*b*d^2*arctan(c*x)^2/e^2/(c^2
*d-e)/(c^4*d^2-2*c^2*d*e+e^2)*(c^2*e*d)^(1/2)-3/4*I*c*b*ln(1-(c^2*d-e)*(1+I
*c*x)^2/(c^2*x^2+1)/(-c^2*d+2*(c^2*e*d)^(1/2)-e))*arctan(c*x)/(c^4*d^2-2*c^
2*d*e+e^2)/(c^2*d-e)*(c^2*e*d)^(1/2)-1/2*I*c^3*b*arctan(c*x)/(c^2*d-e)/e/(c
^2*e*x^2+c^2*d)*d+1/4*I*c*b*(c^2*e*d)^(1/2)/e^2/(c^2*d-e)*arctan(c*x)*ln(1-
(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d-2*(c^2*e*d)^(1/2)-e))-1/8/c*b*(c^
2*e*d)^(1/2)/(c^2*d-e)/d/e*polylog(2,(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^
2*d-2*(c^2*e*d)^(1/2)-e))-1/4/c*b*(c^2*e*d)^(1/2)/(c^2*d-e)/d/e*arctan(c*x)
^2+1/4*c^4*b*(d*e)^(1/2)*d/e^2*arctanh(1/4*(2*(c^2*d-e)*(1+I*c*x)^2/(c^2*x^
2+1)+2*c^2*d+2*e)/c/(d*e)^(1/2))/(c^2*d-e)^2-1/2*I*c^3*b*arctan(c*x)/(c^2*d
-e)/(c^2*e*x^2+c^2*d)*x^2-1/2*c^2*a/e*x/(c^2*e*x^2+c^2*d)-1/4*b*(d*e)^(1/2)
/d*arctanh(1/4*(2*(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)+2*c^2*d+2*e)/c/(d*e)^(1
/2))/(c^2*d-e)^2

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{bx^2 \arctan(cx) + ax^2}{e^2x^4 + 2\,dex^2 + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^2*arctan(c*x) + a*x^2)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atan(c*x))/(e*x**2+d)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \arctan(cx) + a)x^2}{\left(ex^2 + d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)*x^2/(e*x^2 + d)^2, x)
```

3.1163 $\int \frac{a+b\tan^{-1}(cx)}{\left(d+ex^2\right)^2}\,dx$

Optimal. Leaf size=819

$$\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(a+b\tan^{-1}(cx)\right)}{2d^{3/2}\sqrt{e}}+\frac{x\left(a+b\tan^{-1}(cx)\right)}{2d\left(ex^2+d\right)}+\frac{ibc\log\left(\frac{\sqrt{e}\left(1-\sqrt{-c^2}x\right)}{i\sqrt{-c^2}\sqrt{d}+\sqrt{e}}\right)\log\left(1-\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{8\sqrt{-c^2}d^{3/2}\sqrt{e}}-\frac{ibc\log\left(-\frac{\sqrt{e}\left(\sqrt{-c^2}x+1\right)}{i\sqrt{-c^2}\sqrt{d}-\sqrt{e}}\right)\log\left(1\right.}{8\sqrt{-c^2}d^{3/2}\sqrt{e}}$$

```
[Out] (x*(a + b*ArcTan[c*x]))/(2*d*(d + e*x^2)) + ((a + b*ArcTan[c*x])*ArcTan[(Sq
rt[e]*x)/Sqrt[d]])/(2*d^(3/2)*Sqrt[e]) + ((I/8)*b*c*Log[(Sqrt[e]*(1 - Sqrt[
-c^2]*x))/(I*Sqrt[-c^2]*Sqrt[d] + Sqrt[e])]*Log[1 - (I*Sqrt[e]*x)/Sqrt[d]])
/(Sqrt[-c^2]*d^(3/2)*Sqrt[e]) - ((I/8)*b*c*Log[-((Sqrt[e]*(1 + Sqrt[-c^2]*x
))/(I*Sqrt[-c^2]*Sqrt[d] - Sqrt[e]))]*Log[1 - (I*Sqrt[e]*x)/Sqrt[d]])/(Sqrt
[-c^2]*d^(3/2)*Sqrt[e]) - ((I/8)*b*c*Log[-((Sqrt[e]*(1 - Sqrt[-c^2]*x))/(I*
Sqrt[-c^2]*Sqrt[d] - Sqrt[e]))]*Log[1 + (I*Sqrt[e]*x)/Sqrt[d]])/(Sqrt[-c^2]
*d^(3/2)*Sqrt[e]) + ((I/8)*b*c*Log[(Sqrt[e]*(1 + Sqrt[-c^2]*x))/(I*Sqrt[-c^
2]*Sqrt[d] + Sqrt[e])]*Log[1 + (I*Sqrt[e]*x)/Sqrt[d]])/(Sqrt[-c^2]*d^(3/2)*
Sqrt[e]) - (b*c*Log[1 + c^2*x^2])/(4*d*(c^2*d - e)) + (b*c*Log[d + e*x^2])/
(4*d*(c^2*d - e)) + ((I/8)*b*c*PolyLog[2, (Sqrt[-c^2]*(Sqrt[d] - I*Sqrt[e]*
x))/(Sqrt[-c^2]*Sqrt[d] - I*Sqrt[e])])/(Sqrt[-c^2]*d^(3/2)*Sqrt[e]) - ((I/8
)*b*c*PolyLog[2, (Sqrt[-c^2]*(Sqrt[d] - I*Sqrt[e]*x))/(Sqrt[-c^2]*Sqrt[d] +
 I*Sqrt[e])])/(Sqrt[-c^2]*d^(3/2)*Sqrt[e]) + ((I/8)*b*c*PolyLog[2, (Sqrt[-c
^2]*(Sqrt[d] + I*Sqrt[e]*x))/(Sqrt[-c^2]*Sqrt[d] - I*Sqrt[e])])/(Sqrt[-c^2]
*d^(3/2)*Sqrt[e]) - ((I/8)*b*c*PolyLog[2, (Sqrt[-c^2]*(Sqrt[d] + I*Sqrt[e]*
x))/(Sqrt[-c^2]*Sqrt[d] + I*Sqrt[e])])/(Sqrt[-c^2]*d^(3/2)*Sqrt[e])
```

Rubi [A] time = 0.892897, antiderivative size = 819, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 12, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {199, 205, 4912, 6725, 444, 36, 31, 4908, 2409, 2394, 2393, 2391}

$$\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(a+b\tan^{-1}(cx)\right)}{2d^{3/2}\sqrt{e}}+\frac{x\left(a+b\tan^{-1}(cx)\right)}{2d\left(ex^2+d\right)}+\frac{ibc\log\left(\frac{\sqrt{e}\left(1-\sqrt{-c^2}x\right)}{i\sqrt{-c^2}\sqrt{d}+\sqrt{e}}\right)\log\left(1-\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{8\sqrt{-c^2}d^{3/2}\sqrt{e}}-\frac{ibc\log\left(-\frac{\sqrt{e}\left(\sqrt{-c^2}x+1\right)}{i\sqrt{-c^2}\sqrt{d}-\sqrt{e}}\right)\log\left(1\right.}{8\sqrt{-c^2}d^{3/2}\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])/(d + e*x^2)^2,x]
```

```
[Out] (x*(a + b*ArcTan[c*x]))/(2*d*(d + e*x^2)) + ((a + b*ArcTan[c*x])*ArcTan[(Sq
rt[e]*x)/Sqrt[d]])/(2*d^(3/2)*Sqrt[e]) + ((I/8)*b*c*Log[(Sqrt[e]*(1 - Sqrt[
```

```
-c^2]*x))/(I*Sqrt[-c^2]*Sqrt[d] + Sqrt[e])]*Log[1 - (I*Sqrt[e]*x)/Sqrt[d]])
/(Sqrt[-c^2]*d^(3/2)*Sqrt[e]) - ((I/8)*b*c*Log[-((Sqrt[e]*(1 + Sqrt[-c^2]*x
))/(I*Sqrt[-c^2]*Sqrt[d] - Sqrt[e]))]*Log[1 - (I*Sqrt[e]*x)/Sqrt[d]])/(Sqrt
[-c^2]*d^(3/2)*Sqrt[e]) - ((I/8)*b*c*Log[-((Sqrt[e]*(1 - Sqrt[-c^2]*x))/(I*
Sqrt[-c^2]*Sqrt[d] - Sqrt[e]))]*Log[1 + (I*Sqrt[e]*x)/Sqrt[d]])/(Sqrt[-c^2]
*d^(3/2)*Sqrt[e]) + ((I/8)*b*c*Log[(Sqrt[e]*(1 + Sqrt[-c^2]*x))/(I*Sqrt[-c^
2]*Sqrt[d] + Sqrt[e])]*Log[1 + (I*Sqrt[e]*x)/Sqrt[d]])/(Sqrt[-c^2]*d^(3/2)*
Sqrt[e]) - (b*c*Log[1 + c^2*x^2])/(4*d*(c^2*d - e)) + (b*c*Log[d + e*x^2])/
(4*d*(c^2*d - e)) + ((I/8)*b*c*PolyLog[2, (Sqrt[-c^2]*(Sqrt[d] - I*Sqrt[e]*
x))/(Sqrt[-c^2]*Sqrt[d] - I*Sqrt[e])])/(Sqrt[-c^2]*d^(3/2)*Sqrt[e]) - ((I/8
)*b*c*PolyLog[2, (Sqrt[-c^2]*(Sqrt[d] - I*Sqrt[e]*x))/(Sqrt[-c^2]*Sqrt[d] +
 I*Sqrt[e])])/(Sqrt[-c^2]*d^(3/2)*Sqrt[e]) + ((I/8)*b*c*PolyLog[2, (Sqrt[-c
^2]*(Sqrt[d] + I*Sqrt[e]*x))/(Sqrt[-c^2]*Sqrt[d] - I*Sqrt[e])])/(Sqrt[-c^2]
*d^(3/2)*Sqrt[e]) - ((I/8)*b*c*PolyLog[2, (Sqrt[-c^2]*(Sqrt[d] + I*Sqrt[e]*
x))/(Sqrt[-c^2]*Sqrt[d] + I*Sqrt[e])])/(Sqrt[-c^2]*d^(3/2)*Sqrt[e])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 4912

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcTan[c*x], u, x]
- Dist[b*c, Int[u/(1 + c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && (I
ntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
```

```
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 4908

```
Int[ArcTan[(c_.)*(x_)]/((d_.) + (e_.)*(x_)^2), x_Symbol] :> Dist[I/2, Int[L
og[1 - I*c*x]/(d + e*x^2), x], x] - Dist[I/2, Int[Log[1 + I*c*x]/(d + e*x^2
), x], x] /; FreeQ[{c, d, e}, x]
```

Rule 2409

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{a+b\tan^{-1}(cx)}{\left(d+ex^2\right)^2}\,dx = \frac{x\left(a+b\tan^{-1}(cx)\right)}{2d\left(d+ex^2\right)} + \frac{\left(a+b\tan^{-1}(cx)\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}\sqrt{e}} - (bc)\int \frac{\frac{x}{2d\left(d+ex^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}\sqrt{e}}}{1+c^2x^2}\,dx$$

$$= \frac{x\left(a+b\tan^{-1}(cx)\right)}{2d\left(d+ex^2\right)} + \frac{\left(a+b\tan^{-1}(cx)\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}\sqrt{e}} - (bc)\int\left(\frac{x}{2d\left(1+c^2x^2\right)\left(d+ex^2\right)} + \frac{\tan}{2d^{3/2}\sqrt{}}\right.$$

$$= \frac{x\left(a+b\tan^{-1}(cx)\right)}{2d\left(d+ex^2\right)} + \frac{\left(a+b\tan^{-1}(cx)\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}\sqrt{e}} - \frac{(bc)\int \frac{x}{\left(1+c^2x^2\right)\left(d+ex^2\right)}\,dx}{2d} - \frac{(bc)\int \frac{\tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right.}{1+c^2x^2}}{2d^{3/2}\sqrt{e}}$$

$$= \frac{x\left(a+b\tan^{-1}(cx)\right)}{2d\left(d+ex^2\right)} + \frac{\left(a+b\tan^{-1}(cx)\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}\sqrt{e}} - \frac{(bc)\,\mathrm{Subst}\left(\int \frac{1}{\left(1+c^2x\right)(d+ex)}\,dx, x, x^2\right)}{4d} - (ib$$

$$= \frac{x\left(a+b\tan^{-1}(cx)\right)}{2d\left(d+ex^2\right)} + \frac{\left(a+b\tan^{-1}(cx)\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}\sqrt{e}} - \frac{\left(bc^3\right)\mathrm{Subst}\left(\int \frac{1}{1+c^2x}\,dx, x, x^2\right)}{4d\left(c^2d-e\right)} - \frac{(ibc)\int}{}$$

$$= \frac{x\left(a+b\tan^{-1}(cx)\right)}{2d\left(d+ex^2\right)} + \frac{\left(a+b\tan^{-1}(cx)\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}\sqrt{e}} - \frac{bc\log\left(1+c^2x^2\right)}{4d\left(c^2d-e\right)} + \frac{bc\log\left(d+ex^2\right)}{4d\left(c^2d-e\right)} - \frac{(ib}{}$$

$$= \frac{x\left(a+b\tan^{-1}(cx)\right)}{2d\left(d+ex^2\right)} + \frac{\left(a+b\tan^{-1}(cx)\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}\sqrt{e}} + \frac{ibc\log\left(\frac{\sqrt{e}\left(1-\sqrt{-c^2}x\right)}{i\sqrt{-c^2}\sqrt{d}+\sqrt{e}}\right)\log\left(1-\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{8\sqrt{-c^2}d^{3/2}\sqrt{e}} - \frac{ibc}{}$$

$$= \frac{x\left(a+b\tan^{-1}(cx)\right)}{2d\left(d+ex^2\right)} + \frac{\left(a+b\tan^{-1}(cx)\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}\sqrt{e}} + \frac{ibc\log\left(\frac{\sqrt{e}\left(1-\sqrt{-c^2}x\right)}{i\sqrt{-c^2}\sqrt{d}+\sqrt{e}}\right)\log\left(1-\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{8\sqrt{-c^2}d^{3/2}\sqrt{e}} - \frac{ibc}{}$$

$$= \frac{x\left(a+b\tan^{-1}(cx)\right)}{2d\left(d+ex^2\right)} + \frac{\left(a+b\tan^{-1}(cx)\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}\sqrt{e}} + \frac{ibc\log\left(\frac{\sqrt{e}\left(1-\sqrt{-c^2}x\right)}{i\sqrt{-c^2}\sqrt{d}+\sqrt{e}}\right)\log\left(1-\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{8\sqrt{-c^2}d^{3/2}\sqrt{e}} - \frac{ibc}{}$$

Mathematica [A] time = 10.0501, size = 861, normalized size = 1.05

$$\frac{ax}{2d\left(ex^2+d\right)}+\frac{a\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}\sqrt{e}}+\frac{bc\left(\frac{2\log\left(\frac{\left(c^2d-e\right)\cos\left(2\tan^{-1}(cx)\right)}{dc^2+e}+1\right)}{c^2d-e}+\frac{-4\tan^{-1}(cx)\tanh^{-1}\left(\frac{\sqrt{-c^2de}}{cex}\right)+2\cos^{-1}\left(-\frac{dc^2+e}{c^2d-e}\right)\tanh^{-1}\left(\frac{cex}{\sqrt{-c^2de}}\right)-\left(\cos^{-1}\cdots\right.}{\cdots}\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c*x])/(d + e*x^2)^2,x]
```

```
[Out] (a*x)/(2*d*(d + e*x^2)) + (a*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*Sqrt[e
]) + (b*c*((2*Log[1 + ((c^2*d - e)*Cos[2*ArcTan[c*x]])/(c^2*d + e)])/(c^2*d
 - e) + (-4*ArcTan[c*x]*ArcTanh[Sqrt[-(c^2*d*e)]/(c*e*x)] + 2*ArcCos[-((c^2
*d + e)/(c^2*d - e))]*ArcTanh[(c*e*x)/Sqrt[-(c^2*d*e)]] - (ArcCos[-((c^2*d
+ e)/(c^2*d - e))] + (2*I)*ArcTanh[(c*e*x)/Sqrt[-(c^2*d*e)]])*Log[(2*c^2*d*
((-I)*e + Sqrt[-(c^2*d*e)])*(-I + c*x))/((c^2*d - e)*(c^2*d + c*Sqrt[-(c^2*
d*e)]*x))] - (ArcCos[-((c^2*d + e)/(c^2*d - e))] - (2*I)*ArcTanh[(c*e*x)/Sq
rt[-(c^2*d*e)]])*Log[(2*c^2*d*(I*e + Sqrt[-(c^2*d*e)])*(I + c*x))/((c^2*d -
 e)*(c^2*d + c*Sqrt[-(c^2*d*e)]*x))] + (ArcCos[-((c^2*d + e)/(c^2*d - e))]
- (2*I)*(ArcTanh[(c*d)/(Sqrt[-(c^2*d*e)]*x)] + ArcTanh[(c*e*x)/Sqrt[-(c^2*d
*e)]]))*Log[(Sqrt[2]*Sqrt[-(c^2*d*e)])/(Sqrt[c^2*d - e]*E^(I*ArcTan[c*x])*S
qrt[c^2*d + e + (c^2*d - e)*Cos[2*ArcTan[c*x]]])] + (ArcCos[-((c^2*d + e)/(
c^2*d - e))] + (2*I)*(ArcTanh[(c*d)/(Sqrt[-(c^2*d*e)]*x)] + ArcTanh[(c*e*x)
/Sqrt[-(c^2*d*e)]]))*Log[(Sqrt[2]*Sqrt[-(c^2*d*e)]*E^(I*ArcTan[c*x]))/(Sqrt
[c^2*d - e]*Sqrt[c^2*d + e + (c^2*d - e)*Cos[2*ArcTan[c*x]]])] + I*(PolyLog
[2, ((c^2*d + e - (2*I)*Sqrt[-(c^2*d*e)])*(c^2*d - c*Sqrt[-(c^2*d*e)]*x))/(
(c^2*d - e)*(c^2*d + c*Sqrt[-(c^2*d*e)]*x))] - PolyLog[2, ((c^2*d + e + (2*
I)*Sqrt[-(c^2*d*e)])*(c^2*d - c*Sqrt[-(c^2*d*e)]*x))/((c^2*d - e)*(c^2*d +
c*Sqrt[-(c^2*d*e)]*x))]))/Sqrt[-(c^2*d*e)] + (4*ArcTan[c*x]*Sin[2*ArcTan[c*
x]])/(c^2*d + e + (c^2*d - e)*Cos[2*ArcTan[c*x]])))/(8*d)
```

Maple [B] time = 0.767, size = 2315, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/(e*x^2+d)^2,x)
```

[Out] -1/4*I/c*b*(c^2*e*d)^(1/2)/d^2/(c^2*d-e)*arctan(c*x)*ln(1-(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d-2*(c^2*e*d)^(1/2)-e))+3/4*I*c^3*b*ln(1-(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d+2*(c^2*e*d)^(1/2)-e))*arctan(c*x)/(c^4*d^2-2*c^2*d*e+e^2)/(c^2*d-e)*(c^2*e*d)^(1/2)+1/8/c*b*e^2*polylog(2,(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d+2*(c^2*e*d)^(1/2)-e))/d^2/(c^2*d-e)/(c^4*d^2-2*c^2*d*e+e^2)*(c^2*e*d)^(1/2)+1/4/c*b*e^2*arctan(c*x)^2/d^2/(c^2*d-e)/(c^4*d^2-2*c^2*d*e+e^2)*(c^2*e*d)^(1/2)-3/4*c*b*arctan(c*x)^2/d/(c^2*d-e)/(c^4*d^2-2*c^2*d*e+e^2)*(c^2*e*d)^(1/2)*e-1/4*c^5*b*d*arctan(c*x)^2/e/(c^2*d-e)/(c^4*d^2-2*c^2*d*e+e^2)*(c^2*e*d)^(1/2)-3/8*c*b*polylog(2,(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d+2*(c^2*e*d)^(1/2)-e))/d/(c^2*d-e)/(c^4*d^2-2*c^2*d*e+e^2)*(c^2*e*d)^(1/2)*e-1/8*c^5*b*d*polylog(2,(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d+2*(c^2*e*d)^(1/2)-e))/e/(c^2*d-e)/(c^4*d^2-2*c^2*d*e+e^2)*(c^2*e*d)^(1/2)-1/2*c^2*b*arctan(c*x)/d/(c^2*d-e)/(c^2*e*x^2+c^2*d)*x*e+1/8*c*b*(c^2*e*d)^(1/2)/(c^2*d-e)/d/e*polylog(2,(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d-2*(c^2*e*d)^(1/2)-e))+1/4*c^2*b*(d*e)^(1/2)/d/e*arctanh(1/4*(2*(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)+2*c^2*d+2*e)/c/(d*e)^(1/2))/(c^2*d-e)+1/4*c*b*(c^2*e*d)^(1/2)/(c^2*d-e)/d/e*arctan(c*x)^2+1/4*c^3*b/(c^2*d-e)^2*ln((1+I*c*x)^4/(c^2*x^2+1)^2*c^2*d+2*c^2*d*(1+I*c*x)^2/(c^2*x^2+1)-(1+I*c*x)^4/(c^2*x^2+1)^2*e+c^2*d+2*(1+I*c*x)^2/(c^2*x^2+1)*e-e)-c^3*b/(c^2*d-e)^2*ln((1+I*c*x)/(c^2*x^2+1)^(1/2))+1/2*a/d/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))+1/2*I*c^3*b*arctan(c*x)/d/(c^2*d-e)/(c^2*e*x^2+c^2*d)*x^2*e+1/4*I*c*b*(c^2*e*d)^(1/2)/(c^2*d-e)/d/e*arctan(c*x)*ln(1-(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d-2*(c^2*e*d)^(1/2)-e))-1/4*I*c^5*b*ln(1-(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d+2*(c^2*e*d)^(1/2)-e))*arctan(c*x)/(c^2*d-e)/e/(c^4*d^2-2*c^2*d*e+e^2)*(c^2*e*d)^(1/2)*d-3/4*I*c*b*ln(1-(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d+2*(c^2*e*d)^(1/2)-e))*arctan(c*x)*e/d/(c^2*d-e)/(c^4*d^2-2*c^2*d*e+e^2)*(c^2*e*d)^(1/2)+1/4*I/c*b*e^2*ln(1-(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d+2*(c^2*e*d)^(1/2)-e))*arctan(c*x)/d^2/(c^2*d-e)/(c^4*d^2-2*c^2*d*e+e^2)*(c^2*e*d)^(1/2)+1/4*b*(d*e)^(1/2)/d^2*e*arctanh(1/4*(2*(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)+2*c^2*d+2*e)/c/(d*e)^(1/2))/(c^2*d-e)^2-1/4*c*b/d/(c^2*d-e)^2*e*ln((1+I*c*x)^4/(c^2*x^2+1)^2*c^2*d+2*c^2*d*(1+I*c*x)^2/(c^2*x^2+1)-(1+I*c*x)^4/(c^2*x^2+1)^2*e+c^2*d+2*(1+I*c*x)^2/(c^2*x^2+1)*e-e)+1/2*c^4*b*arctan(c*x)/(c^2*d-e)/(c^2*e*x^2+c^2*d)*x+c*b/d/(c^2*d-e)^2*e*ln((1+I*c*x)/(c^2*x^2+1)^(1/2))+3/4*c^3*b*arctan(c*x)^2/(c^4*d^2-2*c^2*d*e+e^2)/(c^2*d-e)*(c^2*e*d)^(1/2)-1/4*c^4*b*(d*e)^(1/2)/e*arctanh(1/4*(2*(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)+2*c^2*d+2*e)/c/(d*e)^(1/2))/(c^2*d-e)^2-1/8/c*b*(c^2*e*d)^(1/2)/d^2/(c^2*d-e)*polylog(2,(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d-2*(c^2*e*d)^(1/2)-e))-1/4/c*b*(c^2*e*d)^(1/2)/d^2/(c^2*d-e)*arctan(c*x)^2+3/8*c^3*b*polylog(2,(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d+2*(c^2*e*d)^(1/2)-e))/(c^4*d^2-2*c^2*d*e+e^2)/(c^2*d-e)*(c^2*e*d)^(1/2)+1/2*I*c^3*b*arctan(c*x)/(c^2*d-e)/(c^2*e*x^2+c^2*d)+1/2*c^2*a*x/d/(c^2*e*x^2+c^2*d)+1/4*b*(d*e)^(1/2)/d^2*arctanh(1/4*(2*(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)+2*c^2*d+2*e)/c/(d*e)^(1/2))/(c^2*d-e)

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b\arctan\left(cx\right)+a}{e^2x^4+2\,dex^2+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*arctan(c*x) + a)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/(e*x**2+d)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\arctan\left(cx\right)+a}{\left(ex^2+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)/(e*x^2 + d)^2, x)
```

3.1164 $\int \frac{a+b\tan^{-1}(cx)}{x^2(d+ex^2)^2}\,dx$

Optimal. Leaf size=1382

result too large to display

```
[Out] -((a + b*ArcTan[c*x])/(d^2*x)) - (e*x*(a + b*ArcTan[c*x]))/(2*d^2*(d + e*x^
2)) - (a*Sqrt[e]*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/d^(5/2) - (Sqrt[e]*(a + b*Arc
Tan[c*x])*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(5/2)) + (b*c*Log[x])/d^2 + ((I
/4)*b*Sqrt[e]*Log[1 + I*c*x]*Log[(c*(Sqrt[-d] - Sqrt[e]*x))/(c*Sqrt[-d] - I
*Sqrt[e])])/(-d)^(5/2) - ((I/4)*b*Sqrt[e]*Log[1 - I*c*x]*Log[(c*(Sqrt[-d] -
 Sqrt[e]*x))/(c*Sqrt[-d] + I*Sqrt[e])])/(-d)^(5/2) + ((I/4)*b*Sqrt[e]*Log[1
 - I*c*x]*Log[(c*(Sqrt[-d] + Sqrt[e]*x))/(c*Sqrt[-d] - I*Sqrt[e])])/(-d)^(5
/2) - ((I/4)*b*Sqrt[e]*Log[1 + I*c*x]*Log[(c*(Sqrt[-d] + Sqrt[e]*x))/(c*Sqr
t[-d] + I*Sqrt[e])])/(-d)^(5/2) - ((I/8)*b*c*Sqrt[e]*Log[(Sqrt[e]*(1 - Sqrt
[-c^2]*x))/(I*Sqrt[-c^2]*Sqrt[d] + Sqrt[e])]*Log[1 - (I*Sqrt[e]*x)/Sqrt[d]]
)/(Sqrt[-c^2]*d^(5/2)) + ((I/8)*b*c*Sqrt[e]*Log[-((Sqrt[e]*(1 + Sqrt[-c^2]*
x))/(I*Sqrt[-c^2]*Sqrt[d] - Sqrt[e]))]*Log[1 - (I*Sqrt[e]*x)/Sqrt[d]])/(Sqr
t[-c^2]*d^(5/2)) + ((I/8)*b*c*Sqrt[e]*Log[-((Sqrt[e]*(1 - Sqrt[-c^2]*x))/(I
*Sqrt[-c^2]*Sqrt[d] - Sqrt[e]))]*Log[1 + (I*Sqrt[e]*x)/Sqrt[d]])/(Sqrt[-c^2
]*d^(5/2)) - ((I/8)*b*c*Sqrt[e]*Log[(Sqrt[e]*(1 + Sqrt[-c^2]*x))/(I*Sqrt[-c
^2]*Sqrt[d] + Sqrt[e])]*Log[1 + (I*Sqrt[e]*x)/Sqrt[d]])/(Sqrt[-c^2]*d^(5/2)
) - (b*c*Log[1 + c^2*x^2])/(2*d^2) + (b*c*e*Log[1 + c^2*x^2])/(4*d^2*(c^2*d
 - e)) - (b*c*e*Log[d + e*x^2])/(4*d^2*(c^2*d - e)) - ((I/4)*b*Sqrt[e]*Poly
Log[2, (Sqrt[e]*(I - c*x))/(c*Sqrt[-d] + I*Sqrt[e])])/(-d)^(5/2) + ((I/4)*b
*Sqrt[e]*PolyLog[2, (Sqrt[e]*(1 - I*c*x))/(I*c*Sqrt[-d] + Sqrt[e])])/(-d)^(
5/2) + ((I/4)*b*Sqrt[e]*PolyLog[2, (Sqrt[e]*(1 + I*c*x))/(I*c*Sqrt[-d] + Sq
rt[e])])/(-d)^(5/2) - ((I/4)*b*Sqrt[e]*PolyLog[2, (Sqrt[e]*(I + c*x))/(c*Sq
rt[-d] + I*Sqrt[e])])/(-d)^(5/2) - ((I/8)*b*c*Sqrt[e]*PolyLog[2, (Sqrt[-c^2
]*(Sqrt[d] - I*Sqrt[e]*x))/(Sqrt[-c^2]*Sqrt[d] - I*Sqrt[e])])/(Sqrt[-c^2]*d
^(5/2)) + ((I/8)*b*c*Sqrt[e]*PolyLog[2, (Sqrt[-c^2]*(Sqrt[d] - I*Sqrt[e]*x)
)/(Sqrt[-c^2]*Sqrt[d] + I*Sqrt[e])])/(Sqrt[-c^2]*d^(5/2)) - ((I/8)*b*c*Sqrt
[e]*PolyLog[2, (Sqrt[-c^2]*(Sqrt[d] + I*Sqrt[e]*x))/(Sqrt[-c^2]*Sqrt[d] - I
*Sqrt[e])])/(Sqrt[-c^2]*d^(5/2)) + ((I/8)*b*c*Sqrt[e]*PolyLog[2, (Sqrt[-c^2
]*(Sqrt[d] + I*Sqrt[e]*x))/(Sqrt[-c^2]*Sqrt[d] + I*Sqrt[e])])/(Sqrt[-c^2]*d
^(5/2))
```

Rubi [A] time = 1.58037, antiderivative size = 1382, normalized size of antiderivative = 1., number of steps used = 50, number of rules used = 17, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.81, Rules used = {4980, 4852, 266, 36, 29, 31, 199, 205, 4912, 6725, 444, 4908, 2409, 2394, 2393, 2391, 4910}

$$-\frac{\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(a+b\tan^{-1}(cx)\right)}{2d^{5/2}}-\frac{a+b\tan^{-1}(cx)}{d^2x}-\frac{ex\left(a+b\tan^{-1}(cx)\right)}{2d^2\left(ex^2+d\right)}-\frac{a\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{5/2}}+\frac{bc\log(x)}{d^2}+\frac{ib\sqrt{e}\log(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])/(x^2*(d + e*x^2)^2),x]
```

```
[Out] -((a + b*ArcTan[c*x])/(d^2*x)) - (e*x*(a + b*ArcTan[c*x]))/(2*d^2*(d + e*x^
2)) - (a*Sqrt[e]*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/d^(5/2) - (Sqrt[e]*(a + b*Arc
Tan[c*x])*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(5/2)) + (b*c*Log[x])/d^2 + ((I
/4)*b*Sqrt[e]*Log[1 + I*c*x]*Log[(c*(Sqrt[-d] - Sqrt[e]*x))/(c*Sqrt[-d] - I
*Sqrt[e])])/(-d)^(5/2) - ((I/4)*b*Sqrt[e]*Log[1 - I*c*x]*Log[(c*(Sqrt[-d] -
 Sqrt[e]*x))/(c*Sqrt[-d] + I*Sqrt[e])])/(-d)^(5/2) + ((I/4)*b*Sqrt[e]*Log[1
 - I*c*x]*Log[(c*(Sqrt[-d] + Sqrt[e]*x))/(c*Sqrt[-d] - I*Sqrt[e])])/(-d)^(5
/2) - ((I/4)*b*Sqrt[e]*Log[1 + I*c*x]*Log[(c*(Sqrt[-d] + Sqrt[e]*x))/(c*Sqr
t[-d] + I*Sqrt[e])])/(-d)^(5/2) - ((I/8)*b*c*Sqrt[e]*Log[(Sqrt[e]*(1 - Sqrt
[-c^2]*x))/(I*Sqrt[-c^2]*Sqrt[d] + Sqrt[e])]*Log[1 - (I*Sqrt[e]*x)/Sqrt[d]]
)/(Sqrt[-c^2]*d^(5/2)) + ((I/8)*b*c*Sqrt[e]*Log[-((Sqrt[e]*(1 + Sqrt[-c^2]*
x))/(I*Sqrt[-c^2]*Sqrt[d] - Sqrt[e]))]*Log[1 - (I*Sqrt[e]*x)/Sqrt[d]])/(Sqr
t[-c^2]*d^(5/2)) + ((I/8)*b*c*Sqrt[e]*Log[-((Sqrt[e]*(1 - Sqrt[-c^2]*x))/(I
*Sqrt[-c^2]*Sqrt[d] - Sqrt[e]))]*Log[1 + (I*Sqrt[e]*x)/Sqrt[d]])/(Sqrt[-c^2
]*d^(5/2)) - ((I/8)*b*c*Sqrt[e]*Log[(Sqrt[e]*(1 + Sqrt[-c^2]*x))/(I*Sqrt[-c
^2]*Sqrt[d] + Sqrt[e])]*Log[1 + (I*Sqrt[e]*x)/Sqrt[d]])/(Sqrt[-c^2]*d^(5/2)
) - (b*c*Log[1 + c^2*x^2])/(2*d^2) + (b*c*e*Log[1 + c^2*x^2])/(4*d^2*(c^2*d
 - e)) - (b*c*e*Log[d + e*x^2])/(4*d^2*(c^2*d - e)) - ((I/4)*b*Sqrt[e]*Poly
Log[2, (Sqrt[e]*(I - c*x))/(c*Sqrt[-d] + I*Sqrt[e])])/(-d)^(5/2) + ((I/4)*b
*Sqrt[e]*PolyLog[2, (Sqrt[e]*(1 - I*c*x))/(I*c*Sqrt[-d] + Sqrt[e])])/(-d)^(
5/2) + ((I/4)*b*Sqrt[e]*PolyLog[2, (Sqrt[e]*(1 + I*c*x))/(I*c*Sqrt[-d] + Sq
rt[e])])/(-d)^(5/2) - ((I/4)*b*Sqrt[e]*PolyLog[2, (Sqrt[e]*(I + c*x))/(c*Sq
rt[-d] + I*Sqrt[e])])/(-d)^(5/2) - ((I/8)*b*c*Sqrt[e]*PolyLog[2, (Sqrt[-c^2
]*(Sqrt[d] - I*Sqrt[e]*x))/(Sqrt[-c^2]*Sqrt[d] - I*Sqrt[e])])/(Sqrt[-c^2]*d
^(5/2)) + ((I/8)*b*c*Sqrt[e]*PolyLog[2, (Sqrt[-c^2]*(Sqrt[d] - I*Sqrt[e]*x)
)/(Sqrt[-c^2]*Sqrt[d] + I*Sqrt[e])])/(Sqrt[-c^2]*d^(5/2)) - ((I/8)*b*c*Sqrt
[e]*PolyLog[2, (Sqrt[-c^2]*(Sqrt[d] + I*Sqrt[e]*x))/(Sqrt[-c^2]*Sqrt[d] - I
*Sqrt[e])])/(Sqrt[-c^2]*d^(5/2)) + ((I/8)*b*c*Sqrt[e]*PolyLog[2, (Sqrt[-c^2
]*(Sqrt[d] + I*Sqrt[e]*x))/(Sqrt[-c^2]*Sqrt[d] + I*Sqrt[e])])/(Sqrt[-c^2]*d
^(5/2))
```

Rule 4980

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
```

```
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 4912

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcTan[c*x], u, x]
- Dist[b*c, Int[u/(1 + c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && (I
ntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 4908

```
Int[ArcTan[(c_.)*(x_)]/((d_.) + (e_.)*(x_)^2), x_Symbol] :> Dist[I/2, Int[L
og[1 - I*c*x]/(d + e*x^2), x], x] - Dist[I/2, Int[Log[1 + I*c*x]/(d + e*x^2
), x], x] /; FreeQ[{c, d, e}, x]
```

Rule 2409

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
```

```
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4910

```
Int[(ArcTan[(c_.)*(x_)]*(b_.) + (a_))/((d_.) + (e_.)*(x_)^2), x_Symbol] :>
Dist[a, Int[1/(d + e*x^2), x], x] + Dist[b, Int[ArcTan[c*x]/(d + e*x^2), x]
, x] /; FreeQ[{a, b, c, d, e}, x]
```

Rubi steps

$$\int \frac{a+b\tan^{-1}(cx)}{x^2\left(d+ex^2\right)^2}\,dx=\int\left(\frac{a+b\tan^{-1}(cx)}{d^2x^2}-\frac{e\left(a+b\tan^{-1}(cx)\right)}{d\left(d+ex^2\right)^2}-\frac{e\left(a+b\tan^{-1}(cx)\right)}{d^2\left(d+ex^2\right)}\right)dx$$

$$=\frac{\int\frac{a+b\tan^{-1}(cx)}{x^2}\,dx}{d^2}-\frac{e\int\frac{a+b\tan^{-1}(cx)}{d+ex^2}\,dx}{d^2}-\frac{e\int\frac{a+b\tan^{-1}(cx)}{\left(d+ex^2\right)^2}\,dx}{d}$$

$$=-\frac{a+b\tan^{-1}(cx)}{d^2x}-\frac{ex\left(a+b\tan^{-1}(cx)\right)}{2d^2\left(d+ex^2\right)}-\frac{\sqrt{e}\left(a+b\tan^{-1}(cx)\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{5/2}}+\frac{(bc)\int\frac{1}{x\left(1+c^2x^2\right)}\,d}{d^2}$$

$$=-\frac{a+b\tan^{-1}(cx)}{d^2x}-\frac{ex\left(a+b\tan^{-1}(cx)\right)}{2d^2\left(d+ex^2\right)}-\frac{a\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{5/2}}-\frac{\sqrt{e}\left(a+b\tan^{-1}(cx)\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{5/2}}$$

$$=-\frac{a+b\tan^{-1}(cx)}{d^2x}-\frac{ex\left(a+b\tan^{-1}(cx)\right)}{2d^2\left(d+ex^2\right)}-\frac{a\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{5/2}}-\frac{\sqrt{e}\left(a+b\tan^{-1}(cx)\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{5/2}}$$

$$=-\frac{a+b\tan^{-1}(cx)}{d^2x}-\frac{ex\left(a+b\tan^{-1}(cx)\right)}{2d^2\left(d+ex^2\right)}-\frac{a\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{5/2}}-\frac{\sqrt{e}\left(a+b\tan^{-1}(cx)\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{5/2}}$$

$$=-\frac{a+b\tan^{-1}(cx)}{d^2x}-\frac{ex\left(a+b\tan^{-1}(cx)\right)}{2d^2\left(d+ex^2\right)}-\frac{a\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{5/2}}-\frac{\sqrt{e}\left(a+b\tan^{-1}(cx)\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{5/2}}$$

$$=-\frac{a+b\tan^{-1}(cx)}{d^2x}-\frac{ex\left(a+b\tan^{-1}(cx)\right)}{2d^2\left(d+ex^2\right)}-\frac{a\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{5/2}}-\frac{\sqrt{e}\left(a+b\tan^{-1}(cx)\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{5/2}}$$

$$=-\frac{a+b\tan^{-1}(cx)}{d^2x}-\frac{ex\left(a+b\tan^{-1}(cx)\right)}{2d^2\left(d+ex^2\right)}-\frac{a\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{5/2}}-\frac{\sqrt{e}\left(a+b\tan^{-1}(cx)\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{5/2}}$$

$$=-\frac{a+b\tan^{-1}(cx)}{d^2x}-\frac{ex\left(a+b\tan^{-1}(cx)\right)}{2d^2\left(d+ex^2\right)}-\frac{a\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{5/2}}-\frac{\sqrt{e}\left(a+b\tan^{-1}(cx)\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{5/2}}$$

$$=-\frac{a+b\tan^{-1}(cx)}{d^2x}-\frac{ex\left(a+b\tan^{-1}(cx)\right)}{2d^2\left(d+ex^2\right)}-\frac{a\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{5/2}}-\frac{\sqrt{e}\left(a+b\tan^{-1}(cx)\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{5/2}}$$

Mathematica [A] time = 12.9503, size = 982, normalized size = 0.71

$$b\left(-\frac{e\sin\left(2\tan^{-1}(cx)\right)\tan^{-1}(cx)}{2c^4d^2\left(dc^2+d\cos\left(2\tan^{-1}(cx)\right)c^2+e-e\cos\left(2\tan^{-1}(cx)\right)\right)}-\frac{\tan^{-1}(cx)}{c^5d^2x}+\frac{\log\left(\frac{cx}{\sqrt{c^2x^2+1}}\right)}{c^4d^2}-\frac{e\log\left(\frac{\left(c^2d-e\right)\cos\left(2\tan^{-1}\right.}{dc^2+e}\right.}{4c^4d^2\left(c^2d-e\right.}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c*x])/(x^2*(d + e*x^2)^2),x]
```

```
[Out] -(a/(d^2*x)) - (a*e*x)/(2*d^2*(d + e*x^2)) - (3*a*Sqrt[e]*ArcTan[(Sqrt[e]*x
)/Sqrt[d]])/(2*d^(5/2)) + b*c^5*(-(ArcTan[c*x]/(c^5*d^2*x)) + Log[(c*x)/Sqr
t[1 + c^2*x^2]]/(c^4*d^2) - (e*Log[1 + ((c^2*d - e)*Cos[2*ArcTan[c*x]])/(c^
2*d + e)])/(4*c^4*d^2*(c^2*d - e)) - (3*e*(4*ArcTan[c*x]*ArcTanh[(c*d)/(Sqr
t[-(c^2*d*e)]*x)] + 2*ArcCos[-((c^2*d + e)/(c^2*d - e))]*ArcTanh[(c*e*x)/Sq
rt[-(c^2*d*e)]] - (ArcCos[-((c^2*d + e)/(c^2*d - e))] - (2*I)*ArcTanh[(c*e*
x)/Sqrt[-(c^2*d*e)]])*Log[1 - ((c^2*d + e - (2*I)*Sqrt[-(c^2*d*e)])*(2*c^2*
d - 2*c*Sqrt[-(c^2*d*e)]*x))/((c^2*d - e)*(2*c^2*d + 2*c*Sqrt[-(c^2*d*e)]*x
))] + (-ArcCos[-((c^2*d + e)/(c^2*d - e))] - (2*I)*ArcTanh[(c*e*x)/Sqrt[-(c
^2*d*e)]])*Log[1 - ((c^2*d + e + (2*I)*Sqrt[-(c^2*d*e)])*(2*c^2*d - 2*c*Sqr
t[-(c^2*d*e)]*x))/((c^2*d - e)*(2*c^2*d + 2*c*Sqrt[-(c^2*d*e)]*x))] + (ArcC
os[-((c^2*d + e)/(c^2*d - e))] - (2*I)*(ArcTanh[(c*d)/(Sqrt[-(c^2*d*e)]*x)]
 + ArcTanh[(c*e*x)/Sqrt[-(c^2*d*e)]]))*Log[(Sqrt[2]*Sqrt[-(c^2*d*e)])/(Sqrt
[c^2*d - e]*E^(I*ArcTan[c*x])*Sqrt[c^2*d + e + (c^2*d - e)*Cos[2*ArcTan[c*x
]]])] + (ArcCos[-((c^2*d + e)/(c^2*d - e))] + (2*I)*(ArcTanh[(c*d)/(Sqrt[-(
c^2*d*e)]*x)] + ArcTanh[(c*e*x)/Sqrt[-(c^2*d*e)]]))*Log[(Sqrt[2]*Sqrt[-(c^2
*d*e)]*E^(I*ArcTan[c*x]))/(Sqrt[c^2*d - e]*Sqrt[c^2*d + e + (c^2*d - e)*Cos
[2*ArcTan[c*x]]])] + I*(PolyLog[2, ((c^2*d + e - (2*I)*Sqrt[-(c^2*d*e)])*(2
*c^2*d - 2*c*Sqrt[-(c^2*d*e)]*x))/((c^2*d - e)*(2*c^2*d + 2*c*Sqrt[-(c^2*d*
e)]*x))] - PolyLog[2, ((c^2*d + e + (2*I)*Sqrt[-(c^2*d*e)])*(2*c^2*d - 2*c*
Sqrt[-(c^2*d*e)]*x))/((c^2*d - e)*(2*c^2*d + 2*c*Sqrt[-(c^2*d*e)]*x))])))/(
8*c^4*d^2*Sqrt[-(c^2*d*e)]) - (e*ArcTan[c*x]*Sin[2*ArcTan[c*x]])/(2*c^4*d^2
*(c^2*d + e + c^2*d*Cos[2*ArcTan[c*x]] - e*Cos[2*ArcTan[c*x]])))
```

Maple [C] time = 0.638, size = 3851, normalized size = 2.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/x^2/(e*x^2+d)^2,x)
```

```
[Out] 3/8*b*c^8*(d*e)^(1/2)*d/e*arctanh(1/4*(2*(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)+
2*c^2*d+2*e)/c/(d*e)^(1/2))/(c^2*d-e)^3-1/8*b*c^4*(d*e)^(1/2)/d*e*arctanh(1
/4*(2*(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)+2*c^2*d+2*e)/c/(d*e)^(1/2))/(c^2*d-
e)^3-3/16*b/c^2*(d*e)^(1/2)/d^4*e^4*arctanh(1/4*(2*(c^2*d-e)*(1+I*c*x)^2/(c
^2*x^2+1)+2*c^2*d+2*e)/c/(d*e)^(1/2))/(c^2*d-e)^3+b*c^2*(d*e)^(1/2)/d^2*e^2
*arctanh(1/4*(2*(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)+2*c^2*d+2*e)/c/(d*e)^(1/2
))/(c^2*d-e)^3+3/8*b*c^4*(d*e)^(1/2)/d/e*arctanh(1/4*(2*(c^2*d-e)*(1+I*c*x)
^2/(c^2*x^2+1)+2*c^2*d+2*e)/c/(d*e)^(1/2))/(c^2*d-e)+3/16*b/c^2*(d*e)^(1/2)
/d^4*e^2*arctanh(1/4*(2*(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)+2*c^2*d+2*e)/c/(d
*e)^(1/2))/(c^2*d-e)+2*b*c^2*(d*e)^(1/2)/d^2*e*arctanh(1/4*(2*(c^2*d-e)*(1+
I*c*x)^2/(c^2*x^2+1)+2*c^2*d+2*e)/c/(d*e)^(1/2))/(c^2*d-e)^2+3/2*b*arctan(c
*x)/(c^2*d-e)/d^2/(c^2*e*x^2+c^2*d)*c^2*x*e^2+b*c^2*arctan(c*x)/(c^2*d-e)/d
/(c^2*e*x^2+c^2*d)/x*e-3/2*b*c^4*arctan(c*x)/d/(c^2*d-e)/(c^2*e*x^2+c^2*d)*
x*e-3/2*I*b*c^3*arctan(c*x)/(c^2*d-e)/d/(c^2*e*x^2+c^2*d)*e-1/8*b*c^5/(c^2*
d-e)^2*ln((1+I*c*x)^4/(c^2*x^2+1)^2*c^2*d+2*c^2*d*(1+I*c*x)^2/(c^2*x^2+1)-(
1+I*c*x)^4/(c^2*x^2+1)^2*e+c^2*d+2*(1+I*c*x)^2/(c^2*x^2+1)*e-e)-19/16*b*c^5
/(c^2*d-e)^3*ln((1+I*c*x)^4/(c^2*x^2+1)^2*c^2*d+2*c^2*d*(1+I*c*x)^2/(c^2*x^
2+1)-(1+I*c*x)^4/(c^2*x^2+1)^2*e+c^2*d+2*(1+I*c*x)^2/(c^2*x^2+1)*e-e)*e-13/
16*b*c^6*(d*e)^(1/2)*arctanh(1/4*(2*(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)+2*c^2
*d+2*e)/c/(d*e)^(1/2))/(c^2*d-e)^3-3/16*b/c/(c^2*d-e)^3/d^3*e^4*ln((1+I*c*x
)^4/(c^2*x^2+1)^2*c^2*d+2*c^2*d*(1+I*c*x)^2/(c^2*x^2+1)-(1+I*c*x)^4/(c^2*x^
2+1)^2*e+c^2*d+2*(1+I*c*x)^2/(c^2*x^2+1)*e-e)-3/8*b/c/(c^2*d-e)/d^3*e^2*sum
((_R1^2*c^2*d-_R1^2*e+3*c^2*d+e)/(_R1^2*c^2*d-_R1^2*e+c^2*d+e)*(I*arctan(c*
x)*ln((_R1-(1+I*c*x)/(c^2*x^2+1)^(1/2))/_R1)+dilog((_R1-(1+I*c*x)/(c^2*x^2+
1)^(1/2))/_R1)),_R1=RootOf((c^2*d-e)*_Z^4+(2*c^2*d+2*e)*_Z^2+c^2*d-e))+1/2*
b*c^4*(d*e)^(1/2)/d*arctanh(1/4*(2*(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)+2*c^2*
d+2*e)/c/(d*e)^(1/2))/(c^2*d-e)^2+5*b*c^3/d/(c^2*d-e)^2*e*ln((1+I*c*x)/(c^2
*x^2+1)^(1/2))+5/16*b*c^3/d/(c^2*d-e)^2*e*ln((1+I*c*x)^4/(c^2*x^2+1)^2*c^2*
d+2*c^2*d*(1+I*c*x)^2/(c^2*x^2+1)-(1+I*c*x)^4/(c^2*x^2+1)^2*e+c^2*d+2*(1+I*
c*x)^2/(c^2*x^2+1)*e-e)+I*b*c^5*arctan(c*x)/(c^2*d-e)/(c^2*e*x^2+c^2*d)-b*c
^4*arctan(c*x)/(c^2*d-e)/(c^2*e*x^2+c^2*d)/x-3/4*b*c^6*(d*e)^(1/2)/e*arctan
h(1/4*(2*(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)+2*c^2*d+2*e)/c/(d*e)^(1/2))/(c^2
*d-e)^2+5/16*b*c^2*(d*e)^(1/2)/d^2*arctanh(1/4*(2*(c^2*d-e)*(1+I*c*x)^2/(c^
2*x^2+1)+2*c^2*d+2*e)/c/(d*e)^(1/2))/(c^2*d-e)-3*c*b/(c^2*d-e)^2/d^2*e^2*ln
((1+I*c*x)/(c^2*x^2+1)^(1/2))-c*b/(c^2*d-e)/d^2*e*ln((1+I*c*x)/(c^2*x^2+1)^
(1/2)-1)-c*b/(c^2*d-e)/d^2*e*ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))-3/8*c*b/(c^2
*d-e)/d^2*e*sum((_R1^2*c^2*d-_R1^2*e-c^2*d+e)/(_R1^2*c^2*d-_R1^2*e+c^2*d+e)
*(I*arctan(c*x)*ln((_R1-(1+I*c*x)/(c^2*x^2+1)^(1/2))/_R1)+dilog((_R1-(1+I*c
*x)/(c^2*x^2+1)^(1/2))/_R1)),_R1=RootOf((c^2*d-e)*_Z^4+(2*c^2*d+2*e)*_Z^2+c
^2*d-e))+3/8*c*b/(c^2*d-e)/d^2*e*sum((_R1^2*c^2*d-_R1^2*e+3*c^2*d+e)/(_R1^2
*c^2*d-_R1^2*e+c^2*d+e)*(I*arctan(c*x)*ln((_R1-(1+I*c*x)/(c^2*x^2+1)^(1/2))
/_R1)+dilog((_R1-(1+I*c*x)/(c^2*x^2+1)^(1/2))/_R1)),_R1=RootOf((c^2*d-e)*_Z
```

```
^4+(2*c^2*d+2*e)*_Z^2+c^2*d-e))-3/8*c*b/(c^2*d-e)^2/d^2*e^2*ln((1+I*c*x)^4/
(c^2*x^2+1)^2*c^2*d+2*c^2*d*(1+I*c*x)^2/(c^2*x^2+1)-(1+I*c*x)^4/(c^2*x^2+1)
^2*e+c^2*d+2*(1+I*c*x)^2/(c^2*x^2+1)*e-e)-1/16*c*b/(c^2*d-e)^3/d^2*e^3*ln((
1+I*c*x)^4/(c^2*x^2+1)^2*c^2*d+2*c^2*d*(1+I*c*x)^2/(c^2*x^2+1)-(1+I*c*x)^4/
(c^2*x^2+1)^2*e+c^2*d+2*(1+I*c*x)^2/(c^2*x^2+1)*e-e)+3/8*b/c/(c^2*d-e)/d^3*
e^2*ln((1+I*c*x)^4/(c^2*x^2+1)^2*c^2*d+2*c^2*d*(1+I*c*x)^2/(c^2*x^2+1)-(1+I
*c*x)^4/(c^2*x^2+1)^2*e+c^2*d+2*(1+I*c*x)^2/(c^2*x^2+1)*e-e)+3/16*b/c/(c^2*
d-e)^2/d^3*e^3*ln((1+I*c*x)^4/(c^2*x^2+1)^2*c^2*d+2*c^2*d*(1+I*c*x)^2/(c^2*
x^2+1)-(1+I*c*x)^4/(c^2*x^2+1)^2*e+c^2*d+2*(1+I*c*x)^2/(c^2*x^2+1)*e-e)+17/
16*b*c^3/(c^2*d-e)^3/d*e^2*ln((1+I*c*x)^4/(c^2*x^2+1)^2*c^2*d+2*c^2*d*(1+I*
c*x)^2/(c^2*x^2+1)-(1+I*c*x)^4/(c^2*x^2+1)^2*e+c^2*d+2*(1+I*c*x)^2/(c^2*x^2
+1)*e-e)+3/8*b/c/(c^2*d-e)/d^3*e^2*sum((_R1^2*c^2*d-_R1^2*e-c^2*d+e)/(_R1^2
*c^2*d-_R1^2*e+c^2*d+e)*(I*arctan(c*x)*ln((_R1-(1+I*c*x)/(c^2*x^2+1)^(1/2))
/_R1)+dilog((_R1-(1+I*c*x)/(c^2*x^2+1)^(1/2))/_R1)),_R1=RootOf((c^2*d-e)*_Z
^4+(2*c^2*d+2*e)*_Z^2+c^2*d-e))-1/4*b*(d*e)^(1/2)/d^3*e^3*arctanh(1/4*(2*(c
^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)+2*c^2*d+2*e)/c/(d*e)^(1/2))/(c^2*d-e)^3-9/8
*b*(d*e)^(1/2)/d^3*e*arctanh(1/4*(2*(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)+2*c^2
*d+2*e)/c/(d*e)^(1/2))/(c^2*d-e)-7/4*b*(d*e)^(1/2)/d^3*e^2*arctanh(1/4*(2*(
c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)+2*c^2*d+2*e)/c/(d*e)^(1/2))/(c^2*d-e)^2-1/
2*a/d^2*e*c^2*x/(c^2*e*x^2+c^2*d)+b*c^3/(c^2*d-e)/d*ln(1+(1+I*c*x)/(c^2*x^2
+1)^(1/2))-1/4*b*c^3/(c^2*d-e)/d*ln((1+I*c*x)^4/(c^2*x^2+1)^2*c^2*d+2*c^2*d
*(1+I*c*x)^2/(c^2*x^2+1)-(1+I*c*x)^4/(c^2*x^2+1)^2*e+c^2*d+2*(1+I*c*x)^2/(c
^2*x^2+1)*e-e)+3/8*b*c^7/(c^2*d-e)^3*d*ln((1+I*c*x)^4/(c^2*x^2+1)^2*c^2*d+2
*c^2*d*(1+I*c*x)^2/(c^2*x^2+1)-(1+I*c*x)^4/(c^2*x^2+1)^2*e+c^2*d+2*(1+I*c*x
)^2/(c^2*x^2+1)*e-e)+b*c^3/(c^2*d-e)/d*ln((1+I*c*x)/(c^2*x^2+1)^(1/2)-1)-3/
2*a/d^2*e/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))+I*b*c^5*arctan(c*x)/(c^2*d-e)
/d/(c^2*e*x^2+c^2*d)*x^2*e-3/2*I*b*arctan(c*x)/(c^2*d-e)/d^2/(c^2*e*x^2+c^2
*d)*c^3*x^2*e^2-a/d^2/x-2*b*c^5/(c^2*d-e)^2*ln((1+I*c*x)/(c^2*x^2+1)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^2/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b\arctan(cx)+a}{e^2x^6+2\,dex^4+d^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^2/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*arctan(c*x) + a)/(e^2*x^6 + 2*d*e*x^4 + d^2*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/x**2/(e*x**2+d)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{b\arctan(cx)+a}{(ex^2+d)^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^2/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)/((e*x^2 + d)^2*x^2), x)
```

3.1165 $\int \frac{x^5\left(a+b\tan^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx$

Optimal. Leaf size=532

$$-\frac{ib\text{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{(1-icx)\left(c\sqrt{-d}-i\sqrt{e}\right)}\right)}{4e^3}-\frac{ib\text{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-d}+\sqrt{e}x\right)}{(1-icx)\left(c\sqrt{-d}+i\sqrt{e}\right)}\right)}{4e^3}+\frac{ib\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2e^3}-\frac{d^2\left(a+b\tan\right.}{4e^3\left(d+ex\right.}$$

```
[Out] -(b*c*d*x)/(8*(c^2*d - e)*e^2*(d + e*x^2)) + (b*c^4*d^2*ArcTan[c*x])/(4*(c^
2*d - e)^2*e^3) - (b*c^2*d*ArcTan[c*x])/((c^2*d - e)*e^3) - (d^2*(a + b*Arc
Tan[c*x]))/(4*e^3*(d + e*x^2)^2) + (d*(a + b*ArcTan[c*x]))/(e^3*(d + e*x^2)
) + (b*c*Sqrt[d]*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/((c^2*d - e)*e^(5/2)) - (b*c*
Sqrt[d]*(3*c^2*d - e)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*(c^2*d - e)^2*e^(5/2)
) - ((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e^3 + ((a + b*ArcTan[c*x])*Log
[(2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/(2*e
^3) + ((a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] +
I*Sqrt[e])*(1 - I*c*x))])/(2*e^3) + ((I/2)*b*PolyLog[2, 1 - 2/(1 - I*c*x)])
/e^3 - ((I/4)*b*PolyLog[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] -
I*Sqrt[e])*(1 - I*c*x))])/e^3 - ((I/4)*b*PolyLog[2, 1 - (2*c*(Sqrt[-d] + Sq
rt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/e^3
```

Rubi [A] time = 0.650102, antiderivative size = 532, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 11, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.524, Rules used = {4980, 4974, 414, 522, 203, 205, 391, 4856, 2402, 2315, 2447}

$$-\frac{ib\text{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{(1-icx)\left(c\sqrt{-d}-i\sqrt{e}\right)}\right)}{4e^3}-\frac{ib\text{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-d}+\sqrt{e}x\right)}{(1-icx)\left(c\sqrt{-d}+i\sqrt{e}\right)}\right)}{4e^3}+\frac{ib\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2e^3}-\frac{d^2\left(a+b\tan\right.}{4e^3\left(d+ex\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*ArcTan[c*x]))/(d + e*x^2)^3,x]
```


```
[Out] -(b*c*d*x)/(8*(c^2*d - e)*e^2*(d + e*x^2)) + (b*c^4*d^2*ArcTan[c*x])/(4*(c^
2*d - e)^2*e^3) - (b*c^2*d*ArcTan[c*x])/((c^2*d - e)*e^3) - (d^2*(a + b*Arc
Tan[c*x]))/(4*e^3*(d + e*x^2)^2) + (d*(a + b*ArcTan[c*x]))/(e^3*(d + e*x^2)
) + (b*c*Sqrt[d]*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/((c^2*d - e)*e^(5/2)) - (b*c*
Sqrt[d]*(3*c^2*d - e)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*(c^2*d - e)^2*e^(5/2)
) - ((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e^3 + ((a + b*ArcTan[c*x])*Log
```

```
[(2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))]])/(2*e
^3) + ((a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] +
I*Sqrt[e])*(1 - I*c*x))]])/(2*e^3) + ((I/2)*b*PolyLog[2, 1 - 2/(1 - I*c*x)])
/e^3 - ((I/4)*b*PolyLog[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] -
I*Sqrt[e])*(1 - I*c*x))]])/e^3 - ((I/4)*b*PolyLog[2, 1 - (2*c*(Sqrt[-d] + Sq
rt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))]])/e^3
```

Rule 4980

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

Rule 4974

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]))/(2*e*(q + 1)), x
] - Dist[(b*c)/(2*e*(q + 1)), Int[(d + e*x^2)^(q + 1)/(1 + c^2*x^2), x], x]
 /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 391

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{x^5\left(a+b\tan^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx=\int\left(\frac{d^2x\left(a+b\tan^{-1}(cx)\right)}{e^2\left(d+ex^2\right)^3}-\frac{2dx\left(a+b\tan^{-1}(cx)\right)}{e^2\left(d+ex^2\right)^2}+\frac{x\left(a+b\tan^{-1}(cx)\right)}{e^2\left(d+ex^2\right)}\right)dx$$

$$=\frac{\int\frac{x\left(a+b\tan^{-1}(cx)\right)}{d+ex^2}\,dx}{e^2}-\frac{(2d)\int\frac{x\left(a+b\tan^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx}{e^2}+\frac{d^2\int\frac{x\left(a+b\tan^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx}{e^2}$$

$$=-\frac{d^2\left(a+b\tan^{-1}(cx)\right)}{4e^3\left(d+ex^2\right)^2}+\frac{d\left(a+b\tan^{-1}(cx)\right)}{e^3\left(d+ex^2\right)}-\frac{(bcd)\int\frac{1}{\left(1+c^2x^2\right)\left(d+ex^2\right)}\,dx}{e^3}+\frac{\left(bcd^2\right)\int\frac{1}{\left(1+c^2x^2\right)(}}{4e^3}$$

$$=-\frac{bcdx}{8\left(c^2d-e\right)e^2\left(d+ex^2\right)}-\frac{d^2\left(a+b\tan^{-1}(cx)\right)}{4e^3\left(d+ex^2\right)^2}+\frac{d\left(a+b\tan^{-1}(cx)\right)}{e^3\left(d+ex^2\right)}+\frac{(bcd)\int\frac{2c^2d-e-c^2ex}{\left(1+c^2x^2\right)\left(d+e\right.}}{8\left(c^2d-e\right)e^3}$$

$$=-\frac{bcdx}{8\left(c^2d-e\right)e^2\left(d+ex^2\right)}-\frac{bc^2d\tan^{-1}(cx)}{\left(c^2d-e\right)e^3}-\frac{d^2\left(a+b\tan^{-1}(cx)\right)}{4e^3\left(d+ex^2\right)^2}+\frac{d\left(a+b\tan^{-1}(cx)\right)}{e^3\left(d+ex^2\right)}+$$

$$=-\frac{bcdx}{8\left(c^2d-e\right)e^2\left(d+ex^2\right)}+\frac{bc^4d^2\tan^{-1}(cx)}{4\left(c^2d-e\right)^2e^3}-\frac{bc^2d\tan^{-1}(cx)}{\left(c^2d-e\right)e^3}-\frac{d^2\left(a+b\tan^{-1}(cx)\right)}{4e^3\left(d+ex^2\right)^2}+$$

$$=-\frac{bcdx}{8\left(c^2d-e\right)e^2\left(d+ex^2\right)}+\frac{bc^4d^2\tan^{-1}(cx)}{4\left(c^2d-e\right)^2e^3}-\frac{bc^2d\tan^{-1}(cx)}{\left(c^2d-e\right)e^3}-\frac{d^2\left(a+b\tan^{-1}(cx)\right)}{4e^3\left(d+ex^2\right)^2}+$$

Mathematica [A] time = 12.4638, size = 589, normalized size = 1.11

$$a\left(\frac{d\left(3d+4ex^2\right)}{\left(d+ex^2\right)^2}+2\log\left(d+ex^2\right)\right)+b\left(-i\text{PolyLog}\left(2,\frac{c\left(\sqrt{d}-i\sqrt{e}x\right)}{c\sqrt{d}-\sqrt{e}}\right)+i\text{PolyLog}\left(2,\frac{c\left(\sqrt{d}-i\sqrt{e}x\right)}{c\sqrt{d}+\sqrt{e}}\right)+i\text{PolyLog}\left(2,\frac{c\left(\sqrt{d}+i\sqrt{e}x\right)}{c\sqrt{d}-\sqrt{e}}\right)-i\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(a + b*ArcTan[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] (a*((d*(3*d + 4*e*x^2))/(d + e*x^2)^2 + 2*Log[d + e*x^2]) + b*(-(c*d*e*x)/(
2*(c^2*d - e)*(d + e*x^2)) + (c^2*d*(-3*c^2*d + 4*e)*ArcTan[c*x])/(-(c^2*d)
 + e)^2 + (d*(3*d + 4*e*x^2)*ArcTan[c*x])/(d + e*x^2)^2 + (c*Sqrt[d]*(5*c^2
*d - 7*e)*Sqrt[e]*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*(-(c^2*d) + e)^2) + 2*Arc
```

```
Tan[c*x]*Log[((-I)*Sqrt[d])/Sqrt[e] + x] + 2*ArcTan[c*x]*Log[(I*Sqrt[d])/Sq
rt[e] + x] + I*Log[((-I)*Sqrt[d])/Sqrt[e] + x]*Log[(Sqrt[e]*(-1 - I*c*x))/(
c*Sqrt[d] - Sqrt[e])] - I*Log[((-I)*Sqrt[d])/Sqrt[e] + x]*Log[(Sqrt[e]*(1 -
 I*c*x))/(c*Sqrt[d] + Sqrt[e])] - I*Log[(I*Sqrt[d])/Sqrt[e] + x]*Log[(Sqrt[
e]*(-1 + I*c*x))/(c*Sqrt[d] - Sqrt[e])] + I*Log[(I*Sqrt[d])/Sqrt[e] + x]*Lo
g[(Sqrt[e]*(1 + I*c*x))/(c*Sqrt[d] + Sqrt[e])] - I*PolyLog[2, (c*(Sqrt[d] -
 I*Sqrt[e]*x))/(c*Sqrt[d] - Sqrt[e])] + I*PolyLog[2, (c*(Sqrt[d] - I*Sqrt[e
]*x))/(c*Sqrt[d] + Sqrt[e])] + I*PolyLog[2, (c*(Sqrt[d] + I*Sqrt[e]*x))/(c*
Sqrt[d] - Sqrt[e])] - I*PolyLog[2, (c*(Sqrt[d] + I*Sqrt[e]*x))/(c*Sqrt[d] +
 Sqrt[e])]))/(4*e^3)
```

Maple [C] time = 0.229, size = 959, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arctan(c*x))/(e*x^2+d)^3,x)
```

```
[Out] 1/2*a/e^3*ln(c^2*e*x^2+c^2*d)-1/4*c^4*a*d^2/e^3/(c^2*e*x^2+c^2*d)^2+c^2*a/e
^3*d/(c^2*e*x^2+c^2*d)+1/2*b*arctan(c*x)/e^3*ln(c^2*e*x^2+c^2*d)-1/4*c^4*b*
arctan(c*x)*d^2/e^3/(c^2*e*x^2+c^2*d)^2+c^2*b*arctan(c*x)/e^3*d/(c^2*e*x^2+
c^2*d)-1/8*c^5*b/e^2*d^2/(c^2*d-e)^2*x/(c^2*e*x^2+c^2*d)+1/8*c^3*b/e*d/(c^2
*d-e)^2*x/(c^2*e*x^2+c^2*d)+5/8*c^3*b/e^2*d^2/(c^2*d-e)^2/(d*e)^(1/2)*arcta
n(e*x/(d*e)^(1/2))-7/8*c*b/e*d/(c^2*d-e)^2/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/
2))-3/4*b*c^4*d^2*arctan(c*x)/(c^2*d-e)^2/e^3+c^2*b/e^2*d/(c^2*d-e)^2*arcta
n(c*x)+1/4*I*b/e^3*dilog((RootOf(e*_Z^2-2*I*_Z*e+c^2*d-e,index=1)-c*x-I)/Ro
otOf(e*_Z^2-2*I*_Z*e+c^2*d-e,index=1))+1/4*I*b/e^3*ln(c*x-I)*ln(c^2*e*x^2+c
^2*d)-1/4*I*b/e^3*ln(c*x+I)*ln(c^2*e*x^2+c^2*d)-1/4*I*b/e^3*dilog((RootOf(e
*_Z^2+2*I*_Z*e+c^2*d-e,index=1)-c*x+I)/RootOf(e*_Z^2+2*I*_Z*e+c^2*d-e,index
=1))+1/4*I*b/e^3*dilog((RootOf(e*_Z^2-2*I*_Z*e+c^2*d-e,index=2)-c*x-I)/Root
Of(e*_Z^2-2*I*_Z*e+c^2*d-e,index=2))-1/4*I*b/e^3*dilog((RootOf(e*_Z^2+2*I*_
Z*e+c^2*d-e,index=2)-c*x+I)/RootOf(e*_Z^2+2*I*_Z*e+c^2*d-e,index=2))-1/4*I*
b/e^3*ln(c*x-I)*ln((RootOf(e*_Z^2+2*I*_Z*e+c^2*d-e,index=1)-c*x+I)/RootOf(e
*_Z^2+2*I*_Z*e+c^2*d-e,index=1))-1/4*I*b/e^3*ln(c*x-I)*ln((RootOf(e*_Z^2+2*
I*_Z*e+c^2*d-e,index=2)-c*x+I)/RootOf(e*_Z^2+2*I*_Z*e+c^2*d-e,index=2))+1/4
*I*b/e^3*ln(c*x+I)*ln((RootOf(e*_Z^2-2*I*_Z*e+c^2*d-e,index=2)-c*x-I)/RootO
f(e*_Z^2-2*I*_Z*e+c^2*d-e,index=2))+1/4*I*b/e^3*ln(c*x+I)*ln((RootOf(e*_Z^2
-2*I*_Z*e+c^2*d-e,index=1)-c*x-I)/RootOf(e*_Z^2-2*I*_Z*e+c^2*d-e,index=1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{4}\,a\left(\frac{4\,dex^2+3\,d^2}{e^5x^4+2\,de^4x^2+d^2e^3}+\frac{2\,\log\left(ex^2+d\right)}{e^3}\right)+2\,b\int\frac{x^5\arctan\left(cx\right)}{2\left(e^3x^6+3\,de^2x^4+3\,d^2ex^2+d^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctan(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/4*a*((4*d*e*x^2 + 3*d^2)/(e^5*x^4 + 2*d*e^4*x^2 + d^2*e^3) + 2*log(e*x^2
+ d)/e^3) + 2*b*integrate(1/2*x^5*arctan(c*x)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^
2*e*x^2 + d^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{bx^5\arctan\left(cx\right)+ax^5}{e^3x^6+3\,de^2x^4+3\,d^2ex^2+d^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctan(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*x^5*arctan(c*x) + a*x^5)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 +
 d^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*atan(c*x))/(e*x**2+d)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\arctan(cx)+a)x^5}{\left(ex^2+d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctan(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)*x^5/(e*x^2 + d)^3, x)
```

3.1166 $\int \frac{x^3(a+b\tan^{-1}(cx))}{(d+ex^2)^3}\,dx$

Optimal. Leaf size=130

$$\frac{x^4\left(a+b\tan^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2}-\frac{bc\left(c^2d-3e\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8\sqrt{d}e^{3/2}\left(c^2d-e\right)^2}+\frac{bcx}{8e\left(c^2d-e\right)\left(d+ex^2\right)}-\frac{b\tan^{-1}(cx)}{4d\left(c^2d-e\right)^2}$$

```
[Out] (b*c*x)/(8*(c^2*d - e)*e*(d + e*x^2)) - (b*ArcTan[c*x])/(4*d*(c^2*d - e)^2)
 + (x^4*(a + b*ArcTan[c*x]))/(4*d*(d + e*x^2)^2) - (b*c*(c^2*d - 3*e)*ArcTa
n[(Sqrt[e]*x)/Sqrt[d]])/(8*Sqrt[d]*(c^2*d - e)^2*e^(3/2))
```

Rubi [A] time = 0.185947, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {264, 4976, 12, 470, 522, 205}

$$\frac{x^4\left(a+b\tan^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2}-\frac{bc\left(c^2d-3e\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8\sqrt{d}e^{3/2}\left(c^2d-e\right)^2}+\frac{bcx}{8e\left(c^2d-e\right)\left(d+ex^2\right)}-\frac{b\tan^{-1}(cx)}{4d\left(c^2d-e\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcTan[c*x]))/(d + e*x^2)^3,x]
```


```
[Out] (b*c*x)/(8*(c^2*d - e)*e*(d + e*x^2)) - (b*ArcTan[c*x])/(4*d*(c^2*d - e)^2)
 + (x^4*(a + b*ArcTan[c*x]))/(4*d*(d + e*x^2)^2) - (b*c*(c^2*d - 3*e)*ArcTa
n[(Sqrt[e]*x)/Sqrt[d]])/(8*Sqrt[d]*(c^2*d - e)^2*e^(3/2))
```


Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 4976

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Dis
t[a + b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&  !(
```

```
ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(IL
tQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILtQ[(m
 - 1)/2, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^3\left(a+b\tan^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx = \frac{x^4\left(a+b\tan^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2} - (bc)\int \frac{x^4}{4\left(d+c^2dx^2\right)\left(d+ex^2\right)^2}\,dx$$
$$= \frac{x^4\left(a+b\tan^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2} - \frac{1}{4}(bc)\int \frac{x^4}{\left(d+c^2dx^2\right)\left(d+ex^2\right)^2}\,dx$$
$$= \frac{bcx}{8\left(c^2d-e\right)e\left(d+ex^2\right)} + \frac{x^4\left(a+b\tan^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2} - \frac{(bc)\int \frac{d^2+d\left(c^2d-2e\right)x^2}{\left(d+c^2dx^2\right)\left(d+ex^2\right)}\,dx}{8d\left(c^2d-e\right)e}$$
$$= \frac{bcx}{8\left(c^2d-e\right)e\left(d+ex^2\right)} + \frac{x^4\left(a+b\tan^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2} - \frac{(bc)\int \frac{1}{d+c^2dx^2}\,dx}{4\left(c^2d-e\right)^2} - \frac{\left(bc\left(c^2d-3e\right)\right)\int \frac{1}{d+ex^2}\,d}{8\left(c^2d-e\right)^2 e}$$
$$= \frac{bcx}{8\left(c^2d-e\right)e\left(d+ex^2\right)} - \frac{b\tan^{-1}(cx)}{4d\left(c^2d-e\right)^2} + \frac{x^4\left(a+b\tan^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2} - \frac{bc\left(c^2d-3e\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8\sqrt{d}\left(c^2d-e\right)^2 e^{3/2}}$$

Mathematica [A] time = 3.18357, size = 158, normalized size = 1.22

$$\frac{\frac{-4ac^2d+4ae+bcex}{\left(c^2d-e\right)\left(d+ex^2\right)} + \frac{2ad}{\left(d+ex^2\right)^2} + \frac{2bc^2\left(c^2d-2e\right)\tan^{-1}(cx)}{\left(e-c^2d\right)^2} - \frac{bc\sqrt{e}\left(c^2d-3e\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\left(e-c^2d\right)^2} - \frac{2b\tan^{-1}(cx)\left(d+2ex^2\right)}{\left(d+ex^2\right)^2}}{8e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcTan[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] ((2*a*d)/(d + e*x^2)^2 + (-4*a*c^2*d + 4*a*e + b*c*e*x)/((c^2*d - e)*(d + e
*x^2)) + (2*b*c^2*(c^2*d - 2*e)*ArcTan[c*x])/(-(c^2*d) + e)^2 - (2*b*(d + 2
*e*x^2)*ArcTan[c*x])/(d + e*x^2)^2 - (b*c*(c^2*d - 3*e)*Sqrt[e]*ArcTan[(Sqr
t[e]*x)/Sqrt[d]])/(Sqrt[d]*(-(c^2*d) + e)^2))/(8*e^2)
```

Maple [B] time = 0.058, size = 297, normalized size = 2.3

$$\frac{ac^4d}{4e^2\left(c^2ex^2+c^2d\right)^2} - \frac{c^2a}{2e^2\left(c^2ex^2+c^2d\right)} + \frac{bc^4\arctan(cx)\,d}{4e^2\left(c^2ex^2+c^2d\right)^2} - \frac{c^2b\arctan(cx)}{2e^2\left(c^2ex^2+c^2d\right)} + \frac{c^5bdx}{8e\left(c^2d-e\right)^2\left(c^2ex^2+c^2d\right)} - \frac{}{8\left(c^2d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctan(c*x))/(e*x^2+d)^3,x)
```

```
[Out] 1/4*c^4*a/e^2*d/(c^2*e*x^2+c^2*d)^2-1/2*c^2*a/e^2/(c^2*e*x^2+c^2*d)+1/4*c^4
*b*arctan(c*x)/e^2*d/(c^2*e*x^2+c^2*d)^2-1/2*c^2*b*arctan(c*x)/e^2/(c^2*e*x
^2+c^2*d)+1/8*c^5*b/e*d/(c^2*d-e)^2*x/(c^2*e*x^2+c^2*d)-1/8*c^3*b/(c^2*d-e)
^2*x/(c^2*e*x^2+c^2*d)-1/8*c^3*b/e*d/(c^2*d-e)^2/(d*e)^(1/2)*arctan(e*x/(d*
e)^(1/2))+3/8*c*b/(c^2*d-e)^2/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))+1/4*c^4*b
/e^2*d/(c^2*d-e)^2*arctan(c*x)-1/2*c^2*b/e/(c^2*d-e)^2*arctan(c*x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.74923, size = 1418, normalized size = 10.91

$$\left[-\frac{4\,ac^4d^4 - 8\,ac^2d^3e + 4\,ad^2e^2 - 2\left(bc^3d^2e^2 - bcde^3\right)x^3 + 8\left(ac^4d^3e - 2\,ac^2d^2e^2 + ade^3\right)x^2 - \left(bc^3d^3 - 3\,bcd^2e + \left(bc^3de^2 - \right.\right.}{16\left(c^4d^5e^2 - 2\,c^2d^4e^3 + d^3e^4 + \left(c\right.\right.} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*a*c^4*d^4 - 8*a*c^2*d^3*e + 4*a*d^2*e^2 - 2*(b*c^3*d^2*e^2 - b*c*
d*e^3)*x^3 + 8*(a*c^4*d^3*e - 2*a*c^2*d^2*e^2 + a*d*e^3)*x^2 - (b*c^3*d^3 -
 3*b*c*d^2*e + (b*c^3*d*e^2 - 3*b*c*e^3)*x^4 + 2*(b*c^3*d^2*e - 3*b*c*d*e^2
)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) - 2*(b*c^3*
d^3*e - b*c*d^2*e^2)*x + 4*(2*b*d*e^3*x^2 + b*d^2*e^2 - (b*c^4*d^2*e^2 - 2*
b*c^2*d*e^3)*x^4)*arctan(c*x))/(c^4*d^5*e^2 - 2*c^2*d^4*e^3 + d^3*e^4 + (c^
4*d^3*e^4 - 2*c^2*d^2*e^5 + d*e^6)*x^4 + 2*(c^4*d^4*e^3 - 2*c^2*d^3*e^4 + d
^2*e^5)*x^2), -1/8*(2*a*c^4*d^4 - 4*a*c^2*d^3*e + 2*a*d^2*e^2 - (b*c^3*d^2*
e^2 - b*c*d*e^3)*x^3 + 4*(a*c^4*d^3*e - 2*a*c^2*d^2*e^2 + a*d*e^3)*x^2 + (b
*c^3*d^3 - 3*b*c*d^2*e + (b*c^3*d*e^2 - 3*b*c*e^3)*x^4 + 2*(b*c^3*d^2*e - 3
```

```
*b*c*d*e^2)*x^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) - (b*c^3*d^3*e - b*c*d^2*e
^2)*x + 2*(2*b*d*e^3*x^2 + b*d^2*e^2 - (b*c^4*d^2*e^2 - 2*b*c^2*d*e^3)*x^4)
*arctan(c*x))/(c^4*d^5*e^2 - 2*c^2*d^4*e^3 + d^3*e^4 + (c^4*d^3*e^4 - 2*c^2
*d^2*e^5 + d*e^6)*x^4 + 2*(c^4*d^4*e^3 - 2*c^2*d^3*e^4 + d^2*e^5)*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atan(c*x))/(e*x**2+d)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 9.2829, size = 506, normalized size = 3.89

$$-\frac{\left(bc^3d - 3\,bce\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}}{8\left(c^4d^2e - 2\,c^2de^2 + e^3\right)\sqrt{d}} + \frac{bc^4dx^4\arctan(cx)\,e^2 - 4\,ac^4d^2x^2e - 2\,bc^2x^4\arctan(cx)\,e^3 + bc^3dx^3e^2 - 2\,ac^4d^3 + b}{4\left(c^4d^2x^4e^4 + 2\,c^4d^3x^2e^3 + c^4d^4e^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] -1/8*(b*c^3*d - 3*b*c*e)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/((c^4*d^2*e - 2
*c^2*d*e^2 + e^3)*sqrt(d)) + 1/4*(b*c^4*d*x^4*arctan(c*x)*e^2 - 4*a*c^4*d^2
*x^2*e - 2*b*c^2*x^4*arctan(c*x)*e^3 + b*c^3*d*x^3*e^2 - 2*a*c^4*d^3 + b*c^
3*d^2*x*e + 8*a*c^2*d*x^2*e^2 - b*c*x^3*e^3 + 4*a*c^2*d^2*e - 2*b*x^2*arcta
n(c*x)*e^3 - b*c*d*x*e^2 - 4*a*x^2*e^3 - b*d*arctan(c*x)*e^2 - 2*a*d*e^2)/(
c^4*d^2*x^4*e^4 + 2*c^4*d^3*x^2*e^3 + c^4*d^4*e^2 - 2*c^2*d*x^4*e^5 - 4*c^2
*d^2*x^2*e^4 - 2*c^2*d^3*e^3 + x^4*e^6 + 2*d*x^2*e^5 + d^2*e^4) - 1/8*(4*a*
c^2*d*x^2*e - b*c*x^3*e^2 + 2*a*c^2*d^2 - b*c*d*x*e - 4*a*x^2*e^2 - 2*a*d*e
)/((c^2*d*e^2 - e^3)*(x^2*e + d)^2)
```

3.1167 $\int \frac{x\left(a+b\tan^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx$

Optimal. Leaf size=131

$$-\frac{a+b\tan^{-1}(cx)}{4e\left(d+ex^2\right)^2}-\frac{bc\left(3c^2d-e\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{3/2}\sqrt{e}\left(c^2d-e\right)^2}-\frac{bcx}{8d\left(c^2d-e\right)\left(d+ex^2\right)}+\frac{bc^4\tan^{-1}(cx)}{4e\left(c^2d-e\right)^2}$$

```
[Out] -(b*c*x)/(8*d*(c^2*d - e)*(d + e*x^2)) + (b*c^4*ArcTan[c*x])/(4*(c^2*d - e)
^2*e) - (a + b*ArcTan[c*x])/(4*e*(d + e*x^2)^2) - (b*c*(3*c^2*d - e)*ArcTan
[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(3/2)*(c^2*d - e)^2*Sqrt[e])
```

Rubi [A] time = 0.113401, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {4974, 414, 522, 203, 205}

$$-\frac{a+b\tan^{-1}(cx)}{4e\left(d+ex^2\right)^2}-\frac{bc\left(3c^2d-e\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{3/2}\sqrt{e}\left(c^2d-e\right)^2}-\frac{bcx}{8d\left(c^2d-e\right)\left(d+ex^2\right)}+\frac{bc^4\tan^{-1}(cx)}{4e\left(c^2d-e\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcTan[c*x]))/(d + e*x^2)^3,x]
```


```
[Out] -(b*c*x)/(8*d*(c^2*d - e)*(d + e*x^2)) + (b*c^4*ArcTan[c*x])/(4*(c^2*d - e)
^2*e) - (a + b*ArcTan[c*x])/(4*e*(d + e*x^2)^2) - (b*c*(3*c^2*d - e)*ArcTan
[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(3/2)*(c^2*d - e)^2*Sqrt[e])
```


Rule 4974

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]))/(2*e*(q + 1)), x
] - Dist[(b*c)/(2*e*(q + 1)), Int[(d + e*x^2)^(q + 1)/(1 + c^2*x^2), x], x]
 /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
```

```
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x\left(a+b\tan^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx &= -\frac{a+b\tan^{-1}(cx)}{4e\left(d+ex^2\right)^2}+\frac{(bc)\int \frac{1}{\left(1+c^2x^2\right)\left(d+ex^2\right)^2}\,dx}{4e}\\
&= -\frac{bcx}{8d\left(c^2d-e\right)\left(d+ex^2\right)}-\frac{a+b\tan^{-1}(cx)}{4e\left(d+ex^2\right)^2}+\frac{(bc)\int \frac{2c^2d-e-c^2ex^2}{\left(1+c^2x^2\right)\left(d+ex^2\right)}\,dx}{8d\left(c^2d-e\right)e}\\
&= -\frac{bcx}{8d\left(c^2d-e\right)\left(d+ex^2\right)}-\frac{a+b\tan^{-1}(cx)}{4e\left(d+ex^2\right)^2}-\frac{\left(bc\left(3c^2d-e\right)\right)\int \frac{1}{d+ex^2}\,dx}{8d\left(c^2d-e\right)^2}+\frac{\left(bc^5\right)\int \frac{1}{1+c^2x^2}\,dx}{4\left(c^2d-e\right)^2e}\\
&= -\frac{bcx}{8d\left(c^2d-e\right)\left(d+ex^2\right)}+\frac{bc^4\tan^{-1}(cx)}{4\left(c^2d-e\right)^2e}-\frac{a+b\tan^{-1}(cx)}{4e\left(d+ex^2\right)^2}-\frac{bc\left(3c^2d-e\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{3/2}\left(c^2d-e\right)^2\sqrt{e}}
\end{aligned}
$$

Mathematica [A] time = 1.02572, size = 131, normalized size = 1.

$$\frac{1}{8}\left(-\frac{\frac{2a}{e}+\frac{bcx(d+ex^2)}{d(c^2d-e)}}{(d+ex^2)^2}-\frac{bc\,(3c^2d-e)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{3/2}\sqrt{e}\,(e-c^2d)^2}+\frac{2b\tan^{-1}(cx)\left(\frac{c^4}{(e-c^2d)^2}-\frac{1}{(d+ex^2)^2}\right)}{e}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcTan[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] (-(((2*a)/e + (b*c*x*(d + e*x^2))/(d*(c^2*d - e)))/(d + e*x^2)^2) + (2*b*(c
^4/(-(c^2*d) + e)^2 - (d + e*x^2)^(-2))*ArcTan[c*x])/e - (b*c*(3*c^2*d - e)
*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(3/2)*Sqrt[e]*(-(c^2*d) + e)^2))/8
```

Maple [A] time = 0.046, size = 216, normalized size = 1.7

$$-\frac{ac^4}{4e\,(c^2ex^2+c^2d)^2}-\frac{bc^4\arctan(cx)}{4e\,(c^2ex^2+c^2d)^2}-\frac{c^5bx}{8\,(c^2d-e)^2\,(c^2ex^2+c^2d)}+\frac{c^3bex}{8\,(c^2d-e)^2\,d\,(c^2ex^2+c^2d)}-\frac{3\,c^3b}{8\,(c^2d-e)^2}\text{arct}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctan(c*x))/(e*x^2+d)^3,x)
```

```
[Out] -1/4*c^4*a/e/(c^2*e*x^2+c^2*d)^2-1/4*c^4*b/e/(c^2*e*x^2+c^2*d)^2*arctan(c*x
)-1/8*c^5*b/(c^2*d-e)^2*x/(c^2*e*x^2+c^2*d)+1/8*c^3*b*e/(c^2*d-e)^2*x/d/(c^
2*e*x^2+c^2*d)-3/8*c^3*b/(c^2*d-e)^2/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))+1/
8*c*b*e/(c^2*d-e)^2/d/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))+1/4*b*c^4*arctan(
c*x)/(c^2*d-e)^2/e
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

[Out] Exception raised: ValueError

Fricas [B] time = 2.52233, size = 1288, normalized size = 9.83

$$\left[-\frac{4\,ac^4d^4 - 8\,ac^2d^3e + 4\,ad^2e^2 + 2\left(bc^3d^2e^2 - bcde^3\right)x^3 - \left(3\,bc^3d^3 - bcd^2e + \left(3\,bc^3de^2 - bce^3\right)x^4 + 2\left(3\,bc^3d^2e - bcde^2\right)x^2\right)}{16\left(c^4d^6e - 2\,c^2d^5e^2 + d^4e^3 + \left(c^4d^4e^3 - 2\,c^2d^3e^4 + d\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*a*c^4*d^4 - 8*a*c^2*d^3*e + 4*a*d^2*e^2 + 2*(b*c^3*d^2*e^2 - b*c*
d*e^3)*x^3 - (3*b*c^3*d^3 - b*c*d^2*e + (3*b*c^3*d*e^2 - b*c*e^3)*x^4 + 2*(
3*b*c^3*d^2*e - b*c*d*e^2)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)
/(e*x^2 + d)) + 2*(b*c^3*d^3*e - b*c*d^2*e^2)*x - 4*(b*c^4*d^2*e^2*x^4 + 2*
b*c^4*d^3*e*x^2 + 2*b*c^2*d^3*e - b*d^2*e^2)*arctan(c*x))/(c^4*d^6*e - 2*c^
2*d^5*e^2 + d^4*e^3 + (c^4*d^4*e^3 - 2*c^2*d^3*e^4 + d^2*e^5)*x^4 + 2*(c^4*
d^5*e^2 - 2*c^2*d^4*e^3 + d^3*e^4)*x^2), -1/8*(2*a*c^4*d^4 - 4*a*c^2*d^3*e
+ 2*a*d^2*e^2 + (b*c^3*d^2*e^2 - b*c*d*e^3)*x^3 + (3*b*c^3*d^3 - b*c*d^2*e
+ (3*b*c^3*d*e^2 - b*c*e^3)*x^4 + 2*(3*b*c^3*d^2*e - b*c*d*e^2)*x^2)*sqrt(d
*e)*arctan(sqrt(d*e)*x/d) + (b*c^3*d^3*e - b*c*d^2*e^2)*x - 2*(b*c^4*d^2*e^
2*x^4 + 2*b*c^4*d^3*e*x^2 + 2*b*c^2*d^3*e - b*d^2*e^2)*arctan(c*x))/(c^4*d^
6*e - 2*c^2*d^5*e^2 + d^4*e^3 + (c^4*d^4*e^3 - 2*c^2*d^3*e^4 + d^2*e^5)*x^4
 + 2*(c^4*d^5*e^2 - 2*c^2*d^4*e^3 + d^3*e^4)*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atan(c*x))/(e*x**2+d)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 2.04461, size = 518, normalized size = 3.95

$$-\frac{\left(3\,bc^3d-bce\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}}{8\left(c^4d^3-2\,c^2d^2e+de^2\right)\sqrt{d}}-\frac{\pi bc^4dx^4e^2\mathrm{sgn}\left(c\right)\mathrm{sgn}\left(x\right)+2\,\pi bc^4d^2x^2e\mathrm{sgn}\left(c\right)\mathrm{sgn}\left(x\right)-bc^4dx^4\arctan\left(cx\right)e^2-2}{4\left(c^4d^3x^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] -1/8*(3*b*c^3*d - b*c*e)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/((c^4*d^3 - 2*c
^2*d^2*e + d*e^2)*sqrt(d)) - 1/4*(pi*b*c^4*d*x^4*e^2*sgn(c)*sgn(x) + 2*pi*b
*c^4*d^2*x^2*e*sgn(c)*sgn(x) - b*c^4*d*x^4*arctan(c*x)*e^2 - 2*b*c^4*d^2*x^
2*arctan(c*x)*e + pi*b*c^4*d^3*sgn(c)*sgn(x) + b*c^3*d*x^3*e^2 + 2*a*c^4*d^
3 + b*c^3*d^2*x*e - 2*b*c^2*d^2*arctan(c*x)*e - b*c*x^3*e^3 - 4*a*c^2*d^2*e
 - b*c*d*x*e^2 + b*d*arctan(c*x)*e^2 + 2*a*d*e^2)/(c^4*d^3*x^4*e^3 + 2*c^4*
d^4*x^2*e^2 + c^4*d^5*e - 2*c^2*d^2*x^4*e^4 - 4*c^2*d^3*x^2*e^3 - 2*c^2*d^4
*e^2 + d*x^4*e^5 + 2*d^2*x^2*e^4 + d^3*e^3) - 1/8*(b*c*x^3*e^2 + 2*a*c^2*d^
2 + b*c*d*x*e - 2*a*d*e)/((c^2*d^2*e - d*e^2)*(x^2*e + d)^2)
```

3.1168 $\int \frac{a+b\tan^{-1}(cx)}{x(d+ex^2)^3}\,dx$

Optimal. Leaf size=574

$$\frac{ib\text{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(1-icx)(c\sqrt{-d}-i\sqrt{e})}\right)}{4d^3}+\frac{ib\text{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(1-icx)(c\sqrt{-d}+i\sqrt{e})}\right)}{4d^3}+\frac{ib\text{PolyLog}(2,-icx)}{2d^3}-\frac{ib\text{PolyLog}(2,icx)}{2d^3}-$$

```
[Out] (b*c*e*x)/(8*d^2*(c^2*d - e)*(d + e*x^2)) - (b*c^4*ArcTan[c*x])/(4*d*(c^2*d
 - e)^2) - (b*c^2*ArcTan[c*x])/(2*d^2*(c^2*d - e)) + (a + b*ArcTan[c*x])/(4
*d*(d + e*x^2)^2) + (a + b*ArcTan[c*x])/(2*d^2*(d + e*x^2)) + (b*c*Sqrt[e]*
ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(5/2)*(c^2*d - e)) + (b*c*(3*c^2*d - e)*S
qrt[e]*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(5/2)*(c^2*d - e)^2) + (a*Log[x])/
d^3 + ((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/d^3 - ((a + b*ArcTan[c*x])*L
og[(2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/(2
*d^3) - ((a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d]
+ I*Sqrt[e])*(1 - I*c*x))])/(2*d^3) + ((I/2)*b*PolyLog[2, (-I)*c*x])/d^3 -
((I/2)*b*PolyLog[2, I*c*x])/d^3 - ((I/2)*b*PolyLog[2, 1 - 2/(1 - I*c*x)])/d
^3 + ((I/4)*b*PolyLog[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*
Sqrt[e])*(1 - I*c*x))])/d^3 + ((I/4)*b*PolyLog[2, 1 - (2*c*(Sqrt[-d] + Sqrt
[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/d^3
```

Rubi [A] time = 0.631111, antiderivative size = 574, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 13, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.619, Rules used = {4980, 4848, 2391, 4974, 414, 522, 203, 205, 391, 4856, 2402, 2315, 2447}

$$\frac{ib\text{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(1-icx)(c\sqrt{-d}-i\sqrt{e})}\right)}{4d^3}+\frac{ib\text{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(1-icx)(c\sqrt{-d}+i\sqrt{e})}\right)}{4d^3}+\frac{ib\text{PolyLog}(2,-icx)}{2d^3}-\frac{ib\text{PolyLog}(2,icx)}{2d^3}-$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])/(x*(d + e*x^2)^3),x]
```

```
[Out] (b*c*e*x)/(8*d^2*(c^2*d - e)*(d + e*x^2)) - (b*c^4*ArcTan[c*x])/(4*d*(c^2*d
 - e)^2) - (b*c^2*ArcTan[c*x])/(2*d^2*(c^2*d - e)) + (a + b*ArcTan[c*x])/(4
*d*(d + e*x^2)^2) + (a + b*ArcTan[c*x])/(2*d^2*(d + e*x^2)) + (b*c*Sqrt[e]*
ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(5/2)*(c^2*d - e)) + (b*c*(3*c^2*d - e)*S
qrt[e]*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(5/2)*(c^2*d - e)^2) + (a*Log[x])/
```

```
d^3 + ((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/d^3 - ((a + b*ArcTan[c*x])*L
og[(2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))]])/(2
*d^3) - ((a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d]
+ I*Sqrt[e])*(1 - I*c*x))]])/(2*d^3) + ((I/2)*b*PolyLog[2, (-I)*c*x])/d^3 -
((I/2)*b*PolyLog[2, I*c*x])/d^3 - ((I/2)*b*PolyLog[2, 1 - 2/(1 - I*c*x)])/d
^3 + ((I/4)*b*PolyLog[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*
Sqrt[e])*(1 - I*c*x))]])/d^3 + ((I/4)*b*PolyLog[2, 1 - (2*c*(Sqrt[-d] + Sqrt
[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))]])/d^3
```

Rule 4980

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4974

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]))/(2*e*(q + 1)), x
] - Dist[(b*c)/(2*e*(q + 1)), Int[(d + e*x^2)^(q + 1)/(1 + c^2*x^2), x], x]
 /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 391

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{a+b\tan^{-1}(cx)}{x\left(d+ex^2\right)^3}\,dx = \int \left(\frac{a+b\tan^{-1}(cx)}{d^3x} - \frac{ex\left(a+b\tan^{-1}(cx)\right)}{d\left(d+ex^2\right)^3} - \frac{ex\left(a+b\tan^{-1}(cx)\right)}{d^2\left(d+ex^2\right)^2} - \frac{ex\left(a+b\tan^{-1}(cx)\right)}{d^3\left(d+ex^2\right)}\right)dx$$

$$= \frac{\int \frac{a+b\tan^{-1}(cx)}{x}\,dx}{d^3} - \frac{e\int \frac{x\left(a+b\tan^{-1}(cx)\right)}{d+ex^2}\,dx}{d^3} - \frac{e\int \frac{x\left(a+b\tan^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx}{d^2} - \frac{e\int \frac{x\left(a+b\tan^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx}{d}$$

$$= \frac{a+b\tan^{-1}(cx)}{4d\left(d+ex^2\right)^2} + \frac{a+b\tan^{-1}(cx)}{2d^2\left(d+ex^2\right)} + \frac{a\log(x)}{d^3} + \frac{(ib)\int \frac{\log(1-icx)}{x}\,dx}{2d^3} - \frac{(ib)\int \frac{\log(1+icx)}{x}\,dx}{2d^3} - \frac{(bc)}{}$$

$$= \frac{bcex}{8d^2\left(c^2d-e\right)\left(d+ex^2\right)} + \frac{a+b\tan^{-1}(cx)}{4d\left(d+ex^2\right)^2} + \frac{a+b\tan^{-1}(cx)}{2d^2\left(d+ex^2\right)} + \frac{a\log(x)}{d^3} + \frac{ib\text{Li}_2(-icx)}{2d^3} - \frac{ib\text{Li}_2}{2d}$$

$$= \frac{bcex}{8d^2\left(c^2d-e\right)\left(d+ex^2\right)} - \frac{bc^2\tan^{-1}(cx)}{2d^2\left(c^2d-e\right)} + \frac{a+b\tan^{-1}(cx)}{4d\left(d+ex^2\right)^2} + \frac{a+b\tan^{-1}(cx)}{2d^2\left(d+ex^2\right)} + \frac{bc\sqrt{e}\tan^{-1}\left(\right.}{2d^{5/2}\left(c^2d-e\right.}$$

$$= \frac{bcex}{8d^2\left(c^2d-e\right)\left(d+ex^2\right)} - \frac{bc^4\tan^{-1}(cx)}{4d\left(c^2d-e\right)^2} - \frac{bc^2\tan^{-1}(cx)}{2d^2\left(c^2d-e\right)} + \frac{a+b\tan^{-1}(cx)}{4d\left(d+ex^2\right)^2} + \frac{a+b\tan^{-1}(cx)}{2d^2\left(d+ex^2\right)}$$

$$= \frac{bcex}{8d^2\left(c^2d-e\right)\left(d+ex^2\right)} - \frac{bc^4\tan^{-1}(cx)}{4d\left(c^2d-e\right)^2} - \frac{bc^2\tan^{-1}(cx)}{2d^2\left(c^2d-e\right)} + \frac{a+b\tan^{-1}(cx)}{4d\left(d+ex^2\right)^2} + \frac{a+b\tan^{-1}(cx)}{2d^2\left(d+ex^2\right)}$$

Mathematica [A] time = 12.8439, size = 645, normalized size = 1.12

$$\frac{2a\left(\frac{d\left(3d+2ex^2\right)}{\left(d+ex^2\right)^2} - 2\log\left(d+ex^2\right) + 4\log(x)\right) + b\left(2i\text{PolyLog}\left(2, \frac{c\left(\sqrt{d}-i\sqrt{ex}\right)}{c\sqrt{d}-\sqrt{e}}\right) - 2i\text{PolyLog}\left(2, \frac{c\left(\sqrt{d}-i\sqrt{ex}\right)}{c\sqrt{d}+\sqrt{e}}\right) - 2i\text{PolyLog}\left(2,\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])/(x*(d + e*x^2)^3),x]
```

```
[Out] (2*a*((d*(3*d + 2*e*x^2))/(d + e*x^2)^2 + 4*Log[x] - 2*Log[d + e*x^2]) + b*
((c*d*e*x)/((c^2*d - e)*(d + e*x^2)) + (2*c^2*d*(-3*c^2*d + 2*e)*ArcTan[c*x
])/(-(c^2*d) + e)^2 + (2*d*(3*d + 2*e*x^2)*ArcTan[c*x])/(d + e*x^2)^2 + (c*
Sqrt[d]*(7*c^2*d - 5*e)*Sqrt[e]*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(-(c^2*d) + e)
^2 + 8*ArcTan[c*x]*Log[x] - 4*ArcTan[c*x]*Log[((-I)*Sqrt[d])/Sqrt[e] + x] -
 4*ArcTan[c*x]*Log[(I*Sqrt[d])/Sqrt[e] + x] - (2*I)*Log[((-I)*Sqrt[d])/Sqrt
[e] + x]*Log[(Sqrt[e]*(-1 - I*c*x))/(c*Sqrt[d] - Sqrt[e])] + (2*I)*Log[((-I
)*Sqrt[d])/Sqrt[e] + x]*Log[(Sqrt[e]*(1 - I*c*x))/(c*Sqrt[d] + Sqrt[e])] +
(2*I)*Log[(I*Sqrt[d])/Sqrt[e] + x]*Log[(Sqrt[e]*(-1 + I*c*x))/(c*Sqrt[d] -
Sqrt[e])] - (2*I)*Log[(I*Sqrt[d])/Sqrt[e] + x]*Log[(Sqrt[e]*(1 + I*c*x))/(c
*Sqrt[d] + Sqrt[e])] - (4*I)*(Log[x]*(Log[1 - I*c*x] - Log[1 + I*c*x]) - Po
lyLog[2, (-I)*c*x] + PolyLog[2, I*c*x]) + (2*I)*PolyLog[2, (c*(Sqrt[d] - I*
Sqrt[e]*x))/(c*Sqrt[d] - Sqrt[e])] - (2*I)*PolyLog[2, (c*(Sqrt[d] - I*Sqrt[
e]*x))/(c*Sqrt[d] + Sqrt[e])] - (2*I)*PolyLog[2, (c*(Sqrt[d] + I*Sqrt[e]*x)
)/(c*Sqrt[d] - Sqrt[e])] + (2*I)*PolyLog[2, (c*(Sqrt[d] + I*Sqrt[e]*x))/(c*
Sqrt[d] + Sqrt[e])]))/(8*d^3)
```

Maple [C] time = 0.219, size = 1041, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/x/(e*x^2+d)^3,x)
```

```
[Out] -1/2*I*b/d^3*dilog(1-I*c*x)-1/4*I*b/d^3*dilog((RootOf(e*_Z^2-2*I*_Z*e+c^2*d
-e,index=2)-c*x-I)/RootOf(e*_Z^2-2*I*_Z*e+c^2*d-e,index=2))+1/2*I*b/d^3*dil
og(1+I*c*x)+1/4*I*b/d^3*dilog((RootOf(e*_Z^2+2*I*_Z*e+c^2*d-e,index=2)-c*x+
I)/RootOf(e*_Z^2+2*I*_Z*e+c^2*d-e,index=2))-1/2*b*arctan(c*x)/d^3*ln(c^2*e*
x^2+c^2*d)+b*arctan(c*x)/d^3*ln(c*x)-1/2*a/d^3*ln(c^2*e*x^2+c^2*d)+a/d^3*ln
(c*x)-1/2*I*b/d^3*ln(c*x)*ln(1-I*c*x)+1/4*b*c^4*arctan(c*x)/d/(c^2*e*x^2+c^
2*d)^2+1/2*b*c^2*arctan(c*x)/d^2/(c^2*e*x^2+c^2*d)+1/2*I*b/d^3*ln(c*x)*ln(1
+I*c*x)-1/4*I*b/d^3*ln(c*x+I)*ln((RootOf(e*_Z^2-2*I*_Z*e+c^2*d-e,index=2)-c
*x-I)/RootOf(e*_Z^2-2*I*_Z*e+c^2*d-e,index=2))+1/4*I*b/d^3*ln(c*x-I)*ln((Ro
otOf(e*_Z^2+2*I*_Z*e+c^2*d-e,index=1)-c*x+I)/RootOf(e*_Z^2+2*I*_Z*e+c^2*d-e
,index=1))-1/4*I*b/d^3*ln(c*x-I)*ln(c^2*e*x^2+c^2*d)-1/4*I*b/d^3*ln(c*x+I)*
ln((RootOf(e*_Z^2-2*I*_Z*e+c^2*d-e,index=1)-c*x-I)/RootOf(e*_Z^2-2*I*_Z*e+c
^2*d-e,index=1))+1/4*I*b/d^3*ln(c*x-I)*ln((RootOf(e*_Z^2+2*I*_Z*e+c^2*d-e,i
ndex=2)-c*x+I)/RootOf(e*_Z^2+2*I*_Z*e+c^2*d-e,index=2))+1/4*I*b/d^3*ln(c*x+
I)*ln(c^2*e*x^2+c^2*d)+1/4*a*c^4/d/(c^2*e*x^2+c^2*d)^2+1/2*a*c^2/d^2/(c^2*e
```

```
*x^2+c^2*d)+1/4*I*b/d^3*dilog((RootOf(e*_Z^2+2*I*_Z*e+c^2*d-e,index=1)-c*x+
I)/RootOf(e*_Z^2+2*I*_Z*e+c^2*d-e,index=1))-1/4*I*b/d^3*dilog((RootOf(e*_Z^
2-2*I*_Z*e+c^2*d-e,index=1)-c*x-I)/RootOf(e*_Z^2-2*I*_Z*e+c^2*d-e,index=1))
+1/2*b*c^2/d^2/(c^2*d-e)^2*arctan(c*x)*e-1/8*b*c^3/d^2/(c^2*d-e)^2*e^2*x/(c
^2*e*x^2+c^2*d)+7/8*b*c^3*e/(c^2*d-e)^2/d/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2
))-5/8*b*c/d^2/(c^2*d-e)^2*e^2/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))+1/8*b*c^
5*e/(c^2*d-e)^2*x/d/(c^2*e*x^2+c^2*d)-3/4*b*c^4*arctan(c*x)/d/(c^2*d-e)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{4}\,a\left(\frac{2\,ex^2+3\,d}{d^2e^2x^4+2\,d^3ex^2+d^4}-\frac{2\,\log\left(ex^2+d\right)}{d^3}+\frac{4\,\log\left(x\right)}{d^3}\right)+2\,b\int\frac{\arctan\left(cx\right)}{2\left(e^3x^7+3\,de^2x^5+3\,d^2ex^3+d^3x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/4*a*((2*e*x^2 + 3*d)/(d^2*e^2*x^4 + 2*d^3*e*x^2 + d^4) - 2*log(e*x^2 + d)
/d^3 + 4*log(x)/d^3) + 2*b*integrate(1/2*arctan(c*x)/(e^3*x^7 + 3*d*e^2*x^5
 + 3*d^2*e*x^3 + d^3*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b\arctan\left(cx\right)+a}{e^3x^7+3\,de^2x^5+3\,d^2ex^3+d^3x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*arctan(c*x) + a)/(e^3*x^7 + 3*d*e^2*x^5 + 3*d^2*e*x^3 + d^3*x),
 x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/x/(e*x**2+d)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \arctan(cx) + a}{(ex^2 + d)^3 x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)/((e*x^2 + d)^3*x), x)
```

3.1169 $\int \frac{a+b\tan^{-1}(cx)}{x^3(d+ex^2)^3}\,dx$

Optimal. Leaf size=629

$$-\frac{3ibe\text{PolyLog}(2,-icx)}{2d^4}+\frac{3ibe\text{PolyLog}(2,icx)}{2d^4}+\frac{3ibe\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2d^4}-\frac{3ibe\text{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(1-icx)(c\sqrt{-d}-i\sqrt{e})}\right)}{4d^4}-$$

```
[Out] -(b*c)/(2*d^3*x) - (b*c*e^2*x)/(8*d^3*(c^2*d - e)*(d + e*x^2)) - (b*c^2*Arc
Tan[c*x])/(2*d^3) + (b*c^4*e*ArcTan[c*x])/(4*d^2*(c^2*d - e)^2) + (b*c^2*e*
ArcTan[c*x])/(d^3*(c^2*d - e)) - (a + b*ArcTan[c*x])/(2*d^3*x^2) - (e*(a +
b*ArcTan[c*x]))/(4*d^2*(d + e*x^2)^2) - (e*(a + b*ArcTan[c*x]))/(d^3*(d + e
*x^2)) - (b*c*e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(7/2)*(c^2*d - e)) -
(b*c*(3*c^2*d - e)*e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(7/2)*(c^2*d -
 e)^2) - (3*a*e*Log[x])/d^4 - (3*e*(a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/
d^4 + (3*e*(a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d
] - I*Sqrt[e])*(1 - I*c*x))])/(2*d^4) + (3*e*(a + b*ArcTan[c*x])*Log[(2*c*(
Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(2*d^4) - (
((3*I)/2)*b*e*PolyLog[2, (-I)*c*x])/d^4 + (((3*I)/2)*b*e*PolyLog[2, I*c*x])
/d^4 + (((3*I)/2)*b*e*PolyLog[2, 1 - 2/(1 - I*c*x)])/d^4 - (((3*I)/4)*b*e*P
olyLog[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I
*c*x))])/d^4 - (((3*I)/4)*b*e*PolyLog[2, 1 - (2*c*(Sqrt[-d] + Sqrt[e]*x))/(
(c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/d^4
```

Rubi [A] time = 0.680135, antiderivative size = 629, normalized size of antiderivative = 1., number of steps used = 27, number of rules used = 15, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {4980, 4852, 325, 203, 4848, 2391, 4974, 414, 522, 205, 391, 4856, 2402, 2315, 2447}

$$-\frac{3ibe\text{PolyLog}(2,-icx)}{2d^4}+\frac{3ibe\text{PolyLog}(2,icx)}{2d^4}+\frac{3ibe\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2d^4}-\frac{3ibe\text{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(1-icx)(c\sqrt{-d}-i\sqrt{e})}\right)}{4d^4}-$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])/(x^3*(d + e*x^2)^3),x]
```



```
[Out] -(b*c)/(2*d^3*x) - (b*c*e^2*x)/(8*d^3*(c^2*d - e)*(d + e*x^2)) - (b*c^2*Arc
Tan[c*x])/(2*d^3) + (b*c^4*e*ArcTan[c*x])/(4*d^2*(c^2*d - e)^2) + (b*c^2*e*
```

```
ArcTan[c*x])/(d^3*(c^2*d - e)) - (a + b*ArcTan[c*x])/(2*d^3*x^2) - (e*(a +
b*ArcTan[c*x]))/(4*d^2*(d + e*x^2)^2) - (e*(a + b*ArcTan[c*x]))/(d^3*(d + e
*x^2)) - (b*c*e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(7/2)*(c^2*d - e)) -
(b*c*(3*c^2*d - e)*e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(7/2)*(c^2*d -
 e)^2) - (3*a*e*Log[x])/d^4 - (3*e*(a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/
d^4 + (3*e*(a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d
] - I*Sqrt[e])*(1 - I*c*x))])/(2*d^4) + (3*e*(a + b*ArcTan[c*x])*Log[(2*c*(
Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(2*d^4) - (
((3*I)/2)*b*e*PolyLog[2, (-I)*c*x])/d^4 + (((3*I)/2)*b*e*PolyLog[2, I*c*x])
/d^4 + (((3*I)/2)*b*e*PolyLog[2, 1 - 2/(1 - I*c*x)])/d^4 - (((3*I)/4)*b*e*P
olyLog[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I
*c*x))])/d^4 - (((3*I)/4)*b*e*PolyLog[2, 1 - (2*c*(Sqrt[-d] + Sqrt[e]*x))/(
(c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/d^4
```

Rule 4980

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4974

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]))/(2*e*(q + 1)), x
] - Dist[(b*c)/(2*e*(q + 1)), Int[(d + e*x^2)^(q + 1)/(1 + c^2*x^2), x], x]
 /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 391

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
```

```
d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{a+b\tan^{-1}(cx)}{x^3\left(d+ex^2\right)^3}\,dx = \int \left(\frac{a+b\tan^{-1}(cx)}{d^3x^3} - \frac{3e\left(a+b\tan^{-1}(cx)\right)}{d^4x} + \frac{e^2x\left(a+b\tan^{-1}(cx)\right)}{d^2\left(d+ex^2\right)^3} + \frac{2e^2x\left(a+b\tan^{-1}(cx)\right)}{d^3\left(d+ex^2\right)^2} +$$

$$= \frac{\int \frac{a+b\tan^{-1}(cx)}{x^3}\,dx}{d^3} - \frac{(3e)\int \frac{a+b\tan^{-1}(cx)}{x}\,dx}{d^4} + \frac{\left(3e^2\right)\int \frac{x\left(a+b\tan^{-1}(cx)\right)}{d+ex^2}\,dx}{d^4} + \frac{\left(2e^2\right)\int \frac{x\left(a+b\tan^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,d}{d^3}$$

$$= -\frac{a+b\tan^{-1}(cx)}{2d^3x^2} - \frac{e\left(a+b\tan^{-1}(cx)\right)}{4d^2\left(d+ex^2\right)^2} - \frac{e\left(a+b\tan^{-1}(cx)\right)}{d^3\left(d+ex^2\right)} - \frac{3ae\log(x)}{d^4} + \frac{(bc)\int \frac{1}{x^2\left(1+c^2x^2\right)}\,d}{2d^3}$$

$$= -\frac{bc}{2d^3x} - \frac{bce^2x}{8d^3\left(c^2d-e\right)\left(d+ex^2\right)} - \frac{a+b\tan^{-1}(cx)}{2d^3x^2} - \frac{e\left(a+b\tan^{-1}(cx)\right)}{4d^2\left(d+ex^2\right)^2} - \frac{e\left(a+b\tan^{-1}(cx)\right)}{d^3\left(d+ex^2\right)}$$

$$= -\frac{bc}{2d^3x} - \frac{bce^2x}{8d^3\left(c^2d-e\right)\left(d+ex^2\right)} - \frac{bc^2\tan^{-1}(cx)}{2d^3} + \frac{bc^2e\tan^{-1}(cx)}{d^3\left(c^2d-e\right)} - \frac{a+b\tan^{-1}(cx)}{2d^3x^2} - \frac{e\left(a+\right.}{4d^2\left(\right.}$$

$$= -\frac{bc}{2d^3x} - \frac{bce^2x}{8d^3\left(c^2d-e\right)\left(d+ex^2\right)} - \frac{bc^2\tan^{-1}(cx)}{2d^3} + \frac{bc^4e\tan^{-1}(cx)}{4d^2\left(c^2d-e\right)^2} + \frac{bc^2e\tan^{-1}(cx)}{d^3\left(c^2d-e\right)} - \frac{a+b\,\mathrm{ta}}{2d}$$

$$= -\frac{bc}{2d^3x} - \frac{bce^2x}{8d^3\left(c^2d-e\right)\left(d+ex^2\right)} - \frac{bc^2\tan^{-1}(cx)}{2d^3} + \frac{bc^4e\tan^{-1}(cx)}{4d^2\left(c^2d-e\right)^2} + \frac{bc^2e\tan^{-1}(cx)}{d^3\left(c^2d-e\right)} - \frac{a+b\,\mathrm{ta}}{2d}$$

Mathematica [A] time = 15.9124, size = 723, normalized size = 1.15

$$\frac{-a\left(\frac{d\left(2d^2+9dex^2+6e^2x^4\right)}{x^2\left(d+ex^2\right)^2} - 6e\log\left(d+ex^2\right) + 12e\log(x)\right) + b\left(-3ie\left(\operatorname{PolyLog}\left(2, \frac{c\left(\sqrt{d}-i\sqrt{e}x\right)}{c\sqrt{d}-\sqrt{e}}\right) + \log\left(x+\frac{i\sqrt{d}}{\sqrt{e}}\right)\log\left(\frac{\sqrt{e}(-1+icx)}{c\sqrt{d}-\sqrt{e}}\right)\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])/(x^3*(d + e*x^2)^3),x]
```

```
[Out] (-(a*((d*(2*d^2 + 9*d*e*x^2 + 6*e^2*x^4))/(x^2*(d + e*x^2)^2) + 12*e*Log[x]
 - 6*e*Log[d + e*x^2])) + b*((-2*c*d)/x - (c*d*e^2*x)/(2*(c^2*d - e)*(d + e
*x^2)) + (c^2*d*(-2*c^4*d^2 + 9*c^2*d*e - 6*e^2)*ArcTan[c*x])/(-(c^2*d) + e
)^2 - (d*(2*d^2 + 9*d*e*x^2 + 6*e^2*x^4)*ArcTan[c*x])/(x^2*(d + e*x^2)^2) +
```

```
(c*Sqrt[d]*e^(3/2)*(-11*c^2*d + 9*e)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*(-(c^
2*d) + e)^2) - 12*e*ArcTan[c*x]*Log[x] + 6*e*ArcTan[c*x]*(Log[((-I)*Sqrt[d]
)/Sqrt[e] + x] + Log[(I*Sqrt[d])/Sqrt[e] + x] - Log[d + e*x^2]) + 6*e*ArcTa
n[c*x]*Log[d + e*x^2] - (6*I)*e*(Log[x]*Log[1 + I*c*x] + PolyLog[2, (-I)*c*
x]) + (6*I)*e*(Log[x]*Log[1 - I*c*x] + PolyLog[2, I*c*x]) - (3*I)*e*(Log[(I
*Sqrt[d])/Sqrt[e] + x]*Log[(Sqrt[e]*(-1 + I*c*x))/(c*Sqrt[d] - Sqrt[e])] +
PolyLog[2, (c*(Sqrt[d] - I*Sqrt[e]*x))/(c*Sqrt[d] - Sqrt[e])]) + (3*I)*e*(L
og[(I*Sqrt[d])/Sqrt[e] + x]*Log[(Sqrt[e]*(1 + I*c*x))/(c*Sqrt[d] + Sqrt[e])
] + PolyLog[2, (c*(Sqrt[d] - I*Sqrt[e]*x))/(c*Sqrt[d] + Sqrt[e])]) + (3*I)*
e*(Log[((-I)*Sqrt[d])/Sqrt[e] + x]*Log[(Sqrt[e]*(-1 - I*c*x))/(c*Sqrt[d] -
Sqrt[e])] + PolyLog[2, (c*(Sqrt[d] + I*Sqrt[e]*x))/(c*Sqrt[d] - Sqrt[e])])
- (3*I)*e*(Log[((-I)*Sqrt[d])/Sqrt[e] + x]*Log[(Sqrt[e]*(1 - I*c*x))/(c*Sqr
t[d] + Sqrt[e])] + PolyLog[2, (c*(Sqrt[d] + I*Sqrt[e]*x))/(c*Sqrt[d] + Sqrt
[e])])))/(4*d^4)
```

Maple [C] time = 0.232, size = 1128, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/x^3/(e*x^2+d)^3,x)
```

```
[Out] 9/4*b*c^4*e*arctan(c*x)/d^2/(c^2*d-e)^2-3/2*I*b/d^4*e*ln(c*x)*ln(1+I*c*x)+3
/2*I*b/d^4*e*ln(c*x)*ln(1-I*c*x)-3/4*I*b/d^4*e*ln(c*x-I)*ln((RootOf(e*_Z^2+
2*I*_Z*e+c^2*d-e,index=1)-c*x+I)/RootOf(e*_Z^2+2*I*_Z*e+c^2*d-e,index=1))-3
/4*I*b/d^4*e*ln(c*x-I)*ln((RootOf(e*_Z^2+2*I*_Z*e+c^2*d-e,index=2)-c*x+I)/R
ootOf(e*_Z^2+2*I*_Z*e+c^2*d-e,index=2))+3/4*I*b/d^4*e*ln(c*x-I)*ln(c^2*e*x^
2+c^2*d)+3/4*I*b/d^4*e*ln(c*x+I)*ln((RootOf(e*_Z^2-2*I*_Z*e+c^2*d-e,index=1
)-c*x-I)/RootOf(e*_Z^2-2*I*_Z*e+c^2*d-e,index=1))-1/4*c^4*b*arctan(c*x)*e/d
^2/(c^2*e*x^2+c^2*d)^2-3/2*c^2*b/d^3/(c^2*d-e)^2*arctan(c*x)*e^2-1/2*b*c/d^
3/x-1/2*a/d^3/x^2-c^2*b*arctan(c*x)*e/d^3/(c^2*e*x^2+c^2*d)+3/4*I*b/d^4*e*l
n(c*x+I)*ln((RootOf(e*_Z^2-2*I*_Z*e+c^2*d-e,index=2)-c*x-I)/RootOf(e*_Z^2-2
*I*_Z*e+c^2*d-e,index=2))-3/4*I*b/d^4*e*ln(c*x+I)*ln(c^2*e*x^2+c^2*d)-1/2*b
*arctan(c*x)/d^3/x^2+3/2*a*e/d^4*ln(c^2*e*x^2+c^2*d)-3*a/d^4*e*ln(c*x)+1/8*
c^3*b/d^3*e^3/(c^2*d-e)^2*x/(c^2*e*x^2+c^2*d)-11/8*c^3*b/d^2/(c^2*d-e)^2*e^
2/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))+9/8*c*b/d^3*e^3/(c^2*d-e)^2/(d*e)^(1/
2)*arctan(e*x/(d*e)^(1/2))-1/8*c^5*b/d^2/(c^2*d-e)^2*e^2*x/(c^2*e*x^2+c^2*d
)-c^2*a*e/d^3/(c^2*e*x^2+c^2*d)-3/2*I*b/d^4*e*dilog(1+I*c*x)+3/2*I*b/d^4*e*
dilog(1-I*c*x)-3/4*I*b/d^4*e*dilog((RootOf(e*_Z^2+2*I*_Z*e+c^2*d-e,index=2)
-c*x+I)/RootOf(e*_Z^2+2*I*_Z*e+c^2*d-e,index=2))-3/4*I*b/d^4*e*dilog((RootO
f(e*_Z^2+2*I*_Z*e+c^2*d-e,index=1)-c*x+I)/RootOf(e*_Z^2+2*I*_Z*e+c^2*d-e,in
```

```
dex=1))+3/4*I*b/d^4*e*dilog((RootOf(e*_Z^2-2*I*_Z*e+c^2*d-e,index=2)-c*x-I)
/RootOf(e*_Z^2-2*I*_Z*e+c^2*d-e,index=2))+3/4*I*b/d^4*e*dilog((RootOf(e*_Z^
2-2*I*_Z*e+c^2*d-e,index=1)-c*x-I)/RootOf(e*_Z^2-2*I*_Z*e+c^2*d-e,index=1))
+3/2*b*arctan(c*x)*e/d^4*ln(c^2*e*x^2+c^2*d)-3*b*arctan(c*x)/d^4*e*ln(c*x)-
1/2*c^6*b/d/(c^2*d-e)^2*arctan(c*x)-1/4*c^4*a*e/d^2/(c^2*e*x^2+c^2*d)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{4}a\left(\frac{6e^2x^4+9dex^2+2d^2}{d^3e^2x^6+2d^4ex^4+d^5x^2}-\frac{6e\log\left(ex^2+d\right)}{d^4}+\frac{12e\log\left(x\right)}{d^4}\right)+2b\int\frac{\arctan\left(cx\right)}{2\left(e^3x^9+3de^2x^7+3d^2ex^5+d^3x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^3/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] -1/4*a*((6*e^2*x^4 + 9*d*e*x^2 + 2*d^2)/(d^3*e^2*x^6 + 2*d^4*e*x^4 + d^5*x^
2) - 6*e*log(e*x^2 + d)/d^4 + 12*e*log(x)/d^4) + 2*b*integrate(1/2*arctan(c
*x)/(e^3*x^9 + 3*d*e^2*x^7 + 3*d^2*e*x^5 + d^3*x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b\arctan\left(cx\right)+a}{e^3x^9+3de^2x^7+3d^2ex^5+d^3x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^3/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*arctan(c*x) + a)/(e^3*x^9 + 3*d*e^2*x^7 + 3*d^2*e*x^5 + d^3*x^3
), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/x**3/(e*x**2+d)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\arctan(cx)+a}{\left(ex^2+d\right)^3x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^3/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)/((e*x^2 + d)^3*x^3), x)
```

3.1170 $\int \frac{x^2\left(a+b\tan^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx$

Optimal. Leaf size=966

result too large to display

```
[Out] (b*c)/(8*(c^2*d - e)*e*(d + e*x^2)) - (x*(a + b*ArcTan[c*x]))/(4*e*(d + e*x
^2)^2) + (x*(a + b*ArcTan[c*x]))/(8*d*e*(d + e*x^2)) + ((a + b*ArcTan[c*x])
*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(3/2)*e^(3/2)) + ((I/32)*b*c*Log[(Sqrt[e
]*(1 - Sqrt[-c^2]*x))/(I*Sqrt[-c^2]*Sqrt[d] + Sqrt[e])]*Log[1 - (I*Sqrt[e]*
x)/Sqrt[d]])/(Sqrt[-c^2]*d^(3/2)*e^(3/2)) - ((I/32)*b*c*Log[-((Sqrt[e]*(1 +
 Sqrt[-c^2]*x))/(I*Sqrt[-c^2]*Sqrt[d] - Sqrt[e]))]*Log[1 - (I*Sqrt[e]*x)/Sq
rt[d]])/(Sqrt[-c^2]*d^(3/2)*e^(3/2)) - ((I/32)*b*c*Log[-((Sqrt[e]*(1 - Sqrt
[-c^2]*x))/(I*Sqrt[-c^2]*Sqrt[d] - Sqrt[e]))]*Log[1 + (I*Sqrt[e]*x)/Sqrt[d]
])/(Sqrt[-c^2]*d^(3/2)*e^(3/2)) + ((I/32)*b*c*Log[(Sqrt[e]*(1 + Sqrt[-c^2]*
x))/(I*Sqrt[-c^2]*Sqrt[d] + Sqrt[e])]*Log[1 + (I*Sqrt[e]*x)/Sqrt[d]])/(Sqrt
[-c^2]*d^(3/2)*e^(3/2)) + (b*c*(5*c^2*d - 3*e)*Log[1 + c^2*x^2])/(16*d*(c^2
*d - e)^2*e) - (b*c*Log[1 + c^2*x^2])/(4*d*(c^2*d - e)*e) - (b*c*(5*c^2*d -
 3*e)*Log[d + e*x^2])/(16*d*(c^2*d - e)^2*e) + (b*c*Log[d + e*x^2])/(4*d*(c
^2*d - e)*e) + ((I/32)*b*c*PolyLog[2, (Sqrt[-c^2]*(Sqrt[d] - I*Sqrt[e]*x))/
(Sqrt[-c^2]*Sqrt[d] - I*Sqrt[e])])/(Sqrt[-c^2]*d^(3/2)*e^(3/2)) - ((I/32)*b
*c*PolyLog[2, (Sqrt[-c^2]*(Sqrt[d] - I*Sqrt[e]*x))/(Sqrt[-c^2]*Sqrt[d] + I*
Sqrt[e])])/(Sqrt[-c^2]*d^(3/2)*e^(3/2)) + ((I/32)*b*c*PolyLog[2, (Sqrt[-c^2
]*(Sqrt[d] + I*Sqrt[e]*x))/(Sqrt[-c^2]*Sqrt[d] - I*Sqrt[e])])/(Sqrt[-c^2]*d
^(3/2)*e^(3/2)) - ((I/32)*b*c*PolyLog[2, (Sqrt[-c^2]*(Sqrt[d] + I*Sqrt[e]*x
))/(Sqrt[-c^2]*Sqrt[d] + I*Sqrt[e])])/(Sqrt[-c^2]*d^(3/2)*e^(3/2))
```

Rubi [A] time = 2.26016, antiderivative size = 966, normalized size of antiderivative = 1., number of steps used = 49, number of rules used = 15, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {4980, 199, 205, 4912, 6725, 571, 77, 4908, 2409, 2394, 2393, 2391, 444, 36, 31}

$$\frac{ib\log\left(\frac{\sqrt{e}\left(1-\sqrt{-c^2}x\right)}{i\sqrt{-c^2}\sqrt{d}+\sqrt{e}}\right)\log\left(1-\frac{i\sqrt{e}x}{\sqrt{d}}\right)c}{32\sqrt{-c^2}d^{3/2}e^{3/2}}-\frac{ib\log\left(-\frac{\sqrt{e}\left(\sqrt{-c^2}x+1\right)}{i\sqrt{-c^2}\sqrt{d}-\sqrt{e}}\right)\log\left(1-\frac{i\sqrt{e}x}{\sqrt{d}}\right)c}{32\sqrt{-c^2}d^{3/2}e^{3/2}}-\frac{ib\log\left(-\frac{\sqrt{e}\left(1-\sqrt{-c^2}x\right)}{i\sqrt{-c^2}\sqrt{d}-\sqrt{e}}\right)\log\left(\frac{i\sqrt{e}x}{\sqrt{d}}+1\right)c}{32\sqrt{-c^2}d^{3/2}e^{3/2}}+\frac{ib}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcTan[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] (b*c)/(8*(c^2*d - e)*e*(d + e*x^2)) - (x*(a + b*ArcTan[c*x]))/(4*e*(d + e*x
^2)^2) + (x*(a + b*ArcTan[c*x]))/(8*d*e*(d + e*x^2)) + ((a + b*ArcTan[c*x])
*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(3/2)*e^(3/2)) + ((I/32)*b*c*Log[(Sqrt[e
]*(1 - Sqrt[-c^2]*x))/(I*Sqrt[-c^2]*Sqrt[d] + Sqrt[e])]*Log[1 - (I*Sqrt[e]*
x)/Sqrt[d]])/(Sqrt[-c^2]*d^(3/2)*e^(3/2)) - ((I/32)*b*c*Log[-((Sqrt[e]*(1 +
 Sqrt[-c^2]*x))/(I*Sqrt[-c^2]*Sqrt[d] - Sqrt[e]))]*Log[1 - (I*Sqrt[e]*x)/Sq
rt[d]])/(Sqrt[-c^2]*d^(3/2)*e^(3/2)) - ((I/32)*b*c*Log[-((Sqrt[e]*(1 - Sqrt
[-c^2]*x))/(I*Sqrt[-c^2]*Sqrt[d] - Sqrt[e]))]*Log[1 + (I*Sqrt[e]*x)/Sqrt[d]
])/(Sqrt[-c^2]*d^(3/2)*e^(3/2)) + ((I/32)*b*c*Log[(Sqrt[e]*(1 + Sqrt[-c^2]*
x))/(I*Sqrt[-c^2]*Sqrt[d] + Sqrt[e])]*Log[1 + (I*Sqrt[e]*x)/Sqrt[d]])/(Sqrt
[-c^2]*d^(3/2)*e^(3/2)) + (b*c*(5*c^2*d - 3*e)*Log[1 + c^2*x^2])/(16*d*(c^2
*d - e)^2*e) - (b*c*Log[1 + c^2*x^2])/(4*d*(c^2*d - e)*e) - (b*c*(5*c^2*d -
 3*e)*Log[d + e*x^2])/(16*d*(c^2*d - e)^2*e) + (b*c*Log[d + e*x^2])/(4*d*(c
^2*d - e)*e) + ((I/32)*b*c*PolyLog[2, (Sqrt[-c^2]*(Sqrt[d] - I*Sqrt[e]*x))/
(Sqrt[-c^2]*Sqrt[d] - I*Sqrt[e])])/(Sqrt[-c^2]*d^(3/2)*e^(3/2)) - ((I/32)*b
*c*PolyLog[2, (Sqrt[-c^2]*(Sqrt[d] - I*Sqrt[e]*x))/(Sqrt[-c^2]*Sqrt[d] + I*
Sqrt[e])])/(Sqrt[-c^2]*d^(3/2)*e^(3/2)) + ((I/32)*b*c*PolyLog[2, (Sqrt[-c^2
]*(Sqrt[d] + I*Sqrt[e]*x))/(Sqrt[-c^2]*Sqrt[d] - I*Sqrt[e])])/(Sqrt[-c^2]*d
^(3/2)*e^(3/2)) - ((I/32)*b*c*PolyLog[2, (Sqrt[-c^2]*(Sqrt[d] + I*Sqrt[e]*x
))/(Sqrt[-c^2]*Sqrt[d] + I*Sqrt[e])])/(Sqrt[-c^2]*d^(3/2)*e^(3/2))
```

Rule 4980

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 4912

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Symb
```

```
ol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcTan[c*x], u, x]
- Dist[b*c, Int[u/(1 + c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && (I
ntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 571

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 4908

```
Int[ArcTan[(c_.)*(x_)]/((d_.) + (e_.)*(x_)^2), x_Symbol] :> Dist[I/2, Int[L
og[1 - I*c*x]/(d + e*x^2), x], x] - Dist[I/2, Int[Log[1 + I*c*x]/(d + e*x^2
), x], x] /; FreeQ[{c, d, e}, x]
```

Rule 2409

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
```

```
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{x^2\left(a+b\tan^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx = \int\left(-\frac{d\left(a+b\tan^{-1}(cx)\right)}{e\left(d+ex^2\right)^3}+\frac{a+b\tan^{-1}(cx)}{e\left(d+ex^2\right)^2}\right)dx$$

$$= \frac{\int \frac{a+b\tan^{-1}(cx)}{\left(d+ex^2\right)^2}\,dx}{e} - \frac{d\int \frac{a+b\tan^{-1}(cx)}{\left(d+ex^2\right)^3}\,dx}{e}$$

$$= -\frac{x\left(a+b\tan^{-1}(cx)\right)}{4e\left(d+ex^2\right)^2}+\frac{x\left(a+b\tan^{-1}(cx)\right)}{8de\left(d+ex^2\right)}+\frac{\left(a+b\tan^{-1}(cx)\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{3/2}e^{3/2}} - \frac{(bc)\int \ldots}{\ldots}$$

$$= -\frac{x\left(a+b\tan^{-1}(cx)\right)}{4e\left(d+ex^2\right)^2}+\frac{x\left(a+b\tan^{-1}(cx)\right)}{8de\left(d+ex^2\right)}+\frac{\left(a+b\tan^{-1}(cx)\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{3/2}e^{3/2}} - \frac{(bc)\int\left(\ldots\right.}{\ldots}$$

$$= -\frac{x\left(a+b\tan^{-1}(cx)\right)}{4e\left(d+ex^2\right)^2}+\frac{x\left(a+b\tan^{-1}(cx)\right)}{8de\left(d+ex^2\right)}+\frac{\left(a+b\tan^{-1}(cx)\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{3/2}e^{3/2}} + \frac{(3bc)\int \ldots}{8d^{3/2}\ldots}$$

$$= -\frac{x\left(a+b\tan^{-1}(cx)\right)}{4e\left(d+ex^2\right)^2}+\frac{x\left(a+b\tan^{-1}(cx)\right)}{8de\left(d+ex^2\right)}+\frac{\left(a+b\tan^{-1}(cx)\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{3/2}e^{3/2}} + \frac{(3ibc)\int \ldots}{16d^{3}\ldots}$$

$$= -\frac{x\left(a+b\tan^{-1}(cx)\right)}{4e\left(d+ex^2\right)^2}+\frac{x\left(a+b\tan^{-1}(cx)\right)}{8de\left(d+ex^2\right)}+\frac{\left(a+b\tan^{-1}(cx)\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{3/2}e^{3/2}} + \frac{(bc)\,\mathrm{Subst}\ldots}{4d\ldots}$$

$$= \frac{bc}{8\left(c^2d-e\right)e\left(d+ex^2\right)}-\frac{x\left(a+b\tan^{-1}(cx)\right)}{4e\left(d+ex^2\right)^2}+\frac{x\left(a+b\tan^{-1}(cx)\right)}{8de\left(d+ex^2\right)}+\frac{\left(a+b\tan^{-1}(cx)\right)\mathrm{ta}\ldots}{8d^{3/2}e^{3/2}}$$

$$= \frac{bc}{8\left(c^2d-e\right)e\left(d+ex^2\right)}-\frac{x\left(a+b\tan^{-1}(cx)\right)}{4e\left(d+ex^2\right)^2}+\frac{x\left(a+b\tan^{-1}(cx)\right)}{8de\left(d+ex^2\right)}+\frac{\left(a+b\tan^{-1}(cx)\right)\mathrm{ta}\ldots}{8d^{3/2}e^{3/2}}$$

$$= \frac{bc}{8\left(c^2d-e\right)e\left(d+ex^2\right)}-\frac{x\left(a+b\tan^{-1}(cx)\right)}{4e\left(d+ex^2\right)^2}+\frac{x\left(a+b\tan^{-1}(cx)\right)}{8de\left(d+ex^2\right)}+\frac{\left(a+b\tan^{-1}(cx)\right)\mathrm{ta}\ldots}{8d^{3/2}e^{3/2}}$$

$$= \frac{bc}{8\left(c^2d-e\right)e\left(d+ex^2\right)}-\frac{x\left(a+b\tan^{-1}(cx)\right)}{4e\left(d+ex^2\right)^2}+\frac{x\left(a+b\tan^{-1}(cx)\right)}{8de\left(d+ex^2\right)}+\frac{\left(a+b\tan^{-1}(cx)\right)\mathrm{ta}\ldots}{8d^{3/2}e^{3/2}}$$

Mathematica [A] time = 12.9692, size = 1894, normalized size = 1.96

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(a + b*ArcTan[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] -(a*x)/(4*e*(d + e*x^2)^2) + (a*x)/(8*d*e*(d + e*x^2)) + (a*ArcTan[(Sqrt[e]
*x)/Sqrt[d]])/(8*d^(3/2)*e^(3/2)) + b*c^3*(-Log[1 + ((c^2*d - e)*Cos[2*ArcT
an[c*x]])/(c^2*d + e)]/(16*c^2*d*(c^2*d - e)^2) - Log[1 + ((c^2*d - e)*Cos[
2*ArcTan[c*x]])/(c^2*d + e)]/(16*(c^2*d - e)^2*e) - (4*ArcTan[c*x]*ArcTanh[
(c*d)/(Sqrt[-(c^2*d*e)]*x)] + 2*ArcCos[-((c^2*d + e)/(c^2*d - e))]*ArcTanh[
(c*e*x)/Sqrt[-(c^2*d*e)]] - (ArcCos[-((c^2*d + e)/(c^2*d - e))] - (2*I)*Arc
Tanh[(c*e*x)/Sqrt[-(c^2*d*e)]])*Log[1 - ((c^2*d + e - (2*I)*Sqrt[-(c^2*d*e)
])*(2*c^2*d - 2*c*Sqrt[-(c^2*d*e)]*x))/((c^2*d - e)*(2*c^2*d + 2*c*Sqrt[-(c
^2*d*e)]*x))] + (-ArcCos[-((c^2*d + e)/(c^2*d - e))] - (2*I)*ArcTanh[(c*e*x
)/Sqrt[-(c^2*d*e)]])*Log[1 - ((c^2*d + e + (2*I)*Sqrt[-(c^2*d*e)])*(2*c^2*d
 - 2*c*Sqrt[-(c^2*d*e)]*x))/((c^2*d - e)*(2*c^2*d + 2*c*Sqrt[-(c^2*d*e)]*x)
)] + (ArcCos[-((c^2*d + e)/(c^2*d - e))] - (2*I)*(ArcTanh[(c*d)/(Sqrt[-(c^2
*d*e)]*x)] + ArcTanh[(c*e*x)/Sqrt[-(c^2*d*e)]]))*Log[(Sqrt[2]*Sqrt[-(c^2*d*
e)])/(Sqrt[c^2*d - e]*E^(I*ArcTan[c*x])*Sqrt[c^2*d + e + (c^2*d - e)*Cos[2*
ArcTan[c*x]]])] + (ArcCos[-((c^2*d + e)/(c^2*d - e))] + (2*I)*(ArcTanh[(c*d
)/(Sqrt[-(c^2*d*e)]*x)] + ArcTanh[(c*e*x)/Sqrt[-(c^2*d*e)]]))*Log[(Sqrt[2]*
Sqrt[-(c^2*d*e)]*E^(I*ArcTan[c*x]))/(Sqrt[c^2*d - e]*Sqrt[c^2*d + e + (c^2*
d - e)*Cos[2*ArcTan[c*x]]])] + I*(PolyLog[2, ((c^2*d + e - (2*I)*Sqrt[-(c^2
*d*e)])*(2*c^2*d - 2*c*Sqrt[-(c^2*d*e)]*x))/((c^2*d - e)*(2*c^2*d + 2*c*Sqr
t[-(c^2*d*e)]*x))] - PolyLog[2, ((c^2*d + e + (2*I)*Sqrt[-(c^2*d*e)])*(2*c^
2*d - 2*c*Sqrt[-(c^2*d*e)]*x))/((c^2*d - e)*(2*c^2*d + 2*c*Sqrt[-(c^2*d*e)]
*x))]))/(32*c^2*d*(c^2*d - e)*Sqrt[-(c^2*d*e)]) + (4*ArcTan[c*x]*ArcTanh[(c
*d)/(Sqrt[-(c^2*d*e)]*x)] + 2*ArcCos[-((c^2*d + e)/(c^2*d - e))]*ArcTanh[(c
*e*x)/Sqrt[-(c^2*d*e)]] - (ArcCos[-((c^2*d + e)/(c^2*d - e))] - (2*I)*ArcTa
nh[(c*e*x)/Sqrt[-(c^2*d*e)]])*Log[1 - ((c^2*d + e - (2*I)*Sqrt[-(c^2*d*e)])
*(2*c^2*d - 2*c*Sqrt[-(c^2*d*e)]*x))/((c^2*d - e)*(2*c^2*d + 2*c*Sqrt[-(c^2
*d*e)]*x))] + (-ArcCos[-((c^2*d + e)/(c^2*d - e))] - (2*I)*ArcTanh[(c*e*x)/
Sqrt[-(c^2*d*e)]])*Log[1 - ((c^2*d + e + (2*I)*Sqrt[-(c^2*d*e)])*(2*c^2*d -
 2*c*Sqrt[-(c^2*d*e)]*x))/((c^2*d - e)*(2*c^2*d + 2*c*Sqrt[-(c^2*d*e)]*x))]
 + (ArcCos[-((c^2*d + e)/(c^2*d - e))] - (2*I)*(ArcTanh[(c*d)/(Sqrt[-(c^2*d
*e)]*x)] + ArcTanh[(c*e*x)/Sqrt[-(c^2*d*e)]]))*Log[(Sqrt[2]*Sqrt[-(c^2*d*e)
])/(Sqrt[c^2*d - e]*E^(I*ArcTan[c*x])*Sqrt[c^2*d + e + (c^2*d - e)*Cos[2*Ar
cTan[c*x]]])] + (ArcCos[-((c^2*d + e)/(c^2*d - e))] + (2*I)*(ArcTanh[(c*d)/
(Sqrt[-(c^2*d*e)]*x)] + ArcTanh[(c*e*x)/Sqrt[-(c^2*d*e)]]))*Log[(Sqrt[2]*Sq
rt[-(c^2*d*e)]*E^(I*ArcTan[c*x]))/(Sqrt[c^2*d - e]*Sqrt[c^2*d + e + (c^2*d
- e)*Cos[2*ArcTan[c*x]]])] + I*(PolyLog[2, ((c^2*d + e - (2*I)*Sqrt[-(c^2*d
```

```
*e)])*(2*c^2*d - 2*c*Sqrt[-(c^2*d*e)]*x))/((c^2*d - e)*(2*c^2*d + 2*c*Sqrt[
-(c^2*d*e)]*x))] - PolyLog[2, ((c^2*d + e + (2*I)*Sqrt[-(c^2*d*e)])*(2*c^2*
d - 2*c*Sqrt[-(c^2*d*e)]*x))/((c^2*d - e)*(2*c^2*d + 2*c*Sqrt[-(c^2*d*e)]*x
))]))/(32*(c^2*d - e)*e*Sqrt[-(c^2*d*e)]) + (ArcTan[c*x]*Sin[2*ArcTan[c*x]]
)/(2*(c^2*d - e)*(c^2*d + e + c^2*d*Cos[2*ArcTan[c*x]] - e*Cos[2*ArcTan[c*x
]])^2) + (-2*c^2*d*e - c^4*d^2*ArcTan[c*x]*Sin[2*ArcTan[c*x]] + e^2*ArcTan[
c*x]*Sin[2*ArcTan[c*x]])/(8*c^2*d*(c^2*d - e)^2*e*(c^2*d + e + c^2*d*Cos[2*
ArcTan[c*x]] - e*Cos[2*ArcTan[c*x]])))
```

Maple [B] time = 0.75, size = 3801, normalized size = 3.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctan(c*x))/(e*x^2+d)^3,x)
```

```
[Out] 1/8*a/d/e/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))-1/8*I*c^5*b/(c^4*d^2-2*c^2*d*
e+e^2)/(c^2*e*x^2+c^2*d)^2*e^2/d*arctan(c*x)*x^4-1/16*I*c^7*b*d^2*ln(1-(c^2
*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d+2*(c^2*e*d)^(1/2)-e))*arctan(c*x)/e^2
/(c^4*d^2-2*c^2*d*e+e^2)^2*(c^2*e*d)^(1/2)+1/4*I*c^5*b*d*ln(1-(c^2*d-e)*(1+
I*c*x)^2/(c^2*x^2+1)/(-c^2*d+2*(c^2*e*d)^(1/2)-e))*arctan(c*x)/e/(c^4*d^2-2
*c^2*d*e+e^2)^2*(c^2*e*d)^(1/2)+1/4*I*c*b*e*ln(1-(c^2*d-e)*(1+I*c*x)^2/(c^2
*x^2+1)/(-c^2*d+2*(c^2*e*d)^(1/2)-e))*arctan(c*x)/d/(c^4*d^2-2*c^2*d*e+e^2)
^2*(c^2*e*d)^(1/2)-1/16*I/c*b*e^2*ln(1-(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-
c^2*d+2*(c^2*e*d)^(1/2)-e))*arctan(c*x)/d^2/(c^4*d^2-2*c^2*d*e+e^2)^2*(c^2*
e*d)^(1/2)-1/8*I*c^7*b/(c^4*d^2-2*c^2*d*e+e^2)/(c^2*e*x^2+c^2*d)^2*e*arctan
(c*x)*x^4-1/4*I*c^7*b/(c^4*d^2-2*c^2*d*e+e^2)/(c^2*e*x^2+c^2*d)^2*d*arctan(
c*x)*x^2-1/8*c^8*b/(c^4*d^2-2*c^2*d*e+e^2)/(c^2*e*x^2+c^2*d)^2/e*d^2*arctan
(c*x)*x+1/8*c^4*b/(c^4*d^2-2*c^2*d*e+e^2)/(c^2*e*x^2+c^2*d)^2*e^2/d*arctan(
c*x)*x^3+1/16*c^6*b*(d*e)^(1/2)*d/e^2*arctanh(1/4*(2*(c^2*d-e)*(1+I*c*x)^2/
(c^2*x^2+1)+2*c^2*d+2*e)/c/(d*e)^(1/2))/(c^4*d^2-2*c^2*d*e+e^2)/(c^2*d-e)-1
/4*I*c^5*b/(c^4*d^2-2*c^2*d*e+e^2)/(c^2*e*x^2+c^2*d)^2*e*arctan(c*x)*x^2+1/
16*I/c*b*(c^2*e*d)^(1/2)/d^2/(c^4*d^2-2*c^2*d*e+e^2)*arctan(c*x)*ln(1-(c^2*
d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d-2*(c^2*e*d)^(1/2)-e))+1/16*I*c^3*b*(c^
2*e*d)^(1/2)/e^2/(c^4*d^2-2*c^2*d*e+e^2)*arctan(c*x)*ln(1-(c^2*d-e)*(1+I*c*
x)^2/(c^2*x^2+1)/(-c^2*d-2*(c^2*e*d)^(1/2)-e))-1/8*I*c^7*b/(c^4*d^2-2*c^2*d
*e+e^2)/(c^2*e*x^2+c^2*d)^2/e*d^2*arctan(c*x)-1/8*c^7*b/(c^4*d^2-2*c^2*d*e+
e^2)/(c^2*e*x^2+c^2*d)^2*e*x^4-1/8*c^7*b/(c^4*d^2-2*c^2*d*e+e^2)/(c^2*e*x^2
+c^2*d)^2*d*x^2-1/8*c^5*b/(c^4*d^2-2*c^2*d*e+e^2)/(c^2*e*x^2+c^2*d)^2*e*x^2
+1/32/c*b*(c^2*e*d)^(1/2)/d^2/(c^4*d^2-2*c^2*d*e+e^2)*polylog(2,(c^2*d-e)*(
1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d-2*(c^2*e*d)^(1/2)-e))+1/32*c^3*b*(c^2*e*d)^(
```

1/2)/e^2/(c^4*d^2-2*c^2*d*e+e^2)*polylog(2,(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1
)/(-c^2*d-2*(c^2*e*d)^(1/2)-e))+1/16/c*b*(c^2*e*d)^(1/2)/d^2/(c^4*d^2-2*c^2
*d*e+e^2)*arctan(c*x)^2-1/16*c^4*b*(d*e)^(1/2)/e^2*arctanh(1/4*(2*(c^2*d-e)
*(1+I*c*x)^2/(c^2*x^2+1)+2*c^2*d+2*e)/c/(d*e)^(1/2))/(c^4*d^2-2*c^2*d*e+e^2
)+1/16*c^3*b*(c^2*e*d)^(1/2)/e^2/(c^4*d^2-2*c^2*d*e+e^2)*arctan(c*x)^2-1/16
*c^2*b*(d*e)^(1/2)/d*arctanh(1/4*(2*(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)+2*c^2
*d+2*e)/c/(d*e)^(1/2))/(c^4*d^2-2*c^2*d*e+e^2)/(c^2*d-e)-1/8*I*c^5*b/(c^4*d
^2-2*c^2*d*e+e^2)/(c^2*e*x^2+c^2*d)^2*d*arctan(c*x)-3/8*I*c^3*b*arctan(c*x)
*ln(1-(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d+2*(c^2*e*d)^(1/2)-e))/(c^4*
d^2-2*c^2*d*e+e^2)^2*(c^2*e*d)^(1/2)-1/8*I*c*b*(c^2*e*d)^(1/2)/d/e/(c^4*d^2
-2*c^2*d*e+e^2)*arctan(c*x)*ln(1-(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d-
2*(c^2*e*d)^(1/2)-e))-1/8*c^5*b/(c^4*d^2-2*c^2*d*e+e^2)/(c^2*e*x^2+c^2*d)^2
*d-1/8*c^4*a/(c^2*e*x^2+c^2*d)^2/e*x+1/8*c^4*a/(c^2*e*x^2+c^2*d)^2/d*x^3-3/
8*c^3*b*arctan(c*x)^2/(c^4*d^2-2*c^2*d*e+e^2)^2*(c^2*e*d)^(1/2)-3/16*c^3*b*
polylog(2,(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d+2*(c^2*e*d)^(1/2)-e))/(
c^4*d^2-2*c^2*d*e+e^2)^2*(c^2*e*d)^(1/2)-1/16*b*(d*e)^(1/2)/d^2*arctanh(1/4
(2(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)+2*c^2*d+2*e)/c/(d*e)^(1/2))/(c^4*d^2-
2*c^2*d*e+e^2)-1/16*b*(d*e)^(1/2)*e/d^2*arctanh(1/4*(2*(c^2*d-e)*(1+I*c*x)^
2/(c^2*x^2+1)+2*c^2*d+2*e)/c/(d*e)^(1/2))/(c^4*d^2-2*c^2*d*e+e^2)/(c^2*d-e)
+1/4*c^5*b*d*arctan(c*x)^2/e/(c^4*d^2-2*c^2*d*e+e^2)^2*(c^2*e*d)^(1/2)-1/4*
c^6*b/(c^4*d^2-2*c^2*d*e+e^2)/(c^2*e*x^2+c^2*d)^2*e*arctan(c*x)*x^3+1/4*c^6
*b/(c^4*d^2-2*c^2*d*e+e^2)/(c^2*e*x^2+c^2*d)^2*d*arctan(c*x)*x-1/8*c^4*b/(c
^4*d^2-2*c^2*d*e+e^2)/(c^2*e*x^2+c^2*d)^2*e*arctan(c*x)*x-1/8*c*b*(c^2*e*d)
^(1/2)/d/e/(c^4*d^2-2*c^2*d*e+e^2)*arctan(c*x)^2-1/4*c*b/(c^4*d^2-2*c^2*d*e
+e^2)*e/d/(c^2*d-e)*ln((1+I*c*x)/(c^2*x^2+1)^(1/2))-1/8*c^2*b*(d*e)^(1/2)/d
/e*arctanh(1/4*(2*(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)+2*c^2*d+2*e)/c/(d*e)^(1
/2))/(c^4*d^2-2*c^2*d*e+e^2)+1/16*c^4*b*(d*e)^(1/2)/e*arctanh(1/4*(2*(c^2*d
-e)*(1+I*c*x)^2/(c^2*x^2+1)+2*c^2*d+2*e)/c/(d*e)^(1/2))/(c^4*d^2-2*c^2*d*e+
e^2)/(c^2*d-e)+1/4*c*b*arctan(c*x)^2*e/d/(c^4*d^2-2*c^2*d*e+e^2)^2*(c^2*e*d
)^(1/2)-1/16*c^7*b*d^2*arctan(c*x)^2/e^2/(c^4*d^2-2*c^2*d*e+e^2)^2*(c^2*e*d
)^(1/2)+1/8*c*b*e*polylog(2,(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d+2*(c^
2*e*d)^(1/2)-e))/d/(c^4*d^2-2*c^2*d*e+e^2)^2*(c^2*e*d)^(1/2)-1/32/c*b*e^2*p
olylog(2,(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d+2*(c^2*e*d)^(1/2)-e))/d^
2/(c^4*d^2-2*c^2*d*e+e^2)^2*(c^2*e*d)^(1/2)-1/32*c^7*b*d^2*polylog(2,(c^2*d
-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d+2*(c^2*e*d)^(1/2)-e))/e^2/(c^4*d^2-2*c^
2*d*e+e^2)^2*(c^2*e*d)^(1/2)+1/8*c^8*b/(c^4*d^2-2*c^2*d*e+e^2)/(c^2*e*x^2+c
^2*d)^2*d*arctan(c*x)*x^3-1/16/c*b*e^2*arctan(c*x)^2/d^2/(c^4*d^2-2*c^2*d*e
+e^2)^2*(c^2*e*d)^(1/2)+1/16*c*b/(c^4*d^2-2*c^2*d*e+e^2)*e/d/(c^2*d-e)*ln((
1+I*c*x)^4/(c^2*x^2+1)^2*c^2*d+2*c^2*d*(1+I*c*x)^2/(c^2*x^2+1)-(1+I*c*x)^4/
(c^2*x^2+1)^2*e+c^2*d+2*(1+I*c*x)^2/(c^2*x^2+1)*e-e)+1/4*c^5*b/(c^4*d^2-2*c
^2*d*e+e^2)/e*d/(c^2*d-e)*ln((1+I*c*x)/(c^2*x^2+1)^(1/2))-1/16*c*b*(c^2*e*d
)^(1/2)/d/e/(c^4*d^2-2*c^2*d*e+e^2)*polylog(2,(c^2*d-e)*(1+I*c*x)^2/(c^2*x^
2+1)/(-c^2*d-2*(c^2*e*d)^(1/2)-e))-1/16*c^5*b/(c^4*d^2-2*c^2*d*e+e^2)/e*d/(
c^2*d-e)*ln((1+I*c*x)^4/(c^2*x^2+1)^2*c^2*d+2*c^2*d*(1+I*c*x)^2/(c^2*x^2+1)
-(1+I*c*x)^4/(c^2*x^2+1)^2*e+c^2*d+2*(1+I*c*x)^2/(c^2*x^2+1)*e-e)+1/8*c^5*b

```
*d*polylog(2,(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d+2*(c^2*e*d)^(1/2)-e)
)/e/(c^4*d^2-2*c^2*d*e+e^2)^2*(c^2*e*d)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{bx^2\arctan(cx) + ax^2}{e^3x^6 + 3\,de^2x^4 + 3\,d^2ex^2 + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*x^2*arctan(c*x) + a*x^2)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 +
 d^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atan(c*x))/(e*x**2+d)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\arctan(cx)+a)x^2}{\left(ex^2+d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)*x^2/(e*x^2 + d)^3, x)
```

3.1171 $\int \frac{a+b\tan^{-1}(cx)}{\left(d+ex^2\right)^3}\,dx$

Optimal. Leaf size=893

$$\frac{3ib\log\left(\frac{\sqrt{e}\left(1-\sqrt{-c^2}x\right)}{i\sqrt{-c^2}\sqrt{d}+\sqrt{e}}\right)\log\left(1-\frac{i\sqrt{e}x}{\sqrt{d}}\right)c}{32\sqrt{-c^2}d^{5/2}\sqrt{e}}-\frac{3ib\log\left(-\frac{\sqrt{e}\left(\sqrt{-c^2}x+1\right)}{i\sqrt{-c^2}\sqrt{d}-\sqrt{e}}\right)\log\left(1-\frac{i\sqrt{e}x}{\sqrt{d}}\right)c}{32\sqrt{-c^2}d^{5/2}\sqrt{e}}-\frac{3ib\log\left(-\frac{\sqrt{e}\left(1-\sqrt{-c^2}x\right)}{i\sqrt{-c^2}\sqrt{d}-\sqrt{e}}\right)\log\left(\frac{i\sqrt{e}x}{\sqrt{d}}+1\right)c}{32\sqrt{-c^2}d^{5/2}\sqrt{e}}$$

```
[Out] -(b*c)/(8*d*(c^2*d - e)*(d + e*x^2)) + (x*(a + b*ArcTan[c*x]))/(4*d*(d + e*
x^2)^2) + (3*x*(a + b*ArcTan[c*x]))/(8*d^2*(d + e*x^2)) + (3*(a + b*ArcTan[
c*x])*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(5/2)*Sqrt[e]) + (((3*I)/32)*b*c*Lo
g[(Sqrt[e]*(1 - Sqrt[-c^2]*x))/(I*Sqrt[-c^2]*Sqrt[d] + Sqrt[e])]*Log[1 - (I
*Sqrt[e]*x)/Sqrt[d]])/(Sqrt[-c^2]*d^(5/2)*Sqrt[e]) - (((3*I)/32)*b*c*Log[-(
(Sqrt[e]*(1 + Sqrt[-c^2]*x))/(I*Sqrt[-c^2]*Sqrt[d] - Sqrt[e]))]*Log[1 - (I*
Sqrt[e]*x)/Sqrt[d]])/(Sqrt[-c^2]*d^(5/2)*Sqrt[e]) - (((3*I)/32)*b*c*Log[-((
Sqrt[e]*(1 - Sqrt[-c^2]*x))/(I*Sqrt[-c^2]*Sqrt[d] - Sqrt[e]))]*Log[1 + (I*S
qrt[e]*x)/Sqrt[d]])/(Sqrt[-c^2]*d^(5/2)*Sqrt[e]) + (((3*I)/32)*b*c*Log[(Sqr
t[e]*(1 + Sqrt[-c^2]*x))/(I*Sqrt[-c^2]*Sqrt[d] + Sqrt[e])]*Log[1 + (I*Sqrt[
e]*x)/Sqrt[d]])/(Sqrt[-c^2]*d^(5/2)*Sqrt[e]) - (b*c*(5*c^2*d - 3*e)*Log[1 +
 c^2*x^2])/(16*d^2*(c^2*d - e)^2) + (b*c*(5*c^2*d - 3*e)*Log[d + e*x^2])/(1
6*d^2*(c^2*d - e)^2) + (((3*I)/32)*b*c*PolyLog[2, (Sqrt[-c^2]*(Sqrt[d] - I*
Sqrt[e]*x))/(Sqrt[-c^2]*Sqrt[d] - I*Sqrt[e])])/(Sqrt[-c^2]*d^(5/2)*Sqrt[e])
 - (((3*I)/32)*b*c*PolyLog[2, (Sqrt[-c^2]*(Sqrt[d] - I*Sqrt[e]*x))/(Sqrt[-c
^2]*Sqrt[d] + I*Sqrt[e])])/(Sqrt[-c^2]*d^(5/2)*Sqrt[e]) + (((3*I)/32)*b*c*P
olyLog[2, (Sqrt[-c^2]*(Sqrt[d] + I*Sqrt[e]*x))/(Sqrt[-c^2]*Sqrt[d] - I*Sqrt
[e])])/(Sqrt[-c^2]*d^(5/2)*Sqrt[e]) - (((3*I)/32)*b*c*PolyLog[2, (Sqrt[-c^2
]*(Sqrt[d] + I*Sqrt[e]*x))/(Sqrt[-c^2]*Sqrt[d] + I*Sqrt[e])])/(Sqrt[-c^2]*d
^(5/2)*Sqrt[e])
```

Rubi [A] time = 0.948515, antiderivative size = 893, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 11, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.611, Rules used = {199, 205, 4912, 6725, 571, 77, 4908, 2409, 2394, 2393, 2391}

$$\frac{3ib\log\left(\frac{\sqrt{e}\left(1-\sqrt{-c^2}x\right)}{i\sqrt{-c^2}\sqrt{d}+\sqrt{e}}\right)\log\left(1-\frac{i\sqrt{e}x}{\sqrt{d}}\right)c}{32\sqrt{-c^2}d^{5/2}\sqrt{e}}-\frac{3ib\log\left(-\frac{\sqrt{e}\left(\sqrt{-c^2}x+1\right)}{i\sqrt{-c^2}\sqrt{d}-\sqrt{e}}\right)\log\left(1-\frac{i\sqrt{e}x}{\sqrt{d}}\right)c}{32\sqrt{-c^2}d^{5/2}\sqrt{e}}-\frac{3ib\log\left(-\frac{\sqrt{e}\left(1-\sqrt{-c^2}x\right)}{i\sqrt{-c^2}\sqrt{d}-\sqrt{e}}\right)\log\left(\frac{i\sqrt{e}x}{\sqrt{d}}+1\right)c}{32\sqrt{-c^2}d^{5/2}\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])/(d + e*x^2)^3,x]
```

```
[Out] -(b*c)/(8*d*(c^2*d - e)*(d + e*x^2)) + (x*(a + b*ArcTan[c*x]))/(4*d*(d + e*
x^2)^2) + (3*x*(a + b*ArcTan[c*x]))/(8*d^2*(d + e*x^2)) + (3*(a + b*ArcTan[
c*x])*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(5/2)*Sqrt[e]) + (((3*I)/32)*b*c*Lo
g[(Sqrt[e]*(1 - Sqrt[-c^2]*x))/(I*Sqrt[-c^2]*Sqrt[d] + Sqrt[e])]*Log[1 - (I
*Sqrt[e]*x)/Sqrt[d]])/(Sqrt[-c^2]*d^(5/2)*Sqrt[e]) - (((3*I)/32)*b*c*Log[-(
(Sqrt[e]*(1 + Sqrt[-c^2]*x))/(I*Sqrt[-c^2]*Sqrt[d] - Sqrt[e]))]*Log[1 - (I*
Sqrt[e]*x)/Sqrt[d]])/(Sqrt[-c^2]*d^(5/2)*Sqrt[e]) - (((3*I)/32)*b*c*Log[-((
Sqrt[e]*(1 - Sqrt[-c^2]*x))/(I*Sqrt[-c^2]*Sqrt[d] - Sqrt[e]))]*Log[1 + (I*S
qrt[e]*x)/Sqrt[d]])/(Sqrt[-c^2]*d^(5/2)*Sqrt[e]) + (((3*I)/32)*b*c*Log[(Sqr
t[e]*(1 + Sqrt[-c^2]*x))/(I*Sqrt[-c^2]*Sqrt[d] + Sqrt[e])]*Log[1 + (I*Sqrt[
e]*x)/Sqrt[d]])/(Sqrt[-c^2]*d^(5/2)*Sqrt[e]) - (b*c*(5*c^2*d - 3*e)*Log[1 +
 c^2*x^2])/(16*d^2*(c^2*d - e)^2) + (b*c*(5*c^2*d - 3*e)*Log[d + e*x^2])/(1
6*d^2*(c^2*d - e)^2) + (((3*I)/32)*b*c*PolyLog[2, (Sqrt[-c^2]*(Sqrt[d] - I*
Sqrt[e]*x))/(Sqrt[-c^2]*Sqrt[d] - I*Sqrt[e])])/(Sqrt[-c^2]*d^(5/2)*Sqrt[e])
 - (((3*I)/32)*b*c*PolyLog[2, (Sqrt[-c^2]*(Sqrt[d] - I*Sqrt[e]*x))/(Sqrt[-c
^2]*Sqrt[d] + I*Sqrt[e])])/(Sqrt[-c^2]*d^(5/2)*Sqrt[e]) + (((3*I)/32)*b*c*P
olyLog[2, (Sqrt[-c^2]*(Sqrt[d] + I*Sqrt[e]*x))/(Sqrt[-c^2]*Sqrt[d] - I*Sqrt
[e])])/(Sqrt[-c^2]*d^(5/2)*Sqrt[e]) - (((3*I)/32)*b*c*PolyLog[2, (Sqrt[-c^2
]*(Sqrt[d] + I*Sqrt[e]*x))/(Sqrt[-c^2]*Sqrt[d] + I*Sqrt[e])])/(Sqrt[-c^2]*d
^(5/2)*Sqrt[e])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 4912

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcTan[c*x], u, x]
- Dist[b*c, Int[u/(1 + c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && (I
ntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 571

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 4908

```
Int[ArcTan[(c_.)*(x_)]/((d_.) + (e_.)*(x_)^2), x_Symbol] :> Dist[I/2, Int[L
og[1 - I*c*x]/(d + e*x^2), x], x] - Dist[I/2, Int[Log[1 + I*c*x]/(d + e*x^2
), x], x] /; FreeQ[{c, d, e}, x]
```

Rule 2409

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
```

```
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{a+b\tan^{-1}(cx)}{\left(d+ex^2\right)^3}\,dx = \frac{x\left(a+b\tan^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2} + \frac{3x\left(a+b\tan^{-1}(cx)\right)}{8d^2\left(d+ex^2\right)} + \frac{3\left(a+b\tan^{-1}(cx)\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{5/2}\sqrt{e}} - (bc)\int \frac{\frac{x}{4d(d+ex\cdots}}{\cdots}$$

$$= \frac{x\left(a+b\tan^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2} + \frac{3x\left(a+b\tan^{-1}(cx)\right)}{8d^2\left(d+ex^2\right)} + \frac{3\left(a+b\tan^{-1}(cx)\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{5/2}\sqrt{e}} - (bc)\int\left(\frac{\cdots}{8d^2\left(1\cdots\right.}\right.$$

$$= \frac{x\left(a+b\tan^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2} + \frac{3x\left(a+b\tan^{-1}(cx)\right)}{8d^2\left(d+ex^2\right)} + \frac{3\left(a+b\tan^{-1}(cx)\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{5/2}\sqrt{e}} - \frac{(bc)\int\frac{x(5d+\cdots}{(1+c^2x^2)\cdots}}{8d^2\cdots}$$

$$= \frac{x\left(a+b\tan^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2} + \frac{3x\left(a+b\tan^{-1}(cx)\right)}{8d^2\left(d+ex^2\right)} + \frac{3\left(a+b\tan^{-1}(cx)\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{5/2}\sqrt{e}} - \frac{(bc)\,\mathrm{Subst}\left(\int\cdots\right.}{\cdots}$$

$$= \frac{x\left(a+b\tan^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2} + \frac{3x\left(a+b\tan^{-1}(cx)\right)}{8d^2\left(d+ex^2\right)} + \frac{3\left(a+b\tan^{-1}(cx)\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{5/2}\sqrt{e}} - \frac{(bc)\,\mathrm{Subst}\left(\int\cdots\right.}{\cdots}$$

$$= -\frac{bc}{8d\left(c^2d-e\right)\left(d+ex^2\right)} + \frac{x\left(a+b\tan^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2} + \frac{3x\left(a+b\tan^{-1}(cx)\right)}{8d^2\left(d+ex^2\right)} + \frac{3\left(a+b\tan^{-1}(cx)\right)\tan\cdots}{8d^{5/2}\sqrt{e}}$$

$$= -\frac{bc}{8d\left(c^2d-e\right)\left(d+ex^2\right)} + \frac{x\left(a+b\tan^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2} + \frac{3x\left(a+b\tan^{-1}(cx)\right)}{8d^2\left(d+ex^2\right)} + \frac{3\left(a+b\tan^{-1}(cx)\right)\tan\cdots}{8d^{5/2}\sqrt{e}}$$

$$= -\frac{bc}{8d\left(c^2d-e\right)\left(d+ex^2\right)} + \frac{x\left(a+b\tan^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2} + \frac{3x\left(a+b\tan^{-1}(cx)\right)}{8d^2\left(d+ex^2\right)} + \frac{3\left(a+b\tan^{-1}(cx)\right)\tan\cdots}{8d^{5/2}\sqrt{e}}$$

$$= -\frac{bc}{8d\left(c^2d-e\right)\left(d+ex^2\right)} + \frac{x\left(a+b\tan^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2} + \frac{3x\left(a+b\tan^{-1}(cx)\right)}{8d^2\left(d+ex^2\right)} + \frac{3\left(a+b\tan^{-1}(cx)\right)\tan\cdots}{8d^{5/2}\sqrt{e}}$$

Mathematica [A] time = 12.1938, size = 1745, normalized size = 1.95

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c*x])/(d + e*x^2)^3,x]
```

```
[Out] (a*x)/(4*d*(d + e*x^2)^2) + (3*a*x)/(8*d^2*(d + e*x^2)) + (3*a*ArcTan[(Sqrt
[e]*x)/Sqrt[d]])/(8*d^(5/2)*Sqrt[e]) + (b*c*(10*c^2*d*Log[1 + ((c^2*d - e)*
Cos[2*ArcTan[c*x]])/(c^2*d + e)] - 6*e*Log[1 + ((c^2*d - e)*Cos[2*ArcTan[c*
x]])/(c^2*d + e)] + (3*c^2*d*(c^2*d - e)*(-4*ArcTan[c*x]*ArcTanh[Sqrt[-(c^2
*d*e)]/(c*e*x)] + 2*ArcCos[-((c^2*d + e)/(c^2*d - e))]*ArcTanh[(c*e*x)/Sqrt
[-(c^2*d*e)]] - (ArcCos[-((c^2*d + e)/(c^2*d - e))] + (2*I)*ArcTanh[(c*e*x)
/Sqrt[-(c^2*d*e)]])*Log[(2*c^2*d*((-I)*e + Sqrt[-(c^2*d*e)])*(-I + c*x))/((
c^2*d - e)*(c^2*d + c*Sqrt[-(c^2*d*e)]*x))] - (ArcCos[-((c^2*d + e)/(c^2*d
- e))] - (2*I)*ArcTanh[(c*e*x)/Sqrt[-(c^2*d*e)]])*Log[(2*c^2*d*(I*e + Sqrt[
-(c^2*d*e)])*(I + c*x))/((c^2*d - e)*(c^2*d + c*Sqrt[-(c^2*d*e)]*x))] + (Ar
cCos[-((c^2*d + e)/(c^2*d - e))] - (2*I)*(ArcTanh[(c*d)/(Sqrt[-(c^2*d*e)]*x
)] + ArcTanh[(c*e*x)/Sqrt[-(c^2*d*e)]]))*Log[(Sqrt[2]*Sqrt[-(c^2*d*e)])/(Sq
rt[c^2*d - e]*E^(I*ArcTan[c*x])*Sqrt[c^2*d + e + (c^2*d - e)*Cos[2*ArcTan[c
*x]]])] + (ArcCos[-((c^2*d + e)/(c^2*d - e))] + (2*I)*(ArcTanh[(c*d)/(Sqrt[
-(c^2*d*e)]*x)] + ArcTanh[(c*e*x)/Sqrt[-(c^2*d*e)]]))*Log[(Sqrt[2]*Sqrt[-(c
^2*d*e)]*E^(I*ArcTan[c*x]))/(Sqrt[c^2*d - e]*Sqrt[c^2*d + e + (c^2*d - e)*C
os[2*ArcTan[c*x]]])] + I*(PolyLog[2, ((c^2*d + e - (2*I)*Sqrt[-(c^2*d*e)])*
(c^2*d - c*Sqrt[-(c^2*d*e)]*x))/((c^2*d - e)*(c^2*d + c*Sqrt[-(c^2*d*e)]*x)
)] - PolyLog[2, ((c^2*d + e + (2*I)*Sqrt[-(c^2*d*e)])*(c^2*d - c*Sqrt[-(c^2
*d*e)]*x))/((c^2*d - e)*(c^2*d + c*Sqrt[-(c^2*d*e)]*x))])))/Sqrt[-(c^2*d*e)
] - (3*(c^2*d - e)*e*(-4*ArcTan[c*x]*ArcTanh[Sqrt[-(c^2*d*e)]/(c*e*x)] + 2*
ArcCos[-((c^2*d + e)/(c^2*d - e))]*ArcTanh[(c*e*x)/Sqrt[-(c^2*d*e)]] - (Arc
Cos[-((c^2*d + e)/(c^2*d - e))] + (2*I)*ArcTanh[(c*e*x)/Sqrt[-(c^2*d*e)]])*
Log[(2*c^2*d*((-I)*e + Sqrt[-(c^2*d*e)])*(-I + c*x))/((c^2*d - e)*(c^2*d +
c*Sqrt[-(c^2*d*e)]*x))] - (ArcCos[-((c^2*d + e)/(c^2*d - e))] - (2*I)*ArcTa
nh[(c*e*x)/Sqrt[-(c^2*d*e)]])*Log[(2*c^2*d*(I*e + Sqrt[-(c^2*d*e)])*(I + c*
x))/((c^2*d - e)*(c^2*d + c*Sqrt[-(c^2*d*e)]*x))] + (ArcCos[-((c^2*d + e)/(
c^2*d - e))] - (2*I)*(ArcTanh[(c*d)/(Sqrt[-(c^2*d*e)]*x)] + ArcTanh[(c*e*x)
/Sqrt[-(c^2*d*e)]]))*Log[(Sqrt[2]*Sqrt[-(c^2*d*e)])/(Sqrt[c^2*d - e]*E^(I*A
rcTan[c*x])*Sqrt[c^2*d + e + (c^2*d - e)*Cos[2*ArcTan[c*x]]])] + (ArcCos[-(
(c^2*d + e)/(c^2*d - e))] + (2*I)*(ArcTanh[(c*d)/(Sqrt[-(c^2*d*e)]*x)] + Ar
cTanh[(c*e*x)/Sqrt[-(c^2*d*e)]]))*Log[(Sqrt[2]*Sqrt[-(c^2*d*e)]*E^(I*ArcTan
[c*x]))/(Sqrt[c^2*d - e]*Sqrt[c^2*d + e + (c^2*d - e)*Cos[2*ArcTan[c*x]]])]
 + I*(PolyLog[2, ((c^2*d + e - (2*I)*Sqrt[-(c^2*d*e)])*(c^2*d - c*Sqrt[-(c^
2*d*e)]*x))/((c^2*d - e)*(c^2*d + c*Sqrt[-(c^2*d*e)]*x))] - PolyLog[2, ((c^
2*d + e + (2*I)*Sqrt[-(c^2*d*e)])*(c^2*d - c*Sqrt[-(c^2*d*e)]*x))/((c^2*d -
```

```
e)*(c^2*d + c*Sqrt[-(c^2*d*e)]*x))]])))/Sqrt[-(c^2*d*e)] - (16*c^2*d*(c^2*d
 - e)*e*ArcTan[c*x]*Sin[2*ArcTan[c*x]])/(c^2*d + e + (c^2*d - e)*Cos[2*ArcT
an[c*x]])^2 + (8*c^2*d*e + 4*(5*c^4*d^2 - 8*c^2*d*e + 3*e^2)*ArcTan[c*x]*Si
n[2*ArcTan[c*x]])/(c^2*d + e + (c^2*d - e)*Cos[2*ArcTan[c*x]])))/(32*d^2*(-
(c^2*d) + e)^2)
```

Maple [B] time = 0.971, size = 4027, normalized size = 4.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/(e*x^2+d)^3,x)
```

```
[Out] 1/8*c^5*b/(c^4*d^2-2*c^2*d*e+e^2)/(c^2*e*x^2+c^2*d)^2*e+5/16*c^5*b/(c^4*d^2
-2*c^2*d*e+e^2)/(c^2*d-e)*ln((1+I*c*x)^4/(c^2*x^2+1)^2*c^2*d+2*c^2*d*(1+I*c
*x)^2/(c^2*x^2+1)-(1+I*c*x)^4/(c^2*x^2+1)^2*e+c^2*d+2*(1+I*c*x)^2/(c^2*x^2+
1)*e-e)-5/4*c^5*b/(c^4*d^2-2*c^2*d*e+e^2)/(c^2*d-e)*ln((1+I*c*x)/(c^2*x^2+1
)^(1/2))+3/4*c^5*b*arctan(c*x)^2/(c^4*d^2-2*c^2*d*e+e^2)^2*(c^2*e*d)^(1/2)+
3/8*c^5*b*polylog(2,(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d+2*(c^2*e*d)^(
1/2)-e))/(c^4*d^2-2*c^2*d*e+e^2)^2*(c^2*e*d)^(1/2)-3/4*c^6*b/(c^4*d^2-2*c^2
*d*e+e^2)/(c^2*e*x^2+c^2*d)^2*e^2/d*arctan(c*x)*x^3+3/8*c^4*b/(c^4*d^2-2*c^
2*d*e+e^2)/(c^2*e*x^2+c^2*d)^2/d^2*arctan(c*x)*x^3*e^3+5/8*c^4*b/(c^4*d^2-2
*c^2*d*e+e^2)/(c^2*e*x^2+c^2*d)^2/d*arctan(c*x)*x*e^2+5/16*c^2*b*(d*e)^(1/2
)*e/d^2*arctanh(1/4*(2*(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)+2*c^2*d+2*e)/c/(d*
e)^(1/2))/(c^4*d^2-2*c^2*d*e+e^2)/(c^2*d-e)-3/8*I*c*b*(c^2*e*d)^(1/2)/(c^4*
d^2-2*c^2*d*e+e^2)/d^2*arctan(c*x)*ln(1-(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(
-c^2*d-2*(c^2*e*d)^(1/2)-e))+5/4*I*c^7*b/(c^4*d^2-2*c^2*d*e+e^2)/(c^2*e*x^2
+c^2*d)^2*arctan(c*x)*x^2*e+3/8*a/d^2/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))-3
/16*I*c^7*b*ln(1-(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d+2*(c^2*e*d)^(1/2
)-e))*arctan(c*x)/e/(c^4*d^2-2*c^2*d*e+e^2)^2*(c^2*e*d)^(1/2)*d+3/16*I*c^3*
b*(c^2*e*d)^(1/2)/(c^4*d^2-2*c^2*d*e+e^2)/e/d*arctan(c*x)*ln(1-(c^2*d-e)*(1
+I*c*x)^2/(c^2*x^2+1)/(-c^2*d-2*(c^2*e*d)^(1/2)-e))-9/8*I*c^3*b*ln(1-(c^2*d
-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d+2*(c^2*e*d)^(1/2)-e))*arctan(c*x)*e/d/(
c^4*d^2-2*c^2*d*e+e^2)^2*(c^2*e*d)^(1/2)+3/4*I*c*b*ln(1-(c^2*d-e)*(1+I*c*x)
^2/(c^2*x^2+1)/(-c^2*d+2*(c^2*e*d)^(1/2)-e))*arctan(c*x)*e^2/d^2/(c^4*d^2-2
*c^2*d*e+e^2)^2*(c^2*e*d)^(1/2)+3/16*I/c*b*(c^2*e*d)^(1/2)/d^3*e/(c^4*d^2-2
*c^2*d*e+e^2)*arctan(c*x)*ln(1-(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d-2*
(c^2*e*d)^(1/2)-e))-3/16*I/c*b*e^3*ln(1-(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(
-c^2*d+2*(c^2*e*d)^(1/2)-e))*arctan(c*x)/d^3/(c^4*d^2-2*c^2*d*e+e^2)^2*(c^2
*e*d)^(1/2)+5/8*I*c^7*b/(c^4*d^2-2*c^2*d*e+e^2)/(c^2*e*x^2+c^2*d)^2/d*arcta
n(c*x)*x^4*e^2-3/8*I*c^5*b/(c^4*d^2-2*c^2*d*e+e^2)/(c^2*e*x^2+c^2*d)^2/d^2*
```

arctan(c*x)*x^4*e^3-3/4*I*c^5*b/(c^4*d^2-2*c^2*d*e+e^2)/(c^2*e*x^2+c^2*d)^2
/d*arctan(c*x)*x^2*e^2+3/8*c^2*a/d^2*x/(c^2*e*x^2+c^2*d)+1/4*c^4*a*x/d/(c^2
*e*x^2+c^2*d)^2+1/8*c^7*b/(c^4*d^2-2*c^2*d*e+e^2)/(c^2*e*x^2+c^2*d)^2*e*x^2
-3/16*c*b*(c^2*e*d)^(1/2)/d^2/(c^4*d^2-2*c^2*d*e+e^2)*polylog(2,(c^2*d-e)*(
1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d-2*(c^2*e*d)^(1/2)-e))+1/8*c^2*b*(d*e)^(1/2)/
d^2*arctanh(1/4*(2*(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)+2*c^2*d+2*e)/c/(d*e)^(
1/2))/(c^4*d^2-2*c^2*d*e+e^2)-3/8*c*b*(c^2*e*d)^(1/2)/d^2/(c^4*d^2-2*c^2*d*
e+e^2)*arctan(c*x)^2-3/16*b*(d*e)^(1/2)/d^3*e*arctanh(1/4*(2*(c^2*d-e)*(1+I
*c*x)^2/(c^2*x^2+1)+2*c^2*d+2*e)/c/(d*e)^(1/2))/(c^4*d^2-2*c^2*d*e+e^2)-5/4
*c^6*b/(c^4*d^2-2*c^2*d*e+e^2)/(c^2*e*x^2+c^2*d)^2*e*arctan(c*x)*x-3/16/c*b
*e^3*arctan(c*x)^2/d^3/(c^4*d^2-2*c^2*d*e+e^2)^2*(c^2*e*d)^(1/2)-3/32*c^7*b
*d*polylog(2,(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d+2*(c^2*e*d)^(1/2)-e)
)/e/(c^4*d^2-2*c^2*d*e+e^2)^2*(c^2*e*d)^(1/2)+3/16*c^4*b*(d*e)^(1/2)/d*arct
anh(1/4*(2*(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)+2*c^2*d+2*e)/c/(d*e)^(1/2))/(c
^4*d^2-2*c^2*d*e+e^2)/(c^2*d-e)+3/4*c*b*e^2*arctan(c*x)^2/d^2/(c^4*d^2-2*c^
2*d*e+e^2)^2*(c^2*e*d)^(1/2)-1/2*c^3*b/(c^4*d^2-2*c^2*d*e+e^2)*e/d/(c^2*d-e
)*ln((1+I*c*x)^4/(c^2*x^2+1)^2*c^2*d+2*c^2*d*(1+I*c*x)^2/(c^2*x^2+1)-(1+I*c
*x)^4/(c^2*x^2+1)^2*e+c^2*d+2*(1+I*c*x)^2/(c^2*x^2+1)*e-e)+5/8*I*c^7*b/(c^4
*d^2-2*c^2*d*e+e^2)/(c^2*e*x^2+c^2*d)^2*d*arctan(c*x)-3/8*I*c^5*b/(c^4*d^2-
2*c^2*d*e+e^2)/(c^2*e*x^2+c^2*d)^2*e*arctan(c*x)+3/4*I*c^5*b*arctan(c*x)*ln
(1-(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d+2*(c^2*e*d)^(1/2)-e))/(c^4*d^2
-2*c^2*d*e+e^2)^2*(c^2*e*d)^(1/2)+1/8*c^5*b/(c^4*d^2-2*c^2*d*e+e^2)/(c^2*e*
x^2+c^2*d)^2/d*x^2*e^2+1/8*c^7*b/(c^4*d^2-2*c^2*d*e+e^2)/(c^2*e*x^2+c^2*d)^
2/d*x^4*e^2-3/16*b*(d*e)^(1/2)/d^3*e^2*arctanh(1/4*(2*(c^2*d-e)*(1+I*c*x)^2
/(c^2*x^2+1)+2*c^2*d+2*e)/c/(d*e)^(1/2))/(c^4*d^2-2*c^2*d*e+e^2)/(c^2*d-e)+
5/16*c^4*b*(d*e)^(1/2)/d/e*arctanh(1/4*(2*(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)
+2*c^2*d+2*e)/c/(d*e)^(1/2))/(c^4*d^2-2*c^2*d*e+e^2)-5/16*c^6*b*(d*e)^(1/2)
/e*arctanh(1/4*(2*(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)+2*c^2*d+2*e)/c/(d*e)^(1
/2))/(c^4*d^2-2*c^2*d*e+e^2)/(c^2*d-e)-9/8*c^3*b*arctan(c*x)^2*e/d/(c^4*d^2
-2*c^2*d*e+e^2)^2*(c^2*e*d)^(1/2)-9/16*c^3*b*e*polylog(2,(c^2*d-e)*(1+I*c*x
)^2/(c^2*x^2+1)/(-c^2*d+2*(c^2*e*d)^(1/2)-e))/d/(c^4*d^2-2*c^2*d*e+e^2)^2*(
c^2*e*d)^(1/2)+3/8*c*b*e^2*polylog(2,(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^
2*d+2*(c^2*e*d)^(1/2)-e))/d^2/(c^4*d^2-2*c^2*d*e+e^2)^2*(c^2*e*d)^(1/2)+3/1
6*c*b/(c^4*d^2-2*c^2*d*e+e^2)/d^2*e^2/(c^2*d-e)*ln((1+I*c*x)^4/(c^2*x^2+1)^
2*c^2*d+2*c^2*d*(1+I*c*x)^2/(c^2*x^2+1)-(1+I*c*x)^4/(c^2*x^2+1)^2*e+c^2*d+2
*(1+I*c*x)^2/(c^2*x^2+1)*e-e)-3/4*c*b/(c^4*d^2-2*c^2*d*e+e^2)/d^2*e^2/(c^2*
d-e)*ln((1+I*c*x)/(c^2*x^2+1)^(1/2))+3/16/c*b*(c^2*e*d)^(1/2)/d^3*e/(c^4*d^
2-2*c^2*d*e+e^2)*arctan(c*x)^2+3/32/c*b*(c^2*e*d)^(1/2)/d^3*e/(c^4*d^2-2*c^
2*d*e+e^2)*polylog(2,(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d-2*(c^2*e*d)^
(1/2)-e))-3/32/c*b*e^3*polylog(2,(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d+
2*(c^2*e*d)^(1/2)-e))/d^3/(c^4*d^2-2*c^2*d*e+e^2)^2*(c^2*e*d)^(1/2)+3/32*c^
3*b*(c^2*e*d)^(1/2)/d/e/(c^4*d^2-2*c^2*d*e+e^2)*polylog(2,(c^2*d-e)*(1+I*c*
x)^2/(c^2*x^2+1)/(-c^2*d-2*(c^2*e*d)^(1/2)-e))-3/16*c^7*b*d*arctan(c*x)^2/e
/(c^4*d^2-2*c^2*d*e+e^2)^2*(c^2*e*d)^(1/2)+3/16*c^3*b*(c^2*e*d)^(1/2)/d/e/(
c^4*d^2-2*c^2*d*e+e^2)*arctan(c*x)^2+2*c^3*b/(c^4*d^2-2*c^2*d*e+e^2)*e/d/(c

```
^2*d-e)*ln((1+I*c*x)/(c^2*x^2+1)^(1/2))+3/8*c^8*b/(c^4*d^2-2*c^2*d*e+e^2)/(
c^2*e*x^2+c^2*d)^2*e*arctan(c*x)*x^3+5/8*c^8*b/(c^4*d^2-2*c^2*d*e+e^2)/(c^2
*e*x^2+c^2*d)^2*d*arctan(c*x)*x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b \arctan(cx) + a}{e^3x^6 + 3\,de^2x^4 + 3\,d^2ex^2 + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*arctan(c*x) + a)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3), x
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/(e*x**2+d)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \arctan (cx) + a}{\left(ex^2 + d\right)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)/(e*x^2 + d)^3, x)
```

3.1172 $\int \frac{a+b\tan^{-1}(cx)}{x^2(d+ex^2)^3}\,dx$

Optimal. Leaf size=1518

result too large to display

```
[Out] (b*c*e)/(8*d^2*(c^2*d - e)*(d + e*x^2)) - (a + b*ArcTan[c*x])/(d^3*x) - (e*
x*(a + b*ArcTan[c*x]))/(4*d^2*(d + e*x^2)^2) - (7*e*x*(a + b*ArcTan[c*x]))/
(8*d^3*(d + e*x^2)) - (a*Sqrt[e]*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/d^(7/2) - (7*
Sqrt[e]*(a + b*ArcTan[c*x])*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(7/2)) + (b*c
*Log[x])/d^3 - ((I/4)*b*Sqrt[e]*Log[1 + I*c*x]*Log[(c*(Sqrt[-d] - Sqrt[e]*x
))/(c*Sqrt[-d] - I*Sqrt[e])])/(-d)^(7/2) + ((I/4)*b*Sqrt[e]*Log[1 - I*c*x]*
Log[(c*(Sqrt[-d] - Sqrt[e]*x))/(c*Sqrt[-d] + I*Sqrt[e])])/(-d)^(7/2) - ((I/
4)*b*Sqrt[e]*Log[1 - I*c*x]*Log[(c*(Sqrt[-d] + Sqrt[e]*x))/(c*Sqrt[-d] - I*
Sqrt[e])])/(-d)^(7/2) + ((I/4)*b*Sqrt[e]*Log[1 + I*c*x]*Log[(c*(Sqrt[-d] +
Sqrt[e]*x))/(c*Sqrt[-d] + I*Sqrt[e])])/(-d)^(7/2) - (((7*I)/32)*b*c*Sqrt[e]
*Log[(Sqrt[e]*(1 - Sqrt[-c^2]*x))/(I*Sqrt[-c^2]*Sqrt[d] + Sqrt[e])]*Log[1 -
 (I*Sqrt[e]*x)/Sqrt[d]])/(Sqrt[-c^2]*d^(7/2)) + (((7*I)/32)*b*c*Sqrt[e]*Log
[-((Sqrt[e]*(1 + Sqrt[-c^2]*x))/(I*Sqrt[-c^2]*Sqrt[d] - Sqrt[e]))]*Log[1 -
(I*Sqrt[e]*x)/Sqrt[d]])/(Sqrt[-c^2]*d^(7/2)) + (((7*I)/32)*b*c*Sqrt[e]*Log[
-((Sqrt[e]*(1 - Sqrt[-c^2]*x))/(I*Sqrt[-c^2]*Sqrt[d] - Sqrt[e]))]*Log[1 + (
I*Sqrt[e]*x)/Sqrt[d]])/(Sqrt[-c^2]*d^(7/2)) - (((7*I)/32)*b*c*Sqrt[e]*Log[(
Sqrt[e]*(1 + Sqrt[-c^2]*x))/(I*Sqrt[-c^2]*Sqrt[d] + Sqrt[e])]*Log[1 + (I*Sq
rt[e]*x)/Sqrt[d]])/(Sqrt[-c^2]*d^(7/2)) - (b*c*Log[1 + c^2*x^2])/(2*d^3) +
(b*c*(5*c^2*d - 3*e)*e*Log[1 + c^2*x^2])/(16*d^3*(c^2*d - e)^2) + (b*c*e*Lo
g[1 + c^2*x^2])/(4*d^3*(c^2*d - e)) - (b*c*(5*c^2*d - 3*e)*e*Log[d + e*x^2]
)/(16*d^3*(c^2*d - e)^2) - (b*c*e*Log[d + e*x^2])/(4*d^3*(c^2*d - e)) + ((I
/4)*b*Sqrt[e]*PolyLog[2, (Sqrt[e]*(I - c*x))/(c*Sqrt[-d] + I*Sqrt[e])])/(-d
)^(7/2) - ((I/4)*b*Sqrt[e]*PolyLog[2, (Sqrt[e]*(1 - I*c*x))/(I*c*Sqrt[-d] +
 Sqrt[e])])/(-d)^(7/2) - ((I/4)*b*Sqrt[e]*PolyLog[2, (Sqrt[e]*(1 + I*c*x))/
(I*c*Sqrt[-d] + Sqrt[e])])/(-d)^(7/2) + ((I/4)*b*Sqrt[e]*PolyLog[2, (Sqrt[e
]*(I + c*x))/(c*Sqrt[-d] + I*Sqrt[e])])/(-d)^(7/2) - (((7*I)/32)*b*c*Sqrt[e
]*PolyLog[2, (Sqrt[-c^2]*(Sqrt[d] - I*Sqrt[e]*x))/(Sqrt[-c^2]*Sqrt[d] - I*S
qrt[e])])/(Sqrt[-c^2]*d^(7/2)) + (((7*I)/32)*b*c*Sqrt[e]*PolyLog[2, (Sqrt[-
c^2]*(Sqrt[d] - I*Sqrt[e]*x))/(Sqrt[-c^2]*Sqrt[d] + I*Sqrt[e])])/(Sqrt[-c^2
]*d^(7/2)) - (((7*I)/32)*b*c*Sqrt[e]*PolyLog[2, (Sqrt[-c^2]*(Sqrt[d] + I*Sq
rt[e]*x))/(Sqrt[-c^2]*Sqrt[d] - I*Sqrt[e])])/(Sqrt[-c^2]*d^(7/2)) + (((7*I)
/32)*b*c*Sqrt[e]*PolyLog[2, (Sqrt[-c^2]*(Sqrt[d] + I*Sqrt[e]*x))/(Sqrt[-c^2
]*Sqrt[d] + I*Sqrt[e])])/(Sqrt[-c^2]*d^(7/2))
```

Rubi [A] time = 2.63596, antiderivative size = 1518, normalized size of antiderivative =

1., number of steps used = 73, number of rules used = 19, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.905, Rules used = {4980, 4852, 266, 36, 29, 31, 199, 205, 4912, 6725, 571, 77, 4908, 2409, 2394, 2393, 2391, 444, 4910}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])/(x^2*(d + e*x^2)^3),x]
```

```
[Out] (b*c*e)/(8*d^2*(c^2*d - e)*(d + e*x^2)) - (a + b*ArcTan[c*x])/(d^3*x) - (e*
x*(a + b*ArcTan[c*x]))/(4*d^2*(d + e*x^2)^2) - (7*e*x*(a + b*ArcTan[c*x]))/
(8*d^3*(d + e*x^2)) - (a*Sqrt[e]*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/d^(7/2) - (7*
Sqrt[e]*(a + b*ArcTan[c*x])*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(7/2)) + (b*c
*Log[x])/d^3 - ((I/4)*b*Sqrt[e]*Log[1 + I*c*x]*Log[(c*(Sqrt[-d] - Sqrt[e]*x
))/(c*Sqrt[-d] - I*Sqrt[e])])/(-d)^(7/2) + ((I/4)*b*Sqrt[e]*Log[1 - I*c*x]*
Log[(c*(Sqrt[-d] - Sqrt[e]*x))/(c*Sqrt[-d] + I*Sqrt[e])])/(-d)^(7/2) - ((I/
4)*b*Sqrt[e]*Log[1 - I*c*x]*Log[(c*(Sqrt[-d] + Sqrt[e]*x))/(c*Sqrt[-d] - I*
Sqrt[e])])/(-d)^(7/2) + ((I/4)*b*Sqrt[e]*Log[1 + I*c*x]*Log[(c*(Sqrt[-d] +
Sqrt[e]*x))/(c*Sqrt[-d] + I*Sqrt[e])])/(-d)^(7/2) - (((7*I)/32)*b*c*Sqrt[e]
*Log[(Sqrt[e]*(1 - Sqrt[-c^2]*x))/(I*Sqrt[-c^2]*Sqrt[d] + Sqrt[e])]*Log[1 -
 (I*Sqrt[e]*x)/Sqrt[d]])/(Sqrt[-c^2]*d^(7/2)) + (((7*I)/32)*b*c*Sqrt[e]*Log
[-((Sqrt[e]*(1 + Sqrt[-c^2]*x))/(I*Sqrt[-c^2]*Sqrt[d] - Sqrt[e]))]*Log[1 -
(I*Sqrt[e]*x)/Sqrt[d]])/(Sqrt[-c^2]*d^(7/2)) + (((7*I)/32)*b*c*Sqrt[e]*Log[
-((Sqrt[e]*(1 - Sqrt[-c^2]*x))/(I*Sqrt[-c^2]*Sqrt[d] - Sqrt[e]))]*Log[1 + (
I*Sqrt[e]*x)/Sqrt[d]])/(Sqrt[-c^2]*d^(7/2)) - (((7*I)/32)*b*c*Sqrt[e]*Log[(
Sqrt[e]*(1 + Sqrt[-c^2]*x))/(I*Sqrt[-c^2]*Sqrt[d] + Sqrt[e])]*Log[1 + (I*Sq
rt[e]*x)/Sqrt[d]])/(Sqrt[-c^2]*d^(7/2)) - (b*c*Log[1 + c^2*x^2])/(2*d^3) +
(b*c*(5*c^2*d - 3*e)*e*Log[1 + c^2*x^2])/(16*d^3*(c^2*d - e)^2) + (b*c*e*Lo
g[1 + c^2*x^2])/(4*d^3*(c^2*d - e)) - (b*c*(5*c^2*d - 3*e)*e*Log[d + e*x^2]
)/(16*d^3*(c^2*d - e)^2) - (b*c*e*Log[d + e*x^2])/(4*d^3*(c^2*d - e)) + ((I
/4)*b*Sqrt[e]*PolyLog[2, (Sqrt[e]*(I - c*x))/(c*Sqrt[-d] + I*Sqrt[e])])/(-d
)^(7/2) - ((I/4)*b*Sqrt[e]*PolyLog[2, (Sqrt[e]*(1 - I*c*x))/(I*c*Sqrt[-d] +
 Sqrt[e])])/(-d)^(7/2) - ((I/4)*b*Sqrt[e]*PolyLog[2, (Sqrt[e]*(1 + I*c*x))/
(I*c*Sqrt[-d] + Sqrt[e])])/(-d)^(7/2) + ((I/4)*b*Sqrt[e]*PolyLog[2, (Sqrt[e
]*(I + c*x))/(c*Sqrt[-d] + I*Sqrt[e])])/(-d)^(7/2) - (((7*I)/32)*b*c*Sqrt[e
]*PolyLog[2, (Sqrt[-c^2]*(Sqrt[d] - I*Sqrt[e]*x))/(Sqrt[-c^2]*Sqrt[d] - I*S
qrt[e])])/(Sqrt[-c^2]*d^(7/2)) + (((7*I)/32)*b*c*Sqrt[e]*PolyLog[2, (Sqrt[-
c^2]*(Sqrt[d] - I*Sqrt[e]*x))/(Sqrt[-c^2]*Sqrt[d] + I*Sqrt[e])])/(Sqrt[-c^2
]*d^(7/2)) - (((7*I)/32)*b*c*Sqrt[e]*PolyLog[2, (Sqrt[-c^2]*(Sqrt[d] + I*Sq
rt[e]*x))/(Sqrt[-c^2]*Sqrt[d] - I*Sqrt[e])])/(Sqrt[-c^2]*d^(7/2)) + (((7*I)
/32)*b*c*Sqrt[e]*PolyLog[2, (Sqrt[-c^2]*(Sqrt[d] + I*Sqrt[e]*x))/(Sqrt[-c^2
]*Sqrt[d] + I*Sqrt[e])])/(Sqrt[-c^2]*d^(7/2))
```

Rule 4980

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 4912

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcTan[c*x], u, x]
- Dist[b*c, Int[u/(1 + c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && (I
ntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 571

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 4908

```
Int[ArcTan[(c_.)*(x_)]/((d_.) + (e_.)*(x_)^2), x_Symbol] :> Dist[I/2, Int[L
og[1 - I*c*x]/(d + e*x^2), x], x] - Dist[I/2, Int[Log[1 + I*c*x]/(d + e*x^2
), x], x] /; FreeQ[{c, d, e}, x]
```

Rule 2409

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
```

```
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 4910

```
Int[(ArcTan[(c_.)*(x_)]*(b_.) + (a_))/((d_.) + (e_.)*(x_)^2), x_Symbol] :>
Dist[a, Int[1/(d + e*x^2), x], x] + Dist[b, Int[ArcTan[c*x]/(d + e*x^2), x]
, x] /; FreeQ[{a, b, c, d, e}, x]
```

Rubi steps

$$\int \frac{a+b\tan^{-1}(cx)}{x^2\left(d+ex^2\right)^3}\,dx = \int\left(\frac{a+b\tan^{-1}(cx)}{d^3x^2}-\frac{e\left(a+b\tan^{-1}(cx)\right)}{d\left(d+ex^2\right)^3}-\frac{e\left(a+b\tan^{-1}(cx)\right)}{d^2\left(d+ex^2\right)^2}-\frac{e\left(a+b\tan^{-1}(cx)\right)}{d^3\left(d+ex^2\right)}\right)dx$$

$$=\frac{\int\frac{a+b\tan^{-1}(cx)}{x^2}\,dx}{d^3}-\frac{e\int\frac{a+b\tan^{-1}(cx)}{d+ex^2}\,dx}{d^3}-\frac{e\int\frac{a+b\tan^{-1}(cx)}{\left(d+ex^2\right)^2}\,dx}{d^2}-\frac{e\int\frac{a+b\tan^{-1}(cx)}{\left(d+ex^2\right)^3}\,dx}{d}$$

$$=-\frac{a+b\tan^{-1}(cx)}{d^3x}-\frac{ex\left(a+b\tan^{-1}(cx)\right)}{4d^2\left(d+ex^2\right)^2}-\frac{7ex\left(a+b\tan^{-1}(cx)\right)}{8d^3\left(d+ex^2\right)}-\frac{7\sqrt{e}\left(a+b\tan^{-1}(cx)\right)\tan^{-1}}{8d^{7/2}}$$

$$=-\frac{a+b\tan^{-1}(cx)}{d^3x}-\frac{ex\left(a+b\tan^{-1}(cx)\right)}{4d^2\left(d+ex^2\right)^2}-\frac{7ex\left(a+b\tan^{-1}(cx)\right)}{8d^3\left(d+ex^2\right)}-\frac{a\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{7/2}}-7\sqrt{e}\left(a+\right.$$

$$=-\frac{a+b\tan^{-1}(cx)}{d^3x}-\frac{ex\left(a+b\tan^{-1}(cx)\right)}{4d^2\left(d+ex^2\right)^2}-\frac{7ex\left(a+b\tan^{-1}(cx)\right)}{8d^3\left(d+ex^2\right)}-\frac{a\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{7/2}}-7\sqrt{e}\left(a+\right.$$

$$=-\frac{a+b\tan^{-1}(cx)}{d^3x}-\frac{ex\left(a+b\tan^{-1}(cx)\right)}{4d^2\left(d+ex^2\right)^2}-\frac{7ex\left(a+b\tan^{-1}(cx)\right)}{8d^3\left(d+ex^2\right)}-\frac{a\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{7/2}}-7\sqrt{e}\left(a+\right.$$

$$=-\frac{a+b\tan^{-1}(cx)}{d^3x}-\frac{ex\left(a+b\tan^{-1}(cx)\right)}{4d^2\left(d+ex^2\right)^2}-\frac{7ex\left(a+b\tan^{-1}(cx)\right)}{8d^3\left(d+ex^2\right)}-\frac{a\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{7/2}}-7\sqrt{e}\left(a+\right.$$

$$=\frac{bce}{8d^2\left(c^2d-e\right)\left(d+ex^2\right)}-\frac{a+b\tan^{-1}(cx)}{d^3x}-\frac{ex\left(a+b\tan^{-1}(cx)\right)}{4d^2\left(d+ex^2\right)^2}-\frac{7ex\left(a+b\tan^{-1}(cx)\right)}{8d^3\left(d+ex^2\right)}-a\sqrt{e}$$

$$=\frac{bce}{8d^2\left(c^2d-e\right)\left(d+ex^2\right)}-\frac{a+b\tan^{-1}(cx)}{d^3x}-\frac{ex\left(a+b\tan^{-1}(cx)\right)}{4d^2\left(d+ex^2\right)^2}-\frac{7ex\left(a+b\tan^{-1}(cx)\right)}{8d^3\left(d+ex^2\right)}-a\sqrt{e}$$

$$=\frac{bce}{8d^2\left(c^2d-e\right)\left(d+ex^2\right)}-\frac{a+b\tan^{-1}(cx)}{d^3x}-\frac{ex\left(a+b\tan^{-1}(cx)\right)}{4d^2\left(d+ex^2\right)^2}-\frac{7ex\left(a+b\tan^{-1}(cx)\right)}{8d^3\left(d+ex^2\right)}-a\sqrt{e}$$

$$=\frac{bce}{\quad}-\frac{a+b\tan^{-1}(cx)}{\quad}-\frac{ex\left(a+b\tan^{-1}(cx)\right)}{\quad}-\frac{7ex\left(a+b\tan^{-1}(cx)\right)}{\quad}-a\sqrt{e}$$

Mathematica [A] time = 13.3256, size = 1985, normalized size = 1.31

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c*x])/(x^2*(d + e*x^2)^3),x]
```

```
[Out] -(a/(d^3*x)) - (a*e*x)/(4*d^2*(d + e*x^2)^2) - (7*a*e*x)/(8*d^3*(d + e*x^2)
) - (15*a*Sqrt[e]*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(7/2)) + b*c^7*(-(ArcTa
n[c*x]/(c^7*d^3*x)) + Log[(c*x)/Sqrt[1 + c^2*x^2]]/(c^6*d^3) - (9*e*Log[1 +
 ((c^2*d - e)*Cos[2*ArcTan[c*x]])/(c^2*d + e)])/(16*c^4*d^2*(c^2*d - e)^2)
+ (7*e^2*Log[1 + ((c^2*d - e)*Cos[2*ArcTan[c*x]])/(c^2*d + e)])/(16*c^6*d^3
*(c^2*d - e)^2) - (15*e*(4*ArcTan[c*x]*ArcTanh[(c*d)/(Sqrt[-(c^2*d*e)]*x)]
+ 2*ArcCos[-((c^2*d + e)/(c^2*d - e))]*ArcTanh[(c*e*x)/Sqrt[-(c^2*d*e)]] -
(ArcCos[-((c^2*d + e)/(c^2*d - e))] - (2*I)*ArcTanh[(c*e*x)/Sqrt[-(c^2*d*e)
]])*Log[1 - ((c^2*d + e - (2*I)*Sqrt[-(c^2*d*e)])*(2*c^2*d - 2*c*Sqrt[-(c^2
*d*e)]*x))/((c^2*d - e)*(2*c^2*d + 2*c*Sqrt[-(c^2*d*e)]*x))] + (-ArcCos[-((
c^2*d + e)/(c^2*d - e))] - (2*I)*ArcTanh[(c*e*x)/Sqrt[-(c^2*d*e)]])*Log[1 -
 ((c^2*d + e + (2*I)*Sqrt[-(c^2*d*e)])*(2*c^2*d - 2*c*Sqrt[-(c^2*d*e)]*x))/
((c^2*d - e)*(2*c^2*d + 2*c*Sqrt[-(c^2*d*e)]*x))] + (ArcCos[-((c^2*d + e)/(
c^2*d - e))] - (2*I)*(ArcTanh[(c*d)/(Sqrt[-(c^2*d*e)]*x)] + ArcTanh[(c*e*x)
/Sqrt[-(c^2*d*e)]]))*Log[(Sqrt[2]*Sqrt[-(c^2*d*e)])/(Sqrt[c^2*d - e]*E^(I*A
rcTan[c*x])*Sqrt[c^2*d + e + (c^2*d - e)*Cos[2*ArcTan[c*x]]])] + (ArcCos[-(
(c^2*d + e)/(c^2*d - e))] + (2*I)*(ArcTanh[(c*d)/(Sqrt[-(c^2*d*e)]*x)] + Ar
cTanh[(c*e*x)/Sqrt[-(c^2*d*e)]]))*Log[(Sqrt[2]*Sqrt[-(c^2*d*e)]*E^(I*ArcTan
[c*x]))/(Sqrt[c^2*d - e]*Sqrt[c^2*d + e + (c^2*d - e)*Cos[2*ArcTan[c*x]]])]
 + I*(PolyLog[2, ((c^2*d + e - (2*I)*Sqrt[-(c^2*d*e)])*(2*c^2*d - 2*c*Sqrt[
-(c^2*d*e)]*x))/((c^2*d - e)*(2*c^2*d + 2*c*Sqrt[-(c^2*d*e)]*x))] - PolyLog
[2, ((c^2*d + e + (2*I)*Sqrt[-(c^2*d*e)])*(2*c^2*d - 2*c*Sqrt[-(c^2*d*e)]*x
))/((c^2*d - e)*(2*c^2*d + 2*c*Sqrt[-(c^2*d*e)]*x))])))/(32*c^4*d^2*(c^2*d
- e)*Sqrt[-(c^2*d*e)]) + (15*e^2*(4*ArcTan[c*x]*ArcTanh[(c*d)/(Sqrt[-(c^2*d
*e)]*x)] + 2*ArcCos[-((c^2*d + e)/(c^2*d - e))]*ArcTanh[(c*e*x)/Sqrt[-(c^2*
d*e)]] - (ArcCos[-((c^2*d + e)/(c^2*d - e))] - (2*I)*ArcTanh[(c*e*x)/Sqrt[-
(c^2*d*e)]])*Log[1 - ((c^2*d + e - (2*I)*Sqrt[-(c^2*d*e)])*(2*c^2*d - 2*c*S
qrt[-(c^2*d*e)]*x))/((c^2*d - e)*(2*c^2*d + 2*c*Sqrt[-(c^2*d*e)]*x))] + (-A
rcCos[-((c^2*d + e)/(c^2*d - e))] - (2*I)*ArcTanh[(c*e*x)/Sqrt[-(c^2*d*e)]]
)*Log[1 - ((c^2*d + e + (2*I)*Sqrt[-(c^2*d*e)])*(2*c^2*d - 2*c*Sqrt[-(c^2*d
*e)]*x))/((c^2*d - e)*(2*c^2*d + 2*c*Sqrt[-(c^2*d*e)]*x))] + (ArcCos[-((c^2
*d + e)/(c^2*d - e))] - (2*I)*(ArcTanh[(c*d)/(Sqrt[-(c^2*d*e)]*x)] + ArcTan
h[(c*e*x)/Sqrt[-(c^2*d*e)]]))*Log[(Sqrt[2]*Sqrt[-(c^2*d*e)])/(Sqrt[c^2*d -
e]*E^(I*ArcTan[c*x])*Sqrt[c^2*d + e + (c^2*d - e)*Cos[2*ArcTan[c*x]]])] + (
ArcCos[-((c^2*d + e)/(c^2*d - e))] + (2*I)*(ArcTanh[(c*d)/(Sqrt[-(c^2*d*e)]
*x)] + ArcTanh[(c*e*x)/Sqrt[-(c^2*d*e)]]))*Log[(Sqrt[2]*Sqrt[-(c^2*d*e)]*E^
```

```
(I*ArcTan[c*x]))/(Sqrt[c^2*d - e]*Sqrt[c^2*d + e + (c^2*d - e)*Cos[2*ArcTan
[c*x]]])] + I*(PolyLog[2, ((c^2*d + e - (2*I)*Sqrt[-(c^2*d*e)])*(2*c^2*d -
2*c*Sqrt[-(c^2*d*e)]*x))/((c^2*d - e)*(2*c^2*d + 2*c*Sqrt[-(c^2*d*e)]*x))]
- PolyLog[2, ((c^2*d + e + (2*I)*Sqrt[-(c^2*d*e)])*(2*c^2*d - 2*c*Sqrt[-(c^
2*d*e)]*x))/((c^2*d - e)*(2*c^2*d + 2*c*Sqrt[-(c^2*d*e)]*x))])))/(32*c^6*d^
3*(c^2*d - e)*Sqrt[-(c^2*d*e)]) + (e^2*ArcTan[c*x]*Sin[2*ArcTan[c*x]])/(2*c
^4*d^2*(c^2*d - e)*(c^2*d + e + c^2*d*Cos[2*ArcTan[c*x]] - e*Cos[2*ArcTan[c
*x]])^2) + (-2*c^2*d*e^2 - 9*c^4*d^2*e*ArcTan[c*x]*Sin[2*ArcTan[c*x]] + 16*
c^2*d*e^2*ArcTan[c*x]*Sin[2*ArcTan[c*x]] - 7*e^3*ArcTan[c*x]*Sin[2*ArcTan[c
*x]])/(8*c^6*d^3*(c^2*d - e)^2*(c^2*d + e + c^2*d*Cos[2*ArcTan[c*x]] - e*Co
s[2*ArcTan[c*x]])))
```

Maple [C] time = 0.939, size = 6655, normalized size = 4.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/x^2/(e*x^2+d)^3,x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^2/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b\arctan(cx)+a}{e^{3}x^{8}+3\,de^{2}x^{6}+3\,d^{2}ex^{4}+d^{3}x^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^2/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*arctan(c*x) + a)/(e^3*x^8 + 3*d*e^2*x^6 + 3*d^2*e*x^4 + d^3*x^2
), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/x**2/(e*x**2+d)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \arctan (cx) + a}{\left(ex^2 + d\right)^3 x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^2/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)/((e*x^2 + d)^3*x^2), x)
```

3.1173 $\int x^3\sqrt{d+ex^2}\left(a+b\tan^{-1}(cx)\right)\,dx$

Optimal. Leaf size=223

$$\frac{\left(d+ex^2\right)^{5/2}\left(a+b\tan^{-1}(cx)\right)}{5e^2}-\frac{d\left(d+ex^2\right)^{3/2}\left(a+b\tan^{-1}(cx)\right)}{3e^2}+\frac{b\left(15c^4d^2+20c^2de-24e^2\right)\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{120c^5e^{3/2}}+\frac{b\left(c^2d\right.}{}$$

```
[Out] -(b*(c^2*d - 12*e)*x*Sqrt[d + e*x^2])/(120*c^3*e) - (b*x*(d + e*x^2)^(3/2))
/(20*c*e) - (d*(d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]))/(3*e^2) + ((d + e*x^2
)^(5/2)*(a + b*ArcTan[c*x]))/(5*e^2) + (b*(c^2*d - e)^(3/2)*(2*c^2*d + 3*e)
*ArcTan[(Sqrt[c^2*d - e]*x)/Sqrt[d + e*x^2]])/(15*c^5*e^2) + (b*(15*c^4*d^2
 + 20*c^2*d*e - 24*e^2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(120*c^5*e^(3
/2))
```

Rubi [A] time = 0.367628, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {266, 43, 4976, 12, 528, 523, 217, 206, 377, 203}

$$\frac{\left(d+ex^2\right)^{5/2}\left(a+b\tan^{-1}(cx)\right)}{5e^2}-\frac{d\left(d+ex^2\right)^{3/2}\left(a+b\tan^{-1}(cx)\right)}{3e^2}+\frac{b\left(15c^4d^2+20c^2de-24e^2\right)\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{120c^5e^{3/2}}+\frac{b\left(c^2d\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]),x]
```

```
[Out] -(b*(c^2*d - 12*e)*x*Sqrt[d + e*x^2])/(120*c^3*e) - (b*x*(d + e*x^2)^(3/2))
/(20*c*e) - (d*(d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]))/(3*e^2) + ((d + e*x^2
)^(5/2)*(a + b*ArcTan[c*x]))/(5*e^2) + (b*(c^2*d - e)^(3/2)*(2*c^2*d + 3*e)
*ArcTan[(Sqrt[c^2*d - e]*x)/Sqrt[d + e*x^2]])/(15*c^5*e^2) + (b*(15*c^4*d^2
 + 20*c^2*d*e - 24*e^2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(120*c^5*e^(3
/2))
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 4976

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Dis
t[a + b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&  !(
ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(IL
tQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILtQ[(m
 - 1)/2, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int x^3\sqrt{d+ex^2}\left(a+b\tan^{-1}(cx)\right)dx = -\frac{d\left(d+ex^2\right)^{3/2}\left(a+b\tan^{-1}(cx)\right)}{3e^2}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\tan^{-1}(cx)\right)}{5e^2}-(bc)\int\frac{\left(d+e\ldots\right.}{1\ldots}$$

$$= -\frac{d\left(d+ex^2\right)^{3/2}\left(a+b\tan^{-1}(cx)\right)}{3e^2}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\tan^{-1}(cx)\right)}{5e^2}-\frac{(bc)\int\frac{\left(d+ex^2\right)^{3\ldots}}{1\ldots}}{15\ldots}$$

$$= -\frac{bx\left(d+ex^2\right)^{3/2}}{20ce}-\frac{d\left(d+ex^2\right)^{3/2}\left(a+b\tan^{-1}(cx)\right)}{3e^2}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\tan^{-1}(cx\ldots\right.}{5e^2}$$

$$= -\frac{b\left(c^2d-12e\right)x\sqrt{d+ex^2}}{120c^3e}-\frac{bx\left(d+ex^2\right)^{3/2}}{20ce}-\frac{d\left(d+ex^2\right)^{3/2}\left(a+b\tan^{-1}(cx)\right)}{3e^2}+\ldots$$

$$= -\frac{b\left(c^2d-12e\right)x\sqrt{d+ex^2}}{120c^3e}-\frac{bx\left(d+ex^2\right)^{3/2}}{20ce}-\frac{d\left(d+ex^2\right)^{3/2}\left(a+b\tan^{-1}(cx)\right)}{3e^2}+\ldots$$

$$= -\frac{b\left(c^2d-12e\right)x\sqrt{d+ex^2}}{120c^3e}-\frac{bx\left(d+ex^2\right)^{3/2}}{20ce}-\frac{d\left(d+ex^2\right)^{3/2}\left(a+b\tan^{-1}(cx)\right)}{3e^2}+\ldots$$

$$= -\frac{b\left(c^2d-12e\right)x\sqrt{d+ex^2}}{120c^3e}-\frac{bx\left(d+ex^2\right)^{3/2}}{20ce}-\frac{d\left(d+ex^2\right)^{3/2}\left(a+b\tan^{-1}(cx)\right)}{3e^2}+\ldots$$

Mathematica [C] time = 0.491348, size = 391, normalized size = 1.75

$$\frac{-c^2\sqrt{d+ex^2}\left(8ac^3\left(2d^2-dex^2-3e^2x^4\right)+bex\left(c^2\left(7d+6ex^2\right)-12e\right)\right)+b\sqrt{e}\left(15c^4d^2+20c^2de-24e^2\right)\log\left(\sqrt{e}\sqrt{d+ex^2}+\ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]),x]
```

```
[Out] (-(c^2*Sqrt[d + e*x^2]*(8*a*c^3*(2*d^2 - d*e*x^2 - 3*e^2*x^4) + b*e*x*(-12*
e + c^2*(7*d + 6*e*x^2)))) - 8*b*c^5*Sqrt[d + e*x^2]*(2*d^2 - d*e*x^2 - 3*e
^2*x^4)*ArcTan[c*x] - (4*I)*b*(c^2*d - e)^(3/2)*(2*c^2*d + 3*e)*Log[((-60*I
)*c^6*e^2*(c*d - I*e*x + Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*(c^2*d - e)^(
5/2)*(2*c^2*d + 3*e)*(I + c*x))] + (4*I)*b*(c^2*d - e)^(3/2)*(2*c^2*d + 3*e
)*Log[((60*I)*c^6*e^2*(c*d + I*e*x + Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*(
c^2*d - e)^(5/2)*(2*c^2*d + 3*e)*(-I + c*x))] + b*Sqrt[e]*(15*c^4*d^2 + 20*
c^2*d*e - 24*e^2)*Log[e*x + Sqrt[e]*Sqrt[d + e*x^2]])/(120*c^5*e^2)
```

Maple [F] time = 0.965, size = 0, normalized size = 0.

$$\int x^3\sqrt{ex^2+d}\,(a+b\arctan(cx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x^2+d)^(1/2)*(a+b*arctan(c*x)),x)
```

```
[Out] int(x^3*(e*x^2+d)^(1/2)*(a+b*arctan(c*x)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^(1/2)*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 26.3063, size = 2657, normalized size = 11.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^(1/2)*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] [-1/240*((15*b*c^4*d^2 + 20*b*c^2*d*e - 24*b*e^2)*sqrt(e)*log(-2*e*x^2 + 2*
sqrt(e*x^2 + d)*sqrt(e)*x - d) + 4*(2*b*c^4*d^2 + b*c^2*d*e - 3*b*e^2)*sqrt
(-c^2*d + e)*log(((c^4*d^2 - 8*c^2*d*e + 8*e^2)*x^4 - 2*(3*c^2*d^2 - 4*d*e)
*x^2 - 4*((c^2*d - 2*e)*x^3 - d*x)*sqrt(-c^2*d + e)*sqrt(e*x^2 + d) + d^2)/
(c^4*x^4 + 2*c^2*x^2 + 1)) - 2*(24*a*c^5*e^2*x^4 + 8*a*c^5*d*e*x^2 - 6*b*c^
4*e^2*x^3 - 16*a*c^5*d^2 - (7*b*c^4*d*e - 12*b*c^2*e^2)*x + 8*(3*b*c^5*e^2*
x^4 + b*c^5*d*e*x^2 - 2*b*c^5*d^2)*arctan(c*x))*sqrt(e*x^2 + d))/(c^5*e^2),
 1/240*(8*(2*b*c^4*d^2 + b*c^2*d*e - 3*b*e^2)*sqrt(c^2*d - e)*arctan(1/2*sq
rt(c^2*d - e)*((c^2*d - 2*e)*x^2 - d)*sqrt(e*x^2 + d)/((c^2*d*e - e^2)*x^3
+ (c^2*d^2 - d*e)*x)) - (15*b*c^4*d^2 + 20*b*c^2*d*e - 24*b*e^2)*sqrt(e)*lo
g(-2*e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(e)*x - d) + 2*(24*a*c^5*e^2*x^4 + 8*a*c
^5*d*e*x^2 - 6*b*c^4*e^2*x^3 - 16*a*c^5*d^2 - (7*b*c^4*d*e - 12*b*c^2*e^2)*
x + 8*(3*b*c^5*e^2*x^4 + b*c^5*d*e*x^2 - 2*b*c^5*d^2)*arctan(c*x))*sqrt(e*x
^2 + d))/(c^5*e^2), -1/120*((15*b*c^4*d^2 + 20*b*c^2*d*e - 24*b*e^2)*sqrt(-
e)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) + 2*(2*b*c^4*d^2 + b*c^2*d*e - 3*b*e^
2)*sqrt(-c^2*d + e)*log(((c^4*d^2 - 8*c^2*d*e + 8*e^2)*x^4 - 2*(3*c^2*d^2 -
 4*d*e)*x^2 - 4*((c^2*d - 2*e)*x^3 - d*x)*sqrt(-c^2*d + e)*sqrt(e*x^2 + d)
+ d^2)/(c^4*x^4 + 2*c^2*x^2 + 1)) - (24*a*c^5*e^2*x^4 + 8*a*c^5*d*e*x^2 - 6
*b*c^4*e^2*x^3 - 16*a*c^5*d^2 - (7*b*c^4*d*e - 12*b*c^2*e^2)*x + 8*(3*b*c^5
*e^2*x^4 + b*c^5*d*e*x^2 - 2*b*c^5*d^2)*arctan(c*x))*sqrt(e*x^2 + d))/(c^5*
e^2), 1/120*(4*(2*b*c^4*d^2 + b*c^2*d*e - 3*b*e^2)*sqrt(c^2*d - e)*arctan(1
/2*sqrt(c^2*d - e)*((c^2*d - 2*e)*x^2 - d)*sqrt(e*x^2 + d)/((c^2*d*e - e^2)
*x^3 + (c^2*d^2 - d*e)*x)) - (15*b*c^4*d^2 + 20*b*c^2*d*e - 24*b*e^2)*sqrt(
-e)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) + (24*a*c^5*e^2*x^4 + 8*a*c^5*d*e*x^
2 - 6*b*c^4*e^2*x^3 - 16*a*c^5*d^2 - (7*b*c^4*d*e - 12*b*c^2*e^2)*x + 8*(3*
b*c^5*e^2*x^4 + b*c^5*d*e*x^2 - 2*b*c^5*d^2)*arctan(c*x))*sqrt(e*x^2 + d))/
(c^5*e^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 (a + b \operatorname{atan}(cx)) \sqrt{d + ex^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x**2+d)**(1/2)*(a+b*atan(c*x)),x)
```

```
[Out] Integral(x**3*(a + b*atan(c*x))*sqrt(d + e*x**2), x)
```

Giac [A] time = 1.32272, size = 362, normalized size = 1.62

$$\frac{1}{15}\left(3\left(x^2e+d\right)^{\frac{5}{2}}-5\left(x^2e+d\right)^{\frac{3}{2}}d\right)ae^{(-2)}+\frac{1}{240}\left(16\left(3\left(x^2e+d\right)^{\frac{5}{2}}-5\left(x^2e+d\right)^{\frac{3}{2}}d\right)\arctan\left(cx\right)e^{(-2)}-\left(2\sqrt{x^2e+d}x\left(\frac{6x^2}{c^2}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^(1/2)*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] 1/15*(3*(x^2*e + d)^(5/2) - 5*(x^2*e + d)^(3/2)*d)*a*e^(-2) + 1/240*(16*(3*
(x^2*e + d)^(5/2) - 5*(x^2*e + d)^(3/2)*d)*arctan(c*x)*e^(-2) - (2*sqrt(x^2
*e + d)*x*(6*x^2/c^2 + (7*c^10*d*e^2 - 12*c^8*e^3)*e^(-3)/c^12) + (15*c^4*d
^2 + 20*c^2*d*e - 24*e^2)*e^(-3/2)*log((x*e^(1/2) - sqrt(x^2*e + d))^2)/c^6
 + 16*(2*c^6*d^3*e^(1/2) - c^4*d^2*e^(3/2) - 4*c^2*d*e^(5/2) + 3*e^(7/2))*a
rctan(1/2*((x*e^(1/2) - sqrt(x^2*e + d))^2*c^2 - c^2*d + 2*e)*e^(-1/2)/sqrt
(c^2*d - e))*e^(-5/2)/(sqrt(c^2*d - e)*c^6))*c)*b
```

3.1174 $\int x^2\sqrt{d+ex^2}\left(a+b\tan^{-1}(cx)\right)\,dx$

Optimal. Leaf size=96

$$b\text{Unintegrable}\left(x^2\tan^{-1}(cx)\sqrt{d+ex^2},x\right)-\frac{ad^2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{8e^{3/2}}+\frac{1}{4}ax^3\sqrt{d+ex^2}+\frac{adx\sqrt{d+ex^2}}{8e}$$

```
[Out] (a*d*x*Sqrt[d + e*x^2])/(8*e) + (a*x^3*Sqrt[d + e*x^2])/4 - (a*d^2*ArcTanh[
(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(8*e^(3/2)) + b*Unintegrable[x^2*Sqrt[d + e*x
^2]*ArcTan[c*x], x]
```

Rubi [A] time = 0.158155, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^2\sqrt{d+ex^2}\left(a+b\tan^{-1}(cx)\right)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^2*Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]),x]
```

```
[Out] (a*d*x*Sqrt[d + e*x^2])/(8*e) + (a*x^3*Sqrt[d + e*x^2])/4 - (a*d^2*ArcTanh[
(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(8*e^(3/2)) + b*Defer[Int][x^2*Sqrt[d + e*x^2
]*ArcTan[c*x], x]
```

Rubi steps

$$\begin{aligned}
\int x^2\sqrt{d+ex^2}\left(a+b\tan^{-1}(cx)\right)\,dx &= a\int x^2\sqrt{d+ex^2}\,dx+b\int x^2\sqrt{d+ex^2}\tan^{-1}(cx)\,dx\\
&=\frac{1}{4}ax^3\sqrt{d+ex^2}+b\int x^2\sqrt{d+ex^2}\tan^{-1}(cx)\,dx+\frac{1}{4}(ad)\int\frac{x^2}{\sqrt{d+ex^2}}\,dx\\
&=\frac{adx\sqrt{d+ex^2}}{8e}+\frac{1}{4}ax^3\sqrt{d+ex^2}+b\int x^2\sqrt{d+ex^2}\tan^{-1}(cx)\,dx-\frac{\left(ad^2\right)\int\frac{1}{\sqrt{d+ex^2}}\,dx}{8e}\\
&=\frac{adx\sqrt{d+ex^2}}{8e}+\frac{1}{4}ax^3\sqrt{d+ex^2}+b\int x^2\sqrt{d+ex^2}\tan^{-1}(cx)\,dx-\frac{\left(ad^2\right)\text{Subst}\left(\int\ldots\right.}{8\ldots}\\
&=\frac{adx\sqrt{d+ex^2}}{8e}+\frac{1}{4}ax^3\sqrt{d+ex^2}-\frac{ad^2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{8e^{3/2}}+b\int x^2\sqrt{d+ex^2}\tan^{-1}(cx\ldots
\end{aligned}$$

Mathematica [A] time = 10.9487, size = 0, normalized size = 0.

$$\int x^2\sqrt{d+ex^2}\left(a+b\tan^{-1}(cx)\right)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^2*Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]),x]
```

```
[Out] Integrate[x^2*Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]), x]
```

Maple [A] time = 0.787, size = 0, normalized size = 0.

$$\int x^2\sqrt{ex^2+d}\,(a+b\arctan(cx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^2+d)^(1/2)*(a+b*arctan(c*x)),x)
```

```
[Out] int(x^2*(e*x^2+d)^(1/2)*(a+b*arctan(c*x)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^(1/2)*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(bx^2\arctan(cx)+ax^2\right)\sqrt{ex^2+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^(1/2)*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] integral((b*x^2*arctan(c*x) + a*x^2)*sqrt(e*x^2 + d), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int x^2 (a + b \operatorname{atan}(cx)) \sqrt{d + ex^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d)**(1/2)*(a+b*atan(c*x)),x)
```

```
[Out] Integral(x**2*(a + b*atan(c*x))*sqrt(d + e*x**2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ex^2 + d}(b \arctan(cx) + a)x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^(1/2)*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)*(b*arctan(c*x) + a)*x^2, x)
```

3.1175 $\int x\sqrt{d+ex^2}\left(a+b\tan^{-1}(cx)\right)\,dx$

Optimal. Leaf size=140

$$\frac{\left(d+ex^2\right)^{3/2}\left(a+b\tan^{-1}(cx)\right)}{3e}-\frac{b\left(c^2d-e\right)^{3/2}\tan^{-1}\left(\frac{x\sqrt{c^2d-e}}{\sqrt{d+ex^2}}\right)}{3c^3e}-\frac{b\left(3c^2d-2e\right)\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{6c^3\sqrt{e}}-\frac{bx\sqrt{d+ex^2}}{6c}$$

```
[Out] -(b*x*Sqrt[d + e*x^2])/(6*c) + ((d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]))/(3*e
) - (b*(c^2*d - e)^(3/2)*ArcTan[(Sqrt[c^2*d - e]*x)/Sqrt[d + e*x^2]])/(3*c^
3*e) - (b*(3*c^2*d - 2*e)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(6*c^3*Sqrt
[e])
```

Rubi [A] time = 0.142284, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4974, 416, 523, 217, 206, 377, 203}

$$\frac{\left(d+ex^2\right)^{3/2}\left(a+b\tan^{-1}(cx)\right)}{3e}-\frac{b\left(c^2d-e\right)^{3/2}\tan^{-1}\left(\frac{x\sqrt{c^2d-e}}{\sqrt{d+ex^2}}\right)}{3c^3e}-\frac{b\left(3c^2d-2e\right)\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{6c^3\sqrt{e}}-\frac{bx\sqrt{d+ex^2}}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]),x]
```

```
[Out] -(b*x*Sqrt[d + e*x^2])/(6*c) + ((d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]))/(3*e
) - (b*(c^2*d - e)^(3/2)*ArcTan[(Sqrt[c^2*d - e]*x)/Sqrt[d + e*x^2]])/(3*c^
3*e) - (b*(3*c^2*d - 2*e)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(6*c^3*Sqrt
[e])
```

Rule 4974

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]))/(2*e*(q + 1)), x
] - Dist[(b*c)/(2*e*(q + 1)), Int[(d + e*x^2)^(q + 1)/(1 + c^2*x^2), x], x]
 /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
```

```
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int x\sqrt{d+ex^2}\left(a+b\tan^{-1}(cx)\right)dx = \frac{\left(d+ex^2\right)^{3/2}\left(a+b\tan^{-1}(cx)\right)}{3e} - \frac{(bc)\int\frac{\left(d+ex^2\right)^{3/2}}{1+c^2x^2}dx}{3e}$$

$$= -\frac{bx\sqrt{d+ex^2}}{6c} + \frac{\left(d+ex^2\right)^{3/2}\left(a+b\tan^{-1}(cx)\right)}{3e} - \frac{b\int\frac{d\left(2c^2d-e\right)+\left(3c^2d-2e\right)ex^2}{\left(1+c^2x^2\right)\sqrt{d+ex^2}}dx}{6ce}$$

$$= -\frac{bx\sqrt{d+ex^2}}{6c} + \frac{\left(d+ex^2\right)^{3/2}\left(a+b\tan^{-1}(cx)\right)}{3e} - \frac{\left(b\left(3c^2d-2e\right)\right)\int\frac{1}{\sqrt{d+ex^2}}dx}{6c^3} - \frac{(b}{}$$

$$= -\frac{bx\sqrt{d+ex^2}}{6c} + \frac{\left(d+ex^2\right)^{3/2}\left(a+b\tan^{-1}(cx)\right)}{3e} - \frac{\left(b\left(3c^2d-2e\right)\right)\text{Subst}\left(\int\frac{1}{1-ex^2}dx\right.}{6c^3}$$

$$= -\frac{bx\sqrt{d+ex^2}}{6c} + \frac{\left(d+ex^2\right)^{3/2}\left(a+b\tan^{-1}(cx)\right)}{3e} - \frac{b\left(c^2d-e\right)^{3/2}\tan^{-1}\left(\frac{\sqrt{c^2d-ex}}{\sqrt{d+ex^2}}\right)}{3c^3e} - \frac{b}{}$$

Mathematica [C] time = 0.517287, size = 279, normalized size = 1.99

$$\frac{c^2\sqrt{d+ex^2}\left(2ac\left(d+ex^2\right)-bex\right) - ib\left(c^2d-e\right)^{3/2}\log\left(\frac{12c^4e\left(-i\sqrt{c^2d-e}\sqrt{d+ex^2}-icd+ex\right)}{b(cx-i)\left(c^2d-e\right)^{5/2}}\right) + ib\left(c^2d-e\right)^{3/2}\log\left(\frac{12c^4e\left(i\sqrt{c^2d-e}\sqrt{d+ex^2}\right.}{b(cx+i)\left(c^2d-e\right)}\right.}{6c^3e}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]),x]
```

```
[Out] (c^2*Sqrt[d + e*x^2]*(-(b*e*x) + 2*a*c*(d + e*x^2)) + 2*b*c^3*(d + e*x^2)^(
3/2)*ArcTan[c*x] - I*b*(c^2*d - e)^(3/2)*Log[(12*c^4*e*((-I)*c*d + e*x - I*
Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*(c^2*d - e)^(5/2)*(-I + c*x))] + I*b*(
c^2*d - e)^(3/2)*Log[(12*c^4*e*(I*c*d + e*x + I*Sqrt[c^2*d - e]*Sqrt[d + e*
x^2]))/(b*(c^2*d - e)^(5/2)*(I + c*x))] + b*Sqrt[e]*(-3*c^2*d + 2*e)*Log[e*
x + Sqrt[e]*Sqrt[d + e*x^2]])/(6*c^3*e)
```

Maple [F] time = 0.806, size = 0, normalized size = 0.

$$\int x\sqrt{ex^2+d}\,(a+b\arctan(cx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d)^(1/2)*(a+b*arctan(c*x)),x)
```

```
[Out] int(x*(e*x^2+d)^(1/2)*(a+b*arctan(c*x)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^(1/2)*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 6.76812, size = 1947, normalized size = 13.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^(1/2)*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] [-1/12*((3*b*c^2*d - 2*b*e)*sqrt(e)*log(-2*e*x^2 - 2*sqrt(e*x^2 + d)*sqrt(e
)*x - d) + (b*c^2*d - b*e)*sqrt(-c^2*d + e)*log(((c^4*d^2 - 8*c^2*d*e + 8*e
^2)*x^4 - 2*(3*c^2*d^2 - 4*d*e)*x^2 + 4*((c^2*d - 2*e)*x^3 - d*x)*sqrt(-c^2
*d + e)*sqrt(e*x^2 + d) + d^2)/(c^4*x^4 + 2*c^2*x^2 + 1)) - 2*(2*a*c^3*e*x^
2 + 2*a*c^3*d - b*c^2*e*x + 2*(b*c^3*e*x^2 + b*c^3*d)*arctan(c*x))*sqrt(e*x
^2 + d))/(c^3*e), -1/12*(2*(b*c^2*d - b*e)*sqrt(c^2*d - e)*arctan(1/2*sqrt(
c^2*d - e)*((c^2*d - 2*e)*x^2 - d)*sqrt(e*x^2 + d)/((c^2*d*e - e^2)*x^3 + (
c^2*d^2 - d*e)*x)) + (3*b*c^2*d - 2*b*e)*sqrt(e)*log(-2*e*x^2 - 2*sqrt(e*x^
2 + d)*sqrt(e)*x - d) - 2*(2*a*c^3*e*x^2 + 2*a*c^3*d - b*c^2*e*x + 2*(b*c^3
*e*x^2 + b*c^3*d)*arctan(c*x))*sqrt(e*x^2 + d))/(c^3*e), 1/12*(2*(3*b*c^2*d
 - 2*b*e)*sqrt(-e)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) - (b*c^2*d - b*e)*sqr
t(-c^2*d + e)*log(((c^4*d^2 - 8*c^2*d*e + 8*e^2)*x^4 - 2*(3*c^2*d^2 - 4*d*e
)*x^2 + 4*((c^2*d - 2*e)*x^3 - d*x)*sqrt(-c^2*d + e)*sqrt(e*x^2 + d) + d^2)
/(c^4*x^4 + 2*c^2*x^2 + 1)) + 2*(2*a*c^3*e*x^2 + 2*a*c^3*d - b*c^2*e*x + 2*
(b*c^3*e*x^2 + b*c^3*d)*arctan(c*x))*sqrt(e*x^2 + d))/(c^3*e), -1/6*((b*c^2
*d - b*e)*sqrt(c^2*d - e)*arctan(1/2*sqrt(c^2*d - e)*((c^2*d - 2*e)*x^2 - d
)*sqrt(e*x^2 + d)/((c^2*d*e - e^2)*x^3 + (c^2*d^2 - d*e)*x)) - (3*b*c^2*d -
```

```
 2*b*e)*sqrt(-e)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) - (2*a*c^3*e*x^2 + 2*a*
c^3*d - b*c^2*e*x + 2*(b*c^3*e*x^2 + b*c^3*d)*arctan(c*x))*sqrt(e*x^2 + d))
/(c^3*e)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\left(a+b\operatorname{atan}\left(cx\right)\right)\sqrt{d+ex^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d)**(1/2)*(a+b*atan(c*x)),x)
```

```
[Out] Integral(x*(a + b*atan(c*x))*sqrt(d + e*x**2), x)
```

Giac [A] time = 1.29327, size = 251, normalized size = 1.79

$$\frac{1}{3}\left(x^{2}e+d\right)^{\frac{3}{2}}ae^{(-1)}+\frac{1}{12}\left(4\left(x^{2}e+d\right)^{\frac{3}{2}}\arctan\left(cx\right)e^{(-1)}-c\left(\frac{2\sqrt{x^{2}e+d}x}{c^{2}}-\frac{\left(3c^{2}d-2e\right)e^{\left(-\frac{1}{2}\right)}\log\left(\left(xe^{\frac{1}{2}}-\sqrt{x^{2}e+d}\right)^{2}\right)}{c^{4}}-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^(1/2)*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] 1/3*(x^2*e + d)^(3/2)*a*e^(-1) + 1/12*(4*(x^2*e + d)^(3/2)*arctan(c*x)*e^(-
1) - c*(2*sqrt(x^2*e + d)*x/c^2 - (3*c^2*d - 2*e)*e^(-1/2)*log((x*e^(1/2) -
 sqrt(x^2*e + d))^2)/c^4 - 4*(c^4*d^2*e^(1/2) - 2*c^2*d*e^(3/2) + e^(5/2))*
arctan(1/2*((x*e^(1/2) - sqrt(x^2*e + d))^2*c^2 - c^2*d + 2*e)*e^(-1/2)/sqr
t(c^2*d - e))*e^(-3/2)/(sqrt(c^2*d - e)*c^4)))*b
```

3.1176 $\int \sqrt{d+ex^2}\left(a+b\tan^{-1}(cx)\right)\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\sqrt{d+ex^2}\left(a+b\tan^{-1}(cx)\right),x\right)$$

```
[Out] Unintegrable[Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]), x]
```

Rubi [A] time = 0.0238289, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \sqrt{d+ex^2}\left(a+b\tan^{-1}(cx)\right)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]),x]
```

```
[Out] Defer[Int][Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]), x]
```

Rubi steps

$$\int \sqrt{d+ex^2}\left(a+b\tan^{-1}(cx)\right)\,dx=\int \sqrt{d+ex^2}\left(a+b\tan^{-1}(cx)\right)\,dx$$

Mathematica [A] time = 4.702, size = 0, normalized size = 0.

$$\int \sqrt{d+ex^2}\left(a+b\tan^{-1}(cx)\right)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]),x]
```

```
[Out] Integrate[Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]), x]
```

Maple [A] time = 1.524, size = 0, normalized size = 0.

$$\int \sqrt{e x^2 + d}\, (a + b \arctan (c x))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(1/2)*(a+b*arctan(c*x)),x)
```

```
[Out] int((e*x^2+d)^(1/2)*(a+b*arctan(c*x)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{e x^2 + d}(b \arctan (c x) + a), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x^2 + d)*(b*arctan(c*x) + a), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int (a + b \operatorname{atan} (c x)) \sqrt{d + e x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(1/2)*(a+b*atan(c*x)),x)
```

```
[Out] Integral((a + b*atan(c*x))*sqrt(d + e*x**2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ex^2 + d}(b \arctan(cx) + a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)*(b*arctan(c*x) + a), x)
```

3.1177 $\int \frac{\sqrt{d+ex^2}\left(a+b\tan^{-1}(cx)\right)}{x}\,dx$

Optimal. Leaf size=63

$$b\text{Unintegrable}\left(\frac{\tan^{-1}(cx)\sqrt{d+ex^2}}{x},x\right)+a\sqrt{d+ex^2}+a\left(-\sqrt{d}\right)\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)$$

```
[Out] a*Sqrt[d + e*x^2] - a*Sqrt[d]*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]] + b*Unintegr
able[(Sqrt[d + e*x^2]*ArcTan[c*x])/x, x]
```

Rubi [A] time = 0.16305, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sqrt{d+ex^2}\left(a+b\tan^{-1}(cx)\right)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]))/x,x]
```

```
[Out] a*Sqrt[d + e*x^2] - a*Sqrt[d]*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]] + b*Defer[In
t][(Sqrt[d + e*x^2]*ArcTan[c*x])/x, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{d+ex^2}\left(a+b\tan^{-1}(cx)\right)}{x}\,dx &= a\int \frac{\sqrt{d+ex^2}}{x}\,dx+b\int \frac{\sqrt{d+ex^2}\tan^{-1}(cx)}{x}\,dx\\
&= \frac{1}{2}a\,\text{Subst}\left(\int \frac{\sqrt{d+ex}}{x}\,dx,x,x^2\right)+b\int \frac{\sqrt{d+ex^2}\tan^{-1}(cx)}{x}\,dx\\
&= a\sqrt{d+ex^2}+b\int \frac{\sqrt{d+ex^2}\tan^{-1}(cx)}{x}\,dx+\frac{1}{2}(ad)\,\text{Subst}\left(\int \frac{1}{x\sqrt{d+ex}}\,dx,x,x^2\right)\\
&= a\sqrt{d+ex^2}+b\int \frac{\sqrt{d+ex^2}\tan^{-1}(cx)}{x}\,dx+\frac{(ad)\,\text{Subst}\left(\int \frac{1}{-\frac{d}{e}+\frac{x^2}{e}}\,dx,x,\sqrt{d+ex^2}\right)}{e}\\
&= a\sqrt{d+ex^2}-a\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)+b\int \frac{\sqrt{d+ex^2}\tan^{-1}(cx)}{x}\,dx
\end{aligned}$$

Mathematica [A] time = 71.1609, size = 0, normalized size = 0.

$$\int \frac{\sqrt{d+ex^2}\left(a+b\tan^{-1}(cx)\right)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]))/x,x]
```

```
[Out] Integrate[(Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]))/x, x]
```

Maple [A] time = 0.771, size = 0, normalized size = 0.

$$\int \frac{a+b\arctan(cx)}{x}\sqrt{ex^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x,x)
```

```
[Out] int((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{ex^2+d}(b\arctan(cx)+a)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x,x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x^2 + d)*(b*arctan(c*x) + a)/x, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b\operatorname{atan}(cx))\sqrt{d + ex^2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(1/2)*(a+b*atan(c*x))/x,x)
```

```
[Out] Integral((a + b*atan(c*x))*sqrt(d + e*x**2)/x, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex^2 + d}(b\arctan(cx) + a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)*(b*arctan(c*x) + a)/x, x)
```

3.1178 $\int \frac{\sqrt{d+ex^2}\left(a+b\tan^{-1}(cx)\right)}{x^2}\,dx$

Optimal. Leaf size=67

$$b\text{Unintegrable}\left(\frac{\tan^{-1}(cx)\sqrt{d+ex^2}}{x^2},x\right)-\frac{a\sqrt{d+ex^2}}{x}+a\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)$$

```
[Out] -((a*Sqrt[d + e*x^2])/x) + a*Sqrt[e]*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]] +
 b*Unintegrable[(Sqrt[d + e*x^2]*ArcTan[c*x])/x^2, x]
```

Rubi [A] time = 0.146594, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sqrt{d+ex^2}\left(a+b\tan^{-1}(cx)\right)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]))/x^2,x]
```

```
[Out] -((a*Sqrt[d + e*x^2])/x) + a*Sqrt[e]*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]] +
 b*Defer[Int][(Sqrt[d + e*x^2]*ArcTan[c*x])/x^2, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{d+ex^2}\left(a+b\tan^{-1}(cx)\right)}{x^2}\,dx &= a\int \frac{\sqrt{d+ex^2}}{x^2}\,dx+b\int \frac{\sqrt{d+ex^2}\tan^{-1}(cx)}{x^2}\,dx\\
&= -\frac{a\sqrt{d+ex^2}}{x}+b\int \frac{\sqrt{d+ex^2}\tan^{-1}(cx)}{x^2}\,dx+(ae)\int \frac{1}{\sqrt{d+ex^2}}\,dx\\
&= -\frac{a\sqrt{d+ex^2}}{x}+b\int \frac{\sqrt{d+ex^2}\tan^{-1}(cx)}{x^2}\,dx+(ae)\,\text{Subst}\left(\int \frac{1}{1-ex^2}\,dx,x,\frac{x}{\sqrt{d+ex^2}}\right)\\
&= -\frac{a\sqrt{d+ex^2}}{x}+a\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)+b\int \frac{\sqrt{d+ex^2}\tan^{-1}(cx)}{x^2}\,dx
\end{aligned}$$

Mathematica [A] time = 8.54584, size = 0, normalized size = 0.

$$\int \frac{\sqrt{d+ex^2}\left(a+b\tan^{-1}(cx)\right)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]))/x^2,x]
```

```
[Out] Integrate[(Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]))/x^2, x]
```

Maple [A] time = 0.758, size = 0, normalized size = 0.

$$\int \frac{a+b\arctan(cx)}{x^2}\sqrt{ex^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x^2,x)
```

```
[Out] int((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{ex^2+d}(b\arctan(cx)+a)}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x^2 + d)*(b*arctan(c*x) + a)/x^2, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b\,\mathrm{atan}\,(cx))\,\sqrt{d + ex^2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(1/2)*(a+b*atan(c*x))/x**2,x)
```

```
[Out] Integral((a + b*atan(c*x))*sqrt(d + e*x**2)/x**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex^2 + d}(b\arctan\,(cx) + a)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)*(b*arctan(c*x) + a)/x^2, x)
```

3.1179 $\int \frac{\sqrt{d+ex^2}\left(a+b\tan^{-1}(cx)\right)}{x^3}\,dx$

Optimal. Leaf size=72

$$b\text{Unintegrable}\left(\frac{\tan^{-1}(cx)\sqrt{d+ex^2}}{x^3},x\right)-\frac{a\sqrt{d+ex^2}}{2x^2}-\frac{ae\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{2\sqrt{d}}$$

```
[Out] -(a*Sqrt[d + e*x^2])/(2*x^2) - (a*e*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/(2*Sq
rt[d]) + b*Unintegrable[(Sqrt[d + e*x^2]*ArcTan[c*x])/x^3, x]
```

Rubi [A] time = 0.165242, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sqrt{d+ex^2}\left(a+b\tan^{-1}(cx)\right)}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]))/x^3,x]
```

```
[Out] -(a*Sqrt[d + e*x^2])/(2*x^2) - (a*e*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/(2*Sq
rt[d]) + b*Defer[Int][(Sqrt[d + e*x^2]*ArcTan[c*x])/x^3, x]
```

Rubi steps

$$\int \frac{\sqrt{d+ex^2}\left(a+b\tan^{-1}(cx)\right)}{x^3}\,dx = a\int \frac{\sqrt{d+ex^2}}{x^3}\,dx + b\int \frac{\sqrt{d+ex^2}\tan^{-1}(cx)}{x^3}\,dx$$
$$= \frac{1}{2}a\,\text{Subst}\left(\int \frac{\sqrt{d+ex}}{x^2}\,dx, x, x^2\right) + b\int \frac{\sqrt{d+ex^2}\tan^{-1}(cx)}{x^3}\,dx$$
$$= -\frac{a\sqrt{d+ex^2}}{2x^2} + b\int \frac{\sqrt{d+ex^2}\tan^{-1}(cx)}{x^3}\,dx + \frac{1}{4}(ae)\,\text{Subst}\left(\int \frac{1}{x\sqrt{d+ex}}\,dx, x, x^2\right)$$
$$= -\frac{a\sqrt{d+ex^2}}{2x^2} + \frac{1}{2}a\,\text{Subst}\left(\int \frac{1}{-\frac{d}{e}+\frac{x^2}{e}}\,dx, x, \sqrt{d+ex^2}\right) + b\int \frac{\sqrt{d+ex^2}\tan^{-1}(cx)}{x^3}\,dx$$
$$= -\frac{a\sqrt{d+ex^2}}{2x^2} - \frac{ae\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{2\sqrt{d}} + b\int \frac{\sqrt{d+ex^2}\tan^{-1}(cx)}{x^3}\,dx$$

Mathematica [A] time = 47.6666, size = 0, normalized size = 0.

$$\int \frac{\sqrt{d+ex^2}\left(a+b\tan^{-1}(cx)\right)}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]))/x^3,x]
```

```
[Out] Integrate[(Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]))/x^3, x]
```

Maple [A] time = 0.77, size = 0, normalized size = 0.

$$\int \frac{a+b\arctan(cx)}{x^3}\sqrt{ex^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x^3,x)
```

```
[Out] int((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x^3,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{ex^2+d}(b\arctan(cx)+a)}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x^2 + d)*(b*arctan(c*x) + a)/x^3, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{(a+b\,\mathrm{atan}\,(cx))\sqrt{d+ex^2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(1/2)*(a+b*atan(c*x))/x**3,x)
```

```
[Out] Integral((a + b*atan(c*x))*sqrt(d + e*x**2)/x**3, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{ex^2+d}(b\arctan(cx)+a)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)*(b*arctan(c*x) + a)/x^3, x)
```

3.1180 $\int \frac{\sqrt{d+ex^2}\left(a+b\tan^{-1}(cx)\right)}{x^4}\,dx$

Optimal. Leaf size=137

$$-\frac{\left(d+ex^2\right)^{3/2}\left(a+b\tan^{-1}(cx)\right)}{3dx^3}-\frac{b\left(c^2d-e\right)^{3/2}\tanh^{-1}\left(\frac{c\sqrt{d+ex^2}}{\sqrt{c^2d-e}}\right)}{3d}+\frac{bc\left(2c^2d-3e\right)\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{6\sqrt{d}}-\frac{bc\sqrt{d+ex^2}}{6x^2}$$

```
[Out] -(b*c*Sqrt[d + e*x^2])/(6*x^2) - ((d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]))/(3
*d*x^3) + (b*c*(2*c^2*d - 3*e)*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/(6*Sqrt[d]
) - (b*(c^2*d - e)^(3/2)*ArcTanh[(c*Sqrt[d + e*x^2])/Sqrt[c^2*d - e]])/(3*d
)
```

Rubi [A] time = 0.279043, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {264, 4976, 12, 446, 98, 156, 63, 208}

$$-\frac{\left(d+ex^2\right)^{3/2}\left(a+b\tan^{-1}(cx)\right)}{3dx^3}-\frac{b\left(c^2d-e\right)^{3/2}\tanh^{-1}\left(\frac{c\sqrt{d+ex^2}}{\sqrt{c^2d-e}}\right)}{3d}+\frac{bc\left(2c^2d-3e\right)\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{6\sqrt{d}}-\frac{bc\sqrt{d+ex^2}}{6x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]))/x^4,x]
```

```
[Out] -(b*c*Sqrt[d + e*x^2])/(6*x^2) - ((d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]))/(3
*d*x^3) + (b*c*(2*c^2*d - 3*e)*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/(6*Sqrt[d]
) - (b*(c^2*d - e)^(3/2)*ArcTanh[(c*Sqrt[d + e*x^2])/Sqrt[c^2*d - e]])/(3*d
)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 4976

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Dis
t[a + b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2
```

```
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&  !(
ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(IL
tQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILtQ[(m
 - 1)/2, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{d+ex^2}\left(a+b\tan^{-1}(cx)\right)}{x^4}\,dx = -\frac{\left(d+ex^2\right)^{3/2}\left(a+b\tan^{-1}(cx)\right)}{3dx^3}-(bc)\int \frac{\left(d+ex^2\right)^{3/2}}{3x^3\left(-d-c^2dx^2\right)}\,dx$$

$$= -\frac{\left(d+ex^2\right)^{3/2}\left(a+b\tan^{-1}(cx)\right)}{3dx^3}-\frac{1}{3}(bc)\int \frac{\left(d+ex^2\right)^{3/2}}{x^3\left(-d-c^2dx^2\right)}\,dx$$

$$= -\frac{\left(d+ex^2\right)^{3/2}\left(a+b\tan^{-1}(cx)\right)}{3dx^3}-\frac{1}{6}(bc)\,\mathrm{Subst}\left(\int \frac{(d+ex)^{3/2}}{x^2\left(-d-c^2dx\right)}\,dx,x,x^2\right)$$

$$= -\frac{bc\sqrt{d+ex^2}}{6x^2}-\frac{\left(d+ex^2\right)^{3/2}\left(a+b\tan^{-1}(cx)\right)}{3dx^3}-\frac{(bc)\,\mathrm{Subst}\left(\int \frac{-\frac{1}{2}d^2\left(2c^2d-3e\right)-\frac{1}{2}d\left(c^2d-\right.}{x\left(-d-c^2dx\right)\sqrt{d+ex}}\right.}{6d}$$

$$= -\frac{bc\sqrt{d+ex^2}}{6x^2}-\frac{\left(d+ex^2\right)^{3/2}\left(a+b\tan^{-1}(cx)\right)}{3dx^3}-\frac{1}{12}\left(bc\left(2c^2d-3e\right)\right)\mathrm{Subst}\left(\int \frac{}{x\sqrt{}}\right.$$

$$= -\frac{bc\sqrt{d+ex^2}}{6x^2}-\frac{\left(d+ex^2\right)^{3/2}\left(a+b\tan^{-1}(cx)\right)}{3dx^3}-\frac{\left(bc\left(2c^2d-3e\right)\right)\mathrm{Subst}\left(\int \frac{1}{-\frac{d}{e}+\frac{x^2}{e}}\,d\right.}{6e}$$

$$= -\frac{bc\sqrt{d+ex^2}}{6x^2}-\frac{\left(d+ex^2\right)^{3/2}\left(a+b\tan^{-1}(cx)\right)}{3dx^3}+\frac{bc\left(2c^2d-3e\right)\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{6\sqrt{d}}-$$

Mathematica [C] time = 0.637386, size = 288, normalized size = 2.1

$$-\frac{\sqrt{d+ex^2}\left(2a\left(d+ex^2\right)+bcdx\right)+bc\sqrt{d}x^3\log(x)\left(2c^2d-3e\right)-bc\sqrt{d}x^3\left(2c^2d-3e\right)\log\left(\sqrt{d}\sqrt{d+ex^2}+d\right)+bx^3\left(c^2d-\right.}{6dx^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]))/x^4,x]
```

```
[Out] -(Sqrt[d + e*x^2]*(b*c*d*x + 2*a*(d + e*x^2)) + 2*b*(d + e*x^2)^(3/2)*ArcTa
n[c*x] + b*c*Sqrt[d]*(2*c^2*d - 3*e)*x^3*Log[x] - b*c*Sqrt[d]*(2*c^2*d - 3*
e)*x^3*Log[d + Sqrt[d]*Sqrt[d + e*x^2]] + b*(c^2*d - e)^(3/2)*x^3*Log[(12*c
```

```
*d*(c*d - I*e*x + Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*(c^2*d - e)^(5/2)*(I
 + c*x))] + b*(c^2*d - e)^(3/2)*x^3*Log[(12*c*d*(c*d + I*e*x + Sqrt[c^2*d -
 e]*Sqrt[d + e*x^2]))/(b*(c^2*d - e)^(5/2)*(-I + c*x))])/(6*d*x^3)
```

Maple [F] time = 0.829, size = 0, normalized size = 0.

$$\int \frac{a + b\arctan(cx)}{x^4}\sqrt{ex^2 + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x^4,x)
```

```
[Out] int((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x^4,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.74001, size = 1940, normalized size = 14.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] [-1/12*((b*c^2*d - b*e)*sqrt(c^2*d - e)*x^3*log((c^4*e^2*x^4 + 8*c^4*d^2 -
8*c^2*d*e + 2*(4*c^4*d*e - 3*c^2*e^2)*x^2 + 4*(c^3*e*x^2 + 2*c^3*d - c*e)*s
qrt(c^2*d - e)*sqrt(e*x^2 + d) + e^2)/(c^4*x^4 + 2*c^2*x^2 + 1)) + (2*b*c^3
*d - 3*b*c*e)*sqrt(d)*x^3*log(-(e*x^2 - 2*sqrt(e*x^2 + d)*sqrt(d) + 2*d)/x^
```

```
2) + 2*(b*c*d*x + 2*a*e*x^2 + 2*a*d + 2*(b*e*x^2 + b*d)*arctan(c*x))*sqrt(e
*x^2 + d))/(d*x^3), -1/12*(2*(b*c^2*d - b*e)*sqrt(-c^2*d + e)*x^3*arctan(-1
/2*(c^2*e*x^2 + 2*c^2*d - e)*sqrt(-c^2*d + e)*sqrt(e*x^2 + d)/(c^3*d^2 - c*
d*e + (c^3*d*e - c*e^2)*x^2)) + (2*b*c^3*d - 3*b*c*e)*sqrt(d)*x^3*log(-(e*x
^2 - 2*sqrt(e*x^2 + d)*sqrt(d) + 2*d)/x^2) + 2*(b*c*d*x + 2*a*e*x^2 + 2*a*d
 + 2*(b*e*x^2 + b*d)*arctan(c*x))*sqrt(e*x^2 + d))/(d*x^3), -1/12*(2*(2*b*c
^3*d - 3*b*c*e)*sqrt(-d)*x^3*arctan(sqrt(-d)/sqrt(e*x^2 + d)) + (b*c^2*d -
b*e)*sqrt(c^2*d - e)*x^3*log((c^4*e^2*x^4 + 8*c^4*d^2 - 8*c^2*d*e + 2*(4*c^
4*d*e - 3*c^2*e^2)*x^2 + 4*(c^3*e*x^2 + 2*c^3*d - c*e)*sqrt(c^2*d - e)*sqrt
(e*x^2 + d) + e^2)/(c^4*x^4 + 2*c^2*x^2 + 1)) + 2*(b*c*d*x + 2*a*e*x^2 + 2*
a*d + 2*(b*e*x^2 + b*d)*arctan(c*x))*sqrt(e*x^2 + d))/(d*x^3), -1/6*((b*c^2
*d - b*e)*sqrt(-c^2*d + e)*x^3*arctan(-1/2*(c^2*e*x^2 + 2*c^2*d - e)*sqrt(-
c^2*d + e)*sqrt(e*x^2 + d)/(c^3*d^2 - c*d*e + (c^3*d*e - c*e^2)*x^2)) + (2*
b*c^3*d - 3*b*c*e)*sqrt(-d)*x^3*arctan(sqrt(-d)/sqrt(e*x^2 + d)) + (b*c*d*x
 + 2*a*e*x^2 + 2*a*d + 2*(b*e*x^2 + b*d)*arctan(c*x))*sqrt(e*x^2 + d))/(d*x
^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b\operatorname{atan}(cx))\sqrt{d + ex^2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(1/2)*(a+b*atan(c*x))/x**4,x)
```

```
[Out] Integral((a + b*atan(c*x))*sqrt(d + e*x**2)/x**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex^2 + d}(b\arctan(cx) + a)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)*(b*arctan(c*x) + a)/x^4, x)
```

3.1181 $\int \frac{\sqrt{d+ex^2}\left(a+b\tan^{-1}(cx)\right)}{x^5}\,dx$

Optimal. Leaf size=97

$$b\text{Unintegrable}\left(\frac{\tan^{-1}(cx)\sqrt{d+ex^2}}{x^5},x\right)+\frac{ae^2\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{8d^{3/2}}-\frac{ae\sqrt{d+ex^2}}{8dx^2}-\frac{a\sqrt{d+ex^2}}{4x^4}$$

```
[Out] -(a*Sqrt[d + e*x^2])/(4*x^4) - (a*e*Sqrt[d + e*x^2])/(8*d*x^2) + (a*e^2*Arc
Tanh[Sqrt[d + e*x^2]/Sqrt[d]])/(8*d^(3/2)) + b*Unintegrable[(Sqrt[d + e*x^2
]*ArcTan[c*x])/x^5, x]
```

Rubi [A] time = 0.175813, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sqrt{d+ex^2}\left(a+b\tan^{-1}(cx)\right)}{x^5}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]))/x^5,x]
```

```
[Out] -(a*Sqrt[d + e*x^2])/(4*x^4) - (a*e*Sqrt[d + e*x^2])/(8*d*x^2) + (a*e^2*Arc
Tanh[Sqrt[d + e*x^2]/Sqrt[d]])/(8*d^(3/2)) + b*Defer[Int][(Sqrt[d + e*x^2]*
ArcTan[c*x])/x^5, x]
```

Rubi steps

$$\int \frac{\sqrt{d+ex^2}\left(a+b\tan^{-1}(cx)\right)}{x^5}\,dx = a\int \frac{\sqrt{d+ex^2}}{x^5}\,dx + b\int \frac{\sqrt{d+ex^2}\tan^{-1}(cx)}{x^5}\,dx$$
$$= \frac{1}{2}a\,\text{Subst}\left(\int \frac{\sqrt{d+ex}}{x^3}\,dx,x,x^2\right) + b\int \frac{\sqrt{d+ex^2}\tan^{-1}(cx)}{x^5}\,dx$$
$$= -\frac{a\sqrt{d+ex^2}}{4x^4} + b\int \frac{\sqrt{d+ex^2}\tan^{-1}(cx)}{x^5}\,dx + \frac{1}{8}(ae)\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{d+ex}}\,dx,x,x^2\right)$$
$$= -\frac{a\sqrt{d+ex^2}}{4x^4} - \frac{ae\sqrt{d+ex^2}}{8dx^2} + b\int \frac{\sqrt{d+ex^2}\tan^{-1}(cx)}{x^5}\,dx - \frac{\left(ae^2\right)\text{Subst}\left(\int \frac{1}{x\sqrt{d+ex}}\,d\right.}{16d}$$
$$= -\frac{a\sqrt{d+ex^2}}{4x^4} - \frac{ae\sqrt{d+ex^2}}{8dx^2} + b\int \frac{\sqrt{d+ex^2}\tan^{-1}(cx)}{x^5}\,dx - \frac{(ae)\,\text{Subst}\left(\int \frac{1}{-\frac{d}{e}+\frac{x^2}{e}}\,dx\right.}{8d}$$
$$= -\frac{a\sqrt{d+ex^2}}{4x^4} - \frac{ae\sqrt{d+ex^2}}{8dx^2} + \frac{ae^2\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{8d^{3/2}} + b\int \frac{\sqrt{d+ex^2}\tan^{-1}(cx)}{x^5}\,dx$$

Mathematica [A] time = 50.3483, size = 0, normalized size = 0.

$$\int \frac{\sqrt{d+ex^2}\left(a+b\tan^{-1}(cx)\right)}{x^5}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]))/x^5,x]
```

```
[Out] Integrate[(Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]))/x^5, x]
```

Maple [A] time = 0.785, size = 0, normalized size = 0.

$$\int \frac{a+b\arctan(cx)}{x^5}\sqrt{ex^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x^5,x)
```

```
[Out] int((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x^5,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{ex^2+d}(b\arctan(cx)+a)}{x^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x^5,x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x^2 + d)*(b*arctan(c*x) + a)/x^5, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{(a+b\,\mathrm{atan}\,(cx))\,\sqrt{d+ex^2}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(1/2)*(a+b*atan(c*x))/x**5,x)
```

```
[Out] Integral((a + b*atan(c*x))*sqrt(d + e*x**2)/x**5, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{ex^2+d}(b\arctan(cx)+a)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x^5,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)*(b*arctan(c*x) + a)/x^5, x)
```

3.1182 $\int \frac{\sqrt{d+ex^2}\left(a+b\tan^{-1}(cx)\right)}{x^6}\,dx$

Optimal. Leaf size=224

$$\frac{2e\left(d+ex^2\right)^{3/2}\left(a+b\tan^{-1}(cx)\right)}{15d^2x^3}-\frac{\left(d+ex^2\right)^{3/2}\left(a+b\tan^{-1}(cx)\right)}{5dx^5}-\frac{bc\left(24c^4d^2-20c^2de-15e^2\right)\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{120d^{3/2}}+\frac{b\left(3c\right.}{}$$

```
[Out] (b*c*(12*c^2*d - e)*Sqrt[d + e*x^2])/(120*d*x^2) - (b*c*(d + e*x^2)^(3/2))/
(20*d*x^4) - ((d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]))/(5*d*x^5) + (2*e*(d +
e*x^2)^(3/2)*(a + b*ArcTan[c*x]))/(15*d^2*x^3) - (b*c*(24*c^4*d^2 - 20*c^2*
d*e - 15*e^2)*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/(120*d^(3/2)) + (b*(c^2*d -
 e)^(3/2)*(3*c^2*d + 2*e)*ArcTanh[(c*Sqrt[d + e*x^2])/Sqrt[c^2*d - e]])/(15
*d^2)
```

Rubi [A] time = 0.352288, antiderivative size = 224, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {271, 264, 4976, 12, 573, 149, 156, 63, 208}

$$\frac{2e\left(d+ex^2\right)^{3/2}\left(a+b\tan^{-1}(cx)\right)}{15d^2x^3}-\frac{\left(d+ex^2\right)^{3/2}\left(a+b\tan^{-1}(cx)\right)}{5dx^5}-\frac{bc\left(24c^4d^2-20c^2de-15e^2\right)\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{120d^{3/2}}+\frac{b\left(3c\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]))/x^6,x]
```

```
[Out] (b*c*(12*c^2*d - e)*Sqrt[d + e*x^2])/(120*d*x^2) - (b*c*(d + e*x^2)^(3/2))/
(20*d*x^4) - ((d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]))/(5*d*x^5) + (2*e*(d +
e*x^2)^(3/2)*(a + b*ArcTan[c*x]))/(15*d^2*x^3) - (b*c*(24*c^4*d^2 - 20*c^2*
d*e - 15*e^2)*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/(120*d^(3/2)) + (b*(c^2*d -
 e)^(3/2)*(3*c^2*d + 2*e)*ArcTanh[(c*Sqrt[d + e*x^2])/Sqrt[c^2*d - e]])/(15
*d^2)
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 4976

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Dis
t[a + b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&  !(
ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(IL
tQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILtQ[(m
 - 1)/2, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 573

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simpl
ify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /;
 FreeQ[{a, b, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[Simplify[(m + 1)/n
]]
```

Rule 149

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegerQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
```

```
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{d+ex^2}\left(a+b\tan^{-1}(cx)\right)}{x^6}\,dx = -\frac{\left(d+ex^2\right)^{3/2}\left(a+b\tan^{-1}(cx)\right)}{5dx^5} + \frac{2e\left(d+ex^2\right)^{3/2}\left(a+b\tan^{-1}(cx)\right)}{15d^2x^3} - (bc)\int \frac{\left(d+e\right.}{15}$$

$$= -\frac{\left(d+ex^2\right)^{3/2}\left(a+b\tan^{-1}(cx)\right)}{5dx^5} + \frac{2e\left(d+ex^2\right)^{3/2}\left(a+b\tan^{-1}(cx)\right)}{15d^2x^3} - \frac{(bc)\int \frac{\left(d+ex^2\right)^{3}}{x^5(}}{15}$$

$$= -\frac{\left(d+ex^2\right)^{3/2}\left(a+b\tan^{-1}(cx)\right)}{5dx^5} + \frac{2e\left(d+ex^2\right)^{3/2}\left(a+b\tan^{-1}(cx)\right)}{15d^2x^3} - \frac{(bc)\,\mathrm{Subst}\left(\int\right.}{}$$

$$= -\frac{bc\left(d+ex^2\right)^{3/2}}{20dx^4} - \frac{\left(d+ex^2\right)^{3/2}\left(a+b\tan^{-1}(cx)\right)}{5dx^5} + \frac{2e\left(d+ex^2\right)^{3/2}\left(a+b\tan^{-1}(cx)\right)}{15d^2x^3}$$

$$= \frac{bc\left(12c^2d-e\right)\sqrt{d+ex^2}}{120dx^2} - \frac{bc\left(d+ex^2\right)^{3/2}}{20dx^4} - \frac{\left(d+ex^2\right)^{3/2}\left(a+b\tan^{-1}(cx)\right)}{5dx^5} + \frac{2e\left(d\right.}{}$$

$$= \frac{bc\left(12c^2d-e\right)\sqrt{d+ex^2}}{120dx^2} - \frac{bc\left(d+ex^2\right)^{3/2}}{20dx^4} - \frac{\left(d+ex^2\right)^{3/2}\left(a+b\tan^{-1}(cx)\right)}{5dx^5} + \frac{2e\left(d\right.}{}$$

$$= \frac{bc\left(12c^2d-e\right)\sqrt{d+ex^2}}{120dx^2} - \frac{bc\left(d+ex^2\right)^{3/2}}{20dx^4} - \frac{\left(d+ex^2\right)^{3/2}\left(a+b\tan^{-1}(cx)\right)}{5dx^5} + \frac{2e\left(d\right.}{}$$

$$= \frac{bc\left(12c^2d-e\right)\sqrt{d+ex^2}}{120dx^2} - \frac{bc\left(d+ex^2\right)^{3/2}}{20dx^4} - \frac{\left(d+ex^2\right)^{3/2}\left(a+b\tan^{-1}(cx)\right)}{5dx^5} + \frac{2e\left(d\right.}{}$$

Mathematica [C] time = 0.508839, size = 413, normalized size = 1.84

$$\frac{-\sqrt{d+ex^2}\left(8a\left(3d^2+dex^2-2e^2x^4\right)+bcdx\left(d\left(6-12c^2x^2\right)+7ex^2\right)\right)+bc\sqrt{d}x^5\log(x)\left(24c^4d^2-20c^2de-15e^2\right)-bc\sqrt{d}x}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]))/x^6,x]
```

```
[Out] (-(Sqrt[d + e*x^2]*(8*a*(3*d^2 + d*e*x^2 - 2*e^2*x^4) + b*c*d*x*(7*e*x^2 +
d*(6 - 12*c^2*x^2)))) - 8*b*Sqrt[d + e*x^2]*(3*d^2 + d*e*x^2 - 2*e^2*x^4)*A
rcTan[c*x] + b*c*Sqrt[d]*(24*c^4*d^2 - 20*c^2*d*e - 15*e^2)*x^5*Log[x] - b*
c*Sqrt[d]*(24*c^4*d^2 - 20*c^2*d*e - 15*e^2)*x^5*Log[d + Sqrt[d]*Sqrt[d + e
```

```
*x^2]] + 4*b*(c^2*d - e)^(3/2)*(3*c^2*d + 2*e)*x^5*Log[(-60*c*d^2*(c*d - I*
e*x + Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*(c^2*d - e)^(5/2)*(3*c^2*d + 2*e
)*(I + c*x))] + 4*b*(c^2*d - e)^(3/2)*(3*c^2*d + 2*e)*x^5*Log[(-60*c*d^2*(c
*d + I*e*x + Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*(c^2*d - e)^(5/2)*(3*c^2*
d + 2*e)*(-I + c*x))])/(120*d^2*x^5)
```

Maple [F] time = 0.881, size = 0, normalized size = 0.

$$\int \frac{a + b\arctan(cx)}{x^6}\sqrt{ex^2 + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x^6,x)
```

```
[Out] int((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x^6,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 6.67949, size = 2593, normalized size = 11.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x^6,x, algorithm="fricas")
```

```
[Out] [-1/240*(4*(3*b*c^4*d^2 - b*c^2*d*e - 2*b*e^2)*sqrt(c^2*d - e)*x^5*log((c^4
*e^2*x^4 + 8*c^4*d^2 - 8*c^2*d*e + 2*(4*c^4*d*e - 3*c^2*e^2)*x^2 - 4*(c^3*e
```

```
*x^2 + 2*c^3*d - c*e)*sqrt(c^2*d - e)*sqrt(e*x^2 + d) + e^2)/(c^4*x^4 + 2*c
^2*x^2 + 1)) + (24*b*c^5*d^2 - 20*b*c^3*d*e - 15*b*c*e^2)*sqrt(d)*x^5*log(-
(e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(d) + 2*d)/x^2) - 2*(16*a*e^2*x^4 - 6*b*c*d^
2*x - 8*a*d*e*x^2 + (12*b*c^3*d^2 - 7*b*c*d*e)*x^3 - 24*a*d^2 + 8*(2*b*e^2*
x^4 - b*d*e*x^2 - 3*b*d^2)*arctan(c*x))*sqrt(e*x^2 + d))/(d^2*x^5), 1/240*(
8*(3*b*c^4*d^2 - b*c^2*d*e - 2*b*e^2)*sqrt(-c^2*d + e)*x^5*arctan(-1/2*(c^2
*e*x^2 + 2*c^2*d - e)*sqrt(-c^2*d + e)*sqrt(e*x^2 + d)/(c^3*d^2 - c*d*e + (
c^3*d*e - c*e^2)*x^2)) - (24*b*c^5*d^2 - 20*b*c^3*d*e - 15*b*c*e^2)*sqrt(d)
*x^5*log(-(e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(d) + 2*d)/x^2) + 2*(16*a*e^2*x^4
- 6*b*c*d^2*x - 8*a*d*e*x^2 + (12*b*c^3*d^2 - 7*b*c*d*e)*x^3 - 24*a*d^2 + 8
*(2*b*e^2*x^4 - b*d*e*x^2 - 3*b*d^2)*arctan(c*x))*sqrt(e*x^2 + d))/(d^2*x^5
), 1/120*((24*b*c^5*d^2 - 20*b*c^3*d*e - 15*b*c*e^2)*sqrt(-d)*x^5*arctan(sq
rt(-d)/sqrt(e*x^2 + d)) - 2*(3*b*c^4*d^2 - b*c^2*d*e - 2*b*e^2)*sqrt(c^2*d
- e)*x^5*log((c^4*e^2*x^4 + 8*c^4*d^2 - 8*c^2*d*e + 2*(4*c^4*d*e - 3*c^2*e^
2)*x^2 - 4*(c^3*e*x^2 + 2*c^3*d - c*e)*sqrt(c^2*d - e)*sqrt(e*x^2 + d) + e^
2)/(c^4*x^4 + 2*c^2*x^2 + 1)) + (16*a*e^2*x^4 - 6*b*c*d^2*x - 8*a*d*e*x^2 +
 (12*b*c^3*d^2 - 7*b*c*d*e)*x^3 - 24*a*d^2 + 8*(2*b*e^2*x^4 - b*d*e*x^2 - 3
*b*d^2)*arctan(c*x))*sqrt(e*x^2 + d))/(d^2*x^5), 1/120*(4*(3*b*c^4*d^2 - b*
c^2*d*e - 2*b*e^2)*sqrt(-c^2*d + e)*x^5*arctan(-1/2*(c^2*e*x^2 + 2*c^2*d -
e)*sqrt(-c^2*d + e)*sqrt(e*x^2 + d)/(c^3*d^2 - c*d*e + (c^3*d*e - c*e^2)*x^
2)) + (24*b*c^5*d^2 - 20*b*c^3*d*e - 15*b*c*e^2)*sqrt(-d)*x^5*arctan(sqrt(-
d)/sqrt(e*x^2 + d)) + (16*a*e^2*x^4 - 6*b*c*d^2*x - 8*a*d*e*x^2 + (12*b*c^3
*d^2 - 7*b*c*d*e)*x^3 - 24*a*d^2 + 8*(2*b*e^2*x^4 - b*d*e*x^2 - 3*b*d^2)*ar
ctan(c*x))*sqrt(e*x^2 + d))/(d^2*x^5)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b\operatorname{atan}(cx))\sqrt{d + ex^2}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(1/2)*(a+b*atan(c*x))/x**6,x)
```

```
[Out] Integral((a + b*atan(c*x))*sqrt(d + e*x**2)/x**6, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex^2 + d}(b\arctan(cx) + a)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x^6,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)*(b*arctan(c*x) + a)/x^6, x)
```

3.1183 $\int x^3 \left(d+ex^2\right)^{3/2} \left(a+b\tan^{-1}(cx)\right)\,dx$

Optimal. Leaf size=279

$$\frac{\left(d+ex^2\right)^{7/2}\left(a+b\tan^{-1}(cx)\right)}{7e^2}-\frac{d\left(d+ex^2\right)^{5/2}\left(a+b\tan^{-1}(cx)\right)}{5e^2}+\frac{bx\left(3c^4d^2+54c^2de-40e^2\right)\sqrt{d+ex^2}}{560c^5e}+\frac{b\left(70c^4d^2e+\right.}{}$$

```
[Out] (b*(3*c^4*d^2 + 54*c^2*d*e - 40*e^2)*x*Sqrt[d + e*x^2])/(560*c^5*e) - (b*(1
3*c^2*d - 30*e)*x*(d + e*x^2)^(3/2))/(840*c^3*e) - (b*x*(d + e*x^2)^(5/2))/
(42*c*e) - (d*(d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]))/(5*e^2) + ((d + e*x^2)
^(7/2)*(a + b*ArcTan[c*x]))/(7*e^2) + (b*(c^2*d - e)^(5/2)*(2*c^2*d + 5*e)*
ArcTan[(Sqrt[c^2*d - e]*x)/Sqrt[d + e*x^2]])/(35*c^7*e^2) + (b*(35*c^6*d^3
+ 70*c^4*d^2*e - 168*c^2*d*e^2 + 80*e^3)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2
]])/(560*c^7*e^(3/2))
```

Rubi [A] time = 0.461088, antiderivative size = 279, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {266, 43, 4976, 12, 528, 523, 217, 206, 377, 203}

$$\frac{\left(d+ex^2\right)^{7/2}\left(a+b\tan^{-1}(cx)\right)}{7e^2}-\frac{d\left(d+ex^2\right)^{5/2}\left(a+b\tan^{-1}(cx)\right)}{5e^2}+\frac{bx\left(3c^4d^2+54c^2de-40e^2\right)\sqrt{d+ex^2}}{560c^5e}+\frac{b\left(70c^4d^2e+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]),x]
```

```
[Out] (b*(3*c^4*d^2 + 54*c^2*d*e - 40*e^2)*x*Sqrt[d + e*x^2])/(560*c^5*e) - (b*(1
3*c^2*d - 30*e)*x*(d + e*x^2)^(3/2))/(840*c^3*e) - (b*x*(d + e*x^2)^(5/2))/
(42*c*e) - (d*(d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]))/(5*e^2) + ((d + e*x^2)
^(7/2)*(a + b*ArcTan[c*x]))/(7*e^2) + (b*(c^2*d - e)^(5/2)*(2*c^2*d + 5*e)*
ArcTan[(Sqrt[c^2*d - e]*x)/Sqrt[d + e*x^2]])/(35*c^7*e^2) + (b*(35*c^6*d^3
+ 70*c^4*d^2*e - 168*c^2*d*e^2 + 80*e^3)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2
]])/(560*c^7*e^(3/2))
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 4976

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Dis
t[a + b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&  !(
ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(IL
tQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILtQ[(m
 - 1)/2, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int x^3\left(d+ex^2\right)^{3/2}\left(a+b\tan^{-1}(cx)\right)dx=-\frac{d\left(d+ex^2\right)^{5/2}\left(a+b\tan^{-1}(cx)\right)}{5e^2}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\tan^{-1}(cx)\right)}{7e^2}-(bc)\int\frac{\left(d\right.}{}$$

$$=-\frac{d\left(d+ex^2\right)^{5/2}\left(a+b\tan^{-1}(cx)\right)}{5e^2}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\tan^{-1}(cx)\right)}{7e^2}-\frac{(bc)\int\frac{(d+}{}}{}$$

$$=-\frac{bx\left(d+ex^2\right)^{5/2}}{42ce}-\frac{d\left(d+ex^2\right)^{5/2}\left(a+b\tan^{-1}(cx)\right)}{5e^2}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\tan^{-1}\right.}{7e^2}$$

$$=-\frac{b\left(13c^2d-30e\right)x\left(d+ex^2\right)^{3/2}}{840c^3e}-\frac{bx\left(d+ex^2\right)^{5/2}}{42ce}-\frac{d\left(d+ex^2\right)^{5/2}\left(a+b\tan^{-1}\right.}{5e^2}$$

$$=\frac{b\left(3c^4d^2+54c^2de-40e^2\right)x\sqrt{d+ex^2}}{560c^5e}-\frac{b\left(13c^2d-30e\right)x\left(d+ex^2\right)^{3/2}}{840c^3e}-\frac{bx\left(d\right.}{4}$$

$$=\frac{b\left(3c^4d^2+54c^2de-40e^2\right)x\sqrt{d+ex^2}}{560c^5e}-\frac{b\left(13c^2d-30e\right)x\left(d+ex^2\right)^{3/2}}{840c^3e}-\frac{bx\left(d\right.}{4}$$

$$=\frac{b\left(3c^4d^2+54c^2de-40e^2\right)x\sqrt{d+ex^2}}{560c^5e}-\frac{b\left(13c^2d-30e\right)x\left(d+ex^2\right)^{3/2}}{840c^3e}-\frac{bx\left(d\right.}{4}$$

$$=\frac{b\left(3c^4d^2+54c^2de-40e^2\right)x\sqrt{d+ex^2}}{560c^5e}-\frac{b\left(13c^2d-30e\right)x\left(d+ex^2\right)^{3/2}}{840c^3e}-\frac{bx\left(d\right.}{4}$$

Mathematica [C] time = 0.652957, size = 418, normalized size = 1.5

$$-\frac{c^2\sqrt{d+ex^2}\left(48ac^5\left(2d-5ex^2\right)\left(d+ex^2\right)^2+bex\left(c^4\left(57d^2+106dex^2+40e^2x^4\right)-6c^2e\left(37d+10ex^2\right)+120e^2\right)\right)-3b\sqrt{e}\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]),x]
```

```
[Out] -(c^2*Sqrt[d + e*x^2]*(48*a*c^5*(2*d - 5*e*x^2)*(d + e*x^2)^2 + b*e*x*(120*
e^2 - 6*c^2*e*(37*d + 10*e*x^2) + c^4*(57*d^2 + 106*d*e*x^2 + 40*e^2*x^4)))
 + 48*b*c^7*(2*d - 5*e*x^2)*(d + e*x^2)^(5/2)*ArcTan[c*x] + (24*I)*b*(c^2*d
 - e)^(5/2)*(2*c^2*d + 5*e)*Log[((-140*I)*c^8*e^2*(c*d - I*e*x + Sqrt[c^2*d
 - e]*Sqrt[d + e*x^2]))/(b*(c^2*d - e)^(7/2)*(2*c^2*d + 5*e)*(I + c*x))] -
(24*I)*b*(c^2*d - e)^(5/2)*(2*c^2*d + 5*e)*Log[((140*I)*c^8*e^2*(c*d + I*e*
x + Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*(c^2*d - e)^(7/2)*(2*c^2*d + 5*e)*
(-I + c*x))] - 3*b*Sqrt[e]*(35*c^6*d^3 + 70*c^4*d^2*e - 168*c^2*d*e^2 + 80*
e^3)*Log[e*x + Sqrt[e]*Sqrt[d + e*x^2]])/(1680*c^7*e^2)
```

Maple [F] time = 0.654, size = 0, normalized size = 0.

$$\int x^3\left(ex^2+d\right)^{\frac{3}{2}}(a+b\arctan(cx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x^2+d)^(3/2)*(a+b*arctan(c*x)),x)
```

```
[Out] int(x^3*(e*x^2+d)^(3/2)*(a+b*arctan(c*x)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^(3/2)*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 117.58, size = 3487, normalized size = 12.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^(3/2)*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] [1/3360*(3*(35*b*c^6*d^3 + 70*b*c^4*d^2*e - 168*b*c^2*d*e^2 + 80*b*e^3)*sqr
t(e)*log(-2*e*x^2 - 2*sqrt(e*x^2 + d)*sqrt(e)*x - d) + 24*(2*b*c^6*d^3 + b*
c^4*d^2*e - 8*b*c^2*d*e^2 + 5*b*e^3)*sqrt(-c^2*d + e)*log(((c^4*d^2 - 8*c^2
*d*e + 8*e^2)*x^4 - 2*(3*c^2*d^2 - 4*d*e)*x^2 + 4*((c^2*d - 2*e)*x^3 - d*x)
*sqrt(-c^2*d + e)*sqrt(e*x^2 + d) + d^2)/(c^4*x^4 + 2*c^2*x^2 + 1)) + 2*(24
0*a*c^7*e^3*x^6 + 384*a*c^7*d*e^2*x^4 - 40*b*c^6*e^3*x^5 + 48*a*c^7*d^2*e*x
^2 - 96*a*c^7*d^3 - 2*(53*b*c^6*d*e^2 - 30*b*c^4*e^3)*x^3 - 3*(19*b*c^6*d^2
*e - 74*b*c^4*d*e^2 + 40*b*c^2*e^3)*x + 48*(5*b*c^7*e^3*x^6 + 8*b*c^7*d*e^2
*x^4 + b*c^7*d^2*e*x^2 - 2*b*c^7*d^3)*arctan(c*x))*sqrt(e*x^2 + d))/(c^7*e^
2), 1/3360*(48*(2*b*c^6*d^3 + b*c^4*d^2*e - 8*b*c^2*d*e^2 + 5*b*e^3)*sqrt(c
^2*d - e)*arctan(1/2*sqrt(c^2*d - e)*((c^2*d - 2*e)*x^2 - d)*sqrt(e*x^2 + d
)/((c^2*d*e - e^2)*x^3 + (c^2*d^2 - d*e)*x)) + 3*(35*b*c^6*d^3 + 70*b*c^4*d
^2*e - 168*b*c^2*d*e^2 + 80*b*e^3)*sqrt(e)*log(-2*e*x^2 - 2*sqrt(e*x^2 + d)
*sqrt(e)*x - d) + 2*(240*a*c^7*e^3*x^6 + 384*a*c^7*d*e^2*x^4 - 40*b*c^6*e^3
*x^5 + 48*a*c^7*d^2*e*x^2 - 96*a*c^7*d^3 - 2*(53*b*c^6*d*e^2 - 30*b*c^4*e^3
)*x^3 - 3*(19*b*c^6*d^2*e - 74*b*c^4*d*e^2 + 40*b*c^2*e^3)*x + 48*(5*b*c^7*
e^3*x^6 + 8*b*c^7*d*e^2*x^4 + b*c^7*d^2*e*x^2 - 2*b*c^7*d^3)*arctan(c*x))*s
qrt(e*x^2 + d))/(c^7*e^2), -1/1680*(3*(35*b*c^6*d^3 + 70*b*c^4*d^2*e - 168*
b*c^2*d*e^2 + 80*b*e^3)*sqrt(-e)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) - 12*(2
*b*c^6*d^3 + b*c^4*d^2*e - 8*b*c^2*d*e^2 + 5*b*e^3)*sqrt(-c^2*d + e)*log(((
c^4*d^2 - 8*c^2*d*e + 8*e^2)*x^4 - 2*(3*c^2*d^2 - 4*d*e)*x^2 + 4*((c^2*d -
2*e)*x^3 - d*x)*sqrt(-c^2*d + e)*sqrt(e*x^2 + d) + d^2)/(c^4*x^4 + 2*c^2*x^
2 + 1)) - (240*a*c^7*e^3*x^6 + 384*a*c^7*d*e^2*x^4 - 40*b*c^6*e^3*x^5 + 48*
a*c^7*d^2*e*x^2 - 96*a*c^7*d^3 - 2*(53*b*c^6*d*e^2 - 30*b*c^4*e^3)*x^3 - 3*
(19*b*c^6*d^2*e - 74*b*c^4*d*e^2 + 40*b*c^2*e^3)*x + 48*(5*b*c^7*e^3*x^6 +
8*b*c^7*d*e^2*x^4 + b*c^7*d^2*e*x^2 - 2*b*c^7*d^3)*arctan(c*x))*sqrt(e*x^2
+ d))/(c^7*e^2), 1/1680*(24*(2*b*c^6*d^3 + b*c^4*d^2*e - 8*b*c^2*d*e^2 + 5*
b*e^3)*sqrt(c^2*d - e)*arctan(1/2*sqrt(c^2*d - e)*((c^2*d - 2*e)*x^2 - d)*s
qrt(e*x^2 + d)/((c^2*d*e - e^2)*x^3 + (c^2*d^2 - d*e)*x)) - 3*(35*b*c^6*d^3
 + 70*b*c^4*d^2*e - 168*b*c^2*d*e^2 + 80*b*e^3)*sqrt(-e)*arctan(sqrt(-e)*x/
sqrt(e*x^2 + d)) + (240*a*c^7*e^3*x^6 + 384*a*c^7*d*e^2*x^4 - 40*b*c^6*e^3*
x^5 + 48*a*c^7*d^2*e*x^2 - 96*a*c^7*d^3 - 2*(53*b*c^6*d*e^2 - 30*b*c^4*e^3)
```

```
*x^3 - 3*(19*b*c^6*d^2*e - 74*b*c^4*d*e^2 + 40*b*c^2*e^3)*x + 48*(5*b*c^7*e
^3*x^6 + 8*b*c^7*d*e^2*x^4 + b*c^7*d^2*e*x^2 - 2*b*c^7*d^3)*arctan(c*x))*sq
rt(e*x^2 + d))/(c^7*e^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x**2+d)**(3/2)*(a+b*atan(c*x)),x)

[Out] Timed out
```

Giac [B] time = 1.58018, size = 852, normalized size = 3.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^(3/2)*(a+b*arctan(c*x)),x, algorithm="giac")

[Out] 1/15*(3*(x^2*e + d)^(5/2) - 5*(x^2*e + d)^(3/2)*d)*a*d*e^(-2) + 1/240*(16*(
3*(x^2*e + d)^(5/2) - 5*(x^2*e + d)^(3/2)*d)*arctan(c*x)*e^(-2) - (2*sqrt(x
^2*e + d)*x*(6*x^2/c^2 + (7*c^10*d*e^2 - 12*c^8*e^3)*e^(-3)/c^12) + (15*c^4
*d^2 + 20*c^2*d*e - 24*e^2)*e^(-3/2)*log((x*e^(1/2) - sqrt(x^2*e + d))^2)/c
^6 + 16*(2*c^6*d^3*e^(1/2) - c^4*d^2*e^(3/2) - 4*c^2*d*e^(5/2) + 3*e^(7/2))
*arctan(1/2*((x*e^(1/2) - sqrt(x^2*e + d))^2*c^2 - c^2*d + 2*e)*e^(-1/2)/sq
rt(c^2*d - e))*e^(-5/2)/(sqrt(c^2*d - e)*c^6))*c)*b*d + 1/3360*(32*(15*(x^2
*e + d)^(7/2) - 42*(x^2*e + d)^(5/2)*d + 35*(x^2*e + d)^(3/2)*d^2)*a*e^(-3)
 + (32*(15*(x^2*e + d)^(7/2) - 42*(x^2*e + d)^(5/2)*d + 35*(x^2*e + d)^(3/2
)*d^2)*arctan(c*x)*e^(-3) - (2*(2*x^2*(20*x^2/c^2 + (11*c^18*d*e^6 - 30*c^1
6*e^7)*e^(-7)/c^20) - (41*c^18*d^2*e^5 + 54*c^16*d*e^6 - 120*c^14*e^7)*e^(-
7)/c^20)*sqrt(x^2*e + d)*x - (105*c^6*d^3 + 70*c^4*d^2*e + 168*c^2*d*e^2 -
240*e^3)*e^(-5/2)*log((x*e^(1/2) - sqrt(x^2*e + d))^2)/c^8 - 32*(8*c^8*d^4*
e^(1/2) - 4*c^6*d^3*e^(3/2) - c^4*d^2*e^(5/2) - 18*c^2*d*e^(7/2) + 15*e^(9/
2))*arctan(1/2*((x*e^(1/2) - sqrt(x^2*e + d))^2*c^2 - c^2*d + 2*e)*e^(-1/2)
/sqrt(c^2*d - e))*e^(-7/2)/(sqrt(c^2*d - e)*c^8))*c)*b)*e
```

3.1184 $\int x^2 \left(d+ex^2\right)^{3/2} \left(a+b\tan^{-1}(cx)\right)\,dx$

Optimal. Leaf size=118

$$b\text{Unintegrable}\left(x^2\tan^{-1}(cx)\left(d+ex^2\right)^{3/2},x\right)-\frac{ad^3\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{16e^{3/2}}+\frac{ad^2x\sqrt{d+ex^2}}{16e}+\frac{1}{8}adx^3\sqrt{d+ex^2}+\frac{1}{6}ax^3\left(d+e\right.$$

```
[Out] (a*d^2*x*Sqrt[d + e*x^2])/(16*e) + (a*d*x^3*Sqrt[d + e*x^2])/8 + (a*x^3*(d
+ e*x^2)^(3/2))/6 - (a*d^3*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(16*e^(3/2
)) + b*Unintegrable[x^2*(d + e*x^2)^(3/2)*ArcTan[c*x], x]
```

Rubi [A] time = 0.19486, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^2 \left(d+ex^2\right)^{3/2} \left(a+b\tan^{-1}(cx)\right)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^2*(d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]),x]
```

```
[Out] (a*d^2*x*Sqrt[d + e*x^2])/(16*e) + (a*d*x^3*Sqrt[d + e*x^2])/8 + (a*x^3*(d
+ e*x^2)^(3/2))/6 - (a*d^3*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(16*e^(3/2
)) + b*Defer[Int][x^2*(d + e*x^2)^(3/2)*ArcTan[c*x], x]
```

Rubi steps

$$\int x^2\left(d+ex^2\right)^{3/2}\left(a+b\tan^{-1}(cx)\right)dx = a\int x^2\left(d+ex^2\right)^{3/2}dx + b\int x^2\left(d+ex^2\right)^{3/2}\tan^{-1}(cx)\,dx$$
$$= \frac{1}{6}ax^3\left(d+ex^2\right)^{3/2} + b\int x^2\left(d+ex^2\right)^{3/2}\tan^{-1}(cx)\,dx + \frac{1}{2}(ad)\int x^2\sqrt{d+ex^2}\,dx$$
$$= \frac{1}{8}adx^3\sqrt{d+ex^2} + \frac{1}{6}ax^3\left(d+ex^2\right)^{3/2} + b\int x^2\left(d+ex^2\right)^{3/2}\tan^{-1}(cx)\,dx + \frac{1}{8}\left(ad^2\right.$$
$$= \frac{ad^2x\sqrt{d+ex^2}}{16e} + \frac{1}{8}adx^3\sqrt{d+ex^2} + \frac{1}{6}ax^3\left(d+ex^2\right)^{3/2} + b\int x^2\left(d+ex^2\right)^{3/2}\tan^{-}$$
$$= \frac{ad^2x\sqrt{d+ex^2}}{16e} + \frac{1}{8}adx^3\sqrt{d+ex^2} + \frac{1}{6}ax^3\left(d+ex^2\right)^{3/2} + b\int x^2\left(d+ex^2\right)^{3/2}\tan^{-}$$
$$= \frac{ad^2x\sqrt{d+ex^2}}{16e} + \frac{1}{8}adx^3\sqrt{d+ex^2} + \frac{1}{6}ax^3\left(d+ex^2\right)^{3/2} - \frac{ad^3\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{16e^{3/2}} + b$$

Mathematica [A] time = 10.7402, size = 0, normalized size = 0.

$$\int x^2\left(d+ex^2\right)^{3/2}\left(a+b\tan^{-1}(cx)\right)dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^2*(d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]),x]
```

```
[Out] Integrate[x^2*(d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]), x]
```

Maple [A] time = 0.593, size = 0, normalized size = 0.

$$\int x^2\left(ex^2+d\right)^{\frac{3}{2}}\left(a+b\arctan(cx)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^2+d)^(3/2)*(a+b*arctan(c*x)),x)
```

```
[Out] int(x^2*(e*x^2+d)^(3/2)*(a+b*arctan(c*x)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^(3/2)*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(aex^4 + adx^2 + \left(bex^4 + bdx^2\right)\arctan\left(cx\right)\right)\sqrt{ex^2 + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^(3/2)*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] integral((a*e*x^4 + a*d*x^2 + (b*e*x^4 + b*d*x^2)*arctan(c*x))*sqrt(e*x^2 +
 d), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int x^2 \left(a + b \operatorname{atan}\left(cx\right)\right) \left(d + ex^2\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d)**(3/2)*(a+b*atan(c*x)),x)
```

```
[Out] Integral(x**2*(a + b*atan(c*x))*(d + e*x**2)**(3/2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(ex^2 + d\right)^{\frac{3}{2}} (b \arctan\left(cx\right) + a)x^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^(3/2)*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^(3/2)*(b*arctan(c*x) + a)*x^2, x)
```

3.1185 $\int x\left(d+ex^2\right)^{3/2}\left(a+b\tan^{-1}(cx)\right)\,dx$

Optimal. Leaf size=181

$$\frac{\left(d+ex^2\right)^{5/2}\left(a+b\tan^{-1}(cx)\right)}{5e}-\frac{b\left(15c^4d^2-20c^2de+8e^2\right)\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{40c^5\sqrt{e}}-\frac{bx\left(7c^2d-4e\right)\sqrt{d+ex^2}}{40c^3}-\frac{b\left(c^2d-e\right)^{5/2}\text{t}}{5c}$$

```
[Out] -(b*(7*c^2*d - 4*e)*x*Sqrt[d + e*x^2])/(40*c^3) - (b*x*(d + e*x^2)^(3/2))/(
20*c) + ((d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]))/(5*e) - (b*(c^2*d - e)^(5/2
)*ArcTan[(Sqrt[c^2*d - e]*x)/Sqrt[d + e*x^2]])/(5*c^5*e) - (b*(15*c^4*d^2 -
 20*c^2*d*e + 8*e^2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(40*c^5*Sqrt[e])
```

Rubi [A] time = 0.233115, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {4974, 416, 528, 523, 217, 206, 377, 203}

$$\frac{\left(d+ex^2\right)^{5/2}\left(a+b\tan^{-1}(cx)\right)}{5e}-\frac{b\left(15c^4d^2-20c^2de+8e^2\right)\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{40c^5\sqrt{e}}-\frac{bx\left(7c^2d-4e\right)\sqrt{d+ex^2}}{40c^3}-\frac{b\left(c^2d-e\right)^{5/2}\text{t}}{5c}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]),x]
```


```
[Out] -(b*(7*c^2*d - 4*e)*x*Sqrt[d + e*x^2])/(40*c^3) - (b*x*(d + e*x^2)^(3/2))/(
20*c) + ((d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]))/(5*e) - (b*(c^2*d - e)^(5/2
)*ArcTan[(Sqrt[c^2*d - e]*x)/Sqrt[d + e*x^2]])/(5*c^5*e) - (b*(15*c^4*d^2 -
 20*c^2*d*e + 8*e^2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(40*c^5*Sqrt[e])
```


Rule 4974

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]))/(2*e*(q + 1)), x
] - Dist[(b*c)/(2*e*(q + 1)), Int[(d + e*x^2)^(q + 1)/(1 + c^2*x^2), x], x]
 /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
```

```
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int x\left(d+ex^2\right)^{3/2}\left(a+b\tan^{-1}(cx)\right)dx = \frac{\left(d+ex^2\right)^{5/2}\left(a+b\tan^{-1}(cx)\right)}{5e} - \frac{(bc)\int\frac{\left(d+ex^2\right)^{5/2}}{1+c^2x^2}dx}{5e}$$

$$= -\frac{bx\left(d+ex^2\right)^{3/2}}{20c} + \frac{\left(d+ex^2\right)^{5/2}\left(a+b\tan^{-1}(cx)\right)}{5e} - \frac{b\int\frac{\sqrt{d+ex^2}\left(d\left(4c^2d-e\right)+\left(7c^2d-4e\right)\right.}{1+c^2x^2}}{20ce}$$

$$= -\frac{b\left(7c^2d-4e\right)x\sqrt{d+ex^2}}{40c^3} - \frac{bx\left(d+ex^2\right)^{3/2}}{20c} + \frac{\left(d+ex^2\right)^{5/2}\left(a+b\tan^{-1}(cx)\right)}{5e} -$$

$$= -\frac{b\left(7c^2d-4e\right)x\sqrt{d+ex^2}}{40c^3} - \frac{bx\left(d+ex^2\right)^{3/2}}{20c} + \frac{\left(d+ex^2\right)^{5/2}\left(a+b\tan^{-1}(cx)\right)}{5e} -$$

$$= -\frac{b\left(7c^2d-4e\right)x\sqrt{d+ex^2}}{40c^3} - \frac{bx\left(d+ex^2\right)^{3/2}}{20c} + \frac{\left(d+ex^2\right)^{5/2}\left(a+b\tan^{-1}(cx)\right)}{5e} -$$

$$= -\frac{b\left(7c^2d-4e\right)x\sqrt{d+ex^2}}{40c^3} - \frac{bx\left(d+ex^2\right)^{3/2}}{20c} + \frac{\left(d+ex^2\right)^{5/2}\left(a+b\tan^{-1}(cx)\right)}{5e} -$$

Mathematica [C] time = 0.425465, size = 313, normalized size = 1.73

$$\frac{c^2\sqrt{d+ex^2}\left(8ac^3\left(d+ex^2\right)^2+bex\left(4e-c^2\left(9d+2ex^2\right)\right)\right)-b\sqrt{e}\left(15c^4d^2-20c^2de+8e^2\right)\log\left(\sqrt{e}\sqrt{d+ex^2}+ex\right)-4ib\left(c\right.}{40}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]),x]
```

```
[Out] (c^2*Sqrt[d + e*x^2]*(8*a*c^3*(d + e*x^2)^2 + b*e*x*(4*e - c^2*(9*d + 2*e*x
^2))) + 8*b*c^5*(d + e*x^2)^(5/2)*ArcTan[c*x] - (4*I)*b*(c^2*d - e)^(5/2)*L
og[(20*c^6*e*((-I)*c*d + e*x - I*Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*(c^2*
d - e)^(7/2)*(-I + c*x))] + (4*I)*b*(c^2*d - e)^(5/2)*Log[(20*c^6*e*(I*c*d
+ e*x + I*Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*(c^2*d - e)^(7/2)*(I + c*x))
] - b*Sqrt[e]*(15*c^4*d^2 - 20*c^2*d*e + 8*e^2)*Log[e*x + Sqrt[e]*Sqrt[d +
e*x^2]])/(40*c^5*e)
```

Maple [F] time = 0.619, size = 0, normalized size = 0.

$$\int x\left(ex^2+d\right)^{\frac{3}{2}}\left(a+b\arctan\left(cx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d)^(3/2)*(a+b*arctan(c*x)),x)
```

```
[Out] int(x*(e*x^2+d)^(3/2)*(a+b*arctan(c*x)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^(3/2)*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 29.7106, size = 2603, normalized size = 14.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^(3/2)*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] [1/80*((15*b*c^4*d^2 - 20*b*c^2*d*e + 8*b*e^2)*sqrt(e)*log(-2*e*x^2 + 2*sqr
t(e*x^2 + d)*sqrt(e)*x - d) + 4*(b*c^4*d^2 - 2*b*c^2*d*e + b*e^2)*sqrt(-c^2
*d + e)*log(((c^4*d^2 - 8*c^2*d*e + 8*e^2)*x^4 - 2*(3*c^2*d^2 - 4*d*e)*x^2
- 4*((c^2*d - 2*e)*x^3 - d*x)*sqrt(-c^2*d + e)*sqrt(e*x^2 + d) + d^2)/(c^4*
x^4 + 2*c^2*x^2 + 1)) + 2*(8*a*c^5*e^2*x^4 + 16*a*c^5*d*e*x^2 - 2*b*c^4*e^2
*x^3 + 8*a*c^5*d^2 - (9*b*c^4*d*e - 4*b*c^2*e^2)*x + 8*(b*c^5*e^2*x^4 + 2*b
*c^5*d*e*x^2 + b*c^5*d^2)*arctan(c*x))*sqrt(e*x^2 + d))/(c^5*e), -1/80*(8*(
b*c^4*d^2 - 2*b*c^2*d*e + b*e^2)*sqrt(c^2*d - e)*arctan(1/2*sqrt(c^2*d - e)
*((c^2*d - 2*e)*x^2 - d)*sqrt(e*x^2 + d)/((c^2*d*e - e^2)*x^3 + (c^2*d^2 -
d*e)*x)) - (15*b*c^4*d^2 - 20*b*c^2*d*e + 8*b*e^2)*sqrt(e)*log(-2*e*x^2 + 2
```

```
*sqrt(e*x^2 + d)*sqrt(e)*x - d) - 2*(8*a*c^5*e^2*x^4 + 16*a*c^5*d*e*x^2 - 2
*b*c^4*e^2*x^3 + 8*a*c^5*d^2 - (9*b*c^4*d*e - 4*b*c^2*e^2)*x + 8*(b*c^5*e^2
*x^4 + 2*b*c^5*d*e*x^2 + b*c^5*d^2)*arctan(c*x))*sqrt(e*x^2 + d))/(c^5*e),
1/40*((15*b*c^4*d^2 - 20*b*c^2*d*e + 8*b*e^2)*sqrt(-e)*arctan(sqrt(-e)*x/sq
rt(e*x^2 + d)) + 2*(b*c^4*d^2 - 2*b*c^2*d*e + b*e^2)*sqrt(-c^2*d + e)*log((
(c^4*d^2 - 8*c^2*d*e + 8*e^2)*x^4 - 2*(3*c^2*d^2 - 4*d*e)*x^2 - 4*((c^2*d -
 2*e)*x^3 - d*x)*sqrt(-c^2*d + e)*sqrt(e*x^2 + d) + d^2)/(c^4*x^4 + 2*c^2*x
^2 + 1)) + (8*a*c^5*e^2*x^4 + 16*a*c^5*d*e*x^2 - 2*b*c^4*e^2*x^3 + 8*a*c^5*
d^2 - (9*b*c^4*d*e - 4*b*c^2*e^2)*x + 8*(b*c^5*e^2*x^4 + 2*b*c^5*d*e*x^2 +
b*c^5*d^2)*arctan(c*x))*sqrt(e*x^2 + d))/(c^5*e), -1/40*(4*(b*c^4*d^2 - 2*b
*c^2*d*e + b*e^2)*sqrt(c^2*d - e)*arctan(1/2*sqrt(c^2*d - e)*((c^2*d - 2*e)
*x^2 - d)*sqrt(e*x^2 + d)/((c^2*d*e - e^2)*x^3 + (c^2*d^2 - d*e)*x)) - (15*
b*c^4*d^2 - 20*b*c^2*d*e + 8*b*e^2)*sqrt(-e)*arctan(sqrt(-e)*x/sqrt(e*x^2 +
 d)) - (8*a*c^5*e^2*x^4 + 16*a*c^5*d*e*x^2 - 2*b*c^4*e^2*x^3 + 8*a*c^5*d^2
- (9*b*c^4*d*e - 4*b*c^2*e^2)*x + 8*(b*c^5*e^2*x^4 + 2*b*c^5*d*e*x^2 + b*c^
5*d^2)*arctan(c*x))*sqrt(e*x^2 + d))/(c^5*e)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\left(a+b\operatorname{atan}(cx)\right)\left(d+ex^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d)**(3/2)*(a+b*atan(c*x)),x)
```

```
[Out] Integral(x*(a + b*atan(c*x))*(d + e*x**2)**(3/2), x)
```

Giac [B] time = 1.50523, size = 620, normalized size = 3.43

$$\frac{1}{3}\left(x^2e+d\right)^{\frac{3}{2}}ade^{(-1)}+\frac{1}{12}\left(4\left(x^2e+d\right)^{\frac{3}{2}}\arctan(cx)\,e^{(-1)}-c\left(\frac{2\sqrt{x^2e+d}\,x}{c^2}-\frac{\left(3c^2d-2e\right)e^{\left(-\frac{1}{2}\right)}\log\left(\left(xe^{\frac{1}{2}}-\sqrt{x^2e+d}\right)^2\right)}{c^4}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^(3/2)*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] 1/3*(x^2*e + d)^(3/2)*a*d*e^(-1) + 1/12*(4*(x^2*e + d)^(3/2)*arctan(c*x)*e^
(-1) - c*(2*sqrt(x^2*e + d)*x/c^2 - (3*c^2*d - 2*e)*e^(-1/2)*log((x*e^(1/2)
 - sqrt(x^2*e + d))^2)/c^4 - 4*(c^4*d^2*e^(1/2) - 2*c^2*d*e^(3/2) + e^(5/2)
)*arctan(1/2*((x*e^(1/2) - sqrt(x^2*e + d))^2*c^2 - c^2*d + 2*e)*e^(-1/2)/s
qrt(c^2*d - e))*e^(-3/2)/(sqrt(c^2*d - e)*c^4)))*b*d + 1/240*(16*(3*(x^2*e
+ d)^(5/2) - 5*(x^2*e + d)^(3/2)*d)*a*e^(-2) + (16*(3*(x^2*e + d)^(5/2) - 5
*(x^2*e + d)^(3/2)*d)*arctan(c*x)*e^(-2) - (2*sqrt(x^2*e + d)*x*(6*x^2/c^2
+ (7*c^10*d*e^2 - 12*c^8*e^3)*e^(-3)/c^12) + (15*c^4*d^2 + 20*c^2*d*e - 24*
e^2)*e^(-3/2)*log((x*e^(1/2) - sqrt(x^2*e + d))^2)/c^6 + 16*(2*c^6*d^3*e^(1
/2) - c^4*d^2*e^(3/2) - 4*c^2*d*e^(5/2) + 3*e^(7/2))*arctan(1/2*((x*e^(1/2)
 - sqrt(x^2*e + d))^2*c^2 - c^2*d + 2*e)*e^(-1/2)/sqrt(c^2*d - e))*e^(-5/2)
/(sqrt(c^2*d - e)*c^6))*c)*b)*e
```

3.1186 $\int \left(d+ex^2\right)^{3/2} \left(a+b\tan^{-1}(cx)\right)\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\left(d+ex^2\right)^{3/2} \left(a+b\tan^{-1}(cx)\right),x\right)$$

```
[Out] Unintegrable[(d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]), x]
```

Rubi [A] time = 0.0283884, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \left(d+ex^2\right)^{3/2} \left(a+b\tan^{-1}(cx)\right)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]),x]
```

```
[Out] Defer[Int][(d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]), x]
```

Rubi steps

$$\int \left(d+ex^2\right)^{3/2} \left(a+b\tan^{-1}(cx)\right)\,dx = \int \left(d+ex^2\right)^{3/2} \left(a+b\tan^{-1}(cx)\right)\,dx$$

Mathematica [A] time = 4.91983, size = 0, normalized size = 0.

$$\int \left(d+ex^2\right)^{3/2} \left(a+b\tan^{-1}(cx)\right)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]),x]
```

```
[Out] Integrate[(d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]), x]
```

Maple [A] time = 1.197, size = 0, normalized size = 0.

$$\int \left(e x^{2}+d \right)^{\frac{3}{2}} \left(a +b \arctan \left(c x \right)\right)d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(3/2)*(a+b*arctan(c*x)),x)
```

```
[Out] int((e*x^2+d)^(3/2)*(a+b*arctan(c*x)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a e x^{2} + a d + \left(b e x^{2} + b d\right) \arctan\left(c x\right)\right) \sqrt{e x^{2} + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arctan(c*x))*sqrt(e*x^2 + d), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b \operatorname{atan}{\left(c x \right)}\right) \left(d + e x^{2}\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(3/2)*(a+b*atan(c*x)),x)
```

```
[Out] Integral((a + b*atan(c*x))*(d + e*x**2)**(3/2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(ex^2 + d\right)^{\frac{3}{2}} (b \arctan(cx) + a) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^(3/2)*(b*arctan(c*x) + a), x)
```

3.1187 $\int \frac{(d+ex^2)^{3/2}(a+b\tan^{-1}(cx))}{x}\,dx$

Optimal. Leaf size=80

$$b\text{Unintegrable}\left(\frac{\tan^{-1}(cx)\left(d+ex^2\right)^{3/2}}{x},x\right)-ad^{3/2}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)+ad\sqrt{d+ex^2}+\frac{1}{3}a\left(d+ex^2\right)^{3/2}$$

```
[Out] a*d*Sqrt[d + e*x^2] + (a*(d + e*x^2)^(3/2))/3 - a*d^(3/2)*ArcTanh[Sqrt[d +
e*x^2]/Sqrt[d]] + b*Unintegrable[((d + e*x^2)^(3/2)*ArcTan[c*x])/x, x]
```

Rubi [A] time = 0.192781, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\tan^{-1}(cx)\right)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]))/x,x]
```

```
[Out] a*d*Sqrt[d + e*x^2] + (a*(d + e*x^2)^(3/2))/3 - a*d^(3/2)*ArcTanh[Sqrt[d +
e*x^2]/Sqrt[d]] + b*Defer[Int][((d + e*x^2)^(3/2)*ArcTan[c*x])/x, x]
```

Rubi steps

$$\int \frac{(d+ex^2)^{3/2}(a+b\tan^{-1}(cx))}{x}\,dx = a\int \frac{(d+ex^2)^{3/2}}{x}\,dx + b\int \frac{(d+ex^2)^{3/2}\tan^{-1}(cx)}{x}\,dx$$

$$= \frac{1}{2}a\,\text{Subst}\left(\int \frac{(d+ex)^{3/2}}{x}\,dx, x, x^2\right) + b\int \frac{(d+ex^2)^{3/2}\tan^{-1}(cx)}{x}\,dx$$

$$= \frac{1}{3}a\left(d+ex^2\right)^{3/2} + b\int \frac{(d+ex^2)^{3/2}\tan^{-1}(cx)}{x}\,dx + \frac{1}{2}(ad)\,\text{Subst}\left(\int \frac{\sqrt{d+ex}}{x}\,dx,\right.$$

$$= ad\sqrt{d+ex^2} + \frac{1}{3}a\left(d+ex^2\right)^{3/2} + b\int \frac{(d+ex^2)^{3/2}\tan^{-1}(cx)}{x}\,dx + \frac{1}{2}\left(ad^2\right)\text{Subst}\left(\right.$$

$$= ad\sqrt{d+ex^2} + \frac{1}{3}a\left(d+ex^2\right)^{3/2} + b\int \frac{(d+ex^2)^{3/2}\tan^{-1}(cx)}{x}\,dx + \frac{\left(ad^2\right)\text{Subst}\left(\int\right.}{}$$

$$= ad\sqrt{d+ex^2} + \frac{1}{3}a\left(d+ex^2\right)^{3/2} - ad^{3/2}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right) + b\int \frac{(d+ex^2)^{3/2}\tan}{x}$$

Mathematica [A] time = 71.7397, size = 0, normalized size = 0.

$$\int \frac{(d+ex^2)^{3/2}(a+b\tan^{-1}(cx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]))/x,x]
```

```
[Out] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]))/x, x]
```

Maple [A] time = 0.568, size = 0, normalized size = 0.

$$\int \frac{a+b\arctan(cx)}{x}\left(ex^2+d\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x,x)
```

```
[Out] int((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(aex^2 + ad + \left(bex^2 + bd\right)\arctan\left(cx\right)\right)\sqrt{ex^2 + d}}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x,x, algorithm="fricas")
```

```
[Out] integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arctan(c*x))*sqrt(e*x^2 + d)/x, x
)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b\operatorname{atan}(cx))\left(d + ex^2\right)^{\frac{3}{2}}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(3/2)*(a+b*atan(c*x))/x,x)
```

```
[Out] Integral((a + b*atan(c*x))*(d + e*x**2)**(3/2)/x, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2+d\right)^{\frac{3}{2}}(b\arctan\left(cx\right)+a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^(3/2)*(b*arctan(c*x) + a)/x, x)
```

3.1188 $\int \frac{(d+ex^2)^{3/2}(a+b\tan^{-1}(cx))}{x^2}\,dx$

Optimal. Leaf size=89

$$b\text{Unintegrable}\left(\frac{\tan^{-1}(cx)\left(d+ex^2\right)^{3/2}}{x^2},x\right)-\frac{a\left(d+ex^2\right)^{3/2}}{x}+\frac{3}{2}aex\sqrt{d+ex^2}+\frac{3}{2}ad\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)$$

```
[Out] (3*a*e*x*Sqrt[d + e*x^2])/2 - (a*(d + e*x^2)^(3/2))/x + (3*a*d*Sqrt[e]*ArcT
anh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/2 + b*Unintegrable[((d + e*x^2)^(3/2)*Arc
Tan[c*x])/x^2, x]
```

Rubi [A] time = 0.174019, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\tan^{-1}(cx)\right)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]))/x^2,x]
```

```
[Out] (3*a*e*x*Sqrt[d + e*x^2])/2 - (a*(d + e*x^2)^(3/2))/x + (3*a*d*Sqrt[e]*ArcT
anh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/2 + b*Defer[Int][((d + e*x^2)^(3/2)*ArcTa
n[c*x])/x^2, x]
```

Rubi steps

$$\int \frac{(d+ex^2)^{3/2}(a+b\tan^{-1}(cx))}{x^2}\,dx = a\int \frac{(d+ex^2)^{3/2}}{x^2}\,dx + b\int \frac{(d+ex^2)^{3/2}\tan^{-1}(cx)}{x^2}\,dx$$
$$= -\frac{a(d+ex^2)^{3/2}}{x} + b\int \frac{(d+ex^2)^{3/2}\tan^{-1}(cx)}{x^2}\,dx + (3ae)\int \sqrt{d+ex^2}\,dx$$
$$= \frac{3}{2}aex\sqrt{d+ex^2} - \frac{a(d+ex^2)^{3/2}}{x} + b\int \frac{(d+ex^2)^{3/2}\tan^{-1}(cx)}{x^2}\,dx + \frac{1}{2}(3ade)\int \frac{}{\sqrt{d}}$$
$$= \frac{3}{2}aex\sqrt{d+ex^2} - \frac{a(d+ex^2)^{3/2}}{x} + b\int \frac{(d+ex^2)^{3/2}\tan^{-1}(cx)}{x^2}\,dx + \frac{1}{2}(3ade)\,\text{Subst}$$
$$= \frac{3}{2}aex\sqrt{d+ex^2} - \frac{a(d+ex^2)^{3/2}}{x} + \frac{3}{2}ad\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) + b\int \frac{(d+ex^2)^{3/2}}{x^2}$$

Mathematica [A] time = 8.68745, size = 0, normalized size = 0.

$$\int \frac{(d+ex^2)^{3/2}(a+b\tan^{-1}(cx))}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]))/x^2,x]
```

```
[Out] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]))/x^2, x]
```

Maple [A] time = 0.574, size = 0, normalized size = 0.

$$\int \frac{a+b\arctan(cx)}{x^2}\left(ex^2+d\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x^2,x)
```

```
[Out] int((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(aex^2+ad+\left(bex^2+bd\right)\arctan\left(cx\right)\right)\sqrt{ex^2+d}}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x^2,x, algorithm="fricas")
```

```
[Out] integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arctan(c*x))*sqrt(e*x^2 + d)/x^2,
 x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a+b\,\mathrm{atan}\left(cx\right)\right)\left(d+ex^2\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(3/2)*(a+b*atan(c*x))/x**2,x)
```

```
[Out] Integral((a + b*atan(c*x))*(d + e*x**2)**(3/2)/x**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(ex^2+d\right)^{\frac{3}{2}}\left(b\arctan\left(cx\right)+a\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x^2,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^(3/2)*(b*arctan(c*x) + a)/x^2, x)
```

3.1189 $\int \frac{(d+ex^2)^{3/2}(a+b\tan^{-1}(cx))}{x^3}\,dx$

Optimal. Leaf size=89

$$b\text{Unintegrable}\left(\frac{\tan^{-1}(cx)\left(d+ex^2\right)^{3/2}}{x^3},x\right)-\frac{a\left(d+ex^2\right)^{3/2}}{2x^2}+\frac{3}{2}ae\sqrt{d+ex^2}-\frac{3}{2}a\sqrt{d}e\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)$$

```
[Out] (3*a*e*Sqrt[d + e*x^2])/2 - (a*(d + e*x^2)^(3/2))/(2*x^2) - (3*a*Sqrt[d]*e*
ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/2 + b*Unintegrable[((d + e*x^2)^(3/2)*Arc
Tan[c*x])/x^3, x]
```

Rubi [A] time = 0.204055, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\tan^{-1}(cx)\right)}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]))/x^3,x]
```

```
[Out] (3*a*e*Sqrt[d + e*x^2])/2 - (a*(d + e*x^2)^(3/2))/(2*x^2) - (3*a*Sqrt[d]*e*
ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/2 + b*Defer[Int][((d + e*x^2)^(3/2)*ArcTa
n[c*x])/x^3, x]
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\tan^{-1}(cx)\right)}{x^3}\,dx = a\int \frac{\left(d+ex^2\right)^{3/2}}{x^3}\,dx + b\int \frac{\left(d+ex^2\right)^{3/2}\tan^{-1}(cx)}{x^3}\,dx$$

$$= \frac{1}{2}a\,\text{Subst}\left(\int \frac{(d+ex)^{3/2}}{x^2}\,dx, x, x^2\right) + b\int \frac{\left(d+ex^2\right)^{3/2}\tan^{-1}(cx)}{x^3}\,dx$$

$$= -\frac{a\left(d+ex^2\right)^{3/2}}{2x^2} + b\int \frac{\left(d+ex^2\right)^{3/2}\tan^{-1}(cx)}{x^3}\,dx + \frac{1}{4}(3ae)\,\text{Subst}\left(\int \frac{\sqrt{d+ex}}{x}\,dx\right.$$

$$= \frac{3}{2}ae\sqrt{d+ex^2} - \frac{a\left(d+ex^2\right)^{3/2}}{2x^2} + b\int \frac{\left(d+ex^2\right)^{3/2}\tan^{-1}(cx)}{x^3}\,dx + \frac{1}{4}(3ade)\,\text{Subst}$$

$$= \frac{3}{2}ae\sqrt{d+ex^2} - \frac{a\left(d+ex^2\right)^{3/2}}{2x^2} + b\int \frac{\left(d+ex^2\right)^{3/2}\tan^{-1}(cx)}{x^3}\,dx + \frac{1}{2}(3ad)\,\text{Subst}\Bigg($$

$$= \frac{3}{2}ae\sqrt{d+ex^2} - \frac{a\left(d+ex^2\right)^{3/2}}{2x^2} - \frac{3}{2}a\sqrt{d}e\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right) + b\int \frac{\left(d+ex^2\right)^{3/2}\text{ta}}{x^3}$$

Mathematica [A] time = 47.9191, size = 0, normalized size = 0.

$$\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\tan^{-1}(cx)\right)}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]))/x^3,x]
```

```
[Out] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]))/x^3, x]
```

Maple [A] time = 0.579, size = 0, normalized size = 0.

$$\int \frac{a+b\arctan(cx)}{x^3}\left(ex^2+d\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x^3,x)
```

```
[Out] int((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x^3,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(aex^2 + ad + \left(bex^2 + bd\right)\arctan\left(cx\right)\right)\sqrt{ex^2 + d}}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x^3,x, algorithm="fricas")
```

```
[Out] integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arctan(c*x))*sqrt(e*x^2 + d)/x^3,
 x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b\,\text{atan}\,(cx))\left(d + ex^2\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(3/2)*(a+b*atan(c*x))/x**3,x)
```

```
[Out] Integral((a + b*atan(c*x))*(d + e*x**2)**(3/2)/x**3, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2+d\right)^{\frac{3}{2}}(b\arctan(cx)+a)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x^3,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^(3/2)*(b*arctan(c*x) + a)/x^3, x)
```

3.1190 $\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\tan^{-1}(cx)\right)}{x^4}\,dx$

Optimal. Leaf size=87

$$b\text{Unintegrable}\left(\frac{\tan^{-1}(cx)\left(d+ex^2\right)^{3/2}}{x^4},x\right)+ae^{3/2}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{ae\sqrt{d+ex^2}}{x}-\frac{a\left(d+ex^2\right)^{3/2}}{3x^3}$$

```
[Out] -((a*e*Sqrt[d + e*x^2])/x) - (a*(d + e*x^2)^(3/2))/(3*x^3) + a*e^(3/2)*ArcT
anh[(Sqrt[e]*x)/Sqrt[d + e*x^2]] + b*Unintegrable[((d + e*x^2)^(3/2)*ArcTan
[c*x])/x^4, x]
```

Rubi [A] time = 0.174858, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\tan^{-1}(cx)\right)}{x^4}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]))/x^4,x]
```

```
[Out] -((a*e*Sqrt[d + e*x^2])/x) - (a*(d + e*x^2)^(3/2))/(3*x^3) + a*e^(3/2)*ArcT
anh[(Sqrt[e]*x)/Sqrt[d + e*x^2]] + b*Defer[Int][((d + e*x^2)^(3/2)*ArcTan[c
*x])/x^4, x]
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\tan^{-1}(cx)\right)}{x^4}\,dx = a\int \frac{\left(d+ex^2\right)^{3/2}}{x^4}\,dx + b\int \frac{\left(d+ex^2\right)^{3/2}\tan^{-1}(cx)}{x^4}\,dx$$

$$= -\frac{a\left(d+ex^2\right)^{3/2}}{3x^3} + b\int \frac{\left(d+ex^2\right)^{3/2}\tan^{-1}(cx)}{x^4}\,dx + (ae)\int \frac{\sqrt{d+ex^2}}{x^2}\,dx$$

$$= -\frac{ae\sqrt{d+ex^2}}{x} - \frac{a\left(d+ex^2\right)^{3/2}}{3x^3} + b\int \frac{\left(d+ex^2\right)^{3/2}\tan^{-1}(cx)}{x^4}\,dx + \left(ae^2\right)\int \frac{1}{\sqrt{d+e}}$$

$$= -\frac{ae\sqrt{d+ex^2}}{x} - \frac{a\left(d+ex^2\right)^{3/2}}{3x^3} + b\int \frac{\left(d+ex^2\right)^{3/2}\tan^{-1}(cx)}{x^4}\,dx + \left(ae^2\right)\text{Subst}\Big(\int$$

$$= -\frac{ae\sqrt{d+ex^2}}{x} - \frac{a\left(d+ex^2\right)^{3/2}}{3x^3} + ae^{3/2}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) + b\int \frac{\left(d+ex^2\right)^{3/2}\tan}{x^4}$$

Mathematica [A] time = 31.1651, size = 0, normalized size = 0.

$$\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\tan^{-1}(cx)\right)}{x^4}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]))/x^4,x]
```

```
[Out] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]))/x^4, x]
```

Maple [A] time = 0.586, size = 0, normalized size = 0.

$$\int \frac{a+b\arctan(cx)}{x^4}\left(ex^2+d\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x^4,x)
```

```
[Out] int((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x^4,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(aex^2+ad+\left(bex^2+bd\right)\arctan\left(cx\right)\right)\sqrt{ex^2+d}}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arctan(c*x))*sqrt(e*x^2 + d)/x^4,
 x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a+b\,\mathrm{atan}\left(cx\right)\right)\left(d+ex^2\right)^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(3/2)*(a+b*atan(c*x))/x**4,x)
```

```
[Out] Integral((a + b*atan(c*x))*(d + e*x**2)**(3/2)/x**4, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(ex^2+d\right)^{\frac{3}{2}}\left(b\arctan\left(cx\right)+a\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x^4,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^(3/2)*(b*arctan(c*x) + a)/x^4, x)
```

3.1191 $\int \frac{(d+ex^2)^{3/2}(a+b\tan^{-1}(cx))}{x^5}\,dx$

Optimal. Leaf size=94

$$b\text{Unintegrable}\left(\frac{\tan^{-1}(cx)\left(d+ex^2\right)^{3/2}}{x^5},x\right)-\frac{3ae^2\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{8\sqrt{d}}-\frac{3ae\sqrt{d+ex^2}}{8x^2}-\frac{a\left(d+ex^2\right)^{3/2}}{4x^4}$$

```
[Out] (-3*a*e*Sqrt[d + e*x^2])/(8*x^2) - (a*(d + e*x^2)^(3/2))/(4*x^4) - (3*a*e^2
*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/(8*Sqrt[d]) + b*Unintegrable[((d + e*x^2
)^(3/2)*ArcTan[c*x])/x^5, x]
```

Rubi [A] time = 0.196105, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\tan^{-1}(cx)\right)}{x^5}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]))/x^5,x]
```

```
[Out] (-3*a*e*Sqrt[d + e*x^2])/(8*x^2) - (a*(d + e*x^2)^(3/2))/(4*x^4) - (3*a*e^2
*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/(8*Sqrt[d]) + b*Defer[Int][((d + e*x^2)^
(3/2)*ArcTan[c*x])/x^5, x]
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\tan^{-1}(cx)\right)}{x^5}\,dx = a\int \frac{\left(d+ex^2\right)^{3/2}}{x^5}\,dx + b\int \frac{\left(d+ex^2\right)^{3/2}\tan^{-1}(cx)}{x^5}\,dx$$

$$= \frac{1}{2}a\,\text{Subst}\left(\int \frac{(d+ex)^{3/2}}{x^3}\,dx, x, x^2\right) + b\int \frac{\left(d+ex^2\right)^{3/2}\tan^{-1}(cx)}{x^5}\,dx$$

$$= -\frac{a\left(d+ex^2\right)^{3/2}}{4x^4} + b\int \frac{\left(d+ex^2\right)^{3/2}\tan^{-1}(cx)}{x^5}\,dx + \frac{1}{8}(3ae)\,\text{Subst}\left(\int \frac{\sqrt{d+ex}}{x^2}\,dx\right.$$

$$= -\frac{3ae\sqrt{d+ex^2}}{8x^2} - \frac{a\left(d+ex^2\right)^{3/2}}{4x^4} + b\int \frac{\left(d+ex^2\right)^{3/2}\tan^{-1}(cx)}{x^5}\,dx + \frac{1}{16}\left(3ae^2\right)\text{Sub}$$

$$= -\frac{3ae\sqrt{d+ex^2}}{8x^2} - \frac{a\left(d+ex^2\right)^{3/2}}{4x^4} + b\int \frac{\left(d+ex^2\right)^{3/2}\tan^{-1}(cx)}{x^5}\,dx + \frac{1}{8}(3ae)\,\text{Subst}$$

$$= -\frac{3ae\sqrt{d+ex^2}}{8x^2} - \frac{a\left(d+ex^2\right)^{3/2}}{4x^4} - \frac{3ae^2\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{8\sqrt{d}} + b\int \frac{\left(d+ex^2\right)^{3/2}\tan^{-1}}{x^5}$$

Mathematica [A] time = 49.6443, size = 0, normalized size = 0.

$$\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\tan^{-1}(cx)\right)}{x^5}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]))/x^5,x]
```

```
[Out] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]))/x^5, x]
```

Maple [A] time = 0.594, size = 0, normalized size = 0.

$$\int \frac{a+b\arctan(cx)}{x^5}\left(ex^2+d\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x^5,x)
```

```
[Out] int((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x^5,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(aex^2 + ad + \left(bex^2 + bd\right)\arctan\left(cx\right)\right)\sqrt{ex^2 + d}}{x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x^5,x, algorithm="fricas")
```

```
[Out] integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arctan(c*x))*sqrt(e*x^2 + d)/x^5,
 x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b\,\mathrm{atan}\,(cx))\left(d + ex^2\right)^{\frac{3}{2}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(3/2)*(a+b*atan(c*x))/x**5,x)
```

```
[Out] Integral((a + b*atan(c*x))*(d + e*x**2)**(3/2)/x**5, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2 + d\right)^{\frac{3}{2}} (b \arctan(cx) + a)}{x^5} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x^5,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^(3/2)*(b*arctan(c*x) + a)/x^5, x)
```

3.1192 $\int \frac{(d+ex^2)^{3/2}(a+b\tan^{-1}(cx))}{x^6}\,dx$

Optimal. Leaf size=178

$$-\frac{(d+ex^2)^{5/2}(a+b\tan^{-1}(cx))}{5dx^5}-\frac{bc(8c^4d^2-20c^2de+15e^2)\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{40\sqrt{d}}+\frac{bc(4c^2d-7e)\sqrt{d+ex^2}}{40x^2}+\frac{b(c^2d-e)^{5/2}}{}$$

```
[Out] (b*c*(4*c^2*d - 7*e)*Sqrt[d + e*x^2])/(40*x^2) - (b*c*(d + e*x^2)^(3/2))/(2
0*x^4) - ((d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]))/(5*d*x^5) - (b*c*(8*c^4*d^
2 - 20*c^2*d*e + 15*e^2)*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/(40*Sqrt[d]) + (
b*(c^2*d - e)^(5/2)*ArcTanh[(c*Sqrt[d + e*x^2])/Sqrt[c^2*d - e]])/(5*d)
```

Rubi [A] time = 0.3246, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {264, 4976, 12, 446, 98, 149, 156, 63, 208}

$$-\frac{(d+ex^2)^{5/2}(a+b\tan^{-1}(cx))}{5dx^5}-\frac{bc(8c^4d^2-20c^2de+15e^2)\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{40\sqrt{d}}+\frac{bc(4c^2d-7e)\sqrt{d+ex^2}}{40x^2}+\frac{b(c^2d-e)^{5/2}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]))/x^6,x]
```


```
[Out] (b*c*(4*c^2*d - 7*e)*Sqrt[d + e*x^2])/(40*x^2) - (b*c*(d + e*x^2)^(3/2))/(2
0*x^4) - ((d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]))/(5*d*x^5) - (b*c*(8*c^4*d^
2 - 20*c^2*d*e + 15*e^2)*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/(40*Sqrt[d]) + (
b*(c^2*d - e)^(5/2)*ArcTanh[(c*Sqrt[d + e*x^2])/Sqrt[c^2*d - e]])/(5*d)
```


Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 4976

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Dis
```

```
t[a + b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] && !(
ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(IL
tQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] && !ILtQ[(m
 - 1)/2, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 149

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegerQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
```

```
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\tan^{-1}(cx)\right)}{x^6}\,dx = -\frac{\left(d+ex^2\right)^{5/2}\left(a+b\tan^{-1}(cx)\right)}{5dx^5}-(bc)\int \frac{\left(d+ex^2\right)^{5/2}}{5x^5\left(-d-c^2dx^2\right)}\,dx$$

$$= -\frac{\left(d+ex^2\right)^{5/2}\left(a+b\tan^{-1}(cx)\right)}{5dx^5}-\frac{1}{5}(bc)\int \frac{\left(d+ex^2\right)^{5/2}}{x^5\left(-d-c^2dx^2\right)}\,dx$$

$$= -\frac{\left(d+ex^2\right)^{5/2}\left(a+b\tan^{-1}(cx)\right)}{5dx^5}-\frac{1}{10}(bc)\,\mathrm{Subst}\left(\int \frac{(d+ex)^{5/2}}{x^3\left(-d-c^2dx\right)}\,dx,x,x^2\right)$$

$$= -\frac{bc\left(d+ex^2\right)^{3/2}}{20x^4}-\frac{\left(d+ex^2\right)^{5/2}\left(a+b\tan^{-1}(cx)\right)}{5dx^5}-\frac{(bc)\,\mathrm{Subst}\left(\int \frac{\sqrt{d+ex}\left(-\frac{1}{2}d^2\left(4c^2d-7\right.\right.}{x^2\left(-d-\right.}\right.}{20d}$$

$$= \frac{bc\left(4c^2d-7e\right)\sqrt{d+ex^2}}{40x^2}-\frac{bc\left(d+ex^2\right)^{3/2}}{20x^4}-\frac{\left(d+ex^2\right)^{5/2}\left(a+b\tan^{-1}(cx)\right)}{5dx^5}+\frac{(bc)\,\mathrm{S}}{}$$

$$= \frac{bc\left(4c^2d-7e\right)\sqrt{d+ex^2}}{40x^2}-\frac{bc\left(d+ex^2\right)^{3/2}}{20x^4}-\frac{\left(d+ex^2\right)^{5/2}\left(a+b\tan^{-1}(cx)\right)}{5dx^5}+\frac{1}{10}\left(b\right.$$

$$= \frac{bc\left(4c^2d-7e\right)\sqrt{d+ex^2}}{40x^2}-\frac{bc\left(d+ex^2\right)^{3/2}}{20x^4}-\frac{\left(d+ex^2\right)^{5/2}\left(a+b\tan^{-1}(cx)\right)}{5dx^5}+\frac{\left(bc\left(\right.\right.}{}$$

$$= \frac{bc\left(4c^2d-7e\right)\sqrt{d+ex^2}}{40x^2}-\frac{bc\left(d+ex^2\right)^{3/2}}{20x^4}-\frac{\left(d+ex^2\right)^{5/2}\left(a+b\tan^{-1}(cx)\right)}{5dx^5}-\frac{bc\left(8\right.}{}$$

Mathematica [C] time = 0.463967, size = 334, normalized size = 1.88

$$-\sqrt{d+ex^2}\left(8a\left(d+ex^2\right)^2+bcdx\left(d\left(2-4c^2x^2\right)+9ex^2\right)\right)+bc\sqrt{d}x^5\log(x)\left(8c^4d^2-20c^2de+15e^2\right)-bc\sqrt{d}x^5\left(8c^4d^2-2\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]))/x^6,x]
```

```
[Out] (-(Sqrt[d + e*x^2]*(8*a*(d + e*x^2)^2 + b*c*d*x*(9*e*x^2 + d*(2 - 4*c^2*x^2
)))) - 8*b*(d + e*x^2)^(5/2)*ArcTan[c*x] + b*c*Sqrt[d]*(8*c^4*d^2 - 20*c^2*
d*e + 15*e^2)*x^5*Log[x] - b*c*Sqrt[d]*(8*c^4*d^2 - 20*c^2*d*e + 15*e^2)*x^
5*Log[d + Sqrt[d]*Sqrt[d + e*x^2]] + 4*b*(c^2*d - e)^(5/2)*x^5*Log[(-20*c*d
*(c*d - I*e*x + Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*(c^2*d - e)^(7/2)*(I +
 c*x))] + 4*b*(c^2*d - e)^(5/2)*x^5*Log[(-20*c*d*(c*d + I*e*x + Sqrt[c^2*d
- e]*Sqrt[d + e*x^2]))/(b*(c^2*d - e)^(7/2)*(-I + c*x))])/(40*d*x^5)
```

Maple [F] time = 0.682, size = 0, normalized size = 0.

$$\int \frac{a+b\arctan\left(cx\right)}{x^6}\left(ex^2+d\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x^6,x)
```

```
[Out] int((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x^6,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 6.77062, size = 2538, normalized size = 14.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x^6,x, algorithm="fricas")
```

```
[Out] [1/80*(4*(b*c^4*d^2 - 2*b*c^2*d*e + b*e^2)*sqrt(c^2*d - e)*x^5*log((c^4*e^2
*x^4 + 8*c^4*d^2 - 8*c^2*d*e + 2*(4*c^4*d*e - 3*c^2*e^2)*x^2 + 4*(c^3*e*x^2
 + 2*c^3*d - c*e)*sqrt(c^2*d - e)*sqrt(e*x^2 + d) + e^2)/(c^4*x^4 + 2*c^2*x
^2 + 1)) + (8*b*c^5*d^2 - 20*b*c^3*d*e + 15*b*c*e^2)*sqrt(d)*x^5*log(-(e*x^
2 - 2*sqrt(e*x^2 + d)*sqrt(d) + 2*d)/x^2) - 2*(8*a*e^2*x^4 + 2*b*c*d^2*x +
16*a*d*e*x^2 - (4*b*c^3*d^2 - 9*b*c*d*e)*x^3 + 8*a*d^2 + 8*(b*e^2*x^4 + 2*b
*d*e*x^2 + b*d^2)*arctan(c*x))*sqrt(e*x^2 + d))/(d*x^5), 1/80*(8*(b*c^4*d^2
 - 2*b*c^2*d*e + b*e^2)*sqrt(-c^2*d + e)*x^5*arctan(-1/2*(c^2*e*x^2 + 2*c^2
*d - e)*sqrt(-c^2*d + e)*sqrt(e*x^2 + d)/(c^3*d^2 - c*d*e + (c^3*d*e - c*e^
2)*x^2)) + (8*b*c^5*d^2 - 20*b*c^3*d*e + 15*b*c*e^2)*sqrt(d)*x^5*log(-(e*x^
2 - 2*sqrt(e*x^2 + d)*sqrt(d) + 2*d)/x^2) - 2*(8*a*e^2*x^4 + 2*b*c*d^2*x +
16*a*d*e*x^2 - (4*b*c^3*d^2 - 9*b*c*d*e)*x^3 + 8*a*d^2 + 8*(b*e^2*x^4 + 2*b
*d*e*x^2 + b*d^2)*arctan(c*x))*sqrt(e*x^2 + d))/(d*x^5), 1/40*((8*b*c^5*d^2
 - 20*b*c^3*d*e + 15*b*c*e^2)*sqrt(-d)*x^5*arctan(sqrt(-d)/sqrt(e*x^2 + d))
 + 2*(b*c^4*d^2 - 2*b*c^2*d*e + b*e^2)*sqrt(c^2*d - e)*x^5*log((c^4*e^2*x^4
 + 8*c^4*d^2 - 8*c^2*d*e + 2*(4*c^4*d*e - 3*c^2*e^2)*x^2 + 4*(c^3*e*x^2 + 2
*c^3*d - c*e)*sqrt(c^2*d - e)*sqrt(e*x^2 + d) + e^2)/(c^4*x^4 + 2*c^2*x^2 +
 1)) - (8*a*e^2*x^4 + 2*b*c*d^2*x + 16*a*d*e*x^2 - (4*b*c^3*d^2 - 9*b*c*d*e
)*x^3 + 8*a*d^2 + 8*(b*e^2*x^4 + 2*b*d*e*x^2 + b*d^2)*arctan(c*x))*sqrt(e*x
^2 + d))/(d*x^5), 1/40*(4*(b*c^4*d^2 - 2*b*c^2*d*e + b*e^2)*sqrt(-c^2*d + e
)*x^5*arctan(-1/2*(c^2*e*x^2 + 2*c^2*d - e)*sqrt(-c^2*d + e)*sqrt(e*x^2 + d
)/(c^3*d^2 - c*d*e + (c^3*d*e - c*e^2)*x^2)) + (8*b*c^5*d^2 - 20*b*c^3*d*e
+ 15*b*c*e^2)*sqrt(-d)*x^5*arctan(sqrt(-d)/sqrt(e*x^2 + d)) - (8*a*e^2*x^4
+ 2*b*c*d^2*x + 16*a*d*e*x^2 - (4*b*c^3*d^2 - 9*b*c*d*e)*x^3 + 8*a*d^2 + 8*
(b*e^2*x^4 + 2*b*d*e*x^2 + b*d^2)*arctan(c*x))*sqrt(e*x^2 + d))/(d*x^5)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(3/2)*(a+b*atan(c*x))/x**6,x)
```

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2+d\right)^{\frac{3}{2}}\left(b\arctan\left(cx\right)+a\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x^6,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^(3/2)*(b*arctan(c*x) + a)/x^6, x)
```

3.1193 $\int x^3 \left(d+ex^2\right)^{5/2} \left(a+b\tan^{-1}(cx)\right) dx$

Optimal. Leaf size=345

$$\frac{\left(d+ex^2\right)^{9/2}\left(a+b\tan^{-1}(cx)\right)}{9e^2}-\frac{d\left(d+ex^2\right)^{7/2}\left(a+b\tan^{-1}(cx)\right)}{7e^2}-\frac{bx\left(69c^4d^2-520c^2de+336e^2\right)\left(d+ex^2\right)^{3/2}}{12096c^5e}+\frac{bx(712\ldots}{}$$

```
[Out] (b*(59*c^6*d^3 + 712*c^4*d^2*e - 1104*c^2*d*e^2 + 448*e^3)*x*Sqrt[d + e*x^2
])/(8064*c^7*e) - (b*(69*c^4*d^2 - 520*c^2*d*e + 336*e^2)*x*(d + e*x^2)^(3/
2))/(12096*c^5*e) - (b*(33*c^2*d - 56*e)*x*(d + e*x^2)^(5/2))/(3024*c^3*e)
- (b*x*(d + e*x^2)^(7/2))/(72*c*e) - (d*(d + e*x^2)^(7/2)*(a + b*ArcTan[c*x
]))/(7*e^2) + ((d + e*x^2)^(9/2)*(a + b*ArcTan[c*x]))/(9*e^2) + (b*(c^2*d -
 e)^(7/2)*(2*c^2*d + 7*e)*ArcTan[(Sqrt[c^2*d - e]*x)/Sqrt[d + e*x^2]])/(63*
c^9*e^2) + (b*(315*c^8*d^4 + 840*c^6*d^3*e - 3024*c^4*d^2*e^2 + 2880*c^2*d*
e^3 - 896*e^4)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(8064*c^9*e^(3/2))
```

Rubi [A] time = 0.58331, antiderivative size = 345, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {266, 43, 4976, 12, 528, 523, 217, 206, 377, 203}

$$\frac{\left(d+ex^2\right)^{9/2}\left(a+b\tan^{-1}(cx)\right)}{9e^2}-\frac{d\left(d+ex^2\right)^{7/2}\left(a+b\tan^{-1}(cx)\right)}{7e^2}-\frac{bx\left(69c^4d^2-520c^2de+336e^2\right)\left(d+ex^2\right)^{3/2}}{12096c^5e}+\frac{bx(712\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]),x]
```

```
[Out] (b*(59*c^6*d^3 + 712*c^4*d^2*e - 1104*c^2*d*e^2 + 448*e^3)*x*Sqrt[d + e*x^2
])/(8064*c^7*e) - (b*(69*c^4*d^2 - 520*c^2*d*e + 336*e^2)*x*(d + e*x^2)^(3/
2))/(12096*c^5*e) - (b*(33*c^2*d - 56*e)*x*(d + e*x^2)^(5/2))/(3024*c^3*e)
- (b*x*(d + e*x^2)^(7/2))/(72*c*e) - (d*(d + e*x^2)^(7/2)*(a + b*ArcTan[c*x
]))/(7*e^2) + ((d + e*x^2)^(9/2)*(a + b*ArcTan[c*x]))/(9*e^2) + (b*(c^2*d -
 e)^(7/2)*(2*c^2*d + 7*e)*ArcTan[(Sqrt[c^2*d - e]*x)/Sqrt[d + e*x^2]])/(63*
c^9*e^2) + (b*(315*c^8*d^4 + 840*c^6*d^3*e - 3024*c^4*d^2*e^2 + 2880*c^2*d*
e^3 - 896*e^4)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(8064*c^9*e^(3/2))
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 4976

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Dis
t[a + b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&  !(
ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(IL
tQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILtQ[(m
 - 1)/2, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int x^3\left(d+ex^2\right)^{5/2}\left(a+b\tan^{-1}(cx)\right)dx=-\frac{d\left(d+ex^2\right)^{7/2}\left(a+b\tan^{-1}(cx)\right)}{7e^2}+\frac{\left(d+ex^2\right)^{9/2}\left(a+b\tan^{-1}(cx)\right)}{9e^2}-(bc)\int\frac{(d$$

$$=-\frac{d\left(d+ex^2\right)^{7/2}\left(a+b\tan^{-1}(cx)\right)}{7e^2}+\frac{\left(d+ex^2\right)^{9/2}\left(a+b\tan^{-1}(cx)\right)}{9e^2}-\frac{(bc)\int\frac{(d+}{}}{}$$

$$=-\frac{bx\left(d+ex^2\right)^{7/2}}{72ce}-\frac{d\left(d+ex^2\right)^{7/2}\left(a+b\tan^{-1}(cx)\right)}{7e^2}+\frac{\left(d+ex^2\right)^{9/2}\left(a+b\tan^{-1}\right.}{9e^2}$$

$$=-\frac{b\left(33c^2d-56e\right)x\left(d+ex^2\right)^{5/2}}{3024c^3e}-\frac{bx\left(d+ex^2\right)^{7/2}}{72ce}-\frac{d\left(d+ex^2\right)^{7/2}\left(a+b\tan^{-1}\right.}{7e^2}$$

$$=-\frac{b\left(69c^4d^2-520c^2de+336e^2\right)x\left(d+ex^2\right)^{3/2}}{12096c^5e}-\frac{b\left(33c^2d-56e\right)x\left(d+ex^2\right)^{5/2}}{3024c^3e}$$

$$=\frac{b\left(59c^6d^3+712c^4d^2e-1104c^2de^2+448e^3\right)x\sqrt{d+ex^2}}{8064c^7e}-\frac{b\left(69c^4d^2-520c^2de\right.}{120}$$

$$=\frac{b\left(59c^6d^3+712c^4d^2e-1104c^2de^2+448e^3\right)x\sqrt{d+ex^2}}{8064c^7e}-\frac{b\left(69c^4d^2-520c^2de\right.}{120}$$

$$=\frac{b\left(59c^6d^3+712c^4d^2e-1104c^2de^2+448e^3\right)x\sqrt{d+ex^2}}{8064c^7e}-\frac{b\left(69c^4d^2-520c^2de\right.}{120}$$

$$=\frac{b\left(59c^6d^3+712c^4d^2e-1104c^2de^2+448e^3\right)x\sqrt{d+ex^2}}{8064c^7e}-\frac{b\left(69c^4d^2-520c^2de\right.}{120}$$

Mathematica [C] time = 0.869719, size = 470, normalized size = 1.36

$$-\frac{c^2\sqrt{d+ex^2}\left(384ac^7\left(2d-7ex^2\right)\left(d+ex^2\right)^3+bex\left(3c^6\left(558d^2ex^2+187d^3+424de^2x^4+112e^3x^6\right)-8c^4e\left(453d^2+242d\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]),x]
```

```
[Out] -(c^2*Sqrt[d + e*x^2]*(384*a*c^7*(2*d - 7*e*x^2)*(d + e*x^2)^3 + b*e*x*(-13
44*e^3 + 48*c^2*e^2*(83*d + 14*e*x^2) - 8*c^4*e*(453*d^2 + 242*d*e*x^2 + 56
*e^2*x^4) + 3*c^6*(187*d^3 + 558*d^2*e*x^2 + 424*d*e^2*x^4 + 112*e^3*x^6)))
```

```
 + 384*b*c^9*(2*d - 7*e*x^2)*(d + e*x^2)^(7/2)*ArcTan[c*x] + (192*I)*b*(c^2
*d - e)^(7/2)*(2*c^2*d + 7*e)*Log[((-252*I)*c^10*e^2*(c*d - I*e*x + Sqrt[c^
2*d - e]*Sqrt[d + e*x^2]))/(b*(c^2*d - e)^(9/2)*(2*c^2*d + 7*e)*(I + c*x))]
 - (192*I)*b*(c^2*d - e)^(7/2)*(2*c^2*d + 7*e)*Log[((252*I)*c^10*e^2*(c*d +
 I*e*x + Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*(c^2*d - e)^(9/2)*(2*c^2*d +
7*e)*(-I + c*x))] + 3*b*Sqrt[e]*(-315*c^8*d^4 - 840*c^6*d^3*e + 3024*c^4*d^
2*e^2 - 2880*c^2*d*e^3 + 896*e^4)*Log[e*x + Sqrt[e]*Sqrt[d + e*x^2]])/(2419
2*c^9*e^2)
```

Maple [F] time = 0.652, size = 0, normalized size = 0.

$$\int x^3 \left(ex^2+d\right)^{\frac{5}{2}} (a+b\arctan(cx))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x^2+d)^(5/2)*(a+b*arctan(c*x)),x)
```

```
[Out] int(x^3*(e*x^2+d)^(5/2)*(a+b*arctan(c*x)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^(5/2)*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^(5/2)*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x**2+d)**(5/2)*(a+b*atan(c*x)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.93645, size = 1481, normalized size = 4.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^(5/2)*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] 1/15*(3*(x^2*e + d)^(5/2) - 5*(x^2*e + d)^(3/2)*d)*a*d^2*e^(-2) + 1/240*(16
*(3*(x^2*e + d)^(5/2) - 5*(x^2*e + d)^(3/2)*d)*arctan(c*x)*e^(-2) - (2*sqrt
(x^2*e + d)*x*(6*x^2/c^2 + (7*c^10*d*e^2 - 12*c^8*e^3)*e^(-3)/c^12) + (15*c
^4*d^2 + 20*c^2*d*e - 24*e^2)*e^(-3/2)*log((x*e^(1/2) - sqrt(x^2*e + d))^2)
/c^6 + 16*(2*c^6*d^3*e^(1/2) - c^4*d^2*e^(3/2) - 4*c^2*d*e^(5/2) + 3*e^(7/2
))*arctan(1/2*((x*e^(1/2) - sqrt(x^2*e + d))^2*c^2 - c^2*d + 2*e)*e^(-1/2)/
sqrt(c^2*d - e))*e^(-5/2)/(sqrt(c^2*d - e)*c^6))*c)*b*d^2 + 1/241920*(768*(
35*(x^2*e + d)^(9/2) - 135*(x^2*e + d)^(7/2)*d + 189*(x^2*e + d)^(5/2)*d^2
- 105*(x^2*e + d)^(3/2)*d^3)*a*e^(-4) + (768*(35*(x^2*e + d)^(9/2) - 135*(x
^2*e + d)^(7/2)*d + 189*(x^2*e + d)^(5/2)*d^2 - 105*(x^2*e + d)^(3/2)*d^3)*
arctan(c*x)*e^(-4) - (2*(2*(20*x^2*(42*x^2/c^2 + (15*c^28*d*e^11 - 56*c^26*
e^12)*e^(-12)/c^30) - (423*c^28*d^2*e^10 + 520*c^26*d*e^11 - 1680*c^24*e^12
)*e^(-12)/c^30)*x^2 + 3*(551*c^28*d^3*e^9 + 584*c^26*d^2*e^10 + 880*c^24*d*
e^11 - 2240*c^22*e^12)*e^(-12)/c^30)*sqrt(x^2*e + d)*x + 3*(1575*c^8*d^4 +
840*c^6*d^3*e + 1008*c^4*d^2*e^2 + 2880*c^2*d*e^3 - 4480*e^4)*e^(-7/2)*log(
(x*e^(1/2) - sqrt(x^2*e + d))^2)/c^10 + 768*(16*c^10*d^5*e^(1/2) - 8*c^8*d^
4*e^(3/2) - 2*c^6*d^3*e^(5/2) - c^4*d^2*e^(7/2) - 40*c^2*d*e^(9/2) + 35*e^(
11/2))*arctan(1/2*((x*e^(1/2) - sqrt(x^2*e + d))^2*c^2 - c^2*d + 2*e)*e^(-1
/2)/sqrt(c^2*d - e))*e^(-9/2)/(sqrt(c^2*d - e)*c^10))*c)*b)*e^2 + 1/1680*(3
```

```
2*(15*(x^2*e + d)^(7/2) - 42*(x^2*e + d)^(5/2)*d + 35*(x^2*e + d)^(3/2)*d^2
)*a*d*e^(-3) + (32*(15*(x^2*e + d)^(7/2) - 42*(x^2*e + d)^(5/2)*d + 35*(x^2
*e + d)^(3/2)*d^2)*arctan(c*x)*e^(-3) - (2*(2*x^2*(20*x^2/c^2 + (11*c^18*d*
e^6 - 30*c^16*e^7)*e^(-7)/c^20) - (41*c^18*d^2*e^5 + 54*c^16*d*e^6 - 120*c^
14*e^7)*e^(-7)/c^20)*sqrt(x^2*e + d)*x - (105*c^6*d^3 + 70*c^4*d^2*e + 168*
c^2*d*e^2 - 240*e^3)*e^(-5/2)*log((x*e^(1/2) - sqrt(x^2*e + d))^2)/c^8 - 32
*(8*c^8*d^4*e^(1/2) - 4*c^6*d^3*e^(3/2) - c^4*d^2*e^(5/2) - 18*c^2*d*e^(7/2
) + 15*e^(9/2))*arctan(1/2*((x*e^(1/2) - sqrt(x^2*e + d))^2*c^2 - c^2*d + 2
*e)*e^(-1/2)/sqrt(c^2*d - e))*e^(-7/2)/(sqrt(c^2*d - e)*c^8))*c)*b*d)*e
```

3.1194 $\int x^2 \left(d+ex^2\right)^{5/2} \left(a+b\tan^{-1}(cx)\right) dx$

Optimal. Leaf size=140

$$b\text{Unintegrable}\left(x^2\tan^{-1}(cx)\left(d+ex^2\right)^{5/2},x\right)-\frac{5ad^4\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{128e^{3/2}}+\frac{5ad^3x\sqrt{d+ex^2}}{128e}+\frac{5}{64}ad^2x^3\sqrt{d+ex^2}+\frac{5}{48}adx^3$$

```
[Out] (5*a*d^3*x*Sqrt[d + e*x^2])/(128*e) + (5*a*d^2*x^3*Sqrt[d + e*x^2])/64 + (5
*a*d*x^3*(d + e*x^2)^(3/2))/48 + (a*x^3*(d + e*x^2)^(5/2))/8 - (5*a*d^4*Arc
Tanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(128*e^(3/2)) + b*Unintegrable[x^2*(d +
e*x^2)^(5/2)*ArcTan[c*x], x]
```

Rubi [A] time = 0.227219, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^2 \left(d+ex^2\right)^{5/2} \left(a+b\tan^{-1}(cx)\right) dx$$

Verification is Not applicable to the result.

```
[In] Int[x^2*(d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]),x]
```

```
[Out] (5*a*d^3*x*Sqrt[d + e*x^2])/(128*e) + (5*a*d^2*x^3*Sqrt[d + e*x^2])/64 + (5
*a*d*x^3*(d + e*x^2)^(3/2))/48 + (a*x^3*(d + e*x^2)^(5/2))/8 - (5*a*d^4*Arc
Tanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(128*e^(3/2)) + b*Defer[Int][x^2*(d + e*
x^2)^(5/2)*ArcTan[c*x], x]
```

Rubi steps

$$\int x^2 \left(d+ex^2\right)^{5/2} \left(a+b\tan^{-1}(cx)\right) dx = a\int x^2 \left(d+ex^2\right)^{5/2} dx + b\int x^2 \left(d+ex^2\right)^{5/2} \tan^{-1}(cx)\, dx$$
$$= \frac{1}{8}ax^3 \left(d+ex^2\right)^{5/2} + b\int x^2 \left(d+ex^2\right)^{5/2} \tan^{-1}(cx)\, dx + \frac{1}{8}(5ad)\int x^2 \left(d+ex^2\right)^{3/2}$$
$$= \frac{5}{48}adx^3 \left(d+ex^2\right)^{3/2} + \frac{1}{8}ax^3 \left(d+ex^2\right)^{5/2} + b\int x^2 \left(d+ex^2\right)^{5/2} \tan^{-1}(cx)\, dx + \frac{1}{16}$$
$$= \frac{5}{64}ad^2x^3\sqrt{d+ex^2} + \frac{5}{48}adx^3 \left(d+ex^2\right)^{3/2} + \frac{1}{8}ax^3 \left(d+ex^2\right)^{5/2} + b\int x^2 \left(d+ex^2\right)$$
$$= \frac{5ad^3x\sqrt{d+ex^2}}{128e} + \frac{5}{64}ad^2x^3\sqrt{d+ex^2} + \frac{5}{48}adx^3 \left(d+ex^2\right)^{3/2} + \frac{1}{8}ax^3 \left(d+ex^2\right)^{5/2}$$
$$= \frac{5ad^3x\sqrt{d+ex^2}}{128e} + \frac{5}{64}ad^2x^3\sqrt{d+ex^2} + \frac{5}{48}adx^3 \left(d+ex^2\right)^{3/2} + \frac{1}{8}ax^3 \left(d+ex^2\right)^{5/2}$$
$$= \frac{5ad^3x\sqrt{d+ex^2}}{128e} + \frac{5}{64}ad^2x^3\sqrt{d+ex^2} + \frac{5}{48}adx^3 \left(d+ex^2\right)^{3/2} + \frac{1}{8}ax^3 \left(d+ex^2\right)^{5/2}$$

Mathematica [A] time = 11.443, size = 0, normalized size = 0.

$$\int x^2 \left(d+ex^2\right)^{5/2} \left(a+b\tan^{-1}(cx)\right) dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^2*(d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]),x]
```

```
[Out] Integrate[x^2*(d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]), x]
```

Maple [A] time = 0.586, size = 0, normalized size = 0.

$$\int x^2 \left(ex^2+d\right)^{\frac{5}{2}} (a+b\arctan(cx))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^2+d)^(5/2)*(a+b*arctan(c*x)),x)
```

```
[Out] int(x^2*(e*x^2+d)^(5/2)*(a+b*arctan(c*x)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^(5/2)*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(ae^2x^6 + 2\,adex^4 + ad^2x^2 + \left(be^2x^6 + 2\,bdex^4 + bd^2x^2\right)\arctan\left(cx\right)\right)\sqrt{ex^2 + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^(5/2)*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] integral((a*e^2*x^6 + 2*a*d*e*x^4 + a*d^2*x^2 + (b*e^2*x^6 + 2*b*d*e*x^4 +
b*d^2*x^2)*arctan(c*x))*sqrt(e*x^2 + d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d)**(5/2)*(a+b*atan(c*x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(ex^2 + d\right)^{\frac{5}{2}}(b\arctan\left(cx\right) + a)x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^(5/2)*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^(5/2)*(b*arctan(c*x) + a)*x^2, x)
```

3.1195 $\int x\left(d+ex^2\right)^{5/2}\left(a+b\tan^{-1}(cx)\right)\,dx$

Optimal. Leaf size=233

$$\frac{\left(d+ex^2\right)^{7/2}\left(a+b\tan^{-1}(cx)\right)}{7e}-\frac{bx\left(19c^4d^2-22c^2de+8e^2\right)\sqrt{d+ex^2}}{112c^5}-\frac{b\left(-70c^4d^2e+35c^6d^3+56c^2de^2-16e^3\right)\tanh^{-1}}{112c^7\sqrt{e}}$$

```
[Out] -(b*(19*c^4*d^2 - 22*c^2*d*e + 8*e^2)*x*Sqrt[d + e*x^2])/(112*c^5) - (b*(11
*c^2*d - 6*e)*x*(d + e*x^2)^(3/2))/(168*c^3) - (b*x*(d + e*x^2)^(5/2))/(42*
c) + ((d + e*x^2)^(7/2)*(a + b*ArcTan[c*x]))/(7*e) - (b*(c^2*d - e)^(7/2)*A
rcTan[(Sqrt[c^2*d - e]*x)/Sqrt[d + e*x^2]])/(7*c^7*e) - (b*(35*c^6*d^3 - 70
*c^4*d^2*e + 56*c^2*d*e^2 - 16*e^3)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(
112*c^7*Sqrt[e])
```

Rubi [A] time = 0.330622, antiderivative size = 233, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {4974, 416, 528, 523, 217, 206, 377, 203}

$$\frac{\left(d+ex^2\right)^{7/2}\left(a+b\tan^{-1}(cx)\right)}{7e}-\frac{bx\left(19c^4d^2-22c^2de+8e^2\right)\sqrt{d+ex^2}}{112c^5}-\frac{b\left(-70c^4d^2e+35c^6d^3+56c^2de^2-16e^3\right)\tanh^{-1}}{112c^7\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]),x]
```



```
[Out] -(b*(19*c^4*d^2 - 22*c^2*d*e + 8*e^2)*x*Sqrt[d + e*x^2])/(112*c^5) - (b*(11
*c^2*d - 6*e)*x*(d + e*x^2)^(3/2))/(168*c^3) - (b*x*(d + e*x^2)^(5/2))/(42*
c) + ((d + e*x^2)^(7/2)*(a + b*ArcTan[c*x]))/(7*e) - (b*(c^2*d - e)^(7/2)*A
rcTan[(Sqrt[c^2*d - e]*x)/Sqrt[d + e*x^2]])/(7*c^7*e) - (b*(35*c^6*d^3 - 70
*c^4*d^2*e + 56*c^2*d*e^2 - 16*e^3)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(
112*c^7*Sqrt[e])
```


Rule 4974

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]))/(2*e*(q + 1)), x
] - Dist[(b*c)/(2*e*(q + 1)), Int[(d + e*x^2)^(q + 1)/(1 + c^2*x^2), x], x]
 /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int x\left(d+ex^2\right)^{5/2}\left(a+b\tan^{-1}(cx)\right)dx=\frac{\left(d+ex^2\right)^{7/2}\left(a+b\tan^{-1}(cx)\right)}{7e}-\frac{(bc)\int\frac{\left(d+ex^2\right)^{7/2}}{1+c^2x^2}dx}{7e}$$

$$=-\frac{bx\left(d+ex^2\right)^{5/2}}{42c}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\tan^{-1}(cx)\right)}{7e}-\frac{b\int\frac{\left(d+ex^2\right)^{3/2}\left(d\left(6c^2d-e\right)+\left(11c^2d\right.\right.}{1+c^2x^2}}{42ce}$$

$$=-\frac{b\left(11c^2d-6e\right)x\left(d+ex^2\right)^{3/2}}{168c^3}-\frac{bx\left(d+ex^2\right)^{5/2}}{42c}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\tan^{-1}(cx)\right.}{7e}$$

$$=-\frac{b\left(19c^4d^2-22c^2de+8e^2\right)x\sqrt{d+ex^2}}{112c^5}-\frac{b\left(11c^2d-6e\right)x\left(d+ex^2\right)^{3/2}}{168c^3}-\frac{bx\left(d+\right.}{4}$$

$$=-\frac{b\left(19c^4d^2-22c^2de+8e^2\right)x\sqrt{d+ex^2}}{112c^5}-\frac{b\left(11c^2d-6e\right)x\left(d+ex^2\right)^{3/2}}{168c^3}-\frac{bx\left(d+\right.}{4}$$

$$=-\frac{b\left(19c^4d^2-22c^2de+8e^2\right)x\sqrt{d+ex^2}}{112c^5}-\frac{b\left(11c^2d-6e\right)x\left(d+ex^2\right)^{3/2}}{168c^3}-\frac{bx\left(d+\right.}{4}$$

$$=-\frac{b\left(19c^4d^2-22c^2de+8e^2\right)x\sqrt{d+ex^2}}{112c^5}-\frac{b\left(11c^2d-6e\right)x\left(d+ex^2\right)^{3/2}}{168c^3}-\frac{bx\left(d+\right.}{4}$$

Mathematica [C] time = 0.521259, size = 353, normalized size = 1.52

$$c^2\sqrt{d+ex^2}\left(48ac^5\left(d+ex^2\right)^3-bex\left(c^4\left(87d^2+38dex^2+8e^2x^4\right)-6c^2e\left(13d+2ex^2\right)+24e^2\right)\right)+3b\sqrt{e}\left(70c^4d^2e-35c^6d^3\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]),x]
```

```
[Out] (c^2*Sqrt[d + e*x^2]*(48*a*c^5*(d + e*x^2)^3 - b*e*x*(24*e^2 - 6*c^2*e*(13*
d + 2*e*x^2) + c^4*(87*d^2 + 38*d*e*x^2 + 8*e^2*x^4))) + 48*b*c^7*(d + e*x^
2)^(7/2)*ArcTan[c*x] - (24*I)*b*(c^2*d - e)^(7/2)*Log[(28*c^8*e*((-I)*c*d +
```

```
 e*x - I*Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*(c^2*d - e)^(9/2)*(-I + c*x))
] + (24*I)*b*(c^2*d - e)^(7/2)*Log[(28*c^8*e*(I*c*d + e*x + I*Sqrt[c^2*d -
e]*Sqrt[d + e*x^2]))/(b*(c^2*d - e)^(9/2)*(I + c*x))] + 3*b*Sqrt[e]*(-35*c^
6*d^3 + 70*c^4*d^2*e - 56*c^2*d*e^2 + 16*e^3)*Log[e*x + Sqrt[e]*Sqrt[d + e*
x^2]])/(336*c^7*e)
```

Maple [F] time = 0.606, size = 0, normalized size = 0.

$$\int x\left(ex^2+d\right)^{\frac{5}{2}}\left(a+b\arctan\left(cx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d)^(5/2)*(a+b*arctan(c*x)),x)
```

```
[Out] int(x*(e*x^2+d)^(5/2)*(a+b*arctan(c*x)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^(5/2)*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 95.808, size = 3430, normalized size = 14.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^(5/2)*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] [-1/672*(3*(35*b*c^6*d^3 - 70*b*c^4*d^2*e + 56*b*c^2*d*e^2 - 16*b*e^3)*sqrt
(e)*log(-2*e*x^2 - 2*sqrt(e*x^2 + d)*sqrt(e)*x - d) + 24*(b*c^6*d^3 - 3*b*c
```

```
^4*d^2*e + 3*b*c^2*d*e^2 - b*e^3)*sqrt(-c^2*d + e)*log(((c^4*d^2 - 8*c^2*d*
e + 8*e^2)*x^4 - 2*(3*c^2*d^2 - 4*d*e)*x^2 + 4*((c^2*d - 2*e)*x^3 - d*x)*sq
rt(-c^2*d + e)*sqrt(e*x^2 + d) + d^2)/(c^4*x^4 + 2*c^2*x^2 + 1)) - 2*(48*a*
c^7*e^3*x^6 + 144*a*c^7*d*e^2*x^4 - 8*b*c^6*e^3*x^5 + 144*a*c^7*d^2*e*x^2 +
 48*a*c^7*d^3 - 2*(19*b*c^6*d*e^2 - 6*b*c^4*e^3)*x^3 - 3*(29*b*c^6*d^2*e -
26*b*c^4*d*e^2 + 8*b*c^2*e^3)*x + 48*(b*c^7*e^3*x^6 + 3*b*c^7*d*e^2*x^4 + 3
*b*c^7*d^2*e*x^2 + b*c^7*d^3)*arctan(c*x))*sqrt(e*x^2 + d))/(c^7*e), -1/672
*(48*(b*c^6*d^3 - 3*b*c^4*d^2*e + 3*b*c^2*d*e^2 - b*e^3)*sqrt(c^2*d - e)*ar
ctan(1/2*sqrt(c^2*d - e)*((c^2*d - 2*e)*x^2 - d)*sqrt(e*x^2 + d)/((c^2*d*e
- e^2)*x^3 + (c^2*d^2 - d*e)*x)) + 3*(35*b*c^6*d^3 - 70*b*c^4*d^2*e + 56*b*
c^2*d*e^2 - 16*b*e^3)*sqrt(e)*log(-2*e*x^2 - 2*sqrt(e*x^2 + d)*sqrt(e)*x -
d) - 2*(48*a*c^7*e^3*x^6 + 144*a*c^7*d*e^2*x^4 - 8*b*c^6*e^3*x^5 + 144*a*c^
7*d^2*e*x^2 + 48*a*c^7*d^3 - 2*(19*b*c^6*d*e^2 - 6*b*c^4*e^3)*x^3 - 3*(29*b
*c^6*d^2*e - 26*b*c^4*d*e^2 + 8*b*c^2*e^3)*x + 48*(b*c^7*e^3*x^6 + 3*b*c^7*
d*e^2*x^4 + 3*b*c^7*d^2*e*x^2 + b*c^7*d^3)*arctan(c*x))*sqrt(e*x^2 + d))/(c
^7*e), 1/336*(3*(35*b*c^6*d^3 - 70*b*c^4*d^2*e + 56*b*c^2*d*e^2 - 16*b*e^3)
*sqrt(-e)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) - 12*(b*c^6*d^3 - 3*b*c^4*d^2*
e + 3*b*c^2*d*e^2 - b*e^3)*sqrt(-c^2*d + e)*log(((c^4*d^2 - 8*c^2*d*e + 8*e
^2)*x^4 - 2*(3*c^2*d^2 - 4*d*e)*x^2 + 4*((c^2*d - 2*e)*x^3 - d*x)*sqrt(-c^2
*d + e)*sqrt(e*x^2 + d) + d^2)/(c^4*x^4 + 2*c^2*x^2 + 1)) + (48*a*c^7*e^3*x
^6 + 144*a*c^7*d*e^2*x^4 - 8*b*c^6*e^3*x^5 + 144*a*c^7*d^2*e*x^2 + 48*a*c^7
*d^3 - 2*(19*b*c^6*d*e^2 - 6*b*c^4*e^3)*x^3 - 3*(29*b*c^6*d^2*e - 26*b*c^4*
d*e^2 + 8*b*c^2*e^3)*x + 48*(b*c^7*e^3*x^6 + 3*b*c^7*d*e^2*x^4 + 3*b*c^7*d^
2*e*x^2 + b*c^7*d^3)*arctan(c*x))*sqrt(e*x^2 + d))/(c^7*e), -1/336*(24*(b*c
^6*d^3 - 3*b*c^4*d^2*e + 3*b*c^2*d*e^2 - b*e^3)*sqrt(c^2*d - e)*arctan(1/2*
sqrt(c^2*d - e)*((c^2*d - 2*e)*x^2 - d)*sqrt(e*x^2 + d)/((c^2*d*e - e^2)*x^
3 + (c^2*d^2 - d*e)*x)) - 3*(35*b*c^6*d^3 - 70*b*c^4*d^2*e + 56*b*c^2*d*e^2
 - 16*b*e^3)*sqrt(-e)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) - (48*a*c^7*e^3*x^
6 + 144*a*c^7*d*e^2*x^4 - 8*b*c^6*e^3*x^5 + 144*a*c^7*d^2*e*x^2 + 48*a*c^7*
d^3 - 2*(19*b*c^6*d*e^2 - 6*b*c^4*e^3)*x^3 - 3*(29*b*c^6*d^2*e - 26*b*c^4*d
*e^2 + 8*b*c^2*e^3)*x + 48*(b*c^7*e^3*x^6 + 3*b*c^7*d*e^2*x^4 + 3*b*c^7*d^2
*e*x^2 + b*c^7*d^3)*arctan(c*x))*sqrt(e*x^2 + d))/(c^7*e)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d)**(5/2)*(a+b*atan(c*x)),x)

[Out] Timed out
```

Giac [B] time = 1.82125, size = 1115, normalized size = 4.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^(5/2)*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] 1/3*(x^2*e + d)^(3/2)*a*d^2*e^(-1) + 1/12*(4*(x^2*e + d)^(3/2)*arctan(c*x)*
e^(-1) - c*(2*sqrt(x^2*e + d)*x/c^2 - (3*c^2*d - 2*e)*e^(-1/2)*log((x*e^(1/
2) - sqrt(x^2*e + d))^2)/c^4 - 4*(c^4*d^2*e^(1/2) - 2*c^2*d*e^(3/2) + e^(5/
2))*arctan(1/2*((x*e^(1/2) - sqrt(x^2*e + d))^2*c^2 - c^2*d + 2*e)*e^(-1/2)
/sqrt(c^2*d - e))*e^(-3/2)/(sqrt(c^2*d - e)*c^4)))*b*d^2 + 1/3360*(32*(15*(
x^2*e + d)^(7/2) - 42*(x^2*e + d)^(5/2)*d + 35*(x^2*e + d)^(3/2)*d^2)*a*e^(
-3) + (32*(15*(x^2*e + d)^(7/2) - 42*(x^2*e + d)^(5/2)*d + 35*(x^2*e + d)^(
3/2)*d^2)*arctan(c*x)*e^(-3) - (2*(2*x^2*(20*x^2/c^2 + (11*c^18*d*e^6 - 30*
c^16*e^7)*e^(-7)/c^20) - (41*c^18*d^2*e^5 + 54*c^16*d*e^6 - 120*c^14*e^7)*e
^(-7)/c^20)*sqrt(x^2*e + d)*x - (105*c^6*d^3 + 70*c^4*d^2*e + 168*c^2*d*e^2
 - 240*e^3)*e^(-5/2)*log((x*e^(1/2) - sqrt(x^2*e + d))^2)/c^8 - 32*(8*c^8*d
^4*e^(1/2) - 4*c^6*d^3*e^(3/2) - c^4*d^2*e^(5/2) - 18*c^2*d*e^(7/2) + 15*e^
(9/2))*arctan(1/2*((x*e^(1/2) - sqrt(x^2*e + d))^2*c^2 - c^2*d + 2*e)*e^(-1
/2)/sqrt(c^2*d - e))*e^(-7/2)/(sqrt(c^2*d - e)*c^8))*c)*b)*e^2 + 1/120*(16*
(3*(x^2*e + d)^(5/2) - 5*(x^2*e + d)^(3/2)*d)*a*d*e^(-2) + (16*(3*(x^2*e +
d)^(5/2) - 5*(x^2*e + d)^(3/2)*d)*arctan(c*x)*e^(-2) - (2*sqrt(x^2*e + d)*x
*(6*x^2/c^2 + (7*c^10*d*e^2 - 12*c^8*e^3)*e^(-3)/c^12) + (15*c^4*d^2 + 20*c
^2*d*e - 24*e^2)*e^(-3/2)*log((x*e^(1/2) - sqrt(x^2*e + d))^2)/c^6 + 16*(2*
c^6*d^3*e^(1/2) - c^4*d^2*e^(3/2) - 4*c^2*d*e^(5/2) + 3*e^(7/2))*arctan(1/2
*((x*e^(1/2) - sqrt(x^2*e + d))^2*c^2 - c^2*d + 2*e)*e^(-1/2)/sqrt(c^2*d -
e))*e^(-5/2)/(sqrt(c^2*d - e)*c^6))*c)*b*d)*e
```

3.1196 $\int \left(d+ex^2\right)^{5/2} \left(a+b\tan^{-1}(cx)\right) \, dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\left(d+ex^2\right)^{5/2} \left(a+b\tan^{-1}(cx)\right),x\right)$$

```
[Out] Unintegrable[(d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]), x]
```

Rubi [A] time = 0.0278654, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \left(d+ex^2\right)^{5/2} \left(a+b\tan^{-1}(cx)\right) \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]),x]
```

```
[Out] Defer[Int][(d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]), x]
```

Rubi steps

$$\int \left(d+ex^2\right)^{5/2} \left(a+b\tan^{-1}(cx)\right) \, dx = \int \left(d+ex^2\right)^{5/2} \left(a+b\tan^{-1}(cx)\right) \, dx$$

Mathematica [A] time = 5.00274, size = 0, normalized size = 0.

$$\int \left(d+ex^2\right)^{5/2} \left(a+b\tan^{-1}(cx)\right) \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]),x]
```

```
[Out] Integrate[(d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]), x]
```

Maple [A] time = 1.19, size = 0, normalized size = 0.

$$\int \left(e x^{2}+d\right)^{\frac{5}{2}} \left(a +b \arctan \left(c x\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(5/2)*(a+b*arctan(c*x)),x)
```

```
[Out] int((e*x^2+d)^(5/2)*(a+b*arctan(c*x)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(5/2)*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a e^{2} x^{4} + 2 \, a d e x^{2} + a d^{2} + \left(b e^{2} x^{4} + 2 \, b d e x^{2} + b d^{2}\right) \arctan\left(c x\right)\right) \sqrt{e x^{2} + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(5/2)*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] integral((a*e^2*x^4 + 2*a*d*e*x^2 + a*d^2 + (b*e^2*x^4 + 2*b*d*e*x^2 + b*d^
2)*arctan(c*x))*sqrt(e*x^2 + d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(5/2)*(a+b*atan(c*x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(ex^2+d\right)^{\frac{5}{2}}(b\arctan\left(cx\right)+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(5/2)*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^(5/2)*(b*arctan(c*x) + a), x)
```

3.1197 $\int \frac{\left(d+ex^2\right)^{5/2}\left(a+b\tan^{-1}(cx)\right)}{x}\,dx$

Optimal. Leaf size=99

$$b\text{Unintegrable}\left(\frac{\tan^{-1}(cx)\left(d+ex^2\right)^{5/2}}{x},x\right)+ad^2\sqrt{d+ex^2}-ad^{5/2}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)+\frac{1}{3}ad\left(d+ex^2\right)^{3/2}+\frac{1}{5}a\left(d+ex^2\right)^{5/2}$$

```
[Out] a*d^2*Sqrt[d + e*x^2] + (a*d*(d + e*x^2)^(3/2))/3 + (a*(d + e*x^2)^(5/2))/5
 - a*d^(5/2)*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]] + b*Unintegrable[((d + e*x^2)
^(5/2)*ArcTan[c*x])/x, x]
```

Rubi [A] time = 0.20259, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(d+ex^2\right)^{5/2}\left(a+b\tan^{-1}(cx)\right)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]))/x,x]
```

```
[Out] a*d^2*Sqrt[d + e*x^2] + (a*d*(d + e*x^2)^(3/2))/3 + (a*(d + e*x^2)^(5/2))/5
 - a*d^(5/2)*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]] + b*Defer[Int][((d + e*x^2)^(
5/2)*ArcTan[c*x])/x, x]
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^{5/2}\left(a+b\tan^{-1}(cx)\right)}{x}\,dx = a\int \frac{\left(d+ex^2\right)^{5/2}}{x}\,dx + b\int \frac{\left(d+ex^2\right)^{5/2}\tan^{-1}(cx)}{x}\,dx$$
$$= \frac{1}{2}a\,\text{Subst}\left(\int \frac{(d+ex)^{5/2}}{x}\,dx, x, x^2\right) + b\int \frac{\left(d+ex^2\right)^{5/2}\tan^{-1}(cx)}{x}\,dx$$
$$= \frac{1}{5}a\left(d+ex^2\right)^{5/2} + b\int \frac{\left(d+ex^2\right)^{5/2}\tan^{-1}(cx)}{x}\,dx + \frac{1}{2}(ad)\,\text{Subst}\left(\int \frac{(d+ex)^{3/2}}{x}\,d\right.$$
$$= \frac{1}{3}ad\left(d+ex^2\right)^{3/2} + \frac{1}{5}a\left(d+ex^2\right)^{5/2} + b\int \frac{\left(d+ex^2\right)^{5/2}\tan^{-1}(cx)}{x}\,dx + \frac{1}{2}\left(ad^2\right)\text{Su}$$
$$= ad^2\sqrt{d+ex^2} + \frac{1}{3}ad\left(d+ex^2\right)^{3/2} + \frac{1}{5}a\left(d+ex^2\right)^{5/2} + b\int \frac{\left(d+ex^2\right)^{5/2}\tan^{-1}(cx)}{x}\,d$$
$$= ad^2\sqrt{d+ex^2} + \frac{1}{3}ad\left(d+ex^2\right)^{3/2} + \frac{1}{5}a\left(d+ex^2\right)^{5/2} + b\int \frac{\left(d+ex^2\right)^{5/2}\tan^{-1}(cx)}{x}\,d$$
$$= ad^2\sqrt{d+ex^2} + \frac{1}{3}ad\left(d+ex^2\right)^{3/2} + \frac{1}{5}a\left(d+ex^2\right)^{5/2} - ad^{5/2}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right) + b$$

Mathematica [A] time = 71.1817, size = 0, normalized size = 0.

$$\int \frac{\left(d+ex^2\right)^{5/2}\left(a+b\tan^{-1}(cx)\right)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]))/x,x]
```

```
[Out] Integrate[((d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]))/x, x]
```

Maple [A] time = 0.566, size = 0, normalized size = 0.

$$\int \frac{a+b\arctan(cx)}{x}\left(ex^2+d\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(5/2)*(a+b*arctan(c*x))/x,x)
```

```
[Out] int((e*x^2+d)^(5/2)*(a+b*arctan(c*x))/x,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(5/2)*(a+b*arctan(c*x))/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(ae^2x^4 + 2\,adex^2 + ad^2 + \left(be^2x^4 + 2\,bdex^2 + bd^2\right)\arctan\left(cx\right)\right)\sqrt{ex^2 + d}}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(5/2)*(a+b*arctan(c*x))/x,x, algorithm="fricas")
```

```
[Out] integral((a*e^2*x^4 + 2*a*d*e*x^2 + a*d^2 + (b*e^2*x^4 + 2*b*d*e*x^2 + b*d^
2)*arctan(c*x))*sqrt(e*x^2 + d)/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(5/2)*(a+b*atan(c*x))/x,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2+d\right)^{\frac{5}{2}}\left(b\arctan\left(cx\right)+a\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(5/2)*(a+b*arctan(c*x))/x,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^(5/2)*(b*arctan(c*x) + a)/x, x)
```

3.1198 $\int \frac{(d+ex^2)^{5/2}(a+b\tan^{-1}(cx))}{x^2}\,dx$

Optimal. Leaf size=110

$$b\text{Unintegrable}\left(\frac{\tan^{-1}(cx)\left(d+ex^2\right)^{5/2}}{x^2},x\right)+\frac{15}{8}ad^2\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{a\left(d+ex^2\right)^{5/2}}{x}+\frac{5}{4}aex\left(d+ex^2\right)^{3/2}+\frac{15}{8}ade$$

```
[Out] (15*a*d*e*x*Sqrt[d + e*x^2])/8 + (5*a*e*x*(d + e*x^2)^(3/2))/4 - (a*(d + e*
x^2)^(5/2))/x + (15*a*d^2*Sqrt[e]*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/8 +
 b*Unintegrable[((d + e*x^2)^(5/2)*ArcTan[c*x])/x^2, x]
```

Rubi [A] time = 0.186994, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\left(d+ex^2\right)^{5/2}\left(a+b\tan^{-1}(cx)\right)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]))/x^2,x]
```

```
[Out] (15*a*d*e*x*Sqrt[d + e*x^2])/8 + (5*a*e*x*(d + e*x^2)^(3/2))/4 - (a*(d + e*
x^2)^(5/2))/x + (15*a*d^2*Sqrt[e]*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/8 +
 b*Defer[Int][((d + e*x^2)^(5/2)*ArcTan[c*x])/x^2, x]
```

Rubi steps

$$\int \frac{(d+ex^2)^{5/2}(a+b\tan^{-1}(cx))}{x^2}\,dx = a\int \frac{(d+ex^2)^{5/2}}{x^2}\,dx + b\int \frac{(d+ex^2)^{5/2}\tan^{-1}(cx)}{x^2}\,dx$$
$$= -\frac{a(d+ex^2)^{5/2}}{x} + b\int \frac{(d+ex^2)^{5/2}\tan^{-1}(cx)}{x^2}\,dx + (5ae)\int (d+ex^2)^{3/2}\,dx$$
$$= \frac{5}{4}aex(d+ex^2)^{3/2} - \frac{a(d+ex^2)^{5/2}}{x} + b\int \frac{(d+ex^2)^{5/2}\tan^{-1}(cx)}{x^2}\,dx + \frac{1}{4}(15ade)\int$$
$$= \frac{15}{8}adex\sqrt{d+ex^2} + \frac{5}{4}aex(d+ex^2)^{3/2} - \frac{a(d+ex^2)^{5/2}}{x} + b\int \frac{(d+ex^2)^{5/2}\tan^{-1}(c}{x^2}$$
$$= \frac{15}{8}adex\sqrt{d+ex^2} + \frac{5}{4}aex(d+ex^2)^{3/2} - \frac{a(d+ex^2)^{5/2}}{x} + b\int \frac{(d+ex^2)^{5/2}\tan^{-1}(c}{x^2}$$
$$= \frac{15}{8}adex\sqrt{d+ex^2} + \frac{5}{4}aex(d+ex^2)^{3/2} - \frac{a(d+ex^2)^{5/2}}{x} + \frac{15}{8}ad^2\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}}{\sqrt{d+}}\right.$$

Mathematica [A] time = 9.01508, size = 0, normalized size = 0.

$$\int \frac{(d+ex^2)^{5/2}(a+b\tan^{-1}(cx))}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]))/x^2,x]
```

```
[Out] Integrate[((d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]))/x^2, x]
```

Maple [A] time = 0.585, size = 0, normalized size = 0.

$$\int \frac{a+b\arctan(cx)}{x^2}(ex^2+d)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(5/2)*(a+b*arctan(c*x))/x^2,x)
```

```
[Out] int((e*x^2+d)^(5/2)*(a+b*arctan(c*x))/x^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(5/2)*(a+b*arctan(c*x))/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(ae^2x^4+2\,adex^2+ad^2+\left(be^2x^4+2\,bdex^2+bd^2\right)\arctan\left(cx\right)\right)\sqrt{ex^2+d}}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(5/2)*(a+b*arctan(c*x))/x^2,x, algorithm="fricas")
```

```
[Out] integral((a*e^2*x^4 + 2*a*d*e*x^2 + a*d^2 + (b*e^2*x^4 + 2*b*d*e*x^2 + b*d^
2)*arctan(c*x))*sqrt(e*x^2 + d)/x^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(5/2)*(a+b*atan(c*x))/x**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2+d\right)^{\frac{5}{2}}\left(b\arctan\left(cx\right)+a\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(5/2)*(a+b*arctan(c*x))/x^2,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^(5/2)*(b*arctan(c*x) + a)/x^2, x)
```

3.1199 $\int \frac{\left(d+ex^2\right)^{5/2}\left(a+b\tan^{-1}(cx)\right)}{x^3}\,dx$

Optimal. Leaf size=107

$$b\text{Unintegrable}\left(\frac{\tan^{-1}(cx)\left(d+ex^2\right)^{5/2}}{x^3},x\right)-\frac{5}{2}ad^{3/2}e\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)-\frac{a\left(d+ex^2\right)^{5/2}}{2x^2}+\frac{5}{6}ae\left(d+ex^2\right)^{3/2}+\frac{5}{2}ade\sqrt{d+}$$

```
[Out] (5*a*d*e*Sqrt[d + e*x^2])/2 + (5*a*e*(d + e*x^2)^(3/2))/6 - (a*(d + e*x^2)^
(5/2))/(2*x^2) - (5*a*d^(3/2)*e*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/2 + b*Uni
ntegrable[((d + e*x^2)^(5/2)*ArcTan[c*x])/x^3, x]
```

Rubi [A] time = 0.208068, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(d+ex^2\right)^{5/2}\left(a+b\tan^{-1}(cx)\right)}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]))/x^3,x]
```

```
[Out] (5*a*d*e*Sqrt[d + e*x^2])/2 + (5*a*e*(d + e*x^2)^(3/2))/6 - (a*(d + e*x^2)^
(5/2))/(2*x^2) - (5*a*d^(3/2)*e*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/2 + b*Def
er[Int][((d + e*x^2)^(5/2)*ArcTan[c*x])/x^3, x]
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^{5/2}\left(a+b\tan^{-1}(cx)\right)}{x^3}\,dx = a\int \frac{\left(d+ex^2\right)^{5/2}}{x^3}\,dx + b\int \frac{\left(d+ex^2\right)^{5/2}\tan^{-1}(cx)}{x^3}\,dx$$

$$= \frac{1}{2}a\,\text{Subst}\left(\int \frac{(d+ex)^{5/2}}{x^2}\,dx, x, x^2\right) + b\int \frac{\left(d+ex^2\right)^{5/2}\tan^{-1}(cx)}{x^3}\,dx$$

$$= -\frac{a\left(d+ex^2\right)^{5/2}}{2x^2} + b\int \frac{\left(d+ex^2\right)^{5/2}\tan^{-1}(cx)}{x^3}\,dx + \frac{1}{4}(5ae)\,\text{Subst}\left(\int \frac{(d+ex)^{3/2}}{x}\right.$$

$$= \frac{5}{6}ae\left(d+ex^2\right)^{3/2} - \frac{a\left(d+ex^2\right)^{5/2}}{2x^2} + b\int \frac{\left(d+ex^2\right)^{5/2}\tan^{-1}(cx)}{x^3}\,dx + \frac{1}{4}(5ade)\,\text{Sub}$$

$$= \frac{5}{2}ade\sqrt{d+ex^2} + \frac{5}{6}ae\left(d+ex^2\right)^{3/2} - \frac{a\left(d+ex^2\right)^{5/2}}{2x^2} + b\int \frac{\left(d+ex^2\right)^{5/2}\tan^{-1}(cx)}{x^3}$$

$$= \frac{5}{2}ade\sqrt{d+ex^2} + \frac{5}{6}ae\left(d+ex^2\right)^{3/2} - \frac{a\left(d+ex^2\right)^{5/2}}{2x^2} + b\int \frac{\left(d+ex^2\right)^{5/2}\tan^{-1}(cx)}{x^3}$$

$$= \frac{5}{2}ade\sqrt{d+ex^2} + \frac{5}{6}ae\left(d+ex^2\right)^{3/2} - \frac{a\left(d+ex^2\right)^{5/2}}{2x^2} - \frac{5}{2}ad^{3/2}e\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)$$

Mathematica [A] time = 48.5479, size = 0, normalized size = 0.

$$\int \frac{\left(d+ex^2\right)^{5/2}\left(a+b\tan^{-1}(cx)\right)}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]))/x^3,x]
```

```
[Out] Integrate[((d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]))/x^3, x]
```

Maple [A] time = 0.587, size = 0, normalized size = 0.

$$\int \frac{a+b\arctan(cx)}{x^3}\left(ex^2+d\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(5/2)*(a+b*arctan(c*x))/x^3,x)
```

```
[Out] int((e*x^2+d)^(5/2)*(a+b*arctan(c*x))/x^3,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(5/2)*(a+b*arctan(c*x))/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(ae^2x^4+2\,adex^2+ad^2+\left(be^2x^4+2\,bdex^2+bd^2\right)\arctan\left(cx\right)\right)\sqrt{ex^2+d}}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(5/2)*(a+b*arctan(c*x))/x^3,x, algorithm="fricas")
```

```
[Out] integral((a*e^2*x^4 + 2*a*d*e*x^2 + a*d^2 + (b*e^2*x^4 + 2*b*d*e*x^2 + b*d^
2)*arctan(c*x))*sqrt(e*x^2 + d)/x^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(5/2)*(a+b*atan(c*x))/x**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2+d\right)^{\frac{5}{2}}\left(b\arctan\left(cx\right)+a\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(5/2)*(a+b*arctan(c*x))/x^3,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^(5/2)*(b*arctan(c*x) + a)/x^3, x)
```

3.1200 $\int \frac{\left(d+ex^2\right)^{5/2}\left(a+b\tan^{-1}(cx)\right)}{x^4}\,dx$

Optimal. Leaf size=113

$$b\text{Unintegrable}\left(\frac{\tan^{-1}(cx)\left(d+ex^2\right)^{5/2}}{x^4},x\right)+\frac{5}{2}ae^2x\sqrt{d+ex^2}+\frac{5}{2}ade^{3/2}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{a\left(d+ex^2\right)^{5/2}}{3x^3}-\frac{5ae\left(d+e\right.}{3x}$$

```
[Out] (5*a*e^2*x*Sqrt[d + e*x^2])/2 - (5*a*e*(d + e*x^2)^(3/2))/(3*x) - (a*(d + e
*x^2)^(5/2))/(3*x^3) + (5*a*d*e^(3/2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])
/2 + b*Unintegrable[((d + e*x^2)^(5/2)*ArcTan[c*x])/x^4, x]
```

Rubi [A] time = 0.187136, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(d+ex^2\right)^{5/2}\left(a+b\tan^{-1}(cx)\right)}{x^4}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]))/x^4,x]
```

```
[Out] (5*a*e^2*x*Sqrt[d + e*x^2])/2 - (5*a*e*(d + e*x^2)^(3/2))/(3*x) - (a*(d + e
*x^2)^(5/2))/(3*x^3) + (5*a*d*e^(3/2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])
/2 + b*Defer[Int][((d + e*x^2)^(5/2)*ArcTan[c*x])/x^4, x]
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^{5/2}\left(a+b\tan^{-1}(cx)\right)}{x^4}\,dx = a\int \frac{\left(d+ex^2\right)^{5/2}}{x^4}\,dx + b\int \frac{\left(d+ex^2\right)^{5/2}\tan^{-1}(cx)}{x^4}\,dx$$
$$= -\frac{a\left(d+ex^2\right)^{5/2}}{3x^3} + b\int \frac{\left(d+ex^2\right)^{5/2}\tan^{-1}(cx)}{x^4}\,dx + \frac{1}{3}(5ae)\int \frac{\left(d+ex^2\right)^{3/2}}{x^2}\,dx$$
$$= -\frac{5ae\left(d+ex^2\right)^{3/2}}{3x} - \frac{a\left(d+ex^2\right)^{5/2}}{3x^3} + b\int \frac{\left(d+ex^2\right)^{5/2}\tan^{-1}(cx)}{x^4}\,dx + \left(5ae^2\right)\int \sqrt{}$$
$$= \frac{5}{2}ae^2x\sqrt{d+ex^2} - \frac{5ae\left(d+ex^2\right)^{3/2}}{3x} - \frac{a\left(d+ex^2\right)^{5/2}}{3x^3} + b\int \frac{\left(d+ex^2\right)^{5/2}\tan^{-1}(cx)}{x^4}$$
$$= \frac{5}{2}ae^2x\sqrt{d+ex^2} - \frac{5ae\left(d+ex^2\right)^{3/2}}{3x} - \frac{a\left(d+ex^2\right)^{5/2}}{3x^3} + b\int \frac{\left(d+ex^2\right)^{5/2}\tan^{-1}(cx)}{x^4}$$
$$= \frac{5}{2}ae^2x\sqrt{d+ex^2} - \frac{5ae\left(d+ex^2\right)^{3/2}}{3x} - \frac{a\left(d+ex^2\right)^{5/2}}{3x^3} + \frac{5}{2}ade^{3/2}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)$$

Mathematica [A] time = 8.94192, size = 0, normalized size = 0.

$$\int \frac{\left(d+ex^2\right)^{5/2}\left(a+b\tan^{-1}(cx)\right)}{x^4}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]))/x^4,x]
```

```
[Out] Integrate[((d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]))/x^4, x]
```

Maple [A] time = 0.595, size = 0, normalized size = 0.

$$\int \frac{a+b\arctan(cx)}{x^4}\left(ex^2+d\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(5/2)*(a+b*arctan(c*x))/x^4,x)
```

```
[Out] int((e*x^2+d)^(5/2)*(a+b*arctan(c*x))/x^4,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(5/2)*(a+b*arctan(c*x))/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(ae^2x^4+2\,adex^2+ad^2+\left(be^2x^4+2\,bdex^2+bd^2\right)\arctan\left(cx\right)\right)\sqrt{ex^2+d}}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(5/2)*(a+b*arctan(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] integral((a*e^2*x^4 + 2*a*d*e*x^2 + a*d^2 + (b*e^2*x^4 + 2*b*d*e*x^2 + b*d^
2)*arctan(c*x))*sqrt(e*x^2 + d)/x^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(5/2)*(a+b*atan(c*x))/x**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(ex^2+d\right)^{\frac{5}{2}}\left(b\arctan\left(cx\right)+a\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(5/2)*(a+b*arctan(c*x))/x^4,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^(5/2)*(b*arctan(c*x) + a)/x^4, x)
```

3.1201 $\int \frac{x^3(a+b\tan^{-1}(cx))}{\sqrt{d+ex^2}}\,dx$

Optimal. Leaf size=176

$$\frac{(d+ex^2)^{3/2}\left(a+b\tan^{-1}(cx)\right)}{3e^2}-\frac{d\sqrt{d+ex^2}\left(a+b\tan^{-1}(cx)\right)}{e^2}+\frac{b\sqrt{c^2d-e}\left(2c^2d+e\right)\tan^{-1}\left(\frac{x\sqrt{c^2d-e}}{\sqrt{d+ex^2}}\right)}{3c^3e^2}+\frac{b\left(3c^2d+2e\right)\tan}{6c^3e^3}$$

```
[Out] -(b*x*Sqrt[d + e*x^2])/(6*c*e) - (d*Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]))/e^
2 + ((d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]))/(3*e^2) + (b*Sqrt[c^2*d - e]*(2
*c^2*d + e)*ArcTan[(Sqrt[c^2*d - e]*x)/Sqrt[d + e*x^2]])/(3*c^3*e^2) + (b*(
3*c^2*d + 2*e)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(6*c^3*e^(3/2))
```

Rubi [A] time = 0.247594, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {266, 43, 4976, 12, 528, 523, 217, 206, 377, 203}

$$\frac{(d+ex^2)^{3/2}\left(a+b\tan^{-1}(cx)\right)}{3e^2}-\frac{d\sqrt{d+ex^2}\left(a+b\tan^{-1}(cx)\right)}{e^2}+\frac{b\sqrt{c^2d-e}\left(2c^2d+e\right)\tan^{-1}\left(\frac{x\sqrt{c^2d-e}}{\sqrt{d+ex^2}}\right)}{3c^3e^2}+\frac{b\left(3c^2d+2e\right)\tan}{6c^3e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcTan[c*x]))/Sqrt[d + e*x^2],x]
```

```
[Out] -(b*x*Sqrt[d + e*x^2])/(6*c*e) - (d*Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]))/e^
2 + ((d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]))/(3*e^2) + (b*Sqrt[c^2*d - e]*(2
*c^2*d + e)*ArcTan[(Sqrt[c^2*d - e]*x)/Sqrt[d + e*x^2]])/(3*c^3*e^2) + (b*(
3*c^2*d + 2*e)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(6*c^3*e^(3/2))
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 4976

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Dis
t[a + b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&  !(
ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(IL
tQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILtQ[(m
 - 1)/2, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{x^3\left(a+b\tan^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx=-\frac{d\sqrt{d+ex^2}\left(a+b\tan^{-1}(cx)\right)}{e^2}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\tan^{-1}(cx)\right)}{3e^2}-(bc)\int\frac{\left(-2d+ex^2\right)\sqrt{d+\cdots}}{3e^2\left(1+c^2x^2\right)}$$

$$=-\frac{d\sqrt{d+ex^2}\left(a+b\tan^{-1}(cx)\right)}{e^2}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\tan^{-1}(cx)\right)}{3e^2}-\frac{(bc)\int\frac{\left(-2d+ex^2\right)\sqrt{d+ex^2}}{1+c^2x^2}\,dx}{3e^2}$$

$$=-\frac{bx\sqrt{d+ex^2}}{6ce}-\frac{d\sqrt{d+ex^2}\left(a+b\tan^{-1}(cx)\right)}{e^2}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\tan^{-1}(cx)\right)}{3e^2}-\frac{b\int\frac{-d\left(4c^2d+\cdots\right.}{\left(1+\cdots\right.}}{\cdots}$$

$$=-\frac{bx\sqrt{d+ex^2}}{6ce}-\frac{d\sqrt{d+ex^2}\left(a+b\tan^{-1}(cx)\right)}{e^2}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\tan^{-1}(cx)\right)}{3e^2}+\frac{\left(b\left(c^2d-e\cdots\right.\right.}{\cdots}$$

$$=-\frac{bx\sqrt{d+ex^2}}{6ce}-\frac{d\sqrt{d+ex^2}\left(a+b\tan^{-1}(cx)\right)}{e^2}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\tan^{-1}(cx)\right)}{3e^2}+\frac{\left(b\left(c^2d-e\cdots\right.\right.}{\cdots}$$

$$=-\frac{bx\sqrt{d+ex^2}}{6ce}-\frac{d\sqrt{d+ex^2}\left(a+b\tan^{-1}(cx)\right)}{e^2}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\tan^{-1}(cx)\right)}{3e^2}+\frac{b\sqrt{c^2d-e}\left(\cdots\right.}{\cdots}$$

Mathematica [C] time = 0.484655, size = 377, normalized size = 2.14

$$\frac{-\frac{\sqrt{d+ex^2}\left(ac\left(4d-2ex^2\right)+bex\right)}{c}-\frac{ib\left(2c^4d^2-c^2de-e^2\right)\log\left(\frac{12ic^4e^2\left(\sqrt{c^2d-e}\sqrt{d+ex^2}+cd-iex\right)}{b(cx+i)\sqrt{c^2d-e}\left(-2c^4d^2+c^2de+e^2\right)}\right)}{c^3\sqrt{c^2d-e}}+\frac{ib\left(2c^4d^2-c^2de-e^2\right)\log\left(-\frac{12ic^4e^2\left(\sqrt{c^2d-e}\sqrt{d+ex^2}+cd+iex\right)}{b(cx-i)\sqrt{c^2d-e}\left(-2c^4d^2+c^2de+e^2\right)}\right)}{c^3\sqrt{c^2d-e}}+\frac{b\sqrt{e}\left(3c^2d\cdots\right.}{\cdots}}{6e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcTan[c*x]))/Sqrt[d + e*x^2],x]
```

```
[Out] (-((Sqrt[d + e*x^2]*(b*e*x + a*c*(4*d - 2*e*x^2)))/c) + 2*b*(-2*d + e*x^2)*
Sqrt[d + e*x^2]*ArcTan[c*x] - (I*b*(2*c^4*d^2 - c^2*d*e - e^2)*Log[((12*I)*
c^4*e^2*(c*d - I*e*x + Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*Sqrt[c^2*d - e]
*(-2*c^4*d^2 + c^2*d*e + e^2)*(I + c*x))])/(c^3*Sqrt[c^2*d - e]) + (I*b*(2*
c^4*d^2 - c^2*d*e - e^2)*Log[((-12*I)*c^4*e^2*(c*d + I*e*x + Sqrt[c^2*d - e
]*Sqrt[d + e*x^2]))/(b*Sqrt[c^2*d - e]*(-2*c^4*d^2 + c^2*d*e + e^2)*(-I + c
*x))])/(c^3*Sqrt[c^2*d - e]) + (b*Sqrt[e]*(3*c^2*d + 2*e)*Log[e*x + Sqrt[e]
*Sqrt[d + e*x^2]])/c^3)/(6*e^2)
```

Maple [F] time = 0.831, size = 0, normalized size = 0.

$$\int x^3 (a + b \arctan (cx)) \frac{1}{\sqrt{ex^2 + d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctan(c*x))/(e*x^2+d)^(1/2),x)
```

```
[Out] int(x^3*(a+b*arctan(c*x))/(e*x^2+d)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 7.63185, size = 1976, normalized size = 11.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*((3*b*c^2*d + 2*b*e)*sqrt(e)*log(-2*e*x^2 - 2*sqrt(e*x^2 + d)*sqrt(e)
*x - d) + (2*b*c^2*d + b*e)*sqrt(-c^2*d + e)*log(((c^4*d^2 - 8*c^2*d*e + 8*
e^2)*x^4 - 2*(3*c^2*d^2 - 4*d*e)*x^2 + 4*((c^2*d - 2*e)*x^3 - d*x)*sqrt(-c^
2*d + e)*sqrt(e*x^2 + d) + d^2)/(c^4*x^4 + 2*c^2*x^2 + 1)) + 2*(2*a*c^3*e*x
^2 - 4*a*c^3*d - b*c^2*e*x + 2*(b*c^3*e*x^2 - 2*b*c^3*d)*arctan(c*x))*sqrt(
e*x^2 + d))/(c^3*e^2), 1/12*(2*(2*b*c^2*d + b*e)*sqrt(c^2*d - e)*arctan(1/2
*sqrt(c^2*d - e)*((c^2*d - 2*e)*x^2 - d)*sqrt(e*x^2 + d)/((c^2*d*e - e^2)*x
^3 + (c^2*d^2 - d*e)*x)) + (3*b*c^2*d + 2*b*e)*sqrt(e)*log(-2*e*x^2 - 2*sqr
t(e*x^2 + d)*sqrt(e)*x - d) + 2*(2*a*c^3*e*x^2 - 4*a*c^3*d - b*c^2*e*x + 2*
(b*c^3*e*x^2 - 2*b*c^3*d)*arctan(c*x))*sqrt(e*x^2 + d))/(c^3*e^2), -1/12*(2
*(3*b*c^2*d + 2*b*e)*sqrt(-e)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) - (2*b*c^2
*d + b*e)*sqrt(-c^2*d + e)*log(((c^4*d^2 - 8*c^2*d*e + 8*e^2)*x^4 - 2*(3*c^
2*d^2 - 4*d*e)*x^2 + 4*((c^2*d - 2*e)*x^3 - d*x)*sqrt(-c^2*d + e)*sqrt(e*x^
2 + d) + d^2)/(c^4*x^4 + 2*c^2*x^2 + 1)) - 2*(2*a*c^3*e*x^2 - 4*a*c^3*d - b
*c^2*e*x + 2*(b*c^3*e*x^2 - 2*b*c^3*d)*arctan(c*x))*sqrt(e*x^2 + d))/(c^3*e
^2), 1/6*((2*b*c^2*d + b*e)*sqrt(c^2*d - e)*arctan(1/2*sqrt(c^2*d - e)*((c^
2*d - 2*e)*x^2 - d)*sqrt(e*x^2 + d)/((c^2*d*e - e^2)*x^3 + (c^2*d^2 - d*e)*
x)) - (3*b*c^2*d + 2*b*e)*sqrt(-e)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) + (2*
a*c^3*e*x^2 - 4*a*c^3*d - b*c^2*e*x + 2*(b*c^3*e*x^2 - 2*b*c^3*d)*arctan(c*
x))*sqrt(e*x^2 + d))/(c^3*e^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 (a + b \operatorname{atan}(cx))}{\sqrt{d + ex^2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atan(c*x))/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**3*(a + b*atan(c*x))/sqrt(d + e*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \arctan(cx) + a)x^3}{\sqrt{ex^2 + d}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)*x^3/sqrt(e*x^2 + d), x)
```

3.1202 $\int \frac{x^2(a+b\tan^{-1}(cx))}{\sqrt{d+ex^2}}\,dx$

Optimal. Leaf size=74

$$b\text{Unintegrable}\left(\frac{x^2\tan^{-1}(cx)}{\sqrt{d+ex^2}},x\right)-\frac{ad\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2e^{3/2}}+\frac{ax\sqrt{d+ex^2}}{2e}$$

```
[Out] (a*x*Sqrt[d + e*x^2])/(2*e) - (a*d*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2
*e^(3/2)) + b*Unintegrable[(x^2*ArcTan[c*x])/Sqrt[d + e*x^2], x]
```

Rubi [A] time = 0.155581, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^2\left(a+b\tan^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^2*(a + b*ArcTan[c*x]))/Sqrt[d + e*x^2],x]
```

```
[Out] (a*x*Sqrt[d + e*x^2])/(2*e) - (a*d*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2
*e^(3/2)) + b*Defer[Int][(x^2*ArcTan[c*x])/Sqrt[d + e*x^2], x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2\left(a+b\tan^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx &= a\int \frac{x^2}{\sqrt{d+ex^2}}\,dx+b\int \frac{x^2\tan^{-1}(cx)}{\sqrt{d+ex^2}}\,dx\\
&= \frac{ax\sqrt{d+ex^2}}{2e}+b\int \frac{x^2\tan^{-1}(cx)}{\sqrt{d+ex^2}}\,dx-\frac{(ad)\int \frac{1}{\sqrt{d+ex^2}}\,dx}{2e}\\
&= \frac{ax\sqrt{d+ex^2}}{2e}+b\int \frac{x^2\tan^{-1}(cx)}{\sqrt{d+ex^2}}\,dx-\frac{(ad)\,\text{Subst}\left(\int \frac{1}{1-ex^2}\,dx,x,\frac{x}{\sqrt{d+ex^2}}\right)}{2e}\\
&= \frac{ax\sqrt{d+ex^2}}{2e}-\frac{ad\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2e^{3/2}}+b\int \frac{x^2\tan^{-1}(cx)}{\sqrt{d+ex^2}}\,dx
\end{aligned}$$

Mathematica [A] time = 9.11565, size = 0, normalized size = 0.

$$\int \frac{x^2 \left(a + b \tan^{-1}(cx)\right)}{\sqrt{d + ex^2}} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^2*(a + b*ArcTan[c*x]))/Sqrt[d + e*x^2],x]
```

```
[Out] Integrate[(x^2*(a + b*ArcTan[c*x]))/Sqrt[d + e*x^2], x]
```

Maple [A] time = 0.785, size = 0, normalized size = 0.

$$\int x^2 \left(a + b \arctan(cx)\right) \frac{1}{\sqrt{ex^2 + d}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctan(c*x))/(e*x^2+d)^(1/2),x)
```

```
[Out] int(x^2*(a+b*arctan(c*x))/(e*x^2+d)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{bx^2 \arctan(cx) + ax^2}{\sqrt{ex^2 + d}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^2*arctan(c*x) + a*x^2)/sqrt(e*x^2 + d), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 (a + b \operatorname{atan}(cx))}{\sqrt{d + ex^2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atan(c*x))/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**2*(a + b*atan(c*x))/sqrt(d + e*x**2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \arctan(cx) + a)x^2}{\sqrt{ex^2 + d}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)*x^2/sqrt(e*x^2 + d), x)
```

3.1203 $\int \frac{x\left(a+b\tan^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$

Optimal. Leaf size=103

$$\frac{\sqrt{d+ex^2}\left(a+b\tan^{-1}(cx)\right)}{e}-\frac{b\sqrt{c^2d-e}\tan^{-1}\left(\frac{x\sqrt{c^2d-e}}{\sqrt{d+ex^2}}\right)}{ce}-\frac{b\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c\sqrt{e}}$$

```
[Out] (Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]))/e - (b*Sqrt[c^2*d - e]*ArcTan[(Sqrt[c
^2*d - e]*x)/Sqrt[d + e*x^2]])/(c*e) - (b*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^
2]])/(c*Sqrt[e])
```

Rubi [A] time = 0.0986641, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4974, 402, 217, 206, 377, 203}

$$\frac{\sqrt{d+ex^2}\left(a+b\tan^{-1}(cx)\right)}{e}-\frac{b\sqrt{c^2d-e}\tan^{-1}\left(\frac{x\sqrt{c^2d-e}}{\sqrt{d+ex^2}}\right)}{ce}-\frac{b\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcTan[c*x]))/Sqrt[d + e*x^2],x]
```

```
[Out] (Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]))/e - (b*Sqrt[c^2*d - e]*ArcTan[(Sqrt[c
^2*d - e]*x)/Sqrt[d + e*x^2]])/(c*e) - (b*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^
2]])/(c*Sqrt[e])
```

Rule 4974

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]))/(2*e*(q + 1)), x
] - Dist[(b*c)/(2*e*(q + 1)), Int[(d + e*x^2)^(q + 1)/(1 + c^2*x^2), x], x]
 /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
```

```
GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x\left(a+b\tan^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx &= \frac{\sqrt{d+ex^2}\left(a+b\tan^{-1}(cx)\right)}{e} - \frac{(bc)\int \frac{\sqrt{d+ex^2}}{1+c^2x^2}\,dx}{e}\\
&= \frac{\sqrt{d+ex^2}\left(a+b\tan^{-1}(cx)\right)}{e} - \frac{b\int \frac{1}{\sqrt{d+ex^2}}\,dx}{c} + \frac{\left(b\left(-c^2d+e\right)\right)\int \frac{1}{\left(1+c^2x^2\right)\sqrt{d+ex^2}}\,dx}{ce}\\
&= \frac{\sqrt{d+ex^2}\left(a+b\tan^{-1}(cx)\right)}{e} - \frac{b\,\text{Subst}\left(\int \frac{1}{1-ex^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{c} + \frac{\left(b\left(-c^2d+e\right)\right)\text{Subst}\left(\int \frac{}{1-(-}\right.}{ce}\\
&= \frac{\sqrt{d+ex^2}\left(a+b\tan^{-1}(cx)\right)}{e} - \frac{b\sqrt{c^2d-e}\tan^{-1}\left(\frac{\sqrt{c^2d-e}x}{\sqrt{d+ex^2}}\right)}{ce} - \frac{b\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c\sqrt{e}}
\end{aligned}$$

Mathematica [C] time = 0.371652, size = 251, normalized size = 2.44

$$\frac{2ac\sqrt{d+ex^2} - ib\sqrt{c^2d-e}\log\left(\frac{4c^2e\left(-i\sqrt{c^2d-e}\sqrt{d+ex^2}-icd+ex\right)}{b(cx-i)\left(c^2d-e\right)^{3/2}}\right) + ib\sqrt{c^2d-e}\log\left(\frac{4c^2e\left(i\sqrt{c^2d-e}\sqrt{d+ex^2}+icd+ex\right)}{b(cx+i)\left(c^2d-e\right)^{3/2}}\right) + 2bc\tan^{-1}(cx)\sqrt{d\cdots}}{2ce}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcTan[c*x]))/Sqrt[d + e*x^2],x]
```

```
[Out] (2*a*c*Sqrt[d + e*x^2] + 2*b*c*Sqrt[d + e*x^2]*ArcTan[c*x] - I*b*Sqrt[c^2*d
 - e]*Log[(4*c^2*e*((-I)*c*d + e*x - I*Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b
*(c^2*d - e)^(3/2)*(-I + c*x))] + I*b*Sqrt[c^2*d - e]*Log[(4*c^2*e*(I*c*d +
 e*x + I*Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*(c^2*d - e)^(3/2)*(I + c*x))]
 - 2*b*Sqrt[e]*Log[e*x + Sqrt[e]*Sqrt[d + e*x^2]])/(2*c*e)
```

Maple [F] time = 0.806, size = 0, normalized size = 0.

$$\int x\left(a+b\arctan\left(cx\right)\right)\frac{1}{\sqrt{ex^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctan(c*x))/(e*x^2+d)^(1/2),x)
```

```
[Out] int(x*(a+b*arctan(c*x))/(e*x^2+d)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.57512, size = 1461, normalized size = 14.18

$$\left[\frac{2\,b\sqrt{e}\log\left(-2\,ex^2+2\,\sqrt{ex^2+d}\sqrt{ex}-d\right)+\sqrt{-c^2d+e}b\log\left(\frac{\left(c^4d^2-8\,c^2de+8\,e^2\right)x^4-2\left(3\,c^2d^2-4\,de\right)x^2-4\left(\left(c^2d-2\,e\right)x^3-dx\right)\sqrt{-c^2d+e}\sqrt{ex^2+d}+d}{c^4x^4+2\,c^2x^2+1}\right.}{4\,ce}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*b*sqrt(e)*log(-2*e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(e)*x - d) + sqrt(-c
^2*d + e)*b*log(((c^4*d^2 - 8*c^2*d*e + 8*e^2)*x^4 - 2*(3*c^2*d^2 - 4*d*e)*
x^2 - 4*((c^2*d - 2*e)*x^3 - d*x)*sqrt(-c^2*d + e)*sqrt(e*x^2 + d) + d^2)/(
c^4*x^4 + 2*c^2*x^2 + 1)) + 4*sqrt(e*x^2 + d)*(b*c*arctan(c*x) + a*c))/(c*e
), -1/2*(sqrt(c^2*d - e)*b*arctan(1/2*sqrt(c^2*d - e)*((c^2*d - 2*e)*x^2 -
d)*sqrt(e*x^2 + d)/((c^2*d*e - e^2)*x^3 + (c^2*d^2 - d*e)*x)) - b*sqrt(e)*l
og(-2*e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(e)*x - d) - 2*sqrt(e*x^2 + d)*(b*c*arc
tan(c*x) + a*c))/(c*e), 1/4*(4*b*sqrt(-e)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)
) + sqrt(-c^2*d + e)*b*log(((c^4*d^2 - 8*c^2*d*e + 8*e^2)*x^4 - 2*(3*c^2*d^
2 - 4*d*e)*x^2 - 4*((c^2*d - 2*e)*x^3 - d*x)*sqrt(-c^2*d + e)*sqrt(e*x^2 +
d) + d^2)/(c^4*x^4 + 2*c^2*x^2 + 1)) + 4*sqrt(e*x^2 + d)*(b*c*arctan(c*x) +
 a*c))/(c*e), -1/2*(sqrt(c^2*d - e)*b*arctan(1/2*sqrt(c^2*d - e)*((c^2*d -
2*e)*x^2 - d)*sqrt(e*x^2 + d)/((c^2*d*e - e^2)*x^3 + (c^2*d^2 - d*e)*x)) -
2*b*sqrt(-e)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) - 2*sqrt(e*x^2 + d)*(b*c*ar
ctan(c*x) + a*c))/(c*e)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x\left(a+b\,\mathrm{atan}\,(cx)\right)}{\sqrt{d+ex^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atan(c*x))/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral(x*(a + b*atan(c*x))/sqrt(d + e*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \arctan(cx) + a)x}{\sqrt{ex^2 + d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)*x/sqrt(e*x^2 + d), x)
```

3.1204 $\int \frac{a+b\tan^{-1}(cx)}{\sqrt{d+ex^2}}\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{a+b\tan^{-1}(cx)}{\sqrt{d+ex^2}},x\right)$$

```
[Out] Unintegrable[(a + b*ArcTan[c*x])/Sqrt[d + e*x^2], x]
```

Rubi [A] time = 0.0249126, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{a+b\tan^{-1}(cx)}{\sqrt{d+ex^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcTan[c*x])/Sqrt[d + e*x^2],x]
```

```
[Out] Defer[Int][(a + b*ArcTan[c*x])/Sqrt[d + e*x^2], x]
```

Rubi steps

$$\int \frac{a+b\tan^{-1}(cx)}{\sqrt{d+ex^2}}\,dx=\int \frac{a+b\tan^{-1}(cx)}{\sqrt{d+ex^2}}\,dx$$

Mathematica [A] time = 3.43226, size = 0, normalized size = 0.

$$\int \frac{a+b\tan^{-1}(cx)}{\sqrt{d+ex^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcTan[c*x])/Sqrt[d + e*x^2],x]
```

```
[Out] Integrate[(a + b*ArcTan[c*x])/Sqrt[d + e*x^2], x]
```

Maple [A] time = 1.367, size = 0, normalized size = 0.

$$\int (a+b\arctan (cx))\frac{1}{\sqrt{e x^{2}+d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/(e*x^2+d)^(1/2),x)
```

```
[Out] int((a+b*arctan(c*x))/(e*x^2+d)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b\arctan (cx)+a}{\sqrt{e x^{2}+d}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*arctan(c*x) + a)/sqrt(e*x^2 + d), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{a+b\,\mathrm{atan}\,(cx)}{\sqrt{d+e x^{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*atan(c*x))/sqrt(d + e*x**2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\arctan(cx) + a}{\sqrt{ex^2 + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)/sqrt(e*x^2 + d), x)
```

3.1205 $\int \frac{a+b\tan^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx$

Optimal. Leaf size=50

$$b\text{Unintegrable}\left(\frac{\tan^{-1}(cx)}{x\sqrt{d+ex^2}},x\right)-\frac{a\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{\sqrt{d}}$$

```
[Out] -((a*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/Sqrt[d]) + b*Unintegrable[ArcTan[c*x
]/(x*Sqrt[d + e*x^2]), x]
```

Rubi [A] time = 0.163019, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{a+b\tan^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcTan[c*x])/(x*Sqrt[d + e*x^2]),x]
```

```
[Out] -((a*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/Sqrt[d]) + b*Defer[Int][ArcTan[c*x]/
(x*Sqrt[d + e*x^2]), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\tan^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx &= a\int \frac{1}{x\sqrt{d+ex^2}}\,dx+b\int \frac{\tan^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx\\
&= \frac{1}{2}a\,\text{Subst}\left(\int \frac{1}{x\sqrt{d+ex}}\,dx,x,x^2\right)+b\int \frac{\tan^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx\\
&= b\int \frac{\tan^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx+\frac{a\,\text{Subst}\left(\int \frac{1}{-\frac{d}{e}+\frac{x^2}{e}}\,dx,x,\sqrt{d+ex^2}\right)}{e}\\
&= -\frac{a\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{\sqrt{d}}+b\int \frac{\tan^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx
\end{aligned}$$

Mathematica [A] time = 5.31851, size = 0, normalized size = 0.

$$\int \frac{a+b\tan^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcTan[c*x])/(x*Sqrt[d + e*x^2]),x]
```

```
[Out] Integrate[(a + b*ArcTan[c*x])/(x*Sqrt[d + e*x^2]), x]
```

Maple [A] time = 0.783, size = 0, normalized size = 0.

$$\int \frac{a+b\arctan(cx)}{x}\frac{1}{\sqrt{ex^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/x/(e*x^2+d)^(1/2),x)
```

```
[Out] int((a+b*arctan(c*x))/x/(e*x^2+d)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{ex^2+d}(b\arctan(cx)+a)}{ex^3+dx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x^2 + d)*(b*arctan(c*x) + a)/(e*x^3 + d*x), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b\,\mathrm{atan}\,(cx)}{x\sqrt{d + ex^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/x/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*atan(c*x))/(x*sqrt(d + e*x**2)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\arctan(cx) + a}{\sqrt{ex^2 + d}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)/(sqrt(e*x^2 + d)*x), x)
```

3.1206 $\int \frac{a+b\tan^{-1}(cx)}{x^2\sqrt{d+ex^2}}\,dx$

Optimal. Leaf size=100

$$-\frac{\sqrt{d+ex^2}\left(a+b\tan^{-1}(cx)\right)}{dx}+\frac{b\sqrt{c^2d-e}\tanh^{-1}\left(\frac{c\sqrt{d+ex^2}}{\sqrt{c^2d-e}}\right)}{d}-\frac{bc\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{\sqrt{d}}$$

```
[Out] -((Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]))/(d*x)) - (b*c*ArcTanh[Sqrt[d + e*x^
2]/Sqrt[d]])/Sqrt[d] + (b*Sqrt[c^2*d - e]*ArcTanh[(c*Sqrt[d + e*x^2])/Sqrt[
c^2*d - e]])/d
```

Rubi [A] time = 0.176807, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {264, 4976, 446, 83, 63, 208}

$$-\frac{\sqrt{d+ex^2}\left(a+b\tan^{-1}(cx)\right)}{dx}+\frac{b\sqrt{c^2d-e}\tanh^{-1}\left(\frac{c\sqrt{d+ex^2}}{\sqrt{c^2d-e}}\right)}{d}-\frac{bc\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])/(x^2*Sqrt[d + e*x^2]),x]
```

```
[Out] -((Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]))/(d*x)) - (b*c*ArcTanh[Sqrt[d + e*x^
2]/Sqrt[d]])/Sqrt[d] + (b*Sqrt[c^2*d - e]*ArcTanh[(c*Sqrt[d + e*x^2])/Sqrt[
c^2*d - e]])/d
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 4976

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Dis
t[a + b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&  !(
ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(IL
```

```
tQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILtQ[(m
 - 1)/2, 0]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 83

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(a + b*x)
, x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(c + d*x), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{a+b\tan^{-1}(cx)}{x^2\sqrt{d+ex^2}}\,dx = -\frac{\sqrt{d+ex^2}\left(a+b\tan^{-1}(cx)\right)}{dx} - (bc)\int \frac{\sqrt{d+ex^2}}{x\left(-d-c^2dx^2\right)}\,dx$$
$$= -\frac{\sqrt{d+ex^2}\left(a+b\tan^{-1}(cx)\right)}{dx} - \frac{1}{2}(bc)\,\text{Subst}\left(\int \frac{\sqrt{d+ex}}{x\left(-d-c^2dx\right)}\,dx, x, x^2\right)$$
$$= -\frac{\sqrt{d+ex^2}\left(a+b\tan^{-1}(cx)\right)}{dx} + \frac{1}{2}(bc)\,\text{Subst}\left(\int \frac{1}{x\sqrt{d+ex}}\,dx, x, x^2\right) + \frac{1}{2}\left(bc\left(c^2d-e\right)\right)\text{Subst}\left(\int\right.$$
$$= -\frac{\sqrt{d+ex^2}\left(a+b\tan^{-1}(cx)\right)}{dx} + \frac{(bc)\,\text{Subst}\left(\int \frac{1}{-\frac{d}{e}+\frac{x^2}{e}}\,dx, x, \sqrt{d+ex^2}\right)}{e} + \frac{\left(bc\left(c^2d-e\right)\right)\text{Subst}\left(\int\right.}{}$$
$$= -\frac{\sqrt{d+ex^2}\left(a+b\tan^{-1}(cx)\right)}{dx} - \frac{bc\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{\sqrt{d}} + \frac{b\sqrt{c^2d-e}\tanh^{-1}\left(\frac{c\sqrt{d+ex^2}}{\sqrt{c^2d-e}}\right)}{d}$$

Mathematica [C] time = 0.418306, size = 247, normalized size = 2.47

$$\frac{-2a\sqrt{d+ex^2} + bx\sqrt{c^2d-e}\log\left(-\frac{4cd\left(\sqrt{c^2d-e}\sqrt{d+ex^2}+cd-iex\right)}{b(cx+i)\left(c^2d-e\right)^{3/2}}\right) + bx\sqrt{c^2d-e}\log\left(-\frac{4cd\left(\sqrt{c^2d-e}\sqrt{d+ex^2}+cd+iex\right)}{b(cx-i)\left(c^2d-e\right)^{3/2}}\right) - 2bc\sqrt{d}x\log\left(\sqrt{d}\right.}{2dx}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])/(x^2*Sqrt[d + e*x^2]),x]
```

```
[Out] (-2*a*Sqrt[d + e*x^2] - 2*b*Sqrt[d + e*x^2]*ArcTan[c*x] + 2*b*c*Sqrt[d]*x*L
og[x] - 2*b*c*Sqrt[d]*x*Log[d + Sqrt[d]*Sqrt[d + e*x^2]] + b*Sqrt[c^2*d - e
]*x*Log[(-4*c*d*(c*d - I*e*x + Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*(c^2*d
- e)^(3/2)*(I + c*x))] + b*Sqrt[c^2*d - e]*x*Log[(-4*c*d*(c*d + I*e*x + Sqr
t[c^2*d - e]*Sqrt[d + e*x^2]))/(b*(c^2*d - e)^(3/2)*(-I + c*x))])/(2*d*x)
```

Maple [F] time = 0.81, size = 0, normalized size = 0.

$$\int \frac{a+b\arctan(cx)}{x^2}\frac{1}{\sqrt{ex^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/x^2/(e*x^2+d)^(1/2),x)
```

```
[Out] int((a+b*arctan(c*x))/x^2/(e*x^2+d)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^2/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.4444, size = 1504, normalized size = 15.04

$$\left[\frac{2\,bc\sqrt{d}x\log\left(-\frac{ex^2-2\,\sqrt{ex^2+d}\sqrt{d}+2\,d}{x^2}\right)+\sqrt{c^2d-e}\,bx\log\left(\frac{c^4e^2x^4+8\,c^4d^2-8\,c^2de+2\,\left(4\,c^4de-3\,c^2e^2\right)x^2+4\,\left(c^3ex^2+2\,c^3d-ce\right)\sqrt{c^2d-e}\sqrt{ex^2+d}+e^2}{c^4x^4+2\,c^2x^2+1}\right)-4}{4\,dx}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^2/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*b*c*sqrt(d)*x*log(-(e*x^2 - 2*sqrt(e*x^2 + d)*sqrt(d) + 2*d)/x^2) +
 sqrt(c^2*d - e)*b*x*log((c^4*e^2*x^4 + 8*c^4*d^2 - 8*c^2*d*e + 2*(4*c^4*d*
e - 3*c^2*e^2)*x^2 + 4*(c^3*e*x^2 + 2*c^3*d - c*e)*sqrt(c^2*d - e)*sqrt(e*x
^2 + d) + e^2)/(c^4*x^4 + 2*c^2*x^2 + 1)) - 4*sqrt(e*x^2 + d)*(b*arctan(c*x
) + a))/(d*x), 1/2*(b*c*sqrt(d)*x*log(-(e*x^2 - 2*sqrt(e*x^2 + d)*sqrt(d) +
 2*d)/x^2) + sqrt(-c^2*d + e)*b*x*arctan(-1/2*(c^2*e*x^2 + 2*c^2*d - e)*sqr
t(-c^2*d + e)*sqrt(e*x^2 + d)/(c^3*d^2 - c*d*e + (c^3*d*e - c*e^2)*x^2)) -
2*sqrt(e*x^2 + d)*(b*arctan(c*x) + a))/(d*x), 1/4*(4*b*c*sqrt(-d)*x*arctan(
sqrt(-d)/sqrt(e*x^2 + d)) + sqrt(c^2*d - e)*b*x*log((c^4*e^2*x^4 + 8*c^4*d^
2 - 8*c^2*d*e + 2*(4*c^4*d*e - 3*c^2*e^2)*x^2 + 4*(c^3*e*x^2 + 2*c^3*d - c*
e)*sqrt(c^2*d - e)*sqrt(e*x^2 + d) + e^2)/(c^4*x^4 + 2*c^2*x^2 + 1)) - 4*sq
rt(e*x^2 + d)*(b*arctan(c*x) + a))/(d*x), 1/2*(2*b*c*sqrt(-d)*x*arctan(sqrt
(-d)/sqrt(e*x^2 + d)) + sqrt(-c^2*d + e)*b*x*arctan(-1/2*(c^2*e*x^2 + 2*c^2
*d - e)*sqrt(-c^2*d + e)*sqrt(e*x^2 + d)/(c^3*d^2 - c*d*e + (c^3*d*e - c*e^
```

```
2)*x^2)) - 2*sqrt(e*x^2 + d)*(b*arctan(c*x) + a))/(d*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b \operatorname{atan}(cx)}{x^2 \sqrt{d + ex^2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/x**2/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*atan(c*x))/(x**2*sqrt(d + e*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \arctan(cx) + a}{\sqrt{ex^2 + d}x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^2/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)/(sqrt(e*x^2 + d)*x^2), x)
```

3.1207 $\int \frac{a+b\tan^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx$

Optimal. Leaf size=75

$$b\text{Unintegrable}\left(\frac{\tan^{-1}(cx)}{x^3\sqrt{d+ex^2}},x\right)+\frac{ae\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{2d^{3/2}}-\frac{a\sqrt{d+ex^2}}{2dx^2}$$

```
[Out] -(a*Sqrt[d + e*x^2])/(2*d*x^2) + (a*e*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/(2*
d^(3/2)) + b*Unintegrable[ArcTan[c*x]/(x^3*Sqrt[d + e*x^2]), x]
```

Rubi [A] time = 0.172151, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{a+b\tan^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcTan[c*x])/(x^3*Sqrt[d + e*x^2]),x]
```

```
[Out] -(a*Sqrt[d + e*x^2])/(2*d*x^2) + (a*e*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/(2*
d^(3/2)) + b*Defer[Int][ArcTan[c*x]/(x^3*Sqrt[d + e*x^2]), x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+b\tan^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx &= a\int \frac{1}{x^3\sqrt{d+ex^2}}\,dx + b\int \frac{\tan^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx \\
&= \frac{1}{2}a\,\mathrm{Subst}\left(\int \frac{1}{x^2\sqrt{d+ex}}\,dx, x, x^2\right) + b\int \frac{\tan^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx \\
&= -\frac{a\sqrt{d+ex^2}}{2dx^2} + b\int \frac{\tan^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx - \frac{(ae)\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{d+ex}}\,dx, x, x^2\right)}{4d} \\
&= -\frac{a\sqrt{d+ex^2}}{2dx^2} + b\int \frac{\tan^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx - \frac{a\,\mathrm{Subst}\left(\int \frac{1}{-\frac{d}{e}+\frac{x^2}{e}}\,dx, x, \sqrt{d+ex^2}\right)}{2d} \\
&= -\frac{a\sqrt{d+ex^2}}{2dx^2} + \frac{ae\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{2d^{3/2}} + b\int \frac{\tan^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx
\end{aligned}
$$

Mathematica [A] time = 50.9552, size = 0, normalized size = 0.

$$
\int \frac{a+b\tan^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx
$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcTan[c*x])/(x^3*Sqrt[d + e*x^2]),x]
```

```
[Out] Integrate[(a + b*ArcTan[c*x])/(x^3*Sqrt[d + e*x^2]), x]
```

Maple [A] time = 0.776, size = 0, normalized size = 0.

$$
\int \frac{a+b\arctan(cx)}{x^3}\frac{1}{\sqrt{ex^2+d}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/x^3/(e*x^2+d)^(1/2),x)
```

```
[Out] int((a+b*arctan(c*x))/x^3/(e*x^2+d)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^3/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{ex^2+d}(b\arctan(cx)+a)}{ex^5+dx^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^3/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x^2 + d)*(b*arctan(c*x) + a)/(e*x^5 + d*x^3), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{a+b\,\text{atan}(cx)}{x^3\sqrt{d+ex^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/x**3/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*atan(c*x))/(x**3*sqrt(d + e*x**2)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{b\arctan(cx)+a}{\sqrt{ex^2+d}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^3/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)/(sqrt(e*x^2 + d)*x^3), x)
```

3.1208 $\int \frac{a+b\tan^{-1}(cx)}{x^4\sqrt{d+ex^2}}\,dx$

Optimal. Leaf size=179

$$\frac{2e\sqrt{d+ex^2}\left(a+b\tan^{-1}(cx)\right)}{3d^2x}-\frac{\sqrt{d+ex^2}\left(a+b\tan^{-1}(cx)\right)}{3dx^3}+\frac{bc\left(2c^2d+3e\right)\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{6d^{3/2}}-\frac{b\sqrt{c^2d-e}\left(c^2d+2e\right)\text{ta}}{3d^2}$$

```
[Out] -(b*c*Sqrt[d + e*x^2])/(6*d*x^2) - (Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]))/(3
*d*x^3) + (2*e*Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]))/(3*d^2*x) + (b*c*(2*c^2
*d + 3*e)*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/(6*d^(3/2)) - (b*Sqrt[c^2*d - e
]*(c^2*d + 2*e)*ArcTanh[(c*Sqrt[d + e*x^2])/Sqrt[c^2*d - e]])/(3*d^2)
```

Rubi [A] time = 0.266089, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {271, 264, 4976, 12, 573, 149, 156, 63, 208}

$$\frac{2e\sqrt{d+ex^2}\left(a+b\tan^{-1}(cx)\right)}{3d^2x}-\frac{\sqrt{d+ex^2}\left(a+b\tan^{-1}(cx)\right)}{3dx^3}+\frac{bc\left(2c^2d+3e\right)\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{6d^{3/2}}-\frac{b\sqrt{c^2d-e}\left(c^2d+2e\right)\text{ta}}{3d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])/(x^4*Sqrt[d + e*x^2]),x]
```


```
[Out] -(b*c*Sqrt[d + e*x^2])/(6*d*x^2) - (Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]))/(3
*d*x^3) + (2*e*Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]))/(3*d^2*x) + (b*c*(2*c^2
*d + 3*e)*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/(6*d^(3/2)) - (b*Sqrt[c^2*d - e
]*(c^2*d + 2*e)*ArcTanh[(c*Sqrt[d + e*x^2])/Sqrt[c^2*d - e]])/(3*d^2)
```


Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
```

```
p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 4976

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Dis
t[a + b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&  !(
ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(IL
tQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILtQ[(m
 - 1)/2, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 573

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simpl
ify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /;
 FreeQ[{a, b, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[Simplify[(m + 1)/n
]]
```

Rule 149

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegerQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{a+b\tan^{-1}(cx)}{x^4\sqrt{d+ex^2}}\,dx = -\frac{\sqrt{d+ex^2}\left(a+b\tan^{-1}(cx)\right)}{3dx^3}+\frac{2e\sqrt{d+ex^2}\left(a+b\tan^{-1}(cx)\right)}{3d^2x}-(bc)\int\frac{\sqrt{d+ex^2}\left(-d+2ex^2\right)}{3d^2x^3\left(1+c^2x^2\right)}\,d$$

$$= -\frac{\sqrt{d+ex^2}\left(a+b\tan^{-1}(cx)\right)}{3dx^3}+\frac{2e\sqrt{d+ex^2}\left(a+b\tan^{-1}(cx)\right)}{3d^2x}-\frac{(bc)\int\frac{\sqrt{d+ex^2}\left(-d+2ex^2\right)}{x^3\left(1+c^2x^2\right)}\,dx}{3d^2}$$

$$= -\frac{\sqrt{d+ex^2}\left(a+b\tan^{-1}(cx)\right)}{3dx^3}+\frac{2e\sqrt{d+ex^2}\left(a+b\tan^{-1}(cx)\right)}{3d^2x}-\frac{(bc)\,\mathrm{Subst}\left(\int\frac{\sqrt{d+ex}(-d+2ex)}{x^2\left(1+c^2x\right)}\,dx,\right.}{6d^2}$$

$$= -\frac{bc\sqrt{d+ex^2}}{6dx^2}-\frac{\sqrt{d+ex^2}\left(a+b\tan^{-1}(cx)\right)}{3dx^3}+\frac{2e\sqrt{d+ex^2}\left(a+b\tan^{-1}(cx)\right)}{3d^2x}-\frac{(bc)\,\mathrm{Subst}\left(\int\frac{1}{2}\right.}{}$$

$$= -\frac{bc\sqrt{d+ex^2}}{6dx^2}-\frac{\sqrt{d+ex^2}\left(a+b\tan^{-1}(cx)\right)}{3dx^3}+\frac{2e\sqrt{d+ex^2}\left(a+b\tan^{-1}(cx)\right)}{3d^2x}+\frac{\left(bc\left(c^2d-e\right)\left(c^2\right.\right.}{}$$

$$= -\frac{bc\sqrt{d+ex^2}}{6dx^2}-\frac{\sqrt{d+ex^2}\left(a+b\tan^{-1}(cx)\right)}{3dx^3}+\frac{2e\sqrt{d+ex^2}\left(a+b\tan^{-1}(cx)\right)}{3d^2x}+\frac{\left(bc\left(c^2d-e\right)\left(c^2\right.\right.}{}$$

$$= -\frac{bc\sqrt{d+ex^2}}{6dx^2}-\frac{\sqrt{d+ex^2}\left(a+b\tan^{-1}(cx)\right)}{3dx^3}+\frac{2e\sqrt{d+ex^2}\left(a+b\tan^{-1}(cx)\right)}{3d^2x}+\frac{bc\left(2c^2d+3e\right)\mathrm{t}}{6d}$$

Mathematica [C] time = 0.485693, size = 372, normalized size = 2.08

$$-\frac{\frac{\sqrt{d+ex^2}\left(2a\left(d-2ex^2\right)+bcdx\right)}{x^3}+\frac{b\left(c^4d^2+c^2de-2e^2\right)\log\left(\frac{12cd^2\left(\sqrt{c^2d-e}\sqrt{d+ex^2}+cd-iex\right)}{b(cx+i)\sqrt{c^2d-e}\left(c^4d^2+c^2de-2e^2\right)}\right)}{\sqrt{c^2d-e}}+\frac{b\left(c^4d^2+c^2de-2e^2\right)\log\left(\frac{12cd^2\left(\sqrt{c^2d-e}\sqrt{d+ex^2}+cd+iex\right)}{b(cx-i)\sqrt{c^2d-e}\left(c^4d^2+c^2de-2e^2\right)}\right)}{\sqrt{c^2d-e}}-bc\sqrt{d}\left(2c^2\right.}{6d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])/(x^4*Sqrt[d + e*x^2]),x]
```

```
[Out] -((Sqrt[d + e*x^2]*(b*c*d*x + 2*a*(d - 2*e*x^2)))/x^3 + (2*b*(d - 2*e*x^2)*
Sqrt[d + e*x^2]*ArcTan[c*x])/x^3 + b*c*Sqrt[d]*(2*c^2*d + 3*e)*Log[x] - b*c
*Sqrt[d]*(2*c^2*d + 3*e)*Log[d + Sqrt[d]*Sqrt[d + e*x^2]] + (b*(c^4*d^2 + c
^2*d*e - 2*e^2)*Log[(12*c*d^2*(c*d - I*e*x + Sqrt[c^2*d - e]*Sqrt[d + e*x^2
]))/(b*Sqrt[c^2*d - e]*(c^4*d^2 + c^2*d*e - 2*e^2)*(I + c*x))])/Sqrt[c^2*d
- e] + (b*(c^4*d^2 + c^2*d*e - 2*e^2)*Log[(12*c*d^2*(c*d + I*e*x + Sqrt[c^2
*d - e]*Sqrt[d + e*x^2]))/(b*Sqrt[c^2*d - e]*(c^4*d^2 + c^2*d*e - 2*e^2)*(-
I + c*x))])/Sqrt[c^2*d - e])/(6*d^2)
```

Maple [F] time = 0.837, size = 0, normalized size = 0.

$$\int \frac{a + b\arctan(cx)}{x^4} \frac{1}{\sqrt{ex^2 + d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/x^4/(e*x^2+d)^(1/2),x)
```

```
[Out] int((a+b*arctan(c*x))/x^4/(e*x^2+d)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^4/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.8863, size = 1971, normalized size = 11.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^4/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*((b*c^2*d + 2*b*e)*sqrt(c^2*d - e)*x^3*log((c^4*e^2*x^4 + 8*c^4*d^2 -
 8*c^2*d*e + 2*(4*c^4*d*e - 3*c^2*e^2)*x^2 - 4*(c^3*e*x^2 + 2*c^3*d - c*e)*
sqrt(c^2*d - e)*sqrt(e*x^2 + d) + e^2)/(c^4*x^4 + 2*c^2*x^2 + 1)) + (2*b*c^
3*d + 3*b*c*e)*sqrt(d)*x^3*log(-(e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(d) + 2*d)/x
^2) - 2*(b*c*d*x - 4*a*e*x^2 + 2*a*d - 2*(2*b*e*x^2 - b*d)*arctan(c*x))*sqr
t(e*x^2 + d))/(d^2*x^3), -1/12*(2*(b*c^2*d + 2*b*e)*sqrt(-c^2*d + e)*x^3*ar
ctan(-1/2*(c^2*e*x^2 + 2*c^2*d - e)*sqrt(-c^2*d + e)*sqrt(e*x^2 + d)/(c^3*d
^2 - c*d*e + (c^3*d*e - c*e^2)*x^2)) - (2*b*c^3*d + 3*b*c*e)*sqrt(d)*x^3*lo
g(-(e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(d) + 2*d)/x^2) + 2*(b*c*d*x - 4*a*e*x^2
+ 2*a*d - 2*(2*b*e*x^2 - b*d)*arctan(c*x))*sqrt(e*x^2 + d))/(d^2*x^3), -1/1
2*(2*(2*b*c^3*d + 3*b*c*e)*sqrt(-d)*x^3*arctan(sqrt(-d)/sqrt(e*x^2 + d)) -
(b*c^2*d + 2*b*e)*sqrt(c^2*d - e)*x^3*log((c^4*e^2*x^4 + 8*c^4*d^2 - 8*c^2*
d*e + 2*(4*c^4*d*e - 3*c^2*e^2)*x^2 - 4*(c^3*e*x^2 + 2*c^3*d - c*e)*sqrt(c^
2*d - e)*sqrt(e*x^2 + d) + e^2)/(c^4*x^4 + 2*c^2*x^2 + 1)) + 2*(b*c*d*x - 4
*a*e*x^2 + 2*a*d - 2*(2*b*e*x^2 - b*d)*arctan(c*x))*sqrt(e*x^2 + d))/(d^2*x
^3), -1/6*((b*c^2*d + 2*b*e)*sqrt(-c^2*d + e)*x^3*arctan(-1/2*(c^2*e*x^2 +
2*c^2*d - e)*sqrt(-c^2*d + e)*sqrt(e*x^2 + d)/(c^3*d^2 - c*d*e + (c^3*d*e -
 c*e^2)*x^2)) + (2*b*c^3*d + 3*b*c*e)*sqrt(-d)*x^3*arctan(sqrt(-d)/sqrt(e*x
^2 + d)) + (b*c*d*x - 4*a*e*x^2 + 2*a*d - 2*(2*b*e*x^2 - b*d)*arctan(c*x))*
sqrt(e*x^2 + d))/(d^2*x^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b\,\mathrm{atan}\,(cx)}{x^4\sqrt{d + ex^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/x**4/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*atan(c*x))/(x**4*sqrt(d + e*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\arctan\,(cx) + a}{\sqrt{ex^2 + dx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^4/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)/(sqrt(e*x^2 + d)*x^4), x)
```

3.1209 $\int \frac{x^3\left(a+b\tan^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=137

$$\frac{\sqrt{d+ex^2}\left(a+b\tan^{-1}(cx)\right)}{e^2}+\frac{d\left(a+b\tan^{-1}(cx)\right)}{e^2\sqrt{d+ex^2}}-\frac{b\left(2c^2d-e\right)\tan^{-1}\left(\frac{x\sqrt{c^2d-e}}{\sqrt{d+ex^2}}\right)}{ce^2\sqrt{c^2d-e}}-\frac{b\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{ce^{3/2}}$$

```
[Out] (d*(a + b*ArcTan[c*x]))/(e^2*Sqrt[d + e*x^2]) + (Sqrt[d + e*x^2]*(a + b*Arc
Tan[c*x]))/e^2 - (b*(2*c^2*d - e)*ArcTan[(Sqrt[c^2*d - e]*x)/Sqrt[d + e*x^2
]])/(c*Sqrt[c^2*d - e]*e^2) - (b*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(c*e
^(3/2))
```

Rubi [A] time = 0.184364, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {266, 43, 4976, 12, 523, 217, 206, 377, 203}

$$\frac{\sqrt{d+ex^2}\left(a+b\tan^{-1}(cx)\right)}{e^2}+\frac{d\left(a+b\tan^{-1}(cx)\right)}{e^2\sqrt{d+ex^2}}-\frac{b\left(2c^2d-e\right)\tan^{-1}\left(\frac{x\sqrt{c^2d-e}}{\sqrt{d+ex^2}}\right)}{ce^2\sqrt{c^2d-e}}-\frac{b\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{ce^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcTan[c*x]))/(d + e*x^2)^(3/2),x]
```

```
[Out] (d*(a + b*ArcTan[c*x]))/(e^2*Sqrt[d + e*x^2]) + (Sqrt[d + e*x^2]*(a + b*Arc
Tan[c*x]))/e^2 - (b*(2*c^2*d - e)*ArcTan[(Sqrt[c^2*d - e]*x)/Sqrt[d + e*x^2
]])/(c*Sqrt[c^2*d - e]*e^2) - (b*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(c*e
^(3/2))
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 4976

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Dis
t[a + b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&  !(
ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(IL
tQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILtQ[(m
 - 1)/2, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{x^3\left(a+b\tan^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx = \frac{d\left(a+b\tan^{-1}(cx)\right)}{e^2\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}\left(a+b\tan^{-1}(cx)\right)}{e^2} - (bc)\int \frac{2d+ex^2}{e^2\left(1+c^2x^2\right)\sqrt{d+ex^2}}\,dx$$

$$= \frac{d\left(a+b\tan^{-1}(cx)\right)}{e^2\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}\left(a+b\tan^{-1}(cx)\right)}{e^2} - \frac{(bc)\int \frac{2d+ex^2}{\left(1+c^2x^2\right)\sqrt{d+ex^2}}\,dx}{e^2}$$

$$= \frac{d\left(a+b\tan^{-1}(cx)\right)}{e^2\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}\left(a+b\tan^{-1}(cx)\right)}{e^2} - \frac{b\int \frac{1}{\sqrt{d+ex^2}}\,dx}{ce} - \frac{\left(bc\left(2d-\frac{e}{c^2}\right)\right)\int \frac{}{(1+c^2}}{e^2}$$

$$= \frac{d\left(a+b\tan^{-1}(cx)\right)}{e^2\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}\left(a+b\tan^{-1}(cx)\right)}{e^2} - \frac{b\,\text{Subst}\left(\int \frac{1}{1-ex^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{ce} - \frac{(bc}{}$$

$$= \frac{d\left(a+b\tan^{-1}(cx)\right)}{e^2\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}\left(a+b\tan^{-1}(cx)\right)}{e^2} - \frac{bc\left(2d-\frac{e}{c^2}\right)\tan^{-1}\left(\frac{\sqrt{c^2d-ex}}{\sqrt{d+ex^2}}\right)}{\sqrt{c^2d-e}\,e^2} - \frac{b\tanh}{}$$

Mathematica [C] time = 0.615544, size = 321, normalized size = 2.34

$$\frac{\frac{2a\left(2d+ex^2\right)}{\sqrt{d+ex^2}} - \frac{ib\left(2c^2d-e\right)\log\left(\frac{4c^2e^2\left(-i\sqrt{c^2d-e}\sqrt{d+ex^2}-icd+ex\right)}{b(cx-i)\sqrt{c^2d-e}\left(2c^2d-e\right)}\right)}{c\sqrt{c^2d-e}} + \frac{ib\left(2c^2d-e\right)\log\left(\frac{4c^2e^2\left(i\sqrt{c^2d-e}\sqrt{d+ex^2}+icd+ex\right)}{b(cx+i)\sqrt{c^2d-e}\left(2c^2d-e\right)}\right)}{c\sqrt{c^2d-e}} - \frac{2b\sqrt{e}\log\left(\sqrt{e}\sqrt{d+ex^2}+ex\right)}{c} + \frac{2b\tan^{-1}(cx)(2}{\sqrt{d+ex^2}}}{2e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcTan[c*x]))/(d + e*x^2)^(3/2),x]
```

```
[Out] ((2*a*(2*d + e*x^2))/Sqrt[d + e*x^2] + (2*b*(2*d + e*x^2)*ArcTan[c*x])/Sqrt
[d + e*x^2] - (I*b*(2*c^2*d - e)*Log[(4*c^2*e^2*((-I)*c*d + e*x - I*Sqrt[c^
2*d - e]*Sqrt[d + e*x^2]))/(b*Sqrt[c^2*d - e]*(2*c^2*d - e)*(-I + c*x))]])/(
c*Sqrt[c^2*d - e]) + (I*b*(2*c^2*d - e)*Log[(4*c^2*e^2*(I*c*d + e*x + I*Sqr
t[c^2*d - e]*Sqrt[d + e*x^2]))/(b*Sqrt[c^2*d - e]*(2*c^2*d - e)*(I + c*x))]
)/(c*Sqrt[c^2*d - e]) - (2*b*Sqrt[e]*Log[e*x + Sqrt[e]*Sqrt[d + e*x^2]])/c)
/(2*e^2)
```

Maple [F] time = 0.605, size = 0, normalized size = 0.

$$\int x^3 (a + b \arctan (cx)) \left(e x^2 + d\right)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctan(c*x))/(e*x^2+d)^(3/2),x)
```

```
[Out] int(x^3*(a+b*arctan(c*x))/(e*x^2+d)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 5.2971, size = 2724, normalized size = 19.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*(b*c^2*d^2 - b*d*e + (b*c^2*d*e - b*e^2)*x^2)*sqrt(e)*log(-2*e*x^2
+ 2*sqrt(e*x^2 + d)*sqrt(e)*x - d) + (2*b*c^2*d^2 - b*d*e + (2*b*c^2*d*e -
b*e^2)*x^2)*sqrt(-c^2*d + e)*log(((c^4*d^2 - 8*c^2*d*e + 8*e^2)*x^4 - 2*(3*
c^2*d^2 - 4*d*e)*x^2 - 4*((c^2*d - 2*e)*x^3 - d*x)*sqrt(-c^2*d + e)*sqrt(e*
x^2 + d) + d^2)/(c^4*x^4 + 2*c^2*x^2 + 1)) + 4*(2*a*c^3*d^2 - 2*a*c*d*e + (
a*c^3*d*e - a*c*e^2)*x^2 + (2*b*c^3*d^2 - 2*b*c*d*e + (b*c^3*d*e - b*c*e^2)
*x^2)*arctan(c*x))*sqrt(e*x^2 + d))/(c^3*d^2*e^2 - c*d*e^3 + (c^3*d*e^3 - c
*e^4)*x^2), -1/2*((2*b*c^2*d^2 - b*d*e + (2*b*c^2*d*e - b*e^2)*x^2)*sqrt(c^
2*d - e)*arctan(1/2*sqrt(c^2*d - e)*((c^2*d - 2*e)*x^2 - d)*sqrt(e*x^2 + d)
```

```
/((c^2*d*e - e^2)*x^3 + (c^2*d^2 - d*e)*x)) - (b*c^2*d^2 - b*d*e + (b*c^2*d
*e - b*e^2)*x^2)*sqrt(e)*log(-2*e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(e)*x - d) -
2*(2*a*c^3*d^2 - 2*a*c*d*e + (a*c^3*d*e - a*c*e^2)*x^2 + (2*b*c^3*d^2 - 2*b
*c*d*e + (b*c^3*d*e - b*c*e^2)*x^2)*arctan(c*x))*sqrt(e*x^2 + d))/(c^3*d^2*
e^2 - c*d*e^3 + (c^3*d*e^3 - c*e^4)*x^2), 1/4*(4*(b*c^2*d^2 - b*d*e + (b*c^
2*d*e - b*e^2)*x^2)*sqrt(-e)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) + (2*b*c^2*
d^2 - b*d*e + (2*b*c^2*d*e - b*e^2)*x^2)*sqrt(-c^2*d + e)*log(((c^4*d^2 - 8
*c^2*d*e + 8*e^2)*x^4 - 2*(3*c^2*d^2 - 4*d*e)*x^2 - 4*((c^2*d - 2*e)*x^3 -
d*x)*sqrt(-c^2*d + e)*sqrt(e*x^2 + d) + d^2)/(c^4*x^4 + 2*c^2*x^2 + 1)) + 4
*(2*a*c^3*d^2 - 2*a*c*d*e + (a*c^3*d*e - a*c*e^2)*x^2 + (2*b*c^3*d^2 - 2*b*
c*d*e + (b*c^3*d*e - b*c*e^2)*x^2)*arctan(c*x))*sqrt(e*x^2 + d))/(c^3*d^2*e
^2 - c*d*e^3 + (c^3*d*e^3 - c*e^4)*x^2), -1/2*((2*b*c^2*d^2 - b*d*e + (2*b*
c^2*d*e - b*e^2)*x^2)*sqrt(c^2*d - e)*arctan(1/2*sqrt(c^2*d - e)*((c^2*d -
2*e)*x^2 - d)*sqrt(e*x^2 + d)/((c^2*d*e - e^2)*x^3 + (c^2*d^2 - d*e)*x)) -
2*(b*c^2*d^2 - b*d*e + (b*c^2*d*e - b*e^2)*x^2)*sqrt(-e)*arctan(sqrt(-e)*x/
sqrt(e*x^2 + d)) - 2*(2*a*c^3*d^2 - 2*a*c*d*e + (a*c^3*d*e - a*c*e^2)*x^2 +
 (2*b*c^3*d^2 - 2*b*c*d*e + (b*c^3*d*e - b*c*e^2)*x^2)*arctan(c*x))*sqrt(e*
x^2 + d))/(c^3*d^2*e^2 - c*d*e^3 + (c^3*d*e^3 - c*e^4)*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 (a + b \operatorname{atan}(cx))}{\left(d + ex^2\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atan(c*x))/(e*x**2+d)**(3/2),x)
```

```
[Out] Integral(x**3*(a + b*atan(c*x))/(d + e*x**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \arctan(cx) + a)x^3}{\left(ex^2 + d\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)*x^3/(e*x^2 + d)^(3/2), x)
```

3.1210 $\int \frac{x^2(a+b\tan^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx$

Optimal. Leaf size=68

$$b\text{Unintegrable}\left(\frac{x^2\tan^{-1}(cx)}{(d+ex^2)^{3/2}},x\right)+\frac{a\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{e^{3/2}}-\frac{ax}{e\sqrt{d+ex^2}}$$

```
[Out] -((a*x)/(e*Sqrt[d + e*x^2])) + (a*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/e^(
3/2) + b*Unintegrable[(x^2*ArcTan[c*x])/(d + e*x^2)^(3/2), x]
```

Rubi [A] time = 0.167533, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^2\left(a+b\tan^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^2*(a + b*ArcTan[c*x]))/(d + e*x^2)^(3/2),x]
```

```
[Out] -((a*x)/(e*Sqrt[d + e*x^2])) + (a*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/e^(
3/2) + b*Defer[Int][(x^2*ArcTan[c*x])/(d + e*x^2)^(3/2), x]
```

Rubi steps

$$\int \frac{x^2 \left(a+b\tan^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx = a\int \frac{x^2}{\left(d+ex^2\right)^{3/2}}\,dx + b\int \frac{x^2\tan^{-1}(cx)}{\left(d+ex^2\right)^{3/2}}\,dx$$

$$= -\frac{ax}{e\sqrt{d+ex^2}} + b\int \frac{x^2\tan^{-1}(cx)}{\left(d+ex^2\right)^{3/2}}\,dx + \frac{a\int \frac{1}{\sqrt{d+ex^2}}\,dx}{e}$$

$$= -\frac{ax}{e\sqrt{d+ex^2}} + b\int \frac{x^2\tan^{-1}(cx)}{\left(d+ex^2\right)^{3/2}}\,dx + \frac{a\,\text{Subst}\left(\int \frac{1}{1-ex^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{e}$$

$$= -\frac{ax}{e\sqrt{d+ex^2}} + \frac{a\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{e^{3/2}} + b\int \frac{x^2\tan^{-1}(cx)}{\left(d+ex^2\right)^{3/2}}\,dx$$

Mathematica [A] time = 18.2179, size = 0, normalized size = 0.

$$\int \frac{x^2 \left(a+b\tan^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^2*(a + b*ArcTan[c*x]))/(d + e*x^2)^(3/2),x]
```

```
[Out] Integrate[(x^2*(a + b*ArcTan[c*x]))/(d + e*x^2)^(3/2), x]
```

Maple [A] time = 0.598, size = 0, normalized size = 0.

$$\int x^2 \left(a+b\arctan(cx)\right)\left(ex^2+d\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctan(c*x))/(e*x^2+d)^(3/2),x)
```

```
[Out] int(x^2*(a+b*arctan(c*x))/(e*x^2+d)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^2\arctan\left(cx\right)+ax^2\right)\sqrt{ex^2+d}}{e^2x^4+2\,dex^2+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^2*arctan(c*x) + a*x^2)*sqrt(e*x^2 + d)/(e^2*x^4 + 2*d*e*x^2 +
 d^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atan(c*x))/(e*x**2+d)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(b\arctan\left(cx\right)+a\right)x^2}{\left(ex^2+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)*x^2/(e*x^2 + d)^(3/2), x)
```

3.1211 $\int \frac{x\left(a+b\tan^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=71

$$\frac{bc\tan^{-1}\left(\frac{x\sqrt{c^2d-e}}{\sqrt{d+ex^2}}\right)}{e\sqrt{c^2d-e}}-\frac{a+b\tan^{-1}(cx)}{e\sqrt{d+ex^2}}$$

```
[Out] -((a + b*ArcTan[c*x])/(e*Sqrt[d + e*x^2])) + (b*c*ArcTan[(Sqrt[c^2*d - e]*x
)/Sqrt[d + e*x^2]])/(Sqrt[c^2*d - e]*e)
```

Rubi [A] time = 0.0739139, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4974, 377, 203}

$$\frac{bc\tan^{-1}\left(\frac{x\sqrt{c^2d-e}}{\sqrt{d+ex^2}}\right)}{e\sqrt{c^2d-e}}-\frac{a+b\tan^{-1}(cx)}{e\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcTan[c*x]))/(d + e*x^2)^(3/2),x]
```

```
[Out] -((a + b*ArcTan[c*x])/(e*Sqrt[d + e*x^2])) + (b*c*ArcTan[(Sqrt[c^2*d - e]*x
)/Sqrt[d + e*x^2]])/(Sqrt[c^2*d - e]*e)
```

Rule 4974

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]))/(2*e*(q + 1)), x
] - Dist[(b*c)/(2*e*(q + 1)), Int[(d + e*x^2)^(q + 1)/(1 + c^2*x^2), x], x]
 /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{x\left(a+b\tan^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx=-\frac{a+b\tan^{-1}(cx)}{e\sqrt{d+ex^2}}+\frac{(bc)\int\frac{1}{\left(1+c^2x^2\right)\sqrt{d+ex^2}}\,dx}{e}$$

$$=-\frac{a+b\tan^{-1}(cx)}{e\sqrt{d+ex^2}}+\frac{(bc)\,\mathrm{Subst}\left(\int\frac{1}{1-\left(-c^2d+e\right)x^2}\,dx,x,\frac{x}{\sqrt{d+ex^2}}\right)}{e}$$

$$=-\frac{a+b\tan^{-1}(cx)}{e\sqrt{d+ex^2}}+\frac{bc\tan^{-1}\left(\frac{\sqrt{c^2d-ex}}{\sqrt{d+ex^2}}\right)}{\sqrt{c^2d-ee}}$$

Mathematica [C] time = 0.36672, size = 210, normalized size = 2.96

$$-\frac{\frac{2a}{\sqrt{d+ex^2}}+\frac{ibc\log\left(-\frac{4ie\left(\sqrt{c^2d-e}\sqrt{d+ex^2}+cd-iex\right)}{b(cx+i)\sqrt{c^2d-e}}\right)}{\sqrt{c^2d-e}}-\frac{ibc\log\left(\frac{4ie\left(\sqrt{c^2d-e}\sqrt{d+ex^2}+cd+iex\right)}{b(cx-i)\sqrt{c^2d-e}}\right)}{\sqrt{c^2d-e}}+\frac{2b\tan^{-1}(cx)}{\sqrt{d+ex^2}}}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcTan[c*x]))/(d + e*x^2)^(3/2),x]
```

```
[Out] -((2*a)/Sqrt[d + e*x^2] + (2*b*ArcTan[c*x])/Sqrt[d + e*x^2] + (I*b*c*Log[((
-4*I)*e*(c*d - I*e*x + Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*Sqrt[c^2*d - e]
*(I + c*x))])/Sqrt[c^2*d - e] - (I*b*c*Log[((4*I)*e*(c*d + I*e*x + Sqrt[c^2
*d - e]*Sqrt[d + e*x^2]))/(b*Sqrt[c^2*d - e]*(-I + c*x))])/Sqrt[c^2*d - e])
/(2*e)
```

Maple [F] time = 0.611, size = 0, normalized size = 0.

$$\int x\left(a+b\arctan(cx)\right)\left(ex^2+d\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctan(c*x))/(e*x^2+d)^(3/2),x)
```

```
[Out] int(x*(a+b*arctan(c*x))/(e*x^2+d)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.34946, size = 791, normalized size = 11.14

$$\left[-\frac{\left(bcex^2+bcd\right)\sqrt{-c^2d+e}\log\left(\frac{\left(c^4d^2-8\,c^2de+8\,e^2\right)x^4-2\left(3\,c^2d^2-4\,de\right)x^2-4\left(\left(c^2d-2\,e\right)x^3-dx\right)\sqrt{-c^2d+e}\sqrt{ex^2+d}+d^2}{c^4x^4+2\,c^2x^2+1}\right)+4\left(ac^2d-ae+\left(bc^2d-\right.\right.}{4\left(c^2d^2e-de^2+\left(c^2de^2-e^3\right)x^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((b*c*e*x^2 + b*c*d)*sqrt(-c^2*d + e)*log(((c^4*d^2 - 8*c^2*d*e + 8*e
^2)*x^4 - 2*(3*c^2*d^2 - 4*d*e)*x^2 - 4*((c^2*d - 2*e)*x^3 - d*x)*sqrt(-c^2
*d + e)*sqrt(e*x^2 + d) + d^2)/(c^4*x^4 + 2*c^2*x^2 + 1)) + 4*(a*c^2*d - a*
e + (b*c^2*d - b*e)*arctan(c*x))*sqrt(e*x^2 + d))/(c^2*d^2*e - d*e^2 + (c^2
*d*e^2 - e^3)*x^2), 1/2*((b*c*e*x^2 + b*c*d)*sqrt(c^2*d - e)*arctan(1/2*sqr
t(c^2*d - e)*((c^2*d - 2*e)*x^2 - d)*sqrt(e*x^2 + d)/((c^2*d*e - e^2)*x^3 +
 (c^2*d^2 - d*e)*x)) - 2*(a*c^2*d - a*e + (b*c^2*d - b*e)*arctan(c*x))*sqrt
(e*x^2 + d))/(c^2*d^2*e - d*e^2 + (c^2*d*e^2 - e^3)*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(a+b\operatorname{atan}(cx)\right)}{\left(d+ex^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atan(c*x))/(e*x**2+d)**(3/2),x)
```

```
[Out] Integral(x*(a + b*atan(c*x))/(d + e*x**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\arctan(cx)+a)x}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)*x/(e*x^2 + d)^(3/2), x)
```

3.1212 $\int \frac{a+b\tan^{-1}(cx)}{\left(d+ex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=70

$$\frac{x\left(a+b\tan^{-1}(cx)\right)}{d\sqrt{d+ex^2}}+\frac{b\tanh^{-1}\left(\frac{c\sqrt{d+ex^2}}{\sqrt{c^2d-e}}\right)}{d\sqrt{c^2d-e}}$$

```
[Out] (x*(a + b*ArcTan[c*x]))/(d*Sqrt[d + e*x^2]) + (b*ArcTanh[(c*Sqrt[d + e*x^2]
)/Sqrt[c^2*d - e]])/(d*Sqrt[c^2*d - e])
```

Rubi [A] time = 0.0769833, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {191, 4912, 12, 444, 63, 208}

$$\frac{x\left(a+b\tan^{-1}(cx)\right)}{d\sqrt{d+ex^2}}+\frac{b\tanh^{-1}\left(\frac{c\sqrt{d+ex^2}}{\sqrt{c^2d-e}}\right)}{d\sqrt{c^2d-e}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])/(d + e*x^2)^(3/2),x]
```

```
[Out] (x*(a + b*ArcTan[c*x]))/(d*Sqrt[d + e*x^2]) + (b*ArcTanh[(c*Sqrt[d + e*x^2]
)/Sqrt[c^2*d - e]])/(d*Sqrt[c^2*d - e])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 4912

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcTan[c*x], u, x]
- Dist[b*c, Int[u/(1 + c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && (I
ntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\tan^{-1}(cx)}{\left(d+ex^2\right)^{3/2}}\,dx &= \frac{x\left(a+b\tan^{-1}(cx)\right)}{d\sqrt{d+ex^2}}-(bc)\int \frac{x}{d\left(1+c^2x^2\right)\sqrt{d+ex^2}}\,dx\\
&= \frac{x\left(a+b\tan^{-1}(cx)\right)}{d\sqrt{d+ex^2}}-\frac{(bc)\int \frac{x}{\left(1+c^2x^2\right)\sqrt{d+ex^2}}\,dx}{d}\\
&= \frac{x\left(a+b\tan^{-1}(cx)\right)}{d\sqrt{d+ex^2}}-\frac{(bc)\,\text{Subst}\left(\int \frac{1}{\left(1+c^2x\right)\sqrt{d+ex}}\,dx,x,x^2\right)}{2d}\\
&= \frac{x\left(a+b\tan^{-1}(cx)\right)}{d\sqrt{d+ex^2}}-\frac{(bc)\,\text{Subst}\left(\int \frac{1}{1-\frac{c^2d}{e}+\frac{c^2x^2}{e}}\,dx,x,\sqrt{d+ex^2}\right)}{de}\\
&= \frac{x\left(a+b\tan^{-1}(cx)\right)}{d\sqrt{d+ex^2}}+\frac{b\tanh^{-1}\left(\frac{c\sqrt{d+ex^2}}{\sqrt{c^2d-e}}\right)}{d\sqrt{c^2d-e}}
\end{aligned}$$

Mathematica [C] time = 0.265082, size = 202, normalized size = 2.89

$$\frac{\frac{2ax}{\sqrt{d+ex^2}}+\frac{b\log\left(-\frac{4cd\left(\sqrt{c^2d-e}\sqrt{d+ex^2}+cd-iex\right)}{b(cx+i)\sqrt{c^2d-e}}\right)}{\sqrt{c^2d-e}}+\frac{b\log\left(-\frac{4cd\left(\sqrt{c^2d-e}\sqrt{d+ex^2}+cd+iex\right)}{b(cx-i)\sqrt{c^2d-e}}\right)}{\sqrt{c^2d-e}}+\frac{2bx\tan^{-1}(cx)}{\sqrt{d+ex^2}}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])/(d + e*x^2)^(3/2),x]
```

```
[Out] ((2*a*x)/Sqrt[d + e*x^2] + (2*b*x*ArcTan[c*x])/Sqrt[d + e*x^2] + (b*Log[(-4
*c*d*(c*d - I*e*x + Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*Sqrt[c^2*d - e]*(I
 + c*x))])/Sqrt[c^2*d - e] + (b*Log[(-4*c*d*(c*d + I*e*x + Sqrt[c^2*d - e]*
Sqrt[d + e*x^2]))/(b*Sqrt[c^2*d - e]*(-I + c*x))])/Sqrt[c^2*d - e])/(2*d)
```

Maple [F] time = 1.188, size = 0, normalized size = 0.

$$\int (a+b\arctan(cx))\left(ex^2+d\right)^{-\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/(e*x^2+d)^(3/2),x)
```

```
[Out] int((a+b*arctan(c*x))/(e*x^2+d)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.35641, size = 810, normalized size = 11.57

$$\left[\frac{\left(bex^2+bd\right)\sqrt{c^2d-e}\log\left(\frac{c^4e^2x^4+8c^4d^2-8c^2de+2\left(4c^4de-3c^2e^2\right)x^2+4\left(c^3ex^2+2c^3d-ce\right)\sqrt{c^2d-e}\sqrt{ex^2+d}+e^2}{c^4x^4+2c^2x^2+1}\right)+4\sqrt{ex^2+d}\left(\left(bc^2d-be\right)x\operatorname{arct}\right.}{4\left(c^2d^3-d^2e+\left(c^2d^2e-de^2\right)x^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*((b*e*x^2 + b*d)*sqrt(c^2*d - e)*log((c^4*e^2*x^4 + 8*c^4*d^2 - 8*c^2*
d*e + 2*(4*c^4*d*e - 3*c^2*e^2)*x^2 + 4*(c^3*e*x^2 + 2*c^3*d - c*e)*sqrt(c^
2*d - e)*sqrt(e*x^2 + d) + e^2)/(c^4*x^4 + 2*c^2*x^2 + 1)) + 4*sqrt(e*x^2 +
 d)*((b*c^2*d - b*e)*x*arctan(c*x) + (a*c^2*d - a*e)*x))/(c^2*d^3 - d^2*e +
 (c^2*d^2*e - d*e^2)*x^2), 1/2*((b*e*x^2 + b*d)*sqrt(-c^2*d + e)*arctan(-1/
2*(c^2*e*x^2 + 2*c^2*d - e)*sqrt(-c^2*d + e)*sqrt(e*x^2 + d)/(c^3*d^2 - c*d
*e + (c^3*d*e - c*e^2)*x^2)) + 2*sqrt(e*x^2 + d)*((b*c^2*d - b*e)*x*arctan(
c*x) + (a*c^2*d - a*e)*x))/(c^2*d^3 - d^2*e + (c^2*d^2*e - d*e^2)*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/(e*x**2+d)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\arctan(cx)+a}{\left(ex^2+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)/(e*x^2 + d)^(3/2), x)
```

3.1213 $\int \frac{a+b\tan^{-1}(cx)}{x(d+ex^2)^{3/2}}\,dx$

Optimal. Leaf size=66

$$b\text{Unintegrable}\left(\frac{\tan^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}},x\right)-\frac{a\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{d^{3/2}}+\frac{a}{d\sqrt{d+ex^2}}$$

```
[Out] a/(d*Sqrt[d + e*x^2]) - (a*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/d^(3/2) + b*Un
integrable[ArcTan[c*x]/(x*(d + e*x^2)^(3/2)), x]
```

Rubi [A] time = 0.176169, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{a+b\tan^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcTan[c*x])/(x*(d + e*x^2)^(3/2)),x]
```

```
[Out] a/(d*Sqrt[d + e*x^2]) - (a*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/d^(3/2) + b*De
fer[Int][ArcTan[c*x]/(x*(d + e*x^2)^(3/2)), x]
```

Rubi steps

$$\int \frac{a+b\tan^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx = a\int \frac{1}{x\left(d+ex^2\right)^{3/2}}\,dx + b\int \frac{\tan^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx$$

$$= \frac{1}{2}a\,\text{Subst}\left(\int \frac{1}{x(d+ex)^{3/2}}\,dx,x,x^2\right) + b\int \frac{\tan^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx$$

$$= \frac{a}{d\sqrt{d+ex^2}} + b\int \frac{\tan^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx + \frac{a\,\text{Subst}\left(\int \frac{1}{x\sqrt{d+ex}}\,dx,x,x^2\right)}{2d}$$

$$= \frac{a}{d\sqrt{d+ex^2}} + b\int \frac{\tan^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx + \frac{a\,\text{Subst}\left(\int \frac{1}{-\frac{d}{e}+\frac{x^2}{e}}\,dx,x,\sqrt{d+ex^2}\right)}{de}$$

$$= \frac{a}{d\sqrt{d+ex^2}} - \frac{a\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{d^{3/2}} + b\int \frac{\tan^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx$$

Mathematica [A] time = 49.9619, size = 0, normalized size = 0.

$$\int \frac{a+b\tan^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcTan[c*x])/(x*(d + e*x^2)^(3/2)),x]
```

```
[Out] Integrate[(a + b*ArcTan[c*x])/(x*(d + e*x^2)^(3/2)), x]
```

Maple [A] time = 0.558, size = 0, normalized size = 0.

$$\int \frac{a+b\arctan(cx)}{x}\left(ex^2+d\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/x/(e*x^2+d)^(3/2),x)
```

```
[Out] int((a+b*arctan(c*x))/x/(e*x^2+d)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{ex^2+d}(b\arctan(cx)+a)}{e^2x^5+2\,dex^3+d^2x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x^2 + d)*(b*arctan(c*x) + a)/(e^2*x^5 + 2*d*e*x^3 + d^2*x),
 x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/x/(e*x**2+d)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \arctan(cx) + a}{\left(ex^2 + d\right)^{\frac{3}{2}} x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x/(e*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)/((e*x^2 + d)^(3/2)*x), x)
```

3.1214 $\int \frac{a+b\tan^{-1}(cx)}{x^2(d+ex^2)^{3/2}}\,dx$

Optimal. Leaf size=135

$$-\frac{2ex\left(a+b\tan^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}}-\frac{a+b\tan^{-1}(cx)}{dx\sqrt{d+ex^2}}+\frac{b\left(c^2d-2e\right)\tanh^{-1}\left(\frac{c\sqrt{d+ex^2}}{\sqrt{c^2d-e}}\right)}{d^2\sqrt{c^2d-e}}-\frac{bc\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{d^{3/2}}$$

```
[Out] -((a + b*ArcTan[c*x])/(d*x*Sqrt[d + e*x^2])) - (2*e*x*(a + b*ArcTan[c*x]))/
(d^2*Sqrt[d + e*x^2]) - (b*c*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/d^(3/2) + (b
*(c^2*d - 2*e)*ArcTanh[(c*Sqrt[d + e*x^2])/Sqrt[c^2*d - e]])/(d^2*Sqrt[c^2*
d - e])
```

Rubi [A] time = 0.229967, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {271, 191, 4976, 12, 573, 156, 63, 208}

$$-\frac{2ex\left(a+b\tan^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}}-\frac{a+b\tan^{-1}(cx)}{dx\sqrt{d+ex^2}}+\frac{b\left(c^2d-2e\right)\tanh^{-1}\left(\frac{c\sqrt{d+ex^2}}{\sqrt{c^2d-e}}\right)}{d^2\sqrt{c^2d-e}}-\frac{bc\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])/(x^2*(d + e*x^2)^(3/2)),x]
```

```
[Out] -((a + b*ArcTan[c*x])/(d*x*Sqrt[d + e*x^2])) - (2*e*x*(a + b*ArcTan[c*x]))/
(d^2*Sqrt[d + e*x^2]) - (b*c*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/d^(3/2) + (b
*(c^2*d - 2*e)*ArcTanh[(c*Sqrt[d + e*x^2])/Sqrt[c^2*d - e]])/(d^2*Sqrt[c^2*
d - e])
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 4976

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Dis
t[a + b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&  !(
ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(IL
tQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILtQ[(m
 - 1)/2, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 573

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simpl
ify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /;
 FreeQ[{a, b, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[Simplify[(m + 1)/n
]]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{a+b\tan^{-1}(cx)}{x^2\left(d+ex^2\right)^{3/2}}\,dx = -\frac{a+b\tan^{-1}(cx)}{dx\sqrt{d+ex^2}}-\frac{2ex\left(a+b\tan^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}}-(bc)\int\frac{-d-2ex^2}{d^2x\left(1+c^2x^2\right)\sqrt{d+ex^2}}\,dx$$

$$= -\frac{a+b\tan^{-1}(cx)}{dx\sqrt{d+ex^2}}-\frac{2ex\left(a+b\tan^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}}-\frac{(bc)\int\frac{-d-2ex^2}{x\left(1+c^2x^2\right)\sqrt{d+ex^2}}\,dx}{d^2}$$

$$= -\frac{a+b\tan^{-1}(cx)}{dx\sqrt{d+ex^2}}-\frac{2ex\left(a+b\tan^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}}-\frac{(bc)\,\text{Subst}\left(\int\frac{-d-2ex}{x\left(1+c^2x\right)\sqrt{d+ex}}\,dx,x,x^2\right)}{2d^2}$$

$$= -\frac{a+b\tan^{-1}(cx)}{dx\sqrt{d+ex^2}}-\frac{2ex\left(a+b\tan^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}}+\frac{(bc)\,\text{Subst}\left(\int\frac{1}{x\sqrt{d+ex}}\,dx,x,x^2\right)}{2d}-\frac{\left(bc\left(c^2d-2e\right)\right)\text{S}}{}$$

$$= -\frac{a+b\tan^{-1}(cx)}{dx\sqrt{d+ex^2}}-\frac{2ex\left(a+b\tan^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}}+\frac{(bc)\,\text{Subst}\left(\int\frac{1}{-\frac{d}{e}+\frac{x^2}{e}}\,dx,x,\sqrt{d+ex^2}\right)}{de}-\frac{\left(bc\left(c^2d-\right.\right.}{}$$

$$= -\frac{a+b\tan^{-1}(cx)}{dx\sqrt{d+ex^2}}-\frac{2ex\left(a+b\tan^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}}-\frac{bc\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{d^{3/2}}+\frac{b\left(c^2d-2e\right)\tanh^{-1}\left(\frac{c\sqrt{d+ex^2}}{\sqrt{c^2d-e}}\right.}{d^2\sqrt{c^2d-e}}$$

Mathematica [C] time = 0.6409, size = 306, normalized size = 2.27

$$\frac{-\frac{2a\left(d+2ex^2\right)}{x\sqrt{d+ex^2}}+\frac{b\left(c^2d-2e\right)\log\left(-\frac{4cd^2\left(\sqrt{c^2d-e}\sqrt{d+ex^2}+cd-iex\right)}{b(cx+i)\left(c^2d-2e\right)\sqrt{c^2d-e}}\right)}{\sqrt{c^2d-e}}+\frac{b\left(c^2d-2e\right)\log\left(-\frac{4cd^2\left(\sqrt{c^2d-e}\sqrt{d+ex^2}+cd+iex\right)}{b(cx-i)\left(c^2d-2e\right)\sqrt{c^2d-e}}\right)}{\sqrt{c^2d-e}}-2bc\sqrt{d}\log\left(\sqrt{d}\sqrt{d+ex^2}+d\right)-\frac{2b}{}}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])/(x^2*(d + e*x^2)^(3/2)),x]
```

```
[Out] ((-2*a*(d + 2*e*x^2))/(x*Sqrt[d + e*x^2]) - (2*b*(d + 2*e*x^2)*ArcTan[c*x])
/(x*Sqrt[d + e*x^2]) + 2*b*c*Sqrt[d]*Log[x] - 2*b*c*Sqrt[d]*Log[d + Sqrt[d]
*Sqrt[d + e*x^2]] + (b*(c^2*d - 2*e)*Log[(-4*c*d^2*(c*d - I*e*x + Sqrt[c^2*
d - e]*Sqrt[d + e*x^2]))/(b*(c^2*d - 2*e)*Sqrt[c^2*d - e]*(I + c*x))])/Sqrt
[c^2*d - e] + (b*(c^2*d - 2*e)*Log[(-4*c*d^2*(c*d + I*e*x + Sqrt[c^2*d - e]
*Sqrt[d + e*x^2]))/(b*(c^2*d - 2*e)*Sqrt[c^2*d - e]*(-I + c*x))])/Sqrt[c^2*
```

```
d - e])/(2*d^2)
```

Maple [F] time = 0.599, size = 0, normalized size = 0.

$$\int \frac{a+b\arctan(cx)}{x^2}\left(ex^2+d\right)^{-\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/x^2/(e*x^2+d)^(3/2),x)
```

```
[Out] int((a+b*arctan(c*x))/x^2/(e*x^2+d)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^2/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.60906, size = 2758, normalized size = 20.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^2/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(((b*c^2*d*e - 2*b*e^2)*x^3 + (b*c^2*d^2 - 2*b*d*e)*x)*sqrt(c^2*d - e
)*log((c^4*e^2*x^4 + 8*c^4*d^2 - 8*c^2*d*e + 2*(4*c^4*d*e - 3*c^2*e^2)*x^2
- 4*(c^3*e*x^2 + 2*c^3*d - c*e)*sqrt(c^2*d - e)*sqrt(e*x^2 + d) + e^2)/(c^4
*x^4 + 2*c^2*x^2 + 1)) - 2*((b*c^3*d*e - b*c*e^2)*x^3 + (b*c^3*d^2 - b*c*d*
e)*x)*sqrt(d)*log(-(e*x^2 - 2*sqrt(e*x^2 + d)*sqrt(d) + 2*d)/x^2) + 4*(a*c^
2*d^2 - a*d*e + 2*(a*c^2*d*e - a*e^2)*x^2 + (b*c^2*d^2 - b*d*e + 2*(b*c^2*d
```

```
*e - b*e^2)*x^2)*arctan(c*x))*sqrt(e*x^2 + d))/((c^2*d^3*e - d^2*e^2)*x^3 +
 (c^2*d^4 - d^3*e)*x), 1/2*(((b*c^2*d*e - 2*b*e^2)*x^3 + (b*c^2*d^2 - 2*b*d
*e)*x)*sqrt(-c^2*d + e)*arctan(-1/2*(c^2*e*x^2 + 2*c^2*d - e)*sqrt(-c^2*d +
 e)*sqrt(e*x^2 + d)/(c^3*d^2 - c*d*e + (c^3*d*e - c*e^2)*x^2)) + ((b*c^3*d*
e - b*c*e^2)*x^3 + (b*c^3*d^2 - b*c*d*e)*x)*sqrt(d)*log(-(e*x^2 - 2*sqrt(e*
x^2 + d)*sqrt(d) + 2*d)/x^2) - 2*(a*c^2*d^2 - a*d*e + 2*(a*c^2*d*e - a*e^2)
*x^2 + (b*c^2*d^2 - b*d*e + 2*(b*c^2*d*e - b*e^2)*x^2)*arctan(c*x))*sqrt(e*
x^2 + d))/((c^2*d^3*e - d^2*e^2)*x^3 + (c^2*d^4 - d^3*e)*x), 1/4*(4*((b*c^3
*d*e - b*c*e^2)*x^3 + (b*c^3*d^2 - b*c*d*e)*x)*sqrt(-d)*arctan(sqrt(-d)/sqr
t(e*x^2 + d)) - ((b*c^2*d*e - 2*b*e^2)*x^3 + (b*c^2*d^2 - 2*b*d*e)*x)*sqrt(
c^2*d - e)*log((c^4*e^2*x^4 + 8*c^4*d^2 - 8*c^2*d*e + 2*(4*c^4*d*e - 3*c^2*
e^2)*x^2 - 4*(c^3*e*x^2 + 2*c^3*d - c*e)*sqrt(c^2*d - e)*sqrt(e*x^2 + d) +
e^2)/(c^4*x^4 + 2*c^2*x^2 + 1)) - 4*(a*c^2*d^2 - a*d*e + 2*(a*c^2*d*e - a*e
^2)*x^2 + (b*c^2*d^2 - b*d*e + 2*(b*c^2*d*e - b*e^2)*x^2)*arctan(c*x))*sqrt
(e*x^2 + d))/((c^2*d^3*e - d^2*e^2)*x^3 + (c^2*d^4 - d^3*e)*x), 1/2*(((b*c^
2*d*e - 2*b*e^2)*x^3 + (b*c^2*d^2 - 2*b*d*e)*x)*sqrt(-c^2*d + e)*arctan(-1/
2*(c^2*e*x^2 + 2*c^2*d - e)*sqrt(-c^2*d + e)*sqrt(e*x^2 + d)/(c^3*d^2 - c*d
*e + (c^3*d*e - c*e^2)*x^2)) + 2*((b*c^3*d*e - b*c*e^2)*x^3 + (b*c^3*d^2 -
b*c*d*e)*x)*sqrt(-d)*arctan(sqrt(-d)/sqrt(e*x^2 + d)) - 2*(a*c^2*d^2 - a*d*
e + 2*(a*c^2*d*e - a*e^2)*x^2 + (b*c^2*d^2 - b*d*e + 2*(b*c^2*d*e - b*e^2)*
x^2)*arctan(c*x))*sqrt(e*x^2 + d))/((c^2*d^3*e - d^2*e^2)*x^3 + (c^2*d^4 -
d^3*e)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/x**2/(e*x**2+d)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \arctan(cx) + a}{(ex^2 + d)^{\frac{3}{2}} x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^2/(e*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)/((e*x^2 + d)^(3/2)*x^2), x)
```

3.1215 $\int \frac{a+b\tan^{-1}(cx)}{x^3(d+ex^2)^{3/2}}\,dx$

Optimal. Leaf size=95

$$b\text{Unintegrable}\left(\frac{\tan^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}},x\right)-\frac{3ae}{2d^2\sqrt{d+ex^2}}+\frac{3ae\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{2d^{5/2}}-\frac{a}{2dx^2\sqrt{d+ex^2}}$$

```
[Out] (-3*a*e)/(2*d^2*Sqrt[d + e*x^2]) - a/(2*d*x^2*Sqrt[d + e*x^2]) + (3*a*e*Arc
Tanh[Sqrt[d + e*x^2]/Sqrt[d]])/(2*d^(5/2)) + b*Unintegrable[ArcTan[c*x]/(x^
3*(d + e*x^2)^(3/2)), x]
```

Rubi [A] time = 0.196432, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{a+b\tan^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcTan[c*x])/(x^3*(d + e*x^2)^(3/2)),x]
```

```
[Out] a/(d*x^2*Sqrt[d + e*x^2]) - (3*a*Sqrt[d + e*x^2])/(2*d^2*x^2) + (3*a*e*ArcT
anh[Sqrt[d + e*x^2]/Sqrt[d]])/(2*d^(5/2)) + b*Defer[Int][ArcTan[c*x]/(x^3*(
d + e*x^2)^(3/2)), x]
```

Rubi steps

$$\int \frac{a+b\tan^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx = a\int \frac{1}{x^3\left(d+ex^2\right)^{3/2}}\,dx + b\int \frac{\tan^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx$$
$$= \frac{1}{2}a\,\text{Subst}\left(\int \frac{1}{x^2(d+ex)^{3/2}}\,dx, x, x^2\right) + b\int \frac{\tan^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx$$
$$= \frac{a}{dx^2\sqrt{d+ex^2}} + b\int \frac{\tan^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx + \frac{(3a)\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{d+ex}}\,dx, x, x^2\right)}{2d}$$
$$= \frac{a}{dx^2\sqrt{d+ex^2}} - \frac{3a\sqrt{d+ex^2}}{2d^2x^2} + b\int \frac{\tan^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx - \frac{(3ae)\,\text{Subst}\left(\int \frac{1}{x\sqrt{d+ex}}\,dx, x, x^2\right)}{4d^2}$$
$$= \frac{a}{dx^2\sqrt{d+ex^2}} - \frac{3a\sqrt{d+ex^2}}{2d^2x^2} + b\int \frac{\tan^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx - \frac{(3a)\,\text{Subst}\left(\int \frac{1}{-\frac{d}{e}+\frac{x^2}{e}}\,dx, x, \sqrt{d+ex^2}\right)}{2d^2}$$
$$= \frac{a}{dx^2\sqrt{d+ex^2}} - \frac{3a\sqrt{d+ex^2}}{2d^2x^2} + \frac{3ae\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{2d^{5/2}} + b\int \frac{\tan^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx$$

Mathematica [A] time = 53.0281, size = 0, normalized size = 0.

$$\int \frac{a+b\tan^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcTan[c*x])/(x^3*(d + e*x^2)^(3/2)),x]
```

```
[Out] Integrate[(a + b*ArcTan[c*x])/(x^3*(d + e*x^2)^(3/2)), x]
```

Maple [A] time = 0.599, size = 0, normalized size = 0.

$$\int \frac{a+b\arctan(cx)}{x^3}\left(ex^2+d\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/x^3/(e*x^2+d)^(3/2),x)
```

```
[Out] int((a+b*arctan(c*x))/x^3/(e*x^2+d)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^3/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{ex^2+d}(b\arctan(cx)+a)}{e^2x^7+2\,dex^5+d^2x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^3/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x^2 + d)*(b*arctan(c*x) + a)/(e^2*x^7 + 2*d*e*x^5 + d^2*x^3
), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/x**3/(e*x**2+d)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\arctan(cx) + a}{\left(ex^2 + d\right)^{\frac{3}{2}} x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^3/(e*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)/((e*x^2 + d)^(3/2)*x^3), x)
```

3.1216 $\int \frac{a+b\tan^{-1}(cx)}{x^4(d+ex^2)^{3/2}}\,dx$

Optimal. Leaf size=249

$$\frac{8e^2x\left(a+b\tan^{-1}(cx)\right)}{3d^3\sqrt{d+ex^2}}+\frac{4e\left(a+b\tan^{-1}(cx)\right)}{3d^2x\sqrt{d+ex^2}}-\frac{a+b\tan^{-1}(cx)}{3dx^3\sqrt{d+ex^2}}-\frac{b\left(c^4d^2+4c^2de-8e^2\right)\tanh^{-1}\left(\frac{c\sqrt{d+ex^2}}{\sqrt{c^2d-e}}\right)}{3d^3\sqrt{c^2d-e}}+\frac{bc\left(c^2d+\right.}{}$$

```
[Out] -(b*c*Sqrt[d + e*x^2])/(6*d^2*x^2) - (a + b*ArcTan[c*x])/(3*d*x^3*Sqrt[d +
e*x^2]) + (4*e*(a + b*ArcTan[c*x]))/(3*d^2*x*Sqrt[d + e*x^2]) + (8*e^2*x*(a
 + b*ArcTan[c*x]))/(3*d^3*Sqrt[d + e*x^2]) + (b*c*e*ArcTanh[Sqrt[d + e*x^2]
/Sqrt[d]])/(6*d^(5/2)) + (b*c*(c^2*d + 4*e)*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]
])/(3*d^(5/2)) - (b*(c^4*d^2 + 4*c^2*d*e - 8*e^2)*ArcTanh[(c*Sqrt[d + e*x^2
])/Sqrt[c^2*d - e]])/(3*d^3*Sqrt[c^2*d - e])
```

Rubi [A] time = 0.873621, antiderivative size = 249, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {271, 191, 4976, 12, 6725, 266, 51, 63, 208, 444}

$$\frac{8e^2x\left(a+b\tan^{-1}(cx)\right)}{3d^3\sqrt{d+ex^2}}+\frac{4e\left(a+b\tan^{-1}(cx)\right)}{3d^2x\sqrt{d+ex^2}}-\frac{a+b\tan^{-1}(cx)}{3dx^3\sqrt{d+ex^2}}-\frac{b\left(c^4d^2+4c^2de-8e^2\right)\tanh^{-1}\left(\frac{c\sqrt{d+ex^2}}{\sqrt{c^2d-e}}\right)}{3d^3\sqrt{c^2d-e}}+\frac{bc\left(c^2d+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])/(x^4*(d + e*x^2)^(3/2)),x]
```


```
[Out] -(b*c*Sqrt[d + e*x^2])/(6*d^2*x^2) - (a + b*ArcTan[c*x])/(3*d*x^3*Sqrt[d +
e*x^2]) + (4*e*(a + b*ArcTan[c*x]))/(3*d^2*x*Sqrt[d + e*x^2]) + (8*e^2*x*(a
 + b*ArcTan[c*x]))/(3*d^3*Sqrt[d + e*x^2]) + (b*c*e*ArcTanh[Sqrt[d + e*x^2]
/Sqrt[d]])/(6*d^(5/2)) + (b*c*(c^2*d + 4*e)*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]
])/(3*d^(5/2)) - (b*(c^4*d^2 + 4*c^2*d*e - 8*e^2)*ArcTanh[(c*Sqrt[d + e*x^2
])/Sqrt[c^2*d - e]])/(3*d^3*Sqrt[c^2*d - e])
```


Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 4976

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Dis
t[a + b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&  !(
ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(IL
tQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILtQ[(m
 - 1)/2, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\int \frac{a+b\tan^{-1}(cx)}{x^4\left(d+ex^2\right)^{3/2}}\,dx = -\frac{a+b\tan^{-1}(cx)}{3dx^3\sqrt{d+ex^2}} + \frac{4e\left(a+b\tan^{-1}(cx)\right)}{3d^2x\sqrt{d+ex^2}} + \frac{8e^2x\left(a+b\tan^{-1}(cx)\right)}{3d^3\sqrt{d+ex^2}} - (bc)\int \frac{-d^2+4dex^2+8e}{3d^3x^3\left(1+c^2x^2\right)\sqrt{d}}$$

$$= -\frac{a+b\tan^{-1}(cx)}{3dx^3\sqrt{d+ex^2}} + \frac{4e\left(a+b\tan^{-1}(cx)\right)}{3d^2x\sqrt{d+ex^2}} + \frac{8e^2x\left(a+b\tan^{-1}(cx)\right)}{3d^3\sqrt{d+ex^2}} - \frac{(bc)\int \frac{-d^2+4dex^2+8e^2x^4}{x^3\left(1+c^2x^2\right)\sqrt{d+ex^2}}\,dx}{3d^3}$$

$$= -\frac{a+b\tan^{-1}(cx)}{3dx^3\sqrt{d+ex^2}} + \frac{4e\left(a+b\tan^{-1}(cx)\right)}{3d^2x\sqrt{d+ex^2}} + \frac{8e^2x\left(a+b\tan^{-1}(cx)\right)}{3d^3\sqrt{d+ex^2}} - \frac{(bc)\int\left(-\frac{d^2}{x^3\sqrt{d+ex^2}} + \frac{d\left(c^2d+4e\right.}{x\sqrt{d+ex^2}}\right.}{3d^3}$$

$$= -\frac{a+b\tan^{-1}(cx)}{3dx^3\sqrt{d+ex^2}} + \frac{4e\left(a+b\tan^{-1}(cx)\right)}{3d^2x\sqrt{d+ex^2}} + \frac{8e^2x\left(a+b\tan^{-1}(cx)\right)}{3d^3\sqrt{d+ex^2}} + \frac{(bc)\int \frac{1}{x^3\sqrt{d+ex^2}}\,dx}{3d} - \frac{\left(bc\left(c^2\right.\right.}{}$$

$$= -\frac{a+b\tan^{-1}(cx)}{3dx^3\sqrt{d+ex^2}} + \frac{4e\left(a+b\tan^{-1}(cx)\right)}{3d^2x\sqrt{d+ex^2}} + \frac{8e^2x\left(a+b\tan^{-1}(cx)\right)}{3d^3\sqrt{d+ex^2}} + \frac{(bc)\,\mathrm{Subst}\left(\int \frac{1}{x^2\sqrt{d+ex}}\,dx, x\right.}{6d}$$

$$= -\frac{bc\sqrt{d+ex^2}}{6d^2x^2} - \frac{a+b\tan^{-1}(cx)}{3dx^3\sqrt{d+ex^2}} + \frac{4e\left(a+b\tan^{-1}(cx)\right)}{3d^2x\sqrt{d+ex^2}} + \frac{8e^2x\left(a+b\tan^{-1}(cx)\right)}{3d^3\sqrt{d+ex^2}} - \frac{(bce)\,\mathrm{Subst}\left(\right.}{}$$

$$= -\frac{bc\sqrt{d+ex^2}}{6d^2x^2} - \frac{a+b\tan^{-1}(cx)}{3dx^3\sqrt{d+ex^2}} + \frac{4e\left(a+b\tan^{-1}(cx)\right)}{3d^2x\sqrt{d+ex^2}} + \frac{8e^2x\left(a+b\tan^{-1}(cx)\right)}{3d^3\sqrt{d+ex^2}} + \frac{bc\left(c^2d+4e\right)}{}$$

$$= -\frac{bc\sqrt{d+ex^2}}{6d^2x^2} - \frac{a+b\tan^{-1}(cx)}{3dx^3\sqrt{d+ex^2}} + \frac{4e\left(a+b\tan^{-1}(cx)\right)}{3d^2x\sqrt{d+ex^2}} + \frac{8e^2x\left(a+b\tan^{-1}(cx)\right)}{3d^3\sqrt{d+ex^2}} + \frac{bce\tanh^{-1}\left(\right.}{6d^{5/2}}$$

<u>Mathematica</u> [C] time = 0.711319, size = 405, normalized size = 1.63

$$-\frac{\frac{2a\left(d^2-4dex^2-8e^2x^4\right)+bcdx\left(d+ex^2\right)}{x^3\sqrt{d+ex^2}} + \frac{b\left(c^4d^2+4c^2de-8e^2\right)\log\left(\frac{12cd^3\left(\sqrt{c^2d-e}\sqrt{d+ex^2}+cd-iex\right)}{b(cx+i)\sqrt{c^2d-e}\left(c^4d^2+4c^2de-8e^2\right)}\right)}{\sqrt{c^2d-e}} + \frac{b\left(c^4d^2+4c^2de-8e^2\right)\log\left(\frac{12cd^3\left(\sqrt{c^2d-e}\sqrt{d+ex^2}+cd+iex\right)}{b(cx-i)\sqrt{c^2d-e}\left(c^4d^2+4c^2de-8e^2\right)}\right)}{\sqrt{c^2d-e}} - bc}{6d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])/(x^4*(d + e*x^2)^(3/2)),x]
```

```
[Out] -((b*c*d*x*(d + e*x^2) + 2*a*(d^2 - 4*d*e*x^2 - 8*e^2*x^4))/(x^3*Sqrt[d + e
*x^2]) + (2*b*(d^2 - 4*d*e*x^2 - 8*e^2*x^4)*ArcTan[c*x])/(x^3*Sqrt[d + e*x^
2]) + b*c*Sqrt[d]*(2*c^2*d + 9*e)*Log[x] - b*c*Sqrt[d]*(2*c^2*d + 9*e)*Log[
```

```
d + Sqrt[d]*Sqrt[d + e*x^2]] + (b*(c^4*d^2 + 4*c^2*d*e - 8*e^2)*Log[(12*c*d
^3*(c*d - I*e*x + Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*Sqrt[c^2*d - e]*(c^4
*d^2 + 4*c^2*d*e - 8*e^2)*(I + c*x))])/Sqrt[c^2*d - e] + (b*(c^4*d^2 + 4*c^
2*d*e - 8*e^2)*Log[(12*c*d^3*(c*d + I*e*x + Sqrt[c^2*d - e]*Sqrt[d + e*x^2]
))/(b*Sqrt[c^2*d - e]*(c^4*d^2 + 4*c^2*d*e - 8*e^2)*(-I + c*x))])/Sqrt[c^2*
d - e])/(6*d^3)
```

Maple [F] time = 0.598, size = 0, normalized size = 0.

$$\int \frac{a+b\arctan(cx)}{x^4}\left(ex^2+d\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/x^4/(e*x^2+d)^(3/2),x)
```

```
[Out] int((a+b*arctan(c*x))/x^4/(e*x^2+d)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^4/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 5.39429, size = 3976, normalized size = 15.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^4/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(((b*c^4*d^2*e + 4*b*c^2*d*e^2 - 8*b*e^3)*x^5 + (b*c^4*d^3 + 4*b*c^2
*d^2*e - 8*b*d*e^2)*x^3)*sqrt(c^2*d - e)*log((c^4*e^2*x^4 + 8*c^4*d^2 - 8*c
```

```
^2*d*e + 2*(4*c^4*d*e - 3*c^2*e^2)*x^2 + 4*(c^3*e*x^2 + 2*c^3*d - c*e)*sqrt
(c^2*d - e)*sqrt(e*x^2 + d) + e^2)/(c^4*x^4 + 2*c^2*x^2 + 1)) - ((2*b*c^5*d
^2*e + 7*b*c^3*d*e^2 - 9*b*c*e^3)*x^5 + (2*b*c^5*d^3 + 7*b*c^3*d^2*e - 9*b*
c*d*e^2)*x^3)*sqrt(d)*log(-(e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(d) + 2*d)/x^2) +
 2*(2*a*c^2*d^3 - 16*(a*c^2*d*e^2 - a*e^3)*x^4 - 2*a*d^2*e + (b*c^3*d^2*e -
 b*c*d*e^2)*x^3 - 8*(a*c^2*d^2*e - a*d*e^2)*x^2 + (b*c^3*d^3 - b*c*d^2*e)*x
 + 2*(b*c^2*d^3 - 8*(b*c^2*d*e^2 - b*e^3)*x^4 - b*d^2*e - 4*(b*c^2*d^2*e -
b*d*e^2)*x^2)*arctan(c*x))*sqrt(e*x^2 + d))/((c^2*d^4*e - d^3*e^2)*x^5 + (c
^2*d^5 - d^4*e)*x^3), -1/12*(2*((b*c^4*d^2*e + 4*b*c^2*d*e^2 - 8*b*e^3)*x^5
 + (b*c^4*d^3 + 4*b*c^2*d^2*e - 8*b*d*e^2)*x^3)*sqrt(-c^2*d + e)*arctan(-1/
2*(c^2*e*x^2 + 2*c^2*d - e)*sqrt(-c^2*d + e)*sqrt(e*x^2 + d)/(c^3*d^2 - c*d
*e + (c^3*d*e - c*e^2)*x^2)) - ((2*b*c^5*d^2*e + 7*b*c^3*d*e^2 - 9*b*c*e^3)
*x^5 + (2*b*c^5*d^3 + 7*b*c^3*d^2*e - 9*b*c*d*e^2)*x^3)*sqrt(d)*log(-(e*x^2
 + 2*sqrt(e*x^2 + d)*sqrt(d) + 2*d)/x^2) + 2*(2*a*c^2*d^3 - 16*(a*c^2*d*e^2
 - a*e^3)*x^4 - 2*a*d^2*e + (b*c^3*d^2*e - b*c*d*e^2)*x^3 - 8*(a*c^2*d^2*e
- a*d*e^2)*x^2 + (b*c^3*d^3 - b*c*d^2*e)*x + 2*(b*c^2*d^3 - 8*(b*c^2*d*e^2
- b*e^3)*x^4 - b*d^2*e - 4*(b*c^2*d^2*e - b*d*e^2)*x^2)*arctan(c*x))*sqrt(e
*x^2 + d))/((c^2*d^4*e - d^3*e^2)*x^5 + (c^2*d^5 - d^4*e)*x^3), -1/12*(2*((
2*b*c^5*d^2*e + 7*b*c^3*d*e^2 - 9*b*c*e^3)*x^5 + (2*b*c^5*d^3 + 7*b*c^3*d^2
*e - 9*b*c*d*e^2)*x^3)*sqrt(-d)*arctan(sqrt(-d)/sqrt(e*x^2 + d)) + ((b*c^4*
d^2*e + 4*b*c^2*d*e^2 - 8*b*e^3)*x^5 + (b*c^4*d^3 + 4*b*c^2*d^2*e - 8*b*d*e
^2)*x^3)*sqrt(c^2*d - e)*log((c^4*e^2*x^4 + 8*c^4*d^2 - 8*c^2*d*e + 2*(4*c^
4*d*e - 3*c^2*e^2)*x^2 + 4*(c^3*e*x^2 + 2*c^3*d - c*e)*sqrt(c^2*d - e)*sqrt
(e*x^2 + d) + e^2)/(c^4*x^4 + 2*c^2*x^2 + 1)) + 2*(2*a*c^2*d^3 - 16*(a*c^2*
d*e^2 - a*e^3)*x^4 - 2*a*d^2*e + (b*c^3*d^2*e - b*c*d*e^2)*x^3 - 8*(a*c^2*d
^2*e - a*d*e^2)*x^2 + (b*c^3*d^3 - b*c*d^2*e)*x + 2*(b*c^2*d^3 - 8*(b*c^2*d
*e^2 - b*e^3)*x^4 - b*d^2*e - 4*(b*c^2*d^2*e - b*d*e^2)*x^2)*arctan(c*x))*s
qrt(e*x^2 + d))/((c^2*d^4*e - d^3*e^2)*x^5 + (c^2*d^5 - d^4*e)*x^3), -1/6*(
((b*c^4*d^2*e + 4*b*c^2*d*e^2 - 8*b*e^3)*x^5 + (b*c^4*d^3 + 4*b*c^2*d^2*e -
 8*b*d*e^2)*x^3)*sqrt(-c^2*d + e)*arctan(-1/2*(c^2*e*x^2 + 2*c^2*d - e)*sqr
t(-c^2*d + e)*sqrt(e*x^2 + d)/(c^3*d^2 - c*d*e + (c^3*d*e - c*e^2)*x^2)) +
((2*b*c^5*d^2*e + 7*b*c^3*d*e^2 - 9*b*c*e^3)*x^5 + (2*b*c^5*d^3 + 7*b*c^3*d
^2*e - 9*b*c*d*e^2)*x^3)*sqrt(-d)*arctan(sqrt(-d)/sqrt(e*x^2 + d)) + (2*a*c
^2*d^3 - 16*(a*c^2*d*e^2 - a*e^3)*x^4 - 2*a*d^2*e + (b*c^3*d^2*e - b*c*d*e^
2)*x^3 - 8*(a*c^2*d^2*e - a*d*e^2)*x^2 + (b*c^3*d^3 - b*c*d^2*e)*x + 2*(b*c
^2*d^3 - 8*(b*c^2*d*e^2 - b*e^3)*x^4 - b*d^2*e - 4*(b*c^2*d^2*e - b*d*e^2)*
x^2)*arctan(c*x))*sqrt(e*x^2 + d))/((c^2*d^4*e - d^3*e^2)*x^5 + (c^2*d^5 -
d^4*e)*x^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/x**4/(e*x**2+d)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \arctan(cx) + a}{(ex^2 + d)^{\frac{3}{2}} x^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^4/(e*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)/((e*x^2 + d)^(3/2)*x^4), x)
```

3.1217 $\int \frac{x^4(a+b\tan^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx$

Optimal. Leaf size=90

$$b\text{Unintegrable}\left(\frac{x^4\tan^{-1}(cx)}{(d+ex^2)^{5/2}},x\right)-\frac{ax}{e^2\sqrt{d+ex^2}}+\frac{a\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{e^{5/2}}-\frac{ax^3}{3e(d+ex^2)^{3/2}}$$

```
[Out] -(a*x^3)/(3*e*(d + e*x^2)^(3/2)) - (a*x)/(e^2*Sqrt[d + e*x^2]) + (a*ArcTanh
[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/e^(5/2) + b*Unintegrable[(x^4*ArcTan[c*x])/(
d + e*x^2)^(5/2), x]
```

Rubi [A] time = 0.179546, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^4\left(a+b\tan^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^4*(a + b*ArcTan[c*x]))/(d + e*x^2)^(5/2),x]
```

```
[Out] -(a*x^3)/(3*e*(d + e*x^2)^(3/2)) - (a*x)/(e^2*Sqrt[d + e*x^2]) + (a*ArcTanh
[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/e^(5/2) + b*Defer[Int][(x^4*ArcTan[c*x])/(d
+ e*x^2)^(5/2), x]
```

Rubi steps

$$\int \frac{x^4(a+b\tan^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = a\int \frac{x^4}{(d+ex^2)^{5/2}}\,dx + b\int \frac{x^4\tan^{-1}(cx)}{(d+ex^2)^{5/2}}\,dx$$

$$= -\frac{ax^3}{3e(d+ex^2)^{3/2}} + b\int \frac{x^4\tan^{-1}(cx)}{(d+ex^2)^{5/2}}\,dx + \frac{a\int \frac{x^2}{(d+ex^2)^{3/2}}\,dx}{e}$$

$$= -\frac{ax^3}{3e(d+ex^2)^{3/2}} - \frac{ax}{e^2\sqrt{d+ex^2}} + b\int \frac{x^4\tan^{-1}(cx)}{(d+ex^2)^{5/2}}\,dx + \frac{a\int \frac{1}{\sqrt{d+ex^2}}\,dx}{e^2}$$

$$= -\frac{ax^3}{3e(d+ex^2)^{3/2}} - \frac{ax}{e^2\sqrt{d+ex^2}} + b\int \frac{x^4\tan^{-1}(cx)}{(d+ex^2)^{5/2}}\,dx + \frac{a\,\mathrm{Subst}\left(\int \frac{1}{1-ex^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{e^2}$$

$$= -\frac{ax^3}{3e(d+ex^2)^{3/2}} - \frac{ax}{e^2\sqrt{d+ex^2}} + \frac{a\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{e^{5/2}} + b\int \frac{x^4\tan^{-1}(cx)}{(d+ex^2)^{5/2}}\,dx$$

Mathematica [A] time = 11.4376, size = 0, normalized size = 0.

$$\int \frac{x^4(a+b\tan^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^4*(a + b*ArcTan[c*x]))/(d + e*x^2)^(5/2),x]
```

```
[Out] Integrate[(x^4*(a + b*ArcTan[c*x]))/(d + e*x^2)^(5/2), x]
```

Maple [A] time = 0.608, size = 0, normalized size = 0.

$$\int x^4(a+b\arctan(cx))\left(ex^2+d\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arctan(c*x))/(e*x^2+d)^(5/2),x)
```

```
[Out] int(x^4*(a+b*arctan(c*x))/(e*x^2+d)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctan(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^4\arctan\left(cx\right)+ax^4\right)\sqrt{ex^2+d}}{e^3x^6+3\,de^2x^4+3\,d^2ex^2+d^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctan(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^4*arctan(c*x) + a*x^4)*sqrt(e*x^2 + d)/(e^3*x^6 + 3*d*e^2*x^4
 + 3*d^2*e*x^2 + d^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*atan(c*x))/(e*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\arctan(cx) + a)x^4}{\left(ex^2 + d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctan(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)*x^4/(e*x^2 + d)^(5/2), x)
```

3.1218 $\int \frac{x^3(a+b\tan^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx$

Optimal. Leaf size=143

$$-\frac{a+b\tan^{-1}(cx)}{e^2\sqrt{d+ex^2}}+\frac{d\left(a+b\tan^{-1}(cx)\right)}{3e^2\left(d+ex^2\right)^{3/2}}+\frac{bc\left(2c^2d-3e\right)\tan^{-1}\left(\frac{x\sqrt{c^2d-e}}{\sqrt{d+ex^2}}\right)}{3e^2\left(c^2d-e\right)^{3/2}}+\frac{bcx}{3e\left(c^2d-e\right)\sqrt{d+ex^2}}$$

```
[Out] (b*c*x)/(3*(c^2*d - e)*e*Sqrt[d + e*x^2]) + (d*(a + b*ArcTan[c*x]))/(3*e^2*
(d + e*x^2)^(3/2)) - (a + b*ArcTan[c*x])/(e^2*Sqrt[d + e*x^2]) + (b*c*(2*c^
2*d - 3*e)*ArcTan[(Sqrt[c^2*d - e]*x)/Sqrt[d + e*x^2]])/(3*(c^2*d - e)^(3/2
)*e^2)
```

Rubi [A] time = 0.208724, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {266, 43, 4976, 12, 527, 377, 203}

$$-\frac{a+b\tan^{-1}(cx)}{e^2\sqrt{d+ex^2}}+\frac{d\left(a+b\tan^{-1}(cx)\right)}{3e^2\left(d+ex^2\right)^{3/2}}+\frac{bc\left(2c^2d-3e\right)\tan^{-1}\left(\frac{x\sqrt{c^2d-e}}{\sqrt{d+ex^2}}\right)}{3e^2\left(c^2d-e\right)^{3/2}}+\frac{bcx}{3e\left(c^2d-e\right)\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcTan[c*x]))/(d + e*x^2)^(5/2),x]
```

```
[Out] (b*c*x)/(3*(c^2*d - e)*e*Sqrt[d + e*x^2]) + (d*(a + b*ArcTan[c*x]))/(3*e^2*
(d + e*x^2)^(3/2)) - (a + b*ArcTan[c*x])/(e^2*Sqrt[d + e*x^2]) + (b*c*(2*c^
2*d - 3*e)*ArcTan[(Sqrt[c^2*d - e]*x)/Sqrt[d + e*x^2]])/(3*(c^2*d - e)^(3/2
)*e^2)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 4976

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Dis
t[a + b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&  !(
ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(IL
tQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILtQ[(m
 - 1)/2, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{x^3\left(a+b\tan^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx = \frac{d\left(a+b\tan^{-1}(cx)\right)}{3e^2\left(d+ex^2\right)^{3/2}} - \frac{a+b\tan^{-1}(cx)}{e^2\sqrt{d+ex^2}} - (bc)\int \frac{-2d-3ex^2}{3e^2\left(1+c^2x^2\right)\left(d+ex^2\right)^{3/2}}\,dx$$

$$= \frac{d\left(a+b\tan^{-1}(cx)\right)}{3e^2\left(d+ex^2\right)^{3/2}} - \frac{a+b\tan^{-1}(cx)}{e^2\sqrt{d+ex^2}} - \frac{(bc)\int \frac{-2d-3ex^2}{\left(1+c^2x^2\right)\left(d+ex^2\right)^{3/2}}\,dx}{3e^2}$$

$$= \frac{bcx}{3\left(c^2d-e\right)e\sqrt{d+ex^2}} + \frac{d\left(a+b\tan^{-1}(cx)\right)}{3e^2\left(d+ex^2\right)^{3/2}} - \frac{a+b\tan^{-1}(cx)}{e^2\sqrt{d+ex^2}} + \frac{(bc)\int \frac{d\left(2c^2d-3e\right)}{\left(1+c^2x^2\right)\sqrt{d+ex^2}}\,dx}{3d\left(c^2d-e\right)e^2}$$

$$= \frac{bcx}{3\left(c^2d-e\right)e\sqrt{d+ex^2}} + \frac{d\left(a+b\tan^{-1}(cx)\right)}{3e^2\left(d+ex^2\right)^{3/2}} - \frac{a+b\tan^{-1}(cx)}{e^2\sqrt{d+ex^2}} + \frac{\left(bc\left(2c^2d-3e\right)\right)\int \frac{1}{\left(1+c^2x^2\right)\cdots}}{3\left(c^2d-e\right)e^2}$$

$$= \frac{bcx}{3\left(c^2d-e\right)e\sqrt{d+ex^2}} + \frac{d\left(a+b\tan^{-1}(cx)\right)}{3e^2\left(d+ex^2\right)^{3/2}} - \frac{a+b\tan^{-1}(cx)}{e^2\sqrt{d+ex^2}} + \frac{\left(bc\left(2c^2d-3e\right)\right)\text{Subst}\left(\int \cdots\right.}{3\left(c^2d \cdots\right.}$$

$$= \frac{bcx}{3\left(c^2d-e\right)e\sqrt{d+ex^2}} + \frac{d\left(a+b\tan^{-1}(cx)\right)}{3e^2\left(d+ex^2\right)^{3/2}} - \frac{a+b\tan^{-1}(cx)}{e^2\sqrt{d+ex^2}} + \frac{bc\left(2c^2d-3e\right)\tan^{-1}\left(\frac{\sqrt{c^2d \cdots}}{\sqrt{d+\cdots}}\right.}{3\left(c^2d-e\right)^{3/2}e^2}$$

Mathematica [C] time = 0.537823, size = 326, normalized size = 2.28

$$\frac{2\sqrt{c^2d-e}\left(bcex\left(d+ex^2\right)-a\left(c^2d-e\right)\left(2d+3ex^2\right)\right)-ibc\left(2c^2d-3e\right)\left(d+ex^2\right)^{3/2}\log\left(-\frac{12ie^2\sqrt{c^2d-e}\left(\sqrt{c^2d-e}\sqrt{d+ex^2}+cd-iex\right)}{b(cx+i)\left(2c^2d-3e\right)}\right)+\cdots}{6e^2\left(c^2d-e\right)^{3/2}\left(d+ex^2\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcTan[c*x]))/(d + e*x^2)^(5/2),x]
```

```
[Out] (2*Sqrt[c^2*d - e]*(b*c*e*x*(d + e*x^2) - a*(c^2*d - e)*(2*d + 3*e*x^2)) -
2*b*(c^2*d - e)^(3/2)*(2*d + 3*e*x^2)*ArcTan[c*x] - I*b*c*(2*c^2*d - 3*e)*(
d + e*x^2)^(3/2)*Log[((-12*I)*Sqrt[c^2*d - e]*e^2*(c*d - I*e*x + Sqrt[c^2*d
 - e]*Sqrt[d + e*x^2]))/(b*(2*c^2*d - 3*e)*(I + c*x))] + I*b*c*(2*c^2*d - 3
*e)*(d + e*x^2)^(3/2)*Log[((12*I)*Sqrt[c^2*d - e]*e^2*(c*d + I*e*x + Sqrt[c
^2*d - e]*Sqrt[d + e*x^2]))/(b*(2*c^2*d - 3*e)*(-I + c*x))])/(6*(c^2*d - e)
^(3/2)*e^2*(d + e*x^2)^(3/2))
```

Maple [F] time = 0.601, size = 0, normalized size = 0.

$$\int x^3 (a + b \arctan (cx)) \left(e x^2 + d\right)^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctan(c*x))/(e*x^2+d)^(5/2),x)
```

```
[Out] int(x^3*(a+b*arctan(c*x))/(e*x^2+d)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 5.67279, size = 1781, normalized size = 12.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/12*((2*b*c^3*d^3 - 3*b*c*d^2*e + (2*b*c^3*d*e^2 - 3*b*c*e^3)*x^4 + 2*(2
*b*c^3*d^2*e - 3*b*c*d*e^2)*x^2)*sqrt(-c^2*d + e)*log((((c^4*d^2 - 8*c^2*d*e
 + 8*e^2)*x^4 - 2*(3*c^2*d^2 - 4*d*e)*x^2 - 4*((c^2*d - 2*e)*x^3 - d*x)*sqr
t(-c^2*d + e)*sqrt(e*x^2 + d) + d^2)/(c^4*x^4 + 2*c^2*x^2 + 1)) + 4*(2*a*c^
4*d^3 - 4*a*c^2*d^2*e + 2*a*d*e^2 - (b*c^3*d*e^2 - b*c*e^3)*x^3 + 3*(a*c^4*
d^2*e - 2*a*c^2*d*e^2 + a*e^3)*x^2 - (b*c^3*d^2*e - b*c*d*e^2)*x + (2*b*c^4
*d^3 - 4*b*c^2*d^2*e + 2*b*d*e^2 + 3*(b*c^4*d^2*e - 2*b*c^2*d*e^2 + b*e^3)*
x^2)*arctan(c*x))*sqrt(e*x^2 + d))/(c^4*d^4*e^2 - 2*c^2*d^3*e^3 + d^2*e^4 +
 (c^4*d^2*e^4 - 2*c^2*d*e^5 + e^6)*x^4 + 2*(c^4*d^3*e^3 - 2*c^2*d^2*e^4 + d
```

```
*e^5)*x^2), 1/6*((2*b*c^3*d^3 - 3*b*c*d^2*e + (2*b*c^3*d*e^2 - 3*b*c*e^3)*x
^4 + 2*(2*b*c^3*d^2*e - 3*b*c*d*e^2)*x^2)*sqrt(c^2*d - e)*arctan(1/2*sqrt(c
^2*d - e)*((c^2*d - 2*e)*x^2 - d)*sqrt(e*x^2 + d)/((c^2*d*e - e^2)*x^3 + (c
^2*d^2 - d*e)*x)) - 2*(2*a*c^4*d^3 - 4*a*c^2*d^2*e + 2*a*d*e^2 - (b*c^3*d*e
^2 - b*c*e^3)*x^3 + 3*(a*c^4*d^2*e - 2*a*c^2*d*e^2 + a*e^3)*x^2 - (b*c^3*d^
2*e - b*c*d*e^2)*x + (2*b*c^4*d^3 - 4*b*c^2*d^2*e + 2*b*d*e^2 + 3*(b*c^4*d^
2*e - 2*b*c^2*d*e^2 + b*e^3)*x^2)*arctan(c*x))*sqrt(e*x^2 + d))/(c^4*d^4*e^
2 - 2*c^2*d^3*e^3 + d^2*e^4 + (c^4*d^2*e^4 - 2*c^2*d*e^5 + e^6)*x^4 + 2*(c^
4*d^3*e^3 - 2*c^2*d^2*e^4 + d*e^5)*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 (a + b \operatorname{atan}(cx))}{(d + ex^2)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atan(c*x))/(e*x**2+d)**(5/2),x)
```

```
[Out] Integral(x**3*(a + b*atan(c*x))/(d + e*x**2)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \arctan(cx) + a)x^3}{(ex^2 + d)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)*x^3/(e*x^2 + d)^(5/2), x)
```

3.1219 $\int \frac{x^2(a+b\tan^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx$

Optimal. Leaf size=109

$$\frac{x^3\left(a+b\tan^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}+\frac{bc}{3e\left(c^2d-e\right)\sqrt{d+ex^2}}-\frac{b\tanh^{-1}\left(\frac{c\sqrt{d+ex^2}}{\sqrt{c^2d-e}}\right)}{3d\left(c^2d-e\right)^{3/2}}$$

```
[Out] (b*c)/(3*(c^2*d - e)*e*Sqrt[d + e*x^2]) + (x^3*(a + b*ArcTan[c*x]))/(3*d*(d
 + e*x^2)^(3/2)) - (b*ArcTanh[(c*Sqrt[d + e*x^2])/Sqrt[c^2*d - e]])/(3*d*(c
^2*d - e)^(3/2))
```

Rubi [A] time = 0.199304, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {264, 4976, 446, 78, 63, 208}

$$\frac{x^3\left(a+b\tan^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}+\frac{bc}{3e\left(c^2d-e\right)\sqrt{d+ex^2}}-\frac{b\tanh^{-1}\left(\frac{c\sqrt{d+ex^2}}{\sqrt{c^2d-e}}\right)}{3d\left(c^2d-e\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcTan[c*x]))/(d + e*x^2)^(5/2),x]
```

```
[Out] (b*c)/(3*(c^2*d - e)*e*Sqrt[d + e*x^2]) + (x^3*(a + b*ArcTan[c*x]))/(3*d*(d
 + e*x^2)^(3/2)) - (b*ArcTanh[(c*Sqrt[d + e*x^2])/Sqrt[c^2*d - e]])/(3*d*(c
^2*d - e)^(3/2))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 4976

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Dis
t[a + b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2
```

```
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&  !(
ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(IL
tQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILtQ[(m
 - 1)/2, 0]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{x^2\left(a+b\tan^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx=\frac{x^3\left(a+b\tan^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}-(bc)\int\frac{x^3}{\left(3d+3c^2dx^2\right)\left(d+ex^2\right)^{3/2}}\,dx$$
$$=\frac{x^3\left(a+b\tan^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}-\frac{1}{2}(bc)\,\text{Subst}\left(\int\frac{x}{\left(3d+3c^2dx\right)(d+ex)^{3/2}}\,dx,x,x^2\right)$$
$$=\frac{bc}{3\left(c^2d-e\right)e\sqrt{d+ex^2}}+\frac{x^3\left(a+b\tan^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}+\frac{(bc)\,\text{Subst}\left(\int\frac{1}{\left(3d+3c^2dx\right)\sqrt{d+ex}}\,dx,x,x^2\right)}{2\left(c^2d-e\right)}$$
$$=\frac{bc}{3\left(c^2d-e\right)e\sqrt{d+ex^2}}+\frac{x^3\left(a+b\tan^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}+\frac{(bc)\,\text{Subst}\left(\int\frac{1}{3d-\frac{3c^2d^2}{e}+\frac{3c^2dx^2}{e}}\,dx,x,\sqrt{d+ex^2}\right)}{\left(c^2d-e\right)e}$$
$$=\frac{bc}{3\left(c^2d-e\right)e\sqrt{d+ex^2}}+\frac{x^3\left(a+b\tan^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}-\frac{b\tanh^{-1}\left(\frac{c\sqrt{d+ex^2}}{\sqrt{c^2d-e}}\right)}{3d\left(c^2d-e\right)^{3/2}}$$

Mathematica [C] time = 1.06281, size = 252, normalized size = 2.31

$$-\frac{-\frac{2\left(ax\left(c^2d-e\right)+bcd\right)}{e\left(c^2d-e\right)\sqrt{d+ex^2}}+\frac{2adx}{e\left(d+ex^2\right)^{3/2}}+\frac{b\log\left(\frac{12cd\sqrt{c^2d-e}\left(\sqrt{c^2d-e}\sqrt{d+ex^2}+cd-iex\right)}{b(cx+i)}\right)}{\left(c^2d-e\right)^{3/2}}+\frac{b\log\left(\frac{12cd\sqrt{c^2d-e}\left(\sqrt{c^2d-e}\sqrt{d+ex^2}+cd+iex\right)}{b(cx-i)}\right)}{\left(c^2d-e\right)^{3/2}}-\frac{2bx^3\tan^{-1}(cx)}{\left(d+ex^2\right)^{3/2}}}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*ArcTan[c*x]))/(d + e*x^2)^(5/2),x]
```

```
[Out] -((2*a*d*x)/(e*(d + e*x^2)^(3/2)) - (2*(b*c*d + a*(c^2*d - e)*x))/((c^2*d -
 e)*e*Sqrt[d + e*x^2]) - (2*b*x^3*ArcTan[c*x])/(d + e*x^2)^(3/2) + (b*Log[(
12*c*d*Sqrt[c^2*d - e]*(c*d - I*e*x + Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*
(I + c*x))])/(c^2*d - e)^(3/2) + (b*Log[(12*c*d*Sqrt[c^2*d - e]*(c*d + I*e*
x + Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*(-I + c*x))])/(c^2*d - e)^(3/2))/(
6*d)
```

Maple [F] time = 0.604, size = 0, normalized size = 0.

$$\int x^2\left(a+b\arctan(cx)\right)\left(ex^2+d\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctan(c*x))/(e*x^2+d)^(5/2),x)
```

```
[Out] int(x^2*(a+b*arctan(c*x))/(e*x^2+d)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{3}\,a\left(\frac{x}{\left(ex^2+d\right)^{\frac{3}{2}}e}-\frac{x}{\sqrt{ex^2+d}de}\right)+2\,b\int\frac{x^2\arctan\left(cx\right)}{2\left(e^2x^4+2\,dex^2+d^2\right)\sqrt{ex^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*a*(x/((e*x^2 + d)^(3/2)*e) - x/(sqrt(e*x^2 + d)*d*e)) + 2*b*integrate(
1/2*x^2*arctan(c*x)/((e^2*x^4 + 2*d*e*x^2 + d^2)*sqrt(e*x^2 + d)), x)
```

Fricas [B] time = 3.3693, size = 1386, normalized size = 12.72

$$\left[-\frac{\left(be^3x^4+2\,bde^2x^2+bd^2e\right)\sqrt{c^2d-e}\log\left(\frac{c^4e^2x^4+8\,c^4d^2-8\,c^2de+2\left(4\,c^4de-3\,c^2e^2\right)x^2+4\left(c^3ex^2+2\,c^3d-ce\right)\sqrt{c^2d-e}\sqrt{ex^2+d}+e^2}{c^4x^4+2\,c^2x^2+1}\right)-4\left(bc^3d^3-bc\right.}{12\left(c^4d^5e-2\,c^2d^4e^2+d^3e^3+\left(c^4d^3e^3-2\,c^2d^2e^4+d\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/12*((b*e^3*x^4 + 2*b*d*e^2*x^2 + b*d^2*e)*sqrt(c^2*d - e)*log((c^4*e^2*
x^4 + 8*c^4*d^2 - 8*c^2*d*e + 2*(4*c^4*d*e - 3*c^2*e^2)*x^2 + 4*(c^3*e*x^2
+ 2*c^3*d - c*e)*sqrt(c^2*d - e)*sqrt(e*x^2 + d) + e^2)/(c^4*x^4 + 2*c^2*x^
2 + 1)) - 4*(b*c^3*d^3 - b*c*d^2*e + (b*c^4*d^2*e - 2*b*c^2*d*e^2 + b*e^3)*
x^3*arctan(c*x) + (a*c^4*d^2*e - 2*a*c^2*d*e^2 + a*e^3)*x^3 + (b*c^3*d^2*e
- b*c*d*e^2)*x^2)*sqrt(e*x^2 + d))/(c^4*d^5*e - 2*c^2*d^4*e^2 + d^3*e^3 + (
c^4*d^3*e^3 - 2*c^2*d^2*e^4 + d*e^5)*x^4 + 2*(c^4*d^4*e^2 - 2*c^2*d^3*e^3 +
 d^2*e^4)*x^2), -1/6*((b*e^3*x^4 + 2*b*d*e^2*x^2 + b*d^2*e)*sqrt(-c^2*d + e
)*arctan(-1/2*(c^2*e*x^2 + 2*c^2*d - e)*sqrt(-c^2*d + e)*sqrt(e*x^2 + d)/(c
```

```
^3*d^2 - c*d*e + (c^3*d*e - c*e^2)*x^2)) - 2*(b*c^3*d^3 - b*c*d^2*e + (b*c^
4*d^2*e - 2*b*c^2*d*e^2 + b*e^3)*x^3*arctan(c*x) + (a*c^4*d^2*e - 2*a*c^2*d
*e^2 + a*e^3)*x^3 + (b*c^3*d^2*e - b*c*d*e^2)*x^2)*sqrt(e*x^2 + d))/(c^4*d^
5*e - 2*c^2*d^4*e^2 + d^3*e^3 + (c^4*d^3*e^3 - 2*c^2*d^2*e^4 + d*e^5)*x^4 +
 2*(c^4*d^4*e^2 - 2*c^2*d^3*e^3 + d^2*e^4)*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atan(c*x))/(e*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\arctan(cx)+a)x^2}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)*x^2/(e*x^2 + d)^(5/2), x)
```

3.1220 $\int \frac{x\left(a+b\tan^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=110

$$-\frac{a+b\tan^{-1}(cx)}{3e\left(d+ex^2\right)^{3/2}}-\frac{bcx}{3d\left(c^2d-e\right)\sqrt{d+ex^2}}+\frac{bc^3\tan^{-1}\left(\frac{x\sqrt{c^2d-e}}{\sqrt{d+ex^2}}\right)}{3e\left(c^2d-e\right)^{3/2}}$$

```
[Out] -(b*c*x)/(3*d*(c^2*d - e)*Sqrt[d + e*x^2]) - (a + b*ArcTan[c*x])/(3*e*(d +
e*x^2)^(3/2)) + (b*c^3*ArcTan[(Sqrt[c^2*d - e]*x)/Sqrt[d + e*x^2]])/(3*(c^2
*d - e)^(3/2)*e)
```

Rubi [A] time = 0.093573, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {4974, 382, 377, 203}

$$-\frac{a+b\tan^{-1}(cx)}{3e\left(d+ex^2\right)^{3/2}}-\frac{bcx}{3d\left(c^2d-e\right)\sqrt{d+ex^2}}+\frac{bc^3\tan^{-1}\left(\frac{x\sqrt{c^2d-e}}{\sqrt{d+ex^2}}\right)}{3e\left(c^2d-e\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcTan[c*x]))/(d + e*x^2)^(5/2),x]
```

```
[Out] -(b*c*x)/(3*d*(c^2*d - e)*Sqrt[d + e*x^2]) - (a + b*ArcTan[c*x])/(3*e*(d +
e*x^2)^(3/2)) + (b*c^3*ArcTan[(Sqrt[c^2*d - e]*x)/Sqrt[d + e*x^2]])/(3*(c^2
*d - e)^(3/2)*e)
```

Rule 4974

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]))/(2*e*(q + 1)), x
] - Dist[(b*c)/(2*e*(q + 1)), Int[(d + e*x^2)^(q + 1)/(1 + c^2*x^2), x], x]
 /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
```

```
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x\left(a+b\tan^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx &= -\frac{a+b\tan^{-1}(cx)}{3e\left(d+ex^2\right)^{3/2}}+\frac{(bc)\int \frac{1}{\left(1+c^2x^2\right)\left(d+ex^2\right)^{3/2}}\,dx}{3e}\\
&= -\frac{bcx}{3d\left(c^2d-e\right)\sqrt{d+ex^2}}-\frac{a+b\tan^{-1}(cx)}{3e\left(d+ex^2\right)^{3/2}}+\frac{\left(bc^3\right)\int \frac{1}{\left(1+c^2x^2\right)\sqrt{d+ex^2}}\,dx}{3\left(c^2d-e\right)e}\\
&= -\frac{bcx}{3d\left(c^2d-e\right)\sqrt{d+ex^2}}-\frac{a+b\tan^{-1}(cx)}{3e\left(d+ex^2\right)^{3/2}}+\frac{\left(bc^3\right)\text{Subst}\left(\int \frac{1}{1-\left(-c^2d+e\right)x^2}\,dx,x,\frac{x}{\sqrt{d+ex^2}}\right)}{3\left(c^2d-e\right)e}\\
&= -\frac{bcx}{3d\left(c^2d-e\right)\sqrt{d+ex^2}}-\frac{a+b\tan^{-1}(cx)}{3e\left(d+ex^2\right)^{3/2}}+\frac{bc^3\tan^{-1}\left(\frac{\sqrt{c^2d-e}x}{\sqrt{d+ex^2}}\right)}{3\left(c^2d-e\right)^{3/2}e}
\end{aligned}
$$

Mathematica [C] time = 0.725135, size = 259, normalized size = 2.35

$$
\frac{1}{6}\left(-\frac{2a}{e\left(d+ex^2\right)^{3/2}}-\frac{2bcx}{\left(c^2d^2-de\right)\sqrt{d+ex^2}}-\frac{ibc^3\log\left(-\frac{12ie\sqrt{c^2d-e}\left(\sqrt{c^2d-e}\sqrt{d+ex^2}+cd-iex\right)}{bc^2(cx+i)}\right)}{e\left(c^2d-e\right)^{3/2}}+\frac{ibc^3\log\left(\frac{12ie\sqrt{c^2d-e}\left(\sqrt{c^2d-e}\sqrt{d+ex^2}\right.}{bc^2(cx-i)}\right.}{e\left(c^2d-e\right)^{3/2}}\right.
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcTan[c*x]))/(d + e*x^2)^(5/2),x]
```

```
[Out] ((-2*a)/(e*(d + e*x^2)^(3/2)) - (2*b*c*x)/((c^2*d^2 - d*e)*Sqrt[d + e*x^2])
 - (2*b*ArcTan[c*x])/(e*(d + e*x^2)^(3/2)) - (I*b*c^3*Log[((-12*I)*Sqrt[c^2
*d - e]*e*(c*d - I*e*x + Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*c^2*(I + c*x)
)])/((c^2*d - e)^(3/2)*e) + (I*b*c^3*Log[((12*I)*Sqrt[c^2*d - e]*e*(c*d + I
*e*x + Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*c^2*(-I + c*x))])/((c^2*d - e)^
(3/2)*e))/6
```

Maple [F] time = 0.607, size = 0, normalized size = 0.

$$\int x\left(a+b\arctan\left(cx\right)\right)\left(ex^{2}+d\right)^{-\frac{5}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctan(c*x))/(e*x^2+d)^(5/2),x)
```

```
[Out] int(x*(a+b*arctan(c*x))/(e*x^2+d)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 5.24779, size = 1385, normalized size = 12.59

$$\left[\frac{\left(bc^{3}de^{2}x^{4}+2\,bc^{3}d^{2}ex^{2}+bc^{3}d^{3}\right)\sqrt{-c^{2}d+e}\log\left(\frac{\left(c^{4}d^{2}-8\,c^{2}de+8\,e^{2}\right)x^{4}-2\left(3\,c^{2}d^{2}-4\,de\right)x^{2}+4\left(\left(c^{2}d-2\,e\right)x^{3}-dx\right)\sqrt{-c^{2}d+e}\sqrt{ex^{2}+d}+d^{2}}{c^{4}x^{4}+2\,c^{2}x^{2}+1}\right)-4\left(ac^{4}\cdots\right.}{12\left(c^{4}d^{5}e-2\,c^{2}d^{4}e^{2}+d^{3}e^{3}+\left(c^{4}d^{3}e^{3}-2\,c^{2}d^{2}e^{4}+\cdots\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*((b*c^3*d*e^2*x^4 + 2*b*c^3*d^2*e*x^2 + b*c^3*d^3)*sqrt(-c^2*d + e)*l
og(((c^4*d^2 - 8*c^2*d*e + 8*e^2)*x^4 - 2*(3*c^2*d^2 - 4*d*e)*x^2 + 4*((c^2
*d - 2*e)*x^3 - d*x)*sqrt(-c^2*d + e)*sqrt(e*x^2 + d) + d^2)/(c^4*x^4 + 2*c
^2*x^2 + 1)) - 4*(a*c^4*d^3 - 2*a*c^2*d^2*e + a*d*e^2 + (b*c^3*d*e^2 - b*c*
e^3)*x^3 + (b*c^3*d^2*e - b*c*d*e^2)*x + (b*c^4*d^3 - 2*b*c^2*d^2*e + b*d*e
^2)*arctan(c*x))*sqrt(e*x^2 + d))/(c^4*d^5*e - 2*c^2*d^4*e^2 + d^3*e^3 + (c
^4*d^3*e^3 - 2*c^2*d^2*e^4 + d*e^5)*x^4 + 2*(c^4*d^4*e^2 - 2*c^2*d^3*e^3 +
d^2*e^4)*x^2), 1/6*((b*c^3*d*e^2*x^4 + 2*b*c^3*d^2*e*x^2 + b*c^3*d^3)*sqrt(
c^2*d - e)*arctan(1/2*sqrt(c^2*d - e)*((c^2*d - 2*e)*x^2 - d)*sqrt(e*x^2 +
d)/((c^2*d*e - e^2)*x^3 + (c^2*d^2 - d*e)*x)) - 2*(a*c^4*d^3 - 2*a*c^2*d^2*
e + a*d*e^2 + (b*c^3*d*e^2 - b*c*e^3)*x^3 + (b*c^3*d^2*e - b*c*d*e^2)*x + (
b*c^4*d^3 - 2*b*c^2*d^2*e + b*d*e^2)*arctan(c*x))*sqrt(e*x^2 + d))/(c^4*d^5
*e - 2*c^2*d^4*e^2 + d^3*e^3 + (c^4*d^3*e^3 - 2*c^2*d^2*e^4 + d*e^5)*x^4 +
2*(c^4*d^4*e^2 - 2*c^2*d^3*e^3 + d^2*e^4)*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\,(a + b\,\mathrm{atan}\,(cx))}{\left(d + ex^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atan(c*x))/(e*x**2+d)**(5/2),x)
```

```
[Out] Integral(x*(a + b*atan(c*x))/(d + e*x**2)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\arctan{(cx)} + a)x}{\left(ex^2 + d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)*x/(e*x^2 + d)^(5/2), x)
```

3.1221 $\int \frac{a+b\tan^{-1}(cx)}{\left(d+ex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=144

$$\frac{2x\left(a+b\tan^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}}+\frac{x\left(a+b\tan^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}+\frac{b\left(3c^2d-2e\right)\tanh^{-1}\left(\frac{c\sqrt{d+ex^2}}{\sqrt{c^2d-e}}\right)}{3d^2\left(c^2d-e\right)^{3/2}}-\frac{bc}{3d\left(c^2d-e\right)\sqrt{d+ex^2}}$$

```
[Out] -(b*c)/(3*d*(c^2*d - e)*Sqrt[d + e*x^2]) + (x*(a + b*ArcTan[c*x]))/(3*d*(d
+ e*x^2)^(3/2)) + (2*x*(a + b*ArcTan[c*x]))/(3*d^2*Sqrt[d + e*x^2]) + (b*(3
*c^2*d - 2*e)*ArcTanh[(c*Sqrt[d + e*x^2])/Sqrt[c^2*d - e]])/(3*d^2*(c^2*d -
 e)^(3/2))
```

Rubi [A] time = 0.314336, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.45, Rules used = {192, 191, 4912, 6688, 12, 571, 78, 63, 208}

$$\frac{2x\left(a+b\tan^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}}+\frac{x\left(a+b\tan^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}+\frac{b\left(3c^2d-2e\right)\tanh^{-1}\left(\frac{c\sqrt{d+ex^2}}{\sqrt{c^2d-e}}\right)}{3d^2\left(c^2d-e\right)^{3/2}}-\frac{bc}{3d\left(c^2d-e\right)\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])/(d + e*x^2)^(5/2),x]
```


```
[Out] -(b*c)/(3*d*(c^2*d - e)*Sqrt[d + e*x^2]) + (x*(a + b*ArcTan[c*x]))/(3*d*(d
+ e*x^2)^(3/2)) + (2*x*(a + b*ArcTan[c*x]))/(3*d^2*Sqrt[d + e*x^2]) + (b*(3
*c^2*d - 2*e)*ArcTanh[(c*Sqrt[d + e*x^2])/Sqrt[c^2*d - e]])/(3*d^2*(c^2*d -
 e)^(3/2))
```


Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 4912

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcTan[c*x], u, x]
- Dist[b*c, Int[u/(1 + c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && (I
ntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 571

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{a+b\tan^{-1}(cx)}{\left(d+ex^2\right)^{5/2}}\,dx = \frac{x\left(a+b\tan^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}+\frac{2x\left(a+b\tan^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}}-(bc)\int \frac{\frac{x}{3d\left(d+ex^2\right)^{3/2}}+\frac{2x}{3d^2\sqrt{d+ex^2}}}{1+c^2x^2}\,dx$$

$$= \frac{x\left(a+b\tan^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}+\frac{2x\left(a+b\tan^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}}-(bc)\int \frac{x\left(3d+2ex^2\right)}{3d^2\left(1+c^2x^2\right)\left(d+ex^2\right)^{3/2}}\,dx$$

$$= \frac{x\left(a+b\tan^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}+\frac{2x\left(a+b\tan^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}}-\frac{(bc)\int \frac{x\left(3d+2ex^2\right)}{\left(1+c^2x^2\right)\left(d+ex^2\right)^{3/2}}\,dx}{3d^2}$$

$$= \frac{x\left(a+b\tan^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}+\frac{2x\left(a+b\tan^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}}-\frac{(bc)\,\text{Subst}\left(\int \frac{3d+2ex}{(1+c^2x)(d+ex)^{3/2}}\,dx,x,x^2\right)}{6d^2}$$

$$= -\frac{bc}{3d\left(c^2d-e\right)\sqrt{d+ex^2}}+\frac{x\left(a+b\tan^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}+\frac{2x\left(a+b\tan^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}}-\frac{\left(bc\left(3c^2d-2e\right)\right)\text{Subst}}{6d^2\,(}$$

$$= -\frac{bc}{3d\left(c^2d-e\right)\sqrt{d+ex^2}}+\frac{x\left(a+b\tan^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}+\frac{2x\left(a+b\tan^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}}-\frac{\left(bc\left(3c^2d-2e\right)\right)\text{Subst}}{3d^2}$$

$$= -\frac{bc}{3d\left(c^2d-e\right)\sqrt{d+ex^2}}+\frac{x\left(a+b\tan^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}+\frac{2x\left(a+b\tan^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}}+\frac{b\left(3c^2d-2e\right)\tanh^{-1}\left(\right.}{3d^2\left(c^2d-e\right)^{3/2}}$$

Mathematica [C] time = 0.552874, size = 317, normalized size = 2.2

$$\frac{2\sqrt{c^2d-e}\left(ax\left(c^2d-e\right)\left(3d+2ex^2\right)-bcd\left(d+ex^2\right)\right)+b\left(3c^2d-2e\right)\left(d+ex^2\right)^{3/2}\log\left(-\frac{12cd^2\sqrt{c^2d-e}\left(\sqrt{c^2d-e}\sqrt{d+ex^2}+cd-iex\right)}{b(cx+i)\left(3c^2d-2e\right)}\right)+}{6d^2\left(c^2d-e\right)^{3/2}\left(d+e\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])/(d + e*x^2)^(5/2),x]
```

```
[Out] (2*Sqrt[c^2*d - e]*(-(b*c*d*(d + e*x^2)) + a*(c^2*d - e)*x*(3*d + 2*e*x^2))
 + 2*b*(c^2*d - e)^(3/2)*x*(3*d + 2*e*x^2)*ArcTan[c*x] + b*(3*c^2*d - 2*e)*
(d + e*x^2)^(3/2)*Log[(-12*c*d^2*Sqrt[c^2*d - e]*(c*d - I*e*x + Sqrt[c^2*d
- e]*Sqrt[d + e*x^2]))/(b*(3*c^2*d - 2*e)*(I + c*x))] + b*(3*c^2*d - 2*e)*(
d + e*x^2)^(3/2)*Log[(-12*c*d^2*Sqrt[c^2*d - e]*(c*d + I*e*x + Sqrt[c^2*d -
 e]*Sqrt[d + e*x^2]))/(b*(3*c^2*d - 2*e)*(-I + c*x))])/(6*d^2*(c^2*d - e)^(
3/2)*(d + e*x^2)^(3/2))
```

Maple [F] time = 1.197, size = 0, normalized size = 0.

$$\int (a + b \arctan (cx)) \left(ex^2 + d\right)^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/(e*x^2+d)^(5/2),x)
```

```
[Out] int((a+b*arctan(c*x))/(e*x^2+d)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{3}a\left(\frac{2x}{\sqrt{ex^2 + d}d^2} + \frac{x}{\left(ex^2 + d\right)^{\frac{3}{2}}d}\right) + 2b\int \frac{\arctan (cx)}{2\left(e^2x^4 + 2dex^2 + d^2\right)\sqrt{ex^2 + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*a*(2*x/(sqrt(e*x^2 + d)*d^2) + x/((e*x^2 + d)^(3/2)*d)) + 2*b*integrate
(1/2*arctan(c*x)/((e^2*x^4 + 2*d*e*x^2 + d^2)*sqrt(e*x^2 + d)), x)
```

Fricas [B] time = 5.92089, size = 1778, normalized size = 12.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*((3*b*c^2*d^3 + (3*b*c^2*d*e^2 - 2*b*e^3)*x^4 - 2*b*d^2*e + 2*(3*b*c^
2*d^2*e - 2*b*d*e^2)*x^2)*sqrt(c^2*d - e)*log((c^4*e^2*x^4 + 8*c^4*d^2 - 8*
c^2*d*e + 2*(4*c^4*d*e - 3*c^2*e^2)*x^2 + 4*(c^3*e*x^2 + 2*c^3*d - c*e)*sqr
t(c^2*d - e)*sqrt(e*x^2 + d) + e^2)/(c^4*x^4 + 2*c^2*x^2 + 1)) - 4*(b*c^3*d
^3 - b*c*d^2*e - 2*(a*c^4*d^2*e - 2*a*c^2*d*e^2 + a*e^3)*x^3 + (b*c^3*d^2*e
 - b*c*d*e^2)*x^2 - 3*(a*c^4*d^3 - 2*a*c^2*d^2*e + a*d*e^2)*x - (2*(b*c^4*d
^2*e - 2*b*c^2*d*e^2 + b*e^3)*x^3 + 3*(b*c^4*d^3 - 2*b*c^2*d^2*e + b*d*e^2)
*x)*arctan(c*x))*sqrt(e*x^2 + d))/(c^4*d^6 - 2*c^2*d^5*e + d^4*e^2 + (c^4*d
^4*e^2 - 2*c^2*d^3*e^3 + d^2*e^4)*x^4 + 2*(c^4*d^5*e - 2*c^2*d^4*e^2 + d^3*
e^3)*x^2), 1/6*((3*b*c^2*d^3 + (3*b*c^2*d*e^2 - 2*b*e^3)*x^4 - 2*b*d^2*e +
2*(3*b*c^2*d^2*e - 2*b*d*e^2)*x^2)*sqrt(-c^2*d + e)*arctan(-1/2*(c^2*e*x^2
+ 2*c^2*d - e)*sqrt(-c^2*d + e)*sqrt(e*x^2 + d)/(c^3*d^2 - c*d*e + (c^3*d*e
 - c*e^2)*x^2)) - 2*(b*c^3*d^3 - b*c*d^2*e - 2*(a*c^4*d^2*e - 2*a*c^2*d*e^2
 + a*e^3)*x^3 + (b*c^3*d^2*e - b*c*d*e^2)*x^2 - 3*(a*c^4*d^3 - 2*a*c^2*d^2*
e + a*d*e^2)*x - (2*(b*c^4*d^2*e - 2*b*c^2*d*e^2 + b*e^3)*x^3 + 3*(b*c^4*d^
3 - 2*b*c^2*d^2*e + b*d*e^2)*x)*arctan(c*x))*sqrt(e*x^2 + d))/(c^4*d^6 - 2*
c^2*d^5*e + d^4*e^2 + (c^4*d^4*e^2 - 2*c^2*d^3*e^3 + d^2*e^4)*x^4 + 2*(c^4*
d^5*e - 2*c^2*d^4*e^2 + d^3*e^3)*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/(e*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \arctan(cx) + a}{(ex^2 + d)^{\frac{5}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)/(e*x^2 + d)^(5/2), x)
```

3.1222 $\int \frac{a+b\tan^{-1}(cx)}{x(d+ex^2)^{5/2}}\,dx$

Optimal. Leaf size=85

$$b\text{Unintegrable}\left(\frac{\tan^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}},x\right)+\frac{a}{d^2\sqrt{d+ex^2}}-\frac{a\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{d^{5/2}}+\frac{a}{3d\left(d+ex^2\right)^{3/2}}$$

```
[Out] a/(3*d*(d + e*x^2)^(3/2)) + a/(d^2*Sqrt[d + e*x^2]) - (a*ArcTanh[Sqrt[d + e
*x^2]/Sqrt[d]])/d^(5/2) + b*Unintegrable[ArcTan[c*x]/(x*(d + e*x^2)^(5/2)),
 x]
```

Rubi [A] time = 0.193208, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{a+b\tan^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcTan[c*x])/(x*(d + e*x^2)^(5/2)),x]
```

```
[Out] a/(3*d*(d + e*x^2)^(3/2)) + a/(d^2*Sqrt[d + e*x^2]) - (a*ArcTanh[Sqrt[d + e
*x^2]/Sqrt[d]])/d^(5/2) + b*Defer[Int][ArcTan[c*x]/(x*(d + e*x^2)^(5/2)), x
]
```

Rubi steps

$$\int \frac{a+b\tan^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx = a\int \frac{1}{x\left(d+ex^2\right)^{5/2}}\,dx + b\int \frac{\tan^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx$$

$$= \frac{1}{2}a\,\text{Subst}\left(\int \frac{1}{x(d+ex)^{5/2}}\,dx, x, x^2\right) + b\int \frac{\tan^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx$$

$$= \frac{a}{3d\left(d+ex^2\right)^{3/2}} + b\int \frac{\tan^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx + \frac{a\,\text{Subst}\left(\int \frac{1}{x(d+ex)^{3/2}}\,dx, x, x^2\right)}{2d}$$

$$= \frac{a}{3d\left(d+ex^2\right)^{3/2}} + \frac{a}{d^2\sqrt{d+ex^2}} + b\int \frac{\tan^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx + \frac{a\,\text{Subst}\left(\int \frac{1}{x\sqrt{d+ex}}\,dx, x, x^2\right)}{2d^2}$$

$$= \frac{a}{3d\left(d+ex^2\right)^{3/2}} + \frac{a}{d^2\sqrt{d+ex^2}} + b\int \frac{\tan^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx + \frac{a\,\text{Subst}\left(\int \frac{1}{-\frac{d}{e}+\frac{x^2}{e}}\,dx, x, \sqrt{d+ex^2}\right)}{d^2e}$$

$$= \frac{a}{3d\left(d+ex^2\right)^{3/2}} + \frac{a}{d^2\sqrt{d+ex^2}} - \frac{a\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{d^{5/2}} + b\int \frac{\tan^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx$$

Mathematica [A] time = 50.6108, size = 0, normalized size = 0.

$$\int \frac{a+b\tan^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcTan[c*x])/(x*(d + e*x^2)^(5/2)),x]

[Out] Integrate[(a + b*ArcTan[c*x])/(x*(d + e*x^2)^(5/2)), x]
```

Maple [A] time = 0.599, size = 0, normalized size = 0.

$$\int \frac{a+b\arctan(cx)}{x}\left(ex^2+d\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/x/(e*x^2+d)^(5/2),x)
```

```
[Out] int((a+b*arctan(c*x))/x/(e*x^2+d)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{ex^2+d}(b\arctan(cx)+a)}{e^3x^7+3\,de^2x^5+3\,d^2ex^3+d^3x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x^2 + d)*(b*arctan(c*x) + a)/(e^3*x^7 + 3*d*e^2*x^5 + 3*d^2
*e*x^3 + d^3*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/x/(e*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \arctan(cx) + a}{(ex^2 + d)^{\frac{5}{2}} x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x/(e*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)/((e*x^2 + d)^(5/2)*x), x)
```

3.1223 $\int \frac{a+b\tan^{-1}(cx)}{x^2\left(d+ex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=274

$$-\frac{8ex\left(a+b\tan^{-1}(cx)\right)}{3d^3\sqrt{d+ex^2}}-\frac{4ex\left(a+b\tan^{-1}(cx)\right)}{3d^2\left(d+ex^2\right)^{3/2}}-\frac{a+b\tan^{-1}(cx)}{dx\left(d+ex^2\right)^{3/2}}-\frac{b\left(3c^4d^2-12c^2de+8e^2\right)}{3cd^3\left(c^2d-e\right)\sqrt{d+ex^2}}+\frac{b\left(3c^4d^2-12c^2de+8e\right.}{3d^3\left(c^2d\right.}$$

```
[Out] (b*c)/(d^2*Sqrt[d + e*x^2]) - (8*b*e)/(3*c*d^3*Sqrt[d + e*x^2]) - (b*(3*c^4
*d^2 - 12*c^2*d*e + 8*e^2))/(3*c*d^3*(c^2*d - e)*Sqrt[d + e*x^2]) - (a + b*
ArcTan[c*x])/(d*x*(d + e*x^2)^(3/2)) - (4*e*x*(a + b*ArcTan[c*x]))/(3*d^2*(
d + e*x^2)^(3/2)) - (8*e*x*(a + b*ArcTan[c*x]))/(3*d^3*Sqrt[d + e*x^2]) - (
b*c*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/d^(5/2) + (b*(3*c^4*d^2 - 12*c^2*d*e
+ 8*e^2)*ArcTanh[(c*Sqrt[d + e*x^2])/Sqrt[c^2*d - e]])/(3*d^3*(c^2*d - e)^(
3/2))
```

Rubi [A] time = 0.93222, antiderivative size = 274, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 12, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.522, Rules used = {271, 192, 191, 4976, 12, 6725, 266, 51, 63, 208, 261, 444}

$$-\frac{8ex\left(a+b\tan^{-1}(cx)\right)}{3d^3\sqrt{d+ex^2}}-\frac{4ex\left(a+b\tan^{-1}(cx)\right)}{3d^2\left(d+ex^2\right)^{3/2}}-\frac{a+b\tan^{-1}(cx)}{dx\left(d+ex^2\right)^{3/2}}-\frac{b\left(3c^4d^2-12c^2de+8e^2\right)}{3cd^3\left(c^2d-e\right)\sqrt{d+ex^2}}+\frac{b\left(3c^4d^2-12c^2de+8e\right.}{3d^3\left(c^2d\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])/(x^2*(d + e*x^2)^(5/2)),x]
```

```
[Out] (b*c)/(d^2*Sqrt[d + e*x^2]) - (8*b*e)/(3*c*d^3*Sqrt[d + e*x^2]) - (b*(3*c^4
*d^2 - 12*c^2*d*e + 8*e^2))/(3*c*d^3*(c^2*d - e)*Sqrt[d + e*x^2]) - (a + b*
ArcTan[c*x])/(d*x*(d + e*x^2)^(3/2)) - (4*e*x*(a + b*ArcTan[c*x]))/(3*d^2*(
d + e*x^2)^(3/2)) - (8*e*x*(a + b*ArcTan[c*x]))/(3*d^3*Sqrt[d + e*x^2]) - (
b*c*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/d^(5/2) + (b*(3*c^4*d^2 - 12*c^2*d*e
+ 8*e^2)*ArcTanh[(c*Sqrt[d + e*x^2])/Sqrt[c^2*d - e]])/(3*d^3*(c^2*d - e)^(
3/2))
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
```

```
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 4976

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Dis
t[a + b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&  !(
ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(IL
tQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILtQ[(m
 - 1)/2, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\int \frac{a+b\tan^{-1}(cx)}{x^2\left(d+ex^2\right)^{5/2}}\,dx = -\frac{a+b\tan^{-1}(cx)}{dx\left(d+ex^2\right)^{3/2}} - \frac{4ex\left(a+b\tan^{-1}(cx)\right)}{3d^2\left(d+ex^2\right)^{3/2}} - \frac{8ex\left(a+b\tan^{-1}(cx)\right)}{3d^3\sqrt{d+ex^2}} - (bc)\int \frac{-3d^2-12dex^2-8}{3d^3x\left(1+c^2x^2\right)\left(d+\right.}$$

$$= -\frac{a+b\tan^{-1}(cx)}{dx\left(d+ex^2\right)^{3/2}} - \frac{4ex\left(a+b\tan^{-1}(cx)\right)}{3d^2\left(d+ex^2\right)^{3/2}} - \frac{8ex\left(a+b\tan^{-1}(cx)\right)}{3d^3\sqrt{d+ex^2}} - \frac{(bc)\int \frac{-3d^2-12dex^2-8e^2x^4}{x\left(1+c^2x^2\right)\left(d+ex^2\right)^{3/2}}\,dx}{3d^3}$$

$$= -\frac{a+b\tan^{-1}(cx)}{dx\left(d+ex^2\right)^{3/2}} - \frac{4ex\left(a+b\tan^{-1}(cx)\right)}{3d^2\left(d+ex^2\right)^{3/2}} - \frac{8ex\left(a+b\tan^{-1}(cx)\right)}{3d^3\sqrt{d+ex^2}} - \frac{(bc)\int\left(-\frac{3d^2}{x\left(d+ex^2\right)^{3/2}} - \frac{8e^2}{c^2\left(d+ex\right.}\right.}{3}$$

$$= -\frac{a+b\tan^{-1}(cx)}{dx\left(d+ex^2\right)^{3/2}} - \frac{4ex\left(a+b\tan^{-1}(cx)\right)}{3d^2\left(d+ex^2\right)^{3/2}} - \frac{8ex\left(a+b\tan^{-1}(cx)\right)}{3d^3\sqrt{d+ex^2}} + \frac{(bc)\int \frac{1}{x\left(d+ex^2\right)^{3/2}}\,dx}{d} + \frac{\left(8be^2\right.}{}$$

$$= -\frac{8be}{3cd^3\sqrt{d+ex^2}} - \frac{a+b\tan^{-1}(cx)}{dx\left(d+ex^2\right)^{3/2}} - \frac{4ex\left(a+b\tan^{-1}(cx)\right)}{3d^2\left(d+ex^2\right)^{3/2}} - \frac{8ex\left(a+b\tan^{-1}(cx)\right)}{3d^3\sqrt{d+ex^2}} + \frac{(bc)\,\mathrm{Subst}}{}$$

$$= \frac{bc}{d^2\sqrt{d+ex^2}} - \frac{8be}{3cd^3\sqrt{d+ex^2}} - \frac{b\left(3c^4d^2-12c^2de+8e^2\right)}{3cd^3\left(c^2d-e\right)\sqrt{d+ex^2}} - \frac{a+b\tan^{-1}(cx)}{dx\left(d+ex^2\right)^{3/2}} - \frac{4ex\left(a+b\tan^{-1}(c\right.}{3d^2\left(d+ex^2\right)^{3/2}}$$

$$= \frac{bc}{d^2\sqrt{d+ex^2}} - \frac{8be}{3cd^3\sqrt{d+ex^2}} - \frac{b\left(3c^4d^2-12c^2de+8e^2\right)}{3cd^3\left(c^2d-e\right)\sqrt{d+ex^2}} - \frac{a+b\tan^{-1}(cx)}{dx\left(d+ex^2\right)^{3/2}} - \frac{4ex\left(a+b\tan^{-1}(c\right.}{3d^2\left(d+ex^2\right)^{3/2}}$$

$$= \frac{bc}{d^2\sqrt{d+ex^2}} - \frac{8be}{3cd^3\sqrt{d+ex^2}} - \frac{b\left(3c^4d^2-12c^2de+8e^2\right)}{3cd^3\left(c^2d-e\right)\sqrt{d+ex^2}} - \frac{a+b\tan^{-1}(cx)}{dx\left(d+ex^2\right)^{3/2}} - \frac{4ex\left(a+b\tan^{-1}(c\right.}{3d^2\left(d+ex^2\right)^{3/2}}$$

Mathematica [C] time = 1.21761, size = 418, normalized size = 1.53

$$\frac{\frac{2e\left(5ax\left(e-c^2d\right)+bcd\right)}{\left(c^2d-e\right)\sqrt{d+ex^2}} - \frac{6a\sqrt{d+ex^2}}{x} - \frac{2adex}{\left(d+ex^2\right)^{3/2}} + \frac{b\left(3c^4d^2-12c^2de+8e^2\right)\log\left(-\frac{12cd^3\sqrt{c^2d-e}\left(\sqrt{c^2d-e}\sqrt{d+ex^2}+cd-iex\right)}{b(cx+i)\left(3c^4d^2-12c^2de+8e^2\right)}\right)}{\left(c^2d-e\right)^{3/2}} + \frac{b\left(3c^4d^2-12c^2de+8e^2\right)\log\left(-\frac{12cd^3\sqrt{c^2d-e}}{b(cx-i)(}\right.}{\left(c^2d-e\right)^{3/2}}}{6d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])/(x^2*(d + e*x^2)^(5/2)),x]
```

```
[Out] ((-2*a*d*e*x)/(d + e*x^2)^(3/2) + (2*e*(b*c*d + 5*a*(-(c^2*d) + e)*x))/((c^
2*d - e)*Sqrt[d + e*x^2]) - (6*a*Sqrt[d + e*x^2])/x - (2*b*(3*d^2 + 12*d*e*
x^2 + 8*e^2*x^4)*ArcTan[c*x])/(x*(d + e*x^2)^(3/2)) + 6*b*c*Sqrt[d]*Log[x]
- 6*b*c*Sqrt[d]*Log[d + Sqrt[d]*Sqrt[d + e*x^2]] + (b*(3*c^4*d^2 - 12*c^2*d
*e + 8*e^2)*Log[(-12*c*d^3*Sqrt[c^2*d - e]*(c*d - I*e*x + Sqrt[c^2*d - e]*S
qrt[d + e*x^2]))/(b*(3*c^4*d^2 - 12*c^2*d*e + 8*e^2)*(I + c*x))])/(c^2*d -
e)^(3/2) + (b*(3*c^4*d^2 - 12*c^2*d*e + 8*e^2)*Log[(-12*c*d^3*Sqrt[c^2*d -
e]*(c*d + I*e*x + Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*(3*c^4*d^2 - 12*c^2*
d*e + 8*e^2)*(-I + c*x))])/(c^2*d - e)^(3/2))/(6*d^3)
```

Maple [F] time = 0.584, size = 0, normalized size = 0.

$$\int \frac{a + b\arctan(cx)}{x^2}\left(ex^2 + d\right)^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/x^2/(e*x^2+d)^(5/2),x)
```

```
[Out] int((a+b*arctan(c*x))/x^2/(e*x^2+d)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^2/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 12.2379, size = 5621, normalized size = 20.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^2/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(((3*b*c^4*d^2*e^2 - 12*b*c^2*d*e^3 + 8*b*e^4)*x^5 + 2*(3*b*c^4*d^3*
e - 12*b*c^2*d^2*e^2 + 8*b*d*e^3)*x^3 + (3*b*c^4*d^4 - 12*b*c^2*d^3*e + 8*b
*d^2*e^2)*x)*sqrt(c^2*d - e)*log((c^4*e^2*x^4 + 8*c^4*d^2 - 8*c^2*d*e + 2*(
4*c^4*d*e - 3*c^2*e^2)*x^2 - 4*(c^3*e*x^2 + 2*c^3*d - c*e)*sqrt(c^2*d - e)*
sqrt(e*x^2 + d) + e^2)/(c^4*x^4 + 2*c^2*x^2 + 1)) - 6*((b*c^5*d^2*e^2 - 2*b
*c^3*d*e^3 + b*c*e^4)*x^5 + 2*(b*c^5*d^3*e - 2*b*c^3*d^2*e^2 + b*c*d*e^3)*x
^3 + (b*c^5*d^4 - 2*b*c^3*d^3*e + b*c*d^2*e^2)*x)*sqrt(d)*log(-(e*x^2 - 2*s
qrt(e*x^2 + d)*sqrt(d) + 2*d)/x^2) + 4*(3*a*c^4*d^4 - 6*a*c^2*d^3*e + 3*a*d
^2*e^2 + 8*(a*c^4*d^2*e^2 - 2*a*c^2*d*e^3 + a*e^4)*x^4 - (b*c^3*d^2*e^2 - b
*c*d*e^3)*x^3 + 12*(a*c^4*d^3*e - 2*a*c^2*d^2*e^2 + a*d*e^3)*x^2 - (b*c^3*d
^3*e - b*c*d^2*e^2)*x + (3*b*c^4*d^4 - 6*b*c^2*d^3*e + 3*b*d^2*e^2 + 8*(b*c
^4*d^2*e^2 - 2*b*c^2*d*e^3 + b*e^4)*x^4 + 12*(b*c^4*d^3*e - 2*b*c^2*d^2*e^2
 + b*d*e^3)*x^2)*arctan(c*x))*sqrt(e*x^2 + d))/((c^4*d^5*e^2 - 2*c^2*d^4*e^
3 + d^3*e^4)*x^5 + 2*(c^4*d^6*e - 2*c^2*d^5*e^2 + d^4*e^3)*x^3 + (c^4*d^7 -
 2*c^2*d^6*e + d^5*e^2)*x), 1/6*(((3*b*c^4*d^2*e^2 - 12*b*c^2*d*e^3 + 8*b*e
^4)*x^5 + 2*(3*b*c^4*d^3*e - 12*b*c^2*d^2*e^2 + 8*b*d*e^3)*x^3 + (3*b*c^4*d
^4 - 12*b*c^2*d^3*e + 8*b*d^2*e^2)*x)*sqrt(-c^2*d + e)*arctan(-1/2*(c^2*e*x
^2 + 2*c^2*d - e)*sqrt(-c^2*d + e)*sqrt(e*x^2 + d)/(c^3*d^2 - c*d*e + (c^3*
d*e - c*e^2)*x^2)) + 3*((b*c^5*d^2*e^2 - 2*b*c^3*d*e^3 + b*c*e^4)*x^5 + 2*(
b*c^5*d^3*e - 2*b*c^3*d^2*e^2 + b*c*d*e^3)*x^3 + (b*c^5*d^4 - 2*b*c^3*d^3*e
 + b*c*d^2*e^2)*x)*sqrt(d)*log(-(e*x^2 - 2*sqrt(e*x^2 + d)*sqrt(d) + 2*d)/x
^2) - 2*(3*a*c^4*d^4 - 6*a*c^2*d^3*e + 3*a*d^2*e^2 + 8*(a*c^4*d^2*e^2 - 2*a
*c^2*d*e^3 + a*e^4)*x^4 - (b*c^3*d^2*e^2 - b*c*d*e^3)*x^3 + 12*(a*c^4*d^3*e
 - 2*a*c^2*d^2*e^2 + a*d*e^3)*x^2 - (b*c^3*d^3*e - b*c*d^2*e^2)*x + (3*b*c^
4*d^4 - 6*b*c^2*d^3*e + 3*b*d^2*e^2 + 8*(b*c^4*d^2*e^2 - 2*b*c^2*d*e^3 + b*
e^4)*x^4 + 12*(b*c^4*d^3*e - 2*b*c^2*d^2*e^2 + b*d*e^3)*x^2)*arctan(c*x))*s
qrt(e*x^2 + d))/((c^4*d^5*e^2 - 2*c^2*d^4*e^3 + d^3*e^4)*x^5 + 2*(c^4*d^6*e
 - 2*c^2*d^5*e^2 + d^4*e^3)*x^3 + (c^4*d^7 - 2*c^2*d^6*e + d^5*e^2)*x), 1/1
2*(12*((b*c^5*d^2*e^2 - 2*b*c^3*d*e^3 + b*c*e^4)*x^5 + 2*(b*c^5*d^3*e - 2*b
*c^3*d^2*e^2 + b*c*d*e^3)*x^3 + (b*c^5*d^4 - 2*b*c^3*d^3*e + b*c*d^2*e^2)*x
)*sqrt(-d)*arctan(sqrt(-d)/sqrt(e*x^2 + d)) - ((3*b*c^4*d^2*e^2 - 12*b*c^2*
d*e^3 + 8*b*e^4)*x^5 + 2*(3*b*c^4*d^3*e - 12*b*c^2*d^2*e^2 + 8*b*d*e^3)*x^3
 + (3*b*c^4*d^4 - 12*b*c^2*d^3*e + 8*b*d^2*e^2)*x)*sqrt(c^2*d - e)*log((c^4
*e^2*x^4 + 8*c^4*d^2 - 8*c^2*d*e + 2*(4*c^4*d*e - 3*c^2*e^2)*x^2 - 4*(c^3*e
*x^2 + 2*c^3*d - c*e)*sqrt(c^2*d - e)*sqrt(e*x^2 + d) + e^2)/(c^4*x^4 + 2*c
^2*x^2 + 1)) - 4*(3*a*c^4*d^4 - 6*a*c^2*d^3*e + 3*a*d^2*e^2 + 8*(a*c^4*d^2*
e^2 - 2*a*c^2*d*e^3 + a*e^4)*x^4 - (b*c^3*d^2*e^2 - b*c*d*e^3)*x^3 + 12*(a*
c^4*d^3*e - 2*a*c^2*d^2*e^2 + a*d*e^3)*x^2 - (b*c^3*d^3*e - b*c*d^2*e^2)*x
+ (3*b*c^4*d^4 - 6*b*c^2*d^3*e + 3*b*d^2*e^2 + 8*(b*c^4*d^2*e^2 - 2*b*c^2*d
*e^3 + b*e^4)*x^4 + 12*(b*c^4*d^3*e - 2*b*c^2*d^2*e^2 + b*d*e^3)*x^2)*arcta
n(c*x))*sqrt(e*x^2 + d))/((c^4*d^5*e^2 - 2*c^2*d^4*e^3 + d^3*e^4)*x^5 + 2*(
c^4*d^6*e - 2*c^2*d^5*e^2 + d^4*e^3)*x^3 + (c^4*d^7 - 2*c^2*d^6*e + d^5*e^2
)*x), 1/6*(((3*b*c^4*d^2*e^2 - 12*b*c^2*d*e^3 + 8*b*e^4)*x^5 + 2*(3*b*c^4*d
^3*e - 12*b*c^2*d^2*e^2 + 8*b*d*e^3)*x^3 + (3*b*c^4*d^4 - 12*b*c^2*d^3*e +
8*b*d^2*e^2)*x)*sqrt(-c^2*d + e)*arctan(-1/2*(c^2*e*x^2 + 2*c^2*d - e)*sqrt
```

```
(-c^2*d + e)*sqrt(e*x^2 + d)/(c^3*d^2 - c*d*e + (c^3*d*e - c*e^2)*x^2)) + 6
*((b*c^5*d^2*e^2 - 2*b*c^3*d*e^3 + b*c*e^4)*x^5 + 2*(b*c^5*d^3*e - 2*b*c^3*
d^2*e^2 + b*c*d*e^3)*x^3 + (b*c^5*d^4 - 2*b*c^3*d^3*e + b*c*d^2*e^2)*x)*sqr
t(-d)*arctan(sqrt(-d)/sqrt(e*x^2 + d)) - 2*(3*a*c^4*d^4 - 6*a*c^2*d^3*e + 3
*a*d^2*e^2 + 8*(a*c^4*d^2*e^2 - 2*a*c^2*d*e^3 + a*e^4)*x^4 - (b*c^3*d^2*e^2
 - b*c*d*e^3)*x^3 + 12*(a*c^4*d^3*e - 2*a*c^2*d^2*e^2 + a*d*e^3)*x^2 - (b*c
^3*d^3*e - b*c*d^2*e^2)*x + (3*b*c^4*d^4 - 6*b*c^2*d^3*e + 3*b*d^2*e^2 + 8*
(b*c^4*d^2*e^2 - 2*b*c^2*d*e^3 + b*e^4)*x^4 + 12*(b*c^4*d^3*e - 2*b*c^2*d^2
*e^2 + b*d*e^3)*x^2)*arctan(c*x))*sqrt(e*x^2 + d))/((c^4*d^5*e^2 - 2*c^2*d^
4*e^3 + d^3*e^4)*x^5 + 2*(c^4*d^6*e - 2*c^2*d^5*e^2 + d^4*e^3)*x^3 + (c^4*d
^7 - 2*c^2*d^6*e + d^5*e^2)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/x**2/(e*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \arctan(cx) + a}{(ex^2 + d)^{\frac{5}{2}} x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^2/(e*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)/((e*x^2 + d)^(5/2)*x^2), x)
```

3.1224 $\int \frac{a+b\tan^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=115

$$b\text{Unintegrable}\left(\frac{\tan^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}},x\right)-\frac{5ae}{2d^3\sqrt{d+ex^2}}-\frac{5ae}{6d^2\left(d+ex^2\right)^{3/2}}+\frac{5ae\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{2d^{7/2}}-\frac{a}{2dx^2\left(d+ex^2\right)^{3/2}}$$

```
[Out] (-5*a*e)/(6*d^2*(d + e*x^2)^(3/2)) - a/(2*d*x^2*(d + e*x^2)^(3/2)) - (5*a*e
)/(2*d^3*Sqrt[d + e*x^2]) + (5*a*e*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/(2*d^(
7/2)) + b*Unintegrable[ArcTan[c*x]/(x^3*(d + e*x^2)^(5/2)), x]
```

Rubi [A] time = 0.208684, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{a+b\tan^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcTan[c*x])/(x^3*(d + e*x^2)^(5/2)),x]
```

```
[Out] a/(3*d*x^2*(d + e*x^2)^(3/2)) + (5*a)/(3*d^2*x^2*Sqrt[d + e*x^2]) - (5*a*Sq
rt[d + e*x^2])/(2*d^3*x^2) + (5*a*e*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/(2*d^
(7/2)) + b*Defer[Int][ArcTan[c*x]/(x^3*(d + e*x^2)^(5/2)), x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+b\tan^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx &= a\int \frac{1}{x^3\left(d+ex^2\right)^{5/2}}\,dx + b\int \frac{\tan^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx \\
&= \frac{1}{2}a\,\text{Subst}\left(\int \frac{1}{x^2(d+ex)^{5/2}}\,dx, x, x^2\right) + b\int \frac{\tan^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx \\
&= \frac{a}{3dx^2\left(d+ex^2\right)^{3/2}} + b\int \frac{\tan^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx + \frac{(5a)\,\text{Subst}\left(\int \frac{1}{x^2(d+ex)^{3/2}}\,dx, x, x^2\right)}{6d} \\
&= \frac{a}{3dx^2\left(d+ex^2\right)^{3/2}} + \frac{5a}{3d^2x^2\sqrt{d+ex^2}} + b\int \frac{\tan^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx + \frac{(5a)\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{d+ex}}\,dx, x, x^2\right.}{2d^2} \\
&= \frac{a}{3dx^2\left(d+ex^2\right)^{3/2}} + \frac{5a}{3d^2x^2\sqrt{d+ex^2}} - \frac{5a\sqrt{d+ex^2}}{2d^3x^2} + b\int \frac{\tan^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx - \frac{(5ae)\,\text{Subst}\left(\int \right.}{4d} \\
&= \frac{a}{3dx^2\left(d+ex^2\right)^{3/2}} + \frac{5a}{3d^2x^2\sqrt{d+ex^2}} - \frac{5a\sqrt{d+ex^2}}{2d^3x^2} + b\int \frac{\tan^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx - \frac{(5a)\,\text{Subst}\left(\int \right.}{} \\
&= \frac{a}{3dx^2\left(d+ex^2\right)^{3/2}} + \frac{5a}{3d^2x^2\sqrt{d+ex^2}} - \frac{5a\sqrt{d+ex^2}}{2d^3x^2} + \frac{5ae\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{2d^{7/2}} + b\int \frac{\tan^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}
\end{aligned}
$$

Mathematica [A] time = 53.7482, size = 0, normalized size = 0.

$$\int \frac{a+b\tan^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcTan[c*x])/(x^3*(d + e*x^2)^(5/2)),x]
```

```
[Out] Integrate[(a + b*ArcTan[c*x])/(x^3*(d + e*x^2)^(5/2)), x]
```

Maple [A] time = 0.595, size = 0, normalized size = 0.

$$\int \frac{a+b\arctan(cx)}{x^3}\left(ex^2+d\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/x^3/(e*x^2+d)^(5/2),x)
```

```
[Out] int((a+b*arctan(c*x))/x^3/(e*x^2+d)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^3/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{ex^2+d}(b\arctan(cx)+a)}{e^3x^9+3\,de^2x^7+3\,d^2ex^5+d^3x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^3/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x^2 + d)*(b*arctan(c*x) + a)/(e^3*x^9 + 3*d*e^2*x^7 + 3*d^2
*e*x^5 + d^3*x^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/x**3/(e*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \arctan(cx) + a}{(ex^2 + d)^{\frac{5}{2}} x^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^3/(e*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)/((e*x^2 + d)^(5/2)*x^3), x)
```

3.1225 $\int \frac{a+b\tan^{-1}(cx)}{x^4\left(d+ex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=423

$$\frac{16e^2x\left(a+b\tan^{-1}(cx)\right)}{3d^4\sqrt{d+ex^2}}+\frac{8e^2x\left(a+b\tan^{-1}(cx)\right)}{3d^3\left(d+ex^2\right)^{3/2}}+\frac{2e\left(a+b\tan^{-1}(cx)\right)}{d^2x\left(d+ex^2\right)^{3/2}}-\frac{a+b\tan^{-1}(cx)}{3dx^3\left(d+ex^2\right)^{3/2}}+\frac{b\left(c^2d-2e\right)\left(c^4d^2+8c^2de\right.}{3cd^4\left(c^2d-e\right)\sqrt{d+e}}$$

```
[Out] -(b*c*e)/(2*d^3*Sqrt[d + e*x^2]) + (16*b*e^2)/(3*c*d^4*Sqrt[d + e*x^2]) - (
b*c*(c^2*d + 6*e))/(3*d^3*Sqrt[d + e*x^2]) + (b*(c^2*d - 2*e)*(c^4*d^2 + 8*
c^2*d*e - 8*e^2))/(3*c*d^4*(c^2*d - e)*Sqrt[d + e*x^2]) - (b*c)/(6*d^2*x^2*
Sqrt[d + e*x^2]) - (a + b*ArcTan[c*x])/(3*d*x^3*(d + e*x^2)^(3/2)) + (2*e*(
a + b*ArcTan[c*x]))/(d^2*x*(d + e*x^2)^(3/2)) + (8*e^2*x*(a + b*ArcTan[c*x]
))/(3*d^3*(d + e*x^2)^(3/2)) + (16*e^2*x*(a + b*ArcTan[c*x]))/(3*d^4*Sqrt[d
 + e*x^2]) + (b*c*e*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/(2*d^(7/2)) + (b*c*(c
^2*d + 6*e)*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/(3*d^(7/2)) - (b*(c^2*d - 2*e
)*(c^4*d^2 + 8*c^2*d*e - 8*e^2)*ArcTanh[(c*Sqrt[d + e*x^2])/Sqrt[c^2*d - e]
])/(3*d^4*(c^2*d - e)^(3/2))
```

Rubi [A] time = 1.09543, antiderivative size = 425, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 12, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.522, Rules used = {271, 192, 191, 4976, 12, 6725, 266, 51, 63, 208, 261, 444}

$$\frac{16e^2x\left(a+b\tan^{-1}(cx)\right)}{3d^4\sqrt{d+ex^2}}+\frac{8e^2x\left(a+b\tan^{-1}(cx)\right)}{3d^3\left(d+ex^2\right)^{3/2}}+\frac{2e\left(a+b\tan^{-1}(cx)\right)}{d^2x\left(d+ex^2\right)^{3/2}}-\frac{a+b\tan^{-1}(cx)}{3dx^3\left(d+ex^2\right)^{3/2}}+\frac{b\left(c^2d-2e\right)\left(c^4d^2+8c^2de\right.}{3cd^4\left(c^2d-e\right)\sqrt{d+e}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])/(x^4*(d + e*x^2)^(5/2)),x]
```

```
[Out] (16*b*e^2)/(3*c*d^4*Sqrt[d + e*x^2]) - (b*c*(c^2*d + 6*e))/(3*d^3*Sqrt[d +
e*x^2]) + (b*(c^2*d - 2*e)*(c^4*d^2 + 8*c^2*d*e - 8*e^2))/(3*c*d^4*(c^2*d -
 e)*Sqrt[d + e*x^2]) + (b*c)/(3*d^2*x^2*Sqrt[d + e*x^2]) - (b*c*Sqrt[d + e*
x^2])/(2*d^3*x^2) - (a + b*ArcTan[c*x])/(3*d*x^3*(d + e*x^2)^(3/2)) + (2*e*
(a + b*ArcTan[c*x]))/(d^2*x*(d + e*x^2)^(3/2)) + (8*e^2*x*(a + b*ArcTan[c*x
]))/(3*d^3*(d + e*x^2)^(3/2)) + (16*e^2*x*(a + b*ArcTan[c*x]))/(3*d^4*Sqrt[
d + e*x^2]) + (b*c*e*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/(2*d^(7/2)) + (b*c*(
c^2*d + 6*e)*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/(3*d^(7/2)) - (b*(c^2*d - 2*
```

```
e)*(c^4*d^2 + 8*c^2*d*e - 8*e^2)*ArcTanh[(c*Sqrt[d + e*x^2])/Sqrt[c^2*d - e
]])/(3*d^4*(c^2*d - e)^(3/2))
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 4976

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Dis
t[a + b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&  !(
ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(IL
tQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILtQ[(m
 - 1)/2, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\int \frac{a+b\tan^{-1}(cx)}{x^4\left(d+ex^2\right)^{5/2}}\,dx = -\frac{a+b\tan^{-1}(cx)}{3dx^3\left(d+ex^2\right)^{3/2}} + \frac{2e\left(a+b\tan^{-1}(cx)\right)}{d^2x\left(d+ex^2\right)^{3/2}} + \frac{8e^2x\left(a+b\tan^{-1}(cx)\right)}{3d^3\left(d+ex^2\right)^{3/2}} + \frac{16e^2x\left(a+b\tan^{-1}(cx)\right)}{3d^4\sqrt{d+ex^2}}$$

$$= -\frac{a+b\tan^{-1}(cx)}{3dx^3\left(d+ex^2\right)^{3/2}} + \frac{2e\left(a+b\tan^{-1}(cx)\right)}{d^2x\left(d+ex^2\right)^{3/2}} + \frac{8e^2x\left(a+b\tan^{-1}(cx)\right)}{3d^3\left(d+ex^2\right)^{3/2}} + \frac{16e^2x\left(a+b\tan^{-1}(cx)\right)}{3d^4\sqrt{d+ex^2}}$$

$$= -\frac{a+b\tan^{-1}(cx)}{3dx^3\left(d+ex^2\right)^{3/2}} + \frac{2e\left(a+b\tan^{-1}(cx)\right)}{d^2x\left(d+ex^2\right)^{3/2}} + \frac{8e^2x\left(a+b\tan^{-1}(cx)\right)}{3d^3\left(d+ex^2\right)^{3/2}} + \frac{16e^2x\left(a+b\tan^{-1}(cx)\right)}{3d^4\sqrt{d+ex^2}}$$

$$= -\frac{a+b\tan^{-1}(cx)}{3dx^3\left(d+ex^2\right)^{3/2}} + \frac{2e\left(a+b\tan^{-1}(cx)\right)}{d^2x\left(d+ex^2\right)^{3/2}} + \frac{8e^2x\left(a+b\tan^{-1}(cx)\right)}{3d^3\left(d+ex^2\right)^{3/2}} + \frac{16e^2x\left(a+b\tan^{-1}(cx)\right)}{3d^4\sqrt{d+ex^2}}$$

$$= \frac{16be^2}{3cd^4\sqrt{d+ex^2}} - \frac{a+b\tan^{-1}(cx)}{3dx^3\left(d+ex^2\right)^{3/2}} + \frac{2e\left(a+b\tan^{-1}(cx)\right)}{d^2x\left(d+ex^2\right)^{3/2}} + \frac{8e^2x\left(a+b\tan^{-1}(cx)\right)}{3d^3\left(d+ex^2\right)^{3/2}} + \frac{16e^2x\left(a\right.}{3d^4}$$

$$= \frac{16be^2}{3cd^4\sqrt{d+ex^2}} - \frac{bc\left(c^2d+6e\right)}{3d^3\sqrt{d+ex^2}} + \frac{b\left(c^2d-2e\right)\left(c^4d^2+8c^2de-8e^2\right)}{3cd^4\left(c^2d-e\right)\sqrt{d+ex^2}} + \frac{bc}{3d^2x^2\sqrt{d+ex^2}} - \frac{a+b\,\mathrm{t}}{3dx^3\left(d\right.}$$

$$= \frac{16be^2}{3cd^4\sqrt{d+ex^2}} - \frac{bc\left(c^2d+6e\right)}{3d^3\sqrt{d+ex^2}} + \frac{b\left(c^2d-2e\right)\left(c^4d^2+8c^2de-8e^2\right)}{3cd^4\left(c^2d-e\right)\sqrt{d+ex^2}} + \frac{bc}{3d^2x^2\sqrt{d+ex^2}} - \frac{bc\sqrt{d+}}{2d^3x}$$

$$= \frac{16be^2}{3cd^4\sqrt{d+ex^2}} - \frac{bc\left(c^2d+6e\right)}{3d^3\sqrt{d+ex^2}} + \frac{b\left(c^2d-2e\right)\left(c^4d^2+8c^2de-8e^2\right)}{3cd^4\left(c^2d-e\right)\sqrt{d+ex^2}} + \frac{bc}{3d^2x^2\sqrt{d+ex^2}} - \frac{bc\sqrt{d+}}{2d^3x}$$

$$= \frac{16be^2}{3cd^4\sqrt{d+ex^2}} - \frac{bc\left(c^2d+6e\right)}{3d^3\sqrt{d+ex^2}} + \frac{b\left(c^2d-2e\right)\left(c^4d^2+8c^2de-8e^2\right)}{3cd^4\left(c^2d-e\right)\sqrt{d+ex^2}} + \frac{bc}{3d^2x^2\sqrt{d+ex^2}} - \frac{bc\sqrt{d+}}{2d^3x}$$

Mathematica [C] time = 2.13869, size = 510, normalized size = 1.21

$$-\frac{\frac{2a\left(-6d^2ex^2+d^3-24de^2x^4-16e^3x^6\right)}{x^3\left(d+ex^2\right)^{3/2}} + \frac{b\left(6c^4d^2e+c^6d^3-24c^2de^2+16e^3\right)\log\left(\frac{12cd^4\sqrt{c^2d-e}\left(\sqrt{c^2d-e}\sqrt{d+ex^2}+cd-iex\right)}{b(cx+i)\left(6c^4d^2e+c^6d^3-24c^2de^2+16e^3\right)}\right)}{\left(c^2d-e\right)^{3/2}} + \frac{b\left(6c^4d^2e+c^6d^3-24c^2de^2+16e^3\right)\log\left(\frac{12cd^4\sqrt{c}}{b(cx-i)(}\right.}{\left(c^2d-e\right)^{3/2}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])/(x^4*(d + e*x^2)^(5/2)),x]
```

```
[Out] -((2*a*(d^3 - 6*d^2*e*x^2 - 24*d*e^2*x^4 - 16*e^3*x^6))/(x^3*(d + e*x^2)^(3
/2)) + (b*c*d*(e*(-d + e*x^2) + c^2*d*(d + e*x^2)))/((c^2*d - e)*x^2*Sqrt[d
 + e*x^2]) + (2*b*(d^3 - 6*d^2*e*x^2 - 24*d*e^2*x^4 - 16*e^3*x^6)*ArcTan[c*
x])/(x^3*(d + e*x^2)^(3/2)) + b*c*Sqrt[d]*(2*c^2*d + 15*e)*Log[x] - b*c*Sqr
t[d]*(2*c^2*d + 15*e)*Log[d + Sqrt[d]*Sqrt[d + e*x^2]] + (b*(c^6*d^3 + 6*c^
4*d^2*e - 24*c^2*d*e^2 + 16*e^3)*Log[(12*c*d^4*Sqrt[c^2*d - e]*(c*d - I*e*x
 + Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*(c^6*d^3 + 6*c^4*d^2*e - 24*c^2*d*e
^2 + 16*e^3)*(I + c*x))])/(c^2*d - e)^(3/2) + (b*(c^6*d^3 + 6*c^4*d^2*e - 2
4*c^2*d*e^2 + 16*e^3)*Log[(12*c*d^4*Sqrt[c^2*d - e]*(c*d + I*e*x + Sqrt[c^2
*d - e]*Sqrt[d + e*x^2]))/(b*(c^6*d^3 + 6*c^4*d^2*e - 24*c^2*d*e^2 + 16*e^3
)*(-I + c*x))])/(c^2*d - e)^(3/2))/(6*d^4)
```

Maple [F] time = 0.601, size = 0, normalized size = 0.

$$\int \frac{a+b\arctan(cx)}{x^4}\left(ex^2+d\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/x^4/(e*x^2+d)^(5/2),x)
```

```
[Out] int((a+b*arctan(c*x))/x^4/(e*x^2+d)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^4/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 20.367, size = 7200, normalized size = 17.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^4/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(((b*c^6*d^3*e^2 + 6*b*c^4*d^2*e^3 - 24*b*c^2*d*e^4 + 16*b*e^5)*x^7
+ 2*(b*c^6*d^4*e + 6*b*c^4*d^3*e^2 - 24*b*c^2*d^2*e^3 + 16*b*d*e^4)*x^5 + (
b*c^6*d^5 + 6*b*c^4*d^4*e - 24*b*c^2*d^3*e^2 + 16*b*d^2*e^3)*x^3)*sqrt(c^2*
d - e)*log((c^4*e^2*x^4 + 8*c^4*d^2 - 8*c^2*d*e + 2*(4*c^4*d*e - 3*c^2*e^2)
*x^2 + 4*(c^3*e*x^2 + 2*c^3*d - c*e)*sqrt(c^2*d - e)*sqrt(e*x^2 + d) + e^2)
/(c^4*x^4 + 2*c^2*x^2 + 1)) - ((2*b*c^7*d^3*e^2 + 11*b*c^5*d^2*e^3 - 28*b*c
^3*d*e^4 + 15*b*c*e^5)*x^7 + 2*(2*b*c^7*d^4*e + 11*b*c^5*d^3*e^2 - 28*b*c^3
*d^2*e^3 + 15*b*c*d*e^4)*x^5 + (2*b*c^7*d^5 + 11*b*c^5*d^4*e - 28*b*c^3*d^3
*e^2 + 15*b*c*d^2*e^3)*x^3)*sqrt(d)*log(-(e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(d)
 + 2*d)/x^2) + 2*(2*a*c^4*d^5 - 4*a*c^2*d^4*e - 32*(a*c^4*d^2*e^3 - 2*a*c^2
*d*e^4 + a*e^5)*x^6 + 2*a*d^3*e^2 + (b*c^5*d^3*e^2 - b*c*d*e^4)*x^5 - 48*(a
*c^4*d^3*e^2 - 2*a*c^2*d^2*e^3 + a*d*e^4)*x^4 + 2*(b*c^5*d^4*e - b*c^3*d^3*
e^2)*x^3 - 12*(a*c^4*d^4*e - 2*a*c^2*d^3*e^2 + a*d^2*e^3)*x^2 + (b*c^5*d^5
- 2*b*c^3*d^4*e + b*c*d^3*e^2)*x + 2*(b*c^4*d^5 - 2*b*c^2*d^4*e - 16*(b*c^4
*d^2*e^3 - 2*b*c^2*d*e^4 + b*e^5)*x^6 + b*d^3*e^2 - 24*(b*c^4*d^3*e^2 - 2*b
*c^2*d^2*e^3 + b*d*e^4)*x^4 - 6*(b*c^4*d^4*e - 2*b*c^2*d^3*e^2 + b*d^2*e^3)
*x^2)*arctan(c*x))*sqrt(e*x^2 + d))/((c^4*d^6*e^2 - 2*c^2*d^5*e^3 + d^4*e^4
)*x^7 + 2*(c^4*d^7*e - 2*c^2*d^6*e^2 + d^5*e^3)*x^5 + (c^4*d^8 - 2*c^2*d^7*
e + d^6*e^2)*x^3), -1/12*(2*((b*c^6*d^3*e^2 + 6*b*c^4*d^2*e^3 - 24*b*c^2*d*
e^4 + 16*b*e^5)*x^7 + 2*(b*c^6*d^4*e + 6*b*c^4*d^3*e^2 - 24*b*c^2*d^2*e^3 +
 16*b*d*e^4)*x^5 + (b*c^6*d^5 + 6*b*c^4*d^4*e - 24*b*c^2*d^3*e^2 + 16*b*d^2
*e^3)*x^3)*sqrt(-c^2*d + e)*arctan(-1/2*(c^2*e*x^2 + 2*c^2*d - e)*sqrt(-c^2
*d + e)*sqrt(e*x^2 + d)/(c^3*d^2 - c*d*e + (c^3*d*e - c*e^2)*x^2)) - ((2*b*
c^7*d^3*e^2 + 11*b*c^5*d^2*e^3 - 28*b*c^3*d*e^4 + 15*b*c*e^5)*x^7 + 2*(2*b*
c^7*d^4*e + 11*b*c^5*d^3*e^2 - 28*b*c^3*d^2*e^3 + 15*b*c*d*e^4)*x^5 + (2*b*
c^7*d^5 + 11*b*c^5*d^4*e - 28*b*c^3*d^3*e^2 + 15*b*c*d^2*e^3)*x^3)*sqrt(d)*
log(-(e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(d) + 2*d)/x^2) + 2*(2*a*c^4*d^5 - 4*a*
c^2*d^4*e - 32*(a*c^4*d^2*e^3 - 2*a*c^2*d*e^4 + a*e^5)*x^6 + 2*a*d^3*e^2 +
(b*c^5*d^3*e^2 - b*c*d*e^4)*x^5 - 48*(a*c^4*d^3*e^2 - 2*a*c^2*d^2*e^3 + a*d
*e^4)*x^4 + 2*(b*c^5*d^4*e - b*c^3*d^3*e^2)*x^3 - 12*(a*c^4*d^4*e - 2*a*c^2
*d^3*e^2 + a*d^2*e^3)*x^2 + (b*c^5*d^5 - 2*b*c^3*d^4*e + b*c*d^3*e^2)*x + 2
*(b*c^4*d^5 - 2*b*c^2*d^4*e - 16*(b*c^4*d^2*e^3 - 2*b*c^2*d*e^4 + b*e^5)*x^
6 + b*d^3*e^2 - 24*(b*c^4*d^3*e^2 - 2*b*c^2*d^2*e^3 + b*d*e^4)*x^4 - 6*(b*c
^4*d^4*e - 2*b*c^2*d^3*e^2 + b*d^2*e^3)*x^2)*arctan(c*x))*sqrt(e*x^2 + d))/
((c^4*d^6*e^2 - 2*c^2*d^5*e^3 + d^4*e^4)*x^7 + 2*(c^4*d^7*e - 2*c^2*d^6*e^2
 + d^5*e^3)*x^5 + (c^4*d^8 - 2*c^2*d^7*e + d^6*e^2)*x^3), -1/12*(2*((2*b*c^
```

```
7*d^3*e^2 + 11*b*c^5*d^2*e^3 - 28*b*c^3*d*e^4 + 15*b*c*e^5)*x^7 + 2*(2*b*c^
7*d^4*e + 11*b*c^5*d^3*e^2 - 28*b*c^3*d^2*e^3 + 15*b*c*d*e^4)*x^5 + (2*b*c^
7*d^5 + 11*b*c^5*d^4*e - 28*b*c^3*d^3*e^2 + 15*b*c*d^2*e^3)*x^3)*sqrt(-d)*a
rctan(sqrt(-d)/sqrt(e*x^2 + d)) + ((b*c^6*d^3*e^2 + 6*b*c^4*d^2*e^3 - 24*b*
c^2*d*e^4 + 16*b*e^5)*x^7 + 2*(b*c^6*d^4*e + 6*b*c^4*d^3*e^2 - 24*b*c^2*d^2
*e^3 + 16*b*d*e^4)*x^5 + (b*c^6*d^5 + 6*b*c^4*d^4*e - 24*b*c^2*d^3*e^2 + 16
*b*d^2*e^3)*x^3)*sqrt(c^2*d - e)*log((c^4*e^2*x^4 + 8*c^4*d^2 - 8*c^2*d*e +
 2*(4*c^4*d*e - 3*c^2*e^2)*x^2 + 4*(c^3*e*x^2 + 2*c^3*d - c*e)*sqrt(c^2*d -
 e)*sqrt(e*x^2 + d) + e^2)/(c^4*x^4 + 2*c^2*x^2 + 1)) + 2*(2*a*c^4*d^5 - 4*
a*c^2*d^4*e - 32*(a*c^4*d^2*e^3 - 2*a*c^2*d*e^4 + a*e^5)*x^6 + 2*a*d^3*e^2
+ (b*c^5*d^3*e^2 - b*c*d*e^4)*x^5 - 48*(a*c^4*d^3*e^2 - 2*a*c^2*d^2*e^3 + a
*d*e^4)*x^4 + 2*(b*c^5*d^4*e - b*c^3*d^3*e^2)*x^3 - 12*(a*c^4*d^4*e - 2*a*c
^2*d^3*e^2 + a*d^2*e^3)*x^2 + (b*c^5*d^5 - 2*b*c^3*d^4*e + b*c*d^3*e^2)*x +
 2*(b*c^4*d^5 - 2*b*c^2*d^4*e - 16*(b*c^4*d^2*e^3 - 2*b*c^2*d*e^4 + b*e^5)*
x^6 + b*d^3*e^2 - 24*(b*c^4*d^3*e^2 - 2*b*c^2*d^2*e^3 + b*d*e^4)*x^4 - 6*(b
*c^4*d^4*e - 2*b*c^2*d^3*e^2 + b*d^2*e^3)*x^2)*arctan(c*x))*sqrt(e*x^2 + d)
)/((c^4*d^6*e^2 - 2*c^2*d^5*e^3 + d^4*e^4)*x^7 + 2*(c^4*d^7*e - 2*c^2*d^6*e
^2 + d^5*e^3)*x^5 + (c^4*d^8 - 2*c^2*d^7*e + d^6*e^2)*x^3), -1/6*(((b*c^6*d
^3*e^2 + 6*b*c^4*d^2*e^3 - 24*b*c^2*d*e^4 + 16*b*e^5)*x^7 + 2*(b*c^6*d^4*e
+ 6*b*c^4*d^3*e^2 - 24*b*c^2*d^2*e^3 + 16*b*d*e^4)*x^5 + (b*c^6*d^5 + 6*b*c
^4*d^4*e - 24*b*c^2*d^3*e^2 + 16*b*d^2*e^3)*x^3)*sqrt(-c^2*d + e)*arctan(-1
/2*(c^2*e*x^2 + 2*c^2*d - e)*sqrt(-c^2*d + e)*sqrt(e*x^2 + d)/(c^3*d^2 - c*
d*e + (c^3*d*e - c*e^2)*x^2)) + ((2*b*c^7*d^3*e^2 + 11*b*c^5*d^2*e^3 - 28*b
*c^3*d*e^4 + 15*b*c*e^5)*x^7 + 2*(2*b*c^7*d^4*e + 11*b*c^5*d^3*e^2 - 28*b*c
^3*d^2*e^3 + 15*b*c*d*e^4)*x^5 + (2*b*c^7*d^5 + 11*b*c^5*d^4*e - 28*b*c^3*d
^3*e^2 + 15*b*c*d^2*e^3)*x^3)*sqrt(-d)*arctan(sqrt(-d)/sqrt(e*x^2 + d)) + (
2*a*c^4*d^5 - 4*a*c^2*d^4*e - 32*(a*c^4*d^2*e^3 - 2*a*c^2*d*e^4 + a*e^5)*x^
6 + 2*a*d^3*e^2 + (b*c^5*d^3*e^2 - b*c*d*e^4)*x^5 - 48*(a*c^4*d^3*e^2 - 2*a
*c^2*d^2*e^3 + a*d*e^4)*x^4 + 2*(b*c^5*d^4*e - b*c^3*d^3*e^2)*x^3 - 12*(a*c
^4*d^4*e - 2*a*c^2*d^3*e^2 + a*d^2*e^3)*x^2 + (b*c^5*d^5 - 2*b*c^3*d^4*e +
b*c*d^3*e^2)*x + 2*(b*c^4*d^5 - 2*b*c^2*d^4*e - 16*(b*c^4*d^2*e^3 - 2*b*c^2
*d*e^4 + b*e^5)*x^6 + b*d^3*e^2 - 24*(b*c^4*d^3*e^2 - 2*b*c^2*d^2*e^3 + b*d
*e^4)*x^4 - 6*(b*c^4*d^4*e - 2*b*c^2*d^3*e^2 + b*d^2*e^3)*x^2)*arctan(c*x))
*sqrt(e*x^2 + d))/((c^4*d^6*e^2 - 2*c^2*d^5*e^3 + d^4*e^4)*x^7 + 2*(c^4*d^7
*e - 2*c^2*d^6*e^2 + d^5*e^3)*x^5 + (c^4*d^8 - 2*c^2*d^7*e + d^6*e^2)*x^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/x**4/(e*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\arctan(cx)+a}{(ex^2+d)^{\frac{5}{2}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^4/(e*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)/((e*x^2 + d)^(5/2)*x^4), x)
```

3.1226 $\int \frac{\tan^{-1}(ax)}{\left(c+dx^2\right)^{7/2}}\,dx$

Optimal. Leaf size=208

$$\frac{\left(15a^4c^2-20a^2cd+8d^2\right)\tanh^{-1}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c-d}}\right)}{15c^3\left(a^2c-d\right)^{5/2}}-\frac{a\left(7a^2c-4d\right)}{15c^2\left(a^2c-d\right)^2\sqrt{c+dx^2}}-\frac{a}{15c\left(a^2c-d\right)\left(c+dx^2\right)^{3/2}}+\frac{8x\tan^{-1}(ax)}{15c^3\sqrt{c+dx^2}}+\frac{4\ldots}{15\ldots}$$

```
[Out] -a/(15*c*(a^2*c - d)*(c + d*x^2)^(3/2)) - (a*(7*a^2*c - 4*d))/(15*c^2*(a^2*
c - d)^2*Sqrt[c + d*x^2]) + (x*ArcTan[a*x])/(5*c*(c + d*x^2)^(5/2)) + (4*x*
ArcTan[a*x])/(15*c^2*(c + d*x^2)^(3/2)) + (8*x*ArcTan[a*x])/(15*c^3*Sqrt[c
+ d*x^2]) + ((15*a^4*c^2 - 20*a^2*c*d + 8*d^2)*ArcTanh[(a*Sqrt[c + d*x^2])/
Sqrt[a^2*c - d]])/(15*c^3*(a^2*c - d)^(5/2))
```

Rubi [A] time = 0.995861, antiderivative size = 208, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {192, 191, 4912, 6688, 12, 6715, 897, 1261, 208}

$$\frac{\left(15a^4c^2-20a^2cd+8d^2\right)\tanh^{-1}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c-d}}\right)}{15c^3\left(a^2c-d\right)^{5/2}}-\frac{a\left(7a^2c-4d\right)}{15c^2\left(a^2c-d\right)^2\sqrt{c+dx^2}}-\frac{a}{15c\left(a^2c-d\right)\left(c+dx^2\right)^{3/2}}+\frac{8x\tan^{-1}(ax)}{15c^3\sqrt{c+dx^2}}+\frac{4\ldots}{15\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]/(c + d*x^2)^(7/2),x]
```

```
[Out] -a/(15*c*(a^2*c - d)*(c + d*x^2)^(3/2)) - (a*(7*a^2*c - 4*d))/(15*c^2*(a^2*
c - d)^2*Sqrt[c + d*x^2]) + (x*ArcTan[a*x])/(5*c*(c + d*x^2)^(5/2)) + (4*x*
ArcTan[a*x])/(15*c^2*(c + d*x^2)^(3/2)) + (8*x*ArcTan[a*x])/(15*c^3*Sqrt[c
+ d*x^2]) + ((15*a^4*c^2 - 20*a^2*c*d + 8*d^2)*ArcTanh[(a*Sqrt[c + d*x^2])/
Sqrt[a^2*c - d]])/(15*c^3*(a^2*c - d)^(5/2))
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 4912

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcTan[c*x], u, x]
- Dist[b*c, Int[u/(1 + c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && (I
ntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)}{(c+dx^2)^{7/2}}\,dx = \frac{x\tan^{-1}(ax)}{5c(c+dx^2)^{5/2}} + \frac{4x\tan^{-1}(ax)}{15c^2(c+dx^2)^{3/2}} + \frac{8x\tan^{-1}(ax)}{15c^3\sqrt{c+dx^2}} - a\int \frac{\frac{x}{5c(c+dx^2)^{5/2}} + \frac{4x}{15c^2(c+dx^2)^{3/2}} + \frac{8x}{15c^3\sqrt{c+dx^2}}}{1+a^2x^2}\,dx$$

$$= \frac{x\tan^{-1}(ax)}{5c(c+dx^2)^{5/2}} + \frac{4x\tan^{-1}(ax)}{15c^2(c+dx^2)^{3/2}} + \frac{8x\tan^{-1}(ax)}{15c^3\sqrt{c+dx^2}} - a\int \frac{x(15c^2+20cdx^2+8d^2x^4)}{15c^3(1+a^2x^2)(c+dx^2)^{5/2}}\,dx$$

$$= \frac{x\tan^{-1}(ax)}{5c(c+dx^2)^{5/2}} + \frac{4x\tan^{-1}(ax)}{15c^2(c+dx^2)^{3/2}} + \frac{8x\tan^{-1}(ax)}{15c^3\sqrt{c+dx^2}} - \frac{a\int \frac{x(15c^2+20cdx^2+8d^2x^4)}{(1+a^2x^2)(c+dx^2)^{5/2}}\,dx}{15c^3}$$

$$= \frac{x\tan^{-1}(ax)}{5c(c+dx^2)^{5/2}} + \frac{4x\tan^{-1}(ax)}{15c^2(c+dx^2)^{3/2}} + \frac{8x\tan^{-1}(ax)}{15c^3\sqrt{c+dx^2}} - \frac{a\,\mathrm{Subst}\left(\int \frac{15c^2+20cdx+8d^2x^2}{(1+a^2x)(c+dx)^{5/2}}\,dx, x, x^2\right)}{30c^3}$$

$$= \frac{x\tan^{-1}(ax)}{5c(c+dx^2)^{5/2}} + \frac{4x\tan^{-1}(ax)}{15c^2(c+dx^2)^{3/2}} + \frac{8x\tan^{-1}(ax)}{15c^3\sqrt{c+dx^2}} - \frac{a\,\mathrm{Subst}\left(\int \frac{3c^2+4cx^2+8x^4}{x^4\left(\frac{-a^2c+d}{d}+\frac{a^2x^2}{d}\right)}\,dx, x, \sqrt{c+dx^2}\right)}{15c^3d}$$

$$= \frac{x\tan^{-1}(ax)}{5c(c+dx^2)^{5/2}} + \frac{4x\tan^{-1}(ax)}{15c^2(c+dx^2)^{3/2}} + \frac{8x\tan^{-1}(ax)}{15c^3\sqrt{c+dx^2}} - \frac{a\,\mathrm{Subst}\left(\int\left(\frac{3c^2d}{(-a^2c+d)x^4} - \frac{c(7a^2c-4d)d}{(-a^2c+d)^2x^2} + \frac{d(15a^4c^2}{(-a^2c+d)}\right.\right.}{15c^3d}$$

$$= -\frac{a}{15c(a^2c-d)(c+dx^2)^{3/2}} - \frac{a(7a^2c-4d)}{15c^2(a^2c-d)^2\sqrt{c+dx^2}} + \frac{x\tan^{-1}(ax)}{5c(c+dx^2)^{5/2}} + \frac{4x\tan^{-1}(ax)}{15c^2(c+dx^2)^{3/2}} + \frac{8x\,\mathrm{t}}{15c^3}$$

$$= -\frac{a}{15c(a^2c-d)(c+dx^2)^{3/2}} - \frac{a(7a^2c-4d)}{15c^2(a^2c-d)^2\sqrt{c+dx^2}} + \frac{x\tan^{-1}(ax)}{5c(c+dx^2)^{5/2}} + \frac{4x\tan^{-1}(ax)}{15c^2(c+dx^2)^{3/2}} + \frac{8x\,\mathrm{t}}{15c^3}$$

Mathematica [C] time = 0.814048, size = 345, normalized size = 1.66

$$\frac{\frac{\left(15a^4c^2-20a^2cd+8d^2\right)\log\left(-\frac{60ac^3\left(a^2c-d\right)^{3/2}\left(\sqrt{a^2c-d}\sqrt{c+dx^2}+ac-idx\right)}{(ax+i)\left(15a^4c^2-20a^2cd+8d^2\right)}\right)}{\left(a^2c-d\right)^{5/2}}+\frac{\left(15a^4c^2-20a^2cd+8d^2\right)\log\left(-\frac{60ac^3\left(a^2c-d\right)^{3/2}\left(\sqrt{a^2c-d}\sqrt{c+dx^2}+ac+idx\right)}{(ax-i)\left(15a^4c^2-20a^2cd+8d^2\right)}\right)}{\left(a^2c-d\right)^{5/2}}-\frac{2ac\left(a^2c\left(8c+7dx^2\right.\right.}{\left(d-a^2c\right)^2\left(c\right.}}{30c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]/(c + d*x^2)^(7/2),x]
```

```
[Out] ((-2*a*c*(-(d*(5*c + 4*d*x^2)) + a^2*c*(8*c + 7*d*x^2)))/((-(a^2*c) + d)^2*
(c + d*x^2)^(3/2)) + (2*x*(15*c^2 + 20*c*d*x^2 + 8*d^2*x^4)*ArcTan[a*x])/(c
 + d*x^2)^(5/2) + ((15*a^4*c^2 - 20*a^2*c*d + 8*d^2)*Log[(-60*a*c^3*(a^2*c
- d)^(3/2)*(a*c - I*d*x + Sqrt[a^2*c - d]*Sqrt[c + d*x^2]))/((15*a^4*c^2 -
20*a^2*c*d + 8*d^2)*(I + a*x))])/(a^2*c - d)^(5/2) + ((15*a^4*c^2 - 20*a^2*
c*d + 8*d^2)*Log[(-60*a*c^3*(a^2*c - d)^(3/2)*(a*c + I*d*x + Sqrt[a^2*c - d
]*Sqrt[c + d*x^2]))/((15*a^4*c^2 - 20*a^2*c*d + 8*d^2)*(-I + a*x))])/(a^2*c
 - d)^(5/2))/(30*c^3)
```

Maple [F] time = 0.763, size = 0, normalized size = 0.

$$\int \arctan\left(ax\right)\left(dx^2+c\right)^{-\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)/(d*x^2+c)^(7/2),x)
```

```
[Out] int(arctan(a*x)/(d*x^2+c)^(7/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/(d*x^2+c)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.71288, size = 2568, normalized size = 12.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/(d*x^2+c)^(7/2),x, algorithm="fricas")
```

```
[Out] [1/60*((15*a^4*c^5 - 20*a^2*c^4*d + (15*a^4*c^2*d^3 - 20*a^2*c*d^4 + 8*d^5)
*x^6 + 8*c^3*d^2 + 3*(15*a^4*c^3*d^2 - 20*a^2*c^2*d^3 + 8*c*d^4)*x^4 + 3*(1
5*a^4*c^4*d - 20*a^2*c^3*d^2 + 8*c^2*d^3)*x^2)*sqrt(a^2*c - d)*log((a^4*d^2
*x^4 + 8*a^4*c^2 - 8*a^2*c*d + 2*(4*a^4*c*d - 3*a^2*d^2)*x^2 + 4*(a^3*d*x^2
 + 2*a^3*c - a*d)*sqrt(a^2*c - d)*sqrt(d*x^2 + c) + d^2)/(a^4*x^4 + 2*a^2*x
^2 + 1)) - 4*(8*a^5*c^5 - 13*a^3*c^4*d + 5*a*c^3*d^2 + (7*a^5*c^3*d^2 - 11*
a^3*c^2*d^3 + 4*a*c*d^4)*x^4 + 3*(5*a^5*c^4*d - 8*a^3*c^3*d^2 + 3*a*c^2*d^3
)*x^2 - (8*(a^6*c^3*d^2 - 3*a^4*c^2*d^3 + 3*a^2*c*d^4 - d^5)*x^5 + 20*(a^6*
c^4*d - 3*a^4*c^3*d^2 + 3*a^2*c^2*d^3 - c*d^4)*x^3 + 15*(a^6*c^5 - 3*a^4*c^
4*d + 3*a^2*c^3*d^2 - c^2*d^3)*x)*arctan(a*x))*sqrt(d*x^2 + c))/(a^6*c^9 -
3*a^4*c^8*d + 3*a^2*c^7*d^2 - c^6*d^3 + (a^6*c^6*d^3 - 3*a^4*c^5*d^4 + 3*a^
2*c^4*d^5 - c^3*d^6)*x^6 + 3*(a^6*c^7*d^2 - 3*a^4*c^6*d^3 + 3*a^2*c^5*d^4 -
 c^4*d^5)*x^4 + 3*(a^6*c^8*d - 3*a^4*c^7*d^2 + 3*a^2*c^6*d^3 - c^5*d^4)*x^2
), 1/30*((15*a^4*c^5 - 20*a^2*c^4*d + (15*a^4*c^2*d^3 - 20*a^2*c*d^4 + 8*d^
5)*x^6 + 8*c^3*d^2 + 3*(15*a^4*c^3*d^2 - 20*a^2*c^2*d^3 + 8*c*d^4)*x^4 + 3*
(15*a^4*c^4*d - 20*a^2*c^3*d^2 + 8*c^2*d^3)*x^2)*sqrt(-a^2*c + d)*arctan(-1
/2*(a^2*d*x^2 + 2*a^2*c - d)*sqrt(-a^2*c + d)*sqrt(d*x^2 + c)/(a^3*c^2 - a*
c*d + (a^3*c*d - a*d^2)*x^2)) - 2*(8*a^5*c^5 - 13*a^3*c^4*d + 5*a*c^3*d^2 +
 (7*a^5*c^3*d^2 - 11*a^3*c^2*d^3 + 4*a*c*d^4)*x^4 + 3*(5*a^5*c^4*d - 8*a^3*
c^3*d^2 + 3*a*c^2*d^3)*x^2 - (8*(a^6*c^3*d^2 - 3*a^4*c^2*d^3 + 3*a^2*c*d^4
- d^5)*x^5 + 20*(a^6*c^4*d - 3*a^4*c^3*d^2 + 3*a^2*c^2*d^3 - c*d^4)*x^3 + 1
5*(a^6*c^5 - 3*a^4*c^4*d + 3*a^2*c^3*d^2 - c^2*d^3)*x)*arctan(a*x))*sqrt(d*
x^2 + c))/(a^6*c^9 - 3*a^4*c^8*d + 3*a^2*c^7*d^2 - c^6*d^3 + (a^6*c^6*d^3 -
 3*a^4*c^5*d^4 + 3*a^2*c^4*d^5 - c^3*d^6)*x^6 + 3*(a^6*c^7*d^2 - 3*a^4*c^6*
d^3 + 3*a^2*c^5*d^4 - c^4*d^5)*x^4 + 3*(a^6*c^8*d - 3*a^4*c^7*d^2 + 3*a^2*c
^6*d^3 - c^5*d^4)*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)/(d*x**2+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.18414, size = 275, normalized size = 1.32

$$-\frac{1}{15}a\left(\frac{\left(15a^4c^2-20a^2cd+8d^2\right)\arctan\left(\frac{\sqrt{dx^2+c}a}{\sqrt{-a^2c+d}}\right)}{\left(a^4c^5-2a^2c^4d+c^3d^2\right)\sqrt{-a^2c+d}a}+\frac{7\left(dx^2+c\right)a^2c+a^2c^2-4\left(dx^2+c\right)d-cd}{\left(a^4c^4-2a^2c^3d+c^2d^2\right)\left(dx^2+c\right)^{\frac{3}{2}}}\right)+\frac{\left(4x^2\left(\frac{2d^2x^2}{c^3}+\frac{5d}{c^2}\right)+\right.}{15\left(dx^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/(d*x^2+c)^(7/2),x, algorithm="giac")
```

```
[Out] -1/15*a*((15*a^4*c^2 - 20*a^2*c*d + 8*d^2)*arctan(sqrt(d*x^2 + c)*a/sqrt(-a
^2*c + d))/((a^4*c^5 - 2*a^2*c^4*d + c^3*d^2)*sqrt(-a^2*c + d)*a) + (7*(d*x
^2 + c)*a^2*c + a^2*c^2 - 4*(d*x^2 + c)*d - c*d)/((a^4*c^4 - 2*a^2*c^3*d +
c^2*d^2)*(d*x^2 + c)^(3/2))) + 1/15*(4*x^2*(2*d^2*x^2/c^3 + 5*d/c^2) + 15/c
)*x*arctan(a*x)/(d*x^2 + c)^(5/2)
```

3.1227 $\int \frac{\tan^{-1}(ax)}{(c+dx^2)^{9/2}}\,dx$

Optimal. Leaf size=293

$$-\frac{a\left(19a^4c^2-22a^2cd+8d^2\right)}{35c^3\left(a^2c-d\right)^3\sqrt{c+dx^2}}+\frac{\left(-70a^4c^2d+35a^6c^3+56a^2cd^2-16d^3\right)\tanh^{-1}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c-d}}\right)}{35c^4\left(a^2c-d\right)^{7/2}}-\frac{a\left(11a^2c-6d\right)}{105c^2\left(a^2c-d\right)^2\left(c+dx^2\right)^{3/2}}$$

```
[Out] -a/(35*c*(a^2*c - d)*(c + d*x^2)^(5/2)) - (a*(11*a^2*c - 6*d))/(105*c^2*(a^
2*c - d)^2*(c + d*x^2)^(3/2)) - (a*(19*a^4*c^2 - 22*a^2*c*d + 8*d^2))/(35*c
^3*(a^2*c - d)^3*Sqrt[c + d*x^2]) + (x*ArcTan[a*x])/(7*c*(c + d*x^2)^(7/2))
 + (6*x*ArcTan[a*x])/(35*c^2*(c + d*x^2)^(5/2)) + (8*x*ArcTan[a*x])/(35*c^3
*(c + d*x^2)^(3/2)) + (16*x*ArcTan[a*x])/(35*c^4*Sqrt[c + d*x^2]) + ((35*a^
6*c^3 - 70*a^4*c^2*d + 56*a^2*c*d^2 - 16*d^3)*ArcTanh[(a*Sqrt[c + d*x^2])/S
qrt[a^2*c - d]])/(35*c^4*(a^2*c - d)^(7/2))
```

Rubi [A] time = 1.23099, antiderivative size = 293, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {192, 191, 4912, 6688, 12, 6715, 1619, 63, 208}

$$-\frac{a\left(19a^4c^2-22a^2cd+8d^2\right)}{35c^3\left(a^2c-d\right)^3\sqrt{c+dx^2}}+\frac{\left(-70a^4c^2d+35a^6c^3+56a^2cd^2-16d^3\right)\tanh^{-1}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c-d}}\right)}{35c^4\left(a^2c-d\right)^{7/2}}-\frac{a\left(11a^2c-6d\right)}{105c^2\left(a^2c-d\right)^2\left(c+dx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]/(c + d*x^2)^(9/2),x]
```


```
[Out] -a/(35*c*(a^2*c - d)*(c + d*x^2)^(5/2)) - (a*(11*a^2*c - 6*d))/(105*c^2*(a^
2*c - d)^2*(c + d*x^2)^(3/2)) - (a*(19*a^4*c^2 - 22*a^2*c*d + 8*d^2))/(35*c
^3*(a^2*c - d)^3*Sqrt[c + d*x^2]) + (x*ArcTan[a*x])/(7*c*(c + d*x^2)^(7/2))
 + (6*x*ArcTan[a*x])/(35*c^2*(c + d*x^2)^(5/2)) + (8*x*ArcTan[a*x])/(35*c^3
*(c + d*x^2)^(3/2)) + (16*x*ArcTan[a*x])/(35*c^4*Sqrt[c + d*x^2]) + ((35*a^
6*c^3 - 70*a^4*c^2*d + 56*a^2*c*d^2 - 16*d^3)*ArcTanh[(a*Sqrt[c + d*x^2])/S
qrt[a^2*c - d]])/(35*c^4*(a^2*c - d)^(7/2))
```


Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
```

```
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 4912

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcTan[c*x], u, x]
- Dist[b*c, Int[u/(1 + c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && (I
ntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 1619

```
Int[((Px_)*((c_.) + (d_.)*(x_))^(n_.))/((a_.) + (b_.)*(x_)), x_Symbol] :> I
nt[ExpandIntegrand[1/Sqrt[c + d*x], (Px*(c + d*x)^(n + 1/2))/(a + b*x), x],
 x] /; FreeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && ILtQ[n + 1/2, 0] && GtQ
[Expon[Px, x], 2]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)}{\left(c+dx^2\right)^{9/2}}\,dx = \frac{x\tan^{-1}(ax)}{7c\left(c+dx^2\right)^{7/2}} + \frac{6x\tan^{-1}(ax)}{35c^2\left(c+dx^2\right)^{5/2}} + \frac{8x\tan^{-1}(ax)}{35c^3\left(c+dx^2\right)^{3/2}} + \frac{16x\tan^{-1}(ax)}{35c^4\sqrt{c+dx^2}} - a\int \frac{\frac{x}{7c\left(c+dx^2\right)^{7/2}} + \frac{6x}{35c^2\left(c+d\right.}}{}$$

$$= \frac{x\tan^{-1}(ax)}{7c\left(c+dx^2\right)^{7/2}} + \frac{6x\tan^{-1}(ax)}{35c^2\left(c+dx^2\right)^{5/2}} + \frac{8x\tan^{-1}(ax)}{35c^3\left(c+dx^2\right)^{3/2}} + \frac{16x\tan^{-1}(ax)}{35c^4\sqrt{c+dx^2}} - a\int \frac{x\left(35c^3+70c^2dx^2+\right.}{35c^4\left(1+a^2x^2\right.}$$

$$= \frac{x\tan^{-1}(ax)}{7c\left(c+dx^2\right)^{7/2}} + \frac{6x\tan^{-1}(ax)}{35c^2\left(c+dx^2\right)^{5/2}} + \frac{8x\tan^{-1}(ax)}{35c^3\left(c+dx^2\right)^{3/2}} + \frac{16x\tan^{-1}(ax)}{35c^4\sqrt{c+dx^2}} - \frac{a\int \frac{x\left(35c^3+70c^2dx^2+56cd^2x^4+\right.}{\left(1+a^2x^2\right)\left(c+dx^2\right)^{7/2}}}{35c^4}$$

$$= \frac{x\tan^{-1}(ax)}{7c\left(c+dx^2\right)^{7/2}} + \frac{6x\tan^{-1}(ax)}{35c^2\left(c+dx^2\right)^{5/2}} + \frac{8x\tan^{-1}(ax)}{35c^3\left(c+dx^2\right)^{3/2}} + \frac{16x\tan^{-1}(ax)}{35c^4\sqrt{c+dx^2}} - \frac{a\,\mathrm{Subst}\left(\int \frac{35c^3+70c^2dx+56}{\left(1+a^2x\right)(c}\right.}{70c^4}$$

$$= \frac{x\tan^{-1}(ax)}{7c\left(c+dx^2\right)^{7/2}} + \frac{6x\tan^{-1}(ax)}{35c^2\left(c+dx^2\right)^{5/2}} + \frac{8x\tan^{-1}(ax)}{35c^3\left(c+dx^2\right)^{3/2}} + \frac{16x\tan^{-1}(ax)}{35c^4\sqrt{c+dx^2}} - \frac{a\,\mathrm{Subst}\left(\int\left(-\frac{5c^3d}{\left(a^2c-d\right)(c+dx}\right.\right.}{}$$

$$= -\frac{a}{35c\left(a^2c-d\right)\left(c+dx^2\right)^{5/2}} - \frac{a\left(11a^2c-6d\right)}{105c^2\left(a^2c-d\right)^2\left(c+dx^2\right)^{3/2}} - \frac{a\left(19a^4c^2-22a^2cd+8d^2\right)}{35c^3\left(a^2c-d\right)^3\sqrt{c+dx^2}} + \frac{x\tan^{-1}}{7c\left(c+d\right.}$$

$$= -\frac{a}{35c\left(a^2c-d\right)\left(c+dx^2\right)^{5/2}} - \frac{a\left(11a^2c-6d\right)}{105c^2\left(a^2c-d\right)^2\left(c+dx^2\right)^{3/2}} - \frac{a\left(19a^4c^2-22a^2cd+8d^2\right)}{35c^3\left(a^2c-d\right)^3\sqrt{c+dx^2}} + \frac{x\tan^{-1}}{7c\left(c+d\right.}$$

$$= -\frac{a}{35c\left(a^2c-d\right)\left(c+dx^2\right)^{5/2}} - \frac{a\left(11a^2c-6d\right)}{105c^2\left(a^2c-d\right)^2\left(c+dx^2\right)^{3/2}} - \frac{a\left(19a^4c^2-22a^2cd+8d^2\right)}{35c^3\left(a^2c-d\right)^3\sqrt{c+dx^2}} + \frac{x\tan^{-1}}{7c\left(c+d\right.}$$

Mathematica [C] time = 1.34959, size = 450, normalized size = 1.54

$$\frac{-\frac{2ac\left(3\left(19a^4c^2-22a^2cd+8d^2\right)\left(c+dx^2\right)^2+3c^2\left(d-a^2c\right)^2+c\left(11a^2c-6d\right)\left(a^2c-d\right)\left(c+dx^2\right)\right)}{\left(a^2c-d\right)^3\left(c+dx^2\right)^{5/2}}+\frac{3\left(-70a^4c^2d+35a^6c^3+56a^2cd^2-16d^3\right)\log\left(-\frac{140ac^4\left(a^2c-d\right)^{5/2}\left(\sqrt{a^2c-d}\sqrt{c+\cdots}\right.}{(ax+i)\left(-70a^4c^2d+35a^6c^3+56a\cdots\right.}\right.}{\left(a^2c-d\right)^{7/2}}}{210c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]/(c + d*x^2)^(9/2),x]
```

```
[Out] ((-2*a*c*(3*c^2*(-(a^2*c) + d)^2 + c*(11*a^2*c - 6*d)*(a^2*c - d)*(c + d*x^
2) + 3*(19*a^4*c^2 - 22*a^2*c*d + 8*d^2)*(c + d*x^2)^2))/((a^2*c - d)^3*(c
+ d*x^2)^(5/2)) + (6*x*(35*c^3 + 70*c^2*d*x^2 + 56*c*d^2*x^4 + 16*d^3*x^6)*
ArcTan[a*x])/(c + d*x^2)^(7/2) + (3*(35*a^6*c^3 - 70*a^4*c^2*d + 56*a^2*c*d
^2 - 16*d^3)*Log[(-140*a*c^4*(a^2*c - d)^(5/2)*(a*c - I*d*x + Sqrt[a^2*c -
d]*Sqrt[c + d*x^2]))/((35*a^6*c^3 - 70*a^4*c^2*d + 56*a^2*c*d^2 - 16*d^3)*(
I + a*x))])/(a^2*c - d)^(7/2) + (3*(35*a^6*c^3 - 70*a^4*c^2*d + 56*a^2*c*d^
2 - 16*d^3)*Log[(-140*a*c^4*(a^2*c - d)^(5/2)*(a*c + I*d*x + Sqrt[a^2*c - d
]*Sqrt[c + d*x^2]))/((35*a^6*c^3 - 70*a^4*c^2*d + 56*a^2*c*d^2 - 16*d^3)*(-
I + a*x))])/(a^2*c - d)^(7/2))/(210*c^4)
```

Maple [F] time = 0.664, size = 0, normalized size = 0.

$$\int \arctan(ax)\left(dx^2+c\right)^{-\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)/(d*x^2+c)^(9/2),x)
```

```
[Out] int(arctan(a*x)/(d*x^2+c)^(9/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/(d*x^2+c)^(9/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 6.0733, size = 4096, normalized size = 13.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/(d*x^2+c)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/420*(3*(35*a^6*c^7 - 70*a^4*c^6*d + 56*a^2*c^5*d^2 + (35*a^6*c^3*d^4 - 7
0*a^4*c^2*d^5 + 56*a^2*c*d^6 - 16*d^7)*x^8 - 16*c^4*d^3 + 4*(35*a^6*c^4*d^3
 - 70*a^4*c^3*d^4 + 56*a^2*c^2*d^5 - 16*c*d^6)*x^6 + 6*(35*a^6*c^5*d^2 - 70
*a^4*c^4*d^3 + 56*a^2*c^3*d^4 - 16*c^2*d^5)*x^4 + 4*(35*a^6*c^6*d - 70*a^4*
c^5*d^2 + 56*a^2*c^4*d^3 - 16*c^3*d^4)*x^2)*sqrt(a^2*c - d)*log((a^4*d^2*x^
4 + 8*a^4*c^2 - 8*a^2*c*d + 2*(4*a^4*c*d - 3*a^2*d^2)*x^2 + 4*(a^3*d*x^2 +
2*a^3*c - a*d)*sqrt(a^2*c - d)*sqrt(d*x^2 + c) + d^2)/(a^4*x^4 + 2*a^2*x^2
+ 1)) - 4*(71*a^7*c^7 - 160*a^5*c^6*d + 122*a^3*c^5*d^2 - 33*a*c^4*d^3 + 3*
(19*a^7*c^4*d^3 - 41*a^5*c^3*d^4 + 30*a^3*c^2*d^5 - 8*a*c*d^6)*x^6 + (182*a
^7*c^5*d^2 - 397*a^5*c^4*d^3 + 293*a^3*c^3*d^4 - 78*a*c^2*d^5)*x^4 + (196*a
^7*c^6*d - 434*a^5*c^5*d^2 + 325*a^3*c^4*d^3 - 87*a*c^3*d^4)*x^2 - 3*(16*(a
^8*c^4*d^3 - 4*a^6*c^3*d^4 + 6*a^4*c^2*d^5 - 4*a^2*c*d^6 + d^7)*x^7 + 56*(a
^8*c^5*d^2 - 4*a^6*c^4*d^3 + 6*a^4*c^3*d^4 - 4*a^2*c^2*d^5 + c*d^6)*x^5 + 7
0*(a^8*c^6*d - 4*a^6*c^5*d^2 + 6*a^4*c^4*d^3 - 4*a^2*c^3*d^4 + c^2*d^5)*x^3
 + 35*(a^8*c^7 - 4*a^6*c^6*d + 6*a^4*c^5*d^2 - 4*a^2*c^4*d^3 + c^3*d^4)*x)*
arctan(a*x))*sqrt(d*x^2 + c))/(a^8*c^12 - 4*a^6*c^11*d + 6*a^4*c^10*d^2 - 4
*a^2*c^9*d^3 + c^8*d^4 + (a^8*c^8*d^4 - 4*a^6*c^7*d^5 + 6*a^4*c^6*d^6 - 4*a
^2*c^5*d^7 + c^4*d^8)*x^8 + 4*(a^8*c^9*d^3 - 4*a^6*c^8*d^4 + 6*a^4*c^7*d^5
- 4*a^2*c^6*d^6 + c^5*d^7)*x^6 + 6*(a^8*c^10*d^2 - 4*a^6*c^9*d^3 + 6*a^4*c^
8*d^4 - 4*a^2*c^7*d^5 + c^6*d^6)*x^4 + 4*(a^8*c^11*d - 4*a^6*c^10*d^2 + 6*a
^4*c^9*d^3 - 4*a^2*c^8*d^4 + c^7*d^5)*x^2), 1/210*(3*(35*a^6*c^7 - 70*a^4*c
^6*d + 56*a^2*c^5*d^2 + (35*a^6*c^3*d^4 - 70*a^4*c^2*d^5 + 56*a^2*c*d^6 - 1
6*d^7)*x^8 - 16*c^4*d^3 + 4*(35*a^6*c^4*d^3 - 70*a^4*c^3*d^4 + 56*a^2*c^2*d
^5 - 16*c*d^6)*x^6 + 6*(35*a^6*c^5*d^2 - 70*a^4*c^4*d^3 + 56*a^2*c^3*d^4 -
16*c^2*d^5)*x^4 + 4*(35*a^6*c^6*d - 70*a^4*c^5*d^2 + 56*a^2*c^4*d^3 - 16*c^
3*d^4)*x^2)*sqrt(-a^2*c + d)*arctan(-1/2*(a^2*d*x^2 + 2*a^2*c - d)*sqrt(-a^
2*c + d)*sqrt(d*x^2 + c)/(a^3*c^2 - a*c*d + (a^3*c*d - a*d^2)*x^2)) - 2*(71
*a^7*c^7 - 160*a^5*c^6*d + 122*a^3*c^5*d^2 - 33*a*c^4*d^3 + 3*(19*a^7*c^4*d
^3 - 41*a^5*c^3*d^4 + 30*a^3*c^2*d^5 - 8*a*c*d^6)*x^6 + (182*a^7*c^5*d^2 -
397*a^5*c^4*d^3 + 293*a^3*c^3*d^4 - 78*a*c^2*d^5)*x^4 + (196*a^7*c^6*d - 43
```

```
4*a^5*c^5*d^2 + 325*a^3*c^4*d^3 - 87*a*c^3*d^4)*x^2 - 3*(16*(a^8*c^4*d^3 -
4*a^6*c^3*d^4 + 6*a^4*c^2*d^5 - 4*a^2*c*d^6 + d^7)*x^7 + 56*(a^8*c^5*d^2 -
4*a^6*c^4*d^3 + 6*a^4*c^3*d^4 - 4*a^2*c^2*d^5 + c*d^6)*x^5 + 70*(a^8*c^6*d
- 4*a^6*c^5*d^2 + 6*a^4*c^4*d^3 - 4*a^2*c^3*d^4 + c^2*d^5)*x^3 + 35*(a^8*c^
7 - 4*a^6*c^6*d + 6*a^4*c^5*d^2 - 4*a^2*c^4*d^3 + c^3*d^4)*x)*arctan(a*x))*
sqrt(d*x^2 + c))/(a^8*c^12 - 4*a^6*c^11*d + 6*a^4*c^10*d^2 - 4*a^2*c^9*d^3
+ c^8*d^4 + (a^8*c^8*d^4 - 4*a^6*c^7*d^5 + 6*a^4*c^6*d^6 - 4*a^2*c^5*d^7 +
c^4*d^8)*x^8 + 4*(a^8*c^9*d^3 - 4*a^6*c^8*d^4 + 6*a^4*c^7*d^5 - 4*a^2*c^6*d
^6 + c^5*d^7)*x^6 + 6*(a^8*c^10*d^2 - 4*a^6*c^9*d^3 + 6*a^4*c^8*d^4 - 4*a^2
*c^7*d^5 + c^6*d^6)*x^4 + 4*(a^8*c^11*d - 4*a^6*c^10*d^2 + 6*a^4*c^9*d^3 -
4*a^2*c^8*d^4 + c^7*d^5)*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)/(d*x**2+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21842, size = 454, normalized size = 1.55

$$-\frac{1}{105}a\left(\frac{3\left(35a^6c^3-70a^4c^2d+56a^2cd^2-16d^3\right)\arctan\left(\frac{\sqrt{dx^2+c}a}{\sqrt{-a^2c+d}}\right)}{\left(a^6c^7-3a^4c^6d+3a^2c^5d^2-c^4d^3\right)\sqrt{-a^2c+d}a}+\frac{57\left(dx^2+c\right)^2a^4c^2+11\left(dx^2+c\right)a^4c^3+3a^4c^4-6}{\left(a\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/(d*x^2+c)^(9/2),x, algorithm="giac")
```

```
[Out] -1/105*a*(3*(35*a^6*c^3 - 70*a^4*c^2*d + 56*a^2*c*d^2 - 16*d^3)*arctan(sqrt
(d*x^2 + c)*a/sqrt(-a^2*c + d))/((a^6*c^7 - 3*a^4*c^6*d + 3*a^2*c^5*d^2 - c
^4*d^3)*sqrt(-a^2*c + d)*a) + (57*(d*x^2 + c)^2*a^4*c^2 + 11*(d*x^2 + c)*a^
4*c^3 + 3*a^4*c^4 - 66*(d*x^2 + c)^2*a^2*c*d - 17*(d*x^2 + c)*a^2*c^2*d - 6
*a^2*c^3*d + 24*(d*x^2 + c)^2*d^2 + 6*(d*x^2 + c)*c*d^2 + 3*c^2*d^2)/((a^6*
c^6 - 3*a^4*c^5*d + 3*a^2*c^4*d^2 - c^3*d^3)*(d*x^2 + c)^(5/2))) + 1/35*(2*
```

```
(4*x^2*(2*d^3*x^2/c^4 + 7*d^2/c^3) + 35*d/c^2)*x^2 + 35/c)*x*arctan(a*x)/(d
*x^2 + c)^(7/2)
```

3.1228 $\int x^m \left(d+ex^2\right)^3 \left(a+b\tan^{-1}(cx)\right) dx$

Optimal. Leaf size=378

$$\frac{bx^{m+2}\left(3c^4d^2e\left(m^3+13m^2+47m+35\right)+c^6\left(-d^3\right)\left(m^3+15m^2+71m+105\right)-3c^2de^2\left(m^3+11m^2+31m+21\right)+e^3\left(\right.\right.}{c^5(m+1)(m+2)(m+3)(m+5)(m+7)}$$

```
[Out] -((b*e*(e^2*(15 + 8*m + m^2) - 3*c^2*d*e*(21 + 10*m + m^2) + 3*c^4*d^2*(35
+ 12*m + m^2))*x^(2 + m))/(c^5*(2 + m)*(3 + m)*(5 + m)*(7 + m))) + (b*e^2*(
e*(5 + m) - 3*c^2*d*(7 + m))*x^(4 + m))/(c^3*(4 + m)*(5 + m)*(7 + m)) - (b*
e^3*x^(6 + m))/(c*(6 + m)*(7 + m)) + (d^3*x^(1 + m)*(a + b*ArcTan[c*x]))/(1
 + m) + (3*d^2*e*x^(3 + m)*(a + b*ArcTan[c*x]))/(3 + m) + (3*d*e^2*x^(5 + m
)*(a + b*ArcTan[c*x]))/(5 + m) + (e^3*x^(7 + m)*(a + b*ArcTan[c*x]))/(7 + m
) + (b*(e^3*(15 + 23*m + 9*m^2 + m^3) - 3*c^2*d*e^2*(21 + 31*m + 11*m^2 + m
^3) + 3*c^4*d^2*e*(35 + 47*m + 13*m^2 + m^3) - c^6*d^3*(105 + 71*m + 15*m^2
 + m^3))*x^(2 + m)*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, -(c^2*x^2)])/
(c^5*(1 + m)*(2 + m)*(3 + m)*(5 + m)*(7 + m))
```

Rubi [A] time = 1.98322, antiderivative size = 374, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {270, 4976, 1802, 364}

$$\frac{3d^2ex^{m+3}\left(a+b\tan^{-1}(cx)\right)}{m+3}+\frac{d^3x^{m+1}\left(a+b\tan^{-1}(cx)\right)}{m+1}+\frac{3de^2x^{m+5}\left(a+b\tan^{-1}(cx)\right)}{m+5}+\frac{e^3x^{m+7}\left(a+b\tan^{-1}(cx)\right)}{m+7}+\frac{bx}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(d + e*x^2)^3*(a + b*ArcTan[c*x]),x]
```

```
[Out] -((b*e*(e^2*(15 + 8*m + m^2) - 3*c^2*d*e*(21 + 10*m + m^2) + 3*c^4*d^2*(35
+ 12*m + m^2))*x^(2 + m))/(c^5*(2 + m)*(3 + m)*(5 + m)*(7 + m))) - (b*e^2*(
(3*c^2*d)/(5 + m) - e/(7 + m))*x^(4 + m))/(c^3*(4 + m)) - (b*e^3*x^(6 + m))
/(c*(6 + m)*(7 + m)) + (d^3*x^(1 + m)*(a + b*ArcTan[c*x]))/(1 + m) + (3*d^2
*e*x^(3 + m)*(a + b*ArcTan[c*x]))/(3 + m) + (3*d*e^2*x^(5 + m)*(a + b*ArcTa
n[c*x]))/(5 + m) + (e^3*x^(7 + m)*(a + b*ArcTan[c*x]))/(7 + m) + (b*(e^3*(1
5 + 23*m + 9*m^2 + m^3) - 3*c^2*d*e^2*(21 + 31*m + 11*m^2 + m^3) + 3*c^4*d^
2*e*(35 + 47*m + 13*m^2 + m^3) - c^6*d^3*(105 + 71*m + 15*m^2 + m^3))*x^(2
+ m)*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, -(c^2*x^2)])/(c^5*(1 + m)*(
2 + m)*(3 + m)*(5 + m)*(7 + m))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 4976

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Dis
t[a + b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&  !(
ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(IL
tQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILtQ[(m
 - 1)/2, 0]))
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int x^m \left(d+ex^2\right)^3 \left(a+b\tan^{-1}(cx)\right) dx = \frac{d^3x^{1+m}\left(a+b\tan^{-1}(cx)\right)}{1+m} + \frac{3d^2ex^{3+m}\left(a+b\tan^{-1}(cx)\right)}{3+m} + \frac{3de^2x^{5+m}\left(a+b\tan^{-1}\right.}{5+m}$$

$$= \frac{d^3x^{1+m}\left(a+b\tan^{-1}(cx)\right)}{1+m} + \frac{3d^2ex^{3+m}\left(a+b\tan^{-1}(cx)\right)}{3+m} + \frac{3de^2x^{5+m}\left(a+b\tan^{-1}\right.}{5+m}$$

$$= -\frac{be\left(e^2\left(15+8m+m^2\right)-3c^2de\left(21+10m+m^2\right)+3c^4d^2\left(35+12m+m^2\right)\right)x^{2+}}{c^5(2+m)(3+m)(5+m)(7+m)}$$

$$= -\frac{be\left(e^2\left(15+8m+m^2\right)-3c^2de\left(21+10m+m^2\right)+3c^4d^2\left(35+12m+m^2\right)\right)x^{2+}}{c^5(2+m)(3+m)(5+m)(7+m)}$$

Mathematica [A] time = 0.581661, size = 264, normalized size = 0.7

$$x^{m+1}\left(-\frac{3bcd^2ex^3\text{Hypergeometric2F1}\left(1,\frac{m+4}{2},\frac{m+6}{2},-c^2x^2\right)}{m^2+7m+12}-\frac{bcd^3x\text{Hypergeometric2F1}\left(1,\frac{m+2}{2},\frac{m+4}{2},-c^2x^2\right)}{m^2+3m+2}-\frac{3bc\cdots}{\cdots}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(d + e*x^2)^3*(a + b*ArcTan[c*x]),x]
```

```
[Out] x^(1 + m)*((d^3*(a + b*ArcTan[c*x]))/(1 + m) + (3*d^2*e*x^2*(a + b*ArcTan[c
*x]))/(3 + m) + (3*d*e^2*x^4*(a + b*ArcTan[c*x]))/(5 + m) + (e^3*x^6*(a + b
*ArcTan[c*x]))/(7 + m) - (b*c*e^3*x^7*Hypergeometric2F1[1, 4 + m/2, 5 + m/2
, -(c^2*x^2)])/((7 + m)*(8 + m)) - (b*c*d^3*x*Hypergeometric2F1[1, (2 + m)/
2, (4 + m)/2, -(c^2*x^2)])/(2 + 3*m + m^2) - (3*b*c*d^2*e*x^3*Hypergeometri
c2F1[1, (4 + m)/2, (6 + m)/2, -(c^2*x^2)])/(12 + 7*m + m^2) - (3*b*c*d*e^2*
x^5*Hypergeometric2F1[1, (6 + m)/2, (8 + m)/2, -(c^2*x^2)])/((5 + m)*(6 + m
)))
```

Maple [F] time = 1.137, size = 0, normalized size = 0.

$$\int x^m\left(ex^2+d\right)^3\left(a+b\arctan\left(cx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(e*x^2+d)^3*(a+b*arctan(c*x)),x)
```

```
[Out] int(x^m*(e*x^2+d)^3*(a+b*arctan(c*x)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*x^2+d)^3*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(ae^3x^6 + 3\,ade^2x^4 + 3\,ad^2ex^2 + ad^3 + \left(be^3x^6 + 3\,bde^2x^4 + 3\,bd^2ex^2 + bd^3\right)\arctan\left(cx\right)\right)x^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*x^2+d)^3*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] integral((a*e^3*x^6 + 3*a*d*e^2*x^4 + 3*a*d^2*e*x^2 + a*d^3 + (b*e^3*x^6 +
3*b*d*e^2*x^4 + 3*b*d^2*e*x^2 + b*d^3)*arctan(c*x))*x^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^m\left(a + b\,\text{atan}\left(cx\right)\right)\left(d + ex^2\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(e*x**2+d)**3*(a+b*atan(c*x)),x)
```

```
[Out] Integral(x**m*(a + b*atan(c*x))*(d + e*x**2)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(ex^2 + d\right)^3\left(b\arctan\left(cx\right) + a\right)x^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*x^2+d)^3*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^3*(b*arctan(c*x) + a)*x^m, x)
```

3.1229 $\int x^m \left(d+ex^2\right)^2 \left(a+b\tan^{-1}(cx)\right)\,dx$

Optimal. Leaf size=230

$$-\frac{bx^{m+2}\left(c^4d^2\left(m^2+8m+15\right)-2c^2de\left(m^2+6m+5\right)+e^2\left(m^2+4m+3\right)\right)\text{Hypergeometric2F1}\left(1,\frac{m+2}{2},\frac{m+4}{2},-c^2x^2\right)}{c^3(m+1)(m+2)(m+3)(m+5)}$$

```
[Out] (b*e*(e*(3 + m) - 2*c^2*d*(5 + m))*x^(2 + m))/(c^3*(2 + m)*(3 + m)*(5 + m))
 - (b*e^2*x^(4 + m))/(c*(4 + m)*(5 + m)) + (d^2*x^(1 + m)*(a + b*ArcTan[c*x
]))/(1 + m) + (2*d*e*x^(3 + m)*(a + b*ArcTan[c*x]))/(3 + m) + (e^2*x^(5 + m
)*(a + b*ArcTan[c*x]))/(5 + m) - (b*(e^2*(3 + 4*m + m^2) - 2*c^2*d*e*(5 + 6
*m + m^2) + c^4*d^2*(15 + 8*m + m^2))*x^(2 + m)*Hypergeometric2F1[1, (2 + m
)/2, (4 + m)/2, -(c^2*x^2)])/(c^3*(1 + m)*(2 + m)*(3 + m)*(5 + m))
```

Rubi [A] time = 0.293679, antiderivative size = 226, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {270, 4976, 1261, 364}

$$\frac{d^2x^{m+1}\left(a+b\tan^{-1}(cx)\right)}{m+1}+\frac{2dex^{m+3}\left(a+b\tan^{-1}(cx)\right)}{m+3}+\frac{e^2x^{m+5}\left(a+b\tan^{-1}(cx)\right)}{m+5}-\frac{bx^{m+2}\left(c^4d^2\left(m^2+8m+15\right)-2c^2\right.}{c}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(d + e*x^2)^2*(a + b*ArcTan[c*x]),x]
```

```
[Out] -((b*e*((2*c^2*d)/(3 + m) - e/(5 + m))*x^(2 + m))/(c^3*(2 + m))) - (b*e^2*x
^(4 + m))/(c*(4 + m)*(5 + m)) + (d^2*x^(1 + m)*(a + b*ArcTan[c*x]))/(1 + m)
 + (2*d*e*x^(3 + m)*(a + b*ArcTan[c*x]))/(3 + m) + (e^2*x^(5 + m)*(a + b*Ar
cTan[c*x]))/(5 + m) - (b*(e^2*(3 + 4*m + m^2) - 2*c^2*d*e*(5 + 6*m + m^2) +
 c^4*d^2*(15 + 8*m + m^2))*x^(2 + m)*Hypergeometric2F1[1, (2 + m)/2, (4 + m
)/2, -(c^2*x^2)])/(c^3*(1 + m)*(2 + m)*(3 + m)*(5 + m))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 4976

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Dis
t[a + b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] && !(
ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(IL
tQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] && !ILtQ[(m
 - 1)/2, 0]))
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int x^m \left(d+ex^2\right)^2 \left(a+b\tan^{-1}(cx)\right)dx = \frac{d^2x^{1+m}\left(a+b\tan^{-1}(cx)\right)}{1+m} + \frac{2dex^{3+m}\left(a+b\tan^{-1}(cx)\right)}{3+m} + \frac{e^2x^{5+m}\left(a+b\tan^{-1}(cx\right.}{5+m}$$

$$= \frac{d^2x^{1+m}\left(a+b\tan^{-1}(cx)\right)}{1+m} + \frac{2dex^{3+m}\left(a+b\tan^{-1}(cx)\right)}{3+m} + \frac{e^2x^{5+m}\left(a+b\tan^{-1}(cx\right.}{5+m}$$

$$= -\frac{be\left(\frac{2c^2d}{3+m}-\frac{e}{5+m}\right)x^{2+m}}{c^3(2+m)} - \frac{be^2x^{4+m}}{c(4+m)(5+m)} + \frac{d^2x^{1+m}\left(a+b\tan^{-1}(cx)\right)}{1+m} + \frac{2dex^{3+}}{}$$

$$= -\frac{be\left(\frac{2c^2d}{3+m}-\frac{e}{5+m}\right)x^{2+m}}{c^3(2+m)} - \frac{be^2x^{4+m}}{c(4+m)(5+m)} + \frac{d^2x^{1+m}\left(a+b\tan^{-1}(cx)\right)}{1+m} + \frac{2dex^{3+}}{}$$

Mathematica [A] time = 0.205338, size = 193, normalized size = 0.84

$$x^{m+1}\left(-\frac{bcd^2x\text{Hypergeometric2F1}\left(1,\frac{m+2}{2},\frac{m+4}{2},-c^2x^2\right)}{m^2+3m+2} - \frac{2bcdex^3\text{Hypergeometric2F1}\left(1,\frac{m+4}{2},\frac{m+6}{2},-c^2x^2\right)}{m^2+7m+12} - \frac{bce^2x^5}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(d + e*x^2)^2*(a + b*ArcTan[c*x]),x]
```

```
[Out] x^(1 + m)*((d^2*(a + b*ArcTan[c*x]))/(1 + m) + (2*d*e*x^2*(a + b*ArcTan[c*x
]))/(3 + m) + (e^2*x^4*(a + b*ArcTan[c*x]))/(5 + m) - (b*c*d^2*x*Hypergeome
tric2F1[1, (2 + m)/2, (4 + m)/2, -(c^2*x^2)])/(2 + 3*m + m^2) - (2*b*c*d*e*
x^3*Hypergeometric2F1[1, (4 + m)/2, (6 + m)/2, -(c^2*x^2)])/(12 + 7*m + m^2
) - (b*c*e^2*x^5*Hypergeometric2F1[1, (6 + m)/2, (8 + m)/2, -(c^2*x^2)])/((
5 + m)*(6 + m)))
```

Maple [F] time = 0.935, size = 0, normalized size = 0.

$$\int x^{m}\left(ex^{2}+d\right)^{2}\left(a+b\arctan\left(cx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(e*x^2+d)^2*(a+b*arctan(c*x)),x)
```

```
[Out] int(x^m*(e*x^2+d)^2*(a+b*arctan(c*x)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*x^2+d)^2*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(ae^{2}x^{4}+2\,adex^{2}+ad^{2}+\left(be^{2}x^{4}+2\,bdex^{2}+bd^{2}\right)\arctan\left(cx\right)\right)x^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*x^2+d)^2*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] integral((a*e^2*x^4 + 2*a*d*e*x^2 + a*d^2 + (b*e^2*x^4 + 2*b*d*e*x^2 + b*d^
2)*arctan(c*x))*x^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^{m}\left(a+b\operatorname{atan}\left(cx\right)\right)\left(d+ex^{2}\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(e*x**2+d)**2*(a+b*atan(c*x)),x)
```

```
[Out] Integral(x**m*(a + b*atan(c*x))*(d + e*x**2)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(ex^{2}+d\right)^{2}(b\arctan\left(cx\right)+a)x^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*x^2+d)^2*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^2*(b*arctan(c*x) + a)*x^m, x)
```

3.1230 $\int x^m \left(d+ex^2\right)\left(a+b\tan^{-1}(cx)\right)dx$

Optimal. Leaf size=122

$$-\frac{bx^{m+2}\left(\frac{c^2d}{m+1}-\frac{e}{m+3}\right)\text{Hypergeometric2F1}\left(1,\frac{m+2}{2},\frac{m+4}{2},-c^2x^2\right)}{c(m+2)}+\frac{dx^{m+1}\left(a+b\tan^{-1}(cx)\right)}{m+1}+\frac{ex^{m+3}\left(a+b\tan^{-1}(cx)\right)}{m+3}$$

```
[Out] -((b*e*x^(2 + m))/(c*(6 + 5*m + m^2))) + (d*x^(1 + m)*(a + b*ArcTan[c*x]))/
(1 + m) + (e*x^(3 + m)*(a + b*ArcTan[c*x]))/(3 + m) - (b*((c^2*d)/(1 + m) -
 e/(3 + m))*x^(2 + m)*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, -(c^2*x^2)
])/(c*(2 + m))
```

Rubi [A] time = 0.126968, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {14, 4976, 459, 364}

$$\frac{dx^{m+1}\left(a+b\tan^{-1}(cx)\right)}{m+1}+\frac{ex^{m+3}\left(a+b\tan^{-1}(cx)\right)}{m+3}-\frac{bx^{m+2}\left(\frac{c^2d}{m+1}-\frac{e}{m+3}\right){}_2F_1\left(1,\frac{m+2}{2};\frac{m+4}{2};-c^2x^2\right)}{c(m+2)}-\frac{bex^{m+2}}{c\left(m^2+5m+6\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(d + e*x^2)*(a + b*ArcTan[c*x]),x]
```


```
[Out] -((b*e*x^(2 + m))/(c*(6 + 5*m + m^2))) + (d*x^(1 + m)*(a + b*ArcTan[c*x]))/
(1 + m) + (e*x^(3 + m)*(a + b*ArcTan[c*x]))/(3 + m) - (b*((c^2*d)/(1 + m) -
 e/(3 + m))*x^(2 + m)*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, -(c^2*x^2)
])/(c*(2 + m))
```


Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 4976

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Dis
t[a + b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&  !(
```

```
ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(IL
tQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] && !ILtQ[(m
 - 1)/2, 0]))
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int x^m\left(d+ex^2\right)\left(a+b\tan^{-1}(cx)\right)dx=\frac{dx^{1+m}\left(a+b\tan^{-1}(cx)\right)}{1+m}+\frac{ex^{3+m}\left(a+b\tan^{-1}(cx)\right)}{3+m}-(bc)\int\frac{x^{1+m}\left(\frac{d}{1+m}+\frac{ex^2}{3+m}\right)}{1+c^2x^2}$$

$$=-\frac{bex^{2+m}}{c\left(6+5m+m^2\right)}+\frac{dx^{1+m}\left(a+b\tan^{-1}(cx)\right)}{1+m}+\frac{ex^{3+m}\left(a+b\tan^{-1}(cx)\right)}{3+m}+\left(bc\left(-\right.\right.$$

$$=-\frac{bex^{2+m}}{c\left(6+5m+m^2\right)}+\frac{dx^{1+m}\left(a+b\tan^{-1}(cx)\right)}{1+m}+\frac{ex^{3+m}\left(a+b\tan^{-1}(cx)\right)}{3+m}-\frac{bc\left(\frac{d}{1+}\right.}{}$$

Mathematica [A] time = 0.172725, size = 119, normalized size = 0.98

$$x^{m+1}\left(\frac{\frac{\left(d(m+3)+e(m+1)x^2\right)\left(a+b\tan^{-1}(cx)\right)}{m+1}-\frac{bcex^3\text{Hypergeometric2F1}\left(1,\frac{m+4}{2},\frac{m+6}{2},-c^2x^2\right)}{m+4}}{m+3}-\frac{bcdx\text{Hypergeometric2F1}\left(1,\frac{m+2}{2},\frac{m+4}{2},-c^2\right.}{m^2+3m+2}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(d + e*x^2)*(a + b*ArcTan[c*x]),x]
```

```
[Out] x^(1 + m)*(-((b*c*d*x*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, -(c^2*x^2)
])/(2 + 3*m + m^2)) + (((d*(3 + m) + e*(1 + m)*x^2)*(a + b*ArcTan[c*x]))/(1
 + m) - (b*c*e*x^3*Hypergeometric2F1[1, (4 + m)/2, (6 + m)/2, -(c^2*x^2)])/
(4 + m))/(3 + m))
```

Maple [F] time = 0.678, size = 0, normalized size = 0.

$$\int x^{m} \left(e x^{2}+d\right) \left(a +b \arctan \left(c x \right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(e*x^2+d)*(a+b*arctan(c*x)),x)
```

```
[Out] int(x^m*(e*x^2+d)*(a+b*arctan(c*x)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*x^2+d)*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a e x^{2} + a d + \left(b e x^{2} + b d\right) \arctan\left(c x\right)\right) x^{m}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*x^2+d)*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arctan(c*x))*x^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^m (a + b \operatorname{atan}(cx)) \left(d + ex^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(e*x**2+d)*(a+b*atan(c*x)),x)
```

```
[Out] Integral(x**m*(a + b*atan(c*x))*(d + e*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(ex^2 + d\right)(b \arctan(cx) + a)x^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*x^2+d)*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*(b*arctan(c*x) + a)*x^m, x)
```

3.1231 $\int \frac{x^m(a+b\tan^{-1}(cx))}{d+ex^2}\,dx$

Optimal. Leaf size=62

$$\frac{ax^{m+1}\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\frac{ex^2}{d}\right)}{d(m+1)}+b\text{Unintegrable}\left(\frac{x^m\tan^{-1}(cx)}{d+ex^2},x\right)$$

```
[Out] (a*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((e*x^2)/d)])/(d*(
1 + m)) + b*Unintegrable[(x^m*ArcTan[c*x])/(d + e*x^2), x]
```

Rubi [A] time = 0.120963, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m\left(a+b\tan^{-1}(cx)\right)}{d+ex^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*(a + b*ArcTan[c*x]))/(d + e*x^2),x]
```

```
[Out] (a*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((e*x^2)/d)])/(d*(
1 + m)) + b*Defer[Int][(x^m*ArcTan[c*x])/(d + e*x^2), x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^m\left(a+b\tan^{-1}(cx)\right)}{d+ex^2}\,dx &= a\int \frac{x^m}{d+ex^2}\,dx+b\int \frac{x^m\tan^{-1}(cx)}{d+ex^2}\,dx\\ &= \frac{ax^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{ex^2}{d}\right)}{d(1+m)}+b\int \frac{x^m\tan^{-1}(cx)}{d+ex^2}\,dx\end{aligned}$$

Mathematica [A] time = 2.22537, size = 0, normalized size = 0.

$$\int \frac{x^m\left(a+b\tan^{-1}(cx)\right)}{d+ex^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*(a + b*ArcTan[c*x]))/(d + e*x^2),x]
```

```
[Out] Integrate[(x^m*(a + b*ArcTan[c*x]))/(d + e*x^2), x]
```

Maple [A] time = 1.5, size = 0, normalized size = 0.

$$\int \frac{x^m (a + b \arctan(cx))}{ex^2 + d} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a+b*arctan(c*x))/(e*x^2+d),x)
```

```
[Out] int(x^m*(a+b*arctan(c*x))/(e*x^2+d),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \arctan(cx) + a)x^m}{ex^2 + d} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arctan(c*x))/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] integrate((b*arctan(c*x) + a)*x^m/(e*x^2 + d), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(b \arctan(cx) + a)x^m}{ex^2 + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arctan(c*x))/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*arctan(c*x) + a)*x^m/(e*x^2 + d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a+b*atan(c*x))/(e*x**2+d),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \arctan(cx) + a)x^m}{ex^2 + d} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arctan(c*x))/(e*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)*x^m/(e*x^2 + d), x)
```

3.1232 $\int \frac{x^m\left(a+b\tan^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$

Optimal. Leaf size=62

$$\frac{ax^{m+1}\text{Hypergeometric2F1}\left(2,\frac{m+1}{2},\frac{m+3}{2},-\frac{ex^2}{d}\right)}{d^2(m+1)}+b\text{Unintegrable}\left(\frac{x^m\tan^{-1}(cx)}{\left(d+ex^2\right)^2},x\right)$$

```
[Out] (a*x^(1 + m)*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, -((e*x^2)/d)])/(d^2
*(1 + m)) + b*Unintegrable[(x^m*ArcTan[c*x])/(d + e*x^2)^2, x]
```

Rubi [A] time = 0.116845, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m\left(a+b\tan^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*(a + b*ArcTan[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] (a*x^(1 + m)*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, -((e*x^2)/d)])/(d^2
*(1 + m)) + b*Defer[Int][(x^m*ArcTan[c*x])/(d + e*x^2)^2, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^m\left(a+b\tan^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx &= a\int \frac{x^m}{\left(d+ex^2\right)^2}\,dx+b\int \frac{x^m\tan^{-1}(cx)}{\left(d+ex^2\right)^2}\,dx\\ &= \frac{ax^{1+m}\,{}_2F_1\left(2,\frac{1+m}{2};\frac{3+m}{2};-\frac{ex^2}{d}\right)}{d^2(1+m)}+b\int \frac{x^m\tan^{-1}(cx)}{\left(d+ex^2\right)^2}\,dx\end{aligned}$$

Mathematica [A] time = 5.5736, size = 0, normalized size = 0.

$$\int \frac{x^m\left(a+b\tan^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*(a + b*ArcTan[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] Integrate[(x^m*(a + b*ArcTan[c*x]))/(d + e*x^2)^2, x]
```

Maple [A] time = 0.701, size = 0, normalized size = 0.

$$\int \frac{x^{m} \left(a + b \arctan\left(c x\right)\right)}{\left(e x^{2} + d\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a+b*arctan(c*x))/(e*x^2+d)^2,x)
```

```
[Out] int(x^m*(a+b*arctan(c*x))/(e*x^2+d)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b \arctan\left(c x\right) + a\right) x^{m}}{\left(e x^{2} + d\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arctan(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*arctan(c*x) + a)*x^m/(e*x^2 + d)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b \arctan\left(c x\right) + a\right) x^{m}}{e^{2} x^{4} + 2\, d e x^{2} + d^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arctan(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*arctan(c*x) + a)*x^m/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a+b*atan(c*x))/(e*x**2+d)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\arctan(cx) + a)x^m}{\left(ex^2 + d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arctan(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)*x^m/(e*x^2 + d)^2, x)
```

3.1233 $\int x^m \left(d+ex^2\right)^{5/2} \left(a+b\tan^{-1}(cx)\right) dx$

Optimal. Leaf size=75

$$\frac{ax^{m+1}\left(d+ex^2\right)^{7/2}\text{Hypergeometric2F1}\left(1,\frac{m+8}{2},\frac{m+3}{2},-\frac{ex^2}{d}\right)}{d(m+1)}+b\text{Unintegrable}\left(x^m\tan^{-1}(cx)\left(d+ex^2\right)^{5/2},x\right)$$

```
[Out] (a*x^(1 + m)*(d + e*x^2)^(7/2)*Hypergeometric2F1[1, (8 + m)/2, (3 + m)/2, -
((e*x^2)/d)])/(d*(1 + m)) + b*Unintegrable[x^m*(d + e*x^2)^(5/2)*ArcTan[c*x
], x]
```

Rubi [A] time = 0.176079, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int x^m \left(d+ex^2\right)^{5/2} \left(a+b\tan^{-1}(cx)\right) dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*(d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]),x]
```

```
[Out] (a*d^2*x^(1 + m)*Sqrt[d + e*x^2]*Hypergeometric2F1[-5/2, (1 + m)/2, (3 + m)
/2, -((e*x^2)/d)])/((1 + m)*Sqrt[1 + (e*x^2)/d]) + b*Defer[Int][x^m*(d + e*
x^2)^(5/2)*ArcTan[c*x], x]
```

Rubi steps

$$\begin{aligned}
\int x^m \left(d+ex^2\right)^{5/2} \left(a+b\tan^{-1}(cx)\right) dx &= a\int x^m \left(d+ex^2\right)^{5/2} dx+b\int x^m \left(d+ex^2\right)^{5/2}\tan^{-1}(cx)\,dx\\
&= b\int x^m \left(d+ex^2\right)^{5/2}\tan^{-1}(cx)\,dx+\frac{\left(ad^2\sqrt{d+ex^2}\right)\int x^m\left(1+\frac{ex^2}{d}\right)^{5/2} dx}{\sqrt{1+\frac{ex^2}{d}}}\\
&= \frac{ad^2x^{1+m}\sqrt{d+ex^2}\,{}_2F_1\left(-\frac{5}{2},\frac{1+m}{2};\frac{3+m}{2};-\frac{ex^2}{d}\right)}{(1+m)\sqrt{1+\frac{ex^2}{d}}}+b\int x^m\left(d+ex^2\right)^{5/2}\tan^{-1}(cx)\,dx
\end{aligned}$$

Mathematica [A] time = 3.94408, size = 0, normalized size = 0.

$$\int x^m \left(d+ex^2\right)^{5/2} \left(a+b\tan^{-1}(cx)\right)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*(d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]),x]
```

```
[Out] Integrate[x^m*(d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]), x]
```

Maple [A] time = 0.615, size = 0, normalized size = 0.

$$\int x^m \left(ex^2+d\right)^{\frac{5}{2}} \left(a+b\arctan(cx)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(e*x^2+d)^(5/2)*(a+b*arctan(c*x)),x)
```

```
[Out] int(x^m*(e*x^2+d)^(5/2)*(a+b*arctan(c*x)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(ex^2+d\right)^{\frac{5}{2}} (b\arctan(cx)+a)x^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*x^2+d)^(5/2)*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^(5/2)*(b*arctan(c*x) + a)*x^m, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(ae^2x^4+2\,adex^2+ad^2+\left(be^2x^4+2\,bdex^2+bd^2\right)\arctan(cx)\right)\sqrt{ex^2+d}x^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*x^2+d)^(5/2)*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] integral((a*e^2*x^4 + 2*a*d*e*x^2 + a*d^2 + (b*e^2*x^4 + 2*b*d*e*x^2 + b*d^
2)*arctan(c*x))*sqrt(e*x^2 + d)*x^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(e*x**2+d)**(5/2)*(a+b*atan(c*x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(ex^2+d\right)^{\frac{5}{2}}(b\arctan(cx)+a)x^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*x^2+d)^(5/2)*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^(5/2)*(b*arctan(c*x) + a)*x^m, x)
```

3.1234 $\int x^m \left(d+ex^2\right)^{3/2} \left(a+b\tan^{-1}(cx)\right) dx$

Optimal. Leaf size=75

$$\frac{ax^{m+1}\left(d+ex^2\right)^{5/2}\text{Hypergeometric2F1}\left(1,\frac{m+6}{2},\frac{m+3}{2},-\frac{ex^2}{d}\right)}{d(m+1)}+b\text{Unintegrable}\left(x^m\tan^{-1}(cx)\left(d+ex^2\right)^{3/2},x\right)$$

```
[Out] (a*x^(1 + m)*(d + e*x^2)^(5/2)*Hypergeometric2F1[1, (6 + m)/2, (3 + m)/2, -
((e*x^2)/d)])/(d*(1 + m)) + b*Unintegrable[x^m*(d + e*x^2)^(3/2)*ArcTan[c*x
], x]
```

Rubi [A] time = 0.171575, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^m \left(d+ex^2\right)^{3/2} \left(a+b\tan^{-1}(cx)\right) dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*(d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]),x]
```

```
[Out] (a*d*x^(1 + m)*Sqrt[d + e*x^2]*Hypergeometric2F1[-3/2, (1 + m)/2, (3 + m)/2
, -((e*x^2)/d)])/((1 + m)*Sqrt[1 + (e*x^2)/d]) + b*Defer[Int][x^m*(d + e*x^
2)^(3/2)*ArcTan[c*x], x]
```

Rubi steps

$$\begin{aligned}
\int x^m \left(d+ex^2\right)^{3/2} \left(a+b\tan^{-1}(cx)\right) dx &= a\int x^m \left(d+ex^2\right)^{3/2} dx+b\int x^m \left(d+ex^2\right)^{3/2}\tan^{-1}(cx)\,dx\\
&= b\int x^m \left(d+ex^2\right)^{3/2}\tan^{-1}(cx)\,dx+\frac{\left(ad\sqrt{d+ex^2}\right)\int x^m\left(1+\frac{ex^2}{d}\right)^{3/2} dx}{\sqrt{1+\frac{ex^2}{d}}}\\
&= \frac{adx^{1+m}\sqrt{d+ex^2}\,{}_2F_1\left(-\frac{3}{2},\frac{1+m}{2};\frac{3+m}{2};-\frac{ex^2}{d}\right)}{(1+m)\sqrt{1+\frac{ex^2}{d}}}+b\int x^m \left(d+ex^2\right)^{3/2}\tan^{-1}(cx)\,dx
\end{aligned}$$

Mathematica [A] time = 0.123576, size = 0, normalized size = 0.

$$\int x^m \left(d+ex^2\right)^{3/2} \left(a+b\tan^{-1}(cx)\right) \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*(d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]),x]
```

```
[Out] Integrate[x^m*(d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]), x]
```

Maple [A] time = 0.607, size = 0, normalized size = 0.

$$\int x^m \left(ex^2+d\right)^{\frac{3}{2}} (a+b\arctan(cx)) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(e*x^2+d)^(3/2)*(a+b*arctan(c*x)),x)
```

```
[Out] int(x^m*(e*x^2+d)^(3/2)*(a+b*arctan(c*x)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(ex^2+d\right)^{\frac{3}{2}} (b\arctan(cx)+a)x^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*x^2+d)^(3/2)*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^(3/2)*(b*arctan(c*x) + a)*x^m, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(aex^2+ad+\left(bex^2+bd\right)\arctan(cx)\right)\sqrt{ex^2+d}x^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*x^2+d)^(3/2)*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arctan(c*x))*sqrt(e*x^2 + d)*x^m,
 x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(e*x**2+d)**(3/2)*(a+b*atan(c*x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(ex^2+d\right)^{\frac{3}{2}}(b\arctan(cx)+a)x^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*x^2+d)^(3/2)*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^(3/2)*(b*arctan(c*x) + a)*x^m, x)
```

3.1235 $\int x^m \sqrt{d+ex^2}\left(a+b\tan^{-1}(cx)\right)\,dx$

Optimal. Leaf size=75

$$\frac{ax^{m+1}\left(d+ex^2\right)^{3/2}\text{Hypergeometric2F1}\left(1,\frac{m+4}{2},\frac{m+3}{2},-\frac{ex^2}{d}\right)}{d(m+1)}+b\text{Unintegrable}\left(x^m\tan^{-1}(cx)\sqrt{d+ex^2},x\right)$$

```
[Out] (a*x^(1 + m)*(d + e*x^2)^(3/2)*Hypergeometric2F1[1, (4 + m)/2, (3 + m)/2, -
((e*x^2)/d)])/(d*(1 + m)) + b*Unintegrable[x^m*Sqrt[d + e*x^2]*ArcTan[c*x],
 x]
```

Rubi [A] time = 0.153588, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^m\sqrt{d+ex^2}\left(a+b\tan^{-1}(cx)\right)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]),x]
```

```
[Out] (a*x^(1 + m)*Sqrt[d + e*x^2]*Hypergeometric2F1[-1/2, (1 + m)/2, (3 + m)/2,
-((e*x^2)/d)])/((1 + m)*Sqrt[1 + (e*x^2)/d]) + b*Defer[Int][x^m*Sqrt[d + e*
x^2]*ArcTan[c*x], x]
```

Rubi steps

$$\begin{aligned}\int x^m\sqrt{d+ex^2}\left(a+b\tan^{-1}(cx)\right)\,dx &= a\int x^m\sqrt{d+ex^2}\,dx+b\int x^m\sqrt{d+ex^2}\tan^{-1}(cx)\,dx\\ &= b\int x^m\sqrt{d+ex^2}\tan^{-1}(cx)\,dx+\frac{\left(a\sqrt{d+ex^2}\right)\int x^m\sqrt{1+\frac{ex^2}{d}}\,dx}{\sqrt{1+\frac{ex^2}{d}}}\\ &= \frac{ax^{1+m}\sqrt{d+ex^2}\,{}_2F_1\left(-\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};-\frac{ex^2}{d}\right)}{(1+m)\sqrt{1+\frac{ex^2}{d}}}+b\int x^m\sqrt{d+ex^2}\tan^{-1}(cx)\,dx\end{aligned}$$

Mathematica [A] time = 0.0903251, size = 0, normalized size = 0.

$$\int x^m\sqrt{d+ex^2}\left(a+b\tan^{-1}(cx)\right)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]),x]
```

```
[Out] Integrate[x^m*Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]), x]
```

Maple [A] time = 0.816, size = 0, normalized size = 0.

$$\int x^m\sqrt{ex^2+d}\,(a+b\arctan(cx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(e*x^2+d)^(1/2)*(a+b*arctan(c*x)),x)
```

```
[Out] int(x^m*(e*x^2+d)^(1/2)*(a+b*arctan(c*x)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{ex^2+d}(b\arctan(cx)+a)x^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*x^2+d)^(1/2)*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x^2 + d)*(b*arctan(c*x) + a)*x^m, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{ex^2+d}(b\arctan(cx)+a)x^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*x^2+d)^(1/2)*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x^2 + d)*(b*arctan(c*x) + a)*x^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(e*x**2+d)**(1/2)*(a+b*atan(c*x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ex^2 + d}(b \arctan(cx) + a)x^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*x^2+d)^(1/2)*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)*(b*arctan(c*x) + a)*x^m, x)
```

3.1236 $\int \frac{x^m\left(a+b\tan^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$

Optimal. Leaf size=75

$$\frac{ax^{m+1}\sqrt{d+ex^2}\text{Hypergeometric2F1}\left(1,\frac{m+2}{2},\frac{m+3}{2},-\frac{ex^2}{d}\right)}{d(m+1)}+b\text{Unintegrable}\left(\frac{x^m\tan^{-1}(cx)}{\sqrt{d+ex^2}},x\right)$$

```
[Out] (a*x^(1 + m)*Sqrt[d + e*x^2]*Hypergeometric2F1[1, (2 + m)/2, (3 + m)/2, -((
e*x^2)/d)])/(d*(1 + m)) + b*Unintegrable[(x^m*ArcTan[c*x])/Sqrt[d + e*x^2],
 x]
```

Rubi [A] time = 0.156405, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m\left(a+b\tan^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*(a + b*ArcTan[c*x]))/Sqrt[d + e*x^2],x]
```

```
[Out] (a*x^(1 + m)*Sqrt[1 + (e*x^2)/d]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/
2, -((e*x^2)/d)])/((1 + m)*Sqrt[d + e*x^2]) + b*Defer[Int][(x^m*ArcTan[c*x]
)/Sqrt[d + e*x^2], x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^m\left(a+b\tan^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx &= a\int \frac{x^m}{\sqrt{d+ex^2}}\,dx+b\int \frac{x^m\tan^{-1}(cx)}{\sqrt{d+ex^2}}\,dx\\
&= b\int \frac{x^m\tan^{-1}(cx)}{\sqrt{d+ex^2}}\,dx+\frac{\left(a\sqrt{1+\frac{ex^2}{d}}\right)\int \frac{x^m}{\sqrt{1+\frac{ex^2}{d}}}\,dx}{\sqrt{d+ex^2}}\\
&= \frac{ax^{1+m}\sqrt{1+\frac{ex^2}{d}}\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};-\frac{ex^2}{d}\right)}{(1+m)\sqrt{d+ex^2}}+b\int \frac{x^m\tan^{-1}(cx)}{\sqrt{d+ex^2}}\,dx
\end{aligned}$$

Mathematica [A] time = 3.40774, size = 0, normalized size = 0.

$$\int \frac{x^m \left(a + b\tan^{-1}(cx)\right)}{\sqrt{d + ex^2}}\, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*(a + b*ArcTan[c*x]))/Sqrt[d + e*x^2],x]
```

```
[Out] Integrate[(x^m*(a + b*ArcTan[c*x]))/Sqrt[d + e*x^2], x]
```

Maple [A] time = 0.797, size = 0, normalized size = 0.

$$\int x^m \left(a + b\arctan(cx)\right) \frac{1}{\sqrt{ex^2 + d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a+b*arctan(c*x))/(e*x^2+d)^(1/2),x)
```

```
[Out] int(x^m*(a+b*arctan(c*x))/(e*x^2+d)^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\arctan(cx) + a)x^m}{\sqrt{ex^2 + d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arctan(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arctan(c*x) + a)*x^m/sqrt(e*x^2 + d), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(b\arctan(cx) + a)x^m}{\sqrt{ex^2 + d}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arctan(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*arctan(c*x) + a)*x^m/sqrt(e*x^2 + d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a+b*atan(c*x))/(e*x**2+d)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \arctan(cx) + a)x^m}{\sqrt{ex^2 + d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arctan(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)*x^m/sqrt(e*x^2 + d), x)
```

3.1237 $\int \frac{x^m(a+b\tan^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx$

Optimal. Leaf size=73

$$\frac{ax^{m+1}\text{Hypergeometric2F1}\left(1,\frac{m}{2},\frac{m+3}{2},-\frac{ex^2}{d}\right)}{d(m+1)\sqrt{d+ex^2}}+b\text{Unintegrable}\left(\frac{x^m\tan^{-1}(cx)}{(d+ex^2)^{3/2}},x\right)$$

```
[Out] (a*x^(1 + m)*Hypergeometric2F1[1, m/2, (3 + m)/2, -((e*x^2)/d)])/(d*(1 + m)
*Sqrt[d + e*x^2]) + b*Unintegrable[(x^m*ArcTan[c*x])/(d + e*x^2)^(3/2), x]
```

Rubi [A] time = 0.173692, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m\left(a+b\tan^{-1}(cx)\right)}{(d+ex^2)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*(a + b*ArcTan[c*x]))/(d + e*x^2)^(3/2),x]
```

```
[Out] (a*x^(1 + m)*Sqrt[1 + (e*x^2)/d]*Hypergeometric2F1[3/2, (1 + m)/2, (3 + m)/
2, -((e*x^2)/d)])/(d*(1 + m)*Sqrt[d + e*x^2]) + b*Defer[Int][(x^m*ArcTan[c*
x])/(d + e*x^2)^(3/2), x]
```

Rubi steps

$$\int \frac{x^m \left(a+b\tan^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx = a\int \frac{x^m}{\left(d+ex^2\right)^{3/2}}\,dx + b\int \frac{x^m\tan^{-1}(cx)}{\left(d+ex^2\right)^{3/2}}\,dx$$

$$= b\int \frac{x^m\tan^{-1}(cx)}{\left(d+ex^2\right)^{3/2}}\,dx + \frac{\left(a\sqrt{1+\frac{ex^2}{d}}\right)\int \frac{x^m}{\left(1+\frac{ex^2}{d}\right)^{3/2}}\,dx}{d\sqrt{d+ex^2}}$$

$$= \frac{ax^{1+m}\sqrt{1+\frac{ex^2}{d}}\,{}_2F_1\left(\frac{3}{2},\frac{1+m}{2};\frac{3+m}{2};-\frac{ex^2}{d}\right)}{d(1+m)\sqrt{d+ex^2}} + b\int \frac{x^m\tan^{-1}(cx)}{\left(d+ex^2\right)^{3/2}}\,dx$$

Mathematica [A] time = 4.37088, size = 0, normalized size = 0.

$$\int \frac{x^m \left(a+b\tan^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*(a + b*ArcTan[c*x]))/(d + e*x^2)^(3/2),x]
```

```
[Out] Integrate[(x^m*(a + b*ArcTan[c*x]))/(d + e*x^2)^(3/2), x]
```

Maple [A] time = 0.619, size = 0, normalized size = 0.

$$\int x^m\left(a+b\arctan(cx)\right)\left(ex^2+d\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a+b*arctan(c*x))/(e*x^2+d)^(3/2),x)
```

```
[Out] int(x^m*(a+b*arctan(c*x))/(e*x^2+d)^(3/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\arctan(cx)+a)x^m}{\left(ex^2+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arctan(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arctan(c*x) + a)*x^m/(e*x^2 + d)^(3/2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{ex^2+d}(b\arctan(cx)+a)x^m}{e^2x^4+2\,dex^2+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arctan(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x^2 + d)*(b*arctan(c*x) + a)*x^m/(e^2*x^4 + 2*d*e*x^2 + d^2
), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a+b*atan(c*x))/(e*x**2+d)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\arctan(cx)+a)x^m}{\left(ex^2+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arctan(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)*x^m/(e*x^2 + d)^(3/2), x)
```

3.1238 $\int \frac{x^m(a+b\tan^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx$

Optimal. Leaf size=75

$$\frac{ax^{m+1}\text{Hypergeometric2F1}\left(1,\frac{m-2}{2},\frac{m+3}{2},-\frac{ex^2}{d}\right)}{d(m+1)\left(d+ex^2\right)^{3/2}}+b\text{Unintegrable}\left(\frac{x^m\tan^{-1}(cx)}{\left(d+ex^2\right)^{5/2}},x\right)$$

```
[Out] (a*x^(1 + m)*Hypergeometric2F1[1, (-2 + m)/2, (3 + m)/2, -((e*x^2)/d)])/(d*
(1 + m)*(d + e*x^2)^(3/2)) + b*Unintegrable[(x^m*ArcTan[c*x])/(d + e*x^2)^(
5/2), x]
```

Rubi [A] time = 0.176407, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m\left(a+b\tan^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*(a + b*ArcTan[c*x]))/(d + e*x^2)^(5/2),x]
```

```
[Out] (a*x^(1 + m)*Sqrt[1 + (e*x^2)/d]*Hypergeometric2F1[5/2, (1 + m)/2, (3 + m)/
2, -((e*x^2)/d)])/(d^2*(1 + m)*Sqrt[d + e*x^2]) + b*Defer[Int][(x^m*ArcTan[
c*x])/(d + e*x^2)^(5/2), x]
```

Rubi steps

$$\int \frac{x^m\left(a+b\tan^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx = a\int \frac{x^m}{\left(d+ex^2\right)^{5/2}}\,dx + b\int \frac{x^m\tan^{-1}(cx)}{\left(d+ex^2\right)^{5/2}}\,dx$$

$$= b\int \frac{x^m\tan^{-1}(cx)}{\left(d+ex^2\right)^{5/2}}\,dx + \frac{\left(a\sqrt{1+\frac{ex^2}{d}}\right)\int \frac{x^m}{\left(1+\frac{ex^2}{d}\right)^{5/2}}\,dx}{d^2\sqrt{d+ex^2}}$$

$$= \frac{ax^{1+m}\sqrt{1+\frac{ex^2}{d}}\,{}_2F_1\left(\frac{5}{2},\frac{1+m}{2};\frac{3+m}{2};-\frac{ex^2}{d}\right)}{d^2(1+m)\sqrt{d+ex^2}} + b\int \frac{x^m\tan^{-1}(cx)}{\left(d+ex^2\right)^{5/2}}\,dx$$

Mathematica [A] time = 5.77045, size = 0, normalized size = 0.

$$\int \frac{x^m\left(a+b\tan^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*(a + b*ArcTan[c*x]))/(d + e*x^2)^(5/2),x]
```

```
[Out] Integrate[(x^m*(a + b*ArcTan[c*x]))/(d + e*x^2)^(5/2), x]
```

Maple [A] time = 0.606, size = 0, normalized size = 0.

$$\int x^m\left(a+b\arctan(cx)\right)\left(ex^2+d\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a+b*arctan(c*x))/(e*x^2+d)^(5/2),x)
```

```
[Out] int(x^m*(a+b*arctan(c*x))/(e*x^2+d)^(5/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\arctan(cx)+a)x^m}{\left(ex^2+d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arctan(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arctan(c*x) + a)*x^m/(e*x^2 + d)^(5/2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{ex^2+d}(b\arctan(cx)+a)x^m}{e^3x^6+3\,de^2x^4+3\,d^2ex^2+d^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arctan(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x^2 + d)*(b*arctan(c*x) + a)*x^m/(e^3*x^6 + 3*d*e^2*x^4 + 3
*d^2*e*x^2 + d^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a+b*atan(c*x))/(e*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{(b\arctan(cx)+a)x^m}{\left(ex^2+d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arctan(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)*x^m/(e*x^2 + d)^(5/2), x)
```

3.1239 $\int x^m \left(d+ex^2\right)^p \left(a+b\tan^{-1}(cx)\right)\,dx$

Optimal. Leaf size=76

$$\frac{ax^{m+1}\left(d+ex^2\right)^{p+1}\text{Hypergeometric2F1}\left(1,\frac{1}{2}(m+2p+3),\frac{m+3}{2},-\frac{ex^2}{d}\right)}{d(m+1)}+b\text{Unintegrable}\left(x^m\tan^{-1}(cx)\left(d+ex^2\right)^p,x\right)$$

```
[Out] (a*x^(1 + m)*(d + e*x^2)^(1 + p)*Hypergeometric2F1[1, (3 + m + 2*p)/2, (3 +
 m)/2, -((e*x^2)/d)])/(d*(1 + m)) + b*Unintegrable[x^m*(d + e*x^2)^p*ArcTan
[c*x], x]
```

Rubi [A] time = 0.11844, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^m\left(d+ex^2\right)^p\left(a+b\tan^{-1}(cx)\right)dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*(d + e*x^2)^p*(a + b*ArcTan[c*x]),x]
```

```
[Out] (a*x^(1 + m)*(d + e*x^2)^p*Hypergeometric2F1[(1 + m)/2, -p, (3 + m)/2, -((e
*x^2)/d)])/((1 + m)*(1 + (e*x^2)/d)^p) + b*Defer[Int][x^m*(d + e*x^2)^p*Arc
Tan[c*x], x]
```

Rubi steps

$$\begin{aligned}\int x^m\left(d+ex^2\right)^p\left(a+b\tan^{-1}(cx)\right)dx &= a\int x^m\left(d+ex^2\right)^p dx+b\int x^m\left(d+ex^2\right)^p\tan^{-1}(cx)\,dx\\ &= b\int x^m\left(d+ex^2\right)^p\tan^{-1}(cx)\,dx+\left(a\left(d+ex^2\right)^p\left(1+\frac{ex^2}{d}\right)^{-p}\right)\int x^m\left(1+\frac{ex^2}{d}\right)^p d\\ &= \frac{ax^{1+m}\left(d+ex^2\right)^p\left(1+\frac{ex^2}{d}\right)^{-p}{}_2F_1\left(\frac{1+m}{2},-p;\frac{3+m}{2};-\frac{ex^2}{d}\right)}{1+m}+b\int x^m\left(d+ex^2\right)^p\tan\end{aligned}$$

Mathematica [A] time = 3.05692, size = 0, normalized size = 0.

$$\int x^m\left(d+ex^2\right)^p\left(a+b\tan^{-1}(cx)\right)dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*(d + e*x^2)^p*(a + b*ArcTan[c*x]),x]
```

```
[Out] Integrate[x^m*(d + e*x^2)^p*(a + b*ArcTan[c*x]), x]
```

Maple [A] time = 0.886, size = 0, normalized size = 0.

$$\int x^{m} \left(e x^{2}+d\right)^{p} \left(a+b \arctan \left(c x\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(e*x^2+d)^p*(a+b*arctan(c*x)),x)
```

```
[Out] int(x^m*(e*x^2+d)^p*(a+b*arctan(c*x)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int {\left(b \arctan \left(c x\right) + a\right)} {\left(e x^{2} + d\right)}^{p} x^{m}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*x^2+d)^p*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((b*arctan(c*x) + a)*(e*x^2 + d)^p*x^m, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$${\rm integral}\left({\left(b \arctan \left(c x\right) + a\right)} {\left(e x^{2} + d\right)}^{p} x^{m}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*x^2+d)^p*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] integral((b*arctan(c*x) + a)*(e*x^2 + d)^p*x^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(e*x**2+d)**p*(a+b*atan(c*x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (b\arctan(cx)+a)\left(ex^2+d\right)^p x^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*x^2+d)^p*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)*(e*x^2 + d)^p*x^m, x)
```

3.1240 $\int x^{-2-2p}\left(d+ex^2\right)^p\left(a+b\tan^{-1}(cx)\right)\,dx$

Optimal. Leaf size=80

$$b\text{Unintegrable}\left(x^{-2p-2}\tan^{-1}(cx)\left(d+ex^2\right)^p,x\right)-\frac{ax^{-2p-1}\left(d+ex^2\right)^{p+1}\text{Hypergeometric2F1}\left(\frac{1}{2},1,\frac{1}{2}(1-2p),-\frac{ex^2}{d}\right)}{d(2p+1)}$$

```
[Out] -((a*x^(-1 - 2*p)*(d + e*x^2)^(1 + p)*Hypergeometric2F1[1/2, 1, (1 - 2*p)/2
, -((e*x^2)/d)])/(d*(1 + 2*p))) + b*Unintegrable[x^(-2 - 2*p)*(d + e*x^2)^p
*ArcTan[c*x], x]
```

Rubi [A] time = 0.14238, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^{-2-2p}\left(d+ex^2\right)^p\left(a+b\tan^{-1}(cx)\right)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^(-2 - 2*p)*(d + e*x^2)^p*(a + b*ArcTan[c*x]),x]
```

```
[Out] -((a*x^(-1 - 2*p)*(d + e*x^2)^p*Hypergeometric2F1[(-1 - 2*p)/2, -p, (1 - 2*
p)/2, -((e*x^2)/d)])/((1 + 2*p)*(1 + (e*x^2)/d)^p)) + b*Defer[Int][x^(-2 -
2*p)*(d + e*x^2)^p*ArcTan[c*x], x]
```

Rubi steps

$$\begin{aligned}\int x^{-2-2p}\left(d+ex^2\right)^p\left(a+b\tan^{-1}(cx)\right)\,dx &= a\int x^{-2-2p}\left(d+ex^2\right)^p\,dx+b\int x^{-2-2p}\left(d+ex^2\right)^p\tan^{-1}(cx)\,dx\\ &= b\int x^{-2-2p}\left(d+ex^2\right)^p\tan^{-1}(cx)\,dx+\left(a\left(d+ex^2\right)^p\left(1+\frac{ex^2}{d}\right)^{-p}\right)\int x^{-2-2p}\left(1\right.\\ &= -\frac{ax^{-1-2p}\left(d+ex^2\right)^p\left(1+\frac{ex^2}{d}\right)^{-p}{}_2F_1\left(\frac{1}{2}(-1-2p),-p;\frac{1}{2}(1-2p);-\frac{ex^2}{d}\right)}{1+2p}+b\int x\end{aligned}$$

Mathematica [A] time = 3.01047, size = 0, normalized size = 0.

$$\int x^{-2-2p}\left(d+ex^2\right)^p\left(a+b\tan^{-1}(cx)\right)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^(-2 - 2*p)*(d + e*x^2)^p*(a + b*ArcTan[c*x]),x]
```

```
[Out] Integrate[x^(-2 - 2*p)*(d + e*x^2)^p*(a + b*ArcTan[c*x]), x]
```

Maple [A] time = 0.871, size = 0, normalized size = 0.

$$\int x^{-2-2p} \left(ex^2+d\right)^p (a+b\arctan(cx))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-2-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x)
```

```
[Out] int(x^(-2-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (b\arctan(cx)+a)\left(ex^2+d\right)^p x^{-2p-2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((b*arctan(c*x) + a)*(e*x^2 + d)^p*x^(-2*p - 2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b\arctan(cx)+a)\left(ex^2+d\right)^p x^{-2p-2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] integral((b*arctan(c*x) + a)*(e*x^2 + d)^p*x^(-2*p - 2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-2-2*p)*(e*x**2+d)**p*(a+b*atan(c*x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (b\arctan(cx)+a)\left(ex^2+d\right)^p x^{-2p-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)*(e*x^2 + d)^p*x^(-2*p - 2), x)
```

3.1241 $\int x^{-3-2p}\left(d+ex^2\right)^p\left(a+b\tan^{-1}(cx)\right)\,dx$

Optimal. Leaf size=129

$$-\frac{x^{-2(p+1)}\left(d+ex^2\right)^{p+1}\left(a+b\tan^{-1}(cx)\right)}{2d(p+1)}-\frac{bcx^{-2p-1}\left(d+ex^2\right)^p\left(\frac{ex^2}{d}+1\right)^{-p}F_1\left(\frac{1}{2}(-2p-1);1,-p-1;\frac{1}{2}(1-2p);-c^2x^2,-\frac{e}{}\right.}{2\left(2p^2+3p+1\right)}$$

```
[Out] -(b*c*x^(-1 - 2*p)*(d + e*x^2)^p*AppellF1[(-1 - 2*p)/2, 1, -1 - p, (1 - 2*p
)/2, -(c^2*x^2), -((e*x^2)/d)])/(2*(1 + 3*p + 2*p^2)*(1 + (e*x^2)/d)^p) - (
(d + e*x^2)^(1 + p)*(a + b*ArcTan[c*x]))/(2*d*(1 + p)*x^(2*(1 + p)))
```

Rubi [A] time = 0.166492, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {264, 4976, 12, 511, 510}

$$-\frac{x^{-2(p+1)}\left(d+ex^2\right)^{p+1}\left(a+b\tan^{-1}(cx)\right)}{2d(p+1)}-\frac{bcx^{-2p-1}\left(d+ex^2\right)^p\left(\frac{ex^2}{d}+1\right)^{-p}F_1\left(\frac{1}{2}(-2p-1);1,-p-1;\frac{1}{2}(1-2p);-c^2x^2,-\frac{e}{}\right.}{2\left(2p^2+3p+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-3 - 2*p)*(d + e*x^2)^p*(a + b*ArcTan[c*x]),x]
```


```
[Out] -(b*c*x^(-1 - 2*p)*(d + e*x^2)^p*AppellF1[(-1 - 2*p)/2, 1, -1 - p, (1 - 2*p
)/2, -(c^2*x^2), -((e*x^2)/d)])/(2*(1 + 3*p + 2*p^2)*(1 + (e*x^2)/d)^p) - (
(d + e*x^2)^(1 + p)*(a + b*ArcTan[c*x]))/(2*d*(1 + p)*x^(2*(1 + p)))
```


Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 4976

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Dis
t[a + b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&  !(
ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(IL
```

```
tQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILtQ[(m
 - 1)/2, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int x^{-3-2p}\left(d+ex^2\right)^p\left(a+b\tan^{-1}(cx)\right)dx=-\frac{x^{-2(1+p)}\left(d+ex^2\right)^{1+p}\left(a+b\tan^{-1}(cx)\right)}{2d(1+p)}-(bc)\int-\frac{x^{-2(1+p)}\left(d+ex^2\right)^{1+p}}{2d(1+p)\left(1+c^2x^2\right)}\,dx$$

$$=-\frac{x^{-2(1+p)}\left(d+ex^2\right)^{1+p}\left(a+b\tan^{-1}(cx)\right)}{2d(1+p)}+\frac{(bc)\int\frac{x^{-2(1+p)}\left(d+ex^2\right)^{1+p}}{1+c^2x^2}\,dx}{2d(1+p)}$$

$$=-\frac{x^{-2(1+p)}\left(d+ex^2\right)^{1+p}\left(a+b\tan^{-1}(cx)\right)}{2d(1+p)}+\frac{\left(bc\left(d+ex^2\right)^p\left(1+\frac{ex^2}{d}\right)^{-p}\right)\int\frac{x^{-2(1-}}{}}{2(1+p)}$$

$$=-\frac{bcx^{-1-2p}\left(d+ex^2\right)^p\left(1+\frac{ex^2}{d}\right)^{-p}F_1\left(\frac{1}{2}(-1-2p);1,-1-p;\frac{1}{2}(1-2p);-c^2x^2,-\right.}{2\left(1+3p+2p^2\right)}$$

Mathematica [A] time = 0.434221, size = 166, normalized size = 1.29

$$-\frac{x^{-2(p+1)}\left(d+ex^2\right)^p\left(\frac{ex^2}{d}+1\right)^{-p}\left(bex\text{Hypergeometric2F1}\left(-p-\frac{1}{2},-p,\frac{1}{2}-p,-\frac{ex^2}{d}\right)+c(2p+1)\left(d+ex^2\right)\left(\frac{ex^2}{d}+1\right)^p\left(a\right.\right.}{2cd(p+1)(2p+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^(-3 - 2*p)*(d + e*x^2)^p*(a + b*ArcTan[c*x]),x]
```

```
[Out] -((d + e*x^2)^p*(b*(c^2*d - e)*x*AppellF1[-1/2 - p, -p, 1, 1/2 - p, -((e*x^
2)/d), -(c^2*x^2)] + c*(1 + 2*p)*(d + e*x^2)*(1 + (e*x^2)/d)^p*(a + b*ArcTa
n[c*x]) + b*e*x*Hypergeometric2F1[-1/2 - p, -p, 1/2 - p, -((e*x^2)/d)]))/(2
*c*d*(1 + p)*(1 + 2*p)*x^(2*(1 + p))*(1 + (e*x^2)/d)^p)
```

Maple [F] time = 0.886, size = 0, normalized size = 0.

$$\int x^{-3-2p}\left(ex^2+d\right)^p(a+b\arctan(cx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-3-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x)
```

```
[Out] int(x^(-3-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$b\int\frac{\arctan(cx)\,e^{\left(p\log\left(ex^2+d\right)-2\,p\log(x)\right)}}{x^3}\,dx-\frac{\left(ex^2+d\right)ae^{\left(p\log\left(ex^2+d\right)-2\,p\log(x)\right)}}{2\,d\left(p+1\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] b*integrate(arctan(c*x)*e^(p*log(e*x^2 + d) - 2*p*log(x))/x^3, x) - 1/2*(e*
x^2 + d)*a*e^(p*log(e*x^2 + d) - 2*p*log(x))/(d*(p + 1)*x^2)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b\arctan(cx)+a)\left(ex^2+d\right)^p x^{-2p-3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] integral((b*arctan(c*x) + a)*(e*x^2 + d)^p*x^(-2*p - 3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-3-2*p)*(e*x**2+d)**p*(a+b*atan(c*x)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\arctan(cx)+a)\left(ex^2+d\right)^p x^{-2p-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)*(e*x^2 + d)^p*x^(-2*p - 3), x)
```

3.1242 $\int x^{-4-2p}\left(d+ex^2\right)^p\left(a+b\tan^{-1}(cx)\right)\,dx$

Optimal. Leaf size=80

$$b\text{Unintegrable}\left(x^{-2p-4}\tan^{-1}(cx)\left(d+ex^2\right)^p,x\right)-\frac{ax^{-2p-3}\left(d+ex^2\right)^{p+1}\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2}(-2p-1),-\frac{ex^2}{d}\right)}{d(2p+3)}$$

```
[Out] -((a*x^(-3 - 2*p)*(d + e*x^2)^(1 + p)*Hypergeometric2F1[-1/2, 1, (-1 - 2*p)
/2, -((e*x^2)/d)])/(d*(3 + 2*p))) + b*Unintegrable[x^(-4 - 2*p)*(d + e*x^2)
^p*ArcTan[c*x], x]
```

Rubi [A] time = 0.140094, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^{-4-2p}\left(d+ex^2\right)^p\left(a+b\tan^{-1}(cx)\right)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^(-4 - 2*p)*(d + e*x^2)^p*(a + b*ArcTan[c*x]),x]
```

```
[Out] -((a*x^(-3 - 2*p)*(d + e*x^2)^p*Hypergeometric2F1[(-3 - 2*p)/2, -p, (-1 - 2
*p)/2, -((e*x^2)/d)])/((3 + 2*p)*(1 + (e*x^2)/d)^p)) + b*Defer[Int][x^(-4 -
 2*p)*(d + e*x^2)^p*ArcTan[c*x], x]
```

Rubi steps

$$\begin{aligned}\int x^{-4-2p}\left(d+ex^2\right)^p\left(a+b\tan^{-1}(cx)\right)\,dx &= a\int x^{-4-2p}\left(d+ex^2\right)^p\,dx+b\int x^{-4-2p}\left(d+ex^2\right)^p\tan^{-1}(cx)\,dx\\ &= b\int x^{-4-2p}\left(d+ex^2\right)^p\tan^{-1}(cx)\,dx+\left(a\left(d+ex^2\right)^p\left(1+\frac{ex^2}{d}\right)^{-p}\right)\int x^{-4-2p}\left(1\right.\\ &= -\frac{ax^{-3-2p}\left(d+ex^2\right)^p\left(1+\frac{ex^2}{d}\right)^{-p}{}_2F_1\left(\frac{1}{2}(-3-2p),-p;\frac{1}{2}(-1-2p);-\frac{ex^2}{d}\right)}{3+2p}+b\end{aligned}$$

Mathematica [A] time = 3.30148, size = 0, normalized size = 0.

$$\int x^{-4-2p}\left(d+ex^2\right)^p\left(a+b\tan^{-1}(cx)\right)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^(-4 - 2*p)*(d + e*x^2)^p*(a + b*ArcTan[c*x]),x]
```

```
[Out] Integrate[x^(-4 - 2*p)*(d + e*x^2)^p*(a + b*ArcTan[c*x]), x]
```

Maple [A] time = 0.874, size = 0, normalized size = 0.

$$\int x^{-4-2p} \left(e x^2 + d\right)^p (a + b \arctan(cx))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-4-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x)
```

```
[Out] int(x^(-4-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (b \arctan(cx) + a)\left(e x^2 + d\right)^p x^{-2p-4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-4-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((b*arctan(c*x) + a)*(e*x^2 + d)^p*x^(-2*p - 4), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b \arctan(cx) + a)\left(e x^2 + d\right)^p x^{-2p-4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-4-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] integral((b*arctan(c*x) + a)*(e*x^2 + d)^p*x^(-2*p - 4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-4-2*p)*(e*x**2+d)**p*(a+b*atan(c*x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (b\arctan(cx) + a)\left(ex^2 + d\right)^p x^{-2p-4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-4-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)*(e*x^2 + d)^p*x^(-2*p - 4), x)
```

3.1243 $\int x^{-5-2p}\left(d+ex^2\right)^p\left(a+b\tan^{-1}(cx)\right)\,dx$

Optimal. Leaf size=285

$$\frac{bex^{-2p-3}\left(d+ex^2\right)^p\left(\frac{ex^2}{d}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2}(-2p-3),-p-1,\frac{1}{2}(-2p-1),-\frac{ex^2}{d}\right)}{2cd\left(2p^3+9p^2+13p+6\right)}+\frac{ex^{-2(p+1)}\left(d+ex^2\right)^{p+1}\left(a-\right.}{2d^2(p+1)(p+}$$

```
[Out] -(b*(e + c^2*d*(1 + p))*x^(-3 - 2*p)*(d + e*x^2)^p*AppellF1[(-3 - 2*p)/2, 1
, -1 - p, (-1 - 2*p)/2, -(c^2*x^2), -((e*x^2)/d)])/(2*c*d*(1 + p)*(2 + p)*(
3 + 2*p)*(1 + (e*x^2)/d)^p) + (e*(d + e*x^2)^(1 + p)*(a + b*ArcTan[c*x]))/(
2*d^2*(1 + p)*(2 + p)*x^(2*(1 + p))) - ((d + e*x^2)^(1 + p)*(a + b*ArcTan[c
*x]))/(2*d*(2 + p)*x^(2*(2 + p))) + (b*e*x^(-3 - 2*p)*(d + e*x^2)^p*Hyperge
ometric2F1[(-3 - 2*p)/2, -1 - p, (-1 - 2*p)/2, -((e*x^2)/d)])/(2*c*d*(6 + 1
3*p + 9*p^2 + 2*p^3)*(1 + (e*x^2)/d)^p)
```

Rubi [A] time = 0.374795, antiderivative size = 285, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {271, 264, 4976, 12, 584, 365, 364, 511, 510}

$$\frac{ex^{-2(p+1)}\left(d+ex^2\right)^{p+1}\left(a+b\tan^{-1}(cx)\right)}{2d^2(p+1)(p+2)}-\frac{x^{-2(p+2)}\left(d+ex^2\right)^{p+1}\left(a+b\tan^{-1}(cx)\right)}{2d(p+2)}-\frac{bx^{-2p-3}\left(c^2d(p+1)+e\right)\left(d+ex^2\right)^p\left(\frac{e}{}\right.}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-5 - 2*p)*(d + e*x^2)^p*(a + b*ArcTan[c*x]),x]
```


```
[Out] -(b*(e + c^2*d*(1 + p))*x^(-3 - 2*p)*(d + e*x^2)^p*AppellF1[(-3 - 2*p)/2, 1
, -1 - p, (-1 - 2*p)/2, -(c^2*x^2), -((e*x^2)/d)])/(2*c*d*(1 + p)*(2 + p)*(
3 + 2*p)*(1 + (e*x^2)/d)^p) + (e*(d + e*x^2)^(1 + p)*(a + b*ArcTan[c*x]))/(
2*d^2*(1 + p)*(2 + p)*x^(2*(1 + p))) - ((d + e*x^2)^(1 + p)*(a + b*ArcTan[c
*x]))/(2*d*(2 + p)*x^(2*(2 + p))) + (b*e*x^(-3 - 2*p)*(d + e*x^2)^p*Hyperge
ometric2F1[(-3 - 2*p)/2, -1 - p, (-1 - 2*p)/2, -((e*x^2)/d)])/(2*c*d*(6 + 1
3*p + 9*p^2 + 2*p^3)*(1 + (e*x^2)/d)^p)
```


Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
```

tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 4976

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Dis
t[a + b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] && !(
ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(IL
tQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] && !ILtQ[(m
 - 1)/2, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int x^{-5-2p}\left(d+ex^2\right)^p\left(a+b\tan^{-1}(cx)\right)dx = \frac{ex^{-2(1+p)}\left(d+ex^2\right)^{1+p}\left(a+b\tan^{-1}(cx)\right)}{2d^2(1+p)(2+p)} - \frac{x^{-2(2+p)}\left(d+ex^2\right)^{1+p}\left(a+b\tan^{-1}(\right.}{2d(2+p)}$$

$$= \frac{ex^{-2(1+p)}\left(d+ex^2\right)^{1+p}\left(a+b\tan^{-1}(cx)\right)}{2d^2(1+p)(2+p)} - \frac{x^{-2(2+p)}\left(d+ex^2\right)^{1+p}\left(a+b\tan^{-1}(\right.}{2d(2+p)}$$

$$= \frac{ex^{-2(1+p)}\left(d+ex^2\right)^{1+p}\left(a+b\tan^{-1}(cx)\right)}{2d^2(1+p)(2+p)} - \frac{x^{-2(2+p)}\left(d+ex^2\right)^{1+p}\left(a+b\tan^{-1}(\right.}{2d(2+p)}$$

$$= \frac{ex^{-2(1+p)}\left(d+ex^2\right)^{1+p}\left(a+b\tan^{-1}(cx)\right)}{2d^2(1+p)(2+p)} - \frac{x^{-2(2+p)}\left(d+ex^2\right)^{1+p}\left(a+b\tan^{-1}(\right.}{2d(2+p)}$$

$$= \frac{ex^{-2(1+p)}\left(d+ex^2\right)^{1+p}\left(a+b\tan^{-1}(cx)\right)}{2d^2(1+p)(2+p)} - \frac{x^{-2(2+p)}\left(d+ex^2\right)^{1+p}\left(a+b\tan^{-1}(\right.}{2d(2+p)}$$

$$= -\frac{b\left(e+c^2d(1+p)\right)x^{-3-2p}\left(d+ex^2\right)^p\left(1+\frac{ex^2}{d}\right)^{-p}F_1\left(\frac{1}{2}(-3-2p);1,-1-p;\frac{1}{2}(\right.}{2cd(1+p)(2+p)(3+2p)}$$

Mathematica [F] time = 4.08405, size = 0, normalized size = 0.

$$\int x^{-5-2p}\left(d+ex^2\right)^p\left(a+b\tan^{-1}(cx)\right)dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^(-5 - 2*p)*(d + e*x^2)^p*(a + b*ArcTan[c*x]),x]
```

```
[Out] Integrate[x^(-5 - 2*p)*(d + e*x^2)^p*(a + b*ArcTan[c*x]), x]
```

Maple [F] time = 0.898, size = 0, normalized size = 0.

$$\int x^{-5-2p} \left(e x^2+d\right)^p \left(a+b \arctan \left(c x\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-5-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x)
```

```
[Out] int(x^(-5-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$b \int \frac{\arctan \left(c x\right) e^{\left(p \log\left(e x^2+d\right)-2 p \log\left(x\right)\right)}}{x^5}\, dx + \frac{\left(e^2 x^4 - d e p x^2 - d^2 \left(p+1\right)\right) a e^{\left(p \log\left(e x^2+d\right)-2 p \log\left(x\right)\right)}}{2 \left(p^2+3 p+2\right) d^2 x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-5-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] b*integrate(arctan(c*x)*e^(p*log(e*x^2 + d) - 2*p*log(x))/x^5, x) + 1/2*(e^
2*x^4 - d*e*p*x^2 - d^2*(p + 1))*a*e^(p*log(e*x^2 + d) - 2*p*log(x))/((p^2
+ 3*p + 2)*d^2*x^4)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b \arctan \left(c x\right)+a\right)\left(e x^2+d\right)^p x^{-2 p-5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-5-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] integral((b*arctan(c*x) + a)*(e*x^2 + d)^p*x^(-2*p - 5), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-5-2*p)*(e*x**2+d)**p*(a+b*atan(c*x)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \arctan (cx) + a)\left(ex^2 + d\right)^p x^{-2p-5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-5-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)*(e*x^2 + d)^p*x^(-2*p - 5), x)
```

3.1244 $\int x^{-6-2p}\left(d+ex^2\right)^p\left(a+b\tan^{-1}(cx)\right)\,dx$

Optimal. Leaf size=80

$$b\text{Unintegrable}\left(x^{-2p-6}\tan^{-1}(cx)\left(d+ex^2\right)^p,x\right)-\frac{ax^{-2p-5}\left(d+ex^2\right)^{p+1}\text{Hypergeometric2F1}\left(-\frac{3}{2},1,\frac{1}{2}(-2p-3),-\frac{ex^2}{d}\right)}{d(2p+5)}$$

```
[Out] -((a*x^(-5 - 2*p)*(d + e*x^2)^(1 + p)*Hypergeometric2F1[-3/2, 1, (-3 - 2*p)
/2, -((e*x^2)/d)])/(d*(5 + 2*p))) + b*Unintegrable[x^(-6 - 2*p)*(d + e*x^2)
^p*ArcTan[c*x], x]
```

Rubi [A] time = 0.143778, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^{-6-2p}\left(d+ex^2\right)^p\left(a+b\tan^{-1}(cx)\right)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^(-6 - 2*p)*(d + e*x^2)^p*(a + b*ArcTan[c*x]),x]
```

```
[Out] -((a*x^(-5 - 2*p)*(d + e*x^2)^p*Hypergeometric2F1[(-5 - 2*p)/2, -p, (-3 - 2
*p)/2, -((e*x^2)/d)])/((5 + 2*p)*(1 + (e*x^2)/d)^p)) + b*Defer[Int][x^(-6 -
 2*p)*(d + e*x^2)^p*ArcTan[c*x], x]
```

Rubi steps

$$\begin{aligned}\int x^{-6-2p}\left(d+ex^2\right)^p\left(a+b\tan^{-1}(cx)\right)\,dx &= a\int x^{-6-2p}\left(d+ex^2\right)^p\,dx+b\int x^{-6-2p}\left(d+ex^2\right)^p\tan^{-1}(cx)\,dx\\ &= b\int x^{-6-2p}\left(d+ex^2\right)^p\tan^{-1}(cx)\,dx+\left(a\left(d+ex^2\right)^p\left(1+\frac{ex^2}{d}\right)^{-p}\right)\int x^{-6-2p}\left(1\right.\\ &= -\frac{ax^{-5-2p}\left(d+ex^2\right)^p\left(1+\frac{ex^2}{d}\right)^{-p}{}_2F_1\left(\frac{1}{2}(-5-2p),-p;\frac{1}{2}(-3-2p);-\frac{ex^2}{d}\right)}{5+2p}+b\end{aligned}$$

Mathematica [A] time = 3.42695, size = 0, normalized size = 0.

$$\int x^{-6-2p}\left(d+ex^2\right)^p\left(a+b\tan^{-1}(cx)\right)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^(-6 - 2*p)*(d + e*x^2)^p*(a + b*ArcTan[c*x]),x]
```

```
[Out] Integrate[x^(-6 - 2*p)*(d + e*x^2)^p*(a + b*ArcTan[c*x]), x]
```

Maple [A] time = 0.881, size = 0, normalized size = 0.

$$\int x^{-6-2p} \left(ex^2+d\right)^p (a+b\arctan(cx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-6-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x)
```

```
[Out] int(x^(-6-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (b\arctan(cx)+a)\left(ex^2+d\right)^p x^{-2p-6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-6-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((b*arctan(c*x) + a)*(e*x^2 + d)^p*x^(-2*p - 6), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b\arctan(cx)+a)\left(ex^2+d\right)^p x^{-2p-6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-6-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] integral((b*arctan(c*x) + a)*(e*x^2 + d)^p*x^(-2*p - 6), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-6-2*p)*(e*x**2+d)**p*(a+b*atan(c*x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (b\arctan (cx) + a)\left(ex^2 + d\right)^p x^{-2\,p-6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-6-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)*(e*x^2 + d)^p*x^(-2*p - 6), x)
```

3.1245 $\int x^{-7-2p}\left(d+ex^2\right)^p\left(a+b\tan^{-1}(cx)\right)\,dx$

Optimal. Leaf size=466

$$\frac{bex^{-2p-5}\left(c^2d(p+1)+e\right)\left(d+ex^2\right)^p\left(\frac{ex^2}{d}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2}(-2p-5),-p-1,\frac{1}{2}(-2p-3),-\frac{ex^2}{d}\right)}{c^3d^2(p+1)(p+2)(p+3)(2p+5)}-\frac{be^2x^{-2p-3}}{}$$

```
[Out] -(b*(2*e^2 + 2*c^2*d*e*(1 + p) + c^4*d^2*(2 + 3*p + p^2))*x^(-5 - 2*p)*(d +
 e*x^2)^p*AppellF1[(-5 - 2*p)/2, 1, -1 - p, (-3 - 2*p)/2, -(c^2*x^2), -((e*
x^2)/d)])/(2*c^3*d^2*(1 + p)*(2 + p)*(3 + p)*(5 + 2*p)*(1 + (e*x^2)/d)^p) -
 (e^2*(d + e*x^2)^(1 + p)*(a + b*ArcTan[c*x]))/(d^3*(1 + p)*(2 + p)*(3 + p)
*x^(2*(1 + p))) + (e*(d + e*x^2)^(1 + p)*(a + b*ArcTan[c*x]))/(d^2*(2 + p)*
(3 + p)*x^(2*(2 + p))) - ((d + e*x^2)^(1 + p)*(a + b*ArcTan[c*x]))/(2*d*(3
+ p)*x^(2*(3 + p))) + (b*e*(e + c^2*d*(1 + p))*x^(-5 - 2*p)*(d + e*x^2)^p*H
ypergeometric2F1[(-5 - 2*p)/2, -1 - p, (-3 - 2*p)/2, -((e*x^2)/d)])/(c^3*d^
2*(1 + p)*(2 + p)*(3 + p)*(5 + 2*p)*(1 + (e*x^2)/d)^p) - (b*e^2*x^(-3 - 2*p
)*(d + e*x^2)^p*Hypergeometric2F1[(-3 - 2*p)/2, -1 - p, (-1 - 2*p)/2, -((e*
x^2)/d)])/(c*d^2*(1 + p)*(2 + p)*(3 + p)*(3 + 2*p)*(1 + (e*x^2)/d)^p)
```

Rubi [A] time = 1.42578, antiderivative size = 466, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {271, 264, 4976, 12, 6725, 365, 364, 511, 510}

$$-\frac{e^2x^{-2(p+1)}\left(d+ex^2\right)^{p+1}\left(a+b\tan^{-1}(cx)\right)}{d^3(p+1)(p+2)(p+3)}+\frac{ex^{-2(p+2)}\left(d+ex^2\right)^{p+1}\left(a+b\tan^{-1}(cx)\right)}{d^2(p+2)(p+3)}-\frac{x^{-2(p+3)}\left(d+ex^2\right)^{p+1}\left(a+b\tan^{-1}\right.}{2d(p+3)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-7 - 2*p)*(d + e*x^2)^p*(a + b*ArcTan[c*x]),x]
```


```
[Out] -(b*(2*e^2 + 2*c^2*d*e*(1 + p) + c^4*d^2*(2 + 3*p + p^2))*x^(-5 - 2*p)*(d +
 e*x^2)^p*AppellF1[(-5 - 2*p)/2, 1, -1 - p, (-3 - 2*p)/2, -(c^2*x^2), -((e*
x^2)/d)])/(2*c^3*d^2*(1 + p)*(2 + p)*(3 + p)*(5 + 2*p)*(1 + (e*x^2)/d)^p) -
 (e^2*(d + e*x^2)^(1 + p)*(a + b*ArcTan[c*x]))/(d^3*(1 + p)*(2 + p)*(3 + p)
*x^(2*(1 + p))) + (e*(d + e*x^2)^(1 + p)*(a + b*ArcTan[c*x]))/(d^2*(2 + p)*
(3 + p)*x^(2*(2 + p))) - ((d + e*x^2)^(1 + p)*(a + b*ArcTan[c*x]))/(2*d*(3
+ p)*x^(2*(3 + p))) + (b*e*(e + c^2*d*(1 + p))*x^(-5 - 2*p)*(d + e*x^2)^p*H
ypergeometric2F1[(-5 - 2*p)/2, -1 - p, (-3 - 2*p)/2, -((e*x^2)/d)])/(c^3*d^
2*(1 + p)*(2 + p)*(3 + p)*(5 + 2*p)*(1 + (e*x^2)/d)^p) - (b*e^2*x^(-3 - 2*p
)*(d + e*x^2)^p*Hypergeometric2F1[(-3 - 2*p)/2, -1 - p, (-1 - 2*p)/2, -((e*
```

```
x^2)/d)])/(c*d^2*(1 + p)*(2 + p)*(3 + p)*(3 + 2*p)*(1 + (e*x^2)/d)^p)
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 4976

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Dis
t[a + b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&  !(
ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(IL
tQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILtQ[(m
 - 1)/2, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int x^{-7-2p}\left(d+ex^2\right)^p\left(a+b\tan^{-1}(cx)\right)dx = -\frac{e^2x^{-2(1+p)}\left(d+ex^2\right)^{1+p}\left(a+b\tan^{-1}(cx)\right)}{d^3(1+p)(2+p)(3+p)}+\frac{ex^{-2(2+p)}\left(d+ex^2\right)^{1+p}\left(a+b\tan\right.}{d^2(2+p)(3+p)}$$

$$= -\frac{e^2x^{-2(1+p)}\left(d+ex^2\right)^{1+p}\left(a+b\tan^{-1}(cx)\right)}{d^3(1+p)(2+p)(3+p)}+\frac{ex^{-2(2+p)}\left(d+ex^2\right)^{1+p}\left(a+b\tan\right.}{d^2(2+p)(3+p)}$$

$$= -\frac{e^2x^{-2(1+p)}\left(d+ex^2\right)^{1+p}\left(a+b\tan^{-1}(cx)\right)}{d^3(1+p)(2+p)(3+p)}+\frac{ex^{-2(2+p)}\left(d+ex^2\right)^{1+p}\left(a+b\tan\right.}{d^2(2+p)(3+p)}$$

$$= -\frac{e^2x^{-2(1+p)}\left(d+ex^2\right)^{1+p}\left(a+b\tan^{-1}(cx)\right)}{d^3(1+p)(2+p)(3+p)}+\frac{ex^{-2(2+p)}\left(d+ex^2\right)^{1+p}\left(a+b\tan\right.}{d^2(2+p)(3+p)}$$

$$= -\frac{e^2x^{-2(1+p)}\left(d+ex^2\right)^{1+p}\left(a+b\tan^{-1}(cx)\right)}{d^3(1+p)(2+p)(3+p)}+\frac{ex^{-2(2+p)}\left(d+ex^2\right)^{1+p}\left(a+b\tan\right.}{d^2(2+p)(3+p)}$$

$$= -\frac{b\left(2e^2+2c^2de(1+p)+c^4d^2\left(2+3p+p^2\right)\right)x^{-5-2p}\left(d+ex^2\right)^p\left(1+\frac{ex^2}{d}\right)^{-p}F_1}{2c^3d^2(1+p)(2+p)(3+p)(5+}$$

Mathematica [F] time = 4.4568, size = 0, normalized size = 0.

$$\int x^{-7-2p}\left(d+ex^2\right)^p\left(a+b\tan^{-1}(cx)\right)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^(-7 - 2*p)*(d + e*x^2)^p*(a + b*ArcTan[c*x]),x]
```

```
[Out] Integrate[x^(-7 - 2*p)*(d + e*x^2)^p*(a + b*ArcTan[c*x]), x]
```

Maple [F] time = 0.874, size = 0, normalized size = 0.

$$\int x^{-7-2p}\left(ex^2+d\right)^p\left(a+b\arctan(cx)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-7-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x)
```

```
[Out] int(x^(-7-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$b\int\frac{\arctan(cx)\,e^{\left(p\log\left(ex^2+d\right)-2\,p\log(x)\right)}}{x^7}\,dx-\frac{\left(2\,e^3x^6-2\,de^2px^4+\left(p^2+p\right)d^2ex^2+\left(p^2+3\,p+2\right)d^3\right)ae^{\left(p\log\left(ex^2+d\right)-2\,p\log(x)\right)}}{2\left(p^3+6\,p^2+11\,p+6\right)d^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-7-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] b*integrate(arctan(c*x)*e^(p*log(e*x^2 + d) - 2*p*log(x))/x^7, x) - 1/2*(2*
e^3*x^6 - 2*d*e^2*p*x^4 + (p^2 + p)*d^2*e*x^2 + (p^2 + 3*p + 2)*d^3)*a*e^(p
*log(e*x^2 + d) - 2*p*log(x))/((p^3 + 6*p^2 + 11*p + 6)*d^3*x^6)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\arctan(cx)+a\right)\left(ex^2+d\right)^p x^{-2\,p-7},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-7-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] integral((b*arctan(c*x) + a)*(e*x^2 + d)^p*x^(-2*p - 7), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-7-2*p)*(e*x**2+d)**p*(a+b*atan(c*x)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \arctan(cx) + a)\left(ex^2 + d\right)^p x^{-2p-7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-7-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)*(e*x^2 + d)^p*x^(-2*p - 7), x)
```

3.1246 $\int x^{-8-2p}\left(d+ex^2\right)^p\left(a+b\tan^{-1}(cx)\right)dx$

Optimal. Leaf size=80

$$b\text{Unintegrable}\left(x^{-2p-8}\tan^{-1}(cx)\left(d+ex^2\right)^p,x\right)-\frac{ax^{-2p-7}\left(d+ex^2\right)^{p+1}\text{Hypergeometric2F1}\left(-\frac{5}{2},1,\frac{1}{2}(-2p-5),-\frac{ex^2}{d}\right)}{d(2p+7)}$$

```
[Out] -((a*x^(-7 - 2*p)*(d + e*x^2)^(1 + p)*Hypergeometric2F1[-5/2, 1, (-5 - 2*p)
/2, -((e*x^2)/d)])/(d*(7 + 2*p))) + b*Unintegrable[x^(-8 - 2*p)*(d + e*x^2)
^p*ArcTan[c*x], x]
```

Rubi [A] time = 0.140934, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^{-8-2p}\left(d+ex^2\right)^p\left(a+b\tan^{-1}(cx)\right)dx$$

Verification is Not applicable to the result.

```
[In] Int[x^(-8 - 2*p)*(d + e*x^2)^p*(a + b*ArcTan[c*x]),x]
```

```
[Out] -((a*x^(-7 - 2*p)*(d + e*x^2)^p*Hypergeometric2F1[(-7 - 2*p)/2, -p, (-5 - 2
*p)/2, -((e*x^2)/d)])/((7 + 2*p)*(1 + (e*x^2)/d)^p)) + b*Defer[Int][x^(-8 -
 2*p)*(d + e*x^2)^p*ArcTan[c*x], x]
```

Rubi steps

$$\begin{aligned}
\int x^{-8-2p}\left(d+ex^2\right)^p\left(a+b\tan^{-1}(cx)\right)dx &= a\int x^{-8-2p}\left(d+ex^2\right)^p\,dx+b\int x^{-8-2p}\left(d+ex^2\right)^p\tan^{-1}(cx)\,dx\\
&= b\int x^{-8-2p}\left(d+ex^2\right)^p\tan^{-1}(cx)\,dx+\left(a\left(d+ex^2\right)^p\left(1+\frac{ex^2}{d}\right)^{-p}\right)\int x^{-8-2p}\left(1\right.\\
&= -\frac{ax^{-7-2p}\left(d+ex^2\right)^p\left(1+\frac{ex^2}{d}\right)^{-p}{}_2F_1\left(\frac{1}{2}(-7-2p),-p;\frac{1}{2}(-5-2p);-\frac{ex^2}{d}\right)}{7+2p}+b
\end{aligned}$$

Mathematica [A] time = 2.75171, size = 0, normalized size = 0.

$$\int x^{-8-2p}\left(d+ex^2\right)^p\left(a+b\tan^{-1}(cx)\right)dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^(-8 - 2*p)*(d + e*x^2)^p*(a + b*ArcTan[c*x]),x]
```

```
[Out] Integrate[x^(-8 - 2*p)*(d + e*x^2)^p*(a + b*ArcTan[c*x]), x]
```

Maple [A] time = 0.904, size = 0, normalized size = 0.

$$\int x^{-8-2p}\left(ex^2+d\right)^p\left(a+b\arctan\left(cx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-8-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x)
```

```
[Out] int(x^(-8-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int\left(b\arctan\left(cx\right)+a\right)\left(ex^2+d\right)^p x^{-2p-8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-8-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((b*arctan(c*x) + a)*(e*x^2 + d)^p*x^(-2*p - 8), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b\arctan\left(cx\right)+a\right)\left(ex^2+d\right)^p x^{-2p-8},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-8-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] integral((b*arctan(c*x) + a)*(e*x^2 + d)^p*x^(-2*p - 8), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-8-2*p)*(e*x**2+d)**p*(a+b*atan(c*x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (b \arctan (cx) + a)\left(ex^2 + d\right)^p x^{-2\,p-8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-8-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)*(e*x^2 + d)^p*x^(-2*p - 8), x)
```

3.1247 $\int x^3 (d+ex^2) (a+b\tan^{-1}(cx))^2 \, dx$

Optimal. Leaf size=271

$$\frac{abdx}{2c^3}-\frac{d(a+b\tan^{-1}(cx))^2}{4c^4}+\frac{bex^3(a+b\tan^{-1}(cx))}{9c^3}-\frac{abex}{3c^5}+\frac{e(a+b\tan^{-1}(cx))^2}{6c^6}+\frac{1}{4}dx^4(a+b\tan^{-1}(cx))^2-\frac{bdx^3(\ldots}{\ldots}$$

```
[Out] (a*b*d*x)/(2*c^3) - (a*b*e*x)/(3*c^5) + (b^2*d*x^2)/(12*c^2) - (4*b^2*e*x^2
)/(45*c^4) + (b^2*e*x^4)/(60*c^2) + (b^2*d*x*ArcTan[c*x])/(2*c^3) - (b^2*e*
x*ArcTan[c*x])/(3*c^5) - (b*d*x^3*(a + b*ArcTan[c*x]))/(6*c) + (b*e*x^3*(a
+ b*ArcTan[c*x]))/(9*c^3) - (b*e*x^5*(a + b*ArcTan[c*x]))/(15*c) - (d*(a +
b*ArcTan[c*x])^2)/(4*c^4) + (e*(a + b*ArcTan[c*x])^2)/(6*c^6) + (d*x^4*(a +
 b*ArcTan[c*x])^2)/4 + (e*x^6*(a + b*ArcTan[c*x])^2)/6 - (b^2*d*Log[1 + c^2
*x^2])/(3*c^4) + (23*b^2*e*Log[1 + c^2*x^2])/(90*c^6)
```

Rubi [A] time = 0.651159, antiderivative size = 271, normalized size of antiderivative = 1., number of steps used = 29, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {4980, 4852, 4916, 266, 43, 4846, 260, 4884}

$$\frac{abdx}{2c^3}-\frac{d(a+b\tan^{-1}(cx))^2}{4c^4}+\frac{bex^3(a+b\tan^{-1}(cx))}{9c^3}-\frac{abex}{3c^5}+\frac{e(a+b\tan^{-1}(cx))^2}{6c^6}+\frac{1}{4}dx^4(a+b\tan^{-1}(cx))^2-\frac{bdx^3(\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + e*x^2)*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] (a*b*d*x)/(2*c^3) - (a*b*e*x)/(3*c^5) + (b^2*d*x^2)/(12*c^2) - (4*b^2*e*x^2
)/(45*c^4) + (b^2*e*x^4)/(60*c^2) + (b^2*d*x*ArcTan[c*x])/(2*c^3) - (b^2*e*
x*ArcTan[c*x])/(3*c^5) - (b*d*x^3*(a + b*ArcTan[c*x]))/(6*c) + (b*e*x^3*(a
+ b*ArcTan[c*x]))/(9*c^3) - (b*e*x^5*(a + b*ArcTan[c*x]))/(15*c) - (d*(a +
b*ArcTan[c*x])^2)/(4*c^4) + (e*(a + b*ArcTan[c*x])^2)/(6*c^6) + (d*x^4*(a +
 b*ArcTan[c*x])^2)/4 + (e*x^6*(a + b*ArcTan[c*x])^2)/6 - (b^2*d*Log[1 + c^2
*x^2])/(3*c^4) + (23*b^2*e*Log[1 + c^2*x^2])/(90*c^6)
```

Rule 4980

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
```

```
 IntegerQ[m])
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
```

```
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rubi steps

$$\int x^3\left(d+ex^2\right)\left(a+b\tan^{-1}(cx)\right)^2\,dx=\int\left(dx^3\left(a+b\tan^{-1}(cx)\right)^2+ex^5\left(a+b\tan^{-1}(cx)\right)^2\right)dx$$

$$=d\int x^3\left(a+b\tan^{-1}(cx)\right)^2\,dx+e\int x^5\left(a+b\tan^{-1}(cx)\right)^2\,dx$$

$$=\frac{1}{4}dx^4\left(a+b\tan^{-1}(cx)\right)^2+\frac{1}{6}ex^6\left(a+b\tan^{-1}(cx)\right)^2-\frac{1}{2}(bcd)\int\frac{x^4\left(a+b\tan^{-1}(c\right.}{1+c^2x^2}$$

$$=\frac{1}{4}dx^4\left(a+b\tan^{-1}(cx)\right)^2+\frac{1}{6}ex^6\left(a+b\tan^{-1}(cx)\right)^2-\frac{(bd)\int x^2\left(a+b\tan^{-1}(cx)\right)}{2c}$$

$$=-\frac{bdx^3\left(a+b\tan^{-1}(cx)\right)}{6c}-\frac{bex^5\left(a+b\tan^{-1}(cx)\right)}{15c}+\frac{1}{4}dx^4\left(a+b\tan^{-1}(cx)\right)^2+\frac{1}{6}$$

$$=\frac{abdx}{2c^3}-\frac{bdx^3\left(a+b\tan^{-1}(cx)\right)}{6c}+\frac{bex^3\left(a+b\tan^{-1}(cx)\right)}{9c^3}-\frac{bex^5\left(a+b\tan^{-1}(cx)\right)}{15c}$$

$$=\frac{abdx}{2c^3}-\frac{abex}{3c^5}+\frac{b^2dx\tan^{-1}(cx)}{2c^3}-\frac{bdx^3\left(a+b\tan^{-1}(cx)\right)}{6c}+\frac{bex^3\left(a+b\tan^{-1}(cx)\right)}{9c^3}$$

$$=\frac{abdx}{2c^3}-\frac{abex}{3c^5}+\frac{b^2dx^2}{12c^2}-\frac{b^2ex^2}{30c^4}+\frac{b^2ex^4}{60c^2}+\frac{b^2dx\tan^{-1}(cx)}{2c^3}-\frac{b^2ex\tan^{-1}(cx)}{3c^5}-$$

$$=\frac{abdx}{2c^3}-\frac{abex}{3c^5}+\frac{b^2dx^2}{12c^2}-\frac{4b^2ex^2}{45c^4}+\frac{b^2ex^4}{60c^2}+\frac{b^2dx\tan^{-1}(cx)}{2c^3}-\frac{b^2ex\tan^{-1}(cx)}{3c^5}-$$

Mathematica [A] time = 0.235419, size = 240, normalized size = 0.89

$$\frac{cx\left(15a^2c^5x^3\left(3d+2ex^2\right)-2ab\left(3c^4\left(5dx^2+2ex^4\right)-5c^2\left(9d+2ex^2\right)+30e\right)+b^2cx\left(3c^2\left(5d+ex^2\right)-16e\right)\right)+2b\tan^{-1}(cx}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + e*x^2)*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] (c*x*(15*a^2*c^5*x^3*(3*d + 2*e*x^2) + b^2*c*x*(-16*e + 3*c^2*(5*d + e*x^2)
) - 2*a*b*(30*e - 5*c^2*(9*d + 2*e*x^2) + 3*c^4*(5*d*x^2 + 2*e*x^4))) + 2*b
*(b*c*x*(-30*e + 5*c^2*(9*d + 2*e*x^2) - 3*c^4*(5*d*x^2 + 2*e*x^4)) + 15*a*
(-3*c^2*d + 2*e + c^6*(3*d*x^4 + 2*e*x^6)))*ArcTan[c*x] + 15*b^2*(-3*c^2*d
+ 2*e + c^6*(3*d*x^4 + 2*e*x^6))*ArcTan[c*x]^2 + 2*b^2*(-30*c^2*d + 23*e)*L
```

```
og[1 + c^2*x^2])/(180*c^6)
```

Maple [A] time = 0.053, size = 329, normalized size = 1.2

$$-\frac{b^2 d \ln \left(c^2 x^2+1\right)}{3 c^4}-\frac{a b e x^5}{15 c}+\frac{a b \arctan \left(c x\right) e}{3 c^6}+\frac{b^2 \arctan \left(c x\right) x^3 e}{9 c^3}-\frac{b^2 \arctan \left(c x\right) e x^5}{15 c}-\frac{a b d x^3}{6 c}+\frac{a b x^3 e}{9 c^3}+\frac{a b \arctan}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x^2+d)*(a+b*arctan(c*x))^2,x)
```

```
[Out] -1/3*b^2*d*ln(c^2*x^2+1)/c^4-1/15/c*a*b*e*x^5+1/3/c^6*a*b*arctan(c*x)*e+1/9
/c^3*b^2*arctan(c*x)*x^3*e-1/15/c*b^2*arctan(c*x)*e*x^5-1/6/c*a*b*d*x^3+1/9
/c^3*a*b*x^3*e+1/3*a*b*arctan(c*x)*e*x^6+1/2*a*b*arctan(c*x)*x^4*d-4/45*b^2
*e*x^2/c^4+1/60*b^2*e*x^4/c^2+23/90*b^2*e*ln(c^2*x^2+1)/c^6-1/6/c*b^2*arcta
n(c*x)*d*x^3+1/6*a^2*e*x^6+1/4*a^2*x^4*d-1/2/c^4*a*b*arctan(c*x)*d-1/4/c^4*
b^2*arctan(c*x)^2*d+1/4*b^2*arctan(c*x)^2*x^4*d+1/6*b^2*arctan(c*x)^2*e*x^6
+1/6/c^6*b^2*arctan(c*x)^2*e-1/3*a*b*e*x/c^5-1/3*b^2*e*x*arctan(c*x)/c^5+1/
2*a*b*d*x/c^3+1/2*b^2*d*x*arctan(c*x)/c^3+1/12*b^2*d*x^2/c^2
```

Maxima [A] time = 1.59891, size = 413, normalized size = 1.52

$$\frac{1}{6} b^2 e x^6 \arctan \left(c x\right)^2+\frac{1}{6} a^2 e x^6+\frac{1}{4} b^2 d x^4 \arctan \left(c x\right)^2+\frac{1}{4} a^2 d x^4+\frac{1}{6} \left(3 x^4 \arctan \left(c x\right)-c \left(\frac{c^2 x^3-3 x}{c^4}+\frac{3 \arctan \left(c x\right)}{c^5}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/6*b^2*e*x^6*arctan(c*x)^2 + 1/6*a^2*e*x^6 + 1/4*b^2*d*x^4*arctan(c*x)^2 +
 1/4*a^2*d*x^4 + 1/6*(3*x^4*arctan(c*x) - c*((c^2*x^3 - 3*x)/c^4 + 3*arctan
(c*x)/c^5))*a*b*d - 1/12*(2*c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5)*arc
tan(c*x) - (c^2*x^2 + 3*arctan(c*x)^2 - 4*log(c^2*x^2 + 1))/c^4)*b^2*d + 1/
45*(15*x^6*arctan(c*x) - c*((3*c^4*x^5 - 5*c^2*x^3 + 15*x)/c^6 - 15*arctan(
c*x)/c^7))*a*b*e - 1/180*(4*c*((3*c^4*x^5 - 5*c^2*x^3 + 15*x)/c^6 - 15*arct
an(c*x)/c^7)*arctan(c*x) - (3*c^4*x^4 - 16*c^2*x^2 - 30*arctan(c*x)^2 + 46*
log(c^2*x^2 + 1))/c^6)*b^2*e
```

Fricas [A] time = 1.8157, size = 652, normalized size = 2.41

$$\frac{30\,a^2c^6ex^6 - 12\,abc^5ex^5 + 3\left(15\,a^2c^6d + b^2c^4e\right)x^4 - 10\left(3\,abc^5d - 2\,abc^3e\right)x^3 + \left(15\,b^2c^4d - 16\,b^2c^2e\right)x^2 + 15\left(2\,b^2c^6ex^6 + 3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/180*(30*a^2*c^6*e*x^6 - 12*a*b*c^5*e*x^5 + 3*(15*a^2*c^6*d + b^2*c^4*e)*x
^4 - 10*(3*a*b*c^5*d - 2*a*b*c^3*e)*x^3 + (15*b^2*c^4*d - 16*b^2*c^2*e)*x^2
 + 15*(2*b^2*c^6*e*x^6 + 3*b^2*c^6*d*x^4 - 3*b^2*c^2*d + 2*b^2*e)*arctan(c*
x)^2 + 30*(3*a*b*c^3*d - 2*a*b*c*e)*x + 2*(30*a*b*c^6*e*x^6 + 45*a*b*c^6*d*
x^4 - 6*b^2*c^5*e*x^5 - 45*a*b*c^2*d - 5*(3*b^2*c^5*d - 2*b^2*c^3*e)*x^3 +
30*a*b*e + 15*(3*b^2*c^3*d - 2*b^2*c*e)*x)*arctan(c*x) - 2*(30*b^2*c^2*d -
23*b^2*e)*log(c^2*x^2 + 1))/c^6
```

Sympy [A] time = 6.14251, size = 398, normalized size = 1.47

$$\begin{cases} \frac{a^2dx^4}{4} + \frac{a^2ex^6}{6} + \frac{abdx^4\operatorname{atan}(cx)}{2} + \frac{abex^6\operatorname{atan}(cx)}{3} - \frac{abdx^3}{6c} - \frac{abex^5}{15c} + \frac{abdx}{2c^3} + \frac{abex^3}{9c^3} - \frac{abd\operatorname{atan}(cx)}{2c^4} - \frac{abex}{3c^5} + \frac{abe\operatorname{atan}(cx)}{3c^6} + \frac{b^2dx^4\operatorname{atan}^2(cx)}{4} \\ a^2\left(\frac{dx^4}{4} + \frac{ex^6}{6}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x**2+d)*(a+b*atan(c*x))**2,x)
```

```
[Out] Piecewise((a**2*d*x**4/4 + a**2*e*x**6/6 + a*b*d*x**4*atan(c*x)/2 + a*b*e*x
**6*atan(c*x)/3 - a*b*d*x**3/(6*c) - a*b*e*x**5/(15*c) + a*b*d*x/(2*c**3) +
 a*b*e*x**3/(9*c**3) - a*b*d*atan(c*x)/(2*c**4) - a*b*e*x/(3*c**5) + a*b*e*
atan(c*x)/(3*c**6) + b**2*d*x**4*atan(c*x)**2/4 + b**2*e*x**6*atan(c*x)**2/
6 - b**2*d*x**3*atan(c*x)/(6*c) - b**2*e*x**5*atan(c*x)/(15*c) + b**2*d*x**
2/(12*c**2) + b**2*e*x**4/(60*c**2) + b**2*d*x*atan(c*x)/(2*c**3) + b**2*e*
x**3*atan(c*x)/(9*c**3) - b**2*d*log(x**2 + c**(-2))/(3*c**4) - b**2*d*atan
(c*x)**2/(4*c**4) - 4*b**2*e*x**2/(45*c**4) - b**2*e*x*atan(c*x)/(3*c**5) +
 23*b**2*e*log(x**2 + c**(-2))/(90*c**6) + b**2*e*atan(c*x)**2/(6*c**6), Ne
(c, 0)), (a**2*(d*x**4/4 + e*x**6/6), True))
```

Giac [A] time = 1.42377, size = 490, normalized size = 1.81

$$\frac{30\,b^2c^6x^6\arctan(cx)^2e+60\,abc^6x^6\arctan(cx)e+45\,b^2c^6dx^4\arctan(cx)^2+30\,a^2c^6x^6e+90\,abc^6dx^4\arctan(cx)-12}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

```
[Out] 1/180*(30*b^2*c^6*x^6*arctan(c*x)^2*e + 60*a*b*c^6*x^6*arctan(c*x)*e + 45*b
^2*c^6*d*x^4*arctan(c*x)^2 + 30*a^2*c^6*x^6*e + 90*a*b*c^6*d*x^4*arctan(c*x
) - 12*b^2*c^5*x^5*arctan(c*x)*e + 45*a^2*c^6*d*x^4 - 12*a*b*c^5*x^5*e - 30
*b^2*c^5*d*x^3*arctan(c*x) - 30*a*b*c^5*d*x^3 + 3*b^2*c^4*x^4*e + 20*b^2*c^
3*x^3*arctan(c*x)*e + 15*b^2*c^4*d*x^2 + 20*a*b*c^3*x^3*e + 90*b^2*c^3*d*x*
arctan(c*x) + 90*a*b*c^3*d*x - 45*b^2*c^2*d*arctan(c*x)^2 - 16*b^2*c^2*x^2*
e - 90*a*b*c^2*d*arctan(c*x) - 60*b^2*c*x*arctan(c*x)*e - 60*b^2*c^2*d*log(
c^2*x^2 + 1) - 60*pi*a*b*e*sgn(c)*sgn(x) - 60*a*b*c*x*e + 30*b^2*arctan(c*x
)^2*e + 60*a*b*arctan(c*x)*e + 46*b^2*e*log(c^2*x^2 + 1))/c^6
```

3.1248 $\int x^2 \left(d+ex^2\right) \left(a+b\tan^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=323

$$-\frac{ib^2d\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{3c^3}+\frac{ib^2e\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{5c^5}-\frac{id\left(a+b\tan^{-1}(cx)\right)^2}{3c^3}-\frac{2bd\log\left(\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{3c^3}+$$

```
[Out] (b^2*d*x)/(3*c^2) - (3*b^2*e*x)/(10*c^4) + (b^2*e*x^3)/(30*c^2) - (b^2*d*Ar
cTan[c*x])/(3*c^3) + (3*b^2*e*ArcTan[c*x])/(10*c^5) - (b*d*x^2*(a + b*ArcTa
n[c*x]))/(3*c) + (b*e*x^2*(a + b*ArcTan[c*x]))/(5*c^3) - (b*e*x^4*(a + b*Ar
cTan[c*x]))/(10*c) - ((I/3)*d*(a + b*ArcTan[c*x])^2)/c^3 + ((I/5)*e*(a + b*
ArcTan[c*x])^2)/c^5 + (d*x^3*(a + b*ArcTan[c*x])^2)/3 + (e*x^5*(a + b*ArcTa
n[c*x])^2)/5 - (2*b*d*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(3*c^3) + (2*
b*e*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(5*c^5) - ((I/3)*b^2*d*PolyLog[
2, 1 - 2/(1 + I*c*x)])/c^3 + ((I/5)*b^2*e*PolyLog[2, 1 - 2/(1 + I*c*x)])/c^
5
```

Rubi [A] time = 0.590095, antiderivative size = 323, normalized size of antiderivative = 1., number of steps used = 25, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {4980, 4852, 4916, 321, 203, 4920, 4854, 2402, 2315, 302}

$$-\frac{ib^2d\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{3c^3}+\frac{ib^2e\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{5c^5}-\frac{id\left(a+b\tan^{-1}(cx)\right)^2}{3c^3}-\frac{2bd\log\left(\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{3c^3}+$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + e*x^2)*(a + b*ArcTan[c*x])^2,x]
```


```
[Out] (b^2*d*x)/(3*c^2) - (3*b^2*e*x)/(10*c^4) + (b^2*e*x^3)/(30*c^2) - (b^2*d*Ar
cTan[c*x])/(3*c^3) + (3*b^2*e*ArcTan[c*x])/(10*c^5) - (b*d*x^2*(a + b*ArcTa
n[c*x]))/(3*c) + (b*e*x^2*(a + b*ArcTan[c*x]))/(5*c^3) - (b*e*x^4*(a + b*Ar
cTan[c*x]))/(10*c) - ((I/3)*d*(a + b*ArcTan[c*x])^2)/c^3 + ((I/5)*e*(a + b*
ArcTan[c*x])^2)/c^5 + (d*x^3*(a + b*ArcTan[c*x])^2)/3 + (e*x^5*(a + b*ArcTa
n[c*x])^2)/5 - (2*b*d*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(3*c^3) + (2*
b*e*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(5*c^5) - ((I/3)*b^2*d*PolyLog[
2, 1 - 2/(1 + I*c*x)])/c^3 + ((I/5)*b^2*e*PolyLog[2, 1 - 2/(1 + I*c*x)])/c^
5
```


Rule 4980

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\int x^2 \left(d+ex^2\right) \left(a+b\tan^{-1}(cx)\right)^2 \, dx = \int \left(dx^2 \left(a+b\tan^{-1}(cx)\right)^2 + ex^4 \left(a+b\tan^{-1}(cx)\right)^2\right) dx$$

$$= d\int x^2 \left(a+b\tan^{-1}(cx)\right)^2 \, dx + e\int x^4 \left(a+b\tan^{-1}(cx)\right)^2 \, dx$$

$$= \frac{1}{3}dx^3 \left(a+b\tan^{-1}(cx)\right)^2 + \frac{1}{5}ex^5 \left(a+b\tan^{-1}(cx)\right)^2 - \frac{1}{3}(2bcd)\int \frac{x^3 \left(a+b\tan^{-1}\right.}{1+c^2x^2}$$

$$= \frac{1}{3}dx^3 \left(a+b\tan^{-1}(cx)\right)^2 + \frac{1}{5}ex^5 \left(a+b\tan^{-1}(cx)\right)^2 - \frac{(2bd)\int x \left(a+b\tan^{-1}(cx)\right.}{3c}$$

$$= -\frac{bdx^2 \left(a+b\tan^{-1}(cx)\right)}{3c} - \frac{bex^4 \left(a+b\tan^{-1}(cx)\right)}{10c} - \frac{id \left(a+b\tan^{-1}(cx)\right)^2}{3c^3} + \frac{1}{3}d$$

$$= \frac{b^2dx}{3c^2} - \frac{bdx^2 \left(a+b\tan^{-1}(cx)\right)}{3c} + \frac{bex^2 \left(a+b\tan^{-1}(cx)\right)}{5c^3} - \frac{bex^4 \left(a+b\tan^{-1}(cx\right.}{10c}$$

$$= \frac{b^2dx}{3c^2} - \frac{3b^2ex}{10c^4} + \frac{b^2ex^3}{30c^2} - \frac{b^2d\tan^{-1}(cx)}{3c^3} - \frac{bdx^2 \left(a+b\tan^{-1}(cx)\right)}{3c} + \frac{bex^2 \left(a+b\right.}{5c}$$

$$= \frac{b^2dx}{3c^2} - \frac{3b^2ex}{10c^4} + \frac{b^2ex^3}{30c^2} - \frac{b^2d\tan^{-1}(cx)}{3c^3} + \frac{3b^2e\tan^{-1}(cx)}{10c^5} - \frac{bdx^2 \left(a+b\tan^{-1}(c\right.}{3c}$$

$$= \frac{b^2dx}{3c^2} - \frac{3b^2ex}{10c^4} + \frac{b^2ex^3}{30c^2} - \frac{b^2d\tan^{-1}(cx)}{3c^3} + \frac{3b^2e\tan^{-1}(cx)}{10c^5} - \frac{bdx^2 \left(a+b\tan^{-1}(c\right.}{3c}$$

Mathematica [A] time = 0.809411, size = 287, normalized size = 0.89

$$\frac{2ib^2\left(5c^2d-3e\right)\text{PolyLog}\left(2,-e^{2i\tan^{-1}(cx)}\right)+10a^2c^5dx^3+6a^2c^5ex^5-b\tan^{-1}(cx)\left(-4ac^5x^3\left(5d+3ex^2\right)+b\left(c^2x^2+1\right)\left(c\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(d + e*x^2)*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] (9*a*b*e + 10*b^2*c^3*d*x - 9*b^2*c*e*x - 10*a*b*c^4*d*x^2 + 6*a*b*c^2*e*x^
2 + 10*a^2*c^5*d*x^3 + b^2*c^3*e*x^3 - 3*a*b*c^4*e*x^4 + 6*a^2*c^5*e*x^5 +
2*b^2*((5*I)*c^2*d - (3*I)*e + c^5*(5*d*x^3 + 3*e*x^5))*ArcTan[c*x]^2 - b*A
rcTan[c*x]*(-4*a*c^5*x^3*(5*d + 3*e*x^2) + b*(1 + c^2*x^2)*(-9*e + c^2*(10*
d + 3*e*x^2)) + 4*b*(5*c^2*d - 3*e)*Log[1 + E^((2*I)*ArcTan[c*x])]) + 10*a*
b*c^2*d*Log[1 + c^2*x^2] - 6*a*b*e*Log[1 + c^2*x^2] + (2*I)*b^2*(5*c^2*d -
3*e)*PolyLog[2, -E^((2*I)*ArcTan[c*x])])/(30*c^5)
```

Maple [B] time = 0.132, size = 667, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^2+d)*(a+b*arctan(c*x))^2,x)
```

```
[Out] -1/12*I/c^3*b^2*ln(c*x-I)^2*d+1/20*I/c^5*b^2*ln(c*x-I)^2*e-1/10*I/c^5*b^2*d
ilog(1/2*I*(c*x-I))*e-1/6*I/c^3*b^2*dilog(-1/2*I*(c*x+I))*d-1/10/c*a*b*e*x^
4+1/5/c^3*b^2*arctan(c*x)*x^2*e+1/3/c^3*b^2*arctan(c*x)*ln(c^2*x^2+1)*d-1/3
/c*b^2*arctan(c*x)*d*x^2-1/5/c^5*a*b*ln(c^2*x^2+1)*e+1/5/c^3*a*b*x^2*e-3/10
*b^2*e*x/c^4+1/30*b^2*e*x^3/c^2+3/10*b^2*e*arctan(c*x)/c^5+1/10*I/c^5*b^2*l
n(c*x+I)*ln(c^2*x^2+1)*e+1/10*I/c^5*b^2*ln(c*x-I)*ln(-1/2*I*(c*x+I))*e+1/6*
I/c^3*b^2*ln(c*x-I)*ln(c^2*x^2+1)*d+1/6*I/c^3*b^2*ln(c*x+I)*ln(1/2*I*(c*x-I
))*d-1/6*I/c^3*b^2*ln(c*x-I)*ln(-1/2*I*(c*x+I))*d-1/10*I/c^5*b^2*ln(c*x-I)*
ln(c^2*x^2+1)*e-1/10*I/c^5*b^2*ln(c*x+I)*ln(1/2*I*(c*x-I))*e-1/6*I/c^3*b^2*
ln(c*x+I)*ln(c^2*x^2+1)*d+1/5*a^2*e*x^5+1/3*a^2*d*x^3-1/3*b^2*d*arctan(c*x)
/c^3+1/5*b^2*arctan(c*x)^2*e*x^5+1/3*b^2*arctan(c*x)^2*d*x^3-1/5/c^5*b^2*ar
ctan(c*x)*ln(c^2*x^2+1)*e+1/3*b^2*d*x/c^2-1/10/c*b^2*arctan(c*x)*e*x^4-1/3/
c*a*b*d*x^2+2/3*a*b*arctan(c*x)*d*x^3+2/5*a*b*arctan(c*x)*e*x^5+1/3/c^3*a*b
*ln(c^2*x^2+1)*d-1/20*I/c^5*b^2*ln(c*x+I)^2*e+1/12*I/c^3*b^2*ln(c*x+I)^2*d+
1/10*I/c^5*b^2*dilog(-1/2*I*(c*x+I))*e+1/6*I/c^3*b^2*dilog(1/2*I*(c*x-I))*d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{5}\,a^2ex^5+\frac{1}{3}\,a^2dx^3+\frac{1}{3}\left(2\,x^3\arctan\left(cx\right)-c\left(\frac{x^2}{c^2}-\frac{\log\left(c^2x^2+1\right)}{c^4}\right)\right)abd+\frac{1}{10}\left(4\,x^5\arctan\left(cx\right)-c\left(\frac{c^2x^4-2\,x^2}{c^4}+\frac{2\log\left(c^2\right.}{c^{\ldots}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/5*a^2*e*x^5 + 1/3*a^2*d*x^3 + 1/3*(2*x^3*arctan(c*x) - c*(x^2/c^2 - log(c
^2*x^2 + 1)/c^4))*a*b*d + 1/10*(4*x^5*arctan(c*x) - c*((c^2*x^4 - 2*x^2)/c^
4 + 2*log(c^2*x^2 + 1)/c^6))*a*b*e + 1/60*(3*b^2*e*x^5 + 5*b^2*d*x^3)*arcta
n(c*x)^2 - 1/240*(3*b^2*e*x^5 + 5*b^2*d*x^3)*log(c^2*x^2 + 1)^2 + integrate
(1/240*(180*(b^2*c^2*e*x^6 + b^2*d*x^2 + (b^2*c^2*d + b^2*e)*x^4)*arctan(c*
x)^2 + 15*(b^2*c^2*e*x^6 + b^2*d*x^2 + (b^2*c^2*d + b^2*e)*x^4)*log(c^2*x^2
```

```
 + 1)^2 - 8*(3*b^2*c*e*x^5 + 5*b^2*c*d*x^3)*arctan(c*x) + 4*(3*b^2*c^2*e*x^
6 + 5*b^2*c^2*d*x^4)*log(c^2*x^2 + 1))/(c^2*x^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(a^2ex^4 + a^2dx^2 + \left(b^2ex^4 + b^2dx^2\right)\arctan(cx)^2 + 2\left(abex^4 + abdx^2\right)\arctan(cx), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(a^2*e*x^4 + a^2*d*x^2 + (b^2*e*x^4 + b^2*d*x^2)*arctan(c*x)^2 + 2*
(a*b*e*x^4 + a*b*d*x^2)*arctan(c*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2(a + b\,\text{atan}(cx))^2\left(d + ex^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d)*(a+b*atan(c*x))**2,x)
```

```
[Out] Integral(x**2*(a + b*atan(c*x))**2*(d + e*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(ex^2 + d\right)(b\arctan(cx) + a)^2x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*(b*arctan(c*x) + a)^2*x^2, x)
```

3.1249 $\int x\left(d+ex^2\right)\left(a+b\tan^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=199

$$\frac{d\left(a+b\tan^{-1}(cx)\right)^2}{2c^2}+\frac{abex}{2c^3}-\frac{e\left(a+b\tan^{-1}(cx)\right)^2}{4c^4}+\frac{1}{2}dx^2\left(a+b\tan^{-1}(cx)\right)^2-\frac{abdx}{c}+\frac{1}{4}ex^4\left(a+b\tan^{-1}(cx)\right)^2-\frac{bex^3\left(\right.}{}$$

```
[Out] -((a*b*d*x)/c) + (a*b*e*x)/(2*c^3) + (b^2*e*x^2)/(12*c^2) - (b^2*d*x*ArcTan
[c*x])/c + (b^2*e*x*ArcTan[c*x])/(2*c^3) - (b*e*x^3*(a + b*ArcTan[c*x]))/(6
*c) + (d*(a + b*ArcTan[c*x])^2)/(2*c^2) - (e*(a + b*ArcTan[c*x])^2)/(4*c^4)
 + (d*x^2*(a + b*ArcTan[c*x])^2)/2 + (e*x^4*(a + b*ArcTan[c*x])^2)/4 + (b^2
*d*Log[1 + c^2*x^2])/(2*c^2) - (b^2*e*Log[1 + c^2*x^2])/(3*c^4)
```

Rubi [A] time = 0.398327, antiderivative size = 199, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {4980, 4852, 4916, 4846, 260, 4884, 266, 43}

$$\frac{d\left(a+b\tan^{-1}(cx)\right)^2}{2c^2}+\frac{abex}{2c^3}-\frac{e\left(a+b\tan^{-1}(cx)\right)^2}{4c^4}+\frac{1}{2}dx^2\left(a+b\tan^{-1}(cx)\right)^2-\frac{abdx}{c}+\frac{1}{4}ex^4\left(a+b\tan^{-1}(cx)\right)^2-\frac{bex^3\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + e*x^2)*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] -((a*b*d*x)/c) + (a*b*e*x)/(2*c^3) + (b^2*e*x^2)/(12*c^2) - (b^2*d*x*ArcTan
[c*x])/c + (b^2*e*x*ArcTan[c*x])/(2*c^3) - (b*e*x^3*(a + b*ArcTan[c*x]))/(6
*c) + (d*(a + b*ArcTan[c*x])^2)/(2*c^2) - (e*(a + b*ArcTan[c*x])^2)/(4*c^4)
 + (d*x^2*(a + b*ArcTan[c*x])^2)/2 + (e*x^4*(a + b*ArcTan[c*x])^2)/4 + (b^2
*d*Log[1 + c^2*x^2])/(2*c^2) - (b^2*e*Log[1 + c^2*x^2])/(3*c^4)
```

Rule 4980

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x\left(d+ex^2\right)\left(a+b\tan^{-1}(cx)\right)^2\,dx=\int\left(dx\left(a+b\tan^{-1}(cx)\right)^2+ex^3\left(a+b\tan^{-1}(cx)\right)^2\right)dx$$
$$=d\int x\left(a+b\tan^{-1}(cx)\right)^2\,dx+e\int x^3\left(a+b\tan^{-1}(cx)\right)^2\,dx$$
$$=\frac{1}{2}dx^2\left(a+b\tan^{-1}(cx)\right)^2+\frac{1}{4}ex^4\left(a+b\tan^{-1}(cx)\right)^2-(bcd)\int\frac{x^2\left(a+b\tan^{-1}(cx)\right)}{1+c^2x^2}$$
$$=\frac{1}{2}dx^2\left(a+b\tan^{-1}(cx)\right)^2+\frac{1}{4}ex^4\left(a+b\tan^{-1}(cx)\right)^2-\frac{(bd)\int\left(a+b\tan^{-1}(cx)\right)dx}{c}$$
$$=-\frac{abdx}{c}-\frac{bex^3\left(a+b\tan^{-1}(cx)\right)}{6c}+\frac{d\left(a+b\tan^{-1}(cx)\right)^2}{2c^2}+\frac{1}{2}dx^2\left(a+b\tan^{-1}(cx)\right)^2$$
$$=-\frac{abdx}{c}+\frac{abex}{2c^3}-\frac{b^2dx\tan^{-1}(cx)}{c}-\frac{bex^3\left(a+b\tan^{-1}(cx)\right)}{6c}+\frac{d\left(a+b\tan^{-1}(cx)\right)^2}{2c^2}$$
$$=-\frac{abdx}{c}+\frac{abex}{2c^3}-\frac{b^2dx\tan^{-1}(cx)}{c}+\frac{b^2ex\tan^{-1}(cx)}{2c^3}-\frac{bex^3\left(a+b\tan^{-1}(cx)\right)}{6c}+\frac{d(}{}$$
$$=-\frac{abdx}{c}+\frac{abex}{2c^3}+\frac{b^2ex^2}{12c^2}-\frac{b^2dx\tan^{-1}(cx)}{c}+\frac{b^2ex\tan^{-1}(cx)}{2c^3}-\frac{bex^3\left(a+b\tan^{-1}(c\right.}{6c}$$

Mathematica [A] time = 0.173684, size = 179, normalized size = 0.9

$$\frac{cx\left(3a^2c^3x\left(2d+ex^2\right)-2abc^2\left(6d+ex^2\right)+6abe+b^2cex\right)+2b\tan^{-1}(cx)\left(3ac^4\left(2dx^2+ex^4\right)+6ac^2d-3ae-bc^3x\left(6d+e\right.\right.}{12c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + e*x^2)*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] (c*x*(6*a*b*e + b^2*c*e*x + 3*a^2*c^3*x*(2*d + e*x^2) - 2*a*b*c^2*(6*d + e*
x^2)) + 2*b*(6*a*c^2*d - 3*a*e + 3*b*c*e*x - b*c^3*x*(6*d + e*x^2) + 3*a*c^
4*(2*d*x^2 + e*x^4))*ArcTan[c*x] + 3*b^2*(2*c^2*d - e + c^4*(2*d*x^2 + e*x^
4))*ArcTan[c*x]^2 + 2*b^2*(3*c^2*d - 2*e)*Log[1 + c^2*x^2])/(12*c^4)
```

Maple [A] time = 0.049, size = 249, normalized size = 1.3

$$\frac{a^2ex^4}{4}+\frac{a^2x^2d}{2}+\frac{b^2\left(\arctan(cx)\right)^2ex^4}{4}+\frac{b^2\left(\arctan(cx)\right)^2dx^2}{2}-\frac{b^2\arctan(cx)\,x^3e}{6c}-\frac{b^2dx\arctan(cx)}{c}+\frac{b^2ex\arctan(c}{2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d)*(a+b*arctan(c*x))^2,x)
```

```
[Out] 1/4*a^2*e*x^4+1/2*a^2*x^2*d+1/4*b^2*arctan(c*x)^2*e*x^4+1/2*b^2*arctan(c*x)
^2*d*x^2-1/6/c*b^2*arctan(c*x)*x^3*e-b^2*d*x*arctan(c*x)/c+1/2*b^2*e*x*arct
an(c*x)/c^3+1/2/c^2*b^2*arctan(c*x)^2*d-1/4/c^4*b^2*arctan(c*x)^2*e+1/12*b^
2*e*x^2/c^2+1/2*b^2*d*ln(c^2*x^2+1)/c^2-1/3*b^2*e*ln(c^2*x^2+1)/c^4+1/2*a*b
*arctan(c*x)*e*x^4+a*b*arctan(c*x)*d*x^2-1/6/c*a*b*x^3*e-a*b*d*x/c+1/2*a*b*
e*x/c^3+1/c^2*a*b*arctan(c*x)*d-1/2/c^4*a*b*arctan(c*x)*e
```

Maxima [A] time = 1.61354, size = 333, normalized size = 1.67

$$\frac{1}{4}b^2ex^4\arctan(cx)^2+\frac{1}{4}a^2ex^4+\frac{1}{2}b^2dx^2\arctan(cx)^2+\frac{1}{2}a^2dx^2+\left(x^2\arctan(cx)-c\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)\right)abd-\frac{1}{2}\Big(2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/4*b^2*e*x^4*arctan(c*x)^2 + 1/4*a^2*e*x^4 + 1/2*b^2*d*x^2*arctan(c*x)^2 +
 1/2*a^2*d*x^2 + (x^2*arctan(c*x) - c*(x/c^2 - arctan(c*x)/c^3))*a*b*d - 1/
2*(2*c*(x/c^2 - arctan(c*x)/c^3)*arctan(c*x) + (arctan(c*x)^2 - log(c^2*x^2
 + 1))/c^2)*b^2*d + 1/6*(3*x^4*arctan(c*x) - c*((c^2*x^3 - 3*x)/c^4 + 3*arc
tan(c*x)/c^5))*a*b*e - 1/12*(2*c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5)*
arctan(c*x) - (c^2*x^2 + 3*arctan(c*x)^2 - 4*log(c^2*x^2 + 1))/c^4)*b^2*e
```

Fricas [A] time = 2.04188, size = 471, normalized size = 2.37

$$\frac{3a^2c^4ex^4-2abc^3ex^3+\left(6a^2c^4d+b^2c^2e\right)x^2+3\left(b^2c^4ex^4+2b^2c^4dx^2+2b^2c^2d-b^2e\right)\arctan(cx)^2-6\left(2abc^3d-abce\right)x}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/12*(3*a^2*c^4*e*x^4 - 2*a*b*c^3*e*x^3 + (6*a^2*c^4*d + b^2*c^2*e)*x^2 + 3
*(b^2*c^4*e*x^4 + 2*b^2*c^4*d*x^2 + 2*b^2*c^2*d - b^2*e)*arctan(c*x)^2 - 6*
(2*a*b*c^3*d - a*b*c*e)*x + 2*(3*a*b*c^4*e*x^4 + 6*a*b*c^4*d*x^2 - b^2*c^3*
e*x^3 + 6*a*b*c^2*d - 3*a*b*e - 3*(2*b^2*c^3*d - b^2*c*e)*x)*arctan(c*x) +
```

```
2*(3*b^2*c^2*d - 2*b^2*e)*log(c^2*x^2 + 1))/c^4
```

Sympy [A] time = 3.61472, size = 296, normalized size = 1.49

$$\begin{cases} \frac{a^2dx^2}{2} + \frac{a^2ex^4}{4} + abdx^2\operatorname{atan}(cx) + \frac{abex^4\operatorname{atan}(cx)}{2} - \frac{abdx}{c} - \frac{abex^3}{6c} + \frac{abd\operatorname{atan}(cx)}{c^2} + \frac{abex}{2c^3} - \frac{abe\operatorname{atan}(cx)}{2c^4} + \frac{b^2dx^2\operatorname{atan}^2(cx)}{2} + \frac{b^2ex^4\operatorname{atan} \\ a^2\left(\frac{dx^2}{2} + \frac{ex^4}{4}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d)*(a+b*atan(c*x))**2,x)
```

```
[Out] Piecewise((a**2*d*x**2/2 + a**2*e*x**4/4 + a*b*d*x**2*atan(c*x) + a*b*e*x**
4*atan(c*x)/2 - a*b*d*x/c - a*b*e*x**3/(6*c) + a*b*d*atan(c*x)/c**2 + a*b*e
*x/(2*c**3) - a*b*e*atan(c*x)/(2*c**4) + b**2*d*x**2*atan(c*x)**2/2 + b**2*
e*x**4*atan(c*x)**2/4 - b**2*d*x*atan(c*x)/c - b**2*e*x**3*atan(c*x)/(6*c)
+ b**2*d*log(x**2 + c**(-2))/(2*c**2) + b**2*d*atan(c*x)**2/(2*c**2) + b**2
*e*x**2/(12*c**2) + b**2*e*x*atan(c*x)/(2*c**3) - b**2*e*log(x**2 + c**(-2)
)/(3*c**4) - b**2*e*atan(c*x)**2/(4*c**4), Ne(c, 0)), (a**2*(d*x**2/2 + e*x
**4/4), True))
```

Giac [A] time = 1.28589, size = 382, normalized size = 1.92

$$\frac{3\,b^2c^4x^4\arctan(cx)^2e + 6\,abc^4x^4\arctan(cx)e + 6\,b^2c^4dx^2\arctan(cx)^2 + 3\,a^2c^4x^4e + 12\,abc^4dx^2\arctan(cx) - 2\,b^2c^3x^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

```
[Out] 1/12*(3*b^2*c^4*x^4*arctan(c*x)^2*e + 6*a*b*c^4*x^4*arctan(c*x)*e + 6*b^2*c
^4*d*x^2*arctan(c*x)^2 + 3*a^2*c^4*x^4*e + 12*a*b*c^4*d*x^2*arctan(c*x) - 2
*b^2*c^3*x^3*arctan(c*x)*e + 6*a^2*c^4*d*x^2 - 2*a*b*c^3*x^3*e - 12*b^2*c^3
*d*x*arctan(c*x) - 12*pi*a*b*c^2*d*sgn(c)*sgn(x) - 12*a*b*c^3*d*x + 6*b^2*c
^2*d*arctan(c*x)^2 + b^2*c^2*x^2*e + 12*a*b*c^2*d*arctan(c*x) + 6*b^2*c*x*a
rctan(c*x)*e + 6*b^2*c^2*d*log(c^2*x^2 + 1) + 6*a*b*c*x*e - 3*b^2*arctan(c*
x)^2*e - 6*a*b*arctan(c*x)*e - 4*b^2*e*log(c^2*x^2 + 1))/c^4
```

3.1250 $\int \left(d+ex^2\right)\left(a+b\tan^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=231

$$-\frac{ib^2e\mathrm{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{3c^3}+\frac{ib^2d\mathrm{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c}-\frac{ie\left(a+b\tan^{-1}(cx)\right)^2}{3c^3}-\frac{2be\log\left(\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{3c^3}+$$

```
[Out] (b^2*e*x)/(3*c^2) - (b^2*e*ArcTan[c*x])/(3*c^3) - (b*e*x^2*(a + b*ArcTan[c*
x]))/(3*c) + (I*d*(a + b*ArcTan[c*x])^2)/c - ((I/3)*e*(a + b*ArcTan[c*x])^2
)/c^3 + d*x*(a + b*ArcTan[c*x])^2 + (e*x^3*(a + b*ArcTan[c*x])^2)/3 + (2*b*
d*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/c - (2*b*e*(a + b*ArcTan[c*x])*Lo
g[2/(1 + I*c*x)])/(3*c^3) + (I*b^2*d*PolyLog[2, 1 - 2/(1 + I*c*x)])/c - ((I
/3)*b^2*e*PolyLog[2, 1 - 2/(1 + I*c*x)])/c^3
```

Rubi [A] time = 0.35754, antiderivative size = 231, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 10, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {4914, 4846, 4920, 4854, 2402, 2315, 4852, 4916, 321, 203}

$$-\frac{ib^2e\mathrm{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{3c^3}+\frac{ib^2d\mathrm{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c}-\frac{ie\left(a+b\tan^{-1}(cx)\right)^2}{3c^3}-\frac{2be\log\left(\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{3c^3}+$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)*(a + b*ArcTan[c*x])^2,x]
```


```
[Out] (b^2*e*x)/(3*c^2) - (b^2*e*ArcTan[c*x])/(3*c^3) - (b*e*x^2*(a + b*ArcTan[c*
x]))/(3*c) + (I*d*(a + b*ArcTan[c*x])^2)/c - ((I/3)*e*(a + b*ArcTan[c*x])^2
)/c^3 + d*x*(a + b*ArcTan[c*x])^2 + (e*x^3*(a + b*ArcTan[c*x])^2)/3 + (2*b*
d*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/c - (2*b*e*(a + b*ArcTan[c*x])*Lo
g[2/(1 + I*c*x)])/(3*c^3) + (I*b^2*d*PolyLog[2, 1 - 2/(1 + I*c*x)])/c - ((I
/3)*b^2*e*PolyLog[2, 1 - 2/(1 + I*c*x)])/c^3
```


Rule 4914

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (d + e*x^2)^q, x], x
] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[q] && IGtQ[p, 0]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \left(d+ex^2\right)\left(a+b\tan^{-1}(cx)\right)^2\,dx = \int \left(d\left(a+b\tan^{-1}(cx)\right)^2+ex^2\left(a+b\tan^{-1}(cx)\right)^2\right)dx$$

$$= d\int \left(a+b\tan^{-1}(cx)\right)^2\,dx + e\int x^2\left(a+b\tan^{-1}(cx)\right)^2\,dx$$

$$= dx\left(a+b\tan^{-1}(cx)\right)^2+\frac{1}{3}ex^3\left(a+b\tan^{-1}(cx)\right)^2-(2bcd)\int \frac{x\left(a+b\tan^{-1}(cx)\right)}{1+c^2x^2}\,dx$$

$$= \frac{id\left(a+b\tan^{-1}(cx)\right)^2}{c}+dx\left(a+b\tan^{-1}(cx)\right)^2+\frac{1}{3}ex^3\left(a+b\tan^{-1}(cx)\right)^2+(2bd)\int$$

$$= -\frac{bex^2\left(a+b\tan^{-1}(cx)\right)}{3c}+\frac{id\left(a+b\tan^{-1}(cx)\right)^2}{c}-\frac{ie\left(a+b\tan^{-1}(cx)\right)^2}{3c^3}+dx\left(a+\right.$$

$$= \frac{b^2ex}{3c^2}-\frac{bex^2\left(a+b\tan^{-1}(cx)\right)}{3c}+\frac{id\left(a+b\tan^{-1}(cx)\right)^2}{c}-\frac{ie\left(a+b\tan^{-1}(cx)\right)^2}{3c^3}+d$$

$$= \frac{b^2ex}{3c^2}-\frac{b^2e\tan^{-1}(cx)}{3c^3}-\frac{bex^2\left(a+b\tan^{-1}(cx)\right)}{3c}+\frac{id\left(a+b\tan^{-1}(cx)\right)^2}{c}-\frac{ie\left(a+b\right.}{}$$

$$= \frac{b^2ex}{3c^2}-\frac{b^2e\tan^{-1}(cx)}{3c^3}-\frac{bex^2\left(a+b\tan^{-1}(cx)\right)}{3c}+\frac{id\left(a+b\tan^{-1}(cx)\right)^2}{c}-\frac{ie\left(a+b\right.}{}$$

Mathematica [A] time = 0.433747, size = 208, normalized size = 0.9

$$\frac{-ib^2\left(3c^2d-e\right)\text{PolyLog}\left(2,-e^{2i\tan^{-1}(cx)}\right)+3a^2c^3dx+a^2c^3ex^3-b\tan^{-1}(cx)\left(-2ac^3x\left(3d+ex^2\right)+2b\left(e-3c^2d\right)\log\left(1-\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x^2)*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] (3*a^2*c^3*d*x + b^2*c*e*x - a*b*c^2*e*x^2 + a^2*c^3*e*x^3 + b^2*((-3*I)*c^
2*d + I*e + c^3*(3*d*x + e*x^3))*ArcTan[c*x]^2 - b*ArcTan[c*x]*(-2*a*c^3*x*
(3*d + e*x^2) + b*(e + c^2*e*x^2) + 2*b*(-3*c^2*d + e)*Log[1 + E^((2*I)*Arc
Tan[c*x])]) - 3*a*b*c^2*d*Log[1 + c^2*x^2] + a*b*e*Log[1 + c^2*x^2] - I*b^2
*(3*c^2*d - e)*PolyLog[2, -E^((2*I)*ArcTan[c*x])])/(3*c^3)
```

Maple [B] time = 0.128, size = 570, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(a+b*arctan(c*x))^2,x)
```

```
[Out] -1/3/c*a*b*x^2*e-1/c*a*b*ln(c^2*x^2+1)*d+1/3/c^3*b^2*arctan(c*x)*ln(c^2*x^2
+1)*e+1/3/c^3*a*b*ln(c^2*x^2+1)*e-1/c*b^2*arctan(c*x)*ln(c^2*x^2+1)*d-1/3/c
*b^2*arctan(c*x)*x^2*e+1/6*I/c^3*b^2*dilog(1/2*I*(c*x-I))*e-1/4*I/c*b^2*ln(
c*x+I)^2*d-1/6*I/c^3*b^2*dilog(-1/2*I*(c*x+I))*e-1/2*I/c*b^2*dilog(1/2*I*(c
*x-I))*d-1/12*I/c^3*b^2*ln(c*x-I)^2*e+1/12*I/c^3*b^2*ln(c*x+I)^2*e-1/2*I/c*
b^2*ln(c*x+I)*ln(1/2*I*(c*x-I))*d+1/2*I/c*b^2*ln(c*x+I)*ln(c^2*x^2+1)*d+1/2
*I/c*b^2*ln(c*x-I)*ln(-1/2*I*(c*x+I))*d-1/2*I/c*b^2*ln(c*x-I)*ln(c^2*x^2+1)
*d+1/6*I/c^3*b^2*ln(c*x+I)*ln(1/2*I*(c*x-I))*e-1/6*I/c^3*b^2*ln(c*x+I)*ln(c
^2*x^2+1)*e-1/6*I/c^3*b^2*ln(c*x-I)*ln(-1/2*I*(c*x+I))*e+1/6*I/c^3*b^2*ln(c
*x-I)*ln(c^2*x^2+1)*e+b^2*arctan(c*x)^2*d*x+1/3*b^2*arctan(c*x)^2*x^3*e+1/4
*I/c*b^2*ln(c*x-I)^2*d+1/3*b^2*e*x/c^2-1/3*b^2*e*arctan(c*x)/c^3+1/3*a^2*x^
3*e+a^2*d*x+2/3*a*b*arctan(c*x)*x^3*e+1/2*I/c*b^2*dilog(-1/2*I*(c*x+I))*d+2
*a*b*arctan(c*x)*d*x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{3}\,a^2ex^3 + 36\,b^2c^2e\int \frac{x^4\arctan\left(cx\right)^2}{48\left(c^2x^2+1\right)}\,dx + 3\,b^2c^2e\int \frac{x^4\log\left(c^2x^2+1\right)^2}{48\left(c^2x^2+1\right)}\,dx + 4\,b^2c^2e\int \frac{x^4\log\left(c^2x^2+1\right)}{48\left(c^2x^2+1\right)}\,dx + 36\,b^2c^2d\int$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*e*x^3 + 36*b^2*c^2*e*integrate(1/48*x^4*arctan(c*x)^2/(c^2*x^2 + 1)
, x) + 3*b^2*c^2*e*integrate(1/48*x^4*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x)
+ 4*b^2*c^2*e*integrate(1/48*x^4*log(c^2*x^2 + 1)/(c^2*x^2 + 1), x) + 36*b^
2*c^2*d*integrate(1/48*x^2*arctan(c*x)^2/(c^2*x^2 + 1), x) + 3*b^2*c^2*d*in
tegrate(1/48*x^2*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) + 12*b^2*c^2*d*integr
ate(1/48*x^2*log(c^2*x^2 + 1)/(c^2*x^2 + 1), x) + 1/4*b^2*d*arctan(c*x)^3/c
 - 8*b^2*c*e*integrate(1/48*x^3*arctan(c*x)/(c^2*x^2 + 1), x) - 24*b^2*c*d*
integrate(1/48*x*arctan(c*x)/(c^2*x^2 + 1), x) + 1/3*(2*x^3*arctan(c*x) - c
*(x^2/c^2 - log(c^2*x^2 + 1)/c^4))*a*b*e + a^2*d*x + 36*b^2*e*integrate(1/4
8*x^2*arctan(c*x)^2/(c^2*x^2 + 1), x) + 3*b^2*e*integrate(1/48*x^2*log(c^2*
x^2 + 1)^2/(c^2*x^2 + 1), x) + 3*b^2*d*integrate(1/48*log(c^2*x^2 + 1)^2/(c
^2*x^2 + 1), x) + (2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*a*b*d/c + 1/12*(b^
2*e*x^3 + 3*b^2*d*x)*arctan(c*x)^2 - 1/48*(b^2*e*x^3 + 3*b^2*d*x)*log(c^2*x
^2 + 1)^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(a^2ex^2 + a^2d + \left(b^2ex^2 + b^2d\right)\arctan\left(cx\right)^2 + 2\left(abex^2 + abd\right)\arctan\left(cx\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(a^2*e*x^2 + a^2*d + (b^2*e*x^2 + b^2*d)*arctan(c*x)^2 + 2*(a*b*e*x
^2 + a*b*d)*arctan(c*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\,\mathrm{atan}\left(cx\right)\right)^2\left(d + ex^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(a+b*atan(c*x))**2,x)
```

```
[Out] Integral((a + b*atan(c*x))**2*(d + e*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(ex^2+d\right)(b\arctan(cx)+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*(b*arctan(c*x) + a)^2, x)
```

3.1251 $\int \frac{(d+ex^2)(a+b\tan^{-1}(cx))^2}{x}\,dx$

Optimal. Leaf size=217

$$-ibd\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)+ibd\text{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)-\frac{1}{2}b^2d\text{PolyLog}\Big(3,$$

```
[Out] -((a*b*e*x)/c) - (b^2*e*x*ArcTan[c*x])/c + (e*(a + b*ArcTan[c*x])^2)/(2*c^2
) + (e*x^2*(a + b*ArcTan[c*x])^2)/2 + 2*d*(a + b*ArcTan[c*x])^2*ArcTanh[1 -
 2/(1 + I*c*x)] + (b^2*e*Log[1 + c^2*x^2])/(2*c^2) - I*b*d*(a + b*ArcTan[c*
x])*PolyLog[2, 1 - 2/(1 + I*c*x)] + I*b*d*(a + b*ArcTan[c*x])*PolyLog[2, -1
 + 2/(1 + I*c*x)] - (b^2*d*PolyLog[3, 1 - 2/(1 + I*c*x)])/2 + (b^2*d*PolyLo
g[3, -1 + 2/(1 + I*c*x)])/2
```

Rubi [A] time = 0.442359, antiderivative size = 217, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {4980, 4850, 4988, 4884, 4994, 6610, 4852, 4916, 4846, 260}

$$-ibd\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)+ibd\text{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)-\frac{1}{2}b^2d\text{PolyLog}\Big(3,$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)*(a + b*ArcTan[c*x])^2)/x,x]
```


```
[Out] -((a*b*e*x)/c) - (b^2*e*x*ArcTan[c*x])/c + (e*(a + b*ArcTan[c*x])^2)/(2*c^2
) + (e*x^2*(a + b*ArcTan[c*x])^2)/2 + 2*d*(a + b*ArcTan[c*x])^2*ArcTanh[1 -
 2/(1 + I*c*x)] + (b^2*e*Log[1 + c^2*x^2])/(2*c^2) - I*b*d*(a + b*ArcTan[c*
x])*PolyLog[2, 1 - 2/(1 + I*c*x)] + I*b*d*(a + b*ArcTan[c*x])*PolyLog[2, -1
 + 2/(1 + I*c*x)] - (b^2*d*PolyLog[3, 1 - 2/(1 + I*c*x)])/2 + (b^2*d*PolyLo
g[3, -1 + 2/(1 + I*c*x)])/2
```


Rule 4980

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

Rule 4850

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTan[c*x])^(p - 1)*ArcTanh[1 - 2/(1 + I*c*x)])/(1 + c^2*x^2), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4988

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTan[c*x])^p)/(d + e
*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTan[c*x])^p)/(d + e*x^2
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
```

```
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)\left(a+b\tan^{-1}(cx)\right)^2}{x}\,dx = \int \left(\frac{d\left(a+b\tan^{-1}(cx)\right)^2}{x}+ex\left(a+b\tan^{-1}(cx)\right)^2\right)dx$$

$$= d\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x}\,dx + e\int x\left(a+b\tan^{-1}(cx)\right)^2\,dx$$

$$= \frac{1}{2}ex^2\left(a+b\tan^{-1}(cx)\right)^2 + 2d\left(a+b\tan^{-1}(cx)\right)^2\tanh^{-1}\left(1-\frac{2}{1+icx}\right) - (4bcd)\int$$

$$= \frac{1}{2}ex^2\left(a+b\tan^{-1}(cx)\right)^2 + 2d\left(a+b\tan^{-1}(cx)\right)^2\tanh^{-1}\left(1-\frac{2}{1+icx}\right) + (2bcd)\int$$

$$= -\frac{abex}{c} + \frac{e\left(a+b\tan^{-1}(cx)\right)^2}{2c^2} + \frac{1}{2}ex^2\left(a+b\tan^{-1}(cx)\right)^2 + 2d\left(a+b\tan^{-1}(cx)\right)^2\,\mathrm{t}$$

$$= -\frac{abex}{c} - \frac{b^2ex\tan^{-1}(cx)}{c} + \frac{e\left(a+b\tan^{-1}(cx)\right)^2}{2c^2} + \frac{1}{2}ex^2\left(a+b\tan^{-1}(cx)\right)^2 + 2d\,(a$$

$$= -\frac{abex}{c} - \frac{b^2ex\tan^{-1}(cx)}{c} + \frac{e\left(a+b\tan^{-1}(cx)\right)^2}{2c^2} + \frac{1}{2}ex^2\left(a+b\tan^{-1}(cx)\right)^2 + 2d\,(a$$

Mathematica [A] time = 0.364247, size = 263, normalized size = 1.21

$$iabd(\mathrm{PolyLog}(2,-icx)-\mathrm{PolyLog}(2,icx))+b^2d\left(i\tan^{-1}(cx)\mathrm{PolyLog}\left(2,e^{-2i\tan^{-1}(cx)}\right)+i\tan^{-1}(cx)\mathrm{PolyLog}\left(2,-e^{2i\,\mathrm{ta}}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + e*x^2)*(a + b*ArcTan[c*x])^2)/x,x]
```

```
[Out] (a^2*e*x^2)/2 + (a*b*e*(-(c*x) + (1 + c^2*x^2)*ArcTan[c*x]))/c^2 + a^2*d*Lo
g[x] + (b^2*e*(-2*c*x*ArcTan[c*x] + (1 + c^2*x^2)*ArcTan[c*x]^2 + Log[1 + c
^2*x^2]))/(2*c^2) + I*a*b*d*(PolyLog[2, (-I)*c*x] - PolyLog[2, I*c*x]) + b^
2*d*((-I/24)*Pi^3 + ((2*I)/3)*ArcTan[c*x]^3 + ArcTan[c*x]^2*Log[1 - E^((-2*
I)*ArcTan[c*x])] - ArcTan[c*x]^2*Log[1 + E^((2*I)*ArcTan[c*x])] + I*ArcTan[
c*x]*PolyLog[2, E^((-2*I)*ArcTan[c*x])] + I*ArcTan[c*x]*PolyLog[2, -E^((2*I
)*ArcTan[c*x])] + PolyLog[3, E^((-2*I)*ArcTan[c*x])]/2 - PolyLog[3, -E^((2*
I)*ArcTan[c*x])]/2)
```

Maple [C] time = 1.72, size = 1284, normalized size = 5.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(a+b*arctan(c*x))^2/x,x)
```

```
[Out] 1/2*I*b^2*d*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I/((1+I*c*x)^2/(c^2
*x^2+1)+1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))
*arctan(c*x)^2-a*b*e*x/c-b^2*e*x*arctan(c*x)/c+a^2*d*ln(c*x)+1/2*a^2*x^2*e+
2*b^2*d*polylog(3,(1+I*c*x)/(c^2*x^2+1)^(1/2))+2*b^2*d*polylog(3,-(1+I*c*x)
/(c^2*x^2+1)^(1/2))-1/2*b^2*d*polylog(3,-(1+I*c*x)^2/(c^2*x^2+1))+1/2*b^2/c
^2*arctan(c*x)^2*e+b^2*arctan(c*x)^2*d*ln(c*x)-b^2/c^2*e*ln((1+I*c*x)^2/(c^
2*x^2+1)+1)-b^2*d*arctan(c*x)^2*ln((1+I*c*x)^2/(c^2*x^2+1)-1)+b^2*d*arctan(
c*x)^2*ln(1-(1+I*c*x)/(c^2*x^2+1)^(1/2))+b^2*d*arctan(c*x)^2*ln(1+(1+I*c*x)
/(c^2*x^2+1)^(1/2))+1/2*b^2*arctan(c*x)^2*x^2*e+a*b/c^2*arctan(c*x)*e-1/2*I
*b^2*d*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+
1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2+a*b*arctan(c*x)*x^2*e+2*
a*b*arctan(c*x)*d*ln(c*x)+I*a*b*d*dilog(1+I*c*x)-I*a*b*d*dilog(1-I*c*x)+I*b
^2*d*arctan(c*x)*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))+I*b^2/c^2*arctan(c*x)*
e-2*I*b^2*d*arctan(c*x)*polylog(2,(1+I*c*x)/(c^2*x^2+1)^(1/2))+1/2*I*b^2*d*
Pi*arctan(c*x)^2-2*I*b^2*d*arctan(c*x)*polylog(2,-(1+I*c*x)/(c^2*x^2+1)^(1/
2))+1/2*I*b^2*d*Pi*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1
)+1))^3*arctan(c*x)^2-1/2*I*b^2*d*Pi*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I
*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2+1/2*I*b^2*d*Pi*csgn(I*((1+I*c*x)^2/
(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*arctan(c*x)^2+I*a*b*d*ln(c*x)
*ln(1+I*c*x)-I*a*b*d*ln(c*x)*ln(1-I*c*x)+1/2*I*b^2*d*Pi*csgn(I*((1+I*c*x)^2
/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-
```

```
1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)^2-1/2*I*b^2*d*Pi*csgn(I/((1+I*c
*x)^2/(c^2*x^2+1)+1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*
x^2+1)+1))^2*arctan(c*x)^2-1/2*I*b^2*d*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1
)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^
2/(c^2*x^2+1)+1))^2*arctan(c*x)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{8}b^2ex^2\arctan(cx)^2-\frac{1}{32}b^2ex^2\log\left(c^2x^2+1\right)^2+12\,b^2c^2e\int\frac{x^4\arctan(cx)^2}{16\left(c^2x^3+x\right)}\,dx+b^2c^2e\int\frac{x^4\log\left(c^2x^2+1\right)^2}{16\left(c^2x^3+x\right)}\,dx+32\,a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arctan(c*x))^2/x,x, algorithm="maxima")
```

```
[Out] 1/8*b^2*e*x^2*arctan(c*x)^2 - 1/32*b^2*e*x^2*log(c^2*x^2 + 1)^2 + 12*b^2*c^
2*e*integrate(1/16*x^4*arctan(c*x)^2/(c^2*x^3 + x), x) + b^2*c^2*e*integrat
e(1/16*x^4*log(c^2*x^2 + 1)^2/(c^2*x^3 + x), x) + 32*a*b*c^2*e*integrate(1/
16*x^4*arctan(c*x)/(c^2*x^3 + x), x) + 2*b^2*c^2*e*integrate(1/16*x^4*log(c
^2*x^2 + 1)/(c^2*x^3 + x), x) + 12*b^2*c^2*d*integrate(1/16*x^2*arctan(c*x)
^2/(c^2*x^3 + x), x) + 32*a*b*c^2*d*integrate(1/16*x^2*arctan(c*x)/(c^2*x^3
 + x), x) + 1/96*b^2*d*log(c^2*x^2 + 1)^3 + 1/2*a^2*e*x^2 - 4*b^2*c*e*integ
rate(1/16*x^3*arctan(c*x)/(c^2*x^3 + x), x) + 12*b^2*e*integrate(1/16*x^2*a
rctan(c*x)^2/(c^2*x^3 + x), x) + 32*a*b*e*integrate(1/16*x^2*arctan(c*x)/(c
^2*x^3 + x), x) + 12*b^2*d*integrate(1/16*arctan(c*x)^2/(c^2*x^3 + x), x) +
 b^2*d*integrate(1/16*log(c^2*x^2 + 1)^2/(c^2*x^3 + x), x) + 32*a*b*d*integ
rate(1/16*arctan(c*x)/(c^2*x^3 + x), x) + 1/96*b^2*e*log(c^2*x^2 + 1)^3/c^2
 + a^2*d*log(x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{a^2ex^2+a^2d+\left(b^2ex^2+b^2d\right)\arctan(cx)^2+2\left(abex^2+abd\right)\arctan(cx)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arctan(c*x))^2/x,x, algorithm="fricas")
```

```
[Out] integral((a^2*e*x^2 + a^2*d + (b^2*e*x^2 + b^2*d)*arctan(c*x)^2 + 2*(a*b*e*
x^2 + a*b*d)*arctan(c*x))/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b\,\mathrm{atan}\,(cx))^2 \left(d + ex^2\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(a+b*atan(c*x))**2/x,x)
```

```
[Out] Integral((a + b*atan(c*x))**2*(d + e*x**2)/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2 + d\right)(b\arctan(cx) + a)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arctan(c*x))^2/x,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*(b*arctan(c*x) + a)^2/x, x)
```

3.1252 $\int \frac{(d+ex^2)(a+b\tan^{-1}(cx))^2}{x^2}\,dx$

Optimal. Leaf size=172

$$-ib^2cd\text{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)+\frac{ib^2e\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c}-icd\left(a+b\tan^{-1}(cx)\right)^2-\frac{d\left(a+b\tan^{-1}(cx)\right)^2}{x}+2bcd\,\mathrm{l}$$

```
[Out] (-I)*c*d*(a + b*ArcTan[c*x])^2 + (I*e*(a + b*ArcTan[c*x])^2)/c - (d*(a + b*
ArcTan[c*x])^2)/x + e*x*(a + b*ArcTan[c*x])^2 + (2*b*e*(a + b*ArcTan[c*x])*
Log[2/(1 + I*c*x)])/c + 2*b*c*d*(a + b*ArcTan[c*x])*Log[2 - 2/(1 - I*c*x)]
- I*b^2*c*d*PolyLog[2, -1 + 2/(1 - I*c*x)] + (I*b^2*e*PolyLog[2, 1 - 2/(1 +
 I*c*x)])/c
```

Rubi [A] time = 0.326766, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {4980, 4846, 4920, 4854, 2402, 2315, 4852, 4924, 4868, 2447}

$$-ib^2cd\text{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)+\frac{ib^2e\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c}-icd\left(a+b\tan^{-1}(cx)\right)^2-\frac{d\left(a+b\tan^{-1}(cx)\right)^2}{x}+2bcd\,\mathrm{l}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)*(a + b*ArcTan[c*x])^2)/x^2,x]
```


```
[Out] (-I)*c*d*(a + b*ArcTan[c*x])^2 + (I*e*(a + b*ArcTan[c*x])^2)/c - (d*(a + b*
ArcTan[c*x])^2)/x + e*x*(a + b*ArcTan[c*x])^2 + (2*b*e*(a + b*ArcTan[c*x])*
Log[2/(1 + I*c*x)])/c + 2*b*c*d*(a + b*ArcTan[c*x])*Log[2 - 2/(1 - I*c*x)]
- I*b^2*c*d*PolyLog[2, -1 + 2/(1 - I*c*x)] + (I*b^2*e*PolyLog[2, 1 - 2/(1 +
 I*c*x)])/c
```


Rule 4980

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)\left(a+b\tan^{-1}(cx)\right)^2}{x^2}\,dx = \int \left(e\left(a+b\tan^{-1}(cx)\right)^2+\frac{d\left(a+b\tan^{-1}(cx)\right)^2}{x^2}\right)dx$$

$$= d\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^2}\,dx + e\int \left(a+b\tan^{-1}(cx)\right)^2\,dx$$

$$= -\frac{d\left(a+b\tan^{-1}(cx)\right)^2}{x} + ex\left(a+b\tan^{-1}(cx)\right)^2 + (2bcd)\int \frac{a+b\tan^{-1}(cx)}{x\left(1+c^2x^2\right)}\,dx - (2$$

$$= -icd\left(a+b\tan^{-1}(cx)\right)^2 + \frac{ie\left(a+b\tan^{-1}(cx)\right)^2}{c} - \frac{d\left(a+b\tan^{-1}(cx)\right)^2}{x} + ex\left(a+b\right.$$

$$= -icd\left(a+b\tan^{-1}(cx)\right)^2 + \frac{ie\left(a+b\tan^{-1}(cx)\right)^2}{c} - \frac{d\left(a+b\tan^{-1}(cx)\right)^2}{x} + ex\left(a+b\right.$$

$$= -icd\left(a+b\tan^{-1}(cx)\right)^2 + \frac{ie\left(a+b\tan^{-1}(cx)\right)^2}{c} - \frac{d\left(a+b\tan^{-1}(cx)\right)^2}{x} + ex\left(a+b\right.$$

$$= -icd\left(a+b\tan^{-1}(cx)\right)^2 + \frac{ie\left(a+b\tan^{-1}(cx)\right)^2}{c} - \frac{d\left(a+b\tan^{-1}(cx)\right)^2}{x} + ex\left(a+b\right.$$

Mathematica [A] time = 0.272527, size = 204, normalized size = 1.19

$$-b^2cd\left(icx\left(\tan^{-1}(cx)^2+\text{PolyLog}\left(2,e^{2i\tan^{-1}(cx)}\right)\right)+\tan^{-1}(cx)^2-2cx\tan^{-1}(cx)\log\left(1-e^{2i\tan^{-1}(cx)}\right)\right)+b^2ex\left(\tan^{-1}(c\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + e*x^2)*(a + b*ArcTan[c*x])^2)/x^2,x]
```

```
[Out] (-(a^2*c*d) + a^2*c*e*x^2 + a*b*c*d*(-2*ArcTan[c*x] + c*x*(2*Log[c*x] - Log
[1 + c^2*x^2])) + a*b*e*x*(2*c*x*ArcTan[c*x] - Log[1 + c^2*x^2]) + b^2*e*x*
(ArcTan[c*x]*((-I + c*x)*ArcTan[c*x] + 2*Log[1 + E^((2*I)*ArcTan[c*x])]) -
I*PolyLog[2, -E^((2*I)*ArcTan[c*x])]) - b^2*c*d*(ArcTan[c*x]^2 - 2*c*x*ArcT
an[c*x]*Log[1 - E^((2*I)*ArcTan[c*x])] + I*c*x*(ArcTan[c*x]^2 + PolyLog[2,
E^((2*I)*ArcTan[c*x])])))/(c*x)
```

Maple [B] time = 0.132, size = 597, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(a+b*arctan(c*x))^2/x^2,x)
```

```
[Out] 1/2*I*c*b^2*dilog(-1/2*I*(c*x+I))*d-1/2*I*b^2/c*ln(c*x-I)*ln(c^2*x^2+1)*e-1
/2*I*b^2/c*ln(c*x+I)*ln(1/2*I*(c*x-I))*e+1/2*I*c*b^2*ln(c*x-I)*ln(-1/2*I*(c
*x+I))*d-1/2*I*c*b^2*ln(c*x-I)*ln(c^2*x^2+1)*d+I*c*b^2*d*ln(c*x)*ln(1+I*c*x
)+1/2*I*b^2/c*ln(c*x-I)*ln(-1/2*I*(c*x+I))*e+a^2*e*x-a^2*d/x+1/2*I*b^2/c*ln
(c*x+I)*ln(c^2*x^2+1)*e-I*c*b^2*d*ln(c*x)*ln(1-I*c*x)+1/2*I*c*b^2*ln(c*x+I)
*ln(c^2*x^2+1)*d-1/2*I*c*b^2*ln(c*x+I)*ln(1/2*I*(c*x-I))*d-1/4*I*c*b^2*ln(c
*x+I)^2*d+1/4*I*c*b^2*ln(c*x-I)^2*d-I*c*b^2*d*dilog(1-I*c*x)-b^2/c*arctan(c
*x)*ln(c^2*x^2+1)*e-a*b/c*ln(c^2*x^2+1)*e+I*c*b^2*d*dilog(1+I*c*x)-2*a*b*ar
ctan(c*x)*d/x+2*a*b*arctan(c*x)*e*x-c*a*b*ln(c^2*x^2+1)*d+2*c*b^2*arctan(c*
x)*d*ln(c*x)-c*b^2*arctan(c*x)*ln(c^2*x^2+1)*d+2*c*a*b*d*ln(c*x)+1/4*I*b^2/
c*ln(c*x-I)^2*e+1/2*I*b^2/c*dilog(-1/2*I*(c*x+I))*e-1/4*I*b^2/c*ln(c*x+I)^2
*e-1/2*I*b^2/c*dilog(1/2*I*(c*x-I))*e-1/2*I*c*b^2*dilog(1/2*I*(c*x-I))*d-b^
2*arctan(c*x)^2*d/x+b^2*arctan(c*x)^2*e*x
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arctan(c*x))^2/x^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{a^2ex^2 + a^2d + \left(b^2ex^2 + b^2d\right)\arctan\left(cx\right)^2 + 2\left(abex^2 + abd\right)\arctan\left(cx\right)}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arctan(c*x))^2/x^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*e*x^2 + a^2*d + (b^2*e*x^2 + b^2*d)*arctan(c*x)^2 + 2*(a*b*e*
x^2 + a*b*d)*arctan(c*x))/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + b\operatorname{atan}\left(cx\right)\right)^2\left(d + ex^2\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(a+b*atan(c*x))**2/x**2,x)
```

```
[Out] Integral((a + b*atan(c*x))**2*(d + e*x**2)/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2 + d\right)\left(b\arctan\left(cx\right) + a\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arctan(c*x))^2/x^2,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*(b*arctan(c*x) + a)^2/x^2, x)
```

3.1253 $\int \frac{(d+ex^2)(a+b\tan^{-1}(cx))^2}{x^3}\,dx$

Optimal. Leaf size=220

$$-ibe\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)+ibe\text{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)-\frac{1}{2}b^2e\text{PolyLog}\left(3,1-\right.$$

```
[Out] -((b*c*d*(a + b*ArcTan[c*x]))/x) - (c^2*d*(a + b*ArcTan[c*x])^2)/2 - (d*(a
+ b*ArcTan[c*x])^2)/(2*x^2) + 2*e*(a + b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1 +
I*c*x)] + b^2*c^2*d*Log[x] - (b^2*c^2*d*Log[1 + c^2*x^2])/2 - I*b*e*(a + b*
ArcTan[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)] + I*b*e*(a + b*ArcTan[c*x])*Poly
Log[2, -1 + 2/(1 + I*c*x)] - (b^2*e*PolyLog[3, 1 - 2/(1 + I*c*x)])/2 + (b^2
*e*PolyLog[3, -1 + 2/(1 + I*c*x)])/2
```

Rubi [A] time = 0.460768, antiderivative size = 220, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 12, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {4980, 4852, 4918, 266, 36, 29, 31, 4884, 4850, 4988, 4994, 6610}

$$-ibe\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)+ibe\text{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)-\frac{1}{2}b^2e\text{PolyLog}\left(3,1-\right.$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)*(a + b*ArcTan[c*x])^2)/x^3,x]
```

```
[Out] -((b*c*d*(a + b*ArcTan[c*x]))/x) - (c^2*d*(a + b*ArcTan[c*x])^2)/2 - (d*(a
+ b*ArcTan[c*x])^2)/(2*x^2) + 2*e*(a + b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1 +
I*c*x)] + b^2*c^2*d*Log[x] - (b^2*c^2*d*Log[1 + c^2*x^2])/2 - I*b*e*(a + b*
ArcTan[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)] + I*b*e*(a + b*ArcTan[c*x])*Poly
Log[2, -1 + 2/(1 + I*c*x)] - (b^2*e*PolyLog[3, 1 - 2/(1 + I*c*x)])/2 + (b^2
*e*PolyLog[3, -1 + 2/(1 + I*c*x)])/2
```

Rule 4980

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4918

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/(d + e*x^2),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4850

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTan[c*x])^(p - 1)*ArcTanh[1 - 2/(1 + I*c*x)])/(1 + c^2*x^2), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4988

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTan[c*x])^p)/(d + e
*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTan[c*x])^p)/(d + e*x^2
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{(d+ex^2)\left(a+b\tan^{-1}(cx)\right)^2}{x^3}\,dx = \int \left(\frac{d\left(a+b\tan^{-1}(cx)\right)^2}{x^3}+\frac{e\left(a+b\tan^{-1}(cx)\right)^2}{x}\right)dx$$

$$= d\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^3}\,dx + e\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x}\,dx$$

$$= -\frac{d\left(a+b\tan^{-1}(cx)\right)^2}{2x^2} + 2e\left(a+b\tan^{-1}(cx)\right)^2\tanh^{-1}\left(1-\frac{2}{1+icx}\right) + (bcd)\int \frac{a+}{x^2}$$

$$= -\frac{d\left(a+b\tan^{-1}(cx)\right)^2}{2x^2} + 2e\left(a+b\tan^{-1}(cx)\right)^2\tanh^{-1}\left(1-\frac{2}{1+icx}\right) + (bcd)\int \frac{a+}{}$$

$$= -\frac{bcd\left(a+b\tan^{-1}(cx)\right)}{x} - \frac{1}{2}c^2d\left(a+b\tan^{-1}(cx)\right)^2 - \frac{d\left(a+b\tan^{-1}(cx)\right)^2}{2x^2} + 2e\left(a+\right.$$

$$= -\frac{bcd\left(a+b\tan^{-1}(cx)\right)}{x} - \frac{1}{2}c^2d\left(a+b\tan^{-1}(cx)\right)^2 - \frac{d\left(a+b\tan^{-1}(cx)\right)^2}{2x^2} + 2e\left(a+\right.$$

$$= -\frac{bcd\left(a+b\tan^{-1}(cx)\right)}{x} - \frac{1}{2}c^2d\left(a+b\tan^{-1}(cx)\right)^2 - \frac{d\left(a+b\tan^{-1}(cx)\right)^2}{2x^2} + 2e\left(a+\right.$$

$$= -\frac{bcd\left(a+b\tan^{-1}(cx)\right)}{x} - \frac{1}{2}c^2d\left(a+b\tan^{-1}(cx)\right)^2 - \frac{d\left(a+b\tan^{-1}(cx)\right)^2}{2x^2} + 2e\left(a+\right.$$

Mathematica [A] time = 0.34584, size = 273, normalized size = 1.24

$$iabe(\text{PolyLog}(2,-icx)-\text{PolyLog}(2,icx)) + \frac{1}{24}b^2e\left(24i\tan^{-1}(cx)\text{PolyLog}\left(2,e^{-2i\tan^{-1}(cx)}\right)+24i\tan^{-1}(cx)\text{PolyLog}\left(2\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + e*x^2)*(a + b*ArcTan[c*x])^2)/x^3,x]
```

```
[Out] -(a^2*d)/(2*x^2) - (a*b*d*(ArcTan[c*x] + c*x*(1 + c*x*ArcTan[c*x])))/x^2 +
a^2*e*Log[x] - (b^2*d*(2*c*x*ArcTan[c*x] + (1 + c^2*x^2)*ArcTan[c*x]^2 - 2*
c^2*x^2*Log[(c*x)/Sqrt[1 + c^2*x^2]]))/(2*x^2) + I*a*b*e*(PolyLog[2, (-I)*c
*x] - PolyLog[2, I*c*x]) + (b^2*e*((-I)*Pi^3 + (16*I)*ArcTan[c*x]^3 + 24*Ar
cTan[c*x]^2*Log[1 - E^((-2*I)*ArcTan[c*x])] - 24*ArcTan[c*x]^2*Log[1 + E^((
2*I)*ArcTan[c*x])] + (24*I)*ArcTan[c*x]*PolyLog[2, E^((-2*I)*ArcTan[c*x])]
+ (24*I)*ArcTan[c*x]*PolyLog[2, -E^((2*I)*ArcTan[c*x])] + 12*PolyLog[3, E^(
```

```
(-2*I)*ArcTan[c*x])] - 12*PolyLog[3, -E^((2*I)*ArcTan[c*x])]))/24
```

Maple [C] time = 4.318, size = 1313, normalized size = 6.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(a+b*arctan(c*x))^2/x^3,x)
```

```
[Out] 1/2*I*b^2*e*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I/((1+I*c*x)^2/(c^2
*x^2+1)+1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))
*arctan(c*x)^2+I*a*b*e*ln(c*x)*ln(1+I*c*x)-I*a*b*e*ln(c*x)*ln(1-I*c*x)+1/2*
I*b^2*e*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^
3*arctan(c*x)^2-1/2*I*b^2*e*Pi*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^
2/(c^2*x^2+1)+1))^2*arctan(c*x)^2+1/2*I*b^2*e*Pi*csgn(((1+I*c*x)^2/(c^2*x^2
+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*arctan(c*x)^2-1/2*a^2*d/x^2-1/2*c^2*b
^2*arctan(c*x)^2*d+c^2*b^2*d*ln((1+I*c*x)/(c^2*x^2+1)^(1/2)-1)+c^2*b^2*d*ln
(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))-b^2*e*arctan(c*x)^2*ln((1+I*c*x)^2/(c^2*x^2
+1)-1)+b^2*e*arctan(c*x)^2*ln(1-(1+I*c*x)/(c^2*x^2+1)^(1/2))+b^2*e*arctan(c
*x)^2*ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))+b^2*arctan(c*x)^2*e*ln(c*x)-1/2*b^2
*arctan(c*x)^2*d/x^2-1/2*I*b^2*e*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+
I*c*x)^2/(c^2*x^2+1)+1))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2
*x^2+1)+1))^2*arctan(c*x)^2-1/2*I*b^2*e*Pi*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+
1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arcta
n(c*x)^2+1/2*I*b^2*e*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^
2*x^2+1)+1))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*
arctan(c*x)^2-1/2*I*b^2*e*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I*((1
+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2+a^2*e
*ln(c*x)-1/2*b^2*e*polylog(3,-(1+I*c*x)^2/(c^2*x^2+1))+2*b^2*e*polylog(3,(1
+I*c*x)/(c^2*x^2+1)^(1/2))+2*b^2*e*polylog(3,-(1+I*c*x)/(c^2*x^2+1)^(1/2))-
c*a*b*d/x-a*b*arctan(c*x)*d/x^2+I*a*b*e*dilog(1+I*c*x)+I*b^2*e*arctan(c*x)*
polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))+2*a*b*arctan(c*x)*e*ln(c*x)-c^2*a*b*arc
tan(c*x)*d-c*b^2*arctan(c*x)*d/x-2*I*b^2*e*arctan(c*x)*polylog(2,(1+I*c*x)/
(c^2*x^2+1)^(1/2))-2*I*b^2*e*arctan(c*x)*polylog(2,-(1+I*c*x)/(c^2*x^2+1)^(
1/2))+1/2*I*b^2*e*Pi*arctan(c*x)^2-I*a*b*e*dilog(1-I*c*x)-I*c^2*b^2*arctan(
c*x)*d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arctan(c*x))^2/x^3,x, algorithm="maxima")
```

```
[Out] -((c*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*a*b*d + a^2*e*log(x) - 1/2*a^2
*d/x^2 - 1/96*(12*b^2*d*arctan(c*x)^2 - 3*b^2*d*log(c^2*x^2 + 1)^2 - (1152*
b^2*c^2*e*integrate(1/16*x^4*arctan(c*x)^2/(c^2*x^5 + x^3), x) + 3072*a*b*c
^2*e*integrate(1/16*x^4*arctan(c*x)/(c^2*x^5 + x^3), x) + 1152*b^2*c^2*d*in
tegrate(1/16*x^2*arctan(c*x)^2/(c^2*x^5 + x^3), x) + 96*b^2*c^2*d*integrate
(1/16*x^2*log(c^2*x^2 + 1)^2/(c^2*x^5 + x^3), x) - 192*b^2*c^2*d*integrate(
1/16*x^2*log(c^2*x^2 + 1)/(c^2*x^5 + x^3), x) + b^2*e*log(c^2*x^2 + 1)^3 +
384*b^2*c*d*integrate(1/16*x*arctan(c*x)/(c^2*x^5 + x^3), x) + 1152*b^2*e*i
ntegrate(1/16*x^2*arctan(c*x)^2/(c^2*x^5 + x^3), x) + 96*b^2*e*integrate(1/
16*x^2*log(c^2*x^2 + 1)^2/(c^2*x^5 + x^3), x) + 3072*a*b*e*integrate(1/16*x
^2*arctan(c*x)/(c^2*x^5 + x^3), x) + 1152*b^2*d*integrate(1/16*arctan(c*x)^
2/(c^2*x^5 + x^3), x) + 96*b^2*d*integrate(1/16*log(c^2*x^2 + 1)^2/(c^2*x^5
 + x^3), x))*x^2)/x^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{a^2ex^2 + a^2d + \left(b^2ex^2 + b^2d\right)\arctan\left(cx\right)^2 + 2\left(abex^2 + abd\right)\arctan\left(cx\right)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arctan(c*x))^2/x^3,x, algorithm="fricas")
```

```
[Out] integral((a^2*e*x^2 + a^2*d + (b^2*e*x^2 + b^2*d)*arctan(c*x)^2 + 2*(a*b*e*
x^2 + a*b*d)*arctan(c*x))/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + b\,\mathrm{atan}\left(cx\right)\right)^2 \left(d + ex^2\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(a+b*atan(c*x))**2/x**3,x)
```

```
[Out] Integral((a + b*atan(c*x))**2*(d + e*x**2)/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2 + d)(b\arctan(cx) + a)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arctan(c*x))^2/x^3,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*(b*arctan(c*x) + a)^2/x^3, x)
```

3.1254 $\int x^3 \left(d+ex^2\right)^2 \left(a+b\tan^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=502

$$\frac{abd^2x}{2c^3}-\frac{d^2\left(a+b\tan^{-1}(cx)\right)^2}{4c^4}+\frac{2bdex^3\left(a+b\tan^{-1}(cx)\right)}{9c^3}-\frac{2abdex}{3c^5}+\frac{de\left(a+b\tan^{-1}(cx)\right)^2}{3c^6}+\frac{be^2x^5\left(a+b\tan^{-1}(cx)\right)}{20c^3}$$

```
[Out] (a*b*d^2*x)/(2*c^3) - (2*a*b*d*e*x)/(3*c^5) + (a*b*e^2*x)/(4*c^7) + (b^2*d^
2*x^2)/(12*c^2) - (8*b^2*d*e*x^2)/(45*c^4) + (71*b^2*e^2*x^2)/(840*c^6) + (
b^2*d*e*x^4)/(30*c^2) - (3*b^2*e^2*x^4)/(140*c^4) + (b^2*e^2*x^6)/(168*c^2)
 + (b^2*d^2*x*ArcTan[c*x])/(2*c^3) - (2*b^2*d*e*x*ArcTan[c*x])/(3*c^5) + (b
^2*e^2*x*ArcTan[c*x])/(4*c^7) - (b*d^2*x^3*(a + b*ArcTan[c*x]))/(6*c) + (2*
b*d*e*x^3*(a + b*ArcTan[c*x]))/(9*c^3) - (b*e^2*x^3*(a + b*ArcTan[c*x]))/(1
2*c^5) - (2*b*d*e*x^5*(a + b*ArcTan[c*x]))/(15*c) + (b*e^2*x^5*(a + b*ArcTa
n[c*x]))/(20*c^3) - (b*e^2*x^7*(a + b*ArcTan[c*x]))/(28*c) - (d^2*(a + b*Ar
cTan[c*x])^2)/(4*c^4) + (d*e*(a + b*ArcTan[c*x])^2)/(3*c^6) - (e^2*(a + b*A
rcTan[c*x])^2)/(8*c^8) + (d^2*x^4*(a + b*ArcTan[c*x])^2)/4 + (d*e*x^6*(a +
b*ArcTan[c*x])^2)/3 + (e^2*x^8*(a + b*ArcTan[c*x])^2)/8 - (b^2*d^2*Log[1 +
c^2*x^2])/(3*c^4) + (23*b^2*d*e*Log[1 + c^2*x^2])/(45*c^6) - (22*b^2*e^2*Lo
g[1 + c^2*x^2])/(105*c^8)
```

Rubi [A] time = 1.14011, antiderivative size = 502, normalized size of antiderivative = 1., number of steps used = 50, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {4980, 4852, 4916, 266, 43, 4846, 260, 4884}

$$\frac{abd^2x}{2c^3}-\frac{d^2\left(a+b\tan^{-1}(cx)\right)^2}{4c^4}+\frac{2bdex^3\left(a+b\tan^{-1}(cx)\right)}{9c^3}-\frac{2abdex}{3c^5}+\frac{de\left(a+b\tan^{-1}(cx)\right)^2}{3c^6}+\frac{be^2x^5\left(a+b\tan^{-1}(cx)\right)}{20c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + e*x^2)^2*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] (a*b*d^2*x)/(2*c^3) - (2*a*b*d*e*x)/(3*c^5) + (a*b*e^2*x)/(4*c^7) + (b^2*d^
2*x^2)/(12*c^2) - (8*b^2*d*e*x^2)/(45*c^4) + (71*b^2*e^2*x^2)/(840*c^6) + (
b^2*d*e*x^4)/(30*c^2) - (3*b^2*e^2*x^4)/(140*c^4) + (b^2*e^2*x^6)/(168*c^2)
 + (b^2*d^2*x*ArcTan[c*x])/(2*c^3) - (2*b^2*d*e*x*ArcTan[c*x])/(3*c^5) + (b
^2*e^2*x*ArcTan[c*x])/(4*c^7) - (b*d^2*x^3*(a + b*ArcTan[c*x]))/(6*c) + (2*
b*d*e*x^3*(a + b*ArcTan[c*x]))/(9*c^3) - (b*e^2*x^3*(a + b*ArcTan[c*x]))/(1
2*c^5) - (2*b*d*e*x^5*(a + b*ArcTan[c*x]))/(15*c) + (b*e^2*x^5*(a + b*ArcTa
n[c*x]))/(20*c^3) - (b*e^2*x^7*(a + b*ArcTan[c*x]))/(28*c) - (d^2*(a + b*Ar
cTan[c*x])^2)/(4*c^4) + (d*e*(a + b*ArcTan[c*x])^2)/(3*c^6) - (e^2*(a + b*A
```

```
rcTan[c*x])^2)/(8*c^8) + (d^2*x^4*(a + b*ArcTan[c*x])^2)/4 + (d*e*x^6*(a +
b*ArcTan[c*x])^2)/3 + (e^2*x^8*(a + b*ArcTan[c*x])^2)/8 - (b^2*d^2*Log[1 +
c^2*x^2])/(3*c^4) + (23*b^2*d*e*Log[1 + c^2*x^2])/(45*c^6) - (22*b^2*e^2*Lo
g[1 + c^2*x^2])/(105*c^8)
```

Rule 4980

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
```

```
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rubi steps

$$\int x^3\left(d+ex^2\right)^2\left(a+b\tan^{-1}(cx)\right)^2\,dx=\int\left(d^2x^3\left(a+b\tan^{-1}(cx)\right)^2+2dex^5\left(a+b\tan^{-1}(cx)\right)^2+e^2x^7\left(a+b\tan^{-1}(cx)\right.\right.$$

$$=d^2\int x^3\left(a+b\tan^{-1}(cx)\right)^2\,dx+(2de)\int x^5\left(a+b\tan^{-1}(cx)\right)^2\,dx+e^2\int x^7\left(a\right.$$

$$=\frac{1}{4}d^2x^4\left(a+b\tan^{-1}(cx)\right)^2+\frac{1}{3}dex^6\left(a+b\tan^{-1}(cx)\right)^2+\frac{1}{8}e^2x^8\left(a+b\tan^{-1}(cx)\right.$$

$$=\frac{1}{4}d^2x^4\left(a+b\tan^{-1}(cx)\right)^2+\frac{1}{3}dex^6\left(a+b\tan^{-1}(cx)\right)^2+\frac{1}{8}e^2x^8\left(a+b\tan^{-1}(cx)\right.$$

$$=-\frac{bd^2x^3\left(a+b\tan^{-1}(cx)\right)}{6c}-\frac{2bdex^5\left(a+b\tan^{-1}(cx)\right)}{15c}-\frac{be^2x^7\left(a+b\tan^{-1}(cx)\right.}{28c}$$

$$=\frac{abd^2x}{2c^3}-\frac{bd^2x^3\left(a+b\tan^{-1}(cx)\right)}{6c}+\frac{2bdex^3\left(a+b\tan^{-1}(cx)\right)}{9c^3}-\frac{2bdex^5\left(a+b\,\mathrm{t}\right.}{15c}$$

$$=\frac{abd^2x}{2c^3}-\frac{2abdex}{3c^5}+\frac{b^2d^2x\tan^{-1}(cx)}{2c^3}-\frac{bd^2x^3\left(a+b\tan^{-1}(cx)\right)}{6c}+\frac{2bdex^3\left(a+b\right.}{9c}$$

$$=\frac{abd^2x}{2c^3}-\frac{2abdex}{3c^5}+\frac{abe^2x}{4c^7}+\frac{b^2d^2x^2}{12c^2}-\frac{b^2dex^2}{15c^4}+\frac{b^2e^2x^2}{56c^6}+\frac{b^2dex^4}{30c^2}-\frac{b^2e^2x^4}{112c^4}+$$

$$=\frac{abd^2x}{2c^3}-\frac{2abdex}{3c^5}+\frac{abe^2x}{4c^7}+\frac{b^2d^2x^2}{12c^2}-\frac{8b^2dex^2}{45c^4}+\frac{3b^2e^2x^2}{70c^6}+\frac{b^2dex^4}{30c^2}-\frac{3b^2e^2x^4}{140c^4}$$

$$=\frac{abd^2x}{2c^3}-\frac{2abdex}{3c^5}+\frac{abe^2x}{4c^7}+\frac{b^2d^2x^2}{12c^2}-\frac{8b^2dex^2}{45c^4}+\frac{71b^2e^2x^2}{840c^6}+\frac{b^2dex^4}{30c^2}-\frac{3b^2e^2x}{140c^4}$$

Mathematica [A] time = 0.455738, size = 414, normalized size = 0.82

$$\frac{cx\left(105a^2c^7x^3\left(6d^2+8dex^2+3e^2x^4\right)-2ab\left(3c^6\left(70d^2x^2+56dex^4+15e^2x^6\right)-7c^4\left(90d^2+40dex^2+9e^2x^4\right)+105c^2e\left(8d\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + e*x^2)^2*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] (c*x*(105*a^2*c^7*x^3*(6*d^2 + 8*d*e*x^2 + 3*e^2*x^4) + b^2*c*x*(213*e^2 -
2*c^2*e*(224*d + 27*e*x^2) + 3*c^4*(70*d^2 + 28*d*e*x^2 + 5*e^2*x^4)) - 2*a
*b*(-315*e^2 + 105*c^2*e*(8*d + e*x^2) - 7*c^4*(90*d^2 + 40*d*e*x^2 + 9*e^2
*x^4) + 3*c^6*(70*d^2*x^2 + 56*d*e*x^4 + 15*e^2*x^6))) + 2*b*(b*c*x*(315*e^
2 - 105*c^2*e*(8*d + e*x^2) + 7*c^4*(90*d^2 + 40*d*e*x^2 + 9*e^2*x^4) - 3*c
^6*(70*d^2*x^2 + 56*d*e*x^4 + 15*e^2*x^6)) + 105*a*(-6*c^4*d^2 + 8*c^2*d*e
- 3*e^2 + c^8*(6*d^2*x^4 + 8*d*e*x^6 + 3*e^2*x^8)))*ArcTan[c*x] + 105*b^2*(
-6*c^4*d^2 + 8*c^2*d*e - 3*e^2 + c^8*(6*d^2*x^4 + 8*d*e*x^6 + 3*e^2*x^8))*A
rcTan[c*x]^2 - 8*b^2*(105*c^4*d^2 - 161*c^2*d*e + 66*e^2)*Log[1 + c^2*x^2])
/(2520*c^8)
```

Maple [A] time = 0.053, size = 621, normalized size = 1.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x^2+d)^2*(a+b*arctan(c*x))^2,x)
```

```
[Out] 2/3/c^6*a*b*arctan(c*x)*d*e+2/9/c^3*b^2*arctan(c*x)*x^3*d*e+2/3*a*b*arctan(
c*x)*e*d*x^6-2/15/c*a*b*e*d*x^5+2/9/c^3*a*b*x^3*d*e-2/15/c*b^2*arctan(c*x)*
e*d*x^5+1/2*a*b*arctan(c*x)*d^2*x^4+1/3/c^6*b^2*arctan(c*x)^2*d*e-1/2/c^4*a
*b*arctan(c*x)*d^2-1/6/c*a*b*d^2*x^3-1/12/c^5*a*b*x^3*e^2+1/20/c^3*a*b*x^5*
e^2-1/28/c*a*b*e^2*x^7+1/3*b^2*arctan(c*x)^2*e*d*x^6+1/4*a*b*arctan(c*x)*e^
2*x^8-1/6/c*b^2*arctan(c*x)*d^2*x^3-1/12/c^5*b^2*arctan(c*x)*x^3*e^2-1/28/c
*b^2*arctan(c*x)*e^2*x^7+1/20/c^3*b^2*arctan(c*x)*x^5*e^2-1/4/c^8*a*b*arcta
n(c*x)*e^2-22/105*b^2*e^2*ln(c^2*x^2+1)/c^8+71/840*b^2*e^2*x^2/c^6-3/140*b^
2*e^2*x^4/c^4+1/168*b^2*e^2*x^6/c^2+1/8*a^2*e^2*x^8+1/4*a^2*x^4*d^2+1/3*a^2
*e*d*x^6+1/8*b^2*arctan(c*x)^2*e^2*x^8+1/4*b^2*arctan(c*x)^2*d^2*x^4-1/8/c^
8*b^2*arctan(c*x)^2*e^2-1/4/c^4*b^2*arctan(c*x)^2*d^2+1/2*a*b*d^2*x/c^3+1/2
*b^2*d^2*x*arctan(c*x)/c^3+1/4*a*b*e^2*x/c^7-8/45*b^2*d*e*x^2/c^4+1/30*b^2*
d*e*x^4/c^2+1/4*b^2*e^2*x*arctan(c*x)/c^7+1/12*b^2*d^2*x^2/c^2-1/3*b^2*d^2*
ln(c^2*x^2+1)/c^4-2/3*a*b*d*e*x/c^5-2/3*b^2*d*e*x*arctan(c*x)/c^5+23/45*b^2
```

```
*d*e*ln(c^2*x^2+1)/c^6
```

Maxima [A] time = 1.71036, size = 697, normalized size = 1.39

$$\frac{1}{8}b^2e^2x^8\arctan(cx)^2+\frac{1}{8}a^2e^2x^8+\frac{1}{3}b^2dex^6\arctan(cx)^2+\frac{1}{3}a^2dex^6+\frac{1}{4}b^2d^2x^4\arctan(cx)^2+\frac{1}{4}a^2d^2x^4+\frac{1}{6}\Big(3x^4\operatorname{arc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^2*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/8*b^2*e^2*x^8*arctan(c*x)^2 + 1/8*a^2*e^2*x^8 + 1/3*b^2*d*e*x^6*arctan(c*
x)^2 + 1/3*a^2*d*e*x^6 + 1/4*b^2*d^2*x^4*arctan(c*x)^2 + 1/4*a^2*d^2*x^4 +
1/6*(3*x^4*arctan(c*x) - c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5))*a*b*d
^2 - 1/12*(2*c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5)*arctan(c*x) - (c^2
*x^2 + 3*arctan(c*x)^2 - 4*log(c^2*x^2 + 1))/c^4)*b^2*d^2 + 2/45*(15*x^6*ar
ctan(c*x) - c*((3*c^4*x^5 - 5*c^2*x^3 + 15*x)/c^6 - 15*arctan(c*x)/c^7))*a*
b*d*e - 1/90*(4*c*((3*c^4*x^5 - 5*c^2*x^3 + 15*x)/c^6 - 15*arctan(c*x)/c^7)
*arctan(c*x) - (3*c^4*x^4 - 16*c^2*x^2 - 30*arctan(c*x)^2 + 46*log(c^2*x^2
+ 1))/c^6)*b^2*d*e + 1/420*(105*x^8*arctan(c*x) - c*((15*c^6*x^7 - 21*c^4*x
^5 + 35*c^2*x^3 - 105*x)/c^8 + 105*arctan(c*x)/c^9))*a*b*e^2 - 1/840*(2*c*(
(15*c^6*x^7 - 21*c^4*x^5 + 35*c^2*x^3 - 105*x)/c^8 + 105*arctan(c*x)/c^9)*a
rctan(c*x) - (5*c^6*x^6 - 18*c^4*x^4 + 71*c^2*x^2 + 105*arctan(c*x)^2 - 176
*log(c^2*x^2 + 1))/c^8)*b^2*e^2
```

Fricas [A] time = 2.13067, size = 1168, normalized size = 2.33

$$\frac{315a^2c^8e^2x^8-90abc^7e^2x^7+15\left(56a^2c^8de+b^2c^6e^2\right)x^6-42\left(8abc^7de-3abc^5e^2\right)x^5+6\left(105a^2c^8d^2+14b^2c^6de-9b^2c^4\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^2*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/2520*(315*a^2*c^8*e^2*x^8 - 90*a*b*c^7*e^2*x^7 + 15*(56*a^2*c^8*d*e + b^2
*c^6*e^2)*x^6 - 42*(8*a*b*c^7*d*e - 3*a*b*c^5*e^2)*x^5 + 6*(105*a^2*c^8*d^2
 + 14*b^2*c^6*d*e - 9*b^2*c^4*e^2)*x^4 - 70*(6*a*b*c^7*d^2 - 8*a*b*c^5*d*e
+ 3*a*b*c^3*e^2)*x^3 + (210*b^2*c^6*d^2 - 448*b^2*c^4*d*e + 213*b^2*c^2*e^2
)*x^2 + 105*(3*b^2*c^8*e^2*x^8 + 8*b^2*c^8*d*e*x^6 + 6*b^2*c^8*d^2*x^4 - 6*
```

```
b^2*c^4*d^2 + 8*b^2*c^2*d*e - 3*b^2*e^2)*arctan(c*x)^2 + 210*(6*a*b*c^5*d^2
 - 8*a*b*c^3*d*e + 3*a*b*c*e^2)*x + 2*(315*a*b*c^8*e^2*x^8 + 840*a*b*c^8*d*
e*x^6 - 45*b^2*c^7*e^2*x^7 + 630*a*b*c^8*d^2*x^4 - 630*a*b*c^4*d^2 + 840*a*
b*c^2*d*e - 21*(8*b^2*c^7*d*e - 3*b^2*c^5*e^2)*x^5 - 315*a*b*e^2 - 35*(6*b^
2*c^7*d^2 - 8*b^2*c^5*d*e + 3*b^2*c^3*e^2)*x^3 + 105*(6*b^2*c^5*d^2 - 8*b^2
*c^3*d*e + 3*b^2*c*e^2)*x)*arctan(c*x) - 8*(105*b^2*c^4*d^2 - 161*b^2*c^2*d
*e + 66*b^2*e^2)*log(c^2*x^2 + 1))/c^8
```

Sympy [A] time = 14.1747, size = 758, normalized size = 1.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x**2+d)**2*(a+b*atan(c*x))**2,x)
```

```
[Out] Piecewise((a**2*d**2*x**4/4 + a**2*d*e*x**6/3 + a**2*e**2*x**8/8 + a*b*d**2
*x**4*atan(c*x)/2 + 2*a*b*d*e*x**6*atan(c*x)/3 + a*b*e**2*x**8*atan(c*x)/4
- a*b*d**2*x**3/(6*c) - 2*a*b*d*e*x**5/(15*c) - a*b*e**2*x**7/(28*c) + a*b*
d**2*x/(2*c**3) + 2*a*b*d*e*x**3/(9*c**3) + a*b*e**2*x**5/(20*c**3) - a*b*d
**2*atan(c*x)/(2*c**4) - 2*a*b*d*e*x/(3*c**5) - a*b*e**2*x**3/(12*c**5) + 2
*a*b*d*e*atan(c*x)/(3*c**6) + a*b*e**2*x/(4*c**7) - a*b*e**2*atan(c*x)/(4*c
**8) + b**2*d**2*x**4*atan(c*x)**2/4 + b**2*d*e*x**6*atan(c*x)**2/3 + b**2*
e**2*x**8*atan(c*x)**2/8 - b**2*d**2*x**3*atan(c*x)/(6*c) - 2*b**2*d*e*x**5
*atan(c*x)/(15*c) - b**2*e**2*x**7*atan(c*x)/(28*c) + b**2*d**2*x**2/(12*c*
*2) + b**2*d*e*x**4/(30*c**2) + b**2*e**2*x**6/(168*c**2) + b**2*d**2*x*ata
n(c*x)/(2*c**3) + 2*b**2*d*e*x**3*atan(c*x)/(9*c**3) + b**2*e**2*x**5*atan(
c*x)/(20*c**3) - b**2*d**2*log(x**2 + c**(-2))/(3*c**4) - b**2*d**2*atan(c*
x)**2/(4*c**4) - 8*b**2*d*e*x**2/(45*c**4) - 3*b**2*e**2*x**4/(140*c**4) -
2*b**2*d*e*x*atan(c*x)/(3*c**5) - b**2*e**2*x**3*atan(c*x)/(12*c**5) + 23*b
**2*d*e*log(x**2 + c**(-2))/(45*c**6) + b**2*d*e*atan(c*x)**2/(3*c**6) + 71
*b**2*e**2*x**2/(840*c**6) + b**2*e**2*x*atan(c*x)/(4*c**7) - 22*b**2*e**2*
log(x**2 + c**(-2))/(105*c**8) - b**2*e**2*atan(c*x)**2/(8*c**8), Ne(c, 0))
, (a**2*(d**2*x**4/4 + d*e*x**6/3 + e**2*x**8/8), True))
```

Giac [A] time = 2.53245, size = 879, normalized size = 1.75

$$\frac{315\, b^2 c^8 x^8 \arctan(cx)^2 e^2 + 630\, abc^8 x^8 \arctan(cx)\, e^2 + 840\, b^2 c^8 d x^6 \arctan(cx)^2 e + 315\, a^2 c^8 x^8 e^2 + 1680\, abc^8 dx^6 \operatorname{arcta}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^2*(a+b*arctan(c*x))^2,x, algorithm="giac")

[Out] 1/2520*(315*b^2*c^8*x^8*arctan(c*x)^2*e^2 + 630*a*b*c^8*x^8*arctan(c*x)*e^2
 + 840*b^2*c^8*d*x^6*arctan(c*x)^2*e + 315*a^2*c^8*x^8*e^2 + 1680*a*b*c^8*d
*x^6*arctan(c*x)*e + 630*b^2*c^8*d^2*x^4*arctan(c*x)^2 - 90*b^2*c^7*x^7*arc
tan(c*x)*e^2 + 840*a^2*c^8*d*x^6*e + 1260*a*b*c^8*d^2*x^4*arctan(c*x) - 90*
a*b*c^7*x^7*e^2 - 336*b^2*c^7*d*x^5*arctan(c*x)*e + 630*a^2*c^8*d^2*x^4 - 3
36*a*b*c^7*d*x^5*e - 420*b^2*c^7*d^2*x^3*arctan(c*x) + 15*b^2*c^6*x^6*e^2 -
 420*a*b*c^7*d^2*x^3 + 126*b^2*c^5*x^5*arctan(c*x)*e^2 + 84*b^2*c^6*d*x^4*e
 + 126*a*b*c^5*x^5*e^2 + 560*b^2*c^5*d*x^3*arctan(c*x)*e + 210*b^2*c^6*d^2*
x^2 + 560*a*b*c^5*d*x^3*e + 1260*b^2*c^5*d^2*x*arctan(c*x) - 54*b^2*c^4*x^4
*e^2 + 1260*a*b*c^5*d^2*x - 630*b^2*c^4*d^2*arctan(c*x)^2 - 210*b^2*c^3*x^3
*arctan(c*x)*e^2 - 448*b^2*c^4*d*x^2*e - 1260*a*b*c^4*d^2*arctan(c*x) - 210
*a*b*c^3*x^3*e^2 - 1680*b^2*c^3*d*x*arctan(c*x)*e - 840*b^2*c^4*d^2*log(c^2
*x^2 + 1) - 1680*pi*a*b*c^2*d*e*sgn(c)*sgn(x) - 1680*a*b*c^3*d*x*e + 840*b^
2*c^2*d*arctan(c*x)^2*e + 213*b^2*c^2*x^2*e^2 + 1680*a*b*c^2*d*arctan(c*x)*
e + 1288*b^2*c^2*d*e*log(c^2*x^2 + 1) + 630*b^2*c*x*arctan(c*x)*e^2 + 630*a
*b*c*x*e^2 - 315*b^2*arctan(c*x)^2*e^2 - 630*a*b*arctan(c*x)*e^2 - 528*b^2*
e^2*log(c^2*x^2 + 1))/c^8
```

3.1255 $\int x^2 \left(d+ex^2\right)^2 \left(a+b\tan^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=580

$$-\frac{ib^2d^2\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{3c^3}+\frac{2ib^2de\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{5c^5}-\frac{ib^2e^2\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{7c^7}-\frac{id^2\left(a+b\tan^{-1}(cx)\right)^2}{3c^3}-$$

```
[Out] (b^2*d^2*x)/(3*c^2) - (3*b^2*d*e*x)/(5*c^4) + (11*b^2*e^2*x)/(42*c^6) + (b^
2*d*e*x^3)/(15*c^2) - (5*b^2*e^2*x^3)/(126*c^4) + (b^2*e^2*x^5)/(105*c^2) -
 (b^2*d^2*ArcTan[c*x])/(3*c^3) + (3*b^2*d*e*ArcTan[c*x])/(5*c^5) - (11*b^2*
e^2*ArcTan[c*x])/(42*c^7) - (b*d^2*x^2*(a + b*ArcTan[c*x]))/(3*c) + (2*b*d*
e*x^2*(a + b*ArcTan[c*x]))/(5*c^3) - (b*e^2*x^2*(a + b*ArcTan[c*x]))/(7*c^5
) - (b*d*e*x^4*(a + b*ArcTan[c*x]))/(5*c) + (b*e^2*x^4*(a + b*ArcTan[c*x]))
/(14*c^3) - (b*e^2*x^6*(a + b*ArcTan[c*x]))/(21*c) - ((I/3)*d^2*(a + b*ArcT
an[c*x])^2)/c^3 + (((2*I)/5)*d*e*(a + b*ArcTan[c*x])^2)/c^5 - ((I/7)*e^2*(a
 + b*ArcTan[c*x])^2)/c^7 + (d^2*x^3*(a + b*ArcTan[c*x])^2)/3 + (2*d*e*x^5*(
a + b*ArcTan[c*x])^2)/5 + (e^2*x^7*(a + b*ArcTan[c*x])^2)/7 - (2*b*d^2*(a +
 b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(3*c^3) + (4*b*d*e*(a + b*ArcTan[c*x])*
Log[2/(1 + I*c*x)])/(5*c^5) - (2*b*e^2*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x
)])/(7*c^7) - ((I/3)*b^2*d^2*PolyLog[2, 1 - 2/(1 + I*c*x)])/c^3 + (((2*I)/5
)*b^2*d*e*PolyLog[2, 1 - 2/(1 + I*c*x)])/c^5 - ((I/7)*b^2*e^2*PolyLog[2, 1
- 2/(1 + I*c*x)])/c^7
```

Rubi [A] time = 1.06815, antiderivative size = 580, normalized size of antiderivative = 1., number of steps used = 44, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {4980, 4852, 4916, 321, 203, 4920, 4854, 2402, 2315, 302}

$$-\frac{ib^2d^2\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{3c^3}+\frac{2ib^2de\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{5c^5}-\frac{ib^2e^2\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{7c^7}-\frac{id^2\left(a+b\tan^{-1}(cx)\right)^2}{3c^3}-$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + e*x^2)^2*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] (b^2*d^2*x)/(3*c^2) - (3*b^2*d*e*x)/(5*c^4) + (11*b^2*e^2*x)/(42*c^6) + (b^
2*d*e*x^3)/(15*c^2) - (5*b^2*e^2*x^3)/(126*c^4) + (b^2*e^2*x^5)/(105*c^2) -
 (b^2*d^2*ArcTan[c*x])/(3*c^3) + (3*b^2*d*e*ArcTan[c*x])/(5*c^5) - (11*b^2*
e^2*ArcTan[c*x])/(42*c^7) - (b*d^2*x^2*(a + b*ArcTan[c*x]))/(3*c) + (2*b*d*
e*x^2*(a + b*ArcTan[c*x]))/(5*c^3) - (b*e^2*x^2*(a + b*ArcTan[c*x]))/(7*c^5
) - (b*d*e*x^4*(a + b*ArcTan[c*x]))/(5*c) + (b*e^2*x^4*(a + b*ArcTan[c*x]))
```

```
/(14*c^3) - (b*e^2*x^6*(a + b*ArcTan[c*x]))/(21*c) - ((I/3)*d^2*(a + b*ArcT
an[c*x])^2)/c^3 + (((2*I)/5)*d*e*(a + b*ArcTan[c*x])^2)/c^5 - ((I/7)*e^2*(a
 + b*ArcTan[c*x])^2)/c^7 + (d^2*x^3*(a + b*ArcTan[c*x])^2)/3 + (2*d*e*x^5*(
a + b*ArcTan[c*x])^2)/5 + (e^2*x^7*(a + b*ArcTan[c*x])^2)/7 - (2*b*d^2*(a +
 b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(3*c^3) + (4*b*d*e*(a + b*ArcTan[c*x])*
Log[2/(1 + I*c*x)])/(5*c^5) - (2*b*e^2*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x
)])/(7*c^7) - ((I/3)*b^2*d^2*PolyLog[2, 1 - 2/(1 + I*c*x)])/c^3 + (((2*I)/5
)*b^2*d*e*PolyLog[2, 1 - 2/(1 + I*c*x)])/c^5 - ((I/7)*b^2*e^2*PolyLog[2, 1
- 2/(1 + I*c*x)])/c^7
```

Rule 4980

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\int x^2\left(d+ex^2\right)^2\left(a+b\tan^{-1}(cx)\right)^2\,dx = \int\left(d^2x^2\left(a+b\tan^{-1}(cx)\right)^2+2dex^4\left(a+b\tan^{-1}(cx)\right)^2+e^2x^6\left(a+b\tan^{-1}(cx)\right.\right.$$
$$= d^2\int x^2\left(a+b\tan^{-1}(cx)\right)^2\,dx+(2de)\int x^4\left(a+b\tan^{-1}(cx)\right)^2\,dx+e^2\int x^6\left(a\right.$$
$$= \frac{1}{3}d^2x^3\left(a+b\tan^{-1}(cx)\right)^2+\frac{2}{5}dex^5\left(a+b\tan^{-1}(cx)\right)^2+\frac{1}{7}e^2x^7\left(a+b\tan^{-1}(cx)\right.$$
$$= \frac{1}{3}d^2x^3\left(a+b\tan^{-1}(cx)\right)^2+\frac{2}{5}dex^5\left(a+b\tan^{-1}(cx)\right)^2+\frac{1}{7}e^2x^7\left(a+b\tan^{-1}(cx)\right.$$
$$= -\frac{bd^2x^2\left(a+b\tan^{-1}(cx)\right)}{3c}-\frac{bdex^4\left(a+b\tan^{-1}(cx)\right)}{5c}-\frac{be^2x^6\left(a+b\tan^{-1}(cx)\right)}{21c}$$
$$= \frac{b^2d^2x}{3c^2}-\frac{bd^2x^2\left(a+b\tan^{-1}(cx)\right)}{3c}+\frac{2bdex^2\left(a+b\tan^{-1}(cx)\right)}{5c^3}-\frac{bdex^4\left(a+b\,\mathrm{ta}\right.}{5c}$$
$$= \frac{b^2d^2x}{3c^2}-\frac{3b^2dex}{5c^4}+\frac{b^2e^2x}{21c^6}+\frac{b^2dex^3}{15c^2}-\frac{b^2e^2x^3}{63c^4}+\frac{b^2e^2x^5}{105c^2}-\frac{b^2d^2\tan^{-1}(cx)}{3c^3}-\frac{bd^2}{}$$
$$= \frac{b^2d^2x}{3c^2}-\frac{3b^2dex}{5c^4}+\frac{11b^2e^2x}{42c^6}+\frac{b^2dex^3}{15c^2}-\frac{5b^2e^2x^3}{126c^4}+\frac{b^2e^2x^5}{105c^2}-\frac{b^2d^2\tan^{-1}(cx)}{3c^3}+$$
$$= \frac{b^2d^2x}{3c^2}-\frac{3b^2dex}{5c^4}+\frac{11b^2e^2x}{42c^6}+\frac{b^2dex^3}{15c^2}-\frac{5b^2e^2x^3}{126c^4}+\frac{b^2e^2x^5}{105c^2}-\frac{b^2d^2\tan^{-1}(cx)}{3c^3}+$$
$$= \frac{b^2d^2x}{3c^2}-\frac{3b^2dex}{5c^4}+\frac{11b^2e^2x}{42c^6}+\frac{b^2dex^3}{15c^2}-\frac{5b^2e^2x^3}{126c^4}+\frac{b^2e^2x^5}{105c^2}-\frac{b^2d^2\tan^{-1}(cx)}{3c^3}+$$

Mathematica [A] time = 1.57, size = 513, normalized size = 0.88

$$\frac{6ib^2\left(35c^4d^2-42c^2de+15e^2\right)\mathrm{PolyLog}\left(2,-e^{2i\tan^{-1}(cx)}\right)+210a^2c^7d^2x^3+252a^2c^7dex^5+90a^2c^7e^2x^7-3b\tan^{-1}(cx)\left(-4\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(d + e*x^2)^2*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] (378*a*b*c^2*d*e - 165*a*b*e^2 + 210*b^2*c^5*d^2*x - 378*b^2*c^3*d*e*x + 16
5*b^2*c*e^2*x - 210*a*b*c^6*d^2*x^2 + 252*a*b*c^4*d*e*x^2 - 90*a*b*c^2*e^2*
x^2 + 210*a^2*c^7*d^2*x^3 + 42*b^2*c^5*d*e*x^3 - 25*b^2*c^3*e^2*x^3 - 126*a
*b*c^6*d*e*x^4 + 45*a*b*c^4*e^2*x^4 + 252*a^2*c^7*d*e*x^5 + 6*b^2*c^5*e^2*x
^5 - 30*a*b*c^6*e^2*x^6 + 90*a^2*c^7*e^2*x^7 + 6*b^2*((35*I)*c^4*d^2 - (42*
```

```
I)*c^2*d*e + (15*I)*e^2 + c^7*(35*d^2*x^3 + 42*d*e*x^5 + 15*e^2*x^7))*ArcTa
n[c*x]^2 - 3*b*ArcTan[c*x]*(-4*a*c^7*x^3*(35*d^2 + 42*d*e*x^2 + 15*e^2*x^4)
 + b*(1 + c^2*x^2)*(55*e^2 - c^2*e*(126*d + 25*e*x^2) + 2*c^4*(35*d^2 + 21*
d*e*x^2 + 5*e^2*x^4)) + 4*b*(35*c^4*d^2 - 42*c^2*d*e + 15*e^2)*Log[1 + E^((
2*I)*ArcTan[c*x])]) + 210*a*b*c^4*d^2*Log[1 + c^2*x^2] - 252*a*b*c^2*d*e*Lo
g[1 + c^2*x^2] + 90*a*b*e^2*Log[1 + c^2*x^2] + (6*I)*b^2*(35*c^4*d^2 - 42*c
^2*d*e + 15*e^2)*PolyLog[2, -E^((2*I)*ArcTan[c*x])])/(630*c^7)
```

Maple [B] time = 0.139, size = 1158, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^2+d)^2*(a+b*arctan(c*x))^2,x)
```

```
[Out] -1/3/c*b^2*arctan(c*x)*x^2*d^2+1/3*a^2*d^2*x^3+1/7*a^2*e^2*x^7-1/14*I/c^7*b
^2*ln(c*x-I)*ln(-1/2*I*(c*x+I))*e^2+1/14*I/c^7*b^2*ln(c*x+I)*ln(1/2*I*(c*x-
I))*e^2+1/14*I/c^7*b^2*ln(c*x-I)*ln(c^2*x^2+1)*e^2-1/10*I/c^5*b^2*ln(c*x+I)
^2*d*e-2/5/c^5*b^2*arctan(c*x)*ln(c^2*x^2+1)*d*e-1/5/c*b^2*arctan(c*x)*x^4*
d*e-1/5/c*a*b*x^4*d*e+2/5/c^3*a*b*x^2*d*e+11/42*b^2*e^2*x/c^6-5/126*b^2*e^2
*x^3/c^4+1/105*b^2*e^2*x^5/c^2-11/42*b^2*e^2*arctan(c*x)/c^7+1/6*I/c^3*b^2*
ln(c*x+I)*ln(1/2*I*(c*x-I))*d^2-1/6*I/c^3*b^2*ln(c*x-I)*ln(-1/2*I*(c*x+I))*
d^2+1/6*I/c^3*b^2*ln(c*x-I)*ln(c^2*x^2+1)*d^2-1/5*I/c^5*b^2*dilog(1/2*I*(c*
x-I))*d*e+1/10*I/c^5*b^2*ln(c*x-I)^2*d*e-1/14*I/c^7*b^2*ln(c*x+I)*ln(c^2*x^
2+1)*e^2+1/5*I/c^5*b^2*dilog(-1/2*I*(c*x+I))*d*e+2/5/c^3*b^2*arctan(c*x)*x^
2*d*e+1/3*b^2*arctan(c*x)^2*d^2*x^3+1/7*b^2*arctan(c*x)^2*e^2*x^7+2/5*a^2*e
*d*x^5-1/6*I/c^3*b^2*dilog(-1/2*I*(c*x+I))*d^2+1/12*I/c^3*b^2*ln(c*x+I)^2*d
^2+1/14*I/c^7*b^2*dilog(1/2*I*(c*x-I))*e^2+1/6*I/c^3*b^2*dilog(1/2*I*(c*x-I
))*d^2-1/14*I/c^7*b^2*dilog(-1/2*I*(c*x+I))*e^2+1/28*I/c^7*b^2*ln(c*x+I)^2*
e^2-1/28*I/c^7*b^2*ln(c*x-I)^2*e^2+1/3/c^3*b^2*arctan(c*x)*ln(c^2*x^2+1)*d^
2-1/7/c^5*b^2*arctan(c*x)*x^2*e^2-1/21/c*b^2*arctan(c*x)*e^2*x^6+1/3/c^3*a*
b*ln(c^2*x^2+1)*d^2-1/12*I/c^3*b^2*ln(c*x-I)^2*d^2+1/14/c^3*b^2*arctan(c*x)
*x^4*e^2+2/3*a*b*arctan(c*x)*d^2*x^3+2/7*a*b*arctan(c*x)*e^2*x^7+2/5*b^2*ar
ctan(c*x)^2*e*d*x^5+1/7/c^7*a*b*ln(c^2*x^2+1)*e^2+1/5*I/c^5*b^2*ln(c*x+I)*l
n(c^2*x^2+1)*d*e+1/5*I/c^5*b^2*ln(c*x-I)*ln(-1/2*I*(c*x+I))*d*e-1/5*I/c^5*b
^2*ln(c*x-I)*ln(c^2*x^2+1)*d*e-1/5*I/c^5*b^2*ln(c*x+I)*ln(1/2*I*(c*x-I))*d*
e-3/5*b^2*d*e*x/c^4+1/15*b^2*d*e*x^3/c^2+3/5*b^2*d*e*arctan(c*x)/c^5+1/7/c^
7*b^2*arctan(c*x)*ln(c^2*x^2+1)*e^2-1/3/c*a*b*x^2*d^2-1/21/c*a*b*e^2*x^6+1/
14/c^3*a*b*x^4*e^2-1/7/c^5*a*b*x^2*e^2-2/5/c^5*a*b*ln(c^2*x^2+1)*d*e+4/5*a*
b*arctan(c*x)*e*d*x^5-1/6*I/c^3*b^2*ln(c*x+I)*ln(c^2*x^2+1)*d^2+1/3*b^2*d^2
*x/c^2-1/3*b^2*d^2*arctan(c*x)/c^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{7}a^2e^2x^7+\frac{2}{5}a^2dex^5+\frac{1}{3}a^2d^2x^3+\frac{1}{3}\left(2x^3\arctan(cx)-c\left(\frac{x^2}{c^2}-\frac{\log(c^2x^2+1)}{c^4}\right)\right)abd^2+\frac{1}{5}\left(4x^5\arctan(cx)-c\left(\frac{c^2x^4-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^2*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/7*a^2*e^2*x^7 + 2/5*a^2*d*e*x^5 + 1/3*a^2*d^2*x^3 + 1/3*(2*x^3*arctan(c*x
) - c*(x^2/c^2 - log(c^2*x^2 + 1)/c^4))*a*b*d^2 + 1/5*(4*x^5*arctan(c*x) -
c*((c^2*x^4 - 2*x^2)/c^4 + 2*log(c^2*x^2 + 1)/c^6))*a*b*d*e + 1/42*(12*x^7*
arctan(c*x) - c*((2*c^4*x^6 - 3*c^2*x^4 + 6*x^2)/c^6 - 6*log(c^2*x^2 + 1)/c
^8))*a*b*e^2 + 1/420*(15*b^2*e^2*x^7 + 42*b^2*d*e*x^5 + 35*b^2*d^2*x^3)*arc
tan(c*x)^2 - 1/1680*(15*b^2*e^2*x^7 + 42*b^2*d*e*x^5 + 35*b^2*d^2*x^3)*log(
c^2*x^2 + 1)^2 + integrate(1/1680*(1260*(b^2*c^2*e^2*x^8 + (2*b^2*c^2*d*e +
 b^2*e^2)*x^6 + b^2*d^2*x^2 + (b^2*c^2*d^2 + 2*b^2*d*e)*x^4)*arctan(c*x)^2
+ 105*(b^2*c^2*e^2*x^8 + (2*b^2*c^2*d*e + b^2*e^2)*x^6 + b^2*d^2*x^2 + (b^2
*c^2*d^2 + 2*b^2*d*e)*x^4)*log(c^2*x^2 + 1)^2 - 8*(15*b^2*c*e^2*x^7 + 42*b^
2*c*d*e*x^5 + 35*b^2*c*d^2*x^3)*arctan(c*x) + 4*(15*b^2*c^2*e^2*x^8 + 42*b^
2*c^2*d*e*x^6 + 35*b^2*c^2*d^2*x^4)*log(c^2*x^2 + 1))/(c^2*x^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(a^2e^2x^6+2a^2dex^4+a^2d^2x^2+\left(b^2e^2x^6+2b^2dex^4+b^2d^2x^2\right)\arctan(cx)^2+2\left(abe^2x^6+2abdex^4+abd^2x^2\right)\text{arc}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^2*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(a^2*e^2*x^6 + 2*a^2*d*e*x^4 + a^2*d^2*x^2 + (b^2*e^2*x^6 + 2*b^2*d
*e*x^4 + b^2*d^2*x^2)*arctan(c*x)^2 + 2*(a*b*e^2*x^6 + 2*a*b*d*e*x^4 + a*b*
d^2*x^2)*arctan(c*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\left(a+b\,\text{atan}(cx)\right)^2\left(d+ex^2\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d)**2*(a+b*atan(c*x))**2,x)
```

```
[Out] Integral(x**2*(a + b*atan(c*x))**2*(d + e*x**2)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(ex^2+d\right)^2 (b \arctan(cx)+a)^2 x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^2*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^2*(b*arctan(c*x) + a)^2*x^2, x)
```

3.1256 $\int x(d+ex^2)^2(a+b\tan^{-1}(cx))^2\,dx$

Optimal. Leaf size=380

$$\frac{d^2(a+b\tan^{-1}(cx))^2}{2c^2}+\frac{abdex}{c^3}-\frac{de(a+b\tan^{-1}(cx))^2}{2c^4}+\frac{be^2x^3(a+b\tan^{-1}(cx))}{9c^3}-\frac{abe^2x}{3c^5}+\frac{e^2(a+b\tan^{-1}(cx))^2}{6c^6}+\frac{1}{2}$$

```
[Out] -((a*b*d^2*x)/c) + (a*b*d*e*x)/c^3 - (a*b*e^2*x)/(3*c^5) + (b^2*d*e*x^2)/(6
*c^2) - (4*b^2*e^2*x^2)/(45*c^4) + (b^2*e^2*x^4)/(60*c^2) - (b^2*d^2*x*ArcT
an[c*x])/c + (b^2*d*e*x*ArcTan[c*x])/c^3 - (b^2*e^2*x*ArcTan[c*x])/(3*c^5)
- (b*d*e*x^3*(a + b*ArcTan[c*x]))/(3*c) + (b*e^2*x^3*(a + b*ArcTan[c*x]))/(
9*c^3) - (b*e^2*x^5*(a + b*ArcTan[c*x]))/(15*c) + (d^2*(a + b*ArcTan[c*x])^
2)/(2*c^2) - (d*e*(a + b*ArcTan[c*x])^2)/(2*c^4) + (e^2*(a + b*ArcTan[c*x])
^2)/(6*c^6) + (d^2*x^2*(a + b*ArcTan[c*x])^2)/2 + (d*e*x^4*(a + b*ArcTan[c*
x])^2)/2 + (e^2*x^6*(a + b*ArcTan[c*x])^2)/6 + (b^2*d^2*Log[1 + c^2*x^2])/(
2*c^2) - (2*b^2*d*e*Log[1 + c^2*x^2])/(3*c^4) + (23*b^2*e^2*Log[1 + c^2*x^2
])/(90*c^6)
```

Rubi [A] time = 0.75363, antiderivative size = 380, normalized size of antiderivative = 1., number of steps used = 35, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {4980, 4852, 4916, 4846, 260, 4884, 266, 43}

$$\frac{d^2(a+b\tan^{-1}(cx))^2}{2c^2}+\frac{abdex}{c^3}-\frac{de(a+b\tan^{-1}(cx))^2}{2c^4}+\frac{be^2x^3(a+b\tan^{-1}(cx))}{9c^3}-\frac{abe^2x}{3c^5}+\frac{e^2(a+b\tan^{-1}(cx))^2}{6c^6}+\frac{1}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + e*x^2)^2*(a + b*ArcTan[c*x])^2,x]
```


```
[Out] -((a*b*d^2*x)/c) + (a*b*d*e*x)/c^3 - (a*b*e^2*x)/(3*c^5) + (b^2*d*e*x^2)/(6
*c^2) - (4*b^2*e^2*x^2)/(45*c^4) + (b^2*e^2*x^4)/(60*c^2) - (b^2*d^2*x*ArcT
an[c*x])/c + (b^2*d*e*x*ArcTan[c*x])/c^3 - (b^2*e^2*x*ArcTan[c*x])/(3*c^5)
- (b*d*e*x^3*(a + b*ArcTan[c*x]))/(3*c) + (b*e^2*x^3*(a + b*ArcTan[c*x]))/(
9*c^3) - (b*e^2*x^5*(a + b*ArcTan[c*x]))/(15*c) + (d^2*(a + b*ArcTan[c*x])^
2)/(2*c^2) - (d*e*(a + b*ArcTan[c*x])^2)/(2*c^4) + (e^2*(a + b*ArcTan[c*x])
^2)/(6*c^6) + (d^2*x^2*(a + b*ArcTan[c*x])^2)/2 + (d*e*x^4*(a + b*ArcTan[c*
x])^2)/2 + (e^2*x^6*(a + b*ArcTan[c*x])^2)/6 + (b^2*d^2*Log[1 + c^2*x^2])/(
2*c^2) - (2*b^2*d*e*Log[1 + c^2*x^2])/(3*c^4) + (23*b^2*e^2*Log[1 + c^2*x^2
])/(90*c^6)
```


Rule 4980

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x\left(d+ex^2\right)^2\left(a+b\tan^{-1}(cx)\right)^2\,dx=\int\left(d^2x\left(a+b\tan^{-1}(cx)\right)^2+2dex^3\left(a+b\tan^{-1}(cx)\right)^2+e^2x^5\left(a+b\tan^{-1}(cx)\right)^2\right.$$

$$=d^2\int x\left(a+b\tan^{-1}(cx)\right)^2\,dx+(2de)\int x^3\left(a+b\tan^{-1}(cx)\right)^2\,dx+e^2\int x^5\left(a+\right.$$

$$=\frac{1}{2}d^2x^2\left(a+b\tan^{-1}(cx)\right)^2+\frac{1}{2}dex^4\left(a+b\tan^{-1}(cx)\right)^2+\frac{1}{6}e^2x^6\left(a+b\tan^{-1}(cx)\right)$$

$$=\frac{1}{2}d^2x^2\left(a+b\tan^{-1}(cx)\right)^2+\frac{1}{2}dex^4\left(a+b\tan^{-1}(cx)\right)^2+\frac{1}{6}e^2x^6\left(a+b\tan^{-1}(cx)\right)$$

$$=-\frac{abd^2x}{c}-\frac{bdex^3\left(a+b\tan^{-1}(cx)\right)}{3c}-\frac{be^2x^5\left(a+b\tan^{-1}(cx)\right)}{15c}+\frac{d^2\left(a+b\tan^{-1}\right.}{2c^2}$$

$$=-\frac{abd^2x}{c}+\frac{abdex}{c^3}-\frac{b^2d^2x\tan^{-1}(cx)}{c}-\frac{bdex^3\left(a+b\tan^{-1}(cx)\right)}{3c}+\frac{be^2x^3\left(a+b\,\mathrm{ta}\right.}{9c^3}$$

$$=-\frac{abd^2x}{c}+\frac{abdex}{c^3}-\frac{abe^2x}{3c^5}-\frac{b^2d^2x\tan^{-1}(cx)}{c}+\frac{b^2dex\tan^{-1}(cx)}{c^3}-\frac{bdex^3\left(a+b\right.}{3c}$$

$$=-\frac{abd^2x}{c}+\frac{abdex}{c^3}-\frac{abe^2x}{3c^5}+\frac{b^2dex^2}{6c^2}-\frac{b^2e^2x^2}{30c^4}+\frac{b^2e^2x^4}{60c^2}-\frac{b^2d^2x\tan^{-1}(cx)}{c}+$$

$$=-\frac{abd^2x}{c}+\frac{abdex}{c^3}-\frac{abe^2x}{3c^5}+\frac{b^2dex^2}{6c^2}-\frac{4b^2e^2x^2}{45c^4}+\frac{b^2e^2x^4}{60c^2}-\frac{b^2d^2x\tan^{-1}(cx)}{c}+$$

Mathematica [A] time = 0.315931, size = 317, normalized size = 0.83

$$cx\left(30a^2c^5x\left(3d^2+3dex^2+e^2x^4\right)-4ab\left(3c^4\left(15d^2+5dex^2+e^2x^4\right)-5c^2e\left(9d+ex^2\right)+15e^2\right)+b^2cex\left(3c^2\left(10d+ex^2\right)-\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + e*x^2)^2*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] (c*x*(30*a^2*c^5*x*(3*d^2 + 3*d*e*x^2 + e^2*x^4) + b^2*c*e*x*(-16*e + 3*c^2
*(10*d + e*x^2)) - 4*a*b*(15*e^2 - 5*c^2*e*(9*d + e*x^2) + 3*c^4*(15*d^2 +
5*d*e*x^2 + e^2*x^4))) + 4*b*(-(b*c*x*(15*e^2 - 5*c^2*e*(9*d + e*x^2) + 3*c
^4*(15*d^2 + 5*d*e*x^2 + e^2*x^4))) + 15*a*(3*c^4*d^2 - 3*c^2*d*e + e^2 + c
^6*(3*d^2*x^2 + 3*d*e*x^4 + e^2*x^6)))*ArcTan[c*x] + 30*b^2*(3*c^4*d^2 - 3*
c^2*d*e + e^2 + c^6*(3*d^2*x^2 + 3*d*e*x^4 + e^2*x^6))*ArcTan[c*x]^2 + 2*b^
2*(45*c^4*d^2 - 60*c^2*d*e + 23*e^2)*Log[1 + c^2*x^2])/(180*c^6)
```

Maple [A] time = 0.053, size = 484, normalized size = 1.3

$$-\frac{b^2\arctan(cx)\,x^5e^2}{15\,c}+ab\arctan(cx)\,x^2d^2+\frac{b^2(\arctan(cx))^2\,x^4de}{2}+\frac{ab\arctan(cx)\,e^2x^6}{3}+\frac{abx^3e^2}{9\,c^3}+\frac{ab\arctan(cx)\,e^2}{3\,c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d)^2*(a+b*arctan(c*x))^2,x)
```

```
[Out] -1/15/c*b^2*arctan(c*x)*x^5*e^2+a*b*arctan(c*x)*x^2*d^2+1/2*b^2*arctan(c*x)
^2*x^4*d*e+1/3*a*b*arctan(c*x)*e^2*x^6+1/9/c^3*a*b*x^3*e^2+1/3/c^6*a*b*arct
an(c*x)*e^2+1/9/c^3*b^2*arctan(c*x)*x^3*e^2-1/15/c*a*b*x^5*e^2-1/2/c^4*b^2*
arctan(c*x)^2*d*e+1/c^2*a*b*arctan(c*x)*d^2+23/90*b^2*e^2*ln(c^2*x^2+1)/c^6
-4/45*b^2*e^2*x^2/c^4+1/60*b^2*e^2*x^4/c^2+1/2*a^2*x^2*d^2+1/6*a^2*e^2*x^6-
1/3/c*b^2*arctan(c*x)*x^3*d*e-1/3/c*a*b*x^3*d*e+a*b*arctan(c*x)*x^4*d*e-1/c
^4*a*b*arctan(c*x)*d*e+1/2*a^2*x^4*d*e+1/6*b^2*arctan(c*x)^2*e^2*x^6+1/2*b^
2*arctan(c*x)^2*x^2*d^2+1/2/c^2*b^2*arctan(c*x)^2*d^2+1/6/c^6*b^2*arctan(c*
x)^2*e^2-a*b*d^2*x/c-b^2*d^2*x*arctan(c*x)/c+1/2*b^2*d^2*ln(c^2*x^2+1)/c^2-
1/3*a*b*e^2*x/c^5+1/6*b^2*d*e*x^2/c^2-1/3*b^2*e^2*x*arctan(c*x)/c^5-2/3*b^2
*d*e*ln(c^2*x^2+1)/c^4+a*b*d*e*x/c^3+b^2*d*e*x*arctan(c*x)/c^3
```

Maxima [A] time = 1.62129, size = 585, normalized size = 1.54

$$\frac{1}{6}\,b^2e^2x^6\arctan(cx)^2+\frac{1}{6}\,a^2e^2x^6+\frac{1}{2}\,b^2dex^4\arctan(cx)^2+\frac{1}{2}\,a^2dex^4+\frac{1}{2}\,b^2d^2x^2\arctan(cx)^2+\frac{1}{2}\,a^2d^2x^2+\Big(x^2\arctan($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^2*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/6*b^2*e^2*x^6*arctan(c*x)^2 + 1/6*a^2*e^2*x^6 + 1/2*b^2*d*e*x^4*arctan(c*
x)^2 + 1/2*a^2*d*e*x^4 + 1/2*b^2*d^2*x^2*arctan(c*x)^2 + 1/2*a^2*d^2*x^2 +
```

```
(x^2*arctan(c*x) - c*(x/c^2 - arctan(c*x)/c^3))*a*b*d^2 - 1/2*(2*c*(x/c^2 -
 arctan(c*x)/c^3)*arctan(c*x) + (arctan(c*x)^2 - log(c^2*x^2 + 1))/c^2)*b^2
*d^2 + 1/3*(3*x^4*arctan(c*x) - c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5)
)*a*b*d*e - 1/6*(2*c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5)*arctan(c*x)
- (c^2*x^2 + 3*arctan(c*x)^2 - 4*log(c^2*x^2 + 1))/c^4)*b^2*d*e + 1/45*(15*
x^6*arctan(c*x) - c*((3*c^4*x^5 - 5*c^2*x^3 + 15*x)/c^6 - 15*arctan(c*x)/c^
7))*a*b*e^2 - 1/180*(4*c*((3*c^4*x^5 - 5*c^2*x^3 + 15*x)/c^6 - 15*arctan(c*
x)/c^7)*arctan(c*x) - (3*c^4*x^4 - 16*c^2*x^2 - 30*arctan(c*x)^2 + 46*log(c
^2*x^2 + 1))/c^6)*b^2*e^2
```

Fricas [A] time = 2.30814, size = 894, normalized size = 2.35

$$\frac{30\,a^2c^6e^2x^6 - 12\,abc^5e^2x^5 + 3\left(30\,a^2c^6de + b^2c^4e^2\right)x^4 - 20\left(3\,abc^5de - abc^3e^2\right)x^3 + 2\left(45\,a^2c^6d^2 + 15\,b^2c^4de - 8\,b^2c^2e^2\right)x^2 \cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^2*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/180*(30*a^2*c^6*e^2*x^6 - 12*a*b*c^5*e^2*x^5 + 3*(30*a^2*c^6*d*e + b^2*c^
4*e^2)*x^4 - 20*(3*a*b*c^5*d*e - a*b*c^3*e^2)*x^3 + 2*(45*a^2*c^6*d^2 + 15*
b^2*c^4*d*e - 8*b^2*c^2*e^2)*x^2 + 30*(b^2*c^6*e^2*x^6 + 3*b^2*c^6*d*e*x^4
+ 3*b^2*c^6*d^2*x^2 + 3*b^2*c^4*d^2 - 3*b^2*c^2*d*e + b^2*e^2)*arctan(c*x)^
2 - 60*(3*a*b*c^5*d^2 - 3*a*b*c^3*d*e + a*b*c*e^2)*x + 4*(15*a*b*c^6*e^2*x^
6 + 45*a*b*c^6*d*e*x^4 - 3*b^2*c^5*e^2*x^5 + 45*a*b*c^6*d^2*x^2 + 45*a*b*c^
4*d^2 - 45*a*b*c^2*d*e + 15*a*b*e^2 - 5*(3*b^2*c^5*d*e - b^2*c^3*e^2)*x^3 -
 15*(3*b^2*c^5*d^2 - 3*b^2*c^3*d*e + b^2*c*e^2)*x)*arctan(c*x) + 2*(45*b^2*
c^4*d^2 - 60*b^2*c^2*d*e + 23*b^2*e^2)*log(c^2*x^2 + 1))/c^6
```

Sympy [A] time = 7.65463, size = 575, normalized size = 1.51

$$\begin{cases} \frac{a^2d^2x^2}{2} + \frac{a^2dex^4}{2} + \frac{a^2e^2x^6}{6} + abd^2x^2\operatorname{atan}(cx) + abdex^4\operatorname{atan}(cx) + \frac{abe^2x^6\operatorname{atan}(cx)}{3} - \frac{abd^2x}{c} - \frac{abdex^3}{3c} - \frac{abe^2x^5}{15c} + \frac{abd^2\operatorname{atan}(cx)}{c^2} + \cdots \\ a^2\left(\frac{d^2x^2}{2} + \frac{dex^4}{2} + \frac{e^2x^6}{6}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d)**2*(a+b*atan(c*x))**2,x)
```

```
[Out] Piecewise((a**2*d**2*x**2/2 + a**2*d*e*x**4/2 + a**2*e**2*x**6/6 + a*b*d**2
*x**2*atan(c*x) + a*b*d*e*x**4*atan(c*x) + a*b*e**2*x**6*atan(c*x)/3 - a*b*
d**2*x/c - a*b*d*e*x**3/(3*c) - a*b*e**2*x**5/(15*c) + a*b*d**2*atan(c*x)/c
**2 + a*b*d*e*x/c**3 + a*b*e**2*x**3/(9*c**3) - a*b*d*e*atan(c*x)/c**4 - a*
b*e**2*x/(3*c**5) + a*b*e**2*atan(c*x)/(3*c**6) + b**2*d**2*x**2*atan(c*x)*
*2/2 + b**2*d*e*x**4*atan(c*x)**2/2 + b**2*e**2*x**6*atan(c*x)**2/6 - b**2*
d**2*x*atan(c*x)/c - b**2*d*e*x**3*atan(c*x)/(3*c) - b**2*e**2*x**5*atan(c*
x)/(15*c) + b**2*d**2*log(x**2 + c**(-2))/(2*c**2) + b**2*d**2*atan(c*x)**2
/(2*c**2) + b**2*d*e*x**2/(6*c**2) + b**2*e**2*x**4/(60*c**2) + b**2*d*e*x*
atan(c*x)/c**3 + b**2*e**2*x**3*atan(c*x)/(9*c**3) - 2*b**2*d*e*log(x**2 +
c**(-2))/(3*c**4) - b**2*d*e*atan(c*x)**2/(2*c**4) - 4*b**2*e**2*x**2/(45*c
**4) - b**2*e**2*x*atan(c*x)/(3*c**5) + 23*b**2*e**2*log(x**2 + c**(-2))/(9
0*c**6) + b**2*e**2*atan(c*x)**2/(6*c**6), Ne(c, 0)), (a**2*(d**2*x**2/2 +
d*e*x**4/2 + e**2*x**6/6), True))
```

Giac [A] time = 1.62472, size = 716, normalized size = 1.88

$$\frac{30\,b^2c^6x^6\arctan(cx)^2e^2 + 60\,abc^6x^6\arctan(cx)\,e^2 + 90\,b^2c^6dx^4\arctan(cx)^2e + 30\,a^2c^6x^6e^2 + 180\,abc^6dx^4\arctan(cx)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^2*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

```
[Out] 1/180*(30*b^2*c^6*x^6*arctan(c*x)^2*e^2 + 60*a*b*c^6*x^6*arctan(c*x)*e^2 +
90*b^2*c^6*d*x^4*arctan(c*x)^2*e + 30*a^2*c^6*x^6*e^2 + 180*a*b*c^6*d*x^4*a
rctan(c*x)*e + 90*b^2*c^6*d^2*x^2*arctan(c*x)^2 - 12*b^2*c^5*x^5*arctan(c*x
)*e^2 + 90*a^2*c^6*d*x^4*e + 180*a*b*c^6*d^2*x^2*arctan(c*x) - 12*a*b*c^5*x
^5*e^2 - 60*b^2*c^5*d*x^3*arctan(c*x)*e + 90*a^2*c^6*d^2*x^2 - 60*a*b*c^5*d
*x^3*e - 180*b^2*c^5*d^2*x*arctan(c*x) + 3*b^2*c^4*x^4*e^2 - 180*pi*a*b*c^4
*d^2*sgn(c)*sgn(x) - 180*a*b*c^5*d^2*x + 90*b^2*c^4*d^2*arctan(c*x)^2 + 20*
b^2*c^3*x^3*arctan(c*x)*e^2 + 30*b^2*c^4*d*x^2*e + 180*a*b*c^4*d^2*arctan(c
*x) + 20*a*b*c^3*x^3*e^2 + 180*b^2*c^3*d*x*arctan(c*x)*e + 90*b^2*c^4*d^2*l
og(c^2*x^2 + 1) + 180*a*b*c^3*d*x*e - 90*b^2*c^2*d*arctan(c*x)^2*e - 16*b^2
*c^2*x^2*e^2 - 180*a*b*c^2*d*arctan(c*x)*e - 120*b^2*c^2*d*e*log(c^2*x^2 +
1) - 60*b^2*c*x*arctan(c*x)*e^2 - 60*pi*a*b*e^2*sgn(c)*sgn(x) - 60*a*b*c*x*
e^2 + 30*b^2*arctan(c*x)^2*e^2 + 60*a*b*arctan(c*x)*e^2 + 46*b^2*e^2*log(c^
2*x^2 + 1))/c^6
```

3.1257 $\int \left(d+ex^2\right)^2 \left(a+b\tan^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=442

$$-\frac{2ib^2de\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{3c^3}+\frac{ib^2e^2\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{5c^5}+\frac{ib^2d^2\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c}-\frac{2ide\left(a+b\tan^{-1}(cx)\right)^2}{3c^3}$$

```
[Out] (2*b^2*d*e*x)/(3*c^2) - (3*b^2*e^2*x)/(10*c^4) + (b^2*e^2*x^3)/(30*c^2) - (
2*b^2*d*e*ArcTan[c*x])/(3*c^3) + (3*b^2*e^2*ArcTan[c*x])/(10*c^5) - (2*b*d*
e*x^2*(a + b*ArcTan[c*x]))/(3*c) + (b*e^2*x^2*(a + b*ArcTan[c*x]))/(5*c^3)
- (b*e^2*x^4*(a + b*ArcTan[c*x]))/(10*c) + (I*d^2*(a + b*ArcTan[c*x])^2)/c
- (((2*I)/3)*d*e*(a + b*ArcTan[c*x])^2)/c^3 + ((I/5)*e^2*(a + b*ArcTan[c*x]
)^2)/c^5 + d^2*x*(a + b*ArcTan[c*x])^2 + (2*d*e*x^3*(a + b*ArcTan[c*x])^2)/
3 + (e^2*x^5*(a + b*ArcTan[c*x])^2)/5 + (2*b*d^2*(a + b*ArcTan[c*x])*Log[2/
(1 + I*c*x)])/c - (4*b*d*e*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(3*c^3)
+ (2*b*e^2*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(5*c^5) + (I*b^2*d^2*Pol
yLog[2, 1 - 2/(1 + I*c*x)])/c - (((2*I)/3)*b^2*d*e*PolyLog[2, 1 - 2/(1 + I*
c*x)])/c^3 + ((I/5)*b^2*e^2*PolyLog[2, 1 - 2/(1 + I*c*x)])/c^5
```

Rubi [A] time = 0.689419, antiderivative size = 442, normalized size of antiderivative = 1., number of steps used = 30, number of rules used = 11, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.55, Rules used = {4914, 4846, 4920, 4854, 2402, 2315, 4852, 4916, 321, 203, 302}

$$-\frac{2ib^2de\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{3c^3}+\frac{ib^2e^2\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{5c^5}+\frac{ib^2d^2\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c}-\frac{2ide\left(a+b\tan^{-1}(cx)\right)^2}{3c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^2*(a + b*ArcTan[c*x])^2,x]
```



```
[Out] (2*b^2*d*e*x)/(3*c^2) - (3*b^2*e^2*x)/(10*c^4) + (b^2*e^2*x^3)/(30*c^2) - (
2*b^2*d*e*ArcTan[c*x])/(3*c^3) + (3*b^2*e^2*ArcTan[c*x])/(10*c^5) - (2*b*d*
e*x^2*(a + b*ArcTan[c*x]))/(3*c) + (b*e^2*x^2*(a + b*ArcTan[c*x]))/(5*c^3)
- (b*e^2*x^4*(a + b*ArcTan[c*x]))/(10*c) + (I*d^2*(a + b*ArcTan[c*x])^2)/c
- (((2*I)/3)*d*e*(a + b*ArcTan[c*x])^2)/c^3 + ((I/5)*e^2*(a + b*ArcTan[c*x]
)^2)/c^5 + d^2*x*(a + b*ArcTan[c*x])^2 + (2*d*e*x^3*(a + b*ArcTan[c*x])^2)/
3 + (e^2*x^5*(a + b*ArcTan[c*x])^2)/5 + (2*b*d^2*(a + b*ArcTan[c*x])*Log[2/
(1 + I*c*x)])/c - (4*b*d*e*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(3*c^3)
+ (2*b*e^2*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(5*c^5) + (I*b^2*d^2*Pol
yLog[2, 1 - 2/(1 + I*c*x)])/c - (((2*I)/3)*b^2*d*e*PolyLog[2, 1 - 2/(1 + I*
```

c*x)])/c^3 + ((I/5)*b^2*e^2*PolyLog[2, 1 - 2/(1 + I*c*x)])/c^5

Rule 4914

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (d + e*x^2)^q, x], x
] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[q] && IGtQ[p, 0]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
```

```
erQ[m]) && NeQ[m, -1]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\int \left(d+ex^2\right)^2 \left(a+b\tan^{-1}(cx)\right)^2\,dx = \int \left(d^2\left(a+b\tan^{-1}(cx)\right)^2 + 2dex^2\left(a+b\tan^{-1}(cx)\right)^2 + e^2x^4\left(a+b\tan^{-1}(cx)\right)^2\right)dx$$

$$= d^2\int \left(a+b\tan^{-1}(cx)\right)^2 dx + (2de)\int x^2\left(a+b\tan^{-1}(cx)\right)^2 dx + e^2\int x^4\left(a+b\,\mathrm{ta}\right.$$

$$= d^2x\left(a+b\tan^{-1}(cx)\right)^2 + \frac{2}{3}dex^3\left(a+b\tan^{-1}(cx)\right)^2 + \frac{1}{5}e^2x^5\left(a+b\tan^{-1}(cx)\right)^2 - (2$$

$$= \frac{id^2\left(a+b\tan^{-1}(cx)\right)^2}{c} + d^2x\left(a+b\tan^{-1}(cx)\right)^2 + \frac{2}{3}dex^3\left(a+b\tan^{-1}(cx)\right)^2 + \frac{1}{5}e^2x$$

$$= -\frac{2bdex^2\left(a+b\tan^{-1}(cx)\right)}{3c} - \frac{be^2x^4\left(a+b\tan^{-1}(cx)\right)}{10c} + \frac{id^2\left(a+b\tan^{-1}(cx)\right)^2}{c} - \frac{2i}{}$$

$$= \frac{2b^2dex}{3c^2} - \frac{2bdex^2\left(a+b\tan^{-1}(cx)\right)}{3c} + \frac{be^2x^2\left(a+b\tan^{-1}(cx)\right)}{5c^3} - \frac{be^2x^4\left(a+b\tan^{-1}\right.}{10c}$$

$$= \frac{2b^2dex}{3c^2} - \frac{3b^2e^2x}{10c^4} + \frac{b^2e^2x^3}{30c^2} - \frac{2b^2de\tan^{-1}(cx)}{3c^3} - \frac{2bdex^2\left(a+b\tan^{-1}(cx)\right)}{3c} + \frac{be^2x^2}{}$$

$$= \frac{2b^2dex}{3c^2} - \frac{3b^2e^2x}{10c^4} + \frac{b^2e^2x^3}{30c^2} - \frac{2b^2de\tan^{-1}(cx)}{3c^3} + \frac{3b^2e^2\tan^{-1}(cx)}{10c^5} - \frac{2bdex^2\left(a+b\,\mathrm{t}\right.}{3c}$$

$$= \frac{2b^2dex}{3c^2} - \frac{3b^2e^2x}{10c^4} + \frac{b^2e^2x^3}{30c^2} - \frac{2b^2de\tan^{-1}(cx)}{3c^3} + \frac{3b^2e^2\tan^{-1}(cx)}{10c^5} - \frac{2bdex^2\left(a+b\,\mathrm{t}\right.}{3c}$$

Mathematica [A] time = 1.08642, size = 391, normalized size = 0.88

$$\frac{-2ib^2\left(15c^4d^2-10c^2de+3e^2\right)\mathrm{PolyLog}\left(2,-e^{2i\tan^{-1}(cx)}\right)+30a^2c^5d^2x+20a^2c^5dex^3+6a^2c^5e^2x^5+b\tan^{-1}(cx)\left(4ac^5x\left(15\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x^2)^2*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] (9*a*b*e^2 + 30*a^2*c^5*d^2*x + 20*b^2*c^3*d*e*x - 9*b^2*c*e^2*x - 20*a*b*c
^4*d*e*x^2 + 6*a*b*c^2*e^2*x^2 + 20*a^2*c^5*d*e*x^3 + b^2*c^3*e^2*x^3 - 3*a
*b*c^4*e^2*x^4 + 6*a^2*c^5*e^2*x^5 + 2*b^2*((-15*I)*c^4*d^2 + (10*I)*c^2*d*
e - (3*I)*e^2 + c^5*(15*d^2*x + 10*d*e*x^3 + 3*e^2*x^5))*ArcTan[c*x]^2 + b*
ArcTan[c*x]*(4*a*c^5*x*(15*d^2 + 10*d*e*x^2 + 3*e^2*x^4) - b*e*(1 + c^2*x^2
)*(-9*e + c^2*(20*d + 3*e*x^2)) + 4*b*(15*c^4*d^2 - 10*c^2*d*e + 3*e^2)*Log
[1 + E^((2*I)*ArcTan[c*x])]) - 30*a*b*c^4*d^2*Log[1 + c^2*x^2] + 20*a*b*c^2
*d*e*Log[1 + c^2*x^2] - 6*a*b*e^2*Log[1 + c^2*x^2] - (2*I)*b^2*(15*c^4*d^2
```

```
- 10*c^2*d*e + 3*e^2)*PolyLog[2, -E^((2*I)*ArcTan[c*x])])/(30*c^5)
```

Maple [B] time = 0.129, size = 1005, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(a+b*arctan(c*x))^2,x)
```

```
[Out] -1/4*I/c*b^2*ln(c*x+I)^2*d^2+1/2*I/c*b^2*ln(c*x-I)*ln(-1/2*I*(c*x+I))*d^2-1
/2*I/c*b^2*ln(c*x-I)*ln(c^2*x^2+1)*d^2-2/3/c*b^2*arctan(c*x)*x^2*d*e+1/10*I
/c^5*b^2*ln(c*x-I)*ln(-1/2*I*(c*x+I))*e^2+1/3*I/c^3*b^2*ln(c*x-I)*ln(c^2*x^
2+1)*d*e+1/3*I/c^3*b^2*ln(c*x+I)*ln(1/2*I*(c*x-I))*d*e-1/3*I/c^3*b^2*ln(c*x
-I)*ln(-1/2*I*(c*x+I))*d*e-1/3*I/c^3*b^2*ln(c*x+I)*ln(c^2*x^2+1)*d*e+1/5*a^
2*x^5*e^2+a^2*d^2*x-3/10*b^2*e^2*x/c^4+1/30*b^2*e^2*x^3/c^2+3/10*b^2*e^2*ar
ctan(c*x)/c^5+1/5/c^3*b^2*arctan(c*x)*x^2*e^2-1/5/c^5*a*b*ln(c^2*x^2+1)*e^2
-1/10/c*a*b*x^4*e^2+1/5/c^3*a*b*x^2*e^2+2/3*b^2*arctan(c*x)^2*x^3*d*e+2/5*a
*b*arctan(c*x)*x^5*e^2+2*a*b*arctan(c*x)*d^2*x+1/10*I/c^5*b^2*dilog(-1/2*I*
(c*x+I))*e^2+1/20*I/c^5*b^2*ln(c*x-I)^2*e^2-1/c*a*b*ln(c^2*x^2+1)*d^2-1/10/
c*b^2*arctan(c*x)*x^4*e^2-1/10*I/c^5*b^2*dilog(1/2*I*(c*x-I))*e^2-1/10*I/c^
5*b^2*ln(c*x-I)*ln(c^2*x^2+1)*e^2-1/3*I/c^3*b^2*dilog(-1/2*I*(c*x+I))*d*e-1
/6*I/c^3*b^2*ln(c*x-I)^2*d*e+1/6*I/c^3*b^2*ln(c*x+I)^2*d*e+1/3*I/c^3*b^2*di
log(1/2*I*(c*x-I))*d*e+1/2*I/c*b^2*ln(c*x+I)*ln(c^2*x^2+1)*d^2-1/2*I/c*b^2*
ln(c*x+I)*ln(1/2*I*(c*x-I))*d^2-1/10*I/c^5*b^2*ln(c*x+I)*ln(1/2*I*(c*x-I))*
e^2+1/10*I/c^5*b^2*ln(c*x+I)*ln(c^2*x^2+1)*e^2+4/3*a*b*arctan(c*x)*x^3*d*e+
2/3/c^3*a*b*ln(c^2*x^2+1)*d*e+2/3/c^3*b^2*arctan(c*x)*ln(c^2*x^2+1)*d*e-2/3
/c*a*b*x^2*d*e-1/2*I/c*b^2*dilog(1/2*I*(c*x-I))*d^2-1/c*b^2*arctan(c*x)*ln(
c^2*x^2+1)*d^2+2/3*a^2*x^3*d*e+b^2*arctan(c*x)^2*d^2*x+1/4*I/c*b^2*ln(c*x-I
)^2*d^2+1/2*I/c*b^2*dilog(-1/2*I*(c*x+I))*d^2-1/5/c^5*b^2*arctan(c*x)*ln(c^
2*x^2+1)*e^2-1/20*I/c^5*b^2*ln(c*x+I)^2*e^2+1/5*b^2*arctan(c*x)^2*x^5*e^2+2
/3*b^2*d*e*x/c^2-2/3*b^2*d*e*arctan(c*x)/c^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/5*a^2*e^2*x^5 + 2/3*a^2*d*e*x^3 + 180*b^2*c^2*e^2*integrate(1/240*x^6*arc
tan(c*x)^2/(c^2*x^2 + 1), x) + 15*b^2*c^2*e^2*integrate(1/240*x^6*log(c^2*x
^2 + 1)^2/(c^2*x^2 + 1), x) + 12*b^2*c^2*e^2*integrate(1/240*x^6*log(c^2*x^
2 + 1)/(c^2*x^2 + 1), x) + 360*b^2*c^2*d*e*integrate(1/240*x^4*arctan(c*x)^
2/(c^2*x^2 + 1), x) + 30*b^2*c^2*d*e*integrate(1/240*x^4*log(c^2*x^2 + 1)^2
/(c^2*x^2 + 1), x) + 40*b^2*c^2*d*e*integrate(1/240*x^4*log(c^2*x^2 + 1)/(c
^2*x^2 + 1), x) + 180*b^2*c^2*d^2*integrate(1/240*x^2*arctan(c*x)^2/(c^2*x^
2 + 1), x) + 15*b^2*c^2*d^2*integrate(1/240*x^2*log(c^2*x^2 + 1)^2/(c^2*x^2
 + 1), x) + 60*b^2*c^2*d^2*integrate(1/240*x^2*log(c^2*x^2 + 1)/(c^2*x^2 +
1), x) + 1/4*b^2*d^2*arctan(c*x)^3/c - 24*b^2*c*e^2*integrate(1/240*x^5*arc
tan(c*x)/(c^2*x^2 + 1), x) - 80*b^2*c*d*e*integrate(1/240*x^3*arctan(c*x)/(
c^2*x^2 + 1), x) - 120*b^2*c*d^2*integrate(1/240*x*arctan(c*x)/(c^2*x^2 + 1
), x) + 2/3*(2*x^3*arctan(c*x) - c*(x^2/c^2 - log(c^2*x^2 + 1)/c^4))*a*b*d*
e + 1/10*(4*x^5*arctan(c*x) - c*((c^2*x^4 - 2*x^2)/c^4 + 2*log(c^2*x^2 + 1)
/c^6))*a*b*e^2 + a^2*d^2*x + 180*b^2*e^2*integrate(1/240*x^4*arctan(c*x)^2/
(c^2*x^2 + 1), x) + 15*b^2*e^2*integrate(1/240*x^4*log(c^2*x^2 + 1)^2/(c^2*
x^2 + 1), x) + 360*b^2*d*e*integrate(1/240*x^2*arctan(c*x)^2/(c^2*x^2 + 1),
 x) + 30*b^2*d*e*integrate(1/240*x^2*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) +
 15*b^2*d^2*integrate(1/240*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) + (2*c*x*a
rctan(c*x) - log(c^2*x^2 + 1))*a*b*d^2/c + 1/60*(3*b^2*e^2*x^5 + 10*b^2*d*e
*x^3 + 15*b^2*d^2*x)*arctan(c*x)^2 - 1/240*(3*b^2*e^2*x^5 + 10*b^2*d*e*x^3
+ 15*b^2*d^2*x)*log(c^2*x^2 + 1)^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(a^2e^2x^4 + 2\,a^2dex^2 + a^2d^2 + \left(b^2e^2x^4 + 2\,b^2dex^2 + b^2d^2\right)\arctan\left(cx\right)^2 + 2\left(abe^2x^4 + 2\,abdex^2 + abd^2\right)\arctan\left(cx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(a^2*e^2*x^4 + 2*a^2*d*e*x^2 + a^2*d^2 + (b^2*e^2*x^4 + 2*b^2*d*e*x
^2 + b^2*d^2)*arctan(c*x)^2 + 2*(a*b*e^2*x^4 + 2*a*b*d*e*x^2 + a*b*d^2)*arc
tan(c*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\operatorname{atan}\left(cx\right)\right)^2 \left(d + ex^2\right)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(a+b*atan(c*x))**2,x)
```

```
[Out] Integral((a + b*atan(c*x))**2*(d + e*x**2)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(ex^2 + d\right)^2 (b\arctan\left(cx\right) + a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^2*(b*arctan(c*x) + a)^2, x)
```

3.1258 $\int \frac{(d+ex^2)^2(a+b\tan^{-1}(cx))^2}{x}\,dx$

Optimal. Leaf size=355

$$-ibd^2\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)+ibd^2\text{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)-\frac{1}{2}b^2d^2\text{PolyLog}\Big(3$$

```
[Out] (-2*a*b*d*e*x)/c + (a*b*e^2*x)/(2*c^3) + (b^2*e^2*x^2)/(12*c^2) - (2*b^2*d*
e*x*ArcTan[c*x])/c + (b^2*e^2*x*ArcTan[c*x])/(2*c^3) - (b*e^2*x^3*(a + b*Ar
cTan[c*x]))/(6*c) + (d*e*(a + b*ArcTan[c*x])^2)/c^2 - (e^2*(a + b*ArcTan[c*
x])^2)/(4*c^4) + d*e*x^2*(a + b*ArcTan[c*x])^2 + (e^2*x^4*(a + b*ArcTan[c*x
])^2)/4 + 2*d^2*(a + b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1 + I*c*x)] + (b^2*d*e
*Log[1 + c^2*x^2])/c^2 - (b^2*e^2*Log[1 + c^2*x^2])/(3*c^4) - I*b*d^2*(a +
b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)] + I*b*d^2*(a + b*ArcTan[c*x])*
PolyLog[2, -1 + 2/(1 + I*c*x)] - (b^2*d^2*PolyLog[3, 1 - 2/(1 + I*c*x)])/2
+ (b^2*d^2*PolyLog[3, -1 + 2/(1 + I*c*x)])/2
```

Rubi [A] time = 0.689985, antiderivative size = 355, normalized size of antiderivative = 1., number of steps used = 25, number of rules used = 12, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.522, Rules used = {4980, 4850, 4988, 4884, 4994, 6610, 4852, 4916, 4846, 260, 266, 43}

$$-ibd^2\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)+ibd^2\text{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)-\frac{1}{2}b^2d^2\text{PolyLog}\Big(3$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^2*(a + b*ArcTan[c*x])^2)/x,x]
```


```
[Out] (-2*a*b*d*e*x)/c + (a*b*e^2*x)/(2*c^3) + (b^2*e^2*x^2)/(12*c^2) - (2*b^2*d*
e*x*ArcTan[c*x])/c + (b^2*e^2*x*ArcTan[c*x])/(2*c^3) - (b*e^2*x^3*(a + b*Ar
cTan[c*x]))/(6*c) + (d*e*(a + b*ArcTan[c*x])^2)/c^2 - (e^2*(a + b*ArcTan[c*
x])^2)/(4*c^4) + d*e*x^2*(a + b*ArcTan[c*x])^2 + (e^2*x^4*(a + b*ArcTan[c*x
])^2)/4 + 2*d^2*(a + b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1 + I*c*x)] + (b^2*d*e
*Log[1 + c^2*x^2])/c^2 - (b^2*e^2*Log[1 + c^2*x^2])/(3*c^4) - I*b*d^2*(a +
b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)] + I*b*d^2*(a + b*ArcTan[c*x])*
PolyLog[2, -1 + 2/(1 + I*c*x)] - (b^2*d^2*PolyLog[3, 1 - 2/(1 + I*c*x)])/2
+ (b^2*d^2*PolyLog[3, -1 + 2/(1 + I*c*x)])/2
```


Rule 4980

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

Rule 4850

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTan[c*x])^(p - 1)*ArcTanh[1 - 2/(1 + I*c*x)])/(1 + c^2*x^2), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4988

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTan[c*x])^p)/(d + e
*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTan[c*x])^p)/(d + e*x^2
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
```

```
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(d+ex^2)^2 \left(a+b\tan^{-1}(cx)\right)^2}{x}\,dx = \int \left(\frac{d^2\left(a+b\tan^{-1}(cx)\right)^2}{x} + 2dex\left(a+b\tan^{-1}(cx)\right)^2 + e^2x^3\left(a+b\tan^{-1}(cx)\right)^2\right) d$$

$$= d^2\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x}\,dx + (2de)\int x\left(a+b\tan^{-1}(cx)\right)^2\,dx + e^2\int x^3\left(a+b\tan\right.$$

$$= dex^2\left(a+b\tan^{-1}(cx)\right)^2 + \frac{1}{4}e^2x^4\left(a+b\tan^{-1}(cx)\right)^2 + 2d^2\left(a+b\tan^{-1}(cx)\right)^2 \tanh$$

$$= dex^2\left(a+b\tan^{-1}(cx)\right)^2 + \frac{1}{4}e^2x^4\left(a+b\tan^{-1}(cx)\right)^2 + 2d^2\left(a+b\tan^{-1}(cx)\right)^2 \tanh$$

$$= -\frac{2abdex}{c} - \frac{be^2x^3\left(a+b\tan^{-1}(cx)\right)}{6c} + \frac{de\left(a+b\tan^{-1}(cx)\right)^2}{c^2} + dex^2\left(a+b\tan^{-1}(\right.$$

$$= -\frac{2abdex}{c} + \frac{abe^2x}{2c^3} - \frac{2b^2dex\tan^{-1}(cx)}{c} - \frac{be^2x^3\left(a+b\tan^{-1}(cx)\right)}{6c} + \frac{de\left(a+b\tan\right.}{c^2}$$

$$= -\frac{2abdex}{c} + \frac{abe^2x}{2c^3} - \frac{2b^2dex\tan^{-1}(cx)}{c} + \frac{b^2e^2x\tan^{-1}(cx)}{2c^3} - \frac{be^2x^3\left(a+b\tan^{-1}(c\right.}{6c}$$

$$= -\frac{2abdex}{c} + \frac{abe^2x}{2c^3} + \frac{b^2e^2x^2}{12c^2} - \frac{2b^2dex\tan^{-1}(cx)}{c} + \frac{b^2e^2x\tan^{-1}(cx)}{2c^3} - \frac{be^2x^3\left(a+\right.}{6}$$

Mathematica [A] time = 0.668017, size = 389, normalized size = 1.1

$$iabd^2(\text{PolyLog}(2,-icx) - \text{PolyLog}(2,icx)) + b^2d^2\left(i\tan^{-1}(cx)\text{PolyLog}\left(2,e^{-2i\tan^{-1}(cx)}\right) + i\tan^{-1}(cx)\text{PolyLog}\left(2,-e^{2i}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + e*x^2)^2*(a + b*ArcTan[c*x])^2)/x,x]
```

```
[Out] a^2*d*e*x^2 + (a^2*e^2*x^4)/4 + (2*a*b*d*e*(-(c*x) + (1 + c^2*x^2)*ArcTan[c
*x]))/c^2 + (a*b*e^2*(3*c*x - c^3*x^3 + 3*(-1 + c^4*x^4)*ArcTan[c*x]))/(6*c
^4) + a^2*d^2*Log[x] + (b^2*e^2*(1 + c^2*x^2 + (6*c*x - 2*c^3*x^3)*ArcTan[c
*x] + 3*(-1 + c^4*x^4)*ArcTan[c*x]^2 - 4*Log[1 + c^2*x^2]))/(12*c^4) + (b^2
*d*e*(-2*c*x*ArcTan[c*x] + (1 + c^2*x^2)*ArcTan[c*x]^2 + Log[1 + c^2*x^2]))
/c^2 + I*a*b*d^2*(PolyLog[2, (-I)*c*x] - PolyLog[2, I*c*x]) + b^2*d^2*((-I/
24)*Pi^3 + ((2*I)/3)*ArcTan[c*x]^3 + ArcTan[c*x]^2*Log[1 - E^((-2*I)*ArcTan
[c*x])] - ArcTan[c*x]^2*Log[1 + E^((2*I)*ArcTan[c*x])] + I*ArcTan[c*x]*Poly
```

```
Log[2, E^((-2*I)*ArcTan[c*x])] + I*ArcTan[c*x]*PolyLog[2, -E^((2*I)*ArcTan[
c*x])] + PolyLog[3, E^((-2*I)*ArcTan[c*x])]/2 - PolyLog[3, -E^((2*I)*ArcTan
[c*x])]/2)
```

Maple [C] time = 4.632, size = 1549, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(a+b*arctan(c*x))^2/x,x)
```

```
[Out] a^2*x^2*d*e-1/4*b^2/c^4*arctan(c*x)^2*e^2+b^2*d^2*arctan(c*x)^2*ln(1-(1+I*c
*x)/(c^2*x^2+1)^(1/2))+1/12*b^2*e^2*x^2/c^2-2*a*b*d*e*x/c-2*b^2*d*e*x*arcta
n(c*x)/c+1/2*a*b*e^2*x/c^3+1/2*b^2*e^2*x*arctan(c*x)/c^3+b^2*arctan(c*x)^2*
x^2*d*e+2*a*b*arctan(c*x)*d^2*ln(c*x)+I*a*b*d^2*dilog(1+I*c*x)-I*a*b*d^2*di
log(1-I*c*x)-1/6*a*b/c*x^3*e^2-1/2*I*b^2*d^2*Pi*csgn(I/((1+I*c*x)^2/(c^2*x^
2+1)+1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*
arctan(c*x)^2-1/2*I*b^2*d^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x
)^2/(c^2*x^2+1)+1))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+
1)+1))^2*arctan(c*x)^2+1/2*I*b^2*d^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/
((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/
(c^2*x^2+1)+1))*arctan(c*x)^2-1/2*I*b^2*d^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2
+1)-1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*a
rctan(c*x)^2-1/6*b^2/c*arctan(c*x)*x^3*e^2+1/2*a*b*arctan(c*x)*x^4*e^2-1/2*
a*b/c^4*arctan(c*x)*e^2+b^2/c^2*arctan(c*x)^2*d*e-2*b^2/c^2*e*d*ln((1+I*c*x
)^2/(c^2*x^2+1)+1)+I*b^2*d^2*arctan(c*x)*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1)
)-2*I*b^2*d^2*arctan(c*x)*polylog(2,-(1+I*c*x)/(c^2*x^2+1)^(1/2))+1/2*I*b^2
*d^2*Pi*arctan(c*x)^2-2*I*b^2*d^2*arctan(c*x)*polylog(2,(1+I*c*x)/(c^2*x^2+
1)^(1/2))-2/3*I*b^2/c^4*arctan(c*x)*e^2+2*b^2*d^2*polylog(3,(1+I*c*x)/(c^2*
x^2+1)^(1/2))+2*b^2*d^2*polylog(3,-(1+I*c*x)/(c^2*x^2+1)^(1/2))-1/2*b^2*d^2
*polylog(3,-(1+I*c*x)^2/(c^2*x^2+1))+1/4*a^2*x^4*e^2+b^2*d^2*arctan(c*x)^2*
ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))+b^2*arctan(c*x)^2*d^2*ln(c*x)-b^2*d^2*arc
tan(c*x)^2*ln((1+I*c*x)^2/(c^2*x^2+1)-1)+2/3*b^2/c^4*e^2*ln((1+I*c*x)^2/(c^
2*x^2+1)+1)+1/4*b^2*arctan(c*x)^2*x^4*e^2+2*a*b/c^2*arctan(c*x)*d*e+I*a*b*d
^2*ln(c*x)*ln(1+I*c*x)-I*a*b*d^2*ln(c*x)*ln(1-I*c*x)+1/2*I*b^2*d^2*Pi*csgn(
I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*arctan(c*x)^2-
1/2*I*b^2*d^2*Pi*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+
1))^2*arctan(c*x)^2+1/2*I*b^2*d^2*Pi*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I
*c*x)^2/(c^2*x^2+1)+1))^3*arctan(c*x)^2+2*I*b^2/c^2*arctan(c*x)*e*d+2*a*b*a
rctan(c*x)*x^2*d*e+a^2*d^2*ln(c*x)+1/2*I*b^2*d^2*Pi*csgn(I*((1+I*c*x)^2/(c^
2*x^2+1)-1))*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*((1+I*c*x)^2/(c^2*x
```

```
^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)^2+1/12*b^2/c^4*e^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arctan(c*x))^2/x,x, algorithm="maxima")
```

```
[Out] 1/4*a^2*e^2*x^4 + 12*b^2*c^2*e^2*integrate(1/16*x^6*arctan(c*x)^2/(c^2*x^3
+ x), x) + b^2*c^2*e^2*integrate(1/16*x^6*log(c^2*x^2 + 1)^2/(c^2*x^3 + x),
 x) + 32*a*b*c^2*e^2*integrate(1/16*x^6*arctan(c*x)/(c^2*x^3 + x), x) + b^2
*c^2*e^2*integrate(1/16*x^6*log(c^2*x^2 + 1)/(c^2*x^3 + x), x) + 24*b^2*c^2
*d*e*integrate(1/16*x^4*arctan(c*x)^2/(c^2*x^3 + x), x) + 2*b^2*c^2*d*e*int
egrate(1/16*x^4*log(c^2*x^2 + 1)^2/(c^2*x^3 + x), x) + 64*a*b*c^2*d*e*integ
rate(1/16*x^4*arctan(c*x)/(c^2*x^3 + x), x) + 4*b^2*c^2*d*e*integrate(1/16*
x^4*log(c^2*x^2 + 1)/(c^2*x^3 + x), x) + 12*b^2*c^2*d^2*integrate(1/16*x^2*
arctan(c*x)^2/(c^2*x^3 + x), x) + 32*a*b*c^2*d^2*integrate(1/16*x^2*arctan(
c*x)/(c^2*x^3 + x), x) + 1/96*b^2*d^2*log(c^2*x^2 + 1)^3 + a^2*d*e*x^2 - 2*
b^2*c*e^2*integrate(1/16*x^5*arctan(c*x)/(c^2*x^3 + x), x) - 8*b^2*c*d*e*in
tegrate(1/16*x^3*arctan(c*x)/(c^2*x^3 + x), x) + 12*b^2*e^2*integrate(1/16*
x^4*arctan(c*x)^2/(c^2*x^3 + x), x) + b^2*e^2*integrate(1/16*x^4*log(c^2*x^
2 + 1)^2/(c^2*x^3 + x), x) + 32*a*b*e^2*integrate(1/16*x^4*arctan(c*x)/(c^2
*x^3 + x), x) + 24*b^2*d*e*integrate(1/16*x^2*arctan(c*x)^2/(c^2*x^3 + x),
x) + 64*a*b*d*e*integrate(1/16*x^2*arctan(c*x)/(c^2*x^3 + x), x) + 12*b^2*d
^2*integrate(1/16*arctan(c*x)^2/(c^2*x^3 + x), x) + b^2*d^2*integrate(1/16*
log(c^2*x^2 + 1)^2/(c^2*x^3 + x), x) + 32*a*b*d^2*integrate(1/16*arctan(c*x
)/(c^2*x^3 + x), x) + 1/48*b^2*d*e*log(c^2*x^2 + 1)^3/c^2 + a^2*d^2*log(x)
+ 1/16*(b^2*e^2*x^4 + 4*b^2*d*e*x^2)*arctan(c*x)^2 - 1/64*(b^2*e^2*x^4 + 4*
b^2*d*e*x^2)*log(c^2*x^2 + 1)^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{a^2e^2x^4 + 2\,a^2dex^2 + a^2d^2 + \left(b^2e^2x^4 + 2\,b^2dex^2 + b^2d^2\right)\arctan\left(cx\right)^2 + 2\left(abe^2x^4 + 2\,abdex^2 + abd^2\right)\arctan\left(c\right.}{x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arctan(c*x))^2/x,x, algorithm="fricas")
```

```
[Out] integral((a^2*e^2*x^4 + 2*a^2*d*e*x^2 + a^2*d^2 + (b^2*e^2*x^4 + 2*b^2*d*e*
x^2 + b^2*d^2)*arctan(c*x)^2 + 2*(a*b*e^2*x^4 + 2*a*b*d*e*x^2 + a*b*d^2)*ar
ctan(c*x))/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b\,\mathrm{atan}\,(cx))^2\left(d + ex^2\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(a+b*atan(c*x))**2/x,x)
```

```
[Out] Integral((a + b*atan(c*x))**2*(d + e*x**2)**2/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2 + d\right)^2(b\arctan\,(cx) + a)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arctan(c*x))^2/x,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^2*(b*arctan(c*x) + a)^2/x, x)
```

3.1259 $\int \frac{\left(d+ex^2\right)^2\left(a+b\tan^{-1}(cx)\right)^2}{x^2}\,dx$

Optimal. Leaf size=343

$$-\frac{ib^2e^2\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{3c^3}-ib^2cd^2\text{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)+\frac{2ib^2de\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c}-\frac{ie^2\left(a+b\tan^{-1}(cx\right.}{3c^3}$$

```
[Out] (b^2*e^2*x)/(3*c^2) - (b^2*e^2*ArcTan[c*x])/(3*c^3) - (b*e^2*x^2*(a + b*Arc
Tan[c*x]))/(3*c) - I*c*d^2*(a + b*ArcTan[c*x])^2 + ((2*I)*d*e*(a + b*ArcTan
[c*x])^2)/c - ((I/3)*e^2*(a + b*ArcTan[c*x])^2)/c^3 - (d^2*(a + b*ArcTan[c*
x])^2)/x + 2*d*e*x*(a + b*ArcTan[c*x])^2 + (e^2*x^3*(a + b*ArcTan[c*x])^2)/
3 + (4*b*d*e*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/c - (2*b*e^2*(a + b*Ar
cTan[c*x])*Log[2/(1 + I*c*x)])/(3*c^3) + 2*b*c*d^2*(a + b*ArcTan[c*x])*Log[
2 - 2/(1 - I*c*x)] - I*b^2*c*d^2*PolyLog[2, -1 + 2/(1 - I*c*x)] + ((2*I)*b^
2*d*e*PolyLog[2, 1 - 2/(1 + I*c*x)])/c - ((I/3)*b^2*e^2*PolyLog[2, 1 - 2/(1
 + I*c*x)])/c^3
```

Rubi [A] time = 0.575716, antiderivative size = 343, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 13, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.565, Rules used = {4980, 4846, 4920, 4854, 2402, 2315, 4852, 4924, 4868, 2447, 4916, 321, 203}

$$-\frac{ib^2e^2\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{3c^3}-ib^2cd^2\text{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)+\frac{2ib^2de\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c}-\frac{ie^2\left(a+b\tan^{-1}(cx\right.}{3c^3}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^2*(a + b*ArcTan[c*x])^2)/x^2,x]
```


```
[Out] (b^2*e^2*x)/(3*c^2) - (b^2*e^2*ArcTan[c*x])/(3*c^3) - (b*e^2*x^2*(a + b*Arc
Tan[c*x]))/(3*c) - I*c*d^2*(a + b*ArcTan[c*x])^2 + ((2*I)*d*e*(a + b*ArcTan
[c*x])^2)/c - ((I/3)*e^2*(a + b*ArcTan[c*x])^2)/c^3 - (d^2*(a + b*ArcTan[c*
x])^2)/x + 2*d*e*x*(a + b*ArcTan[c*x])^2 + (e^2*x^3*(a + b*ArcTan[c*x])^2)/
3 + (4*b*d*e*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/c - (2*b*e^2*(a + b*Ar
cTan[c*x])*Log[2/(1 + I*c*x)])/(3*c^3) + 2*b*c*d^2*(a + b*ArcTan[c*x])*Log[
2 - 2/(1 - I*c*x)] - I*b^2*c*d^2*PolyLog[2, -1 + 2/(1 - I*c*x)] + ((2*I)*b^
2*d*e*PolyLog[2, 1 - 2/(1 + I*c*x)])/c - ((I/3)*b^2*e^2*PolyLog[2, 1 - 2/(1
 + I*c*x)])/c^3
```

Rule 4980

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
```

```
erQ[m]) && NeQ[m, -1]
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^2\left(a+b\tan^{-1}(cx)\right)^2}{x^2}\,dx = \int \left(2de\left(a+b\tan^{-1}(cx)\right)^2 + \frac{d^2\left(a+b\tan^{-1}(cx)\right)^2}{x^2} + e^2x^2\left(a+b\tan^{-1}(cx)\right)^2\right)dx$$

$$= d^2\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^2}\,dx + (2de)\int \left(a+b\tan^{-1}(cx)\right)^2\,dx + e^2\int x^2\left(a+b\tan^{-1}\right.$$

$$= -\frac{d^2\left(a+b\tan^{-1}(cx)\right)^2}{x} + 2dex\left(a+b\tan^{-1}(cx)\right)^2 + \frac{1}{3}e^2x^3\left(a+b\tan^{-1}(cx)\right)^2 + (2b$$

$$= -icd^2\left(a+b\tan^{-1}(cx)\right)^2 + \frac{2ide\left(a+b\tan^{-1}(cx)\right)^2}{c} - \frac{d^2\left(a+b\tan^{-1}(cx)\right)^2}{x} + 2dex$$

$$= -\frac{be^2x^2\left(a+b\tan^{-1}(cx)\right)}{3c} - icd^2\left(a+b\tan^{-1}(cx)\right)^2 + \frac{2ide\left(a+b\tan^{-1}(cx)\right)^2}{c} - \frac{ie^2}{}$$

$$= \frac{b^2e^2x}{3c^2} - \frac{be^2x^2\left(a+b\tan^{-1}(cx)\right)}{3c} - icd^2\left(a+b\tan^{-1}(cx)\right)^2 + \frac{2ide\left(a+b\tan^{-1}(cx)\right)}{c}$$

$$= \frac{b^2e^2x}{3c^2} - \frac{b^2e^2\tan^{-1}(cx)}{3c^3} - \frac{be^2x^2\left(a+b\tan^{-1}(cx)\right)}{3c} - icd^2\left(a+b\tan^{-1}(cx)\right)^2 + \frac{2ide}{}$$

$$= \frac{b^2e^2x}{3c^2} - \frac{b^2e^2\tan^{-1}(cx)}{3c^3} - \frac{be^2x^2\left(a+b\tan^{-1}(cx)\right)}{3c} - icd^2\left(a+b\tan^{-1}(cx)\right)^2 + \frac{2ide}{}$$

Mathematica [A] time = 0.766775, size = 349, normalized size = 1.02

$$\frac{1}{3}\left(\frac{b^2e^2\left(i\text{PolyLog}\left(2,-e^{2i\tan^{-1}(cx)}\right)+\left(c^3x^3+i\right)\tan^{-1}(cx)^2-\tan^{-1}(cx)\left(c^2x^2+2\log\left(1+e^{2i\tan^{-1}(cx)}\right)+1\right)+cx\right)}{c^3}+3b^2c\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + e*x^2)^2*(a + b*ArcTan[c*x])^2)/x^2,x]
```

```
[Out] ((-3*a^2*d^2)/x + 6*a^2*d*e*x + a^2*e^2*x^3 + (6*a*b*d*e*(2*c*x*ArcTan[c*x]
 - Log[1 + c^2*x^2]))/c + (a*b*e^2*(-(c^2*x^2) + 2*c^3*x^3*ArcTan[c*x] + Lo
g[1 + c^2*x^2]))/c^3 - (3*a*b*d^2*(2*ArcTan[c*x] + c*x*(-2*Log[c*x] + Log[1
 + c^2*x^2])))/x + (6*b^2*d*e*(ArcTan[c*x]*((-I + c*x)*ArcTan[c*x] + 2*Log[
1 + E^((2*I)*ArcTan[c*x])]) - I*PolyLog[2, -E^((2*I)*ArcTan[c*x])]))/c + (b
```

```
^2*e^2*(c*x + (I + c^3*x^3)*ArcTan[c*x]^2 - ArcTan[c*x]*(1 + c^2*x^2 + 2*Lo
g[1 + E^((2*I)*ArcTan[c*x])]) + I*PolyLog[2, -E^((2*I)*ArcTan[c*x])]))/c^3
+ 3*b^2*c*d^2*(ArcTan[c*x]*((-I - 1/(c*x))*ArcTan[c*x] + 2*Log[1 - E^((2*I)
*ArcTan[c*x])]) - I*PolyLog[2, E^((2*I)*ArcTan[c*x])]))/3
```

Maple [B] time = 0.145, size = 997, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(a+b*arctan(c*x))^2/x^2,x)
```

```
[Out] -1/2*I*c*b^2*dilog(1/2*I*(c*x-I))*d^2+1/3*b^2*e^2*x/c^2-1/3*b^2*e^2*arctan(
c*x)/c^3+1/3*a^2*x^3*e^2-a^2*d^2/x-1/6*I*b^2/c^3*ln(c*x+I)*ln(c^2*x^2+1)*e^
2-2*b^2/c*arctan(c*x)*ln(c^2*x^2+1)*d*e+I*b^2/c*ln(c*x+I)*ln(c^2*x^2+1)*d*e
+I*b^2/c*ln(c*x-I)*ln(-1/2*I*(c*x+I))*d*e-I*b^2/c*ln(c*x-I)*ln(c^2*x^2+1)*d
*e-I*b^2/c*ln(c*x+I)*ln(1/2*I*(c*x-I))*d*e-b^2*arctan(c*x)^2*d^2/x+2*a^2*e*
d*x+1/3*b^2*arctan(c*x)^2*x^3*e^2-1/2*I*b^2/c*ln(c*x+I)^2*d*e-1/6*I*b^2/c^3
*ln(c*x-I)*ln(-1/2*I*(c*x+I))*e^2+1/6*I*b^2/c^3*ln(c*x-I)*ln(c^2*x^2+1)*e^2
+1/6*I*b^2/c^3*ln(c*x+I)*ln(1/2*I*(c*x-I))*e^2+I*b^2/c*dilog(-1/2*I*(c*x+I)
)*d*e+4*a*b*arctan(c*x)*e*d*x-2*a*b/c*ln(c^2*x^2+1)*d*e-1/2*I*c*b^2*ln(c*x+
I)*ln(1/2*I*(c*x-I))*d^2+1/2*I*c*b^2*ln(c*x+I)*ln(c^2*x^2+1)*d^2+I*c*b^2*d^
2*ln(c*x)*ln(1+I*c*x)+1/2*I*b^2/c*ln(c*x-I)^2*d*e-I*b^2/c*dilog(1/2*I*(c*x-
I))*d*e-I*c*b^2*d^2*ln(c*x)*ln(1-I*c*x)+1/2*I*c*b^2*ln(c*x-I)*ln(-1/2*I*(c*
x+I))*d^2-1/2*I*c*b^2*ln(c*x-I)*ln(c^2*x^2+1)*d^2-c*a*b*ln(c^2*x^2+1)*d^2+I
*c*b^2*d^2*dilog(1+I*c*x)+2*b^2*arctan(c*x)^2*e*d*x+2/3*a*b*arctan(c*x)*x^3
*e^2-1/3*a*b/c*x^2*e^2-1/3*b^2/c*arctan(c*x)*x^2*e^2+1/3*b^2/c^3*arctan(c*x
)*ln(c^2*x^2+1)*e^2+1/3*a*b/c^3*ln(c^2*x^2+1)*e^2+1/4*I*c*b^2*ln(c*x-I)^2*d
^2+1/2*I*c*b^2*dilog(-1/2*I*(c*x+I))*d^2-1/4*I*c*b^2*ln(c*x+I)^2*d^2-I*c*b^
2*d^2*dilog(1-I*c*x)-1/12*I*b^2/c^3*ln(c*x-I)^2*e^2+1/6*I*b^2/c^3*dilog(1/2
*I*(c*x-I))*e^2-1/6*I*b^2/c^3*dilog(-1/2*I*(c*x+I))*e^2+1/12*I*b^2/c^3*ln(c
*x+I)^2*e^2-2*a*b*arctan(c*x)*d^2/x+2*c*b^2*arctan(c*x)*d^2*ln(c*x)-c*b^2*a
rctan(c*x)*ln(c^2*x^2+1)*d^2+2*c*a*b*d^2*ln(c*x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arctan(c*x))^2/x^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{a^2e^2x^4 + 2\,a^2dex^2 + a^2d^2 + \left(b^2e^2x^4 + 2\,b^2dex^2 + b^2d^2\right)\arctan\left(cx\right)^2 + 2\left(abe^2x^4 + 2\,abdex^2 + abd^2\right)\arctan\left(cx\right)}{x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arctan(c*x))^2/x^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*e^2*x^4 + 2*a^2*d*e*x^2 + a^2*d^2 + (b^2*e^2*x^4 + 2*b^2*d*e*
x^2 + b^2*d^2)*arctan(c*x)^2 + 2*(a*b*e^2*x^4 + 2*a*b*d*e*x^2 + a*b*d^2)*ar
ctan(c*x))/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + b\,\mathrm{atan}\left(cx\right)\right)^2\left(d + ex^2\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(a+b*atan(c*x))**2/x**2,x)
```

```
[Out] Integral((a + b*atan(c*x))**2*(d + e*x**2)**2/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2 + d\right)^2\left(b\arctan\left(cx\right) + a\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arctan(c*x))^2/x^2,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^2*(b*arctan(c*x) + a)^2/x^2, x)
```

3.1260 $\int \frac{(d+ex^2)^2 (a+b\tan^{-1}(cx))^2}{x^3} dx$

Optimal. Leaf size=320

$$-2ibde\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)+2ibde\text{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)-b^2de\text{PolyLog}\left(\right.$$

```
[Out] -((a*b*e^2*x)/c) - (b^2*e^2*x*ArcTan[c*x])/c - (b*c*d^2*(a + b*ArcTan[c*x])
)/x - (c^2*d^2*(a + b*ArcTan[c*x])^2)/2 + (e^2*(a + b*ArcTan[c*x])^2)/(2*c^
2) - (d^2*(a + b*ArcTan[c*x])^2)/(2*x^2) + (e^2*x^2*(a + b*ArcTan[c*x])^2)/
2 + 4*d*e*(a + b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1 + I*c*x)] + b^2*c^2*d^2*Lo
g[x] - (b^2*c^2*d^2*Log[1 + c^2*x^2])/2 + (b^2*e^2*Log[1 + c^2*x^2])/(2*c^2
) - (2*I)*b*d*e*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)] + (2*I)*b
*d*e*(a + b*ArcTan[c*x])*PolyLog[2, -1 + 2/(1 + I*c*x)] - b^2*d*e*PolyLog[3
, 1 - 2/(1 + I*c*x)] + b^2*d*e*PolyLog[3, -1 + 2/(1 + I*c*x)]
```

Rubi [A] time = 0.608636, antiderivative size = 320, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 15, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.652, Rules used = {4980, 4852, 4918, 266, 36, 29, 31, 4884, 4850, 4988, 4994, 6610, 4916, 4846, 260}

$$-2ibde\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)+2ibde\text{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)-b^2de\text{PolyLog}\left(\right.$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^2*(a + b*ArcTan[c*x])^2)/x^3,x]
```


```
[Out] -((a*b*e^2*x)/c) - (b^2*e^2*x*ArcTan[c*x])/c - (b*c*d^2*(a + b*ArcTan[c*x])
)/x - (c^2*d^2*(a + b*ArcTan[c*x])^2)/2 + (e^2*(a + b*ArcTan[c*x])^2)/(2*c^
2) - (d^2*(a + b*ArcTan[c*x])^2)/(2*x^2) + (e^2*x^2*(a + b*ArcTan[c*x])^2)/
2 + 4*d*e*(a + b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1 + I*c*x)] + b^2*c^2*d^2*Lo
g[x] - (b^2*c^2*d^2*Log[1 + c^2*x^2])/2 + (b^2*e^2*Log[1 + c^2*x^2])/(2*c^2
) - (2*I)*b*d*e*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)] + (2*I)*b
*d*e*(a + b*ArcTan[c*x])*PolyLog[2, -1 + 2/(1 + I*c*x)] - b^2*d*e*PolyLog[3
, 1 - 2/(1 + I*c*x)] + b^2*d*e*PolyLog[3, -1 + 2/(1 + I*c*x)]
```


Rule 4980

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
```

```
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4918

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/(d + e*x^2),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
```

```
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4850

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTan[c*x])^(p - 1)*ArcTanh[1 - 2/(1 + I*c*x)])/(1 + c^2*x^2), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4988

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTan[c*x])^p)/(d + e
*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTan[c*x])^p)/(d + e*x^2
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^2\left(a+b\tan^{-1}(cx)\right)^2}{x^3}\,dx = \int\left(\frac{d^2\left(a+b\tan^{-1}(cx)\right)^2}{x^3}+\frac{2de\left(a+b\tan^{-1}(cx)\right)^2}{x}+e^2x\left(a+b\tan^{-1}(cx)\right)^2\right)dx$$

$$= d^2\int\frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^3}\,dx+(2de)\int\frac{\left(a+b\tan^{-1}(cx)\right)^2}{x}\,dx+e^2\int x\left(a+b\tan\right.$$

$$= -\frac{d^2\left(a+b\tan^{-1}(cx)\right)^2}{2x^2}+\frac{1}{2}e^2x^2\left(a+b\tan^{-1}(cx)\right)^2+4de\left(a+b\tan^{-1}(cx)\right)^2\tanh$$

$$= -\frac{d^2\left(a+b\tan^{-1}(cx)\right)^2}{2x^2}+\frac{1}{2}e^2x^2\left(a+b\tan^{-1}(cx)\right)^2+4de\left(a+b\tan^{-1}(cx)\right)^2\tanh$$

$$= -\frac{abe^2x}{c}-\frac{bcd^2\left(a+b\tan^{-1}(cx)\right)}{x}-\frac{1}{2}c^2d^2\left(a+b\tan^{-1}(cx)\right)^2+\frac{e^2\left(a+b\tan^{-1}(cx\right.}{2c^2}$$

$$= -\frac{abe^2x}{c}-\frac{b^2e^2x\tan^{-1}(cx)}{c}-\frac{bcd^2\left(a+b\tan^{-1}(cx)\right)}{x}-\frac{1}{2}c^2d^2\left(a+b\tan^{-1}(cx)\right)^2+$$

$$= -\frac{abe^2x}{c}-\frac{b^2e^2x\tan^{-1}(cx)}{c}-\frac{bcd^2\left(a+b\tan^{-1}(cx)\right)}{x}-\frac{1}{2}c^2d^2\left(a+b\tan^{-1}(cx)\right)^2+$$

$$= -\frac{abe^2x}{c}-\frac{b^2e^2x\tan^{-1}(cx)}{c}-\frac{bcd^2\left(a+b\tan^{-1}(cx)\right)}{x}-\frac{1}{2}c^2d^2\left(a+b\tan^{-1}(cx)\right)^2+$$

Mathematica [A] time = 0.584555, size = 367, normalized size = 1.15

$$\frac{1}{2}\left(4iabde(\text{PolyLog}(2,-icx)-\text{PolyLog}(2,icx))+\frac{1}{6}b^2de\left(24i\tan^{-1}(cx)\text{PolyLog}\left(2,e^{-2i\tan^{-1}(cx)}\right)+24i\tan^{-1}(cx)\text{PolyI}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + e*x^2)^2*(a + b*ArcTan[c*x])^2)/x^3,x]
```

```
[Out] (-((a^2*d^2)/x^2) + a^2*e^2*x^2 + (2*a*b*e^2*(-(c*x) + (1 + c^2*x^2)*ArcTan
[c*x]))/c^2 - (2*a*b*d^2*(ArcTan[c*x] + c*x*(1 + c*x*ArcTan[c*x])))/x^2 + 4
```

```
*a^2*d*e*Log[x] - (b^2*d^2*(2*c*x*ArcTan[c*x] + (1 + c^2*x^2)*ArcTan[c*x]^2
 - 2*c^2*x^2*Log[(c*x)/Sqrt[1 + c^2*x^2]]))/x^2 + (b^2*e^2*(-2*c*x*ArcTan[c
*x] + (1 + c^2*x^2)*ArcTan[c*x]^2 + Log[1 + c^2*x^2]))/c^2 + (4*I)*a*b*d*e*
(PolyLog[2, (-I)*c*x] - PolyLog[2, I*c*x]) + (b^2*d*e*((-I)*Pi^3 + (16*I)*A
rcTan[c*x]^3 + 24*ArcTan[c*x]^2*Log[1 - E^((-2*I)*ArcTan[c*x])] - 24*ArcTan
[c*x]^2*Log[1 + E^((2*I)*ArcTan[c*x])] + (24*I)*ArcTan[c*x]*PolyLog[2, E^((
-2*I)*ArcTan[c*x])] + (24*I)*ArcTan[c*x]*PolyLog[2, -E^((2*I)*ArcTan[c*x])]
 + 12*PolyLog[3, E^((-2*I)*ArcTan[c*x])] - 12*PolyLog[3, -E^((2*I)*ArcTan[c
*x])]))/6)/2
```

Maple [C] time = 3.498, size = 1511, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(a+b*arctan(c*x))^2/x^3,x)
```

```
[Out] 4*b^2*e*d*polylog(3,(1+I*c*x)/(c^2*x^2+1)^(1/2))+4*b^2*e*d*polylog(3,-(1+I*
c*x)/(c^2*x^2+1)^(1/2))-b^2*e*d*polylog(3,-(1+I*c*x)^2/(c^2*x^2+1))+1/2*b^2
*arctan(c*x)^2*x^2*e^2-1/2*c^2*b^2*arctan(c*x)^2*d^2+c^2*b^2*d^2*ln((1+I*c*
x)/(c^2*x^2+1)^(1/2)-1)+c^2*b^2*d^2*ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))+1/2/c
^2*b^2*arctan(c*x)^2*e^2-1/c^2*b^2*e^2*ln((1+I*c*x)^2/(c^2*x^2+1)+1)-1/2*b^
2*arctan(c*x)^2*d^2/x^2+2*a^2*e*d*ln(c*x)-I*b^2*e*d*Pi*csgn(((1+I*c*x)^2/(c
^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2+2*I*a*b*e*d*ln(c*
x)*ln(1+I*c*x)-2*I*a*b*e*d*ln(c*x)*ln(1-I*c*x)+1/c^2*a*b*arctan(c*x)*e^2-c*
b^2*arctan(c*x)*d^2/x-c^2*a*b*arctan(c*x)*d^2+a*b*arctan(c*x)*x^2*e^2-a*b*a
rctan(c*x)*d^2/x^2+1/2*a^2*x^2*e^2-a*b*e^2*x/c-b^2*e^2*x*arctan(c*x)/c+I*b^
2*e*d*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I/((1+I*c*x)^2/(c^2*x^2+1
)+1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*arcta
n(c*x)^2+I*b^2*e*d*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*
x^2+1)+1))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*ar
ctan(c*x)^2-I*b^2*e*d*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c
^2*x^2+1)+1))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))
^2*arctan(c*x)^2-I*b^2*e*d*Pi*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*((
1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2-I*b^
2*e*d*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1
)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2-1/2*a^2*d^2/x^2+I*b^2*e*d
*Pi*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*arctan(
c*x)^2+I*b^2*e*d*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^
2+1)+1))^3*arctan(c*x)^2-c*a*b*d^2/x-2*I*a*b*e*d*dilog(1-I*c*x)-4*I*b^2*e*d
*arctan(c*x)*polylog(2,(1+I*c*x)/(c^2*x^2+1)^(1/2))-4*I*b^2*e*d*arctan(c*x)
```

```
*polylog(2,-(1+I*c*x)/(c^2*x^2+1)^(1/2))+2*I*b^2*e*d*arctan(c*x)*polylog(2,
-(1+I*c*x)^2/(c^2*x^2+1))+2*I*a*b*e*d*dilog(1+I*c*x)+I*b^2*e*d*Pi*arctan(c*
x)^2+4*a*b*arctan(c*x)*e*d*ln(c*x)-2*b^2*e*d*arctan(c*x)^2*ln((1+I*c*x)^2/(
c^2*x^2+1)-1)+2*b^2*arctan(c*x)^2*e*d*ln(c*x)+2*b^2*e*d*arctan(c*x)^2*ln(1-
(1+I*c*x)/(c^2*x^2+1)^(1/2))+2*b^2*e*d*arctan(c*x)^2*ln(1+(1+I*c*x)/(c^2*x^
2+1)^(1/2))+I/c^2*b^2*arctan(c*x)*e^2-I*c^2*b^2*arctan(c*x)*d^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arctan(c*x))^2/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*e^2*x^2 - ((c*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*a*b*d^2 + 2*a
^2*d*e*log(x) - 1/2*a^2*d^2/x^2 + 1/96*((1152*b^2*c^2*e^2*integrate(1/16*x^
6*arctan(c*x)^2/(c^2*x^5 + x^3), x) + 96*b^2*c^2*e^2*integrate(1/16*x^6*log
(c^2*x^2 + 1)^2/(c^2*x^5 + x^3), x) + 3072*a*b*c^2*e^2*integrate(1/16*x^6*a
rctan(c*x)/(c^2*x^5 + x^3), x) + 192*b^2*c^2*e^2*integrate(1/16*x^6*log(c^2
*x^2 + 1)/(c^2*x^5 + x^3), x) + 2304*b^2*c^2*d*e*integrate(1/16*x^4*arctan(
c*x)^2/(c^2*x^5 + x^3), x) + 6144*a*b*c^2*d*e*integrate(1/16*x^4*arctan(c*x
)/(c^2*x^5 + x^3), x) + 1152*b^2*c^2*d^2*integrate(1/16*x^2*arctan(c*x)^2/(
c^2*x^5 + x^3), x) + 96*b^2*c^2*d^2*integrate(1/16*x^2*log(c^2*x^2 + 1)^2/(
c^2*x^5 + x^3), x) - 192*b^2*c^2*d^2*integrate(1/16*x^2*log(c^2*x^2 + 1)/(c
^2*x^5 + x^3), x) + 2*b^2*d*e*log(c^2*x^2 + 1)^3 - 384*b^2*c*e^2*integrate(
1/16*x^5*arctan(c*x)/(c^2*x^5 + x^3), x) + 384*b^2*c*d^2*integrate(1/16*x*a
rctan(c*x)/(c^2*x^5 + x^3), x) + 1152*b^2*e^2*integrate(1/16*x^4*arctan(c*x
)^2/(c^2*x^5 + x^3), x) + 3072*a*b*e^2*integrate(1/16*x^4*arctan(c*x)/(c^2*
x^5 + x^3), x) + 2304*b^2*d*e*integrate(1/16*x^2*arctan(c*x)^2/(c^2*x^5 + x
^3), x) + 192*b^2*d*e*integrate(1/16*x^2*log(c^2*x^2 + 1)^2/(c^2*x^5 + x^3)
, x) + 6144*a*b*d*e*integrate(1/16*x^2*arctan(c*x)/(c^2*x^5 + x^3), x) + 11
52*b^2*d^2*integrate(1/16*arctan(c*x)^2/(c^2*x^5 + x^3), x) + 96*b^2*d^2*in
tegrate(1/16*log(c^2*x^2 + 1)^2/(c^2*x^5 + x^3), x) + b^2*e^2*log(c^2*x^2 +
 1)^3/c^2)*x^2 + 12*(b^2*e^2*x^4 - b^2*d^2)*arctan(c*x)^2 - 3*(b^2*e^2*x^4
- b^2*d^2)*log(c^2*x^2 + 1)^2)/x^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{a^2e^2x^4 + 2\,a^2dex^2 + a^2d^2 + \left(b^2e^2x^4 + 2\,b^2dex^2 + b^2d^2\right)\arctan\left(cx\right)^2 + 2\left(abe^2x^4 + 2\,abdex^2 + abd^2\right)\arctan\left(c\right.}{x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arctan(c*x))^2/x^3,x, algorithm="fricas")
```

```
[Out] integral((a^2*e^2*x^4 + 2*a^2*d*e*x^2 + a^2*d^2 + (b^2*e^2*x^4 + 2*b^2*d*e*
x^2 + b^2*d^2)*arctan(c*x)^2 + 2*(a*b*e^2*x^4 + 2*a*b*d*e*x^2 + a*b*d^2)*ar
ctan(c*x))/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b\,\mathrm{atan}\,(cx))^2 \left(d + ex^2\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(a+b*atan(c*x))**2/x**3,x)
```

```
[Out] Integral((a + b*atan(c*x))**2*(d + e*x**2)**2/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2 + d\right)^2 (b\arctan\,(cx) + a)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arctan(c*x))^2/x^3,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^2*(b*arctan(c*x) + a)^2/x^3, x)
```

3.1261 $\int \frac{x^3\left(a+b\tan^{-1}(cx)\right)^2}{d+ex^2}\,dx$

Optimal. Leaf size=590

$$-\frac{ibd\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)\left(a+b\tan^{-1}(cx)\right)}{e^2}+\frac{ibd\left(a+b\tan^{-1}(cx)\right)\text{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{(1-icx)\left(c\sqrt{-d}-i\sqrt{e}\right)}\right)}{2e^2}+\frac{ibd\left(a+b\tan\right.}{}$$

```
[Out] -((a*b*x)/(c*e)) - (b^2*x*ArcTan[c*x])/(c*e) + (a + b*ArcTan[c*x])^2/(2*c^2
*e) + (x^2*(a + b*ArcTan[c*x])^2)/(2*e) + (d*(a + b*ArcTan[c*x])^2*Log[2/(1
 - I*c*x)])/e^2 - (d*(a + b*ArcTan[c*x])^2*Log[(2*c*(Sqrt[-d] - Sqrt[e]*x))
/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/(2*e^2) - (d*(a + b*ArcTan[c*x])^
2*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])
/(2*e^2) + (b^2*Log[1 + c^2*x^2])/(2*c^2*e) - (I*b*d*(a + b*ArcTan[c*x])*Po
lyLog[2, 1 - 2/(1 - I*c*x)])/e^2 + ((I/2)*b*d*(a + b*ArcTan[c*x])*PolyLog[2
, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])
/e^2 + ((I/2)*b*d*(a + b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(Sqrt[-d] + Sqrt[
e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/e^2 + (b^2*d*PolyLog[3, 1 -
 2/(1 - I*c*x)])/(2*e^2) - (b^2*d*PolyLog[3, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x
))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/(4*e^2) - (b^2*d*PolyLog[3, 1 -
 (2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(4*e
^2)
```

Rubi [A] time = 0.49888, antiderivative size = 590, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4916, 4852, 4846, 260, 4884, 4980, 4858}

$$-\frac{ibd\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)\left(a+b\tan^{-1}(cx)\right)}{e^2}+\frac{ibd\left(a+b\tan^{-1}(cx)\right)\text{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{(1-icx)\left(c\sqrt{-d}-i\sqrt{e}\right)}\right)}{2e^2}+\frac{ibd\left(a+b\tan\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcTan[c*x])^2)/(d + e*x^2),x]
```


```
[Out] -((a*b*x)/(c*e)) - (b^2*x*ArcTan[c*x])/(c*e) + (a + b*ArcTan[c*x])^2/(2*c^2
*e) + (x^2*(a + b*ArcTan[c*x])^2)/(2*e) + (d*(a + b*ArcTan[c*x])^2*Log[2/(1
 - I*c*x)])/e^2 - (d*(a + b*ArcTan[c*x])^2*Log[(2*c*(Sqrt[-d] - Sqrt[e]*x))
/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/(2*e^2) - (d*(a + b*ArcTan[c*x])^
2*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])
/(2*e^2) + (b^2*Log[1 + c^2*x^2])/(2*c^2*e) - (I*b*d*(a + b*ArcTan[c*x])*Po
```

```
lyLog[2, 1 - 2/(1 - I*c*x)])/e^2 + ((I/2)*b*d*(a + b*ArcTan[c*x])*PolyLog[2
, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])
/e^2 + ((I/2)*b*d*(a + b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(Sqrt[-d] + Sqrt[
e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/e^2 + (b^2*d*PolyLog[3, 1 -
 2/(1 - I*c*x)])/(2*e^2) - (b^2*d*PolyLog[3, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x
))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/(4*e^2) - (b^2*d*PolyLog[3, 1 -
 (2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(4*e
^2)
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4980

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
```

```
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

Rule 4858

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^2/((d_) + (e_.)*(x_)), x_Symbol] :>
-Simp[((a + b*ArcTan[c*x])^2*Log[2/(1 - I*c*x)])/e, x] + (Simp[((a + b*ArcT
an[c*x])^2*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x] + Simp[(I*
b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 - I*c*x)])/e, x] - Simp[(I*b*(a +
 b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/
e, x] - Simp[(b^2*PolyLog[3, 1 - 2/(1 - I*c*x)])/(2*e), x] + Simp[(b^2*Poly
Log[3, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/(2*e), x]) /; FreeQ[
{a, b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rubi steps

$$\int \frac{x^3\left(a+b\tan^{-1}(cx)\right)^2}{d+ex^2}\,dx = \frac{\int x\left(a+b\tan^{-1}(cx)\right)^2\,dx}{e} - \frac{d\int \frac{x\left(a+b\tan^{-1}(cx)\right)^2}{d+ex^2}\,dx}{e}$$

$$= \frac{x^2\left(a+b\tan^{-1}(cx)\right)^2}{2e} - \frac{(bc)\int \frac{x^2\left(a+b\tan^{-1}(cx)\right)}{1+c^2x^2}\,dx}{e} - \frac{d\int\left(-\frac{\left(a+b\tan^{-1}(cx)\right)^2}{2\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)} + \frac{\left(a+b\tan^{-1}(cx)\right)^2}{2\sqrt{e}\left(\sqrt{-d}+\sqrt{e}x\right)}\right)}{e}$$

$$= \frac{x^2\left(a+b\tan^{-1}(cx)\right)^2}{2e} + \frac{d\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{\sqrt{-d}-\sqrt{e}x}\,dx}{2e^{3/2}} - \frac{d\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{\sqrt{-d}+\sqrt{e}x}\,dx}{2e^{3/2}} - \frac{b\int\left(a+b\tan^{-1}(\right.}{ce}$$

$$= -\frac{abx}{ce} + \frac{\left(a+b\tan^{-1}(cx)\right)^2}{2c^2e} + \frac{x^2\left(a+b\tan^{-1}(cx)\right)^2}{2e} + \frac{d\left(a+b\tan^{-1}(cx)\right)^2\log\left(\frac{2}{1-icx}\right)}{e^2} -$$

$$= -\frac{abx}{ce} - \frac{b^2x\tan^{-1}(cx)}{ce} + \frac{\left(a+b\tan^{-1}(cx)\right)^2}{2c^2e} + \frac{x^2\left(a+b\tan^{-1}(cx)\right)^2}{2e} + \frac{d\left(a+b\tan^{-1}(cx\right.}{e^2}$$

$$= -\frac{abx}{ce} - \frac{b^2x\tan^{-1}(cx)}{ce} + \frac{\left(a+b\tan^{-1}(cx)\right)^2}{2c^2e} + \frac{x^2\left(a+b\tan^{-1}(cx)\right)^2}{2e} + \frac{d\left(a+b\tan^{-1}(cx\right.}{e^2}$$

Mathematica [B] time = 9.59908, size = 1567, normalized size = 2.66

result too large to display

Warning: Unable to verify antiderivative.

[In] Integrate[(x^3*(a + b*ArcTan[c*x])^2)/(d + e*x^2),x]

[Out] (2*a^2*e*x^2 - 2*a^2*d*Log[d + e*x^2] + 4*a*b*(-((e*x)/c) - I*d*ArcTan[c*x]^2 + ArcTan[c*x]*(e*(c^(-2) + x^2) + 2*d*Log[1 + E^((2*I)*ArcTan[c*x])]) - I*d*PolyLog[2, -E^((2*I)*ArcTan[c*x])] + (2*d*(-(c^2*d) + e)*((-I)*ArcTan[c*x]^2 + (2*I)*ArcSin[Sqrt[(c^2*d)/(c^2*d - e)]]*ArcTan[(c*e*x)/Sqrt[c^2*d*e]] + (-ArcSin[Sqrt[(c^2*d)/(c^2*d - e)]] + ArcTan[c*x])*Log[1 + ((c^2*d + e + 2*Sqrt[c^2*d*e])*E^((2*I)*ArcTan[c*x]))/(c^2*d - e)] + (ArcSin[Sqrt[(c^2*d)/(c^2*d - e)]] + ArcTan[c*x])*Log[(-2*Sqrt[c^2*d*e]*E^((2*I)*ArcTan[c*x]) + e*(-1 + E^((2*I)*ArcTan[c*x])) + c^2*d*(1 + E^((2*I)*ArcTan[c*x])))/(c^2*d - e)] - (I/2)*(PolyLog[2, -(((c^2*d + e - 2*Sqrt[c^2*d*e])*E^((2*I)*ArcTan[c*x]))/(c^2*d - e))] + PolyLog[2, -(((c^2*d + e + 2*Sqrt[c^2*d*e])*E^((2*I)*ArcTan[c*x]))/(c^2*d - e))])))/(2*c^2*d - 2*e)) + (b^2*(-4*c*e*x*ArcTan[c*x] + 2*e*ArcTan[c*x]^2 + 2*c^2*e*x^2*ArcTan[c*x]^2 + 4*c^2*d*ArcTan[c*x]^2*Log[1 + E^((2*I)*ArcTan[c*x])] - 2*c^2*d*ArcTan[c*x]^2*Log[1 + ((c*Sqrt[d] - Sqrt[e])*E^((2*I)*ArcTan[c*x]))/(c*Sqrt[d] + Sqrt[e])] - 2*c^2*d*ArcTan[c*x]^2*Log[1 + ((c*Sqrt[d] + Sqrt[e])*E^((2*I)*ArcTan[c*x]))/(c*Sqrt[d] - Sqrt[e])] + 2*c^2*d*ArcTan[c*x]^2*Log[1 + ((c^2*d + e - 2*Sqrt[c^2*d*e])*E^((2*I)*ArcTan[c*x]))/(c^2*d - e)] + 4*c^2*d*ArcSin[Sqrt[(c^2*d)/(c^2*d - e)]]*ArcTan[c*x]*Log[1 + ((c^2*d + e + 2*Sqrt[c^2*d*e])*E^((2*I)*ArcTan[c*x]))/(c^2*d - e)] - 2*c^2*d*ArcTan[c*x]^2*Log[1 + ((c^2*d + e + 2*Sqrt[c^2*d*e])*E^((2*I)*ArcTan[c*x]))/(c^2*d - e)] - 4*c^2*d*ArcSin[Sqrt[(c^2*d)/(c^2*d - e)]]*ArcTan[c*x]*Log[(-2*Sqrt[c^2*d*e]*E^((2*I)*ArcTan[c*x]) + e*(-1 + E^((2*I)*ArcTan[c*x])) + c^2*d*(1 + E^((2*I)*ArcTan[c*x])))/(c^2*d - e)] - 4*c^2*d*ArcTan[c*x]^2*Log[(-2*Sqrt[c^2*d*e]*E^((2*I)*ArcTan[c*x]) + e*(-1 + E^((2*I)*ArcTan[c*x])) + c^2*d*(1 + E^((2*I)*ArcTan[c*x])))/(c^2*d - e)] + 4*c^2*d*ArcSin[Sqrt[(c^2*d)/(c^2*d - e)]]*ArcTan[c*x]*Log[((2*I)*c^2*d - (2*I)*Sqrt[c^2*d*e] + 2*c*(-e + Sqrt[c^2*d*e])*x)/((c^2*d - e)*(I + c*x))] + 2*c^2*d*ArcTan[c*x]^2*Log[((2*I)*c^2*d - (2*I)*Sqrt[c^2*d*e] + 2*c*(-e + Sqrt[c^2*d*e])*x)/((c^2*d - e)*(I + c*x))] + 2*e*Log[1 + c^2*x^2] - 4*c^2*d*ArcSin[Sqrt[(c^2*d)/(c^2*d - e)]]*ArcTan[c*x]*Log[1 + ((c^2*d + e + 2*Sqrt[c^2*d*e])*(Cos[2*ArcTan[c*x]] + I*Sin[2*ArcTan[c*x]]))/(c^2*d - e)] + 2*c^2*d*ArcTan[c*x]^2*Log[1 + ((c^2*d + e + 2*Sqrt[c^2*d*e])*(Cos[2*ArcTan[c*x]] + I*Sin[2*ArcTan[c*x]]))/(c^2*d - e)] - (4*I)*c^2*d*ArcTan[c*x]*PolyLog[2, -E^((2*I)*ArcTan[c*x])] + (2*I)*c^2*d*ArcTan[c*x]*PolyLog[2, ((-(c*Sqrt[d]) + Sqrt[e])*E^((2*I)*ArcTan[c*x]))/(c*Sqrt[d] + Sqrt[e])] + (2*I)*c^2*d*ArcTan[c*x]*PolyLog[2, -(((c*Sqrt[d] + Sqrt[e])*E^((2*I)*ArcTan[c*x]))/(c*Sqrt[d] - Sqrt[e]))] + 2*c^2*d*PolyLog[3, -E^((2*I)*ArcTan[c*x])] - c^2*d*PolyLog[3, ((-(c*Sqrt[d]) + Sqrt[e])*E^((2*I)*ArcTan[c*x]))/(c*Sqrt[d] + Sqrt[e])] - c^2*d*PolyLog[3, -(((c*Sqrt[d] + Sqrt[e])*E^((2*I)*ArcTan[c*x]))/(c*Sqrt[d] - Sqrt[e]))]))/c^2)/(4*e^2)

Maple [F] time = 18.961, size = 0, normalized size = 0.

$$\int \frac{x^3 (a + b\arctan(cx))^2}{ex^2 + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctan(c*x))^2/(e*x^2+d),x)
```

```
[Out] int(x^3*(a+b*arctan(c*x))^2/(e*x^2+d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}a^2\left(\frac{x^2}{e} - \frac{d\log\left(ex^2 + d\right)}{e^2}\right) + \int \frac{b^2x^3\arctan(cx)^2 + 2\,abx^3\arctan(cx)}{ex^2 + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))^2/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] 1/2*a^2*(x^2/e - d*log(e*x^2 + d)/e^2) + integrate((b^2*x^3*arctan(c*x)^2 +
 2*a*b*x^3*arctan(c*x))/(e*x^2 + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2x^3\arctan(cx)^2 + 2\,abx^3\arctan(cx) + a^2x^3}{ex^2 + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))^2/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^3*arctan(c*x)^2 + 2*a*b*x^3*arctan(c*x) + a^2*x^3)/(e*x^2 +
 d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atan(c*x))**2/(e*x**2+d),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\arctan(cx)+a)^2x^3}{ex^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))^2/(e*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)^2*x^3/(e*x^2 + d), x)
```

3.1262 $\int \frac{x^2(a+b\tan^{-1}(cx))^2}{d+ex^2}\,dx$

Optimal. Leaf size=554

$$-\frac{ib\sqrt{-d}\left(a+b\tan^{-1}(cx)\right)\text{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{(1-icx)\left(c\sqrt{-d}-i\sqrt{e}\right)}\right)}{2e^{3/2}}+\frac{ib\sqrt{-d}\left(a+b\tan^{-1}(cx)\right)\text{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-d}+\sqrt{e}x\right)}{(1-icx)\left(c\sqrt{-d}+i\sqrt{e}\right)}\right)}{2e^{3/2}}$$

```
[Out] (I*(a + b*ArcTan[c*x])^2)/(c*e) + (x*(a + b*ArcTan[c*x])^2)/e + (2*b*(a + b
*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(c*e) + (Sqrt[-d]*(a + b*ArcTan[c*x])^2*L
og[(2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/(2
*e^(3/2)) - (Sqrt[-d]*(a + b*ArcTan[c*x])^2*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x)
)/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(2*e^(3/2)) + (I*b^2*PolyLog[2,
1 - 2/(1 + I*c*x)])/(c*e) - ((I/2)*b*Sqrt[-d]*(a + b*ArcTan[c*x])*PolyLog[2
, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])
/e^(3/2) + ((I/2)*b*Sqrt[-d]*(a + b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(Sqrt[
-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/e^(3/2) + (b^2*S
qrt[-d]*PolyLog[3, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e
])*(1 - I*c*x))])/(4*e^(3/2)) - (b^2*Sqrt[-d]*PolyLog[3, 1 - (2*c*(Sqrt[-d]
 + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(4*e^(3/2))
```

Rubi [A] time = 0.479909, antiderivative size = 554, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {4916, 4846, 4920, 4854, 2402, 2315, 4914, 4858}

$$-\frac{ib\sqrt{-d}\left(a+b\tan^{-1}(cx)\right)\text{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{(1-icx)\left(c\sqrt{-d}-i\sqrt{e}\right)}\right)}{2e^{3/2}}+\frac{ib\sqrt{-d}\left(a+b\tan^{-1}(cx)\right)\text{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-d}+\sqrt{e}x\right)}{(1-icx)\left(c\sqrt{-d}+i\sqrt{e}\right)}\right)}{2e^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcTan[c*x])^2)/(d + e*x^2),x]
```


```
[Out] (I*(a + b*ArcTan[c*x])^2)/(c*e) + (x*(a + b*ArcTan[c*x])^2)/e + (2*b*(a + b
*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(c*e) + (Sqrt[-d]*(a + b*ArcTan[c*x])^2*L
og[(2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/(2
*e^(3/2)) - (Sqrt[-d]*(a + b*ArcTan[c*x])^2*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x)
)/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(2*e^(3/2)) + (I*b^2*PolyLog[2,
1 - 2/(1 + I*c*x)])/(c*e) - ((I/2)*b*Sqrt[-d]*(a + b*ArcTan[c*x])*PolyLog[2
, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])
/e^(3/2) + ((I/2)*b*Sqrt[-d]*(a + b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(Sqrt[
```

```
-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/e^(3/2) + (b^2*S
qrt[-d]*PolyLog[3, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e
])*(1 - I*c*x))])/(4*e^(3/2)) - (b^2*Sqrt[-d]*PolyLog[3, 1 - (2*c*(Sqrt[-d]
 + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(4*e^(3/2))
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 4914

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (d + e*x^2)^q, x], x
] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[q] && IGtQ[p, 0]
```

Rule 4858

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^2/((d_) + (e_.)*(x_)), x_Symbol] :>
-Simp[((a + b*ArcTan[c*x])^2*Log[2/(1 - I*c*x)])/e, x] + (Simp[((a + b*ArcT
an[c*x])^2*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x] + Simp[(I*
b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 - I*c*x)])/e, x] - Simp[(I*b*(a +
 b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/
e, x] - Simp[(b^2*PolyLog[3, 1 - 2/(1 - I*c*x)])/(2*e), x] + Simp[(b^2*Poly
Log[3, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/(2*e), x]) /; FreeQ[
{a, b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rubi steps

$$\int \frac{x^2\left(a+b\tan^{-1}(cx)\right)^2}{d+ex^2}\,dx = \frac{\int \left(a+b\tan^{-1}(cx)\right)^2 dx}{e} - \frac{d\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{d+ex^2}\,dx}{e}$$

$$= \frac{x\left(a+b\tan^{-1}(cx)\right)^2}{e} - \frac{(2bc)\int \frac{x\left(a+b\tan^{-1}(cx)\right)}{1+c^2x^2}\,dx}{e} - \frac{d\int \left(\frac{\sqrt{-d}\left(a+b\tan^{-1}(cx)\right)^2}{2d\left(\sqrt{-d}-\sqrt{e}x\right)} + \frac{\sqrt{-d}\left(a+b\tan^{-1}(c\right.}{2d\left(\sqrt{-d}+\sqrt{e}x\right.}\right.}{e}$$

$$= \frac{i\left(a+b\tan^{-1}(cx)\right)^2}{ce} + \frac{x\left(a+b\tan^{-1}(cx)\right)^2}{e} + \frac{(2b)\int \frac{a+b\tan^{-1}(cx)}{i-cx}\,dx}{e} - \frac{\sqrt{-d}\int \frac{\left(a+b\tan^{-1}(cx\right.}{\sqrt{-d}-\sqrt{e}x}}{2e}$$

$$= \frac{i\left(a+b\tan^{-1}(cx)\right)^2}{ce} + \frac{x\left(a+b\tan^{-1}(cx)\right)^2}{e} + \frac{2b\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2}{1+icx}\right)}{ce} + \frac{\sqrt{-d}\,(a}{}$$

$$= \frac{i\left(a+b\tan^{-1}(cx)\right)^2}{ce} + \frac{x\left(a+b\tan^{-1}(cx)\right)^2}{e} + \frac{2b\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2}{1+icx}\right)}{ce} + \frac{\sqrt{-d}\,(a}{}$$

$$= \frac{i\left(a+b\tan^{-1}(cx)\right)^2}{ce} + \frac{x\left(a+b\tan^{-1}(cx)\right)^2}{e} + \frac{2b\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2}{1+icx}\right)}{ce} + \frac{\sqrt{-d}\,(a}{}$$

Mathematica [F] time = 180.001, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(x^2*(a + b*ArcTan[c*x])^2)/(d + e*x^2),x]
```

```
[Out] $Aborted
```

Maple [F] time = 2.909, size = 0, normalized size = 0.

$$\int \frac{x^2 (a + b \arctan (cx))^2}{ex^2 + d} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctan(c*x))^2/(e*x^2+d),x)
```

```
[Out] int(x^2*(a+b*arctan(c*x))^2/(e*x^2+d),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))^2/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2x^2 \arctan (cx)^2 + 2\,abx^2 \arctan (cx) + a^2x^2}{ex^2 + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))^2/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^2*arctan(c*x)^2 + 2*a*b*x^2*arctan(c*x) + a^2*x^2)/(e*x^2 +
 d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atan(c*x))**2/(e*x**2+d),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\arctan(cx)+a)^2x^2}{ex^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))^2/(e*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)^2*x^2/(e*x^2 + d), x)
```

3.1263 $\int \frac{x\left(a+b\tan^{-1}(cx)\right)^2}{d+ex^2}\,dx$

Optimal. Leaf size=492

$$-\frac{ib\left(a+b\tan^{-1}(cx)\right)\text{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{(1-icx)\left(c\sqrt{-d}-i\sqrt{e}\right)}\right)}{2e}-\frac{ib\left(a+b\tan^{-1}(cx)\right)\text{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-d}+\sqrt{e}x\right)}{(1-icx)\left(c\sqrt{-d}+i\sqrt{e}\right)}\right)}{2e}+\frac{ib\text{Poly}}{}$$

```
[Out] -(((a + b*ArcTan[c*x])^2*Log[2/(1 - I*c*x)])/e) + ((a + b*ArcTan[c*x])^2*Lo
g[(2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/(2*
e) + ((a + b*ArcTan[c*x])^2*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] +
 I*Sqrt[e])*(1 - I*c*x))])/(2*e) + (I*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 -
2/(1 - I*c*x)])/e - ((I/2)*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(Sqrt[
-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/e - ((I/2)*b*(a
+ b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] +
 I*Sqrt[e])*(1 - I*c*x))])/e - (b^2*PolyLog[3, 1 - 2/(1 - I*c*x)])/(2*e) +
(b^2*PolyLog[3, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*
(1 - I*c*x))])/(4*e) + (b^2*PolyLog[3, 1 - (2*c*(Sqrt[-d] + Sqrt[e]*x))/((c
*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(4*e)
```

Rubi [A] time = 0.249505, antiderivative size = 492, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4980, 4858}

$$-\frac{ib\left(a+b\tan^{-1}(cx)\right)\text{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{(1-icx)\left(c\sqrt{-d}-i\sqrt{e}\right)}\right)}{2e}-\frac{ib\left(a+b\tan^{-1}(cx)\right)\text{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-d}+\sqrt{e}x\right)}{(1-icx)\left(c\sqrt{-d}+i\sqrt{e}\right)}\right)}{2e}+\frac{ib\text{Poly}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcTan[c*x])^2)/(d + e*x^2),x]
```


```
[Out] -(((a + b*ArcTan[c*x])^2*Log[2/(1 - I*c*x)])/e) + ((a + b*ArcTan[c*x])^2*Lo
g[(2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/(2*
e) + ((a + b*ArcTan[c*x])^2*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] +
 I*Sqrt[e])*(1 - I*c*x))])/(2*e) + (I*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 -
2/(1 - I*c*x)])/e - ((I/2)*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(Sqrt[
-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/e - ((I/2)*b*(a
+ b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] +
 I*Sqrt[e])*(1 - I*c*x))])/e - (b^2*PolyLog[3, 1 - 2/(1 - I*c*x)])/(2*e) +
(b^2*PolyLog[3, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*
```

```
(1 - I*c*x))])/(4*e) + (b^2*PolyLog[3, 1 - (2*c*(Sqrt[-d] + Sqrt[e]*x))/((c
*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(4*e)
```

Rule 4980

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

Rule 4858

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^2/((d_) + (e_.)*(x_)), x_Symbol] :>
-Simp[((a + b*ArcTan[c*x])^2*Log[2/(1 - I*c*x)])/e, x] + (Simp[((a + b*ArcT
an[c*x])^2*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x] + Simp[(I*
b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 - I*c*x)])/e, x] - Simp[(I*b*(a +
 b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/
e, x] - Simp[(b^2*PolyLog[3, 1 - 2/(1 - I*c*x)])/(2*e), x] + Simp[(b^2*Poly
Log[3, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/(2*e), x]) /; FreeQ[
{a, b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rubi steps

$$\int \frac{x\left(a+b\tan^{-1}(cx)\right)^2}{d+ex^2}\,dx = \int\left(-\frac{\left(a+b\tan^{-1}(cx)\right)^2}{2\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{\left(a+b\tan^{-1}(cx)\right)^2}{2\sqrt{e}\left(\sqrt{-d}+\sqrt{e}x\right)}\right)dx$$

$$= -\frac{\int\frac{\left(a+b\tan^{-1}(cx)\right)^2}{\sqrt{-d}-\sqrt{e}x}\,dx}{2\sqrt{e}}+\frac{\int\frac{\left(a+b\tan^{-1}(cx)\right)^2}{\sqrt{-d}+\sqrt{e}x}\,dx}{2\sqrt{e}}$$

$$= -\frac{\left(a+b\tan^{-1}(cx)\right)^2\log\left(\frac{2}{1-icx}\right)}{e}+\frac{\left(a+b\tan^{-1}(cx)\right)^2\log\left(\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(c\sqrt{-d}-i\sqrt{e}\right)(1-icx)}\right)}{2e}+\frac{\left(a+b\tan^{-1}\right.}{}$$

Mathematica [B] time = 8.21787, size = 1527, normalized size = 3.1

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(a + b*ArcTan[c*x])^2)/(d + e*x^2),x]
```

```
[Out] ((8*I)*a*b*ArcSin[Sqrt[(c^2*d)/(c^2*d - e)]]*ArcTan[(c*e*x)/Sqrt[c^2*d*e]]
- 8*a*b*ArcTan[c*x]*Log[1 + E^((2*I)*ArcTan[c*x])] - 4*b^2*ArcTan[c*x]^2*Lo
g[1 + E^((2*I)*ArcTan[c*x])] + 2*b^2*ArcTan[c*x]^2*Log[1 + ((c*Sqrt[d] - Sq
rt[e])*E^((2*I)*ArcTan[c*x]))/(c*Sqrt[d] + Sqrt[e])] + 2*b^2*ArcTan[c*x]^2*
Log[1 + ((c*Sqrt[d] + Sqrt[e])*E^((2*I)*ArcTan[c*x]))/(c*Sqrt[d] - Sqrt[e])
] - 2*b^2*ArcTan[c*x]^2*Log[1 + ((c^2*d + e - 2*Sqrt[c^2*d*e])*E^((2*I)*Arc
Tan[c*x]))/(c^2*d - e)] - 4*a*b*ArcSin[Sqrt[(c^2*d)/(c^2*d - e)]]*Log[1 + (
(c^2*d + e + 2*Sqrt[c^2*d*e])*E^((2*I)*ArcTan[c*x]))/(c^2*d - e)] + 4*a*b*A
rcTan[c*x]*Log[1 + ((c^2*d + e + 2*Sqrt[c^2*d*e])*E^((2*I)*ArcTan[c*x]))/(c
^2*d - e)] - 4*b^2*ArcSin[Sqrt[(c^2*d)/(c^2*d - e)]]*ArcTan[c*x]*Log[1 + ((
c^2*d + e + 2*Sqrt[c^2*d*e])*E^((2*I)*ArcTan[c*x]))/(c^2*d - e)] + 2*b^2*Ar
cTan[c*x]^2*Log[1 + ((c^2*d + e + 2*Sqrt[c^2*d*e])*E^((2*I)*ArcTan[c*x]))/(
c^2*d - e)] + 4*a*b*ArcSin[Sqrt[(c^2*d)/(c^2*d - e)]]*Log[(-2*Sqrt[c^2*d*e]
*E^((2*I)*ArcTan[c*x]) + e*(-1 + E^((2*I)*ArcTan[c*x])) + c^2*d*(1 + E^((2*
I)*ArcTan[c*x])))/(c^2*d - e)] + 4*a*b*ArcTan[c*x]*Log[(-2*Sqrt[c^2*d*e]*E^
((2*I)*ArcTan[c*x]) + e*(-1 + E^((2*I)*ArcTan[c*x])) + c^2*d*(1 + E^((2*I)*
ArcTan[c*x])))/(c^2*d - e)] + 4*b^2*ArcSin[Sqrt[(c^2*d)/(c^2*d - e)]]*ArcTa
n[c*x]*Log[(-2*Sqrt[c^2*d*e]*E^((2*I)*ArcTan[c*x]) + e*(-1 + E^((2*I)*ArcTa
n[c*x])) + c^2*d*(1 + E^((2*I)*ArcTan[c*x])))/(c^2*d - e)] + 4*b^2*ArcTan[c
*x]^2*Log[(-2*Sqrt[c^2*d*e]*E^((2*I)*ArcTan[c*x]) + e*(-1 + E^((2*I)*ArcTan
[c*x])) + c^2*d*(1 + E^((2*I)*ArcTan[c*x])))/(c^2*d - e)] - 4*b^2*ArcSin[Sq
rt[(c^2*d)/(c^2*d - e)]]*ArcTan[c*x]*Log[((2*I)*c^2*d - (2*I)*Sqrt[c^2*d*e]
 + 2*c*(-e + Sqrt[c^2*d*e])*x)/((c^2*d - e)*(I + c*x))] - 2*b^2*ArcTan[c*x]
^2*Log[((2*I)*c^2*d - (2*I)*Sqrt[c^2*d*e] + 2*c*(-e + Sqrt[c^2*d*e])*x)/((c
^2*d - e)*(I + c*x))] + 2*a^2*Log[d + e*x^2] + 4*b^2*ArcSin[Sqrt[(c^2*d)/(c
^2*d - e)]]*ArcTan[c*x]*Log[1 + ((c^2*d + e + 2*Sqrt[c^2*d*e])*(Cos[2*ArcTa
n[c*x]] + I*Sin[2*ArcTan[c*x]]))/(c^2*d - e)] - 2*b^2*ArcTan[c*x]^2*Log[1 +
 ((c^2*d + e + 2*Sqrt[c^2*d*e])*(Cos[2*ArcTan[c*x]] + I*Sin[2*ArcTan[c*x]])
)/(c^2*d - e)] + (4*I)*b*(a + b*ArcTan[c*x])*PolyLog[2, -E^((2*I)*ArcTan[c*
x])] - (2*I)*b^2*ArcTan[c*x]*PolyLog[2, ((-(c*Sqrt[d]) + Sqrt[e])*E^((2*I)*
ArcTan[c*x]))/(c*Sqrt[d] + Sqrt[e])] - (2*I)*b^2*ArcTan[c*x]*PolyLog[2, -((
(c*Sqrt[d] + Sqrt[e])*E^((2*I)*ArcTan[c*x]))/(c*Sqrt[d] - Sqrt[e]))] - (2*I
)*a*b*PolyLog[2, -(((c^2*d + e - 2*Sqrt[c^2*d*e])*E^((2*I)*ArcTan[c*x]))/(c
^2*d - e))] - (2*I)*a*b*PolyLog[2, -(((c^2*d + e + 2*Sqrt[c^2*d*e])*E^((2*I
)*ArcTan[c*x]))/(c^2*d - e))] - 2*b^2*PolyLog[3, -E^((2*I)*ArcTan[c*x])] +
b^2*PolyLog[3, ((-(c*Sqrt[d]) + Sqrt[e])*E^((2*I)*ArcTan[c*x]))/(c*Sqrt[d]
+ Sqrt[e])] + b^2*PolyLog[3, -(((c*Sqrt[d] + Sqrt[e])*E^((2*I)*ArcTan[c*x])
)/(c*Sqrt[d] - Sqrt[e]))])/(4*e)
```

Maple [F] time = 6.373, size = 0, normalized size = 0.

$$\int \frac{x(a+b\arctan(cx))^2}{ex^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctan(c*x))^2/(e*x^2+d),x)
```

```
[Out] int(x*(a+b*arctan(c*x))^2/(e*x^2+d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^2 \log\left(ex^2+d\right)}{2\,e} + \int \frac{b^2 x \arctan\left(cx\right)^2 + 2\,abx \arctan\left(cx\right)}{ex^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))^2/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] 1/2*a^2*log(e*x^2 + d)/e + integrate((b^2*x*arctan(c*x)^2 + 2*a*b*x*arctan(
c*x))/(e*x^2 + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2 x \arctan\left(cx\right)^2 + 2\,abx \arctan\left(cx\right) + a^2 x}{ex^2+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))^2/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*x*arctan(c*x)^2 + 2*a*b*x*arctan(c*x) + a^2*x)/(e*x^2 + d), x
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(a + b \operatorname{atan}\left(cx\right)\right)^2}{d + ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atan(c*x))**2/(e*x**2+d),x)
```

```
[Out] Integral(x*(a + b*atan(c*x))**2/(d + e*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \arctan(cx) + a)^2 x}{ex^2 + d} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))^2/(e*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)^2*x/(e*x^2 + d), x)
```

3.1264 $\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{d+ex^2}\,dx$

Optimal. Leaf size=460

$$-\frac{ib\left(a+b\tan^{-1}(cx)\right)\text{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{(1-icx)\left(c\sqrt{-d}-i\sqrt{e}\right)}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{ib\left(a+b\tan^{-1}(cx)\right)\text{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-d}+\sqrt{e}x\right)}{(1-icx)\left(c\sqrt{-d}+i\sqrt{e}\right)}\right)}{2\sqrt{-d}\sqrt{e}}+b^2\text{Po}$$

```
[Out] ((a + b*ArcTan[c*x])^2*Log[(2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sq
rt[e])*(1 - I*c*x))])/(2*Sqrt[-d]*Sqrt[e]) - ((a + b*ArcTan[c*x])^2*Log[(2*
c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(2*Sqrt[
-d]*Sqrt[e]) - ((I/2)*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(Sqrt[-d] -
 Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/(Sqrt[-d]*Sqrt[e]) +
((I/2)*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(Sqrt[-d] + Sqrt[e]*x))/((
c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(Sqrt[-d]*Sqrt[e]) + (b^2*PolyLog[3,
 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/
(4*Sqrt[-d]*Sqrt[e]) - (b^2*PolyLog[3, 1 - (2*c*(Sqrt[-d] + Sqrt[e]*x))/((c
*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(4*Sqrt[-d]*Sqrt[e])
```

Rubi [A] time = 0.246568, antiderivative size = 460, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {4914, 4858}

$$-\frac{ib\left(a+b\tan^{-1}(cx)\right)\text{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{(1-icx)\left(c\sqrt{-d}-i\sqrt{e}\right)}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{ib\left(a+b\tan^{-1}(cx)\right)\text{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-d}+\sqrt{e}x\right)}{(1-icx)\left(c\sqrt{-d}+i\sqrt{e}\right)}\right)}{2\sqrt{-d}\sqrt{e}}+b^2\text{Po}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^2/(d + e*x^2),x]
```

```
[Out] ((a + b*ArcTan[c*x])^2*Log[(2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sq
rt[e])*(1 - I*c*x))])/(2*Sqrt[-d]*Sqrt[e]) - ((a + b*ArcTan[c*x])^2*Log[(2*
c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(2*Sqrt[
-d]*Sqrt[e]) - ((I/2)*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(Sqrt[-d] -
 Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/(Sqrt[-d]*Sqrt[e]) +
((I/2)*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(Sqrt[-d] + Sqrt[e]*x))/((
c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(Sqrt[-d]*Sqrt[e]) + (b^2*PolyLog[3,
 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/
(4*Sqrt[-d]*Sqrt[e]) - (b^2*PolyLog[3, 1 - (2*c*(Sqrt[-d] + Sqrt[e]*x))/((c
```

```
*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(4*Sqrt[-d]*Sqrt[e])
```

Rule 4914

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (d + e*x^2)^q, x], x
] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[q] && IGtQ[p, 0]
```

Rule 4858

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^2/((d_) + (e_.)*(x_)), x_Symbol] :>
-Simp[((a + b*ArcTan[c*x])^2*Log[2/(1 - I*c*x)])/e, x] + (Simp[((a + b*ArcT
an[c*x])^2*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x] + Simp[(I*
b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 - I*c*x)])/e, x] - Simp[(I*b*(a +
 b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/
e, x] - Simp[(b^2*PolyLog[3, 1 - 2/(1 - I*c*x)])/(2*e), x] + Simp[(b^2*Poly
Log[3, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/(2*e), x]) /; FreeQ[
{a, b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{d+ex^2}\,dx = \int \left(\frac{\sqrt{-d}\left(a+b\tan^{-1}(cx)\right)^2}{2d\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{\sqrt{-d}\left(a+b\tan^{-1}(cx)\right)^2}{2d\left(\sqrt{-d}+\sqrt{e}x\right)}\right)dx$$

$$= -\frac{\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{\sqrt{-d}-\sqrt{e}x}\,dx}{2\sqrt{-d}}-\frac{\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{\sqrt{-d}+\sqrt{e}x}\,dx}{2\sqrt{-d}}$$

$$= \frac{\left(a+b\tan^{-1}(cx)\right)^2\log\left(\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(c\sqrt{-d}-i\sqrt{e}\right)(1-icx)}\right)}{2\sqrt{-d}\sqrt{e}}-\frac{\left(a+b\tan^{-1}(cx)\right)^2\log\left(\frac{2c\left(\sqrt{-d}+\sqrt{e}x\right)}{\left(c\sqrt{-d}+i\sqrt{e}\right)(1-icx)}\right)}{2\sqrt{-d}\sqrt{e}}-\frac{ib\left(a+\right.}{}$$

Mathematica [F] time = 180.002, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcTan[c*x])^2/(d + e*x^2),x]
```

```
[Out] $Aborted
```

Maple [B] time = 0.359, size = 2600, normalized size = 5.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^2/(e*x^2+d),x)
```

```
[Out] -I*c*b^2*ln(1-(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d-2*(c^2*e*d)^(1/2)-e
))*arctan(c*x)^2/(c^4*d^2-2*c^2*d*e+e^2)*(c^2*e*d)^(1/2)-1/4*I/c*b^2*(c^2*e
*d)^(1/2)/e/d*polylog(3,(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d+2*(c^2*e*
d)^(1/2)-e))+1/2/c*b^2/d/(c^4*d^2-2*c^2*d*e+e^2)*polylog(2,(c^2*d-e)*(1+I*c
*x)^2/(c^2*x^2+1)/(-c^2*d-2*(c^2*e*d)^(1/2)-e))*arctan(c*x)*(c^2*e*d)^(1/2)
*e+1/c*a*b/d/(c^4*d^2-2*c^2*d*e+e^2)*arctan(c*x)^2*(c^2*e*d)^(1/2)*e+1/2/c*
a*b/d/(c^4*d^2-2*c^2*d*e+e^2)*polylog(2,(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(
-c^2*d-2*(c^2*e*d)^(1/2)-e))*(c^2*e*d)^(1/2)*e+c^3*a*b/e/(c^4*d^2-2*c^2*d*e
+e^2)*arctan(c*x)^2*(c^2*e*d)^(1/2)*d+1/2*c^3*a*b/e/(c^4*d^2-2*c^2*d*e+e^2)
*polylog(2,(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d-2*(c^2*e*d)^(1/2)-e))*
(c^2*e*d)^(1/2)*d+1/4*I/c*b^2*polylog(3,(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(
-c^2*d-2*(c^2*e*d)^(1/2)-e))/d/(c^4*d^2-2*c^2*d*e+e^2)*(c^2*e*d)^(1/2)*e+1/
2*c^3*b^2/e/(c^4*d^2-2*c^2*d*e+e^2)*polylog(2,(c^2*d-e)*(1+I*c*x)^2/(c^2*x^
2+1)/(-c^2*d-2*(c^2*e*d)^(1/2)-e))*arctan(c*x)*(c^2*e*d)^(1/2)*d+1/4*I*c^3*
b^2*polylog(3,(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d-2*(c^2*e*d)^(1/2)-e
))/e/(c^4*d^2-2*c^2*d*e+e^2)*(c^2*e*d)^(1/2)*d-1/2*I/c*b^2*(c^2*e*d)^(1/2)/
e/d*arctan(c*x)^2*ln(1-(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d+2*(c^2*e*d
)^(1/2)-e))-2*I*c*a*b*ln(1-(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d-2*(c^2
*e*d)^(1/2)-e))*arctan(c*x)/(c^4*d^2-2*c^2*d*e+e^2)*(c^2*e*d)^(1/2)-2*c*a*b
/(c^4*d^2-2*c^2*d*e+e^2)*arctan(c*x)^2*(c^2*e*d)^(1/2)-c*a*b/(c^4*d^2-2*c^2
*d*e+e^2)*polylog(2,(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d-2*(c^2*e*d)^(
1/2)-e))*(c^2*e*d)^(1/2)-1/3/c*b^2*(c^2*e*d)^(1/2)/e/d*arctan(c*x)^3-1/2*I*
c^3*b^2*polylog(3,(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d-2*(c^2*e*d)^(1/
2)-e))/(c^4*d^2-2*c^2*d*e+e^2)*d+1/2*I*c*b^2*polylog(3,(c^2*d-e)*(1+I*c*x)^
2/(c^2*x^2+1)/(-c^2*d-2*(c^2*e*d)^(1/2)-e))/(c^4*d^2-2*c^2*d*e+e^2)*(c^2*e*
d)^(1/2)-1/2*I*c*b^2*polylog(3,(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d-2*
(c^2*e*d)^(1/2)-e))/(c^4*d^2-2*c^2*d*e+e^2)*e-2*I*c*b^2*polylog(3,(c^2*d-e)
*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d-2*(c^2*e*d)^(1/2)-e))/(2*c^4*d^2-4*c^2*d*e
+2*e^2)*(c^2*e*d)^(1/2)-c*b^2/(c^4*d^2-2*c^2*d*e+e^2)*polylog(2,(c^2*d-e)*(
1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d-2*(c^2*e*d)^(1/2)-e))*arctan(c*x)*(c^2*e*d)^
(1/2)+I*c*b^2*polylog(3,(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d-2*(c^2*e*
d)^(1/2)-e))/(2*c^4*d^2-4*c^2*d*e+2*e^2)*e+I*c^3*b^2*polylog(3,(c^2*d-e)*(1
+I*c*x)^2/(c^2*x^2+1)/(-c^2*d-2*(c^2*e*d)^(1/2)-e))/(2*c^4*d^2-4*c^2*d*e+2*
e^2)*d+I/c*a*b*ln(1-(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d-2*(c^2*e*d)^(
1/2)-e))*arctan(c*x)/d/(c^4*d^2-2*c^2*d*e+e^2)*(c^2*e*d)^(1/2)*e+I*c^3*a*b*
```

```
ln(1-(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d-2*(c^2*e*d)^(1/2)-e))*arctan
(c*x)/e/(c^4*d^2-2*c^2*d*e+e^2)*(c^2*e*d)^(1/2)*d+a^2/(d*e)^(1/2)*arctan(e*
x/(d*e)^(1/2))+1/2*I*c^3*b^2*ln(1-(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d
-2*(c^2*e*d)^(1/2)-e))*arctan(c*x)^2/e/(c^4*d^2-2*c^2*d*e+e^2)*(c^2*e*d)^(1
/2)*d+1/2*I/c*b^2*ln(1-(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d-2*(c^2*e*d
)^(1/2)-e))*arctan(c*x)^2/d/(c^4*d^2-2*c^2*d*e+e^2)*(c^2*e*d)^(1/2)*e-2/3*c
*b^2/(c^4*d^2-2*c^2*d*e+e^2)*arctan(c*x)^3*(c^2*e*d)^(1/2)-I/c*a*b*(c^2*e*d
)^(1/2)/e/d*arctan(c*x)*ln(1-(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d+2*(c
^2*e*d)^(1/2)-e))-1/2/c*b^2*(c^2*e*d)^(1/2)/e/d*arctan(c*x)*polylog(2,(c^2*
d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d+2*(c^2*e*d)^(1/2)-e))+1/3/c*b^2/d/(c^4
*d^2-2*c^2*d*e+e^2)*arctan(c*x)^3*(c^2*e*d)^(1/2)*e-1/c*a*b*(c^2*e*d)^(1/2)
/e/d*arctan(c*x)^2-1/2/c*a*b*(c^2*e*d)^(1/2)/e/d*polylog(2,(c^2*d-e)*(1+I*c
*x)^2/(c^2*x^2+1)/(-c^2*d+2*(c^2*e*d)^(1/2)-e))+1/3*c^3*b^2/e/(c^4*d^2-2*c^
2*d*e+e^2)*arctan(c*x)^3*(c^2*e*d)^(1/2)*d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2 \arctan (cx)^2 + 2\,ab \arctan (cx) + a^2}{ex^2 + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*arctan(c*x)^2 + 2*a*b*arctan(c*x) + a^2)/(e*x^2 + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b\operatorname{atan}(cx))^2}{d + ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**2/(e*x**2+d),x)
```

```
[Out] Integral((a + b*atan(c*x))**2/(d + e*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\arctan(cx) + a)^2}{ex^2 + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/(e*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)^2/(e*x^2 + d), x)
```

3.1265 $\int \frac{(a+b\tan^{-1}(cx))^2}{x(d+ex^2)}\,dx$

Optimal. Leaf size=637

$$\frac{ib\left(a+b\tan^{-1}(cx)\right)\text{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{(1-icx)\left(c\sqrt{-d}-i\sqrt{e}\right)}\right)}{2d}+\frac{ib\left(a+b\tan^{-1}(cx)\right)\text{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-d}+\sqrt{e}x\right)}{(1-icx)\left(c\sqrt{-d}+i\sqrt{e}\right)}\right)}{2d}-\frac{ib\text{PolyL}}{}$$

```
[Out] (2*(a + b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1 + I*c*x)])/d + ((a + b*ArcTan[c*x
])^2*Log[2/(1 - I*c*x)])/d - ((a + b*ArcTan[c*x])^2*Log[(2*c*(Sqrt[-d] - Sq
rt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/(2*d) - ((a + b*ArcTan[c
*x])^2*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*
x))])/(2*d) - (I*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 - I*c*x)])/d - (
I*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)])/d + (I*b*(a + b*ArcT
an[c*x])*PolyLog[2, -1 + 2/(1 + I*c*x)])/d + ((I/2)*b*(a + b*ArcTan[c*x])*P
olyLog[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I
*c*x))])/d + ((I/2)*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(Sqrt[-d] + S
qrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/d + (b^2*PolyLog[3, 1 -
 2/(1 - I*c*x)])/(2*d) - (b^2*PolyLog[3, 1 - 2/(1 + I*c*x)])/(2*d) + (b^2*P
olyLog[3, -1 + 2/(1 + I*c*x)])/(2*d) - (b^2*PolyLog[3, 1 - (2*c*(Sqrt[-d] -
 Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/(4*d) - (b^2*PolyLog[
3, 1 - (2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))]
)/(4*d)
```

Rubi [A] time = 0.66702, antiderivative size = 637, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4980, 4850, 4988, 4884, 4994, 6610, 4858}

$$\frac{ib\left(a+b\tan^{-1}(cx)\right)\text{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{(1-icx)\left(c\sqrt{-d}-i\sqrt{e}\right)}\right)}{2d}+\frac{ib\left(a+b\tan^{-1}(cx)\right)\text{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-d}+\sqrt{e}x\right)}{(1-icx)\left(c\sqrt{-d}+i\sqrt{e}\right)}\right)}{2d}-\frac{ib\text{PolyL}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^2/(x*(d + e*x^2)),x]
```


```
[Out] (2*(a + b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1 + I*c*x)])/d + ((a + b*ArcTan[c*x
])^2*Log[2/(1 - I*c*x)])/d - ((a + b*ArcTan[c*x])^2*Log[(2*c*(Sqrt[-d] - Sq
rt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/(2*d) - ((a + b*ArcTan[c
*x])^2*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*
```

```
x))])/(2*d) - (I*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 - I*c*x)])/d - (
I*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)])/d + (I*b*(a + b*ArcT
an[c*x])*PolyLog[2, -1 + 2/(1 + I*c*x)])/d + ((I/2)*b*(a + b*ArcTan[c*x])*P
olyLog[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I
*c*x))])/d + ((I/2)*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(Sqrt[-d] + S
qrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/d + (b^2*PolyLog[3, 1 -
 2/(1 - I*c*x)])/(2*d) - (b^2*PolyLog[3, 1 - 2/(1 + I*c*x)])/(2*d) + (b^2*P
olyLog[3, -1 + 2/(1 + I*c*x)])/(2*d) - (b^2*PolyLog[3, 1 - (2*c*(Sqrt[-d] -
 Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/(4*d) - (b^2*PolyLog[
3, 1 - (2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))]
)/(4*d)
```

Rule 4980

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

Rule 4850

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTan[c*x])^(p - 1)*ArcTanh[1 - 2/(1 + I*c*x)])/(1 + c^2*x^2), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4988

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTan[c*x])^p)/(d + e
*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTan[c*x])^p)/(d + e*x^2
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
```

```
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 4858

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^2/((d_) + (e_.)*(x_)), x_Symbol] :>
-Simp[((a + b*ArcTan[c*x])^2*Log[2/(1 - I*c*x)])/e, x] + (Simp[((a + b*ArcT
an[c*x])^2*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x] + Simp[(I*
b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 - I*c*x)])/e, x] - Simp[(I*b*(a +
 b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/
e, x] - Simp[(b^2*PolyLog[3, 1 - 2/(1 - I*c*x)])/(2*e), x] + Simp[(b^2*Poly
Log[3, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/(2*e), x]) /; FreeQ[
{a, b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x\left(d+ex^2\right)}\,dx = \int \left(\frac{\left(a+b\tan^{-1}(cx)\right)^2}{dx} - \frac{ex\left(a+b\tan^{-1}(cx)\right)^2}{d\left(d+ex^2\right)}\right)dx$$

$$= \frac{\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x}\,dx}{d} - \frac{e\int \frac{x\left(a+b\tan^{-1}(cx)\right)^2}{d+ex^2}\,dx}{d}$$

$$= \frac{2\left(a+b\tan^{-1}(cx)\right)^2\tanh^{-1}\left(1-\frac{2}{1+icx}\right)}{d} - \frac{(4bc)\int \frac{\left(a+b\tan^{-1}(cx)\right)\tanh^{-1}\left(1-\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx}{d} - \frac{e\int\left(-\frac{(a+b\,\ldots}{2\sqrt{e}(\ldots}\right.}{\ldots}$$

$$= \frac{2\left(a+b\tan^{-1}(cx)\right)^2\tanh^{-1}\left(1-\frac{2}{1+icx}\right)}{d} + \frac{(2bc)\int \frac{\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx}{d} - \frac{(2bc)\int \frac{(a+b\tan^{-}\ldots}{\ldots}}{\ldots}$$

$$= \frac{2\left(a+b\tan^{-1}(cx)\right)^2\tanh^{-1}\left(1-\frac{2}{1+icx}\right)}{d} + \frac{\left(a+b\tan^{-1}(cx)\right)^2\log\left(\frac{2}{1-icx}\right)}{d} - \frac{\left(a+b\tan^{-1}(cx)\right)^2\ldots}{\ldots}$$

$$= \frac{2\left(a+b\tan^{-1}(cx)\right)^2\tanh^{-1}\left(1-\frac{2}{1+icx}\right)}{d} + \frac{\left(a+b\tan^{-1}(cx)\right)^2\log\left(\frac{2}{1-icx}\right)}{d} - \frac{\left(a+b\tan^{-1}(cx)\right)^2\ldots}{\ldots}$$

Mathematica [B] time = 6.85224, size = 1410, normalized size = 2.21

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c*x])^2/(x*(d + e*x^2)),x]
```

```
[Out] (24*a^2*Log[x] - 12*a^2*Log[d + e*x^2] - 24*a*b*((-I)*ArcTan[c*x]^2 + (2*I)
*ArcSin[Sqrt[(c^2*d)/(c^2*d - e)]]*ArcTan[(c*e*x)/Sqrt[c^2*d*e]] - 2*ArcTan
[c*x]*Log[1 - E^((2*I)*ArcTan[c*x])] + (-ArcSin[Sqrt[(c^2*d)/(c^2*d - e)]]
+ ArcTan[c*x])*Log[1 + ((c^2*d + e + 2*Sqrt[c^2*d*e])*E^((2*I)*ArcTan[c*x])
)/(c^2*d - e)] + (ArcSin[Sqrt[(c^2*d)/(c^2*d - e)]] + ArcTan[c*x])*Log[(-2*
Sqrt[c^2*d*e]*E^((2*I)*ArcTan[c*x]) + e*(-1 + E^((2*I)*ArcTan[c*x])) + c^2*
d*(1 + E^((2*I)*ArcTan[c*x])))/(c^2*d - e)] + I*(ArcTan[c*x]^2 + PolyLog[2,
 E^((2*I)*ArcTan[c*x])]) - (I/2)*(PolyLog[2, -(((c^2*d + e - 2*Sqrt[c^2*d*e
])*E^((2*I)*ArcTan[c*x]))/(c^2*d - e))] + PolyLog[2, -(((c^2*d + e + 2*Sqrt
[c^2*d*e])*E^((2*I)*ArcTan[c*x]))/(c^2*d - e))])) + b^2*((-I)*Pi^3 + (16*I)
*ArcTan[c*x]^3 + 24*ArcTan[c*x]^2*Log[1 - E^((-2*I)*ArcTan[c*x])] - 12*ArcT
an[c*x]^2*Log[1 + ((c*Sqrt[d] - Sqrt[e])*E^((2*I)*ArcTan[c*x]))/(c*Sqrt[d]
+ Sqrt[e])] - 12*ArcTan[c*x]^2*Log[1 + ((c*Sqrt[d] + Sqrt[e])*E^((2*I)*ArcT
an[c*x]))/(c*Sqrt[d] - Sqrt[e])] + 12*ArcTan[c*x]^2*Log[1 + ((c^2*d + e - 2
*Sqrt[c^2*d*e])*E^((2*I)*ArcTan[c*x]))/(c^2*d - e)] + 24*ArcSin[Sqrt[(c^2*d
)/(c^2*d - e)]]*ArcTan[c*x]*Log[1 + ((c^2*d + e + 2*Sqrt[c^2*d*e])*E^((2*I)
*ArcTan[c*x]))/(c^2*d - e)] - 12*ArcTan[c*x]^2*Log[1 + ((c^2*d + e + 2*Sqrt
[c^2*d*e])*E^((2*I)*ArcTan[c*x]))/(c^2*d - e)] - 24*ArcSin[Sqrt[(c^2*d)/(c^
2*d - e)]]*ArcTan[c*x]*Log[(-2*Sqrt[c^2*d*e]*E^((2*I)*ArcTan[c*x]) + e*(-1
+ E^((2*I)*ArcTan[c*x])) + c^2*d*(1 + E^((2*I)*ArcTan[c*x])))/(c^2*d - e)]
- 24*ArcTan[c*x]^2*Log[(-2*Sqrt[c^2*d*e]*E^((2*I)*ArcTan[c*x]) + e*(-1 + E^
((2*I)*ArcTan[c*x])) + c^2*d*(1 + E^((2*I)*ArcTan[c*x])))/(c^2*d - e)] + 24
*ArcSin[Sqrt[(c^2*d)/(c^2*d - e)]]*ArcTan[c*x]*Log[((2*I)*c^2*d - (2*I)*Sqr
t[c^2*d*e] + 2*c*(-e + Sqrt[c^2*d*e])*x)/((c^2*d - e)*(I + c*x))] + 12*ArcT
an[c*x]^2*Log[((2*I)*c^2*d - (2*I)*Sqrt[c^2*d*e] + 2*c*(-e + Sqrt[c^2*d*e])
*x)/((c^2*d - e)*(I + c*x))] - 24*ArcSin[Sqrt[(c^2*d)/(c^2*d - e)]]*ArcTan[
c*x]*Log[1 + ((c^2*d + e + 2*Sqrt[c^2*d*e])*(Cos[2*ArcTan[c*x]] + I*Sin[2*A
rcTan[c*x]]))/(c^2*d - e)] + 12*ArcTan[c*x]^2*Log[1 + ((c^2*d + e + 2*Sqrt[
c^2*d*e])*(Cos[2*ArcTan[c*x]] + I*Sin[2*ArcTan[c*x]]))/(c^2*d - e)] + (24*I
)*ArcTan[c*x]*PolyLog[2, E^((-2*I)*ArcTan[c*x])] + (12*I)*ArcTan[c*x]*PolyL
og[2, ((-(c*Sqrt[d]) + Sqrt[e])*E^((2*I)*ArcTan[c*x]))/(c*Sqrt[d] + Sqrt[e]
)] + (12*I)*ArcTan[c*x]*PolyLog[2, -(((c*Sqrt[d] + Sqrt[e])*E^((2*I)*ArcTan
[c*x]))/(c*Sqrt[d] - Sqrt[e]))] + 12*PolyLog[3, E^((-2*I)*ArcTan[c*x])] - 6
*PolyLog[3, ((-(c*Sqrt[d]) + Sqrt[e])*E^((2*I)*ArcTan[c*x]))/(c*Sqrt[d] + S
qrt[e])] - 6*PolyLog[3, -(((c*Sqrt[d] + Sqrt[e])*E^((2*I)*ArcTan[c*x]))/(c*
Sqrt[d] - Sqrt[e]))]))/(24*d)
```

Maple [F] time = 9.102, size = 0, normalized size = 0.

$$\int \frac{(a+b\arctan(cx))^2}{x(ex^2+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^2/x/(e*x^2+d),x)
```

```
[Out] int((a+b*arctan(c*x))^2/x/(e*x^2+d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2}a^2\left(\frac{\log(ex^2+d)}{d}-\frac{2\log(x)}{d}\right)+\int\frac{b^2\arctan(cx)^2+2ab\arctan(cx)}{ex^3+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] -1/2*a^2*(log(e*x^2 + d)/d - 2*log(x)/d) + integrate((b^2*arctan(c*x)^2 + 2
*a*b*arctan(c*x))/(e*x^3 + d*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2\arctan(cx)^2+2ab\arctan(cx)+a^2}{ex^3+dx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*arctan(c*x)^2 + 2*a*b*arctan(c*x) + a^2)/(e*x^3 + d*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**2/x/(e*x**2+d),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\arctan(cx)+a)^2}{(ex^2+d)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x/(e*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)^2/((e*x^2 + d)*x), x)
```

3.1266 $\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^2\left(d+ex^2\right)}\,dx$

Optimal. Leaf size=553

$$-\frac{ib\sqrt{e}\left(a+b\tan^{-1}(cx)\right)\text{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{(1-icx)\left(c\sqrt{-d}-i\sqrt{e}\right)}\right)}{2(-d)^{3/2}}+\frac{ib\sqrt{e}\left(a+b\tan^{-1}(cx)\right)\text{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-d}+\sqrt{e}x\right)}{(1-icx)\left(c\sqrt{-d}+i\sqrt{e}\right)}\right)}{2(-d)^{3/2}}+\cdots$$

```
[Out] ((-I)*c*(a + b*ArcTan[c*x])^2)/d - (a + b*ArcTan[c*x])^2/(d*x) + (Sqrt[e]*(
a + b*ArcTan[c*x])^2*Log[(2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt
[e])*(1 - I*c*x))])/(2*(-d)^(3/2)) - (Sqrt[e]*(a + b*ArcTan[c*x])^2*Log[(2*
c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(2*(-d)^
(3/2)) + (2*b*c*(a + b*ArcTan[c*x])*Log[2 - 2/(1 - I*c*x)])/d - (I*b^2*c*Po
lyLog[2, -1 + 2/(1 - I*c*x)])/d - ((I/2)*b*Sqrt[e]*(a + b*ArcTan[c*x])*Poly
Log[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*
x))])/(-d)^(3/2) + ((I/2)*b*Sqrt[e]*(a + b*ArcTan[c*x])*PolyLog[2, 1 - (2*c
*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(-d)^(3/2
) + (b^2*Sqrt[e]*PolyLog[3, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] -
 I*Sqrt[e])*(1 - I*c*x))])/(4*(-d)^(3/2)) - (b^2*Sqrt[e]*PolyLog[3, 1 - (2*
c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(4*(-d)^
(3/2))
```

Rubi [A] time = 0.516037, antiderivative size = 553, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4918, 4852, 4924, 4868, 2447, 4914, 4858}

$$-\frac{ib\sqrt{e}\left(a+b\tan^{-1}(cx)\right)\text{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{(1-icx)\left(c\sqrt{-d}-i\sqrt{e}\right)}\right)}{2(-d)^{3/2}}+\frac{ib\sqrt{e}\left(a+b\tan^{-1}(cx)\right)\text{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-d}+\sqrt{e}x\right)}{(1-icx)\left(c\sqrt{-d}+i\sqrt{e}\right)}\right)}{2(-d)^{3/2}}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^2/(x^2*(d + e*x^2)),x]
```

```
[Out] ((-I)*c*(a + b*ArcTan[c*x])^2)/d - (a + b*ArcTan[c*x])^2/(d*x) + (Sqrt[e]*(
a + b*ArcTan[c*x])^2*Log[(2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt
[e])*(1 - I*c*x))])/(2*(-d)^(3/2)) - (Sqrt[e]*(a + b*ArcTan[c*x])^2*Log[(2*
c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(2*(-d)^
(3/2)) + (2*b*c*(a + b*ArcTan[c*x])*Log[2 - 2/(1 - I*c*x)])/d - (I*b^2*c*Po
lyLog[2, -1 + 2/(1 - I*c*x)])/d - ((I/2)*b*Sqrt[e]*(a + b*ArcTan[c*x])*Poly
```

```
Log[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*
x))])/(-d)^(3/2) + ((I/2)*b*Sqrt[e]*(a + b*ArcTan[c*x])*PolyLog[2, 1 - (2*c
*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(-d)^(3/2
) + (b^2*Sqrt[e]*PolyLog[3, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] -
 I*Sqrt[e])*(1 - I*c*x))])/(4*(-d)^(3/2)) - (b^2*Sqrt[e]*PolyLog[3, 1 - (2*
c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(4*(-d)^
(3/2))
```

Rule 4918

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/(d + e*x^2),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4914

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (d + e*x^2)^q, x], x
] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[q] && IGtQ[p, 0]
```

Rule 4858

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^2/((d_) + (e_.)*(x_)), x_Symbol] :>
-Simp[((a + b*ArcTan[c*x])^2*Log[2/(1 - I*c*x)])/e, x] + (Simp[((a + b*ArcT
an[c*x])^2*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x] + Simp[(I*
b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 - I*c*x)])/e, x] - Simp[(I*b*(a +
 b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/
e, x] - Simp[(b^2*PolyLog[3, 1 - 2/(1 - I*c*x)])/(2*e), x] + Simp[(b^2*Poly
Log[3, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/(2*e), x]) /; FreeQ[
{a, b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^2\left(d+ex^2\right)}dx = \frac{\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^2}dx}{d} - \frac{e\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{d+ex^2}dx}{d}$$

$$= -\frac{\left(a+b\tan^{-1}(cx)\right)^2}{dx} + \frac{(2bc)\int \frac{a+b\tan^{-1}(cx)}{x\left(1+c^2x^2\right)}dx}{d} - \frac{e\int \left(\frac{\sqrt{-d}\left(a+b\tan^{-1}(cx)\right)^2}{2d\left(\sqrt{-d}-\sqrt{e}x\right)} + \frac{\sqrt{-d}\left(a+b\tan^{-1}(cx)\right)^2}{2d\left(\sqrt{-d}+\sqrt{e}x\right)}\right) dx}{d}$$

$$= -\frac{ic\left(a+b\tan^{-1}(cx)\right)^2}{d} - \frac{\left(a+b\tan^{-1}(cx)\right)^2}{dx} + \frac{(2ibc)\int \frac{a+b\tan^{-1}(cx)}{x(i+cx)}dx}{d} - \frac{e\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{\sqrt{-d}-\sqrt{e}x}dx}{2(-d)^{3/2}}$$

$$= -\frac{ic\left(a+b\tan^{-1}(cx)\right)^2}{d} - \frac{\left(a+b\tan^{-1}(cx)\right)^2}{dx} + \frac{\sqrt{e}\left(a+b\tan^{-1}(cx)\right)^2\log\left(\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(c\sqrt{-d}-i\sqrt{e}\right)(1-icx)}\right)}{2(-d)^{3/2}} -$$

$$= -\frac{ic\left(a+b\tan^{-1}(cx)\right)^2}{d} - \frac{\left(a+b\tan^{-1}(cx)\right)^2}{dx} + \frac{\sqrt{e}\left(a+b\tan^{-1}(cx)\right)^2\log\left(\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(c\sqrt{-d}-i\sqrt{e}\right)(1-icx)}\right)}{2(-d)^{3/2}} -$$

Mathematica [F] time = 180.002, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcTan[c*x])^2/(x^2*(d + e*x^2)),x]
```

```
[Out] $Aborted
```

Maple [F] time = 3.365, size = 0, normalized size = 0.

$$\int \frac{(a + b\arctan(cx))^2}{x^2\left(ex^2 + d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^2/x^2/(e*x^2+d),x)
```

```
[Out] int((a+b*arctan(c*x))^2/x^2/(e*x^2+d),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^2/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2\arctan(cx)^2 + 2\,ab\arctan(cx) + a^2}{ex^4 + dx^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^2/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*arctan(c*x)^2 + 2*a*b*arctan(c*x) + a^2)/(e*x^4 + d*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**2/x**2/(e*x**2+d),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\arctan(cx) + a)^2}{(ex^2 + d)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^2/(e*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)^2/((e*x^2 + d)*x^2), x)
```

3.1267 $\int \frac{(a+b\tan^{-1}(cx))^2}{x^3(d+ex^2)}\,dx$

Optimal. Leaf size=745

$$\frac{ibe\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)\left(a+b\tan^{-1}(cx)\right)}{d^2}+\frac{ibe\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{d^2}-\frac{ibe\text{PolyLog}\left(2,-1+\frac{2}{1+icx}\right.}{d^2}$$

```
[Out] -((b*c*(a + b*ArcTan[c*x]))/(d*x)) - (c^2*(a + b*ArcTan[c*x])^2)/(2*d) - (a
 + b*ArcTan[c*x])^2/(2*d*x^2) - (2*e*(a + b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1
 + I*c*x)])/d^2 + (b^2*c^2*Log[x])/d - (e*(a + b*ArcTan[c*x])^2*Log[2/(1 -
I*c*x)])/d^2 + (e*(a + b*ArcTan[c*x])^2*Log[(2*c*(Sqrt[-d] - Sqrt[e]*x))/((
c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/(2*d^2) + (e*(a + b*ArcTan[c*x])^2*L
og[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(2
*d^2) - (b^2*c^2*Log[1 + c^2*x^2])/(2*d) + (I*b*e*(a + b*ArcTan[c*x])*PolyL
og[2, 1 - 2/(1 - I*c*x)])/d^2 + (I*b*e*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2
/(1 + I*c*x)])/d^2 - (I*b*e*(a + b*ArcTan[c*x])*PolyLog[2, -1 + 2/(1 + I*c*
x)])/d^2 - ((I/2)*b*e*(a + b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(Sqrt[-d] - S
qrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/d^2 - ((I/2)*b*e*(a + b
*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*
Sqrt[e])*(1 - I*c*x))])/d^2 - (b^2*e*PolyLog[3, 1 - 2/(1 - I*c*x)])/(2*d^2)
 + (b^2*e*PolyLog[3, 1 - 2/(1 + I*c*x)])/(2*d^2) - (b^2*e*PolyLog[3, -1 + 2
/(1 + I*c*x)])/(2*d^2) + (b^2*e*PolyLog[3, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))
/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/(4*d^2) + (b^2*e*PolyLog[3, 1 - (
2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(4*d^2
)
```

Rubi [A] time = 0.923272, antiderivative size = 745, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 13, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.565, Rules used = {4918, 4852, 266, 36, 29, 31, 4884, 4980, 4850, 4988, 4994, 6610, 4858}

$$\frac{ibe\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)\left(a+b\tan^{-1}(cx)\right)}{d^2}+\frac{ibe\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{d^2}-\frac{ibe\text{PolyLog}\left(2,-1+\frac{2}{1+icx}\right.}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^2/(x^3*(d + e*x^2)),x]
```

```
[Out] -((b*c*(a + b*ArcTan[c*x]))/(d*x)) - (c^2*(a + b*ArcTan[c*x])^2)/(2*d) - (a
 + b*ArcTan[c*x])^2/(2*d*x^2) - (2*e*(a + b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1
```

```
 + I*c*x)])/d^2 + (b^2*c^2*Log[x])/d - (e*(a + b*ArcTan[c*x])^2*Log[2/(1 -
I*c*x)])/d^2 + (e*(a + b*ArcTan[c*x])^2*Log[(2*c*(Sqrt[-d] - Sqrt[e]*x))/((
c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/(2*d^2) + (e*(a + b*ArcTan[c*x])^2*L
og[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(2
*d^2) - (b^2*c^2*Log[1 + c^2*x^2])/(2*d) + (I*b*e*(a + b*ArcTan[c*x])*PolyL
og[2, 1 - 2/(1 - I*c*x)])/d^2 + (I*b*e*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2
/(1 + I*c*x)])/d^2 - (I*b*e*(a + b*ArcTan[c*x])*PolyLog[2, -1 + 2/(1 + I*c*
x)])/d^2 - ((I/2)*b*e*(a + b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(Sqrt[-d] - S
qrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/d^2 - ((I/2)*b*e*(a + b
*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*
Sqrt[e])*(1 - I*c*x))])/d^2 - (b^2*e*PolyLog[3, 1 - 2/(1 - I*c*x)])/(2*d^2)
 + (b^2*e*PolyLog[3, 1 - 2/(1 + I*c*x)])/(2*d^2) - (b^2*e*PolyLog[3, -1 + 2
/(1 + I*c*x)])/(2*d^2) + (b^2*e*PolyLog[3, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))
/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/(4*d^2) + (b^2*e*PolyLog[3, 1 - (
2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(4*d^2
)
```

Rule 4918

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/(d + e*x^2),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4980

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

Rule 4850

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTan[c*x])^(p - 1)*ArcTanh[1 - 2/(1 + I*c*x)])/(1 + c^2*x^2), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4988

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTan[c*x])^p)/(d + e
*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTan[c*x])^p)/(d + e*x^2
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
```

```
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 4858

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^2/((d_) + (e_.)*(x_)), x_Symbol] :>
-Simp[((a + b*ArcTan[c*x])^2*Log[2/(1 - I*c*x)])/e, x] + (Simp[((a + b*ArcT
an[c*x])^2*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x] + Simp[(I*
b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 - I*c*x)])/e, x] - Simp[(I*b*(a +
 b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/
e, x] - Simp[(b^2*PolyLog[3, 1 - 2/(1 - I*c*x)])/(2*e), x] + Simp[(b^2*Poly
Log[3, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/(2*e), x]) /; FreeQ[
{a, b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^3\left(d+ex^2\right)}\,dx = \frac{\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^3}\,dx}{d} - \frac{e\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x\left(d+ex^2\right)}\,dx}{d}$$

$$= -\frac{\left(a+b\tan^{-1}(cx)\right)^2}{2dx^2} + \frac{(bc)\int \frac{a+b\tan^{-1}(cx)}{x^2\left(1+c^2x^2\right)}\,dx}{d} - \frac{e\int \left(\frac{\left(a+b\tan^{-1}(cx)\right)^2}{dx} - \frac{ex\left(a+b\tan^{-1}(cx)\right)^2}{d\left(d+ex^2\right)}\right)dx}{d}$$

$$= -\frac{\left(a+b\tan^{-1}(cx)\right)^2}{2dx^2} + \frac{(bc)\int \frac{a+b\tan^{-1}(cx)}{x^2}\,dx}{d} - \frac{\left(bc^3\right)\int \frac{a+b\tan^{-1}(cx)}{1+c^2x^2}\,dx}{d} - \frac{e\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x}\,dx}{d^2}$$

$$= -\frac{bc\left(a+b\tan^{-1}(cx)\right)}{dx} - \frac{c^2\left(a+b\tan^{-1}(cx)\right)^2}{2d} - \frac{\left(a+b\tan^{-1}(cx)\right)^2}{2dx^2} - \frac{2e\left(a+b\tan^{-1}(cx)\right)^2 \text{ta}}{d^2}$$

$$= -\frac{bc\left(a+b\tan^{-1}(cx)\right)}{dx} - \frac{c^2\left(a+b\tan^{-1}(cx)\right)^2}{2d} - \frac{\left(a+b\tan^{-1}(cx)\right)^2}{2dx^2} - \frac{2e\left(a+b\tan^{-1}(cx)\right)^2 \text{ta}}{d^2}$$

$$= -\frac{bc\left(a+b\tan^{-1}(cx)\right)}{dx} - \frac{c^2\left(a+b\tan^{-1}(cx)\right)^2}{2d} - \frac{\left(a+b\tan^{-1}(cx)\right)^2}{2dx^2} - \frac{2e\left(a+b\tan^{-1}(cx)\right)^2 \text{ta}}{d^2}$$

$$= -\frac{bc\left(a+b\tan^{-1}(cx)\right)}{dx} - \frac{c^2\left(a+b\tan^{-1}(cx)\right)^2}{2d} - \frac{\left(a+b\tan^{-1}(cx)\right)^2}{2dx^2} - \frac{2e\left(a+b\tan^{-1}(cx)\right)^2 \text{ta}}{d^2}$$

Mathematica [B] time = 10.9148, size = 1555, normalized size = 2.09

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c*x])^2/(x^3*(d + e*x^2)),x]
```

```
[Out] -((12*a^2*d)/x^2 + (24*a*b*c*d)/x + (24*a*b*d*(1 + c^2*x^2)*ArcTan[c*x])/x^
2 + 24*a^2*e*Log[x] - 12*a^2*e*Log[d + e*x^2] - (24*I)*a*b*e*(ArcTan[c*x]*(
ArcTan[c*x] + (2*I)*Log[1 - E^((2*I)*ArcTan[c*x])]) + PolyLog[2, E^((2*I)*A
rcTan[c*x])]) - (48*a*b*(c^2*d - e)*e*((-I)*ArcTan[c*x]^2 + (2*I)*ArcSin[Sq
rt[(c^2*d)/(c^2*d - e)]]*ArcTan[(c*e*x)/Sqrt[c^2*d*e]] + (-ArcSin[Sqrt[(c^2
*d)/(c^2*d - e)]] + ArcTan[c*x])*Log[1 + ((c^2*d + e + 2*Sqrt[c^2*d*e])*E^(
(2*I)*ArcTan[c*x]))/(c^2*d - e)] + (ArcSin[Sqrt[(c^2*d)/(c^2*d - e)]] + Arc
Tan[c*x])*Log[(-2*Sqrt[c^2*d*e]*E^((2*I)*ArcTan[c*x]) + e*(-1 + E^((2*I)*Ar
cTan[c*x])) + c^2*d*(1 + E^((2*I)*ArcTan[c*x])))/(c^2*d - e)] - (I/2)*(Poly
Log[2, -(((c^2*d + e - 2*Sqrt[c^2*d*e])*E^((2*I)*ArcTan[c*x]))/(c^2*d - e))
] + PolyLog[2, -(((c^2*d + e + 2*Sqrt[c^2*d*e])*E^((2*I)*ArcTan[c*x]))/(c^2
*d - e))])))/(2*c^2*d - 2*e) + b^2*((-I)*e*Pi^3 + (24*c*d*ArcTan[c*x])/x +
(12*d*(1 + c^2*x^2)*ArcTan[c*x]^2)/x^2 + (8*I)*e*ArcTan[c*x]^3 + 24*e*ArcTa
n[c*x]^2*Log[1 - E^((-2*I)*ArcTan[c*x])] - 24*c^2*d*Log[(c*x)/Sqrt[1 + c^2*
x^2]] + (24*I)*e*ArcTan[c*x]*PolyLog[2, E^((-2*I)*ArcTan[c*x])] + 12*e*Poly
Log[3, E^((-2*I)*ArcTan[c*x])]) + 2*b^2*e*((4*I)*ArcTan[c*x]^3 - 6*ArcTan[c
*x]^2*Log[1 + ((c*Sqrt[d] - Sqrt[e])*E^((2*I)*ArcTan[c*x]))/(c*Sqrt[d] + Sq
rt[e])] - 6*ArcTan[c*x]^2*Log[1 + ((c*Sqrt[d] + Sqrt[e])*E^((2*I)*ArcTan[c*
x]))/(c*Sqrt[d] - Sqrt[e])] + 6*ArcTan[c*x]^2*Log[1 + ((c^2*d + e - 2*Sqrt[
c^2*d*e])*E^((2*I)*ArcTan[c*x]))/(c^2*d - e)] + 12*ArcSin[Sqrt[(c^2*d)/(c^2
*d - e)]]*ArcTan[c*x]*Log[1 + ((c^2*d + e + 2*Sqrt[c^2*d*e])*E^((2*I)*ArcTa
n[c*x]))/(c^2*d - e)] - 6*ArcTan[c*x]^2*Log[1 + ((c^2*d + e + 2*Sqrt[c^2*d*
e])*E^((2*I)*ArcTan[c*x]))/(c^2*d - e)] - 12*ArcSin[Sqrt[(c^2*d)/(c^2*d - e
)]]*ArcTan[c*x]*Log[(-2*Sqrt[c^2*d*e]*E^((2*I)*ArcTan[c*x]) + e*(-1 + E^((2
*I)*ArcTan[c*x])) + c^2*d*(1 + E^((2*I)*ArcTan[c*x])))/(c^2*d - e)] - 12*Ar
cTan[c*x]^2*Log[(-2*Sqrt[c^2*d*e]*E^((2*I)*ArcTan[c*x]) + e*(-1 + E^((2*I)*
ArcTan[c*x])) + c^2*d*(1 + E^((2*I)*ArcTan[c*x])))/(c^2*d - e)] + 12*ArcSin
[Sqrt[(c^2*d)/(c^2*d - e)]]*ArcTan[c*x]*Log[((2*I)*c^2*d - (2*I)*Sqrt[c^2*d
*e] + 2*c*(-e + Sqrt[c^2*d*e])*x)/((c^2*d - e)*(I + c*x))] + 6*ArcTan[c*x]^
2*Log[((2*I)*c^2*d - (2*I)*Sqrt[c^2*d*e] + 2*c*(-e + Sqrt[c^2*d*e])*x)/((c^
2*d - e)*(I + c*x))] - 12*ArcSin[Sqrt[(c^2*d)/(c^2*d - e)]]*ArcTan[c*x]*Log
[1 + ((c^2*d + e + 2*Sqrt[c^2*d*e])*(Cos[2*ArcTan[c*x]] + I*Sin[2*ArcTan[c*
x]]))/(c^2*d - e)] + 6*ArcTan[c*x]^2*Log[1 + ((c^2*d + e + 2*Sqrt[c^2*d*e])
*(Cos[2*ArcTan[c*x]] + I*Sin[2*ArcTan[c*x]]))/(c^2*d - e)] + (6*I)*ArcTan[c
*x]*PolyLog[2, ((-(c*Sqrt[d]) + Sqrt[e])*E^((2*I)*ArcTan[c*x]))/(c*Sqrt[d]
+ Sqrt[e])] + (6*I)*ArcTan[c*x]*PolyLog[2, -(((c*Sqrt[d] + Sqrt[e])*E^((2*I
```

```
)*ArcTan[c*x]))/(c*Sqrt[d] - Sqrt[e]))] - 3*PolyLog[3, ((-(c*Sqrt[d]) + Sqr
t[e])*E^((2*I)*ArcTan[c*x]))/(c*Sqrt[d] + Sqrt[e])] - 3*PolyLog[3, -(((c*Sq
rt[d] + Sqrt[e])*E^((2*I)*ArcTan[c*x]))/(c*Sqrt[d] - Sqrt[e]))]))/(24*d^2)
```

Maple [F] time = 26.595, size = 0, normalized size = 0.

$$\int \frac{(a + b \arctan(cx))^2}{x^3 (ex^2 + d)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^2/x^3/(e*x^2+d),x)
```

```
[Out] int((a+b*arctan(c*x))^2/x^3/(e*x^2+d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2} a^2 \left(\frac{e \log(ex^2 + d)}{d^2} - \frac{2 e \log(x)}{d^2} - \frac{1}{dx^2} \right) + \int \frac{b^2 \arctan(cx)^2 + 2 ab \arctan(cx)}{ex^5 + dx^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^3/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] 1/2*a^2*(e*log(e*x^2 + d)/d^2 - 2*e*log(x)/d^2 - 1/(d*x^2)) + integrate((b^
2*arctan(c*x)^2 + 2*a*b*arctan(c*x))/(e*x^5 + d*x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2 \arctan(cx)^2 + 2 ab \arctan(cx) + a^2}{ex^5 + dx^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^3/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*arctan(c*x)^2 + 2*a*b*arctan(c*x) + a^2)/(e*x^5 + d*x^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**2/x**3/(e*x**2+d),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\arctan(cx)+a)^2}{(ex^2+d)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^3/(e*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)^2/((e*x^2 + d)*x^3), x)
```

3.1268 $\int \frac{x^3\left(a+b\tan^{-1}(cx)\right)^2}{\left(d+ex^2\right)^2}\,dx$

Optimal. Leaf size=943

result too large to display

```
[Out] -(c^2*d*(a + b*ArcTan[c*x])^2)/(2*(c^2*d - e)*e^2) + (a + b*ArcTan[c*x])^2/
(4*e^2*(1 - (Sqrt[e]*x)/Sqrt[-d])) + (a + b*ArcTan[c*x])^2/(4*e^2*(1 + (Sqr
t[e]*x)/Sqrt[-d])) - ((a + b*ArcTan[c*x])^2*Log[2/(1 - I*c*x)])/e^2 - (b*c*
Sqrt[-d]*(a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d]
- I*Sqrt[e])*(1 - I*c*x))])/(2*(c^2*d - e)*e^(3/2)) + ((a + b*ArcTan[c*x])^
2*Log[(2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])
/(2*e^2) + (b*c*Sqrt[-d]*(a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x
))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(2*(c^2*d - e)*e^(3/2)) + ((a +
 b*ArcTan[c*x])^2*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e]
)*(1 - I*c*x))])/(2*e^2) + (I*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 - I
*c*x)])/e^2 + ((I/4)*b^2*c*Sqrt[-d]*PolyLog[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]
*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/((c^2*d - e)*e^(3/2)) - ((I/2
)*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqr
t[-d] - I*Sqrt[e])*(1 - I*c*x))])/e^2 - ((I/4)*b^2*c*Sqrt[-d]*PolyLog[2, 1
- (2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/((c
^2*d - e)*e^(3/2)) - ((I/2)*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(Sqrt
[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/e^2 - (b^2*Poly
Log[3, 1 - 2/(1 - I*c*x)])/(2*e^2) + (b^2*PolyLog[3, 1 - (2*c*(Sqrt[-d] - S
qrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/(4*e^2) + (b^2*PolyLog[
3, 1 - (2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))]
)/(4*e^2)
```

Rubi [A] time = 1.75896, antiderivative size = 943, normalized size of antiderivative = 1., number of steps used = 33, number of rules used = 12, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.522, Rules used = {4980, 4978, 4864, 4856, 2402, 2315, 2447, 4984, 4884, 4920, 4854, 4858}

$$\frac{ic\sqrt{-d}\text{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(c\sqrt{-d}-i\sqrt{e}\right)(1-icx)}\right)b^2}{4\left(c^2d-e\right)e^{3/2}}-\frac{ic\sqrt{-d}\text{PolyLog}\left(2,1-\frac{2c\left(\sqrt{e}x+\sqrt{-d}\right)}{\left(\sqrt{-d}c+i\sqrt{e}\right)(1-icx)}\right)b^2}{4\left(c^2d-e\right)e^{3/2}}-\frac{\text{PolyLog}\left(3,1-\frac{2}{1-icx}\right)b^2}{2e^2}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcTan[c*x])^2)/(d + e*x^2)^2,x]
```

```
[Out] -(c^2*d*(a + b*ArcTan[c*x])^2)/(2*(c^2*d - e)*e^2) + (a + b*ArcTan[c*x])^2/
(4*e^2*(1 - (Sqrt[e]*x)/Sqrt[-d])) + (a + b*ArcTan[c*x])^2/(4*e^2*(1 + (Sqr
t[e]*x)/Sqrt[-d])) - ((a + b*ArcTan[c*x])^2*Log[2/(1 - I*c*x)])/e^2 - (b*c*
Sqrt[-d]*(a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d]
- I*Sqrt[e])*(1 - I*c*x))])/(2*(c^2*d - e)*e^(3/2)) + ((a + b*ArcTan[c*x])^
2*Log[(2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])
/(2*e^2) + (b*c*Sqrt[-d]*(a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x
))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(2*(c^2*d - e)*e^(3/2)) + ((a +
 b*ArcTan[c*x])^2*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e]
)*(1 - I*c*x))])/(2*e^2) + (I*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 - I
*c*x)])/e^2 + ((I/4)*b^2*c*Sqrt[-d]*PolyLog[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]
*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/((c^2*d - e)*e^(3/2)) - ((I/2
)*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqr
t[-d] - I*Sqrt[e])*(1 - I*c*x))])/e^2 - ((I/4)*b^2*c*Sqrt[-d]*PolyLog[2, 1
- (2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/((c
^2*d - e)*e^(3/2)) - ((I/2)*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(Sqrt
[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/e^2 - (b^2*Poly
Log[3, 1 - 2/(1 - I*c*x)])/(2*e^2) + (b^2*PolyLog[3, 1 - (2*c*(Sqrt[-d] - S
qrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/(4*e^2) + (b^2*PolyLog[
3, 1 - (2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))]
)/(4*e^2)
```

Rule 4980

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

Rule 4978

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2)^2
, x_Symbol] :> Dist[1/(4*d^2*Rt[-(e/d), 2]), Int[(a + b*ArcTan[c*x])^p/(1 -
 Rt[-(e/d), 2]*x)^2, x], x] - Dist[1/(4*d^2*Rt[-(e/d), 2]), Int[(a + b*ArcT
an[c*x])^p/(1 + Rt[-(e/d), 2]*x)^2, x], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 0]
```

Rule 4864

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x])^p)/(e*(q + 1)), x] - D
ist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4984

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(m_.))/((
d_) + (e_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p
/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGt
Q[p, 0] && EqQ[e, c^2*d] && IGtQ[m, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
```

```
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4858

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^2/((d_) + (e_.)*(x_)), x_Symbol] :>
-Simp[((a + b*ArcTan[c*x])^2*Log[2/(1 - I*c*x)])/e, x] + (Simp[((a + b*ArcT
an[c*x])^2*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x] + Simp[(I*
b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 - I*c*x)])/e, x] - Simp[(I*b*(a +
 b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/
e, x] - Simp[(b^2*PolyLog[3, 1 - 2/(1 - I*c*x)])/(2*e), x] + Simp[(b^2*Poly
Log[3, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/(2*e), x]) /; FreeQ[
{a, b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rubi steps

$$\int \frac{x^3\left(a+b\tan^{-1}(cx)\right)^2}{\left(d+ex^2\right)^2}\,dx = \int \left(-\frac{dx\left(a+b\tan^{-1}(cx)\right)^2}{e\left(d+ex^2\right)^2}+\frac{x\left(a+b\tan^{-1}(cx)\right)^2}{e\left(d+ex^2\right)}\right)dx$$

$$= \frac{\int \frac{x\left(a+b\tan^{-1}(cx)\right)^2}{d+ex^2}\,dx}{e} - \frac{d\int \frac{x\left(a+b\tan^{-1}(cx)\right)^2}{\left(d+ex^2\right)^2}\,dx}{e}$$

$$= \frac{\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{\left(1-\frac{\sqrt{e}x}{\sqrt{-d}}\right)^2}\,dx}{4\sqrt{-d}e^{3/2}} - \frac{\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{\left(1+\frac{\sqrt{e}x}{\sqrt{-d}}\right)^2}\,dx}{4\sqrt{-d}e^{3/2}} + \frac{\int \left(-\frac{\left(a+b\tan^{-1}(cx)\right)^2}{2\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{\left(a+b\tan^{-1}(cx)\right)^2}{2\sqrt{e}\left(\sqrt{-d}+\sqrt{e}x\right)}\right)dx}{e}$$

$$= \frac{\left(a+b\tan^{-1}(cx)\right)^2}{4e^2\left(1-\frac{\sqrt{e}x}{\sqrt{-d}}\right)} + \frac{\left(a+b\tan^{-1}(cx)\right)^2}{4e^2\left(1+\frac{\sqrt{e}x}{\sqrt{-d}}\right)} - \frac{(bc)\int \left(\frac{\sqrt{-d}e\left(a+b\tan^{-1}(cx)\right)}{\left(c^2d-e\right)\left(-\sqrt{-d}+\sqrt{e}x\right)}+\frac{c^2d\left(\sqrt{-d}+\sqrt{e}x\right)\left(a+b\tan^{-1}\right.}{\sqrt{-d}\left(c^2d-e\right)\left(1+c^2x^2\right)}\right.}{2e^2}$$

$$= \frac{\left(a+b\tan^{-1}(cx)\right)^2}{4e^2\left(1-\frac{\sqrt{e}x}{\sqrt{-d}}\right)} + \frac{\left(a+b\tan^{-1}(cx)\right)^2}{4e^2\left(1+\frac{\sqrt{e}x}{\sqrt{-d}}\right)} - \frac{\left(a+b\tan^{-1}(cx)\right)^2\log\left(\frac{2}{1-icx}\right)}{e^2} + \frac{\left(a+b\tan^{-1}(c\right.}{}$$

$$= \frac{\left(a+b\tan^{-1}(cx)\right)^2}{4e^2\left(1-\frac{\sqrt{e}x}{\sqrt{-d}}\right)} + \frac{\left(a+b\tan^{-1}(cx)\right)^2}{4e^2\left(1+\frac{\sqrt{e}x}{\sqrt{-d}}\right)} - \frac{\left(a+b\tan^{-1}(cx)\right)^2\log\left(\frac{2}{1-icx}\right)}{e^2} - \frac{bc\sqrt{-d}\left(a+b\,\mathrm{t}\right.}{}$$

$$= \frac{\left(a+b\tan^{-1}(cx)\right)^2}{4e^2\left(1-\frac{\sqrt{e}x}{\sqrt{-d}}\right)} + \frac{\left(a+b\tan^{-1}(cx)\right)^2}{4e^2\left(1+\frac{\sqrt{e}x}{\sqrt{-d}}\right)} - \frac{\left(a+b\tan^{-1}(cx)\right)^2\log\left(\frac{2}{1-icx}\right)}{e^2} - \frac{bc\sqrt{-d}\left(a+b\,\mathrm{t}\right.}{}$$

$$= -\frac{c^2d\left(a+b\tan^{-1}(cx)\right)^2}{2\left(c^2d-e\right)e^2} + \frac{\left(a+b\tan^{-1}(cx)\right)^2}{4e^2\left(1-\frac{\sqrt{e}x}{\sqrt{-d}}\right)} + \frac{\left(a+b\tan^{-1}(cx)\right)^2}{4e^2\left(1+\frac{\sqrt{e}x}{\sqrt{-d}}\right)} - \frac{\left(a+b\tan^{-1}(cx)\right)^2\,\mathrm{lo}}{e^2}$$

Mathematica [F] time = 18.1535, size = 0, normalized size = 0.

$$\int \frac{x^3\left(a+b\tan^{-1}(cx)\right)^2}{\left(d+ex^2\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^3*(a + b*ArcTan[c*x])^2)/(d + e*x^2)^2,x]
```

```
[Out] Integrate[(x^3*(a + b*ArcTan[c*x])^2)/(d + e*x^2)^2, x]
```

Maple [F] time = 9.468, size = 0, normalized size = 0.

$$\int \frac{x^{3} \left(a + b \arctan \left(c x\right)\right)^{2}}{\left(e x^{2} + d\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctan(c*x))^2/(e*x^2+d)^2,x)
```

```
[Out] int(x^3*(a+b*arctan(c*x))^2/(e*x^2+d)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2} a^{2} \left(\frac{d}{e^{3} x^{2} + d e^{2}} + \frac{\log \left(e x^{2} + d\right)}{e^{2}}\right) + \int \frac{b^{2} x^{3} \arctan \left(c x\right)^{2} + 2 \, a b x^{3} \arctan \left(c x\right)}{e^{2} x^{4} + 2 \, d e x^{2} + d^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))^2/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*(d/(e^3*x^2 + d*e^2) + log(e*x^2 + d)/e^2) + integrate((b^2*x^3*arc
tan(c*x)^2 + 2*a*b*x^3*arctan(c*x))/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$${\rm integral}\left(\frac{b^{2} x^{3} \arctan \left(c x\right)^{2} + 2 \, a b x^{3} \arctan \left(c x\right) + a^{2} x^{3}}{e^{2} x^{4} + 2 \, d e x^{2} + d^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))^2/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^3*arctan(c*x)^2 + 2*a*b*x^3*arctan(c*x) + a^2*x^3)/(e^2*x^4
 + 2*d*e*x^2 + d^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atan(c*x))**2/(e*x**2+d)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \arctan(cx) + a)^2 x^3}{(ex^2 + d)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))^2/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)^2*x^3/(e*x^2 + d)^2, x)
```

3.1269 $\int \frac{x^2\left(a+b\tan^{-1}(cx)\right)^2}{\left(d+ex^2\right)^2}\,dx$

Optimal. Leaf size=1033

result too large to display

```
[Out] ((-I/2)*c*(a + b*ArcTan[c*x])^2)/((c^2*d - e)*e) + (a + b*ArcTan[c*x])^2/(4
*e^(3/2)*(Sqrt[-d] - Sqrt[e]*x)) - (a + b*ArcTan[c*x])^2/(4*e^(3/2)*(Sqrt[-
d] + Sqrt[e]*x)) + (b*c*(a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/((c^2*d - e
)*e) - (b*c*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/((c^2*d - e)*e) - (b*c*
(a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[
e])*(1 - I*c*x))])/(2*(c^2*d - e)*e) + ((a + b*ArcTan[c*x])^2*Log[(2*c*(Sqr
t[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/(4*Sqrt[-d]*e^
(3/2)) - (b*c*(a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt
[-d] + I*Sqrt[e])*(1 - I*c*x))])/(2*(c^2*d - e)*e) - ((a + b*ArcTan[c*x])^2
*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/
(4*Sqrt[-d]*e^(3/2)) - ((I/2)*b^2*c*PolyLog[2, 1 - 2/(1 - I*c*x)])/((c^2*d
- e)*e) - ((I/2)*b^2*c*PolyLog[2, 1 - 2/(1 + I*c*x)])/((c^2*d - e)*e) + ((I
/4)*b^2*c*PolyLog[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt
[e])*(1 - I*c*x))])/((c^2*d - e)*e) - ((I/4)*b*(a + b*ArcTan[c*x])*PolyLog[
2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))]
)/(Sqrt[-d]*e^(3/2)) + ((I/4)*b^2*c*PolyLog[2, 1 - (2*c*(Sqrt[-d] + Sqrt[e]
*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/((c^2*d - e)*e) + ((I/4)*b*(a
 + b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d]
+ I*Sqrt[e])*(1 - I*c*x))])/(Sqrt[-d]*e^(3/2)) + (b^2*PolyLog[3, 1 - (2*c*(
Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/(8*Sqrt[-d]
*e^(3/2)) - (b^2*PolyLog[3, 1 - (2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] +
 I*Sqrt[e])*(1 - I*c*x))])/(8*Sqrt[-d]*e^(3/2))
```

Rubi [A] time = 1.9477, antiderivative size = 1033, normalized size of antiderivative = 1., number of steps used = 38, number of rules used = 12, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.522, Rules used = {4980, 4914, 4864, 4856, 2402, 2315, 2447, 4984, 4884, 4920, 4854, 4858}

$$-\frac{ic\,\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)b^2}{2\left(c^2d-e\right)e}-\frac{ic\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)b^2}{2\left(c^2d-e\right)e}+\frac{ic\,\text{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(c\sqrt{-d}-i\sqrt{e}\right)(1-icx)}\right)b^2}{4\left(c^2d-e\right)e}+\frac{ic\,\text{PolyLog}\left(2,1\right.}{4\left(c\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcTan[c*x])^2)/(d + e*x^2)^2,x]
```

```
[Out] ((-I/2)*c*(a + b*ArcTan[c*x])^2)/((c^2*d - e)*e) + (a + b*ArcTan[c*x])^2/(4
*e^(3/2)*(Sqrt[-d] - Sqrt[e]*x)) - (a + b*ArcTan[c*x])^2/(4*e^(3/2)*(Sqrt[-
d] + Sqrt[e]*x)) + (b*c*(a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/((c^2*d - e
)*e) - (b*c*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/((c^2*d - e)*e) - (b*c*
(a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[
e])*(1 - I*c*x))])/(2*(c^2*d - e)*e) + ((a + b*ArcTan[c*x])^2*Log[(2*c*(Sqr
t[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/(4*Sqrt[-d]*e^
(3/2)) - (b*c*(a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt
[-d] + I*Sqrt[e])*(1 - I*c*x))])/(2*(c^2*d - e)*e) - ((a + b*ArcTan[c*x])^2
*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/
(4*Sqrt[-d]*e^(3/2)) - ((I/2)*b^2*c*PolyLog[2, 1 - 2/(1 - I*c*x)])/((c^2*d
- e)*e) - ((I/2)*b^2*c*PolyLog[2, 1 - 2/(1 + I*c*x)])/((c^2*d - e)*e) + ((I
/4)*b^2*c*PolyLog[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt
[e])*(1 - I*c*x))])/((c^2*d - e)*e) - ((I/4)*b*(a + b*ArcTan[c*x])*PolyLog[
2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))]
)/(Sqrt[-d]*e^(3/2)) + ((I/4)*b^2*c*PolyLog[2, 1 - (2*c*(Sqrt[-d] + Sqrt[e]
*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/((c^2*d - e)*e) + ((I/4)*b*(a
 + b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d]
+ I*Sqrt[e])*(1 - I*c*x))])/(Sqrt[-d]*e^(3/2)) + (b^2*PolyLog[3, 1 - (2*c*(
Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/(8*Sqrt[-d]
*e^(3/2)) - (b^2*PolyLog[3, 1 - (2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] +
 I*Sqrt[e])*(1 - I*c*x))])/(8*Sqrt[-d]*e^(3/2))
```

Rule 4980

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

Rule 4914

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (d + e*x^2)^q, x], x
] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[q] && IGtQ[p, 0]
```

Rule 4864

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x])^p)/(e*(q + 1)), x] - D
ist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4984

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(m_.))/((
d_) + (e_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p
/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGt
Q[p, 0] && EqQ[e, c^2*d] && IGtQ[m, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4858

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^2/((d_) + (e_.)*(x_)), x_Symbol] :>
-Simp[((a + b*ArcTan[c*x])^2*Log[2/(1 - I*c*x)])/e, x] + (Simp[((a + b*ArcT
an[c*x])^2*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x] + Simp[(I*
b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 - I*c*x)])/e, x] - Simp[(I*b*(a +
 b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/
e, x] - Simp[(b^2*PolyLog[3, 1 - 2/(1 - I*c*x)])/(2*e), x] + Simp[(b^2*Poly
Log[3, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/(2*e), x]) /; FreeQ[
{a, b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rubi steps

$$\int \frac{x^2 \left(a+b\tan^{-1}(cx)\right)^2}{\left(d+ex^2\right)^2}dx = \int \left(-\frac{d\left(a+b\tan^{-1}(cx)\right)^2}{e\left(d+ex^2\right)^2}+\frac{\left(a+b\tan^{-1}(cx)\right)^2}{e\left(d+ex^2\right)}\right)dx$$

$$= \frac{\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{d+ex^2}dx}{e} - \frac{d\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{\left(d+ex^2\right)^2}dx}{e}$$

$$= \frac{\int \left(\frac{\sqrt{-d}\left(a+b\tan^{-1}(cx)\right)^2}{2d\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{\sqrt{-d}\left(a+b\tan^{-1}(cx)\right)^2}{2d\left(\sqrt{-d}+\sqrt{e}x\right)}\right)dx}{e} - \frac{d\int \left(-\frac{e\left(a+b\tan^{-1}(cx)\right)^2}{4d\left(\sqrt{-d}\sqrt{e}-ex\right)^2}-\frac{e\left(a+b\tan^{-1}(cx)\right)^2}{4d\left(\sqrt{-d}\sqrt{e}+ex\right)^2}-\cdots\right.}{e}$$

$$= \frac{1}{4}\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{\left(\sqrt{-d}\sqrt{e}-ex\right)^2}dx+\frac{1}{4}\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{\left(\sqrt{-d}\sqrt{e}+ex\right)^2}dx+\frac{1}{2}\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{-de-e^2x^2}dx-\cdots$$

$$= \frac{\left(a+b\tan^{-1}(cx)\right)^2}{4e^{3/2}\left(\sqrt{-d}-\sqrt{e}x\right)}-\frac{\left(a+b\tan^{-1}(cx)\right)^2}{4e^{3/2}\left(\sqrt{-d}+\sqrt{e}x\right)}+\frac{\left(a+b\tan^{-1}(cx)\right)^2\log\left(\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(c\sqrt{-d}-i\sqrt{e}\right)(1-icx)}\right)}{2\sqrt{-d}e^{3/2}}-\cdots$$

$$= \frac{\left(a+b\tan^{-1}(cx)\right)^2}{4e^{3/2}\left(\sqrt{-d}-\sqrt{e}x\right)}-\frac{\left(a+b\tan^{-1}(cx)\right)^2}{4e^{3/2}\left(\sqrt{-d}+\sqrt{e}x\right)}+\frac{\left(a+b\tan^{-1}(cx)\right)^2\log\left(\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(c\sqrt{-d}-i\sqrt{e}\right)(1-icx)}\right)}{2\sqrt{-d}e^{3/2}}-\cdots$$

$$= \frac{\left(a+b\tan^{-1}(cx)\right)^2}{4e^{3/2}\left(\sqrt{-d}-\sqrt{e}x\right)}-\frac{\left(a+b\tan^{-1}(cx)\right)^2}{4e^{3/2}\left(\sqrt{-d}+\sqrt{e}x\right)}+\frac{bc\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2}{1-icx}\right)}{\left(c^2d-e\right)e}-\frac{bc\left(a+b\,\mathrm{ta}\cdots\right.}{\cdots}$$

$$= \frac{\left(a+b\tan^{-1}(cx)\right)^2}{4e^{3/2}\left(\sqrt{-d}-\sqrt{e}x\right)}-\frac{\left(a+b\tan^{-1}(cx)\right)^2}{4e^{3/2}\left(\sqrt{-d}+\sqrt{e}x\right)}+\frac{bc\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2}{1-icx}\right)}{\left(c^2d-e\right)e}-\frac{bc\left(a+b\,\mathrm{ta}\cdots\right.}{\cdots}$$

$$= \frac{\left(a+b\tan^{-1}(cx)\right)^2}{4e^{3/2}\left(\sqrt{-d}-\sqrt{e}x\right)}-\frac{\left(a+b\tan^{-1}(cx)\right)^2}{4e^{3/2}\left(\sqrt{-d}+\sqrt{e}x\right)}+\frac{bc\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2}{1-icx}\right)}{\left(c^2d-e\right)e}-\frac{bc\left(a+b\,\mathrm{ta}\cdots\right.}{\cdots}$$

$$= \frac{\left(a+b\tan^{-1}(cx)\right)^2}{4e^{3/2}\left(\sqrt{-d}-\sqrt{e}x\right)}-\frac{\left(a+b\tan^{-1}(cx)\right)^2}{4e^{3/2}\left(\sqrt{-d}+\sqrt{e}x\right)}+\frac{bc\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2}{1-icx}\right)}{\left(c^2d-e\right)e}-\frac{bc\left(a+b\,\mathrm{ta}\cdots\right.}{\cdots}$$

$$= \frac{\left(a+b\tan^{-1}(cx)\right)^2}{4e^{3/2}\left(\sqrt{-d}-\sqrt{e}x\right)}-\frac{\left(a+b\tan^{-1}(cx)\right)^2}{4e^{3/2}\left(\sqrt{-d}+\sqrt{e}x\right)}+\frac{bc\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2}{1-icx}\right)}{\left(c^2d-e\right)e}-\frac{bc\left(a+b\,\mathrm{ta}\cdots\right.}{\cdots}$$

$$= \frac{\left(a+b\tan^{-1}(cx)\right)^2}{4e^{3/2}\left(\sqrt{-d}-\sqrt{e}x\right)}-\frac{\left(a+b\tan^{-1}(cx)\right)^2}{4e^{3/2}\left(\sqrt{-d}+\sqrt{e}x\right)}+\frac{bc\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2}{1-icx}\right)}{\left(c^2d-e\right)e}-\frac{bc\left(a+b\,\mathrm{ta}\cdots\right.}{\cdots}$$

Mathematica [F] time = 42.9801, size = 0, normalized size = 0.

$$\int \frac{x^2 \left(a + b \tan^{-1}(cx)\right)^2}{\left(d + ex^2\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^2*(a + b*ArcTan[c*x])^2)/(d + e*x^2)^2,x]
```

```
[Out] Integrate[(x^2*(a + b*ArcTan[c*x])^2)/(d + e*x^2)^2, x]
```

Maple [C] time = 1.473, size = 6575, normalized size = 6.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctan(c*x))^2/(e*x^2+d)^2,x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))^2/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b^2x^2\arctan(cx)^2 + 2\,abx^2\arctan(cx) + a^2x^2}{e^2x^4 + 2\,dex^2 + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))^2/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^2*arctan(c*x)^2 + 2*a*b*x^2*arctan(c*x) + a^2*x^2)/(e^2*x^4
 + 2*d*e*x^2 + d^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atan(c*x))**2/(e*x**2+d)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \arctan(cx) + a)^2 x^2}{(ex^2 + d)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))^2/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)^2*x^2/(e*x^2 + d)^2, x)
```

3.1270 $\int \frac{x\left(a+b\tan^{-1}(cx)\right)^2}{\left(d+ex^2\right)^2}\,dx$

Optimal. Leaf size=457

$$\frac{ib^2c\,\text{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{(1-icx)\left(c\sqrt{-d}-i\sqrt{e}\right)}\right)}{4\sqrt{-d}\sqrt{e}\left(c^2d-e\right)}-\frac{ib^2c\,\text{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-d}+\sqrt{e}x\right)}{(1-icx)\left(c\sqrt{-d}+i\sqrt{e}\right)}\right)}{4\sqrt{-d}\sqrt{e}\left(c^2d-e\right)}+\frac{c^2\left(a+b\tan^{-1}(cx)\right)^2}{2e\left(c^2d-e\right)}-\frac{bc\left(a+b\tan^{-1}(\cdots\right.}{2\sqrt{-\cdots}}$$

```
[Out] (c^2*(a + b*ArcTan[c*x])^2)/(2*(c^2*d - e)*e) - (a + b*ArcTan[c*x])^2/(4*d*
e*(1 - (Sqrt[e]*x)/Sqrt[-d])) - (a + b*ArcTan[c*x])^2/(4*d*e*(1 + (Sqrt[e]*
x)/Sqrt[-d])) - (b*c*(a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] - Sqrt[e]*x))/(
(c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/(2*Sqrt[-d]*(c^2*d - e)*Sqrt[e]) +
(b*c*(a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*
Sqrt[e])*(1 - I*c*x))])/(2*Sqrt[-d]*(c^2*d - e)*Sqrt[e]) + ((I/4)*b^2*c*Pol
yLog[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c
*x))])/(Sqrt[-d]*(c^2*d - e)*Sqrt[e]) - ((I/4)*b^2*c*PolyLog[2, 1 - (2*c*(S
qrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(Sqrt[-d]*(c
^2*d - e)*Sqrt[e])
```

Rubi [A] time = 1.08768, antiderivative size = 457, normalized size of antiderivative = 1., number of steps used = 27, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {4978, 4864, 4856, 2402, 2315, 2447, 4984, 4884, 4920, 4854}

$$\frac{ib^2c\,\text{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{(1-icx)\left(c\sqrt{-d}-i\sqrt{e}\right)}\right)}{4\sqrt{-d}\sqrt{e}\left(c^2d-e\right)}-\frac{ib^2c\,\text{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-d}+\sqrt{e}x\right)}{(1-icx)\left(c\sqrt{-d}+i\sqrt{e}\right)}\right)}{4\sqrt{-d}\sqrt{e}\left(c^2d-e\right)}+\frac{c^2\left(a+b\tan^{-1}(cx)\right)^2}{2e\left(c^2d-e\right)}-\frac{bc\left(a+b\tan^{-1}(\cdots\right.}{2\sqrt{-\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcTan[c*x])^2)/(d + e*x^2)^2,x]
```

```
[Out] (c^2*(a + b*ArcTan[c*x])^2)/(2*(c^2*d - e)*e) - (a + b*ArcTan[c*x])^2/(4*d*
e*(1 - (Sqrt[e]*x)/Sqrt[-d])) - (a + b*ArcTan[c*x])^2/(4*d*e*(1 + (Sqrt[e]*
x)/Sqrt[-d])) - (b*c*(a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] - Sqrt[e]*x))/(
(c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/(2*Sqrt[-d]*(c^2*d - e)*Sqrt[e]) +
(b*c*(a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*
Sqrt[e])*(1 - I*c*x))])/(2*Sqrt[-d]*(c^2*d - e)*Sqrt[e]) + ((I/4)*b^2*c*Pol
yLog[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c
```

```
*x))])/(Sqrt[-d]*(c^2*d - e)*Sqrt[e]) - ((I/4)*b^2*c*PolyLog[2, 1 - (2*c*(S
qrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(Sqrt[-d]*(c
^2*d - e)*Sqrt[e])
```

Rule 4978

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2)^2
, x_Symbol] :> Dist[1/(4*d^2*Rt[-(e/d), 2]), Int[(a + b*ArcTan[c*x])^p/(1 -
 Rt[-(e/d), 2]*x)^2, x], x] - Dist[1/(4*d^2*Rt[-(e/d), 2]), Int[(a + b*ArcT
an[c*x])^p/(1 + Rt[-(e/d), 2]*x)^2, x], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 0]
```

Rule 4864

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x])^p)/(e*(q + 1)), x] - D
ist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
```

```
x][[2]], Expon[Pq, x]]
```

Rule 4984

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(m_.))/((
d_) + (e_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p
/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGt
Q[p, 0] && EqQ[e, c^2*d] && IGtQ[m, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rubi steps

$$\int \frac{x\left(a+b\tan^{-1}(cx)\right)^2}{\left(d+ex^2\right)^2}\,dx = \frac{\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{\left(1-\frac{\sqrt{e}x}{\sqrt{-d}}\right)^2}\,dx}{4(-d)^{3/2}\sqrt{e}} - \frac{\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{\left(1+\frac{\sqrt{e}x}{\sqrt{-d}}\right)^2}\,dx}{4(-d)^{3/2}\sqrt{e}}$$

$$= -\frac{\left(a+b\tan^{-1}(cx)\right)^2}{4de\left(1-\frac{\sqrt{e}x}{\sqrt{-d}}\right)} - \frac{\left(a+b\tan^{-1}(cx)\right)^2}{4de\left(1+\frac{\sqrt{e}x}{\sqrt{-d}}\right)} + \frac{(bc)\int\left(\frac{\sqrt{-d}e\left(a+b\tan^{-1}(cx)\right)}{\left(c^2d-e\right)\left(-\sqrt{-d}+\sqrt{e}x\right)} + \frac{c^2d\left(\sqrt{-d}+\sqrt{e}x\right)\left(a+b\tan\right.}{\sqrt{-d}\left(c^2d-e\right)\left(1+c^2\right.}\right.}{2de}$$

$$= -\frac{\left(a+b\tan^{-1}(cx)\right)^2}{4de\left(1-\frac{\sqrt{e}x}{\sqrt{-d}}\right)} - \frac{\left(a+b\tan^{-1}(cx)\right)^2}{4de\left(1+\frac{\sqrt{e}x}{\sqrt{-d}}\right)} - \frac{(bc)\int \frac{a+b\tan^{-1}(cx)}{-\sqrt{-d}+\sqrt{e}x}\,dx}{2\sqrt{-d}\left(c^2d-e\right)} + \frac{(bc)\int \frac{a+b\tan^{-1}(cx)}{\sqrt{-d}+\sqrt{e}x}\,dx}{2\sqrt{-d}\left(c^2d-e\right)}$$

$$= -\frac{\left(a+b\tan^{-1}(cx)\right)^2}{4de\left(1-\frac{\sqrt{e}x}{\sqrt{-d}}\right)} - \frac{\left(a+b\tan^{-1}(cx)\right)^2}{4de\left(1+\frac{\sqrt{e}x}{\sqrt{-d}}\right)} - \frac{bc\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(c\sqrt{-d}-i\sqrt{e}\right)(1-icx)}\right)}{2\sqrt{-d}\left(c^2d-e\right)\sqrt{e}} + \cdots$$

$$= -\frac{\left(a+b\tan^{-1}(cx)\right)^2}{4de\left(1-\frac{\sqrt{e}x}{\sqrt{-d}}\right)} - \frac{\left(a+b\tan^{-1}(cx)\right)^2}{4de\left(1+\frac{\sqrt{e}x}{\sqrt{-d}}\right)} - \frac{bc\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(c\sqrt{-d}-i\sqrt{e}\right)(1-icx)}\right)}{2\sqrt{-d}\left(c^2d-e\right)\sqrt{e}} + \cdots$$

$$= \frac{c^2\left(a+b\tan^{-1}(cx)\right)^2}{2\left(c^2d-e\right)e} - \frac{\left(a+b\tan^{-1}(cx)\right)^2}{4de\left(1-\frac{\sqrt{e}x}{\sqrt{-d}}\right)} - \frac{\left(a+b\tan^{-1}(cx)\right)^2}{4de\left(1+\frac{\sqrt{e}x}{\sqrt{-d}}\right)} - \frac{bc\left(a+b\tan^{-1}(cx)\right)\log}{2\sqrt{-d}\left(c^2d\right.}$$

Mathematica [A] time = 8.77836, size = 885, normalized size = 1.94

$$-\frac{a^2}{2e\left(ex^2+d\right)} + 2bc^2\left(\frac{c\tan^{-1}(cx) - \frac{\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}}}{2e\left(c^3d-ce\right)} - \frac{\tan^{-1}(cx)}{2e\left(ex^2c^2+dc^2\right)}\right)a + \frac{b^2c^2\left(\frac{4\tan^{-1}(cx)^2}{dc^2+e+\left(c^2d-e\right)\cos\left(2\tan^{-1}(cx)\right)} + \frac{4\tan^{-1}(cx)\tan}{\cdots}\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(a + b*ArcTan[c*x])^2)/(d + e*x^2)^2,x]
```

```
[Out] -a^2/(2*e*(d + e*x^2)) + 2*a*b*c^2*(-ArcTan[c*x]/(2*e*(c^2*d + c^2*e*x^2))
+ (c*ArcTan[c*x] - (Sqrt[e]*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[d])/(2*e*(c^3
```

```
*d - c*e))) + (b^2*c^2*((4*ArcTan[c*x]^2)/(c^2*d + e + (c^2*d - e)*Cos[2*Ar
cTan[c*x]]) + (4*ArcTan[c*x]*ArcTanh[Sqrt[-(c^2*d*e)]/(c*e*x)] - 2*ArcCos[-
((c^2*d + e)/(c^2*d - e))]*ArcTanh[(c*e*x)/Sqrt[-(c^2*d*e)]] + (ArcCos[-((c
^2*d + e)/(c^2*d - e))] + (2*I)*ArcTanh[(c*e*x)/Sqrt[-(c^2*d*e)]])*Log[(2*c
^2*d*((-I)*e + Sqrt[-(c^2*d*e)])*(-I + c*x))/((c^2*d - e)*(c^2*d + c*Sqrt[-
(c^2*d*e)]*x))] + (ArcCos[-((c^2*d + e)/(c^2*d - e))] - (2*I)*ArcTanh[(c*e*
x)/Sqrt[-(c^2*d*e)]])*Log[(2*c^2*d*(I*e + Sqrt[-(c^2*d*e)])*(I + c*x))/((c^
2*d - e)*(c^2*d + c*Sqrt[-(c^2*d*e)]*x))] - (ArcCos[-((c^2*d + e)/(c^2*d -
e))] - (2*I)*(ArcTanh[(c*d)/(Sqrt[-(c^2*d*e)]*x)] + ArcTanh[(c*e*x)/Sqrt[-(
c^2*d*e)]]))*Log[(Sqrt[2]*Sqrt[-(c^2*d*e)])/(Sqrt[c^2*d - e]*E^(I*ArcTan[c*
x])*Sqrt[c^2*d + e + (c^2*d - e)*Cos[2*ArcTan[c*x]]])] - (ArcCos[-((c^2*d +
 e)/(c^2*d - e))] + (2*I)*(ArcTanh[(c*d)/(Sqrt[-(c^2*d*e)]*x)] + ArcTanh[(c
*e*x)/Sqrt[-(c^2*d*e)]]))*Log[(Sqrt[2]*Sqrt[-(c^2*d*e)]*E^(I*ArcTan[c*x]))/
(Sqrt[c^2*d - e]*Sqrt[c^2*d + e + (c^2*d - e)*Cos[2*ArcTan[c*x]]])] - I*(Po
lyLog[2, ((c^2*d + e - (2*I)*Sqrt[-(c^2*d*e)])*(c^2*d - c*Sqrt[-(c^2*d*e)]*
x))/((c^2*d - e)*(c^2*d + c*Sqrt[-(c^2*d*e)]*x))] - PolyLog[2, ((c^2*d + e
+ (2*I)*Sqrt[-(c^2*d*e)])*(c^2*d - c*Sqrt[-(c^2*d*e)]*x))/((c^2*d - e)*(c^2
*d + c*Sqrt[-(c^2*d*e)]*x))]))/Sqrt[-(c^2*d*e)]))/(4*(c^2*d - e))
```

Maple [B] time = 0.363, size = 1185, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctan(c*x))^2/(e*x^2+d)^2,x)
```

```
[Out] -1/2*c^2*a^2/e/(c^2*e*x^2+c^2*d)-1/2*c^2*b^2/e/(c^2*e*x^2+c^2*d)*arctan(c*x
)^2+c^2*b^2/(c^2*d-e)/(c^4*d^2-2*c^2*d*e+e^2)*arctan(c*x)^2*(c^2*e*d)^(1/2)
+1/2*c^2*b^2/(c^2*d-e)/(c^4*d^2-2*c^2*d*e+e^2)*polylog(2,(c^2*d-e)*(1+I*c*x
)^2/(c^2*x^2+1)/(-c^2*d-2*(c^2*e*d)^(1/2)-e))*(c^2*e*d)^(1/2)-1/2*b^2*e/(c^
2*d-e)/d/(c^4*d^2-2*c^2*d*e+e^2)*arctan(c*x)^2*(c^2*e*d)^(1/2)-1/4*b^2*e/(c
^2*d-e)/d/(c^4*d^2-2*c^2*d*e+e^2)*polylog(2,(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+
1)/(-c^2*d-2*(c^2*e*d)^(1/2)-e))*(c^2*e*d)^(1/2)-1/2*c^4*b^2/e/(c^2*d-e)/(c
^4*d^2-2*c^2*d*e+e^2)*arctan(c*x)^2*(c^2*e*d)^(1/2)*d-1/4*c^4*b^2/e/(c^2*d-
e)/(c^4*d^2-2*c^2*d*e+e^2)*polylog(2,(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^
2*d-2*(c^2*e*d)^(1/2)-e))*(c^2*e*d)^(1/2)*d+1/2*b^2/e*(c^2*e*d)^(1/2)/(c^2*
d-e)/d*arctan(c*x)^2+I*c^2*b^2*ln(1-(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2
*d-2*(c^2*e*d)^(1/2)-e))*arctan(c*x)/(c^2*d-e)/(c^4*d^2-2*c^2*d*e+e^2)*(c^2
*e*d)^(1/2)-1/2*I*c^4*b^2/e*ln(1-(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d-
2*(c^2*e*d)^(1/2)-e))*arctan(c*x)/(c^2*d-e)/(c^4*d^2-2*c^2*d*e+e^2)*(c^2*e*
d)^(1/2)*d-1/2*I*b^2*e*ln(1-(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d-2*(c^
```

```
2*e*d)^(1/2)-e))*arctan(c*x)/d/(c^2*d-e)/(c^4*d^2-2*c^2*d*e+e^2)*(c^2*e*d)^
(1/2)+1/4*b^2/e*(c^2*e*d)^(1/2)/(c^2*d-e)/d*polylog(2,(c^2*d-e)*(1+I*c*x)^2
/(c^2*x^2+1)/(-c^2*d+2*(c^2*e*d)^(1/2)-e))+1/2*I*b^2/e*(c^2*e*d)^(1/2)/(c^2
*d-e)/d*arctan(c*x)*ln(1-(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d+2*(c^2*e
*d)^(1/2)-e))+1/2*c^2*b^2/e/(c^2*d-e)*arctan(c*x)^2-c^2*a*b/e/(c^2*e*x^2+c^
2*d)*arctan(c*x)-c*a*b/(c^2*d-e)/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))+c^2*a*
b/e/(c^2*d-e)*arctan(c*x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))^2/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2x\arctan(cx)^2+2\,abx\arctan(cx)+a^2x}{e^2x^4+2\,dex^2+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))^2/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*x*arctan(c*x)^2 + 2*a*b*x*arctan(c*x) + a^2*x)/(e^2*x^4 + 2*d
*e*x^2 + d^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atan(c*x))**2/(e*x**2+d)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\arctan(cx)+a)^2 x}{\left(ex^2+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))^2/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)^2*x/(e*x^2 + d)^2, x)
```

3.1271 $\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{\left(d+ex^2\right)^2}\,dx$

Optimal. Leaf size=1039

result too large to display

```
[Out] ((I/2)*c*(a + b*ArcTan[c*x])^2)/(d*(c^2*d - e)) - (a + b*ArcTan[c*x])^2/(4*
d*Sqrt[e]*(Sqrt[-d] - Sqrt[e]*x)) + (a + b*ArcTan[c*x])^2/(4*d*Sqrt[e]*(Sqr
t[-d] + Sqrt[e]*x)) - (b*c*(a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/(d*(c^2*
d - e)) + (b*c*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(d*(c^2*d - e)) + (b
*c*(a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sq
rt[e])*(1 - I*c*x))])/(2*d*(c^2*d - e)) - ((a + b*ArcTan[c*x])^2*Log[(2*c*(
Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/(4*(-d)^(3/
2)*Sqrt[e]) + (b*c*(a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c
*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(2*d*(c^2*d - e)) + ((a + b*ArcTan[c*
x])^2*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x
))])/(4*(-d)^(3/2)*Sqrt[e]) + ((I/2)*b^2*c*PolyLog[2, 1 - 2/(1 - I*c*x)])/(
d*(c^2*d - e)) + ((I/2)*b^2*c*PolyLog[2, 1 - 2/(1 + I*c*x)])/(d*(c^2*d - e)
) - ((I/4)*b^2*c*PolyLog[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] -
 I*Sqrt[e])*(1 - I*c*x))])/(d*(c^2*d - e)) + ((I/4)*b*(a + b*ArcTan[c*x])*P
olyLog[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I
*c*x))])/((-d)^(3/2)*Sqrt[e]) - ((I/4)*b^2*c*PolyLog[2, 1 - (2*c*(Sqrt[-d]
+ Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(d*(c^2*d - e)) - ((
I/4)*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*
Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/((-d)^(3/2)*Sqrt[e]) - (b^2*PolyLog[3,
 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/
(8*(-d)^(3/2)*Sqrt[e]) + (b^2*PolyLog[3, 1 - (2*c*(Sqrt[-d] + Sqrt[e]*x))/(
(c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(8*(-d)^(3/2)*Sqrt[e])
```

Rubi [A] time = 1.32887, antiderivative size = 1039, normalized size of antiderivative = 1., number of steps used = 32, number of rules used = 11, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.55, Rules used = {4914, 4864, 4856, 2402, 2315, 2447, 4984, 4884, 4920, 4854, 4858}

$$\frac{ic\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)b^2}{2d\left(c^2d-e\right)}+\frac{ic\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)b^2}{2d\left(c^2d-e\right)}-\frac{ic\text{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(c\sqrt{-d}-i\sqrt{e}\right)(1-icx)}\right)b^2}{4d\left(c^2d-e\right)}-\frac{ic\text{PolyLog}\left(2,1-\right.}{4d\left(c^{\cdots}\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^2/(d + e*x^2)^2,x]
```

```
[Out] ((I/2)*c*(a + b*ArcTan[c*x])^2)/(d*(c^2*d - e)) - (a + b*ArcTan[c*x])^2/(4*
d*Sqrt[e]*(Sqrt[-d] - Sqrt[e]*x)) + (a + b*ArcTan[c*x])^2/(4*d*Sqrt[e]*(Sqr
t[-d] + Sqrt[e]*x)) - (b*c*(a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/(d*(c^2*
d - e)) + (b*c*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(d*(c^2*d - e)) + (b
*c*(a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sq
rt[e])*(1 - I*c*x))])/(2*d*(c^2*d - e)) - ((a + b*ArcTan[c*x])^2*Log[(2*c*(
Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/(4*(-d)^(3/
2)*Sqrt[e]) + (b*c*(a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c
*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(2*d*(c^2*d - e)) + ((a + b*ArcTan[c*
x])^2*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x
))])/(4*(-d)^(3/2)*Sqrt[e]) + ((I/2)*b^2*c*PolyLog[2, 1 - 2/(1 - I*c*x)])/(
d*(c^2*d - e)) + ((I/2)*b^2*c*PolyLog[2, 1 - 2/(1 + I*c*x)])/(d*(c^2*d - e)
) - ((I/4)*b^2*c*PolyLog[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] -
 I*Sqrt[e])*(1 - I*c*x))])/(d*(c^2*d - e)) + ((I/4)*b*(a + b*ArcTan[c*x])*P
olyLog[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I
*c*x))])/((-d)^(3/2)*Sqrt[e]) - ((I/4)*b^2*c*PolyLog[2, 1 - (2*c*(Sqrt[-d]
+ Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(d*(c^2*d - e)) - ((
I/4)*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*
Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/((-d)^(3/2)*Sqrt[e]) - (b^2*PolyLog[3,
 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/
(8*(-d)^(3/2)*Sqrt[e]) + (b^2*PolyLog[3, 1 - (2*c*(Sqrt[-d] + Sqrt[e]*x))/(
(c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(8*(-d)^(3/2)*Sqrt[e])
```

Rule 4914

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (d + e*x^2)^q, x], x
] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[q] && IGtQ[p, 0]
```

Rule 4864

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x])^p)/(e*(q + 1)), x] - D
ist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4984

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(m_.))/((
d_) + (e_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p
/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGt
Q[p, 0] && EqQ[e, c^2*d] && IGtQ[m, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
```

```
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4858

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^2/((d_) + (e_.)*(x_)), x_Symbol] :>
-Simp[((a + b*ArcTan[c*x])^2*Log[2/(1 - I*c*x)])/e, x] + (Simp[((a + b*ArcT
an[c*x])^2*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x] + Simp[(I*
b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 - I*c*x)])/e, x] - Simp[(I*b*(a +
 b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/
e, x] - Simp[(b^2*PolyLog[3, 1 - 2/(1 - I*c*x)])/(2*e), x] + Simp[(b^2*Poly
Log[3, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/(2*e), x]) /; FreeQ[
{a, b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{\left(d+ex^2\right)^2}\,dx = \int\left(-\frac{e\left(a+b\tan^{-1}(cx)\right)^2}{4d\left(\sqrt{-d}\sqrt{e}-ex\right)^2}-\frac{e\left(a+b\tan^{-1}(cx)\right)^2}{4d\left(\sqrt{-d}\sqrt{e}+ex\right)^2}-\frac{e\left(a+b\tan^{-1}(cx)\right)^2}{2d\left(-de-e^2x^2\right)}\right)dx$$

$$= -\frac{e\int\frac{\left(a+b\tan^{-1}(cx)\right)^2}{\left(\sqrt{-d}\sqrt{e}-ex\right)^2}\,dx}{4d}-\frac{e\int\frac{\left(a+b\tan^{-1}(cx)\right)^2}{\left(\sqrt{-d}\sqrt{e}+ex\right)^2}\,dx}{4d}-\frac{e\int\frac{\left(a+b\tan^{-1}(cx)\right)^2}{-de-e^2x^2}\,dx}{2d}$$

$$= -\frac{\left(a+b\tan^{-1}(cx)\right)^2}{4d\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{\left(a+b\tan^{-1}(cx)\right)^2}{4d\sqrt{e}\left(\sqrt{-d}+\sqrt{e}x\right)}-\frac{(bc)\int\left(\frac{\sqrt{e}\left(a+b\tan^{-1}(cx)\right)}{\left(-c^2d+e\right)\left(\sqrt{-d}+\sqrt{e}x\right)}-\frac{c^2\left(-\sqrt{-d}+\sqrt{e}x\right)\left(a+b\tan\cdots\right.}{\sqrt{e}\left(-c^2d+e\right)\left(1+c^2\cdots\right.}\cdots\right.}{2d}$$

$$= -\frac{\left(a+b\tan^{-1}(cx)\right)^2}{4d\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{\left(a+b\tan^{-1}(cx)\right)^2}{4d\sqrt{e}\left(\sqrt{-d}+\sqrt{e}x\right)}+\frac{\int\frac{\left(a+b\tan^{-1}(cx)\right)^2}{\sqrt{-d}-\sqrt{e}x}\,dx}{4(-d)^{3/2}}+\frac{\int\frac{\left(a+b\tan^{-1}(cx)\right)^2}{\sqrt{-d}+\sqrt{e}x}\,dx}{4(-d)^{3/2}}-\cdots$$

$$= -\frac{\left(a+b\tan^{-1}(cx)\right)^2}{4d\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{\left(a+b\tan^{-1}(cx)\right)^2}{4d\sqrt{e}\left(\sqrt{-d}+\sqrt{e}x\right)}-\frac{bc\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2}{1-icx}\right)}{d\left(c^2d-e\right)}+\frac{bc\left(a+b\,\mathrm{ta}\cdots\right.}{\cdots}$$

$$= -\frac{\left(a+b\tan^{-1}(cx)\right)^2}{4d\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{\left(a+b\tan^{-1}(cx)\right)^2}{4d\sqrt{e}\left(\sqrt{-d}+\sqrt{e}x\right)}-\frac{bc\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2}{1-icx}\right)}{d\left(c^2d-e\right)}+\frac{bc\left(a+b\,\mathrm{ta}\cdots\right.}{\cdots}$$

$$= -\frac{\left(a+b\tan^{-1}(cx)\right)^2}{4d\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{\left(a+b\tan^{-1}(cx)\right)^2}{4d\sqrt{e}\left(\sqrt{-d}+\sqrt{e}x\right)}-\frac{bc\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2}{1-icx}\right)}{d\left(c^2d-e\right)}+\frac{bc\left(a+b\,\mathrm{ta}\cdots\right.}{\cdots}$$

$$= -\frac{\left(a+b\tan^{-1}(cx)\right)^2}{4d\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{\left(a+b\tan^{-1}(cx)\right)^2}{4d\sqrt{e}\left(\sqrt{-d}+\sqrt{e}x\right)}-\frac{bc\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2}{1-icx}\right)}{d\left(c^2d-e\right)}+\frac{bc\left(a+b\,\mathrm{ta}\cdots\right.}{\cdots}$$

$$= -\frac{\left(a+b\tan^{-1}(cx)\right)^2}{4d\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{\left(a+b\tan^{-1}(cx)\right)^2}{4d\sqrt{e}\left(\sqrt{-d}+\sqrt{e}x\right)}-\frac{bc\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2}{1-icx}\right)}{d\left(c^2d-e\right)}+\frac{bc\left(a+b\,\mathrm{ta}\cdots\right.}{\cdots}$$

$$= -\frac{\left(a+b\tan^{-1}(cx)\right)^2}{4d\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{\left(a+b\tan^{-1}(cx)\right)^2}{4d\sqrt{e}\left(\sqrt{-d}+\sqrt{e}x\right)}-\frac{bc\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2}{1-icx}\right)}{d\left(c^2d-e\right)}+\frac{bc\left(a+b\,\mathrm{ta}\cdots\right.}{\cdots}$$

Mathematica [F] time = 23.0303, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{\left(d+ex^2\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcTan[c*x])^2/(d + e*x^2)^2,x]
```

```
[Out] Integrate[(a + b*ArcTan[c*x])^2/(d + e*x^2)^2, x]
```

Maple [C] time = 1.447, size = 6575, normalized size = 6.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^2/(e*x^2+d)^2,x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b^2\arctan\left(cx\right)^2+2\,ab\arctan\left(cx\right)+a^2}{e^2x^4+2\,dex^2+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctan(c*x)^2 + 2*a*b*arctan(c*x) + a^2)/(e^2*x^4 + 2*d*e*x^2
 + d^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**2/(e*x**2+d)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\arctan(cx)+a)^2}{(ex^2+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)^2/(e*x^2 + d)^2, x)
```

3.1272 $\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x\left(d+ex^2\right)^2}\,dx$

Optimal. Leaf size=1087

result too large to display

```
[Out] -(c^2*(a + b*ArcTan[c*x])^2)/(2*d*(c^2*d - e)) + (a + b*ArcTan[c*x])^2/(4*d
^2*(1 - (Sqrt[e]*x)/Sqrt[-d])) + (a + b*ArcTan[c*x])^2/(4*d^2*(1 + (Sqrt[e]
*x)/Sqrt[-d])) + (2*(a + b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1 + I*c*x)])/d^2 +
 ((a + b*ArcTan[c*x])^2*Log[2/(1 - I*c*x)])/d^2 - (b*c*Sqrt[e]*(a + b*ArcTa
n[c*x])*Log[(2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c
*x))])/(2*(-d)^(3/2)*(c^2*d - e)) - ((a + b*ArcTan[c*x])^2*Log[(2*c*(Sqrt[-
d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/(2*d^2) + (b*c*Sq
rt[e]*(a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I
*Sqrt[e])*(1 - I*c*x))])/(2*(-d)^(3/2)*(c^2*d - e)) - ((a + b*ArcTan[c*x])^
2*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])
/(2*d^2) - (I*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 - I*c*x)])/d^2 - (I
*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)])/d^2 + (I*b*(a + b*Arc
Tan[c*x])*PolyLog[2, -1 + 2/(1 + I*c*x)])/d^2 + ((I/4)*b^2*c*Sqrt[e]*PolyLo
g[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x)
)])/((-d)^(3/2)*(c^2*d - e)) + ((I/2)*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 -
(2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/d^2 -
 ((I/4)*b^2*c*Sqrt[e]*PolyLog[2, 1 - (2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[
-d] + I*Sqrt[e])*(1 - I*c*x))])/((-d)^(3/2)*(c^2*d - e)) + ((I/2)*b*(a + b*
ArcTan[c*x])*PolyLog[2, 1 - (2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*S
qrt[e])*(1 - I*c*x))])/d^2 + (b^2*PolyLog[3, 1 - 2/(1 - I*c*x)])/(2*d^2) -
(b^2*PolyLog[3, 1 - 2/(1 + I*c*x)])/(2*d^2) + (b^2*PolyLog[3, -1 + 2/(1 + I
*c*x)])/(2*d^2) - (b^2*PolyLog[3, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt
[-d] - I*Sqrt[e])*(1 - I*c*x))])/(4*d^2) - (b^2*PolyLog[3, 1 - (2*c*(Sqrt[-
d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(4*d^2)
```

Rubi [A] time = 1.86097, antiderivative size = 1087, normalized size of antiderivative = 1., number of steps used = 39, number of rules used = 16, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.696, Rules used = {4980, 4850, 4988, 4884, 4994, 6610, 4978, 4864, 4856, 2402, 2315, 2447, 4984, 4920, 4854, 4858}

$$\frac{ic\sqrt{e}\text{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(c\sqrt{-d}-i\sqrt{e}\right)(1-icx)}\right)b^2}{4(-d)^{3/2}\left(c^2d-e\right)}-\frac{ic\sqrt{e}\text{PolyLog}\left(2,1-\frac{2c\left(\sqrt{e}x+\sqrt{-d}\right)}{\left(\sqrt{-d}c+i\sqrt{e}\right)(1-icx)}\right)b^2}{4(-d)^{3/2}\left(c^2d-e\right)}+\frac{\text{PolyLog}\left(3,1-\frac{2}{1-icx}\right)b^2}{2d^2}-\text{Poly}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^2/(x*(d + e*x^2)^2),x]
```

```
[Out] -(c^2*(a + b*ArcTan[c*x])^2)/(2*d*(c^2*d - e)) + (a + b*ArcTan[c*x])^2/(4*d
^2*(1 - (Sqrt[e]*x)/Sqrt[-d])) + (a + b*ArcTan[c*x])^2/(4*d^2*(1 + (Sqrt[e]
*x)/Sqrt[-d])) + (2*(a + b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1 + I*c*x)])/d^2 +
 ((a + b*ArcTan[c*x])^2*Log[2/(1 - I*c*x)])/d^2 - (b*c*Sqrt[e]*(a + b*ArcTa
n[c*x])*Log[(2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c
*x))])/(2*(-d)^(3/2)*(c^2*d - e)) - ((a + b*ArcTan[c*x])^2*Log[(2*c*(Sqrt[-
d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/(2*d^2) + (b*c*Sq
rt[e]*(a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I
*Sqrt[e])*(1 - I*c*x))])/(2*(-d)^(3/2)*(c^2*d - e)) - ((a + b*ArcTan[c*x])^
2*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])
/(2*d^2) - (I*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 - I*c*x)])/d^2 - (I
*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)])/d^2 + (I*b*(a + b*Arc
Tan[c*x])*PolyLog[2, -1 + 2/(1 + I*c*x)])/d^2 + ((I/4)*b^2*c*Sqrt[e]*PolyLo
g[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x)
)])/((-d)^(3/2)*(c^2*d - e)) + ((I/2)*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 -
(2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/d^2 -
 ((I/4)*b^2*c*Sqrt[e]*PolyLog[2, 1 - (2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[
-d] + I*Sqrt[e])*(1 - I*c*x))])/((-d)^(3/2)*(c^2*d - e)) + ((I/2)*b*(a + b*
ArcTan[c*x])*PolyLog[2, 1 - (2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*S
qrt[e])*(1 - I*c*x))])/d^2 + (b^2*PolyLog[3, 1 - 2/(1 - I*c*x)])/(2*d^2) -
(b^2*PolyLog[3, 1 - 2/(1 + I*c*x)])/(2*d^2) + (b^2*PolyLog[3, -1 + 2/(1 + I
*c*x)])/(2*d^2) - (b^2*PolyLog[3, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt
[-d] - I*Sqrt[e])*(1 - I*c*x))])/(4*d^2) - (b^2*PolyLog[3, 1 - (2*c*(Sqrt[-
d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(4*d^2)
```

Rule 4980

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

Rule 4850

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTan[c*x])^(p - 1)*ArcTanh[1 - 2/(1 + I*c*x)])/(1 + c^2*x^2), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4988

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTan[c*x])^p)/(d + e
*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTan[c*x])^p)/(d + e*x^2
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 4978

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2)^2
, x_Symbol] :> Dist[1/(4*d^2*Rt[-(e/d), 2]), Int[(a + b*ArcTan[c*x])^p/(1 -
 Rt[-(e/d), 2]*x)^2, x], x] - Dist[1/(4*d^2*Rt[-(e/d), 2]), Int[(a + b*ArcT
an[c*x])^p/(1 + Rt[-(e/d), 2]*x)^2, x], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 0]
```

Rule 4864

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x])^p)/(e*(q + 1)), x] - D
ist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4984

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(m_.))/((
d_) + (e_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p
/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGt
Q[p, 0] && EqQ[e, c^2*d] && IGtQ[m, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
```

```
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4858

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^2/((d_) + (e_.)*(x_)), x_Symbol] :>
-Simp[((a + b*ArcTan[c*x])^2*Log[2/(1 - I*c*x)])/e, x] + (Simp[((a + b*ArcT
an[c*x])^2*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x] + Simp[(I*
b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 - I*c*x)])/e, x] - Simp[(I*b*(a +
 b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/
e, x] - Simp[(b^2*PolyLog[3, 1 - 2/(1 - I*c*x)])/(2*e), x] + Simp[(b^2*Poly
Log[3, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/(2*e), x]) /; FreeQ[
{a, b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x\left(d+ex^2\right)^2}\,dx = \int \left(\frac{\left(a+b\tan^{-1}(cx)\right)^2}{d^2x} - \frac{ex\left(a+b\tan^{-1}(cx)\right)^2}{d\left(d+ex^2\right)^2} - \frac{ex\left(a+b\tan^{-1}(cx)\right)^2}{d^2\left(d+ex^2\right)}\right)dx$$

$$= \frac{\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x}\,dx}{d^2} - \frac{e\int \frac{x\left(a+b\tan^{-1}(cx)\right)^2}{d+ex^2}\,dx}{d^2} - \frac{e\int \frac{x\left(a+b\tan^{-1}(cx)\right)^2}{\left(d+ex^2\right)^2}\,dx}{d}$$

$$= \frac{2\left(a+b\tan^{-1}(cx)\right)^2\tanh^{-1}\left(1-\frac{2}{1+icx}\right)}{d^2} - \frac{(4bc)\int \frac{\left(a+b\tan^{-1}(cx)\right)\tanh^{-1}\left(1-\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx}{d^2} + \frac{\sqrt{e}\int \frac{(a+b}{(}}{4(}$$

$$= \frac{\left(a+b\tan^{-1}(cx)\right)^2}{4d^2\left(1-\frac{\sqrt{e}x}{\sqrt{-d}}\right)} + \frac{\left(a+b\tan^{-1}(cx)\right)^2}{4d^2\left(1+\frac{\sqrt{e}x}{\sqrt{-d}}\right)} + \frac{2\left(a+b\tan^{-1}(cx)\right)^2\tanh^{-1}\left(1-\frac{2}{1+icx}\right)}{d^2} - \frac{(bc)\int}{}$$

$$= \frac{\left(a+b\tan^{-1}(cx)\right)^2}{4d^2\left(1-\frac{\sqrt{e}x}{\sqrt{-d}}\right)} + \frac{\left(a+b\tan^{-1}(cx)\right)^2}{4d^2\left(1+\frac{\sqrt{e}x}{\sqrt{-d}}\right)} + \frac{2\left(a+b\tan^{-1}(cx)\right)^2\tanh^{-1}\left(1-\frac{2}{1+icx}\right)}{d^2} + \frac{(a+b}{}$$

$$= \frac{\left(a+b\tan^{-1}(cx)\right)^2}{4d^2\left(1-\frac{\sqrt{e}x}{\sqrt{-d}}\right)} + \frac{\left(a+b\tan^{-1}(cx)\right)^2}{4d^2\left(1+\frac{\sqrt{e}x}{\sqrt{-d}}\right)} + \frac{2\left(a+b\tan^{-1}(cx)\right)^2\tanh^{-1}\left(1-\frac{2}{1+icx}\right)}{d^2} + \frac{(a+b}{}$$

$$= \frac{\left(a+b\tan^{-1}(cx)\right)^2}{4d^2\left(1-\frac{\sqrt{e}x}{\sqrt{-d}}\right)} + \frac{\left(a+b\tan^{-1}(cx)\right)^2}{4d^2\left(1+\frac{\sqrt{e}x}{\sqrt{-d}}\right)} + \frac{2\left(a+b\tan^{-1}(cx)\right)^2\tanh^{-1}\left(1-\frac{2}{1+icx}\right)}{d^2} + \frac{(a+b}{}$$

$$= -\frac{c^2\left(a+b\tan^{-1}(cx)\right)^2}{2d\left(c^2d-e\right)} + \frac{\left(a+b\tan^{-1}(cx)\right)^2}{4d^2\left(1-\frac{\sqrt{e}x}{\sqrt{-d}}\right)} + \frac{\left(a+b\tan^{-1}(cx)\right)^2}{4d^2\left(1+\frac{\sqrt{e}x}{\sqrt{-d}}\right)} + \frac{2\left(a+b\tan^{-1}(cx)\right)^2\,\mathrm{ta}}{d^2}$$

Mathematica [F] time = 15.9039, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x\left(d+ex^2\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcTan[c*x])^2/(x*(d + e*x^2)^2),x]
```

```
[Out] Integrate[(a + b*ArcTan[c*x])^2/(x*(d + e*x^2)^2), x]
```

Maple [F] time = 10.853, size = 0, normalized size = 0.

$$\int \frac{(a+b\arctan(cx))^2}{x\left(ex^2+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^2/x/(e*x^2+d)^2,x)
```

```
[Out] int((a+b*arctan(c*x))^2/x/(e*x^2+d)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}a^2\left(\frac{1}{dex^2+d^2}-\frac{\log\left(ex^2+d\right)}{d^2}+\frac{2\log(x)}{d^2}\right)+\int \frac{b^2\arctan(cx)^2+2\,ab\arctan(cx)}{e^2x^5+2\,dex^3+d^2x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*(1/(d*e*x^2 + d^2) - log(e*x^2 + d)/d^2 + 2*log(x)/d^2) + integrate
((b^2*arctan(c*x)^2 + 2*a*b*arctan(c*x))/(e^2*x^5 + 2*d*e*x^3 + d^2*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2\arctan(cx)^2+2\,ab\arctan(cx)+a^2}{e^2x^5+2\,dex^3+d^2x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctan(c*x)^2 + 2*a*b*arctan(c*x) + a^2)/(e^2*x^5 + 2*d*e*x^3
 + d^2*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**2/x/(e*x**2+d)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \arctan(cx) + a)^2}{(ex^2 + d)^2 x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)^2/((e*x^2 + d)^2*x), x)
```

3.1273 $\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^2\left(d+ex^2\right)^2}\,dx$

Optimal. Leaf size=1141

result too large to display

```
[Out] ((-I)*c*(a + b*ArcTan[c*x])^2)/d^2 - ((I/2)*c*e*(a + b*ArcTan[c*x])^2)/(d^2
*(c^2*d - e)) - (a + b*ArcTan[c*x])^2/(d^2*x) + (Sqrt[e]*(a + b*ArcTan[c*x]
)^2)/(4*d^2*(Sqrt[-d] - Sqrt[e]*x)) - (Sqrt[e]*(a + b*ArcTan[c*x])^2)/(4*d^
2*(Sqrt[-d] + Sqrt[e]*x)) + (b*c*e*(a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/
(d^2*(c^2*d - e)) - (b*c*e*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(d^2*(c^
2*d - e)) - (b*c*e*(a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] - Sqrt[e]*x))/((c
*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/(2*d^2*(c^2*d - e)) - (3*Sqrt[e]*(a +
 b*ArcTan[c*x])^2*Log[(2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e]
)*(1 - I*c*x))])/(4*(-d)^(5/2)) - (b*c*e*(a + b*ArcTan[c*x])*Log[(2*c*(Sqrt
[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(2*d^2*(c^2*d -
 e)) + (3*Sqrt[e]*(a + b*ArcTan[c*x])^2*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((
c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(4*(-d)^(5/2)) + (2*b*c*(a + b*ArcTa
n[c*x])*Log[2 - 2/(1 - I*c*x)])/d^2 - ((I/2)*b^2*c*e*PolyLog[2, 1 - 2/(1 -
I*c*x)])/(d^2*(c^2*d - e)) - (I*b^2*c*PolyLog[2, -1 + 2/(1 - I*c*x)])/d^2 -
 ((I/2)*b^2*c*e*PolyLog[2, 1 - 2/(1 + I*c*x)])/(d^2*(c^2*d - e)) + ((I/4)*b
^2*c*e*PolyLog[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e]
)*(1 - I*c*x))])/(d^2*(c^2*d - e)) + (((3*I)/4)*b*Sqrt[e]*(a + b*ArcTan[c*x
])*PolyLog[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1
 - I*c*x))])/(-d)^(5/2) + ((I/4)*b^2*c*e*PolyLog[2, 1 - (2*c*(Sqrt[-d] + Sq
rt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(d^2*(c^2*d - e)) - (((3
*I)/4)*b*Sqrt[e]*(a + b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(Sqrt[-d] + Sqrt[e
]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(-d)^(5/2) - (3*b^2*Sqrt[e]*
PolyLog[3, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 -
I*c*x))])/(8*(-d)^(5/2)) + (3*b^2*Sqrt[e]*PolyLog[3, 1 - (2*c*(Sqrt[-d] + S
qrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(8*(-d)^(5/2))
```

Rubi [A] time = 2.0471, antiderivative size = 1141, normalized size of antiderivative = 1., number of steps used = 42, number of rules used = 15, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.652, Rules used = {4980, 4852, 4924, 4868, 2447, 4914, 4864, 4856, 2402, 2315, 4984, 4884, 4920, 4854, 4858}

$$-\frac{ice\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)b^2}{2d^2\left(c^2d-e\right)}-\frac{ic\text{PolyLog}\left(2,\frac{2}{1-icx}-1\right)b^2}{d^2}-\frac{ice\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)b^2}{2d^2\left(c^2d-e\right)}+\frac{ice\text{PolyLog}\left(2,1-\frac{2c(\sqrt{-\ldots}}{(c\sqrt{-d}-i\ldots}\right.}{4d^2\left(c^2d-e\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^2/(x^2*(d + e*x^2)^2),x]
```

```
[Out] ((-I)*c*(a + b*ArcTan[c*x])^2)/d^2 - ((I/2)*c*e*(a + b*ArcTan[c*x])^2)/(d^2
*(c^2*d - e)) - (a + b*ArcTan[c*x])^2/(d^2*x) + (Sqrt[e]*(a + b*ArcTan[c*x]
)^2)/(4*d^2*(Sqrt[-d] - Sqrt[e]*x)) - (Sqrt[e]*(a + b*ArcTan[c*x])^2)/(4*d^
2*(Sqrt[-d] + Sqrt[e]*x)) + (b*c*e*(a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/
(d^2*(c^2*d - e)) - (b*c*e*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(d^2*(c^
2*d - e)) - (b*c*e*(a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] - Sqrt[e]*x))/((c
*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/(2*d^2*(c^2*d - e)) - (3*Sqrt[e]*(a +
 b*ArcTan[c*x])^2*Log[(2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e]
)*(1 - I*c*x))])/(4*(-d)^(5/2)) - (b*c*e*(a + b*ArcTan[c*x])*Log[(2*c*(Sqrt
[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(2*d^2*(c^2*d -
 e)) + (3*Sqrt[e]*(a + b*ArcTan[c*x])^2*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((
c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(4*(-d)^(5/2)) + (2*b*c*(a + b*ArcTa
n[c*x])*Log[2 - 2/(1 - I*c*x)])/d^2 - ((I/2)*b^2*c*e*PolyLog[2, 1 - 2/(1 -
I*c*x)])/(d^2*(c^2*d - e)) - (I*b^2*c*PolyLog[2, -1 + 2/(1 - I*c*x)])/d^2 -
 ((I/2)*b^2*c*e*PolyLog[2, 1 - 2/(1 + I*c*x)])/(d^2*(c^2*d - e)) + ((I/4)*b
^2*c*e*PolyLog[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e]
)*(1 - I*c*x))])/(d^2*(c^2*d - e)) + (((3*I)/4)*b*Sqrt[e]*(a + b*ArcTan[c*x
])*PolyLog[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1
 - I*c*x))])/(-d)^(5/2) + ((I/4)*b^2*c*e*PolyLog[2, 1 - (2*c*(Sqrt[-d] + Sq
rt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(d^2*(c^2*d - e)) - (((3
*I)/4)*b*Sqrt[e]*(a + b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(Sqrt[-d] + Sqrt[e
]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(-d)^(5/2) - (3*b^2*Sqrt[e]*
PolyLog[3, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 -
I*c*x))])/(8*(-d)^(5/2)) + (3*b^2*Sqrt[e]*PolyLog[3, 1 - (2*c*(Sqrt[-d] + S
qrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(8*(-d)^(5/2))
```

Rule 4980

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4914

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (d + e*x^2)^q, x], x
] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[q] && IGtQ[p, 0]
```

Rule 4864

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x])^p)/(e*(q + 1)), x] - D
ist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
```

```
c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 4984

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(m_.))/((
d_) + (e_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p
/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGt
Q[p, 0] && EqQ[e, c^2*d] && IGtQ[m, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4858

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^2/((d_) + (e_.)*(x_)), x_Symbol] :>
-Simp[((a + b*ArcTan[c*x])^2*Log[2/(1 - I*c*x)])/e, x] + (Simp[((a + b*ArcT
an[c*x])^2*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x] + Simp[(I*
```

```
b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 - I*c*x)])/e, x] - Simp[(I*b*(a +
 b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/
e, x] - Simp[(b^2*PolyLog[3, 1 - 2/(1 - I*c*x)])/(2*e), x] + Simp[(b^2*Poly
Log[3, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/(2*e), x]) /; FreeQ[
{a, b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^2\left(d+ex^2\right)^2}\,dx = \int\left(\frac{\left(a+b\tan^{-1}(cx)\right)^2}{d^2x^2} - \frac{e\left(a+b\tan^{-1}(cx)\right)^2}{d\left(d+ex^2\right)^2} - \frac{e\left(a+b\tan^{-1}(cx)\right)^2}{d^2\left(d+ex^2\right)}\right)dx$$

$$= \frac{\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^2}\,dx}{d^2} - \frac{e\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{d+ex^2}\,dx}{d^2} - \frac{e\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{\left(d+ex^2\right)^2}\,dx}{d}$$

$$= -\frac{\left(a+b\tan^{-1}(cx)\right)^2}{d^2x} + \frac{(2bc)\int \frac{a+b\tan^{-1}(cx)}{x\left(1+c^2x^2\right)}\,dx}{d^2} - \frac{e\int\left(\frac{\sqrt{-d}\left(a+b\tan^{-1}(cx)\right)^2}{2d\left(\sqrt{-d}-\sqrt{e}x\right)} + \frac{\sqrt{-d}\left(a+b\tan^{-1}(cx)\right)^2}{2d\left(\sqrt{-d}+\sqrt{e}x\right)}\right)}{d^2}$$

$$= -\frac{ic\left(a+b\tan^{-1}(cx)\right)^2}{d^2} - \frac{\left(a+b\tan^{-1}(cx)\right)^2}{d^2x} + \frac{(2ibc)\int \frac{a+b\tan^{-1}(cx)}{x(i+cx)}\,dx}{d^2} + \frac{e\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{\sqrt{-d}-\sqrt{e}x}\,d}{2(-d)^{5/2}}$$

$$= -\frac{ic\left(a+b\tan^{-1}(cx)\right)^2}{d^2} - \frac{\left(a+b\tan^{-1}(cx)\right)^2}{d^2x} + \frac{\sqrt{e}\left(a+b\tan^{-1}(cx)\right)^2}{4d^2\left(\sqrt{-d}-\sqrt{e}x\right)} - \frac{\sqrt{e}\left(a+b\tan^{-1}(cx)\right.}{4d^2\left(\sqrt{-d}+\sqrt{e}x\right)}$$

$$= -\frac{ic\left(a+b\tan^{-1}(cx)\right)^2}{d^2} - \frac{\left(a+b\tan^{-1}(cx)\right)^2}{d^2x} + \frac{\sqrt{e}\left(a+b\tan^{-1}(cx)\right)^2}{4d^2\left(\sqrt{-d}-\sqrt{e}x\right)} - \frac{\sqrt{e}\left(a+b\tan^{-1}(cx)\right.}{4d^2\left(\sqrt{-d}+\sqrt{e}x\right)}$$

$$= -\frac{ic\left(a+b\tan^{-1}(cx)\right)^2}{d^2} - \frac{\left(a+b\tan^{-1}(cx)\right)^2}{d^2x} + \frac{\sqrt{e}\left(a+b\tan^{-1}(cx)\right)^2}{4d^2\left(\sqrt{-d}-\sqrt{e}x\right)} - \frac{\sqrt{e}\left(a+b\tan^{-1}(cx)\right.}{4d^2\left(\sqrt{-d}+\sqrt{e}x\right)}$$

$$= -\frac{ic\left(a+b\tan^{-1}(cx)\right)^2}{d^2} - \frac{\left(a+b\tan^{-1}(cx)\right)^2}{d^2x} + \frac{\sqrt{e}\left(a+b\tan^{-1}(cx)\right)^2}{4d^2\left(\sqrt{-d}-\sqrt{e}x\right)} - \frac{\sqrt{e}\left(a+b\tan^{-1}(cx)\right.}{4d^2\left(\sqrt{-d}+\sqrt{e}x\right)}$$

$$= -\frac{ic\left(a+b\tan^{-1}(cx)\right)^2}{d^2} - \frac{\left(a+b\tan^{-1}(cx)\right)^2}{d^2x} + \frac{\sqrt{e}\left(a+b\tan^{-1}(cx)\right)^2}{4d^2\left(\sqrt{-d}-\sqrt{e}x\right)} - \frac{\sqrt{e}\left(a+b\tan^{-1}(cx)\right.}{4d^2\left(\sqrt{-d}+\sqrt{e}x\right)}$$

$$= -\frac{ic\left(a+b\tan^{-1}(cx)\right)^2}{d^2} - \frac{\left(a+b\tan^{-1}(cx)\right)^2}{d^2x} + \frac{\sqrt{e}\left(a+b\tan^{-1}(cx)\right)^2}{4d^2\left(\sqrt{-d}-\sqrt{e}x\right)} - \frac{\sqrt{e}\left(a+b\tan^{-1}(cx)\right.}{4d^2\left(\sqrt{-d}+\sqrt{e}x\right)}$$

$$= -\frac{ic\left(a+b\tan^{-1}(cx)\right)^2}{d^2} - \frac{\left(a+b\tan^{-1}(cx)\right)^2}{d^2x} + \frac{\sqrt{e}\left(a+b\tan^{-1}(cx)\right)^2}{4d^2\left(\sqrt{-d}-\sqrt{e}x\right)} - \frac{\sqrt{e}\left(a+b\tan^{-1}(cx)\right.}{4d^2\left(\sqrt{-d}+\sqrt{e}x\right)}$$

$$= -\frac{ic\left(a+b\tan^{-1}(cx)\right)^2}{d^2} - \frac{\left(a+b\tan^{-1}(cx)\right)^2}{d^2x} + \frac{\sqrt{e}\left(a+b\tan^{-1}(cx)\right)^2}{4d^2\left(\sqrt{-d}-\sqrt{e}x\right)} - \frac{\sqrt{e}\left(a+b\tan^{-1}(cx)\right.}{4d^2\left(\sqrt{-d}+\sqrt{e}x\right)}$$

Mathematica [F] time = 180.004, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcTan[c*x])^2/(x^2*(d + e*x^2)^2),x]
```

```
[Out] $Aborted
```

Maple [F] time = 3.653, size = 0, normalized size = 0.

$$\int \frac{(a+b\arctan(cx))^2}{x^2(ex^2+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^2/x^2/(e*x^2+d)^2,x)
```

```
[Out] int((a+b*arctan(c*x))^2/x^2/(e*x^2+d)^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^2/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2\arctan(cx)^2+2\,ab\arctan(cx)+a^2}{e^2x^6+2\,dex^4+d^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^2/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctan(c*x)^2 + 2*a*b*arctan(c*x) + a^2)/(e^2*x^6 + 2*d*e*x^4
 + d^2*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**2/x**2/(e*x**2+d)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\arctan(cx)+a)^2}{(ex^2+d)^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^2/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)^2/((e*x^2 + d)^2*x^2), x)
```

3.1274 $\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^3\left(d+ex^2\right)^2}\,dx$

Optimal. Leaf size=1181

result too large to display

```
[Out] -((b*c*(a + b*ArcTan[c*x]))/(d^2*x)) - (c^2*(a + b*ArcTan[c*x])^2)/(2*d^2)
+ (c^2*e*(a + b*ArcTan[c*x])^2)/(2*d^2*(c^2*d - e)) - (a + b*ArcTan[c*x])^2
/(2*d^2*x^2) - (e*(a + b*ArcTan[c*x])^2)/(4*d^3*(1 - (Sqrt[e]*x)/Sqrt[-d]))
 - (e*(a + b*ArcTan[c*x])^2)/(4*d^3*(1 + (Sqrt[e]*x)/Sqrt[-d])) - (4*e*(a +
 b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1 + I*c*x)])/d^3 + (b^2*c^2*Log[x])/d^2 -
(2*e*(a + b*ArcTan[c*x])^2*Log[2/(1 - I*c*x)])/d^3 - (b*c*e^(3/2)*(a + b*Ar
cTan[c*x])*Log[(2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 -
I*c*x))])/(2*(-d)^(5/2)*(c^2*d - e)) + (e*(a + b*ArcTan[c*x])^2*Log[(2*c*(S
qrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/d^3 + (b*c*e
^(3/2)*(a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] +
I*Sqrt[e])*(1 - I*c*x))])/(2*(-d)^(5/2)*(c^2*d - e)) + (e*(a + b*ArcTan[c*x
])^2*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x)
)])/d^3 - (b^2*c^2*Log[1 + c^2*x^2])/(2*d^2) + ((2*I)*b*e*(a + b*ArcTan[c*x
])*PolyLog[2, 1 - 2/(1 - I*c*x)])/d^3 + ((2*I)*b*e*(a + b*ArcTan[c*x])*Poly
Log[2, 1 - 2/(1 + I*c*x)])/d^3 - ((2*I)*b*e*(a + b*ArcTan[c*x])*PolyLog[2,
-1 + 2/(1 + I*c*x)])/d^3 + ((I/4)*b^2*c*e^(3/2)*PolyLog[2, 1 - (2*c*(Sqrt[-
d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/((-d)^(5/2)*(c^2*
d - e)) - (I*b*e*(a + b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e
]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/d^3 - ((I/4)*b^2*c*e^(3/2)*P
olyLog[2, 1 - (2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I
*c*x))])/((-d)^(5/2)*(c^2*d - e)) - (I*b*e*(a + b*ArcTan[c*x])*PolyLog[2, 1
 - (2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/d^
3 - (b^2*e*PolyLog[3, 1 - 2/(1 - I*c*x)])/d^3 + (b^2*e*PolyLog[3, 1 - 2/(1
+ I*c*x)])/d^3 - (b^2*e*PolyLog[3, -1 + 2/(1 + I*c*x)])/d^3 + (b^2*e*PolyLo
g[3, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x)
)])/(2*d^3) + (b^2*e*PolyLog[3, 1 - (2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-
d] + I*Sqrt[e])*(1 - I*c*x))])/(2*d^3)
```

Rubi [A] time = 2.017, antiderivative size = 1181, normalized size of antiderivative = 1., number of steps used = 47, number of rules used = 22, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.956, Rules used = {4980, 4852, 4918, 266, 36, 29, 31, 4884, 4850, 4988, 4994, 6610, 4978, 4864, 4856, 2402, 2315, 2447, 4984, 4920, 4854, 4858}

$$\frac{c^2\log(x)b^2}{d^2}-\frac{c^2\log\left(c^2x^2+1\right)b^2}{2d^2}+\frac{ice^{3/2}\text{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(c\sqrt{-d}-i\sqrt{e}\right)(1-icx)}\right)b^2}{4(-d)^{5/2}\left(c^2d-e\right)}-\frac{ice^{3/2}\text{PolyLog}\left(2,1-\frac{2c\left(\sqrt{e}x+\sqrt{-d}\right)}{\left(\sqrt{-d}c+i\sqrt{e}\right)(1-icx)}\right)}{4(-d)^{5/2}\left(c^2d-e\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^2/(x^3*(d + e*x^2)^2),x]
```

```
[Out] -((b*c*(a + b*ArcTan[c*x]))/(d^2*x)) - (c^2*(a + b*ArcTan[c*x])^2)/(2*d^2)
+ (c^2*e*(a + b*ArcTan[c*x])^2)/(2*d^2*(c^2*d - e)) - (a + b*ArcTan[c*x])^2
/(2*d^2*x^2) - (e*(a + b*ArcTan[c*x])^2)/(4*d^3*(1 - (Sqrt[e]*x)/Sqrt[-d]))
 - (e*(a + b*ArcTan[c*x])^2)/(4*d^3*(1 + (Sqrt[e]*x)/Sqrt[-d])) - (4*e*(a +
 b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1 + I*c*x)])/d^3 + (b^2*c^2*Log[x])/d^2 -
(2*e*(a + b*ArcTan[c*x])^2*Log[2/(1 - I*c*x)])/d^3 - (b*c*e^(3/2)*(a + b*Ar
cTan[c*x])*Log[(2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 -
I*c*x))])/(2*(-d)^(5/2)*(c^2*d - e)) + (e*(a + b*ArcTan[c*x])^2*Log[(2*c*(S
qrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/d^3 + (b*c*e
^(3/2)*(a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] +
I*Sqrt[e])*(1 - I*c*x))])/(2*(-d)^(5/2)*(c^2*d - e)) + (e*(a + b*ArcTan[c*x
])^2*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x)
)])/d^3 - (b^2*c^2*Log[1 + c^2*x^2])/(2*d^2) + ((2*I)*b*e*(a + b*ArcTan[c*x
])*PolyLog[2, 1 - 2/(1 - I*c*x)])/d^3 + ((2*I)*b*e*(a + b*ArcTan[c*x])*Poly
Log[2, 1 - 2/(1 + I*c*x)])/d^3 - ((2*I)*b*e*(a + b*ArcTan[c*x])*PolyLog[2,
-1 + 2/(1 + I*c*x)])/d^3 + ((I/4)*b^2*c*e^(3/2)*PolyLog[2, 1 - (2*c*(Sqrt[-
d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/((-d)^(5/2)*(c^2*
d - e)) - (I*b*e*(a + b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e
]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/d^3 - ((I/4)*b^2*c*e^(3/2)*P
olyLog[2, 1 - (2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I
*c*x))])/((-d)^(5/2)*(c^2*d - e)) - (I*b*e*(a + b*ArcTan[c*x])*PolyLog[2, 1
 - (2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/d^
3 - (b^2*e*PolyLog[3, 1 - 2/(1 - I*c*x)])/d^3 + (b^2*e*PolyLog[3, 1 - 2/(1
+ I*c*x)])/d^3 - (b^2*e*PolyLog[3, -1 + 2/(1 + I*c*x)])/d^3 + (b^2*e*PolyLo
g[3, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x)
)])/(2*d^3) + (b^2*e*PolyLog[3, 1 - (2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-
d] + I*Sqrt[e])*(1 - I*c*x))])/(2*d^3)
```

Rule 4980

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
```

```
IntegerQ[m])
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4918

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/(d + e*x^2),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4850

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTan[c*x])^(p - 1)*ArcTanh[1 - 2/(1 + I*c*x)])/(1 + c^2*x^2), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4988

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTan[c*x])^p)/(d + e
*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTan[c*x])^p)/(d + e*x^2
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 4978

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2)^2
, x_Symbol] :> Dist[1/(4*d^2*Rt[-(e/d), 2]), Int[(a + b*ArcTan[c*x])^p/(1 -
 Rt[-(e/d), 2]*x)^2, x], x] - Dist[1/(4*d^2*Rt[-(e/d), 2]), Int[(a + b*ArcT
an[c*x])^p/(1 + Rt[-(e/d), 2]*x)^2, x], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 0]
```

Rule 4864

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x])^p)/(e*(q + 1)), x] - D
ist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4984

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(m_.))/((
d_) + (e_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p
/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGt
Q[p, 0] && EqQ[e, c^2*d] && IGtQ[m, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
```

```
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4858

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^2/((d_) + (e_.)*(x_)), x_Symbol] :>
-Simp[((a + b*ArcTan[c*x])^2*Log[2/(1 - I*c*x)])/e, x] + (Simp[((a + b*ArcT
an[c*x])^2*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x] + Simp[(I*
b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 - I*c*x)])/e, x] - Simp[(I*b*(a +
 b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/
e, x] - Simp[(b^2*PolyLog[3, 1 - 2/(1 - I*c*x)])/(2*e), x] + Simp[(b^2*Poly
Log[3, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/(2*e), x]) /; FreeQ[
{a, b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^3\left(d+ex^2\right)^2}\,dx = \int\left(\frac{\left(a+b\tan^{-1}(cx)\right)^2}{d^2x^3} - \frac{2e\left(a+b\tan^{-1}(cx)\right)^2}{d^3x} + \frac{e^2x\left(a+b\tan^{-1}(cx)\right)^2}{d^2\left(d+ex^2\right)^2} + \frac{2e^2x\left(a+b\tan^{-1}(\right.}{d^3\left(d+ex^2\right)}\right.$$

$$= \frac{\int\frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^3}\,dx}{d^2} - \frac{(2e)\int\frac{\left(a+b\tan^{-1}(cx)\right)^2}{x}\,dx}{d^3} + \frac{\left(2e^2\right)\int\frac{x\left(a+b\tan^{-1}(cx)\right)^2}{d+ex^2}\,dx}{d^3} + \frac{e^2\int\frac{x\left(a+b\tan^{-1}(cx\right.}{\left(d+ex^2\right)^2}}{d^2}$$

$$= -\frac{\left(a+b\tan^{-1}(cx)\right)^2}{2d^2x^2} - \frac{4e\left(a+b\tan^{-1}(cx)\right)^2\tanh^{-1}\left(1-\frac{2}{1+icx}\right)}{d^3} + \frac{(bc)\int\frac{a+b\tan^{-1}(cx)}{x^2\left(1+c^2x^2\right)}\,dx}{d^2} + \frac{(8b}{}$$

$$= -\frac{\left(a+b\tan^{-1}(cx)\right)^2}{2d^2x^2} - \frac{e\left(a+b\tan^{-1}(cx)\right)^2}{4d^3\left(1-\frac{\sqrt{e}x}{\sqrt{-d}}\right)} - \frac{e\left(a+b\tan^{-1}(cx)\right)^2}{4d^3\left(1+\frac{\sqrt{e}x}{\sqrt{-d}}\right)} - \frac{4e\left(a+b\tan^{-1}(cx)\right)^2\tan}{d^3}$$

$$= -\frac{bc\left(a+b\tan^{-1}(cx)\right)}{d^2x} - \frac{c^2\left(a+b\tan^{-1}(cx)\right)^2}{2d^2} - \frac{\left(a+b\tan^{-1}(cx)\right)^2}{2d^2x^2} - \frac{e\left(a+b\tan^{-1}(cx)\right)^2}{4d^3\left(1-\frac{\sqrt{e}x}{\sqrt{-d}}\right)} -$$

$$= -\frac{bc\left(a+b\tan^{-1}(cx)\right)}{d^2x} - \frac{c^2\left(a+b\tan^{-1}(cx)\right)^2}{2d^2} - \frac{\left(a+b\tan^{-1}(cx)\right)^2}{2d^2x^2} - \frac{e\left(a+b\tan^{-1}(cx)\right)^2}{4d^3\left(1-\frac{\sqrt{e}x}{\sqrt{-d}}\right)} -$$

$$= -\frac{bc\left(a+b\tan^{-1}(cx)\right)}{d^2x} - \frac{c^2\left(a+b\tan^{-1}(cx)\right)^2}{2d^2} - \frac{\left(a+b\tan^{-1}(cx)\right)^2}{2d^2x^2} - \frac{e\left(a+b\tan^{-1}(cx)\right)^2}{4d^3\left(1-\frac{\sqrt{e}x}{\sqrt{-d}}\right)} -$$

$$= -\frac{bc\left(a+b\tan^{-1}(cx)\right)}{d^2x} - \frac{c^2\left(a+b\tan^{-1}(cx)\right)^2}{2d^2} + \frac{c^2e\left(a+b\tan^{-1}(cx)\right)^2}{2d^2\left(c^2d-e\right)} - \frac{\left(a+b\tan^{-1}(cx)\right)^2}{2d^2x^2}$$

Mathematica [F] time = 28.9067, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^3\left(d+ex^2\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcTan[c*x])^2/(x^3*(d + e*x^2)^2),x]
```

```
[Out] Integrate[(a + b*ArcTan[c*x])^2/(x^3*(d + e*x^2)^2), x]
```

Maple [F] time = 16.436, size = 0, normalized size = 0.

$$\int \frac{(a+b\arctan(cx))^2}{x^3\left(ex^2+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^2/x^3/(e*x^2+d)^2,x)
```

```
[Out] int((a+b*arctan(c*x))^2/x^3/(e*x^2+d)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2}\,a^2\left(\frac{2\,ex^2+d}{d^2ex^4+d^3x^2}-\frac{2\,e\log\left(ex^2+d\right)}{d^3}+\frac{4\,e\log(x)}{d^3}\right)+\int \frac{b^2\arctan(cx)^2+2\,ab\arctan(cx)}{e^2x^7+2\,dex^5+d^2x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^3/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] -1/2*a^2*((2*e*x^2 + d)/(d^2*e*x^4 + d^3*x^2) - 2*e*log(e*x^2 + d)/d^3 + 4*
e*log(x)/d^3) + integrate((b^2*arctan(c*x)^2 + 2*a*b*arctan(c*x))/(e^2*x^7
+ 2*d*e*x^5 + d^2*x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b^2\arctan(cx)^2+2\,ab\arctan(cx)+a^2}{e^2x^7+2\,dex^5+d^2x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^3/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctan(c*x)^2 + 2*a*b*arctan(c*x) + a^2)/(e^2*x^7 + 2*d*e*x^5
 + d^2*x^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**2/x**3/(e*x**2+d)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\arctan(cx)+a)^2}{(ex^2+d)^2 x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^3/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)^2/((e*x^2 + d)^2*x^3), x)
```

3.1275 $\int x^4 \tan^{-1}(x) \log\left(1+x^2\right) dx$

Optimal. Leaf size=111

$$\frac{9x^4}{200} - \frac{77x^2}{300} - \frac{1}{20}\log^2\left(x^2+1\right) - \frac{1}{20}x^4\log\left(x^2+1\right) + \frac{1}{10}x^2\log\left(x^2+1\right) + \frac{137}{300}\log\left(x^2+1\right) - \frac{2}{25}x^5\tan^{-1}(x) + \frac{2}{15}x^3\tan$$

```
[Out] (-77*x^2)/300 + (9*x^4)/200 - (2*x*ArcTan[x])/5 + (2*x^3*ArcTan[x])/15 - (2
*x^5*ArcTan[x])/25 + ArcTan[x]^2/5 + (137*Log[1 + x^2])/300 + (x^2*Log[1 +
x^2])/10 - (x^4*Log[1 + x^2])/20 + (x^5*ArcTan[x]*Log[1 + x^2])/5 - Log[1 +
 x^2]^2/20
```

Rubi [A] time = 0.444091, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 14, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.167, Rules used = {4852, 266, 43, 5021, 6725, 446, 77, 4916, 4846, 260, 4884, 2475, 2390, 2301}

$$\frac{9x^4}{200} - \frac{77x^2}{300} - \frac{1}{20}\log^2\left(x^2+1\right) - \frac{1}{20}x^4\log\left(x^2+1\right) + \frac{1}{10}x^2\log\left(x^2+1\right) + \frac{137}{300}\log\left(x^2+1\right) - \frac{2}{25}x^5\tan^{-1}(x) + \frac{2}{15}x^3\tan$$

Antiderivative was successfully verified.

```
[In] Int[x^4*ArcTan[x]*Log[1 + x^2],x]
```


```
[Out] (-77*x^2)/300 + (9*x^4)/200 - (2*x*ArcTan[x])/5 + (2*x^3*ArcTan[x])/15 - (2
*x^5*ArcTan[x])/25 + ArcTan[x]^2/5 + (137*Log[1 + x^2])/300 + (x^2*Log[1 +
x^2])/10 - (x^4*Log[1 + x^2])/20 + (x^5*ArcTan[x]*Log[1 + x^2])/5 - Log[1 +
 x^2]^2/20
```


Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5021

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*(
e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(a + b*ArcTan[c*x]), x
]}, Dist[d + e*Log[f + g*x^2], u, x] - Dist[2*e*g, Int[ExpandIntegrand[(x*u
)/(f + g*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IntegerQ[m
] && NeQ[m, -1]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
```

```
e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rubi steps

$$\int x^4\tan^{-1}(x)\log\left(1+x^2\right)\,dx = \frac{1}{10}x^2\log\left(1+x^2\right)-\frac{1}{20}x^4\log\left(1+x^2\right)+\frac{1}{5}x^5\tan^{-1}(x)\log\left(1+x^2\right)-\frac{1}{10}\log^2\left(1+x^2\right)-$$

$$=\frac{1}{10}x^2\log\left(1+x^2\right)-\frac{1}{20}x^4\log\left(1+x^2\right)+\frac{1}{5}x^5\tan^{-1}(x)\log\left(1+x^2\right)-\frac{1}{10}\log^2\left(1+x^2\right)-$$

$$=\frac{1}{10}x^2\log\left(1+x^2\right)-\frac{1}{20}x^4\log\left(1+x^2\right)+\frac{1}{5}x^5\tan^{-1}(x)\log\left(1+x^2\right)-\frac{1}{10}\log^2\left(1+x^2\right)-$$

$$=\frac{1}{10}x^2\log\left(1+x^2\right)-\frac{1}{20}x^4\log\left(1+x^2\right)+\frac{1}{5}x^5\tan^{-1}(x)\log\left(1+x^2\right)-\frac{1}{10}\log^2\left(1+x^2\right)+$$

$$=\frac{1}{10}x^2\log\left(1+x^2\right)-\frac{1}{20}x^4\log\left(1+x^2\right)+\frac{1}{5}x^5\tan^{-1}(x)\log\left(1+x^2\right)-\frac{1}{20}\log^2\left(1+x^2\right)+$$

$$=-\frac{2}{25}x^5\tan^{-1}(x)+\frac{1}{10}x^2\log\left(1+x^2\right)-\frac{1}{20}x^4\log\left(1+x^2\right)+\frac{1}{5}x^5\tan^{-1}(x)\log\left(1+x^2\right)-$$

$$=-\frac{3x^2}{20}+\frac{x^4}{40}+\frac{2}{15}x^3\tan^{-1}(x)-\frac{2}{25}x^5\tan^{-1}(x)+\frac{3}{20}\log\left(1+x^2\right)+\frac{1}{10}x^2\log\left(1+x^2\right)-$$

$$=-\frac{3x^2}{20}+\frac{x^4}{40}-\frac{2}{5}x\tan^{-1}(x)+\frac{2}{15}x^3\tan^{-1}(x)-\frac{2}{25}x^5\tan^{-1}(x)+\frac{1}{5}\tan^{-1}(x)^2+\frac{3}{20}\log\left(1\right.$$

$$=-\frac{19x^2}{100}+\frac{9x^4}{200}-\frac{2}{5}x\tan^{-1}(x)+\frac{2}{15}x^3\tan^{-1}(x)-\frac{2}{25}x^5\tan^{-1}(x)+\frac{1}{5}\tan^{-1}(x)^2+\frac{39}{100}\log$$

$$=-\frac{77x^2}{300}+\frac{9x^4}{200}-\frac{2}{5}x\tan^{-1}(x)+\frac{2}{15}x^3\tan^{-1}(x)-\frac{2}{25}x^5\tan^{-1}(x)+\frac{1}{5}\tan^{-1}(x)^2+\frac{137}{300}\log$$

Mathematica [A] time = 0.0245255, size = 79, normalized size = 0.71

$$\frac{1}{600}\left(\left(27x^2-154\right)x^2-30\log^2\left(x^2+1\right)+\left(-30x^4+60x^2+274\right)\log\left(x^2+1\right)+8x\left(-6x^4+10x^2+15x^4\log\left(x^2+1\right)-30\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*ArcTan[x]*Log[1 + x^2],x]
```

```
[Out] (x^2*(-154 + 27*x^2) + 120*ArcTan[x]^2 + (274 + 60*x^2 - 30*x^4)*Log[1 + x^
2] - 30*Log[1 + x^2]^2 + 8*x*ArcTan[x]*(-30 + 10*x^2 - 6*x^4 + 15*x^4*Log[1
 + x^2]))/600
```

Maple [C] time = 1.822, size = 3626, normalized size = 32.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arctan(x)*ln(x^2+1),x)
```

```
[Out] 1/5*x^2*ln(2)-1/5*ln((1+I*x)^2/(x^2+1)+1)*x^2+1/10*ln((1+I*x)^2/(x^2+1)+1)*
x^4+46/75*I*arctan(x)+3/10*ln(2)+9/200*x^4-77/300*x^2+3/20*I*Pi*csgn(I*(1+I
*x)/(x^2+1)^(1/2))*csgn(I*(1+I*x)^2/(x^2+1))^2+3/40*I*Pi*csgn(I/((1+I*x)^2/
(x^2+1)+1)^2)*csgn(I*(1+I*x)^2/(x^2+1)/((1+I*x)^2/(x^2+1)+1)^2)^2+3/40*I*Pi
*csgn(I*((1+I*x)^2/(x^2+1)+1))^2*csgn(I*((1+I*x)^2/(x^2+1)+1)^2)-2/5*x*arct
an(x)+2/15*x^3*arctan(x)-2/25*x^5*arctan(x)-3/40*I*Pi*csgn(I*(1+I*x)^2/(x^2
+1)/((1+I*x)^2/(x^2+1)+1)^2)^3-2/5*I*ln(2)*arctan(x)-1/10*I*Pi*ln((1+I*x)^2
/(x^2+1)+1)*csgn(I/((1+I*x)^2/(x^2+1)+1)^2)*csgn(I*(1+I*x)^2/(x^2+1))*csgn(
I*(1+I*x)^2/(x^2+1)/((1+I*x)^2/(x^2+1)+1)^2)+1/10*I*arctan(x)*Pi*csgn(I*((1
+I*x)^2/(x^2+1)+1))^2*csgn(I*((1+I*x)^2/(x^2+1)+1)^2)*x^5-1/5*I*arctan(x)*P
i*csgn(I*((1+I*x)^2/(x^2+1)+1))*csgn(I*((1+I*x)^2/(x^2+1)+1)^2)^2*x^5+1/40*
I*Pi*csgn(I*(1+I*x)^2/(x^2+1))*csgn(I*(1+I*x)^2/(x^2+1)/((1+I*x)^2/(x^2+1)+
1)^2)*csgn(I/((1+I*x)^2/(x^2+1)+1)^2)*x^4-1/20*I*Pi*csgn(I*(1+I*x)^2/(x^2+1
))*csgn(I*(1+I*x)^2/(x^2+1)/((1+I*x)^2/(x^2+1)+1)^2)*csgn(I/((1+I*x)^2/(x^2
+1)+1)^2)*x^2+1/5*I*arctan(x)*Pi*csgn(I*(1+I*x)^2/(x^2+1))^2*csgn(I*(1+I*x)
/(x^2+1)^(1/2))*x^5+1/10*I*arctan(x)*Pi*csgn(I*(1+I*x)^2/(x^2+1))*csgn(I*(1
+I*x)^2/(x^2+1)/((1+I*x)^2/(x^2+1)+1)^2)^2*x^5-1/10*I*arctan(x)*Pi*csgn(I*(
1+I*x)^2/(x^2+1))*csgn(I*(1+I*x)/(x^2+1)^(1/2))^2*x^5+1/10*I*arctan(x)*Pi*c
sgn(I*(1+I*x)^2/(x^2+1)/((1+I*x)^2/(x^2+1)+1)^2)^2*csgn(I/((1+I*x)^2/(x^2+1
)+1)^2)*x^5-1/10*I*arctan(x)*Pi*csgn(I*(1+I*x)^2/(x^2+1))*csgn(I*(1+I*x)^2/
(x^2+1)/((1+I*x)^2/(x^2+1)+1)^2)*csgn(I/((1+I*x)^2/(x^2+1)+1)^2)*x^5+1/10*(
-4*I*arctan(x)+4*x^5*arctan(x)+4*ln((1+I*x)^2/(x^2+1)+1)+3+2*x^2-x^4)*ln((1
+I*x)/(x^2+1)^(1/2))-3/20*I*Pi*csgn(I*((1+I*x)^2/(x^2+1)+1))*csgn(I*((1+I*x
)^2/(x^2+1)+1)^2)^2-3/40*I*Pi*csgn(I*(1+I*x)/(x^2+1)^(1/2))^2*csgn(I*(1+I*x
)^2/(x^2+1))+3/40*I*Pi*csgn(I*(1+I*x)^2/(x^2+1))*csgn(I*(1+I*x)^2/(x^2+1)/(
(1+I*x)^2/(x^2+1)+1)^2)^2+1/40*I*Pi*csgn(I*(1+I*x)^2/(x^2+1))^3*x^4+1/40*I*
Pi*csgn(I*(1+I*x)^2/(x^2+1)/((1+I*x)^2/(x^2+1)+1)^2)^3*x^4-1/40*I*Pi*csgn(I
*((1+I*x)^2/(x^2+1)+1)^2)^3*x^4-1/20*I*Pi*csgn(I*(1+I*x)^2/(x^2+1))^3*x^2+1
/5*arctan(x)*Pi*csgn(I*(1+I*x)^2/(x^2+1))^2*csgn(I*(1+I*x)/(x^2+1)^(1/2))+1
/10*arctan(x)*Pi*csgn(I*(1+I*x)^2/(x^2+1))*csgn(I*(1+I*x)^2/(x^2+1)/((1+I*x
)^2/(x^2+1)+1)^2)^2-1/10*arctan(x)*Pi*csgn(I*(1+I*x)^2/(x^2+1))*csgn(I*(1+I
*x)/(x^2+1)^(1/2))^2+1/10*I*Pi*ln((1+I*x)^2/(x^2+1)+1)*csgn(I/((1+I*x)^2/(x
^2+1)+1)^2)*csgn(I*(1+I*x)^2/(x^2+1)/((1+I*x)^2/(x^2+1)+1)^2)^2-1/10*I*Pi*l
n((1+I*x)^2/(x^2+1)+1)*csgn(I*(1+I*x)/(x^2+1)^(1/2))^2*csgn(I*(1+I*x)^2/(x^
2+1))+1/10*I*Pi*ln((1+I*x)^2/(x^2+1)+1)*csgn(I*((1+I*x)^2/(x^2+1)+1))^2*csg
n(I*((1+I*x)^2/(x^2+1)+1)^2)-1/5*I*Pi*ln((1+I*x)^2/(x^2+1)+1)*csgn(I*((1+I*
x)^2/(x^2+1)+1))*csgn(I*((1+I*x)^2/(x^2+1)+1)^2)^2+1/5*I*Pi*ln((1+I*x)^2/(x
^2+1)+1)*csgn(I*(1+I*x)/(x^2+1)^(1/2))*csgn(I*(1+I*x)^2/(x^2+1))^2+1/10*I*P
i*ln((1+I*x)^2/(x^2+1)+1)*csgn(I*(1+I*x)^2/(x^2+1))*csgn(I*(1+I*x)^2/(x^2+1
)/((1+I*x)^2/(x^2+1)+1)^2)^2-1/20*I*Pi*csgn(I*(1+I*x)^2/(x^2+1))*csgn(I*(1+
```

```
I*x)/(x^2+1)^(1/2))^2*x^2+1/20*I*Pi*csgn(I*(1+I*x)^2/(x^2+1)/((1+I*x)^2/(x^
2+1)+1)^2)^2*csgn(I/((1+I*x)^2/(x^2+1)+1)^2)*x^2+1/20*I*Pi*csgn(I*((1+I*x)^
2/(x^2+1)+1))^2*csgn(I*((1+I*x)^2/(x^2+1)+1)^2)*x^2-1/10*I*Pi*csgn(I*((1+I*
x)^2/(x^2+1)+1))*csgn(I*((1+I*x)^2/(x^2+1)+1)^2)^2*x^2-1/10*I*arctan(x)*Pi*
csgn(I*(1+I*x)^2/(x^2+1))^3*x^5-1/10*I*arctan(x)*Pi*csgn(I*(1+I*x)^2/(x^2+1
)/((1+I*x)^2/(x^2+1)+1)^2)^3*x^5+1/10*I*arctan(x)*Pi*csgn(I*((1+I*x)^2/(x^2
+1)+1)^2)^3*x^5-1/20*I*Pi*csgn(I*(1+I*x)^2/(x^2+1))^2*csgn(I*(1+I*x)/(x^2+1
)^(1/2))*x^4-1/40*I*Pi*csgn(I*(1+I*x)^2/(x^2+1)/((1+I*x)^2/(x^2+1)+1)^2)^2*
csgn(I/((1+I*x)^2/(x^2+1)+1)^2)*x^4-1/40*I*Pi*csgn(I*((1+I*x)^2/(x^2+1)+1))
^2*csgn(I*((1+I*x)^2/(x^2+1)+1)^2)*x^4+1/20*I*Pi*csgn(I*((1+I*x)^2/(x^2+1)+
1))*csgn(I*((1+I*x)^2/(x^2+1)+1)^2)^2*x^4+1/10*I*Pi*csgn(I*(1+I*x)^2/(x^2+1
))^2*csgn(I*(1+I*x)/(x^2+1)^(1/2))*x^2+1/20*I*Pi*csgn(I*(1+I*x)^2/(x^2+1))*
csgn(I*(1+I*x)^2/(x^2+1)/((1+I*x)^2/(x^2+1)+1)^2)^2*x^2-3/40*I*Pi*csgn(I/((
1+I*x)^2/(x^2+1)+1)^2)*csgn(I*(1+I*x)^2/(x^2+1))*csgn(I*(1+I*x)^2/(x^2+1)/(
(1+I*x)^2/(x^2+1)+1)^2)-1/10*arctan(x)*Pi*csgn(I*(1+I*x)^2/(x^2+1))*csgn(I*
(1+I*x)^2/(x^2+1)/((1+I*x)^2/(x^2+1)+1)^2)*csgn(I/((1+I*x)^2/(x^2+1)+1)^2)-
1/40*I*Pi*csgn(I*(1+I*x)^2/(x^2+1))*csgn(I*(1+I*x)^2/(x^2+1)/((1+I*x)^2/(x^
2+1)+1)^2)^2*x^4+1/40*I*Pi*csgn(I*(1+I*x)^2/(x^2+1))*csgn(I*(1+I*x)/(x^2+1)
^(1/2))^2*x^4-1/10*ln(2)*x^4-137/150*ln((1+I*x)^2/(x^2+1)+1)-1/5*ln((1+I*x)
^2/(x^2+1)+1)^2-3/40*I*Pi*csgn(I*(1+I*x)^2/(x^2+1))^3-1/10*arctan(x)*Pi*csg
n(I*(1+I*x)^2/(x^2+1))^3-1/10*arctan(x)*Pi*csgn(I*(1+I*x)^2/(x^2+1)/((1+I*x
)^2/(x^2+1)+1)^2)^3+1/10*arctan(x)*Pi*csgn(I*((1+I*x)^2/(x^2+1)+1)^2)^3-2/5
*arctan(x)*ln((1+I*x)^2/(x^2+1)+1)*x^5+2/5*arctan(x)*ln(2)*x^5-1/10*I*Pi*ln
((1+I*x)^2/(x^2+1)+1)*csgn(I*(1+I*x)^2/(x^2+1))^3-1/10*I*Pi*ln((1+I*x)^2/(x
^2+1)+1)*csgn(I*(1+I*x)^2/(x^2+1)/((1+I*x)^2/(x^2+1)+1)^2)^3+1/10*I*Pi*ln((
1+I*x)^2/(x^2+1)+1)*csgn(I*((1+I*x)^2/(x^2+1)+1)^2)^3-1/20*I*Pi*csgn(I*(1+I
*x)^2/(x^2+1)/((1+I*x)^2/(x^2+1)+1)^2)^3*x^2+1/20*I*Pi*csgn(I*((1+I*x)^2/(x
^2+1)+1)^2)^3*x^2+3/40*I*Pi*csgn(I*((1+I*x)^2/(x^2+1)+1)^2)^3+1/10*arctan(x
)*Pi*csgn(I*(1+I*x)^2/(x^2+1)/((1+I*x)^2/(x^2+1)+1)^2)^2*csgn(I/((1+I*x)^2/
(x^2+1)+1)^2)+1/10*arctan(x)*Pi*csgn(I*((1+I*x)^2/(x^2+1)+1))^2*csgn(I*((1+
I*x)^2/(x^2+1)+1)^2)-1/5*arctan(x)*Pi*csgn(I*((1+I*x)^2/(x^2+1)+1))*csgn(I*
((1+I*x)^2/(x^2+1)+1)^2)^2-181/600+2/5*ln(2)*ln((1+I*x)^2/(x^2+1)+1)
```

Maxima [A] time = 1.64656, size = 108, normalized size = 0.97

$$\frac{9}{200}\,x^4 - \frac{77}{300}\,x^2 + \frac{1}{75}\left(15\,x^5\log\left(x^2+1\right) - 6\,x^5 + 10\,x^3 - 30\,x + 30\,\arctan\left(x\right)\right)\arctan\left(x\right) - \frac{1}{5}\,\arctan\left(x\right)^2 - \frac{1}{300}\left(15\,x^4\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctan(x)*log(x^2+1),x, algorithm="maxima")
```

```
[Out] 9/200*x^4 - 77/300*x^2 + 1/75*(15*x^5*log(x^2 + 1) - 6*x^5 + 10*x^3 - 30*x
```

```
+ 30*arctan(x))*arctan(x) - 1/5*arctan(x)^2 - 1/300*(15*x^4 - 30*x^2 - 137)
*log(x^2 + 1) - 1/20*log(x^2 + 1)^2
```

Fricas [A] time = 1.18781, size = 227, normalized size = 2.05

$$\frac{9}{200}x^4 - \frac{77}{300}x^2 - \frac{2}{75}\left(3x^5 - 5x^3 + 15x\right)\arctan(x) + \frac{1}{5}\arctan(x)^2 + \frac{1}{300}\left(60x^5\arctan(x) - 15x^4 + 30x^2 + 137\right)\mathrm{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctan(x)*log(x^2+1),x, algorithm="fricas")
```

```
[Out] 9/200*x^4 - 77/300*x^2 - 2/75*(3*x^5 - 5*x^3 + 15*x)*arctan(x) + 1/5*arctan
(x)^2 + 1/300*(60*x^5*arctan(x) - 15*x^4 + 30*x^2 + 137)*log(x^2 + 1) - 1/2
0*log(x^2 + 1)^2
```

Sympy [A] time = 8.40477, size = 107, normalized size = 0.96

$$\frac{x^5\log\left(x^2+1\right)\operatorname{atan}(x)}{5} - \frac{2x^5\operatorname{atan}(x)}{25} - \frac{x^4\log\left(x^2+1\right)}{20} + \frac{9x^4}{200} + \frac{2x^3\operatorname{atan}(x)}{15} + \frac{x^2\log\left(x^2+1\right)}{10} - \frac{77x^2}{300} - \frac{2x\operatorname{atan}(x)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*atan(x)*ln(x**2+1),x)
```

```
[Out] x**5*log(x**2 + 1)*atan(x)/5 - 2*x**5*atan(x)/25 - x**4*log(x**2 + 1)/20 +
9*x**4/200 + 2*x**3*atan(x)/15 + x**2*log(x**2 + 1)/10 - 77*x**2/300 - 2*x*
atan(x)/5 - log(x**2 + 1)**2/20 + 137*log(x**2 + 1)/300 + atan(x)**2/5
```

Giac [A] time = 1.10394, size = 227, normalized size = 2.05

$$\frac{1}{10}\pi x^5\log\left(x^2+1\right)\operatorname{sgn}(x) - \frac{1}{5}x^5\arctan\left(\frac{1}{x}\right)\log\left(x^2+1\right) - \frac{1}{25}\pi x^5\operatorname{sgn}(x) + \frac{2}{25}x^5\arctan\left(\frac{1}{x}\right) - \frac{1}{20}x^4\log\left(x^2+1\right) +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctan(x)*log(x^2+1),x, algorithm="giac")
```

```
[Out] 1/10*pi*x^5*log(x^2 + 1)*sgn(x) - 1/5*x^5*arctan(1/x)*log(x^2 + 1) - 1/25*p
i*x^5*sgn(x) + 2/25*x^5*arctan(1/x) - 1/20*x^4*log(x^2 + 1) + 1/15*pi*x^3*s
gn(x) + 9/200*x^4 - 2/15*x^3*arctan(1/x) + 1/10*x^2*log(x^2 + 1) - 3/10*pi^
2*sgn(x) - 1/5*pi*x*sgn(x) - 1/5*pi*arctan(1/x)*sgn(x) + 1/10*pi^2 - 77/300
*x^2 + 1/5*pi*arctan(x) + 1/5*pi*arctan(1/x) + 2/5*x*arctan(1/x) + 1/5*arct
an(1/x)^2 - 1/20*log(x^2 + 1)^2 + 137/300*log(x^2 + 1)
```

3.1276 $\int x^3 \tan^{-1}(x) \log(1 + x^2)\, dx$

Optimal. Leaf size=88

$$\frac{7x^3}{72} - \frac{1}{12}x^3 \log(x^2+1) + \frac{1}{4}x \log(x^2+1) - \frac{1}{8}x^4 \tan^{-1}(x) + \frac{1}{4}x^2 \tan^{-1}(x) + \frac{1}{4}x^4 \log(x^2+1) \tan^{-1}(x) - \frac{1}{4}\log(x^2+1)$$

```
[Out] (-25*x)/24 + (7*x^3)/72 + (25*ArcTan[x])/24 + (x^2*ArcTan[x])/4 - (x^4*ArcT
an[x])/8 + (x*Log[1 + x^2])/4 - (x^3*Log[1 + x^2])/12 - (ArcTan[x]*Log[1 +
x^2])/4 + (x^4*ArcTan[x]*Log[1 + x^2])/4
```

Rubi [A] time = 0.117512, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 12, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {4852, 302, 203, 2454, 2395, 43, 5019, 459, 321, 2471, 2448, 2455}

$$\frac{7x^3}{72} - \frac{1}{12}x^3 \log(x^2+1) + \frac{1}{4}x \log(x^2+1) - \frac{1}{8}x^4 \tan^{-1}(x) + \frac{1}{4}x^2 \tan^{-1}(x) + \frac{1}{4}x^4 \log(x^2+1) \tan^{-1}(x) - \frac{1}{4}\log(x^2+1)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcTan[x]*Log[1 + x^2],x]
```


```
[Out] (-25*x)/24 + (7*x^3)/72 + (25*ArcTan[x])/24 + (x^2*ArcTan[x])/4 - (x^4*ArcT
an[x])/8 + (x*Log[1 + x^2])/4 - (x^3*Log[1 + x^2])/12 - (ArcTan[x]*Log[1 +
x^2])/4 + (x^4*ArcTan[x]*Log[1 + x^2])/4
```


Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5019

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*(
e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*Log[f + g*x^2])
, x]}, Dist[a + b*ArcTan[c*x], u, x] - Dist[b*c, Int[ExpandIntegrand[u/(1 +
 c^2*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[(m + 1)/2
, 0]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2471

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\int x^3 \tan^{-1}(x) \log\left(1+x^2\right) dx = \frac{1}{4}x^2 \tan^{-1}(x) - \frac{1}{8}x^4 \tan^{-1}(x) - \frac{1}{4}\tan^{-1}(x)\log\left(1+x^2\right) + \frac{1}{4}x^4 \tan^{-1}(x)\log\left(1+x^2\right) - \int$$

$$= \frac{1}{4}x^2 \tan^{-1}(x) - \frac{1}{8}x^4 \tan^{-1}(x) - \frac{1}{4}\tan^{-1}(x)\log\left(1+x^2\right) + \frac{1}{4}x^4 \tan^{-1}(x)\log\left(1+x^2\right) + \frac{1}{8}$$

$$= \frac{x^3}{24} + \frac{1}{4}x^2 \tan^{-1}(x) - \frac{1}{8}x^4 \tan^{-1}(x) - \frac{1}{4}\tan^{-1}(x)\log\left(1+x^2\right) + \frac{1}{4}x^4 \tan^{-1}(x)\log\left(1+x^2\right.$$

$$= -\frac{3x}{8} + \frac{x^3}{24} + \frac{1}{4}x^2 \tan^{-1}(x) - \frac{1}{8}x^4 \tan^{-1}(x) - \frac{1}{4}\tan^{-1}(x)\log\left(1+x^2\right) + \frac{1}{4}x^4 \tan^{-1}(x)\log$$

$$= -\frac{3x}{8} + \frac{x^3}{24} + \frac{3}{8}\tan^{-1}(x) + \frac{1}{4}x^2 \tan^{-1}(x) - \frac{1}{8}x^4 \tan^{-1}(x) + \frac{1}{4}x\log\left(1+x^2\right) - \frac{1}{12}x^3\log\left(1\right.$$

$$= -\frac{7x}{8} + \frac{x^3}{24} + \frac{3}{8}\tan^{-1}(x) + \frac{1}{4}x^2 \tan^{-1}(x) - \frac{1}{8}x^4 \tan^{-1}(x) + \frac{1}{4}x\log\left(1+x^2\right) - \frac{1}{12}x^3\log\left(1\right.$$

$$= -\frac{25x}{24} + \frac{7x^3}{72} + \frac{7}{8}\tan^{-1}(x) + \frac{1}{4}x^2 \tan^{-1}(x) - \frac{1}{8}x^4 \tan^{-1}(x) + \frac{1}{4}x\log\left(1+x^2\right) - \frac{1}{12}x^3\log$$

$$= -\frac{25x}{24} + \frac{7x^3}{72} + \frac{25}{24}\tan^{-1}(x) + \frac{1}{4}x^2 \tan^{-1}(x) - \frac{1}{8}x^4 \tan^{-1}(x) + \frac{1}{4}x\log\left(1+x^2\right) - \frac{1}{12}x^3\,\mathrm{lo}$$

Mathematica [A] time = 0.0228806, size = 56, normalized size = 0.64

$$\frac{1}{72}\left(x\left(7x^2-6\left(x^2-3\right)\log\left(x^2+1\right)-75\right)+3\left(-3x^4+6x^2+6\left(x^4-1\right)\log\left(x^2+1\right)+25\right)\tan^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcTan[x]*Log[1 + x^2],x]
```

```
[Out] (x*(-75 + 7*x^2 - 6*(-3 + x^2)*Log[1 + x^2]) + 3*ArcTan[x]*(25 + 6*x^2 - 3*
x^4 + 6*(-1 + x^4)*Log[1 + x^2]))/72
```

Maple [C] time = 1.192, size = 2849, normalized size = 32.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctan(x)*ln(x^2+1),x)
```

[Out] -25/24*x+7/72*x^3-1/2*arctan(x)*ln((1+I*x)^2/(x^2+1)+1)*x^4+1/2*arctan(x)*l
n(2)*x^4+41/24*arctan(x)+1/2*ln(2)*x+1/4*x^2*arctan(x)-1/8*x^4*arctan(x)+1/
6*Pi*csgn(I*(1+I*x)/(x^2+1)^(1/2))^2*csgn(I*(1+I*x)^2/(x^2+1))-1/3*Pi*csgn(
I*(1+I*x)/(x^2+1)^(1/2))*csgn(I*(1+I*x)^2/(x^2+1))^2-1/6*Pi*csgn(I/((1+I*x)
^2/(x^2+1)+1)^2)*csgn(I*(1+I*x)^2/(x^2+1)/((1+I*x)^2/(x^2+1)+1)^2)^2-1/6*Pi
csgn(I(1+I*x)^2/(x^2+1))*csgn(I*(1+I*x)^2/(x^2+1)/((1+I*x)^2/(x^2+1)+1)^2
)^2-1/6*Pi*csgn(I*((1+I*x)^2/(x^2+1)+1))^2*csgn(I*((1+I*x)^2/(x^2+1)+1)^2)+
1/3*Pi*csgn(I*((1+I*x)^2/(x^2+1)+1))*csgn(I*((1+I*x)^2/(x^2+1)+1)^2)^2+1/24
*I*Pi*csgn(I*(1+I*x)^2/(x^2+1))^3*x^3+1/24*I*Pi*csgn(I*(1+I*x)^2/(x^2+1)/((
1+I*x)^2/(x^2+1)+1)^2)^3*x^3-1/24*I*Pi*csgn(I*((1+I*x)^2/(x^2+1)+1)^2)^3*x^
3+1/8*I*arctan(x)*Pi*csgn(I*(1+I*x)^2/(x^2+1))^3+1/8*I*arctan(x)*Pi*csgn(I*
(1+I*x)^2/(x^2+1)/((1+I*x)^2/(x^2+1)+1)^2)^3-1/8*I*arctan(x)*Pi*csgn(I*((1+
I*x)^2/(x^2+1)+1)^2)^3-1/8*I*Pi*csgn(I*(1+I*x)^2/(x^2+1))^3*x+1/6*(3*I*arct
an(x)-3*x*arctan(x)-3*I*arctan(x)*x^2+3*x^3*arctan(x)+4+I*x-x^2)*(x+I)*ln((
1+I*x)/(x^2+1)^(1/2))-1/8*I*Pi*csgn(I*(1+I*x)^2/(x^2+1)/((1+I*x)^2/(x^2+1)+
1)^2)^3*x+1/8*I*Pi*csgn(I*((1+I*x)^2/(x^2+1)+1)^2)^3*x+1/8*I*arctan(x)*Pi*c
sgn(I/((1+I*x)^2/(x^2+1)+1)^2)*csgn(I*(1+I*x)^2/(x^2+1)/((1+I*x)^2/(x^2+1)+
1)^2)^2*x^4+1/4*I*arctan(x)*Pi*csgn(I*(1+I*x)^2/(x^2+1))^2*csgn(I*(1+I*x)/(
x^2+1)^(1/2))*x^4+1/8*I*arctan(x)*Pi*csgn(I*(1+I*x)^2/(x^2+1))*csgn(I*(1+I*
x)^2/(x^2+1)/((1+I*x)^2/(x^2+1)+1)^2)^2*x^4-1/8*I*arctan(x)*Pi*csgn(I*(1+I*
x)^2/(x^2+1))*csgn(I*(1+I*x)/(x^2+1)^(1/2))^2*x^4+1/8*I*arctan(x)*Pi*csgn(I
*((1+I*x)^2/(x^2+1)+1))^2*csgn(I*((1+I*x)^2/(x^2+1)+1)^2)*x^4-1/4*I*arctan(
x)*Pi*csgn(I*((1+I*x)^2/(x^2+1)+1))*csgn(I*((1+I*x)^2/(x^2+1)+1)^2)^2*x^4+1
/24*I*Pi*csgn(I/((1+I*x)^2/(x^2+1)+1)^2)*csgn(I*(1+I*x)^2/(x^2+1))*csgn(I*(
1+I*x)^2/(x^2+1)/((1+I*x)^2/(x^2+1)+1)^2)*x^3-1/8*I*arctan(x)*Pi*csgn(I*((1
+I*x)^2/(x^2+1)+1))^2*csgn(I*((1+I*x)^2/(x^2+1)+1)^2)+1/4*I*arctan(x)*Pi*cs
gn(I*((1+I*x)^2/(x^2+1)+1))*csgn(I*((1+I*x)^2/(x^2+1)+1)^2)^2+1/8*I*Pi*csgn
(I/((1+I*x)^2/(x^2+1)+1)^2)*csgn(I*(1+I*x)^2/(x^2+1)/((1+I*x)^2/(x^2+1)+1)^
2)^2*x+1/4*I*Pi*csgn(I*(1+I*x)^2/(x^2+1))^2*csgn(I*(1+I*x)/(x^2+1)^(1/2))*x
+1/8*I*Pi*csgn(I*(1+I*x)^2/(x^2+1))*csgn(I*(1+I*x)^2/(x^2+1)/((1+I*x)^2/(x^
2+1)+1)^2)^2*x-1/8*I*Pi*csgn(I*(1+I*x)^2/(x^2+1))*csgn(I*(1+I*x)/(x^2+1)^(1
/2))^2*x+1/8*I*Pi*csgn(I*((1+I*x)^2/(x^2+1)+1))^2*csgn(I*((1+I*x)^2/(x^2+1)
+1)^2)*x-1/4*I*Pi*csgn(I*((1+I*x)^2/(x^2+1)+1))*csgn(I*((1+I*x)^2/(x^2+1)+1
)^2)^2*x-1/8*I*arctan(x)*Pi*csgn(I*(1+I*x)^2/(x^2+1))^3*x^4-1/8*I*arctan(x)
*Pi*csgn(I*(1+I*x)^2/(x^2+1)/((1+I*x)^2/(x^2+1)+1)^2)^3*x^4+1/8*I*arctan(x)
*Pi*csgn(I*((1+I*x)^2/(x^2+1)+1)^2)^3*x^4-1/24*I*Pi*csgn(I/((1+I*x)^2/(x^2+
1)+1)^2)*csgn(I*(1+I*x)^2/(x^2+1)/((1+I*x)^2/(x^2+1)+1)^2)^2*x^3-1/12*I*Pi*
csgn(I*(1+I*x)^2/(x^2+1))^2*csgn(I*(1+I*x)/(x^2+1)^(1/2))*x^3-1/24*I*Pi*csg
n(I*(1+I*x)^2/(x^2+1))*csgn(I*(1+I*x)^2/(x^2+1)/((1+I*x)^2/(x^2+1)+1)^2)^2*
x^3+1/24*I*Pi*csgn(I*(1+I*x)^2/(x^2+1))*csgn(I*(1+I*x)/(x^2+1)^(1/2))^2*x^3
-1/24*I*Pi*csgn(I*((1+I*x)^2/(x^2+1)+1))^2*csgn(I*((1+I*x)^2/(x^2+1)+1)^2)*
x^3+1/12*I*Pi*csgn(I*((1+I*x)^2/(x^2+1)+1))*csgn(I*((1+I*x)^2/(x^2+1)+1)^2)
^2*x^3-1/8*I*arctan(x)*Pi*csgn(I/((1+I*x)^2/(x^2+1)+1)^2)*csgn(I*(1+I*x)^2/
(x^2+1)/((1+I*x)^2/(x^2+1)+1)^2)^2-1/4*I*arctan(x)*Pi*csgn(I*(1+I*x)^2/(x^2
+1))^2*csgn(I*(1+I*x)/(x^2+1)^(1/2))-1/8*I*arctan(x)*Pi*csgn(I*(1+I*x)^2/(x

```
^2+1))*csgn(I*(1+I*x)^2/(x^2+1)/((1+I*x)^2/(x^2+1)+1)^2)^2-1/8*I*arctan(x)*
Pi*csgn(I/((1+I*x)^2/(x^2+1)+1)^2)*csgn(I*(1+I*x)^2/(x^2+1))*csgn(I*(1+I*x)
^2/(x^2+1)/((1+I*x)^2/(x^2+1)+1)^2)*x^4+2/3*I*ln(2)-1/6*Pi*csgn(I*((1+I*x)^
2/(x^2+1)+1)^2)^3+1/6*Pi*csgn(I/((1+I*x)^2/(x^2+1)+1)^2)*csgn(I*(1+I*x)^2/(
x^2+1))*csgn(I*(1+I*x)^2/(x^2+1)/((1+I*x)^2/(x^2+1)+1)^2)-41/36*I+1/8*I*arc
tan(x)*Pi*csgn(I/((1+I*x)^2/(x^2+1)+1)^2)*csgn(I*(1+I*x)^2/(x^2+1))*csgn(I*
(1+I*x)^2/(x^2+1)/((1+I*x)^2/(x^2+1)+1)^2)-1/8*I*Pi*csgn(I/((1+I*x)^2/(x^2+
1)+1)^2)*csgn(I*(1+I*x)^2/(x^2+1))*csgn(I*(1+I*x)^2/(x^2+1)/((1+I*x)^2/(x^2
+1)+1)^2)*x+1/6*Pi*csgn(I*(1+I*x)^2/(x^2+1)/((1+I*x)^2/(x^2+1)+1)^2)^3+1/6*
Pi*csgn(I*(1+I*x)^2/(x^2+1))^3-1/6*x^3*ln(2)+1/6*ln((1+I*x)^2/(x^2+1)+1)*x^
3+1/2*arctan(x)*ln((1+I*x)^2/(x^2+1)+1)-1/2*arctan(x)*ln(2)-1/2*ln((1+I*x)^
2/(x^2+1)+1)*x+1/8*I*arctan(x)*Pi*csgn(I*(1+I*x)^2/(x^2+1))*csgn(I*(1+I*x)/
(x^2+1)^(1/2))^2
```

Maxima [A] time = 1.52241, size = 84, normalized size = 0.95

$$\frac{7}{72}x^3+\frac{1}{8}\left(2x^4\log\left(x^2+1\right)-x^4+2x^2-2\log\left(x^2+1\right)\right)\arctan\left(x\right)-\frac{1}{12}\left(x^3-3x\right)\log\left(x^2+1\right)-\frac{25}{24}x+\frac{25}{24}\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(x)*log(x^2+1),x, algorithm="maxima")
```

```
[Out] 7/72*x^3 + 1/8*(2*x^4*log(x^2 + 1) - x^4 + 2*x^2 - 2*log(x^2 + 1))*arctan(x
) - 1/12*(x^3 - 3*x)*log(x^2 + 1) - 25/24*x + 25/24*arctan(x)
```

Fricas [A] time = 1.37501, size = 154, normalized size = 1.75

$$\frac{7}{72}x^3-\frac{1}{24}\left(3x^4-6x^2-25\right)\arctan\left(x\right)-\frac{1}{12}\left(x^3-3\left(x^4-1\right)\arctan\left(x\right)-3x\right)\log\left(x^2+1\right)-\frac{25}{24}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(x)*log(x^2+1),x, algorithm="fricas")
```

```
[Out] 7/72*x^3 - 1/24*(3*x^4 - 6*x^2 - 25)*arctan(x) - 1/12*(x^3 - 3*(x^4 - 1)*ar
ctan(x) - 3*x)*log(x^2 + 1) - 25/24*x
```

Sympy [A] time = 4.91633, size = 83, normalized size = 0.94

$$\frac{x^4\log\left(x^2+1\right)\operatorname{atan}(x)}{4}-\frac{x^4\operatorname{atan}(x)}{8}-\frac{x^3\log\left(x^2+1\right)}{12}+\frac{7x^3}{72}+\frac{x^2\operatorname{atan}(x)}{4}+\frac{x\log\left(x^2+1\right)}{4}-\frac{25x}{24}-\frac{\log\left(x^2+1\right)\operatorname{ata}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atan(x)*ln(x**2+1),x)
```

```
[Out] x**4*log(x**2 + 1)*atan(x)/4 - x**4*atan(x)/8 - x**3*log(x**2 + 1)/12 + 7*x
**3/72 + x**2*atan(x)/4 + x*log(x**2 + 1)/4 - 25*x/24 - log(x**2 + 1)*atan(
x)/4 + 25*atan(x)/24
```

Giac [A] time = 1.11212, size = 167, normalized size = 1.9

$$\frac{1}{8}\pi x^4\log\left(x^2+1\right)\operatorname{sgn}(x)-\frac{1}{4}x^4\arctan\left(\frac{1}{x}\right)\log\left(x^2+1\right)-\frac{1}{16}\pi x^4\operatorname{sgn}(x)+\frac{1}{8}x^4\arctan\left(\frac{1}{x}\right)-\frac{1}{12}x^3\log\left(x^2+1\right)+\frac{1}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(x)*log(x^2+1),x, algorithm="giac")
```

```
[Out] 1/8*pi*x^4*log(x^2 + 1)*sgn(x) - 1/4*x^4*arctan(1/x)*log(x^2 + 1) - 1/16*pi
*x^4*sgn(x) + 1/8*x^4*arctan(1/x) - 1/12*x^3*log(x^2 + 1) + 1/8*pi*x^2*sgn(
x) + 7/72*x^3 - 1/4*x^2*arctan(1/x) - 1/8*pi*log(x^2 + 1)*sgn(x) + 1/4*x*lo
g(x^2 + 1) + 1/4*arctan(1/x)*log(x^2 + 1) - 25/24*pi*sgn(x) - 25/24*x + 25/
24*arctan(x)
```

3.1277 $\int x^2 \tan^{-1}(x) \log\left(1+x^2\right) dx$

Optimal. Leaf size=82

$$\frac{5x^2}{18}+\frac{1}{12}\log^2\left(x^2+1\right)-\frac{1}{6}x^2\log\left(x^2+1\right)-\frac{11}{18}\log\left(x^2+1\right)-\frac{2}{9}x^3\tan^{-1}(x)+\frac{1}{3}x^3\log\left(x^2+1\right)\tan^{-1}(x)+\frac{2}{3}x\tan^{-1}(x)-$$

```
[Out] (5*x^2)/18 + (2*x*ArcTan[x])/3 - (2*x^3*ArcTan[x])/9 - ArcTan[x]^2/3 - (11*
Log[1 + x^2])/18 - (x^2*Log[1 + x^2])/6 + (x^3*ArcTan[x]*Log[1 + x^2])/3 +
Log[1 + x^2]^2/12
```

Rubi [A] time = 0.325158, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 12, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {4852, 266, 43, 5021, 6725, 4916, 4846, 260, 4884, 2475, 2390, 2301}

$$\frac{5x^2}{18}+\frac{1}{12}\log^2\left(x^2+1\right)-\frac{1}{6}x^2\log\left(x^2+1\right)-\frac{11}{18}\log\left(x^2+1\right)-\frac{2}{9}x^3\tan^{-1}(x)+\frac{1}{3}x^3\log\left(x^2+1\right)\tan^{-1}(x)+\frac{2}{3}x\tan^{-1}(x)-$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTan[x]*Log[1 + x^2],x]
```

```
[Out] (5*x^2)/18 + (2*x*ArcTan[x])/3 - (2*x^3*ArcTan[x])/9 - ArcTan[x]^2/3 - (11*
Log[1 + x^2])/18 - (x^2*Log[1 + x^2])/6 + (x^3*ArcTan[x]*Log[1 + x^2])/3 +
Log[1 + x^2]^2/12
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5021

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*(
e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(a + b*ArcTan[c*x]), x
]}, Dist[d + e*Log[f + g*x^2], u, x] - Dist[2*e*g, Int[ExpandIntegrand[(x*u
)/(f + g*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IntegerQ[m
] && NeQ[m, -1]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rubi steps

$$\int x^2\tan^{-1}(x)\log\left(1+x^2\right)dx = -\frac{1}{6}x^2\log\left(1+x^2\right)+\frac{1}{3}x^3\tan^{-1}(x)\log\left(1+x^2\right)+\frac{1}{6}\log^2\left(1+x^2\right)-2\int\left(\frac{x^3\left(-1+2x\tan\right.}{6\left(1+x^2\right.}\right.$$

$$= -\frac{1}{6}x^2\log\left(1+x^2\right)+\frac{1}{3}x^3\tan^{-1}(x)\log\left(1+x^2\right)+\frac{1}{6}\log^2\left(1+x^2\right)-\frac{1}{3}\int\frac{x^3\left(-1+2x\tan\right.}{1+x^2}$$

$$= -\frac{1}{6}x^2\log\left(1+x^2\right)+\frac{1}{3}x^3\tan^{-1}(x)\log\left(1+x^2\right)+\frac{1}{6}\log^2\left(1+x^2\right)-\frac{1}{6}\,\mathrm{Subst}\left(\int\frac{\log(1+}{1+x}\right.$$

$$= -\frac{1}{6}x^2\log\left(1+x^2\right)+\frac{1}{3}x^3\tan^{-1}(x)\log\left(1+x^2\right)+\frac{1}{6}\log^2\left(1+x^2\right)-\frac{1}{6}\,\mathrm{Subst}\left(\int\frac{\log(x)}{x}\,d\right.$$

$$= -\frac{1}{6}x^2\log\left(1+x^2\right)+\frac{1}{3}x^3\tan^{-1}(x)\log\left(1+x^2\right)+\frac{1}{12}\log^2\left(1+x^2\right)+\frac{1}{6}\,\mathrm{Subst}\left(\int\frac{x}{1+x}\,d\right.$$

$$= -\frac{2}{9}x^3\tan^{-1}(x)-\frac{1}{6}x^2\log\left(1+x^2\right)+\frac{1}{3}x^3\tan^{-1}(x)\log\left(1+x^2\right)+\frac{1}{12}\log^2\left(1+x^2\right)+\frac{1}{6}\,\mathrm{S}$$

$$= \frac{x^2}{6}+\frac{2}{3}x\tan^{-1}(x)-\frac{2}{9}x^3\tan^{-1}(x)-\frac{1}{3}\tan^{-1}(x)^2-\frac{1}{6}\log\left(1+x^2\right)-\frac{1}{6}x^2\log\left(1+x^2\right)+\frac{1}{3}$$

$$= \frac{x^2}{6}+\frac{2}{3}x\tan^{-1}(x)-\frac{2}{9}x^3\tan^{-1}(x)-\frac{1}{3}\tan^{-1}(x)^2-\frac{1}{2}\log\left(1+x^2\right)-\frac{1}{6}x^2\log\left(1+x^2\right)+\frac{1}{3}$$

$$= \frac{5x^2}{18}+\frac{2}{3}x\tan^{-1}(x)-\frac{2}{9}x^3\tan^{-1}(x)-\frac{1}{3}\tan^{-1}(x)^2-\frac{11}{18}\log\left(1+x^2\right)-\frac{1}{6}x^2\log\left(1+x^2\right)+$$

Mathematica [A] time = 0.0195742, size = 64, normalized size = 0.78

$$\frac{1}{36}\left(10x^2+3\log^2\left(x^2+1\right)-2\left(3x^2+11\right)\log\left(x^2+1\right)+4x\left(-2x^2+3x^2\log\left(x^2+1\right)+6\right)\tan^{-1}(x)-12\tan^{-1}(x)^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTan[x]*Log[1 + x^2],x]
```

```
[Out] (10*x^2 - 12*ArcTan[x]^2 - 2*(11 + 3*x^2)*Log[1 + x^2] + 3*Log[1 + x^2]^2 +
 4*x*ArcTan[x]*(6 - 2*x^2 + 3*x^2*Log[1 + x^2]))/36
```

Maple [C] time = 1.144, size = 3041, normalized size = 37.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(x)*ln(x^2+1),x)
```

```
[Out] -1/3*x^2*ln(2)+1/3*ln((1+I*x)^2/(x^2+1)+1)*x^2-1/3*ln(2)-1/12*I*Pi*csgn(I*(
(1+I*x)^2/(x^2+1)+1)^2)^3*x^2+1/12*I*Pi*csgn(I*(1+I*x)^2/(x^2+1)/((1+I*x)^2
/(x^2+1)+1)^2)^3*x^2+1/12*I*Pi*csgn(I*(1+I*x)^2/(x^2+1))^3*x^2-1/12*I*Pi*cs
gn(I*(1+I*x)^2/(x^2+1))*csgn(I*(1+I*x)^2/(x^2+1)/((1+I*x)^2/(x^2+1)+1)^2)^2
-1/6*I*Pi*csgn(I*(1+I*x)/(x^2+1)^(1/2))*csgn(I*(1+I*x)^2/(x^2+1))^2+1/12*I*
Pi*csgn(I*(1+I*x)/(x^2+1)^(1/2))^2*csgn(I*(1+I*x)^2/(x^2+1))-1/12*I*Pi*csgn
(I/((1+I*x)^2/(x^2+1)+1)^2)*csgn(I*(1+I*x)^2/(x^2+1)/((1+I*x)^2/(x^2+1)+1)^
2)^2+1/6*I*Pi*csgn(I*((1+I*x)^2/(x^2+1)+1))*csgn(I*((1+I*x)^2/(x^2+1)+1)^2)
^2-1/12*I*Pi*csgn(I*((1+I*x)^2/(x^2+1)+1))^2*csgn(I*((1+I*x)^2/(x^2+1)+1)^2
)-1/6*I*Pi*ln((1+I*x)^2/(x^2+1)+1)*csgn(I*((1+I*x)^2/(x^2+1)+1)^2)^3+5/18*x
^2+1/12*I*Pi*csgn(I*(1+I*x)^2/(x^2+1)/((1+I*x)^2/(x^2+1)+1)^2)^3+1/12*I*Pi*
csgn(I*(1+I*x)^2/(x^2+1))^3+2/3*ln(2)*arctan(x)*x^3-2/3*arctan(x)*ln((1+I*x
)^2/(x^2+1)+1)*x^3+5/18+2/3*x*arctan(x)-2/9*x^3*arctan(x)-1/6*I*Pi*csgn(I*(
1+I*x)^2/(x^2+1))^2*csgn(I*(1+I*x)/(x^2+1)^(1/2))*x^2+1/12*I*Pi*csgn(I*(1+I
*x)^2/(x^2+1))*csgn(I*(1+I*x)/(x^2+1)^(1/2))^2*x^2+1/12*I*Pi*csgn(I/((1+I*x
)^2/(x^2+1)+1)^2)*csgn(I*(1+I*x)^2/(x^2+1))*csgn(I*(1+I*x)^2/(x^2+1)/((1+I*
x)^2/(x^2+1)+1)^2)+1/6*I*Pi*ln((1+I*x)^2/(x^2+1)+1)*csgn(I*(1+I*x)/(x^2+1)^
(1/2))^2*csgn(I*(1+I*x)^2/(x^2+1))-1/3*I*Pi*ln((1+I*x)^2/(x^2+1)+1)*csgn(I*
(1+I*x)/(x^2+1)^(1/2))*csgn(I*(1+I*x)^2/(x^2+1))^2-1/6*I*Pi*ln((1+I*x)^2/(x
^2+1)+1)*csgn(I*((1+I*x)^2/(x^2+1)+1))^2*csgn(I*((1+I*x)^2/(x^2+1)+1)^2)+1/
3*I*Pi*ln((1+I*x)^2/(x^2+1)+1)*csgn(I*((1+I*x)^2/(x^2+1)+1))*csgn(I*((1+I*x
)^2/(x^2+1)+1)^2)^2-1/6*I*Pi*ln((1+I*x)^2/(x^2+1)+1)*csgn(I/((1+I*x)^2/(x^2
```

```
+1)+1)^2)*csgn(I*(1+I*x)^2/(x^2+1)/((1+I*x)^2/(x^2+1)+1)^2)^2-1/6*I*Pi*ln((
1+I*x)^2/(x^2+1)+1)*csgn(I*(1+I*x)^2/(x^2+1))*csgn(I*(1+I*x)^2/(x^2+1)/((1+
I*x)^2/(x^2+1)+1)^2)^2+1/6*I*arctan(x)*Pi*csgn(I*((1+I*x)^2/(x^2+1)+1)^2)^3
*x^3-1/6*I*arctan(x)*Pi*csgn(I*(1+I*x)^2/(x^2+1)/((1+I*x)^2/(x^2+1)+1)^2)^3
*x^3-1/6*I*arctan(x)*Pi*csgn(I*(1+I*x)^2/(x^2+1))^3*x^3+1/6*I*Pi*csgn(I*((1
+I*x)^2/(x^2+1)+1)^2)^2*csgn(I*((1+I*x)^2/(x^2+1)+1))*x^2-1/12*I*Pi*csgn(I*
((1+I*x)^2/(x^2+1)+1)^2)*csgn(I*((1+I*x)^2/(x^2+1)+1))^2*x^2-1/12*I*Pi*csgn
(I*(1+I*x)^2/(x^2+1)/((1+I*x)^2/(x^2+1)+1)^2)^2*csgn(I*(1+I*x)^2/(x^2+1))*x
^2+1/6*I*ln((1+I*x)^2/(x^2+1)+1)*Pi*csgn(I*(1+I*x)^2/(x^2+1))^3+1/6*I*ln((1
+I*x)^2/(x^2+1)+1)*Pi*csgn(I*(1+I*x)^2/(x^2+1)/((1+I*x)^2/(x^2+1)+1)^2)^3-1
/12*I*Pi*csgn(I*((1+I*x)^2/(x^2+1)+1)^2)^3+2/3*I*ln(2)*arctan(x)-8/9*I*arct
an(x)-1/6*I*arctan(x)*Pi*csgn(I*(1+I*x)^2/(x^2+1)/((1+I*x)^2/(x^2+1)+1)^2)*
csgn(I*(1+I*x)^2/(x^2+1))*csgn(I/((1+I*x)^2/(x^2+1)+1)^2)*x^3-1/3*arctan(x)
*Pi*csgn(I*(1+I*x)^2/(x^2+1))^2*csgn(I*(1+I*x)/(x^2+1)^(1/2))-1/6*arctan(x)
*Pi*csgn(I*(1+I*x)^2/(x^2+1))*csgn(I*(1+I*x)^2/(x^2+1)/((1+I*x)^2/(x^2+1)+1
)^2)^2+1/6*arctan(x)*Pi*csgn(I*(1+I*x)^2/(x^2+1))*csgn(I*(1+I*x)/(x^2+1)^(1
/2))^2+1/6*arctan(x)*Pi*csgn(I*(1+I*x)^2/(x^2+1))*csgn(I*(1+I*x)^2/(x^2+1)/
((1+I*x)^2/(x^2+1)+1)^2)*csgn(I/((1+I*x)^2/(x^2+1)+1)^2)+11/9*ln((1+I*x)^2/
(x^2+1)+1)+1/3*ln((1+I*x)^2/(x^2+1)+1)^2+1/6*arctan(x)*Pi*csgn(I*(1+I*x)^2/
(x^2+1))^3+1/6*arctan(x)*Pi*csgn(I*(1+I*x)^2/(x^2+1)/((1+I*x)^2/(x^2+1)+1)^
2)^3-1/6*arctan(x)*Pi*csgn(I*((1+I*x)^2/(x^2+1)+1)^2)^3-1/6*arctan(x)*Pi*cs
gn(I*(1+I*x)^2/(x^2+1)/((1+I*x)^2/(x^2+1)+1)^2)^2*csgn(I/((1+I*x)^2/(x^2+1)
+1)^2)-1/6*arctan(x)*Pi*csgn(I*((1+I*x)^2/(x^2+1)+1))^2*csgn(I*((1+I*x)^2/(
x^2+1)+1)^2)+1/3*arctan(x)*Pi*csgn(I*((1+I*x)^2/(x^2+1)+1))*csgn(I*((1+I*x)
^2/(x^2+1)+1)^2)^2+1/3*(2*x^3*arctan(x)+2*I*arctan(x)-x^2-2*ln((1+I*x)^2/(x
^2+1)+1)-1)*ln((1+I*x)/(x^2+1)^(1/2))-2/3*ln(2)*ln((1+I*x)^2/(x^2+1)+1)-1/1
2*I*Pi*csgn(I*(1+I*x)^2/(x^2+1)/((1+I*x)^2/(x^2+1)+1)^2)^2*csgn(I/((1+I*x)^
2/(x^2+1)+1)^2)*x^2-1/3*I*arctan(x)*Pi*csgn(I*((1+I*x)^2/(x^2+1)+1)^2)^2*cs
gn(I*((1+I*x)^2/(x^2+1)+1))*x^3+1/6*I*arctan(x)*Pi*csgn(I*((1+I*x)^2/(x^2+1
)+1)^2)*csgn(I*((1+I*x)^2/(x^2+1)+1))^2*x^3+1/6*I*arctan(x)*Pi*csgn(I*(1+I*
x)^2/(x^2+1)/((1+I*x)^2/(x^2+1)+1)^2)^2*csgn(I*(1+I*x)^2/(x^2+1))*x^3+1/6*I
*arctan(x)*Pi*csgn(I*(1+I*x)^2/(x^2+1)/((1+I*x)^2/(x^2+1)+1)^2)^2*csgn(I/((
1+I*x)^2/(x^2+1)+1)^2)*x^3+1/3*I*arctan(x)*Pi*csgn(I*(1+I*x)^2/(x^2+1))^2*c
sgn(I*(1+I*x)/(x^2+1)^(1/2))*x^3+1/6*I*Pi*ln((1+I*x)^2/(x^2+1)+1)*csgn(I/((
1+I*x)^2/(x^2+1)+1)^2)*csgn(I*(1+I*x)^2/(x^2+1))*csgn(I*(1+I*x)^2/(x^2+1)/(
(1+I*x)^2/(x^2+1)+1)^2)-1/6*I*arctan(x)*Pi*csgn(I*(1+I*x)^2/(x^2+1))*csgn(I
*(1+I*x)/(x^2+1)^(1/2))^2*x^3+1/12*I*Pi*csgn(I*(1+I*x)^2/(x^2+1)/((1+I*x)^2
/(x^2+1)+1)^2)*csgn(I*(1+I*x)^2/(x^2+1))*csgn(I/((1+I*x)^2/(x^2+1)+1)^2)*x^
2
```

Maxima [A] time = 1.48757, size = 88, normalized size = 1.07

$$\frac{5}{18}\,x^2+\frac{1}{9}\left(3\,x^3\log\left(x^2+1\right)-2\,x^3+6\,x-6\,\arctan\left(x\right)\right)\arctan\left(x\right)+\frac{1}{3}\,\arctan\left(x\right)^2-\frac{1}{18}\left(3\,x^2+11\right)\log\left(x^2+1\right)+\frac{1}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(x)*log(x^2+1),x, algorithm="maxima")
```

```
[Out] 5/18*x^2 + 1/9*(3*x^3*log(x^2 + 1) - 2*x^3 + 6*x - 6*arctan(x))*arctan(x) +
 1/3*arctan(x)^2 - 1/18*(3*x^2 + 11)*log(x^2 + 1) + 1/12*log(x^2 + 1)^2
```

Fricas [A] time = 1.31985, size = 174, normalized size = 2.12

$$\frac{5}{18}\,x^2-\frac{2}{9}\left(x^3-3\,x\right)\arctan\left(x\right)-\frac{1}{3}\,\arctan\left(x\right)^2+\frac{1}{18}\left(6\,x^3\arctan\left(x\right)-3\,x^2-11\right)\log\left(x^2+1\right)+\frac{1}{12}\,\log\left(x^2+1\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(x)*log(x^2+1),x, algorithm="fricas")
```

```
[Out] 5/18*x^2 - 2/9*(x^3 - 3*x)*arctan(x) - 1/3*arctan(x)^2 + 1/18*(6*x^3*arctan
(x) - 3*x^2 - 11)*log(x^2 + 1) + 1/12*log(x^2 + 1)^2
```

Sympy [A] time = 2.89118, size = 78, normalized size = 0.95

$$\frac{x^3\log\left(x^2+1\right)\operatorname{atan}\left(x\right)}{3}-\frac{2x^3\operatorname{atan}\left(x\right)}{9}-\frac{x^2\log\left(x^2+1\right)}{6}+\frac{5x^2}{18}+\frac{2x\operatorname{atan}\left(x\right)}{3}+\frac{\log\left(x^2+1\right)^2}{12}-\frac{11\log\left(x^2+1\right)}{18}-\operatorname{ata}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(x)*ln(x**2+1),x)
```

```
[Out] x**3*log(x**2 + 1)*atan(x)/3 - 2*x**3*atan(x)/9 - x**2*log(x**2 + 1)/6 + 5*
x**2/18 + 2*x*atan(x)/3 + log(x**2 + 1)**2/12 - 11*log(x**2 + 1)/18 - atan(
x)**2/3
```

Giac [B] time = 1.11747, size = 182, normalized size = 2.22

$$\frac{1}{6}\,\pi x^3 \log\left(x^2+1\right) \operatorname{sgn}\left(x\right) - \frac{1}{3}\,x^3 \arctan\left(\frac{1}{x}\right) \log\left(x^2+1\right) - \frac{1}{9}\,\pi x^3 \operatorname{sgn}\left(x\right) + \frac{2}{9}\,x^3 \arctan\left(\frac{1}{x}\right) - \frac{1}{6}\,x^2 \log\left(x^2+1\right) + \frac{1}{6}\,\pi^2 \mathrm{sg}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(x)*log(x^2+1),x, algorithm="giac")
```

```
[Out] 1/6*pi*x^3*log(x^2 + 1)*sgn(x) - 1/3*x^3*arctan(1/x)*log(x^2 + 1) - 1/9*pi*
x^3*sgn(x) + 2/9*x^3*arctan(1/x) - 1/6*x^2*log(x^2 + 1) + 1/6*pi^2*sgn(x) +
 1/3*pi*x*sgn(x) + 1/3*pi*arctan(1/x)*sgn(x) - 1/6*pi^2 + 5/18*x^2 - 1/3*pi
*arctan(x) - 1/3*pi*arctan(1/x) - 2/3*x*arctan(1/x) - 1/3*arctan(1/x)^2 + 1
/12*log(x^2 + 1)^2 - 11/18*log(x^2 + 1)
```

3.1278 $\int x \tan^{-1}(x) \log\left(1 + x^2\right) dx$

Optimal. Leaf size=49

$$-\frac{1}{2}x\log\left(x^2+1\right)-\frac{1}{2}x^2\tan^{-1}(x)+\frac{1}{2}\left(x^2+1\right)\log\left(x^2+1\right)\tan^{-1}(x)+\frac{3x}{2}-\frac{3}{2}\tan^{-1}(x)$$

```
[Out] (3*x)/2 - (3*ArcTan[x])/2 - (x^2*ArcTan[x])/2 - (x*Log[1 + x^2])/2 + ((1 + 
x^2)*ArcTan[x]*Log[1 + x^2])/2
```

Rubi [A] time = 0.0499575, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 8, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.8, Rules used = {4852, 321, 203, 2454, 2389, 2295, 5019, 2448}

$$-\frac{1}{2}x\log\left(x^2+1\right)-\frac{1}{2}x^2\tan^{-1}(x)+\frac{1}{2}\left(x^2+1\right)\log\left(x^2+1\right)\tan^{-1}(x)+\frac{3x}{2}-\frac{3}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTan[x]*Log[1 + x^2],x]
```

```
[Out] (3*x)/2 - (3*ArcTan[x])/2 - (x^2*ArcTan[x])/2 - (x*Log[1 + x^2])/2 + ((1 + 
x^2)*ArcTan[x]*Log[1 + x^2])/2
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 5019

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*(
e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*Log[f + g*x^2])
, x]}, Dist[a + b*ArcTan[c*x], u, x] - Dist[b*c, Int[ExpandIntegrand[u/(1 +
 c^2*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[(m + 1)/2
, 0]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rubi steps

$$\int x\tan^{-1}(x)\log\left(1+x^2\right)\,dx = -\frac{1}{2}x^2\tan^{-1}(x)+\frac{1}{2}\left(1+x^2\right)\tan^{-1}(x)\log\left(1+x^2\right)-\int\left(-\frac{x^2}{2\left(1+x^2\right)}+\frac{1}{2}\log\left(1+x^2\right)\right)d$$

$$= -\frac{1}{2}x^2\tan^{-1}(x)+\frac{1}{2}\left(1+x^2\right)\tan^{-1}(x)\log\left(1+x^2\right)+\frac{1}{2}\int\frac{x^2}{1+x^2}\,dx-\frac{1}{2}\int\log\left(1+x^2\right)$$

$$= \frac{x}{2}-\frac{1}{2}x^2\tan^{-1}(x)-\frac{1}{2}x\log\left(1+x^2\right)+\frac{1}{2}\left(1+x^2\right)\tan^{-1}(x)\log\left(1+x^2\right)-\frac{1}{2}\int\frac{1}{1+x^2}\,dx$$

$$= \frac{3x}{2}-\frac{1}{2}\tan^{-1}(x)-\frac{1}{2}x^2\tan^{-1}(x)-\frac{1}{2}x\log\left(1+x^2\right)+\frac{1}{2}\left(1+x^2\right)\tan^{-1}(x)\log\left(1+x^2\right)-$$

$$= \frac{3x}{2}-\frac{3}{2}\tan^{-1}(x)-\frac{1}{2}x^2\tan^{-1}(x)-\frac{1}{2}x\log\left(1+x^2\right)+\frac{1}{2}\left(1+x^2\right)\tan^{-1}(x)\log\left(1+x^2\right)$$

Mathematica [A] time = 0.0168273, size = 38, normalized size = 0.78

$$\frac{1}{2}\left(x^2\left(-\tan^{-1}(x)\right)+\log\left(x^2+1\right)\left(\left(x^2+1\right)\tan^{-1}(x)-x\right)+3x-3\tan^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTan[x]*Log[1 + x^2],x]
```

```
[Out] (3*x - 3*ArcTan[x] - x^2*ArcTan[x] + (-x + (1 + x^2)*ArcTan[x])*Log[1 + x^2
])/2
```

Maple [C] time = 0.581, size = 2240, normalized size = 45.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(x)*ln(x^2+1),x)
```

```
[Out] 3/2*x+1/4*I*csgn(I*(1+I*x)/(x^2+1)^(1/2))^2*csgn(I*(1+I*x)^2/(x^2+1))*Pi*x-
5/2*arctan(x)+(-I*arctan(x)+x*arctan(x)-1)*(x+I)*ln((1+I*x)/(x^2+1)^(1/2))-
1/4*I*arctan(x)*csgn(I*(1+I*x)/(x^2+1)^(1/2))^2*csgn(I*(1+I*x)^2/(x^2+1))*P
i+1/2*I*arctan(x)*csgn(I*(1+I*x)/(x^2+1)^(1/2))*csgn(I*(1+I*x)^2/(x^2+1))^2
*Pi-1/2*I*arctan(x)*csgn(I*((1+I*x)^2/(x^2+1)+1)^2)^2*csgn(I*((1+I*x)^2/(x^
2+1)+1))*Pi+1/4*I*arctan(x)*csgn(I*((1+I*x)^2/(x^2+1)+1)^2)*csgn(I*((1+I*x)
^2/(x^2+1)+1))^2*Pi-1/4*I*csgn(I*(1+I*x)^2/(x^2+1)/((1+I*x)^2/(x^2+1)+1)^2)
```

```
^2*csgn(I/((1+I*x)^2/(x^2+1)+1)^2)*Pi*x-1/4*I*csgn(I*(1+I*x)^2/(x^2+1)/((1+
I*x)^2/(x^2+1)+1)^2)^2*csgn(I*(1+I*x)^2/(x^2+1))*Pi*x+3/2*I-1/2*I*csgn(I*(1
+I*x)/(x^2+1)^(1/2))*csgn(I*(1+I*x)^2/(x^2+1))^2*Pi*x+1/2*I*csgn(I*((1+I*x)
^2/(x^2+1)+1)^2)^2*csgn(I*((1+I*x)^2/(x^2+1)+1))*Pi*x-1/4*I*csgn(I*((1+I*x)
^2/(x^2+1)+1)^2)*csgn(I*((1+I*x)^2/(x^2+1)+1))^2*Pi*x-1/4*I*arctan(x)*csgn(
I*(1+I*x)^2/(x^2+1)/((1+I*x)^2/(x^2+1)+1)^2)^3*Pi*x^2-1/4*I*arctan(x)*csgn(
I*(1+I*x)^2/(x^2+1))^3*Pi*x^2+1/4*I*arctan(x)*csgn(I*((1+I*x)^2/(x^2+1)+1)^
2)^3*Pi*x^2+1/4*I*arctan(x)*csgn(I*(1+I*x)^2/(x^2+1)/((1+I*x)^2/(x^2+1)+1)^
2)^2*csgn(I/((1+I*x)^2/(x^2+1)+1)^2)*Pi+1/4*I*arctan(x)*csgn(I*(1+I*x)^2/(x
^2+1)/((1+I*x)^2/(x^2+1)+1)^2)^2*csgn(I*(1+I*x)^2/(x^2+1))*Pi-ln(2)*x-1/2*x
^2*arctan(x)-1/4*I*arctan(x)*csgn(I*(1+I*x)^2/(x^2+1)/((1+I*x)^2/(x^2+1)+1)
^2)*csgn(I/((1+I*x)^2/(x^2+1)+1)^2)*csgn(I*(1+I*x)^2/(x^2+1))*Pi*x^2-arctan
(x)*ln((1+I*x)^2/(x^2+1)+1)*x^2+arctan(x)*ln(2)*x^2-I*ln(2)-1/4*I*arctan(x)
*csgn(I*(1+I*x)^2/(x^2+1)/((1+I*x)^2/(x^2+1)+1)^2)^3*Pi-1/4*I*arctan(x)*csg
n(I*(1+I*x)^2/(x^2+1))^3*Pi+1/4*I*arctan(x)*csgn(I*((1+I*x)^2/(x^2+1)+1)^2)
^3*Pi+1/4*I*csgn(I*(1+I*x)^2/(x^2+1)/((1+I*x)^2/(x^2+1)+1)^2)^3*Pi*x+1/4*I*
csgn(I*(1+I*x)^2/(x^2+1))^3*Pi*x-1/4*I*csgn(I*((1+I*x)^2/(x^2+1)+1)^2)^3*Pi
*x-1/4*Pi*csgn(I*(1+I*x)/(x^2+1)^(1/2))^2*csgn(I*(1+I*x)^2/(x^2+1))+1/2*Pi*
csgn(I*(1+I*x)/(x^2+1)^(1/2))*csgn(I*(1+I*x)^2/(x^2+1))^2+1/4*Pi*csgn(I/((1
+I*x)^2/(x^2+1)+1)^2)*csgn(I*(1+I*x)^2/(x^2+1)/((1+I*x)^2/(x^2+1)+1)^2)^2+1
/4*Pi*csgn(I*(1+I*x)^2/(x^2+1))*csgn(I*(1+I*x)^2/(x^2+1)/((1+I*x)^2/(x^2+1)
+1)^2)^2+1/4*Pi*csgn(I*((1+I*x)^2/(x^2+1)+1))^2*csgn(I*((1+I*x)^2/(x^2+1)+1
)^2)-1/2*Pi*csgn(I*((1+I*x)^2/(x^2+1)+1))*csgn(I*((1+I*x)^2/(x^2+1)+1)^2)^2
+1/4*I*arctan(x)*csgn(I*(1+I*x)^2/(x^2+1)/((1+I*x)^2/(x^2+1)+1)^2)^2*csgn(I
/((1+I*x)^2/(x^2+1)+1)^2)*Pi*x^2+1/4*I*arctan(x)*csgn(I*(1+I*x)^2/(x^2+1)/(
(1+I*x)^2/(x^2+1)+1)^2)^2*csgn(I*(1+I*x)^2/(x^2+1))*Pi*x^2-1/4*I*arctan(x)*
csgn(I*(1+I*x)/(x^2+1)^(1/2))^2*csgn(I*(1+I*x)^2/(x^2+1))*Pi*x^2+1/2*I*arct
an(x)*csgn(I*(1+I*x)/(x^2+1)^(1/2))*csgn(I*(1+I*x)^2/(x^2+1))^2*Pi*x^2-1/2*
I*arctan(x)*csgn(I*((1+I*x)^2/(x^2+1)+1)^2)^2*csgn(I*((1+I*x)^2/(x^2+1)+1))
*Pi*x^2+1/4*I*arctan(x)*csgn(I*((1+I*x)^2/(x^2+1)+1)^2)*csgn(I*((1+I*x)^2/(
x^2+1)+1))^2*Pi*x^2-1/4*I*arctan(x)*csgn(I*(1+I*x)^2/(x^2+1)/((1+I*x)^2/(x^
2+1)+1)^2)*csgn(I/((1+I*x)^2/(x^2+1)+1)^2)*csgn(I*(1+I*x)^2/(x^2+1))*Pi+1/4
*I*csgn(I*(1+I*x)^2/(x^2+1)/((1+I*x)^2/(x^2+1)+1)^2)*csgn(I/((1+I*x)^2/(x^2
+1)+1)^2)*csgn(I*(1+I*x)^2/(x^2+1))*Pi*x+1/4*Pi*csgn(I*((1+I*x)^2/(x^2+1)+1
)^2)^3-1/4*Pi*csgn(I/((1+I*x)^2/(x^2+1)+1)^2)*csgn(I*(1+I*x)^2/(x^2+1))*csg
n(I*(1+I*x)^2/(x^2+1)/((1+I*x)^2/(x^2+1)+1)^2)-1/4*Pi*csgn(I*(1+I*x)^2/(x^2
+1)/((1+I*x)^2/(x^2+1)+1)^2)^3-1/4*Pi*csgn(I*(1+I*x)^2/(x^2+1))^3-arctan(x)
*ln((1+I*x)^2/(x^2+1)+1)+arctan(x)*ln(2)+ln((1+I*x)^2/(x^2+1)+1)*x
```

Maxima [A] time = 1.89245, size = 53, normalized size = 1.08

$$-\frac{1}{2}\left(x^2-\left(x^2+1\right)\log\left(x^2+1\right)+1\right)\arctan\left(x\right)-\frac{1}{2}\,x\log\left(x^2+1\right)+\frac{3}{2}\,x-\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(x)*log(x^2+1),x, algorithm="maxima")
```

```
[Out] -1/2*(x^2 - (x^2 + 1)*log(x^2 + 1) + 1)*arctan(x) - 1/2*x*log(x^2 + 1) + 3/
2*x - arctan(x)
```

Fricas [A] time = 1.40267, size = 107, normalized size = 2.18

$$-\frac{1}{2}\left(x^2+3\right)\arctan\left(x\right)+\frac{1}{2}\left(\left(x^2+1\right)\arctan\left(x\right)-x\right)\log\left(x^2+1\right)+\frac{3}{2}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(x)*log(x^2+1),x, algorithm="fricas")
```

```
[Out] -1/2*(x^2 + 3)*arctan(x) + 1/2*((x^2 + 1)*arctan(x) - x)*log(x^2 + 1) + 3/2
*x
```

Sympy [A] time = 1.589, size = 56, normalized size = 1.14

$$\frac{x^2\log\left(x^2+1\right)\operatorname{atan}\left(x\right)}{2}-\frac{x^2\operatorname{atan}\left(x\right)}{2}-\frac{x\log\left(x^2+1\right)}{2}+\frac{3x}{2}+\frac{\log\left(x^2+1\right)\operatorname{atan}\left(x\right)}{2}-\frac{3\operatorname{atan}\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(x)*ln(x**2+1),x)
```

```
[Out] x**2*log(x**2 + 1)*atan(x)/2 - x**2*atan(x)/2 - x*log(x**2 + 1)/2 + 3*x/2 +
 log(x**2 + 1)*atan(x)/2 - 3*atan(x)/2
```

Giac [B] time = 1.10753, size = 116, normalized size = 2.37

$$\frac{1}{4}\,\pi x^2\log\left(x^2+1\right)\operatorname{sgn}\left(x\right)-\frac{1}{2}\,x^2\arctan\left(\frac{1}{x}\right)\log\left(x^2+1\right)-\frac{1}{4}\,\pi x^2\operatorname{sgn}\left(x\right)+\frac{1}{2}\,x^2\arctan\left(\frac{1}{x}\right)+\frac{1}{4}\,\pi\log\left(x^2+1\right)\operatorname{sgn}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(x)*log(x^2+1),x, algorithm="giac")
```

```
[Out] 1/4*pi*x^2*log(x^2 + 1)*sgn(x) - 1/2*x^2*arctan(1/x)*log(x^2 + 1) - 1/4*pi*
x^2*sgn(x) + 1/2*x^2*arctan(1/x) + 1/4*pi*log(x^2 + 1)*sgn(x) - 1/2*x*log(x
^2 + 1) - 1/2*arctan(1/x)*log(x^2 + 1) + 3/2*x - 3/2*arctan(x)
```

3.1279 $\int \tan^{-1}(x) \log\left(1+x^2\right) dx$

Optimal. Leaf size=38

$$-\frac{1}{4}\log^2\left(x^2+1\right)+\log\left(x^2+1\right)+x\log\left(x^2+1\right)\tan^{-1}(x)+\tan^{-1}(x)^2-2x\tan^{-1}(x)$$

```
[Out] -2*x*ArcTan[x] + ArcTan[x]^2 + Log[1 + x^2] + x*ArcTan[x]*Log[1 + x^2] - Lo
g[1 + x^2]^2/4
```

Rubi [A] time = 0.106084, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.889, Rules used = {4846, 260, 5009, 2475, 2390, 2301, 4916, 4884}

$$-\frac{1}{4}\log^2\left(x^2+1\right)+\log\left(x^2+1\right)+x\log\left(x^2+1\right)\tan^{-1}(x)+\tan^{-1}(x)^2-2x\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[x]*Log[1 + x^2],x]
```

```
[Out] -2*x*ArcTan[x] + ArcTan[x]^2 + Log[1 + x^2] + x*ArcTan[x]*Log[1 + x^2] - Lo
g[1 + x^2]^2/4
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5009

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*(
e_.)), x_Symbol] :> Simp[x*(d + e*Log[f + g*x^2])*(a + b*ArcTan[c*x]), x] +
 (-Dist[b*c, Int[(x*(d + e*Log[f + g*x^2]))/(1 + c^2*x^2), x], x] - Dist[2*
e*g, Int[(x^2*(a + b*ArcTan[c*x]))/(f + g*x^2), x], x]) /; FreeQ[{a, b, c,
d, e, f, g}, x]
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rubi steps

$$\int \tan^{-1}(x)\log\left(1+x^2\right)dx = x\tan^{-1}(x)\log\left(1+x^2\right) - 2\int \frac{x^2\tan^{-1}(x)}{1+x^2}dx - \int \frac{x\log\left(1+x^2\right)}{1+x^2}dx$$
$$= x\tan^{-1}(x)\log\left(1+x^2\right) - \frac{1}{2}\text{Subst}\left(\int \frac{\log(1+x)}{1+x}dx, x, x^2\right) - 2\int \tan^{-1}(x)\,dx + 2\int \frac{\tan}{1\ }$$
$$= -2x\tan^{-1}(x) + \tan^{-1}(x)^2 + x\tan^{-1}(x)\log\left(1+x^2\right) - \frac{1}{2}\text{Subst}\left(\int \frac{\log(x)}{x}dx, x, 1+x^2\right) +$$
$$= -2x\tan^{-1}(x) + \tan^{-1}(x)^2 + \log\left(1+x^2\right) + x\tan^{-1}(x)\log\left(1+x^2\right) - \frac{1}{4}\log^2\left(1+x^2\right)$$

Mathematica [A] time = 0.0083078, size = 38, normalized size = 1.

$$-\frac{1}{4}\log^2\left(x^2+1\right) + \log\left(x^2+1\right) + x\log\left(x^2+1\right)\tan^{-1}(x) + \tan^{-1}(x)^2 - 2x\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[x]*Log[1 + x^2],x]
```

```
[Out] -2*x*ArcTan[x] + ArcTan[x]^2 + Log[1 + x^2] + x*ArcTan[x]*Log[1 + x^2] - Lo
g[1 + x^2]^2/4
```

Maple [C] time = 0.582, size = 1913, normalized size = 50.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x)*ln(x^2+1),x)
```

```
[Out] -2*I*ln(2)*arctan(x)-2*arctan(x)*ln((1+I*x)^2/(x^2+1)+1)*x+2*arctan(x)*ln(2
)*x-2*x*arctan(x)-1/2*I*arctan(x)*Pi*csgn(I*(1+I*x)^2/(x^2+1))*csgn(I*(1+I*
x)^2/(x^2+1)/((1+I*x)^2/(x^2+1)+1)^2)*csgn(I/((1+I*x)^2/(x^2+1)+1)^2)*x-1/2
*I*ln((1+I*x)^2/(x^2+1)+1)*Pi*csgn(I*(1+I*x)^2/(x^2+1))^3-1/2*I*ln((1+I*x)^
2/(x^2+1)+1)*Pi*csgn(I*(1+I*x)^2/(x^2+1)/((1+I*x)^2/(x^2+1)+1)^2)^3+1/2*I*l
n((1+I*x)^2/(x^2+1)+1)*Pi*csgn(I*((1+I*x)^2/(x^2+1)+1)^2)^3+1/2*I*arctan(x)
*Pi*csgn(I*((1+I*x)^2/(x^2+1)+1)^2)^3*x-1/2*I*arctan(x)*Pi*csgn(I*(1+I*x)^2
/(x^2+1))^3*x-1/2*I*arctan(x)*Pi*csgn(I*(1+I*x)^2/(x^2+1)/((1+I*x)^2/(x^2+1
)+1)^2)^3*x+1/2*I*ln((1+I*x)^2/(x^2+1)+1)*Pi*csgn(I*(1+I*x)^2/(x^2+1))*csgn
```

```
(I*(1+I*x)^2/(x^2+1)/((1+I*x)^2/(x^2+1)+1)^2)^2+1/2*I*Pi*ln((1+I*x)^2/(x^2+
1)+1)*csgn(I*((1+I*x)^2/(x^2+1)+1))^2*csgn(I*((1+I*x)^2/(x^2+1)+1)^2)+1/2*I
*Pi*ln((1+I*x)^2/(x^2+1)+1)*csgn(I/((1+I*x)^2/(x^2+1)+1)^2)*csgn(I*(1+I*x)^
2/(x^2+1)/((1+I*x)^2/(x^2+1)+1)^2)^2-I*ln((1+I*x)^2/(x^2+1)+1)*Pi*csgn(I*((
1+I*x)^2/(x^2+1)+1))*csgn(I*((1+I*x)^2/(x^2+1)+1)^2)^2-1/2*I*Pi*ln((1+I*x)^
2/(x^2+1)+1)*csgn(I*(1+I*x)/(x^2+1)^(1/2))^2*csgn(I*(1+I*x)^2/(x^2+1))+I*Pi
*ln((1+I*x)^2/(x^2+1)+1)*csgn(I*(1+I*x)/(x^2+1)^(1/2))*csgn(I*(1+I*x)^2/(x^
2+1))^2-1/2*I*ln((1+I*x)^2/(x^2+1)+1)*Pi*csgn(I/((1+I*x)^2/(x^2+1)+1)^2)*cs
gn(I*(1+I*x)^2/(x^2+1))*csgn(I*(1+I*x)^2/(x^2+1)/((1+I*x)^2/(x^2+1)+1)^2)+1
/2*I*arctan(x)*Pi*csgn(I*(1+I*x)^2/(x^2+1))*csgn(I*(1+I*x)^2/(x^2+1)/((1+I*
x)^2/(x^2+1)+1)^2)^2*x-1/2*I*arctan(x)*Pi*csgn(I*(1+I*x)^2/(x^2+1))*csgn(I*
(1+I*x)/(x^2+1)^(1/2))^2*x+2*(-I*arctan(x)+x*arctan(x)+ln((1+I*x)^2/(x^2+1)
+1))*ln((1+I*x)/(x^2+1)^(1/2))+1/2*I*arctan(x)*Pi*csgn(I*(1+I*x)^2/(x^2+1)/
((1+I*x)^2/(x^2+1)+1)^2)^2*csgn(I/((1+I*x)^2/(x^2+1)+1)^2)*x-I*arctan(x)*Pi
*csgn(I*((1+I*x)^2/(x^2+1)+1)^2)^2*csgn(I*((1+I*x)^2/(x^2+1)+1))*x+1/2*I*ar
ctan(x)*Pi*csgn(I*((1+I*x)^2/(x^2+1)+1)^2)*csgn(I*((1+I*x)^2/(x^2+1)+1))^2*
x+I*arctan(x)*Pi*csgn(I*(1+I*x)^2/(x^2+1))^2*csgn(I*(1+I*x)/(x^2+1)^(1/2))*
x+arctan(x)*Pi*csgn(I*(1+I*x)^2/(x^2+1))^2*csgn(I*(1+I*x)/(x^2+1)^(1/2))+1/
2*arctan(x)*Pi*csgn(I*(1+I*x)^2/(x^2+1))*csgn(I*(1+I*x)^2/(x^2+1)/((1+I*x)^
2/(x^2+1)+1)^2)^2-1/2*arctan(x)*Pi*csgn(I*(1+I*x)^2/(x^2+1))*csgn(I*(1+I*x)
/(x^2+1)^(1/2))^2-1/2*arctan(x)*Pi*csgn(I*(1+I*x)^2/(x^2+1))*csgn(I*(1+I*x)
^2/(x^2+1)/((1+I*x)^2/(x^2+1)+1)^2)*csgn(I/((1+I*x)^2/(x^2+1)+1)^2)-2*ln((1
+I*x)^2/(x^2+1)+1)-ln((1+I*x)^2/(x^2+1)+1)^2-1/2*arctan(x)*Pi*csgn(I*(1+I*x
)^2/(x^2+1))^3-1/2*arctan(x)*Pi*csgn(I*(1+I*x)^2/(x^2+1)/((1+I*x)^2/(x^2+1)
+1)^2)^3+1/2*arctan(x)*Pi*csgn(I*((1+I*x)^2/(x^2+1)+1)^2)^3+1/2*arctan(x)*P
i*csgn(I*(1+I*x)^2/(x^2+1)/((1+I*x)^2/(x^2+1)+1)^2)^2*csgn(I/((1+I*x)^2/(x^
2+1)+1)^2)+1/2*arctan(x)*Pi*csgn(I*((1+I*x)^2/(x^2+1)+1))^2*csgn(I*((1+I*x)
^2/(x^2+1)+1)^2)-arctan(x)*Pi*csgn(I*((1+I*x)^2/(x^2+1)+1))*csgn(I*((1+I*x)
^2/(x^2+1)+1)^2)^2+2*I*arctan(x)+2*ln(2)*ln((1+I*x)^2/(x^2+1)+1)
```

Maxima [A] time = 1.44473, size = 57, normalized size = 1.5

$$\left(x\log\left(x^2+1\right)-2\,x+2\,\arctan\left(x\right)\right)\arctan\left(x\right)-\arctan\left(x\right)^2-\frac{1}{4}\,\log\left(x^2+1\right)^2+\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)*log(x^2+1),x, algorithm="maxima")
```

```
[Out] (x*log(x^2 + 1) - 2*x + 2*arctan(x))*arctan(x) - arctan(x)^2 - 1/4*log(x^2
+ 1)^2 + log(x^2 + 1)
```

Fricas [A] time = 1.43342, size = 113, normalized size = 2.97

$$-2\,x\arctan\left(x\right)+\arctan\left(x\right)^{2}+\left(x\arctan\left(x\right)+1\right)\log\left(x^{2}+1\right)-\frac{1}{4}\,\log\left(x^{2}+1\right)^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)*log(x^2+1),x, algorithm="fricas")
```

```
[Out] -2*x*arctan(x) + arctan(x)^2 + (x*arctan(x) + 1)*log(x^2 + 1) - 1/4*log(x^2
 + 1)^2
```

Sympy [A] time = 0.94249, size = 39, normalized size = 1.03

$$x\log\left(x^{2}+1\right)\operatorname{atan}\left(x\right)-2x\operatorname{atan}\left(x\right)-\frac{\log\left(x^{2}+1\right)^{2}}{4}+\log\left(x^{2}+1\right)+\operatorname{atan}^{2}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x)*ln(x**2+1),x)
```

```
[Out] x*log(x**2 + 1)*atan(x) - 2*x*atan(x) - log(x**2 + 1)**2/4 + log(x**2 + 1)
+ atan(x)**2
```

Giac [B] time = 1.09061, size = 124, normalized size = 3.26

$$\frac{1}{2}\,\pi x\log\left(x^{2}+1\right)\operatorname{sgn}\left(x\right)-x\arctan\left(\frac{1}{x}\right)\log\left(x^{2}+1\right)-\frac{3}{2}\,\pi^{2}\operatorname{sgn}\left(x\right)-\pi x\operatorname{sgn}\left(x\right)-\pi\arctan\left(\frac{1}{x}\right)\operatorname{sgn}\left(x\right)+\frac{1}{2}\,\pi^{2}+\pi\,\mathrm{ar}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)*log(x^2+1),x, algorithm="giac")
```

```
[Out] 1/2*pi*x*log(x^2 + 1)*sgn(x) - x*arctan(1/x)*log(x^2 + 1) - 3/2*pi^2*sgn(x)
 - pi*x*sgn(x) - pi*arctan(1/x)*sgn(x) + 1/2*pi^2 + pi*arctan(x) + pi*arcta
n(1/x) + 2*x*arctan(1/x) + arctan(1/x)^2 - 1/4*log(x^2 + 1)^2 + log(x^2 + 1
)
```

3.1280 $\int \frac{\tan^{-1}(x)\log(1+x^2)}{x}\,dx$

Optimal. Leaf size=189

$$-\frac{1}{2}i\left(-\log\left(x^2+1\right)+\log(1-ix)+\log(1+ix)\right)\text{PolyLog}(2,-ix)+\frac{1}{2}i\left(-\log\left(x^2+1\right)+\log(1-ix)+\log(1+ix)\right)\text{PolyLo}$$

```
[Out] (-I/2)*Log[1 + I*x]^2*Log[(-I)*x] + (I/2)*Log[1 - I*x]^2*Log[I*x] + I*Log[1
 - I*x]*PolyLog[2, 1 - I*x] - I*Log[1 + I*x]*PolyLog[2, 1 + I*x] - (I/2)*(L
og[1 - I*x] + Log[1 + I*x] - Log[1 + x^2])*PolyLog[2, (-I)*x] + (I/2)*(Log[
1 - I*x] + Log[1 + I*x] - Log[1 + x^2])*PolyLog[2, I*x] - I*PolyLog[3, 1 -
I*x] + I*PolyLog[3, 1 + I*x]
```

Rubi [A] time = 0.18104, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {4848, 2391, 5011, 2396, 2433, 2374, 6589}

$$-\frac{1}{2}i\left(-\log\left(x^2+1\right)+\log(1-ix)+\log(1+ix)\right)\text{PolyLog}(2,-ix)+\frac{1}{2}i\left(-\log\left(x^2+1\right)+\log(1-ix)+\log(1+ix)\right)\text{PolyLo}$$

Antiderivative was successfully verified.

```
[In] Int[(ArcTan[x]*Log[1 + x^2])/x,x]
```


```
[Out] (-I/2)*Log[1 + I*x]^2*Log[(-I)*x] + (I/2)*Log[1 - I*x]^2*Log[I*x] + I*Log[1
 - I*x]*PolyLog[2, 1 - I*x] - I*Log[1 + I*x]*PolyLog[2, 1 + I*x] - (I/2)*(L
og[1 - I*x] + Log[1 + I*x] - Log[1 + x^2])*PolyLog[2, (-I)*x] + (I/2)*(Log[
1 - I*x] + Log[1 + I*x] - Log[1 + x^2])*PolyLog[2, I*x] - I*PolyLog[3, 1 -
I*x] + I*PolyLog[3, 1 + I*x]
```


Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5011

```
Int[(ArcTan[(c_.)*(x_)]*Log[(f_.) + (g_.)*(x_)^2])/(x_), x_Symbol] :> Dist[
Log[f + g*x^2] - Log[1 - I*c*x] - Log[1 + I*c*x], Int[ArcTan[c*x]/x, x], x]
 + (Dist[I/2, Int[Log[1 - I*c*x]^2/x, x], x] - Dist[I/2, Int[Log[1 + I*c*x]
^2/x, x], x]) /; FreeQ[{c, f, g}, x] && EqQ[g, c^2*f]
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\tan^{-1}(x)\log\left(1+x^2\right)}{x}\,dx = \frac{1}{2}i\int \frac{\log^2(1-ix)}{x}\,dx - \frac{1}{2}i\int \frac{\log^2(1+ix)}{x}\,dx + \left(-\log(1-ix)-\log(1+ix)+\log\left(1+x^2\right)\right)$$
$$= -\frac{1}{2}i\log^2(1+ix)\log(-ix) + \frac{1}{2}i\log^2(1-ix)\log(ix) + \frac{1}{2}\left(i\left(\log(1-ix)+\log(1+ix)-\log\left(1\right.\right.\right.$$
$$= -\frac{1}{2}i\log^2(1+ix)\log(-ix) + \frac{1}{2}i\log^2(1-ix)\log(ix) - \frac{1}{2}i\left(\log(1-ix)+\log(1+ix)-\log\left(1-\right.\right.$$
$$= -\frac{1}{2}i\log^2(1+ix)\log(-ix) + \frac{1}{2}i\log^2(1-ix)\log(ix) + i\log(1-ix)\mathrm{Li}_2(1-ix) - i\log(1+ix)\mathrm{I}$$
$$= -\frac{1}{2}i\log^2(1+ix)\log(-ix) + \frac{1}{2}i\log^2(1-ix)\log(ix) + i\log(1-ix)\mathrm{Li}_2(1-ix) - i\log(1+ix)\mathrm{I}$$

Mathematica [F] time = 2.64211, size = 0, normalized size = 0.

$$\int \frac{\tan^{-1}(x)\log\left(1+x^2\right)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(ArcTan[x]*Log[1 + x^2])/x,x]
```

```
[Out] Integrate[(ArcTan[x]*Log[1 + x^2])/x, x]
```

Maple [C] time = 0.974, size = 5237, normalized size = 27.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x)*ln(x^2+1)/x,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(x)\log\left(x^2+1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)*log(x^2+1)/x,x, algorithm="maxima")
```

```
[Out] integrate(arctan(x)*log(x^2 + 1)/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\arctan\left(x\right)\log\left(x^{2}+1\right)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)*log(x^2+1)/x,x, algorithm="fricas")
```

```
[Out] integral(arctan(x)*log(x^2 + 1)/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(x^{2}+1\right)\operatorname{atan}\left(x\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x)*ln(x**2+1)/x,x)
```

```
[Out] Integral(log(x**2 + 1)*atan(x)/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan\left(x\right)\log\left(x^{2}+1\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)*log(x^2+1)/x,x, algorithm="giac")
```

```
[Out] integrate(arctan(x)*log(x^2 + 1)/x, x)
```

3.1281 $\int \frac{\tan^{-1}(x)\log(1+x^2)}{x^2}\,dx$

Optimal. Leaf size=41

$$-\frac{1}{2}\mathrm{PolyLog}\left(2,-x^2\right)-\frac{1}{4}\log^2\left(x^2+1\right)-\frac{\log\left(x^2+1\right)\tan^{-1}(x)}{x}+\tan^{-1}(x)^2$$

```
[Out] ArcTan[x]^2 - (ArcTan[x]*Log[1 + x^2])/x - Log[1 + x^2]^2/4 - PolyLog[2, -x
^2]/2
```

Rubi [A] time = 0.125174, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 12, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {4852, 266, 36, 29, 31, 5017, 2475, 2410, 2390, 2301, 2391, 4884}

$$-\frac{1}{2}\mathrm{PolyLog}\left(2,-x^2\right)-\frac{1}{4}\log^2\left(x^2+1\right)-\frac{\log\left(x^2+1\right)\tan^{-1}(x)}{x}+\tan^{-1}(x)^2$$

Antiderivative was successfully verified.

```
[In] Int[(ArcTan[x]*Log[1 + x^2])/x^2,x]
```

```
[Out] ArcTan[x]^2 - (ArcTan[x]*Log[1 + x^2])/x - Log[1 + x^2]^2/4 - PolyLog[2, -x
^2]/2
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
```

```
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 5017

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*(
e_.))*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*(d + e*Log[f + g*x^2])*(a +
b*ArcTan[c*x]))/(m + 1), x] + (-Dist[(b*c)/(m + 1), Int[(x^(m + 1)*(d + e*L
og[f + g*x^2]))/(1 + c^2*x^2), x], x] - Dist[(2*e*g)/(m + 1), Int[(x^(m + 2
)*(a + b*ArcTan[c*x]))/(f + g*x^2), x], x]) /; FreeQ[{a, b, c, d, e, f, g},
 x] && ILtQ[m/2, 0]
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 2410

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symb
ol] :> Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ
[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^{-1}(x)\log\left(1+x^2\right)}{x^2}\,dx &= -\frac{\tan^{-1}(x)\log\left(1+x^2\right)}{x}+2\int \frac{\tan^{-1}(x)}{1+x^2}\,dx+\int \frac{\log\left(1+x^2\right)}{x\left(1+x^2\right)}\,dx\\
&= \tan^{-1}(x)^2-\frac{\tan^{-1}(x)\log\left(1+x^2\right)}{x}+\frac{1}{2}\,\text{Subst}\left(\int \frac{\log(1+x)}{x(1+x)}\,dx,x,x^2\right)\\
&= \tan^{-1}(x)^2-\frac{\tan^{-1}(x)\log\left(1+x^2\right)}{x}+\frac{1}{2}\,\text{Subst}\left(\int \left(\frac{\log(1+x)}{-1-x}+\frac{\log(1+x)}{x}\right)dx,x,x^2\right)\\
&= \tan^{-1}(x)^2-\frac{\tan^{-1}(x)\log\left(1+x^2\right)}{x}+\frac{1}{2}\,\text{Subst}\left(\int \frac{\log(1+x)}{-1-x}\,dx,x,x^2\right)+\frac{1}{2}\,\text{Subst}\left(\int \frac{\log
\right.\\
&= \tan^{-1}(x)^2-\frac{\tan^{-1}(x)\log\left(1+x^2\right)}{x}-\frac{\text{Li}_2\left(-x^2\right)}{2}-\frac{1}{2}\,\text{Subst}\left(\int \frac{\log(x)}{x}\,dx,x,1+x^2\right)\\
&= \tan^{-1}(x)^2-\frac{\tan^{-1}(x)\log\left(1+x^2\right)}{x}-\frac{1}{4}\log^2\left(1+x^2\right)-\frac{\text{Li}_2\left(-x^2\right)}{2}
\end{aligned}$$

Mathematica [A] time = 0.0097258, size = 41, normalized size = 1.

$$-\frac{1}{2}\text{PolyLog}\left(2,-x^2\right)-\frac{1}{4}\log^2\left(x^2+1\right)-\frac{\log\left(x^2+1\right)\tan^{-1}(x)}{x}+\tan^{-1}(x)^2$$

Antiderivative was successfully verified.

```
[In] Integrate[(ArcTan[x]*Log[1 + x^2])/x^2,x]
```

```
[Out] ArcTan[x]^2 - (ArcTan[x]*Log[1 + x^2])/x - Log[1 + x^2]^2/4 - PolyLog[2, -x
^2]/2
```

Maple [F] time = 1.354, size = 0, normalized size = 0.

$$\int \frac{\arctan(x)\ln\left(x^2+1\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x)*ln(x^2+1)/x^2,x)
```

```
[Out] int(arctan(x)*ln(x^2+1)/x^2,x)
```

Maxima [A] time = 1.47988, size = 78, normalized size = 1.9

$$-\left(\frac{\log\left(x^2+1\right)}{x}-2\arctan(x)\right)\arctan(x)-\arctan(x)^2+\frac{1}{2}\log\left(-x^2\right)\log\left(x^2+1\right)-\frac{1}{4}\log\left(x^2+1\right)^2+\frac{1}{2}\,\mathrm{Li}_2\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)*log(x^2+1)/x^2,x, algorithm="maxima")
```

```
[Out] -(log(x^2 + 1)/x - 2*arctan(x))*arctan(x) - arctan(x)^2 + 1/2*log(-x^2)*log
(x^2 + 1) - 1/4*log(x^2 + 1)^2 + 1/2*dilog(x^2 + 1)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\arctan(x)\log\left(x^2+1\right)}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)*log(x^2+1)/x^2,x, algorithm="fricas")
```

```
[Out] integral(arctan(x)*log(x^2 + 1)/x^2, x)
```

Sympy [C] time = 87.8367, size = 37, normalized size = 0.9

$$-\frac{\log\left(x^2+1\right)^2}{4}+\operatorname{atan}^2(x)-\frac{\operatorname{Li}_2\left(x^2e^{i\pi}\right)}{2}-\frac{\log\left(x^2+1\right)\operatorname{atan}(x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x)*ln(x**2+1)/x**2,x)
```

```
[Out] -log(x**2 + 1)**2/4 + atan(x)**2 - polylog(2, x**2*exp_polar(I*pi))/2 - log
(x**2 + 1)*atan(x)/x
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(x)\log\left(x^2+1\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)*log(x^2+1)/x^2,x, algorithm="giac")
```

```
[Out] integrate(arctan(x)*log(x^2 + 1)/x^2, x)
```

3.1282 $\int \frac{\tan^{-1}(x)\log(1+x^2)}{x^3}\,dx$

Optimal. Leaf size=69

$$\frac{1}{2}i\text{PolyLog}(2,-ix)-\frac{1}{2}i\text{PolyLog}(2,ix)-\frac{\log\left(x^2+1\right)}{2x}-\frac{\log\left(x^2+1\right)\tan^{-1}(x)}{2x^2}-\frac{1}{2}\log\left(x^2+1\right)\tan^{-1}(x)+\tan^{-1}(x)$$

```
[Out] ArcTan[x] - Log[1 + x^2]/(2*x) - (ArcTan[x]*Log[1 + x^2])/2 - (ArcTan[x]*Lo
g[1 + x^2])/(2*x^2) + (I/2)*PolyLog[2, (-I)*x] - (I/2)*PolyLog[2, I*x]
```

Rubi [A] time = 0.0752657, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4852, 325, 203, 5021, 4848, 2391}

$$\frac{1}{2}i\text{PolyLog}(2,-ix)-\frac{1}{2}i\text{PolyLog}(2,ix)-\frac{\log\left(x^2+1\right)}{2x}-\frac{\log\left(x^2+1\right)\tan^{-1}(x)}{2x^2}-\frac{1}{2}\log\left(x^2+1\right)\tan^{-1}(x)+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(ArcTan[x]*Log[1 + x^2])/x^3,x]
```

```
[Out] ArcTan[x] - Log[1 + x^2]/(2*x) - (ArcTan[x]*Log[1 + x^2])/2 - (ArcTan[x]*Lo
g[1 + x^2])/(2*x^2) + (I/2)*PolyLog[2, (-I)*x] - (I/2)*PolyLog[2, I*x]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 5021

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*(
e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(a + b*ArcTan[c*x]), x
]}, Dist[d + e*Log[f + g*x^2], u, x] - Dist[2*e*g, Int[ExpandIntegrand[(x*u
)/(f + g*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IntegerQ[m
] && NeQ[m, -1]
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\tan^{-1}(x)\log\left(1+x^2\right)}{x^3}\,dx = -\frac{\log\left(1+x^2\right)}{2x} - \frac{1}{2}\tan^{-1}(x)\log\left(1+x^2\right) - \frac{\tan^{-1}(x)\log\left(1+x^2\right)}{2x^2} - 2\int\left(-\frac{1}{2\left(1+x^2\right)} - \frac{\text{ta}}{}\right.$$

$$= -\frac{\log\left(1+x^2\right)}{2x} - \frac{1}{2}\tan^{-1}(x)\log\left(1+x^2\right) - \frac{\tan^{-1}(x)\log\left(1+x^2\right)}{2x^2} + \int\frac{1}{1+x^2}\,dx + \int\frac{\tan}{}$$

$$= \tan^{-1}(x) - \frac{\log\left(1+x^2\right)}{2x} - \frac{1}{2}\tan^{-1}(x)\log\left(1+x^2\right) - \frac{\tan^{-1}(x)\log\left(1+x^2\right)}{2x^2} + \frac{1}{2}i\int\frac{\log(1}{x}$$

$$= \tan^{-1}(x) - \frac{\log\left(1+x^2\right)}{2x} - \frac{1}{2}\tan^{-1}(x)\log\left(1+x^2\right) - \frac{\tan^{-1}(x)\log\left(1+x^2\right)}{2x^2} + \frac{1}{2}i\text{Li}_2(-ix)$$

Mathematica [A] time = 0.0286804, size = 49, normalized size = 0.71

$$\frac{1}{2}i(\text{PolyLog}(2,-ix)-\text{PolyLog}(2,ix)) - \frac{\log\left(x^2+1\right)\left(x^2\tan^{-1}(x)+x+\tan^{-1}(x)\right)}{2x^2} + \tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(ArcTan[x]*Log[1 + x^2])/x^3,x]
```

```
[Out] ArcTan[x] - ((x + ArcTan[x] + x^2*ArcTan[x])*Log[1 + x^2])/(2*x^2) + (I/2)*
(PolyLog[2, (-I)*x] - PolyLog[2, I*x])
```

Maple [F] time = 2.244, size = 0, normalized size = 0.

$$\int \frac{\arctan(x)\ln\left(x^2+1\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x)*ln(x^2+1)/x^3,x)
```

```
[Out] int(arctan(x)*ln(x^2+1)/x^3,x)
```

Maxima [A] time = 1.63935, size = 95, normalized size = 1.38

$$\frac{4\,x^2\arctan(x)\log(x)+4\,x^2\arctan(x)-2i\,x^2\mathrm{Li}_2\,(i\,x+1)+2i\,x^2\mathrm{Li}_2\,(-i\,x+1)-\left(\pi x^2+2\left(x^2+1\right)\arctan(x)+2\,x\right)\log}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)*log(x^2+1)/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*(4*x^2*arctan(x)*log(x) + 4*x^2*arctan(x) - 2*I*x^2*dilog(I*x + 1) + 2*
I*x^2*dilog(-I*x + 1) - (pi*x^2 + 2*(x^2 + 1)*arctan(x) + 2*x)*log(x^2 + 1)
)/x^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\arctan(x)\log\left(x^2+1\right)}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)*log(x^2+1)/x^3,x, algorithm="fricas")
```

```
[Out] integral(arctan(x)*log(x^2 + 1)/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(x^2+1\right)\operatorname{atan}(x)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x)*ln(x**2+1)/x**3,x)
```

```
[Out] Integral(log(x**2 + 1)*atan(x)/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(x)\log\left(x^2+1\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)*log(x^2+1)/x^3,x, algorithm="giac")
```

```
[Out] integrate(arctan(x)*log(x^2 + 1)/x^3, x)
```

3.1283 $\int \frac{\tan^{-1}(x)\log(1+x^2)}{x^4}\,dx$

Optimal. Leaf size=81

$$\frac{1}{6}\text{PolyLog}\left(2,-x^2\right)+\frac{1}{12}\log^2\left(x^2+1\right)-\frac{\log\left(x^2+1\right)}{6x^2}-\frac{1}{2}\log\left(x^2+1\right)-\frac{\log\left(x^2+1\right)\tan^{-1}(x)}{3x^3}+\log(x)-\frac{1}{3}\tan^{-1}(x)^2-$$

```
[Out] (-2*ArcTan[x])/(3*x) - ArcTan[x]^2/3 + Log[x] - Log[1 + x^2]/2 - Log[1 + x^
2]/(6*x^2) - (ArcTan[x]*Log[1 + x^2])/(3*x^3) + Log[1 + x^2]^2/12 + PolyLog
[2, -x^2]/6
```

Rubi [A] time = 0.209636, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 15, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.25, Rules used = {4852, 266, 44, 5017, 2475, 2410, 2395, 36, 29, 31, 2391, 2390, 2301, 4918, 4884}

$$\frac{1}{6}\text{PolyLog}\left(2,-x^2\right)+\frac{1}{12}\log^2\left(x^2+1\right)-\frac{\log\left(x^2+1\right)}{6x^2}-\frac{1}{2}\log\left(x^2+1\right)-\frac{\log\left(x^2+1\right)\tan^{-1}(x)}{3x^3}+\log(x)-\frac{1}{3}\tan^{-1}(x)^2-$$

Antiderivative was successfully verified.

```
[In] Int[(ArcTan[x]*Log[1 + x^2])/x^4,x]
```


```
[Out] (-2*ArcTan[x])/(3*x) - ArcTan[x]^2/3 + Log[x] - Log[1 + x^2]/2 - Log[1 + x^
2]/(6*x^2) - (ArcTan[x]*Log[1 + x^2])/(3*x^3) + Log[1 + x^2]^2/12 + PolyLog
[2, -x^2]/6
```


Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 5017

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*(
e_.))*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*(d + e*Log[f + g*x^2])*(a +
b*ArcTan[c*x]))/(m + 1), x] + (-Dist[(b*c)/(m + 1), Int[(x^(m + 1)*(d + e*L
og[f + g*x^2]))/(1 + c^2*x^2), x], x] - Dist[(2*e*g)/(m + 1), Int[(x^(m + 2
)*(a + b*ArcTan[c*x]))/(f + g*x^2), x], x]) /; FreeQ[{a, b, c, d, e, f, g},
 x] && ILtQ[m/2, 0]
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 2410

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symb
ol] :> Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ
[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 4918

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/(d + e*x^2),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\tan^{-1}(x)\log\left(1+x^2\right)}{x^4}\,dx = -\frac{\tan^{-1}(x)\log\left(1+x^2\right)}{3x^3} + \frac{1}{3}\int \frac{\log\left(1+x^2\right)}{x^3\left(1+x^2\right)}\,dx + \frac{2}{3}\int \frac{\tan^{-1}(x)}{x^2\left(1+x^2\right)}\,dx$$

$$= -\frac{\tan^{-1}(x)\log\left(1+x^2\right)}{3x^3} + \frac{1}{6}\,\mathrm{Subst}\left(\int \frac{\log(1+x)}{x^2(1+x)}\,dx, x, x^2\right) + \frac{2}{3}\int \frac{\tan^{-1}(x)}{x^2}\,dx - \frac{2}{3}\int \frac{\mathrm{ta}}{}$$

$$= -\frac{2\tan^{-1}(x)}{3x} - \frac{1}{3}\tan^{-1}(x)^2 - \frac{\tan^{-1}(x)\log\left(1+x^2\right)}{3x^3} + \frac{1}{6}\,\mathrm{Subst}\left(\int\left(\frac{\log(1+x)}{x^2} - \frac{\log(1+}{x}\right.\right.$$

$$= -\frac{2\tan^{-1}(x)}{3x} - \frac{1}{3}\tan^{-1}(x)^2 - \frac{\tan^{-1}(x)\log\left(1+x^2\right)}{3x^3} + \frac{1}{6}\,\mathrm{Subst}\left(\int \frac{\log(1+x)}{x^2}\,dx, x, x^2\right) -$$

$$= -\frac{2\tan^{-1}(x)}{3x} - \frac{1}{3}\tan^{-1}(x)^2 - \frac{\log\left(1+x^2\right)}{6x^2} - \frac{\tan^{-1}(x)\log\left(1+x^2\right)}{3x^3} + \frac{\mathrm{Li}_2\left(-x^2\right)}{6} + \frac{1}{6}\,\mathrm{Sub}$$

$$= -\frac{2\tan^{-1}(x)}{3x} - \frac{1}{3}\tan^{-1}(x)^2 + \frac{2\log(x)}{3} - \frac{1}{3}\log\left(1+x^2\right) - \frac{\log\left(1+x^2\right)}{6x^2} - \frac{\tan^{-1}(x)\log\left(1\right.}{3x^3}$$

$$= -\frac{2\tan^{-1}(x)}{3x} - \frac{1}{3}\tan^{-1}(x)^2 + \log(x) - \frac{1}{2}\log\left(1+x^2\right) - \frac{\log\left(1+x^2\right)}{6x^2} - \frac{\tan^{-1}(x)\log\left(1+\right.}{3x^3}$$

Mathematica [A] time = 0.0127778, size = 81, normalized size = 1.

$$\frac{1}{6}\mathrm{PolyLog}\left(2,-x^2\right) + \frac{1}{12}\log^2\left(x^2+1\right) - \frac{\log\left(x^2+1\right)}{6x^2} - \frac{1}{2}\log\left(x^2+1\right) - \frac{\log\left(x^2+1\right)\tan^{-1}(x)}{3x^3} + \log(x) - \frac{1}{3}\tan^{-1}(x)^2 -$$

Antiderivative was successfully verified.

```
[In] Integrate[(ArcTan[x]*Log[1 + x^2])/x^4,x]
```

```
[Out] (-2*ArcTan[x])/(3*x) - ArcTan[x]^2/3 + Log[x] - Log[1 + x^2]/2 - Log[1 + x^
2]/(6*x^2) - (ArcTan[x]*Log[1 + x^2])/(3*x^3) + Log[1 + x^2]^2/12 + PolyLog
[2, -x^2]/6
```

Maple [F] time = 1.478, size = 0, normalized size = 0.

$$\int \frac{\arctan(x)\ln\left(x^2+1\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x)*ln(x^2+1)/x^4,x)
```

```
[Out] int(arctan(x)*ln(x^2+1)/x^4,x)
```

Maxima [A] time = 1.64977, size = 128, normalized size = 1.58

$$-\frac{1}{3}\left(\frac{2}{x}+\frac{\log\left(x^2+1\right)}{x^3}+2\arctan\left(x\right)\right)\arctan\left(x\right)+\frac{4x^2\arctan\left(x\right)^2+x^2\log\left(x^2+1\right)^2-2x^2\mathrm{Li}_2\left(x^2+1\right)+12x^2\log\left(x\right)\cdots}{12x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)*log(x^2+1)/x^4,x, algorithm="maxima")
```

```
[Out] -1/3*(2/x + log(x^2 + 1)/x^3 + 2*arctan(x))*arctan(x) + 1/12*(4*x^2*arctan(
x)^2 + x^2*log(x^2 + 1)^2 - 2*x^2*dilog(x^2 + 1) + 12*x^2*log(x) - 2*(x^2*l
og(-x^2) + 3*x^2 + 1)*log(x^2 + 1))/x^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\arctan\left(x\right)\log\left(x^2+1\right)}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)*log(x^2+1)/x^4,x, algorithm="fricas")
```

```
[Out] integral(arctan(x)*log(x^2 + 1)/x^4, x)
```

Sympy [C] time = 35.9001, size = 97, normalized size = 1.2

$$\frac{2\log\left(x\right)}{3}+\frac{\log\left(2x^2\right)}{6}+\frac{\log\left(x^2+1\right)^2}{12}-\frac{\log\left(x^2+1\right)}{3}-\frac{\log\left(2x^2+2\right)}{6}-\frac{\operatorname{atan}^2\left(x\right)}{3}+\frac{\mathrm{Li}_2\left(x^2e^{i\pi}\right)}{6}-\frac{2\operatorname{atan}\left(x\right)}{3x}-\frac{\log\left(x^2+\cdots\right.}{6x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x)*ln(x**2+1)/x**4,x)
```

```
[Out] 2*log(x)/3 + log(2*x**2)/6 + log(x**2 + 1)**2/12 - log(x**2 + 1)/3 - log(2*
x**2 + 2)/6 - atan(x)**2/3 + polylog(2, x**2*exp_polar(I*pi))/6 - 2*atan(x)
/(3*x) - log(x**2 + 1)/(6*x**2) - log(x**2 + 1)*atan(x)/(3*x**3)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(x)\log\left(x^2+1\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)*log(x^2+1)/x^4,x, algorithm="giac")
```

```
[Out] integrate(arctan(x)*log(x^2 + 1)/x^4, x)
```

3.1284 $\int \frac{\tan^{-1}(x)\log(1+x^2)}{x^5}\,dx$

Optimal. Leaf size=102

$$-\frac{1}{4}i\text{PolyLog}(2,-ix)+\frac{1}{4}i\text{PolyLog}(2,ix)+\frac{\log\left(x^2+1\right)}{4x}-\frac{\log\left(x^2+1\right)}{12x^3}-\frac{\tan^{-1}(x)}{4x^2}-\frac{\log\left(x^2+1\right)\tan^{-1}(x)}{4x^4}+\frac{1}{4}\log\left(x^2\right.$$

```
[Out] -5/(12*x) - (11*ArcTan[x])/12 - ArcTan[x]/(4*x^2) - Log[1 + x^2]/(12*x^3) +
 Log[1 + x^2]/(4*x) + (ArcTan[x]*Log[1 + x^2])/4 - (ArcTan[x]*Log[1 + x^2])
/(4*x^4) - (I/4)*PolyLog[2, (-I)*x] + (I/4)*PolyLog[2, I*x]
```

Rubi [A] time = 0.129238, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {4852, 325, 203, 5021, 453, 4980, 4848, 2391}

$$-\frac{1}{4}i\text{PolyLog}(2,-ix)+\frac{1}{4}i\text{PolyLog}(2,ix)+\frac{\log\left(x^2+1\right)}{4x}-\frac{\log\left(x^2+1\right)}{12x^3}-\frac{\tan^{-1}(x)}{4x^2}-\frac{\log\left(x^2+1\right)\tan^{-1}(x)}{4x^4}+\frac{1}{4}\log\left(x^2\right.$$

Antiderivative was successfully verified.

```
[In] Int[(ArcTan[x]*Log[1 + x^2])/x^5,x]
```


```
[Out] -5/(12*x) - (11*ArcTan[x])/12 - ArcTan[x]/(4*x^2) - Log[1 + x^2]/(12*x^3) +
 Log[1 + x^2]/(4*x) + (ArcTan[x]*Log[1 + x^2])/4 - (ArcTan[x]*Log[1 + x^2])
/(4*x^4) - (I/4)*PolyLog[2, (-I)*x] + (I/4)*PolyLog[2, I*x]
```


Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 5021

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*(
e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(a + b*ArcTan[c*x]), x
]}, Dist[d + e*Log[f + g*x^2], u, x] - Dist[2*e*g, Int[ExpandIntegrand[(x*u
)/(f + g*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IntegerQ[m
] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 4980

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\tan^{-1}(x)\log\left(1+x^2\right)}{x^5}\,dx = -\frac{\log\left(1+x^2\right)}{12x^3}+\frac{\log\left(1+x^2\right)}{4x}+\frac{1}{4}\tan^{-1}(x)\log\left(1+x^2\right)-\frac{\tan^{-1}(x)\log\left(1+x^2\right)}{4x^4}-2\int\Bigg(\frac{}{12}$$

$$=-\frac{\log\left(1+x^2\right)}{12x^3}+\frac{\log\left(1+x^2\right)}{4x}+\frac{1}{4}\tan^{-1}(x)\log\left(1+x^2\right)-\frac{\tan^{-1}(x)\log\left(1+x^2\right)}{4x^4}-\frac{1}{6}\int\frac{-}{x^2}$$

$$=-\frac{1}{6x}-\frac{\log\left(1+x^2\right)}{12x^3}+\frac{\log\left(1+x^2\right)}{4x}+\frac{1}{4}\tan^{-1}(x)\log\left(1+x^2\right)-\frac{\tan^{-1}(x)\log\left(1+x^2\right)}{4x^4}-\frac{1}{2}$$

$$=-\frac{1}{6x}-\frac{2}{3}\tan^{-1}(x)-\frac{\log\left(1+x^2\right)}{12x^3}+\frac{\log\left(1+x^2\right)}{4x}+\frac{1}{4}\tan^{-1}(x)\log\left(1+x^2\right)-\frac{\tan^{-1}(x)\log}{4x^4}$$

$$=-\frac{1}{6x}-\frac{2}{3}\tan^{-1}(x)-\frac{\tan^{-1}(x)}{4x^2}-\frac{\log\left(1+x^2\right)}{12x^3}+\frac{\log\left(1+x^2\right)}{4x}+\frac{1}{4}\tan^{-1}(x)\log\left(1+x^2\right)-$$

$$=-\frac{5}{12x}-\frac{2}{3}\tan^{-1}(x)-\frac{\tan^{-1}(x)}{4x^2}-\frac{\log\left(1+x^2\right)}{12x^3}+\frac{\log\left(1+x^2\right)}{4x}+\frac{1}{4}\tan^{-1}(x)\log\left(1+x^2\right)-$$

$$=-\frac{5}{12x}-\frac{11}{12}\tan^{-1}(x)-\frac{\tan^{-1}(x)}{4x^2}-\frac{\log\left(1+x^2\right)}{12x^3}+\frac{\log\left(1+x^2\right)}{4x}+\frac{1}{4}\tan^{-1}(x)\log\left(1+x^2\right)-$$

Mathematica [A] time = 0.0350324, size = 98, normalized size = 0.96

$$-\frac{1}{4}i(\mathrm{PolyLog}(2,-ix)-\mathrm{PolyLog}(2,ix))+\frac{1}{2}\left(\frac{1}{2}\left(-\frac{1}{x}-\tan^{-1}(x)\right)-\frac{\tan^{-1}(x)}{2x^2}\right)+\frac{\log\left(x^2+1\right)\left(3x^3+3x^4\tan^{-1}(x)-x-3\right.}{12x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(ArcTan[x]*Log[1 + x^2])/x^5,x]
```

```
[Out] -1/(6*x) - (2*ArcTan[x])/3 + ((-x^(-1) - ArcTan[x])/2 - ArcTan[x]/(2*x^2))/
2 + ((-x + 3*x^3 - 3*ArcTan[x] + 3*x^4*ArcTan[x])*Log[1 + x^2])/(12*x^4) -
(I/4)*(PolyLog[2, (-I)*x] - PolyLog[2, I*x])
```

Maple [F] time = 3.61, size = 0, normalized size = 0.

$$\int \frac{\arctan(x)\ln\left(x^2+1\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x)*ln(x^2+1)/x^5,x)
```

```
[Out] int(arctan(x)*ln(x^2+1)/x^5,x)
```

Maxima [A] time = 1.68755, size = 120, normalized size = 1.18

$$-\frac{12\,x^4\arctan\left(x\right)\log\left(x\right)-6i\,x^4\mathrm{Li}_2\left(i\,x+1\right)+6i\,x^4\mathrm{Li}_2\left(-i\,x+1\right)+10\,x^3+2\left(11\,x^4+3\,x^2\right)\arctan\left(x\right)-\left(3\,\pi x^4+6\,x^3+\right.}{24\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)*log(x^2+1)/x^5,x, algorithm="maxima")
```

```
[Out] -1/24*(12*x^4*arctan(x)*log(x) - 6*I*x^4*dilog(I*x + 1) + 6*I*x^4*dilog(-I*
x + 1) + 10*x^3 + 2*(11*x^4 + 3*x^2)*arctan(x) - (3*pi*x^4 + 6*x^3 + 6*(x^4
 - 1)*arctan(x) - 2*x)*log(x^2 + 1))/x^4
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\arctan\left(x\right)\log\left(x^2+1\right)}{x^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)*log(x^2+1)/x^5,x, algorithm="fricas")
```

```
[Out] integral(arctan(x)*log(x^2 + 1)/x^5, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(x^2+1\right)\operatorname{atan}\left(x\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x)*ln(x**2+1)/x**5,x)
```

```
[Out] Integral(log(x**2 + 1)*atan(x)/x**5, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(x)\log\left(x^2+1\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)*log(x^2+1)/x^5,x, algorithm="giac")
```

```
[Out] integrate(arctan(x)*log(x^2 + 1)/x^5, x)
```

3.1285 $\int \frac{\tan^{-1}(x)\log(1+x^2)}{x^6}\,dx$

Optimal. Leaf size=114

$$-\frac{1}{10}\text{PolyLog}\left(2,-x^2\right)-\frac{7}{60x^2}-\frac{1}{20}\log^2\left(x^2+1\right)+\frac{\log\left(x^2+1\right)}{10x^2}-\frac{\log\left(x^2+1\right)}{20x^4}+\frac{5}{12}\log\left(x^2+1\right)-\frac{2\tan^{-1}(x)}{15x^3}-\frac{\log\left(x\right.}{}$$

```
[Out] -7/(60*x^2) - (2*ArcTan[x])/(15*x^3) + (2*ArcTan[x])/(5*x) + ArcTan[x]^2/5
- (5*Log[x])/6 + (5*Log[1 + x^2])/12 - Log[1 + x^2]/(20*x^4) + Log[1 + x^2]
/(10*x^2) - (ArcTan[x]*Log[1 + x^2])/(5*x^5) - Log[1 + x^2]^2/20 - PolyLog[
2, -x^2]/10
```

Rubi [A] time = 0.279356, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 26, number of rules used = 15, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.25, Rules used = {4852, 266, 44, 5017, 2475, 2410, 2390, 2301, 2395, 36, 29, 31, 2391, 4918, 4884}

$$-\frac{1}{10}\text{PolyLog}\left(2,-x^2\right)-\frac{7}{60x^2}-\frac{1}{20}\log^2\left(x^2+1\right)+\frac{\log\left(x^2+1\right)}{10x^2}-\frac{\log\left(x^2+1\right)}{20x^4}+\frac{5}{12}\log\left(x^2+1\right)-\frac{2\tan^{-1}(x)}{15x^3}-\frac{\log\left(x\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(ArcTan[x]*Log[1 + x^2])/x^6,x]
```



```
[Out] -7/(60*x^2) - (2*ArcTan[x])/(15*x^3) + (2*ArcTan[x])/(5*x) + ArcTan[x]^2/5
- (5*Log[x])/6 + (5*Log[1 + x^2])/12 - Log[1 + x^2]/(20*x^4) + Log[1 + x^2]
/(10*x^2) - (ArcTan[x]*Log[1 + x^2])/(5*x^5) - Log[1 + x^2]^2/20 - PolyLog[
2, -x^2]/10
```


Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 5017

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*(
e_.))*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*(d + e*Log[f + g*x^2])*(a +
b*ArcTan[c*x]))/(m + 1), x] + (-Dist[(b*c)/(m + 1), Int[(x^(m + 1)*(d + e*L
og[f + g*x^2]))/(1 + c^2*x^2), x], x] - Dist[(2*e*g)/(m + 1), Int[(x^(m + 2
)*(a + b*ArcTan[c*x]))/(f + g*x^2), x], x]) /; FreeQ[{a, b, c, d, e, f, g},
 x] && ILtQ[m/2, 0]
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 2410

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symb
ol] :> Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ
[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4918

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/(d + e*x^2),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\tan^{-1}(x)\log\left(1+x^2\right)}{x^6}\,dx = -\frac{\tan^{-1}(x)\log\left(1+x^2\right)}{5x^5}+\frac{1}{5}\int \frac{\log\left(1+x^2\right)}{x^5\left(1+x^2\right)}\,dx+\frac{2}{5}\int \frac{\tan^{-1}(x)}{x^4\left(1+x^2\right)}\,dx$$

$$= -\frac{\tan^{-1}(x)\log\left(1+x^2\right)}{5x^5}+\frac{1}{10}\text{Subst}\left(\int \frac{\log(1+x)}{x^3(1+x)}\,dx,x,x^2\right)+\frac{2}{5}\int \frac{\tan^{-1}(x)}{x^4}\,dx-\frac{2}{5}\int \frac{\text{t}}{x^2}$$

$$= -\frac{2\tan^{-1}(x)}{15x^3}-\frac{\tan^{-1}(x)\log\left(1+x^2\right)}{5x^5}+\frac{1}{10}\text{Subst}\left(\int \left(\frac{\log(1+x)}{-1-x}+\frac{\log(1+x)}{x^3}-\frac{\log(1+}{x^2}\right.\right.$$

$$= -\frac{2\tan^{-1}(x)}{15x^3}+\frac{2\tan^{-1}(x)}{5x}+\frac{1}{5}\tan^{-1}(x)^2-\frac{\tan^{-1}(x)\log\left(1+x^2\right)}{5x^5}+\frac{1}{15}\text{Subst}\left(\int \frac{1}{x^2(1+}\right.$$

$$= -\frac{2\tan^{-1}(x)}{15x^3}+\frac{2\tan^{-1}(x)}{5x}+\frac{1}{5}\tan^{-1}(x)^2-\frac{\log\left(1+x^2\right)}{20x^4}+\frac{\log\left(1+x^2\right)}{10x^2}-\frac{\tan^{-1}(x)\log(1}{5x^5}$$

$$= -\frac{1}{15x^2}-\frac{2\tan^{-1}(x)}{15x^3}+\frac{2\tan^{-1}(x)}{5x}+\frac{1}{5}\tan^{-1}(x)^2-\frac{2\log(x)}{15}+\frac{1}{15}\log\left(1+x^2\right)-\frac{\log(1+}{20x^4}$$

$$= -\frac{7}{60x^2}-\frac{2\tan^{-1}(x)}{15x^3}+\frac{2\tan^{-1}(x)}{5x}+\frac{1}{5}\tan^{-1}(x)^2-\frac{5\log(x)}{6}+\frac{5}{12}\log\left(1+x^2\right)-\frac{\log(1+}{20x^4}$$

Mathematica [A] time = 0.023778, size = 114, normalized size = 1.

$$-\frac{1}{10}\text{PolyLog}\left(2,-x^2\right)-\frac{7}{60x^2}-\frac{1}{20}\log^2\left(x^2+1\right)+\frac{\log\left(x^2+1\right)}{10x^2}-\frac{\log\left(x^2+1\right)}{20x^4}+\frac{5}{12}\log\left(x^2+1\right)-\frac{2\tan^{-1}(x)}{15x^3}-\frac{\log(x^2}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(ArcTan[x]*Log[1 + x^2])/x^6,x]
```

```
[Out] -7/(60*x^2) - (2*ArcTan[x])/(15*x^3) + (2*ArcTan[x])/(5*x) + ArcTan[x]^2/5
- (5*Log[x])/6 + (5*Log[1 + x^2])/12 - Log[1 + x^2]/(20*x^4) + Log[1 + x^2]
/(10*x^2) - (ArcTan[x]*Log[1 + x^2])/(5*x^5) - Log[1 + x^2]^2/20 - PolyLog[
2, -x^2]/10
```

Maple [F] time = 1.354, size = 0, normalized size = 0.

$$\int \frac{\arctan(x)\ln\left(x^2+1\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x)*ln(x^2+1)/x^6,x)
```

```
[Out] int(arctan(x)*ln(x^2+1)/x^6,x)
```

Maxima [A] time = 1.49956, size = 155, normalized size = 1.36

$$\frac{1}{15}\left(\frac{2\left(3x^2-1\right)}{x^3}-\frac{3\log\left(x^2+1\right)}{x^5}+6\arctan(x)\right)\arctan(x)-\frac{12x^4\arctan(x)^2+3x^4\log\left(x^2+1\right)^2-6x^4\mathrm{Li}_2\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)*log(x^2+1)/x^6,x, algorithm="maxima")
```

```
[Out] 1/15*(2*(3*x^2 - 1)/x^3 - 3*log(x^2 + 1)/x^5 + 6*arctan(x))*arctan(x) - 1/6
0*(12*x^4*arctan(x)^2 + 3*x^4*log(x^2 + 1)^2 - 6*x^4*dilog(x^2 + 1) + 50*x^
4*log(x) + 7*x^2 - (6*x^4*log(-x^2) + 25*x^4 + 6*x^2 - 3)*log(x^2 + 1))/x^4
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\arctan(x)\log\left(x^2+1\right)}{x^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)*log(x^2+1)/x^6,x, algorithm="fricas")
```

```
[Out] integral(arctan(x)*log(x^2 + 1)/x^6, x)
```

Sympy [C] time = 85.7021, size = 134, normalized size = 1.18

$$-\frac{8\log(x)}{15}-\frac{\log\left(x^2\right)}{20}-\frac{\log\left(2x^2\right)}{10}-\frac{\log\left(x^2+1\right)^2}{20}+\frac{19\log\left(x^2+1\right)}{60}+\frac{\log\left(2x^2+2\right)}{10}+\frac{\operatorname{atan}^2(x)}{5}-\frac{\mathrm{Li}_2\left(x^2e^{i\pi}\right)}{10}+\frac{2\,\mathrm{ata}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x)*ln(x**2+1)/x**6,x)
```

```
[Out] -8*log(x)/15 - log(x**2)/20 - log(2*x**2)/10 - log(x**2 + 1)**2/20 + 19*log
(x**2 + 1)/60 + log(2*x**2 + 2)/10 + atan(x)**2/5 - polylog(2, x**2*exp_pol
ar(I*pi))/10 + 2*atan(x)/(5*x) + log(x**2 + 1)/(10*x**2) - 7/(60*x**2) - 2*
atan(x)/(15*x**3) - log(x**2 + 1)/(20*x**4) - log(x**2 + 1)*atan(x)/(5*x**5
)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(x)\log\left(x^2+1\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)*log(x^2+1)/x^6,x, algorithm="giac")
```

```
[Out] integrate(arctan(x)*log(x^2 + 1)/x^6, x)
```

3.1286 $\int x^4 \left(a+b\tan^{-1}(cx)\right)\left(d+e\log\left(1+c^2x^2\right)\right)\,dx$

Optimal. Leaf size=278

$$\frac{1}{5}x^5\left(a+b\tan^{-1}(cx)\right)\left(e\log\left(c^2x^2+1\right)+d\right)+\frac{2aex^3}{15c^2}-\frac{2aex}{5c^4}+\frac{2ae\tan^{-1}(cx)}{5c^5}-\frac{2}{25}aex^5-\frac{bx^4\left(e\log\left(c^2x^2+1\right)+d\right)}{20c}+$$

```
[Out] (-2*a*e*x)/(5*c^4) - (77*b*e*x^2)/(300*c^3) + (2*a*e*x^3)/(15*c^2) + (9*b*e
*x^4)/(200*c) - (2*a*e*x^5)/25 + (2*a*e*ArcTan[c*x])/(5*c^5) - (2*b*e*x*Arc
Tan[c*x])/(5*c^4) + (2*b*e*x^3*ArcTan[c*x])/(15*c^2) - (2*b*e*x^5*ArcTan[c*
x])/25 + (b*e*ArcTan[c*x]^2)/(5*c^5) + (137*b*e*Log[1 + c^2*x^2])/(300*c^5)
 + (b*e*Log[1 + c^2*x^2]^2)/(20*c^5) + (b*x^2*(d + e*Log[1 + c^2*x^2]))/(10
*c^3) - (b*x^4*(d + e*Log[1 + c^2*x^2]))/(20*c) + (x^5*(a + b*ArcTan[c*x])*
(d + e*Log[1 + c^2*x^2]))/5 - (b*Log[1 + c^2*x^2]*(d + e*Log[1 + c^2*x^2]))
/(10*c^5)
```

Rubi [A] time = 0.692885, antiderivative size = 278, normalized size of antiderivative = 1., number of steps used = 26, number of rules used = 15, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.577, Rules used = {4852, 266, 43, 5021, 6725, 1802, 635, 203, 260, 4916, 4846, 4884, 2475, 2390, 2301}

$$\frac{1}{5}x^5\left(a+b\tan^{-1}(cx)\right)\left(e\log\left(c^2x^2+1\right)+d\right)+\frac{2aex^3}{15c^2}-\frac{2aex}{5c^4}+\frac{2ae\tan^{-1}(cx)}{5c^5}-\frac{2}{25}aex^5-\frac{bx^4\left(e\log\left(c^2x^2+1\right)+d\right)}{20c}+$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]),x]
```



```
[Out] (-2*a*e*x)/(5*c^4) - (77*b*e*x^2)/(300*c^3) + (2*a*e*x^3)/(15*c^2) + (9*b*e
*x^4)/(200*c) - (2*a*e*x^5)/25 + (2*a*e*ArcTan[c*x])/(5*c^5) - (2*b*e*x*Arc
Tan[c*x])/(5*c^4) + (2*b*e*x^3*ArcTan[c*x])/(15*c^2) - (2*b*e*x^5*ArcTan[c*
x])/25 + (b*e*ArcTan[c*x]^2)/(5*c^5) + (137*b*e*Log[1 + c^2*x^2])/(300*c^5)
 + (b*e*Log[1 + c^2*x^2]^2)/(20*c^5) + (b*x^2*(d + e*Log[1 + c^2*x^2]))/(10
*c^3) - (b*x^4*(d + e*Log[1 + c^2*x^2]))/(20*c) + (x^5*(a + b*ArcTan[c*x])*
(d + e*Log[1 + c^2*x^2]))/5 - (b*Log[1 + c^2*x^2]*(d + e*Log[1 + c^2*x^2]))
/(10*c^5)
```


Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
```

```
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5021

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*(
e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(a + b*ArcTan[c*x]), x
]}, Dist[d + e*Log[f + g*x^2], u, x] - Dist[2*e*g, Int[ExpandIntegrand[(x*u
)/(f + g*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IntegerQ[m
] && NeQ[m, -1]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rubi steps

$$\int x^4\left(a+b\tan^{-1}(cx)\right)\left(d+e\log\left(1+c^2x^2\right)\right)dx=\frac{bx^2\left(d+e\log\left(1+c^2x^2\right)\right)}{10c^3}-\frac{bx^4\left(d+e\log\left(1+c^2x^2\right)\right)}{20c}+\frac{1}{5}x^5\left(a+b\,\mathrm{ta}\right.$$

$$=\frac{bx^2\left(d+e\log\left(1+c^2x^2\right)\right)}{10c^3}-\frac{bx^4\left(d+e\log\left(1+c^2x^2\right)\right)}{20c}+\frac{1}{5}x^5\left(a+b\,\mathrm{ta}\right.$$

$$=\frac{bx^2\left(d+e\log\left(1+c^2x^2\right)\right)}{10c^3}-\frac{bx^4\left(d+e\log\left(1+c^2x^2\right)\right)}{20c}+\frac{1}{5}x^5\left(a+b\,\mathrm{ta}\right.$$

$$=\frac{bx^2\left(d+e\log\left(1+c^2x^2\right)\right)}{10c^3}-\frac{bx^4\left(d+e\log\left(1+c^2x^2\right)\right)}{20c}+\frac{1}{5}x^5\left(a+b\,\mathrm{ta}\right.$$

$$=\frac{be\log^2\left(1+c^2x^2\right)}{20c^5}+\frac{bx^2\left(d+e\log\left(1+c^2x^2\right)\right)}{10c^3}-\frac{bx^4\left(d+e\log\left(1+c^2\right.\right.}{20c}$$

$$=-\frac{2aex}{5c^4}-\frac{3bex^2}{20c^3}+\frac{2aex^3}{15c^2}+\frac{bex^4}{40c}-\frac{2}{25}aex^5-\frac{2}{25}bex^5\tan^{-1}(cx)+\frac{be\,\mathrm{lo}}{}$$

$$=-\frac{2aex}{5c^4}-\frac{3bex^2}{20c^3}+\frac{2aex^3}{15c^2}+\frac{bex^4}{40c}-\frac{2}{25}aex^5+\frac{2bex^3\tan^{-1}(cx)}{15c^2}-\frac{2}{25}be$$

$$=-\frac{2aex}{5c^4}-\frac{3bex^2}{20c^3}+\frac{2aex^3}{15c^2}+\frac{bex^4}{40c}-\frac{2}{25}aex^5+\frac{2ae\tan^{-1}(cx)}{5c^5}-\frac{2bex\,\mathrm{tan}}{5c}$$

$$=-\frac{2aex}{5c^4}-\frac{19bex^2}{100c^3}+\frac{2aex^3}{15c^2}+\frac{9bex^4}{200c}-\frac{2}{25}aex^5+\frac{2ae\tan^{-1}(cx)}{5c^5}-\frac{2bex\,\mathrm{t}}{}$$

$$=-\frac{2aex}{5c^4}-\frac{77bex^2}{300c^3}+\frac{2aex^3}{15c^2}+\frac{9bex^4}{200c}-\frac{2}{25}aex^5+\frac{2ae\tan^{-1}(cx)}{5c^5}-\frac{2bex}{}$$

Mathematica [A] time = 0.173072, size = 214, normalized size = 0.77

$$\frac{cx\left(8a\left(15c^4dx^4-2e\left(3c^4x^4-5c^2x^2+15\right)\right)+bcx\left(e\left(27c^2x^2-154\right)-30d\left(c^2x^2-2\right)\right)\right)+\log\left(c^2x^2+1\right)\left(120ac^5ex^5+2be\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]),x]
```

```
[Out] (c*x*(b*c*x*(-30*d*(-2 + c^2*x^2) + e*(-154 + 27*c^2*x^2)) + 8*a*(15*c^4*d*
x^4 - 2*e*(15 - 5*c^2*x^2 + 3*c^4*x^4))) + 120*b*e*ArcTan[c*x]^2 + (-60*b*d
 + 120*a*c^5*e*x^5 + 2*b*e*(137 + 30*c^2*x^2 - 15*c^4*x^4))*Log[1 + c^2*x^2
] - 30*b*e*Log[1 + c^2*x^2]^2 + 8*ArcTan[c*x]*(30*a*e + 15*b*c^5*d*x^5 - 2*
b*c*e*x*(15 - 5*c^2*x^2 + 3*c^4*x^4) + 15*b*c^5*e*x^5*Log[1 + c^2*x^2]))/(6
00*c^5)
```

Maple [C] time = 2.08, size = 4941, normalized size = 17.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arctan(c*x))*(d+e*ln(c^2*x^2+1)),x)
```

```
[Out] 9/200*b*e*x^4/c-2/25*a*e*x^5-2/5*b*e*x*arctan(c*x)/c^4+2/15*b*e*x^3*arctan(
c*x)/c^2-1/10*I*b*arctan(c*x)*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*csgn(I*(1+
I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*csgn(I/((1+I*c*x)^2/(c^
2*x^2+1)+1)^2)*x^5*e-1/20*I/c^3*b*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*csgn(I
*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*csgn(I/((1+I*c*x)^2
/(c^2*x^2+1)+1)^2)*x^2*e+1/40*I/c*b*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*csgn
(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*csgn(I/((1+I*c*x)
^2/(c^2*x^2+1)+1)^2)*x^4*e-1/10*I/c^5*b*Pi*ln((1+I*c*x)^2/(c^2*x^2+1)+1)*e*
csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*csgn(
I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)-1/40*I/c*b*Pi*csgn
(I*(1+I*c*x)^2/(c^2*x^2+1))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^
2*x^2+1)+1)^2)^2*x^4*e-1/20*I/c*b*Pi*csgn(I*(1+I*c*x)/(c^2*x^2+1)^(1/2))*cs
gn(I*(1+I*c*x)^2/(c^2*x^2+1))^2*x^4*e-1/10/c^5*b*arctan(c*x)*Pi*e*csgn(I/((
1+I*c*x)^2/(c^2*x^2+1)+1)^2)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*csgn(I*(1+I*c*
x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)+1/20*I/c*b*Pi*csgn(I*((1+I*
c*x)^2/(c^2*x^2+1)+1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^2*x^4*e-1/40*I
/c*b*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^2*csg
n(I/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*x^4*e+1/20*I/c^3*b*Pi*csgn(I*(1+I*c*x)^2
/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^2*csgn(I/((1+I*c*x)^2/(c^2*x^2+
1)+1)^2)*x^2*e+1/10*I/c^5*b*Pi*ln((1+I*c*x)^2/(c^2*x^2+1)+1)*e*csgn(I*((1+I
*c*x)^2/(c^2*x^2+1)+1))^2*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)+1/10*I/c^5*
b*Pi*ln((1+I*c*x)^2/(c^2*x^2+1)+1)*e*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*csgn(I
*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^2-3/40*I/c^5*b*Pi*e
*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*csgn
(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)-2/5*a*e*x/c^4-77/
```

300*b*e*x^2/c^3+2/15*a*e*x^3/c^2+2/5*a*e*arctan(c*x)/c^5-2/25*b*e*x^5*arcta
n(c*x)+1/5*x^5*a*d+1/5/c^5*b*ln((1+I*c*x)^2/(c^2*x^2+1)+1)*d+1/5*b*arctan(c
*x)*x^5*d+1/5*a*e*x^5*ln(c^2*x^2+1)+1/10/c^3*b*d*x^2-1/20/c*b*x^4*d-137/150
/c^5*b*e*ln((1+I*c*x)^2/(c^2*x^2+1)+1)-1/5/c^5*b*e*ln((1+I*c*x)^2/(c^2*x^2+
1)+1)^2+3/20/c^5*b*d-181/600/c^5*b*e+1/10*I*b*arctan(c*x)*Pi*csgn(I*((1+I*c
*x)^2/(c^2*x^2+1)+1)^2)^3*x^5*e-1/10*I*b*arctan(c*x)*Pi*csgn(I*(1+I*c*x)^2/
(c^2*x^2+1))^3*x^5*e-1/10*I*b*arctan(c*x)*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)
/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^3*x^5*e+3/40*I/c^5*b*csgn(I*(1+I*c*x)^2/(c^
2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^2*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*P
i*e-3/40*I/c^5*b*Pi*e*csgn(I*(1+I*c*x)/(c^2*x^2+1)^(1/2))^2*csgn(I*(1+I*c*x
)^2/(c^2*x^2+1))-3/20*I/c^5*b*Pi*e*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn
(I*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^2+1/10/c^5*b*csgn(I*(1+I*c*x)^2/(c^2*x^2+
1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^2*arctan(c*x)*Pi*e*csgn(I/((1+I*c*x)^2/(c
^2*x^2+1)+1)^2)+1/10/c^5*b*arctan(c*x)*Pi*e*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))
csgn(I(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^2+1/10/c^5*b
csgn(I((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1))^
2*arctan(c*x)*Pi*e-1/10/c^5*b*arctan(c*x)*Pi*e*csgn(I*(1+I*c*x)/(c^2*x^2+1)
^(1/2))^2*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))-1/10*I/c^5*b*Pi*ln((1+I*c*x)^2/(c
^2*x^2+1)+1)*e*csgn(I*(1+I*c*x)/(c^2*x^2+1)^(1/2))^2*csgn(I*(1+I*c*x)^2/(c^
2*x^2+1))+1/5*I/c^5*b*Pi*ln((1+I*c*x)^2/(c^2*x^2+1)+1)*e*csgn(I*(1+I*c*x)/(
c^2*x^2+1)^(1/2))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))^2+1/10*I/c^5*b*Pi*ln((1+I
*c*x)^2/(c^2*x^2+1)+1)*e*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*csgn(I*(1+I*
c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^2-1/5*I/c^5*b*Pi*ln((1+I*
c*x)^2/(c^2*x^2+1)+1)*e*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*((1+I*c*
x)^2/(c^2*x^2+1)+1)^2)^2+1/20*I/c^3*b*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1)
)^2*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*x^2*e-1/20*I/c^3*b*Pi*csgn(I*(1+I
*c*x)/(c^2*x^2+1)^(1/2))^2*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*x^2*e+1/20*I/c^3
*b*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*
c*x)^2/(c^2*x^2+1)+1)^2)^2*x^2*e+1/10*I/c^3*b*Pi*csgn(I*(1+I*c*x)/(c^2*x^2+
1)^(1/2))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))^2*x^2*e-1/10*I/c^3*b*Pi*csgn(I*((
1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^2*x^2*e+1/
10*I*b*arctan(c*x)*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*csgn(I*(1+I*c*x)^2/(c
^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^2*x^5*e+1/10*I*b*arctan(c*x)*Pi*cs
gn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^2*csgn(I/((1+I*
c*x)^2/(c^2*x^2+1)+1)^2)*x^5*e-1/10*I*b*arctan(c*x)*Pi*csgn(I*(1+I*c*x)/(c^
2*x^2+1)^(1/2))^2*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*x^5*e+1/10*I*b*arctan(c*x
)*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1))^2*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+
1)^2)*x^5*e-1/5*I*b*arctan(c*x)*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn
(I*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^2*x^5*e+1/5*I*b*arctan(c*x)*Pi*csgn(I*(1+
I*c*x)/(c^2*x^2+1)^(1/2))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))^2*x^5*e-1/40*I/c*
b*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1))^2*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+
1)^2)*x^4*e+1/40*I/c*b*Pi*csgn(I*(1+I*c*x)/(c^2*x^2+1)^(1/2))^2*csgn(I*(1+I
*c*x)^2/(c^2*x^2+1))*x^4*e+3/10/c^5*b*ln(2)*e-1/5/c^5*b*csgn(I*((1+I*c*x)^2
/(c^2*x^2+1)+1)^2)^2*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)*Pi*e+1
/5/c^5*b*arctan(c*x)*Pi*e*csgn(I*(1+I*c*x)/(c^2*x^2+1)^(1/2))*csgn(I*(1+I*c

```
*x)^2/(c^2*x^2+1))^2+3/20*I/c^5*b*Pi*e*csgn(I*(1+I*c*x)/(c^2*x^2+1)^(1/2))*
csgn(I*(1+I*c*x)^2/(c^2*x^2+1))^2+3/40*I/c^5*b*Pi*e*csgn(I/((1+I*c*x)^2/(c^
2*x^2+1)+1)^2)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2
)^2+1/10*I/c^5*b*Pi*ln((1+I*c*x)^2/(c^2*x^2+1)+1)*e*csgn(I*((1+I*c*x)^2/(c^
2*x^2+1)+1)^2)^3-1/10*I/c^5*b*Pi*ln((1+I*c*x)^2/(c^2*x^2+1)+1)*e*csgn(I*(1+
I*c*x)^2/(c^2*x^2+1))^3-1/10*I/c^5*b*Pi*ln((1+I*c*x)^2/(c^2*x^2+1)+1)*e*csg
n(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^3-1/40*I/c*b*Pi*
csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^3*x^4*e+1/40*I/c*b*Pi*csgn(I*(1+I*c*x
)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^3*x^4*e+1/40*I/c*b*Pi*csgn(I
*(1+I*c*x)^2/(c^2*x^2+1))^3*x^4*e-1/20*I/c^3*b*Pi*csgn(I*(1+I*c*x)^2/(c^2*x
^2+1))^3*x^2*e-1/20*I/c^3*b*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/
(c^2*x^2+1)+1)^2)^3*x^2*e+1/20*I/c^3*b*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1
)^2)^3*x^2*e+3/40*I/c^5*b*Pi*e*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1))^2*csgn(I
*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)-1/10/c^5*b*arctan(c*x)*Pi*e*csgn(I*(1+I*c*x
)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^3-1/10/c^5*b*arctan(c*x)*Pi*
e*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))^3+1/10/c^5*b*arctan(c*x)*Pi*e*csgn(I*((1+
I*c*x)^2/(c^2*x^2+1)+1)^2)^3-3/40*I/c^5*b*Pi*e*csgn(I*(1+I*c*x)^2/(c^2*x^2+
1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^3+3/40*I/c^5*b*Pi*e*csgn(I*((1+I*c*x)^2/(
c^2*x^2+1)+1)^2)^3-3/40*I/c^5*b*Pi*e*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))^3-2/5*
I/c^5*b*ln(2)*e*arctan(c*x)+2/5*b*arctan(c*x)*ln(2)*x^5*e-2/5*b*arctan(c*x)
*ln((1+I*c*x)^2/(c^2*x^2+1)+1)*x^5*e+2/5/c^5*b*ln(2)*ln((1+I*c*x)^2/(c^2*x^
2+1)+1)*e-1/10/c*b*ln(2)*x^4*e+1/5/c^3*b*ln(2)*x^2*e+1/10/c*b*ln((1+I*c*x)^
2/(c^2*x^2+1)+1)*x^4*e-1/5/c^3*b*ln((1+I*c*x)^2/(c^2*x^2+1)+1)*x^2*e-1/5*I/
c^5*b*arctan(c*x)*d+46/75*I/c^5*b*e*arctan(c*x)+1/10/c^5*b*e*(4*arctan(c*x)
*x^5*c^5-c^4*x^4-4*I*arctan(c*x)+2*c^2*x^2+4*ln((1+I*c*x)^2/(c^2*x^2+1)+1)+
3)*ln((1+I*c*x)/(c^2*x^2+1)^(1/2))
```

Maxima [A] time = 1.50644, size = 346, normalized size = 1.24

$$\frac{1}{5}\,adx^5 + \frac{1}{75}\left(15\,x^5\log\left(c^2x^2+1\right) - 2\,c^2\left(\frac{3\,c^4x^5 - 5\,c^2x^3 + 15\,x}{c^6} - \frac{15\,\arctan\left(cx\right)}{c^7}\right)\right)be\arctan\left(cx\right) + \frac{1}{20}\left(4\,x^5\arctan\left(cx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctan(c*x))*(d+e*log(c^2*x^2+1)),x, algorithm="maxima")
```

```
[Out] 1/5*a*d*x^5 + 1/75*(15*x^5*log(c^2*x^2 + 1) - 2*c^2*((3*c^4*x^5 - 5*c^2*x^3
 + 15*x)/c^6 - 15*arctan(c*x)/c^7))*b*e*arctan(c*x) + 1/20*(4*x^5*arctan(c*
x) - c*((c^2*x^4 - 2*x^2)/c^4 + 2*log(c^2*x^2 + 1)/c^6))*b*d + 1/75*(15*x^5
*log(c^2*x^2 + 1) - 2*c^2*((3*c^4*x^5 - 5*c^2*x^3 + 15*x)/c^6 - 15*arctan(c
*x)/c^7))*a*e + 1/600*(27*c^4*x^4 - 154*c^2*x^2 - 120*arctan(c*x)^2 - 2*(15
*c^4*x^4 - 30*c^2*x^2 - 137)*log(c^2*x^2 + 1) - 30*log(c^2*x^2 + 1)^2)*b*e/
```

```
c^5
```

Fricas [A] time = 1.37734, size = 541, normalized size = 1.95

$$\frac{80\,ac^3ex^3 + 24\left(5\,ac^5d - 2\,ac^5e\right)x^5 - 3\left(10\,bc^4d - 9\,bc^4e\right)x^4 - 240\,acex + 120\,be\arctan\left(cx\right)^2 - 30\,be\log\left(c^2x^2 + 1\right)^2 + 2\left(3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctan(c*x))*(d+e*log(c^2*x^2+1)),x, algorithm="fricas")
```

```
[Out] 1/600*(80*a*c^3*e*x^3 + 24*(5*a*c^5*d - 2*a*c^5*e)*x^5 - 3*(10*b*c^4*d - 9*
b*c^4*e)*x^4 - 240*a*c*e*x + 120*b*e*arctan(c*x)^2 - 30*b*e*log(c^2*x^2 + 1
)^2 + 2*(30*b*c^2*d - 77*b*c^2*e)*x^2 + 8*(10*b*c^3*e*x^3 + 3*(5*b*c^5*d -
2*b*c^5*e)*x^5 - 30*b*c*e*x + 30*a*e)*arctan(c*x) + 2*(60*b*c^5*e*x^5*arcta
n(c*x) + 60*a*c^5*e*x^5 - 15*b*c^4*e*x^4 + 30*b*c^2*e*x^2 - 30*b*d + 137*b*
e)*log(c^2*x^2 + 1))/c^5
```

Sympy [A] time = 35.4914, size = 338, normalized size = 1.22

$$\begin{cases} \frac{adx^5}{5} + \frac{aex^5\log\left(c^2x^2+1\right)}{5} - \frac{2aex^5}{25} + \frac{2aex^3}{15c^2} - \frac{2aex}{5c^4} + \frac{2ae\,\mathrm{atan}\,(cx)}{5c^5} + \frac{bdx^5\,\mathrm{atan}\,(cx)}{5} + \frac{bex^5\log\left(c^2x^2+1\right)\mathrm{atan}\,(cx)}{5} - \frac{2bex^5\,\mathrm{atan}\,(cx)}{25} - \frac{bdx^4}{20c} - \frac{be}{} \\ \frac{adx^5}{5} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*atan(c*x))*(d+e*ln(c**2*x**2+1)),x)
```

```
[Out] Piecewise((a*d*x**5/5 + a*e*x**5*log(c**2*x**2 + 1)/5 - 2*a*e*x**5/25 + 2*a
*e*x**3/(15*c**2) - 2*a*e*x/(5*c**4) + 2*a*e*atan(c*x)/(5*c**5) + b*d*x**5*
atan(c*x)/5 + b*e*x**5*log(c**2*x**2 + 1)*atan(c*x)/5 - 2*b*e*x**5*atan(c*x
)/25 - b*d*x**4/(20*c) - b*e*x**4*log(c**2*x**2 + 1)/(20*c) + 9*b*e*x**4/(2
00*c) + 2*b*e*x**3*atan(c*x)/(15*c**2) + b*d*x**2/(10*c**3) + b*e*x**2*log(
c**2*x**2 + 1)/(10*c**3) - 77*b*e*x**2/(300*c**3) - 2*b*e*x*atan(c*x)/(5*c*
*4) - b*d*log(c**2*x**2 + 1)/(10*c**5) - b*e*log(c**2*x**2 + 1)**2/(20*c**5
) + 137*b*e*log(c**2*x**2 + 1)/(300*c**5) + b*e*atan(c*x)**2/(5*c**5), Ne(c
, 0)), (a*d*x**5/5, True))
```

Giac [A] time = 1.655, size = 618, normalized size = 2.22

$$\frac{60\,\pi bc^5x^5e\log\left(c^2x^2+1\right)\mathrm{sgn}\left(c\right)\mathrm{sgn}\left(x\right)-24\,\pi bc^5x^5e\mathrm{sgn}\left(c\right)\mathrm{sgn}\left(x\right)-120\,bc^5x^5\arctan\left(\frac{1}{cx}\right)e\log\left(c^2x^2+1\right)+120\,bc^5d}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctan(c*x))*(d+e*log(c^2*x^2+1)),x, algorithm="giac")
```

```
[Out] 1/600*(60*pi*b*c^5*x^5*e*log(c^2*x^2 + 1)*sgn(c)*sgn(x) - 24*pi*b*c^5*x^5*e
*sgn(c)*sgn(x) - 120*b*c^5*x^5*arctan(1/(c*x))*e*log(c^2*x^2 + 1) + 120*b*c
^5*d*x^5*arctan(c*x) + 48*b*c^5*x^5*arctan(1/(c*x))*e + 120*a*c^5*x^5*e*log
(c^2*x^2 + 1) + 120*a*c^5*d*x^5 - 48*a*c^5*x^5*e - 30*b*c^4*x^4*e*log(c^2*x
^2 + 1) + 40*pi*b*c^3*x^3*e*sgn(c)*sgn(x) - 30*b*c^4*d*x^4 + 27*b*c^4*x^4*e
 - 80*b*c^3*x^3*arctan(1/(c*x))*e + 80*a*c^3*x^3*e + 60*b*c^2*x^2*e*log(c^2
*x^2 + 1) - 120*pi*b*c*x*e*sgn(c)*sgn(x) + 60*b*c^2*d*x^2 - 154*b*c^2*x^2*e
 - 180*pi^2*b*e*sgn(c)*sgn(x) - 120*pi*b*arctan(1/(c*x))*e*sgn(c)*sgn(x) +
240*b*c*x*arctan(1/(c*x))*e - 240*pi*a*e*sgn(c)*sgn(x) + 60*pi^2*b*e - 240*
a*c*x*e + 120*pi*b*arctan(c*x)*e + 120*pi*b*arctan(1/(c*x))*e + 120*b*arcta
n(1/(c*x))^2*e - 30*b*e*log(c^2*x^2 + 1)^2 + 240*a*arctan(c*x)*e - 60*b*d*l
og(c^2*x^2 + 1) + 274*b*e*log(c^2*x^2 + 1))/c^5
```

3.1287 $\int x^3 \left(a+b\tan^{-1}(cx)\right)\left(d+e\log\left(1+c^2x^2\right)\right)\,dx$

Optimal. Leaf size=221

$$\frac{1}{4}x^4\left(a+b\tan^{-1}(cx)\right)\left(e\log\left(c^2x^2+1\right)+d\right)+\frac{ex^2\left(a+b\tan^{-1}(cx)\right)}{4c^2}-\frac{e\log\left(c^2x^2+1\right)\left(a+b\tan^{-1}(cx)\right)}{4c^4}-\frac{1}{8}ex^4\left(a+b\,\text{ta}\right.$$

```
[Out] (b*(2*d - 3*e)*x)/(8*c^3) - (2*b*e*x)/(3*c^3) - (b*(2*d - e)*x^3)/(24*c) +
(b*e*x^3)/(18*c) - (b*(2*d - 3*e)*ArcTan[c*x])/(8*c^4) + (2*b*e*ArcTan[c*x]
)/(3*c^4) + (e*x^2*(a + b*ArcTan[c*x]))/(4*c^2) - (e*x^4*(a + b*ArcTan[c*x]
))/8 + (b*e*x*Log[1 + c^2*x^2])/(4*c^3) - (b*e*x^3*Log[1 + c^2*x^2])/(12*c)
 - (e*(a + b*ArcTan[c*x])*Log[1 + c^2*x^2])/(4*c^4) + (x^4*(a + b*ArcTan[c*
x])*(d + e*Log[1 + c^2*x^2]))/4
```

Rubi [A] time = 0.243993, antiderivative size = 221, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 11, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {2454, 2395, 43, 5019, 459, 321, 203, 2471, 2448, 2455, 302}

$$\frac{1}{4}x^4\left(a+b\tan^{-1}(cx)\right)\left(e\log\left(c^2x^2+1\right)+d\right)+\frac{ex^2\left(a+b\tan^{-1}(cx)\right)}{4c^2}-\frac{e\log\left(c^2x^2+1\right)\left(a+b\tan^{-1}(cx)\right)}{4c^4}-\frac{1}{8}ex^4\left(a+b\,\text{ta}\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]),x]
```


```
[Out] (b*(2*d - 3*e)*x)/(8*c^3) - (2*b*e*x)/(3*c^3) - (b*(2*d - e)*x^3)/(24*c) +
(b*e*x^3)/(18*c) - (b*(2*d - 3*e)*ArcTan[c*x])/(8*c^4) + (2*b*e*ArcTan[c*x]
)/(3*c^4) + (e*x^2*(a + b*ArcTan[c*x]))/(4*c^2) - (e*x^4*(a + b*ArcTan[c*x]
))/8 + (b*e*x*Log[1 + c^2*x^2])/(4*c^3) - (b*e*x^3*Log[1 + c^2*x^2])/(12*c)
 - (e*(a + b*ArcTan[c*x])*Log[1 + c^2*x^2])/(4*c^4) + (x^4*(a + b*ArcTan[c*
x])*(d + e*Log[1 + c^2*x^2]))/4
```


Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5019

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*(
e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*Log[f + g*x^2])
, x]}, Dist[a + b*ArcTan[c*x], u, x] - Dist[b*c, Int[ExpandIntegrand[u/(1 +
 c^2*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[(m + 1)/2
, 0]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2471

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\int x^3\left(a+b\tan^{-1}(cx)\right)\left(d+e\log\left(1+c^2x^2\right)\right)dx=\frac{ex^2\left(a+b\tan^{-1}(cx)\right)}{4c^2}-\frac{1}{8}ex^4\left(a+b\tan^{-1}(cx)\right)-\frac{e\left(a+b\tan^{-1}(cx)\right)}{4c^4}$$

$$=\frac{ex^2\left(a+b\tan^{-1}(cx)\right)}{4c^2}-\frac{1}{8}ex^4\left(a+b\tan^{-1}(cx)\right)-\frac{e\left(a+b\tan^{-1}(cx)\right)}{4c^4}$$

$$=-\frac{b(2d-e)x^3}{24c}+\frac{ex^2\left(a+b\tan^{-1}(cx)\right)}{4c^2}-\frac{1}{8}ex^4\left(a+b\tan^{-1}(cx)\right)-\frac{e(}{}$$

$$=\frac{b(2d-3e)x}{8c^3}-\frac{b(2d-e)x^3}{24c}+\frac{ex^2\left(a+b\tan^{-1}(cx)\right)}{4c^2}-\frac{1}{8}ex^4\left(a+b\tan\right.$$

$$=\frac{b(2d-3e)x}{8c^3}-\frac{b(2d-e)x^3}{24c}-\frac{b(2d-3e)\tan^{-1}(cx)}{8c^4}+\frac{ex^2\left(a+b\tan^{-}\right.}{4c^2}$$

$$=\frac{b(2d-3e)x}{8c^3}-\frac{bex}{2c^3}-\frac{b(2d-e)x^3}{24c}-\frac{b(2d-3e)\tan^{-1}(cx)}{8c^4}+\frac{ex^2\left(a+\right.}{}$$

$$=\frac{b(2d-3e)x}{8c^3}-\frac{2bex}{3c^3}-\frac{b(2d-e)x^3}{24c}+\frac{bex^3}{18c}-\frac{b(2d-3e)\tan^{-1}(cx)}{8c^4}+$$

$$=\frac{b(2d-3e)x}{8c^3}-\frac{2bex}{3c^3}-\frac{b(2d-e)x^3}{24c}+\frac{bex^3}{18c}-\frac{b(2d-3e)\tan^{-1}(cx)}{8c^4}+$$

Mathematica [A] time = 0.152217, size = 164, normalized size = 0.74

$$\frac{cx\left(18ac^3dx^3-9acex\left(c^2x^2-2\right)-6bd\left(c^2x^2-3\right)+be\left(7c^2x^2-75\right)\right)-6e\log\left(c^2x^2+1\right)\left(a\left(3-3c^4x^4\right)+bcx\left(c^2x^2-3\right)\right)}{72c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]),x]
```

```
[Out] (c*x*(18*a*c^3*d*x^3 - 6*b*d*(-3 + c^2*x^2) - 9*a*c*e*x*(-2 + c^2*x^2) + b*
e*(-75 + 7*c^2*x^2)) - 6*e*(b*c*x*(-3 + c^2*x^2) + a*(3 - 3*c^4*x^4))*Log[1
 + c^2*x^2] + 3*b*ArcTan[c*x]*(e*(25 + 6*c^2*x^2 - 3*c^4*x^4) + 6*d*(-1 + c
^4*x^4) + 6*e*(-1 + c^4*x^4)*Log[1 + c^2*x^2]))/(72*c^4)
```

Maple [C] time = 1.831, size = 3897, normalized size = 17.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctan(c*x))*(d+e*ln(c^2*x^2+1)),x)
```

```
[Out] -1/8*I/c^4*b*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^
2*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*arctan(c*x)*Pi*e+1/8*I/c^4*b*csgn(I*(1+I*
c*x)^2/(c^2*x^2+1))*csgn(I*(1+I*c*x)/(c^2*x^2+1)^(1/2))^2*arctan(c*x)*Pi*e-
1/8*I/c^4*b*arctan(c*x)*Pi*e*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1))^2*csgn(I*(
(1+I*c*x)^2/(c^2*x^2+1)+1)^2)+1/4*I/c^4*b*arctan(c*x)*Pi*e*csgn(I*((1+I*c*x
)^2/(c^2*x^2+1)+1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^2-1/24*I/c*b*csgn
(I/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)
^2/(c^2*x^2+1)+1)^2)^2*Pi*x^3*e-1/12*I/c*b*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))^
2*csgn(I*(1+I*c*x)/(c^2*x^2+1)^(1/2))*Pi*x^3*e+1/4*b*d*x/c^3-1/12*b*d*x^3/c
-1/4*b*d*arctan(c*x)/c^4-25/24*b*e*x/c^3+41/24*b*e*arctan(c*x)/c^4+7/72*b*e
*x^3/c-1/24*I/c*b*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*csgn(I*(1+I*c*x)^2/(c^2*x
^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^2*Pi*x^3*e+1/24*I/c*b*csgn(I*(1+I*c*x)
^2/(c^2*x^2+1))*csgn(I*(1+I*c*x)/(c^2*x^2+1)^(1/2))^2*Pi*x^3*e+1/8*I*b*arct
an(c*x)*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1
)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^2*Pi*x^4*e+1/4*I*b*arctan(c*x)*csgn(I*(1+I
*c*x)^2/(c^2*x^2+1))^2*csgn(I*(1+I*c*x)/(c^2*x^2+1)^(1/2))*Pi*x^4*e+1/8*I*b
*arctan(c*x)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)
/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^2*Pi*x^4*e-1/8*I*b*arctan(c*x)*csgn(I*(1+I*
c*x)^2/(c^2*x^2+1))*csgn(I*(1+I*c*x)/(c^2*x^2+1)^(1/2))^2*Pi*x^4*e+1/8*I*b*
arctan(c*x)*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1))^2*csgn(I*((1+I*c*x)^2/(c^2*
x^2+1)+1)^2)*Pi*x^4*e-1/4*I*b*arctan(c*x)*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1
))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^2*Pi*x^4*e-1/24*I/c*b*csgn(I*((1+I
*c*x)^2/(c^2*x^2+1)+1))^2*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*Pi*x^3*e+1/
12*I/c*b*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1
)+1)^2)^2*Pi*x^3*e+1/8*I/c^3*b*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*csgn(I
*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^2*Pi*x*e+1/4*I/c^3*
b*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))^2*csgn(I*(1+I*c*x)/(c^2*x^2+1)^(1/2))*Pi*
x*e+1/8*I/c^3*b*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*csgn(I*(1+I*c*x)^2/(c^2*x^2
+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^2*Pi*x*e-1/8*I/c^3*b*csgn(I*(1+I*c*x)^2/
(c^2*x^2+1))*csgn(I*(1+I*c*x)/(c^2*x^2+1)^(1/2))^2*Pi*x*e+1/8*I/c^3*b*csgn(
I*((1+I*c*x)^2/(c^2*x^2+1)+1))^2*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*Pi*x
*e-1/4*I/c^3*b*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*((1+I*c*x)^2/(c^2
*x^2+1)+1)^2)^2*Pi*x*e-1/8*I/c^4*b*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x
)^2/(c^2*x^2+1)+1)^2)^2*arctan(c*x)*Pi*e*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1)
^2)-1/4*I/c^4*b*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))^2*csgn(I*(1+I*c*x)/(c^2*x^2
+1)^(1/2))*arctan(c*x)*Pi*e-1/8*a*e*x^4-1/8*I*b*arctan(c*x)*csgn(I/((1+I*c*
x)^2/(c^2*x^2+1)+1)^2)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*csgn(I*(1+I*c*x)^2/(
c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*Pi*x^4*e+1/24*I/c*b*csgn(I/((1+I*
c*x)^2/(c^2*x^2+1)+1)^2)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*csgn(I*(1+I*c*x)^2
/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*Pi*x^3*e-1/8*I/c^3*b*csgn(I/((1
```

```
+I*c*x)^2/(c^2*x^2+1)+1)^2)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*csgn(I*(1+I*c*x
)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*Pi*x*e+1/8*I/c^4*b*arctan(c*
x)*Pi*e*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1
))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)+1/4*a*e/c^
2*x^2-1/4*a*e/c^4*ln(c^2*x^2+1)+1/4*b*arctan(c*x)*x^4*d-1/8*b*arctan(c*x)*x
^4*e-1/2/c^3*b*ln((1+I*c*x)^2/(c^2*x^2+1)+1)*x*e+1/2/c^3*b*ln(2)*x*e+1/2/c^
4*b*ln((1+I*c*x)^2/(c^2*x^2+1)+1)*e*arctan(c*x)-1/2/c^4*b*ln(2)*e*arctan(c*
x)+1/6/c^4*b*e*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))^3+1/6/c^4*b*e*Pi*csgn(I/(
(1+I*c*x)^2/(c^2*x^2+1)+1)^2)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*csgn(I*(1+I*c
*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)+1/24*I/c*b*csgn(I*(1+I*c*x
)^2/(c^2*x^2+1))^3*Pi*x^3*e+1/24*I/c*b*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I
*c*x)^2/(c^2*x^2+1)+1)^2)^3*Pi*x^3*e-1/24*I/c*b*csgn(I*((1+I*c*x)^2/(c^2*x^
2+1)+1)^2)^3*Pi*x^3*e-1/8*I/c^3*b*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))^3*Pi*x*e-
1/8*I/c^3*b*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^3
*Pi*x*e+1/8*I/c^3*b*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^3*Pi*x*e+1/8*I/c^
4*b*arctan(c*x)*Pi*e*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))^3+1/8*I/c^4*b*arctan(c
*x)*Pi*e*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^3-1/
8*I/c^4*b*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^3*arctan(c*x)*Pi*e-1/8*I*b*
arctan(c*x)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))^3*Pi*x^4*e-1/8*I*b*arctan(c*x)*
csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^3*Pi*x^4*e+1/
8*I*b*arctan(c*x)*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^3*Pi*x^4*e+1/6/c^4*
b*e*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^3-1/6/
c^4*b*e*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^3-1/2*b*arctan(c*x)*ln((1+
I*c*x)^2/(c^2*x^2+1)+1)*x^4*e+1/2*b*arctan(c*x)*ln(2)*x^4*e+1/6/c*b*ln((1+I
*c*x)^2/(c^2*x^2+1)+1)*x^3*e-1/6/c*b*ln(2)*x^3*e+2/3*I/c^4*b*ln(2)*e+1/4/c^
2*b*arctan(c*x)*x^2*e+1/4*x^4*a*e*ln(c^2*x^2+1)+1/3*I/c^4*b*d-41/36*I/c^4*b
*e+1/4*x^4*a*d+1/6/c^4*b*e*(3*arctan(c*x)*x^3*c^3-3*I*arctan(c*x)*x^2*c^2-c
^2*x^2-3*arctan(c*x)*x*c+I*c*x+3*I*arctan(c*x)+4)*(c*x+I)*ln((1+I*c*x)/(c^2
*x^2+1)^(1/2))-1/6/c^4*b*e*Pi*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*csgn(I*
(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^2-1/3/c^4*b*e*Pi*csg
n(I*(1+I*c*x)/(c^2*x^2+1)^(1/2))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))^2-1/6/c^4*
b*e*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I
*c*x)^2/(c^2*x^2+1)+1)^2)^2+1/6/c^4*b*e*Pi*csgn(I*(1+I*c*x)/(c^2*x^2+1)^(1/
2))^2*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))-1/6/c^4*b*e*Pi*csgn(I*((1+I*c*x)^2/(c
^2*x^2+1)+1))^2*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)+1/3/c^4*b*e*Pi*csgn(I
*((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^2
```

Maxima [A] time = 1.49209, size = 302, normalized size = 1.37

$$\frac{1}{4}adx^4 + \frac{1}{72}bce\left(\frac{7c^2x^3 - 6\left(c^2x^3 - 3x\right)\log\left(c^2x^2+1\right) - 75x}{c^4} + \frac{75\arctan(cx)}{c^5}\right) + \frac{1}{8}\left(2x^4\log\left(c^2x^2+1\right) - c^2\left(\frac{c^2x^4 - 2}{c^4}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))*(d+e*log(c^2*x^2+1)),x, algorithm="maxima")
```

```
[Out] 1/4*a*d*x^4 + 1/72*b*c*e*((7*c^2*x^3 - 6*(c^2*x^3 - 3*x)*log(c^2*x^2 + 1) -
 75*x)/c^4 + 75*arctan(c*x)/c^5) + 1/8*(2*x^4*log(c^2*x^2 + 1) - c^2*((c^2*
x^4 - 2*x^2)/c^4 + 2*log(c^2*x^2 + 1)/c^6))*b*e*arctan(c*x) + 1/12*(3*x^4*a
rctan(c*x) - c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5))*b*d + 1/8*(2*x^4*
log(c^2*x^2 + 1) - c^2*((c^2*x^4 - 2*x^2)/c^4 + 2*log(c^2*x^2 + 1)/c^6))*a*
e
```

Fricas [A] time = 1.38576, size = 405, normalized size = 1.83

$$\frac{18\,ac^2ex^2 + 9\left(2\,ac^4d - ac^4e\right)x^4 - \left(6\,bc^3d - 7\,bc^3e\right)x^3 + 3\,(6\,bcd - 25\,bce)x + 3\left(6\,bc^2ex^2 + 3\left(2\,bc^4d - bc^4e\right)x^4 - 6\,bd + 25}{72\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))*(d+e*log(c^2*x^2+1)),x, algorithm="fricas")
```

```
[Out] 1/72*(18*a*c^2*e*x^2 + 9*(2*a*c^4*d - a*c^4*e)*x^4 - (6*b*c^3*d - 7*b*c^3*e
)*x^3 + 3*(6*b*c*d - 25*b*c*e)*x + 3*(6*b*c^2*e*x^2 + 3*(2*b*c^4*d - b*c^4*
e)*x^4 - 6*b*d + 25*b*e)*arctan(c*x) + 6*(3*a*c^4*e*x^4 - b*c^3*e*x^3 + 3*b
*c*e*x - 3*a*e + 3*(b*c^4*e*x^4 - b*e)*arctan(c*x))*log(c^2*x^2 + 1))/c^4
```

Sympy [A] time = 22.3741, size = 279, normalized size = 1.26

$$\begin{cases} \frac{adx^4}{4} + \frac{aex^4\log\left(c^2x^2+1\right)}{4} - \frac{aex^4}{8} + \frac{aex^2}{4c^2} - \frac{ae\log\left(c^2x^2+1\right)}{4c^4} + \frac{bdx^4\operatorname{atan}(cx)}{4} + \frac{bex^4\log\left(c^2x^2+1\right)\operatorname{atan}(cx)}{4} - \frac{bex^4\operatorname{atan}(cx)}{8} - \frac{bdx^3}{12c} - \frac{bex^3\log\left(c^2x\right.}{12c} \\ \frac{adx^4}{4} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atan(c*x))*(d+e*ln(c**2*x**2+1)),x)
```

```
[Out] Piecewise((a*d*x**4/4 + a*e*x**4*log(c**2*x**2 + 1)/4 - a*e*x**4/8 + a*e*x*
*2/(4*c**2) - a*e*log(c**2*x**2 + 1)/(4*c**4) + b*d*x**4*atan(c*x)/4 + b*e*
x**4*log(c**2*x**2 + 1)*atan(c*x)/4 - b*e*x**4*atan(c*x)/8 - b*d*x**3/(12*c
) - b*e*x**3*log(c**2*x**2 + 1)/(12*c) + 7*b*e*x**3/(72*c) + b*e*x**2*atan(
c*x)/(4*c**2) + b*d*x/(4*c**3) + b*e*x*log(c**2*x**2 + 1)/(4*c**3) - 25*b*e
*x/(24*c**3) - b*d*atan(c*x)/(4*c**4) - b*e*log(c**2*x**2 + 1)*atan(c*x)/(4
```

```
*c**4) + 25*b*e*atan(c*x)/(24*c**4), Ne(c, 0)), (a*d*x**4/4, True))
```

Giac [A] time = 1.54895, size = 479, normalized size = 2.17

$$\frac{18\,\pi bc^4x^4e\log\left(c^2x^2+1\right)\mathrm{sgn}\left(c\right)\mathrm{sgn}\left(x\right)-9\,\pi bc^4x^4e\mathrm{sgn}\left(c\right)\mathrm{sgn}\left(x\right)-36\,bc^4x^4\arctan\left(\frac{1}{cx}\right)e\log\left(c^2x^2+1\right)+36\,bc^4dx^4\ldots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))*(d+e*log(c^2*x^2+1)),x, algorithm="giac")
```

```
[Out] 1/144*(18*pi*b*c^4*x^4*e*log(c^2*x^2 + 1)*sgn(c)*sgn(x) - 9*pi*b*c^4*x^4*e*
sgn(c)*sgn(x) - 36*b*c^4*x^4*arctan(1/(c*x))*e*log(c^2*x^2 + 1) + 36*b*c^4*
d*x^4*arctan(c*x) + 18*b*c^4*x^4*arctan(1/(c*x))*e + 36*a*c^4*x^4*e*log(c^2
*x^2 + 1) + 36*a*c^4*d*x^4 - 18*a*c^4*x^4*e - 12*b*c^3*x^3*e*log(c^2*x^2 +
1) + 18*pi*b*c^2*x^2*e*sgn(c)*sgn(x) - 12*b*c^3*d*x^3 + 14*b*c^3*x^3*e - 36
*b*c^2*x^2*arctan(1/(c*x))*e + 36*a*c^2*x^2*e - 18*pi*b*e*log(c^2*x^2 + 1)*
sgn(c)*sgn(x) + 36*b*c*x*e*log(c^2*x^2 + 1) - 150*pi*b*e*sgn(c)*sgn(x) + 36
*b*c*d*x - 150*b*c*x*e + 36*b*arctan(1/(c*x))*e*log(c^2*x^2 + 1) - 36*b*d*a
rctan(c*x) + 150*b*arctan(c*x)*e - 36*a*e*log(c^2*x^2 + 1))/c^4
```

3.1288 $\int x^2 \left(a+b\tan^{-1}(cx)\right)\left(d+e\log\left(1+c^2x^2\right)\right)\,dx$

Optimal. Leaf size=213

$$\frac{1}{3}x^3\left(a+b\tan^{-1}(cx)\right)\left(e\log\left(c^2x^2+1\right)+d\right)+\frac{2aex}{3c^2}-\frac{2ae\tan^{-1}(cx)}{3c^3}-\frac{2}{9}aex^3-\frac{bx^2\left(e\log\left(c^2x^2+1\right)+d\right)}{6c}+\frac{b\log\left(c^2x^2+\right.}{}$$

```
[Out] (2*a*e*x)/(3*c^2) + (5*b*e*x^2)/(18*c) - (2*a*e*x^3)/9 - (2*a*e*ArcTan[c*x]
)/(3*c^3) + (2*b*e*x*ArcTan[c*x])/(3*c^2) - (2*b*e*x^3*ArcTan[c*x])/9 - (b*
e*ArcTan[c*x]^2)/(3*c^3) - (11*b*e*Log[1 + c^2*x^2])/(18*c^3) - (b*e*Log[1
+ c^2*x^2]^2)/(12*c^3) - (b*x^2*(d + e*Log[1 + c^2*x^2]))/(6*c) + (x^3*(a +
 b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]))/3 + (b*Log[1 + c^2*x^2]*(d + e*Lo
g[1 + c^2*x^2]))/(6*c^3)
```

Rubi [A] time = 0.569288, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 15, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.577, Rules used = {4852, 266, 43, 5021, 6725, 801, 635, 203, 260, 4916, 4846, 4884, 2475, 2390, 2301}

$$\frac{1}{3}x^3\left(a+b\tan^{-1}(cx)\right)\left(e\log\left(c^2x^2+1\right)+d\right)+\frac{2aex}{3c^2}-\frac{2ae\tan^{-1}(cx)}{3c^3}-\frac{2}{9}aex^3-\frac{bx^2\left(e\log\left(c^2x^2+1\right)+d\right)}{6c}+\frac{b\log\left(c^2x^2+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]),x]
```

```
[Out] (2*a*e*x)/(3*c^2) + (5*b*e*x^2)/(18*c) - (2*a*e*x^3)/9 - (2*a*e*ArcTan[c*x]
)/(3*c^3) + (2*b*e*x*ArcTan[c*x])/(3*c^2) - (2*b*e*x^3*ArcTan[c*x])/9 - (b*
e*ArcTan[c*x]^2)/(3*c^3) - (11*b*e*Log[1 + c^2*x^2])/(18*c^3) - (b*e*Log[1
+ c^2*x^2]^2)/(12*c^3) - (b*x^2*(d + e*Log[1 + c^2*x^2]))/(6*c) + (x^3*(a +
 b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]))/3 + (b*Log[1 + c^2*x^2]*(d + e*Lo
g[1 + c^2*x^2]))/(6*c^3)
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5021

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*(
e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(a + b*ArcTan[c*x]), x
]}, Dist[d + e*Log[f + g*x^2], u, x] - Dist[2*e*g, Int[ExpandIntegrand[(x*u
)/(f + g*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IntegerQ[m
] && NeQ[m, -1]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rubi steps

$$\int x^2 \left(a+b\tan^{-1}(cx)\right)\left(d+e\log\left(1+c^2x^2\right)\right)dx = -\frac{bx^2\left(d+e\log\left(1+c^2x^2\right)\right)}{6c}+\frac{1}{3}x^3\left(a+b\tan^{-1}(cx)\right)\left(d+e\log\left(1+c\right.\right.$$

$$= -\frac{bx^2\left(d+e\log\left(1+c^2x^2\right)\right)}{6c}+\frac{1}{3}x^3\left(a+b\tan^{-1}(cx)\right)\left(d+e\log\left(1+c\right.\right.$$

$$= -\frac{bx^2\left(d+e\log\left(1+c^2x^2\right)\right)}{6c}+\frac{1}{3}x^3\left(a+b\tan^{-1}(cx)\right)\left(d+e\log\left(1+c\right.\right.$$

$$= -\frac{bx^2\left(d+e\log\left(1+c^2x^2\right)\right)}{6c}+\frac{1}{3}x^3\left(a+b\tan^{-1}(cx)\right)\left(d+e\log\left(1+c\right.\right.$$

$$= -\frac{be\log^2\left(1+c^2x^2\right)}{12c^3}-\frac{bx^2\left(d+e\log\left(1+c^2x^2\right)\right)}{6c}+\frac{1}{3}x^3\left(a+b\tan^{-1}(\right.$$

$$= \frac{2aex}{3c^2}+\frac{bex^2}{6c}-\frac{2}{9}aex^3-\frac{2}{9}bex^3\tan^{-1}(cx)-\frac{be\log^2\left(1+c^2x^2\right)}{12c^3}-\frac{bx^2}{}$$

$$= \frac{2aex}{3c^2}+\frac{bex^2}{6c}-\frac{2}{9}aex^3+\frac{2bex\tan^{-1}(cx)}{3c^2}-\frac{2}{9}bex^3\tan^{-1}(cx)-\frac{be\tan}{3}$$

$$= \frac{2aex}{3c^2}+\frac{bex^2}{6c}-\frac{2}{9}aex^3-\frac{2ae\tan^{-1}(cx)}{3c^3}+\frac{2bex\tan^{-1}(cx)}{3c^2}-\frac{2}{9}bex^3\,\mathrm{ta}$$

$$= \frac{2aex}{3c^2}+\frac{5bex^2}{18c}-\frac{2}{9}aex^3-\frac{2ae\tan^{-1}(cx)}{3c^3}+\frac{2bex\tan^{-1}(cx)}{3c^2}-\frac{2}{9}bex^3\,\mathrm{t}$$

Mathematica [A] time = 0.131023, size = 171, normalized size = 0.8

$$\frac{2cx\left(6ac^2dx^2-4ae\left(c^2x^2-3\right)+bcx(5e-3d)\right)+2\log\left(c^2x^2+1\right)\left(6ac^3ex^3-be\left(3c^2x^2+11\right)+3bd\right)-4\tan^{-1}(cx)\left(6ae+\right.}{36c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]),x]
```

```
[Out] (2*c*x*(b*c*(-3*d + 5*e)*x + 6*a*c^2*d*x^2 - 4*a*e*(-3 + c^2*x^2)) - 12*b*e
*ArcTan[c*x]^2 + 2*(3*b*d + 6*a*c^3*e*x^3 - b*e*(11 + 3*c^2*x^2))*Log[1 + c
^2*x^2] + 3*b*e*Log[1 + c^2*x^2]^2 - 4*ArcTan[c*x]*(6*a*e + b*c*x*(-6*e - 3
*c^2*d*x^2 + 2*c^2*e*x^2) - 3*b*c^3*e*x^3*Log[1 + c^2*x^2]))/(36*c^3)
```

Maple [C] time = 1.43, size = 4146, normalized size = 19.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctan(c*x))*(d+e*ln(c^2*x^2+1)),x)
```

```
[Out] -1/6*b*d*x^2/c+2/3*a*e*x/c^2-2/3*a*e*arctan(c*x)/c^3-2/9*b*e*x^3*arctan(c*x
)-1/6*I*b*arctan(c*x)*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x
^2+1)+1)^2)^3*x^3*e+1/12*I/c^3*b*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*csgn(I*(1+
I*c*x)/(c^2*x^2+1)^(1/2))^2*Pi*e-1/6*I/c^3*b*Pi*e*csgn(I*(1+I*c*x)/(c^2*x^2
+1)^(1/2))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))^2-1/12*I/c^3*b*Pi*e*csgn(I*(1+I*
c*x)^2/(c^2*x^2+1))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)
+1)^2)^2-1/12*I/c^3*b*Pi*e*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1))^2*csgn(I*((1
+I*c*x)^2/(c^2*x^2+1)+1)^2)+1/6*I/c^3*b*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1)^
2)^2*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1))*Pi*e-1/12*I/c^3*b*csgn(I*(1+I*c*x)
^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^2*Pi*e*csgn(I/((1+I*c*x)^2/(c
^2*x^2+1)+1)^2)+1/12*I/c*b*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))^3*x^2*e-1/12*
I/c*b*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^3*x^2*e+1/12*I/c*b*Pi*csgn(I
*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^3*x^2*e+1/6*I/c^3*b
*Pi*ln((1+I*c*x)^2/(c^2*x^2+1)+1)*e*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))^3-1/6*I
/c^3*b*Pi*ln((1+I*c*x)^2/(c^2*x^2+1)+1)*e*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1
)^2)^3+5/18*b*e*x^2/c-1/3/c^3*b*ln(2)*e+1/3*x^3*a*e*ln(c^2*x^2+1)+1/3/c^3*b
*e*ln((1+I*c*x)^2/(c^2*x^2+1)+1)^2-1/3/c^3*b*ln((1+I*c*x)^2/(c^2*x^2+1)+1)*
d+11/9/c^3*b*e*ln((1+I*c*x)^2/(c^2*x^2+1)+1)+1/3*b*arctan(c*x)*x^3*d-2/9*a*
e*x^3+1/12*I/c*b*Pi*csgn(I*(1+I*c*x)/(c^2*x^2+1)^(1/2))^2*csgn(I*(1+I*c*x)^
2/(c^2*x^2+1))*x^2*e-1/6*I/c*b*Pi*csgn(I*(1+I*c*x)/(c^2*x^2+1)^(1/2))*csgn(
I*(1+I*c*x)^2/(c^2*x^2+1))^2*x^2*e-1/12*I/c*b*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^
2+1))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^2*x^2*e
-1/12*I/c*b*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1))^2*csgn(I*((1+I*c*x)^2/(c
^2*x^2+1)+1)^2)*x^2*e+1/6*I/c*b*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn
(I*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^2*x^2*e-1/12*I/c*b*Pi*csgn(I/((1+I*c*x)^2
/(c^2*x^2+1)+1)^2)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+
1)^2)^2*x^2*e+1/6*I/c^3*b*Pi*ln((1+I*c*x)^2/(c^2*x^2+1)+1)*e*csgn(I*(1+I*c*
x)/(c^2*x^2+1)^(1/2))^2*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))-1/3*I/c^3*b*Pi*ln((
1+I*c*x)^2/(c^2*x^2+1)+1)*e*csgn(I*(1+I*c*x)/(c^2*x^2+1)^(1/2))*csgn(I*(1+I
*c*x)^2/(c^2*x^2+1))^2-1/6*I/c^3*b*Pi*ln((1+I*c*x)^2/(c^2*x^2+1)+1)*e*csgn(
I*(1+I*c*x)^2/(c^2*x^2+1))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2
*x^2+1)+1)^2)^2-1/6*I/c^3*b*Pi*ln((1+I*c*x)^2/(c^2*x^2+1)+1)*e*csgn(I*((1+I
*c*x)^2/(c^2*x^2+1)+1))^2*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)+1/3*I/c^3*b
*Pi*ln((1+I*c*x)^2/(c^2*x^2+1)+1)*e*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1))*csg
n(I*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^2-1/6*I/c^3*b*Pi*ln((1+I*c*x)^2/(c^2*x^2
```

+1)+1)*e*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^2+1/12*I/c^3*b*Pi*e*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)-1/6*I*b*arctan(c*x)*Pi*csgn(I*(1+I*c*x)/(c^2*x^2+1)^(1/2))^2*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*x^3*e+1/3*I*b*arctan(c*x)*Pi*csgn(I*(1+I*c*x)/(c^2*x^2+1)^(1/2))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))^2*x^3*e+1/6*I*b*arctan(c*x)*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^2*x^3*e+1/6*I*b*arctan(c*x)*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1))^2*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*x^3*e-1/3*I*b*arctan(c*x)*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^2*x^3*e+1/6*I*b*arctan(c*x)*Pi*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^2*x^3*e+1/6/c^3*b*arctan(c*x)*Pi*e*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)+1/3*x^3*a*d+2/3*b*e*x*arctan(c*x)/c^2+2/3*b*arctan(c*x)*ln(2)*x^3*e-2/3*b*arctan(c*x)*ln((1+I*c*x)^2/(c^2*x^2+1)+1)*x^3*e-1/3/c*b*ln(2)*x^2*e+1/3/c*b*ln((1+I*c*x)^2/(c^2*x^2+1)+1)*x^2*e-2/3/c^3*b*ln(2)*ln((1+I*c*x)^2/(c^2*x^2+1)+1)*e+1/3*I/c^3*b*d*arctan(c*x)-8/9*I/c^3*b*arctan(c*x)*e-1/6/c^3*b*d+5/18/c^3*b*e-1/6*I*b*arctan(c*x)*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*x^3*e+1/12*I/c*b*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*x^2*e+1/6*I/c^3*b*Pi*ln((1+I*c*x)^2/(c^2*x^2+1)+1)*e*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)+1/6/c^3*b*arctan(c*x)*Pi*e*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))^3-1/6/c^3*b*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^3*arctan(c*x)*Pi*e+1/6/c^3*b*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^3*arctan(c*x)*Pi*e+1/12*I/c^3*b*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))^3*Pi*e-1/12*I/c^3*b*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^3*Pi*e+1/12*I/c^3*b*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^3*Pi*e+2/3*I/c^3*b*ln(2)*arctan(c*x)*e+1/6*I/c^3*b*Pi*ln((1+I*c*x)^2/(c^2*x^2+1)+1)*e*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^3-1/3/c^3*b*arctan(c*x)*Pi*e*csgn(I*(1+I*c*x)/(c^2*x^2+1)^(1/2))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))^2-1/6/c^3*b*arctan(c*x)*Pi*e*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^2-1/6/c^3*b*arctan(c*x)*Pi*e*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1))^2*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)+1/3/c^3*b*arctan(c*x)*Pi*e*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^2-1/6/c^3*b*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^2*arctan(c*x)*Pi*e*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)+1/6/c^3*b*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*csgn(I*(1+I*c*x)/(c^2*x^2+1)^(1/2))^2*arctan(c*x)*Pi*e-1/6*I*b*arctan(c*x)*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))^3*x^3*e+1/6*I*b*arctan(c*x)*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^3*x^3*e+1/3/c^3*b*e*(2*arctan(c*x)*x^3*c^3-c^2*x^2+2*I*arctan(c*x)-2*ln((1+I*c*x)^2/(c^2*x^2+1)+1)-1)*ln((1+I*c*x)/(c^2*x^2+1)^(1/2))

Maxima [A] time = 1.50211, size = 286, normalized size = 1.34

$$\frac{1}{3}adx^3 + \frac{1}{9}\left(3x^3\log\left(c^2x^2+1\right) - 2c^2\left(\frac{c^2x^3-3x}{c^4} + \frac{3\arctan(cx)}{c^5}\right)\right)be\arctan(cx) + \frac{1}{6}\left(2x^3\arctan(cx) - c\left(\frac{x^2}{c^2} - \frac{\log\left(c^2\right.}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))*(d+e*log(c^2*x^2+1)),x, algorithm="maxima")
```

```
[Out] 1/3*a*d*x^3 + 1/9*(3*x^3*log(c^2*x^2 + 1) - 2*c^2*((c^2*x^3 - 3*x)/c^4 + 3*
arctan(c*x)/c^5))*b*e*arctan(c*x) + 1/6*(2*x^3*arctan(c*x) - c*(x^2/c^2 - l
og(c^2*x^2 + 1)/c^4))*b*d + 1/9*(3*x^3*log(c^2*x^2 + 1) - 2*c^2*((c^2*x^3 -
 3*x)/c^4 + 3*arctan(c*x)/c^5))*a*e + 1/36*(10*c^2*x^2 + 12*arctan(c*x)^2 -
 2*(3*c^2*x^2 + 11)*log(c^2*x^2 + 1) + 3*log(c^2*x^2 + 1)^2)*b*e/c^3
```

Fricas [A] time = 1.29493, size = 406, normalized size = 1.91

$$\frac{24\,acex + 4\left(3\,ac^3d - 2\,ac^3e\right)x^3 - 12\,be\arctan(cx)^2 + 3\,be\log\left(c^2x^2+1\right)^2 - 2\left(3\,bc^2d - 5\,bc^2e\right)x^2 + 4\left(6\,bcex + \left(3\,bc^3d - 2\right.\right.}{36\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))*(d+e*log(c^2*x^2+1)),x, algorithm="fricas")
```

```
[Out] 1/36*(24*a*c*e*x + 4*(3*a*c^3*d - 2*a*c^3*e)*x^3 - 12*b*e*arctan(c*x)^2 + 3
*b*e*log(c^2*x^2 + 1)^2 - 2*(3*b*c^2*d - 5*b*c^2*e)*x^2 + 4*(6*b*c*e*x + (3
*b*c^3*d - 2*b*c^3*e)*x^3 - 6*a*e)*arctan(c*x) + 2*(6*b*c^3*e*x^3*arctan(c*
x) + 6*a*c^3*e*x^3 - 3*b*c^2*e*x^2 + 3*b*d - 11*b*e)*log(c^2*x^2 + 1))/c^3
```

Sympy [A] time = 12.9119, size = 258, normalized size = 1.21

$$\begin{cases} \frac{adx^3}{3} + \frac{aex^3\log\left(c^2x^2+1\right)}{3} - \frac{2aex^3}{9} + \frac{2aex}{3c^2} - \frac{2ae\,\mathrm{atan}\,(cx)}{3c^3} + \frac{bdx^3\,\mathrm{atan}\,(cx)}{3} + \frac{bex^3\log\left(c^2x^2+1\right)\mathrm{atan}\,(cx)}{3} - \frac{2bex^3\,\mathrm{atan}\,(cx)}{9} - \frac{bdx^2}{6c} - \frac{bex^2\log\left(c^2\right.}{6c} \\ \frac{adx^3}{3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atan(c*x))*(d+e*ln(c**2*x**2+1)),x)
```

```
[Out] Piecewise((a*d*x**3/3 + a*e*x**3*log(c**2*x**2 + 1)/3 - 2*a*e*x**3/9 + 2*a*
e*x/(3*c**2) - 2*a*e*atan(c*x)/(3*c**3) + b*d*x**3*atan(c*x)/3 + b*e*x**3*l
og(c**2*x**2 + 1)*atan(c*x)/3 - 2*b*e*x**3*atan(c*x)/9 - b*d*x**2/(6*c) - b
*e*x**2*log(c**2*x**2 + 1)/(6*c) + 5*b*e*x**2/(18*c) + 2*b*e*x*atan(c*x)/(3
*c**2) + b*d*log(c**2*x**2 + 1)/(6*c**3) + b*e*log(c**2*x**2 + 1)**2/(12*c*
*3) - 11*b*e*log(c**2*x**2 + 1)/(18*c**3) - b*e*atan(c*x)**2/(3*c**3), Ne(c
, 0)), (a*d*x**3/3, True))
```

Giac [A] time = 1.33773, size = 486, normalized size = 2.28

$$\frac{6\,\pi bc^3x^3e\log\left(c^2x^2+1\right)\operatorname{sgn}(c)\operatorname{sgn}(x)-4\,\pi bc^3x^3e\operatorname{sgn}(c)\operatorname{sgn}(x)-12\,bc^3x^3\arctan\left(\frac{1}{cx}\right)e\log\left(c^2x^2+1\right)+12\,bc^3dx^3\,\text{ar}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))*(d+e*log(c^2*x^2+1)),x, algorithm="giac")
```

```
[Out] 1/36*(6*pi*b*c^3*x^3*e*log(c^2*x^2 + 1)*sgn(c)*sgn(x) - 4*pi*b*c^3*x^3*e*sg
n(c)*sgn(x) - 12*b*c^3*x^3*arctan(1/(c*x))*e*log(c^2*x^2 + 1) + 12*b*c^3*d*
x^3*arctan(c*x) + 8*b*c^3*x^3*arctan(1/(c*x))*e + 12*a*c^3*x^3*e*log(c^2*x^
2 + 1) + 12*a*c^3*d*x^3 - 8*a*c^3*x^3*e - 6*b*c^2*x^2*e*log(c^2*x^2 + 1) +
12*pi*b*c*x*e*sgn(c)*sgn(x) - 6*b*c^2*d*x^2 + 10*b*c^2*x^2*e + 6*pi^2*b*e*s
gn(c)*sgn(x) + 12*pi*b*arctan(1/(c*x))*e*sgn(c)*sgn(x) - 24*b*c*x*arctan(1/
(c*x))*e - 6*pi^2*b*e + 24*a*c*x*e - 12*pi*b*arctan(c*x)*e - 12*pi*b*arctan
(1/(c*x))*e - 12*b*arctan(1/(c*x))^2*e + 3*b*e*log(c^2*x^2 + 1)^2 - 24*a*ar
ctan(c*x)*e + 6*b*d*log(c^2*x^2 + 1) - 22*b*e*log(c^2*x^2 + 1))/c^3
```

3.1289 $\int x\left(a+b\tan^{-1}(cx)\right)\left(d+e\log\left(1+c^2x^2\right)\right)\,dx$

Optimal. Leaf size=137

$$\frac{e\left(c^2x^2+1\right)\log\left(c^2x^2+1\right)\left(a+b\tan^{-1}(cx)\right)}{2c^2}+\frac{1}{2}dx^2\left(a+b\tan^{-1}(cx)\right)-\frac{1}{2}ex^2\left(a+b\tan^{-1}(cx)\right)+\frac{b(d-e)\tan^{-1}(cx)}{2c^2}-\frac{b}{}$$

```
[Out] -(b*(d - e)*x)/(2*c) + (b*e*x)/c + (b*(d - e)*ArcTan[c*x])/(2*c^2) - (b*e*A
rcTan[c*x])/c^2 + (d*x^2*(a + b*ArcTan[c*x]))/2 - (e*x^2*(a + b*ArcTan[c*x]
))/2 - (b*e*x*Log[1 + c^2*x^2])/(2*c) + (e*(1 + c^2*x^2)*(a + b*ArcTan[c*x]
)*Log[1 + c^2*x^2])/(2*c^2)
```

Rubi [A] time = 0.110792, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {2454, 2389, 2295, 5019, 321, 203, 2448}

$$\frac{e\left(c^2x^2+1\right)\log\left(c^2x^2+1\right)\left(a+b\tan^{-1}(cx)\right)}{2c^2}+\frac{1}{2}dx^2\left(a+b\tan^{-1}(cx)\right)-\frac{1}{2}ex^2\left(a+b\tan^{-1}(cx)\right)+\frac{b(d-e)\tan^{-1}(cx)}{2c^2}-\frac{b}{}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]),x]
```


```
[Out] -(b*(d - e)*x)/(2*c) + (b*e*x)/c + (b*(d - e)*ArcTan[c*x])/(2*c^2) - (b*e*A
rcTan[c*x])/c^2 + (d*x^2*(a + b*ArcTan[c*x]))/2 - (e*x^2*(a + b*ArcTan[c*x]
))/2 - (b*e*x*Log[1 + c^2*x^2])/(2*c) + (e*(1 + c^2*x^2)*(a + b*ArcTan[c*x]
)*Log[1 + c^2*x^2])/(2*c^2)
```


Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 5019

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*(
e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*Log[f + g*x^2])
, x]}, Dist[a + b*ArcTan[c*x], u, x] - Dist[b*c, Int[ExpandIntegrand[u/(1 +
 c^2*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[(m + 1)/2
, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rubi steps

$$\int x\left(a+b\tan^{-1}(cx)\right)\left(d+e\log\left(1+c^2x^2\right)\right)dx = \frac{1}{2}dx^2\left(a+b\tan^{-1}(cx)\right)-\frac{1}{2}ex^2\left(a+b\tan^{-1}(cx)\right)+\frac{e\left(1+c^2x^2\right)\left(a+b\,\mathrm{t}\right.}{}$$

$$= \frac{1}{2}dx^2\left(a+b\tan^{-1}(cx)\right)-\frac{1}{2}ex^2\left(a+b\tan^{-1}(cx)\right)+\frac{e\left(1+c^2x^2\right)\left(a+b\,\mathrm{t}\right.}{}$$

$$= -\frac{b(d-e)x}{2c}+\frac{1}{2}dx^2\left(a+b\tan^{-1}(cx)\right)-\frac{1}{2}ex^2\left(a+b\tan^{-1}(cx)\right)-\frac{bex\,\mathrm{lo}}{}$$

$$= -\frac{b(d-e)x}{2c}+\frac{bex}{c}+\frac{b(d-e)\tan^{-1}(cx)}{2c^2}+\frac{1}{2}dx^2\left(a+b\tan^{-1}(cx)\right)-\frac{1}{2}ex$$

$$= -\frac{b(d-e)x}{2c}+\frac{bex}{c}+\frac{b(d-e)\tan^{-1}(cx)}{2c^2}-\frac{be\tan^{-1}(cx)}{c^2}+\frac{1}{2}dx^2\left(a+b\,\mathrm{ta}\right.$$

Mathematica [A] time = 0.0892493, size = 105, normalized size = 0.77

$$\frac{e\log\left(c^2x^2+1\right)\left(ac^2x^2+a-bcx\right)+cx(acx(d-e)-b(d-3e))+b\tan^{-1}(cx)\left(c^2dx^2-e\left(c^2x^2+3\right)+\left(c^2ex^2+e\right)\log\left(c^2x^2\right.\right.}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]),x]
```

```
[Out] (c*x*(-(b*(d - 3*e)) + a*c*(d - e)*x) + e*(a - b*c*x + a*c^2*x^2)*Log[1 + c
^2*x^2] + b*ArcTan[c*x]*(d + c^2*d*x^2 - e*(3 + c^2*x^2) + (e + c^2*e*x^2)*
Log[1 + c^2*x^2]))/(2*c^2)
```

Maple [C] time = 0.785, size = 3074, normalized size = 22.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctan(c*x))*(d+e*ln(c^2*x^2+1)),x)
```

```
[Out] -1/4*I/c^2*b*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))^3*Pi*e*arctan(c*x)-1/2*b*d*x/c
+1/2*b*d*arctan(c*x)/c^2+1/2*b*arctan(c*x)*x^2*d-1/2*b*arctan(c*x)*x^2*e-1/
2*I/c^2*b*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^2*csgn(I*((1+I*c*x)^2/(c^2*
x^2+1)+1))*Pi*e*arctan(c*x)+1/2*I/c^2*b*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))^2*c
```

sgn(I*(1+I*c*x)/(c^2*x^2+1)^(1/2))*Pi*e*arctan(c*x)+1/4*I/c^2*b*csgn(I*(1+I
*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^2*csgn(I*(1+I*c*x)^2/(c^
2*x^2+1))*Pi*e*arctan(c*x)-1/4*I/c^2*b*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*csgn
(I*(1+I*c*x)/(c^2*x^2+1)^(1/2))^2*Pi*e*arctan(c*x)+1/4*I/c^2*b*csgn(I*(1+I*
c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^2*csgn(I/((1+I*c*x)^2/(c^
2*x^2+1)+1)^2)*Pi*e*arctan(c*x)+1/4*I*b*arctan(c*x)*Pi*csgn(I*((1+I*c*x)^2/
(c^2*x^2+1)+1))^2*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*x^2*e-1/2*I*b*arcta
n(c*x)*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+
1)+1)^2)^2*x^2*e+1/2*I*b*arctan(c*x)*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))^2*c
sgn(I*(1+I*c*x)/(c^2*x^2+1)^(1/2))*x^2*e+1/4*I*b*arctan(c*x)*Pi*csgn(I*(1+I
*c*x)^2/(c^2*x^2+1))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1
)+1)^2)^2*x^2*e-1/4*I*b*arctan(c*x)*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*csgn
(I*(1+I*c*x)/(c^2*x^2+1)^(1/2))^2*x^2*e+1/4*I*b*arctan(c*x)*Pi*csgn(I*(1+I*
c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^2*csgn(I/((1+I*c*x)^2/(c^
2*x^2+1)+1)^2)*x^2*e-1/2*a*x^2*e-1/4*I/c^2*b*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)
/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^3*Pi*e*arctan(c*x)-1/4/c^2*b*e*Pi*csgn(I/((
1+I*c*x)^2/(c^2*x^2+1)+1)^2)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*csgn(I*(1+I*c*
x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)-5/2/c^2*b*e*arctan(c*x)+3/2
*b*e*x/c+1/2*x^2*a*d+1/4/c^2*b*e*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^3
-1/4/c^2*b*e*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))^3-1/4/c^2*b*e*Pi*csgn(I*(1+
I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^3+1/c*b*ln((1+I*c*x)^2/
(c^2*x^2+1)+1)*x*e-1/c*b*ln(2)*x*e-1/c^2*b*ln((1+I*c*x)^2/(c^2*x^2+1)+1)*e*
arctan(c*x)+1/c^2*b*ln(2)*e*arctan(c*x)-b*arctan(c*x)*ln((1+I*c*x)^2/(c^2*x
^2+1)+1)*x^2*e+b*arctan(c*x)*ln(2)*x^2*e-1/2*a*e/c^2+1/2*x^2*a*e*ln(c^2*x^2
+1)+1/2*a*e/c^2*ln(c^2*x^2+1)+1/4*I*b*arctan(c*x)*Pi*csgn(I*((1+I*c*x)^2/(c
^2*x^2+1)+1)^2)^3*x^2*e-1/4*I*b*arctan(c*x)*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+
1))^3*x^2*e-1/4*I*b*arctan(c*x)*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x
)^2/(c^2*x^2+1)+1)^2)^3*x^2*e-1/4*I/c*b*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+
1)^2)^3*x*e+1/4*I/c*b*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))^3*x*e+1/4*I/c*b*Pi
csgn(I(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^3*x*e+1/4*I/
c^2*b*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^3*Pi*e*arctan(c*x)-1/4*I*b*arct
an(c*x)*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/(
(1+I*c*x)^2/(c^2*x^2+1)+1)^2)*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*x^2*e+1
/4*I/c*b*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/
((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*x*e-1/
4*I/c^2*b*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*csg
n(I*(1+I*c*x)^2/(c^2*x^2+1))*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*Pi*e*arc
tan(c*x)-1/4*I/c*b*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1))^2*csgn(I*((1+I*c*
x)^2/(c^2*x^2+1)+1)^2)*x*e+1/2*I/c*b*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1))
csgn(I((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^2*x*e-1/2*I/c*b*Pi*csgn(I*(1+I*c*x)^
2/(c^2*x^2+1))^2*csgn(I*(1+I*c*x)/(c^2*x^2+1)^(1/2))*x*e-1/4*I/c*b*Pi*csgn(
I*(1+I*c*x)^2/(c^2*x^2+1))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2
*x^2+1)+1)^2)^2*x*e+1/4*I/c*b*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*csgn(I*(1+
I*c*x)/(c^2*x^2+1)^(1/2))^2*x*e-1/4*I/c*b*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)
/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^2*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*x*e

```
+1/4*I/c^2*b*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*csgn(I*((1+I*c*x)^2/(c^2
*x^2+1)+1))^2*Pi*e*arctan(c*x)-I/c^2*b*e*ln(2)-1/2*I/c^2*b*d+3/2*I/c^2*b*e+
1/4/c^2*b*e*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1))^2*csgn(I*((1+I*c*x)^2/(c
^2*x^2+1)+1)^2)-1/2/c^2*b*e*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*(
(1+I*c*x)^2/(c^2*x^2+1)+1)^2)^2+1/2/c^2*b*e*Pi*csgn(I*(1+I*c*x)/(c^2*x^2+1)
^(1/2))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))^2+1/4/c^2*b*e*Pi*csgn(I*(1+I*c*x)^2
/(c^2*x^2+1))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)
^2-1/4/c^2*b*e*Pi*csgn(I*(1+I*c*x)/(c^2*x^2+1)^(1/2))^2*csgn(I*(1+I*c*x)^2/
(c^2*x^2+1))+1/4/c^2*b*e*Pi*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*csgn(I*(1
+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^2+1/c^2*b*e*(arctan(c*
x)*x*c-I*arctan(c*x)-1)*(c*x+I)*ln((1+I*c*x)/(c^2*x^2+1)^(1/2))
```

Maxima [A] time = 1.4501, size = 201, normalized size = 1.47

$$\frac{1}{2}\,adx^2 + \frac{1}{2}\left(x^2\arctan(cx) - c\left(\frac{x}{c^2} - \frac{\arctan(cx)}{c^3}\right)\right)bd - \frac{\left(x\log\left(c^2x^2+1\right) - 3\,x + \frac{2\arctan(cx)}{c}\right)be}{2\,c} - \frac{\left(c^2x^2 - \left(c^2x^2+1\right)\log\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))*(d+e*log(c^2*x^2+1)),x, algorithm="maxima")
```

```
[Out] 1/2*a*d*x^2 + 1/2*(x^2*arctan(c*x) - c*(x/c^2 - arctan(c*x)/c^3))*b*d - 1/2
*(x*log(c^2*x^2 + 1) - 3*x + 2*arctan(c*x)/c)*b*e/c - 1/2*(c^2*x^2 - (c^2*x
^2 + 1)*log(c^2*x^2 + 1) + 1)*b*e*arctan(c*x)/c^2 - 1/2*(c^2*x^2 - (c^2*x^2
 + 1)*log(c^2*x^2 + 1) + 1)*a*e/c^2
```

Fricas [A] time = 1.44882, size = 262, normalized size = 1.91

$$\frac{\left(ac^2d - ac^2e\right)x^2 - (bcd - 3\,bce)x + \left(\left(bc^2d - bc^2e\right)x^2 + bd - 3\,be\right)\arctan(cx) + \left(ac^2ex^2 - bcex + ae + \left(bc^2ex^2 + be\right)\mathrm{arctar}\right.}{2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))*(d+e*log(c^2*x^2+1)),x, algorithm="fricas")
```

```
[Out] 1/2*((a*c^2*d - a*c^2*e)*x^2 - (b*c*d - 3*b*c*e)*x + ((b*c^2*d - b*c^2*e)*x
^2 + b*d - 3*b*e)*arctan(c*x) + (a*c^2*e*x^2 - b*c*e*x + a*e + (b*c^2*e*x^2
 + b*e)*arctan(c*x))*log(c^2*x^2 + 1))/c^2
```

Sympy [A] time = 6.0819, size = 202, normalized size = 1.47

$$\begin{cases} \frac{adx^2}{2} + \frac{aex^2\log\left(c^2x^2+1\right)}{2} - \frac{aex^2}{2} + \frac{ae\log\left(c^2x^2+1\right)}{2c^2} + \frac{bdx^2\operatorname{atan}(cx)}{2} + \frac{bex^2\log\left(c^2x^2+1\right)\operatorname{atan}(cx)}{2} - \frac{bex^2\operatorname{atan}(cx)}{2} - \frac{bdx}{2c} - \frac{bex\log\left(c^2x^2+1\right)}{2c} + \\ \frac{adx^2}{2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atan(c*x))*(d+e*ln(c**2*x**2+1)),x)
```

```
[Out] Piecewise((a*d*x**2/2 + a*e*x**2*log(c**2*x**2 + 1)/2 - a*e*x**2/2 + a*e*lo
g(c**2*x**2 + 1)/(2*c**2) + b*d*x**2*atan(c*x)/2 + b*e*x**2*log(c**2*x**2 +
 1)*atan(c*x)/2 - b*e*x**2*atan(c*x)/2 - b*d*x/(2*c) - b*e*x*log(c**2*x**2
+ 1)/(2*c) + 3*b*e*x/(2*c) + b*d*atan(c*x)/(2*c**2) + b*e*log(c**2*x**2 + 1
)*atan(c*x)/(2*c**2) - 3*b*e*atan(c*x)/(2*c**2), Ne(c, 0)), (a*d*x**2/2, Tr
ue))
```

Giac [B] time = 1.31502, size = 398, normalized size = 2.91

$$-\frac{3\,b\arctan(cx)\,e}{2\,c^2} + \frac{(\pi be + 2\,ae)\log\left(c^2x^2+1\right)}{4\,c^2} + \frac{\pi bc^2x^2e\log\left(c^2x^2+1\right)\operatorname{sgn}(c)\operatorname{sgn}(x) - \pi bc^2x^2e\operatorname{sgn}(c)\operatorname{sgn}(x) - 2\,bc}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))*(d+e*log(c^2*x^2+1)),x, algorithm="giac")
```

```
[Out] -3/2*b*arctan(c*x)*e/c^2 + 1/4*(pi*b*e + 2*a*e)*log(c^2*x^2 + 1)/c^2 + 1/4*
(pi*b*c^2*x^2*e*log(c^2*x^2 + 1)*sgn(c)*sgn(x) - pi*b*c^2*x^2*e*sgn(c)*sgn(
x) - 2*b*c^2*x^2*arctan(1/(c*x))*e*log(c^2*x^2 + 1) + 2*b*c^2*d*x^2*arctan(
c*x) + 2*b*c^2*x^2*arctan(1/(c*x))*e + 2*a*c^2*x^2*e*log(c^2*x^2 + 1) + 2*a
*c^2*d*x^2 - 2*a*c^2*x^2*e + pi*b*e*log(c^2*x^2 + 1)*sgn(c)*sgn(x) - 2*b*c*
x*e*log(c^2*x^2 + 1) - 2*pi*b*d*sgn(c)*sgn(x) - 2*b*c*d*x + 6*b*c*x*e - pi*
b*e*log(c^2*x^2 + 1) - 2*b*arctan(1/(c*x))*e*log(c^2*x^2 + 1) + 2*b*d*arcta
n(c*x))/c^2
```

3.1290 $\int \left(a+b\tan^{-1}(cx)\right)\left(d+e\log\left(1+c^2x^2\right)\right)\,dx$

Optimal. Leaf size=100

$$x\left(a+b\tan^{-1}(cx)\right)\left(e\log\left(c^2x^2+1\right)+d\right)+\frac{e\left(a+b\tan^{-1}(cx)\right)^2}{bc}-2aex-\frac{b\left(e\log\left(c^2x^2+1\right)+d\right)^2}{4ce}+\frac{be\log\left(c^2x^2+1\right)}{c}-2$$

```
[Out] -2*a*e*x - 2*b*e*x*ArcTan[c*x] + (e*(a + b*ArcTan[c*x])^2)/(b*c) + (b*e*Log
[1 + c^2*x^2])/c + x*(a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]) - (b*(d +
 e*Log[1 + c^2*x^2])^2)/(4*c*e)
```

Rubi [A] time = 0.189008, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {5009, 2475, 2390, 2301, 4916, 4846, 260, 4884}

$$x\left(a+b\tan^{-1}(cx)\right)\left(e\log\left(c^2x^2+1\right)+d\right)+\frac{e\left(a+b\tan^{-1}(cx)\right)^2}{bc}-2aex-\frac{b\left(e\log\left(c^2x^2+1\right)+d\right)^2}{4ce}+\frac{be\log\left(c^2x^2+1\right)}{c}-2$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]),x]
```

```
[Out] -2*a*e*x - 2*b*e*x*ArcTan[c*x] + (e*(a + b*ArcTan[c*x])^2)/(b*c) + (b*e*Log
[1 + c^2*x^2])/c + x*(a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]) - (b*(d +
 e*Log[1 + c^2*x^2])^2)/(4*c*e)
```

Rule 5009

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*(
e_.)), x_Symbol] :> Simp[x*(d + e*Log[f + g*x^2])*(a + b*ArcTan[c*x]), x] +
 (-Dist[b*c, Int[(x*(d + e*Log[f + g*x^2]))/(1 + c^2*x^2), x], x] - Dist[2*
e*g, Int[(x^2*(a + b*ArcTan[c*x]))/(f + g*x^2), x], x]) /; FreeQ[{a, b, c,
d, e, f, g}, x]
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
```

```
|| IGtQ[q, 0])
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule steps

$$\int \left(a+b\tan^{-1}(cx)\right)\left(d+e\log\left(1+c^2x^2\right)\right)dx = x\left(a+b\tan^{-1}(cx)\right)\left(d+e\log\left(1+c^2x^2\right)\right)-(bc)\int \frac{x\left(d+e\log\left(1+c^2x^2\right)\right.}{1+c^2x^2}$$

$$= x\left(a+b\tan^{-1}(cx)\right)\left(d+e\log\left(1+c^2x^2\right)\right)-\frac{1}{2}(bc)\,\text{Subst}\left(\int \frac{d+e\log\left(1\right.}{1+c^2}\right.$$

$$= -2aex+\frac{e\left(a+b\tan^{-1}(cx)\right)^2}{bc}+x\left(a+b\tan^{-1}(cx)\right)\left(d+e\log\left(1+c^2x^2\right)\right)$$

$$= -2aex-2bex\tan^{-1}(cx)+\frac{e\left(a+b\tan^{-1}(cx)\right)^2}{bc}+x\left(a+b\tan^{-1}(cx)\right)\left(d\right.$$

$$= -2aex-2bex\tan^{-1}(cx)+\frac{e\left(a+b\tan^{-1}(cx)\right)^2}{bc}+\frac{be\log\left(1+c^2x^2\right)}{c}+x\left(\right.$$

Mathematica [A] time = 0.0157285, size = 138, normalized size = 1.38

$$aex\log\left(c^2x^2+1\right)+\frac{2ae\tan^{-1}(cx)}{c}+adx-2aex-\frac{bd\log\left(c^2x^2+1\right)}{2c}-\frac{be\log^2\left(c^2x^2+1\right)}{4c}+\frac{be\log\left(c^2x^2+1\right)}{c}+bex\log\left(\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]),x]
```

```
[Out] a*d*x - 2*a*e*x + (2*a*e*ArcTan[c*x])/c + b*d*x*ArcTan[c*x] - 2*b*e*x*ArcTa
n[c*x] + (b*e*ArcTan[c*x]^2)/c - (b*d*Log[1 + c^2*x^2])/(2*c) + (b*e*Log[1
+ c^2*x^2])/c + a*e*x*Log[1 + c^2*x^2] + b*e*x*ArcTan[c*x]*Log[1 + c^2*x^2]
 - (b*e*Log[1 + c^2*x^2]^2)/(4*c)
```

Maple [A] time = 0.156, size = 192, normalized size = 1.9

$$axd+bd\arctan(cx)x-\frac{bd\ln\left(c^2x^2+1\right)}{2c}+\frac{be}{c}\ln\left(2\left(1+\frac{-c^2x^2+1}{c^2x^2+1}\right)^{-1}\right)+\frac{b(\arctan(cx))^2e}{c}-\frac{be}{4c}\left(\ln\left(2\left(1+\frac{-c^2x^2+1}{c^2x^2+1}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))*(d+e*ln(c^2*x^2+1)),x)
```

```
[Out] a*x*d+b*d*arctan(c*x)*x-1/2/c*b*d*ln(c^2*x^2+1)+1/c*b*e*ln(2/(1+(-c^2*x^2+1
)/(c^2*x^2+1)))+1/c*b*arctan(c*x)^2*e-1/4/c*b*e*ln(2/(1+(-c^2*x^2+1)/(c^2*x
```

```
^2+1)))^2-2*b*e*x*arctan(c*x)+b*e*arctan(c*x)*x*ln(2/(1+(-c^2*x^2+1)/(c^2*x
^2+1)))+a*x*e*ln(c^2*x^2+1)-2*a*e*x+2*a*e/c*arctan(c*x)
```

Maxima [A] time = 1.49276, size = 207, normalized size = 2.07

$$-\left(2\,c^2\left(\frac{x}{c^2}-\frac{\arctan\left(cx\right)}{c^3}\right)-x\log\left(c^2x^2+1\right)\right)be\arctan\left(cx\right)-\left(2\,c^2\left(\frac{x}{c^2}-\frac{\arctan\left(cx\right)}{c^3}\right)-x\log\left(c^2x^2+1\right)\right)ae+adx+\frac{(2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))*(d+e*log(c^2*x^2+1)),x, algorithm="maxima")
```

```
[Out] -(2*c^2*(x/c^2 - arctan(c*x)/c^3) - x*log(c^2*x^2 + 1))*b*e*arctan(c*x) - (
2*c^2*(x/c^2 - arctan(c*x)/c^3) - x*log(c^2*x^2 + 1))*a*e + a*d*x + 1/2*(2*
c*x*arctan(c*x) - log(c^2*x^2 + 1))*b*d/c - 1/4*(4*arctan(c*x)^2 + log(c^2*
x^2 + 1)^2 - 4*log(c^2*x^2 + 1))*b*e/c
```

Fricas [A] time = 1.43958, size = 263, normalized size = 2.63

$$\frac{4\,be\arctan\left(cx\right)^2-be\log\left(c^2x^2+1\right)^2+4\left(acd-2\,ace\right)x+4\left(2\,ae+\left(bcd-2\,bce\right)x\right)\arctan\left(cx\right)+2\left(2\,bcex\arctan\left(cx\right)+\right.}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))*(d+e*log(c^2*x^2+1)),x, algorithm="fricas")
```

```
[Out] 1/4*(4*b*e*arctan(c*x)^2 - b*e*log(c^2*x^2 + 1)^2 + 4*(a*c*d - 2*a*c*e)*x +
 4*(2*a*e + (b*c*d - 2*b*c*e)*x)*arctan(c*x) + 2*(2*b*c*e*x*arctan(c*x) + 2
*a*c*e*x - b*d + 2*b*e)*log(c^2*x^2 + 1))/c
```

Sympy [A] time = 2.9767, size = 148, normalized size = 1.48

$$\begin{cases} adx+aex\log\left(c^2x^2+1\right)-2aex+\frac{2ae\operatorname{atan}\left(cx\right)}{c}+bdx\operatorname{atan}\left(cx\right)+bex\log\left(c^2x^2+1\right)\operatorname{atan}\left(cx\right)-2bex\operatorname{atan}\left(cx\right)-\frac{bd\log(c}{2c} \\ adx \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))*(d+e*ln(c**2*x**2+1)),x)
```

```
[Out] Piecewise((a*d*x + a*e*x*log(c**2*x**2 + 1) - 2*a*e*x + 2*a*e*atan(c*x)/c +
 b*d*x*atan(c*x) + b*e*x*log(c**2*x**2 + 1)*atan(c*x) - 2*b*e*x*atan(c*x) -
 b*d*log(c**2*x**2 + 1)/(2*c) - b*e*log(c**2*x**2 + 1)**2/(4*c) + b*e*log(c
**2*x**2 + 1)/c + b*e*atan(c*x)**2/c, Ne(c, 0)), (a*d*x, True))
```

Giac [B] time = 1.19696, size = 354, normalized size = 3.54

$$\frac{2\,\pi bcxe\log\left(c^2x^2+1\right)\operatorname{sgn}(c)\operatorname{sgn}(x) - 4\,\pi bcxe\operatorname{sgn}(c)\operatorname{sgn}(x) - 4\,bcx\arctan\left(\frac{1}{cx}\right)e\log\left(c^2x^2+1\right) - 6\,\pi^2 be\operatorname{sgn}(c)\operatorname{sgn}(x)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))*(d+e*log(c^2*x^2+1)),x, algorithm="giac")
```

```
[Out] 1/4*(2*pi*b*c*x*e*log(c^2*x^2 + 1)*sgn(c)*sgn(x) - 4*pi*b*c*x*e*sgn(c)*sgn(
x) - 4*b*c*x*arctan(1/(c*x))*e*log(c^2*x^2 + 1) - 6*pi^2*b*e*sgn(c)*sgn(x)
- 4*pi*b*arctan(1/(c*x))*e*sgn(c)*sgn(x) + 4*b*c*d*x*arctan(c*x) + 8*b*c*x*
arctan(1/(c*x))*e + 4*a*c*x*e*log(c^2*x^2 + 1) - 8*pi*a*e*sgn(c)*sgn(x) + 4
*a*c*d*x + 2*pi^2*b*e - 8*a*c*x*e + 4*pi*b*arctan(c*x)*e + 4*pi*b*arctan(1/
(c*x))*e + 4*b*arctan(1/(c*x))^2*e - b*e*log(c^2*x^2 + 1)^2 + 8*a*arctan(c*
x)*e - 2*b*d*log(c^2*x^2 + 1) + 4*b*e*log(c^2*x^2 + 1))/c
```

3.1291 $\int \frac{(a+b\tan^{-1}(cx))(d+e\log(1+c^2x^2))}{x}\,dx$

Optimal. Leaf size=282

$$-\frac{1}{2}ae\text{PolyLog}\left(2,-c^2x^2\right)-\frac{1}{2}ibe\left(-\log\left(c^2x^2+1\right)+\log(1-icx)+\log(1+icx)\right)\text{PolyLog}(2,-icx)+\frac{1}{2}ibe\left(-\log\left(c^2x^2+\right.\right.$$

```
[Out] a*d*Log[x] + (I/2)*b*e*Log[I*c*x]*Log[1 - I*c*x]^2 - (I/2)*b*e*Log[(-I)*c*x
]*Log[1 + I*c*x]^2 + (I/2)*b*d*PolyLog[2, (-I)*c*x] - (I/2)*b*e*(Log[1 - I*
c*x] + Log[1 + I*c*x] - Log[1 + c^2*x^2])*PolyLog[2, (-I)*c*x] - (I/2)*b*d*
PolyLog[2, I*c*x] + (I/2)*b*e*(Log[1 - I*c*x] + Log[1 + I*c*x] - Log[1 + c^
2*x^2])*PolyLog[2, I*c*x] - (a*e*PolyLog[2, -(c^2*x^2)])/2 + I*b*e*Log[1 -
I*c*x]*PolyLog[2, 1 - I*c*x] - I*b*e*Log[1 + I*c*x]*PolyLog[2, 1 + I*c*x] -
 I*b*e*PolyLog[3, 1 - I*c*x] + I*b*e*PolyLog[3, 1 + I*c*x]
```

Rubi [A] time = 0.341867, antiderivative size = 282, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {5015, 4848, 2391, 5013, 5011, 2396, 2433, 2374, 6589}

$$-\frac{1}{2}ae\text{PolyLog}\left(2,-c^2x^2\right)-\frac{1}{2}ibe\left(-\log\left(c^2x^2+1\right)+\log(1-icx)+\log(1+icx)\right)\text{PolyLog}(2,-icx)+\frac{1}{2}ibe\left(-\log\left(c^2x^2+\right.\right.$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]))/x,x]
```


```
[Out] a*d*Log[x] + (I/2)*b*e*Log[I*c*x]*Log[1 - I*c*x]^2 - (I/2)*b*e*Log[(-I)*c*x
]*Log[1 + I*c*x]^2 + (I/2)*b*d*PolyLog[2, (-I)*c*x] - (I/2)*b*e*(Log[1 - I*
c*x] + Log[1 + I*c*x] - Log[1 + c^2*x^2])*PolyLog[2, (-I)*c*x] - (I/2)*b*d*
PolyLog[2, I*c*x] + (I/2)*b*e*(Log[1 - I*c*x] + Log[1 + I*c*x] - Log[1 + c^
2*x^2])*PolyLog[2, I*c*x] - (a*e*PolyLog[2, -(c^2*x^2)])/2 + I*b*e*Log[1 -
I*c*x]*PolyLog[2, 1 - I*c*x] - I*b*e*Log[1 + I*c*x]*PolyLog[2, 1 + I*c*x] -
 I*b*e*PolyLog[3, 1 - I*c*x] + I*b*e*PolyLog[3, 1 + I*c*x]
```


Rule 5015

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(Log[(f_.) + (g_.)*(x_)^2]*(e_.) +
(d_)))/(x_), x_Symbol] :> Dist[d, Int[(a + b*ArcTan[c*x])/x, x], x] + Dist[
e, Int[(Log[f + g*x^2]*(a + b*ArcTan[c*x]))/x, x], x] /; FreeQ[{a, b, c, d,
 e, f, g}, x]
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5013

```
Int[(Log[(f_.) + (g_.)*(x_)^2]*(ArcTan[(c_.)*(x_)]*(b_.) + (a_)))/(x_), x_S
ymbol] :> Dist[a, Int[Log[f + g*x^2]/x, x], x] + Dist[b, Int[(Log[f + g*x^2
]*ArcTan[c*x])/x, x], x] /; FreeQ[{a, b, c, f, g}, x]
```

Rule 5011

```
Int[(ArcTan[(c_.)*(x_)]*Log[(f_.) + (g_.)*(x_)^2])/(x_), x_Symbol] :> Dist[
Log[f + g*x^2] - Log[1 - I*c*x] - Log[1 + I*c*x], Int[ArcTan[c*x]/x, x], x]
 + (Dist[I/2, Int[Log[1 - I*c*x]^2/x, x], x] - Dist[I/2, Int[Log[1 + I*c*x]
^2/x, x], x]) /; FreeQ[{c, f, g}, x] && EqQ[g, c^2*f]
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
```

```
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}(cx)\right)\left(d+e\log\left(1+c^2x^2\right)\right)}{x}\,dx = d\int \frac{a+b\tan^{-1}(cx)}{x}\,dx + e\int \frac{\left(a+b\tan^{-1}(cx)\right)\log\left(1+c^2x^2\right)}{x}\,dx$$

$$= ad\log(x)+\frac{1}{2}(ibd)\int \frac{\log(1-icx)}{x}\,dx-\frac{1}{2}(ibd)\int \frac{\log(1+icx)}{x}\,dx+(ae)$$

$$= ad\log(x)+\frac{1}{2}ibd\mathrm{Li}_2(-icx)-\frac{1}{2}ibd\mathrm{Li}_2(icx)-\frac{1}{2}ae\mathrm{Li}_2\left(-c^2x^2\right)+\frac{1}{2}(ibe)\int$$

$$= ad\log(x)+\frac{1}{2}ibe\log(icx)\log^2(1-icx)-\frac{1}{2}ibe\log(-icx)\log^2(1+icx)+$$

$$= ad\log(x)+\frac{1}{2}ibe\log(icx)\log^2(1-icx)-\frac{1}{2}ibe\log(-icx)\log^2(1+icx)+$$

$$= ad\log(x)+\frac{1}{2}ibe\log(icx)\log^2(1-icx)-\frac{1}{2}ibe\log(-icx)\log^2(1+icx)+$$

$$= ad\log(x)+\frac{1}{2}ibe\log(icx)\log^2(1-icx)-\frac{1}{2}ibe\log(-icx)\log^2(1+icx)+$$

Mathematica [F] time = 0.204386, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\tan^{-1}(cx)\right)\left(d+e\log\left(1+c^2x^2\right)\right)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]))/x,x]
```

```
[Out] Integrate[((a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]))/x, x]
```

Maple [C] time = 2.036, size = 6931, normalized size = 24.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))*(d+e*ln(c^2*x^2+1))/x,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$ad\log(x) + \frac{1}{2}\int \frac{2\left(bd\arctan(cx) + (be\arctan(cx) + ae)\log\left(c^2x^2 + 1\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))*(d+e*log(c^2*x^2+1))/x,x, algorithm="maxima")
```

```
[Out] a*d*log(x) + 1/2*integrate(2*(b*d*arctan(c*x) + (b*e*arctan(c*x) + a*e)*log
(c^2*x^2 + 1))/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{bd\arctan(cx) + ad + (be\arctan(cx) + ae)\log\left(c^2x^2 + 1\right)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))*(d+e*log(c^2*x^2+1))/x,x, algorithm="fricas")
```

```
[Out] integral((b*d*arctan(c*x) + a*d + (b*e*arctan(c*x) + a*e)*log(c^2*x^2 + 1))
/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))*(d+e*ln(c**2*x**2+1))/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\arctan(cx)+a)\left(e\log\left(c^2x^2+1\right)+d\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))*(d+e*log(c^2*x^2+1))/x,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)*(e*log(c^2*x^2 + 1) + d)/x, x)
```

3.1292 $\int \frac{\left(a+b\tan^{-1}(cx)\right)\left(d+e\log\left(1+c^2x^2\right)\right)}{x^2}\,dx$

Optimal. Leaf size=100

$$-\frac{1}{2}bce\text{PolyLog}\left(2,\frac{1}{c^2x^2+1}\right)-\frac{\left(a+b\tan^{-1}(cx)\right)\left(e\log\left(c^2x^2+1\right)+d\right)}{x}+\frac{ce\left(a+b\tan^{-1}(cx)\right)^2}{b}+\frac{1}{2}bc\log\left(1-\frac{1}{c^2x^2+1}\right)$$

```
[Out] (c*e*(a + b*ArcTan[c*x])^2)/b - ((a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2
]))/x + (b*c*(d + e*Log[1 + c^2*x^2])*Log[1 - (1 + c^2*x^2)^(-1)])/2 - (b*c
*e*PolyLog[2, (1 + c^2*x^2)^(-1)])/2
```

Rubi [A] time = 0.24991, antiderivative size = 92, normalized size of antiderivative = 0.92, number of steps used = 8, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {5017, 2475, 2411, 2344, 2301, 2316, 2315, 4884}

$$-\frac{1}{2}bce\text{PolyLog}\left(2,-c^2x^2\right)-\frac{\left(a+b\tan^{-1}(cx)\right)\left(e\log\left(c^2x^2+1\right)+d\right)}{x}+\frac{ce\left(a+b\tan^{-1}(cx)\right)^2}{b}-\frac{bc\left(e\log\left(c^2x^2+1\right)+d\right)^2}{4e}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]))/x^2,x]
```

```
[Out] (c*e*(a + b*ArcTan[c*x])^2)/b + b*c*d*Log[x] - ((a + b*ArcTan[c*x])*(d + e*
Log[1 + c^2*x^2]))/x - (b*c*(d + e*Log[1 + c^2*x^2])^2)/(4*e) - (b*c*e*Poly
Log[2, -(c^2*x^2)])/2
```

Rule 5017

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*(
e_.))*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*(d + e*Log[f + g*x^2])*(a +
b*ArcTan[c*x]))/(m + 1), x] + (-Dist[(b*c)/(m + 1), Int[(x^(m + 1)*(d + e*L
og[f + g*x^2]))/(1 + c^2*x^2), x], x] - Dist[(2*e*g)/(m + 1), Int[(x^(m + 2
)*(a + b*ArcTan[c*x]))/(f + g*x^2), x], x]) /; FreeQ[{a, b, c, d, e, f, g},
 x] && ILtQ[m/2, 0]
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
```

```
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2316

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[
((a + b*Log[-((c*d)/e)])*Log[d + e*x])/e, x] + Dist[b, Int[Log[-((e*x)/d)]/
(d + e*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[-((c*d)/e), 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}(cx)\right)\left(d+e\log\left(1+c^2x^2\right)\right)}{x^2}\,dx = -\frac{\left(a+b\tan^{-1}(cx)\right)\left(d+e\log\left(1+c^2x^2\right)\right)}{x} + (bc)\int \frac{d+e\log\left(1+c^2x^2\right)}{x\left(1+c^2x^2\right)}\,d$$

$$= \frac{ce\left(a+b\tan^{-1}(cx)\right)^2}{b} - \frac{\left(a+b\tan^{-1}(cx)\right)\left(d+e\log\left(1+c^2x^2\right)\right)}{x} + \frac{1}{2}(bc)$$

$$= \frac{ce\left(a+b\tan^{-1}(cx)\right)^2}{b} - \frac{\left(a+b\tan^{-1}(cx)\right)\left(d+e\log\left(1+c^2x^2\right)\right)}{x} + \frac{b\,\mathrm{Sub}}{}$$

$$= \frac{ce\left(a+b\tan^{-1}(cx)\right)^2}{b} - \frac{\left(a+b\tan^{-1}(cx)\right)\left(d+e\log\left(1+c^2x^2\right)\right)}{x} + \frac{b\,\mathrm{Sub}}{}$$

$$= \frac{ce\left(a+b\tan^{-1}(cx)\right)^2}{b} + bcd\log(x) - \frac{\left(a+b\tan^{-1}(cx)\right)\left(d+e\log\left(1+c^2\right.\right.}{x}$$

$$= \frac{ce\left(a+b\tan^{-1}(cx)\right)^2}{b} + bcd\log(x) - \frac{\left(a+b\tan^{-1}(cx)\right)\left(d+e\log\left(1+c^2\right.\right.}{x}$$

Mathematica [A] time = 0.0897007, size = 113, normalized size = 1.13

$$bc\left(\frac{1}{2}\left(e\mathrm{PolyLog}\left(2,c^2x^2+1\right)+\log\left(-c^2x^2\right)\left(e\log\left(c^2x^2+1\right)+d\right)\right)-\frac{\left(e\log\left(c^2x^2+1\right)+d\right)^2}{4e}\right)-\frac{\left(a+b\tan^{-1}(cx)\right)\left(e\log\left(\right.\right.}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]))/x^2,x]
```

```
[Out] (c*e*(a + b*ArcTan[c*x])^2)/b - ((a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2
]))/x + b*c*(-(d + e*Log[1 + c^2*x^2])^2/(4*e) + (Log[-(c^2*x^2)]*(d + e*Lo
g[1 + c^2*x^2]) + e*PolyLog[2, 1 + c^2*x^2])/2)
```

Maple [F] time = 3.487, size = 0, normalized size = 0.

$$\int \frac{(a+b\arctan(cx))\left(d+e\ln\left(c^2x^2+1\right)\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))*(d+e*ln(c^2*x^2+1))/x^2,x)
```

```
[Out] int((a+b*arctan(c*x))*(d+e*ln(c^2*x^2+1))/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2}\left(c\left(\log\left(c^2x^2+1\right)-\log\left(x^2\right)\right)+\frac{2\arctan\left(cx\right)}{x}\right)bd+\left(2\,c\arctan\left(cx\right)-\frac{\log\left(c^2x^2+1\right)}{x}\right)ae+be\int\frac{\arctan\left(cx\right)\log\left(c^2\right.}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))*(d+e*log(c^2*x^2+1))/x^2,x, algorithm="maxima")
```

```
[Out] -1/2*(c*(log(c^2*x^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*b*d + (2*c*arctan(
c*x) - log(c^2*x^2 + 1)/x)*a*e + b*e*integrate(arctan(c*x)*log(c^2*x^2 + 1)
/x^2, x) - a*d/x
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{bd\arctan\left(cx\right)+ad+\left(be\arctan\left(cx\right)+ae\right)\log\left(c^2x^2+1\right)}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))*(d+e*log(c^2*x^2+1))/x^2,x, algorithm="fricas")
```

```
[Out] integral((b*d*arctan(c*x) + a*d + (b*e*arctan(c*x) + a*e)*log(c^2*x^2 + 1))
/x^2, x)
```

Sympy [A] time = 140.782, size = 160, normalized size = 1.6

$$-\frac{ad}{x}+\frac{2ae\,\mathrm{atan}\left(\frac{x}{\sqrt{\frac{1}{c^2}}}\right)}{\sqrt{\frac{1}{c^2}}}-\frac{ae\log\left(c^2x^2+1\right)}{x}-bc^3e\left(\begin{cases}0 & \text{for}\ c^2=0\\ \frac{\log\left(c^2x^2+1\right)^2}{4c^2} & \text{otherwise}\end{cases}\right)+4bc^2e\left(\begin{cases}0 & \text{for}\ c=0\\ \frac{\mathrm{atan}^2\left(cx\right)}{4c} & \text{otherwise}\end{cases}\right)-\frac{bca}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))*(d+e*ln(c**2*x**2+1))/x**2,x)
```

```
[Out] -a*d/x + 2*a*e*atan(x/sqrt(c**(-2)))/sqrt(c**(-2)) - a*e*log(c**2*x**2 + 1)
/x - b*c**3*e*Piecewise((0, Eq(c**2, 0)), (log(c**2*x**2 + 1)**2/(4*c**2),
True)) + 4*b*c**2*e*Piecewise((0, Eq(c, 0)), (atan(c*x)**2/(4*c), True)) -
b*c*d*log(c**2 + x**(-2))/2 - b*c*e*polylog(2, c**2*x**2*exp_polar(I*pi))/2
 - b*d*atan(c*x)/x - b*e*log(c**2*x**2 + 1)*atan(c*x)/x
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\arctan(cx) + a)\left(e\log\left(c^2x^2 + 1\right) + d\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))*(d+e*log(c^2*x^2+1))/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)*(e*log(c^2*x^2 + 1) + d)/x^2, x)
```

3.1293 $\int \frac{(a+b\tan^{-1}(cx))(d+e\log(1+c^2x^2))}{x^3}\,dx$

Optimal. Leaf size=154

$$\frac{1}{2}ibc^2e\text{PolyLog}(2,-icx)-\frac{1}{2}ibc^2e\text{PolyLog}(2,icx)-\frac{(a+b\tan^{-1}(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{2x^2}-\frac{1}{2}ac^2e\log\left(c^2x^2+1\right)+ac$$

```
[Out] b*c^2*e*ArcTan[c*x] + a*c^2*e*Log[x] - (a*c^2*e*Log[1 + c^2*x^2])/2 - (b*c*
(d + e*Log[1 + c^2*x^2]))/(2*x) - (b*c^2*ArcTan[c*x]*(d + e*Log[1 + c^2*x^2
]))/2 - ((a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]))/(2*x^2) + (I/2)*b*c^
2*e*PolyLog[2, (-I)*c*x] - (I/2)*b*c^2*e*PolyLog[2, I*c*x]
```

Rubi [A] time = 0.140145, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {4852, 325, 203, 5021, 801, 635, 260, 4848, 2391}

$$\frac{1}{2}ibc^2e\text{PolyLog}(2,-icx)-\frac{1}{2}ibc^2e\text{PolyLog}(2,icx)-\frac{(a+b\tan^{-1}(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{2x^2}-\frac{1}{2}ac^2e\log\left(c^2x^2+1\right)+ac$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]))/x^3,x]
```

```
[Out] b*c^2*e*ArcTan[c*x] + a*c^2*e*Log[x] - (a*c^2*e*Log[1 + c^2*x^2])/2 - (b*c*
(d + e*Log[1 + c^2*x^2]))/(2*x) - (b*c^2*ArcTan[c*x]*(d + e*Log[1 + c^2*x^2
]))/2 - ((a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]))/(2*x^2) + (I/2)*b*c^
2*e*PolyLog[2, (-I)*c*x] - (I/2)*b*c^2*e*PolyLog[2, I*c*x]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
```

```
b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 5021

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*(
e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(a + b*ArcTan[c*x]), x
]}, Dist[d + e*Log[f + g*x^2], u, x] - Dist[2*e*g, Int[ExpandIntegrand[(x*u
)/(f + g*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IntegerQ[m
] && NeQ[m, -1]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}(cx)\right)\left(d+e\log\left(1+c^2x^2\right)\right)}{x^3}\,dx = -\frac{bc\left(d+e\log\left(1+c^2x^2\right)\right)}{2x} - \frac{1}{2}bc^2\tan^{-1}(cx)\left(d+e\log\left(1+c^2x^2\right)\right) - \frac{(a}{}$$

$$= -\frac{bc\left(d+e\log\left(1+c^2x^2\right)\right)}{2x} - \frac{1}{2}bc^2\tan^{-1}(cx)\left(d+e\log\left(1+c^2x^2\right)\right) - \frac{(a}{}$$

$$= -\frac{bc\left(d+e\log\left(1+c^2x^2\right)\right)}{2x} - \frac{1}{2}bc^2\tan^{-1}(cx)\left(d+e\log\left(1+c^2x^2\right)\right) - \frac{(a}{}$$

$$= ac^2e\log(x) - \frac{bc\left(d+e\log\left(1+c^2x^2\right)\right)}{2x} - \frac{1}{2}bc^2\tan^{-1}(cx)\left(d+e\log\left(1+\right.\right.$$

$$= ac^2e\log(x) - \frac{bc\left(d+e\log\left(1+c^2x^2\right)\right)}{2x} - \frac{1}{2}bc^2\tan^{-1}(cx)\left(d+e\log\left(1+\right.\right.$$

$$= bc^2e\tan^{-1}(cx) + ac^2e\log(x) - \frac{1}{2}ac^2e\log\left(1+c^2x^2\right) - \frac{bc\left(d+e\log\left(1+\right.\right.}{2x}$$

Mathematica [A] time = 0.11815, size = 189, normalized size = 1.23

$$-\frac{-ibc^2ex^2\text{PolyLog}(2,-icx)+ibc^2ex^2\text{PolyLog}(2,icx)-2ac^2ex^2\log(x)+ac^2ex^2\log\left(c^2x^2+1\right)+ae\log\left(c^2x^2+1\right)+ad}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]))/x^3,x]
```

```
[Out] -(a*d + b*c*d*x + b*d*ArcTan[c*x] + b*c^2*d*x^2*ArcTan[c*x] - 2*b*c^2*e*x^2
*ArcTan[c*x] - 2*a*c^2*e*x^2*Log[x] + a*e*Log[1 + c^2*x^2] + b*c*e*x*Log[1
+ c^2*x^2] + a*c^2*e*x^2*Log[1 + c^2*x^2] + b*e*ArcTan[c*x]*Log[1 + c^2*x^2
] + b*c^2*e*x^2*ArcTan[c*x]*Log[1 + c^2*x^2] - I*b*c^2*e*x^2*PolyLog[2, (-I
)*c*x] + I*b*c^2*e*x^2*PolyLog[2, I*c*x])/(2*x^2)
```

Maple [F] time = 11.976, size = 0, normalized size = 0.

$$\int \frac{(a+b\arctan(cx))\left(d+e\ln\left(c^2x^2+1\right)\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))*(d+e*ln(c^2*x^2+1))/x^3,x)
```

```
[Out] int((a+b*arctan(c*x))*(d+e*ln(c^2*x^2+1))/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2}\left(\left(c\arctan(cx)+\frac{1}{x}\right)c+\frac{\arctan(cx)}{x^2}\right)bd-\frac{1}{2}\left(c^2\left(\log\left(c^2x^2+1\right)-\log\left(x^2\right)\right)+\frac{\log\left(c^2x^2+1\right)}{x^2}\right)ae+\frac{\left(2\,c^4x^2\int\frac{x\arctan(cx}{c^2x^2+1}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))*(d+e*log(c^2*x^2+1))/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*((c*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*b*d - 1/2*(c^2*(log(c^2*x^
2 + 1) - log(x^2)) + log(c^2*x^2 + 1)/x^2)*a*e + 1/2*(4*c^4*x^2*integrate(1
/2*x*arctan(c*x)/(c^2*x^2 + 1), x) + 2*c^2*x^2*arctan(c*x) + 4*c^2*x^2*inte
grate(1/2*arctan(c*x)/(c^2*x^3 + x), x) - (c*x + (c^2*x^2 + 1)*arctan(c*x))
*log(c^2*x^2 + 1))*b*e/x^2 - 1/2*a*d/x^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{bd\arctan(cx)+ad+(be\arctan(cx)+ae)\log\left(c^2x^2+1\right)}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))*(d+e*log(c^2*x^2+1))/x^3,x, algorithm="fricas")
```

```
[Out] integral((b*d*arctan(c*x) + a*d + (b*e*arctan(c*x) + a*e)*log(c^2*x^2 + 1))
/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(a+b\,\text{atan}(cx))\left(d+e\log\left(c^2x^2+1\right)\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))*(d+e*ln(c**2*x**2+1))/x**3,x)
```

```
[Out] Integral((a + b*atan(c*x))*(d + e*log(c**2*x**2 + 1))/x**3, x)
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\arctan(cx) + a)\left(e\log\left(c^2x^2 + 1\right) + d\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))*(d+e*log(c^2*x^2+1))/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)*(e*log(c^2*x^2 + 1) + d)/x^3, x)
```

3.1294 $\int \frac{(a+b\tan^{-1}(cx))(d+e\log(1+c^2x^2))}{x^4}\,dx$

Optimal. Leaf size=189

$$\frac{1}{6}bc^3e\text{PolyLog}\left(2,\frac{1}{c^2x^2+1}\right)-\frac{\left(a+b\tan^{-1}(cx)\right)\left(e\log\left(c^2x^2+1\right)+d\right)}{3x^3}-\frac{c^3e\left(a+b\tan^{-1}(cx)\right)^2}{3b}-\frac{2c^2e\left(a+b\tan^{-1}(cx)\right)}{3x}$$

```
[Out] (-2*c^2*e*(a + b*ArcTan[c*x]))/(3*x) - (c^3*e*(a + b*ArcTan[c*x])^2)/(3*b)
+ b*c^3*e*Log[x] - (b*c^3*e*Log[1 + c^2*x^2])/3 - (b*c*(1 + c^2*x^2)*(d + e
*Log[1 + c^2*x^2]))/(6*x^2) - ((a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2])
)/(3*x^3) - (b*c^3*(d + e*Log[1 + c^2*x^2])*Log[1 - (1 + c^2*x^2)^(-1)])/6
+ (b*c^3*e*PolyLog[2, (1 + c^2*x^2)^(-1)])/6
```

Rubi [A] time = 0.428442, antiderivative size = 186, normalized size of antiderivative = 0.98, number of steps used = 17, number of rules used = 16, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {5017, 2475, 2411, 2347, 2344, 2301, 2316, 2315, 2314, 31, 4918, 4852, 266, 36, 29, 4884}

$$\frac{1}{6}bc^3e\text{PolyLog}\left(2,-c^2x^2\right)-\frac{\left(a+b\tan^{-1}(cx)\right)\left(e\log\left(c^2x^2+1\right)+d\right)}{3x^3}-\frac{c^3e\left(a+b\tan^{-1}(cx)\right)^2}{3b}-\frac{2c^2e\left(a+b\tan^{-1}(cx)\right)}{3x}+$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]))/x^4,x]
```

```
[Out] (-2*c^2*e*(a + b*ArcTan[c*x]))/(3*x) - (c^3*e*(a + b*ArcTan[c*x])^2)/(3*b)
- (b*c^3*d*Log[x])/3 + b*c^3*e*Log[x] - (b*c^3*e*Log[1 + c^2*x^2])/3 - (b*c
*(1 + c^2*x^2)*(d + e*Log[1 + c^2*x^2]))/(6*x^2) - ((a + b*ArcTan[c*x])*(d
+ e*Log[1 + c^2*x^2]))/(3*x^3) + (b*c^3*(d + e*Log[1 + c^2*x^2])^2)/(12*e)
+ (b*c^3*e*PolyLog[2, -(c^2*x^2)])/6
```

Rule 5017

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*(
e_.))*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*(d + e*Log[f + g*x^2])*(a +
b*ArcTan[c*x]))/(m + 1), x] + (-Dist[(b*c)/(m + 1), Int[(x^(m + 1)*(d + e*L
og[f + g*x^2]))/(1 + c^2*x^2), x], x] - Dist[(2*e*g)/(m + 1), Int[(x^(m + 2
)*(a + b*ArcTan[c*x]))/(f + g*x^2), x], x]) /; FreeQ[{a, b, c, d, e, f, g},
 x] && ILtQ[m/2, 0]
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2347

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2316

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[
((a + b*Log[-((c*d)/e)])*Log[d + e*x])/e, x] + Dist[b, Int[Log[-((e*x)/d)]/
(d + e*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[-((c*d)/e), 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
```

```
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b
*n)/d, Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 4918

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/(d + e*x^2),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}(cx)\right)\left(d+e\log\left(1+c^2x^2\right)\right)}{x^4}\,dx = -\frac{\left(a+b\tan^{-1}(cx)\right)\left(d+e\log\left(1+c^2x^2\right)\right)}{3x^3}+\frac{1}{3}(bc)\int\frac{d+e\log\left(1+c^2x^2\right)}{x^3\left(1+c^2x^2\right)}$$

$$= -\frac{\left(a+b\tan^{-1}(cx)\right)\left(d+e\log\left(1+c^2x^2\right)\right)}{3x^3}+\frac{1}{6}(bc)\,\mathrm{Subst}\left(\int\frac{d+e\log\left(\right.}{x^2\left(1+\right.}\right.$$

$$= -\frac{2c^2e\left(a+b\tan^{-1}(cx)\right)}{3x}-\frac{c^3e\left(a+b\tan^{-1}(cx)\right)^2}{3b}-\frac{\left(a+b\tan^{-1}(cx)\right)\left(\right.}{3}$$

$$= -\frac{2c^2e\left(a+b\tan^{-1}(cx)\right)}{3x}-\frac{c^3e\left(a+b\tan^{-1}(cx)\right)^2}{3b}-\frac{\left(a+b\tan^{-1}(cx)\right)\left(\right.}{3}$$

$$= -\frac{2c^2e\left(a+b\tan^{-1}(cx)\right)}{3x}-\frac{c^3e\left(a+b\tan^{-1}(cx)\right)^2}{3b}-\frac{bc\left(1+c^2x^2\right)\left(d+e\right.}{6x^2}$$

$$= -\frac{2c^2e\left(a+b\tan^{-1}(cx)\right)}{3x}-\frac{c^3e\left(a+b\tan^{-1}(cx)\right)^2}{3b}-\frac{1}{3}bc^3d\log(x)+bc^3$$

$$= -\frac{2c^2e\left(a+b\tan^{-1}(cx)\right)}{3x}-\frac{c^3e\left(a+b\tan^{-1}(cx)\right)^2}{3b}-\frac{1}{3}bc^3d\log(x)+bc^3$$

Mathematica [A] time = 0.140352, size = 181, normalized size = 0.96

$$\frac{1}{12}\left(-2bc^3\left(e\mathrm{PolyLog}\left(2,c^2x^2+1\right)+\log\left(-c^2x^2\right)\left(e\log\left(c^2x^2+1\right)+d\right)\right)-\frac{4\left(a+b\tan^{-1}(cx)\right)\left(e\log\left(c^2x^2+1\right)+d\right)}{x^3}-\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]))/x^4,x]
```

```
[Out] ((-8*c^2*e*(a + b*ArcTan[c*x]))/x - (4*c^3*e*(a + b*ArcTan[c*x])^2)/b + 6*b
*c^3*e*(2*Log[x] - Log[1 + c^2*x^2]) - (2*b*c*(d + e*Log[1 + c^2*x^2]))/x^2
 - (4*(a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]))/x^3 + (b*c^3*(d + e*Log
[1 + c^2*x^2])^2)/e - 2*b*c^3*(Log[-(c^2*x^2)]*(d + e*Log[1 + c^2*x^2]) + e
*PolyLog[2, 1 + c^2*x^2]))/12
```

Maple [F] time = 6.51, size = 0, normalized size = 0.

$$\int \frac{(a + b\arctan(cx))\left(d + e\ln\left(c^2x^2+1\right)\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))*(d+e*ln(c^2*x^2+1))/x^4,x)
```

```
[Out] int((a+b*arctan(c*x))*(d+e*ln(c^2*x^2+1))/x^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{6}\left(\left(c^2\log\left(c^2x^2+1\right) - c^2\log\left(x^2\right) - \frac{1}{x^2}\right)c - \frac{2\arctan(cx)}{x^3}\right)bd - \frac{1}{3}\left(2\left(c\arctan(cx) + \frac{1}{x}\right)c^2 + \frac{\log\left(c^2x^2+1\right)}{x^3}\right)ae + be\int$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))*(d+e*log(c^2*x^2+1))/x^4,x, algorithm="maxima")
```

```
[Out] 1/6*((c^2*log(c^2*x^2 + 1) - c^2*log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^3)*b
*d - 1/3*(2*(c*arctan(c*x) + 1/x)*c^2 + log(c^2*x^2 + 1)/x^3)*a*e + b*e*int
egrate(arctan(c*x)*log(c^2*x^2 + 1)/x^4, x) - 1/3*a*d/x^3
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{bd\arctan(cx) + ad + (be\arctan(cx) + ae)\log\left(c^2x^2+1\right)}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))*(d+e*log(c^2*x^2+1))/x^4,x, algorithm="fricas")
```

```
[Out] integral((b*d*arctan(c*x) + a*d + (b*e*arctan(c*x) + a*e)*log(c^2*x^2 + 1))
/x^4, x)
```

Sympy [A] time = 53.4573, size = 428, normalized size = 2.26

$$-\frac{2ac^2e\,\mathrm{atan}\left(\frac{x}{\sqrt{\frac{1}{c^2}}}\right)}{3\sqrt{\frac{1}{c^2}}}-\frac{2ac^2e}{3x}-\frac{ad}{3x^3}-\frac{ae\log\left(c^2x^2+1\right)}{3x^3}-2bc^7e\left(\begin{cases}\frac{x^2}{12c^2}-\frac{\log\left(c^2x^2+1\right)}{12c^4} & \text{for } c=0\\ \frac{\log\left(c^2x^2+1\right)^2}{24c^4} & \text{otherwise}\end{cases}\right)+\frac{bc^5d\left(\begin{cases}x^2\\ \frac{\log\left(c^2x^2+1\right)}{c^2}\end{cases}\right.}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))*(d+e*ln(c**2*x**2+1))/x**4,x)
```

```
[Out] -2*a*c**2*e*atan(x/sqrt(c**(-2)))/(3*sqrt(c**(-2))) - 2*a*c**2*e/(3*x) - a*
d/(3*x**3) - a*e*log(c**2*x**2 + 1)/(3*x**3) - 2*b*c**7*e*Piecewise((x**2/(
12*c**2) - log(c**2*x**2 + 1)/(12*c**4), Eq(c, 0)), (log(c**2*x**2 + 1)**2/
(24*c**4), True)) + b*c**5*d*Piecewise((x**2, Eq(c**2, 0)), (log(c**2*x**2
+ 1)/c**2, True))/6 + b*c**5*e*Piecewise((x**2, Eq(c**2, 0)), (log(c**2*x**
2 + 1)/c**2, True))*log(c**2*x**2 + 1)/6 - b*c**3*d*log(x**2)/6 + b*c**3*e*
log(x)/3 - b*c**3*e*log(c**2*x**2 + 1)/6 - b*c**3*e*log(6*c**2*sqrt(c**(-2)
) + 6*sqrt(c**(-2))/x**2)/3 + b*c**3*e*atan(x/sqrt(c**(-2)))**2/3 + b*c**3*
e*polylog(2, c**2*x**2*exp_polar(I*pi))/6 - 2*b*c**2*e*atan(c*x)*atan(x/sqr
t(c**(-2)))/(3*sqrt(c**(-2))) - 2*b*c**2*e*atan(c*x)/(3*x) - b*c*d/(6*x**2)
 - b*c*e*log(c**2*x**2 + 1)/(6*x**2) - b*d*atan(c*x)/(3*x**3) - b*e*log(c**
2*x**2 + 1)*atan(c*x)/(3*x**3)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(b\arctan(cx)+a)\left(e\log\left(c^2x^2+1\right)+d\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))*(d+e*log(c^2*x^2+1))/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)*(e*log(c^2*x^2 + 1) + d)/x^4, x)
```

3.1295 $\int \frac{(a+b\tan^{-1}(cx))(d+e\log(1+c^2x^2))}{x^5}\,dx$

Optimal. Leaf size=225

$$-\frac{1}{4}ibc^4e\text{PolyLog}(2,-icx)+\frac{1}{4}ibc^4e\text{PolyLog}(2,icx)-\frac{(a+b\tan^{-1}(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{4x^4}-\frac{ac^2e}{4x^2}+\frac{1}{4}ac^4e\log\left(c^2x^2+\right.$$

```
[Out] -(a*c^2*e)/(4*x^2) - (5*b*c^3*e)/(12*x) - (11*b*c^4*e*ArcTan[c*x])/12 - (b*
c^2*e*ArcTan[c*x])/(4*x^2) - (a*c^4*e*Log[x])/2 + (a*c^4*e*Log[1 + c^2*x^2]
)/4 - (b*c*(d + e*Log[1 + c^2*x^2]))/(12*x^3) + (b*c^3*(d + e*Log[1 + c^2*x
^2]))/(4*x) + (b*c^4*ArcTan[c*x]*(d + e*Log[1 + c^2*x^2]))/4 - ((a + b*ArcT
an[c*x])*(d + e*Log[1 + c^2*x^2]))/(4*x^4) - (I/4)*b*c^4*e*PolyLog[2, (-I)*
c*x] + (I/4)*b*c^4*e*PolyLog[2, I*c*x]
```

Rubi [A] time = 0.255709, antiderivative size = 225, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {4852, 325, 203, 5021, 1802, 635, 260, 4980, 4848, 2391}

$$-\frac{1}{4}ibc^4e\text{PolyLog}(2,-icx)+\frac{1}{4}ibc^4e\text{PolyLog}(2,icx)-\frac{(a+b\tan^{-1}(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{4x^4}-\frac{ac^2e}{4x^2}+\frac{1}{4}ac^4e\log\left(c^2x^2+\right.$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]))/x^5,x]
```


```
[Out] -(a*c^2*e)/(4*x^2) - (5*b*c^3*e)/(12*x) - (11*b*c^4*e*ArcTan[c*x])/12 - (b*
c^2*e*ArcTan[c*x])/(4*x^2) - (a*c^4*e*Log[x])/2 + (a*c^4*e*Log[1 + c^2*x^2]
)/4 - (b*c*(d + e*Log[1 + c^2*x^2]))/(12*x^3) + (b*c^3*(d + e*Log[1 + c^2*x
^2]))/(4*x) + (b*c^4*ArcTan[c*x]*(d + e*Log[1 + c^2*x^2]))/4 - ((a + b*ArcT
an[c*x])*(d + e*Log[1 + c^2*x^2]))/(4*x^4) - (I/4)*b*c^4*e*PolyLog[2, (-I)*
c*x] + (I/4)*b*c^4*e*PolyLog[2, I*c*x]
```


Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 5021

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*(
e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(a + b*ArcTan[c*x]), x
]}, Dist[d + e*Log[f + g*x^2], u, x] - Dist[2*e*g, Int[ExpandIntegrand[(x*u
)/(f + g*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IntegerQ[m
] && NeQ[m, -1]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4980

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}(cx)\right)\left(d+e\log\left(1+c^2x^2\right)\right)}{x^5}\,dx = -\frac{bc\left(d+e\log\left(1+c^2x^2\right)\right)}{12x^3}+\frac{bc^3\left(d+e\log\left(1+c^2x^2\right)\right)}{4x}+\frac{1}{4}bc^4\tan^{-1}(cx)$$

$$= -\frac{bc\left(d+e\log\left(1+c^2x^2\right)\right)}{12x^3}+\frac{bc^3\left(d+e\log\left(1+c^2x^2\right)\right)}{4x}+\frac{1}{4}bc^4\tan^{-1}(cx)$$

$$= -\frac{bc\left(d+e\log\left(1+c^2x^2\right)\right)}{12x^3}+\frac{bc^3\left(d+e\log\left(1+c^2x^2\right)\right)}{4x}+\frac{1}{4}bc^4\tan^{-1}(cx)$$

$$= -\frac{ac^2e}{4x^2}-\frac{bc^3e}{6x}-\frac{1}{2}ac^4e\log(x)-\frac{bc\left(d+e\log\left(1+c^2x^2\right)\right)}{12x^3}+\frac{bc^3\left(d+e\log\right.}{4x}$$

$$= -\frac{ac^2e}{4x^2}-\frac{bc^3e}{6x}-\frac{bc^2e\tan^{-1}(cx)}{4x^2}-\frac{1}{2}ac^4e\log(x)-\frac{bc\left(d+e\log\left(1+c^2x^2\right)\right)}{12x^3}$$

$$= -\frac{ac^2e}{4x^2}-\frac{5bc^3e}{12x}-\frac{2}{3}bc^4e\tan^{-1}(cx)-\frac{bc^2e\tan^{-1}(cx)}{4x^2}-\frac{1}{2}ac^4e\log(x)+\frac{1}{4}a$$

$$= -\frac{ac^2e}{4x^2}-\frac{5bc^3e}{12x}-\frac{11}{12}bc^4e\tan^{-1}(cx)-\frac{bc^2e\tan^{-1}(cx)}{4x^2}-\frac{1}{2}ac^4e\log(x)+\frac{1}{4}$$

Mathematica [A] time = 0.17545, size = 260, normalized size = 1.16

$$-\frac{3ibc^4ex^4\text{PolyLog}(2,-icx)-3ibc^4ex^4\text{PolyLog}(2,icx)+3ac^2ex^2+6ac^4ex^4\log(x)-3ac^4ex^4\log\left(c^2x^2+1\right)+3ae\log\left(c^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]))/x^5,x]
```

```
[Out] -(3*a*d + b*c*d*x + 3*a*c^2*e*x^2 - 3*b*c^3*d*x^3 + 5*b*c^3*e*x^3 + 3*b*d*A
rcTan[c*x] + 3*b*c^2*e*x^2*ArcTan[c*x] - 3*b*c^4*d*x^4*ArcTan[c*x] + 11*b*c
```

```
^4*e*x^4*ArcTan[c*x] + 6*a*c^4*e*x^4*Log[x] + 3*a*e*Log[1 + c^2*x^2] + b*c*
e*x*Log[1 + c^2*x^2] - 3*b*c^3*e*x^3*Log[1 + c^2*x^2] - 3*a*c^4*e*x^4*Log[1
 + c^2*x^2] + 3*b*e*ArcTan[c*x]*Log[1 + c^2*x^2] - 3*b*c^4*e*x^4*ArcTan[c*x
]*Log[1 + c^2*x^2] + (3*I)*b*c^4*e*x^4*PolyLog[2, (-I)*c*x] - (3*I)*b*c^4*e
*x^4*PolyLog[2, I*c*x])/(12*x^4)
```

Maple [F] time = 9.67, size = 0, normalized size = 0.

$$\int \frac{(a+b\arctan(cx))\left(d+e\ln\left(c^2x^2+1\right)\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))*(d+e*ln(c^2*x^2+1))/x^5,x)
```

```
[Out] int((a+b*arctan(c*x))*(d+e*ln(c^2*x^2+1))/x^5,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{12}\left(\left(3\,c^3\arctan(cx)+\frac{3\,c^2x^2-1}{x^3}\right)c-\frac{3\,\arctan(cx)}{x^4}\right)bd+\frac{1}{4}\left(\left(c^2\log\left(c^2x^2+1\right)-c^2\log\left(x^2\right)-\frac{1}{x^2}\right)c^2-\frac{\log\left(c^2x^2+1\right)}{x^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))*(d+e*log(c^2*x^2+1))/x^5,x, algorithm="maxima")
```

```
[Out] 1/12*((3*c^3*arctan(c*x) + (3*c^2*x^2 - 1)/x^3)*c - 3*arctan(c*x)/x^4)*b*d
+ 1/4*((c^2*log(c^2*x^2 + 1) - c^2*log(x^2) - 1/x^2)*c^2 - log(c^2*x^2 + 1)
/x^4)*a*e - 1/12*(72*c^6*x^4*integrate(1/12*x*arctan(c*x)/(c^2*x^2 + 1), x)
 + 8*c^4*x^4*arctan(c*x) - 72*c^2*x^4*integrate(1/12*arctan(c*x)/(c^2*x^5 +
 x^3), x) + 2*c^3*x^3 - (3*c^3*x^3 - c*x + 3*(c^4*x^4 - 1)*arctan(c*x))*log
(c^2*x^2 + 1))*b*e/x^4 - 1/4*a*d/x^4
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{bd\arctan(cx)+ad+(be\arctan(cx)+ae)\log\left(c^2x^2+1\right)}{x^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))*(d+e*log(c^2*x^2+1))/x^5,x, algorithm="fricas")
```

```
[Out] integral((b*d*arctan(c*x) + a*d + (b*e*arctan(c*x) + a*e)*log(c^2*x^2 + 1))
/x^5, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b\,\mathrm{atan}\,(cx))\left(d + e\log\left(c^2x^2 + 1\right)\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))*(d+e*ln(c**2*x**2+1))/x**5,x)
```

```
[Out] Integral((a + b*atan(c*x))*(d + e*log(c**2*x**2 + 1))/x**5, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\arctan(cx) + a)\left(e\log\left(c^2x^2 + 1\right) + d\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))*(d+e*log(c^2*x^2+1))/x^5,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)*(e*log(c^2*x^2 + 1) + d)/x^5, x)
```

3.1296 $\int \frac{\left(a+b\tan^{-1}(cx)\right)\left(d+e\log\left(1+c^2x^2\right)\right)}{x^6}\,dx$

Optimal. Leaf size=248

$$-\frac{1}{10}bc^5e\text{PolyLog}\left(2,\frac{1}{c^2x^2+1}\right)-\frac{\left(a+b\tan^{-1}(cx)\right)\left(e\log\left(c^2x^2+1\right)+d\right)}{5x^5}-\frac{2c^2e\left(a+b\tan^{-1}(cx)\right)}{15x^3}+\frac{c^5e\left(a+b\tan^{-1}(c\right.}{5b}$$

```
[Out] (-7*b*c^3*e)/(60*x^2) - (2*c^2*e*(a + b*ArcTan[c*x]))/(15*x^3) + (2*c^4*e*(
a + b*ArcTan[c*x]))/(5*x) + (c^5*e*(a + b*ArcTan[c*x])^2)/(5*b) - (5*b*c^5*
e*Log[x])/6 + (19*b*c^5*e*Log[1 + c^2*x^2])/60 - (b*c*(d + e*Log[1 + c^2*x^
2]))/(20*x^4) + (b*c^3*(1 + c^2*x^2)*(d + e*Log[1 + c^2*x^2]))/(10*x^2) - (
(a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]))/(5*x^5) + (b*c^5*(d + e*Log[1
 + c^2*x^2])*Log[1 - (1 + c^2*x^2)^(-1)])/10 - (b*c^5*e*PolyLog[2, (1 + c^2
*x^2)^(-1)])/10
```

Rubi [A] time = 0.625143, antiderivative size = 245, normalized size of antiderivative = 0.99, number of steps used = 26, number of rules used = 18, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {5017, 2475, 2411, 2347, 2344, 2301, 2316, 2315, 2314, 31, 2319, 44, 4918, 4852, 266, 36, 29, 4884}

$$-\frac{1}{10}bc^5e\text{PolyLog}\left(2,-c^2x^2\right)-\frac{\left(a+b\tan^{-1}(cx)\right)\left(e\log\left(c^2x^2+1\right)+d\right)}{5x^5}-\frac{2c^2e\left(a+b\tan^{-1}(cx)\right)}{15x^3}+\frac{c^5e\left(a+b\tan^{-1}(cx)\right)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]))/x^6,x]
```

```
[Out] (-7*b*c^3*e)/(60*x^2) - (2*c^2*e*(a + b*ArcTan[c*x]))/(15*x^3) + (2*c^4*e*(
a + b*ArcTan[c*x]))/(5*x) + (c^5*e*(a + b*ArcTan[c*x])^2)/(5*b) + (b*c^5*d*
Log[x])/5 - (5*b*c^5*e*Log[x])/6 + (19*b*c^5*e*Log[1 + c^2*x^2])/60 - (b*c*
(d + e*Log[1 + c^2*x^2]))/(20*x^4) + (b*c^3*(1 + c^2*x^2)*(d + e*Log[1 + c^
2*x^2]))/(10*x^2) - ((a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]))/(5*x^5)
- (b*c^5*(d + e*Log[1 + c^2*x^2])^2)/(20*e) - (b*c^5*e*PolyLog[2, -(c^2*x^2
)])/10
```

Rule 5017

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*(
e_.))*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*(d + e*Log[f + g*x^2])*(a +
b*ArcTan[c*x]))/(m + 1), x] + (-Dist[(b*c)/(m + 1), Int[(x^(m + 1)*(d + e*L
```

```
og[f + g*x^2]))/(1 + c^2*x^2), x], x] - Dist[(2*e*g)/(m + 1), Int[(x^(m + 2
)*(a + b*ArcTan[c*x]))/(f + g*x^2), x], x]) /; FreeQ[{a, b, c, d, e, f, g},
 x] && ILtQ[m/2, 0]
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2347

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2316

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[
((a + b*Log[-((c*d)/e)])*Log[d + e*x])/e, x] + Dist[b, Int[Log[-((e*x)/d)]/
```

```
(d + e*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[-((c*d)/e), 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b
*n)/d, Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2319

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/(e*(q + 1)), x]
- Dist[(b*n*p)/(e*(q + 1)), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 4918

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/(d + e*x^2),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}(cx)\right)\left(d+e\log\left(1+c^2x^2\right)\right)}{x^6}\,dx = -\frac{\left(a+b\tan^{-1}(cx)\right)\left(d+e\log\left(1+c^2x^2\right)\right)}{5x^5}+\frac{1}{5}(bc)\int \frac{d+e\log\left(1+c^2x^2\right)}{x^5\left(1+c^2x^2\right)}$$

$$= -\frac{\left(a+b\tan^{-1}(cx)\right)\left(d+e\log\left(1+c^2x^2\right)\right)}{5x^5}+\frac{1}{10}(bc)\,\text{Subst}\left(\int \frac{d+e\log}{x^3\left(1\right.}\right.$$

$$= -\frac{2c^2e\left(a+b\tan^{-1}(cx)\right)}{15x^3}-\frac{\left(a+b\tan^{-1}(cx)\right)\left(d+e\log\left(1+c^2x^2\right)\right)}{5x^5}+\frac{b}{}$$

$$= -\frac{2c^2e\left(a+b\tan^{-1}(cx)\right)}{15x^3}+\frac{2c^4e\left(a+b\tan^{-1}(cx)\right)}{5x}+\frac{c^5e\left(a+b\tan^{-1}(cx\right.}{5b}$$

$$= -\frac{2c^2e\left(a+b\tan^{-1}(cx)\right)}{15x^3}+\frac{2c^4e\left(a+b\tan^{-1}(cx)\right)}{5x}+\frac{c^5e\left(a+b\tan^{-1}(cx\right.}{5b}$$

$$= -\frac{bc^3e}{15x^2}-\frac{2c^2e\left(a+b\tan^{-1}(cx)\right)}{15x^3}+\frac{2c^4e\left(a+b\tan^{-1}(cx)\right)}{5x}+\frac{c^5e\left(a+b\,t\right.}{5}$$

$$= -\frac{7bc^3e}{60x^2}-\frac{2c^2e\left(a+b\tan^{-1}(cx)\right)}{15x^3}+\frac{2c^4e\left(a+b\tan^{-1}(cx)\right)}{5x}+\frac{c^5e\left(a+b\right.}{}$$

$$= -\frac{7bc^3e}{60x^2}-\frac{2c^2e\left(a+b\tan^{-1}(cx)\right)}{15x^3}+\frac{2c^4e\left(a+b\tan^{-1}(cx)\right)}{5x}+\frac{c^5e\left(a+b\right.}{}$$

Mathematica [A] time = 0.246785, size = 259, normalized size = 1.04

$$\frac{1}{60}\left(6bc^5e\text{PolyLog}\left(2,c^2x^2+1\right)-\frac{12\left(a+b\tan^{-1}(cx)\right)\left(e\log\left(c^2x^2+1\right)+d\right)}{x^5}-\frac{8c^2e\left(a+b\tan^{-1}(cx)\right)}{x^3}+\frac{12c^5e\left(a+b\,\text{ta}\right.}{b}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]))/x^6,x]
```

```
[Out] ((-8*c^2*e*(a + b*ArcTan[c*x]))/x^3 + (24*c^4*e*(a + b*ArcTan[c*x]))/x + (1
2*c^5*e*(a + b*ArcTan[c*x])^2)/b - 18*b*c^5*e*(2*Log[x] - Log[1 + c^2*x^2])
 + 7*b*c^3*e*(-x^(-2) - 2*c^2*Log[x] + c^2*Log[1 + c^2*x^2]) - (3*b*c*(d +
```

```
e*Log[1 + c^2*x^2]))/x^4 + (6*b*c^3*(d + e*Log[1 + c^2*x^2]))/x^2 - (12*(a
+ b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]))/x^5 + 6*b*c^5*Log[-(c^2*x^2)]*(d
 + e*Log[1 + c^2*x^2]) - (3*b*c^5*(d + e*Log[1 + c^2*x^2])^2)/e + 6*b*c^5*e
*PolyLog[2, 1 + c^2*x^2])/60
```

Maple [F] time = 14.653, size = 0, normalized size = 0.

$$\int \frac{(a + b\arctan(cx))\left(d + e\ln\left(c^2x^2 + 1\right)\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))*(d+e*ln(c^2*x^2+1))/x^6,x)
```

```
[Out] int((a+b*arctan(c*x))*(d+e*ln(c^2*x^2+1))/x^6,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{20}\left(\left(2\,c^4\log\left(c^2x^2+1\right)-2\,c^4\log\left(x^2\right)-\frac{2\,c^2x^2-1}{x^4}\right)c+\frac{4\,\arctan(cx)}{x^5}\right)bd+\frac{1}{15}\left(2\left(3\,c^3\arctan(cx)+\frac{3\,c^2x^2-1}{x^3}\right)c^2-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))*(d+e*log(c^2*x^2+1))/x^6,x, algorithm="maxima")
```

```
[Out] -1/20*((2*c^4*log(c^2*x^2 + 1) - 2*c^4*log(x^2) - (2*c^2*x^2 - 1)/x^4)*c +
4*arctan(c*x)/x^5)*b*d + 1/15*(2*(3*c^3*arctan(c*x) + (3*c^2*x^2 - 1)/x^3)*
c^2 - 3*log(c^2*x^2 + 1)/x^5)*a*e + b*e*integrate(arctan(c*x)*log(c^2*x^2 +
 1)/x^6, x) - 1/5*a*d/x^5
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{bd\arctan(cx) + ad + (be\arctan(cx) + ae)\log\left(c^2x^2 + 1\right)}{x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))*(d+e*log(c^2*x^2+1))/x^6,x, algorithm="fricas")
```

```
[Out] integral((b*d*arctan(c*x) + a*d + (b*e*arctan(c*x) + a*e)*log(c^2*x^2 + 1))
/x^6, x)
```

Sympy [A] time = 87.3639, size = 474, normalized size = 1.91

$$\frac{2ac^4e\,\mathrm{atan}\left(\frac{x}{\sqrt{\frac{1}{c^2}}}\right)}{5\sqrt{\frac{1}{c^2}}}+\frac{2ac^4e}{5x}-\frac{2ac^2e}{15x^3}-\frac{ad}{5x^5}-\frac{ae\log\left(c^2x^2+1\right)}{5x^5}+4bc^9e\left(\begin{cases}\frac{x^2}{40c^2}-\frac{\log\left(c^2x^2+1\right)}{40c^4} & \text{for } c=0\\ \frac{\log\left(c^2x^2+1\right)^2}{80c^4} & \text{otherwise}\end{cases}\right)-\frac{bc^7d\left(\begin{cases}x^2\\ \log(\ldots\end{cases}\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))*(d+e*ln(c**2*x**2+1))/x**6,x)
```

```
[Out] 2*a*c**4*e*atan(x/sqrt(c**(-2)))/(5*sqrt(c**(-2))) + 2*a*c**4*e/(5*x) - 2*a
*c**2*e/(15*x**3) - a*d/(5*x**5) - a*e*log(c**2*x**2 + 1)/(5*x**5) + 4*b*c*
*9*e*Piecewise((x**2/(40*c**2) - log(c**2*x**2 + 1)/(40*c**4), Eq(c, 0)), (
log(c**2*x**2 + 1)**2/(80*c**4), True)) - b*c**7*d*Piecewise((x**2, Eq(c**2
, 0)), (log(c**2*x**2 + 1)/c**2, True))/10 - b*c**7*e*Piecewise((x**2, Eq(c
**2, 0)), (log(c**2*x**2 + 1)/c**2, True))*log(c**2*x**2 + 1)/10 + b*c**5*d
*log(x**2)/10 - 5*b*c**5*e*log(x)/6 + 5*b*c**5*e*log(c**2*x**2 + 1)/12 - b*
c**5*e*atan(x/sqrt(c**(-2)))**2/5 - b*c**5*e*polylog(2, c**2*x**2*exp_polar
(I*pi))/10 + 2*b*c**4*e*atan(c*x)*atan(x/sqrt(c**(-2)))/(5*sqrt(c**(-2))) +
 2*b*c**4*e*atan(c*x)/(5*x) + b*c**3*d/(10*x**2) + b*c**3*e*log(c**2*x**2 +
 1)/(10*x**2) - 7*b*c**3*e/(60*x**2) - 2*b*c**2*e*atan(c*x)/(15*x**3) - b*c
*d/(20*x**4) - b*c*e*log(c**2*x**2 + 1)/(20*x**4) - b*d*atan(c*x)/(5*x**5)
- b*e*log(c**2*x**2 + 1)*atan(c*x)/(5*x**5)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\arctan(cx)+a)\left(e\log\left(c^2x^2+1\right)+d\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))*(d+e*log(c^2*x^2+1))/x^6,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)*(e*log(c^2*x^2 + 1) + d)/x^6, x)
```

3.1297 $\int x\left(a+b\tan^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)\,dx$

Optimal. Leaf size=562

$$\frac{ibe\left(c^2f-g\right)\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2c^2g}-\frac{ibe\left(c^2f-g\right)\text{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-f}-\sqrt{g}x\right)}{(1-icx)\left(c\sqrt{-f}-i\sqrt{g}\right)}\right)}{4c^2g}-\frac{ibe\left(c^2f-g\right)\text{PolyLog}\left(2,1-\right.}{4c^2g}$$

```
[Out] -(b*(d - e)*x)/(2*c) + (b*e*x)/c + (b*(d - e)*ArcTan[c*x])/(2*c^2) + (d*x^2
*(a + b*ArcTan[c*x]))/2 - (e*x^2*(a + b*ArcTan[c*x]))/2 - (b*e*Sqrt[f]*ArcT
an[(Sqrt[g]*x)/Sqrt[f]])/(c*Sqrt[g]) - (b*e*(c^2*f - g)*ArcTan[c*x]*Log[2/(
1 - I*c*x)])/(c^2*g) + (b*e*(c^2*f - g)*ArcTan[c*x]*Log[(2*c*(Sqrt[-f] - Sq
rt[g]*x))/((c*Sqrt[-f] - I*Sqrt[g])*(1 - I*c*x))])/(2*c^2*g) + (b*e*(c^2*f
- g)*ArcTan[c*x]*Log[(2*c*(Sqrt[-f] + Sqrt[g]*x))/((c*Sqrt[-f] + I*Sqrt[g])
*(1 - I*c*x))])/(2*c^2*g) - (b*e*x*Log[f + g*x^2])/(2*c) - (b*e*(c^2*f - g)
*ArcTan[c*x]*Log[f + g*x^2])/(2*c^2*g) + (e*(f + g*x^2)*(a + b*ArcTan[c*x])
*Log[f + g*x^2])/(2*g) + ((I/2)*b*e*(c^2*f - g)*PolyLog[2, 1 - 2/(1 - I*c*x
)])/(c^2*g) - ((I/4)*b*e*(c^2*f - g)*PolyLog[2, 1 - (2*c*(Sqrt[-f] - Sqrt[g
]*x))/((c*Sqrt[-f] - I*Sqrt[g])*(1 - I*c*x))])/(c^2*g) - ((I/4)*b*e*(c^2*f
- g)*PolyLog[2, 1 - (2*c*(Sqrt[-f] + Sqrt[g]*x))/((c*Sqrt[-f] + I*Sqrt[g])*
(1 - I*c*x))])/(c^2*g)
```

Rubi [A] time = 0.711949, antiderivative size = 562, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 16, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.727, Rules used = {2454, 2389, 2295, 5019, 321, 203, 2528, 2448, 205, 2470, 12, 4928, 4856, 2402, 2315, 2447}

$$\frac{ibe\left(c^2f-g\right)\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2c^2g}-\frac{ibe\left(c^2f-g\right)\text{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-f}-\sqrt{g}x\right)}{(1-icx)\left(c\sqrt{-f}-i\sqrt{g}\right)}\right)}{4c^2g}-\frac{ibe\left(c^2f-g\right)\text{PolyLog}\left(2,1-\right.}{4c^2g}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcTan[c*x])*(d + e*Log[f + g*x^2]),x]
```


```
[Out] -(b*(d - e)*x)/(2*c) + (b*e*x)/c + (b*(d - e)*ArcTan[c*x])/(2*c^2) + (d*x^2
*(a + b*ArcTan[c*x]))/2 - (e*x^2*(a + b*ArcTan[c*x]))/2 - (b*e*Sqrt[f]*ArcT
an[(Sqrt[g]*x)/Sqrt[f]])/(c*Sqrt[g]) - (b*e*(c^2*f - g)*ArcTan[c*x]*Log[2/(
1 - I*c*x)])/(c^2*g) + (b*e*(c^2*f - g)*ArcTan[c*x]*Log[(2*c*(Sqrt[-f] - Sq
rt[g]*x))/((c*Sqrt[-f] - I*Sqrt[g])*(1 - I*c*x))])/(2*c^2*g) + (b*e*(c^2*f
- g)*ArcTan[c*x]*Log[(2*c*(Sqrt[-f] + Sqrt[g]*x))/((c*Sqrt[-f] + I*Sqrt[g])
```

```
*(1 - I*c*x))])/(2*c^2*g) - (b*e*x*Log[f + g*x^2])/(2*c) - (b*e*(c^2*f - g)
*ArcTan[c*x]*Log[f + g*x^2])/(2*c^2*g) + (e*(f + g*x^2)*(a + b*ArcTan[c*x])
*Log[f + g*x^2])/(2*g) + ((I/2)*b*e*(c^2*f - g)*PolyLog[2, 1 - 2/(1 - I*c*x
)])/(c^2*g) - ((I/4)*b*e*(c^2*f - g)*PolyLog[2, 1 - (2*c*(Sqrt[-f] - Sqrt[g
]*x))/((c*Sqrt[-f] - I*Sqrt[g])*(1 - I*c*x))])/(c^2*g) - ((I/4)*b*e*(c^2*f
- g)*PolyLog[2, 1 - (2*c*(Sqrt[-f] + Sqrt[g]*x))/((c*Sqrt[-f] + I*Sqrt[g])*
(1 - I*c*x))])/(c^2*g)
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 5019

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*(
e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*Log[f + g*x^2])
, x]}, Dist[a + b*ArcTan[c*x], u, x] - Dist[b*c, Int[ExpandIntegrand[u/(1 +
 c^2*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[(m + 1)/2
, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4928

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
```

```
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int x\left(a+b\tan^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)dx = \frac{1}{2}dx^2\left(a+b\tan^{-1}(cx)\right)-\frac{1}{2}ex^2\left(a+b\tan^{-1}(cx)\right)+\frac{e\left(f+gx^2\right)\left(a+b\right.}{}$$

$$=\frac{1}{2}dx^2\left(a+b\tan^{-1}(cx)\right)-\frac{1}{2}ex^2\left(a+b\tan^{-1}(cx)\right)+\frac{e\left(f+gx^2\right)\left(a+b\right.}{}$$

$$=-\frac{b(d-e)x}{2c}+\frac{1}{2}dx^2\left(a+b\tan^{-1}(cx)\right)-\frac{1}{2}ex^2\left(a+b\tan^{-1}(cx)\right)+\frac{e\left(f\right.}{}$$

$$=-\frac{b(d-e)x}{2c}+\frac{b(d-e)\tan^{-1}(cx)}{2c^2}+\frac{1}{2}dx^2\left(a+b\tan^{-1}(cx)\right)-\frac{1}{2}ex^2\left(a+\right.$$

$$=-\frac{b(d-e)x}{2c}+\frac{b(d-e)\tan^{-1}(cx)}{2c^2}+\frac{1}{2}dx^2\left(a+b\tan^{-1}(cx)\right)-\frac{1}{2}ex^2\left(a+\right.$$

$$=-\frac{b(d-e)x}{2c}+\frac{bex}{c}+\frac{b(d-e)\tan^{-1}(cx)}{2c^2}+\frac{1}{2}dx^2\left(a+b\tan^{-1}(cx)\right)-\frac{1}{2}e$$

$$=-\frac{b(d-e)x}{2c}+\frac{bex}{c}+\frac{b(d-e)\tan^{-1}(cx)}{2c^2}+\frac{1}{2}dx^2\left(a+b\tan^{-1}(cx)\right)-\frac{1}{2}e$$

$$=-\frac{b(d-e)x}{2c}+\frac{bex}{c}+\frac{b(d-e)\tan^{-1}(cx)}{2c^2}+\frac{1}{2}dx^2\left(a+b\tan^{-1}(cx)\right)-\frac{1}{2}e$$

$$=-\frac{b(d-e)x}{2c}+\frac{bex}{c}+\frac{b(d-e)\tan^{-1}(cx)}{2c^2}+\frac{1}{2}dx^2\left(a+b\tan^{-1}(cx)\right)-\frac{1}{2}e$$

$$=-\frac{b(d-e)x}{2c}+\frac{bex}{c}+\frac{b(d-e)\tan^{-1}(cx)}{2c^2}+\frac{1}{2}dx^2\left(a+b\tan^{-1}(cx)\right)-\frac{1}{2}e$$

$$=-\frac{b(d-e)x}{2c}+\frac{bex}{c}+\frac{b(d-e)\tan^{-1}(cx)}{2c^2}+\frac{1}{2}dx^2\left(a+b\tan^{-1}(cx)\right)-\frac{1}{2}e$$

Mathematica [B] time = 6.82376, size = 1138, normalized size = 2.02

$$\frac{2adgx^2c^2-2aegx^2c^2+2bdgx^2\tan^{-1}(cx)c^2-2begx^2\tan^{-1}(cx)c^2+4ibef\sin^{-1}\left(\sqrt{\frac{c^2f}{c^2f-g}}\right)\tan^{-1}\left(\frac{cgx}{\sqrt{c^2fg}}\right)c^2-4bef\tan^{-1}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*(a + b*ArcTan[c*x])*(d + e*Log[f + g*x^2]),x]
```

```
[Out] (-2*b*c*d*g*x + 6*b*c*e*g*x + 2*a*c^2*d*g*x^2 - 2*a*c^2*e*g*x^2 + 2*b*d*g*A
rcTan[c*x] - 2*b*e*g*ArcTan[c*x] + 2*b*c^2*d*g*x^2*ArcTan[c*x] - 2*b*c^2*e*
g*x^2*ArcTan[c*x] - 4*b*c*e*Sqrt[f]*Sqrt[g]*ArcTan[(Sqrt[g]*x)/Sqrt[f]] + (
4*I)*b*c^2*e*f*ArcSin[Sqrt[(c^2*f)/(c^2*f - g)]]*ArcTan[(c*g*x)/Sqrt[c^2*f*
g]] - (4*I)*b*e*g*ArcSin[Sqrt[(c^2*f)/(c^2*f - g)]]*ArcTan[(c*g*x)/Sqrt[c^2
*f*g]] - 4*b*c^2*e*f*ArcTan[c*x]*Log[1 + E^((2*I)*ArcTan[c*x])] + 4*b*e*g*A
rcTan[c*x]*Log[1 + E^((2*I)*ArcTan[c*x])] + 2*b*c^2*e*f*ArcSin[Sqrt[(c^2*f)
/(c^2*f - g)]]*Log[(c^2*(1 + E^((2*I)*ArcTan[c*x]))*f + (-1 + E^((2*I)*ArcT
an[c*x]))*g - 2*E^((2*I)*ArcTan[c*x])*Sqrt[c^2*f*g])/(c^2*f - g)] - 2*b*e*g
*ArcSin[Sqrt[(c^2*f)/(c^2*f - g)]]*Log[(c^2*(1 + E^((2*I)*ArcTan[c*x]))*f +
 (-1 + E^((2*I)*ArcTan[c*x]))*g - 2*E^((2*I)*ArcTan[c*x])*Sqrt[c^2*f*g])/(c
^2*f - g)] + 2*b*c^2*e*f*ArcTan[c*x]*Log[(c^2*(1 + E^((2*I)*ArcTan[c*x]))*f
 + (-1 + E^((2*I)*ArcTan[c*x]))*g - 2*E^((2*I)*ArcTan[c*x])*Sqrt[c^2*f*g])/
(c^2*f - g)] - 2*b*e*g*ArcTan[c*x]*Log[(c^2*(1 + E^((2*I)*ArcTan[c*x]))*f +
 (-1 + E^((2*I)*ArcTan[c*x]))*g - 2*E^((2*I)*ArcTan[c*x])*Sqrt[c^2*f*g])/(c
^2*f - g)] - 2*b*c^2*e*f*ArcSin[Sqrt[(c^2*f)/(c^2*f - g)]]*Log[1 + (E^((2*I
)*ArcTan[c*x])*(c^2*f + g + 2*Sqrt[c^2*f*g]))/(c^2*f - g)] + 2*b*e*g*ArcSin
[Sqrt[(c^2*f)/(c^2*f - g)]]*Log[1 + (E^((2*I)*ArcTan[c*x])*(c^2*f + g + 2*S
qrt[c^2*f*g]))/(c^2*f - g)] + 2*b*c^2*e*f*ArcTan[c*x]*Log[1 + (E^((2*I)*Arc
Tan[c*x])*(c^2*f + g + 2*Sqrt[c^2*f*g]))/(c^2*f - g)] - 2*b*e*g*ArcTan[c*x]
*Log[1 + (E^((2*I)*ArcTan[c*x])*(c^2*f + g + 2*Sqrt[c^2*f*g]))/(c^2*f - g)]
 + 2*a*c^2*e*f*Log[f + g*x^2] - 2*b*c*e*g*x*Log[f + g*x^2] + 2*a*c^2*e*g*x^
2*Log[f + g*x^2] + 2*b*e*g*ArcTan[c*x]*Log[f + g*x^2] + 2*b*c^2*e*g*x^2*Arc
Tan[c*x]*Log[f + g*x^2] + (2*I)*b*e*(c^2*f - g)*PolyLog[2, -E^((2*I)*ArcTan
[c*x])] - I*b*e*(c^2*f - g)*PolyLog[2, -((E^((2*I)*ArcTan[c*x])*(c^2*f + g
- 2*Sqrt[c^2*f*g]))/(c^2*f - g))] - I*b*c^2*e*f*PolyLog[2, -((E^((2*I)*ArcT
an[c*x])*(c^2*f + g + 2*Sqrt[c^2*f*g]))/(c^2*f - g))] + I*b*e*g*PolyLog[2,
-((E^((2*I)*ArcTan[c*x])*(c^2*f + g + 2*Sqrt[c^2*f*g]))/(c^2*f - g))])/(4*c
^2*g)
```

Maple [C] time = 3.667, size = 21442, normalized size = 38.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctan(c*x))*(d+e*ln(g*x^2+f)),x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))*(d+e*log(g*x^2+f)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(bdx\arctan(cx) + adx + (bex\arctan(cx) + aex)\log\left(gx^2 + f\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))*(d+e*log(g*x^2+f)),x, algorithm="fricas")
```

```
[Out] integral(b*d*x*arctan(c*x) + a*d*x + (b*e*x*arctan(c*x) + a*e*x)*log(g*x^2
+ f), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atan(c*x))*(d+e*ln(g*x**2+f)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\arctan(cx) + a)\left(e\log\left(gx^2 + f\right) + d\right)x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))*(d+e*log(g*x^2+f)),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)*(e*log(g*x^2 + f) + d)*x, x)
```

3.1298 $\int \left(a+b\tan^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)\,dx$

Optimal. Leaf size=656

$$-\frac{be\text{PolyLog}\left(2,\frac{c^2(f+gx^2)}{c^2f-g}\right)}{2c}-\frac{ibe\sqrt{-f}\text{PolyLog}\left(2,\frac{\sqrt{g}(-cx+i)}{c\sqrt{-f}+i\sqrt{g}}\right)}{2\sqrt{g}}+\frac{ibe\sqrt{-f}\text{PolyLog}\left(2,\frac{\sqrt{g}(1-icx)}{\sqrt{g}+ic\sqrt{-f}}\right)}{2\sqrt{g}}+\frac{ibe\sqrt{-f}\text{PolyLog}\left(2,\ldots\right.}{2\sqrt{g}}$$

```
[Out] -2*a*e*x - 2*b*e*x*ArcTan[c*x] + (2*a*e*Sqrt[f]*ArcTan[(Sqrt[g]*x)/Sqrt[f]]
)/Sqrt[g] + ((I/2)*b*e*Sqrt[-f]*Log[1 + I*c*x]*Log[(c*(Sqrt[-f] - Sqrt[g]*x
))/(c*Sqrt[-f] - I*Sqrt[g])])/Sqrt[g] - ((I/2)*b*e*Sqrt[-f]*Log[1 - I*c*x]*
Log[(c*(Sqrt[-f] - Sqrt[g]*x))/(c*Sqrt[-f] + I*Sqrt[g])])/Sqrt[g] + ((I/2)*
b*e*Sqrt[-f]*Log[1 - I*c*x]*Log[(c*(Sqrt[-f] + Sqrt[g]*x))/(c*Sqrt[-f] - I*
Sqrt[g])])/Sqrt[g] - ((I/2)*b*e*Sqrt[-f]*Log[1 + I*c*x]*Log[(c*(Sqrt[-f] +
Sqrt[g]*x))/(c*Sqrt[-f] + I*Sqrt[g])])/Sqrt[g] + (b*e*Log[1 + c^2*x^2])/c +
 x*(a + b*ArcTan[c*x])*(d + e*Log[f + g*x^2]) - (b*Log[-((g*(1 + c^2*x^2))/
(c^2*f - g))]*(d + e*Log[f + g*x^2]))/(2*c) - ((I/2)*b*e*Sqrt[-f]*PolyLog[2
, (Sqrt[g]*(I - c*x))/(c*Sqrt[-f] + I*Sqrt[g])])/Sqrt[g] + ((I/2)*b*e*Sqrt[
-f]*PolyLog[2, (Sqrt[g]*(1 - I*c*x))/(I*c*Sqrt[-f] + Sqrt[g])])/Sqrt[g] + (
(I/2)*b*e*Sqrt[-f]*PolyLog[2, (Sqrt[g]*(1 + I*c*x))/(I*c*Sqrt[-f] + Sqrt[g]
)])/Sqrt[g] - ((I/2)*b*e*Sqrt[-f]*PolyLog[2, (Sqrt[g]*(I + c*x))/(c*Sqrt[-f
] + I*Sqrt[g])])/Sqrt[g] - (b*e*PolyLog[2, (c^2*(f + g*x^2))/(c^2*f - g)])/
(2*c)
```

Rubi [A] time = 0.830916, antiderivative size = 656, normalized size of antiderivative = 1., number of steps used = 28, number of rules used = 12, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5009, 2475, 2394, 2393, 2391, 4916, 4846, 260, 4910, 205, 4908, 2409}

$$-\frac{be\text{PolyLog}\left(2,\frac{c^2(f+gx^2)}{c^2f-g}\right)}{2c}-\frac{ibe\sqrt{-f}\text{PolyLog}\left(2,\frac{\sqrt{g}(-cx+i)}{c\sqrt{-f}+i\sqrt{g}}\right)}{2\sqrt{g}}+\frac{ibe\sqrt{-f}\text{PolyLog}\left(2,\frac{\sqrt{g}(1-icx)}{\sqrt{g}+ic\sqrt{-f}}\right)}{2\sqrt{g}}+\frac{ibe\sqrt{-f}\text{PolyLog}\left(2,\ldots\right.}{2\sqrt{g}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])*(d + e*Log[f + g*x^2]),x]
```

```
[Out] -2*a*e*x - 2*b*e*x*ArcTan[c*x] + (2*a*e*Sqrt[f]*ArcTan[(Sqrt[g]*x)/Sqrt[f]]
)/Sqrt[g] + ((I/2)*b*e*Sqrt[-f]*Log[1 + I*c*x]*Log[(c*(Sqrt[-f] - Sqrt[g]*x
))/(c*Sqrt[-f] - I*Sqrt[g])])/Sqrt[g] - ((I/2)*b*e*Sqrt[-f]*Log[1 - I*c*x]*
Log[(c*(Sqrt[-f] - Sqrt[g]*x))/(c*Sqrt[-f] + I*Sqrt[g])])/Sqrt[g] + ((I/2)*
b*e*Sqrt[-f]*Log[1 - I*c*x]*Log[(c*(Sqrt[-f] + Sqrt[g]*x))/(c*Sqrt[-f] - I*
```

```
Sqrt[g])])/Sqrt[g] - ((I/2)*b*e*Sqrt[-f]*Log[1 + I*c*x]*Log[(c*(Sqrt[-f] +
Sqrt[g]*x))/(c*Sqrt[-f] + I*Sqrt[g])])/Sqrt[g] + (b*e*Log[1 + c^2*x^2])/c +
 x*(a + b*ArcTan[c*x])*(d + e*Log[f + g*x^2]) - (b*Log[-((g*(1 + c^2*x^2))/
(c^2*f - g))]*(d + e*Log[f + g*x^2]))/(2*c) - ((I/2)*b*e*Sqrt[-f]*PolyLog[2
, (Sqrt[g]*(I - c*x))/(c*Sqrt[-f] + I*Sqrt[g])])/Sqrt[g] + ((I/2)*b*e*Sqrt[
-f]*PolyLog[2, (Sqrt[g]*(1 - I*c*x))/(I*c*Sqrt[-f] + Sqrt[g])])/Sqrt[g] + (
(I/2)*b*e*Sqrt[-f]*PolyLog[2, (Sqrt[g]*(1 + I*c*x))/(I*c*Sqrt[-f] + Sqrt[g]
)])/Sqrt[g] - ((I/2)*b*e*Sqrt[-f]*PolyLog[2, (Sqrt[g]*(I + c*x))/(c*Sqrt[-f
] + I*Sqrt[g])])/Sqrt[g] - (b*e*PolyLog[2, (c^2*(f + g*x^2))/(c^2*f - g)])/
(2*c)
```

Rule 5009

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*(
e_.)), x_Symbol] :> Simp[x*(d + e*Log[f + g*x^2])*(a + b*ArcTan[c*x]), x] +
 (-Dist[b*c, Int[(x*(d + e*Log[f + g*x^2]))/(1 + c^2*x^2), x], x] - Dist[2*
e*g, Int[(x^2*(a + b*ArcTan[c*x]))/(f + g*x^2), x], x]) /; FreeQ[{a, b, c,
d, e, f, g}, x]
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
```

```
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4910

```
Int[(ArcTan[(c_.)*(x_)]*(b_.) + (a_))/((d_.) + (e_.)*(x_)^2), x_Symbol] :>
Dist[a, Int[1/(d + e*x^2), x], x] + Dist[b, Int[ArcTan[c*x]/(d + e*x^2), x]
, x] /; FreeQ[{a, b, c, d, e}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 4908

```
Int[ArcTan[(c_.)*(x_)]/((d_.) + (e_.)*(x_)^2), x_Symbol] :> Dist[I/2, Int[L
og[1 - I*c*x]/(d + e*x^2), x], x] - Dist[I/2, Int[Log[1 + I*c*x]/(d + e*x^2
), x], x] /; FreeQ[{c, d, e}, x]
```

Rule 2409

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rubi steps

$$\int \left(a+b\tan^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)\,dx = x\left(a+b\tan^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)-(bc)\int \frac{x\left(d+e\log\left(f+gx^2\right)\right)}{1+c^2x^2}$$

$$= x\left(a+b\tan^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)-\frac{1}{2}(bc)\,\text{Subst}\left(\int \frac{d+e\log(f+}{1+c^2x}\right.$$

$$= -2aex+x\left(a+b\tan^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)-\frac{b\log\left(-\frac{g\left(1+c^2x^2\right)}{c^2f-g}\right)\left(d\right.}{2c}$$

$$= -2aex-2bex\tan^{-1}(cx)+\frac{2ae\sqrt{f}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{g}}+x\left(a+b\tan^{-1}(cx)\right)\left(d+\right.$$

$$= -2aex-2bex\tan^{-1}(cx)+\frac{2ae\sqrt{f}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{g}}+\frac{be\log\left(1+c^2x^2\right)}{c}+x\left(a\right.$$

$$= -2aex-2bex\tan^{-1}(cx)+\frac{2ae\sqrt{f}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{g}}+\frac{be\log\left(1+c^2x^2\right)}{c}+x\left(a\right.$$

$$= -2aex-2bex\tan^{-1}(cx)+\frac{2ae\sqrt{f}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{g}}+\frac{ibe\sqrt{-f}\log(1+icx)\log}{2\sqrt{g}}$$

$$= -2aex-2bex\tan^{-1}(cx)+\frac{2ae\sqrt{f}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{g}}+\frac{ibe\sqrt{-f}\log(1+icx)\log}{2\sqrt{g}}$$

$$= -2aex-2bex\tan^{-1}(cx)+\frac{2ae\sqrt{f}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{g}}+\frac{ibe\sqrt{-f}\log(1+icx)\log}{2\sqrt{g}}$$

Mathematica [B] time = 3.48472, size = 1362, normalized size = 2.08

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])*(d + e*Log[f + g*x^2]),x]
```

```
[Out] a*d*x - 2*a*e*x + b*d*x*ArcTan[c*x] + (2*a*e*Sqrt[f]*ArcTan[(Sqrt[g]*x)/Sqr
t[f]])/Sqrt[g] - (b*d*Log[1 + c^2*x^2])/(2*c) + a*e*x*Log[f + g*x^2] + b*e*
(x*ArcTan[c*x] - Log[1 + c^2*x^2]/(2*c))*Log[f + g*x^2] + (b*e*g*(((-Log[(-
I)/c + x] - Log[I/c + x] + Log[1 + c^2*x^2])*Log[f + g*x^2])/(2*g) + (Log[(
-I)/c + x]*Log[1 - (Sqrt[g]*((-I)/c + x))/((-I)*Sqrt[f] - (I*Sqrt[g])/c)] +
 PolyLog[2, (Sqrt[g]*((-I)/c + x))/((-I)*Sqrt[f] - (I*Sqrt[g])/c)])/(2*g) +
 (Log[(-I)/c + x]*Log[1 - (Sqrt[g]*((-I)/c + x))/(I*Sqrt[f] - (I*Sqrt[g])/c
)] + PolyLog[2, (Sqrt[g]*((-I)/c + x))/(I*Sqrt[f] - (I*Sqrt[g])/c)])/(2*g)
+ (Log[I/c + x]*Log[1 - (Sqrt[g]*(I/c + x))/((-I)*Sqrt[f] + (I*Sqrt[g])/c)]
 + PolyLog[2, (Sqrt[g]*(I/c + x))/((-I)*Sqrt[f] + (I*Sqrt[g])/c)])/(2*g) +
(Log[I/c + x]*Log[1 - (Sqrt[g]*(I/c + x))/(I*Sqrt[f] + (I*Sqrt[g])/c)] + Po
lyLog[2, (Sqrt[g]*(I/c + x))/(I*Sqrt[f] + (I*Sqrt[g])/c)])/(2*g)))/c - (b*e
*(4*c*x*ArcTan[c*x] + 4*Log[1/Sqrt[1 + c^2*x^2]] + (c^2*f*(4*ArcTan[c*x]*Ar
cTanh[Sqrt[-(c^2*f*g)]/(c*g*x)] - 2*ArcCos[-((c^2*f + g)/(c^2*f - g))]*ArcT
anh[(c*g*x)/Sqrt[-(c^2*f*g)]] - (ArcCos[-((c^2*f + g)/(c^2*f - g))] - (2*I)
*ArcTanh[(c*g*x)/Sqrt[-(c^2*f*g)]])*Log[(-2*c^2*f*(I*g + Sqrt[-(c^2*f*g)])*
(-I + c*x))/((c^2*f - g)*(c^2*f - c*Sqrt[-(c^2*f*g)]*x))] - (ArcCos[-((c^2*
f + g)/(c^2*f - g))] + (2*I)*ArcTanh[(c*g*x)/Sqrt[-(c^2*f*g)]])*Log[((2*I)*
c^2*f*(g + I*Sqrt[-(c^2*f*g)])*(I + c*x))/((c^2*f - g)*(c^2*f - c*Sqrt[-(c^
2*f*g)]*x))] + (ArcCos[-((c^2*f + g)/(c^2*f - g))] - (2*I)*ArcTanh[Sqrt[-(c
^2*f*g)]/(c*g*x)] + (2*I)*ArcTanh[(c*g*x)/Sqrt[-(c^2*f*g)]])*Log[(Sqrt[2]*S
qrt[-(c^2*f*g)])/(E^(I*ArcTan[c*x])*Sqrt[-(c^2*f) + g]*Sqrt[-(c^2*f) - g +
(-(c^2*f) + g)*Cos[2*ArcTan[c*x]]])] + (ArcCos[-((c^2*f + g)/(c^2*f - g))]
+ (2*I)*ArcTanh[Sqrt[-(c^2*f*g)]/(c*g*x)] - (2*I)*ArcTanh[(c*g*x)/Sqrt[-(c^
2*f*g)]])*Log[(Sqrt[2]*E^(I*ArcTan[c*x])*Sqrt[-(c^2*f*g)])/(Sqrt[-(c^2*f) +
 g]*Sqrt[-(c^2*f) - g + (-(c^2*f) + g)*Cos[2*ArcTan[c*x]]])] + I*(-PolyLog[
2, ((c^2*f + g - (2*I)*Sqrt[-(c^2*f*g)])*(c^2*f + c*Sqrt[-(c^2*f*g)]*x))/((
c^2*f - g)*(c^2*f - c*Sqrt[-(c^2*f*g)]*x))] + PolyLog[2, ((c^2*f + g + (2*I
)*Sqrt[-(c^2*f*g)])*(c^2*f + c*Sqrt[-(c^2*f*g)]*x))/((c^2*f - g)*(c^2*f - c
*Sqrt[-(c^2*f*g)]*x))])))/Sqrt[-(c^2*f*g)]))/(2*c)
```

Maple [F] time = 3.373, size = 0, normalized size = 0.

$$\int (a + b \arctan (cx)) \left(d + e \ln \left(g x^2 + f\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))*(d+e*ln(g*x^2+f)),x)
```

```
[Out] int((a+b*arctan(c*x))*(d+e*ln(g*x^2+f)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))*(d+e*log(g*x^2+f)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(bd\arctan\left(cx\right)+ad+\left(be\arctan\left(cx\right)+ae\right)\log\left(gx^2+f\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))*(d+e*log(g*x^2+f)),x, algorithm="fricas")
```

```
[Out] integral(b*d*arctan(c*x) + a*d + (b*e*arctan(c*x) + a*e)*log(g*x^2 + f), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))*(d+e*ln(g*x**2+f)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\arctan\left(cx\right)+a\right)\left(e\log\left(gx^2+f\right)+d\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))*(d+e*log(g*x^2+f)),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)*(e*log(g*x^2 + f) + d), x)
```

3.1299 $\int \frac{(a+b\tan^{-1}(cx))(d+e\log(f+gx^2))}{x}\,dx$

Optimal. Leaf size=101

$$\frac{1}{2}ae\text{PolyLog}\left(2,\frac{gx^2}{f}+1\right)+be\text{CannotIntegrate}\left(\frac{\tan^{-1}(cx)\log\left(f+gx^2\right)}{x},x\right)+\frac{1}{2}ibd\text{PolyLog}(2,-icx)-\frac{1}{2}ibd\text{PolyLog}$$

```
[Out] b*e*CannotIntegrate[(ArcTan[c*x]*Log[f + g*x^2])/x, x] + a*d*Log[x] + (a*e*
Log[-((g*x^2)/f)]*Log[f + g*x^2])/2 + (I/2)*b*d*PolyLog[2, (-I)*c*x] - (I/2
)*b*d*PolyLog[2, I*c*x] + (a*e*PolyLog[2, 1 + (g*x^2)/f])/2
```

Rubi [A] time = 0.28075, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(a+b\tan^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((a + b*ArcTan[c*x])*(d + e*Log[f + g*x^2]))/x,x]
```

```
[Out] a*d*Log[x] + (a*e*Log[-((g*x^2)/f)]*Log[f + g*x^2])/2 + (I/2)*b*d*PolyLog[2
, (-I)*c*x] - (I/2)*b*d*PolyLog[2, I*c*x] + (a*e*PolyLog[2, 1 + (g*x^2)/f])
/2 + b*e*Defer[Int][(ArcTan[c*x]*Log[f + g*x^2])/x, x]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{x}\,dx = d\int \frac{a+b\tan^{-1}(cx)}{x}\,dx + e\int \frac{\left(a+b\tan^{-1}(cx)\right)\log\left(f+gx^2\right)}{x}\,dx$$

$$= ad\log(x) + \frac{1}{2}(ibd)\int \frac{\log(1-icx)}{x}\,dx - \frac{1}{2}(ibd)\int \frac{\log(1+icx)}{x}\,dx + (ae)$$

$$= ad\log(x) + \frac{1}{2}ibd\,\text{Li}_2(-icx) - \frac{1}{2}ibd\,\text{Li}_2(icx) + \frac{1}{2}(ae)\,\text{Subst}\left(\int \frac{\log(f+g}{x}\right.$$

$$= ad\log(x) + \frac{1}{2}ae\log\left(-\frac{gx^2}{f}\right)\log\left(f+gx^2\right) + \frac{1}{2}ibd\,\text{Li}_2(-icx) - \frac{1}{2}ibd\,\text{Li}_2(i$$

$$= ad\log(x) + \frac{1}{2}ae\log\left(-\frac{gx^2}{f}\right)\log\left(f+gx^2\right) + \frac{1}{2}ibd\,\text{Li}_2(-icx) - \frac{1}{2}ibd\,\text{Li}_2(i$$

Mathematica [A] time = 0.192716, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\tan^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((a + b*ArcTan[c*x])*(d + e*Log[f + g*x^2]))/x,x]
```

```
[Out] Integrate[((a + b*ArcTan[c*x])*(d + e*Log[f + g*x^2]))/x, x]
```

Maple [A] time = 1.146, size = 0, normalized size = 0.

$$\int \frac{(a+b\arctan(cx))\left(d+e\ln\left(gx^2+f\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))*(d+e*ln(g*x^2+f))/x,x)
```

```
[Out] int((a+b*arctan(c*x))*(d+e*ln(g*x^2+f))/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$ad\log\left(x\right)+\frac{1}{2}\int\frac{2\left(bd\arctan\left(cx\right)+\left(be\arctan\left(cx\right)+ae\right)\log\left(gx^{2}+f\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))*(d+e*log(g*x^2+f))/x,x, algorithm="maxima")
```

```
[Out] a*d*log(x) + 1/2*integrate(2*(b*d*arctan(c*x) + (b*e*arctan(c*x) + a*e)*log
(g*x^2 + f))/x, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{bd\arctan\left(cx\right)+ad+\left(be\arctan\left(cx\right)+ae\right)\log\left(gx^{2}+f\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))*(d+e*log(g*x^2+f))/x,x, algorithm="fricas")
```

```
[Out] integral((b*d*arctan(c*x) + a*d + (b*e*arctan(c*x) + a*e)*log(g*x^2 + f))/x
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))*(d+e*ln(g*x**2+f))/x,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(b\arctan\left(cx\right)+a\right)\left(e\log\left(gx^{2}+f\right)+d\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))*(d+e*log(g*x^2+f))/x,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)*(e*log(g*x^2 + f) + d)/x, x)
```

3.1300 $\int \frac{(a+b\tan^{-1}(cx))(d+e\log(f+gx^2))}{x^2}\,dx$

Optimal. Leaf size=672

$$-\frac{1}{2}bce\text{PolyLog}\left(2,\frac{c^2\left(f+gx^2\right)}{c^2f-g}\right)+\frac{1}{2}bce\text{PolyLog}\left(2,\frac{gx^2}{f}+1\right)+\frac{ibe\sqrt{g}\text{PolyLog}\left(2,\frac{\sqrt{g}(-cx+i)}{c\sqrt{-f}+i\sqrt{g}}\right)}{2\sqrt{-f}}-\frac{ibe\sqrt{g}\text{PolyLog}\left(2,\frac{\sqrt{g}(}{\sqrt{g}+}\right.}{2\sqrt{-f}}$$

```
[Out] (2*a*e*Sqrt[g]*ArcTan[(Sqrt[g]*x)/Sqrt[f]])/Sqrt[f] - ((I/2)*b*e*Sqrt[g]*Lo
g[1 + I*c*x]*Log[(c*(Sqrt[-f] - Sqrt[g]*x))/(c*Sqrt[-f] - I*Sqrt[g])])/Sqrt
[-f] + ((I/2)*b*e*Sqrt[g]*Log[1 - I*c*x]*Log[(c*(Sqrt[-f] - Sqrt[g]*x))/(c*
Sqrt[-f] + I*Sqrt[g])])/Sqrt[-f] - ((I/2)*b*e*Sqrt[g]*Log[1 - I*c*x]*Log[(c
*(Sqrt[-f] + Sqrt[g]*x))/(c*Sqrt[-f] - I*Sqrt[g])])/Sqrt[-f] + ((I/2)*b*e*S
qrt[g]*Log[1 + I*c*x]*Log[(c*(Sqrt[-f] + Sqrt[g]*x))/(c*Sqrt[-f] + I*Sqrt[g
])])/Sqrt[-f] - ((a + b*ArcTan[c*x])*(d + e*Log[f + g*x^2]))/x + (b*c*Log[-
((g*x^2)/f)]*(d + e*Log[f + g*x^2]))/2 - (b*c*Log[-((g*(1 + c^2*x^2))/(c^2*
f - g))]*(d + e*Log[f + g*x^2]))/2 + ((I/2)*b*e*Sqrt[g]*PolyLog[2, (Sqrt[g]
*(I - c*x))/(c*Sqrt[-f] + I*Sqrt[g])])/Sqrt[-f] - ((I/2)*b*e*Sqrt[g]*PolyLo
g[2, (Sqrt[g]*(1 - I*c*x))/(I*c*Sqrt[-f] + Sqrt[g])])/Sqrt[-f] - ((I/2)*b*e
*Sqrt[g]*PolyLog[2, (Sqrt[g]*(1 + I*c*x))/(I*c*Sqrt[-f] + Sqrt[g])])/Sqrt[-
f] + ((I/2)*b*e*Sqrt[g]*PolyLog[2, (Sqrt[g]*(I + c*x))/(c*Sqrt[-f] + I*Sqrt
[g])])/Sqrt[-f] - (b*c*e*PolyLog[2, (c^2*(f + g*x^2))/(c^2*f - g)])/2 + (b*
c*e*PolyLog[2, 1 + (g*x^2)/f])/2
```

Rubi [A] time = 0.756769, antiderivative size = 672, normalized size of antiderivative = 1., number of steps used = 28, number of rules used = 14, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {5017, 2475, 36, 29, 31, 2416, 2394, 2315, 2393, 2391, 4910, 205, 4908, 2409}

$$-\frac{1}{2}bce\text{PolyLog}\left(2,\frac{c^2\left(f+gx^2\right)}{c^2f-g}\right)+\frac{1}{2}bce\text{PolyLog}\left(2,\frac{gx^2}{f}+1\right)+\frac{ibe\sqrt{g}\text{PolyLog}\left(2,\frac{\sqrt{g}(-cx+i)}{c\sqrt{-f}+i\sqrt{g}}\right)}{2\sqrt{-f}}-\frac{ibe\sqrt{g}\text{PolyLog}\left(2,\frac{\sqrt{g}(}{\sqrt{g}+}\right.}{2\sqrt{-f}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*ArcTan[c*x])*(d + e*Log[f + g*x^2]))/x^2,x]
```

```
[Out] (2*a*e*Sqrt[g]*ArcTan[(Sqrt[g]*x)/Sqrt[f]])/Sqrt[f] - ((I/2)*b*e*Sqrt[g]*Lo
g[1 + I*c*x]*Log[(c*(Sqrt[-f] - Sqrt[g]*x))/(c*Sqrt[-f] - I*Sqrt[g])])/Sqrt
[-f] + ((I/2)*b*e*Sqrt[g]*Log[1 - I*c*x]*Log[(c*(Sqrt[-f] - Sqrt[g]*x))/(c*
Sqrt[-f] + I*Sqrt[g])])/Sqrt[-f] - ((I/2)*b*e*Sqrt[g]*Log[1 - I*c*x]*Log[(c
```

```
*(Sqrt[-f] + Sqrt[g]*x))/(c*Sqrt[-f] - I*Sqrt[g])])/Sqrt[-f] + ((I/2)*b*e*S
qrt[g]*Log[1 + I*c*x]*Log[(c*(Sqrt[-f] + Sqrt[g]*x))/(c*Sqrt[-f] + I*Sqrt[g
])])/Sqrt[-f] - ((a + b*ArcTan[c*x])*(d + e*Log[f + g*x^2]))/x + (b*c*Log[-
((g*x^2)/f)]*(d + e*Log[f + g*x^2]))/2 - (b*c*Log[-((g*(1 + c^2*x^2))/(c^2*
f - g))]*(d + e*Log[f + g*x^2]))/2 + ((I/2)*b*e*Sqrt[g]*PolyLog[2, (Sqrt[g]
*(I - c*x))/(c*Sqrt[-f] + I*Sqrt[g])])/Sqrt[-f] - ((I/2)*b*e*Sqrt[g]*PolyLo
g[2, (Sqrt[g]*(1 - I*c*x))/(I*c*Sqrt[-f] + Sqrt[g])])/Sqrt[-f] - ((I/2)*b*e
*Sqrt[g]*PolyLog[2, (Sqrt[g]*(1 + I*c*x))/(I*c*Sqrt[-f] + Sqrt[g])])/Sqrt[-
f] + ((I/2)*b*e*Sqrt[g]*PolyLog[2, (Sqrt[g]*(I + c*x))/(c*Sqrt[-f] + I*Sqrt
[g])])/Sqrt[-f] - (b*c*e*PolyLog[2, (c^2*(f + g*x^2))/(c^2*f - g)])/2 + (b*
c*e*PolyLog[2, 1 + (g*x^2)/f])/2
```

<u>Rule 5017</u>

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*(
e_.))*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*(d + e*Log[f + g*x^2])*(a +
b*ArcTan[c*x]))/(m + 1), x] + (-Dist[(b*c)/(m + 1), Int[(x^(m + 1)*(d + e*L
og[f + g*x^2]))/(1 + c^2*x^2), x], x] - Dist[(2*e*g)/(m + 1), Int[(x^(m + 2
)*(a + b*ArcTan[c*x]))/(f + g*x^2), x], x]) /; FreeQ[{a, b, c, d, e, f, g},
 x] && ILtQ[m/2, 0]
```

<u>Rule 2475</u>

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

<u>Rule 36</u>

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

<u>Rule 29</u>

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

<u>Rule 31</u>

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4910

```
Int[(ArcTan[(c_.)*(x_)]*(b_.) + (a_))/((d_.) + (e_.)*(x_)^2), x_Symbol] :>
Dist[a, Int[1/(d + e*x^2), x], x] + Dist[b, Int[ArcTan[c*x]/(d + e*x^2), x]
, x] /; FreeQ[{a, b, c, d, e}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 4908

```
Int[ArcTan[(c_.)*(x_)]/((d_.) + (e_.)*(x_)^2), x_Symbol] :> Dist[I/2, Int[L
og[1 - I*c*x]/(d + e*x^2), x], x] - Dist[I/2, Int[Log[1 + I*c*x]/(d + e*x^2
), x], x] /; FreeQ[{c, d, e}, x]
```

Rule 2409

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{x^2}\,dx = -\frac{\left(a+b\tan^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{x} + (bc)\int \frac{d+e\log\left(f+gx^2\right)}{x\left(1+c^2x^2\right)}\,d$$

$$= -\frac{\left(a+b\tan^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{x} + \frac{1}{2}(bc)\,\mathrm{Subst}\left(\int \frac{d+e\log(f}{x\left(1+c\right.}\right.$$

$$= \frac{2ae\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}} - \frac{\left(a+b\tan^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{x} + \frac{1}{2}(bc)\,\mathrm{S}$$

$$= \frac{2ae\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}} - \frac{\left(a+b\tan^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{x} + \frac{1}{2}(bc)\,\mathrm{S}$$

$$= \frac{2ae\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}} - \frac{\left(a+b\tan^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{x} + \frac{1}{2}bc\log$$

$$= \frac{2ae\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}} - \frac{ibe\sqrt{g}\log(1+icx)\log\left(\frac{c\left(\sqrt{-f}-\sqrt{g}x\right)}{c\sqrt{-f}-i\sqrt{g}}\right)}{2\sqrt{-f}} + \frac{ibe\sqrt{g}\log(}{}$$

$$= \frac{2ae\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}} - \frac{ibe\sqrt{g}\log(1+icx)\log\left(\frac{c\left(\sqrt{-f}-\sqrt{g}x\right)}{c\sqrt{-f}-i\sqrt{g}}\right)}{2\sqrt{-f}} + \frac{ibe\sqrt{g}\log(}{}$$

$$= \frac{2ae\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}} - \frac{ibe\sqrt{g}\log(1+icx)\log\left(\frac{c\left(\sqrt{-f}-\sqrt{g}x\right)}{c\sqrt{-f}-i\sqrt{g}}\right)}{2\sqrt{-f}} + \frac{ibe\sqrt{g}\log(}{}$$

Mathematica [A] time = 0.902962, size = 552, normalized size = 0.82

$$\frac{1}{2}\left(\frac{e\sqrt{g}\left(ib\sqrt{f}\left(\text{PolyLog}\left(2,\frac{\sqrt{g}(-cx+i)}{c\sqrt{-f}+i\sqrt{g}}\right)+\log(1+icx)\log\left(\frac{c\left(\sqrt{-f}+\sqrt{g}x\right)}{c\sqrt{-f}+i\sqrt{g}}\right)\right)-ib\sqrt{f}\left(\text{PolyLog}\left(2,\frac{\sqrt{g}(1-icx)}{\sqrt{g}+ic\sqrt{-f}}\right)+\log(1-icx)\log\right.\right.}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*ArcTan[c*x])*(d + e*Log[f + g*x^2]))/x^2,x]
```

```
[Out] ((-2*(a + b*ArcTan[c*x])*(d + e*Log[f + g*x^2]))/x + (e*Sqrt[g]*(4*a*Sqrt[-
f]*ArcTan[(Sqrt[g]*x)/Sqrt[f]] + I*b*Sqrt[f]*(Log[1 + I*c*x]*Log[(c*(Sqrt[-
f] + Sqrt[g]*x))/(c*Sqrt[-f] + I*Sqrt[g])] + PolyLog[2, (Sqrt[g]*(I - c*x))
/(c*Sqrt[-f] + I*Sqrt[g])]) - I*b*Sqrt[f]*(Log[1 - I*c*x]*Log[(c*(Sqrt[-f]
+ Sqrt[g]*x))/(c*Sqrt[-f] - I*Sqrt[g])] + PolyLog[2, (Sqrt[g]*(1 - I*c*x))/
(I*c*Sqrt[-f] + Sqrt[g])]) - I*b*Sqrt[f]*(Log[1 + I*c*x]*Log[(c*(Sqrt[-f] -
 Sqrt[g]*x))/(c*Sqrt[-f] - I*Sqrt[g])] + PolyLog[2, (Sqrt[g]*(1 + I*c*x))/(
I*c*Sqrt[-f] + Sqrt[g])]) + I*b*Sqrt[f]*(Log[1 - I*c*x]*Log[(c*(Sqrt[-f] -
Sqrt[g]*x))/(c*Sqrt[-f] + I*Sqrt[g])] + PolyLog[2, (Sqrt[g]*(I + c*x))/(c*S
qrt[-f] + I*Sqrt[g])])))/Sqrt[-f^2] + b*c*((Log[-((g*x^2)/f)] - Log[-((g*(1
 + c^2*x^2))/(c^2*f - g))])*(d + e*Log[f + g*x^2]) - e*PolyLog[2, (c^2*(f +
 g*x^2))/(c^2*f - g)] + e*PolyLog[2, 1 + (g*x^2)/f]))/2
```

Maple [F] time = 1.717, size = 0, normalized size = 0.

$$\int \frac{(a+b\arctan(cx))\left(d+e\ln\left(gx^2+f\right)\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))*(d+e*ln(g*x^2+f))/x^2,x)
```

```
[Out] int((a+b*arctan(c*x))*(d+e*ln(g*x^2+f))/x^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))*(d+e*log(g*x^2+f))/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{bd\arctan(cx)+ad+(be\arctan(cx)+ae)\log\left(gx^2+f\right)}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))*(d+e*log(g*x^2+f))/x^2,x, algorithm="fricas")
```

```
[Out] integral((b*d*arctan(c*x) + a*d + (b*e*arctan(c*x) + a*e)*log(g*x^2 + f))/x
^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))*(d+e*ln(g*x**2+f))/x**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(b\arctan(cx)+a)\left(e\log\left(gx^2+f\right)+d\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))*(d+e*log(g*x^2+f))/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)*(e*log(g*x^2 + f) + d)/x^2, x)
```

3.1301 $\int \frac{(a+b\tan^{-1}(cx))(d+e\log(f+gx^2))}{x^3}\,dx$

Optimal. Leaf size=528

$$\frac{ibe\left(c^2f-g\right)\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2f}-\frac{ibe\left(c^2f-g\right)\text{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-f}-\sqrt{g}x\right)}{(1-icx)\left(c\sqrt{-f}-i\sqrt{g}\right)}\right)}{4f}-\frac{ibe\left(c^2f-g\right)\text{PolyLog}\left(2,1-\frac{2}{(1-}\right.}{4f}$$

```
[Out] (b*c*e*Sqrt[g]*ArcTan[(Sqrt[g]*x)/Sqrt[f]])/Sqrt[f] + (a*e*g*Log[x])/f - (b
*e*(c^2*f - g)*ArcTan[c*x]*Log[2/(1 - I*c*x)])/f + (b*e*(c^2*f - g)*ArcTan[
c*x]*Log[(2*c*(Sqrt[-f] - Sqrt[g]*x))/((c*Sqrt[-f] - I*Sqrt[g])*(1 - I*c*x)
)])/(2*f) + (b*e*(c^2*f - g)*ArcTan[c*x]*Log[(2*c*(Sqrt[-f] + Sqrt[g]*x))/(
(c*Sqrt[-f] + I*Sqrt[g])*(1 - I*c*x))])/(2*f) - (a*e*g*Log[f + g*x^2])/(2*f
) - (b*c*(d + e*Log[f + g*x^2]))/(2*x) - (b*c^2*ArcTan[c*x]*(d + e*Log[f +
g*x^2]))/2 - ((a + b*ArcTan[c*x])*(d + e*Log[f + g*x^2]))/(2*x^2) + ((I/2)*
b*e*g*PolyLog[2, (-I)*c*x])/f - ((I/2)*b*e*g*PolyLog[2, I*c*x])/f + ((I/2)*
b*e*(c^2*f - g)*PolyLog[2, 1 - 2/(1 - I*c*x)])/f - ((I/4)*b*e*(c^2*f - g)*P
olyLog[2, 1 - (2*c*(Sqrt[-f] - Sqrt[g]*x))/((c*Sqrt[-f] - I*Sqrt[g])*(1 - I
*c*x))])/f - ((I/4)*b*e*(c^2*f - g)*PolyLog[2, 1 - (2*c*(Sqrt[-f] + Sqrt[g]
*x))/((c*Sqrt[-f] + I*Sqrt[g])*(1 - I*c*x))])/f
```

Rubi [A] time = 0.769275, antiderivative size = 528, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 18, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75, Rules used = {4852, 325, 203, 5021, 801, 635, 205, 260, 446, 72, 6725, 4848, 2391, 4928, 4856, 2402, 2315, 2447}

$$\frac{ibe\left(c^2f-g\right)\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2f}-\frac{ibe\left(c^2f-g\right)\text{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-f}-\sqrt{g}x\right)}{(1-icx)\left(c\sqrt{-f}-i\sqrt{g}\right)}\right)}{4f}-\frac{ibe\left(c^2f-g\right)\text{PolyLog}\left(2,1-\frac{2}{(1-}\right.}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*ArcTan[c*x])*(d + e*Log[f + g*x^2]))/x^3,x]
```

```
[Out] (b*c*e*Sqrt[g]*ArcTan[(Sqrt[g]*x)/Sqrt[f]])/Sqrt[f] + (a*e*g*Log[x])/f - (b
*e*(c^2*f - g)*ArcTan[c*x]*Log[2/(1 - I*c*x)])/f + (b*e*(c^2*f - g)*ArcTan[
c*x]*Log[(2*c*(Sqrt[-f] - Sqrt[g]*x))/((c*Sqrt[-f] - I*Sqrt[g])*(1 - I*c*x)
)])/(2*f) + (b*e*(c^2*f - g)*ArcTan[c*x]*Log[(2*c*(Sqrt[-f] + Sqrt[g]*x))/(
(c*Sqrt[-f] + I*Sqrt[g])*(1 - I*c*x))])/(2*f) - (a*e*g*Log[f + g*x^2])/(2*f
) - (b*c*(d + e*Log[f + g*x^2]))/(2*x) - (b*c^2*ArcTan[c*x]*(d + e*Log[f +
g*x^2]))/2 - ((a + b*ArcTan[c*x])*(d + e*Log[f + g*x^2]))/(2*x^2) + ((I/2)*
```

```
b*e*g*PolyLog[2, (-I)*c*x])/f - ((I/2)*b*e*g*PolyLog[2, I*c*x])/f + ((I/2)*
b*e*(c^2*f - g)*PolyLog[2, 1 - 2/(1 - I*c*x)])/f - ((I/4)*b*e*(c^2*f - g)*P
olyLog[2, 1 - (2*c*(Sqrt[-f] - Sqrt[g]*x))/((c*Sqrt[-f] - I*Sqrt[g])*(1 - I
*c*x))])/f - ((I/4)*b*e*(c^2*f - g)*PolyLog[2, 1 - (2*c*(Sqrt[-f] + Sqrt[g]
*x))/((c*Sqrt[-f] + I*Sqrt[g])*(1 - I*c*x))])/f
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 5021

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*(
e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(a + b*ArcTan[c*x]), x
]}, Dist[d + e*Log[f + g*x^2], u, x] - Dist[2*e*g, Int[ExpandIntegrand[(x*u
)/(f + g*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IntegerQ[m
] && NeQ[m, -1]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4928

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{x^3}\,dx = -\frac{bc\left(d+e\log\left(f+gx^2\right)\right)}{2x} - \frac{1}{2}bc^2\tan^{-1}(cx)\left(d+e\log\left(f+gx^2\right)\right) - \frac{(a+b}{}$$

$$= -\frac{bc\left(d+e\log\left(f+gx^2\right)\right)}{2x} - \frac{1}{2}bc^2\tan^{-1}(cx)\left(d+e\log\left(f+gx^2\right)\right) - \frac{(a+b}{}$$

$$= -\frac{bc\left(d+e\log\left(f+gx^2\right)\right)}{2x} - \frac{1}{2}bc^2\tan^{-1}(cx)\left(d+e\log\left(f+gx^2\right)\right) - \frac{(a+b}{}$$

$$= \frac{aeg\log(x)}{f} - \frac{bc\left(d+e\log\left(f+gx^2\right)\right)}{2x} - \frac{1}{2}bc^2\tan^{-1}(cx)\left(d+e\log\left(f+gx\right.\right.$$

$$= \frac{aeg\log(x)}{f} - \frac{bc\left(d+e\log\left(f+gx^2\right)\right)}{2x} - \frac{1}{2}bc^2\tan^{-1}(cx)\left(d+e\log\left(f+gx\right.\right.$$

$$= \frac{bce\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}} + \frac{aeg\log(x)}{f} - \frac{aeg\log\left(f+gx^2\right)}{2f} - \frac{bc\left(d+e\log\left(f+\right.\right.}{2x}$$

$$= \frac{bce\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}} + \frac{aeg\log(x)}{f} - \frac{be\left(c^2f-g\right)\tan^{-1}(cx)\log\left(\frac{2}{1-icx}\right)}{f} + \cdots$$

$$= \frac{bce\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}} + \frac{aeg\log(x)}{f} - \frac{be\left(c^2f-g\right)\tan^{-1}(cx)\log\left(\frac{2}{1-icx}\right)}{f} + \cdots$$

$$= \frac{bce\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}} + \frac{aeg\log(x)}{f} - \frac{be\left(c^2f-g\right)\tan^{-1}(cx)\log\left(\frac{2}{1-icx}\right)}{f} + \cdots$$

<u>Mathematica</u> [B] time = 5.67443, size = 1213, normalized size = 2.3

$$-\frac{2bc^2df\tan^{-1}(cx)x^2 - 4bce\sqrt{f}\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)x^2 - 4ibc^2ef\sin^{-1}\left(\sqrt{\frac{c^2f}{c^2f-g}}\right)\tan^{-1}\left(\frac{cgx}{\sqrt{c^2fg}}\right)x^2 + 4ibeg\sin^{-1}\left(\sqrt{\frac{c^2f}{c^2f-g}}\right)\tan\cdots}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*ArcTan[c*x])*(d + e*Log[f + g*x^2]))/x^3,x]
```

```
[Out] -(2*a*d*f + 2*b*c*d*f*x + 2*b*d*f*ArcTan[c*x] + 2*b*c^2*d*f*x^2*ArcTan[c*x]
 - 4*b*c*e*Sqrt[f]*Sqrt[g]*x^2*ArcTan[(Sqrt[g]*x)/Sqrt[f]] - (4*I)*b*c^2*e*
```

```
f*x^2*ArcSin[Sqrt[(c^2*f)/(c^2*f - g)]]*ArcTan[(c*g*x)/Sqrt[c^2*f*g]] + (4*
I)*b*e*g*x^2*ArcSin[Sqrt[(c^2*f)/(c^2*f - g)]]*ArcTan[(c*g*x)/Sqrt[c^2*f*g]
] - 4*b*e*g*x^2*ArcTan[c*x]*Log[1 - E^((2*I)*ArcTan[c*x])] + 4*b*c^2*e*f*x^
2*ArcTan[c*x]*Log[1 + E^((2*I)*ArcTan[c*x])] - 2*b*c^2*e*f*x^2*ArcSin[Sqrt[
(c^2*f)/(c^2*f - g)]]*Log[(c^2*(1 + E^((2*I)*ArcTan[c*x]))*f + (-1 + E^((2*
I)*ArcTan[c*x]))*g - 2*E^((2*I)*ArcTan[c*x])*Sqrt[c^2*f*g])/(c^2*f - g)] +
2*b*e*g*x^2*ArcSin[Sqrt[(c^2*f)/(c^2*f - g)]]*Log[(c^2*(1 + E^((2*I)*ArcTan
[c*x]))*f + (-1 + E^((2*I)*ArcTan[c*x]))*g - 2*E^((2*I)*ArcTan[c*x])*Sqrt[c
^2*f*g])/(c^2*f - g)] - 2*b*c^2*e*f*x^2*ArcTan[c*x]*Log[(c^2*(1 + E^((2*I)*
ArcTan[c*x]))*f + (-1 + E^((2*I)*ArcTan[c*x]))*g - 2*E^((2*I)*ArcTan[c*x])*
Sqrt[c^2*f*g])/(c^2*f - g)] + 2*b*e*g*x^2*ArcTan[c*x]*Log[(c^2*(1 + E^((2*I
)*ArcTan[c*x]))*f + (-1 + E^((2*I)*ArcTan[c*x]))*g - 2*E^((2*I)*ArcTan[c*x]
)*Sqrt[c^2*f*g])/(c^2*f - g)] + 2*b*c^2*e*f*x^2*ArcSin[Sqrt[(c^2*f)/(c^2*f
- g)]]*Log[1 + (E^((2*I)*ArcTan[c*x])*(c^2*f + g + 2*Sqrt[c^2*f*g]))/(c^2*f
 - g)] - 2*b*e*g*x^2*ArcSin[Sqrt[(c^2*f)/(c^2*f - g)]]*Log[1 + (E^((2*I)*Ar
cTan[c*x])*(c^2*f + g + 2*Sqrt[c^2*f*g]))/(c^2*f - g)] - 2*b*c^2*e*f*x^2*Ar
cTan[c*x]*Log[1 + (E^((2*I)*ArcTan[c*x])*(c^2*f + g + 2*Sqrt[c^2*f*g]))/(c^
2*f - g)] + 2*b*e*g*x^2*ArcTan[c*x]*Log[1 + (E^((2*I)*ArcTan[c*x])*(c^2*f +
 g + 2*Sqrt[c^2*f*g]))/(c^2*f - g)] - 4*a*e*g*x^2*Log[x] + 2*a*e*f*Log[f +
g*x^2] + 2*b*c*e*f*x*Log[f + g*x^2] + 2*a*e*g*x^2*Log[f + g*x^2] + 2*b*e*f*
ArcTan[c*x]*Log[f + g*x^2] + 2*b*c^2*e*f*x^2*ArcTan[c*x]*Log[f + g*x^2] - (
2*I)*b*c^2*e*f*x^2*PolyLog[2, -E^((2*I)*ArcTan[c*x])] + (2*I)*b*e*g*x^2*Pol
yLog[2, E^((2*I)*ArcTan[c*x])] + I*b*c^2*e*f*x^2*PolyLog[2, -((E^((2*I)*Arc
Tan[c*x])*(c^2*f + g - 2*Sqrt[c^2*f*g]))/(c^2*f - g))] - I*b*e*g*x^2*PolyLo
g[2, -((E^((2*I)*ArcTan[c*x])*(c^2*f + g - 2*Sqrt[c^2*f*g]))/(c^2*f - g))]
+ I*b*c^2*e*f*x^2*PolyLog[2, -((E^((2*I)*ArcTan[c*x])*(c^2*f + g + 2*Sqrt[c
^2*f*g]))/(c^2*f - g))] - I*b*e*g*x^2*PolyLog[2, -((E^((2*I)*ArcTan[c*x])*(
c^2*f + g + 2*Sqrt[c^2*f*g]))/(c^2*f - g))])/(4*f*x^2)
```

Maple [F] time = 4.83, size = 0, normalized size = 0.

$$\int \frac{(a + b \arctan(cx)) \left(d + e \ln\left(gx^2 + f\right)\right)}{x^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))*(d+e*ln(g*x^2+f))/x^3,x)
```

```
[Out] int((a+b*arctan(c*x))*(d+e*ln(g*x^2+f))/x^3,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))*(d+e*log(g*x^2+f))/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{bd\arctan(cx) + ad + (be\arctan(cx) + ae)\log\left(gx^2 + f\right)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))*(d+e*log(g*x^2+f))/x^3,x, algorithm="fricas")
```

```
[Out] integral((b*d*arctan(c*x) + a*d + (b*e*arctan(c*x) + a*e)*log(g*x^2 + f))/x
^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))*(d+e*ln(g*x**2+f))/x**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\arctan(cx) + a)\left(e\log\left(gx^2 + f\right) + d\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))*(d+e*log(g*x^2+f))/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)*(e*log(g*x^2 + f) + d)/x^3, x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
```

```
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
```

```

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
```

```
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
```

```
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
```

```
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
```

```
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
```

```
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)
   ))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type
   (expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
```

```
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType, Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
```

```
#               Albert Rich to use with Sagemath. This is used to
#               grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#               'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len(
    flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                     #since this estimate of leaf count is bit lower than
                     #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

```

```
def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
    ']

def is_appell_function(func):
```

```
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
```

```
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, " leaf_count_optimal=",leaf_count_optimal)
```

```


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
   expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```